

# On the Statistical Behaviour of the Degree of Polarization in Pulsars

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## ABSTRACT

In the *IRAS* [12]–[25], [25]–[60] colour–colour diagram, RV Tauri stars are found to populate cooler temperature regions ( $T < 600$  K), distinctly different from those occupied by the oxygen and carbon Miras. The *IRAS* fluxes are consistent with the dust density in the envelope varying as the inverse square of the radial distance, implying that the grain formation processes in these objects are most probably continuous and not sporadic. It is found that the spectroscopic subgroups A and B are well separated in the far-infrared two-colour diagram, with group B objects having systematically cooler dust envelopes. We interpret this as being due to a difference in the nature of grains, including the chemical composition, in the two cases.

**Key words:** circumstellar matter – infrared: stars.

## 1 INTRODUCTION

Of the many astrophysical sources, pulsars tend to be those with high degrees of polarization encountered, in the observed radio emission. Typical polarisation percentages for any other celestial radio source, than a pulsar, are usually less than about 20%, although a recent NRAO/VLA survey seem to have thrown up radio sources with slightly higher polarisation percentages (Ref :). The pulsars remain a distinct set amongst the radio sources due to their radio radiation being so highly polarised.

As the underlying magnetic fields near the pulsar surface are known to be of very high magnitude  $10^{10}$  Gauss on the lowest end and  $10^{14} - 10^{16}$  Gauss at the Magnetar end, it is not very surprising that a high degree of linear polarization is encountered in pulsars. The intriguing aspects come from the behaviour of the linear polarization under the profile of the pulsars, as also the presence and behaviour of circular polarization, seen to be present in many pulsars.

Most detailed models for pulsar polarization use a constant value of high degree of polarization for all pulsars. Depolarization is then assumed to be the cause for the differences in the degree of polarization seen from pulsar to pulsar. One of the causes for depolarization in pulsars is well known to be the phenomenon of orthogonal polarization modes which has been observed in many pulsars. There seems to be some debate, whether, the presence of orthogonal power

under the pulsar profile is the cause of depolarisation in average profiles (Stinebring et al 1984) or, whether, low linear polarisation arising from some intrinsic cause gives rise to the presence of an orthogonal mode of emission in pulsars (Xu and Qiao 2000).

In general the position angle of linear polarization dramatically follows an S shaped curve as predicted by the celebrated rotating vector model of Radhakrishnan and Cooke (1969). In some pulsars, observations of single pulses throw up instances where the position angle of linear polarization is 90 degrees away from the general average S-shaped curve of the swing of the position angle of linear polarization, under the profile of the pulsar. In some pulsars, the presence of this orthogonal mode of emission is manifested in the average pulsar profiles, where for some phase angles, the average position angle of the linear polarisation is seen to consistently remain in an orthogonal mode. (Examples from EPN)

What is not clear from the studies so far is whether orthogonal polarization modes are the main cause for depolarization, and whether the defining degree of polarization observed in different pulsars is dependent on any physical parameter of these pulsars.

The current study is an attempt to compile a database of pulsar degree of polarization values using online pulsar databases and statistically analyze the sample to check for any such dependencies or, otherwise, to confirm their absence through a rigorous analysis.

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**Table 1.** Data on the RV Tauri stars detected by *IRAS*.

Name		Flux density (Jy) <sup>a</sup>							
Variable	<i>IRAS</i>	12 $\mu$ m	25 $\mu$ m	60 $\mu$ m	100 $\mu$ m	Sp. group	Period (d)	Light- curve type	$T_0$ (K)
TW Cam	04166+5719	8.27	5.62	1.82	<1.73	A	85.6	a	555
RV Tau	04440+2605	22.53	18.08	6.40	2.52	A	78.9	b	460
DY Ori	06034+1354	12.44	14.93	4.12	<11.22	B	60.3		295
CT Ori	06072+0953	6.16	5.57	1.22	<1.54	B	135.6		330
SU Gem	06108+2734	7.90	5.69	2.16	<11.66	A	50.1	b	575
UY CMa	06160–1701	3.51	2.48	0.57	<1.00	B	113.9	a	420
U Mon	07284–0940	124.30	88.43	26.28	9.24	A	92.3	b	480
AR Pup	08011–3627	131.33	94.32	25.81	11.65	B	75.0	b	450
IW Car	09256–6324	101/06	96.24	34.19	13.07	B	67.5	b	395
GK Car	11118–5726	2.87	2.48	0.78	<12.13	B	55.6		405
RU Cen	12067–4508	5.36	11.02	5.57	2.01	B	64.7		255
SX Cen	12185–4856	5.95	3.62	1.09	<1.50	B	32.9	b	590
AI Sco	17530–3348	17.68	11.46	2.88	<45.62	A	71.0	b	480
AC Her	18281+2149	41.47	65.33	21.12	7.79	B	75.5	a	260
R Sct	18448–0545	20.88	9.30	8.10	<138.78	A	140.2	a	
R Sge	20117+1634	10.63	7.57	2.10	<1.66	A	70.6	b	455
V Vul	20343+2625	12.39	5.72	1.29	<6.96	A	75.7	a	690

<sup>a</sup> Observed by *IRAS*.

## 2 DEGREE OF POLARISATION FROM THE EUROPEAN PULSAR NETWORK DATABASE

In the present analysis, the entire contents made available on the European Pulsar Network site <http://www.jb.man.ac.uk/research/pulsar/Resources/epn/browser.html> were downloaded locally and used as one single dataset to be analysed using MATLAB software. The Pulsar data used from the EPN database is the average profile i.e the stokes parameters  $I, Q, U, V$  at the corresponding pulsar phase as given in the ASCII data against each pulsar for the available frequencies. For the present study, mainly the data at around 1400 MHz has been used. The parameters - like periods, period derivatives, magnetic field strength etc. were obtained from the ATNF database, for the corresponding pulsars. The MATLAB code downloaded the entire EPN data base and performed offline analysis on the dataset. Data from a pulsar at a particular frequency was included from the data base, if (1) How were the pulsars chosen and identified how were the frequencies identified The pulse profiles in terms of total power  $I$ , linear polarisation  $L$ , circular polarisation  $V$  and the position angle of linear polarisation  $PA$  were reproduced for the pulsars using MATLAB software after extracting the relevant pulsars and data at frequencies of interest. The data as expected had noise which had to be removed, to account for this, only significant data was chosen by deciding for each pulsar the cut off which gave us the degree of linear polarization physically acceptable. The degree of linear polarization was calculated for each of the time instances of the significant data was available for and it was averaged. The calculated dpi values were then plotted against different parameters like magnetic field, age, energy, time period.

### 2.1 Colour-colour diagram

The IR colour is defined as

$$[\nu_1] - [\nu_2] = -2.5 \log[f(\nu_1)/f(\nu_2)],$$

where  $\nu_1$  and  $\nu_2$  are any two wavebands and  $f(\nu_1)$  and  $f(\nu_2)$  are the corresponding flux densities assuming a flat energy spectrum for the source. In Fig. 1, we have plotted the [25]–[60] colours of RV Tauri stars against their corresponding [12]–[25] colours derived from the *IRAS* data. Filled circles represent stars of group A and open circles stars of group B. The two sets of near-parallel lines represent the loci of constant inner shell temperature  $T_0$  and the quantity  $Q$  defined above. The models correspond to the case of absorption efficiency  $Q_{\text{abs}}(\nu)$  varying as  $\nu$  (with  $\gamma = 1$  and hence  $\beta = -0.4$ ). We have omitted R Sct in Fig. 1 because it shows a large deviation from the average relation shown by all the other objects. R Sct has a comparatively large excess at 60  $\mu$ m, but the extent of a possible contamination by the infrared cirrus (Low et al. 1984) is unknown. Goldsmith et al. (1987) found no evidence of the presence of a dust envelope at near-IR wavelengths and the spectrum was consistent with a stellar continuum. This explains why R Sct lies well below the mean relation shown by stars of groups A and C between the [3.6]–[11.3] colour excess and the photometrically determined (Fe/H) (Dawson 1979). R Sct has the longest period of 140 d among the RV Tauri stars detected at far-infrared wavelengths and does not have the 10- $\mu$ m emission feature seen in other objects (Gerhz 1972; Olmon & Raimond 1986). R Sct is probably the most irregular RV Tauri star known (McLaughlin 1932).

The inner shell temperatures ( $T_0$ ) derived for the various objects are also given in Table 1 and we find the majority of them to have temperatures in the narrow range 400–600 K. If the dependences of  $Q_{\text{abs}}(\nu)$  on  $\nu$  and  $\rho(r)$  on  $r$  are similar in all the objects considered, then in the colour–

**Figure 1.** Plot of [25]–[60] colours of RV Tauri stars against their [12]–[25] colours after normalizing as indicated in Beichman et al. (1985b). Some of the objects are identified by their variable-star names. Typical error bars are shown in the bottom right-hand corner. The lines represent the loci for constant inner shell temperature and the quantity  $Q$ . Note the separation of group A and B stars at  $T_0 \sim 460$  K. Positions occupied by a sample of carbon and oxygen Miras are also shown. The  $Q = 1.0$  line differs from the blackbody line by a maximum of  $\sim 0.05$ .

colour diagram they all should lie along a line corresponding to different values of  $T_0$  and in Fig. 1 we find that this is essentially the case. In view of the quoted uncertainties in the flux measurements, we cannot attach much significance to the scatter in Fig. 1.

At  $100\,\mu\text{m}$  the infrared sky is characterized by emission, called infrared cirrus, from interstellar dust on all spatial scales (Low et al. 1984), thereby impairing the measurements at far-infrared wavelengths. In Fig. 2, we have plotted the [60]–[100] colours of the six RV Tauri stars detected at  $100\,\mu\text{m}$  against their [25]–[60] colours, along with the grid showing the regions of different values for inner shell temperature  $T_0$  and the quantity  $Q$ , as in Fig. 1. The results indicated by Fig. 2 are consistent with those derived from Fig. 1. AR Pup shows a large excess at  $100\,\mu\text{m}$  but, in view of the large values for the cirrus flags given in the catalogue, the intrinsic flux at  $100\,\mu\text{m}$  is uncertain.

## 2.2 Radial distribution of dust

From Fig. 1, it is evident that all RV Tauri stars lie between the lines corresponding to  $Q = 1.5$  and  $0.5$ . With

$$\alpha = -(1 + Q)\beta - 3,$$

these values suggest limits of  $r^{-2.0}$  and  $r^{-2.4}$  for the dust density variation, indicating a near-constant mass-loss rate. Jura (1986) has suggested that the density in the circumstellar envelope around RV Tauri stars varies as  $r^{-1}$ , implying a mass-loss rate that was greater in the past than it is currently. By fitting a power law to the observed fluxes, such that  $f(\nu)$  varies as  $\nu^q$ , values of  $q$  determined by him for the various objects given in Table 1 lie in the range  $0.6$ – $1.2$ , with a mean  $\bar{q} = 0.98$ . The assumption of a power law corresponds to the case of  $X_0 = 0$  in equation (3) and hence we get

$$q = 2 + \gamma - Q.$$

Since we assume that  $Q_{\text{abs}}(\nu)$  varies as  $\nu$ , the resulting value for  $Q=2.0$ . None of the objects is found to lie in the corresponding region in the colour–colour diagram. Even this extreme value for  $Q$  implies a density which varies as  $r^{-1.8}$ .

Goldsmith et al. (1987) have reported that the simultaneous optical and near-IR data of AC Her can be fitted by a combination of two blackbodies at  $5680$  and  $1800$  K, representing, respectively, the stellar and dust shell temperatures, and suggested that in RV Tauri stars the grain formation is a sporadic phenomenon and not a continuous process. Apparently, they have been influenced by the remark by Gerhz & Woolf (1970) that their data in the  $3.5$ – $11\,\mu\text{m}$  region of AC Her indicated a dust temperature of  $\sim 300$  K. We find that the  $K$ – $L$  colours given by Gerhz (1972), Lloyd Evans (1985) and Goldsmith et al. (1987) are all consistent with each other. Surely, hot dust ( $\sim 1800$  K), if present at the time of observations by Goldsmith et al. (1987), would have affected the  $K$ – $L$  colour significantly. AC Her, like other members of its class, is found to execute elongated loops in the  $(U-B)$ ,  $(B-V)$  plane (Preston et al. 1963), indicating that significant departure of the stellar continuum from the blackbody is to be expected. Further, their data show only a marginal excess at the near-IR wavelengths. We feel that the case for the existence of hot dust around AC Her and hence for the sporadic grain formation around RV Tauri stars is not strong. In Fig. 3 we find that AC Her and RU Cen lie very close to R Sct which, according to Goldsmith et al. (1987), shows no evidence for the presence of a hot dust envelope.

### 2.2.1 Comparison with oxygen and carbon Miras

In Fig. 1 we have also shown the positions of a sample of oxygen-rich and carbon-rich Miras. At the low temperatures characteristic of the Miras, a part of the emission at  $12\,\mu\text{m}$  comes from the photosphere. For a blackbody at  $2000$  K, the ratio of fluxes at wavelengths of  $12$  and  $2\,\mu\text{m}$  ( $f_{12}/f_2$ )  $\sim 0.18$ . The Miras shown in Fig. 1 have ( $f_{12}/f_2$ ) ratios larger than twice the above value. It is clear that the three groups of objects populate three different regions of the diagram. Hackington et al. (1985) have already noticed that there are distinct differences between the *IRAS* colours of oxygen-rich and carbon-rich objects. On the basis of an analysis, using a bigger sample of bright giant stars in the *IRAS* catalogue, this has been interpreted by Zuckerman & Dyck (1986) as being due to a systematic difference in the dust grain emissivity index. U Mon shows the  $10\text{-}\mu\text{m}$  silicate emission convincingly and, in most of the other objects for which low-resolution spectra in the near-infrared have been reported (Gerhz 1972;

**Figure 2.** Plot of the  $[60]-[100]$  colours of RV Tauri stars against their  $[25]-[60]$  colours after normalizing as indicated in Beichman et al. (1985b). The solid lines represent the loci for constant inner shell temperature and the quantity  $Q$ . The dashed line shows the locus for a blackbody distribution.

Olson & Raimond 1986), the 10- $\mu$ m emission may be partly attributed to silicates. Hence it is reasonable to expect that, in the envelopes around at least some of the RV Tauri stars, the dust grains are predominantly of silicates, as in the case of oxygen Miras (Rowan-Robinson & Harris 1983a). The fact that none of the RV Tauri stars is found in the region of the two-colour diagram occupied by the oxygen Miras indicates that the emissivity indices of the silicate grains in the two cases are different. Because of the higher temperatures and luminosities, the environment of grain formation will be different in RV Tauri stars.

### 2.2.2 Correlation with subgroups

Preston et al. (1963) have identified three spectroscopic subgroups, which are designated as groups A, B and C. Objects of group A are metal-rich; group C are metal-poor; group B objects are also metal-poor, but show carbon enhancements (Preston et al. 1963; Lloyd Evans 1974; Dawson 1979; Baird 1981). It is interesting to see that Table 1 contains no group C objects and that in Fig. 1 there is a clear separation of the two spectroscopic subgroups A and B, with the demarcation occurring at an inner shell temperature of about 450 K, group B stars having lower temperatures than group A. SX Cen is the only exception. Lloyd Evans (1974) has reported that metal lines are stronger in SX Cen than in other group B objects. It may be worth noting that SX Cen has the shortest period among the 100 or so objects with the RV Tauri classification. RU Cen has the coolest inner shell temperature, as already suggested by the near-infrared spectrum (Gerhz & Ney 1972).

Group B objects follow a different mean relationship from those of group A, having systematically larger 11- $\mu$ m excess for a given excess at 3  $\mu$ m (Lloyd Evans 1985). For a general sample of RV Tauri stars, the distinction between the oxygen-rich and carbon-rich objects is not that apparent in the  $JHK_L$  bands. In Fig. 3 we have plotted the near-IR magnitudes of the objects given in Table 1 (except V Vul which has no available measurements) in the  $J-K$ ,  $K-L$  plane. The colours, taken from Lloyd Evans (1985) and Goldsmith et al. (1987), are averaged if more than one observation exists,

**Figure 3.** Plot of  $(K-L)$  colours of RV Tauri stars detected by *IRAS* against their corresponding  $(J-K)$  colours. The position of AR Pup is indicated. The three objects lying close to the black-body line are AC Her, RU Cen and R Sct.

because the internal agreements are found to be often of the order of observational uncertainties, in accordance with the earlier finding by Gerhz (1972) that variability has relatively little effect on colours. Barring RU Cen and AC Her, it is evident that stars belonging to group B show systematically larger excesses at  $L$  band for a given excess at  $K$ . The low excesses at near-IR wavelengths for AC Her and RU Cen are consistent with the very low dust temperatures indicated by the far-infrared colours.

It is already well established that from *UBV* photometry one can distinguish between groups A and B, members of group A being significantly redder than those of group B (Preston et al. 1963). Similarly, Dawson (1979) has found that the two spectroscopic groups are well separated in the DDO colour-colour diagrams when mean colours are used for the individual objects.

The clear separation of the spectroscopic subgroups A and B in the IR two-colour diagram suggests that the natures of dust grains in the envelopes in the two cases are not

Landscape figure to go here. This figure was not part of the original paper and is inserted here for illustrative purposes.  
See the author guide for details (section 2.2 of `mn2eguide.tex`) on how to handle landscape figures or tables.

**Figure 4.**

identical. This is to be expected because of the differences in the physical properties of the stars themselves. The average colours of group B stars are bluer than group A, but the envelope dust temperatures of B are cooler than those of A. The near-IR spectra of AC Her and RU Cen are extremely similar (Gerhz & Ney 1972). The striking similarities in the optical spectra of AC Her and RU Cen have been pointed out by Bidelman (O'Connell 1961). We feel that the physical properties, including the chemical composition, of the grains formed in the circumstellar envelope strongly depend on those of the embedded star. This, probably, explains the diversity of the energy distributions of RV Tauri stars in the near-infrared found by Gerhz & Ney (1972). On the basis of the observed differences in chemical abundances and space distribution of RV Tauri stars, Lloyd Evans (1985) has already pointed out that there is no direct evolutionary connection between group A and group B objects, thus ruling out the possibility that group B objects are the evolutionary successors of group A, in which grain formation has stopped and the cooler temperatures for the former are caused by an envelope expansion.

Kukarkin et al. (1969) have subdivided RV Tauri stars into two classes, RVa and RVb, on the basis of their light curves; the former shows a constant mean brightness, whereas the latter shows a cyclically varying mean brightness. Extensive observations in the near-infrared show that, on average, RVb stars are redder than RVa stars, and Lloyd Evans (1985) has suggested that in RVb stars dust shells are denser in the inner regions and hence radiate strongly in the 1–3  $\mu\text{m}$  region. Fig. 3 confirms this; RVb objects show systematically larger ( $J-K$ ) and ( $K-L$ ) colours than RVa objects. Apparently, there is no distinction between objects of the two light-curve types at far-infrared wavelengths (Fig. 1).

### 3 CONCLUSIONS

In the [12]–[25], [25]–[60] colour diagram, RV Tauri stars populate cooler temperature regions ( $T < 600\text{ K}$ ), distinctly different from those occupied by the oxygen and carbon Miras. Using a simple model in which

- (i) the envelope is spherically symmetric,
- (ii) the IR-emitting grains are predominantly of the same kind, and
- (iii) in the infrared the absorption efficiency  $Q_{\text{abs}}(\nu) \propto \nu$ ,

we find that the *IRAS* fluxes are consistent with the density in the envelope  $\rho(r) \propto r^{-2}$ , where  $r$  is the radial distance. Such a dependence for the dust density implies that the mass-loss rates in RV Tauri stars have not reduced considerably during the recent past, contrary to the suggestion by Jura (1986). In the two-colour diagram, the blackbody line and the line corresponding to  $\rho(r) \propto r^{-2.2}$  nearly overlap and the present data are insufficient to resolve between the two cases. The latter case is more physically reasonable, however.

The spectroscopic subgroups A and B are well separated in the *IRAS* two-colour diagram, with group B objects having systematically cooler dust envelopes. If we consider only the objects detected by *IRAS*, we find that stars belonging to group B show systematically larger excess at  $L$  band

for a given excess at  $K$ . Apparently, there is no correlation between the light-curve types (RVa and RVb) and the far-infrared behaviour of these objects. It is fairly certain that the physical properties, including the chemical composition, of the embedded stars are directly reflected by those of the dust grains. Most probably, the grain formation process in RV Tauri stars is continuous and not sporadic as suggested by Goldsmith et al. (1987).

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### APPENDIX A: LARGE GAPS IN $\text{Ly}\alpha$ FORESTS DUE TO FLUCTUATIONS IN LINE DISTRIBUTION

(This appendix was not part of the original paper by A.V. Raveendran and is included here just for illustrative purposes. The references are not relevant to the text of the

appendix, they are references from the bibliography used to illustrate text before and after citations.)

Spectroscopic observations of bright quasars show that the mean number density of Ly $\alpha$  forest lines, which satisfy certain criteria, evolves like  $dN/dz = A(1+z)^\gamma$ , where  $A$  and  $\gamma$  are two constants. Given the above intrinsic line distribution we examine the probability of finding large gaps in the Ly $\alpha$  forests. We concentrate here only on the statistics and neglect all observational complications such as the line blending effect (see Harvey et al. 1979, for example).

Suppose we have observed a Ly $\alpha$  forest between redshifts  $z_1$  and  $z_2$  and found  $N - 1$  lines. For high-redshift quasars  $z_2$  is usually the emission redshift  $z_{\text{em}}$  and  $z_1$  is set to  $(\lambda_{\text{Ly}\beta}/\lambda_{\text{Ly}\alpha})(1+z_{\text{em}}) = 0.844(1+z_{\text{em}})$  to avoid contamination by Ly $\beta$  lines. We want to know whether the largest gaps observed in the forest are significantly inconsistent with the above line distribution. To do this we introduce a new variable  $x$ :

$$x = \frac{(1+z)^{\gamma+1} - (1+z_1)^{\gamma+1}}{(1+z_2)^{\gamma+1} - (1+z_1)^{\gamma+1}}. \quad (\text{A1})$$

$x$  varies from 0 to 1. We then have  $dN/dx = \lambda$ , where  $\lambda$  is the mean number of lines between  $z_1$  and  $z_2$  and is given by

$$\lambda \equiv \frac{A[(1+z_2)^{\gamma+1} - (1+z_1)^{\gamma+1}]}{\gamma+1}. \quad (\text{A2})$$

This means that the Ly $\alpha$  forest lines are uniformly distributed in  $x$ . The probability of finding  $N - 1$  lines between  $z_1$  and  $z_2$ ,  $P_{N-1}$ , is assumed to be the Poisson distribution.

**Figure A1.**  $P(> x_{\text{gap}})$  as a function of  $x_{\text{gap}}$  for, from left to right,  $N = 160, 150, 140, 110, 100, 90, 50, 45$  and  $40$ . Compare this with Lloyd Evans (1985).

#### A1 Subsection title

We plot in Fig. A1  $P(> x_{\text{gap}})$  for several  $N$  values. We see that, for  $N = 100$  and  $x_{\text{gap}} = 0.06$ ,  $P(> 0.06) \approx 20$  per cent. This means that the probability of finding a gap with a size larger than six times the mean separation is not significantly small. When the mean number of lines is large,  $\lambda \sim N \gg 1$ , our  $P(> x_{\text{gap}})$  approaches the result obtained by Rowan-Robinson & Harris (1983b, fig. 4) for small (but still very large if measured in units of the mean separation)  $x_{\text{gap}}$ , i.e.,  $P(> x_{\text{gap}}) \sim N(1 - x_{\text{gap}})^{N-1} \sim N \exp(-\lambda x_{\text{gap}})$ .

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