NCERT solutions for class 10 maths chapter 1 Real Numbers

Excercise: 1.1

Q1 (1) Use Euclid's division algorithm to find the HCF of 135 and 225

Answer:

225 > 135. Applying Euclid's Division algorithm we get

$225 = 135 \times 1 + 90$

since remainder $\neq$ 0 we again apply the algorithm

$135 = 90 \times 1 + 45$

since remainder $\neq$ 0 we again apply the algorithm

$90 = 45 \times 2$

since remainder = 0 we conclude the HCF of 135 and 225 is 45.

Q1 (2) Use Euclid's division algorithm to find the HCF of 196 and 38220

Answer:

38220 > 196. Applying Euclid's Division algorithm we get

$38220 = 196 \times 195 + 0$

since remainder = 0 we conclude the HCF of 38220 and 196 is 196.

Q1 (3) Use Euclid's division algorithm to find the HCF of 867 and 255

Answer:

867 > 225. Applying Euclid's Division algorithm we get

$867 = 255 \times 3 + 102$

since remainder $\neq$ 0 we apply the algorithm again.

since 255 > 102

$255 = 102 \times 2 + 51$

since remainder $\neq$ 0 we apply the algorithm again.

since 102 > 51

$102 = 51 \times 2 + 0$

since remainder = 0 we conclude the HCF of 867 and 255 is 51.

Q2 Show that any positive odd integer is of the form 6q + 1, or 6q + 3, or 6q + 5, where q is some integer.

Answer:

Let p be any positive integer. It can be expressed as

p = 6q + r

where $q \geq 0$ and $0 \leq r < 6$

but for r = 0, 2 or 4 p will be an even number therefore all odd positive integers can be written in the form 6q + 1, 6q + 3 or 6q + 5.

Q3 An army contingent of 616 members is to march behind an army band of 32 members in a parade. The two groups are to march in the same number of columns. What is the maximum number of columns in which they can march?

Answer:

The maximum number of columns in which they can march = HCF (32, 616)

Since 616 > 32, applying Euclid's Division Algorithm we have

$616 = 32 \times 19 + 8$

Since remainder $\neq$ 0 we again apply Euclid's Division Algorithm

Since 32 > 8

$32 = 8 \times 4 + 0$

Since remainder = 0 we conclude, 8 is the HCF of 616 and 32.

The maximum number of columns in which they can march is 8.

Q4 Use Euclid’s division lemma to show that the square of any positive integer is either of the form 3m or 3m + 1 for some integer m.

[Hint : Let x be any positive integer then it is of the form 3q, 3q + 1 or 3q + 2. Now square each of these and show that they can be rewritten in the form 3m or 3m + 1]

Answer:

Let x be any positive integer.

It can be written in the form 3q + r where $q \geq 0$ and r = 0, 1 or 2

Case 1:

For r = 0 we have

$x^2 = (3q)^2$

$x^2 = 9q^2$

$x^2 = 3(3q^2)$

$x^2 = 3m$

Case 2:

For r = 1 we have

$x^2 = (3q+1)^2$

$x^2 = 9q^2 + 6q + 1$

$x^2 = 3(3q^2 + 2q) + 1$

$x^2 = 3m + 1$

Case 3:

For r = 2 we have

$x^2 = (3q+2)^2$

$x^2 = 9q^2 + 12q + 4$

$x^2 = 3(3q^2 + 4q + 1) + 1$

$x^2 = 3m + 1$

Hence proved.

Q5 Use Euclid's division lemma to show that the cube of any positive integer is of the form 9m, 9m + 1 or 9m + 8.

Answer:

Let x be any positive integer.

It can be written in the form 3q + r where $q \geq 0$ and r = 0, 1 or 2

Case 1:

For r = 0 we have

$x^3 = (3q)^3$

$x^3 = 27q^3$

$x^3 = 9(3q^3)$

$x^3 = 9m$

Case 2:

For r = 1 we have

$x^3 = (3q+1)^3$

$x^3 = 27q^3 + 27q^2 + 9q + 1$

$x^3 = 9(3q^3 + 3q^2 + q) + 1$

$x^3 = 3m + 1$

Case 3:

For r = 2 we have

$x^3 = (3q+2)^3$

$x^3 = 27q^3 + 54q^2 + 36q + 8$

$x^3 = 9(3q^3 + 6q^2 + 4q) + 8$

$x^3 = 3m + 8$

Hence proved.

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Excercise: 1.2

Q1 (1) Express each number as a product of its prime factors: 140

Answer:

The number can be as a product of its prime factors as follows

$$140 = 2 \times 2 \times 5 \times 7$$
$$140 = 2^2 \times 5 \times 7$$

Q 1 (2) Express each number as a product of its prime factors: 156

Answer:

The given number can be expressed as follows

$$156 = 2 \times 2 \times 3 \times 13$$
$$156 = 2^2 \times 3 \times 13$$

Q1 (3) Express each number as a product of its prime factors: 3825

Answer:

The number is expressed as the product of the prime factors as follows

$$3825 = 3 \times 3 \times 5 \times 5 \times 17$$
$$3825 = 3^2 \times 5^2 \times 17$$

Q1 (4) Express each number as a product of its prime factors: 5005

Answer:

The given number can be expressed as the product of its prime factors as follows.

$5005 = 5 \times 7 \times 11 \times 13$

Q1 (5) Express each number as a product of its prime factors: 7429

Answer:

The given number can be expressed as the product of their prime factors as follows

$7429 = 17 \times 19 \times 23$

Q2 (1) Find the LCM and HCF of the following pairs of integers and verify that LCM $\times$ HCF = product of the two numbers: 26 and 91

Answer:

26 = 2 x 13

91 = 7 x 13

HCF(26,91) = 13

LCM(26,91) = 2 x 7 x 13 = 182

HCF x LCM = 13 x 182 = 2366

26 x 91 = 2366

26 x 91 = HCF x LCM

Hence Verified

Q2 (2) Find the LCM and HCF of the following pairs of integers and verify that LCM × HCF = product of the two numbers. 510 and 92

Answer:

The number can be expressed as the product of prime factors as

510 = 2 x 3 x 5 x 17

92 = 2^2 x 23

HCF(510,92) = 2

LCM(510,92) = 2^2 x 3 x 5 x 17 x 23 = 23460

HCF x LCM = 2 x 23460 = 46920

510 x 92 = 46920

510 x 92 = HCF x LCM

Hence Verified

Q2 (3) Find the LCM and HCF of the following pairs of integers and verify that LCM × HCF = product of the two numbers. 336 and 54

Answer:

336 is expressed as the product of its prime factor as

336 = 2^4 x 3 x 7

54 is expressed as the product of its prime factor as

54 = 2 x 3^3

HCF(336,54) = 2 x 3 = 6

LCM(336,54) = 2^4 x 3^3 x 7 = 3024

HCF x LCM = 6 x 3024 = 18144

336 x 54 = 18144

336 x 54 = HCF x LCM

Hence Verified

Q3 (1) Find the LCM and HCF of the following integers by applying the prime factorization method. 12, 15 and 21

Answer:

The numbers can be written as the product of their prime factors as follows

$12 = 2^2 \times 3$

$15 = 3 \times 5$

$21 = 3 \times 7$

HCF = 3

$LCM = 2^2 \times 3 \times 5 \times 7 = 420$

Q3 (2) Find the LCM and HCF of the following integers by applying the prime factorisation method. 17, 23 and 29

Answer:

The given numbers are written as the product of their prime factors as follows

$17 = 1 \times 17$

$23 = 1 \times 23$

29 = 1 x 29

HCF = 1

LCM = 17 x 23 x 29 = 11339

Q3 (3) Find the LCM and HCF of the following integers by applying the prime factorization method. 8, 9 and 25

Answer:

The given numbers are written as the product of their prime factors as follows

$8 = 2^3$

$9 = 3^2$

$25 = 5^2$

HCF = 1

LCM = $2^3 \times 3^2 \times 5^2 = 1800$

Q4 Given that HCF (306, 657) = 9, find LCM (306, 657).

Answer:

As we know the product of HCF and LCM of two numbers is equal to the product of the two numbers we have

HCF (306, 657) x LCM (306, 657) = 306 x 657

Q5 Check whether 6^n can end with the digit 0 for any natural number n.

Answer:

By prime factorizing we have

$6^n = 2^n \times 3^n$

A number will end with 0 if it has at least 1 as the power of both 2 and 5 in its prime factorization. Since the power of 5 is 0 in the prime factorization of 6^n we can conclude that for no value of n 6^n will end with the digit 0.

Q6 Explain why $7 \times 11 \times 13 + 13$ and $7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5$ are composite numbers.

Answer:

$7 \times 11 \times 13 + 13$

$= (7 \times 11 + 1) \times 13$

$= 78 \times 13$

$= 2 \times 3 \times 13^2$

$7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5$

$= (7 \times 6 \times 4 \times 3 \times 2 \times 1 + 1) \times 5$

$= 5 \times 1008$

After Solving we observed that both the number are even numbers and the number rule says that we can take atleast two common out of two numbers. So that the number is composite number.

Q7 There is a circular path around a sports field. Sonia takes 18 minutes to drive one round of the field, while Ravi takes 12 minutes for the same. Suppose they both start at the same point and at the same time and go in the same direction. After how many minutes will they meet again at the starting point?

Answer:

The time after which they meet again at the starting point will be equal to the LCM of the times they individually take to complete one round.

Time taken by Sonia = 18 = 2 x 3 ^{2}

Time taken by Ravi = 12 = 2 ^{2} x 3

LCM(18,12) = 2 ^{2} x 3 ^{2} = 36

Therefore they would again meet at the starting point after 36 minutes.

NCERT solutions for class 10 maths chapter 1 Real Numbers Excercise: 1.3

Q1 Prove that $\sqrt{5}$ is irrational.

Answer:

Let us assume $\sqrt{5}$ is rational.

It means $\sqrt{5}$ can be written in the form $\frac{p}{q}$ where p and q are co-primes and $q \neq 0$

$$\sqrt{5} = \frac{p}{q}$$

Squaring both sides we obtain

From the above equation, we can see that p^2 is divisible by 5, Therefore p will also be divisible by 5 as 5 is a prime number. (i)

Therefore p can be written as 5r

$p = 5r$

$p^2 = (5r)^2$

$5q^2 = 25r^2$

$q^2 = 5r^2$

From the above equation, we can see that q^2 is divisible by 5, Therefore q will also be divisible by 5 as 5 is a prime number. (ii)

From (i) and (ii) we can see that both p and q are divisible by 5. This implies that p and q are not co-primes. This contradiction arises because our initial assumption that $\sqrt{5}$ is rational was wrong. Hence proved that $\sqrt{5}$ is irrational.

Q2 Prove that $3+2\sqrt{5}$ is irrational.

Answer:

Let us assume $3+2\sqrt{5}$ is rational.

This means $3+2\sqrt{5}$ can be wriiten in the form $\frac{p}{q}$ where p and q are co-prime integers.

$$3+2\sqrt{5}=\frac{p}{q}$$
$$2\sqrt{5}=\frac{p}{q}-3$$
$$\sqrt{5}=\frac{p-3q}{2q}$$

As p and q are integers $\frac{p-3q}{2q}$ would be rational, this contradicts the fact that $\sqrt{5}$ is irrational. This contradiction arises because our initial assumption that $3+2\sqrt{5}$ is rational was wrong. Therefore $3+2\sqrt{5}$ is irrational.

Q3 Prove that the following are irrationals :

(i) $1/\sqrt{2}$

Answer:

Let us assume $\frac{1}{\sqrt{2}}$ is rational.

This means $\frac{1}{\sqrt{2}}$ can be written in the form $\frac{p}{q}$ where p and q are co-prime integers.

$$\frac{1}{\sqrt{2}}=\frac{p}{q}$$
$$\sqrt{2}=\frac{q}{p}$$

Since p and q are co-prime integers $\frac{q}{p}$ will be rational, this contradicts the fact that $\sqrt{2}$ is irrational. This contradiction arises because our initial assumption that $\frac{1}{\sqrt{2}}$ is rational was wrong. Therefore $\frac{1}{\sqrt{2}}$ is irrational.

Q3 (2) Prove that the following are irrationals :

(ii) $7\sqrt{5}$

Answer:

Let us assume $7\sqrt{5}$ is rational.

This means $7\sqrt{5}$ can be written in the form $\frac{p}{q}$ where p and q are co-prime integers.

$$7\sqrt{5} = \frac{p}{q}$$
$$\sqrt{5} = \frac{p}{7q}$$

As p and q are integers $\frac{p}{7q}$ would be rational, this contradicts the fact that $\sqrt{5}$ is irrational. This contradiction arises because our initial assumption that $7\sqrt{5}$ is rational was wrong. Therefore $7\sqrt{5}$ is irrational.

Q3 (3) Prove that the following are irrationals : $6 + \sqrt{2}$

Answer:

Let us assume $6 + \sqrt{2}$ is rational.

This means $6 + \sqrt{2}$ can be written in the form $\frac{p}{q}$ where p and q are co-prime integers.

$$6 + \sqrt{2} = \frac{p}{q}$$
$$\sqrt{2} = \frac{p}{q} - 6$$
$$\sqrt{2} = \frac{p - 6q}{q}$$

As p and q are integers $\frac{p-6q}{q}$ would be rational, this contradicts the fact that $\sqrt{2}$ is irrational. This contradiction arises because our initial assumption that $6 + \sqrt{2}$ is rational was wrong. Therefore $6 + \sqrt{2}$ is irrational.

NCERT solutions for class 10 maths chapter 1 Real Numbers Excercise: 1.4

Q1 (1) Without actually performing the long division, state whether the following rational numbers will have a terminating decimal expansion or a non-terminating repeating decimal expansion:13/3125

Answer:

$$\frac{13}{3125} = \frac{13}{5^5}$$

The denominator is of the form $2^a \times 5^b$ where a = 0 and b = 5. Therefore the given rational number will have a terminating decimal expansion.

Q1 (2) Without actually performing the long division, state whether the following rational numbers will have a terminating decimal expansion or a non-terminating repeating decimal expansion:17 / 8

Answer:

$$\frac{17}{8} = \frac{17}{2^3}$$

The denominator is of the form $2^a \times 5^b$ where a = 3 and b = 0. Therefore the given rational number will have a terminating decimal expansion.

Q1 (3) Without actually performing the long division, state whether the following rational numbers will have a terminating decimal expansion or a non-terminating repeating decimal expansion: 64 / 455

Answer:

$$\frac{64}{455} = \frac{64}{5 \times 7 \times 13}$$

The denominator is not of the form $2^a \times 5^b$. Therefore the given rational number will have a non-terminating repeating decimal expansion.

Q 1 (4) Without actually performing the long division, state whether the following rational numbers will have a terminating decimal expansion or a non-terminating repeating decimal expansion: 15 / 1600

Answer:

$$\frac{15}{1600} = \frac{3 \times 5}{2^6 \times 5^2}$$
$$\frac{15}{1600} = \frac{3}{2^6 \times 5}$$

The denominator is of the form 2 ^{a} x 5 ^{b} where a = 6 and b = 1. Therefore the given rational number will have a terminating decimal expansion.

Q 1 (5) Without actually performing the long division, state whether the following rational numbers will have a terminating decimal expansion or a non-terminating repeating decimal expansion: 29 / 343

Answer:

$$\frac{29}{343} = \frac{29}{7^3}$$

The denominator is not of the form 2 ^{a} x 5 ^{b} . Therefore the given rational number will have a non-terminating repeating decimal expansion.

Q1 (6) Without actually performing the long division, state whether the following rational numbers will have a terminating decimal expansion or a non-terminating repeating decimal expansion: $\frac{23}{2^3 5^2}$

Answer:

$$\frac{23}{2^3 \times 5^2}$$

The denominator is of the form 2 ^{a} x 5 ^{b} where a = 3 and b = 2. Therefore the given rational number will have a terminating decimal expansion.

Q1 (7) Without actually performing the long division, state whether the following rational numbers will have a terminating decimal expansion or a non-terminating repeating decimal expansion: $\frac{129}{2^2 7^5 5^7}$

Answer:

$$\frac{129}{2^2 \times 7^5 \times 5^7}$$

The denominator is not of the form $2^a \times 5^b$. Therefore the given rational number will have a non-terminating repeating decimal expansion.

Q1 (8) Without actually performing the long division, state whether the following rational numbers will have a terminating decimal expansion or a non-terminating repeating decimal expansion: 6 / 15

Answer:

$$\frac{6}{15} = \frac{2 \times 3}{3 \times 5}$$
$$\frac{6}{15} = \frac{2}{5}$$

The denominator is of the form $2^a \times 5^b$ where a = 0 and b = 1. Therefore the given rational number will have a terminating decimal expansion.

Q1 (9) Without actually performing the long division, state whether the following rational numbers will have a terminating decimal expansion or a non-terminating repeating decimal expansion: 35 / 50

Answer:

$$\frac{35}{50} = \frac{5 \times 7}{2 \times 5^2}$$
$$\frac{35}{50} = \frac{7}{2 \times 5}$$

The denominator is of the form $2^a \times 5^b$ where a = 1 and b = 1. Therefore the given rational number will have a terminating decimal expansion.

Q1 (10) Without actually performing the long division, state whether the following rational numbers will have a terminating decimal expansion or a non-terminating repeating decimal expansion: 77 / 210

Answer:

$$\frac{77}{210} = \frac{7 \times 11}{2 \times 3 \times 5 \times 7}$$
$$\frac{77}{210} = \frac{11}{2 \times 3 \times 5}$$

The denominator is not of the form $2^{a} \times 5^{b}$. Therefore the given rational number will have a non-terminating repeating decimal expansion.

Q2 Write down the decimal expansions of those rational numbers in Question 1 above which have terminating decimal expansions.

Answer:

decimal expansions of rational numbers are

(i) $\frac{13}{3125} = 0.00416$

(ii)&nbsnbsp; $\frac{17}{8} = 2.125$

(iv) $\frac{15}{1600} = 0.009375$

(vI) $\frac{23}{2^3 \times 5^2} = \frac{23}{200} = 0.115$

(viii) $\frac{6}{15} = \frac{2}{5} = 0.4$

(ix) $\frac{35}{50} = \frac{7}{2 \times 5} = 0.7$

Q3 (1) The following real numbers have decimal expansions as given below. In each case, decide whether they are rational or not. If they are rational, and of the form, p / q what can you say about the prime factors of q? 43.123456789

Answer:

$$43.123456789 = \frac{43123456789}{10^9}$$
$$43.123456789 = \frac{43123456789}{2^9 \times 5^9}$$

The denominator is of the form $2^a \times 5^b$ where a = 9 and b = 9. Therefore the given number is rational and has a terminating decimal expansion.

Q3 (2) The following real numbers have decimal expansions as given below. In each case, decide whether they are rational or not. If they are rational, and of the form, p / q what can you say about the prime factors of q? 0.120120012000120000

Answer:

Since the decimal part of the given number is non-terminating and non-repeating we can conclude that the given number is irrational and cannot be written in the form $\frac{p}{q}$ where p and q are integers.

Q3 (3) The following real numbers have decimal expansions as given below. In each case, decide whether they are rational or not. If they are rational, and of the form, p/q what can you say about the prime factors of q?

$43.\overline{123456789}$

Answer:

As the decimal part of the given number is non-terminating and repeating, the number is rational but its denominator will have factors other than 2 and 5.