

Program-M8

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Write and execute a FORTRAN program for computing motion of a particle trapped in a 3-dimensional box.

1 Theory

1.1 Particle in a 3D box

The particle moves freely for $0 < x < L_x$, $0 < y < L_y$ and $0 < z < L_z$; when it reaches the boundaries $x = 0$, $x = L_x$, $y = 0$, $y = L_y$, $z = 0$ and $z = L_z$, it bounces and its velocity instantly reverses. The potential energy is given by (1), which has a shape of infinitely deep well.

$$V(x, y, z) = \begin{cases} 0 & 0 < x < L_x, 0 < y < L_y, 0 < z < L_z \\ +\infty & \text{elsewhere} \end{cases} \quad (1)$$

Initially, we know that the particle's position is (x_0, y_0, z_0) with velocity (v_x, v_y, v_z) at time t_0 .

Equation of motions are:

$$\begin{aligned} x(t + dt) &= x(t) + v_x(t)dt \\ y(t + dt) &= y(t) + v_y(t)dt \\ z(t + dt) &= z(t) + v_z(t)dt \end{aligned} \quad (2)$$

When the particle bounces of the boundary: $v(t) \rightarrow -v(t)$.

2 Numerical Solution

Assuming,

$$\begin{aligned} x_0 &= y_0 = z_0 = 0 \\ v_x &= v_y = v_z = 1 \text{ m/s} \\ t_0 &= 0 \\ L_x &= 10 \text{ m} \\ L_y &= 8 \text{ m} \\ L_z &= 6 \text{ m} \end{aligned} \quad (3)$$

We expect

$$\begin{aligned} v_x(t) &\rightarrow -v_x(t) \text{ at } x = 10, 20, 30, \dots \\ v_y(t) &\rightarrow -v_y(t) \text{ at } y = 8, 16, 24, 32, \dots \\ v_z(t) &\rightarrow -v_z(t) \text{ at } z = 6, 12, 18, 24, \dots \end{aligned} \quad (4)$$

3 Program Algorithm

NOTE: Blue-colored text represents variables in the algorithm, eg. `variable`.

1. Program open.
2. Define variables (`x0`, `y0`, `z0`, `t0`, `tf`, `dt`, `t`, `x`, `y`, `z`, `vx`, `vy`, `vz`, `lx0`, `lx1`, `ly0`, `ly1`, `lz0`, `lz1`, `fmt1`).
3. Open a writable data file.
4. Get input from user for initial position (`x0`, `y0`, `z0`), initial velocity(`vx`, `vy`, `vz`), bounds (`lx0`, `lx1`, `ly0`, `ly1`, `lz0`, `lz1`) and time period(`t0`, `tf`, `dt`).
5. Print parameters to stdout for the user.
6. Write appropriate comments in the data file and initialize other parameters.
7. Compute `x`, `y`, `z` and `t` for the initial position.
8. Define a do while loop with index `t` which runs from `t0` to `tf`.
9. Compute the parameters `x`, `y`.
10. Write the parameters to stdout and data file.
11. If `x` reaches `lx0` or `lx1` \Rightarrow `vx` = -`vx`.
12. If `y` reaches `ly0` or `ly1` \Rightarrow `vy` = -`vy`.
13. Increment the index according to `t` = `t` + `dt`
14. End do-while loop.
15. Close data file.
16. Program close.

4 Program

4.1 Fortran program:

For computing the parameters

```
=====
! particle3d.f90
! Author: Devansh Shukla
!-----
program particle_3D
    ! Program to compute motion of a particle trapped in a 3d box

    implicit none
    real*8 :: x0, y0, z0, t0, tf, dt, t
    real*8 :: x, y, z, vx, vy, vz, lx0, lx1, ly0, ly1, lz0, lz1
    character(len=*), parameter :: fmt1 = "(F10.4,x,F10.4,x,F10.4,x,F10.4,x,F10.4,F10.4,x,F10.4)"

    open(unit=8, file="Particle3D.dat")

    print *, "-----"
    print *, "Enter initial position x0, y0, z0"
    read *, x0, y0, z0

    print *, "Enter velocity vx, vy, vz"
    read *, vx, vy, vz

    print *, "Enter bounds lx0, lx1"
    read *, lx0, lx1

    print *, "Enter bounds ly0, ly1"
    read *, ly0, ly1

    print *, "Enter bounds lz0, lz1"
    read *, lz0, lz1

    print *, "Enter t0, tf, dt"
    read *, t0, tf, dt
    print *, "-----"

    print "(x,A,F10.4,F10.4,F10.4)", "x0, y0, z0=", x0, y0, z0
    print "(x,A,F10.4,F10.4)", "lx0, lx1=", lx0, lx1
    print "(x,A,F10.4,F10.4)", "ly0, ly1=", ly0, ly1
    print "(x,A,F10.4,F10.4)", "lz0, lz1=", lz0, lz1
    print "(x,A,F10.4,F10.4,F10.4)", "t0, tf, dt=", t0, tf, dt
    print *, "-----"

    print "(A10,A10,A10,xA10,xA10,xA10,xA10)", "time", "x(t)", "y(t)", "z(t)", "vx(t)", "vy(t)", "vz(t)"
    ! Formatting 10, 11 to our standards for fortran floating point arithmetic
    lx1 = lx1 - dt
    ly1 = ly1 - dt
    lz1 = lz1 - dt
    t = t0
    x = x0
    y = y0
    z = z0
    do while (t <= tf)
        write (*, fmt1) t, x, y, z, vx, vy, vz
        write (8, fmt1) t, x, y, z, vx, vy, vz
        x = x + vx * dt
        y = y + vy * dt
        z = z + vz * dt
        t = t + dt
        if (x < lx0 .or. x > lx1) vx = -vx
        if (y < ly0 .or. y > ly1) vy = -vy
        if (z < lz0 .or. z > lz1) vz = -vz
    enddo
    print *, "-----"
    close(8)
end program particle_3D
```

4.2 Python program: Plots

```
#!/usr/bin/env python
"""
Author: Devansh Shukla
"""
# In[0]
```

```

import pandas as pd
import numpy as np
import matplotlib as mpl
import matplotlib.pyplot as plt
import matplotlib.gridspec as gridspec

custom_rcparams = {
    "axes.labelsize": 8,
    "axes.titlesize": 10,
    "axes.titleweight": "normal",
    "axes.grid": True,
    # Figure
    "figure.autolayout": True,
    "figure.figsize": (9, 6),
    "figure.titlesize": 10,
    "savefig.format": "pdf",
    "lines.linewidth": 1,
    # Legend
    "legend.fontsize": 8,
    "legend.frameon": True,
    # Ticks
    "xtick.labelsize": 8,
    "ytick.labelsize": 8,
    "xtick.minor.visible": True,
    "xtick.direction": "in",
    "ytick.direction": "in",
    "ytick.minor.visible": True,
    # TeX
    "pgf.texsystem": "lualatex",
}
mpl.rcParams.update(custom_rcparams)
mpl.use("pgf")
plt.ioff()

# t, x, y, z, vx, vy, vz
df = pd.read_csv("Particle3D.dat", engine="python", delimiter=" ", header=None, skipinitialspace=True, comment="#")

fig = plt.figure(figsize=(10,4))
gs = gridspec.GridSpec(1, 2)
ax = fig.add_subplot(gs[0, 0])
ax.plot(df[0], df[1], "o-", markersize=1.5, color="C0", label=r"$x(t)$")
ax.plot(df[0], df[2], "o-", markersize=1.5, color="C1", label=r"$y(t)$")
ax.plot(df[0], df[3], "o-", markersize=1.5, color="C2", label=r"$z(t)$")
ax.set_xlim(left=0)
ax.set_ylim(0, 12)
ax.set_xlabel(r"$Time(s)$")
ax.set_ylabel(r"$Position(m)$")
ax.legend(loc="upper right")
plt.title("Position")

ax = fig.add_subplot(gs[0, 1])
ax.plot(df[0], df[4], "o-", markersize=1.5, color="C0", label=r"$v_x(t)$")
ax.plot(df[0], df[5], "o-", markersize=1.5, color="C1", label=r"$v_y(t)$")
ax.plot(df[0], df[6], "o-", markersize=1.5, color="C2", label=r"$v_z(t)$")
ax.set_xlim(left=0)
ax.set_ylim(-1.5, 1.5)
ax.set_xlabel(r"$Time(s)$")
ax.set_ylabel(r"$Velocity(m/s)$")
ax.legend(loc="center right")
plt.title("Velocity")

plt.savefig("plots/3d.pdf")

fig = plt.figure(figsize=(6,6))
ax = fig.add_subplot(111, projection="3d")
ax.plot3D(df[1], df[2], df[3], "o-", markersize=1.5, color="C0", label=r"$trace$")
ax.set_xlim3d([df[1].values.min(), df[1].values.max()])
ax.set_xlabel(r"$X$")
ax.set_ylim3d([df[2].values.min(), df[2].values.max()])
ax.set_ylabel(r"$Y$")
ax.set_zlim3d([df[3].values.min(), df[3].values.max()])
ax.set_zlabel(r"$Z$")
xline = np.linspace(df[1].values.min(), df[1].values.max())
ax.plot(xline, [df[2].values.min()*len(xline), df[3].values.min()*len(xline), "red", label=rf"$x={df[1].min()}, {df[1].max()}$")
ax.plot(xline, [df[2].values.max()*len(xline), df[3].values.min()*len(xline), "red")
ax.plot(xline, [df[2].values.min()*len(xline), df[3].values.max()*len(xline), "red")
ax.plot(xline, [df[2].values.max()*len(xline), df[3].values.max()*len(xline), "red")

```

```

yline = np.linspace(df[2].values.min(), df[2].values.max())
ax.plot([df[1].values.min()*len(yline), yline, [df[3].values.min()*len(yline), "red", label=rf"$y={df[2].min()}, {df[2].max()}$")
ax.plot([df[1].values.max()*len(yline), yline, [df[3].values.min()*len(yline), "red")
ax.plot([df[1].values.min()*len(yline), yline, [df[3].values.max()*len(yline), "red")
ax.plot([df[1].values.max()*len(yline), yline, [df[3].values.max()*len(yline), "red")

zline = np.linspace(df[3].values.min(), df[3].values.max())
ax.plot([df[1].values.min()*len(zline), [df[2].values.min()*len(zline), zline, "red", label=rf"$z={df[3].min()}, {df[3].max()}$")
ax.plot([df[1].values.max()*len(zline), [df[2].values.min()*len(zline), zline, "red")
ax.plot([df[1].values.min()*len(zline), [df[2].values.max()*len(zline), zline, "red")
ax.plot([df[1].values.max()*len(zline), [df[2].values.max()*len(zline), zline, "red")
ax.legend()
ax.view_init(15, 60)
ax.set_title("Trajectory")

# plt.show()
plt.savefig("plots/3d_traj.pdf")

# %%

```

4.3 Python program: Animation

```

#!/usr/bin/env python
"""
Author: Devansh Shukla
"""
import pandas as pd
import numpy as np
import matplotlib as mpl
import matplotlib.pyplot as plt
from matplotlib.animation import FuncAnimation, FFMpegWriter
import matplotlib.gridspec as gridspec

custom_rcparams = {
    "axes.labelsize": 7,
    "axes.titlesize": 8,
    "axes.grid": True,
    # Figure
    "figure.autolayout": True,
    "figure.titlesize": 9,
    "figure.figsize": (10, 5),
    "savefig.format": "pdf",
    "lines.linewidth": 1,
    # Legend
    "legend.fontsize": 8,
    "legend.frameon": True,
    # Ticks
    "xtick.labelsize": 8,
    "ytick.labelsize": 8,
    "xtick.minor.visible": True,
    "xtick.direction": "in",
    "ytick.direction": "in",
    "ytick.minor.visible": True,
}
mpl.rcParams.update(custom_rcparams)

# t, x, y, z, vx, vy, vz
df = pd.read_csv("Particle3D.dat", engine="python", delimiter=" ", header=None, skipinitialspace=True, comment="#")
print(df)

angle = np.linspace(30, 360, 300)

time = df[0].values
pos_x = df[1].values
pos_y = df[2].values
pos_z = df[3].values
vel_x = df[4].values
vel_y = df[5].values
vel_z = df[6].values

gs = gridspec.GridSpec(1, 2, width_ratios=[2, 1], hspace=0)
fig = plt.figure()
ax1 = fig.add_subplot(gs[0, 0], projection="3d")
ax2 = fig.add_subplot(gs[0, 1])

line1, = ax1.plot3D([], [], [], 'o', lw=2, label="particle")
trace, = ax1.plot3D([], [], [], '-', lw=1, label="trace")

```

```

time_template = "time = %.2fs"
time_text = ax1.text2D(0.05, 0.85, '', transform=ax1.transAxes)

line_vx, = ax2.plot([], [], '-', lw=2, label=r"$v_x(t)$")
line_vy, = ax2.plot([], [], '-', lw=2, label=r"$v_y(t)$")
line_vz, = ax2.plot([], [], '-', lw=2, label=r"$v_z(t)$")
ax2.legend()

line = [line1, line_vx, line_vy, line_vz,]
ax1.set_xlim3d([pos_x.min(), pos_x.max()])
ax1.set_xlabel("X", labelpad=0)
ax1.set_ylim3d([pos_y.min(), pos_y.max()])
ax1.set_ylabel("Y", labelpad=0)
ax1.set_zlim3d([pos_z.min(), pos_z.max()])
ax1.set_zlabel("Z", labelpad=0)

xline = np.linspace(pos_x.min(), pos_x.max())
ax1.plot(xline, [pos_y.min()*len(xline), [pos_z.min()*len(xline), "red", label=rf"$x={pos_x.min()},{pos_x.max()}$")
ax1.plot(xline, [pos_y.max()*len(xline), [pos_z.min()*len(xline), "red")
ax1.plot(xline, [pos_y.min()*len(xline), [pos_z.max()*len(xline), "red")
ax1.plot(xline, [pos_y.max()*len(xline), [pos_z.max()*len(xline), "red")

yline = np.linspace(pos_y.min(), pos_y.max())
ax1.plot([pos_x.min()*len(yline), yline, [pos_z.min()*len(yline), "red", label=rf"$y={pos_y.min()},{pos_y.max()}$")
ax1.plot([pos_x.max()*len(yline), yline, [pos_z.min()*len(yline), "red")
ax1.plot([pos_x.min()*len(yline), yline, [pos_z.max()*len(yline), "red")
ax1.plot([pos_x.max()*len(yline), yline, [pos_z.max()*len(yline), "red")

zline = np.linspace(pos_z.min(), pos_z.max())
ax1.plot([pos_x.min()*len(zline), [pos_y.min()*len(zline), zline, "red", label=rf"$z={pos_z.min()},{pos_z.max()}$")
ax1.plot([pos_x.max()*len(zline), [pos_y.min()*len(zline), zline, "red")
ax1.plot([pos_x.min()*len(zline), [pos_y.max()*len(zline), zline, "red")
ax1.plot([pos_x.max()*len(zline), [pos_y.max()*len(zline), zline, "red")

ax1.legend(loc="upper right")
ax1.view_init(30, 30)

ax2.set_xlim(0, 30) #time[-1]+5)
ax2.set_ylim(-1.5, 1.5)
ax2.set_aspect(6)
ax2.set_xlabel(r"$Time(s)$")
ax2.set_ylabel(r"$Velocity(m/s)$")

def init():
    line[0].set_data([], [])
    trace.set_data([], [])
    return line, trace

def animate(i):
    global angle, ax1
    global time, pos_x, pos_y, pos_z, vel_x, vel_y, vel_z

    line[0].set_data(pos_x[i], pos_y[i])
    line[0].set_3d_properties(pos_z[i])
    trace.set_data(pos_x[i], pos_y[i])
    trace.set_3d_properties(pos_z[i])
    time_text.set_text(time_template % (time[i]))

    ax1.view_init(15, angle[i % 300])

    line[1].set_data(time[:i], vel_x[:i])
    line[2].set_data(time[:i], vel_y[:i])
    line[3].set_data(time[:i], vel_z[:i])

    if time[i] == 30.0:
        ax2.set_xlim(0, 60)
        ax2.set_aspect(12)
        captures = np.arange(2.0, 59.9, 2.0)
        captures = np.insert(captures, 0, 0.5)
        if time[i] in captures:
            toggle_capture()

    return line, trace, time_text

def toggle_capture(*args, **kwargs):
    global ani, capture_no
    ani.pause()
    plt.gcf().savefig(f"plots/3d_{capture_no}.pdf")
    capture_no += 1

```

```

ani.resume()

capture_no = 0
ani = FuncAnimation(fig, animate, frames=len(time), interval=10, init_func=init, blit=False, repeat=False)
fig.canvas.mpl_connect('button_press_event', toggle_capture)
writer = FFMpegWriter(fps=10)
ani.save('animation.mp4', writer=writer)
plt.show()

```

5 Results

5.1 Terminal Output

```

-----
Enter initial position x0, y0, z0
0.0 0.0 0.0
Enter velocity vx, vy, vz
1.0 1.0 1.0
Enter bounds lx0, lx1
0.0 10.0
Enter bounds ly0, ly1
0.0 8.0
Enter bounds lz0, lz1
0.0 6.0
Enter t0, tf, dt
0.0 30.0 0.1
-----
x0, y0, z0=      0.0000      0.0000      0.0000
lx0, lx1=    0.0000    10.0000
ly0, ly1=    0.0000      8.0000
lz0, lz1=    0.0000      6.0000
t0, tf, dt=    0.0000    30.0000    0.1000
-----

```

time	x(t)	y(t)	z(t)	vx(t)	vy(t)	vz(t)
0.0000	0.0000	0.0000	0.0000	1.0000	1.0000	1.0000
0.1000	0.1000	0.1000	0.1000	1.0000	1.0000	1.0000
0.2000	0.2000	0.2000	0.2000	1.0000	1.0000	1.0000
0.3000	0.3000	0.3000	0.3000	1.0000	1.0000	1.0000
0.4000	0.4000	0.4000	0.4000	1.0000	1.0000	1.0000
0.5000	0.5000	0.5000	0.5000	1.0000	1.0000	1.0000
0.6000	0.6000	0.6000	0.6000	1.0000	1.0000	1.0000
0.7000	0.7000	0.7000	0.7000	1.0000	1.0000	1.0000
0.8000	0.8000	0.8000	0.8000	1.0000	1.0000	1.0000
0.9000	0.9000	0.9000	0.9000	1.0000	1.0000	1.0000
1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1.1000	1.1000	1.1000	1.1000	1.0000	1.0000	1.0000
1.2000	1.2000	1.2000	1.2000	1.0000	1.0000	1.0000
1.3000	1.3000	1.3000	1.3000	1.0000	1.0000	1.0000
1.4000	1.4000	1.4000	1.4000	1.0000	1.0000	1.0000
1.5000	1.5000	1.5000	1.5000	1.0000	1.0000	1.0000
1.6000	1.6000	1.6000	1.6000	1.0000	1.0000	1.0000
1.7000	1.7000	1.7000	1.7000	1.0000	1.0000	1.0000
1.8000	1.8000	1.8000	1.8000	1.0000	1.0000	1.0000
1.9000	1.9000	1.9000	1.9000	1.0000	1.0000	1.0000
2.0000	2.0000	2.0000	2.0000	1.0000	1.0000	1.0000
2.1000	2.1000	2.1000	2.1000	1.0000	1.0000	1.0000
2.2000	2.2000	2.2000	2.2000	1.0000	1.0000	1.0000
2.3000	2.3000	2.3000	2.3000	1.0000	1.0000	1.0000
2.4000	2.4000	2.4000	2.4000	1.0000	1.0000	1.0000
2.5000	2.5000	2.5000	2.5000	1.0000	1.0000	1.0000
2.6000	2.6000	2.6000	2.6000	1.0000	1.0000	1.0000
2.7000	2.7000	2.7000	2.7000	1.0000	1.0000	1.0000
2.8000	2.8000	2.8000	2.8000	1.0000	1.0000	1.0000
2.9000	2.9000	2.9000	2.9000	1.0000	1.0000	1.0000
3.0000	3.0000	3.0000	3.0000	1.0000	1.0000	1.0000
3.1000	3.1000	3.1000	3.1000	1.0000	1.0000	1.0000
3.2000	3.2000	3.2000	3.2000	1.0000	1.0000	1.0000
3.3000	3.3000	3.3000	3.3000	1.0000	1.0000	1.0000
3.4000	3.4000	3.4000	3.4000	1.0000	1.0000	1.0000
3.5000	3.5000	3.5000	3.5000	1.0000	1.0000	1.0000
3.6000	3.6000	3.6000	3.6000	1.0000	1.0000	1.0000
3.7000	3.7000	3.7000	3.7000	1.0000	1.0000	1.0000
3.8000	3.8000	3.8000	3.8000	1.0000	1.0000	1.0000
3.9000	3.9000	3.9000	3.9000	1.0000	1.0000	1.0000
4.0000	4.0000	4.0000	4.0000	1.0000	1.0000	1.0000
4.1000	4.1000	4.1000	4.1000	1.0000	1.0000	1.0000
4.2000	4.2000	4.2000	4.2000	1.0000	1.0000	1.0000
4.3000	4.3000	4.3000	4.3000	1.0000	1.0000	1.0000
4.4000	4.4000	4.4000	4.4000	1.0000	1.0000	1.0000

4.5000	4.5000	4.5000	4.5000	1.0000	1.0000	1.0000
4.6000	4.6000	4.6000	4.6000	1.0000	1.0000	1.0000
4.7000	4.7000	4.7000	4.7000	1.0000	1.0000	1.0000
4.8000	4.8000	4.8000	4.8000	1.0000	1.0000	1.0000
4.9000	4.9000	4.9000	4.9000	1.0000	1.0000	1.0000
5.0000	5.0000	5.0000	5.0000	1.0000	1.0000	1.0000
5.1000	5.1000	5.1000	5.1000	1.0000	1.0000	1.0000
5.2000	5.2000	5.2000	5.2000	1.0000	1.0000	1.0000
5.3000	5.3000	5.3000	5.3000	1.0000	1.0000	1.0000
5.4000	5.4000	5.4000	5.4000	1.0000	1.0000	1.0000
5.5000	5.5000	5.5000	5.5000	1.0000	1.0000	1.0000
5.6000	5.6000	5.6000	5.6000	1.0000	1.0000	1.0000
5.7000	5.7000	5.7000	5.7000	1.0000	1.0000	1.0000
5.8000	5.8000	5.8000	5.8000	1.0000	1.0000	1.0000
5.9000	5.9000	5.9000	5.9000	1.0000	1.0000	1.0000
6.0000	6.0000	6.0000	6.0000	1.0000	1.0000	-1.0000
6.1000	6.1000	6.1000	5.9000	1.0000	1.0000	-1.0000
6.2000	6.2000	6.2000	5.8000	1.0000	1.0000	-1.0000
6.3000	6.3000	6.3000	5.7000	1.0000	1.0000	-1.0000
6.4000	6.4000	6.4000	5.6000	1.0000	1.0000	-1.0000
6.5000	6.5000	6.5000	5.5000	1.0000	1.0000	-1.0000
6.6000	6.6000	6.6000	5.4000	1.0000	1.0000	-1.0000
6.7000	6.7000	6.7000	5.3000	1.0000	1.0000	-1.0000
6.8000	6.8000	6.8000	5.2000	1.0000	1.0000	-1.0000
6.9000	6.9000	6.9000	5.1000	1.0000	1.0000	-1.0000
7.0000	7.0000	7.0000	5.0000	1.0000	1.0000	-1.0000
7.1000	7.1000	7.1000	4.9000	1.0000	1.0000	-1.0000
7.2000	7.2000	7.2000	4.8000	1.0000	1.0000	-1.0000
7.3000	7.3000	7.3000	4.7000	1.0000	1.0000	-1.0000
7.4000	7.4000	7.4000	4.6000	1.0000	1.0000	-1.0000
7.5000	7.5000	7.5000	4.5000	1.0000	1.0000	-1.0000
7.6000	7.6000	7.6000	4.4000	1.0000	1.0000	-1.0000
7.7000	7.7000	7.7000	4.3000	1.0000	1.0000	-1.0000
7.8000	7.8000	7.8000	4.2000	1.0000	1.0000	-1.0000
7.9000	7.9000	7.9000	4.1000	1.0000	1.0000	-1.0000
8.0000	8.0000	8.0000	4.0000	1.0000	-1.0000	-1.0000
8.1000	8.1000	7.9000	3.9000	1.0000	-1.0000	-1.0000
8.2000	8.2000	7.8000	3.8000	1.0000	-1.0000	-1.0000
8.3000	8.3000	7.7000	3.7000	1.0000	-1.0000	-1.0000
8.4000	8.4000	7.6000	3.6000	1.0000	-1.0000	-1.0000
8.5000	8.5000	7.5000	3.5000	1.0000	-1.0000	-1.0000
8.6000	8.6000	7.4000	3.4000	1.0000	-1.0000	-1.0000
8.7000	8.7000	7.3000	3.3000	1.0000	-1.0000	-1.0000
8.8000	8.8000	7.2000	3.2000	1.0000	-1.0000	-1.0000
8.9000	8.9000	7.1000	3.1000	1.0000	-1.0000	-1.0000
9.0000	9.0000	7.0000	3.0000	1.0000	-1.0000	-1.0000
9.1000	9.1000	6.9000	2.9000	1.0000	-1.0000	-1.0000
9.2000	9.2000	6.8000	2.8000	1.0000	-1.0000	-1.0000
9.3000	9.3000	6.7000	2.7000	1.0000	-1.0000	-1.0000
9.4000	9.4000	6.6000	2.6000	1.0000	-1.0000	-1.0000
9.5000	9.5000	6.5000	2.5000	1.0000	-1.0000	-1.0000
9.6000	9.6000	6.4000	2.4000	1.0000	-1.0000	-1.0000
9.7000	9.7000	6.3000	2.3000	1.0000	-1.0000	-1.0000
9.8000	9.8000	6.2000	2.2000	1.0000	-1.0000	-1.0000
9.9000	9.9000	6.1000	2.1000	1.0000	-1.0000	-1.0000
10.0000	10.0000	6.0000	2.0000	-1.0000	-1.0000	-1.0000
10.1000	9.9000	5.9000	1.9000	-1.0000	-1.0000	-1.0000
10.2000	9.8000	5.8000	1.8000	-1.0000	-1.0000	-1.0000
10.3000	9.7000	5.7000	1.7000	-1.0000	-1.0000	-1.0000
10.4000	9.6000	5.6000	1.6000	-1.0000	-1.0000	-1.0000
10.5000	9.5000	5.5000	1.5000	-1.0000	-1.0000	-1.0000
10.6000	9.4000	5.4000	1.4000	-1.0000	-1.0000	-1.0000
10.7000	9.3000	5.3000	1.3000	-1.0000	-1.0000	-1.0000
10.8000	9.2000	5.2000	1.2000	-1.0000	-1.0000	-1.0000
10.9000	9.1000	5.1000	1.1000	-1.0000	-1.0000	-1.0000
11.0000	9.0000	5.0000	1.0000	-1.0000	-1.0000	-1.0000
11.1000	8.9000	4.9000	0.9000	-1.0000	-1.0000	-1.0000
11.2000	8.8000	4.8000	0.8000	-1.0000	-1.0000	-1.0000
11.3000	8.7000	4.7000	0.7000	-1.0000	-1.0000	-1.0000
11.4000	8.6000	4.6000	0.6000	-1.0000	-1.0000	-1.0000
11.5000	8.5000	4.5000	0.5000	-1.0000	-1.0000	-1.0000
11.6000	8.4000	4.4000	0.4000	-1.0000	-1.0000	-1.0000
11.7000	8.3000	4.3000	0.3000	-1.0000	-1.0000	-1.0000
11.8000	8.2000	4.2000	0.2000	-1.0000	-1.0000	-1.0000
11.9000	8.1000	4.1000	0.1000	-1.0000	-1.0000	-1.0000
12.0000	8.0000	4.0000	-0.0000	-1.0000	-1.0000	1.0000
12.1000	7.9000	3.9000	0.1000	-1.0000	-1.0000	1.0000
12.2000	7.8000	3.8000	0.2000	-1.0000	-1.0000	1.0000
12.3000	7.7000	3.7000	0.3000	-1.0000	-1.0000	1.0000

12.4000	7.6000	3.6000	0.4000	-1.0000	-1.0000	1.0000
12.5000	7.5000	3.5000	0.5000	-1.0000	-1.0000	1.0000
12.6000	7.4000	3.4000	0.6000	-1.0000	-1.0000	1.0000
12.7000	7.3000	3.3000	0.7000	-1.0000	-1.0000	1.0000
12.8000	7.2000	3.2000	0.8000	-1.0000	-1.0000	1.0000
12.9000	7.1000	3.1000	0.9000	-1.0000	-1.0000	1.0000
13.0000	7.0000	3.0000	1.0000	-1.0000	-1.0000	1.0000
13.1000	6.9000	2.9000	1.1000	-1.0000	-1.0000	1.0000
13.2000	6.8000	2.8000	1.2000	-1.0000	-1.0000	1.0000
13.3000	6.7000	2.7000	1.3000	-1.0000	-1.0000	1.0000
13.4000	6.6000	2.6000	1.4000	-1.0000	-1.0000	1.0000
13.5000	6.5000	2.5000	1.5000	-1.0000	-1.0000	1.0000
13.6000	6.4000	2.4000	1.6000	-1.0000	-1.0000	1.0000
13.7000	6.3000	2.3000	1.7000	-1.0000	-1.0000	1.0000
13.8000	6.2000	2.2000	1.8000	-1.0000	-1.0000	1.0000
13.9000	6.1000	2.1000	1.9000	-1.0000	-1.0000	1.0000
14.0000	6.0000	2.0000	2.0000	-1.0000	-1.0000	1.0000
14.1000	5.9000	1.9000	2.1000	-1.0000	-1.0000	1.0000
14.2000	5.8000	1.8000	2.2000	-1.0000	-1.0000	1.0000
14.3000	5.7000	1.7000	2.3000	-1.0000	-1.0000	1.0000
14.4000	5.6000	1.6000	2.4000	-1.0000	-1.0000	1.0000
14.5000	5.5000	1.5000	2.5000	-1.0000	-1.0000	1.0000
14.6000	5.4000	1.4000	2.6000	-1.0000	-1.0000	1.0000
14.7000	5.3000	1.3000	2.7000	-1.0000	-1.0000	1.0000
14.8000	5.2000	1.2000	2.8000	-1.0000	-1.0000	1.0000
14.9000	5.1000	1.1000	2.9000	-1.0000	-1.0000	1.0000
15.0000	5.0000	1.0000	3.0000	-1.0000	-1.0000	1.0000
15.1000	4.9000	0.9000	3.1000	-1.0000	-1.0000	1.0000
15.2000	4.8000	0.8000	3.2000	-1.0000	-1.0000	1.0000
15.3000	4.7000	0.7000	3.3000	-1.0000	-1.0000	1.0000
15.4000	4.6000	0.6000	3.4000	-1.0000	-1.0000	1.0000
15.5000	4.5000	0.5000	3.5000	-1.0000	-1.0000	1.0000
15.6000	4.4000	0.4000	3.6000	-1.0000	-1.0000	1.0000
15.7000	4.3000	0.3000	3.7000	-1.0000	-1.0000	1.0000
15.8000	4.2000	0.2000	3.8000	-1.0000	-1.0000	1.0000
15.9000	4.1000	0.1000	3.9000	-1.0000	-1.0000	1.0000
16.0000	4.0000	-0.0000	4.0000	-1.0000	1.0000	1.0000
16.1000	3.9000	0.1000	4.1000	-1.0000	1.0000	1.0000
16.2000	3.8000	0.2000	4.2000	-1.0000	1.0000	1.0000
16.3000	3.7000	0.3000	4.3000	-1.0000	1.0000	1.0000
16.4000	3.6000	0.4000	4.4000	-1.0000	1.0000	1.0000
16.5000	3.5000	0.5000	4.5000	-1.0000	1.0000	1.0000
16.6000	3.4000	0.6000	4.6000	-1.0000	1.0000	1.0000
16.7000	3.3000	0.7000	4.7000	-1.0000	1.0000	1.0000
16.8000	3.2000	0.8000	4.8000	-1.0000	1.0000	1.0000
16.9000	3.1000	0.9000	4.9000	-1.0000	1.0000	1.0000
17.0000	3.0000	1.0000	5.0000	-1.0000	1.0000	1.0000
17.1000	2.9000	1.1000	5.1000	-1.0000	1.0000	1.0000
17.2000	2.8000	1.2000	5.2000	-1.0000	1.0000	1.0000
17.3000	2.7000	1.3000	5.3000	-1.0000	1.0000	1.0000
17.4000	2.6000	1.4000	5.4000	-1.0000	1.0000	1.0000
17.5000	2.5000	1.5000	5.5000	-1.0000	1.0000	1.0000
17.6000	2.4000	1.6000	5.6000	-1.0000	1.0000	1.0000
17.7000	2.3000	1.7000	5.7000	-1.0000	1.0000	1.0000
17.8000	2.2000	1.8000	5.8000	-1.0000	1.0000	1.0000
17.9000	2.1000	1.9000	5.9000	-1.0000	1.0000	1.0000
18.0000	2.0000	2.0000	6.0000	-1.0000	1.0000	-1.0000
18.1000	1.9000	2.1000	5.9000	-1.0000	1.0000	-1.0000
18.2000	1.8000	2.2000	5.8000	-1.0000	1.0000	-1.0000
18.3000	1.7000	2.3000	5.7000	-1.0000	1.0000	-1.0000
18.4000	1.6000	2.4000	5.6000	-1.0000	1.0000	-1.0000
18.5000	1.5000	2.5000	5.5000	-1.0000	1.0000	-1.0000
18.6000	1.4000	2.6000	5.4000	-1.0000	1.0000	-1.0000
18.7000	1.3000	2.7000	5.3000	-1.0000	1.0000	-1.0000
18.8000	1.2000	2.8000	5.2000	-1.0000	1.0000	-1.0000
18.9000	1.1000	2.9000	5.1000	-1.0000	1.0000	-1.0000
19.0000	1.0000	3.0000	5.0000	-1.0000	1.0000	-1.0000
19.1000	0.9000	3.1000	4.9000	-1.0000	1.0000	-1.0000
19.2000	0.8000	3.2000	4.8000	-1.0000	1.0000	-1.0000
19.3000	0.7000	3.3000	4.7000	-1.0000	1.0000	-1.0000
19.4000	0.6000	3.4000	4.6000	-1.0000	1.0000	-1.0000
19.5000	0.5000	3.5000	4.5000	-1.0000	1.0000	-1.0000
19.6000	0.4000	3.6000	4.4000	-1.0000	1.0000	-1.0000
19.7000	0.3000	3.7000	4.3000	-1.0000	1.0000	-1.0000
19.8000	0.2000	3.8000	4.2000	-1.0000	1.0000	-1.0000
19.9000	0.1000	3.9000	4.1000	-1.0000	1.0000	-1.0000
20.0000	-0.0000	4.0000	4.0000	1.0000	1.0000	-1.0000
20.1000	0.1000	4.1000	3.9000	1.0000	1.0000	-1.0000
20.2000	0.2000	4.2000	3.8000	1.0000	1.0000	-1.0000

20.3000	0.3000	4.3000	3.7000	1.0000	1.0000	-1.0000
20.4000	0.4000	4.4000	3.6000	1.0000	1.0000	-1.0000
20.5000	0.5000	4.5000	3.5000	1.0000	1.0000	-1.0000
20.6000	0.6000	4.6000	3.4000	1.0000	1.0000	-1.0000
20.7000	0.7000	4.7000	3.3000	1.0000	1.0000	-1.0000
20.8000	0.8000	4.8000	3.2000	1.0000	1.0000	-1.0000
20.9000	0.9000	4.9000	3.1000	1.0000	1.0000	-1.0000
21.0000	1.0000	5.0000	3.0000	1.0000	1.0000	-1.0000
21.1000	1.1000	5.1000	2.9000	1.0000	1.0000	-1.0000
21.2000	1.2000	5.2000	2.8000	1.0000	1.0000	-1.0000
21.3000	1.3000	5.3000	2.7000	1.0000	1.0000	-1.0000
21.4000	1.4000	5.4000	2.6000	1.0000	1.0000	-1.0000
21.5000	1.5000	5.5000	2.5000	1.0000	1.0000	-1.0000
21.6000	1.6000	5.6000	2.4000	1.0000	1.0000	-1.0000
21.7000	1.7000	5.7000	2.3000	1.0000	1.0000	-1.0000
21.8000	1.8000	5.8000	2.2000	1.0000	1.0000	-1.0000
21.9000	1.9000	5.9000	2.1000	1.0000	1.0000	-1.0000
22.0000	2.0000	6.0000	2.0000	1.0000	1.0000	-1.0000
22.1000	2.1000	6.1000	1.9000	1.0000	1.0000	-1.0000
22.2000	2.2000	6.2000	1.8000	1.0000	1.0000	-1.0000
22.3000	2.3000	6.3000	1.7000	1.0000	1.0000	-1.0000
22.4000	2.4000	6.4000	1.6000	1.0000	1.0000	-1.0000
22.5000	2.5000	6.5000	1.5000	1.0000	1.0000	-1.0000
22.6000	2.6000	6.6000	1.4000	1.0000	1.0000	-1.0000
22.7000	2.7000	6.7000	1.3000	1.0000	1.0000	-1.0000
22.8000	2.8000	6.8000	1.2000	1.0000	1.0000	-1.0000
22.9000	2.9000	6.9000	1.1000	1.0000	1.0000	-1.0000
23.0000	3.0000	7.0000	1.0000	1.0000	1.0000	-1.0000
23.1000	3.1000	7.1000	0.9000	1.0000	1.0000	-1.0000
23.2000	3.2000	7.2000	0.8000	1.0000	1.0000	-1.0000
23.3000	3.3000	7.3000	0.7000	1.0000	1.0000	-1.0000
23.4000	3.4000	7.4000	0.6000	1.0000	1.0000	-1.0000
23.5000	3.5000	7.5000	0.5000	1.0000	1.0000	-1.0000
23.6000	3.6000	7.6000	0.4000	1.0000	1.0000	-1.0000
23.7000	3.7000	7.7000	0.3000	1.0000	1.0000	-1.0000
23.8000	3.8000	7.8000	0.2000	1.0000	1.0000	-1.0000
23.9000	3.9000	7.9000	0.1000	1.0000	1.0000	-1.0000
24.0000	4.0000	8.0000	-0.0000	1.0000	-1.0000	1.0000
24.1000	4.1000	7.9000	0.1000	1.0000	-1.0000	1.0000
24.2000	4.2000	7.8000	0.2000	1.0000	-1.0000	1.0000
24.3000	4.3000	7.7000	0.3000	1.0000	-1.0000	1.0000
24.4000	4.4000	7.6000	0.4000	1.0000	-1.0000	1.0000
24.5000	4.5000	7.5000	0.5000	1.0000	-1.0000	1.0000
24.6000	4.6000	7.4000	0.6000	1.0000	-1.0000	1.0000
24.7000	4.7000	7.3000	0.7000	1.0000	-1.0000	1.0000
24.8000	4.8000	7.2000	0.8000	1.0000	-1.0000	1.0000
24.9000	4.9000	7.1000	0.9000	1.0000	-1.0000	1.0000
25.0000	5.0000	7.0000	1.0000	1.0000	-1.0000	1.0000
25.1000	5.1000	6.9000	1.1000	1.0000	-1.0000	1.0000
25.2000	5.2000	6.8000	1.2000	1.0000	-1.0000	1.0000
25.3000	5.3000	6.7000	1.3000	1.0000	-1.0000	1.0000
25.4000	5.4000	6.6000	1.4000	1.0000	-1.0000	1.0000
25.5000	5.5000	6.5000	1.5000	1.0000	-1.0000	1.0000
25.6000	5.6000	6.4000	1.6000	1.0000	-1.0000	1.0000
25.7000	5.7000	6.3000	1.7000	1.0000	-1.0000	1.0000
25.8000	5.8000	6.2000	1.8000	1.0000	-1.0000	1.0000
25.9000	5.9000	6.1000	1.9000	1.0000	-1.0000	1.0000
26.0000	6.0000	6.0000	2.0000	1.0000	-1.0000	1.0000
26.1000	6.1000	5.9000	2.1000	1.0000	-1.0000	1.0000
26.2000	6.2000	5.8000	2.2000	1.0000	-1.0000	1.0000
26.3000	6.3000	5.7000	2.3000	1.0000	-1.0000	1.0000
26.4000	6.4000	5.6000	2.4000	1.0000	-1.0000	1.0000
26.5000	6.5000	5.5000	2.5000	1.0000	-1.0000	1.0000
26.6000	6.6000	5.4000	2.6000	1.0000	-1.0000	1.0000
26.7000	6.7000	5.3000	2.7000	1.0000	-1.0000	1.0000
26.8000	6.8000	5.2000	2.8000	1.0000	-1.0000	1.0000
26.9000	6.9000	5.1000	2.9000	1.0000	-1.0000	1.0000
27.0000	7.0000	5.0000	3.0000	1.0000	-1.0000	1.0000
27.1000	7.1000	4.9000	3.1000	1.0000	-1.0000	1.0000
27.2000	7.2000	4.8000	3.2000	1.0000	-1.0000	1.0000
27.3000	7.3000	4.7000	3.3000	1.0000	-1.0000	1.0000
27.4000	7.4000	4.6000	3.4000	1.0000	-1.0000	1.0000
27.5000	7.5000	4.5000	3.5000	1.0000	-1.0000	1.0000
27.6000	7.6000	4.4000	3.6000	1.0000	-1.0000	1.0000
27.7000	7.7000	4.3000	3.7000	1.0000	-1.0000	1.0000
27.8000	7.8000	4.2000	3.8000	1.0000	-1.0000	1.0000
27.9000	7.9000	4.1000	3.9000	1.0000	-1.0000	1.0000
28.0000	8.0000	4.0000	4.0000	1.0000	-1.0000	1.0000
28.1000	8.1000	3.9000	4.1000	1.0000	-1.0000	1.0000

28.2000	8.2000	3.8000	4.2000	1.0000	-1.0000	1.0000
28.3000	8.3000	3.7000	4.3000	1.0000	-1.0000	1.0000
28.4000	8.4000	3.6000	4.4000	1.0000	-1.0000	1.0000
28.5000	8.5000	3.5000	4.5000	1.0000	-1.0000	1.0000
28.6000	8.6000	3.4000	4.6000	1.0000	-1.0000	1.0000
28.7000	8.7000	3.3000	4.7000	1.0000	-1.0000	1.0000
28.8000	8.8000	3.2000	4.8000	1.0000	-1.0000	1.0000
28.9000	8.9000	3.1000	4.9000	1.0000	-1.0000	1.0000
29.0000	9.0000	3.0000	5.0000	1.0000	-1.0000	1.0000
29.1000	9.1000	2.9000	5.1000	1.0000	-1.0000	1.0000
29.2000	9.2000	2.8000	5.2000	1.0000	-1.0000	1.0000
29.3000	9.3000	2.7000	5.3000	1.0000	-1.0000	1.0000
29.4000	9.4000	2.6000	5.4000	1.0000	-1.0000	1.0000
29.5000	9.5000	2.5000	5.5000	1.0000	-1.0000	1.0000
29.6000	9.6000	2.4000	5.6000	1.0000	-1.0000	1.0000
29.7000	9.7000	2.3000	5.7000	1.0000	-1.0000	1.0000
29.8000	9.8000	2.2000	5.8000	1.0000	-1.0000	1.0000
29.9000	9.9000	2.1000	5.9000	1.0000	-1.0000	1.0000

5.2 Plots

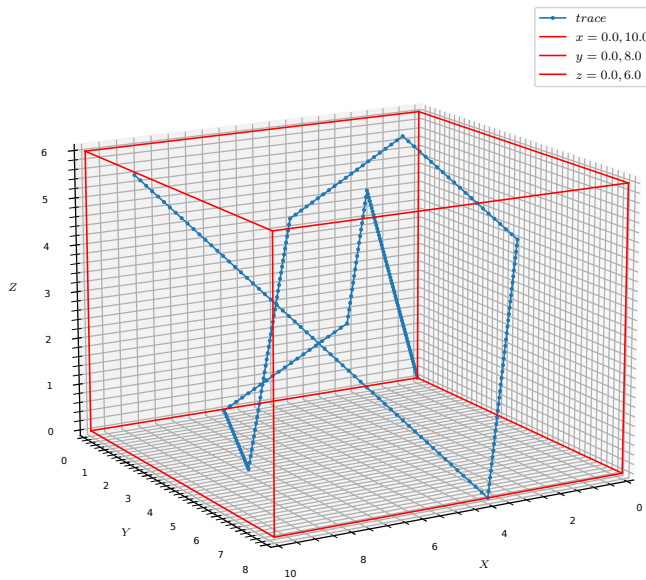
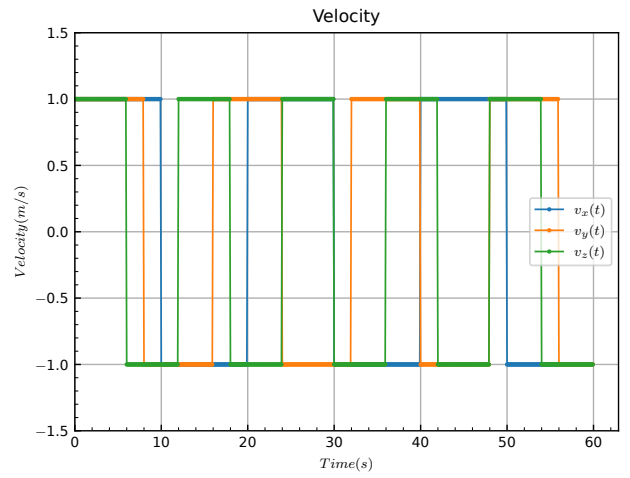
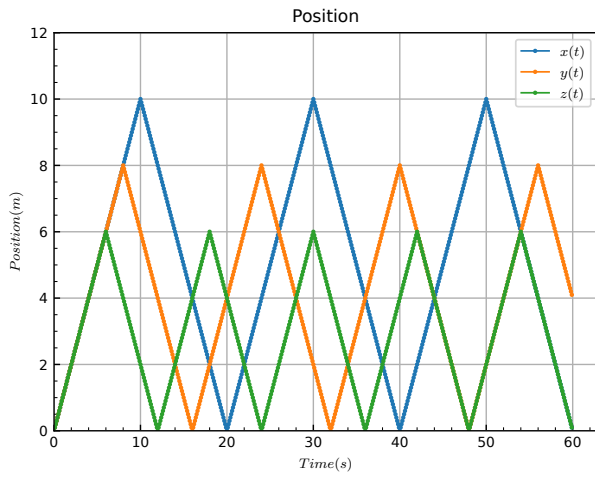


Figure 1: Trajectory: $t_f = 30\text{ s}$

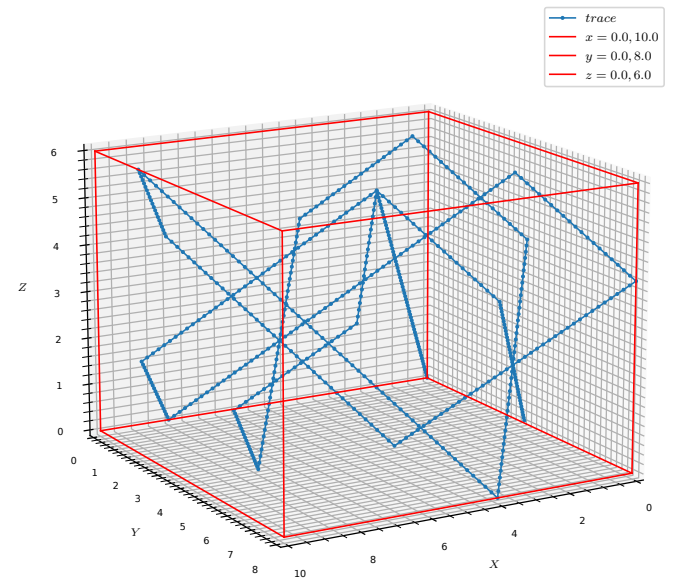
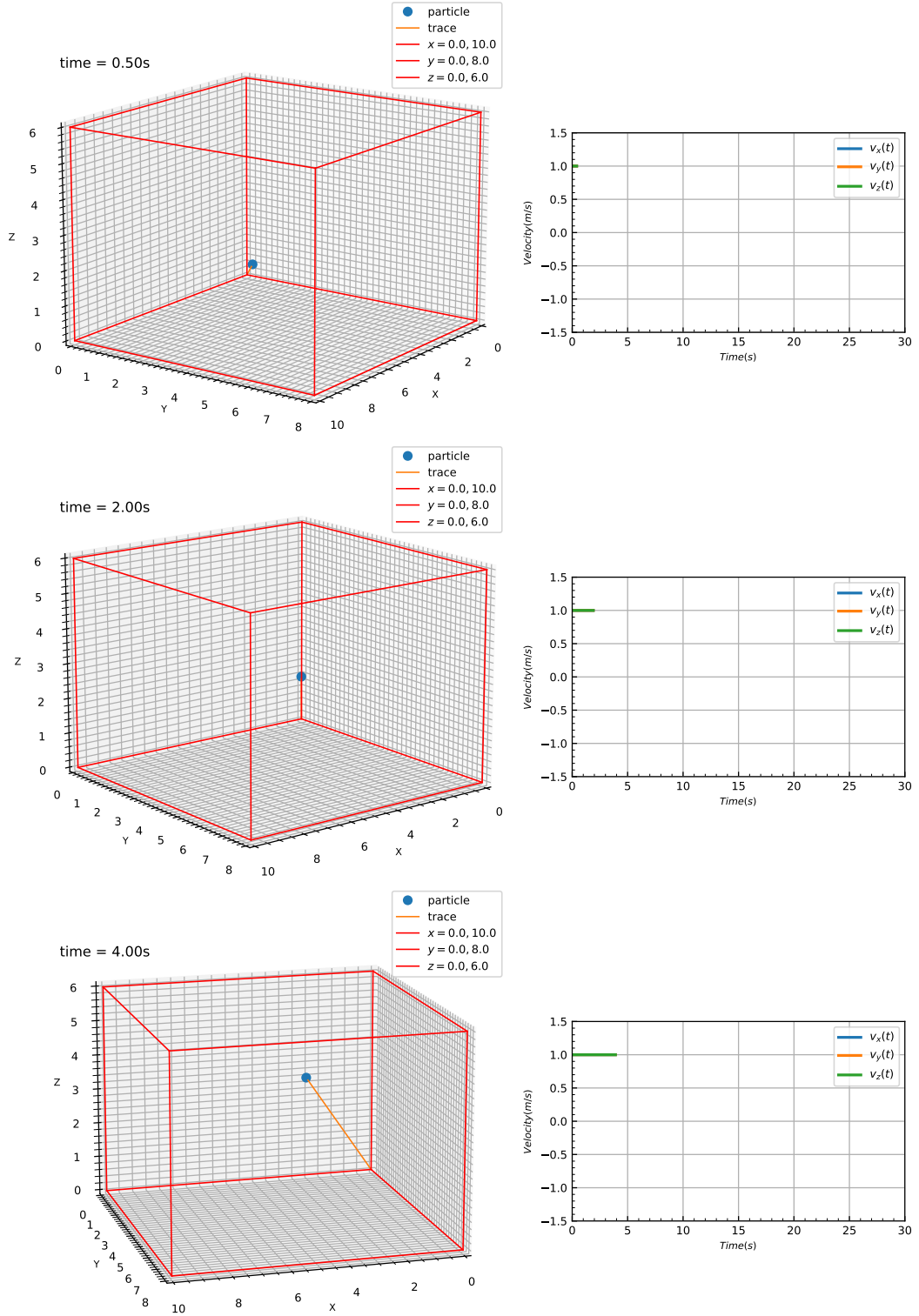


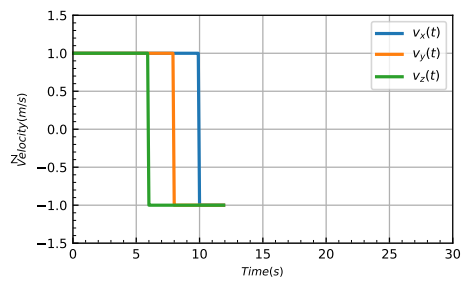
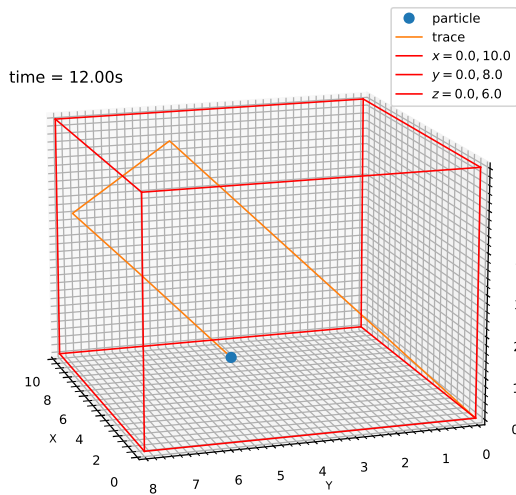
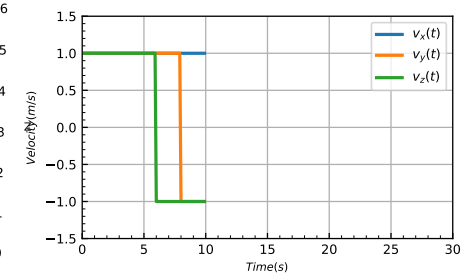
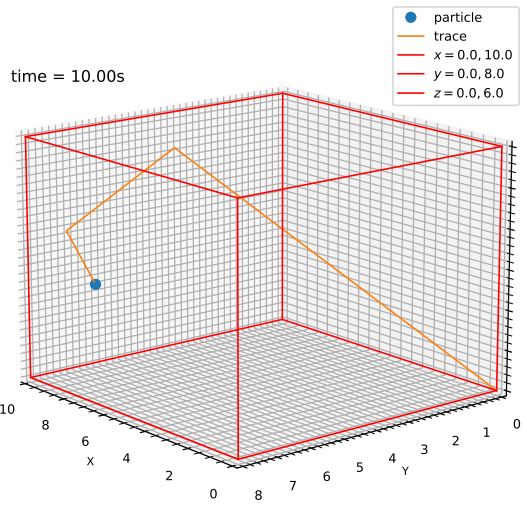
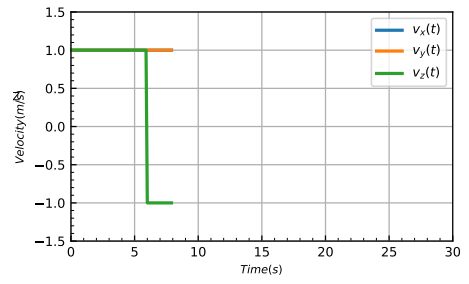
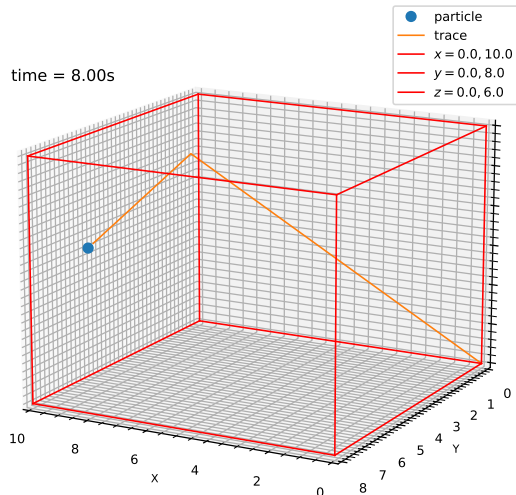
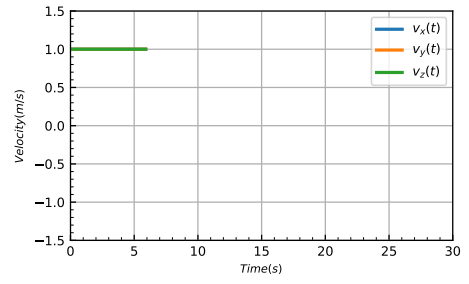
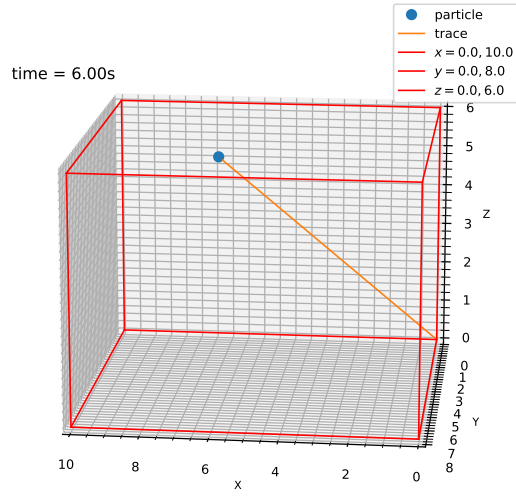
Figure 2: Trajectory: $t_f = 60\text{ s}$

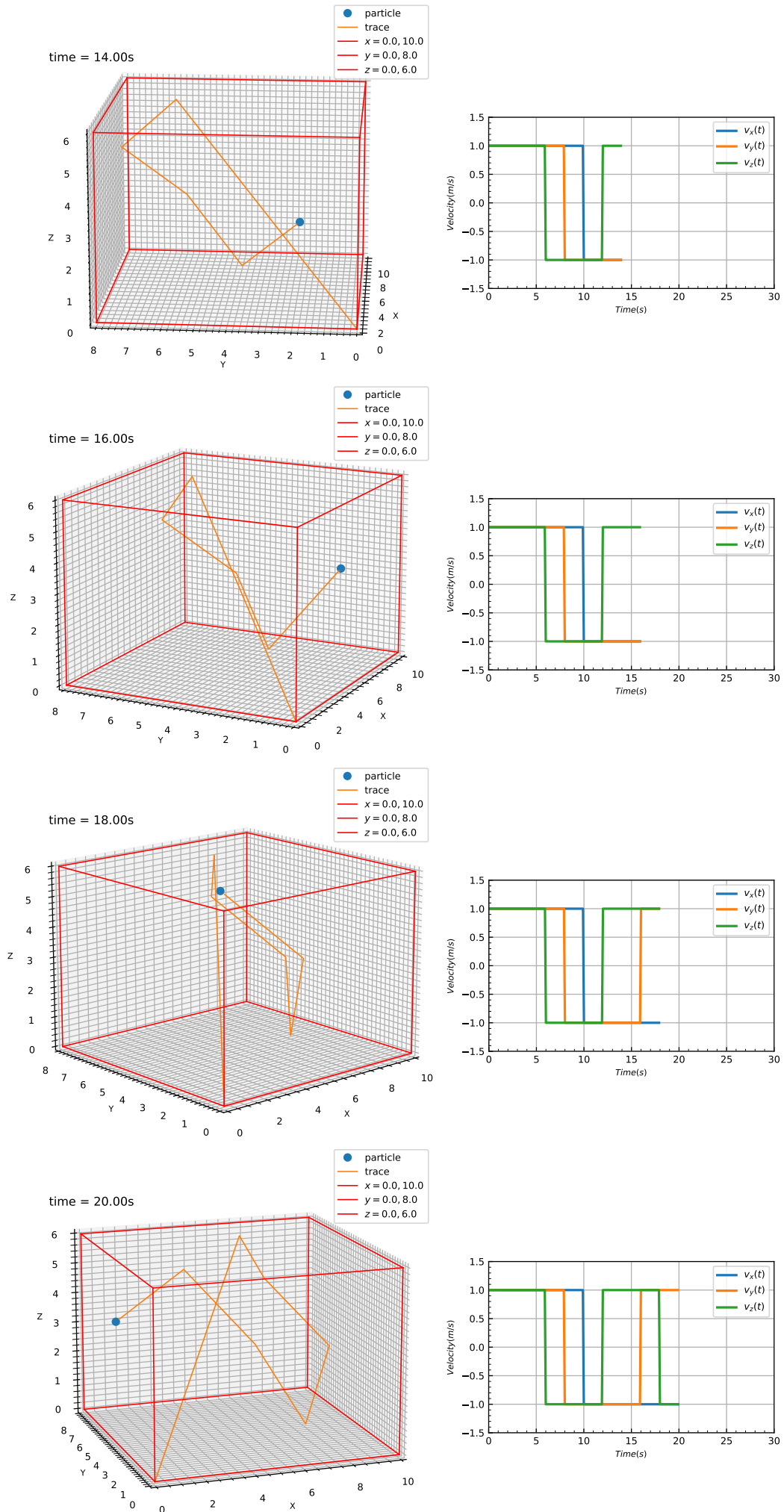
5.3 Animation

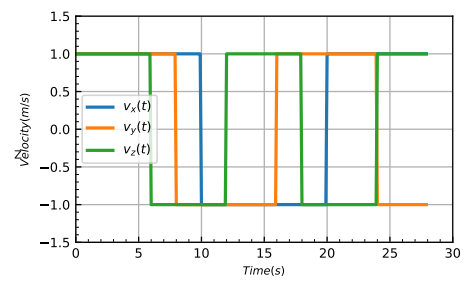
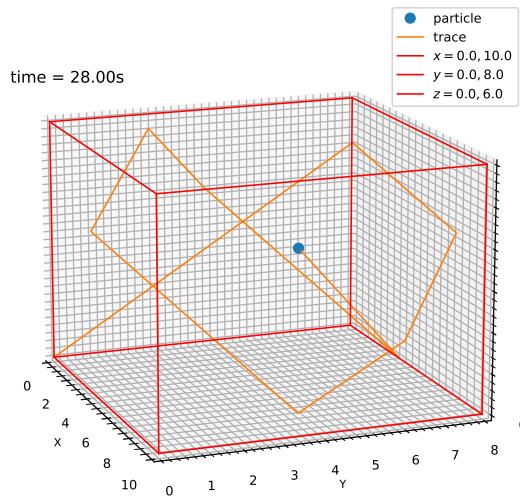
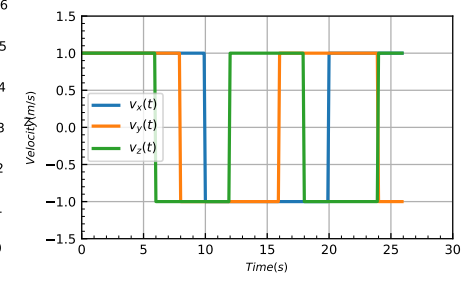
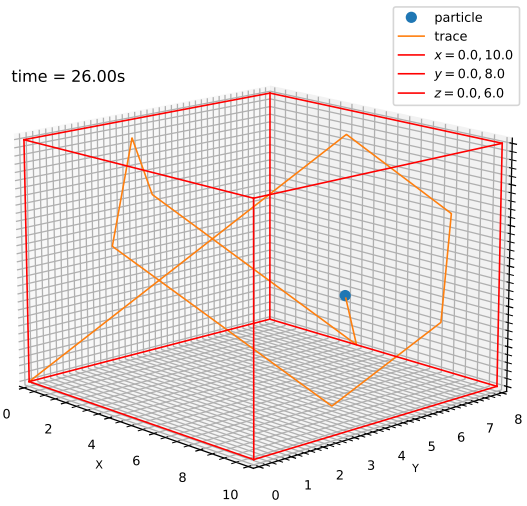
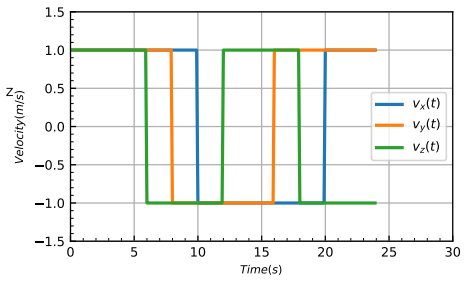
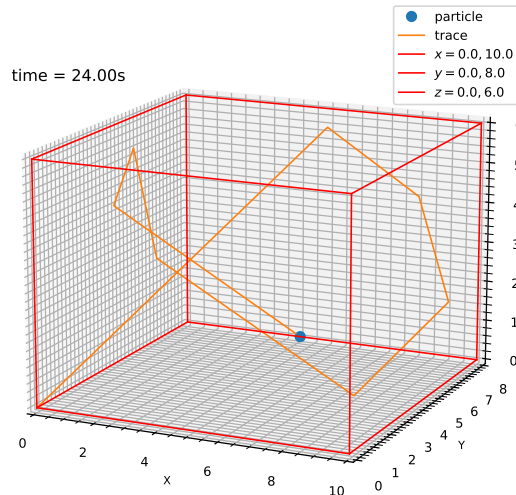
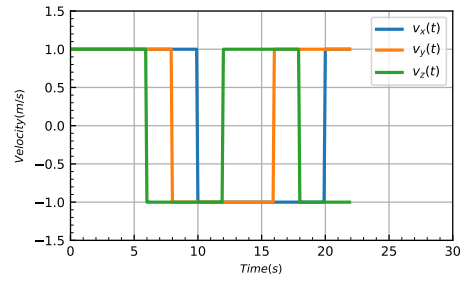
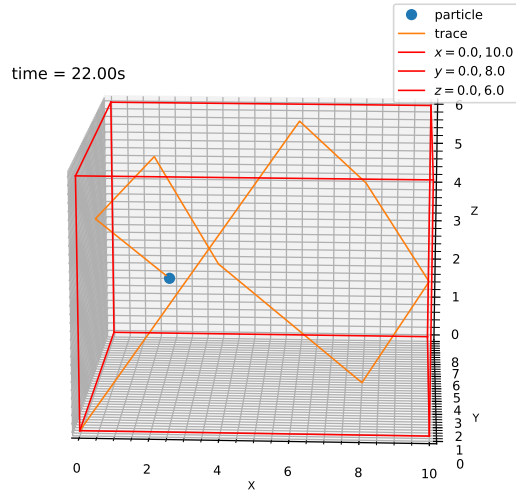
Note: Input parameters,

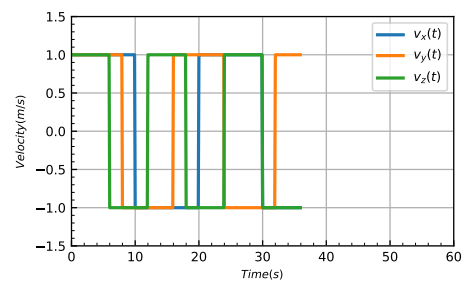
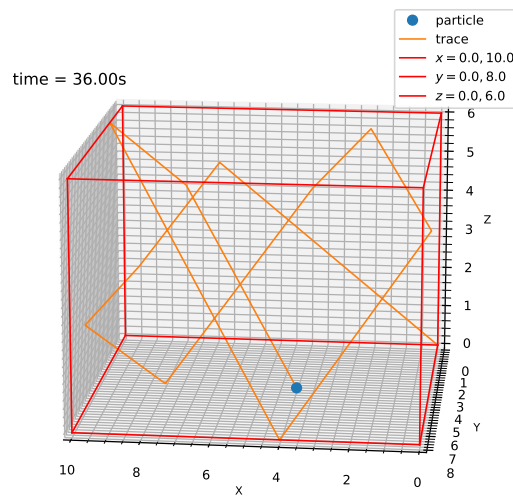
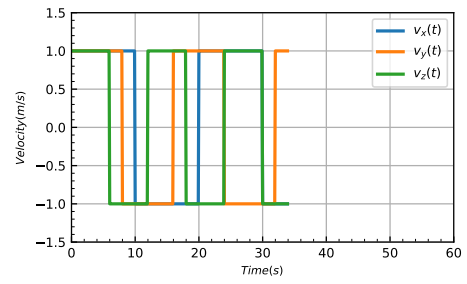
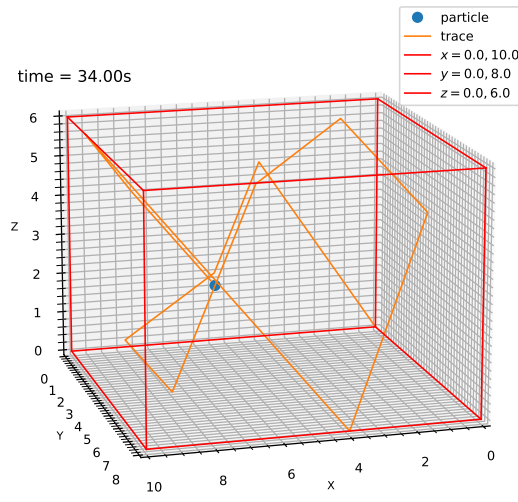
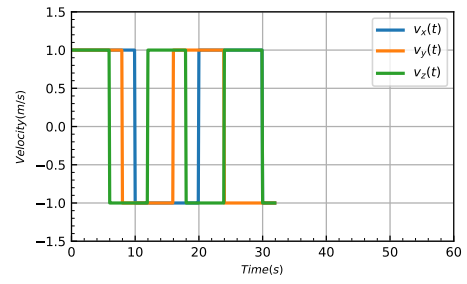
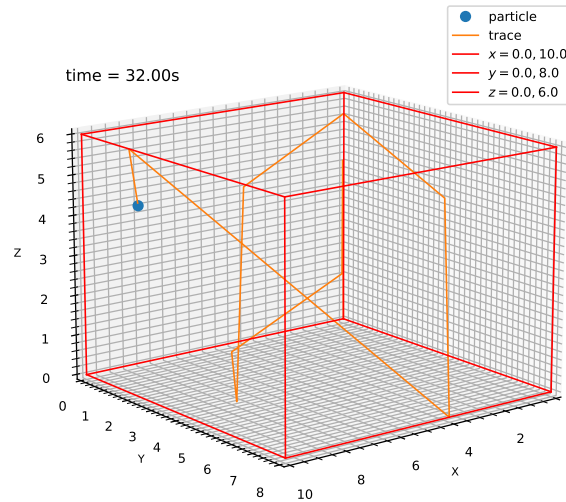
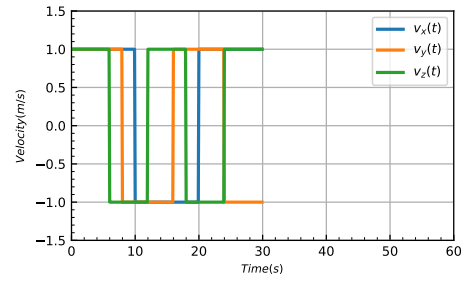
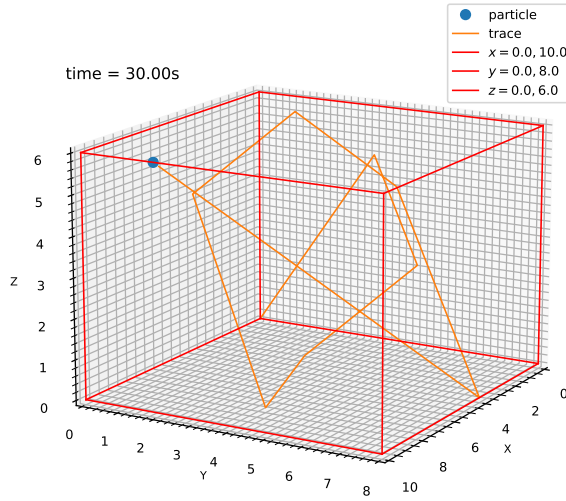
$$\begin{aligned}
 x_0 &= y_0 = z_0 = 0.0 \\
 t_0 &= 0.0; t_f = 30.0; dt = 0.1 \\
 v_x &= v_y = v_z = 1.0; \\
 lx_0 &= ly_0 = lz_0 = 0.0; \\
 lx_1 &= 10.0; ly_1 = 8.0; lz_1 = 6.0
 \end{aligned}
 \tag{5}$$

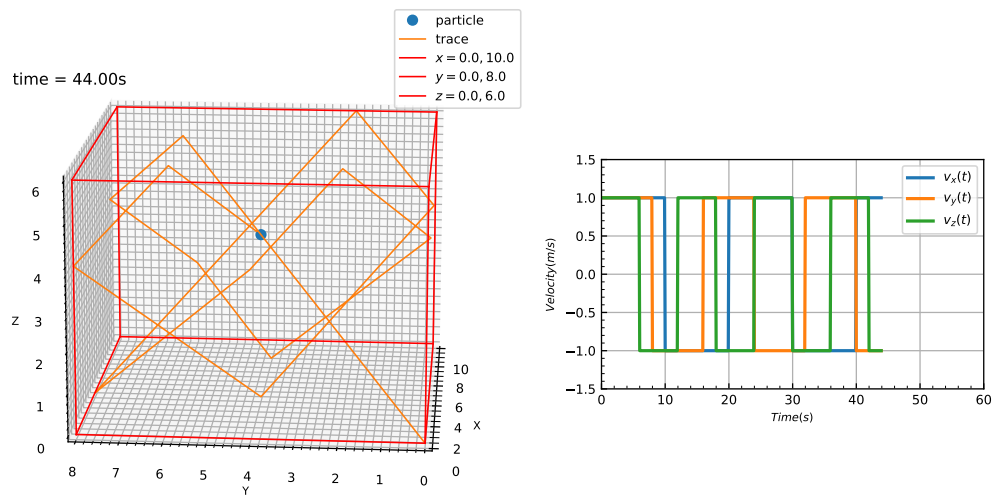
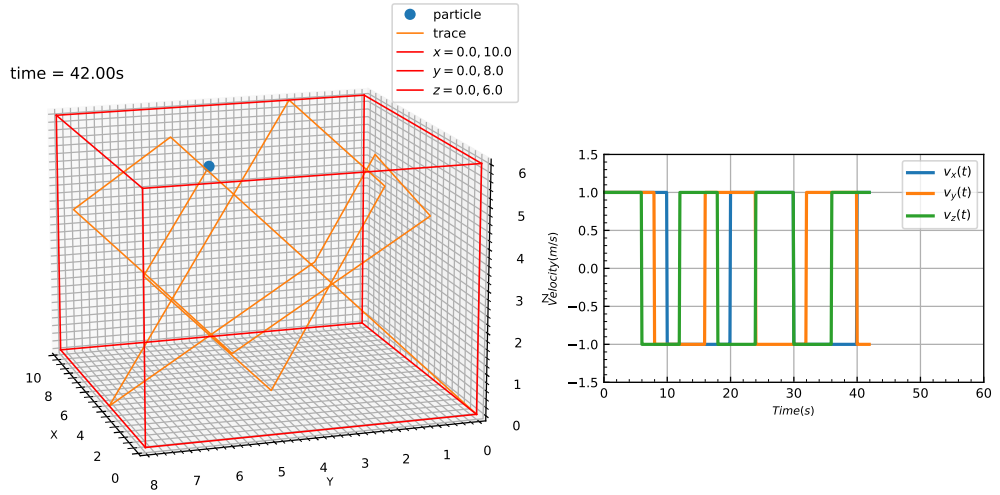
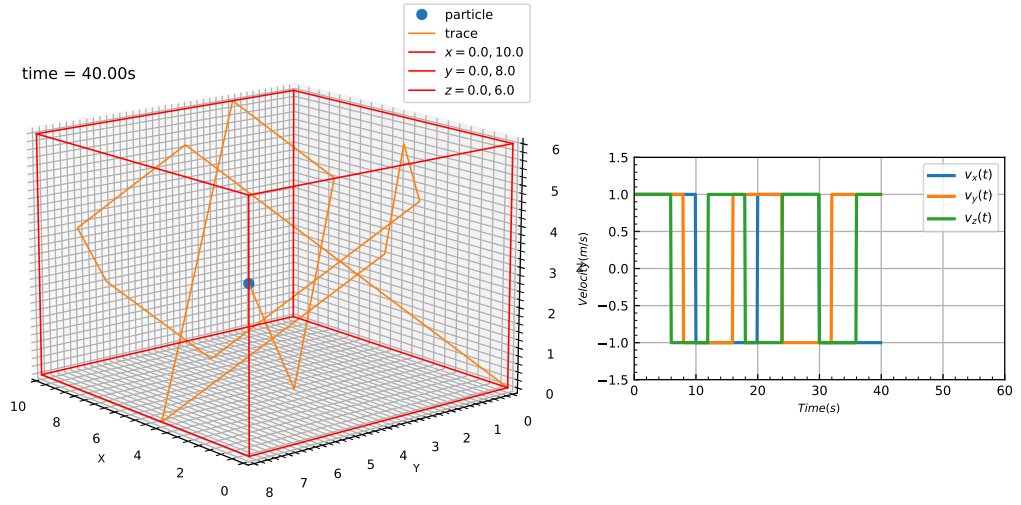
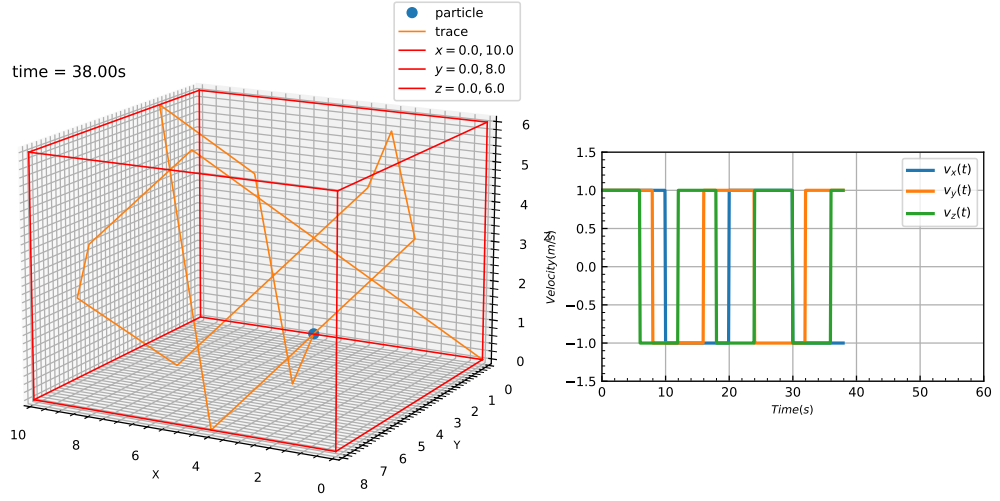


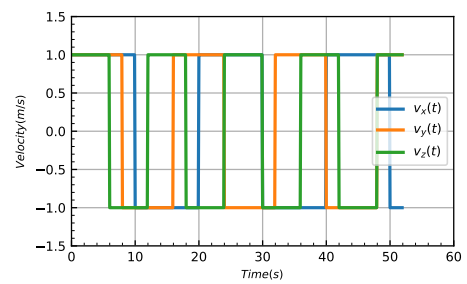
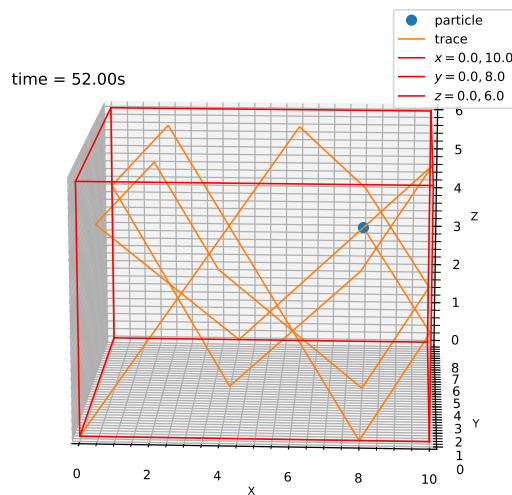
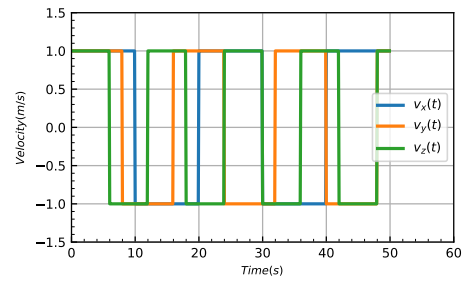
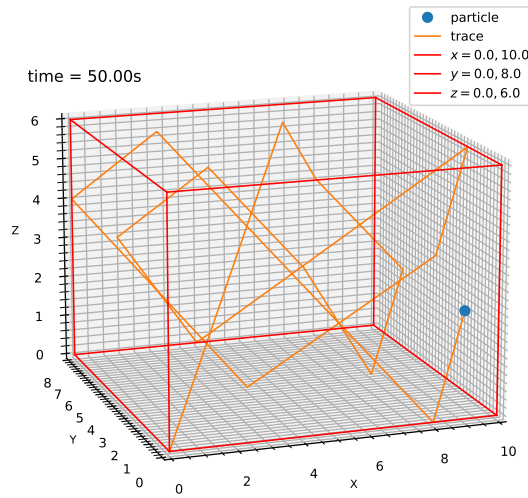
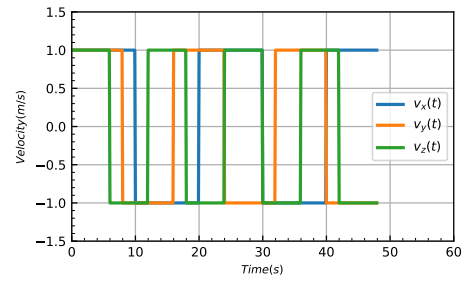
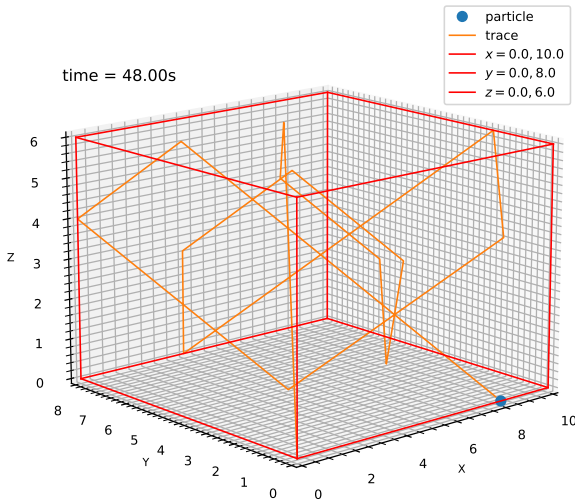
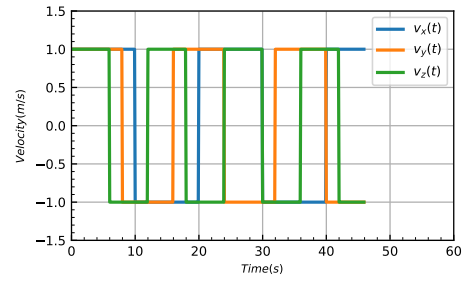
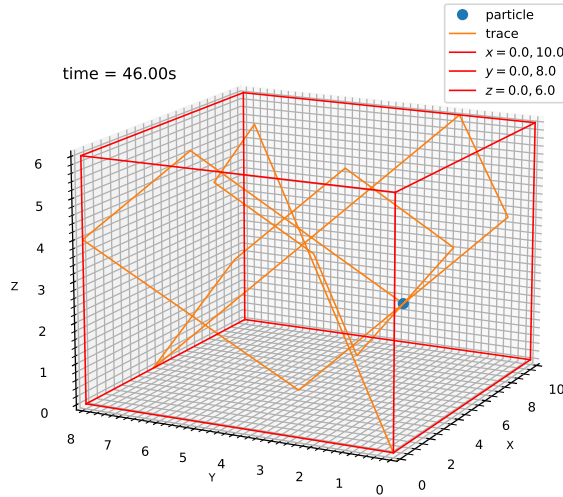


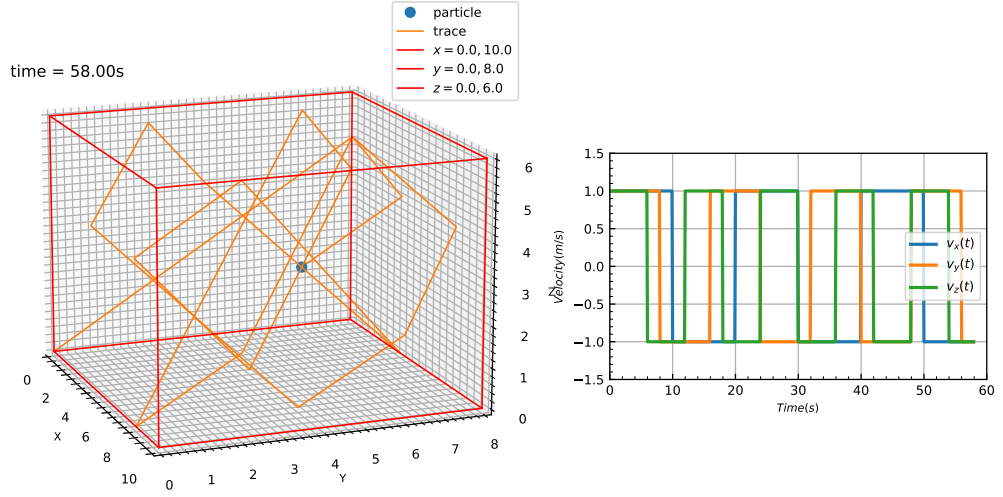
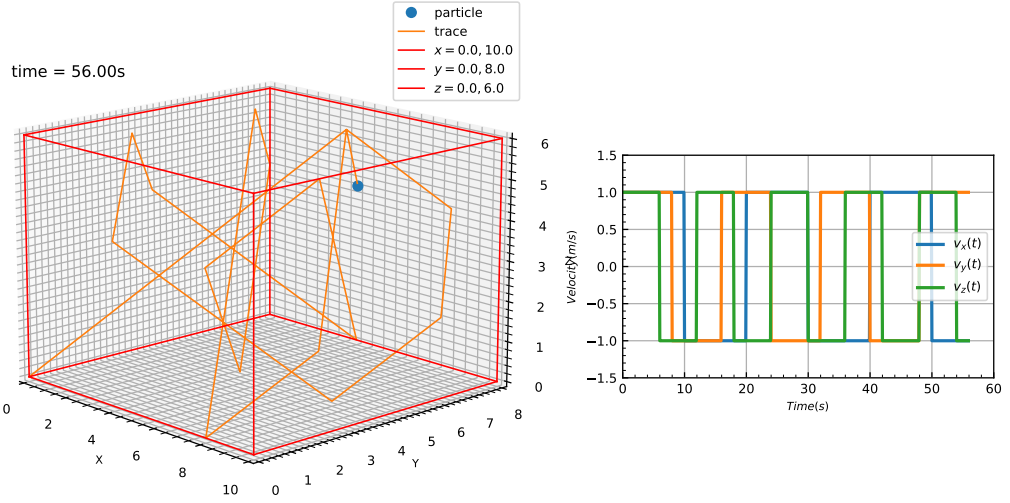
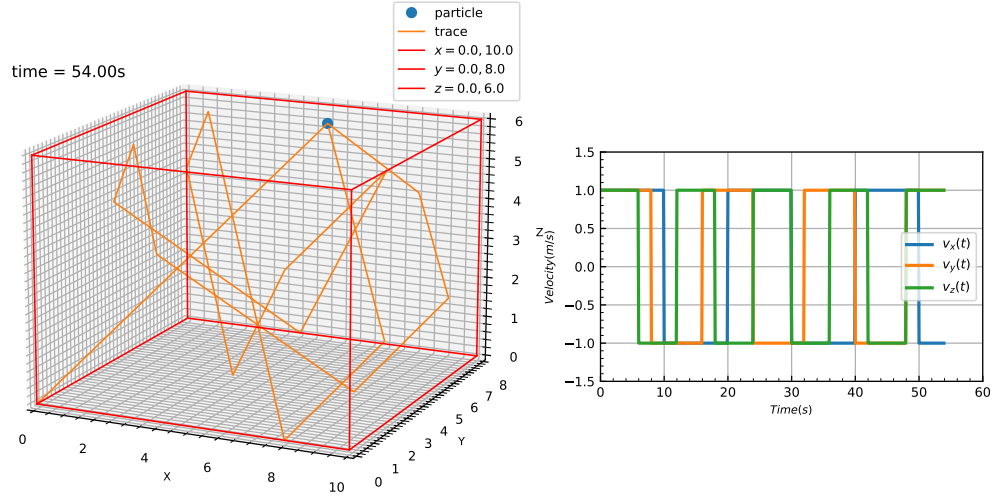












6 Remarks

The programs can be used to numerically trace and simulate the motion of a particle trapped in a 3D-box, provided the required parameters are defined.

The parameters computed numerically and via the programs are in agreement.

With some modifications, this program can be used to solve worm hole problem, wack-a-mole game etc.