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Today's Topic

Time & Work

Topics to be covered

- ✓ Proportionality & Chain Rule concept
- ✓ Chain rule Examples
- ✓ Time & work concept
- ✓ Frequently asked Examples
- ✓ Pipes & cisterns Examples
- ✓ Practice examples

Chain Rule Concept

Quick Puzzle: 5 monkeys can eat 5 bananas in 5 minutes. Then how many bananas can 1 monkey eat in 1 minute? 

To understand chain rule concept, lets try to answer few more questions.

1) **If 5 men can make 5 pencils in certain time, then in the same time12 men can make more or less pencils? → MORE**

$$M \propto W \rightarrow M = K * W \rightarrow \frac{M}{W} = K \text{ (constant)} \rightarrow \frac{M1}{W1} = \frac{M2}{W2}$$

2) **If 5 men take 10 hours to make certain number of pencils, then to make same number of pencils, 12 men will take more or less time? → LESS**

$$M \propto \frac{1}{T} \rightarrow M = \frac{k}{T} \rightarrow M * T = K \text{ (constant)} \rightarrow M1 * T1 = M2 * T2$$

Considering both scenario together we can say $\frac{M * T}{W}$ = Constant

$$\rightarrow \frac{M1 * T1}{W1} = \frac{M2 * T2}{W2}$$

Now solving above puzzle by this equation ,

$\frac{5 * 5}{5} = \frac{1 * 1}{x} \rightarrow x = \frac{1}{5}$, So, 1 monkey will eat $\frac{1}{5}$ banana in 1 minute

Practice Examples

Que: If 20 men can build a wall 56 m long in 20 days then what length of similar wall can be built by 35 men in 3 days?

Sol:

$$\frac{M1 * T1}{W1} = \frac{M2 * T2}{W2} \rightarrow \frac{20 * 20}{56} = \frac{35 * 3}{x}$$

→ x = 14.7 m

Que: If 400 men can complete a project in 60 days working 9 hours a day then in what time 200 men can complete the same task working 8 hours a day?

Sol:

$$\frac{M1 * T1}{W1} = \frac{M2 * T2}{W2} \rightarrow \frac{400 * 60 * 9}{1} = \frac{200 * x * 8}{1} \rightarrow x = 135$$

Practice Examples

Que: A garrison of 2000 men has provisions for 54 days. At the end of 15 days, a reinforcement arrives, and it is now found that the provisions will last only for 20 days more. What is the reinforcement strength?

Sol:

We can think like this, If the reinforcement would not have arrived after 15 days then the provisions would have lasted for 39(54 – 15) more days. But the same provision lasted for 20 days because reinforcement arrived.

$$\frac{2000 * 39}{P} = \frac{x * 20}{P}$$

X = 3900

So reinforcement was of 3900 – 2000 = 1900

Practice Examples

Que: A contract is to be completed in 46 days & 117 men are employed for the same, each working 8 hours a day. After 33 days $\frac{4}{7}$ of the work was completed, how many more men are required so that contract is completed in time now each working 9 hours a day.

Options: A. 112 B. 198 C. 91 D. none

Sol:

After 33 days work remaining = $1-\frac{4}{7} = \frac{3}{7}$

$$\frac{117 * 33 * 8}{\frac{4}{7}} = \frac{x * 13 * 9}{\frac{3}{7}} \rightarrow x = 198$$

198 is total men, but already there are 117 working so

Extra 198 – 117 = 81 men

Time & Work concept

Time Concept → A can make a wall in 10 hours and B can make same wall in 15 hours then, if they work together , what time they will take to build the wall?

10 + 15 = 25 hours? → is this correct? ✖

$\frac{10+15}{2}$ = 12.5 hours? → is this correct? ✖

Work concept → A can make 10 pencils in 1 hour and B can make 15 pencils in 1 hour, if they work together then how many pencils will be made in 1 hour?

10 + 15 = 25 → is this correct? ✔

So, in time & work concept we can add work done in same time but we can't add time with same work.

So, when data given in terms of Time we need to convert that in work & then calculate.

Time & Work concept

Que: A can make a wall in 10 hours and B can make same wall in 15 hours then, if they work together , what time they will take to build the wall?

Sol:

For A: 10 hours → 1 work

1 hour → ? → $\frac{1}{10}$ work done by A in 1 hour (called work rate)

For B: in 15 hours → 1 work

1 hour → ? → $\frac{1}{15}$ work done by B in 1 hour

Now we have works of both persons, now we can add those.

$\frac{1}{10} + \frac{1}{15} = \frac{1}{6}$ is total work done by A & B together in 1 hour.

For A & B: $\frac{1}{6}$ work → 1 hour

1 work → ?? → 6 hour to complete 1 work.

Practice Example

Que: If A can complete a work in 5 days working 9 hours a day, along with B they can complete the same work in 3 days working 10 hours a day. Then in how many days B alone can complete the same work working 6 hours a day?

Options: A. 90 days B. 20 days C. 15 days D. none

Sol:

$\frac{1}{45} + \frac{1}{x} = \frac{1}{30}$ → x = 90 hours.

B works 6 hours a day so $\frac{90}{6} = 15$ days.

Practice Example

Que: A takes 20 days to complete a work. B takes 30 days for same. If they work together for 8 days and then B leaves,How long will A take to finish the remaining work ?

Sol:

A's work rate $= \frac{1}{20}$

B's work $= \frac{1}{30}$

Together they work for 8 days then work completed in 8 days will be $\rightarrow 8 * (\frac{1}{20} + \frac{1}{30}) = 8 * \frac{1}{12} = \frac{2}{3}$

Remaining work $= 1 - \frac{2}{3} = \frac{1}{3}$

$\frac{1}{20} \rightarrow 1$

$\frac{1}{3} \rightarrow$

Ans: $\frac{20}{3}$ Days

Practice Example

Que: X & Y can complete a piece of work in 10 & 30 days respectively. After working for 2 days together Y runs away and remaining work is completed by X alone. Fine time to complete the whole work.

Options: A. 7 days **B. $\frac{22}{3}$ days** **C. $\frac{28}{3}$ days** **D. none**

Sol:

$\frac{1}{10} + \frac{1}{30} = \frac{4}{30}$ → $\frac{2}{15}$ work completed by X & Y together in 1 day

As they have worked together for 2 days,

Work completed = 2 * $\frac{2}{15} = \frac{4}{15}$, so work remaining = 1 - $\frac{4}{15} = \frac{11}{15}$

The remaining work must be completed by X alone

$\frac{1}{10}$ - 1

$\frac{11}{15}$ - ? → $\frac{22}{3}$ days, Be careful this is not the answer.

We are asked total time, so $\frac{22}{3}$ + 2 = $\frac{28}{3}$ Days

Practice Example

Que: A is twice efficient than B. Together they can complete a piece of work in 30 days, then find how many days B alone will take to complete the same work?

Options: A. 120 days B. 50 days C. 45 days D. 90 days

Sol:

	A	:	B
Ratio of Efficiency →	2	:	1
Ratio of time taken →	1	:	2

So time taken by A → x

time taken by B → 2x

$\frac{1}{x} + \frac{1}{2x} = \frac{1}{30}$ → solving we get x = 45

B takes 2 * x days → 2 * 45 = 90 days

Practice Example

Que: A & B together can do a piece of work in 60 days, B & C can do the same work in 90 days, A & C can do the same work in 120 days. Find in how many days A alone can do that work?

Options: A. 160 days B. 144 days C. 182 days D.None

Sol: As give,

$$\frac{1}{A}+\frac{1}{B}=\frac{1}{60} \quad , \quad \frac{1}{B}+\frac{1}{C}=\frac{1}{90} \quad , \quad \frac{1}{A}+\frac{1}{C}=\frac{1}{120}$$

Adding all three equations we get,

$$2 * \left(\frac{1}{A}+\frac{1}{B}+\frac{1}{C}\right)=\frac{13}{360} \rightarrow \left(\frac{1}{A}+\frac{1}{B}+\frac{1}{C}\right)=\frac{13}{360*2}$$

$$\frac{1}{A}=\left(\frac{1}{A}+\frac{1}{B}+\frac{1}{C}\right) - \left(\frac{1}{B}+\frac{1}{C}\right) = \frac{13}{360*2} - \frac{1}{90} = \frac{5}{720}$$

So A = 144 days

Practice Example

Que: A & B take a piece of work for ₹ 600, A alone can do it in 6 days and B alone can do it in 8 days, with the help of C they complete the work in 3 days. Find the share of each.

Sol:

$\frac{1}{6}+\frac{1}{8}+\frac{1}{c}=\frac{1}{3}$ → **c = 24, C alone can complete the task in 3 days**

Amount will be distributed as per work done in same time.

So it's same as work efficiency

	A	:	B	:	C
Ratio of time taken →	6	:	8	:	24
Ratio of efficiency →	$\frac{1}{6}$	:	$\frac{1}{8}$	:	$\frac{1}{24}$
→	4	:	3	:	1

A = 600 * $\frac{4}{8}$ = ₹ 300 , A = 600 * $\frac{3}{8}$ = ₹ 225 , A = 600 * $\frac{1}{8}$ = ₹ 75

Practice Example

Que: 3 Men or 2 Women can complete a task in 6 days. In how many days 4 men and 3 women can complete the same task?

Sol:

3 Men can do one task → 6 days then,

1 Man can do the same → 18 days

So 1 man's 1 day's work → $\frac{1}{18}$

Similarly,

2 women can do one task → 6 days then,

1 Woman can do the same → 12 days

So 1 man's 1 day's work → $\frac{1}{12}$

Now, 4 men and 3 women so, work done in 1 day will be

$4\left(\frac{1}{18}\right) + 3\left(\frac{1}{12}\right) = \frac{17}{36}$, So, Time taken to complete the work = $\frac{36}{17}$

Practice Example

Que: 4 men and 6 women can complete a work in 8 days, while 3 men and 7 women can complete it in 10 days. In how many days will 10 women complete it ?

Sol:

Lets assume 1 man's 1 day work = $\frac{1}{m}$

Lets assume 1 woman's 1 day work = $\frac{1}{w}$

$$\frac{4}{m} + \frac{6}{w} = \frac{1}{8}$$
$$\frac{3}{m} + \frac{7}{w} = \frac{1}{10}$$

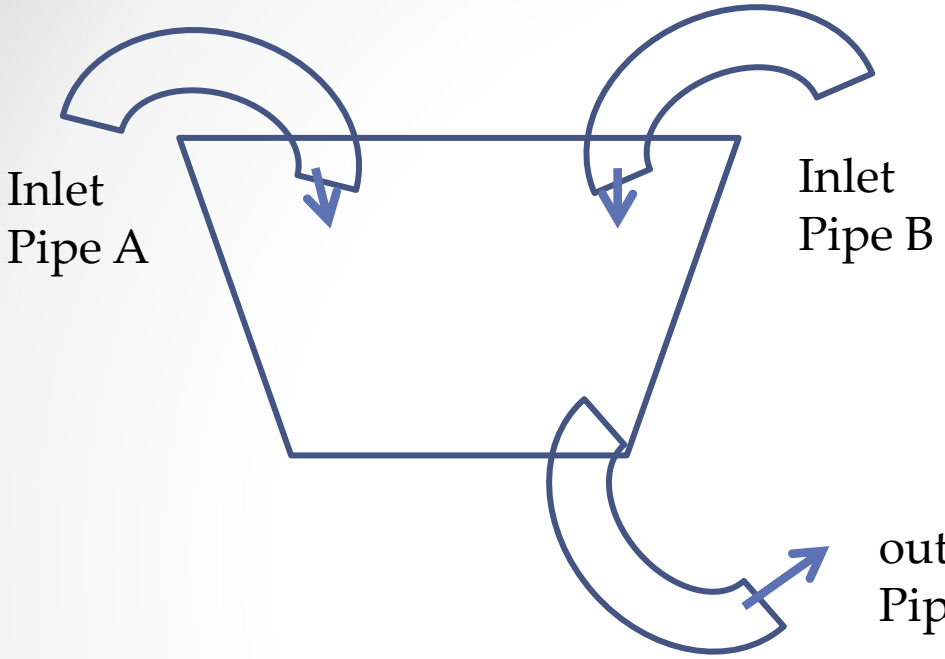
To solve for W

$$\left(\frac{4}{m} + \frac{6}{w} = \frac{1}{8}\right) * 3 = \frac{12}{m} + \frac{18}{w} = \frac{3}{8}$$

$$\left(\frac{3}{m} + \frac{7}{w} = \frac{1}{10}\right) * 4 = \frac{12}{m} + \frac{28}{w} = \frac{4}{10}$$

Solving we get, $\frac{10}{w} = \frac{1}{40}$ → 10 women can complete the work in 40 days

Concept of Pipes & Cisterns



→ As shown **Pipe A** inlet pipe and can fill an empty tank in **X** hours alone.

→ **Pipe B** is also Inlet pipe and can fill the empty tank in **Y** hours

→ **Pipe C** is outlet pipe and can empty the full tank in **Z** hours.

IF all three pipes function simultaneously then work rate per hour will be,

$\frac{1}{x} + \frac{1}{y} - \frac{1}{z}$ **, At this work rate after some time**

→ either tank will be full or

→ Tank will be empty depending upon values of x, y & z

Practice Example

Que: Pipe A can fill the tank in 12 hours, Pipe B can fill the same tank in 18 hours, both pipes were functioning together hence the tank was filled in 15 hours While Pipe C , the outlet pipe was also working from the beginning. Then in what time Pipe C alone can empty the full tank?

Options: A. 15 $\frac{11}{12}$ hours B. 14 $\frac{9}{17}$ hours C. $13\frac{11}{13}$ hours D. None

Sol:

Work rate of Pipe A = $\frac{1}{12}$

Work rate of Pipe B = $\frac{1}{18}$

Work rate of Pipe C = $\frac{1}{C}$

$\frac{1}{12} + \frac{1}{18} - \frac{1}{C} = \frac{1}{15}$ → solving $\frac{1}{c} = \frac{13}{180}$ → **C = $13\frac{11}{13}$**

Practice Example

Que: Because of Leak, a tank which usually filled in 3 hours, takes 3.5 hours to be filled. In what time the leak alone will fill empty the full tank?

Sol:

$$\rightarrow \frac{1}{3} - \frac{1}{x} = \frac{1}{3.5}$$

$$\rightarrow \frac{1}{x} = \frac{1}{21}$$

→ **X = 21 hours**

Practice Example

Que: Four pipes P, Q, R and S can fill a cistern in 20, 25, 40 and 50 hours respectively. The first pipe P was opened at 6:00 am, Q at 8:00 am, R at 9:00 am and S at 10:00 am. When will the Cistern be full?

Sol:

Lets assume that tank will be filled in x hours then,

Pipe P (rate $\frac{1}{20}$) will work for all x hours → part of tank filled by P → $\frac{x}{20}$

Pipe Q (rate $\frac{1}{25}$) will work for all x – 2 hours → part of tank filled by Q → $\frac{x-2}{25}$

Pipe R (rate $\frac{1}{40}$) will work for all x – 3 hours → part of tank filled by R → $\frac{x-3}{40}$

Pipe S (rate $\frac{1}{50}$) will work for all x – 4 hours → part of tank filled by S → $\frac{x-4}{50}$

So, $\frac{x}{20} + \frac{x-2}{25} + \frac{x-3}{40} + \frac{x-4}{50} = 1$

Solving we get x = $9\frac{4}{27}$ hours → tank will be full after $9\frac{4}{27}$ hours from 6 AM

So, at 3:09 PM approx

Practice Example

Que: Two pipes A & B can fill the tank in 24 minutes & 32 minutes respectively working alone. If both pipes are opened simultaneously, after how much time B should be closed so Bucket is full in 18 Minutes?

Sol:

Pipe A's work rate = $\frac{1}{24}$ → Part of tank filled in 1 minute

Pipe B's work rate = $\frac{1}{32}$ → Part of tank filled in 1 minute

Now after some time B should be closed, but A will keep on working so we can say A will work for 18 minutes,

IN 18 minutes part of tank filled by A will be = 18 * $\frac{1}{24}$ = $\frac{3}{4}$

So $\frac{3}{4}$ of the tank will be filled by A, remaining = 1 - $\frac{3}{4}$ = $\frac{1}{4}$ will be filled by B

$\frac{1}{32}$ → 1 min

$\frac{1}{4}$ → ?? → 8 minutes. So B works for 8 minutes and hence

B will be closed after 8 minutes

Practice Example

Que: A takes 30 days to complete a work. B takes 40 days for same. If they start working together and b leaves 10 days before the work is finished, find total time to complete the work.

Sol:

→ If total time to complete the work is X days then & total work to be completed is 1

→So we can say Work done by A + Work done by B = 1

→ $X * \frac{1}{30} + (x-10) * \frac{1}{40} = 1$

→ Solving we get x = $\frac{150}{7}$, Ans = $21\frac{3}{7}$ days

Practice Example

Que: Twenty women can do a work in sixteen days. Sixteen men can complete the same work in fifteen days. What is the ratio of efficiency of a man and a woman?

Sol:

One woman's one day work = $\frac{1}{20*16}$

One man's one day work = $\frac{1}{16*15}$

Ratio of man:women = $\frac{20*16}{16*15}$ **= 4:3**

Practice Example

Que:A is thrice as good as workman as B and therefore is able to finish a job in 60 days less than B. Working together, they can do it in___

Sol:

	A	:	B
Ratio of Efficiency →	3	:	1
Ratio of time taken →	1	:	3

Time taken by A = x, & B = 3x → given difference is 3x – x = 60

→ X = 30 → Time taken by A = 30 and B = 90

So $\frac{1}{30} + \frac{1}{90} = \frac{2}{45}$

Ans: = $\frac{45}{2}$ = 22.5 days

Practice Example

Que: A & B working separately can complete a work in 9 & 12 days respectively. If they work on alternate days with A starting the work, then in how many days work will be completed?

Option: A. $5\frac{1}{7}$ days B. $10\frac{2}{7}$ days C. $10\frac{1}{4}\ days$ D. None

Work done by A on 1st day = $\frac{1}{9}$

Work done by B on 2nd day = $\frac{1}{12}$

Work completed in 2 days = $\frac{1}{9}+\frac{1}{12}=\frac{7}{36}$

→ So one cycle of 2 days completes $\frac{7}{36}$ work.

→ 5 such cycles of 10 days will complete $\frac{7}{36}*5=\frac{35}{36}\ of\ the\ work$

→ After 10 days remaining work is $1-\frac{35}{36}=\frac{1}{36}$

→ This remaining work will be completed by A, because it's A's turn on 11th day

→ Now, A's work rate is $\frac{1}{9}$ → 1 day

$\frac{1}{36}$ → ?? → $\frac{1}{4}$ days So total days = $10\frac{1}{4}$ days

Practice Example

Que: 3 men, 4 women and 6 children can complete a work in 7 days. A woman does double the work a man does and a child does half the work a man does. How many women alone can complete this work in 7 days ?

Sol: As given,

Work of 3 men + work of 4 women + work of 6 children = 1 work done in 7 days

Also given as,

→A woman does double the work a man does → so we can say that

work done by 3 men = work done by 1.5 women

→ A child does half the work a man does → so we can say that

work done by 6 children = work done by 3 men = work done by 1.5 women

Work of 1.5 women + work of 4 women + work of 1.5 women = 1 work done in 7 days

Total 7 women can do same work in 7 days.

Practice Example

Que: Nal takes 20 days to complete the work. Neel takes 30 days to complete same work & Angad takes 40 days for same. If Nal is helped by Neel and Angad on every third day, how long will they take to finish the work ?

Sol:

Sequence of work done will be

Nal alone + Nal alone + (Nal, Nil & Angad) + Nal alone + ...

$\frac{1}{20} + \frac{1}{20} + \left(\frac{1}{20} + \frac{1}{30} + \frac{1}{40}\right) + \frac{1}{20} + \ldots\ldots$

So , work done in three days will be

$\frac{1}{20} + \frac{1}{20} + \left(\frac{1}{20} + \frac{1}{30} + \frac{1}{40}\right) = \frac{1}{20} + \frac{1}{20} + \frac{13}{120} = \frac{25}{120}$ → work done in 3 days

So, Work done in 3 * 4 = 12 days will be $\frac{25}{120} * 4 = \frac{100}{120} = \frac{5}{6}$

Remaining work is $1-\frac{5}{6} = \frac{1}{6}$, here we have reached to 13th day

On 13th & 14th day Nal will work alone so work done in these two days will be $\frac{1}{20} + \frac{1}{20} = \frac{1}{10}$, now remaining work is $\frac{1}{6} - \frac{1}{10} = \frac{1}{15}$ now, we have reached 15th day when all three of them will work together.

$\frac{13}{120}$ → 1

$\frac{1}{15}$ → ? → $\frac{8}{13}$ days. So total days = $14\frac{8}{13}$

Practice Example

Que: P,Q,R & S starts working on a project, Q can finish the task in 25 days working 12 hours a day, R can finish in 50 days working 12 hours a day. Q worked 12 hours a day but took a sick leave on first two days at beginning. R worked 18 hours a day on all days. What is ratio of work done by Q & R after 7 days from start of the project?

Sol:

Work rate of Q = $\frac{1}{25*12}$ one day's one hour's work

Work rate of R = $\frac{1}{50*12}$ one day's one hour's work

Work done by Q: $\frac{5 * 12}{25 * 12}$

Work done by R: $\frac{7 * 18}{50 * 12}$

Ratio = 20:21

Practice Example

Que: A tank is filled in 10 minutes with a pipe having circular cross section. Another tank, twice the size of first tank is filled by 3 other pipes having diameter In the ratio 1:2:3. If the smallest of these three pipes have 4 times the cross section than the pipe in first tank & speed of inflow of water in second tank is half that of in first tank then find the time taken to fill second tank.

Sol:

Ratio of Diameters of three pipes = 1 : 2 : 3

Ratio of Cross sections of three pipes = 1 : 4 : 9 (As cross section = $\pi\left(\frac{d}{2}\right)^2$)

Now if we assume the cross section of first pipe to be k, then

Cross sections of the three pipes of second tank = 4k , 16k , 36k

So total cross section working in second tank = 4k + 16k + 36k = 56k

As we know " Larger the cross section , Lesser the time to fill the tank"

First tank has cross section working = k → time taken = 10 minutes

Second tank has cross section working = 56k time taken → $\frac{10}{56}$ minutes

But tank size is double & speed of inflow is half so time = $\frac{10}{56}$ * 2 * 2

Time = $\frac{5}{7}$ minutes

Doubts?

- Do you particularly require revision of any topic ?
- Did you not understand any particular example?
- Or any thing else?

- Feel free to ask.