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(Q1) Use central difference scheme solve coupled system of DE. to know $y(x)$ and $z(x)$

step size $\boxed{h = 0.2}$

$$y'' + (x-1)y' - 6z = x^2 \quad \text{—— (I)}$$

$$z'' - 2z' + xy = 4x+1 \quad \text{—— (II)}$$

$$y(0) = y'(2) = 0$$

$$z(0) = z'(2) = 0$$

Discretizing (I) using finite difference scheme.

$$\left(\frac{y_{i-1} - 2y_i + y_{i+1}}{h^2}\right) + (x-1)\left(\frac{y_{i+1} - y_{i-1}}{2h}\right) - 6z_i = x_i^2$$

$$\left[\begin{array}{l} y_{i-1}\left(\frac{1}{h^2} - \frac{(x-1)}{2h}\right) + y_i\left(-\frac{2}{h^2}\right) + y_{i+1}\left(\frac{1}{h^2} + \frac{(x-1)}{2h}\right) \\ +(0)\,z_{i-1} + (-6)z_i + (0)\,z_{i+1} \end{array}\right] = x_i^2$$

Discretizing (II) using finite difference scheme

$$\left(\frac{z_{i-1} - 2z_i + z_{i+1}}{h^2}\right) - 2\left(\frac{z_{i+1} - z_{i-1}}{2h}\right) + x_i y_i = 4x_i + 1.$$

$$\left[\begin{array}{l} y_{i-1}(0) + y_i(x_i) + y_{i+1}(0) \\ + z_{i-1}\left(\frac{1}{h^2} + \frac{1}{h}\right) + z_i\left(-\frac{2}{h^2}\right) + z_{i+1}\left(\frac{1}{h^2} - \frac{1}{h}\right) \end{array}\right] = 4x_i + 1$$

$$\begin{bmatrix} \left(\frac{1}{h^2} - \frac{(x-1)}{2h}\right) & 0 \\ 0 & \left(\frac{1}{h^2} + \frac{1}{h}\right) \end{bmatrix} \begin{bmatrix} y_{j-1} \\ z_{j-1} \end{bmatrix} + \begin{bmatrix} \left(-\frac{2}{h^2}\right) & (-6) \\ (x_j) & \left(-\frac{2}{h^2}\right) \end{bmatrix} \begin{bmatrix} y_j \\ z_j \end{bmatrix} + \begin{bmatrix} \left(\frac{1}{h^2} + \frac{(x-1)}{2h}\right) & 0 \\ 0 & \left(\frac{1}{h^2} - \frac{1}{h}\right) \end{bmatrix} \begin{bmatrix} y_{j+1} \\ z_{j+1} \end{bmatrix}$$

Q2 Solve Non Linear BVP using finite difference scheme and Newtons Linearization $h = 0.1$

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$$y''(y-y') = e^x - 1$$
$$y(0) = 1 \quad y(1) = 0$$

$$y''y - y''y' = e^x - 1$$

Discretizing it we get using finite difference scheme.

$$\left(\frac{y_{i-1} - 2y_i + y_{i+1}}{h^2}\right)\left(y_i - \left(\frac{y_{i+1} - y_{i-1}}{2h}\right)\right) = e^{x_i} - 1$$

$y_{i-1} = Y_m$
$y_i = Y_i$
$y_{i+1} = Y_p$

Substitute value of $h = 0.1$

$$100\left(y_{i-1} - 2y_i + y_{i+1}\right)\left(y_i - 5\left(y_{i+1} - y_{i-1}\right)\right) = e^{x_i} - 1$$

$$F = \begin{bmatrix} 100y_{i-1}y_i - 500y_{i-1}y_{i+1} + 500y_{i-1}^2 \\ -200y_i^2 + 1000y_iy_{i+1} - 1000y_iy_{i-1} \\ + 100y_{i+1}y_i - 500y_{i+1}^2 + 500y_{i+1}y_{i-1} \end{bmatrix} - \left(e^{x_i} - 1\right) = 0$$

Lets find value of

$$a_i \Rightarrow \left(\frac{\partial F}{\partial y_{i-1}}\right) = \left(100y_i \cancel{- 500y_{i+1}} + 1000y_{i-1} - 1000y_i \cancel{+ 500y_{i+1}}\right)$$
$$= \left(1000y_{i-1} - 900y_i\right)$$

$$b_i \Rightarrow \left(\frac{\partial F}{\partial y_i}\right) = 100y_{i-1} - 400y_i + 1000y_{i+1} - 1000y_{i-1} + 100y_{i+1}$$
$$= \left(-900y_{i-1} - 400y_i + 1100y_{i+1}\right)$$

$$c_i \Rightarrow \left(\frac{\partial F}{\partial y_{i+1}}\right) = \cancel{-500y_{i-1}} + 1000y_i + 100y_i \cancel{+ 500y_{i-1}} - 1000y_{i+1}$$
$$= \left(1100y_i - 1000y_{i+1}\right)$$

$$\left(\frac{y_{i-1}y_i}{h^2} - \cancel{\frac{y_{i-1}y_{i+1}}{2h^3}} + \frac{y_{i-1}^2}{2h^3}\right) - \frac{y_i^2}{h^2} + \frac{y_i y_{i+1}}{h^2} - \frac{y_i y_{i+1}}{h^3}$$

$$\frac{y_{i+1}y_i}{h^2} - \frac{y_{i+1}^2}{2h^3} + \cancel{\frac{y_{i-1}y_{i+1}}{2h^3}}$$

In terms of h

$$\frac{\partial F}{\partial y_{i-1}} = \frac{y_i}{h^2} + \frac{y_{i-1}}{h^3} - \frac{y_i}{h^3}.$$

$$y_i\left(\frac{1}{h^2} - \frac{1}{h^3}\right) + y_{i-1}\left(\frac{1}{h^3}\right)$$

$$\frac{\partial F}{\partial y_i} = \frac{y_{i-1}}{h^2} - \frac{4y_i}{h^2} + \frac{y_{i+1}}{h^3} - \frac{y_{i-1}}{h^3} + \frac{y_{i+1}}{h^2}$$

$$y_{i-1}\left(\frac{1}{h^2} - \frac{1}{h^3}\right) + y_i\left(-\frac{4}{h^2}\right) + y_{i+1}\left(\frac{1}{h^3} + \frac{1}{h^2}\right)$$

$$\frac{\partial F}{\partial y_{i+1}} = \frac{y_i}{h^3} + \frac{y_i}{h^2} - \frac{y_{i+1}}{h^3}.$$

$$y_i\left(\frac{1}{h^3} + \frac{1}{h^2}\right) + y_{i+1}\left(-\frac{1}{h^3}\right)$$