NCERT solutions for class 10 maths chapter 13 Surface Areas and Volumes Excercise: 13.1

Q1 2 cubes each of volume 64 cm ^{3} are joined end to end. Find the surface area of the resulting cuboid.

Answer:

We are given that volume of the cube $= \ 64 \ cm^3$

Also, the volume of a cube is given by $= \ a^3$ (here a is the edge of the cube)

Thus : $a^3 \ = \ 64$

$a \ = \ 4 \ cm$

Now according to the question we have combined the two cubes that edge lengths of the formed cuboid are 4 cm, 4 cm, and 8 cm.

The surface area of a cuboid is : $= \ 2\left (lb \ + \ bh \ + \ hl \right)$

or $= \ 2\left (8\times 4 \ + 4\times 4 \ + \ 4\times 8 \right)$

or $= \ 2\left (80 \right)$

or $= \ 160 \ cm^2$

Thus the area of the formed cuboid is 160 cm ^{2} .

Q2 A vessel is in the form of a hollow hemisphere mounted by a hollow cylinder. The diameter of the hemisphere is 14 cm and the total height of the vessel is 13 cm. Find the inner surface area of the vessel.

Answer:

Since the vessel consists of hemisphere and cylinder, thus its area is given by :

Area of vessel = Inner area of the cylinder(curved) + Inner area of hemisphere

The inner surface area of the hemisphere is :

$= \ 2\pi r^2$

or $= \ 2 \times \left(\frac{22}{7}\right) \times 7^2$

or $= \ 308 \ cm^2$

And the surface area of the cylinder is :

$= \ 2\pi rh$

or $= \ 2 \times \frac{22}{7} \times 7 \times 6$

or $= \ 264 \ cm^2$

Thus the inner surface area of the vessel is $= 308 + 264 = 572 \, cm^2$.

Q3 A toy is in the form of a cone of radius 3.5 cm mounted on a hemisphere of the same radius. The total height of the toy is 15.5 cm. Find the total surface area of the toy.

Answer:

The required surface area of the toy is given by :

Area of toy = Surface area of hemisphere + Surface area of the cone

Firstly consider the hemisphere :

The surface area of a hemisphere is $= \ 2\pi r^2$

or $= 2 \times \frac{22}{7} \times (3.5)^2$

or $= 77\ cm^2$

Now for cone we have :

The surface area of a cone $= \pi r l$

Thus we need to calculate the slant of the cone.

We know that :

$l^2 = h^2 + r^2$

or $= 12^2 + 3.5^2$

or $= \frac{625}{4}$

or $l = \frac{25}{2} = 12.5\ cm$

Thus surface area of a cone $= \pi r l$

or $= \frac{22}{7} \times 3.5 \times 12.5$

or $= 137.5\ cm^2$

Hence the total surface area of toy = $= 77 + 137.5 = 214.5\ cm^2$

Q4 A cubical block of side 7 cm is surmounted by a hemisphere. What is the greatest diameter the hemisphere can have? Find the surface area of the solid.

Answer:

It is given that the hemisphere is mounted on the cuboid, thus the hemisphere can take on complete as its diameter (which is maximum).

Thus the greatest diameter of the hemisphere is 7 cm.

Now, the total surface area of solid = Surface area of cube + Surface area of the hemisphere - Area of the base of a hemisphere (as this is counted on one side of the cube)

The surface area of the cube is :

$$= 6a^3$$

$$= 6 \times 7^3 = 294\ cm^2$$

Now the area of a hemisphere is

$$= 2\pi r^2$$

$$= 2 \times \frac{22}{7} \times \left(\frac{7}{2}\right)^2 = 77\ cm^2$$

And the area of the base of a hemisphere is

$$= \pi r^2 = \frac{22}{7} \times \left(\frac{7}{2}\right)^2 = 38.5\ cm^2$$

Hence the surface area of solid is $= 294 + 77 - 38.5 = 332.5\ cm^2$.

Q5 A hemispherical depression is cut out from one face of a cubical wooden block such that the diameter l of the hemisphere is equal to the edge of the cube. Determine the surface area of the remaining solid.

Answer:

It is given that the diameter of the hemisphere is equal to the edge length of the cube.

The total surface area of solid is given by :

The surface area of solid = Surface area of cube + Surface area of the hemisphere - Area of the base of the hemisphere

The surface area of the cube $= \ 6l^2$

And surface area of the hemisphere:

$$= \ 2\pi r^2 \ = \ 2\pi\left(\frac{l}{2}\right)^2$$

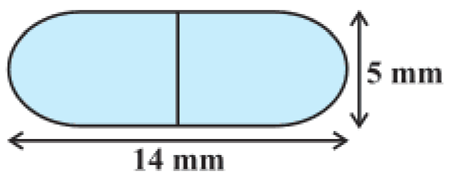
Area of base of the hemisphere:

$$= \ \pi r^2 \ = \ \pi\left(\frac{l}{2}\right)^2$$

Thus the area of solid is:

$$= \ 6l^2 \ + \ \pi\left(\frac{l}{2}\right)^2 \ unit^2$$

Q6 A medicine capsule is in the shape of a cylinder with two hemispheres stuck to each of its ends (see Fig.). The length of the entire capsule is 14 mm and the diameter of the capsule is 5 mm. Find its surface area.



Answer:

It is clear from the figure that the capsule has hemisphere and cylinder structure.

The surface area of capsule = 2 (Area of the hemisphere) + Area of the cylindrical part

Area of hemisphere $= \ 2\pi r^2$

or $= \ 2\pi \times \left(\frac{5}{2}\right)^2$

or $= \ \frac{25}{2}\pi \ mm^2$

And the area of the cylinder $= \ 2\pi rh$

or $= \ 2\pi \times \frac{5}{2} \times 9$

or $= \ 45\pi \ mm^2$

Thus the area of the solid is :

$$= \ 2\left(\frac{25}{2}\right)\pi \ + \ 45\pi$$

$$= \ 25\pi \ + \ 45\pi$$

$$= \ 70\pi$$

$$= \ 220 \ mm^2$$

Q7 A tent is in the shape of a cylinder surmounted by a conical top. If the height and diameter of the cylindrical part are 2.1 m and 4 m respectively, and the slant height of the top is 2.8 m, find the area of the canvas used for making the tent. Also, find the cost of the canvas of the tent at the rate of Rs 500 per m ^{2}. (Note that the base of the tent will not be covered with canvas.)

Answer:

The canvas will cover the cylindrical part as well as the conical part.

So, the area of canvas = Area of cylindrical part (curved) + Area of the conical part

Now, the area of the cylindrical part is $= \ 2\pi rh$

or $= \ 2\pi \times 2 \times 2.1$

or $= \ 8.4\pi \ m^2$

And the area of the cone is $= \ \pi rl$

or $= \ \pi \times 2 \times 2.8$

or $= \ 5.6\pi \ m^2$

Thus, the area of the canvas $= \ 8.4\pi \ + \ 5.6\pi$

or $= \ 14\pi \ = \ 44 \ m^2$

Further, it is given that the rate of canvas per m ^{2} is = Rs. 500.

Thus the required money is $= \ 500 \times 44 \ = \ Rs. \ 22,000$

Q8 From a solid cylinder whose height is 2.4 cm and diameter 1.4 cm, a conical cavity of the same height and same diameter is hollowed out. Find the total surface area of the remaining solid to the nearest cm ^{2}.

Answer:

Firstly we need to calculate the slant height of the cone :

$l^2 \ = \ r^2 \ + \ h^2$

or $= \ (0.7)^2 \ + \ (2.4)^2$

or $l^2 = 6.25$

or $l = 2.5\ cm$

Now, the total surface area of solid can be calculated as :

The surface area of solid = Surface area of cylinder + Surface area of cone + Area of base of the cylinder

The surface area of the cylinder is $= 2\pi rh$

or $= 2\pi \times 0.7 \times 2.4$

or $= 10.56\ cm^2$

Now, the surface area of a cone $= \pi rl$

or $= \pi \times 0.7 \times 2.5$

or $= 5.50\ cm^2$

And the area of the base of the cylinder is $= \pi r^2$

or $= \pi \times 0.7 \times 0.7$

or $= 1.54\ cm^2$

Thus required area of solid = 10. 56 + 5.50 + 1.54 = 17.60 cm ^{2}.

Thus total surface area of remaining solid to nearest cm ^{2} is 18 cm ^{2}.

Q9 A wooden article was made by scooping out a hemisphere from each end of a solid cylinder, as shown in Fig. 13.11. If the height of the cylinder is 10 cm, and its base is of radius 3.5 cm, find the total surface area of the article.

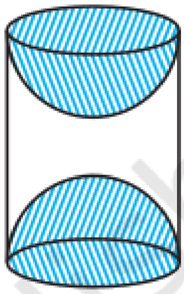


Fig. 13.11

Answer:

The required surface area is given by :

The surface area of article = Surface area of cylindrical part + 2 (Surface area of the hemisphere)

Now, the area of the cylinder $= \ 2\pi rh$

or $= \ 2\pi \times 3.5 \times 10$

or $= \ 70\pi \ cm^2$

And the surface area of the hemisphere : $= \ 2\pi r^2$

or $= \ 2\pi \times 3.5 \times 3.5$

or $= \ 24.5\pi \ cm^2$

Thus the required area $= \ 70\pi \ + \ 2(24.5\pi) \ = \ 374 \ cm^2$

NCERT solutions for class 10 maths chapter 13 Surface Areas and Volumes

Excercise: 13.2

Q1 A solid is in the shape of a cone standing on a hemisphere with both their radii being equal to 1 cm and the height of the cone is equal to its radius. Find the volume of the solid in terms of π .

Answer:

The volume of the solid is given by :

The volume of solid = Volume of cone + Volume of a hemisphere

The volume of cone :

$$= \frac{1}{3}\pi r^2 h$$

or $= \frac{1}{3}\pi \times 1^2 \times 1$

or $= \frac{\pi}{3}\ cm^3$

And the volume of the hemisphere :

$$= \frac{2}{3}\pi r^3$$

or $= \frac{2}{3}\pi \times 1^3$

or $= \frac{2\pi}{3}\ cm^3$

Hence the volume of solid is :

$$= \frac{\pi}{3} + \frac{2\pi}{3} = \pi\ cm^3$$

Q2 Rachel, an engineering student, was asked to make a model shaped like a cylinder with two cones attached at its two ends by using a thin aluminum sheet. The diameter of the model is 3 cm and its length is 12 cm. If each cone has a height of 2 cm, find the volume of air contained in the model that Rachel made. (Assume the outer and inner dimensions of the model to be nearly the same.)

Answer:

The volume of air present = Volume of cylinder + 2 (Volume of a cone)

Now, the volume of a cylinder : $= \pi r^2 h$

or $= \pi \left(\frac{3}{2}\right)^2 \times 8$

or $= 18\pi \ cm^3$

And the volume of a cone is :

$= \frac{1}{3}\pi r^2 h$

or $= \frac{1}{3}\pi \times \left(\frac{3}{2}\right)^2 \times 2$

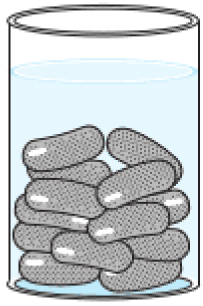
or $= \frac{3}{2}\pi \ cm^3$

Thus the volume of air is :

$= 18\pi + 2 \times \frac{3}{2}\pi = 21\pi$

or $= 66 \ cm^3$

Q3 A gulab jamun contains sugar syrup up to about 30% of its volume. Find approximately how much syrup would be found in 45 gulab jamuns, each shaped like a cylinder with two hemispherical ends with length 5 cm and diameter 2.8 cm (see Fig).



Answer:

It is clear from the figure that gulab jamun has one cylindrical part and two hemispherical parts.

Thus, the volume of gulab jamun is = Volume of cylindrical part + 2 (Volume of the hemisphere)

Now, the volume of the cylinder is $= \pi r^2 h$

or $= \pi \times 1.4^2 \times 2.2$

or $= 13.55\ cm^3$

And the volume of a hemisphere is :

$= \frac{2}{3}\pi r^3$

$= \frac{2}{3} \times \pi \times (1.4)^3$

$= 5.75\ cm^3$

Thus the volume of 1 gulab jamun is $= 13.55 + 2(5.75) = 25.05cm^3$.

Hence the volume of 45 gulab jamun $= 45(25.05)\ cm^3 = 1127.25\ cm^3$

Further, it is given that one gulab jamun contains sugar syrup upto $30\ \%$.

So, the total volume of sugar present :

$$= \frac{30}{100} \times 1127.25 = 338\ cm^3$$

Q4 A pen stand made of wood is in the shape of a cuboid with four conical depressions to hold pens. The dimensions of the cuboid are 15 cm by 10 cm by 3.5 cm. The radius of each of the depressions is 0.5 cm and the depth is 1.4 cm. Find the volume of wood in the entire stand (see Fig. 13.16).

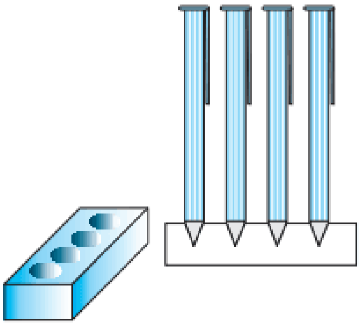


Fig. 13.16

Answer:

The volume of wood is given by = volume of the cuboid - the volume of four cones.

Firstly, the volume of cuboid : $= \ lbh$

or $= \ 15 \times 10 \times 3.5$

or $= \ 525\ cm^3$

And, the volume of cone :

$$= \frac{1}{3}\pi r^2 h$$
$$= \frac{1}{3}\pi \times (0.5)^2 \times 1.4$$
$$= 0.3665\ cm^3$$

Thus the volume of wood is $= 525 + 4(0.3665) = 523.53\, cm^3$

Q5 A vessel is in the form of an inverted cone. Its height is 8 cm and the radius of its top, which is open, is 5 cm. It is filled with water up to the brim. When lead shots, each of which is a sphere of radius 0.5 cm are dropped into the vessel, one-fourth of the water flows out. Find the number of lead shots dropped in the vessel.

Answer:

According to the question :

Water spilled from the container = Volume of lead balls.

Let us assume the number of lead balls to be n.

Thus the equation becomes :

$$\frac{1}{4} \times Volume_{cone} = n \times \frac{4}{3}\pi r^3$$

or $\frac{1}{4} \times \frac{1}{3}\pi \times 5^2 \times 8 = n \times \frac{4}{3}\pi \times 0.5^3$

or $n = \frac{25 \times 8}{16 \times \left(\frac{1}{2}\right)^3}$

or $n = 100$

Hence the number of lead shots dropped is 100.

Q6 A solid iron pole consists of a cylinder of height 220 cm and base diameter 24 cm, which is surmounted by another cylinder of height 60 cm and radius 8 cm. Find the mass of the pole, given that 1 cm 3 of iron has approximately 8g mass. (Use $\pi = 3.14$)

Answer:

The pole can be divided into one large cylinder and one small cylinder.

Thus, the volume of pole = volume of large cylinder + volume of a small cylinder

$= \pi r_l^2 h_l + \pi r_s^2 h_s$

or $= \pi \times 12^2 \times 220 + \pi \times 8^2 \times 60$

or $= \pi \times (144 \times 220 + 64 \times 60)$

or $= 3.14 \times 35520$

or $= 111532.5\ cm^3$

Now, according to question mass of the pole is :

$= 8 \times 111532.5$

or $= 892262.4\ g = 892.262\ Kg$

Q7 A solid consisting of a right circular cone of height 120 cm and radius 60 cm standing on a hemisphere of radius 60 cm is placed upright in a right circular cylinder full of water such that it touches the bottom. Find the volume of water left in the cylinder, if the radius of the cylinder is 60 cm and its height is 180 cm.

Answer:

It is clear from the question that the required volume is :

The volume of water (left) =Volume of a cylinder - Volume of solid

Now the volume of the cylinder is $= \pi r^2 h$

or $= \pi \times (60)^2 \times 180\ cm^3$

And the volume of solid is :

Thus the volume of water left :

Q8 A spherical glass vessel has a cylindrical neck 8 cm long, 2 cm in diameter; the diameter of the spherical part is 8.5 cm. By measuring the amount of water it holds, a child finds its volume to be 345 cm 3 . Check whether she is correct, taking the above as the inside measurements, and $\pi = 3.14$.

Answer:

The volume of the vessel is given by :

The volume of vessel = Volume of sphere + Volume of the cylindrical part

Now, the volume of the sphere is :

$$= \frac{4}{3}\pi r^3$$
$$= \frac{4}{3}\pi\left(\frac{8.5}{2}\right)^3$$
$$= 321.55\ cm^3$$

And the volume of the cylinder is:-

$$= \pi r^2 h$$

$$= \pi \times (1)^2 \times 8$$

$$= 25.13\ cm^3$$

Thus the volume of the vessel is $= 321.55 + 25.13 = 346.68\, cm^3$

NCERT solutions for class 10 maths chapter 13 Surface Areas and Volumes

Excercise: 13.3

Q1 A metallic sphere of radius 4.2 cm is melted and recast into the shape of a cylinder of radius 6 cm. Find the height of the cylinder.

Answer:

Let us assume the height of the cylinder to be h.

Since the material is melted and recast thus its volume will remain the same.

So, Volume of sphere = Volume of obtained cylinder.

$$\frac{4}{3}\pi r_s^3 \;=\; \pi r_c^2 h$$

$$\frac{4}{3}\pi \times 4.2^3 \;=\; \pi \times 6^2 \times h$$

$$h \;=\; \frac{4}{3} \times \frac{4.2 \times 4.2 \times 4.2}{36}$$

$$h \;=\; 2.74\ cm$$

Hence the height of the cylinder is 2.74 cm.

Q2 Metallic spheres of radii 6 cm, 8 cm, and 10 cm, respectively, are melted to form a single solid sphere. Find the radius of the resulting sphere.

Answer:

According to the question, small spheres are melted and converted into a bigger sphere. Thus the sum of their volume is equal to the volume of the bigger sphere.

The volume of 3 small spheres = Volume of bigger sphere

Let us assume the radius of the bigger sphere is r.

$$\frac{4}{3}\pi\left(r_1^3 + r_2^3 + r_3^3\right) = \frac{4}{3}\pi r^3$$

$$r_1^3 + r_2^3 + r_3^3 = r^3$$

$$r^3 = 6^3 + 8^3 + 10^3$$

$$r = 12\ cm$$

Hence the radius of the sphere obtained is 12 cm.

Q3 A 20 m deep well with diameter 7 m is dug and the earth from digging is evenly spread out to form a platform 22 m by 14 m. Find the height of the platform.

Answer:

According to the question, the volume of soil dug will be equal to the volume of the platform created.

Thus we can write :

The volume of soil dug = Volume of platform

Thus the height of the platform created is 2.5 m.

Q4 A well of diameter 3 m is dug 14 m deep. The earth taken out of it has been spread evenly all around it in the shape of a circular ring of width 4 m to form an embankment. Find the height of the embankment.

Answer:

According to the question, the volume is conserved here :

The volume of soil dug out = Volume of the embankment made.

Let the height of the embankment is h.

Hence the height of the embankment made is 1.125 m.

Q5 A container shaped like a right circular cylinder having a diameter of 12 cm and a height 15 cm is full of ice cream. The ice cream is to be filled into cones of height 12 cm and diameter 6 cm, having a hemispherical shape on the top. Find the number of such cones which can be filled with ice cream.

Answer:

Let the number of cones that can be filled with ice cream be n.

Then we can write :

The volume of a cylinder containing ice cream = n (volume of 1 ice cream cone)

Hence the number of cones that can be filled is 10.

Q6 How many silver coins, 1.75 cm in diameter and of thickness 2 mm, must be melted to form a cuboid of dimensions 5.5 cm × 10 cm × 3.5 cm?

Answer:

Let us assume the number of coins that need to be melted be n.

Then we can write :

The volume of n coins = Volume of cuboid formed.

$$n\left(\pi r^2 h\right) = lbh'$$

$$n\left(\pi \times \left(\frac{1.75}{2}\right)^2 \times 0.2\right) = 5.5 \times 10 \times 3.5$$

$$n = 400$$

Thus the required number of coins is 400.

Q7 A cylindrical bucket, 32 cm high and with radius of base 18 cm, is filled with sand. This bucket is emptied on the ground and a conical heap of sand is formed. If the height of the conical heap is 24 cm, find the radius and slant height of the heap.

Answer:

According to question volume will remain constant thus we can write :

The volume of bucket = Volume of heap formed.

$$\pi r_1^2 h_1 = \frac{1}{3}\pi r_2^2 h_2$$

Let the radius of heap be r.

$$\pi \times 18^2 \times 32 = \frac{1}{3} \times \pi \times r^2 \times 24$$

$$r = 18 \times 2 = 36\ cm$$

And thus the slant height will be

$$l = \sqrt{r^2 + h^2}$$

$$= \sqrt{36^2 + 24^2}$$

$$= 12\sqrt{13}\ cm$$

Hence the radius of heap made is 36 cm and its slant height is $12\sqrt{13}\ cm$.

Q8 Water in a canal, 6 m wide and 1.5 m deep, is flowing with a speed of 10 km/h. How much area will it irrigate in 30 minutes, if 8 cm of standing water is needed?

Answer:

Speed of water is: 10 Km/hr

And the volume of water flow in 1 minute is :

$$= 9 \times \frac{10000}{60} = 1500\ m^3$$

Thus the volume of water flow in 30 minutes will be : $= 1500 \times 30 = 45000\ m^3$

Let us assume irrigated area be A. Now we can equation the expression of volumes as the volume will remain the same.

$$45000 = \frac{A \times 8}{100}$$

$$A = 562500\ m^2$$

Thus the irrigated area is $562500\ m^2$.

Q9 A farmer connects a pipe of internal diameter 20 cm from a canal into a cylindrical tank in her field, which is 10 m in diameter and 2 m deep. If water flows through the pipe at the rate of 3 km/h, in how much time will the tank be filled?

Answer:

Area of the cross-section of pipe is $= \pi r^2$

$$= \pi(0.1)^2 = 0.01\pi\ m^2$$

Speed of water is given to be = 3 km/hr

Thus, the volume of water flowing through a pipe in 1 min. is

$$= \frac{3000}{60} \times 0.01\pi$$

$$= 0.5\pi \ m^3$$

Now let us assume that the tank will be completely filled after t minutes.

Then we write :

$$t \times 0.5\pi \ = \ \pi r^2 h$$

$$t \times 0.5 \ = \ 5^2 \times 2$$

$$t \ = \ 100$$

Hence the time required for filling the tank completely in 100 minutes.

NCERT solutions for class 10 maths chapter 13 Surface Areas and Volumes Excercise: 13.4

Q1 A drinking glass is in the shape of a frustum of a cone of height 14 cm. The diameters of its two circular ends are 4 cm and 2 cm. Find the capacity of the glass.

Answer:

The capacity of glass is the same as the volume of glass.

Thus the volume of glass :

$$= \frac{1}{3}\pi h \left(r_1^2 \ + \ r_2^2 \ + \ r_1 r_2\right)$$

$$= \frac{1}{3}\pi h \left(2^2 \ + \ 1^2 \ + \ 2 \times 1\right)$$

$$= \frac{308}{3} \ cm^3$$

= Capacity of glass

Q2 The slant height of a frustum of a cone is 4 cm and the perimeters (circumference) of its circular ends are 18 cm and 6 cm. Find the curved surface area of the frustum.

Answer:

We are given the perimeter of upper and lower ends thus we can find r_1 and r_2.

$$2\pi r_1 = 18$$

$$r_1 = \frac{9}{\pi} cm$$

And,

$$2\pi r_2 = 6$$

$$r_2 = \frac{3}{\pi} cm$$

Thus curved surface area of the frustum is given by : $= \pi (r_1 + r_2) l$

$$= \pi \left(\frac{9}{\pi} + \frac{3}{\pi}\right) 4$$

$$= 48\ cm^2$$

Q3 A *fez* , the cap used by the Turks, is shaped like the frustum of a cone (see Fig.). If its radius on the open side is 10 cm, radius at the upper base is 4 cm and its slant height is 15 cm, find the area of material used for making it .

Answer:

The area of material used is given by :

Area of material = Curved surface area of a frustum of cone + Area of upper end

$= \pi (r_1 + r_2) l + \pi r_2^2$

$= \pi (10 + 4) 15 + \pi \times 4^2$

$= \dfrac{226 \times 22}{7}$

$= 710.28 \; cm^2$

Q4 A container opened from the top and made up of a metal sheet, is in the form of a frustum of a cone of height 16 cm with radii of its lower and upper ends as 8 cm and 20 cm, respectively. Find the cost of the milk which can completely fill the container, at the rate of Rs 20 per litre. Also find the cost of metal sheet used to make the container, if it costs Rs 8 per 100 cm 2 . (Take $\pi = 3.14$)

Answer:

Firstly we will calculate the slant height of the cone :

$l^2 = (r_1^2 - r_2^2) + h^2$

$l^2 = (20^2 - 8^2) + 16^2$

$l = 20\ cm$

Now, the volume of the frustum is :

$$= \frac{1}{3}\pi h\left(r_1^2 + r_2^2 + r_1 r_2\right)$$

$$= \frac{1}{3}\pi \times 16\left(20^2 + 8^2 + 20 \times 8\right)$$

$$= 10449.92\ cm^3$$

= Capacity of the container.

Now, the cost of 1-liter milk is Rs. 20.

Then the cost of 10.449-liter milk will be $= 10.45 \times 20 = Rs.\ 209$

The metal sheet required for the container is :

$$= \pi(r_1 + r_2)l + \pi r_2^2$$

$$= \pi(20 + 8)20 + \pi \times 8^2$$

$$= 624\pi\ cm^2$$

Thus cost for metal sheet is

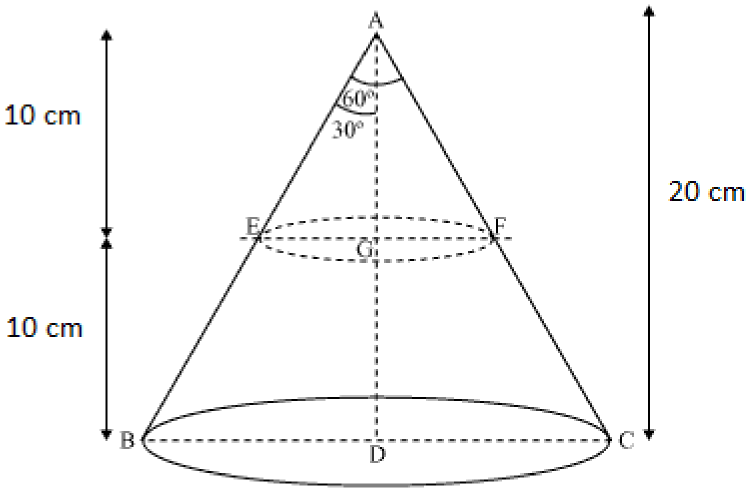
$$= 624\pi \times \frac{8}{100}$$

$$= Rs.\ 156.57$$

Q5 A metallic right circular cone 20 cm high and whose vertical angle is 60° is cut into two parts at the middle of its height by a plane parallel to its base. If the frustum so obtained be drawn into a wire of diameter $\frac{1}{16}\text{cm}$, find the length of the wire.

Answer:

The figure for the problem is shown below :



Using geometry we can write :

$EG = \frac{10\sqrt{3}}{3}\ cm$ and $BD = \frac{20\sqrt{3}}{3}\ cm$

Thus the volume of the frustum is given by :

Now, the radius of the wire is :

$$= \frac{1}{16} \times \frac{1}{2} = \frac{1}{32}\ cm$$

Thus the volume of wire is given by : $= \pi r^2 \times l$

$$= \pi \times \left(\frac{1}{32}\right)^2 \times l$$

Now equating volume of frustum and wire, we get :

$$\frac{22000}{9} = \pi \times \left(\frac{1}{32}\right)^2 \times l$$

$$l = 796444.44\ cm$$

$$l = 7964.44\ m$$

Thus the length of wire drawn is 7964.44 m.

NCERT solutions for class 10 maths chapter 13 Surface Areas and Volumes

Excercise: 13.5

Q1 A copper wire, 3 mm in diameter, is wound about a cylinder whose length is 12 cm, and diameter 10 cm, so as to cover the curved surface of the cylinder. Find the length and mass of the wire, assuming the density of copper to be 8.88 g per cm ^{3} .

Answer:

A number of rounds are calculated by :

$$= \frac{Height\ of\ cylinder}{Diameter\ of\ wire}$$

$$= \frac{12}{0.3} = 40\ rounds$$

Thus the length of wire in 40 rounds will be $= 40 \times 2\pi \times 5 = 400\pi\ cm = 12.57\ m$

And the volume of wire is: Area of cross-section $\times$ Length of wire

$$= \pi \times (0.15)^2 \times 1257.14$$

$$= 88.89\ cm^3$$

Hence the mass of wire is. $= \ 88.89 \ \times 8.88 \ = \ 789.41 \ gm$

Q2 A right triangle, whose sides are 3 cm and 4 cm (other than hypotenuse) is made to revolve about its hypotenuse. Find the volume and surface area of the double cone so formed. (Choose value of π as found appropriate.)

Answer:

The volume of the double cone will be = Volume of cone 1 + Volume of cone 2.

$$= \frac{1}{3}\pi r^2 h_1 + \frac{1}{3}\pi r^2 h_2$$

$= \frac{1}{3}\pi \times 2.4^2 \times 5$ (Note that sum of heights of both the cone is 5 cm - hypotenuse).

$$= 30.14 \ cm^3$$

Now the surface area of a double cone is :

$$= \pi r l_1 + \pi r l_2$$

$$= \pi \times 2.4\,(4 + 3)$$

$$= 52.8 \ cm^2$$

Q3 A cistern, internally measuring 150 cm × 120 cm × 110 cm, has 129600 cm ^{3} of water in it. Porous bricks are placed in the water until the cistern is full to the brim. Each brick absorbs one-seventeenth of its own volume of water. How many bricks can be put in without overflowing the water, each brick being 22.5 cm × 7.5 cm × 6.5 cm?

Answer:

The total volume of the cistern is : $= \ 150 \times 120 \times 110 \ = \ 1980000 \ cm^3$

And the volume to be filled in it is $= 1980000 - 129600 \; = \; 1850400 \; cm^3$

Now let the number of bricks be n.

Then the volume of bricks : $= n \times 22.5 \times 7.5 \times 6.5 \; = \; 1096.87n \; cm^3$

Further, it is given that brick absorbs one-seventeenth of its own volume of water.

Thus water absorbed :

$$= \frac{1}{17} \times 1096.87n \; cm^3$$

Hence we write :

$$1850400 \; + \; \frac{1}{17}(1096.87n) \; = \; 1096.87n$$

$$n \; = \; 1792.41$$

Thus the total number of bricks is 1792.

Q4 In one fortnight of a given month, there was a rainfall of 10 cm in a river valley. If the area of the valley is 7280 km ^{2}, show that the total rainfall was approximately equivalent to the addition to the normal water of three rivers each 1072 km long, 75 m wide and 3 m deep.

Answer:

Firstly we will calculate the volume of rainfall :

Volume of rainfall :

$$= \; 7280 \times \frac{10}{100 \times 1000}$$

$$= \; 0.7280 \; Km^3$$

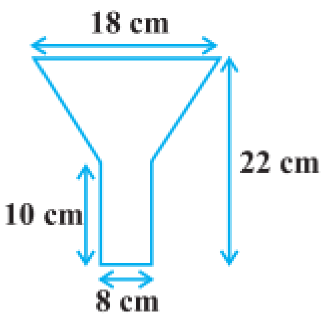
And the volume of the three rivers is :

$$= 3\left(1072 \times \frac{75}{1000} \times \frac{3}{1000}\right)\ Km^3$$

$$= 0.7236\ Km^3$$

It can be seen that both volumes are approximately equal to each other.

Q5 An oil funnel made of tin sheet consists of a 10 cm long cylindrical portion attached to a frustum of a cone. If the total height is 22 cm, the diameter of the cylindrical portion is 8 cm and the diameter of the top of the funnel is 18 cm, find the area of the tin sheet required to make the funnel (see Fig.).



Answer:

From this, we can write the values of both the radius (upper and lower) and the height of the frustum.

Thus slant height of frustum is :

$$= \sqrt{(r_1 - r_2)^2 + h^2}$$

$$= \sqrt{(9 - 4)^2 + 12^2}$$

$$= 13\ cm$$

Now, the area of the tin shed required :

= Area of frustum + Area of the cylinder

$$= \pi(r_1 + r_2)l + 2\pi r_2 h$$

$$= \pi(9 + 4)13 + 2\pi \times 4 \times 10$$

$$= 782.57\ cm^2$$

Q6 Derive the formula for the curved surface area and total surface area of the frustum of a cone, given to you in Section 13.5, using the symbols as explained.

Answer:

In the case of the frustum, we can consider:- removing a smaller cone (upper part) from a larger cone.

So the CSA of frustum becomes:- CSA of bigger cone - CSA of the smaller cone

And the total surface area of the frustum is = CSA of frustum + Area of upper circle and area of lower circle.

$$= \pi l(r_1 + r_2) + \pi r_1^2 + \pi r_2^2$$

Q7 Derive the formula for the volume of the frustum of a cone, given to you in Section 13.5, using the symbols as explained.

Answer:

Similar to how we find the surface area of the frustum.

The volume of the frustum is given by-

=Volume of the bigger cone - Volume of the smaller cone

$$= \frac{1}{3}\pi r_1^2 h_1 \ - \ \frac{1}{3}\pi r_2^2 (h_1 \ - \ h)$$

$$= \frac{\pi}{3}\left(r_1^2 h_1 \ - \ r_2^2 (h_1 \ - \ h)\right)$$

$$= \frac{\pi}{3}\left(\frac{hr_1^3}{r_1 \ - \ r_2} \ - \ \frac{hr_2^3}{r_1 \ - \ r_2}\right)$$

$$= \frac{\pi}{3} h \left(\frac{r_1^3 \ - \ r_2^3}{r_1 \ - \ r_2}\right)$$

$$= \frac{1}{3}\pi h \left(r_1^2 \ + \ r_2^2 \ + \ r_1 r_2\right)$$