Morphological Image Processing

Reference:
Digital Image Processing
Rafael C. Gonzalez, Richard E. Woods

What is Morphology

- Morphological image processing (or *morphology*) describes a range of image processing techniques that deal with the shape (or morphology) of features in an image

- Morphological operations are typically applied to remove imperfections introduced during segmentation, and so typically operate on bi-level images

Quick Example

Introduction

- Morphological image processing is used to:
  - extract image components for representation and description of region shape, such boundaries, skeletons, and the convex hull.
  - edge detection, noise removal, image enhancement and image segmentation.

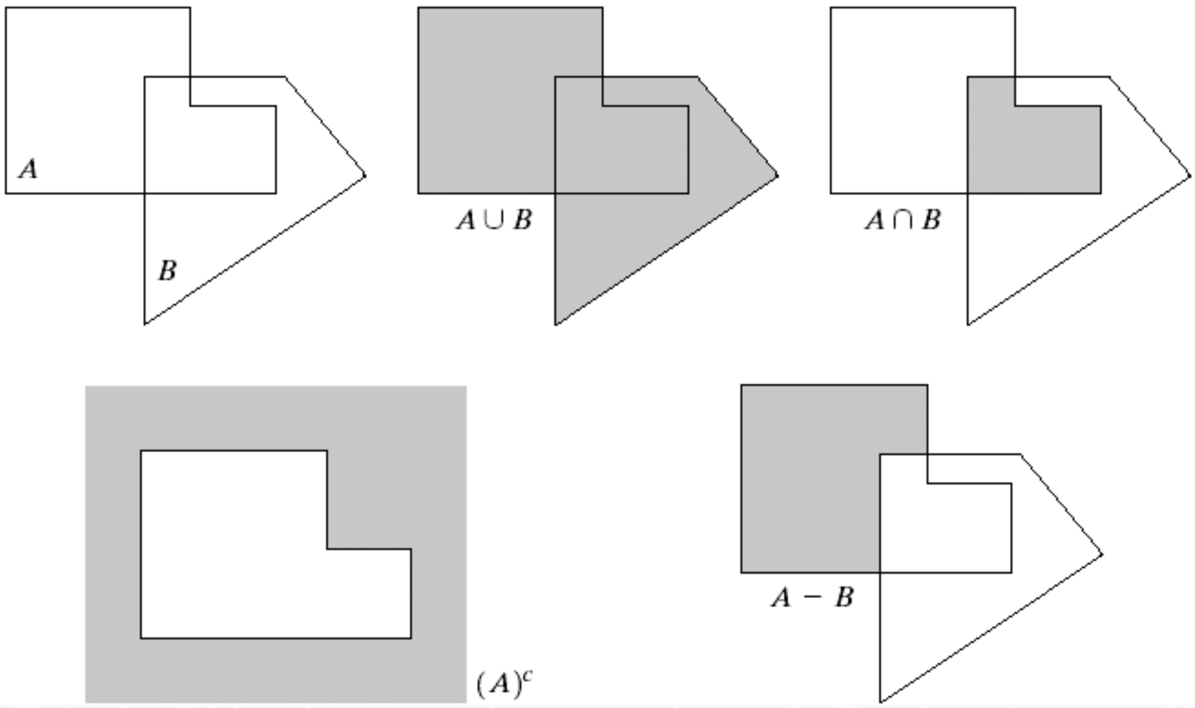
Introduction

- Morphological techniques probe an image with a small shape or template known as a **structuring element.**
  - The structuring element is positioned at all possible locations in the image.
  - It is compared with the corresponding neighborhood of pixels.
  - Morphological operations differ in how they carry out this comparison.

Basic Set Theory

- If a=(a1,a2) is an element of A: $a \in A$
- If a is not an element of A : $a \notin A$
- The set with no elements is called the *null* or *empty set* and is denoted by the symbol $\varnothing$.
- If every element of a set A is also an element of another set B, then a is said to be a subset of B: $A \subseteq B$
- The **union** of two sets A and B is the set of all elements belonging to either A,B or both: $C = A \cup B$
- The **intersection** of two sets A and B is the set of all elements belonging to both A and B: $D = A \cap B$
- Two sets A and B are **disjoint or mutually exclusive** if they have no common elements. $A \cap B = \varnothing$

Basic Set Theory

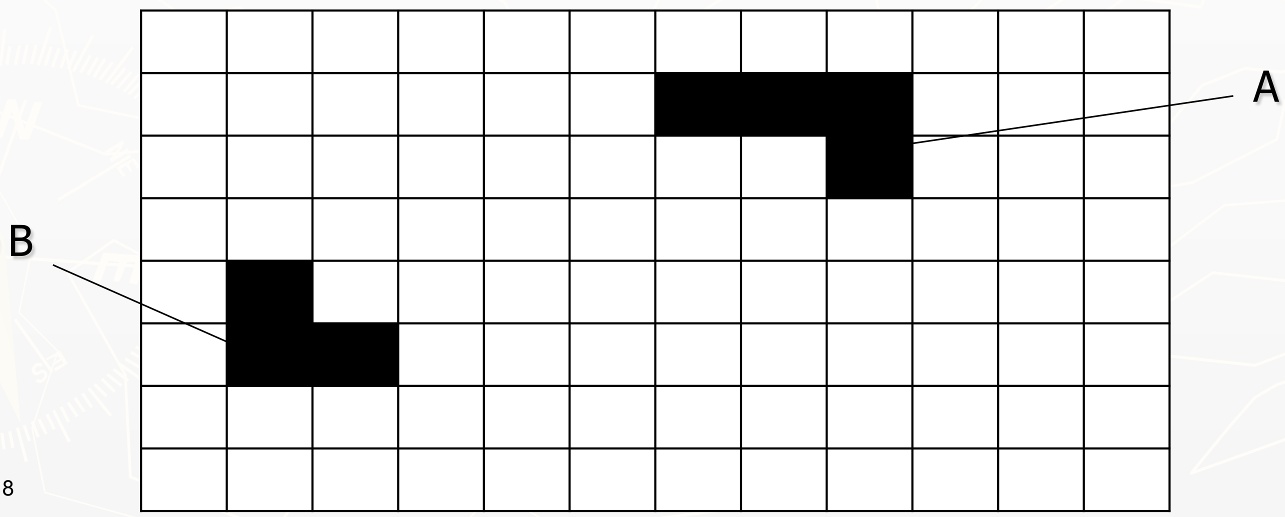


a b c
d e

FIGURE 9.1 (a) Two sets A and B. (b) The union of A and B. (c) The intersection of A and B. (d) The complement of A. (e) The difference between A and B.

Preliminaries (1)

- We define an image as:
  - an (amplitude) function of two discrete variables *a* [*m*,*n*] or,
  - a set (or collection) of discrete coordinates. The set corresponds to the pixels belonging to objects in the image.



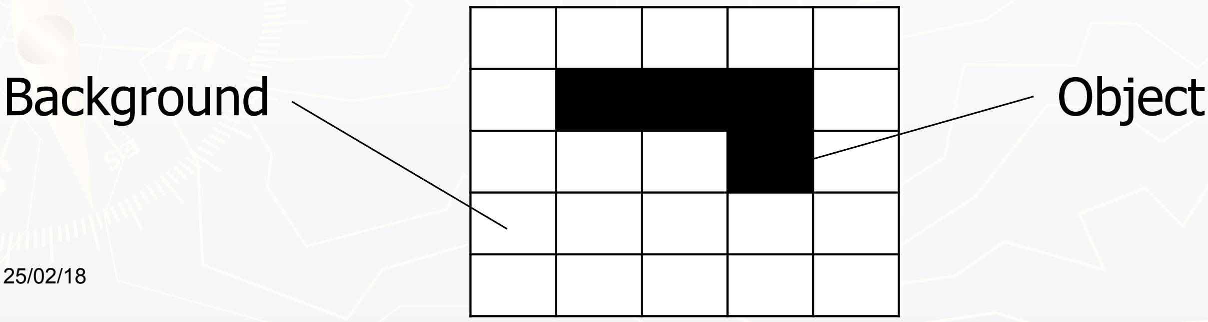
Preliminaries (2)

- The object ***A*** consists of those pixels α that share some common property:

$$A = \{\alpha \mid property(\alpha) == TRUE\}$$

- The background of ***A*** is given by ***A****ᶜ* (the *complement* of ***A***) which is defined as those elements that are not in ***A.***

$$A^c = \{\alpha \mid \alpha \notin A\}$$



Preliminaries (3)

- The **intersection** of any two binary images A and B , denoted by A ∩ B, is the binary image which is 1 at all pixels *p* which are 1 in both A and B:

$$A \cap B = \{p \mid p \in A \text{ and } p \in B\}$$

- The **union** of A and B, denoted by A∪B, is the binary image which is 1 at all pixels p which are 1 in A or 1 in B or 1 in both:

$$A \cup B = \{p \mid p \in A \text{ or } p \in B\}$$

Preliminaries (4)

- Reflection

The reflection of a set B, denoted by $\hat{B}$,

is defined as,

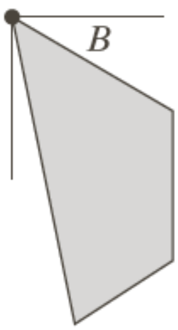
$$\hat{B} = \{w \mid w = -b, \text{ for } b \in B\}$$

- Translation

The translation of a set B by point $z = (z_1, z_2)$, denoted $(B)_Z$,
is defined as

$$(B)_Z = \{c \mid c = b + z, \text{for } b \in B\}$$

Example: Reflection and Translation



a b c

FIGURE 9.1
(a) A set, (b) its reflection, and (c) its translation by z.

Logic Operations

p	q	p AND q (also $p \cdot q$)	p OR q (also $p + q$)	NOT (p) (also $\bar{p}$)
0	0	0	0	1
0	1	0	1	1
1	0	0	1	0
1	1	1	1	0

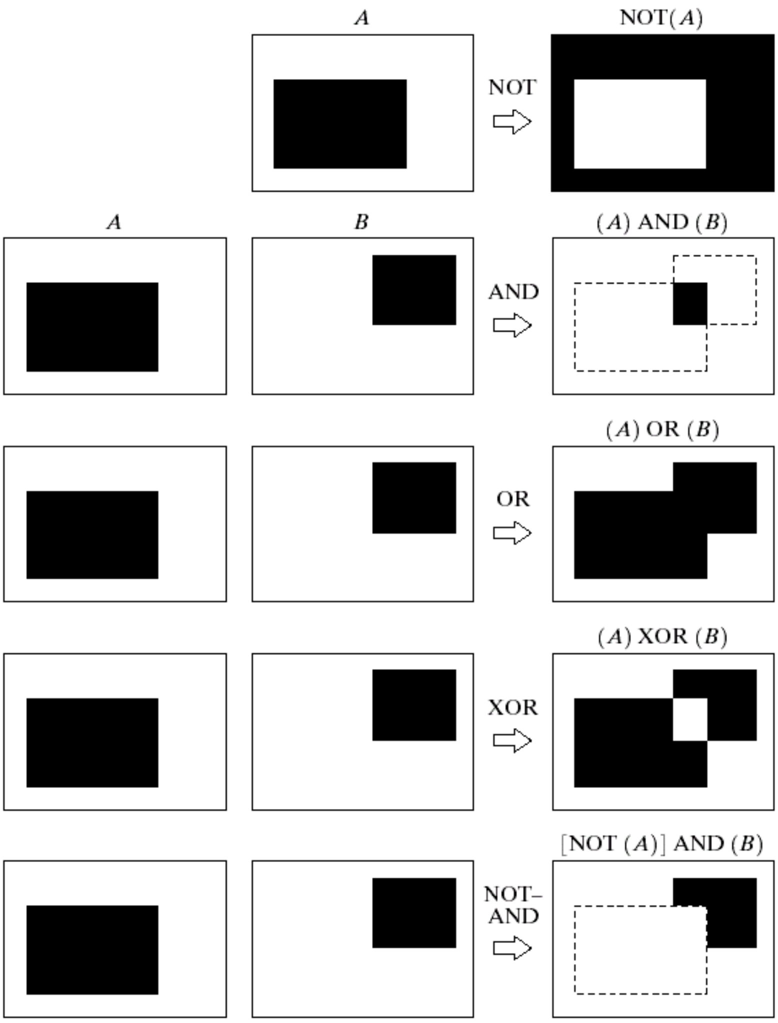


FIGURE 9.3 Some logic operations between binary images. Black represents binary 1s and white binary 0s in this example.

Structuring Element (SE)

- Small set/sub-image to probe an image under study
- Consists of a pattern specified as the coordinates of a number of discrete points relative to some origin.
- A convenient way of representing the element is as a small image on a rectangular grid.
- The origin does not have to be in the center of the structuring element, but often it is.
- Structuring elements that fit into a 3×3 grid with its origin at the center are most common.

Example: Structuring Element

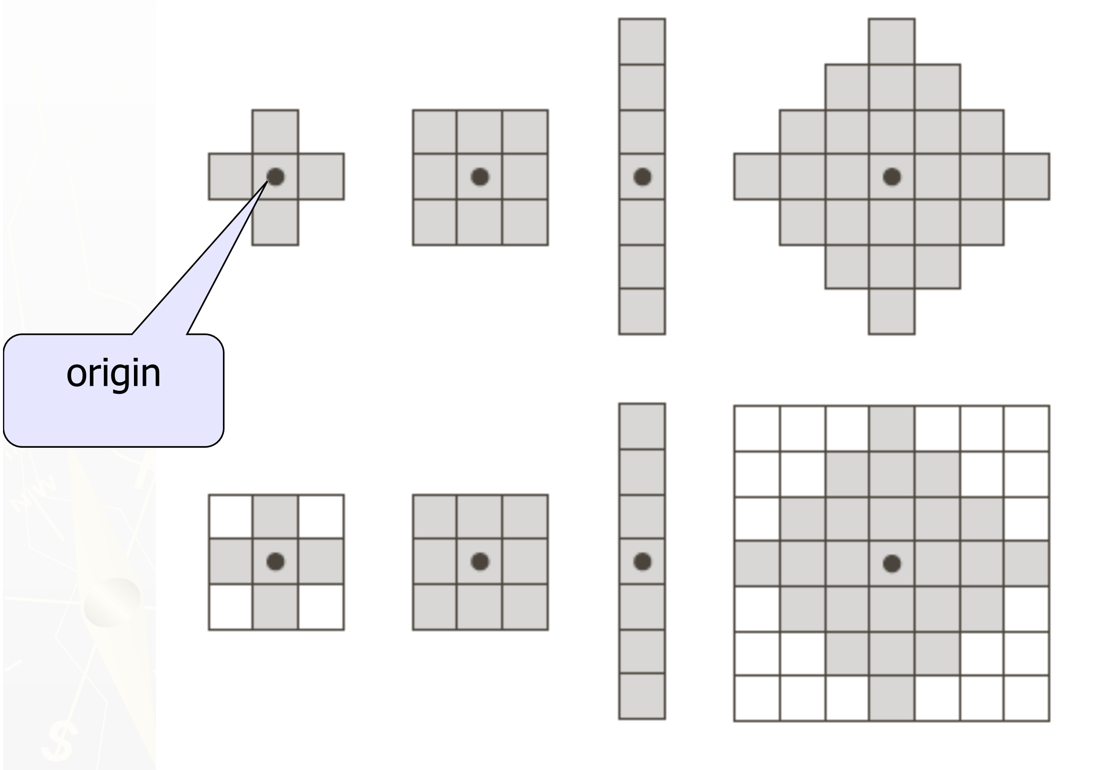
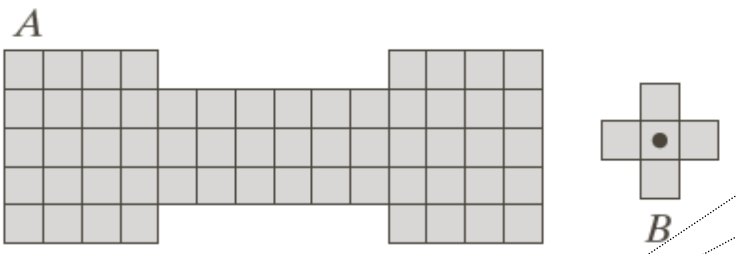


FIGURE 9.2 First row: Examples of structuring elements. Second row: Structuring elements converted to rectangular arrays. The dots denote the centers of the SEs.

Example: Structuring Elements

Accommodate the entire structuring elements when its origin is on the border of the original set A



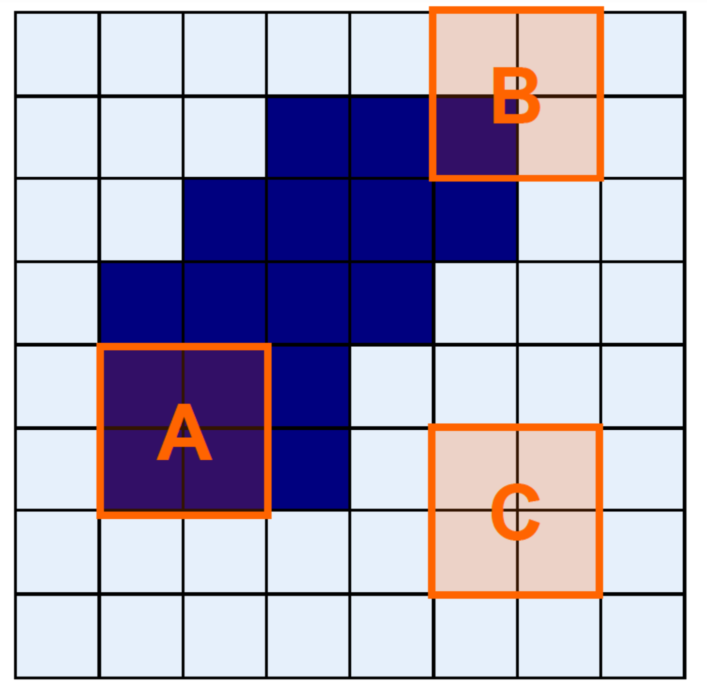
Origin of B visits every element of A

At each location of the origin of B, if B is completely contained in A, then the location is a member of the new set, otherwise it is not a member of the new set.

a b
c d e

FIGURE 9.3 (a) A set (each shaded square is a member of the set). (b) A structuring element. (c) The set padded with background elements to form a rectangular array and provide a background border. (d) Structuring element as a rectangular array. (e) Set processed by the structuring element.

Structuring Elements, Hits & Fits



- **Fit:** All on pixels in the structuring element cover on pixels in the image
- **Hit:** Any on pixel in the structuring element covers an on pixel in the image

Basic Morphological Operations

- Erosion 

- Dilation 

- Combine to

  Keep general shape but smooth with respect to
  - Opening → object
  - Closing → background

Erosion

- Does the structuring element **fit the set?**

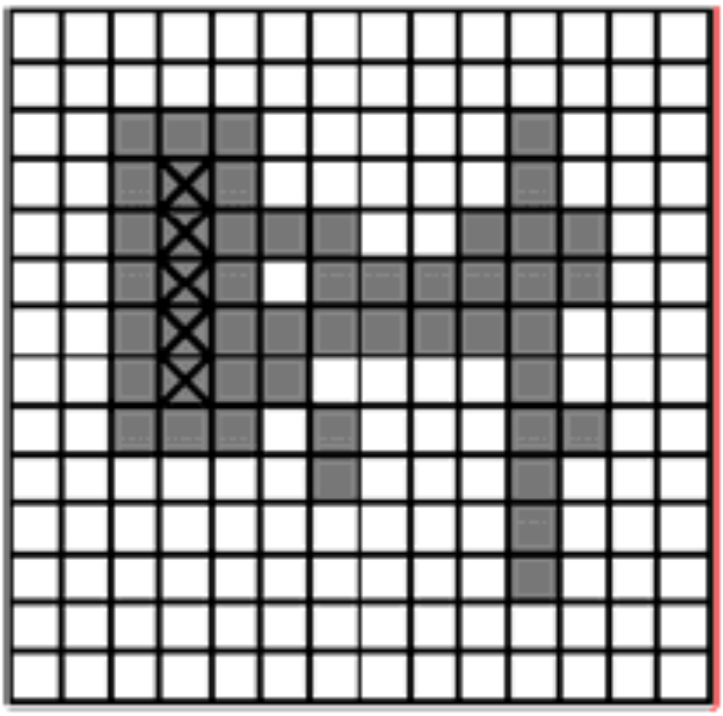
Erosion of a set A by structuring element B: all z in A such that B is in A when origin of B=z

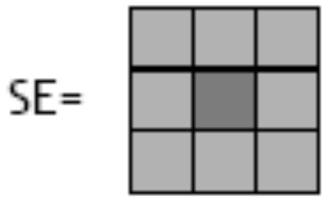
$$A \ominus B = \left\{ z \mid (B)_Z \subseteq A \right\}$$

Erosion: How it works

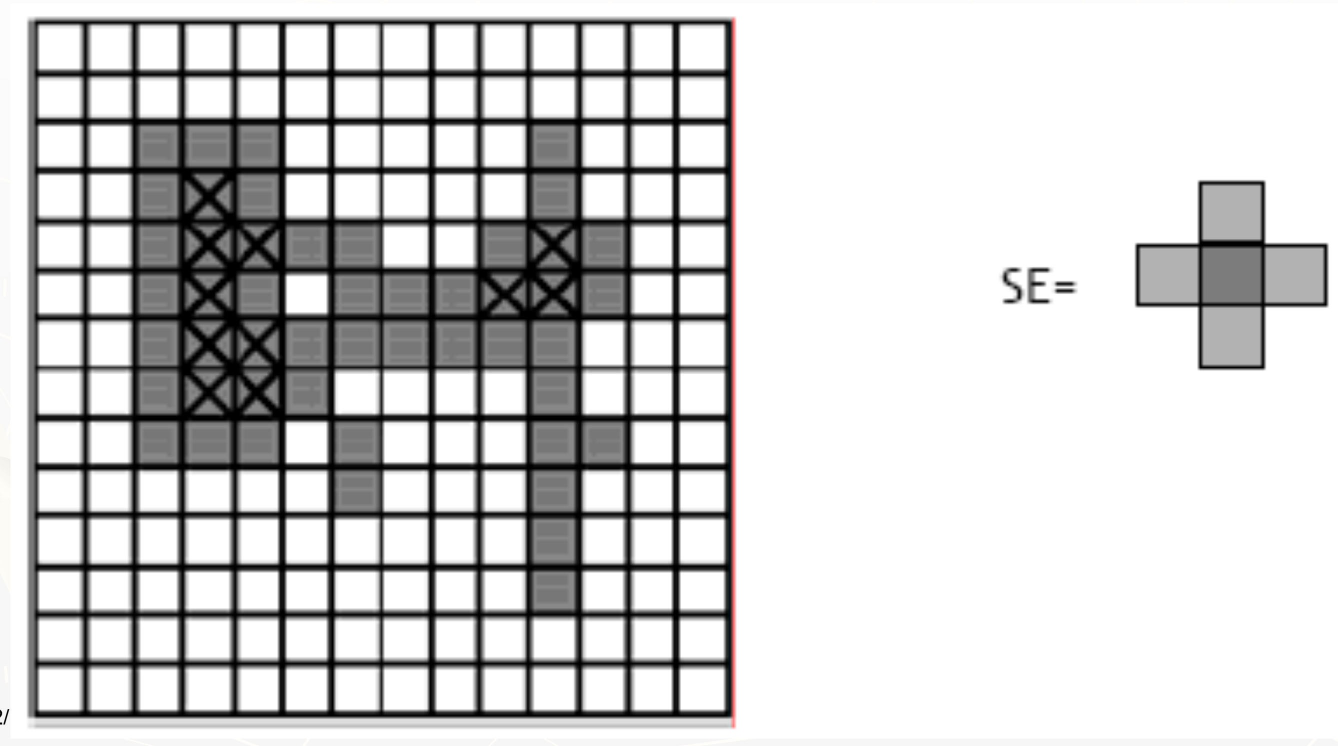
- For each foreground pixel, superimpose the SE on the input image such that the origin of the SE coincides with the input pixel coordinates.
- If for *every* pixel in the SE, the corresponding pixel in the image underneath is a foreground pixel, then the input pixel is left as it is.
- If any of the corresponding pixels in the image are background, however, the input pixel is also set to background value.

Erosion: How it works





Erosion: How it works



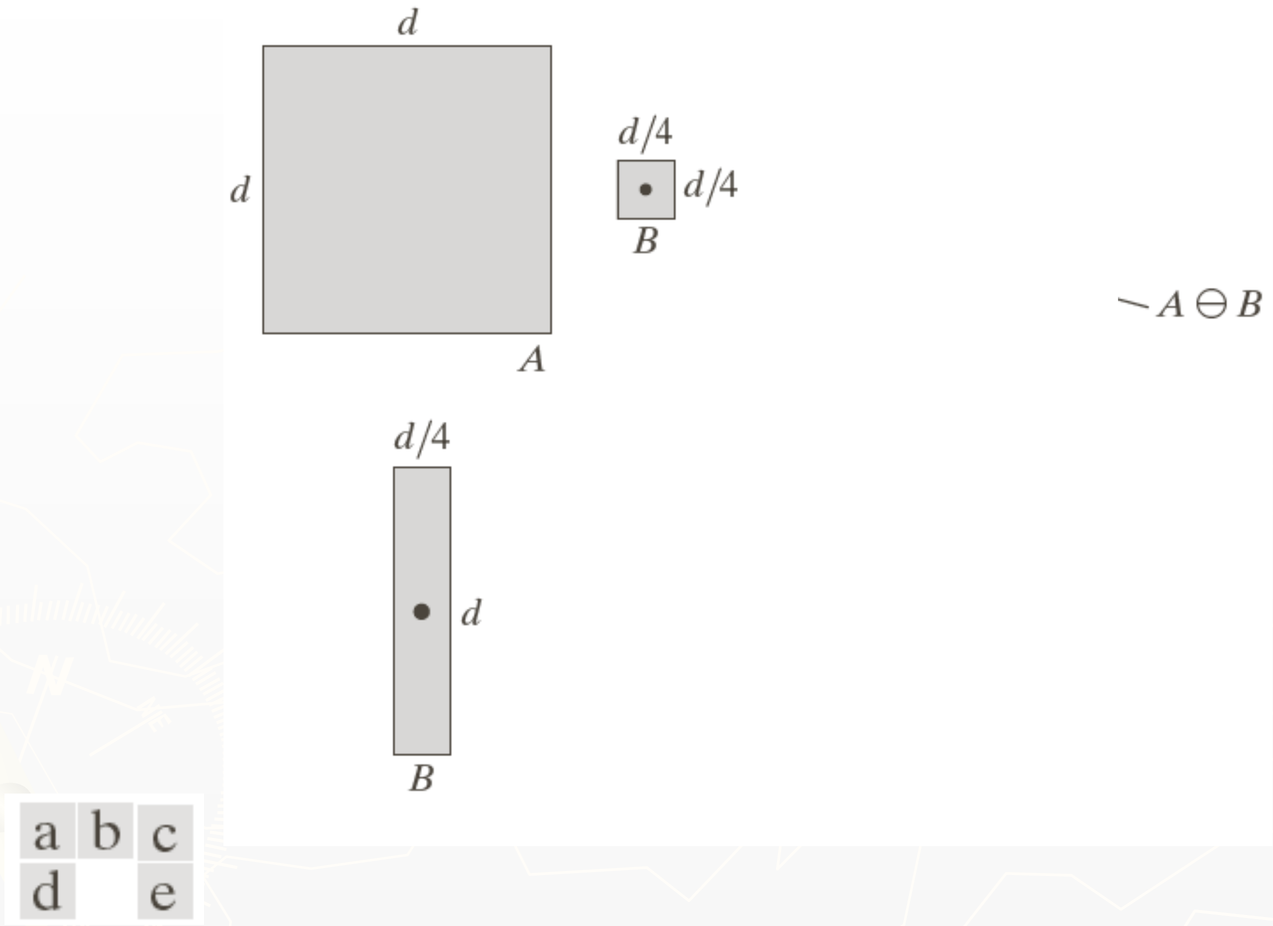
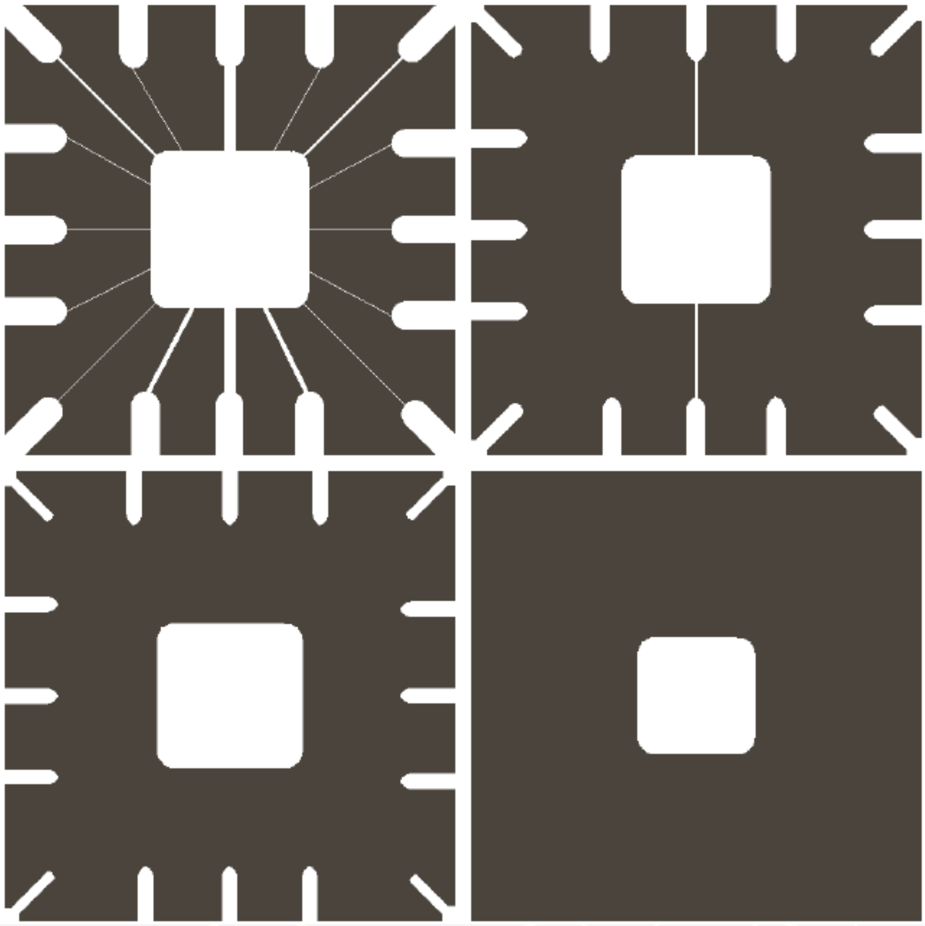


FIGURE 9.4 (a) Set A. (b) Square structuring element, B. (c) Erosion of A by B, shown shaded. (d) Elongated structuring element. (e) Erosion of A by B using this element. The dotted border in (c) and (e) is the boundary of set A, shown only for reference.

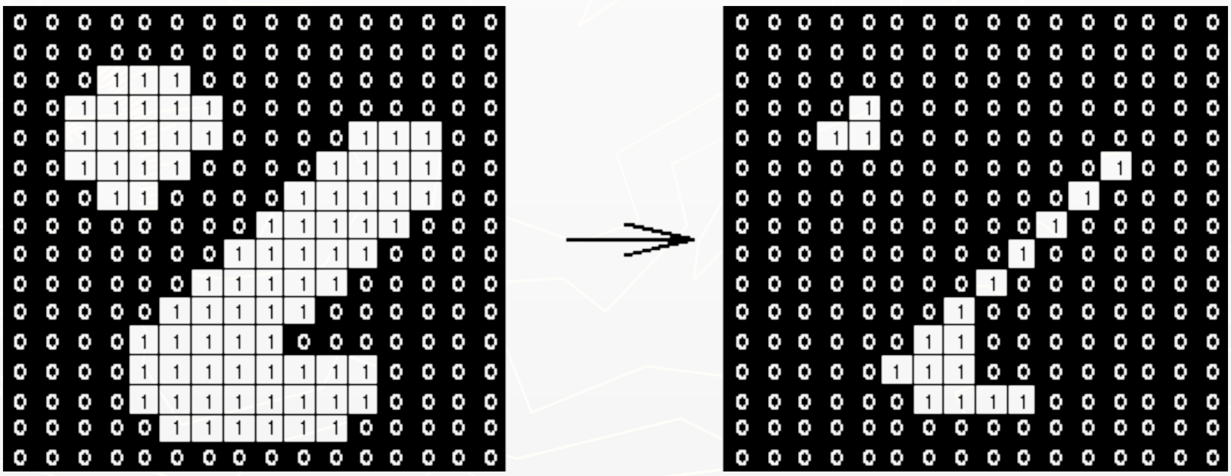


a	b
c	d

FIGURE 9.5 Using erosion to remove image components. (a) A 486×486 binary image of a wire-bond mask. (b)–(d) Image eroded using square structuring elements of sizes 11×11, 15×15, and 45×45, respectively. The elements of the SEs were all 1s.

Erosion

- The basic effect of the operator on a binary image is to **erode away the boundaries of regions of foreground pixels** (*i.e.* white pixels, typically).
- Thus, areas of foreground pixels shrink in size, and holes within those areas become larger.

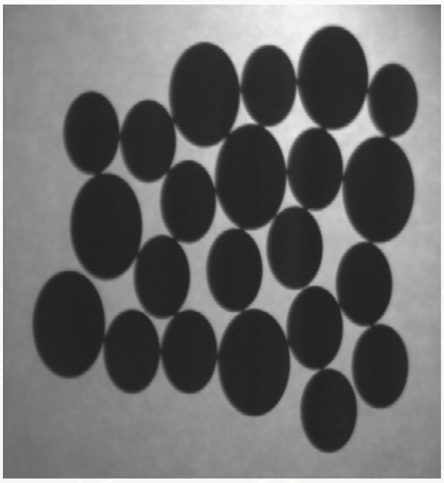


Erosion

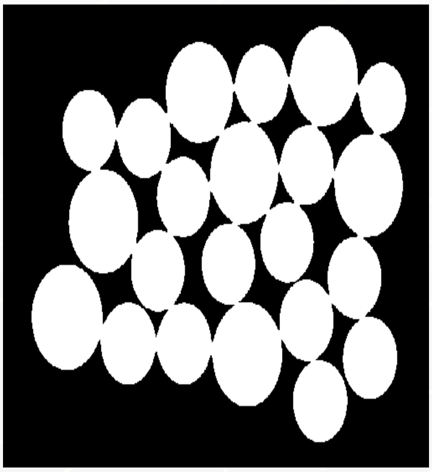
- Erosions can be made directional by using less symmetrical structuring elements.
- For example, a structuring element that is 10 pixels wide and 1 pixel high will erode in a horizontal direction only.
- Similarly, a 3×3 square structuring element with the origin in the middle of the top row rather than the center, will erode the bottom of a region more severely than the top.

Erosion

- There are many specialist uses for erosion.
- One of the more common is to separate touching objects in a binary image so that they can be counted using a **labeling algorithm**.



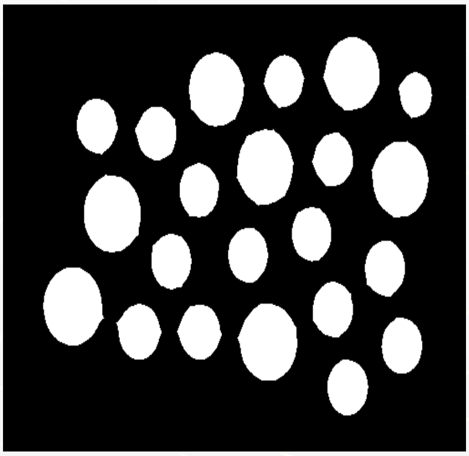
A no. of dark disks



Thresholding at 90

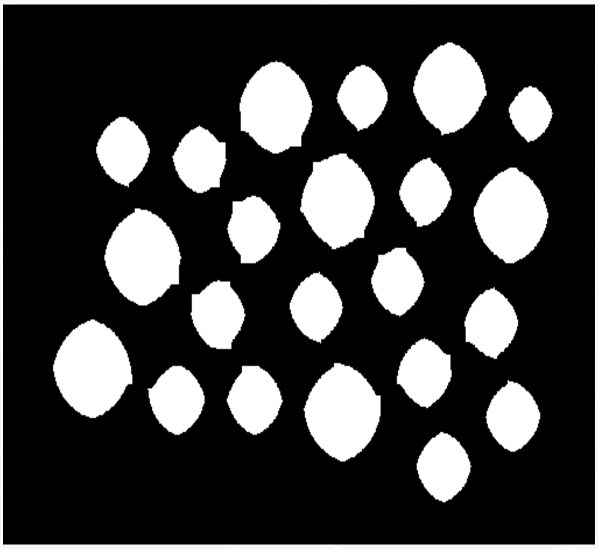
Erosion

- Result of eroding twice using a disk shaped structuring element 11 pixels in diameter.
- Now the coins can be easily counted.
- The relative sizes of the coins can be used to distinguish the various types.



Erosion

- 9×9 square structuring element is used instead of a disk .
- The square structuring element has led to distortion of the shapes, which in some situations could cause problems in identifying the regions after erosion.



Erosion

- Erosion can also be used to remove small spurious bright spots (**‘salt noise’**) in images.
- We can also use erosion for **edge detection** by taking the erosion of an image and then **subtracting** it away from the original image, thus highlighting just those pixels at the edges of objects that were removed by the erosion.
- Erosion is also used as the basis for many other mathematical morphology operators.

Dilation

With A and B as sets in Z^2, the dilation of A by B, denoted $A \oplus B$, is defined as

$$A \oplus B = \{z \mid (\hat{B})_Z \cap A \neq \phi\}$$

Dilation

- Does the structuring element **hit the set?**

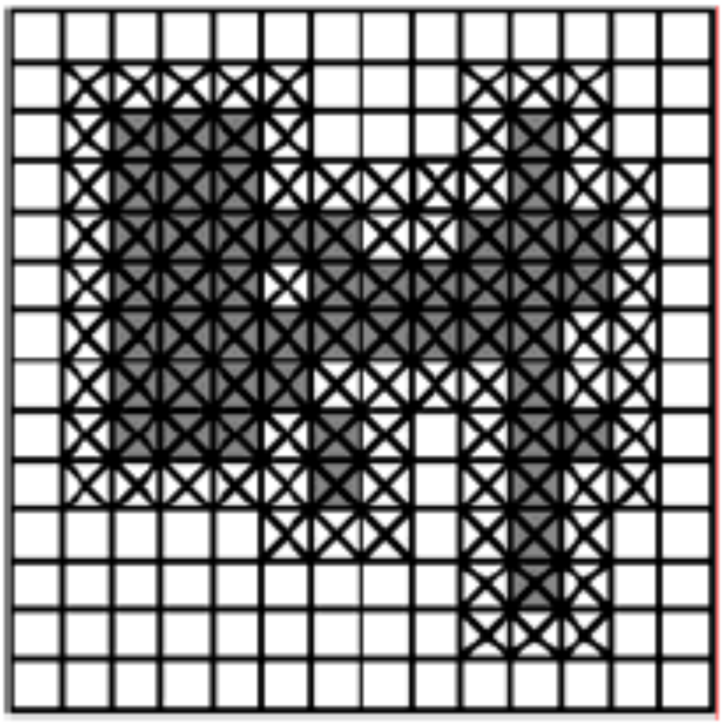
Dilation of a set A by structuring element B: all z in A such that B hits A when origin of B=z

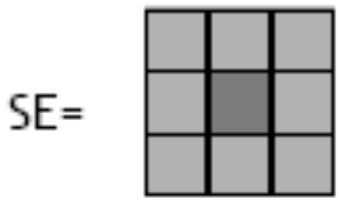
$$A \oplus B = \{ z \mid (\hat{B})_z \cap A \neq \varphi \}$$

Dilation: How it works

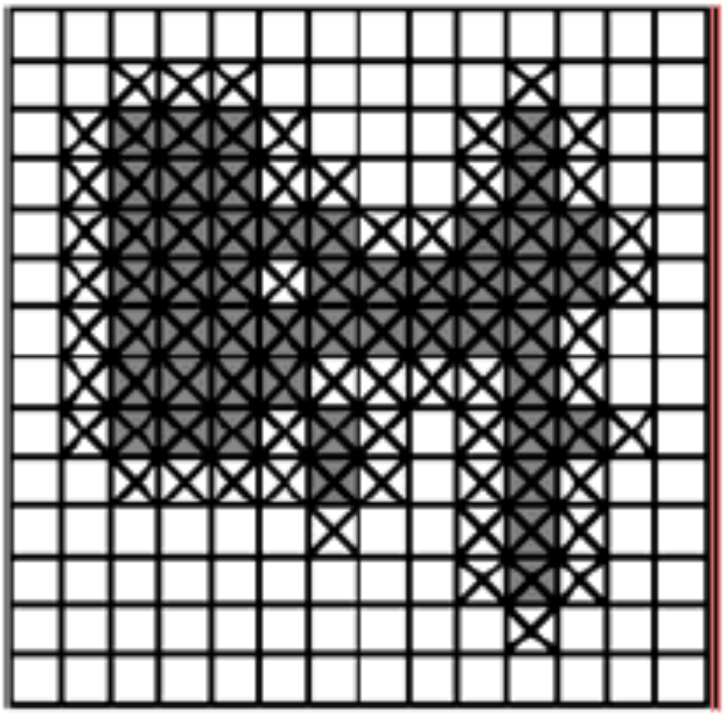
- For each background pixel (which we will call the *input pixel*) we superimpose the structuring element on top of the input image so that the origin of the structuring element coincides with the input pixel position.
- If *at least one* pixel in the structuring element coincides with a foreground pixel in the image underneath, then the input pixel is set to the foreground value.
- If all the corresponding pixels in the image are background, however, the input pixel is left at the background value.

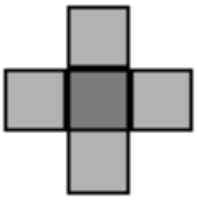
Dilation: How it works



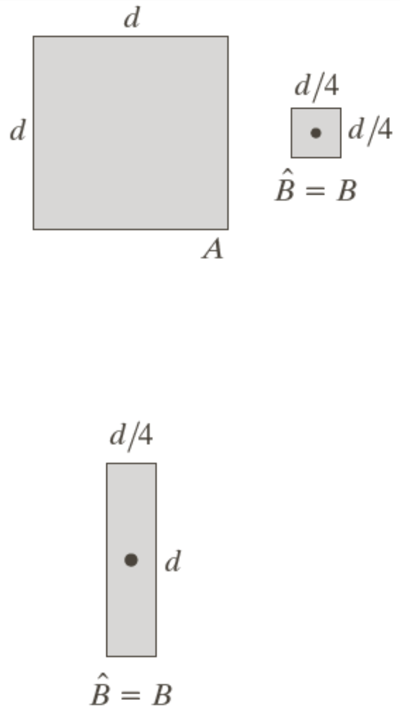


Dilation: How it works





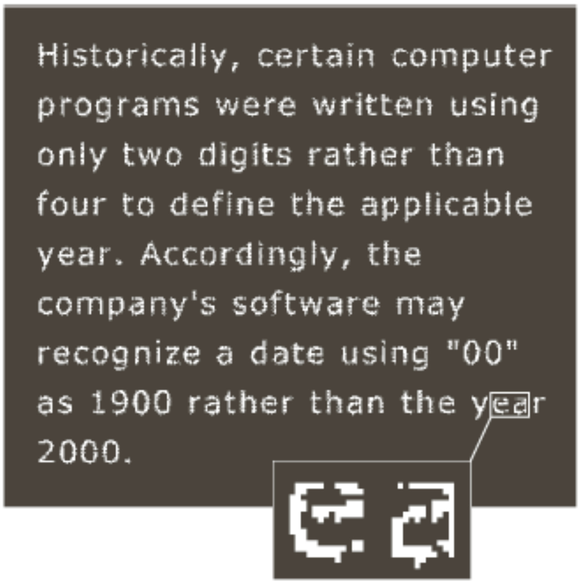
Examples of Dilation (1)



a b c
d e

FIGURE 9.6
(a) Set A.
(b) Square structuring element (the dot denotes the origin).
(c) Dilation of A by B, shown shaded.
(d) Elongated structuring element. (e) Dilation of A using this element. The dotted border in (c) and (e) is the boundary of set A, shown only for reference

Examples of Dilation (2)


Historically, certain computer programs were written using only two digits rather than four to define the applicable year. Accordingly, the company's software may recognize a date using "00" as 1900 rather than the year 2000.

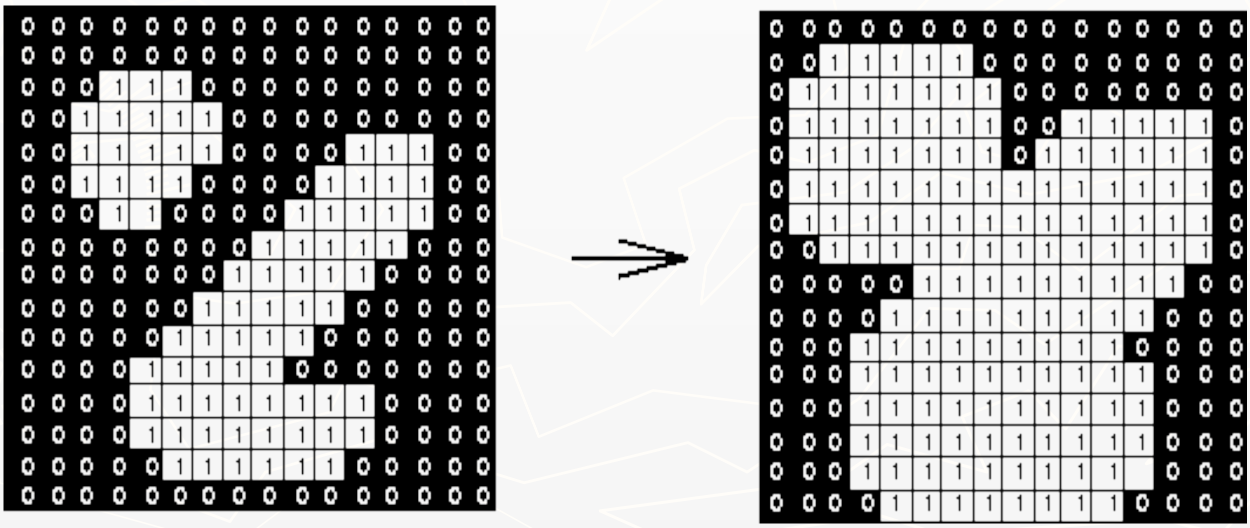
0	1	0
1	1	1
0	1	0

a c
b

FIGURE 9.7
(a) Sample text of poor resolution with broken characters (see magnified view).
(b) Structuring element.
(c) Dilation of (a) by (b). Broken segments were joined.

Dilation

- The basic effect of the operator on a binary image is to **gradually enlarge the boundaries of regions of foreground pixels** (*i.e.* white pixels, typically). Thus areas of foreground pixels grow in size while holes within those regions become smaller.



Dilation

- There are many specialist uses for dilation. For instance it can be used to fill in small spurious holes (`**pepper noise'**) in images.
- Result of dilating image with a 3×3 square structuring element. Although the noise has been effectively removed, the image has been degraded significantly.

Duality

- Erosion and dilation are duals of each other with respect to set complementation and reflection

$$(A \ominus B)^c = A^c \oplus \hat{B}$$

and

$$(A \oplus B)^c = A^c \ominus \hat{B}$$

Duality

- Erosion and dilation are duals of each other with respect to set complementation and reflection

$$(A \ominus B)^c = \{z \mid (B)_z \subseteq A\}^c$$

$$= \{z \mid (B)_z \cap A^c = \phi\}^c$$

$$= \{z \mid (B)_z \cap A^c \neq \phi\}$$

$$= A^c \oplus \hat{B}$$

Duality

- Erosion and dilation are duals of each other with respect to set complementation and reflection

$$(A \oplus B)^c = \{z \mid (\hat{B})_Z \cap A \neq \phi\}^c$$

$$= \{z \mid (\hat{B})_Z \cap A^c = \phi\}$$

$$= A^c \Theta \hat{B}$$

Opening and Closing

- Opening generally smoothes the contour of an object, breaks narrow isthmuses, and eliminates thin protrusions.

- Closing tends to smooth sections of contours but it generates fuses narrow breaks and long thin gulfs, eliminates small holes, and fills gaps in the contour.

Opening and Closing

The opening of set A by structuring element B, denoted $A \circ B$, is defined as

$$A \circ B = \left(A \ominus B\right) \oplus B$$

The closing of set A by structuring element B, denoted by $A \bullet B$, is defined as

$$A \bullet B = (A \oplus B) \Theta B$$

Opening

The opening of set A by structuring element B, denoted $A \circ B$, is defined as

$$A \circ B = \bigcup \left\{ (B)_Z \mid (B)_Z \subseteq A \right\}$$

Opening

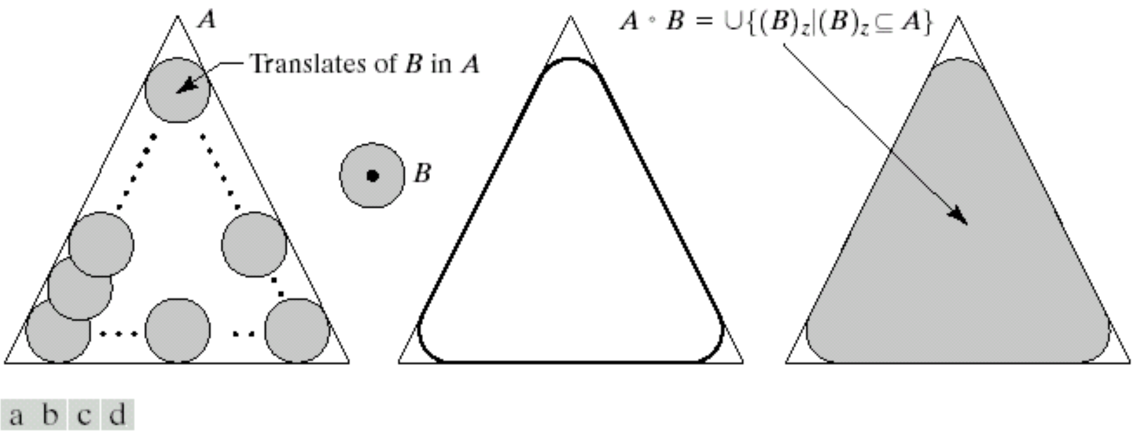


FIGURE 9.8 (a) Structuring element B "rolling" along the inner boundary of A (the dot indicates the origin of B). (c) The heavy line is the outer boundary of the opening. (d) Complete opening (shaded).

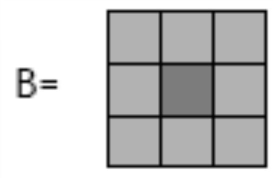
Opening

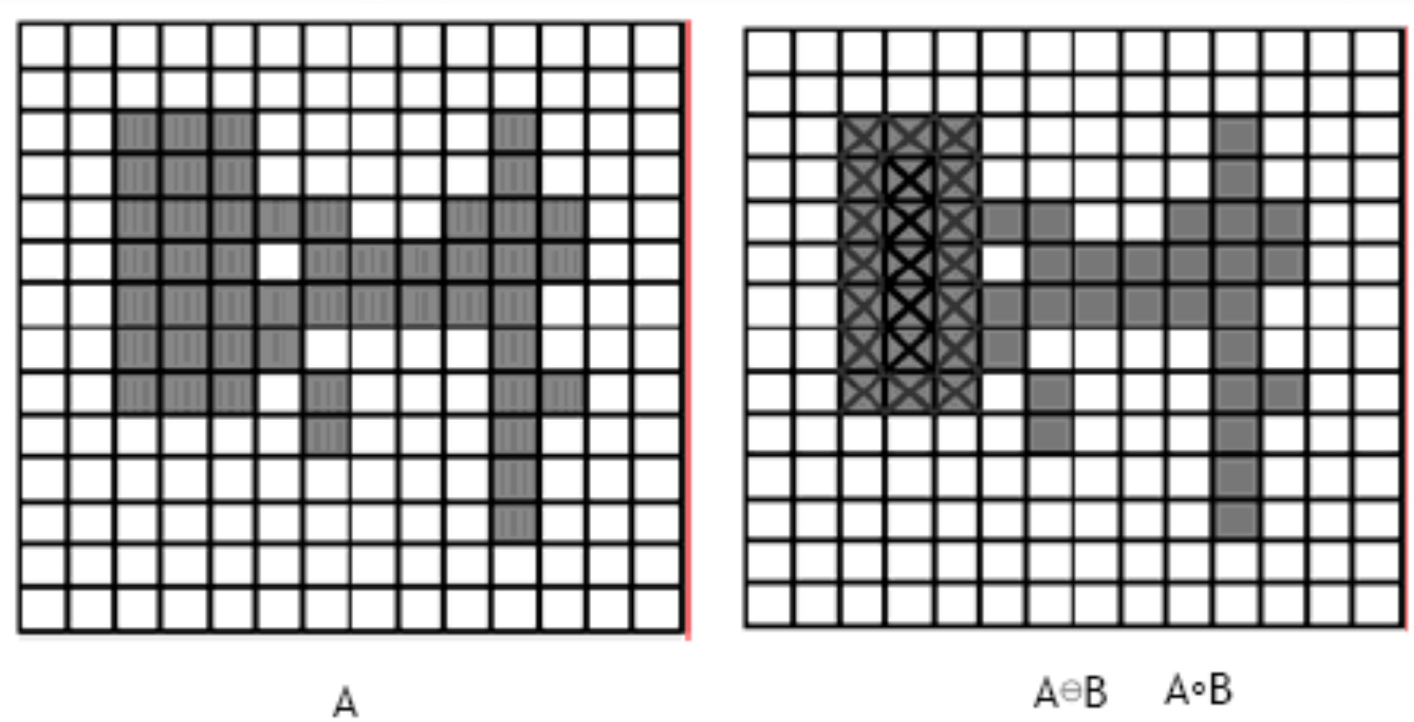
- The basic effect of an **opening** is somewhat like **erosion.**
  - It tends to remove some of the foreground (bright) pixels from the edges of regions of foreground pixels. However it is less destructive than erosion in general.
- The exact operation is determined by a structuring element.
- The effect of the operator is to preserve *foreground* regions that have a similar shape to this structuring element, or that can completely contain the structuring element, while eliminating all other regions of foreground pixels.

Opening

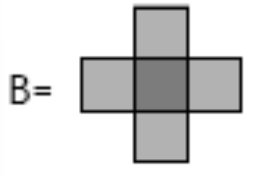
- Erosion has the big disadvantage that it affects *all* regions of foreground pixels indiscriminately. Opening gets around this by performing both an erosion and a dilation on the image.
- Imagine taking the structuring element and sliding it around *inside* each foreground region, without changing its orientation.
- All pixels which can be covered by the structuring element with the structuring element being entirely within the foreground region will be preserved.
- All foreground pixels which cannot be reached by the structuring element without parts of it moving out of the foreground region will be eroded away.
- The new boundaries of foreground regions will all be such that the structuring element fits inside them, and so further openings with the same element have no effect. The property is known as *idempotence*.

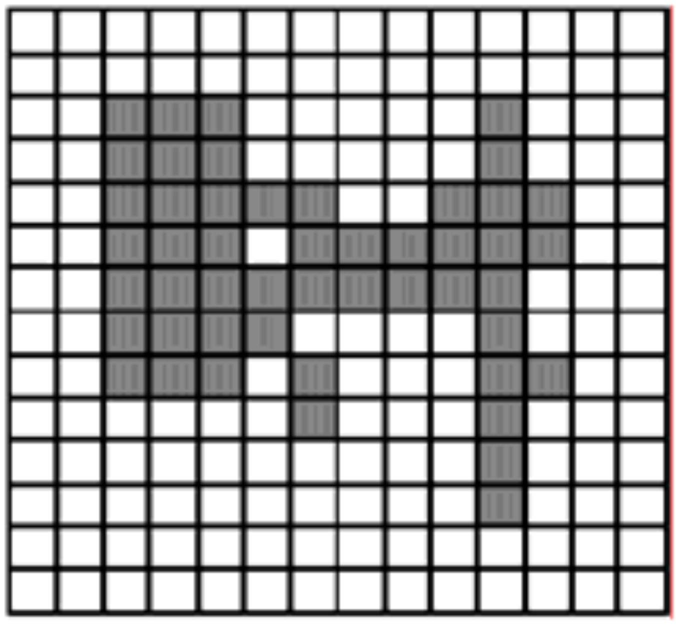
Opening



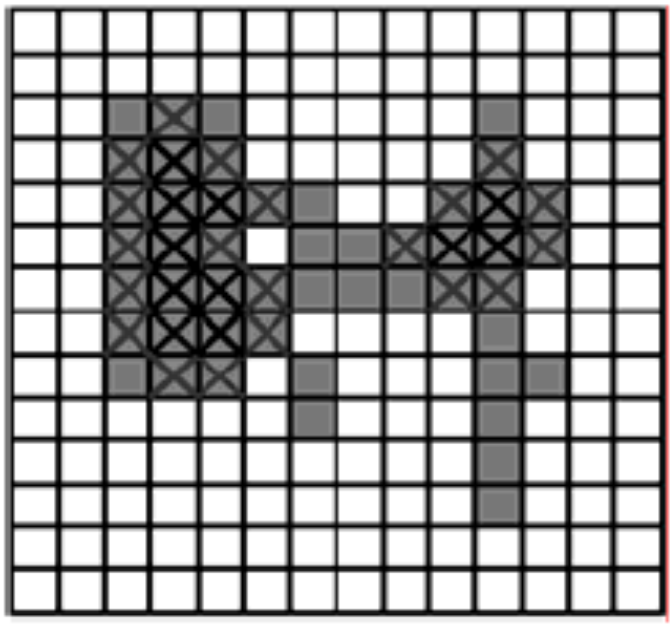


Opening





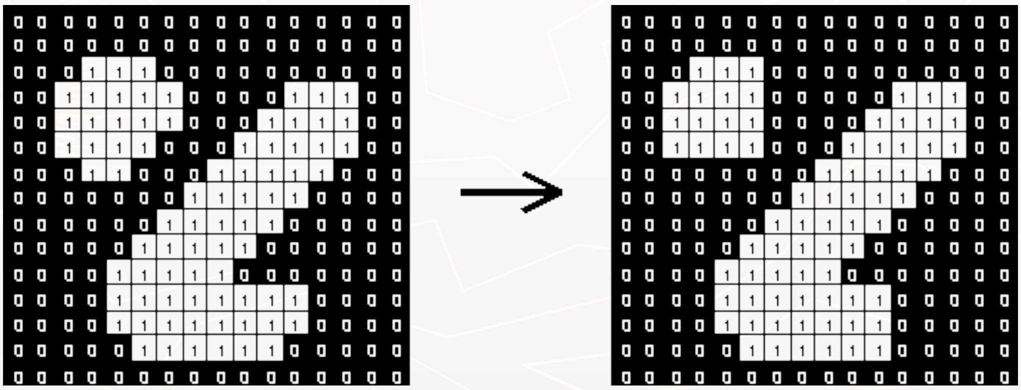
A



A⊖B A∘B

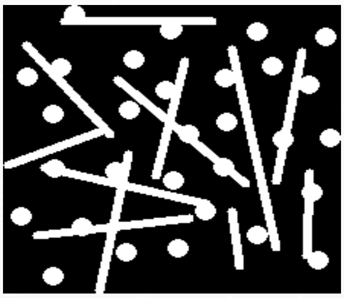
Opening

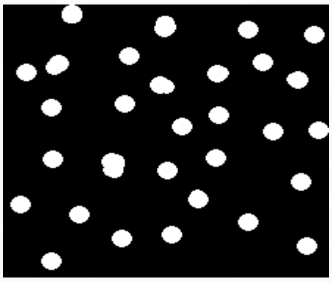
- As with erosion and dilation, it is very common to use the 3×3 structuring element.
- The effect in the figure is rather subtle since the structuring element is quite compact and so it fits into the foreground boundaries quite well even before the opening operation.
- To increase the effect, multiple erosions are often performed with this element followed by the same number of dilations. This effectively performs an opening with a larger square structuring element.



Opening

- A binary image containing a mixture of circles and lines. Suppose that we want to separate out the circles from the lines, so that they can be counted.
- Opening with a disk shaped structuring element 11 pixels in diameter:

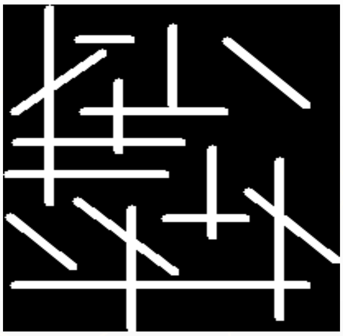




- Some of the circles are slightly distorted, but in general, the lines have been almost completely removed while the circles remain almost completely unaffected.

Opening

► We wish to separately extract the horizontal and vertical lines.



Opening with a 3×9 vertically oriented structuring element

Opening with a 9×3 horizontally oriented structuring element

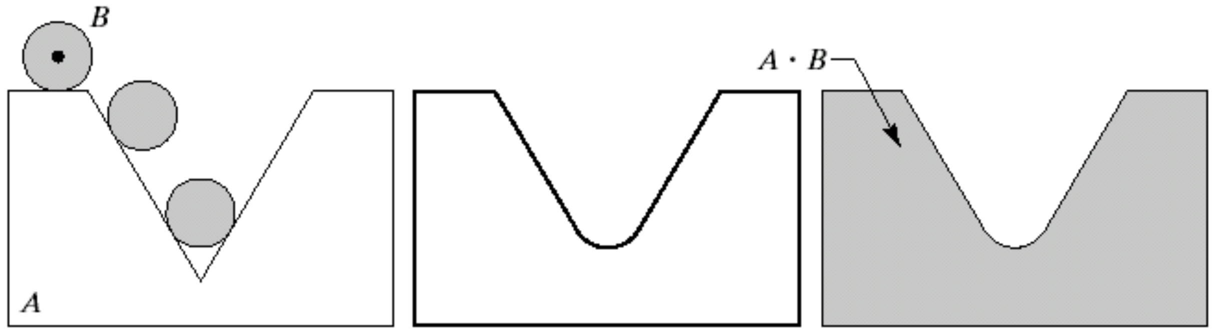
Closing

- Closing is similar in some ways to **dilation:**
  - it tends to enlarge the boundaries of foreground (bright) regions in an image (and shrink background color holes in such regions).
  - but it is less destructive of the original boundary shape.
- The effect of the operator is to preserve *background* regions that have a similar shape to this structuring element, or that can completely contain the structuring element, while eliminating all other regions of background pixels.

Closing

- The effect of closing can be quite easily visualized.
  - Imagine taking the structuring element and sliding it around *outside* each foreground region, without changing its orientation.
  - For any background boundary point, if the structuring element can be made to touch that point, without any part of the element being inside a foreground region, then that point remains background.
  - If this is not possible, then the pixel is set to foreground.
  - After the closing has been carried out the background region will be such that the structuring element can be made to cover any point in the background without any part of it also covering a foreground point, and so further closings will have no effect. This property is known as *idempotence*

Closing

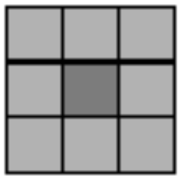


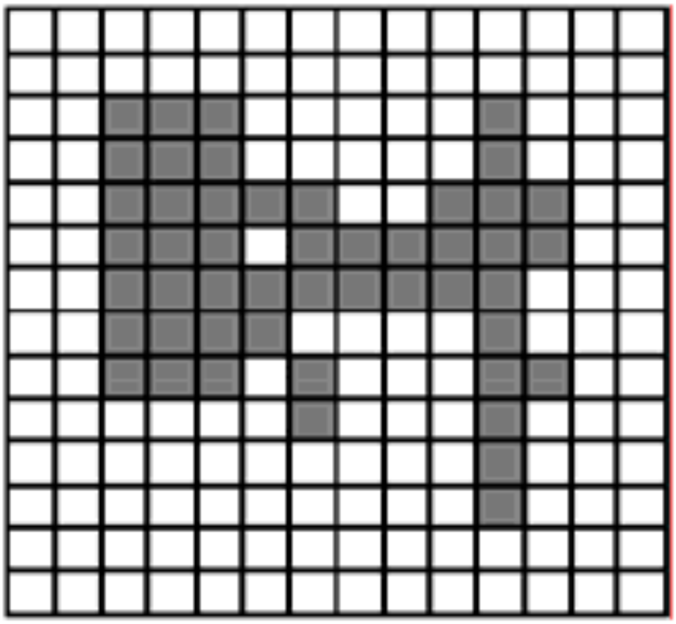
a b c

FIGURE 9.9 (a) Structuring element B "rolling" on the outer boundary of set A. (b) Heavy line is the outer boundary of the closing. (c) Complete closing (shaded).

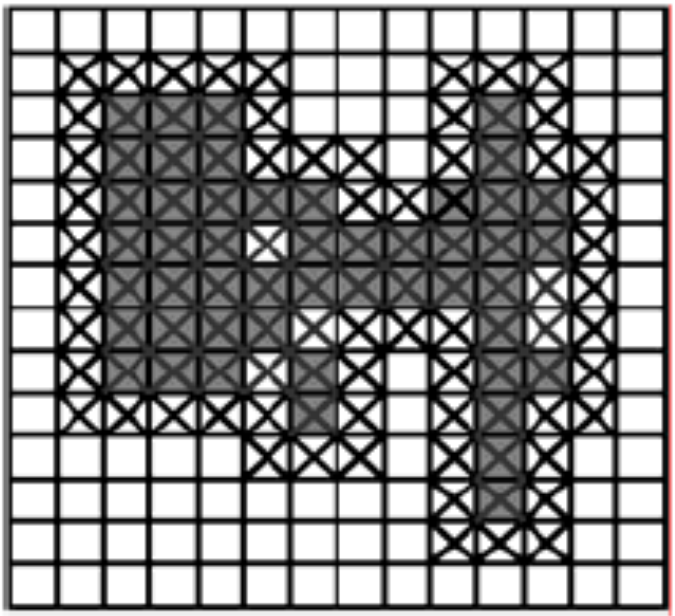
Closing

B=



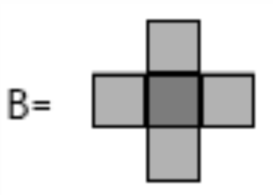


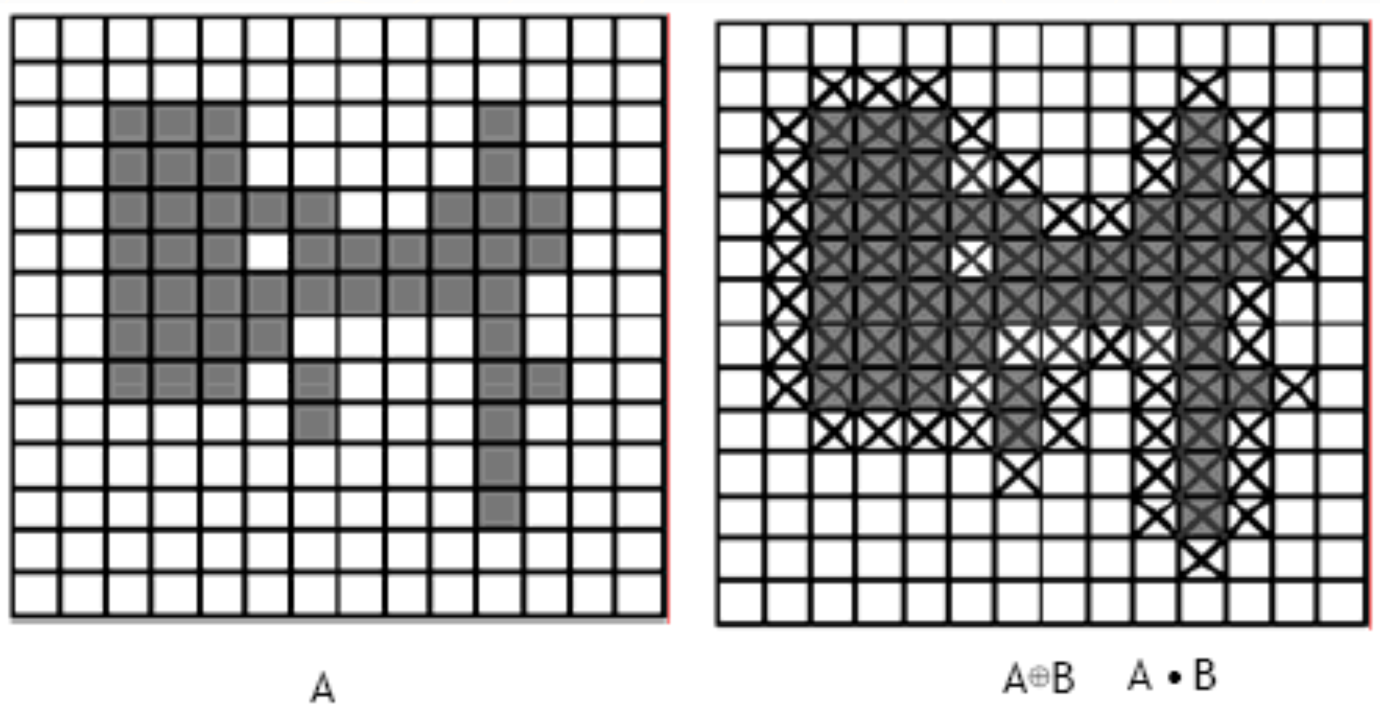
A



$A \oplus B$ $A \bullet B$

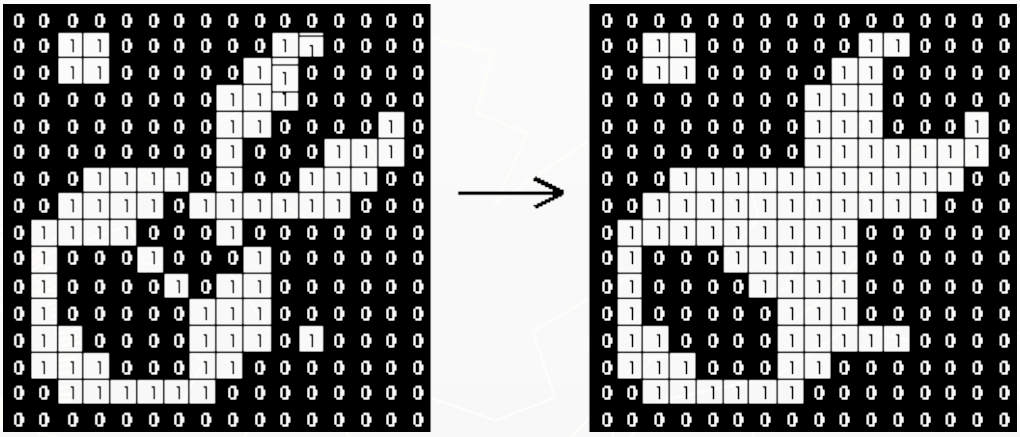
Closing





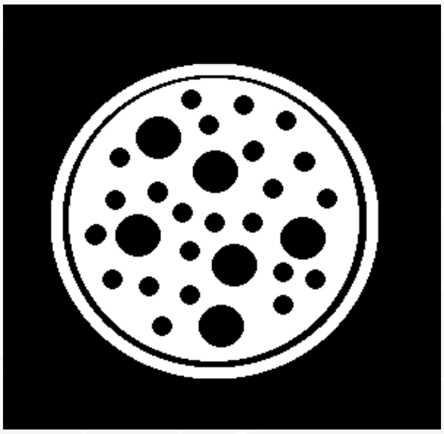
Closing

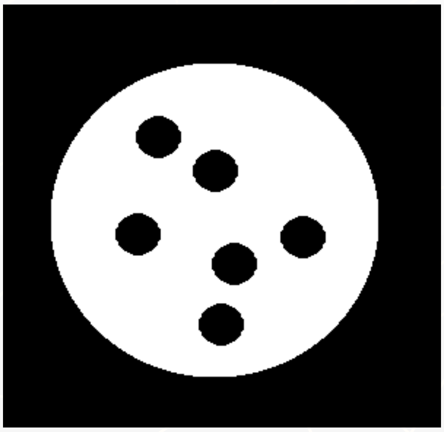
- Effect of closing using a 3x3 square structuring element.



Closing

- An image containing large holes and small holes.
  - If it is desired to remove the small holes while retaining the large holes, then we can simply perform **a closing with a disk-shaped structuring element with a diameter larger than the smaller holes, but smaller than the large holes.**





Closing with 22 pixel dia. disk

The Properties of Opening and Closing

- Properties of Opening

  (a) $A \circ B$ is a subset (subimage) of A

  (b) If C is a subset of D, then $C \circ B$ is a subset of $D \circ B$

  (c) $(A \circ B) \circ B = A \circ B$

- Properties of Closing

  (a) A is a subset (subimage) of $A \bullet B$

  (b) If C is a subset of D, then $C \bullet B$ is a subset of $D \bullet B$

  (c) $(A \bullet B) \bullet B = A \bullet B$