

Program-M2

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Write and execute a FORTRAN program to simulate motion of a particle in uniform circular motion in cartesian coordinates.

1 Theory

1.1 Motion of a particle in uniform circular motion

Degrees of freedom, $dof = 1$

Choosing θ as the generalized coordinate.

$$\begin{aligned}\text{Lagrangian, } \mathcal{L} &= T - V \\ \mathcal{L} &= \frac{1}{2}mr^2\dot{\theta}^2\end{aligned}\tag{1}$$

Using Euler-Lagrange equation

$$\frac{\partial \mathcal{L}}{\partial \theta} = \frac{\partial}{\partial t} \frac{\partial \mathcal{L}}{\partial \dot{\theta}}\tag{2}$$

$$\boxed{\ddot{\theta} = 0}\tag{3}$$

Therefore, equation of motion is $\ddot{\theta} = 0$ or $\dot{\theta} = \omega = \text{constant}$

1.1.1 In Cartesian coordinates

$$\begin{aligned}x &= R \cos(\theta) \\ y &= R \sin(\theta) \\ \therefore \theta &= \omega t \\ \implies x &= R \cos(\omega t) \\ y &= R \sin(\omega t)\end{aligned}\tag{4}$$

$$\boxed{\begin{aligned}v_x = \dot{x} &= -R\omega \sin(\omega t) \\ v_y = \dot{y} &= R\omega \cos(\omega t) \\ a_x = \ddot{x} &= -R\omega^2 \cos(\omega t) \\ a_y = \ddot{y} &= -R\omega^2 \sin(\omega t)\end{aligned}}\tag{5}$$

2 Numerical Solution

For $\omega = 0.5$, $R = 1.0$, time period, T will be:

$$\begin{aligned}T &= \frac{2\pi}{\omega} \\ T &= 12.5664 \text{ s}\end{aligned}\tag{6}$$

3 Program Algorithm

NOTE: Blue-colored text represents variables in the algorithm, eg. `variable`.

1. Program open.
2. Define variables (`PI`, `x0`, `y0`, `R`, `omega`, `t0`, `tf`, `dt`, `t`, `fmt1`).
3. Define functions (`x(t)`, `y(t)`, `dx(t)`, `dy(t)`, `ddx(t)`, `ddy(t)`).
4. Open a writable data file.
5. Get input from user for angular velocity (`omega`), radius of the circle(`R`), initial position (`x0`, `y0`) and time period(`t0`, `tf`, `dt`).
6. Print parameters to stdout for the user.
7. Write appropriate comments in the data file and initialize other parameters.

8. Define a do while loop with index t which runs from t_0 to t_f .
9. Compute the parameters using the functions $x(t)$, $y(t)$, $dx(t)$, $dy(t)$, $ddx(t)$, $ddy(t)$.
10. Write the parameters to stdout and data file.
11. Increment the index according to $t = t + dt$
12. End do-while loop.
13. Close data file.
14. Program close.

4 Program

4.1 Fortran program:

For computing the parameters

```

=====
! Uniform circular motion in cartesian coordinates
! Author: Devansh Shukla
=====
program CircularMotion_Cartesian
    ! Program to compute motion of a particle moving with uniform angular velocity in cartesian coordinates.

    implicit none
    real, parameter :: PI=3.141593
    real :: x0=0.0, y0=0.0, R=0.0, omega=0.0
    real :: x, y, dx, dy, ddx, ddy
    real :: t0=0.0, tf=0.0, dt=0.0, t=0.0
    character(len=*), parameter :: fmt1 = "(F10.4,x,F10.4,x,F10.4,x,F10.4,x,F10.4,x,F10.4,x,F10.4)"

    x(t) = x0 + R*cos(omega*(t-t0))
    y(t) = y0 + R*sin(omega*(t-t0))

    dx(t) = -R*omega*sin(omega*(t-t0))
    dy(t) = R*omega*cos(omega*(t-t0))

    ddx(t) = -R*omega*omega*cos(omega*(t-t0))
    ddy(t) = -R*omega*omega*sin(omega*(t-t0))

    open(UNIT=8, FILE="CircleCartesian.dat")

    ! input
    print *, "Enter angular velocity(omega) and radius (R)"
    read *, omega, R
    if (omega .le. 0.0) stop "Illegal value of omega"
    if (R .le. 0.0) stop "Illegal value of R"

    print *, "Enter the value of x0, y0"
    read *, x0, y0

    print *, "Enter t0, tf, dt"
    read *, t0, tf, dt
    if (dt .le. 0.0) stop "Illegal value of dt"

    print *, "-----"
    print "(x,A,F10.4,F10.4)", "omega, R =", omega, R
    print "(x,A,F10.4)", "T =", 2.0*PI/omega
    print "(x,A,F10.4,F10.4)", "x0, y0 =", x0, y0
    print "(x,A,F10.4,F10.4,F10.4)", "t0, tf, dt =", t0, tf, dt
    print *, "-----"

    write (8, *) "# x0=", x0
    write (8, *) "# y0=", y0
    write (8, *) "# t0=", t0
    write (8, *) "# t x y dx dy ddx ddy"
    print "(xA10,A10,xA10,A10,A10,xA10,xA10)", "time", "x(t)", "y(t)", "vx(t)", "vy(t)", "ax(t)", "ay(t)"
    ! Computing
    t = t0
    do while (t <= tf)
        write (*, fmt1) t, x(t), y(t), dx(t), dy(t), ddx(t), ddy(t)
        write (8, fmt1) t, x(t), y(t), dx(t), dy(t), ddx(t), ddy(t)
        t = t + dt
    enddo
    print *, "-----"
    close(8)

end program CircularMotion_Cartesian

```

4.2 Python program: Plots

```
#!/usr/bin/env python
"""
Author: Devansh Shukla
"""
# In[0]
import pandas as pd
import numpy as np
import matplotlib as mpl
import matplotlib.pyplot as plt
import matplotlib.gridspec as gridspec

custom_rcparams = {
    "axes.labelsize": 8,
    "axes.titlesize": 10,
    "axes.grid": True,
    # Figure
    "figure.autolayout": True,
    "figure.figsize": (8, 8),
    "figure.titlesize": 11,
    "savefig.format": "pdf",
    "lines.linewidth": 1,
    # Legend
    "legend.fontsize": 8,
    "legend.frameon": True,
    # Ticks
    "xtick.labelsize": 6,
    "ytick.labelsize": 6,
    "xtick.minor.visible": True,
    "xtick.direction": "in",
    "ytick.direction": "in",
    "ytick.minor.visible": True,
    # TeX
    "pgf.texsystem": "lualatex",
}
mpl.rcParams.update(custom_rcparams)
mpl.use("pgf")
plt.ioff()

# t x y dx dy ddx ddy
df = pd.read_csv("CircleCartesian.dat", engine="python", delimiter=" ", header=None, skipinitialspace=True, comment="#")
print(df)

gs = gridspec.GridSpec(2, 2)

fig = plt.figure()
ax = plt.subplot(gs[0, 0])
plt.plot(df[0], df[1], "o-", markersize=1.5, color="C0", label=r"$x(t)$")
plt.plot(df[0], df[2], "o-", markersize=1.5, color="C1", label=r"$y(t)$")
plt.title("Position")
plt.xlim(left=0)
ax.set_xlabel(r"$Time(s)$")
ax.set_ylabel(r"$Position(m)$")
plt.legend(loc="upper right")

ax = plt.subplot(gs[0, 1])
plt.plot(df[1], df[2], "o-", markersize=1.5, color="C1", label="trace")
plt.title("Trajectory")
ax.set_aspect("equal")
ax.set_xlabel("X")
ax.set_ylabel("Y")
plt.legend(loc="upper right")

ax = plt.subplot(gs[1, 0])
plt.plot(df[0], df[3], "o-", markersize=1.5, color="C0", label=r"$v_x(t)$")
plt.plot(df[0], df[4], "o-", markersize=1.5, color="C1", label=r"$v_y(t)$")
plt.title("Velocity")
plt.xlim(left=0)
ax.set_xlabel(r"$Time(s)$")
ax.set_ylabel(r"$Velocity(m/s)$")
plt.legend(loc="upper right")

ax = plt.subplot(gs[1, 1])
plt.plot(df[0], df[5], "o-", markersize=1.5, color="C0", label=r"$a_x(t)$")
plt.plot(df[0], df[6], "o-", markersize=1.5, color="C1", label=r"$a_y(t)$")
plt.title("Acceleration")
plt.xlim(left=0)
ax.set_xlabel(r"$Time(s)$")
```

```

ax.set_ylabel(r"$Acceleration(m/s^2)$")
plt.legend(loc="upper right")

plt.suptitle("Circular Motion", fontsize=12)
plt.savefig("plots/i_params.pdf")

# plt.show()
# %%

```

4.3 Python program: Animation

```

#!/usr/bin/env python
"""
Author: Devansh Shukla
"""
import pandas as pd
import numpy as np
import matplotlib as mpl
import matplotlib.pyplot as plt
from matplotlib.animation import FuncAnimation, FFMpegWriter

custom_rcparams = {
    "axes.labelsize": 6,
    "axes.titlesize": 8,
    "axes.grid": True,
    # Figure
    "figure.autolayout": True,
    "figure.titlesize": 9,
    # "figure.dpi": 200,
    "figure.figsize": (8, 3),
    "savefig.format": "pdf",
    "lines.linewidth": 1,
    # Legend
    "legend.fontsize": 8,
    "legend.frameon": True,
    # Ticks
    "xtick.labelsize": 8,
    "ytick.labelsize": 8,
    "xtick.minor.visible": True,
    "xtick.direction": "in",
    "ytick.direction": "in",
    "ytick.minor.visible": True,
}
mpl.rcParams.update(custom_rcparams)

df = pd.read_csv("CircleCartesian.dat", engine="python", delimiter=" ", header=None, skipinitialspace=True, comment="#")

# Extract data
time = df[0].values[:,1]
x0 = df[1].iloc[0] - 1
y0 = df[2].iloc[0]
pos_x = df[1].values[:,1]
pos_y = df[2].values[:,1]

# Plot
fig, (ax1, ax2) = plt.subplots(1, 2)
line1, = ax1.plot([], [], 'o', lw=2, label="particle")
line2, = ax2.plot([], [], '-', lw=2, label=r"$x(t)$")
line3, = ax2.plot([], [], '-', lw=2, label=r"$y(t)$")
trace, = ax1.plot([], [], '-', lw=1, label="trace")
time_template = "time = %.1fs"
time_text = ax1.text(0.05, 0.9, '', transform=ax1.transAxes)

line = [line1, line2, line3,]

ax1.set_xlim(-2, 2)
ax1.set_ylim(-2, 2)
ax1.set_aspect("equal")
ax1.set_xlabel("X", labelpad=0)
ax1.set_ylabel("Y", labelpad=0)
ax1.legend()

ax2.set_xlim(left=0, right=time[-1])
ax2.set_ylim(-1.25, 1.25)
ax2.set_xlabel("Time(s)", labelpad=0)
ax2.set_ylabel("Position", labelpad=0)
ax2.set_aspect(5)
ax2.legend(loc="upper right")

def init():

```

```

    line[0].set_data([], [])
    line[1].set_data([], [])
    line[2].set_data([], [])
    trace.set_data([], [])
    return line, trace

def animate(i):
    global time, pos_x, pos_y, theta
    line[0].set_data(pos_x[i], pos_y[i])
    line[1].set_data(time[:i], pos_x[:i])
    line[2].set_data(time[:i], pos_y[:i])
    trace.set_data(pos_x[:i], pos_y[:i])
    time_text.set_text(time_template % (time[i]))
    return line, trace, time_text

def toggle_capture(*args, **kwargs):
    global ani, capture_no
    ani.pause()
    plt.gcf().savefig(f"plots/circle_cart_capture_{capture_no}.pdf")
    capture_no += 1
    ani.resume()

capture_no = 0
ani = FuncAnimation(fig, animate, frames=len(time), interval=1, init_func=init, blit=False, repeat=False)
fig.canvas.mpl_connect('button_press_event', toggle_capture)
writer = FFMpegWriter(fps=10)
ani.save('animation.mp4', writer=writer)
plt.show()

```

5 Results

5.1 Terminal Output

```

Enter angular velocity(omega) and radius (R)
0.5 1.0
Enter the value of x0, y0
0.0 0.0
Enter t0, tf, dt
0.0 20.0 0.1
-----
omega, R =    0.5000    1.0000
T =    12.5664
x0, y0 =    0.0000    0.0000
t0, tf, dt =    0.0000    20.0000    0.1000
-----

```

| time | x(t) | y(t) | vx(t) | vy(t) | ax(t) | ay(t) |
|--------|---------|--------|---------|---------|---------|---------|
| 0.0000 | 1.0000 | 0.0000 | -0.0000 | 0.5000 | -0.2500 | -0.0000 |
| 0.1000 | 0.9988 | 0.0500 | -0.0250 | 0.4994 | -0.2497 | -0.0125 |
| 0.2000 | 0.9950 | 0.0998 | -0.0499 | 0.4975 | -0.2488 | -0.0250 |
| 0.3000 | 0.9888 | 0.1494 | -0.0747 | 0.4944 | -0.2472 | -0.0374 |
| 0.4000 | 0.9801 | 0.1987 | -0.0993 | 0.4900 | -0.2450 | -0.0497 |
| 0.5000 | 0.9689 | 0.2474 | -0.1237 | 0.4845 | -0.2422 | -0.0619 |
| 0.6000 | 0.9553 | 0.2955 | -0.1478 | 0.4777 | -0.2388 | -0.0739 |
| 0.7000 | 0.9394 | 0.3429 | -0.1714 | 0.4697 | -0.2348 | -0.0857 |
| 0.8000 | 0.9211 | 0.3894 | -0.1947 | 0.4605 | -0.2303 | -0.0974 |
| 0.9000 | 0.9004 | 0.4350 | -0.2175 | 0.4502 | -0.2251 | -0.1087 |
| 1.0000 | 0.8776 | 0.4794 | -0.2397 | 0.4388 | -0.2194 | -0.1199 |
| 1.1000 | 0.8525 | 0.5227 | -0.2613 | 0.4263 | -0.2131 | -0.1307 |
| 1.2000 | 0.8253 | 0.5646 | -0.2823 | 0.4127 | -0.2063 | -0.1412 |
| 1.3000 | 0.7961 | 0.6052 | -0.3026 | 0.3980 | -0.1990 | -0.1513 |
| 1.4000 | 0.7648 | 0.6442 | -0.3221 | 0.3824 | -0.1912 | -0.1611 |
| 1.5000 | 0.7317 | 0.6816 | -0.3408 | 0.3658 | -0.1829 | -0.1704 |
| 1.6000 | 0.6967 | 0.7174 | -0.3587 | 0.3484 | -0.1742 | -0.1793 |
| 1.7000 | 0.6600 | 0.7513 | -0.3756 | 0.3300 | -0.1650 | -0.1878 |
| 1.8000 | 0.6216 | 0.7833 | -0.3917 | 0.3108 | -0.1554 | -0.1958 |
| 1.9000 | 0.5817 | 0.8134 | -0.4067 | 0.2908 | -0.1454 | -0.2034 |
| 2.0000 | 0.5403 | 0.8415 | -0.4207 | 0.2702 | -0.1351 | -0.2104 |
| 2.1000 | 0.4976 | 0.8674 | -0.4337 | 0.2488 | -0.1244 | -0.2169 |
| 2.2000 | 0.4536 | 0.8912 | -0.4456 | 0.2268 | -0.1134 | -0.2228 |
| 2.3000 | 0.4085 | 0.9128 | -0.4564 | 0.2042 | -0.1021 | -0.2282 |
| 2.4000 | 0.3624 | 0.9320 | -0.4660 | 0.1812 | -0.0906 | -0.2330 |
| 2.5000 | 0.3153 | 0.9490 | -0.4745 | 0.1577 | -0.0788 | -0.2372 |
| 2.6000 | 0.2675 | 0.9636 | -0.4818 | 0.1337 | -0.0669 | -0.2409 |
| 2.7000 | 0.2190 | 0.9757 | -0.4879 | 0.1095 | -0.0548 | -0.2439 |
| 2.8000 | 0.1700 | 0.9854 | -0.4927 | 0.0850 | -0.0425 | -0.2464 |
| 2.9000 | 0.1205 | 0.9927 | -0.4964 | 0.0603 | -0.0301 | -0.2482 |
| 3.0000 | 0.0707 | 0.9975 | -0.4987 | 0.0354 | -0.0177 | -0.2494 |
| 3.1000 | 0.0208 | 0.9998 | -0.4999 | 0.0104 | -0.0052 | -0.2499 |
| 3.2000 | -0.0292 | 0.9996 | -0.4998 | -0.0146 | 0.0073 | -0.2499 |

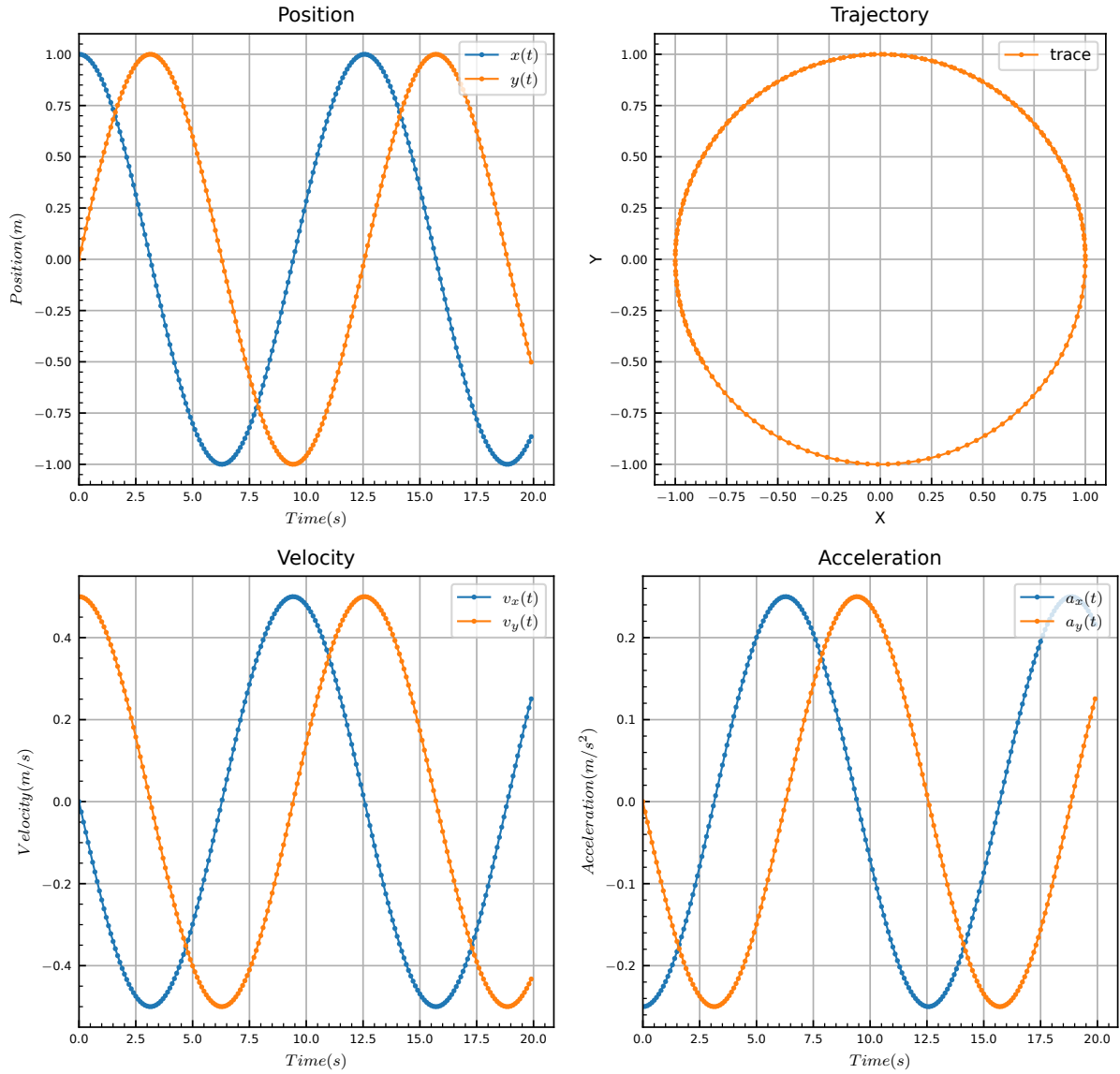
| | | | | | | |
|---------|---------|---------|---------|---------|---------|---------|
| 3.3000 | -0.0791 | 0.9969 | -0.4984 | -0.0396 | 0.0198 | -0.2492 |
| 3.4000 | -0.1288 | 0.9917 | -0.4958 | -0.0644 | 0.0322 | -0.2479 |
| 3.5000 | -0.1782 | 0.9840 | -0.4920 | -0.0891 | 0.0446 | -0.2460 |
| 3.6000 | -0.2272 | 0.9738 | -0.4869 | -0.1136 | 0.0568 | -0.2435 |
| 3.7000 | -0.2756 | 0.9613 | -0.4806 | -0.1378 | 0.0689 | -0.2403 |
| 3.8000 | -0.3233 | 0.9463 | -0.4732 | -0.1616 | 0.0808 | -0.2366 |
| 3.9000 | -0.3702 | 0.9290 | -0.4645 | -0.1851 | 0.0925 | -0.2322 |
| 4.0000 | -0.4161 | 0.9093 | -0.4546 | -0.2081 | 0.1040 | -0.2273 |
| 4.1000 | -0.4611 | 0.8874 | -0.4437 | -0.2305 | 0.1153 | -0.2218 |
| 4.2000 | -0.5048 | 0.8632 | -0.4316 | -0.2524 | 0.1262 | -0.2158 |
| 4.3000 | -0.5474 | 0.8369 | -0.4184 | -0.2737 | 0.1368 | -0.2092 |
| 4.4000 | -0.5885 | 0.8085 | -0.4042 | -0.2943 | 0.1471 | -0.2021 |
| 4.5000 | -0.6282 | 0.7781 | -0.3890 | -0.3141 | 0.1570 | -0.1945 |
| 4.6000 | -0.6663 | 0.7457 | -0.3729 | -0.3331 | 0.1666 | -0.1864 |
| 4.7000 | -0.7027 | 0.7115 | -0.3557 | -0.3514 | 0.1757 | -0.1779 |
| 4.8000 | -0.7374 | 0.6755 | -0.3377 | -0.3687 | 0.1843 | -0.1689 |
| 4.9000 | -0.7702 | 0.6378 | -0.3189 | -0.3851 | 0.1926 | -0.1594 |
| 5.0000 | -0.8011 | 0.5985 | -0.2992 | -0.4006 | 0.2003 | -0.1496 |
| 5.1000 | -0.8301 | 0.5577 | -0.2788 | -0.4150 | 0.2075 | -0.1394 |
| 5.2000 | -0.8569 | 0.5155 | -0.2578 | -0.4284 | 0.2142 | -0.1289 |
| 5.3000 | -0.8816 | 0.4720 | -0.2360 | -0.4408 | 0.2204 | -0.1180 |
| 5.4000 | -0.9041 | 0.4274 | -0.2137 | -0.4520 | 0.2260 | -0.1068 |
| 5.5000 | -0.9243 | 0.3817 | -0.1908 | -0.4622 | 0.2311 | -0.0954 |
| 5.6000 | -0.9422 | 0.3350 | -0.1675 | -0.4711 | 0.2356 | -0.0837 |
| 5.7000 | -0.9578 | 0.2875 | -0.1437 | -0.4789 | 0.2394 | -0.0719 |
| 5.8000 | -0.9710 | 0.2393 | -0.1196 | -0.4855 | 0.2427 | -0.0598 |
| 5.9000 | -0.9817 | 0.1904 | -0.0952 | -0.4909 | 0.2454 | -0.0476 |
| 6.0000 | -0.9900 | 0.1411 | -0.0706 | -0.4950 | 0.2475 | -0.0353 |
| 6.1000 | -0.9958 | 0.0915 | -0.0457 | -0.4979 | 0.2490 | -0.0229 |
| 6.2000 | -0.9991 | 0.0416 | -0.0208 | -0.4996 | 0.2498 | -0.0104 |
| 6.3000 | -1.0000 | -0.0084 | 0.0042 | -0.5000 | 0.2500 | 0.0021 |
| 6.4000 | -0.9983 | -0.0584 | 0.0292 | -0.4991 | 0.2496 | 0.0146 |
| 6.5000 | -0.9941 | -0.1082 | 0.0541 | -0.4971 | 0.2485 | 0.0270 |
| 6.6000 | -0.9875 | -0.1577 | 0.0789 | -0.4937 | 0.2469 | 0.0394 |
| 6.7000 | -0.9784 | -0.2069 | 0.1035 | -0.4892 | 0.2446 | 0.0517 |
| 6.8000 | -0.9668 | -0.2555 | 0.1278 | -0.4834 | 0.2417 | 0.0639 |
| 6.9000 | -0.9528 | -0.3035 | 0.1518 | -0.4764 | 0.2382 | 0.0759 |
| 7.0000 | -0.9365 | -0.3508 | 0.1754 | -0.4682 | 0.2341 | 0.0877 |
| 7.1000 | -0.9178 | -0.3971 | 0.1986 | -0.4589 | 0.2294 | 0.0993 |
| 7.2000 | -0.8968 | -0.4425 | 0.2213 | -0.4484 | 0.2242 | 0.1106 |
| 7.3000 | -0.8735 | -0.4868 | 0.2434 | -0.4368 | 0.2184 | 0.1217 |
| 7.4000 | -0.8481 | -0.5298 | 0.2649 | -0.4241 | 0.2120 | 0.1325 |
| 7.5000 | -0.8206 | -0.5716 | 0.2858 | -0.4103 | 0.2051 | 0.1429 |
| 7.6000 | -0.7910 | -0.6119 | 0.3059 | -0.3955 | 0.1977 | 0.1530 |
| 7.7000 | -0.7594 | -0.6506 | 0.3253 | -0.3797 | 0.1899 | 0.1627 |
| 7.8000 | -0.7259 | -0.6878 | 0.3439 | -0.3630 | 0.1815 | 0.1719 |
| 7.9000 | -0.6907 | -0.7232 | 0.3616 | -0.3453 | 0.1727 | 0.1808 |
| 8.0000 | -0.6536 | -0.7568 | 0.3784 | -0.3268 | 0.1634 | 0.1892 |
| 8.1000 | -0.6150 | -0.7885 | 0.3943 | -0.3075 | 0.1538 | 0.1971 |
| 8.2000 | -0.5748 | -0.8183 | 0.4091 | -0.2874 | 0.1437 | 0.2046 |
| 8.3000 | -0.5332 | -0.8460 | 0.4230 | -0.2666 | 0.1333 | 0.2115 |
| 8.4000 | -0.4903 | -0.8716 | 0.4358 | -0.2451 | 0.1226 | 0.2179 |
| 8.5000 | -0.4461 | -0.8950 | 0.4475 | -0.2230 | 0.1115 | 0.2237 |
| 8.6000 | -0.4008 | -0.9162 | 0.4581 | -0.2004 | 0.1002 | 0.2290 |
| 8.7000 | -0.3545 | -0.9351 | 0.4675 | -0.1773 | 0.0886 | 0.2338 |
| 8.8000 | -0.3073 | -0.9516 | 0.4758 | -0.1537 | 0.0768 | 0.2379 |
| 8.9000 | -0.2594 | -0.9658 | 0.4829 | -0.1297 | 0.0648 | 0.2414 |
| 9.0000 | -0.2108 | -0.9775 | 0.4888 | -0.1054 | 0.0527 | 0.2444 |
| 9.1000 | -0.1617 | -0.9868 | 0.4934 | -0.0808 | 0.0404 | 0.2467 |
| 9.2000 | -0.1122 | -0.9937 | 0.4968 | -0.0561 | 0.0280 | 0.2484 |
| 9.3000 | -0.0623 | -0.9981 | 0.4990 | -0.0312 | 0.0156 | 0.2495 |
| 9.4000 | -0.0124 | -0.9999 | 0.5000 | -0.0062 | 0.0031 | 0.2500 |
| 9.5000 | 0.0376 | -0.9993 | 0.4996 | 0.0188 | -0.0094 | 0.2498 |
| 9.6000 | 0.0875 | -0.9962 | 0.4981 | 0.0437 | -0.0219 | 0.2490 |
| 9.7000 | 0.1372 | -0.9905 | 0.4953 | 0.0686 | -0.0343 | 0.2476 |
| 9.8000 | 0.1865 | -0.9825 | 0.4912 | 0.0933 | -0.0466 | 0.2456 |
| 9.9000 | 0.2354 | -0.9719 | 0.4860 | 0.1177 | -0.0588 | 0.2430 |
| 10.0000 | 0.2837 | -0.9589 | 0.4795 | 0.1418 | -0.0709 | 0.2397 |
| 10.1000 | 0.3312 | -0.9435 | 0.4718 | 0.1656 | -0.0828 | 0.2359 |
| 10.2000 | 0.3780 | -0.9258 | 0.4629 | 0.1890 | -0.0945 | 0.2315 |
| 10.3000 | 0.4238 | -0.9058 | 0.4529 | 0.2119 | -0.1059 | 0.2264 |
| 10.4000 | 0.4685 | -0.8835 | 0.4417 | 0.2343 | -0.1171 | 0.2209 |
| 10.5000 | 0.5121 | -0.8589 | 0.4295 | 0.2560 | -0.1280 | 0.2147 |
| 10.6000 | 0.5544 | -0.8323 | 0.4161 | 0.2772 | -0.1386 | 0.2081 |
| 10.7000 | 0.5953 | -0.8035 | 0.4018 | 0.2976 | -0.1488 | 0.2009 |
| 10.8000 | 0.6347 | -0.7728 | 0.3864 | 0.3173 | -0.1587 | 0.1932 |
| 10.9000 | 0.6725 | -0.7401 | 0.3700 | 0.3363 | -0.1681 | 0.1850 |
| 11.0000 | 0.7087 | -0.7055 | 0.3528 | 0.3543 | -0.1772 | 0.1764 |
| 11.1000 | 0.7430 | -0.6692 | 0.3346 | 0.3715 | -0.1858 | 0.1673 |

| | | | | | | |
|---------|---------|---------|---------|---------|---------|---------|
| 11.2000 | 0.7756 | -0.6313 | 0.3156 | 0.3878 | -0.1939 | 0.1578 |
| 11.3000 | 0.8061 | -0.5917 | 0.2959 | 0.4031 | -0.2015 | 0.1479 |
| 11.4000 | 0.8347 | -0.5507 | 0.2753 | 0.4174 | -0.2087 | 0.1377 |
| 11.5000 | 0.8612 | -0.5083 | 0.2541 | 0.4306 | -0.2153 | 0.1271 |
| 11.6000 | 0.8855 | -0.4646 | 0.2323 | 0.4428 | -0.2214 | 0.1161 |
| 11.7000 | 0.9076 | -0.4198 | 0.2099 | 0.4538 | -0.2269 | 0.1049 |
| 11.8000 | 0.9275 | -0.3739 | 0.1869 | 0.4637 | -0.2319 | 0.0935 |
| 11.9000 | 0.9450 | -0.3271 | 0.1635 | 0.4725 | -0.2363 | 0.0818 |
| 12.0000 | 0.9602 | -0.2794 | 0.1397 | 0.4801 | -0.2400 | 0.0699 |
| 12.1000 | 0.9729 | -0.2311 | 0.1155 | 0.4865 | -0.2432 | 0.0578 |
| 12.2000 | 0.9833 | -0.1822 | 0.0911 | 0.4916 | -0.2458 | 0.0455 |
| 12.3000 | 0.9911 | -0.1328 | 0.0664 | 0.4956 | -0.2478 | 0.0332 |
| 12.4000 | 0.9965 | -0.0831 | 0.0415 | 0.4983 | -0.2491 | 0.0208 |
| 12.5000 | 0.9994 | -0.0332 | 0.0166 | 0.4997 | -0.2499 | 0.0083 |
| 12.6000 | 0.9999 | 0.0168 | -0.0084 | 0.4999 | -0.2500 | -0.0042 |
| 12.7000 | 0.9978 | 0.0668 | -0.0334 | 0.4989 | -0.2494 | -0.0167 |
| 12.8000 | 0.9932 | 0.1166 | -0.0583 | 0.4966 | -0.2483 | -0.0291 |
| 12.9000 | 0.9861 | 0.1660 | -0.0830 | 0.4931 | -0.2465 | -0.0415 |
| 13.0000 | 0.9766 | 0.2151 | -0.1076 | 0.4883 | -0.2441 | -0.0538 |
| 13.1000 | 0.9646 | 0.2637 | -0.1318 | 0.4823 | -0.2412 | -0.0659 |
| 13.2000 | 0.9502 | 0.3115 | -0.1558 | 0.4751 | -0.2376 | -0.0779 |
| 13.3000 | 0.9335 | 0.3587 | -0.1793 | 0.4667 | -0.2334 | -0.0897 |
| 13.4000 | 0.9144 | 0.4049 | -0.2024 | 0.4572 | -0.2286 | -0.1012 |
| 13.5000 | 0.8930 | 0.4501 | -0.2250 | 0.4465 | -0.2233 | -0.1125 |
| 13.6000 | 0.8694 | 0.4941 | -0.2471 | 0.4347 | -0.2173 | -0.1235 |
| 13.7000 | 0.8436 | 0.5370 | -0.2685 | 0.4218 | -0.2109 | -0.1342 |
| 13.8000 | 0.8157 | 0.5784 | -0.2892 | 0.4079 | -0.2039 | -0.1446 |
| 13.9000 | 0.7858 | 0.6185 | -0.3092 | 0.3929 | -0.1964 | -0.1546 |
| 14.0000 | 0.7539 | 0.6570 | -0.3285 | 0.3769 | -0.1885 | -0.1642 |
| 14.1000 | 0.7201 | 0.6939 | -0.3469 | 0.3601 | -0.1800 | -0.1735 |
| 14.2000 | 0.6845 | 0.7290 | -0.3645 | 0.3423 | -0.1711 | -0.1822 |
| 14.3000 | 0.6473 | 0.7623 | -0.3811 | 0.3236 | -0.1618 | -0.1906 |
| 14.4000 | 0.6083 | 0.7937 | -0.3968 | 0.3042 | -0.1521 | -0.1984 |
| 14.5000 | 0.5679 | 0.8231 | -0.4115 | 0.2840 | -0.1420 | -0.2058 |
| 14.6000 | 0.5261 | 0.8504 | -0.4252 | 0.2630 | -0.1315 | -0.2126 |
| 14.7000 | 0.4829 | 0.8757 | -0.4378 | 0.2415 | -0.1207 | -0.2189 |
| 14.8000 | 0.4385 | 0.8987 | -0.4494 | 0.2193 | -0.1096 | -0.2247 |
| 14.9000 | 0.3931 | 0.9195 | -0.4598 | 0.1965 | -0.0983 | -0.2299 |
| 15.0000 | 0.3466 | 0.9380 | -0.4690 | 0.1733 | -0.0867 | -0.2345 |
| 15.1000 | 0.2993 | 0.9542 | -0.4771 | 0.1497 | -0.0748 | -0.2385 |
| 15.2000 | 0.2512 | 0.9679 | -0.4840 | 0.1256 | -0.0628 | -0.2420 |
| 15.3000 | 0.2026 | 0.9793 | -0.4896 | 0.1013 | -0.0506 | -0.2448 |
| 15.4000 | 0.1534 | 0.9882 | -0.4941 | 0.0767 | -0.0383 | -0.2470 |
| 15.5000 | 0.1038 | 0.9946 | -0.4973 | 0.0519 | -0.0259 | -0.2486 |
| 15.6000 | 0.0539 | 0.9985 | -0.4993 | 0.0270 | -0.0135 | -0.2496 |
| 15.7000 | 0.0040 | 1.0000 | -0.5000 | 0.0020 | -0.0010 | -0.2500 |
| 15.8000 | -0.0460 | 0.9989 | -0.4995 | -0.0230 | 0.0115 | -0.2497 |
| 15.9000 | -0.0959 | 0.9954 | -0.4977 | -0.0479 | 0.0240 | -0.2488 |
| 16.0000 | -0.1455 | 0.9894 | -0.4947 | -0.0728 | 0.0364 | -0.2473 |
| 16.1000 | -0.1948 | 0.9808 | -0.4904 | -0.0974 | 0.0487 | -0.2452 |
| 16.2000 | -0.2436 | 0.9699 | -0.4849 | -0.1218 | 0.0609 | -0.2425 |
| 16.3000 | -0.2917 | 0.9565 | -0.4783 | -0.1459 | 0.0729 | -0.2391 |
| 16.4000 | -0.3392 | 0.9407 | -0.4704 | -0.1696 | 0.0848 | -0.2352 |
| 16.5000 | -0.3858 | 0.9226 | -0.4613 | -0.1929 | 0.0964 | -0.2306 |
| 16.6000 | -0.4314 | 0.9022 | -0.4511 | -0.2157 | 0.1078 | -0.2255 |
| 16.7000 | -0.4759 | 0.8795 | -0.4397 | -0.2380 | 0.1190 | -0.2199 |
| 16.8000 | -0.5193 | 0.8546 | -0.4273 | -0.2597 | 0.1298 | -0.2136 |
| 16.9000 | -0.5614 | 0.8276 | -0.4138 | -0.2807 | 0.1403 | -0.2069 |
| 17.0000 | -0.6020 | 0.7985 | -0.3992 | -0.3010 | 0.1505 | -0.1996 |
| 17.1000 | -0.6412 | 0.7674 | -0.3837 | -0.3206 | 0.1603 | -0.1918 |
| 17.2000 | -0.6787 | 0.7344 | -0.3672 | -0.3394 | 0.1697 | -0.1836 |
| 17.3000 | -0.7146 | 0.6995 | -0.3498 | -0.3573 | 0.1786 | -0.1749 |
| 17.4000 | -0.7487 | 0.6630 | -0.3315 | -0.3743 | 0.1872 | -0.1657 |
| 17.5000 | -0.7809 | 0.6247 | -0.3124 | -0.3904 | 0.1952 | -0.1562 |
| 17.6000 | -0.8111 | 0.5849 | -0.2925 | -0.4056 | 0.2028 | -0.1462 |
| 17.7000 | -0.8393 | 0.5436 | -0.2718 | -0.4197 | 0.2098 | -0.1359 |
| 17.8000 | -0.8654 | 0.5010 | -0.2505 | -0.4327 | 0.2164 | -0.1253 |
| 17.9000 | -0.8894 | 0.4571 | -0.2286 | -0.4447 | 0.2224 | -0.1143 |
| 18.0000 | -0.9111 | 0.4121 | -0.2061 | -0.4556 | 0.2278 | -0.1030 |
| 18.1000 | -0.9306 | 0.3661 | -0.1830 | -0.4653 | 0.2326 | -0.0915 |
| 18.2000 | -0.9477 | 0.3191 | -0.1595 | -0.4739 | 0.2369 | -0.0798 |
| 18.3000 | -0.9625 | 0.2713 | -0.1357 | -0.4812 | 0.2406 | -0.0678 |
| 18.4000 | -0.9748 | 0.2229 | -0.1114 | -0.4874 | 0.2437 | -0.0557 |
| 18.5000 | -0.9848 | 0.1739 | -0.0869 | -0.4924 | 0.2462 | -0.0435 |
| 18.6000 | -0.9922 | 0.1244 | -0.0622 | -0.4961 | 0.2481 | -0.0311 |
| 18.7000 | -0.9972 | 0.0747 | -0.0373 | -0.4986 | 0.2493 | -0.0187 |
| 18.8000 | -0.9997 | 0.0248 | -0.0124 | -0.4998 | 0.2499 | -0.0062 |
| 18.9000 | -0.9997 | -0.0252 | 0.0126 | -0.4998 | 0.2499 | 0.0063 |
| 19.0000 | -0.9972 | -0.0752 | 0.0376 | -0.4986 | 0.2493 | 0.0188 |

| | | | | | | |
|---------|---------|---------|--------|---------|--------|--------|
| 19.1000 | -0.9922 | -0.1249 | 0.0625 | -0.4961 | 0.2480 | 0.0312 |
| 19.2000 | -0.9847 | -0.1743 | 0.0872 | -0.4923 | 0.2462 | 0.0436 |
| 19.3000 | -0.9747 | -0.2233 | 0.1117 | -0.4874 | 0.2437 | 0.0558 |
| 19.4000 | -0.9624 | -0.2718 | 0.1359 | -0.4812 | 0.2406 | 0.0679 |
| 19.5000 | -0.9476 | -0.3195 | 0.1598 | -0.4738 | 0.2369 | 0.0799 |
| 19.6000 | -0.9304 | -0.3665 | 0.1832 | -0.4652 | 0.2326 | 0.0916 |
| 19.7000 | -0.9109 | -0.4125 | 0.2063 | -0.4555 | 0.2277 | 0.1031 |
| 19.8000 | -0.8892 | -0.4576 | 0.2288 | -0.4446 | 0.2223 | 0.1144 |
| 19.9000 | -0.8652 | -0.5014 | 0.2507 | -0.4326 | 0.2163 | 0.1254 |

5.2 Plots

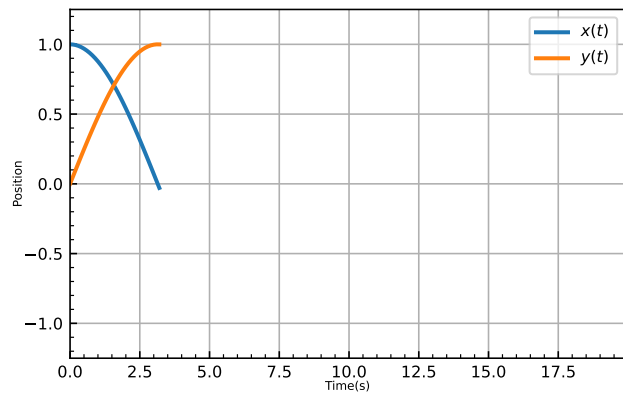
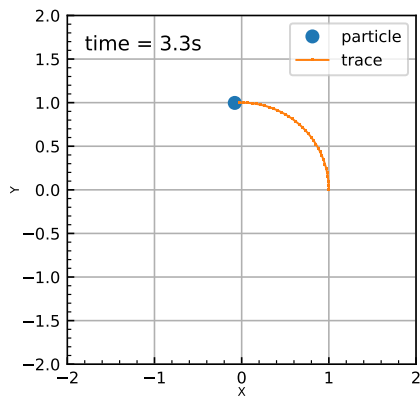
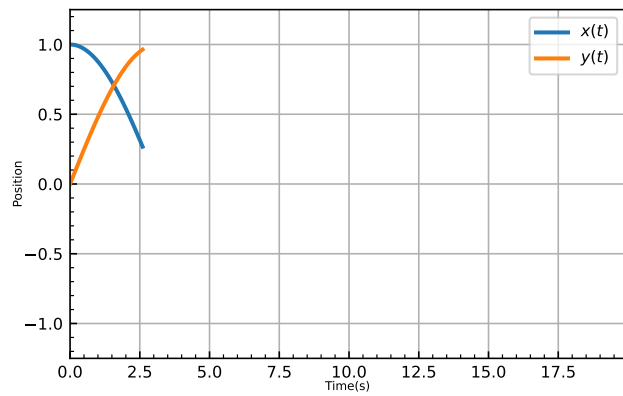
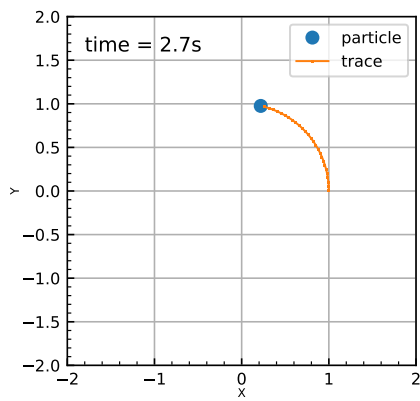
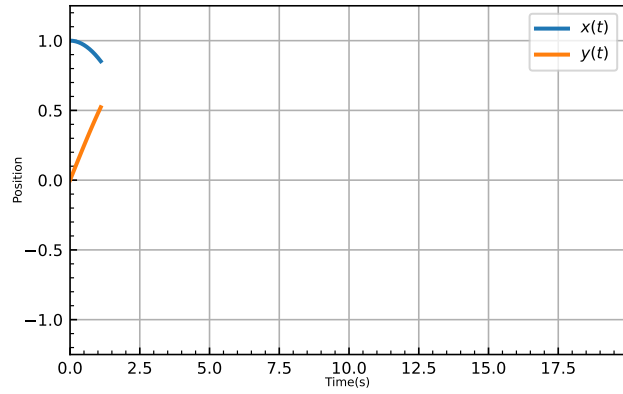
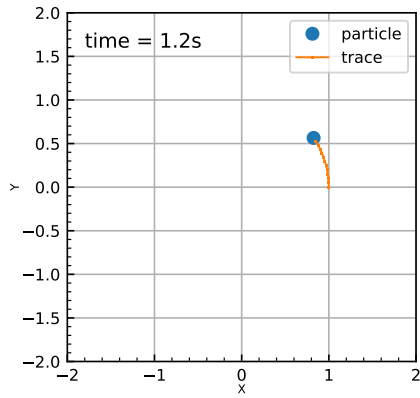
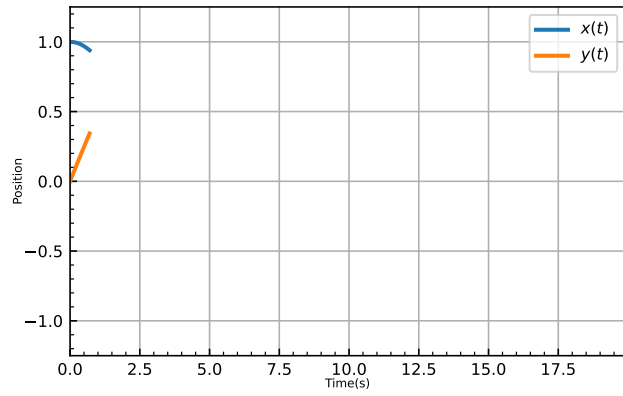
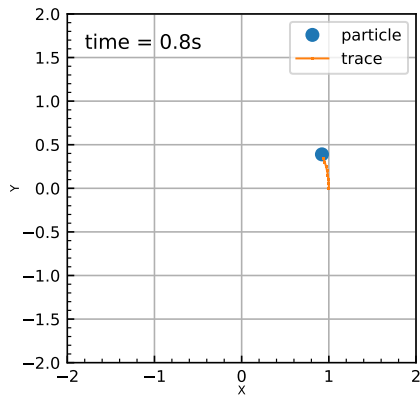
Circular Motion

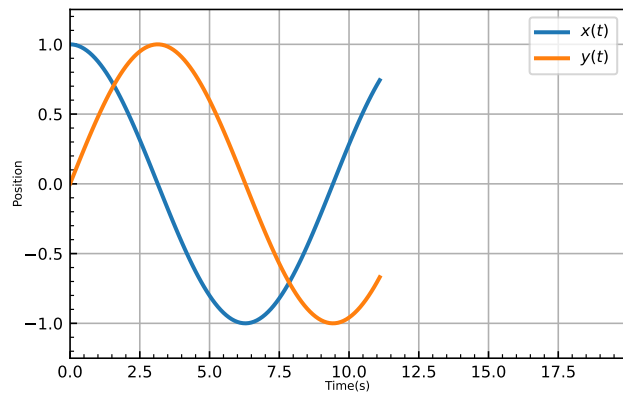
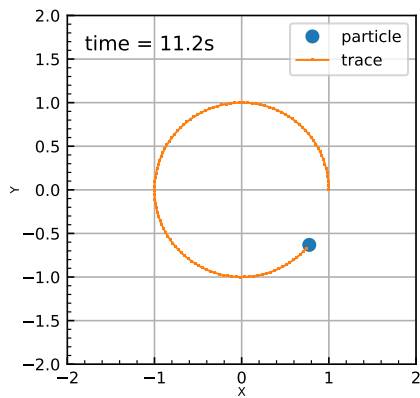
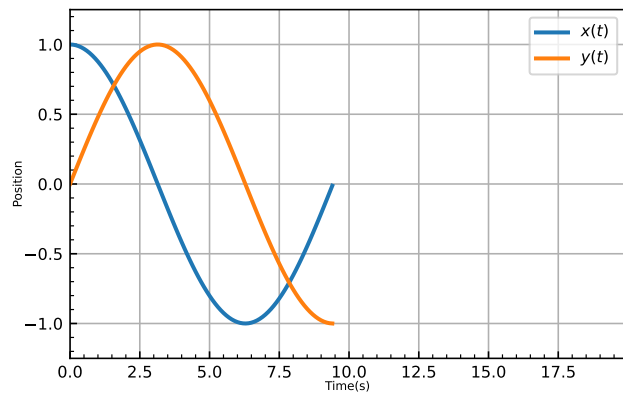
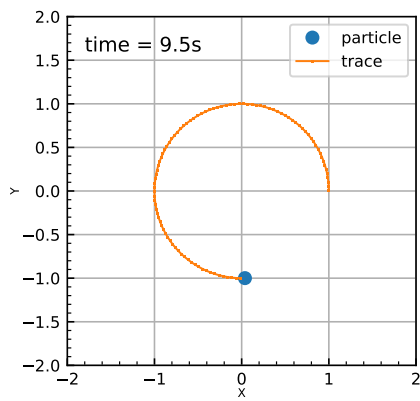
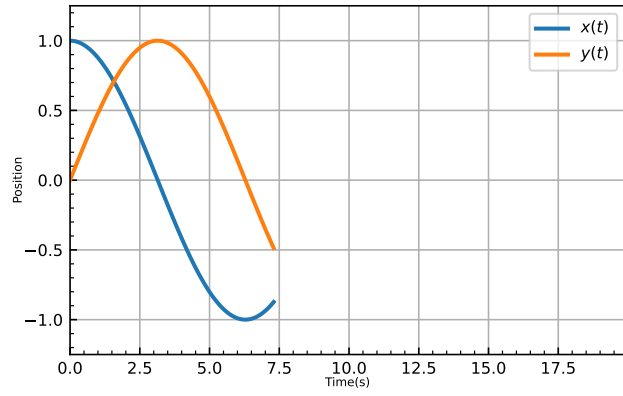
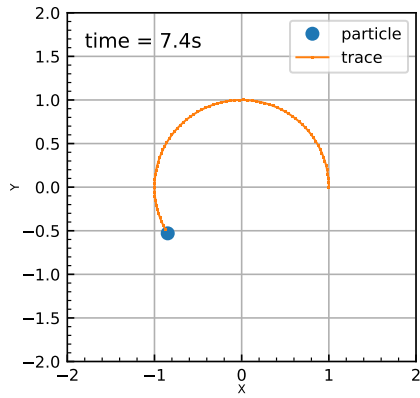
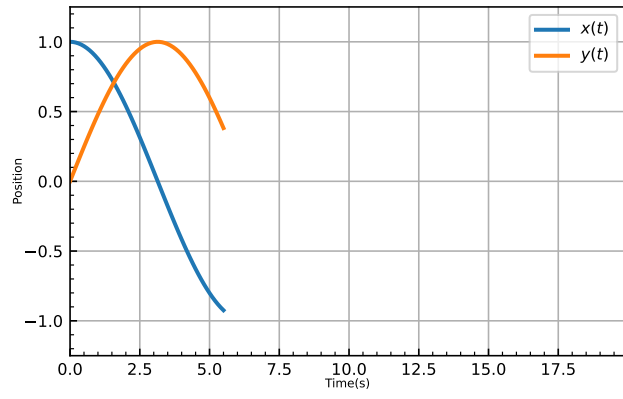
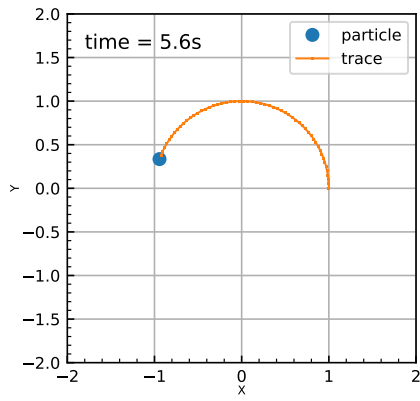


5.3 Animation

Note: Input parameters,

$$\begin{aligned}
 \omega &= 0.5 \text{ rad/s} \\
 R &= 1 \text{ m} \\
 \Rightarrow T &= 12.5664 \text{ s}
 \end{aligned}
 \tag{7}$$





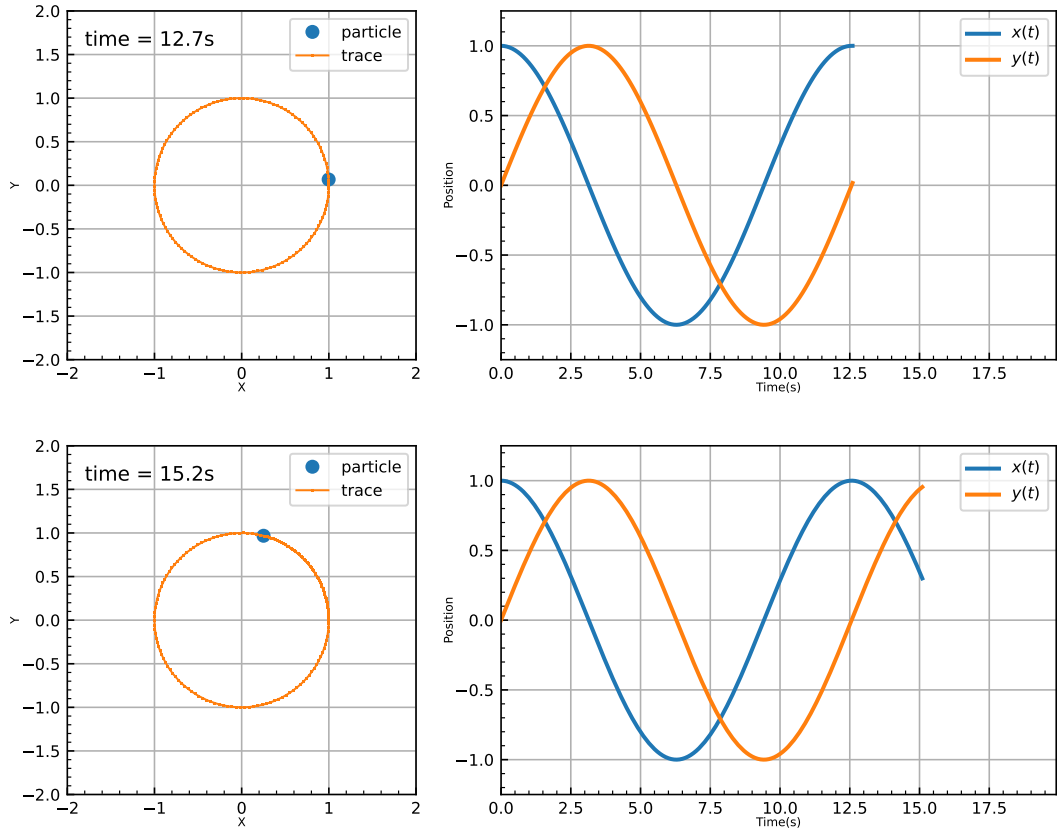


Figure 1: Animation for uniform circular motion in cartesian coords.

6 Remarks

The programs can be used to trace and simulate the motion of any particle in uniform circular motion by defining the required parameters.

The parameters computed numerically and via the programs are in agreement.