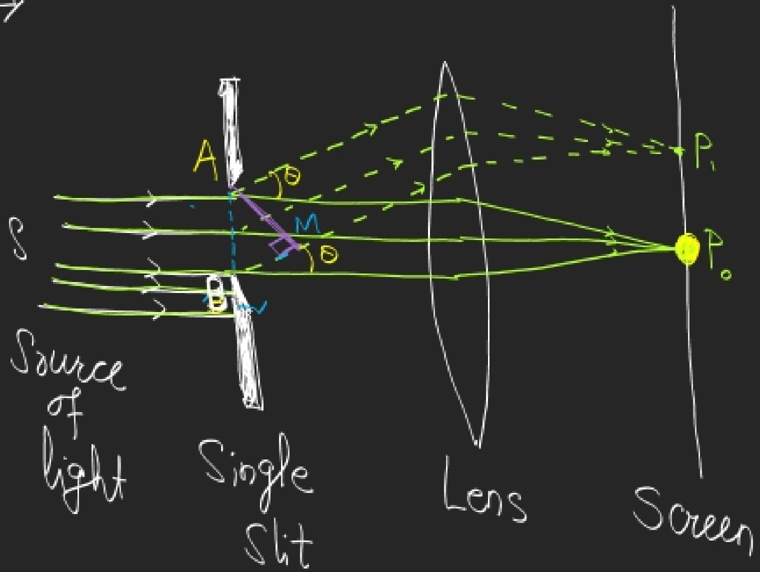
30.01.24

2. Diffraction of light

* Diffraction due to Single Slit (5M-M.IMP)

(Derivation)

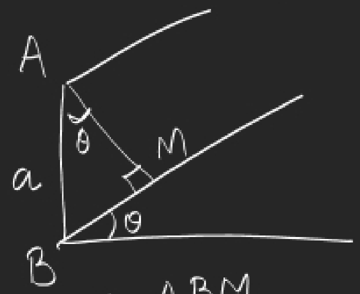
⇒



Slit width = AB = a

Angle of diffraction = θ

Path difference between diffracted rays. BM



$\triangle ABM$.

$$\sin\theta = \frac{BM}{AB} = \frac{BM}{a}$$

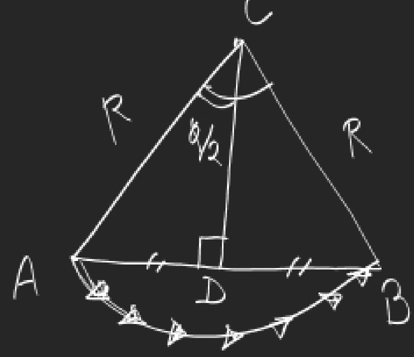
$$\Rightarrow \boxed{BM = a\sin\theta} \quad (1)$$

$\lambda \longrightarrow 2\pi$

$a\sin\theta \longrightarrow \phi$ (say)

$$\boxed{\phi = \frac{2\pi}{\lambda}a\sin\theta} \quad (2)$$

$\phi \rightarrow$ phase diff.



$\angle ACB = \phi$

$AC = CB = R$ = radius of curvature

E_θ = Resultant amplitude = Chord AB

E_m = Maximum amplitude = Arc AB

Arc AB = Radius × angle

$$\boxed{E_m = R \times \phi} \quad (3)$$

$\triangle ACD \cong \triangle BCD$

$AD = DB = \frac{1}{2}$ chord AB

$$\boxed{AD = \frac{E_\theta}{2}} \quad (4)$$

$$\sin\frac{\phi}{2} = \frac{AD}{AC} = \frac{(E_\theta)/2}{R}$$

$$\boxed{E_\theta = 2R\sin\frac{\phi}{2}} \quad (5)$$

$\because R = \frac{E_m}{\phi}$ put in eq^n (5)

$$E_\theta = 2\frac{E_m}{\phi} \cdot \sin\frac{\phi}{2}$$

$$= \frac{E_m}{\phi/2} \cdot \sin\frac{\phi}{2}$$

$$= E_m \frac{\sin\frac{\phi}{2}}{\frac{\phi}{2}}$$

$$\boxed{E_\theta = E_m \frac{\sin\alpha}{\alpha}} \quad \text{— (6)}$$

Let $\alpha = \frac{\phi}{2} = \frac{\pi}{\lambda} a \sin\theta$

(phase angle)

$$E_\theta^2 = E_m^2 \left(\frac{\sin\alpha}{\alpha}\right)^2$$

$$\boxed{I_\theta = I_m \left(\frac{\sin\alpha}{\alpha}\right)^2} \quad \text{— (7)}$$

$\because$ Intensity is proportional to square of amplitude

Case (i) Principal Maxima

$$\lim_{\alpha \to 0} \frac{\sin\alpha}{\alpha} = 1$$

$$\Rightarrow \boxed{I_\theta = I_m}$$ Maximum intensity

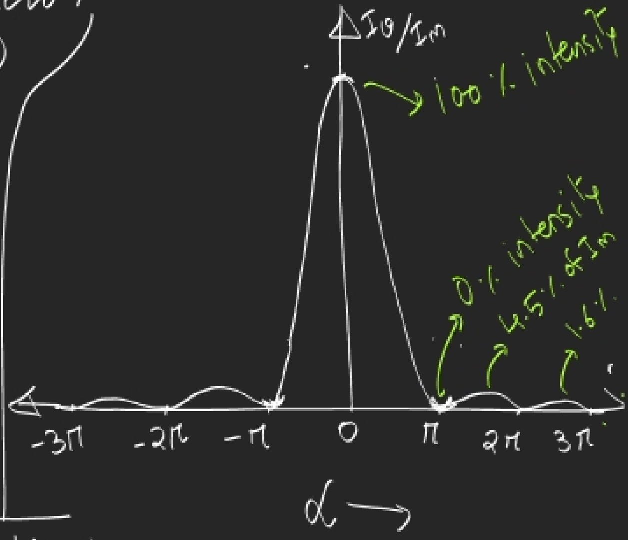
Case (ii) Minima (Zero) intensity (Dark)

$$\alpha = \pm n\pi$$

$n = 1, 2, 3 \ldots$

$$\Rightarrow I_\theta = I_m \cdot 0$$

$$\boxed{I_\theta = 0}$$



Case (iii) Secondary Maxima (moderate intensity)

$$\alpha = (2n+1)\frac{\pi}{2}$$

$n = 1, 2, 3 \ldots$

$$I_\theta = I_m(0.045, 0.016, 0.008, \ldots)$$

4.5% 1.6% 0.8%

N = 500 lines

N = 1

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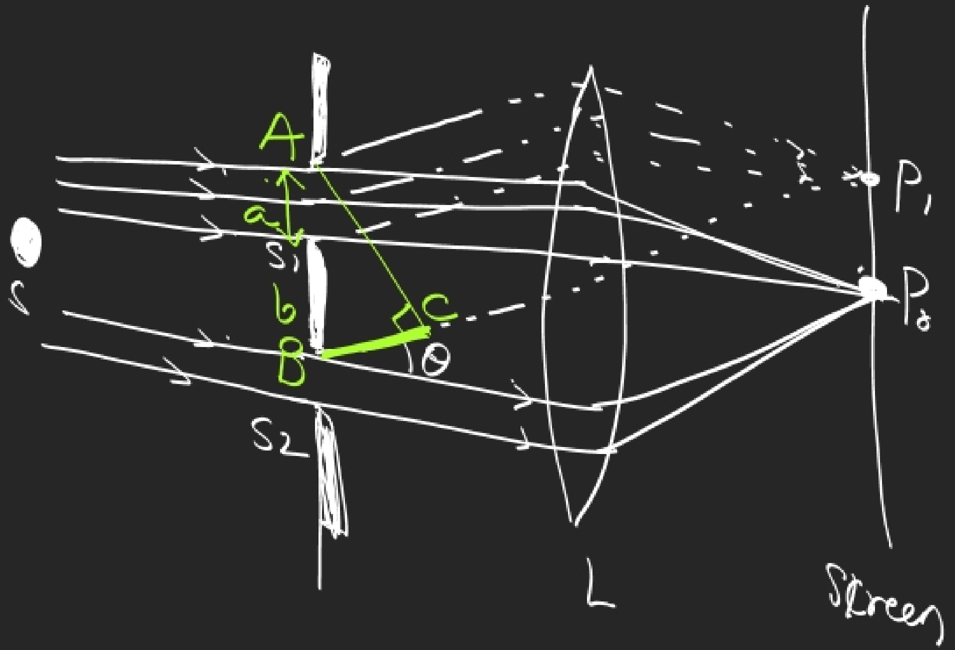
Diffraction due to double slits

IMP - 5M To discuss Conditions of principal Maxima, Minima, Secondary Maxima



$$O.P.d = BC = (a+b)\sin\theta$$

Resultant Intensity for double slits

$$I_\theta = I_m\left(\frac{\sin\alpha}{\alpha}\right)^2 \cos^2\beta \quad (1)$$

where $\beta = \frac{\phi}{2} = \frac{\pi}{\lambda}(a+b)\sin\theta$

and $\phi = \frac{2\pi}{\lambda}(a+b)\sin\theta$

$\alpha = \frac{\pi}{\lambda} a\sin\theta$

Condition - 1 Principal Maxima

$\beta = 0 \Rightarrow \cos\beta = 1$

$\lim_{\alpha\to 0} \frac{\sin\alpha}{\alpha} = 1$

$\Rightarrow \boxed{I_\theta = I_m}$

Maximum Intensity at P_0

Condition 2 Minima

$\alpha = \pm n\pi$

$\beta = (2n+1)\frac{\pi}{2}$

$\Rightarrow I_\theta = I_m \cdot 0 \cdot 0$

$\boxed{I_\theta = 0}$

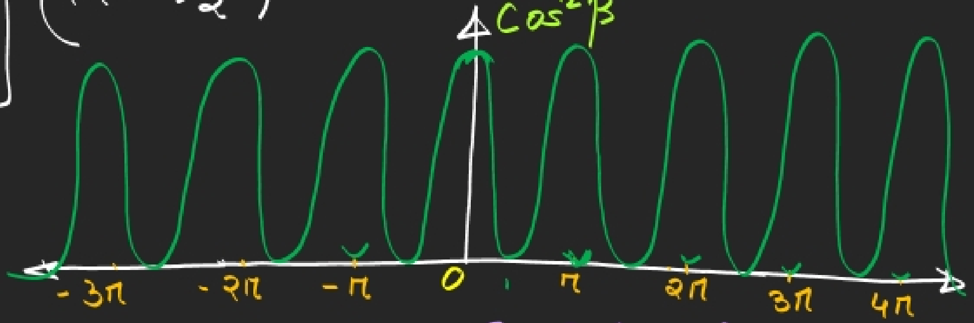
Resultant intensity is Zero (minimum)

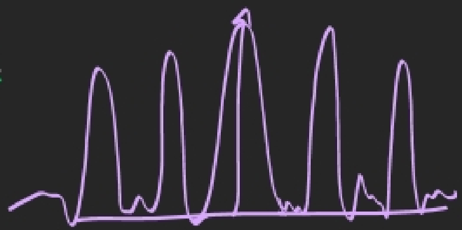
for $\alpha = (2n+1)\frac{\pi}{2} \Rightarrow \left(\frac{\sin\alpha}{\alpha}\right)^2 = \frac{1}{\left((2n+1)\frac{\pi}{2}\right)^2} = 0.045,\ 0.016,\ 0.008, \ldots$

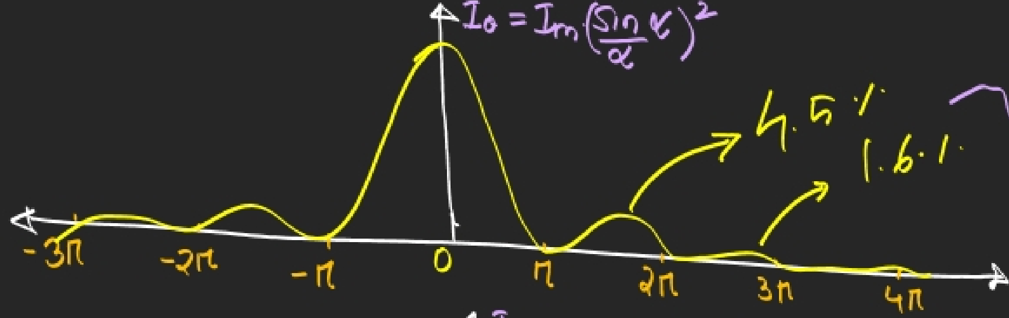
& $\boxed{\beta = \pm n\pi \Rightarrow \cos^2\beta = 1}$

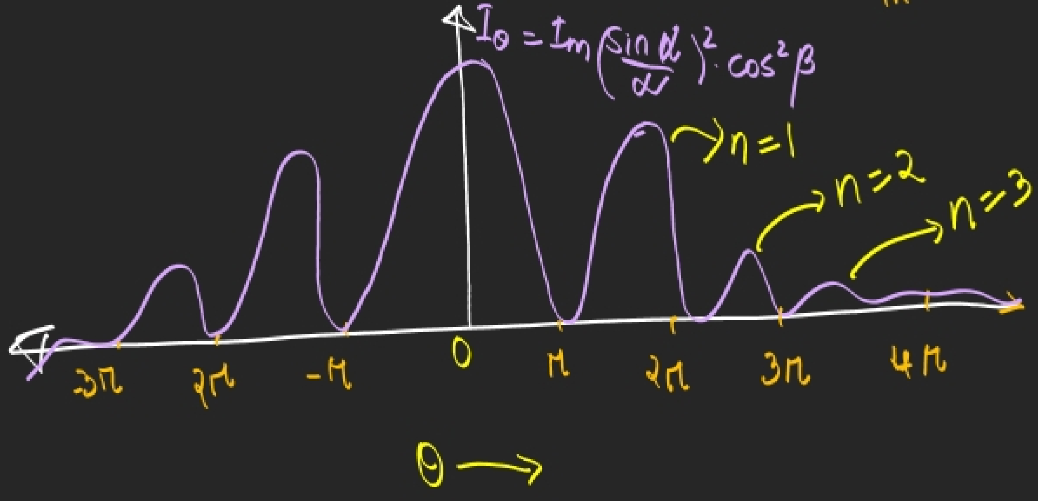
$\boxed{0 < I_\theta < I_m}$

Moderate intensity.





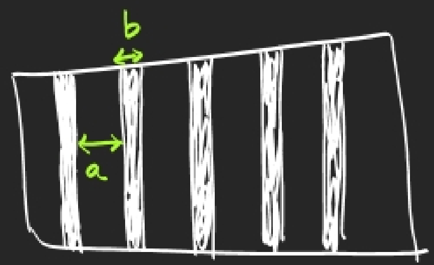




n = order of diffraction

Diffraction Grating

(N-slits)



a = Slit width

b = line width

Diffraction Maxima Condition (double slit)

$$\boxed{(a+b)\sin\theta = n\lambda}$$

$N = \dfrac{\text{no. of lines}}{\text{unit length}}$

$N = \dfrac{1}{a+b}$

a+b = grating element

$\theta \rightarrow$ angle of diffraction

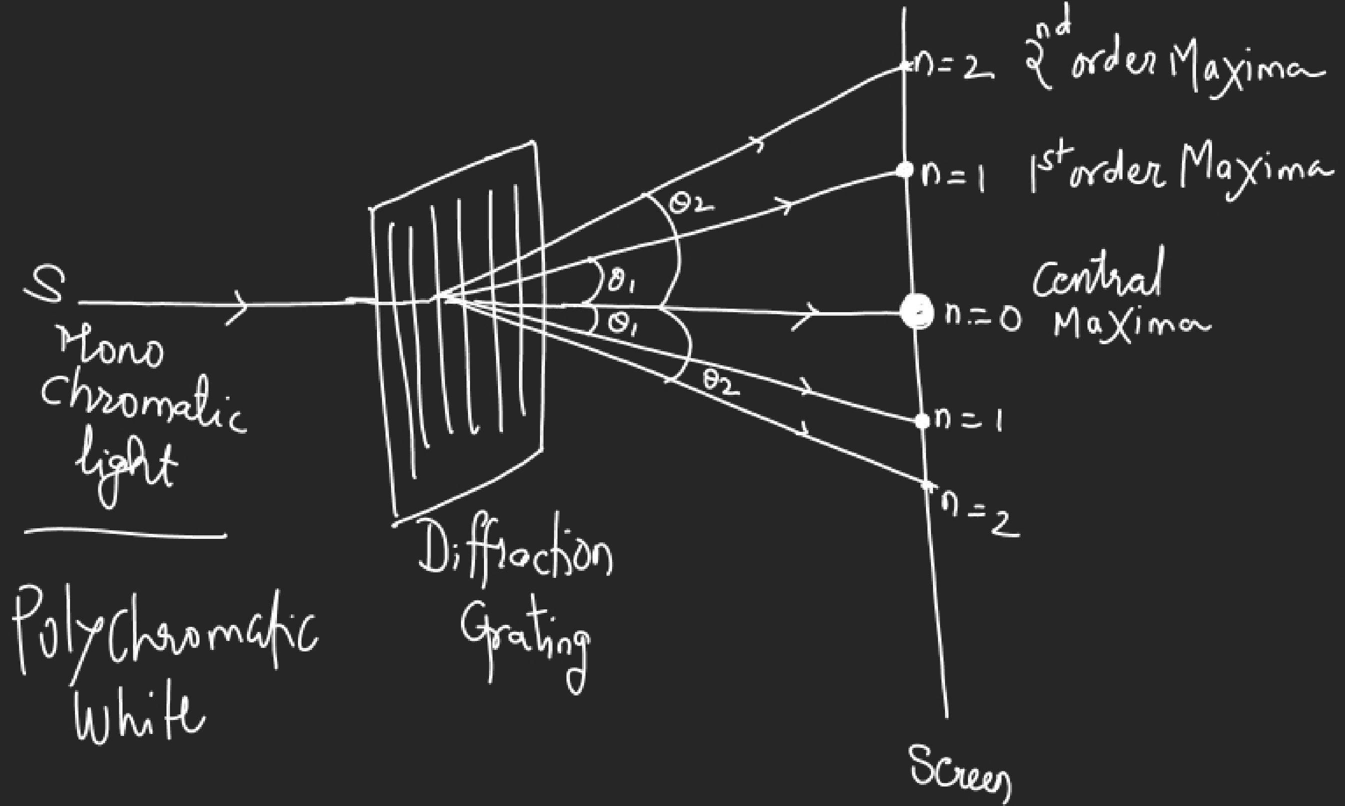
$n \rightarrow$ order of diffraction

$\lambda \rightarrow$ wavelength of light

$(a+b)\sin\theta = n\lambda \longrightarrow$ cm

8|2|24

Grating for highest order :-



Diffraction Maxima for double Slits is given by,

$$(a+b)\sin\theta = n\lambda$$

for $n = n_{max}$, $\theta = \pi/2$ or 90°

$$\boxed{n_{max} = \frac{(a+b)}{\lambda} = \frac{1}{N.\lambda}}$$

① Given -: $\lambda_1 = 6000\,Å = 6000 \times 10^{-7}$ mm.

$\lambda_2 = 4800\,Å =$

$\theta = 35°$

To find -: $N \left(\frac{\text{no. of Lines}}{\text{mm}}\right)$

Formula -: $(a+b)\sin\theta = n\lambda_1$ —— ①

& $(a+b)\sin\theta = (n+1)\lambda_2$ —— ②

$\Rightarrow n\lambda_1 = (n+1)\lambda_2$

$$n = \frac{\lambda_2}{\lambda_1 - \lambda_2} = \frac{4800\,Å}{(6000-4800)\,Å}$$

$$\boxed{n = 4}$$

$$\boxed{N = \frac{\sin\theta}{n\lambda_1}} = \frac{\sin 35°}{4 \times 6000 \times 10^{-7}\,\text{mm}}$$

$$\boxed{N = 238.99 \simeq 238 \ \text{lines/mm}}$$