Indian Institute of Technology Kharagpur
Department of Mathematics
MA39110-Advanced Numerical Techniques Lab
Lab sheet-9
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1. Consider a linear wave moving with a velocity a along x axis in a non-viscous medium. Initially at time $t = 0$ the wave follows the profile of the sinusoidal wave $u(x, 0) = \sin x$. Then write a Matlab program of first order upwind scheme for a user defined velocity of the linear wave and plot the corresponding wave profile at $t = 0.1$ by considering the periodic boundary conditions in the domain $(x, t) \in ([0, 2\pi] \times [0, 1])$ with $\Delta x = \pi/64$ and $\Delta t = 0.001$ and compare it with the exact solution in the same graph. Note that you have to consider all possible cases of user defined velocity in your Matlab code. For what choices of user defined velocity, the upwind scheme is unstable?

2. Let us consider a solute of bulk concentration $c(x, t)$ dissolved in a solvent in a very thin film, where diffusion and capillarity effects are negligible. Further, let us assume that during the flow the concentration gradient remains unchanged and thus the solute concentration moves with a constant velocity v along x axis. Initially at time $t = 0$ the concentration profile is given by the Gaussian distribution $u(x, 0) = exp(-200(x - x_c)^2),\ 0 < x < 1$, where $x_c = 0.25$ is the center of the Gaussian profile. Then write a Matlab program of first order upwind scheme for a user defined velocity of the concentration and plot the corresponding concentration profile at $t = 0.3$ in the domain $(x, t) \in ([0, 1] \times [0, 1])$ with $\Delta x = 0.1$ and $\Delta t = 0.009$. Also, plot the graph of exact solution in the same plot and compare the approximate and exact solutions. Note that you have to consider all possible cases of user defined velocity in your Matlab code.