EXAMPLE 2.1

The system data for a load flow solution are given in table-2.1.1 and table-2.1.2. Determine the voltages at the end of first iteration by Gauss-Seidel method. Take $\alpha = 1.6$.

Table 2.1.1 : Line admittances

Buscode	Admittance
1 - 2	$2 - j8$
1 - 3	$1 - j4$
2 - 3	$0.666 - j2.664$
2 - 4	$1 - j4$
3 - 4	$2 - j8$

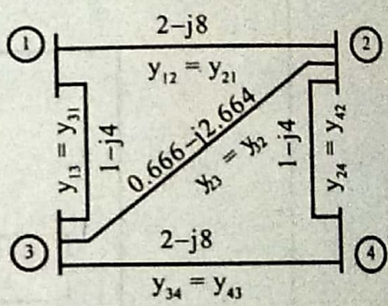
Table 2.1.2 : Bus Specifications

Buscode	P	Q	V	Remarks
1	-	-	$1.06\angle 0°$	Slack
2	0.5	0.2	-	PQ
3	0.4	0.3	-	PQ
4	0.3	0.1	-	PQ

SOLUTION

Using the data given in table-1 the single line diagram of the power system can be drawn as shown in fig 2.1.1.

The bus admittance matrix can be directly formed from fig 2.1.1. The elements of bus admittance matrix Y_{jk} are obtained from line admittances y_{jk} as shown below.



Note : All elements are admittances in p.u.

Fig 2.1.1.

$$Y_{11} = y_{12} + y_{13} = 2 - j8 + 1 - j4 = 3 - j12$$

$$Y_{22} = y_{12} + y_{23} + y_{24} = 2 - j8 + 0.666 - j2.664 + 1 - j4 = 3.666 - j14.664$$

$$Y_{33} = y_{13} + y_{23} + y_{34} = 1 - j4 + 0.666 - j2.664 + 2 - j8 = 3.666 - j14.664$$

$$Y_{44} = y_{24} + y_{34} = 1 - j4 + 2 - j8 = 3 - j12$$

$$Y_{12} = Y_{21} = -y_{12} = -(2 - j8) = -2 + j8$$

$$Y_{13} = Y_{31} = -y_{13} = -(1 - j4) = -1 + j4$$

$$Y_{14} = Y_{41} = 0$$

$$Y_{23} = Y_{32} = -y_{23} = -(0.666 - j2.664) = -0.666 + j2.664$$

$$Y_{24} = Y_{42} = -y_{24} = -(1 - j4) = -1 + j4$$

$$Y_{34} = Y_{43} = -y_{34} = -(2 - j8) = -2 + j8$$

$$\mathbf{Y}_{bus} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & Y_{14} \\ Y_{21} & Y_{22} & Y_{23} & Y_{24} \\ Y_{31} & Y_{32} & Y_{33} & Y_{34} \\ Y_{41} & Y_{42} & Y_{43} & Y_{44} \end{bmatrix} = \begin{bmatrix} 3 - j12 & -2 + j8 & -1 + j4 & 0 \\ -2 + j8 & 3.666 - j14.664 & -0.666 + j2.664 & -1 + j4 \\ -1 + j4 & -0.666 + j2.664 & 3.666 - j14.664 & -2 + j8 \\ 0 & -1 + j4 & -2 + j8 & 3 - j12 \end{bmatrix}$$

The initial values of bus voltages are considered as 1 p.u. except the slack bus.

$$\therefore V_2^{\circ} = 1+j0 \quad ; \quad V_3^{\circ} = 1+j0 \quad ; \quad V_4^{\circ} = 1+j0$$

The bus-1 is a slack bus and so its voltage remain at the specified value for all iterations.

$$\text{i.e., } V_1^{\circ} = V_1^{1} = \dots\dots\dots\dots = V_1^{k} = V_1 = 1.06 + j0\,\text{p.u.}$$

Since the buses are PQ buses the specified real and reactive powers are considered as load powers. Therefore a negative sign is attached to the specified power.

The $(k+1)^{th}$ iteration voltage of a PQ (load) bus-p is given by

$$V_p^{k+1} = \frac{1}{Y_{pp}}\left[\frac{P_p - jQ_q}{(V_p^k)^*} - \sum_{q=1}^{p-1} Y_{pq}V_q^{k+1} - \sum_{q=p+1}^{n} Y_{pq}V_q^{k}\right]$$

For first iteration, k = 0. The system has four buses and so p will take values from 1 to 4. Here all the buses are load buses except bus-1. The calculation of bus voltages for first iteration are shown below.

$$V_1^1 = V_1^{\circ} = 1.06 + j0 \text{ p.u.} \qquad (\because \text{Bus-1 is a slack bus})$$

$$V_2^1 = \frac{1}{Y_{22}}\left[\frac{P_2 - jQ_2}{(V_2^{\circ})^*} - Y_{21}V_1^1 - Y_{23}V_3^{\circ} - Y_{24}V_4^{\circ}\right]$$

$$= \frac{1}{3.666 - j14.664}\left[\frac{-0.5 + j0.2}{1 - j0} - (-2+j8)(1.06) - (-0.666 + j2.664)(1+j0) - (-1+j4)(1+j0)\right]$$

$$= \frac{-0.5 + j0.2 + 2.12 - j8.48 + 0.666 - j2.664 + 1 - j4}{3.666 - j14.664}$$

$$= \frac{3.286 - j14.944}{3.666 - j14.664} = \frac{15.3010\angle -77.6^{\circ}}{15.1153\angle -75.96^{\circ}} = 1.0123\angle -1.64^{\circ} = 1.0119 - j0.0290 \text{ p.u.}$$

$$\boxed{V_{2,acc}^1 = V_2^{\circ} + \alpha(V_2^1 - V_2^{\circ})} = 1 + 1.6\,(1.0119 - j0.0290 - 1)$$

$$= 1 + 1.6(0.0119 - j0.0290) = 1.0190 - j0.0464$$

$$\text{Now, } V_2^1 = V_{2,acc}^1 = 1.0190 - j0.0464 \text{ p.u.} = 1.0201\angle -2.61^{\circ} \text{ p.u.}$$

$$V_3^1 = \frac{1}{Y_{33}}\left[\frac{P_3 - jQ_3}{(V_3^{\circ})^*} - Y_{31}V_1^1 - Y_{32}V_2^1 - Y_{34}V_4^{\circ}\right]$$

$$= \frac{1}{3.666 - j14.664}\left[\frac{-0.4 + j0.3}{1 - j0} - (-1+j4)(1.06) - (-0.666 + j2.664)(1.0190 - j0.0464) - (-2+j8)(1+j0)\right]$$

$$= \frac{-0.4 + j0.3 + 1.06 - j4.24 - (-0.5550 + j2.7455) + 2 - j8}{3.666 - j14.664}$$

$$= \frac{3.215 - j14.6855}{3.666 - j14.664} = \frac{15.0333\angle -77.65^{\circ}}{15.1153\angle -75.96^{\circ}} = 0.9946\angle -1.69^{\circ} = 0.9942 - j0.0293 \text{ p.u.}$$

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$$\boxed{V^1_{3,acc} = V^o_3 + \alpha(V^1_3 - V^o_3)}$$
$$= 1 + 1.6(0.9942 - j0.0293 - 1) = 1 + 1.6(-0.0058 - j0.0293)$$
$$= 0.9907 - j0.0469$$

Now, $V^1_3 = V^1_{3,acc} = 0.9907 - j0.0469$ p.u.

$$= 0.9918\angle -2.71^\circ \text{ p.u.}$$

$$V^1_4 = \frac{1}{Y_{44}}\left[\frac{P_4 - jQ_4}{(V^o_4)^*} - Y_{41}V^1_1 - Y_{42}V^1_2 - Y_{43}V^1_3\right]$$

$$= \frac{1}{3 - j12}\left[\frac{-0.3 + j0.1}{1 - j0} - (0 \times 1.06) - (-1 + j4)(1.0190 - j0.0464) - (-2 + j8)(0.9907 - j0.0469)\right]$$

$$= \frac{-0.3 + j0.1 - (-0.8334 + j4.1224) - (-1.6062 + j8.0194)}{3 - j12}$$

$$= \frac{2.1396 - j12.0418}{3 - j12} = \frac{12.2304\angle -79.92^\circ}{12.3693\angle -75.96^\circ} = 0.9888\angle -3.96^\circ = 0.9864 - j0.0683 \text{ p.u.}$$

$$\boxed{V^1_{4,acc} = V^o_4 + \alpha(V^1_4 - V^o_4)}$$
$$= 1 + 1.6(0.9864 - j0.0683 - 1) = 1 + 1.6(-0.0136 - j0.0683) = 0.9782 - j0.1093$$

Now, $V^1_4 = V^1_{4,acc} = 0.9782 - j0.1093$ p.u.

$$= 0.9843\angle -6.38^\circ \text{ p.u.}$$

RESULT

The bus voltages at the end of first iteration are

$V_1^1 = 1.06 + j0 = 1.06\angle 0^\circ$ p.u

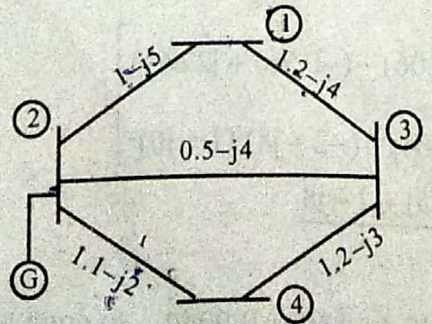
$V_2^1 = 1.019 - j0.0464 = 1.0201\angle -2.61^\circ$ p.u

$V_3^1 = 0.9907 - j0.0469 = 0.9918\angle -2.7^\circ$ p.u

$V_4^1 = 0.9782 - j0.1093 = 0.9843\angle -6.38^\circ$ p.u

EXAMPLE 2.2

For the system shown in fig 2.2.2, determine the voltages at the end of first iteration by Gauss-Seidel method. Take $\alpha = 1$ and bus specifications are given in table-2.2.1.



Note : All elements are admittances in p.u.

Fig : 2.2.2.

Table-2.2.1 : Bus Specifications

Buscode	P	Q	V	Remarks
1	-	-	$1.06\angle 0^\circ$	Slack
2	0.5	$0.1 \le Q_2 \le 1$	1.04	PV
3	0.4	0.3	-	PQ
4	0.2	0.1	-	PQ

SOLUTION

The line admittances are, (Refer fig 2.2.2)

$$y_{12} = y_{21} = 1 - j5 \qquad y_{24} = y_{42} = 1.1 - j2$$
$$y_{13} = y_{31} = 1.2 - j4 \qquad y_{34} = y_{43} = 1.2 - j3$$
$$y_{23} = y_{32} = 0.5 - j4$$

The elements of bus admittance matrix Y_{jk} are obtained from line admittance y_{jk} as shown below

$$Y_{11} = y_{12} + y_{13} = 1 - j5 + 1.2 - j4 = 2.2 - j9$$
$$Y_{22} = y_{12} + y_{23} + y_{24} = 1 - j5 + 0.5 - j4 + 1.1 - j2 = 2.6 - j11$$
$$Y_{33} = y_{13} + y_{23} + y_{34} = 1.2 - j4 + 0.5 - j4 + 1.2 - j3 = 2.9 - j11$$
$$Y_{44} = y_{24} + y_{34} = 1.1 - j2 + 1.2 - j3 = 2.3 - j5$$
$$Y_{12} = Y_{21} = -y_{12} = -(1 - j5) = -1 + j5$$
$$Y_{13} = Y_{31} = -y_{13} = -(1.2 - j4) = -1.2 + j4$$
$$Y_{14} = Y_{41} = 0$$
$$Y_{23} = Y_{32} = -y_{23} = -(0.5 - j4) = -0.5 + j4$$
$$Y_{24} = Y_{42} = -y_{24} = -(1.1 - j2) = -1.1 + j2$$
$$Y_{34} = Y_{43} = -y_{34} = -(1.2 - j3) = -1.2 + j3$$

$$\mathbf{Y_{bus}} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & Y_{14} \\ Y_{21} & Y_{22} & Y_{23} & Y_{24} \\ Y_{31} & Y_{32} & Y_{33} & Y_{34} \\ Y_{41} & Y_{42} & Y_{43} & Y_{44} \end{bmatrix} = \begin{bmatrix} 2.2 - j9 & -1 + j5 & -1.2 + j4 & 0 \\ -1 + j5 & 2.6 - j11 & -0.5 + j4 & -1.1 + j2 \\ -1.2 + j4 & -0.5 + j4 & 2.9 - j11 & -1.2 + j3 \\ 0 & -1.1 + j2 & -1.2 + j3 & 2.3 - j5 \end{bmatrix}$$

In the given system bus-1 is slack bus, bus-2 is generator bus and bus-3 and bus-4 are load buses. The initial voltages of load buses are assumed as 1 + j0 p.u. For slack and generator buses the specified voltages are used as initial values.

$$\therefore V_1^{\circ} = V_1^{1} = \ldots\ldots = V_1^{k} = V_1 = 1.06 + j0 \text{ p.u. (slack bus)}$$
$$V_2^{\circ} = 1.04 + j0 \text{ p.u. (Generator bus) (Initial phase in assumed as zero)}$$
$$V_3^{\circ} = 1 + j0 \text{ p.u. (Load bus)}$$
$$V_4^{\circ} = 1 + j0 \text{ p.u. (Load bus)}$$

For generator bus the specified powers are considered as positive powers but for load buses the specified powers are considered as negative powers.

For first iteration, k = 0. In each iteration the slack bus voltage need not be recalculated. In each iteration the reactive power for generator bus has to be calculated and checked for violation of the specified limits. If the limits are violated then it is treated as load bus.

The calculations of bus voltages for first iteration are shown below

$$V_1^{1} = V_1^{\circ} = 1.06 + j0 \text{ p.u. (Bus-1 is a slack bus).}$$

The bus-2 is a generator bus and so calculate its reactive power, Q_2.

$$Q_{p,cal}^{k+1} = -1 \times \text{Im}\left\{(V_p^k)^* \left[\sum_{q=1}^{n-1} Y_{pq} V_q^{k+1} + \sum_{q=p}^{n} Y_{pq} V_q^k\right]\right\}$$

Here $p = 2, k = 0, n = 4$

$$\therefore Q_{2,cal}^1 = -1 \times \text{Im}\left\{(V_2^o)^* \left[Y_{21}V_1^1 + Y_{22}V_2^o + Y_{23}V_3^o + Y_{24}V_4^o\right]\right\}$$

Note : *Here $|V_2^\circ|$ is same as $|V_2|_{spec}$ and so V_2° is used for calculation as such. If it is not same then we have to replace $|V_2^\circ|$ with $|V_2|_{spec}$.*

$$\therefore Q_{2,cal}^1 = -1 \times \text{Im}\left\{1.04\begin{bmatrix}(-1+j5)\,(1.06+j0)+(2.6-j11)\,(1.04+j0)\\ +(-0.5+j4)\,(1+j0)+(-1.1+j2)\,(1+j0)\end{bmatrix}\right\}$$

$$= -1 \times \text{Im}\left\{1.04\left[-1.06+j5.3+2.704-j11.44-0.5+j4-1.1+j2\right]\right\}$$

$$= -1 \times \text{Im}\left\{0.0458 - j0.1456\right\}$$

$$= 0.1456 \text{ p.u.}$$

The specified range for Q_2 is, $0.1 \le Q_2 \le 1.0$. The calculated value of Q_2 is within this range and so the reactive power limit is not violated. Therefore the bus can be treated as generator bus.

Now, $P_2 = 0.5, \quad Q_2 = 0.1456, \quad V_2^\circ = 1.04 + j0$

Since the bus-2, is treated as generator bus, the $|V_2^1| = |V_2|_{spec}$ and phase of V_2^1 is given by the phase of $V_{2,temp}^1$.

$$V_{p,temp}^{k+1} = \frac{1}{Y_{pp}}\left[\frac{P_p - jQ_p}{(V_p^k)^*} - \sum_{q=1}^{p-1} Y_{pq}V_q^{k+1} - \sum_{q=p+1}^{n} Y_{pq}V_q^k\right]$$

$$V_{2,temp}^1 = \frac{1}{Y_{22}}\left[\frac{P_2 - jQ_2}{(V_2^o)^*} - Y_{21}V_1^1 - Y_{23}V_3^o - Y_{24}V_4^o\right]$$

$$= \frac{1}{2.6-j11}\begin{bmatrix}\dfrac{0.5-j0.1456}{1.04-j0}-(-1+j5)\,(1.06+j0)\\ -(-0.5+j4)\,(1+j0)-(-1.1+j2)\,(1+j0)\end{bmatrix}$$

$$= \frac{1}{2.6-j11}\left[0.4808-j0.14+1.06-j5.3+0.5-j4+1.1-j2\right]$$

$$= \frac{3.1408-j11.44}{2.6-j11} = \frac{11.8633\angle-74.65^\circ}{11.3031\angle-76.70^\circ} = 1.0496\angle 2.05^\circ \text{ p.u.}$$

$$\therefore \delta_2^1 = \angle V_{2,temp}^1 = 2.05^\circ$$

$$V_2^1 = |V_2|_{spec}\angle\delta_2^1 = 1.04\angle 2.05^\circ \text{ p.u.} = 1.0393 + j0.0372 \text{ p.u.}$$

The bus-3 and bus-4 are load buses. The voltages of load bus are calculated using the following equation.

$$V_p^{k+1} = \frac{1}{Y_{pp}}\left[\frac{P_p - jQ_p}{(V_p^k)^*} - \sum_{q=1}^{p-1} Y_{pq} V_q^{k+1} - \sum_{q=p+1}^{n} Y_{pq} V_q^k\right]$$

$$V_3^1 = \frac{1}{Y_{33}}\left[\frac{P_3 - Q_3}{(V_3^o)^*} - Y_{31}V_1^1 - Y_{32}V_2^1 - Y_{34}V_4^o\right]$$

$$= \frac{1}{2.9 - j11}\left[\begin{array}{l}\dfrac{-0.4 + j0.3}{1 - j0} - (-1.2 + j4)\,(1.06 + j0) - (-0.5 + j4) \\ \qquad (1.0393 + j0.0372) - (-1.2 + j3)\,(1 + j0)\end{array}\right]$$

$$= \frac{1}{2.9 - j11}\left[-0.4 + j0.3 + 1.272 - j4.24 - (-0.6685 + j4.1386) + 1.2 - j3\right]$$

$$= \frac{2.7405 - j11.0786}{2.9 - j11} = \frac{11.4125\angle -76.11^\circ}{11.3759\angle -75.23^\circ} = 1.0032\angle -0.88^\circ \text{ p.u.}$$

$$= 1.0031 - j0.0154 \text{ p.u.}$$

$$V_4^1 = \frac{1}{Y_{44}}\left[\frac{P_4 - Q_4}{(V_4^o)^*} - Y_{41}V_1^1 - Y_{42}V_2^1 - Y_{43}V_3^1\right]$$

$$= \frac{1}{2.3 - j5}\left[\begin{array}{l}\dfrac{-0.2 + j0.1}{1 - j0} - (0 \times 1.06) - (-1.1 + j2)\,\,(1.0393 + j0.0372) \\ \qquad - (-1.2 + j3)\,(1.0031 - j0.0154)\end{array}\right]$$

$$= \frac{1}{2.3 - j5}\left[-0.2 + j0.1 - (-1.2176 + j2.0377) - (-1.1575 + j3.0278)\right]$$

$$= \frac{2.1751 - j4.9655}{2.3 - j5} = \frac{5.4210\angle -66.34^\circ}{5.5036\angle -65.30^\circ}$$

$$= 0.9850\angle -1.04^\circ \text{ p.u.} = 0.9848 - j0.0179 \text{ p.u.}$$

RESULT

The bus voltages at the end of first iteration are,

$V_1^1 = 1.06 + j0 = 1.06\angle 0^\circ$ p.u

$V_2^1 = 1.0393 + j0.0371 = 1.04\angle 2.05^\circ$ p.u

$V_3^1 = 1.0031 - j0.0154 = 1.0032\angle -0.88^\circ$ p.u

$V_4^1 = 0.9848 - j0.0179 = 0.9850\angle -1.04^\circ$ p.u.

EXAMPLE 2.3

Let the reactive power constraint on generator bus-2 in example 2.2 be changed to $0.2 \le Q_2 \le 1.0$. With other data of example 2.2 remaining same, solve the voltages at the end of first iteration by Gauss-Seidel method.

SOLUTION

The formation of bus impedance matrix and calculation of $Q^1_{2,cal}$ are same as in example 2.2. The $Q^1_{2,cal}$ corresponding to initial guess $V_2^\circ = 1.04 + j0$ is 0.1456 p.u. This value of Q_2 violates the lower limit of the specified range for Q_2. Therefore the reactive power generation for bus-2 is fixed at 0.2 (lower limit) and the bus-2 is treated as load bus for 1st iteration. Now $V_2^\circ = 1 + j0.0$, similar to other load buses for first iteration. But P and Q are considered as positive for bus-2 and P and Q are negative for other load buses. The bus voltages are calculated using the following equation.

$$V_p^{k+1} = \frac{1}{Y_{pp}}\left[\frac{P_p - jQ_p}{(V_p^k)^*} - \sum_{q=1}^{p-1} Y_{pq}V_q^{k+1} - \sum_{q=p+1}^{n} Y_{pq}V_q^k\right]$$

$V_1^1 = V_1 = 1.06 + j0$ p.u. (∵ Bus - 1 is a slack bus)

$$V_2^1 = \frac{1}{Y_{22}}\left[\frac{P_2 - jQ_2}{(V_2^\circ)^*} - Y_{21}V_1^1 - Y_{23}V_3^\circ - Y_{24}V_4^\circ\right]$$

$$V_2^1 = \frac{1}{2.6 - j11}\left[\frac{0.5 - j0.2}{1 - j0} - (-1 + j5)(1.06 + j0) - (-0.5 + j4)(1 + j0) - (-1.1 + j2)(1 + j0)\right]$$

$$= \frac{1}{2.6 - j11}[0.5 - j0.2 + 1.06 - j5.3 + 0.5 - j4 + 1.1 - j2]$$

$$= \frac{3.16 - j11.5}{2.6 - j11} = \frac{11.9263\angle -74.64^\circ}{11.3031\angle -76.70^\circ} = 1.0551\angle 2.06^\circ \text{ p.u.} = 1.0544 + j0.0379 \text{ p.u.}$$

$$V_3^1 = \frac{1}{Y_{33}}\left[\frac{P_3 - jQ_3}{(V_3^\circ)^*} - Y_{31}V_1^1 - Y_{32}V_2^1 - Y_{34}V_4^\circ\right]$$

$$= \frac{1}{2.9 - j11}\left[\frac{-0.4 + j0.3}{1 - j0} - (-1.2 + j4)(1.06 + j0) - (-0.5 + j4)(1.0544 + j0.0379) - (-1.2 + j3)(1 + j0)\right]$$

$$= \frac{1}{2.9 - j11}[-0.4 + j0.3 + 1.272 - j4.24 - (-0.6788 + j4.1987) + 1.2 - j3]$$

$$= \frac{2.7508 - j8.4387}{2.9 - j11} = \frac{8.8757\angle -71.95^\circ}{11.3759\angle -75.23^\circ} = 0.7802\angle 3.28^\circ \text{ p.u.}$$

$$= 0.7789 + j0.0446 \text{ p.u}$$

$$V_4^1 = \frac{1}{Y_{44}}\left[\frac{P_4 - jQ_4}{(V_4^o)^*} - Y_{41}V_1^1 - Y_{42}V_2^1 - Y_{43}V_3^1\right]$$

$$= \frac{1}{2.3 - j5}\left[\frac{-0.2 + j0.1}{1 - j0} - (0 \times 1.06) - (-1.1 + j2)(1.0544 + j0.0379) - (-1.2 + j3)(0.7789 + j0.0446)\right]$$

$$= \frac{1}{2.3 - j5}\left[-0.2 + j0.1 - (-1.2356 + j2.0671) - (-1.0685 + j2.2832)\right]$$

$$= \frac{2.1041 - j4.2503}{2.3 - j5} = \frac{4.7426\angle -63.66^\circ}{5.5036\angle -65.30^\circ}$$

$$= 0.8617\angle 1.64^\circ \text{ p.u.} = 0.8613 + j0.0247 \text{ p.u.}$$

RESULT

The voltages at the end of first iteration are,

$V_1^1 = 1.06 + j0 = 1.06\angle 0^\circ$ p.u

$V_2^1 = 1.0544 + j0.0379 = 1.0551\angle 2.06^\circ$ p.u

$V_3^1 = 0.7789 + j0.0446 = 0.7802\angle 3.28^\circ$ p.u

$V_4^1 = 0.8613 + j0.0247 = 0.8617\angle 1.64^\circ$ p.u

EXAMPLE 2.4

Let the reactive power constraint on generator bus-2 in example 2.2. be changed to $0.05 \le Q_2 \le 0.12$. With other data of example 2.2 remaining same, solve the voltages at the end of first iteration by Gauss-Seidel method.

SOLUTION

The formation of bus impedance matrix and calculation of $Q_{2,cal}^1$ are same as in example 2.2. The $Q_{2,cal}^1$ corresponding to initial guess $V_2^\circ = 1.04 + j0$ is 0.1456 p.u. This value of Q_2 violates the upper limit of the specified range for Q_2. Therefore the reactive power generation for bus-2 is fixed at 0.12 (upper limit) and the bus-2 is treated as load bus for 1st iteration.

Now $V_2^\circ = 1 + j0.0$, similar to other load buses for first iteration. But P & Q are considered as positive for bus-2 and P & Q are negative for other load buses.

The bus voltages are calculated using the following equation.

$$V_p^{k+1} = \frac{1}{Y_{pp}}\left[\frac{P_p - jQ_p}{(V_p^k)^*} - \sum_{q=1}^{p-1} Y_{pq}V_q^{k+1} - \sum_{q=p+1}^{n} Y_{pq}V_q^k\right]$$

$V_1^1 = V_1 = 1.06 + j0$ p.u. $\qquad$ ($\because$ Bus - 1 is a slack bus)

$$V_2^1 = \frac{1}{Y_{22}}\left[\frac{P_2 - jQ_2}{(V_2^o)^*} - Y_{21}V_1^1 - Y_{23}V_3^o - Y_{24}V_4^o\right]$$

$$V_2^1 = \frac{1}{2.6 - j11}\left[\frac{0.5 - j0.12}{1 - j0} - (-1 + j5)(1.06 + j0) - (-0.5 + j4)(1 + j0) - (-1.1 + j2)(1 + j0)\right]$$

$$= \frac{1}{2.6 - j11}\left[0.5 - j0.12 + 1.06 - j5.3 + 0.5 - j4 + 1.1 - j2\right]$$

$$= \frac{3.16 - j11.42}{2.6 - j11} = \frac{11.8491\angle -74.53^\circ}{11.3031\angle -76.70^\circ}$$

$$= 1.0483\angle 2.17^\circ \text{ p.u.} = 1.0475 + j0.0397 \text{ p.u.}$$

$$V_3^1 = \frac{1}{Y_{33}}\left[\frac{P_3 - jQ_3}{(V_3^o)^*} - Y_{31}V_1^1 - Y_{32}V_2^1 - Y_{34}V_4^o\right]$$

$$= \frac{1}{2.9 - j11}\left[\frac{-0.4 + j0.3}{1 - j0} - (-1.2 + j4)(1.06 + j0) - (-0.5 + j4)(1.0475 + j0.0397) - (-1.2 + j3)(1 + j0)\right]$$

$$= \frac{1}{2.9 - j11}\left[-0.4 + j0.3 + 1.272 - j4.24 - (-0.6826 + j4.1702) + 1.2 - j3\right]$$

$$= \frac{2.7546 - j8.4102}{2.9 - j11} = \frac{8.8498\angle -71.86^\circ}{11.3759\angle -75.23^\circ} = 0.7779\angle 3.37^\circ \text{ p.u.} = 0.7766 + j0.0457 \text{ p.u}$$

$$V_4^1 = \frac{1}{Y_{44}}\left[\frac{P_4 - jQ_4}{(V_4^o)^*} - Y_{41}V_1^1 - Y_{42}V_2^1 - Y_{43}V_3^1\right]$$

$$= \frac{1}{2.3 - j5}\left[\frac{-0.2 + j0.1}{1 - j0} - (0 \times 1.06) - (-1.1 + j2)(1.0475 + j0.0397) - (-1.2 + j3)(0.7766 + j0.0457)\right]$$

$$= \frac{1}{2.3 - j5}\left[-0.2 + j0.1 - (-1.2317 + j2.0513) - (-1.0690 + j2.2750)\right]$$

$$= \frac{2.1007 - j4.2263}{2.3 - j5} = \frac{4.7196\angle -63.57^\circ}{5.5036\angle -65.30^\circ}$$

$$= 0.8575\angle 1.73^\circ \text{ p.u.} = 0.8571 + j0.0259 \text{ p.u.}$$

RESULT

The voltages at the end of first iteration are,

$V_1^1 = 1.06 + j0 = 1.06\angle 0^\circ$ p.u

$V_2^1 = 1.0475 + j0.0397 = 1.0483\angle 2.17^\circ$ p.u

$V_3^1 = 0.7766 + j0.0457 = 0.7779\angle 3.37^\circ$ p.u

$V_4^1 = 0.8571 + j0.0259 = 0.8575\angle 1.73^\circ$ p.u

EXAMPLE 2.5

In the system shown in fig 2.5.1, generators are connected to all the four buses, while loads are at buses 2 and 3. The specifications of the buses are given in table 2.5.1. and line impedances in table 2.5.2. Assume that all the buses other than slack bus are PQ type. By taking a flat voltage profile, determine the bus voltages at the end of first Gauss-Seidel iteration.

Table 2.5.1 : Bus specifications

Buscode	P	Q	V	Remarks
1	-	-	$1.05\angle 0°$	Slack bus
2	0.5	−0.2	-	PQ bus
3	−1.0	0.5	-	PQ bus
4	0.3	−0.1	-	PQ bus

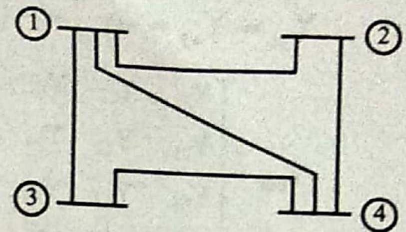


Table-2.5.2 : Line Impedances

Line	R in p.u.	X in p.u.
1-2	0.05	0.15
1-3	0.10	0.30
1-4	0.20	0.40
2-4	0.10	0.30
3-4	0.05	0.15

SOLUTION

First convert the line impedances to line admittances. The line admittances are given by the inverse of the line impedances.

From the table 2.5.2 we get the following line impedances.

$z_{12} = 0.05 + j0.15$ p.u. $\quad z_{24} = 0.10 + j0.30$ p.u.

$z_{13} = 0.10 + j0.30$ p.u. $\quad z_{34} = 0.05 + j0.15$ p.u.

$z_{14} = 0.20 + j0.40$ p.u.

$$\therefore y_{12} = \frac{1}{z_{12}} = \frac{1}{0.05 + j0.15} = 2 - j6 \quad ; \quad y_{13} = \frac{1}{z_{13}} = \frac{1}{0.10 + j0.30} = 1 - j3$$

$$y_{14} = \frac{1}{z_{14}} = \frac{1}{0.20 + j0.40} = 1 - j2 \quad ; \quad y_{24} = \frac{1}{z_{24}} = \frac{1}{0.10 + j0.30} = 1 - j3$$

$$y_{34} = \frac{1}{z_{34}} = \frac{1}{0.05 + j0.15} = 2 - j6$$

Note : *The inverse of a complex number can be easily obtained using a calculator in complex mode.*

The elements of bus admittance matrix can be obtained from the line admittances as shown below.

$$Y_{11} = y_{12} + y_{13} + y_{14} = 2 - j6 + 1 - j3 + 1 - j2 = 4 - j11$$
$$Y_{22} = y_{12} + y_{24} = 2 - j6 + 1 - j3 = 3 - j9$$
$$Y_{33} = y_{13} + y_{34} = 1 - j3 + 2 - j6 = 3 - j9$$
$$Y_{44} = y_{14} + y_{24} + y_{34} = 1 - j2 + 1 - j3 + 2 - j6 = 4 - j11$$

$$Y_{12} = Y_{21} = -y_{12} = -(2 - j6) = -2 + j6$$
$$Y_{13} = Y_{31} = -y_{13} = -(1 - j3) = -1 + j3$$
$$Y_{14} = Y_{41} = -y_{14} = -(1 - j2) = -1 + j2$$
$$Y_{23} = Y_{32} = 0$$
$$Y_{24} = Y_{42} = -y_{24} = -(1 - j3) = -1 + j3$$
$$Y_{34} = Y_{43} = -y_{34} = -(2 - j6) = -2 + j6$$

$$\mathbf{Y}_{bus} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & Y_{14} \\ Y_{21} & Y_{22} & Y_{23} & Y_{24} \\ Y_{31} & Y_{32} & Y_{33} & Y_{34} \\ Y_{41} & Y_{42} & Y_{43} & Y_{44} \end{bmatrix} = \begin{bmatrix} 4 - j11 & -2 + j6 & -1 + j3 & -1 + j2 \\ -2 + j6 & 3 - j9 & 0 & -1 + j3 \\ -1 + j3 & 0 & 3 - j9 & -2 + j6 \\ -1 + j2 & -1 + j3 & -2 + j6 & 4 - j11 \end{bmatrix}$$

The initial value of load bus voltages can be assumed as $1 + j0$ p.u.

$$\therefore V_2^{\circ} = 1 + j0 \quad ; \quad V_3^{\circ} = 1 + j0 \quad ; \quad V_4^{\circ} = 1 + j0$$

The slack bus (bus-1) voltage is not modified in any iteration

$$\therefore V_1^{\circ} = V_1^{1} = \ldots\ldots\ldots\ldots\ldots = V_1^{n} = V_1 = 1.05 + j0 \text{ p.u.}$$

The load bus voltages can be calculated using the following equation. For the first iteration $k = 0$. Here the table 2.5.1, provides the net power and so the values of P and Q are used as such in the following equation.

$$V_p^{k+1} = \frac{1}{Y_{pp}} \left[\frac{P_p - jQ_p}{(V_p^k)^*} - \sum_{q=1}^{p-1} Y_{pq} V_q^{k+1} - \sum_{q=p+1}^{n} Y_{pq} V_q^k \right]$$

$$V_1^1 = V_1 = 1.05 + j0 = 1.05\angle 0^{\circ} \qquad (\because \text{Bus-1 is slack bus})$$

$$V_2^1 = \frac{1}{Y_{22}} \left[\frac{P_2 - jQ_2}{(V_2^0)^*} - Y_{21} V_1^1 - Y_{23} V_3^0 - Y_{24} V_4^0 \right]$$

$$= \frac{1}{3 - j9} \left[\frac{0.5 + j0.2}{1 - j0} - (-2 + j6)\,(1.05 + j0) - 0 \times (1 + j0) - (-1 + j3)\,(1 + j0) \right]$$

$$= \frac{1}{3 - j9} \left[0.5 + j0.2 + 2.1 - j6.3 + 1 - j3 \right]$$

$$= \frac{3.6 - j9.1}{3 - j9} = \frac{9.7862\angle -68.42^{\circ}}{9.4868\angle -71.57^{\circ}} = 1.0316\angle 3.15^{\circ} = 1.0300 + j0.0567 \text{ p.u.}$$

$$V_3^1 = \frac{1}{Y_{33}} \left[\frac{P_3 - jQ_3}{(V_3^0)^*} - Y_{31} V_1^1 - Y_{32} V_2^1 - Y_{34} V_4^0 \right]$$

$$= \frac{1}{3 - j9} \left[\frac{-1 - j0.5}{1 - j0} - (-1 + j3)\,(1.05 + j0) - (0 \times V_2^1) - (-2 + j6)\,(1 + j0) \right]$$

$$= \frac{1}{3-j9}\left[-1-j0.5+1.05-j3.15+2-j6\right]$$

$$= \frac{2.05-j9.65}{3-j9} = \frac{9.8653\angle-78.01°}{9.4868\angle-71.57°} = 1.0399\angle-6.44° = 1.0333-j0.1166 \text{ p.u.}$$

$$V_4^1 = \frac{1}{Y_{44}}\left[\frac{P_4-jQ_4}{(V_4^0)^*} - Y_{41}V_1^1 - Y_{42}V_2^1 - Y_{43}V_3^1\right]$$

$$= \frac{1}{4-j11}\left[\frac{0.3+j0.1}{1-j0} - (-1+j2)(1.05+j0) - (-1+j3)(1.03+j0.0567) - (-2+j6)(1.0333-j0.1166)\right]$$

$$= \frac{1}{4-j11}\left[0.3+j0.1+1.05-j2.1-(-1.2001+j3.0333)-(-1.367+j6.433)\right]$$

$$= \frac{3.9171-j11.4663}{4-j11} = \frac{12.1169\angle-71.14°}{11.7047\angle-70.02°}$$

$$= 1.0352\angle-1.12° = 1.0350-j0.0202 \text{ p.u.}$$

RESULT

The bus voltages at the end of first Gauss-Seidel iteration are,

$V_1^1 = 1.05 + j0 = 1.05\angle 0°$ p.u

$V_2^1 = 1.0300 + j0.0567 = 1.0316\angle 3.15°$ p.u

$V_3^1 = 1.0333 - j0.1166 = 1.0399\angle -6.44°$ p.u

$V_4^1 = 1.0350 - j0.0202 = 1.0352\angle -1.12°$ p.u

EXAMPLE 2.6

In example 2.5 let the bus-2 be a PV bus (Generator bus) with $|V_2| = 1.07$ p.u. The reactive power constraint of the generator bus is $0.3 \le Q_2 \le 1.0$. With other data of example 2.5 remaining same (except Q_2), calculate the bus voltages at the end of first G-S iteration.

SOLUTION

The bus admittance matrix is same as that of example 2.5. The initial value of PQ bus (load bus) voltages can be assumed as 1 + j0 p.u.

$\therefore V_3^{\circ} = 1 + j0$; $V_4^{\circ} = 1 + j0$

The bus-1 is a slack bus,

$\therefore V_1^{\circ} = V_1^1 = \ldots\ldots\ldots\ldots = V_1^n = V_1 = 1.05 + j0$

The bus-2 is a generator bus, $\therefore V_2^{\circ} = 1.07 + j0$

Since the bus-2 is a generator bus its reactive power has to be calculated and checked for violation of specified limits. (In first iteration k = 0).

$$Q^1_{2,cal} = (-1) \times \text{Im}\left\{(V_2^o)^*\left[Y_{21}V_1^1 + Y_{22}V_2^o + Y_{23}V_3^o + Y_{24}V_4^o\right]\right\}$$

$$= -1 \times \text{Im}\left\{(1.07 - j0)\left[\begin{matrix}(-2 + j6)\ (1.05 + j0) + (3 - j9)\ (1.07 + j0) \\ + (0 \times V_3^o) + (-1 + j3)\ (1 + j0)\end{matrix}\right]\right\}$$

$$= -1 \times \text{Im}\left\{1.07\left[-2.1 + j6.3 + 3.21 - j9.63 - 1 + j3\right]\right\}$$

$$= -1 \times \text{Im}\left\{1.07[0.11 - j0.33]\right\} = 0.3531 \text{ p.u.}$$

The calculated value of reactive power, $Q^1_{2,cal}$ is within the specified limits. Hence the bus-2 can be treated as generator bus. Now, $Q_2 = Q^1_{2,cal} = 0.3531$ p.u., $P_2 = 0.5$ and $|V_2| = 1.07$. Since the bus-2 is a generator bus, magnitude of the bus-2 voltage r emains at 1.07 p.u. The phase of bus-2 voltage can be calculated using the following equation.

$$V^{k+1}_{p,temp} = \frac{1}{Y_{pp}}\left[\frac{P_p - jQ_p}{(V_p^k)^*} - \sum_{q=1}^{p-1} Y_{pq}V_q^{k+1} - \sum_{q=p+1}^{n} Y_{pq}V_q^k\right]$$

$$V^1_{2,temp} = \frac{1}{Y_{22}}\left[\frac{P_2 - jQ_2}{(V_2^o)^*} - Y_{21}V_1^1 - Y_{23}V_3^o - Y_{24}V_4^o\right]$$

$$= \frac{1}{3 - j9}\left[\frac{0.5 - j0.3531}{1.07 - j0} - (-2 + j6)\ (1.05 + j0) - (0 \times V_3^o) - (-1 + j3)\ (1 + j0)\right]$$

$$= \frac{1}{3 - j9}\ [0.4673 - j0.33 + 2.1 - j6.3 + 1 - j3]$$

$$= \frac{3.5673 - j9.63}{3 - j9} = \frac{10.2695\angle -69.67°}{9.4868\angle -71.57°} = 1.0825\angle 1.9°$$

$$\therefore \delta_2^1 = \angle V^1_{2,temp} = 1.9°$$

$$\therefore V_2^1 = |V_2|_{spec} \angle\delta_2^1 = 1.07\angle 1.9° \text{ p.u.} = 1.0694 + j0.0355 \text{ p.u.}$$

The voltage of bus-3 and bus-4 are calculated using the following equation.

$$V_p^{k+1} = \frac{1}{Y_{pp}}\left[\frac{P_p - jQ_p}{(V_p^k)^*} - \sum_{q=1}^{p-1} Y_{pq}V_q^{k+1} - \sum_{q=p+1}^{n} Y_{pq}V_q^k\right]$$

$$V_3^1 = \frac{1}{Y_{33}} = \left[\frac{P_3 - jQ_3}{(V_3^o)^*} - Y_{31}V_1^1 - Y_{32}V_2^1 - Y_{34}V_4^o\right]$$

$$= \frac{1}{3 - j9}\left[\frac{-1 - j0.5}{1 - j0} - (-1 + j3)\ (1.05 + j0) - (0 \times V_2^1) - (-2 + j6)\ (1 + j0)\right]$$

$$= \frac{1}{3 - j9}\ [-1 - j0.5 + 1.05 - j3.15 + 2 - j6] = \frac{2.05 - j9.65}{3 - j9} = \frac{9.8653\angle -78.01°}{9.4868\angle -71.57°}$$

$$= 1.0399\angle -6.44° = 1.0333 - j0.1166 \text{ p.u.}$$

$$V_4^1 = \frac{1}{Y_{44}} \left[\frac{P_4 - jQ_4}{(V_4^o)^*} - Y_{41}V_1^1 - Y_{42}V_2^1 - Y_{43}V_3^1 \right]$$

$$= \frac{1}{4 - j11} \left[\frac{0.3 + j0.1}{1 - j0} - (-1 + j2)\,(1.05 + j0) - (-1 + j3)\,(1.0694 + j0.0355) - (-2 + j6)\,(1.0333 - j0.1166) \right]$$

$$= \frac{1}{4 - j11} \left[0.3 + j0.1 + 1.05 - j2.1 - (-1.1759 + j3.1727) - (-1.367 + j6.433)\right]$$

$$= \frac{3.8929 - j11.6057}{4 - j11} = \frac{12.2412\angle -71.46^\circ}{11.7047\angle -70.02^\circ}$$

$$= 1.0458\angle -1.44^\circ = 1.0455 - j0.0263 \text{ p.u.}$$

RESULT

The bus voltages at the end of first Gauss-Seidel iteration are,

$V_1{}^1 = 1.05 + j0 = 1.05\angle 0^\circ$ p.u

$V_2{}^1 = 1.0694 + j0.0355 = 1.07\angle 1.9^\circ$ p.u

$V_3{}^1 = 1.0333 - j0.1166 = 1.0399\angle -6.44^\circ$ p.u

$V_4{}^1 = 1.0455 - j0.0263 = 1.0458\angle -1.44^\circ$ p.u

EXAMPLE 2.7

In example 2.6 let the reactive power constraint of the generator bus be changed to $0.4 \le Q_2 \le 1.0$. Now calculate the bus voltages at the end of first G.S. iteration.

SOLUTION

The bus admittance matrix and calculated value of reactive power are same as that of example 2.6. The calculated value of reactive power with initial voltage $V_2{}^o = 1.07 + j0$ p.u. is 0.3531 p.u. The calculated value is less than the specified lower limit and so the reactive power generation of bus-2 is fixed at 0.4 (the lower limit). Now the bus-2 is treated as load bus for this iteration.

Now, $Q_2 = 0.4$ p.u. ; $P_2 = 0.5$ and $V_2{}^o = 1 + j0$

The initial value of PQ bus (load bus) voltages are assumed as $1 + j0$ p.u.

$\therefore V_3{}^o = 1 + j0$; $V_4{}^o = 1 + j0$

The bus-1 is a slack bus,

$\therefore V_1{}^o = V_1{}^1 = \ldots\ldots\ldots\ldots = V_1{}^n = V_1 = 1.05 + j0$

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The voltages of the load buses are calculated using the following equation.

$$V_p^{k+1} = \frac{1}{Y_{pp}}\left[\frac{P_p - jQ_p}{(V_p^k)^*} - \sum_{q=1}^{p-1} Y_{pq}V_q^{k+1} - \sum_{q=p+1}^{n} Y_{pq}V_q^k\right]$$

$$V_2^1 = \frac{1}{Y_{22}}\left[\frac{P_2 - jQ_2}{(V_2^o)^*} - Y_{21}V_1^1 - Y_{23}V_3^o - Y_{24}V_4^o\right]$$

$$= \frac{1}{3-j9}\left[\frac{0.5-j0.4}{1-j0} - (-2+j6)\,(1.05+j0) - (0 \times V_3^o) - (-1+j3)\,(1+j0)\right]$$

$$= \frac{1}{3-j9}\,[0.5 - j0.4 + 2.1 - j6.3 + 1 - j3]$$

$$= \frac{3.6-j9.7}{3-j9} = \frac{10.3465\angle -69.64^\circ}{9.4868\angle -71.57^\circ}$$

$$= 1.0906\angle 1.93^\circ = 1.0900 + j0.0367 \text{ p.u.}$$

$$V_3^1 = 1.0399\angle -6.44^\circ = 1.0333 - j0.1166 \text{ p.u.}$$

***(Note :** Value of $V_3^{\,1}$ remains same as in example 2.6)*

$$V_4^1 = \frac{1}{Y_{44}} = \left[\frac{P_4 - jQ_4}{(V_4^o)^*} - Y_{41}V_1^1 - Y_{42}V_2^1 - Y_{43}V_3^1\right]$$

$$= \frac{1}{4-j11}\left[\begin{array}{l}\dfrac{0.3+j0.1}{1-j0} - (-1+j2)\,(1.05+j0) - (-1+j3)\,(1.09+j0.0367) \\ \qquad - (-2+j6)\,(1.0333-j0.1166)\end{array}\right]$$

$$= \frac{1}{4-j11}\,[0.3 + j0.1 + 1.05 - j2.1 - (-1.2001 + j3.2333) - (-1.367 + j6.433)]$$

$$= \frac{3.9171 - j11.6663}{4-j11} = \frac{12.3063\angle -71.44^\circ}{11.7047\angle -70.02^\circ}$$

$$= 1.0514\angle -1.42^\circ = 1.0511 - j0.0261 \text{ p.u.}$$

RESULT

The bus voltages at the end of first Gauss-Seidel iteration are,

$V_1^{\,1} = 1.05 + j0 = 1.05\angle 0^\circ$ p.u

$V_2^{\,1} = 1.09 + j0.0367 = 1.0906\angle 1.93^\circ$ p.u

$V_3^{\,1} = 1.0333 - j0.1166 = 1.0399\angle -6.44^\circ$ p.u

$V_4^{\,1} = 1.0511 - j0.0261 = 1.0514\angle -1.42^\circ$ p.u

EXAMPLE 2.8

In example 2.6 let the reactive power constraint of the generator bus be changed to $0.1 \le Q_2 \le 0.3$. Now calculate the bus voltages at the end of first G.S. iteration.

SOLUTION

The bus admittance matrix and calculated value of reactive power are same as that of example 2.6. The calculated value of reactive power with initial voltage $V_2^{\circ} = 1.07 + j0$ p.u. is 0.3531 p.u. The calculated value is greater than the specified upper limit and so the reactive power generation of bus-2 is fixed at 0.3 (the lower limit). Now bus-2 is treated as load bus for this iteration.

Now, $Q_2 = 0.3$ p.u. ; $P_2 = 0.5$ and $V_2^{\circ} = 1 + j0$

The initial value of PQ bus (load bus) voltages are assumed as $1 + j0$ p.u.

$\therefore V_3^{\circ} = 1 + j0$; $V_4^{\circ} = 1 + j0$

The bus-1 is a slack bus,

$\therefore V_1^{\circ} = V_1^{1} = \ldots\ldots\ldots\ldots = V_1^{n} = V_1 = 1.05 + j0$

The voltages of the load buses are calculated using the following equation.

$$V_p^{k+1} = \frac{1}{Y_{pp}}\left[\frac{P_p - jQ_p}{(V_p^k)^*} - \sum_{q=1}^{p-1} Y_{pq}V_q^{k+1} - \sum_{q=p+1}^{n} Y_{pq}V_q^k\right]$$

$$V_2^1 = \frac{1}{Y_{22}}\left[\frac{P_2 - jQ_2}{(V_2^0)^*} - Y_{21}V_1^1 - Y_{23}V_3^0 - Y_{24}V_4^0\right]$$

$$= \frac{1}{3-j9}\left[\frac{0.5-j0.3}{1-j0} - (-2+j6)(1.05+j0) - (0 \times V_3^0) - (-1+j3)(1+j0)\right]$$

$$= \frac{1}{3-j9}\left[0.5 - j0.3 + 2.1 - j6.3 + 1 - j3\right]$$

$$= \frac{3.6-j9.6}{3-j9} = \frac{10.2528\angle -69.44^{\circ}}{9.4868\angle -71.57^{\circ}}$$

$$= 1.0807\angle 2.13^{\circ} = 1.0800 + j0.0402 \text{ p.u.}$$

$$V_3^1 = 1.0399\angle -6.44^{\circ} = 1.0333 - j0.1166 \text{ p.u.}$$

(Note : Value of V_3^1 remains same as in example 2.6)

$$V_4^1 = \frac{1}{Y_{44}} = \left[\frac{P_4 - jQ_4}{(V_4^0)^*} - Y_{41}V_1^1 - Y_{42}V_2^1 - Y_{43}V_3^1\right]$$

$$= \frac{1}{4 - j11}\left[\frac{0.3 + j0.1}{1 - j0} - (-1 + j2)\,(1.05 + j0) - (-1 + j3)\,(1.08 + j0.0402) - (-2 + j6)\,(1.0333 - j0.1166)\right]$$

$$= \frac{1}{4 - j11}\left[0.3 + j0.1 + 1.05 - j2.1 - (-1.2006 + j3.1998) - (-1.367 + j6.433)\right]$$

$$= \frac{3.9176 - j11.6328}{4 - j11} = \frac{12.2748\angle -71.39°}{11.7047\angle -70.02°}$$

$$= 1.0487\angle -1.37° = 1.0484 - j0.0251 \text{ p.u.}$$

RESULT

The bus voltages at the end of first Gauss-Seidel iteration are,

$V_1^{\,1} = 1.05 + j0 = 1.05\angle 0°$ p.u

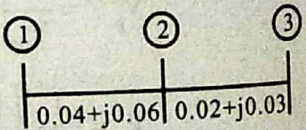
$V_2^{\,1} = 1.08 + j0.0402 = 1.0807\angle 2.13°$ p.u

$V_3^{\,1} = 1.0333 - j0.1166 = 1.0399\angle -6.44°$ p.u

$V_4^{\,1} = 1.0484 - j0.0251 = 1.0487\angle -1.37°$ p.u

EXAMPLE 2.9

For the network shown in fig 2.9.1, obtain the complex bus bar voltages at the end of first iteration, using G-S method. Bus-1 is a slack bus with $V_1 = 1.0\angle 0°$. Take $P_2 + jQ_2 = -5.96 + j1.46$, $P_3 = 6.02$ and $|V_3| = 1.02$. Assume, $V_3^{\circ} = 1.02\angle 0°$ and $V_2^{\circ} = 1\angle 0°$.



Note : Line impedances are in p.u.

Fig 2.9.1.

SOLUTION

From fig 2.9.1, the line impedances are

$z_{12} = 0.04 + j0.06$ p.u.

$z_{23} = 0.02 + j0.03$ p.u.

The line admittances are given by the inverse of line impedances.

$$y_{12} = \frac{1}{z_{12}} = \frac{1}{0.04 + j0.06} = 7.692 - j11.538 \text{ p.u.}$$

$$y_{23} = \frac{1}{z_{23}} = \frac{1}{0.02 + j0.03} = 15.385 - j23.077 \text{ p.u.}$$

The elements of bus admittance matrix are formed from the line admittances as shown below.

$Y_{11} = y_{12} = 7.692 - j11.538$
$Y_{22} = y_{12} + y_{23} = 7.692 - j11.538 + 15.385 - j23.077 = 23.077 - j34.615$
$Y_{33} = y_{23} = 15.385 - j23.077$
$Y_{12} = Y_{21} = -y_{12} = -(-7.692 - j11.538) = -7.692 + j11.538$
$Y_{13} = Y_{31} = 0$
$Y_{23} = Y_{32} = -y_{23} = -(15.385 - j23.077) = -15.385 + j23.077$

$$Y_{bus} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} \\ Y_{21} & Y_{22} & Y_{23} \\ Y_{31} & Y_{32} & Y_{33} \end{bmatrix} = \begin{bmatrix} 7.692 - j11.538 & -7.692 + j11.538 & 0 \\ -7.692 + j11.538 & 23.077 - j34.615 & -15.385 + j23.077 \\ 0 & -15.385 + j23.077 & 15.385 - j23.077 \end{bmatrix}$$

In the given system, bus-1 is slack bus, the bus-2 is load bus and bus-3 is generator bus. The initial value of the phase of V_3 is assumed as zero.

$\therefore V_1^{\circ} = 1.0\angle 0^{\circ}$; $V_2^{\circ} = 1.0\angle 0^{\circ}$; $V_3^{\circ} = 1.02\angle 0^{\circ}$

The bus-1 is a slack bus and so it's voltage will not change.

$\therefore V_1^1 = V_1^{\circ} = V_1 = 1.0\angle 0^{\circ} = 1 + j0$ p.u.

The bus-2 is a load bus. The voltage of bus-2 is calculated using the following equation. For first iteration, k = 0.

$$V_p^{k+1} = \frac{1}{Y_{pp}}\left[\frac{P_p - jQ_p}{(V_p^k)^*} - \sum_{q=1}^{p-1} Y_{pq}V_q^{k+1} - \sum_{q=p+1}^{n} Y_{pq}V_q^k\right]$$

$$V_2^1 = \frac{1}{Y_{22}}\left[\frac{P_2 - jQ_2}{(V_2^o)^*} - Y_{21}V_1^1 - Y_{23}V_3^o\right]$$

$$= \frac{1}{23.077 - j34.615}\left[\frac{-5.96 - j1.46}{1 - j0} - (-7.692 + j11.538)(1 + j0) - (-15.385 + j23.077)(1.02 + j0)\right]$$

$$= \frac{1}{23.077 - j34.615}[-5.96 - j1.46 + 7.692 - j11.538 + 15.6927 - j23.5385]$$

$$= \frac{17.4247 - j36.5365}{23.077 - j34.615} = \frac{40.4788\angle -64.50^{\circ}}{41.6022\angle -56.31^{\circ}}$$

$$= 0.973\angle -8.19^{\circ}$$

$$= 0.96307 - j0.13861 \text{ p.u.}$$

The bus-3 is a generator bus. The magnitude of the voltage does not change in a generator bus. The phase of the voltage can be calculated from the following equation.

$$V_{p,temp}^{k+1} = \frac{1}{Y_{pp}}\left[\frac{P_p - jQ_p}{(V_p^k)^*} - \sum_{q=1}^{p-1} Y_{pq}V_q^{k+1} - \sum_{q=p+1}^{n} Y_{pq}V_q^k\right]$$

The phase of V_3^1 is the phase of $V^1_{3,temp}$ and the magnitude of V_3^1 is the specified voltage.

i.e., $|V_3^1| = |V_3|_{spec} = 1.02$ p.u.

Let initial value of bus-2 voltage, $V_3^o = 1.02 + j0$

In order to calculate the phase of V_3^1 we require the value of Q_3 which can be calculated using the following equation.

$$Q^1_{3,cal} = -1 \times \text{Im}\left\{(V_3^o)^* \left[Y_{31}V_1^1 + Y_{32}V_2^1 + Y_{33}V_3^o\right]\right\}$$

$$= -1 \times \text{Im}\left\{(1.02 - j0)\begin{bmatrix}(0 \times V_1^1) + (-15.385 + j23.077)(0.96307 - j0.13861) \\ + (15.385 - j23.077)(1.02 + j0)\end{bmatrix}\right\}$$

$$= -1 \times \text{Im}\left\{1.02\left[-11.6181 + j24.3573 + 15.6927 - j23.5385\right]\right\}$$

$$= -0.8351 \text{ p.u.}$$

***Note :** For the generator bus the limits of reactive power are not specified and so the calculated reactive power can be used as such.*

$$V^1_{3,temp} = \frac{1}{Y_{33}}\left[\frac{P_3 - jQ_3}{(V_3^o)^*} - Y_{31}V_1^1 - Y_{32}V_2^1\right]$$

$$= \frac{1}{15.385 - j23.077}\left[\frac{6.02 + j0.8351}{1.02 - j0} - (0 \times V_1^1) - (-15.385 + j23.077)(0.96307 - j0.13861)\right]$$

$$= \frac{1}{15.385 - j23.077}\left[5.90196 + j0.81872 - (-11.6181 + j24.3566)\right]$$

$$= \frac{17.52006 - j23.53788}{15.385 - j23.077} = \frac{29.3425\angle -53.34^\circ}{27.73529\angle -56.31^\circ} = 1.05795\angle 2.97^\circ$$

$$\therefore \delta_3^1 = \angle V^1_{3,temp} = 2.97^\circ$$

$$\therefore V_3^1 = |V_3|_{spec}\angle\delta_3^1 = 1.02\angle 2.97^\circ \text{ p.u.} = 1.01863 + j0.05285 \text{ p.u.}$$

RESULT

The bus voltages at the end of first iteration Gauss-Seidel iteration are,

$V_1^1 = 1 + j0 = 1.0\angle 0^\circ$ p.u

$V_2^1 = 0.96307 - j0.13861 = 0.973\angle -8.19^\circ$ p.u

$V_3^1 = 1.01863 + j0.05285 = 1.02\angle 2.97^\circ$ p.u.

EXAMPLE 2.10

Fig 2.10.1 shows a three bus power system.

Bus 1 : Slack bus, $V = 1.05\angle 0°$ p.u.

Bus 2 : PV bus, $|V| = 1.0$ p.u, $P_g = 3$ p.u.

Bus 3 : PQ bus, $P_L = 4$ p.u, $Q_L = 2$ p.u.

Carry out one iteration of load flow solution by Gauss-Seidel method. Neglect limits on reactive power generation.

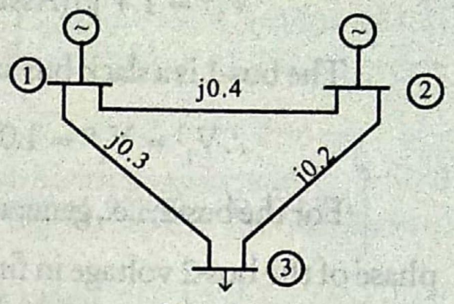


Fig 2.10.1

SOLUTION

From fig 2.10.1, the line impedances are,

$$z_{12} = j0.4 \text{ p.u.}$$
$$z_{13} = j0.3 \text{ p.u.}$$
$$z_{23} = j0.2 \text{ p.u.}$$

The line admittances are given by the inverse of line impedances.

$$y_{12} = \frac{1}{z_{12}} = \frac{1}{j0.4} = -j2.5 \text{ p.u.}$$

$$y_{13} = \frac{1}{z_{13}} = \frac{1}{j0.3} = -j3.333 \text{ p.u.}$$

$$y_{23} = \frac{1}{z_{23}} = \frac{1}{j0.2} = -j5 \text{ p.u.}$$

The elements of bus admittance matrix are formed from the line admittances as shown below.

$$Y_{11} = y_{12} + y_{13} = -j2.5 - j3.33 = -j5.833$$
$$Y_{22} = y_{12} + y_{23} = -j2.5 - j5 = -j7.5$$
$$Y_{33} = y_{13} + y_{23} = -j3.333 - j5 = -j8.333$$
$$Y_{12} = Y_{21} = -y_{12} = -(-j2.5) = j2.5$$
$$Y_{13} = Y_{31} = -y_{13} = -(-j3.333) = j3.333$$
$$Y_{23} = Y_{32} = -y_{23} = -(j5) = j5$$

$$Y_{bus} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} \\ Y_{21} & Y_{22} & Y_{23} \\ Y_{31} & Y_{32} & Y_{33} \end{bmatrix} = \begin{bmatrix} -j5.833 & j2.5 & j3.333 \\ j2.5 & -j7.5 & j5 \\ j3.333 & j5 & -j8.333 \end{bmatrix}$$

In the given system bus-1 is slack bus, the bus-2 is generator bus and bus-3 is load bus. The initial values of bus voltages are as follows

$$V_1^{\circ} = 1.05\angle 0° = 1.05 + j0 \text{ p.u. (Specified value)}$$

$V_2^{\circ} = 1.0\angle 0^{\circ} = 1.0 + j0$ p.u. (Initial phase is assumed zero)

$V_3^{\circ} = 1 + j0$ (Assumed value)

The bus-1 is a slack bus and so its voltage will not change in any iteration.

$\therefore V_1^1 = V_1^{\circ} = 1.05\angle 0^{\circ} = 1.05 + j0$ p.u.

For the bus-2, i.e., generator bus the magnitude of the voltage is the specified value. The phase of the bus-2 voltage in first iteration is given by phase of $V_{p,temp}^{k+1}$, when p = 2 & k = 0.

$$V_{p,temp}^{k+1} = \frac{1}{Y_{pp}}\left[\frac{P_p - jQ_p}{(V_p^k)^*} - \sum_{q=1}^{p-1} Y_{pq}V_q^{k+1} - \sum_{q=p+1}^{n} Y_{pq}V_q^k\right]$$

To calculate the phase using the above equation we have to estimate the reactive power P_2. The reactive power of bus-2 is given by the following equation.

$$Q_{p,cal}^{k+1} = (-1) \times \text{Im}\left\{(V_p^k)^*\left[\sum_{q=1}^{p-1} Y_{pq}V_q^{k+1} + \sum_{q=p}^{n} Y_{pq}V_q^k\right]\right\}$$

$$\therefore Q_{2,cal}^1 = (-1) \times \text{Im}\left\{(V_2^o)^*\left[Y_{21}V_1^1 + Y_{22}V_2^o + Y_{23}V_3^o\right]\right\}$$

$$= -1 \times \text{Im}\left\{(1 - j0)\left[j2.5(1.05 + j0) + (-j7.5)(1 + j0) + j5(1 + j0)\right]\right\}$$

$$= -1 \times \text{Im}\left\{j2.625 - j7.5 + j5\right\} = -0.125 \text{ p.u.}$$

$$\text{Now}, Q_2 = -0.125, \quad P_2 = 3, \quad V_2^o = 1 + j0, \quad |V_2|_{spec} = 1.0$$

$$\therefore V_{2,temp}^1 = \frac{1}{Y_{22}}\left[\frac{P_2 - jQ_2}{(V_2^o)^*} - Y_{21}V_1^1 - Y_{23}V_3^o\right]$$

$$= \frac{1}{-j7.5}\left[\frac{3 + j0.125}{1 - j0} - (j2.5)(1.05 + j0) - (j5)(1 + j0)\right]$$

$$= \frac{1}{-j7.5}\left[3 + j0.125 - j2.625 - j5\right] = \frac{1}{-j7.5}\left[3 - j7.5\right]$$

$$= 1 + j0.4 = 1.077\angle 21.8^{\circ}$$

$$\therefore \delta_2^1 = \angle V_{2,temp}^1 = 21.8^{\circ}$$

$$\therefore V_2^1 = |V_2|_{spec}\ \angle\delta_2^1 = 1.0\angle 21.8^{\circ} = 0.92849 + j0.37137$$

The bus-3 in load bus and its voltage in first iteration is given by the following equation when p = 3 and k = 0. The specified power are load powers, and so they are considered as negative powers.

$\therefore P_3 = -P_L = -4$ and $Q_3 = -Q_L = -2$

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$$V_p^{k+1} = \frac{1}{Y_{pp}}\left[\frac{P_p - jQ_p}{(V_p^k)^*} - \sum_{q=1}^{p-1} Y_{pq}V_q^{k+1} - \sum_{q=p+1}^{n} Y_{pq}V_q^k\right]$$

$$\therefore V_3^1 = \frac{1}{Y_{33}}\left[\frac{P_3 - jQ_3}{(V_3^k)^*} - Y_{31}V_1^1 - Y_{32}V_2^1\right]$$

$$= \frac{1}{-j8.333}\left[\frac{-4+j2}{1-j0} - (j3.333)\,(1.05+j0) - (j5)\,(0.92849+j0.37137)\right]$$

$$= \frac{1}{-j8.333}\,[-4+j2-j3.49965+1.85685-j4.64245]$$

$$= \frac{-2.14315-j6.1421}{-j8.3333} = \frac{6.50527\angle-109.24°}{8.3333\angle-90°}$$

$$= 0.78064\angle-19.24° = 0.73704 - j0.25724 \text{ p.u.}$$

RESULT

The bus voltages at the end of first Gauss-Seidel iteration are,

$V_1^1 = 1.05 + j0 = 1.05\angle 0°$ p.u

$V_2^1 = 0.92849 + j0.37137 = 1.0\angle 21.8°$ p.u

$V_3^1 = 0.73704 - j0.25724 = 0.78064\angle -19.24°$ p.u.