Theory of computation.

TOC

3/Jan/2018.

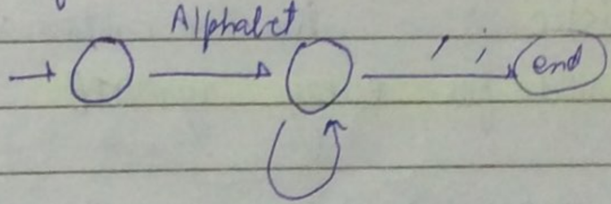
Formal Language and Autometa Theory
By Peter Linz.

One scheduled
Surprise Quize.

Theory of computation

High Level Language ⟶ Low Level Language.
conversion → check that language syntax correct.

Different Level of Automata.



Check Syntax.

drawing flow chart → Autometa.
conversion → compiler

Automata ⟶ ① finite Automata. — Least powerful.
② push down Automata (FA+Stack)
③ Linear bounded Automata (FA+finite tape)
④ Turing Machine (FA+ Infinite Tape).

Capable of Processing any kind of language

Finite Automata

defined as 5 tuple, Q, Σ, δ, q_0, F

Q → Set of states

q_0 → initial state.

F → set of final states.

δ → transition function.

Σ → input symbol.

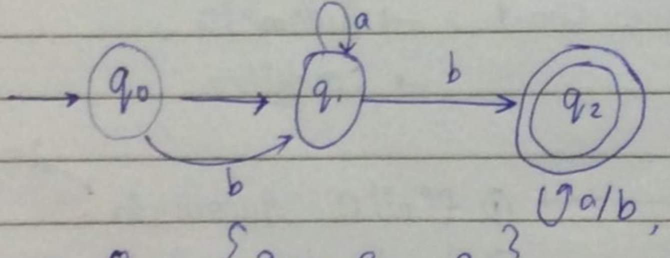
Finite Automata → deterministic (DFA) / Non deterministic (NFA)

deterministic : Are the automata which have move defined for every symbol.

Move not defined are considered as dead move.

DFA.

$$\delta : Q \times \Sigma \longrightarrow Q$$



$Q = \{q_0, q_1, q_2\}$

$q_0 = \{q_0\}$

$F = \{q_2\}$

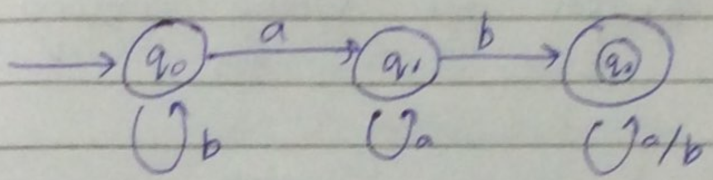
$\Sigma = \{a, b\}$

	a	b
→ q_0	q_1	q_1
q_1	q_1	q_2
q_2	q_2	q_2

$\delta(q_0, a) \rightarrow q_1$
$\delta(q_0, b) \rightarrow q_1$
$\delta(q_1, a) \rightarrow q_1$
$\delta(q_1, b) \rightarrow q_2$
$\delta(q_2, a) \rightarrow q_2$
$\delta(q_2, b) \rightarrow q_2$.

- Design a finite automata over $\{a, b\}$ that accepts string containing ab as substring.

bag ✗ aab ✓



baa ✗ babbab ✓

Acceptance by Final State.

If a string reaches final state then acceptable.

✗ baa

$(q_0, baa) \vdash (q_0, aa) \vdash (q_1, a) \vdash (q_1, a)$
Not accepted.

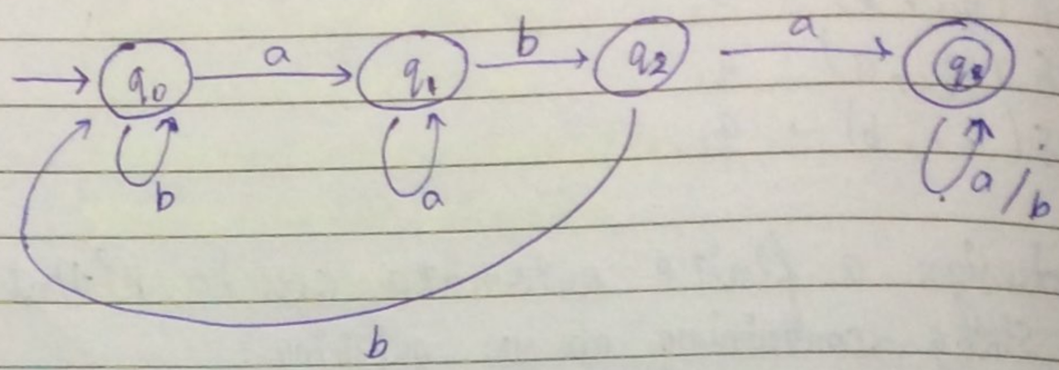
✓ ababa.

~~$(q_0, a) \vdash (q_1, b) \vdash (q_2, a) \vdash (q_2, b) \vdash (q_2, a)$~~

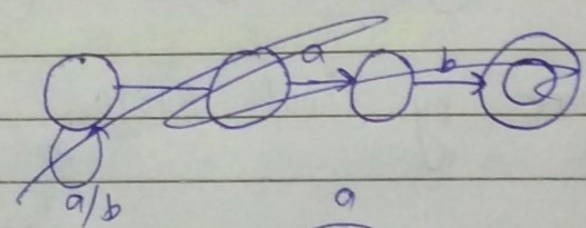
$(q_0, ababa) \vdash (q_1, baba) \vdash (q_2, aba) \vdash (q_2, ba) \vdash$
$(q_2, a) \vdash (q_2, a)$

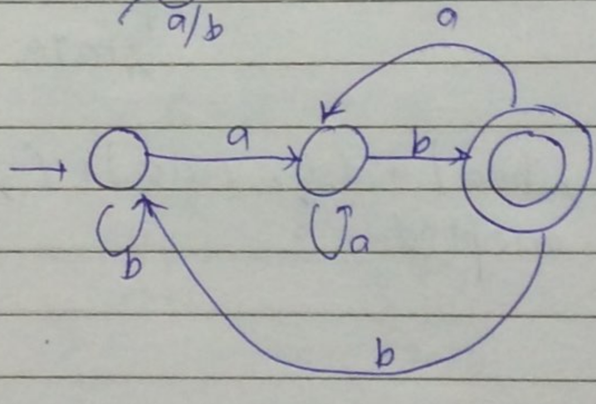
acceptable

o Design with aba as subscript

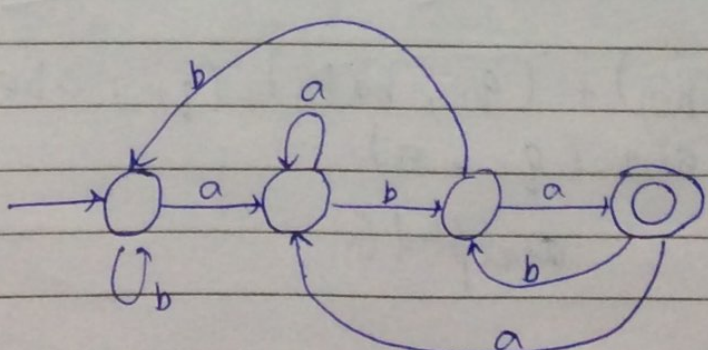


* o FA that accepts all string ending with ab





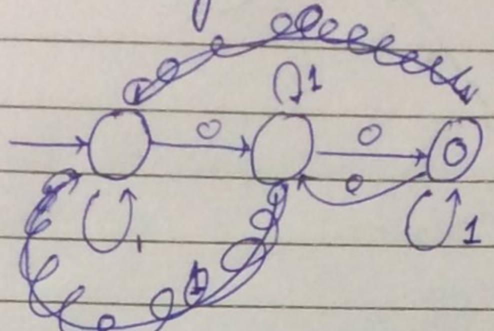
o Design end with aba

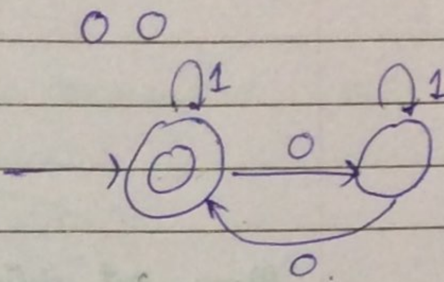


abab

○ $\Sigma = \{0, 1\}$ (no of 0 ≥ 1)

* Containing 0 in even number.





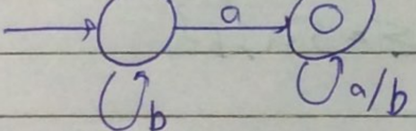
○ $\Sigma = \{a, b\}$

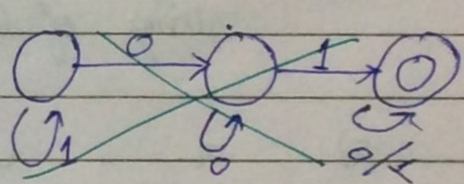
~~* Exactly one a~~ **

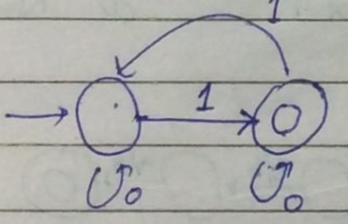
a) atleast one a.

b) 0 followed by 1. (*whenever 0 then 1)

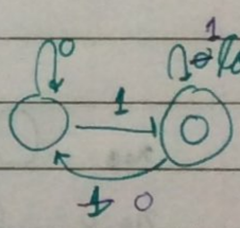
c) odd number of 1.

a) 

b) 

c) 

1 1 1

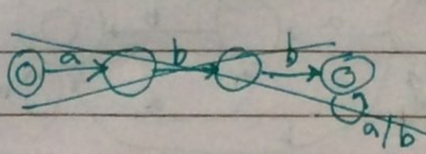


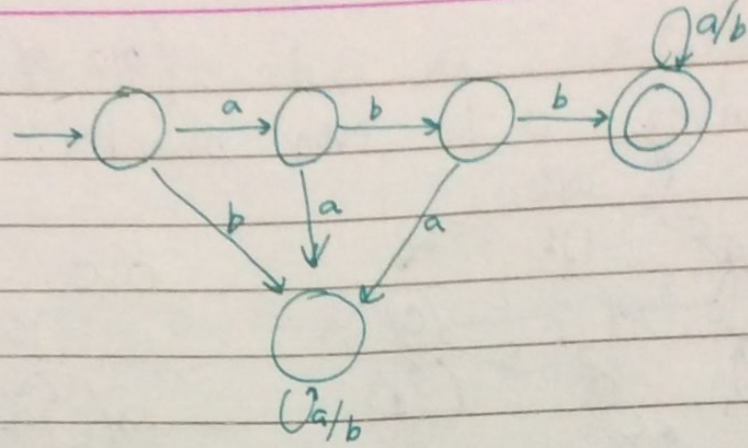
5/Jan/2018. <u>Dead State</u> : Is a state which has all outgoing moves defined to itself

i.e $\delta(q_d, a) \rightarrow q_d$

$\forall a \in \Sigma$

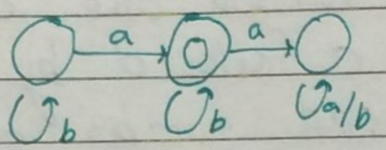
○ Design a FA that accepts string starting with abb





Moves not defined are considered as dead move.

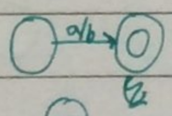
o $\Sigma = \{a, b\}$ accept all string containing exactly one a



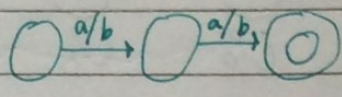
o What will be the min number of state required to accept string of length x over $\Sigma = \{a, b\}$

a) x. ~~b)~~ $x+1$ c) $x-1$ d) None of these

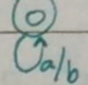
$x = 1$



$x = 2$



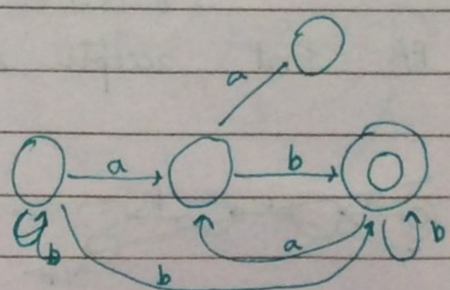
$x = 0$

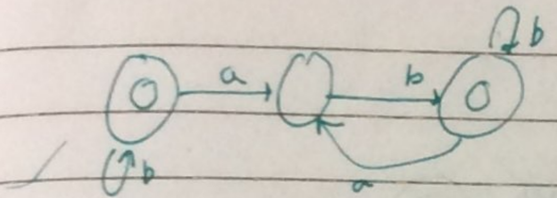


o $\Sigma = \{a, b\}$

* accept all string having a followed by b.

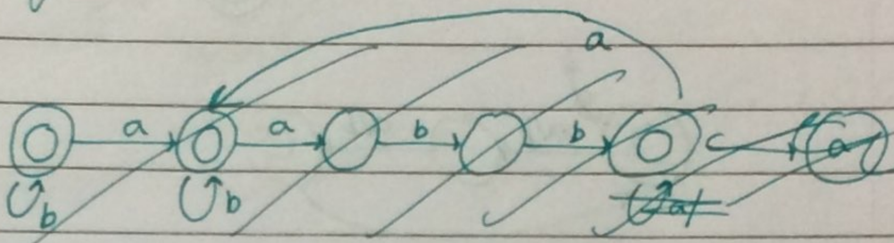
ab



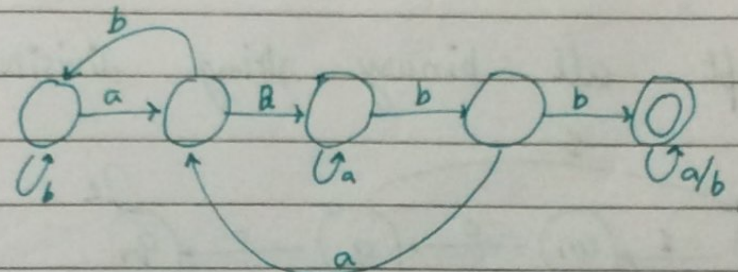


o Accept all the string containing two consecutive a followed by two consecutive b.

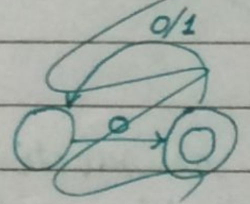
Atleast

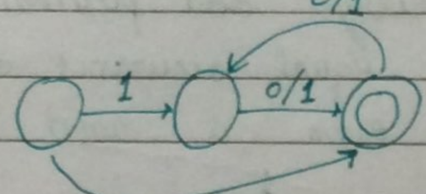


abaabb
bb

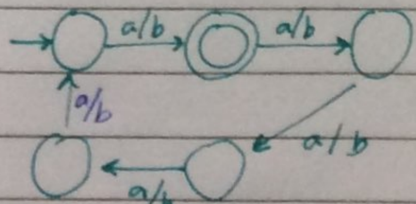


o All string ~~stra~~ start with 0 have odd length
* and strat with 1 have even length.





o Draw a FA over $\Sigma = \{a, b\}$ where $|\omega| \bmod 5 = 1$



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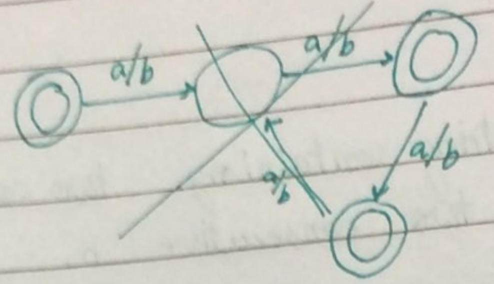
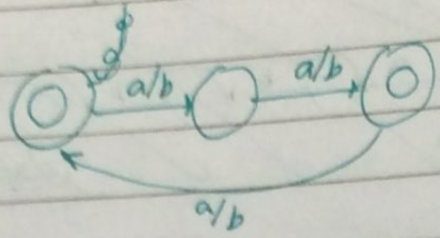
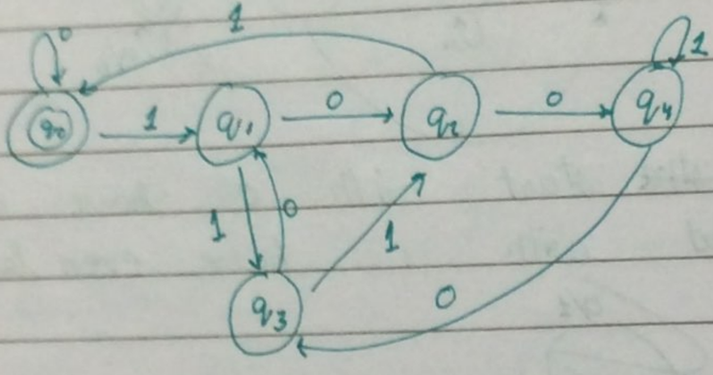
$|w| \bmod 3 \neq 1$

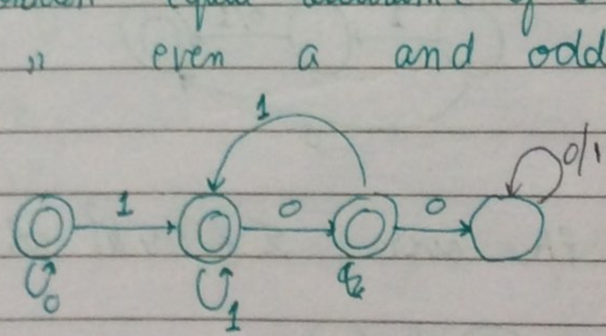




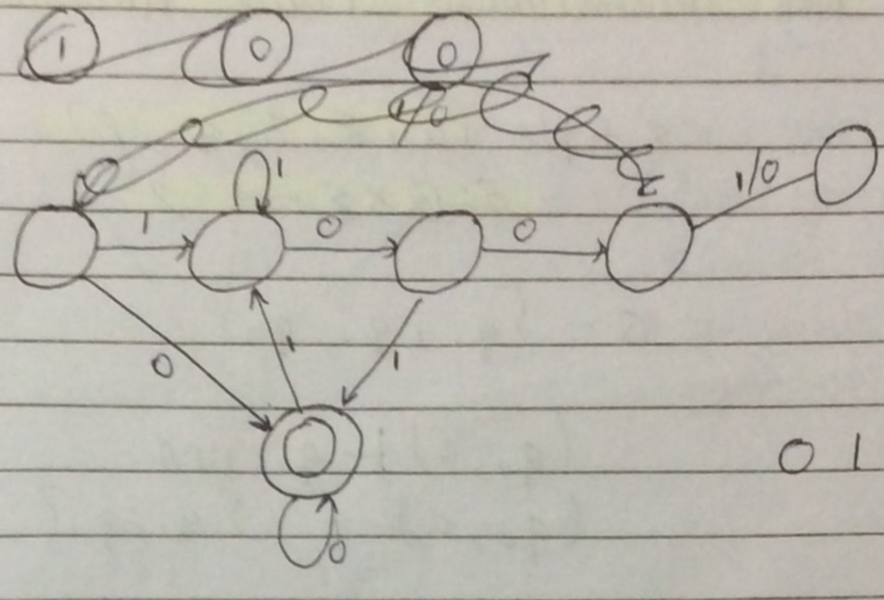
Accept all binary string divisible by 5.



Q1. All string not containing 100 as substring
Q. All string where every odd position is 1.
Q. All string contain equal occurance of 01 or 10.
Q. ,, ,, ,, even a and odd b.

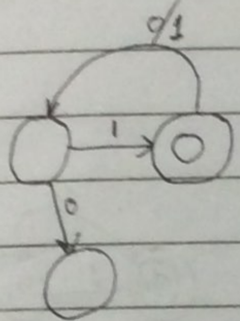


1.

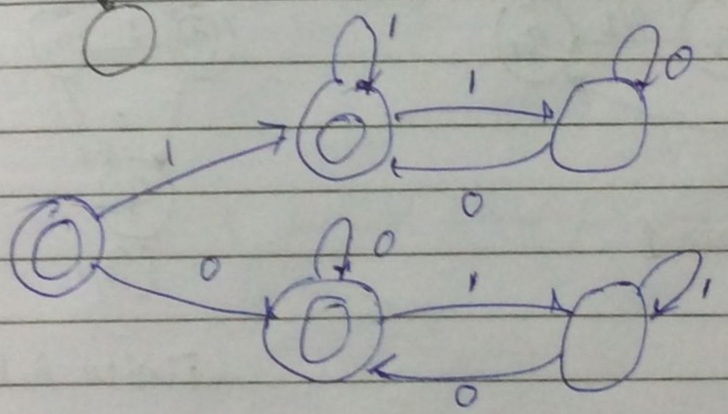


0 1 0 0 1

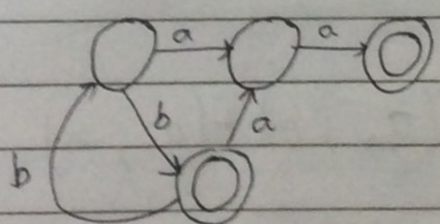
2.



3.



4.



15/Jan/2018

Non-deterministic Finite Automata

5 tuple $(Q, \Sigma, \delta, q_0, F)$

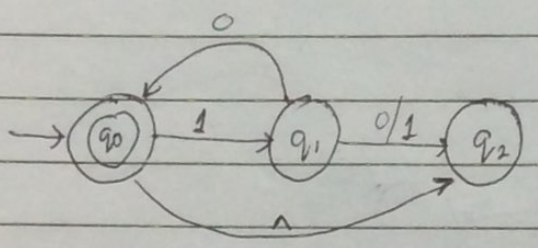
$\delta : Q \times \Sigma \longrightarrow 2^Q$

$Q = \{q_1, q_2, q_3\}$

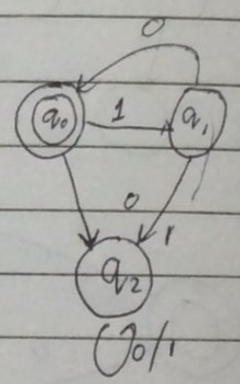
$(q_0, a) \vdash q_1$ DFA

$(q_0, a) \vdash \{q_1, q_2\}$

Acceptance of string.



Finite Automata 1.



Finite Automata 2

1010

$(q_0, 1010)$

$\vdash (q_1, 010)$

$\vdash (q_2, 10) \vdash (q_2, 0) \vdash (q_2, \wedge)$

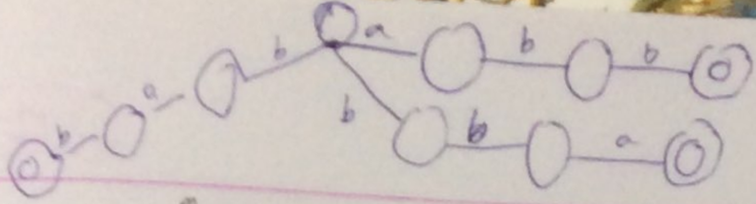
$(q_0, 1010)$

$\vdash (q_1, 010)$

$\vdash (q_0, 10)$

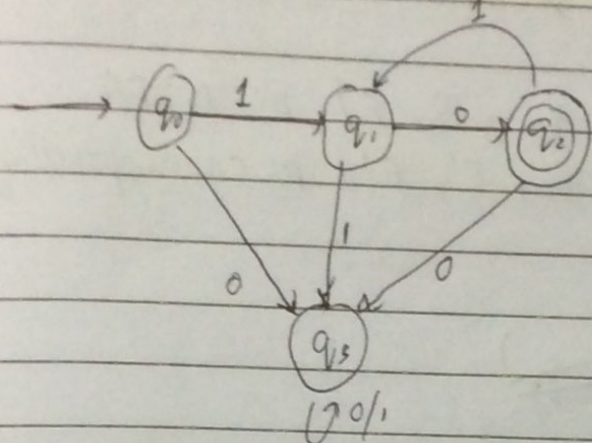
$\vdash (q_1, 0)$

$\vdash (q_0, \wedge)$



1a 1b

$(10)^n$, $n > 0$

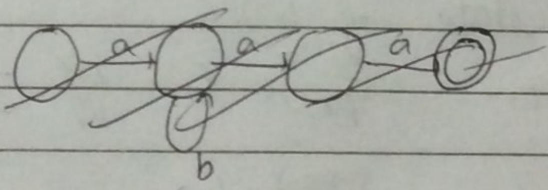


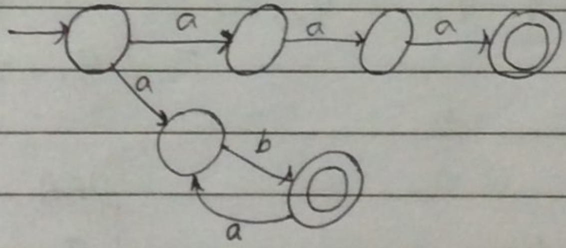
NFA

→ There exists many ~~input~~ path for a specific input

→ Use empty string transition.

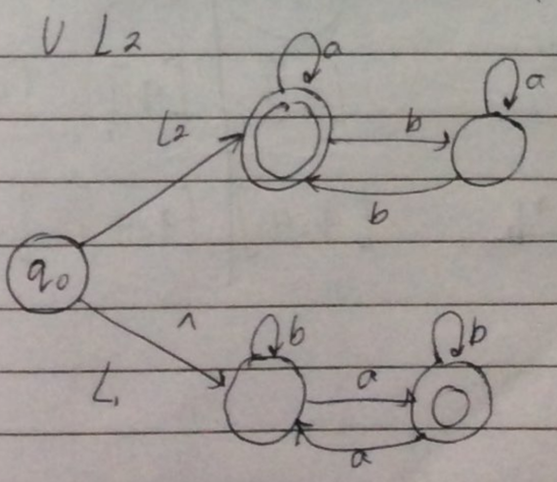
$L = (a^3 \cup (ab)^n \quad n \geq 0)$





2) L_1 : a in odd no. L_2 : b in even no.

$L_1 \cup L_2$



NDFA to DFA

$M = (Q, \Sigma, q_0, \delta, F)$ be NDFA
$M' = (Q', \Sigma, q_0', \delta', F')$ be corresponding DFA

~~$\delta'([q_1, q_2 \ldots q_i$~~

$Q' = [q_0, q_1, \ldots q_i]$ where $q_0, q_1 \ldots q_i \in Q$
$q_0' = [q_0]$
$\Sigma' = \Sigma$
F' = it is state that contains at least one final state of NDFA.
$[q_0, q_1 \ldots q_i] \in F'$ iff $\exists q_i \in F$

$\delta'([q_1, q_2 \ldots q_i], a) + (q_1, q_2, \ldots q_k)$
where $\delta(q_1, a) \cup \delta(q_2, a) \ldots \cup (q_i, a)$
$= \{q_1, q_2, \ldots q_n\}$

NDFA

	a	b
$\rightarrow q_0$	q_0	q_1
q_1	q_1	$q_0 q_1$

DFA

	a	b
$\rightarrow [q_0]$	$[q_0]$	$[q_1]$
$[q_1]$	$[q_1]$	$[q_0 q_1]$
$[q_0 q_1]$	$[q_0 q_1]$	$[q_1, q_0]$

~~$a^* \cup (ab)^* b$~~

NDFA

	a	b
$\to q_0$	$q_0 q_1$	q_2
q_1	q_0	q_1
q_2		$q_0 q_1$

NDFA

DFA

	a	b
$\to$ [q_0]	[$q_0 q_1$]	[q_2]
~~[q_1]~~	~~[q_0]~~	~~[q_1]~~
[q_2]		[$q_0 q_1$]
[$q_0 q_1$]	[$q_0 q_1$]	[$q_1 q_2$]
[$q_1 q_2$]	[q_0]	[$q_0 q_1$]

NDFA

	0	1	2
$\to q_0$	$q_1 q_4$	q_4	$q_2 q_3$
q_1		q_4	
q_2	q_2		q_3
q_3		$q_4 q_1$	
q_4	q_1	q_2	$q_1 q_2$

DFA

	0	1	2
$\to$ [q_0]	[q_1, q_4]	[q_4]	[q_2, q_3]
[q_4]	[q_1]	[q_2]	[$q_1 q_3$]
[q_1]		[q_4]	
[q_2]	[q_2]		[q_3]
[q_3]		[$q_4 q_1$]	
[$q_1 q_4$]	[q_1]	[$q_2 q_4$]	[$q_1 q_3$]
[q_2, q_4]	[$q_1 q_2$]	[q_2]	[$q_1 q_2 q_3$]
[$q_2 q_3$]	[q_2]	[$q_1 q_4$]	[q_3]
[$q_1 q_2 q_3$]	[q_2]	[$q_1 q_4$]	[q_3]

NDFA

	a	b
$\to q_0$	$q_1 q_2$	q_2
q_1	q_2	$q_1 q_3$
q_2	q_0	$q_1 q_0$
q_3	q_2	$q_1 q_2$

17/Jun/2018

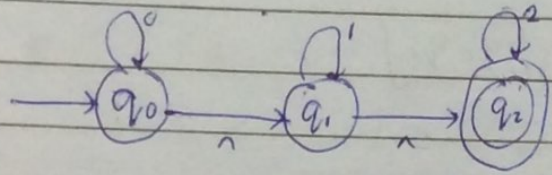
NFA with ^ transition

NFA to DFA with empty string transition.

1. Λ - moves
2. NFA to DFA
3. Minimize it
4. Implement it

$q_1 \xrightarrow{\wedge} q_2$

1. Duplicate all moves from q_2 to q_1 itself
2. If q_2 is FS then make q_1 also as final state.



Final

	0	1	2	^
→ q_0	q_0	q_1	q_2	-
q_1		q_1	q_2	
q_2			q_2	

Step 1

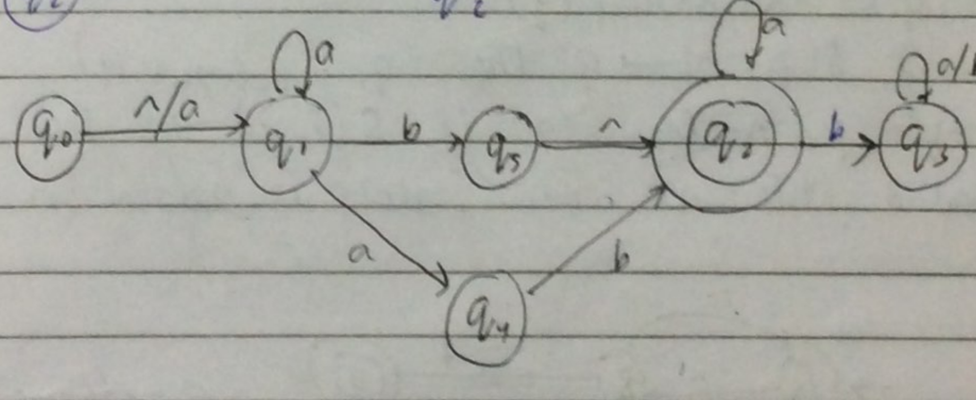
	0	1	2	^
q_0	q_0			q_1
q_1		q_1		q_2
q_2			q_2	

q

	0	1	2	^
→q_0	q_0	q_1		q_2
q_1		q_1	q_2	q_2
q_2			q_2	

	0	1	2	^
→q_0	q_0	q_1		q_2
q_1		q_1	q_2	
q_2			q_2	

	0	1	2	^
→q_0	q_0	q_1	q_2	
q_1		q_1	q_2	
q_2			q_2	



	a	b	^
q_0	q_1		q_1
q_1	$q_1 q_4$	q_5	
q_2	q_2	q_3	
q_3	q_3	q_3	
q_4	q_2		
q_5			q_2

	a	b	$\wedge$
$\rightarrow q_0$	q_1, q_4	q_5	
q_1	q_1, q_4	q_5	
q_2	q_2	q_3	
Final q_3	q_3	q_3	
q_4		q_2	
q_5	q_2	q_3	

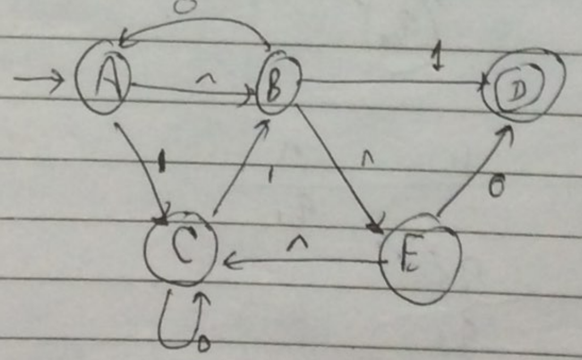
ϵ - closure (q)

- Is the set of state possible to reach from q with $\wedge$- moves

ϵ - closure (q)

1. $q \in$ c-closure (q)
2. if $\delta(q, \wedge) \rightarrow q_i$ then $q_i \in$ ϵclosure (q)
3. $\forall$ $p \in \wedge$-closure (q) if $\delta(p, \wedge) \rightarrow r$ then $r \in \wedge$-closure(q)

Repeat 3 till no new state is added in $\wedge$-closure(q)



ϵ - closure(A)
$= \{A, B, E, C\}$

δ(q, ω)

ϵ-closure(q) $\xrightarrow{\delta(\epsilon\text{-closure}(q),\ a)}$ P

↓

ϵ - closure(P) = R

↓ δ(R, a)

S

↓

ϵ closure(S)

~~δ(B, 101)~~

~~= (D, 01)~~

~~= {D}~~

{BEC}, 1

{DBEC}, 0

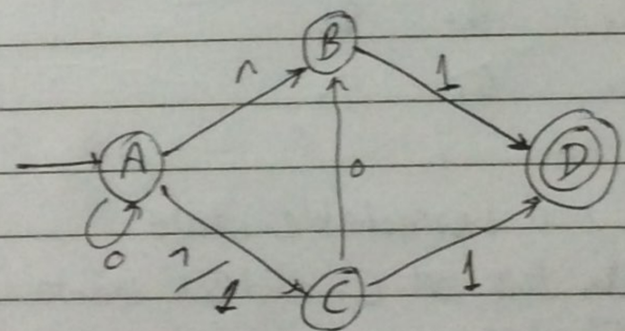
{ADCBE}, 1

{CBD1}

δ(A, 01)

({A, B, E, C}, 01)

→ {ADC} → ({ABECD}, 1) → {CDB} → {CDEB}



Task 1 δ(A, 01)

Task 2 Remove Λ Move

Task 3 Convert NFA to DFA.

a b

22/Jan/2018

Minimization of finite Automata.

✓ State Equivalence

q_1 and q_2 are equivalent if

$(q_1, x) \rightarrow q_f$

than

$(q_2, x) \rightarrow q_f$

$\forall\ x \in \Sigma$

$(q_1, x) \vdash q_5$ $\quad q_5 \notin F$

$(q_2, x) \vdash q_k$ $\quad q_k \in K$

	a	b	c
$\rightarrow q_1$	q_1	q_2	q_4
q_2	q_1	q_3	q_4
q_3	q_4	q_2	q_4
q_4	q_1	q_1	q_4

1. Remove all dead and unreachable state
2. Divide set Q into two set Q_1 and Q_2 containing set of NF and Final State.
3. Check equivalence for every state in Q_1 and Q_2 if the state in set are not equivalent split the set
   Repeat above till all states in set are equivalent

	$q = 0$	$q = 1$
$\rightarrow q_0$	q_1	q_5
q_1	q_6	q_2
q_2	q_0	q_2
~~q_3~~	~~q_2~~	~~q_6~~
q_4	q_7	q_5
q_5	q_2	q_6
q_6	q_6	q_4
q_7	q_6	q_2
q_8	q_8	q_8

~~$q_0 q_1 q_5 q_6 q_2 q_4 q_7$~~

~~$Q_1 (q_0, q$~~

$P_1 (q_0, q_1, q_4, q_5, q_6, q_7)$ $P_2 (q_2)$

q_0 a

$P_1 (q_0, q_1, q_4, q_5, q_6, q_7, q_8)$ $P_2 (q_2)$

$P_3 (q_0, q_4, q_6)$ $P_4 (q_1, q_5, q_7)$

(q_0, q_4) P_5 (q_6) P_6 (q_1, q_7) P_7 q_5 P_8

q_0 a $\to P_1$
q_0 b $\to P_1$
q_1 a $\to P_1$
q_1 b $\to P_2$
q_4 a $\to P_1$
q_4 b $\to P_1$
q_5 a $\to P_2$
q_5 b $\to P_3$
q_6 a $\to P_3$
q_6 b $\to P_3$
q_7 a $\to P_3$
q_7 b $\to P_2$

q_0, a $\to P_4$
q_0, b $\to P_4$
q_4, a $\to P_4$
q_4, b $\to P_4$
q_6, a $\to P_3$
q_6, b $\to P_3$.

q_1, a $\to P_6$
q_1, b $\to P_2$
q_5, a $\to P_2$
q_5, b $\to P_6$
q_7, a $\to P_6$
q_7, b $\to P_2$.

q_0, a $\to P_7$
q_0, b $\to P_8$
q_4, a $\to P_7$
q_4, b $\to P_8$

q_1, a P_6
q_1, b P_2
q_7, a P_6
q_7, b P_2.

	a	b
$\to P_5$	P_7	P_8
P_6	P_6	P_5
P_7	P_6	P_2
P_8	P_2	P_6
P_2	P_5	P_2

	a	b	
$\rightarrow q_0$	q_1	q_2	
q_1	q_0	q_2	
q_2	q_3	q_1	
q_3	q_3	q_0	
q_4	q_3	q_5	x
q_6	q_5	q_6	x
q_7	q_6	q_3	x
q_5	q_6	q_4	x

$P_1(q_0\ q_1\ q_2\ \cancel{q_3})$ $P_2(q_3)$

q_2

$P_3(q_0, q_1)$ $P_4(q_2)$

$P_5(q_0)$ $P_6(q_1)$

q_0	a	P_1
q_0	b	P_1
q_1	a	P_1
q_1	b	P_1
q_2	a	P_2
q_2	b	P_1
$\cancel{q_3}$	$\cancel{a}$	
$\cancel{q_3}$	$\cancel{b}$	

q_0 a

q_0 b

q_1 a

q_1 b

^

	a	b
$\rightarrow q_0$	q_0	$q_0 q_2$
~~q_1~~	~~q_3~~	~~$q_2 q_3$~~
q_2	q_4	-
~~q_3~~	~~q_4~~	~~-~~
q_4	-	-

$q_0 q_2 q_4$

	a	b
$[q_0]$	$[q_0]$	$[q_0 q_2]$
$[q_0 q_2]$	$[q_0 q_4]$	$[q_0 q_2]$
$[q_0 q_4]$	$[q_0]$	$[q_0 q_2]$

29/Jan/2018 **Regular Expression**

→ It is an algebra notation for the language accepted by FA

→ it includes three symbol +, ·, *

+ union
· concatenation
* kleene closure

$a^* =$ 0 or more occurance of a

$L = a^*$

$= \{ \wedge, a, aaa \_\_\_\_ \}$

$(a+b) =$ Either a or b

$a \cdot b$

$\Sigma = \{a, b\}$

(1) L = all string containing abb as substring abb

~~abb~~

$(a+b)^* = \{ \wedge, a, b, ab, aa, ba, bb \ldots$

Ans $(a+b)^* abb (a+b)^*$

(2) All string ending with abb

$(a+b)^* abb$

(3) Even no of a.

$(b^* a b^* a b^*)^*$

$b^* (a b^* a)^* b^*$

(4) All string containing at least 3 occurance of a.

$\cancel{b^* a b^* a b^* a b^* a^*}$

$(b^* a b^* a b^* a)(a+b)^*$

(5) Exactly 3

$b^* a b^* a b^* a b^*$

→ L = a.b+c

$a.(b+c)$ ✗ $(ab+c)$ ✓

(ab, ac) (ab, c)

~~ab~~

~~abc~~

~~c~~

Precedence → Star $\quad * \quad *$
→ Concatenation $\quad \cdot$
→ Union $\quad + \quad +$

→ $L = a^{2m} b^{2n+1} ab \qquad m>0 \quad n>0$

*

$R_1: (aa)^*(bb)^* b\ ab$ ✗
$R_2: aa(aa)^* b(bb)^* bab$ ✓
$(aa)^+ (bb)^+ bab$

$aa^* = a^+$ = +ve kleene closure
$RR^* = R^+$

⑥ At least one pair of consecutive zero

$(0+1)^* 00 (0+1)^*$

⑦ No pair of consecutive 0.

*

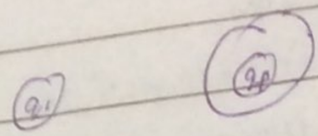
~~$1^*(010)^*1^*$~~
$(1+01)^*(0+\wedge)$

Kleene Theorem
→ For every R.E. there exists a FA to accept it.

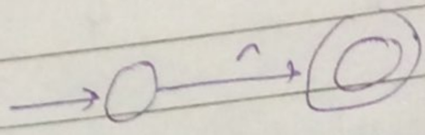
Lang
↓
RE
↓
ϵ -NFA
↓
NFA
↓
DFA
↓
minimize

Proof.

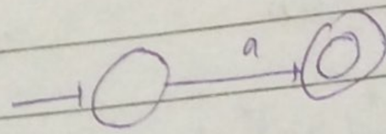
$R = \phi \rightarrow$



$R = \wedge$



$R = a$

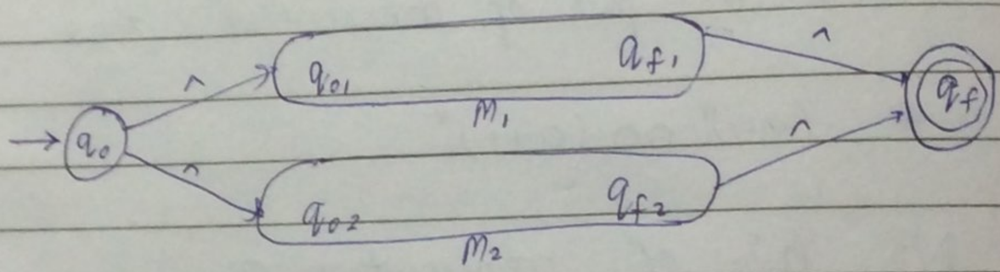


There exist FA for RE R_1 and R_2

R_1 : it is $M_1(Q_1, q_{01}, \Sigma_1, \delta_1, q_{f_1})$

R_2 : " " $M_2(Q_2, q_{02}, \Sigma_2, \delta_2, q_f)$

$R = R_1 + R_2$



$M(Q, \Sigma, \delta, q_0, q_f)$

$Q = Q_1 \cup Q_2 \cup \{q_0, q_f\}$

$\Sigma = \Sigma_1 \cup \Sigma_2$

$\delta = \delta_1 \cup \delta_2 \cup \quad q_0, \wedge \rightarrow \{q_{01}, q_{02}\}$

$\cup \quad q_{f_1}, \wedge \rightarrow q_f$

$\cup \quad q_{f_2} \wedge \rightarrow q_f$

30/Jan/2018.

Finite Automata to Regular Expression.

Identies

1. $\Phi + R = R$
2. $\Phi \cdot R = \Phi$
3. $\wedge \cdot R = R$
4. $\wedge^* = \wedge$
5. $R^* R = RR^* = R^+$
6. $R^* R^* = R^*$
7. $R + R = R$
8. $\wedge + RR^* = R^*$
9. $R(PQ)^* P = PQPQ \_\_\_ PQP = P(QP)^*$
10. $(P+Q)^* = (P^* Q^*)^* = (P^* + Q^*)^*$
11. $(P+Q)R = PR + QR$
12. $\Phi^* = \epsilon \quad \Phi^* = \epsilon$

8 Feb
17 Feb
Quiz

Arden Theorem

$R = Q + RP$ and $P \neq \wedge$

then $R = QP^*$

$R = QP^*$

$R = Q + QP^* P = Q(\wedge + P^* P) = QP^*$

$R = Q + RP = Q + (Q+RP)P = Q + QP + RP^2$

$= Q + QP + (Q+RP)P^2 = Q + QP + QP^2 + RP^3$

$R = \underbrace{(Q + QP + QP^2 + \_\_\_\_ QP^i)}_{A} + \underbrace{RP^{i+1}}_{B}$

Let $\omega \in R$ and $|\omega| = i$

$\therefore R \in (Q + QP + \_\_\_ QP^i)$

or

$R \in RP^{i+1}$

$P \neq \wedge$ So $|RP^{i+1}| \geq i+1$

therefore $\omega \notin RP^{i+1}$

$\omega \in Q + QP + \_\_ QP^i$

So

$$R = Q + QP + \_\_ QP^i$$

$$= Q(\wedge + P + \_\_ P^i)$$

$$R = QP^*$$

Change Automata to Regular Expression.

Write equation for each state $q_i \in Q$ describing its reachability from other states.

$$q_1 = q_{11} q_1 + q_{12} q_2 + q_{13} q_3 + \_\_ q_{1n} q_n$$

$$q_2 = q_{21} q_1 + q_{22} q_2 + q_{23} q_3 + \_\_ + q_{2n} q_n$$

$$q_n = q_{n1} q_1 + q_{n2} q_2 + \_\_ + q_{nn} q_n$$

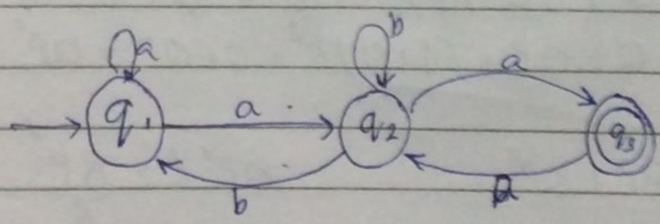
Apply identifiers and Arden Theorem and generate

$q_f = q_0 q_{ij}$

Remove q_0.

This is the final automata.

Q.



$$q_1 = q_1 a + b q_2 \quad — ①$$

$$q_2 = a q_1 + b q_2 + a q_3 — ②$$

$$q_3 = a q_2 \quad — ③$$

$q_2 = q_1 a + q_2 b + q_2 aa$

$\underset{R}{q_2} = \underset{Q}{q_1 a} + \underset{R}{q_2} \underset{P}{(b+aa)}$

$q_2 = q_1 a(b+aa)^* \text{ — } ④$

① and ④

$q_1 = q_1 a + q_1 a(b+aa)^* b$

$\underset{R}{q_1} = \underset{R}{q_1} \underset{P}{(a + a(b+aa)^* b)} + \underset{Q}{\wedge} \qquad R = RP + Q$

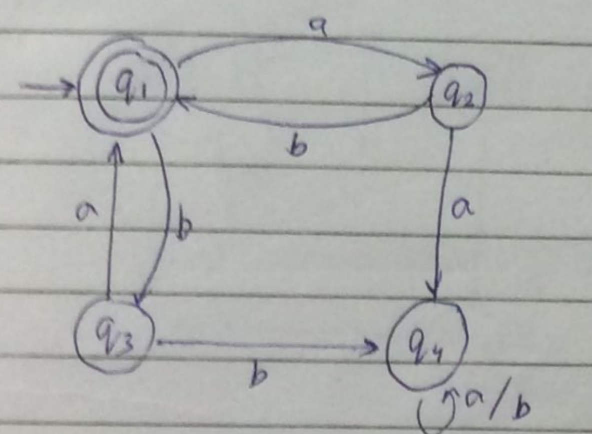
$q_1 = \wedge(a + a(b+aa)^* b)^*$ $\qquad R = QP^*$

$q_1 = (a + a(b+aa)^* b)^* \text{ — } ⑤$

eq ③ $q_3 = q_1 a(b+aa)^* a$

⑤ $q_3 = (a + a(b+aa)^* b)^* a(b+aa)^* a$

Q



$q_1 = bq_2 + aq_3 \qquad = baq_1 + abq_1 = q_1$

$q_2 = aq_1 \qquad = abq_2 + aaq_3 \qquad = q_1(ab+ba) + \wedge$

$q_3 = bq_1 \qquad = bbq_2 + baq_3 \qquad = (ab+ba)^*$

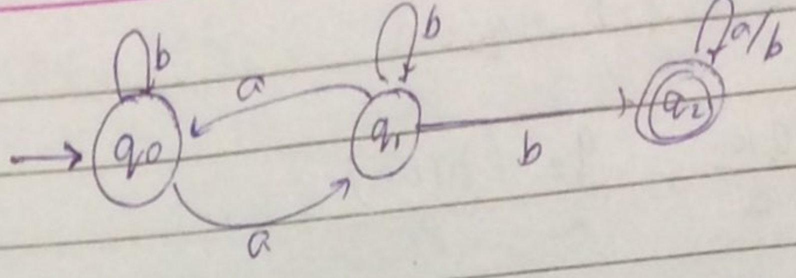
$q_4 = aq_4 + bq_4 + bq_3 + aq_2$

$\cancel{q_4 = aq_4 + bq_4 + bbq_1 + aaq_1}$

$\cancel{q_2 = abq_2 + aaq_3}$

$\cancel{q_3 = bbq_2 + baq_3} \qquad = \cancel{bb(abq_2 + aaq_3) + baq_3}$

0.



$q_0 = q_0 b + q_1 a + \cancel{q_1 b}$ — (1)

$q_1 = q_1 b + q_0 a$ — (2)

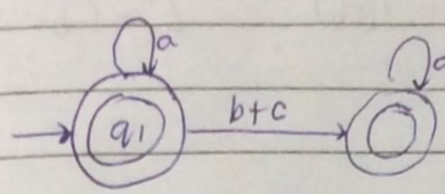
$q_2 = a q_2 + b q_2 + b q_1$ — (3)

$q_1 = q_1 b + ab q_0 + aa q_1 + ab q_1$

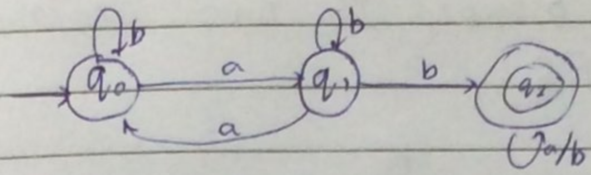
$q_2 = (a+b) q_2 + b q_1$

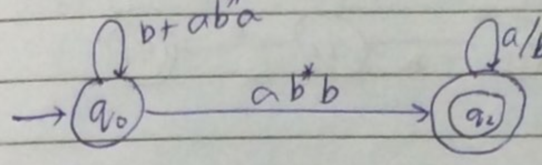
$q_2 = (a+b) q_2 + bb q_1 + ba q_0$

$(q_0 + q_1) = (q_0 + q_1)(a+b)$

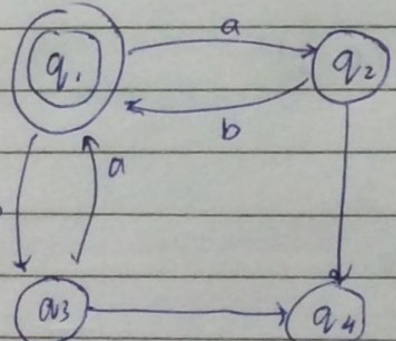


$(a^* + (b+c)a^*)^*$

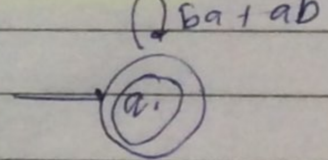




$(b + ab^*a)^* \; ab^*b \; (a+b)^*$



$(ab + ba)^*$



Closure propertie on regular language

L_1 and L_2 are Regular language

Union $L = L_1 \cup L_2$

Concatenation $L = L_1 \cdot L_2$

kleeve closure

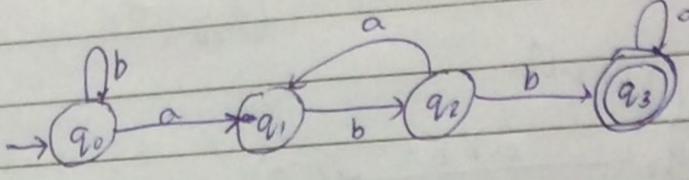
$L = L_1^*$

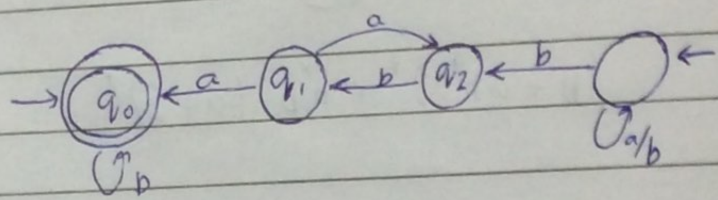
Reversal $L = L_1^R$ $L_1 = (ab)^*$ $L^R = (ba)^*$

Procedure

1. Reverse the move
2. Make IS ~~as~~ as FS (Final State).
3. Make FS as IS if there are more than one FS then create a new IS and ∧- move to all the FS.

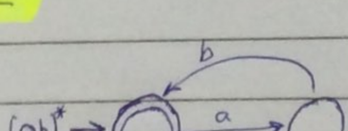
$\boxed{L_1} \Rightarrow$

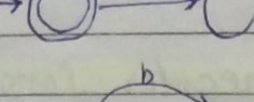




$(a+b)^* \, bba \, (a+b)^*$

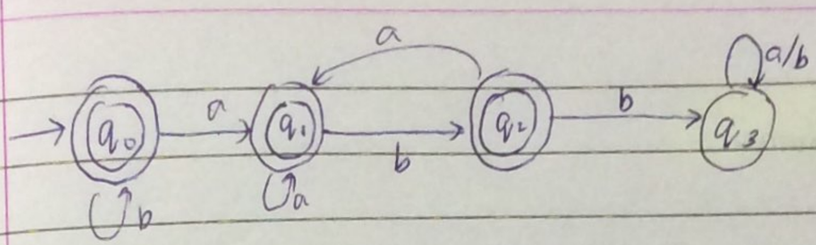
Complement





$L = \overline{L_1}$ Procedure

1. Make FS as NFS (Non final state)
2. Make all non-FS as FS



Intersection

$L = L_1 \cap L_2 = \overline{\overline{L_1} \cup \overline{L_2}}$ $\quad L_1 = \{q_{01}, q_{f1}, \Sigma, \delta_1, Q_1\}$

Even 0 and odd 1 $\quad L_2 = \{q_{02}, q_{f2}, \Sigma, \delta_2, Q_2\}$

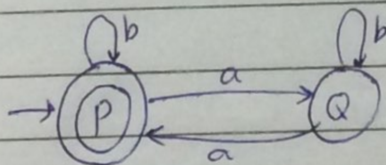
Procedure

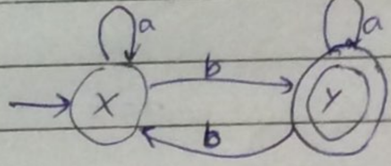
→ Make $[q_{01}, q_{02}]$ as initial state for FAL

→ $\delta[q_{12}, q_{26}], a = \{\delta_1(q_{1i}, a), \delta_2(q_{2i}, a)\}$

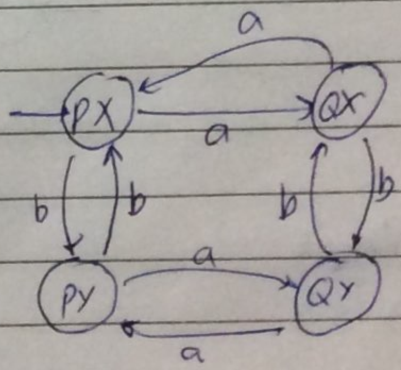
→ $[q_{f1}, q_{f2}]$ is FS

$Q = [q_i, q_j]$ where $q_i \in Q_1$, $q_j \in Q_2$.





	a	b
→PX	QX	PY
QX	PX	QY
PY	QY	PX
QY	PY	QX



$L = L_1 - L_2$
$= L_1 \cap \overline{L_2}$

1/Feb/2018

Pumping Lemma.

$Q : \Sigma, \delta, q_0, F$

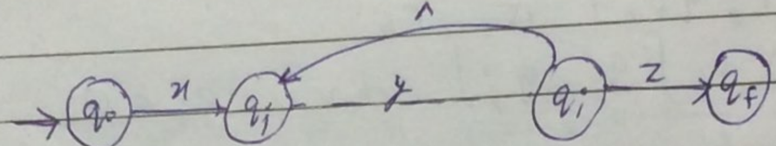
if L is regular then each string $w \in L$ can be written $x\ y\ z$ i.e $w \in xyz$

where $|y| \geq 1$ $\quad |w| \geq m$

and

$|xy| \leq M$ — No of stage.

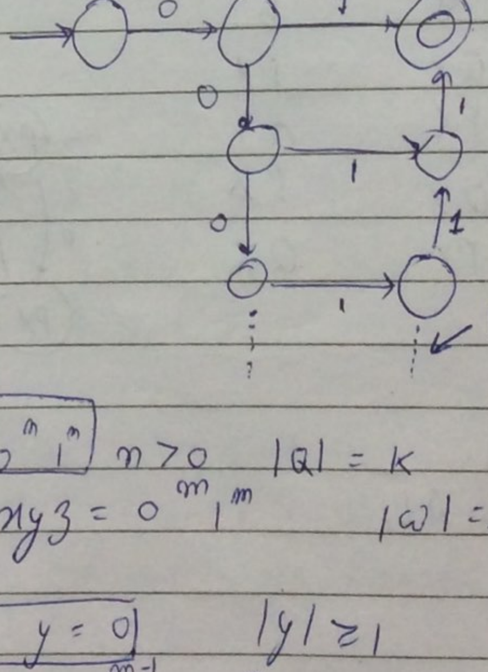
then $\forall i \geq 0$ $xy^i z \in L$



$L = 0^n 1^n \quad n > 0$

$n = 1 \quad w = 01$

$n = 2 \quad w = 0011$



~~Any string is~~ .

Not finite.

k = no of states.

~~Let~~ $\boxed{L = 0^n 1^n}$ $n > 0$ $\quad |Q| = k \quad K \geq M$

$w = xyz = 0^m 1^m \quad |w| = 2m > k$

$\boxed{\text{Let } y = 0}$ $\quad |y| \geq 1 \quad |xy| \leq k$

$w = \underset{x}{0^{m-1}}\ \underset{y}{0}\ \underset{z}{1^m}$ $\quad$ Non Regular.

$i = 0 \qquad w = 0^{m-1} 1^{m} \notin L$

$i = 1 \qquad w = 0^{m} 1^{m}$

$i = 2 \qquad w = 0^{m-1} 0^{2} 1^{m} = 0^{m+1} 1^{m} \notin L$

$\boxed{\text{Let } y = 1} \qquad 0^{m} 1 1^{m-1}$

Non regular. $\quad i = 2 \qquad 0^{m} 1^{2} 1^{m-1} \notin L$

$\boxed{\text{Let } y = 01}$

$$w = \underbrace{0^{m-1}}_{x} \underbrace{01}_{y} \underbrace{1^{m-1}}_{z} \in L$$

Non Regular.

$i = 2 \qquad 0^{m-1} (01)^{2} y^{m-1}$

$0^{m-1} \, 0101 \, y^{m-1} = 0^{m} 10 1^{m} \notin L$

$\boxed{L = 0^{2n}} \qquad n \geq 0$

$y = 00 \qquad (00)^{i} 0^{2(n-1)} \in L$

$x = \wedge$

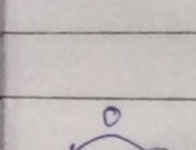
$z = 0^{2(n-1)}$

Regular

$x = 0^{2(n-1)} \qquad K = 2n.$

$y = 00$

$z = \wedge$



a^{*}

5/Feb/2018

Regular Grammar

$S \rightarrow NVN$
$N \rightarrow$ Ram / apple / Book.
$V \rightarrow$ ate / Read.

$G(V, \Sigma, P, S)$

$V = \{S, N, V\}$

$V \rightarrow$ is set of NT
$\Sigma \rightarrow$ set of terminal
$P \rightarrow$ production
$S \rightarrow$ Start symbol

$\Sigma = \{$Ram, apple, book, ate, read$\}$

P:
$S \Rightarrow NVN$
$N \Rightarrow$ Ram
$V \Rightarrow$ ate
$V \rightarrow$ read
$N \rightarrow$ apple
$N \rightarrow$ book.

Ram ate apple.
$S \rightarrow NVN$
$\rightarrow$ Ram V N
$\rightarrow$ Ram ate N.
$\rightarrow$ Ram ate apple.

A grammer G is regular if all the production are of form

$A \rightarrow a \mid aB$ (Right linear)
or
$A \rightarrow a \mid Ba$ (left linear)

$S \rightarrow abA$
$A \rightarrow aA \mid a$

$S \rightarrow abA$
$S \rightarrow aba$

$L = abaa^*$ (Right Linear)

$S \rightarrow abA$
$\rightarrow abaA$
$\rightarrow abaaA$

$S \rightarrow Aba$
$A \rightarrow a \mid Ba$
$B \rightarrow Ba \mid a$

~~$L = aa^*ba$~~ or $L = a^*aba$ (Left Linear)

$S \rightarrow aA$
$A \rightarrow Bb \mid b$
$B \rightarrow aB \mid a$

ab
$aaBb$ $\quad a^*ab$

$S \rightarrow aA$
$A \rightarrow Bb \mid b$
$B \rightarrow aA$

aBb $\quad ab$
$aaAb$ $\quad aabb$ $\quad aaBbb$
$aaaAbb$
$a^n b^n \; \forall n > 0$ $\quad aaabbb$

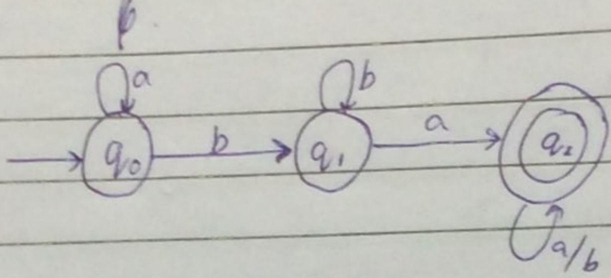
If all ~~Left~~ Left Linear or Right Linear grammar regular ~~or not~~

Otherwise (mixed) can be regular or not

FA to regular ~~gr~~ grammar

$\delta(q_i, a) \longrightarrow q_j$ when $q_j \notin F$ — P: $A_i \to aA_j$

$\delta(q_i, a) \to q_j$, $q_j \in F$ — P: $A_i \to aA_j | a$



$(q_0, a) \vdash (q_0, a)$

P: $A_0 \to aA_0$

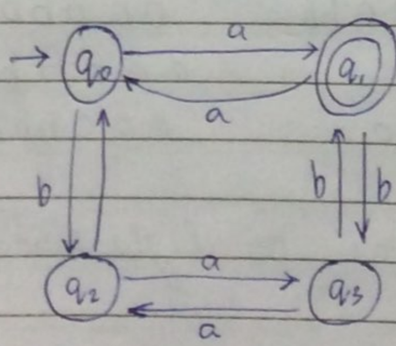
$A_0 \to bA_1$ $\quad (q_0, b) \to q_1$

$A_1 \to bA_1$

$A_1 \to aA_2 | a$

$A_2 \to aA_2 | a$

$A_2 \to bA_2 | b$



P:

$A_0 \to aA_1 | a$ $\qquad A_2 \to bA_0 | b$

$A_0 \to bA_2$ $\qquad A_3 \to aA_2$

$A_2 \to aA_3$ $\qquad A_3 \to bA_1$

$A_1 \rightarrow a A_2$ then $\delta(q_1, a) \rightarrow q_2$

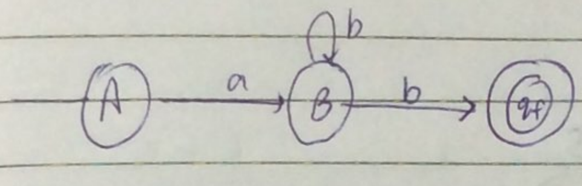
$A_1 \rightarrow a$ then $\delta(q_1, a) \rightarrow q_f$

$q_f \in F$

Eg

$A \rightarrow aB$

$B \rightarrow bB | b$.



Eg.

$A \rightarrow aB / bC$

$B \rightarrow bB / b$

$C \rightarrow aA / a$.

6/Feb/2018.

Context free grammar

→ A grammar $G(V_N, \Sigma, P, S)$ is CFG if all the production are of form

$A \rightarrow \alpha$ where $\alpha \in (V_N \cup \Sigma)^*$

$S \rightarrow AB$

$A \rightarrow aaA$

$A \rightarrow \wedge$

$B \rightarrow Bb$

$B \rightarrow \wedge$

Eg aaab

$S \rightarrow AB$

$S \rightarrow aaAB$

$S \rightarrow aaaaAB$

does not belong to this automata.

Eg

aab

↓

belong to grammar.

$S \rightarrow AB$

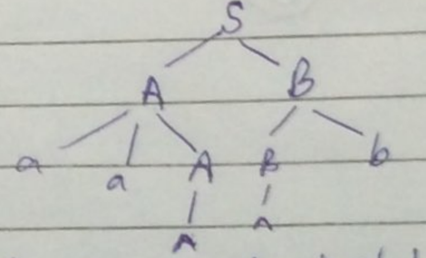
$\rightarrow aaAB$ $(A \rightarrow aaA)$

$\rightarrow aaB$ $(A \rightarrow \wedge)$

$\rightarrow aaBb$ $(B \rightarrow Bb)$

$\rightarrow aab$ $(B \rightarrow \wedge)$

derivation tree



⇒ string: ababcbcbb. ⟶ belong to the grammar

$S \longrightarrow abScB$

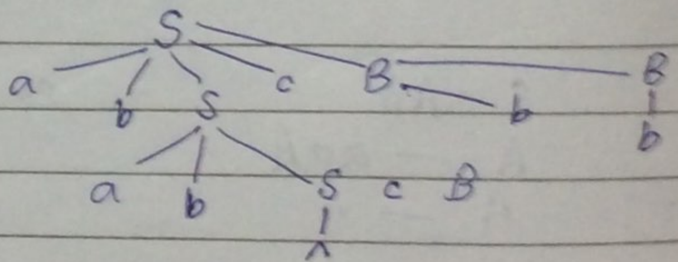
$S \longrightarrow \wedge$

$B \longrightarrow bB \mid b$

~~ababScB~~

~~ababcB~~

derivation tree.



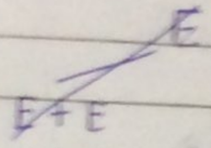
Ambiguous grammar

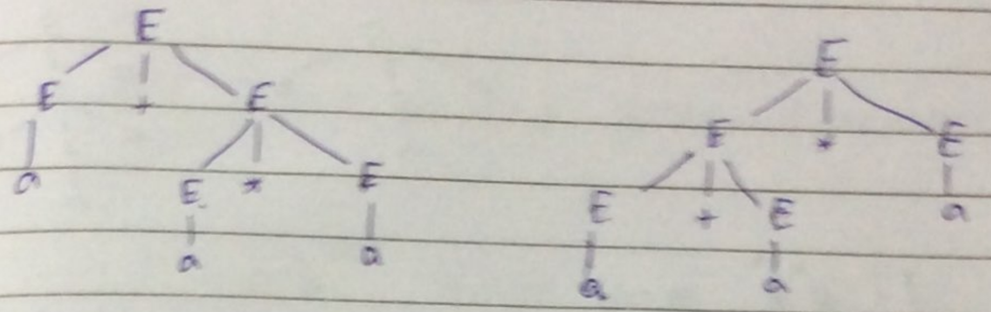
⇒ $E \rightarrow E+E \mid E*E \mid a$

string $a+a*a$.

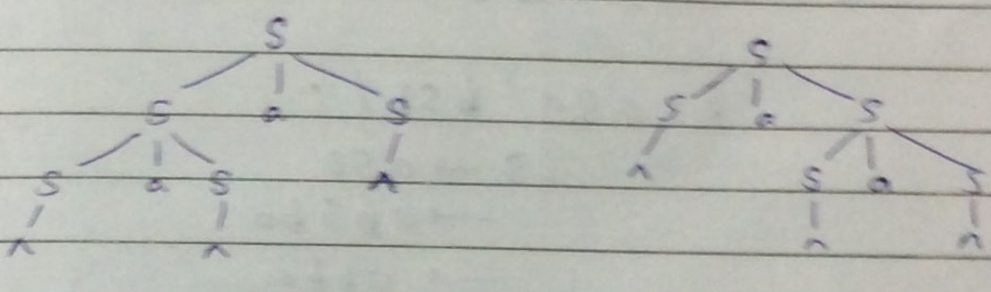
$E+E$

$E+E*E$





⇒ $S \rightarrow SaS \mid \wedge$



⇒ $S \rightarrow 0A \mid 1B$

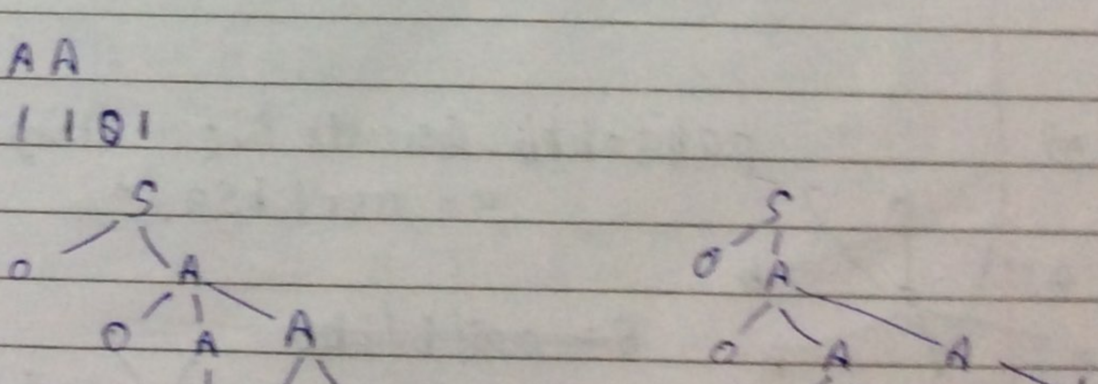
$A \rightarrow 0AA \mid 1S \mid 1$

$B \rightarrow 1BB \mid 0S \mid 0$

001101

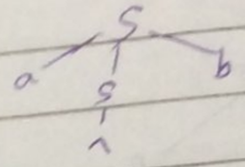
00AA

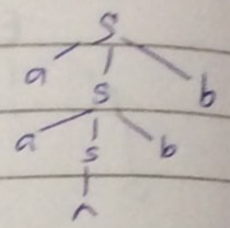
001101



⇒ $L = a^n b^n \quad n \geq 0$

$S \to aSb \mid \Lambda$





$L = a^n b^n \quad n > 0$

$S \to aSb \mid ab$

⇒ $L = ww^R \quad w \in \{a, b\}^*$

$S = aSa \mid bSb \mid \Lambda$

$S \to aSa$

$\to abSba$

$\to abba$

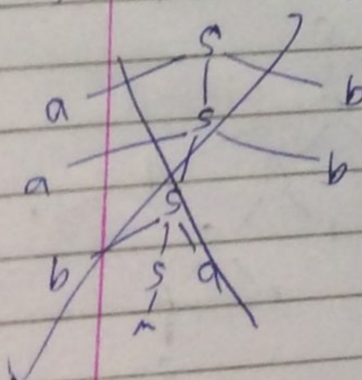
⇒ $L = wcw^R \quad w \in \{a, b, c\}^*$

$S = aSa \mid bSb \mid c$

L = all string having equal number of a & b

⇒ aabaabbb. Generate this String from gramm

$S = aSb \mid bSa \mid \Lambda$



~~S = aaS / b / bbS~~

aab~~aa~~b bb

⇒ baab ~~for~~ from $S \to aSb \,/\, bSa \,/\, \wedge \,/\, SS$

• L = all string having unequal number of a & b

7/Feb/2018

$L = a^n b^m \quad n \geq m$

$S \to aSb \,/\, A$
$A \to aA \,/\, a$

⇒ $L_1 = a^n b^{2m} \qquad L_2 = a^n b^{2n}$

$a^*(bb)^* \qquad S \to aSbb \,/\, \wedge$

⇒ $L = a^n b^n c^m d^m$

$S \to AS \qquad m\,n > 0$
$A \to aAb \,/\, ab$
$B \to cBd \,/\, cd$

⇒ $L = a^m b^{2m} c^n \quad m, n \geq 0$

$S \to AC$
$A \to aAbb \,/\, \wedge$
$C \to cC \,/\, \wedge$

⇒ $L = a^n b^{m+n} c^m = a^n b^n b^m c^m$

$L = a^n b^m c^k \quad n = m \quad m \leq k$

$L_1 = n = m \quad a^n b^n c^* \quad L = L_1 \cup L_2$

$L_2 = m \leq k \quad a^* b^m c^{m+n}$

$S = S_1 / S_1$
$S_1 \longrightarrow AC$
$A \rightarrow aAb / \wedge$
$C \longrightarrow cC / \wedge$
$S_2 \rightarrow BD$
$B \rightarrow aB / \wedge$
$D \longrightarrow aB / \wedge$

$L = \omega(a, b)$ where ~~$x^{(\omega)}$~~

no. of as in $\omega \neq$ no. of bs in ω

$S \rightarrow aSb / bSA / SS / A / B / abS / Sab / Sb$

$A \rightarrow aA / a$
$B \rightarrow bB / b$

9/ Feb/2018. **Closure properties of CFL.**

Let L_1 and L_2 be CFL

$L_1 : G_1 (S_1, \Sigma_1, V_1, P_1)$
$L_2 : G_2 (S_2, \Sigma, V_2, P_2)$

$L = L_1 \cup L_2$

$L_1 = a^n b^n c^m \quad m, n \geq 0$
$L_2 = a^n b^m c^m \quad m, n \geq 0$

$S \rightarrow S_1 / S_2$

$S_1 \longrightarrow AC$
$A \longrightarrow aAb / \wedge$
$C \longrightarrow cC / \wedge$

$S_2 \longrightarrow PB$
$P \longrightarrow aP / \wedge$
$B \longrightarrow bBc / \wedge$

Concatenation

doubt
73 → 0 followed by 1
→ all string exactly one a. Pg 74
75 → aabb ans

$L = L_1 . L_2$

$\underline{a b a^2 b^2 c a^3 b^3 c}\ \underline{b^4 c^4 b c a b c^2}$

Kleene Closure

If L_1 is context free $\quad L = L_1^*$ $\quad S \to SS_1 / \wedge$

L_1^* will also be context free.

$a b c a^2 b^2 a b a^3 b^3 c$

Reversal (transpose)

$L = L_1^R$

$L_1 = a^n b^m c^m \quad n, m \geq 0$

$L = c^m b^n a^n \quad n, m \geq 0$

$S \to CA$

$A \to bAa / \wedge$ $\qquad G(V, \Sigma, P, S)$

$C \to Cc / \wedge$ $\qquad P = P^T$

⇒ $L = L_1 \cap L_2$

$L_1 = a^n b^m c^m d^k \quad n, m, k \geq 0$

$L_2 = a^n b^p c^q d^n \quad n, p, q \geq 0$ $\qquad$ follow CFL

$a = d.$

$L = a^n b^m c^m d^n \quad m, n \geq 0$

⇒ $L = L_1 \cap L_2$

$L_1 = a^n b^n c^r$ $\qquad$ not follow CFL

$L_2 = a^m b^n c^n \quad L = L_1 \cap L_2$

$\overline{\overline{L_1} \cup \overline{L_2}}$

Complement

$\overline{\overline{L_1} \cup \overline{L_2}} = L_1 \cap L_2$

CFL are closed under complementation

$L_1 / \overline{L_1} \rightarrow CFL$

$L_2 / \overline{L_1} \rightarrow CFL$

$\overline{\overline{L_1} \cup \overline{L_2}} \rightarrow CFL$

$\overline{L_1} \cup \overline{L_2}$

Simplification of CFL.

Elimination of useless symbol

Eg

$S \rightarrow BD$

$A \rightarrow aAB / C$

$C \rightarrow BC$

$D \rightarrow a$

$B \rightarrow a$

$B \rightarrow bB / L$

$E \rightarrow a/b$

Non Reachable

$S \rightarrow AB / BC$

$A \rightarrow aA / SC$

$B \rightarrow C / \Lambda$

$C \rightarrow bC$

$D \rightarrow b/c$

Non Generating Non Terminal

Non Reachable

W - set of All non terminal that are generating.

$W_1 = \{A / A \rightarrow \alpha$ and $\alpha \in \Sigma^*\}$

$W_{i+1} = W_i \cup \{A \mid A \rightarrow \alpha \in P$ and $\alpha \in (\Sigma \cup W_i)^*\}$

$W_i = W_{i+1}$ $\quad V_N' = W_i$

P' = all production from W_i

Q. $C \to d$
$S \to AB/BC$
$A \to aA$
$C \to dC$
$B \to bC/SA$
$E \to b/C$

Q. $S \to AB/a$
$A \to BC/b$ $W = \{S\ A\}$
$B \to ab/C \to$ Remove
$C \to aC/B \to$ Remove
Remove all containg A B & C
$\left.\begin{array}{l} S \to a \\ A \to b \end{array}\right\} \Rightarrow \boxed{S \to a}$

W. ⟶ One who derives only terminal.

$W_1 = \{C, E\}$
$W_2 = \{E, C\} \cup \{B\} = \{E\ C\ B\}$
$W_3 = \{E\ C\ B\} \cup \{S\} = \{E\ C\ B\ S\}$

$C \to d$ ✓
$C \to dC$ ✓
$S \to BC$
$B \to bC$
$E \to b/C$

⇒ $S \to AB/CaD$
$A \to Bc/Aa/b$
$B \to CB$
$C \to Ac/d$
$D \to SA/Hb$
$G \to a$
$H \to aG$

$W_1 = \{G\ A\ C\}$.
$W_2 = \{G\ A\ C\ H$
$W_3 = \{A, C, G, H, D, S\}$
S⇒
$A \to Aa$
$A \to b$
$C \to d$
$C \to Ac$
$G \to a$
$H = aG$.

* If S not included Grammar is empty *

12/feb/2018.

Eliminating Useless Symbol

→ Not Reachable

Method.

$W_1 = \{S\}$

$W_{i+1} = W_i \cup \{A \mid B \to A \text{ and } B \in W_i\}$

Repeat till $W_i = W_{i+1}$.

$V_N' = V_N \cap W_i$

$\Sigma' = \Sigma \cap W_i$

P' = include all producti on which have T and NT only from W_i

Q

$S \to aAa$

$A \to Sb / bCC$

$C \to aC / b$

$E \to a/b$

$W_1 = \{S\}$

$W_2 = \{S\} \cup \{a, A\} = \{S, a, A\}$

$W_3 = \{S, a, A\} \cup \{b, C\} = \{S, a, A, b, C\}$

$W_4 = \{S, a, A, b, C\} \cup \{a, C, b\}$

$= \{S, a, A, b, C\}$

$V_N' = \{S, A, C, E\} \cap W_i$

$= \{S, A, C\}$

$\Sigma' = \{a, b\} \cap W_i = \{a, b\}$

P' = $S \to aAa$ $\quad$ $C \to aC$

$A \to Sb$ $\quad$ $C \to b$

$A \to bCC$

Q. $S \to Aa / f$
$A \to SbB$
$B \to bCC$
$C \to eD / Ec / d$
$E \to aA / bB$
$F \to bE / c$

$W_1 = \{S\}$
$W_2 = \{S\} \cup \{A\,a\,f\}$
$W_3 = \{S\} \cup \{Aaf\} \cup \{SbB\}$
$W_4 = \{S\} \cup \{Aaf\} \cup \{SbB\} \cup \{bCC\}$

$\{S, C.$

Elimination of Null production

$\cancel{S \to aS / \wedge}$
$\cancel{S \to aS / a}$
$A \to aAb / \wedge$

W_i = find set of all nullable variable.
$A \to \wedge$, then add A to W_1
$W_1 = \{A / A \to \wedge \in P\}$
$W_{i+1} = W_i \cup \{B / B \to A$ and $A \in W_i\}$

$A \to \wedge$
$\times\ B \to aA$
$\times\ B \to AC$
$C \to b$
$B \to A$

$P' =$ if $A_1 \in W_i$
and $B \to A_1 A_2 \ldots A_n \in P$
$P' = B \to A_2 \ldots A_n / A_1 A_2 \ldots A$

= write new P including & excluding nullable variable

$S \to aS / AB$
$A \to \wedge / aD$
$B \to \wedge / bDa$
$D \to b$

$W_1 = \{A, B\}$
$W_2 = \{A, B\} \cup \{S\} = \{ABS\}$

$S \to aS / a$
$S \to AB / A / AB$
$A \to aD$
$B \to bDa$
$D \to b$

$\Rightarrow$ $S \rightarrow ABC$

$A \rightarrow aAb / \wedge$

$B \rightarrow bC / CA$

$C \rightarrow aC / \wedge$

$W_1 = \{A\ C\}$

$W_2 = \{AC\} \cup \{B\}$

$W_3 = \{A\ C\} \cup \{B\} \cup \{S\}$

$S = ABC / AB / BC / AC / A / B / C$

$A \rightarrow aAb / ab$

$B \rightarrow bC / b / C / A / CA$

$C \rightarrow aC / a$

$\Rightarrow$ Elimination of unit Production

$$\left.\begin{array}{l} A \rightarrow B \\ B \rightarrow C \\ C \rightarrow a \end{array}\right\} \begin{array}{l} A \rightarrow a \\ B \rightarrow a \\ C \rightarrow a \end{array}$$

Method

construction of $W_i(A)$ set defining at NT derived from A. unit production form.

$W_1(A) = A$

$W_{i+1}(A) = W_i(A) \cup \{B / A \rightarrow B \ \& \ B \in W_i(A)\}$

if $B \in W_i(A)$ and $B \rightarrow \alpha \in P$ then
$A \rightarrow \alpha \in P'$

$$\left.\begin{array}{l} A \rightarrow B \\ B \rightarrow \alpha \end{array}\right\} \begin{array}{l} A \rightarrow \alpha \\ B \rightarrow \alpha \end{array}$$

$S \to AB$ $\quad W_i(S) = \{S\}$

$A \to a$ $\quad W_i(A) = \{A\}$

$B \to C / b$ $\quad W_i(B) = \{B, C, D, E\}$

$C \to D$ $\quad W_i(C) = \{C, D, E\}$

$D \to E / bc$ $\quad W_i(D) = \{D, E\}$

$E \to a.$ $\quad W_i(E) = \{E\}$

Start Writing from down.

$S \to AB$

$A \to a$

$B \to b$

$E \to a$

$D \to a / bC$

$C \to a / bC$

$B \to a / bC$

$A \to BC / B$

$B \to e / bd$

$C \to aC / d$

$A \to BC / e / bd$

$B \to e / bd$

$C \to eC / d$

Null

↓

Unit

↓

Useless — Non Generating

Useless — Non Reachable.

12/Feb/2018

Non-Reachable

$W_1 = \{S\}$

$W_{i+1} = W_i \cup \{A \mid B \to A \text{ and } B \in W_i\}$

$V_N' = V_N \cap W_i \quad \Sigma' = \Sigma \cap W_i$

P' = all production P that include $\{V_i \in W$

$S \to aA/b$

$A \to aCb/e$

$C \to Be$

$B \to cB/d$

$E \to a/b$

$S \to aA/b$

$A \to aCb/e$

$C \to Be$

$B \to cb/d.$

$W_1 = \{S\}$

$W_2 = \{S\} \cup \{A, a, b\}$

$W_3 = \{S, A, a, b, e, C\}$

$W_4 = \{S, A, a, b, e, c, B\}$

$W_5 = \{S, A, a, b, e, C, B, c, d\}$

$V_N' = V_N \cap W_i$

$= \{S, A, B, C, E\} \cap W_i$

$= \{S, A, B, C\}$

$\Sigma' = \Sigma' \cap W_i$

$= \{a, b, c, d, e\} \cap W_i$

$= \{a, b, c, d, e\}$

12/Feb/2018.

Non-Reachable

$W_1 = \{S\}$

$W_{i+1} = W_i \cup \{A / B \rightarrow A \text{ and } B \in W_i\}$

$V_N' = V_N \cap W_i \quad \Sigma' = \Sigma \cap W_i$

P' = all production P that include $\{V_n \in W$

$S \rightarrow aA/b$
$A \rightarrow aCb/e$
$C \rightarrow Be$
$B \rightarrow cB/d$
$E \rightarrow a/b.$

$S \rightarrow aA/b$
$A \rightarrow aCb/e$
$C \rightarrow Be$
$B \rightarrow cB/d.$

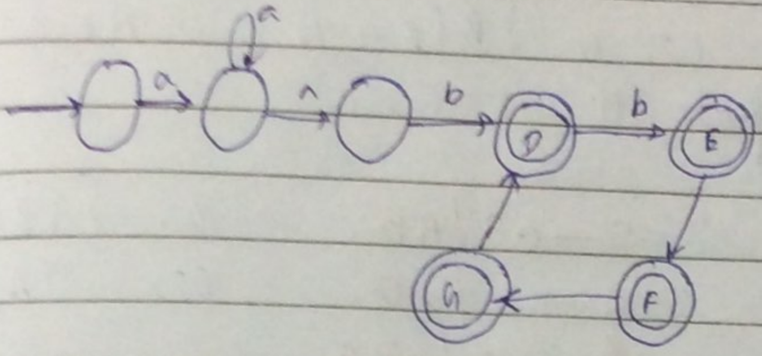
$W_1 = \{S\}$
$W_2 = \{S\} \cup \{A, a, b\}$
$W_3 = \{S, A, a, b, e, C\}$
$W_4 = \{S, A, a, b, e, C, B\}$
$W_5 = \{S, A, a, b, e, C, B, c, d\}$

$V_N' = V_N \cap W_i$
$= \{S, A, B, C, E\} \cap W_i$
$= \{S, A, B, C\}$

$\Sigma' = \Sigma' \cap W_i$
$= \{a, b, c, d, e\} \cap W_i$
$= \{a, b, c, d, e\}$

Make context free grammar

→ $L = a^m b^n$ $m > 0$ n is not multiple of 5.



$A \to aA \,/\, aC$

$C \to bD \,/\, b$

$D \to bE \,/\, b$

$E \to bF \,/\, b$

$F \to bG \,/\, b$

$G \to bC$

→ Every opening bracket have a closing bracket

$S \to e \,/\, SS \,/\, [S]$

$G = (V, \Sigma, R, S)$ with set of variable $V = \{S\}$, where S is the start variable, set of terminals $\Sigma = \{[,]\}$; and rules

→ Find CFG that generate the language

$L(G) = \{ a^n b^m \mid 0 \le n \le m \le 2n \}$

$S \to aSb \,/\, aSbb \,/\, \wedge$

Elimination of NULL Production

W_i = Set of all nullable variable

$A \to \wedge \longrightarrow$ add A to W_i

$W_{i+1} = W_i \cup \{B \mid B \to A \text{ and } A \in W_i\}$

Repeat till $W_i = W_{i+1}$

① $S \to aS / AB$ $\qquad W_1 = \{A B\}$

$A \to \wedge / aD$ $\qquad W_2 = \{A, B, S\}$

$B \to \wedge / bDa$

$D \to b$

$S \longrightarrow aS / A / AB / B$

$A \longrightarrow aD$

$B \longrightarrow bDa$

$D \longrightarrow b$

$S \to ABC$ $\qquad W_1 = \{A, C\}$

$A \to aAb / \wedge$ $\qquad W_2 = \{A, C\} \cup \{B\}$

$B \to bc / CA$ $\qquad W_3 = \{A, C, B, S\}$

$C \to aC / \wedge$

$S \to ABC / AB / BC / CA / A / B / C$

$A \to aAB / ab$

$B \longrightarrow bC / b$

$B \longrightarrow CA / C / A$

$C \longrightarrow aC / a.$

~~Normal Form~~.

$(Q, \Sigma, \delta, q_0, F)$

$Q \rightarrow$ Set of states
$q_0 \rightarrow$ initial state
$F \rightarrow$ set of final states
$\delta \rightarrow$ transition function
$\Sigma \rightarrow$ input symbol.

DFA → moves defined for every symbol.

Chomsky Normal Form (CNF)

Grammar G is in CNF if all the production are in following form:

$A \rightarrow \underbrace{BC}$ — 2 non terminal

or

$A \rightarrow a$ — 1 terminal.

Procedure

~~Simply~~

Simplify grammar 112 116 120 118

Removing intermediate terminals for each terminal.

Q
$S \rightarrow AB/a$
$A \rightarrow b/\cancel{c}D/eD/d$
$B \rightarrow cD/eD/dC$
$C \rightarrow eDdB/a$
$D \rightarrow g/c/b$

$X \rightarrow c \quad Y \rightarrow e \quad Z \rightarrow d$

$S \rightarrow AB/a$
$A \rightarrow b$
~~$A \rightarrow cD$~~
$A \rightarrow XD$
$A \rightarrow YD$
$A \rightarrow d$

$B \rightarrow XD$
$B \rightarrow YD$
$B \rightarrow ZC$

$C \rightarrow YDZB$
$C \rightarrow a$
$D \rightarrow g/c/b.$

Gribeach Normal Form (GNF)

A grammar G is in GNF of all the production are of form

$$A \rightarrow a\alpha_1\alpha_2 \text{ -- } \alpha_n / a$$
$$a \in \Sigma$$
$$\alpha_1, \alpha_2, \text{ -- } \alpha_n \in NT$$

Start with T followed by any no. NT.

if $A \rightarrow A\alpha/\beta$

$$A \rightarrow \beta A'/\beta$$
$$A' \rightarrow \alpha A'/\alpha$$

Q $S \rightarrow CA/BB$
$B \rightarrow b/SB$
$C \rightarrow b$
$A \rightarrow a$

↓

Already in chomsky normal form.

STEP 1 Change the names of the Non-terminal Symbol into some A_i in ascending order of i.

$S \rightarrow A_1$
$C \rightarrow A_2$
$A \rightarrow A_3$
$B \rightarrow A_4$.

$A_1 \rightarrow A_2A_3 / A_4A_4$
$A_4 \rightarrow b / A_1A_4$
$A_2 \rightarrow b$
$A_3 \rightarrow a$

STEP 4. Alter the rules so that the Non-Terminals are in ascending order, s.t. if the production is of the form

$A_i \rightarrow A_jX$, then
$i < j$ & should never be $\underbrace{i = j}_{\Downarrow \text{ removed later}}$

$A_1 \rightarrow A_2A_3 / A_4A_4$ ✓ since $i < j$ Here.
$A_4 \rightarrow b / A_1A_4$ $i = 4 \longrightarrow j \longrightarrow$ X
$A_2 \rightarrow b$
$A_3 \rightarrow a$.

$A_4 \longrightarrow b / A_2A_3A_4 / A_4A_4A_4 \longrightarrow$ X
$A_4 \longrightarrow b / bA_3A_4 / A_4A_4A_4$.
✓ ✓ X

~~A_4~~ This condition is left recurrsion.

Step Remove Left Recursion

$A_4 \rightarrow b / bA_3A_4 / A_4A_4A_4$

$\cancel{A_4}$ $Z \rightarrow A_4A_4Z / A_4A_4$

$A_4 \rightarrow b / bA_3A_4 / bZ / bA_3A_4Z$

$A_1 \rightarrow bA_3 / bA_4 / bA_3A_4A_4 / bZA_4 / bA_3A_4ZA_4$
$A_4 \rightarrow b / bA_3A_4 / bZ / bA_3A_4Z$
$Z \rightarrow bA_4 / bA_3A_4A_4 / bZA_4 / bA_3A_4 / bZA_4 /$
$bA_4Z / bA_3A_4A_4Z / bZA_4Z / bA_3A_4ZA_4Z$

eg

$S \rightarrow AB / b$
$A \rightarrow CBx / gF$
$F \rightarrow a$
$B \rightarrow bB / C$

(have not removed C because we've to again add it later on).

$S \to ABa$

$S \to b$

$A \to aaA$

$A \to B$

$B \to bBb$

$B \to \wedge$

$X \to AB$

$Z \to a$

$S \to XZ$

$X \to AB$

$Z \to a$

$S \to b$

$A \to \wedge$

~~$A \to BBb$~~ $A \to bQ$.

~~$B \to$~~ $\wedge$.

~~$B \to bBB$~~

$B \to \wedge$

$B \to bQ$

$Q \to bB$

$S \to XZ/b$

$X \to AB$

b^{2n}

$P \to bQ/\wedge$

$Q \to bP$

bbbb

X

Z

A

B