In[300]:=

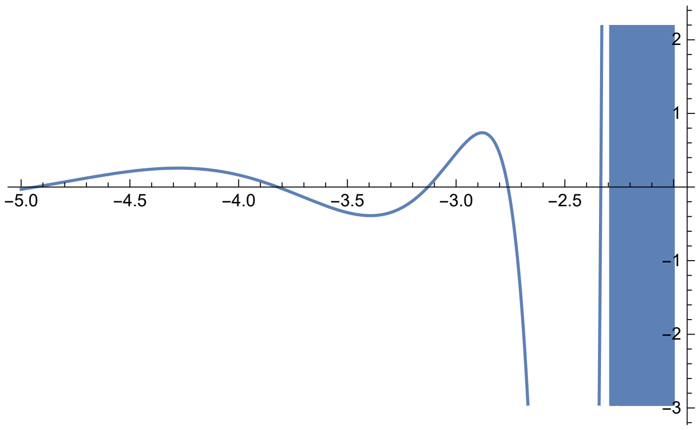
In[301]:=

```
(*Note that,
  the Schrodinger-Poisson equation written earlier in the readme file has now been
    written in its expanded form for practical purpose. We consider the value
    of \Phi(r = 0) as the parameter p. Then we plot ψ(r) at boundary value)
 denoted as f(p) as a function of the paramter p*. In this computation,
we start with black mass zero or that it is absent,
which is why another term V(r) is not explicit below*)


f[p_? NumberQ] :=
 First[ψ[6] /. NDSolve[{(1/2) * ψ''[r] + (1/r) ψ'[r] - 2 ψ[r] / r^2 == Φ[r] × ψ[r],
     Φ''[r] + (2/r) Φ'[r] == (1/2) ψ[r]^2, Φ[0.01] == p,
     ψ[0.01] == 0, ψ'[0.01] == 1, Φ'[0.01] == 0}, {ψ, Φ}, {r, 0.01, 6}]]

(*It is important to note that in general f(p) may have many
 roots as displayed below. However for the problem of interest we
 are interested only in the ground state or lowest energy solution,
which is obtained when the minimum value of the root in magnitude is taken. Thus,
it is crucial to choose the range of p in a sensible manner
*)
```

In[302]:=

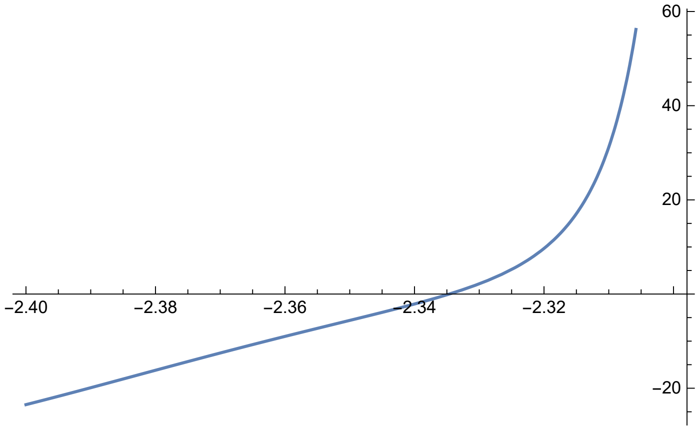
```
Plot[f[p], {p, 0 - 2, -5}]
```

Out[302]=



In[303]:=
```
(*From above plot, it is evident that the first true root appears before -2.5,
hence to locate the exact point we need to modify our
 range appropriately. It is worth mentioning that this process of
 choosing th range involves a reasonable amount of trial and error,
which is precisely why the method is called "shooting" method. The default
 amount of educated guess work involved seems unavoidable to us*)

Plot[f[p], {p, -2.3, -2.4}]
```

Out[303]=



In[304]:=

In[305]:=

In[306]:=
```
(*From the above plot it is evident that the exct value of p lies somewhere between -
 2.3 and -2.4, and this value is given by the following command*)
```

In[307]:=
```
FindRoot[f[p], {p, -2.3, -2.4}]
```

Out[307]= {p → -2.33463}

In[308]:=
```
{p → -2.3346327581951587`}

(*Using the above value of p,
we can then solve the Schrodinger-Poisson equation exactly*)
```

Out[308]= {p → -2.33463}

In[309]:=
```
sol = NDSolve[{(1/2) * ψ''[r] + (1/r) ψ'[r] - 2 ψ[r] / r^2 == ψ[r] × Φ[r],
    Φ''[r] + (2/r) Φ'[r] == (1/2) ψ[r]^2, Φ[0.01] == p, ψ[0.01] == 0,
    ψ'[0.01] == 1, Φ'[0.01] == 0} /. %, {ψ, Φ}, {r, 0.01, 6}]
```

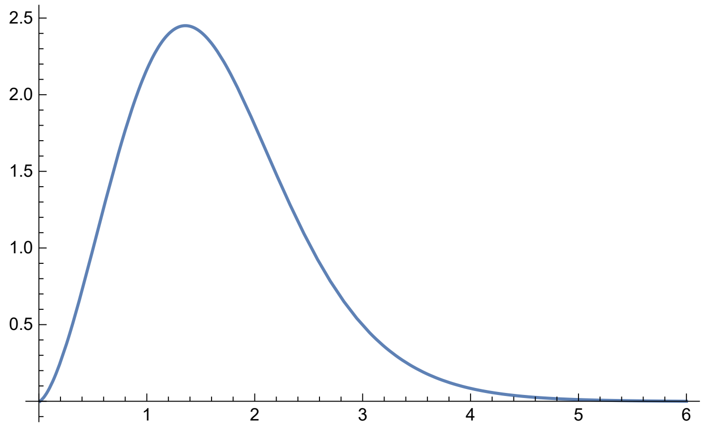
Out[309]=
```
{{ψ → InterpolatingFunction[ Domain: {{0.01, 6.}} Output: scalar ],
  Φ → InterpolatingFunction[ Domain: {{0.01, 6.}} Output: scalar ]}}
```

In[310]:=

In[311]:=

In[312]:=
```
Plot[ψ[r] /. sol, {r, 0.01, 6}, PlotRange → Automatic]
```

Out[312]=



In[313]:=

In[314]:=
```
(*As expected, one notices that the function goes to zero at large distance which is
 in agreement with the boundary condition we earier considered while finding the
 root. The above process is then repeated with the presence of black hole whose mass
 defined as a parameter α is changed and correspondingly the plots are obtained*)
```

In[315]:=

In[316]:=

In[317]:=

In[318]:=

In[319]:=

In[320]:=

In[321]:=

In[322]:=

In[323]:=