Homework 8

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Part I: Optimization Framework

The optimal cost $F(t)$ is given by :

$$F(t){=}min \begin{cases} K_t + cD_t + F(t-1) \\ \min_{0\leq j\leq t}[K_j + c\sum_{k=j}^{t} D_k \sum_{k=j}^{t-1}\sum_{l=j+1}^{t} h_j d_l] \end{cases}$$

I use the "Finding Least-Cost Paths" framework from Thomas Sargent's book chapter on dynamic programming using least costly path. Following the logic of this chapter, the equivalence with the current problem can be thought of as having to reach the final time period T, starting from initial time period t=0 (with the obvious constraints of meeting demand in each intervening time period). This can be done by producing in all or some of theintervening time periods. In network terms, if $c(\nu, w)$ is the cost of travelling from node ν to node w, $J(\nu)$ is the minimum cost-to-go from node node ν. Therefore, the problem to be solved at the current node ν is one where we can move to w such that $J(\nu) = \min_{w\in F_\nu}[c(\nu, w) + J(w)]$.The lot sizing problem can be represented as a network, where each node t represents a period and an arc from node $t^{'}$ to node t represents the fact that we order (or produce) in both periods $t^{'}$and t but not in periods in between.

Part II: Description of the problem

I solve the problem of planning production for 6 periods, where the demand is forecasted for all six periods. The problem at hand is to decide in which periods to produce, and how much. The conseuence of this decision is that we also get to know how much inventory to carry from period to period. The key parameters for this optimization function are:

- K= Fixed Cost of production (100)
- c=per unit production cost (7)
- h=per unit inventory holding cost (1)
- max inventory= cacacity contraint on inventory that can be held. For this problem, I have set it high enough to make it an uncapacited lot-sizing problem, but it can be changed. (1000)
- max production=capacity contraint on the production volume. Again, I have set this high enough so that results represent the uncapacited problem. (1000)

- starting inventory is the inventory at t=1 or the start of the planning horizon. I set this to be equal to zero. But can be changed in the program. (0)

Demand=[70, 90, 140, 150, 120, 130]. From the analytical proof provided in the HW7, the minimum production cost=5490 and optimal schedule is:

t	1	2	3	4	5	6
# of periods supplied	2	0	1	1	1	1
# Production amount	160	0	140	150	120	130
# Inventory amount	90	0	0	0	0	0

This means that except for period 2, the production should be done in their respective periods. In period 1, production should also be done for period 2, and that inventory should be held.

Part III: Computational Solution Method

In a nutshell, I solve for this problem by starting at the last time period (=t) and iterate over all possible values of the production/invetory to figure out the minimum cost of reaching to the next period(=t-1). This process is updated untill we reach t=1 (start of planning horizon). The output of production and inventory policy (essentially similar to the above) is below:

FIGURE 1: **Production Policy Function**

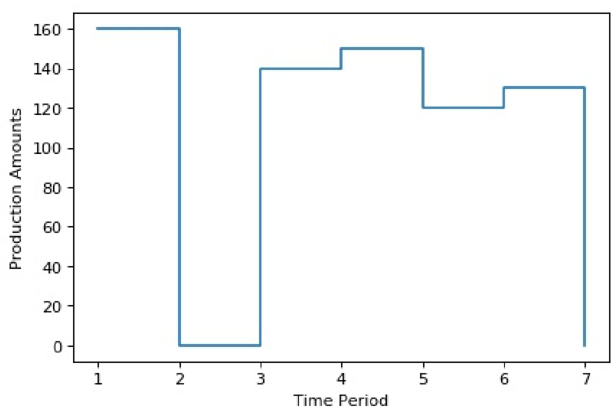
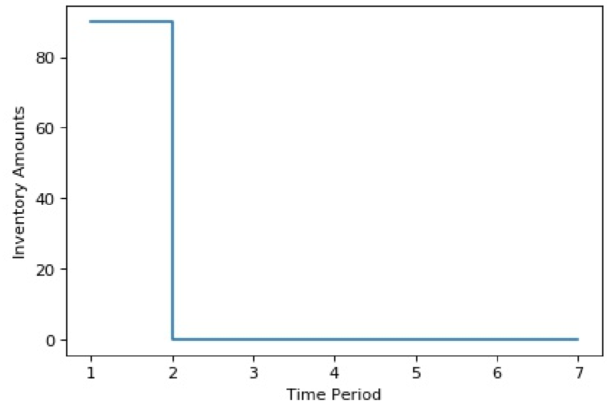
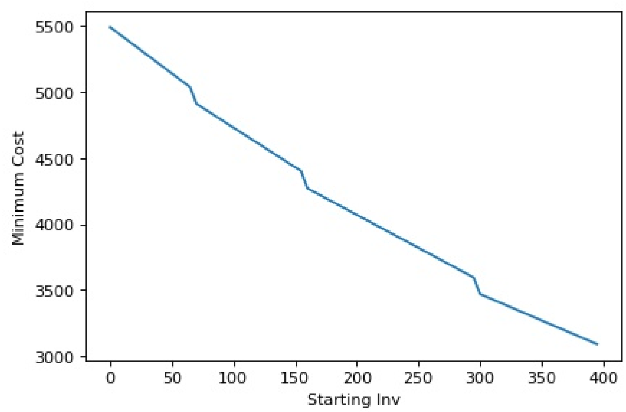


Figure 2: **Inventory Policy Function**



Although I start with an inventory of 0, one interesting question is what would the optimal (min) cost have been for higher starting inventory (subject to the inventory capacity). I iterate over the values of starting inventory to get the following relatioship:

Figure 3: **Cost vs Starting Inventory**



Also, another important parameter is the holding cost. Many planners try to find ways to minimize their per unit holding costs by locating storage facilities strategically and thus relying more on storage rather than production (since production incurs a fixed cost). I plot the optimal cost vs. the holding cost and it can be seen that the optimal cost flattens out beyond h=1

FIGURE 4: **Cost vs Per unit holding cost**