

Indian Institute of Technology Kharagpur
Department of Mathematics
MA39110-Advanced Numerical Techniques Lab
Lab sheet-8
Spring 2022

1. Consider a thin metal wire of length two meter with no heat exchange with its surroundings. Initially the wire is heated with a temperature distribution $x^2(L^3 - x^3)$ at time $t = 0$ while both ends of the wire are kept at a fixed temperature of 0° c for all times t . Then solve the following one-dimensional heat equation $\frac{\partial u}{\partial t} = 0.3 \frac{\partial^2 u}{\partial x^2}$ for $(x, t) \in ([0, 2] \times [0, 1])$ using the Crank-Nicolson method for finding the temperature distribution of the wire at a later time t by solving a tridiagonal system of linear equations (use Thomas Algorithm). Take no. of space steps as 100 and no. of time steps as 2500 to plot the solution at specific time points $t = 3/25, 6/25, 11/25, 16/25$ unit in a single graph and then plot the temperature distribution with change in x and t .
2. Use Crank-Nicolson discretization method to solve the heat equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ for $(x, t) \in ([0, 1] \times [0, 1])$ by solving a tridiagonal system of linear equations (Use Thomas Algorithm) noting the initial and Neumann boundary conditions as follows:

$$u(x, 0) = 1, \text{ for } 0 < x < 1, \quad u_x(0, t) = 3u, \quad u_x(1, t) = -3u, \text{ for } 0 < t < 1.$$

Take $\Delta x = 0.2$ unit and $\Delta t = 0.001$ unit to plot the solution at specific time points $t = 0.003, 0.036, 0.054, 0.081$ unit and then plot the temperature distribution with change in x and t (use forward difference in space for handling the left Neumann boundary condition while backward difference in space for right boundary condition.)