EXAMPLE 1.14

For the network shown in fig 1.14.1. form the bus admittance matrix. Determine the reduced admittance matrix by eliminating node 4. The values are marked in p.u.

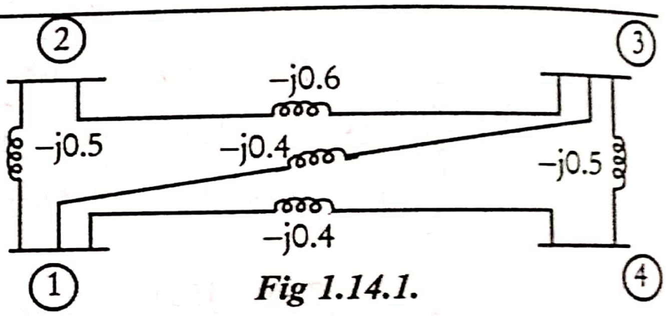


Fig 1.14.1.

SOLUTION

The Y_{bus} matrix of the network is,

$$Y_{bus} = \begin{bmatrix} -(j0.5+j0.4+j0.4) & j0.5 & j0.4 & j0.4 \\ j0.5 & -(j0.5+j0.6) & j0.6 & 0 \\ j0.4 & j0.6 & -(j0.6+j0.5+j0.4) & j0.5 \\ j0.4 & 0 & j0.5 & -(j0.5+j0.4) \end{bmatrix}$$

$$Y_{bus} = \begin{bmatrix} -j1.3 & j0.5 & j0.4 & j0.4 \\ j0.5 & -j1.1 & j0.6 & 0 \\ j0.4 & j0.6 & -j1.5 & j0.5 \\ j0.4 & 0 & j0.5 & -j0.9 \end{bmatrix}$$

The elements of new bus admittance matrix after eliminating the 4th row and 4th column is given by

$$Y_{jk,\,new} = Y_{jk} - \frac{Y_{jn}\,Y_{nk}}{Y_{nn}} \quad ; \text{ where } n = 4\,;\, j = 1, 2, 3 \text{ and } k = 1, 2, 3,$$

The bus admittance matrix is symmetrical, $\therefore Y_{kj,\,new} = Y_{jk,\,new}$

$$Y_{11,\,new} = Y_{11} - \frac{Y_{14}\,Y_{41}}{Y_{44}} = -j1.3 - \frac{(j0.4)\,(j0.4)}{-j0.9} = -j1.12$$

$$Y_{12,\,new} = Y_{12} - \frac{Y_{14}\,Y_{42}}{Y_{44}} = j0.5 - \frac{(j0.4 \times 0)}{-j0.9} = j0.5$$

$$Y_{13,\,new} = Y_{13} - \frac{Y_{14}\,Y_{43}}{Y_{44}} = j0.4 - \frac{(j0.4)\,(j0.5)}{-j0.9} = j0.622$$

$$Y_{21,\,new} = Y_{12,\,new} = j0.5$$

$$Y_{22,\,new} = Y_{22} - \frac{Y_{24}\,Y_{42}}{Y_{44}} = -j1.1 - \frac{(0)\,(0)}{-j0.9} = -j1.1$$

$$Y_{23,\,new} = Y_{23} - \frac{Y_{24}\,Y_{43}}{Y_{44}} = j0.6 - \frac{(0)\,(j0.5)}{-j0.9} = j0.6$$

$$Y_{31,\,new} = Y_{13,\,new} = j0.622$$

$$Y_{32,\,new} = Y_{23,new} = j0.6$$

$$Y_{33,\,new} = Y_{33} - \frac{Y_{34}\,Y_{43}}{Y_{44}} = -j1.5 - \frac{(j0.5)\,(j0.5)}{-j0.9} = -j1.222$$

The reduced bus admittance matrix after eliminating 4th row is shown below.

$$Y_{bus} = \begin{bmatrix} -j1.12 & j0.5 & j0.622 \\ j0.5 & -j1.1 & j0.6 \\ j0.622 & j0.6 & -j1.222 \end{bmatrix}$$

EXAMPLE 1.15

Eliminate buses 3 and 4 in the given bus admittance matrix and form new bus admittance matrix.

$$Y_{bus} = \begin{bmatrix} -j9.8 & 0.0 & j4.0 & j5.0 \\ 0.0 & -j8.3 & j2.5 & j5.0 \\ j4.0 & j2.5 & -j14 & j8.0 \\ j5.0 & j5.0 & j8.0 & -j18.0 \end{bmatrix}$$

SOLUTION

4 First let us eliminate 4th bus, $\therefore Y_{nn} = Y_{44} = -j18.0$

The elements of new bus admittance after eliminating 4^{th} row and 4^{th} column is given by

$$Y_{jk,\,new} = Y_{jk} - \frac{Y_{jn}\,Y_{nk}}{Y_{nn}} \quad \text{where } n = 4\ ;\ j = 1,\,2,\,3,\ \text{and } k = 1,\,2,\,3$$

$$Y_{11,\,new} = Y_{11} - \frac{Y_{14}\,Y_{41}}{Y_{44}} = -j9.8 - \frac{j5.0 \times j5.0}{-j18.0} = -j8.411$$

$$Y_{12,\,new} = Y_{12} - \frac{Y_{14}\,Y_{42}}{Y_{44}} = 0.0 - \frac{j5.0 \times j5.0}{-j18.0} = j1.388$$

$$Y_{13,\,new} = Y_{13} - \frac{Y_{14}\,Y_{43}}{Y_{44}} = j4.0 - \frac{j5 \times j8}{-j18.0} = j6.222$$

$$Y_{21,\,new} = Y_{12,\,new} = j1.388$$

$$Y_{22,\,new} = Y_{22} - \frac{Y_{24}\,Y_{42}}{Y_{44}} = -j8.3 - \frac{j5 \times j5}{-j18.0} = -j6.911$$

$$Y_{23,\,new} = Y_{23} - \frac{Y_{24}\,Y_{43}}{Y_{44}} = j2.5 - \frac{j5 \times j8}{-j18} = j4.722$$

$$Y_{31,\,new} = Y_{13,\,new} = j6.222$$

$$Y_{32,\,new} = Y_{23,\,new} = j4.722$$

$$Y_{33,\,new} = Y_{33} - \frac{Y_{34}\,Y_{43}}{Y_{44}} = -j14 - \frac{j8 \times j8}{-j18.0} = -j10.444$$

The reduced bus admittance matrix after eliminating 4^{th} node is given by

$$Y_{bus} = \begin{bmatrix} -j8.411 & j1.3889 & j6.222 \\ j1.388 & -j6.911 & j4.722 \\ j6.222 & j4.722 & -j10.444 \end{bmatrix}$$

<u>Elimination of node 3 :</u> $Y_{nn} = Y_{33} = -j10.444$

The other elements of the reduced bus admittance matrix can be formed from the equation

$$Y_{jk,\,new} = Y_{jk} - \frac{Y_{jn}\,Y_{nk}}{Y_{nn}} \quad \text{where } n = 3\ ;\ j = 1,\,2\ \text{and } k = 1,\,2$$

$$Y_{11,\,new} = Y_{11} - \frac{Y_{13}\,Y_{31}}{Y_{33}} = -j8.411 - \frac{j6.222 \times j6.222}{-j10.444} = -j4.7043$$

$$Y_{12,\,new} = Y_{12} - \frac{Y_{13}\,Y_{32}}{Y_{33}} = j1.388 - \frac{j6.222 \times j4.722}{-j10.444} = j4.2011$$

$$Y_{21,\,new} = Y_{12,\,new} = j4.2011$$

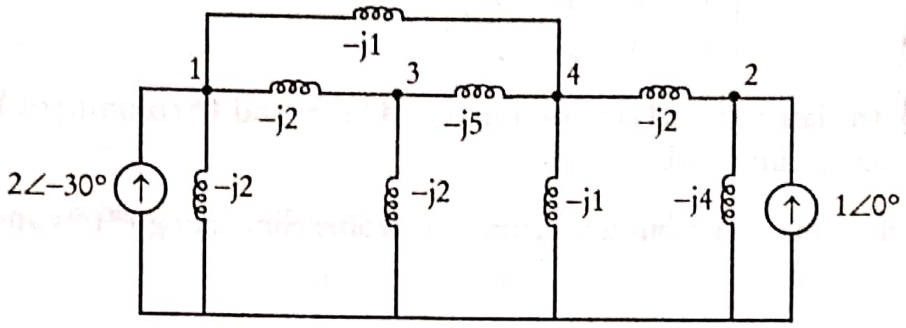
$$Y_{22,\,new} = Y_{22} - \frac{Y_{23}\,Y_{32}}{Y_{33}} = -j6.911 - \frac{j4.722 \times j4.722}{-j10.444} = -j4.7761$$

The reduced bus admittance matrix after eliminating node 3 and 4 is

$$Y_{bus} = \begin{bmatrix} -j4.7043 & j4.2011 \\ j4.2011 & -j4.7761 \end{bmatrix}$$

EXAMPLE 1.16

Determine the bus admittance matrix of the system whose reactance diagram is shown in fig 1.16.1. The currents and admittances are given in p.u. Determine the reduced bus admittance matrix after eliminating node-3.



SOLUTION

Fig 1.16.1.

The bus admittance matrix can be formed by inspection using the following guidelines.

1. The diagonal element Y_{jj} is given by sum of all the admittances connected to node-j.
2. The off-diagonal element Y_{jk} is given by negative of the sum of all the admittances connected between node-j and node-k.

$$\therefore \mathbf{Y}_{\text{bus}} = \begin{bmatrix} -j2-j2-j1 & 0 & j2 & j1 \\ 0 & -j2-j4 & 0 & j2 \\ j2 & 0 & -j2-j2-j5 & j5 \\ j1 & j2 & j5 & -j1-j5-j2-j1 \end{bmatrix}$$

$$\therefore \mathbf{Y}_{\text{bus}} = \begin{bmatrix} -j5 & 0 & j2 & j1 \\ 0 & -j6 & 0 & j2 \\ j2 & 0 & -j9 & j5 \\ j1 & j2 & j5 & -j9 \end{bmatrix} \qquad .....(1.16.1)$$

For eliminating node-3, the bus admittance matrix is rearranged by interchanging row-3 & row-4, and then interchanging column-3 & column - 4.

After interchanging row - 3 & row - 4 of $\mathbf{Y}_{\text{bus}}$ matrix of equ(1.16.1) we get,

$$\mathbf{Y}_{\text{bus}} = \begin{bmatrix} -j5 & 0 & j2 & j1 \\ 0 & -j6 & 0 & j2 \\ j1 & j2 & j5 & -j9 \\ j2 & 0 & -j9 & j5 \end{bmatrix} \qquad .....(1.16.2)$$

After interchanging column - 3 & column - 4 of Y_{bus} matrix of equ(1.16.2) we get,

$$Y_{bus} = \begin{bmatrix} -j5 & 0 & j1 & j2 \\ 0 & -j6 & j2 & 0 \\ j1 & j2 & -j9 & j5 \\ j2 & 0 & j5 & -j9 \end{bmatrix} \quad .....(1.16.3)$$

Now the last row and last column [i.e., 4th row and 4th column] of Y_{bus} matrix of equ(1.16.3) can be eliminated.

The elements of new bus admittance matrix after eliminating 4th row and 4th column is given by

$$Y_{jk,\,new} = Y_{jk} - \frac{Y_{jn}\,Y_{nk}}{Y_{nn}} \quad \text{where } n = 4 \;\; ; \; j = 1,2,3 \;\text{ and } k = 1,2,3$$

$$Y_{11,\,new} = Y_{11} - \frac{Y_{14}\,Y_{41}}{Y_{44}} = -j5 - \frac{(j2)\,(j2)}{-j9} = -j4.556$$

$$Y_{12,\,new} = Y_{12} - \frac{Y_{14}\,Y_{42}}{Y_{44}} = 0 - \frac{j2 \times 0}{-j9} = 0$$

$$Y_{13,\,new} = Y_{13} - \frac{Y_{14}\,Y_{43}}{Y_{44}} = j1 - \frac{(j2)\,(j5)}{-j9} = j2.111$$

$$Y_{21,\,new} = Y_{12,\,new} = 0$$

$$Y_{22,\,new} = Y_{22} - \frac{Y_{24}\,Y_{42}}{Y_{44}} = -j6 - \frac{0 \times 0}{-j9} = -j6$$

$$Y_{23,\,new} = Y_{23} - \frac{Y_{24}\,Y_{43}}{Y_{44}} = j2 - \frac{0 \times j5}{-j9} = j2$$

$$Y_{31,\,new} = Y_{13,\,new} = j2.111$$

$$Y_{32,\,new} = Y_{23,\,new} = j2$$

$$Y_{33,\,new} = Y_{33} - \frac{Y_{34}\,Y_{43}}{Y_{44}} = -j9 - \frac{j5 \times j5}{-j9} = -j6.222$$

The reduced bus admittance matrix after eliminating bus-3 is given by,

$$Y_{bus} = \begin{bmatrix} -j4.556 & 0 & j2.111 \\ 0 & -j6 & j2 \\ j2.111 & j2 & -j6.222 \end{bmatrix}$$

EXAMPLE 1.17

Determine Z_{bus} for system whose reactance diagram is shown in fig 1.17.1. where the impedance is given in p.u. Preserve all the three nodes.

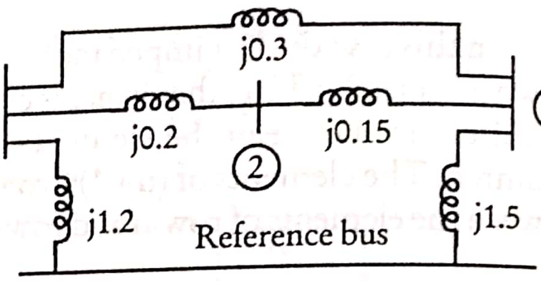


Fig 1.17.1.

SOLUTION

Step 1 : Consider the branch with impedance j1.2 p.u. connected between bus-1 and reference as shown in fig 1.17.2. The system shown in fig 1.17.2. has a single bus and so the order of bus impedance matrix is one, as shown below.

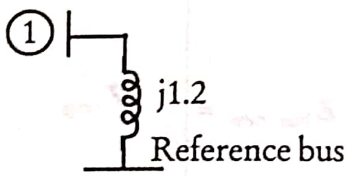


Fig 1.17.2

$$Z_{bus} = [j1.2]$$

Step 2 : Connect bus-2 to bus-1 through an impedance j0.2 as shown in fig 1.17.3. This is case-2 modification and so the order of bus impedance matrix increases by one. In the new bus impedance matrix, the elements of 1st column are copied as elements of 2nd column and the elements of 1st row are copied as elements of 2nd row. The diagonal element is given by $Z_{11} + Z_b$ where $Z_b = j0.2$.

$$\therefore \mathbf{Z}_{bus} = \begin{bmatrix} j1.2 & j1.2 \\ j1.2 & j1.2 + j0.2 \end{bmatrix} = \begin{bmatrix} j1.2 & j1.2 \\ j1.2 & j1.4 \end{bmatrix}$$

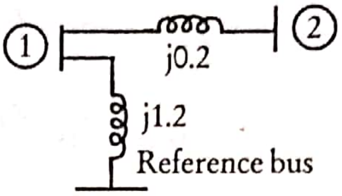


Fig 1.17.3

Step 3 : Connect the bus-3 to bus-2 through an impedance j0.15 as shown in fig 1.17.4. This is case-2 modification and so the order of the bus impedance matrix increases by one. In the new bus impedance matrix the elements of 2nd column are copied as elements of 3rd column and the elements of 2nd row are copied as elements of 3rd row. The diagonal element is given by $Z_{22} + Z_b$ where Z_b = j0.15.

$$\therefore \mathbf{Z}_{bus} = \begin{bmatrix} j1.2 & j1.2 & j1.2 \\ j1.2 & j1.4 & j1.4 \\ j1.2 & j1.4 & j1.4 + j0.15 \end{bmatrix}$$

$$= \begin{bmatrix} j1.2 & j1.2 & j1.2 \\ j1.2 & j1.4 & j1.4 \\ j1.2 & j1.4 & j1.55 \end{bmatrix}$$

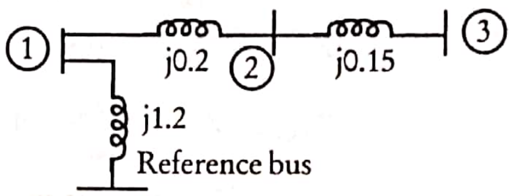


Fig 1.17.4

Step 4 : Connect the impedance j1.5 from bus-3 to reference bus as shown in fig 1.17.5. This is case-3 modification. In case-3 modification the new bus impedance matrix is formed as that of case-2 and then the last row and column are eliminated by node elimination techniques.

In new bus impedance matrix the elements of 3rd column are copied as elements of 4th column and the elements of 3rd row are copied as elements of 4th row. The diagonal element is given by $Z_{33} + Z_b$ where Z_b = j1.5.

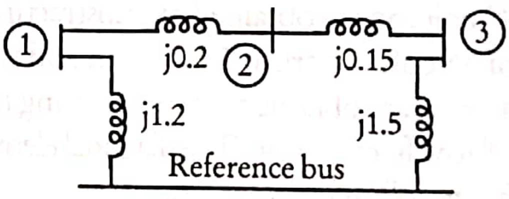


Fig 1.17.5

$$\therefore \mathbf{Z}_{bus} = \begin{bmatrix} j1.2 & j1.2 & j1.2 & j1.2 \\ j1.2 & j1.4 & j1.4 & j1.4 \\ j1.2 & j1.4 & j1.55 & j1.55 \\ j1.2 & j1.4 & j1.55 & j1.55 + j1.5 \end{bmatrix} = \begin{bmatrix} j1.2 & j1.2 & j1.2 & j1.2 \\ j1.2 & j1.4 & j1.4 & j.14 \\ j1.2 & j1.4 & j1.55 & j1.55 \\ j1.2 & j1.4 & j1.55 & j3.05 \end{bmatrix}$$

The actual new bus impedance matrix is obtained by eliminating the 4th row and 4th column. The element Z_{jk} of the actual new bus impedance matrix is given by,

$$\therefore \mathbf{Z}_{jk,act} = \mathbf{Z}_{jk} - \frac{Z_{j(n+1)}\, Z_{(n+1)k}}{Z_{(n+1)\,(n+1)}} \qquad \text{where } n = 3 \;;\; j = 1, 2, 3 \text{ and } k = 1, 2, 3$$

$$Z_{11,\,act} = Z_{11} - \frac{Z_{14}\, Z_{41}}{Z_{44}} = j1.2 - \frac{j1.2 \times j1.2}{j3.05} = j0.728$$

$$Z_{12,\,act} = Z_{12} - \frac{Z_{14}\,Z_{42}}{Z_{44}} = j1.2 - \frac{j1.2 \times j1.4}{j3.05} = j0.649$$

$$Z_{13,\,act} = Z_{13} - \frac{Z_{14}\,Z_{43}}{Z_{44}} = j1.2 - \frac{j1.2 \times j1.55}{j3.05} = j0.590$$

$$Z_{21,\,act} = Z_{12,act} = j0.649$$

$$Z_{22,\,act} = Z_{22} - \frac{Z_{24}\,Z_{42}}{Z_{44}} = j1.4 - \frac{j1.4 \times j1.4}{j3.05} = j0.757$$

$$Z_{23,\,act} = Z_{23} - \frac{Z_{24}\,Z_{43}}{Z_{44}} = j1.4 - \frac{j1.4 \times j1.55}{j3.05} = j0.689$$

$$Z_{31,\,act} = Z_{13,act} = j0.590$$

$$Z_{32,act} = Z_{23,act} = j0.689$$

$$Z_{33,\,act} = Z_{33} - \frac{Z_{34}Z_{43}}{Z_{44}} = j1.55 - \frac{j1.55 \times j1.55}{j3.05} = j0.762$$

$$\therefore \mathbf{Z}_{bus} = \begin{bmatrix} j0.728 & j0.649 & j0.590 \\ j0.649 & j0.757 & j0.689 \\ j0.590 & j0.689 & j0.762 \end{bmatrix}$$

Step 5 : Connect the impedance j0.3 between bus-1 and bus-3 as shown in fig 1.17.6. This is case-4 modification.

In new bus impedance matrix, the elements of 4th column are obtained by substracting the elements of 3rd column from 1st column and the elements of 4th row are obtained by substracting the elements of 3rd row from 1st row. The diagonal element Z_{44} is given by the following equation.

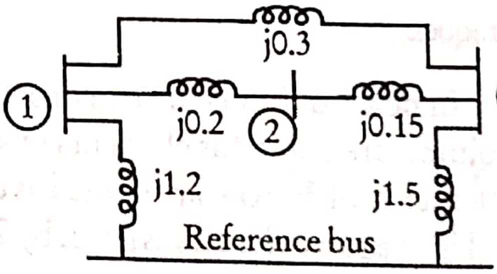


Fig 1.17.6

$$Z_{44} = Z_b + Z_{11} + Z_{33} - 2Z_{13}$$

where $Z_b = j0.3$

$$\therefore Z_{44} = j0.3 + j0.728 + j0.762 - 2(j0.59) = j0.61$$

$$Z_{bus} = \begin{bmatrix} j0.728 & j0.649 & j0.590 & j0.728 - j0.59 \\ j0.649 & j0.757 & j0.689 & j0.649 - j0.689 \\ j0.59 & j0.689 & j0.762 & j0.59 - j0.762 \\ j0.728 - j0.59 & j0.649 - j0.689 & j0.59 - j0.762 & j0.61 \end{bmatrix}$$

$$\mathbf{Z}_{bus} = \begin{bmatrix} j0.728 & j0.649 & j0.590 & j0.138 \\ j0.649 & j0.757 & j0.689 & -0.04 \\ j0.59 & j0.689 & j0.762 & -j0.172 \\ j0.138 & -0.04 & -j0.172 & j0.61 \end{bmatrix}$$

Since this modification does not add a new bus, the 4th row and column has to be eliminated using node elimination technique, to determine the actual new bus impedance matrix. The element Z_{jk} of actual new bus impedance matrix is given by,

$$\therefore Z_{jk,act} = Z_{jk} - \frac{Z_{j(n+1)}\, Z_{(n+1)k}}{Z_{(n+1)(n+1)}} \qquad \text{where } n = 3 \ ;\ j = 1,2,3 \text{ and } k = 1,2,3$$

$$Z_{11,act} = Z_{11} - \frac{Z_{14}\, Z_{41}}{Z_{44}} = j0.728 - \frac{j0.138 \times j0.138}{j0.61} = j0.697$$

$$Z_{12,act} = Z_{12} - \frac{Z_{14}\, Z_{42}}{Z_{44}} = j0.649 - \frac{j0.138 \times (-j0.04)}{j0.61} = j0.658$$

$$Z_{13,act} = Z_{13} - \frac{Z_{14}\, Z_{43}}{Z_{44}} = j0.59 - \frac{j0.138 \times (-j0.172)}{j0.61} = j0.629$$

$$Z_{21,act} = Z_{12,act} = j0.658$$

$$Z_{22,act} = Z_{22} - \frac{Z_{24}\, Z_{42}}{Z_{44}} = j0.757 - \frac{(-j0.04)\,(-j0.04)}{j0.61} = j0.754$$

$$Z_{23,act} = Z_{23} - \frac{Z_{24}\, Z_{43}}{Z_{44}} = j0.689 - \frac{(-j0.04)\,(-j0.172)}{j0.61} = j0.678$$

$$Z_{31,act} = Z_{13,act} = j0.629$$

$$Z_{32,act} = Z_{23,act} = j0.678$$

$$Z_{33,act} = Z_{33} - \frac{Z_{34} Z_{43}}{Z_{44}} = j0.762 - \frac{(-j0.172)\,(-j0.172)}{j0.61} = j0.714$$

$$\therefore Z_{bus} = \begin{bmatrix} j0.697 & j0.658 & j0.629 \\ j0.658 & j0.754 & j0.678 \\ j0.629 & j0.678 & j0.714 \end{bmatrix}$$

EXAMPLE 1.18

Find the bus impedance matrix for the system whose reactance diagram is shown in fig 1.18.1. All the impedances are in p.u.

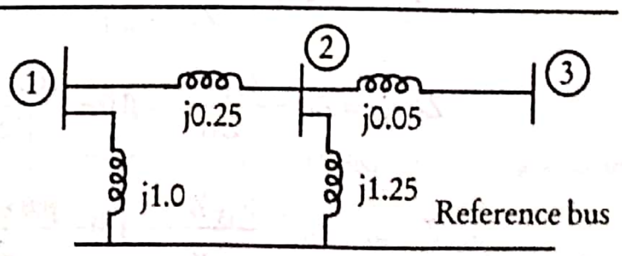


Fig 1.18.1.

SOLUTION

Step 1 : Consider the branch with impedance j1 p.u. connected between bus-1 and reference as shown in fig 1.18.2. The system shown in fig 1.18.2. has a single bus and so the order of bus impedance matrix is one, as shown below.

$$Z_{bus} = [j1.0]$$

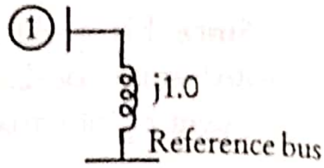


Fig 1.18.2

Step 2 : Connect bus-2 to bus-1 through an impedance j0.25 as shown in fig 1.18.3. This is case-2 modification and so the order of bus impedance matrix increases by one.

In the new bus impedance matrix, the elements of 1st column are copied as elements of 2nd column and the elements of 1st row are copied as elements of 2nd row. The diagonal element is given by $Z_{11} + Z_b$ where $Z_b = j0.25$.

$$\therefore Z_{bus} = \begin{bmatrix} j1.0 & j1.0 \\ j1.0 & j1.0 + j0.25 \end{bmatrix} = \begin{bmatrix} j1.0 & j1.0 \\ j1.0 & j1.25 \end{bmatrix}$$

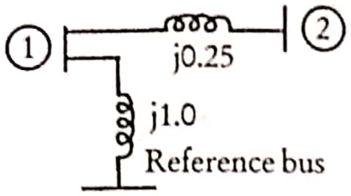


Fig 1.18.3

Step 3 : Connect the impedance j1.25 from bus-2 to reference bus as shown in fig 1.18.4. This is case-3 modification. In case-3 modification the new bus impedance matrix is formed as that of case-2 and then the last row and column are eliminated by node elimination technique.

In the new bus impedance matrix the elements of 2nd column are copied as elements of 3rd column and the elements of 2nd row are copied as elements of 3rd row. The diagonal element is given by $Z_{22} + Z_b$ where $Z_b = j1.25$.

$$\therefore Z_{bus} = \begin{bmatrix} j1.0 & j1.0 & j1.0 \\ j1.0 & j1.25 & j1.25 \\ j1.0 & j1.25 & j1.25 + j1.25 \end{bmatrix}$$

$$= \begin{bmatrix} j1.0 & j1.0 & j1.0 \\ j1.0 & j1.25 & j1.25 \\ j1.0 & j1.25 & j2.5 \end{bmatrix}$$

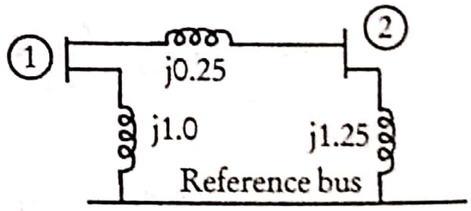


Fig 1.18.4

The actual new bus impedance matrix is obtained by eliminating the 3rd row and 3rd column. The element Z_{jk} of the actual new bus impedance matrix is given by,

$$\therefore Z_{jk,act} = Z_{jk} - \frac{Z_{j(n+1)}\, Z_{(n+1)k}}{Z_{(n+1)\,(n+1)}} \quad \text{where } n = 2 \;;\; j = 1, 2, \text{ and } k = 1, 2,$$

$$Z_{11,act} = Z_{11} - \frac{Z_{13}\, Z_{31}}{Z_{33}} = j1.0 - \frac{j1.0 \times j1.0}{j2.5} = j0.6$$

$$Z_{12,act} = Z_{12} - \frac{Z_{13}\, Z_{32}}{Z_{33}} = j1.0 - \frac{j1.0 \times j1.25}{j2.5} = j0.5$$

$$Z_{21,act} = Z_{12,act} = j0.5$$

$$Z_{22,act} = Z_{22} - \frac{Z_{23}\, Z_{32}}{Z_{33}} = j1.25 - \frac{j1.25 \times j1.25}{j2.5} = j0.625$$

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$$\therefore \mathbf{Z}_{bus} = \begin{bmatrix} j0.6 & j0.5 \\ j0.5 & j0.625 \end{bmatrix}$$

Step 4 : Connect the bus-3 to bus-2 through an impedance j0.05 as shown in fig 1.18.5. This is case-2 modification and so the order of the bus impedance matrix increases by one.

In the new bus impedance matrix, the elements of 2nd column are copied as elements of 3rd column and the elements of 2nd row are copied as elements of 3rd row. The diagonal element is given by $Z_{22} + Z_b$ where Z_b = j0.5.

$$\mathbf{Z}_{bus} = \begin{bmatrix} j0.6 & j0.5 & j0.5 \\ j0.5 & j0.625 & j0.625 \\ j0.5 & j0.625 & j0.625 + j0.05 \end{bmatrix}$$

$$\therefore \mathbf{Z}_{bus} = \begin{bmatrix} j0.6 & j0.5 & j0.5 \\ j0.5 & j0.625 & j0.625 \\ j0.5 & j0.625 & j0.675 \end{bmatrix}$$

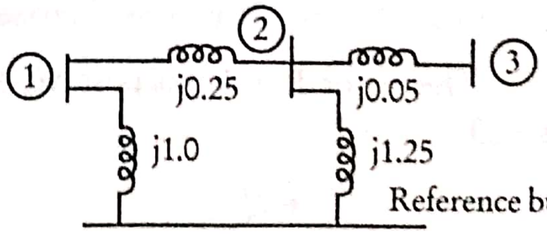

Fig 1.18.5