Chapter 4 - Quadratic Equations Exercise Ex. 4.1

Solution 1

(i) $(x+1)^2 = 2(x-3) \Rightarrow x^2 + 2x + 1 = 2x - 6 \Rightarrow x^2 + 7 = 0$

It is of the form $ax^2 + bx + c = 0$.
Hence, the given equation is a quadratic equation.

(ii) $x^2 - 2x = (-2)(3-x) \Rightarrow x^2 - 2x = -6 + 2x \Rightarrow x^2 - 4x + 6 = 0$

It is of the form $ax^2 + bx + c = 0$.
Hence, the given equation is a quadratic equation.

(iii) $(x-2)(x+1) = (x-1)(x+3) \Rightarrow x^2 - x - 2 = x^2 + 2x - 3 \Rightarrow 3x - 1 = 0$

It is not of the form $ax^2 + bx + c = 0$.
Hence, the given equation is not a quadratic equation.

(iv) $(x-3)(2x+1) = x(x+5) \Rightarrow 2x^2 - 5x - 3 = x^2 + 5x \Rightarrow x^2 - 10x - 3 = 0$

It is of the form $ax^2 + bx + c = 0$.
Hence, the given equation is a quadratic equation.

(v) $(2x-1)(x-3) = (x+5)(x-1) \Rightarrow 2x^2 - 7x + 3 = x^2 + 4x - 5 \Rightarrow x^2 - 11x + 8 = 0$

It is of the form $ax^2 + bx + c = 0$.
Hence, the given equation is a quadratic equation.

(vi) $x^2 + 3x + 1 = (x-2)^2 \Rightarrow x^2 + 3x + 1 = x^2 + 4 - 4x \Rightarrow 7x - 3 = 0$

It is not of the form $ax^2 + bx + c = 0$.
Hence, the given equation is not a quadratic equation.

(vii) $(x+2)^3 = 2x(x^2-1) \Rightarrow x^3 + 8 + 6x^2 + 12x = 2x^3 - 2x \Rightarrow x^3 - 14x - 6x^2 - 8 = 0$

It is not of the form $ax^2 + bx + c = 0$.
Hence, the given equation is not a quadratic equation.

(viii) $x^3 - 4x^2 - x + 1 = (x-2)^3 \Rightarrow x^3 - 4x^2 - x + 1 = x^3 - 8 - 6x^2 + 12x \Rightarrow 2x^2 - 13x + 9 = 0$

It is of the form $ax^2 + bx + c = 0$.
Hence, the given equation is a quadratic equation.

Concept Insight: Remember the general form of a quadratic equation $ax^2+bx+c=0$ where a, b, c are real numbers and a is non zero.

Solution 2

(i)
Let the breadth of the plot be x m.
Hence, the length of the plot is $(2x + 1)$ m.
Area of a rectangle = Length × Breadth $\therefore 528 = x(2x + 1)$

$\Rightarrow 2x^2 + x - 528 = 0$

(ii)
Let the consecutive integers be x and $x + 1$.
It is given that their product is 306. $\therefore x(x+1) = 306 \Rightarrow x^2 + x - 306 = 0$

(iii)
Let Rohan's age be x.
Hence, his mother's age = $x + 26$
3 years hence,
Rohan's age = $x + 3$
Mother's age = $x + 26 + 3 = x + 29$
It is given that the product of their ages after 3 years is 360.

$\therefore (x+3)(x+29) = 360$

$\Rightarrow x^2 + 32x - 273 = 0$

(iv) Let the speed of train be x km/h.

Time taken to travel 480 km = $\frac{480}{x}$ hrs

In second condition, let the speed of train = $(x-8)$ km/h
It is also given that the train will take 3 more hours to cover the same distance.

Therefore, time taken to travel 480 km = $\left(\frac{480}{x} + 3\right)$ hrs

Speed × Time = Distance

$$(x-8)\left(\frac{480}{x} + 3\right) = 480$$

$$\Rightarrow 480 + 3x - \frac{3840}{x} - 24 = 480$$

$$\Rightarrow 3x - \frac{3840}{x} = 24$$

$$\Rightarrow 3x^2 - 24x - 3840 = 0$$

$$\Rightarrow x^2 - 8x - 1280 = 0$$

Concept Insight: Read the question carefully and choose the variable to represent what needs to found. Only one variable must be there in the final equation.

Chapter 4 - Quadratic Equations Exercise Ex. 4.2

Solution 1

(i) $x^2 - 3x - 10$

$= x^2 - 5x + 2x - 10$

$= x(x-5) + 2(x-5)$

$= (x-5)(x+2)$

Roots of this equation are the values for which $(x-5)(x+2) = 0$

$\therefore$ $x - 5 = 0$ or $x + 2 = 0$

i.e., $x = 5$ or $x = -2$

(ii) $2x^2 + x - 6$

$= 2x^2 + 4x - 3x - 6$

$= 2x(x+2) - 3(x+2)$

$= (x+2)(2x-3)$

Roots of this equation are the values for which $(x+2)(2x-3) = 0$

$\therefore$ $x + 2 = 0$ or $2x - 3 = 0$

i.e., $x = -2$ or $x = \frac{3}{2}$

(iii) $\sqrt{2}x^2 + 7x + 5\sqrt{2}$

$= \sqrt{2}x^2 + 5x + 2x + 5\sqrt{2}$

$= x(\sqrt{2}x + 5) + \sqrt{2}(\sqrt{2}x + 5)$

$= (\sqrt{2}x + 5)(x + \sqrt{2})$

Roots of this equation are the values for which $(\sqrt{2}x + 5)(x + \sqrt{2}) = 0$

$\therefore$ $\sqrt{2}x + 5 = 0$ or $x + \sqrt{2} = 0$

i.e., $x = \frac{-5}{\sqrt{2}}$ or $x = -\sqrt{2}$

(iv) $2x^2 - x + \frac{1}{8}$

$= \frac{1}{8}(16x^2 - 8x + 1)$

$= \frac{1}{8}(16x^2 - 4x - 4x + 1)$

$= \frac{1}{8}(4x(4x-1) - 1(4x-1))$

$= \frac{1}{8}(4x-1)^2$

Roots of this equation are the values for which $(4x-1)^2 = 0$

Therefore, $(4x-1) = 0$ or $(4x-1) = 0$

i.e., $x = \frac{1}{4}$ or $x = \frac{1}{4}$

(v) $100x^2 - 20x + 1$

$= 100x^2 - 10x - 10x + 1$

$= 10x(10x-1) - 1(10x-1)$

$= (10x-1)^2$

Roots of this equation are the values for which $(10x-1)^2 = 0$

Therefore, $(10x-1) = 0$ or $(10x-1) = 0$

i.e., $x = \frac{1}{10}$ or $x = \frac{1}{10}$

Concept Insight: Two solve a quadratic equation by factorisation method two

Solution 3

Let the first number be x and the second number is $27 - x$.
Therefore, their product = $x\,(27 - x)$
It is given that the product of these numbers is 182.

Therefore, $x\left(27 - x\right) = 182$

$\Rightarrow x^2 - 27x + 182 = 0$

$\Rightarrow x^2 - 13x - 14x + 182 = 0$

$\Rightarrow x\left(x - 13\right) - 14\left(x - 13\right) = 0$

$\Rightarrow \left(x - 13\right)\left(x - 14\right) = 0$

Either $x - 13 = 0$ or $x - 14 = 0$
i.e., $x = 13$ or $x = 14$
If first number = 13, then
Other number = $27 - 13 = 14$
If first number = 14, then
Other number = $27 - 14 = 13$
Therefore, the numbers are 13 and 14.

Solution 4

Given that $x^2 + \left(x + 1\right)^2 = 365$

$\Rightarrow x^2 + x^2 + 1 + 2x = 365$

$\Rightarrow 2x^2 + 2x - 364 = 0$

$\Rightarrow x^2 + x - 182 = 0$

$\Rightarrow x^2 + 14x - 13x - 182 = 0$

$\Rightarrow x\left(x + 14\right) - 13\left(x + 14\right) = 0$

$\Rightarrow \left(x + 14\right)\left(x - 13\right) = 0$

Either $x + 14 = 0$ or $x - 13 = 0$, i.e., $x = -14$ or $x = 13$
Since the integers are positive, x can only be 13.
$\therefore x + 1 = 13 + 1 = 14$
Therefore, two consecutive positive integers will be 13 and 14.

Solution 5

Let the base of the right triangle be x cm.
Its altitude = $(x - 7)$ cm
From pythagoras theorem,
$\text{Base}^2 + \text{Altitude}^2 = \text{Hypotenuse}^2$

$\therefore x^2 + (x-7)^2 = 13^2$

$\Rightarrow x^2 + x^2 + 49 - 14x = 169$

$\Rightarrow 2x^2 - 14x - 120 = 0$

$\Rightarrow x^2 - 7x - 60 = 0$

$\Rightarrow x^2 - 12x + 5x - 60 = 0$

$\Rightarrow x(x-12) + 5(x-12) = 0$

$\Rightarrow (x-12)(x+5) = 0$

Either $x - 12 = 0$ or $x + 5 = 0$, i.e., $x = 12$ or $x = -5$
Since sides are positive, x can only be 12.
Therefore, the base of the given triangle is 12 cm and the altitude of this triangle will be $(12 - 7)$ cm = 5 cm.

Concept Insights: Apply Pythagoras Theorem to form the equation

Solution 6
Let the number of articles produced be x.
Therefore, cost of production of each article = Rs $(2x + 3)$
It is given that the total production is Rs 90.

$\therefore x(2x+3) = 90$

$\Rightarrow 2x^2 + 3x - 90 = 0$

$\Rightarrow 2x^2 + 15x - 12x - 90 = 0$

$\Rightarrow x(2x+15) - 6(2x+15) = 0$

$\Rightarrow (2x+15)(x-6) = 0$

Either $2x + 15 = 0$ or $x - 6 = 0$, i.e., $x = \frac{-15}{2}$ or $x = 6$

As the number of articles produced can only be a positive integer, therefore, x can only be 6.
Hence, number of articles produced = 6
Cost of each article = $2 \times 6 + 3$ = Rs 15

Chapter 4 - Quadratic Equations Exercise Ex. 4.3
Solution 1

(i) $2x^2 - 7x + 3 = 0$

$\Rightarrow 2x^2 - 7x = -3$

On dividing both sides of the equation by 2, we obtain

$\Rightarrow x^2 - \frac{7}{2}x = -\frac{3}{2}$

$\Rightarrow x^2 - 2 \times x \times \frac{7}{4} = -\frac{3}{2}$

On adding $\left(\frac{7}{4}\right)^2$ to both sidesof equation, we obtain

$\Rightarrow (x)^2 - 2 \times x \times \frac{7}{4} + \left(\frac{7}{4}\right)^2 = \left(\frac{7}{4}\right)^2 - \frac{3}{2}$

$\Rightarrow \left(x - \frac{7}{4}\right)^2 = \frac{49}{16} - \frac{3}{2}$

$\Rightarrow \left(x - \frac{7}{4}\right)^2 = \frac{25}{16}$

$\Rightarrow \left(x - \frac{7}{4}\right) = \pm\frac{5}{4}$

$\Rightarrow x = \frac{7}{4} \pm \frac{5}{4}$

$\Rightarrow x = \frac{7}{4} + \frac{5}{4}$ or $x = \frac{7}{4} - \frac{5}{4}$

$\Rightarrow x = \frac{12}{4}$ or $x = \frac{2}{4}$

$\Rightarrow x = 3$ or $\frac{1}{2}$

(ii) $2x^2 + x - 4 = 0$

$\Rightarrow 2x^2 + x = 4$

On dividing both sides of the equation by 2, we obtain

$\Rightarrow x^2 + \frac{1}{2}x = 2$

On adding $\left(\frac{1}{4}\right)^2$ to both sides of the equation, we obtain

$\Rightarrow (x)^2 + 2 \times x \times \frac{1}{4} + \left(\frac{1}{4}\right)^2 = 2 + \left(\frac{1}{4}\right)^2$

$\Rightarrow \left(x + \frac{1}{4}\right)^2 = \frac{33}{16}$

$\Rightarrow x + \frac{1}{4} = \pm\frac{\sqrt{33}}{4}$

$\Rightarrow x = \pm\frac{\sqrt{33}}{4} - \frac{1}{4}$

$\Rightarrow x = \frac{\pm\sqrt{33} - 1}{4}$

$\Rightarrow x = \frac{\sqrt{33} - 1}{4}$ or $\frac{-\sqrt{33} - 1}{4}$

(iii) $4x^2 + 4\sqrt{3}x + 3 = 0$

$\Rightarrow (2x)^2 + 2 \times 2x \times \sqrt{3} + \left(\sqrt{3}\right)^2 = 0$

$\Rightarrow \left(2x + \sqrt{3}\right)^2 = 0$

$\Rightarrow \left(2x + \sqrt{3}\right) = 0$ and $\left(2x + \sqrt{3}\right) = 0$

(iv) $2x^2 + x + 4 = 0$

$\Rightarrow 2x^2 + x = -4$

On dividing both sides of the equation by 2, we obtain

$\Rightarrow x^2 + \frac{1}{2}x = -2$

$\Rightarrow x^2 + 2 \times x \times \frac{1}{4} = -2$

On adding $\left(\frac{1}{4}\right)^2$ to both sides of the equation, we obtain

$\Rightarrow (x)^2 + 2 \times x \times \frac{1}{4} + \left(\frac{1}{4}\right)^2 = \left(\frac{1}{4}\right)^2 - 2$

$\Rightarrow \left(x + \frac{1}{4}\right)^2 = \frac{1}{16} - 2$

$\Rightarrow \left(x + \frac{1}{4}\right)^2 = -\frac{31}{16}$

However, the square of a number cannot be negative.

Therefore, there is no real root for the given equation.

Concept insight: Before completing the square of any quadratic equation make sure to make the coefficient of square term x^2 or y^2 unity by dividing the whole equation appropriate real number.
Once this is done add and subtract the square of half the coefficient of x or y.

Solution 2

(i) $2x^2 - 7x + 3 = 0$

On comparing this equation with $ax^2 + bx + c = 0$, we obtain

$a = 2, b = -7, c = 3$

By using quadratic formula, we obtain

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Rightarrow x = \frac{7 \pm \sqrt{49 - 24}}{4}$$

$$\Rightarrow x = \frac{7 \pm \sqrt{25}}{4}$$

$$\Rightarrow x = \frac{7 \pm 5}{4}$$

$$\Rightarrow x = \frac{7 + 5}{4} \text{ or } \frac{7 - 5}{4}$$

$$\Rightarrow x = \frac{12}{4} \text{ or } \frac{2}{4}$$

$$\therefore \ x = 3 \text{ or } \frac{1}{2}$$

(ii) $2x^2 + x - 4 = 0$

On comparing this equation with $ax^2 + bx + c = 0$, we obtain

$a = 2, b = 1, c = -4$

By using quadratic formula, we obtain

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Rightarrow x = \frac{-1 \pm \sqrt{1 + 32}}{4}$$

$$\Rightarrow x = \frac{-1 \pm \sqrt{33}}{4}$$

$$\therefore \ x = \frac{-1 + \sqrt{33}}{4} \text{ or } \frac{-1 - \sqrt{33}}{4}$$

(iii) $4x^2 + 4\sqrt{3}x + 3 = 0$

On comparing this equation with $ax^2 + bx + c = 0$, we obtain

$a = 4, b = 4\sqrt{3}, c = 3$

By using quadratic formula, we obtain

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Rightarrow x = \frac{-4\sqrt{3} \pm \sqrt{48 - 48}}{8}$$

$$\Rightarrow x = \frac{-4\sqrt{3} \pm 0}{8}$$

$$\therefore \ x = \frac{-\sqrt{3}}{2} \text{ or } \frac{-\sqrt{3}}{2}$$

(iv) $2x^2 + x + 4 = 0$

On comparing this equation with $ax^2 + bx + c = 0$, we obtain

$a = 2, b = 1, c = 4$

By using quadratic formula, we obtain

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Rightarrow x = \frac{-1 \pm \sqrt{1 - 32}}{4}$$

Solution 3

(i) $x - \frac{1}{x} = 3 \Rightarrow \quad x^2 - 3x - 1 = 0$

On comparing this equation with $ax^2 + bx + c = 0$, we obtain

$a = 1, b = -3, c = -1$

By using quadratic formula, we obtain

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Rightarrow x = \frac{3 \pm \sqrt{9 + 4}}{2}$$

$$\Rightarrow x = \frac{3 \pm \sqrt{13}}{2}$$

Therefore, $x = \frac{3 + \sqrt{13}}{2}$ or $\frac{3 - \sqrt{13}}{2}$

(ii) $\frac{1}{x + 4} - \frac{1}{x - 7} = \frac{11}{30}$

$$\Rightarrow \frac{x - 7 - x - 4}{(x + 4)(x - 7)} = \frac{11}{30}$$

$$\Rightarrow \frac{-11}{(x + 4)(x - 7)} = \frac{11}{30}$$

$$\Rightarrow (x + 4)(x - 7) = -30$$

$$\Rightarrow x^2 - 3x - 28 = -30$$

$$\Rightarrow x^2 - 3x + 2 = 0$$

$$\Rightarrow x^2 - 2x - x + 2 = 0$$

$$\Rightarrow x(x - 2) - 1(x - 2) = 0$$

$$\Rightarrow (x - 2)(x - 1) = 0$$

$$\Rightarrow x = 1 \text{ or } 2$$

Concept Insights: Apply the arithmetical simplifications appropriately to reduce the equation to quadratic form.

Solution 4

Let the present age of Rehman be x years.
Three years ago, his age was (x − 3) years.
Five years hence, his age will be (x + 5) years.
It is given that the sum of the reciprocals of Rehman's ages 3 years ago and 5 years from now is $\frac{1}{3}$.

$\therefore \frac{1}{x-3}+\frac{1}{x+5}=\frac{1}{3}$

$\frac{x+5+x-3}{(x-3)(x+5)}=\frac{1}{3}$

$\frac{2x+2}{(x-3)(x+5)}=\frac{1}{3}$

$\Rightarrow 3(2x+2)=(x-3)(x+5)$

$\Rightarrow 6x+6=x^2+2x-15$

$\Rightarrow x^2-4x-21=0$

$\Rightarrow x^2-7x+3x-21=0$

$\Rightarrow x(x-7)+3(x-7)=0$

$\Rightarrow (x-7)(x+3)=0$

$\Rightarrow x=7,-3$

However, age cannot be negative.
Therefore, Rehman's present age is 7 years.

Solution 5

Let the marks in Maths be x.
Then, the marks in English will be 30 − x.
According to the given question,

$(x+2)(30-x-3)=210$

$(x+2)(27-x)=210$

$\Rightarrow -x^2+25x+54=210$

$\Rightarrow x^2-25x+156=0$

$\Rightarrow x^2-12x-13x+156=0$

$\Rightarrow x(x-12)-13(x-12)=0$

$\Rightarrow (x-12)(x-13)=0$

$\Rightarrow x=12,13$

If the marks in Maths are 12, then marks in English will be 30 − 12 = 18
If the marks in Maths are 13, then marks in English will be 30 − 13 = 17

Solution 6

Let the shorter side of the rectangle be x m.
Then, larger side of the rectangle = $(x + 30)$ m

Diagonal of the rectangle = $\sqrt{x^2+(x+30)^2}$

It is given that the diagonal of the rectangle is 60 m more than the shorter side

$\therefore \sqrt{x^2+(x+30)^2} = x+60$

$\Rightarrow x^2+(x+30)^2 = (x+60)^2$

$\Rightarrow x^2+x^2+900+60x = x^2+3600+120x$

$\Rightarrow x^2-60x-2700 = 0$

$\Rightarrow x^2-90x+30x-2700 = 0$

$\Rightarrow x(x-90)+30(x-90)$

$\Rightarrow (x-90)(x+30) = 0$

$\Rightarrow x = 90,-30$

However, side cannot be negative. Therefore, the length of the shorter side will be 90 m. Hence, length of the larger side will be (90 + 30) m = 120 m

Solution 7

Let the larger and smaller number be x and y respectively.
According to the given question,

$x^2-y^2 = 180$ and $y^2 = 8x$

$\Rightarrow x^2-8x = 180$

$\Rightarrow x^2-8x-180 = 0$

$\Rightarrow x^2-18x+10x-180 = 0$

$\Rightarrow x(x-18)+10(x-18) = 0$

$\Rightarrow (x-18)(x+10) = 0$

$\Rightarrow x = 18,-10$

However, the larger number cannot be negative as 8 times of the larger number will be negative and hence, the square of the smaller number will be negative which is not possible.
Therefore, the larger number will be 18 only.

$x = 18$

$\therefore\ y^2 = 8x = 8\times 18 = 144$

$\Rightarrow\ y = \pm\sqrt{144} = \pm 12$

$\therefore$ Smaller number = ± 12

Therefore, the numbers are 18 and 12 or 18 and −12.

Solution 8

Let the speed of the train be x km/hr.

Time taken to cover 360 km $= \frac{360}{x}$ hr

According to the given question,

$$(x+5)\left(\frac{360}{x}-1\right)=360$$

$$\Rightarrow (x+5)\left(\frac{360}{x}-1\right)=360$$

$$\Rightarrow 360-x+\frac{1800}{x}-5=360$$

$$\Rightarrow x^2+5x-1800=0$$

$$\Rightarrow x^2+45x-40x-1800=0$$

$$\Rightarrow x(x+45)-40(x+45)=0$$

$$\Rightarrow (x+45)(x-40)=0$$

$$\Rightarrow x=40,-45$$

However, speed cannot be negative.

Therefore, the speed of train is 40 km/h

Concept Insight: Use the relation s =d/t to crack this question and remember here distance is constant so speed and time will vary inversely.

Solution 9

Let the time taken by the smaller pipe to fill the tank be x hr.
Time taken by the larger pipe = $(x - 10)$ hr

Part of tank filled by smaller pipe in 1 hour = $\frac{1}{x}$

Part of tank filled by larger pipe in 1 hour = $\frac{1}{x-10}$

It is given that the tank can be filled in $9\frac{3}{8} = \frac{75}{8}$ hours by both the pipes together. Therefore,

$$\frac{1}{x} + \frac{1}{x-10} = \frac{8}{75}$$

$$\frac{x-10+x}{x(x-10)} = \frac{8}{75}$$

$$\Rightarrow \frac{2x-10}{x(x-10)} = \frac{8}{75}$$

$$\Rightarrow 75(2x-10) = 8x^2 - 80x$$

$$\Rightarrow 150x - 750 = 8x^2 - 80x$$

$$\Rightarrow 8x^2 - 230x + 750 = 0$$

$$\Rightarrow 8x^2 - 200x - 30x + 750 = 0$$

$$\Rightarrow 8x(x-25) - 30(x-25) = 0$$

$$\Rightarrow (x-25)(8x-30) = 0$$

i.e., $x = 25, \frac{30}{8}$

Time taken by the smaller pipe cannot be $\frac{30}{8} = 3.75$ hours. As in this case, the time taken by the larger pipe will be negative, which is logically not possible.
Therefore, time taken individually by the smaller pipe and the larger pipe will be 25 and 25 − 10 = 15 hours respectively.

Concept Insight: Concept of work done and time taken will be applicable here. Unitary method will be used to find the time taken in an hour.

Solution 10

Let the average speed of passenger train be x km/h.
Average speed of express train = $(x + 11)$ km/h
It is given that the time taken by the express train to cover 132 km is 1 hour less than the passenger train to cover the same distance.

$$\therefore \frac{132}{x} - \frac{132}{x+11} = 1$$
$$\Rightarrow 132\left[\frac{x+11-x}{x(x+11)}\right] = 1$$
$$\Rightarrow \frac{132\times 11}{x(x+11)} = 1$$
$$\Rightarrow 132\times 11 = x(x+11)$$
$$\Rightarrow x^2 + 11x - 1452 = 0$$
$$\Rightarrow x^2 + 44x - 33x - 1452 = 0$$
$$\Rightarrow x(x+44) - 33(x+44) = 0$$
$$\Rightarrow (x+44)(x-33) = 0$$
$$\Rightarrow x = -44, 33$$

Speed cannot be negative.Therefore, the speed of the passenger train will be 33 km/h and thus, the speed of the express train will be 33 + 11 = 44 km/h.

Solution 11

Let the sides of the two squares be x m and y m. Therefore, their perimeter will be $4x$ and $4y$ respectively and their areas will be x^2 and y^2 respectively.
It is given that, $4x - 4y = 24$
$x - y = 6$
$x = y + 6$

$$\text{Also, } x^2 + y^2 = 468$$
$$\Rightarrow (6+y)^2 + y^2 = 468$$
$$\Rightarrow 36 + y^2 + 12y + y^2 = 468$$
$$\Rightarrow 2y^2 + 12y - 432 = 0$$
$$\Rightarrow y^2 + 6y - 216 = 0$$
$$\Rightarrow y^2 + 18y - 12y - 216 = 0$$
$$\Rightarrow y(y+18) - 12(y+18) = 0$$
$$\Rightarrow (y+18)(y-12) = 0$$
$$\Rightarrow y = -18 \text{ or } 12.$$

However, side of a square cannot be negative.
Hence, the sides of the squares are 12 m and (12 + 6) m = 18 m

Chapter 4 - Quadratic Equations Exercise Ex. 4.4

Solution 1

We know that for a quadratic equation $ax^2 + bx + c = 0$,
discriminant is $b^2 - 4ac$.

(A) If $b^2 - 4ac > 0$ implies two distinct real roots

(B) If $b^2 - 4ac = 0$ implies two equal real roots

(C) If $b^2 - 4ac < 0$ implies imaginary roots

(I) $2x^2 - 3x + 5 = 0$

Comparing this equation with $ax^2 + bx + c = 0$, we obtain

$a = 2, b = -3, c = 5$

Discriminant $= b^2 - 4ac = (-3)^2 - 4(2)(5) = 9 - 40$

$= -31$

As $b^2 - 4ac < 0$,

Therefore, no real root is possible for the given equation.

(II) $3x^2 - 4\sqrt{3}x + 4 = 0$

Comparing this equation with $ax^2 + bx + c = 0$, we obtain

$a = 3, \ b = -4\sqrt{3}, c = 4$

Discriminant $= b^2 - 4ac = \left(-4\sqrt{3}\right)^2 - 4(3)(4)$

$= 48 - 48 = 0$

As $b^2 - 4ac = 0$,

Therefore, real roots exist for the given equation and they are equal to each other.

And the roots will be $\frac{-b}{2a}$ and $\frac{-b}{2a}$.

$$\frac{-b}{2a} = \frac{-\left(-4\sqrt{3}\right)}{2 \times 3} = \frac{4\sqrt{3}}{6} = \frac{2\sqrt{3}}{3} = \frac{2}{\sqrt{3}}$$

Therefore, the roots are $\frac{2}{\sqrt{3}}$ and $\frac{2}{\sqrt{3}}$.

(III) $2x^2 - 6x + 3 = 0$

Comparing this equation with $ax^2 + bx + c = 0$, we obtain

$a = 2, b = -6, c = 3$

Discriminant $= b^2 - 4ac = (-6)^2 - 4(2)(3)$

$= 36 - 24 = 12$

As $b^2 - 4ac > 0$,

Therefore, distinct real roots exist for this equation as follows.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-6) \pm \sqrt{(-6)^2 - 4(2)(3)}}{2(2)}$$

$$= \frac{6 \pm \sqrt{12}}{4} = \frac{6 \pm 2\sqrt{3}}{4}$$

$$= \frac{3 \pm \sqrt{3}}{2}$$

Therefore, the roots are $\frac{3+\sqrt{3}}{2}$ or $\frac{3-\sqrt{3}}{2}$.

Solution 2

We know that if an equation $ax^2 + bx + c = 0$ has two equal roots, its discriminant ($b^2 - 4ac$) will be 0.

(I) $2x^2 + kx + 3 = 0$
Comparing equation with $ax^2 + bx + c = 0$, we obtain
$a = 2, b = k, c = 3$

Discriminant $= b^2 - 4ac = (k)^2 - 4(2)(3)$
$= k^2 - 24$
For equal roots,

Discriminant $= 0$
$k^2 - 24 = 0$
$k^2 = 24$
$k = \pm\sqrt{24} = \pm 2\sqrt{6}$

(II) $kx(x - 2) + 6 = 0$
or $kx^2 - 2kx + 6 = 0$
Comparing this equation with $ax^2 + bx + c = 0$, we obtain
$a = k, b = -2k, c = 6$

Discriminant $= b^2 - 4ac = (-2k)^2 - 4(k)(6)$
$= 4k^2 - 24k$
For equal roots,
$b^2 - 4ac = 0$
$4k^2 - 24k = 0$
$4k(k - 6) = 0$
Either $4k = 0$ or $k - 6 = 0$
$k = 0$ or $k = 6$
However, if $k = 0$, then the equation will not have the terms 'x^2' and 'x'.
Therefore, if this equation has two equal roots, k should be 6 only.

Solution 3

Let the breadth of mango grove be l.
Length of mango grove will be $2l$.
Area of mango grove $= (2l)(l)$
$= 2l^2$
$2l^2 = 800$
$l^2 = \frac{800}{2} = 400$
$l^2 - 400 = 0$
Comparing this equation with $al^2 + bl + c = 0$, we obtain

$a = 1\ b = 0, c = -400$

Discriminant $= b^2 - 4ac = (0)^2 - 4 \times (1) \times (-400) = 1600$

Here, $b^2 - 4ac > 0$

Therefore, the equation will have real roots. And hence, the desired rectangular mango grove can be designed.
$l = \pm 20$

However, length cannot be negative.
Therefore, breadth of mango grove = 20 m
Length of mango grove = 2 × 20 = 40 m

Concept Insights: After setting the equation check the discriminant if the equation has real roots then only the situation is possible.

Solution 4

Let the age of one friend be x years.
Age of the other friend will be $(20 - x)$ years.
4 years ago, age of 1st friend = $(x - 4)$ years

And, age of 2nd friend = $(20 - x - 4)$
$= (16 - x)$ years

Given that,

$(x - 4)\ (16 - x) = 48$
$16x - 64 - x^2 + 4x = 48$
$-x^2 + 20x - 112 = 0$
$x^2 - 20x + 112 = 0$

Comparing this equation with $ax^2 + bx + c = 0$, we obtain
$a = 1, b = -20, c = 112$

Discriminant $= b^2 - 4ac = (-20)^2 - 4\ (1)\ (112)$
$= 400 - 448 = -48$

As $b^2 - 4ac < 0$,
Therefore, no real root is possible for this equation and hence, this situation is not possible.

Solution 5

Let the length and breadth of the park be l and b.
Perimeter = 2 (l + b) = 80

$l + b = 40$
Or, $b = 40 - l$
Area = $l \times b = l\,(40 - l) = 40l - l^2$
$40l - l^2 = 400$

$l^2 - 40l + 400 = 0$

Comparing this equation with
$al^2 + bl + c = 0$, we obtain
$a = 1, b = -40, c = 400$

Discriminate = $b^2 - 4ac = (-40)^2 - 4\,(1)\,(400)$
$= 1600 - 1600 = 0$

As $b^2 - 4ac = 0$,
Therefore, this equation has equal real roots. And hence, this situation is possible.
Root of this equation,

$$l = -\frac{b}{2a}$$

$$l = -\frac{(-40)}{2(1)} = \frac{40}{2} = 20$$

Therefore, length of park, l = 20 m
And breadth of park, $b = 40 - l = 40 - 20 = 20$ m