CIS 455 – Homework #3

Due on myCourses *before* 11:59pm on Sunday, October 18th, 2015

Instructions: There are 7 problems worth a total of 100 points.

Submit on myCourses: solutions to the written parts, any source code and screenshots of sample runs which thoroughly verify the correctness of your code.

Document and indent your programs properly. You will be graded on both your solutions and your ability to show their correctness.

If you feel it would help, you are encouraged to work together on homework. But remember that you must submit your own work, as the point of the homework is to learn the material. *If you do work with others on homework, you must write the names of those you worked with on your homework.*

Late Homework will be penalized!

Problem 3-1. (10 points) Jones & Pevzner, (*modified version* of Problem 5.1, page 143). Suppose you have a maximization algorithm, A, that has an approximation ratio of ¼. When run on some input π, $A(\pi) = 12$.

1. What can you say about the true (correct) answer OPT = OPT(π)?

    a. OPT(π) >= 3;
    b. OPT(π) <= 3;
    c. OPT(π) >= 12;
    d. OPT(π) <= 12;
    e. OPT(π) >= 48;
    f. OPT(π) <= 48;

2. What if A is a minimization algorithm? (*Hint:* This is a trick question)

Problem 3-2. (28 points) Jones & Pevzner, (*modified version* of Problem 5.4, page 143).

1. Perform the *ImprovedBreakpointReversalSort* algorithm with π = 3 4 6 5 8 1 7 2 (Remember to start with **0** 3 4 6 5 8 1 7 2 **9** and to follow the algorithm below, **particularly line 3**).

    For each step (similar to the class activity):
    - Show the reversal you have chosen
    - Show the resulting sequence
    - Show the number of breakpoints that remain after performing the reversal

ImprovedBreakpointReversalSort($\boldsymbol{\pi}$)
1 **while** $b(\boldsymbol{\pi}) > 0$
2   **if** $\boldsymbol{\pi}$ has a decreasing strip
3   Among all possible reversals, choose reversal ρ that minimizes $b(\boldsymbol{\pi}\cdot\rho)$
4   **else**
5   Choose a reversal ρ that flips an increasing strip in $\boldsymbol{\pi}$
6     $\boldsymbol{\pi} \leftarrow \boldsymbol{\pi}\cdot\rho$
7   **output** $\boldsymbol{\pi}$
8 **return**

Problem 3-2. (continued) Jones & Pevzner, (*modified version* of Problem 5.4, page 143).

2. The *if-test* in line 2 of the *ImprovedBreakpointReversalSort* algorithm ensures that the algorithm never gets stuck in a situation were there is no way to eventually decrease the number of breakpoints.

   Again using π = 3 4 6 5 8 1 7 2, construct a permutation σ where this if-test is needed.
   (Find a step that *ImprovedBreakpointReversalSort* might have ended up with where there are no decreasing strips, and no reversal that reduces the number of breakpoints).
   You do not need to solve it from this step on, just give an example of such a permutation.

3. Since this is an approximation algorithm, there might be a sequence of reversals that is shorter than the one found by *ImprovedBreakpointReversalSort*.

   Again using π = 3 4 6 5 8 1 7 2, construct a permutation σ for which this is the case.
   (Instead of following the *ImprovedBreakpointReversalSort* algorithm, perform any series of reversals—starting from π to the identity permutation—in fewer steps than it took you in part 1 when you used *ImprovedBreakpointReversalSort*).

Problem 3-3. (10 points) Jones & Pevzner, Problem 5.5, page 143.
Find a permutation with no decreasing strips for which there exists a reversal that reduces the number of breakpoints.

Problem 3-4. (12 points) Jones & Pevzner, Problem 5.13, page 144 (*read the description between 5.11 and 5.12*).

Given permutations π and σ, a breakpoint between π and σ is defined as a pair of adjacent elements π_i and π_{i+1} in π that are separated in σ. For example, if $\pi = 143256$ and $\sigma = 123465$, then $\pi_1 = 1$ and $\pi_2 = 4$ in π form a breakpoint between π and σ since 1 and 4 are separated in σ. The number of breakpoints between π=01432567 and σ=01234657 is three (14, 25 and 67), while the number of breakpoints between σ and π is also three (12, 46 and 57).

Given permutations π^1 = 124356, π^2 = 143256, and π^3 = 123465 , compute the number of breakpoints between:

(1) π^1 and π^2
(2) π^1 and π^3
(3) π^2 and π^3

(Use the first permutation as the given sorting order. For example, given π^3 = 123465, 4 and 6 are adjacent and 4 and 5 are not.)

Problem 3-5. (10 points) Jones & Pevzner, Problem 6.4, page 212.
Modify *DPChange* (on page 151) to return not only the smallest *number* of coins but also the correct combination of coins.

Problem 3-6. (10 points) Jones & Pevzner, Problem 6.6, page 212.
Find the number of different paths from *source* (0,0) to *sink* (*n*,*m*) in an *n* by *m* rectangular grid.

Problem 3-7. (20 points) Jones & Pevzner, (*modified version* of Problem 6.14, page 213).
Two players play the following rock game with two piles of rocks of heights *n* and *m*.
At every turn a player must take two rocks from one pile (either the first pile or the second pile) **and** one rock from the other. The player who cannot complete their turn loses.

1. Who will win? Using dynamic programming, describe the winning strategy for each *n* and *m*.

2. Show the details of the winning strategy for *n*=*m*=6.