Power Flow Analysis

Well known as : Load Flow

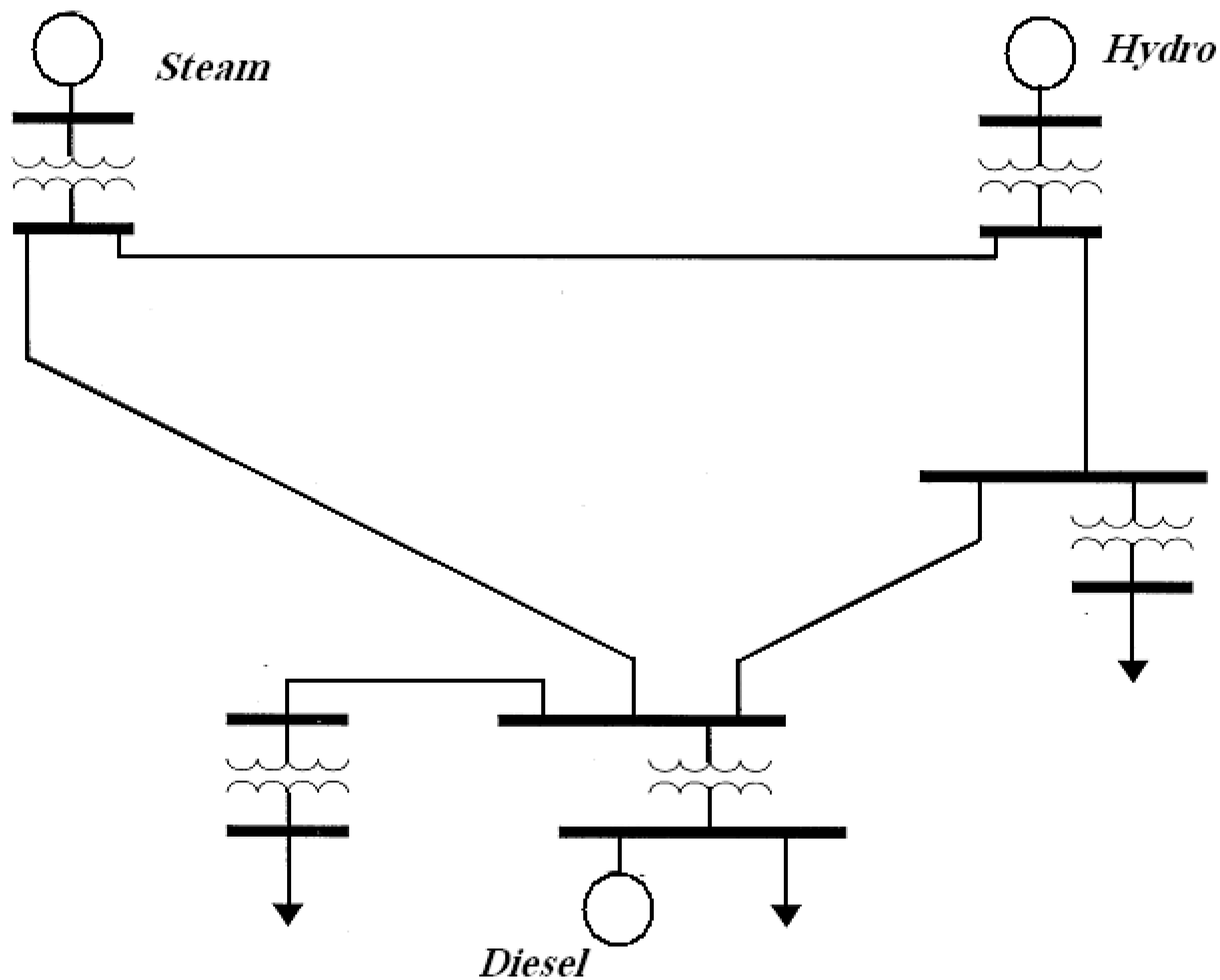
The Power Flow Problem

- Power flow analysis is fundamental to the study of power systems.
- In fact, power flow forms the core of power system analysis.
- power flow study plays a key role in the planning of additions or expansions to transmission and generation facilities.
- A power flow solution is often the starting point for many other types of power system analyses.
- In addition, power flow analysis is at the heart of contingency analysis and the implementation of real-time monitoring systems.

Problem Statement

For a given power network, with known complex power loads and some set of specifications or restrictions on power generations and voltages, **solve** for any unknown bus voltages and unspecified generation and finally for the complex power flow in the network components.

Network Structure



Power Flow Study Steps

1. Determine element values for passive network components.
2. Determine locations and values of all complex power loads.
3. Determine generation specifications and constraints.
4. Develop a mathematical model describing power flow in the network.
5. Solve for the voltage profile of the network.
6. Solve for the power flows and losses in the network.
7. Check for constraint violations.

Formulation of the Bus Admittance Matrix

- The first step in developing the mathematical model describing the power flow in the network is the formulation of the bus admittance matrix.
- The bus admittance matrix is an n*n matrix (where n is the number of buses in the system) constructed from the admittances of the equivalent circuit elements of the segments making up the power system.
- Most system segments are represented by a combination of shunt elements (connected between a bus and the reference node) and series elements (connected between two system buses).

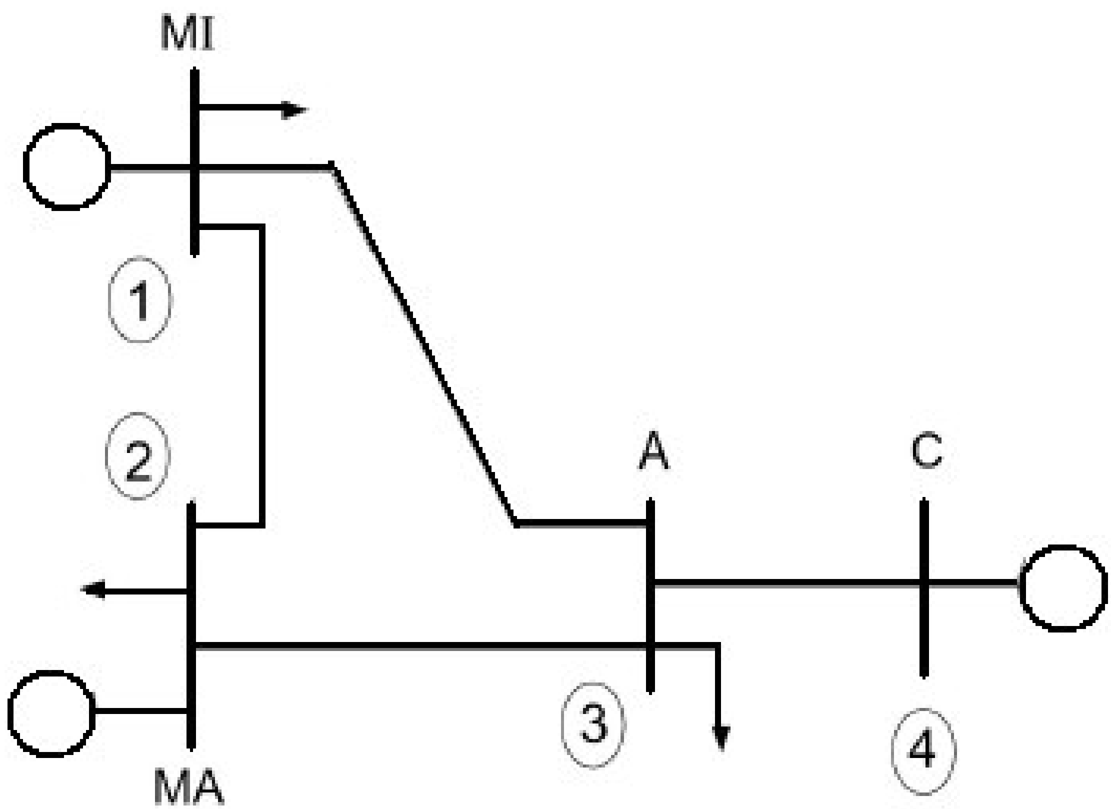
Bus Admittance Matrix

Formulation of the bus admittance matrix follows two simple rules:

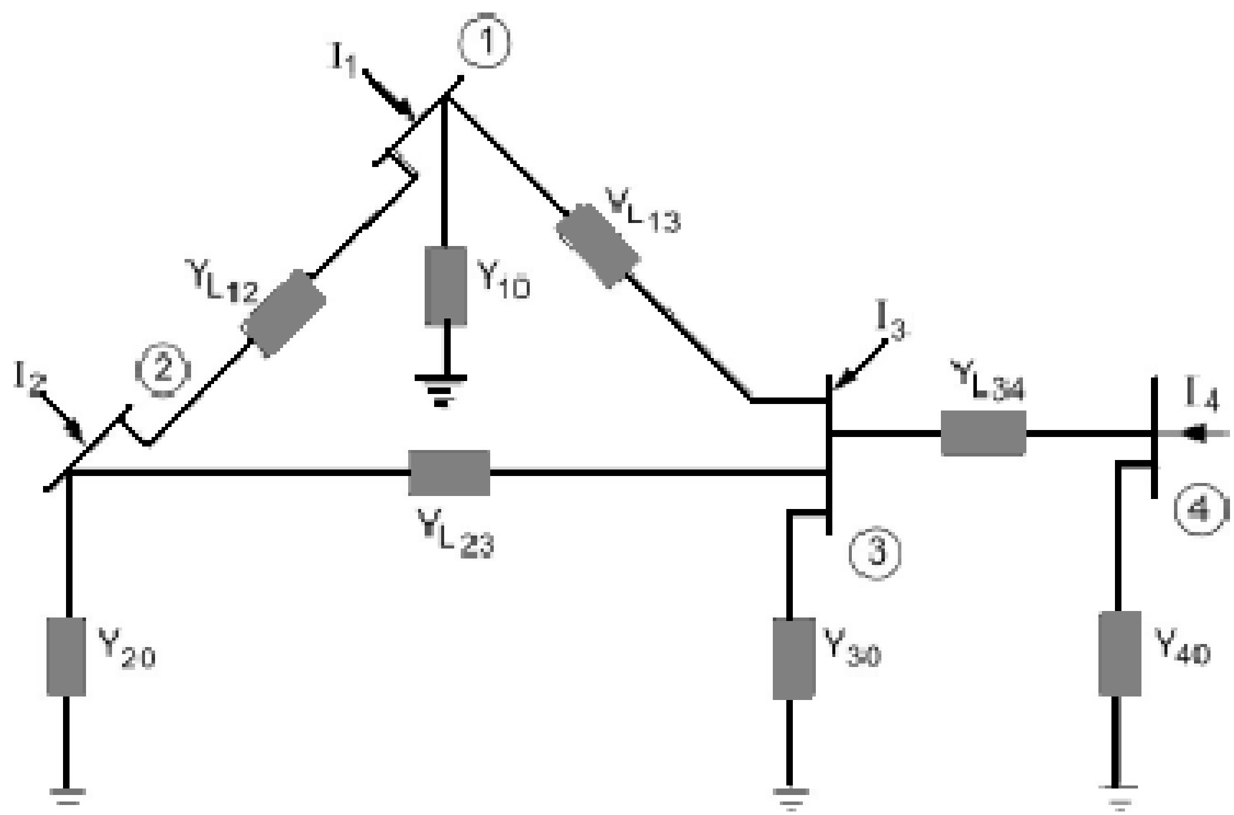
1. The admittance of elements connected between node k and reference is added to the (k, k) entry of the admittance matrix.
2. The admittance of elements connected between nodes j and k is added to the (j, j) and (k, k) entries of the admittance matrix.

- The negative of the admittance is added to the (j, k) and (k, j) entries of the admittance matrix.

Bus Admittance Matrix



Bus Admittance Matrix



Node-Voltage Equations

Applying KCL at each node yields:

$$
\begin{aligned}
I_1 &= V_1 Y_{10} + (V_1 - V_2) Y_{L_{12}} + (V_1 - V_3) Y_{L_{13}} \\
I_2 &= V_2 Y_{20} + (V_2 - V_1) Y_{L_{12}} + (V_2 - V_3) Y_{L_{23}} \\
I_3 &= V_3 Y_{30} + (V_3 - V_1) Y_{L_{13}} + (V_3 - V_4) Y_{L_{34}} + (V_3 - V_2) Y_{L_{23}} \\
I_4 &= V_4 Y_{40} + (V_4 - V_3) Y_{L_{34}}
\end{aligned}
$$

Defining the Y's as

$$
\begin{aligned}
Y_{11} &= Y_{10} + Y_{L_{12}} + Y_{L_{13}} \\
Y_{22} &= Y_{20} + Y_{L_{12}} + Y_{L_{23}} \\
Y_{33} &= Y_{30} + Y_{L_{13}} + Y_{L_{23}} + Y_{L_{34}} \\
Y_{44} &= Y_{40} + Y_{L_{34}} \\
Y_{12} &= Y_{21} = -Y_{L_{12}} \\
Y_{13} &= Y_{31} = -Y_{L_{13}} \\
Y_{23} &= Y_{32} = -Y_{L_{23}} \\
Y_{34} &= Y_{43} = -Y_{L_{34}}
\end{aligned}
$$

The Y-Bus

The current equations reduced to

$$
\boxed{\begin{aligned}
I_1 &= Y_{11}V_1 + Y_{12}V_2 + Y_{13}V_3 + 0V_4 \\
I_2 &= Y_{21}V_1 + Y_{22}V_2 + Y_{23}V_3 + 0V_4 \\
I_3 &= Y_{13}V_1 + Y_{23}V_2 + Y_{33}V_3 + Y_{34}V_4 \\
I_4 &= 0V_1 + 0V_2 + Y_{43}V_3 + Y_{44}V_4
\end{aligned}}
$$

Where,

$$
\mathbf{I}_{\text{bus}} = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} \qquad \mathbf{V}_{\text{bus}} = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix}
$$

In a compact form

$$
\boxed{\mathbf{I}_{\text{bus}} = \mathbf{Y}_{\text{bus}} \mathbf{V}_{\text{bus}}}
$$

$$
\mathbf{Y}_{\text{bus}} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & Y_{14} \\ Y_{12} & Y_{22} & Y_{23} & Y_{24} \\ Y_{13} & Y_{23} & Y_{33} & Y_{34} \\ Y_{14} & Y_{24} & Y_{34} & Y_{44} \end{bmatrix}
$$

The General Form of the Load-Flow Equations

- In Practice, bus powers S_i *is specified* rather than the bus currents I_i .

$$I^* = \frac{S_i}{V_i}$$

- As a result, we have

$$P_i - jQ_i = V_i^* I_i = V^*_i \sum_{n=1}^{N} (Y_{in} V_n) = \sum_{n=1}^{N} |Y_{in} V_i V_n| \angle(\theta_{in} + \delta_n - \delta_i)$$

$$Y_{ij} = |Y_{ij}| \angle \theta_{ij} = |Y_{ij}| \cos\theta_{ij} + j|Y_{ij}| \sin\theta_{ij} = G_{ij} + jB_{ij}$$

Load-Flow Equations

- These are the static power flow equations. Each equation is complex, and therefore we have 2*n* real equations. The nodal admittance matrix current equation can be written in the power form:

$$P_i - jQ_i = V_i^* I_i = V^*_i \sum_{n=1}^{N} (Y_{in} V_n) = \sum_{n=1}^{N} |Y_{in} V_i V_n| \angle(\theta_{in} + \delta_n - \delta_i)$$

Let,

$$Y_{ij} = |Y_{ij}| \angle \theta_{ij} = |Y_{ij}| \cos\theta_{ij} + j|Y_{ij}| \sin\theta_{ij} = G_{ij} + jB_{ij}$$

Load-Flow Equations

- **Finally,**

$$P_i = |V_i|^2 G_{ii} + \sum_{\substack{n=1 \\ n \neq i}}^{N} |V_i V_n Y_{in}| \cos(\theta_{in} + \delta_n - \delta_i)$$

$$Q_i = -|V_i|^2 B_{ii} - \sum_{\substack{n=1 \\ n \neq i}}^{N} |V_i V_n Y_{in}| \sin(\theta_{in} + \delta_n - \delta_i)$$

- **This is known as NR (Newton – Raphson) formulation**

Gauss Power Flow

We first need to put the equation in the correct form

$$S_i = V_i I_i^* = V_i \left(\sum_{k=1}^{n} Y_{ik} V_k \right)^* = V_i \sum_{k=1}^{n} Y_{ik}^* V_k^*$$

$$S_i^* = V_i^* I_i = V_i^* \sum_{k=1}^{n} Y_{ik} V_k = V_i^* \sum_{k=1}^{n} Y_{ik} V_k$$

$$\frac{S_i^*}{V_i^*} = \sum_{k=1}^{n} Y_{ik} V_k = Y_{ii} V_i + \sum_{k=1, k \neq i}^{n} Y_{ik} V_k$$

$$V_i = \frac{1}{Y_{ii}} \left(\frac{S_i^*}{V_i^*} - \sum_{k=1, k \neq i}^{n} Y_{ik} V_k \right)$$

Difficulties

- Unless the generation equals the load at every bus, the complex power outputs of the generators cannot be arbitrarily selected.
- In fact, the complex power output of at least one of the generators must be calculated last, since it must take up the unknown “slack” due to the uncalculated network losses.
- Further, losses cannot be calculated until the voltages are known.
- Also, it is not possible to solve these equations for the absolute phase angles of the phasor voltages. This simply means that the problem can only be solved to some arbitrary phase angle reference.

Difficulties

- For a 4- bus system, suppose that S_{G4} is arbitrarily allowed to float or swing (in order to take up the necessary slack caused by the losses) and that S_{G1}, S_{G2}, S_{G3} are specified.

$$\overline{S}^*_{G1} - \overline{S}^*_{D1} = \overline{V}^*_1\left[\overline{Y}_{11}\overline{V}_1 + \overline{Y}_{12}\overline{V}_2 + \overline{Y}_{13}\overline{V}_3 + \overline{Y}_{14}\overline{V}_4\right]$$

$$\overline{S}^*_{G2} - \overline{S}^*_{D2} = \overline{V}^*_2\left[\overline{Y}_{21}\overline{V}_1 + \overline{Y}_{22}\overline{V}_2 + \overline{Y}_{23}\overline{V}_3 + \overline{Y}_{24}\overline{V}_4\right]$$

$$\overline{S}^*_{G3} - \overline{S}^*_{D3} = \overline{V}^*_3\left[\overline{Y}_{31}\overline{V}_1 + \overline{Y}_{32}\overline{V}_2 + \overline{Y}_{33}\overline{V}_3 + \overline{Y}_{34}\overline{V}_4\right]$$

$$\overline{S}^*_{G4} - \overline{S}^*_{D4} = \overline{V}^*_4\left[\overline{Y}_{41}\overline{V}_1 + \overline{Y}_{42}\overline{V}_2 + \overline{Y}_{43}\overline{V}_3 + \overline{Y}_{44}\overline{V}_4\right]$$

Remedies

- Now, with the loads known, the equations are seen as four simultaneous nonlinear equations with complex coefficients in five unknowns. (V_1, V_2, V_3, V_4 and S_{G4}).

- Designating bus 4 as the slack bus and specifying the voltage V_4 reduces the problem to four equations in four unknowns.

Remedies

- The slack bus is chosen as the phase reference for all phasor calculations, its magnitude is constrained, and the complex power generation at this bus is free to take up the slack necessary in order to account for the system real and reactive power losses.

- Systems of nonlinear equations, cannot (except in rare cases) be solved by closed-form techniques.

Load Flow Solution

- There are four quantities of interest associated with each bus:
  1. **Real Power, P**
  2. **Reactive Power, Q**
  3. **Voltage Magnitude, V**
  4. **Voltage Angle, δ**
- At every bus of the system, two of these four quantities will be specified and the remaining two will be unknowns.
- Each of the system buses may be classified in accordance with which of the two quantities are specified

Bus Classifications

Slack Bus — The slack bus for the system is a single bus for which the voltage magnitude and angle are specified.

- The real and reactive power are unknowns.
- The bus selected as the slack bus must have a source of both real and reactive power, since the injected power at this bus must “swing” to take up the “slack” in the solution.
- The best choice for the slack bus (since, in most power systems, many buses have real and reactive power sources) requires experience with the particular system under study.
- The behavior of the solution is often influenced by the bus chosen.

Bus Classifications

Load Bus (P-Q Bus) : A load bus is defined as any bus of the system for which the real and reactive power are specified.

- Load buses may contain generators with specified real and reactive power outputs;
- however, it is often convenient to designate any bus with specified injected complex power as a load bus.

Voltage Controlled Bus (P-V Bus) : Any bus for which the voltage magnitude and the injected real power are specified is classified as a voltage controlled (or P-V) bus.

- The injected reactive power is a variable (with specified upper and lower bounds) in the power flow analysis.
- (A P-V bus must have a variable source of reactive power such as a generator.)

Solution Methods

- The solution of the simultaneous nonlinear power flow equations requires the use of iterative techniques for even the simplest power systems.
- There are many methods for solving nonlinear equations, such as:

- Gauss Seidel.
- Newton Raphson.
- Fast Decoupled.

Guess Solution

- It is important to have a good approximation to the load-flow solution, which is then used as a starting estimate (or initial guess) in the iterative procedure.
- A fairly simple process can be used to evaluate a good approximation to the unknown voltages and phase angles.
- The process is implemented in two stages: the first calculates the approximate angles, and the second calculates the approximate voltage magnitudes.

Gauss Iteration Method

With the Gauss method we need to rewrite our equation in an implicit form: $x = h(x)$

To iterate we first make an initial guess of x, $x^{(0)}$, and then iteratively solve $x^{(v+1)} = \boldsymbol{h}(\boldsymbol{x}^{(v)})$ until we find a "fixed point", $\hat{x}$, such that $\hat{x} = \boldsymbol{h}(\hat{x})$.

Gauss Iteration Example

Example: Solve $x - \sqrt{x} - 1 = 0$

$$x^{(v+1)} = 1 + \sqrt{x^{(v)}}$$

Let k = 0 and arbitrarily guess $x^{(0)} = 1$ and solve

k	$x^{(v)}$	k	$x^{(v)}$
0	1	5	2.61185
1	2	6	2.61612
2	2.41421	7	2.61744
3	2.55538	8	2.61785
4	2.59805	9	2.61798

Stopping Criteria

A key problem to address is when to stop the iteration. With the Guass iteration we stop when

$$\left|\Delta x^{(v)}\right| < \varepsilon \qquad \text{with} \quad \Delta x^{(v)} \equiv x^{(v+1)} - x^{(v)}$$

If x is a scalar this is clear, but if x is a vector we need to generalize the absolute value by using a norm

$$\left\|\Delta x^{(v)}\right\|_j < \varepsilon$$

Gauss Power Flow

We first need to put the equation in the correct form

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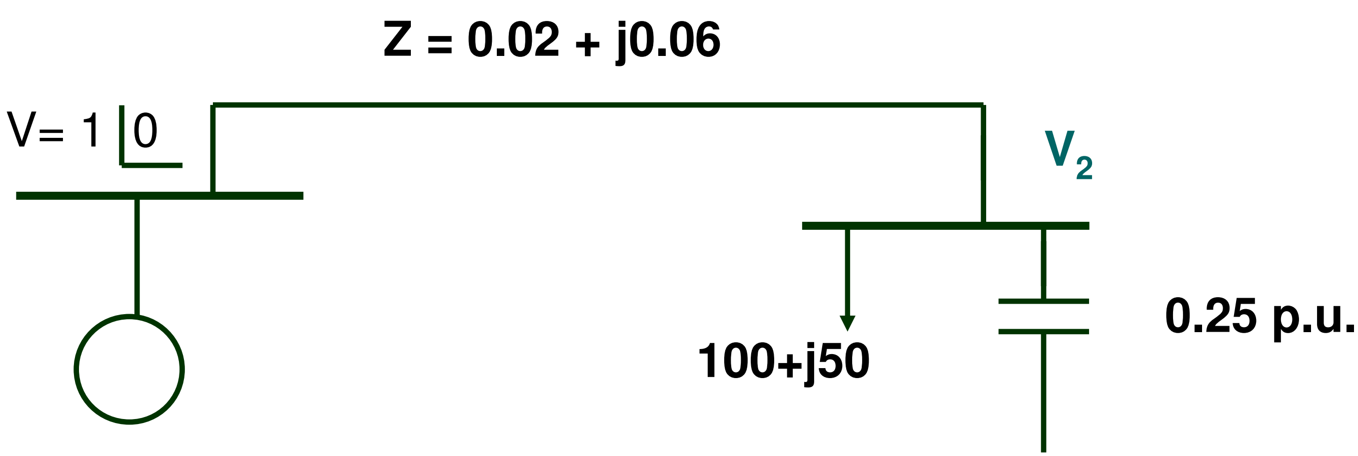
$$\frac{S_i^*}{V_i^*} = \sum_{k=1}^{n} Y_{ik} V_k = Y_{ii} V_i + \sum_{k=1, k \neq i}^{n} Y_{ik} V_k$$

$$V_i = \frac{1}{Y_{ii}} \left(\frac{S_i^*}{V_i^*} - \sum_{k=1, k \neq i}^{n} Y_{ik} V_k \right)$$

Example

A 100 MW, 50 Mvar load is connected to a generator through a line with z = 0.02 + j0.06 p.u. and line charging of 0.05 p.u on each end (100 MVA base). Also, there is a 0.25 p.u. capacitance at bus 2. If the generator voltage is 1.0 p.u., what is V_2?



Y-Bus

The unknown is the complex load voltage, V_2.

To determine V_2 we need to know the $\mathbf{Y}_{\text{bus}}$.

$$\frac{1}{0.02 + j0.06} = 5 - j15$$

$$\text{Hence } \mathbf{Y}_{\text{bus}} = \begin{bmatrix} 5 - j14.95 & -5 + j15 \\ -5 + j15 & 5 - j14.70 \end{bmatrix}$$

$$(\text{Note } B_{22} = -j15 + j0.05 + j0.25)$$

Solution

$$V_2 = \frac{1}{Y_{22}}\left(\frac{S_2^*}{V_2^*} - \sum_{k=1,k\neq i}^{n} Y_{ik}V_k\right)$$

$$V_2 = \frac{1}{5-j14.70}\left(\frac{-1+j0.5}{V_2^*} - (-5+j15)(1.0\angle 0)\right)$$

Guess $V_2^{(0)} = 1.0\angle 0$ (this is known as a flat start)

v	$V_2^{(v)}$	v	$V_2^{(v)}$
0	$1.000 + j0.000$	3	$0.9622 - j0.0556$
1	$0.9671 - j0.0568$	4	$0.9622 - j0.0556$
2	$0.9624 - j0.0553$		

Solution (cont.)

$$V_2 = 0.9622 - j0.0556 = 0.9638\angle -3.3°$$

Once the voltages are known all other values can be determined, such as the generator powers and the line flows

$$S_1^* = V_1^*(Y_{11}V_1 + Y_{12}V_2) = 1.023 - j0.239$$

In actual units $P_1 = 102.3$ MW, $Q_1 = 23.9$ Mvar

Gauss-Seidel Iteration

Immediately use the new voltage estimates:

$$
\begin{aligned}
V_2^{(v+1)} &= h_2(V_1, V_2^{(v)}, V_3^{(v)}, \ldots, V_n^{(v)}) \\
V_3^{(v+1)} &= h_2(V_1, V_2^{(v+1)}, V_3^{(v)}, \ldots, V_n^{(v)}) \\
V_4^{(v+1)} &= h_2(V_1, V_2^{(v+1)}, V_3^{(v+1)}, V_4^{(v)} \ldots, V_n^{(v)}) \\
&\vdots \\
V_n^{(v+1)} &= h_2(V_1, V_2^{(v+1)}, V_3^{(v+1)}, V_4^{(v+1)} \ldots, V_n^{(v)})
\end{aligned}
$$

The Gauss-Seidel works better than the Gauss, and is actually easier to implement. It is used instead of Gauss.