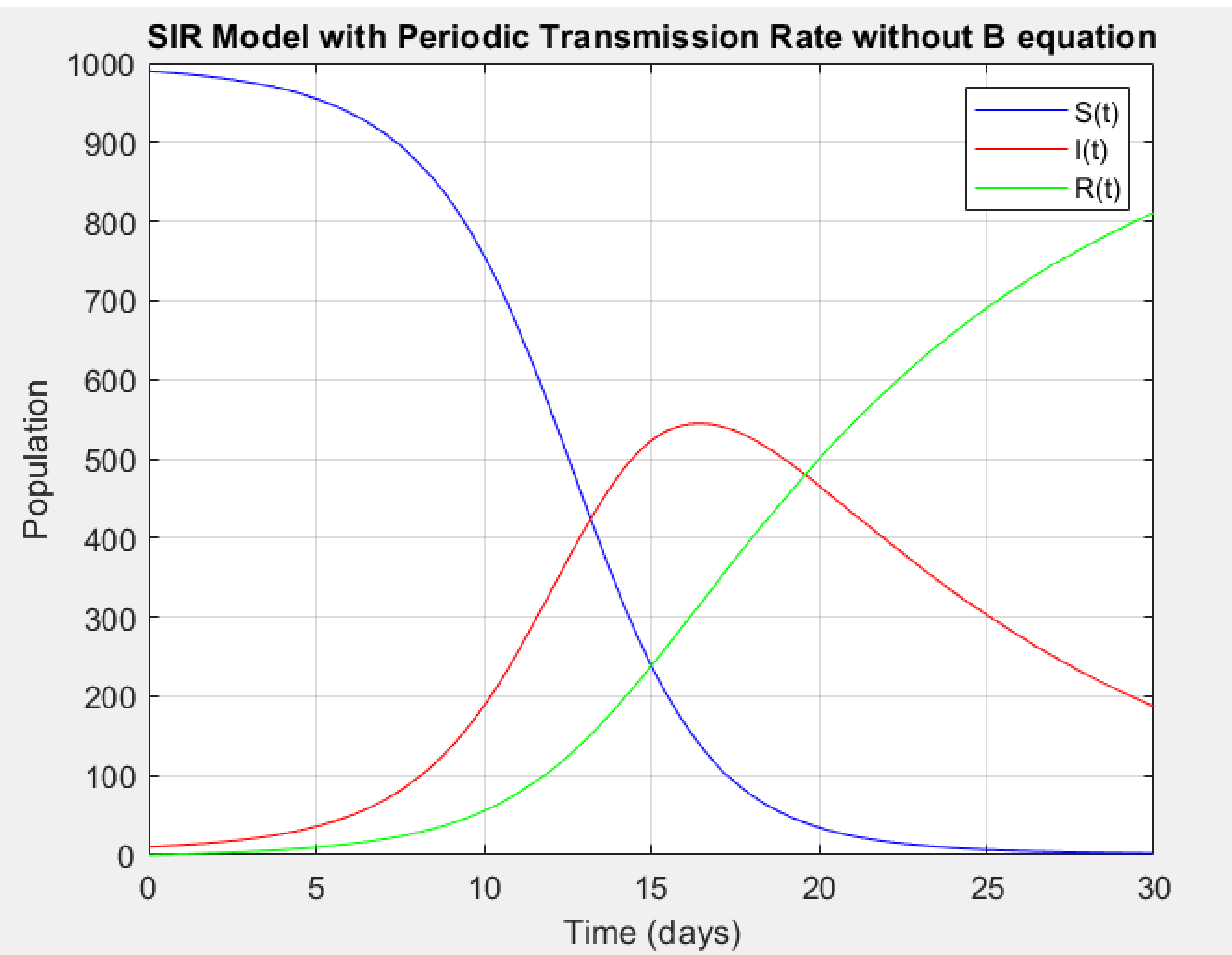
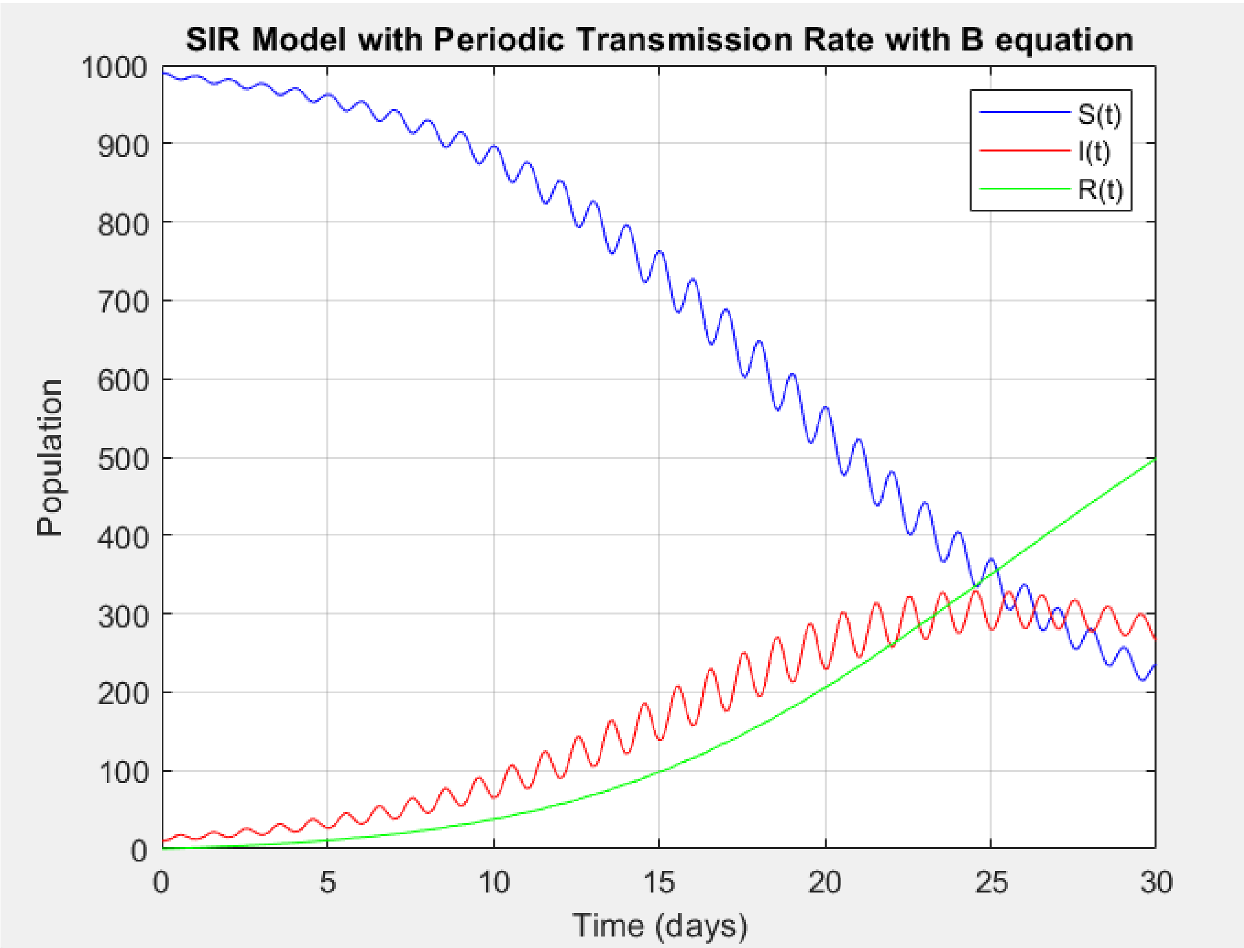
```
% Code for Part IV, Step 1 with Plot
beta0 = 0.3;      % Base transmission rate
A = 5;            % Amplitude of variation
omega= 2 * pi / 365; % Angular frequency for daily periodicity
gamma = 0.1;      % Recovery rate
h = 0.1;          % Time step (days)
T = 30 ;          % Total simulation time (days)
S0 = 990;         % Initial susceptible population
I0 = 10;          % Initial infected population
R0 = 0;           % Initial recovered population
N= S0 +I0 +R0; % Total population

% Time vector
time =0 :h:T;

% Initialize arrays for S, I, and R
S = zeros( size( time ));
I = zeros(size(time));
R = zeros( size(time ));

% Set initial conditions
S(1) = S0;
I(1) = I0;
R(1) = R0;

for t = 1 :length( time )  - 1
   % Compute beta(t) with periodic variation
   beta_t = beta0 * (1 + A * sin(omega * time(t)));

   k1_S = -beta_t * S(t) * I(t) / N;
   k1_I =beta_t * S(t) * I(t) / N - gamma * I(t);
   k1_R= gamma * I ( t);

   k2_S = -beta_t * (S(t) + h/2 * k1_S) * (I(t) + h/2 * k1_I) / N;
   k2_I = beta_t * (S(t) + h/2 * k1_S) * (I(t) + h/2 * k1_I) / N - gamma * (I( t) + h/2 * k1_I);
   k2_R = gamma * (I(t) + h/2 * k1_I);

   k3_S = -beta_t * (S(t) + h/2 * k2_S) * (I(t) + h/2 * k2_I )/ N;
   k3_I = beta_t * (S(t) + h/2 * k2_S) * (I(t) + h/2 * k2_I) / N - gamma * (I(t) + h/2 * k2_I);
   k3_R = gamma * (I(t) + h/2 * k2_I);

   k4_S = -beta_t * (S(t) + h * k3_S) * (I(t) + h * k3_I)/ N;
   k4_I = beta_t * (S(t) + h * k3_S) * (I(t) + h * k3_I) / N - gamma * (I(t) + h * k3_I);
   k4_R = gamma * (I(t) + h * k3_I);
```

```
    S(t+1) = S(t) + h/ 6 * (k1_S + 2*k2_S + 2*k3_S + k4_S);
    I(t+1) = I(t) + h/6 * (k1_I + 2 *k2_I + 2*k3_I + k4_I);
    R(t+1) = R(t) + h/6 * (k1_R + 2* k2_R + 2*k3_R + k4_R);
end

% Plot
figure;
plot(time, S, 'b', 'DisplayName', 'S(t)'); hold on;
plot(time, I, 'r', 'DisplayName', 'I(t)');
plot(time, R, 'g', 'DisplayName', 'R(t)');
xlabel('Time (days)');
ylabel('Population');
legend;
title('SIR Model with Periodic Transmission Rate without B equation');
grid on;
```



```
% Code for Part IV, Step 2 with the B equation
beta0 =0.3;      % Base transmission rate
A = 5;           % Amplitude of variation
omega =2*pi;   % Angular frequency for daily periodicity
gamma = 0.1;      % Recovery rate
h = 0.1;          % Time step (days)
T = 30 ;           % Total simulation time (days)
S0 = 990;         % Initial susceptible population
I0 = 10;          % Initial infected population
R0 = 0;           % Initial recovered population
N = S0+I0 +R0; % Total population
time = 0: h: T;

% Initialize arrays
S = zeros(size(time));
I = zeros(size(time));
R = zeros(size(time));

% Initial conditions
S(1)=  S0;
I(1) = I0;
R(1) = R0;

% Runge-Kutta 4th order method to solve the ODEs
for t = 1 : length( time) - 1
   % Compute beta(t)
   beta_t = beta0 * (1 + A * sin(omega * time(t)));
   % Compute k-values for RK4
   k1_S = -beta_t * S(t) * I(t) / N;
   k1_I = beta_t * S(t) * I(t) / N - gamma * I(t);
   k1_R = gamma * I(t);

   k2_S = -beta_t*(S(t)+ h/2 * k1_S) * (I(t) + h/2 * k1_I) / N;
   k2_I = beta_t * (S(t) +h/2 * k1_S) * (I(t) + h/2 * k1_I) / N - gamma * (I(t) + h/2 * k1_I);
   k2_R = gamma * (I(t) + h/2 * k1_I);

   k3_S = -beta_t * (S(t) + h/2* k2_S) * (I(t) +h/2 * k2_I) / N;
   k3_I = beta_t * (S(t) + h/2* k2_S) * (I(t) + h/2 * k2_I) / N - gamma *(I(t)+h/2 * k2_I);
   k3_R = gamma * (I(t) + h/2 *k2_I);

   k4_S = -beta_t * (S(t) + h * k3_S) * (I(t) + h * k3_I) / N;
   k4_I = beta_t * (S(t) + h * k3_S) * ( I(t) + h * k3_I) / N - gamma * (I(t) + h * k3_I);
   k4_R = gamma * (I(t) + h * k3_I);
```

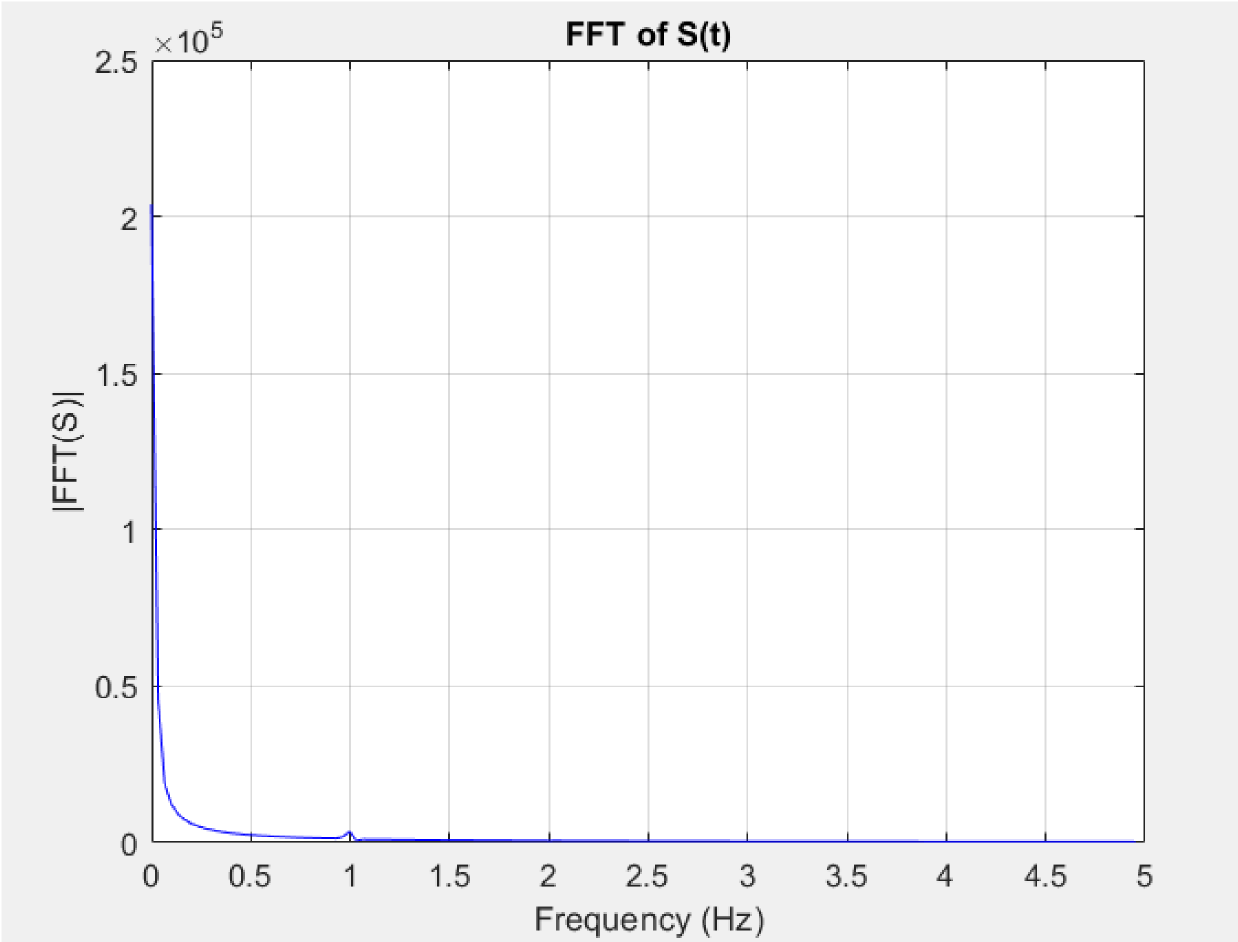
```
    S(t+1) = S(t) + h/6 * (k1_S + 2*k2_S + 2*k3_S + k4_S);
    I(t+1) = I(t) + h/6 * (k1_I + 2*k2_I + 2*k3_I + k4_I);
    R(t+1) = R(t) + h/6 * (k1_R + 2*k2_R + 2*k3_R + k4_R);
end

% Plot
figure;
plot( time, S, 'b','DisplayName','S(t)'); hold on;
plot(time, I, 'r', 'DisplayName', 'I(t)');
plot(time, R, 'g', 'DisplayName', 'R(t)');
xlabel( 'Time (days)');
ylabel ( 'Population');
legend;
title( 'SIR Model with Periodic Transmission Rate with B equation');
grid on;
```

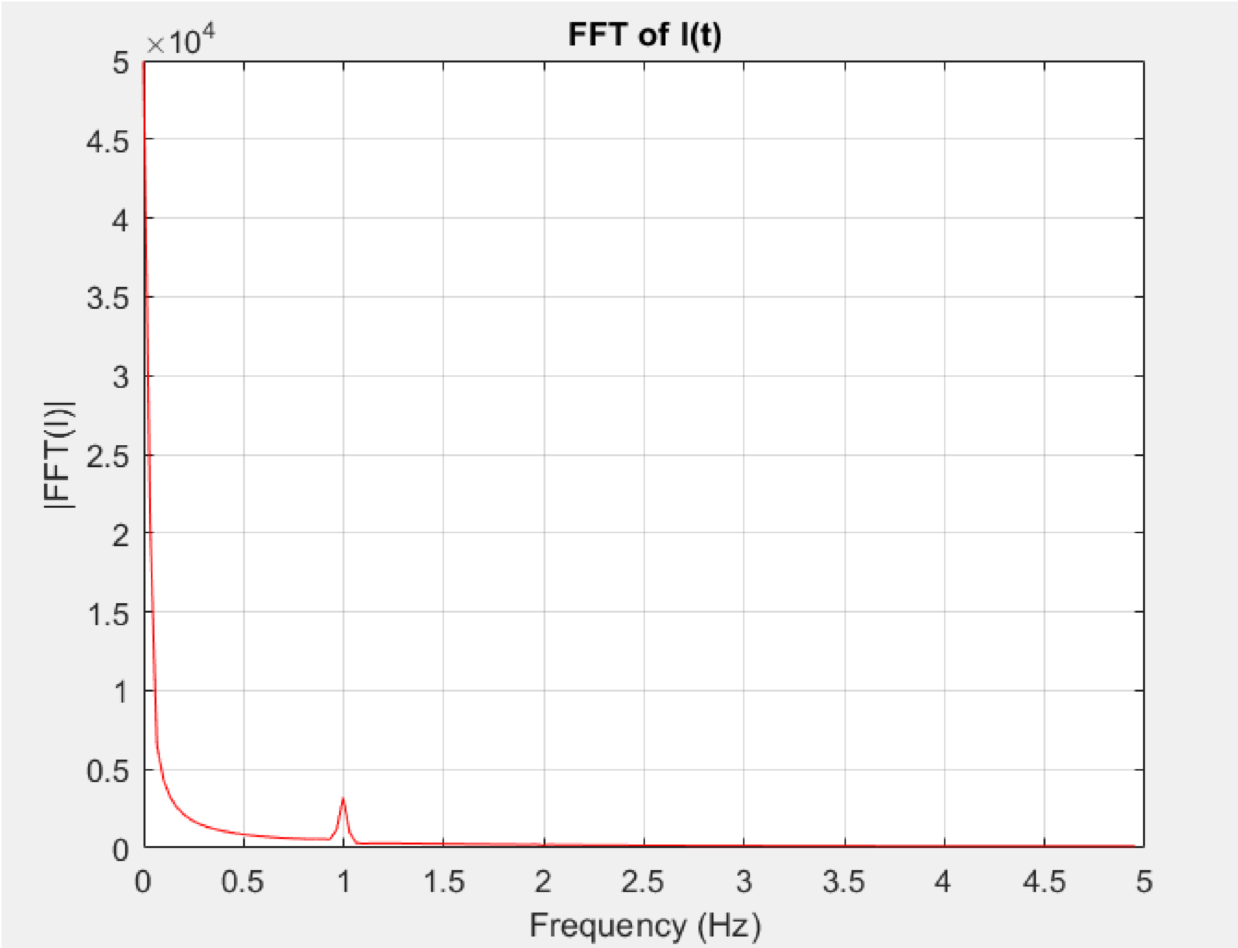


Do you observe any periodic fluctuations in the signals due to periodicity of β?

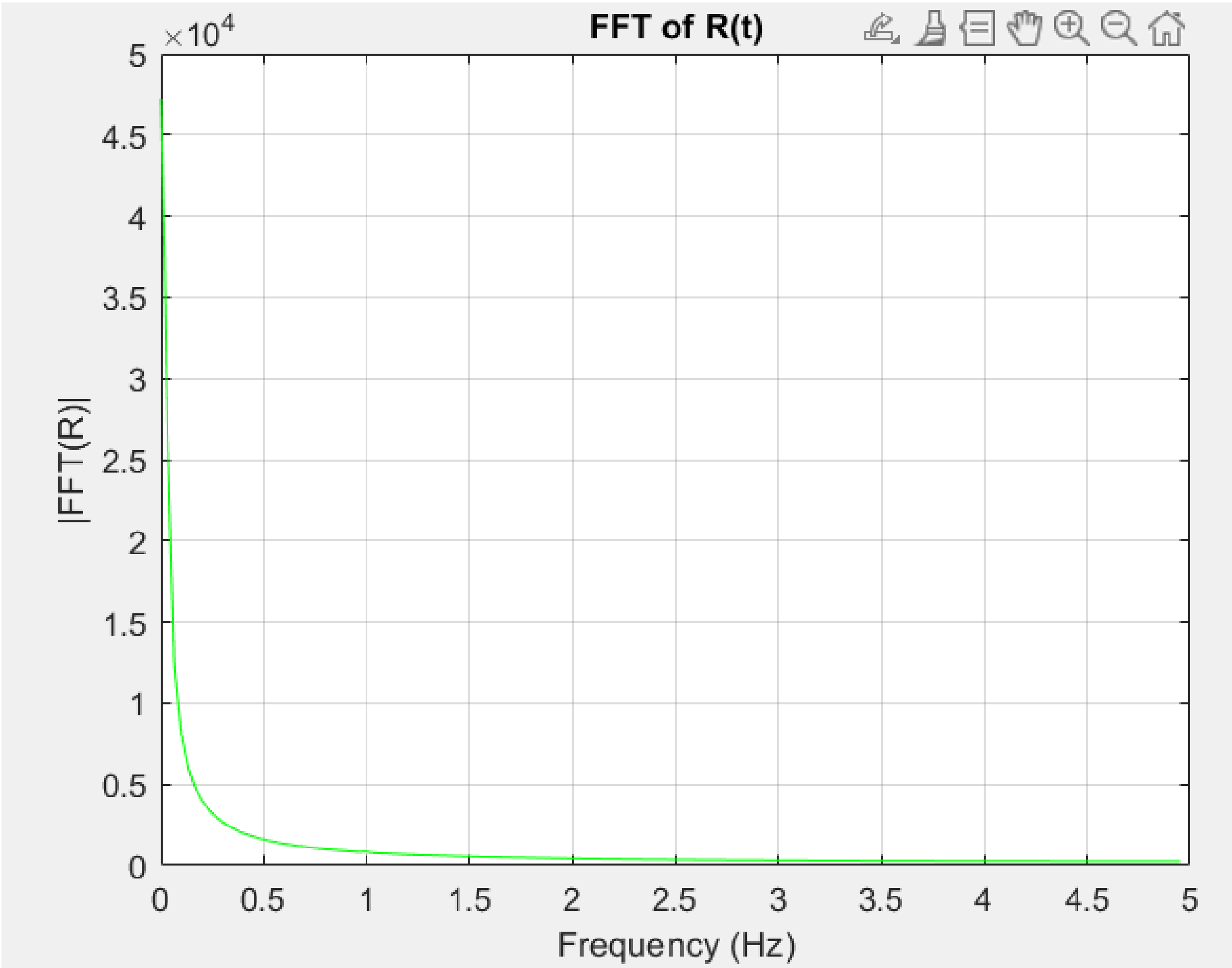
The periodic changes in (S(t)) (susceptible individuals) and (I(t)) (infected individuals) happen because the transmission rate, (beta(t)), changes in a pattern, following the equation (beta_0 (1 + A sin(omega t))). When the (beta(t)) is high, infections spread faster, that decreases (S(t)) and increases (I(t)) and when (beta(t)) is low, the spread slows down which causes (I (t)) to drop and (S(t)) to level off. However, these fluctuations don't affect (R(t)) (recovered individuals) much because they only grow steadily over time, smoothing out short-term changes. This matches the SIR model's prediction when the transmission rate varies over time.

```
% FFT for I(t) with specified frequency vector and plotting
T = time(end); % T = 30 days
N = length(time);
FFT_I = fft(I);
f = (0:1/T:1/2);

% Compute magnitude of FFT (absolute value of Fourier coefficients)
FFT_I_mag = abs(FFT_I(1:length(f)));

% Plot the spectrum for I(t)
figure;
plot(f, FFT_I_mag, 'r', 'LineWidth', 1.5);
xlabel('Frequency (1/days)');
ylabel('|FFT(I)|');
title('Frequency Spectrum of I(t)');
grid on;
```


FFT of S(t)
×10^5
2.5
2
1.5
1
0.5
0
|FFT(S)|
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
Frequency (Hz)



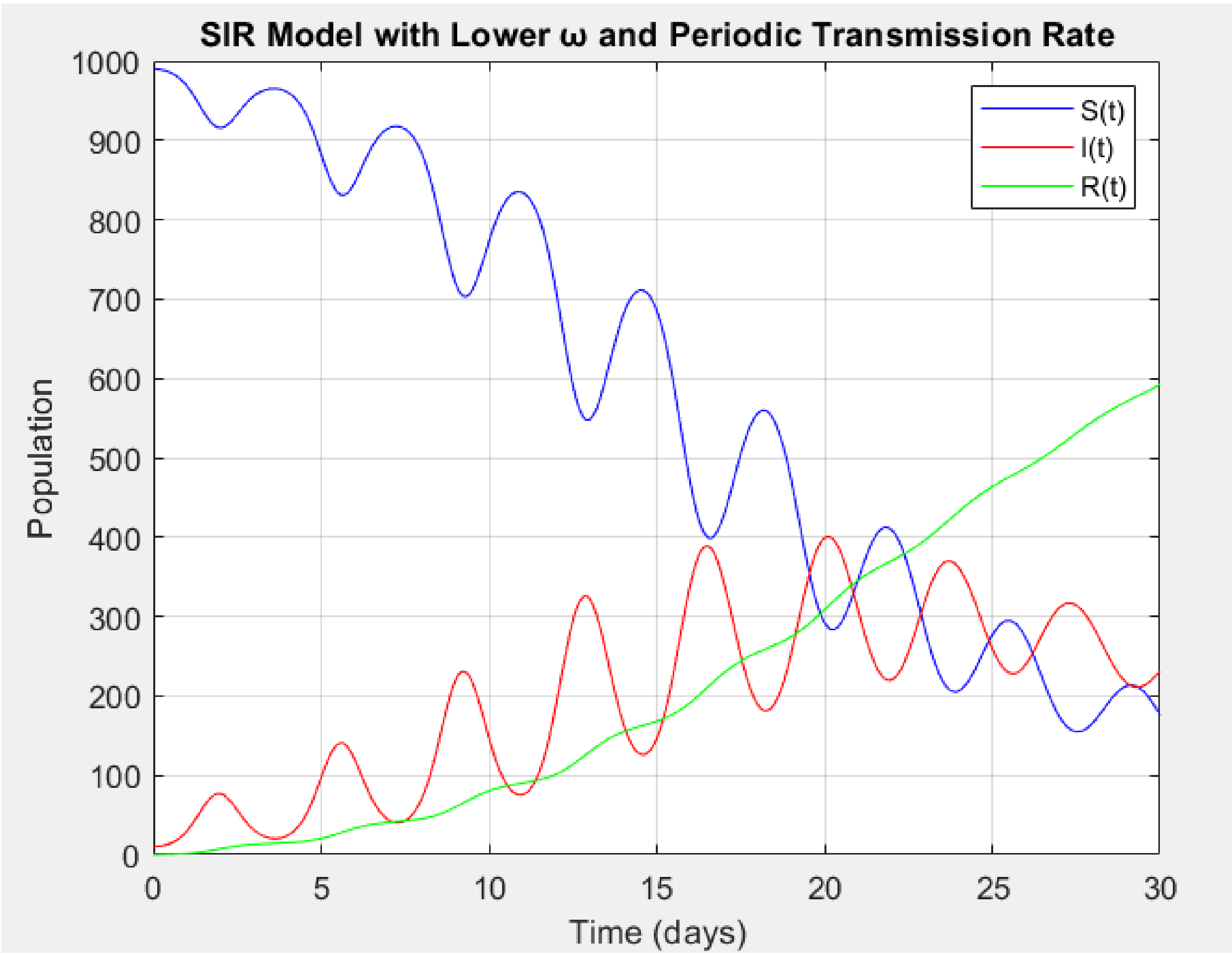
FFT of I(t)
×10^4
5
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
|FFT(I)|
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
Frequency (Hz)

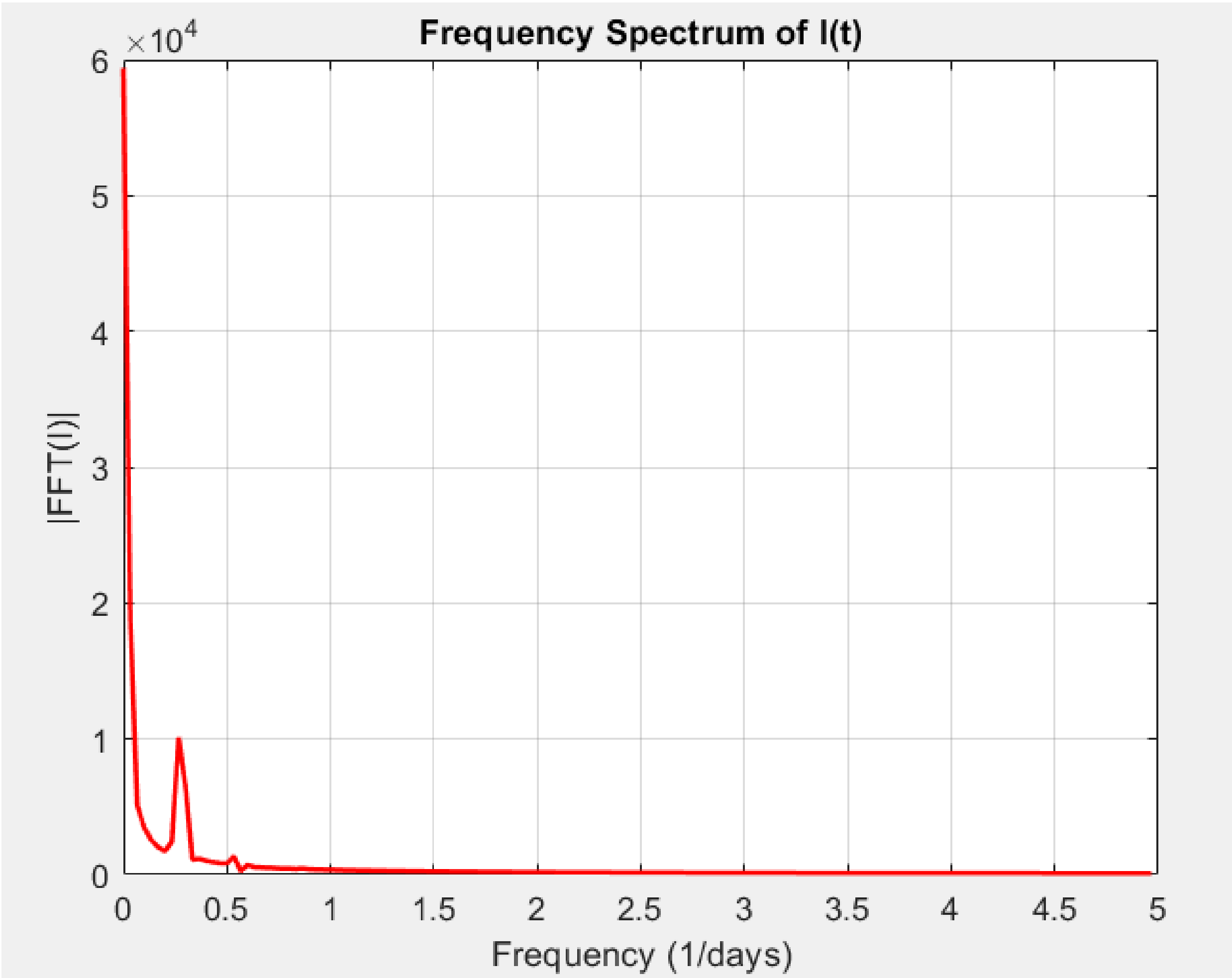



```
% Code for SIR Model with Lower ω and FFT I(t)
% Parameters
beta0 = 0.3;       % Base transmission rate
A = 5;             % Amplitude of variation
omega = 2 * pi * 100 / 365; % Lower angular frequency for weekly variation
gamma = 0.1;       % Recovery rate
h = 0.1;           % Time step (days)
T = 30;            % Total simulation time (days)
S0 = 990;          % Initial susceptible population
I0 = 10;           % Initial infected population
R0 = 0;            % Initial recovered population
N = S0 + I0 + R0; % Total population
time = 0:h:T;
% Initialize arrays
S = zeros(size(time));
I = zeros(size(time));
R = zeros(size(time));
% Initial conditions
S(1) = S0;
I(1) = I0;
```

```
R(1) = R0;
for t = 1:length(time) - 1
   % Compute beta(t) with periodic varation
   beta_t =beta0 * (1 + A * sin(omega * time(t)));

   k1_S = -beta_t * S(t) * I(t) / N;
   k1_I = beta_t * S(t) * I(t) / N - gamma * I( t);
   k1_R =gamma * I(t);

   k2_S = -beta_t * (S(t) + h/2 * k1_S) * (I(t) + h/ 2 * k1_I ) / N;
   k2_I = beta_t * (S(t) + h/ 2 * k1_S) * ( I(t) + h/2 * k1_I) / N - gamma * (
I(t) + h/2 * k1_I);
   k2_R = gamma * (I(t) + h /2 * k1_I);

   k3_S = -beta_t* (S(t) + h/2 * k2_S) * (I(t) + h/2 * k2_I) / N;
   k3_I = beta_t * ( S(t) + h/2 * k2_S) * (I(t) + h/2 * k2_I) / N - gamma *
(I(t) + h/2 * k2_I );
   k3_R = gamma * ( I(t) + h/2 * k2_I);

   k4_S = -beta_t * (S(t) + h * k3_S) * (I(t) + h * k3_I) / N;
   k4_I = beta_t * (S(t) + h * k3_S) * (I( t ) + h *  k3_I) / N - gamma * (I(t)
+ h * k3_I);
   k4_R = gamma * (I(t) + h * k3_I);

   S(t+1) = S(t) + h /6 * (k1_S + 2*k2_S + 2*k3_S + k4_S);
   I(t+1) = I(t) + h/6 * (k1_I + 2*k2_I + 2*k3_I + k4_I);
   R(t+1) = R(t) +  h /6 * (k1_R + 2*k2_R + 2*k3_R + k4_R);
end
% Plot the results of the SIR model
figure;
plot(time, S, 'b', 'DisplayName', 'S(t)'); hold on;
plot(time, I, 'r', ' DisplayName', 'I(t)');
plot(time,R, 'g', 'DisplayName', 'R(t)');
xlabel('Time (days)');
ylabel(' Population');
legend;
title('SIR Model with Lower ω and Periodic Transmission Rate');
grid on;
T_signal = time(end ); % T = 30 days
N = length( time);
FFT_I = fft(I);
f = (0:N/2-1) / T; % Correct frequncy vector definition
% Compute magnitude of FFT (absolute value of Fourier coefficients)
FFT_I_mag = abs (FFT_I(1:N/2)); % Considr only the first half of FFT results
% Plot the spectrum for I(t)
figure ;
plot(f, FFT_I_mag, 'r', 'LineWidth', 1.5);
xlabel('Frequency (1/days)');
ylabel('|FFT(I)|');
```

```
title(' Frequency Spectrum of I(t) with Lower ω');
```





Frequency Spectrum of I(t)
×10^4
6
5
4
3
2
1
0
|FFT(I)|
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
Frequency (1/days)