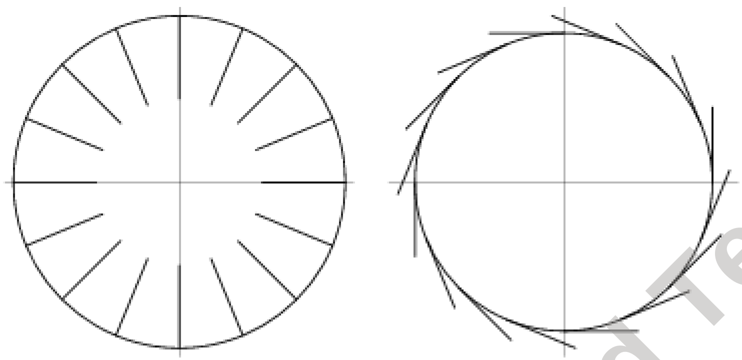
NCERT solutions for class 10 maths chapter 10 Circles Excercise: 10.1

Q1 How many tangents can a circle have?

Answer:

The lines that intersect the circle exactly at one single point are called tangents. In a circle, there can be infinitely many tangents.



Q2 Fill in the blanks :

(1) A tangent to a circle intersects it in __________ point (s).

(2) A line intersecting a circle in two points is called a ______.

(3) A circle can have _________ parallel tangents at the most.

(4) The common point of a tangent to a circle and the circle ______

Answer:

(a) one

A tangent of a circle intersects the circle exactly in one single point.

(b) secant

It is a line that intersects the circle at two points.

(c) Two,

There can be only two parallel tangents to a circle.

(d) point of contact

The common point of a tangent and a circle.

Q3 A tangent PQ at a point P of a circle of radius 5 cm meets a line through the center O at a point Q so that OQ = 12 cm. Length PQ is :

(A) 12 cm

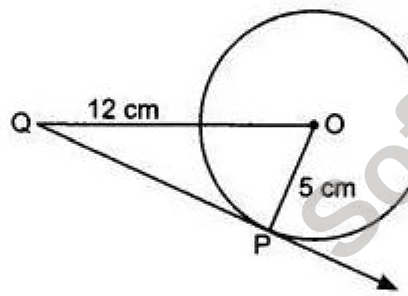
(B) 13 cm

(C) 8.5 cm

(D) $\sqrt{119}$ cm.

Answer:

The correct option is (d) = $\sqrt{119}$ cm



It is given that the radius of the circle is 5 cm. OQ = 12 cm

According to question,

We know that $\angle QPO = 90^0$

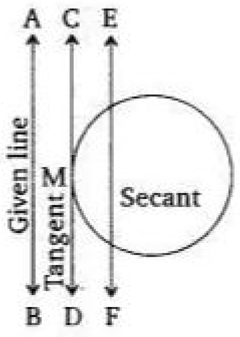
So, triangle OPQ is a right-angle triangle. By using Pythagoras theorem,

$PQ = \sqrt{OQ^2 - OP^2} = \sqrt{144 - 25}$

$= \sqrt{119}$ cm

Q4 Draw a circle and two lines parallel to a given line such that one is a tangent and the other, a secant to the circle.

Answer:



AB is the given line and the line CD is the tangent to a circle at point M and parallels to the line AB. The line EF is a secant parallel to the AB

NCERT solutions for class 10 maths chapter 10 Circles Excercise: 10.2

Q1 From a point Q, the length of the tangent to a circle is 24 cm and the distance of Q from the center is 25 cm. The radius of the circle is

(A) 7 cm

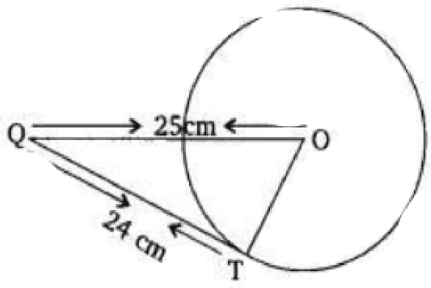
(B) 12 cm

(C) 15 cm

(D) 24.5 cm

Answer:

The correct option is (A) = 7 cm



Given that,

The length of the tangent (QT) is 24 cm and the length of OQ is 25 cm.

Suppose the length of the radius OT be l cm.

We know that ΔOTQ is a right angle triangle. So, by using Pythagoras theorem-

$$OQ^2 = TQ^2 + OT^2$$

$$l = \sqrt{25^2 - 24^2}$$

$$OT = l = \sqrt{49}$$

OT = 7 cm

Q2 In Fig. 10.11, if TP and TQ are the two tangents to a circle with center O so that $\angle$ POQ = 110°, then $\angle$ PTQ is equal to

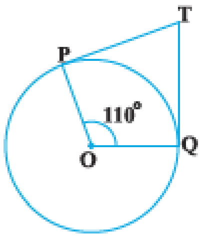


Fig. 10.11

(A) 60°

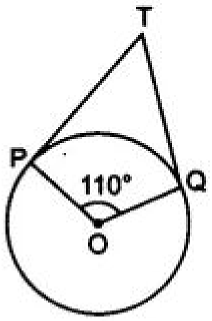
(B) 70°

(C) 80°

(D) $90°$

Answer:

The correct option is (b)



In figure, $\angle POQ = 110^0$

Since POQT is quadrilateral. Therefore the sum of the opposite angles are 180

$\Rightarrow \angle PTQ + \angle POQ = 180^0$

$\Rightarrow \angle PTQ = 180^0 - \angle POQ$

$= 180^0 - 100^0$

$= 70^0$

Q3 If tangents PA and PB from a point P to a circle with center O are inclined to each other at an angle of $80°$, then $\angle$ POA is equal to

(A) 50°

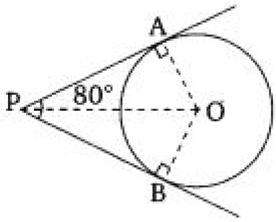
(B) 60°

(C) 70°

(D) 80°

Answer:

The correct option is (A)



It is given that, tangent PA and PB from point P inclined at $\angle APB = 80^0$

In triangle $\triangle$ OAP and $\triangle$ OBP

$\angle OAP = \angle OBP = 90$

OA =OB (radii of the circle)

PA = PB (tangents of the circle)

Therefore, by SAS congruence

$\therefore \Delta OAP \cong \Delta OBP$

By CPCT, $\angle OPA = \angle OPB$

Now, $\angle$ OPA = 80/4 = 40

In $\triangle$ PAO,

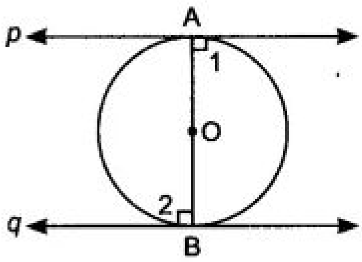
$\angle P + \angle A + \angle O = 180$

$\angle O = 180 - 130$

= 50

Q4 Prove that the tangents drawn at the ends of a diameter of a circle are parallel.

Answer:



Let line p and line q are two tangents of a circle and AB is the diameter of the circle.

OA and OB are perpendicular to the tangents p and q respectively.

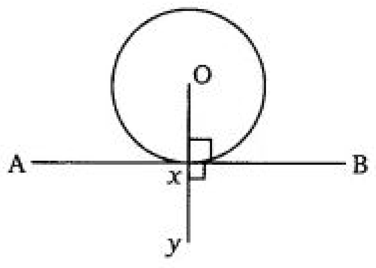
therefore,

$\angle 1 = \angle 2 = 90^0$

$\Rightarrow$ p || q {$\angle$ ∵ 1 & $\angle$ 2 are alternate angles}

Q5 Prove that the perpendicular at the point of contact to the tangent to a circle passes through the center.

Answer:



In the above figure, the line AXB is the tangent to a circle with center O. Here, OX is the perpendicular to the tangent AXB ($OX \perp AXB$) at point of contact X.

Therefore, we have,

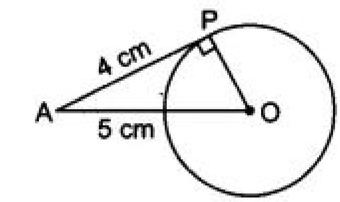
$\angle$ BXO + $\angle$ YXB = $90^0 + 90^0 = 180^0$

∴ OXY is a collinear

$\Rightarrow$ OX is passing through the center of the circle.

Q6 The length of a tangent from a point A at distance 5 cm from the center of the circle is 4 cm. Find the radius of the circle.

Answer:



Given that,

the length of the tangent from the point A (AP) is 4 cm and the length of OA is 5 cm.

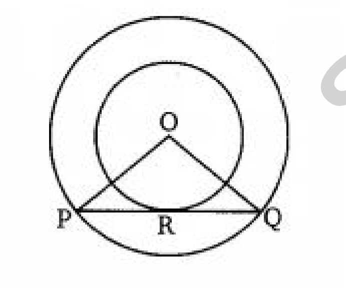
Since ∠ APO = 90 ^{0}

Therefore, △ APO is a right-angle triangle. By using Pythagoras theorem;

$$OA^2 = AP^2 + OP^2$$
$$5^2 = 4^2 + OP^2$$
$$OP = \sqrt{25 - 16} = \sqrt{9}$$
$$OP = 3cm$$

Q7 Two concentric circles are of radii 5 cm and 3 cm. Find the length of the chord of the larger circle which touches the smaller circle.

Answer:



In the above figure, Pq is the chord to the larger circle, which is also tangent to a smaller circle at the point of contact R.

We have,

radius of the larger circle OP = OQ = 5 cm

radius of the small circle (OR) = 3 cm

OR ⊥ PQ [since PQ is tangent to a smaller circle]

According to question,

In $\triangle$ OPR and $\triangle$ OQR

$\angle$ PRO = $\angle$ QRO {both 90^0 }

OR = OR {common}

OP = OQ {both radii}

By RHS congruence $\triangle$ OPR $\cong$ $\triangle$ OQR

So, by CPCT

PR = RQ

Now, In $\triangle$ OPR,

by using pythagoras theorem,

$$PR = \sqrt{25 - 9} = \sqrt{16}$$

PR = 4 cm

Hence, PQ = 2.PR = 8 cm

Q8 A quadrilateral ABCD is drawn to circumscribe a circle (see Fig. 10.12). Prove that AB + CD = AD + BC

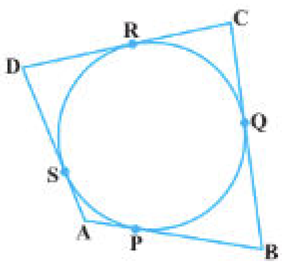


Fig. 10.12

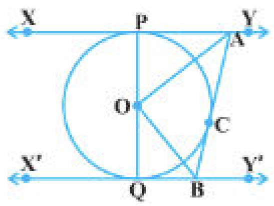
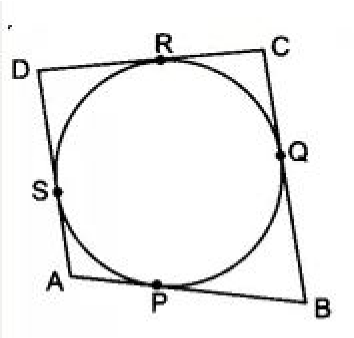


Fig. 10.13

Answer:



To prove- AB + CD = AD + BC

Proof-

We have,

Since the length of the tangents drawn from an external point to a circle are equal

AP =AS ....(i)

BP = BQ.........(ii)

AS = AP...........(iii)

CR = CQ ....(iv)

By adding all the equations, we get;

AP + BP +RD+ CR = AS +DS +BQ +CQ

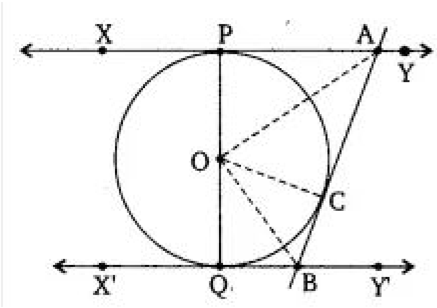
⇒ (AP + BP) + (RD + CR) = (AS+DS)+(BQ + CQ)

⇒ AB + CD = AD + BC

Hence proved.

Q9 In Fig. 10.13, XY and X'Y'are two parallel tangents to a circle with center O and another tangent AB with a point of contact C intersecting XY at A and X'Y' at B. Prove that ∠ AOB = 90°.

Answer:



To prove- $\angle$ AOB = 90^0

Proof-

In $\triangle$ AOP and $\triangle$ AOC,

OA =OA [Common]

OP = OC [Both radii]

AP =AC [tangents from external point A]

Therefore by SSS congruence, $\triangle$ AOP $\cong$ $\triangle$ AOC

and by CPCT, $\angle$ PAO = $\angle$ OAC

$\Rightarrow \angle PAC = 2\angle OAC$ ....(i)

Similarly, from $\triangle$ OBC and $\triangle$ OBQ, we get;

$\angle$ QBC = 2. $\angle$ OBC.............(ii)

Adding eq (1) and eq (2)

$\angle$ PAC + $\angle$ QBC = 180

2($\angle$ OBC + $\angle$ OAC) = 180

($\angle$ OBC + $\angle$ OAC) = 90

Now, in $\triangle$ OAB,

Sum of interior angle is 180.

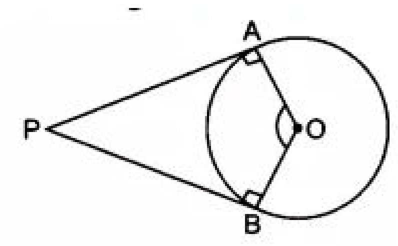
So, $\angle$ OBC + $\angle$ OAC + $\angle$ AOB = 180

$\therefore \angle$ AOB = 90

hence proved.

Q10 Prove that the angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line-segment joining the points of contact at the center.

Answer:



To prove - $\angle APB + \angle AOB = 180^0$

Proof-

We have, PA and PB are two tangents, B and A are the point of contacts of the tangent to a circle. And $OA \perp PA$, $OB \perp PB$ (since tangents and radius are perpendiculars)

According to question,

In quadrilateral PAOB,

$\angle$ OAP + $\angle$ APB + $\angle$ PBO + $\angle$ BOA = 360^0

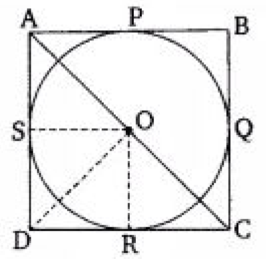
90 + $\angle$ APB + 90 + $\angle$ BOA = 360

$\angle APB + \angle AOB = 180^0$

Hence proved .

Q11 Prove that the parallelogram circumscribing a circle is a rhombus.

Answer:



To prove - the parallelogram circumscribing a circle is a rhombus

Proof-

ABCD is a parallelogram that circumscribes a circle with center O.

P, Q, R, S are the points of contacts on sides AB, BC, CD, and DA respectively

AB = CD .and AD = BC...........(i)

It is known that tangents drawn from an external point are equal in length.

RD = DS ....(ii)

RC = QC...........(iii)

BP = BQ...........(iv)

AP = AS ....(v)

By adding eq (ii) to eq (v) we get;

(RD + RC) + (BP + AP) = (DS + AS) + (BQ + QC)

CD + AB = AD + BC

$\Rightarrow$ 2AB = 2AD [from equation (i)]

$\Rightarrow$ AB = AD

Now, AB = AD and AB = CD

$\therefore$ AB = AD = CD = BC

Hence ABCD is a rhombus.

Q12 A triangle ABC is drawn to circumscribe a circle of radius 4 cm such that the segments BD and DC into which BC is divided by the point of contact D are of lengths 8 cm and 6 cm respectively (see Fig. 10.14). Find the sides AB and AC.

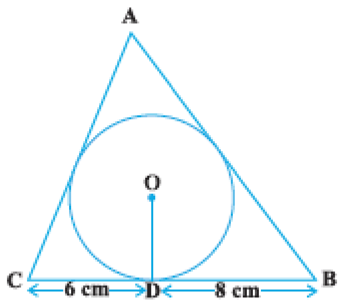
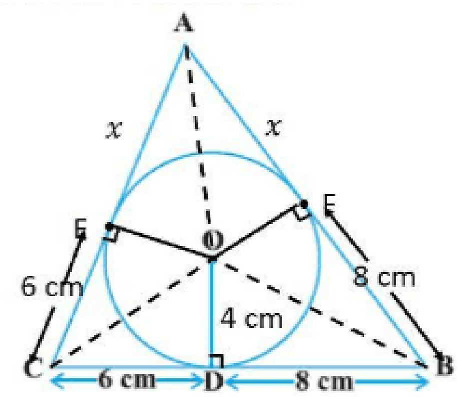


Fig. 10.14

Answer:



Consider the above figure. Assume center O touches the sides AB and AC of the triangle at point E and F respectively.

Let the length of AE is x.

Now in $\triangle ABC$,

$CF = CD = 6$ (tangents on the circle from point C)

$BE = BD = 6$ (tangents on the circle from point B)

$AE = AF = x$ (tangents on the circle from point A)

Now AB = AE + EB

Now

Area of triangle $\triangle ABC$

Now the area of $\triangle OBC$

$= (1/2) * OD * BC$

$= (1/2) * 4 * 14$

$= 56/2 = 28$

Area of $\triangle OCA$

$= (1/2) * OF * AC$

$= (1/2) * 4 * (6 + x)$

$= 2(6 + x)$

$= 12 + 2x$

Area of $\triangle OAB$

$= (1/2) * OE * AB$

$= (1/2) * 4 * (8 + x)$

$= 2(8 + x)$

$= 16 + 2x$

Now Area of the $\triangle ABC$ = Area of $\triangle OBC$ + Area of $\triangle OCA$ + Area of $\triangle OAB$

On squaring both the side, we get

Hence

AB = x + 8

=> AB = 7+8

=> AB = 15

AC = 6 + x

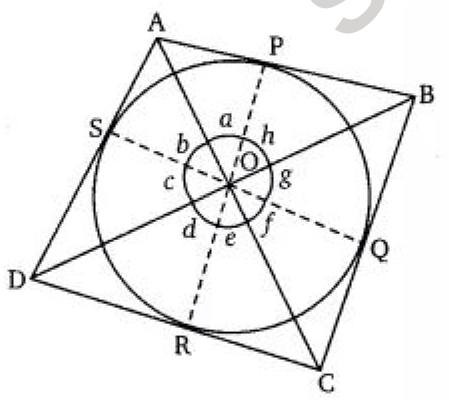
=> AC = 6 + 7

=> AC = 13

Answer- AB = 15 and AC = 13

Q13 Prove that opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the center of the circle.

Answer:



Given- ABCD is a quadrilateral circumscribing a circle. P, Q, R, S are the point of contact on sides AB, BC, CD, and DA respectively.

To prove-

$\angle AOB + \angle COD = 180^0$

$\angle AOD + \angle BOC = 180^0$

Proof -

Join OP, OQ, OR and OS

In triangle Δ DOS and Δ DOR,

OD =OD [common]

OS = OR [radii of same circle]

DR = DS [length of tangents drawn from an external point are equal]

By SSS congruency, Δ DOS $\cong$ Δ DOR,

and by CPCT, $\angle$ DOS = $\angle$ DOR

$\angle c = \angle d$ ....(i)

Similarily,

$\angle a = \angle b$

$\angle e = \angle f$

$\angle g = \angle h$ ....(2, 3, 4)

$\therefore 2(\angle a + \angle e + \angle h + \angle d) = 360^0$

$(\angle a + \angle e) + (\angle h + \angle d) = 180^0$

$\angle AOB + \angle DOC = 180^0$

SImilarily, $\angle AOD + \angle BOC = 180^0$

Hence proved.