SOLVED PROBLEMS

Problem 1.1. *For network shown in figure 1.17 draw the graph, from that find A′, A, B′, B, C′, C, K*

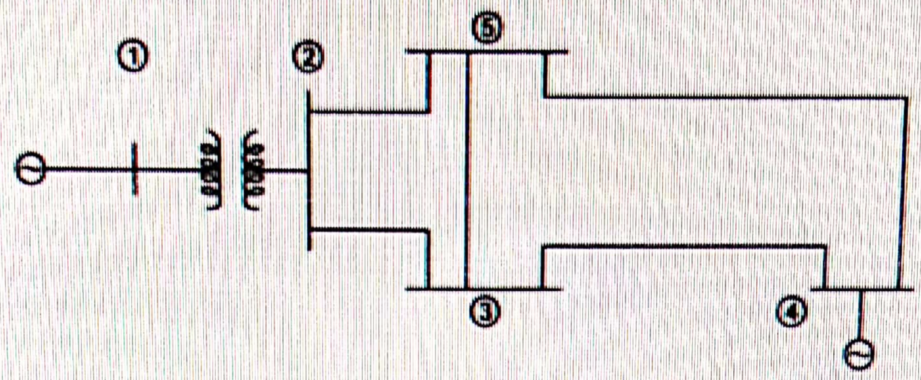

Fig. 1.17

Sol. For this graph

Number of Elements, $e = 9$

Number of Nodes, $n = 5 + 1 = 6$

Number of Branches, $b = 5$

Number of Links, $l = 4$

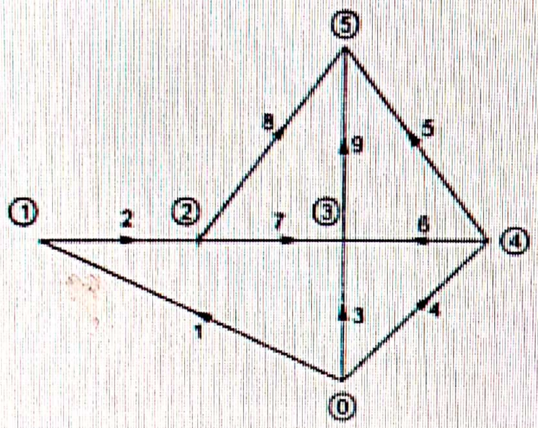

Fig. 1.17 (a) *Graph*

Element node incidence matrix (A′) elements can be obtained from figure 1.17 (*a*) as follows :

$a_{ij} = 1$, if the i^{th} element is incidence and oriented away from the j^{th} node.

$a_{ij} = -1$, if the i^{th} element is incidence and oriented towards the j^{th} node.

$a_{ij} = 0$, if the i^{th} element is not incidence to the j^{th} node.

A′ =

element \ node	0	1	2	3	4	5
1	1	−1	0	0	0	0
2	0	1	−1	0	0	0
3	1	0	0	−1	0	0
4	1	0	0	0	−1	0
5	0	0	0	0	1	−1
6	0	0	0	−1	1	0
7	0	0	1	−1	0	0
8	0	0	1	0	0	−1
9	0	0	0	1	0	−1

For obtaining bus incidence matrix (A), eliminate the reference node column in A'.

∴ **Bus incidence matrix**

A =

element \ node	①	②	③	④	⑤
1	-1	0	0	0	0
2	1	-1	0	0	0
3	0	0	-1	0	0
4	0	0	0	-1	0
5	0	0	0	1	-1
6	0	0	-1	1	0
7	0	1	-1	0	0
8	0	1	0	0	-1
9	0	0	1	0	-1

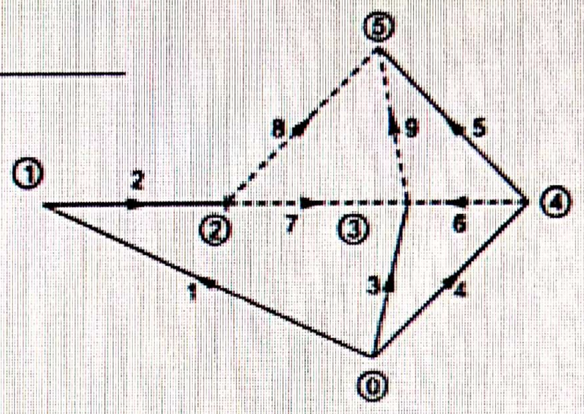

Fig. 1.17 (b) *Tree*

Branch path incidence matrix (K) elements are obtained from figure. 1.17 (*b*).

K =

Branch \ path	①	②	③	④	⑤
1	-1	0	0	0	0
2	1	-1	0	0	0
3	0	0	-1	0	0
4	0	0	0	-1	0
5	0	0	0	1	-1

Basic cut-set (B)

The elements in this matrix can be found as follows from figure. 1.17 (*c*)

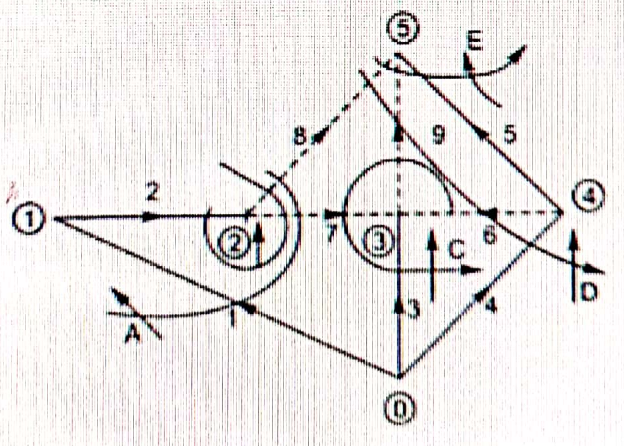

Fig. 1.17 (c) *Cut-set*

B_{ij} = 1, if the i^{th} element is incidence to and oriented in the same direction as the j^{th} basic cut-set

B_{ij} = - 1, if the i^{th} element is incidence to and oriented in the opposite direction as the j^{th} basic cut-set

B_{ij} = 0, if the i^{th} element is not incidence with the j^{th} basic cut-set

Branch \ Cut-set	A	B	C	D	E
1	1	0	0	0	0
2	0	1	0	0	0
3	0	0	1	0	0
4	0	0	0	1	0
5	0	0	0	0	1
6	0	0	1	–1	0
7	–1	–1	1	0	0
8	–1	–1	0	1	1
9	0	0	–1	1	1

B = (the matrix above)

Augmented cut-set matrix (B')

Element \ Cut-set	A	B	C	D	E	F	G	H	I
1	1	0	0	0	0	0	0	0	0
2	0	1	0	0	0	0	0	0	0
3	0	0	1	0	0	0	0	0	0
4	0	0	0	1	0	0	0	0	0
5	0	0	0	0	1	0	0	– 1	– 1
6	0	0	1	–1	0	1	0	0	0
7	– 1	– 1	1	0	0	0	1	0	0
8	– 1	– 1	0	1	1	0	0	1	0
9	0	0	– 1	1	1	0	0	0	1

B' = (the matrix above)

Basic loop incidence matrix (C)

The elements can be found as follows from figure 1.17 (*d*)

C_{ij} = 1, If the element is incidence to and oriented in the same direction as the j^{th} basic loop

C_{ij} = – 1, If the element is incidence to and oriented in the opposite direction as the j^{th} basic loop

C_{ij} = 0, If the element is not incidence to the j^{th} basic loop

G : 1, 2, 7, 3	7 as link
I : 9, 5, 4, 7	9 as link
H : 1, 2, 8, 5, 4	8 as link
F : 3, 4, 6	6 as link

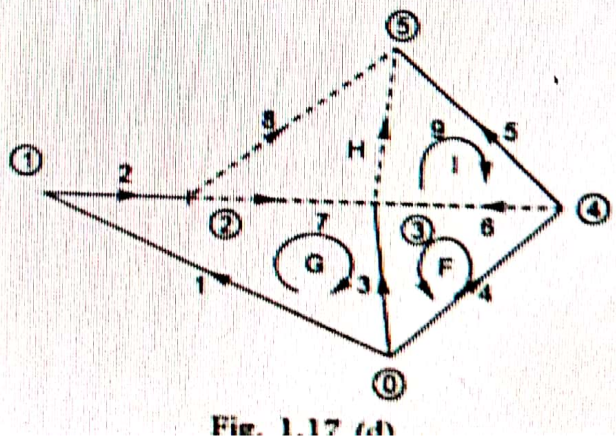

Fig. 1.17 (d)

C =

Loop / element	F	G	H	I
1	0	1	1	0
2	0	1	1	0
3	−1	−1	0	1
4	1	0	−1	−1
5	0	0	−1	−1
6	1	0	0	0
7	0	1	0	0
8	0	0	1	0
9	0	0	0	1

Augmented loop incidence matrix (C′) elements are is obtained from figure 1.17 (*d*)

C′ =

Loop / Element	A	B	C	D	E	F	G	H	I
1	1	0	0	0	0	0	1	1	0
2	0	1	0	0	0	0	1	1	0
3	0	0	1	0	0	−1	−1	0	1
4	0	0	0	1	0	1	0	−1	−1
5	0	0	0	0	1	0	0	−1	−1
6	0	0	0	0	0	1	0	0	0
7	0	0	0	0	0	0	1	0	0
8	0	0	0	0	0	0	0	1	0
9	0	0	0	0	0	0	0	0	1

Problem 1.2. *For the network shown in figure 1.18 draw the graph and tree. Also determine the 'Y_{Bus}' matrix by direct inspection method. All the mentioned values are impedances in p.u.*

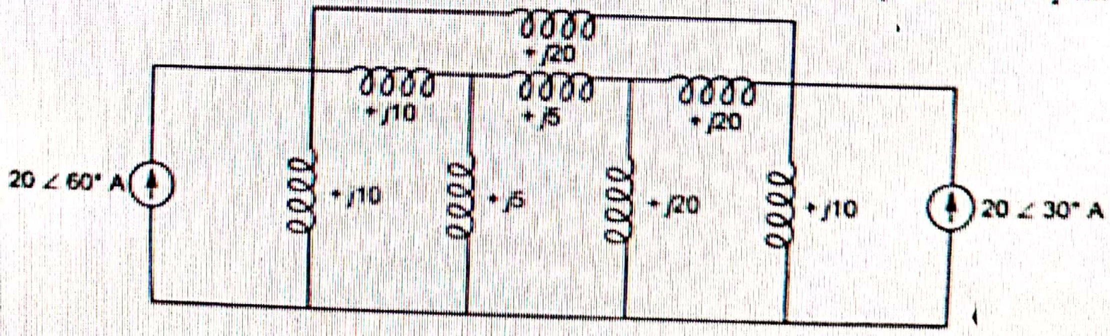

Fig. 1.18

Sol.

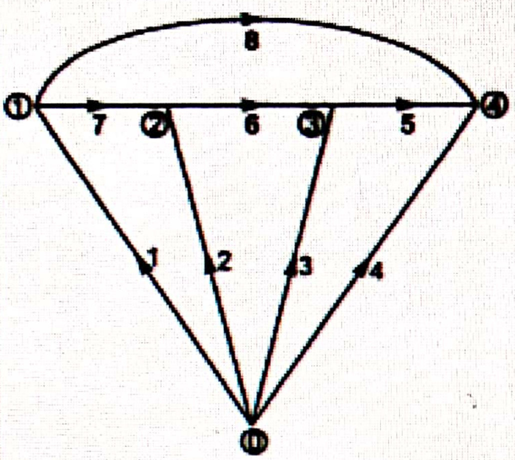

Fig. 1.19 (a) *Graph*

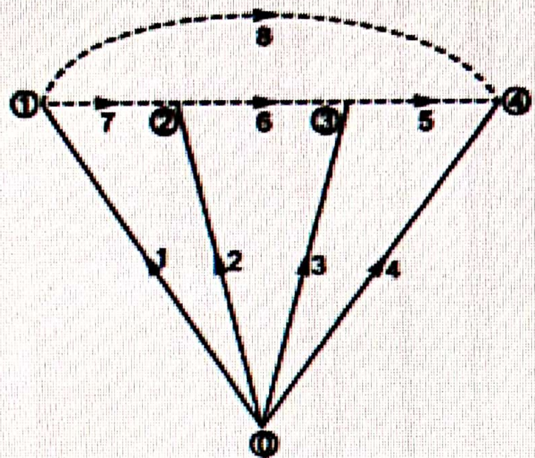

Fig. 1.19 (b) *Tree*

From the graph shown in figure 1.19 (*a*)

Number of nodes, $n = 5$

Number of elements, $e = 8$

Number of branches, $b = n - 1 = 5 - 1 = 4$

Number of links, $l = e - n + 1 = 8 - 5 + 1 = 4$

Therefore 'Y_{Bus}' has the dimension of '4×4'

Given values are impedances and are given in p.u quantities. We can determine admittance values and then form the 'Y_{Bus}'.

The elements of 'Y_{Bus}' can calculate as follows

Y_{ii} = Sum of the admittances of the elements that are creating the node at bus '*i*'

Y_{ij} = Negative sign of admittance between the adjacent busses '*i*' and '*j*'

For $i, j = 1, 2, 3$ and 4

∴ Diagonal elements :

$$Y_{11} = j0.1 + j0.1 + j0.05 = j0.25 \text{ p.u}$$
$$Y_{22} = j0.1 + j0.2 + j0.2 = j0.5 \text{ p.u}$$
$$Y_{33} = j0.1 + j0.2 + j0.05 = j0.35 \text{ p.u}$$
$$Y_{44} = j0.05 + j0.05 + j0.05 = j0.15 \text{ p.u}$$

Off-diagonal elements :

$$Y_{12} = Y_{21} = -j0.1 \text{ p.u}$$
$$Y_{13} = Y_{31} = 0.0 \text{ p.u}$$
$$Y_{14} = Y_{41} = -j0.05 \text{ p.u}$$
$$Y_{23} = Y_{32} = -j0.2 \text{ p.u}$$
$$Y_{24} = Y_{42} = 0 \text{ p.u}$$
$$Y_{34} = Y_{43} = -j0.05 \text{ p.u}$$

$$\therefore \qquad Y_{Bus} = \begin{bmatrix} j0.25 & -j0.1 & 0 & -j0.05 \\ -j0.1 & j0.5 & -j0.2 & 0 \\ 0 & -j0.2 & j0.35 & -j0.05 \\ -j0.05 & 0 & -j0.05 & j0.15 \end{bmatrix}$$

Problem 1.3. *For problem 1.2, determine the elements of 'Z_{Loop}' by direct inspection method.*

Sol. Here we have to replace the current sources into equivalent voltage sources when applying the direct inspection method.

The elements of loop impedance matrix as follows :

Z_{ii} = Sum of the impedances of the elements forming the loop '*i*'

Z_{ij} = Negative sign of impedance that are common to the loops '*i*' and '*j*'

∴ Diagonal elements are

$$Z_{11} = +j25 \text{ p.u}$$
$$Z_{22} = +j20 \text{ p.u}$$
$$Z_{33} = +j50 \text{ p.u}$$
$$Z_{44} = +j50 \text{ p.u}$$

and off-diagonal elements are

$$Z_{12} = Z_{21} = -j5 \text{ p.u}$$
$$Z_{13} = Z_{31} = 0 \text{ p.u}$$
$$Z_{14} = Z_{41} = -j10 \text{ p.u}$$
$$Z_{23} = Z_{32} = -j10 \text{ p.u}$$
$$Z_{24} = Z_{42} = -j5 \text{ p.u}$$
$$Z_{34} = Z_{43} = -j20 \text{ p.u}$$

$$\therefore \qquad Z_{Loop} = \begin{bmatrix} +j25 & -j5 & 0 & -j10 \\ -j5 & +j20 & -j10 & -j5 \\ 0 & -j10 & +j50 & -j20 \\ -j10 & -j5 & -j20 & +j50 \end{bmatrix}$$

Problem 1.4. *Determine the incidence matrices A, B, B', C, C' and K. From that verify the following relations shown in figure 1.20. Take 1 as ground bus.*

(i) $C_b = -B_l^T$ *(ii)* $C'B'T = U$

(iii) $A_b K^T = U$ *and* *(iv)* $B_l = A_l K^T$

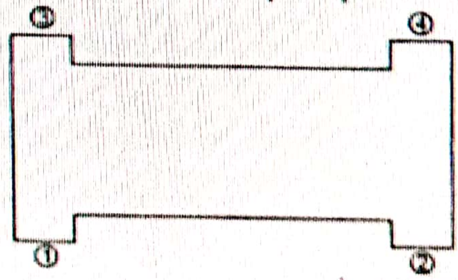

Fig. 1.20

Sol. Number of nodes, $n = 4$

Number of branches, $b = n - 1$

$$= 4 - 1 = 3$$

Number of links, $l = e - n + 1$

$$= 4 - 4 + 1 = 1.$$

Fig. 1.20 (a) *Graph*

From figure 1.20 (*b*), Bus Incidence matrix [A]

$$A = \begin{array}{c|ccc} e \backslash \text{bus} & ② & ③ & ④ \\ \hline 1 & -1 & 0 & 0 \\ 2 & 0 & -1 & 0 \\ 3 & 0 & 1 & -1 \\ \hdashline 4 & 1 & 0 & -1 \end{array} \quad \begin{array}{l} \left.\begin{array}{c} \\ \\ \\ \end{array}\right\} A_b \\ \}\ A_l \end{array}$$

$$\therefore \qquad A_b = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 1 & -1 \end{bmatrix}$$

and $$A_l = [1 \quad 0 \quad -1]$$

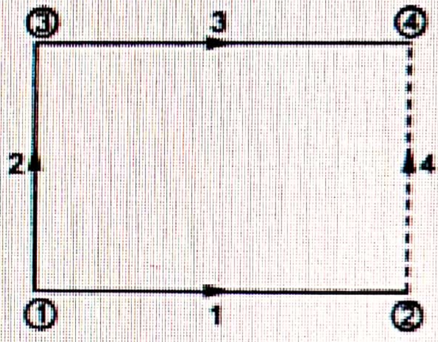

Fig. 1.20 (b) *Tree*

From figure 1.20 (*b*), Branch path matrix (K)

$$K = \begin{array}{c|ccc} \text{branch} \backslash \text{path} & ② & ③ & ④ \\ \hline 1 & -1 & 0 & 0 \\ 2 & 0 & -1 & -1 \\ 3 & 0 & 0 & -1 \end{array}$$

From figure 1.20 (*c*), basic cut-set incidence matrix (B)

$$B = \begin{array}{c|ccc} \text{element} \backslash \text{cut-set} & A & B & C \\ \hline 1 & 1 & 0 & 0 \\ 2 & 0 & 1 & 0 \\ 3 & 0 & 0 & 1 \\ 4 & -1 & 1 & 1 \end{array} \quad \begin{array}{l} \left.\begin{array}{c} \\ \\ \\ \end{array}\right\} B_b \\ \}\ B_l \end{array}$$

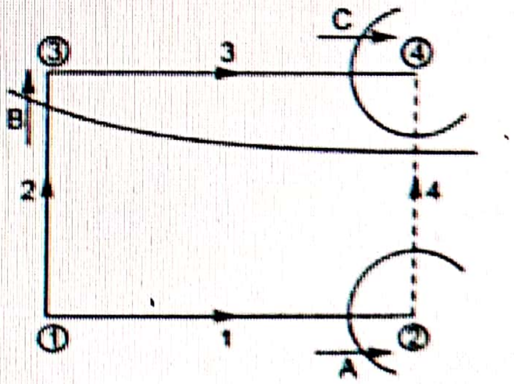

Fig. 1.20 (c)

$$\therefore \qquad B_b = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

and $$B_l = [-1 \quad 1 \quad 1]$$

From figure 1.20 (*d*). Augmented cut-set incidence matrix (B′)

$$B' = \begin{array}{c|ccc:c} \text{element} \backslash \text{cut-set} & A & B & C & D \\ \hline 1 & 1 & 0 & 0 & 0 \\ 2 & 0 & 1 & 0 & 0 \\ 3 & 0 & 0 & 1 & 0 \\ \hdashline 4 & -1 & 1 & 1 & 1 \end{array} = \left[\begin{array}{c|c} U_b & 0 \\ \hline B_l & U_l \end{array}\right]$$

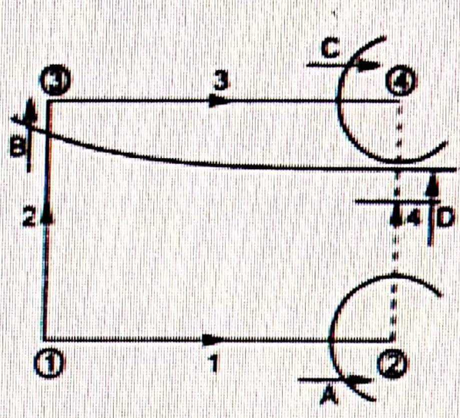

Fig. 1.20 (d)

$\therefore$ $$B_l = [-1\ 1\ 1]$$

and $$U_l = [1]$$

From figure 1.20 (*e*), basic loop incidence matrix (C)

$$C = \begin{array}{c|c} \text{element} \backslash \text{loop} & D \\ \hline 1 & 1 \\ 2 & -1 \\ 3 & -1 \\ \hdashline 4 & 1 \end{array} \begin{array}{l} \left.\vphantom{\begin{array}{c}1\\1\\1\end{array}}\right\} C_b \\ \ \}\ C_l \end{array}$$

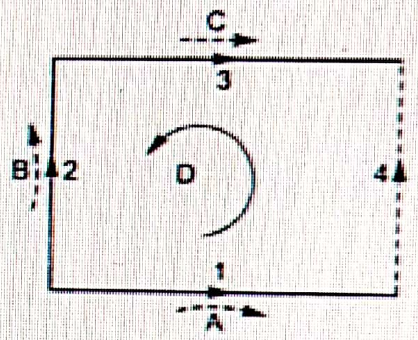

Fig. 1.20 (e)

$\therefore$ $$C_b = \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix}$$

and $$C_l = [1]$$

From figure 1.20 (*e*). Augmented loop incidence matrix (C′)

$$C' = \begin{array}{c|ccc:c} \text{element} \backslash \text{loop} & A & B & C & D \\ \hline 1 & 1 & 0 & 0 & 1 \\ 2 & 0 & 1 & 0 & -1 \\ 3 & 0 & 0 & 1 & -1 \\ \hdashline 4 & 0 & 0 & 0 & 1 \end{array} = \left[\begin{array}{c|c} U_b & C_b \\ \hline 0 & U_l \end{array}\right]$$

$\therefore$ $$C_b = \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix}$$

Verification :

(*i*) $C_b = -B_l^T$

From the matrices C_b and B_l values

$\therefore$ $$C_b = -B_l^T$$

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(*ii*) $C'B'^T = U$

$$C' = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$B' = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -1 & 1 & 1 & 1 \end{bmatrix}$$

$$B'^T = \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$C'(B')^T = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = U$$

$\therefore$ $C'(B')^T = U$

(*iii*) $A_b K^T = U$

$$A_b = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 1 & -1 \end{bmatrix}$$

$$K = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & -1 \\ 0 & 0 & -1 \end{bmatrix} \Rightarrow K^T = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & -1 & -1 \end{bmatrix}$$

$$A_b \cdot K^T = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 1 & -1 \end{bmatrix}\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & -1 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\therefore \quad A_b K^T = U$

(*iv*) $B_l = A_l K^T$

$$A_l = [1 \quad 0 \quad -1]$$

$$K^T = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & -1 & -1 \end{bmatrix}$$

$$A_l K^T = [1 \quad 0 \quad -1] \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & -1 & -1 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix} = B_l$$

Problem 1.5. *For the graph shown in figure 1.21, form the necessary incidence matrices and hence verify the following relations :*

(*i*) $A_b K^T = U$ (*ii*) $B_l = A_l K^T$

(*iii*) $C_b = -B_l^T$ (*iv*) $C'(B')^T = U$

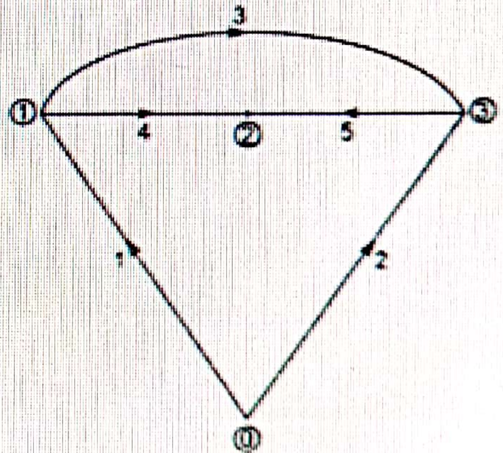

Fig. 1.21 *Graph*

Sol. Number of branches, $b = n - 1$

$$b = 4 - 1$$
$$= 3$$

Number of links $= e - n + 1$

$$= 5 - 4 + 1 = 2$$

From figure 1.21, Bus incidence matrix (A)

	e \ b	①	②	③	
	1	−1	0	0	A_b
	2	0	0	−1	
A =	4	1	−1	0	
	3	1	0	−1	A_l
	5	0	−1	1	

$$\therefore \qquad A_b = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & -1 \\ 1 & -1 & 0 \end{bmatrix}$$

and

$$A_l = \begin{bmatrix} 1 & 0 & -1 \\ 0 & -1 & 1 \end{bmatrix}$$

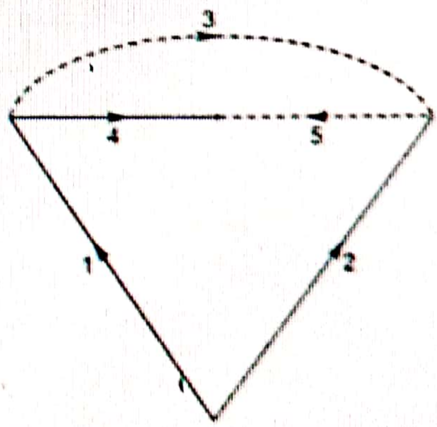

Fig. 1.21 (a) *Tree*

From figure 1.21 (*a*), branch path matrix (K)

$$K = \begin{array}{c|ccc} b \backslash \text{path} & ① & ② & ③ \\ \hline 1 & -1 & -1 & 0 \\ 2 & 0 & 0 & -1 \\ 4 & 0 & -1 & 0 \end{array}$$

From figure 1.21 (*b*), basic cut-set incidence matrix (B)

$$B = \begin{array}{c|ccc} e \backslash \text{cut-set} & A & B & C \\ \hline 1 & 1 & 0 & 0 \\ 2 & 0 & 1 & 0 \\ 4 & 0 & 0 & 1 \\ \hdashline 3 & -1 & 1 & 0 \\ 5 & 1 & -1 & 1 \end{array} \quad \begin{array}{l} \left.\begin{array}{c} \\ \\ \\ \end{array}\right\} U_b \\ \left.\begin{array}{c} \\ \\ \end{array}\right\} B_l \end{array}$$

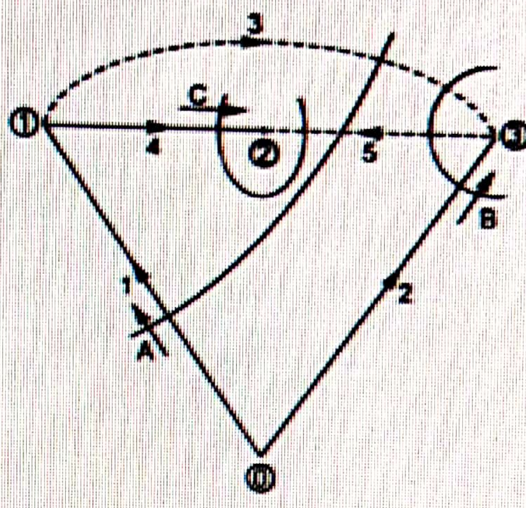

Fig. 1.21 (b)

$$\therefore \qquad B_l = \begin{bmatrix} -1 & 1 & 0 \\ 1 & -1 & 1 \end{bmatrix}$$

From figure 1.21 (*b*), Augmented cut-set incidence matrix (B′)

$$B' = \begin{array}{c|ccc|cc} e \backslash \text{Cut-set} & A & B & C & D & E \\ \hline 1 & 1 & 0 & 0 & 0 & 0 \\ 2 & 0 & 1 & 0 & 0 & 0 \\ 4 & 0 & 0 & 1 & 0 & 0 \\ \hdashline 3 & -1 & 1 & 0 & 1 & 0 \\ 5 & 1 & -1 & 1 & 0 & 1 \end{array} = \left[\begin{array}{c|c} U_b & 0 \\ \hline B_l & U_l \end{array}\right]$$

$$B_l = \begin{bmatrix} -1 & 1 & 0 \\ 1 & -1 & 1 \end{bmatrix}$$

From figure 1.21 (*c*), Basic loop incidence matrix (C)

$$C = \begin{array}{c|cc} e \backslash \text{loop} & D & E \\ \hline 1 & 1 & -1 \\ 2 & -1 & 1 \\ 4 & 0 & -1 \\ \hdashline 3 & 1 & 0 \\ 5 & 0 & 1 \end{array} \quad \begin{array}{l} \left.\begin{array}{c} \\ \\ \\ \end{array}\right\} C_b \\ \left.\begin{array}{c} \\ \\ \end{array}\right\} U_l \end{array}$$

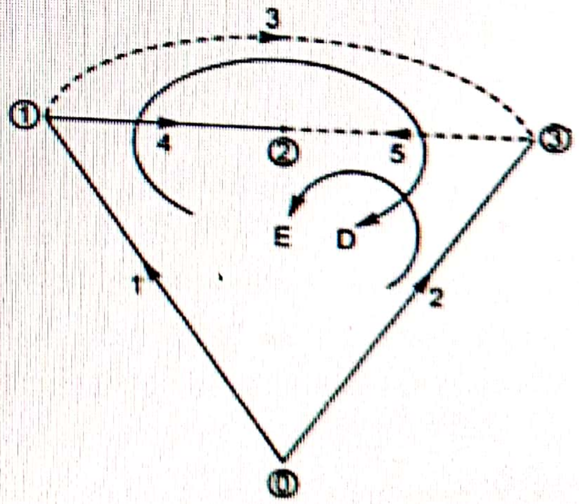

Fig. 1.21 (c)

$$C_b = \begin{bmatrix} 1 & -1 \\ -1 & 1 \\ 0 & -1 \end{bmatrix}$$

From figure 1.21 (*c*), Augmented loop incidence matrix (C′)

loop / e	A	B	C	D	E
1	1	0	0	1	−1
2	0	1	0	−1	1
4	0	0	1	0	−1
3	0	0	0	1	0
5	0	0	0	0	1

$$= \left[\begin{array}{c|c} U_b & C_b \\ \hline 0 & U_l \end{array}\right]$$

Verification :

(*i*) $A_b K^T = U$

$$A_b = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & -1 \\ 1 & -1 & 0 \end{bmatrix}$$

$$K = \begin{bmatrix} -1 & 0 & -1 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \Rightarrow K^T = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ -1 & 0 & -1 \end{bmatrix}$$

$$A_b \times K^T = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & -1 \\ 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ -1 & 0 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = U = \text{Unit matrix.}$$

(*ii*) $B_l = A_l K^T$

R.H.S. $\quad A_l = \begin{bmatrix} 1 & 1 & -1 \\ 0 & -1 & 0 \end{bmatrix}$

$$K^T = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ -1 & 0 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ -1 & 0 & -1 \end{bmatrix} = \begin{bmatrix} -1 & 1 & 0 \\ 1 & -1 & 1 \end{bmatrix} = B_l$$

R.H.S. = L.H.S.

(*iii*) $C_b = -B_l^T$

$$B_l = \begin{bmatrix} -1 & 1 & 0 \\ 1 & -1 & 1 \end{bmatrix}$$

$$-B_l^T = \begin{bmatrix} 1 & 1 \\ -1 & 1 \\ 0 & 1 \end{bmatrix} = C_b.$$

(*iv*) $C'(B')^T = U$

$$C' = \begin{bmatrix} 1 & 0 & 0 & 1 & -1 \\ 0 & 1 & 0 & -1 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$(B')^T = \begin{bmatrix} 1 & 0 & 0 & -1 & 1 \\ 0 & 1 & 0 & 1 & -1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$C'(B')^T = \begin{bmatrix} 1 & 0 & 0 & 1 & -1 \\ 0 & 1 & 0 & -1 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -1 & 1 \\ 0 & 1 & 0 & 1 & -1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} = U = \text{Unit matrix.}$$

Problem 1.6. *The incidence matrix is given below. From that draw the oriented graph.*

Branches →

nodes ↓

$A^T =$

	1	2	3	4	5	6	7	8
1	1	0	0	0	1	0	0	1
2	0	1	0	0	−1	1	0	0
3	0	0	1	0	0	−1	1	−1
4	0	0	0	1	0	0	−1	0

Sol.

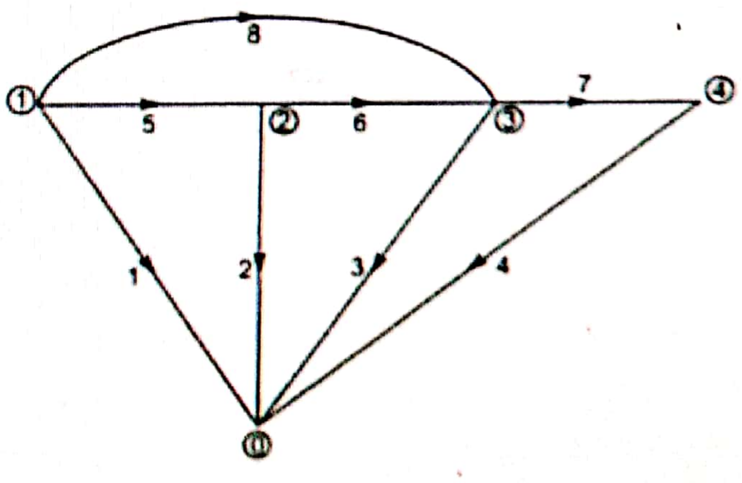

Fig. 1.22

Problem 1.7. *For figure 1.23, the impedance data is given in table 1.1. All the impedance values are in p.u. Values. Determine Y_{Bus} matrices by Singular Transformation method.*

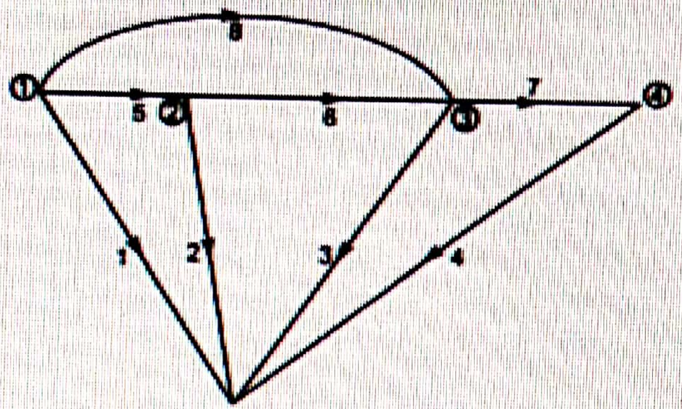

Fig. 1.23

Table 1.1

Element	*Bus Code*	*Self Impedance In p.u*
1	0 – 1	0.1
2	0 – 2	0.2
3	0 – 3	0.25
4	0 – 4	0.5
5	1 – 2	0.1
6	2 – 3	0.4
7	3 – 4	0.3
8	1 – 3	0.6

Sol. From figure 1.23, the Bus incidence matrix A is

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 \\ 1 & 0 & -1 & 0 \end{bmatrix}$$

Primitive impedance matrix is

$$z = \begin{array}{c|cccccccc} \downarrow \text{elements} \; \longrightarrow & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ \hline 1 & 0.1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & 0 & 0.2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 3 & 0 & 0 & 0.25 & 0 & 0 & 0 & 0 & 0 \\ 4 & 0 & 0 & 0 & 0.5 & 0 & 0 & 0 & 0 \\ 5 & 0 & 0 & 0 & 0 & 0.1 & 0 & 0 & 0 \\ 6 & 0 & 0 & 0 & 0 & 0 & 0.4 & 0 & 0 \\ 7 & 0 & 0 & 0 & 0 & 0 & 0 & 0.3 & 0 \\ 8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.6 \end{array}$$

Primitive admittance is given by

$$y = [z]^{-1}$$

$$y = \begin{bmatrix} 0.1 & 0 & 0 & & & & & \\ 0 & 0.2 & 0 & & & & & \\ 0 & 0 & 0.25 & & & & & \\ 0 & 0 & 0 & 0.5 & & & & \\ 0 & 0 & 0 & 0 & 0.1 & & & \\ 0 & 0 & 0 & 0 & 0 & 0.4 & & \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.3 & \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.6 \end{bmatrix}^{-1}$$

$$\therefore \quad y = \begin{bmatrix} 10 & & & & & & & \\ 0 & 5 & & & & & & \\ 0 & 0 & 4 & & & & & \\ 0 & 0 & 0 & 2 & & & & \\ 0 & 0 & 0 & 0 & 10 & & & \\ 0 & 0 & 0 & 0 & 0 & 2.5 & & \\ 0 & 0 & 0 & 0 & 0 & 0 & 6.7 & \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3.33 \end{bmatrix}$$

$$Y_{Bus} = [A^T]\,[y]\,[A]$$

$$[y]\,[A] = \begin{bmatrix} 10 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 10 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2.5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 6.7 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3.33 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 \\ 1 & 0 & -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 10 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 \\ 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 2 \\ 10 & -10 & 0 & 0 \\ 0 & +2.5 & -2.5 & 0 \\ 0 & 0 & 6.67 & -6.67 \\ +3.33 & 0 & -3.33 & 0 \end{bmatrix}$$

$$[A^T]\,[y]\,[A] = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 & 1 & -1 \\ 0 & 0 & 0 & 1 & 0 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} 10 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 \\ 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 2 \\ 10 & -10 & 0 & 0 \\ 0 & 2.5 & -2.5 & 0 \\ 0 & 0 & 6.67 & -6.67 \\ 3.33 & 0 & -3.33 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} (10+10+3.33) & -10 & -3.33 & 0 \\ -10 & (5+10+2.5) & -2.5 & 0 \\ -33 & -2.5 & (4+2.5+6.67+3.33) & -6.67 \\ 0 & 0 & -6.67 & (2+6.67) \end{bmatrix}$$

$$\therefore \qquad Y_{Bus} = \begin{bmatrix} 20.33 & -10 & -3.33 & 0 \\ -10 & 17.5 & -2.5 & 0 \\ -3.33 & -2.5 & 16.5 & -6.67 \\ 0 & 0 & -6.67 & 8.67 \end{bmatrix}$$

Problem 1.8. *For the system shown in figure 1.24, construct Y_{Bus} by singular transformation method. The parameters of various elements are given in table 1.2. Take node '6' as refereneće node.*

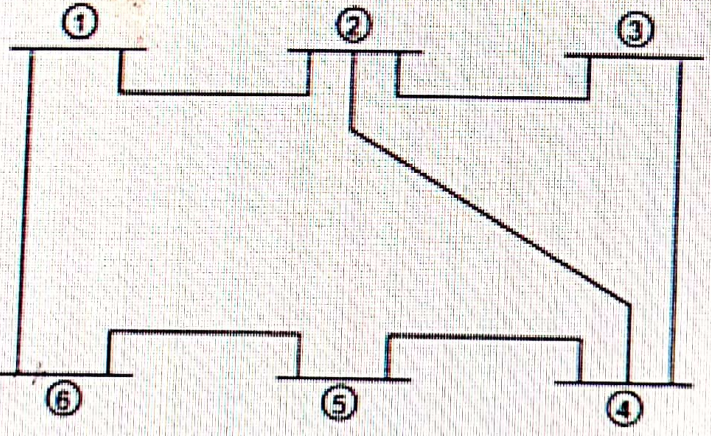

Fig. 1.24

Table 1.2

Element	*Reactances in p.u*
1 — 2	0.04
1 — 6	0.02
2 — 4	0.03
2 — 3	0.02
3 — 4	0.08
4 — 5	0.06
5 — 6	0.05

Sol.

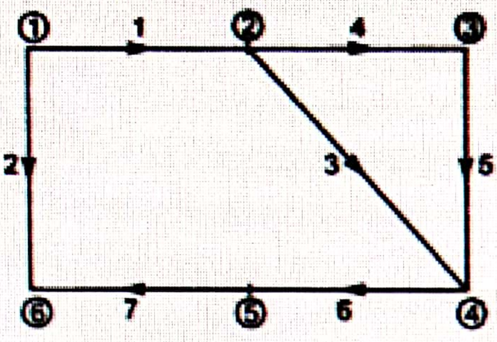

Fig. 1.24 (a) *Oriented graph*

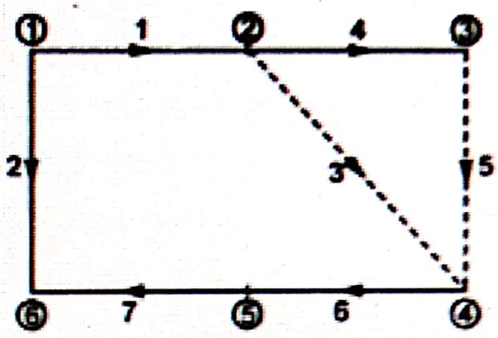

Fig. 1.24 (b) *Tree*

Element node incidence matrix (A′)

A′ =

e \ n	1	2	3	4	5	6
1	1	−1	0	0	0	0
2	1	0	0	0	0	−1
3	0	1	0	−1	0	0
4	0	1	−1	0	0	0
5	0	0	1	−1	0	0
6	0	0	0	1	−1	0
7	0	0	0	0	1	−1

From the element node incidence matrix bus incidence matrix can be determined, *i.e.*, by deleting the row corresponding to reference node (6).

Bus incidence matrix

A =

e \ n	1	2	3	4	5
1	1	−1	0	0	0
2	1	0	0	0	0
3	0	1	0	−1	0
4	0	1	−1	0	0
5	0	0	1	−1	0
6	0	0	0	1	−1
7	0	0	0	0	1

The bus incidence matrix (A) is rearranged by separating branches and links as follows :

A =

e \ n	1	2	3	4	5	
1	1	−1	0	0	0	A_b (Branch)
2	1	0	0	0	0	
4	0	1	−1	0	0	
6	0	0	0	1	−1	
7	0	0	0	0	1	
3	0	1	0	−1	0	A_l (Link)
5	0	0	1	−1	0	

Primitive impedance matrix is

$$z = \begin{bmatrix} j0.04 & & & & & & \\ 0 & j0.06 & & & & & \\ 0 & 0 & j0.03 & & & & \\ 0 & 0 & 0 & j0.02 & & & \\ 0 & 0 & 0 & 0 & j0.08 & & \\ 0 & 0 & 0 & 0 & 0 & j0.06 & \\ 0 & 0 & 0 & 0 & 0 & 0 & j0.05 \end{bmatrix}$$

Primitive admittance matrix is

$$y = [z]^{-1}$$

$$y = \begin{bmatrix} -j25 & & & & & & \\ 0 & -j16.67 & & & & & \\ 0 & 0 & -j33.39 & & & & \\ 0 & 0 & 0 & -j50 & & & \\ 0 & 0 & 0 & 0 & -j12.5 & & \\ 0 & 0 & 0 & 0 & 0 & -j16.67 & \\ 0 & 0 & 0 & 0 & 0 & 0 & -j20 \end{bmatrix}$$

$$Y_{Bus} = [A^T]\,[y]\,[A]$$

$$[A]^T\,[y] = -j \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} 25 & & & & & & \\ 0 & 16.67 & & & & & \\ 0 & 0 & 33.33 & & & & \\ 0 & 0 & 0 & 50 & & & \\ 0 & 0 & 0 & 0 & 12.5 & & \\ 0 & 0 & 0 & 0 & 0 & 16.67 & \\ 0 & 0 & 0 & 0 & 0 & 0 & 20 \end{bmatrix}$$

$$= -j \begin{bmatrix} 25 & 16.67 & 0 & 0 & 0 & 0 & 0 \\ -25 & 0 & 33.33 & 50 & 0 & 0 & 0 \\ 0 & 0 & 0 & -50 & 12.5 & 0 & 0 \\ 0 & 0 & -33.33 & 0 & -12.5 & 16.67 & 0 \\ 0 & 0 & 0 & 0 & 0 & -16.67 & 20 \end{bmatrix}$$

$$[A^T]\,[y]\,[A] = -j \begin{bmatrix} 25 & 16.67 & 0 & 0 & 0 & 0 & 0 \\ -25 & 0 & 33.33 & 50 & 0 & 0 & 0 \\ 0 & 0 & 0 & -50 & 12.5 & 0 & 0 \\ 0 & 0 & -33.33 & 0 & -12.5 & 16.67 & 0 \\ 0 & 0 & 0 & 0 & 0 & -16.67 & 20 \end{bmatrix} \begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$Y_{Bus} = -j\begin{bmatrix} (25+16.67) & -25 & 0 & 0 & 0 \\ -25 & (25+33.33+50) & 0 & 0 & 0 \\ 0 & -50 & (50+12.5) & 0 & 0 \\ 0 & -33.33 & -12.5 & (33.33+12.5+16.67) & -16.67 \\ 0 & 0 & 0 & -16.67 & (16.67+20) \end{bmatrix}$$

$$\therefore \; Y_{Bus} = -j\begin{bmatrix} 41.67 & -25 & 0 & 0 & 0 \\ -25 & 108.33 & -50 & -33.33 & 0 \\ 0 & -50 & 62.5 & -12.5 & 0 \\ 0 & -33.33 & -12.5 & 62.5 & -16.67 \\ 0 & 0 & 0 & -16.67 & 36.67 \end{bmatrix}$$

Problem 1.9. *Determine the Y_{Bus} matrix by singular transformation method for the network shown in figure 1.25. The parameters are given in table 1.3.*

Table 1.3

Element	*Self Impedance in p.u*		*Mutual Impedance in p.u*	
	Bus Code $p-q$	*Impedance* $Zpq-pq$	*Bus Code* $r-s$	*Impedance* $Zpq-rs$
1	*1 – 2 (1)*	*0.2*	—	—
2	*1 – 3*	*0.4*	*1-2 (1)*	*0.05*
3	*3 – 4*	*0.5*	—	—
4	*1 – 2 (2)*	*0.25*	*1-2 (1)*	*0.1*
5	*2 – 4*	*0.2*	—	—

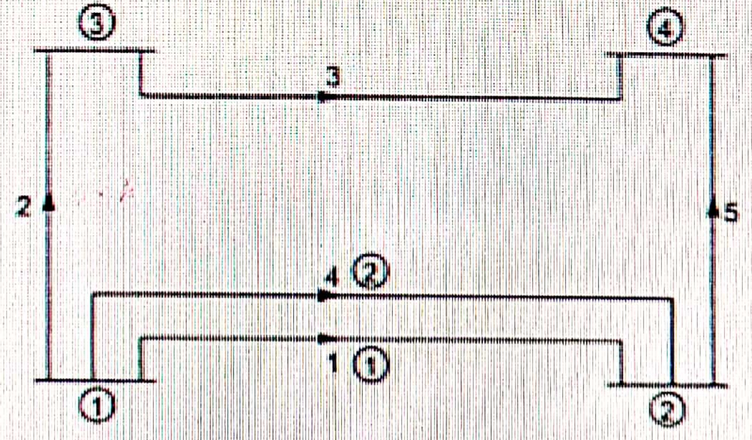

Fig. 1.25

Sol. Number of elements, $e = 5$

Number of busses $\quad b = n - 1 = 4 - 1 = 3$

Number of links, $\quad l = e - n + 1 = 5 - 4 + 1 = 2$

Element-node incidence matrix (A')

$$A' = \begin{array}{c|cccc} e \backslash n & ① & ② & ③ & ④ \\ \hline 1 & 1 & -1 & 0 & 0 \\ 2 & 1 & 0 & -1 & 0 \\ 3 & 0 & 0 & 1 & -1 \\ 4 & 1 & -1 & 0 & 0 \\ 5 & 0 & 1 & 0 & -1 \end{array}$$

Bus-incidence matrix (A)

$$A = \begin{array}{c|ccc} e \backslash n & ② & ③ & ④ \\ \hline 1 & -1 & 0 & 0 \\ 2 & 0 & -1 & 0 \\ 3 & 0 & 1 & -1 \\ 4 & -1 & 0 & 0 \\ 5 & 1 & 0 & -1 \end{array}$$

(∵ ① is reference node)

$$A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 0 \\ 1 & 0 & -1 \end{bmatrix}$$

Primitive impedance matrix [z]

$$z = \begin{array}{c} e/e \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 0.2 & 0.05 & 0 & 0.1 & 0 \\ 0.05 & 0.4 & 0 & 0 & 0 \\ 0 & 0 & 0.5 & 0 & 0 \\ 0.1 & 0 & 0 & 0.25 & 0 \\ 0 & 0 & 0 & 0 & 0.2 \end{bmatrix} \end{array}$$

Primitive admittance matrix $y = [z]^{-1}$

$$y = [z]^{-1} = \begin{bmatrix} 0.2 & 0.05 & 0 & 0.1 & 0 \\ 0.05 & 0.4 & 0 & 0 & 0 \\ 0 & 0 & 0.5 & 0 & 0 \\ 0.1 & 0 & 0 & 0.25 & 0 \\ 0 & 0 & 0 & 0 & 0.2 \end{bmatrix}^{-1}$$

To calculate z^{-1} there are two methods :

Method 1 :

$$y = [z]^{-1} = \begin{bmatrix} 0.2 & 0.05 & 0 & 0.1 & 0 \\ 0.05 & 0.4 & 0 & 0 & 0 \\ 0 & 0 & 0.5 & 0 & 0 \\ 0.1 & 0 & 0 & 0.25 & 0 \\ 0 & 0 & 0 & 0 & 0.2 \end{bmatrix}^{-1}$$

Step 1. Interchange row 3 and 4

$$y' = \begin{bmatrix} 0.2 & 0.05 & 0 & 0.1 & 0 \\ 0.05 & 0.4 & 0 & 0 & 0 \\ 0.1 & 0 & 0 & 0.25 & 0 \\ 0 & 0 & 0.5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.2 \end{bmatrix}^{-1}$$

Step 2. Interchange column 3 and 4

$$y'' = \left[\begin{array}{ccc:cc} 0.2 & 0.05 & 0.1 & 0 & 0 \\ 0.05 & 0.4 & 0 & 0 & 0 \\ 0.1 & 0 & 0.25 & 0 & 0 \\ \hdashline 0 & 0 & 0 & 0.5 & 0 \\ 0 & 0 & 0 & 0 & 0.2 \end{array}\right]^{-1} = \left[\begin{array}{c:c} A_1 & A_2 \\ \hdashline A_3 & A_4 \end{array}\right]$$

Step 3. Above matrix can divide into 4 submatrices

$$A_1 = \begin{bmatrix} 0.2 & 0.05 & 0.1 \\ 0.05 & 0.4 & 0 \\ 0.1 & 0 & 0.25 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \quad A_3 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$A_4 = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.2 \end{bmatrix}$$

$$A_1^{-1} = \begin{bmatrix} 6.5 & -0.81 & -2.6 \\ -0.81 & 2.6 & 0.33 \\ -2.6 & 0.33 & 5.0 \end{bmatrix}$$

$$A_4^{-1} = \begin{bmatrix} 2 & 0 \\ 0 & 5 \end{bmatrix}$$

Step 4.

$$y' = \left[\begin{array}{c:c} A_1^{-1} & A_2 \\ \hdashline A_3 & A_4^{-1} \end{array}\right]$$

$$y' = \begin{bmatrix} 6.5 & -0.81 & -2.6 & 0 & 0 \\ -0.81 & 2.6 & 0.33 & 0 & 0 \\ -2.6 & 0.33 & 5 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 5 \end{bmatrix}$$

Step 5. Interchange column 3 and 4

$$y' = \begin{bmatrix} 6.5 & -0.81 & 0 & -2.6 & 0 \\ -0.81 & 2.6 & 0 & 0.33 & 0 \\ -2.6 & 0.33 & 0 & 5 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 5 \end{bmatrix}$$

Step 6. Interchange row 3 and 4

$$y = [z^{-1}] = \begin{bmatrix} 6.5 & -0.81 & 0 & -2.6 & 0 \\ 0.81 & 2.6 & 0 & 0.33 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ -2.6 & 0.33 & 0 & 5 & 0 \\ 0 & 0 & 0 & 0 & 5 \end{bmatrix}$$

Limitations : This method is applicable only the matrices A_2 and A_3 are having the elements zero's only. Otherwise it is not applicable.

Method 2 :

$$y = [z^{-1}] = \left[\begin{array}{ccc:cc} 0.2 & 0.05 & 0 & 0.1 & 0 \\ 0.05 & 0.4 & 0 & 0 & 0 \\ 0 & 0 & 0.5 & 0 & 0 \\ \hdashline 0.1 & 0 & 0 & 0.25 & 0 \\ 0 & 0 & 0 & 0 & 0.2 \end{array}\right]^{-1} = \left[\begin{array}{c:c} A_1 & A_2 \\ \hdashline A_3 & A_4 \end{array}\right]^{-1}$$

Let

$$\begin{bmatrix} A_1 & A_2 \\ A_3 & A_4 \end{bmatrix}^{-1} = \begin{bmatrix} B_1 & B_2 \\ B_3 & B_4 \end{bmatrix}$$

The values of matrices B_1, B_2, B_3 and B_4 are

$$B_1 = [A_1 - (A_2 A_4^{-1} A_3)]^{-1}$$

$$A_4^{-1} = \begin{bmatrix} 0.25 & 0 \\ 0 & 0.2 \end{bmatrix}^{-1} = \begin{bmatrix} 4 & 0 \\ 0 & 5 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} 0.1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$A_2 A_4^{-1} A_3 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 4 & 0 \\ 0 & 5 \end{bmatrix} \begin{bmatrix} 0.1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0.4 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0.1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0.04 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$[A_1 - (A_2 A_4^{-1} A_3)]^{-1} = \left\{ \begin{bmatrix} 0.2 & 0.05 & 0 \\ 0.05 & 0.4 & 0 \\ 0 & 0 & 0.5 \end{bmatrix} - \begin{bmatrix} 0.04 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \right\}^{-1}$$

$$= \begin{bmatrix} 0.16 & 0.05 & 0 \\ 0.05 & 0.4 & 0 \\ 0 & 0 & 0.5 \end{bmatrix}^{-1}$$

$$B_1 = \begin{bmatrix} 6.5 & -0.81 & 0 \\ -0.81 & 2.6 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$B_2 = -B_1 A_2 A_4^{-1}$$

$$= \begin{bmatrix} -6.5 & 0.81 & 0 \\ 0.81 & -2.6 & 0 \\ 0 & 0 & -2 \end{bmatrix} \begin{bmatrix} 0.1 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 4 & 0 \\ 0 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} -6.5 & 0.81 & 0 \\ 0.81 & -2.6 & 0 \\ 0 & 0 & -2 \end{bmatrix} \begin{bmatrix} 0.4 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$B_2 = \begin{bmatrix} -2.6 & 0 \\ 0.324 & 0 \\ 0 & 0 \end{bmatrix}$$

$$B_3 = B_2^T = \begin{bmatrix} -2.6 & 0.324 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$B_4 = A_4^{-1} - (A_4^{-1} A_3 B_2)$$

$$A_4^{-1} A_3 B_2 = \begin{bmatrix} 4 & 0 \\ 0 & 5 \end{bmatrix} \begin{bmatrix} 0.1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} -2.6 & 0 \\ 0.324 & 0 \\ 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0.4 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} -2.6 & 0 \\ 0.324 & 0 \\ 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -1.04 & 0 \\ 0 & 0 \end{bmatrix}$$

$$B_4 = A_4^{-1} - (A_4^{-1} A_3 B_2)$$

$$= \begin{bmatrix} 4 & 0 \\ 0 & 5 \end{bmatrix} - \begin{bmatrix} -1.04 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 5.04 & 0 \\ 0 & 5 \end{bmatrix}$$

$$\therefore \qquad y = z^{-1} = \begin{bmatrix} B_1 & B_2 \\ B_3 & B_4 \end{bmatrix} = \begin{bmatrix} 6.5 & -0.81 & 0 & -2.6 & 0 \\ -0.81 & 2.6 & 0 & 0.324 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ -2.6 & 0.324 & 0 & 5.04 & 0 \\ 0 & 0 & 0 & 0 & 5 \end{bmatrix}$$

Next after calculating primitive admittance matrix.

$$Y_{Bus} = [A^T]\,[y]\,[A]$$

$$Y_{Bus} = \begin{bmatrix} -1 & 0 & 0 & -1 & 1 \\ 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & -1 \end{bmatrix} \begin{bmatrix} 0.6 & -0.81 & 0 & -2.6 & 0 \\ -0.81 & 2.6 & 0 & 0.33 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ -2.6 & 0.33 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 5 \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 0 \\ 1 & 0 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} -0.6+2.6 & 0.81-0.33 & 0 & 2.6 & 5 \\ 0.81 & -2.6 & 2 & -0.33 & 0 \\ 0 & 0 & -2 & 0 & -5 \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 0 \\ 1 & 0 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 0.48 & 0 & 2.6 & 5 \\ 0.81 & -2.6 & 2 & -0.33 & 0 \\ 0 & 0 & -2 & 0 & -5 \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 0 \\ 1 & 0 & -1 \end{bmatrix}$$

$$Y_{Bus} = \begin{bmatrix} 0.4 & -0.48 & -5 \\ -0.48 & 4.6 & -2 \\ -5 & -2 & 7 \end{bmatrix}$$

$$Z_{Bus} = [Y_{Bus}]^{-1} = \{[A^T]\,[y]\,[A]\}^{-1}$$

$$= \begin{bmatrix} 0.4 & -0.48 & -5 \\ -0.48 & 4.6 & -2 \\ -5 & -2 & 7 \end{bmatrix}$$

$$Z_{Bus} = \begin{bmatrix} -0.25 & -0.116 & -0.208 \\ -0.116 & -0.193 & -0.028 \\ -0.208 & -0.028 & -0.014 \end{bmatrix}$$

Problem 1.10. *Determine Y_{Bus} matrix for problem 1.7 by direct inspection method.*

Sol.

Diagonal elements

$$Y_{11} = \frac{1}{0.1} + \frac{1}{0.1} + \frac{1}{0.6} = 10 + 10 + 1.67 = 21.67$$

$$Y_{22} = \frac{1}{0.2} + \frac{1}{0.1} + \frac{1}{0.4} = 5 + 10 + 2.5 = 17.5$$

$$Y_{33} = \frac{1}{0.25} + \frac{1}{0.4} + \frac{1}{0.3} = 4 + 2.5 + 3.33 = 9.83$$

$$Y_{44} = \frac{1}{0.5} + \frac{1}{0.3} = 2 + 3.33 = 5.33.$$

Off-diagonal elements

$$Y_{12} = Y_{21} = -\frac{1}{0.1} = -10$$

$$Y_{13} = Y_{31} = -\frac{1}{0.6} = -1.67$$

$$Y_{14} = Y_{41} = 0$$

$$Y_{23} = Y_{32} = -\frac{1}{0.4} = -2.5$$

$$Y_{24} = Y_{42} = 0$$

$$Y_{34} = Y_{43} = -\frac{1}{0.3} = -3.33$$

$$Y_{Bus} = \begin{bmatrix} 21.67 & -10 & -1.67 & 0 \\ -10 & 17.5 & -2.5 & 0 \\ -1.67 & -2.5 & 9.83 & -3.33 \\ 0 & 0 & -3.33 & 5.33 \end{bmatrix}$$

Problem 1.11. *The network shown in figure 1.26. Draw graph and tree. Determine the Y_{Bus} by direct inspection method and verify Y_{Bus} by nodal equation analysis. The admittance values are given in p.u quantities.*

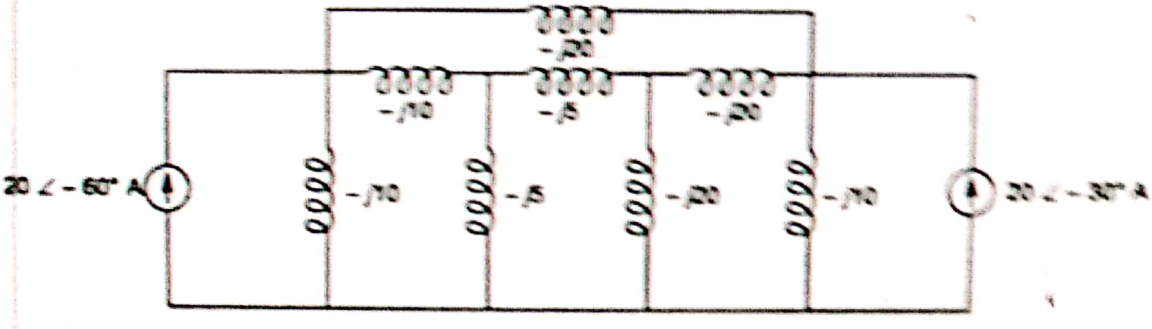

Fig. 1.26

Sol.

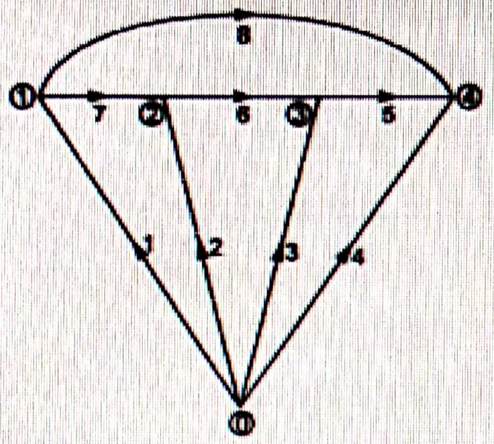

Fig. 1.26 (a) *Graph*

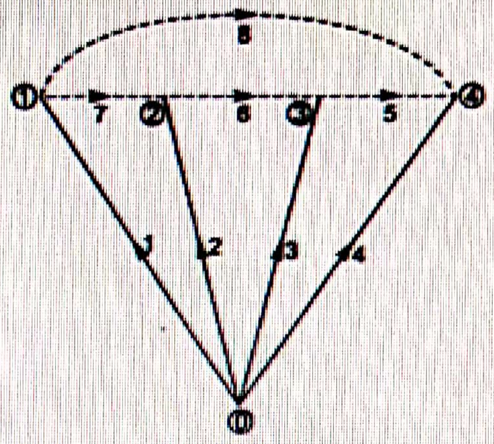

Fig. 1.26 (b) *Tree*

By Direct Inspection Method

Diagonal elements

$$Y_{11} = -j10 + (-j10) + (-j20) = -j40$$
$$Y_{22} = -j5 + (-j5) + (-j10) = -j20$$
$$Y_{33} = -j5 + (-j20) + (-j20) = -j45$$
$$Y_{44} = -j20 + (-j20) + (-j10) = -j50$$

Off-diagonal elements

$$Y_{21} = Y_{12} = -(-j10) = +j10$$
$$Y_{13} = Y_{31} = 0$$
$$Y_{14} = Y_{41} = -(-j20) = j20$$
$$Y_{23} = Y_{32} = -(-j5) = j5$$
$$Y_{24} = Y_{42} = 0$$
$$Y_{34} = Y_{43} = -(-j20) = j20$$

$$\therefore \quad Y_{Bus} = \begin{bmatrix} -j40 & j10 & 0 & j20 \\ j10 & -j20 & j5 & 0 \\ 0 & j5 & -j45 & j20 \\ j20 & 0 & j20 & -j50 \end{bmatrix}$$

Verification by Nodal Analysis Method

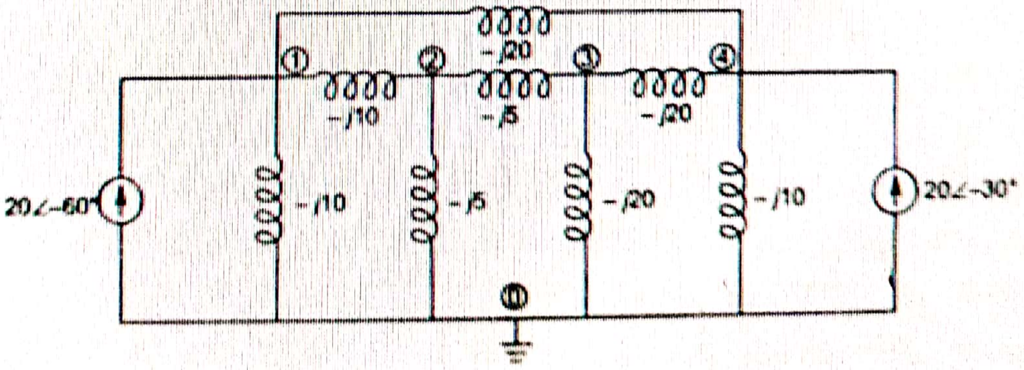

Fig. 1.26 (c)

Let voltages at node ① = V_1

node ② = V_2

node ③ = V_3

node ④ = V_4

Writing nodal equations for figure 1.26 (*c*)

At node ①

$$(V_1 - 0)(-j10) + (V_1 - V_2)(-j10) + (V_1 - V_4)(-j20) = 20\angle -60^\circ$$

$$V_1[-j10 - j10 - j20) + V_2(j10) + V_4(j20) = 20\angle -60^\circ$$

$$-j40\,V_1 + j10\,V_2 + j20\,V_4 = 20\angle -60^\circ \qquad ...(1.85)$$

At node ②

$$(V_2 - V_1)(-j10) + (V_2 - V_3)(-j5) + (V_2 - 0)(-j5) = 0$$

$$j10\,V_1 + (-j10 - j5 - j5)\,V_2 + j_5\,V_3$$

$$j10\,V_1 + (-j20)\,V_2 + j5\,V_3 = 0 \qquad ...(1.86)$$

At node ③

$$(V_3 - V_2)(-j5) + (V_3 - 0)(-j20) + (V_3 - V_4)(-j20) = 0$$

$$j5\,V_2 + (-j5 - j20 - j20)\,V_3 + j20\,V_4 = 0$$

$$j5\,V_2 + (-j45)\,V_3 + j20\,V_4 = 0 \qquad ...(1.87)$$

At node ④

$$V_2(-j10) + (V_4 - V_1)(-j20) + (V_4 - V_3)(-j20) = 20\angle -30^\circ$$

$$+j20\,V_1 + j20\,V_3 + (-j10 - j20 - j20)\,V_4 = 20\angle -30^\circ$$

$$+j20\,V_1 + j20\,V_3 + (-j50)\,V_4 = 20\angle -30^\circ \qquad ...(1.88)$$

Writing the equations (1.85), (1.86), (1.87) and (1.88) in matrix form.

$$\begin{bmatrix} 20\angle -60^\circ \\ 0 \\ 0 \\ 20\angle -30^\circ \end{bmatrix} = \begin{bmatrix} -j40 & j10 & 0 & j20 \\ j10 & -j20 & j5 & 0 \\ 0 & j5 & -j45 & j20 \\ j20 & 0 & j20 & -j50 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix}$$

$$[I_{Bus}] = [Y_{Bus}]\,[V_{Bus}]$$

$$\therefore \quad Y_{Bus} = \begin{bmatrix} -j40 & j10 & 0 & j20 \\ j10 & -j20 & j5 & 0 \\ 0 & j5 & -j45 & j20 \\ j20 & 0 & j20 & -j50 \end{bmatrix}$$