Digital Image Fundamentals

Simple Image Formation Model

- For an image generated from a physical process, its value $f(x,y)$ are proportional to energy radiated by the physical source and hence

  $f(x,y)$ must be non-zero and finite i.e. $0 < f(x,y) < \infty$

- The two components of $f(x,y)$ are
  - The amount of source illumination incident on the scene $i(x,y)$ – illumination component
  - The amount of illumination reflected by the objects in the scene $r(x,y)$ – reflectance component

Simple Image Formation Model

- The two combine as a product to form the image

$$f(x,y) = i(x,y)\,r(x,y)$$

where

$$0 < i(x,y) < \infty$$

and

$$0 < r(x,y) < 1$$

- The intensity of a monochrome image at any coordinate (x_i,y_i) is the *gray level* (l).

$$L_{\min} \le l \le L_{\max}$$

- The interval $[L_{\min}, L_{\max}]$ is called the *gray scale*.

Image Formation

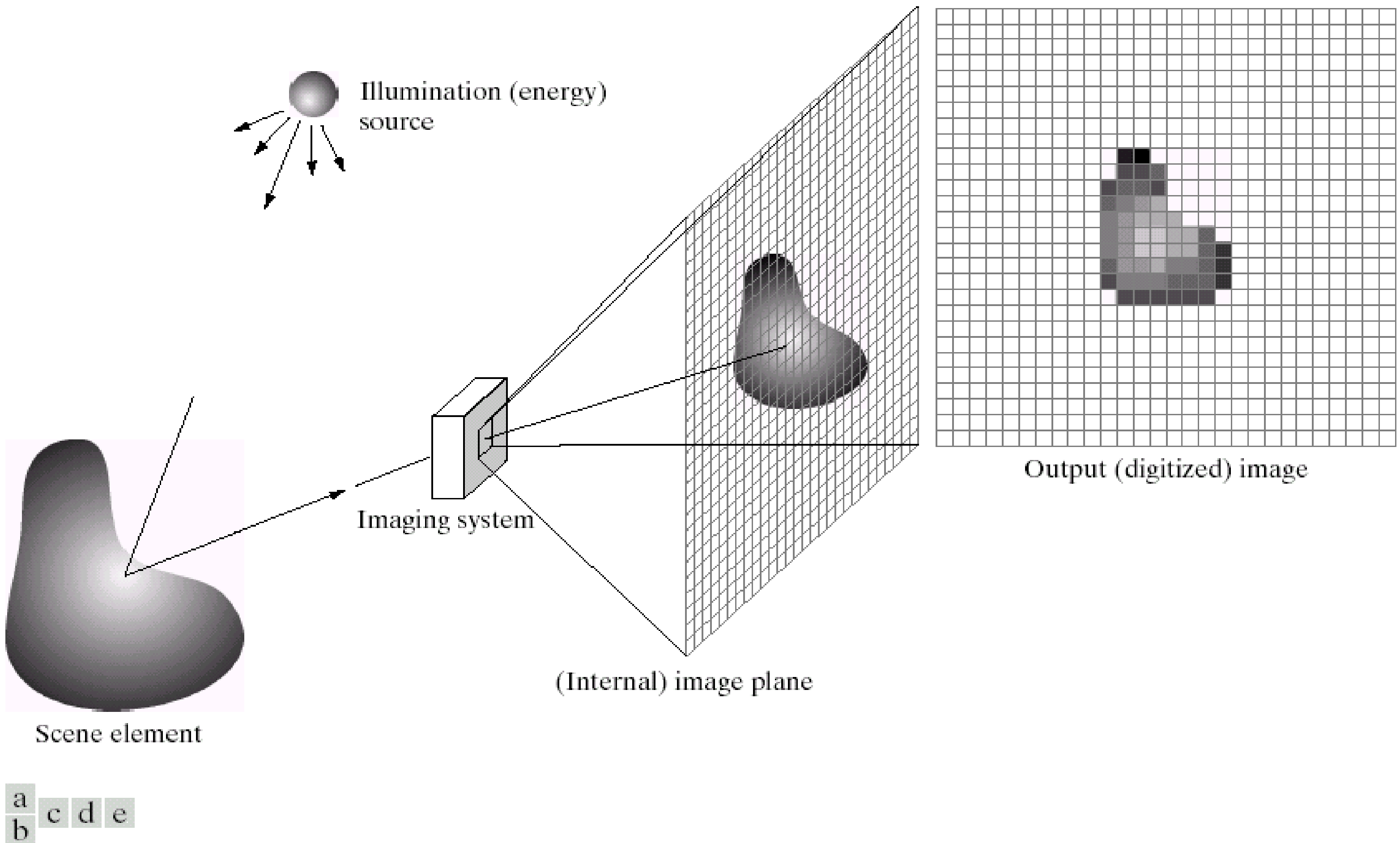


FIGURE 2.15 An example of the digital image acquisition process. (a) Energy ("illumination") source. (b) An element of a scene. (c) Imaging system. (d) Projection of the scene onto the image plane. (e) Digitized image.

Image Sampling & Quantization

- The output of most sensors is a continuous current or voltage signal whose spatial behavior and amplitude is related to the physical phenomena being sensed.

- This signal is converted into digital form by
  - Sampling – digitizing the co-ordinate values
  - Quantization – digitizing the amplitude.

Image Sampling & Quantization

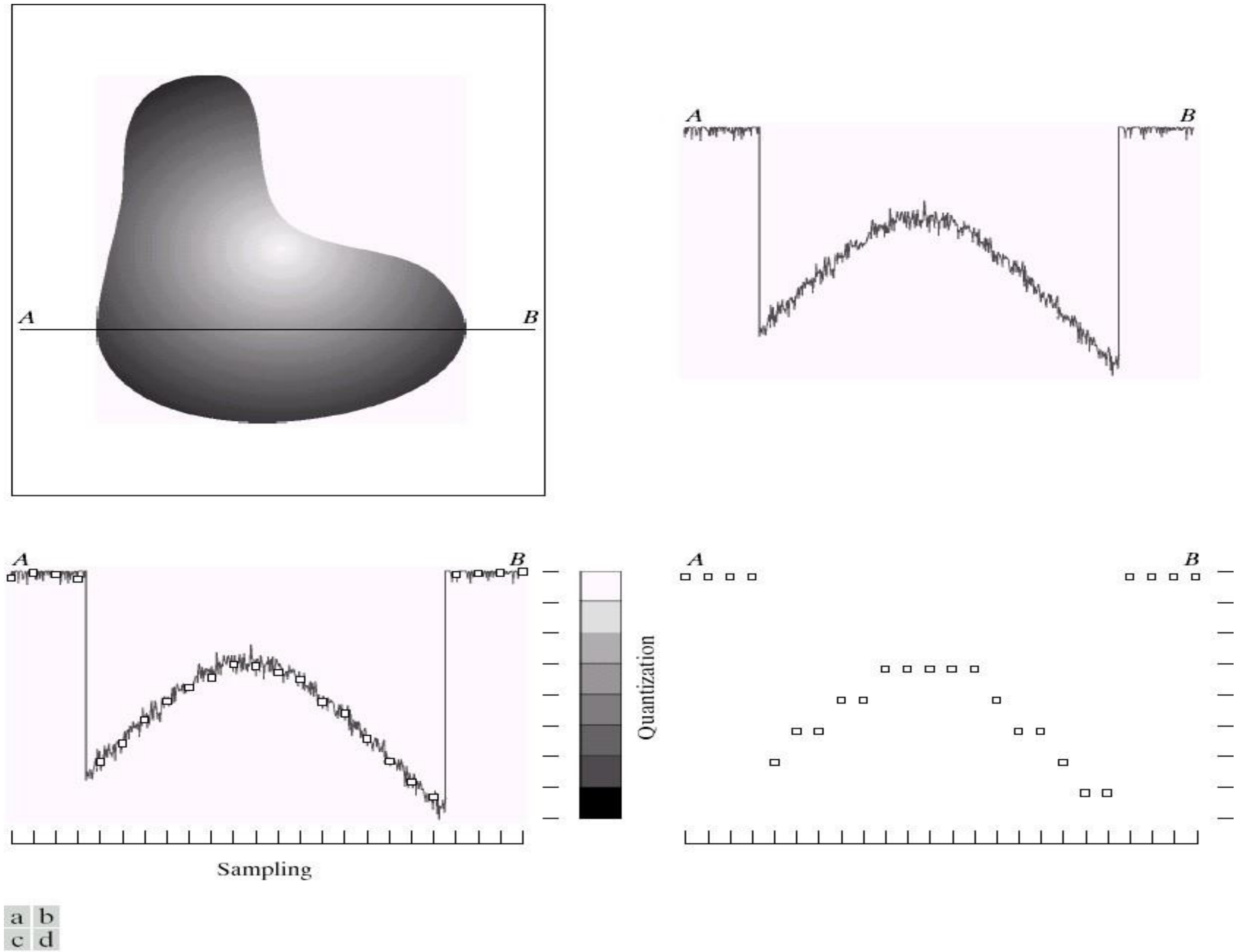


FIGURE 2.16 Generating a digital image. (a) Continuous image. (b) A scan line from A to B in the continuous image, used to illustrate the concepts of sampling and quantization. (c) Sampling and quantization. (d) Digital scan line.

Digital Image Representation

- A digital Image is an image $f(x,y)$ that has been discretized both in spatial coordinates and brightness value.
- The elements of the digital array are called *image elements*, *picture elements*, *pixels* or *pels*

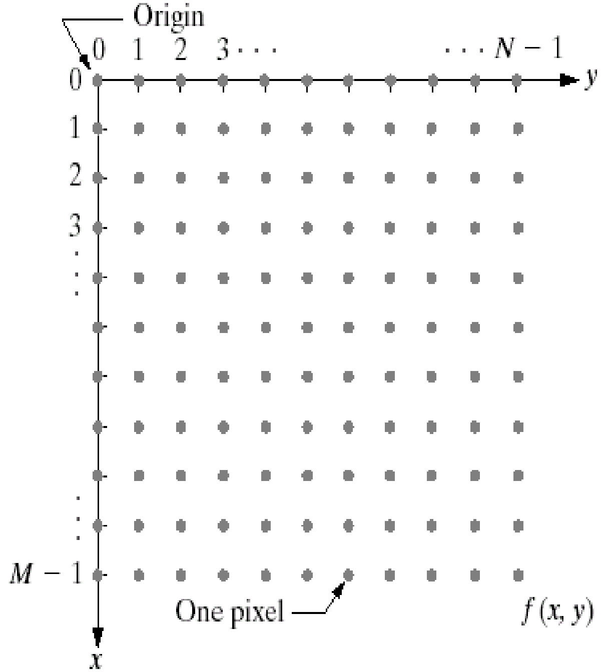


FIGURE 2.18 Coordinate convention used in this book to represent digital images.

Digital Image Representation

- The resulting sampled and quantized image is a matrix of numbers having *M* rows and *N* columns.

  NOTE: By convention in image representation the indices of the matrix are represented using integer values.
- The range of values spanned by the *gray scale* is called the *dynamic range* of the image usually expressed as [0, *L*].
- The number gray levels is typically

$$L = 2^k$$

- The number of bits required to store a digitized image is

$$b = M \times N \times k$$

Image Resolution

- Image resolution is the degree of discernible detail of an image
- It depends on

  -The number of samples in an image

  -The number of gray levels in an image

- Spatial resolution

  - A measure of the smallest discernible detail in an image

  - Stated with *line pairs per unit distance*, *dots (pixels) per unit distance*, *dots per inch (dpi)*

- Gray Level resolution

  - The smallest discernible change in intensity level

  - Stated with *8 bits*, *12 bits*, *16 bits*, *etc.*

Spatial Resolution

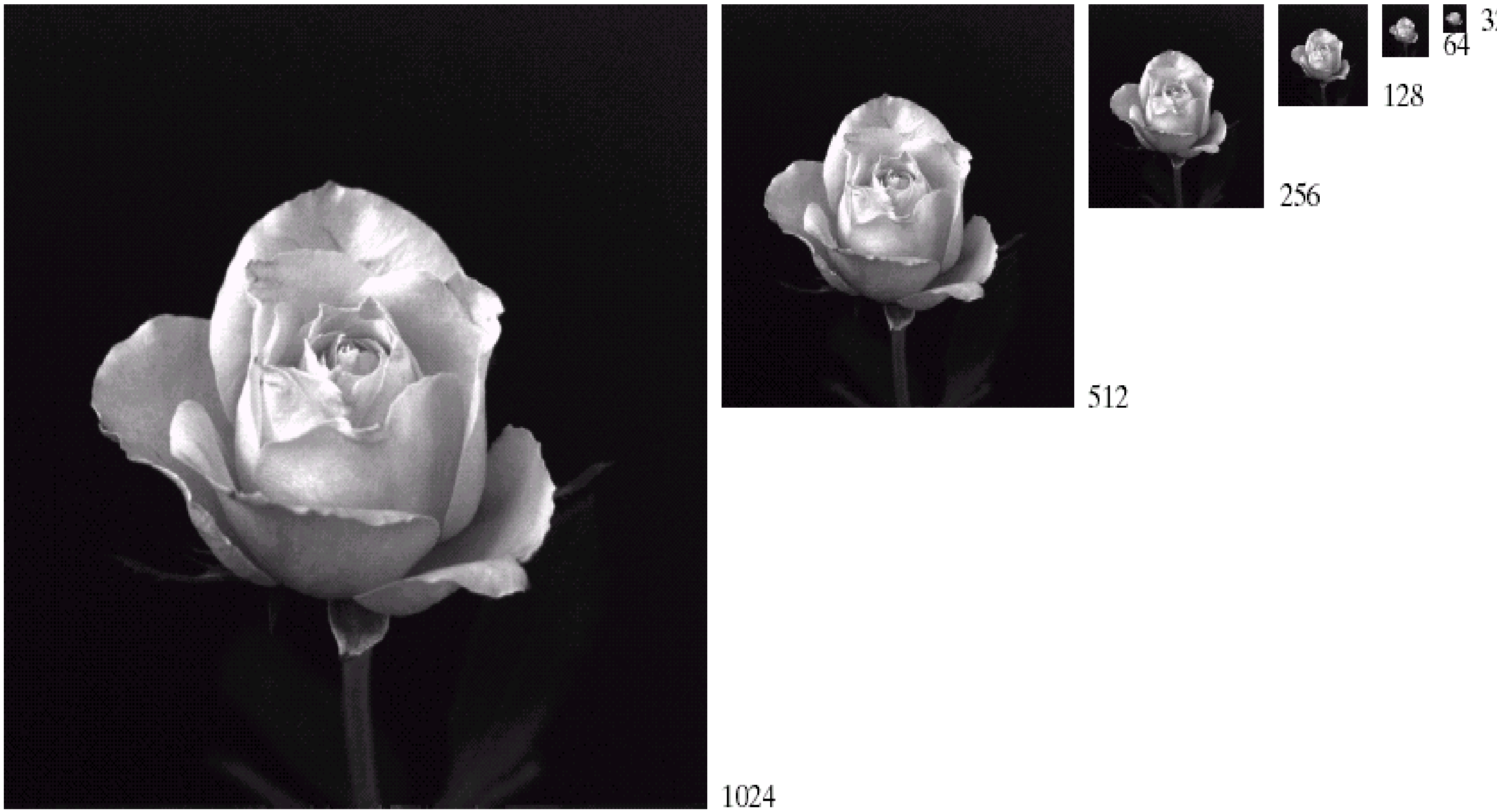
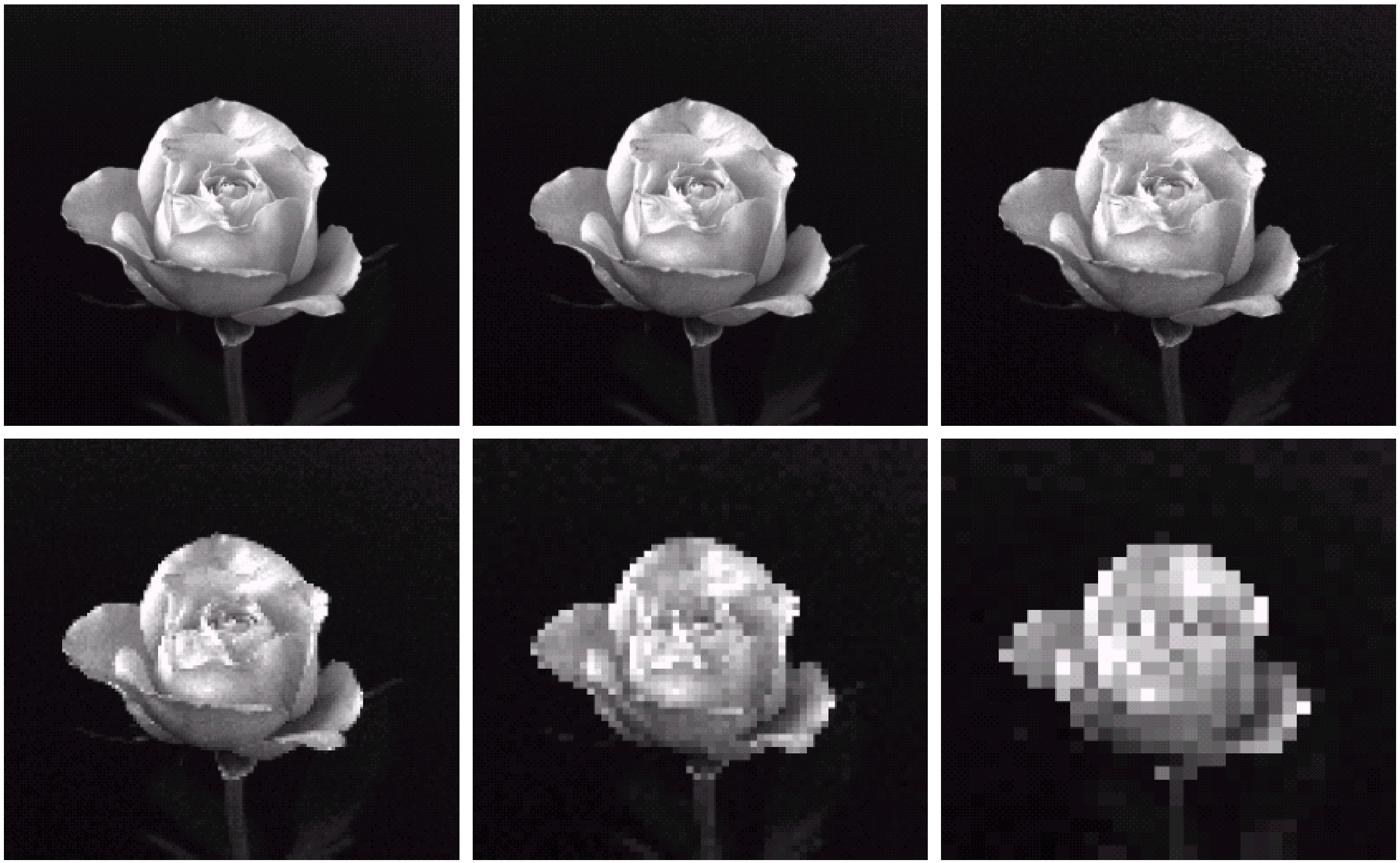


FIGURE 2.19 A 1024 × 1024, 8-bit image subsampled down to size 32 × 32 pixels. The number of allowable gray levels was kept at 256.

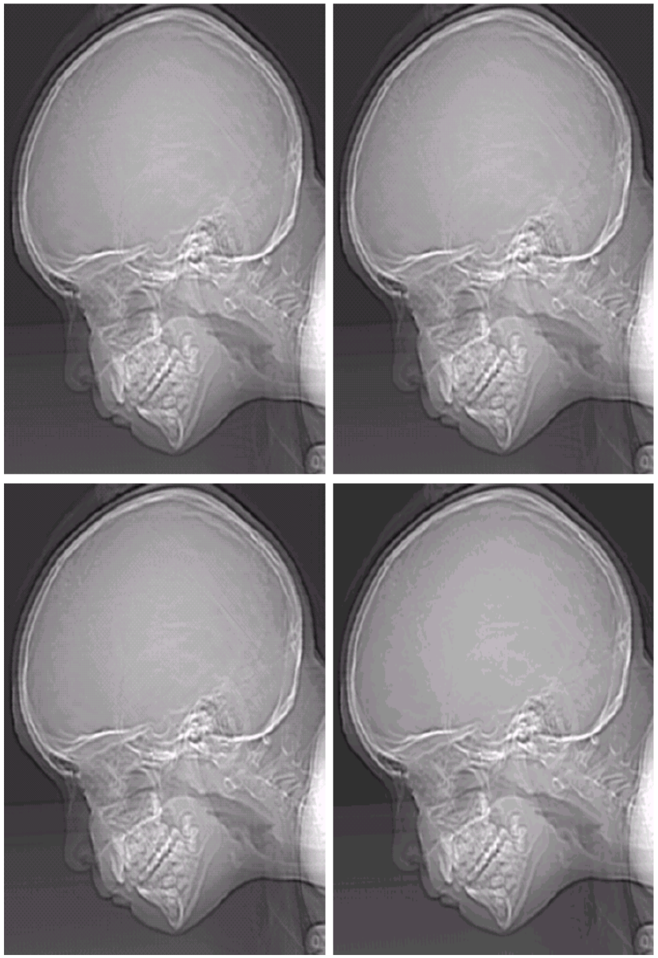
Effects Reducing Spatial Resolution



a	b	c
d	e	f

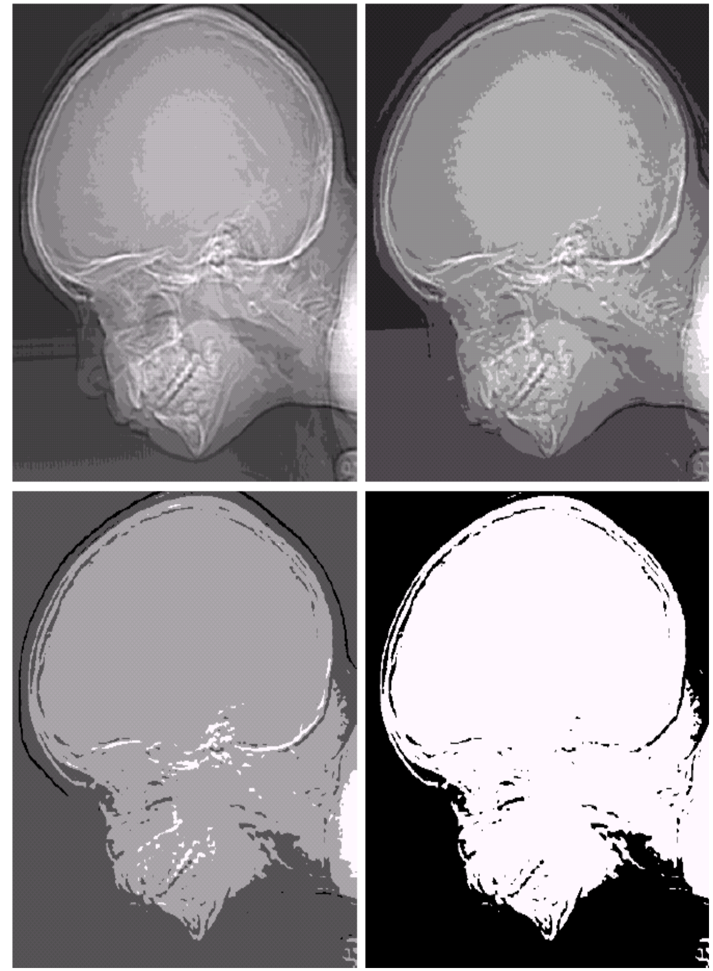
FIGURE 2.20 (a) 1024×1024, 8-bit image. (b) 512×512 image resampled into 1024×1024 pixels by row and column duplication. (c) through (f) 256×256, 128×128, 64×64, and 32×32 images resampled into 1024×1024 pixels.

Gray Level Resolution



a b
c d

FIGURE 2.21 (a) 452 × 374, 256-level image. (b)–(d) Image displayed in 128, 64, and 32 gray levels, while keeping the spatial resolution constant.



e f
g h

FIGURE 2.21 *(Continued)* (e)–(h) Image displayed in 16, 8, 4, and 2 gray levels. (Original courtesy of Dr. David R. Pickens, Department of Radiology & Radiological Sciences, Vanderbilt University Medical Center.)

Image Interpolation

- **Interpolation** — Process of using known data to estimate unknown values

  e.g., zooming, shrinking, rotating, and geometric correction

- **Interpolation** (sometimes called ***resampling***) — an imaging method to increase (or decrease) the number of pixels in a digital image.

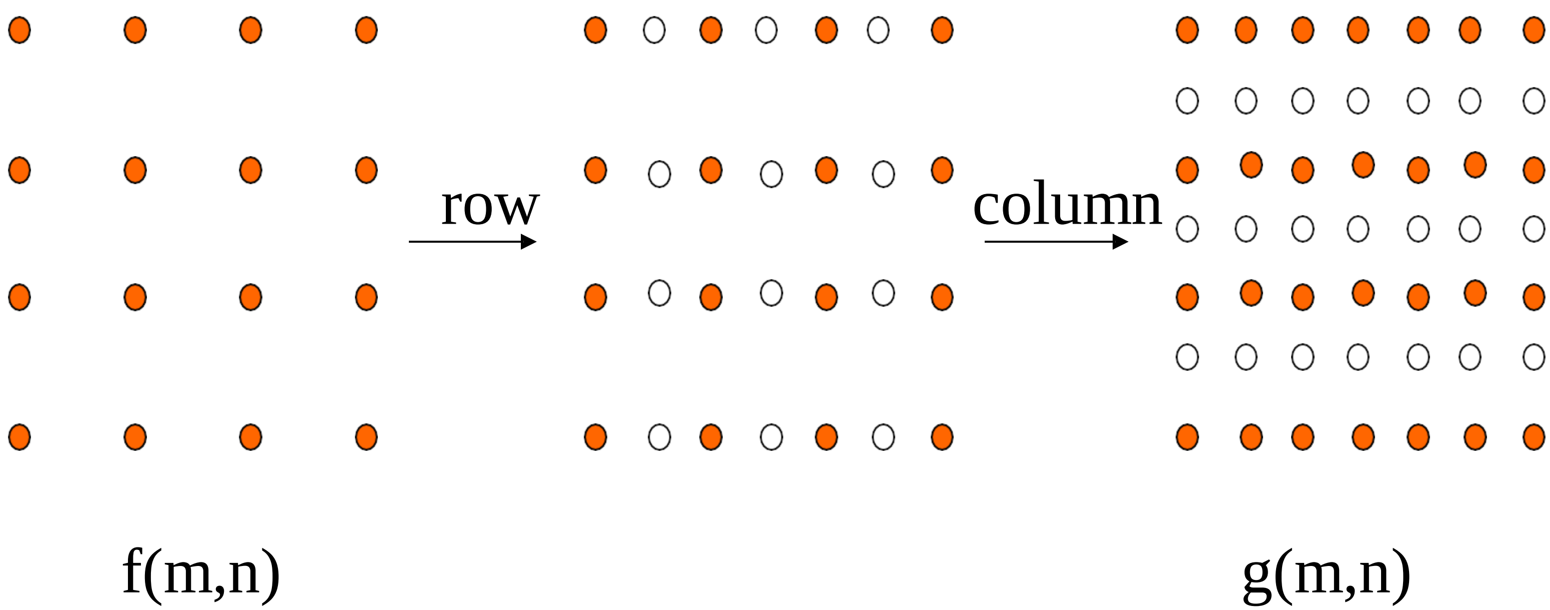
  Some digital cameras use interpolation to produce a larger image than the sensor captured or to create digital zoom

Zooming & Shrinking Images

- Zooming an image can be achieved by
  - Nearest neighbour interpolation
  - Pixel replication
  - Bilinear interpolation

  NOTE: Once the image is digitized, there is no information available of the gray level values between two pixels in the image.
- Shrinking the image can be achieved by
  - Subsampling the image
  - Row-column deletion

Examples



Examples

a	b
c	d

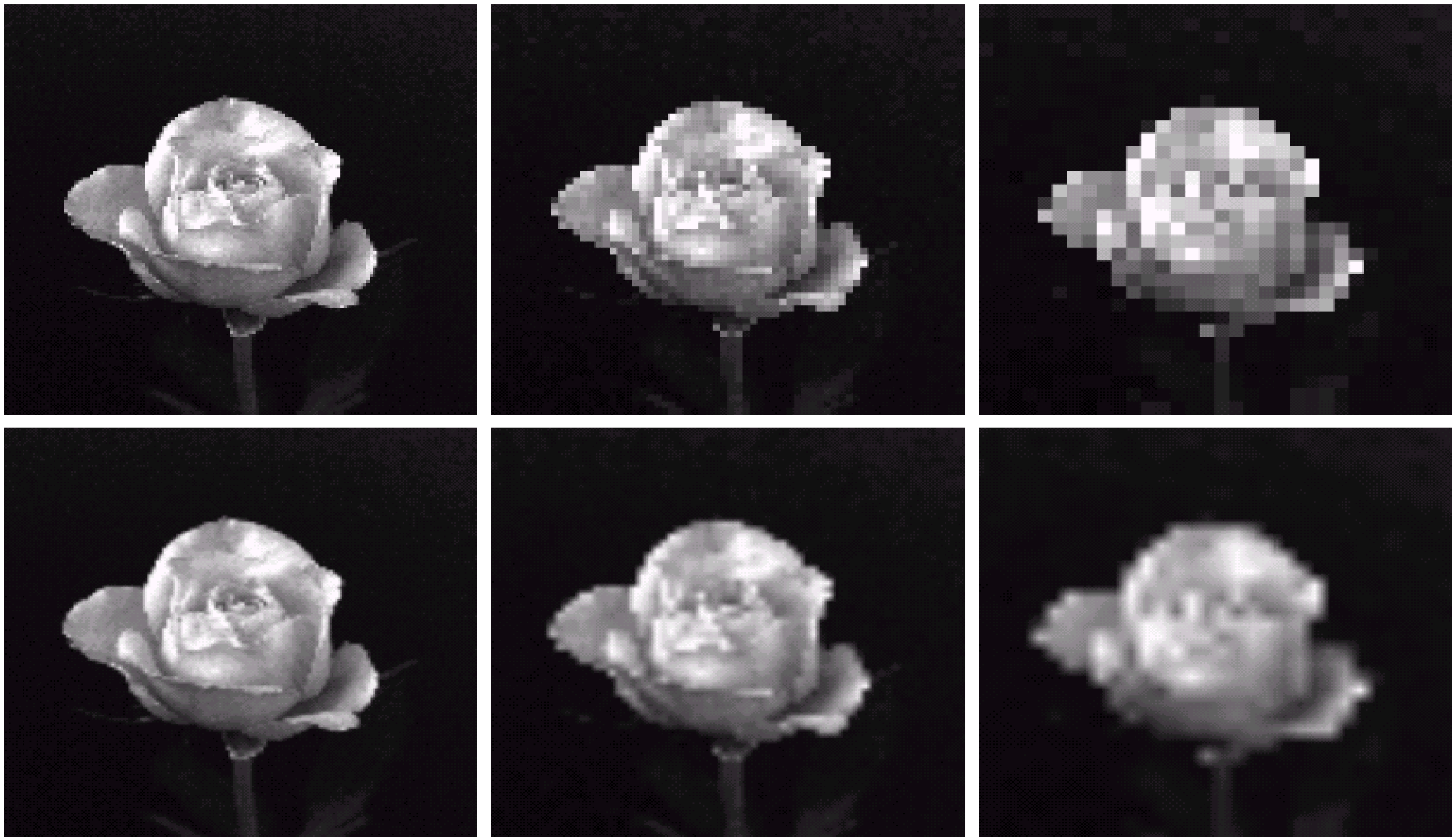
zero-order

a	a	b	b
a	a	b	b
c	c	d	d
c	c	d	d

first-order

a	(a+b)/2	b
(a+c)/2	(a+b+c+d)/4	(b+d)/2
c	(c+d)/	d

Zooming & Shrinking Images



a b c
d e f

FIGURE 2.25 Top row: images zoomed from 128 × 128, 64 × 64, and 32 × 32 pixels to 1024 × 1024 pixels, using nearest neighbor gray-level interpolation. Bottom row: same sequence, but using bilinear interpolation.

Pixel Neighborhood

- A pixel p at coordinates (x,y)
  - Has four horizontal and vertical neighbors $(x-1,y)$, $(x+1,y)$ and $(x,y-1)$, $(x,y+1)$
    - These are called the 4-neighbors of p denoted by $N_4(p)$
  - And four *diagonal* neighbors $(x-1,y-1)$, $(x+1,y+1)$ and $(x- 1,y+1)$, $(x+1,y-1)$
    - These are denoted by $N_D(p)$
- These points together define the 8-neighbors of p ($N_8(p)$).

Connectivity of Pixels

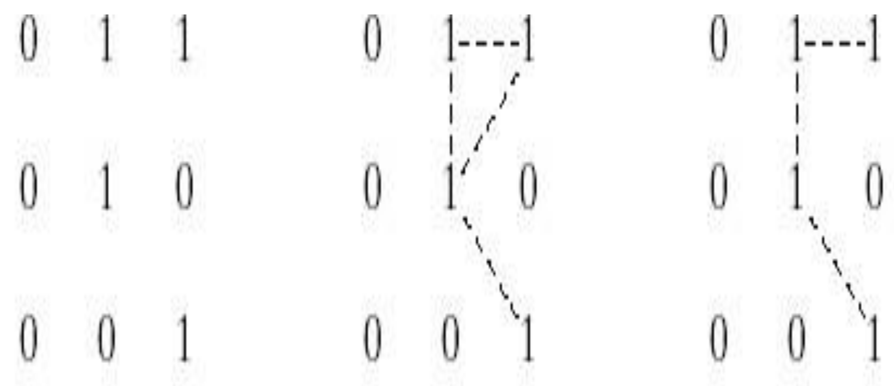
- Connectivity between pixels is a fundamental concept that simplifies the definition of regions and boundaries.
- It is useful for:
  - Establishing object boundaries
  - Defining image components/regions.

- Two pixels are connected if
  - They are neighbors and
  - Their gray levels satisfy a specified criterion of similarity.

Adjacency of Pixels

- Two pixels p and q with values from set V (set of gray level values used to define adjacency) are *4-adjacent* if q is in the set $N_4(p)$.
- Two pixels p and q with values from set V are *8-adjacent* if q is in the set $N_8(p)$.
- Two pixels p and q with values from set V are *m-adjacent* if
  - q is in the set $N_4(p)$ or
  - q is in $N_D(p)$ and the set $N_4(p) \cap N_4(q)$ has no pixels whose values are from V
- Two image subsets S_1 and S_2 are adjacent if some pixel in S_1 is adjacent to some pixel in S_2.

Adjacency of Pixels



a b c

FIGURE 2.26 (a) Arrangement of pixels; (b) pixels that are 8-adjacent (shown dashed) to the center pixel; (c) *m*-adjacency.

Digital Path

- A digital path or curve from pixel *p* with co-ordinates (x_0,y_0) to pixel *q* with co-ordinates (x_n,y_n) is a sequence of distinct pixels with co-ordinates (x_0,y_0), (x_1,y_1),…, (x_n,y_n) and pixels (x_i,y_i) and (x_{i-1},y_{i-1}) are adjacent for $1 \leq i \leq n$.
- n is the length of the path.

  NOTE: If $(x_0,y_0) = (x_n,y_n)$ then the path is a closed path.
- We can define 4-, 8-, and m-paths based on the type of adjacency used.

Examples: Adjacency and Path

$$\begin{array}{ccc} 0_{1,1} & 1_{1,2} & 1_{1,3} \\ 0_{2,1} & 1_{2,2} & 0_{2,3} \\ 0_{3,1} & 0_{3,2} & 1_{3,3} \end{array}$$

$$\begin{array}{ccc} 0 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array}$$

8-adjacent

$$\begin{array}{ccc} 0 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array}$$

m-adjacent

The 8-path from (1,3) to (3,3):

(i) (1,3), (1,2), (2,2), (3,3)

(ii) (1,3), (2,2), (3,3)

The m-path from (1,3) to (3,3):

(1,3), (1,2), (2,2), (3,3)

Connected Pixels

- If S is a subset of pixels in an image, then two pixels p and q are said to be connected in S if there exists a path between them consisting entirely of pixels in S.

- For any pixel p in S, the set of pixels that are connected to it in S is called a connected component of S

  NOTE: If it has only one connected component, then set S is called a *connected set*.

Image Regions & Boundaries

- R (a subset of pixels in an image) is a region of the image if R is a connected set.
- The boundary (border or contour) of the region R is the set of pixels in the region that have one or more neighbors that are not in R.

Distance Measures

- For pixels p, q and z, with coordinates (x,y), (s,t) and (v,w), respectively D is a *distance function* or *metric* if

1. $D(p,q) \geq 0$
2. $D(p,q) = D(q,p)$
3. $D(p,z) \leq D(p,q) + D(q,z)$

Distance Measures

- Some of the distance measures that are used in image processing are
  - *Euclidean distance*

$$D_e(p,q) = \sqrt{(x-s)^2 + (y-t)^2}$$

  - *City-block distance or D_4 distance*

$$D_4(p,q) = |x-s| + |y-t|$$

		2		
	2	1	2	
2	1	0	1	2
	2	1	2	
		2		

  - *Chessboard distance or D_8 distance*

$$D_8(p,q) = \max(|x-s|, |y-t|)$$

2	2	2	2	2
2	1	1	1	2
2	1	0	1	2
2	1	1	1	2
2	2	2	2	2

Distance Measures

- D_m distance between two points is defined as the shortest *m*-path between the points.

	p3	p4
p1	p2	
p		

Image Operations on Pixels

- Division
- Addition
- Subtraction
- Multiplication
- Linear and Nonlinear operations:
  - $H(af+bg) = aH(f) + bH(g)$