Program-M7

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Write and execute a FORTRAN program for computing motion of a particle trapped in a 2-dimensional box.

1 Theory

1.1 Particle in a 2D box

The particle moves freely on the xy plane for $0 < x < L_x$ and $0 < y < L_y$; when it reaches the boundries $x = 0$, $x = L_x$, $y = 0$ and $y = L_y$, it bounces and its velocity instantly reverses. The potential energy is given by (1),

$$V(x,\ y) = \begin{cases} 0 & 0 < x < L_x,\ 0 < y < L_y \\ +\infty & \text{elsewhere} \end{cases} \tag{1}$$

Initially, we know that the particle's position $(x_0,\ y_0)$ with velocity $(v_x,\ v_y)$ at time t_0. Equation of motions are:

$$\begin{aligned} x(t\ +\ dt) &= x(t)\ +\ v_x(t)dt \\ y(t\ +\ dt) &= y(t)\ +\ v_y(t)dt \end{aligned} \tag{2}$$

When the particle bounces the boundary the velocity's direction reverses, $v(t) \to -v(t)$.

2 Numerical Solution

Assuming,

$$\begin{aligned} x_0 &= y_0 = 0 \\ v_x &= v_y = 1 \text{ m/s} \\ t_0 &= 0 \\ L_x &= 10\ m \\ L_y &= 8\ m \end{aligned} \tag{3}$$

We expect

$$v_x(t) \to -v_x(t) \text{ at } x = 10,\ 20,\ 30,\ \ldots \tag{4}$$

$$v_y(t) \to -v_y(t) \text{ at } y = 8,\ 16,\ 24,\ 32,\ \ldots \tag{5}$$

3 Program Algorithm

NOTE: Blue-colored text represents variables in the algorithm, eg. variable.

1. Program open.
2. Define variables (x0, y0, t0, tf, dt, t, x, y, vx, vy, lx0, lx1, ly0, ly1).
3. Open a writable data file.
4. Get input from user for initial position (x0, y0), inital velocity(vx, vy), bounds (lx0, lx1, ly0, ly1) and time period(t0, tf, dt).
5. Print parameters to stdout for the user.
6. Write appropiate comments in the data file and initialize other parameters.
7. Compute x, y and t for the initial position.
8. Define a do while loop with index t which runs from t0 to tf.
9. Compute the parameters x, y.
10. Write the parameters to stdout and data file.
11. If x reaches lx0 or lx1 $\implies$ vx = -vx.
12. If y reaches lx0 or ly1 $\implies$ vy = -vy.
13. Increment the index according to t = t + dt
14. End do-while loop.
15. Close data file.
16. Program close.

4 Program

4.1 Fortran program:

For computing the parameters

```
!=========================================
! particle2d.f90
! Author: Devansh Shukla
!-----------------------------------------
program particle_2D
    ! Program to compute motion of a particle moving in a circle with uniform velocity

    implicit none
    real*8 :: x0, y0, t0, tf, dt, t
    real*8 :: x, y, vx, vy, lx0, lx1, ly0, ly1
    character(len=*), parameter :: fmt1 = "(F10.4,x,F10.4,x,F10.4,x,F10.4,x,F10.4)"

    open(unit=8, file="Particle2D.dat")

    print *, "---------------------------------------"
    print *, "Enter initial position x0, y0"
    read *, x0, y0

    print *, "Enter velocity vx, vy"
    read *, vx, vy

    print *, "Enter bounds lx0, lx1"
    read *, lx0, lx1

    print *, "Enter bounds ly0, ly1"
    read *, ly0, ly1

    print *, "Enter t0, tf, dt"
    read *, t0, tf, dt
    print *, "---------------------------------------"

    print "(x,A,F10.4,F10.4)", "x0, y0=", x0, y0
    print "(x,A,F10.4,F10.4)", "lx0, lx1=", lx0, lx1
    print "(x,A,F10.4,F10.4)", "ly0, ly1=", ly0, ly1
    print "(x,A,F10.4,F10.4,F10.4)", "t0, tf, dt=", t0, tf, dt
    print *, "---------------------------------------"

    print "(A10,A10,A10,xA10,xxA10)", "time", "x(t)", "y(t)", "vx(t)", "vy(t)"
    t = t0
    x = x0
    y = y0
    ! Formatting l0, l1 to our standards for fortran floating point arithmatic
    lx1 = lx1 - dt
    ly1 = ly1 - dt
    do while (t <= tf)
        write (*, fmt1) t, x, y, vx, vy
        write (8, fmt1) t, x, y, vx, vy
        x = x + vx * dt
        y = y + vy * dt
        t = t + dt
        if (x < lx0 .or. x > lx1) vx = -vx
        if (y < ly0 .or. y > ly1) vy = -vy
    enddo
    print *, "---------------------------------------"
    close(8)
end program particle_2D
```

4.2 Python program: Plots

```
#!/usr/bin/env python
"""
Author: Devansh Shukla
"""
# In[0]
import pandas as pd
import numpy as np
import matplotlib as mpl
import matplotlib.pyplot as plt
import matplotlib.gridspec as gridspec

custom_rcparams = {
    "axes.labelsize": 8,
```

```
    "axes.titlesize": 10,
    "axes.grid": True,
    # Figure
    "figure.figsize": (9, 6),
    "figure.autolayout": True,
    "figure.titlesize": 10,
    "savefig.format": "pdf",
    "lines.linewidth": 1,
    # Legend
    "legend.fontsize": 8,
    "legend.frameon": True,
    # Ticks
    "xtick.labelsize": 7,
    "ytick.labelsize": 7,
    "xtick.minor.visible": True,
    "xtick.direction": "in",
    "ytick.direction": "in",
    "ytick.minor.visible": True,
    # TeX
    "pgf.texsystem": "lualatex",
}
mpl.rcParams.update(custom_rcparams)
mpl.use("pgf")
plt.ioff()

# t, x, y, vx, vy
df = pd.read_csv("Particle2D.dat", engine="python", delimiter=" ", header=None, skipinitialspace=True, comment="#")

time = df[0].values
pos_x = df[1].values
pos_y = df[2].values

gs = gridspec.GridSpec(2, 2)

ax = plt.subplot(gs[0, 0])
ax.plot(df[0], df[1], "o-", markersize=1.5, color="C0", label=r"$x(t)$")
ax.plot(df[0], df[2], "o-", markersize=1.5, color="C1", label=r"$y(t)$")
ax.set_xlim(left=0)
ax.set_ylim(0, 12)
ax.set_xlabel(r"$Time(s)$")
ax.set_ylabel(r"$Position(m)$")
ax.legend(loc="upper left")
plt.title("Position")

ax = plt.subplot(gs[0, 1])
ax.plot(df[0], df[3], "o-", markersize=1.5, color="C0", label=r"$v_{x}(t)$")
ax.plot(df[0], df[4], "o-", markersize=1.5, color="C1", label=r"$v_{y}(t)$")
ax.set_xlim(left=0)
ax.set_ylim(-1.5, 1.5)
ax.set_xlabel(r"$Time(s)$")
ax.set_ylabel(r"$Velocity(m/s)$")
ax.legend(loc="center right")
plt.title("Velocity")

ax = plt.subplot(gs[1, :])
ax.plot(df[1], df[2], "o-", markersize=1.5, color="C0", label="trace")
ax.set_xlim(left=pos_x.min()-1, right=pos_x.max()+1)
ax.set_ylim(pos_y.min()-1, pos_y.max()+1)
ax.vlines(pos_x.max(), pos_y.min(), pos_y.max(), "red", label=rf"$x={pos_x.min()},{pos_x.max()}$")
ax.vlines(pos_x.min(), pos_y.min(), pos_y.max(), "red")
ax.hlines(pos_y.min(), pos_x.min(), pos_x.max(), "red", label=rf"$y={pos_y.min()},{pos_y.max()}$")
ax.hlines(pos_y.max(), pos_x.min(), pos_x.max(), "red")
ax.legend(loc="lower right")
ax.set_xlabel(r"$X$")
ax.set_ylabel(r"$Y$")
plt.title("Trajectory")

plt.savefig("plots/2d.pdf")
# plt.show()

# %%
```

4.3 Python program: Animation

```
#!/usr/bin/env python
"""
Author: Devansh Shukla
"""
import pandas as pd
import numpy as np
```

```
import matplotlib as mpl
import matplotlib.pyplot as plt
from matplotlib.animation import FuncAnimation, FFMpegWriter
import matplotlib.gridspec as gridspec

custom_rcparams = {
    "axes.labelsize": 7,
    "axes.titlesize": 8,
    "axes.grid": True,
    # Figure
    "figure.autolayout": True,
    "figure.titlesize": 9,
    "figure.figsize": (10, 4),
    "savefig.format": "pdf",
    "lines.linewidth": 1,
    # Legend
    "legend.fontsize": 8,
    "legend.frameon": True,
    # Ticks
    "xtick.labelsize": 8,
    "ytick.labelsize": 8,
    "xtick.minor.visible": True,
    "xtick.direction": "in",
    "ytick.direction": "in",
    "ytick.minor.visible": True,
}
mpl.rcParams.update(custom_rcparams)

# t, x, y, vx, vy
df = pd.read_csv("Particle2D.dat", engine="python", delimiter=" ", header=None, skipinitialspace=True, comment="#")

time = df[0].values
pos_x = df[1].values
pos_y = df[2].values
vel_x = df[3].values
vel_y = df[4].values

gs = gridspec.GridSpec(1, 2, width_ratios=[2, 1], hspace=0)

fig = plt.figure()
ax1 = plt.subplot(gs[0, 0])
ax2 = plt.subplot(gs[0, 1])
plt.tight_layout()

line1, = ax1.plot([], [], 'o', lw=2, label="particle")
trace, = ax1.plot([], [], '-', lw=1, label="trace")
time_template = "time = %.2fs"
time_text = ax1.text(0.05, 0.8, '', transform=ax1.transAxes)
line_arrow = ax1.plot([], [], "-", color="C4", label=r"$v$")

patch = plt.Arrow(pos_x[0], pos_y[0], vel_x[0], vel_y[0], width=0.15, color="C4")
ax1.add_patch(patch)

line_vx, = ax2.plot([], [], '-', lw=2, label=r"$v_{x}(t)$")
line_vy, = ax2.plot([], [], '-', lw=2, label=r"$v_{y}(t)$")
ax2.legend(loc="upper right")

line = [line1, line_vx, line_vy,]
ax1.set_xlim(left=pos_x.min()-1, right=pos_x.max()+1)
ax1.set_ylim(pos_y.min()-1, pos_y.max()+1)
ax1.vlines(pos_x.max(), pos_y.min(), pos_y.max(), "red", label=rf"$x={pos_x.min()},{pos_x.max()}$")
ax1.vlines(pos_x.min(), pos_y.min(), pos_y.max(), "red")
ax1.hlines(pos_y.min(), pos_x.min(), pos_x.max(), "red", label=rf"$y={pos_y.min()},{pos_y.max()}$")
ax1.hlines(pos_y.max(), pos_x.min(), pos_x.max(), "red")
ax1.set_xlabel("X")
ax1.set_ylabel("Y")
ax1.legend(loc="lower right")

ax2.set_xlim(0, time[-1]+1)
ax2.set_ylim(-2, 2)
ax2.set_ylabel(r"$v(m/s)$")
ax2.set_xlabel("Time(s)")

def init():
    line[0].set_data([], [])
    trace.set_data([], [])
    return line, trace

def animate(i):
```

```
    global time, pos_x, pos_y, vel_x, vel_y, ax2
    line[0].set_data(pos_x[i], pos_y[i])
    trace.set_data(pos_x[:i], pos_y[:i])
    time_text.set_text(time_template % (time[i]))

    line[1].set_data(time[:i], vel_x[:i])
    line[2].set_data(time[:i], vel_y[:i])
    global ax1, patch
    ax1.patches.remove(patch)
    patch = plt.Arrow(pos_x[i], pos_y[i], vel_x[i], vel_y[i], width=0.15, color="C4")
    ax1.add_patch(patch)

    if time[i] in [0.5, 2.0, 4.0, 6.7, 8.9, 12.2, 14.0, 19.6, 21.7, 25.0, 28.2, 29.9, 32.7, 36.0, 42.2, 46.0, 50.0, 52.0,
          55.0, 58.0, 59.9]:
        toggle_capture()

    if time[i] == 30.0:
        ax2.set_xlim(0, 60)
    return line, trace, time_text

def toggle_capture(*args, **kwargs):
    global ani, capture_no
    ani.pause()
    plt.gcf().savefig(f"plots/2d_{capture_no}.pdf")
    capture_no += 1
    ani.resume()

capture_no = 0
ani = FuncAnimation(fig, animate, frames=len(time), interval=1, init_func=init, blit=False, repeat=False)
fig.canvas.mpl_connect('button_press_event', toggle_capture)
writer = FFMpegWriter(fps=10)
ani.save('animation.mp4', writer=writer)
plt.show()
```

5 Results

5.1 Terminal Output

```
 ----------------------------------------
 Enter initial position x0, y0
0.0 0.0
 Enter velocity vx, vy
1.0 1.0
 Enter bounds lx0, lx1
0.0 10.0
 Enter bounds ly0, ly1
0.0 8.0
 Enter t0, tf, dt
0.0 30.0 0.1
 ----------------------------------------
 x0, y0=    0.0000    0.0000
 lx0, lx1=    0.0000   10.0000
 ly0, ly1=    0.0000    8.0000
 t0, tf, dt=    0.0000   30.0000    0.1000
 ----------------------------------------
      time      x(t)      y(t)      vx(t)       vy(t)
    0.0000    0.0000    0.0000     1.0000     1.0000
    0.1000    0.1000    0.1000     1.0000     1.0000
    0.2000    0.2000    0.2000     1.0000     1.0000
    0.3000    0.3000    0.3000     1.0000     1.0000
    0.4000    0.4000    0.4000     1.0000     1.0000
    0.5000    0.5000    0.5000     1.0000     1.0000
    0.6000    0.6000    0.6000     1.0000     1.0000
    0.7000    0.7000    0.7000     1.0000     1.0000
    0.8000    0.8000    0.8000     1.0000     1.0000
    0.9000    0.9000    0.9000     1.0000     1.0000
    1.0000    1.0000    1.0000     1.0000     1.0000
    1.1000    1.1000    1.1000     1.0000     1.0000
    1.2000    1.2000    1.2000     1.0000     1.0000
    1.3000    1.3000    1.3000     1.0000     1.0000
    1.4000    1.4000    1.4000     1.0000     1.0000
    1.5000    1.5000    1.5000     1.0000     1.0000
    1.6000    1.6000    1.6000     1.0000     1.0000
    1.7000    1.7000    1.7000     1.0000     1.0000
    1.8000    1.8000    1.8000     1.0000     1.0000
    1.9000    1.9000    1.9000     1.0000     1.0000
    2.0000    2.0000    2.0000     1.0000     1.0000
    2.1000    2.1000    2.1000     1.0000     1.0000
    2.2000    2.2000    2.2000     1.0000     1.0000
```

```
   2.3000    2.3000    2.3000    1.0000    1.0000
   2.4000    2.4000    2.4000    1.0000    1.0000
   2.5000    2.5000    2.5000    1.0000    1.0000
   2.6000    2.6000    2.6000    1.0000    1.0000
   2.7000    2.7000    2.7000    1.0000    1.0000
   2.8000    2.8000    2.8000    1.0000    1.0000
   2.9000    2.9000    2.9000    1.0000    1.0000
   3.0000    3.0000    3.0000    1.0000    1.0000
   3.1000    3.1000    3.1000    1.0000    1.0000
   3.2000    3.2000    3.2000    1.0000    1.0000
   3.3000    3.3000    3.3000    1.0000    1.0000
   3.4000    3.4000    3.4000    1.0000    1.0000
   3.5000    3.5000    3.5000    1.0000    1.0000
   3.6000    3.6000    3.6000    1.0000    1.0000
   3.7000    3.7000    3.7000    1.0000    1.0000
   3.8000    3.8000    3.8000    1.0000    1.0000
   3.9000    3.9000    3.9000    1.0000    1.0000
   4.0000    4.0000    4.0000    1.0000    1.0000
   4.1000    4.1000    4.1000    1.0000    1.0000
   4.2000    4.2000    4.2000    1.0000    1.0000
   4.3000    4.3000    4.3000    1.0000    1.0000
   4.4000    4.4000    4.4000    1.0000    1.0000
   4.5000    4.5000    4.5000    1.0000    1.0000
   4.6000    4.6000    4.6000    1.0000    1.0000
   4.7000    4.7000    4.7000    1.0000    1.0000
   4.8000    4.8000    4.8000    1.0000    1.0000
   4.9000    4.9000    4.9000    1.0000    1.0000
   5.0000    5.0000    5.0000    1.0000    1.0000
   5.1000    5.1000    5.1000    1.0000    1.0000
   5.2000    5.2000    5.2000    1.0000    1.0000
   5.3000    5.3000    5.3000    1.0000    1.0000
   5.4000    5.4000    5.4000    1.0000    1.0000
   5.5000    5.5000    5.5000    1.0000    1.0000
   5.6000    5.6000    5.6000    1.0000    1.0000
   5.7000    5.7000    5.7000    1.0000    1.0000
   5.8000    5.8000    5.8000    1.0000    1.0000
   5.9000    5.9000    5.9000    1.0000    1.0000
   6.0000    6.0000    6.0000    1.0000    1.0000
   6.1000    6.1000    6.1000    1.0000    1.0000
   6.2000    6.2000    6.2000    1.0000    1.0000
   6.3000    6.3000    6.3000    1.0000    1.0000
   6.4000    6.4000    6.4000    1.0000    1.0000
   6.5000    6.5000    6.5000    1.0000    1.0000
   6.6000    6.6000    6.6000    1.0000    1.0000
   6.7000    6.7000    6.7000    1.0000    1.0000
   6.8000    6.8000    6.8000    1.0000    1.0000
   6.9000    6.9000    6.9000    1.0000    1.0000
   7.0000    7.0000    7.0000    1.0000    1.0000
   7.1000    7.1000    7.1000    1.0000    1.0000
   7.2000    7.2000    7.2000    1.0000    1.0000
   7.3000    7.3000    7.3000    1.0000    1.0000
   7.4000    7.4000    7.4000    1.0000    1.0000
   7.5000    7.5000    7.5000    1.0000    1.0000
   7.6000    7.6000    7.6000    1.0000    1.0000
   7.7000    7.7000    7.7000    1.0000    1.0000
   7.8000    7.8000    7.8000    1.0000    1.0000
   7.9000    7.9000    7.9000    1.0000    1.0000
   8.0000    8.0000    8.0000    1.0000   -1.0000
   8.1000    8.1000    7.9000    1.0000   -1.0000
   8.2000    8.2000    7.8000    1.0000   -1.0000
   8.3000    8.3000    7.7000    1.0000   -1.0000
   8.4000    8.4000    7.6000    1.0000   -1.0000
   8.5000    8.5000    7.5000    1.0000   -1.0000
   8.6000    8.6000    7.4000    1.0000   -1.0000
   8.7000    8.7000    7.3000    1.0000   -1.0000
   8.8000    8.8000    7.2000    1.0000   -1.0000
   8.9000    8.9000    7.1000    1.0000   -1.0000
   9.0000    9.0000    7.0000    1.0000   -1.0000
   9.1000    9.1000    6.9000    1.0000   -1.0000
   9.2000    9.2000    6.8000    1.0000   -1.0000
   9.3000    9.3000    6.7000    1.0000   -1.0000
   9.4000    9.4000    6.6000    1.0000   -1.0000
   9.5000    9.5000    6.5000    1.0000   -1.0000
   9.6000    9.6000    6.4000    1.0000   -1.0000
   9.7000    9.7000    6.3000    1.0000   -1.0000
   9.8000    9.8000    6.2000    1.0000   -1.0000
   9.9000    9.9000    6.1000    1.0000   -1.0000
  10.0000   10.0000    6.0000   -1.0000   -1.0000
  10.1000    9.9000    5.9000   -1.0000   -1.0000
```

```
  10.2000    9.8000    5.8000   -1.0000   -1.0000
  10.3000    9.7000    5.7000   -1.0000   -1.0000
  10.4000    9.6000    5.6000   -1.0000   -1.0000
  10.5000    9.5000    5.5000   -1.0000   -1.0000
  10.6000    9.4000    5.4000   -1.0000   -1.0000
  10.7000    9.3000    5.3000   -1.0000   -1.0000
  10.8000    9.2000    5.2000   -1.0000   -1.0000
  10.9000    9.1000    5.1000   -1.0000   -1.0000
  11.0000    9.0000    5.0000   -1.0000   -1.0000
  11.1000    8.9000    4.9000   -1.0000   -1.0000
  11.2000    8.8000    4.8000   -1.0000   -1.0000
  11.3000    8.7000    4.7000   -1.0000   -1.0000
  11.4000    8.6000    4.6000   -1.0000   -1.0000
  11.5000    8.5000    4.5000   -1.0000   -1.0000
  11.6000    8.4000    4.4000   -1.0000   -1.0000
  11.7000    8.3000    4.3000   -1.0000   -1.0000
  11.8000    8.2000    4.2000   -1.0000   -1.0000
  11.9000    8.1000    4.1000   -1.0000   -1.0000
  12.0000    8.0000    4.0000   -1.0000   -1.0000
  12.1000    7.9000    3.9000   -1.0000   -1.0000
  12.2000    7.8000    3.8000   -1.0000   -1.0000
  12.3000    7.7000    3.7000   -1.0000   -1.0000
  12.4000    7.6000    3.6000   -1.0000   -1.0000
  12.5000    7.5000    3.5000   -1.0000   -1.0000
  12.6000    7.4000    3.4000   -1.0000   -1.0000
  12.7000    7.3000    3.3000   -1.0000   -1.0000
  12.8000    7.2000    3.2000   -1.0000   -1.0000
  12.9000    7.1000    3.1000   -1.0000   -1.0000
  13.0000    7.0000    3.0000   -1.0000   -1.0000
  13.1000    6.9000    2.9000   -1.0000   -1.0000
  13.2000    6.8000    2.8000   -1.0000   -1.0000
  13.3000    6.7000    2.7000   -1.0000   -1.0000
  13.4000    6.6000    2.6000   -1.0000   -1.0000
  13.5000    6.5000    2.5000   -1.0000   -1.0000
  13.6000    6.4000    2.4000   -1.0000   -1.0000
  13.7000    6.3000    2.3000   -1.0000   -1.0000
  13.8000    6.2000    2.2000   -1.0000   -1.0000
  13.9000    6.1000    2.1000   -1.0000   -1.0000
  14.0000    6.0000    2.0000   -1.0000   -1.0000
  14.1000    5.9000    1.9000   -1.0000   -1.0000
  14.2000    5.8000    1.8000   -1.0000   -1.0000
  14.3000    5.7000    1.7000   -1.0000   -1.0000
  14.4000    5.6000    1.6000   -1.0000   -1.0000
  14.5000    5.5000    1.5000   -1.0000   -1.0000
  14.6000    5.4000    1.4000   -1.0000   -1.0000
  14.7000    5.3000    1.3000   -1.0000   -1.0000
  14.8000    5.2000    1.2000   -1.0000   -1.0000
  14.9000    5.1000    1.1000   -1.0000   -1.0000
  15.0000    5.0000    1.0000   -1.0000   -1.0000
  15.1000    4.9000    0.9000   -1.0000   -1.0000
  15.2000    4.8000    0.8000   -1.0000   -1.0000
  15.3000    4.7000    0.7000   -1.0000   -1.0000
  15.4000    4.6000    0.6000   -1.0000   -1.0000
  15.5000    4.5000    0.5000   -1.0000   -1.0000
  15.6000    4.4000    0.4000   -1.0000   -1.0000
  15.7000    4.3000    0.3000   -1.0000   -1.0000
  15.8000    4.2000    0.2000   -1.0000   -1.0000
  15.9000    4.1000    0.1000   -1.0000   -1.0000
  16.0000    4.0000   -0.0000   -1.0000    1.0000
  16.1000    3.9000    0.1000   -1.0000    1.0000
  16.2000    3.8000    0.2000   -1.0000    1.0000
  16.3000    3.7000    0.3000   -1.0000    1.0000
  16.4000    3.6000    0.4000   -1.0000    1.0000
  16.5000    3.5000    0.5000   -1.0000    1.0000
  16.6000    3.4000    0.6000   -1.0000    1.0000
  16.7000    3.3000    0.7000   -1.0000    1.0000
  16.8000    3.2000    0.8000   -1.0000    1.0000
  16.9000    3.1000    0.9000   -1.0000    1.0000
  17.0000    3.0000    1.0000   -1.0000    1.0000
  17.1000    2.9000    1.1000   -1.0000    1.0000
  17.2000    2.8000    1.2000   -1.0000    1.0000
  17.3000    2.7000    1.3000   -1.0000    1.0000
  17.4000    2.6000    1.4000   -1.0000    1.0000
  17.5000    2.5000    1.5000   -1.0000    1.0000
  17.6000    2.4000    1.6000   -1.0000    1.0000
  17.7000    2.3000    1.7000   -1.0000    1.0000
  17.8000    2.2000    1.8000   -1.0000    1.0000
  17.9000    2.1000    1.9000   -1.0000    1.0000
  18.0000    2.0000    2.0000   -1.0000    1.0000
```

```
  18.1000    1.9000    2.1000   -1.0000    1.0000
  18.2000    1.8000    2.2000   -1.0000    1.0000
  18.3000    1.7000    2.3000   -1.0000    1.0000
  18.4000    1.6000    2.4000   -1.0000    1.0000
  18.5000    1.5000    2.5000   -1.0000    1.0000
  18.6000    1.4000    2.6000   -1.0000    1.0000
  18.7000    1.3000    2.7000   -1.0000    1.0000
  18.8000    1.2000    2.8000   -1.0000    1.0000
  18.9000    1.1000    2.9000   -1.0000    1.0000
  19.0000    1.0000    3.0000   -1.0000    1.0000
  19.1000    0.9000    3.1000   -1.0000    1.0000
  19.2000    0.8000    3.2000   -1.0000    1.0000
  19.3000    0.7000    3.3000   -1.0000    1.0000
  19.4000    0.6000    3.4000   -1.0000    1.0000
  19.5000    0.5000    3.5000   -1.0000    1.0000
  19.6000    0.4000    3.6000   -1.0000    1.0000
  19.7000    0.3000    3.7000   -1.0000    1.0000
  19.8000    0.2000    3.8000   -1.0000    1.0000
  19.9000    0.1000    3.9000   -1.0000    1.0000
  20.0000   -0.0000    4.0000    1.0000    1.0000
  20.1000    0.1000    4.1000    1.0000    1.0000
  20.2000    0.2000    4.2000    1.0000    1.0000
  20.3000    0.3000    4.3000    1.0000    1.0000
  20.4000    0.4000    4.4000    1.0000    1.0000
  20.5000    0.5000    4.5000    1.0000    1.0000
  20.6000    0.6000    4.6000    1.0000    1.0000
  20.7000    0.7000    4.7000    1.0000    1.0000
  20.8000    0.8000    4.8000    1.0000    1.0000
  20.9000    0.9000    4.9000    1.0000    1.0000
  21.0000    1.0000    5.0000    1.0000    1.0000
  21.1000    1.1000    5.1000    1.0000    1.0000
  21.2000    1.2000    5.2000    1.0000    1.0000
  21.3000    1.3000    5.3000    1.0000    1.0000
  21.4000    1.4000    5.4000    1.0000    1.0000
  21.5000    1.5000    5.5000    1.0000    1.0000
  21.6000    1.6000    5.6000    1.0000    1.0000
  21.7000    1.7000    5.7000    1.0000    1.0000
  21.8000    1.8000    5.8000    1.0000    1.0000
  21.9000    1.9000    5.9000    1.0000    1.0000
  22.0000    2.0000    6.0000    1.0000    1.0000
  22.1000    2.1000    6.1000    1.0000    1.0000
  22.2000    2.2000    6.2000    1.0000    1.0000
  22.3000    2.3000    6.3000    1.0000    1.0000
  22.4000    2.4000    6.4000    1.0000    1.0000
  22.5000    2.5000    6.5000    1.0000    1.0000
  22.6000    2.6000    6.6000    1.0000    1.0000
  22.7000    2.7000    6.7000    1.0000    1.0000
  22.8000    2.8000    6.8000    1.0000    1.0000
  22.9000    2.9000    6.9000    1.0000    1.0000
  23.0000    3.0000    7.0000    1.0000    1.0000
  23.1000    3.1000    7.1000    1.0000    1.0000
  23.2000    3.2000    7.2000    1.0000    1.0000
  23.3000    3.3000    7.3000    1.0000    1.0000
  23.4000    3.4000    7.4000    1.0000    1.0000
  23.5000    3.5000    7.5000    1.0000    1.0000
  23.6000    3.6000    7.6000    1.0000    1.0000
  23.7000    3.7000    7.7000    1.0000    1.0000
  23.8000    3.8000    7.8000    1.0000    1.0000
  23.9000    3.9000    7.9000    1.0000    1.0000
  24.0000    4.0000    8.0000    1.0000   -1.0000
  24.1000    4.1000    7.9000    1.0000   -1.0000
  24.2000    4.2000    7.8000    1.0000   -1.0000
  24.3000    4.3000    7.7000    1.0000   -1.0000
  24.4000    4.4000    7.6000    1.0000   -1.0000
  24.5000    4.5000    7.5000    1.0000   -1.0000
  24.6000    4.6000    7.4000    1.0000   -1.0000
  24.7000    4.7000    7.3000    1.0000   -1.0000
  24.8000    4.8000    7.2000    1.0000   -1.0000
  24.9000    4.9000    7.1000    1.0000   -1.0000
  25.0000    5.0000    7.0000    1.0000   -1.0000
  25.1000    5.1000    6.9000    1.0000   -1.0000
  25.2000    5.2000    6.8000    1.0000   -1.0000
  25.3000    5.3000    6.7000    1.0000   -1.0000
  25.4000    5.4000    6.6000    1.0000   -1.0000
  25.5000    5.5000    6.5000    1.0000   -1.0000
  25.6000    5.6000    6.4000    1.0000   -1.0000
  25.7000    5.7000    6.3000    1.0000   -1.0000
  25.8000    5.8000    6.2000    1.0000   -1.0000
  25.9000    5.9000    6.1000    1.0000   -1.0000
```

```
 26.0000     6.0000     6.0000     1.0000    -1.0000
 26.1000     6.1000     5.9000     1.0000    -1.0000
 26.2000     6.2000     5.8000     1.0000    -1.0000
 26.3000     6.3000     5.7000     1.0000    -1.0000
 26.4000     6.4000     5.6000     1.0000    -1.0000
 26.5000     6.5000     5.5000     1.0000    -1.0000
 26.6000     6.6000     5.4000     1.0000    -1.0000
 26.7000     6.7000     5.3000     1.0000    -1.0000
 26.8000     6.8000     5.2000     1.0000    -1.0000
 26.9000     6.9000     5.1000     1.0000    -1.0000
 27.0000     7.0000     5.0000     1.0000    -1.0000
 27.1000     7.1000     4.9000     1.0000    -1.0000
 27.2000     7.2000     4.8000     1.0000    -1.0000
 27.3000     7.3000     4.7000     1.0000    -1.0000
 27.4000     7.4000     4.6000     1.0000    -1.0000
 27.5000     7.5000     4.5000     1.0000    -1.0000
 27.6000     7.6000     4.4000     1.0000    -1.0000
 27.7000     7.7000     4.3000     1.0000    -1.0000
 27.8000     7.8000     4.2000     1.0000    -1.0000
 27.9000     7.9000     4.1000     1.0000    -1.0000
 28.0000     8.0000     4.0000     1.0000    -1.0000
 28.1000     8.1000     3.9000     1.0000    -1.0000
 28.2000     8.2000     3.8000     1.0000    -1.0000
 28.3000     8.3000     3.7000     1.0000    -1.0000
 28.4000     8.4000     3.6000     1.0000    -1.0000
 28.5000     8.5000     3.5000     1.0000    -1.0000
 28.6000     8.6000     3.4000     1.0000    -1.0000
 28.7000     8.7000     3.3000     1.0000    -1.0000
 28.8000     8.8000     3.2000     1.0000    -1.0000
 28.9000     8.9000     3.1000     1.0000    -1.0000
 29.0000     9.0000     3.0000     1.0000    -1.0000
 29.1000     9.1000     2.9000     1.0000    -1.0000
 29.2000     9.2000     2.8000     1.0000    -1.0000
 29.3000     9.3000     2.7000     1.0000    -1.0000
 29.4000     9.4000     2.6000     1.0000    -1.0000
 29.5000     9.5000     2.5000     1.0000    -1.0000
 29.6000     9.6000     2.4000     1.0000    -1.0000
 29.7000     9.7000     2.3000     1.0000    -1.0000
 29.8000     9.8000     2.2000     1.0000    -1.0000
 29.9000     9.9000     2.1000     1.0000    -1.0000
----------------------------------------
```

5.2 Plots

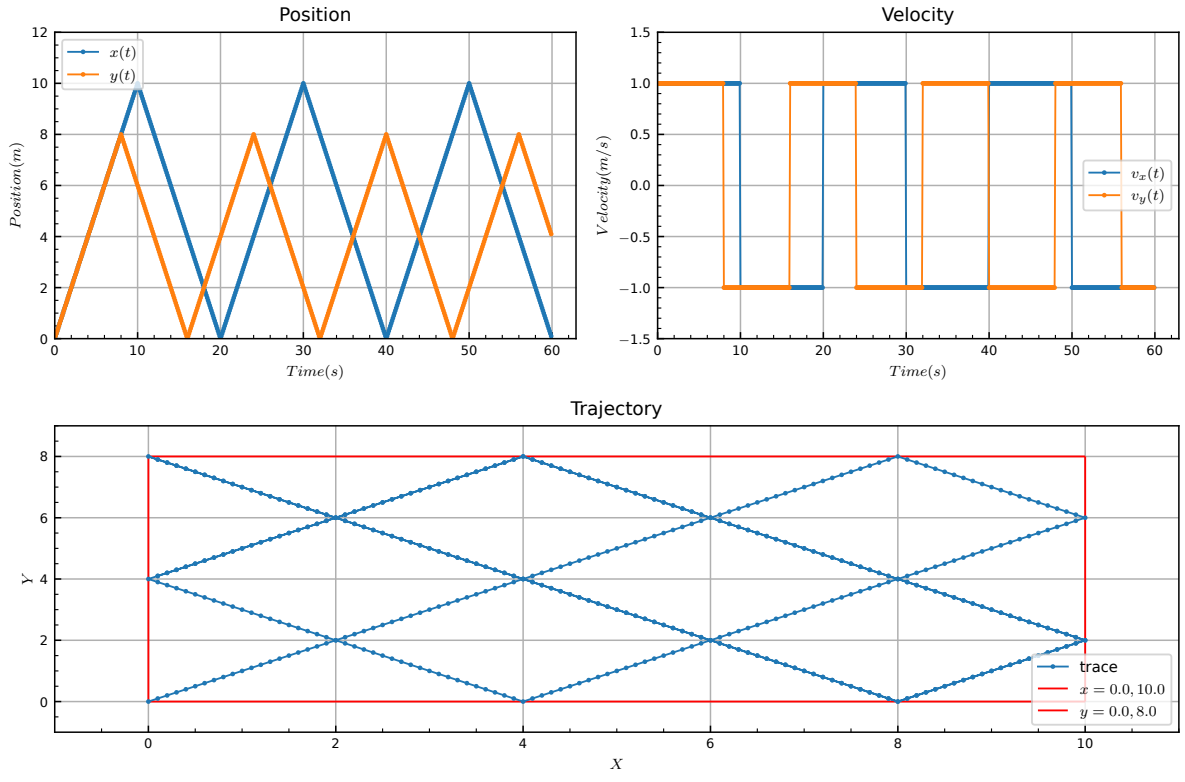
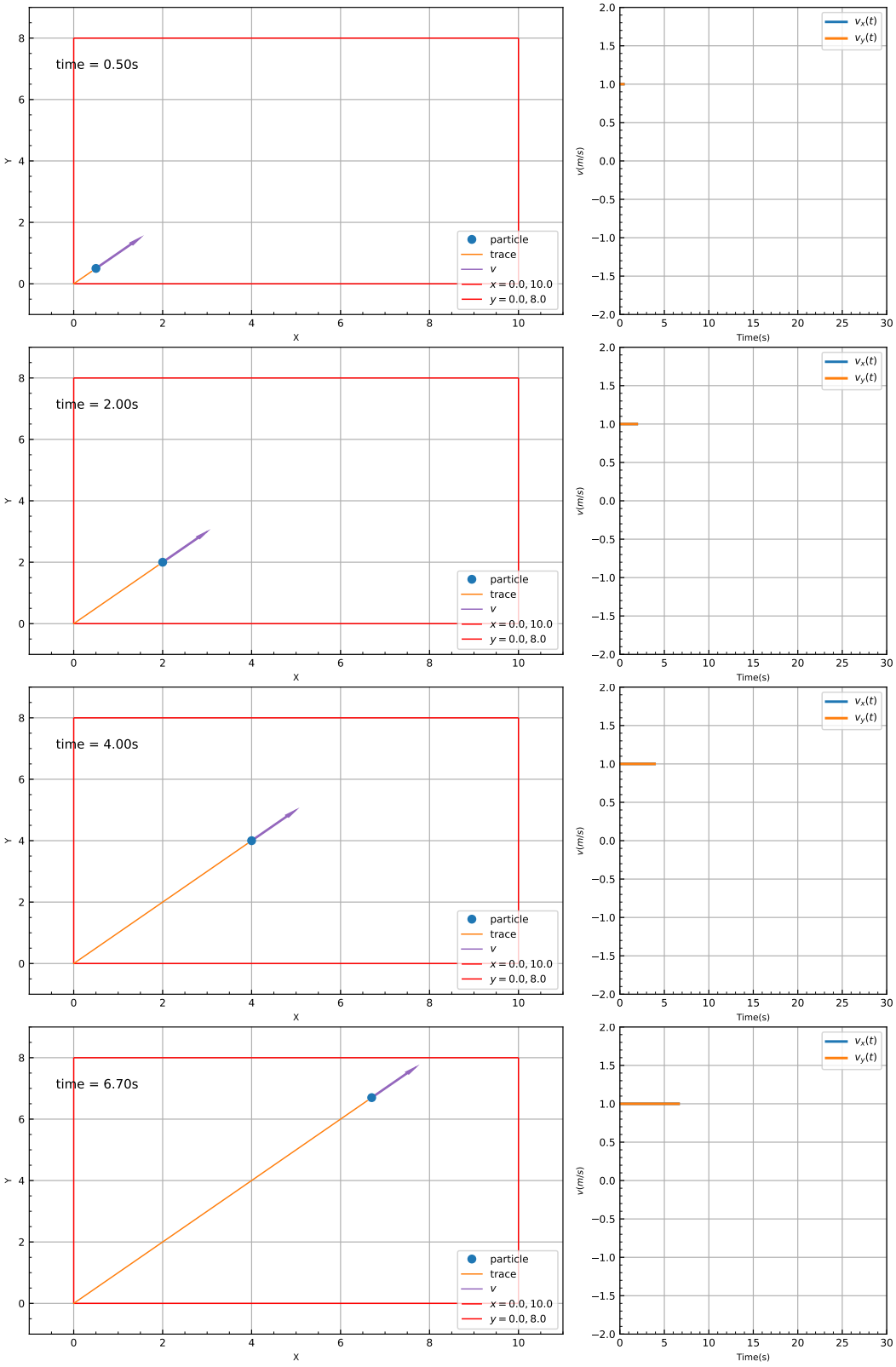


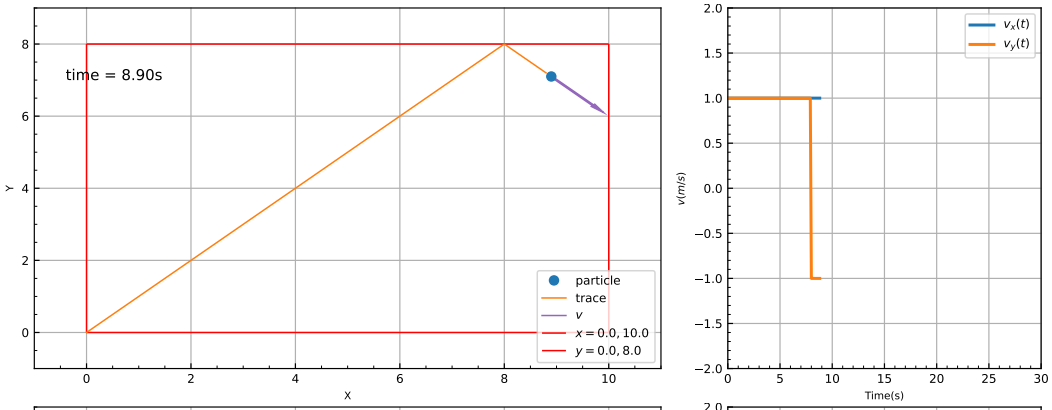
Figure 1: Particle in a 1D box

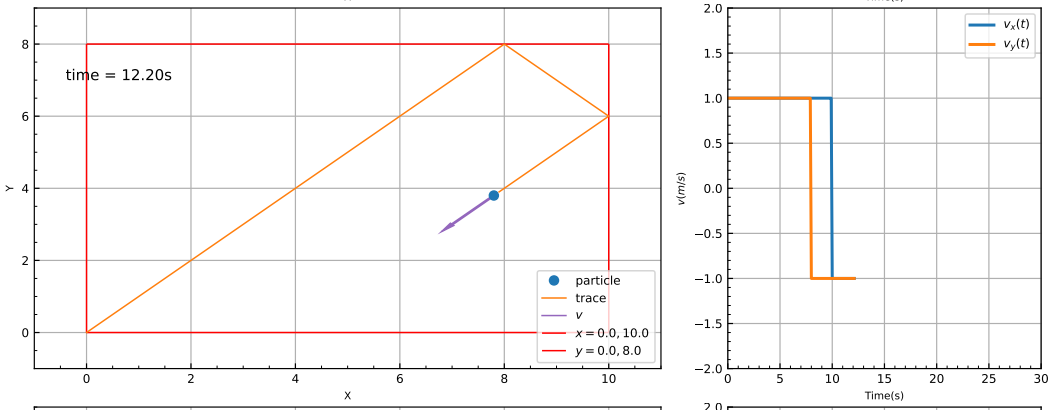
5.3 Animation

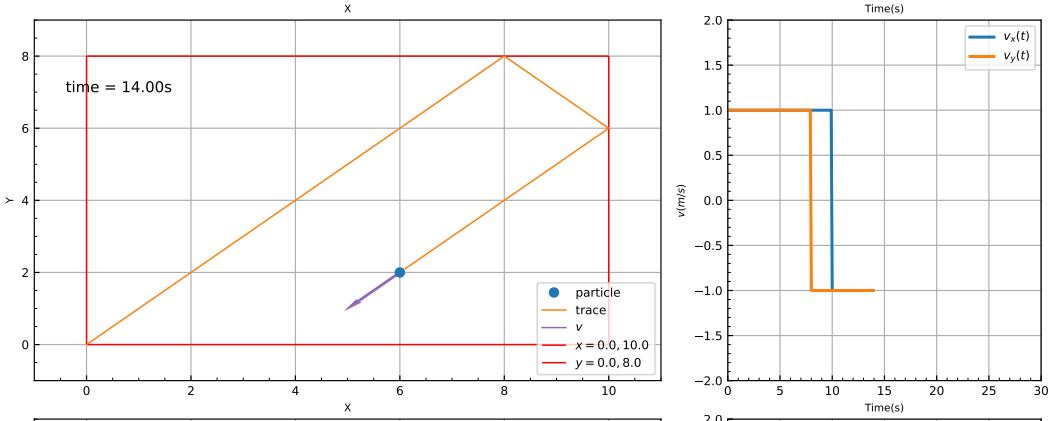
Note: Input parameters,

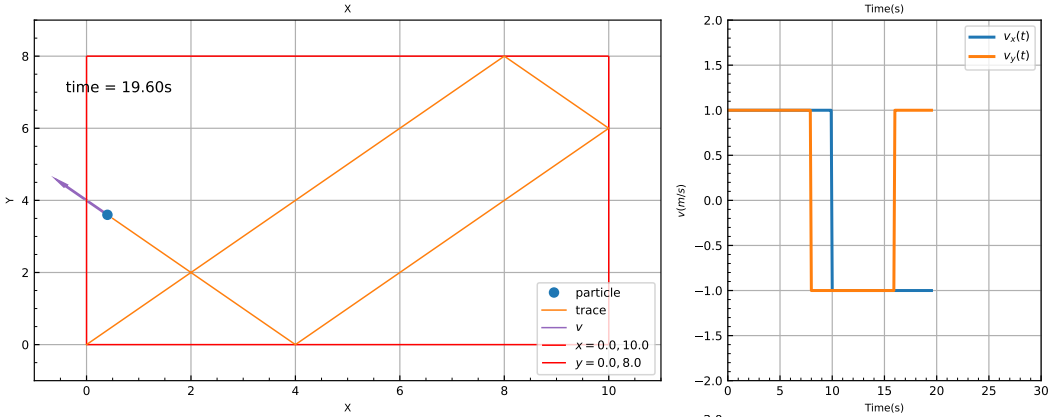
$$
\begin{aligned}
x_0 &= y_0 = 0 \\
t_0 &= 0.0;\ t_f = 30.0;\ dt = 0.1 \\
v_x &= v_y = 1.0 \\
lx0 &= 0.0;\ lx1 = 10.0 \\
ly0 &= 0.0;\ ly1 = 8.0
\end{aligned}
\tag{6}
$$

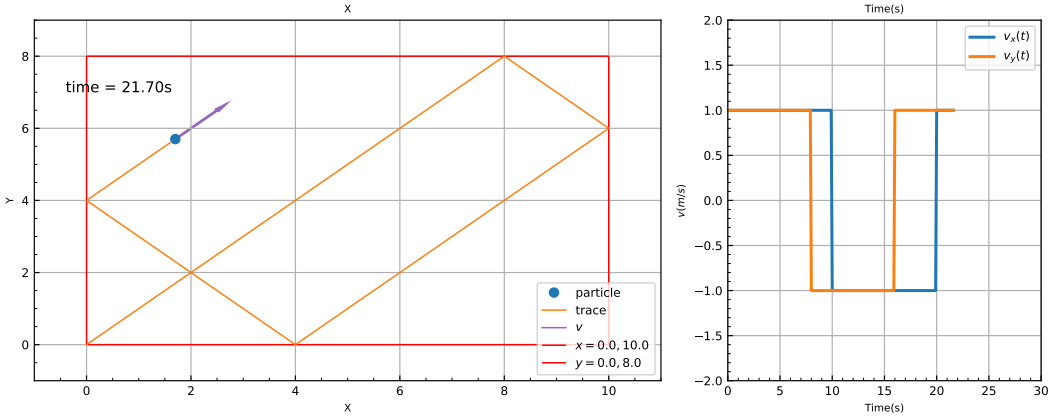


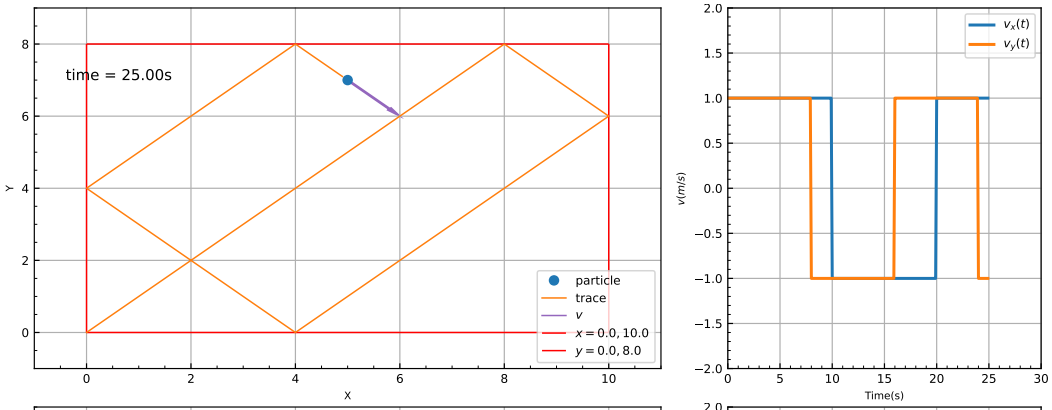

time = 8.90s
particle
trace
v
x = 0.0, 10.0
y = 0.0, 8.0
X
Y
$v_x(t)$
$v_y(t)$
v(m/s)
Time(s)

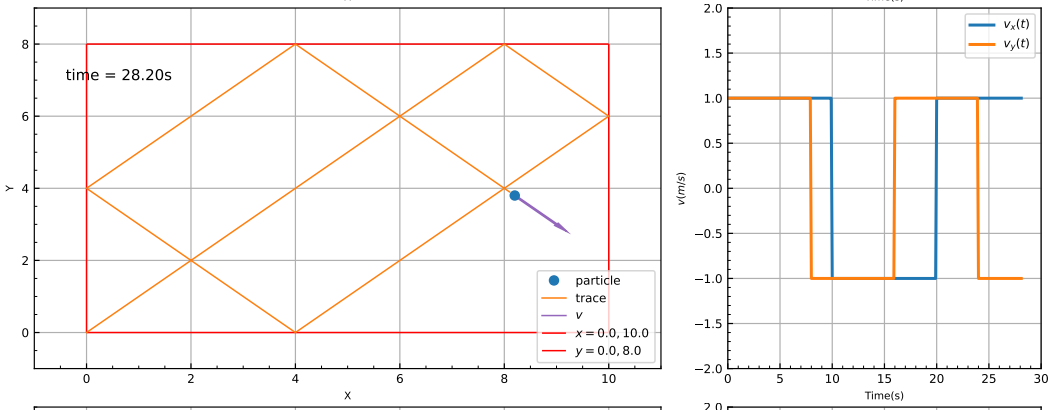

time = 12.20s
particle
trace
v
x = 0.0, 10.0
y = 0.0, 8.0
X
Y
$v_x(t)$
$v_y(t)$
v(m/s)
Time(s)

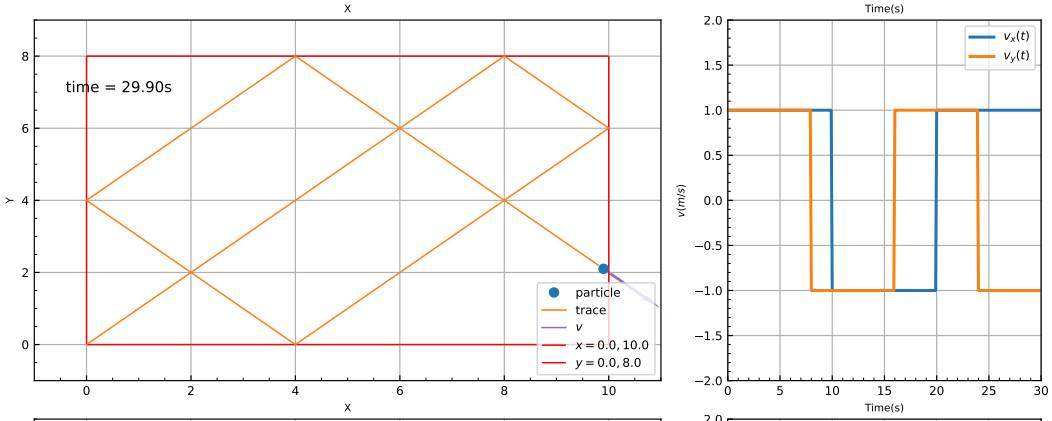

time = 14.00s
particle
trace
v
x = 0.0, 10.0
y = 0.0, 8.0
X
Y
$v_x(t)$
$v_y(t)$
v(m/s)
Time(s)

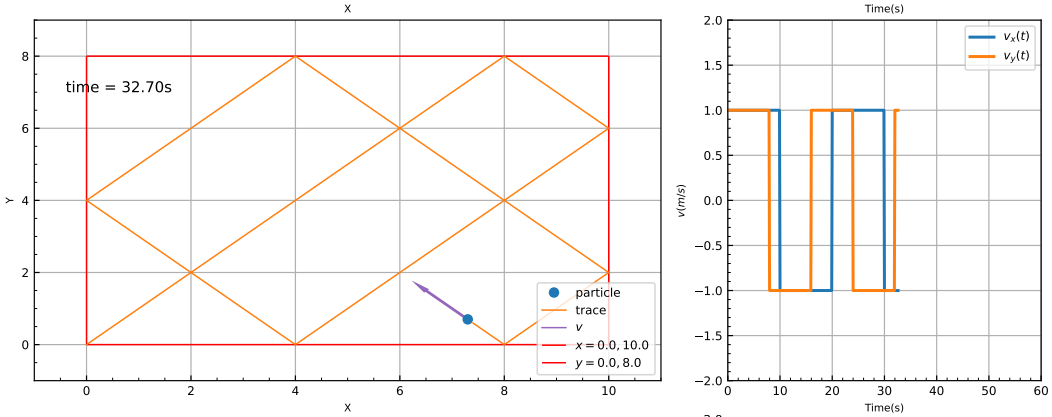

time = 19.60s
particle
trace
v
x = 0.0, 10.0
y = 0.0, 8.0
X
Y
$v_x(t)$
$v_y(t)$
v(m/s)
Time(s)

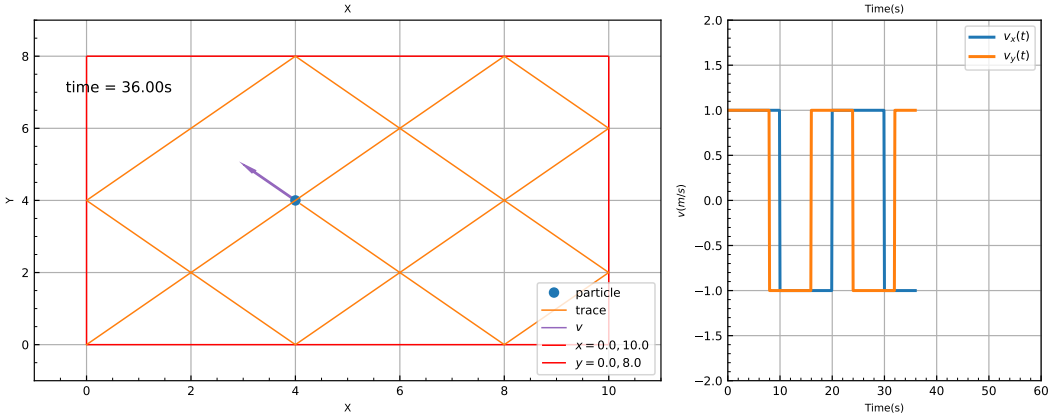

time = 21.70s
particle
trace
v
x = 0.0, 10.0
y = 0.0, 8.0
X
Y
$v_x(t)$
$v_y(t)$
v(m/s)
Time(s)

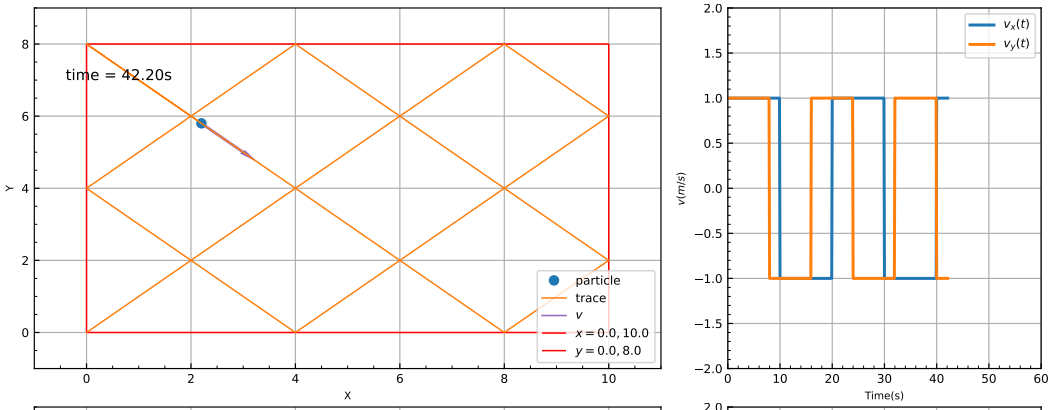

time = 25.00s
particle
trace
v
x = 0.0, 10.0
y = 0.0, 8.0
X
Y
$v_x(t)$
$v_y(t)$
v(m/s)
Time(s)

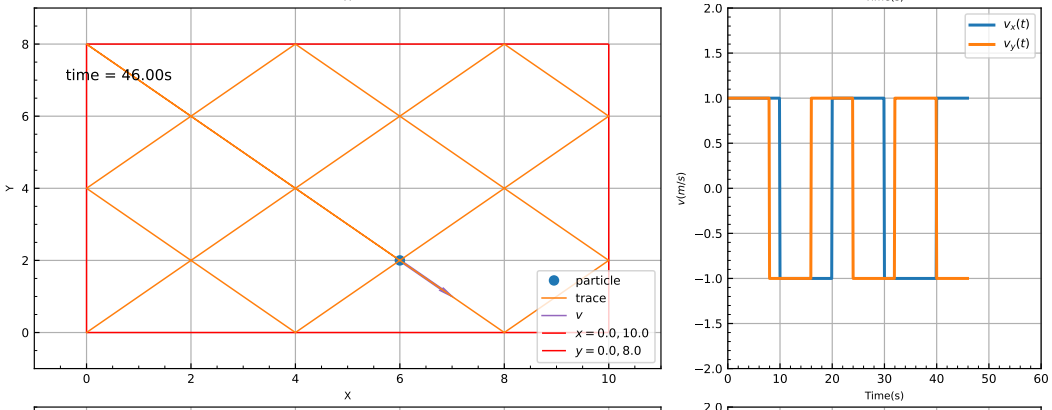

time = 28.20s
particle
trace
v
x = 0.0, 10.0
y = 0.0, 8.0
X
Y
$v_x(t)$
$v_y(t)$
v(m/s)
Time(s)

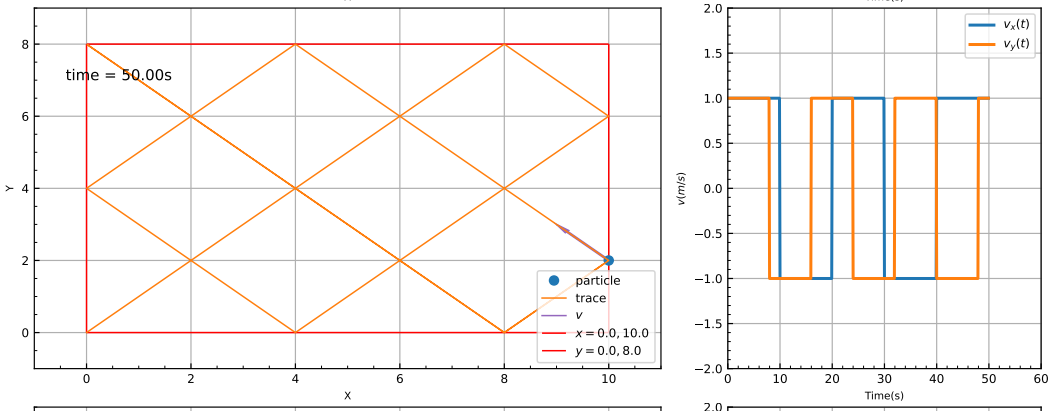

time = 29.90s
particle
trace
v
x = 0.0, 10.0
y = 0.0, 8.0
X
Y
$v_x(t)$
$v_y(t)$
v(m/s)
Time(s)

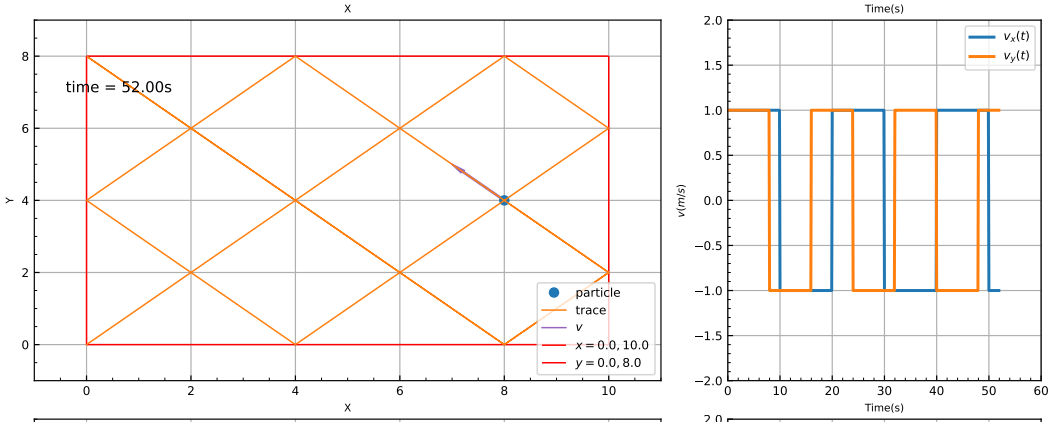

time = 32.70s
particle
trace
v
x = 0.0, 10.0
y = 0.0, 8.0
X
Y
$v_x(t)$
$v_y(t)$
v(m/s)
Time(s)

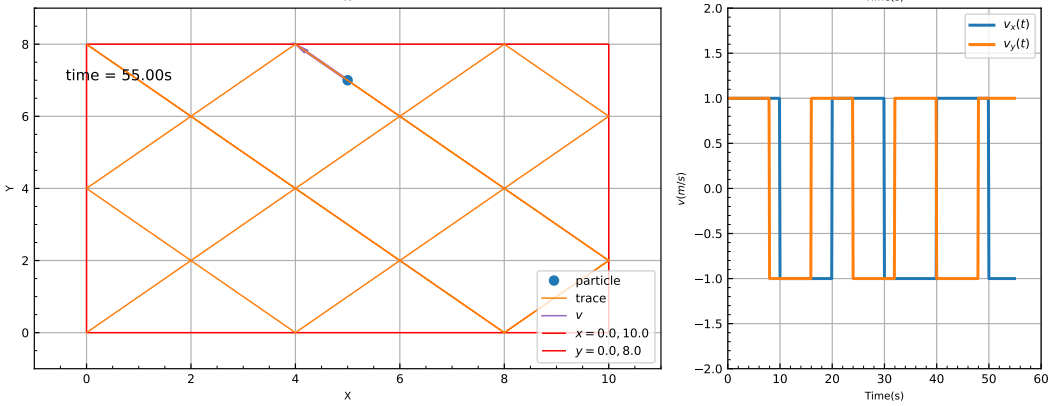

time = 36.00s
particle
trace
v
x = 0.0, 10.0
y = 0.0, 8.0
X
Y
$v_x(t)$
$v_y(t)$
v(m/s)
Time(s)

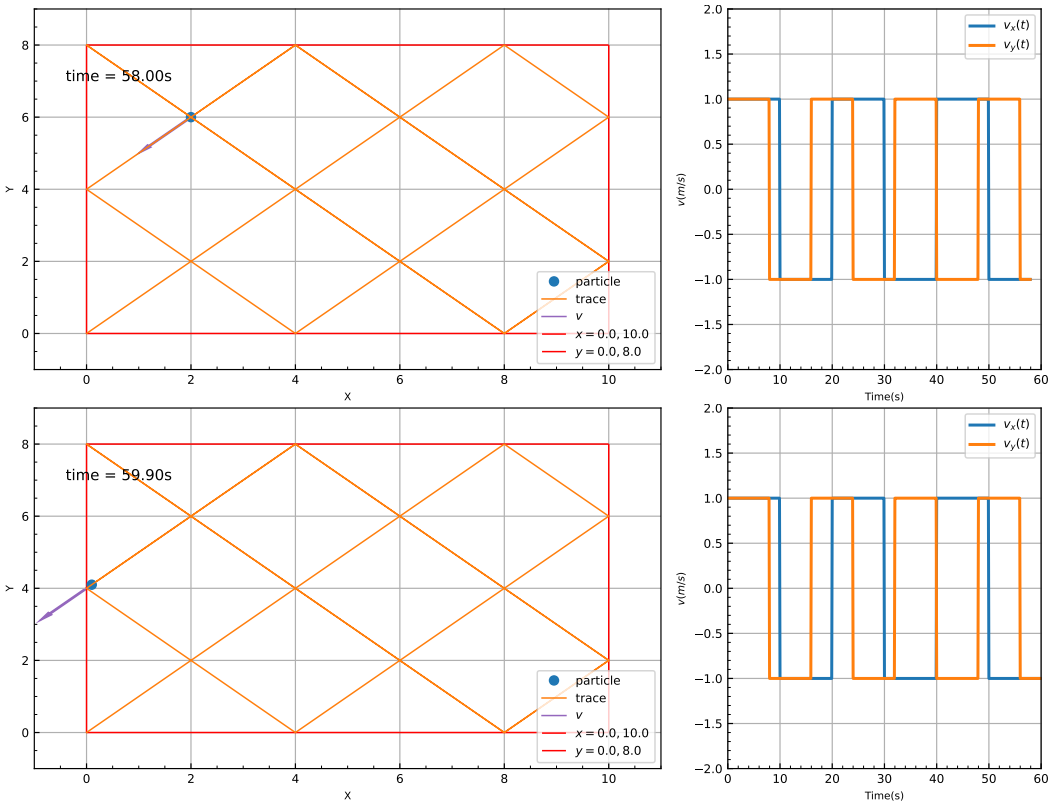

time = 42.20s
particle
trace
v
x = 0.0, 10.0
y = 0.0, 8.0
X
Y
$v_x(t)$
$v_y(t)$
v(m/s)
Time(s)


time = 46.00s
particle
trace
v
x = 0.0, 10.0
y = 0.0, 8.0
X
Y
$v_x(t)$
$v_y(t)$
v(m/s)
Time(s)


time = 50.00s
particle
trace
v
x = 0.0, 10.0
y = 0.0, 8.0
X
Y
$v_x(t)$
$v_y(t)$
v(m/s)
Time(s)


time = 52.00s
particle
trace
v
x = 0.0, 10.0
y = 0.0, 8.0
X
Y
$v_x(t)$
$v_y(t)$
v(m/s)
Time(s)


time = 55.00s
particle
trace
v
x = 0.0, 10.0
y = 0.0, 8.0
X
Y
$v_x(t)$
$v_y(t)$
v(m/s)
Time(s)



6 Remarks

The programs can be used to numerically trace and simulate the motion of a particle trapped in a 2D box, provided the required parameters are defined.

The parameters computed numerically and via the programs are in agreement.

With some modifications this program can be used to solve problems for 2D games such as mini-golf etc.