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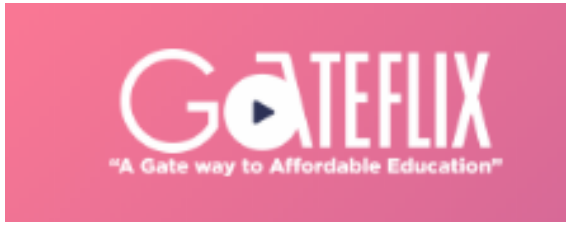


Today's Topic

Ratio & Proportion

Topics to be covered

- ✓ Introduction Ratio
- ✓ Basic Examples & frequently asked Examples
- ✓ Introduction Proportion
- ✓ Basic Examples & frequently asked Examples
- ✓ Practice Examples

Introduction To Ratio

- Ratio: The concept of ratio is used to **compare** two or more quantities of similar kind.
- The ratio of A to B tells us what **multiple of fraction** of A is B.
- Suppose A:B = 2:3 → $\frac{A}{B} = \frac{2}{3}$ → A = 2, B = 3 ✗

$$\frac{A}{B} = \frac{2\,k}{3\,k} \leftarrow \text{A} = 2\,\text{k}\,,\ \text{B} = 3\,\text{k}$$

- This means Ratio just gives **comparative** values and **not absolute / exact** values.
- We can interpret below given things from Ratio A:B = 2:3

✓ B:A = 3:2

✓ A = $\frac{2}{3}$ B , B = $\frac{3}{2}$ A

✓ A is 66.66% of B , A is 33.33% less than B

✓ B is 150% of A , B is 50% more than A

Note: Ratio should be in its lowest form. i.e. if $\frac{a}{b} = \frac{8}{12}$

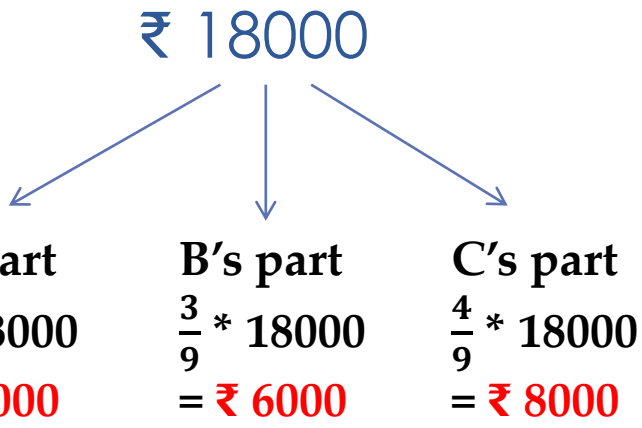
We can say ratio is a:b = 2:3

Introduction To Ratio

Que: If ₹ 18000 is to be divided among three children A,B & C in the ratio 2:3:4, then find the share of each.

Sol:

Method 1: As the ratio is 2:3:4, total parts must be 2+3+4 =9 and out of those2, 3, 4 parts are to be given to A, B, & C respectively. So,



Method 2: The exact values of parts 2:3:4 can be assumed as 2k, 3k, & 4k & total of those values must be ₹18000. So, 2k + 3k + 4k = ₹18000, 9k = 18000 → k = 2000

So, A = 2k = ₹ 4000, B = 3k = ₹ 6000, C = 4k = ₹ 8000

Basic Examples

Que: 5 kg of wheat flour is mixed with 500 gm of sugar. What will be the ratio of sugar to rest of the mixture after adding 1.5 kg of water.

Options: A. 1:10 B. 1:14 C.1:11 D. None

Sol:

5.0 kg Wheat flour

0.5 kg Sugar

1.5 kg water

Total = 7 kg

Now, sugar = 0.5 kg, rest of mixture = 6.5 kg

So Ratio = 0.5:6.5 = Ans: 1:13

Basic Examples

Que: A father decided to distribute his cash property of among his four children Rahul ,Rajesh, Rishi, & Raj in the ratio 3:5:4:7 respectively, find difference of amount received by Rahul and Raj, if Rishi received ₹ 9,20,000.

Sol:

Rahul : Rajesh : Rishi : Raj = 3:5:4:7

→ exact values cane assumed as 3k, 5k, 4k, 7k.

→ So, Rishi = 4k → 4k = 920000 → k = 230000

→ So Difference between Rahul & Raj = 7k – 3k = 4k

→ 4 * 230000 = 9,20,000

Short cut: Rishi is 4k and Difference is also 7k – 3k = 4k

→ So same Ans: 9,20,000

Basic Examples

Que: if x:y = 2:3, then find 3x + 2y : 2x + 5y

Sol: Put x = 2k & y = 3k & solve

$$\frac{3*2k+2*3k}{2*2k+5*3k} = \frac{12}{19}$$

Que: Arrange given ratios in ascending order.

1. A) $\frac{11}{13}$, B) $\frac{13}{15}$, C) $\frac{15}{17}$,D)$\frac{17}{19}$

2. A) $\frac{7}{3}$, B) $\frac{11}{7}$, C) $\frac{15}{11}$,D)$\frac{19}{15}$

Sol: Consider this

If x<y in $\frac{x}{y}$ then $\frac{x}{y} < \frac{x+a}{y+a}$ & $\frac{x}{y} > \frac{x-a}{y-a}$ if a being +ve

If x>y in $\frac{x}{y}$ then $\frac{x}{y} > \frac{x+a}{y+a}$ & $\frac{x}{y} < \frac{x-a}{y-a}$ if a being +ve

So according to this in

1. A<B<C<D
2. A>B>C>D

Comparison of two ratios: $\frac{3}{7}$ & $\frac{11}{23}$

Just cross multiply in upwards direction.

$\frac{3}{7}$ $\frac{11}{23}$ 69 <77 so $\frac{3}{7} < \frac{11}{23}$

Basic Examples

Que: Kirinda and Mirinda are two countries engaged in a war, having number of tanks in the ratio 5:6 respectively. A third country helps both countries with 5 tanks each. Then whose strength in terms of number of tanks is increased more?

Options: A. Kirinda B. Mirinda C. Both Equal D. Can't say

Sol:

Initially the ratio is 5:6, now it will be 10:11

Clearly $\frac{10}{11} > \frac{5}{6}$

So kirinda'S strength is increased more.

Note: Also try with % increase.

Basic Examples

Que: There are total 312 students in a class. There are more girls than boys in the class then which of the below options can not be ratio of number of girls to boys?

Options: A. 31:21 B. 7:6 C. 6:5 D. 13:11

Sol:

As there are total 312 students, total parts of any ratio must exactly divide 312 as number of boys or girls can't be in fractions. So lets Check

A. 31:21 → 31+21 = 52 → 312/52 = 6 Divisible

B. 7:6 → 7+6 = 13 → 312/13 = 24 Divisible

C. 6:5 → 6+5 = 11 → 312/11 = 28.36 Not divisible

D. 13:11 → 13+11 = 24 → 312/24 = 13 Divisible

Ans: 6:5 can not be the ratio

Frequently asked Examples

Que: What common must be subtracted from Numerator and Denominator to convert fraction $\frac{6}{7}$ to $\frac{16}{21}$?

Sol: Lets say the common factor to be subtracted is x then,

$$\frac{6 - x}{7 - x} = \frac{16}{21}$$

→ x = 2.8

Frequently asked Examples

Que: The incomes of A & B are in ratio 3:2 and their expenditures are in the ratio 5:3. If each of them saves ₹2000 then find their incomes.

Sol:

Method 1: Incomes can be assumed as 3x & 2x

Expenditures can be assumed as 5y & 3y

Then as Savings = Income – Exp

$3x - 5y = 2000$ -------Eq 1

$2x - 3y = 2000$ -------Eq 2

Solving x = 4000, So Incomes **A = ₹ 12000 & B = ₹ 8000**

Method 2: As we know Savings = Income – Expenditure

So Expenditure = Income – Savings

$\frac{5}{3} = \frac{3x - 2000}{2x - 2000}$ Solving → x = 4000, so, **A = ₹ 12000 & B = ₹ 8000**

Frequently asked Examples

Que: A bag contains 50 paisa, 25 paisa & 10 paisa coins in the ratio 5:9:4 respectively , amounting to ₹ 206. Find number of coins of each type.

Sol: we can assume like

Number of 50 p coins = 5x → amounting = 5x * 50 paisa

Number of 25 p coins = 9x → amounting = 9x * 25 paisa

Number of 10 p coins = 4x → amounting = 4x * 10 paisa

But we are given amount so,

5x * 50 + 9x * 25 + 4x * 10 = 20600, solving we get x = 40. So,

Number of 50 p coins = 5x → 5 * 40 = 200

Number of 25 p coins = 9x → 9 * 40 = 360

Number of 10 p coins = 4x → 4 * 40 = 160

Frequently asked Examples

Que: A's salary and B's salary is in the ratio 3:4, where as C's salary and B's salary is in the ratio 6:5. Find ratio of salaries of A,B & C.

Options: A. 3:6:5 B. 9:12:10 C. 15:20:24 D. none

Sol:

A:B = 3 : 4

B:C = 5 : 6

We need this B same in both ratios

So,

A:B = (3 : 4) * 5 → A:B = 15 : 20

B:C = (5 : 6) * 4 → B:C = 20 : 24

So now we can write A:B:C = 15:20:24

Frequently asked Examples

Que: Two tanks of similar volume are full of a mixture of oil and water. In the first, the ratio of oil and water is 5:8 and in the second, it is 7:19. If both these tanks are poured in a larger tank, what would be the resultant ratio of oil and water?

Sol: as given that they are same in volume so, we have two options

Option 1: we take 1 litre liquid of both tanks

Tank 1 → oil = $\frac{5}{13}$, Water = $\frac{8}{13}$ Tank 2 → Oil = $\frac{7}{26}$, Water = $\frac{19}{26}$

- Now in new tank $Oil_1 + Oil_2 : Water_1 + Water_2 \rightarrow \frac{5}{13} + \frac{7}{26} : \frac{8}{13} + \frac{19}{26}$

$\frac{17}{26} : \frac{35}{26}$ = Ans: 17:35

Option 2:

To avoid fractions we can take LCM of (5+8) 13 & (7+19) 26

i.e. 26 litres, so take 26 litres from both tanks and find the answer.

Direct & Indirect Proportion

E.g: If we can buy 5 chocolates in ₹ 25, then how much will it cost to buy 25 chocolates?

Sol: 5 → 25

25 → ?? → $\frac{25 * 25}{5}$ → Ans: ₹ 125, why so simple?

→ Because it's direct Proportion. As number of Choc. Increases the Rupees will also increase.

$X \propto Y \rightarrow X = K * Y \rightarrow \frac{X}{Y} = K \text{ (constant)} \rightarrow \frac{X1}{Y1} = \frac{X2}{Y2}$

→ In our example X1 = 5 ,Y1 = 25 ,X2 =25, Y2 = ? Y2 = $\frac{X2 * Y1}{X1} = \frac{25 * 25}{5}$ = ₹ 125

E.g: If 10 Men can complete work in 20 days then same work can be completed by 15 men in ____ days.

Sol: 10 → 20

15 → ?? → $\frac{15 * 20}{10}$ → Ans: 30 days. Is this correct? ← Incorrect

→ Because it's indirect Proportion. As number of men Increases the time Decreases.

$X \propto \frac{1}{y} \rightarrow X = \frac{k}{Y} \rightarrow X * Y = K \text{ (constant)} \rightarrow X1 * Y1 = X2 * Y2$

In our example X1 = 10 ,Y1 = 20 ,X2 = 15, Y2 = ? Y2 = $\frac{X1 * Y1}{X2} = \frac{10 * 20}{15}$ = $13\frac{1}{3}$ days

Practice Examples

Que: The height of a person is proportional to square root of his age. What will be the height of Khali at the age of 16 years if his height was 5 feet at the age of 9 years.

Sol: as given

→ $H \propto \sqrt{Age}$ → $H_1 = k\sqrt{Age_1}$ → $\frac{H1}{H2} = \frac{\sqrt{Age1}}{\sqrt{Age2}}$

→ $H2 = 5 * \frac{4}{3} = 6\frac{2}{3}$ feet

Practice Examples

Que: The value of Diamond is directly proportional to square of its weight. If a diamond of value ₹ 6800, breaks down in 3 pieces with ratio of their weight as 5: 7: 8, then find if there is loss or profit due to this breakage & of what value ?

Sol: As given $V \propto W^2 \rightarrow V = k * W^2$

Now, total weight doesn't change after breakage so total weight of diamond is 5x + 7x + 8x = 20x

So $6800 = k\,(20x)^2 \rightarrow k = \frac{6800}{(20x)^2}$

Now after breakage,

$V1 + V2 + V3 = k(W1^2 + W2^2 + W3^2)$

$= \frac{6800}{(20x)^2} [\,(5x)^2 + (7x)^2 + (8x)^2]$

$= \frac{6800}{400} * [25 + 49 + 64]$

= ₹ 2346 → loss of 6800 – 2346 = **Ans: ₹ 4454**

Practice Examples

Que: The reduction in the speed of a Goods Train is directly proportional to the square root of the number of wagons attached to it. The speed of the train is 42 km/hr when there are no wagons and 24 kmph when there are nine wagons attached. What is the maximum number of wagons that can be attached to the train so that it can move?

Options: A. 24 B. 40 C. 49 D. 48

Sol: Reduction of speed = k $\sqrt{W}$

Now as given the speed is reduced by 42 – 24 = 18 kmph when we attach 9 wagons, so 18 = k $\sqrt{9}$ → k = 6

Now we need to find the maximum wagons so we take 42 Kmph

42 = k $\sqrt{W}$ → 42 = 6 * $\sqrt{W}$ → W = 49

That means if 49 wagons are attached then speed of the train will reduce by 42 kmps → train will not move.

So highest we can attach 48 wagons

Practice Examples

Que: Total expenses of a boarding house are partly fixed and partly varying linearly with the number of boarders. The average expense per boarder is Rs 700 when there are 25 boarders and Rs 600 when there are 50 boarders. What is the average expense per boarder when there are 100 boarders?

Sol:

Total exp = fixed exp + no. of boarder * Per boarder exp

Average exp is 700 when 25 boarders so total exp = 700 * 25

$700 * 25 = F + 25 * k$

Similarly $600 * 50 = F + 50 * k$

Solving above two equation we get $k = 500$

Ans = ₹ 500

Practice Examples

Que: X & Y earns in the ratio 5:7 and spends in the ratio 5:7, then their saving will be in the ratio _____

Option: A. 1:1 B. 7:5 C. 5:7 D: Can't be determined

Sol:

Saving = Earning – spending

Earnings can be assumed by 5a & 7a

Spending can be assumed by 5b & 7b

So Savings ratio = $\frac{5a-5b}{7a-7b} = \frac{5}{7}$ ← Ans, Is this correct?

NO..! why? As because while cancelling (a-b), we igonored the possibility of (a-b) being zero.

→means what if they spend everything whatever they earn, so either it can be $\frac{5}{7}$ or $\frac{0}{0}$

Means, can't be determined

Doubts?

- Do you particularly require revision of any topic ?
- Did you not understand any particular example?
- Or any thing else?

- Feel free to ask.