Program-M4

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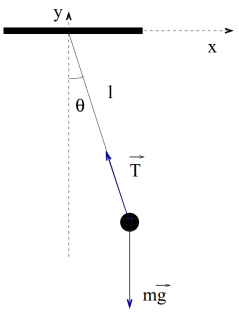
Write and execute a FORTRAN program to simulate motion of a simple pendulum.

1 Theory

1.1 Motion of a Simple Pendulum

Assuming the string is massless and inextendable.
Choosing θ as the generalized coordinate:



$$\mathcal{L} = T - V$$
$$\mathcal{L} = \frac{1}{2}mr^2\dot{\theta}^2 - mgh$$
$$\mathcal{L} = \frac{1}{2}mr^2\dot{\theta}^2 - mgr(1 - \cos(\theta)) \tag{1}$$

Let the length of the string be l

$$\mathcal{L} = \frac{1}{2}ml^2\dot{\theta}^2 - mgl(1 - \cos(\theta)) \tag{2}$$

$$\text{Using Euler-Lagrange equation, } \frac{\partial}{\partial t}\frac{\partial \mathcal{L}}{\partial \dot{\theta}} = \frac{\partial \mathcal{L}}{\partial \theta}$$
$$ml^2\ddot{\theta} = mgl \sin\theta$$
$$\ddot{\theta} = \frac{g}{l} \sin\theta$$

$$\text{which has a solution of the form, } \theta = A\cos(\omega t);\ \omega = \sqrt{\frac{g}{l}}$$

$$\boxed{\begin{aligned} &\therefore \theta = A\cos(\omega t) \\ &T = 2\pi\frac{l}{g} \\ &\dot{\theta} = -A\omega \sin(\omega t) \\ &\ddot{\theta} = -A\omega^2 \cos(\omega t) \end{aligned}} \tag{3}$$

2 Numerical Solution

For length, $l = 2.0$; $\theta_0 = \pi/4 = 0.785$; the time-period will be:

$$\begin{aligned} \omega &= \sqrt{\frac{g}{l}} \\ \omega &= 2.2143\ s^{-1} \\ T &= \frac{2\pi}{\omega} \\ T &= 2.8375\ s \end{aligned} \tag{4}$$

3 Program Algorithm

NOTE: Blue-colored text represents variables in the algorithm, eg. variable.

1. Program open.

2. Define variables (PI , g , omega , theta0 , l , t0 , tf , dt , t , FMT , fmt1).
3. Define functions (theta(t) , dtheta(t) , ddtheta(t) , x(t) , y(t) , vx(t) , vy(t)).
4. Open a writable data file.
5. Get input from user for angular velocity (omega), length(l), initial position (theta0) and time period(t0 , tf , dt).
6. If the time-increment is less-than or equal to 0, terminate the program with message Illegal value of dt
7. Print parameters to stdout for the user.
8. Write appropiate comments in the data file and initialize other parameters.
9. Define a do while loop with index t which runs from t0 to tf .
10. Compute the parameters using the functions theta(t) , dtheta(t) , x(t) , y(t) , vx(t) , vy(t) .
11. Write the parameters to stdout and data file.
12. Increment the index according to t = t + dt
13. End do-while loop.
14. Close data file.
15. Program close.

4 Program

4.1 Fortran program:

For computing the parameters

```
!=============================================================
! Simple Pendulum
! Author: Devansh Shukla
!=============================================================
program simple_pendulum
    ! Program to compute motion of a simple pendulum.

    implicit none
    real*8, parameter :: PI=3.141593, g=9.80665 !m/s^2
    real*8 :: l=0.0, omega=0.0, theta0=PI/4.0, theta, dtheta, ddtheta
    real*8 :: t0=0.0, tf=0.0, dt=0.0, t=0.0
    real*8 :: x, y, vx, vy
    character(LEN=*), parameter :: FMT = "(F10.6, x, F10.6, x, F10.6, x, F10.6, x, F10.6, x, F10.6, x, F10.6, x, F10.6,
        x, F10.6)"
    character(len=*), parameter :: fmt1="(A10, A10, A10, xA10, xxxxA10, A10, A10, A10, xA10)"

    theta(t) = theta0*cos(omega*(t-t0))
    dtheta(t) = -omega * theta0 * sin(omega*(t-t0))
    ddtheta(t) = -omega * omega * theta0 * cos(omega*(t-t0))
    x(t) = l * sin(theta(t))
    y(t) = -l * cos(theta(t))
    vx(t) = l * dtheta(t) * cos(theta(t))
    vy(t) = l * dtheta(t) * sin(theta(t))

    open(UNIT=8, FILE="SimplePendulum.dat")

    print *, "Enter length (l)"
    read *, l
    omega = sqrt(g/l)

    print *, "Enter the value of theta0"
    read *, theta0

    print *, "Enter time: initial (t0), final (tf), increment (dt)"
    read *, t0, tf, dt
    if (dt .le. 0.0) stop "Illegal value of dt"

    print *, "-------------------------------------------------------"
    print "(x,A,F10.4,F10.4,F10.4)", "l, omega, T =", l, omega, 2.0*PI / omega
    print "(x,A,F10.4,F10.4,F10.4)", "theta0 =", theta0
    print "(x,A,F10.4,F10.4,F10.4)", "t0, tf, dt =", t0, tf, dt
    print *, "-------------------------------------------------------"
    print *, "Computing..."

    write (8, *) "# t0=", t0
    write (8, *) "# t l theta(t) dtheta(t) ddtheta(t) x(t) y(t) vx(t) vy(t)"

    print fmt1, "time", "length", "theta", "dtheta", "ddtheta(t)", "x(t)", "y(t)", "vx(t)", "vy(t)"
    t = t0
    do while (t <= tf)
        write (*, FMT) t, l, theta(t), dtheta(t), ddtheta(t), x(t), y(t), vx(t), vy(t)
```

```
        write (8, FMT) t, l, theta(t), dtheta(t), ddtheta(t), x(t), y(t), vx(t), vy(t)
        t = t + dt
    enddo
    print *, "-----------------------------------------------------"
    close(8)

end program simple_pendulum
```

4.2 Python program: Plots

```
#!/usr/bin/env python
"""
Author: Devansh Shukla
"""
# In[0]
import pandas as pd
import numpy as np
import matplotlib as mpl
import matplotlib.pyplot as plt
import matplotlib.gridspec as gridspec

custom_rcparams = {
    "axes.labelsize": 7,
    "axes.titlesize": 8,
    "axes.grid": True,
    # Figure
    "figure.autolayout": True,
    "figure.titlesize": 9,
    "figure.figsize": (8, 8),
    "savefig.format": "pdf",
    "lines.linewidth": 1,
    # Legend
    "legend.fontsize": 8,
    "legend.frameon": True,
    # Ticks
    "xtick.labelsize": 6,
    "ytick.labelsize": 6,
    "xtick.minor.visible": True,
    "xtick.direction": "in",
    "ytick.direction": "in",
    "ytick.minor.visible": True,
    # TeX
    "pgf.texsystem": "lualatex",
}
mpl.rcParams.update(custom_rcparams)
mpl.use("pgf")
plt.ioff()

# Read data file
df = pd.read_csv("SimplePendulum.dat", engine="python", delimiter=" ", header=None, skipinitialspace=True, comment="#")
print(df)

# Plot
gs = gridspec.GridSpec(3, 2)

fig = plt.figure()
ax = plt.subplot(gs[0, 0])
plt.plot(df[0], df[5], "o-", markersize=1.5, color="C0", label=r"$x(t)$")
plt.plot(df[0], df[6], "o-", markersize=1.5, color="C1", label=r"$y(t)$")
plt.title("Position")
ax.set_xlim(left=0)
ax.set_xlabel(r"$Time(s)$")
ax.set_ylabel(r"$Position(m)$")
plt.legend(loc="upper right")

ax = plt.subplot(gs[0, 1])
plt.plot(df[0], df[7], "o-", markersize=1.5, label=r"$v_{x}(t)$")
plt.plot(df[0], df[8], "o-", markersize=1.5, label=r"$v_{y}(t)$")
plt.title("Velocity")
ax.set_xlim(left=0)
ax.set_xlabel(r"$Time(s)$")
ax.set_ylabel(r"$Velocity(m/s)$")
plt.legend(loc="upper right")

ax = plt.subplot(gs[1, 0])
plt.plot(df[0], df[2], "o-", markersize=1.5, color="C0", label=r"$\theta(t)$")
plt.title(r"$\theta$ vs $t$")
ax.set_xlim(left=0)
```

```
ax.set_ylim(-1.0, 1.0)
ax.set_xlabel(r"$Time(s)$")
ax.set_ylabel(r"$\theta(rad)$")
plt.legend(loc="upper right")

ax = plt.subplot(gs[1, 1])
plt.plot(df[0], df[3], "o-", markersize=1.5, color="C0", label=r"$\theta'(t)$")
plt.title(r"$\theta'$ vs $t$")
ax.set_xlim(left=0)
ax.set_xlabel(r"$Time(s)$")
ax.set_ylabel(r"$\theta'(rad/s)$")
plt.legend(loc="upper right")

ax = plt.subplot(gs[2, 0])
plt.plot(df[0], df[4], "o-", markersize=1.5, color="C0", label=r"$\theta''(t)$")
plt.title(r"$\theta''$ vs $t$")
ax.set_xlim(left=0)
ax.set_xlabel(r"$Time(s)$")
ax.set_ylabel(r"$\theta''(rad/s^2)$")
plt.legend(loc="upper right")

ax = plt.subplot(gs[2, 1])
ax.spines['top'].set_color('none')
ax.spines['bottom'].set_position('zero')
ax.spines['left'].set_position('zero')
ax.spines['right'].set_color('none')
ax.set_xlim(-2.5, 2.5)
ax.set_ylim(-2.5,0)
ax.text(0.05, 1.025, 'X', transform=ax.transAxes)
ax.text(-0.025, 0.5, 'Y', transform=ax.transAxes)
# ax.set_aspect("equal")
plt.plot(df[5], df[6], "o-", markersize=1.5, color="C1", label="trace")
plt.title("Trajectory trace")
plt.legend(loc="upper right")


plt.suptitle("Simple Pendulum")
plt.savefig("plots/simple_params.pdf")
# plt.show()

# %%
```

4.3 Python program: Animation

```
#!/usr/bin/env python
"""
Author: Devansh Shukla
"""
import pandas as pd
import numpy as np
import matplotlib as mpl
import matplotlib.pyplot as plt
from matplotlib.animation import FuncAnimation, FFMpegWriter

custom_rcparams = {
    "axes.labelsize": 6,
    "axes.titlesize": 8,
    "axes.grid": True,
    # Figure
    "figure.autolayout": True,
    "figure.titlesize": 9,
    "figure.figsize": (10, 3.5),
    # "figure.dpi": 150,
    "savefig.format": "pdf",
    "lines.linewidth": 1,
    # Legend
    "legend.fontsize": 8,
    "legend.frameon": True,
    # Ticks
    "xtick.labelsize": 8,
    "ytick.labelsize": 8,
    "xtick.minor.visible": True,
    "xtick.direction": "in",
    "ytick.direction": "in",
    "ytick.minor.visible": True,
}
mpl.rcParams.update(custom_rcparams)

df = pd.read_csv("SimplePendulum.dat", engine="python", delimiter=" ", header=None, skipinitialspace=True, comment="#")
```

```
time = df[0].values
theta = df[2].values
pos_x = df[5].values
pos_y = df[6].values
vel_x = df[7].values
vel_y = df[8].values

gs = mpl.gridspec.GridSpec(1, 2, width_ratios=[1, 1])
fig = plt.figure()
ax1 = fig.add_subplot(gs[0, 0])
ax2 = fig.add_subplot(gs[0, 1])

line1, = ax1.plot([], [], 'o-', lw=2, label="Pendulum")
trace, = ax1.plot([], [], ',-', lw=1, label="trace")
_, = ax1.plot([], [], ',-', color="C4", lw=1, label="velocity vector")
time_template = "time = %.1fs"
time_text = ax1.text(0.05, 0.8, '', transform=ax1.transAxes)

patch = plt.Arrow(pos_x[0], pos_y[0], 0.1, 0.1, width=0.1, color="C4")
ax1.add_patch(patch)
ax1.legend()

line_theta, = ax2.plot([], [], '-', lw=2, label=r"$\theta(t)$")
ax2.legend()

line = [line1, line_theta,]

ax1.spines["top"].set_color("none")
ax1.spines["bottom"].set_position("zero")
ax1.spines["left"].set_position("zero")
ax1.spines["right"].set_color("none")
ax1.set_ylim(top=0.0, bottom=(min(pos_y)-1.0))
ax1.set_xlim(left=-(abs(max(pos_x))+1.0), right=(abs(max(pos_x))+1.0))
ax1.set_aspect("equal")
ax1.text(0.05, 1.025, "X", transform=ax1.transAxes)
ax1.text(-0.025, 0.5, "Y", transform=ax1.transAxes)

ax2.set_xlim(0, time[-1])
ax2.set_aspect(4)
ax2.set_ylabel(r"$\theta(rad)$")
ax2.set_xlabel("Time(s)")
ax2.set_ylim(-1.2, 1.2)
ax2.set_title(r"$\theta(t)$")

def init():
    line[0].set_data([], [])
    trace.set_data([], [])
    return line, trace

def animate(i):
    global time, pos_x, pos_y, vel_x, vel_y

    line[0].set_data([0, pos_x[i]], [0, pos_y[i]])
    trace.set_data(pos_x[:i], pos_y[:i])
    time_text.set_text(time_template % (time[i]))

    line[1].set_data(time[:i], theta[:i])
    global ax1, patch
    ax1.patches.remove(patch)
    patch = plt.Arrow(pos_x[i], pos_y[i], vel_x[i]/10, vel_y[i]/10, width=0.1, color="C4") # Velocity rescaled for arrow
          length
    ax1.add_patch(patch)

    global captures
    if time[i] in captures:
        toggle_capture()
    return line, trace, time_text

def toggle_capture(*args, **kwargs):
    global ani, capture_no
    ani.pause()
    plt.gcf().savefig(f"plots/simple_capture_{capture_no}.pdf")
    capture_no += 1
    ani.resume()

capture_no = 0
captures = np.arange(2.5, 18.0, 1.0)
captures = np.insert(captures, 0, [0.5, 1.0, 1.5])
```

```
ani = FuncAnimation(fig, animate, frames=len(time), interval=10, init_func=init, blit=False, repeat=False)
fig.canvas.mpl_connect('button_press_event', toggle_capture)
writer = FFMpegWriter(fps=10)
ani.save('animation.mp4', writer=writer)
plt.show()
```

5 Results

5.1 Terminal Output

```
 Enter length (l)
2.0
 Enter the value of theta0
0.785
 Enter time: initial (t0), final (tf), increment (dt)
0.0 20.0 0.1
 ------------------------------------------------------
 l, omega, T =    2.0000    2.2143    2.8375
 theta0 =    0.7850
 t0, tf, dt =    0.0000   20.0000    0.1000
 ------------------------------------------------------
 Computing...
      time     length      theta      dtheta    ddtheta(t)       x(t)       y(t)      vx(t)      vy(t)
  0.000000   2.000000   0.785000  -0.000000  -3.849110   1.413650  -1.414777  -0.000000  -0.000000
  0.100000   2.000000   0.765833  -0.381773  -3.755128   1.386275  -1.441611  -0.550368  -0.529243
  0.200000   2.000000   0.709268  -0.744903  -3.477771   1.302557  -1.517678  -1.130523  -0.970278
  0.300000   2.000000   0.618067  -1.071657  -3.030583   1.158921  -1.630000  -1.746801  -1.241966
  0.400000   2.000000   0.496684  -1.346078  -2.435402   0.953025  -1.758335  -2.366857  -1.282847
  0.500000   2.000000   0.351046  -1.554767  -1.721292   0.687760  -1.878027  -2.919894  -1.069307
  0.600000   2.000000   0.188265  -1.687530  -0.923126   0.374310  -1.964661  -3.315425  -0.631660
  0.700000   2.000000   0.016291  -1.737887  -0.079881   0.032581  -1.999735  -3.475312  -0.056622
  0.800000   2.000000  -0.156478  -1.703376   0.767265  -0.311681  -1.975564  -3.365130   0.530911
  0.900000   2.000000  -0.321607  -1.585685   1.576943  -0.632183  -1.897457  -3.008769   1.002443
  1.000000   2.000000  -0.471030  -1.390559   2.309613  -0.907609  -1.782203  -2.478258   1.262084
  1.100000   2.000000  -0.597451  -1.127528   2.929498  -1.125074  -1.653544  -1.864417   1.268552
  1.200000   2.000000  -0.694697  -0.809436   3.406326  -1.280306  -1.536495  -1.243694   1.036325
  1.300000   2.000000  -0.758019  -0.451816   3.716812  -1.374968  -1.452399  -0.656217   0.621233
  1.400000   2.000000  -0.784324  -0.072133   3.845795  -1.412693  -1.415732  -0.102121   0.101902
  1.500000   2.000000  -0.772328   0.311072   3.786974  -1.395609  -1.432577   0.445635  -0.434135
  1.600000   2.000000  -0.722617   0.679087   3.543224  -1.322699  -1.500156   1.018737  -0.898228
  1.700000   2.000000  -0.637618   1.013940   3.126446  -1.190566  -1.607033   1.629435  -1.207162
  1.800000   2.000000  -0.521482   1.299279   2.556994  -0.996331  -1.734164   2.253163  -1.294512
  1.900000   2.000000  -0.379880   1.521170   1.862676  -0.741618  -1.857418   2.825449  -1.128128
  2.000000   2.000000  -0.219728   1.668777   1.077397  -0.435928  -1.951914   3.257309  -0.727467
  2.100000   2.000000  -0.048846   1.734893   0.239506  -0.097652  -1.997615   3.465647  -0.169416
  2.200000   2.000000   0.124422   1.716288  -0.610082   0.248203  -1.984539   3.406040   0.425987
  2.300000   2.000000   0.291614   1.613871  -1.429877   0.574996  -1.915562   3.091471   0.927970
  2.400000   2.000000   0.444565   1.432644  -2.179846   0.860130  -1.805596   2.586775   1.232260
  2.500000   2.000000   0.575807   1.181456  -2.823367   1.089023  -1.677507   1.981900   1.286633
  2.600000   2.000000   0.678930   0.872574  -3.329014   1.255921  -1.556490   1.358153   1.095884
  2.700000   2.000000   0.748899   0.521081  -3.672094   1.361665  -1.464878   0.763320   0.709538
  2.800000   2.000000   0.782296   0.144142  -3.835854   1.409820  -1.418593   0.204479   0.203214
  2.900000   2.000000   0.777492  -0.239836  -3.812296   1.402989  -1.425350  -0.341850  -0.336487
  3.000000   2.000000   0.734720  -0.612101  -3.602573   1.340759  -1.484037  -0.908381  -0.820681
  3.100000   2.000000   0.656070  -0.954476  -3.216923   1.220015  -1.584792  -1.512646  -1.164475
  3.200000   2.000000   0.545381  -1.250241  -2.674182   1.037488  -1.709859  -2.137736  -1.297110
  3.300000   2.000000   0.408060  -1.484953  -2.000851   0.793659  -1.835785  -2.726053  -1.178546
  3.400000   2.000000   0.250812  -1.647149  -1.229812   0.496381  -1.937422  -3.191224  -0.817613
  3.500000   2.000000   0.081316  -1.728910  -0.398718   0.162452  -1.993391  -3.446394  -0.280865
  3.600000   2.000000  -0.092151  -1.726243   0.451848  -0.184042  -1.991514  -3.437836   0.317701
  3.700000   2.000000  -0.261118  -1.639277   1.280347  -0.516322  -1.932204  -3.167417   0.846395
  3.800000   2.000000  -0.417334  -1.472261   2.046324  -0.810649  -1.828346  -2.691801   1.193487
  3.900000   2.000000  -0.553170  -1.233349   2.712372  -1.050774  -1.701727  -2.098823   1.295971
  4.000000   2.000000  -0.661993  -0.934209   3.245966  -1.229380  -1.577538  -1.473749   1.148497
  4.100000   2.000000  -0.738488  -0.589448   3.621049  -1.346342  -1.478974  -0.871778   0.793598
  4.200000   2.000000  -0.778921  -0.215903   3.819304  -1.405024  -1.423343  -0.307304   0.303349
  4.300000   2.000000  -0.781317   0.168186   3.831051  -1.408430  -1.419973   0.238819  -0.236878
  4.400000   2.000000  -0.745558   0.544061   3.655715  -1.356764  -1.469418   0.799454  -0.738163
  4.500000   2.000000  -0.673392   0.893368   3.301859  -1.247282  -1.563422   1.396712  -1.114282
  4.600000   2.000000  -0.568341   1.199049   2.786762  -1.076470  -1.685590   2.021106  -1.290740
  4.700000   2.000000  -0.435537   1.446177   2.135579  -0.843794  -1.813287   2.622335  -1.220276
  4.800000   2.000000  -0.281464   1.622683   1.380108  -0.555524  -1.921300   3.117661  -0.901440
  4.900000   2.000000  -0.113646   1.719949   0.557243  -0.226803  -1.987099   3.417707  -0.390089
  5.000000   2.000000   0.059722   1.733223  -0.292835   0.119372  -1.996434   3.460267   0.206899
  5.100000   2.000000   0.230173   1.661859  -1.128612   0.456292  -1.947254   3.236062   0.758292
  5.200000   2.000000   0.389384   1.509341  -1.909276   0.759237  -1.850286   2.792713   1.145948
  5.300000   2.000000   0.529580   1.283117  -2.596704   1.010342  -1.726038   2.214709   1.296387
  5.400000   2.000000   0.643915   0.994234  -3.157326   1.200663  -1.599503   1.590280   1.193739
  5.500000   2.000000   0.726806   0.656799  -3.563766   1.328972  -1.494601   0.981653   0.872868
```

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5.900000	2.000000	0.689554	-0.830721	-3.381106	1.272386	-1.543060	-1.281853	-1.056998
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18.900000   2.000000  -0.417230   1.472404   2.045816  -0.810460  -1.828430   2.692188  -1.193325
19.000000   2.000000  -0.261003   1.639367   1.279782  -0.516099  -1.932263   3.167689  -0.846076
19.100000   2.000000  -0.092030   1.726274   0.451252  -0.183800  -1.991536   3.437938  -0.317289
19.200000   2.000000   0.081437   1.728882  -0.399314   0.162695  -1.993372   3.446304   0.281280
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19.600000   2.000000   0.656137   0.954250  -3.217253   1.220121  -1.584710   1.512210   1.164301
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5.2 Plots

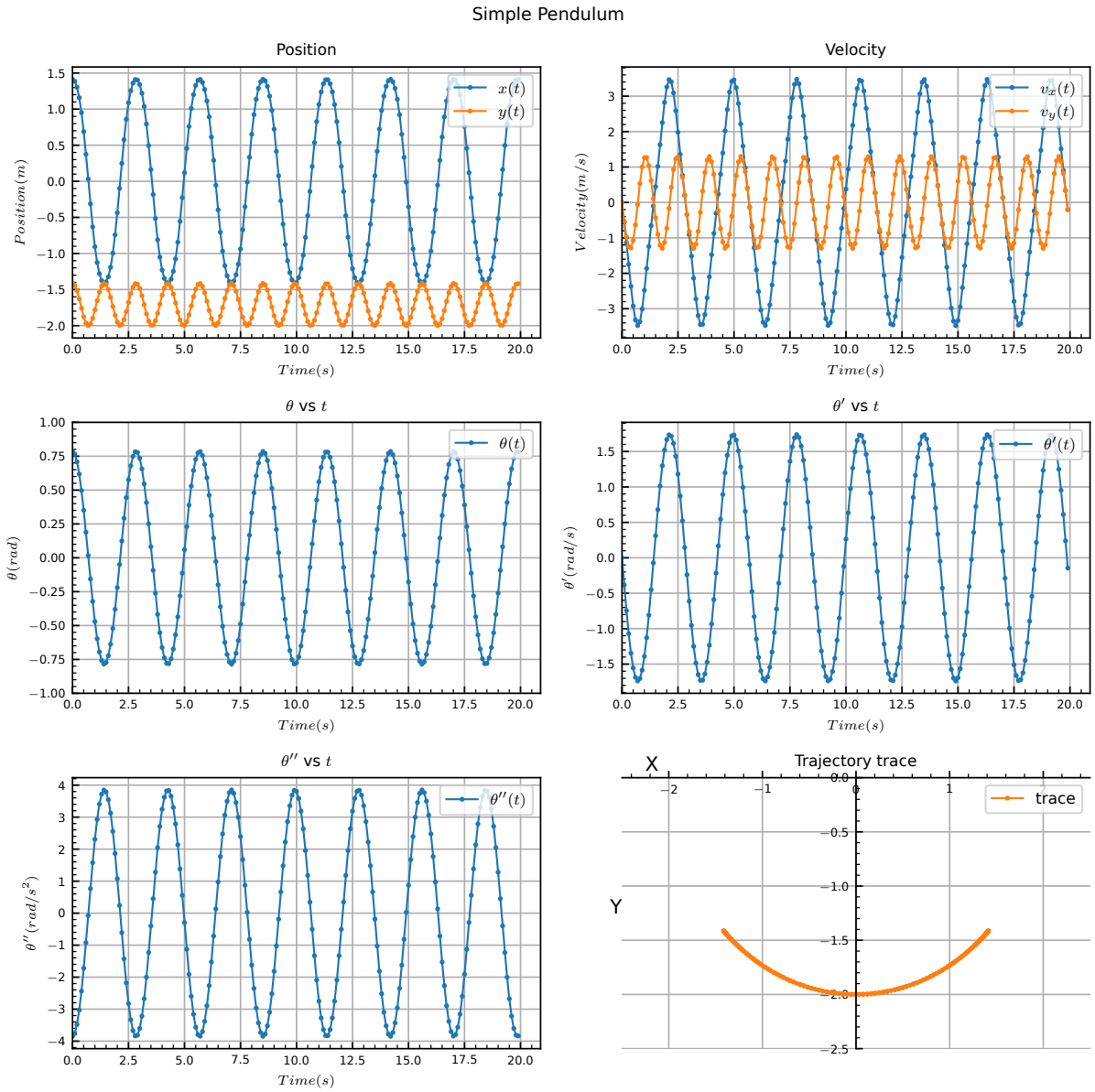
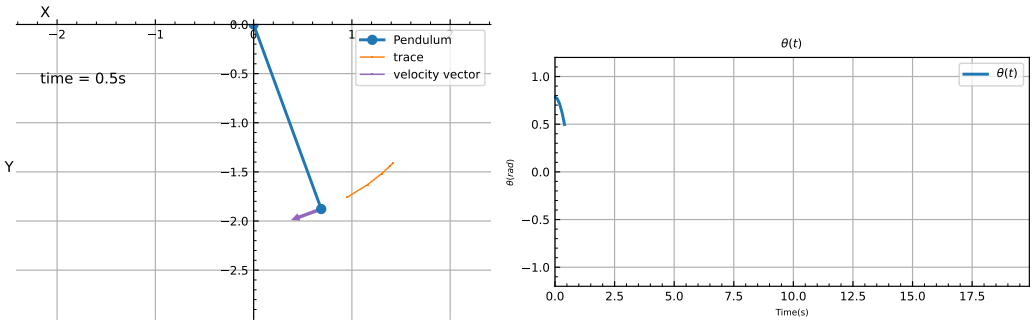


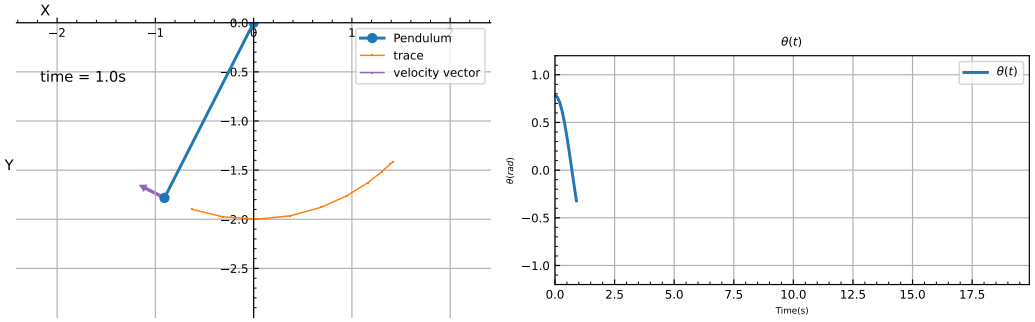
Figure 1: Simple Pendulum

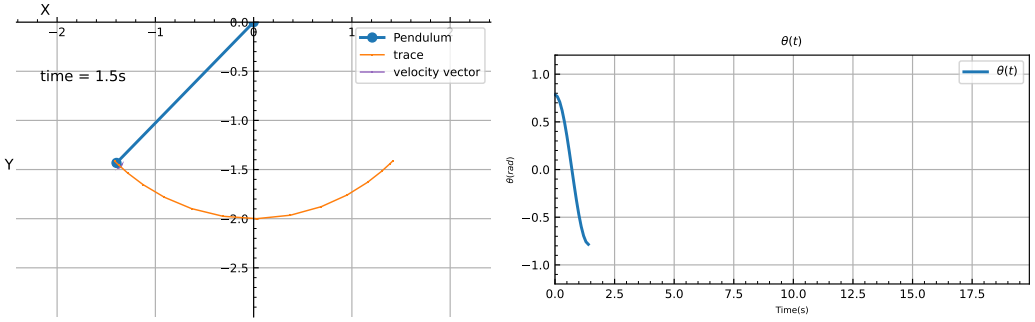
5.3 Animation

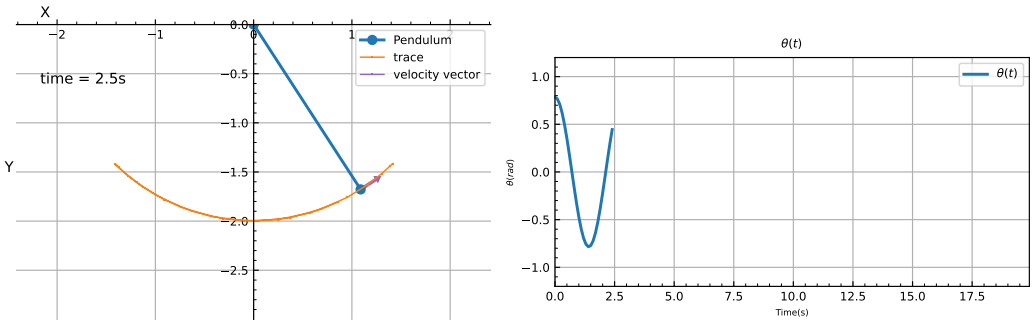
Note: Input parameters,

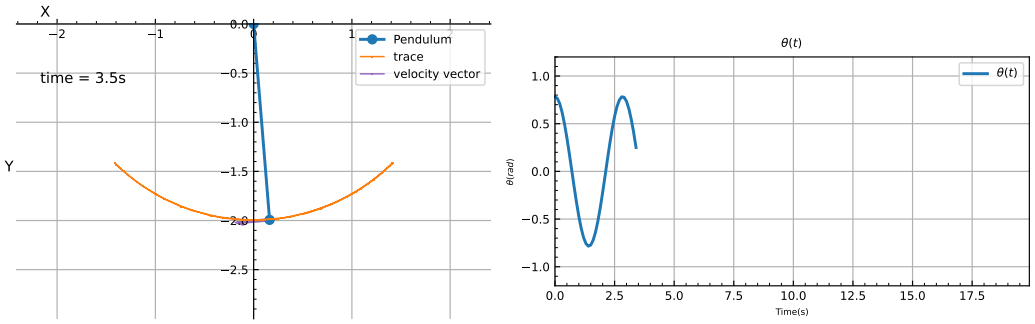
$$\begin{aligned} \theta_0 &= \frac{\pi}{4} = 0.785 \\ \text{length, } l &= 2\ m \\ \implies T &= 2.837\ s \end{aligned} \tag{5}$$

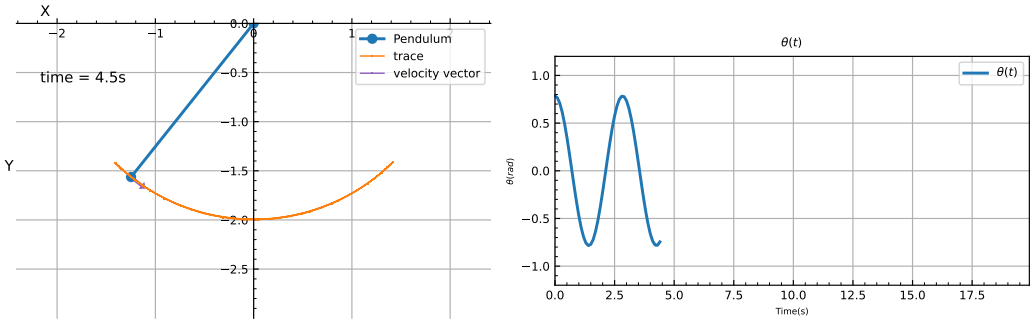


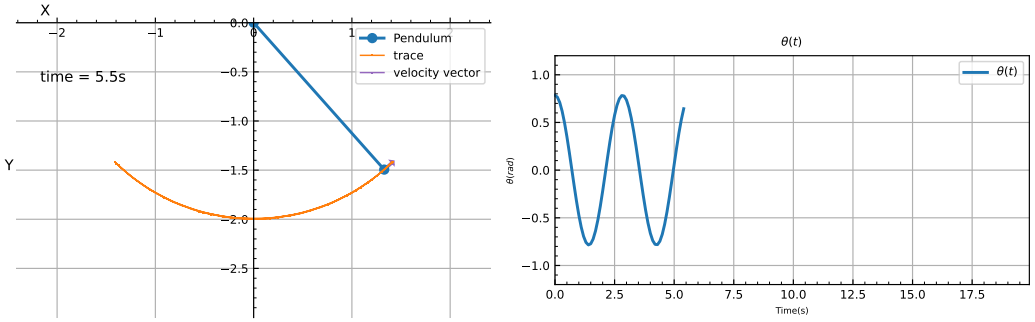

X
Y
time = 1.0s
Pendulum
trace
velocity vector
θ(t)
θ(rad)
Time(s)

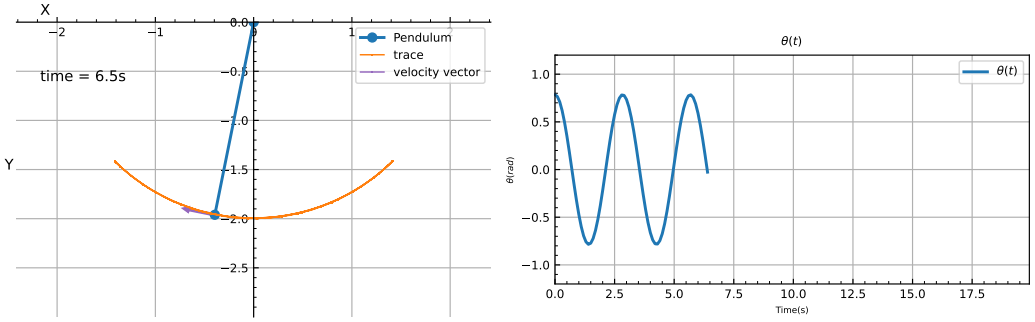

X
Y
time = 1.5s
Pendulum
trace
velocity vector
θ(t)
θ(rad)
Time(s)

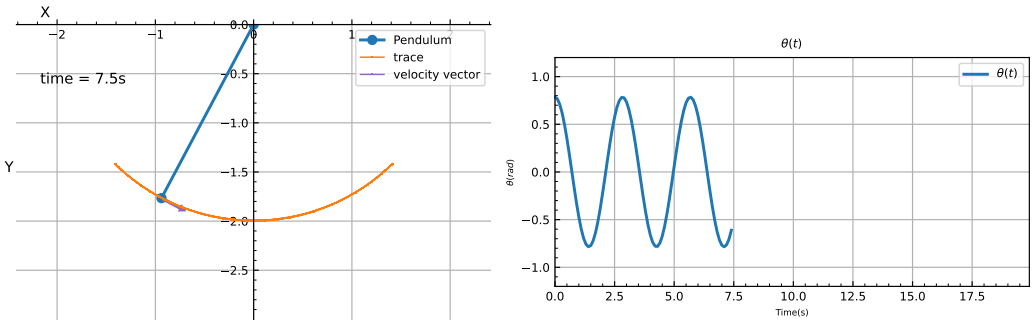

X
Y
time = 2.5s
Pendulum
trace
velocity vector
θ(t)
θ(rad)
Time(s)

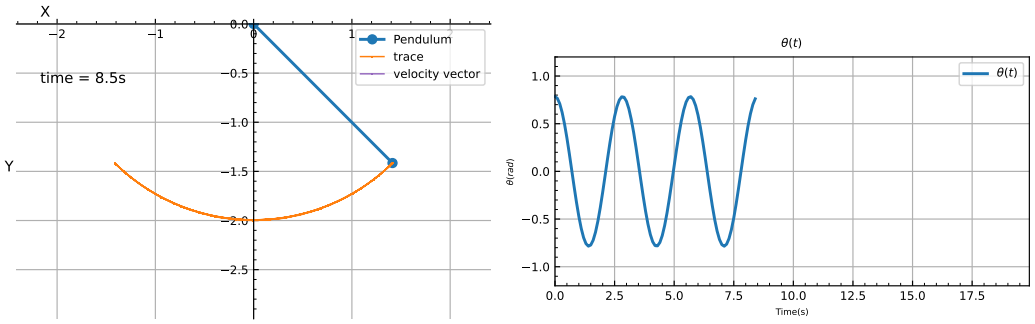

X
Y
time = 3.5s
Pendulum
trace
velocity vector
θ(t)
θ(rad)
Time(s)

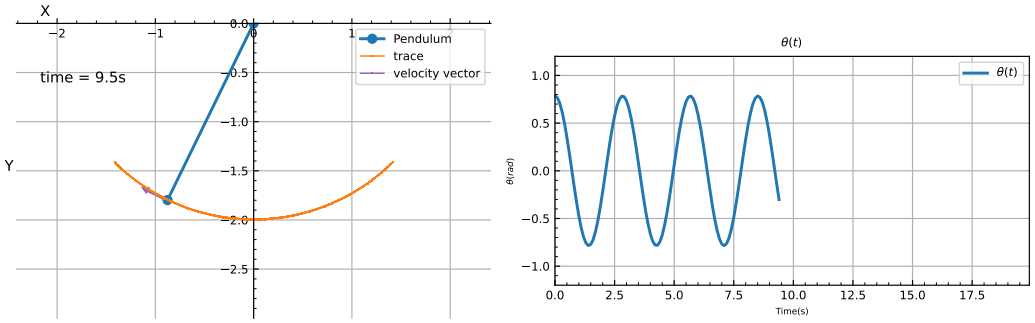

X
Y
time = 4.5s
Pendulum
trace
velocity vector
θ(t)
θ(rad)
Time(s)

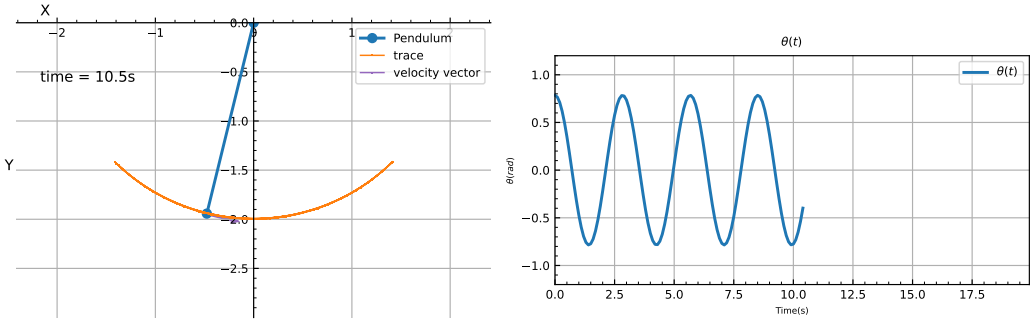

X
Y
time = 5.5s
Pendulum
trace
velocity vector
θ(t)
θ(t)
θ(rad)
Time(s)

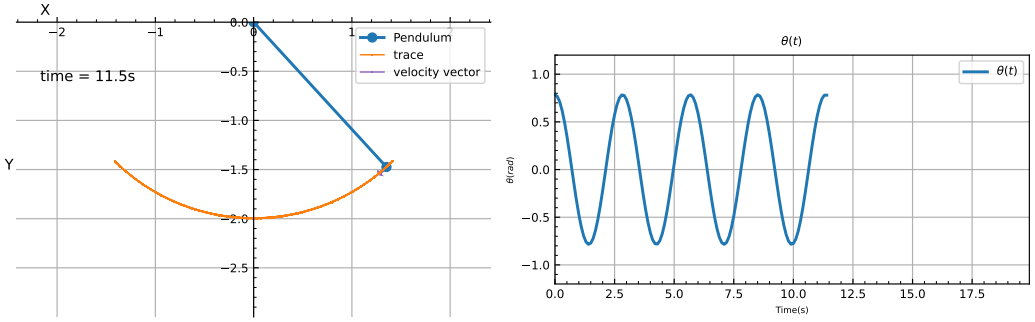

X
Y
time = 6.5s
Pendulum
trace
velocity vector
θ(t)
θ(t)
θ(rad)
Time(s)

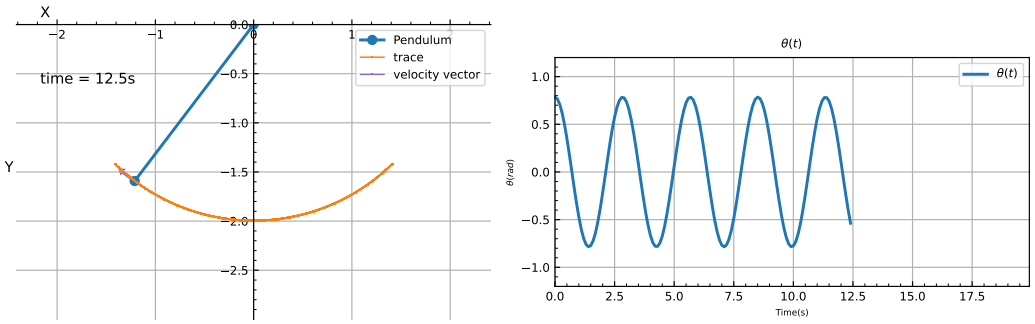

X
Y
time = 7.5s
Pendulum
trace
velocity vector
θ(t)
θ(t)
θ(rad)
Time(s)

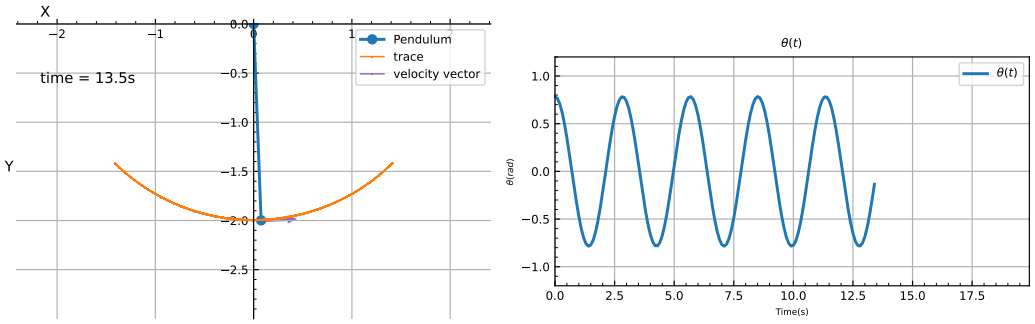

X
Y
time = 8.5s
Pendulum
trace
velocity vector
θ(t)
θ(t)
θ(rad)
Time(s)

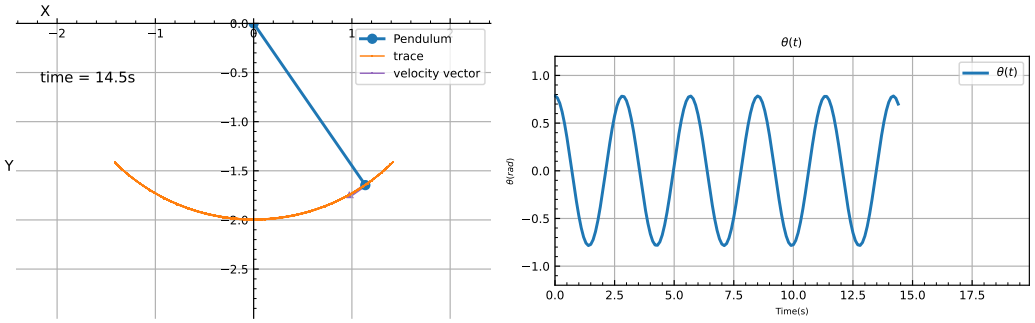

X
Y
time = 9.5s
Pendulum
trace
velocity vector
θ(t)
θ(t)
θ(rad)
Time(s)


X
Y
time = 10.5s
Pendulum
trace
velocity vector
θ(t)
θ(rad)
Time(s)


X
Y
time = 11.5s
Pendulum
trace
velocity vector
θ(t)
θ(rad)
Time(s)


X
Y
time = 12.5s
Pendulum
trace
velocity vector
θ(t)
θ(rad)
Time(s)


X
Y
time = 13.5s
Pendulum
trace
velocity vector
θ(t)
θ(rad)
Time(s)


X
Y
time = 14.5s
Pendulum
trace
velocity vector
θ(t)
θ(rad)
Time(s)

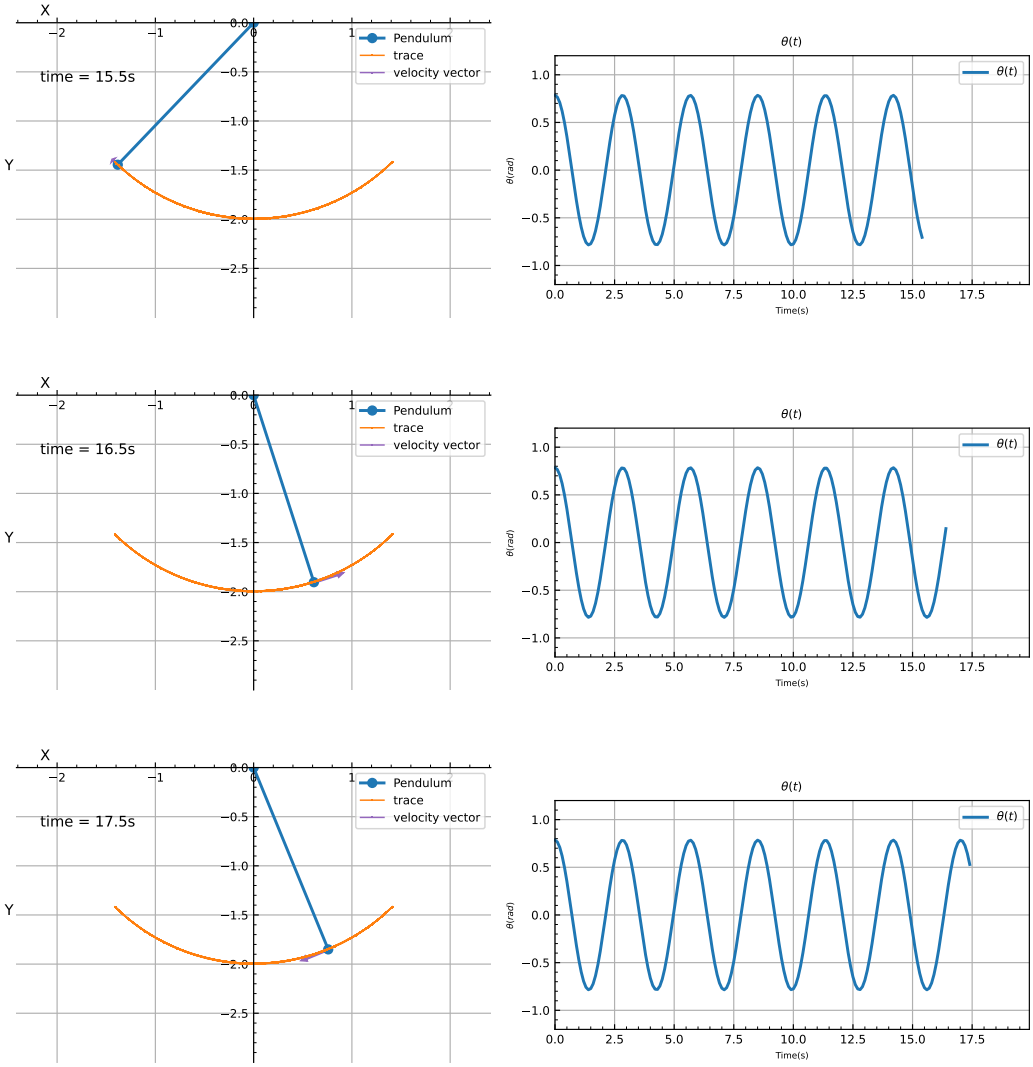


Figure 2: Animation of simple pendulum

6 Remarks

The programs can be used to numerically trace and simulate the motion of any simple pendulum or any simple harmonic motion, provided the required parameters are defined.

The parameters computed numerically and via the programs are in agreement.

With some modifications, programs can be used to simulate motion of forced damped and damped harmonic motion.