

NUMERICAL APTITUDE

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1. PROBLEMS ON NUMBERS

Q1. Simplify: (i) $8888 + 888 + 88 + 8$

(ii) $11992 - 7823 - 456$

Sol. i) 8888

$$\begin{array}{r} 888 \\ 88 \\ + 8 \\ \hline 9872 \end{array}$$

$$\begin{aligned} \text{ii) } 11992 - 7823 - 456 &= 11992 - (7823 + 456) \\ &= 11992 - 8279 = 3713 \end{aligned}$$

$$\begin{array}{r} 7823 \\ + 456 \\ \hline 8279 \end{array}$$

$$\begin{array}{r} 11992 \\ - 8279 \\ \hline 3713 \end{array}$$

Q2. What value will replace the question mark in each of the following equations?

(i) $? - 1936248 = 1635773$

(ii) $8597 - ? = 7429 - 4358$

Sol. (i) Let $x - 1936248 = 1635773$.

$$\text{Then, } x = 1635773 + 1936248 = 3572021.$$

(ii) Let $8597 - x = 7429 - 4358$.

$$\text{Then, } x = (8597 + 4358) - 7429 = 12955 - 7429 = 5526.$$

Q3. What could be the maximum value of Q in the following equation?

$$5P9 + 3R7 + 2Q8 = 1114$$

Sol. We may analyze the given equation as shown:

$$\text{Clearly, } 2 + P + R + Q = 11.$$

So, the maximum value of Q can be

$$(11 - 2) \text{ i.e., } 9 \text{ (when } P = 0, R = 0);$$

$$\begin{array}{r} 1 \ 2 \\ 5 \ P \ 9 \\ 3 \ R \ 7 \\ \hline 2 \ Q \ 8 \\ \hline 11 \ 1 \ 4 \end{array}$$

Q4. Simplify: (i) 5793405×9999 (ii) 839478×625

Sol.

$$\text{i) } 5793405 \times 9999 = 5793405(10000 - 1) = 57934050000 - 5793405 = 57928256595.$$

$$\text{ii) } 839478 \times 625 = 839478 \times 5^4 = \frac{8394780000}{16} = 524673750.$$

Q5. Evaluate: (i) $986 \times 237 + 986 \times 863$ (ii) $983 \times 207 - 983 \times 107$

Sol.

$$\text{(i) } 986 \times 137 + 986 \times 863 = 986 \times (137 + 863) = 986 \times 1000 = 986000.$$

$$\text{(ii) } 983 \times 207 - 983 \times 107 = 983 \times (207 - 107) = 983 \times 100 = 98300.$$

Q6. Simplify: (i) 1605×1605 (ii) 1398×1398

Sol.

$$\begin{aligned} \text{i) } 1605 \times 1605 &= (1605)^2 = (1600 + 5)^2 = (1600)^2 + (5)^2 + 2 \times 1600 \times 5 \\ &= 2560000 + 25 + 16000 = 2576025. \end{aligned}$$

$$\begin{aligned} \text{(ii) } 1398 \times 1398 &= (1398)^2 = (1400 - 2)^2 = (1400)^2 + (2)^2 - 2 \times 1400 \times 2 \\ &= 1960000 + 4 - 5600 = 1954404. \end{aligned}$$

Q7. Evaluate: $(313 \times 313 + 287 \times 287)$.

Sol.

$$(a^2 + b^2) = \frac{1}{2} [(a + b)^2 + (a - b)^2]$$

$$(313)^2 + (287)^2 = \frac{1}{2} [(313 + 287)^2 + (313 - 287)^2] = \frac{1}{2} [(600)^2 + (26)^2]$$
$$= \frac{1}{2} (360000 + 676) = 180338.$$

Q8. Which of the following are prime numbers?

(i) 241 (ii) 337 (Hi) 391 (iv) 571

Sol.

1. Clearly, $16 > 241$. Prime numbers less than 16 are 2, 3, 5, 7, 11, 13.

241 is not divisible by any one of them.

241 is a prime number.

2. Clearly, $19 > 337$. Prime numbers less than 19 are 2, 3, 5, 7, 11, 13, 17.

337 is not divisible by any one of them.

337 is a prime number.

3. Clearly, $20 > 391$. Prime numbers less than 20 are 2, 3, 5, 7, 11, 13, 17, 19.

We find that 391 is divisible by 17.

391 is not prime.

4. Clearly, $24 > 571$. Prime numbers less than 24 are 2, 3, 5, 7, 11, 13, 17, 19, 23.

571 is not divisible by any one of them.

571 is a prime number.

Q9. Find the unit's digit in the product $(2467)^{153} \times (341)^{72}$.

Sol. Clearly, unit's digit in the given product = unit's digit in $7^{153} \times 1^{72}$.

Now, 7^4 gives unit digit 1.

7^{152} gives unit digit 1,

$\therefore 7^{153}$ gives unit digit $(1 \times 7) = 7$. Also, 1^{72} gives unit digit 1.

Hence, unit's digit in the product = $(7 \times 1) = 7$.

Q10. Find the unit's digit in $(264)^{102} + (264)^{103}$

Sol. Required unit's digit = unit's digit in $(4)^{102} + (4)^{103}$.

Now, 4^2 gives unit digit 6.

$\therefore (4)^{102}$ gives unit digit 6.

$\therefore (4)^{103}$ gives unit digit of the product (6×4) i.e., 4.

Hence, unit's digit is $(6 + 4) = 0$.

Q11. Find the total number of prime factors in the expression $(4)^{11} \times (7)^5 \times (11)^2$.

Sol. $(4)^{11} \times (7)^5 \times (11)^2 = (2 \times 2)^{11} \times (7)^5 \times (11)^2 = 2^{22} \times 2^{11} \times 7^5 \times 11^2 = 2^{22} \times 7^5 \times 11^2$

Total number of prime factors = $(22 + 5 + 2) = 29$.

Q12. Simplify: (i) $896 \times 896 - 204 \times 204$

(ii) $387 \times 387 + 114 \times 114 + 2 \times 387 \times 114$

(iii) $81 \times 81 + 68 \times 68 - 2 \times 81 \times 68$.

Sol.

- (i) Given $\exp = (896)^2 - (204)^2 = (896 + 204)(896 - 204) = 1100 \times 692 = 761200$.
- (ii) Given $\exp = (387)^2 + (114)^2 + (2 \times 387 \times 114)$
 $= a^2 + b^2 + 2ab$, where $a = 387$, $b = 114$
 $= (a+b)^2 = (387 + 114)^2 = (501)^2 = 251001$.
- (iii) Given $\exp = (81)^2 + (68)^2 - 2 \times 81 \times 68 = a^2 + b^2 - 2ab$, Where $a = 81$, $b = 68$
 $= (a-b)^2 = (81 - 68)^2 = (13)^2 = 169$.

Q13. Which of the following numbers is divisible by 3?

(i) 541326

(ii) 5967013

Sol.

(i) Sum of digits in 541326 = $(5 + 4 + 1 + 3 + 2 + 6) = 21$, which is divisible by 3.
Hence, 541326 is divisible by 3.

(ii) Sum of digits in 5967013 = $(5 + 9 + 6 + 7 + 0 + 1 + 3) = 31$, which is not divisible by 3.
Hence, 5967013 is not divisible by 3.

Q14. What least value must be assigned to * so that the number 197*5462 is r 9?

Sol.

Let the missing digit be x.

Sum of digits = $(1 + 9 + 7 + x + 5 + 4 + 6 + 2) = (34 + x)$.

For $(34 + x)$ to be divisible by 9, x must be replaced by 2.

Hence, the digit in place of * must be 2.

Q15. Which of the following numbers is divisible by 4?

(i) 67920594

(ii) 618703572

Sol.

(i) The number formed by the last two digits in the given number is 94, which is not divisible by 4.

Hence, 67920594 is not divisible by 4.

(ii) The number formed by the last two digits in the given number is 72, which is divisible by 4.
Hence, 618703572 is divisible by 4.

Q16. Which digits should come in place of * and \$ if the number 62684*\$ is divisible by both 8 and 5?

Sol.

Since the given number is divisible by 5, so 0 or 5 must come in place of \$. But, a number ending with 5 is never divisible by 8. So, 0 will replace \$.

Now, the number formed by the last three digits is $4*0$, which becomes divisible by 8, if * is replaced by 4.

Hence, digits in place of * and \$ are 4 and 0 respectively.

Q17. Show that 4832718 is divisible by 11.

Sol. (Sum of digits at odd places) - (Sum of digits at even places)
= $(8 + 7 + 3 + 4) - (1 + 2 + 8) = 11$, which is divisible by 11.
Hence, 4832718 is divisible by 11.

Q18. Is 52563744 divisible by 24?

Sol. $24 = 3 \times 8$, where 3 and 8 are co-primes.

The sum of the digits in the given number is 36, which is divisible by 3. So, the given number is divisible by 3.

The number formed by the last 3 digits of the given number is 744, which is divisible by 8. So, the given number is divisible by 8.

Thus, the given number is divisible by both 3 and 8, where 3 and 8 are co-primes.
So, it is divisible by 3×8 , i.e., 24.

Q19. What least number must be added to 3000 to obtain a number exactly divisible by 19?

Sol. On dividing 3000 by 19, we get 17 as remainder.

\therefore Number to be added = $(19 - 17) = 2$.

Q20. What least number must be subtracted from 2000 to get a number exactly divisible by 17?

Sol. On dividing 2000 by 17, we get 11 as remainder.

\therefore Required number to be subtracted = 11.

Q21. Find the number which is nearest to 3105 and is exactly divisible by 21.

Sol. On dividing 3105 by 21, we get 18 as remainder.

\therefore Number to be added to 3105 = $(21 - 18) = 3$.

Hence, required number = $3105 + 3 = 3108$.

Q22. Find the smallest number of 6 digits which is exactly divisible by 111.

Sol. Smallest number of 6 digits is 100000.

On dividing 100000 by 111, we get 100 as remainder.

\therefore Number to be added = $(111 - 100) = 11$.

Hence, required number = 100011.

Q23. On dividing 15968 by a certain number, the quotient is 89 and the remainder is 37. Find the divisor.

Sol.
$$\begin{array}{rcl} \text{Dividend} - \text{Remainder} & 15968 - 37 & \\ \text{Divisor} = \frac{\quad}{\text{Quotient}} & = \frac{\quad}{89} & = 179. \end{array}$$

Q24. A number when divided by 342 gives a remainder 47. When the same number is divided by 19, what would be the remainder?

Sol. On dividing the given number by 342, let k be the quotient and 47 as remainder.

Then, number - $342k + 47 = (19 \times 18k + 19 \times 2 + 9) = 19(18k + 2) + 9$.

\therefore The given number when divided by 19, gives $(18k + 2)$ as quotient and 9 as remainder.

Q25. A number being successively divided by 3, 5 and 8 leaves remainders 1, 4 and 7 respectively. Find the respective remainders if the order of divisors be reversed

Sol.

$$\begin{array}{r|l} 3 & X \\ \hline 5 & y - 1 \\ \hline 8 & z - 4 \\ \hline & 1 - 7 \end{array}$$

$$\therefore z = (8 \times 1 + 7) = 15; y = \{5z + 4\} = (5 \times 15 + 4) = 79; x = (3y + 1) = (3 \times 79 + 1) = 238.$$

Now,

$$\begin{array}{r|l} 8 & 238 \\ \hline 5 & 29 - 6 \\ \hline 3 & 5 - 4 \\ \hline & 1 - 9, \end{array}$$

\therefore Respective remainders are 6, 4, 2.

Q26. Find the remainder when 2^{31} is divided by 5.

Sol. $2^{10} = 1024$. Unit digit of $2^{10} \times 2^{10} \times 2^{10}$ is 4 [as $4 \times 4 \times 4$ gives unit digit 4].

\therefore Unit digit of 2^{31} is 8.

Now, 8 when divided by 5, gives 3 as remainder.

Hence, 2^{31} when divided by 5, gives 3 as remainder.

Q27. How many numbers between 11 and 90 are divisible by 7?

Sol. The required numbers are 14, 21, 28, 35, ..., 77, 84.

This is an A.P. with $a = 14$ and $d = (21 - 14) = 7$.

Let it contain n terms.

$$\text{Then, } T_n = 84 \Rightarrow a + (n - 1)d = 84$$

$$\Rightarrow 14 + (n - 1) \times 7 = 84 \text{ or } n = 11.$$

\therefore Required number of terms = 11.

Q28. Find the sum of all odd numbers upto 100.

Sol. The given numbers are 1, 3, 5, 7, ..., 99.

This is an A.P. with $a = 1$ and $d = 2$.

Let it contain n terms. Then,

$$1 + (n - 1) \times 2 = 99 \text{ or } n = 50.$$

\therefore Required sum = $\frac{n}{2}$ (first term + last term)

$$= \frac{50}{2} (1 + 99) = 2500.$$

Q29. Find the sum of all 2 digit numbers divisible by 3.

Sol. All 2 digit numbers divisible by 3 are:

12, 15, 18, 21, ..., 99.

This is an A.P. with $a = 12$ and $d = 3$.

Let it contain n terms. Then,

$$12 + (n - 1) \times 3 = 99 \text{ or } n = 30.$$

$$\therefore \text{Required sum} = \frac{30}{2} \times (12 + 99) = 1665.$$

Q30. How many terms are there in 2, 4, 8, 16, ..., 1024?

Sol. Clearly 2, 4, 8, 16, ..., 1024 form a GP. With $a=2$ and $r = 4/2 = 2$.

Let the number of terms be n . Then

$$2 \times 2^{n-1} = 1024 \text{ or } 2^{n-1} = 512 = 2^9.$$

$$\therefore n-1=9 \text{ or } n=10.$$

Q31. $2 + 2^2 + 2^3 + \dots + 2^8 = ?$

Sol. Given series is a G.P. with $a = 2$, $r = 2$ and $n = 8$.

$$\therefore \text{sum} = \frac{a(r^n - 1)}{(r - 1)} = \frac{2 \times (2^8 - 1)}{(2 - 1)} = (2 \times 255) = 510$$

Q32. A number is as much greater than 36 as is less than 86. Find the number.

Sol. Let the number be x . Then, $x - 36 = 86 - x \Rightarrow 2x = 86 + 36 = 122 \Rightarrow x = 61$.

Hence, the required number is 61.

Q33. Find a number such that when 15 is subtracted from 7 times the number, the Result is 10 more than twice the number.

Sol. Let the number be x . Then, $7x - 15 = 2x + 10 \Rightarrow 5x = 25 \Rightarrow x = 5$.

Hence, the required number is 5.

Q34. The sum of a rational number and its reciprocal is $13/6$. Find the number.

Sol. Let the number be x .

$$\begin{aligned} \text{Then, } x + (1/x) &= 13/6 \Rightarrow (x^2 + 1)/x = 13/6 \Rightarrow 6x^2 - 13x + 6 = 0 \\ &\Rightarrow 6x^2 - 9x - 4x + 6 = 0 \Rightarrow (3x - 2)(2x - 3) = 0 \\ &\Rightarrow x = 2/3 \text{ or } x = 3/2 \end{aligned}$$

Hence the required number is $2/3$ or $3/2$.

Q35 The sum of two numbers is 184. If one-third of the one exceeds one-seventh of the other by 8, find the smaller number.

Sol. Let the numbers be x and $(184 - x)$. Then,

$$(X/3) - ((184 - x)/7) = 8 \Rightarrow 7x - 3(184 - x) = 168 \Rightarrow 10x = 720 \Rightarrow x = 72.$$

So, the numbers are 72 and 112. Hence, smaller number = 72.

Q36. The difference of two numbers is 11 and one-fifth of their sum is 9. Find the numbers.

Sol. Let the number be x and y . Then,

$$x - y = 11 \quad \text{----(i)} \quad \text{and } 1/5 (x + y) = 9 \Rightarrow x + y = 45 \quad \text{----(ii)}$$

Adding (i) and (ii), we get: $2x = 56$ or $x = 28$. Putting $x = 28$ in (i), we get: $y = 17$.

Hence, the numbers are 28 and 17.

Q37. If the sum of two numbers is 42 and their product is 437, then find the absolute difference between the numbers.

Sol. Let the numbers be x and y . Then, $x + y = 42$ and $xy = 437$

$$x - y = \sqrt{(x + y)^2 - 4xy} = \sqrt{[(42)^2 - 4 \times 437]} = \sqrt{1764 - 1748} = \sqrt{16} = 4.$$

Required difference = 4.

Q38. The sum of two numbers is 16 and the sum of their squares is 113. Find the numbers.

Sol. Let the numbers be x and $(15 - x)$.

$$\begin{aligned} \text{Then, } x^2 + (15 - x)^2 &= 113 & \Rightarrow & x^2 + 225 + X^2 - 30x = 113 \\ \Rightarrow 2x^2 - 30x + 112 &= 0 & \Rightarrow & x^2 - 15x + 56 = 0 \\ \Rightarrow (x - 7)(x - 8) &= 0 & \Rightarrow & x = 7 \text{ or } x = 8. \end{aligned}$$

So, the numbers are 7 and 8.

Q39. The average of four consecutive even numbers is 27. Find the largest of these numbers.

Sol. Let the four consecutive even numbers be $x, x + 2, x + 4$ and $x + 6$.

$$\text{Then, sum of these numbers} = (27 \times 4) = 108.$$

$$\text{So, } x + (x + 2) + (x + 4) + (x + 6) = 108 \text{ or } 4x = 96 \text{ or } x = 24.$$

$$\therefore \text{Largest number} = (x + 6) = 30.$$

Q40. The sum of the squares of three consecutive odd numbers is 2531. Find the numbers.

Sol. Let the numbers be $x, x + 2$ and $x + 4$.

$$\begin{aligned} \text{Then, } X^2 + (x + 2)^2 + (x + 4)^2 &= 2531 \Rightarrow 3x^2 + 12x - 2511 = 0 \\ \Rightarrow X^2 + 4x - 837 &= 0 \Rightarrow (x - 27)(x + 31) = 0 \Rightarrow x = 27. \end{aligned}$$

Hence, the required numbers are 27, 29 and 31.

Q41. Of two numbers, 4 times the smaller one is less than 3 times the larger one by 5. If the sum of the numbers is larger than 6 times their difference by 6, find the two numbers.

Sol. Let the numbers be x and y , such that $x > y$

$$\text{Then, } 3x - 4y = 5 \dots(i) \text{ and } (x + y) - 6(x - y) = 6 \Rightarrow -5x + 7y = 6 \dots(ii)$$

$$\text{Solving (i) and (ii), we get: } x = 59 \text{ and } y = 43.$$

$$\text{Hence, the required numbers are 59 and 43.}$$

Q42. The ratio between a two-digit number and the sum of the digits of that number is 4 : 1. If the digit in the unit's place is 3 more than the digit in the ten's place, what is the number?

Sol. Let the ten's digit be x . Then, unit's digit = $(x + 3)$.

$$\text{Sum of the digits} = x + (x + 3) = 2x + 3. \text{ Number} = 10x + (x + 3) = 11x + 3.$$

$$11x + 3 / 2x + 3 = 4 / 1 \Rightarrow 11x + 3 = 4(2x + 3) \Rightarrow 3x = 9 \Rightarrow x = 3.$$

$$\text{Hence, required number} = 11x + 3 = 36.$$

Q43. A number consists of two digits. The sum of the digits is 9. If 63 is subtracted from the number, its digits are interchanged. Find the number.

Sol. Let the ten's digit be x . Then, unit's digit = $(9 - x)$.

$$\text{Number} = 10x + (9 - x) = 9x + 9.$$

$$\text{Number obtained by reversing the digits} = 10(9 - x) + x = 90 - 9x.$$

$$\text{therefore, } (9x + 9) - 63 = 90 - 9x \Rightarrow 18x = 144 \Rightarrow x = 8.$$

$$\text{So, ten's digit} = 8 \text{ and unit's digit} = 1.$$

$$\text{Hence, the required number is 81.}$$

Q44. A fraction becomes $\frac{2}{3}$ when 1 is added to both, its numerator and denominator.

And, it becomes $\frac{1}{2}$ when 1 is subtracted from both the numerator and denominator. Find the fraction.

Sol. Let the required fraction be $\frac{x}{y}$. Then,
 $\frac{x+1}{y+1} = \frac{2}{3} \Rightarrow 3x - 2y = -1 \dots(i)$ and $\frac{x-1}{y-1} = \frac{1}{2}$
 $\Rightarrow 2x - y = 1 \dots(ii)$
Solving (i) and (ii), we get : $x = 3$, $y = 5$
therefore, Required fraction = $\frac{3}{5}$.

Q45. 50 is divided into two parts such that the sum of their reciprocals is $\frac{1}{12}$. Find the two parts.

Sol. Let the two parts be x and $(50 - x)$.
Then, $\frac{1}{x} + \frac{1}{(50 - x)} = \frac{1}{12} \Rightarrow \frac{(50 - x + x)}{x(50 - x)} = \frac{1}{12}$
 $\Rightarrow x^2 - 50x + 600 = 0 \Rightarrow (x - 30)(x - 20) = 0 \Rightarrow x = 30$ or $x = 20$.
So, the parts are 30 and 20.

Q46. If three numbers are added in pairs, the sums equal 10, 19 and 21. Find the numbers)

Sol. Let the numbers be x , y and z . Then,
 $x + y = 10 \dots(i)$ $y + z = 19 \dots(ii)$ $x + z = 21 \dots(iii)$
Adding (i), (ii) and (iii), we get: $2(x + y + z) = 50$ or $(x + y + z) = 25$.
Thus, $x = (25 - 19) = 6$; $y = (25 - 21) = 4$; $z = (25 - 10) = 15$.
Hence, the required numbers are 6, 4 and 15.

2. PROBLEMS ON LCM & HCF

Q1. Find the H.C.F. of $2^3 \times 3^2 \times 5 \times 7^4$, $2^2 \times 3^5 \times 5^2 \times 7^3$, $2^3 \times 5^3 \times 7^2$

Sol. The prime numbers common to given numbers are 2, 5 and 7.

$$\text{H.C.F.} = 2^2 \times 5 \times 7^2 = 980.$$

Q2. Find the H.C.F. of 108, 288 and 360.

Sol. $108 = 2^2 \times 3^3$, $288 = 2^5 \times 3^2$ and $360 = 2^3 \times 5 \times 3^2$.

$$\text{H.C.F.} = 2^2 \times 3^2 = 36.$$

Q3. Find the H.C.F. of 513, 1134 and 1215.

Sol. _____

$$1134 \overline{) 1215} (1$$

$$\begin{array}{r} 1134 \\ \underline{1134} \\ 81 \overline{) 1134} (14 \\ \underline{81} \\ 324 \\ \underline{324} \\ \underline{0} \end{array}$$

\therefore H.C.F. of 1134 and 1215 is 81.

So, Required H.C.F. = H.C.F. of 513 and 81.

$$\begin{array}{r} 81 \overline{) 513} (6 \\ \underline{486} \\ 27 \overline{) 81} (3 \\ \underline{81} \\ \underline{0} \end{array}$$

H.C.F. of given numbers = 27.

Q4. Reduce $\frac{391}{667}$ to lowest terms.

Sol. H.C.F. of 391 and 667 is 23.

On dividing the numerator and denominator by 23, we get :

$$\frac{391}{667} = \frac{391 \div 23}{667 \div 23} = \frac{17}{29}$$

Q5. Find the L.C.M. of $2^2 \times 3^3 \times 5 \times 7^2$, $2^3 \times 3^2 \times 5^2 \times 7^4$, $2 \times 3 \times 5^3 \times 7 \times 11$.

Sol. L.C.M. = Product of highest powers of 2, 3, 5, 7 and 11 = $2^3 \times 3^3 \times 5^3 \times 7^4 \times 11$

Q6. Find the L.C.M. of 72, 108 and 2100.

Sol. $72 = 2^3 \times 3^2$, $108 = 3^3 \times 2^2$, $2100 = 2^2 \times 5^2 \times 3 \times 7$. L.C.M. = $2^3 \times 3^3 \times 5^2 \times 7 = 37800$.

Q7. Find the L.C.M. of 16, 24, 36 and 54.

Sol.

2	16	-	24	-	36	-	54
2	8	-	12	-	18	-	27
2	4	-	6	-	9	-	27
3	2	-	3	-	9	-	27
3	2	-	1	-	3	-	9
	2	-	1	-	1	-	3

$$\therefore \text{L.C.M.} = 2 \times 2 \times 2 \times 3 \times 3 \times 2 \times 3 = 432.$$

Q8. Find the H.C.F. and L.C.M. of $\frac{2}{3}$, $\frac{8}{9}$, $\frac{16}{81}$ and $\frac{10}{27}$.

$$\text{Sol. H.C.F. of given fractions} = \frac{\text{H.C.F. of } 2, 8, 16, 10}{\text{L.C.M. of } 3, 9, 81, 27} = \frac{2}{81}$$

$$\text{L.C.M of given fractions} = \frac{\text{L.C.M. of } 2, 8, 16, 10}{\text{H.C.F. of } 3, 9, 81, 27} = \frac{80}{3}$$

Q9. Find the H.C.F. and L.C.M. of 0.63, 1.05 and 2.1.

Sol. Making the same number of decimal places, the given numbers are 0.63, 1.05 and 2.10. Without decimal places, these numbers are 63, 105 and 210.

Now, H.C.F. of 63, 105 and 210 is 21.

H.C.F. of 0.63, 1.05 and 2.1 is 0.21.

L.C.M. of 63, 105 and 210 is 630.

L.C.M. of 0.63, 1.05 and 2.1 is 6.30.

Q10. Two numbers are in the ratio of 15:11. If their H.C.F. is 13, find the numbers.

Sol. Let the required numbers be $15x$ and $11x$.

Then, their H.C.F. is x . So, $x = 13$.

The numbers are (15×13) and (11×13) i.e., 195 and 143.

Q11. The H.C.F. of two numbers is 11 and their L.C.M. is 693. If one of the numbers is 77, find the other.

$$\text{Sol. Other number} = \frac{11 \times 693}{77} = 99$$

Q12. Find the greatest possible length which can be used to measure exactly the lengths 4 m 95 cm, 9 m and 16 m 65 cm.

Sol. Required length = H.C.F. of 495 cm, 900 cm and 1665 cm.

$$495 = 3^2 \times 5 \times 11, 900 = 2^2 \times 3^2 \times 5^2, 1665 = 3^2 \times 5 \times 37.$$

$$\therefore \text{H.C.F.} = 3^2 \times 5 = 45.$$

Hence, required length = 45 cm.

Q13. Find the greatest number which on dividing 1657 and 2037 leaves remainders 6 and 5 respectively.

Sol. Required number = H.C.F. of $(1657 - 6)$ and $(2037 - 5)$ = H.C.F. of 1651 and 2032

$$\begin{array}{r}
 1651 \overline{) 2032} \quad (1 \quad 1651 \\
 \underline{1651} \\
 381 \quad 1651 \quad (4 \\
 \underline{1524} \\
 127 \quad 381 \quad (3 \\
 \underline{381} \\
 0
 \end{array}$$

Required number = 127.

Q14. Find the largest number which divides 62, 132 and 237 to leave the same remainder in each case.

Sol . Required number = H.C.F. of (132 - 62), (237 - 132) and (237 - 62)
= H.C.F. of 70, 105 and 175 = 35.

Q15. Find the least number exactly divisible by 12, 15, 20, 27.

Sol.

3	12	-	15	-	20	-	27
4	4	-	5	-	20	-	9
5	1	-	5	-	5	-	9
	1	-	1	-	1	-	9

Q16. Find the least number which when divided by 6, 7, 8, 9, and 12 leave the same remainder 1 in each case

Sol. Required number = (L.C.M OF 6, 7, 8, 9, 12) + 1

3	6	-	7	-	8	-	9	-	12
4	2	-	7	-	8	-	3	-	4
5	1	-	7	-	4	-	3	-	2
	1	-	7	-	2	-	3	-	1

\therefore L.C.M = $3 \times 2 \times 2 \times 7 \times 2 \times 3 = 504$.

Hence required number = $(504 + 1) = 505$.

Q17. Find the largest number of four digits exactly divisible by 12, 15, 18 and 27.

Sol. The Largest number of four digits is 9999.

Required number must be divisible by L.C.M. of 12, 15, 18, 27 i.e. 540.

On dividing 9999 by 540, we get 279 as remainder .

\therefore Required number = $(9999 - 279) = 9720$.

Q18. Find the smallest number of five digits exactly divisible by 16, 24, 36 and 54.

Sol. Smallest number of five digits is 10000.

Required number must be divisible by L.C.M. of 16,24,36,54 i.e 432,
 On dividing 10000 by 432, we get 64 as remainder.
 \therefore Required number = $10000 + (432 - 64) = 10368$.

Q19. Find the least number which when divided by 20,25,35 and 40 leaves remainders 14,19,29 and 34 respectively.

Sol. Here, $(20-14) = 6, (25-19)=6, (35-29)=6$ and $(40-34)=6$.
 \therefore Required number = $(\text{L.C.M. of } 20,25,35,40) - 6 = 1394$.

Q20. Find the least number which when divided by 5,6,7, and 8 leaves a remainder 3, but when divided by 9 leaves no remainder .

Sol. L.C.M. of 5,6,7,8 = 840.

\therefore Required number is of the form $840k + 3$

Least value of k for which $(840k + 3)$ is divisible by 9 is $k = 2$.

\therefore Required number = $(840 \times 2 + 3) = 1683$

Q21. The traffic lights at three different road crossings change after every 48 sec., 72 sec and 108 sec. respectively .If they all change simultaneously at 8:20:00 hours, then at what time they again change simultaneously .

Sol. Interval of change = $(\text{L.C.M of } 48,72,108)\text{sec.} = 432\text{sec.}$

So, the lights will again change simultaneously after every 432 seconds i.e, 7 min. 12sec

Hence, next simultaneous change will take place at 8:27:12 hrs.

Q22. Arrange the fractions $\frac{17}{18}, \frac{31}{36}, \frac{43}{45}, \frac{59}{60}$ in the ascending order.

Sol. L.C.M. of 18,36,45 and 60 = 180.

Now, $\frac{17}{18} = \frac{17 \times 10}{18 \times 10} = \frac{170}{180}$; $\frac{31}{36} = \frac{31 \times 5}{36 \times 5} = \frac{155}{180}$;

$\frac{43}{45} = \frac{43 \times 4}{45 \times 4} = \frac{172}{180}$; $\frac{59}{60} = \frac{59 \times 3}{60 \times 3} = \frac{177}{180}$;

Since, $155 < 170 < 172 < 177$, so, $\frac{155}{180} < \frac{170}{180} < \frac{172}{180} < \frac{177}{180}$

Hence, $\frac{31}{36} < \frac{17}{18} < \frac{43}{45} < \frac{59}{60}$

3. PROBLEMS ON DECIMAL FRACTIONS

Q1. Convert the following into vulgar fraction:

(i) 0.75 (ii) 3.004 (iii) 0.0056

Sol. (i). $0.75 = 75/100 = 3/4$ (ii) $3.004 = 3004/1000 = 751/250$ (iii) $0.0056 = 56/10000 = 7/1250$

Q2. Arrange the fractions $5/8$, $7/12$, $13/16$, $16/29$ and $3/4$ in ascending order of magnitude.

Sol. Converting each of the given fractions into decimal form, we get:

$5/8 = 0.624$, $7/12 = 0.8125$, $16/29 = 0.5517$, and $3/4 = 0.75$

Now, $0.5517 < 0.5833 < 0.625 < 0.75 < 0.8125$

$\therefore 16/29 < 7/12 < 5/8 < 3/4 < 13/16$

Q3. Arrange the fractions $3/5$, $4/7$, $8/9$, and $9/11$ in their descending order.

Sol. Clearly, $3/5 = 0.6$, $4/7 = 0.571$, $8/9 = 0.88$, $9/11 = 0.818$.

Now, $0.88 > 0.818 > 0.6 > 0.571$

$\therefore 8/9 > 9/11 > 3/4 > 13/16$

Q4. Evaluate: (i) $6202.5 + 620.25 + 62.025 + 6.2025 + 0.62025$

(ii) $5.064 + 3.98 + 0.7036 + 7.6 + 0.3 + 2$

Sol. (i) 6202.5

620.25

62.025

6.2025

+ 0.62025

6891.59775

(ii) 5.064

3.98

0.7036

7.6

0.3

2.0

19.6476

Q5. Evaluate: (i) $31.004 - 17.2368$

(ii) $13 - 5.1967$

Sol. (i) 31.0040

- 17.2386

13.7654

(ii) 31.0000

- 5.1967

7.8033

Q6. What value will replace the question mark in the following equations?

(i) $5172.49 + 378.352 + ? = 9318.678$

(ii) $? - 7328.96 + 5169.38$

Sol. (i) Let $5172.49 + 378.352 + x = 9318.678$

Then, $x = 9318.678 - (5172.49 + 378.352) = 9318.678 - 5550.842 = 3767.836$

(ii) Let $x - 7328.96 = 5169.38$. Then, $x = 5169.38 + 7328.96 = 12498.34$.

Q7. Find the products: (i) $6.3204 * 100$

(ii) $0.069 * 10000$

Sol. (i) $6.3204 * 1000 = 632.04$

(ii) $0.069 * 10000 = 0.0690 * 10000 = 690$

Q8. Find the product:

- (i) $2.61 * 1.3$ (ii) $2.1693 * 1.4$ (iii) $0.4 * 0.04 * 0.004 * 40$

Sol. (i) $261 \ 8 \ 13 = 3393$. Sum of decimal places of given numbers = $(2+1) = 3$.

$$2.61 * 1.3 = 3.393.$$

(ii) $21693 * 14 = 303702$. Sum of decimal places = $(4+1) = 5$

$$2.1693 * 1.4 = 3.03702.$$

(iii) $4 * 4 * 4 * 40 = 2560$. Sum of decimal places = $(1 + 2 + 3) = 6$

$$0.4 * 0.04 * 0.004 * 40 = 0.002560.$$

Q9. Given that $268 * 74 = 19832$, find the values of $2.68 * 0.74$.

Sol. Sum of decimal places = $(2 + 2) = 4$

$$2.68 * 0.74 = 1.9832.$$

Q10. Find the quotient:

- (i) $0.63 / 9$ (ii) $0.0204 / 17$ (iii) $3.1603 / 13$

Sol. (i) $63 / 9 = 7$. Dividend contains 2 places decimal.

$$0.63 / 9 = 0.7.$$

(ii) $204 / 17 = 12$. Dividend contains 4 places of decimal.

$$0.2040 / 17 = 0.0012.$$

(iii) $31603 / 13 = 2431$. Dividend contains 4 places of decimal.

$$3.1603 / 13 = 0.2431.$$

Q11. Evaluate:

- (i) $35 + 0.07$ (ii) $2.5 + 0.0005$
(iii) $136.09 + 43.9$

Sol. (i) $35/0.07 = (35*100) / (0.07*100) = (3500 / 7) = 500$

(ii) $25/0.0005 = (25*10000) / (0.0005*10000) = 25000 / 5 = 5000$

(iii) $136.09/43.9 = (136.09*10) / (43.9*10) = 1360.9 / 439 = 3.1$

Q12. What value will come in place of question mark in the following equation?

- (i) $0.006 + ? = 0.6$ (ii) $? + 0.025 = 80$

Sol. (i) Let $0.006 / x = 0.6$, Then, $x = (0.006 / 0.6) = (0.006*10) / (0.6*10) = 0.06/6 = 0.01$

(ii) Let $x / 0.025 = 80$, Then, $x = 80 * 0.025 = 2$

Q13. If $(1 / 3.718) = 0.2689$, Then find the value of $(1 / 0.0003718)$.

Sol. $(1 / 0.0003718) = (10000 / 3.718) = 10000 * (1 / 3.718) = 10000 * 0.2689 = 2689$.

Q14. Express as vulgar fractions: (i) $0.\overline{37}$ (ii) $0.\overline{053}$ (iii) $3.\overline{142857}$

Sol. (i) $\overline{0.37} = 37 / 99$. (ii) $\overline{0.053} = 53 / 999$

(iv) $\overline{3.142857} = 3 + \overline{0.142857} = 3 + (142857 / 999999) = 3 (142857/999999)$

Q15. Express as vulgar fractions: (i) $0.\overline{17}$ (ii) $0.12\overline{54}$ (iii) $2.53\overline{6}$

Sol. (i) $0.\overline{17} = (17 - 1)/90 = 16 / 90 = 8/45$

$$(ii) 0.12\overline{54} = (1254 - 12) / 9900 = 1242 / 9900 = 69 / 550$$

$$(iii) 2.53\overline{6} = 2 + 0.53\overline{6} = 2 + (536 - 53) / 900 = 2 + (483/900) = 2 + (161/300) = 2 (161/300)$$

Q16. Simplify: $\frac{0.05 * 0.05 * 0.05 + 0.04 * 0.04 * 0.04}{0.05 * 0.05 - 0.05 * 0.04 + 0.04 * 0.04}$

Sol. Given expression = $(a^3 + b^3) / (a^2 - ab + b^2)$, where $a = 0.05$, $b = 0.04$
 $= (a + b) = (0.05 + 0.04) = 0.09$

4. SIMPLIFICATION

Q1. Simplify: (i) $5005-5000+10$ (ii) $18800+470+20$

Sol. (i) $5005-5000+10=5005-(5000/10)=5005-500=4505$.

(ii) $18800+470+20=(18800/470)+20=40/20=2$.

Q2. Simplify: $b-[b-(a+b)-\{b-(b-a-b)\}+2a]$

Sol. Given expression $=b-[b-(a+b)-\{b-(b-a-b)\}+2a]$
 $=b-[b-a-b-\{b-2b+a\}+2a]$
 $=b-[-a-\{b-2b+a+2a\}]$
 $=b-[-a-\{-b+3a\}]=b-[-a+b-3a]$
 $=b-[-4a+b]=b+4a-b=4a$.

Q3. What value will replace the question mark in the following equation?

$$4\frac{1}{2} + 3\frac{1}{6} + ? + 2\frac{1}{3} = 13\frac{2}{5}$$

Sol. Let $9/2+19/6+x+7/3=67/5$

$$\text{Then } x = (67/5) - (9/2 + 19/6 + 7/3) \Leftrightarrow x = (67/5) - ((27+19+14)/6) = ((67/5) - (60/6))$$

$$\Leftrightarrow x = ((67/5) - 10) = 17/5 = 3\frac{2}{5}$$

Hence, missing fractions $= 3\frac{2}{5}$

Q4. $4/15$ of $5/7$ of a number is greater than $4/9$ of $2/5$ of the same number by 8. What is half of that number?

Sol. Let the number be x. then $4/15$ of $5/7$ of x $- 4/9$ of $2/5$ of x $= 8 \Leftrightarrow 4/21x - 8/45x = 8$

$$\Leftrightarrow (4/21 - 8/45)x = 8 \Leftrightarrow (60-56)/315x = 8 \Leftrightarrow 4/315x = 8$$

$$\Leftrightarrow x = (8 \times 315)/4 = 630 \Leftrightarrow 1/2x = 315$$

Hence required number = 315.

Q5. Simplify: $3\frac{1}{4} \div \{1\frac{1}{4} - 1/2(2\frac{1}{2} - 1/4 - 1/6)\}$

Sol. Given exp. $= [13/4 \div \{5/4 - 1/2(5/2 - (3-2)/12)\}] = [13/4 \div \{5/4 - 1/2(5/2 - 1/12)\}]$
 $= [13/4 \div \{5/4 - 1/2((30-1)/12)\}] = [13/4 \div \{5/4 - 29/24\}]$
 $= [13/4 \div \{(30-39)/24\}] = [13/4 \div 1/24] = [(13/4) \times 24] = 78$

Q6. Simplify: $108 \div 36$ of $1\frac{2}{5} \times 3\frac{1}{4}$

Sol. Given exp. $= 108 \div 9 + 2\frac{13}{5} \times \frac{13}{4} = \frac{108}{9} + \frac{13}{10} \left(\frac{=133}{10} \right) = 13\frac{3}{10}$

Q7 Simplify: $\frac{(7/2) \div (5/2) * (3/2)}{(7/2) \div (5/2) \text{ of } (3/2)} \div 5.25$

sol.

Given exp. $\frac{(7/2) \times (2/5) \times (3/2)}{(7/2) \div (15/4)} \div 5.25 = \frac{(21/10)}{(525/100)} = \frac{(21/10) \times (15/14)}{(7/2) \div (15/4)}$

Q8. Simplify: (i) $12.05 * 5.4 + 0.6$ (ii) $0.6 * 0.6 + 0.6 * 0.6$ (Bank P.O 2003)

Sol. (i) Given exp. $= 12.05 * (5.4/0.6) = (12.05 * 9) = 108.45$

(ii) Given exp. $= 0.6 * 0.6 + (0.6 * 6) = 0.36 + 0.1 = 0.46$

Q9. Find the value of x in each of the following equation:

(i) $[(17.28/x) / (3.6 * 0.2)] = 2$

(ii) $3648.24 + 364.824 + x - 36.4824 = 3794.1696$

(iii) $8.5 - \{ 5 \frac{1}{2} - [7 \frac{1}{2} + 2.8]/x \} * 4.25 / (0.2)^2 = 306$ (Hotel Management, 1997)

Sol. (i) $(17.28/x) = 2 * 3.6 * 0.2 \Leftrightarrow x = (17.28/1.44) = (1728/14) = 12.$

(ii) $(364.824/x) = (3794.1696 + 36.4824) - 3648.24 = 3830.652 - 3648.24 = 182.412.$

$\Leftrightarrow x = (364.824/182.412) = 2.$

(iii) $8.5 - \{ 5.5 - (7.5 + (2.8/x)) \} * (4.25/0.04) = 306$

$\Leftrightarrow 8.5 - \{ 5.5 - \{ (7.5x + 2.8)/x \} \} * (425/4) = 306$

$\Leftrightarrow 8.5 - \{ (5.5x - 7.5x - 2.8)/x \} * (425/4) = 306$

$\Leftrightarrow 8.5 - \{ (-2x - 2.8)/x \} * 106.25 = 306$

$\Leftrightarrow 8.5 - \{ (-212.5x - 297.5)/x \} = 306$

$\Leftrightarrow (306 - 221)x = 297.5 \Leftrightarrow x = (297.5/85) = 3.5.$

Q10. If $(x/y) = (6/5)$, find the value $(x^2 + y^2)/(x^2 - y^2)$

Sol. $(x^2 + y^2)/(x^2 - y^2) = (x^2/y^2 + 1)/(x^2/y^2 - 1) = [(6/5)^2 + 1] / [(6/5)^2 - 1]$
 $= [(36/25) + 1] / [(36/25) - 1] = (61 * 25)/(25 * 11) = 61/11$

Q11. Find the value of $4 - \frac{5}{1 + \frac{1}{3 + \frac{1}{2 + \frac{1}{4}}}}$

Sol. Given exp. $= 4 - \frac{5}{1 + \frac{1}{3 + \frac{1}{2 + \frac{1}{4}}}} = 4 - \frac{5}{1 + \frac{1}{3 + \frac{4}{9}}} = 4 - \frac{5}{1 + \frac{1}{(31/9)}}$

$= 4 - \frac{5}{1 + \frac{9}{31}} = 4 - \frac{5}{(40/31)} = 4 - (5 * 31)/40 = 4 - (31/8) = 1/8$

Q12. If $\frac{2x}{1 + \frac{1}{1 + \frac{x}{1-x}}} = 1$., then find the value of x .

Sol. We have : $\frac{2x}{1 + \frac{1}{\frac{(1-x)-x}{1-x}}} = 1 \Leftrightarrow \frac{2x}{1 + \frac{1}{[1/(1-x)]}} = 1 \Leftrightarrow \frac{2x}{1+(1-x)} = 1$
 $\Leftrightarrow 2x = 2-x \Leftrightarrow 3x = 2 \Leftrightarrow x = (2/3).$

Q13. (i)If $a/b=3/4$ and $8a+5b=22$,then find the value of a.

(ii)if $x/4-x-3/6=1$,then find the value of x.

Sol. (i) $(a/b)=3/4 \Rightarrow b=(4/3) a.$

$$\therefore 8a+5b=22 \Rightarrow 8a+5*(4/3) a=22 \Rightarrow 8a+(20/3) a=22$$

$$\Rightarrow 44a = 66 \Rightarrow a=(66/44)=3/2$$

(ii) $(x/4)-((x-3)/6)=1 \Leftrightarrow (3x-2(x-3))/12 = 1 \Leftrightarrow 3x-2x+6=12 \Leftrightarrow x=6.$

Q14.If $2x+3y=34$ and $((x+y)/y)=13/8$,then find the value of $5y+7x$.

Sol. The given equations are:

$$2x+3y=34 \dots(i) \text{ and, } ((x+y)/y)=13/8 \Rightarrow 8x+8y=13y \Rightarrow 8x-5y=0 \dots(ii)$$

Multiplying (i) by 5,(ii) by 3 and adding, we get : $34x=170$ or $x=5.$

Putting $x=5$ in (i), we get: $y=8.$

$$\therefore 5y+7x=((5*8)+(7*5))=40+35=75$$

Q15.If $2x+3y+z=55,x-y=4$ and $y-x+z=12$,then what are the values of x , y and z?

Sol. The given equations are:

$$2x+3y+z=55 \dots(i); x+z-y=4 \dots(ii); y-x+z=12 \dots(iii)$$

Subtracting (ii) from (i), we get: $x+4y=51 \dots(iv)$

Subtracting (iii) from (i), we get: $3x+2y=43 \dots(v)$

Multiplying (v) by 2 and subtracting (iv) from it, we get: $5x=35$ or $x=7.$

Putting $x=7$ in (iv), we get: $4y=44$ or $y=11.$

Putting $x=7,y=11$ in (i), we get: $z=8.$

Q16.Find the value of $(1-(1/3))(1-(1/4))(1-(1/5))....(1-(1/100)).$

Sol. Given expression = $(2/3)*(3/4)*(4/5) * * (99/100) = 2/100 = 1/50.$

Q17. Find the value of $(1/(2*3))+(1/(3*4))+(1/(4*5))+(1/(5*6))+.....+ ((1/(9*10)).$

Sol. Given expression= $((1/2)-(1/3))+((1/3)-(1/4))+((1/4)-(1/5))+$

$$((1/5)-(1/6))+....+ ((1/9)-(1/10))$$

$$=((1/2)-(1/10))=4/10 = 2/5.$$

Q18.Simplify: $99^{48}/_{49} * 245.$

Sol. Given expression = $(100-1/49) * 245=(4899/49) * 245 = 4899 * 5=24495.$

Q19. A board 7ft. 9 inches long is divided into 3 equal parts . What is the length of each part?

Sol. Length of board=7ft. 9 inches=(7*12+9)inches=93 inches.

∴ Length of each part = (93/3) inches = 31 inches = 2ft. 7 inches

20.A man divides Rs. Among 5 sons,4daughters and 2 nephews .If each daughter receives four times as much as each nephews and each son receives five times as much as each nephews ,how much does each daughter receive?

Let the share of each nephews be Rs.x.

Then,share of each daughter=rs4x;share of each son=Rs.5x;

So, $5*5x+4*4x+2*x=8600$

$25x+16x+2x=8600$

$=43x=8600$

$x=200$;

Q 21. A man spends 2/5 of his salary on house rent,3/10 of his salary on food and 1/8 of his salary on conveyence.if he has Rs.1400 left with him,find his expenditure on food and conveyance?

Part of salary left= $1-(2/5+3/10+1/8)$

Let the monthly salary be Rs.x

Then, $7/40$ of $x=1400$

$X=(1400*40/7)$

$=8600$

Expenditure on food=Rs.(3/10*800)=Rs.2400

Expenditure on conveyence=Rs.(1/8*8000)=Rs.1000

Q 22. A third of Arun's marks in mathematics exceeds a half of his marks in english by 80.if he got 240 marks In two subjects together how many marks did he got inh english?

Let Arun's marks in mathematics and english be x and y

Then $1/3x-1/2y=30$

$2x-3y=180$>(1)

$x+y=240$>(2)

solving (1) and (2)

$x=180$

and $y=60$

Q23. A tin of oil was 4/5full.when 6 bottles of oil were taken out and four bottles of oil were poured into it, it was 3/4 full. how many bottles of oil can the tin contain?

Suppose x bottles can fill the tin completely

Then $4/5x-3/4x=6-4$

$X/20=2$

$X=40$

Therefore required no of bottles =40

Q 24. If 1/8 of a pencil is black 1/2 of the remaining is white and the remaining 3 1/2 is blue find the total length of the pencil?

Let the total length be xm

Then black part =x/8cm

The remaining part $= (x - x/8) \text{ cm} = 7x/8 \text{ cm}$
 White part $= (1/2 * 7x/8) = 7x/16 \text{ cm}$
 Remaining part $= (7x/8 - 7x/16) = 7x/16 \text{ cm}$
 $7x/16 = 7/2$
 $x = 8 \text{ cm}$

Q25. in a certain office $1/3$ of the workers are women $1/2$ of the women are married and $1/3$ of the married women have children if $3/4$ of the men are married and $2/3$ of the married men have children what part of workers are without children?

Let the total no of workers be x
 No of women $= x/3$
 No of men $= x - (x/3) = 2x/3$
 No of women having children $= 1/3$ of $1/2$ of $x/3 = x/18$
 No of men having children $= 2/3$ of $3/4$ of $2x/3 = x/3$
 No of workers having children $= x/8 + x/3 = 7x/18$
 Workers having no children $= x - 7x/18 = 11x/18 = 11/18$ of all workers

Q26. a crate of mangoes contains one bruised mango for every thirty mango in the crate. If three out of every four bruised mango are considerably unsaleable and there are 12 unsaleable mangoes in the crate then how many mango are there in the crate?

Let the total no of mangoes in the crate be x
 Then the no of bruised mango $= 1/30 x$
 Let the no of unsaleable mangoes $= 3/4 (1/30 x)$
 $1/40 x = 12$
 $x = 480$

Q27. a train starts full of passengers at the first station it drops $1/3$ of the passengers and takes 280 more at the second station it drops one half the new total and takes twelve more. on arriving at the third station it is found to have 248 passengers. Find the no of passengers in the beginning?

Let no of passengers in the beginning be x
 After first station no passengers $= (x - x/3) + 280 = 2x/3 + 280$
 After second station no passengers $= 1/2(2x/3 + 280) + 12$
 $1/2(2x/3 + 280) + 12 = 248$
 $2x/3 + 280 = 2 * 236$
 $2x/3 = 192$
 $x = 288$

Q28. if $a^2 + b^2 = 177$ and $ab = 54$ then find the value of $a + b/a - b$?

$(a + b)^2 = a^2 + b^2 + 2ab = 177 + 2 * 54 = 225$
 $a + b = 15$
 $(a - b)^2 = a^2 + b^2 - 2ab = 177 - 2 * 54$
 $a - b = 3$
 $a + b/a - b = 15/3 = 5$

Q 29.find the value of (75983*75983- 45983*45983/30000)

Given expression= $(75983)^2-(45983)^2/(75983-45983)$

$$=(a-b)^2/(a-b)$$

$$=(a+b)(a-b)/(a-b)$$

$$=(a+b)$$

$$=75983+45983$$

$$=121966$$

Q30.find the value of $\left[\frac{343*343*343-113*113*113}{343*343+343*113+113*113} \right]$

Given expression= $\frac{(a^3-b^3)}{a^2+ab+b^2}$

$$=(a-b)$$

$$=(343-113)$$

$$.=230$$

Q31.Village X has a population of 68000,which is decreasing at the rate of 1200 per year.VillagyY has a population of 42000,which is increasing at the rate of 800 per year .in how many years will the population of the two villages be equal?

Let the population of two villages be equal after p years

$$\text{Then, } 68000-1200p=42000+800p$$

$$2000p=26000$$

$$p=13$$

Q32.From a group of boys and girls,15 girls leave.There are then left 2 boys for each girl.After this,45 boys leave.There are then 5 girls for each boy.Find the number of girls in the beginning?

Let at present there be x boys.

$$\text{Then, no of girls at present}=5x$$

$$\text{Before the boys had left: no of boys}=x+45$$

$$\text{And no of girls}=5x$$

$$X+45=2*5x$$

$$9x=45$$

$$x=5$$

$$\text{no of girls in the beginning}=25+15=40$$

Q33.An employer pays Rs.20 for each day a worker works and for feits Rs.3 for each day is ideal at the end of sixty days a worker gets Rs.280. for how many days did the worker remain ideal?

Suppose a worker remained ideal for x days then he worked for 60-x days

$$20*(60-x)-3x=280$$

$$1200-23x=280$$

$$23x=920$$

$$x=40$$

Q34. kiran had 85 currency notes in all , some of which were of Rs.100 denomination and the remaining of Rs.50 denomination the total amount of all these currency note was Rs.5000.how much amount did she have in the denomination of Rs.50?

Let the no of fifty rupee notes be x

Then,no of 100 rupee notes =(85-x)

$$50x + 100(85-x) = 5000$$

$$x + 2(85-x) = 100$$

$$x = 70$$

so,,required amount=Rs.(50*70)= Rs.3500

Q35. When an amount was distributed among 14 boys, each of them got rs 80 more than the amount received by each boy when the same amount is distributed equally among 18 boys. What was the amount?

Sol. Let the total amount be Rs. X the,

$$\frac{x}{14} - \frac{x}{18} = 80 \Leftrightarrow \frac{2x}{126} = 80 \Leftrightarrow \frac{x}{63} = 80 \Leftrightarrow x = 5040.$$

Hence the total amount is 5040.

Q36. Mr. Bhaskar is on tour and he has Rs. 360 for his expenses. If he exceeds his tour by 4 days, he must cut down his daily expenses by Rs. 3. for how many days is Mr. Bhaskar on tour?

Sol. Suppose Mr. Bhaskar is on tour for x days. Then,

$$\frac{360}{x} - \frac{360}{x+4} = 3 \Leftrightarrow \frac{1}{x} - \frac{1}{x+4} = \frac{1}{120} \Leftrightarrow x(x+4) = 4 \times 120 = 480$$

$$\Leftrightarrow x^2 + 4x - 480 = 0 \Leftrightarrow (x+24)(x-20) = 0 \Leftrightarrow x = 20.$$

Hence Mr. Bhaskar is on tour for 20 days.

Q37. Two pens and three pencils cost Rs 86. four Pens and a pencil cost Rs. 112. find the cost of a pen and that of a pencil.

Sol. Let the cost of a pen and a pencil be Rs. X and Rs. Y respectively.

$$\text{Then, } 2x + 3y = 86 \dots(i) \text{ and } 4x + y = 112.$$

Solving (i) and (ii), we get: x = 25 and y = 12.

Cost of a pen =Rs. 25 and the cost of a pencil =Rs. 12.

Q38. Arjun and Sajal are friends . each has some money. If Arun gives Rs. 30 to Sajal, the Sajal will have twice the money left with Arjun. But, if Sajal gives Rs. 10 to Arjun, the Arjun will have thrice as much as is left with Sajal. How much money does each have?

Sol. Suppose Arun has Rs. X and Sjal has Rs. Y. then,

$$2(x-30) = y+30 \Rightarrow 2x-y = 90 \dots(i)$$

$$\text{and } x+10 = 3(y-10) \Rightarrow x-3y = -40 \dots(ii)$$

Solving (i) and (ii), we get x =62 and y =34.

Arun has Rs. 62 and Sajal has Rs. 34.

Q39. In a caravan, in addition to 50 hens there are 45 goats and 8 camels with some keepers. If the total number of feet be 224 more than the number of heads, find the number of keepers.

Sol. Let the number of keepers be x then,

$$\text{Total number of heads} = (50 + 45 + 8 + x) = (103 + x).$$

$$\text{Total number of feet} = (45 + 8) \times 4 + (50 + x) \times 2 = (312 + 2x).$$

$$(312 + 2x) - (103 + x) = 224 \Leftrightarrow x = 15.$$

Hence, number of keepers = 15.

5. PROBLEMS ON SQUARE ROOTS AND CUBE ROOTS

Q1. Evaluate $\sqrt{6084}$ by factorization method

Sol. Method: Express the given number as the product of prime factors.
Now, take the product of these prime factors choosing one out of every pair of the same primes. This product gives the square root of the given number.
Thus, resolving 6084 into prime factors, we get:
 $6084 = 2^2 \times 3^2 \times 13^2$
 $\therefore \sqrt{6084} = (2 \times 3 \times 13) = 78.$

$$\begin{array}{r|l} 2 & 6084 \\ 2 & 3042 \\ 3 & 1521 \\ 3 & 507 \\ 13 & 169 \\ & 13 \end{array}$$

Q2. Find the square root of 1471369.

Sol. Explanation: In the given number, mark off the digits in pairs starting from the unit's digit. Each pair and the remaining one digit is called a period.
Now, $1^2 = 1$. On subtracting, we get 0 as remainder.
Now, bring down the next period i.e., 47.
Now, trial divisor is $1 \times 2 = 2$ and trial dividend is 47.
So, we take 22 as divisor and put 2 as quotient.
The remainder is 3.
Next, we bring down the next period which is 13.
Now, trial divisor is $12 \times 2 = 24$ and trial dividend is 313. So, we take 241 as dividend and 1 as quotient.
The remainder is 72.
Bring down the next period i.e., 69.
Now, the trial divisor is $121 \times 2 = 242$ and the trial dividend is 7269. So, we take 3 as quotient and 2423 as divisor. The remainder is then zero.
Hence, $\sqrt{1471369} = 1213.$

$$\begin{array}{r|l} 1 & 1471369 \text{ (1213)} \\ & 1 \\ \hline 22 & 47 \\ & 44 \\ \hline 241 & 313 \\ & 241 \\ \hline 2423 & 7269 \\ & 7269 \\ \hline & x \end{array}$$

Q3. Evaluate: $\sqrt{248 + \sqrt{51 + \sqrt{169}}}$.

Sol. Given expression = $\sqrt{248 + \sqrt{51 + 13}} = \sqrt{248 + \sqrt{64}} = \sqrt{248 + 8} = \sqrt{256} = 16.$

Q4. If $a * b * c = \sqrt{(a+2)(b+3)/(c+1)}$, find the value of $6 * 15 * 3$.

Sol. $6 * 15 * 3 = \sqrt{(6+2)(15+3)/(3+1)} = \sqrt{8 * 18 / 4} = \sqrt{144 / 4} = 12 / 4 = 3.$

Q5. Find the value of $\sqrt{25/16}$.

Sol. $\sqrt{25/16} = \sqrt{25} / \sqrt{16} = 5 / 4$

Q6. What is the square root of 0.0009?

Sol. $\sqrt{0.0009} = \sqrt{9 / 1000} = 3 / 100 = 0.03.$

Q7. Evaluate $\sqrt{175.2976}$.

Sol. Method: We make even number of decimal places by affixing a zero, if necessary. Now, we mark off periods and extract the square root as shown.

$$\therefore \sqrt{175.2976} = 13.24$$

1	175.2976 (13.24
1	1
23	75
	69
262	629
	524
2644	10576
	10576
	x

Q8. What will come in place of question mark in each of the following questions?

(i) $\sqrt{32.4 / ?} = 2$

(ii) $\sqrt{86.49} + \sqrt{5 + (?)^2} = 12.3$.

Sol. (i) Let $\sqrt{32.4 / x} = 2$. Then, $32.4/x = 4 \Leftrightarrow 4x = 32.4 \Leftrightarrow x = 8.1$.

(ii) Let $\sqrt{86.49} + \sqrt{5 + x^2} = 12.3$.

Then, $9.3 + \sqrt{5 + x^2} = 12.3 \Leftrightarrow \sqrt{5 + x^2} = 12.3 - 9.3 = 3$

$\Leftrightarrow 5 + x^2 = 9 \Leftrightarrow x^2 = 9 - 5 = 4 \Leftrightarrow x = \sqrt{4} = 2$.

Q9. Find the value of $\sqrt{0.289 / 0.00121}$.

Sol. $\sqrt{0.289 / 0.00121} = \sqrt{0.28900 / 0.00121} = \sqrt{28900 / 121} = 170 / 11$.

Q10. If $\sqrt{1 + (x / 144)} = 13 / 12$, the find the value of x.

Sol. $\sqrt{1 + (x / 144)} = 13 / 12 \Rightarrow (1 + (x / 144)) = (13 / 12)^2 = 169 / 144$

$\Rightarrow x / 144 = (169 / 144) - 1$

$\Rightarrow x / 144 = 25/144 \Rightarrow x = 25$.

Q11. Find the value of $\sqrt{3}$ up to three places of decimal.

Sol.

1	3.000000 (1.732
1	1
27	200
	189
343	1100
	1029
3462	7100
	6924

$$\therefore \sqrt{3} = 1.732.$$

Q12. If $\sqrt{3} = 1.732$, find the value of $\sqrt{192 - \frac{1}{2}\sqrt{48} - \sqrt{75}}$ correct to 3 places of decimal.

$$\begin{aligned}\text{Sol. } \sqrt{192 - (1/2)\sqrt{48} - \sqrt{75}} &= \sqrt{64 * 3 - (1/2)\sqrt{16 * 3} - \sqrt{25 * 3}} \\ &= 8\sqrt{3} - (1/2) * 4\sqrt{3} - 5\sqrt{3} \\ &= 3\sqrt{3} - 2\sqrt{3} = \sqrt{3} = 1.732\end{aligned}$$

Q13. Evaluate: $\sqrt{(9.5 * 0.0085 * 18.9) / (0.0017 * 1.9 * 0.021)}$

Sol. Given exp. = $\sqrt{(9.5 * 0.0085 * 18.9) / (0.0017 * 1.9 * 0.021)}$

Now, since the sum of decimal places in the numerator and denominator under the radical sign is the same, we remove the decimal.

\therefore Given exp = $\sqrt{(95 * 85 * 18900) / (17 * 19 * 21)} = \sqrt{5 * 5 * 900} = 5 * 30 = 150.$

Q14. Simplify: $\sqrt{[(12.1)^2 - (8.1)^2] / [(0.25)^2 + (0.25)(19.95)]}$

$$\begin{aligned}\text{Sol. } \text{Given exp.} &= \sqrt{[(12.1 + 8.1)(12.1 - 8.1)] / [(0.25)(0.25 + 19.95)]} \\ &= \sqrt{(20.2 * 4) / (0.25 * 20.2)} = \sqrt{4 / 0.25} = \sqrt{400 / 25} = \sqrt{16} = 4.\end{aligned}$$

Q15. If $x = 1 + \sqrt{2}$ and $y = 1 - \sqrt{2}$, find the value of $(x^2 + y^2)$.

Sol. $x^2 + y^2 = (1 + \sqrt{2})^2 + (1 - \sqrt{2})^2 = 2[(1)^2 + (\sqrt{2})^2] = 2 * 3 = 6.$

Q16. Evaluate: $\sqrt{0.9}$ up to 3 places of decimal.

Sol.

9	0.900000(0.948	
	81	
184	900	
	736	
1888	16400	
	15104	

$\therefore \sqrt{0.9} = 0.948$

Q17. If $\sqrt{15} = 3.88$, find the value of $\sqrt{(5/3)}$.

Sol. $\sqrt{(5/3)} = \sqrt{(5 * 3) / (3 * 3)} = \sqrt{15} / 3 = 3.88 / 3 = 1.2933.... = 1.29\bar{3}.$

Q18. Find the least square number which is exactly divisible by 10,12,15 and 18.

Sol. L.C.M. of 10, 12, 15, 18 = 180. Now, $180 = 2 * 2 * 3 * 3 * 5 = 2^2 * 3^2 * 5.$

To make it a perfect square, it must be multiplied by 5.

\therefore Required number = $(2^2 * 3^2 * 5^2) = 900.$

Q19. Find the greatest number of five digits which is a perfect square.

Sol. Greatest number of 5 digits is 99999.

$$\begin{array}{r}
 3 \overline{) 99999(316} \\
 \underline{9} \\
 61 \\
 \underline{61} \\
 626 \\
 \underline{3899} \\
 3756 \\
 \underline{143} \\
 0
 \end{array}$$

∴ Required number == (99999 - 143) = 99856.

Q20. Find the smallest number that must be added to 1780 to make it a perfect square.

Sol.

$$\begin{array}{r}
 4 \overline{) 1780(42} \\
 \underline{16} \\
 82 \\
 \underline{180} \\
 164 \\
 \underline{16} \\
 0
 \end{array}$$

∴ Number to be added = $(43)^2 - 1780 = 1849 - 1780 = 69$.

Q21. $\sqrt{2} = 1.4142$, find the value of $\sqrt{2} / (2 + \sqrt{2})$.

Sol. $\sqrt{2} / (2 + \sqrt{2}) = \sqrt{2} / (2 + \sqrt{2}) * (2 - \sqrt{2}) / (2 - \sqrt{2}) = (2\sqrt{2} - 2) / (4 - 2)$
 $= 2(\sqrt{2} - 1) / 2 = \sqrt{2} - 1 = 0.4142$.

Q22. If $x = (\sqrt{5} + \sqrt{3}) / (\sqrt{5} - \sqrt{3})$ and $y = (\sqrt{5} - \sqrt{3}) / (\sqrt{5} + \sqrt{3})$, find the value of $(x^2 + y^2)$.

Sol.

$$x = [(\sqrt{5} + \sqrt{3}) / (\sqrt{5} - \sqrt{3})] * [(\sqrt{5} + \sqrt{3}) / (\sqrt{5} + \sqrt{3})] = (\sqrt{5} + \sqrt{3})^2 / (5 - 3) \\
 = (5 + 3 + 2\sqrt{15}) / 2 = 4 + \sqrt{15}$$

$$y = [(\sqrt{5} - \sqrt{3}) / (\sqrt{5} + \sqrt{3})] * [(\sqrt{5} - \sqrt{3}) / (\sqrt{5} - \sqrt{3})] = (\sqrt{5} - \sqrt{3})^2 / (5 - 3) \\
 = (5 + 3 - 2\sqrt{15}) / 2 = 4 - \sqrt{15}$$

$$\therefore x^2 + y^2 = (4 + \sqrt{15})^2 + (4 - \sqrt{15})^2 = 2[(4)^2 + (\sqrt{15})^2] = 2 * 31 = 62.$$

Q23. Find the cube root of 2744.

Sol. Method: Resolve the given number as the product of prime factors and take the product of prime factors, choosing one out of three of the same prime factors. Resolving 2744 as the product of prime factors, we get:

$$\begin{array}{r}
 2 \overline{) 2744} \\
 \underline{2} \\
 2 \overline{) 1372} \\
 \underline{2} \\
 2 \overline{) 686} \\
 \underline{7} \\
 7 \overline{) 343} \\
 \underline{7} \\
 7 \overline{) 49} \\
 \underline{7} \\
 0
 \end{array}$$

$$2744 = 2^3 \times 7^3.$$

$$\therefore \sqrt[3]{2744} = 2 \times 7 = 14.$$

Q24. By what least number 4320 be multiplied to obtain a number which is a perfect cube?

Sol. Clearly, $4320 = 2^3 * 3^3 * 2^2 * 5$.

To make it a perfect cube, it must be multiplied by $2 * 5^2$ i.e., 50.

6.PROBLEMS ON AVERAGE

Q1. Find the average of all prime numbers between 30 and 50?

Sol: there are five prime numbers between 30 and 50.

They are 31,37,41,43 and 47.

Therefore the required average= $(31+37+41+43+47)/5 \Leftrightarrow 199/5 \Leftrightarrow 39.8$.

Q2. Find the average of first 40 natural numbers?

Sol: sum of first n natural numbers= $n(n+1)/2$;

So, sum of 40 natural numbers= $(40*41)/2 \Leftrightarrow 820$.

Therefore the required average= $(820/40) \Leftrightarrow 20.5$.

Q3. Find the average of first 20 multiples of 7?

Sol: Required average = $7(1+2+3+.....+20)/20 \Leftrightarrow (7*20*21)/(20*2) \Leftrightarrow (147/2)=73.5$.

Q4. The average of four consecutive even numbers is 27. find the largest of these numbers?

Sol: let the numbers be x,x+2,x+4 and x+6. then,

$$(x+(x+2)+(x+4)+(x+6))/4 = 27$$

$$\Leftrightarrow (4x+12)/4 = 27$$

$$\Leftrightarrow x+3=27 \quad \Leftrightarrow x=24.$$

Therefore the largest number= $(x+6)=24+6=30$.

Q5. There are two sections A and B of a class consisting of 36 and 44 students respectively. If the average weight of section A is 40kg and that of section B is 35kg, find the average weight of the whole class?

Sol: total weight of $(36+44)$ students= $(36*40+44*35)$ kg =2980kg.

Therefore weight of the total class= $(2980/80)$ kg =37.25kg.

Q6.Nine persons went to a hotel for taking their meals 8 of them spent Rs.12 each on their meals and the ninth spent Rs.8 more than the average expenditure of all the nine.What was the total money spent by them?

Sol: Let the average expenditure of all nine be Rs.x

Then $12*8+(x+8)=9x$ or $8x=104$ or $x=13$.

Total money spent = $9x$ =Rs. $(9*13)$ =Rs.117.

Q7. Of the three numbers, second is twice the first and is also thrice the third.If the average of the three numbers is 44.Find the largest number.

Sol: Let the third number be x.

Then second number = $3x$.

First number= $3x/2$.

Therefore $x+3x+(3x/2)=(44*3)$ or $x=24$

So largest number= 2nd number= $3x=72$.

Q8. The average of 25 result is 18.The average of 1st 12 of them is 14 & that of last 12 is 17.Find the 13th result.

Sol: Clearly 13th result= $(\text{sum of 25 results})-(\text{sum of 24 results})$

$$\begin{aligned}
 &= (18 \times 25) - (14 \times 12) + (17 \times 12) \\
 &= 450 - (168 + 204) \\
 &= 450 - 372 = 78.
 \end{aligned}$$

Q9. The Average of 11 results is 16, if the average of the 1st 6 results is 58 & that of the last 63. Find the 6th result.

Sol: 6th result = $(58 \times 6 + 63 \times 6 - 60 \times 11) = 66$

Q10. The average waight of A,B,C is 45 Kg. The avg wgt of A & B be 40Kg & that of B,C be 43Kg. Find the wgt of B.

Sol. Let A,B,c represent their individual wgts.

Then,

$$A + B + C = (45 \times 3) \text{Kg} = 135 \text{Kg}$$

$$A + B = (40 \times 2) \text{Kg} = 80 \text{Kg} \text{ \& } B + C = (43 \times 2) \text{Kg} = 86 \text{Kg}$$

$$B = (A + B) + (B + C) - (A + B + C)$$

$$= (80 + 86 - 135) \text{Kg}$$

$$= 31 \text{Kg}.$$

Q11. The average age of a class of 39 students is 15 years. If the age of the teacher be included, then the average increases by 3 months. Find the age of the teacher.

Sol. Total age of 39 persons = (39×15) years
 $= 585$ years.

Average age of 40 persons = 15 yrs 3 months
 $= 61/4$ years.

Total age of 40 persons = $(\frac{61}{4} \times 40)$ years = 610 years.

\therefore Age of the teacher = $(610 - 585)$ years = 25 years.

Q12. The average weight of 10 oarsmen in a boat is increased by 1.8 kg when one of the crew, who weighs 53 kg is replaced by a new man. Find the weight of the new man.

Sol. Total weight increased = (1.8×10) kg = 18 kg.

\therefore Weight of the new man = $(53 + 18)$ kg = 71 kg.

Q13. There were 35 students in a hostel. Due to the admission of 7 new students, ;he expenses of the mess were increased by Rs. 42 per day while the average expenditure per head diminished by Rs 1. What was the original expenditure of the mess?

Sol. Let the original average expenditure be Rs. x. Then,

$$42(x - 1) - 35x = 42 \Leftrightarrow 7x = 84 \Leftrightarrow x = 12.$$

Original expenditure = Rs. (35×12) = Rs. 420. .

Q14. A batsman makes a score of 87 runs in the 17th inning and thus increases his avg by 3. Find his average after 17th inning.

Sol. Let the average after 17th inning = x.

Then, average after 16th inning = $(x - 3)$.

$\therefore 16(x - 3) + 87 = 17x$ or $x = (87 - 48) = 39$.

Q15. Distance between two stations A and B is 778 km. A train covers the journey from A to B at 84 km per hour and returns back to A with a uniform speed of 56 km perhour. Find the average speed of the train during the whole journey.

Sol. Required average speed = $((2xy)/(x+y))$ km / hr

$$=(2 \times 84 \times 56)/(84+56)\text{km/hr}$$

$$= (2*84*56)/140 \text{ km/hr}$$

$$=67.2 \text{ km/hr.}$$

7. PROBLEMS ON AGES

Q1. Rajeev's age after 15 years will be 5 times his age 5 years back. What is the present age of Rajeev ?

Sol. Let Rajeev's present age be x years. Then,

Rajeev's age after 15 years = $(x + 15)$ years.

Rajeev's age 5 years back = $(x - 5)$ years.

$$\therefore x + 15 = 5(x - 5) \Leftrightarrow x + 15 = 5x - 25 \Leftrightarrow 4x = 40 \Leftrightarrow x = 10.$$

Hence, Rajeev's present age = 10 years.

Q2. The ages of two persons differ by 16 years. If 6 years ago, the elder one be 3 times as old as the younger one, find their present ages.

Sol. Let the age of the younger person be x years.

Then, age of the elder person = $(x + 16)$ years.

$$\therefore 3(x - 6) = (x + 16 - 6) \Leftrightarrow 3x - 18 = x + 10 \Leftrightarrow 2x = 28 \Leftrightarrow x = 14.$$

Hence, their present ages are 14 years and 30 years.

Q3. The product of the ages of Ankit and Nikita is 240. If twice the age of Nikita is more than Ankit's age by 4 years, what is Nikita's age?

Sol. Let Ankit's age be x years. Then, Nikita's age = $240/x$ years.

$$\therefore 2 \times (240/x) - x = 4 \Leftrightarrow 480 - x^2 = 4x \Leftrightarrow x^2 + 4x - 480 = 0$$

$$\Leftrightarrow (x+24)(x-20) = 0 \Leftrightarrow x = 20.$$

Hence, Nikita's age = (220) years = 12 years.

Q4. The present age of a father is 3 years more than three times the age of his son. Three years hence, father's age will be 10 years more than twice the age of the son. Find the present age of the father.

Sol. Let the son's present age be x years. Then, father's present age = $(3x + 3)$ years

$$\therefore (3x + 3 + 3) = 2(x + 3) + 10 \Leftrightarrow 3x + 6 = 2x + 16 \Leftrightarrow x = 10.$$

Hence, father's present age = $(3x + 3) = ((3 \times 10) + 3)$ years = 33 years.

Q5. Rohit was 4 times as old as his son 8 years ago. After 8 years, Rohit will be twice as old as his son. What are their present ages?

Sol. Let son's age 8 years ago be x years. Then, Rohit's age 8 years ago = $4x$ years.

Son's age after 8 years = $(x + 8) + 8 = (x + 16)$ years.

Rohit's age after 8 years = $(4x + 8) + 8 = (4x + 16)$ years.

$$\therefore 2(x + 16) = 4x + 16 \Leftrightarrow 2x = 16 \Leftrightarrow x = 8.$$

Hence, son's 'present age = $(x + 8) = 16$ years.

Rohit's present age = $(4x + 8) = 40$ years.

Q6. One year ago, the ratio of Gaurav's and Sachin's age was 6: 7 respectively. Four years hence, this ratio would become 7: 8. How old is Sachin ?

Sol:

Let Gaurav's and Sachin's ages one year ago be $6x$ and $7x$ years respectively. Then, Gaurav's age 4 years hence = $(6x + 1) + 4 = (6x + 5)$ years.

Sachin's age 4 years hence = $(7x + 1) + 4 = (7x + 5)$ years.

$$\frac{6x+5}{7x+5} = \frac{7}{8} \Leftrightarrow 8(6x+5) = 7(7x+5) \Leftrightarrow 48x + 40 = 49x + 35 \Leftrightarrow x = 5.$$

Hence, Sachin's present age = $(7x + 1) = 36$ years.

Q7. Abhay's age after six years will be three-seventh of his fathers age. Ten years ago the ratio of their ages was 1 : 5. What is Abhay's father's age at present?

Sol. Let the ages of Abhay and his father 10 years ago be x and $5x$ years respectively. Then,

Abhay's age after 6 years = $(x + 10) + 6 = (x + 16)$ years.

Father's age after 6 years = $(5x + 10) + 6 = (5x + 16)$ years.

$$\frac{x + 16}{5x + 16} = \frac{3}{7} \Leftrightarrow 7(x + 16) = 3(5x + 16) \Leftrightarrow 7x + 112 = 15x + 48$$

$$\Leftrightarrow 8x = 64 \Leftrightarrow x = 8.$$

Hence, Abhay's father's present age = $(5x + 10) = 50$ years.

8.SURDS AND INDICES

Q1. Simplify: (i) $(27)^{2/3}$ (ii) $(1024)^{-4/5}$ (iii) $(8/125)^{-4/3}$

Sol. (i) $(27)^{2/3} = (3^3)^{2/3} = 3^{(3 \times (2/3))} = 3^2 = 9$
 (ii) $(1024)^{-4/5} = (4^5)^{-4/5} = 4^{\{5 \times (-4/5)\}} = 4^{-4} = 1/4^4 = 1/256$
 (iii) $(8/125)^{-4/3} = \{(2/5)^3\}^{-4/3} = (2/5)^{\{3 \times (-4/3)\}} = (2/5)^{-4} = (5/2)^4 = 5^4/2^4 = 625/16$

Q2. Evaluate: (i) $(.00032)^{3/5}$ (ii) $(256)^{0.16} \times (16)^{0.18}$.

Sol. (i) $(0.00032)^{3/5} = (32/100000)^{3/5} = (2^5/10^5)^{3/5} = \{(2/10)^5\}^{3/5} = (1/5)^{(5 \times 3/5)} = (1/5)^3 = 1/125$
 (ii) $(256)^{0.16} \times (16)^{0.18} = \{(16)^2\}^{0.16} \times (16)^{0.18} = (16)^{(2 \times 0.16)} \times (16)^{0.18} = (16)^{0.32} \times (16)^{0.18} = (16)^{(0.32+0.18)} = (16)^{0.5} = (16)^{1/2} = 4.$

Q3. What is the quotient when $(x^{-1} - 1)$ is divided by $(x - 1)$?

Sol. $\frac{x^{-1} - 1}{x - 1} = \frac{(1/x) - 1}{x - 1} = \frac{1 - x}{x(x - 1)} = \frac{-1}{x}$

Hence, the required quotient is $-1/x$

Q4. If $2^{x-1} + 2^{x+1} = 1280$, then find the value of x .

Sol. $2^{x-1} + 2^{x+1} = 1280 \Leftrightarrow 2^{x-1}(1 + 2^2) = 1280$

$$\Leftrightarrow 2^{x-1} = 1280/5 = 256 = 2^8 \Leftrightarrow x-1 = 8 \Leftrightarrow x = 9.$$

Hence, $x = 9$.

Q5. Find the value of $[5(8^{1/3} + 27^{1/3})^3]^{1/4}$

Sol. $[5(8^{1/3} + 27^{1/3})^3]^{1/4} = [5\{(2^3)^{1/3} + (3^3)^{1/3}\}^3]^{1/4} = [5\{(2^{3 \times 1/3})^{1/3} + (3^{3 \times 1/3})^{1/3}\}^3]^{1/4} = [5(2+3)^3]^{1/4} = (5 \times 5^3)^{1/4} = 5^{(4 \times 1/4)} = 5^1 = 5.$

Q6. Find the Value of $\{(16)^{3/2} + (16)^{-3/2}\}$

Sol. $[(16)^{3/2} + (16)^{-3/2}] = (4^2)^{3/2} + (4^2)^{-3/2} = 4^{(2 \times 3/2)} + 4^{\{2 \times (-3/2)\}} = 4^3 + 4^{-3} = 4^3 + (1/4^3) = (64 + (1/64)) = 4097/64.$

Q7. If $(1/5)^{3y} = 0.008$, then find the value of $(0.25)^y$.

Sol. $(1/5)^{3y} = 0.008 = 8/1000 = 1/125 = (1/5)^3 \Leftrightarrow 3y = 3 \Leftrightarrow Y = 1.$

$$\therefore (0.25)^y = (0.25)^1 = 0.25.$$

Q8. Find the value of $\frac{(243)^{n/5} \times 3^{2n+1}}{9^n \times 3^{n-1}}$.

Sol. $\frac{(243)^{n/5} \times 3^{2n+1}}{(3^2)^n \times 3^{n-1}} = \frac{3^{(5 \times n/5)} \times 3^{2n+1}}{3^{2n} \times 3^{n-1}} = \frac{3^n \times 3^{2n+1}}{3^{2n} \times 3^{n-1}} = \frac{3^{n+(2n+1)}}{3^{2n+n-1}} = \frac{3^{(3n+1)}}{3^{(3n-1)}} = 3^{(3n+1)-(3n-1)} = 3^2 = 9.$

Q9. Find the value Of $(2^{1/4}-1)(2^{3/4}+2^{1/2}+2^{1/4}+1)$

Sol. Putting $2^{1/4} = x$, we get :

$$\begin{aligned}(2^{1/4}-1)(2^{3/4}+2^{1/2}+2^{1/4}+1) &= (x-1)(x^3+x^2+x+1), \text{ where } x = 2^{1/4} \\ &= (x-1)[x^2(x+1)+(x+1)] \\ &= (x-1)(x+1)(x^2+1) = (x^2-1)(x^2+1) \\ &= (x^4-1) = [(2^{1/4})^4-1] = [2^{(1/4 \times 4)}-1] = (2-1) = 1.\end{aligned}$$

Q10. Find the value of $\frac{6^{2/3} \times \sqrt[3]{6^7}}{\sqrt[3]{6^6}}$

$$\text{Sol. } \frac{6^{2/3} \times \sqrt[3]{6^7}}{\sqrt[3]{6^6}} = \frac{6^{2/3} \times (6^7)^{1/3}}{(6^6)^{1/3}} = \frac{6^{2/3} \times 6^{(7 \times 1/3)}}{6^{(6 \times 1/3)}} = \frac{6^{2/3} \times 6^{(7/3)}}{6^2}$$

$$= 6^{2/3} \times 6^{((7/3)-2)} = 6^{2/3} \times 6^{1/3} = 6^1 = 6.$$

Q11. If $x=y^a$, $y=z^b$ and $z=x^c$, then find the value of abc .

$$\begin{aligned}\text{Sol. } z^1 &= x^c = (y^a)^c \quad [\text{since } x = y^a] \\ &= y^{(ac)} = (z^b)^{ac} \quad [\text{since } y = z^b] \\ &= z^{b(ac)} = z^{abc}\end{aligned}$$

$$\therefore abc = 1.$$

Q12. Simplify $[(x^a / x^b)^{(a^2+b^2+ab)}] * [(x^b / x^c)^{(b^2+c^2+bc)}] * [(x^c/x^a)^{(c^2+a^2+ca)]$

Sol. Given Expression

$$\begin{aligned}&= [\{ x^{(a-b)} \}^{(a^2+b^2+ab)}] \cdot [\{ x^{(b-c)} \}^{(b^2+c^2+bc)}] \cdot [\{ x^{(c-a)} \}^{(c^2+a^2+ca)}] \\ &= [x^{(a-b)(a^2+b^2+ab)} \cdot x^{(b-c)(b^2+c^2+bc)} \cdot x^{(c-a)(c^2+a^2+ca)}] \\ &= [x^{(a^3-b^3)}] \cdot [x^{(b^3-c^3)}] \cdot [x^{(c^3-a^3)}] = x^{(a^3-b^3+b^3-c^3+c^3-a^3)} = x^0 = 1.\end{aligned}$$

Q13. Which is larger $\sqrt{2}$ or $\sqrt[3]{3}$?

Sol. Given surds are of order 2 and 3. Their L.C.M. is 6. Changing each to a surd of order 6, we get:

$$\begin{aligned}\sqrt{2} &= 2^{1/2} = 2^{((1/2) \times (3/3))} = 2^{3/6} = 8^{1/6} = \sqrt[6]{8} \\ \sqrt[3]{3} &= 3^{1/3} = 3^{((1/3) \times (2/2))} = 3^{2/6} = (3^2)^{1/6} = (9)^{1/6} = \sqrt[6]{9}.\end{aligned}$$

Clearly, $\sqrt[6]{9} > \sqrt[6]{8}$ and hence $\sqrt[3]{3} > \sqrt{2}$.

Q14. Find the largest from among $4\sqrt{6}$, $\sqrt{2}$ and $\sqrt[3]{4}$.

Sol. Given surds are of order 4, 2 and 3 respectively. Their L.C.M, is 12, Changing each to a surd of order 12, we get:

$$\begin{aligned}4\sqrt{6} &= 6^{1/4} = 6^{((1/4) \times (3/3))} = 6^{3/12} = (6^3)^{1/12} = (216)^{1/12}. \\ \sqrt{2} &= 2^{1/2} = 2^{((1/2) \times (6/6))} = 2^{6/12} = (2^6)^{1/12} = (64)^{1/12}. \\ \sqrt[3]{4} &= 4^{1/3} = 4^{((1/3) \times (4/4))} = 4^{4/12} = (4^4)^{1/12} = (256)^{1/12}.\end{aligned}$$

Clearly, $(256)^{1/12} > (216)^{1/12} > (64)^{1/12}$

Largest one is $(256)^{1/12}$. i.e. $\sqrt[3]{4}$.

9. PROBLEMS ON PERCENTAGE

Q1. Express each of the following as a fraction:

- (i) 56% (ii) 4% (iii) 0.6% (iv) 0.008%

Sol. (i) $56\% = 56/100 = 14/25$. (ii) $4\% = 4/100 = 1/25$.
(iii) $0.6 = 6/1000 = 3/500$. (iv) $0.008 = 8/1000 = 1/1250$.

Q2. Express each of the following as a Decimal :

- (i) 6% (ii) 28% (iii) 0.2% (iv) 0.04%

Sol. (i) $6\% = 6/100 = 0.06$. (ii) $28\% = 28/100 = 0.28$.
(iii) $0.2\% = 0.2/100 = 0.002$. (iv) $0.04\% = 0.04/100 = 0.0004$.

Q3. Express each of the following as rate percent :

- (i) $23/36$ (ii) $6 \frac{3}{4}$ (iii) 0.004

Sol. (i) $23/36 = [(23/36) \times 100]\% = [575/9]\% = 63 \frac{8}{9}\%$.

(ii) $0.004 = [(4/1000) \times 100]\% = 0.4\%$.

(iii) $6 \frac{3}{4} = 27/4 = [(27/4) \times 100]\% = 675\%$.

Q4. Evaluate: (i) 28% of 450 + 45% of 280

(ii) $16 \frac{2}{3}\%$ of 600 gm - $33 \frac{1}{3}\%$ of 180 gm

Sol. (i) $28\% \text{ of } 450 + 45\% \text{ of } 280 = [(28/100) \times 450 + (45/100) \times 280] = (126 + 126) = 252$.

(ii) $16 \frac{2}{3}\% \text{ of } 600 \text{ gm} - 33 \frac{1}{3}\% \text{ of } 180 \text{ gm} = [((50/3) \times (1/100) \times 600) - ((100/3) \times (1/3) \times 280)] \text{ gm} = (100 - 60) \text{ gm} = 40 \text{ gm}$.

Q5.

(i) 2 is what percent of 50 ?

(ii) $\frac{1}{2}$ is what percent of $\frac{1}{3}$?

(iii) What percent of 8 is 64 ?

(iv) What percent of 2 metric tonnes is 40 quintals ?

(v) What percent of 6.5 litres is 130 ml?

Sol.

(i) Required Percentage = $[(2/50) \times 100]\% = 4\%$.

(ii) Required Percentage = $[(1/2) \times (3/1) \times 100]\% = 150\%$.

(iii) Required Percentage = $[(84/7) \times 100]\% = 1200\%$.

(i) 1 metric tonne = 10 quintals.

Required percentage = $[(40/(2 \times 10)) \times 100]\% = 200\%$.

(ii) Required Percentage = $[(130/(6.5 \times 1000)) \times 100]\% = 2\%$.

Q6. Find the missing figures :

(i) ?% of 25 = 2.125

(ii) 9% of ? = 63

(iii) 0.25% of ? = 0.04

Sol.

(i) Let x% of 25 = 2.125. Then, $(x/100) \times 25 = 2.125$

$$X = (2.125 \times 4) = 8.5.$$

(ii) Let 9% of x = 6.3. Then, $9 \times x/100 = 6.3$

$$X = [(6.3 \times 100)/9] = 70.$$

(iii) Let 0.25% of x = 0.04. Then, $0.25 \times x/100 = 0.04$

$$X = [(0.04 \times 100)/0.25] = 16.$$

Q7. Which is greatest in $16 \frac{2}{3}\%$, $2/5$ and 0.17 ?

Sol. $16 \frac{2}{3}\% = [(50/3) \times 1/100] = 1/6 = 0.166$, $2/5 = 0.4$. Clearly, 0.17 is the greatest.

Q8. If the sales tax reduced from $3 \frac{1}{2}\%$ to $3 \frac{1}{3}\%$, then what difference does it make to a person who purchases an article with market price of Rs. 8400?

Sol. Required difference = $[3 \frac{1}{2}\% \text{ of Rs.8400}] - [3 \frac{1}{3}\% \text{ of Rs.8400}]$

$$= [(7/20 - (10/3))\% \text{ of Rs.8400}] = 1/6\% \text{ of Rs.8400}$$

$$= \text{Rs. } [(1/6) \times (1/100) \times 8400] = \text{Rs. } 14.$$

Q9. An inspector rejects 0.08% of the meters as defective. How many will be examine to project?

Sol. Let the number of meters to be examined be x.

Then, 0.08% of x = 2

$$[(8/100) \times (1/100) \times x] = 2$$

$$x = [(2 \times 100 \times 100)/8] = 2500.$$

Q10. Sixty five percent of a number is 21 less than four fifth of that number. What is the number?

Sol. Let the number be x .

Then, $4x/5 - (65\% \text{ of } x) = 21$

$$4x/5 - 65x/100 = 21$$

$$5x = 2100$$

$$x = 140.$$

Q11. Difference of two numbers is 1660. If 7.5% of the number is 12.5% of the other number, find the number?

Sol. Let the numbers be x and y . Then, $7.5\% \text{ of } x = 12.5\% \text{ of } y$

$$x = 125y/75 = 5y/3.$$

$$\text{Now, } x - y = 1660$$

$$5y/3 - y = 1660$$

$$2y/3 = 1660$$

$$y = [(1660 \times 3)/2] = 2490.$$

$$\text{One number} = 2490, \text{ Second number} = 5y/3 = 4150.$$

Q12. In expressing a length 810472 km as nearly as possible with three significant digits, find the percentage error.

Sol. Error = $(81.5 - 81.472) \text{ km} = 0.028$.

$$\text{Required percentage} = [(0.028/81.472) \times 100]\% = 0.034\%.$$

Q13. In an election between two candidates, 75% of the voters cast thier votes, out of which 2% of the votes were declared invalid. A candidate got 9261 votes which were 75% of the total valid votes. Find the total number of votes enrolled in that election.

Sol. Let the number of votes enrolled be x . Then,

Number of votes cast = 75% of x . Valid votes = 98% of (75% of x).

$$75\% \text{ of } (98\% \text{ of } (75\% \text{ of } x)) = 9261.$$

$$[(75/100) \times (98/100) \times (75/100) \times x] = 9261.$$

$$x = [(9261 \times 100 \times 100 \times 100)/(75 \times 98 \times 75)] = 16800.$$

Q14. Shobha's mathematics test had 75 problems i.e.10 arithmetic, 30 algebra and 35 geometry problems. Although she answered 70% of the arithmetic ,40% of the algebra,

and 60% of the geometry problems correctly. she did not pass the test because she got less than 60% of the problems right. How many more questions she would have to answer correctly to earn 60% of the passing grade?

Sol. Number of questions attempted correctly = $(70\% \text{ of } 10 + 40\% \text{ of } 30 + 60\% \text{ of } 35)$
 $= 7 + 12 + 21 = 45$

questions to be answered correctly for 60% grade = $60\% \text{ of } 75 = 45$

therefore required number of questions = $(45 - 40) = 5$.

Q15. if 50% of $(x - y) = 30\%$ of $(x + y)$ then what percent of x is y ?

Sol. $50\% \text{ of } (x - y) = 30\% \text{ of } (x + y) \Leftrightarrow (50/100)(x - y) = (30/100)(x + y)$

$$\Leftrightarrow 5(x - y) = 3(x + y) \Leftrightarrow 2x = 8y \Leftrightarrow x = 4y$$

$$\text{therefore required percentage} = ((y/x) \times 100)\% = ((y/4y) \times 100) = 25\%$$

Q16. Mr. Jones gave 40% of the money he had to his wife. he also gave 20% of the remaining amount to his 3 sons. half of the amount now left was spent on miscellaneous items and the remaining amount of Rs.12000 was deposited in the bank. how much money did Mr. Jones have initially?

Sol. Let the initial amount with Mr. Jones be Rs. x then,

Money given to wife = $\text{Rs. } (40/100)x = \text{Rs. } 2x/5$. Balance = $\text{Rs. } (x - (2x/5)) = \text{Rs. } 3x/5$.

Money given to 3 sons = $\text{Rs. } (3 \times ((20/100) \times (3x/5))) = \text{Rs. } 9x/5$.

Balance = $\text{Rs. } ((3x/5) - (9x/25)) = \text{Rs. } 6x/25$.

Amount deposited in bank = $\text{Rs. } (1/2 \times 6x/25) = \text{Rs. } 3x/25$.

$$\text{Therefore } 3x/25 = 12000 \Leftrightarrow x = ((12000 \times 25)/3) = 100000$$

So Mr. Jones initially had Rs. 1,00,000 with him.

Short-cut Method : Let the initial amount with Mr. Jones be Rs. x

Then, $(1/2)[100 - (3 \times 20)]\%$ of $x = 12000$

$$\Leftrightarrow (1/2) \times (40/100) \times (60/100) \times x = 12000$$

$$\Leftrightarrow x = ((12000 \times 25)/3) = 100000$$

Q17 10% of the inhabitants of village having died of cholera., a panic set in , during which 25% of the remaining inhabitants left the village. The population is then reduced to 4050. Find the number of original inhabitants.

Sol:

Let the total number of original inhabitants be x .

$$((75/100)) \times (90/100) \times x = 4050 \Leftrightarrow (27/40) \times x = 4050$$

$$\Leftrightarrow x = ((4050 \times 40)/27) = 6000.$$

Q18 A salesman's commission is 5% on all sales upto Rs.10,000 and 4% on all sales exceeding this. He remits Rs.31,100 to his parent company after deducting his commission . Find the total sales.

Sol: Let his total sales be Rs.x. Now (Total sales) – (Commission) = Rs.31,100

$$x - [(5\% \text{ of } 10000 + 4\% \text{ of } (x - 10000))] = 31,100$$

$$x - [(5/100) * 10000 + (4/100) * (x - 10000)] = 31,100$$

$$\Leftrightarrow x - 500 - ((x - 10000)/25) = 31,100$$

$$\Leftrightarrow x - (x/25) = 31200 \Leftrightarrow 24x/25 = 31200 \Leftrightarrow x = [(31200 * 25)/24] = 32,500.$$

Total sales = Rs.32,500

Q19 Raman's salary was decreased by 50% and subsequently increased by 50%. How much percent does he lose?

Sol: Let the original salary = Rs.100

New final salary = 150% of (50% of Rs.100) =

$$\text{Rs.}((150/100) * (50/100) * 100) = \text{Rs.}75.$$

Decrease = 25%

Q20 Paulson spends 75% of his income. His income is increased by 20% and he increased his expenditure by 10%. Find the percentage increase in his savings .

Sol:

Let the original income = Rs.100 . Then , expenditure = Rs.75 and savings = Rs.25

New income = Rs.120 , New expenditure =

$$\text{Rs.}((110/100) * 75) = \text{Rs.}165/2$$

$$\text{New savings} = \text{Rs.}(120 - (165/2)) = \text{Rs.}75/2$$

$$\text{Increase in savings} = \text{Rs.}((75/2) - 25) = \text{Rs.}25/2$$

$$\text{Increase \%} = ((25/2) * (1/25) * 100)\% = 50\%.$$

Q21. The salary of a person was reduced by 10% .By what percent should his reduced salary be raised so as to bring it at par with his original salary ?

Sol: Let the original salary be Rs.100 . New salary = Rs.90.

$$\text{Increase on } 90 = 10, \text{ Increase on } 100 = ((10/90) * 100)\%$$

$$= (100/9)\%$$

Q22 When the price of a product was decreased by 10% , the number sold increased by 30%. What was the effect on the total revenue ?

Sol: Let the price of the product be Rs.100 and let original sale be 100 pieces.

$$\text{Then , Total Revenue} = \text{Rs.}(100 * 100) = \text{Rs.}10000.$$

$$\text{New revenue} = \text{Rs.}(90 * 130) = \text{Rs.}11700.$$

$$\text{Increase in revenue} = ((1700/10000) * 100)\% = 17\%.$$

Q23 . If the numerator of a fraction be increased by 15% and its denominator be diminished by 8% , the value of the fraction is 15/16. Find the original fraction.

Sol: Let the original fraction be x/y.

$$\text{Then } (115\% \text{ of } x) / (92\% \text{ of } y) = 15/16 \Rightarrow (115x/92y) = 15/16$$

$$\Rightarrow ((15/16) * (92/115)) = 3/4$$

Q24 In the new budget, the price of kerosene oil rose by 25%. By how much percent must a person reduce his consumption so that his expenditure on it does not increase?

Sol: Reduction in consumption = $[(R/(100+R))*100]\%$
 $\Rightarrow [(25/125)*100]\%=20\%$.

Q25 The population of a town is 1,76,400 . If it increases at the rate of 5% per annum , what will be its population 2 years hence ? What was it 2 years ago ?

Sol: Population after 2 years = $176400*[1+(5/100)]^2$
 $=[176400*(21/20)*(21/20)]$
 $= 194481$.

Population 2 years ago = $176400/[1+(5/100)]^2$
 $=[176400*(20/21)*(20/21)]= 160000$.

Q26. The value of a machine depreciates at the rate of 10% per annum. If its present is Rs.1,62,000 what will be its worth after 2 years ? What was the value of the machine 2 years ago?

Sol. Value of the machine after 2 years
 $=Rs.[162000*(1-(10/100))^2] = Rs.[162000*(9/10)*(9/10)]$
 $=Rs. 131220$
Value of the machine 2 years ago
 $= Rs.[162000/(1-(10/100)^2)]=Rs.[162000*(10/9)*(10/9)]=Rs.200000$

Q27. During one year, the population of town increased by 5% . If the total population is 9975 at the end of the second year , then what was the population size in the beginning of the first year ?

Sol: Population in the beginning of the first year
 $= 9975/[1+(5/100)]*[1-(5/100)] = [9975*(20/21)*(20/19)]=10000$.

Q28 If A earns 99/3% more than B,how much percent does B earn less than A ?

Sol: Required Percentage = $[(100/3)*100]/[100+(100/3)]\%$
 $=[(100/400)*100]\%=25\%$

Q29 If A`s salary is 20% less then B`s salary , by how much percent is B`s salary more than A`s ?

Sol: Required percentage = $[(20*100)/(100-20)]\%=25\%$.

Q30 .How many kg of pure salt must be added to 30kg of 2% solution of salt and water to increase it to 10% solution ?

Sol: Amount of salt in 30kg solution = $[(20/100)*30]kg=0.6kg$
Let x kg of pure salt be added
Then , $(0.6+x)/(30+x)=10/100 \Leftrightarrow 60+100x=300+10x$
 $\Leftrightarrow 90x=240 \Leftrightarrow x=8/3$.

Q31. Due to reduction of 25/4% in the price of sugar , a man is able to buy 1kg more for Rs.120. Find the original and reduced rate of sugar.

Sol: Let the original rate be Rs.x per kg.

Reduced rate = Rs. $[(100-(25/4))*(1/100)*x]$ = Rs.15x/16 per kg

$$120/(15x/16)-(120/x)=1 \Leftrightarrow (128/x)-(120/x)=1$$

$$\Leftrightarrow x=8.$$

So, the original rate = Rs.8 per kg

Reduce rate = Rs. $[(15/16)*8]$ per kg = Rs.7.50 per kg

Q32 In an examination, 35% of total students failed in Hindi, 45% failed in English and 20% in both. Find the percentage of those who passed in both subjects.

Sol:

Let A and B be the sets of students who failed in Hindi and English respectively.

Then , $n(A) = 35$, $n(B)=45$, $n(A \cap B)=20$.

So , $n(A \cup B)=n(A)+n(B)- n(A \cap B)=35+45-20=60$.

Percentage failed in Hindi and English or both=60%

Hence , percentage passed = $(100-60)\%=40\%$

Q33. In an examination, 80% of the students passed in English, 85% in Mathematics and 75% in both English and Mathematics. If 40 students failed in both the subjects, find the total number of students.

Sol:

Let the total number of students be x .

Let A and B represent the sets of students who passed in English and Mathematics respectively.

Then , number of students passed in one or both the subjects

$$= n(A \cup B)=n(A)+n(B)- n(A \cap B)=80\% \text{ of } x + 85\% \text{ of } x - 75\% \text{ of } x$$

$$=[(80/100)x+(85/100)x-(75/100)x]=(90/100)x=(9/10)x$$

$$\text{Students who failed in both the subjects} = [x-(9x/10)]=x/10.$$

So, $x/10=40$ of $x=400$.

Hence ,total number of students = 400.

10. PROBLEMS ON PROFIT AND LOSS

Q1 A man buys an article for rs.27.50 and sells it for rs.28.50. find his gain %.

sol. $cp=rs27.50$, $sp=rs 28.50$

$gain=rs(28.50 -27.50)=rs1.10$

so $gain\%=\{(1.10/27.50)*100\}=4\%$

Q2. If the a radio is sold for rs 490 and sold for rs 465.50.find loss%.

sol. $cp=rs490$, $sp= 465.50$.

$loss=rs(490-465.50)=rs 24.50$.

$loss\%=[(24.50/490)*100]\%=5\%$

Q3.find S.P when

(i) $CP=56.25$, $gain=20\%$.

sol.

(i) $SP =20\%$ of rs 56.25 , $=rs\{(120/100)*56.25\}=rs67.50$.

(ii) $CP=rs 80.40$, $loss=5\%$

sol: $sp=85\%$ of rs 80.40

$=rs \{(85/100)*80.40\}=rs 68.34$.

Q4 find cp when:

(i) $sp =rs 40.60$: $gain=16\%$

(ii) $sp=rs51.70$: $loss=12\%$

(i) $cp=rs\{(100/116)*40.60\}=rs 35$.

(ii) $cp=rs\{(100/88)*51.87\}=rs58.75$.

Q5 A person incures loss for by selling a watch for rs1140.at what price should the watch be sold to earn a 5% profit ?

sol. let the new sp be rsx.then

$$(100-loss\%) : (1^{st} \text{ sp}) = (100+gain\%) : (2^{nd} \text{ sp})$$
$$\Rightarrow \{(100-5)/1140\} = \{(100+5)/x\} \Rightarrow x = \{(105*1140)/95\} = 1260.$$

Q6 A book was sold for rs 27.50 with a profit of 10%. if it were sold for rs25.75, then what would be % of profit or loss?

sol. $SP=rs 27.50$: $profit =10\%$.

sol. $CP=rs \{(100/110)*27.50\}=rs 25$.

When $sp =Rs25.75$, $profit =Rs(25.75-25)=Rs 0.75$

$Profit\%=\{(0.75/25)*100\}\%=25/6\%=3\%$

Q7 .If the cost price is 96% of sp then whqt is the profit %

Sol. $sp=Rs100$: then $cp=Rs 96$: $profit =Rs 4$.

$Profit=\{(4/96)*100\}\%=4.17\%$

Q8. The cp of 21 articles is equal to sp of 18 articles.find gain or loss %

CP of each article be Rs 1

Sol. CP of 18 articles =Rs18 ,sp of 18 articles =Rs 21.

Gain%=[(3/18)*100]% =50/3%

Q9. By selling 33 metres of cloth , one gains the selling price of 11 metres . Find the gain percent .

Sol: (SP of 33m)-(CP of 33m)=Gain=SP of 11m

SP of 22m = CP of 33m

Let CP of each metre be Re.1 , Then, CP of 22m= Rs.22,SP of 22m=Rs.33.

Gain%=[(11/22)*100]% =50%

Q10.A vendor bought bananas at 6 for Rs.10 and sold them at Rs.4 for Rs.6 .Find his gain or loss percent .

Sol: Suppose , number of bananas bought = LCM of 6 and 4=12

CP=Rs.[(10/6)*12]=Rs.20 ; SP= Rs[(6/4)*12]=Rs.18

Loss%=[(2/20)*100]% =10%

Q11. A man brought toffees at for a rupee. How many for a rupee must he sell to gain 50%?

Sol. C.P of 3 toffees=Re 1; S.P of 3 toffees =150% of Re.1=3/2.

For Rs.3/2, toffees sold =3, for Re.1, toffees sold = [3*(2/3)] = 2.

Q12. A grocer purchased 80 kg of sugar at Rs.13.50 per kg and mixed it with 120kg sugar at Rs.16per kg. At what rate should he sell the mixer to gain 16%?

Sol .C.P of 200 kg of mixture = Rs. (80 * 13.50+120*16) = Rs.3000.

S.P =116% Of Rs.3000 =Rs.[(116/200) *3000]=Rs.3480.

∴ Rate of S.P of the mixture =Rs.[3480/200] per kg =Rs.17.40 per kg.

Q13. Pure ghee cost Rs.100 per kg. After adulterating it with vegetable oil costing Rs.50 per kg, A shopkeeper sells the mixture at the rate of Rs.96 per kg, thereby making a profit of 20%.In What ratio does he mix the two?

Sol. Mean cost price =Rs. [(100/120)*96] =Rs.80 per kg.

By the rate of allegation :

C.P of 1kg ghee

100

C.P of 1kg oil

50

Mean price

80

30

20

∴ Required ratio =30:20 =3:2.

Q14. A dishonest dealer professes to sell his goods at cost price but uses a weight of 960 gms for a kg weight . Find his gain percent.

Sol .Gain% = $\left[\frac{\text{Error}}{(\text{error value})-(\text{error})} * 100 \right] \% = \left[\frac{(40/960)*100}{6} \right] \% = 4 \frac{1}{6} \%$

Q15. If the manufacturer gains 10%,the wholesale dealer 15% and the retailer 25% ,then find the cost of production of a ,the retail price of which is Rs.1265?

Sol:

Let the cost of production of the table be Rs x

The ,125% of 115% of 110% of x=1265

$$\Rightarrow 125/100 * 115/100 * 110/100 * x = 1265 \Rightarrow 253/160 * x = 1265 \Rightarrow x = (1265 * 160 / 253) = \text{Rs.}800$$

Q16 . Monika purchased a pressure cooker at 9/10th of its selling price and sold it at 8% more than its S.P .find her gain percent.

Sol:

Let the s.p be Rs. X .then C.P = Rs.9x/10,Receipt=108% of rs.x=Rs 27x/25

$$\text{Gain} = \text{Rs} (27x/25 - 9x/10) = \text{Rs} (108x - 90x/100) = \text{Rs} 18x/100$$

$$\text{Gain}\% = (18x/100 * 10/9x * 100)\% = 20\%$$

Q17 An article is sold at certain price. By selling it at 2/3 of its price one losses 10%,find the gain at original price ?

sol: let the original s.p be Rs x. then now S.P=Rs2x/3,loss=10% now

$$\text{C.P} = \text{Rs} 20x/27 * 27/20x * 100\% = 35\%$$

Q18. A tradesman sold an article at a loss of 20%.if the selling price has been increased by Rs100,ther would have been a gain of 5%.what was the cost price of the article?

Sol:

Let C.P be Rs x. then (105% of x)-(80 % of x)=100 or 25% of x=100

$$\Rightarrow x/4 = 100 \text{ or } x = 400$$

$$\Rightarrow \text{so,C.P} = \text{Rs } 400$$

Q19. A man sells an article at a profit of 25%if he had bought it 20% less and sold it for Rs 10.50 less,he would have gained 30%find the cost price of the article.

Sol:

Let the C.P be Rs x

$$1^{\text{st}} \text{ S.P} = 125\% \text{ of } x = 125x/100 = 5x/4; 2^{\text{nd}} \text{ S.P} = 80\% \text{ of } x = 80x/100 = 4x/5$$

$$2^{\text{nd}} \text{ S.P} = 130\% \text{ of } 4x/5 = (130/100 * 4x/5) = 26x/25$$

$$\Rightarrow 5x/4 - 26x/25 = 10.50 \Leftrightarrow x = (10.50 * 100) / 21 = 50$$

hence C.P=Rs.50

Q20.The price of the jewel,passing through three hands,rises on the whole by65%.if the first and the second sellers 20%and25% profit respectively find the percentage profit earned by the third seller.

Sol: Let the original price of the jewel be Rs p and let the profit earned by the thrid seller be x%

Then, $(100+x)\%$ of 125% OF 120% OF $P=165\%$ OF P
 $\Rightarrow ((100+X)/100 \times 125/100 \times 120/100 \times P) = (165/100 \times P)$
 $\Rightarrow (100+X) = (165 \times 100 \times 100)/(125 \times 120) = 110 \Rightarrow X = 10\%$

Q21 . A man 2 flats for Rs 675958 each.on one he gains 16% while on the other he losses 16%. How much does he gain/loss in the whole transaction?

Sol:

In this case there will be always loss. The selling price is immaterial
Hence, loss % = $(\text{common loss and gain})^2 / 10 = (16/10)\% = (64/25)\% = 2.56\%$

Q22. A dealer sold three-fourth of his article at a gain of 20% and remaining at a cost price. Find the gain earned by him at the two transaction.

Sol:

Let the C.P of the whole be Rs x
C.P of $3/4^{\text{th}}$ = Rs $3x/4$, C.P of $1/4^{\text{th}}$ = Rs $x/4$
 \Rightarrow total S.P = Rs $[(120\% \text{ of } 3x/4) + x/4] = \text{Rs}(9x/10 + x/4) = \text{Rs } 23x/20$
 \Rightarrow gain = Rs $(23x/20 - x) = \text{Rs } 3x/20$
 \Rightarrow gain% = $3x/20 \times 1/x \times 100\% = 15\%$

Q23. A man bought a horse and a carriage for Rs 3000.he sold the horse at a gain of 20% and the carriage at a loss of 10%,thereby gaining 2% on the whole.find the cost of the horse.

Sol:

Let the C.p of the horse be Rs. x , then C.P of the carriage = Rs $(3000-x)$
 $20\% \text{ of } x - 10\% \text{ of } (3000-x) = 2\% \text{ of } 3000$
 $\Rightarrow x/5 - (3000-x)/10 = 60 \Rightarrow 2x - 3000 + x = 600 \Rightarrow 3x + 3600 \Rightarrow x = 1200$
 \Rightarrow hence, C.P of the horse = Rs 1200

Q24. Find the single discount equivalent to a series discount of 20% ,10% and 5%'

sol:

let the marked price be Rs 100
then ,net S.P = $95\% \text{ of } 90\% \text{ of } 80\% \text{ of Rs } 100$
 $= \text{Rs}(95/100 \times 90/100 \times 80/100 \times 100) = \text{Rs}68.40$

Q25 After getting 2 successive discounts, a shirt with a list price of Rs 150 is available at Rs 105. If the second discount is 12.55,find the first discount.

Sol:

Let the first discount be $x\%$
Then, $87.5\% \text{ of } (100-x)\% \text{ of } 150 = 105$
 $\Rightarrow 87.5/100 \times (100-x)/100 \times 150 = 105 \Rightarrow 105 \Rightarrow 100-x = (105 \times 100 \times 100)/(150 \times 87.5) = 80$
 $\Rightarrow x = (100-80) = 20$
 \Rightarrow first discount = 20%

Q26 An uneducated retailer marks all its goods at 50% above the cost price and thinking that he will still make 25% profit,offers a discount of 25% on the marked price.what is the actual profit on the sales?

Sol:

Let C.P =Rs 100.then ,marked price =Rs100
S.P=75% of Rs 150=Rs112.50
Hence,gain%=12.50%

Q27 .A retailer buys 40 pens at the market price of 36 pens from a wholesaler ,if he sells these pens giving a discount of 1% ,what is the profit % ?

Sol:

let the market price of each pen be Rs 1
then,C.P of 40 pens = Rs 36 S.P of 40 pens =99% of Rs 40=Rs 39.60
profit %=((3.60*100)/36) % =10%

Q28 . At what % above C.P must an article be marked so as to gain 33% after allowing a customer a discount of 5%?

Sol

Let C.P be Rs 100.then S.P be Rs 133
Let the market price be Rs x
Then 90% of x=133=>95x/100=133=>x=(133*100/95)=140
Market price = 40% above C.P

Q29 When a producer allows 36% commission on retail price of his product, he earns a profit of 8.8%. what would be his profit % if the commision is reduced by 24%?

Sol:

Let the retail price =Rs 100.then, commission=Rs 36
S.P=Rs(100-36)=Rs 64
But, profit=8.8%
C.P=Rs(100/108.8*64)=Rs 1000/17
New commission =Rs12. New S.P=Rs(100-12)Rs 88
Gain=Rs(88-1000/17)=Rs 496/17
Gain%=(496/17*17/1000*100)%=49.6%

11. PROBLEMS ON RATIO AND PROPORTION

Q1. If $a : b = 5 : 9$ and $b : c = 4 : 7$, find $a : b : c$.

Sol. $a:b=5:9$ and $b:c=4:7=(4 \times 9/4):(7 \times 9/4) = 9:63/4$
 $a:b:c = 5:9:63/4 = 20:36:63$.

Q2. Find:

(i) the fourth proportional to 4, 9, 12;

(ii) the third proportional to 16 and 36;

(iii) the mean proportional between 0.08 and 0.18.

Sol.

i) Let the fourth proportional to 4, 9, 12 be x .

Then, $4 : 9 :: 12 : x \Leftrightarrow 4 \times x = 9 \times 12 \Leftrightarrow x = (9 \times 12)/4 = 27$;

Fourth proportional to 4, 9, 12 is 27.

(ii) Let the third proportional to 16 and 36 be x .

Then, $16 : 36 :: 36 : x \Leftrightarrow 16 \times x = 36 \times 36 \Leftrightarrow x = (36 \times 36)/16 = 81$

Third proportional to 16 and 36 is 81.

(iii) Mean proportional between 0.08 and 0.18

$$\sqrt{0.08 \times 0.18} = \sqrt{8/100 \times 18/100} = \sqrt{144/(100 \times 100)} = 12/100 = 0.12$$

Q3. If $x : y = 3 : 4$, find $(4x + 5y) : (5x - 2y)$.

Sol. $X/Y = 3/4 \Leftrightarrow (4x+5y)/(5x+2y) = (4(x/y)+5)/(5(x/y)-2) = (4(3/4)+5)/(5(3/4)-2)$
 $= (3+5)/(7/4) = 32/7$

Q4. Divide Rs. 672 in the ratio 5 : 3.

Sol. Sum of ratio terms = $(5 + 3) = 8$.

First part = Rs. $(672 \times (5/8)) = \text{Rs. } 420$; Second part = Rs. $(672 \times (3/8)) = \text{Rs. } 252$.

Q5. Divide Rs. 1162 among A, B, C in the ratio 35 : 28 : 20.

Sol. Sum of ratio terms = $(35 + 28 + 20) = 83$.

A's share = Rs. $(1162 \times (35/83)) = \text{Rs. } 490$; B's share = Rs. $(1162 \times (28/83)) = \text{Rs. } 392$;

C's share = Rs. $(1162 \times (20/83)) = \text{Rs. } 280$.

Q6. A bag contains 50 p, 25 P and 10 p coins in the ratio 5: 9: 4, amounting to

Rs. 206. Find the number of coins of each type.

Sol. Let the number of 50 p, 25 P and 10 p coins be $5x$, $9x$ and $4x$ respectively.

$$(5x/2) + (9x/4) + (4x/10) = 206 \Leftrightarrow 50x + 45x + 8x = 4120 \Leftrightarrow 103x = 4120 \Leftrightarrow x = 40$$

Number of 50 p coins = $(5 \times 40) = 200$; Number of 25 p coins = $(9 \times 40) = 360$;

Number of 10 p coins = $(4 \times 40) = 160$.

Q7. A mixture contains alcohol and water in the ratio 4:3. If 5 litres of water is added to the mixture, the ratio becomes 4: 5. Find the quantity of alcohol in the given mixture

Sol. Let the quantity of alcohol and water be $4x$ litres and $3x$ litres respectively

$$4x/(3x+5) = 4/5 \Leftrightarrow 20x = 4(3x+5) \Leftrightarrow 8x = 20 \Leftrightarrow x = 2.5$$

Quantity of alcohol = (4×2.5) litres = 10 litres.

12. PARTNERSHIP

Q1. A, B and C started a business by investing Rs. 1,20,000, Rs. 1,35,000 and ,Rs.1,50,000 respectively. Find the share of each, out of an annual profit of Rs. 56,700.

Sol. Ratio of shares of A, Band C = Ratio of their investments

$$= 120000 : 135000 : 150000 = 8 : 9 : 10.$$

$$A's \text{ share} = \text{Rs. } (56700 \times (8/27)) = \text{Rs. } 16800.$$

$$B's \text{ share} = \text{Rs. } (56700 \times (9/27)) = \text{Rs. } 18900.$$

$$C's \text{ share} = \text{Rs. } (56700 \times (10/27)) = \text{Rs. } 21000.$$

Q2. Alfred started a business investing Rs. 45,000. After 3 months, Peter joined him with a capital of Rs. 60,000. After another 6 months, Ronald joined them with a capital of Rs. 90,000. At the end of the year, they made a profit of Rs. 16,500. Find the lire of each.

Sol. Clearly, Alfred invested his capital for 12 months, Peter for 9 months and Ronald for 3 months. So, ratio of their capitals = $(45000 \times 12) : (60000 \times 9) : (90000 \times 3)$
 $= 540000 : 540000 : 270000 = 2 : 2 : 1.$

$$\text{Alfred's share} = \text{Rs. } (16500 \times (2/5)) = \text{Rs. } 6600$$

$$\text{Peter's share} = \text{Rs. } (16500 \times (2/5)) = \text{Rs. } 6600$$

$$\text{Ronald's share} = \text{Rs. } (16500 \times (1/5)) = \text{Rs. } 3300.$$

Q3. A, Band C start a business each investing Rs. 20,000. After 5 months A withdrew Rs.6000 B withdrew Rs. 4000 and C invests Rs. 6000 more. At the end of the year, a total profit of Rs. 69,900 was recorded. Find the share of each.

Sol. Ratio of the capitals of A, Band C

$$= 20000 \times 5 + 15000 \times 7 : 20000 \times 5 + 16000 \times 7 : 20000 \times 5 + 26000 \times 7$$

$$= 205000 : 212000 : 282000 = 205 : 212 : 282.$$

$$A's \text{ share} = \text{Rs. } 69900 \times (205/699) = \text{Rs. } 20500$$

$$B's \text{ share} = \text{Rs. } 69900 \times (212/699) = \text{Rs. } 21200;$$

$$C's \text{ share} = \text{Rs. } 69900 \times (282/699) = \text{Rs. } 28200.$$

Q4. A, Band C enter into partnership. A invests 3 times as much as B and B invests two-third of what C invests. At the end of the year, the profit earned is Rs. 6600. What is the share of B ?

Sol. Let C's capital = Rs. x. Then, B's capital = Rs. $(2/3)x$

$$A's \text{ capital} = \text{Rs. } (3 \times (2/3).x) = \text{Rs. } 2x.$$

$$\text{Ratio of their capitals} = 2x : (2/3)x : x = 6 : 2 : 3.$$

$$\text{Hence, B's share} = \text{Rs. } (6600 \times (2/11)) = \text{Rs. } 1200.$$

Q5. Four milkmen rented a pasture. A grazed 24 cows for 3 months; B 10 for 5 months; C 35 cows for 4 months and D 21 cows for 3 months. If A's share of rent is Rs. 720, find the total rent of the field.

Sol. Ratio of shares of A, B, C, D = $(24 \times 3) : (10 \times 5) : (35 \times 4) : (21 \times 3) = 72 : 50 : 140 : 63.$

$$\text{Let total rent be Rs. } x. \text{ Then, } A's \text{ share} = \text{Rs. } (72x)/325$$

$$(72x)/325 = 720 \Rightarrow x = (720 \times 325)/72 = 3250$$

$$\text{Hence, total rent of the field is Rs. } 3250.$$

Q6. A invested Rs. 76,000 in a business. After few months, B joined him Rs. 57,000. At the end of the year, the total profit was divided between them in ratio 2 : 1. After how many months did B join?

Sol. Suppose B joined after x months. Then, B's money was invested for (12 - x)

$$(76000 \times 12) / (57000 \times (12 - x)) = 2/1 \quad \square \quad 912000 = 114000(12 - x)$$

$$114(12 - x) = 912 \quad \square \quad 12 - x = 8 \quad \square \quad x = 4 \quad \text{Hence, B joined after 4 months.}$$

Q7. A, B and C enter into a partnership by investing in the ratio of 3 : 2 : 4. After 1 year, B invests another Rs. 2,70,000 and C, at the end of 2 years, also invests Rs. 2,70,000. At the end of three years, profits are shared in the ratio of 3 : 4 : 5. Find initial investment of each.

Sol. Let the initial investments of A, B and C be Rs. 3x, Rs. 2x and Rs. 4x respectively. Then,

$$(3x \times 36) : [(2x \times 12) + (2x + 270000) \times 24] : [(4x \times 24) + (4x + 270000) \times 12] = 3 : 4 : 5$$

$$108x : (72x + 6480000) : (144x + 3240000) = 3 : 4 : 5$$

$$108x / (72x + 6480000) = 3/4 \quad \square \quad 432x = 216x + 19440000$$

$$\square \quad 216x = 19440000$$

$$x = 90000$$

Hence, A's initial investment = 3x = Rs. 2,70,000;

B's initial investment = 2x = Rs. 1,80,000;

C's initial investment = 4x = Rs. 3,60,000.

13. CHAIN RULE

Q1. If 15 toys cost Rs, 234, what do 35 toys cost?

Sol. Let the required cost be Rs. x. Then,

More toys, More cost (Direct Proportion)

$$\frac{14}{15} : 35 :: 234 : x \Leftrightarrow (15 \times x) = (35 \times 234) \Leftrightarrow x = (35 \times 234) / 15 = 546$$

15 Hence, the cost of 35 toys is Rs. 546.

Q2. If 36 men can do a piece of work in 25 hours, in how many hours will 15 men do it ?

Sol. Let the required number of hours be x. Then,

Less men, More hours (Indirect Proportion)

$$15 : 36 :: 25 : x \Leftrightarrow (15 \times x) = (36 \times 25) \Leftrightarrow (36 \times 25) / 15 = 60$$

Hence, 15 men can do it in 60 hours.

Q3. If the wages of 6 men for 15 days be Rs.2100, then find the wages of for 12 days.

Sol. Let the required wages be Rs. x.

More men, More wages (Direct Proportion)

Less days, Less wages (Direct Proportion)

$$\text{Men } 6: 9 :: 2100: x$$

$$\text{Days } 15: 12$$

$$\text{Therefore } (6 \times 15 \times x) = (9 \times 12 \times 2100) \Leftrightarrow x = (9 \times 12 \times 2100) / (6 \times 15) = 2520$$

Hence the required wages are Rs. 2520.

Q4. If 20 men can build a wall 66 metres long in 6 days, what length of a similar can be built by 86 men in 8 days?

Sol. Let the required length be x metres

More men, More length built (Direct Proportion)

Less days, Less length built (Direct Proportion)

$$\text{Men } 20: 35$$

$$\text{Days } 6: 3 :: 56 : x$$

$$\text{Therefore } (20 \times 6 \times x) = (35 \times 3 \times 56) \Leftrightarrow x = (35 \times 3 \times 56) / 120 = 49$$

Hence, the required length is 49 m.

Q5. If 15 men, working 9 hours a day, can reap a field in 16 days, in how many days will 18 men reap the field, working 8 hours a day?

Sol. Let the required number of days be x.

More men, Less days (indirect proportion)

Less hours per day, More days (indirect proportion)

$$\text{Men } 18 : 15$$

$$\text{Hours per day } 8: 9 \} :: 16 : x$$

$$(18 \times 8 \times x) = (15 \times 9 \times 16) \Leftrightarrow x = (44 \times 15) / 144 = 15$$

Hence, required number of days = 15.

Q6. If 9 engines consume 24 metric tonnes of coal, when each is working 8 hours day, how much coal will be required for 8 engines, each running 13 hours a day, it being given that 3 engines of former type consume as much as 4 engines of latter type?

Sol. Let 3 engines of former type consume 1 unit in 1 hour.

Then, 4 engines of latter type consume 1 unit in 1 hour.

Therefore 1 engine of former type consumes $(1/3)$ unit in 1 hour.

1 engine of latter type consumes $(1/4)$ unit in 1 hour.

Let the required consumption of coal be x units.

Less engines, Less coal consumed (direct proportion)

More working hours, More coal consumed (direct proportion)

Less rate of consumption, Less coal consumed (direct proportion)

Number of engines 9: 8

Working hours 8 : 13 } :: 24 : x

Rate of consumption $(1/3):(1/4)$

$[9 \times 8 \times (1/3) \times x] = (8 \times 13 \times (1/4) \times 24) \Leftrightarrow 24x = 624 \Leftrightarrow x = 26.$

Hence, the required consumption of coal = 26 metric tonnes.

Q7. A contract is to be completed in 46 days and 117 men were said to work 8 hours a day. After 33 days, $(4/7)$ of the work is completed. How many additional men may be employed so that the work may be completed in time, each man now working 9 hours a day?

Sol. Remaining work = $(1 - (4/7)) = (3/7)$

Remaining period = $(46 - 33)$ days = 13 days

Let the total men working at it be x .

Less work, Less men (Direct Proportion)

Less days, More men (Indirect Proportion)

More Hours per Day, Less men (Indirect Proportion)

Work $(4/7): (3/7)$

Days 13:33 } :: 117: x

Hrs/day 9 : 8

Therefore $(4/7) \times 13 \times 9 \times x = (3/7) \times 33 \times 8 \times 117$ or $x = (3 \times 33 \times 8 \times 117) / (4 \times 13 \times 9) = 198$

Additional men to be employed = $(198 - 117) = 81.$

Q8. A garrison of 3300 men had provisions for 32 days, when given at the rate of 860 gms per head. At the end of 7 days, a reinforcement arrives and it was found that the provisions will last 17 days more, when given at the rate of 826 gms per head, What is the strength of the reinforcement?

Sol. The problem becomes:

3300 men taking 850 gms per head have provisions for $(32 - 7)$ or 25 days,

How many men taking 825 gms each have provisions for 17 days?

Less ration per head, more men (Indirect Proportion)

Less days, More men (Indirect Proportion)

Ration 825 : 850

Days 17: 25 } :: 3300 : x

$(825 \times 17 \times x) = 850 \times 25 \times 3300$ or $x = (850 \times 25 \times 3300) / (825 \times 17) = 5000$

Strength of reinforcement = $(5000 - 3300) = 1700.$

14. TIME AND WORK

Q1. Worker A takes 8 hours to do a job. Worker B takes 10 hours to do the same Job. How long should it take both A and B, working together but independently, to do the same job?

Sol. A's 1 hour's work = $1/8$

B's 1 hour's work = $1/10$

(A + B)'s 1 hour's work = $(1/8) + (1/10) = 9/40$

Both A and B will finish the work in $40/9$ days.

Q2. A and B together can complete a piece of work in 4 days. If A alone can complete the same work in 12 days, in how many days can B alone complete that work?

Sol. (A + B)'s 1 day's work = $(1/4)$. A's 1 day's work = $(1/12)$.

B's 1 day's work = $((1/4) - (1/12)) = (1/6)$

Hence, B alone can complete the work in 6 days.

Q3. A can do a piece of work in 7 days of 9 hours each and B can do it in 6 days of 7 hours each. How long will they take to do it, working together 8 hours a day?

Sol. A can complete the work in $(7 \times 9) = 63$ hours.

B can complete the work in $(6 \times 7) = 42$ hours.

A's 1 hour's work = $(1/63)$ and B's 1 hour's work = $(1/42)$

(A + B)'s 1 hour's work = $(1/63) + (1/42) = (5/126)$

Both will finish the work in $(126/5)$ hrs.

Number of days. of $(42/5)$ hrs each = $(126 \times 5)/(5 \times 42) = 3$ days

Q4. A and B can do a piece of work in 18 days; Band C can do it in 24 days A and C can do it in 36 days. In how many days will A, Band C finish it together and separately?

Sol. (A + B)'s 1 day's work = $(1/18)$ (B + C)'s 1 day's work = $(1/24)$

and (A + C)'s 1 day's work = $(1/36)$

Adding, we get: $2(A + B + C)$'s 1 day's work = $(1/18 + 1/24 + 1/36)$
 $= 9/72 = 1/8$

(A + B + C)'s 1 day's work = $1/16$ Thus, A, Band C together can finish the work in 16 days.

Now, A's 1 day's work = [(A + B + C)'s 1 day's work] - [(B + C)'s 1 day work:

$= (1/16 - 1/24) = 1/48$ A alone can finish the work in 48 days.

Similarly, B's 1 day's work = $(1/16 - 1/36) = 5/144$

B alone can finish the work in $144/5 = 28 \frac{4}{5}$ days

And C's 1 day work = $(1/16 - 1/18) = 1/144$

Hence C alone can finish the work in 144 days.

Q6. A is twice as good a workman as B and together they finish a piece in 18 days. In how many days will A alone finish the work?

Sol. (A's 1 day's work):(B's 1 days work) = $2 : 1$.

(A + B)'s 1 day's work = $1/18$

Divide $1/18$ in the ratio $2 : 1$.

\therefore A's 1 day's work = $(1/18 \times 2/3) = 1/27$

Hence, A alone can finish the work in 27 days.

Q6. A can do a certain job in 12 days. B is 60% more efficient than A. How many days does B alone take to do the same job?

Sol. Ratio of times taken by A and B = $160 : 100 = 8 : 5$.

Suppose B alone takes x days to do the job.

Then, $8 : 5 :: 12 : x = 8x = 5 \times 12 \Rightarrow x = 7 \frac{1}{2}$ days.

Q7. A can do a piece of work in 80 days. He works at it for 10 days B alone finishes the remaining work in 42 days. In how much time will A and B working together, finish the work?

Sol. Work done by A in 10 days $= (1/80 \times 10) = 1/8$

Remaining work $= (1 - 1/8) = 7/8$

Now, $7/8$ work is done by B in 42 days.

Whole work will be done by B in $(42 \times 8/7) = 48$ days.

A's 1 day's work $= 1/80$ and B's 1 day's work $= 1/48$

(A+B)'s 1 day's work $= (1/80 + 1/48) = 8/240 = 1/30$

Hence, both will finish the work in 30 days.

Q8. A and B undertake to do a piece of work for Rs. 600. A alone can do it in 6 days while B alone can do it in 8 days. With the help of C, they finish it in 3 days. !find the share of each.

Sol : C's 1 day's work $= 1/3 - (1/6 + 1/8) = 1/24$

A : B : C = Ratio of their 1 day's work $= 1/6 : 1/8 : 1/24 = 4 : 3 : 1$.

A's share = Rs. $(600 \times 4/8) = \text{Rs. } 300$, B's share = Rs. $(600 \times 3/8) = \text{Rs. } 225$.

C's share = Rs. $[600 - (300 + 225)] = \text{Rs. } 75$.

Q 9. A and B working separately can do a piece of work in 9 and 12 days respectively, If they work for a day alternately, A beginning, in how many days, the work will be completed?

Sol: (A + B)'s 2 days' work $= (1/9 + 1/12) = 7/36$

Work done in 5 pairs of days $= (5 \times 7/36) = 35/36$

Remaining work $= (1 - 35/36) = 1/36$

On 11th day, it is A's turn. $1/9$ work is done by him in 1 day.

$1/36$ work is done by him in $(9 \times 1/36) = 1/4$ day

Total time taken $= (10 + 1/4)$ days $= 10 \frac{1}{4}$ days.

Q10. 45 men can complete a work in 16 days. Six days after they started working, 30 more men joined them. How many days will they now take to complete the remaining work?

Sol: (45×16) men can complete the work in 1 day.

1 man's 1 day's work $= 1/720$

45 men's 6 days' work $= (1/16 \times 6) = 3/8$

Remaining work $= (1 - 3/8) = 5/8$

75 men's 1 day's work $= 75/720 = 5/48$

Now, 5 work is done by them in 1 day.

48

5 work is done by them in $(\frac{48}{5} \times \frac{5}{8}) = 6$ days.

8

5 8

Q11. 2 men and 3 boys can do a piece of work in 10 days while 3 men and 2 boys can do the same work in 8 days. In how many days can 2 men and 1 boy do the work?

Soln: Let 1 man's 1 day's work = x and 1 boy's 1 day's work = y.

Then, $2x + 3y = \frac{1}{10}$ and $3x + 2y = \frac{1}{8}$

Solving, we get: $x = \frac{7}{200}$ and $y = \frac{1}{100}$

(2 men + 1 boy)'s 1 day's work = $(2 \times \frac{7}{200} + 1 \times \frac{1}{100}) = \frac{16}{200} = \frac{2}{25}$

So, 2 men and 1 boy together can finish the work in $\frac{25}{2} = 12 \frac{1}{2}$ days

15. PIPES AND CISTERNS

Q1. Two pipes A and B can fill a tank in 36 hours and 46 hours respectively. If both the pipes are opened simultaneously, how much time will be taken to fill the tank?

Sol: Part filled by A in 1 hour = $(1/36)$;

Part filled by B in 1 hour = $(1/45)$;

Part filled by (A + B) In 1 hour = $(1/36) + (1/45) = (9/180) = (1/20)$

Hence, both the pipes together will fill the tank in 20 hours.

Q2. Two pipes can fill a tank in 10 hours and 12 hours respectively while a third, pipe empties the full tank in 20 hours. If all the three pipes operate simultaneously, in how much time will the tank be filled?

Sol: Net part filled In 1 hour = $(1/10) + (1/12) - (1/20) = (8/60) = (2/15)$.

The tank will be full in $\frac{15}{2}$ hrs = 7 hrs 30 min.

Q3. If two pipes function simultaneously, the reservoir will be filled in 12 hours. One pipe fills the reservoir 10 hours faster than the other. How many hours does it take the second pipe to fill the reservoir?

Sol: let the reservoir be filled by first pipe in x hours.

Then, second pipe fill it in $(x+10)$ hrs.

Therefore $(1/x) + (1/(x+10)) = (1/12) \Leftrightarrow (x+10+x)/(x(x+10)) = (1/12)$.

$\Leftrightarrow x^2 - 14x - 120 = 0 \Leftrightarrow (x-20)(x+6) = 0$

$\Leftrightarrow x = 20$ [neglecting the negative value of x]

so, the second pipe will take $(20+10)$ hrs. (i.e) 30 hours to fill the reservoir

Q4. A cistern has two taps which fill it in 12 minutes and 15 minutes respectively. There is also a waste pipe in the cistern. When all the 3 are opened, the empty cistern is full in 20 minutes. How long will the waste pipe take to empty the full cistern?

Sol: Work done by the waste pipe in 1 min

= $(1/20) - (1/12) + (1/15) = -1/10$ [negative sign means emptying]

therefore the waste pipe will empty the full cistern in 10 min

Q5: An electric pump can fill a tank in 3 hours. Because of a leak in, the tank it took $3\frac{1}{2}$ hours to fill the tank. If the tank is full, how much time will the leak take to empty it?

Sol: work done by the leak in 1 hour = $(1/3) - (1/(7/2)) = (1/3) - (2/7) = (1/21)$.

The leak will empty the tank in 21 hours.

Q6. Two pipes can fill a cistern in 14 hours and 16 hours respectively. The pipes are opened simultaneously and it is found that due to leakage in the bottom it took 32 minutes more to fill the cistern. When the cistern is full, in what time will the leak empty it?

Sol: Work done by the two pipes in 1 hour = $(1/14) + (1/16) = (15/112)$.

Time taken by these pipes to fill the tank = $(112/15)$ hrs = 7 hrs 28 min.

Due to leakage, time taken = 7 hrs 28 min + 32 min = 8 hrs

Work done by (two pipes + leak) in 1 hour = $(1/8)$.

Work done by the leak in 1 hour = $(15/112) - (1/8) = (1/112)$.

Leak will empty the full cistern in 112 hours.

Q7. Two pipes A and B can fill a tank in 36 min. and 45 min. respectively. A water pipe C can empty the tank in 30 min. First A and B are opened. after 7 min, C is also opened. In how much time, the tank is full?

Sol: Part filled in 7 min. = $7 * ((1/36) + (1/45)) = (7/20)$.

Remaining part = $(1 - (7/20)) = (13/20)$.

Net part filled in 1 min. when A, B and C are opened = $(1/36) + (1/45) - (1/30) = (1/60)$.

Now, $(1/60)$ part is filled in one minute.

$(13/20)$ part is filled in $(60 * (13/20)) = 39$ minutes.

Q8. Two pipes A, B can fill a tank in 24 min. and 32 min. respectively. If both the pipes are opened simultaneously, after how much time B should be closed so that the tank is full in 18 min.?

Sol: let B be closed after x min. then ,

Part filled by (A+B) in x min. + part filled by A in $(18-x)$ min. = 1

Therefore $x * ((1/24) + (1/32)) + (18-x) * (1/24) = 1 \Leftrightarrow (7x/96) + ((18-x)/24) = 1 \Leftrightarrow 7x + 4 * (18-x) = 96$. Hence, B must be closed after 8 min.

16. TIME AND DISTANCE

Q1. How many minutes does Aditya take to cover a distance of 400 m, if he runs at a speed of 20 km/hr?

Sol. Aditya's speed = 20 km/hr = $\{20 * \frac{5}{18}\}$ m/sec = $\frac{50}{9}$ m/sec

$$\therefore \text{Time taken to cover 400 m} = \{400 * \frac{9}{50}\} \text{ sec} = 72 \text{ sec} = 1 \frac{12}{60} \text{ min} = 1 \frac{1}{5} \text{ min.}$$

Q2. A cyclist covers a distance of 750 m in 2 min 30 sec. What is the speed in km/hr of the cyclist?

Sol. Speed = $\{ \frac{750}{150} \}$ m/sec = 5 m/sec = $\{ 5 * \frac{18}{5} \}$ km/hr = 18 km/hr

Q3. A dog takes 4 leaps for every 5 leaps of a hare but 3 leaps of a dog are equal to 4 leaps of the hare. Compare their speeds.

Sol. Let the distance covered in 1 leap of the dog be x and that covered in 1 leap of the hare by y.

Then, $3x = 4y \Rightarrow x = \frac{4}{3}y \Rightarrow 4x = \frac{16}{3}y$.

$$\therefore \text{Ratio of speeds of dog and hare} = \text{Ratio of distances covered by them in the same time} \\ = 4x : 5y = \frac{16}{3}y : 5y = \frac{16}{3} : 5 = 16:15$$

Q4. While covering a distance of 24 km, a man noticed that after walking for 1 hour and 40 minutes, the distance covered by him was $\frac{5}{7}$ of the remaining distance. What was his speed in metres per second?

Sol. Let the speed be x km/hr.

Then, distance covered in 1 hr. 40 min. i.e., $1 \frac{2}{3}$ hrs = $\frac{5x}{3}$ km

Remaining distance = $\{24 - \frac{5x}{3}\}$ km.

$$\therefore \frac{5x}{3} = \frac{5}{7} \{24 - \frac{5x}{3}\} \Leftrightarrow \frac{5x}{3} = \frac{5}{7} \{ \frac{72-5x}{3} \} \Leftrightarrow 7x = 72 - 5x \\ \Leftrightarrow 12x = 72 \Leftrightarrow x = 6$$

Hence speed = 6 km/hr = $\{6 * \frac{5}{18}\}$ m/sec = $\frac{5}{3}$ m/sec = $1 \frac{2}{3}$

Q5. Peter can cover a certain distance in 1 hr. 24 min. by covering two-third of the distance at 4 kmph and the rest at 5 kmph. Find the total distance.

Sol. Let the total distance be x km. Then,

$$\frac{\frac{2}{3}x}{4} + \frac{\frac{1}{3}x}{5} = \frac{7}{5} \Leftrightarrow \frac{x}{6} + \frac{x}{15} = \frac{7}{5} \Leftrightarrow 7x = 42 \Leftrightarrow x = 6$$

Q6. A man traveled from the village to the post-office at the rate of 25 kmph and walked back at the rate of 4 kmph. If the whole journey took 5 hours 48 minutes, find the distance of the post-office from the village.

Sol. Average speed = $\left\{ \frac{2xy}{x+y} \right\}$ km/hr = $\left\{ \frac{2 \times 25 \times 4}{25+4} \right\}$ km/hr = $\frac{200}{29}$ km/hr

Distance traveled in 5 hours 48 minutes i.e., $5 \frac{4}{5}$ hrs. = $\left\{ \frac{200}{29} \times \frac{29}{5} \right\}$ km = 40 km

Distance of the post-office from the village = $\left\{ \frac{40}{2} \right\}$ = 20 km

Q7. An aeroplane flies along the four sides of a square at the speeds of 200, 400, 600 and 800 km/hr. Find the average speed of the plane around the field.

Sol. :

Let each side of the square be x km and let the average speed of the plane around the field be y km per hour then ,

$$x/200 + x/400 + x/600 + x/800 = 4x/y \Leftrightarrow 25x/2500 \Leftrightarrow 4x/y \Leftrightarrow y = (2400 \times 4/25) = 384$$

hence average speed = 384 km/hr

Q8. Walking at $\frac{5}{7}$ of its usual speed, a train is 10 minutes too late. Find its usual time to cover the journey.

Sol. : New speed = $\frac{5}{6}$ of the usual speed

New time taken = $\frac{6}{5}$ of the usual time

So, $(\frac{6}{5} \text{ of the usual time}) - (\text{usual time}) = 10 \text{ minutes.}$

$\Rightarrow \frac{1}{5} \text{ of the usual time} = 10 \text{ minutes.}$

$\Rightarrow \text{usual time} = 10 \text{ minutes}$

Q9. If a man walks at the rate of 5 kmph, he misses a train by 7 minutes. However, if he walks at the rate of 6 kmph, he reaches the station 5 minutes before the arrival of the train. Find the distance covered by him to reach the station.

Sol. Let the required distance be x km

Difference in the time taken at two speeds = 1 min = $\frac{1}{2}$ hr

Hence $x/5 - x/6 = \frac{1}{5} \Leftrightarrow 6x - 5x = 6$

$\Leftrightarrow x = 6$

Hence, the required distance is 6 km

Q10. A and B are two stations 390 km apart. A train starts from A at 10 a.m. and travels towards B at 65 kmph. Another train starts from B at 11 a.m. and travels towards A at 35 kmph. At what time do they meet?

Sol. Suppose they meet x hours after 10 a.m. Then,

(Distance moved by first in x hrs) + [Distance moved by second in (x-1) hrs] = 390.

$$65x + 35(x-1) = 390 \Rightarrow 100x = 425 \Rightarrow x = 17/4$$

So, they meet 4 hrs. 15 min. after 10 a.m i.e., at 2.15 p.m.

Q11. A goods train leaves a station at a certain time and at a fixed speed. After ^hours, an express train leaves the same station and moves in the same direction at a uniform speed of

90 kmph. This train catches up the goods train in 4 hours. Find the speed of the goods train.

Sol. Let the speed of the goods train be x kmph.

Distance covered by goods train in 10 hours = Distance covered by express train in 4 hours

$$10x = 4 \times 90 \text{ or } x = 36.$$

So, speed of goods train = 36kmph.

Q12. A thief is spotted by a policeman from a distance of 100 metres. When the policeman starts the chase, the thief also starts running. If the speed of the thief be 8km/hr and that of the policeman 10 km/hr, how far the thief will have run before he is overtaken?

Sol. Relative speed of the policeman = $(10-8)$ km/hr = 2 km/hr.

Time taken by police man to cover 100m $\left(\frac{100}{1000} \times \frac{1}{2} \right) \text{hr} = \frac{1}{20} \text{ hr}.$

$$\text{In } \frac{1}{20} \text{ hrs, the thief covers a distance of } 8 \times \frac{1}{20} \text{ km} = \frac{2}{5} \text{ km} = 400 \text{ m}$$

Q13. I walk a certain distance and ride back taking a total time of 37 minutes. I could walk both ways in 55 minutes. How long would it take me to ride both ways?

Sol. Let the distance be x km. Then,

(Time taken to walk x km) + (time taken to ride x km) = 37 min.

(Time taken to walk $2x$ km) + (time taken to ride $2x$ km) = 74 min.

But, the time taken to walk $2x$ km = 55 min.

Time taken to ride $2x$ km = $(74-55)\text{min} = 19 \text{ min}.$

17. PROBLEMS ON TRAINS

Q 1. A train 100 m long is running at the speed of 30 km / hr. Find the time taken by it to pass a man standing near the railway line.

Sol. Speed of the train = $(30 \times \frac{5}{18})$ m / sec = $(\frac{25}{3})$ m/ sec. Distance moved in passing the standing man = 100 m. Required time taken = $\frac{100}{(\frac{25}{3})} = (100 \times \frac{3}{25})$ sec = 12 sec

Q2. A train is moving at a speed of 132 km/hr. If the length of the train is 110 metres, how long will it take to cross a railway platform 165 metres long?

Sol. Speed of train = $132 \times (\frac{5}{18})$ m/sec = $110/3$ m/sec.

Distance covered in passing the platform = $(110 + 165)$ m = 275 m.

Time taken = $275 \times (3/110)$ sec = $15/2$ sec = $7 \frac{1}{2}$ sec

Q3. A man is standing on a railway bridge which is 180 m long. He finds that a train crosses the bridge in 20 seconds but himself in 8 seconds. Find the length of the train and its speed?

Sol. Let the length of the train be x metres,

Then, the train covers x metres in 8 seconds and $(x + 180)$ metres in 20 sec

$$x/8 = (x+180)/20 \Leftrightarrow 20x = 8(x + 180) \quad \Leftrightarrow \quad x = 120.$$

Length of the train = 120 m.

Speed of the train = $(\frac{120}{8})$ m / sec = m / sec = $(15 \times \frac{18}{5})$ kmph = 54 km

Q4. A train 150 m long is running with a speed of 68 kmph. In what time will it pass a man who is running at 8 kmph in the same direction in which the train is going?

Sol: Speed of the train relative to man = $(68 - 8)$ kmph

$$= (60 \times \frac{5}{18}) \text{ m/sec} = (\frac{50}{3}) \text{ m/sec}$$

Time taken by the train to cross the man

$$= \frac{150}{50/3} \text{ sec} = 150 \times \frac{3}{50} \text{ sec} = 9 \text{ sec}$$

Q5. A train 220 m long is running with a speed of 59 kmph.. In what will

it pass a man who is running at 7 kmph in the direction opposite to that in which the train is going?

Sol. Speed of the train relative to man = $(59 + 7)$ kmph
= $66 \times \frac{5}{18}$ m/sec = $55/3$ m/sec.

Time taken by the train to cross the man = Time taken by it to cover 220 m at $(\frac{55}{3})$ m / sec = $(220 \times \frac{3}{55})$ sec = 12 sec

Q6. Two trains 137 metres and 163 metres in length are running towards each other on parallel lines, one at the rate of 42 kmph and another at 48 kmph. In what time will they be clear of each other from the moment they meet?

Sol. Relative speed of the trains = $(42 + 48)$ kmph = 90 kmph
= $(90 \times \frac{5}{18})$ m / sec = 25 m /sec.

Time taken by the trains to pass each other

= Time taken to cover $(137 + 163)$ m at 25 m /sec = $(\frac{300}{25})$ sec = 12 sec

Q7. Two trains 100 metres and 120 metres long are running in the same direction with speeds of 72 km/hr, In how much time will the first train cross the second?

Sol: Relative speed of the trains = $(72 - 54)$ km/hr = 18 km/hr
= $(18 \times \frac{5}{18})$ m/sec = 5 m/sec.

Time taken by the trains to cross each other

= Time taken to cover $(100 + 120)$ m at 5 m /sec = $(\frac{220}{5})$ sec = 44 sec.

Q8. A train 100 metres long takes 6 seconds to cross a man walking at 5 kmph in the direction opposite to that of the train. Find the speed of the train.?

Sol: Let the speed of the train be x kmph.

Speed of the train relative to man = $(x + 5)$ kmph = $(x + 5) \times \frac{5}{18}$ m/sec.

Therefore $100 / ((x+5) \times \frac{5}{18}) = 6 \Leftrightarrow 30(x + 5) = 1800 \Leftrightarrow x = 55$

Speed of the train is 55 kmph.

Q9. A train running at 54 kmph takes 20 seconds to pass a platform. Next it takes 12 sec to pass a man walking at 6 kmph in the same direction in which the train is going. Find the length of the train and the length of the platform.

Sol: Let the length of train be x metres and length of platform be y metres.

Speed of the train relative to man = $(54 - 6)$ kmph = 48 kmph

= $48 \times (\frac{5}{18})$ m/sec = $40/3$ m/sec.

In passing a man, the train covers its own length with relative speed.

Length of train = (Relative speed * Time) = ($\frac{40}{3}$)*12 m = 160 m.

Also, speed of the train = $54 \times \frac{5}{18}$ m / sec = 15 m / sec.

$(x+y)/15 = 20 \Leftrightarrow x + y = 300 \Leftrightarrow Y = (300 - 160) \text{ m} = 140 \text{ m}.$

Q10. A man sitting in a train which is traveling at 50 kmph observes that a goods train, traveling in opposite direction, takes 9 seconds to pass him. If the goods train is 280 m long, find its speed.?

Sol: Relative speed = $280/9 \text{ m / sec} = ((280/9) \times (18/5)) \text{ kmph} = 112 \text{ kmph}.$

Speed of goods train = $(112 - 50) \text{ kmph} = 62 \text{ kmph}.$

18. BOATS AND STREAMS

Q1. A man can row upstream at 7 kmph and downstream at 10kmph.find man's rate in still water and the rate of current.

Sol. Rate in still water= $\frac{1}{2}(10+7)$ km/hr=8.5 km/hr.

Rate of current= $\frac{1}{2}(10-7)$ km/hr=1.5 km/hr.

Q2. A man takes 3 hours 45 minutes to row a boat 15 km downstream of a river and 2hours30minutes to cover a distance of 5km upstream. find the speed of the river current in km/hr.

Sol. rate downstream= $(15/3 \frac{3}{4})$ km/hr= $(15*4/15)$ km/hr=4km/hr.

Rate upstream= $(5/2 \frac{1}{2})$ km/hr= $(5*2/5)$ km/hr=2km/hr.

Speed of current= $\frac{1}{2}(4-2)$ km/hr=1km/hr

Q3. a man can row 18 kmph in still water.it takes him thrice as long to row up as to row down the river.find the rate of stream.

Sol. Let man's rate upstream be x kmph.then ,his rate downstream=3xkmph.

So,2x=18 or x=9.

Rate upstream=9 km/hr,rate downstream=27 km/hr.

Hence,rate of stream= $\frac{1}{2}(27-9)$ km/hr=9 km/hr.

Q4. there is a road beside a river.two friends started from a place A,moved to a temple situated at another place B and then returned to A again.one of them moves on a cycle at a speed of 12 km/hr,while the other sails on a boat at a speed of 10 km/hr.if the river flows at the speed of 4 km/hr,which of the two friends will return to placeA first?

Sol. Clearly the cyclist moves both ways at a speed of 12 km/hr.

The boat sailor moves downstream @ $(10+4)$ i.e.,14 km/hr and upstream @ $(10-4)$ i.e., 6km/hr.

So,average speed of the boat sailor= $(2*14*6/14+6)$ km/hr

$=42/5$ km/hr=8.4 km/hr.

since the average speed of the cyclist is greater ,he will return ta A first.

Q5. A man can row $7 \frac{1}{2}$ kmph in still water.if in a river running at 1.5 km/hr an hour,it takes him 50 minutes to row to a place and back,how far off is the place?

Sol. Speed downstream $= (7.5+1.5)$ km/hr=9 km/hr;

Speed upstream= $(7.5-1.5)$ kmph=6kmph.

Let the required distance be x km.then,

$x/9+x/6=50/60$.

$2x+3x=(5/6*18)$

$5x=15$

$x=3$.

Hence,the required distance is 3km.

Q6. In a stream running at 2kmph,a motar boat goes 6km upstream and back again to the starting point in 33 minutes.find the speed of the motarboat in still water.

Sol.let the speed of the motarboat in still water be x kmph.then,

$$6/x+2 +6/x-2=33/60$$

$$11x^2-240x-44=0$$

$$11x^2-242x+2x-44=0$$

$$(x-22)(11x+2)=0$$

$$x=22.$$

Q7.A man can row 40km upstream and 55km downstream in 13 hours also, he can row 30km upstream and 44km downstream in 10 hours.find the speed of the man in still water and the speed of the current.

Sol.let rate upstream=x km/hr and rate downstream=y km/hr.

Then, $40/x +55/y =13$...(i) and $30/x +44/y =10$

Multiplying (ii) by 4 and (i) by 3 and subtracting ,we get: $11/y=1$ or $y=11$.

Substituting $y=11$ in (i),we get: $x=5$.

Rate in still water $=1/2(11+5)$ kmph=8kmph.

Rate of current= $1/2(11-5)$ kmph=3kmph

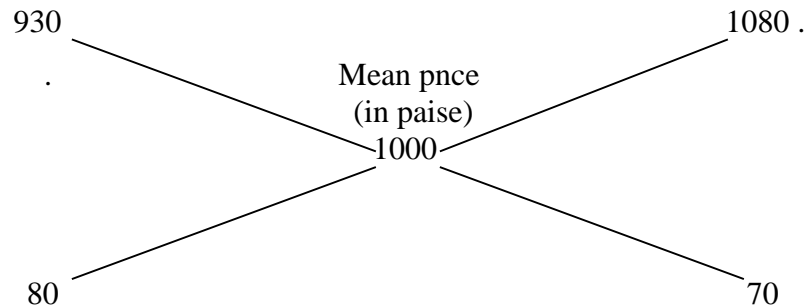
19. ALLIGATION OR MIXTURE

Q1. In what ratio must rice at Rs. 9.30 per kg be mixed with rice at Rs. 10.80 per kg so that the mixture be worth Rs. 10 per kg ?

Sol. By the rule of alligation, we have:

C.P. of 1 kg rice of 1st kind (in paise)

C.P. of 1 kg rice of 2nd kind (in paise)



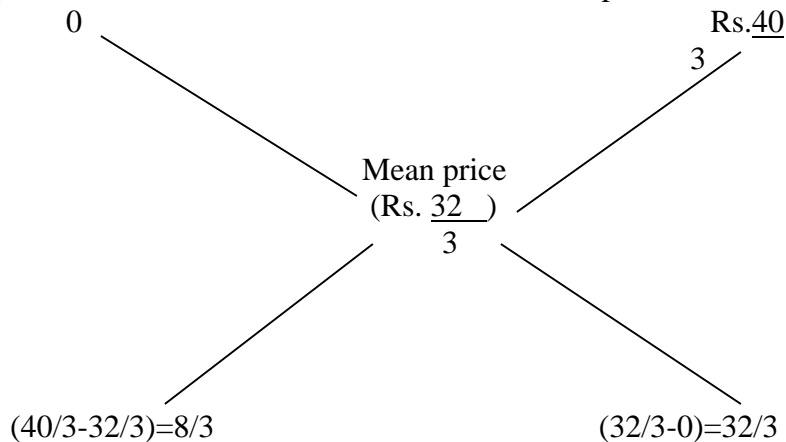
\therefore Required ratio = 80 : 70 = 8 : 7.

Q2. How much water must be added to 60 litres of milk at $1\frac{1}{2}$ litres for Rs. 2 So as to have a mixture worth Rs. $10\frac{2}{3}$ a litre ?

Sol. C.P. of 1 litre of milk = Rs. $(20 \times \frac{2}{3}) = \text{Rs. } \frac{40}{3}$

c.p of 1 litre of milk

c.p of 1 litre of milk



\therefore Ratio of water and milk = $\frac{8}{3} : \frac{32}{3} = 8 : 32 = 1 : 4$

\therefore Quantity of water to be added to 60 litres of milk = $[\frac{1}{4} \times 60]$ litres = 15 litre

Q3. In what ratio must water be mixed with milk to gain 20 % by selling the mixture at cost price?

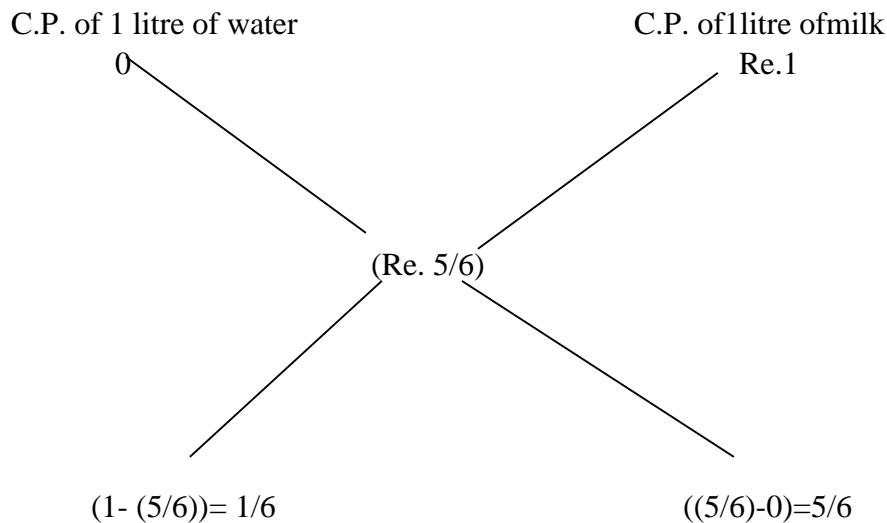
Sol. Let C.P. of milk be Re. 1 per litre.

Then, S.P. of 1 litre of mixture = Re. 1.

Gain obtained = 20%.

\therefore C.P. of 1 litre of mixture = Rs. $[(100/120) * 1] = \text{Rs. } 5/6$

By the rule of alligation, we have:



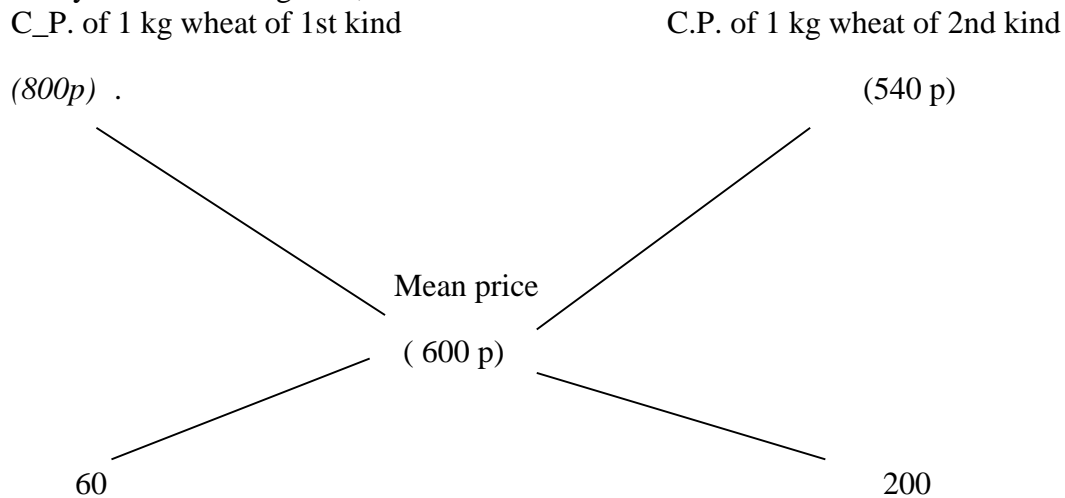
\therefore Ratio of water and milk = $1/6 : 5/6 =$

Q4. How many kgs. of wheat costing Rs. 8 per kg must be mixed with 86 kg of rice costing Rs. 6.40 per kg so that 20% gain may be obtained by Selling the mixture at Rs. 7.20 per kg ?

Sol. S.P. of 1 kg mixture = Rs. 7.20, Gain = 20%.

\therefore C.P. of 1 kg mixture = Rs. $[(100/120) * 7.20] = \text{Rs. } 6$.

By the rule of alligation, we have:



Wheat of 1st kind: Wheat of 2nd kind = $60 : 200 = 3 : 10$.

Let x kg of wheat of 1st kind be mixed with 36 kg of wheat of 2nd kind.

Then, $3 : 10 = x : 36$ or $10x = 3 * 36$ or $x = 10.8$ kg.

Q5. The milk and water in two vessels A and B are in the ratio 4 : 3 and 2: 3 respectively. In what ratio, the liquids in both the vessels be mixed to obtain a new mixture in vessel C containing half milk and half water?

Sol. Let the C.P. of milk be Re. 1 per litre

Milk in 1 litre mixture of A = $\frac{4}{7}$ litre; Milk in 1 litre mixture of B = $\frac{2}{5}$ litre;

Milk in 1 litre mixture of C = $\frac{1}{2}$ litre

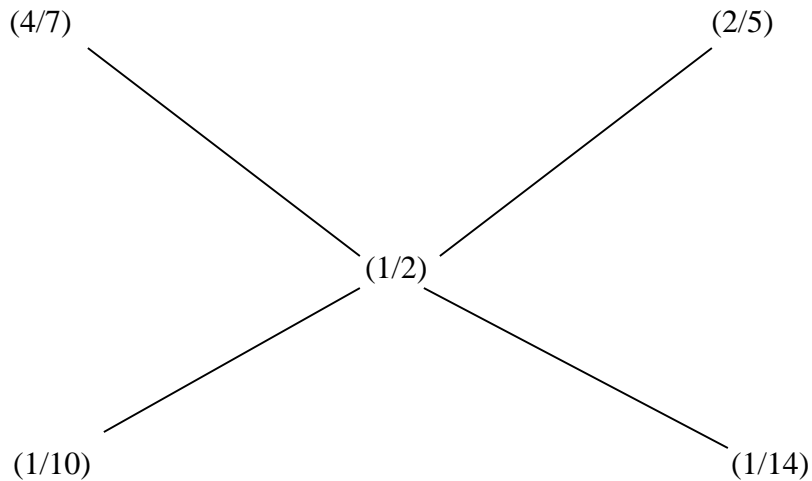
C.P. of 1 litre mixture in A = Re. $\frac{4}{7}$; C.P. of 1 litre mixture in B = Re. $\frac{2}{5}$

Mean price = Re. $\frac{1}{2}$

By the rule of alligation, we have:

C.P. of 1 litre mix. in A

C.P. of 1 litre mix. in B



Required ratio = $\frac{1}{10} : \frac{1}{14} = 7 : 5$

20. SIMPLE INTEREST

Q1. Find the simple interest on Rs. 68,000 at 16 $\frac{2}{3}$ % per annum for 9 months.

Sol. $P = \text{Rs.} 68000, R = 16\frac{2}{3}\% \text{ p.a}$ and $T = 9/12 \text{ years} = 3/4 \text{ years}$.

$$\therefore \text{S.I.} = (P \times R \times T)/100 = \text{Rs.} (68,000 \times (16\frac{2}{3}) \times (3/4) \times (1/100)) = \text{Rs.} 8500$$

Q2. Find the simple interest on Rs. 3000 at 6 $\frac{1}{4}$ % per annum for the period from 4th Feb., 2005 to 18th April, 2005.

Sol. Time = $(24+31+18)\text{days} = 73 \text{ days} = 73/365 \text{ years} = 1/5 \text{ years}$.

$P = \text{Rs.} 3000$ and $R = 6\frac{1}{4}\% \text{ p.a} = 25/4\% \text{ p.a}$

$$\therefore \text{S.I.} = \text{Rs.} (3,000 \times (25/4) \times (1/5) \times (1/100)) = \text{Rs.} 37.50.$$

Remark : The day on which money is deposited is not counted while the day on which money is withdrawn is counted .

Q3. A sum at simple interests at 13 $\frac{1}{2}$ % per annum amounts to Rs.2502.50 after 4 years find the sum.

Sol. Let sum be Rs. x then , $\text{S.I.} = \text{Rs.} (x \times (13\frac{1}{2}) \times 4 \times (1/100)) = \text{Rs.} 27x/50$

$$\therefore \text{amount} = (\text{Rs. } x + (27x/50)) = \text{Rs.} 77x/50$$

$$\therefore 77x/50 = 2502.50 \Leftrightarrow x = \frac{2502.50 \times 50}{77} = 1625$$

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Hence , sum = Rs.1625.

Q4. A sum of Rs. 800 amounts to Rs. 920 in 8 years at simple interest rate is increased by 8%, it would amount to bow much ?

Sol. $\text{S.I.} = \text{Rs.} (920 - 800) = \text{Rs.} 120; P = \text{Rs.} 800, T = 8 \text{ yrs.}$ _

$$\therefore R = ((100 \times 120)/(800 \times 8)) \% = 1.5\%.$$

$$\text{New rate} = (1.5 + 8)\% = 9.5\%.$$

$$\text{New S.I.} = \text{Rs.} (800 \times 9.5 \times 8)/100 = \text{Rs.} 608.$$

$$\therefore \text{New amount} = \text{Rs.} (800 + 608) = \text{Rs.} 1408.$$

Q5. Adam borrowed some money at the rate of 6% p.a. for the first two years , at the rate of 9% p.a. for the next three years , and at the rate of 14% p.a. for the period beyond five years. 1£ he pays a total interest of Rs. 11, 400 at the end of nine years how much money did he borrow ?

Sol. Let the sum borrowed be x. Then,

$$(x \times 6 \times 2)/100 + (x \times 9 \times 3)/100 + (x \times 14 \times 4)/100 = 11400$$

$$\Leftrightarrow (3x/5 + 27x/10 + 56x/25) = 11400 \quad \Leftrightarrow 95x/10 = 11400 \Leftrightarrow x = (11400 \times 10)/95 = 12000.$$

Hence , sum borrowed = Rs.12,000.

Q6. A certain sum of money amounts to Rs. 1008 in 2 years and to Rs.1164 in 3 $\frac{1}{2}$ years. Find the sum and rate of interests.

Sol. S.I. for $1\frac{1}{2}$ years = Rs.(1164-1008) = Rs.156.

S.I. for 2 years = Rs.(156*(2/3)*2)=Rs.208

Principal = Rs. (1008 - 208) = Rs. 800.

Now, P = 800, T = 2 and S.I. = 208.

Rate =(100* 208)/(800*2)% = 13%

Q7. At what rate percent per annum will a sum of money double in 16 years.

Sol. Let principal = P. Then, S.I. = P and T = 16 yrs.

\therefore Rate = (100 x P)/(P*16)% = $6\frac{1}{4}$ % p.a.

Q8. The simple interest on a sum of money is $\frac{4}{9}$ of the principal .Find the rate percent and time, if both are numerically equal.

Sol. Let sum = Rs. x. Then, S.I. = Rs. $\frac{4x}{9}$

Let rate = R% and time = R years.

Then, $(x*R*R)/100=4x/9$ or $R^2=400/9$ or $R = 20/3 = 6\frac{2}{3}$.

\therefore Rate = $6\frac{2}{3}$ % and Time = $6\frac{2}{3}$ years = 6 years 8 months.

Q9. The simple interest on a certain sum of money for $2\frac{1}{2}$ years at 12% per annum is Rs. 40 less than the simple interest on the same sum for $3\frac{1}{2}$ years at 10% per annum. Find the sum.

Sol. Let the sum be Rs. x Then, $((x*10*7)/(100*2)) - ((x*12*5)/(100*2)) = 40$

$\Leftrightarrow (7x/20)-(3x/10)=40$

$\Leftrightarrow x = (40 * 20) = 800$.

Hence, the sum is Rs. 800.

Q10. A sum was put at simple interest at a certain rate for 3 years. Had it been put at 2% higher rate, it would have fetched Rs. 360 more. Find the sum.

Sol. Let sum = P and original rate = R.

Then, $[(P*(R+2)*3)/100] - [(P*R*3)/100] = 360$.

$\Leftrightarrow 3PR + 6P - 3PR = 36000 \Leftrightarrow 6P=36000 \Leftrightarrow P=6000$

Hence, sum = Rs. 6000.

Q11. What annual instalment will discharge a debt of Rs. 1092 due in 3 years at 12% simple interest?

Sol. Let each Instalment be Rs. x

Then, $(x + ((x*12*1)/100)) + (x + ((x*12*2)/100)) + x = 1092$

$$\Leftrightarrow ((28x/25) + (31x/25) + x) = 1092 \Leftrightarrow (28x+31x+25x)=(1092*25)$$

$$\Leftrightarrow x = (1092*25)/84 = \text{Rs.} 325.$$

\therefore Each instalment = Rs. 325.

Q12. A sum of Rs. 1550 is lent out into two parts, one at 8% and another one at 6%. If the total annual income is Rs. 106, find the money lent at each rate.

Sol. Let the sum lent at 8% be Rs. x and that at 6% be Rs. $(1550 - x)$.

$$\therefore ((x*8*1)/100) + ((1550-x)*6*1)/100 = 106$$

$$\Leftrightarrow 8x + 9300 - 6x = 10600 \Leftrightarrow 2x = 1300 \Leftrightarrow x = 650.$$

\therefore Money lent at 8% = Rs. 650. Money lent at 6% = Rs. $(1550 - 650) = \text{Rs.} 900$.

21. COMPOUND INTEREST

Q1. Find compound interest on Rs. 7500 at 4% per annum for 2 years, compounded annually.

Sol.

Amount = Rs $[7500 \times (1 + (4/100))^2]$ = Rs $(7500 \times \frac{26}{25} \times \frac{26}{25})$ = Rs. 8112.
therefore, C.I. = Rs. (8112 - 7500) = Rs. 612.

Q2. Find compound interest on Rs. 8000 at 15% per annum for 2 years 4 months, compounded annually.

Sol. Time = 2 years 4 months = $2(4/12)$ years = $2(1/3)$ years.

Amount = Rs'. $[8000 \times (1 + (15/100))^2 \times (1 + ((1/3) \times 15/100))]$

= Rs. $[8000 \times (23/20) \times (23/20) \times (21/20)]$

= Rs. 11109.

∴ C.I. = Rs. (11109 - 8000) = Rs. 3109.

Q3. Find the compound interest on Rs. 10,000 in 2 years at 4% per annum, the interest being compounded half-yearly.

Sol.

Principal = Rs. 10000; Rate = 2% per half-year; Time = 2 years = 4 half-years.

Amount =

Rs $[10000 \times (1 + (2/100))^4]$ = Rs $(10000 \times (51/50) \times (51/50) \times (51/50) \times (51/50))$

= Rs. 10824.32.

∴ C.I. = Rs. (10824.32 - 10000) = Rs. 824.32.

Q4. Find the compound interest on Rs. 16,000 at 20% per annum for 9 months, compounded quarterly.

Sol. Principal = Rs. 16000; Time = 9 months = 3 quarters;

Rate = 20% per annum = 5% per quarter.

Amount = Rs. $[16000 \times (1 + (5/100))^3]$ = Rs. 18522.

C.I. = Rs. (18522 - 16000) = Rs. 2522.

Q5. If the simple interest on a sum of money at 5% per annum for 3 years is Rs. 1200, find the compound interest on the same sum for the same period at the same rate.

Sol.

Clearly, Rate = 5% p.a., Time = 3 years, S.I. = Rs. 1200. . .

So principal = RS $[100 \times 1200] / 3 \times 5$ = RS 8000

Amount = Rs. $8000 \times [1 + 5/100]^3$ = Rs. 9261.

C.I. = Rs. (9261 - 8000) = Rs. 1261.

Q6. In what time will Rs. 1000 become Rs. 1331 at 10% per annum compounded annually?

Sol. Principal = Rs. 1000; Amount = Rs. 1331; Rate = 10% p.a. Let the time be n years. Then,
 $[1000 (1 + (10/100))^n] = 1331$ or $(11/10)^n = (1331/1000) = (11/10)^3$

n = 3 years.

Q7. If Rs. 600 amounts to Rs. 683.20 in two years compounded annually, find the rate of interest per annum.

Sol. Principal = Rs. 500; Amount = Rs. 583.20; Time = 2 years.

Let the rate be R% per annum.. Then,

$$[500(1+(R/100))^2] = 583.20 \text{ or } [1 + (R/100)]^2 = 5832/5000 = 11664/10000$$

$$[1 + (R/100)]^2 = (108/100)^2 \text{ or } 1 + (R/100) = 108/100 \text{ or } R = 8$$

So, rate = 8% p.a.

Q8. If the compound interest on a certain sum at 16 (2/3)% to 3 years is Rs.1270, find the simple interest on the same sum at the same rate and for the same period.

Sol. Let the sum be Rs. x. Then,

$$\text{C.I.} = [x * (1 + ((50/(3*100)))^3 - x] = ((343x / 216) - x) = 127x / 216$$

$$127x / 216 = 1270 \text{ or } x = (1270 * 216) / 127 = 2160.$$

Thus, the sum is Rs. 2160

$$\text{S.I.} = \text{Rs. } (2160 * (50/3) * 3 * (1/100)) = \text{Rs. } 1080.$$

Q9. The difference between the compound interest and simple interest on a certain sum at 10% per annum for 2 years is Rs. 631. Find the sum.

Sol. Let the sum be Rs. x. Then,

$$\text{C.I.} = x(1 + (10/100))^2 - x = 21x / 100,$$

$$\text{S.I.} = ((x * 10 * 2) / 100) = x / 5$$

$$(\text{C.I.}) - (\text{S.I.}) = ((21x / 100) - (x / 5)) = x / 100$$

$$(x / 100) = 632 \Rightarrow x = 63100.$$

Hence, the sum is Rs.63,100.

Q10. The difference between the compound interest and the simple interest accrued on an amount of Rs. 18,000 in 2 years was Rs. 405. What was the rate of interest p.c.p.a. ?

Sol. Let the rate be R% p.a. then,

$$[18000(1 + (R/100))^2 - 18000] - ((18000 * R * 2) / 100) = 405$$

$$18000[(100 + (R/100))^2 / 10000 - 1 - (2R/100)] = 405$$

$$18000[(100 + R)^2 - 10000 - 200R] / 10000 = 405$$

$$9R^2 / 5 = 405 \Rightarrow R^2 = ((405 * 5) / 9) = 225$$

$$R = 15. \text{ Rate} = 15\%.$$

Q11. Divide Rs. 1301 between A and B, so that the amount of A after 7 years is equal to the amount of B after 9 years, the interest being compounded at 4% per annum.

Sol. Let the two parts be Rs. x and Rs. (1301 - x).

$$x(1+4/100)^7 = (1301-x)(1+4/100)^9$$

$$x/(1301-x) = (1+4/100)^2 = (26/25 * 26/25)$$

$$625x = 676(1301-x)$$

$$1301x = 676 * 1301$$

$$x = 676.$$

So, the parts are rs.676 and rs.(1301-676) i.e rs.676 and rs.625.

Q12. a certain sum amounts to rs.7350 in 2 years and to rs.8575 in 3 years.find the sum and rate percent.

S.I on rs.7350 for 1 year=rs.(8575-7350)=rs.1225.

Rate=(100*1225/7350*1)%=16 2/3%

Let the sum be rs.x.then,

$$X(1+50/3*100)^2=7350$$

$$X*7/6*7/6=7350$$

$$X=(7350*36/49)=5400.$$

Sum=rs.5400.

Q13.a sum of money amounts to rs.6690 after 3 years and to rs.10,035 after 6 years on compound interest.find the sum.

Sol. Let the sum be rs.P.then

$$P(1+R/100)^3=6690...(i) \text{ and } P(1+R/100)^6=10035...(ii)$$

On dividing,we get $(1+R/100)^3=10025/6690=3/2$.

Substituting this value in (i),we get:

$$P*3/2=6690 \text{ or } P=(6690*2/3)=4460$$

Hence,the sum is rs.4460.

Q14. a sum of money doubles itself at compound interest in 15 years.in how many years will it beco,e eight times?

$$P(1+R/100)^{15}=2P$$

$$(1+R/100)^{15}=2P/P=2$$

$$\text{LET } P(1+R/100)^n=8P$$

$$(1+R/100)^n=8=2^3=\{(1+R/100)^{15}\}^3[\text{USING (I)}]$$

$$(1+R/100)^N=(1+R/100)^{45}$$

$$n=45.$$

Thus,the required time=45 years.

Q15.What annual payment will discharge a debt of Rs.7620 due in 3years at 16 2/3% per annum interest?

Sol. Let each installment beRs.x.

Then,(P.W. of Rs.x due 1 year hence)+(P>W of Rs.x due 2 years hence)+(P.W of Rs. X due 3 years hence)=7620.

$$\therefore x/(1+(50/3*100))+ x/(1+(50/3*100))^2 + x/(1+(50/3*100))^3=7620$$

$$\Leftrightarrow (6x/7)+(936x/49)+(216x/343)=7620.$$

$$\Leftrightarrow 294x+252x+216x=7620*343.$$

$$\Leftrightarrow x=(7620*343/762)=3430.$$

\therefore Amount of each installment=Rs.3430.

22. LOGARITHMS

Q 1.Evaluate:

(1) $\log_3 27$

(2) $\log_7 (1/343)$

(3) $\log_{100}(0.01)$

SOLUTION:

(1) let $\log_3 27 = 3^n$ or $n=3$.
ie, $\log_3 27 = 3$.

(2) Let $\log_7 (1/343) = n$.

Then, $7^n = 1/343$

$= 1/7^3$

$n = -3$.

ie,

$\log_7 (1/343) = -3$.

(3) let $\log_{100}(0.01) = n$.

Then, $(100)^n = 0.01 = 1/100 = 100^{-1}$ Or $n = -1$

Q2. evaluate

(i) $\log_7 1 = 0$

(ii) $\log_{34} 34$

(iii) $36^{\log_6 4}$

solution:

i) we know that $\log_a 1 = 0$, so $\log_7 1 = 0$.

ii) we know that $\log_a a = 1$, so $\log_{34} 34 = 1$.

iii) We know that $a^{\log_a x} = x$.

• now $36^{\log_6 4} = (6^2)^{\log_6 4} = 6^{2 \log_6 4} = 6^{\log_6 16} = 16$.

Q3. if $\log_{\sqrt{8}} x = 3$ (1/3), find the value of x.

$\log_{\sqrt{8}} x = 10/3$, $x = (\sqrt{8})^{10/3} = (2^{3/2})^{10/3} = 2^{(3/2 \cdot 10/3)} = 2^5 = 32$.

Q4: Evaluate: (i) $\log_5 3 \cdot \log_{27} 25$ (ii) $\log 27 - \log_{27} 9$

(i) $\log_5 3 \cdot \log_{27} 25 = (\log 3 / \log 5) \cdot (\log 25 / \log 27)$
 $= (\log 3 / \log 5) \cdot (\log 5^2 / \log 3^3)$
 $= (\log 3 / \log 5) \cdot (2 \log 5 / 3 \log 3)$
 $= 2/3$

(ii) Let $\log_9 27 = n$

Then,

$9^n = 27 \Leftrightarrow 3^{2n} = 3^3 \Leftrightarrow 2n = 3 \Leftrightarrow n = 3/2$

Again, let $\log_{27} 9 = m$

Then,

$27^m = 9 \Leftrightarrow 3^{3m} = 3^2 \Leftrightarrow 3m = 2 \Leftrightarrow m = 2/3$

$\Rightarrow \log_9 27 - \log_{27} 9 = (n - m) = (3/2 - 2/3) = 5/6$

Q 5. Simplify :($\log 75/16 - 2 \log 5/9 + \log 32/243$)

Sol: $\log 75/16 - 2 \log 5/9 + \log 32/243$
 $= \log 75/16 - \log(5/9)^2 + \log 32/243$
 $= \log 75/16 - \log 25/81 + \log 32/243$
 $= \log(75/16 * 32/243 * 81/25) = \log 2$

Q6. Find the value of x which satisfies the relation

$\log_{10} 3 + \log_{10} (4x+1) = \log_{10} (x+1) + 1$

Sol: $\log_{10} 3 + \log_{10} (4x+1) = \log_{10} (x+1) + 1$
 $\log_{10} 3 + \log_{10} (4x+1) = \log_{10} (x+1) + \log_{10} 10$
 $\log_{10} (3(4x+1)) = \log_{10} (10(x+1))$
 $= 3(4x+1) = 10(x+1) = 12x+3$
 $= 10x+10$
 $= 2x=7 \Rightarrow x=7/2$

Q7. Simplify: $[1/\log_{xy}(xyz) + 1/\log_{yz}(xyz) + 1/\log_{zx}(xyz)]$

Given expression: $\log_{xyz} xy + \log_{xyz} yz + \log_{xyz} zx$
 $= \log_{xyz} (xy * yz * zx) = \log_{xyz} (xyz)^2$
 $2 \log_{xyz} (xyz) = 2 * 1 = 2$

Q8. If $\log_{10} 2 = 0.30103$, find the value of $\log_{10} 50$.

Soln. $\log_{10} 50 = \log_{10} (100/2) = \log_{10} 100 - \log_{10} 2 = 2 - 0.30103 = 1.69897$.

Q 9. If $\log 2 = 0.3010$ and $\log 3 = 0.4771$, find the values of:

i) $\log 25$ ii) $\log 4.5$

Soln.

i) $\log 25 = \log(100/4) = \log 100 - \log 4 = 2 - 2 \log 2 = (2 - 2 * .3010) = 1.398$.
ii) $\log 4.5 = \log(9/2) = \log 9 - \log 2 = 2 \log 3 - \log 2$
 $= (2 * 0.4771 - .3010) = .6532$

Q10. If $\log 2 = .30103$, find the number of digits in 2^{56} .

Soln. $\log 2^{56} = 56 \log 2 = (56 * 0.30103) = 16.85768$.

Its characteristics is 16.

Hence, the number of digits in 2^{56} is 17

23. AREA

Q1. One side of a rectangular field is 15 m and one of its diagonals is 17 m. Find the area of the field.

Sol. Other side = $((17)^2 - (15)^2)^{(1/2)} = (289 - 225)^{(1/2)} = (64)^{(1/2)} = 8$ m.

Area = $(15 \times 8) \text{ m}^2 = 120 \text{ m}^2$.

Q2. A lawn is in the form of a rectangle having its sides in the ratio 2: 3. The area of the lawn is (1/6) hectares. Find the length and breadth of the lawn.

Sol. Let length = $2x$ metres and breadth = $3x$ metre.

Now, area = $(1/6) \times 1000 \text{ m}^2 = 5000/3 \text{ m}^2$

So, $2x \times 3x = 5000/3 \Leftrightarrow x^2 = 2500/9 \Leftrightarrow x = 50/3$

therefore Length = $2x = (100/3) \text{ m} = 33(1/3) \text{ m}$ and Breadth = $3x = 3(50/3) \text{ m} = 50 \text{ m}$.

Q3. Find the cost of carpeting a room 13 m long and 9 m broad with a carpet 75 cm wide at the rate of Rs. 12.40 per square metre.

Sol. Area of the carpet = Area of the room = $(13 \times 9) \text{ m}^2 = 117 \text{ m}^2$.

Length of the carpet = (area/breadth) = $117 \times (4/3) \text{ m} = 156 \text{ m}$.

Therefore Cost of carpeting = Rs. $(156 \times 12.40) = \text{Rs. } 1934.40$.

Q4. If the diagonal of a rectangle is 17 cm long and its perimeter is 46 cm, find the area of the rectangle.

Sol. Let length = x and breadth = y . Then,

$2(x + y) = 46$ or $x + y = 23$ and $x^2 + y^2 = (17)^2 = 289$.

Now, $(x + y)^2 = (23)^2 \Leftrightarrow (x^2 + y^2) + 2xy = 529 \Leftrightarrow 289 + 2xy = 529 \Leftrightarrow xy = 120$

Area = $xy = 120 \text{ cm}^2$.

Q5. The length of a rectangle is twice its breadth. If its length is decreased by 5 cm and breadth is increased by 5 cm, the area of the rectangle is increased by 75 sqcm. Find the length of the rectangle.

Sol. Let breadth = x . Then, length = $2x$. Then,

$(2x - 5)(x + 5) - 2x \times x = 75 \Leftrightarrow 5x - 25 = 75 \Leftrightarrow x = 20$.

\therefore Length of the rectangle = 20 cm.

Q6. In measuring the sides of a rectangle, one side is taken 5% in excess, and the other 4% in deficit. Find the error percent in the area calculated from these measurements.

Sol. Let x and y be the sides of the rectangle. Then, Correct area = xy .

Calculated area = $(105/100) \times x \times (96/100) \times y = (504/500)(xy)$

Error In measurement = $(504/500)xy - xy = (4/500)xy$

Error % = $[(4/500)xy \times (1/xy) \times 100] \% = (4/5) \% = 0.8\%$.

Q7. A rectangular grassy plot 110 m. by 65 m has a gravel path 2.5 m wide all round it on the inside. Find the cost of gravelling the path at 80 paise per sqmetre.

Sol. Area of the plot = $(110 \times 65) \text{ m}^2 = 7150 \text{ m}^2$

Area of the plot excluding the path = $[(110 - 5) \times (65 - 5)] \text{ m}^2 = 6300 \text{ m}^2$.

Area of the path = $(7150 - 6300) \text{ m}^2 = 850 \text{ m}^2$.
 Cost of gravelling the path = $\text{Rs.} 850 * (80/100) = \text{Rs.} 680$

Q8. The perimeters of two squares are 40 cm and 32 cm. Find the perimeter of a third square whose area is equal to the difference of the areas of the two squares. (S.S.C. 2003)

Sol. Side of first square = $(40/4) = 10 \text{ cm}$;
 Side of second square = $(32/4) \text{ cm} = 8 \text{ cm}$.
 Area of third square = $[(10)^2 - (8)^2] \text{ cm}^2 = (100 - 64) \text{ cm}^2 = 36 \text{ cm}^2$.
 Side of third square = $(36)^{(1/2)} \text{ cm} = 6 \text{ cm}$.
 Required perimeter = $(6 \times 4) \text{ cm} = 24 \text{ cm}$.

Q9. A room 5m 55cm long and 3m 74 cm broad is to be paved with square tiles. Find the least number of square tiles required to cover the floor.

Sol. Area of the room = $(544 \times 374) \text{ cm}^2$.
 Size of largest square tile = H.C.F. of 544 cm and 374 cm = 34 cm.
 Area of 1 tile = $(34 \times 34) \text{ cm}^2$.
 Number of tiles required = $(544 \times 374) / (34 \times 34) = 176$

Q10. Find the area of a square, one of whose diagonals is 3.8 m long.

Sol. Area of the square = $(1/2) * (\text{diagonal})^2 = [(1/2) * 3.8 * 3.8] \text{ m}^2 = 7.22 \text{ m}^2$.

Q11. The diagonals of two squares are in the ratio of 2 : 5. Find the ratio of their areas. (Section Officers', 2003)

Sol. Let the diagonals of the squares be $2x$ and $5x$ respectively.
 Ratio of their areas = $(1/2) * (2x)^2 : (1/2) * (5x)^2 = 4x^2 : 25x^2 = 4 : 25$.

Q12. If each side of a square is increased by 25%, find the percentage change in its area.

Sol. Let each side of the square be a . Then, area = a^2 .
 New side = $(125a/100) = (5a/4)$. New area = $(5a/4)^2 = (25a^2)/16$.
 Increase in area = $((25a^2)/16) - a^2 = (9a^2)/16$.
 Increase% = $[(9a^2)/16 * (1/a^2) * 100] \% = 56.25\%$.

Q13. If the length of a certain rectangle is decreased by 4 cm and the width is increased by 3 cm, a square with the same area as the original rectangle would result. Find the perimeter of the original rectangle.

Sol. Let x and y be the length and breadth of the rectangle respectively.
 Then, $x - 4 = y + 3$ or $x - y = 7$ ----(i)
 Area of the rectangle = xy ; Area of the square = $(x - 4)(y + 3)$
 $(x - 4)(y + 3) = xy \Leftrightarrow 3x - 4y = 12$ ----(ii)
 Solving (i) and (ii), we get $x = 16$ and $y = 9$.
 Perimeter of the rectangle = $2(x + y) = [2(16 + 9)] \text{ cm} = 50 \text{ cm}$.

Q14. A room is half as long again as it is broad. The cost of carpeting the at Rs. 5 per sQm is Rs. 270 and the cost of papering the four walls at Rs. 10 per m² is Rs. 1720. If a door and 2 windows occupy 8 sQm, find the dimensions of the room.

Sol. Let breadth = x metres, length = $3x$ metres, height = H metres.
 Area of the floor = $(\text{Total cost of carpeting}) / (\text{Rate}/\text{m}^2) = (270/5) \text{ m}^2 = 54 \text{ m}^2$.

$$x * (3x/2) = 54 \Leftrightarrow x^2 = (54 * 2/3) = 36 \Leftrightarrow x = 6.$$

So, breadth = 6 m and length = $(3/2) * 6 = 9$ m.

Now, papered area = $(1720/10) \text{ m}^2 = 172 \text{ m}^2$.

Area of 1 door and 2 windows = 8 m^2 .

Total area of 4 walls = $(172 + 8) \text{ m}^2 = 180 \text{ m}^2$

$$2 * (9 + 6) * H = 180 \Leftrightarrow H = 180/30 = 6 \text{ m}.$$

Q15. Find the area of a triangle whose sides measure 13 cm, 14 cm and 15 cm.

Sol. Let $a = 13$, $b = 14$ and $c = 15$. Then, $S = (1/2)(a + b + c) = 21$.

$(s - a) = 8$, $(s - b) = 7$ and $(s - c) = 6$.

$$\text{Area} = (s(s - a)(s - b)(s - c))^{(1/2)} = (21 * 8 * 7 * 6)^{(1/2)} = 84 \text{ cm}^2.$$

Q16. Find the area of a right-angled triangle whose base is 12 cm and hypotenuse is 13 cm.

Sol. Height of the triangle = $[(13)^2 - (12)^2]^{(1/2)} \text{ cm} = (25)^{(1/2)} \text{ cm} = 5 \text{ cm}$.

$$\text{Its area} = (1/2) * \text{Base} * \text{Height} = ((1/2) * 12 * 5) \text{ cm}^2 = 30 \text{ cm}^2.$$

Q17. The base of a triangular field is three times its altitude. If the cost of cultivating the field at Rs. 24.68 per hectare be Rs. 333.18, find its base and height.

Sol. Area of the field = Total cost/rate = $(333.18/25.6) \text{ hectares} = 13.5 \text{ hectares}$

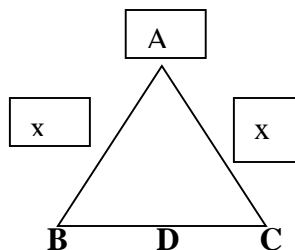
$$\Leftrightarrow (13.5 * 10000) \text{ m}^2 = 135000 \text{ m}^2.$$

Let altitude = x metres and base = $3x$ metres.

$$\text{Then, } (1/2) * 3x * x = 135000 \Leftrightarrow x^2 = 90000 \Leftrightarrow x = 300.$$

Base = 900 m and Altitude = 300 m.

Q18. The altitude drawn to the base of an isosceles triangle is 8 cm and the perimeter is 32 cm. Find the area of the triangle.



Sol. Let ABC be the isosceles triangle and AD be the altitude.

Let $AB = AC = x$. Then, $BC = (32 - 2x)$.

Since, in an isosceles triangle, the altitude bisects the base,

so $BD = DC = (16 - x)$.

$$\text{In triangle ADC, } AC^2 = AD^2 + DC^2 \Rightarrow x^2 = (8^2) + (16 - x)^2$$

$$\Rightarrow 32x = 320 \Rightarrow x = 10.$$

$$BC = (32 - 2x) = (32 - 20) \text{ cm} = 12 \text{ cm}.$$

$$\text{Hence, required area} = ((1/2) * x * BC * AD) = ((1/2) * 12 * 10) \text{ cm}^2 = 60 \text{ cm}^2.$$

Q19. Find the length of the altitude of an equilateral triangle of side $3\sqrt{3}$ cm.

Sol. Area of the triangle = $(\sqrt{3}/4) * (3\sqrt{3})^2 = 27\sqrt{3}$. Let the height be h .

$$\text{Then, } (1/2) * 3\sqrt{3} * h = (27\sqrt{3}/4) * (2/\sqrt{3}) = 4.5 \text{ cm}.$$

Q20. In two triangles, the ratio of the areas is 4 : 3 and the ratio of their heights is 3 : 4. Find the ratio of their bases.

Sol. Let the bases of the two triangles be x and y and their heights be $3h$ and $4h$ respectively.
Then, $((1/2) \times x \times 3h) / ((1/2) \times y \times 4h) = 4/3 \Leftrightarrow x/y = (4/3 \times 4/3) = 16/9$
Required ratio = 16 : 9.

Q21. The base of a parallelogram is twice its height. If the area of the parallelogram is 72 sq cm, find its height.

Sol. Let the height of the parallelogram be x cm. Then, base = $(2x)$ cm.
 $2x \times x = 72 \Leftrightarrow 2x^2 = 72 \Leftrightarrow x^2 = 36 \Leftrightarrow x = 6$
Hence, height of the parallelogram = 6 cm.

Q22. Find the area of a rhombus one side of which measures 20 cm and one diagonal 24 cm.

Sol. Let other diagonal = $2x$ cm.
Since diagonals of a rhombus bisect each other at right angles, we have:
 $(20)^2 = (12)^2 + (x)^2 \Rightarrow x = \sqrt{(20)^2 - (12)^2} = \sqrt{256} = 16$ cm. \therefore
So, other diagonal = 32 cm.
Area of rhombus = $(1/2) \times (\text{Product of diagonals}) = ((1/2) \times 24 \times 32) \text{ cm}^2 = 384 \text{ cm}^2$

Q23. The difference between two parallel sides of a trapezium is 4 cm. perpendicular distance between them is 19 cm. If the area of the trapezium is 475 find the lengths of the parallel sides. (R.R.B. 2002)

Sol. Let the two parallel sides of the trapezium be a cm and b cm.
Then, $a - b = 4$
And, $(1/2) \times (a + b) \times 19 = 475 \Leftrightarrow (a + b) = ((475 \times 2)/19) \Leftrightarrow a + b = 50$
Solving (i) and (ii), we get: $a = 27$, $b = 23$.
So, the two parallel sides are 27 cm and 23 cm.

Q24. Find the length of a rope by which a cow must be tethered in order that it may be able to graze an area of 9856 sq metres.

Sol. Clearly, the cow will graze a circular field of area 9856 sq metres and radius equal to the length of the rope.
Let the length of the rope be R metres.
Then, $\pi(R)^2 = (9856 \times (7/22)) = 3136 \Leftrightarrow R = 56$.
Length of the rope = 56 m.

Q25. The area of a circular field is 13.86 hectares. Find the cost of fencing it at the rate of Rs. 4.40 per metre.

Sol. Area = $(13.86 \times 10000) \text{ m}^2 = 138600 \text{ m}^2$.
 $\pi(R^2) = 138600 \Leftrightarrow (R)^2 = (138600 \times (7/22)) \Leftrightarrow R = 210$ m.
Circumference = $2\pi R = (2 \times (22/7) \times 210) \text{ m} = 1320$ m.
Cost of fencing = Rs. $(1320 \times 4.40) = \text{Rs. } 5808$.

Q26. The diameter of the driving wheel of a bus is 140 cm. How many revolutions, per minute must the wheel make in order to keep a speed of 66 kmph ?

Sol. Distance to be covered in 1 min. = $(66 \times 1000)/(60) \text{ m} = 1100$ m.

Circumference of the wheel = $(2 \times \frac{22}{7} \times 0.70)$ m = 4.4 m.
 Number of revolutions per min. = $(1100/4.4) = 250$.

Q, 27. A wheel makes 1000 revolutions in covering a distance of 88 km. Find the radius of the wheel.

Sol. Distance covered in one revolution = $((88 \times 1000)/1000) = 88$ m.
 $2\pi R = 88 \Leftrightarrow 2 \times \frac{22}{7} \times R = 88 \Leftrightarrow R = 88 \times \frac{7}{44} = 14$ m.

Q, 28. The inner circumference of a circular race track, 14 m wide, is 440 m. Find radius of the outer circle.

Sol. Let inner radius be r metres. Then, $2\pi r = 440 \Leftrightarrow r = (440 \times \frac{7}{44}) = 70$ m.
 Radius of outer circle = $(70 + 14)$ m = 84 m.

Q 29. Two concentric circles form a ring. The inner and outer circumferences of ring are $(352/7)$ m and $(518/7)$ m respectively. Find the width of the ring.

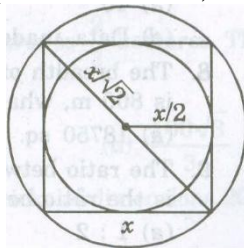
Sol.. Let the inner and outer radii be r and R metres.
 Then $2\pi r = (352/7) \Leftrightarrow r = ((352/7) \times \frac{7}{22} \times \frac{1}{2}) = 8$ m.
 $2\pi R = (528/7) \Leftrightarrow R = ((528/7) \times \frac{7}{22} \times \frac{1}{2}) = 12$ m.
 ,', Width of the ring = $(R - r) = (12 - 8)$ m = 4 m.

Q 30. A sector of 120° , cut out from a circle, has an area of $(66/7)$ sqcm. Find the radius of the circle.

Sol. Let the radius of the circle be r cm. Then,
 $(\pi (r)^2 \theta) / 360 = (66/7) \Leftrightarrow \frac{22}{7} \times (r)^2 \times \frac{120}{360} = (66/7)$
 $\Leftrightarrow (r)^2 = ((66/7) \times \frac{7}{22} \times 3) \Leftrightarrow r = 3$.
 Hence, radius = 3 cm.

Q 31. Find the ratio of the areas of the incircle and circumcircle of a square.

Sol. Let the side of the square be x . Then, its diagonal = $\sqrt{2} x$.



Radius of incircle = $(x/2)$
 Radius of circum circle = $(\sqrt{2}x/2) = (x/\sqrt{2})$
 Required ratio = $((\pi (r)^2)/4 : (\pi (r)^2)/2) = (1/4) : 1/2 = 1 : 2$.

Q32. If the radius of a circle is decreased by 50%, find the percentage decrease in its area.

Sol. Let original radius = R . New radius = $(50/100)R = (R/2)$
 Original area = $\pi (R)^2$ and new area = $\pi ((R/2))^2 = (\pi (R)^2)/4$
 Decrease in area = $((\pi (R)^2)/4 \times (1/\pi (R)^2) \times 100) \% = 75\%$

24. VOLUME AND SURFACE AREA

Q1. Find the volume and surface area of a cuboid 16 m long, 14 m broad and 7 m high.

Sol. Volume = $(16 \times 14 \times 7) \text{ m}^3 = 1568 \text{ m}^3$.

Surface area = $[2 (16 \times 14 + 14 \times 7 + 16 \times 7)] \text{ cm}^2 = (2 \times 434) \text{ cm}^2 = 868 \text{ cm}^2$.

Q2. Find the length of the longest pole that can be placed in a room 12 m long 8m broad and 9m high.

Sol. Length of longest pole = Length of the diagonal of the room
 $= \sqrt{(12^2 + 8^2 + 9^2)} = \sqrt{(289)} = 17 \text{ m}.$

Q3. The volume of a wall, 5 times as high as it is broad and 8 times as long as it is high, is 12.8 cu. metres. Find the breadth of the wall.

Sol. Let the breadth of the wall be x metres.

Then, Height = $5x$ metres and Length = $40x$ metres.

$\therefore x \times 5x \times 40x = 12.8 \Leftrightarrow x^3 = 12.8/200 = 128/2000 = 64/1000$

So, $x = (4/10) \text{ m} = ((4/10) \times 100) \text{ cm} = 40 \text{ cm}$

Q4. Find the number of bricks, each measuring 24 cm x 12 cm x 8 cm, required to construct a wall 24 m long, 8m high and 60 cm thick, if 10% of the wall is filled with mortar?

Sol. Volume of the wall = $(2400 \times 800 \times 60) \text{ cu. cm}.$

Volume of bricks = 90% of the volume of the wall

$= ((90/100) \times 2400 \times 800 \times 60) \text{ cu. cm}.$

Volume of 1 brick = $(24 \times 12 \times 8) \text{ cu. cm}.$

$\therefore \text{Number of bricks} = (90/100) \times (2400 \times 800 \times 60) / (24 \times 12 \times 8) = 45000.$

Q5. Water flows into a tank 200 m x 160 m through a rectangular pipe of 1.5m x 1.25 m @ 20 kmph . In what time (in minutes) will the water rise by 2 metres?

Sol. Volume required in the tank = $(200 \times 150 \times 2) \text{ m}^3 = 60000 \text{ m}^3.$

Length of water column flown in 1 min = $(20 \times 1000) / 60 \text{ m} = 1000/3 \text{ m}$

Volume flown per minute = $1.5 \times 1.25 \times (1000/3) \text{ m}^3 = 625 \text{ m}^3.$

$\therefore \text{Required time} = (60000/625) \text{ min} = 96 \text{ min}$

Q6. The dimensions of an open box are 50 cm, 40 cm and 23 cm. Its thickness is 2 cm. If 1 cubic cm of metal used in the box weighs 0.5 gms, find the weight of the box.

Sol. Volume of the metal used in the box = External Volume - Internal Volume
 $= [(50 \times 40 \times 23) - (44 \times 34 \times 20)] \text{ cm}^3$
 $= 16080 \text{ cm}^3$

$\therefore \text{Weight of the metal} = ((16080 \times 0.5) / 1000) \text{ kg} = 8.04 \text{ kg}.$

Q7. The diagonal of a cube is $6\sqrt{3} \text{ cm}$. Find its volume and surface area.

Sol. Let the edge of the cube be a .

$$\therefore \sqrt[3]{3a} = 6 \Rightarrow a = 6.$$

$$\text{So, Volume} = a^3 = (6 \times 6 \times 6) \text{ cm}^3 = 216 \text{ cm}^3.$$

$$\text{Surface area} = 6a^2 = (6 \times 6 \times 6) \text{ cm}^2 = 216 \text{ cm}^2.$$

Q8. The surface area of a cube is 1734 sq cm. Find its volume.

Sol. Let the edge of the cube be a . Then,

$$6a^2 = 1734 \Rightarrow a^2 = 289 \Rightarrow a = 17 \text{ cm}.$$

$$\therefore \text{Volume} = a^3 = (17)^3 \text{ cm}^3 = 4913 \text{ cm}^3.$$

Q9. A rectangular block 6 cm by 12 cm by 15 cm is cut up into an exact number of equal cubes. Find the least possible number of cubes.

Sol. Volume of the block = $(6 \times 12 \times 15) \text{ cm}^3 = 1080 \text{ cm}^3$.

Side of the largest cube = H.C.F. of 6 cm, 12 cm, 15 cm = 3 cm.

$$\text{Volume of this cube} = (3 \times 3 \times 3) \text{ cm}^3 = 27 \text{ cm}^3.$$

$$\text{Number of cubes} = 1080/27 = 40.$$

Q10. A cube of edge 15 cm is immersed completely in a rectangular vessel containing water. If the dimensions of the base of vessel are 20 cm x 15 cm, find the rise in water level.

Sol. Increase in volume = Volume of the cube = $(15 \times 15 \times 15) \text{ cm}^3$.

$$\therefore \text{Rise in water level} = \text{volume/area} = (15 \times 15 \times 15)/(20 \times 15) \text{ cm} = 11.25 \text{ cm}.$$

Q11. Three solid cubes of sides 1 cm, 6 cm and 8 cm are melted to form a new cube. Find the surface area of the cube so formed.

Sol. Volume of new cube = $(1^3 + 6^3 + 8^3) \text{ cm}^3 = 729 \text{ cm}^3$.

$$\text{Edge of new cube} = \sqrt[3]{729} \text{ cm} = 9 \text{ cm}.$$

$$\therefore \text{Surface area of the new cube} = (6 \times 9 \times 9) \text{ cm}^2 = 486 \text{ cm}^2.$$

Q12. If each edge of a cube is increased by 50%, find the percentage increase in its surface area.

Sol. Let original length of each edge = a .

$$\text{Then, original surface area} = 6a^2.$$

$$\text{New edge} = (150\% \text{ of } a) = (150a/100) = 3a/2$$

$$\text{New surface area} = 6 \times (3a/2)^2 = 27a^2/2$$

$$\text{Increase percent in surface area} = \left(\frac{(15a^2/2)}{6a^2} \times \left(\frac{1}{1} \right) \times 100 \right) \% = 125\%$$

Q13. Two cubes have their volumes in the ratio 1 : 27. Find the ratio of their surface areas.

Sol. Let their edges be a and b . Then, $a^3/b^3 = 1/27$ (or) $(a/b)^3 = (1/3)^3$ (or) $(a/b) = (1/3)$.

$$\therefore \text{Ratio of their surface area} = 6a^2/6b^2 = a^2/b^2 = (a/b)^2 = 1/9, \text{ i.e. } 1:9.$$

Q14. Find the volume, curved surface area and the total surface area of a cylinder with diameter of base 7 cm and height 40 cm.

Sol. Volume = $\pi r^2 h = ((22/7) \times (7/2) \times (7/2) \times 40) = 1540 \text{ cm}^3$.

Curved surface area = $2\pi rh = (2 \times (22/7) \times (7/2) \times 40) = 880 \text{ cm}^2$.

Total surface area = $2\pi rh + 2\pi r^2 = 2\pi r(h + r) = (2 \times (22/7) \times (7/2) \times (40 + 3.5)) \text{ cm}^2 = 957 \text{ cm}^2$

Q15. If the capacity of a cylindrical tank is 1848 m^3 and the diameter of its base is 14 m, then find the depth of the tank.

Sol. Let the depth of the tank be h metres. Then,

$$\pi \times 7^2 \times h = 1848 \Rightarrow h = (1848 \times (7/22) \times (1/49)) = 12 \text{ m}$$

Q16. 2.2 cubic dm of lead is to be drawn into a cylindrical wire 0.50 cm diameter. Find the length of the wire in metres.

Sol. Let the length of the wire be h metres. Then,

$$\pi (0.50/(2 \times 100))^2 \times h = 2.2/1000$$

$$\Rightarrow h = ((2.2/1000) \times (100 \times 100)/(0.25 \times 0.25) \times (7/22)) = 112 \text{ m}.$$

Q17. How many iron rods, each of length 7 m and diameter 2 cm can be made out of 0.88 cubic metre of iron?

Sol. Volume of 1 rod = $((22/7) \times (1/100) \times (1/100) \times 7) \text{ cu.m} = 11/5000 \text{ cu.m}$

Volume of iron = 0.88 cu. m.

$$\text{Number of rods} = (0.88 \times 5000/11) = 400.$$

Q18. The radii of two cylinders are in the ratio 3: 5 and their heights are in the ratio of 2 : 3. Find the ratio of their curved surface areas.

Sol. Let the radii of the cylinders be $3x$, $5x$ and their heights be $2y$, $3y$ respectively. Then

$$\text{Ratio of their curved surface area} = \frac{2\pi \times 3x \times 2y}{2\pi \times 5x \times 3y} = 2/5 = 2.5$$

Q19. If 1 cubic cm of cast iron weighs 21 gms, then find the weight of a cast iron pipe of length 1 metre with a bore of 3 cm and in which thickness of the metal is 1 cm.

Sol. Inner radius = $(3/2) \text{ cm} = 1.5 \text{ cm}$, Outer radius = $(1.5 + 1) = 2.5 \text{ cm}$.

$$\therefore \text{Volume of iron} = [\pi \times (2.5)^2 \times 100 - \pi \times (1.5)^2 \times 100] \text{ cm}^3$$

$$= (22/7) \times 100 \times [(2.5)^2 - (1.5)^2] \text{ cm}^3$$

$$= (8800/7) \text{ cm}^3$$

$$\text{Weight of the pipe} = ((8800/7) \times (21/1000)) \text{ kg} = 26.4 \text{ kg}.$$

Q20. Find the slant height, volume, curved surface area and the whole surface area of a cone of radius 21 cm and height 28 cm.

Sol. Here, $r = 21 \text{ cm}$ and $h = 28 \text{ cm}$.

$$\therefore \text{Slant height, } l = \sqrt{r^2 + h^2} = \sqrt{(21)^2 + (28)^2} = \sqrt{1225} = 35 \text{ cm}$$

Q21. Find the length of canvas 1.25 m wide required to build a conical tent of base radius 7 metres and height 24 metres.

Sol. Here, $r = 7\text{m}$ and $h = 24\text{ m}$.

$$\text{So, } l = \sqrt{(r^2 + h^2)} = \sqrt{(7^2 + 24^2)} = \sqrt{(625)} = 25\text{ m.}$$

$$\text{Area of canvas} = \pi r l = ((22/7) * 7 * 25) \text{m}^2 = 550 \text{ m}^2.$$

$$\text{Length of canvas} = (\text{Area/Width}) = (550/1.25) \text{ m} = 440 \text{ m.}$$

Q22. The heights of two right circular cones are in the ratio 1 : 2 and the perimeters of their bases are in the ratio 3 : 4. Find the ratio of their volumes.

Sol. Let the radii of their bases be r and R and their heights be h and $2h$ respectively.

$$\text{Then, } (2\pi r / 2\pi R) = (3/4) \Rightarrow R = (4/3)r.$$

$$\therefore \text{Ratio of volumes} = (((1/3)\pi r^2 h) / ((1/3)\pi (4/3r)^2 (2h))) = 9 : 32.$$

Q23. The radii of the bases of a cylinder and a cone are in the ratio of 3 : 4 and It heights are in the ratio 2 : 3. Find the ratio of their volumes.

Sol. Let the radii of the cylinder and the cone be $3r$ and $4r$ and their heights be $2h$ and $3h$ respectively.

$$\therefore \text{Volume of cylinder} = \pi \times (3r)^2 * 2h = 9/8 = 9 : 8.$$

$$\text{Volume of cone} = (1/3)\pi r^2 * 3h$$

Q24. A conical vessel, whose internal radius is 12 cm and height 50 cm, is full of liquid. The contents are emptied into a cylindrical vessel with internal radius 10 cm. Find the height to which the liquid rises in the cylindrical vessel

Sol. Volume of the liquid in the cylindrical vessel

= Volume of the conical vessel

$$= ((1/3) * (22/7) * 12 * 12 * 50) \text{ cm}^3 = (22 * 4 * 12 * 50) / 7 \text{ cm}^3.$$

Let the height of the liquid in the vessel be h .

$$\text{Then } (22/7) * 10 * 10 * h = (22 * 4 * 12 * 50) / 7 \text{ or } h = (4 * 12 * 50) / 100 = 24 \text{ cm}$$

Q25. Find the volume and surface area of a sphere of radius 10.5 cm.

$$3 \quad 3$$

$$\text{Sol. Volume} = (4/3)\pi r^3 = (4/3) * (22/7) * (21/2) * (21/2) * (21/2) \text{ cm}^3 = 4851 \text{ cm}^3.$$

$$\text{Surface area} = 4\pi r^2 = (4 * (22/7) * (21/2) * (21/2)) \text{ cm}^2 = 1386 \text{ cm}^2$$

Q26. If the radius of a sphere is increased by 50%, find the increase percent in volume and the increase percent in the surface area.

Sol. Let original radius = R . Then, new radius = $(150/100)R = (3R/2)$

$$\text{Original volume} = (4/3)\pi R^3, \text{ New volume} = (4/3)\pi (3R/2)^3 = (9\pi R^3/2)$$

$$\text{Increase \% in volume} = ((19/6)\pi R^3 * (3/4\pi R^3) * 100) \% = 237.5\%$$

$$\text{Original surface area} = 4\pi R^2, \text{ New surface area} = 4\pi (3R/2)^2 = 9\pi R^2$$

$$\text{Increase \% in surface area} = (5\pi R^2 / 4\pi R^2) * 100 \% = 125\%.$$

Q27. Find the number of lead balls, each 1 cm in diameter that can be a sphere of diameter 12 cm.

Sol. Volume of larger sphere = $(4/3)\pi * 6 * 6 * 6 \text{ cm}^3 = 288\pi \text{ cm}^3$.

Volume of 1 small lead ball = $((4/3)\pi * (1/2) * (1/2) * (1/2)) \text{ cm}^3 = \pi/6 \text{ cm}^3$.

\therefore Number of lead balls = $(288\pi * (6/\pi)) = 1728$.

Q28. How many spherical bullets can be made out of a lead cylinder 28cm high and with radius 6 cm, each bullet being 1.5 cm in diameter ?

Sol. Volume of cylinder = $(\pi * 6 * 6 * 28) \text{ cm}^3 = (9\pi/16) \text{ cm}^3$.

Number of bullet = $\frac{\text{Volume of cylinder}}{\text{Volume of each bullet}} = [(36 * 28)\pi * 16] / 9\pi = 1792$.

Q29. A copper sphere of diameter 18cm is drawn into a wire of diameter 4 mm Find the length of the wire.

Sol. Volume of sphere = $((4\pi/3) * 9 * 9 * 9) \text{ cm}^3 = 972\pi \text{ cm}^3$

Volume of sphere = $(\pi * 0.2 * 0.2 * h) \text{ cm}^3$

$\therefore 972\pi = \pi * (2/10) * (2/10) * h \Rightarrow h = (972 * 5 * 5) \text{ cm} = [(972 * 5 * 5)/100] \text{ m}$
 $= 243 \text{ m}$

Q30. Two metallic right circular cones having their heights 4.1 cm and 4.3 cm and the radii of their bases 2.1 cm each, have been melted together and recast into a sphere. Find the diameter of the sphere.

Sol. Volume of sphere = Volume of 2 cones

$$= \left(\frac{1}{3} \pi * (2.1)^2 * 4.1 + \frac{1}{3} \pi * (2.1)^2 * 4.3 \right)$$

Let the radius of sphere be R

$\therefore (4/3)\pi R^3 = (1/3)\pi (2.1)^3$ or $R = 2.1 \text{ cm}$

Hence, diameter of the sphere = 4.2 cm

Q31. A Cone and a sphere have equal radii and equal volumes. Find the ratio of the sphere of the diameter of the sphere to the height of the cone.

Sol. Let radius of each be R and height of the cone be H.

Then, $(4/3)\pi R^3 = (1/3)\pi R^2 H$ (or) $R/H = 1/4$ (or) $2R/H = 2/4 = 1/2$

\therefore Required ratio = 1:2.

Q32. Find the volume, curved surface area and the total surface area of a hemisphere of radius 10.5 cm.

Sol. Volume = $(2\pi r^3/3) = ((2/3) * (22/7) * (21/2) * (21/2) * (21/2)) \text{ cm}^3$
 $= 2425.5 \text{ cm}^3$

Curved surface area = $2\pi r^2 = (2 * (22/7) * (21/2) * (21/2)) \text{ cm}^2$
 $= 693 \text{ cm}^2$

Total surface area = $3\pi r^2 = (3 * (22/7) * (21/2) * (21/2)) \text{ cm}^2$
 $= 1039.5 \text{ cm}^2$

Q33. Hemispherical bowl of internal radius 9 cm contains a liquid. This liquid is to be filled into cylindrical shaped small bottles of diameter 3 cm and height 4 cm. How many bottles will be needed to empty the bowl ?

Sol. Volume of bowl = $\left(\frac{2\pi}{3} \times 9 \times 9 \times 9\right) \text{ cm}^3 = 486\pi \text{ cm}^3$.

Volume of 1 bottle = $\left(\pi \times \left(\frac{3}{2}\right) \times \left(\frac{3}{2}\right) \times 4\right) \text{ cm}^3 = 9\pi \text{ cm}^3$

Number of bottles = $(486\pi/9\pi) = 54$.

Q34. A Cone, a hemisphere and a cylinder stand on equal bases and have the same height. Find ratio of their volumes.

Sol. Let R be the radius of each

Height of the hemisphere = Its radius = R.

\therefore Height of each = R.

Ratio of volumes = $\left(\frac{1}{3}\right)\pi R^2 \times R : \left(\frac{2}{3}\right)\pi R^3 : \pi R^2 \times R = 1:2:3$

25. RACES AND GAMES

Q1. In a km race, A beats B by 28 metres or 7 seconds. Find A's time over the course.

Sol. Clearly, B covers 28 m in 7 seconds.

∴ B's time over the course = $(278 \times 1000) \text{ sec} = 250 \text{ seconds}$.

∴ A's time over the course = $(250 - 7) \text{ sec} = 243 \text{ sec} = 4 \text{ min. } 3 \text{ sec}$.

Q2. A runs $1\frac{3}{4}$ times as fast as B. if A gives B a start of 84 m, how far must winning post be so that A and B might reach it at the same time?

Sol. Ratio of the rates of A and B = $\frac{7}{4} : 1 = 7 : 4$.

So, in a race of 7 m, A gains 3m over B.

∴ 3 m are gained by A in a race of 7 m.

∴ 84 m are gained by A in a race of $(\frac{7}{3} \times 84) \text{ m} = 196 \text{ m}$.

∴ Winning post must be 196 m away from the starting point.

Q3. A can run 1 km in 3 min. 10 sec. and B can cover the same distance in 3 min. 20 sec. By what distance can A beat B ?

Soln: Clearly, A beats B by 10 sec.

Distance covered by B in 10 sec. = $(1000 \times \frac{10}{200}) \text{ m} = 50 \text{ m}$.

Therefore A beats B by 50 metres.

Q 4 . In a 100 m race, A runs at 8km per hour. If A gives B a start of 4 m and still him by 15 seconds, what is the speed of B ?

Sol: Time taken by A to cover 100 m = $(60 \times 60 / 8000) \times 100 \text{ sec} = 45 \text{ sec}$.

B covers $(100 - 4) \text{ m} = 96 \text{ m}$ in $(45 + 15) \text{ sec} = 60 \text{ sec}$.

B's speed = $(96 \times 60 \times 60 / 60 \times 1000) \text{ km/hr} = 5.76 \text{ km/hr}$.

Q5. A, B and C are three contestants in a km race. If A can give B a start of 40 m and A can give C a start of 64m how many metre's start can B give C ?

Sol: While A covers 1000 m, B covers $(1000 - 40) \text{ m} = 960 \text{ m}$ and

C covers $(1000 - 64) \text{ m}$ or 936 m.

When B covers 960 m, C covers 936 m.

Q 6. In a game of 80 points; A can give B 5 points and C 15 points. Then how many points B can give C in a game of 60 ?

Sol. A : B = 80 : 75, A : C = 80 : 65.

$B/C = (B/A \times A/C) = (75/80 \times 80/65) = 15/13 = 60/52 = 60:5$

Therefore ,In a game of 60, B can give C 8 points.

26. CALENDAR

Q: 1. What was the day of the week on, 16th July, 1776?

Sol: 16th July, 1776 = (1775 years + Period from 1st Jan., 1776 to 16th July, 1776)

Counting of odd days :

1600 years have 0 odd day. 100 years have 5 odd days.

75 years = (18 leap years + 57 ordinary years)

= [(18 x 2) + (57 x 1)] odd days = 93 odd days

= (13 weeks + 2 days) = 2 odd days.

∴ 1775 years have (0 + 5 + 2) odd days = 7 odd days = 0 odd day.

Jan. Feb. March April May June July

31 + 29 + 31 + 30 + 31 + 30 + 16 = 198 days

= (28 weeks + 2 days) = 2 days

∴ Total number of odd days = (0 + 2) = 2. Required day was 'Tuesday'.

Q2. What was the day of the week on 16th August, 1947?

Sol. 15th August, 1947 = (1946 years + Period from 1st Jan., 1947 to 15th

Counting of odd days:

1600 years have 0 odd day. 300 years have 1 odd day.

47 years = (11 leap years + 36 ordinary years)

= [(11 x 2) + (36 x 1)] odd days = 58 odd days = 2 odd days.

Jan. Feb. March April May June July Aug.

31 + 28 + 31 + 30 + 31 + 30 + 31 + 15

= 227 days = (32 weeks + 3 days) = 3,

Total number of odd days = (0 + 1 + 2 + 3) odd days = 6 odd days.

Hence, the required day was 'Saturday'.

Q3. What was the day of the week on 16th April, 2000 ?

Sol. 16th April, 2000 = (1999 years + Period from 1st Jan., 2000 to 16th April)

Counting of odd days:

1600 years have 0 odd day. 300 years have 1 odd day.

99 years = (24 leap years + 75 ordinary years)

= [(24 x 2) + (75 x 1)] odd days = 123 odd days

= (17 weeks + 4 days) = 4 odd days.

Jan. Feb. March April

31 + 29 + 31 + 16 = 107 days = (15 weeks + 2 days) = 2 odd,

Total number of odd days = (0 + 1 + 4 + 2) odd days = 7 odd days = 0 odd day. Hence, the required day was 'Sunday'.

Q4. On what dates of July, 2004 did Monday fall?

Sol . Let us find the day on 1st July, 2004.

2000 years have 0 odd day. 3 ordinary years have 3 odd days.

Jan. Feb. March April May June July

31 + 29 + 31 + 30 + 31 + 30 + 1

= 183 days = (26 weeks + 1 day) = 1 day.

Total number of odd days = $(0 + 3 + 1)$ odd days = 4 odd days.

\therefore 1st July 2004 was 'Thursday',-,-

Thus, 1st Monday in July 2004 was on 5th July.

Hence, during July 2004, Monday fell on 5th, 12th, 19th and 26th. .

Q5. Prove that the calendar for the year 2008 will serve for the year 2011

Sol. In order that the calendar for the year 2003 and 2014 be the same, 1st January of both the years must be on the same day of the week.

For this, the number of odd days between 31st Dec., 2002 and 31st Dec., 2013 must be the same.

We know that an ordinary year has 1 odd day and a leap year has 2 odd. During this period, there are 3 leap years, namely, 2004, 2008 and 2012 and 8 ordinary years.

Total number of odd days = $(6 + 8)$ days = 0 odd day.

Hence, the calendar for 2003 will serve for the year 2014.

Q6. Prove that any date in March of a year is the same day of the week corresponding date in November that year.

Sol. We will show that the number of odd days between last day of February and last day of October is zero.

March April May June July Aug. Sept. Oct.

31 + 30 + 31 + 30 + 31 + 31 + 30 + 31

= 241 days = 35 weeks = 0 odd day. ,Number of odd days during this period = 0.

Thus, 1st March of an year will be the same day as 1st November of that year. Hence, the result follows.

27. CLOCKS

Q 1: Find the angle between the hour hand and the minute hand of a clock when 3.25.

Solution: angle traced by the hour hand in 12 hours = 360°

Angle traced by it in three hours 25 min (ie) $4\frac{1}{2}$ hrs = $(360 \times \frac{41}{12} \times \frac{1}{12})^\circ = 102\frac{1}{2}^\circ$

angle traced by minute hand in 60 min. = 360° .

Angle traced by it in 25 min. = $(\frac{360}{60} \times 25) = 150^\circ$

Required angle = $150^\circ - 102\frac{1}{2}^\circ = 47\frac{1}{2}^\circ$

Q 2: At what time between 2 and 3 o'clock will the hands of a clock be together?

Solution: At 2 o'clock, the hour hand is at 2 and the minute hand is at 12, i.e. they are 10 min spaces apart.

To be together, the minute hand must gain 10 minutes over the hour hand.

Now, 55 minutes are gained by it in 60 min.

10 minutes will be gained in $(\frac{60 \times 10}{55})$ min. = $120/11$ min.

The hands will coincide at $120/11$ min. past 2.

Q3. At what time between 4 and 5 o'clock will the hands of a clock be at right angle?

Sol: At 4 o'clock, the minute hand will be 20 min. spaces behind the hour hand, Now, when the two hands are at right angles, they are 15 min. spaces apart. So, they are at right angles in following two cases.

Case I. When minute hand is 15 min. spaces *behind the hour hand*:

In this case min. hand will have to gain $(20 - 15) = 5$ minute spaces. 55 min. spaces are gained by it in 60 min.

5 min spaces will be gained by it in $\frac{60 \times 5}{55}$ min = $60/11$ min.

\therefore They are at right angles at $60/11$ min. past 4.

Case II. When the minute hand is 15 min. spaces *ahead* of the *hour* hand:

To be in this position, the minute hand will have to gain $(20 + 15) = 35$ minute spaces. 55 min. spaces are gained in 60 min.

35 min spaces are gained in $(\frac{60 \times 35}{55})$ min = $40/11$

\therefore They are at right angles at $40/11$ min. past 4.

Q4. Find at what time between 8 and 9 o'clock will the hands of a clock being the same straight line but not together.

Sol: At 8 o'clock, the hour hand is at 8 and the minute hand is at 12, i.e. the two hands are 20 min. spaces apart.

To be in the same straight line but not together they will be 30 minute spaces apart. So, the minute hand will have to gain $(30 - 20) = 10$ minute spaces over the hour hand.

55 minute spaces are gained in 60 min.

10 minute spaces will be gained in $(\frac{60 \times 10}{55})$ min. = $120/11$ min.

\therefore The hands will be in the same straight line but not together at $120/11$ min.

Q5. At what time between 5 and 6 o'clock are the hands of a clock 3 min apart?

Sol. At 5 o'clock, the minute hand is 25 min. spaces behind the hour hand.

Case I. Minute hand is 3 min. spaces *behind the hour hand*.

In this case, the minute hand has to gain' $(25 - 3) = 22$ minute spaces. 55 min. are gained in 60 min. 22 min. are gained in $(60 \times 22) / 55$ min. = 24 min.

\therefore The hands will be 3 min. apart at 24 min. past 5.

Case II. Minute hand is 3 min. spaces *ahead of the hour hand*.

In this case, the minute hand has to gain $(25 + 3) = 28$ minute spaces. 55 min. are gained in 60 min.

28 min. are gained in $(60 \times 28) / 55 = 346 / 11$

The hands will be 3 min. apart at $346 / 11$ min. past 5.

Q6. The minute hand of a clock overtakes the hour hand at intervals of 65 minutes of the correct time. How much a day does the clock gain or lose?

Sol: In a correct clock, the minute hand gains 55 min. spaces over the hour hand in 60 minutes.

To be together again, the minute hand must gain 60 minutes over the hour hand. 55 min. are gained in 60 min.

60 min are gained in $\frac{60 \times 60 \text{ min}}{55} = 720 / 11$ min.

But, they are together after 65 min.

Gain in 65 min $= 720 / 11 - 65 = 5 / 11$ min.

Gain in 24 hours $= (5 / 11 \times (60 \times 24) / 65)$ min $= 440 / 43$

The clock gains $440 / 43$ minutes in 24 hours.

Q7. A watch which gains uniformly, is 6 min. slow at 8 o'clock in the morning Sunday and it is 6 min. 48 sec. fast at 8 p.m. on following Sunday. When was it correct?

Sol. Time from 8 a.m. on Sunday to 8 p.m. on following Sunday = 7 days 12 hours = 180 hours

The watch gains $(5 + 29 / 5)$ min. or $54 / 5$ min. in 180 hrs.

Now $54 / 5$ min. are gained in 180 hrs.

5 min. are gained in $(180 \times \frac{5}{54} \times 5)$ hrs. = 83 hrs 20 min. = 3 days 11 hrs 20 min.

Watch is correct 3 days 11 hrs 20 min. after 8 a.m. of Sunday.

It will be correct at 20 min. past 7 p.m. on Wednesday.

Q8. A clock is set right at 6 a.m. The clock loses 16 minutes in 24 hours. What will be the true time when the clock indicates 10 p.m. on 4th day?

Sol. Time from 5 a.m. on a day to 10 p.m. on 4th day = 89 hours.

Now 23 hrs 44 min. of this clock = 24 hours of correct clock.

$356 / 15$ hrs of this clock = 24 hours of correct clock.

89 hrs of this clock = $(24 \times \frac{31556}{15} \times 89)$ hrs of correct clock.

= 90 hrs of correct clock.

So, the correct time is 11 p.m.

Q9. A clock is set right at 8 a.m. The clock gains 10 minutes in 24 hours will be the true time when the clock indicates 1 p.m. on the following day?

Sol. Time from 8 a.m. on a day 1 p.m. on the following day = 29 hours.

24 hours 10 min. of this clock = 24 hours of the correct clock.

$145/6$ hrs of this clock = 24 hrs of the correct clock

29 hrs of this clock = $(24 \times \frac{6}{145} \times 29)$ hrs of the correct clock

= 28 hrs 48 min. of correct clock

The correct time is 28 hrs 48 min. after 8 a.m.

This is 48 min. past 12.

28. STOCKS AND SHARES

Q1. Find the cost of:

- (i) **Rs. 7200, 8% stock at 90;**
- (ii) **Rs. 4500, 8.5% stock at 4 premium;**
- (iii) **Rs. 6400, 10% stock at 15 discount.**

Sol. (i) Cost of Rs. 100 stock = Rs. 90

Cost of Rs. 7200 stock = Rs. $(90/100 * 7200)$ = Rs. 6480.

(ii) Cost of Rs. 100 stock = Rs. (100+4)

Cost of Rs. 4500 stock = Rs. $(104/100 * 4500)$ = Rs. 4680

(iii) Cost of Rs. 100 stock = Rs. (100-15)

Cost of Rs. 6400 stock = Rs. $(85/100 * 6400)$ = Rs. 5440.

Q2. Find the cash required to purchase Rs. 3200, 7(1/2) % stock at 107 (brokerage (1/2) %)

Sol. Cash required to purchase Rs. 100 stock = Rs $(107+(1/2))$ = Rs. (215/2).

Cash required to purchase Rs. 100 stock = Rs $[(215/2)*(1/100)*3200]$ = Rs. 3440.

Q3. Find the cash realised by selling Rs. 2440, 9.5% stock at 4 discount (brokerage (1/4) %)

Sol. By selling Rs. 100 stock , cash realised = Rs. $[(100-4)-(1/4)]$ = Rs. (383/4).

By selling Rs. 2400 stock, cash realised = Rs. $[(383/4)*(1/100)*2400]$ = Rs 2298.

Q4. Find the annual income derived from Rs. 2500, 8% stock at 106.

Sol. Income from Rs. 100 stock = Rs. 8.

Income from Rs. 2500 = Rs. $[(8/1000*2500)]$ =Rs. 200.

Q5. Find the annual income derived by investing Rs. 6800 in 10% stock at 136.

Sol. By investing Rs. 136, income obtained = Rs. 10.

By investing Rs. 6800, income obtained = Rs. $[(10/136)*6800]$ = Rs. 500.

Q6. Which is better investment? 7(1/2) % stock at 105 or 6(1/2) % at 94.

Sol. Let the investment in each case be Rs. (105*94).

Case I : 7(1/2) % stock at 105:

On investing Rs. 105, income = Rs. (15/2).

On investing Rs. (105*94), income = Rs. $[(15/2)*(1/105)*105*94]$ = Rs 705.

Case II : 6(1/2) % stock at 94:

On investing Rs. 94, income = Rs. (13/2).

On investing Rs. (105*94), income = Rs. $[(13/2)*(1/94)*105*94]$ = Rs. 682.5.

Clearly, the income from 7(1/2) % stock at 105 is more.

Hence, the investment in 7(1/2) % stock at 105 is better.

Q7. Find the cost of 96 shares of Rs. 10 each at $(3/4)$ discount, brokerage being $(1/4)$ per share.

Sol. Cost of 1 share = Rs. $[(10 - (3/4)) + (1/4)] = \text{Rs. } (19/2)$.

Cost of 96 shares = Rs. $[(19/2) * 96] = \text{Rs. } 912$.

Q8. Find the income derived from 88 shares of Rs. 25 each at 5 premium, brokerage being $(1/4)$ per share and the rate of dividend being $7(1/2)$ % per annum. Also, find the rate of interest on the investment.

Sol. Cost of 1 share = Rs. $[25 + 5 + 1/4] = \text{Rs. } (121/4)$.

Cost of 88 shares = Rs. $[(121/4) * 88] = \text{Rs. } 2662$.

\therefore Investment made = Rs. 2662.

Face value of 88 shares = Rs. $(88 * 25) = \text{Rs. } 2200$.

Dividend on Rs. 100 = $(15/2)$.

Dividend on Rs. 2200 = Rs. $[(15/20) * (1/100) * 2200] = \text{Rs. } 165$.

\therefore Income derived = Rs. 165.

Rate of interest on investment = $[(165/2662) * 100] = 6.2 \%$.

Q9. A man buys Rs. 25 shares in company which pays 9 % dividend. The money invested is such that it gives 10 % on investment. At what price did he buy the shares?

Sol. Suppose he buys each share for Rs. x.

Then, $[25 * (9/100)] = [x * (10/100)]$ or $x = \text{Rs. } 22.50$.

Cost of each share = Rs. 22.50.

Q10. A man sells Rs.5000, 12 % stock at 156 and uinvests the proceeds parity in 8 % stock at 90 and 9 % stock at 108. He hereby increases his income by Rs. 70. How much of the proceeds were invested in each stock?

Sol. S.P of Rs. 5000 stock = Rs. $[(156/100) * 5000] = \text{Rs. } 7800$.

Income from this stock = Rs. $[(12/100) * 5000] = \text{Rs. } 600$.

Let investment in * % stock be x and that in 9 % stock = $(7800 - x)$.

$\therefore [x * (8/90)] + (7800 - x) * (9/108) = (600 + 7)$

$\Leftrightarrow (4x/45) + [(7800 - x)/12] = 670 \Leftrightarrow 16x + 117000 - 15x = (670 * 180) \Leftrightarrow x = 3600$.

\therefore Money invested in 8 % stock at 90 = Rs. 3600.

Money invested in 9 % at 108 = Rs. $(7800 - 3600) = \text{Rs. } 4200$.

29. PERMUTATIONS AND COMBINATIONS

Q1. Evaluate: $30!/28!$

Sol. We have, $30!/28! = 30 \times 29 \times (28!)/28! = (30 \times 29) = 870$.

Q2. Find the value of (i) ${}^{60}P_3$ (ii) 4P_4

Sol. (i) ${}^{60}P_3 = 60!/(60-3)! = 60!/57! = 60 \times 59 \times 58 \times (57!)/57! = (60 \times 59 \times 58) = 205320$.

(ii) ${}^4P_4 = 4! = (4 \times 3 \times 2 \times 1) = 24$.

Q3. Find the value of (i) ${}^{10}C_3$ (ii) ${}^{100}C_{98}$ (iii) ${}^{50}C_{50}$

Sol. (i) ${}^{10}C_3 = 10 \times 9 \times 8/3! = 120$.

(ii) ${}^{100}C_{98} = {}^{100}C_{(100-98)} = 100 \times 99/2! = 4950$.

(iii) ${}^{50}C_{50} = 1$. [${}^nC_n = 1$]

Q4. How many words can be formed by using all letters of the word “BIHAR”

Sol. The word BIHAR contains 5 different letters.

Required number of words = ${}^5P_5 = 5! = (5 \times 4 \times 3 \times 2 \times 1) = 120$.

Q5. How many words can be formed by using all letters of the word ‘DAUGHTER’ so that the vowels always come together?

Sol. Given word contains 8 different letters. When the vowels AUE are always together, we may suppose them to form an entity, treated as one letter.

Then, the letters to be arranged are DGNTR (AUE).

Then 6 letters to be arranged in ${}^6P_6 = 6! = 720$ ways.

The vowels in the group (AUE) may be arranged in $3! = 6$ ways.

Required number of words = $(720 \times 6) = 4320$.

Q6. How many words can be formed from the letters of the word ‘EXTRA’ so that the vowels are never together?

Sol. The given word contains 5 different letters.

Taking the vowels EA together, we treat them as one letter.

Then, the letters to be arranged are XTR (EA).

These letters can be arranged in $4! = 24$ ways.

The vowels EA may be arranged amongst themselves in $2! = 2$ ways.

Number of words, each having vowels together = $(24 \times 2) = 48$ ways.

Total number of words formed by using all the letters of the given words

$= 5! = (5 \times 4 \times 3 \times 2 \times 1) = 120$.

Number of words, each having vowels never together = $(120 - 48) = 72$.

Q7. How many words can be formed from the letters of the word ‘DIRECTOR’

So that the vowels are always together?

Sol. In the given word, we treat the vowels IEO as one letter.

Thus, we have DRCTR (IEO).

This group has 6 letters of which R occurs 2 times and others are different.

Number of ways of arranging these letters = $6!/2! = 360$.

Now 3 vowels can be arranged among themselves in $3! = 6$ ways.
Required number of ways = $(360 \times 6) = 2160$.

Q8. In how many ways can a cricket eleven be chosen out of a batch of 15 players ?

Sol. Required number of ways = ${}^{15}C_{11} = {}^{15}C_{(15-11)} = {}^{11}C_4$
 $= 15 \times 14 \times 13 \times 12 / 4 \times 3 \times 2 \times 1 = 1365$.

Q9. In how many ways, a committee of 5 members can be selected from 6 men and 5 ladies, consisting of 3 men and 2 ladies?

Sol. (3 men out of 6) and (2 ladies out of 5) are to be chosen.

Required number of ways = $({}^6C_3 \times {}^5C_2) = [6 \times 5 \times 4 / 3 \times 2 \times 1] \times [5 \times 4 / 2 \times 1] = 200$.

30. PROBABILITY

Q 1. In a throw of a coin, find the probability of getting a head.

Sol. Here $s=\{h,t\}$ and $e=\{h\}$.

$$P(E)=n(E)/n(S)=1/2$$

Q2. Two unbiased coin are tossed .what is the probability of getting atmost one head?

Sol. Here $s=\{hh,ht,th,tt\}$

Let E =event of getting one head

$$e=\{tt,ht,th\}$$

$$p(e)=n(e)/n(s)=3/4$$

Q3. An unbiased die is tossed .find the probability of getting a multiple of 3

Sol. Here $s=\{1,2,3,4,5,6\}$

Let e be the event of getting the multiple of 3

$$\text{then } e=\{3,6\}$$

$$p(e)=n(e)/n(s)=2/6=1/3$$

Q4. in a simultaneous throw of pair of dice .find the probability of getting the total more than 7

Sol. Here $n(s)=(6*6)=36$

let e =event of getting a total more than 7

$$e=\{(2,6),(3,5),(3,6),(4,4),(4,5),(4,6),(5,3),(5,4),(5,5),(5,6),(6,2),(6,3),(6,4),(6,5),(6,6)\}$$

$$p(e)=n(e)/n(s)=15/36=5/12.$$

Q5. A bag contains 6 white and 4 black balls. 2 balls are drawn at random. find the probability that they are of same color.

Sol. let s be the sample space

$$\text{Then } n(s)=\text{no of ways of drawing 2 balls out of } (6+4)=10c_2=(10*9)/(2*1)=45$$

Let e =event of getting both balls of same colour

Then $n(e)$ =no of ways(2 balls out of six) or(2 balls out of 4)

$$=({}^6C_2+{}^4C_2)=(6*5)/(2*1)+(4*3)/(2*1)=15+6=21$$

$$p(e)=n(e)/n(s)=21/45=7/15$$

Q6. Two dice are thrown together .What is the probability that the sum of the number on the two faces is divided by 4 or 6

sol. Clearly $n(s)=6*6=36$

Let E be the event that the sum of the numbers on the two faces is divided by 4 or 6. Then

$$e=\{(1,3),(1,5),(2,2),(2,4),(2,6),(3,1),(3,3),(3,5),(4,2),(4,4),(5,1),(5,3),(6,2),\\(6,6)\}$$

$$n(e)=14.$$

$$\text{Hence } p(e)=n(e)/n(s)=14/36=7/18$$

Q7. Two cards are drawn at random from a pack of 52 cards. what is the probability that either both are black or both are queen?

sol. We have $n(s) = {}^{52}C_2 = (52 \times 51) / (2 \times 1) = 1326$.

Let A = event of getting both black cards

B = event of getting both queens

$a \cap b$ = event of getting queen of black cards

$n(A) = {}^{26}C_2 = (26 \times 25) / (2 \times 1) = 325$,

$n(b) = {}^4C_2 = (4 \times 3) / (2 \times 1) = 6$ and

$n(a \cap b) = {}^2C_2 = 1$

$p(A) = n(A) / n(S) = 325 / 1326$;

$p(B) = n(B) / n(S) = 6 / 1326$ and

$p(a \cap b) = n(a \cap b) / n(s) = 1 / 1326$

$p(a \cup b) = p(a) + p(b) - p(a \cap b) = (325 + 6 - 1) / 1326 = 330 / 1326 = 55 / 221$

31. TRUE DISCOUNT

Q1. Find the present worth of Rs. 930 due 3 years hence at 8% per annum. Also find the discount.

Sol. $P.W = 100 \times \text{Amount} / [100 + (R \times T)]$

$$= \text{Rs. } 100 \times 930 / 100 + (8 \times 3)$$

$$= (100 \times 930) / 124$$

$$= \text{Rs. } 750,$$

$$T.D. = (\text{Amount}) - (P.W.) = \text{Rs. } (930 - 750) = \text{Rs. } 180.$$

Q2. The true discount on a bill due 9 months hence at 12% per annum is Rs. Find the amount of the bill and its present worth.

Sol. Let amount be Rs. x. Then,

$$x \times R \times T / 100 + (R \times T) = T.D.$$

$$\Rightarrow x \times 12 \times 3 / 4 / [100 + [12 \times 3 / 4]]$$

$$= 540$$

$$x = 540 \times 109 = \text{Rs. } 6540$$

$$\text{Amount} - \text{Rs. } 6540. P.W. = \text{Rs. } (6540 - 540) = \text{Rs. } 6000.$$

Q3. The true discount on a certain sum of money due 3 years hence is Rs. 250 and the simple interest on the same sum for the same time and at the same rate is Rs. 375. Find the sum and the rate percent.

Sol. $T.D. = \text{Rs. } 250$ and $S.I. = \text{Rs. } 375$.

$$\text{Sum due} = S.I. \times T.D. / S.I. - T.D.$$

$$= 375 \times 250 / 375 - 250$$

$$= \text{Rs. } 750.$$

$$\text{Rate} = [100 \times 375 / 750 \times 3] \% = 16 \frac{2}{3} \%$$

Q4. The difference between the simple interest and true discount on a certain sum of money for 6 months at 12—% per annum is Rs. 25. Find the sum.

Sol. Let the sum be Rs. x. Then,

$$T.D. = (x \times 25 / 2 \times 1 / 2) / (100 + (25 / 2 \times 1 / 2)) = x \times 25 / 4 \times 4 / 425 = x / 17$$

$$S.I. = x \times 25 / 2 \times 1 / 2 \times 1 / 100 = x / 16$$

$$x / 16 - x / 17 = 25$$

$$\Rightarrow 17x - 16x = 25 \times 16 \times 17$$

$$\Rightarrow x = 6800$$

Hence, sum due = Rs. 6800.

Q5. A bill falls due in 1 year. The creditor agrees to accept immediate payment of the half and to defer the payment of the other half for 2 years. By this arrangement ins Rs. 40. What is the amount of the bill, if the money be worth 12-z% ?

Sol. Let the sum be Rs. x. Then,

$$[x / 2 + (x / 2 \times 100) / 100 + (25 / 2 \times 2)] - [(x \times 100) / (100 + 25 / 2 \times 1)]$$

$$= 40$$

$$\Rightarrow x / 2 + 2x / 5 - 8x / 9 = 40$$

$$\Rightarrow x = 3600$$

Amount of the bill - Rs. 3600.

32. BANKER'S DISCOUNT

Q1. A bill for Rs. 6000 is drawn on July 14 at 5 months. It is discounted on 5th October at 10%. Find the banker's discount, true discount, banker's gain and the money that the holder of the bill receives.

Sol. Face value of the bill = Rs. 6000.

Date on which the bill was drawn = July 14 at 5 months. Nominally due date = December 14.

Legally due date = December 17.

Date on which the bill was discounted = October 5.

Unexpired time : Oct. Nov. Dec.
 26 + 30 + 17 = 73 days = $1/5$ Years

B.D. = S.I. on Rs. 6000 for $1/5$ year
= Rs. $(6000 \times 10 \times 1/5 \times 1/100)$ = Rs. 120.

T.D. = Rs. $[(6000 \times 10 \times 1/5)/(100 + (10 \times 1/5))]$
= Rs. $(12000/102)$ = Rs. 117.64.

B.G. = (B.D.) - (T.D.) = Rs. $(120 - 117.64)$ = Rs. 2.36.

Money received by the holder of the bill = Rs. $(6000 - 120)$ = Rs. 5880.

Q2. If the true discount on a certain sum due 6 months hence at 15% is Rs. 120, what is the banker's discount on the same sum for the same time and at the same rate?

Sol. B.G. = S.I. on T.D.
= Rs. $(120 \times 15 \times 1/2 \times 1/100)$
= Rs. 9.

(B.D.) - (T.D.) = Rs. 9.

B.D. = Rs. $(120 + 9)$ = Rs. 129.

Q3. The banker's discount on Rs. 1800 at 12% per annum is equal to the true discount on Rs. 1872 for the same time at the same rate. Find the time.

Sol.

S.I. on Rs. 1800 = T.D. on Rs. 1872.

P.W. of Rs. 1872 is Rs. 1800.

Rs. 72 is S.I. on Rs. 1800 at 12%.

Time = $[(100 \times 72)/(12 \times 1800)]$ year

$1/3$ year = 4 months.

Q4. The banker's discount and the true discount on a sum of money due 8 months hence are Rs. 120 and Rs. 110 respectively. Find the sum and the rate percent.

Sol.

Sum = $[(B.D. \times T.D.)/(B.D. - T.D.)]$
= Rs. $[(120 \times 110)/(120 - 110)]$ = Rs. 1320.

Since B.D. is S.I. on sum due, so S.I. on Rs. 1320 for 8 months is Rs. 120.

Rate = $[(100 \times 120)/(1320 \times 2/3)]\%$ = $13 \frac{7}{11}\%$.

Q5. The present worth of a bill due sometime hence is Rs. 1100 and the true discount on the bill is Rs. 110. Find the banker's discount and the banker's gain.

Sol. $T.D. = \sqrt{P.W. \times B.G.}$

$B.G. = (T.D.)^2 / P.W.$

$= Rs. [(110 \times 110) / 1100]$

$= Rs. 11.$

$B.D. = (T.D. + B.G.) = Rs. (110 + 11) = Rs. 121.$

Q6. The banker's discount on Rs. 1650 due a certain time hence is Rs. 165. Find the true discount and the banker's gain.

Sol. $Sum = [(B.D. \times T.D.) / (B.D. - T.D.)]$

$= [(B.D. \times T.D.) / B.G.]$

$T.D. / B.G. = Sum / B.D.$

$= 1650 / 165$

$= 10 / 1$

Thus, if B.G. is Re 1, T.D. = Rs. 10.

If B.D. is Rs. 11, T.D. = Rs. 10.

If B.D. is Rs. 165, T.D. = Rs. $[(10/11) \times 165]$

$= Rs. 150$

And, B.G. = Rs. $(165 - 150) = Rs. 15.$

Q7. What rate percent does a man get for his money when in discounting a bill due 10 months hence, he deducts 10% of the amount of the bill?

Solution: Let amount of the bill = Rs. 100

Money deducted = Rs. 10

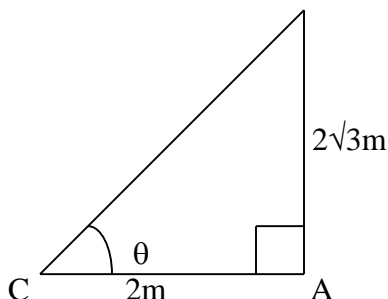
Money received by the holder of the bill = Rs. $100 - 10 = Rs. 90$

SI on Rs. 90 for 10 months = Rs. 10

Rate $= [(100 \times 10) / (90 \times 10 / 12)]\% = 13 \frac{1}{3}\%$

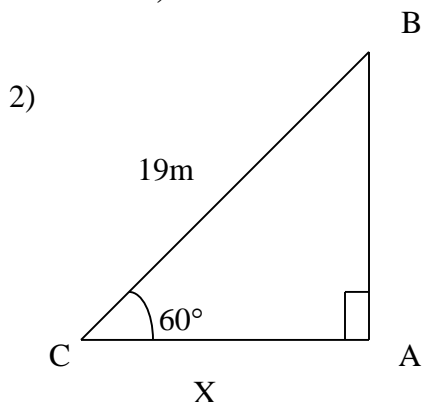
33. HEIGHTS AND DISTANCES

Q1. If the height of a pole is $2\sqrt{3}$ metres and the length of its shadow is 2 metres, find the angle of elevation of the sun.



Sol. Let AB be the pole and AC be its shadow.
Let angle of elevation, $\angle ACB = \theta$.
Then, $AB = 2\sqrt{3}$ m $AC = 2$ m.
 $\tan \theta = AB/AC = 2\sqrt{3}/2 = \sqrt{3} \Rightarrow \theta = 60^\circ$
So, the angle of elevation is 60°

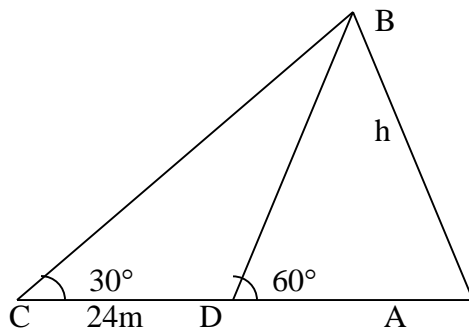
Q2. A ladder leaning against a wall makes an angle of 60° with the ground. If the length of the ladder is 19 m, find the distance of the foot of the ladder from the wall.



Sol. Let AB be the wall and BC be the ladder.
Then, $\angle ACB = 60^\circ$ and $BC = 19$ m.
Let $AC = x$ metres
 $AC/BC = \cos 60^\circ \Rightarrow x/19 = 1/2 \Rightarrow x = 19/2 = 9.5$

\therefore Distance of the foot of the ladder from the wall = 9.5 m

Q3. The angle of elevation of the top of a tower at a point on the ground is 30° . On walking 24 m towards the tower, the angle of elevation becomes 60° . Find the height of the tower.



Sol. Let AB be the tower and C and D be the points of observation. Then,

$$AB/AD = \tan 60^\circ = \sqrt{3} \quad \Rightarrow \quad AD = AB/\sqrt{3} = h/\sqrt{3}$$

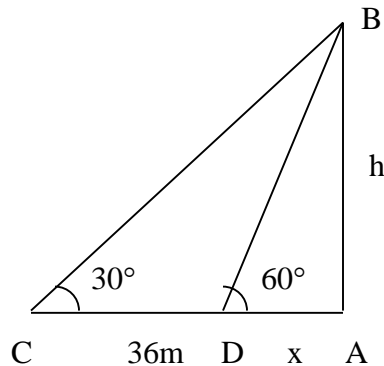
$$AB/AC = \tan 30^\circ = 1/\sqrt{3} \quad AC = AB \times \sqrt{3} = h\sqrt{3}$$

$$CD = (AC - AD) = (h\sqrt{3} - h/\sqrt{3})$$

$$h\sqrt{3} - h/\sqrt{3} = 24 \quad \Rightarrow \quad h = 12\sqrt{3} = (12 \times 1.73) = 20.76$$

Hence, the height of the tower is 20.76 m.

Q4. A man standing on the bank of a river observes that the angle subtended by a tree on the opposite bank is 60° . When he retires 36 m from the bank, he finds the angle to be 30° . Find the breadth of the river.



Sol. Let AB be the tree and AC be the river. Let C and D be the two positions of the man. Then, $\angle ACB = 60^\circ$, $\angle ADB = 30^\circ$ and $CD = 36$ m.

Let $AB = h$ metres and $AC = x$ metres.

Then, $AD = (36 + x)$ metres.

$$AB/AD = \tan 30^\circ = 1/\sqrt{3} \quad \Rightarrow \quad h/(36 + x) = 1/\sqrt{3} \quad \dots(1)$$

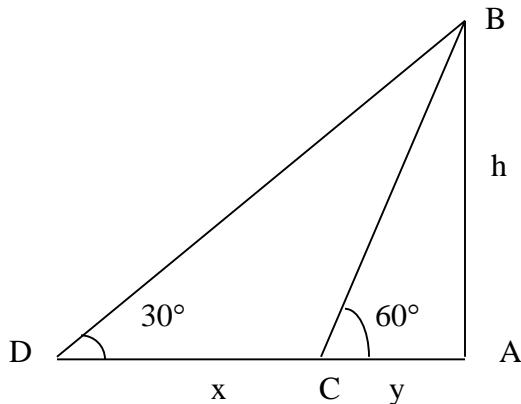
$$AB/AC = \tan 60^\circ = \sqrt{3} \quad \Rightarrow \quad h/x = \sqrt{3} \quad \dots(2)$$

From (i) and (ii), we get:

$$(36+x)/\sqrt{3} = \sqrt{3}x \quad \Rightarrow \quad x=18 \text{ m.}$$

So, the breadth of the river = 18 m.

Q5. A man on the top of a tower, standing on the seashore finds that a boat coming towards him takes 10 minutes for the angle of depression to change from 30° to 60° . Find the time taken by the boat to reach the shore from this position.



Sol. Let AB be the tower and C and D be the two positions of the boat.

Let $AB=h$, $CD=x$ and $AD=y$.

$$h/y = \tan 60^\circ = \sqrt{3} \quad \Rightarrow \quad y = h/\sqrt{3}$$

$$h/(x+y) = \tan 30^\circ = 1/\sqrt{3} \quad \Rightarrow \quad x+y = \sqrt{3}h$$

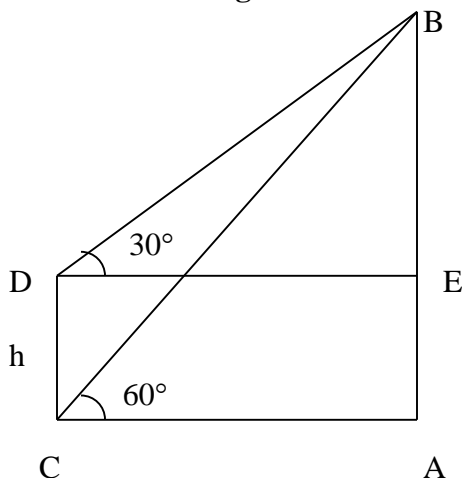
$$x = (x+y) - y = (\sqrt{3}h - h/\sqrt{3}) = 2h/\sqrt{3}$$

Now, $2h/\sqrt{3}$ is covered in 10 min.

$$h/\sqrt{3} \text{ will be covered in } (10 \times (\sqrt{3}/2h) \times (h/\sqrt{3})) = 5 \text{ min}$$

Hence, required time = 5 minutes.

Q6. There are two temples, one on each bank of a river, just opposite to each other. One temple is 54 m high. From the top of this temple, the angles of depression of the top and the foot of the other temple are 30° and 60° respectively. Find the width of the river and the height of the other temple.



Sol. Let AB and CD be the two temples and AC be the river.
Then, AB = 54 m.
Let AC = x metres and CD=h metres.
 $\angle ACB=60^\circ$, $\angle EDB=30^\circ$
 $AB/AC=\tan 60^\circ=\sqrt{3}$
 $AC=AB/\sqrt{3}=54/\sqrt{3}=(54/\sqrt{3}\times\sqrt{3}/\sqrt{3})=18\text{m}$
 $DE=AC=18\sqrt{3}$
 $BE/DE=\tan 30^\circ=1/\sqrt{3}$
 $BE=(18\sqrt{3}\times 1/\sqrt{3})=18\text{ m}$
 $CD=AE=AB-BE=(54-18)\text{ m} = 36\text{ m}.$
So, Width of the river = AC = $18\sqrt{3}\text{ m}=18\times 1.73\text{ m}=31.14\text{m}$
Height of the other temple = CD= 18 m.

34. TABULATION

Q1. The following table gives the sales of batteries manufactured by a company lit the years. Study the table and answer the questions that follow:

NUMBER OF DIFFERENT TYPES OF BATTERIES SOLD BY A COMPANY OVER THE YEARS (NUMBERS N THOUSANDS)

TYPES OF BATTERIES

Year	4AH	7AH	32AH	35AH	55AH	TOTAL
1992	75	144	114	102	108	543
1993	90	126	102	84	426	528
1994	96	114	75	105	135	525
1995	105	90	150	90	75	510
1996	90	75	135	75	90	465
1997	105	60	165	45	120	495
1998	115	85	160	100	145	605

1. The total sales of all the seven years is the maximum for which battery?
(a) 4AH (b) 7AH (c) 32AH (d) 35AH (e) 55AH
2. What is the difference in the number of 35AH batteries sold in 1993 and 1997?
(a) 24000 (b) 28000 (c) 35000 (d) 39000 (e) 42000
3. The percentage of 4AH batteries sold to the total number of batteries sold was maximum in the year:
(a) 1994 . (b) 1995 (c) 1996 (d) 1997 (e) 1998
4. In the case of which battery there was a continuous decrease in sales from 1992 to 1997?
(8) 4AH (b) 7 AH (c) 32AH (d) 35AH (e) 55AH
5. What was the approximate percentage increase in the sales of 55AH batteries in 1998 compared to that in 1992?
(a) 28% (b) 31% (c) 33% (d)34% (e)37%

Sol. 1. (c) : The total sales (in thousands) of all the seven years for various batteries are:

For 4AH = $75 + 90 + 96 + 105 + 90 + 105 + 115 = 676$

For 7AH = $144 + 126 + 114 + 90 + 75 + 60 + 85 = 694$

For 32AH = $114 + 102 + 75 + 150 + 135 + 165 + 160 = 901$

For 35 AH= $102 + 84 + 105 + 90 + 75 + 45 + 100 = 601$

For 55 AH= $108 + 126 + 135 + 75 + 90 + 120 + 145 = 799$.

Clearly, sales are maximum in case of 32AH batteries.

2. (d) : Required difference = $[(84 - 45) \times 1000] = 39000$.

3. (d) : The percentages of sales of 4AH batteries to the total sales in different years are:

For 1992 = $(75 \times 100 / 543)\% = 13.81\%$

For 1993 = $(90 \times 100) / 528\% = 17.05\%$

For 1994 = $(96 \times 100 / 465)\% = 19.35\%$

For 1995 = $(105 \times 100 / 495)\% = 20.59\%$

For 1996 = $(96 \times 100 / 465)\% = 19.35\%$

For 1997 = $(105 \times 100 / 495)\% = 21.21\%$

For 1998 = $(115 \times 100 / 605)\% = 19.01\%$

Clearly, the percentage is maximum in 1997.

4. (b): From the table it is clear that the sales of 7AH batteries have been decreasing continuously from 1992 to 1997.

5. (d): Required Percentage = $(145 - 108) / 108 \times 100 \% = 34.26\% = 34\%$.

Q 2: Study the following table carefully and answer these questions:

NUMBER OF CANDIDATES APPEARED AND QUALIFIED IN A COMPETITIVE EXAMINATION FROM DIFFERENT STATES OVER THE YEAR

	1997		1998		1999		2000		2001	
	App.	Qual.	App.	Qual.	App.	Qual.	App.	Qual.	App.	Qual.
M	5200	720	8500	980	7400	850	6800	775	9500	1125
N	7500	840	9200	1050	8450	920	9200	980	8800	1020
P	6400	780	8800	1020	7800	890	8750	1010	9750	1250
Q	8100	950	9500	1240	8700	980	9700	1200	8950	995
R	7800	870	7600	940	9800	1350	7600	945	7990	885

1. Combining the states P and Q, together in 1998, what is the percentage of the candidates qualified to that of the candidates appeared?

(a) 10.87% (b) 11.49% (c) 12.35% (d) 12.54% (e) 13.50%

2. The percentage of the total number of qualified candidates to the total number appeared candidates among all the five states in 1999 is :

(a) 11.49% (b) 11.84% (c) 12.21% (d) 12.57% (e) 12.7a1

3. What is the percentage of candidates qualified from State N for all the years together, over the candidates appeared from State N during all the years together?

(a) 12.36% (b) 12.16% (c) 11.47% (d) 11.15% (e) None of these

4. What is the average of candidates who appeared from State Q during the given years?

- (a) 8700 (b) 8760 (c) 8810 (d) 8920 (e) 8990

5. In which of the given years the number of candidates appeared from State P has maximum percentage of qualified candidates?

- (a) 1997 (b) 1998 (c) 1999 (d) 2000 (e) 2001

6. Total number of candidates qualified from all the states together in 1997 is approximately what percentage of the total number of candidates qualified from all the states together in 1998?

- (a) 72% (b) 77% (c) 80% (d) 83% (e) 86%

Sol.

$$1.(c) \text{ Required Percentage} = \frac{(1020+1240)}{(8800+9500)} * 100\% = \frac{(2260*100)}{18300}\% = 12.35\%$$

$$2. (b) \text{ Required Percentage} = \frac{(850+920+890+980+1350)}{(7400+8450+7800+8700+9800)} * 100\% \\ = \frac{(4990*100)}{42150}\% = 11.84\%$$

$$3. (d) \text{ Required Percentage} = \frac{(84-1050+920+980+1020)}{(7500+9200+8450+9200+8800)} * 100\% \\ = \frac{(4810*100)}{43150}\% = 11.15\%$$

$$4. (e) \text{ Required average} = \frac{(8100+9500+8700+9700+8950)}{5} \\ = 44950/5 \\ = 8990$$

5. (e) The percentages of candidates qualified to candidates appeared from State P during different years are:

$$\text{For 1997} = \frac{780}{6400} * 100\% = 12.19\%$$

$$\text{for 1998} = \frac{1020*100}{8800}\% = 11.59\%$$

$$\text{For 1999} = \frac{890*100}{7800}\% = 11.41\%;$$

$$\text{For 2000} = \frac{1010*100}{8750}\% = 11.54\%.$$

$$\text{For 2001} = \frac{1250*100}{9750}\% = 12.82\%$$

∴ Maximum percentage is for the year 2001.

$$6. (c) \text{ Required Percentage} = \frac{(720 + 840 + 780 + 950 + 870)}{980+1050+1020+1240+940} * 100 \\ = 80\%$$

Q3. The following table gives the percentage of marks obtained by seven students in six, different subjects in an examination. Study the table and answer the questions based on it. The numbers in

the brackets give the maximum marks in each subject.

(Max. marks) Student	Maths (160)	Chemistry (130)	Physics (120)	Geography (100)	History (60)	Computer Science (40)
Ayush	90	50	90	60	70	80
Aman	100	80	80	40	80	70
Sajal	90	60	70	70	90	70
Rohit	80	65	80	80	60	60
Muskan	80	65	85	95	50	90
Tanvi	70	75	65	85	40	60
Tharun	65	35	50	77	80	80

- What was the aggregate of marks obtained by Sajal in all the six subjects?
(a) 409 (b) 419 (c) 429 (d) 439 (e) 449
- What is the overall percentage of Thrun?
(a) 52.5% (b) 55% (c) 60% (d) 63% (e) 64.5%
- What are the average marks obtained by all the seven students in Physics? (rounded off to two digits after decimal)
(a) 77.26 (b) 89.14 (c) 91.37 (d) 96.11 (e) 103.21
- The number of students who obtained 60% and above marks in all the subjects is :
(a) 1 (b) 2 (c) 3 (d) None (e) None of these
- In which subject is the overall percentage the best?
(a) History (b) Maths (c) Physics (d) Chemistry (e) Geography

Sol.

1(e) : Aggregate marks obtained by Sajal

$$= [(90\% \text{ of } 150) + (60\% \text{ of } 130) + (70\% \text{ of } 120) + (70\% \text{ of } 100) + (90\% \text{ of } 60) + (70\% \text{ of } 40)] = 135 + 78 + 84 + 70 + 54 + 28 = 449.$$

2. (c) : Aggregate marks obtained by Tarun

$$= [(65\% \text{ of } 150) + (35\% \text{ of } 130) + (50\% \text{ of } 120) + (77\% \text{ of } 100) + (80\% \text{ of } 60) + (80\% \text{ of } 40)] = 97.5 + 45.5 + 60 + 77 + 48 + 32 = 360.$$

Total maximum marks (of all the six subjects)

$$= (150 + 130 + 120 + 100 + 60 + 40) = 600.$$

$$\text{Overall percentage of Tarun} = \frac{360 \times 100}{600} \% = 60\%.$$

3. (b) : Average marks obtained in Physics by all the seven students

$$= \frac{1}{7} [(90\% \text{ of } 120) + (80\% \text{ of } 120) + (70\% \text{ of } 120) + (80\% \text{ of } 120) + (85\% \text{ of } 120) + (65\% \text{ of } 120) + (50\% \text{ of } 120)]$$

$$= \frac{1}{7} [(90 + 80 + 70 + 80 + 85 + 65 + 50)\% \text{ of } 120]$$

$$= \frac{1}{7} [520\% \text{ of } 120] = 89.14\%.$$

4. (b) : From the table it is clear that Sajal and Rohit have 60% or more marks in each of the six subjects.

5. (b) : We shall find the overall percentage (for all the seven students) with respect to each subject. The overall percentage for any subject is equal to the average of percentages obtained by all the seven students since the maximum marks for any subject is the same for all the students.

Therefore, overall percentage for:

$$\begin{aligned} \text{(i) Maths} &= \left[\frac{1}{7} (90 + 100 + 90 + 80 + 80 + 70 + 65) \right] \% \\ &= \left[\frac{1}{7} (575) \right] \% = 82.14\%. \end{aligned}$$

$$\begin{aligned} \text{(ii) Chemistry} &= \left[\frac{1}{7} (50 + 80 + 60 + 65 + 65 + 75 + 35) \right] \% \\ &= \left[\frac{1}{7} (430) \right] \% = 61.43\%. \end{aligned}$$

$$\begin{aligned} \text{(iii) Physics} &= \left[\frac{1}{7} (90 + 80 + 70 + 80 + 85 + 65 + 50) \right] \% \\ &= \left[\frac{1}{7} (520) \right] \% = 74.29\%. \end{aligned}$$

$$\begin{aligned} \text{(iv) Geography} &= \left[\frac{1}{7} (60 + 40 + 70 + 80 + 95 + 85 + 77) \right] \% \\ &= \left[\frac{1}{7} (507) \right] \% = 72.43\%. \end{aligned}$$

$$\begin{aligned} \text{(v) History} &= \left[\frac{1}{7} (70 + 80 + 90 + 60 + 50 + 40 + 80) \right] \% \\ &= \left[\frac{1}{7} (470) \right] \% = 67.14\%. \end{aligned}$$

$$\begin{aligned} \text{(vi) Computer Science} &= \left[\frac{1}{7} (80 + 70 + 70 + 60 + 90 + 60 + 80) \right] \% \\ &= \left[\frac{1}{7} (510) \right] \% = 72.86\%. \end{aligned}$$

Clearly this Percentage is highest for Maths.

Q4. Study the following table carefully and answer the questions given below:

CLASSIFICATION OF 100 STUDENTS BASED ON THE MARKS OBTAINED BY THEM IN PHYSICS AND CHEMISTRY IN AN EXAMINATION

Marks out Of 50 Subject	40 and above	30 and Above	20 and above	10 and above	0 and above
physics	9	32	80	92	100
chemistry	4	21	66	81	100
(aggregate Average)	7	27	73	87	100

- The number of students scoring less than 40% marks in aggregate is :
(a) 13 (b) 19 (c) 20 (d) 27 (e) 34
- If at least 60% marks in Physics are required for pursuing higher studies in Physics, how many students will be eligible to pursue higher studies in Physics?
(a) 27 (b) 32 (c) 34 (d) 41 (e) 68
- What is the difference between the number of students passed with 30 as cut-off marks in Chemistry and those passed with: JUAs cut-off marks in aggregate?
(a) 3 (b) 4 (c) 5 (d) 6 (e) 7
- The percentage of the number of students getting at least 60% marks in Chemistry over those getting at least 40% marks in aggregate, is approximately:
(a) 21% (b) 27% (c) 29% (d) 31% (e) 34%
- If it is known that at least 23 students were eligible for a Symposium on Chemistry the minimum qualifying marks in Chemistry for eligibility to Symposium would lie in the range:
(a) 40-50 (b) 30-40 (c) 20-30 (d) Below 20

Sol.

1. (d) : We have $40\% \text{ of } 50 = \frac{(40 \times 50)}{100} = 20$.

$$\begin{aligned} \therefore \text{Required number} &= \text{Number of students scoring less than 20 marks} \\ &\quad \text{in aggregate} \\ &= 100 - \text{number of students scoring 20 and above} \\ &\quad \text{marks in aggregate} = 100 - 73 = 27. \end{aligned}$$

2. (b) : We have $60\% \text{ of } 50 = \frac{(60 \times 50)}{100} = 30$.

$$\therefore \text{Required number} = \text{Number of students scoring 30 and above mark in Physics} = 32.$$

3. (d) : Required difference = (Number of students scoring 30 and above in mark in Chemistry) (Number of students scoring 30 and above marks in aggregate) = $27 - 21 = 6$.

4. (c) : Number of students getting at least 60% marks in Chemistry = Number of students getting 30 and above marks in Chemistry = 21. Number of students getting at least 40% marks in aggregate

= Number of students getting 20 and above marks in aggregate = 73.

\therefore Required Percentage = $\frac{21}{73} \times 100\% = 28.77\% \approx 29\%$.

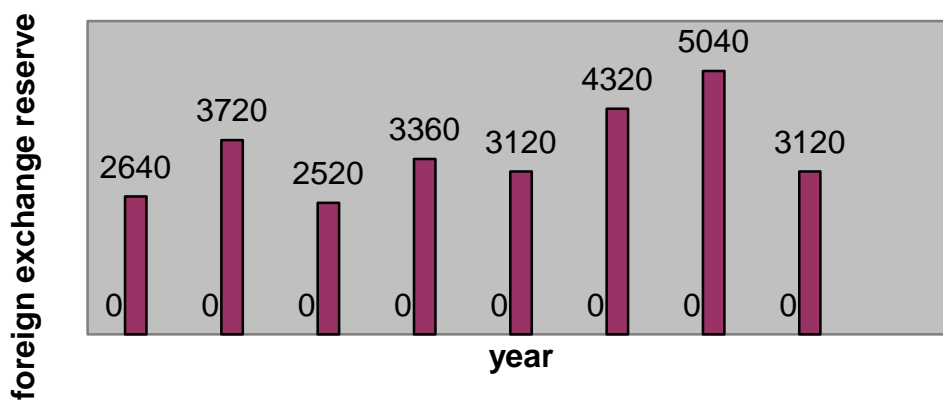
6. (c): Since 66 students get 20 and above marks in Chemistry and out of these 21 students get 30 and above marks, therefore to select top 35 students in Chemistry, the qualifying marks should lie in the range 20-30.

35. BAR GRAPHS

This section comprises of questions in which the data collected in a particular discipline are represented in the form of vertical or horizontal bars drawn by selecting a particular scale. one of the parameters is plotted on the horizontal axis and the other on the vertical axis. The candidate is required to understand the given information and there after answer the given questions on the basis of data analysis.

1. The bar graph given below shows the foreign exchange reserves of a country (in million us\$) from 1991-92 to 1998-99. answer the questions based on this graph.

FOREIGN EXCHANGE RESERVES OF A COUNTRY (IN MILLION US \$)



- The foreign exchange reserves in 1997-98 was how many times that in 1994-95?
(a) 0.7 (b) 1.2 (c) 1.4 (d) 1.5 (e) 1.8
- What was the percentage increase in the foreign exchange reserves in 1997-98 over 1993-94?
(a) 100 (b) 150 (c) 200 (d) 620 (e) 2520
- For which year, the percent increase of foreign exchange reserves over the previous year, is the highest?
(a) 1992-93 (b) 1993-94 (c) 1994-95 (d) 1996-97 (e) 1997-98
- The foreign exchange reserves in 1996-97 were approximately what percent of the average foreign exchange reserves over the period under review?

- (a)95% (b)110% (c)115% (d)125% (e)140%

5. The ratio of the number of years, in which the foreign exchange reserves are above the average reserves, to those in which the reserves are below the average reserves is :

- (a)2:6 (b)3:4 (c)3:5 (d)4:4 (e)5:3

Solutions

1 (d): required ratio = $5040/3360 = 1.5$

2 (a): foreign exchange reserve in 1997-98 = 5040 million us \$

Foreign exchange reserves in 1993-94 = 2520 million us\$

Therefore increase = $(5040 - 2520) = 2520$ million us \$

Therefore percentage increase = $((2520/2520) * 100)\% = 100\%$

3(a): There is an increase in foreign exchange reserves during the years 1992- 93, 1994-95, 1996-97, 1997-98 as compared to previous year (as shown by bar graph)

The percentage increase in reserves during these years compared to previous year are

- (1) for 1992-93 = $[(3720 - 2640)/2640 * 100]\% = 40.91\%$
- (2) for 1994-95 = $[(3360 - 2520)/2520 * 100]\% = 33.33\%$
- (3) for 1996-97 = $[(4320 - 3120)/3120 * 100]\% = 38.46\%$
- (4) for 1997-98 = $[(5040 - 4320)/4320 * 100]\% = 16.67\%$

Clearly, the percentage increase over previous year is highest for 1992-93.

4. (d) : Average foreign exchange reserves over the given period

= $\frac{1}{8} \times (2640 + 3720 + 2520 + 3360 + 3120 + 4320 + 5040 + 3120)$] million US \$

= 3480 million US \$.

Foreign exchange reserves in 1996-97 = 4320 million US \$. . .

Required Percentage = $\frac{4320}{3480} \times 100\% = 124.14\% \approx 125\%$.

3480 .

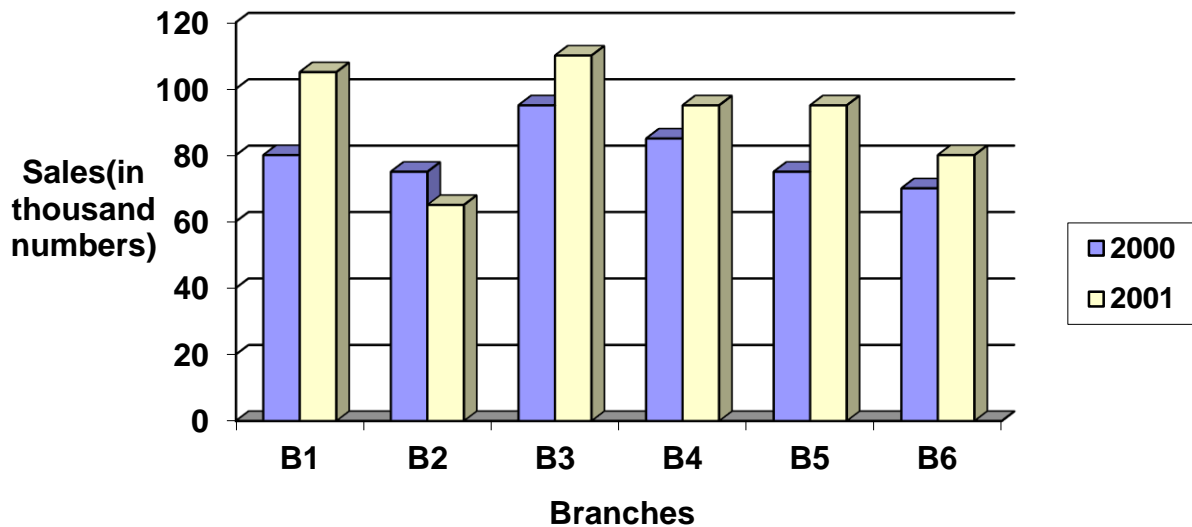
5. (c) : Average foreign exchange reserves over the given period = 3480 million US \$.

The country had reserves above 3480 million US \$ during the years 1992-93, 1996-97 and 1997-98 i.e., for 3 years and below 3480 million US \$ during the years 1991-92, 1993-94, 1994-95, 1995-96 and 1998-99 i.e., for 5 years.

Hence, required ratio = 3 : 5.

Q2. The bar-graph provided on next page gives the sales of books (in thousand numbers) from six branches of a publishing company during two consecutive years 2000 and 2001. Answer the questions based on this bar-graph:

Sales of books (in thousand numbers) from six branches-B1,B2,B3,B4,B5 and B6 of a publishing company in 2000 and 2001



1. Total sales of branches b1, b3 and b5 together for both the years (in thousand numbers) is:

- (a) 250 (b) 310 (c) 435 (d) 560 (e) 585

2. Total sales of branch b6 for both the years is what percent of the total sales of branch b3 for both the years?

- (a) 68.54% (b) 71.11% (c) 73.17% (d) 75.55% (e) 77.26%

3. What is the average sale of all the branches (in thousand numbers) for the year 2000?

- (a) 73 (b) 80 (c) 83 (d) 88 (e) 96

4. What is the ratio of the total sales of branch b2 for both years to the total sales of branch b4 for both years?

- (a) 2:3 (b) 3:5 (c) 4:5 (d) 5:7 (e) 7:9

5. What percent of the average sales of branches b1,b2 and b3 in 2001 is the average sales of branches b1,b3 and b6 in 2000?

- (a) 75% (b) 77.5% (c) 82.5% (d) 85% (e) 87.5%

SOLUTION

1.(d) total sales of branches B1,B3 and B5 for both the years (in thousand numbers) $= (80+105) + (95+110) + (75+95) = 560$

2(c) required percentage $= [(70+80)/(95+110) * 100]\% = (150/205 * 100)\% = 73.17\%$

3(b) average sales of all the six branches (in thousand numbers) for the year 2000 $= 1/6 * (80+75+95+85+75+70) = 80$

4(e) required ratio $= (75+65)/(85+95) = 140/180 = 7/9$

5(e) average sales (in thousand numbers of branches B1, B3, and B6 in

$$2000 = \frac{1}{3} \times (80 + 95 + 70) = \frac{245}{3}$$

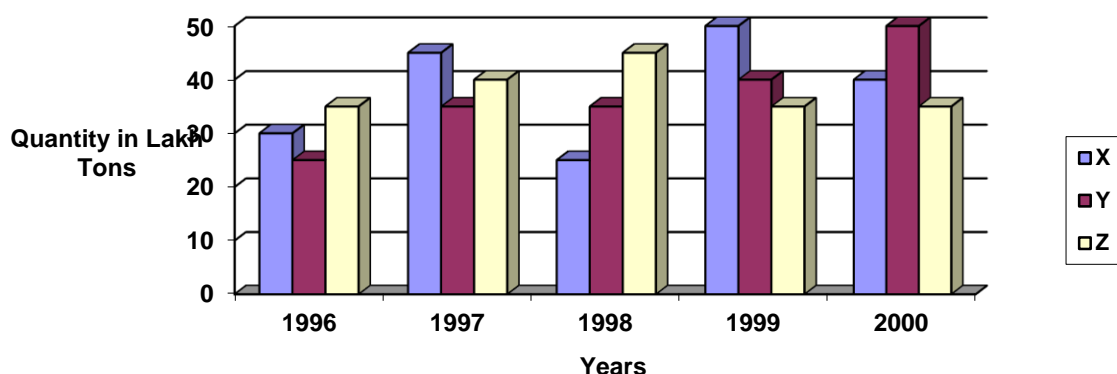
average sales (in thousand numbers of branches B1, B2, and B3 in

$$2001 = \frac{1}{3} \times (105 + 65 + 110) = \frac{280}{3}$$

$$\text{therefore required percentage} = \left[\left(\frac{245/3}{280/3} \right) \times 100 \right] \% = \left(\frac{245}{280} \times 100 \right) \% = 87.5\%$$

Q3. The bar graph provided below gives the data of the production of paper (in thousand tonnes) by three different companies x, y and z over the years. study the graph and answer the questions that follow

production of paper (in lakhs tonnes) by three companies x, y and z over the years



1. What is the difference between the production of the company Z in 1998 and company y in 1996?

- a. 2,00,000 tons
- b. 20,00,000 tons
- c. 20,000 tons
- d. 2,00,00,000 tons
- e. none of these

2. What is the ratio of the average production of company x in the period 1998 to 2000 to the average production of company y in the same period?

- a. 1:1
- b. 15:27
- c. 23:25
- d. 27:29
- e. none of these

3. What is the percentage increase in the production of company y from 1996 to 1999?

- a. 30%
- b. 45%
- c. 50%
- d. 60%
- e. 75%

4. The average production of five years was maximum for which company?

- a.x
- b.y
- c.z
- d.x & y both
- e.x and z both

5. For which of the following years the percentage rise / fall in production from previous year is the maximum for company y?

- a.1997
- b.1998
- c.1999
- d.2000
- e.1997 & 2000

6. In which year was the percentage of production of company z to the production of company y the maximum?

- a.1996
- b.1997
- c.1998
- d.1999
- e.2000

Sol: 1(b): Required difference = $[(45-25)*1,00,000]$ tons=20,00,000 tons

2(c): average production of company x in the period 1998-2000= $[1/3*(25+50+40)]=(115/3)$ lakh tons
average production of company y in the period 1998-2000
 $[1/3*(35+40+50)]=(125/3)$ lakh tons

Therefore req ratio= $(115/3)/(125/3)=115/125=23/25$

3(d): Percentage increase in the production y from 1996-1999= $[(40-25)/25*100]\%=(15/25*100)\%=60\%$

4(e): Average production (in lakh tons)in five years for the three company's are:

for company x= $[1/5*(30+45+25+50+40)]=190/5=38$

for company y= $[1/5*(25+35+35+40+50)]=185/5=37$

for company z= $[1/5*(35+40+45+35+35)]=190/5=38$

therefore the average production of maximum for both the company's x and z

5(a) : Percentage change (rise/fall)in the production of Company Y in comparison to the previous year, for different years are:

For 1997 = $[((32-25)/25)*100]\% = 40\%$

For 1998 = $[((35-35)/25)*100]\% = 0\%$

For 1999 = $[((40-35)/35)*100]\% = 14.29\%$

For 2000 = $[((50-40)/40)*100]\% = 25\%$

Hence, the maximum percentage rise/fall in the production of company Y is for 1997.

6(a) : The percentages of production of company z to the production of company z for various years are:

For 1996 = $((35/25)*100)\% = 140\%$; For 1997 = $((40/35)*100)\% = 114.29\%$

For 1998 = $((45/35)*100)\% = 128.57\%$; For 1999 = $((35/40)*100)\% = 87.5\%$

For 2000 = $((35/50)*100)\% = 70\%$

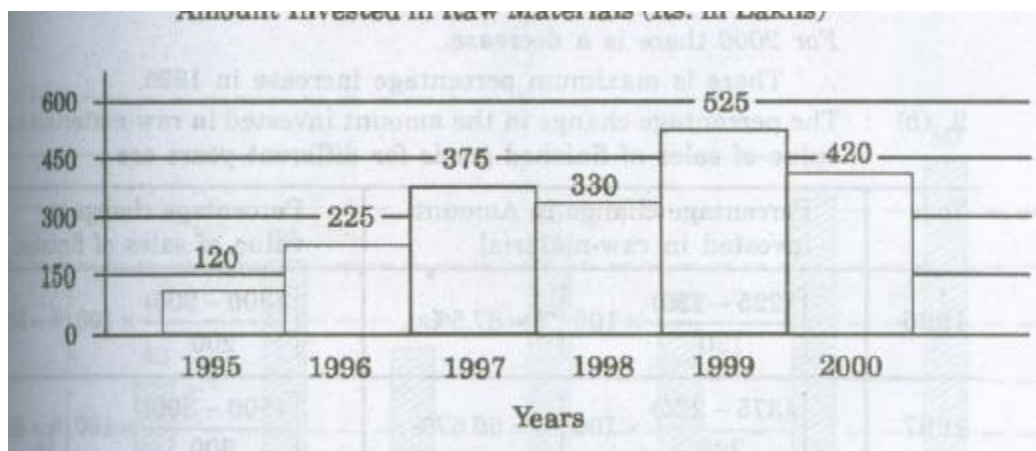
Clearly, this percentage is highest for 1996.

Q4. Out of the two bar graphs provided below, one shows the amounts (in Lakh Rs) invested by a Company in purchasing raw materials over the years and the other shows the values (in Lakh Rs.) of finished goods sold by the Company over the years.

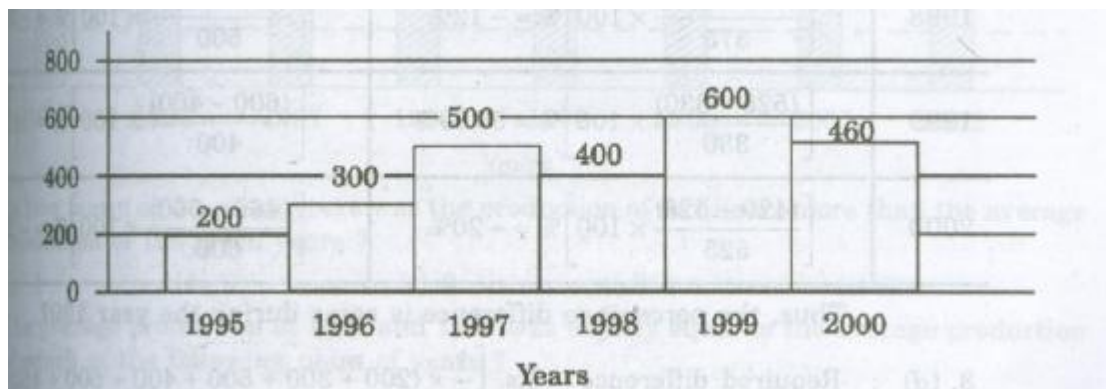
Study the two bar graphs and answer the questions based on them.

Amount Invested in Raw Materials and the Value of Sales of Finished Goods for a Company over the Years

Amount Invested in Raw Materials (Rs. in Lakhs)



Value of Sales of Finished Goods (Rs. in Lakhs)



1. In which year, there has been a maximum percentage increase in the amount invested in Raw Materials as compared to the previous year?

(a) 1996 (b) 1997 (c) 1998 (d) 1999 (e) 2000

2. In which year, the percentage change (compared to the previous year) in the investment on Raw Materials is the same as that in the value of sales of finished goods?

(a) 1996 (b) 1997 (c) 1998 (d) 1999 (e) 2000

3. What was the difference between the average amount invested in Raw Materials during the given period and the average value of sales of finished goods during this period?

(a) Rs. 62.5 lakhs (b) Rs. 68.5 lakhs (c) Rs. 71.5 lakhs
(d) Rs. 77.5 lakhs (e) Rs. 83.5 lakhs

4. The value of sales of finished goods in 1999 was approximately what percent of the average amount invested in Raw Materials in the years 1997, 1998 and 1999?

(a) 33% (b) 37% (c) 45% (d) 49% (e) 53%

5. The maximum difference between the amount invested in Raw Materials and the value of sales of finished goods was during the year:

(a) 1995 (b) 1996 (c) 1997 (d) 1998 (e) 1999

Sol. 1. (a) : The percentage increase in the amount invested in raw-materials as compared to the previous year, for different years are:

For 1996 = $[(225-120)/120] \times 100\% = 87.5\%$

For 1997 = $[(375-225)/225] \times 100\% = 66.67\%$

For 1998 = $[(525-330)/330] \times 100\% = 59.09\%$

For 2000 there is a decrease.

2. (b) The percentage change in the amount invested in raw-materials and in the value of sales of finished goods for different years are:

year	Percentage change in amount invested in raw-materials	Percentage change in value of sales of finished goods
1996	$[(225-120)/120] \times 100\% = 87.5\%$	$[(300-200)/200] \times 100\% = 50\%$
1997	$[(375-225)/225] \times 100\% = 66.7\%$	$[(500-300)/300] \times 100\% = 66.67\%$
1998	$[(525-330)/330] \times 100\% = -12\%$	$[(400-500)/500] \times 100\% = -20\%$
1999	$[(525-330)/330] \times 100\% = 59.09\%$	$[(600-400)/400] \times 100\% = 50\%$
2000	$[(420-525)/525] \times 100\% = -20\%$	$[(460-600)/600] \times 100\% = -23.33\%$

Thus the percentage difference is same during the year 1997.

3. (d) : Required difference = Rs. $[(1/6) \times (200+300+500+400+600+460) - (1/6) \times (120+225+375+330+525+420)]$ lakhs

= Rs. $[(2460/6) - (1995/6)]$ lakhs = Rs. (410-332.5) lakhs = 77.5 lakhs.

4. (d) : Required percentage = $[(600/(375+300+525)) \times 100]\% = 48.78\% \approx 49\%$

5. (c) : The difference between the amount invested in raw-material and the value of sales of finished goods for various years are :

For 1995 = Rs. (200-120) lakhs = Rs. 80 lakhs

For 1996 = Rs.(200-225)lakhs = Rs. 75 lakhs

For 1997 = Rs. (500-375)lakhs = Rs. 125 lakhs

For 1998 = Rs. (400-330)lakhs = Rs. 70 lakhs.

For 1999 = Rs. (600-525)lakhs = Rs. 75 lakhs

For 2000 = Rs. (460-420)lakhs = Rs. 40 lakhs.

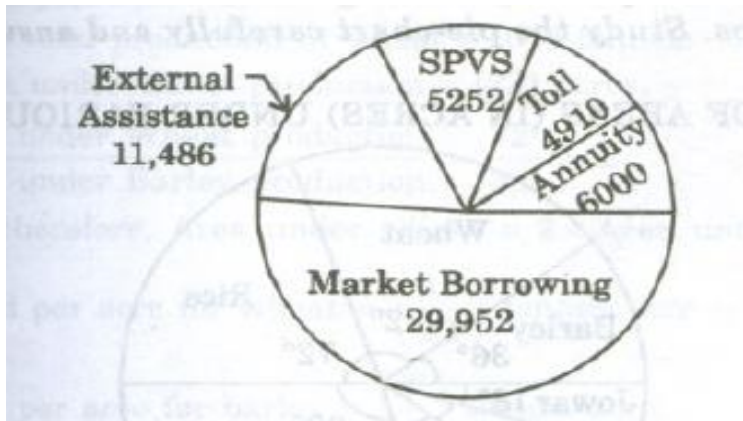
Clearly, maximum difference was during 1997

36. PIE-CHARTS

The procedure of solving problems based on pie-charts will be clear from the following solved examples.

Question. The following pie-chart shows the sources of funds to be collected by the National Highways Authority of India (NHAI) for its Phase II projects. Study the pie-chart and answer the questions that follow.

**SOURCES OF FUNDS TO BE ARRANGED BY NHAI
FOR PHASE II PROJECTS (IN CRORES RS.)**



Total funds to be arranged for Projects (Phase II) = Rs. 57,600 crores.

1. Near about 20% of the funds are to be arranged through:

- (a) SPVS
- (b) External Assistance
- (c) Annuity
- (d) Market Borrowing

2. The central angle corresponding to Market Borrowing is:

- (a) 52°
- (b) 137.8°
- (c) 187.2°
- (d) 192.4°

3. The approximate ratio of the funds to be arranged through Toll and that through Market Borrowing is:

- (a) 2:9
- (b) 1:6
- (c) 3:11
- (d) 2:5

4. If NHAI could receive a total of Rs. 9695 crores as External Assistance, by what percent (approximately) should it increase the Market Borrowings to arrange for the shortage of funds ?

- (a) 4.5% (b) 7.5%
(c) 6% (d) 8%

5.If the toll is to be collected through an outsourced agency by allowing a maximum 10% commission, how much amount should be permitted to be collected by the outsourced agency, so that the project is supported with Rs. 4910 crores ?

- (a) Rs.6213 crores (b) Rs. 5827 crores
(c) Rs. 5401 crores (d) Rs. 5216 crores

SOLUTION

1. (b): 20% of the total funds to be arranged = Rs.(20% of 57600) crores
= Rs.11520 crores Rs.11486 crores.

2. (c):Central angle corresponding to Market Borrowing = $\frac{29952}{57600} \times 360^\circ$
= 187.2°

3. (b): $\frac{4910}{29952} = \frac{1}{6.1} = \frac{1}{6}$
Required ratio = $\frac{4910}{29952} = \frac{1}{6.1} = \frac{1}{6}$

4. (c):Shortage of funds arranged through External Assistance
=Rs.(11486-9695) crores =Rs. 1791 crores.
therefore, Increase required in Market Borrowings =Rs. 1791 crores.

Percentage increase required = $\left| \frac{1791}{29952} \times 100 \right| \% = 5.98 \% = 6\%$

5. (c):Amount permitted = (Funds required from Toll for projects of Phase II) +
(10 % of these funds)
=Rs. 4910 crores + Rs. (10% of 4910) crores
=Rs. (4910 + 491) crores = Rs. 5401 crores.

Question. The pie-chart provided below gives the distribution of land (in a village) under various food crops. Study the pie-chart carefully and answer the questions that follow.

DISTRIBUTION OF AREAS (IN ACRES) UNDER VARIOUS FOOD CROPS

- 2.(a): The area under any of the food crops is proportional to the angle corresponding to that crop.

Let the area under the rice production be x million acres.

$$\text{Then, } 18:72 = 1.5:x \Rightarrow x = (72 \cdot 15 / 18) = 6$$

Thus, the area under rice production be = 6 million acres.

3.(b): Let the total production of barley be T tones and let Z acres of land be put under barley production.

Then, the total production of wheat =(6T) tones.

Also, area under wheat production = (2Z) acres.

$$\therefore \frac{\text{Area Under Wheat Production } 72^\circ}{\text{Area Under Barley Production } 36^\circ} = \frac{\quad}{\quad} = 2$$

And therefore, Area under wheat = 2 * Area under Barley = (2Z) acres

Now, yield per acre for wheat = (6T/2Z) tones/acre = (3T/Z) tones/acre

And yield per acre for barley = (T/Z) tones/acre $3T/Z$

\therefore Required ratio = ----- = 3:1. T/Z

4. (b): Let Z acres of land be put under barley production.

$$\text{Then, } \frac{\text{Area Under Rice Production } 72^\circ}{\text{Area Under Barley Production } 36^\circ} = \frac{\quad}{\quad} = 2.$$

\therefore Area under rice production = 2 * area under barley production = (2Z) acres.

Now, if p tones be the yield per acre of barley then, yield per acre of rice

$$= (p + 50\% \text{ of } p) \text{ tones} = (3/2 p) \text{ tones.}$$

\therefore Total production of rice = (yield per acre) * (area under production)

$$= (3/2 p) * 2Z = (3pZ) \text{ tones.}$$

And, Total production of barley = (pz) tones.

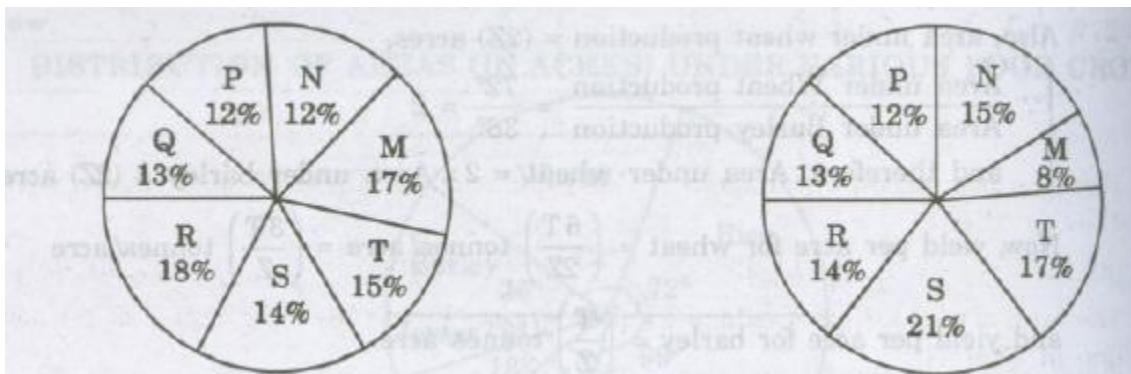
\therefore Percentage production of barley to that rice = $(pZ/3pZ * 100)\% = 33 \frac{1}{3}\%$.

Question. The following pie-charts show the distribution of students of graduate and post graduate levels in seven different institute-M,N,P,Q,R,S and T in a town.

DISTRIBUTION OF STUDENTS AT GRADUATE AND POST-GRADUATE LEVELS IN SEVEN INSTITUTES-M,N,P,Q,R,S AND T.

**Total Number of students of
graduate level**

**Total Number of students of
post graduate level**



- How many students of institutes M and S are studying at graduate level?
(a) 7516 (b) 8463 (c) 9127 (d) 9404
- Total number of students studying at post -graduate level from institutes N and P is:
(a) 5601 (b) 5944 (c) 6669 (d) 7004
- What is the total number of graduate and post-graduate level students in institute R?
(a) 8320 (b) 7916 (c) 9116 (d) 8372
- What is the ratio between the number of students studying at post graduate and graduate levels respectively from institute S?
(a) 14:19 (b) 19:21 (c) 17:21 (d) 19:14
- What is the ratio between the number of students studying post graduate level from institute S and the number of students studying at graduate level from institute Q?
(a) 13:19 (b) 21:13 (c) 13:8 (d) 19:13

SOLUTION

- (b): Students of institute M at graduate level = 17% of 27300 = 4641.
Students of institute S at graduate level = 14% of 27300 = 3822
 \therefore Total number students at graduate level in institutes M and S = 4641+3822=8463
- (c): Required number = (15% of 24700) + (12% of 24700) = 3705 + 2964 = 6669.

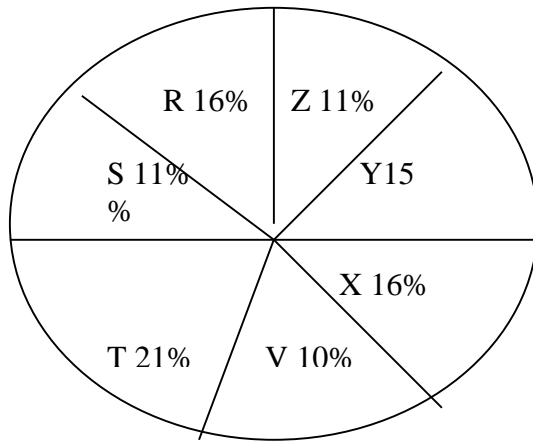
3.(d):Required number = (18% of 27300) + (14% of 24700) = 4914 + 3458 = 8372.

4.(d):Required ratio = $\frac{(21\% \text{ of } 24700)}{(14\% \text{ of } 27300)} = \frac{21 * 24700}{14 * 27300} = \frac{19}{14}$

5.(d):Required ratio = $\frac{(21\% \text{ of } 24700)}{(13\% \text{ of } 27300)} = \frac{21 * 24700}{13 * 27300} = \frac{19}{13}$

Question-4.Study the following pie-chart and the table and the answer the questions based on them.

PROPORTION OF POPULATION OF SEVEN VILLAGES IN 1997



village	% population below poverty
X	38
Y	52
Z	42
R	51
S	49
T	46
V	58

1.Find the population of villages S if the population of village X below poverty line in 1997 is 12160.

- (a). 18500 (b) 20500 (c) 22000 (d) 26000

2.The ratio of the population of the village T below poverty line to that of village Z below poverty line in 1997 is:

- (a) 11:23 (b) 13:11 (c) 23:11 (d) 11:13

3.If the population of village R in 1997 is 32000,then what will be the population of village Y below poverty line in that year?

- (a) 14100 (b)15600 (c) 16500 (d) 17000

4.If in 1998, the population of villages Y and V increases by 10% each and the percentage of population below poverty line remains unchanged for all the villages, then find the population of village V below poverty line in 1998,given that the population of village Y in 1997 was 30000.

- (a) 11250 (b) 12760 (c) 13140 (d) 13780

5.If in 1998,the population of village R increases by 10% while that of village Z reduces by 5% compared that in 1997 and the percentage of population below poverty line remains unchanged for all the village,then find the approximate ratio of population of village R below poverty line for the year 1999.

- (a) 2:1 (b) 3:2 (c) 4:3 (d) 5:4

SOLUTION

1.(c):Let the population of village X be x

$$\text{Then, } 38\% \text{ of } x = 12160 \Rightarrow x = \frac{12160 * 100}{38} = 32000$$

Now ,if s be the population village S,then

$$16:11 = 32000 : s \Rightarrow s = \frac{11 * 32000}{16} = 22000.$$

2.(c): Let N be the total population of all the seven villages.

Then ,population of village T below poverty line = 46% of (21% of N) and population of village Z below poverty line = 42% of (11% of N)

$$\therefore \text{Required ratio} = \frac{46\% \text{ of } (21\% \text{ of } N)}{42\% \text{ of } (11\% \text{ of } N)} = \frac{46 * 21}{42 * 11} = \frac{23}{11}$$

3.(b): Population of village R = 32000(given)

Let the population of village Y be y.

$$\text{Then, } 16:15 = 32000 : y \Rightarrow y = \frac{15 * 32000}{16} = 30000$$

4.(b): Population of village Y in 1997 = 30000(given) .

Let the population village V in 1997 be v.

$$\text{Then, } 15:10 = 30000:v \Rightarrow v = \frac{30000 * 10}{15} = 20000.$$

Now population of village V in 1998 = 20000 + (10% Of 20000) = 22000.

\therefore Population of village V below poverty line in 1998 = 58% of 22000 = 12760.

5.(a) : Let the total population of all the seven villages in 1997 be N.

Then, population of village R in 1997 = 16% of N = $\frac{16}{100} N$

And population of village Z in 1997 = 11% of N = $\frac{11}{100} N$

\therefore Population of village R in 1999 = $\left\{ \frac{16}{100} N + (10\% \text{ of } \frac{16}{100} N) \right\} = \frac{1760}{10000} N$

and population of village Z in 1999 = $\left\{ \frac{11}{100} N - (5\% \text{ of } \frac{11}{100} N) \right\} = \frac{1045}{10000} N$.

Now, population of village R below poverty line for 1999 = 51% of $\left(\frac{1760}{10000} N \right)$

And population of village Z below poverty line 1999 = 42% of $\left(\frac{1045}{10000} N \right)$

$$51\% \text{ of } \left(\frac{1760}{10000} N \right) = \frac{51}{100} * \frac{1760}{10000} N = \frac{9076}{10000} N$$

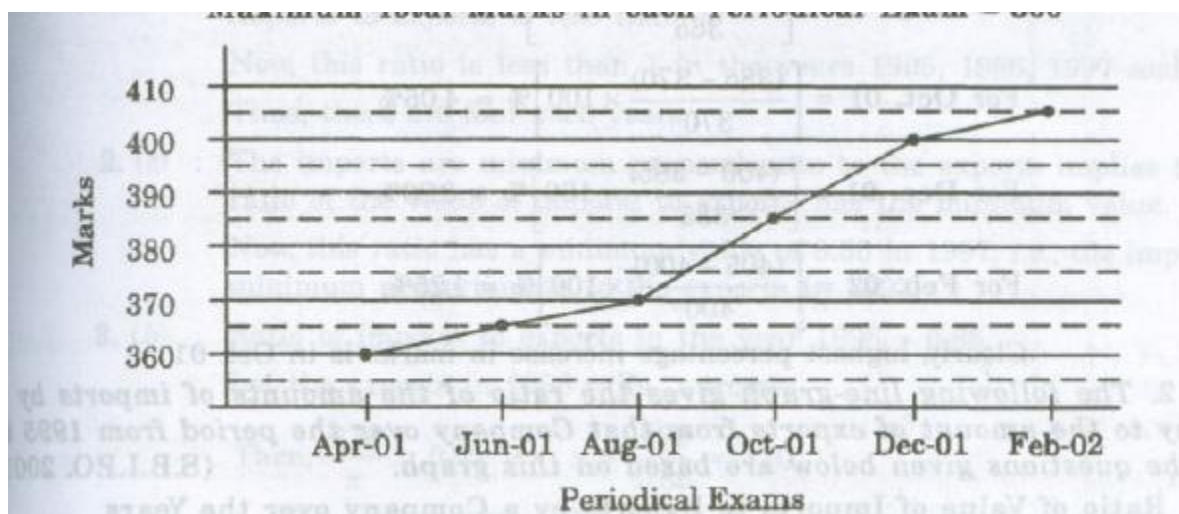
$$42\% \text{ of } \left(\frac{1045}{10000} N \right) = \frac{42}{100} * \frac{1045}{10000} N = \frac{4389}{10000} N$$

37. LINE-GRAPHS

Q1. In a school the periodical examination are held every second month. In a session during Apr. 2001 – Mar. 2002, a student of Class IX appeared for each of the periodical exams. The aggregate marks obtained by him in each periodical exam are represented in the line-graph given below. Study the graph and answer the questions based on it.

**MARKS OBTAINED BY A STUDENT IN SIX PERIODICAL EXAMS HELD IN
EVERY TWO MONTHS DURING THE YEAR IN THE SESSION 2001-02**

Maximum Total Marks In each Periodical Exam = 500



The total number of marks obtained in Feb. 02 is what percent of the total marks obtained in Apr. 01?

- (a) 110% (b) 112.5% (c) 115% (d) 116.5% (e) 117.5%

What are the average marks obtained by the student in all the periodical exams of during the session.

- (a) 373 (b) 379 (c) 381 (d) 385 (e) 389

what is the percentage of marks obtained by the student in the periodical exams of Aug. 01 and Oct. 01 taken together?

- (a) 73.25% (b) 75.5% (c) 77% (d) 78.75% (e) 79.5%

In which periodical exams there is a fall in percentage of marks as compared to the previous periodical exams?

- (a) None (b) Jun. 01 (c) Oct. 01 (d) Feb. 01 (e) None of these

In which periodical exams did the student obtain the highest percentage increase in marks over the previous periodical exams?

- (a) Jun. 01 (b) Aug. 01 (c) Oct. 01 (d) Dec. 01 (e) Feb. 02

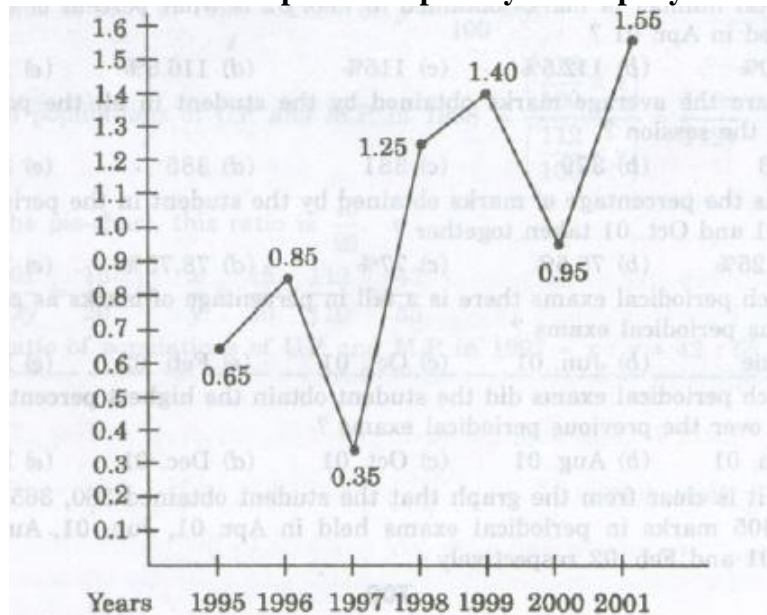
Sol. Here it is clear from the graph that the student obtained 360, 365, 370, 385, 400 and 405 marks in periodical exams held in Apr. 01, Jun. 01, Aug. 01, Oct. 01, Dec. 01 and Feb. 02 respectively.

1. (b) : Required percentage = $[(405/360) \times 100] \% = 112.5 \%$
2. (c) : Average marks obtained in all the periodical exams.

$$= (1/6) \times [360 + 370 + 385 + 400 + 404] = 380.83 \approx 381.$$
3. (d) : Required percentage = $[(370 + 385) / (500 + 500) \times 100] \% = [(755/1000) \times 100] \% = 75.5\%$
4. (a) : As is clear from graph, the total marks obtained in periodical exams, go on increasing. Since, the maximum marks for all the periodical exams are same , it implies that the percentage of marks also goes on increasing. Thus, in none of the periodical exams, there is a fall in percentage of marks compared to the previous exam.
5. (c) : Percentage increases in marks in various periodical exams compared to the previous exams are:
 For **Jun. 01** = $[(365 - 360) / 360 \times 100] \% = 1.39 \%$
 For **Aug. 01** = $[(370 - 365) / 365 \times 100] \% = 1.37 \%$
 For **Oct. 01** = $[(385 - 370) / 370 \times 100] \% = 4.05\%$
 For **Dec. 01** = $[(400 - 385) / 385 \times 100] \% = 3.90 \%$
 For **Feb. 02** = $[(405 - 400) / 400 \times 100] \% = 1.25 \%$

Q2. The following line- graph the ratio of the amounts of imports by a Company to the amount of exports from that Company over the period from 1995 to 2001. The questions given below are based on this graph.

Ratio of value of Import to Export by a Company over the Years



1. In how many of the given years were the exports more than the imports?

- a. 1 b. 2 c. 3 d. 4

2. The imports were minimum proportionate to the exports of the Company in the year:

a..1995 b.1996 c.1997 d.2001

3.If the imports of the Company in 1996 was Rs.272 crores, the exports from the Company in 1996 was:

a. Rs.370 crores b.Rs.320 crores c.Rs.280 crores
d.Rs.275 crores e.Rs.264 crores

4.What was the percentage increase in imports from 1997 to 1998?

a. 72 b.56 c.28 d.None of these e.Data inadequate

5.If the imports in 1998 was Rs.250 crores and the total exports in the years 1998 and 1999 together was Rs.500 crores, then the imports in 1999 was:

a.Rs.250 crores b.Rs.300 crores c.Rs 357 crores
d.Rs 420 crores e.None of these

Sol: 1. d : The exports are more than the imports implies that the ratio of value of imports to exports is less than 1.

Now, this ratio is less than 1 in the years 1995,1996,1997 and 2000.

Thus, there are four such years.

2. c: The imports are minimum proportionate to the exports implies that the ratio of the value of imports to exports has the minimum value.

Now, this ratio has a minimum value of 0.35 in 1997, i.e., the imports are minimum proportionate to the exports in 1997.

3. b: Ratio of imports to exports in the years 1996=0.85.

Let the exports in 1996=Rs.320 crores.

Then, $272/x = 0.85$ implies $x = 272/.85 = 320$.

Exports in 1996 = Rs.320 crores.

4e: The graph gives only the ratio of imports to exports for different years. To find the percentage increase in imports from 1997 to 1998, we require more details such as the value of imports or exports during these years. Hence, the data is inadequate to answer this question.

5. d: The ratio of imports to exports for the years 1998 and 1999 are 1.25 and 1.40 respectively.

Let the exports in the year 1998 = Rs. x crores

Then, the exports in the year 1999=Rs(500-x) crores.

$1.25 = 250/x$ implies $x = 250/1.25 = 200$

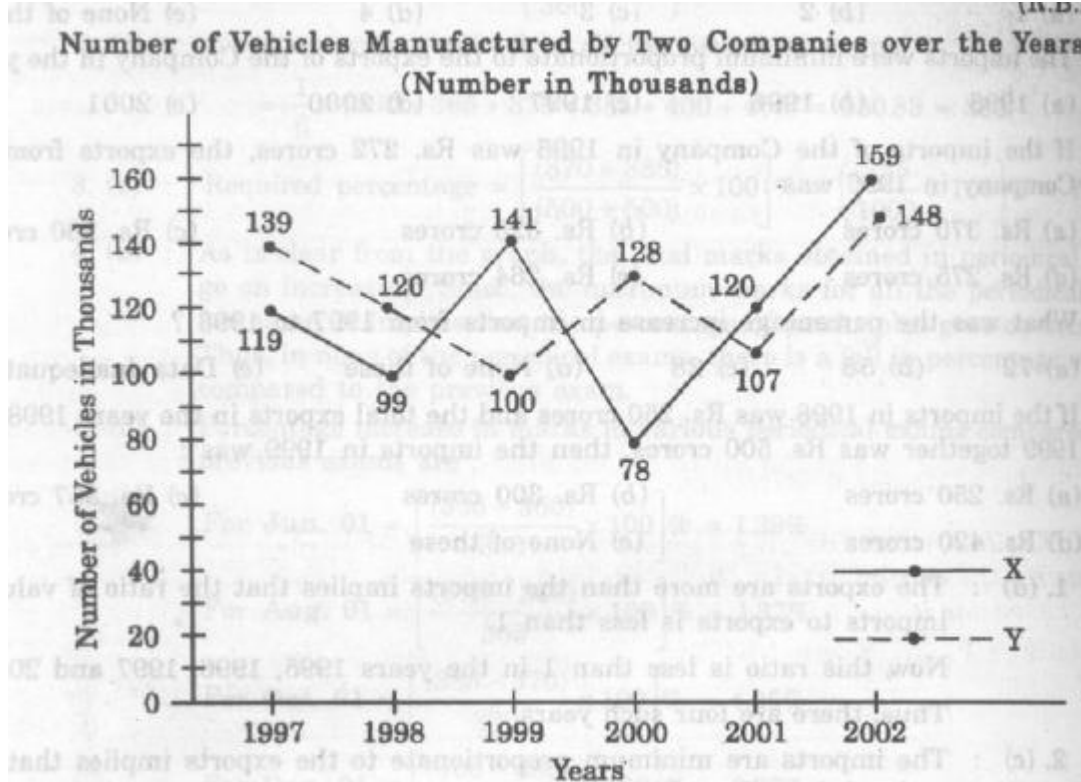
Thus the exports in the year 1999=Rs. (500-200)crores=Rs.300 crores

Let the imports in the year 1999=Rs. y crores

Then, $1.4 = y/300$ implies $y = (300 \times 1.4) = 420$. Imports in the year 1999=Rs.420 crores.

Q3.Study the following line-graph and answer the question based on it.

Number of vehicle Manufactured by Two Companies over the Years
(Numbers in thousands)



- What is the difference between the total productions of the two Companies in the given years?
a. 19000 b. 22000 c. 26000 d. 28000 e. 29000
- What is the difference between the numbers of vehicles manufactured by Company Y in 2000 and 2001?
a. 50000 b. 42000 c. 33000 d. 21000 e. 13000
- What is the average number of vehicles manufactured by Company X over the given period? (rounded off to the nearest integer)
a. 119333 b. 113666 c. 112778 d. 111223 e. None of these
- In which of the following years, the difference between the productions of Companies X and Y was the maximum among the given years?
a. 1997 b. 1998 c. 1999 d. 2000 e. 2001
- The production of Company Y in 2000 was approximately what percent of the production of Company X in the same year?
a. 173. b. 164 c. 132 d. 97 e. 61

Sol: From the line-graph it is clear that the productions of Company X in the years 1997, 1998, 1999, 2000, 2001 and 2002 are 119000, 99000, 141000, 78000, 120000 and 159000 respectively and those of Company Y are 139000, 120000, 100000, 128000, 107000 and 148000 respectively.

1. (c) : Total production of Company X from 1997 to 2002
 $= 119000 + 99000 + 141000 + 78000 + 120000 + 159000 = 716000$
 and total production of Company Y from 1997 to 2002
 $= 139000 + 120000 + 100000 + 128000 + 107000 + 148000 = 742000$
 Difference $= 742000 - 716000 = 26000$.

1. (d) : Required difference $= 128000 - 107000 = 21000$.

2. (a) : Average number of vehicles manufactured by Company X
 $= (91/6) * (119000 + 99000 + 141000 + 78000 + 120000 + 159000) = 119333$.

3. (d) : The difference between the production of Companies X and Y in various years are.

For 1997 $= (139000 - 119000) = 20000$;

For 1998 $= (120000 - 99000) = 21000$;

For 1999 $= (141000 - 100000) = 41000$;

For 2000 $= (128000 - 78000) = 50000$;

For 2001 $= (120000 - 107000) = 13000$;

For 2003 $= (159000 - 148000) = 11000$;

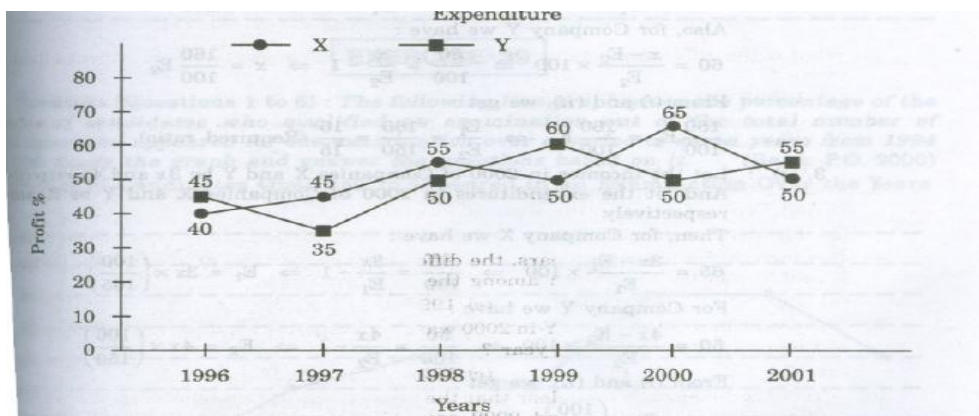
Clearly, maximum difference was in 2000

4. (b) : Required percentage $= [(128000/78000) * 100] \% = 164 \%$.

Q4. The following line-graph gives the percent profit earned by two Companies X and Y during the period 1996 – 2001. Study the line – graph and answer the questions that are based on on it.

Percentage Profit Earned by Two Companies X and Y over the Given years

$\% \text{ profit/ loss} = [(Income - Expenditure) / Expenditure] * 100$



1. If the expenditure of Company Y in 1997 was Rs. 220 crores, what was its income in 1997?

(a). Rs. 312 crores (b). Rs. 297 crores (c) Rs. 283 crores (d) Rs. 275 crores (e)Rs.261 crores

2.If the incomes of the two companies were equal in 1999,then what was the ratio of expenditure of Company X to that of company Y in 1999 ?

(a) 6:5 (b) 5:6 (c) 11:6 (d) 16:15 (e) 15:16

3.The incomes of the companies X and Y in 2000 were in the ratio of 3:4 respectively. What was the respective ratio of their expenditures in 2000?

(a) 7:22 (b) 14:19 (c) 15:22 (d) 27:35 (e) 33:40

4.If the expenditure of companies X and Y in 1996 were equal and the total income of the two companies in 1996 was Rs.342 crores, what was the total profit of the two companies together in 1996 ? (Profit = Income – Expenditure)

(a) Rs.240crores (b) Rs.171crores (c) Rs.120crores (d) Rs.102crores (e) None of these.

5.The expenditure of company X in the year 1998 was Rs.200 crores and the income Company X in 1998 was the same as its expenditure in 2001 was:

(a) Rs.465crores (b) Rs.385crores (c) Rs.295crores (d) Rs.255crores

Sol.1.(b) : Profit percent of company Y in 1997=35.

Let the income of company Y in 1997 be Rs.x crores

$$\text{Then, } 35 = \frac{x-220}{220} \times 100 \Rightarrow x = 297$$

∴ Income of company Y in 1997 = Rs.297crores

2.(d): Let the incomes of the two companies X and Y in 1999 be Rs.x and let the Expenditures of companies X and Y in 1999 be E₁ and E₂ respectively

Then, for Company X we have:

$$50 = \frac{x-E_1}{E_1} \times 100 \Rightarrow \frac{50}{100} = \frac{x}{E_1} - 1 \Rightarrow x = \frac{150}{100} E_1$$

Also, for the Company Y we have:

$$60 = \frac{x-E_2}{E_2} \times 100 \Rightarrow \frac{60}{100} = \frac{x}{E_2} - 1 \Rightarrow x = \frac{160}{100} E_2$$

From (i) and (ii), we get

$$\frac{150}{100} E_1 = \frac{160}{100} E_2 \Rightarrow \frac{E_1}{E_2} = \frac{160}{150} = \frac{16}{15} \text{ (Required ratio)}$$

3.(c): Let the incomes in 2000 of companies X and Y be 3x and 4x respectively. And let the expenditure in 2000 of companies X and Y be E₁ and E₂ respectively.

Then, for company X we have:

$$65 = \frac{3x-E_1}{E_1} \times 100 \Rightarrow \frac{65}{100} = \frac{3x}{E_1} - 1 \Rightarrow E_1 = 3x \times \frac{100}{165}$$

For company Y we have:

$$50 = \frac{4x-E_2}{E_2} \times 100 \Rightarrow \frac{50}{100} = \frac{4x}{E_2} - 1 \Rightarrow E_2 = 4x \times \frac{100}{150}$$

From (i) and (ii) we get:

$$E1 = \frac{3x \cdot (100/165)}{3 \cdot 150} = 15 \text{ (Required ratio)}$$

$$E2 = \frac{4x \cdot (100/150)}{4 \cdot 165} = 22$$

4.(d): Let the expenditures of each of the Companies X and Y in 1996 be Rs.x crores. And let the income of Company X in 1996 be Rs.z crores so that the income of Company Y in 1996 = Rs.(342-z) crores.

Then, for company X we have:

$$40 = \frac{z-x}{x} \cdot 100 \Rightarrow \frac{40}{100} = \frac{z}{x} - 1 \Rightarrow x = \frac{100z}{140}$$

Also for company Y we have:

$$45 = \frac{(342-z)-x}{x} \cdot 100 \Rightarrow \frac{45}{100} = \frac{(342-z)}{x} - 1 \Rightarrow x = \frac{(342-z) \cdot 100}{145}$$

From (i) and (ii) we get:

$$\frac{100z}{140} = \frac{(342-z) \cdot 100}{145} \Rightarrow z = 168$$

Substituting $z=168$ in (i), we get: $x=120$

\therefore Total expenditure of companies X and Y in 1996 = $2x = \text{Rs.}240$ crores.

Total income of companies X and Y in 1996 = Rs.342 crores.

\therefore Total profit = Rs.(342-240) crores = Rs.102 crores

5(a): Let the income of company X in 1998 be Rs.x crores.

$$\text{Then } 55 = \frac{x-200}{200} \cdot 100 \Rightarrow x = 310.$$

\therefore Expenditure of Company X in 2001 = Income of company X in 1998 = Rs.310 crores

Let the income of company X in 2001 be Rs.z crores

$$\text{Then, } 50 = \frac{z-310}{310} \cdot 100 \Rightarrow z = 465.$$

\therefore Income of company X in 2001 = Rs.465 crores.