

## Chapter 3 - Pairs of Linear Equations in Two Variables

### Exercise Ex. 3.1

#### Solution 1

Let the present age of Aftab and his daughter be  $x$  and  $y$  respectively.

Seven years ago,

Age of Aftab =  $x - 7$

Age of his daughter =  $y - 7$

According to the given condition,

$$(x - 7) = 7(y - 7)$$

$$\Rightarrow x - 7 = 7y - 49$$

$$\Rightarrow x - 7y = -42$$

Three years hence,

Age of Aftab =  $x + 3$

Age of his daughter =  $y + 3$

According to the given condition,

$$(x + 3) = 3(y + 3)$$

$$\Rightarrow x + 3 = 3y + 9$$

$$\Rightarrow x - 3y = 6$$

Thus, the given conditions can be algebraically represented as:

$$x - 7y = -42$$

$$x - 3y = 6$$

$$x - 7y = -42 \Rightarrow x = -42 + 7y$$

Three solutions of this equation can be written in a table as follows:

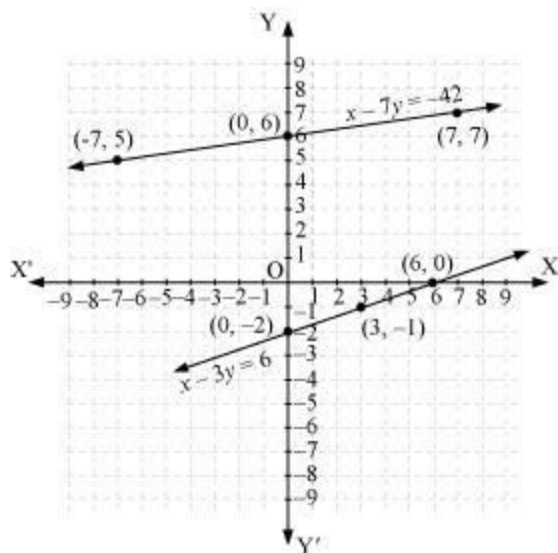
$x$	-7	0	7
$y$	5	6	7

$$x - 3y = 6 \Rightarrow x = 6 + 3y$$

Three solutions of this equation can be written in a table as follows:

$x$	6	3	0
$y$	0	-1	-2

The graphical representatino is as follows:



**Concept insight:** In order to represent a given situation mathematically, first see what we need to find out in the problem. Here, Aftab and his daughter's present age needs to be found so, so the ages will be represented by variables  $x$  and  $y$ . The problem talks about their ages seven years ago and three years from now. Here, the words 'seven years ago' means we have to subtract 7 from their present ages, and 'three years from now' or 'three years hence' means we have to add 3 to their present ages. Remember in order to represent the algebraic equations graphically the solution set of equations must be taken as whole numbers only for the accuracy. Graph of the two linear equations will be represented by a straight line.

## Solution 2

Let the cost of a bat and a ball be Rs  $x$  and Rs  $y$  respectively.

The given conditions can be algebraically represented as:

$$3x + 6y = 3900 \Rightarrow x + 2y = 1300 \quad \dots (i)$$

$$x + 3y = 1300 \quad \dots (ii)$$

From (i), we get

$$x = 1300 - 2y$$

Three solutions of this equation can be written in a table as follows:

$x$	300	100	-100
$y$	500	600	700

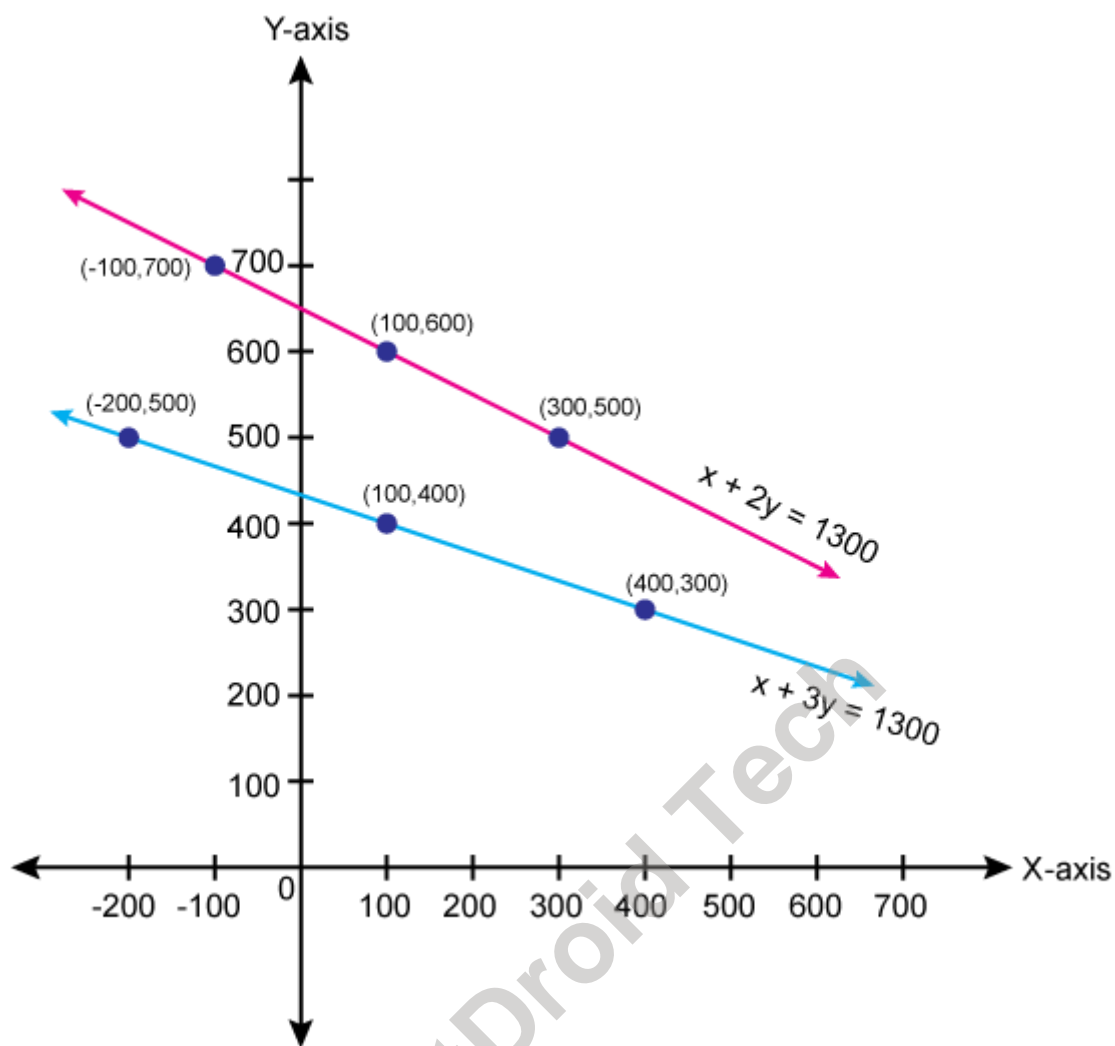
From (ii), we get

$$x = 1300 - 3y$$

Three solutions of this equation can be written in a table as follows:

$x$	100	400	-200
$y$	400	300	500

The graphical representation is as follows:



**Concept insight:** Cost of bats and balls needs to be found so the cost of a ball and bat will be taken as the variables. Applying the conditions of total cost of bats and balls algebraic equations will be obtained. Then, in order to represent the obtained equations graphically take atleast three ordered pairs on both the equations in order to avoid any computational errors.

### Solution 3

Let the cost of 1 kg of apples and 1 kg grapes be Rs.  $x$  and Rs.  $y$ .

The given conditions can be algebraically represented as:

$$2x + y = 160$$

$$4x + 2y = 300$$

$$2x + y = 160 \Rightarrow y = 160 - 2x$$

Three solutions of this equation can be written in a table as follows:

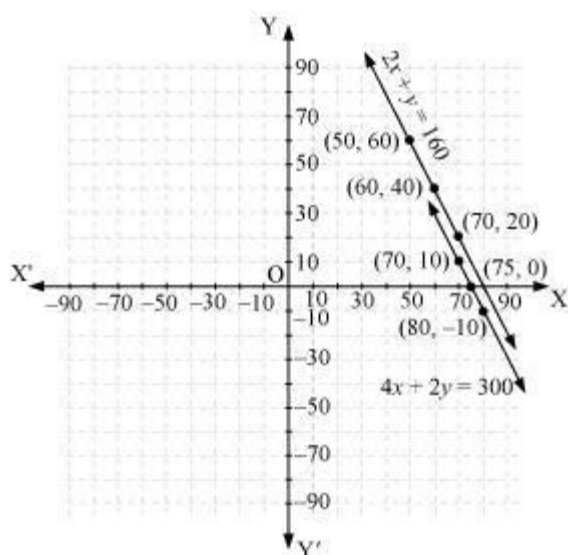
$x$	50	60	70
$y$	60	40	20

$$4x + 2y = 300 \Rightarrow y = \frac{300 - 4x}{2}$$

Three solutions of this equation can be written in a table as follows:

x	70	80	75
y	10	-10	0

The graphical representation is as follows:



**Concept insight:** cost of apples and grapes needs to be found so the cost of 1 kg apples and 1 kg grapes will be taken as the variables. From the given conditions of collective cost of apples and grapes, a pair of linear equations in two variables will be obtained. Then, in order to represent the obtained equations graphically, take the values of variables as whole numbers only. Since these values are large so take the suitable scale like 1 cm = 20.

## Chapter 3 - Pairs of Linear Equations in Two Variables

### Exercise Ex. 3.2

#### Solution 1

(i) Let the number of girls and boys in the class be  $x$  and  $y$  respectively.

According to the given conditions, we have:

$$x + y = 10$$

$$x - y = 4$$

$\Rightarrow$

$$x + y = 10 \quad x = 10 - y$$

Three solutions of this equation can be written in a table as follows:

x	5	4	6
y	5	6	4

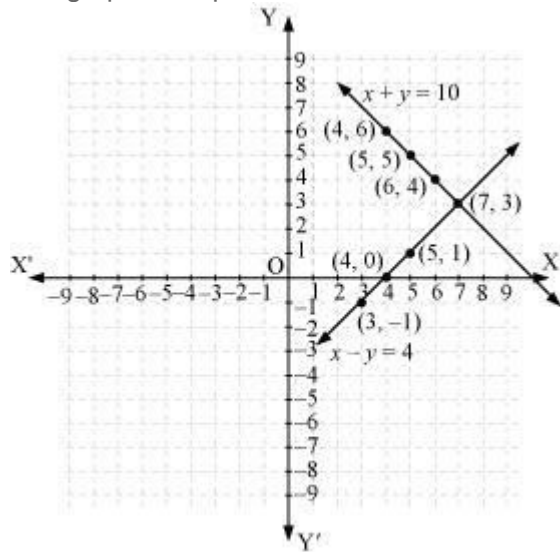
$\Rightarrow$

$$x - y = 4 \quad x = 4 + y$$

Three solutions of this equation can be written in a table as follows:

x	5	4	3
y	1	0	-1

The graphical representation is as follows:



From the graph, it can be observed that the two lines intersect each other at the point (7, 3).  
So,  $x = 7$  and  $y = 3$ .

Thus, the number of girls and boys in the class are 7 and 3 respectively.

(ii) Let the cost of one pencil and one pen be Rs  $x$  and Rs  $y$  respectively.

According to the given conditions, we have:

$$5x + 7y = 50$$

$$7x + 5y = 46$$

$$5x + 7y = 50 \Rightarrow x = \frac{50 - 7y}{5}$$

Three solutions of this equation can be written in a table as follows:

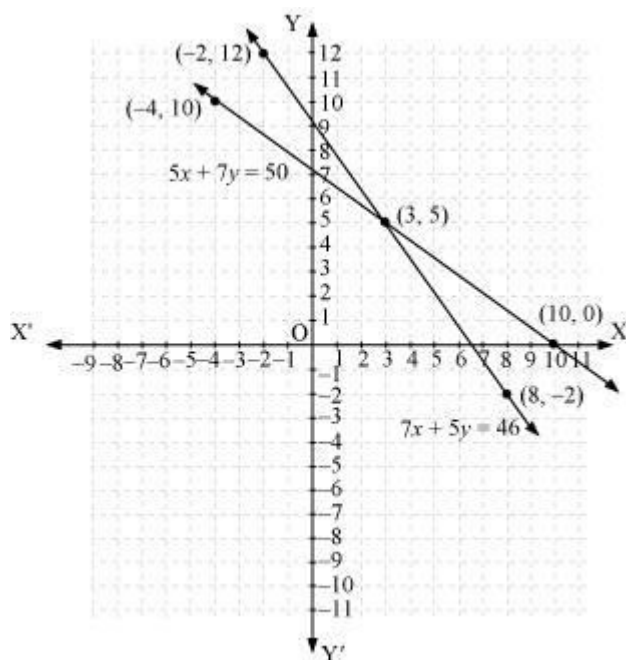
$x$	3	10	-4
$y$	5	0	10

$$7x + 5y = 46 \Rightarrow x = \frac{46 - 5y}{7}$$

Three solutions of this equation can be written in a table as follows:

$x$	8	3	-2
$y$	-2	5	12

The graphical representation is as follows:



From the graph, it can be observed that the two lines intersect each other at the point (3, 5).  
So,  $x = 3$  and  $y = 5$ .

Therefore, the cost of one pencil and one pen are Rs 3 and Rs 5 respectively.

**Concept insight:** Read the question carefully and examine what are the unknowns. Represent the given conditions with the help of equations by taking the unknown quantities as variables. Also carefully state the variables as whole solution is based on it. On the graph paper, mark the points accurately and neatly using a sharp pencil. Also, take at least three points satisfying the two equations in order to obtain the correct straight line of the equation. Since joining any two points gives a straight line and if one of the points is computed incorrect will give a wrong line and taking third point will give a correct line. The point where the two straight lines will intersect will give the values of the two variables, i.e., the solution of the two linear equations. State the solution point.

### Solution 2

(i)  $5x - 4y + 8 = 0$

$7x + 6y - 9 = 0$

Comparing these equations with  $a_1x + b_1y + c_1 = 0$  and  $a_2x + b_2y + c_2 = 0$ , we get:

$$a_1 = 5, \quad b_1 = -4, \quad c_1 = 8$$

$$a_2 = 7, \quad b_2 = 6, \quad c_2 = -9$$

$$\frac{a_1}{a_2} = \frac{5}{7}$$

$$\frac{b_1}{b_2} = \frac{-4}{6} = \frac{-2}{3}$$

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

Since  $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ , the given pair of equations intersect at exactly one point.

(ii)  $9x + 3y + 12 = 0$

$18x + 6y + 24 = 0$

Comparing these equations with  $a_1x + b_1y + c_1 = 0$  and  $a_2x + b_2y + c_2 = 0$ , we get:

$$a_1 = 9, \quad b_1 = 3, \quad c_1 = 12$$

$$a_2 = 18, \quad b_2 = 6, \quad c_2 = 24$$

$$\frac{a_1}{a_2} = \frac{9}{18} = \frac{1}{2}$$

$$\frac{b_1}{b_2} = \frac{3}{6} = \frac{1}{2}$$

$$\frac{c_1}{c_2} = \frac{12}{24} = \frac{1}{2}$$

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

Since  $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ , the given pair of equations are coincident.

$$(iii) \quad 6x - 3y + 10 = 0, \quad 2x - y + 9 = 0$$

Comparing these equations with  $a_1x + b_1y + c_1 = 0$  and  $a_2x + b_2y + c_2 = 0$ , we get:

$$a_1 = 6, \quad b_1 = -3, \quad c_1 = 10$$

$$a_2 = 2, \quad b_2 = -1, \quad c_2 = 9$$

$$\frac{a_1}{a_2} = \frac{6}{2} = 3$$

$$\frac{b_1}{b_2} = \frac{-3}{-1} = 3$$

$$\frac{c_1}{c_2} = \frac{10}{9}$$

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

Since  $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ , the given pair of equations are parallel to each other.

**Concept insight:** In order to answer such questions, remember the condition for the pair of linear equations to be intersecting, parallel or coincident. Also, while writing the coefficients, don't forget to take the signs.

### Solution 3

$$(i) \quad 3x + 2y = 5$$

$$2x - 3y = 7$$

$$\frac{a_1}{a_2} = \frac{3}{2}, \quad \frac{b_1}{b_2} = \frac{-2}{3}, \quad \frac{c_1}{c_2} = \frac{-5}{-7} = \frac{5}{7}$$

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

Since  $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ , the given pair of equations has only one solution.

Thus, the pair of linear equations is consistent.

$$(ii) \quad \begin{aligned} 2x - 3y &= 8 \\ 4x - 6y &= 9 \end{aligned}$$

$$\frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2}, \quad \frac{b_1}{b_2} = \frac{-3}{-6} = \frac{1}{2}, \quad \frac{c_1}{c_2} = \frac{-8}{-9} = \frac{8}{9}$$

Since  $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ , the given pair of equations has no solution.

Thus, the pair of linear equations is inconsistent.

$$(iii) \quad \begin{aligned} \frac{3}{2}x + \frac{5}{3}y &= 7 \\ 9x - 10y &= 14 \end{aligned}$$

$$\frac{a_1}{a_2} = \frac{\frac{3}{2}}{9} = \frac{1}{6}, \quad \frac{b_1}{b_2} = \frac{\frac{5}{3}}{-10} = \frac{-1}{6}, \quad \frac{c_1}{c_2} = \frac{-7}{-14} = \frac{1}{2}$$

Since  $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ , the given pair of equations has only one solution.

Thus, the pair of linear equations is consistent.

$$(iv) \quad \begin{aligned} 5x - 3y &= 11 \\ -10x + 6y &= -22 \end{aligned}$$

$$\frac{a_1}{a_2} = \frac{5}{-10} = \frac{-1}{2}, \quad \frac{b_1}{b_2} = \frac{-3}{6} = \frac{-1}{2}, \quad \frac{c_1}{c_2} = \frac{-11}{-22} = \frac{-1}{2}$$

Since  $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ , the given pair of equations has infinite number of solutions.

Thus, the pair of linear equations is consistent.

$$(v) \quad \begin{aligned} \frac{4}{3}x + 2y &= 8 \\ 2x + 3y &= 12 \end{aligned}$$

$$\frac{a_1}{a_2} = \frac{\frac{4}{3}}{2} = \frac{2}{3}, \quad \frac{b_1}{b_2} = \frac{2}{3}, \quad \frac{c_1}{c_2} = \frac{-8}{-12} = \frac{2}{3}$$

Since  $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ , the given pair of equations has infinite number of solutions.

Thus, the pair of linear equations is consistent.

**Concept Insight:** If a pair of linear equations has one or more than one solution then they are said to be consistent and if they have no solution then they are said to be inconsistent. So, to identify the consistency of a given pair of equations, apply the conditions involving the coefficients of the given pair of equations.

In case, two consistent linear equations are plotted, they will either intersect or overlap each other.

#### Solution 4



(i)  $x + y = 5$   
 $2x + 2y = 10$   
 $\frac{a_1}{a_2} = \frac{1}{2}, \quad \frac{b_1}{b_2} = \frac{1}{2}, \quad \frac{c_1}{c_2} = \frac{-5}{-10} = \frac{1}{2}$

Since  $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ , the given pair of linear equations has infinite number of solutions.

Thus, the pair of linear equations is consistent.

$\Rightarrow$

Now,  $x + y = 5 \Rightarrow x = 5 - y$

Three solutions of this equation can be written in a table as follows:

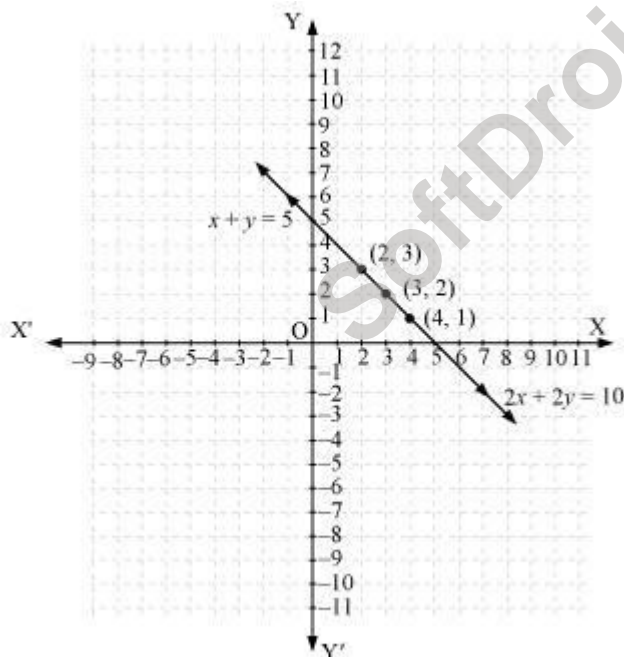
x	4	3	2
y	1	2	3

$$2x + 2y = 10 \Rightarrow x = \frac{10 - 2y}{2}$$

Three solutions of this equation can be written in a table as follows:

x	4	3	2
y	1	2	3

Thus, the graphical representation is as follows:



From the graph, it can be observed that the two lines coincide. Thus, the given pair of equations has infinite solutions.

Let  $x = k$ , then  $y = 5 - k$ . So, the ordered pair  $(k, 5 - k)$ , where  $k$  is a constant, will be the solution of the given pair of linear equations.

(ii)  $x - y = 8$   
 $3x - 3y = 16$   
 $\frac{a_1}{a_2} = \frac{1}{3}, \quad \frac{b_1}{b_2} = \frac{-1}{-3} = \frac{1}{3}, \quad \frac{c_1}{c_2} = \frac{-8}{-16} = \frac{1}{2}$

Since  $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ , the given pair of linear equations has no solution.

Thus, the pair of linear equations is inconsistent.

(iii)  $2x + y - 6 = 0$   
 $4x - 2y - 4 = 0$   
 $\frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2}, \quad \frac{b_1}{b_2} = \frac{-1}{-2} = \frac{1}{2}, \quad \frac{c_1}{c_2} = \frac{-6}{-4} = \frac{3}{2}$

Since  $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ , the given pair of linear equations has only one solution.

Thus, the pair of linear equations is consistent.

$\Rightarrow$   
Now,  $2x + y - 6 = 0 \Rightarrow y = 6 - 2x$

Three solutions of this equation can be written in a table as follows:

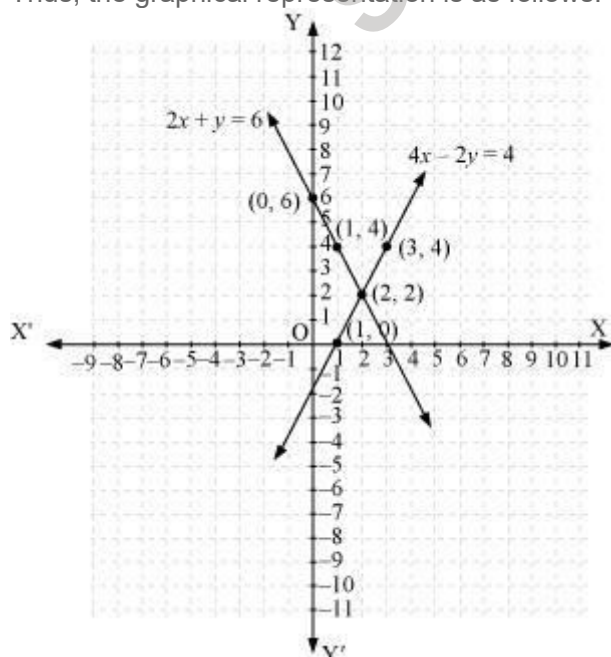
x	0	1	2
y	6	4	2

$4x - 2y - 4 = 0 \Rightarrow y = \frac{4x - 4}{2}$

Three solutions of this equation can be written in a table as follows:

x	1	2	3
y	0	2	4

Thus, the graphical representation is as follows:



From the graph, it can be observed that the two lines intersect each other at the point (2, 2).  
Thus, the solution of the given pair of equations is (2, 2).

$$(iv) \quad \begin{aligned} 2x - 2y - 2 &= 0 \\ 4x - 4y - 5 &= 0 \end{aligned}$$

$$\frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2}, \quad \frac{b_1}{b_2} = \frac{-2}{-4} = \frac{1}{2}, \quad \frac{c_1}{c_2} = \frac{-2}{-5} = \frac{2}{5}$$

Since  $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ , the given pair of linear equations has no solution.

Thus, the pair of linear equations is inconsistent.

**Concept insight:** If a pair of linear equations has one or more than one solutions then they are said to be consistent and if they have no solution then they are said to be inconsistent. The graph of each equation can be plotted by taking at least three ordered pairs which are the solutions of the equations. The point where both the lines intersect will be the solution of the given pair of equations. Remember two overlapping lines intersect each other at infinitely many points. State the solution explicitly.

### Solution 5

Let the width and length of the rectangular garden be x and y respectively.

According to the given conditions,

$$y - x = 4$$

$$y + x = 36$$

$$\Rightarrow y - x = 4 \quad y = x + 4$$

Three solutions of this equation can be written in a table as follows:

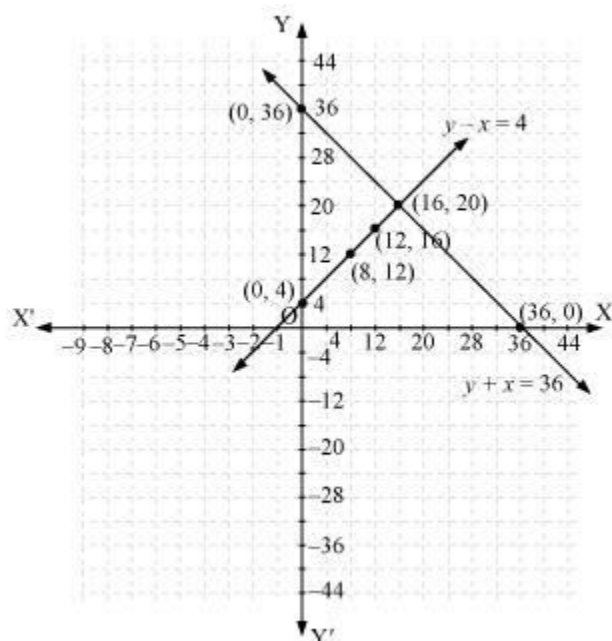
x	0	8	12
y	4	12	16

$$y + x = 36$$

Three solutions of this equation can be written in a table as follows:

x	0	36	16
y	36	0	20

Thus, the graphical representation is as follows:



From the graph, it can be observed that the two lines intersect each other at the point (16, 20).  
So,  $x = 16$  and  $y = 20$ .  
Thus, the length and width of the rectangular garden is 20 m and 16 m respectively.

**Concept insight:** Here dimensions of the rectangular garden needs to be found. Since opposite sides of the rectangle are equal so length and breadth can be taken as variables. Applying conditions given in the problem two linear equations in the 2 variables can be obtained. Now, in order to represent the obtained equations graphically, take the values of variables as whole numbers only, because then it will be easier to represent the values on the graph. The point where the two equations intersect will give the required dimensions. State the dimensions length and breadth from the values of the variables.

### Solution 6

(i) For the two lines  $a_1x + b_1x + c_1 = 0$  and  $a_2x + b_2x + c_2 = 0$ , to be intersecting, we must have

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

So, the other linear equation can be  $5x + 6y - 16 = 0$

$$\text{as } \frac{a_1}{a_2} = \frac{2}{5}, \frac{b_1}{b_2} = \frac{3}{6} = \frac{1}{2}, \frac{c_1}{c_2} = \frac{-8}{-16} = \frac{1}{2}$$

(ii) For the two lines  $a_1x + b_1x + c_1 = 0$  and  $a_2x + b_2x + c_2 = 0$ , to be parallel, we must have

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

So, the other linear equation can be  $6x + 9y + 24 = 0$ ,

$$\text{as } \frac{a_1}{a_2} = \frac{2}{6} = \frac{1}{3}, \frac{b_1}{b_2} = \frac{3}{9} = \frac{1}{3}, \frac{c_1}{c_2} = \frac{-8}{24} = \frac{-1}{3}$$

(iii) For the two lines  $a_1x + b_1x + c_1 = 0$  and  $a_2x + b_2x + c_2 = 0$  to be coincident, we must have

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

So, the other linear equation can be  $8x + 12y - 32 = 0$ ,

$$\text{as } \frac{a_1}{a_2} = \frac{2}{8} = \frac{1}{4}, \quad \frac{b_1}{b_2} = \frac{3}{12} = \frac{1}{4}, \quad \frac{c_1}{c_2} = \frac{-8}{-32} = \frac{1}{4}$$

**Concept insight:** In order to answer such type of problems, just remember the conditions for two lines to be intersecting, parallel, and coincident. This problem will have multiple answers as their can be many equations satisfying the required conditions.

### Solution 7

$$x - y + 1 = 0 \Rightarrow x = y - 1$$

Three solutions of this equation can be written in a table as follows:

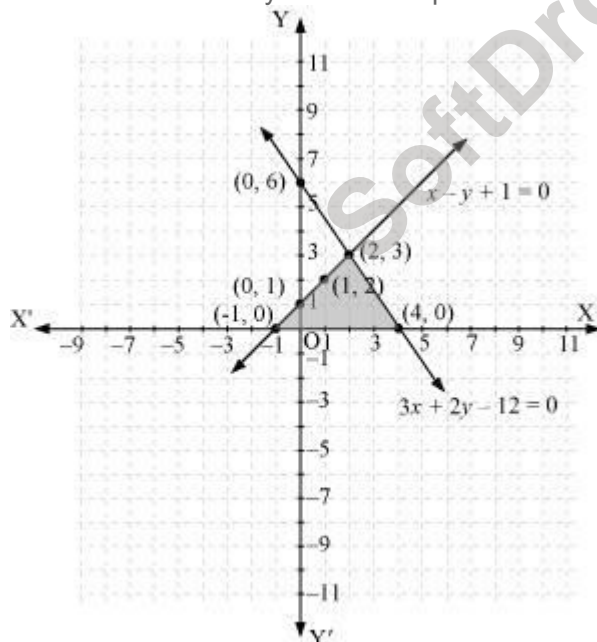
x	0	1	2
y	1	2	3

$$3x + 2y - 12 = 0 \Rightarrow x = \frac{12 - 2y}{3}$$

Three solutions of this equation can be written in a table as follows:

x	4	2	0
y	0	3	6

Now, these equations can be drawn on a graph. The triangle formed by the two lines and the x-axis can be shown by the shaded part as:



From the graph, it can be observed that the coordinates of the vertices of the triangle so formed are (2, 3), (-1, 0), and (4, 0).

**Concept insight:** In order to find the coordinates of the vertices of the triangle so formed, find the points where the two lines intersect the x-axis and also where the two lines intersect each other. Note here that the coordinates of the intersection of lines with x-axis is taken and not with

y-axis, this is because the question says to find the triangle formed by the two lines and the x-axis.

## Chapter 3 - Pairs of Linear Equations in Two Variables

### Exercise Ex. 3.3

**Solution 1**

$$(i) \ x + y = 14 \quad \dots (i)$$

$$x - y = 4 \quad \dots (ii)$$

From (i), we obtain:

$$x = 14 - y \quad \dots (iii)$$

Substituting this value in equation (ii), we obtain:

$$(14 - y) - y = 4$$

$$14 - 2y = 4$$

$$10 = 2y$$

$$y = 5$$

Substituting the value of y in equation (iii), we obtain:

$$x = 9$$

$$\therefore x = 9, y = 5$$

$$(ii) \quad s - t = 3 \quad \dots (i)$$

$$\frac{s}{3} + \frac{t}{2} = 6 \quad \dots (ii)$$

From (i), we obtain:

$$s = t + 3 \quad \dots (iii)$$

Substituting this value in equation (ii), we obtain:

$$\frac{t+3}{3} + \frac{t}{2} = 6$$

$$2t + 6 + 3t = 36$$

$$5t = 30$$

$$t = 6$$

Substituting the value of t in equation (iii), we obtain:

$$s = 9$$

$$\therefore s = 9, t = 6$$

$$(iii) \ 3x - y = 3 \quad \dots (i)$$

$$9x - 3y = 9 \quad \dots (ii)$$

From (i), we obtain

$$y = 3x - 3 \quad \dots (iii)$$

Substituting this value in equation (ii), we obtain:

$$9x - 3(3x - 3) = 9$$

$$9x - 9x + 9 = 9$$

$$9 = 9$$

This is always true.

Thus, the given pair of equations has infinitely many solutions and the relation between these variables can be given by

$$y = 3x - 3$$

So, one of the possible solutions can  $x = 3$ ,  $y = 6$ .

$$(iv) \quad 0.2x + 0.3y = 1.3 \quad \dots (i)$$

$$0.4x + 0.5y = 2.3 \quad \dots (ii)$$

From equation (i), we obtain:

$$x = \frac{1.3 - 0.3y}{0.2} \quad \dots (iii)$$

Substituting this value in equation (ii), we obtain:

$$0.4 \left( \frac{1.3 - 0.3y}{0.2} \right) + 0.5y = 2.3$$

$$2.6 - 0.6y + 0.5y = 2.3$$

$$2.6 - 2.3 = 0.1y$$

$$0.3 = 0.1y$$

$$y = 3$$

Substituting the value of  $y$  in equation (iii), we obtain:

$$x = \frac{1.3 - 0.3 \times 3}{0.2} = \frac{1.3 - 0.9}{0.2} = \frac{0.4}{0.2} = 2$$

$$\therefore x = 2, y = 3$$

$$(v) \quad \sqrt{2}x + \sqrt{3}y = 0 \quad \dots (i)$$

$$\sqrt{3}x - \sqrt{8}y = 0 \quad \dots (ii)$$

From equation (i), we obtain:

$$x = \frac{-\sqrt{3}y}{\sqrt{2}} \quad \dots (iii)$$

Substituting this value in equation (ii), we obtain:

$$\sqrt{3} \left( -\frac{\sqrt{3}y}{\sqrt{2}} \right) - \sqrt{8}y = 0$$

$$-\frac{3y}{\sqrt{2}} - 2\sqrt{2}y = 0$$

$$y \left( -\frac{3}{\sqrt{2}} - 2\sqrt{2} \right) = 0$$

$$y = 0$$

Substituting the value of  $y$  in equation (iii), we obtain:

$$x = 0$$

$$\therefore x = 0, y = 0$$

$$(vi) \quad \frac{3}{2}x - \frac{5}{3}y = -2 \quad \dots (i)$$

$$\frac{x}{3} + \frac{y}{2} = \frac{13}{6} \quad \dots (ii)$$

From equation (i), we obtain:

$$9x - 10y = -12$$

$$\Rightarrow x = \frac{-12 + 10y}{9} \quad \dots (iii)$$

Substituting this value in equation (ii), we obtain:

$$\frac{\frac{-12 + 10y}{9}}{3} + \frac{y}{2} = \frac{13}{6}$$

$$\frac{-12 + 10y}{27} + \frac{y}{2} = \frac{13}{6}$$

$$\frac{-24 + 20y + 27y}{54} = \frac{13}{6}$$

$$-24 + 47y = 13 \times 9$$

$$47y = 117 + 24$$

$$47y = 141$$

$$y = 3$$

Substituting the value of  $y$  in equation (iii), we obtain:

$$x = \frac{-12 + 10 \times 3}{9} = \frac{18}{9} = 2$$

$$\therefore x = 2, y = 3$$

**Concept insight:** In order to solve the given pairs of equations, we need to substitute the value of any one of the variable from any one of the equation. But make sure to substitute the value of that variable which simplifies calculations. For example, in part (iv) it is most convenient to substitute the value of  $x$  from the first equation to the second equation, as the division by 0.2 is more easier than the division by 0.3, 0.4 and 0.5.

**Solution 2**



$$2x + 3y = 11 \quad \dots (i)$$

$$2x - 4y = -24 \quad \dots (ii)$$

From equation (i), we obtain:

$$x = \frac{11-3y}{2} \quad \dots (iii)$$

Substituting this value in equation (ii), we obtain:

$$2\left(\frac{11-3y}{2}\right) - 4y = -24$$

$$11 - 3y - 4y = -24$$

$$-7y = -35$$

$$y = 5$$

Substituting the value of  $y$  in equation (iii), we obtain:

$$x = \frac{11-3 \times 5}{2} = -\frac{4}{2} = -2$$

$$\therefore x = -2, y = 5$$

$$\text{Now, } y = mx + 3$$

$$\Rightarrow 5 = -2m + 3$$

$$\Rightarrow -2m = 2$$

$$\Rightarrow m = -1$$

Thus, the value of  $m$  is  $-1$ .

**Concept insight:** Firstly the solution of the given pair of linear equations can be found out by substituting the value of one variable, say  $x$ , from one equation into the other equation. Then after finding out the values of  $x$  and  $y$ , substitute them in the equation  $y = m x + 3$  in order to find the value of  $m$ .

### Solution 3

(i) Let one number be  $x$  and the other number be  $y$  such that  $y > x$ .

According to the question:

$$y = 3x \quad (1)$$

$$y - x = 26 \quad (2)$$

On substituting the value of  $y$  from equation (1) into equation (2), we obtain

$$3x - x = 26$$

$$x = 13 \quad (3)$$

Substituting this in equation (1), we obtain

$$y = 39$$

Hence, the numbers are 13 and 39.

**Concept insight:** In this problem, two relations between two numbers are given. So, the two numbers have to be found out here. So the two numbers will be represented by variables  $x$  and  $y$  explicitly state the greater variable.

A pair of equations can be obtained from the given conditions. The pair of equations can then be solved by suitable substitution.

(ii) Let the larger angle be  $x$  and smaller angle be  $y$ .

We know that the sum of the measures of angles of a supplementary pair is always  $180^\circ$ .

According to the given information,

$$x + y = 180^\circ \quad (1)$$

$$x - y = 18^\circ \quad (2)$$

From (1), we obtain

$$x = 180^\circ - y \quad (3)$$

Substituting this in equation (2), we obtain

$$180^\circ - y - y = 18^\circ$$

$$162^\circ = 2y$$

$$81^\circ = y \quad (4)$$

Putting this in equation (3), we obtain

$$\begin{aligned} x &= 180^\circ - 81^\circ \\ &= 99^\circ \end{aligned}$$

Hence, the angles are  $99^\circ$  and  $81^\circ$ .

**Concept insight:** This problem talks about the measure of two supplementary angles. So, the two angles will be written as variables. The pair of equations can be formed using the fact that the sum of two supplementary angles is  $180^\circ$  and using the condition given in the problem. The pair of equations can then be solved by suitable substitution.

(iii) Let the cost of a bat and a ball be  $x$  and  $y$  respectively.

According to the given information,

$$7x + 6y = 3800 \quad (1)$$

$$3x + 5y = 1750 \quad (2)$$

From (1), we obtain

$$y = \frac{3800 - 7x}{6} \quad (3)$$

Substituting this value in equation (2), we obtain

$$3x + 5\left(\frac{3800 - 7x}{6}\right) = 1750$$

$$3x + \frac{9500}{3} - \frac{35x}{6} = 1750$$

$$3x - \frac{35x}{6} = 1750 - \frac{9500}{3}$$

$$\frac{18x - 35x}{6} = \frac{5250 - 9500}{3}$$

$$-\frac{17x}{6} = \frac{-4250}{3}$$

$$-17x = -8500$$

$$x = 500 \quad (4)$$

Substituting this in equation (3), we obtain

$$y = \frac{3800 - 7 \times 500}{6}$$

$$= \frac{300}{6} = 50$$

Hence, the cost of a bat is Rs 500 and that of a ball is Rs 50.

**Concept insight:** Cost of bats and balls needs to be found so the cost of a ball and bat will be taken as the variables. Applying the conditions of total cost of bats and balls algebraic equations will be obtained. The pair of equations can then be solved

by suitable substitution.

- (iv) Let the fixed charge be Rs  $x$  and per km charge be Rs  $y$ .  
According to the given information,

$$x + 10y = 105 \quad (1)$$

$$x + 15y = 155 \quad (2)$$

From (1), we obtain

$$x = 105 - 10y \quad (3)$$

Substituting this in equation (2), we obtain

$$105 - 10y + 15y = 155$$

$$5y = 50$$

$$y = 10 \quad (4)$$

Putting this in equation (3), we obtain

$$x = 105 - 10 \times 10$$

$$x = 5$$

Hence, fixed charge = Rs 5

And per km charge = Rs 10

$$\begin{aligned} \text{Charge for 25 km} &= x + 25y \\ &= 5 + 250 = \text{Rs } 255 \end{aligned}$$

**Concept insight:** In this problem, we are required to find out the fixed charge and the charge per km. So, we will represent these two by using different variables. Now, two linear equations can be written by using the conditions given in the problem. The pair of equations can then be solved by suitable substitution.

- (v) Let the fraction be  $\frac{x}{y}$   
According to the given information,

$$\frac{x+2}{y+2} = \frac{9}{11}$$

$$11x + 22 = 9y + 18$$

$$11x - 9y = -4 \quad (1)$$

$$\frac{x+3}{y+3} = \frac{5}{6}$$

$$6x + 18 = 5y + 15$$

$$6x - 5y = -3 \quad (2)$$

$$\text{From equation (1), we obtain } x = \frac{-4+9y}{11} \quad (3)$$

Substituting this in equation (2), we obtain

$$6\left(\frac{-4+9y}{11}\right) - 5y = -3$$

$$-24 + 54y - 55y = -33$$

$$-y = -9$$

$$y = 9 \quad (4)$$

Substituting this in equation (3), we obtain

$$x = \frac{-4+81}{11} = 7$$

Hence, the fraction is  $\frac{7}{9}$ .

**Concept insight:** This problem talks about a fraction which is not known to us. So numerator and denominators will be taken to be variables  $x$  and  $y$  respectively and  $y$  will be strictly non zero. Then, a pair of linear equations can be formed from the given conditions. The pair of equations can then be solved by suitable substitution.

(vi) Let the age of Jacob be  $x$  and the age of his son be  $y$ .  
According to the given information,

$$(x+5) = 3(y+5)$$

$$x - 3y = 10 \quad (1)$$

$$(x-5) = 7(y-5)$$

$$x - 7y = -30 \quad (2)$$

From (1), we obtain

$$x = 3y + 10 \quad (3)$$

Substituting this value in equation (2), we obtain

$$3y + 10 - 7y = -30$$

$$-4y = -40$$

$$y = 10 \quad (4)$$

Substituting this value in equation (3), we obtain

$$x = 3 \times 10 + 10$$

$$= 40$$

Hence, the present age of Jacob is 40 years whereas the present age of his son is 10 years.

**Concept insight:** Here, Jacob's and his son's present age are not known. So, we will write both these in terms of variables. The problem talks about their ages five years ago and five years hence. Here, five years ago means we have to subtract 5 from their present ages, and five years hence means we have to add 3 to their present ages. So, using the given conditions, a pair of linear equations can be formed. The pair of equations can then be solved by suitable substitution.

## Chapter 3 - Pairs of Linear Equations in Two Variables

### Exercise Ex. 3.4

Solution 1

(i) **Elimination method:**

$$x + y = 5 \quad \dots (1)$$

$$2x - 3y = 4 \quad \dots (2)$$

Multiplying equation (1) by 2, we obtain:

$$2x + 2y = 10 \quad \dots (3)$$

Subtracting equation (2) from equation (3), we obtain:

$$5y = 6$$

$$y = \frac{6}{5}$$

Substituting the value of  $y$  in equation (1), we obtain:

$$x = 5 - \frac{6}{5} = \frac{19}{5}$$

$$\therefore x = \frac{19}{5}, y = \frac{6}{5}$$

**Substitution method:**

From equation (1), we obtain:

$$x = 5 - y \quad \dots (4)$$

Putting this value in equation (2), we obtain:

$$2(5 - y) - 3y = 4$$

$$-5y = -6$$

$$y = \frac{6}{5}$$

Substituting the value of  $y$  in equation (4), we obtain:

$$x = 5 - \frac{6}{5} = \frac{19}{5}$$

$$\therefore x = \frac{19}{5}, y = \frac{6}{5}$$

(ii) **Elimination method:**

$$3x + 4y = 10 \quad \dots (1)$$

$$2x - 2y = 2 \quad \dots (2)$$

Multiplying equation (2) by 2, we obtain:

$$4x - 4y = 4 \quad \dots (3)$$

Adding equation (1) and (3), we obtain

$$7x = 14$$

$$x = 2$$

Substituting the value of  $x$  in equation (1), we obtain:

$$6 + 4y = 10$$

$$4y = 4$$

$$y = 1$$

Hence,  $x = 2, y = 1$

**Substitution method:**

From equation (2), we obtain:

$$x = 1 + y \quad \dots (4)$$

Putting this value in equation (1), we obtain:

$$3(1 + y) + 4y = 10.$$

$$7y = 7$$

$$y = 1$$

Substituting the value of y in equation (4), we obtain:

$$x = 1 + 1 = 2$$

$$\therefore x = 2, y = 1$$

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(iii) **Elimination method:**

$$3x - 5y - 4 = 0 \quad \dots (1)$$

$$9x = 2y + 7$$

$$\Rightarrow 9x - 2y - 7 = 0 \quad \dots (2)$$

Multiplying equation (1) by 3, we obtain:

$$9x - 15y - 12 = 0 \quad \dots (3)$$

Subtracting equation (3) from equation (2), we obtain:

$$13y = -5$$

$$y = \frac{-5}{13}$$

Substituting the value of  $y$  in equation (1), we obtain:

$$3x + \frac{25}{13} - 4 = 0$$

$$3x = \frac{27}{13}$$

$$x = \frac{9}{13}$$

$$\therefore x = \frac{9}{13}, y = \frac{-5}{13}$$

**Substitution method:**

From equation (1), we obtain:

$$x = \frac{5y + 4}{3} \quad \dots (4)$$

Putting this value in equation (2), we obtain:

$$9\left(\frac{5y + 4}{3}\right) - 2y - 7 = 0$$

$$13y = -5$$

$$y = \frac{-5}{13}$$

Substituting the value of  $y$  in equation (4), we obtain:

$$x = \frac{5\left(\frac{-5}{13}\right) + 4}{3}$$

$$x = \frac{9}{13}$$

$$\therefore x = \frac{9}{13}, y = \frac{-5}{13}$$

(iv) **Elimination Method:**

$$\frac{x}{2} + \frac{2y}{3} = -1$$

$$\Rightarrow 3x + 4y = -6 \text{ .....(1)}$$

$$x - \frac{y}{3} = 3$$

$$\Rightarrow 3x - y = 9 \text{ .....(2)}$$

Subtracting (2) from (1), we get

$$5y = -15$$

$$\Rightarrow y = -3$$

$$\therefore x = 3 + \frac{y}{3} = 3 + \frac{-1}{3} = 3 - 1 = 2$$

$$\Rightarrow x = 2 \text{ and } y = -3$$

$$\frac{x}{2} + \frac{2y}{3} = -1$$

$$\Rightarrow 3x + 4y = -6 \text{ .....(1)}$$

$$x - \frac{y}{3} = 3$$

$$\Rightarrow 3x - y = 9 \text{ .....(2)}$$

Subtracting (2) from (1), we get

$$5y = -15$$

$$\Rightarrow y = -3$$

$$\therefore x = 3 + \frac{y}{3} = 3 + \frac{-1}{3} = 3 - 1 = 2$$

$$\Rightarrow x = 2 \text{ and } y = -3$$

#### Substitution method:

From equation (2), we obtain:

$$y = 3x - 9 \text{ ..... (3)}$$

Putting this value in equation (1), we obtain:

$$3x + 4(3x - 9) = -6$$

$$15x = 30$$

$$x = 2$$

Substituting the value of x in equation (3), we obtain:

$$y = 6 - 9 = -3$$

$$\therefore x = 2, y = -3$$

**Concept insight:** In order to solve the given pairs of equations by elimination method, it is required to eliminate one variable to get a linear equation in one variable. For elimination, we multiply one equation (or sometimes even both equations) by suitable constant to make the coefficient of one variable same in both the equations. The trick is to eliminate that variable which involves lesser computations.



In order to solve the given pair of equations by substitution method, we need to substitute the value of any one of the variable from any one of the equation. The trick here is to make sure you substitute the value of that variable which simplifies your calculations. Note that the solution must be same in both the cases.

## Solution 2

(i) Let the fraction be  $\frac{x}{y}$

According to the question,

$$\frac{x+1}{y-1} = 1$$

$$\Rightarrow x - y = -2 \quad \dots (1)$$

$$\frac{x}{y+1} = \frac{1}{2}$$

$$\Rightarrow 2x - y = 1 \quad \dots (2)$$

Subtracting equation (1) from equation (2), we obtain:  
 $x = 3$

Substituting this value of x in equation (1), we obtain:

$$3 - y = -2$$

$$-y = -5$$

$$y = 5$$

Hence, the fraction is  $\frac{3}{5}$ .

**Concept insight:** This problem talks about a fraction. The numerator and denominator of the fraction are not known so we represent these as variables x and y respectively where variable y must be non zero. Then, a pair of linear equations can be formed from the given conditions. The pair of equations can then be solved by eliminating a suitable variable.

(ii) Let present age of Nuri and Sonu be x and y respectively.

According to the question,

$$(x-5) = 3(y-5)$$

$$x - 3y = -10 \quad \dots (1)$$

$$(x+10) = 2(y+10)$$

$$x - 2y = 10 \quad \dots (2)$$

Subtracting equation (1) from equation (2), we obtain:  
 $y = 20$

Substituting the value of y in equation (1), we obtain:

$$x - 60 = -10$$

$$x = 50$$

Thus, the age of Nuri and Sonu are 50 years and 20 years respectively.

**Concept insight:** Here, Nuri's and Sonu's present age are not known. So, we will write both these in terms of variables. Then, using the given conditions, a pair of linear equations can be formed. The pair of equations can then be solved by eliminating a suitable variable.

(iii) Let the units digit and tens digit of the number be  $x$  and  $y$  respectively.

$$\text{Number} = 10y + x$$

$$\text{Number after reversing the digits} = 10x + y$$

According to the question,

$$x + y = 9 \quad \dots (1)$$

$$9(10y + x) = 2(10x + y)$$

$$88y - 11x = 0$$

$$-x + 8y = 0 \quad \dots (2)$$

Adding equations (1) and (2), we obtain:

$$9y = 9$$

$$y = 1$$

Substituting the value of  $y$  in equation (1), we obtain:

$$x = 8$$

Thus, the number is  $10y + x = 10 \times 1 + 8 = 18$

**Concept insight:** This problem talks about a two digit number. Here, remember that a two digit number  $xy$  can be expanded as  $10x + y$ . Then, using the two given conditions, a pair of linear equations can be formed which can be solved by eliminating one of the variables.

(iv) Let the number of Rs 50 notes and Rs 100 notes be  $x$  and  $y$  respectively.

According to the question,

$$x + y = 25 \quad \dots (1)$$

$$50x + 100y = 2000 \quad \dots (2)$$

Multiplying equation (1) by 50, we obtain:

$$50x + 50y = 1250 \quad \dots (3)$$

Subtracting equation (3) from equation (2), we obtain:

$$50y = 750$$

$$y = 15$$

Substituting the value of  $y$  in equation (1), we obtain:

$$x = 10$$

Hence, Meena received 10 notes of Rs 50 and 15 notes of Rs 100.

**Concept insight:** This problem talks about two types of notes, Rs 50 notes and Rs 100 notes. And the number of both these notes with Meena is not known. So, we denote the number of Rs 50 notes and Rs 100 notes by variables  $x$  and  $y$  respectively. Now two linear equations can be formed by the given conditions which can be solved by eliminating one of the variables.

(v) Let the fixed charge for first three days and each day charge thereafter be Rs  $x$  and Rs  $y$  respectively.

According to the question,

$$x + 4y = 27 \quad \dots (1)$$

$$x + 2y = 21 \quad \dots (2)$$

Subtracting equation (2) from equation (1), we obtain:

$$2y = 6$$

$$y = 3$$

Substituting the value of  $y$  in equation (1), we obtain:

$$x + 12 = 27$$

$$x = 15$$

Hence, the fixed charge is Rs 15 and the charge per day is Rs 3.

**Concept insight:** Here, the fixed charges for the first three days and per day charges are not known so, they will be represented using two different variables. The two equations can then be obtained by using the given conditions which can be solved by eliminating one of the variables.

## Chapter 3 - Pairs of Linear Equations in Two Variables

### Exercise Ex. 3.5

Solution 1

(i)  $x - 3y - 3 = 0$   
 $3x - 9y - 2 = 0$   
 Here,  $a_1 = 1, b_1 = -3, c_1 = -3$   
 $a_2 = 3, b_2 = -9, c_2 = -2$   
 $\frac{a_1}{a_2} = \frac{1}{3}, \frac{b_1}{b_2} = \frac{-3}{-9} = \frac{1}{3}, \frac{c_1}{c_2} = \frac{-3}{-2} = \frac{3}{2}$   
 $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Thus, the given pair of equations has no solution.

(ii)  $2x + y = 5 \Rightarrow 2x + y - 5 = 0$   
 $3x + 2y = 8 \Rightarrow 3x + 2y - 8 = 0$   
 Here,  $a_1 = 2, b_1 = 1, c_1 = -5$   
 $a_2 = 3, b_2 = 2, c_2 = -8$   
 $\frac{a_1}{a_2} = \frac{2}{3}, \frac{b_1}{b_2} = \frac{1}{2}, \frac{c_1}{c_2} = \frac{-5}{-8}$   
 $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

Thus, the given pair of equations has unique solution.

By cross-multiplication method,

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1}$$

$$\frac{x}{-8 - (-10)} = \frac{y}{-15 - (-16)} = \frac{1}{4 - 3}$$

$$\frac{x}{2} = \frac{y}{1} = \frac{1}{1}$$

$$\frac{x}{2} = 1, \frac{y}{1} = 1$$

$$\therefore x = 2, y = 1$$

$$(iii) \quad 3x - 5y = 20 \Rightarrow 3x - 5y - 20 = 0$$

$$6x - 10y = 40 \Rightarrow 6x - 10y - 40 = 0$$

$$\text{Here, } a_1 = 3, b_1 = -5, c_1 = -20$$

$$a_2 = 6, b_2 = -10, c_2 = -40$$

$$\frac{a_1}{a_2} = \frac{3}{6} = \frac{1}{2}, \quad \frac{b_1}{b_2} = \frac{-5}{-10} = \frac{1}{2}, \quad \frac{c_1}{c_2} = \frac{-20}{-40} = \frac{1}{2}$$

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

Thus, the given pair of equations has infinite solutions.

The solutions can be obtained by assuming the value of  $x$  to be  $k$ , where  $k$  is any

constant. So, the ordered pairs  $\left(k, \frac{3k - 20}{5}\right)$ , where  $k$  is a constant, are the solutions of the given pair of equations.

$$(iv) \quad x - 3y - 7 = 0$$

$$3x - 3y - 15 = 0$$

$$\text{Here, } a_1 = 1, b_1 = -3, c_1 = -7$$

$$a_2 = 3, b_2 = -3, c_2 = -15$$

$$\frac{a_1}{a_2} = \frac{1}{3}, \quad \frac{b_1}{b_2} = \frac{-3}{-3} = 1, \quad \frac{c_1}{c_2} = \frac{-7}{-15} = \frac{7}{15}$$

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

Thus, the given pair of equations has unique solution.

By cross-multiplication,

$$\frac{x}{45 - (21)} = \frac{y}{-21 - (-15)} = \frac{1}{-3 - (-9)}$$

$$\frac{x}{24} = \frac{y}{-6} = \frac{1}{6}$$

$$\frac{x}{24} = \frac{1}{6} \text{ and } \frac{y}{-6} = \frac{1}{6}$$

$$\therefore x = 4, y = -1$$

**Concept Insight:** In order to answer such questions, remember the condition for the pair of linear equations to have unique, infinitely many or no solution relating the coefficients. Also, remember the formula used to solve by cross multiplication method. While applying the formula, be careful about the signs of the coefficients. And above all don't forget to first write the linear equations in standard form which is  $ax + by + c = 0$ .

## Solution 2

(i)  $2x + 3y - 7 = 0$

$$(a-b)x + (a+b)y - (3a+b-2) = 0$$

Here,  $a_1 = 2$ ,  $b_1 = 3$ ,  $c_1 = -7$

$$a_2 = (a-b), b_2 = (a+b), c_2 = -(3a+b-2)$$

$$\frac{a_1}{a_2} = \frac{2}{a-b}, \quad \frac{b_1}{b_2} = \frac{3}{a+b}, \quad \frac{c_1}{c_2} = \frac{-7}{-(3a+b-2)} = \frac{7}{(3a+b-2)}$$

For the equations to have infinitely many solutions, we have:

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\frac{2}{a-b} = \frac{7}{3a+b-2}$$

$$6a + 2b - 4 = 7a - 7b$$

$$a - 9b = -4 \quad \dots (1)$$

$$\frac{2}{a-b} = \frac{3}{a+b}$$

$$2a + 2b = 3a - 3b$$

$$a - 5b = 0 \quad \dots (2)$$

Subtracting (1) from (2), we obtain:

$$4b = 4$$

$$b = 1$$

Substituting the value of  $b$  in equation (2), we obtain:

$$a - 5 \times 1 = 0$$

$$a = 5$$

Thus, the values of  $a$  and  $b$  are 5 and 1 respectively.

$$(ii) \quad 3x + y - 1 = 0$$

$$(2k-1)x + (k-1)y - (2k+1) = 0$$

$$\text{Here, } a_1 = 3, b_1 = 1, c_1 = -1$$

$$a_2 = (2k-1), b_2 = (k-1), c_2 = -(2k+1)$$

$$\frac{a_1}{a_2} = \frac{3}{2k-1}, \quad \frac{b_1}{b_2} = \frac{1}{k-1}, \quad \frac{c_1}{c_2} = \frac{-1}{-2k-1} = \frac{1}{2k+1}$$

For the equations to have no solution, we have:

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

$$\frac{3}{2k-1} = \frac{1}{k-1} \neq \frac{1}{2k+1}$$

$$\frac{3}{2k-1} = \frac{1}{k-1}$$

$$3k-3 = 2k-1$$

$$k = 2$$

Thus, the value of  $k$  is 2.

**Concept insight:** In order to solve such problems, firstly write the linear equations in standard form which is  $ax + by + c = 0$ . To find the value of the unknowns, the key idea is to remember the conditions for a given pair of equations to have infinite solutions and no solution. In case of infinite solutions rule out the values which does not satisfy all the ratios. Also, be careful about the signs of the coefficients.

### Solution 3

**Substitution method:**

$$8x + 5y = 9 \quad \dots (1)$$

$$3x + 2y = 4 \quad \dots (2)$$

From equation (2), we obtain:

$$y = \frac{4-3x}{2} \quad \dots (3)$$

Substituting this value of  $y$  in equation (1), we obtain:

$$8x + 5\left(\frac{4-3x}{2}\right) = 9$$

$$\text{So } x = -2$$

$$\text{using } x = -2 \text{ in (3) we get } y = \left(\frac{4-3 \times -2}{2}\right) = 5$$

$$\therefore x = -2, y = 5$$

**Cross-multiplication method:**

$$8x + 5y - 9 = 0$$

$$3x + 2y - 4 = 0$$

$$\text{Here, } a_1 = 8, b_1 = 5, c_1 = -9$$

$$a_2 = 3, b_2 = 2, c_2 = -4$$

$$\frac{x}{-20 - (-18)} = \frac{y}{-27 - (-32)} = \frac{1}{16 - 15}$$

$$\frac{x}{-2} = \frac{y}{5} = \frac{1}{1}$$

$$\frac{x}{-2} = 1 \text{ and } \frac{y}{5} = 1$$

$$x = -2 \text{ and } y = 5$$

**Concept insight:** In order to solve the given pair of equations by cross multiplication method, remember the formula to be used and convert the system of equations to standard form. While applying the formula, be careful about the signs of the coefficients.

In order to solve the given pair of equations by substitution method, substitute the value of any one of the variable from any one of the equation. Make sure you substitute the value of that variable which simplifies your calculations.

Solution will be same in both cases

#### Solution 4

(i) Let the fixed charge of the food and the charge for food per day be  $x$  and  $y$  respectively.

According to the question,

$$x + 20y = 1000 \quad \dots(1)$$

$$x + 26y = 1180 \quad \dots(2)$$

$$x = 1180 - 26y$$

Substituting the value of  $x$  in equation (1), we obtain

$$6y = 180$$

$$y = 30$$

Substituting the value of  $y$  in equation (1), we obtain:

$$x + 20 \times 30 = 1000$$

$$x = 1000 - 600$$

$$x = 400$$

Thus, the fixed charge of the food and the charge per day are Rs 400 and Rs 30 respectively.

**Concept insight:** Here, the fixed charge of the food and charge for food per day are the unknown quantities. So they are taken as variables  $x$  and  $y$ . The two equations can then be obtained by using the given conditions. You will observe that the variable  $x$  has the same coefficient in both the equations, so it will be easier to find the solution by eliminating  $x$  from both the equations. Also, one can solve the system by other methods.

(ii) Let the fraction be  $\frac{x}{y}$ . According to the question,

According to the question,

$$\frac{x-1}{y} = \frac{1}{3}$$

$$\Rightarrow 3x - y = 3 \quad \dots (1)$$

$$\frac{x}{y+8} = \frac{1}{4}$$

$$\Rightarrow 4x - y = 8 \quad \dots (2)$$

Subtracting equation (1) from equation (2), we obtain:

$$x = 5$$

Putting the value of x in equation (1), we obtain:

$$15 - y = 3$$

$$y = 12$$

Thus, the fraction is  $\frac{5}{12}$ .

**Concept insight:** Since the problem asks for a fraction. The numerator and denominator of the fraction need to be represented by variables. A pair of linear equations can be obtained from the given conditions. Observe that the variable y has the same coefficient in both the equations, so it will be easier to find the solution by eliminating y from both the equations.

(iii) Let the number of right answers and wrong answers be x and y respectively.

According to the question,

$$3x - y = 40 \quad \dots (1)$$

$$4x - 2y = 50$$

$$\Rightarrow 2x - y = 25 \quad \dots (2)$$

Subtracting equation (2) from equation (1), we obtain:

$$x = 15$$

Substituting the value of x in equation (2), we obtain:

$$30 - y = 25$$

$$y = 5$$

Thus, the number of right answers and the number of wrong answers is 15 and 5 respectively.

Therefore, the total number of questions in the test is 20.

**Concept insight:** In this problem, the number of right answers and the number of wrong answers answered by Yash are the unknown quantities. So, they must be represented by two variables. A pair of linear equations can be obtained by applying the given condition. Variable y has the same coefficient in both the equations, so it will be easier to find the solution by eliminating y from both the equations.

(iv) Let the speed of first car and second car be u km/h and v km/h respectively.

Speed of both cars while they are travelling in same direction = (u - v) km/h

Speed of both cars while they are travelling in opposite directions i.e., when they are travelling towards each other = (u + v) km/h



Distance travelled = Speed x Time

According to the question,

$$5(u - v) = 100$$

$$\Rightarrow u - v = 20 \quad \dots(1)$$

$$1(u + v) = 100$$

$$\Rightarrow u + v = 100 \quad \dots(2)$$

Adding equations (1) and (2), we obtain:

$$2u = 120$$

$$u = 60$$

Substituting the value of u in equation (2), we obtain:

$$v = 40$$

Hence, speed of the first car is 60 km/h and speed of the second car is 40 km/h.

**Concept insight:** In this problem, the speeds of the two cars are not known so we will represent them by variables. Remember that when the cars are travel in the same direction, then the speed will be equal to the difference of their speeds and when they travel in opposite direction, then the speed will be equal to the sum of their speeds. Don't miss to write the unit.

(v) Let length and breadth of rectangle be x unit and y unit respectively.

$$\text{Area} = xy$$

According to the question,

According to the question,

$$(x - 5)(y + 3) = xy - 9$$

$$\Rightarrow 3x - 5y - 6 = 0 \quad \dots (1)$$

$$(x + 3)(y + 2) = xy + 67$$

$$\Rightarrow 2x + 3y - 61 = 0 \quad \dots (2)$$

By cross-multiplication method,

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1}$$

we obtain:

$$\frac{x}{305 - (-18)} = \frac{y}{-12 - (-183)} = \frac{1}{9 - (-10)}$$

$$\frac{x}{323} = \frac{y}{171} = \frac{1}{19}$$

$$x = 17, y = 9$$

Thus, the length and breadth of the rectangle are 17 units and 9 units respectively.

**Concept insight:** Here, the length and the breadth of the rectangle will be represented by variables. Then, the pair of equations will be written from the given conditions. Solution can be easily computed using the cross multiplication method.

## Chapter 3 - Pairs of Linear Equations in Two Variables

### Exercise Ex. 3.6

Solution 1

$$(i) \quad \frac{1}{2x} + \frac{1}{3y} = 2$$

$$\frac{1}{3x} + \frac{1}{2y} = \frac{13}{6}$$

$$\text{Let } \frac{1}{x} = p \text{ and } \frac{1}{y} = q$$

The given equations reduce to:

$$\frac{p}{2} + \frac{q}{3} = 2$$

$$\Rightarrow 3p + 2q - 12 = 0 \quad \dots (1)$$

$$\frac{p}{3} + \frac{q}{2} = \frac{13}{6}$$

$$\Rightarrow 2p + 3q - 13 = 0 \quad \dots (2)$$

Using cross-multiplication method, we obtain:

$$\frac{p}{-26 - (-36)} = \frac{q}{-24 - (-39)} = \frac{1}{9 - 4}$$

$$\frac{p}{10} = \frac{q}{15} = \frac{1}{5}$$

$$\frac{p}{10} = \frac{1}{5}, \quad \frac{q}{15} = \frac{1}{5}$$

$$p = 2, q = 3$$

$$\therefore \frac{1}{x} = 2, \quad \frac{1}{y} = 3$$

$$x = \frac{1}{2}, y = \frac{1}{3}$$

**Concept insight:** Here, the two given equations are not linear as the variables x and y both are

in the denominator. By substituting  $\frac{1}{x} = p$  and  $\frac{1}{y} = q$  given equation reduces to

linear equation in variables p and q. Note here that we are assuming the variables x and y to be non-zero, because only then the given equations will make sense. The linear equations can be solved by using any suitable algebraic method. Also, do not forget here that we need to find the values of x and y. So, put the values of p and q to find x and y.

$$(ii) \quad \frac{2}{\sqrt{x}} + \frac{3}{\sqrt{y}} = 2$$

$$\frac{4}{\sqrt{x}} - \frac{9}{\sqrt{y}} = -1$$

$$\text{Let } \frac{1}{\sqrt{x}} = p \text{ and } \frac{1}{\sqrt{y}} = q$$

The given equations reduce to:

$$2p + 3q = 2 \quad \dots (1)$$

$$4p - 9q = -1 \quad \dots (2)$$

Multiplying equation (1) by 3, we obtain:

$$6p + 9q = 6 \quad (3)$$

Adding equation (2) and (3), we obtain:

$$10p = 5$$

$$p = \frac{1}{2}$$

Putting the value of  $p$  in equation (1), we obtain:

$$2 \times \frac{1}{2} + 3q = 2$$

$$3q = 1$$

$$q = \frac{1}{3}$$

$$\therefore p = \frac{1}{\sqrt{x}} = \frac{1}{2}$$

$$\sqrt{x} = 2$$

$$x = 4$$

$$q = \frac{1}{\sqrt{y}} = \frac{1}{3}$$

$$\sqrt{y} = 3$$

$$y = 9$$

$$\therefore x = 4, y = 9$$

**Concept insight:** Here, the two given equations are not linear as it involves  $x^{-1/2}$  and  $y^{-1/2}$ . If we

substitute  $\frac{1}{\sqrt{x}} = p$  and  $\frac{1}{\sqrt{y}} = q$  in the equations, then they will reduce to

linear form in variables  $p$  and  $q$ . Note here that we are assuming the variables  $x$  and  $y$  to be non negative real numbers. The linear equations can be solved by using any suitable algebraic method. Also, do not forget here that we need to find the values of  $x$  and  $y$ . So, put the values of  $p$  and  $q$  to find  $x$  and  $y$ .

$$(iii) \quad \frac{4}{x} + 3y = 14$$

$$\frac{3}{x} - 4y = 23$$

$$\text{Let } \frac{1}{x} = p$$

The given equations reduce to:

$$4p + 3y = 14$$

$$\Rightarrow 4p + 3y - 14 = 0 \quad \dots (1)$$

$$3p - 4y = 23$$

$$\Rightarrow 3p - 4y - 23 = 0 \quad \dots (2)$$

Using cross-multiplication method, we obtain:

$$\frac{p}{-69 - 56} = \frac{y}{-42 - (-92)} = \frac{1}{-16 - 9}$$

$$\frac{p}{-125} = \frac{y}{50} = \frac{-1}{25}$$

$$\frac{p}{-125} = \frac{-1}{25}, \quad \frac{y}{50} = \frac{-1}{25}$$

$$p = 5, y = -2$$

$$\therefore p = \frac{1}{x} = 5$$

$$x = \frac{1}{5}$$

**Concept insight:** Here, the two given equations are not linear as the variable  $x$  is in the

denominator. If we substitute  $\frac{1}{x} = p$  in the equations, then they will reduce to linear form in

variables  $p$  and  $y$ . Note here that we are assuming the variable  $x$  to be non-zero. The linear equations can be solved by using any suitable algebraic method. Also, do not forget here that we need to find the values of  $x$  and  $y$ . So, put the value of  $p$  to find  $x$ .

$$(iv) \quad \frac{5}{x-1} + \frac{1}{y-2} = 2$$

$$\frac{6}{x-1} - \frac{3}{y-2} = 1$$

$$\text{Let } \frac{1}{x-1} = p \text{ and } \frac{1}{y-2} = q$$

The given equations reduce to:

$$5p + q = 2 \quad \dots (1)$$

$$6p - 3q = 1 \quad \dots (2)$$

Multiplying equation (1) by 3, we obtain:

$$15p + 3q = 6 \quad \dots (3)$$

Adding (2) and (3), we obtain:

$$21p = 7$$

$$p = \frac{1}{3}$$

Putting the value of p in equation (1), we obtain:

$$5 \times \frac{1}{3} + q = 2$$

$$q = 2 - \frac{5}{3} = \frac{1}{3}$$

$$p = \frac{1}{x-1} = \frac{1}{3}$$

$$\Rightarrow x-1 = 3$$

$$\Rightarrow x = 4$$

$$q = \frac{1}{y-2} = \frac{1}{3}$$

$$y-2 = 3$$

$$y = 5$$

$$\therefore x = 4, y = 5$$

**Concept insight:** Here, the two given equations are not linear as the expressions on the LHS of

both the equations are not linear. If we substitute  $\frac{1}{x-1} = p$  and  $\frac{1}{y-2} = q$  in the

equations, then they will reduce to linear form in variables p and q. Note here that we are

assuming  $x \neq 1$  and  $y \neq 2$ . The linear equations can be solved by using any suitable algebraic method. Also, do not forget here that we need to find the values of x and y. So, put the values of p and q to find x and y.

$$\begin{aligned}
 \text{(v)} \quad & \frac{7x-2y}{xy} = 5 \\
 & \Rightarrow \frac{7}{y} - \frac{2}{x} = 5 \quad \dots (1) \\
 & \frac{8x+7y}{xy} = 15 \\
 & \Rightarrow \frac{8}{y} + \frac{7}{x} = 15 \quad \dots (2) \\
 & \text{Let } \frac{1}{x} = p \text{ and } \frac{1}{y} = q
 \end{aligned}$$

The given equations reduce to:

$$\begin{aligned}
 & -2p + 7q = 5 \\
 & \Rightarrow -2p + 7q - 5 = 0 \quad \dots (3)
 \end{aligned}$$

$$\begin{aligned}
 & 7p + 8q = 15 \\
 & \Rightarrow 7p + 8q - 15 = 0 \quad \dots (4)
 \end{aligned}$$

Using cross-multiplication method, we obtain:

$$\frac{p}{-105 - (-40)} = \frac{q}{-35 - 30} = \frac{1}{-16 - 49}$$

$$\frac{p}{-65} = \frac{q}{-65} = \frac{1}{-65}$$

$$\frac{p}{-65} = \frac{1}{-65}, \quad \frac{q}{-65} = \frac{1}{-65}$$

$$p = 1, q = 1$$

$$p = \frac{1}{x} = 1, \quad q = \frac{1}{y} = 1$$

$$x = 1, y = 1$$

**Concept insight:** Here, the two given equations are not linear as the expressions on the LHS of both the equations are not linear. So, here our first step will be to break the expression on the

LHS. Thereafter, if we will substitute  $\frac{1}{x} = p$  and  $\frac{1}{y} = q$  in the equations, then they will

reduce to linear form in variables  $p$  and  $q$ . Note here that we are assuming  $x \neq 0$  and  $y \neq 0$ . The linear equations can be solved by using any suitable algebraic method. Also, do not forget here that we need to find the values of  $x$  and  $y$ . So, put the values of  $p$  and  $q$  to find  $x$  and  $y$ .

$$(vi) \quad 6x + 3y = 6xy$$

$$\Rightarrow \frac{6}{y} + \frac{3}{x} = 6 \quad \dots (1)$$

$$2x + 4y = 5xy$$

$$\Rightarrow \frac{2}{y} + \frac{4}{x} = 5 \quad \dots (2)$$

$$\text{Let } \frac{1}{x} = p \text{ and } \frac{1}{y} = q$$

The given equations reduce to:

$$3p + 6q - 6 = 0$$

$$4p + 2q - 5 = 0$$

By cross-multiplication method, we obtain:

$$\frac{p}{-30 - (-12)} = \frac{q}{-24 - (-15)} = \frac{1}{6 - 24}$$

$$\frac{p}{-18} = \frac{q}{-9} = \frac{1}{-18}$$

$$\frac{p}{-18} = \frac{1}{-18}, \quad \frac{q}{-9} = \frac{1}{-18}$$

$$p = 1, \quad q = \frac{1}{2}$$

$$p = \frac{1}{x} = 1, \quad q = \frac{1}{y} = \frac{1}{2}$$

$$x = 1, \quad y = 2$$

**Concept insight:** Here, the two given equations are not linear as the expressions on the LHS of both the equations are not linear. So, first step will be to divide both LHS and RHS by  $xy$ .

$$\frac{1}{x} = p \text{ and } \frac{1}{y} = q$$

Thereafter, substitute  $\frac{1}{x} = p$  and  $\frac{1}{y} = q$  in the equations, then they will reduce to linear

form in variables  $p$  and  $q$ . Note here that we are assuming  $x \neq 0$  and  $y \neq 0$ . The linear equations can be solved by using any suitable algebraic method. Also, do not forget here that we need to find the values of  $x$  and  $y$ . So, put the values of  $p$  and  $q$  to find  $x$  and  $y$ .

$$(vii) \quad \frac{10}{x+y} + \frac{2}{x-y} = 4$$

$$\frac{15}{x+y} - \frac{5}{x-y} = -2$$

$$\text{Let } \frac{1}{x+y} = p \text{ and } \frac{1}{x-y} = q$$

The given equations reduce to:

$$10p + 2q = 4$$

$$\Rightarrow 10p + 2q - 4 = 0 \quad \dots (1)$$

$$15p - 5q = -2$$

$$\Rightarrow 15p - 5q + 2 = 0 \quad \dots (2)$$

Using cross-multiplication method, we obtain:

$$\frac{p}{4 - 20} = \frac{q}{-60 - 20} = \frac{1}{-50 - 30}$$

$$\frac{p}{-16} = \frac{q}{-80} = \frac{1}{-80}$$

$$\frac{p}{-16} = \frac{1}{-80} \text{ and } \frac{q}{-80} = \frac{1}{-80}$$

$$p = \frac{1}{5} \text{ and } q = 1$$

$$p = \frac{1}{x+y} = \frac{1}{5} \text{ and } q = \frac{1}{x-y} = 1$$

$$x + y = 5 \quad \dots (3)$$

$$x - y = 1 \quad \dots (4)$$

Adding equation (3) and (4), we obtain:

$$2x = 6$$

$$x = 3$$

Substituting the value of x in equation (3), we obtain:

$$y = 2$$

$$\therefore x = 3, y = 2$$

**Concept insight:** Here, the two given equations are not linear as the expressions on the LHS

are not linear. If we substitute  $\frac{1}{x+y} = p$  and  $\frac{1}{x-y} = q$  in the equations, then they will

reduce to linear form in variables p and q. Note here that we are assuming  $x + y \neq 0$  and  $x - y \neq 0$ , otherwise the equations will not make sense. The linear equations can then be solved by using any suitable algebraic method. Also, do not forget here that we need to find the values of x and y. So, put the values of p and q and two linear equations  $x + y = 5$  and  $x - y = 1$  will be obtained. And these equations can be solved easily by elimination method.



$$(viii) \quad \frac{1}{3x+y} + \frac{1}{3x-y} = \frac{3}{4}$$

$$\frac{1}{2(3x+y)} - \frac{1}{2(3x-y)} = \frac{-1}{8}$$

$$\text{Let } \frac{1}{3x+y} = p \text{ and } \frac{1}{3x-y} = q$$

The given equations reduce to:

$$p + q = \frac{3}{4} \quad \dots (1)$$

$$\frac{p}{2} - \frac{q}{2} = \frac{-1}{8}$$

$$\Rightarrow p - q = \frac{-1}{4} \quad \dots (2)$$

Adding (1) and (2), we obtain:

$$2p = \frac{3}{4} - \frac{1}{4}$$

$$2p = \frac{1}{2}$$

$$p = \frac{1}{4}$$

Substituting the value of  $p$  in (2), we obtain:

$$\frac{1}{4} - q = \frac{-1}{4}$$

$$q = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$$

$$p = \frac{1}{3x+y} = \frac{1}{4}$$

$$3x + y = 4 \quad \dots (3)$$

$$q = \frac{1}{3x-y} = \frac{1}{2}$$

$$3x - y = 2 \quad \dots (4)$$

Adding equations (3) and (4), we obtain:

$$6x = 6$$

$$x = 1$$

Substituting the value of  $x$  in (3), we obtain:

$$3(1) + y = 4$$

$$y = 1$$

$$\therefore x = 1, y = 1$$

**Concept insight:** Here, the two given equations are not linear as the expressions on the LHS

are not linear. If we substitute  $\frac{1}{3x+y} = p$  and  $\frac{1}{3x-y} = q$  in the equations, then they will

reduce to linear form in variables  $p$  and  $q$ . Note here that we are assuming  $3x + y \neq 0$  and  $3x - y \neq 0$ . The linear equations can be solved by using any suitable algebraic method. Also, do not forget here that we need to find the values of  $x$  and  $y$ . So, put the values of  $p$  and  $q$  to obtain two linear equations  $3x + y = 4$  and  $3x - y = 2$ . These equations can then be solved easily by elimination method.

### Solution 2

(i) Let the speed of Ritu in still water and the speed of stream be  $x$  km/h and  $y$  km/h respectively.

Speed of Ritu while rowing upstream =  $(x - y)$  km/h

Speed of Ritu while rowing downstream =  $(x + y)$  km/h

According to the question,

$$2(x + y) = 20$$

$$\Rightarrow x + y = 10 \quad \dots (1)$$

$$2(x - y) = 4$$

$$\Rightarrow x - y = 2 \quad \dots (2)$$

Adding equations (1) and (2), we obtain:

$$2x = 12$$

$$x = 6$$

Putting the value of  $x$  in equation (1), we obtain:

$$y = 4$$

Thus, Ritu's speed in still water is 6 km/h and the speed of the current is 4 km/h.

**Concept insight:** Two unknown quantities are speed in still water and the speed of the current, which can be represented by variables  $x$  and  $y$  respectively. Then using the given conditions, two linear equations can be formed. Remember the speed while rowing upstream will be  $x + y$  and while rowing downstream, it will be  $x - y$ . Now, the two equations can be solved easily by the substitution method.

(ii) Let the number of days taken by a woman and a man to finish the work be  $x$  and  $y$  respectively.

$$\text{Work done by a woman in 1 day} = \frac{1}{x}$$

$$\text{Work done by a man in 1 day} = \frac{1}{y}$$

According to the question,

$$4\left(\frac{2}{x} + \frac{5}{y}\right) = 1$$

$$\Rightarrow \frac{2}{x} + \frac{5}{y} = \frac{1}{4}$$

$$3\left(\frac{3}{x} + \frac{6}{y}\right) = 1$$

$$\Rightarrow \frac{3}{x} + \frac{6}{y} = \frac{1}{3}$$

$$\text{Let } \frac{1}{x} = p \text{ and } \frac{1}{y} = q$$

The given equations reduce to:

$$2p + 5q = \frac{1}{4}$$

$$\Rightarrow 8p + 20q = 1$$

$$3p + 6q = \frac{1}{3}$$

$$\Rightarrow 9p + 18q = 1$$

Using cross-multiplication, we obtain:

$$\frac{p}{-20 - (-18)} = \frac{q}{-9 - (-8)} = \frac{1}{144 - 180}$$

$$\frac{p}{-2} = \frac{q}{-1} = \frac{1}{-36}$$

$$\frac{p}{-2} = \frac{1}{-36}, \quad \frac{q}{-1} = \frac{1}{-36}$$

$$p = \frac{1}{18}, \quad q = \frac{1}{36}$$

$$p = \frac{1}{x} = \frac{1}{18}, \quad q = \frac{1}{y} = \frac{1}{36}$$

$$x = 18, y = 36$$

Thus, the number of days taken by a woman and a man to finish the work is 18 and 36.

**Concept insight:** Here, some men and women finish an embroidery work in some days, but it is not given in how many days a man or a woman completes the work. So the one day work of a man and woman is assumed to be  $x$  days and  $y$  days respectively. Since work done and time taken are directly proportional to each other so the one day work of

a man and woman will be given by  $\frac{1}{x}$  and  $\frac{1}{y}$ . A pair of equations in  $\frac{1}{x}$  and  $\frac{1}{y}$  which

are not linear can be formed by the given conditions. So first step will be to convert them into linear form by suitable substitution and then solve using an appropriate algebraic method.

- (iii) Let the speed of train and bus be  $u$  km/h and  $v$  km/h respectively.

$$\text{Now, Time taken to travel distance} = \frac{\text{Distance travelled}}{\text{Speed}}$$

According to the question,

$$\frac{60}{u} + \frac{240}{v} = 4 \quad \dots (1)$$

$$\frac{100}{u} + \frac{200}{v} = \frac{25}{6} \quad \dots (2)$$

$$\text{Let } \frac{1}{u} = p \text{ and } \frac{1}{v} = q$$

The given equations reduce to:

$$60p + 240q = 4 \quad \dots (3)$$

$$100p + 200q = \frac{25}{6}$$

$$600p + 1200q = 25 \quad \dots (4)$$

Multiplying equation (3) by 10, we obtain:

$$600p + 2400q = 40 \quad \dots (5)$$

Subtracting equation (4) from equation (5), we obtain:

$$1200q = 15$$

$$q = \frac{15}{1200} = \frac{1}{80}$$

Substituting the value of q in equation (3), we obtain:

$$60p + 3 = 4$$

$$60p = 1$$

$$p = \frac{1}{60}$$

$$\therefore p = \frac{1}{u} = \frac{1}{60}, q = \frac{1}{v} = \frac{1}{80}$$

$$u = 60 \text{ km/h}, v = 80 \text{ km/h}$$

Thus, the speed of train and the speed of bus are 60 km/h and 80 km/h respectively.

**Concept insight:** In this problem it is mentioned that a girl Roohi travels to her home partly by train and partly by bus. So, we represent the speed of the train and bus by variables u and v respectively. Here, to answer this problem, remember the fact that speed is the ratio of distance and time. Using this and the given conditions, formulate pair of linear equations. Observe that the equations obtained are not linear. So, first step will be to convert them into linear form by suitable substitution and then solve it using an appropriate algebraic method.

## Chapter 3 - Pairs of Linear Equations in Two Variables

### Exercise Ex. 3.7

#### Solution 1

The difference between the ages of Ani and Biju is given as 3 years. So, either Biju is 3 years older than Ani or Ani is 3 years older than Biju.

Let the age of Ani and Biju be  $x$  years and  $y$  years respectively.

Age of Dharam =  $2 * x = 2x$  years

Age of Cathy =  $\frac{y}{2}$  years

Case I: Ani is older than Biju by 3 years

$$x - y = 3 \quad \dots (1)$$

$$2x - \frac{y}{2} = 30$$

$$4x - y = 60 \quad \dots (2)$$

Subtracting (1) from (2), we obtain:

$$3x = 60 - 3 = 57$$

$$x = \frac{57}{3} = 19$$

Age of Ani = 19 years

Age of Biju =  $19 - 3 = 16$  years

Case II: Biju is older than Ani by 3 years

$$y - x = 3 \quad \dots (3)$$

$$2x - \frac{y}{2} = 30$$

$$4x - y = 60 \quad \dots (4)$$

Adding (3) and (4), we obtain:

$$3x = 63$$

$$x = 21$$

Age of Ani = 21 years

Age of Biju =  $21 + 3 = 24$  years

**Concept Insight:** In this problem, ages of Ani and Biju are the unknown quantities. So, we represent them by variables  $x$  and  $y$ . Now, note that here it is given that the ages of Ani and Biju differ by 3 years. So, it is not mentioned that which one is older. So, the most important point in this question is to consider both cases Ani is older than Biju and Biju is older than Ani. For second condition the relation on the ages of Dharam and Cathy can be implemented. Pair of linear equations can be solved using a suitable algebraic method.

## Solution 2

Let the money with the first person and second person be Rs  $x$  and Rs  $y$  respectively.

According to the question,

$$x + 100 = 2(y - 100)$$

$$x + 100 = 2y - 200$$

$$x - 2y = -300 \quad \dots (1)$$

$$6(x - 10) = (y + 10)$$

$$6x - 60 = y + 10$$

$$6x - y = 70 \quad \dots (2)$$

Multiplying equation (2) by 2, we obtain:

$$12x - 2y = 140 \quad \dots (3)$$

Subtracting equation (1) from equation (3), we obtain:

$$11x = 140 + 300$$

$$11x = 440$$

$$x = 40$$

Putting the value of x in equation (1), we obtain:

$$40 - 2y = -300$$

$$40 + 300 = 2y$$

$$2y = 340$$

$$y = 170$$

Thus, the two friends had Rs 40 and Rs 170 with them.

**Concept insight:** This problem talks about the amount of capital with two friends. So, we will represent them by variables x and y respectively. Now, using the given conditions, a pair of linear equations can be formed which can then be solved easily using elimination method.

### Solution 3

Let the speed of the train be x km/h and the time taken by train to travel the given distance be t hours and the distance to travel be d km.

$$\text{Now, Speed} = \frac{\text{Distance travelled}}{\text{Time taken to travel that distance}}$$

$$x = \frac{d}{t}$$

$$\text{Or, } d = xt \quad \dots (1)$$

According to the question,

$$(x+10) = \frac{d}{(t-2)}$$

$$(x+10)(t-2) = d$$

$$xt + 10t - 2x - 20 = d$$

By using equation (1), we obtain:

$$-2x + 10t = 20 \quad \dots (2)$$

$$(x-10) = \frac{d}{(t+3)}$$

$$(x-10)(t+3) = d$$

$$xt - 10t + 3x - 30 = d$$

By using equation (1), we obtain:

$$3x - 10t = 30 \quad \dots (3)$$

Adding equations (2) and (3), we obtain:

$$x = 50$$

Substituting the value of x in equation (2), we obtain:

$$(-2) \times (50) + 10t = 20$$

$$-100 + 10t = 20$$

$$10t = 120$$

$$t = 12$$

From equation (1), we obtain:

$$d = xt = 50 \times 12 = 600$$

Thus, the distance covered by the train is 600 km.

**Concept insight:** To solve this problem, it is very important to remember the

relation,  $Speed = \frac{Distance}{Time}$ .

Now, all these three quantities are unknown. So, we will represent these by three different variables. By using the given conditions, a pair of equations will be obtained. Mind one thing that the equations obtained will not be linear. But they can be reduced to linear form by using the fact

that,  $Speed = \frac{Distance}{Time}$ .

Then two linear equations can be formed which can be solved easily by elimination method.

#### Solution 4

Let the number of rows be  $x$  and number of students in a row be  $y$ .

Total number of students in the class = Number of rows  $\times$  Number of students in a row  
 $= xy$

According to the question,

Total number of students =  $(x - 1)(y + 3)$

$$xy = (x - 1)(y + 3)$$

$$= xy - y + 3x - 3$$

$$3x - y - 3 = 0$$

$$3x - y = 3 \quad \dots (1)$$

Total number of students =  $(x + 2)(y - 3)$

$$xy = xy + 2y - 3x - 6$$

$$3x - 2y = -6 \quad \dots (2)$$

Subtracting equation (2) from (1), we obtain:

$$y = 9$$

Substituting the value of  $y$  in equation (1), we obtain:

$$3x - 9 = 3$$

$$3x = 9 + 3 = 12$$

$$x = 4$$

Number of rows =  $x = 4$

Number of students in a row =  $y = 9$

Total number of students in a class =  $xy = 4 \times 9 = 36$

**Concept insight:** This problem talks about some number of students who are made to stand in a row. So, in order to know the total number of students, it is must to know the number of students standing in each row and the total number of rows. So, these two quantities are represented by variables  $x$  and  $y$ . Use the conditions given in the question to obtain a pair of equations. The pair of equations can then be solved easily by eliminating a suitable variable.

#### Solution 5

$$C = 3 \quad B = 2(A + B)$$

$$3B = 2(A + B)$$

$$B = 2A$$

$$2A - B = 0 \quad \dots (1)$$

We know that the sum of the measures of all angles of a triangle is  $180^\circ$ .

$$\angle A + \angle B + \angle C = 180^\circ$$

$$\angle A + \angle B + 3\angle B = 180^\circ$$

$$\angle A + 4\angle B = 180^\circ \dots (2)$$

Multiplying equation (1) by 4, we obtain:

$$4\angle A + 4\angle B = 720^\circ \dots (3)$$

Adding equations (2) and (3), we obtain:

$$5\angle A = 900^\circ \quad \angle A = 180^\circ$$

From equation (2), we obtain:

$$180^\circ + 4\angle B = 180^\circ \quad 4\angle B = 160^\circ$$

$$\angle B = 40^\circ$$

$$\angle C = 3\angle B = 3 \times 40^\circ = 120^\circ$$

Thus, the measure of  $\angle A$ ,  $\angle B$  and  $\angle C$  are  $20^\circ$ ,  $40^\circ$ , and  $120^\circ$  respectively.

**Concept insight:** Here the problem is about three angles of a triangle. By simplifying the given relation, a linear equation is obtained. Using the angle sum property of triangles, i.e., the sum of all the three angles of a triangle is  $180^\circ$  another relation is obtained. So, from these two relations, a pair of linear equations will be obtained which can be easily solved by elimination method.

### Solution 6

$$5x - y = 5 \Rightarrow y = 5x - 5$$

Three solutions of this equation can be written in a table as follows:

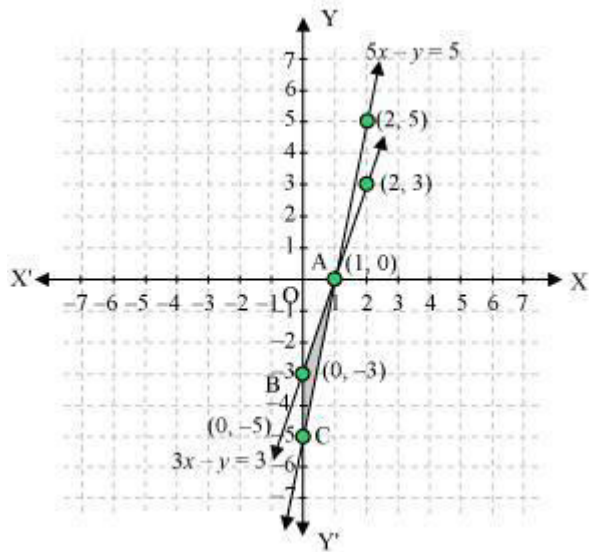
x	0	1	2
y	-5	0	5

$$3x - y = 3 \Rightarrow y = 3x - 3$$

x	0	1	2
y	-3	0	3

The graphical representation of the two lines will be as follows:





It can be observed that the required triangle is  $\triangle ABC$ .  
The coordinates of its vertices are A (1, 0), B (0, -3), C (0, -5).

**Concept insight:** In order to find the coordinates of the vertices of the triangle so formed, find the points where the two lines intersect the y-axis and also where the two lines intersect each other. **Here, note that the coordinates of the intersection of lines with y-axis is taken and not with x-axis, this is because the question says to find the triangle formed by the two lines and the y-axis.**

### Solution 7

(i)

$$px + qy = p - q \quad \dots (1)$$

$$qx - py = p + q \quad \dots (2)$$

Multiplying equation (1) by p and equation (2) by q, we obtain:

$$p^2x + pqy = p^2 - pq \quad \dots (3)$$

$$q^2x - pqy = pq + q^2 \quad \dots (4)$$

Adding equations (3) and (4), we obtain:

$$p^2x + q^2x = p^2 + q^2$$

$$(p^2 + q^2)x = p^2 + q^2$$

$$x = \frac{p^2 + q^2}{p^2 + q^2} = 1$$

Substituting the value of x in equation (1), we obtain:

$$p(1) + qy = p - q$$

$$qy = -q$$

$$y = -1$$

(ii)

$$ax + by = c \quad \dots (1)$$

$$bx + ay = 1 + c \quad \dots (2)$$

Multiplying equation (1) by a and equation (2) by b, we obtain:

$$a^2x + aby = ac \quad \dots (3)$$

$$b^2x + aby = b + bc \quad \dots (4)$$

Subtracting equation (4) from equation (3),

$$(a^2 - b^2)x = ac - bc - b$$

$$x = \frac{c(a-b)-b}{a^2-b^2}$$

Substituting the value of x in equation (1), we obtain:

$$a \left\{ \frac{c(a-b)-b}{a^2-b^2} \right\} + by = c$$

$$\frac{ac(a-b)-ab}{a^2-b^2} + by = c$$

$$by = c - \frac{ac(a-b)-ab}{a^2-b^2}$$

$$by = \frac{a^2c - b^2c - a^2c + abc + ab}{a^2-b^2}$$

$$by = \frac{abc - b^2c + ab}{a^2-b^2}$$

$$by = \frac{bc(a-b) + ab}{a^2-b^2}$$

$$y = \frac{c(a-b) + a}{a^2-b^2}$$

$$(iii) \quad \frac{x}{a} - \frac{y}{b} = 0$$

$$bx - ay = 0 \quad \dots (1)$$

$$ax + by = a^2 + b^2 \quad \dots (2)$$

Multiplying equation (1) and (2) by b and a respectively, we obtain:

$$b^2x - aby = 0 \quad \dots (3)$$

$$a^2x + aby = a^3 + ab^2 \quad \dots (4)$$

Adding equations (3) and (4), we obtain:

$$b^2x + a^2x = a^3 + ab^2$$

$$x(b^2 + a^2) = a(a^2 + b^2)$$

$$x = a$$

Substituting the value of x in equation (1), we obtain:

$$b(a) - ay = 0$$

$$ab - ay = 0$$

$$ay = ab$$

$$y = b$$

(iv)

$$(a-b)x + (a+b)y = a^2 - 2ab - b^2 \quad \dots (1)$$

$$(a+b)(x+y) = a^2 + b^2$$

$$\Rightarrow (a+b)x + (a+b)y = a^2 + b^2 \quad \dots (2)$$

Subtracting equation (2) from (1), we obtain:

$$(a-b)x - (a+b)x = (a^2 - 2ab - b^2) - (a^2 + b^2)$$

$$(a-b-a-b)x = -2ab - 2b^2$$

$$-2bx = -2b(a+b)$$

$$x = a+b$$

Substituting the value of x in equation (1), we obtain:

$$(a-b)(a+b) + (a+b)y = a^2 - 2ab - b^2$$

$$a^2 - b^2 + (a+b)y = a^2 - 2ab - b^2$$

$$(a+b)y = -2ab$$

$$y = \frac{-2ab}{a+b}$$

**Concept insight:** In all the above parts, the given pair of equations can be easily solved by eliminating a suitable variable. Although one can use any of the other methods but the elimination method is the best suited over here.

(v)

$$152x - 378y = -74 \quad \dots (1)$$

$$-378x + 152y = -604 \quad \dots (2)$$

Adding the equations (1) and (2), we obtain:

$$-226x - 226y = -678$$

$$\Rightarrow x + y = 3 \quad \dots (3)$$

Subtracting the equation (2) from equation (1), we obtain:

$$530x - 530y = 530$$

$$\Rightarrow x - y = 1 \quad \dots (4)$$

Adding equations (3) and (4), we obtain:

$$2x = 4$$

$$x = 2$$

Substituting the value of x in equation (3), we obtain:

$$y = 1$$

**Concept insight:** Here, since the coefficients of x and y in the two equations are interchanging this types of problems can be easily solved by once adding the given equations and then subtracting the given equations. By doing this an equation of the form  $x - y = a$  and  $x + y = b$ , where a and b are constants will be obtained. And these equations can now be solved easily by elimination method.

### Solution 8

We know that the sum of the measures of opposite angles in a cyclic quadrilateral is  $180^\circ$ .

$$\angle A + \angle C = 180$$

$$4y + 20 - 4x = 180$$

$$-4x + 4y = 160$$

$$x - y = -40 \quad \dots (1)$$

$$\text{Also, } \angle B + \angle D = 180$$

$$3y - 5 - 7x + 5 = 180$$

$$-7x + 3y = 180 \quad \dots (2)$$

Multiplying equation (1) by 3, we obtain:

$$3x - 3y = -120 \quad \dots (3)$$

Adding equations (2) and (3), we obtain:

$$-4x = 60$$

$$x = -15$$

Substituting the value of x in equation (1), we obtain:

$$-15 - y = -40$$

$$y = -15 + 40 = 25$$

$$\angle A = 4y + 20 = 4(25) + 20 = 120^\circ$$

$$\angle B = 3y - 5 = 3(25) - 5 = 70^\circ$$

$$\angle C = -4x = -4(-15) = 60^\circ$$

$$\angle D = -7x + 5 = -7(-15) + 5 = 110^\circ$$

**Concept insight:** The most important idea to solve this problem is by using the fact that the sum of the measures of opposite angles in a cyclic quadrilateral is  $180^\circ$ . By using this relation, two linear equations can be obtained which can be solved easily by eliminating a suitable variable.

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