**Indian Institute of Technology Kharagpur**
**Department of Mathematics**
**MA39110-Advanced Numerical Techniques Lab**
**Lab sheet-8**
**Spring 2022**

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1. Consider a thin metal wire of length one meter with no heat exchange with its surroundings. Initially the wire is heated with a temperature distribution $x^2(L-x^3)$ at time $t=0$ while both ends of the wire are kept at a fixed temperature of $0^\circ$ c for all times $t$. Then solve the following one-dimensional heat equation $\frac{\partial u}{\partial t} = 0.5\frac{\partial^2 u}{\partial x^2}$ for $(x,t) \in ([0,1]\times[0,1])$ using the Crank-Nicolson method for finding the temperature distribution of the wire at a later time $t$ by solving a tridiagonal system of linear equations (use Thomas Algorithm). Take no. of space steps as 50 and no. of time steps as 2500 to plot the solution at specific time steps $t = 1/25,\ 3/25,\ 6/25,\ 12/25$ unit in a single graph and then plot the temperature distribution with change in $x$ and $t$.

2. Use Crank-Nicolson discretization method to solve the heat equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ for $(x,t) \in ([0,1]\times[0,1])$ by solving a tridiagonal system of linear equations (Use Thomas Algorithm) noting the initial and Neumann boundary conditions as follows:

$$u(x,0) = 1,\ \text{for } 0 < x < 1,\quad u_x(0,t) = u,\quad u_x(1,t) = -u,\ \text{for } 0 < t < 1.$$

   Take $\Delta x = 0.2$ unit and $\Delta t = 0.09$ unit to plot the solution at specific time points $t = 0.18,\ 0.36,\ 0.54,\ 0.81$ unit and then plot the temperature distribution with change in $x$ and $t$ (use forward difference in space for handling the left Neumann boundary condition while backward difference in space for right boundary condition.)