Topics: Normal distribution, Functions of Random Variables

1. The time required for servicing transmissions is normally distributed with μ = 45 minutes and σ = 8 minutes. The service manager plans to have work begin on the transmission of a customer's car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?

   A. 0.3875
   B. 0.2676
   C. 0.5
   D. 0.6987

Ans: Given with μ = 45 minutes and σ = 8 minutes.
Let X = amount of time it takes to complete the repair on a customer's car.
To finish in one hour, you must have X ≤ 50 so the question is to find P (X > 50).
P (X > 50)
= 1 - P (X ≤ 50).
Z = (X -□)/□
= (X - 45)/8
Thus, the question can be answered by using the normal table to find
P (X ≤ 50)
= P (Z ≤ (50 - 45)/8)
= P (Z ≤ 0.625)
= 73.4%
Probability that the service manager will not meet his demand will be
= 100-73.4
= 26.6% or 0.266.

2. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean μ = 38 and Standard deviation σ =6. For each statement below, please specify True/False. If false, briefly explain why.
   A. More employees at the processing center are older than 44 than between 38 and 44.
   B. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Ans:

```
PX44 = 1-stats.norm.cdf(44,loc=38,scale=6)
PX44 = 0.1586
PX4438 = stats.norm.cdf(44,38,6)-stats.norm.cdf(38,38,6)
PX4438 = 0.3413
```

The probability of employee age betweeen 38 and 44 is 34.13 %
The probability of employee age more than 44 is 15.87 %
Answer: False
A training program for employees under the age of 30 at the center would be expected to attract about 36.0 employees

STATEMENT A IS FALSE SINCE FROM ABOVE p(38<X<44)> p(X>44)

```
P30 = stats.norm.cdf(30,38,6)
P30 = 0.0912
ATTENDING EMPLOYEES UNDER 30 FROM 400TOTAL
EXP = 400*stats.norm.cdf(30,38,6)
EXP = 36.48
```

A training program for employees under the age of 30 at the center would be expected to attract about 36 employees so Statement B is TRUE

3. If $X_1 \sim N(\mu, \sigma^2)$ and $X_2 \sim N(\mu, \sigma^2)$ are *iid* normal random variables, then what is the difference between 2 X_1 and $X_1 + X_2$? Discuss both their distributions and parameters.

From the properties of **normal random variables**,
if $X \sim N(\mu_1, \sigma_1^2)$ and $Y \sim N(\mu_2, \sigma_2^2)$ are two independent identically distributed random variables then

- the **sum** of normal random variables is given by
  $X + Y \sim N(\mu_1 + \mu_2, \sigma_1^2 + \sigma_2^2)$,
- and the **difference** of normal random variables is given by
  $X - Y \sim N(\mu_1 - \mu_2, \sigma_1^2 + \sigma_2^2)$
- When $Z = aX$, the **product** of X is given by
  $Z \sim N(a\mu_1, a^2\sigma_1^2)$
- When $Z = aX + bY$, the **linear combination** of X and Y is given by
  $Z \sim N(a\mu_1 + b\mu_2, a^2\sigma_1^2 + b^2\sigma_2^2)$

STEP1: FIND 2X1 WITH PRODUCT FORMULA

$$2X_1 \sim N(2\mu, 2^2\sigma^2) \implies 2X_1 \sim N(2\mu, 4\sigma^2)$$

STEP2: SUM OF X1 AND X2,

$$X_1 + X_2 \sim N(\mu + \mu, \sigma^2 + \sigma^2) \sim N(2\mu, 2\sigma^2)$$

STEP3: DIFFERENCE OF ABOVE TWO VALUES

$$2X_1 - (X_1 + X_2) \sim N(2\mu - 2\mu, 2\sigma_1^2 + 4\sigma_2^2) \sim N(0, 6\sigma^2)$$

MEAN OF $2X_1$ AND X_1 AND X_1+X_2 IS SAME

VARIANCE OF $2X_1$ IS TWICE THAT OF (X_1 AND X_2)

The difference between the two says that the two given variables are **identically** and **independently** distributed.

4. Let X ~ N(100, 20). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.

   A. 90.5, 105.9
   B. 80.2, 119.8
   C. 22, 78
   D. 48.5, 151.5
   E. 90.1, 109.9

Ans. **P(a<x<b) = 0.99**

Mean = 100

Std Dev= 20

Finding z value at 0.5th percentile and 99.5th percentile we get -2.576, +2.576

Z = stats.norm.ppf(0.005), Z = stats.norm.ppf(0.995)

Z= (x-100)/20

X= 20z+100

For 0.5th percentile Zcalc(20*-2.576+100)= 48.5

For 99.5th percentile Zcalc(20*2.576+100)= 151.5

Hence Ans D is correct

5. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions $Profit_1 \sim N(5, 3^2)$ and $Profit_2 \sim N(7, 4^2)$ respectively. Both the profits are in \$ Million. Answer the following questions about the total profit of the company in Rupees. Assume that \$1 = Rs. 45
    A. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
    B. Specify the 5^{th} percentile of profit (in Rupees) for the company
    C. Which of the two divisions has a larger probability of making a loss in a given year?

Ans:

A. Range is Rs (99.01, 982.00) in Millions

B. 5th percentile of profit (in Million Rupees) is 170.0

C. P(Div 1) making loss =0.047,P(Div B) making loss = 0.040 Hence Div 1 has a larger probability of making a loss in a given year