Dynamic Tail Risk Hedging

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April 6, 2024

Abstract

This paper proposes a cost-effective dynamic tail risk hedging strategy. It leverages volatility forecasts from GARCH and APARCH models to measure Value-at-Risk (VaR) spread. A dynamic strategy switches between SPY, an all-weather portfolio, and protective puts based on VaR spread. Backtesting demonstrates outperformance in returns, Sharpe ratio, Sortino ratio, and maximum drawdown compared to a buyand-hold SPY strategy, static put hedging, and even holding gold. Limitations using Black-Scholes for put pricing, exclusion of dividends are also discussed.

1 Topic and Relevance

Risk Management is a pivotal component of all investment strategies. Risk Management aids in better planning and communication. Moreover, a robust risk management framework ensures proper risk framework compliance as well as aids in optimal portfolio management. The five key tenets of managing risk involve identification, assessment, mitigation, monitoring, and reporting. As has been observed time and time again, it is extremely challenging to accurately gauge and act on the risk management framework described above. This task becomes even more arduous when tail risks come into play.

As Nassim Nicholas Taleb puts it in his book, 'Skin in the Game', the best place to hide risks is "in the corners", in burying vulnerabilities to rare events. Fat tails have been many a trader's kryptonite, virtually all market debacles are deeply rooted in excessive, poorly hedged exposure to tail risk, yet, market participants still fail to comprehend the insidious nature of Tail Risk.[11]

It is pivotal for all market participants to cost-effectively hedge against risks that are hidden in the tails - the 'black swan' events that seem to have become uncharacteristically common. Our project titled "Dynamic Tail Risk Hedging" seeks to build a dynamic tail risk hedging strategy that can cost-effectively soften the blow when massive market drawdowns take place. Conventionally, tail risks have been hedged using simple buy-and-hold strategies, however, the core problem with static tail risk hedging is that the exorbitant premium that needs to be paid upfront does not make for an attractive strategy in the long run. Most static hedging strategies end up massively under-performing any market portfolio in the long run, thus, the premium that yields handsome returns during severe downturns cannot be justified by any rational market participant.

In this vein, we seek to build a dynamic tail risk hedging strategy that yields better returns than a market portfolio in the long run by dynamically altering its composition based on a certain signal.

2 Hypothesis

We hypothesize that our Dynamic Tail Risk Hedging Strategy will outperform two traditional benchmarks, the S&P 500 index and a static Protective Put (PP) strategy, over the period from

January 1, 2014, to December 31, 2023. We anticipate that our strategy will achieve a Compound Annual Growth Rate (CAGR) that is 100 basis points higher than that of the S&P 500. Additionally, we expect our strategy to demonstrate superior riskadjusted returns, with an increase of approximately 0.1 in both the Sharpe and Sortino ratios compared to the benchmarks over the 10-year horizon. Furthermore, we predict that our strategy will experience a maximum drawdown that is at least 5% lower than that of the benchmarks. We will conduct an extensive out-of-sample backtest to validate these claims.

3 Dataset

All data for this project was available on Bloomberg. We accessed comprehensive time series data for all Exchange-Traded Funds (ETFs) utilized in our analysis. The ETFs included in our dataset are as follows:

- SPY: An ETF tracking the performance of the S&P 500 index, serving as our benchmark.
- GLD: An ETF representing Gold, providing a hedge against economic uncertainty.
- TLT: An ETF comprised of 20+ Year Treasury Bonds, offering stability and consistent returns in the fixed income segment.
- IEI: An ETF consisting of 3-7 Year Treasury Bonds, further enhancing the fixed income component of our portfolio.
- DBC: An ETF representing a diversified basket of Commodities, contributing to portfolio diversification.
- DIA: An ETF tracking the performance of the Dow Jones index.
- QQQ: An ETF tracking the performace of the top 100 non-financial companies listed on NASDAQ

The inclusion of GLD and various Treasury Bond ETFs aims to construct an 'all-weather' portfolio, capable of withstanding different market conditions. Fixed income instruments such as TLT and IEI are chosen for their ability to deliver consistent returns, while Gold serves as a hedge during periods of economic turmoil. In addition to ETF data, we also incorporated external indicators such as the Volatility Index (ˆVIX) and the Treasury Yield Index for 13-week Treasury Bills (ˆIRX) to calculate the price of put option contracts, further enhancing our risk management strategy.

Our dataset spans from January 1996 to December 2023, depending on the availability of data for each ETF within this timeframe. This comprehensive dataset allows us to conduct a thorough analysis to train and test our models against our benchmark. All data is stored on GitHub and stored as files, facilitating easy access and sharing through a single, standardized link.

4 Literature Review

There has been significant research in the field of Dynamic Tail Risk Hedging over the past decade. We narrowed our review to strategies that focus on the incorporation of fat tailed distributions in order to capture fat tails and their impact on market returns.

In [9], the authors propose a dynamic tail risk hedging strategy on DAX futures. This strategy is based on the computation of the Value-at-Risk (VaR) spread between innovations from a Generalized Pareto Distribution (GPD) and a normal distribution. The GPD is particularly suited to deal with tail risks as it arises as a limit of tail behavior in extreme value theory. Their results indicate that the GPD distribution provides the strongest signals for avoiding tail risks. This is not surprising as the GPD distribution arises as a limit of tail behavior in extreme value theory and therefore is especially suited to deal with tail risks. Out-of-sample backtests on 11 years of DAX futures data, indicate that the dynamic tail-risk protection strategy effectively reduces the tail risk while outperforming traditional portfolio protection strategies. The results are further validated by calculating the statistical significance of the results obtained using bootstrap methods. Several robustness tests including application to other assets further underline the effectiveness of the strategy. Finally, by empirically testing for second-order stochastic dominance, they find that risk-averse investors would be willing to pay a positive premium to move from a static buy-and-hold investment in the DAX future to the tail-risk protection strategy.

Subsequently, in paper [8], the authors *Otto, Philipp and Schmid, Wolfgang* introduce a novel spatial Generalized Autoregressive Conditional Heteroskedasticity (GARCH) process in a unified spatial and spatiotemporal GARCH framework. This framework covers all previously proposed spatial Autoregressive Conditional Heteroskedasticity (ARCH) models, exponential spatial GARCH, and time-series GARCH models. Unlike previous models, this spatial GARCH allows for instantaneous spillovers across all spatial units, capturing the idea that ”near things are more related than distant things”. They demonstrate the use of the model through a Monte Carlo simulation study and an empirical example focusing on real estate prices from 1995 to 2014 across the postal code areas of Berlin. A spatial autoregressive model is applied to the data to illustrate how locally varying model uncertainties (e.g., due to latent regressors) can be captured by the spatial GARCH-type models.

Furhtermore, in paper [1] *Bonato, Matteo* proposes a new multivariate volatility model that combines the appealing properties of the stable Paretian distribution to model the heavy tails with the GARCH model to capture the volatility clustering. Returns on assets are assumed to follow a sub-Gaussian distribution, which is a particular multivariate stable distribution. In this way the characteristic function of the fitted returns has a tractable expression and the density function can be recovered by numerical methods. A multivariate GARCH structure is then adopted to model the covariance matrix of the Gaussian vectors underlying the sub-Gaussian system. The model is applied to a bivariate series of daily U.S. stock returns. Value-at-risk for long and short positions is computed and compared with the one obtained using the multivariate normal and the multivariate Student’s t distribution. Finally, exploiting the recent developments in the vast dimensional time-varying covariances modeling, possible feasible extensions of our model to higher dimensions are suggested and an illustrative example using the Dow Jones index components is presented.

In conclusion, these three research studies present innovative approaches to addressing key challenges in financial modeling and dynamic tail risk management. Packham et al. propose a dynamic tail risk hedging strategy based on the Generalized Pareto Distribution (GPD), demonstrating its effectiveness in mitigating tail risks in DAX futures trading. Otto and Schmid introduce a spatial GARCH framework that captures spatial dependencies in real estate prices, offering valuable insights into localized uncertainties and spatial relationships among markets. Bonato proposes a multivariate volatility model combining stable Paretian distribution and GARCH to capture heavy-tailed returns and volatility clustering, with promising applications in diverse asset classes. Collectively, these papers have been used to gain a deeper understanding of prevailing techniques to develop a dynamic tail risk management strategy, carefully devise our hypothesis and compare our analyses and strategies versus a suitable benchmark.

5 Methodology and Implementation

5.1 GARCH Model

It is a well-documented stylized fact that volatility showcases auto-regressive behavior[2],[7], [4], [12],[5]. In the GARCH model, large shocks to the conditional variance can persist, affecting the conditional variance at all future periods. This is known as volatility clustering. The GARCH (1,1) model can capture this autoregressive behavior to accurately forecast future volatility. Hansen and Lund [6] used over 330 different GARCH volatility models with 7 different criteria including the mean-squared-error, and mean absolute deviation criterion (less sensitive to extreme misprediction) for better comparison between models. They provide compelling evidence that it is difficult to find a model that outperforms the GARCH(1,1) model. Hence, it is a clear choice to forecast future volatility based on daily returns. We assume a normal distribution for our GARCH(1,1) model as it indicates that the standardized residuals are assumed to follow a normal (Gaussian) distribution. Under this assumption, the tails of the distribution are symmetric, and the probability density function (PDF) of the residuals follows a bell-shaped curve. The GARCH(1,1) model can be expressed as follows:

$$y_t = \mu + \epsilon_t$$

$$\epsilon_t = \sigma_t z_t \sim i.i.d\ N(0,1)\ \sigma_t^2 =$$

$$\omega + \alpha\epsilon_{t-1}^2 + \beta\sigma_{t-1}^2$$

where: y_t is the observed series at time t, μ is the constant term or mean of the series. ϵ_t is the error term at time t. z_t is a white noise error term with mean 0 and variance 1. σ_t^2 is the conditional variance of y_t at time t. ω, α, and β are parameters of the model, with $\omega > 0$, $\alpha \geq 0$, and $\beta \geq 0$. $0 \leq \alpha + \beta \leq 1$ is a condition to ensure that the process is covariance stationary [10].

This model specifies that the conditional variance σ_t^2 at time t depends on:

1. A constant term ω.
2. The squared error term ϵ_{t-1}^2 multiplied by a coefficient α, representing the impact of past shocks on the current volatility.
3. The lagged conditional variance σ_{t-1}^2 multiplied by a coefficient β, representing the persistence of volatility.

Now, to forecast volatility at time t+1 using this GARCH(1,1) model, we can substitute the conditional variance equation into itself:

$$\hat{\sigma}_{t+1}^2 = \omega + \alpha\left(\sigma_t z_t\right)^2 + \beta\left(\omega + \alpha\epsilon_t^2 + \beta\hat{\sigma}_t^2\right)$$

where $\hat{\sigma}_{t+1}^2$ represents the forecasted conditional variance at time $t + 1$.

This iterative process allows for forecasting future volatility based on past observations and estimated parameters.

5.2 APARCH Model with Pareto 'Fat-Tail' Shocks

The Asymmetric Power AutoRegressive Conditional Heteroskedasticity (APARCH) is an extension of GARCH. APARCH can capture the asymmetry in volatility meaning it recognizes that positive and negative shocks can affect volatility differently. The markets are always negatively skewed i.e. markets react to bad news more aggressively than they react to good news. Such models are essential for accurately assessing and managing financial risk, especially tail risk. The APARCH model can be expressed as:

$$\epsilon_t = \frac{y_t - \mu_t}{\sigma_t} \sim i.i.d\ GED$$ where μ_t is the conditional mean of y_t

$$\sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i |\epsilon_{t-i}|^{\gamma} + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2$$

where σ_t^2 represents the conditional variance of the time series y_t at time t, ω is the constant term representing the long-term average volatility, α_i and β_j are the parameters to be estimated, p and q are the orders of the ARCH and GARCH terms, ϵ_t represents the standardized residuals. $\gamma > 0$ indicates asymmetry in volatility response to positive and negative shocks.

We use the Generalized Error Distribution (GED) to model the distribution of the error term as it allows us to capture features such as fat tails and skewness. Moreover, the GED's robustness in handling features like fat tails and asymmetry ensures a more accurate estimation of volatility, enhancing the fit of our model.

Pareto Distribution is a probability distribution that models financial scenarios in cases when a small proportion of trading days might witness significant gains or losses, while the majority of days might see more moderate changes in value. The graph in Figure 1 shows us the difference in the PDF of a Normal Distribution and a Pareto 'Fat' Tailed Distribution.

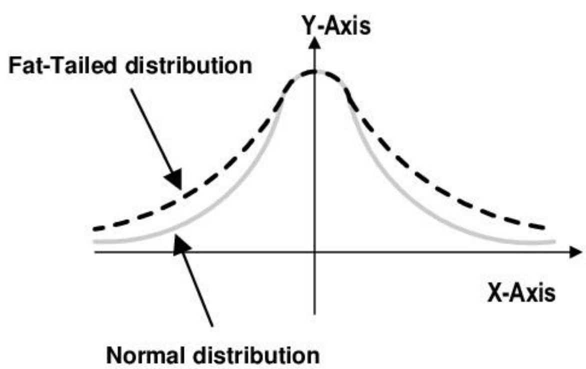


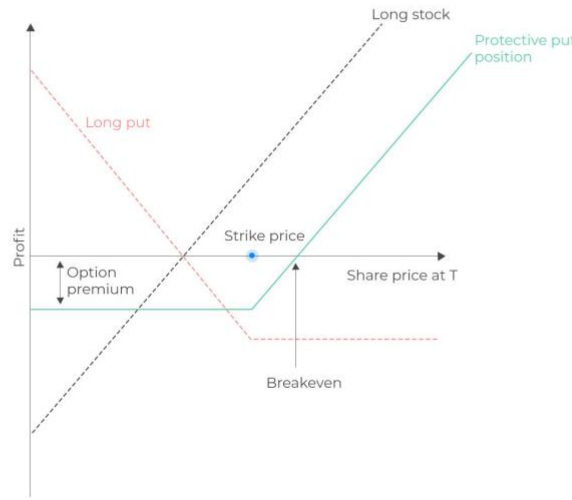
Figure 1: Normal and Pareto Distribution

Its key characteristics include:

• Heavy Tails - The Pareto model we use has heavy end tails, meaning it assigns relatively high probabilities to extreme values. Also referred to as fat tails distribution • Scale invariance: The distribution is scale-invariant, meaning that multiplying all values of x by a constant doesn't change its shape.

5.3 Static Protective Put Strategy

A protective put strategy is an options trading strategy that involves holding an underlying asset (stock or an ETF) while simultaneously purchasing a put option contract on the same asset. The primary objective of this strategy is to provide downside protection for the underlying asset while allowing for potential upside participation. In essence, it serves as an insurance policy against significant losses in the event of a market downturn.



Explanation:
The upward-sloping curve represents the payoff from owning the underlying asset, allowing for unlimited upside potential. The horizontal line segment represents the payoff from the long put option, providing a constant profit if exercised. Unlimited upside potential, reduced by the put option premium. The downside risk is limited to the put option premium, with losses capped at the strike price. Maximum loss occurs at the strike price minus the put premium.

Figure 2: Protective Put Payoff

For this project, our protective put strategy involves taking a long position in 100 shares of the SPDR S&P 500 ETF Trust (SPY), while concurrently implementing a protective put option overlay. An out-of-the-money put option contract on the SPY is purchased with a strike price set at 90% of the underlying SPY's spot price at initiation (t_0). The premium for this put option is determined by the Black-Scholes option pricing model, utilizing the risk-free rate implied by the 13-week U.S. Treasury bill (IRX) and the implied volatility derived from the CBOE Volatility Index (VIX) as inputs.

The put option has an expiration date of three months from the trade initiation date (t_3). Upon expiration, if the put option is in-the-money (ITM), indicating that the SPY's price has breached the strike price to the downside, the investor exercises their right to sell the 100 shares of SPY at the strike price. Subsequently, the investor re-establishes a long position in the SPY by repurchasing 100 shares at the prevailing market price. Conversely, if the put option expires out-of-the-money (OTM), with the SPY's price remaining above the strike price, the investor allows the put option to expire worthless, realizing a loss equal to the premium paid.

In the event of the put option expiring OTM, a new protective put option overlay is implemented by purchasing an out-of-the-money put option contract on the SPY with a strike price set at 90% of the current SPY spot price and a three-month expiration period. This cyclical process is repeated continuously from January 1, 2014, to December 31, 2023.

The protective put strategy provides downside protection for a long position in the underlying asset by granting the holder the right, but not the obligation, to sell the asset at a predetermined strike price. This effectively establishes a floor for potential losses. While offering this downside mitigation, the strategy still allows for upside participation if the asset price appreciates, although the upside gains are reduced by the premium paid for the put option. However, the primary drawback is the recurring cost of purchasing the put option contracts, which erodes potential gains, especially in stagnant or bullish market conditions. Furthermore, the strategy imposes a cap on the maximum potential upside, as any gains are reduced by the premium paid for the put protection.

5.4 Model Implementation

We have gathered data pertaining to the SPY (S&P 500 ETF), which forms the basis of our time series models. Our dataset consists of the prices spanning from January 1, 1996, to December 31, 2023. This timeframe was selected to strike a balance between dataset length, capturing recent market trends, and reflecting post-pandemic market characteristics.

Following data collection, we partitioned the dataset into training and test sets to facilitate model training and evaluation for market volatility prediction. For model training, we utilized daily returns of the SPY ETF spanning from 1996 to 2013. We have also rescaled these daily returns by a factor of 100x, this is since GARCH models fit equity returns better scaled by a factor of 100.

The next step in our approach was to fit a GARCH(1,1) model with normal innovations. This model has been used to forecast volatility based on the assumption that daily returns of the underlying time series has a normally distributed error term or thin tails. This represents market conditions under normal conditions when volatility does not undergo wild fluctuations.

The second step is to fit an APARCH(1,1) model with generalised error distribution innovations as shown above in Figure 3. We then introduce shocks to the fitted model that follow a fat tailed pareto distribution. The pareto distribution arises as a limit of tail behaviour in extreme value theory and can be utilised to capture tail risks since the pareto distribution has fat tails. The parameters of the pareto distribution need to be calibrated before shocks that follow a pareto distribution can be added to the fitted APARCH(1,1) model. For this purpose, we have utilised maximum likelihood estimation in order to compute the mean and variance of the pareto distribution by supplying daily returns of the S&P 500 ETF(SPY). It is important to note that whereas GARCH(1,1) models fit better when the underlying time series is scaled by 100x, fat tailed distributions do not fit well upon scaling, thus we have rescaled the underlying time series back to its original scale. The pareto distribution also has left and right tailed residuals. In order to accurately compute these we have utilised the Hill Double Bootstrap approach on daily returns of the SPY ETF from 1996 till 2013. After fitting the pareto distribution, we use the previously fitted APARCH(1,1) model to forecast volatility that has pareto distribution shocks. This helps in capturing tail events and forecasting volatility that cannot be explained by the normal distribution. This is similar to the approach followed by Danielson et. al [3].

```python
gm = arch_model(spy_returns, vol='aparch', p=1,q=1,o=1, dist ='ged')
res = gm.fit(last_obs = '2013-12-31',disp ='off')
xi_left, xi_right = ph.two_tailed_hill_double_bootstrap(train_data)
model = ph.Phat.fit(train_data, xi_left, xi_right)
# print(model.params)
phat = ph.Phat(model.params[0],model.params[1],0.5,0.5)


def custom_rng(phat):
    def _rng(size):
        shocks = phat.rvs(size=size)
        return shocks
    return _rng

sim= res.forecast(
    start='2014-01-01',
    horizon=1,
    simulations=10000,
    rng=custom_rng(phat),
    method='simulation',
    reindex=False
)
```

Figure 3: Aparch Model with Pareto Shocks

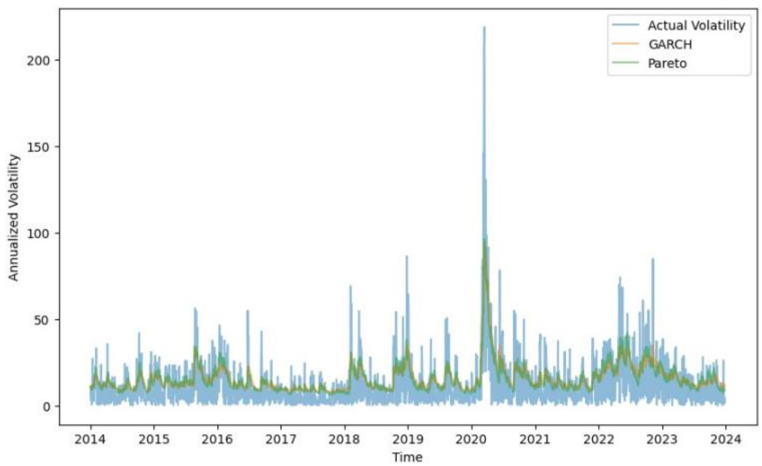


Figure 4: Volatility Comparison of different models with Actual Volatilty

Upon fitting our two volatility prediction models, the next step is to forecast two sets of volatilities, one volatility forecast using the GARCH(1,1) model and another volatility forecast using an APARCH(1,1) model with pareto distribution innovations. Our test set comprises of the daily returns from 2014 to 2024 of the SPY ETF. This timeframe ensures robust evaluation of model performance across distinct time periods, enhancing the reliability of our volatility forecasts. Our predicted volatility for the models is shown in Figure 4.

This led to the generation of our signal, the VaR spread between the two models at a confidence interval of 99% which quantifies the maximum potential loss with a 1% error probability. We are using a parametric approach to estimate daily VaR using the mean and variance from our two fitted models. The daily VaR spread over the test period is shown in figure 5. We can see the spread is highest in times of COVID i.e. March 2020, and in 2022 as well, when the Federal Reserve announced rate hikes to combat with high inflation.

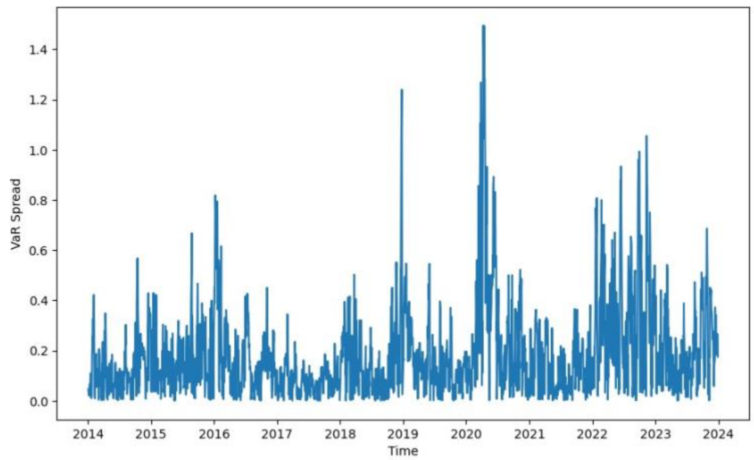


Figure 5: VaR Spread Generated between the GARCH and APARCH w/ Pareto Model

5.4.1 All-weather Portfolio

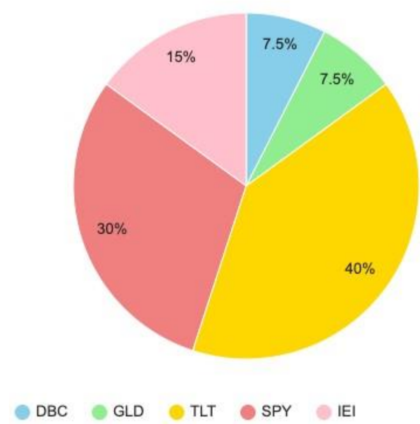


Figure 6: Pie Chart: Weights of All Weather Portfolio

For the purpose of this research we have attempted to create an all-weather portfolio akin to BridgeWater's All-weather portfolio. The construction of the all weather portfolio is 7.5% in the DBC ETF, 7.5% in the GLD ETF, 40% in the TLT ETF, 30% in the SPY ETF and 15% in the IEI ETF. This portfolio aims at performing reasonably well under all market conditions, hence all-weather. A Pie Chart for a clear understanding of the portfolio weights is shown in Figure 6.

The main advantage of this portfolio is that it suffers significantly less maximum drawdown as opposed to conventional ETF's like the DIA and SPY due to its conservative asset allocation.

5.4.2 Dynamic Tail Risk Strategy

```
strike=0.90*spy['Adj Close'].shift(1)
df=pd.DataFrame({'Date':forecast_data.index,'spy_ret': spy_ret.values,
                        'Regime': [2 if spread>=1.4 else 1 if spread>=0.8 else 0 for spread in VaR_diff['h.1'] ],
                       'gold_ret':gold_ret,
                    'weather_ret':weather_ret.values,
                         'rate':tbill['Adj Close'].shift(1).values,'spy':spy['Adj Close']*100,'vol': vix['Adj Close'].shift(1),
                 'Strike':strike})
```

Figure 7: Code Snippet of Signal Generation based on VaR Thresholds

Our Dynamic Tail Risk strategy is as follows, based on historical VaR spread values during tail events and normal conditions we identify three investment regimes. When the forecasted VaR spread is below 0.8 we are in normal conditions and are long the SPY, when the VaR spread is between 0.8 and 1.4 we are fully invested in the aforementioned all-weather portfolio. The code snippet shown in Figure 8 shows the same idea. When the VaR spread exceeds 1.4 we are long the SPY but at the same time we also purchase a 3 month put at 90% of the stike price of the underlying ETF at the time of generation of the signal for a 3 month expiry period. The price of this put option has been calculated using the Black Scholes Option pricing formula. For volatility we have used the VIX index whereas for interest rate we have used the 13 Week treasury bill rate. It might seem counter intuitive to be long the SPY when the VaR spread is high, however by purchasing a protective put we are insuring against severe drawdowns.

5.4.3 Backtest

```
elif(df.loc[i,'Regime']==2):
  protective_put=True
  option_price=bsput(df['spy'][i]/100,0,df['Strike'][i],df['rate'][i]/100,df['vol'][i]/100,90/365)*100
  option_strike=df['Strike'][i]
  option_txn_cost=0.65
  option_price+=option_txn_cost
  option_start_date=df['Date'][i]
  df.loc[i,'spy']-=option_price
  # options.loc[i,'pp']=(options.loc[i,'spy']-option_start_underlying)/(option_start_underlying)+1
  option_start_underlying=df.loc[i,'spy']
  df.loc[i,'strat']=(df.loc[i,'spy']/df.loc[i-1,'spy'])
```

Figure 8: Code Snippet of Protective Put Calculation in Back Test

We have backtested all the aforementioned trading strategies over the time period January 2014 to December 2023. In order to backtest our strategies, we generate daily VaR spread signals and then apply our dynamic tail risk strategy as described above.Moreover, we have shifteed the input to our backtest by one day since our signal needs adjusted close values from the previous day to generate a signal on the next day. We have also considered some transaction costs when constructing our backtest. We have used Fidelity investments' transaction costs. The transaction fee will be $0 for securities and ETFs, while $0.65 for a single options contract, and $1 for any online trading of bills/bonds on the secondary market. Regulatory Fee by FINRA has been omitted for this model due to its variable nature.

6 Results and Analysis

All the data is trained on previous years and is tested from January 1, 2014 to January 1, 2024.

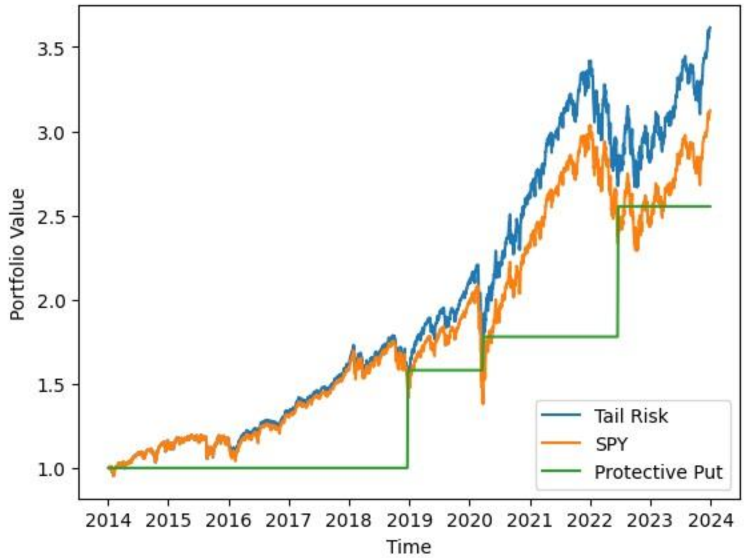


Figure 9: Returns of the the strategies over 10 years

Table 1: Metrics of the 3 Strategies

	Tail Risk Strategy	SPY (Buy and Hold)	Static Protective Put
CAGR	12.85%	11.39%	9.37%
Sharpe	0.72	0.62	0.48
Sortino	0.85	0.73	Nan
Max Drawdown	-0.28	-0.34	0.00

We observe that the dynamic tail risk strategy outperforms both the S&P 500 and the Static Protective Put (SPP), validating our initial hypothesis across various financial metrics. Specifically, our Tail Risk strategy surpasses the S&P 500 by more than 140 basis points, exceeding our prediction by 40 basis points, with an annual Compound Annual Growth Rate (CAGR) of 12.85%. Furthermore, we outperform the SPP by a substantial margin of approximately 350 basis points. Over the 10-year period, our strategy yields approximately 49% higher returns compared to the S&P 500.

Even when considering Sharpe ratios, our strategy demonstrates superior performance compared to both benchmarks. Notably, our hypothesis regarding a marginally superior Sharpe Ratio, beating it by 0.1, holds true. For long-term strategies, achieving a Sharpe ratio of 0.7 is considered optimal. Additionally, we enhance our Sortino Ratio by 0.1, reflecting higher returns for our strategy in contrast to the benchmark. It's worth noting that the Sortino Ratio is 'Nan' for SPP due to the absence of negative drawdowns, given the substantial cost of the puts that minimizes returns. Note that the risk free rate for calculating Sharpe and Sortino ratios has been taken to be 0.5%.

In terms of maximum drawdowns, our hypothesis exceeds S&P 500 drawdowns by a margin of over 5%, as indicated in Table 1. This outcome is attributed to the signals generated by the Value at Risk (VaR) spread prompting a shift towards investment in the all-weather portfolio, which experiences fewer fluctuations. Additionally, the periodic purchase of puts every three months when VaR spread exceeds 1.4 provides downside protection, resulting in reduced negative shocks compared to solely holding the S&P 500. However, it's important to acknowledge that the premium paid for purchasing puts every three months does have a slight impact on our returns.

Nevertheless, we fall short of surpassing the Static Protective Put. The mechanism of the SPP in our scenario ensures the absence of downside risk, although returns are lower. Consequently, our hypothesis concerning beating the maximum drawdown metric is only partially accurate.

Figure 10: Comparing Gold with Tail Risk Strategy

Table 2: Metrics of Gold and All-Weather Portfolio

	Gold	All Weather
CAGR	4.84%	5.59%
Sharpe	0.31	0.64
Sortino	0.44	0.84
Max Drawdown	-0.24	-0.22

The comparison provided in Figure 10 delves into the performance of Gold and the All-Weather portfolio against the standard benchmarks of S&P 500 and the Tail Risk Strategy. Our strategy demonstrates remarkable performance, showcasing a more than twofold increase in Compound Annual Growth Rate (CAGR) compared to both Gold and the AllWeather portfolio. Notably, the All-Weather portfolio outperforms Gold by over 75 basis points in terms of CAGR.

The Sharpe Ratio of our Tail Risk strategy and the S&P 500 doubles that of Gold, underscoring their superior risk-adjusted returns. Additionally, the Sharpe Ratio of the All-

Weather portfolio not only surpasses that of Gold but also exceeds that of the S&P 500, indicating a better risk-to-reward ratio. While the Sharpe Ratio of All-Weather slightly trails behind our Tail Risk Strategy, it still outperforms Gold and the S&P 500.

In terms of the Sortino Ratio, the All-Weather portfolio exhibits double the performance of Gold, with Gold having the lowest Sortino Ratio among the four assets. Remarkably, both the All-Weather portfolio and our Tail Risk Strategy demonstrate comparable Sortino Ratios, indicating an equivalent risk-to-reward ratio. The Sortino Ratio of All-Weather slightly surpasses that of the S&P 500.

Considering maximum drawdowns, the All-Weather portfolio displays the lowest drawdown, indicating its risk-averse nature. Shockingly Gold's drawdown is slightly higher than that of the All-Weather portfolio (considering the fact that Gold is considered the global hedge to inflation), comparisons with the S&P 500 and our Tail Risk Strategy are less relevant due to their substantially higher returns.

Over the 10-year period, Gold yields approximately 62%, while the All-Weather portfolio returns 75%. In contrast, our strategy delivers a remarkable 262% return, surpassing Gold returns by more than fourfold and All-Weather returns by threefold. Notably, the S&P 500 yields 212% over the same period, significantly outperforming Gold and double the returns of the All-Weather portfolio. However, the All-Weather portfolio outperforms Gold in all metrics and surpasses the S&P 500 in three out of four metrics, including Sharpe Ratio, Sortino Ratio, and maximum drawdown. For risk-averse investors, the All-Weather portfolio presents a compelling choice, offering better returns than Gold, superior optimization of Sharpe and Sortino Ratio compared to both Gold and the S&P 500, and a lower drawdown than Gold, S&P and Tail Risk. Moreover, its comparable Sortino Ratio to our Tail Risk Strategy suggests a similar risk-to-reward ratio.

7 Conclusion

We develop a dynamic tail risk investment strategy that is designed to protect against severe market drawdowns whilst still being cost effective in doing so. The main concern with traditional strategies aimed at curtailing tail risk is that their exorbitant premium cannot justify the portfolio protection that they offer during severe market drawdowns.

We utilise a well-documented stylised fact about volatility that it showcases auto-regressive behaviour. Building on this we fit two different volatility forecasting models, a GARCH(1,1) model with normal innovations and an APARCH(1,1) model with pareto innovations. The training data for the same is from 1996 to 2013. Pareto shocks are supplied to the APARCH model forecast to simulate a fat tailed market environment. Using these two models we forecast volatility and subsequently use it to generate a 99% VaR spread between the two. This is the key signal in generating our strategy. The core idea is that the VaR spread between the two models increases as the market approaches turbulent times.

Based on historical VaR spreads we construct our strategy over the time period from January 2014 to December 2023. Historically, it has been observed that a VaR spread of greater than 1.4 signals a high likelihood of a severe market drawdown, VaR spreads between 0.8 and 1.4 signal slightly bearish conditions, whereas those below 0.8 indicate periods of market normalcy.

Based on this, our strategy is to dynamically generate VaR spreads and go long on the SPY if VaR spreads are below 0.8, enter into an 'all-weather' portfolio when VaR spreads are between 0.8 and 1.4 and purchase a protective put on the SPY when spreads exceed 1.4.

We show that the general idea of applying tail risk measures and EVT to identify extreme events yields promising results in tail-risk protection and risk management. An extensive out-of-sample backtest shows that our strategy perfroms better than a simple buy and hold strategy on the SPY ETF over a period of 10 years. Our strategy has a CAGR of 12.85% versus SPY's 11.39%, it has a Sharpe and Sortino Ratio of 0.72 and 0.85 versus 0.62 and 0.73 for the SPY ETF. It also has a better maximum drawdown of -28% versus the SPY ETF's maximum drawdown of -34%. Our strategy also outperforms a static protective put strategy on most key metrics apart from the maximum drawdown since a protective put has a ceiling on the maximum loss. It is also superior to holding Gold over a 10 year horizon apart from gold's maximum drawdown of -24%

Having said that, there are several enhancements that can be made to the model to improve it's accuracy, first, our backtests do not incorporate dividends and certain transaction costs thereby reducing it's overall precision. Moreover, we assume highly liquid markets which might not always be the case during market drawdowns. Additionally, we are using the Black Scholes Model to price puts due to the lack of historical put prices.

8 Limitations and Further Improvement

8.1 Transaction Costs

Regulatory Fee by FINRA has been omitted for this model due to its variable nature. As a result, there will be minor discrepancies in our final results.

8.2 Market Liquidity

We are considering highly liquid markets for the implementation of our backtest and calibration of our model. Since tail risk strategies are more likely to be triggered during times of low liquidity, this might affect the accuracy of our results vis-a-vis an extremely accurate backtest. Thus our model is susceptile to inaccurate Liquidity Risk identification.

8.3 Option Pricing

Due to unavailability of prices of historical put options, we have had to resort to using the Black Scholes Model to price options, we do not expect these prices to be wildly off from actual quotes since we are not using deep OTM put options but there will still be discrepancies in our results.

8.4 Dividends

We have not considered dividends as part of total returns on our positions when performing our backtests, this is a potential limitation that can be addressed in further iterations of our proposed approach.

8.5 Expense Ratio

All ETFs have an expense ratio which ranges from 0.05-0.5% per year, which they take as management fee, which is not included in our backtest.

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