An Analytical Approximation of Gravitational Waves

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Introduction

Gravitational Waves, as predicted by Einstein’s Theory of General Relativity, are an outward flux of energy from a binary system of massive objects such as neutron stars or black holes.[1] These waves propagate at the speed of light and contrary to light waves, have polarizations that differ by 45 degrees instead of 90. These are called the plus polarization and cross polarization, which are aptly named based on appearance. Additionally, they are not characterized by amplitude, but instead by strain, which is a measurement of the displacement of spacetime.[2]

Research Goals

- Translate a pre-existing model from Mathematica to Python in a Jupyter Notebook[1]
- Include detailed physics into the notebook to make the code more accessible for undergraduate students

(For a more detailed explanation of the physics involved, feel free to visit the GitHub page)

Inspiral Model[1,3,4,5,6]

- Utilizes Post-Newtonian (PN) Approximations to determine the PN parameter x
- Kepler’s Third Law allows the angular frequency of the system to be found
- By integrating the frequency, the phase is found
- The radial separation is found by implementing order 3 PN corrections
- Combining those pieces allows for the strain of both the plus polarization, modeled by the real component, and the cross polarization, modeled by the imaginary component to be found

Abstract

The goal of this project is to integrate pre-existing analytical models of gravitational waves into Python so that undergraduate students have access to a model for gravitational waves that is straightforward and easy to understand. To facilitate this, we break the problem into two phases, the inspiral and the merger-ringdown. The inspiral phase is modeled using Post-Newtonian (PN) theory. The merger-ringdown phase utilizes an analytical model called the *Implicit Rotating Source* (IRS) that creates an analytical fit to data created by numerical relativity. To create the final waveform, we use two different matching techniques to combine the merger-ringdown and inspiral waveforms. Future students can use this template we have created to test the generated waveform with experimental data, create solutions for parameters such as non-zero eccentricity, statistically determine the accuracy of the matching technique, and conduct a wide variety of other research projects.

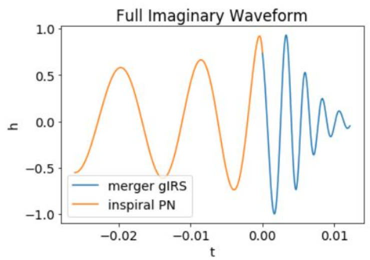
Merger-Ringdown Model[1,3,5]

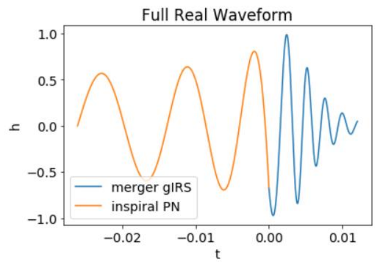
- Utilizes the *Implicit Rotating Source* (IRS) model
- Based off of analytic fits to numerical relativity
- Overall, follows a similar approach to the Inspiral Model in terms of derivations

Matching Techniques

- Manually adjust a parameter until they match by inspection
- Automatically matching the end of the inspiral to an equal value on the merger

Results





Conclusions

Overall, the project was successful in creating a waveform for gravitational waves emitted by a binary system of equal mass objects with a combined mass of 40 solar masses and an eccentricity of 0. While this model has yet to be tested with data collected experimentally and data produced through other analytical methods, the waveform generated shows high similarity when visually compared to other successful models.[1,3,5]

Further Research

- Fix impractical coding practices to shorten the time it takes to run the code
- Modify the code to work with capabilities for handling more mass ratios and non-zero eccentricity
- Perform statistical analysis on matching techniques to determine the best way to match the phases
- Compare results to experimental data
- Implement a separate model for the ringdown phase

References & Suggested Resources

GitHub Link: [AnalyticGravWave/AnalyticGravWaveM40.ipynb at master · devaoneil/AnalyticGravWave · GitHub]

[1]Buskirk and Babiuc. (2018). A Complete Analytic Gravitational Wave Model for Undergraduates. https://arxiv.org/pdf/1810.06160.pdf

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[3]East et al., (2013). Observing complete gravitational wave signals from dynamical capture binaries. Physical Review D. https://arxiv.org/pdf/1212.0837.pdf

[4] Hinder et al., (2010). Comparisons of eccentric binary black hole simulations with post-Newtonian models. Physical Review D. https://arxiv.org/pdf/0806.1037.pd

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