Lab-VIII

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1 Aim

Write and execute an octave program to simulate/solve motion of a relativistic particle in an EM field.

2 Theory

3 Program

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% RelativisticMotionInEM
%
% Program to solve/simulate the motion of a relativistic particle in
% an electromagnetic field
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% 30th March, 2022
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

graphics_toolkit gnuplot
pkg load symbolic


% function to compute parameters for 1st differential eqn
function retval = F1(V, E, B, q, m0)
    pdt_x_m0 = (E(1) + V(2)*B(3) - V(3)*B(2)) * q / m0;
    pdt_y_m0 = (E(2) + V(3)*B(1) - V(1)*B(3)) * q / m0;
    pdt_z_m0 = (E(3) + V(1)*B(2) - V(2)*B(1)) * q / m0;
    retval = [pdt_x_m0, pdt_y_m0, pdt_z_m0];
endfunction

% function to compute parameters for 2nd differential eqn
function retval = F2(V, E, B, q, m0)
    retval = V;
endfunction

% function to solve the two differential eqn using RK45 method
function retval = Compute(t0, tf, steps, X, V, E, B, q, m0)
    increment = (tf - t0) / steps;
    retval = [X];
    for i=t0:increment:tf
        p_m0 = VelocityToMomentum(V);

        k11 = F1(p_m0, E, B, q, m0);
        k21 = F1(p_m0 + increment*k11/2, E, B, q, m0);
        k31 = F1(p_m0 + increment*k21/2, E, B, q, m0);
        k41 = F1(p_m0 + increment*k31, E, B, q, m0);

        k12 = F2(V, E, B, q, m0);
        k22 = F2(V + increment*k12/2, E, B, q, m0);
        k32 = F2(V + increment*k22/2, E, B, q, m0);
        k42 = F2(V + increment*k32, E, B, q, m0);

        p_m0 = p_m0 + (k11 + 2*k21 + 2*k31 + k41)*increment / 6;
        V = MomentumToVelocity(p_m0);

        X = X + (k12 + 2*k22 + 2*k32 + k42)*increment / 6;
        retval = [retval; X];
    endfor
endfunction

% function to get magnitude of vectors
function retval = mag(in_value);
    retval = sqrt(in_value(1)*in_value(1) + in_value(2)*in_value(2) + in_value(3)*in_value(3));
endfunction

% function to transform momentum to velocity
function retval = MomentumToVelocity(p_m0)
    retval = p_m0 ./ sqrt(1 + (mag(p_m0)**2));
endfunction

% function to transform momentum to velocity
function retval = VelocityToMomentum(V)
    vel_mag = mag(V);
    if vel_mag > 1
        printf("***** Solutions warning\n");
    endif
    retval = V / sqrt(1 - vel_mag**2);
endfunction

% Input parameters
printf("Intial parameters\n")
B = [0 0 10]
E = [0 0 0.1]
```

```
pos = [1 0 0]
V = [0 0.95 0.1]
c = 299792458; % m/s
q = 1;
m0 = 1;
t0 = 0;
tf = 10;
steps = 1000;

% computing trajectory
X = Compute(t0, tf, steps, pos, V, E, B, q, m0);

% Plotting trajectory
figure()
set(gca,'XMinorTick','on','YMinorTick','on');
plot3(X(:, 1), X(:, 2), X(:, 3));
xlabel("X");
ylabel("Y");
zlabel("Z");
title("Trajectory");

set(gcf, 'renderer', 'painters');
print("-dpng", "relv_traj.png");
```

4 Results

4.1 Terminal output

```
(escape) devansh@ds:~/GitHub/Vault/OctaveLab/Programs/outputs$ octave ../RelativisticRungaKuttaMotionInEM.m
Intial parameters
B =

    0    0   10

E =

   0.00000   0.00000   0.10000

pos =

   1   0   0

V =

   0.00000   0.95000   0.10000
```

4.2 Plots

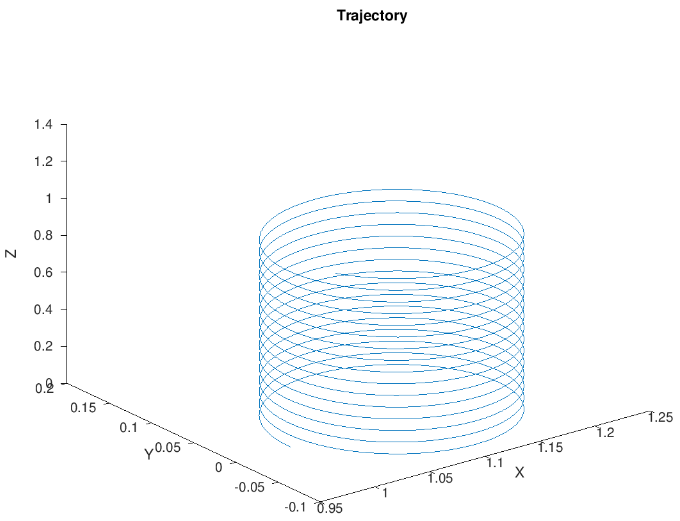

Figure 1: $B = 10\hat{k}$; $E = 0$; $v = 0.95\hat{\jmath}\ +\ 0.1\hat{k}$; $r = \hat{\imath}$; $q = 1C$; $m_0 = 1kg$

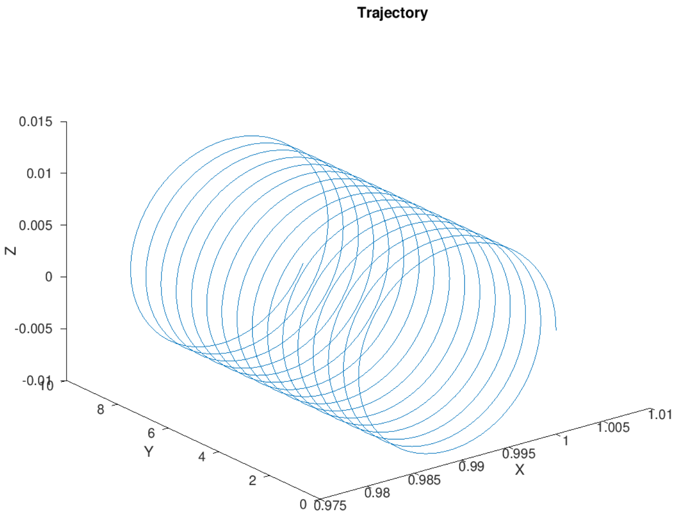

Figure 2: $B = 10\hat{j}$; $E = 0$; $v = 0.95\hat{j}\ +\ 0.1\hat{k}$; $r = \hat{i}$; $q = 1C$; $m_0 = 1kg$

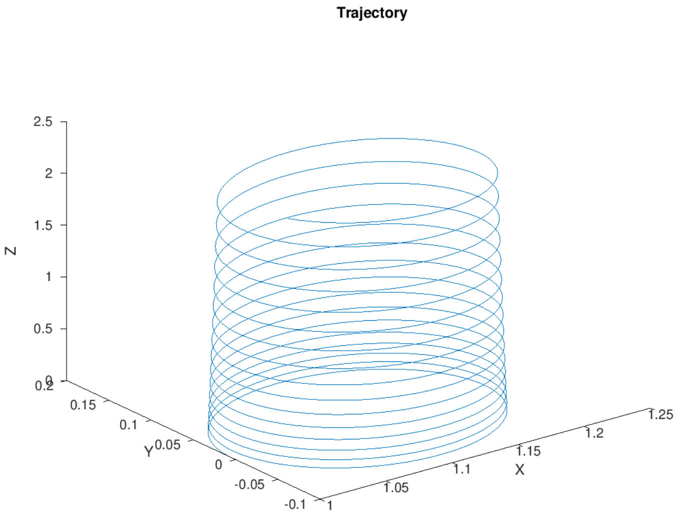

Figure 3: $B = 10\hat{k}$; $E = 0.1\hat{k}$; $v = 0.95\hat{j}\ +\ 0.1\hat{k}$; $r = \hat{i}$; $q = 1C$; $m_0 = 1kg$

5 Remarks

The programs can be used to trace and simulate the motion of any relativistic particle in an electromagnetic field by defining the required parameters.