11.01.24

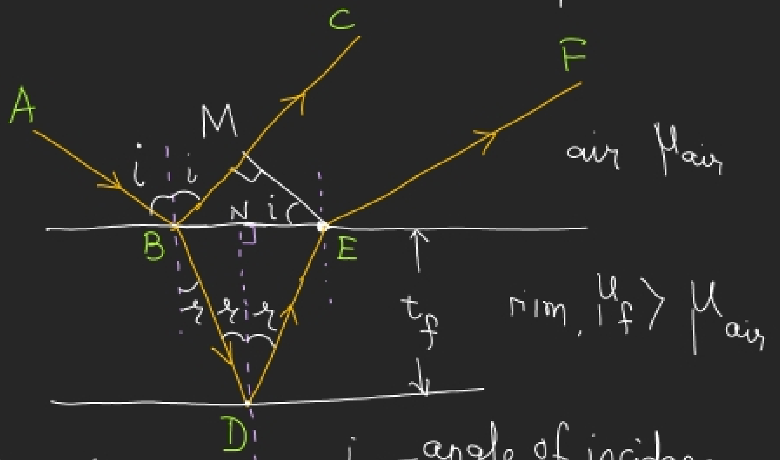
I. Interference in Thin Films

Thin film -:

Thickness of thin film $\simeq$ Wavelength of light incident

Que.: Obtain Interference Conditions for light rays reflected from thin transparent film (Parallel)

Ans-:



AB — Incident ray

BC, EF — Reflected rays

i — angle of incidance

$r \rightarrow$ angle of refraction

$\mu_f \rightarrow$ R.I. of thin film

$t_f \rightarrow$ thickness of thin film

optical Path difference between reflected rays BC & EF,

o.p.d. = Path covered in film − Path covered in air — (1)

$$\text{o.p.d} = \mu_f(BD + DE) - \mu_{air} \cdot BM$$

$\triangle BDN \simeq \triangle EDN$

$$\boxed{BD = DE} \quad \text{— (2)}$$

$$BN = NE = \frac{1}{2} BE$$

$\triangle BDN$, $\cos r = \dfrac{DN}{BD} = \dfrac{t_f}{BD}$

$$\boxed{BD = t_f / \cos r} \quad \text{— (3)}$$

$$\tan r = \frac{BN}{DN} = \frac{\frac{1}{2}BE}{t_f}$$

$$\boxed{BE = 2 t_f \tan r} \quad \text{— (4)}$$

$\triangle BME$,

$$\sin i = \frac{BM}{BE}$$

$$BM = 2 t_f \tan r \cdot \sin i$$

$$= 2 t_f \frac{\sin r}{\cos r} \sin i$$

$$= 2 t_f \frac{\sin^2 r}{\cos r} \left(\frac{\sin i}{\sin r} \right)$$

$$\boxed{BM = \frac{2 \mu_f t_f \sin^2 r}{\cos r}}$$

(using Snell's law)

Put eqⁿ ② & eqⁿ ⑤ into eqⁿ ①

$$\therefore o.p.d. = \frac{2\mu_f t_f}{\cos r} - \frac{2\mu_f t_f \cdot \sin^2 r}{\cos r}$$

$$= \frac{2\mu_f t_f}{\cos r}\left[1 - \sin^2 r\right]$$

$$\boxed{o.p.d. = 2\mu_f t_f \cos r} \quad — 6$$

$\because$ Reflected ray BC is reflected from denser film

$\therefore$ Path diff. of $\frac{\lambda}{2}$ is added

$$\boxed{\therefore o.p.d. = 2\mu_f t_f \cos r + \frac{\lambda}{2}} \quad — 7$$

(A) Maxima Condition

Constructive Interference (Bright)

$$o.p.d = n\lambda$$

$$\therefore 2\mu_f t_f \cos r + \frac{\lambda}{2} = n\lambda$$

$$\boxed{\therefore 2\mu_f t_f \cos r = (2n-1)\frac{\lambda}{2}}$$

$n = 1, 2, 3 \ldots$

Reference book

Engineering Physics – Avadhanulu & Kshirsagar

(B) Minima Condition (Dark)

$$o.p.d = (2n+1)\frac{\lambda}{2} \quad \text{(destructive)}$$

$$2\mu_f t_f \cos r + \frac{\lambda}{2} = (2n+1)\frac{\lambda}{2}$$

$$\boxed{2\mu_f t_f \cos r = n\lambda} \quad — n = 1, 2, 3 \ldots$$

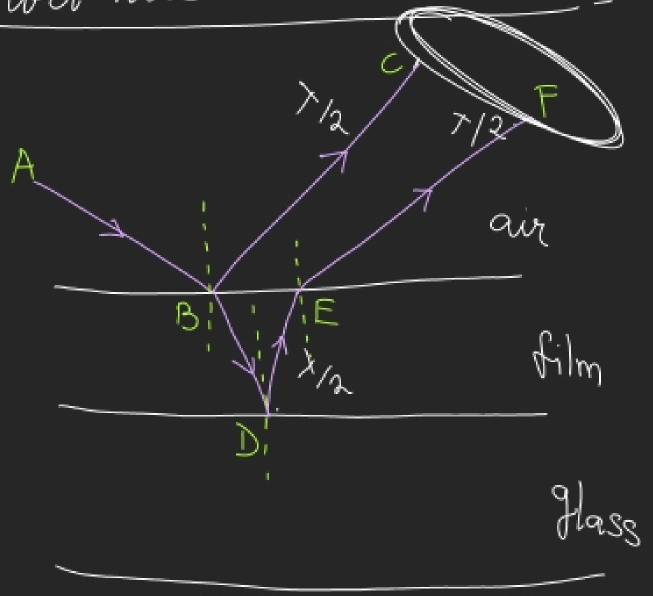
end

12.01.24

1. Interference in Thin Films

Applications -:

Anti reflecting thin film -:
[Short note - 5 Marks - IMP.]



$\mu_{air} < \mu_f < \mu_g$

$t_f \rightarrow$ thickness of film
$\mu_f \rightarrow$ R.I. of thin film

o.p.d. between reflected rays BC & EF,

$$\text{o.p.d.} = 2\mu_f t_f \cos r + \frac{\lambda}{2} + \frac{\lambda}{2}$$

$$\boxed{\text{o.p.d.} \simeq 2\mu_f t_f \cos r}$$

∵ Rays BC & EF meet destructively to form anti-reflecting film.

$$\boxed{2\mu_f t_f \cos r = (2n+1)\frac{\lambda}{2}} \quad (1)$$

For normal incidance of light,
$i = 0^\circ \Rightarrow r = 0^\circ \Rightarrow \cos r = 1$

$$\Rightarrow \boxed{2\mu_f t_f = (2n+1)\frac{\lambda}{2}}$$

for minimum thickness of film,
$n = 0$ ∴

$$2\mu_f t_f = \frac{\lambda}{2}$$

$$\boxed{t_{min} = \frac{\lambda}{4\mu_f}} \quad (2)$$

This is minimum thickness.

Amplitude Condition

$\because$ Reflected rays BC & EF have same amplitudes (nearly)

$$R_1^2 \simeq R_2^2$$

$$\left(\frac{\mu_a-\mu_f}{\mu_a+\mu_f}\right)^2 \simeq \left(\frac{\mu_f-\mu_g}{\mu_f+\mu_g}\right)^2$$

$$\Rightarrow \left(\frac{1-\mu_f}{1+\mu_f}\right)^2 \simeq \left(\frac{\mu_f-\mu_g}{\mu_f+\mu_g}\right)^2$$

$$\mu_f^2 \simeq \mu_g$$

$$\boxed{\mu_f \simeq \sqrt{\mu_g}}$$

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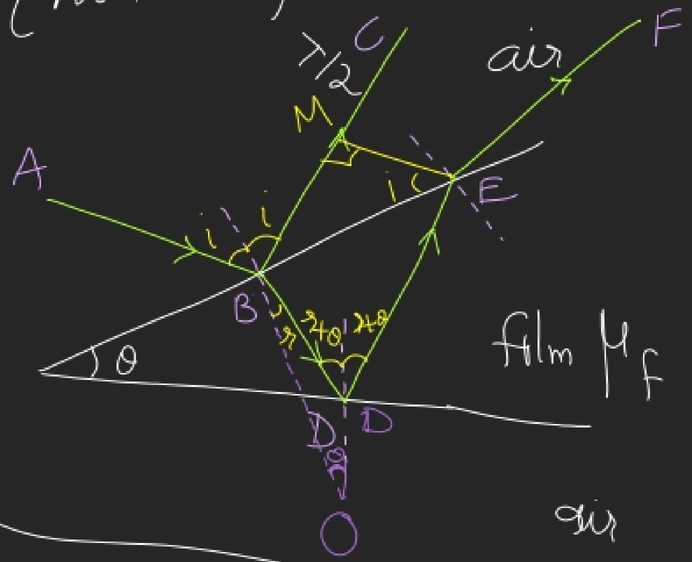
applications - Anti reflecting thin film is used on lenses of optical instruments like Camera, Telescope, Microscope, Solar Panels

* Wedge shaped thin film

(non-uniform thickness)



$$\boxed{\text{O.p.d.} = 2\mu_f t_f \cos(r+\theta) + \frac{\lambda}{2}}$$

For Maxima (Bright)

o.p.d. $= n\lambda$ constructive

$$\therefore 2\mu_f t_f \cos(r+\theta) + \frac{\lambda}{2} = n\lambda$$

$$\therefore \boxed{2\mu_f t_f \cos(r+\theta) = (2n-1)\frac{\lambda}{2}}$$

For minima Condition (Dark) destructive

$$\text{o.p.d} = (2n+1)\frac{\lambda}{2}$$

$$2\mu_f t_f \cos(r+\theta) + \frac{\lambda}{2} = (2n+1)\frac{\lambda}{2}$$

$$\boxed{2\mu_f t_f \cos(r+\theta) = n\lambda}$$

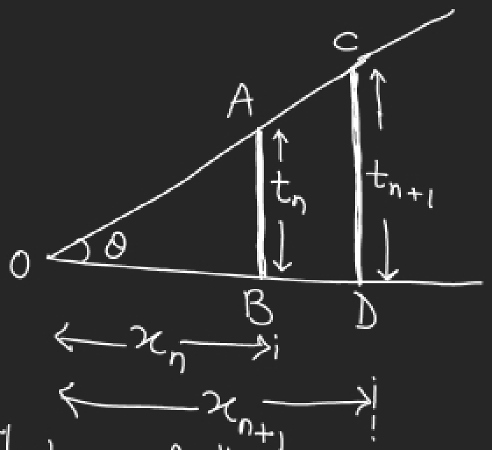
16.01.24

Interference in Thin Films

Bandwidth of Wedge Shaped thin film :-

(5 Marks - IMP. - Derivation)

Soln :-



$t_n \rightarrow$ Thickness of n^{th} dark band

$t_{n+1} \rightarrow$ —— " —— $(n+1)^{th}$ —— " ——

$x_n \rightarrow$ distance of n^{th} dark band from 'O'.

$x_{n+1} \rightarrow$ —— " —— $(n+1)^{th}$ —— " ——

$\mu_f = 1$ for air film.

θ = angle of wedge.

$$\tan\theta = \frac{t_n}{x_n} = \frac{t_{n+1}}{x_{n+1}}$$

$$\boxed{t_n = x_n \cdot \tan\theta} \quad —— (1)$$

$$\boxed{t_{n+1} = x_{n+1} \tan\theta} \quad —— (2)$$

For n^{th} dark band,

$$2\mu_f t_f \cos(r+\theta) + \frac{\lambda}{2} = (2n+1)\frac{\lambda}{2}$$

for normal incidance of light.

$i = 0^\circ \Rightarrow r = 0^\circ \Rightarrow$,

$$2\mu_f t_f \cos\theta = n\lambda$$

$t_f = t_n$ for n^{th} dark band.

$$\boxed{t_n = \frac{n\lambda}{2\mu_f \cos\theta}} \quad —— (3)$$

Put eqn (1) into eqn (3)

$$x_n \cdot \tan\theta = \frac{n\lambda}{2\mu_f \cos\theta}$$

$$\therefore \boxed{x_n \cdot \sin\theta = \frac{n\lambda}{2\mu_f}} \quad —— (4)$$

for $(n+1)^{th}$ dark band,

$$\boxed{x_{n+1} \cdot \sin\theta = \frac{(n+1)\lambda}{2\mu_f}} \quad —— (5)$$

$$(x_{n+1} - x_n) \cdot \sin\theta = \frac{\lambda}{2\mu_f}$$

$$\beta \cdot \sin\theta = \frac{\lambda}{2\mu_f}$$

$$\boxed{\beta = \frac{\lambda}{2\mu_f \sin\theta}} \quad —— (6)$$

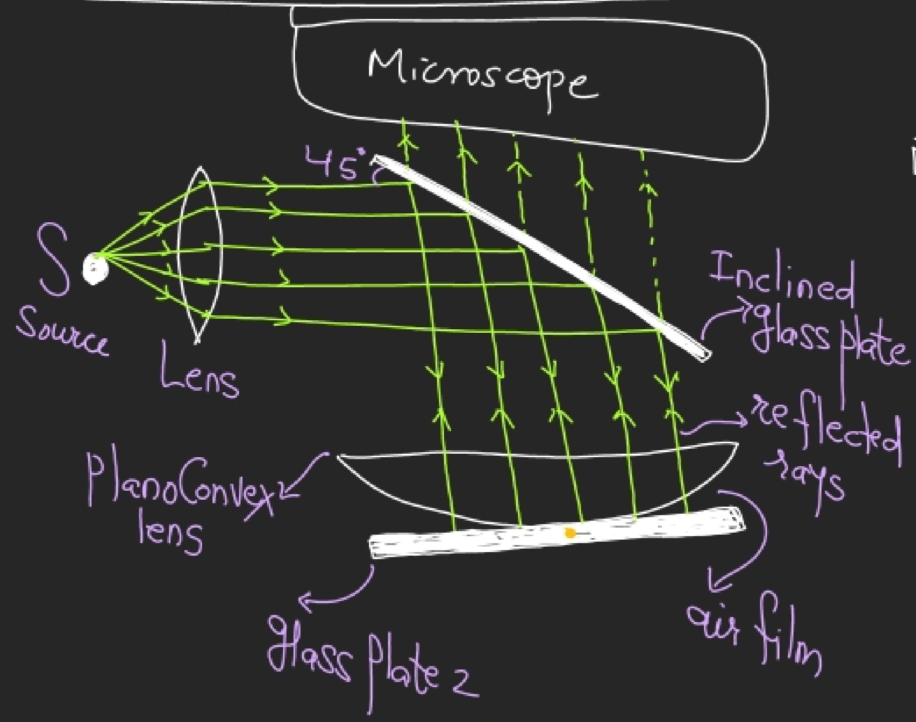
$\because \mu_f, \theta, \lambda$ are constant

$\therefore$ 'β' is constant

* Newton's Rings Experiment -/.

5M - Derivation M. IMP.

Show that diameter of dark ring is proportional to square root of an integer. (i.e. $D_n \propto \sqrt{n}$)



Interference Pattern

i) Centre is dark spot followed by 1st bright ring.

ii) Rings are widely spaced nearby centre and crowded away from Centre.

iii) Concentric Circular rings