



(\* Here, we denote the BH potential as  $V = -GMm/r$  and take the parameter  $\alpha = GMm$  to give the strength of BH \*)

```
sol = ParametricNDSolve [{(1/2)* $\phi$ ''[r] + (1/r) $\phi$ '[r] - 2 $\phi$ [r]/r^2 + ( $\alpha$ /r)* $\phi$ [r] ==  $\psi$ [r]* $\phi$ [r],
 $\psi$ ''[r] + (2/r) $\psi$ '[r] == (1/2) $\phi$ [r]^2,  $\psi$ [0.01] == p,  $\phi$ [0.01] == 0,
 $\phi$ '[0.01] == 1,  $\psi$ '[0.01] == 0}, { $\phi$ ,  $\psi$ }, {r, 0.01, 6}, {p,  $\alpha$ }
```

Out[393]=  $\phi \rightarrow$  ParametricFunction [  Expression :  $\phi$   
Parameters : {p,  $\alpha$ } ],  
  
 $\psi \rightarrow$  ParametricFunction [  Expression :  $\psi$   
Parameters : {p,  $\alpha$ } ] }

```
In[394]:= Plot[Evaluate[{ $\phi$ [-2.33463, 0][r],  $\phi$ [-1.81523, 0.5][r],  $\phi$ [-1.31995, 1][r],
 $\phi$ [-0.850292, 1.5][r],  $\phi$ [-0.407773, 2][r],  $\phi$ [0.00667873, 2.5][r]} /. sol],
{r, 0.01, 7}, PlotLegends -> {" $\alpha=0$ ", " $\alpha=0.5$ ", " $\alpha=1$ ", " $\alpha=1.5$ ", " $\alpha=2$ ", " $\alpha=2.5$ "}]
```

