Program-M3

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Write and execute a FORTRAN program to simulate motion of a particle in uniform circular motion in spherical coordinates.

1 Theory

1.1 Motion of a particle in uniform circular motion

Degrees of freedom, dof = 1

Choosing θ as the generalized coordinate.

Lagrangian,
$$\mathcal{L} = T - V$$

$$\mathcal{L} = \frac{1}{2}mr^2\dot{\theta}^2 \tag{1}$$

Using Euler-Lagrange equation

$$\frac{\partial \mathcal{L}}{\partial \theta} = \frac{\partial}{\partial t} \frac{\partial \mathcal{L}}{\partial \dot{\theta}} \tag{2}$$

$$|\ddot{\theta} = 0 |$$
 (3)

Therefore, Equation of motion is $\ddot{\theta} = 0$ or $\dot{\theta} = \omega = \text{Constant}$

$$\theta = \omega t$$

$$\dot{\theta} = \omega = \text{constant}$$

$$\ddot{\theta} = \alpha = 0$$
(4)

2 Numerical Solution

For $\omega = 0.5$, R = 1.0, time period, T will be:

$$T = \frac{2\pi}{\omega}$$

$$T = 12.5664 s$$
(5)

3 Program Algorithm

NOTE: Blue-colored text represents variables in the algorithm, eg. variable.

- 1. Program open.
- 2. Define variables (PI, theta0, R, omega, t0, tf, dt, t, fmt1).
- 3. Define functions (theta(t), x(t), y(t)).
- 4. Open a writable data file.
- 5. Get input from user for angular velocity (omega), radius of the circle(R), initial position (theta0) and time period(t0, tf, dt).
- 6. If the time-increment is less-than or equal to 0, terminate the program with message Illegal value of dt
- 7. Print parameters to stdout for the user.
- 8. Write appropriate comments in the data file and initialize other parameters.
- 9. Define a do while loop with index t which runs from t0 to tf.
- 10. Compute the parameters using the functions $\, \, theta(t) \, , \, \, x(t) \, , \, \, y(t) \, .$
- 11. Write the parameters to stdout and data file.
- 12. Increment the index according to t = t + dt
- 13. End do-while loop.
- 14. Close data file.
- 15. Program close.

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4 Program

4.1 Fortran program:

For computing the parameters

```
! Uniform circular motion in polar coordinates
! Author: Devansh Shukla
program CircularMotion_Polar
   ! Program to compute motion of a particle moving in a circle with uniform velocity in polar coordinates.
   real*8, parameter :: PI = 3.141593
   real*8 :: R=0.0, omega=0.0, theta0=0.0, theta, x, y
   real*8 :: t0=0.0, tf=0.0, dt=0.0, t=0.0
   character(len=*), parameter :: fmt1 = "(F10.6, x, F10.6, x, F10.6, x, F10.6, x, F10.6)"
   theta(t) = theta0 + omega*(t-t0)
   x(t) = R*cos(theta(t))
   y(t) = R*sin(theta(t))
   open(UNIT=8, FILE="CirclePolar.dat")
   ! Input
   print *, "Enter angular velocity(omega) and radius (R)"
   read *, omega, R
   if (omega .le. 0.0) stop "Illegal value of omega"
   if (R .le. 0.0) stop "Illegal value of R"
   print *, "Enter the value of theta0"
   read *, theta0
   print *, "Enter t0, tf, dt"
   read *, t0, tf, dt
   if (dt .le. 0.0) stop "Illegal value of dt"
   print "(x,A,F10.4)", "omega =", omega
print "(x,A,F10.4)", "T =", 2.0*PI / omega
   print "(x,A,F10.4,F10.4,F10.4)", "theta0 =", theta0
   print "(x,A,F10.4)", "R =", R
   print "(x,A,F10.4,F10.4,F10.4)", "t0, tf, dt =", t0, tf, dt
   print *, "-----
   write (8, *) "# t0=", t0
   write (8, *) "# t R theta x y"
   print "(xA10,A10,xA10,A10,A10)", "time", "R", "theta(t)", "x(t)", "y(t)"
   ! Computing
   t = t0
   do while (t <= tf)</pre>
      write (*, fmt1) t, R, theta(t), x(t), y(t)
       write (8, fmt1) t, R, theta(t), x(t), y(t)
       t = t + dt
   enddo
   print *, "---
   close(8)
end program CircularMotion_Polar
```

4.2 Python program: Plots

```
#!/usr/bin/env python
"""
Author: Devansh Shukla
"""
# In[0]
import pandas as pd
import numpy as np
import matplotlib as mpl
import matplotlib.gridspec as gridspec

custom_rcparams = {
    "axes.labelsize": 7,
    "axes.titlesize": 8,
    "axes.grid": True,
```

```
# Figure
    "figure.autolayout": True,
    "figure.titlesize": 9,
    "figure.dpi": 150,
"savefig.format": "pdf",
    "lines.linewidth": 1,
    # Legend
    "legend.fontsize": 6,
    "legend.frameon": True,
    # Ticks
    "xtick.labelsize": 6,
    "ytick.labelsize": 6,
    "xtick.minor.visible": True,
    "xtick.direction": "in",
    "vtick.direction": "in",
    "ytick.minor.visible": True,
    "pgf.texsystem": "lualatex",
mpl.rcParams.update(custom_rcparams)
mpl.use('pgf')
plt.ioff()
df = pd.read_csv("CirclePolar.dat", engine="python", delimiter=" ", header=None, skipinitialspace=True, comment="#")
# Assuming t0=0, x_at_0=x0+ Rcos(wt0)=1
x0 = df[3].iloc[0] - 1
y0 = df[4].iloc[0]
theta = np.arctan((df[4].values - y0)/(df[3].values - x0))
gs = gridspec.GridSpec(2, 1)
fig = plt.figure()
ax = plt.subplot(gs[0, 0])
plt.plot(df[0], theta, "o-", markersize=2, color="CO", label=r"$\theta(t)$")
plt.hlines(y=np.pi/2, xmin=df[0].values[0], xmax=df[0].values[-1], color="red", label=r"$\pi/2$")
plt.hlines(y=-np.pi/2, xmin=df[0].values[0], xmax=df[0].values[-1], color="red", label=r"\pi/2\")
plt.xlim(left=0)
plt.ylim(-np.pi, np.pi)
plt.title(r"$\theta(t)$")
ax.set_xlabel(r"$Time(s)$")
ax.set_ylabel(r"$\theta(rad)$")
plt.legend(loc="upper right")
ax = plt.subplot(gs[1, 0])
plt.plot(df[0], df[3], "o-", markersize=2, color="CO", label=r"$x(t)$")
\label{localization} plt.plot(df[0],\ df[4],\ "o--",\ markersize=2,\ color="C1",\ label=r"\$y(t)\$")
plt.title("Position")
plt.xlim(left=0)
ax.set_ylim(-1.5, 1.5)
ax.set_xlabel(r"$Time(s)$")
ax.set_ylabel(r"$Position(m)$")
plt.legend(loc="upper right")
plt.tight_layout()
plt.savefig("plots/ii_params.pdf")
fig, ax = plt.subplots(subplot_kw={"projection":"polar"})
plt.plot(df[2], df[1], "o-", markersize=2, color="CO", label="trace")
ax.set_rmax(1.5)
plt.title("Trajectory")
plt.legend(loc="upper right")
plt.tight_layout()
plt.savefig("plots/ii_polar.pdf")
fig, ax = plt.subplots()
plt.plot(df[3], df[4], "o-", markersize=2, color="CO", label="trace")
plt.xlim(-1.5, 1.5)
plt.ylim(-1.5, 1.5)
plt.title("Trajectory")
plt.xlabel("X")
plt.ylabel("Y")
plt.legend(loc="upper right")
ax.set_aspect("equal")
plt.tight_layout()
plt.savefig("plots/ii_cart.pdf")
# plt.show()
# %%
```

4.3 Python program: Animation

```
#!/usr/bin/env python
Author: Devansh Shukla
import pandas as pd
import numpy as np
import matplotlib as mpl
import matplotlib.pyplot as plt
from matplotlib.animation import FuncAnimation, FFMpegWriter
import matplotlib.gridspec as gridspec
custom_rcparams = {
    "axes.labelsize": 6,
    "axes.titlesize": 8,
   "axes.grid": True,
    # Figure
    "figure.autolayout": True,
   "figure.titlesize": 9,
    # "figure.dpi": 200,
    "figure.figsize": (9, 4),
    "savefig.format": "pdf",
    "lines.linewidth": 1,
    # Legend
    "legend.fontsize": 8,
   "legend.frameon": True,
    # Ticks
    "xtick.labelsize": 8,
    "ytick.labelsize": 8,
    "xtick.minor.visible": True,
    "xtick.direction": "in",
    "ytick.direction": "in",
    "ytick.minor.visible": True,
mpl.rcParams.update(custom_rcparams)
# Read data file
df = pd.read_csv("CirclePolar.dat", engine="python", delimiter=" ", header=None, skipinitialspace=True, comment="#")
# Extract data
time = df[0].values[::1]
radius = df[1].values[::1]
df_theta = df[2].values
x0 = df[3].iloc[0] - 1
y0 = df[4].iloc[0]
theta = np.arctan((df[4].values - y0)/(df[3].values - x0))
pos_x = df[3].values[::1]
pos_y = df[4].values[::1]
gs = gridspec.GridSpec(1, 2, width_ratios=[1.5, 1])
# Plot
fig = plt.figure()
ax1 = fig.add_subplot(gs[0, 0], projection="polar")
ax2 = fig.add_subplot(gs[0, 1], )
line1, = ax1.plot([], [], 'o', lw=2, label="particle")
line2, = ax2.plot([], [], '-', lw=2, label=r"$\theta(t)$")
trace, = ax1.plot([], [], ',-', lw=1, label="trace")
time_template = "time = %.1fs"
time_text = ax1.text(0, 1.0, '', transform=ax1.transAxes)
line = [line1, line2]
ax1.set_aspect("equal")
ax1.set_rmax(1.5)
ax1.legend()
ax2.hlines(np.pi/2, 0, 10*np.pi, color="red", label=r"$\theta=\pi/2$")
ax2.hlines(-np.pi/2, 0, 10*np.pi, color="red", label=r"$\theta=-\pii/2$")
ax2.set_ylim(-np.pi, np.pi)
ax2.set_xlim(0, 5*np.pi)
ax2.set_xlabel("Time(s)")
ax2.set_ylabel(r"$\theta(rad)$")
ax2.set_aspect(2)
ax2.legend(loc="upper right")
def init():
   line[0].set_data([], [])
   line[1].set_data([], [])
```

```
trace.set_data([], [])
   return line, trace
def animate(i):
   global time, radius, df_theta, pos_x, pos_y, theta
    # line[0].set_data(pos_x[i], pos_y[i])
   line[0].set_data(df_theta[i], radius[i])
   line[1].set_data(time[:i], theta[:i])
   # trace.set_data(pos_x[:i], pos_y[:i])
   trace.set_data(df_theta[:i], radius[:i])
   time_text.set_text(time_template % (time[i]))
   return line, trace, time_text
def toggle_capture(*args, **kwargs):
   global ani, capture_no
   ani.pause()
   plt.gcf().savefig(f"plots/circle_capture_{capture_no}.pdf")
   capture_no += 1
   ani.resume()
capture_no = 0
ani = FuncAnimation(fig, animate, frames=len(time), interval=1, init_func=init, blit=False, repeat=False)
fig.canvas.mpl_connect('button_press_event', toggle_capture)
writer = FFMpegWriter(fps=10)
ani.save('animation.mp4', writer=writer)
plt.show()
```

5 Results

5.1 Terminal Output

```
Enter angular velocity(omega) and radius (R)
0.5 1.0
Enter the value of theta0
0.0
Enter t0, tf, dt
0.0 20.0 0.1
omega =
          0.5000
T = 12.5664
 theta0 = 0.0000
R = 1.0000
 t0, tf, dt = 0.0000 20.0000 0.1000
      time
               R theta(t) x(t)
                                             y(t)
 0.000000 1.000000 0.000000 1.000000 0.000000
 0.100000 1.000000 0.050000 0.998750 0.049979 0.200000 1.000000 0.100000 0.995004 0.099833
 0.300000 1.000000 0.150000 0.988771
                                             0.149438

    0.400000
    1.000000
    0.200000
    0.980067

    0.500000
    1.000000
    0.250000
    0.968912

                                             0.198669
                                             0.247404
 0.600000 1.000000 0.300000 0.955336 0.295520
 0.700000 1.000000 0.350000 0.939373 0.342898
 0.800000
            1.000000
                       0.400000
                                  0.921061
                                             0.389418
 0.900000 1.000000 0.450000 0.900447
                                             0.434966
  1.000000 1.000000 0.500000 0.877583 0.479426
 1.100000
            1.000000
                      0.550000
                                 0.852525
                                             0.522687
 1.200000 1.000000 0.600000 0.825336
                                             0.564642
  1.300000 1.000000 0.650000 0.796084
                                             0.605186
 1.400000 1.000000 0.700000 0.764842
1.500000 1.000000 0.750000 0.731689
                                             0.644218
                                             0.681639
  1.600000 1.000000 0.800000 0.696707
                                             0.717356
           1.000000 0.850000
1.000000 0.900000
 1.700000
                                 0.659983
                                             0.751280
  1.800000
                                 0.621610
                                             0.783327
           1.000000 0.950000 0.581683
  1.900000
                                             0.813416
           1.000000 1.000000
 2.000000
                                 0.540302
                                             0.841471
 2.100000
            1.000000
                       1.050000
                                  0.497571
                                             0.867423
           1.000000 1.100000
 2.200000
                                 0.453596
                                             0.891207
           1.000000 1.150000 0.408487
 2.300000
                                             0.912764
  2.400000
            1.000000
                       1.200000
                                  0.362358
                                             0.932039
           1.000000 1.250000 0.315322
 2.500000
                                             0.948985
           1.000000 1.300000 0.267499
 2.600000
                                             0.963558
  2.700000
            1.000000
                       1.350000
                                  0.219007
                                             0.975723
           1.000000 1.400000 0.169967
 2.800000
                                             0.985450
 2.900000
           1.000000 1.450000
                                  0.120503
                                             0.992713
 3.000000
            1.000000
                       1.500000
                                  0.070737
                                             0.997495
 3.100000
            1.000000
                       1.550000
                                 0.020795
                                             0.999784
 3.200000
            1.000000
                      1.600000 -0.029200
                                             0.999574
           1.000000
                       1.650000 -0.079121 0.996865
 3.300000
```

3.400000	1.000000	1.700000	-0.128844	0.991665
3.500000	1.000000	1.750000	-0.178246	0.983986
3.600000	1.000000	1.800000	-0.227202	0.973848
3.700000	1.000000	1.850000	-0.275590	0.961275
3.800000	1.000000	1.900000	-0.323290	0.946300
3.900000	1.000000	1.950000	-0.370181	0.928960
4.000000	1.000000	2.000000	-0.416147	0.909297
4.100000	1.000000	2.050000	-0.461073	0.887362
4.100000	1.000000	2.100000	-0.461073	0.863209
4.300000	1.000000	2.150000	-0.547358	0.836899
4.400000	1.000000	2.200000	-0.588501	0.808496
4.500000	1.000000	2.250000	-0.628174	0.778073
4.600000	1.000000	2.300000	-0.666276	0.745705
4.700000	1.000000	2.350000	-0.702713	0.711473
4.800000	1.000000	2.400000	-0.737394	0.675463
4.900000	1.000000	2.450000	-0.770231	0.637765
5.000000	1.000000	2.500000	-0.801144	0.598472
5.100000	1.000000	2.550000	-0.830054	0.557684
5.200000	1.000000	2.600000	-0.856889	0.515501
5.300000	1.000000	2.650000	-0.881582	0.472031
5.400000	1.000000	2.700000	-0.904072	0.427380
5.500000	1.000000	2.750000	-0.924302	0.381661
5.600000	1.000000	2.800000	-0.942222	0.334988
5.700000	1.000000	2.850000	-0.957787	0.287478
5.800000	1.000000	2.900000	-0.970958	0.239249
5.900000	1.000000	2.950000	-0.981702	0.190423
6.000000	1.000000	3.000000	-0.989992	0.141120
6.100000	1.000000	3.050000	-0.995808	0.091465
6.200000	1.000000	3.100000	-0.999135	0.091465
6.300000	1.000000	3.150000	-0.999135	-0.008407
6.400000	1.000000	3.150000	-0.999965	-0.008407
6.500000	1.000000	3.250000	-0.996295	-0.108195
6.600000	1.000000	3.300000	-0.994130	-0.108195
6.700000	1.000000	3.350000	-0.978362	-0.206902
6.800000	1.000000	3.400000	-0.966798	-0.255541
6.900000	1.000000	3.450000	-0.952818	-0.303542
7.000000	1.000000	3.500000	-0.936457	-0.350783
7.100000	1.000000	3.550000	-0.917755	-0.397148
7.200000	1.000000	3.600000	-0.896758	-0.442520
7.300000	1.000000	3.650000	-0.873521	-0.486787
7.400000	1.000000	3.700000	-0.848100	-0.529836
7.500000	1.000000	3.750000	-0.820559	-0.571561
7.600000	1.000000	3.800000	-0.790968	-0.611858
7.700000	1.000000	3.850000	-0.759399	-0.650625
7.800000	1.000000	3.900000	-0.725932	-0.687766
7.900000	1.000000	3.950000	-0.690651	-0.723188
8.000000	1.000000	4.000000	-0.653644	-0.756802
8.100000	1.000000	4.050000	-0.615002	-0.788525
8.200000	1.000000	4.100000	-0.574824	-0.818277
8.300000	1.000000	4.150000	-0.533209	-0.845984
8.400000	1.000000	4.200000	-0.490261	-0.871576
8.500000	1.000000	4.250000	-0.446087	-0.894989
8.600000	1.000000	4.300000	-0.400799	-0.916166
8.700000	1.000000	4.350000	-0.354509	-0.935053
8.800000	1.000000	4.400000	-0.354509	
				-0.951602
8.900000	1.000000	4.450000	-0.259389	-0.965773
9.000000	1.000000	4.500000	-0.210796	-0.977530
9.100000	1.000000	4.550000	-0.161676	-0.986844
9.200000	1.000000	4.600000	-0.112153	-0.993691
9.300000	1.000000	4.650000	-0.062349	-0.998054
9.400000	1.000000	4.700000	-0.012389	-0.999923
9.500000	1.000000	4.750000	0.037602	-0.999293
9.600000	1.000000	4.800000	0.087499	-0.996165
9.700000	1.000000	4.850000	0.137177	-0.990547
9.800000	1.000000	4.900000	0.186512	-0.982453
9.900000	1.000000	4.950000	0.235381	-0.971903
10.000000	1.000000	5.000000	0.283662	-0.958924
10.100000	1.000000	5.050000	0.331234	-0.943549
10.200000	1.000000	5.100000	0.377978	-0.925815
10.300000	1.000000	5.150000	0.423777	-0.905767
10.400000	1.000000	5.200000	0.468517	-0.883455
10.500000	1.000000	5.250000	0.512085	-0.858934
10.600000	1.000000	5.300000	0.554374	-0.832267
10.700000	1.000000	5.350000	0.595278	-0.803520
10.700000	1.000000	5.400000	0.634693	-0.772764
10.900000	1.000000	5.450000 5.500000	0.672522 0.708670	-0.740077 -0.705540
11 000000		2 2000000	U. (USD(U)	-U./UD54U
11.000000				
11.000000 11.100000 11.200000	1.000000	5.550000 5.600000	0.743046 0.775566	-0.669240 -0.631267

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11.300000 11.400000 11.500000 11.600000 11.700000 11.800000 12.000000 12.000000 12.2000000 12.3000000 12.5000000 12.5000000 12.3000000 13.000000 13.000000 13.100000 13.200000 13.300000 13.400000 13.500000 13.700000 13.400000 14.100000 14.100000 14.200000 14.200000 14.300000 15.500000 15.500000 15.500000 15.500000 15.500000 15.500000 15.500000 15.500000 15.500000 15.500000 15.500000 15.500000 15.500000 15.700000 15.500000 15.700000 15.700000 15.500000 15.700000					
11.500000 11.600000 11.700000 11.700000 11.900000 12.0000000 12.1000000 12.3000000 12.4000000 12.5000000 12.6000000 12.7000000 13.0000000 13.1000000 13.2000000 13.4000000 13.5000000 13.6000000 13.7000000 13.7000000 13.7000000 14.0000000 14.1000000 14.2000000 14.3000000 14.3000000 14.5000000 14.5000000 15.5000000 15.1000000 15.1000000 15.1000000 15.1000000 15.1000000 15.1000000 15.5000000 15.7000000 15.6000000 15.7000000000 15.7000000000000000000000000000000000000		1.000000	5.650000		-0.591716
11.600000 11.700000 11.800000 11.900000 12.000000 12.1000000 12.3000000 12.4000000 12.5000000 12.7000000 12.3000000 12.7000000 13.000000 13.000000 13.1000000 13.2000000 13.5000000 13.5000000 13.6000000 13.700000 13.700000 13.4000000 14.1000000 14.2000000 14.3000000 14.3000000 14.4000000 14.5000000 14.7000000 14.5000000 15.5000000 15.1000000 15.1000000 15.5000000 15.5000000 15.5000000 15.6000000 15.700000000 15.7000000000000000000000000000000000000		1.000000	5.700000	0.834713	-0.550686
11.700000 11.800000 11.900000 12.000000 12.100000 12.200000 12.3000000 12.5000000 12.6000000 12.6000000 13.0000000 13.0000000 13.1000000 13.2000000 13.3000000 13.4000000 13.5000000 13.6000000 13.7000000 13.7000000 14.0000000 14.1000000 14.1000000 14.2000000 14.3000000 14.3000000 14.5000000 14.7000000 14.7000000 15.5000000 15.1000000 15.1000000 15.5000000 15.5000000 15.7000000000000000000000000000000000000		1.000000	5.750000	0.861192	-0.508279
11.800000 11.900000 12.000000 12.100000 12.200000 12.3000000 12.5000000 12.6000000 12.7000000 13.0000000 13.0000000 13.1000000 13.3000000 13.4000000 13.6000000 13.7000000 13.7000000 13.60000000 13.60000000 14.0000000 14.1000000 14.2000000 14.3000000 14.3000000 14.4000000 14.5000000 14.7000000 14.5000000 15.1000000000 15.1000000 15.10000000000		1.000000	5.800000	0.885520	-0.464602
11.900000 12.000000 12.100000 12.300000 12.300000 12.400000 12.5000000 12.6000000 12.6000000 12.8000000 13.000000 13.1000000 13.2000000 13.3000000 13.4000000 13.5000000 13.6000000 13.7000000 13.6000000 14.1000000 14.1000000 14.2000000 14.3000000 14.3000000 14.5000000 14.7000000 14.7000000 14.5000000 15.0000000 15.1000000 15.1000000 15.1000000 15.1000000 15.1000000 15.1000000 15.7000000 15.10000000000		1.000000	5.850000	0.907633	-0.419764
12.000000 12.100000 12.200000 12.300000 12.400000 12.5000000 12.6000000 12.7000000 12.8000000 13.000000 13.1000000 13.2000000 13.5000000 13.6000000 13.7000000 13.6000000 14.1000000 14.2000000 14.3000000 14.300000 14.4000000 14.5000000 14.5000000 14.5000000 15.10000000000		1.000000	5.900000	0.927478	-0.373877
12.100000 12.200000 12.300000 12.400000 12.500000 12.6000000 12.7000000 12.8000000 13.1000000 13.1000000 13.2000000 13.5000000 13.6000000 13.7000000 13.9000000 14.0000000 14.1000000 14.2000000 14.3000000 14.4000000 14.5000000 14.7000000 14.5000000 15.10000000000		1.000000	5.950000	0.945005	-0.327055
12.200000 12.300000 12.400000 12.500000 12.6000000 12.7000000 12.8000000 13.000000 13.100000 13.200000 13.400000 13.500000 13.600000 13.700000 13.600000 14.000000 14.000000 14.100000 14.300000 14.400000 14.500000 14.500000 14.700000 15.100000 15.200000 15.200000 15.300000 15.400000 15.500000 15.500000 15.600000 15.700000 15.700000 15.800000 15.900000 15.900000 15.900000 16.0000000 16.1000000 16.1000000 16.200000 16.200000 16.300000 16.300000 16.300000 16.300000 16.700000 17.700000 17.300000 17.300000 17.1000000 17.1000000 17.2000000 17.3000000 17.3000000 17.3000000 17.3000000 17.400000 17.7000000 17.7000000 17.7000000 17.7000000 17.8000000 17.9000000 17.9000000 17.9000000 17.9000000 17.9000000 18.0000000 18.0000000 18.0000000 18.0000000 18.0000000 18.0000000000		1.000000	6.000000	0.960170	-0.279415
12.300000 12.400000 12.500000 12.6000000 12.7000000 12.8000000 13.0000000 13.1000000 13.3000000 13.4000000 13.5000000 13.6000000 13.7000000 13.7000000 14.0000000 14.1000000 14.200000 14.3000000 14.400000 14.5000000 14.5000000 15.0000000 15.1000000 15.1000000 15.1000000 15.1000000 15.1000000 15.1000000 15.1000000 15.1000000 15.1000000 15.1000000 15.1000000 15.1000000 15.1000000 15.1000000 15.1000000 15.1000000 15.7000000 15.1000000 15.70000000 15.7000000000000000000000000000000000000		1.000000	6.050000	0.972935	-0.231078
12.400000 12.500000 12.600000 12.700000 12.8000000 13.000000 13.1000000 13.300000 13.400000 13.500000 13.6000000 13.700000 13.800000 14.000000 14.1000000 14.2000000 14.300000 14.400000 14.5000000 14.700000 14.600000 15.0000000 15.1000000 15.1000000 15.1000000 15.1000000 15.1000000 15.1000000 15.1000000 15.1000000 15.1000000 15.1000000 15.1000000 15.1000000 15.700000 15.6000000 15.700000 15.800000 16.700000 16.300000 16.300000 16.300000 16.300000 17.000000 17.1000000 17.2000000 17.5000000 18.5000000 18.5000000 18.5000000 18.5000000 18.5000000 18.5000000 18.5000000000000000000000000000000000000	12.200000	1.000000	6.100000	0.983268	-0.182163
12.500000 12.600000 12.700000 12.800000 12.900000 13.000000 13.1000000 13.2000000 13.400000 13.500000 13.6000000 13.7000000 14.0000000 14.1000000 14.2000000 14.3000000 14.5000000 14.7000000 14.5000000 15.0000000 15.1000000 15.2000000 15.2000000 15.3000000 15.3000000 15.400000 15.400000 15.600000 15.700000 15.700000 15.6000000 15.7000000 15.700000 16.800000 16.800000 17.7000000 18.7000000000000000000000000000000000000	12.300000	1.000000	6.150000	0.991144	-0.132792
12.600000 12.700000 12.800000 12.900000 13.000000 13.100000 13.1000000 13.3000000 13.4000000 13.5000000 13.7000000 13.7000000 14.0000000 14.0000000 14.3000000 14.4000000 14.5000000 14.50000000 14.5000000 15.1000000 15.1000000 15.2000000 15.3000000 15.5000000 15.5000000 15.6000000 15.7000000 15.7000000 15.6000000 15.7000000 16.7000000 16.7000000 17.7000000 18.7000000 18.7000000 18.7000000 18.7000000 18.7000000 18.7000000 18.7000000 18.7000000 18.7000000 18.7000000 18.7000000 18.7000000 18.7000000 18.7000000 18.7000000 18.7000000 18.7000000000000000000000000000000000000	12.400000	1.000000	6.200000	0.996542	-0.083089
12.700000 12.800000 12.900000 13.000000 13.100000 13.100000 13.3000000 13.4000000 13.5000000 13.6000000 13.9000000 14.0000000 14.1000000 14.2000000 14.3000000 14.4000000 14.5000000 14.7000000 14.6000000 14.7000000 15.1000000 15.2000000 15.2000000 15.3000000 15.5000000 15.5000000 15.6000000 15.7000000 15.7000000 15.6000000 15.7000000 15.7000000 15.7000000 15.7000000 15.7000000 15.7000000 15.7000000 15.7000000 15.7000000 15.7000000 15.7000000 15.7000000 15.7000000 15.7000000 15.7000000 15.7000000 15.700000000 16.7000000 17.7000000 18.0000000 18.0000000 18.0000000 18.0000000 18.5000000 18.5000000 18.5000000 18.5000000 18.5000000 18.5000000 18.5000000 18.5000000 18.5000000 18.5000000 18.5000000 18.5000000 18.50000000 18.50000000 18.50000000 18.5000000000000000000000000000000000000	12.500000	1.000000	6.250000	0.999449	-0.033179
12.800000 12.900000 13.000000 13.100000 13.200000 13.3000000 13.4000000 13.5000000 13.6000000 13.900000 14.000000 14.1000000 14.2000000 14.3000000 14.4000000 14.7000000 14.7000000 14.5000000 15.0000000 15.1000000 16.1000000 16.1000000 16.1000000 16.1000000 16.1000000 16.1000000 16.1000000 16.1000000 16.1000000 16.1000000 16.1000000 16.1000000 16.1000000 16.10000000 16.10000000 16.1000000 16.10000000000	12.600000	1.000000	6.300000	0.999859	0.016814
12.800000 12.900000 13.000000 13.100000 13.200000 13.3000000 13.4000000 13.5000000 13.6000000 13.9000000 14.000000 14.1000000 14.2000000 14.3000000 14.7000000 14.7000000 14.7000000 15.1000000 16.1000000 16.1000000 16.1000000 17.10000000 17.1000000000 17.10000000000	12.700000	1.000000	6.350000	0.997769	0.066765
12.900000 13.000000 13.000000 13.100000 13.200000 13.300000 13.4000000 13.5000000 13.6000000 13.7000000 14.0000000 14.1000000 14.2000000 14.3000000 14.4000000 14.5000000 14.5000000 15.1000000 15.2000000 15.2000000 15.3000000 15.4000000 15.5000000 15.7000000 15.7000000 15.6000000 15.7000000 16.8000000 16.7000000 17.7000000 17.7000000 17.7000000 17.7000000 17.7000000 17.7000000 17.7000000 17.7000000 17.7000000 17.7000000 17.7000000 17.7000000 17.7000000 17.7000000 18.0000000 18.0000000 18.0000000 18.0000000 18.0000000 18.0000000 18.0000000 18.0000000 18.0000000 18.00000000 18.0000000000		1.000000	6.400000	0.993185	0.116549
13.000000 13.100000 13.200000 13.300000 13.400000 13.500000 13.500000 13.600000 13.7000000 13.8000000 14.000000 14.1000000 14.200000 14.3000000 14.4000000 14.5000000 14.5000000 15.0000000 15.1000000 15.2000000 15.3000000 15.4000000 15.5000000 15.6000000 15.7000000 15.8000000 15.8000000 15.9000000 15.9000000 16.1000000 16.1000000 16.1000000 16.1000000 16.1000000 16.1000000 16.1000000 16.1000000 16.1000000 17.1000000 16.3000000 17.10000000 17.10000000000		1.000000	6.450000	0.986119	0.166042
13.100000 13.200000 13.300000 13.400000 13.500000 13.6000000 13.7000000 13.8000000 14.000000 14.1000000 14.2000000 14.3000000 14.4000000 14.5000000 14.5000000 15.00000000 15.1000000 15.2000000 15.4000000 15.4000000 15.4000000 15.6000000 15.7000000 15.7000000 15.7000000 15.6000000 15.7000000 15.7000000 15.7000000 15.7000000 15.7000000 15.7000000 15.7000000 15.7000000 15.7000000 15.7000000 15.7000000 15.7000000 15.7000000 15.7000000 15.7000000 15.7000000 15.7000000 15.7000000 16.7000000 17.7000000 17.7000000 17.7000000 17.7000000 17.7000000 17.7000000 17.7000000 17.8000000 17.9000000 17.9000000 17.9000000 17.8000000 17.9000000 18.0000000 18.0000000 18.0000000 18.0000000 18.0000000 18.0000000 18.0000000 18.0000000 18.0000000 18.00000000 18.0000000000		1.000000	6.500000	0.976588	0.215120
13.200000 13.300000 13.400000 13.500000 13.6000000 13.7000000 13.8000000 14.0000000 14.1000000 14.200000 14.400000 14.5000000 14.6000000 14.7000000 15.0000000 15.1000000 15.2000000 15.3000000 15.4000000 15.5000000 15.400000 15.6000000 15.7000000 15.7000000 15.8000000 15.9000000 16.00000000 16.1000000 16.2000000 16.3000000 16.3000000 16.3000000 16.7000000 16.7000000 17.10000000 17.10000000000		1.000000	6.550000	0.964616	0.263660
13.300000 13.400000 13.500000 13.500000 13.600000 13.700000 13.8000000 14.000000 14.1000000 14.300000 14.400000 14.500000 14.6000000 14.7000000 15.0000000 15.1000000 15.2000000 15.400000 15.5000000 15.400000 15.6000000 15.700000 15.700000 15.600000 15.700000 15.600000 15.700000 15.700000 15.700000 15.700000 15.700000 15.700000 15.700000 15.700000 15.700000 15.700000 15.700000 15.700000 15.700000 15.700000 15.700000 15.700000 16.1000000 16.2000000 16.2000000 16.7000000 17.7000000 17.7000000 17.7000000 17.7000000 17.7000000 17.7000000 17.7000000 17.7000000 17.7000000 17.7000000 17.7000000 17.7000000 17.7000000 17.7000000 17.7000000 17.8000000 17.8000000 18.0000000 18.0000000 18.0000000 18.00000000 18.0000000 18.5000000 18.5000000 18.5000000 18.5000000 18.5000000 18.5000000 18.5000000 18.5000000 18.5000000000000000000000000000000000000		1.000000	6.600000	0.950233	0.311541
13.400000 13.500000 13.600000 13.700000 13.800000 13.900000 14.000000 14.100000 14.100000 14.400000 14.500000 14.600000 14.700000 15.000000 15.1000000 15.1000000 15.2000000 15.400000 15.5000000 15.700000 15.6000000 15.700000 15.600000 16.000000 16.0000000 16.1000000 16.2000000 16.300000 16.300000 16.300000 16.700000 16.300000 16.700000 17.100000000 17.1000000 17.1000000 17.1000000 17.1000000 17.10000000000					
13.500000 13.600000 13.700000 13.700000 13.900000 14.000000 14.1000000 14.200000 14.400000 14.500000 14.5000000 14.7000000 15.0000000 15.1000000 15.2000000 15.3000000 15.400000 15.5000000 15.7000000 15.6000000 15.7000000 16.0000000 16.0000000 16.1000000 16.2000000 16.3000000 16.3000000 16.3000000 16.7000000 16.7000000 16.7000000 17.1000000 17.2000000 17.2000000 17.3000000 17.3000000 17.3000000 17.5000000 17.5000000 17.5000000 17.5000000 17.7000000 17.7000000 17.7000000 17.7000000 17.7000000 17.7000000 17.7000000 17.7000000 17.8000000 17.9000000 18.1000000 18.3000000		1.000000	6.650000	0.933474	0.358644
13.600000 13.700000 13.700000 13.800000 14.000000 14.000000 14.1000000 14.3000000 14.4000000 14.5000000 14.5000000 15.0000000 15.0000000 15.1000000 15.2000000 15.3000000 15.4000000 15.7000000 15.7000000 15.7000000 15.6000000 15.7000000 15.6000000 15.7000000 15.7000000 15.7000000 15.7000000 15.7000000 15.7000000 15.7000000 15.7000000 15.7000000 15.7000000 15.7000000 15.7000000 15.7000000 15.7000000 16.7000000 16.7000000 17.7000000 17.1000000 17.1000000 17.7000000 17.7000000 17.7000000 17.7000000 17.7000000 17.8000000 17.9000000 17.9000000 18.0000000 18.10000000 18.1000000 18.1000000 18.1000000 18.1000000		1.000000	6.700000	0.914383	0.404850
13.700000 13.800000 13.900000 14.000000 14.100000 14.100000 14.3000000 14.4000000 14.5000000 14.6000000 14.7000000 15.0000000 15.0000000 15.1000000 15.2000000 15.3000000 15.4000000 15.7000000 15.7000000 15.7000000 15.6000000 15.7000000 16.1000000 16.3000000 16.3000000 16.4000000 16.5000000 16.7000000 16.7000000 16.7000000 17.1000000 17.1000000 17.1000000 17.1000000 17.5000000 17.5000000 17.7000000 17.7000000 17.7000000 17.8000000 17.7000000 17.9000000 17.9000000 17.9000000 17.9000000 17.9000000 18.0000000 18.0000000 18.1000000 18.3000000 18.50000000 18.50000000 18.5000000000000000000000000000000000000		1.000000	6.750000	0.893006	0.450044
13.800000 13.900000 14.000000 14.100000 14.100000 14.300000 14.400000 14.5000000 14.5000000 14.6000000 15.000000 15.1000000 15.2000000 15.3000000 15.4000000 15.7000000 15.7000000 15.7000000 15.6000000 15.7000000 16.1000000 16.1000000 16.3000000 16.3000000 16.4000000 16.7000000 16.7000000 17.10000000 17.10000000000		1.000000	6.800000	0.869397	0.494113
13.900000 14.000000 14.100000 14.100000 14.200000 14.300000 14.400000 14.5000000 14.6000000 14.7000000 15.0000000 15.1000000 15.2000000 15.3000000 15.4000000 15.5000000 15.7000000 15.6000000 15.7000000 16.0000000 16.1000000 16.2000000 16.3000000 16.4000000 16.5000000 16.7000000 16.7000000 17.100000000 17.10000000000		1.000000	6.850000	0.843616	0.536948
14.000000 14.100000 14.200000 14.300000 14.400000 14.500000 14.6000000 14.7000000 15.1000000 15.1000000 15.2000000 15.3000000 15.4000000 15.5000000 15.6000000 15.7000000 15.7000000 15.8000000 15.9000000 16.0000000 16.1000000 16.2000000 16.3000000 16.3000000 16.3000000 16.7000000 16.7000000 17.1000000 17.1000000 17.2000000 17.3000000 17.4000000 17.5000000 17.5000000 17.5000000 17.7000000 17.7000000 17.7000000 17.8000000 17.9000000 17.9000000 18.0000000 18.0000000 18.0000000 18.0000000 18.0000000 18.0000000000		1.000000	6.900000	0.815725	0.578440
14.100000 14.200000 14.300000 14.400000 14.500000 14.600000 14.7000000 14.9000000 15.1000000 15.2000000 15.3000000 15.4000000 15.7000000 15.7000000 15.7000000 16.0000000 16.1000000 16.2000000 16.3000000 16.4000000 16.5000000 16.7000000 16.7000000 17.1000000 18.1000000 18.1000000 18.1000000 18.1000000 18.5000000 18.5000000 18.5000000 18.5000000 18.5000000 18.9000000		1.000000	6.950000	0.785796	0.618486
14.200000 14.300000 14.400000 14.500000 14.600000 14.700000 14.800000 15.0000000 15.2000000 15.3000000 15.400000 15.5000000 15.7000000 15.7000000 15.7000000 16.0000000 16.1000000 16.2000000 16.3000000 16.3000000 16.400000 16.5000000 16.7000000 16.7000000 17.1000000 18.0000000 18.0000000 18.0000000 18.00000000 18.0000000000	14.000000	1.000000	7.000000	0.753902	0.656987
14.30000 14.40000 14.50000 14.50000 14.60000 14.70000 14.80000 15.00000 15.00000 15.100000 15.30000 15.40000 15.50000 15.600000 15.700000 15.800000 15.900000 16.000000 16.100000 16.200000 16.300000 16.400000 16.500000 16.500000 16.700000 16.700000 17.1000000 17.1000000 17.1000000 17.1000000 17.1000000 17.1000000 17.500000 17.500000 17.600000 17.700000 17.700000 17.800000 17.700000 17.800000 17.900000 18.000000 18.1000000 18.2000000 18.3000000 18.3000000 18.3000000 18.5000000 18.5000000 18.5000000 18.5000000 18.5000000 18.5000000 18.5000000 18.5000000 18.5000000 18.5000000 18.5000000 18.5000000000000000000000000000000000000	14.100000	1.000000	7.050000	0.720124	0.693845
14.40000 14.50000 14.60000 14.70000 14.80000 14.90000 15.00000 15.100000 15.200000 15.40000 15.500000 15.600000 15.700000 15.700000 15.800000 16.000000 16.000000 16.100000 16.200000 16.300000 16.400000 16.500000 16.700000 16.700000 17.000000 17.100000 17.100000 17.200000 17.200000 17.300000 17.500000 17.500000 17.700000 17.700000 17.700000 17.800000 17.700000 17.800000 17.900000 18.000000 18.000000 18.100000 18.2000000 18.3000000 18.3000000 18.4000000 18.5000000 18.5000000 18.5000000 18.5000000 18.50000000 18.5000000 18.5000000 18.5000000 18.5000000 18.5000000 18.5000000 18.5000000 18.5000000 18.5000000 18.5000000 18.5000000 18.5000000 18.5000000 18.5000000000000000000000000000000000000	14.200000	1.000000	7.100000	0.684547	0.728969
14.500000 14.600000 14.700000 14.700000 14.900000 15.0000000 15.1000000 15.2000000 15.400000 15.5000000 15.6000000 15.7000000 15.7000000 16.0000000 16.0000000 16.3000000 16.3000000 16.3000000 16.7000000 16.7000000 17.0000000 17.1000000 17.2000000 17.2000000 17.3000000 17.3000000 17.3000000 17.5000000 17.7000000 17.7000000 17.7000000 17.7000000 17.7000000 17.8000000 17.9000000 17.9000000 18.0000000 18.0000000 18.3000000 18.3000000 18.3000000 18.5000000 18.5000000 18.5000000 18.5000000 18.5000000 18.5000000000000000000000000000000000000	14.300000	1.000000	7.150000	0.647258	0.762271
14.500000 14.600000 14.700000 14.700000 14.900000 15.0000000 15.1000000 15.2000000 15.400000 15.5000000 15.6000000 15.7000000 15.7000000 16.0000000 16.0000000 16.3000000 16.3000000 16.3000000 16.7000000 16.7000000 17.0000000 17.1000000 17.2000000 17.2000000 17.3000000 17.3000000 17.3000000 17.5000000 17.7000000 17.7000000 17.7000000 17.7000000 17.7000000 17.8000000 17.9000000 17.9000000 18.0000000 18.0000000 18.3000000 18.3000000 18.3000000 18.5000000 18.5000000 18.5000000 18.5000000 18.5000000 18.5000000000000000000000000000000000000	14.400000	1.000000	7.200000	0.608351	0.793668
14.600000 14.700000 14.800000 14.900000 15.0000000 15.1000000 15.2000000 15.3000000 15.5000000 15.6000000 15.7000000 15.7000000 16.0000000 16.0000000 16.2000000 16.3000000 16.3000000 16.5000000 16.7000000 16.7000000 17.0000000 17.1000000 17.2000000 17.2000000 17.3000000 17.4000000 17.5000000 17.5000000 17.7000000 17.7000000 17.7000000 17.7000000 17.8000000 17.9000000 17.9000000 18.0000000 18.0000000 18.3000000 18.3000000 18.3000000 18.5000000000000000000000000000000000000		1.000000	7.250000	0.567924	0.823081
14.700000 14.800000 14.900000 15.000000 15.1000000 15.2000000 15.3000000 15.5000000 15.5000000 15.6000000 15.7000000 16.0000000 16.3000000 16.3000000 16.4000000 16.5000000 16.5000000 16.7000000 16.7000000 16.7000000 17.000000 17.1000000 17.1000000 17.2000000 17.3000000 17.4000000 17.5000000 17.5000000 17.7000000 17.7000000 17.8000000 17.9000000 17.9000000 18.00000000 18.10000000 18.10000000 18.2000000 18.3000000 18.5000000 18.5000000 18.5000000 18.5000000 18.5000000 18.5000000 18.7000000 18.7000000 18.7000000 18.7000000 18.8000000 18.9000000 18.9000000 18.9000000		1.000000	7.300000	0.526078	0.850437
14.800000 14.900000 15.000000 15.1000000 15.2000000 15.3000000 15.4000000 15.5000000 15.6000000 15.7000000 15.9000000 16.0000000 16.3000000 16.3000000 16.4000000 16.5000000 16.7000000 16.7000000 16.7000000 17.0000000 17.1000000 17.1000000 17.2000000 17.4000000 17.5000000 17.5000000 17.6000000 17.7000000 17.7000000 17.8000000 17.9000000 17.9000000 18.0000000 18.1000000 18.2000000 18.3000000 18.5000000 18.5000000 18.5000000 18.5000000 18.5000000 18.7000000 18.5000000 18.5000000 18.7000000 18.8000000 18.9000000		1.000000	7.350000	0.482916	0.875667
14.900000 15.000000 15.100000 15.200000 15.300000 15.400000 15.500000 15.6000000 15.7000000 15.8000000 16.0000000 16.1000000 16.3000000 16.3000000 16.4000000 16.5000000 16.7000000 16.7000000 17.0000000 17.1000000 17.1000000 17.200000 17.3000000 17.4000000 17.5000000 17.5000000 17.7000000 17.7000000 17.8000000 17.9000000 17.9000000 18.00000000 18.1000000 18.2000000 18.3000000 18.3000000 18.5000000 18.5000000 18.5000000 18.5000000 18.7000000 18.7000000 18.7000000 18.7000000 18.7000000 18.8000000 18.9000000 18.9000000		1.000000	7.400000	0.438547	0.898708
15.000000 15.100000 15.200000 15.300000 15.300000 15.400000 15.500000 15.600000 15.7000000 15.900000 16.1000000 16.300000 16.300000 16.400000 16.500000 16.600000 16.700000 16.700000 17.000000 17.100000 17.100000 17.200000 17.400000 17.500000 17.500000 17.600000 17.700000 17.700000 17.800000 17.900000 17.900000 18.000000 18.1000000 18.2000000 18.3000000 18.5000000 18.5000000 18.5000000 18.5000000 18.5000000 18.7000000 18.7000000 18.7000000 18.7000000 18.8000000 18.9000000 18.9000000		1.000000	7.450000	0.393083	0.919503
15.100000 15.200000 15.300000 15.400000 15.500000 15.600000 15.700000 15.8000000 16.000000 16.1000000 16.300000 16.400000 16.5000000 16.6000000 16.7000000 17.000000 17.1000000 17.1000000 17.1000000 17.400000 17.500000 17.500000 17.600000 17.700000 17.700000 17.700000 17.800000 17.900000 17.900000 18.000000 18.1000000 18.2000000 18.3000000 18.5000000		1.000000	7.500000	0.346635	0.938000
15.200000 15.300000 15.400000 15.500000 15.600000 15.700000 15.800000 15.900000 16.0000000 16.1000000 16.300000 16.400000 16.500000 16.600000 16.700000 17.000000 17.100000 17.100000 17.200000 17.300000 17.400000 17.500000 17.500000 17.700000 17.700000 17.700000 17.800000 17.900000 18.000000 18.000000 18.1000000 18.2000000 18.3000000 18.5000000 18.5000000 18.5000000 18.5000000 18.7000000 18.7000000 18.7000000 18.7000000 18.7000000 18.7000000 18.7000000 18.7000000 18.7000000 18.8000000 18.9000000		1.000000	7.550000	0.299322	0.954152
15.300000 15.400000 15.400000 15.500000 15.600000 15.700000 15.8000000 15.9000000 16.0000000 16.1000000 16.3000000 16.4000000 16.5000000 16.6000000 16.7000000 17.0000000 17.1000000 17.1000000 17.1000000 17.2000000 17.400000 17.500000 17.500000 17.6000000 17.7000000 17.7000000 17.8000000 17.900000 18.000000 18.1000000 18.2000000 18.3000000 18.5000000 18.5000000 18.5000000 18.5000000 18.5000000 18.7000000 18.7000000 18.7000000 18.7000000 18.7000000 18.7000000 18.8000000 18.9000000		1.000000	7.600000	0.253322	0.967920
15.400000 15.500000 15.600000 15.700000 15.700000 15.800000 15.900000 16.000000 16.1000000 16.2000000 16.300000 16.400000 16.5000000 16.6000000 17.000000 17.1000000 17.2000000 17.2000000 17.3000000 17.400000 17.700000 17.700000 17.700000 17.700000 17.8000000 17.900000 18.000000 18.1000000 18.2000000 18.3000000 18.4000000 18.5000000 18.5000000 18.5000000 18.7000000 18.5000000 18.7000000 18.5000000 18.5000000 18.5000000 18.5000000 18.7000000 18.7000000 18.8000000 18.9000000					
15.500000 15.600000 15.700000 15.700000 15.800000 15.900000 16.0000000 16.1000000 16.300000 16.300000 16.500000 16.6000000 16.7000000 17.0000000 17.1000000 17.2000000 17.2000000 17.3000000 17.400000 17.7000000 17.700000 17.700000 17.700000 17.8000000 17.900000 18.000000 18.000000 18.1000000 18.2000000 18.3000000 18.5000000 18.5000000 18.5000000 18.5000000 18.5000000 18.5000000 18.7000000 18.5000000 18.5000000000000000000000000000000000000		1.000000	7.650000	0.202570	0.979268
15.600000 15.700000 15.700000 15.800000 15.900000 16.000000 16.1000000 16.2000000 16.3000000 16.5000000 16.5000000 16.7000000 17.0000000 17.1000000 17.2000000 17.3000000 17.400000 17.500000 17.500000 17.700000 17.700000 17.800000 17.900000 18.000000 18.1000000 18.3000000 18.3000000 18.5000000 18.5000000 18.5000000 18.5000000 18.7000000 18.5000000 18.5000000 18.5000000 18.5000000000000000000000000000000000000		1.000000	7.700000	0.153374	0.988168
15.700000 15.800000 15.900000 16.000000 16.100000 16.200000 16.300000 16.400000 16.500000 16.600000 16.700000 17.000000 17.100000 17.200000 17.300000 17.400000 17.500000 17.600000 17.700000 17.800000 17.900000 17.900000 18.000000 18.100000 18.200000 18.200000 18.500000 18.500000 18.500000 18.700000 18.700000 18.700000 18.700000 18.700000 18.700000 18.800000 18.900000		1.000000	7.750000	0.103794	0.994599
15.800000 15.900000 16.000000 16.1000000 16.2000000 16.3000000 16.4000000 16.5000000 16.6000000 16.7000000 17.0000000 17.1000000 17.2000000 17.3000000 17.4000000 17.5000000 17.6000000 17.7000000 17.8000000 17.9000000 18.0000000 18.1000000 18.2000000 18.2000000 18.3000000 18.5000000 18.5000000 18.7000000 18.7000000 18.7000000 18.7000000 18.8000000 18.9000000000000000000000000000000000000		1.000000	7.800000	0.053955	0.998543
15.900000 16.000000 16.100000 16.200000 16.300000 16.400000 16.500000 16.6000000 16.7000000 17.000000 17.1000000 17.200000 17.300000 17.400000 17.500000 17.600000 17.700000 17.800000 17.900000 18.000000 18.1000000 18.2000000 18.3000000 18.5000000 18.5000000 18.7000000 18.7000000 18.7000000 18.7000000 18.7000000 18.8000000 18.9000000 18.9000000		1.000000	7.850000	0.003982	0.999992
16.000000 16.100000 16.200000 16.300000 16.400000 16.500000 16.600000 16.700000 17.000000 17.100000 17.200000 17.400000 17.500000 17.600000 17.700000 17.800000 17.900000 18.000000 18.1000000 18.200000 18.300000 18.500000 18.500000 18.700000 18.700000 18.700000 18.700000 18.700000 18.700000 18.900000		1.000000	7.900000	-0.046002	0.998941
16.100000 16.200000 16.300000 16.400000 16.500000 16.600000 16.7000000 16.8000000 17.000000 17.1000000 17.200000 17.300000 17.500000 17.600000 17.700000 17.700000 17.800000 17.900000 18.000000 18.1000000 18.2000000 18.300000 18.5000000 18.5000000 18.7000000 18.7000000 18.7000000 18.7000000 18.9000000 18.9000000		1.000000	7.950000	-0.095871	0.995394
16.200000 16.300000 16.400000 16.500000 16.600000 16.700000 16.800000 17.000000 17.100000 17.200000 17.400000 17.500000 17.600000 17.700000 17.800000 17.900000 18.000000 18.1000000 18.2000000 18.3000000 18.400000 18.5000000 18.5000000 18.7000000 18.7000000 18.7000000 18.8000000 18.9000000 18.9000000	16.000000	1.000000	8.000000	-0.145500	0.989358
16.300000 16.400000 16.500000 16.500000 16.600000 16.700000 16.9000000 17.000000 17.100000 17.200000 17.400000 17.500000 17.600000 17.700000 17.800000 17.900000 18.000000 18.1000000 18.2000000 18.2000000 18.3000000 18.5000000 18.5000000 18.5000000 18.7000000 18.7000000 18.7000000 18.9000000 18.9000000	16.100000	1.000000	8.050000	-0.194765	0.980850
16.40000 16.50000 16.50000 16.60000 16.70000 16.80000 17.00000 17.10000 17.20000 17.30000 17.40000 17.50000 17.60000 17.70000 17.80000 17.90000 18.00000 18.200000 18.200000 18.300000 18.400000 18.500000 18.500000 18.500000 18.700000 18.700000 18.700000	16.200000	1.000000	8.100000	-0.243544	0.969890
16.40000 16.50000 16.50000 16.60000 16.70000 16.80000 17.00000 17.10000 17.20000 17.30000 17.40000 17.50000 17.60000 17.70000 17.80000 17.90000 18.00000 18.200000 18.200000 18.300000 18.400000 18.500000 18.500000 18.500000 18.700000 18.700000 18.700000		1.000000	8.150000	-0.291714	0.956506
16.500000 16.600000 16.700000 16.800000 16.900000 17.000000 17.1000000 17.300000 17.400000 17.500000 17.600000 17.700000 17.800000 17.900000 18.000000 18.2000000 18.2000000 18.3000000 18.4000000 18.5000000 18.5000000 18.7000000 18.7000000 18.7000000 18.8000000 18.9000000		1.000000	8.200000	-0.339155	0.940731
16.600000 16.700000 16.800000 16.900000 17.000000 17.1000000 17.2000000 17.3000000 17.500000 17.6000000 17.700000 17.800000 17.900000 18.000000 18.200000 18.3000000 18.3000000 18.4000000 18.5000000 18.5000000 18.7000000 18.7000000 18.7000000 18.9000000		1.000000	8.250000	-0.385748	0.922604
16.700000 16.800000 16.900000 17.000000 17.100000 17.200000 17.300000 17.400000 17.500000 17.700000 17.800000 17.900000 18.000000 18.2000000 18.3000000 18.3000000 18.4000000 18.5000000 18.7000000 18.7000000 18.7000000 18.7000000 18.9000000		1.000000	8.300000	-0.431377	0.902172
16.800000 16.900000 17.000000 17.100000 17.200000 17.300000 17.400000 17.500000 17.6000000 17.700000 17.8000000 18.000000 18.1000000 18.200000 18.400000 18.500000 18.500000 18.700000 18.7000000 18.7000000 18.9000000		1.000000	8.350000	-0.475928	0.879484
16.900000 17.000000 17.100000 17.200000 17.300000 17.400000 17.500000 17.600000 17.700000 17.8000000 18.000000 18.100000 18.200000 18.300000 18.400000 18.500000 18.600000 18.7000000 18.7000000 18.9000000		1.000000	8.400000	-0.519289	0.854599
17.000000 17.100000 17.200000 17.300000 17.400000 17.500000 17.600000 17.700000 17.8000000 18.0000000 18.1000000 18.200000 18.300000 18.400000 18.500000 18.600000 18.7000000 18.7000000 18.8000000 18.9000000					0.854599
17.100000 17.200000 17.300000 17.400000 17.500000 17.600000 17.700000 17.800000 18.000000 18.1000000 18.2000000 18.3000000 18.4000000 18.5000000 18.6000000 18.7000000 18.7000000 18.8000000 18.9000000		1.000000	8.450000	-0.561352	
17.200000 17.300000 17.400000 17.500000 17.500000 17.600000 17.700000 17.900000 18.0000000 18.1000000 18.2000000 18.3000000 18.4000000 18.5000000 18.6000000 18.7000000 18.8000000 18.9000000000000000000000000000000000000		1.000000	8.500000	-0.602012	0.798487
17.300000 17.400000 17.500000 17.500000 17.600000 17.700000 17.800000 18.000000 18.1000000 18.2000000 18.3000000 18.4000000 18.5000000 18.6000000 18.7000000 18.8000000 18.9000000		1.000000	8.550000	-0.641167	0.767401
17.40000 17.50000 17.60000 17.70000 17.80000 17.90000 18.00000 18.100000 18.200000 18.30000 18.40000 18.50000 18.60000 18.70000 18.80000 18.90000		1.000000	8.600000	-0.678720	0.734397
17.500000 17.600000 17.700000 17.800000 17.900000 18.000000 18.1000000 18.2000000 18.3000000 18.400000 18.5000000 18.6000000 18.7000000 18.8000000 18.9000000		1.000000	8.650000	-0.714576	0.699557
17.600000 17.700000 17.800000 17.900000 18.000000 18.100000 18.2000000 18.300000 18.400000 18.500000 18.700000 18.700000 18.9000000		1.000000	8.700000	-0.748647	0.662969
17.70000 17.80000 17.90000 18.00000 18.10000 18.20000 18.30000 18.40000 18.50000 18.60000 18.70000 18.80000 18.90000		1.000000	8.750000	-0.780846	0.624724
17.800000 17.900000 18.000000 18.100000 18.200000 18.300000 18.400000 18.500000 18.600000 18.700000 18.800000 18.900000		1.000000	8.800000	-0.811093	0.584917
17.900000 18.000000 18.100000 18.200000 18.300000 18.400000 18.5000000 18.7000000 18.8000000 18.9000000	17.700000	1.000000	8.850000	-0.839313	0.543648
18.000000 18.100000 18.200000 18.300000 18.400000 18.5000000 18.7000000 18.8000000 18.9000000	17.800000	1.000000	8.900000	-0.865435	0.501021
18.000000 18.100000 18.200000 18.300000 18.400000 18.5000000 18.700000 18.800000 18.900000		1.000000	8.950000	-0.889394	0.457141
18.10000 18.20000 18.30000 18.40000 18.50000 18.60000 18.70000 18.80000 18.90000		1.000000	9.000000	-0.911130	0.412118
18.200000 18.300000 18.400000 18.500000 18.600000 18.700000 18.800000 18.900000		1.000000	9.050000	-0.930589	0.366066
18.30000 18.40000 18.50000 18.60000 18.70000 18.80000 18.90000		1.000000	9.100000	-0.947722	0.319098
18.40000 18.50000 18.60000 18.70000 18.80000 18.90000		1.000000	9.150000	-0.962485	0.271333
18.500000 18.600000 18.700000 18.800000 18.900000		1.000000	9.200000	-0.974844	0.222890
18.600000 18.700000 18.800000 18.900000		1.000000	9.250000	-0.984765	0.222890
18.700000 18.800000 18.900000		1.000000	9.300000	-0.992225	0.173889
18.800000 18.900000					
18.900000		1.000000	9.350000	-0.997205	0.074708
		1.000000	9.400000	-0.999693	0.024775
19.000000		1.000000	9.450000	-0.999682	-0.025219
	10 000000				
19.100000		1.000000	9.500000 9.550000	-0.997172 -0.992170	-0.075151 -0.124895

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```
19.200000
           1.000000
                      9.600000
                                -0.984688 -0.174327
19.300000
           1.000000
                      9.650000
                                -0.974745
                                           -0.223323
19.400000
           1.000000
                      9.700000
                                -0.962365 -0.271761
19.500000
           1.000000
                      9.750000
                                -0.947580
                                           -0.319519
19.600000
           1.000000
                      9.800000
                                -0.930426
                                           -0.366479
19.700000
           1.000000
                      9.850000
                                -0.910947
                                           -0.412523
19.800000
           1.000000
                      9.900000
                                -0.889191
                                           -0.457536
19.900000
           1.000000
                      9.950000
                                -0.865213
                                           -0.501405
```

5.2 Plots

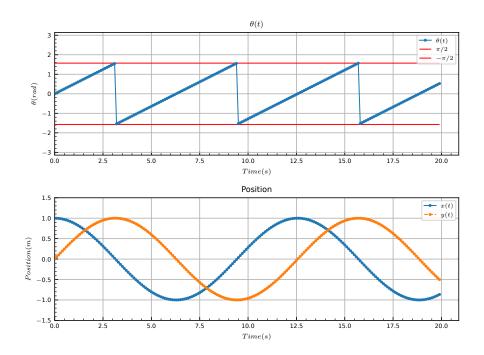


Figure 1: θ and position vs. time

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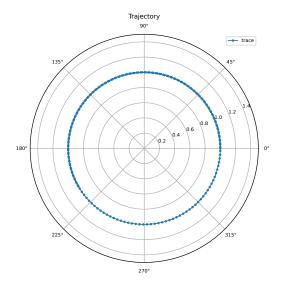


Figure 2: Trajectory in polar coordinates

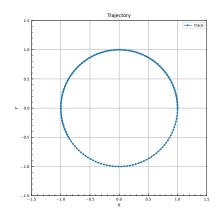


Figure 3: Trajectory in cartesian coordinates

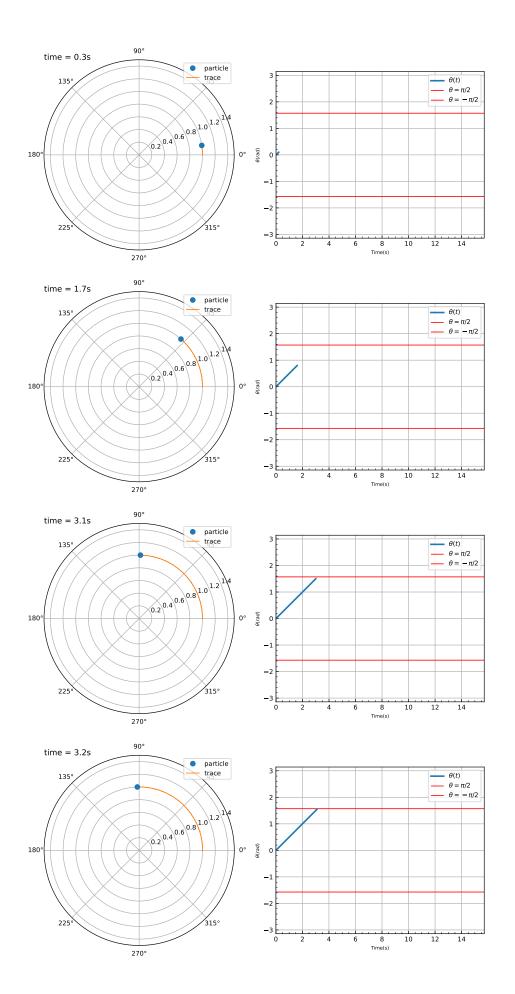
5.3 Animation

Note: Input parameters,

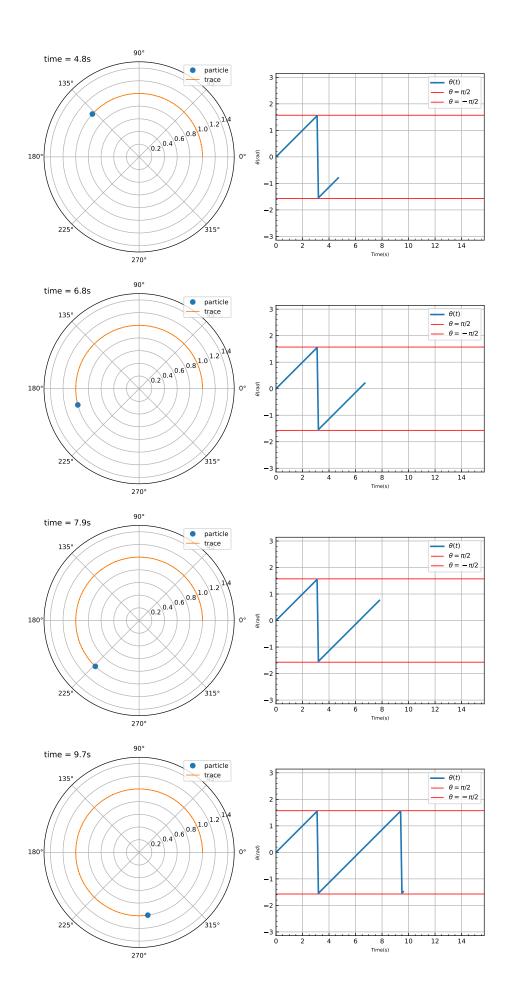
$$\omega = 0.5 \ rad/s$$

$$R = 1 \ m$$

$$\implies T = 12.5664 \ s$$
 (6)



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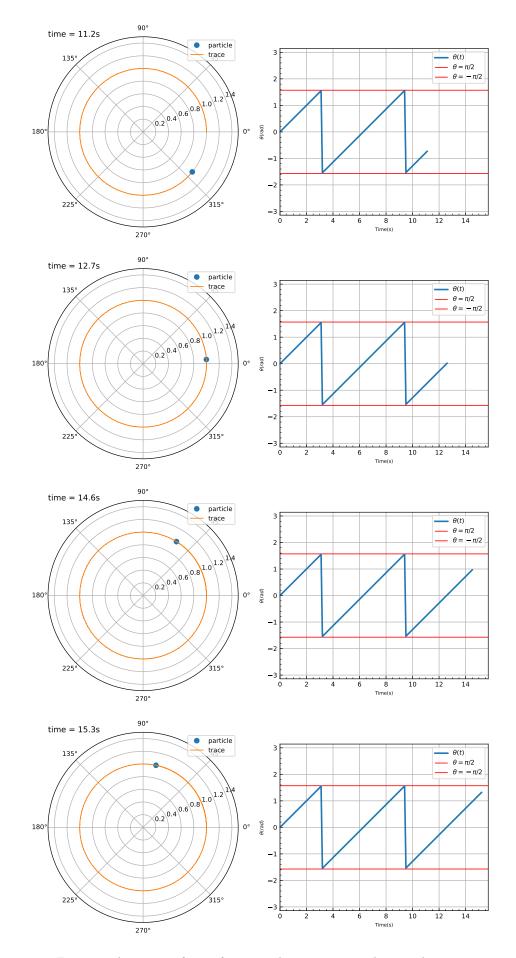


Figure 4: Animation for uniform circular motion in polar coordinates

6 Remarks

The programs can be used to trace and simulate the motion of any particle in uniform circular motion by defining the required parameters.

The parameters computed numerically and via the programs are in agreement.

These programs can be useful in cases of circular motion, and with some modifications with satellite and planetary motions.