ANT_LAB ASSIGNMENT-05-Set-2

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ANT_LAB ASSIGNME...

INDIAN INSTITUTE OF TECHNOLOGY KHARAGPUR

DEPARTMENT OF MATHEMATICS

MA39110 - Advanced Numerical Techniques Lab

1. Use second order FDM(finite difference scheme) and Newton linearization technique, write a MATLAB Code to solve the following BVP (Boundary Value Problem) with step size h=0.1 .Also compare the solution with exact solution and plot the resulting solutions.

$$yy''+y'=2 \qquad x \in [0,1]$$

$$y(0)=0 \quad , \quad y(1)=2$$

2. Use second order FDM(finite difference scheme) and Quasi linearization technique to solve the following BVP with step size h=0.1 . Also compare the solution with exact solution of the given BVP and plot the resulting solution.

$$(y')^2=2yy'' \qquad x \in [1,2]$$

$$y(1)=1, \qquad y(2)=4$$

Q2

$$f = \boxed{(y')^2 - 2yy'' = 0.}$$

$$\left(y^{(k+1)} - y^{(k)}\right)\frac{\partial f}{\partial y} + \left(y'^{(k+1)} - y'^{(k)}\right)\frac{\partial f}{\partial y'} + \left(y''^{(k+1)} - y''^{(k)}\right)\frac{\partial f}{\partial y''} = -f.$$

$$\left(y^{(k+1)} - y^{(k)}\right)\left(-2y''^{(k)}\right) + \left(y'^{(k+1)} - y'^{(k)}\right)\left(2y'^{(k)}\right) + \left(y''^{(k+1)} - y''^{(k)}\right)\left(-2y^{(k)}\right) = 2y^{(k)}y''^{(k)} - y'^{(k)2}.$$

$$-2y''^{(k)}y^{(k+1)} + 2y''^{(k)}y^{(k)} + 2y'^{(k)}y'^{(k+1)} \underset{-y'^{(k)2}}{-2y'^{(k)2}} - 2y^{(k)}y''^{(k+1)} + \cancel{2y^{(k)}y''^{(k)}} = \cancel{2y^{(k)}y''^{(k)}} - y'^{(k)2}$$

$$-2y''^{(k)}y^{(k+1)} + 2y''^{(k)}y^{(k)} + 2y'^{(k)}y'^{(k+1)} - y'^{(k)2} - 2y^{(k)}y''^{(k+1)} = 0.$$

$$-2\left(\frac{y_{i-1}^{(k)} - 2y_i^{(k)} + y_{i+1}^{(k)}}{h^2}\right)\cancel{y_i^{(k+1)}} + 2\underbrace{\left(\frac{y_{i-1}^{(k)} - 2y_i^{(k)} + y_{i+1}^{(k)}}{h^2}\right)}y_i^{(k)} + 2\left(\frac{y_{i+1}^{(k)} - y_{i-1}^{(k)}}{2h}\right)\cancel{\left(\frac{y_{i+1}^{(k+1)} - y_{i-1}^{(k+1)}}{2h}\right)}$$

$$- \left(\frac{y_{i+1}^{(k)} - y_{i-1}^{(k)}}{2h}\right)^2 - 2\cancel{\left(\frac{y_{i-1}^{(k+1)} - 2y_i^{(k+1)} + y_{i+1}^{(k+1)}}{h^2}\right)}y_i^{(k)}$$

$$- \left(\frac{y_{i+1}^{(k)2} + y_{i-1}^{(k)2} - 2y_{i+1}^{(k)}y_{i-1}^{(k)}}{4h^2}\right) = 0.$$

$$\left\lfloor \quad 4h^2 \quad \right\rfloor$$

$$y_{(i-1)}^{(k+1)}\left[-\frac{2y_i^{(k)}}{h^2} - \frac{\left(y_{i+1}^{(k)} - y_{(i-1)}^{(k)}\right)}{2h^2} \right]$$

$$y_i^{(k+1)}\left[-2\left(\frac{y_{i-1}^{(k)} - 2y_i^{(k)} + y_{i+1}^{(k)}}{h^2}\right) + \frac{4}{h^2} y_i^{(k)} \right] = \left(\frac{y_{i+1}^{(k)2} + y_{i-1}^{(k)2} - 2y_{i+1}^{(k)} y_{i-1}^{(k)}}{4h^2}\right) - 2\left(\frac{y_{i-1}^{(k)} - 2y_i^{(k)} + y_{i+1}^{(k)}}{h^2}\right) y_i^{(k)}$$

$$y_{i+1}^{(k+1)}\left[\left(\frac{y_{i+1}^{(k)} - y_{i-1}^{(k)}}{2h^2}\right) - \frac{2y_i^{(k)}}{h^2} \right]$$