NCERT solutions for class 10 maths chapter 5 Arithmetic Progressions Excercise: 5.1

Q1 (i) In which of the following situations, does the list of numbers involved make an arithmetic progression, and why? (i) The taxi fare after each km when the fare is $Rs\ 15$ for the first km and $Rs\ 8$ for each additional km .

Answer:

It is given that

Fare for $1^{st}\ km$ = Rs. 15

And after that $Rs\ 8$ for each additional km

Now,

Fare for $2^{nd}\ km$ = Fare of first km + Additional fare for 1 km

= Rs. 15 + 8 = Rs 23

Fare for $3^{rd}\ km$ = Fare of first km + Fare of additional second km + Fare of additional third km

= Rs. 23 + 8= Rs 31

Fare of n km = $15 + 8 \times (n - 1)$

(We multiplied by n - 1 because the first km was fixed and for rest, we are adding additional fare.

In this, each subsequent term is obtained by adding a fixed number (8) to the previous term.)

Now, we can clearly see that this is an A.P. with the first term (a) = 15 and common difference (d) = 8

Q1 (ii) In which of the following situations, does the list of numbers involved make an arithmetic progression, and why? (ii) The amount of air present in a cylinder when a vacuum pump removes $\frac{1}{4}$ of the air remaining in the cylinder at a time.

Answer:

It is given that

vacum pump removes $\frac{1}{4}$ of the air remaining in the cylinder at a time

Let us take initial quantity of air = 1

Now, the quantity of air removed in first step = 1/4

Remaining quantity after 1 st step

$$= 1 - \frac{1}{4} = \frac{3}{4}$$

Similarly, Quantity removed after 2 nd step = Quantity removed in first step $\times$ Remaining quantity after 1 st step

$$= \frac{3}{4} \times \frac{1}{4} = \frac{3}{16}$$

Now,

Remaining quantity after 2 nd step would be = Remaining quantity after 1 st step - Quantity removed after 2 nd step

$$= \frac{3}{4} - \frac{3}{16} = \frac{12-3}{16} = \frac{9}{16}$$

Now, we can clearly see that

After the second step the difference between second and first and first and initial step is not the same, hence

the common difference (d) is not the same after every step

Therefore, it is not an AP

Q1 (iii) In which of the following situations, does the list of numbers involved make an arithmetic progression, and why? (iii) The cost of digging a well after every meter of digging, when it costs $Rs\ 150$ for the first metre and rises by $Rs\ 50$ for each subsequent meter.

Answer:

It is given that

Cost of digging of 1st meter = Rs 150

and

rises by $Rs\ 50$ for each subsequent meter

Therefore,

Cost of digging of first 2 meters = cost of digging of first meter + cost of digging additional meter

Cost of digging of first 2 meters = 150 + 50

= Rs 200

Similarly,

Cost of digging of first 3 meters = cost of digging of first 2 meters + cost of digging of additional meter

Cost of digging of first 3 meters = 200 + 50

= Rs 250

We can clearly see that 150, 200,250, ... is in AP with each subsequent term is obtained by adding a fixed number (50) to the previous term.

Therefore, it is an AP with first term (a) = 150 and common difference (d) = 50

Q1 (iv) In which of the following situations, does the list of numbers involved make an arithmetic progression, and why? (iv) The amount of money in the account every year, when $Rs\ 10000$ is deposited at compound interest at $8\ \%$ per annum .

Answer:

Amount in the beginning = Rs. 10000

Interest at the end of 1st year at the rate of $8\ \%$

is $8\ \%$ of 10000 = $\frac{8 \times 10000}{100} = 800$

Therefore, amount at the end of 1st year will be

= 10000 + 800

= 10800

Now,

Interest at the end of 2nd year at rate of $8\ \%$

is $8\ \%$ of 10800 = $\frac{8 \times 10800}{100} = 864$

Therefore,, amount at the end of 2 nd year

= 10800 + 864 = 11664

Since each subsequent term is not obtained by adding a unique number to the previous term; hence, it is not an AP

Q2 (i) Write first four terms of the AP, when the first term a and the common difference d are given as follows $a = 10, d = 10$

Answer:

It is given that

$a = 10, d = 10$

Now,

$a_1 = a = 10$

$a_2 = a_1 + d = 10 + 10 = 20$

$a_3 = a_2 + d = 20 + 10 = 30$

$a_4 = a_3 + d = 30 + 10 = 40$

Therefore, the first four terms of the given series are 10,20,30,40

Q2 (ii) Write first four terms of the AP when the first term a and the common difference d are given as follows: $a = -2, d = 0$

Answer:

It is given that

$a = -2, d = 0$

Now,

$a_1 = a = -2$

$a_2 = a_1 + d = -2 + 0 = -2$

$a_3 = a_2 + d = -2 + 0 = -2$

$a_4 = a_3 + d = -2 + 0 = -2$

Therefore, the first four terms of the given series are -2,-2,-2,-2

Q2 (iii) Write first four terms of the AP when the first term a and the common difference d are given as follows $a = 4, d = -3$

Answer:

It is given that

$a = 4, d = -3$

Now,

$a_1 = a = 4$

$a_2 = a_1 + d = 4 - 3 = 1$

$a_3 = a_2 + d = 1 - 3 = -2$

$a_4 = a_3 + d = -2 - 3 = -5$

Therefore, the first four terms of the given series are 4,1,-2,-5

Q2 (iv) Write first four terms of the AP when the first term a and the common difference d are given as follows $a = -1, d = \frac{1}{2}$

Answer:

It is given that

$a = -1, d = \frac{1}{2}$

Now,

$a_1 = a = -1$

$a_2 = a_1 + d = -1 + \frac{1}{2} = -\frac{1}{2}$

$a_3 = a_2 + d = -\frac{1}{2} + \frac{1}{2} = 0$

$a_4 = a_3 + d = 0 + \frac{1}{2} = \frac{1}{2}$

Therefore, the first four terms of the given series are $-1, -\frac{1}{2}, 0, \frac{1}{2}$

Q2 (v) Write first four terms of the AP when the first term a and the common difference d are given as follows $a = -1.25, d = -0.25$

Answer:

It is given that

$a = -1.25, d = -0.25$

Now,

$a_1 = a = -1.25$

$a_2 = a_1 + d = -1.25 - 0.25 = -1.50$

$a_3 = a_2 + d = -1.50 - 0.25 = -1.75$

$a_4 = a_3 + d = -1.75 - 0.25 = -2$

Therefore, the first four terms of the given series are -1.25,-1.50,-1.75,-2

Q3 (i) For the following APs, write the first term and the common difference: $3, 1, -1, -3, \ldots$

Answer:

Given AP series is

$3, 1, -1, -3, \ldots$

Now, first term of this AP series is **3**

Therefore,

First-term of AP series (a) = **3**

Now,

$a_1 = 3 \quad and \quad a_2 = 1$

And common difference (d) = $a_2 - a_1 = 1 - 3 = -2$

Therefore, first term and common difference is **3 and -2** respectively

Q3 (ii) For the following APs, write the first term and the common difference: $-5, -1, 3, 7, \ldots$

Answer:

Given AP series is

$-5, -1, 3, 7, \ldots$

Now, the first term of this AP series is **-5**

Therefore,

First-term of AP series (a) = **-5**

Now,

$a_1 = -5 \quad and \quad a_2 = -1$

And common difference (d) = $a_2 - a_1 = -1 - (-5) = 4$

Therefore, the first term and the common difference is **-5** and **4** respectively

Q3 (iii) For the following APs, write the first term and the common difference: $\frac{1}{3}, \frac{5}{3}, \frac{9}{3}, \frac{13}{3}, \ldots$

Answer:

Given AP series is

$\frac{1}{3}, \frac{5}{3}, \frac{9}{3}, \frac{13}{3}, \ldots$

Now, the first term of this AP series is $\frac{1}{3}$

Therefore,

The first term of AP series (a) = $\frac{1}{3}$

Now,

$a_1 = \frac{1}{3} \quad and \quad a_2 = \frac{5}{3}$

And common difference (d) = $a_2 - a_1 = \frac{5}{3} - \frac{1}{3} = \frac{5-1}{3} = \frac{4}{3}$

Therefore, the first term and the common difference is $\frac{1}{3}$ and $\frac{4}{3}$ respectively

Q3 (iv) For the following APs, write the first term and the common difference: $0.6, 1.7, 2.8, 3.9, \ldots$

Answer:

Given AP series is

$0.6, 1.7, 2.8, 3.9, \ldots$

Now, the first term of this AP series is **0.6**

Therefore,

First-term of AP series (a) = **0.6**

Now,

$a_1 = 0.6 \quad and \quad a_2 = 1.7$

And common difference (d) = $a_2 - a_1 = 1.7 - 0.6 = 1.1$

Therefore, the first term and the common difference is **0.6** and **1.1** respectively.

Q4 (i) Which of the following are APs? If they form an AP, find the common difference d and write three more terms. $2, 4, 8, 12, \ldots$

Answer:

Given series is

$2, 4, 8, 12, \ldots$

Now,

the first term to this series is = **2**

Now,

$a_1 = 2 \quad and \quad a_2 = 4 \quad and \quad a_3 = 8$

$a_2 - a_1 = 4 - 2 = 2$

$a_3 - a_2 = 8 - 4 = 4$

We can clearly see that the difference between terms are not equal

Hence, given series is not an AP

Q4 (ii) Which of the following are APs ? If they form an AP, find the common difference d and write three more terms. $2, \frac{5}{2}, 3, \frac{7}{2}, \ldots$

Answer:

Given series is

$2, \frac{5}{2}, 3, \frac{7}{2}, \ldots$

Now,

first term to this series is = **2**

Now,

$$a_1 = 2 \ \ and \ \ a_2 = \frac{5}{2} \ \ and \ \ a_3 = 3 \ \ and \ \ a_4 = \frac{7}{2}$$

$$a_2 - a_1 = \frac{5}{2} - 2 = \frac{5-4}{2} = \frac{1}{2}$$

$$a_3 - a_2 = 3 - \frac{5}{2} = \frac{6-5}{2} = \frac{1}{2}$$

$$a_4 - a_3 = \frac{7}{2} - 3 = \frac{7-6}{2} = \frac{1}{2}$$

We can clearly see that the difference between terms are equal and equal to $\frac{1}{2}$

Hence, given series is in AP

Now, the next three terms are

$$a_5 = a_4 + d = \frac{7}{2} + \frac{1}{2} = \frac{8}{2} = 4$$

$$a_6 = a_5 + d = 4 + \frac{1}{2} = \frac{8+1}{2} = \frac{9}{2}$$

$$a_7 = a_6 + d = \frac{9}{2} + \frac{1}{2} = \frac{10}{2} = 5$$

Therefore, next three terms of given series are $4, \frac{9}{2}, 5$

Q4 (iii) Which of the following are APs ? If they form an AP, find the common difference d and write three more terms. $-1.2, -3.2, -5.2, -7.2, \ldots$

Answer:

Given series is

$-1.2, -3.2, -5.2, -7.2, \ldots$

Now,

the first term to this series is = **-1.2**

Now,

$$a_1 = -1.2 \ \ and \ \ a_2 = -3.2 \ \ and \ \ a_3 = -5.2 \ \ and \ \ a_4 = -7.2$$

$$a_2 - a_1 = -3.2 - (-1.2) = -3.2 + 1.2 = -2$$

$$a_3 - a_2 = -5.2 - (-3.2) = -5.2 + 3.2 = -2$$

$$a_4 - a_3 = -7.2 - (-5.2) = -7.2 + 5.2 = -2$$

We can clearly see that the difference between terms are equal and equal to **-2**

Hence, given series is in AP

Now, the next three terms are

$a_5 = a_4 + d = -7.2 - 2 = -9.2$

$a_6 = a_5 + d = -9.2 - 2 = -11.2$

$a_7 = a_6 + d = -11.2 - 2 = -13.2$

Therefore, next three terms of given series are -9.2,-11.2,-13.2

Q4 (iv) Which of the following are APs? If they form an AP, find the common difference d and write three more terms. $-10, -6, -2, 2, \ldots$

Answer:

Given series is

$-10, -6, -2, 2, \ldots$

Now,

the first term to this series is = **-10**

Now,

$a_1 = -10 \ \ and \ \ a_2 = -6 \ \ and \ \ a_3 = -2 \ \ and \ \ a_4 = 2$

$a_2 - a_1 = -6 - (-10) = -6 + 10 = 4$

$a_3 - a_2 = -2 - (-6) = -2 + 6 = 4$

$a_4 - a_3 = 2 - (-2) = 2 + 2 = 4$

We can clearly see that the difference between terms are equal and equal to **4**

Hence, given series is in AP

Now, the next three terms are

$a_5 = a_4 + d = 2 + 4 = 6$

$a_6 = a_5 + d = 6 + 4 = 10$

$a_7 = a_6 + d = 10 + 4 = 14$

Therefore, next three terms of given series are 6,10,14

Q4 (v) Which of the following are APs? If they form an AP, find the common difference d and write three more terms. $3, 3+\sqrt{2}, 3+2\sqrt{2}, 3+3\sqrt{2}, \ldots$

Answer:

Given series is

$3, 3+\sqrt{2}, 3+2\sqrt{2}, 3+3\sqrt{2}, \ldots$

Now,

the first term to this series is = **3**

Now,

$a_1 = 3 \;\; and \;\; a_2 = 3+\sqrt{2} \;\; and \;\; a_3 = 3+2\sqrt{2} \;\; and \;\; a_4 = 3+3\sqrt{2}$

$a_2 - a_1 = 3+\sqrt{2}-3 = \sqrt{2}$

$a_3 - a_2 = 3+2\sqrt{2}-3-\sqrt{2} = \sqrt{2}$

$a_4 - a_3 = 3+3\sqrt{2}-3-2\sqrt{2} = \sqrt{2}$

We can clearly see that the difference between terms are equal and equal to $\sqrt{2}$

Hence, given series is in AP

Now, the next three terms are

$a_5 = a_4 + d = 3+3\sqrt{2}+\sqrt{2} = 3+4\sqrt{2}$

$a_6 = a_5 + d = 3+4\sqrt{2}+\sqrt{2} = 3+5\sqrt{2}$

$a_7 = a_6 + d = 3+5\sqrt{2}+\sqrt{2} = 3+6\sqrt{2}$

Therefore, next three terms of given series are $3+4\sqrt{2}, 3+5\sqrt{2}, 3+6\sqrt{2}$

Q4 (vi) Which of the following are APs? If they form an AP, find the common difference d and write three more terms. $0.2, 0.22, 0.222, 0.2222, \ldots$

Answer:

Given series is

$0.2, 0.22, 0.222, 0.2222, \ldots$

Now,

the first term to this series is = 0.2

Now,

$a_1 = 0.2 \ \ and \ \ a_2 = 0.22 \ \ and \ \ a_3 = 0.222 \ \ and \ \ a_4 = 0.2222$

$a_2 - a_1 = 0.22 - 0.2 = 0.02$

$a_3 - a_2 = 0.222 - 0.22 = 0.002$

We can clearly see that the difference between terms are not equal

Hence, given series is not an AP

Q4 (vii) Which of the following are APs? If they form an AP, find the common difference d and write three more terms. $0, -4, -8, -12, \ldots$

Answer:

Given series is

$0, -4, -8, -12, \ldots$

Now,

first term to this series is = 0

Now,

$a_1 = 0 \ \ and \ \ a_2 = -4 \ \ and \ \ a_3 = -8 \ \ and \ \ a_4 = -12$

$a_2 - a_1 = -4 - 0 = -4$

$a_3 - a_2 = -8 - (-4) = -8 + 4 = -4$

$a_4 - a_3 = -12 - (-8) = -12 + 8 = -4$

We can clearly see that the difference between terms are equal and equal to **-4**

Hence, given series is in AP

Now, the next three terms are

$a_5 = a_4 + d = -12 - 4 = -16$

$a_6 = a_5 + d = -16 - 4 = -20$

$a_7 = a_6 + d = -20 - 4 = -24$

Therefore, the next three terms of given series are -16,-20,-24

Q4 (viii) Which of the following are APs? If they form an AP, find the common difference d and write three more terms. $-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \ldots$

Answer:

Given series is

$-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \ldots$

Now,

the first term to this series is = $-\frac{1}{2}$

Now,

$$a_2 - a_1 = -\frac{1}{2} - \left(-\frac{1}{2}\right) = -\frac{1}{2} + \frac{1}{2} = 0$$
$$a_3 - a_2 = -\frac{1}{2} - \left(-\frac{1}{2}\right) = -\frac{1}{2} + \frac{1}{2} = 0$$
$$a_4 - a_3 = -\frac{1}{2} - \left(-\frac{1}{2}\right) = -\frac{1}{2} + \frac{1}{2} = 0$$

We can clearly see that the difference between terms are equal and equal to **0**

Hence, given series is in AP

Now, the next three terms are

$$a_5 = a_4 + d = -\frac{1}{2} + 0 = -\frac{1}{2}$$
$$a_6 = a_5 + d = -\frac{1}{2} + 0 = -\frac{1}{2}$$
$$a_7 = a_6 + d = -\frac{1}{2} + 0 = -\frac{1}{2}$$

Therefore, the next three terms of given series are $-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}$

Q4 (ix) Which of the following are APs? If they form an AP, find the common difference d and write three more terms. $1, 3, 9, 27, \ldots$

Answer:

Given series is

$1, 3, 9, 27, \ldots$

Now,

the first term to this series is = 1

Now,

$a_1 = 1 \ \ and \ \ a_2 = 3 \ \ and \ \ a_3 = 9 \ \ and \ \ a_4 = 27$

$a_2 - a_1 = 3 - 1 = 2$

$a_3 - a_2 = 9 - 3 = 6$

We can clearly see that the difference between terms are not equal

Hence, given series is not an AP

Q4 (x) Which of the following are APs ? If they form an AP, find the common difference d and write three more terms. $a, 2a, 3a, 4a, \ldots$

Answer:

Given series is

$a, 2a, 3a, 4a, \ldots$

Now,

the first term to this series is = **a**

Now,

$a_1 = a \ \ and \ \ a_2 = 2a \ \ and \ \ a_3 = 3a \ \ and \ \ a_4 = 4a$

$a_2 - a_1 = 2a - a = a$

$a_3 - a_2 = 3a - 2a = a$

$a_4 - a_3 = 4a - 3a = a$

We can clearly see that the difference between terms are equal and equal to **a**

Hence, given series is in AP

Now, the next three terms are

$a_5 = a_4 + d = 4a + a = 5a$

$a_6 = a_5 + d = 5a + a = 6a$

$a_7 = a_6 + d = 6a + a = 7a$

Therefore, next three terms of given series are 5a,6a,7a

Q4 (xi) Which of the following are APs ? If they form an AP, find the common difference d and write three more terms. $a, a^2, a^3, a^4, \ldots$

Answer:

Given series is

$a, a^2, a^3, a^4, \ldots$

Now,

the first term to this series is = **a**

Now,

$a_1 = a \quad and \quad a_2 = a^2 \quad and \quad a_3 = a^3 \quad and \quad a_4 = a^4$

$a_2 - a_1 = a^2 - a = a(a-1)$

$a_3 - a_2 = a^3 - a^2 = a^2(a-1)$

We can clearly see that the difference between terms are not equal

Hence, given series is not in AP

Q4 (xii) Which of the following are APs ? If they form an AP, find the common difference d and write three more terms. $\sqrt{2}, \sqrt{8}, \sqrt{18}, \sqrt{32}, \ldots$

Answer:

Given series is

$\sqrt{2}, \sqrt{8}, \sqrt{18}, \sqrt{32}, \ldots$

We can rewrite it as

$\sqrt{2}, 2\sqrt{2}, 3\sqrt{2}, 4\sqrt{2}, \ldots.$

Now,

first term to this series is = **a**

Now,

$a_1 = \sqrt{2} \ \ and \ \ a_2 = 2\sqrt{2} \ \ and \ \ a_3 = 3\sqrt{2} \ \ and \ \ a_4 = 4\sqrt{2}$

$a_2 - a_1 = 2\sqrt{2} - \sqrt{2} = \sqrt{2}$

$a_3 - a_2 = 3\sqrt{2} - 2\sqrt{2} = \sqrt{2}$

$a_4 - a_3 = 4\sqrt{2} - 3\sqrt{2} = \sqrt{2}$

We can clearly see that difference between terms are equal and equal to $\sqrt{2}$

Hence, given series is in AP

Now, the next three terms are

$a_5 = a_4 + d = 4\sqrt{2} + \sqrt{2} = 5\sqrt{2}$

$a_6 = a_5 + d = 5\sqrt{2} + \sqrt{2} = 6\sqrt{2}$

$a_7 = a_6 + d = 6\sqrt{2} + \sqrt{2} = 7\sqrt{2}$

Therefore, next three terms of given series are $5\sqrt{2}, 6\sqrt{2}, 7\sqrt{2}$

That is the next three terms are $\sqrt{50}, \ \sqrt{72}, \ \sqrt{98}$

Q4 (xiii) Which of the following are APs? If they form an AP, find the common difference d and write three more terms. $\sqrt{3}, \sqrt{6}, \sqrt{9}, \sqrt{12}, \ldots$

Answer:

Given series is

$\sqrt{3}, \sqrt{6}, \sqrt{9}, \sqrt{12}, \ldots$

Now,

the first term to this series is = $\sqrt{3}$

Now,

$a_1 = \sqrt{3} \ \ and \ \ a_2 = \sqrt{6} \ \ and \ \ a_3 = \sqrt{9} \ \ and \ \ a_4 = \sqrt{12}$

$a_2 - a_1 = \sqrt{6} - \sqrt{3} = \sqrt{3}(\sqrt{2} - 1)$

$a_3 - a_2 = 3 - \sqrt{3} = \sqrt{3}(\sqrt{3} - 1)$

We can clearly see that the difference between terms are not equal

Hence, given series is not in AP

Q4 (xiv) Which of the following are APs ? If they form an AP, find the common difference d and write three more terms. $1^2, 3^2, 5^2, 7^2, \ldots$

Answer:

Given series is

$1^2, 3^2, 5^2, 7^2, \ldots$

we can rewrite it as

$1, 9, 25, 49, \ldots.$

Now,

the first term to this series is = **1**

Now,

$a_1 = 1 \;\; and \;\; a_2 = 9 \;\; and \;\; a_3 = 25 \;\; and \;\; a_4 = 49$

$a_2 - a_1 = 9 - 1 = 8$

$a_3 - a_2 = 25 - 9 = 16$

We can clearly see that the difference between terms are not equal

Hence, given series is not in AP

Q4 (xv) Which of the following are APs ? If they form an AP, find the common difference d and write three more terms. $1^2, 5^2, 7^2, 73, \ldots$

Answer:

Given series is

$1^2, 5^2, 7^2, 73, \ldots$

we can rewrite it as

$1, 25, 49, 73 \ldots.$

Now,

the first term to this series **is = 1**

Now,

$a_1 = 1 \ \ and \ \ a_2 = 25 \ \ and \ \ a_3 = 49 \ \ and \ \ a_4 = 73$

$a_2 - a_1 = 25 - 1 = 24$

$a_3 - a_2 = 49 - 25 = 24$

$a_4 - a_3 = 73 - 49 = 24$

We can clearly see that the difference between terms are equal and equal to **24**

Hence, given series is in AP

Now, the next three terms are

$a_5 = a_4 + d = 73 + 24 = 97$

$a_6 = a_5 + d = 97 + 24 = 121$

$a_7 = a_6 + d = 121 + 24 = 145$

Therefore, the next three terms of given series are 97,121,145

NCERT solutions for class 10 maths chapter 5 Arithmetic Progressions

Excercise: 5.2

Q1 Fill in the blanks in the following table, given that a is the first term, d the common difference and a_n the n th term of the AP:

	a	d	n	a_n
(i)	7	3	8	...
(ii)	-18	...	10	0
(iii)	...	-3	18	-5
(iv)	-18.9	2.5	...	3.6
(v)	3.5	0	105	...

Answer:

(i)

It is given that

$a = 7, d = 3, n = 8$

Now, we know that

$a_n = a + (n-1)d$

$a_8 = 7 + (8-1)3 = 7 + 7 \times 3 = 7 + 21 = 28$

Therefore,

$a_8 = 28$

(ii) It is given that

$a = -18, n = 10, a_{10} = 0$

Now, we know that

$a_n = a + (n-1)d$

$a_{10} = -18 + (10-1)d$

$0 + 18 = 9d$

$d = \frac{18}{9} = 2$

(iii) It is given that

$d = -3, n = 18, a_{18} = -5$

Now, we know that

$a_n = a + (n-1)d$

$a_{18} = a + (18-1)(-3)$

$-5 = a + 17 \times (-3)$

$a = 51 - 5 = 46$

Therefore,

$a = 46$

(iv) It is given that

$a = -18.9, d = 2.5, a_n = 3.6$

Now, we know that

$a_n = a + (n-1)d$

$a_n = -18.9 + (n-1)2.5$

$3.6 + 18.9 = 2.5n - 2.5$

$n = \frac{22.5 + 2.5}{2.5} = \frac{25}{2.5} = 10$

Therefore,

$n = 10$

(v) It is given that

$a = 3.5, d = 0, n = 105$

Now, we know that

$a_n = a + (n-1)d$

$a_{105} = 3.5 + (105-1)0$

$a_{105} = 3.5$

Therefore,

$a_{105} = 3.5$

Q2 (i) Choose the correct choice in the following and justify: 30 th term of the AP: $10, 7, 4, ...,$ is

(A) 97 (B) 77 (C) -77 (D) -87

Answer:

Given series is

$10, 7, 4, ...,$

Here, $a = 10$

and

$d = 7 - 10 = -3$

Now, we know that

$a_n = a + (n-1)d$

It is given that $n = 30$

Therefore,

$a_{30} = 10 + (30-1)(-3)$

$a_{30} = 10 + (29)(-3)$

$a_{30} = 10 - 87 = -77$

Therefore, 30 th term of the AP: $10, 7, 4, \ldots,$ is **-77**

Hence, Correct answer is (C)

Q2 (ii) Choose the correct choice in the following and justify : 11th term of the AP: $-3, -\frac{1}{2}, 2, \ldots,$ is

(A) 28 (B) 22 (C) -38 (D) $-48\frac{1}{2}$

Answer:

Given series is
$-3, -\frac{1}{2}, 2, \ldots,$

Here, $a = -3$

and
$d = -\frac{1}{2} - (-3) = -\frac{1}{2} + 3 = \frac{-1+6}{2} = \frac{5}{2}$

Now, we know that

$a_n = a + (n-1)d$

It is given that $n = 11$

Therefore,

$a_{11} = -3 + (11-1)\left(\frac{5}{2}\right)$

$a_{11} = -3 + (10)\left(\frac{5}{2}\right)$

$a_{11} = -3 + 5 \times 5 = -3 + 25 = 22$

Therefore, 11th term of the AP: $-3, -\frac{1}{2}, 2, \ldots,$ is **22**

Hence, the Correct answer is (B)

Q3 (i) In the following APs, find the missing terms in the boxes : $2, \square, 26$

Answer:

Given AP series is

$2, \square, 26$

Here, $a = 2, n = 3 \; and \; a_3 = 26$

Now, we know that

$a_n = a + (n-1)d$

$\Rightarrow a_3 = 2 + (3-1)d$

$\Rightarrow 26 - 2 = (2)d$

$\Rightarrow d = \frac{24}{2} = 12$

Now,

$a_2 = a_1 + d$

$a_2 = 2 + 12 = 14$

Therefore, the missing term is **14**

Q3 (ii) In the following APs, find the missing terms in the boxes: $\square, 13, \square, 3$

Answer:

Given AP series is

$\square, 13, \square, 3$

Here, $a_2 = 13, n = 4 \; and \; a_4 = 3$

Now,

$a_2 = a_1 + d$

$a_1 = a = 13 - d$

Now, we know that

$a_n = a + (n-1)d$

$\Rightarrow a_4 = 13 - d + (4-1)d$

$\Rightarrow 3 - 13 = -d + 3d$

$\Rightarrow d = -\frac{10}{2} = -5$

Now,

$a_2 = a_1 + d$

$a_1 = a = 13 - d = 13 - (-5) = 18$

And

$a_3 = a_2 + d$

$a_3 = 13 - 5 = 8$

Therefore, missing terms are **18 and 8**

AP series is 18,13,8,3

Q3 (iii) In the following APs, find the missing terms in the boxes : $5, \square, \square, 9\frac{1}{2}$

Answer:

Given AP series is

$5, \square, \square, 9\frac{1}{2}$

Here, $a = 5, n = 4 \ and \ a_4 = 9\frac{1}{2} = \frac{19}{2}$

Now, we know that

$a_n = a + (n-1)d$

$\Rightarrow a_4 = 5 + (4-1)d$

$\Rightarrow \frac{19}{2} - 5 = 3d$

$\Rightarrow d = \frac{19 - 10}{2 \times 3} = \frac{9}{6} = \frac{3}{2}$

Now,

$a_2 = a_1 + d$

$$a_2 = 5 + \frac{3}{2} = \frac{13}{2}$$

And

$$a_3 = a_2 + d$$
$$a_3 = \frac{13}{2} + \frac{3}{2} = \frac{16}{2} = 8$$

Therefore, missing terms are $\frac{13}{2}$ **and 8**

AP series is $5, \frac{13}{2}, 8, \frac{19}{2}$

Q3 (iv) In the following APs, find the missing terms in the boxes : $-4, \square, \square, \square, \square, 6$

Answer:

Given AP series is

$-4, \square, \square, \square, \square, 6$

Here, $a = -4, n = 6 \; and \; a_6 = 6$

Now, we know that

$$a_n = a + (n-1)d$$
$$\Rightarrow a_6 = -4 + (6-1)d$$
$$\Rightarrow 6 + 4 = 5d$$
$$\Rightarrow d = \frac{10}{5} = 2$$

Now,

$$a_2 = a_1 + d$$
$$a_2 = -4 + 2 = -2$$

And

$$a_3 = a_2 + d$$
$$a_3 = -2 + 2 = 0$$

And

$$a_4 = a_3 + d$$
$$a_4 = 0 + 2 = 2$$

And

$a_5 = a_4 + d$

$a_5 = 2 + 2 = 4$

Therefore, missing terms are **-2,0,2,4**

AP series is -4,-2,0,2,4,6

Q3 (v) In the following APs, find the missing terms in the boxes : $\square, 38, \square, \square, \square, -22$

Answer:

Given AP series is

$\square, 38, \square, \square, \square, -22$

Here, $a_2 = 38, n = 6 \; and \; a_6 = -22$

Now,

$a_2 = a_1 + d$

$a_1 = a = 38 - d \qquad -(i)$

Now, we know that

$a_n = a + (n-1)d$

$\Rightarrow a_6 = 38 - d + (6-1)d \qquad (using \; (i))$

$\Rightarrow -22 - 38- = -d + 5d$

$\Rightarrow d = -\frac{60}{4} = -15$

Now,

$a_2 = a_1 + d$

$a_1 = 38 - (-15) = 38 + 15 = 53$

And

$a_3 = a_2 + d$

$a_3 = 38 - 15 = 23$

And

$a_4 = a_3 + d$

$a_4 = 23 - 15 = 8$

And

$a_5 = a_4 + d$

$a_5 = 8 - 15 = -7$

Therefore, missing terms are **53,23,8,-7**

AP series is 53,38,23,8,-7,-22

Q4 Which term of the AP : $3, 8, 13, 18, \ldots,$ is 78 ?

Answer:

Given AP is

$3, 8, 13, 18, \ldots,$

Let suppose that nth term of AP is 78

Here, $a = 3$

And

$d = a_2 - a_1 = 8 - 3 = 5$

Now, we know that that

$a_n = a + (n-1)d$

$\Rightarrow 78 = 3 + (n-1)5$

$\Rightarrow 78 - 3 = 5n - 5$

$\Rightarrow n = \frac{75+5}{5} = \frac{80}{5} = 16$

Therefore, value of 16th term of given AP is 78

Q5 (i) Find the number of terms in each of the following APs : $7, 13, 19, \ldots, 205$

Answer:

Given AP series is

$7, 13, 19, \ldots, 205$

Let's suppose there are n terms in given AP

Then,

$a = 7, a_n = 205$

And

$d = a_2 - a_1 = 13 - 7 = 6$

Now, we know that

$a_n = a + (n-1)d$

$\Rightarrow 205 = 7 + (n-1)6$

$\Rightarrow 205 - 7 = 6n - 6$

$\Rightarrow n = \frac{198 + 6}{6} = \frac{204}{6} = 34$

Therefore, there are **34** terms in given AP

Q5 (ii) Find the number of terms in each of the following APs : $18, 15\frac{1}{2}, 13, ..., -47$

Answer:

Given AP series is

$18, 15\frac{1}{2}, 13, ..., -47$

suppose there are n terms in given AP

Then,

$a = 18, a_n = -47$

And

$d = a_2 - a_1 = \frac{31}{2} - 18 = \frac{31 - 36}{2} = -\frac{5}{2}$

Now, we know that

$a_n = a + (n-1)d$

$\Rightarrow -47 = 18 + (n-1)\left(-\frac{5}{2}\right)$

$\Rightarrow -47 - 18 = -\frac{5n}{2} + \frac{5}{2}$

$\Rightarrow -\frac{5n}{2} = -65 - \frac{5}{2}$

$\Rightarrow -\frac{5n}{2} = -\frac{135}{2}$

$\Rightarrow n = 27$

Therefore, there are **27** terms in given AP

Q6 Check whether -150 is a term of the AP : $11, 8, 5, 2...$

Answer:

Given AP series is

$11, 8, 5, 2...$

Here, $a = 11$

And

$d = a_2 - a_1 = 8 - 11 = -3$

Now,

suppose -150 is nth term of the given AP

Now, we know that

$a_n = a + (n-1)d$

$\Rightarrow -150 = 11 + (n-1)(-3)$

$\Rightarrow -150 - 11 = -3n + 3$

$\Rightarrow = n = \frac{161+3}{3} = \frac{164}{3} = 54.66$

Value of n is not an integer

Therefore, -150 is not a term of AP $11, 8, 5, 2...$

Q7 Find the 31 st term of an AP whose 11 th term is 38 and the 16 th term is 73 .

Answer:

It is given that

11 th term of an AP is 38 and the 16 th term is 73

Now,

$a_{11} = 38 = a + 10d \qquad -(i)$

And

$a_{16} = 73 = a + 15d \qquad -(ii)$

On solving equation (i) and (ii) we will get

$a = -32 \quad and \quad d = 7$

Now,

$a_{31} = a + 30d = -32 + 30 \times 7 = -32 + 210 = 178$

Therefore, 31st terms of given AP is 178

Q8 An AP consists of 50 terms of which 3 rd term is 12 and the last term is 106 . Find the 29 th term.

Answer:

It is given that

AP consists of 50 terms of which 3 rd term is 12 and the last term is 106

Now,

$a_3 = 12 = a + 2d \qquad -(i)$

And

$a_{50} = 106 = a + 49d \qquad -(ii)$

On solving equation (i) and (ii) we will get

$a = 8 \quad and \quad d = 2$

Now,

$a_{29} = a + 28d = 8 + 28 \times 2 = 8 + 56 = 64$

Therefore, 29th term of given AP is 64

Q9 If the 3 rd and the 9 th terms of an AP are 4 and -8 respectively, which term of this AP is zero?

Answer:

It is given that

3 rd and the 9 th terms of an AP are 4 and -8 respectively

Now,

$a_3 = 4 = a + 2d \qquad -(i)$

And

$a_9 = -8 = a + 8d \qquad -(ii)$

On solving equation (i) and (ii) we will get

$a = 8 \quad and \quad d = -2$

Now,

Let nth term of given AP is 0

Then,

$a_n = a + (n-1)d$

$0 = 8 + (n-1)(-2)$

$2n = 8 + 2 = 10$

$n = \frac{10}{2} = 5$

Therefore, 5th term of given AP is 0

Q10 The 17 th term of an AP exceeds its 10 th term by 7 . Find the common difference.

Answer:

It is given that

17 th term of an AP exceeds its 10 th term by 7

i.e.

$a_{17} = a_{10} + 7$

$\Rightarrow a + 16d = a + 9d + 7$

$\Rightarrow a + 16d - a - 9d = 7$

$\Rightarrow 7d = 7$

$\Rightarrow d = 1$

Therefore, the common difference of AP is 1

Q11 Which term of the AP : $3, 15, 27, 39, \ldots$ will be 132 more than its 54 th term?

Answer:

Given AP is

$3, 15, 27, 39, \ldots$

Here, $a = 3$

And

$d = a_2 - a_1 = 15 - 3 = 12$

Now, let's suppose nth term of given AP is 132 more than its 54 th term

Then,

$a_n = a_{54} + 132$

$\Rightarrow a + (n-1)d = a + 53d + 132$

$\Rightarrow 3 + (n-1)12 = 3 + 53 \times 12 + 132$

$\Rightarrow 12n = 3 + 636 + 132 + 12$

$\Rightarrow 12n = 636 + 132 + 12$

$\Rightarrow n = \frac{780}{12} = 65$

Therefore, 65th term of given AP is 132 more than its 54 th term

Q12 Two APs have the same common difference. The difference between their 100 th terms is 100, what is the difference between their 1000 th terms?

Answer:

It is given that

Two APs have the same common difference and difference between their 100 th terms is 100

i.e.

$a_{100} - a'_{100} = 100$

Let common difference of both the AP's is d

$\Rightarrow a + 99d - a' - 99d = 100$

$\Rightarrow a - a' = 100 \qquad -(i)$

Now, difference between 1000th term is

$a_{1000} - a'_{1000}$

$\Rightarrow a + 999d - a' - 999d$

$\Rightarrow a - a'$

$\Rightarrow 100 \qquad (using\ (i))$

Therefore, difference between 1000th term is 100

Q 13 How many three-digit numbers are divisible by 7 ?

Answer:

We know that the first three digit number divisible by 7 is 105 and last three-digit number divisible by 7 is 994

Therefore,

$a = 105, d = 7\ and\ a_n = 994$

Let there are n three digit numbers divisible by 7

Now, we know that

$a_n = a + (n-1)d$

$\Rightarrow 994 = 105 + (n-1)7$

$\Rightarrow 7n = 896$

$\Rightarrow n = \frac{896}{7} = 128$

Therefore, there are 128 three-digit numbers divisible by 7

Q14 How many multiples of 4 lie between 10 and 250 ?

Answer:

We know that the first number divisible by 4 between 10 to 250 is **12** and last number divisible by 4 is **248**

Therefore,

$a = 12, d = 4\ and\ a_n = 248$

Let there are n numbers divisible by 4

Now, we know that

$a_n = a + (n-1)d$

$\Rightarrow 248 = 12 + (n-1)4$

$\Rightarrow 4n = 240$

$\Rightarrow n = \frac{240}{4} = 60$

Therefore, there are 60 numbers between 10 to 250 that are divisible by 4

Q15 For what value of n , are the n th terms of two APs: $63, 65, 67, \ldots$ and $3, 10, 17, \ldots$ equal?

Answer:

Given two AP's are

$63, 65, 67, \ldots$ and $3, 10, 17, \ldots$

Let first term and the common difference of two AP's are a , a' and d , d'

$a = 63\ , d = a_2 - a_1 = 65 - 63 = 2$

And

$a' = 3\ , d' = a'_2 - a'_1 = 10 - 3 = 7$

Now,

Let nth term of both the AP's are equal

$a_n = a'_n$

$\Rightarrow a + (n-1)d = a' + (n-1)d'$

$\Rightarrow 63 + (n-1)2 = 3 + (n-1)7$

$\Rightarrow 5n = 65$

$\Rightarrow n = \frac{65}{5} = 13$

Therefore, the 13th term of both the AP's are equal

Q16 Determine the AP whose third term is 16 and the 7 th term exceeds the 5 th term by 12 .

Answer:

It is given that

3rd term of AP is 16 and the 7 th term exceeds the 5 th term by 12

i.e.

$a_3 = a + 2d = 16 \qquad -(i)$

And

$a_7 = a_5 + 12$

$a + 6d = a + 4d + 12$

$2d = 12$

$d = 6$

Put the value of d in equation (i) we will get

$a = 4$

Now, AP with first term = 4 and common difference = 6 is

4,10,16,22,.....

Q17 Find the 20 th term from the last term of the AP : $3, 8, 13, \ldots, 253$.

Answer:

Given AP is

$3, 8, 13, \ldots, 253$

Here, $a = 3 \; and \; a_n = 253$

And

$d = a_2 - a_1 = 8 - 3 = 5$

Let suppose there are n terms in the AP

Now, we know that

$a_n = a + (n-1)d$

$253 = 3 + (n-1)5$

$5n = 255$

$n = 51$

So, there are 51 terms in the given AP and 20th term from the last will be 32th term

from the starting

Therefore,

$a_{32} = a + 31d$

$a_{32} = 3 + 31 \times 5 = 3 + 155 = 158$

Therefore, 20th term from the of given AP is **158**

Q18 The sum of the 4 th and 8 th terms of an AP is 24 and the sum of the 6 th and 10 th terms is 44 Find the first three terms of the AP.

Answer:

It is given that

sum of the 4 th and 8 th terms of an AP is 24 and the sum of the 6 th and 10 th terms is 44

i.e.

$a_4 + a_8 = 24$

$\Rightarrow a + 3d + a + 7d = 24$

$\Rightarrow 2a + 10d = 24$

$\Rightarrow a + 5d = 12 \quad -(i)$

And

$a_6 + a_{10} = 44$

$\Rightarrow a + 5d + a + 9d = 44$

$\Rightarrow 2a + 14d = 44$

$\Rightarrow a + 7d = 22 \quad -(ii)$

On solving equation (i) and (ii) we will get

$a = -13 \; and \; d = 5$

Therefore,first three of AP with a = -13 and d = 5 is

-13,-8,-3

Q19 Subba Rao started work in 1995 at an annual salary of Rs 5000 and received an increment of Rs 200 each year. In which year did his income reach Rs 7000?

Answer:

It is given that

Subba Rao started work at an annual salary of Rs 5000 and received an increment of Rs 200 each year

Therefore, $a = 5000 \; and \; d = 200$

Let's suppose after n years his salary will be Rs 7000

Now, we know that

$a_n = a + (n-1)d$

$\Rightarrow 7000 = 5000 + (n-1)200$

$\Rightarrow 2000 = 200n - 200$

$\Rightarrow 200n = 2200$

$\Rightarrow n = 11$

Therefore, after **11** years his salary will be Rs 7000

after 11 years, starting from 1995, his salary will reach to 7000, so we have to add 10 in 1995, because these numbers are in years

Thus , 1995+10 = 2005

Q20 Ramkali saved Rs 5 in the first week of a year and then increased her weekly savings by Rs 1.75 . If in the n th week, her weekly savings become Rs 20.75 , find n

Answer:

It is given that

Ramkali saved Rs 5 in the first week of a year and then increased her weekly savings by Rs 1.75

Therefore, $a = 5 \; and \; d = 1.75$

after n th week, her weekly savings become Rs 20.75

Now, we know that

$a_n = a + (n-1)d$

$\Rightarrow 20.75 = 5 + (n-1)1.75$

$\Rightarrow 15.75 = 1.75n - 1.75$

$\Rightarrow 1.75n = 17.5$

$\Rightarrow n = 10$

Therefore, after 10 weeks her saving will become Rs 20.75

NCERT solutions for class 10 maths chapter 5 Arithmetic Progressions Excercise: 5.3

Q1 (i) Find the sum of the following APs: $2, 7, 12, \ldots,$ to 10 terms.

Answer:

Given AP is

$2, 7, 12, \ldots,$ to 10 terms

Here, $a = 2 \; and \; n = 10$

And

$d = a_2 - a_1 = 7 - 2 = 5$

Now, we know that

$S = \frac{n}{2}\{2a + (n-1)d\}$

$\Rightarrow S = \frac{10}{2}\{2 \times 2 + (10-1)5\}$

$\Rightarrow S = 5\{4 + 45\}$

$\Rightarrow S = 5\{49\}$

$\Rightarrow S = 245$

Therefore, the sum of AP $2, 7, 12, \ldots,$ to 10 terms is **245**

Q1 (ii) Find the sum of the following APs: $-37, -33, -29, \ldots,$ to 12 terms.

Answer:

Given AP is

$-37, -33, -29, \ldots,$ to 12 terms.

Here, $a = -37 \; and \; n = 12$

And

$d = a_2 - a_1 = -33 - (-37) = 4$

Now, we know that

$S = \frac{n}{2}\{2a + (n-1)d\}$

$\Rightarrow S = \frac{12}{2}\{2 \times (-37) + (12-1)4\}$

$\Rightarrow S = 6\{-74 + 44\}$

$\Rightarrow S = 5\{-30\}$

$\Rightarrow S = -180$

Therefore, the sum of AP $-37, -33, -29, \ldots,$ to 12 terms. is **-180**

Q1 (iii) Find the sum of the following APs: $0.6, 1.7, 2.8, \ldots,$ to 100 terms.

Answer:

Given AP is

$0.6, 1.7, 2.8, \ldots,$ to 100 terms..

Here, $a = 0.6 \; and \; n = 100$

And

$d = a_2 - a_1 = 1.7 - 0.6 = 1.1$

Now, we know that

$S = \frac{n}{2}\{2a + (n-1)d\}$

$\Rightarrow S = \frac{100}{2}\{2 \times (0.6) + (100-1)(1.1)\}$

$\Rightarrow S = 50\{1.2 + 108.9\}$

$\Rightarrow S = 50\{110.1\}$

$\Rightarrow S = 5505$

Therefore, the sum of AP $0.6, 1.7, 2.8, \ldots,$ to 100 terms. is **5505**

Q1 (iv) Find the sum of the following APs: $\frac{1}{15}, \frac{1}{12}, \frac{1}{10}, \ldots$ to 11 terms.

Answer:

Given AP is
$\frac{1}{15}, \frac{1}{12}, \frac{1}{10}, \ldots$ to 11 terms.
Here, $a = \frac{1}{15}$ and $n = 11$

And

$$d = a_2 - a_1 = \frac{1}{12} - \frac{1}{15} = \frac{5-4}{60} = \frac{1}{60}$$

Now, we know that

$$S = \frac{n}{2}\{2a + (n-1)d\}$$
$$\Rightarrow S = \frac{11}{2}\left\{2 \times \frac{1}{15} + (11-1)(\frac{1}{60})\right\}$$
$$\Rightarrow S = \frac{11}{2}\left\{\frac{2}{15} + \frac{1}{6}\right\}$$
$$\Rightarrow S = \frac{11}{2}\left\{\frac{9}{30}\right\}$$
$$\Rightarrow S = \frac{99}{60} = \frac{33}{20}$$

Therefore, the sum of AP $\frac{1}{15}, \frac{1}{12}, \frac{1}{10}, \ldots$ to 11 terms. is $\frac{33}{20}$

Q2 (i) Find the sums given below : $7 + 10\frac{1}{2} + 14 + \ldots + 84$

Answer:

Given AP is
$7 + 10\frac{1}{2} + 14 + \ldots + 84$

We first need to find the number of terms

Here, $a = 7$ and $a_n = 84$

And

$$d = a_2 - a_1 = \frac{21}{2} - 7 = \frac{21 - 14}{2} = \frac{7}{2}$$

Let suppose there are n terms in the AP

Now, we know that

$$a_n = a + (n-1)d$$
$$\Rightarrow 84 = 7 + (n-1)\frac{7}{2}$$
$$\Rightarrow \frac{7n}{2} = 77 + \frac{7}{2}$$
$$\Rightarrow n = 23$$

Now, we know that

$$S = \frac{n}{2}\{2a + (n-1)d\}$$
$$\Rightarrow S = \frac{23}{2}\left\{2 \times 7 + (23-1)(\frac{7}{2})\right\}$$
$$\Rightarrow S = \frac{23}{2}\{14 + 77\}$$
$$\Rightarrow S = \frac{23}{2}\{91\}$$
$$\Rightarrow S = \frac{2093}{2} = 1046\frac{1}{2}$$

Therefore, the sum of AP $7 + 10\frac{1}{2} + 14 + ... + 84$ is $1046\frac{1}{2}$

Q2 (ii) Find the sums given below : $34 + 32 + 30 + ... + 10$

Answer:

Given AP is

$$34 + 32 + 30 + ... + 10$$

We first need to find the number of terms

Here, $a = 34 \; and \; a_n = 10$

And

$$d = a_2 - a_1 = 32 - 34 = -2$$

Let suppose there are n terms in the AP

Now, we know that

$$a_n = a + (n-1)d$$

$\Rightarrow 10 = 34 + (n-1)(-2)$

$\Rightarrow -26 = -2n$

$\Rightarrow n = 13$

Now, we know that

$S = \frac{n}{2}\{a + a_n\}$

$\Rightarrow S = \frac{13}{2}\{44\}$

$\Rightarrow S = 13 \times 22 = 286$

Therefore, the sum of AP $34 + 32 + 30 + ... + 10$ is **286**

Q2 (iii) Find the sums given below : $-5 + (-8) + (-11) + ... + (-230)$

Answer:

Given AP is

$-5 + (-8) + (-11) + ... + (-230)$

We first need to find the number of terms

Here, $a = -5 \; and \; a_n = -230$

And

$d = a_2 - a_1 = -8 - (-5) = -3$

Let suppose there are n terms in the AP

Now, we know that

$a_n = a + (n-1)d$

$\Rightarrow -230 = -5 + (n-1)(-3)$

$\Rightarrow -228 = -3n$

$\Rightarrow n = 76$

Now, we know that

$S = \frac{n}{2}\{a + a_n\}$

$\Rightarrow S = \frac{76}{2}\{(-5 - 230)\}$

$\Rightarrow S = 38\{-235\}$

$\Rightarrow S = -8930$

Therefore, the sum of AP $-5 + (-8) + (-11) + ... + (-230)$ is **-8930**

Q3 (i) In an AP: given $a = 5$, $d = 3$, $a_n = 50$, find n and S_n.

Answer:

It is given that

$a = 5, d = 3 \; and \; a_n = 50$

Let suppose there are n terms in the AP

Now, we know that

$a_n = a + (n-1)d$

$\Rightarrow 50 = 5 + (n-1)3$

$\Rightarrow 48 = 3n$

$\Rightarrow n = 16$

Now, we know that

$S = \frac{n}{2}\{2a + (n-1)d\}$

$\Rightarrow S = \frac{16}{2}\{2 \times (5) + (16-1)(3)\}$

$\Rightarrow S = 8\{10 + 45\}$

$\Rightarrow S = 8\{55\}$

$\Rightarrow S = 440$

Therefore, the sum of the given AP is 440

Q3 (ii) In an AP: given $a = 7$, $a_{13} = 35$, find d and S_{13}.

Answer:

It is given that

$a = 7 \; and \; a_{13} = 35$

$a_{13} = a + 12d = 35$

$= 12d = 35 - 7 = 28$

$d = \frac{28}{12} = \frac{7}{3}$

Now, we know that

$S_n = \frac{n}{2}\{2a + (n-1)d\}$

$\Rightarrow S_{13} = \frac{13}{2}\left\{2 \times (7) + (13-1)(\frac{7}{3})\right\}$

$\Rightarrow S_{13} = \frac{13}{2}\{14 + 28\}$

$\Rightarrow S_{13} = \frac{13}{2}\{42\}$

$\Rightarrow S_{13} = 13 \times 21 = 273$

Therefore, the sum of given AP is **273**

Q3 (iii) In an AP: given $a_{12} = 37, d = 3,$ find a and S_{12}.

Answer:

It is given that

$d = 3 \; and \; a_{12} = 37$

$a_{12} = a + 11d = 37$

$= a = 37 - 11 \times 3 = 37 - 33 = 4$

Now, we know that

$S_n = \frac{n}{2}\{2a + (n-1)d\}$

$\Rightarrow S_{12} = \frac{12}{2}\{2 \times (4) + (12-1)3\}$

$\Rightarrow S_{12} = 6\{8 + 33\}$

$\Rightarrow S_{12} = 6\{41\}$

$\Rightarrow S_{12} = 246$

Therefore, the sum of given AP is **246**

Q3 (iv) In an AP: given $a_3 = 15, \; S_{10} = 125,$ find d and S_{10}

Answer:

It is given that

$a_3 = 15, S_{10} = 125$

$a_3 = a + 2d = 15 \qquad -(i)$

Now, we know that

$S_n = \frac{n}{2}\{2a + (n-1)d\}$

$\Rightarrow S_{10} = \frac{10}{2}\{2 \times (a) + (10-1)d\}$

$\Rightarrow 125 = 5\{2a + 9d\}$

$\Rightarrow 2a + 9d = 25 \qquad -(ii)$

On solving equation (i) and (ii) we will get

$a = 17 \; and \; d = -1$

Now,

$a_{10} = a + 9d = 17 + 9(-1) = 17 - 9 = 8$

Therefore, the value of d and 10th terms is -1 and 8 respectively

Q3 (v) In an AP: given $d = 5, S_9 = 75$, find a and a_9 .

Answer:

It is given that

$d = 5, S_9 = 75$

Now, we know that

$S_n = \frac{n}{2}\{2a + (n-1)d\}$

$\Rightarrow S_9 = \frac{9}{2}\{2 \times (a) + (9-1)5\}$

$\Rightarrow 75 = \frac{9}{2}\{2a + 40\}$

$\Rightarrow 150 = 18a + 360$

$\Rightarrow a = -\frac{210}{18} = -\frac{35}{3}$

Now,

$a_9 = a + 8d = -\frac{35}{3} + 8(5) = -\frac{35}{3} + 40 = \frac{-35 + 120}{3} = \frac{85}{3}$

Q3 (vi) In an AP: given $a = 2, d = 8, S_n = 90,$ find n and a_n .

Answer:

It is given that

$a = 2, d = 8, S_n = 90,$

Now, we know that

$$S_n = \frac{n}{2}\{2a + (n-1)d\}$$
$$\Rightarrow 90 = \frac{n}{2}\{2 \times (2) + (n-1)8\}$$
$$\Rightarrow 180 = n\{4 + 8n - 8\}$$
$$\Rightarrow 8n^2 - 4n - 180 = 0$$
$$\Rightarrow 4(2n^2 - n - 45) = 0$$
$$\Rightarrow 2n^2 - n - 45 = 0$$
$$\Rightarrow 2n^2 - 10n + 9n - 45 = 0$$
$$\Rightarrow (n-5)(2n+9) = 0$$
$$\Rightarrow n = 5 \quad and \quad n = -\frac{9}{2}$$

n can not be negative so the only the value of n is **5**

Now,

$$a_5 = a + 4d = 2 + 4 \times 8 = 2 + 32 = 34$$

Therefore, value of n and nth term is 5 and 34 respectively

Q3 (vii) In an AP: given $a = 8, a_n = 62, S_n = 210,$ find n and d .

Answer:

It is given that

$a = 8, a_n = 62, S_n = 210,$

Now, we know that

$$a_n = a + (n-1)d$$
$$62 = 8 + (n-1)d$$
$$(n-1)d = 54 \qquad -(i)$$

Now, we know that

$$S_n = \frac{n}{2}\{2a + (n-1)d\}$$

$$\Rightarrow 210 = \frac{n}{2}\{2 \times (8) + (n-1)d\}$$

$$\Rightarrow 420 = n\{16 + 54\} \qquad (using\ (i))$$

$$\Rightarrow 420 = n\{70\}$$

$$\Rightarrow n = 6$$

Now, put this value in (i) we will get

$$d = \frac{54}{5}$$

Therefore, value of n and d are $6\ and\ \frac{54}{5}$ respectively

Q3 (viii) In an AP: given $a_n = 4, d = 2, S_n = -14,$ find n and a .

Answer:

It is given that

$$a_n = 4, d = 2, S_n = -14,$$

Now, we know that

$$a_n = a + (n-1)d$$

$$4 = a + (n-1)2$$

$$a + 2n = 6 \Rightarrow a = 6 - 2n \qquad -(i)$$

Now, we know that

$$S_n = \frac{n}{2}\{2a + (n-1)d\}$$

$$\Rightarrow -14 = \frac{n}{2}\{2 \times (a) + (n-1)2\}$$

$$\Rightarrow -28 = n\{10 - 2n\}$$

$$\Rightarrow 2n^2 - 10n - 28 = 0$$

$$\Rightarrow 2(n^2 - 5n - 14) = 0$$

$\Rightarrow n^2 - 7n + 2n - 14 = 0$

$\Rightarrow (n+2)(n-7) = 0$

$\Rightarrow n = -2 \ \ and \ \ n = 7$

Value of n cannot be negative so the only the value of n is **7**

Now, put this value in (i) we will get

a = -8

Therefore, the value of n and a are **7** and **-8** respectively

Q3 (ix) In an AP: given $a = 3, n = 8, S = 192,$ find d .

Answer:

It is given that

$a = 3, n = 8, S = 192,$

Now, we know that

$S_n = \frac{n}{2}\{2a + (n-1)d\}$

$\Rightarrow 192 = \frac{8}{2}\{2 \times (3) + (8-1)d\}$

$\Rightarrow 192 = 4\{6 + 7d\}$

$\Rightarrow 7d = 48 - 6$

$\Rightarrow d = \frac{42}{7} = 6$

Therefore, the value of d is 6

Q3 (x) In an AP: given $l = 28, S = 144 \ and \ n = 9$ and there are total 9 terms. Find a .

Answer:

It is given that

$l = 28, S = 144 \ and \ n = 9$

Now, we know that

$l = a_n = a + (n-1)d$

$$28 = a_n = a + (n-1)d \qquad -(i)$$

Now, we know that

$$S_n = \frac{n}{2}\{2a + (n-1)d\}$$

$$\Rightarrow 144 = \frac{9}{2}\{a + a + (n-1)d\}$$

$$\Rightarrow 288 = 9\{a + 28\} \qquad -(using\ (i))$$

$$\Rightarrow a + 28 = 32$$

$$\Rightarrow a = 4$$

Therefore, the value of a is 4

Q4 How many terms of the AP: $9, 17, 25, \ldots$ must be taken to give a sum of 636 ?

Answer:

Given AP is

$9, 17, 25, \ldots$

Here, $a = 9\ and\ d = 8$

And $S_n = 636$

Now , we know that

$$S_n = \frac{n}{2}\{2a + (n-1)d\}$$

$$\Rightarrow 636 = \frac{n}{2}\{18 + (n-1)8\}$$

$$\Rightarrow 1272 = n\{10 + 8n\}$$

$$\Rightarrow 8n^2 + 10n - 1272 = 0$$

$$\Rightarrow 2(4n^2 + 5n - 636) = 0$$

$$\Rightarrow 4n^2 + 53n - 48n - 636 = 0$$

$$\Rightarrow (n-12)(4n+53) = 0$$

$$\Rightarrow n = 12 \ \ and \ \ n = -\frac{53}{4}$$

Value of n can not be negative so the only the value of n is **12**

Therefore, the sum of **12** terms of AP $9, 17, 25, \ldots$ must be taken to give a sum of 636 .

Q5 The first term of an AP is 5, the last term is 45 and the sum is 400. Find the number of terms and the common difference.

Answer:

It is given that

$a = 5, a_n = 45, S_n = 400,$

Now, we know that

$a_n = a + (n-1)d$

$45 = 5 + (n-1)d$

$(n-1)d = 40 \qquad -(i)$

Now, we know that

$S_n = \frac{n}{2}\{2a + (n-1)d\}$

$\Rightarrow 400 = \frac{n}{2}\{2 \times (5) + (n-1)d\}$

$\Rightarrow 800 = n\{10 + 40\} \qquad (using\ (i))$

$\Rightarrow 800 = n\{50\}$

$\Rightarrow n = 16$

Now, put this value in (i) we will get

$d = \frac{40}{15} = \frac{8}{3}$

Therefore, value of n and d are $16\ and\ \frac{8}{3}$ respectively

Q6 The first and the last terms of an AP are 17 and 350 respectively. If the common difference is 9, how many terms are there and what is their sum?

Answer:

It is given that

$a = 17, l = 350, d = 9,$

Now, we know that

$$a_n = a + (n-1)d$$

$$350 = 17 + (n-1)9$$

$$(n-1)9 = 333$$

$$(n-1) = 37$$

$$n = 38$$

Now, we know that

$$S_n = \frac{n}{2}\{2a + (n-1)d\}$$

$$\Rightarrow S_{38} = \frac{38}{2}\{2 \times (17) + (38-1)9\}$$

$$\Rightarrow S_{38} = 19\{34 + 333\}$$

$$\Rightarrow S_{38} = 19\{367\}$$

$$\Rightarrow S_{38} = 6973$$

Therefore, there are **38 terms** and their sun is **6973**

Q7 Find the sum of first 22 terms of an AP in which $d = 7$ and 22 nd term is 149 .

Answer:

It is given that

$$a_{22} = 149, d = 7, n = 22$$

Now, we know that

$$a_{22} = a + 21d$$

$$149 = a + 21 \times 7$$

$$a = 149 - 147 = 2$$

Now, we know that

$$S_n = \frac{n}{2}\{2a + (n-1)d\}$$

$$\Rightarrow S_{22} = \frac{22}{2}\{2 \times (2) + (22-1)7\}$$

$$\Rightarrow S_{22} = 11\{4 + 147\}$$

$$\Rightarrow S_{22} = 11\{151\}$$

$\Rightarrow S_{22} = 1661$

Therefore, there are 22 **terms** and their sum is **1661**

Q8 Find the sum of first 51 terms of an AP whose second and third terms are 14 and 18 respectively.

Answer:

It is given that

$a_2 = 14, a_3 = 18, n = 51$

And $d = a_3 - a_2 = 18 - 14 = 4$

Now,

$a_2 = a + d$

$a = 14 - 4 = 10$

Now, we know that

$S_n = \frac{n}{2}\{2a + (n-1)d\}$

$\Rightarrow S_{51} = \frac{51}{2}\{2 \times (10) + (51-1)4\}$

$\Rightarrow S_{51} = \frac{51}{2}\{20 + 200\}$

$\Rightarrow S_{51} = \frac{51}{2}\{220\}$

$\Rightarrow S_{51} = 51 \times 110$

$\Rightarrow S_{51} = 5304$

Therefore, there are 51 **terms** and their sum is **5610**

Q9 If the sum of first 7 terms of an AP is 49 and that of 17 terms is 289, find the sum of first n terms.

Answer:

It is given that

$S_7 = 49 \ and \ S_{17} = 289$

Now, we know that

$$S_n = \frac{n}{2}\{2a + (n-1)d\}$$

$$\Rightarrow S_7 = \frac{7}{2}\{2 \times (a) + (7-1)d\}$$

$$\Rightarrow 98 = 7\{2a + 6d\}$$

$$\Rightarrow a + 3d = 7 \qquad -(i)$$

Similarly,

$$\Rightarrow S_{17} = \frac{17}{2}\{2 \times (a) + (17-1)d\}$$

$$\Rightarrow 578 = 17\{2a + 16d\}$$

$$\Rightarrow a + 8d = 17 \qquad -(ii)$$

On solving equation (i) and (ii) we will get

a = 1 and d = 2

Now, the sum of first n terms is

$$S_n = \frac{n}{2}\{2 \times 1 + (n-1)2\}$$

$$S_n = \frac{n}{2}\{2 + 2n - 2\}$$

$$S_n = n^2$$

Therefore, the sum of n terms is n^2

Q10 (i) Show that $a_1, a_2, ..., a_n, ...$ form an AP where an is defined as below : $a_n = 3 + 4n$ Also find the sum of the first 15 terms.

Answer:

It is given that

$$a_n = 3 + 4n$$

We will check values of a_n for different values of n

$$a_1 = 3 + 4(1) = 3 + 4 = 7$$

$$a_2 = 3 + 4(2) = 3 + 8 = 11$$

$$a_3 = 3 + 4(3) = 3 + 12 = 15$$

and so on.

From the above, we can clearly see that this is an AP with the first term(a) equals

to **7** and common difference (d) equals to **4**

Now, we know that

$$S_n = \frac{n}{2}\{2a + (n-1)d\}$$

$$\Rightarrow S_{15} = \frac{15}{2}\{2 \times (7) + (15-1)4\}$$

$$\Rightarrow S_{15} = \frac{15}{2}\{14 + 56\}$$

$$\Rightarrow S_{15} = \frac{15}{2}\{70\}$$

$$\Rightarrow S_{15} = 15 \times 35$$

$$\Rightarrow S_{15} = 525$$

Therefore, the sum of 15 terms is **525**

Q10 (ii) Show that $a_1, a_2, ..., a_n, \cdots$ form an AP where an is defined as below : $a_n = 9 - 5n$. Also find the sum of the first 15 terms in each case.

Answer:

It is given that

$$a_n = 9 - 5n$$

We will check values of a_n for different values of n

$$a_1 = 9 - 5(1) = 9 - 5 = 4$$

$$a_2 = 9 - 5(2) = 9 - 10 = -1$$

$$a_3 = 9 - 5(3) = 9 - 15 = -6$$

and so on.

From the above, we can clearly see that this is an AP with the first term(a) equals to **4** and common difference (d) equals to **-5**

Now, we know that

$$S_n = \frac{n}{2}\{2a + (n-1)d\}$$

$$\Rightarrow S_{15} = \frac{15}{2}\{2 \times (4) + (15-1)(-5)\}$$

$$\Rightarrow S_{15} = \frac{15}{2}\{8 - 70\}$$

$$\Rightarrow S_{15} = \frac{15}{2}\{-62\}$$

$\Rightarrow S_{15} = 15 \times (-31)$

$\Rightarrow S_{15} = -465$

Therefore, the sum of 15 terms is **-465**

Q11 If the sum of the first n terms of an AP is $4n - n^2$, what is the first term (that is S_1)? What is the sum of first two terms? What is the second term? Similarly, find the 3 rd, the 10 th and the n th terms

Answer:

It is given that

the sum of the first n terms of an AP is $4n - n^2$

Now,

$\Rightarrow S_n = 4n - n^2$

Now, first term is

$\Rightarrow S_1 = 4(1) - 1^2 = 4 - 1 = 3$

Therefore, first term is **3**

Similarly,

$\Rightarrow S_2 = 4(2) - 2^2 = 8 - 4 = 4$

Therefore, sum of first two terms is **4**

Now, we know that

$\Rightarrow S_n = \frac{n}{2}\{2a + (n-1)d\}$

$\Rightarrow S_2 = \frac{2}{2}\{2 \times 3 + (2-1)d\}$

$\Rightarrow 4 = \{6 + d\}$

$\Rightarrow d = -2$

Now,

$a_2 = a + d = 3 + (-2) = 1$

Similarly,

$a_3 = a + 2d = 3 + 2(-2) = 3 - 4 = -1$

$$a_{10} = a + 9d = 3 + 9(-2) = 3 - 18 = -15$$

$$a_n = a + (n-1)d = 3 + (n-1)(-2) = 5 - 2n$$

Q12 Find the sum of the first 40 positive integers divisible by 6 .

Answer:

Positive integers divisible by 6 are

6,12,18,...

This is an AP with

$$here,\ a = 6\ and\ d = 6$$

Now, we know that

$$S_n = \frac{n}{2}\{2a + (n-1)d\}$$

$$\Rightarrow S_{40} = \frac{40}{2}\{2 \times 6 + (40-1)6\}$$

$$\Rightarrow S_{40} = 20\{12 + 234\}$$

$$\Rightarrow S_{40} = 20\{246\}$$

$$\Rightarrow S_{40} = 4920$$

Therefore, sum of the first 40 positive integers divisible by 6 is **4920**

Q13 Find the sum of the first 15 multiples of 8 .

Answer:

First 15 multiples of 8 are

8,16,24,...

This is an AP with

$$here,\ a = 8\ and\ d = 8$$

Now, we know that

$$S_n = \frac{n}{2}\{2a + (n-1)d\}$$

$$\Rightarrow S_{15} = \frac{15}{2}\{2 \times 8 + (15-1)8\}$$

$$\Rightarrow S_{15} = \frac{15}{2}\{16 + 112\}$$

$$\Rightarrow S_{15} = \frac{15}{2}\{128\}$$

$$\Rightarrow S_{15} = 15 \times 64 = 960$$

Therefore, sum of the first 15 multiple of 8 is **960**

Q14 Find the sum of the odd numbers between 0 and 50 .

Answer:

The odd number between 0 and 50 are

1,3,5,...49

This is an AP with

$$here,\ a = 1\ and\ d = 2$$

There are total **25** odd number between 0 and 50

Now, we know that

$$S_n = \frac{n}{2}\{2a + (n-1)d\}$$

$$\Rightarrow S_{25} = \frac{25}{2}\{2 \times 1 + (25-1)2\}$$

$$\Rightarrow S_{25} = \frac{25}{2}\{2 + 48\}$$

$$\Rightarrow S_{25} = \frac{25}{2} \times 50$$

$$\Rightarrow S_{25} = 25 \times 25 = 625$$

Therefore, sum of the odd numbers between 0 and 50 **625**

Q15 A contract on construction job specifies a penalty for delay of completion beyond a certain date as follows: Rs 200 for the first day, Rs 250 for the second day, Rs 300 for the third day, etc., the penalty for each succeeding day being Rs 50 more than for the preceding day. How much money the contractor has to pay a penalty, if he has delayed the work by 30 days?

Answer:

It is given that

Penalty for delay of completion beyond a certain date is Rs 200 for the first day,

Rs 250 for the second day, Rs 300 for the third day and penalty for each succeeding day being Rs 50 more than for the preceding day

We can clearly see that

200,250,300,..... is an AP with

$a = 200 \; and \; d = 50$

Now, the penalty for 30 days is given by the expression

$S_{30} = \frac{30}{2}\{2 \times 200 + (30-1)50\}$

$S_{30} = 15\,(400 + 1450)$

$S_{30} = 15 \times 1850$

$S_{30} = 27750$

Therefore, the penalty for 30 days is **27750**

Q16 A sum of Rs 700 is to be used to give seven cash prizes to students of a school for their overall academic performance. If each prize is Rs 20 less than its preceding prize, find the value of each of the prizes.

Answer:

It is given that

Each price is decreased by 20 rupees,

Therefore, **d = -20** and there are total 7 prizes so **n = 7** and sum of prize money is Rs 700 so $S_7 = 700$

Let a be the prize money given to the 1st student

Then,

$S_7 = \frac{7}{2}\{2a + (7-1)(-20)\}$

$700 = \frac{7}{2}\{2a - 120\}$

$2a - 120 = 200$

$a = \frac{320}{2} = 160$

Therefore, the prize given to the first student is **Rs 160**

Now,

Let $a_2, a_2, ..., a_7$ is the prize money given to the next 6 students

then,

$a_2 = a + d = 160 + (-20) = 160 - 20 = 140$

$a_3 = a + 2d = 160 + 2(-20) = 160 - 40 = 120$

$a_4 = a + 3d = 160 + 3(-20) = 160 - 60 = 100$

$a_5 = a + 4d = 160 + 4(-20) = 160 - 80 = 80$

$a_6 = a + 5d = 160 + 5(-20) = 160 - 100 = 60$

$a_7 = a + 6d = 160 + 6(-20) = 160 - 120 = 40$

Therefore, prize money given to 1 to 7 student is **160,140,120,100,80,60.40**

Q17 In a school, students thought of planting trees in and around the school to reduce air pollution. It was decided that the number of trees, that each section of each class will plant, will be the same as the class, in which they are studying, e.g., a section of Class I???? will plant 1 tree, a section of Class II will plant 2 trees and so on till Class XII. There are three sections in each class. How many trees will be planted by the students?

Answer:

First there are 12 classes and each class has 3 sections

Since each section of class 1 will plant 1 tree, so 3 trees will be planted by 3 sections of class 1. **Thus every class will plant 3 times the number of their class**

Similarly,

No. of trees planted by 3 sections of class 1 = 3

No. of trees planted by 3 sections of class 2 = 6

No. of trees planted by 3 sections of class 3 = 9

No. of trees planted by 3 sections of class 4 = 12

Its clearly an AP with first term **(a)** = **3** and common difference **(d)** = **3** and total number of classes **(n)** = **12**

Now, number of trees planted by 12 classes is given by

$$S_{12} = \frac{12}{2}\{2 \times 3 + (12-1) \times 3\}$$

$$S_{12} = 6\,(6 + 33)$$

$$S_{12} = 6 \times 39 = 234$$

Therefore, number of trees planted by 12 classes is **234**

Q18 A spiral is made up of successive semicircles, with centres alternately at A and B ??????, starting with centre at A, of radii $0.5\ cm, 1.0\ cm, 1.5\ cm, 2.0\ cm, \ldots$ as shown in Fig. 5.4. What is the total length of such a spiral made up of thirteen consecutive semicircles? (Take $\pi = \frac{22}{7}$)

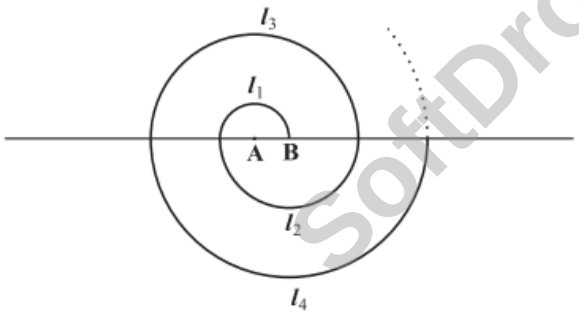

Fig. 5.4

[**Hint** : Length of successive semicircles is $l_1, l_2, l_3, l_4, \ldots$ with centres at $A, B, A, B, \ldots$, respectively.]

Answer:

From the above-given figure

Circumference of 1st semicircle $l_1 = \pi r_1 = 0.5\pi$

Similarly,

Circumference of 2nd semicircle $l_2 = \pi r_2 = \pi$

Circumference of 3rd semicircle $l_3 = \pi r_3 = 1.5\pi$

It is clear that this is an AP with $a = 0.5\pi \; and \; d = 0.5\pi$

Now, sum of length of 13 such semicircles is given by

$$S_{13} = \frac{13}{2}\{2 \times 0.5\pi + (13-1)0.5\pi\}$$
$$S_{13} = \frac{13}{2}(\pi + 6\pi)$$
$$S_{13} = \frac{13}{2} \times 7\pi$$
$$S_{13} = \frac{91\pi}{2} = \frac{91}{2} \times \frac{22}{7} = 143$$

Therefore, sum of length of 13 such semicircles is **143 cm**

Q19 200 logs are stacked in the following manner: 20 logs in the bottom row, 19 in the next row, 18 in the row next to it and so on (see Fig. 5.5). In how many rows are the 200 logs placed and how many logs are in the top row?

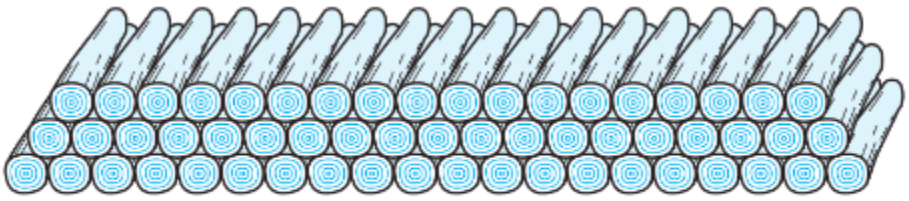

Fig. 5.5

Answer:

As the rows are going up, the no of logs are decreasing,

We can clearly see that 20, 19, 18, ..., is an AP.

and here $a = 20 \; and \; d = -1$

Let suppose 200 logs are arranged in 'n' rows,

Then,

$$S_n = \frac{n}{2}\{2 \times 20 + (n-1)(-1)\}$$

$$200 = \frac{n}{2}\{41 - n\}$$

$$\Rightarrow n^2 - 41n + 400 = 0$$

$$\Rightarrow n^2 - 16n - 25n + 400 = 0$$

$$\Rightarrow (n-16)(n-25) = 0$$

$$\Rightarrow n = 16 \;\; and \;\; n = 25$$

Now,

case (i) **n = 25**

$$a_{25} = a + 24d = 20 + 24 \times (-1) = 20 - 24 = -4$$

But number of rows can not be in negative numbers

Therefore, we will reject the value **n = 25**

case (ii) n = 16

$$a_{16} = a + 15d = 20 + 15 \times (-1) = 20 - 15 = 5$$

Therefore, the number of rows in which 200 logs are arranged is equal to **5**

Q20 In a potato race, a bucket is placed at the starting point, which is 5 m from the first potato, and the other potatoes are placed 3 m apart in a straight line. There are ten potatoes in the line (see Fig. 5.6).

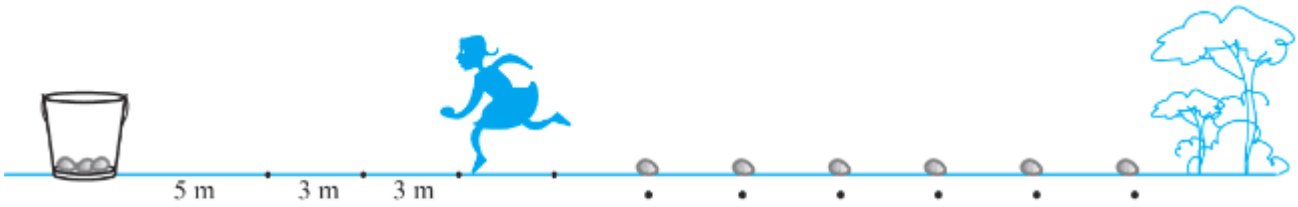

Fig. 5.6

A competitor starts from the bucket, picks up the nearest potato, runs back with it, drops it in the bucket, runs back to pick up the next potato, runs to the bucket to drop it in, and she continues in the same way until all the potatoes are in the bucket. What is the total distance the competitor has to run?

[**Hint** : To pick up the first potato and the second potato, the total distance (in metres) run by a competitor is $2 \times 5 + 2 \times (5 + 3)$]

Answer:

Distance travelled by the competitor in picking and dropping 1st potato $= 2 \times 5 = 10\ m$

Distance travelled by the competitor in picking and dropping 2nd potato $= 2 \times (5 + 3) = 2 \times 8 = 16\ m$

Distance travelled by the competitor in picking and dropping 3rd potato $= 2 \times (5 + 3 + 3) = 2 \times 11 = 22\ m$

and so on

we can clearly see that it is an AP with first term **(a) = 10** and common difference **(d) = 6**

There are 10 potatoes in the line

Therefore, total distance travelled by the competitor in picking and dropping potatoes is

$$S_{10} = \frac{10}{2}\{2 \times 10 + (10 - 1)6\}$$

$$S_{10} = 5\,(20 + 54)$$

$$S_{10} = 5 \times 74 = 370$$

Therefore, the total distance travelled by the competitor in picking and dropping potatoes is **370 m**

NCERT solutions for class 10 maths chapter 5 Arithmetic Progressions Excercise: 5.4

Q1 Which term of the AP: is its first negative term? [**Hint** : Find n for $a_n < 0$]

Answer:

Given AP is

$121, 117, 113, ...,$

Here $a = 121 \; and \; d = -4$

Let suppose nth term of the AP is first negative term

Then,

$a_n = a + (n-1)d$

If nth term is negative then $a_n < 0$

$\Rightarrow 121 + (n-1)(-4) < 0$

$\Rightarrow 125 < 4n$

$\Rightarrow n > \frac{125}{4} = 31.25$

Therefore, first negative term must be 32nd term

Q2 The sum of the third and the seventh terms of an AP is 6 and their product is 8 .

Find the sum of first sixteen terms of the AP.

Answer:

It is given that sum of third and seventh terms of an AP are and their product is 8

$a_3 = a + 2d$

$a_7 = a + 6d$

Now,

$$a_3 + a_7 = a + 2d + a + 6d = 6$$

$$\Rightarrow 2a + 8d = 6$$

$$\Rightarrow a + 4d = 3 \Rightarrow a = 3 - 4d \qquad -(i)$$

And

$$a_3.a_7 = (a + 2d).(a + 6d) = a^2 + 8ad + 12d^2 = 8 \qquad -(ii)$$

put value from equation (i) in (ii) we will get

$$\Rightarrow (3 - 4d)^2 + 8(3 - 4d)d + 12d^2 = 8$$

$$\Rightarrow 9 + 16d^2 - 24d + 24d - 32d^2 + 12d^2 = 8$$

$$\Rightarrow 4d^2 = 1$$

$$\Rightarrow d = \pm\frac{1}{2}$$

Now,

case (i) $d = \frac{1}{2}$

$$a = 3 - 4 \times \frac{1}{2} = 1$$

Then,

$$S_{16} = \frac{16}{2}\left\{2 \times 1 + (16 - 1)\frac{1}{2}\right\}$$

$$S_{16} = 76$$

case (ii) $d = -\frac{1}{2}$

$$a = 3 - 4 \times \left(-\frac{1}{2}\right) = 5$$

Then,

$$S_{16} = \frac{16}{2}\left\{2 \times 1 + (16 - 1)\left(-\frac{1}{2}\right)\right\}$$

$$S_{16} = 20$$

Q3 A ladder has rungs 25 cm apart. (see Fig. 5.7). The rungs decrease uniformly in length from 45 cm at the bottom to 25 cm at the top. If the top and the bottom rungs are $2\frac{1}{2}$ m apart, what is the length of the wood required for the rungs? [**Hint:** Number of rungs $= \frac{250}{25} + 1$]

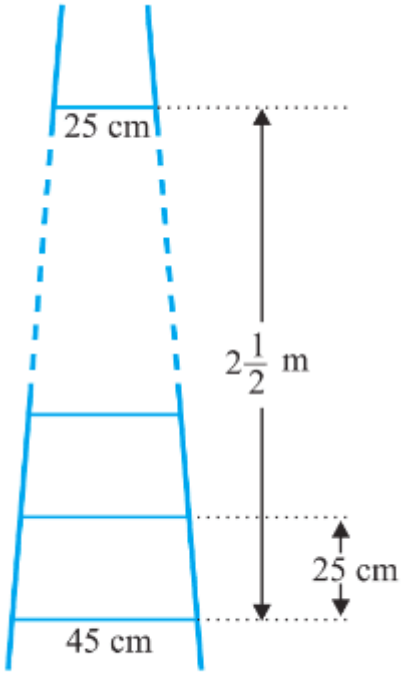

Fig. 5.7

Answer:

It is given that

The total distance between the top and bottom rung $= 2\frac{1}{2}\ m = 250cm$

Distance between any two rungs = 25 cm

Total number of rungs = $\frac{250}{25} + 1 = 11$

And it is also given that bottom-most rungs is of 45 cm length and topmost is of 25 cm length.As it is given that the length of rungs decrease uniformly, it will form an AP with $a = 25, a_{11} = 45\ and\ n = 11$

Now, we know that

$$a_{11} = a + 10d$$

$$\Rightarrow 45 = 25 + 10d$$

$$\Rightarrow d = 2$$

Now, total length of the wood required for the rungs is equal to

$$S_{11} = \frac{11}{2}\{2 \times 25 + (11-1)2\}$$

$$S_{11} = \frac{11}{2}\{50 + 20\}$$

$$S_{11} = \frac{11}{2} \times 70$$

$$S_{11} = 385\ cm$$

Therefore, the total length of the wood required for the rungs is equal to **385 cm**

Q4 The houses of a row are numbered consecutively from 1 to 49. Show that there is a value of x such that the sum of the numbers of the houses preceding the house numbered x is equal to the sum of the numbers of the houses following it. Find this value of x. [**Hint** : $S_{x-1} = S_{49} - S_x$]

Answer:

It is given that the sum of the numbers of the houses preceding the house numbered x is equal to the sum of the numbers of the houses following it

And 1,2,3,.....,49 form an AP with **a = 1 and d = 1**

Now, we know that

$$S_n = \frac{n}{2}\{2a + (n-1)d\}$$

Suppose their exist an n term such that (n < 49)

Now, according to given conditions

Sum of first n - 1 terms of AP = Sum of terms following the nth term

Sum of first n - 1 term of AP = Sum of whole AP - Sum of first m terms of AP

i.e.

$$S_{n-1} = S_{49} - S_n$$

$$\frac{n-1}{2}\{n\} = \frac{49}{2}\{50\} - \frac{n}{2}\{n+1\}$$

$$\frac{n^2}{2} - \frac{n}{2} = 1225 - \frac{n^2}{2} - \frac{n}{2}$$

$$n^2 = 1225$$

$$n = \pm 35$$

Given House number are not negative so we reject n = -35

Therefore, the sum of no of houses preceding the house no 35 is equal to the sum of no of houses following the house no 35

Q5 A small terrace at a football ground comprises of 15 steps each of which is 50 m long and built of solid concrete. Each step has a rise of $\frac{1}{4}\ m$ and a tread of $\frac{1}{2}\ m$. (see Fig. 5.8). Calculate the total volume of concrete required to build the terrace.

[**Hint:** Volume of concrete required to build the first step $= \frac{1}{4} \times \frac{1}{2} \times 50\ m^3$]

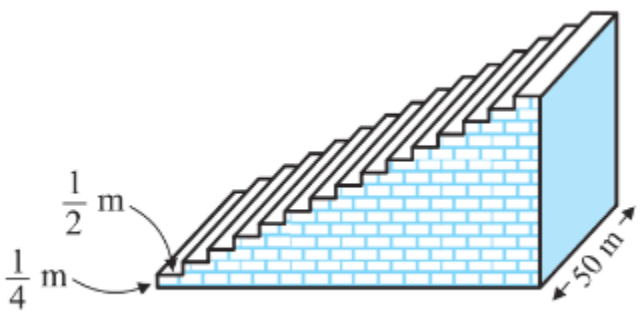

Fig. 5.8

Answer:

It is given that

football ground comprises of 15 steps each of which is 50 m long and Each step has a rise of $\frac{1}{4}\,m$ and a tread of $\frac{1}{2}\,m$

Now,

The volume required to make the first step = $\frac{1}{4} \times \frac{1}{2} \times 50 = 6.25\ m^3$

Similarly,

The volume required to make 2nd step =

And

The volume required to make 3rd step =

And so on

We can clearly see that this is an AP with $a = 6.25\ and\ d = 6.25$

Now, the total volume of concrete required to build the terrace of 15 such step is

$S_{15} = \frac{15}{2}\{2 \times 6.25 + (15-1)6.25\}$

$S_{15} = \frac{15}{2}\{12.5 + 87.5\}$

$S_{15} = \frac{15}{2} \times 100$

$S_{15} = 15 \times 50 = 750$

Therefore, the total volume of concrete required to build the terrace of 15 such steps is $750\ m^3$