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flipping coin large no. of times or head = $\frac{1}{2}$ → limiting value

- set element has some properties :-
1) Ordering does not matter in sets
2) equal A = B ≡ ACB or BCA
3) S or Ω (universal sets), sample space
mutually exclusive / disjoint

Partition : disjoint + union gives back original
cardinality of set = no. of elements in set
$A = \{2, 3\}$ cardinality $|A| = 2$

$$|A \cup B| = |A| + |B| - |A \cap B|$$
$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$$

general →
$|A| + |B|$ - - -
$- |A \cap B|$ - - -
$+ |A \cap B \cap C|$ - - -
$- |A \cap B \cap C \cap D|$ + $|A \cap B \cap C \cap D \cap E|$ - -

Countable & Uncountable sets :
A set S is said to be countable
1) if it is finite
2) if there is a one-one and onto correspondence with the set of natural number. N.
$f: N \to S$

eg.1 set of all even positive Integers →
$f: N \to S$
$f(n) = 2n \quad \forall n \in N$

one - one for two diff o/p we should have 2 diff diff i/p

for one - one
for $f(a) = f(b)$
Then $\underline{a = b}$

$f(1) = f(2)$
but $1 \neq 2$
so it is not one - one

counter part
$1 \neq 2$, Then prove $f(1) \neq f(2)$

---

continuing eg $\rightarrow$ let $f(n) = f(m)$
$2n = 2m$
one - one $\boxed{n = m}$

so thing is that we should make a Right function in this it is $\underline{f(n) = 2n}$

each & every entry should be mapped
For Onto :- if m is even then

$$\boxed{f\left(\frac{m}{2}\right) = \cancel{f}(m)}$$

now this is countable

---

eg. 2 Set of all integers :-
$f: N \rightarrow Z$

$$f(n) = \begin{cases} n/2 & \text{if } n \text{ is even} \\ -\frac{(n-1)}{2} & \text{if } n \text{ is odd} \end{cases}$$

let $n \neq m$
$\frac{n}{2} \neq \frac{m}{2}$ & $-\frac{(n-1)}{2} \neq -\frac{(m-1)}{2}$

for one - one
$$\boxed{f(n) \neq f(m)}$$

we used counter - part.

\# If we take $\rightarrow$ $f(n) = f(m)$
1) when m & n both are odd
2) when m & n both are even

2nd Argument

* In exam just prove this by one argument only

3) m is even, n is odd
4) m is odd, n is even

Taking case - m is odd, n is even

$$-\left(\frac{m-1}{2}\right) = \frac{n}{2} \qquad m+n = 1$$

m and n ~~should~~ are natural nos. ~~odd~~ Min possible values are 1 and 2 $\underline{m+n = 3}$ X so this case is not possible.

→ Taking case n odd, m even

$$-\left(\frac{n-1}{2}\right) = \frac{m}{2} = m+n = 1$$

same as above

→ If both are odd

$$f(m) = f(n)$$

$$-\left(\frac{m-1}{2}\right) = -\left(\frac{n-1}{2}\right) \qquad \underline{m = n}$$

→ if both are even $\boxed{m = n}$

so in this we need to consider all parts.

Onto :-

let $m \in \mathbb{Z}$

If $m = 0$ $\qquad f(1) = 0$

if $m > 0$ $\qquad f(2m) = m$ $\qquad$ In this we covered all the +ve integers

if $m < 0$ $\qquad \underbrace{f(1-2m)}_{\text{i/p}} = -\left(\frac{1-2m-1}{2}\right) = m$

This is basically about the o/p

Total value is going to be positive value

So this is onto

eg 3 Set of positive rational numbers is countable?

what kind of function from set of natural no. to set of Rational numbers.

solution N → Numerator $\quad$ $N+D=2$ in $\frac{1}{1}$
D → Denominator

$N+D$ is 3 in $\frac{1}{2}$, $\frac{2}{1}$

$N+D$ is 4 in $1/3$, $3/1$, $2/2$
$N+D$ is 5 in $1/4$, $4/1$, $2/3$, $3/2$
$N+D$ is 6 in $1/5$, $5/1$, $3/3$, $\frac{2}{4}$, $4/2$

- $1/1$, $1/2$, $2/1$, $1/3$, $2/2$, $3/1$, $1/4$, $2/3$, $3/2$, $4/1$

Arranging them

1/1	1/2	1/3	1/4	1/5	1/6	---
2/1	2/2	2/3	2/4	2/5	2/6	---
3/1	3/2	3/3	3/4	3/5	3/6	---
4/1	4/2	4/3	4/4	4/5	4/6	----

In this Manner we are able to cover all Rational numbers

eg4. set of all Rational numbers countable ?
eg5. set of Real numbers countable ?

$\boxed{\text{Real = union of Rational and Irrational}}$

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Sample space :- set of all possible outcomes of a statistical experiment.
each outcome is called a sample point.

- **3 ways to describe a sample space** :-
List (for finite sample points)
Tree
Statement or rule

any set is subset of S

Subsets :- Facts $A \subset S$, $\phi \subset S$
$S \subseteq S$ → proper subset

- **Event** :- Any subset of sample space S is an event
- **Complement event** :- set of all elements of A that are not present in A.

Basic Principle of counting :-
Multiplication Principle.

Manish Garg's Class

5/1/18

Set: collection of well defined objects
eg S = { set of all presidents } not well defined not set
S = { set of all students } " "
S = {1, 2, 3, 4, 5, 6} set
S = { set of 1 feet tall students in LNMIIT} empty set

operations → $A \cup B$, $A \cap B$, $A - B$ { In A but not B }
In set repetition does not matter. Two sets are unequal if element in one is not present in another.
{2, 4, 3} = { 2, 4, 3, 2 }
For dice S = { 1, 2, 3, 4, 5, 6 } Ω (universal set)
For coin S = { head, tail }

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- **Countable set** :- A set which is either finite or empty or countably infinite is called countable set otherwise called uncountable set

$S_n : n \in N$

countable union → $\bigcup_{n=1}^{\infty} S_n = S_1 \cup S_2 \cup S_3 \text{ --- } \infty = [x : x \in S_n \text{ for some } n]$

countable intersection : $\bigcap_{n=1}^{\infty} S_n = S_1 \cap S_2 \cap S_3 \text{ -- } \infty = [x : x \in S_n \text{ for some } n]$

if $\alpha \in (0,1)$

$\bigcup_{\alpha \in (0,1)} S_\alpha = [x : x \in S_\alpha \text{ for some } \alpha]$

uncountable union

$\bigcap_{\alpha \in (0,1)} S_\alpha = [x : x \in S_\alpha \text{ for some } \alpha]$

uncountable Intersection

- **Countable Infinite set** :- An Infinite set S is called countable Infinite if there exist a Mapping $f : N \to S$ s.t. 'f' is one-one & onto.

eg **set of all positive Integers** :-

Define $f : N \to S$ $\quad f(n) = 2n$

one-one : $f(n) = f(m)$

$2n = 2m$

$\boxed{n = m}$

<u>onto</u> :

$x \in S \ \exists\ n \in N$

s.t. $x = 2n_0$

$\boxed{f(n_0) = x}$

S is countable Infinite set

eg For S = **set of all Integers** :-

Define $f : N \to I$

$$f(n) = \begin{cases} n/2 & \text{if } n \text{ is even} \\ -\frac{(n-1)}{2} & \text{if } n \text{ is odd} \end{cases}$$

(I) for m is even and n is odd

4 cases (It is done before)

Use another Method :-

one-one if $m \neq n$

$\frac{m}{2} \neq \frac{n}{2}$ $\quad -\frac{(m-1)}{2} \neq -\frac{(n-1)}{2} \Rightarrow \boxed{f(m) \neq f(n)}$

For onto : $m \in I$; if $m = 0$ $\quad f(1) = 0$

if $m < 0$ $\quad f(1-2m) = m$

if $m > 0$ $\quad f(2m) = m$ $\quad$ It is onto (every codomain element should have pre Image in domain

Combinational Analysis :-

Basic Principle :- suppose 2 experiments are to be formed suppose exp 1, has 'm' possible outcomes and for each possible outcomes of exp 1, exp 2 has 'n' possible outcomes Then there are mn possible outcomes.

$(1,1)$ $(1,2)$ $(1,3)$ -- $(1,n)$
$(2,1)$ $(2,2)$ $(2,3)$ -- $(2,n)$
$\vdots$
$(m,1)$ $(m,2)$ $(m,3)$ $\quad$ (m,n)

Generalisation :- r- experiment

exp 1 $\to n_1$ $\quad$ exp r $\to n_r$
exp 2 $\to n_2$

Total $n_1 \cdot n_2 \ldots n_r$

Ques 1 :- A small community 10 women each of whom have 3 children. If one women and one of her children are chosen as mother and child of the year Then how many diff choices are possible

Sol^n Total $10 \times 3 = 30$

Q2 A college planning committee consists of 3 freshman, 4-sophomores, 5- Juniors, 2- seniors. A subcomittee of 4 consisting 1 person from each class is to be chosen How many diff sub-committes combination are possible.

Total $= 3 \times 5 \times 2 \times 4 = 120$

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Q-3 How many diff 7 place license plates are possible if the first 3 places are to be occupied by letters & final 4 by numbers.

Ans $(26)^3 \times (10)^4$

If repetition not allowed → $26 \times 25 \times 24 \times 10 \times 9 \times 8 \times$

Q-4 How many different ordered arrangement of a, b, c are possible?

abc, acb, bac, bca, cab, cba

These arrangement is called permutation.

For n objects $n!$ way to arrange.

Q-5 Miss John has 10 books that she is going to put on bookshelf of these 4 are of Maths, 3 - chem, 2 - History, 1 - lang, she wants to arrange s.t. all books dealing same sub are on same place. How many arrangements are possible?

Solⁿ →
4 → Maths → $4!$
3 - chem → $3!$
2 - History → $2!$
1 - language → $1!$

Total → $(4! \times 3! \times 2! \times 1!) \times \underbrace{4!}_{\text{ways of arranging 4 bundles}}$

Q-6 A class in probability consists of 6 men and 4 women. An examination is given, and the students are ranked according to their performance. Assume that no two students obtain the same score.

(i) How many different Ranking are possible?

(ii) if men are ranked themselves and women are also Ranked themselves. How many diff. Ranking are possible?

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Sol (i) (10!) [Total Ranking]

(ii) 6! × 4! [Multiplication due to Basic principle]

Result :- Suppose 'n' objects are given. Out of these n_1 are alike, n_2 are alike ... n_r are same. How many different arrangements are possible

Ans = $\left[\dfrac{(n!)}{n_1!\, n_2! \cdots n_r!} \right]$ diff arrangements when some items are identical

Q PEPPER How many diff. arrangements are possible?

3 − P

2 − E

1 − R → $\dfrac{6!}{3!\,2!\,1!} = \dfrac{\overset{2}{\cancel{4}} \times 5 \times 6}{\cancel{2}} = 60$

Q suppose a chess tournaments has 10 competitors of which 4 are Russian, 3 are from US, 2 are from England, and 1 is from Brazil. If the tournament Results lists just the nationalities of the player in the Order in which they plays. How Many outcomes are possible?

→ Result is in terms of their countries

All 4 Russian can be identified as identical because Result will be like $\dfrac{10!}{4!\,3!\,2!\,1!}$

Winner is from Russia...

\# **COMBINATIONS :-**

How Many diff groups of 3 could be selected from 5 objects A, B, C, D, E ?

Total $\underline{5}$ $\underline{4}$ $\underline{3}$ = 60 $^5C_3$

Ideal arrangement

- In that expression each selection is counted 6 times. ABC, ACB, BAC, BCA, CAB, CBA
so when ordering counts $^5C_3$ - 10 ways

$\boxed{^nC_r \text{ are the ways}}$

$0! = 1$ $\qquad$ $^nC_0 = 1 = {^nC_n}$

$^nC_r = 0$ if $r < 0$, $r > n$

Q From a group of 5 women and 7 men How many different committes consisting of 2 women and 3 Men can be formed?
(ii) What if 2 of the Men are feuding and Refused to serve on the committee together?

Sol :-

W	M	Total
5	7	(i) $^5C_2 \times {^7C_3}$
2	3	

Ans = 350

(ii) 2 feuding

How Many to select 3 Men from 7 $\quad$ $\underline{^7C_3}$

$\frac{7 \times \cancel{6} \times \cancel{5}}{\cancel{3} \times \cancel{2}} = 35$

Two person which cannot be selected together

$\underbrace{^2C_2}_{} \times {^5C_1} = 5$ ways $\rightarrow$ To select 1 More from Men

selecting Both of those two person

By this Those Both person will be in same committee ; Total ways $35 - 5 = 30$ ways
30 ways when Both feuding Men will not be together.

Total ways $= {^5C_2} \times 30$
$= 10 \times 30 = 300.$

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Identity :-

$$\boxed{{}^{n-1}C_{r-1} + {}^{n-1}C_r = {}^nC_r} \qquad 1 \leq r \leq n$$

L.H.S $\quad {}^{n-1}C_{r-1} + {}^{n-1}C_r = \dfrac{(n-1)!}{(r-1)!\,(n-r)!} + \dfrac{(n-1)!}{(r)!\,(n-r-1)!}$

$$= \frac{(n-1)!}{(r-1)!\,(n-r-1)!}\left[\frac{1}{n-r} + \frac{1}{r}\right]$$

$$= \frac{(n-1)!}{(r-1)!\,(n-r-1)!}\left[\frac{n}{(n-r)\,r}\right] = \frac{n!}{r!\,(n-r)!}$$

$$= \underline{{}^nC_r}$$

Result :-

$$(x+y)^n = \sum_{k=0}^{n} {}^nC_k\, x^k\, y^{n-k} \qquad -(1)$$

Proof by MI $\rightarrow$ for $n=1$

R.H.S $\rightarrow \displaystyle\sum_{k=0}^{1} {}^1C_k\, x^k y^{1-k} = {}^1C_0\, x^0 y + {}^1C_1\, x^1 y^0$

$$= \underline{x+y}$$

Suppose (1) is true for $(n-1)$ $\rightarrow$ we need to prove it for n.

$$(x+y)^n = (x+y)\,(x+y)^{n-1}$$

$$(x+y)^n = (x+y)\sum_{k=0}^{n-1} {}^{n-1}C_k\, x^k\, y^{n-1-k}$$

$$= \sum_{k=0}^{n-1} {}^{n-1}C_k\, x^{k+1}\, y^{n-1-k} + \sum_{k=0}^{n-1} {}^{n-1}C_k\, x^k y^{n-k}$$

Putting $i = k+1$ in the first expression and $k = i$ in the second expression.

$$\sum_{i=1}^{n} {}^{n-1}C_{i-1}\, x^i\, y^{n-i} + \sum_{i=0}^{n-1} x^i\, y^{n-i}$$

$$\sum_{i=1}^{n-1} {}^{n-1}C_{i-1}\, x^i y^{n-i} + {}^{n-1}C_{n-1}\, x^n y^0 + {}^{n-1}C_0\, x^0 y^n$$

$$+ \sum_{i=1}^{n-1} x^i y^{n-i}$$

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$\rightarrow \sum_{i=1}^{n-1} \left[{}^{n-1}C_{i-1} + {}^{n-1}C_i \right] x^i y^{n-i} + x^n + y^n$

$\rightarrow x^n + \sum_{i=1}^{n-1} {}^nC_i x^i y^{n-i} + y^n$

$= \boxed{\sum_{i=0}^{n} {}^nC_i \, x^i y^{n-i}} = (x+y)^n$

Q How many subsets are there of a set consisting n elements?

Ans 2^n

Total subsets ${}^nC_0 + {}^nC_1 + {}^nC_2 ---- {}^nC_n$
$= (1+1)^n = 2^n$

* There are $(n-1)!$ arrangements of the n objects around the table.

Q A set of n objects is to be divided into r distinct groups of size $n_1, n_2, --- n_r$ respectively such that $\sum_{i=1}^{r} n_i = n$?
groups
How many diff. divisions are possible?

It is diff from previous one.

$$\boxed{\text{Sol} = \frac{(n)!}{(n_1!)(n_2!) --- (n_r)!}}$$

Order is Important

Q 10 children are to be divided into a team A and Team B of 5 each. The team A will play in one league and team B in another. How many diff. divisions are possible?

10 players are divided into 2 gps of 5 each.

Ans Here teams are seperated ← $\frac{10!}{5!\,5!} =$ $= \frac{10!}{2!\,5!\,5!}$

we are not seperating the gps. clearly

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Q. 10 - policemen

petrolling	station gp	Reserve
5 Men	3 Men	2 Men

All these 3 groups are separated properly.

Sol Then $\frac{10!}{5!\,3!\,2!}$

when Order is not Relevant Then $\frac{10!}{5!\,3!\,2!\,3!}$

<u>Probability</u>

<u>Sample Space</u> → The set of all possible outcomes in an experiment is called sample space.

S or Ω

eg 1) Tossing a coin $S = (\{H\}, \{T\})$

2) Tossing of two coins $S = \{\{HH\}\ \{HT\}\ \{TH\}\ \{TT\})$

3) Rolling of two dices $= S = \{(i, y),\ i, y = 1, 2, 3, 4, 5, 6)$

4) If the experimental measures (in hours) the life of a transistor.

$S = \{\ n : 0 \leq n < \infty \}$

<u>Event</u> :- A subset of a sample space is called event. 'E'.

ex 1) $E = \{H\}$

2) $E = (\{HT\}\ \{TH\})$

3 $E = \{(4,6)\ (5,5)\ (6,4)\}$

→ If the transistor does not last more than 4 hours.

Than event is → $E = \{ 'n' ;\ 0 \leq n \leq 4 \}$

→ Let we have 2 events E and F
Then an another event $\underline{E \cup F}$ → contain all outcomes which exist in either E or F.

$E = \{ (H,H)\ (T,H) \}$
$F = \{ (H,T) \}$
$\underline{E \cup F} = \{ (H,H)\ (T,H)\ (H,T) \}$
at least one head occurs.

\# $\underline{E \cap F}$ → contains all outcomes present in both E and F.

$E = \{ (H,H)\ (T,H)\ (H,T) \}$ → atleast one head
$F = \{ (T,H),\ (T,T)\ (H,T) \}$ → atleast one Tail

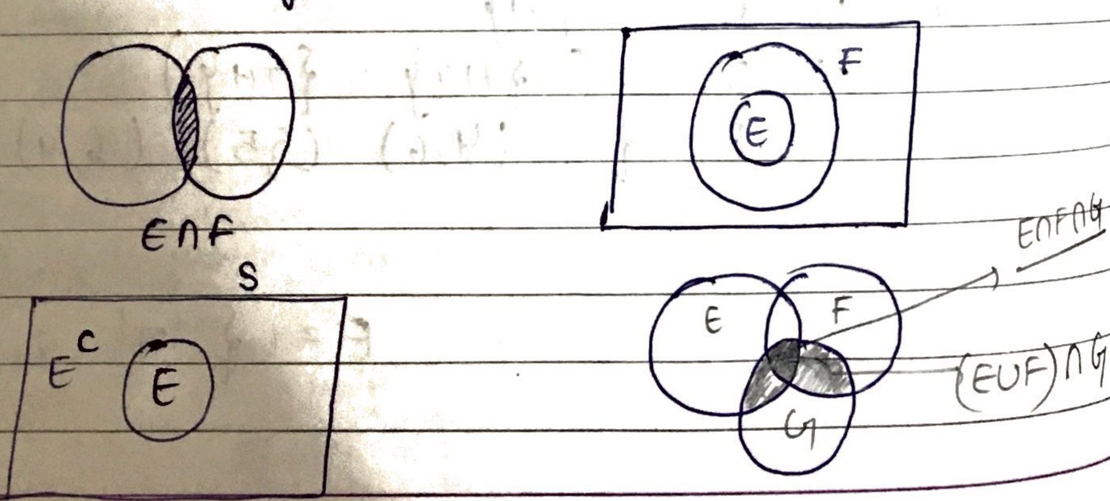
$E \cap F = \{ (H,T)\ (T,H) \}$

→ Intersection can also be written as EF or when Three events are present $E \cap F \cap L$ → EFL

\# **Mutually Exclusive Events** :- Two events are called mutually exclusive if $E \cap F = \phi$
eg $\underline{A \text{ and } A^C}$

\# $E \subset F$ → all the outcomes of E and present in F.

Venn Diagram :-



Some Results :- **De Morgan's Law**

$$\left(\bigcup_{i=1}^{n} E_i\right)^c = \bigcap_{i=1}^{n} E_i^c \quad \text{—— (1)}$$

$$\left(\bigcap_{i=1}^{n} E_i\right)^c = \bigcup_{i=1}^{n} E^c \quad \text{—— (2)}$$

Proof of 1st $\rightarrow$ $x \in \left(\bigcup_{i=1}^{n} E_i\right)^c$

$\Rightarrow$ $x \notin \bigcup_{i=1}^{n} E_i$

$\Rightarrow$ $x \notin E_i$ for any $i = 1, 2, 3 \dots n$.

$\Rightarrow$ $x \in E_i^c$ for all $i = 1, 2, 3 \dots n$

$\Rightarrow$ $\boxed{x \in \bigcap_{i=1}^{n} E_i^c}$

$$\therefore \left(\bigcup_{i=1}^{n} E_i\right)^c \subseteq \bigcap_{i=1}^{n} E_i^c \quad \text{—— (3)}$$

$\rightarrow$ Let $x \in \bigcap_{i=1}^{n} E_i^c$

$x \in E_i^c$, for all $i = 1, 2, \dots n$

$\Rightarrow$ $x \notin E_i$ for any $i = 1, 2 \dots n$

$x \notin \bigcup_{i=1}^{n} E_i$

$x \in \left(\bigcup_{i=1}^{n} E_i\right)^c$

$$\bigcap_{i=1}^{n} E_i^c \subseteq \left(\bigcup_{i=1}^{n} E_i\right)^c \quad \text{— (4)}$$

$$\boxed{\left(\bigcup_{i=1}^{n} E_i\right)^c = \bigcap_{i=1}^{n} E_i^c}$$

OR we can do it $\rightarrow$

$$\left[\left(\bigcup_{i=1}^{n} E_i\right)^c\right]^c = \bigcap_{i=1}^{n} (E_i^c)^c$$

$$\left(\bigcup_{i=1}^{n} E_i^c\right)^c = \bigcap_{i=1}^{n} E_i$$

$\longrightarrow$ Taking complement Again

$$\left(\bigcap_{i=1}^{n} E_i\right)^c = \bigcup_{i=1}^{n} E_i^c$$

\# Probability :- Let S be a sample space & E be the event. We define a number P(E) [called the probability of event E] if it satisfy the following axioms.

1) $0 \leq P(E) \leq 1$
2) $P(S) = 1$
3) For any sequence of mutually exclusive events $E_i, E_2 \dots E_n$ -- $[E_i \cap E_j = \phi, i \neq j]$

$$P\left(\bigcup_{i=1}^{\infty} E_i\right) = \sum_{i=1}^{\infty} P(E_i)$$

ex $P(E^c) = 1 - P(E)$

$E \cap E^c = \phi$ (Mutually exclusive)

$E \cup E^c = S$

$P(E \cup E^c) = P(S)$

$= 1$

By Axiom 3 we have

$P(E) + P(E^c) = 1$

$$\boxed{P(E^c) = 1 - P(E)}$$

result $\boxed{P(\phi) = 0}$

we can write $\cdot\ E \rightarrow E \cup \phi$

$P(E) = P(E \cup \phi)$

$P(E) = P(E) + P(\phi)$

$$\boxed{P(\phi) = 0}$$

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→ if $E \subset F$ then $P(E) \leq P(F)$

since $E \subset F$

$$F = E \cup (E^c \cap F)$$

$$P(F) = P(E \cup (E^c \cap F))$$

$$P(F) = P(E) + P(E^c \cap F)$$

$$P(F) \geq P(E) \quad \text{— from Axiom (1)}$$

→ $P(E \cup F) = P(E) + P(F) - P(E \cap F)$

$$E \cup F = E \cup (E^c \cap F)$$

$$P(E \cup F) = P(E) + P(E^c \cap F) \quad \text{— (1)}$$

$$F = (E \cap F) \cup (E^c \cap F)$$

$$P(F) = P(E \cap F) + P(E^c \cap F) \quad \text{— (2)}$$

(2) in (1)

$$P(E \cup F) = P(E) + P(F) - P(E \cap F)$$

→ suppose $E_1, E_2 \ldots E_n$ are events

$$P(E_1 \cup E_2 \cdots \cup E_n) = \sum_{i=1}^{n} P(E_i) - \sum_{i_1 < i_2} P(E_{i_1} \cap E_{i_2}) \cdots + (-1)^{r-1} \sum_{i_1 < i_2 < i_r} P(E_{i_1} \cap E_{i_2} \cdots \cap E_{i_r}) + (-1)^{n-1} P(E_1 \cap E_2 \cdots \cap E_n)$$

$\sum_{i_1 < i_2 < i_r} P(E_{i_1} \cap E_{i_2} \cdots \cap E_{i_r})$ is taken over all the possible combination ${}^nC_r$ of subset of size r from set n.

eg $P(E \cup F \cup G) = P(E) + P(F) + P(G) - P(E \cap F) - P(E \cap G) - P(G \cap E) + P(E \cap F \cap G)$

one at a time; then two; then three and so on

Normally we assume that o/p's are equally likely.

→ **Sample space having equally likely events :-**

Consider a finite sample space $S = \{1, 2, \ldots n\}$ having equally likely outcome.

$$P(\{1\}) = P(\{2\}) = \cdots\cdots P(\{n\})$$

$$P(\{1\} \cup \{2\} \cup \{3\} \cdots \cup \{n\}) = P(\{1\}) + P(\{2\}) + P(\{3\}) \cdots P(\{n\})$$

→ Because all these are Mutually exclusive events.

$$P(S) = 1 = n\,P(\{1\})$$

$$\boxed{P\{1\} = \frac{1}{n}}$$ → Uniform Discrete Probability Law.

\# E is an event in S.

$$P(E) = \frac{\text{numbers of points in } E}{\text{numbers of points in } S}$$

only for equally likely ← $= \dfrac{\text{number of favourable outcomes to } E}{\text{Total no. of outcomes in the exp.}}$

$$P(\{1, 2, 3\}) = \frac{3}{n}$$

Q-1 If three balls are equally likely Randomly drawn from a bag containing 6 white and 5 Black balls what is the probability That one of the Ball is white and the other 2 are Black.

Solution:

$$\frac{{}^6C_1 \times {}^5C_2}{{}^{11}C_3} = \frac{60}{165}$$

E : Three balls are drawn one - white Two - Black

$$n(S) = {}^{11}C_3 = \frac{\cancel{9}^{3} \times \cancel{10}^{5} \times 11}{\cancel{3} \times \cancel{2}} = 165$$

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Q2 Two dice are rolled. Find the probability of getting an even no. on the 1st dice or a total of 8.

→ Two events

$$\frac{18}{36} + \frac{5}{36} - \frac{3}{36} = \frac{20}{36}$$

we want $P(E_1 \cup E_2)$ in this $P(E_1) + P(E_2) - P(E_1 \cap E_2)$

Q Matching - Problem

1) Let us have n letters corresponding to which there exists n envelopes having different addresses. considering various letters being put in various envelopes at Random. A Match is set to occur if a letter goes into the Right envelope.

2) If in part (b) There are n persons with n different hats. A match is said to occur if in the process of selecting hats at Random. The i^{th} person Rightly gets the i^{th} hat.

3) Suppose n male and female couples are at the party and that the Males and females are Randomly paired for a dance. A Match occur if a couple happens to be paired together.

Q1

→ 1) What is the probability that none of the letter occupies Right envelope.

2) What is the probability that exactly 'r' letters are kept in correct envelope.

Define E_i = a Match occur at ith place
= i^{th} letter goes to i^{th} envelope

only one letter goes to Right place

$$P(E_i) = \frac{(n-1)!}{(n)!} = \frac{1}{n}$$

$i = 1, 2, 3 \ldots$ 1/n ways

none of the letter goes to Right place

$$P(\overline{E_1} \cap \overline{E_2} \cdots \cap \overline{E_n}) = 1 - P(E_1 \cup E_2 \cup E_3 \cdots \cup E_n)$$

$$\Rightarrow 1 - \Big(\sum_{i=1}^{n} P(E_i) - \sum_{i_1 < i_2} P(E_{i_1} \cap E_{i_2})$$

$$+ (-1)^{r-1} \sum_{i_1 < i_2 \cdots i_r} P(E_{i_1} \cap E_{i_2} \cap \cdots \cap E_{i_r})$$

$$+ (-1)^{n-1} P(E_1 \cap E_2 \cdots \cap E_n) \Big] \quad -(1)$$

2 letters are going to Right envelope

$$P(E_1 \cap E_2) = \frac{(n-2)!}{n!} = \frac{1}{n(n-1)}$$

$$P(E_1 \cap E_2 \cap E_3) = \frac{(n-3)!}{n!} = \frac{1}{n(n-1)(n-2)}$$

$$P(E_1 \cap E_2 \cdots E_n) = \frac{1}{n(n-1)(n-2)\cdots 1} = \frac{1}{n!}$$

Putting all values in eq^n (1)

$$\rightarrow P(\overline{E_1} \cap \overline{E_2} \cdots \cap \overline{E_n}) = 1 - \Big(n \cdot \frac{1}{n} - {}^nC_2 \frac{1}{n(n-1)}$$

$$+ {}^nC_3 \frac{1}{n(n-1)(n-2)} \cdots \frac{(-1)^{n-1}}{n!} \Big)$$

$$\Rightarrow 1 - \left[1 - \frac{1}{2!} + \frac{1}{3!} \cdots \frac{(-1)^{n-1}}{n!} \right]$$

$$\boxed{\Rightarrow \sum_{k=0}^{n} \frac{(-1)^k}{k!}}$$ Solution — $k=0$ & $k=1$ term will get cancelled

If n is large $1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} \cdots = e^{-1}$

$= 0.367$

Probability that atleast one letter is kept to Right envelope

$$\Rightarrow 1 - P(\overline{E_1} \cap \overline{E_2} \cap \overline{E_3} \cdots \overline{E_n})$$

$$\Rightarrow 1 - 0.367 = 0.633$$

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Matching Problem :-

E_i = Match occurrence at i^{th} place

$$P(E_i) = \frac{1}{n}$$

$$P(E_1 \cap E_2 \cap \cdots E_n) = \frac{1}{n!}$$ → all letters go into Right place

$$P(\text{no Match}) = \sum_{k=0}^{n} \frac{(-1)^k}{k!}$$

$$P(\text{at least one Match}) = 1 - \sum_{k=0}^{n} \frac{(-1)^k}{k!}$$

Probability (exactly r Matches occurs) :-

P(r letters going to their envelope) =

$$\to \frac{(n-r)!}{n!} = \frac{1}{n(n-1)\cdots(n-r+1)}$$

$$P(\text{remaining } n-r \text{ does not goes to their own envelopes}) = \sum_{k=0}^{n-r} \frac{(-1)^k}{k!}$$

P(only r letters going to their envelopes) =

we are selecting only one set of r from n $\left\{ \frac{1}{n(n-1)\cdots(n-r+1)} \sum_{k=0}^{n-r} \frac{(-1)^k}{k!} \right.$

$$P(\text{required Probability}) = \frac{{}^nC_r}{n(n-1)\cdots(n-r+1)} \sum_{k=0}^{n-r} \frac{(-1)^k}{k!}$$

But their are total ${}^nC_r$ possibilities

$$\Rightarrow \frac{1}{r!} \sum_{k=0}^{n-r} \frac{(-1)^k}{k!}$$

Birthday Problems :-

If n persons are present in room what is the probability that no two persons have their Birthday at the same day of a year ?

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each person can have 365 possibilities to celebrate his/her Bday.

Sample space $\boxed{n(S) = (365)^n}$

$n(E) = (365)^n C_n n!$ → no. of events

→ P(no two person have same B-day) = $\frac{365 C_n \, (n)!}{(365)^n}$

→ $\frac{(365)(364)\cdots(365-n-1)}{(365)^n}$

→ P(at least two person have their b-day at same day) = 1 − P[no two person have b-day at same day]

→ If $n \geq 23$ value of P[no two person have b-day at same day] is less than half then P[at least two person ---] is greater than half.

→ If $\underline{n = 23}$ At least two person from all of them have same Bday $\underline{P > 1/2}$.

→ If $n = 1$ P(Atleast two) = zero
$n = 23$ P(") = 50.2%
$n = 50$ = 97.1%
$n = 70$ = 99.9%
$n = 100$ = 99.99997

→ P = 100%. $\underline{n = 366 \text{ person}}$

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Q? If 10 Married couples are seating at Random around a Round table. Find the prob. That no wife sit next to her husband.

- E_i = i^{th} couple sit next to each other

$n(S) = (19)!$

$$P(\text{no couple}) = 1 - P\left[\bigcup_{i=1}^{n} E_i\right]$$

$$= 1 - \left[\sum_{i=1}^{n} P(E_i) - \sum_{i_1<i_2} P[E_{i_1} \cap E_{i_2}] \cdots + (-1)^{n-1} P[E_1 \cap E_2 \cdots \cap E_n]\right]$$

Probability n couples sit next to each other.

→ $P[E_1 \cap E_2 \cdots \cap E_n]$

Total 20 candidates ; considering each of the couple as a single entity

Only n couples are sitting next to each other

$$\underline{20 - 2n} + n$$

$$= \underline{20 - n}$$

$(19-n)!$ ways That the n couples sit next to each other.

$2^n (19-n)!$ → each person have two choices

example → 3- couples

6 people A, B, C, D, E, F

$n = 2$ couples sit together

$\boxed{CD}$ $\boxed{EF}$

$$6 - 4 + 2 = 4$$

$$20 - 2n + n$$

$$\underset{\text{n couples sitting next to each other}}{P[E_1 \cap E_2 \cdots \cap E_n]} = \frac{2^n (19-n)!}{(19)!}$$

$$P(\text{no couples sit next to each other}) = 1 - \left({}^{10}C_1 \frac{2\cdot(18)!}{(19)!} + {}^{10}C_2 \frac{2^2(17)!}{19!} \right)$$

$$= 0.3395$$

Probability as a continous set function:

- A sequence of events $\{E_n\}_{n \geq 1}$ is called an increasing sequence if
  $E_1 \subset E_2 \subset E_3 \subset \ldots E_n \ldots$

- We define a new event denoted as $\lim_{n \to \infty} E_n$ defined as

$$\boxed{\lim_{n \to \infty} (E_n) = \left(\bigcup_{n=1}^{\infty} E_n \right)}$$

A sequence of events $\{E_n\}_{n \geq 1}$ is called decreasing if $E_1 \supset E_2 \supset E_3 \ldots E_n \supset \ldots$

→ We define a new event denoted as $\lim_{n \to \infty} E_n$ & defined as :-

$$\boxed{\lim_{n \to \infty} (E_n) = \bigcap_{n=1}^{\infty} E_n}$$

<u>Theorem</u> : If $\{E_n\}_{n \geq 1}$ is either Increasing or decreasing sequence of events then

$$\boxed{\lim_{n \to \infty} P(E_n) = P\left(\lim_{n \to \infty} E_n \right)}$$

Proof : Suppose $\{E_n\}_{n \geq 1}$ is an increasing function / sequence. Define an event

$$F_n = E_n \cap \left(\bigcup_{i=1}^{n-1} E_i \right)^c$$

F_n contains those outcomes of E_n which are not in earlier in any of the events.

E_n contains outcomes which are in the previous events.

Clearly event F_n contains of those outcomes of E_n that are not in any of the earlier Event E_i. where $i < n$.

$$F_n = E_n \cap E_{n-1}^c \quad \text{—} \; (1)$$

$$\underline{F_n \cap F_{n+1}} = (E_n \cap E_{n-1}^c) \cap (E_{n+1} \cap E_n^c)$$
$$= \phi$$

F_n are mutually exclusive events

→ • $$\bigcup_{i=1}^{\infty} F_n = \bigcup_{i=1}^{\infty} E_n \quad \text{—} \; (2)$$

→ • $$\bigcup_{i=1}^{n} F_n = \bigcup_{i=1}^{n} E_n \quad \text{—} \; (3)$$

Taking RHS $$P\left(\lim_{n \to \infty} E_n\right) = P\left(\bigcup_{n=1}^{\infty} E_n\right)$$

$$= P\left(\bigcup_{n=1}^{\infty} F_n\right) \rightarrow F_n \text{ are all mutually exclusive}$$

By Axiom 3 → $$\sum_{n=1}^{\infty} P(F_n)$$

$$\rightarrow \underline{\lim_{i \to \infty}} \sum_{n=1}^{i} P(F_n)$$

$$\rightarrow \lim_{i \to \infty} P\left(\bigcup_{n=1}^{i} F_n\right)$$

$$\rightarrow \lim_{i \to \infty} P\left(\bigcup_{n=1}^{i} E_n\right) = \underline{\lim_{i \to \infty} P(E_i)}$$

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Case 2^{nd} :-

If $\{E_n\}_{n \geq 1}$ is decreasing sequence,

$(E_n^c)_{n \geq 1}$ is an increasing sequence

$$P(\lim_{n\to\infty} E_n^c) = P(\bigcup_{i=1}^{\infty} E_i^c) = \lim_{n\to\infty} P(E_n^c)$$

$$= P(\bigcap_{i=1}^{\infty} E_i)^c = \lim_{n\to\infty} (1 - P(E_n))$$

$$= \cancel{1} - P(\bigcap_{i=1}^{\infty} E_i) = \cancel{1} - \lim_{n\to\infty} P(E_n)$$

$$\Rightarrow \quad P(\bigcap_{i=1}^{\infty} E_i) = \lim_{n\to\infty} P(E_n)$$

$$\boxed{P(\lim_{n\to\infty} E_n) = \lim_{n\to\infty} P(E_n)}$$

Conditional Probability :-

ex → Throwing two dice

(A) event → sum is even

$\Omega = (2, 3 - \!\!-\; 12)$

$A = (2, 4, 6, 8, 10, 12)$

$P(A) = \frac{6}{11}$

if is said that ~~no~~ sum is greater than 5.

event : (B) sum is greater than 5

$\Omega = [6, 7, - \; 12]$

$A = \{6, 8, 10, 12\}$

when B has occurred $P(A) = \frac{4}{7}$

event A depends upon event B.

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conditional probability of A when B has occurred

$P(A/B)$ = Probability of A when B has occurred.

$$\boxed{P(A/B) = \frac{P(A \cap B)}{P(B)}}$$

Multiplication Rule :-

$$P(A \cap B) = P(A).\, P(B/A) = P(B)\, P(A/B)$$

suppose $E_1, E_2, \ldots E_n$ events.

$$P(E_1 \cap E_2 \ldots \cap E_n) = P(E_1).\, \frac{P(E_2)}{P(E_1)}.\, \frac{P(E_3)}{P(E_1 \cap E_2)} \ldots \frac{P(E_n)}{P(E_1 \cap E_2 \ldots \cap E_{n-1})}$$

$$\Rightarrow P(E_1).\, P(E_2/E_1).\, P(E_3/E_1 \cap E_2) \ldots P(E_n/(E_1 \cap E_2 \ldots E_{n-1}))$$

Ques Two cards are drawn one after the another from a deck of 52 cards. Find the probability that Both are spade cards if the 1st card is (i) Replaced or (ii) Not Replaced.

$\frac{8}{15}$

Sol : event A = first card is spade & [event B = 2nd card is spade]

$$P(A \cap B) = P(A).\, P(B/A)$$

$$= \frac{13}{52} . \frac{13}{52} = \frac{1}{16}$$

(ii) event A = event B

$$= \frac{\cancel{13}}{\cancel{52}_4} \times \frac{\cancel{12}\,3}{52} = \frac{3}{51}$$

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Q. A die is thrown twice & the sum of the no. appearing is noted to be 8. What is the conditional Prob. that no. 5 has appeared atleast once.

Sol:

A = the no. 5 appeared atleast once

B = the sum is eight.

$\Omega = 36$

1,5
2,6
3,? 4,5
5,1

$$P(A/B) = \cancel{P(A) \cdot P(B)}$$

$$= \frac{P(A \cap B)}{P(B)}$$

2,6
3,5
5,3
4,4
6,2

$$= \cancel{\frac{1/6}{5/}} \quad \frac{2}{5}$$

$A = \{(1,5)\ (2,5)\ (3,5)\ (4,5)\ (6,5)$
$\quad (5,1)\ (5,2)\ (5,3)\ (5,4)\ (5,5)\}$

$B = \{(2,6)\ (3,5),\ (4,4)\ (5,3),\ (6,2)\}$

Total Probability Theorem :-

if $E_1, E_2 \ldots E_n$ are events defined on S than $E_1, E_2, E_3 \ldots E_n$ are called exhaustive if

$$E_1 \cup E_2 \cup E_3 \ldots \cup E_n = S$$

Further; $\boxed{E_i \cap E_j = \phi}$; $i \neq j$

Let $B_1, B_2 \ldots B_n$ are Mutually exclusive and exhaustive events on S with $P(B_n) \neq 0$ $\forall n = 1, 2 \ldots k$ let A be any random event on S

$$\boxed{P(A) = \sum_{n=1}^{k} P(B_n)\, P(A/B_n)}$$

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$A \cap B_1$, $A \cap B_2$ ---- $A \cap B_n$ are M.E.

→ $A = (A \cap B_1) \cup (A \cap B_2) \cup$ ---- $(A \cap B_k)$

$P(A) = \sum_{i=1}^{\infty/k} P(A \cap B_i)$

$= \sum_{i=1}^{\infty/k} P(B_i)\ P(A|B_i)$

Ques Box A contains 6 Red and 4 blue balls
Box B contains 3 Red and 7 Blue.
Two Balls are drawn at Random from Box A and placed in the Box B. Now a Ball is drawn at Random from Box B. Find the prob. that it is a Blue ball.

3 Possibilities

Sol :- B_1 i) Both are Red B_2 (ii) One Red - One Blue
B_3 (iii) Both are Blue

$P(B_1) = \frac{6C_2}{10C_2} = \frac{1}{3}$ $P(B_2) = \frac{6C_1 \times 4C_1}{10C_2} = \frac{8}{15}$

$P(B_3) = \frac{4C_2}{10C_2} = \frac{2}{15}$

Define an event A → drawing a Blue Ball from Box B after the transfer.

- Both Red 5R, 7B → $\frac{7C_1}{12C_1} = \frac{7}{12}$ $P(A/B_1)$
- One Red - One Blue 4R, 8B → $\frac{8}{12}$ $P(A/B_2)$
- Both Blue 3R, 9B = $\frac{9}{12}$ $P(A/B_3)$

$P(A) = \sum_{i=1}^{3} P(B_i)\ P(A/B_i)$

$$\rightarrow \frac{1}{3} \times \frac{7}{12} + \frac{8}{15} \times \frac{8}{12} + \frac{2}{15} \times \frac{9}{12}$$

$$\Rightarrow 0.65$$

Baye's Theorem :-

Let $B_1, B_2 \ldots B_k$ are Mutually exclusive and exhaustive events on S with $P(B_i) \neq 0$ $i = 1, 2 \ldots k$

Let A be any event on S with $P(A) \neq 0$

$$P\left(\frac{B_i}{A}\right) = \frac{P(B_i) \cdot P(A|B_i)}{\sum_{i=1}^{k} P(B_i) \cdot P(A|B_i)}$$

$$\frac{P(B_i \cap A)}{P(A)} =$$

Q A Bag contains 8 White Balls and 4 Black Balls and Bag B → 5W, 6 Black One ball is drawn at Random at from Bag A and placed in B. Now a Ball is drawn at Random from Bag B. It is found that this Ball is white. Find the Prob. That a Black Ball has been transferred from Bag A.

Sol : B_1 = transferring a white Ball to B
B_2 = " " Black Ball to B.] ?

$$P(B_1) = \frac{8}{12} = \frac{2}{3}$$

$$P(B_2) = \frac{4}{12} = \frac{1}{3}$$

A = drawing a white Ball from Box after transfer

$P(A|B_1) = \frac{6}{12} = \frac{1}{2}$

$P(A|B_2) = 5/12$

$$P(B_2|A) = \frac{P(B_2)\,P(A|B_2)}{\sum_{i=1}^{2} P(B_i)\,P(A/B_i)}$$

$$= \underline{5/17}$$

22/1/18 3 cards are drawn at Random from a deck without Replacement. Find the probability that none of the 3 cards ~~are~~ is a heart.

Sol $A_i = \{ i^{th}$ card is not a heart $\}$

$$P(A_1 \cap A_2 \cap A_3) = P(A_1)\,P\left(\frac{A_2}{A_1}\right)\,P\left(\frac{A_3}{A_1 \cap A_2}\right)$$

$$= \left(\frac{39C_1}{52C_1}\right) \times \left(\frac{38C_1}{51C_1}\right) \times \left(\frac{37C_1}{50C_1}\right)$$

Probability Based on Multiplication Theorem $= \left(\frac{39}{52}\right) \times \left(\frac{38}{51}\right) \times \left(\frac{37}{50}\right)$

\# If Probability of A is not dependent on Probability of B Then $\boxed{P(A|B) = P(A)}$; A Independent of B

$P(A \cap B) = P(B)\,P(A/B) = P(B)\,P(A)$

This Relation is also valid for $\underline{P(B) = 0}$

$\Rightarrow$ Two events A & B are Independent if $\boxed{P(A \cap B) = P(A)\,P(B)}$

A, B events are disjoint (non empty)

$A \cap B = \phi$

$P(A \cap B) = 0 \neq P(A)\,P(B)$

$P(A) \neq 0 \qquad P(B) \neq 0$

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→ if A and B are Independent
* A^c and B are also Independent
* A and B^c are also Independent
* A^c and B^c are also Independent.

<u>Prove</u> A^c and B are Independent.

$$B = (A \cap B) \cup (A^c \cap B)$$
$$P(B) = P(A \cap B) + P(A^c \cap B)$$
$$\Rightarrow P(A^c \cap B) = P(B) - P(A)\,P(B)$$
$$\Rightarrow P(A^c \cap B) = P(B)\,(1 - P(A))$$
$$\Rightarrow \boxed{P(A^c \cap B) = P(A^c)\,P(B)}$$

A^c and B are Independent

$P(A^c \cap B^c) = P(A \cup B)^c$
$$= 1 - P(A \cup B)$$
$$= 1 - P(A) - P(B) + P(A \cap B)$$
$$= 1 - P(A) - P(B) + P(A)\,P(B)$$
$$= (1 - P(A))\,(1 - P(B))$$
$$\boxed{P(A^c \cap B^c) = P(A^c)\,P(B^c)}$$

<u>example</u> $\Omega = \{HH, HT, TH, TT\}$

$P(\omega) = \frac{1}{4}$ → prob. of each event

$A = \{HH, HT\}$ $P(A) = 1/2$
$B = \{HH, TH\}$ $P(B) = 1/2$
$C = \{HT, TT\}$ $P(C) = 1/2$

$A \cap B = \{HH\}$ $\Rightarrow P(A \cap B) = 1/4$
$B \cap C = \{\emptyset\}$ $\Rightarrow P(B \cap C) = 0$

$P(A)\,P(B) = \frac{1}{4} = P(A \cap B)$

Independent of each other

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$$P(B)\,P(C) = \frac{1}{4} \neq P(B \cap C)$$

Dependent on each other

Def: We say that events $A_1, A_2 \ldots A_n$ are independent if for any $A_{i_1}, A_{i_2} \ldots A_{i_m}$, $m \geq 2$ from $\{A_1, A_2 \ldots A_n\}$ we have

$$P(A_{i_1} \cap A_{i_2} \ldots \cap A_{i_m}) = P(A_{i_1})\,P(A_{i_2}) \ldots P(A_{i_m})$$

for 3 events A, B, C

$$\left.\begin{aligned} P(A \cap B) &= P(A)\,P(B) \\ P(B \cap C) &= P(B)\,P(C) \\ P(C \cap A) &= P(C)\,P(A) \end{aligned}\right] \text{Pair wise} - (1) \text{ disjoint events}$$

$$\boxed{P(A \cap B \cap C) = P(A)\,P(B)\,P(C)} \quad - (2)$$

example: $\Omega = \{1, 2, 3, 4\}$

$P(i) = \frac{1}{4}$ $\quad i = 1, 2, 3, 4$

$A = \{1, 2\}$ $\quad B = \{1, 3\}$ $\quad C = \{1, 4\}$

$A \cap B = \{1\} \rightarrow P(A \cap B) = 1/4$

$B \cap C = \{1\} \rightarrow P(B \cap C) = 1/4$

$C \cap A = \{1\} \rightarrow P(C \cap A) = 1/4$

$P(A) = 1/2 = P(B) = P(C)$

$A \cap B \cap C = \{1\}$

$P(A \cap B \cap C) = 1/4$

$P(A)\,P(B)\,P(C) = \frac{1}{8}$

→ It Means These events are Pair wise Independent But the eq^n (2) is not satisfied.

Q → consider 2 Independent Roll of a die & consider the following events.

A = { 1^{st} roll is 1, 2 or 3 }
B = { 1^{st} Roll is 3, 4, 5 }
C = { sum of two roll is 9 }

A = { (1, i), (2, i), (3, i) } i = 1, 2, 3 -- 6 }
B = { (3, i) (4, i) (5, i) } i = 1, 2, 3 -- 6 }
C = { (5,4) (4,5) (6,3), (3,6) }

$A \cap B$ = { (3, i) } i = 1, 2, 3 -- 6
$B \cap C$ = { (5,4) (4,5) (3,6) }
$C \cap A$ = { (3, 6) }

$A \cap B \cap C$ = { (3, 6) }

$P(A) = \frac{18}{36} = 1/2$ $P(C) = \frac{4}{36} = \frac{1}{9}$

$P(B) = 18/36 = 1/2$

$P(A \cap B \cap C) = \frac{1}{36}$

$P(A) P(B) P(C) = \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{9} = \frac{1}{36}$

In this eqn ① { Pair conditions are not satisfied }

Q A problem in a question paper is given to 3 students in a class to be solved. The Prob. of there solving the question is 0.5, 0.7, 0.8 Resp. Find the prob that the Problem will be solved.

AB, AC, BC

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$0.5 \times 0.3 \times 0.2 + 0.5 \times 0.7 \times 0.2 + 0.5 \times 0.3 \times 0.8$

sol $0.5 \times 0.3 \times 0.2 + 0.5 \times 0.7 \times 0.2 + 0.5 \times 0.3 \times 0.8 + 0.5 \times 0.7 \times 0.8$
$+ 0.5 \times 0.7 \times 0.2 +$
$+ 0.5 \times 0.7 \times 0.8$

→ $P(\text{Problem will be solved}) = 1 - P(\bar{A} \cap \bar{B} \cap \bar{C})$
$= 1 - P(\bar{A})\, P(\bar{B})\, P(\bar{C})$
$= 1 - (1-P(A))\,(1-P(B))\,(1-P(C))$
$= 1 - (0.5 \times 0.3 \times 0.2)$

Let I be an index set we say $\{A_i : i \in I\}$ is independent if any finite sub collection is independent.

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Random variable :- Random variable is a mapping from the sample space Ω to $\mathbb{R}$.
- function from sample space to Real.

ex-1 Tossing of 3 coin;
X = number of heads occurred
$X = 0, 1, 2, 3$
$X = 0 = \{(T,T,T)\}$
$X = 1 = \{(HTT), (THT), (TTH)\}$ → Prob of this event
$X = 2 = \{(HHT), (HTH), (THH)\}$
$X = 3 = \{HHH\}$

- The value of the Random variable is determined by the possible outcomes.
- $P(X=0) = \frac{1}{8}$
  $P(X=1) = \frac{3}{8}$
  $P(X=2) = 3/8$
  $P(X=3) = 1/8$

These are mutually exclusive

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$$P\left(\bigcup_{i=0}^{3} (x=i)\right) = \sum_{i=0}^{3} P\{x=i\}$$

Q. 3 Balls are to be Randomly selected without Replacement from a bag containing 20 Balls no. 1 - 20. If we bet atleast one of the Ball that are drawn has a number as large as or greater than 17. What is the Prob. that we win the bet?

Sol :- largest no. in the Ball is 17, 18, 19, 20

X = the largest Ball among selected.

X = 3 - 20 · (Because Min no of those 2 Balls can only be 1 or 2)

$$P\{X=i\} = \frac{{}^{i-1}C_2}{{}^{20}C_3} \qquad i = 3, 4, 5 \ldots 20$$

largest selected ball is 17 Then the other two balls can be of Range 1-16

$$P\{X=17\} = \frac{{}^{16}C_2}{{}^{20}C_3}$$

$$P\{X=18\} = \frac{{}^{17}C_2}{{}^{20}C_3}$$

$$P\{X=19\} = \frac{{}^{18}C_2}{{}^{20}C_3}$$

$$P\{X=20\} = \frac{{}^{19}C_2}{{}^{20}C_3}$$

Total Probability = $P(X=17) + P(X=18) + P(X=19) + P(X=20)$ (To win the bet)

Q. 3 Balls are Randomly chosen from a Bag containing 3 W, 3 Red, 5 Black Balls. Suppose that we win \$1 for each white Ball selected and loose \$1 for each Red Ball selected. Find the Probability that we win the Money.

Sol For Black No loss (Nothing)

X = denote the total winning amount

X = 0, ±1, ±2, ±3

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$P\{X=1\} + P\{X=2\} + P\{X=3\}$

$P\{X=1\} = \frac{3C_1 \, 5C_2 + 3C_2 \, 3C_1}{11C_3} = P\{X=-1\}$

$P\{X=2\} = \frac{3C_2 \, 5C_1}{11C_3} = P\{X=-2\}$

$P\{X=3\} = \frac{3C_3}{11C_3} = P\{X=-3\}$

Taking summation of all values
Probability $= 1/3$

Properties of Random variable :-

1) If X and Y are random variables (rv) then $X+Y$, $X-Y$, XY, X/Y $(Y \neq 0)$ are also rv. Moreover $aX + bY$ is rv, $a, b \in R$

ii) if X is rv, $f : R \to R$ is function then $f(X)$ is also rv

iii) if X and Y are two rv and f is a function of two variables then $f(X, Y)$ are also rv.

- Discrete Random variable :- A Random variable whose range is either finite or countably infinite is DRV. Otherwise called continous Random variable. eg measurement of height

- Probability Mass function :- corresponding to only discrete Random function. It is denoted by PMF $\to f_X(\cdot)$
In probability :-
$$f_X(x) = P\{X = x\}$$
Definition :- A real valued function (f) defined by $f(x) = P\{X = x\}$ is called Probability Mass function (pmf of drv) $f_X(x)$

ex Tossing of 2 coins :-

$\Omega = \{ (HH), (TT), (HT), (TH) \}$

$X =$ no. of heads.

$$f_X(x) = \begin{cases} 1/4 & \text{if } x = 0 \\ 2/4 = 1/2 & \text{if } x = 1 \\ 1/4 & \text{if } x = 2 \\ 0 & \text{otherwise} \end{cases}$$

your choice want to define it or not ← 0

This the Prob. Mass function of X.

- **Properties of Probability Mass function :-**

→ Let 'f' be a PMF of a Random variable X then

1) $f(x) \geq 0$ for all $x \in R$
2) $S = \{ x \in R : f(x) > 0 \}$ is either finite or countably Infinite
3) $\sum_{x \in X} f(x) = 1$ → x is taking all the values of X

1). $f_X(x)$ is denoting Prob. which is always greater than zero
2). It is talking about Range of the Random variable which is discrete → and it is always finite or countably Infinite

3). All the Probabilities are Mutually exclusive so Total Prob will always be one.

→ If you have a f^n $R \to R$ which satisfy all the above four properties Then their exist a Random variable X whose PMF is given by $f_X(x)$. So there Must exist a Random variable whose PMF is $f_X(x)$.