CBSE NCERT solutions for class 10 maths chapter 6 Triangles Excercise: 6.1

Q1 (1) Fill in the blanks using the correct word given in brackets: All circles are _______. (congruent, similar)

Answer:

All circles are similar.

Since all the circles have a similar shape. They may have different radius but the shape of all circles is the same.

Therefore, all circles are similar.

Q1 (2) Fill in the blanks using the correct word given in brackets: All squares are _______. (similar, congruent)

Answer:

All squares are similar.

Since all the squares have a similar shape. They may have a different side but the shape of all square is the same.

Therefore, all squares are similar.

Q1 (3) Fill in the blanks using the correct word given in brackets: All _______ triangles are similar. (isosceles, equilateral)

Answer:

All equilateral triangles are similar.

Since all the equilateral triangles have a similar shape. They may have different sides but the shape of all equilateral triangles is the same.

Therefore, all equilateral triangles are similar.

Q1 (4) Fill in the blanks using the correct word given in brackets : (iv) Two polygons of the same number of sides are similar if(a) there corresponding angles are __________ and (b) their corresponding sides are _________. (equal, proportional)

Answer:

Two polygons of the same number of sides are similar if their corresponding angles are equal and their corresponding sides are proportional.

Thus, (a) equal

(b) proportional

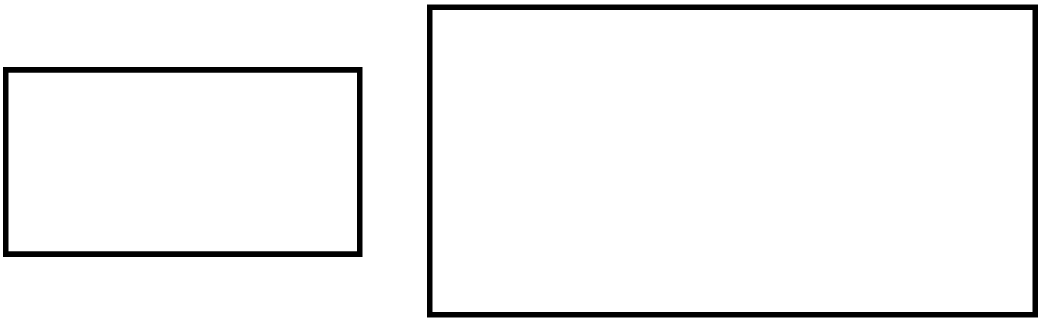
Q2 (1) Give two different examples of a pair of similar figures.

Answer:

The two different examples of a pair of similar figures are :

1. Two circles with different radii.

2. Two rectangles with different breadth and length.

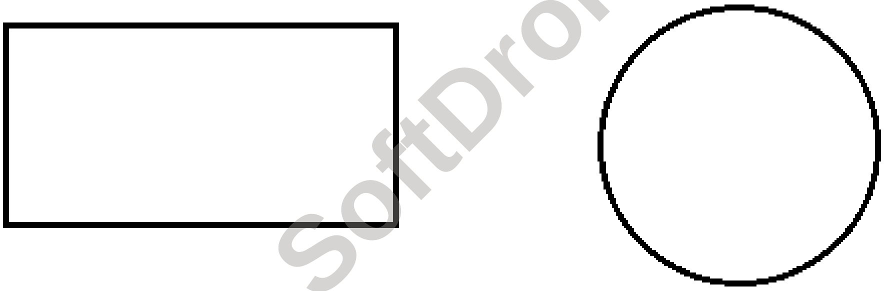


Q2 (2) Give two different examples of a pair of non-similar figures.

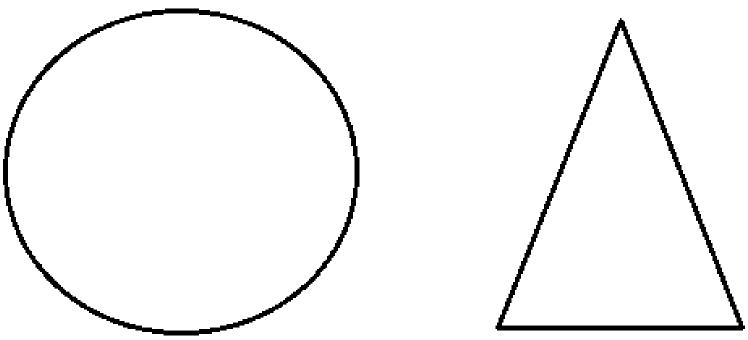
Answer:

The two different examples of a pair of non-similar figures are :

1.Rectangle and circle



2. A circle and a triangle.



Q3 State whether the following quadrilaterals are similar or not:

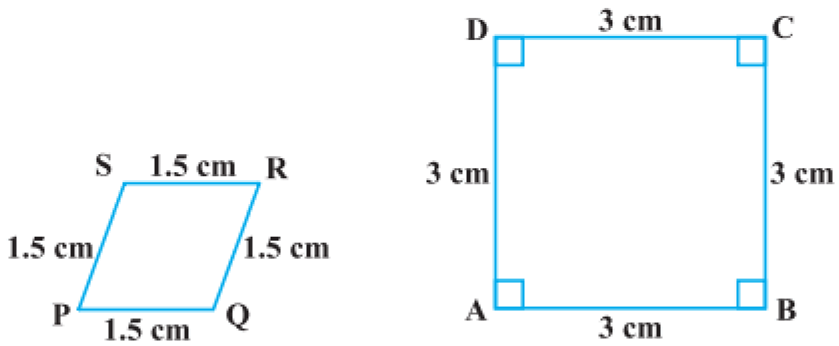


Fig. 6.8

Answer:

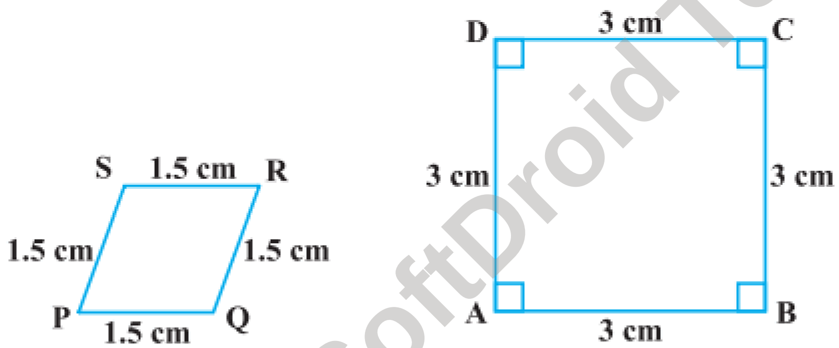


Fig. 6.8

Quadrilateral PQRS and ABCD are not similar as their corresponding sides are proportional i.e. $1:2$ but their corresponding angles are not equal.

NCERT solutions for class 10 maths chapter 6 Triangles Excercise: 6.2

Q1 In Fig. 6.17, (i) and (ii), DE || BC. Find EC in (i) and AD in (ii).

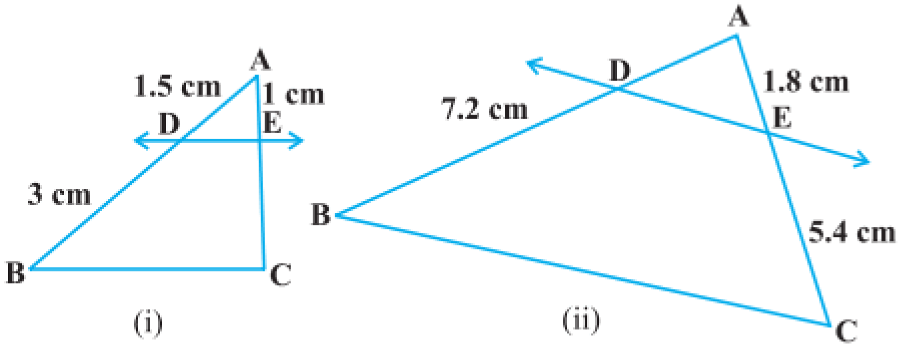
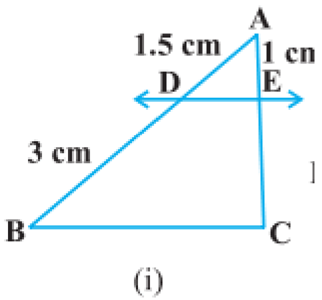


Fig. 6.17

Answer:

(i)



Let EC be x

Given: DE || BC

By using the proportionality theorem, we get

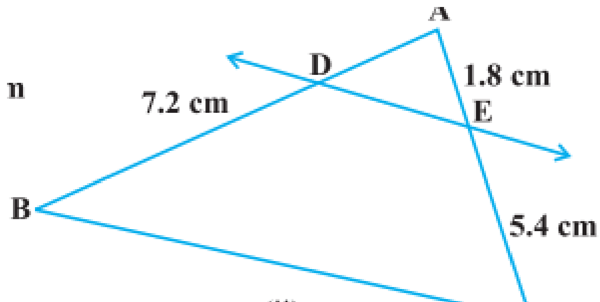
$$\frac{AD}{DB} = \frac{AE}{EC}$$

$$\Rightarrow \frac{1.5}{3} = \frac{1}{x}$$

$$\Rightarrow x = \frac{3}{1.5} = 2\,cm$$

$$\therefore EC = 2\,cm$$

(ii)



(ii)

Let AD be x

Given: DE || BC

By using the proportionality theorem, we get

$$\frac{AD}{DB} = \frac{AE}{EC}$$

$$\Rightarrow \frac{x}{7.2} = \frac{1.8}{5.4}$$

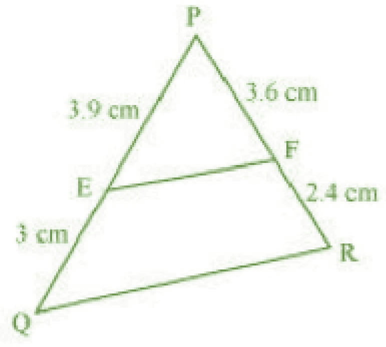
$$\Rightarrow x = \frac{7.2}{3} = 2.4\,cm$$

$$\therefore AD = 2.4\,cm$$

Q2 (1) E and F are points on the sides PQ and PR respectively of a △ PQR. For each of the following cases, state whether EF || QR: PE = 3.9 cm, EQ = 3 cm, PF = 3.6 cm and FR = 2.4 cm

Answer:

(i)



Given :

PE = 3.9 cm, EQ = 3 cm, PF = 3.6 cm and FR = 2.4 cm

$\frac{PE}{EQ} = \frac{3.9}{3} = 1.3\,cm$ and $\frac{PF}{FR} = \frac{3.6}{2.4} = 1.5\,cm$

We have

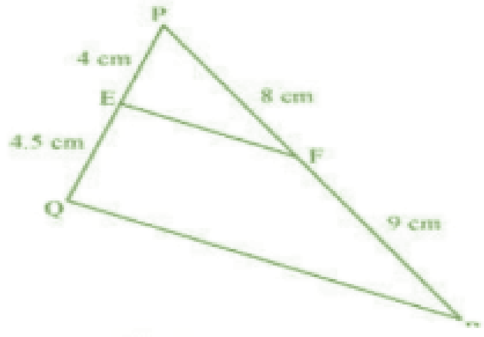
$$\frac{PE}{EQ} \neq \frac{PF}{FR}$$

Hence, EF is not parallel to QR.

Q2 (2) E and F are points on the sides PQ and PR respectively of a triangle PQR. For each of the following cases, state whether EF || QR : PE = 4 cm, QE = 4.5 cm, PF = 8 cm and RF = 9 cm

Answer:

(ii)



Given :

PE = 4 cm, QE = 4.5 cm, PF = 8 cm and RF = 9 cm

$\frac{PE}{EQ} = \frac{4}{4.5} = \frac{8}{9} cm$ and $\frac{PF}{FR} = \frac{8}{9} cm$

We have

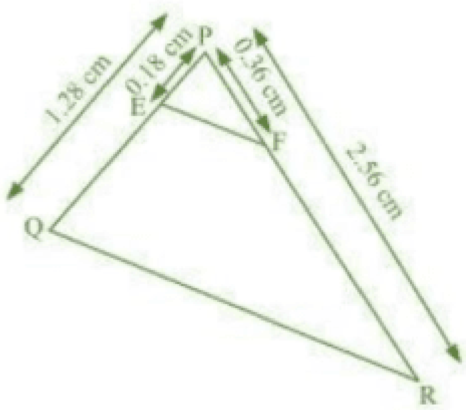
$$\frac{PE}{EQ} = \frac{PF}{FR}$$

Hence, EF is parallel to QR.

Q2 (3) E and F are points on the sides PQ and PR respectively of a triangle PQR. For each of the following cases, state whether EF || QR : PQ = 1.28 cm, PR = 2.56 cm, PE = 0.18 cm and PF = 0.36 cm

Answer:

(iii)



Given :

PQ = 1.28 cm, PR = 2.56 cm, PE = 0.18 cm and PF = 0.36 cm

$\frac{PE}{PQ} = \frac{0.18}{1.28} = \frac{9}{64} cm$ and $\frac{PF}{PR} = \frac{0.36}{2.56} = \frac{9}{64} cm$

We have

$$\frac{PE}{EQ} = \frac{PF}{FR}$$

Hence, EF is parallel to QR.

Q3 In Fig. 6.18, if LM || CB and LN || CD, prove that $\frac{AM}{AB} = \frac{AN}{AD}$

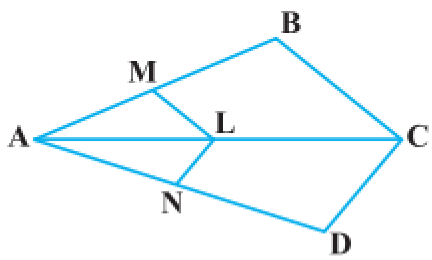


Fig. 6.18

Answer:

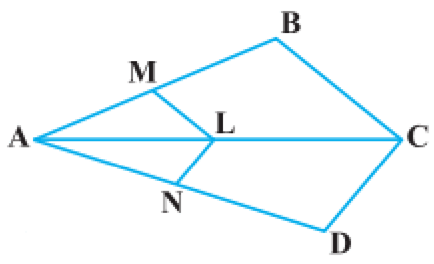


Fig. 6.18

Given : LM || CB and LN || CD

To prove :

$$\frac{AM}{AB} = \frac{AN}{AD}$$

Since , LM || CB so we have

$$\frac{AM}{AB} = \frac{AL}{AC} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 1$$

Also, LN || CD

$$\frac{AL}{AC} = \frac{AN}{AD} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 2$$

From equation 1 and 2, we have

$$\frac{AM}{AB} = \frac{AN}{AD}$$

Hence proved.

Q4 In Fig. 6.19, DE || AC and DF || AE. Prove that BF / FE = BE / EC

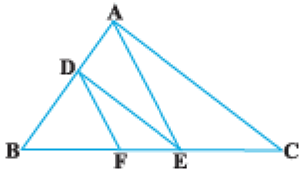


Fig. 6.19

Answer:

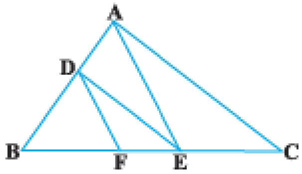


Fig. 6.19

Given : DE || AC and DF || AE.

To prove :

$$\frac{BF}{FE} = \frac{BE}{EC}$$

Since , DE || AC so we have

$$\frac{BD}{DA} = \frac{BE}{EC} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 1$$

Also,DF || AE

$$\frac{BD}{DA} = \frac{BF}{FE} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 2$$

From equation 1 and 2, we have

$$\frac{BF}{FE} = \frac{BE}{EC}$$

Hence proved.

Q5 In Fig. 6.20, DE || OQ and DF || OR. Show that EF || QR.

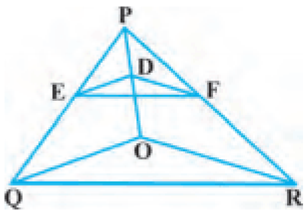


Fig. 6.20

Answer:

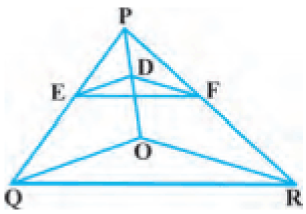


Fig. 6.20

Given : DE || OQ and DF || OR.

To prove EF || QR.

Since DE || OQ so we have

$$\frac{PE}{EQ} = \frac{PD}{DO} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 1$$

Also, DF || OR

$$\frac{PF}{FR} = \frac{PD}{DO} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 2$$

From equation 1 and 2, we have

$$\frac{PE}{EQ} = \frac{PF}{FR}$$

Thus, EF || QR. (converse of basic proportionality theorem)

Hence proved.

Q6 In Fig. 6.21, A, B, and C are points on OP, OQ and OR respectively such that AB || PQ and AC || PR. Show that BC || QR.

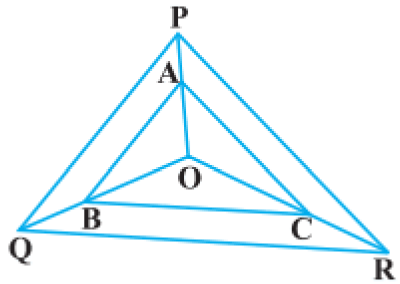


Fig. 6.21

Answer:

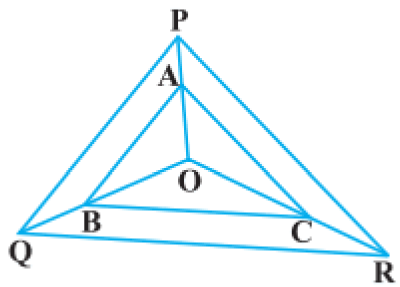


Fig. 6.21

Given : AB || PQ and AC || PR

To prove: BC || QR

Since, AB || PQ so we have

$$\frac{OA}{AP} = \frac{OB}{BQ} \text{...1}$$

Also, AC || PR

$$\frac{OA}{AP} = \frac{OC}{CR} \text{..2}$$

From equation 1 and 2, we have

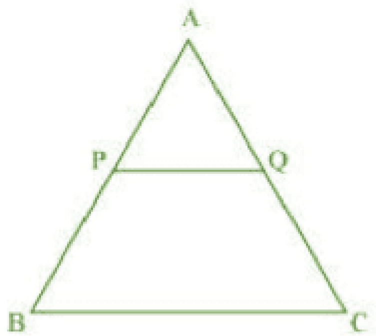
$$\frac{OB}{BQ} = \frac{OC}{CR}$$

Therefore, BC || QR. (converse basic proportionality theorem)

Hence proved.

Q7 Using Theorem 6.1, prove that a line drawn through the mid-point of one side of a triangle parallel to another side bisects the third side. (Recall that you have proved it in Class IX).

Answer:



Let PQ is a line passing through the midpoint of line AB and parallel to line BC intersecting line AC at point Q.

i.e. $PQ||BC$ and $AP = PB$.

Using basic proportionality theorem, we have

$$\frac{AP}{PB} = \frac{AQ}{QC} \text{..........................} 1$$

Since $AP = PB$

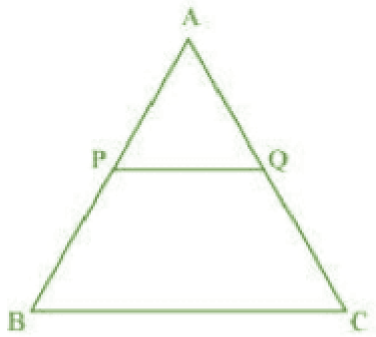
$$\frac{AQ}{QC} = \frac{1}{1}$$

$$\Rightarrow AQ = QC$$

$\therefore$ Q is the midpoint of AC.

Q8 Using Theorem 6.2, prove that the line joining the mid-points of any two sides of a triangle is parallel to the third side. (Recall that you have done it in Class IX).

Answer:



Let P is the midpoint of line AB and Q is the midpoint of line AC.

PQ is the line joining midpoints P and Q of line AB and AC, respectively.

i.e. $AQ = QC$ and $AP = PB$.

we have,

$$\frac{AP}{PB} = \frac{1}{1} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 1$$

$$\frac{AQ}{QC} = \frac{1}{1} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 2$$

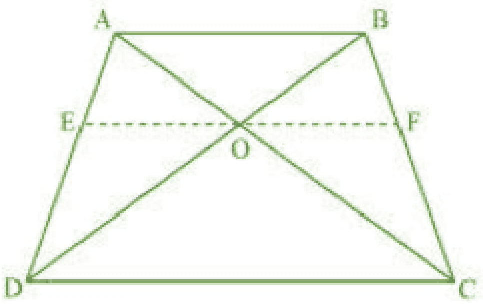
From equation 1 and 2, we get

$$\frac{AQ}{QC} = \frac{AP}{PB}$$

$\therefore$ By basic proportionality theorem, we have $PQ || BC$

Q9 ABCD is a trapezium in which AB || DC and its diagonals intersect each other at the point O. Show that $\frac{AO}{BO} = \frac{CO}{DO}$

Answer:



Draw a line EF passing through point O such that $EO||CD \ and \ FO||CD$

To prove :

$$\frac{AO}{BO} = \frac{CO}{DO}$$

In $\triangle ADC$, we have $CD||EO$

So, by using basic proportionality theorem,

$$\frac{AE}{ED} = \frac{AO}{OC} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 1$$

In $\triangle ABD$, we have $AB||EO$

So, by using basic proportionality theorem,

$$\frac{DE}{EA} = \frac{OD}{BO} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 2$$

Using equation 1 and 2, we get

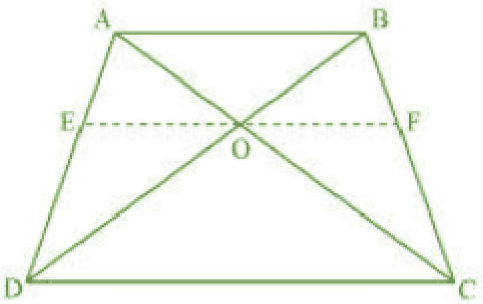
$$\frac{AO}{OC} = \frac{BO}{OD}$$

$$\Rightarrow \frac{AO}{BO} = \frac{CO}{DO}$$

Hence proved.

Q10 The diagonals of a quadrilateral ABCD intersect each other at point O such that $\frac{AO}{BO} = \frac{CO}{DO}$ Show that ABCD is a trapezium.

Answer:



Draw a line EF passing through point O such that $EO || AB$

Given :

$$\frac{AO}{BO} = \frac{CO}{DO}$$

In $\triangle ABD$, we have $AB || EO$

So, by using basic proportionality theorem,

$$\frac{AE}{ED} = \frac{BO}{DO} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 1$$

However, its is given that

$$\frac{AO}{CO} = \frac{BO}{DO} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 2$$

Using equation 1 and 2 , we get

$$\frac{AE}{ED} = \frac{AO}{CO}$$

$\Rightarrow EO||CD$ (By basic proportionality theorem)

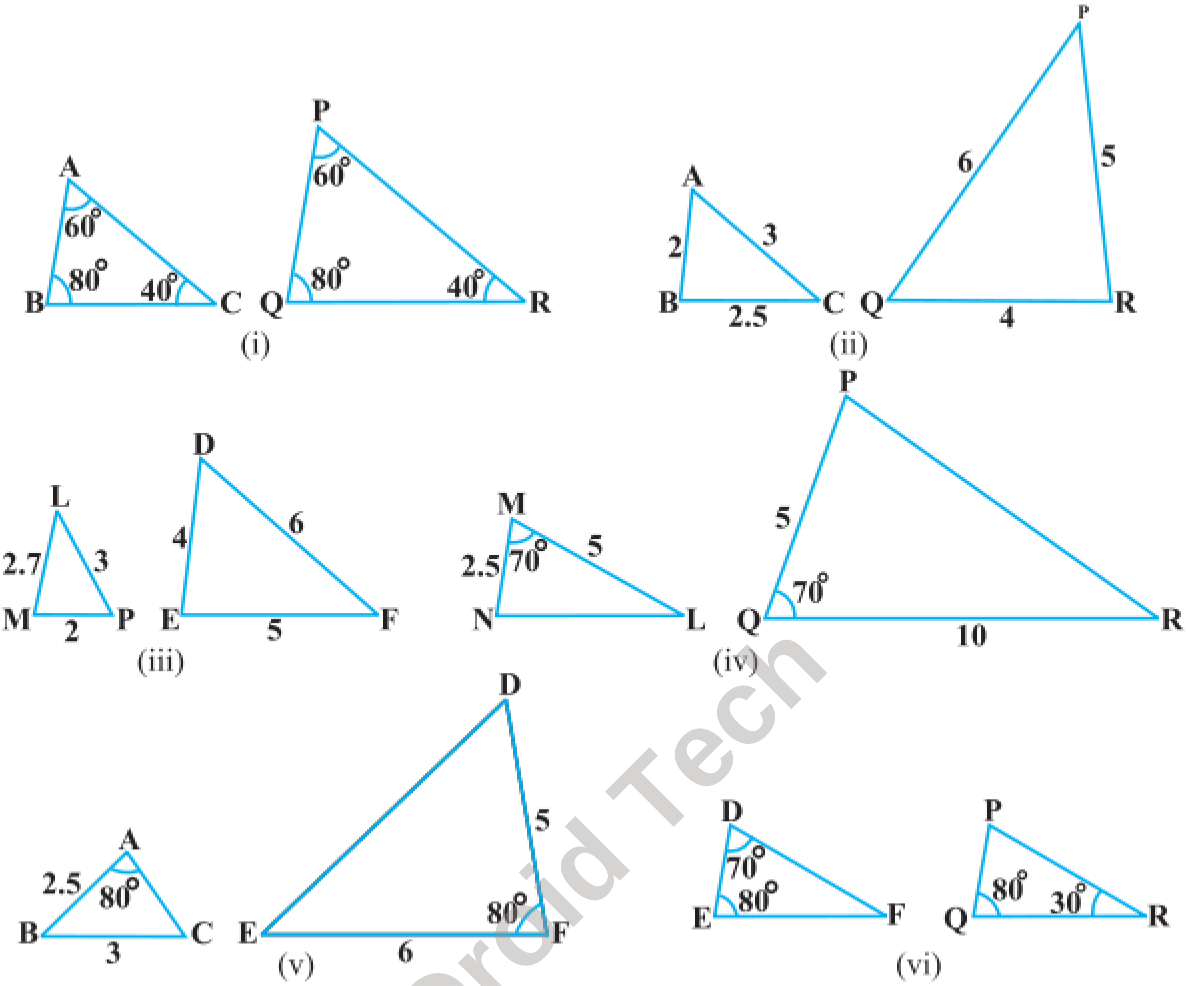
$\Rightarrow AB||EO||CD$

$\Rightarrow AB||CD$

Therefore, ABCD is a trapezium.

NCERT solutions for class 10 maths chapter 6 Triangles Excercise: 6.3

Q1 State which pairs of triangles in Fig. 6.34 are similar. Write the similarity criterion used by you for answering the question and also write the pairs of similar triangles in the symbolic form :



Answer:

(i) $\angle A = \angle P = 60^\circ$

$\angle B = \angle Q = 80^\circ$

$\angle C = \angle R = 40^\circ$

$\therefore \triangle ABC \sim \triangle PQR$ (By AAA)

So , $\dfrac{AB}{QR} = \dfrac{BC}{RP} = \dfrac{CA}{PQ}$

(ii) As corresponding sides of both triangles are proportional.

$\therefore \triangle ABC \sim \triangle PQR$ (By SSS)

(iii) Given triangles are not similar because corresponding sides are not proportional.

(iv) $\triangle MNL \sim \triangle PQR$ by SAS similarity criteria.

(v) Given triangles are not similar because the corresponding angle is not contained by two corresponding sides

(vi) In $\triangle DEF$, we know that

$\angle D + \angle E + \angle F = 180^\circ$

$\Rightarrow 70^\circ + 80^\circ + \angle F = 180^\circ$

$\Rightarrow 150^\circ + \angle F = 180^\circ$

$\Rightarrow \angle F = 180^\circ - 150^\circ = 30^\circ$

In $\triangle PQR$, we know that

$\angle P + \angle Q + \angle R = 180^\circ$

$\Rightarrow 30^\circ + 80^\circ + \angle R = 180^\circ$

$\Rightarrow 110^\circ + \angle R = 180^\circ$

$\Rightarrow \angle R = 180° - 110° = 70°$

$\angle Q = \angle P = 70°$

$\angle E = \angle Q = 80°$

$\angle F = \angle R = 30°$

$\therefore \triangle DEF \sim \triangle PQR$ (By AAA)

Q2 In Fig. 6.35, $\Delta ODC \sim \Delta OBA$, $\angle BOC = 125°$ and $\angle CDO = 70°$. Find $\angle DOC, \angle DCO, \angle OAB$

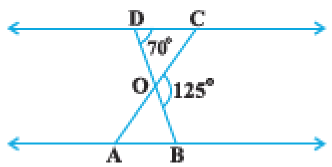


Fig. 6.35

Answer:

Given : $\Delta ODC \sim \Delta OBA$, $\angle BOC = 125°$ and $\angle CDO = 70°$

$\angle DOC + \angle BOC = 180°$ (DOB is a straight line)

$\Rightarrow \angle DOC + 125° = 180°$

$\Rightarrow \angle DOC = 180° - 125°$

$\Rightarrow \angle DOC = 55°$

In ΔODC,

$\angle DOC + \angle ODC + \angle DCO = 180°$

$\Rightarrow 55° + 70° + \angle DCO = 180°$

$\Rightarrow \angle DCO + 125° = 180°$

$\Rightarrow \angle DCO = 180° - 125°$

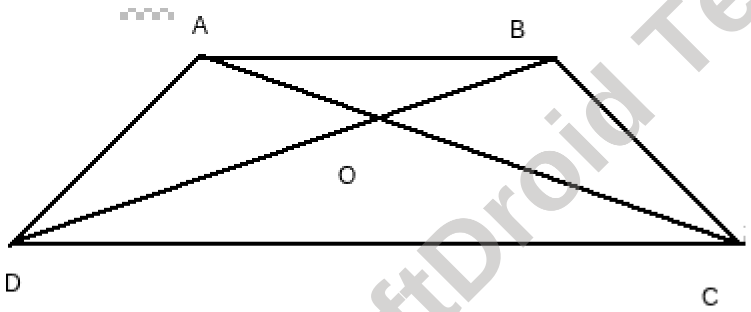
$\Rightarrow \angle DCO = 55°$

Since , $\Delta ODC \sim \Delta OBA$, so

$\Rightarrow \angle OAB = \angle DCO = 55°$ (Corresponding angles are equal in similar triangles).

Q3 Diagonals AC and BD of a trapezium ABCD with AB || DC intersect each other at the point O. Using a similarity criterion for two triangles, show that $\frac{OA}{OC} = \frac{OB}{OD}$

Answer:



In $\triangle DOC \, and \, \triangle BOA$, we have

$\angle CDO = \angle ABO$ (Alternate interior angles as $AB||CD$)

$\angle DCO = \angle BAO$ (Alternate interior angles as $AB||CD$)

$\angle DOC = \angle BOA$ (Vertically opposite angles are equal)

$\therefore \triangle DOC \sim \triangle BOA$ (By AAA)

$\therefore \frac{DO}{BO} = \frac{OC}{OA}$ (corresponding sides are equal)

$\Rightarrow \dfrac{OA}{OC} = \dfrac{OB}{OD}$

Hence proved.

Q4 In Fig. 6.36, $\dfrac{QR}{QS} = \dfrac{QT}{PR}$ and $\angle 1 = \angle 2$. Show that $\Delta PQS \sim \Delta TQR$

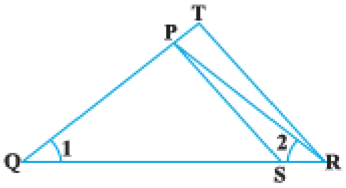


Fig. 6.36

Answer:

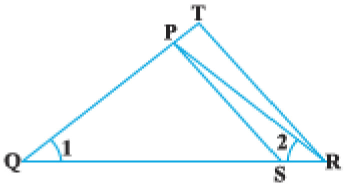


Fig. 6.36

Given : $\dfrac{QR}{QS} = \dfrac{QT}{PR}$ and $\angle 1 = \angle 2$

To prove : $\Delta PQS \sim \Delta TQR$

In ΔPQR , $\angle PQR = \angle PRQ$

$\therefore PQ = PR$

$\dfrac{QR}{QS} = \dfrac{QT}{PR}$ (Given)

$\Rightarrow \dfrac{QR}{QS} = \dfrac{QT}{PQ}$

In $\Delta PQS \, and \, \Delta TQR$,

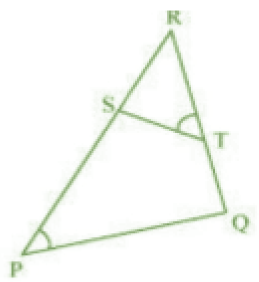
$$\Rightarrow \frac{QR}{QS} = \frac{QT}{PQ}$$

$\angle Q = \angle Q$ (Common)

$\Delta PQS \sim \Delta TQR$ (By SAS)

Q5 S and T are points on sides PR and QR of △ PQR such that ∠ P = ∠ RTS. Show that △ RPQ ~ △ RTS.

Answer:



Given : ∠ P = ∠ RTS

To prove RPQ ~ △ RTS.

In △ RPQ and △ RTS,

∠ P = ∠ RTS (Given)

∠ R = ∠ R (common)

△ RPQ ~ △ RTS. (By AA)

Q6 In Fig. 6.37, if △ ABE ≡ △ ACD, show that △ ADE ~ △ ABC.

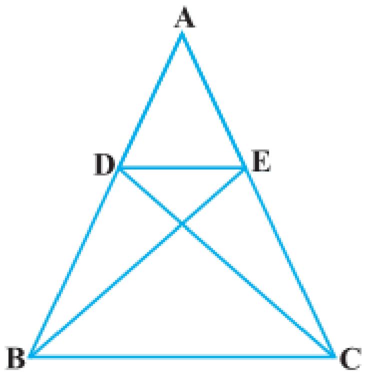


Fig. 6.37

Answer:

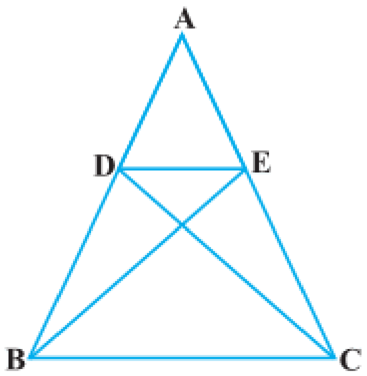


Fig. 6.37

Given : $\triangle ABE \cong \triangle ACD$

To prove ADE ~ $\triangle$ ABC.

Since $\triangle ABE \cong \triangle ACD$

$AB = AC$ (By CPCT)

$AD = AE$ (By CPCT)

In $\triangle$ ADE and $\triangle$ ABC,

$\angle A = \angle A$ (Common)

and

$\frac{AD}{AB} = \frac{AE}{AC}$ ($AB = AC$ and $AD = AE$)

Therefore, $\triangle$ ADE ~ $\triangle$ ABC. (By SAS criteria)

Q7 (1) In Fig. 6.38, altitudes AD and CE of ΔABC intersect each other at the point P. Show that: $\Delta AEP \sim \Delta CDP$

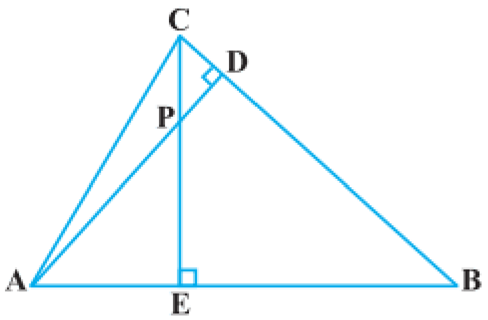


Fig. 6.38

Answer:

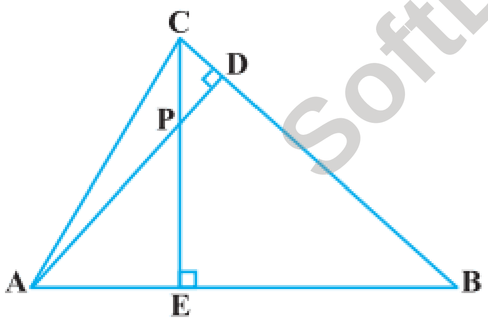


Fig. 6.38

To prove : $\Delta AEP \sim \Delta CDP$

In $\Delta AEP \; and \; \Delta CDP$,

$\angle AEP = \angle CDP$ (Both angles are right angle)

$\angle APE = \angle CPD$ (Vertically opposite angles)

$\Delta AEP \sim \Delta CDP$ (By AA criterion)

Q7 (2) In Fig. 6.38, altitudes AD and CE of ΔABC intersect each other at the point P. Show that: $\Delta ABD \sim \Delta CBE$

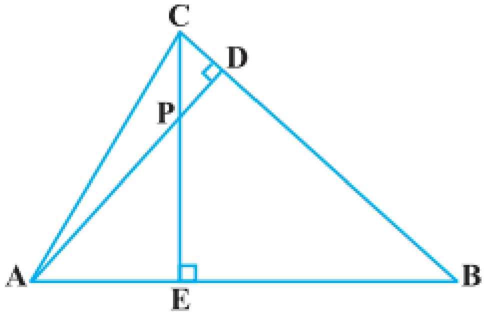


Fig. 6.38

Answer:

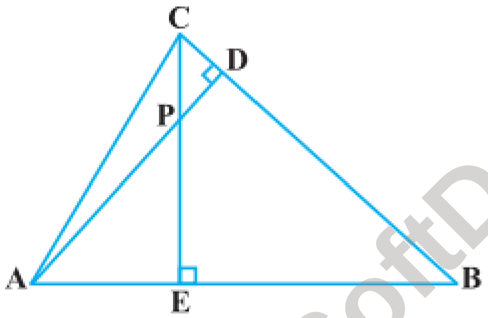


Fig. 6.38

To prove : $\Delta ABD \sim \Delta CBE$

In $\Delta ABD \text{ and } \Delta CBE$,

$\angle ADB = \angle CEB$ (Both angles are right angle)

$\angle ABD = \angle CBE$ (Common)

$\Delta ABD \sim \Delta CBE$ (By AA criterion)

Q7 (3) In Fig. 6.38, altitudes AD and CE of ΔABC intersect each other at the point P. Show that: $\Delta AEP \sim \Delta ADB$

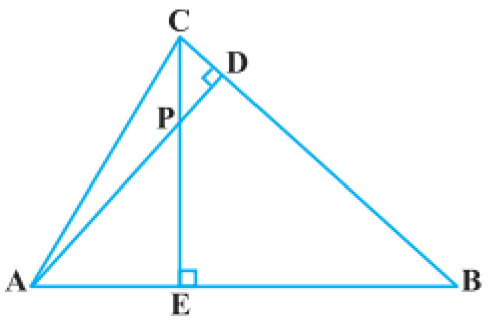


Fig. 6.38

Answer:

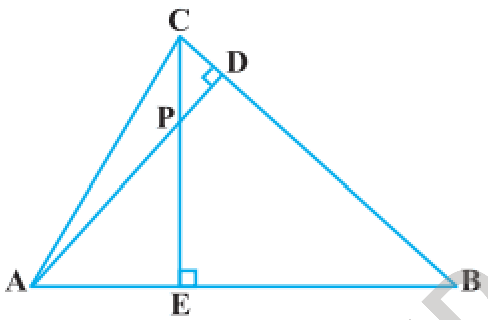


Fig. 6.38

To prove : $\Delta AEP \sim \Delta ADB$

In $\Delta AEP \ \ and \ \Delta ADB$,

$\angle AEP = \angle ADB$ (Both angles are right angle)

$\angle A = \angle A$ (Common)

$\Delta AEP \sim \Delta ADB$ (By AA criterion)

Q7 (4) In Fig. 6.38, altitudes AD and CE of ΔABC intersect each other at the point P. Show that: $\Delta PDC \sim \Delta BEC$

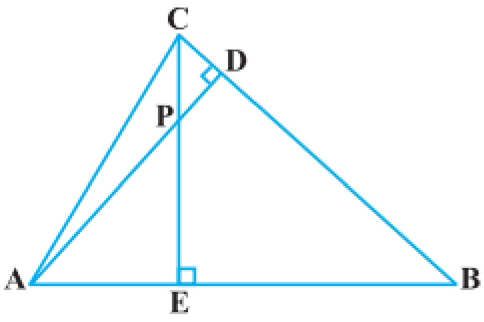


Fig. 6.38

Answer:

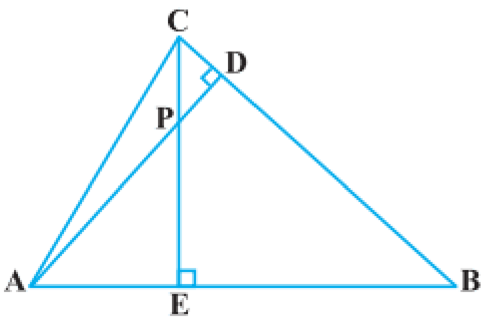


Fig. 6.38

To prove : $\Delta PDC \sim \Delta BEC$

In $\Delta PDC \ and \ \Delta BEC$,

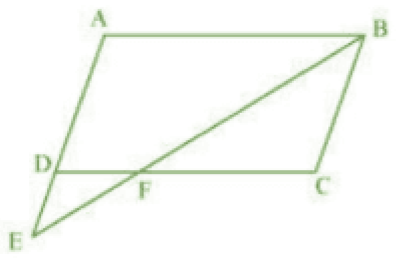
$\angle CDP = \angle CEB$ (Both angles are right angle)

$\angle C = \angle C$ (Common)

$\Delta PDC \sim \Delta BEC$ (By AA criterion)

Q8 E is a point on the side AD produced of a parallelogram ABCD and BE intersects CD at F. Show that $\Delta ABE \sim \Delta CFB$

Answer:



To prove : $\Delta ABE \sim \Delta CFB$

In $\Delta ABE \ and \ \Delta CFB$,

$\angle A = \angle C$ (Opposite angles of a parallelogram are equal)

$\angle AEB = \angle CBF$ (Alternate angles of AE||BC)

$\Delta ABE \sim \Delta CFB$ (By AA criterion)

Q9 (1) In Fig. 6.39, ABC and AMP are two right triangles, right angled at B and M respectively. Prove that: $\Delta ABC \sim \Delta AMP$

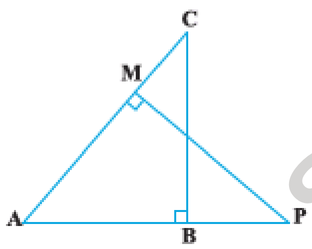


Fig. 6.39

Answer:

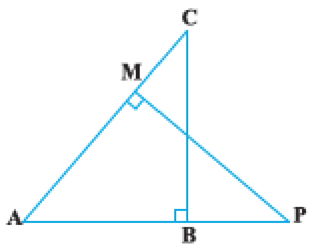


Fig. 6.39

To prove : $\Delta ABC \sim \Delta AMP$

In $\Delta ABC\ and\ \Delta AMP$,

$\angle ABC = \angle AMP$ (Each 90°)

$\angle A = \angle A$ (common)

$\Delta ABC \sim \Delta AMP$ (By AA criterion)

Q9 In Fig. 6.39, ABC and AMP are two right triangles, right angled at B and M respectively. Prove that $\frac{CA}{PA} = \frac{BC}{MP}$

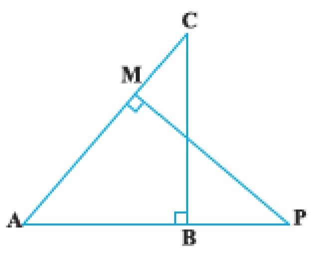


Fig. 6.39

Answer:

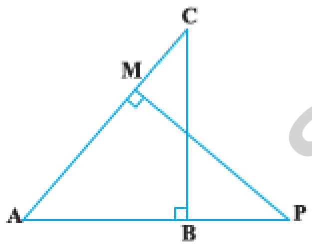


Fig. 6.39

To prove :

$$\frac{CA}{PA} = \frac{BC}{MP}$$

In $\Delta ABC\ and\ \Delta AMP$,

$\angle ABC = \angle AMP$ (Each 90°)

$\angle A = \angle A$ (common)

$\Delta ABC \sim \Delta AMP$ (By AA criterion)

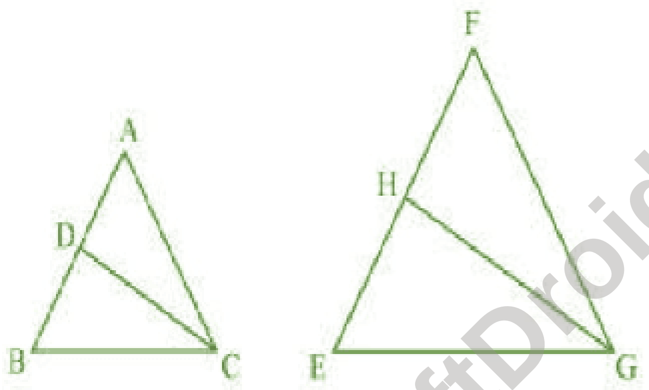
$\frac{CA}{PA} = \frac{BC}{MP}$ (corresponding parts of similar triangles)

Hence proved.

Q10 (1) CD and GH are respectively the bisectors of $\angle ACB$ and $\angle EGF$ such that D and H lie on sides AB and FE of ΔABC and ΔEGF respectively.

If $\Delta ABC \sim \Delta EGF$, show that: $\frac{CD}{GH} = \frac{AC}{FG}$

Answer:



To prove :

$\frac{CD}{GH} = \frac{AC}{FG}$

Given : $\Delta ABC \sim \Delta EGF$

$\angle A = \angle F, \angle B = \angle E \; and \; \angle ACB = \angle FGE, \angle ACB = \angle FGE$

$\therefore \angle ACD = \angle FGH$ (CD and GH are bisectors of equal angles)

$\therefore \angle DCB = \angle HGE$ (CD and GH are bisectors of equal angles)

In $\Delta ACD \; and \; \Delta FGH$

$\therefore \angle ACD = \angle FGH$ (proved above)

$\angle A = \angle F$ (proved above)

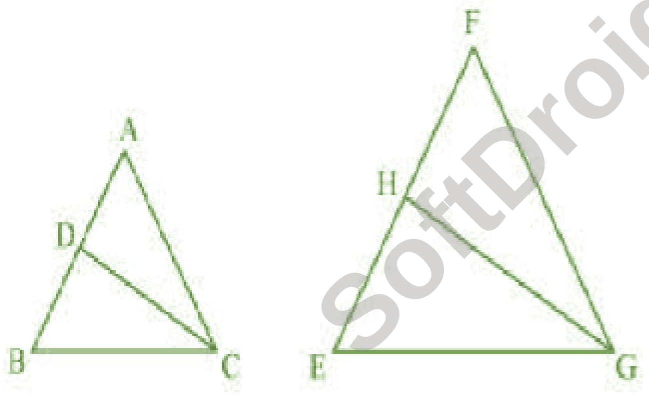
$\Delta ACD \sim \Delta FGH$ (By AA criterion)

$$\Rightarrow \frac{CD}{GH} = \frac{AC}{FG}$$

Hence proved.

Q 10 (2) CD and GH are respectively the bisectors of $\angle ABC \ \ and \ \ \angle EGF$ such that D and H lie on sides AB and FE of $\Delta ABC \ \ and \ \ \Delta EGF$ respectively. If $\Delta ABC \sim \Delta EGF$, show that: $\Delta DCB \sim \Delta HGE$

Answer:



To prove : $\Delta DCB \sim \Delta HGE$

Given : $\Delta ABC \sim \Delta EGF$

In $\Delta DCB \ \ and \ \ \Delta HGE$,

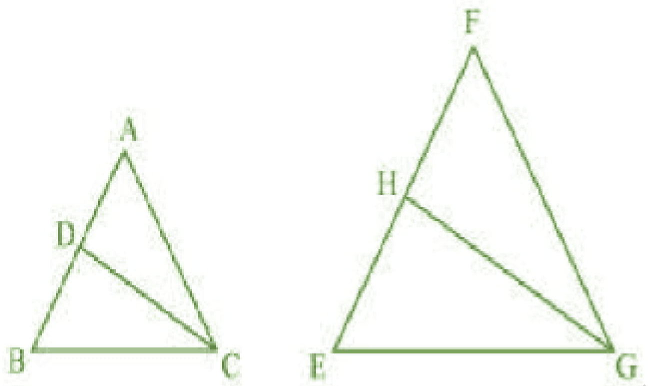
$\therefore \angle DCB = \angle HGE$ (CD and GH are bisectors of equal angles)

$\angle B = \angle E$ ($\Delta ABC \sim \Delta EGF$)

$\Delta DCB \sim \Delta HGE$ (By AA criterion)

Q10 (3) CD and GH are respectively the bisectors of $\angle ABC \ \ and \ \ \angle EGF$ such that D and H lie on sides AB and FE of $\Delta ABC \ \ and \ \ \Delta EGF$ respectively. If $\Delta ABC \sim \Delta EGF$, show that: $\Delta DCA \sim \Delta HGF$

Answer:



To prove : $\Delta DCA \sim \Delta HGF$

Given : $\Delta ABC \sim \Delta EGF$

In $\Delta DCA \ \ and \ \Delta HGF$,

$\therefore \angle ACD = \angle FGH$ (CD and GH are bisectors of equal angles)

$\angle A = \angle F$ ($\Delta ABC \sim \Delta EGF$)

$\Delta DCA \sim \Delta HGF$ (By AA criterion)

Q11 In Fig. 6.40, E is a point on side CB produced of an isosceles triangle ABC with AB = AC. If $AD \perp BC$ and $EF \perp AC$, prove that $\Delta ABD \sim \Delta ECF$

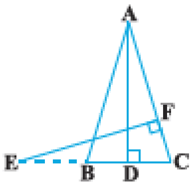


Fig. 6.40

Answer:

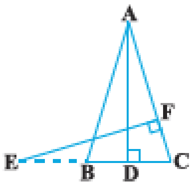


Fig. 6.40

To prove : $\Delta ABD \sim \Delta ECF$

Given: ABC is an isosceles triangle.

$AB = AC \ and \ \angle B = \angle C$

In $\Delta ABD \ and \ \Delta ECF$,

$\angle ABD = \angle ECF$ ($\angle ABD = \angle B = \angle C = \angle ECF$)

$\angle ADB = \angle EFC$ (Each $90°$)

$\Delta ABD \sim \Delta ECF$ (By AA criterion)

Q12 Sides AB and BC and median AD of a triangle ABC are respectively proportional to sides PQ and QR and median PM of ΔPQR (see Fig. 6.41). Show that $\Delta ABC \sim \Delta PQR$

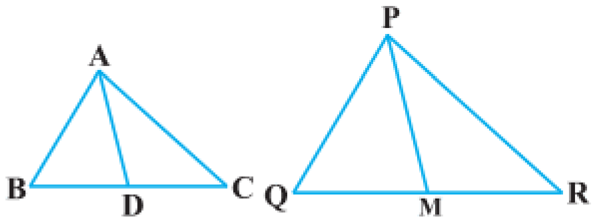


Fig. 6.41

Answer:

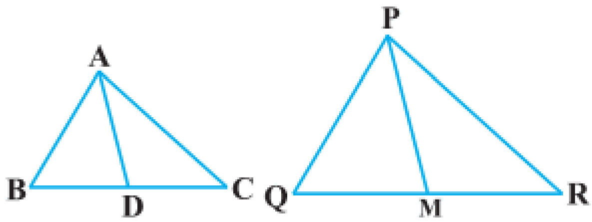


Fig. 6.41

AD and PM are medians of triangles. So,

$$BD = \frac{BC}{2} \, and \, QM = \frac{QR}{2}$$

Given :

$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AD}{PM}$$

$$\Rightarrow \frac{AB}{PQ} = \frac{\frac{1}{2}BC}{\frac{1}{2}QR} = \frac{AD}{PM}$$

$$\Rightarrow \frac{AB}{PQ} = \frac{BD}{QM} = \frac{AD}{PM}$$

In $\triangle ABD \, and \, \triangle PQM,$

$$\frac{AB}{PQ} = \frac{BD}{QM} = \frac{AD}{PM}$$

$\therefore \triangle ABD \sim \triangle PQM,$ (SSS similarity)

$\Rightarrow \angle ABD = \angle PQM$ (Corresponding angles of similar triangles)

In $\triangle ABC \, and \, \triangle PQR,$

$\Rightarrow \angle ABD = \angle PQM$ (proved above)

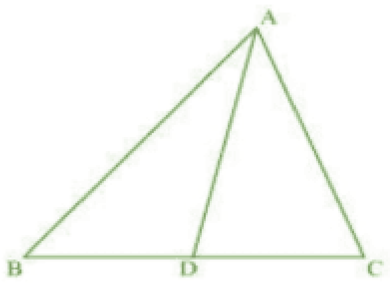
$\frac{AB}{PQ} = \frac{BC}{QR}$

Therefore, $\Delta ABC \sim \Delta PQR$. (SAS similarity)

Q13 D is a point on the side BC of a triangle ABC such that $\angle ADC = \angle BAC$.

Show that $CA^2 = CB.CD.$

Answer:



In, $\triangle ADC \; and \; \triangle BAC,$

$\angle ADC = \angle BAC$ (given)

$\angle ACD = \angle BCA$ (common)

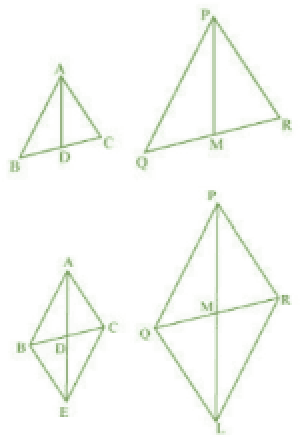
$\triangle ADC \;\; \sim \;\; \triangle BAC,$ (By AA rule)

$\frac{CA}{CB} = \frac{CD}{CA}$ (corresponding sides of similar triangles)

$\Rightarrow CA^2 = CB \times CD$

Q14 Sides AB and AC and median AD of a triangle ABC are respectively proportional to sides PQ and PR and median PM of another triangle PQR. Show that $\Delta ABC \sim \Delta PQR$

Answer:



$\frac{AB}{PQ} = \frac{AC}{PR} = \frac{AD}{PM}$ (given)

Produce AD and PM to E and L such that AD=DE and PM=DE. Now,

join B to E,C to E,Q to L and R to L.

AD and PM are medians of a triangle, therefore

QM=MR and BD=DC

AD = DE (By construction)

PM=ML (By construction)

So, diagonals of ABEC bisecting each other at D,so ABEC is a parallelogram.

Similarly, PQLR is also a parallelogram.

Therefore, AC=BE ,AB=EC and PR=QL,PQ=LR

$\frac{AB}{PQ} = \frac{AC}{PR} = \frac{AD}{PM}$ (Given)

$\Rightarrow \frac{AB}{PQ} = \frac{BE}{QL} = \frac{2.AD}{2.PM}$

$\Rightarrow \frac{AB}{PQ} = \frac{BE}{QL} = \frac{AE}{PL}$

$\triangle ABE \sim \triangle PQL$ (SSS similarity)

$\angle BAE = \angle QPL$ ....1 (Corresponding angles of similar triangles)

Similarity, $\triangle AEC = \triangle PLR$

$\angle CAE = \angle RPL$ 2

Adding equation 1 and 2,

$\angle BAE + \angle CAE = \angle QPL + \angle RPL$

$\angle CAB = \angle RPQ$ 3

In $\triangle ABC \, and \, \triangle PQR,$

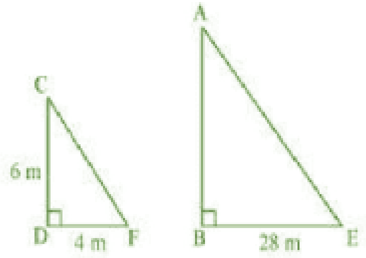
$\frac{AB}{PQ} = \frac{AC}{PR}$ (Given)

$\angle CAB = \angle RPQ$ (From above equation 3)

$\triangle ABC \sim \triangle PQR$ (SAS similarity)

Q15 A vertical pole of length 6 m casts a shadow 4 m long on the ground and at the same time a tower casts a shadow 28 m long. Find the height of the tower.

Answer:



CD = pole

AB = tower

Shadow of pole = DF

Shadow of tower = BE

In $\triangle ABE \ and \triangle CDF,$

$\angle CDF = \angle ABE$ (Each $90°$)

$\angle DCF = \angle BAE$ (Angle of sun at same place)

$\triangle ABE \ \sim \ \triangle CDF,$ (AA similarity)

$$\frac{AB}{CD} = \frac{BE}{QL}$$

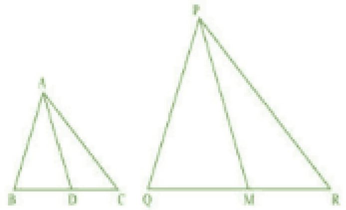
$$\Rightarrow \frac{AB}{6} = \frac{28}{4}$$

$\Rightarrow AB = 42$ cm

Hence, the height of the tower is 42 cm.

Q16 If AD and PM are medians of triangles ABC and PQR, respectively where $\Delta ABC \sim \Delta PQR$, prove that $\frac{AB}{PQ} = \frac{AD}{PM}$

Answer:



$\triangle ABC \sim \triangle PQR$ (Given)

$\frac{AB}{PQ} = \frac{AC}{PR} = \frac{BC}{QR}$ ....1(corresponding sides of similar triangles)

$\angle A = \angle P, \angle B = \angle Q, \angle C = \angle R$ 2

AD and PM are medians of triangle.So,

$BD = \frac{BC}{2} \, and \, QM = \frac{QR}{2}$ 3

From equation 1 and 3, we have

$\frac{AB}{PQ} = \frac{BD}{QM}$ 4

In $\triangle ABD \, and \, \triangle PQM,$

$\angle B = \angle Q$ (From equation 2)

$\frac{AB}{PQ} = \frac{BD}{QM}$ (From equation 4)

$\triangle ABD \sim \triangle PQM,$ (SAS similarity)

$\frac{AB}{PQ} = \frac{BD}{QM} = \frac{AD}{PM}$

NCERT solutions for class 10 maths chapter 6 Triangles Excercise: 6.4

Q1 Let $\Delta ABC \sim \Delta DEF$ and their areas be, respectively, 64 cm^2 and 121 cm^2. If EF = 15.4 cm, find BC.

Answer:

$\Delta ABC \sim \Delta DEF$ (Given)

ar(ABC) = 64 cm^2 and ar(DEF)=121 cm^2 .

EF = 15.4 cm (Given)

$$\frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \frac{AB^2}{DE^2} = \frac{BC^2}{EF^2} = \frac{AC^2}{DF^2}$$

$$\frac{64}{121} = \frac{BC^2}{(15.4)^2}$$

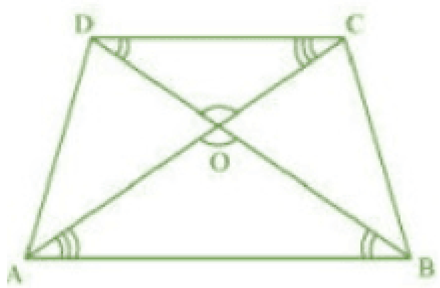
$$\Rightarrow \frac{8}{11} = \frac{BC}{15.4}$$

$$\Rightarrow \frac{8 \times 15.4}{11} = BC$$

$$\Rightarrow BC = 11.2cm$$

Q2 Diagonals of a trapezium ABCD with AB || DC intersect each other at the point O. If AB = 2 CD, find the ratio of the areas of triangles AOB and COD.

Answer:



Given: Diagonals of a trapezium ABCD with AB || DC intersect each other at the point O.

AB = 2 CD (Given)

In $\triangle AOB\, and\, \triangle COD,$

$\angle COD = \angle AOB$ (vertically opposite angles)

$\angle OCD = \angle OAB$ (Alternate angles)

$\angle ODC = \angle OBA$ (Alternate angles)

$\therefore \triangle AOB \sim \triangle COD$ (AAA similarity)

$$\frac{ar(\triangle AOB)}{ar(\triangle COD)} = \frac{AB^2}{CD^2}$$

$$\frac{ar(\triangle AOB)}{ar(\triangle COD)} = \frac{(2CD)^2}{CD^2}$$

$$\frac{ar(\triangle AOB)}{ar(\triangle COD)} = \frac{4.CD^2}{CD^2}$$

$$\Rightarrow \frac{ar(\triangle AOB)}{ar(\triangle COD)} = \frac{4}{1}$$

$$\Rightarrow ar(\triangle AOB) = ar(\triangle COD) = 4:1$$

Q3 In Fig. 6.44, ABC and DBC are two triangles on the same base BC. If AD intersects BC at O, show that $\frac{ar(ABC)}{ar(DBC)} = \frac{AO}{DO}$

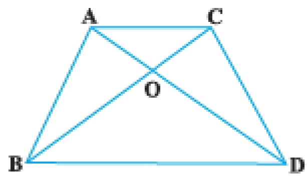
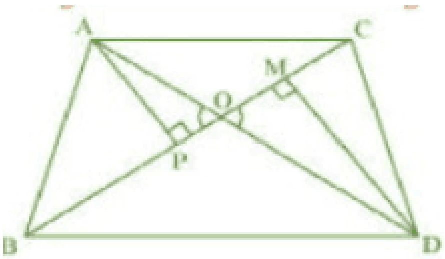


Fig. 6.44

Answer:



Let DM and AP be perpendicular on BC.

$$area\ of\ triangle = \frac{1}{2} \times base \times perpendicular$$

$$\frac{ar(\triangle ABC)}{ar(\triangle BCD)} = \frac{\frac{1}{2} \times BC \times AP}{\frac{1}{2} \times BC \times MD}$$

In $\triangle APO\, and\, \triangle DMO,$

$\angle APO = \angle DMO$ (Each $90°$)

$\angle AOP = \angle MOD$ (Vertically opposite angles)

$\triangle APO \sim \triangle DMO,$ (AA similarity)

$$\frac{AP}{DM} = \frac{AO}{DO}$$

Since

$$\frac{ar(\triangle ABC)}{ar(\triangle BCD)} = \frac{\frac{1}{2} \times BC \times AP}{\frac{1}{2} \times BC \times MD}$$

$$\Rightarrow \frac{ar(\triangle ABC)}{ar(\triangle BCD)} = \frac{AP}{MD} = \frac{AO}{DO}$$

Q4 If the areas of two similar triangles are equal, prove that they are congruent.

Answer:

Let $\triangle ABC \sim \triangle DEF$, , therefore,

$ar(\triangle ABC) = ar(\triangle DEF)$ (Given)

$$\frac{ar(\triangle ABC)}{ar(\triangle DEF)} = \frac{AB^2}{DE^2} = \frac{BC^2}{EF^2} = \frac{AC^2}{DF^2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 1$$

$$\therefore \frac{ar(\triangle ABC)}{ar(\triangle DEF)} = 1$$

$$\Rightarrow \frac{AB^2}{DE^2} = \frac{BC^2}{EF^2} = \frac{AC^2}{DF^2} = 1$$

$AB = DE$

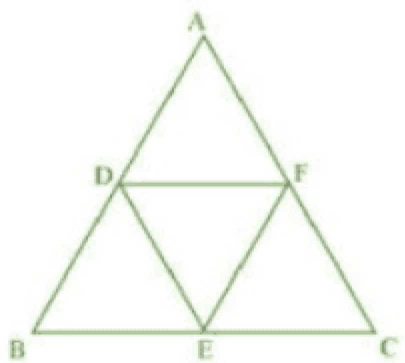
$BC = EF$

$AC = DF$

$\triangle ABC \cong \triangle DEF$ (SSS)

Q5 D, E, and F are respectively the mid-points of sides AB, BC and CA of $\triangle ABC$. Find the ratio of the areas of $\triangle DEF \ and \ \triangle ABC$

Answer:



D, E, and F are respectively the mid-points of sides AB, BC and CA of $\triangle ABC$. (Given)

$DE = \frac{1}{2}AC$ and DE||AC

In $\Delta BED\ and\ \Delta ABC$,

$\angle BED = \angle BCA$ (corresponding angles)

$\angle BDE = \angle BAC$ (corresponding angles)

$\Delta BED\ \sim\ \Delta ABC$ (By AA)

$$\frac{ar(\triangle BED)}{ar(\triangle ABC)} = \frac{DE^2}{AC^2}$$

$$\Rightarrow \frac{ar(\triangle BED)}{ar(\triangle ABC)} = \frac{(\frac{1}{2}AC)^2}{AC^2}$$

$$\Rightarrow \frac{ar(\triangle BED)}{ar(\triangle ABC)} = \frac{1}{4}$$

$$\Rightarrow ar(\triangle BED) = \frac{1}{4} \times ar(\triangle ABC)$$

Let $ar(\triangle ABC)$ be x.

$$\Rightarrow ar(\triangle BED) = \frac{1}{4} \times x$$

Similarly,

$\Rightarrow ar(\triangle CEF) = \frac{1}{4} \times x$ and $\Rightarrow ar(\triangle ADF) = \frac{1}{4} \times x$

$$ar(\triangle ABC) = ar(\triangle ADF) + ar(\triangle BED) + ar(\triangle CEF) + ar(\triangle DEF)$$

$$\Rightarrow x = \frac{x}{4} + \frac{x}{4} + \frac{x}{4} + ar(\triangle DEF)$$

$$\Rightarrow x = \frac{3x}{4} + ar(\triangle DEF)$$

$$\Rightarrow x - \frac{3x}{4} = ar(\triangle DEF)$$

$$\Rightarrow \frac{4x-3x}{4} = ar(\triangle DEF)$$

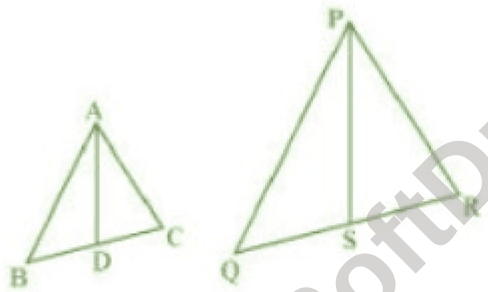
$$\Rightarrow \frac{x}{4} = ar(\triangle DEF)$$

$$\frac{ar(\triangle DEF)}{ar(\triangle ABC)} = \frac{\frac{x}{4}}{x}$$

$$\Rightarrow \frac{ar(\triangle DEF)}{ar(\triangle ABC)} = \frac{1}{4}$$

Q6 Proves that the ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding medians.

Answer:



Let AD and PS be medians of both similar triangles.

$$\triangle ABC \sim \triangle PQR$$

$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 1$$

$$\angle A = \angle P, \angle B = \angle Q, \angle\angle C = \angle R \ldots\ldots\ldots\ldots\ldots 2$$

$$BD = CD = \frac{1}{2}BC \; and \, QS = SR = \frac{1}{2}QR$$

Purring these value in 1,

$$\frac{AB}{PQ}=\frac{BD}{QS}=\frac{AC}{PR}..........................3$$

In $\triangle ABD \, and \, \triangle PQS,$

$\angle B=\angle Q$ (proved above)

$\frac{AB}{PQ}=\frac{BD}{QS}$ (proved above)

$\triangle ABD \sim \triangle PQS$ (SAS)

Therefore,

$$\frac{AB}{PQ}=\frac{BD}{QS}=\frac{AD}{PS}................4$$

$$\frac{ar(\triangle ABC)}{ar(\triangle PQR)}=\frac{AB^2}{PQ^2}=\frac{BC^2}{QR^2}=\frac{AC^2}{PR^2}$$

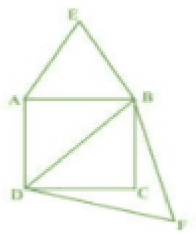
From 1 and 4, we get

$$\frac{AB}{PQ}=\frac{BC}{QR}=\frac{AC}{PR}=\frac{AD}{PS}$$

$$\frac{ar(\triangle ABC)}{ar(\triangle PQR)}=\frac{AD^2}{PS^2}$$

Q7 Prove that the area of an equilateral triangle described on one side of a square is equal to half the area of the equilateral triangle described on one of its diagonals.

Answer:



Let ABCD be a square of side units.

Therefore, diagonal = $\sqrt{2}a$

Triangles form on the side and diagonal are $\triangle$ ABE and $\triangle$ DEF, respectively.

Length of each side of triangle ABE = a units

Length of each side of triangle DEF = $\sqrt{2}a$ units

Both the triangles are equilateral triangles with each angle of 60° .

$\triangle ABE \sim \triangle DBF$ (By AAA)

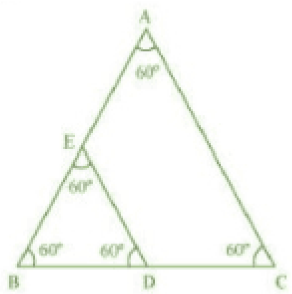
Using area theorem,

$$\frac{ar(\triangle ABC)}{ar(\triangle DBF)} = (\frac{a}{\sqrt{2}a})^2 = \frac{1}{2}$$

Q8 Tick the correct answer and justify : ABC and BDE are two equilateral triangles such that D is the mid-point of BC. The ratio of the areas of triangles ABC and BDE is

(A) 2: 1 (B) 1: 2 (C) 4 : 1 (D) 1: 4

Answer:



Given: ABC and BDE are two equilateral triangles such that D is the mid-point of BC.

All angles of the triangle are 60° .

$\triangle$ ABC $\sim$ $\triangle$ BDE (By AAA)

Let AB=BC=CA = x

then EB=BD=ED= $\frac{x}{2}$

$$\frac{ar(\triangle ABC)}{ar(\triangle BDE)}=(\frac{x}{\frac{x}{2}})^2=\frac{4}{1}$$

Option C is correct.

Q9 Sides of two similar triangles are in the ratio 4: 9. Areas of these triangles are in the ratio

(A) 2 : 3 (B) 4: 9 (C) 81: 16 (D) 16: 81

Answer:

Sides of two similar triangles are in the ratio 4: 9.

Let triangles be ABC and DEF.

We know that

$$\frac{ar(\triangle ABC)}{ar(\triangle DEF)}=\frac{AB^2}{DE^2}=\frac{4^2}{9^2}=\frac{16}{81}$$

Option D is correct.

NCERT solutions for class 10 maths chapter 6 Triangles Excercise: 6.5

Q1 (1) Sides of triangles are given below. Determine which of them are right triangles. In the case of a right triangle, write the length of its hypotenuse. 7 cm, 24 cm, 25 cm

Answer:

In the case of a right triangle, the length of its hypotenuse is highest.

hypotenuse be h.

Taking, 7 cm, 24 cm

By Pythagoras theorem,

$h^2 = 7^2 + 24^2$

$h^2 = 49 + 576$

$h^2 = 625$

$h = 25$ = given third side.

Hence, it is the right triangle with h=25 cm.

Q1 (2) Sides of triangles are given below. Determine which of them are right triangles. In the case of a right triangle, write the length of its hypotenuse. 3 cm, 8 cm, 6 cm

Answer:

In the case of a right triangle, the length of its hypotenuse is highest.

hypotenuse be h.

Taking, 3 cm, 6 cm

By Pythagoras theorem,

$h^2 = 3^2 + 6^2$

$h^2 = 9 + 36$

$h^2 = 45$

$h = \sqrt{45} \neq 8$

Hence, it is not the right triangle.

Q1 (3) Sides of triangles are given below. Determine which of them are right triangles. In the case of a right triangle, write the length of its hypotenuse. 50 cm, 80 cm, 100 cm

Answer:

In the case of a right triangle, the length of its hypotenuse is highest.

hypotenuse be h.

Taking, 50 cm, 80 cm

By Pythagoras theorem,

$h^2 = 50^2 + 80^2$

$h^2 = 2500 + 6400$

$h^2 = 8900$

$h = \sqrt{8900} \neq 100$

Hence, it is not a right triangle.

Q1 (4) Sides of triangles are given below. Determine which of them are right triangles. In case of a right triangle, write the length of its hypotenuse. 13 cm, 12 cm, 5 cm

Answer:

In the case of a right triangle, the length of its hypotenuse is highest.

hypotenuse be h.

Taking, 5cm, 12 cm

By Pythagoras theorem,

$h^2 = 5^2 + 12^2$

$h^2 = 25 + 144$

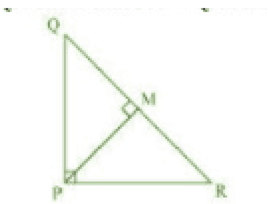
$h^2 = 169$

$h = 13$ = given third side.

Hence, it is a right triangle with h=13 cm.

Q2 PQR is a triangle right angled at P and M is a point on QR such that $PM \perp QR$. Show that $PM^2 = QM.MR.$

Answer:



Let $\angle MPR$ be x

In $\triangle MPR$,

$\angle MRP = 180^\circ - 90^\circ - x$

$\angle MRP = 90^\circ - x$

Similarly,

In $\triangle MPQ$,

$\angle MPQ = 90^\circ - \angle MPR$

$\angle MPQ = 90^\circ - x$

$\angle MQP = 180^\circ - 90^\circ - (90^\circ - x) = x$

In $\triangle QMP \, and \, \triangle PMR,$

$\angle MPQ \ = \angle MRP$

$\angle PMQ \ = \angle RMP$

$\angle MQP \ = \angle MPR$

$\triangle QMP \ \sim \triangle PMR,$ (By AAA)

$$\frac{QM}{PM} = \frac{MP}{MR}$$

$\Rightarrow PM^2 = MQ \times MR$

Hence proved.

Q3 (1) In Fig. 6.53, ABD is a triangle right angled at A and AC $\perp$ BD. Show that $AB^2 = BC.BD.$

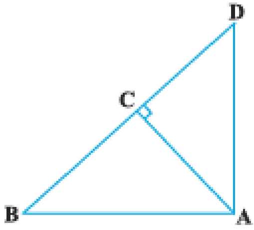


Fig. 6.53

Answer:

In $\triangle ADB \, and \, \triangle ABC,$

$\angle DAB = \angle ACB \quad (Each 90^\circ)$

$\angle ABD = \angle CBA$ (common)

$\triangle ADB \sim \triangle ABC$ (By AA)

$$\Rightarrow \frac{AB}{BC} = \frac{BD}{AB}$$

$\Rightarrow AB^2 = BC.BD$, hence prooved .

Q3 (2) In Fig. 6.53, ABD is a triangle right angled at A and AC $\perp$ BD. Show that $AC^2 = BC.DC.$

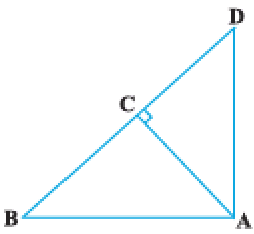


Fig. 6.53

Answer:

Let $\angle CAB$ be x

In $\triangle ABC$,

$\angle CBA = 180^\circ - 90^\circ - x$

$\angle CBA = 90^\circ - x$

Similarly,

In $\triangle CAD$,

$\angle CAD = 90^\circ - \angle CAB$

$\angle CAD = 90^\circ - x$

$\angle CDA = 180^\circ - 90^\circ - (90^\circ - x) = x$

In $\triangle ABC \, and \, \triangle ACD,$

$\angle CBA \ = \angle CAD$

$\angle CAB \ = \angle CDA$

$\angle ACB \ = \angle DCA$ (Each right angle)

$\triangle ABC \ \sim \triangle, ACD$ (By AAA)

$\frac{AC}{DC} = \frac{BC}{AC}$

$\Rightarrow AC^2 = BC \times DC$

Hence proved

Q3 (3) In Fig. 6.53, ABD is a triangle right angled at A and AC $\perp$ BD. Show that $AD^2 = BD.CD.$

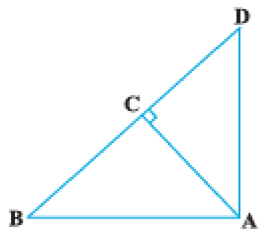


Fig. 6.53

Answer:

In $\triangle ACD \, and \, \triangle ABD,$

$\angle DCA = \angle DAB \quad (Each 90^\circ)$

$\angle CDA = \angle ADB$ (common)

$\triangle ACD \sim \triangle ABD$ (By AA)

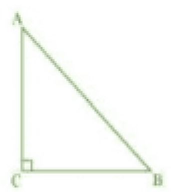
$\Rightarrow \frac{CD}{AD} = \frac{AD}{BD}$

$\Rightarrow AD^2 = BD \times CD$

Hence proved.

Q4 ABC is an isosceles triangle right angled at C. Prove that $AB^2 = 2AC^2$

Answer:



Given: ABC is an isosceles triangle right angled at C.

Let AC=BC

In $\triangle$ ABC,

By Pythagoras theorem

$AB^2 = AC^2 + BC^2$

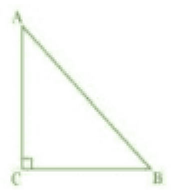
$AB^2 = AC^2 + AC^2$ (AC=BC)

$AB^2 = 2.AC^2$

Hence proved.

Q5 ABC is an isosceles triangle with AC = BC. If $AB^2 = 2AC^2$, prove that ABC is a right triangle.

Answer:



Given: ABC is an isosceles triangle with AC=BC.

In $\triangle$ ABC,

$AB^2 = 2.AC^2$ (Given)

$AB^2 = AC^2 + AC^2$ (AC=BC)

$AB^2 = AC^2 + BC^2$

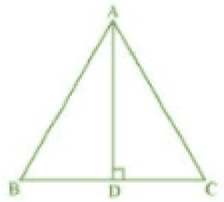
These sides satisfy Pythagoras theorem so ABC is a right-angled triangle.

Hence proved.

Q6 ABC is an equilateral triangle of side 2a. Find each of its altitudes.

Answer:

Given: ABC is an equilateral triangle of side 2a.



AB=BC=AC=2a

AD is perpendicular to BC.

We know that the altitude of an equilateral triangle bisects the opposite side.

So, BD=CD=a

In $\triangle$ ADB,

By Pythagoras theorem,

$AB^2 = AD^2 + BD^2$

$\Rightarrow (2a)^2 = AD^2 + a^2$

$\Rightarrow 4a^2 = AD^2 + a^2$

$\Rightarrow 4a^2 - a^2 = AD^2$

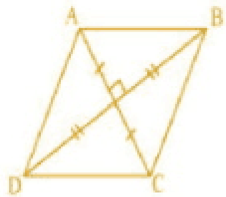
$\Rightarrow 3a^2 = AD^2$

$\Rightarrow AD = \sqrt{3}a$

The length of each altitude is $\sqrt{3}a$.

Q7 Prove that the sum of the squares of the sides of a rhombus is equal to the sum of the squares of its diagonals.

Answer:



In $\triangle$ AOB, by Pythagoras theorem,

$AB^2 = AO^2 + BO^2$.................1

In $\triangle$ BOC, by Pythagoras theorem,

$BC^2 = BO^2 + CO^2$.................2

In $\triangle$ COD, by Pythagoras theorem,

$CD^2 = CO^2 + DO^2$.................3

In $\triangle$ AOD, by Pythagoras theorem,

$AD^2 = AO^2 + DO^2$.................4

Adding equation 1,2,3,4,we get

$AB^2 + BC^2 + CD^2 + AD^2 = AO^2 + BO^2 + BO^2 + CO^2 + CO^2 + DO^2 + AO^2 + DO^2$

$AB^2 + BC^2 + CD^2 + AD^2 = 2(AO^2 + BO^2 + CO^2 + DO^2)$

$\Rightarrow AB^2 + BC^2 + CD^2 + AD^2 = 2(2.AO^2 + 2.BO^2)$ (AO=CO and BO=DO)

$\Rightarrow AB^2 + BC^2 + CD^2 + AD^2 = 4(AO^2 + BO^2)$

$$\Rightarrow AB^2 + BC^2 + CD^2 + AD^2 = 4((\frac{AC}{2})^2 + (\frac{BD}{2})^2)$$

$$\Rightarrow AB^2 + BC^2 + CD^2 + AD^2 = 4((\frac{AC^2}{4}) + (\frac{BD^2}{4}))$$

$$\Rightarrow AB^2 + BC^2 + CD^2 + AD^2 = AC^2 + BD^2$$

Hence proved .

Q8 (1) In Fig. 6.54, O is a point in the interior of a triangle ABC, OD $\perp$ BC, OE $\perp$ AC and OF $\perp$ AB. Show that $OA^2 + OB^2 + OC^2 - OD^2 - OE^2 - OF^2 = AF^2 + BD^2 + CE^2$,

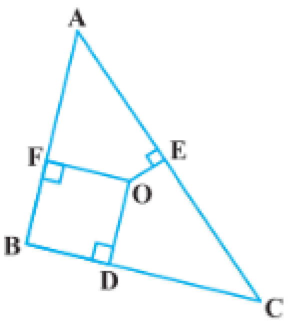
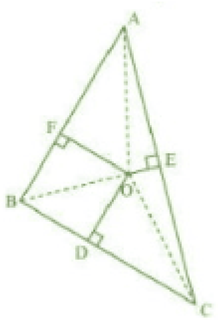


Fig. 6.54

Answer:



Join AO, BO, CO

In $\triangle$ AOF, by Pythagoras theorem,

$OA^2 = OF^2 + AF^2$..................1

In $\triangle$ BOD, by Pythagoras theorem,

$OB^2 = OD^2 + BD^2$..................2

In $\triangle$ COE, by Pythagoras theorem,

$OC^2 = OE^2 + EC^2$..................3

Adding equation 1,2,3,we get

$$OA^2 + OB^2 + OC^2 = OF^2 + AF^2 + OD^2 + BD^2 + OE^2 + EC^2$$

$$\Rightarrow OA^2 + OB^2 + OC^2 - OD^2 - OE^2 - OF^2 = AF^2 + BD^2 + EC^2 \text{....................}4$$

Hence proved

Q8 (2) In Fig. 6.54, O is a point in the interior of a triangle ABC, OD $\perp$ BC, OE $\perp$ AC and OF $\perp$ AB. $AF^2 + BD^2 + CE^2 = AE^2 + CD^2 + BF^2$.

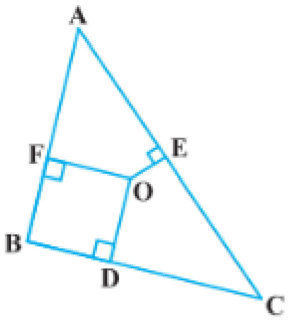
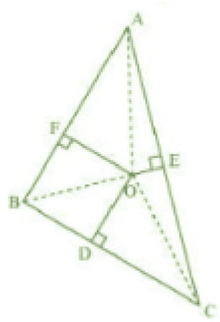


Fig. 6.54

Answer:



Join AO, BO, CO

In △ AOF, by Pythagoras theorem,

$OA^2 = OF^2 + AF^2$..................1

In △ BOD, by Pythagoras theorem,

$OB^2 = OD^2 + BD^2$..................2

In △ COE, by Pythagoras theorem,

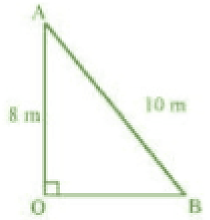
$OC^2 = OE^2 + EC^2$..................3

Adding equation 1,2,3,we get

$$OA^2 + OB^2 + OC^2 = OF^2 + AF^2 + OD^2 + BD^2 + OE^2 + EC^2$$
$$\Rightarrow OA^2 + OB^2 + OC^2 - OD^2 - OE^2 - OF^2 = AF^2 + BD^2 + EC^2 \text{....................}4$$
$$\Rightarrow (OA^2 - OE^2) + (OC^2 - OD^2) + (OB^2 - OF^2) = AF^2 + BD^2 + EC^2$$
$$\Rightarrow AE^2 + CD^2 + BF^2 = AF^2 + BD^2 + EC^2$$

Q9 A ladder 10 m long reaches a window 8 m above the ground. Find the distance of the foot of the ladder from the base of the wall.

Answer:



OA is a wall and AB is a ladder.

In △ AOB, by Pythagoras theorem

$AB^2 = AO^2 + BO^2$

$\Rightarrow 10^2 = 8^2 + BO^2$

$\Rightarrow 100 = 64 + BO^2$

$\Rightarrow 100 - 64 = BO^2$

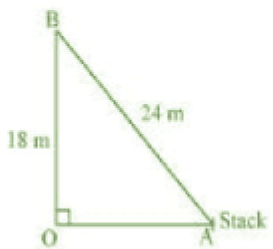
$\Rightarrow 36 = BO^2$

$\Rightarrow BO = 6m$

Hence, the distance of the foot of the ladder from the base of the wall is 6 m.

Q10 A guy wire attached to a vertical pole of height 18 m is 24 m long and has a stake attached to the other end. How far from the base of the pole should the stake be driven so that the wire will be taut?

Answer:



OB is a pole.

In $\triangle$ AOB, by Pythagoras theorem

$AB^2 = AO^2 + BO^2$

$\Rightarrow 24^2 = 18^2 + AO^2$

$\Rightarrow 576 = 324 + AO^2$

$\Rightarrow 576 - 324 = AO^2$

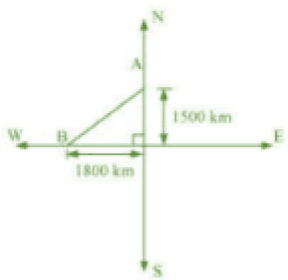
$\Rightarrow 252 = AO^2$

$\Rightarrow AO = 6\sqrt{7}m$

Hence, the distance of the stack from the base of the pole is $6\sqrt{7}$ m.

Q11 An aeroplane leaves an airport and flies due north at a speed of 1000 km per hour. At the same time, another aeroplane leaves the same airport and flies due west at a speed of 1200 km per hour. How far apart will be the two planes after 1 1/2 hours?

Answer:



Distance travelled by the first aeroplane due north in $1\frac{1}{2}$ hours.

$$= 1000 \times \frac{3}{2} = 1500km$$

Distance travelled by second aeroplane due west in $1\frac{1}{2}$ hours.

$$= 1200 \times \frac{3}{2} = 1800km$$

OA and OB are the distance travelled.

By Pythagoras theorem,

$$AB^2 = OA^2 + OB^2$$

$$\Rightarrow AB^2 = 1500^2 + 1800^2$$

$$\Rightarrow AB^2 = 2250000 + 3240000$$

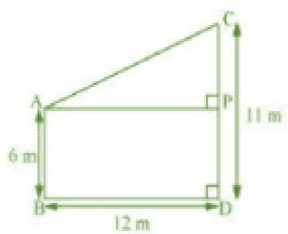
$$\Rightarrow AB^2 = 5490000$$

$$\Rightarrow AB^2 = 300\sqrt{61}km$$

Thus, the distance between the two planes is $300\sqrt{61}km$.

Q12 Two poles of heights 6 m and 11 m stand on a plane ground. If the distance between the feet of the poles is 12 m, find the distance between their top

Answer:



Let AB and CD be poles of heights 6 m and 11 m respectively.

CP=11-6=5 m and AP= 12 m

In $\triangle$ APC,

By Pythagoras theorem,

$AP^2 + PC^2 = AC^2$

$\Rightarrow 12^2 + 5^2 = AC^2$

$\Rightarrow 144 + 25 = AC^2$

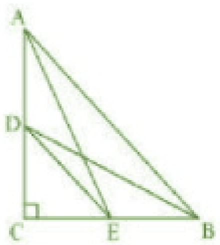
$\Rightarrow 169 = AC^2$

$\Rightarrow AC = 13m$

Hence, the distance between the tops of two poles is 13 m.

Q13 D and E are points on the sides CA and CB respectively of a triangle ABC right angled at C. Prove that $AE^2 + BD^2 = AB^2 + DE^2$.

Answer:



In $\triangle$ ACE, by Pythagoras theorem,

$AE^2 = AC^2 + CE^2$..................1

In $\triangle$ BCD, by Pythagoras theorem,

$DB^2 = BC^2 + CD^2$..................2

From 1 and 2, we get

$AC^2 + CE^2 + BC^2 + CD^2 = AE^2 + DB^2$..................3

In $\triangle$ CDE, by Pythagoras theorem,

$DE^2 = CD^2 + CE^2$..................4

In $\triangle$ ABC, by Pythagoras theorem,

$AB^2 = AC^2 + CB^2$..................5

From 3,4,5 we get

$DE^2 + AB^2 = AE^2 + DB^2$

Q14 The perpendicular from A on side BC of a $\triangle$ ABC intersects BC at D such that DB = 3 CD (see Fig. 6.55). Prove that $2AB^2 = 2AC^2 + BC^2$.

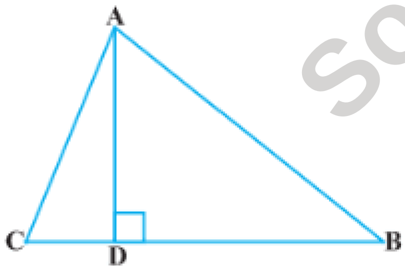


Fig. 6.55

Answer:

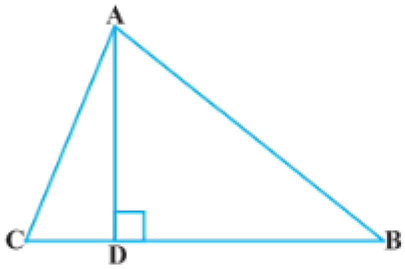


Fig. 6.55

In $\triangle$ ACD, by Pythagoras theorem,

$$AC^2 = AD^2 + DC^2$$

$$AC^2 - DC^2 = AD^2 \ldots\ldots\ldots\ldots\ldots\ldots 1$$

In $\triangle$ ABD, by Pythagoras theorem,

$$AB^2 = AD^2 + BD^2$$

$$AB^2 - BD^2 = AD^2 \ldots\ldots\ldots\ldots\ldots\ldots 2$$

From 1 and 2, we get

$$AC^2 - CD^2 = AB^2 - DB^2 \ldots\ldots\ldots\ldots\ldots\ldots 3$$

Given : 3DC=DB, so

$$CD = \frac{BC}{4}\ and\ BD = \frac{3BC}{4} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 4$$

From 3 and 4, we get

$$AC^2 - (\frac{BC}{4})^2 = AB^2 - (\frac{3BC}{4})^2$$

$$AC^2 - (\frac{BC^2}{16}) = AB^2 - (\frac{9BC^2}{16})$$

$$16AC^2 - BC^2 = 16AB^2 - 9BC^2$$

$16AC^2 = 16AB^2 - 8BC^2$

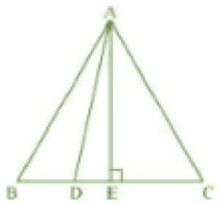
$\Rightarrow 2AC^2 = 2AB^2 - BC^2$

$2AB^2 = 2AC^2 + BC^2.$

Hence proved.

Q15 In an equilateral triangle ABC, D is a point on side BC such that BD = 1/3 BC. Prove that $9AD^2 = 7AB^2$

Answer:



Given: An equilateral triangle ABC, D is a point on side BC such that BD = 1/3 BC.

To prove : $9AD^2 = 7AB^2$

Let AB=BC=CA=a

Draw an altitude AE on BC.

So, $BE = CE = \frac{a}{2}$

In $\triangle$ AEB, by Pythagoras theorem

$AB^2 = AE^2 + BE^2$

$a^2 = AE^2 + (\frac{a}{2})^2$

$\Rightarrow a^2 - (\frac{a^2}{4}) = AE^2$

$$\Rightarrow (\frac{3a^2}{4}) = AE^2$$

$$\Rightarrow AE = (\frac{\sqrt{3}a}{2})$$

Given : BD = 1/3 BC.

$$BD = \frac{a}{3}$$

$$DE = BE = BD = \frac{a}{2} - \frac{a}{3} = \frac{a}{6}$$

In $\triangle$ ADE, by Pythagoras theorem,

$$AD^2 = AE^2 + DE^2$$

$$\Rightarrow AD^2 = (\frac{\sqrt{3}a}{2})^2 + (\frac{a}{6})^2$$

$$\Rightarrow AD^2 = (\frac{3a^2}{4}) + (\frac{a^2}{36})$$

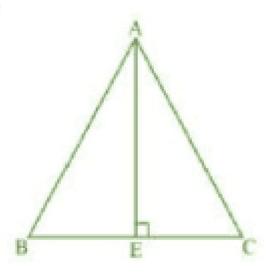
$$\Rightarrow AD^2 = (\frac{7a^2}{9})$$

$$\Rightarrow AD^2 = (\frac{7AB^2}{9})$$

$$\Rightarrow 9AD^2 = 7AB^2$$

Q16 In an equilateral triangle, prove that three times the square of one side is equal to four times the square of one of its altitudes.

Answer:



Given: An equilateral triangle ABC.

Let AB=BC=CA=a

Draw an altitude AE on BC.

So, $BE = CE = \frac{a}{2}$

In $\triangle$ AEB, by Pythagoras theorem

$$AB^2 = AE^2 + BE^2$$

$$a^2 = AE^2 + (\frac{a}{2})^2$$

$$\Rightarrow a^2 - (\frac{a^2}{4}) = AE^2$$

$$\Rightarrow (\frac{3a^2}{4}) = AE^2$$

$$\Rightarrow 3a^2 = 4AE^2$$

$$\Rightarrow 4.(altitude)^2 = 3.(side)^2$$

Q17 Tick the correct answer and justify : In ΔABC AB = $6\sqrt{3}$ cm, AC = 12 cm and BC = 6 cm.

The angle B is :

(A) 120°

(B) 60°

(C) 90°

(D) 45°

Answer:

In ΔABC AB = $6\sqrt{3}$ cm, AC = 12 cm and BC = 6 cm.

$AB^2 + BC^2 = 108 + 36$

$= 144$

$= 12^2$

$= AC^2$

It satisfies the Pythagoras theorem.

Hence, ABC is a right-angled triangle and right-angled at B.

Option C is correct.

NCERT solutions for class 10 maths chapter 6 Triangles Excercise: 6.6

Q1 In Fig. 6.56, PS is the bisector of $\angle QPR \ of \ \Delta PQR$. Prove that $\frac{QS}{SR} = \frac{PQ}{PR}$

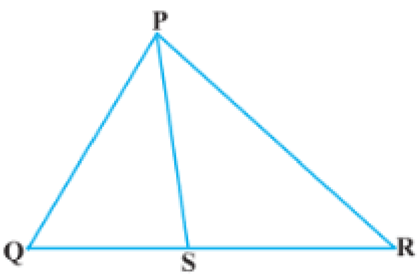
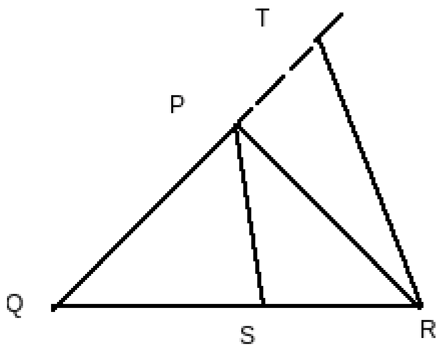


Fig. 6.56

Answer:



A line RT is drawn parallel to SP which intersect QP produced at T.

Given: PS is the bisector of $\angle QPR\ of\ \ \Delta PQR$.

$\angle QPS = \angle SPR$......................................1

By construction,

$\angle SPR = \angle PRT$......................................2 (as PS||TR)

$\angle QPS = \angle QTR$......................................3 (as PS||TR)

From the above equations, we get

$\angle PRT = \angle QTR$

$\therefore PT = PR$

By construction, PS||TR

In $\triangle$ QTR, by Thales theorem,

$$\frac{QS}{SR} = \frac{QP}{PT}$$

$$\frac{QS}{SR} = \frac{PQ}{PR}$$

Hence proved.

Q2 In Fig. 6.57, D is a point on hypotenuse AC of triangle ABC, such that BD $\perp$ AC, DM $\perp$ BC and DN $\perp$ AB. Prove that : $DM^2 = DN.MC$

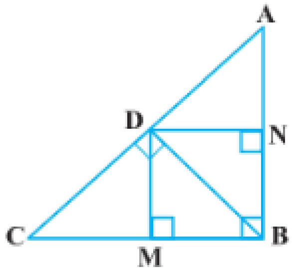
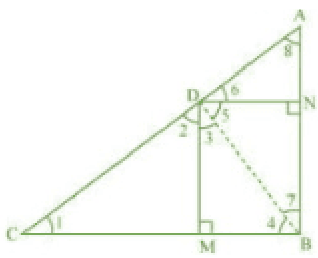


Fig. 6.57

Answer:



Join BD

Given : D is a point on hypotenuse AC of D ABC, such that BD $\perp$ AC, DM $\perp$ BC and DN $\perp$ AB.Also DN || BC, DM||NB

$\angle CDB = 90^{\circ}$

$\Rightarrow \angle 2 + \angle 3 = 90^{\circ}$............................1

In $\triangle$ CDM, $\angle 1 + \angle 2 + \angle DMC = 180^{\circ}$

$\angle 1 + \angle 2 = 90^{\circ}$......................2

In $\triangle$ DMB, $\angle 3 + \angle 4 + \angle DMB = 180^{\circ}$

$\angle 3+\angle 4=90^{\circ}$.......................3

From equation 1 and 2, we get $\angle 1=\angle 3$

From equation 1 and 3, we get $\angle 2=\angle 4$

In $\triangle DCM$ *and* $\triangle BDM$,

$\angle 1=\angle 3$

$\angle 2=\angle 4$

$\triangle DCM \sim \triangle BDM$, (By AA)

$$\Rightarrow \frac{BM}{DM}=\frac{DM}{MC}$$

$$\Rightarrow \frac{DN}{DM}=\frac{DM}{MC}$$ (BM=DN)

$$\Rightarrow DM^2=DN.MC$$

Hence proved

Q2 (2) In Fig. 6.57, D is a point on hypotenuse AC of D ABC, such that BD $\perp$ AC, DM $\perp$ BC and DN $\perp$ AB. Prove that: $DN^2=DM.AN$

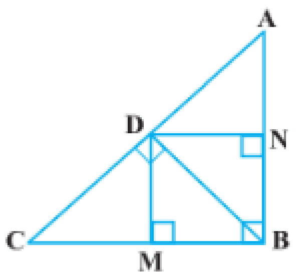
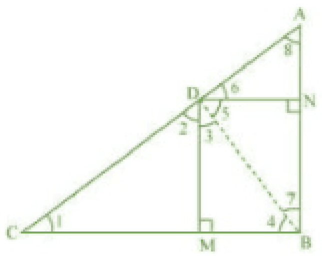


Fig. 6.57

Answer:



In △ DBN,

$\angle 5 + \angle 7 = 90^{\circ}$.......................1

In △ DAN,

$\angle 6 + \angle 8 = 90^{\circ}$.......................2

BD ⊥ AC, $\therefore \angle ADB = 90^{\circ}$

$\angle 5 + \angle 6 = 90^{\circ}$.......................3

From equation 1 and 3, we get $\angle 6 = \angle 7$

From equation 2 and 3, we get $\angle 5 = \angle 8$

In $\triangle DNA \ and \ \triangle BND,$

$\angle 6 = \angle 7$

$\angle 5 = \angle 8$

$\triangle DNA \ \sim \ \triangle BND$ (By AA)

$\Rightarrow \dfrac{AN}{DN} = \dfrac{DN}{NB}$

$\Rightarrow \dfrac{AN}{DN} = \dfrac{DN}{DM}$ (NB=DM)

$\Rightarrow DN^2 = AN.DM$

Hence proved.

Q3 In Fig. 6.58, ABC is a triangle in which $\angle$ ABC > 90° and AD $\perp$ CB produced. Prove that $AC^2 = AB^2 + BC^2 + 2BC.BD.$

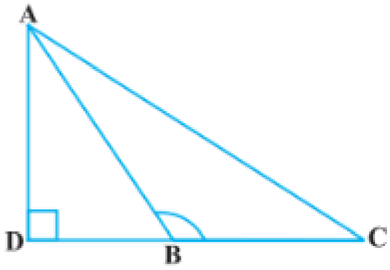


Fig. 6.58

Answer:

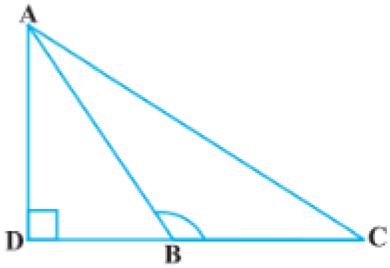


Fig. 6.58

In $\triangle$ ADB, by Pythagoras theorem

$AB^2 = AD^2 + DB^2$.......................1

In $\triangle$ ACD, by Pythagoras theorem

$AC^2 = AD^2 + DC^2$.......................2

$AC^2 = AD^2 + (BD + BC)^2$

$\Rightarrow AC^2 = AD^2 + (BD)^2 + (BC)^2 + 2.BD.BC$

$AC^2 = AB^2 + BC^2 + 2BC.BD.$ (From 1)

Q4 In Fig. 6.59, ABC is a triangle in which $\angle$ ABC < 90° and AD $\perp$ BC. Prove that $AC^2 = AB^2 + BC^2 - 2BC.BD.$

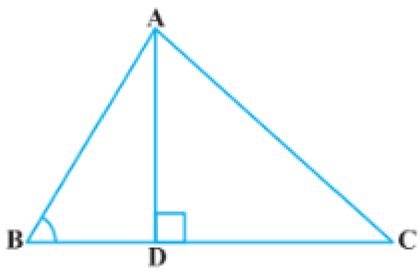


Fig. 6.59

Answer:

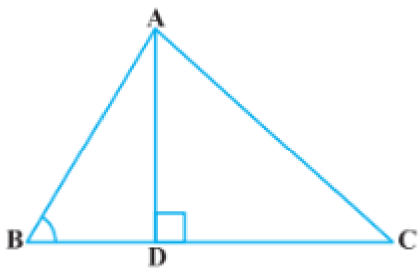


Fig. 6.59

In $\triangle$ ADB, by Pythagoras theorem

$AB^2 = AD^2 + DB^2$

$AD^2 = AB^2 - DB^2$..........................1

In $\triangle$ ACD, by Pythagoras theorem

$AC^2 = AD^2 + DC^2$

$AC^2 = AB^2 - BD^2 + DC^2$ (From 1)

$\Rightarrow AC^2 = AB^2 - BD^2 + (BC - BD)^2$

$\Rightarrow AC^2 = AB^2 - BD^2 + (BC)^2 + (BD)^2 - 2.BD.BC$

$$AC^2 = AB^2 + BC^2 - 2BC.BD.$$

Q5 (1) In Fig. 6.60, AD is a median of a triangle ABC and AM ⊥ BC. Prove that

$$AC^2 = AD^2 + BCDM + \left(\frac{BC}{2}\right)^2$$

:

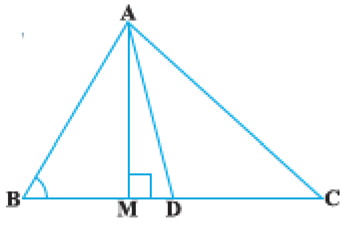


Fig. 6.60

Answer:

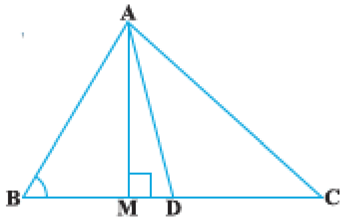


Fig. 6.60

Given: AD is a median of a triangle ABC and AM ⊥ BC.

In △ AMD, by Pythagoras theorem

$$AD^2 = AM^2 + MD^2 \text{......................} 1$$

In △ AMC, by Pythagoras theorem

$$AC^2 = AM^2 + MC^2$$

$$AC^2 = AM^2 + (MD + DC)^2$$

$$\Rightarrow AC^2 = AM^2 + (MD)^2 + (DC)^2 + 2.MD.DC$$

$AC^2 = AD^2 + DC^2 + 2DC.MD.$ (From 1)

$AC^2 = AD^2 + (\frac{BC}{2})^2 + 2(\frac{BC}{2}).MD.$ (BC=2 DC)

$AC^2 = AD^2 + BCDM + \left(\frac{BC}{2}\right)^2$

Q5 (2) In Fig. 6.60, AD is a median of a triangle ABC and AM $\perp$ BC. Prove that $AB^2 = AD^2 - BC.DM + \left(\frac{BC}{2}\right)^2$:

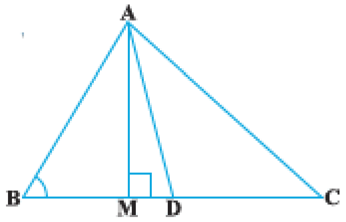


Fig. 6.60

Answer:

In $\triangle$ ABM, by Pythagoras theorem

$AB^2 = AM^2 + MB^2$

$AB^2 = (AD^2 - DM^2) + MB^2$

$\Rightarrow AB^2 = (AD^2 - DM^2) + (BD - MD)^2$

$\Rightarrow AB^2 = AD^2 - DM^2 + (BD)^2 + (MD)^2 - 2.BD.MD$

$\Rightarrow AB^2 = AD^2 + (BD)^2 - 2.BD.MD$

$\Rightarrow AB^2 = AD^2 + (\frac{BC}{2})^2 - 2(\frac{BC}{2}).MD. = AC^2$ (BC=2 BD)

$\Rightarrow AD^2 + (\frac{BC}{2})^2 - BC.MD. = AC^2$

Q5 (3) In Fig. 6.60, AD is a median of a triangle ABC and AM $\perp$ BC. Prove that: $AC^2 + AB^2 = 2AD^2 + \frac{1}{2}BC^2$

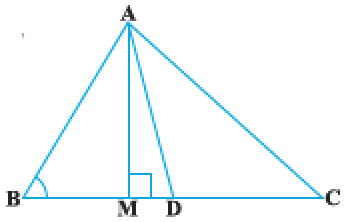


Fig. 6.60

Answer:

In △ ABM, by Pythagoras theorem

$AB^2 = AM^2 + MB^2$.......................1

In △ AMC, by Pythagoras theorem

$AC^2 = AM^2 + MC^2$ 2

Adding equation 1 and 2,

$AB^2 + AC^2 = 2AM^2 + MB^2 + MC^2$

$\Rightarrow AB^2 + AC^2 = 2AM^2 + (BD - DM)^2 + (MD + DC)^2$

$\Rightarrow AB^2 + AC^2 = 2AM^2 + (BD)^2 + (DM)^2 - 2.BD.DM + (MD)^2 + (DC)^2 + 2.MD.DC$

$\Rightarrow AB^2 + AC^2 = 2AM^2 + 2.(DM)^2 + BD^2 + (DC)^2 + 2.MD.(DC - BD)$

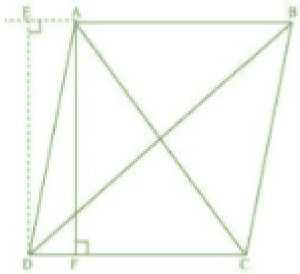
$\Rightarrow AB^2 + AC^2 = 2(AM^2 + (DM)^2) + (\frac{BC}{2})^2 + (\frac{BC}{2})^2 + 2.MD.(\frac{BC}{2} - \frac{BC}{2})$

$\Rightarrow AB^2 + AC^2 = 2(AM^2 + (DM)^2) + (\frac{BC}{2})^2 + (\frac{BC}{2})^2$

$AC^2 + AB^2 = 2AD^2 + \frac{1}{2}BC^2$

Q6 Prove that the sum of the squares of the diagonals of a parallelogram is equal to the sum of the squares of its sides.

Answer:



In parallelogram ABCD, AF and DE are altitudes drawn on DC and produced BA.

In △ DEA, by Pythagoras theorem

$DA^2 = DE^2 + EA^2$.......................1

In △ DEB, by Pythagoras theorem

$DB^2 = DE^2 + EB^2$

$DB^2 = DE^2 + (EA + AB)^2$

$DB^2 = DE^2 + (EA)^2 + (AB)^2 + 2.EA.AB$

$DB^2 = DA^2 + (AB)^2 + 2.EA.AB$ 2

In △ ADF, by Pythagoras theorem

$DA^2 = AF^2 + FD^2$

In △ AFC, by Pythagoras theorem

$AC^2 = AF^2 + FC^2 = AF^2 + (DC - FD)^2$

$\Rightarrow AC^2 = AF^2 + (DC)^2 + (FD)^2 - 2.DC.FD$

$\Rightarrow AC^2 = (AF^2 + FD^2) + (DC)^2 - 2.DC.FD$

$\Rightarrow AC^2 = AD^2 + (DC)^2 - 2.DC.FD$.......................3

Since ABCD is a parallelogram.

SO, AB=CD and BC=AD

In $\triangle DEA \, and \, \triangle ADF,$

$\angle DEA = \angle AFD \quad (each 90^\circ)$

$\angle DAE = \angle ADF$ (AE||DF)

AD=AD (common)

$\triangle DEA \cong \triangle ADF,$ (ASA rule)

$\Rightarrow EA = DF$.......................6

Adding 2 and, we get

$$DA^2 + AB^2 + 2.EA.AB + AD^2 + DC^2 - 2.DC.FD = DB^2 + AC^2$$
$$\Rightarrow DA^2 + AB^2 + AD^2 + DC^2 + 2.EA.AB - 2.DC.FD = DB^2 + AC^2$$

$\Rightarrow BC^2 + AB^2 + AD^2 + 2.EA.AB - 2.AB.EA = DB^2 + AC^2$ (From 4 and 6)

$\Rightarrow BC^2 + AB^2 + CD^2 = DB^2 + AC^2$ \

Q7 (1) In Fig. 6.61, two chords AB and CD intersect each other at point P. Prove that : $\triangle APC \sim \triangle DPB$

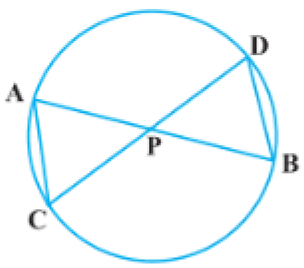
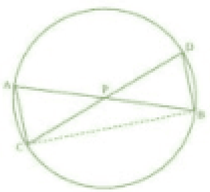


Fig. 6.61

Answer:



Join BC

In $\triangle APC \ and \ \triangle DPB,$

$\angle APC \ = \angle DPB$ (vertically opposite angle)

$\angle CAP \ = \angle BDP$ (Angles in the same segment)

$\triangle APC \ \sim \triangle DPB$ (By AA)

Q7 (2) [In Fig. 6.61, two chords AB and CD intersect each other at point P. Prove that : $AP.PB = CP.DP$]

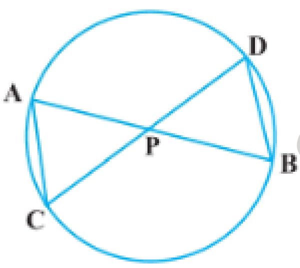
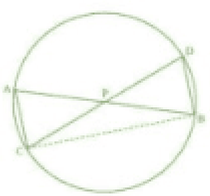


Fig. 6.61

Answer:



Join BC

In $\triangle APC \ and \triangle DPB,$

$\angle APC \ = \angle DPB$ (vertically opposite angle)

$\angle CAP \ = \angle BDP$ (Angles in the same segment)

$\triangle APC \ \sim \triangle DPB$ (By AA)

$\frac{AP}{DP} = \frac{PC}{PB} = \frac{CA}{BD}$ (Corresponding sides of similar triangles are proportional)

$\Rightarrow \frac{AP}{DP} = \frac{PC}{PB}$

$\Rightarrow AP.PB = PC.DP$

Q8 (1) In Fig. 6.62, two chords AB and CD of a circle intersect each other at the point P (when produced) outside the circle. Prove that $\Delta PAC \sim \Delta PDB$

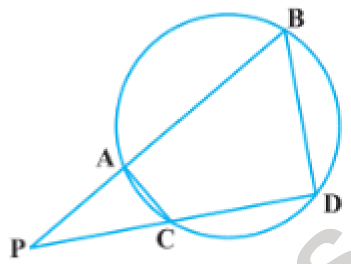


Fig. 6.62

Answer:

In $\Delta PAC \, and \, \Delta PDB,$

$\angle P = \angle P$ (Common)

$\angle PAC = \angle PDB$ (Exterior angle of a cyclic quadrilateral is equal to opposite interior angle)

So, $\Delta PAC \sim \Delta PDB$ (By AA rule)

Q8 (2) In Fig. 6.62, two chords AB and CD of a circle intersect each other at the point P (when produced) outside the circle. Prove that PA. PB = PC. PD

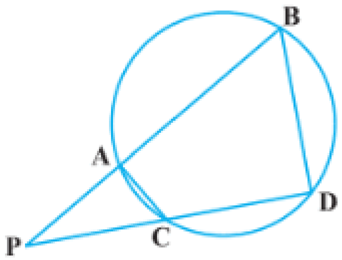


Fig. 6.62

Answer:

In $\Delta PAC \; and \; \Delta PDB,$

$\angle P = \angle P$ (Common)

$\angle PAC = \angle PDB$ (Exterior angle of a cyclic quadrilateral is equal to opposite interior angle)

So, $\Delta PAC \sim \Delta PDB$ (By AA rule)

24440 $\frac{AP}{DP} = \frac{PC}{PB} = \frac{CA}{BD}$ (Corresponding sides of similar triangles are proportional)

$\Rightarrow \frac{AP}{DP} = \frac{PC}{PB}$

$\Rightarrow AP.PB = PC.DP$

Q9 In Fig. 6.63, D is a point on side BC of D ABC such that $\frac{BD}{CD} = \frac{AB}{AC}$ Prove that AD is the bisector of $\angle$ BAC.

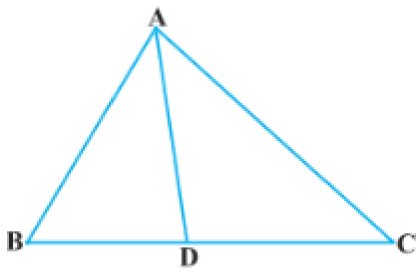
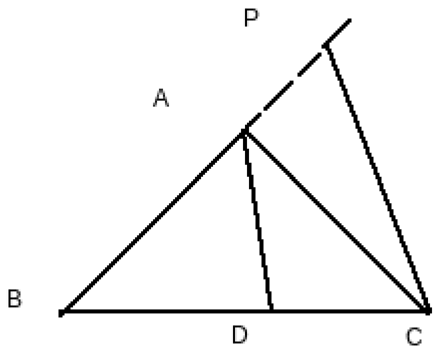


Fig. 6.63

Answer:



Produce BA to P, such that AP=AC and join P to C.

$\frac{BD}{CD} = \frac{AB}{AC}$ (Given)

$\Rightarrow \frac{BD}{CD} = \frac{AP}{AC}$

Using converse of Thales theorem,

AD||PC $\Rightarrow \angle BAD = \angle APC............1$ (Corresponding angles)

$\Rightarrow \angle DAC = \angle ACP............2$ (Alternate angles)

By construction,

AP=AC

$\Rightarrow \angle APC = \angle ACP............3$

From equation 1,2,3, we get

$\Rightarrow \angle BAD = \angle APC$

Thus, AD bisects angle BAC.

Q10 Nazima is fly fishing in a stream. The tip of her fishing rod is 1.8 m above the surface of the water and the fly at the end of the string rests on the water 3.6 m away and 2.4 m from a point directly under the tip of the rod. Assuming that her string (from the tip of her rod to the fly) is taut, how much string does she have out (see Fig. 6.64)? If she pulls in the string at the rate of 5 cm per second, what will be the horizontal distance of the fly from her after 12 seconds?

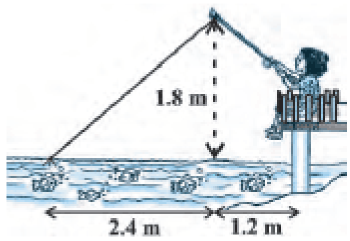
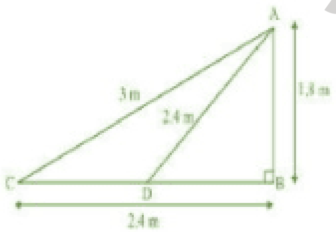


Fig. 6.64

Answer:



Let AB = 1.8 m

BC is a horizontal distance between fly to the tip of the rod.

Then, the length of the string is AC.

In △ ABC, using Pythagoras theorem

$AC^2 = AB^2 + BC^2$

$\Rightarrow AC^2 = (1.8)^2 + (2.4)^2$

$\Rightarrow AC^2 = 3.24 + 5.76$

$\Rightarrow AC^2 = 9.00$

$\Rightarrow AC = 3m$

Hence, the length of the string which is out is 3m.

If she pulls in the string at the rate of 5cm/s, then the distance travelled by fly in 12 seconds.

= $12 \times 5 = 60cm = 0.6m$

Let D be the position of fly after 12 seconds.

Hence, AD is the length of the string that is out after 12 seconds.

Length of string pulled in by nazim=AD=AC-12

=3-0.6=2.4 m

In △ ADB,

$AB^2 + BD^2 = AD^2$

$\Rightarrow (1.8)^2 + BD^2 = (2.4)^2$

$\Rightarrow BD^2 = 5.76 - 3.24 = 2.52m^2$

$\Rightarrow BD = 1.587m$

Horizontal distance travelled by fly = BD+1.2 m

=1.587+1.2=2.787 m

= 2.79 m