POWER SYSTEM ANALYSIS

UNIT – III

POWER FLOW STUDIES - I

Necessity of Power Flow Studies – Data for Power Flow Studies – Derivation of Static load flow equations – Load flow solutions using Gauss Seidel Method: Acceleration Factor, Load flow solution with and without P-V buses, Algorithm and Flowchart. Numerical Load flow Solution for Simple Power Systems (Max. 3-Buses): Determination of Bus Voltages, Injected Active and Reactive Powers (Sample One Iteration only) and finding Line Flows/Losses for the given Bus Voltages.

Necessity of Power Flow Studies

Power flow studies are carried out in order to keep an eye on the current state performance of the power system and also to plan out for the future expansion of the system to meet the increased load demand. The load flow studies help in determining the size and location of the power capacitors that are most favorable. They also help in solving the economic load dispatch problems by dispatching the loads between plants such that the total input cost is less. By carrying out the load flow studies overloaded conditions and poor voltages existing in parts of the power system can be detected. They also help in minimizing the effects of disturbances which can cause the failure of the system. The voltage level of certain buses can be maintained within closed tolerances with the help of data provided with the load flow studies. They also take a major part in the analysis of the stability of any power system.

Data for Power Flow Studies

Load flow studies or power flow studies is nothing but the steady state solution of the power system network. The data obtained from the load flow studies are the magnitude and phase angle of bus voltages, active and reactive power flow in transmission lines. In a power system network, buses are meeting points of various components. The generators feed energy to buses and the loads draw energy from the buses and linked with each bus are four quantities.

(i) Real Power, P
(ii) Reactive Power, Q
(iii) Voltage Magnitude, V
(iv) Phase angle of the voltage, δ

At any bus two out of these four quantities are specified giving rise to 3 types of buses.

1. PQ Bus: PQ Bus is also known as load bus. At this bus the real power (P) and reactive power (Q) are specified and the magnitude and phase angle of the voltage (V and δ) are to determined by solving load flow equations. Most number of buses in a power system is PQ buses (i.e., upto 90% of the total buses).

2. PV Bus: PV Bus is also known as generator bus or voltage controlled bus. At this bus the real power (P) and the voltage magnitude (V) are specified and the reactive power (Q) and phase angle of the voltage (δ) are to be determined by solving load flow equations. These buses are few in number and are located nearby a generator.

3. Slack Bus: Slack Bus is also known as the reference bus or swing bus. At this bus the magnitude and phase angle of the voltage (V and δ) are specified. In a power system network with 'n' number of buses any one bus is selected as reference bus or slack bus to provide the real and reactive power to supply the transmission losses.

System Variables

System variables are nothing but the quantities that are associated with a bus. The system variables are the magnitude and phase angle of voltage, real and reactive power generated, real and reactive power demand. The classification of these system variables that are involved in static load flow equations in performed into 3 groups.

1. State Variables: The magnitude and phase angle of the bus voltage ($|V_i|$ and δ_i) are termed as state variables. These variables are also known as dependent variables since they are dependent on the active and reactive power generated. Voltage magnitude changes with a change in reactive power generated and the phase angle changes with a change in the active power generated.

2. Input Variables: Input variables are the active and reactive power generated (P_{Gi} and Q_{Gi}). They are also known as control variables since the active and reactive power generated can be controlled by varying the turbine power and excitation respectively.

3. Output Variables: The active and reactive power demand variables (P_{Di} and Q_{Di}) are known as output variables. They are also known as disturbance variables or uncontrolled variables.

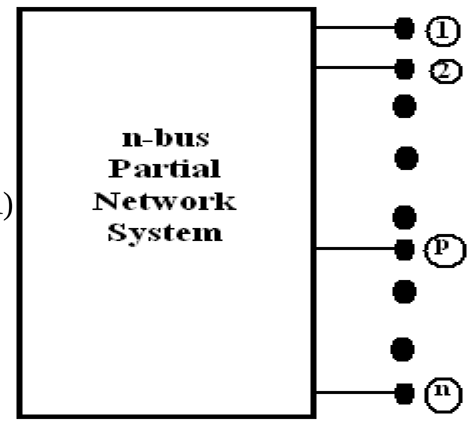
Static Load Flow Equations

Consider an n-bus system as shown in figure with n-bus voltages (V_1, V_2, …, V_n) and n-bus currents (I_1, I_2, …, I_n).

Applying nodal analysis to the system, we obtain

$$\begin{bmatrix} J_1 \\ J_2 \\ J_3 \\ . \\ . \\ . \\ J_n \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & . & . & . & Y_{1n} \\ Y_{21} & Y_{22} & Y_{23} & . & . & . & Y_{2n} \\ Y_{31} & Y_{32} & Y_{33} & . & . & . & Y_{3n} \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ Y_{n1} & Y_{n2} & Y_{n3} & . & . & . & Y_{nn} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ . \\ . \\ . \\ V_n \end{bmatrix} \quad \text{---- (A)}$$

$$\Rightarrow \overline{J_{BUS}} = \overline{Y_{BUS}}.\overline{v_{BUS}}$$



The net complex power injected by source into i^{th} bus is given by

$$S_i = P_i + jQ_i = V_i J_i^* \qquad \text{for i = 1, 2, 3, …., n ---- (1)}$$

where V_i is the voltage at the i^{th} bus with respect to ground and J_i is the source current injected into the bus.

The load flow problem is handled conveniently by use of J_i rather than J_i^*. So, taking the complex conjugate of equation (1), we have

$$P_i - jQ_i = V_i^* J_i \quad \text{for i = 1, 2, 3, … , n} \qquad \text{---- [1(a)]}$$

Considering bus 1,

Complex Power, $S_1 = P_1 + jQ_1 = V_1 I_1^*$

Taking complex conjugate of the above expression, we have

$$S_1^* = P_1 - jQ_1 = V_1^* I_1$$

Substituting the value of I_1 from equation (A) when 3 bus power system is taken into consideration,

$$P_1 - jQ_1 = V_1^* [Y_{11}V_1+Y_{12}V_2+Y_{13}V_3]$$

In polar form,

$V_i = |V_i|\, e^{j\delta_i}$

$Y_{ik} = |Y_{ik}|\, e^{j\theta_{ik}}$

So,

$$P_1 - jQ_1 = \left[|V_1|e^{j\delta_1}\right]^* \left[|Y_{11}|e^{j\theta_{11}}|V_1|e^{j\delta_1} + |Y_{12}|e^{j\theta_{12}}|V_2|e^{j\delta_2} + |Y_{13}|e^{j\theta_{13}}|V_3|e^{j\delta_3}\right]$$

$$P_1 - jQ_1 = |V_1|e^{-j\delta_1}\left[|V_1||Y_{11}|e^{j(\theta_{11}+\delta_1)} + |V_2||Y_{12}|e^{j(\theta_{12}+\delta_2)} + |V_3||Y_{13}|e^{j(\theta_{13}+\delta_3)}\right]$$

$$P_1 - jQ_1 = |V_1|\left[|V_1||Y_{11}|e^{j\theta_{11}} + |V_2||Y_{12}|e^{j(\theta_{12}+\delta_2-\delta_1)} + |V_3||Y_{13}|e^{j(\theta_{13}+\delta_3-\delta_1)}\right]$$

Equating real and imaginary parts,

$$P_1 = |V_1|\left[|V_1||Y_{11}|\cos\theta_{11} + |V_2||Y_{12}|\cos(\theta_{12}+\delta_2-\delta_1) + |V_3||Y_{13}|\cos(\theta_{13}+\delta_3-\delta_1)\right] \quad \text{---- (2)}$$

$$-Q_1 = |V_1|\left[|V_1||Y_{11}|\sin\theta_{11} + |V_2||Y_{12}|\sin(\theta_{12}+\delta_2-\delta_1) + |V_3||Y_{13}|\sin(\theta_{13}+\delta_3-\delta_1)\right] \quad \text{---- (3)}$$

Similarly, the real and reactive power injected into remaining buses can be found out.

The general expressions for real and reactive power can now be expressed from equations (2) and (3) as,

Real Power, $P_i = |V_i| \sum_{k=1}^{n} |V_k||Y_{ik}| \cos(\theta_{ik} + \delta_k - \delta_i)$ for i = 1, 2, 3, …., n ---- (4)

Reactive Power, $Q_i = -|V_i| \sum_{k=1}^{n} |V_k||Y_{ik}| \sin(\theta_{ik} + \delta_k - \delta_i)$ for i = 1, 2, 3, …., n ---- (5)

Equations (4) and (5) are non-linear algebraic equations (bus voltages are involved in product form and sine and cosine terms are present) and therefore solution is not possible. Solution can only be obtained by iterative numerical techniques.

At the cost of solution accuracy, it is possible to linearize load flow equations by making suitable assumptions and approximations so that solutions become possible. Such techniques have value particularly for planning studies, where load flow solutions have to be carried out repeatedly but a high degree of accuracy is not needed.

Approximate Load Flow Solution

Let us make the following assumptions and approximations in the load flow equations (4) and (5).

(i) Line resistance being small is neglected. (Shunt conductance of overhead lines is always negligible), i.e., P_L, the active power loss of the system is zero. Thus in equations (4) and (5) $\theta_{ik} \approx 90^0$ and $\theta_{ii} \approx -90^0$.

(ii) $(\delta_i - \delta_k)$ is small ($<\pi/6$) so that $\sin(\delta_i - \delta_k) \approx (\delta_i - \delta_k)$. This is justified from considerations of stability.

(iii) All buses other than the slack bus (numbered as bus 1) are PV buses, i.e., voltage magnitudes at all the buses including the slack bus are specified.

Equations

Real Power, $P_i = |V_i| \sum_{k=1}^{n} |V_k||Y_{ik}| \cos(\theta_{ik} + \delta_k - \delta_i)$ for i = 1, 2, 3, …., n

Reactive Power, $Q_i = -|V_i| \sum_{k=1}^{n} |V_k||Y_{ik}| \sin(\theta_{ik} + \delta_k - \delta_i)$ for i = 1, 2,…., n

then reduce to

$$P_i = |V_i| \sum_{k=1}^{n} |V_k||Y_{ik}| (\delta_i - \delta_k)$$ for i = 2, 3, …., n ----- (1)

$$Q_i = -|V_i| \sum_{\substack{k=1 \\ k \neq i}}^{n} |V_k||Y_{ik}| \cos(\delta_i - \delta_k) + |V_i|^2 |Y_{ii}|$$ for i = 1, 2,…., n ---- (2)

Since $|V_i|$s are specified, equation (1) represents a set of linear algebraic equations in δ_is which are (n-1) in number as δ_1 is specified at the slack bus ($\delta_1 = 0$).

Gauss-Siedal Method

The Gauss-Siedal (GS) method is an iterative algorithm for solving a set of non-linear algebraic equations. To start with, a solution vector is assumed, based on guidance from practical experience in a physical situation. One of the equations is then used to obtain the revised value of a particular variable by substituting in it the present values of the remaining variables. The solution vector is immediately updated in respect of this variable. The process is then repeated for all the variables thereby completing one iteration. The iterative process is then repeated till the solution vector converges within prescribed accuracy. The convergence is quite sensitive to the starting values assumed. Fortunately, in a load flow study a starting vector close to the final solution can be easily identified with previous experience.

To explain how the GS method is applied to obtain the load flow solution, let it be assumed that all buses other than the slack bus are PQ buses. The slack bus voltage being specified, there are (n - 1) bus voltages starting values of whose magnitudes and angles are assumed. These values are then updated through an iterative process. During the course of any iteration, the revised voltage at the ith bus is obtained as

$$J_i = (P_i - jQ_i)/V_i^* \quad \text{(from equation 1(a))} \qquad \text{---- (1)}$$

Generally, for any n-bus partial network, we know

$$\begin{bmatrix} J_1 \\ J_2 \\ J_3 \\ . \\ . \\ . \\ J_n \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & . & . & . & Y_{1n} \\ Y_{21} & Y_{22} & Y_{23} & . & . & . & Y_{2n} \\ Y_{31} & Y_{32} & Y_{33} & . & . & . & Y_{3n} \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ Y_{n1} & Y_{n2} & Y_{n3} & . & . & . & Y_{nn} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ . \\ . \\ . \\ V_n \end{bmatrix}$$

In compact form, $J_i = \sum_{k=1}^{n} Y_{ik} V_k$ for i = 1, 2, 3, …., n

$$V_i = \frac{1}{Y_{ii}} \left[J_i - \sum_{\substack{k=1 \\ k \neq i}}^{n} Y_{ik} V_k \right] \qquad \text{for i = 1, 2, 3, …., n} \qquad \text{---- (2)}$$

Substituting for J_i from equation (1) into (2),

$$V_i = \frac{1}{Y_{ii}} \left[\frac{P_i - jQ_i}{V_i^*} - \sum_{\substack{k=1 \\ k \neq i}}^{n} Y_{ik} V_k \right] \qquad \text{for i = 2, 3, …., n} \qquad \text{---- (3)}$$

The voltages substituted in the right hand side of equation are the most recently calculated (updated) values for the corresponding buses. During each iteration voltages at buses i = 2,3, ..., n are sequentially updated through use of the above equation. V_1, the slack bus voltage being fixed is not required to be updated. Iterations are repeated till no bus voltage magnitude changes by more than a prescribed value during iteration. The computation process is then said to converge to a solution.

If instead of updating voltages at every step of iteration, updating is carried out at the end of a complete iteration, the process is known as Gauss iteration method. It is much slower to converge and may sometimes fail to do so.

Algorithm: Consider the case where all the buses other than the slack are PQ buses. The steps of the algorithm are

1. With the load profile known at each bus (P_{Di} and Q_{Di}) allocate P_{Gi} and Q_{Gi} to all the generating stations. While active and reactive powers are allocated to the slack bus, these are permitted to vary during iterative computation.
   With this step, bus injections (P_i+jQ_i) are known at all the buses other than slack bus.
2. *Assembly of bus admittance matrix Y_{BUS}:* With the line and shunt admittance data, Y_{BUS} is formed. Alternatively Y_{BUS} is framed where input data are in the form of primitive matrix Y and singular connection matrix A.
3. *Iterative computation of bus voltages (V_i;i=1,2,3,…,n):* A set of initial voltage values is assumed. Initially all the voltages are set equal to (1+j0) except the voltage of the slack bus which is fixed. (n-1) equations in complex numbers are to solved iteratively for finding (n-1) bus voltages $V_2, V_3, \ldots, V_n$. If complex operations are not available in the computer then equation (3) is converted into real unknowns as

$$V_i = e_i + jf_i = |V_i|\, e^{j\delta_i}$$

Define, $A_i = \dfrac{P_i - jQ_i}{Y_{ii}}$ for i = 2, 3, …., n

$B_{ik} = \dfrac{Y_{ik}}{Y_{ii}}$ for i = 2, 3, …., n

for k = 1, 2, 3, …., n and k≠i

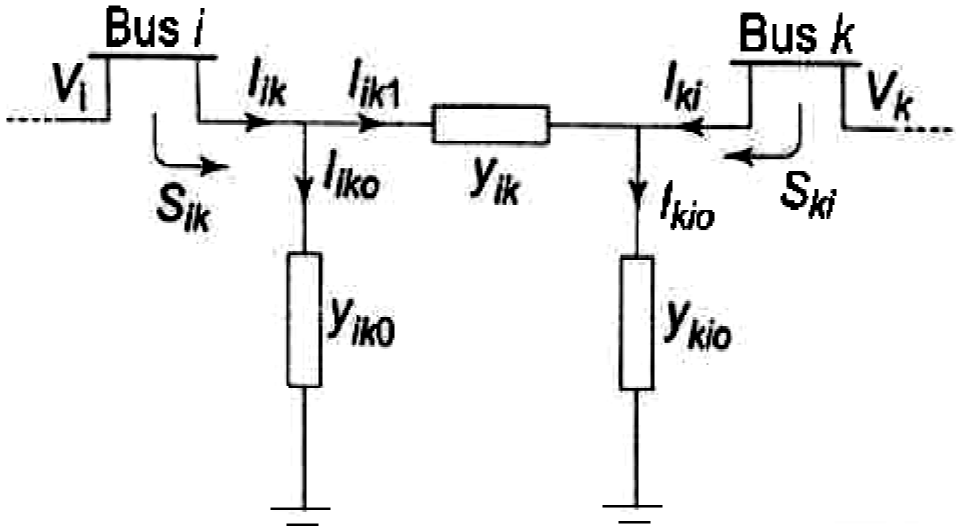
For (r+1)th iteration, equation (3) becomes

$$V_i^{(r+1)} = \frac{A_i}{\left(V_i^{(r)}\right)^*} - \sum_{k=1}^{i-1} B_{ik} V_k^{(r+1)} - \sum_{k=i+1}^{n} B_{ik} V_k^{(r)} \quad \text{for i = 2, 3, …., n} \quad \text{---- (4)}$$

The iteration process is continued till the change in magnitude of bus voltage, $|\Delta V_i^{(r+1)}|$, between two consecutive iterations is less than certain tolerance for all bus voltages i.e.,

$$|\Delta V_i^{(r+1)}| = |V_i^{(r+1)} - V_i^{(r)}| \quad \text{for i = 2, 3, …., n} \quad \text{---- (5)}$$

4. *Computation of slack bus voltage:* Substitution of all bus voltages computed in step 3 along with V_1 gives $S_1^* = P_1 - jQ_1$
5. *Computation of line flows:* The power flows on the various lines of the network are computed. Consider the line connecting buses i and k. The line and transformers at each end can be represented by a circuit with series admittance y_{ik} and two shunt admittances y_{ik0} and y_{ki0} as shown in the figure.



The current fed by bus i into the line is

$I_{ik} = I_{ik1} + I_{ik0} = (V_i – V_k)\, y_{ik} + V_i\, y_{ik0}$ ---- (6)

The power fed into the line from bus i is,

$S_{ik} = P_{ik} + jQ_{ik} = V_i\, I_{ik}^* = V_i(V_i^* – V_k^*)\, y_{ik}^* + V_i\, V_i^*\, y_{ik0}^*$ ---- (7)

Similarly, power fed into the line from bus k is,

$S_{ki} = P_{ki} + jQ_{ki} = V_k\, I_{ki}^* = V_k(V_k^* – V_i^*)\, y_{ki}^* + V_k\, V_k^*\, y_{ki0}^*$ ---- (8)

The power loss in the (i-k)th line is the sum of the power flows determined from equations (7) and (8). Total transmission loss can be computed by summing all the line flows ($S_{ik} + S_{ki}$ for all i, k).

Slack bus power can be found by summing the flows on the lines terminating at the slack bus.

Acceleration of Convergence: Convergence in the GS method can sometimes be speeded up by the use of acceleration factor. For the ith bus, the accelerated value of voltage at the (r+1)th iteration is given by

$$V_i^{(r+1)}{}_{(accelerated)} = V_i^{(r)} + \alpha\, (V_i^{(r+1)} – V_i^{(r)})$$

where α is a real number called the acceleration factor. A suitable value of α for any system can be obtained by trial load flow studies. A generally recommended value is α=1.6. A wrong choice of α may indeed slow down convergence or even cause the method to diverge which concludes the load flow analysis for the case of PQ buses only.

Algorithm Modification when PV Buses are also present: At the PV buses, P and |V| are specified and Q and δ are unknowns to be determined. Therefore, the values of Q and δ are to be updated in every GS iteration through appropriate bus equation which is accomplished in the following steps for the ith PV bus:

1. From equation

$$Q_i = -\mathrm{Im}\left\{V_i^* \sum_{k=1}^{n} Y_{ik} V_k\right\}$$

The revised value of Q_i is obtained from the above equation by substituting most updated values of voltages on the right hand side. In fact, for the (r+1)th iteration the above expression can be written as

$$Q_i^{(r+1)} = -\mathrm{Im}\left\{\left(V_i^{(r)}\right)^* \sum_{k=1}^{i-1} Y_{ik} V_k^{(r+1)} + \left(V_i^{(r)}\right)^* \sum_{k=i}^{n} Y_{ik} V_k^{(r)}\right\} \quad \text{---- (8)}$$

2. The revised value of δ_i is obtained from equation (4).

$$\delta_i^{(r+1)} = \angle V_i^{(r+1)}$$

$$= Angle\left[\frac{A_i^{(r+1)}}{\left(V_i^{(r)}\right)^*} - \sum_{k=1}^{i-1} B_{ik} V_k^{(r+1)} - \sum_{k=i+1}^{n} B_{ik} V_k^{(r)} \right] \quad \text{---- (9)}$$

where

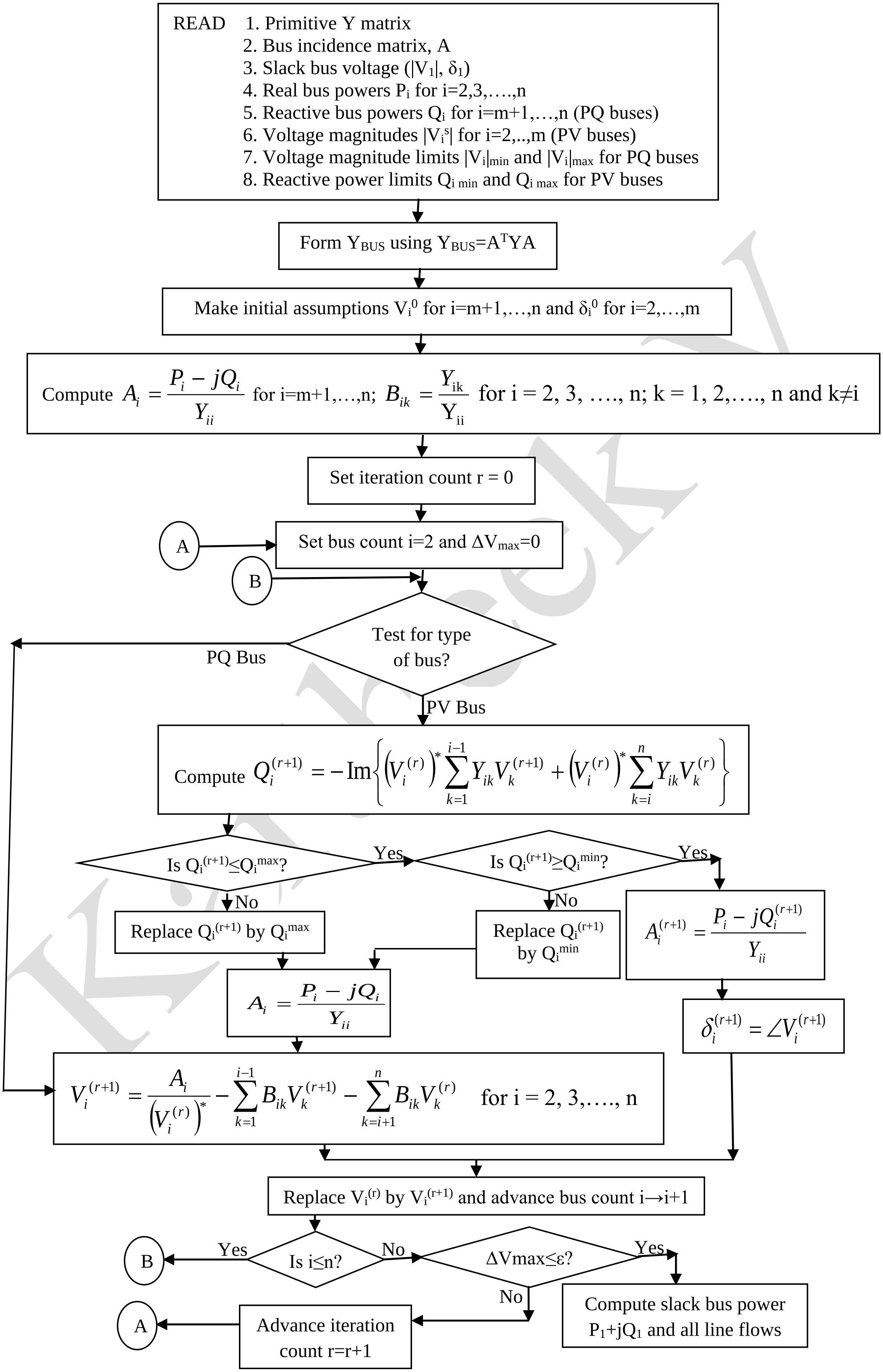
$$A_i^{(r+1)} = \frac{P_i - jQ_i^{(r+1)}}{Y_{ii}} \quad \text{---- (10)}$$

The algorithm for PQ buses remains unchanged.

Physical limitations of Q generation require that Q demand at any bus must be in the range Q_{min} - Q_{max}. If at any stage during the computation, Q at any bus goes outside these limits, it is fixed at Q_{min} or Q_{max} as the case may be, and the bus voltage specification is dropped i.e., the bus is now treated like a PQ bus.

3. If $Q_i^{(r+1)} < Q_{i,min}$, set $Q_i^{(r+1)}=Q_{i,min}$ and treat bus i as a PQ bus. Compute $A_i^{(r+1)}$ and $V_i^{(r+1)}$ from equations (10) and (4). If $Q_i^{(r+1)} > Q_{i,max}$, set $Q_i^{(r+1)}=Q_{i,max}$ and treat bus i as a PQ bus. Compute $A_i^{(r+1)}$ and $V_i^{(r+1)}$ from equations (10) and (4).

The computational steps are summarized in the detailed flowchart. It is assumed that out of n buses, the first is slack as usual, then 2, 3, …, m are PV buses and remaining m+1, …, n are PQ buses.


READ 1. Primitive Y matrix
2. Bus incidence matrix, A
3. Slack bus voltage ($|V_1|$, δ_1)
4. Real bus powers P_i for i=2,3,….,n
5. Reactive bus powers Q_i for i=m+1,…,n (PQ buses)
6. Voltage magnitudes $|V_i^s|$ for i=2,..,m (PV buses)
7. Voltage magnitude limits $|V_i|_{min}$ and $|V_i|_{max}$ for PQ buses
8. Reactive power limits $Q_{i\,min}$ and $Q_{i\,max}$ for PV buses
Form Y_{BUS} using $Y_{BUS}=A^TYA$
Make initial assumptions V_i^0 for i=m+1,…,n and δ_i^0 for i=2,…,m
Compute $A_i = \frac{P_i - jQ_i}{Y_{ii}}$ for i=m+1,…,n; $B_{ik} = \frac{Y_{ik}}{Y_{ii}}$ for i = 2, 3, …., n; k = 1, 2,…., n and k≠i
Set iteration count r = 0
A
Set bus count i=2 and $\Delta V_{max}=0$
B
Test for type of bus?
PQ Bus
PV Bus
Compute $Q_i^{(r+1)} = -\mathrm{Im}\left\{\left(V_i^{(r)}\right)^* \sum_{k=1}^{i-1} Y_{ik}V_k^{(r+1)} + \left(V_i^{(r)}\right)^* \sum_{k=i}^{n} Y_{ik}V_k^{(r)}\right\}$
Is $Q_i^{(r+1)} \le Q_i^{max}$?
Yes
Is $Q_i^{(r+1)} \ge Q_i^{min}$?
Yes
No
Replace $Q_i^{(r+1)}$ by Q_i^{max}
No
Replace $Q_i^{(r+1)}$ by Q_i^{min}
$A_i^{(r+1)} = \frac{P_i - jQ_i^{(r+1)}}{Y_{ii}}$
$A_i = \frac{P_i - jQ_i}{Y_{ii}}$
$\delta_i^{(r+1)} = \angle V_i^{(r+1)}$
$V_i^{(r+1)} = \frac{A_i}{\left(V_i^{(r)}\right)^*} - \sum_{k=1}^{i-1} B_{ik}V_k^{(r+1)} - \sum_{k=i+1}^{n} B_{ik}V_k^{(r)}$ for i = 2, 3,…., n
Replace $V_i^{(r)}$ by $V_i^{(r+1)}$ and advance bus count i→i+1
B
Yes
Is i≤n?
No
ΔVmax≤ε?
Yes
Compute slack bus power P_1+jQ_1 and all line flows
No
A
Advance iteration count r=r+1