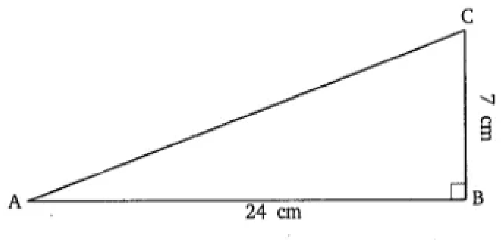
NCERT solutions for class 10 maths chapter 8 Introduction to Trigonometry

Excercise: 8.1

Q1 In $\Delta\ ABC$, right-angled at $B, AB = 24\ cm$, $BC = 7\ cm$. Determine : $(i)\ \sin A, \cos A\ (ii)\ \sin C, \cos C$

Answer:



We have,

In $\Delta\ ABC$, $\angle$ B = 90, and the length of the base (AB) = 24 cm and length of perpendicular (BC) = 7 cm

So, by using Pythagoras theorem,
$AC^2 = AB^2 + BC^2$
$AC = \sqrt{AB^2 + BC^2}$
Therefore, $AC = \sqrt{576 + 49}$

$AC = \sqrt{625}$

AC = 25 cm

Now,
(i) $\sin A = P/H = BC/AB = 7/25$

$\cos A = B/H = BA/AC = 24/25$

(ii) For angle C, AB is perpendicular to the base (BC). Here B indicates to Base and P means perpendicular wrt angle $\angle$ C
So, $\sin C = P/H = BA/AC = 24/25$

and $\cos C = B/H = BC/AC = 7/25$

Q2 In Fig. 8.13, find $\tan P - \cot R$.

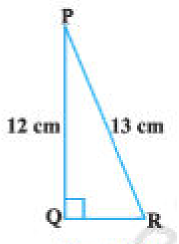
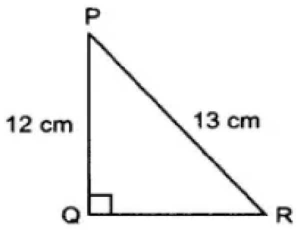


Fig. 8.13

Answer:



We have, $\triangle$ PQR is a right-angled triangle, length of PQ and PR are 12 cm and 13 cm respectively.

So, by using Pythagoras theorem,

$QR = \sqrt{13^2 - 12^2}$

$QR = \sqrt{169 - 144}$

$QR = \sqrt{25} = 5\ cm$

Now, According to question,

$\tan P - \cot R$ = $\frac{RQ}{QP} - \frac{QR}{PQ}$

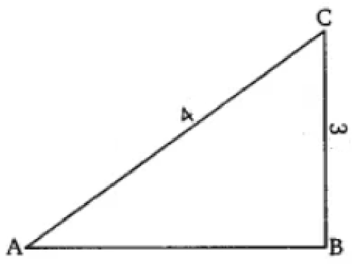
= 5/12 - 5/12 = 0

Q3 If $\sin A = \frac{3}{4}$, calculate $\cos A$ and $\tan A$.

Answer:

Suppose Δ ABC is a right-angled triangle in which $\angle B = 90^0$ and we have $\sin A = \frac{3}{4}$,

So,



Let the length of AB be 4 unit and the length of BC = 3 unit

So, by using Pythagoras theorem,

$AB = \sqrt{16-9} = \sqrt{7}$ units

Therefore,

$\cos A = \frac{AB}{AC} = \frac{\sqrt{7}}{4}$ and $\tan A = \frac{BC}{AB} = \frac{3}{\sqrt{7}}$

Q4 Given $15 \cot A = 8$, find $\sin A$ and $\sec A$.

Answer:

We have,

$15 \cot A = 8, \Rightarrow \cot A = 8/15$

It implies that In the triangle ABC in which $\angle B = 90^0$. The length of AB be 8 units and the length of BC = 15 units

Now, by using Pythagoras theorem,

$AC = \sqrt{64+225} = \sqrt{289}$

$\Rightarrow AC = 17$ units

So, $\sin A = \frac{BC}{AC} = \frac{15}{17}$

and $\sec A = \frac{AC}{AB} = \frac{17}{8}$

Q5 Given $\sec\theta = \frac{13}{12}$, calculate all other trigonometric ratios.

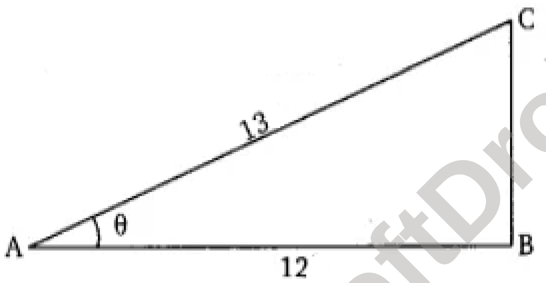
Answer:

We have,

$\sec\theta = \frac{13}{12}$,

It means the Hypotenuse of the triangle is 13 units and the base is 12 units.

Let ABC is a right-angled triangle in which ∠ B is 90 and AB is the base, BC is perpendicular height and AC is the hypotenuse.



By using Pythagoras theorem,

$BC = \sqrt{169 - 144} = \sqrt{25}$

BC = 5 unit

Therefore,

$\sin\theta = \frac{BC}{AC} = \frac{5}{13}$

$\cos\theta = \frac{BA}{AC} = \frac{12}{13}$

$\tan\theta = \frac{BC}{AB} = \frac{5}{12}$

$$\cot\theta = \frac{BA}{BC} = \frac{12}{5}$$

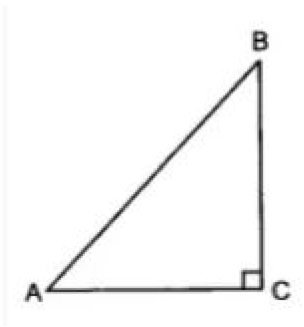
$$\sec\theta = \frac{AC}{AB} = \frac{13}{12}$$

$$\csc\theta = \frac{AC}{BC} = \frac{13}{5}$$

Q6 If $\angle A$ and $\angle B$ are acute angles such that $\cos A = \cos B$, then show that $\angle A = \angle B$.

Answer:

We have, A and B are two acute angles of triangle ABC and $\cos A = \cos B$



According to question, In triangle ABC,

$\cos A = \cos B$

$$\frac{AC}{AB} = \frac{BC}{AB}$$

$\Rightarrow AC = AB$

Therefore, ∠ A = ∠ B [angle opposite to equal sides are equal]

Q7 If $\cot\theta = \frac{7}{8}$, evaluate: $(i)\ \frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}$ $(ii)\ \cot^2\theta$

Answer:

Given that,

$\cot\theta = \frac{7}{8}$

$\therefore$ perpendicular (AB) = 8 units and Base (AB) = 7 units

Draw a right-angled triangle ABC in which $\angle B = 90^0$

Now, By using Pythagoras theorem,

$AC^2 = AB^2 + BC^2$

$AC = \sqrt{64+49} = \sqrt{113}$

So, $\sin\theta = \frac{BC}{AC} = \frac{8}{\sqrt{113}}$

and $\cos\theta = \frac{AB}{AC} = \frac{7}{\sqrt{113}}$

$\Rightarrow \cot\theta = \frac{\cos\theta}{\sin\theta} = \frac{7}{8}$

$(i)\ \frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}$

$\Rightarrow \frac{(1-\sin^2\theta)}{(1-\cos^2\theta)} = \frac{\cos^2\theta}{\sin^2\theta} = \cot^2\theta$

$= (\frac{7}{8})^2 = \frac{49}{64}$

$(ii)\ \cot^2\theta$

$= (\frac{7}{8})^2 = \frac{49}{64}$

Q8 If $3\cot A = 4$, check wether $\frac{1-\tan^2 A}{1+\tan^2 A} = \cos^2 A - \sin^2 A$ or not.

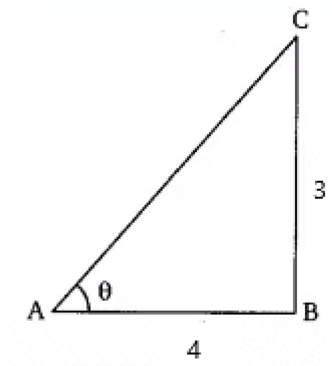
Answer:

Given that,

$3\cot A = 4,$

$\Rightarrow \cot = \frac{4}{3} = \frac{base}{perp.}$

ABC is a right-angled triangle in which $\angle B = 90^0$ and the length of the base AB is 4 units and length of perpendicular is 3 units



By using Pythagoras theorem,

In triangle ABC,

$$AC^2 = AB^2 + BC^2$$
$$AC = \sqrt{16+9}$$
$$AC = \sqrt{25}$$

AC = 5 units

So,

$$\tan A = \frac{BC}{AB} = \frac{3}{4}$$
$$\cos A = \frac{AB}{AC} = \frac{4}{5}$$
$$\sin A = \frac{BC}{AC} = \frac{3}{5}$$
$$\frac{1-\tan^2 A}{1+\tan^2 A} = \cos^2 A - \sin^2 A$$

Put the values of above trigonometric ratios, we get;

$$\Rightarrow \frac{1-9/4}{1+9/4} = \frac{16}{25} - \frac{9}{25}$$
$$\Rightarrow -\frac{5}{13} \neq \frac{7}{25}$$

LHS $\neq$ RHS

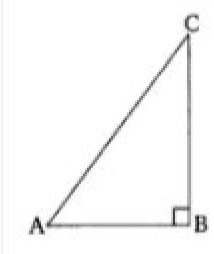
Q9 In triangle ABC , right-angled at B , if $\tan A = \frac{1}{\sqrt{3}}$, find the value of:

$(i) \sin A \ \cos C + \cos A \ \sin C$

$(ii) \cos A \ \cos C + \sin A \ \sin C$

Answer:

Given a triangle ABC, right-angled at B and $\tan A = \frac{1}{\sqrt{3}} \Rightarrow A = 30^0$



According to question, $\tan A = \frac{1}{\sqrt{3}} = \frac{BC}{AB}$

By using Pythagoras theorem,
$AC^2 = AB^2 + BC^2$
$AC = \sqrt{1+3} = \sqrt{4}$

AC = 2

Now,

Therefore,

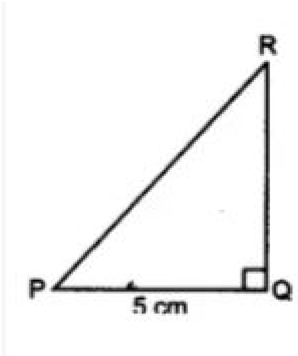
$(i) \sin A \ \cos C + \cos A \ \sin C$

$(ii) \cos A \ \cos C + \sin A \ \sin C$

Q10 In $\Delta \ PQR$, right-angled at Q , $PR + QR = 25 \ cm$ and $PQ = 5 \ cm$.

Determine the values of $\sin P, \cos P \ and \ \tan P.$

Answer:



We have, PR + QR = 25 cm............(i)

PQ = 5 cm

and $\angle Q = 90^0$

According to question,

In triangle Δ PQR,

By using Pythagoras theorem,

$$PR^2 = PQ^2 + QR^2$$
$$PQ^2 = PR^2 - QR^2$$
$$5^2 = (PR - QR)(PR + QR)$$
$$25 = 25(PR - QR)$$

PR - QR = 1........(ii)

From equation(i) and equation(ii), we get;

PR = 13 cm and QR = 12 cm.

therefore,

Q11 State whether the following are true or false. Justify your answer.

(i) The value of $\tan A$ is always less than 1.

(ii) $\sec A = \frac{12}{5}$ for some value of angle A.

(iii) $\cos A$ is the abbreviation used for the cosecant of angle A.

(iv) $\cot A$ is the product of cot and A.

(v) $\sin \Theta = \frac{4}{3}$ for some angle Θ.

Answer:

(i) False,

because $\tan 60 = \sqrt{3}$, which is greater than 1

(ii) TRue,

because $\sec A \geq 1$

(iii) False,

Because $\cos A$ abbreviation is used for cosine A.

(iv) False,

because the term $\cot A$ is a single term, not a product.

(v) False,

because $\sin\theta$ lies between (-1 to +1) [$-1 \leq \sin\theta \leq 1$]

NCERT solutions for class 10 maths chapter 8 Introduction to Trigonometry

Excercise: 8.2

Q1 Evaluate the following :

$(i) \sin 60^o \cos 30^o + \sin 30^o \cos 60^o$

Answer:

$\sin 60^o \cos 30^o + \sin 30^o \cos 60^o$

As we know,

the value of $\sin 60^0 = \sqrt{3}/2 = \cos 30^0$, $\sin 30^0 = 1/2 = \cos 60^0$

$\Rightarrow \frac{\sqrt{3}}{2}.\frac{\sqrt{3}}{2} + \frac{1}{2}.\frac{1}{2}$

$= \frac{3}{4} + \frac{1}{4}$

$= 1$

Q1 Evaluate the following :

$(ii)\ 2\tan^2 45^o + 2\cos^2 30^o - 2\sin^2 60^o$

Answer:

We know the value of

$\tan 45^0 = 1$ and

$\cos 30^0 = \sin 60^0 = \dfrac{\sqrt{3}}{2}$

According to question,

$$= 2\tan^2 45^o + 2\cos^2 30^o - 2\sin^2 60^o$$
$$= 2(1)^2 + (\frac{\sqrt{3}}{2})^2 - (\frac{\sqrt{3}}{2})$$
$$= 2$$

Q1 Evaluate the following :

$(iii)\ \dfrac{\cos 45^o}{\sec 30^o + \csc 30^o}$

Answer:

$$\frac{\cos 45^o}{\sec 30^o + \csc 30^o}$$

we know the value of

$\cos 45 = 1/\sqrt{2}$, $\sec 30^0 = 2/\sqrt{3}$ and $cosec\ 30 = 2$,

After putting these values

$$= \frac{\frac{1}{\sqrt{2}}}{\frac{2}{\sqrt{3}} + 2}$$
$$= \frac{1/\sqrt{3}}{(2 + 2\sqrt{3})/\sqrt{3}}$$

$$= \frac{\sqrt{3}}{2\sqrt{2}+2\sqrt{6}} \times \frac{2\sqrt{2}-2\sqrt{6}}{2\sqrt{2}-2\sqrt{6}}$$
$$= \frac{2\sqrt{6}-2\sqrt{18}}{-16}$$
$$= 2\frac{\sqrt{6}-3\sqrt{3}}{-16} = \frac{3\sqrt{3}-\sqrt{6}}{8}$$

Q1 Evaluate the following :

$$(iv)\ \frac{\sin 30^o + \tan 45^o - cosec 60^o}{\sec 30^o + \cos 60^o + \cot 45^o}$$

Answer:

$$\frac{\sin 30^o + \tan 45^o - cosec 60^o}{\sec 30^o + \cos 60^o + \cot 45^o}$$ ....(i)

It is known that the values of the given trigonometric functions,
$\sin 30^0 = 1/2 = cos 60^0$
$\tan 45^0 = 1 = \cot 45^0$
$\sec 30^0 = 2/\sqrt{3} = cosec 60^0$

Put all these values in equation (i), we get;

Q1 Evaluate the following :

$$(v)\ \frac{5\cos^2 60^o + 4\sec^2 30^o - \tan^2 45^o}{\sin^2 30^o + \cos^2 30^o}$$

Answer:

$$\frac{5\cos^2 60^o + 4\sec^2 30^o - \tan^2 45^o}{\sin^2 30^o + \cos^2 30^o}$$ ....(i)

We know the values of-
$\cos 60^0 = 1/2 = \sin 30^0$
$\sec 30^0 = 2/\sqrt{3}$
$\tan 45^0 = 1$
$\cos 30^0 = \sqrt{3}/2$

By substituting all these values in equation(i), we get;

Q2 Choose the correct option and justify your choice :

$(i)\ \dfrac{2\ \tan 30^o}{1+\tan^2 30^o} =$

$(A)\ \sin 60^o\ (B)\ \cos 60^o\ (C)\ \tan 60^o\ (D)\ \sin 30^o$

Answer:

Put the value of **tan 30** in the given question-

The correct option is (A)

Q2 Choose the correct option and justify your choice :

$(ii)\ \dfrac{1-\tan^2 45^o}{1+\tan^2 45^o} =$

$(A)\ \tan\ 90^o\ (B)\ 1\ (C)\ \sin 45^o\ (D)\ 0$

Answer:

The correct option is (D)

$\dfrac{1-\tan^2 45^o}{1+\tan^2 45^o} =$

We know that $\tan 45 = 1$

So, $\dfrac{1-1}{1+1} = 0$

Q2 Choose the correct option and justify your choice :

$(iii) \sin\ 2A = 2\ \sin A$ is true when A =

$(A) 0^o\ (B)\ 30^o\ (C)\ 45^o\ (D)\ 60^o$

Answer:

The correct option is (A)

$\sin\ 2A = 2\ \sin A$

We know that $\sin 2A = 2 \sin A \cos A$

So, $2 \sin A \cos A = 2 \sin A$
$\cos A = 1$
$A = 0^0$

Q2 Choose the correct option and justify your choice :

$(iv)\dfrac{2\ \tan 30^o}{1 - tan^2\, 30^o} =$

$(A)\ \cos 60^o\ (B)\ \sin 60^o\ (C)\ \tan 60^o\ (D)\ \sin 30^o$

Answer:

Put the value of $tan30^o = \ 1/\sqrt{3}$

The correct option is (C)

Q3 If $\tan(A + B) = \sqrt{3}$ and $\tan(A - B) = \dfrac{1}{\sqrt{3}};\ 0^o < A + B \leq 90^o;\ A > B,$ find $A\ and\ B.$

Answer:

Given that,

$\tan(A + B) = \sqrt{3} = \tan 60^0$

So, $A + B = 60^o$ ....(i)

$\tan(A - B) = 1/\sqrt{3} = \tan 30^0$

therefore, $A - B = 30^o$ ....(ii)

By solving the equation (i) and (ii) we get;

$A = 45^o$ and $B = 15^o$

Q4 State whether the following are true or false. Justify your answer.

$(i)\sin(A+B) = \sin A + \sin B$

(ii) The value of $\sin\theta$ increases as θ increases.

(iii) The value of $\cos\theta$ increases as θ increases.

$(iv)\sin\theta = \cos\theta$ for all values of θ .

$(v)\cot A$ is not defined for $A = 0^o$

Answer:

(i) False,

Let A = B = 45^0

$$\sin(45^0 + 45^0) = \sin 45^0 + \sin 45^0$$
$$\sin 90^{=}1/\sqrt{2} + q/\sqrt{2}$$

Then, $1 \neq \sqrt{2}$

(ii) True,

Take $\theta = 0^0,\ 30^0,\ 45^0$

whent

θ = 0 then zero(0),

θ = 30 then value of $\sin\theta$ is 1/2 = 0.5

θ = 45 then value of $\sin\theta$ is 0.707

(iii) False,

$\cos 0^0 = 1,\ \cos 30^0 = \sqrt{3}/2 = 0.87,\ \cos 45^0 = 1\sqrt{2} = 0.707$

(iv) False,

Let θ = 0

$\sin 0^0 = \cos 0^0$

$0 \neq 1$

(v) True,

$\cot 0^0 = \frac{\cos 0^0}{\sin 0^0} = \frac{1}{0}$ (not defined)

NCERT solutions for class 10 maths chapter 8 Introduction to Trigonometry

Excercise: 8.3

Q1 Evaluate :

$(i) \frac{\sin 18^o}{\cos 72^o}$

Answer:

$\frac{\sin 18^o}{\cos 72^o}$

We can write the above equation as;

$= \frac{\sin(90^0 - 72^0)}{\cos 72^0}$

By using the identity of $\sin(90^o - \theta) = \cos\theta$

Therefore, $\frac{\cos 72^0}{\cos 72^0} = 1$

So, the answer is 1.

Q1 Evaluate :

$(ii) \frac{\tan 26^o}{\cot 64^o}$

Answer:

$$\frac{\tan 26^o}{\cot 64^o}$$

The above equation can be written as ;

$\tan(90^o - 64^o)/\cot 64^o$ ....(i)

It is known that, $\tan(90^o - \theta) = \cot\theta$

Therefore, equation (i) becomes,

$\cot 64^o/\cot 64^o = 1$

So, the answer is 1.

Q1 Evaluate :

$(iii)\cos 48^o - \sin 42^o$

Answer:

$\cos 48^o - \sin 42^o$

The above equation can be written as ;

$\cos(90^o - 42^o) - \sin 42^o$ ....(i)

It is known that $\cos(90^o - \theta) = \sin\theta$

Therefore, equation (i) becomes,

$\sin 42^o - \sin 42^o = 0$

So, the answer is 0.

Q1 Evaluate :

$(iv) cosec\, 31^o - \sec 59^o$

Answer:

$cosec\, 31^o - \sec 59^o$

This equation can be written as;

$cosec 31^o - \sec(90^o - 31^o)$ ....(i)

We know that $\sec(90^o - \theta) = cosec\theta$

Therefore, equation (i) becomes;

$cosec 31^o - cosec\ 31^o$ = 0

So, the answer is 0.

Q2 Show that :

$(i) \tan 48^o \tan 23^o \tan 42^o \tan 67^o = 1$

Answer:

$\tan 48^o \tan 23^o \tan 42^o \tan 67^o = 1$

Taking Left Hand Side (LHS)

= $\tan 48^o \tan 23^o \tan 42^o \tan 67^o$

$\Rightarrow \tan 48^o \tan 23^o \tan(90^o - 48^o) \tan(90^o - 23^o)$

$\Rightarrow \tan 48^o \tan 23^o \cot 48^o \cot 23^o$ [it is known that $\tan(90^0 - \theta = \cot\theta)$ and $\cot\theta \times \tan\theta = 1$

$= 1$

Hence proved.

Q2 Show that :

$(ii) \cos 38^o \cos 52^o - \sin 38^o \sin 52^o = 0$

Answer:

$\cos 38^o \cos 52^o - \sin 38^o \sin 52^o = 0$

Taking Left Hand Side (LHS)

= $\cos 38^o \cos 52^o - \sin 38^o \sin 52^o$

= $\cos 38^o \cos(90^o - 38^o) - \sin 38^o \sin(90^o - 38^o)$

= $\cos 38^o \sin 38^o - \sin 38^o \cos 38^o$ [it is known that $\sin(90^0 - \theta) = \cos\theta$ and $\cos(90^0 - \theta) = \sin\theta$]

= 0

Q3 If $\tan 2A = \cot(A - 18^o)$, where $2A$ is an acute angle, find the value of A .

Answer:

We have,

$\tan$ 2A = $\cot$ (A - 18^0)

we know that, $\cot(90^0 - \theta) = \tan\theta$

Q4 If $\tan A = \cot B$, prove that $A + B = 90^o$.

Answer:

We have,

$\tan A = \cot B$

and we know that $\tan(90^0 - \theta) = \cot\theta$

therefore,

$\tan A = \tan(90^0 - B)$

A = 90 - B

A + B = 90

Hence proved.

Q5 If $\sec 4A = cosec(A - 20^o)$, where $4A$ is an acute angle, find the value of A .

Answer:

We have,

$\sec 4A = cosec(A - 20^o)$, Here 4A is an acute angle

According to question,

We know that $cosec(90^0 - \theta) = \sec\theta$

$cosec(90^0 - 4A) = cosec(A - 20^o)$

$\Rightarrow 90 - 4A = A - 20$

$\Rightarrow 5A = 110$

$\Rightarrow A = \frac{110}{5}$

$\Rightarrow A = 22^o$

Q6 If A, B and C are interior angles of a triangle ABC , then show that

$\sin(\frac{B+C}{2}) = \cos\frac{A}{2}$

Answer:

Given that,

A, B and C are interior angles of ΔABC

To prove - $\sin(\frac{B+C}{2}) = \cos\frac{A}{2}$

Now,

In triangle ΔABC ,

A + B + C = 180^0

$\Rightarrow B + C = 180 - A$

$\Rightarrow B + C/2 = 90^0 - A/2$

$\sin\frac{B+C}{2} = \sin(90^0 - A/2)$

$\sin\frac{B+C}{2} = \cos A/2$

Hence proved.

Q7 Express $sin67^o + \cos 75^o$ in terms of trigonometric ratios of angles between 0^o and 45^o .

Answer:

By using the identity of $\sin\theta$ and $\cos\theta$

$sin67^o + \cos 75^o$

We know that,

$\sin(90-\theta) = \cos\theta$ and $\cos(90-\theta) = \sin\theta$

the above equation can be written as;

$= \sin(90^0 - 23^0) + \cos(90^0 - 15^0)$

$= \sin(15^0) + \cos(23^0)$

NCERT Solutions for Class 10 Maths Chapter 8 Introduction to Trigonometry

Excercise: 8.4

Q1 Express the trigonometric ratios $\sin A$, $\sec A$ and $\tan A$ in terms of $cotA$.

Answer:

We know that $\csc^2 A - \cot^2 A = 1$

(i)

(ii) We know the identity of

(iii) $\tan A = \frac{1}{\cot A}$

Q2 Write all the other trigonometric ratios of $\angle A$ in terms of $\sec A$.

Answer:

We know that the identity $\sin^2 A + \cos^2 = 1$

$\sin^2 A = 1 - \cos^2$

$sin^2 A = 1 - \dfrac{1}{\sec^2 A}$

$= \dfrac{\sec^2 A - 1}{\sec^2 A}$

$\sin A = \sqrt{\dfrac{\sec^2 A - 1}{\sec^2 A}}$

$= \dfrac{\sqrt{\sec^2 A - 1}}{\sec A}$

$$cosecA = \frac{\sec A}{\sqrt{\sec^2 A - 1}}$$

$$\tan A = \frac{\sin A}{\cos A} = \sqrt{\sec^2 A - 1}$$

$$\cot A = \frac{1}{\sqrt{\sec^2 A - 1}}$$

Q3 Evaluate :

$$(i) \frac{\sin^2 63^o + \sin^2 27^o}{\cos^2 17^o + \cos^2 73^o}$$

Answer:

$\dfrac{\sin^2 63^o + \sin^2 27^o}{\cos^2 17^o + \cos^2 73^o}$ ....(i)

The above equation can be written as;

(Since $\sin^2\theta + \cos^2\theta = 1$)

Q3 Evaluate :

$$(ii) \sin 25^o \cos 65^o + \cos 25^o \sin 65^o$$

Answer:

$\sin 25^o \cos 65^o + \cos 25^o \sin 65^o$

We know that
$\sin(90^0 - \theta) = \cos\theta$
$\cos(90^0 - \theta) = \sin\theta$

Therefore,

Q4 Choose the correct option. Justify your choice.

$(i) 9\sec^2 A - 9\tan^2 A =$

(A) 1 (B) 9 (C) 8 (D) 0

Answer:

The correct option is (B) = 9

$9\sec^2 A - 9\tan^2 A = 9(\sec^2 A - \tan^2 A)$ ....(i)

and it is known that $\sec^2\theta - \tan^2 = 1$

Therefore, equation (i) becomes, $9 \times 1 = 9$

Q4 Choose the correct option. Justify your choice.

$(ii)(1 + \tan\theta + \sec\theta)(1 + \cot\theta - cosec\,\theta) =$

(A) 0 (B) 1 (C) 2 (D) –1

Answer:

The correct option is (C)

$(1+\tan\theta+\sec\theta)(1+\cot\theta-cosec\,\theta)$ ....(i)

we can write his above equation as;

= 2

Q4 Choose the correct option. Justify your choice.

$(iii)(\sec A+\tan A)(1-\sin A)=$

$(A)\sec A\,(B)\sin A\,(C)cosecA\,(D)\cos A$

Answer:

The correct option is (D)

$(\sec A+\tan A)(1-\sin A)=$

Q4 Choose the correct option. Justify your choice.

$(iv)\dfrac{1+\tan^2 A}{1+\cot^2 A}=$

$(A)\sec^2 A\,(B)-1\,(C)\cot^2 A\,(D)\tan^2 A$

Answer:

The correct option is (D)

$\dfrac{1+\tan^2 A}{1+\cot^2 A}$ ....eq (i)

The above equation can be written as;

We know that $\cot A=\dfrac{1}{\tan A}$

therefore,

Q5 Prove the following identities, where the angles involved are acute angles for which the expressions are defined.

$$(i)(\csc\theta-\cot\theta)^2=\frac{1-\cos\theta}{1+\cos\theta}$$

Answer:

We need to prove-

$$(\csc\theta-\cot\theta)^2=\frac{1-\cos\theta}{1+\cos\theta}$$

Now, taking LHS,

$$(\csc\theta-\cot\theta)^2=(\frac{1}{\sin\theta}-\frac{\cos\theta}{\sin\theta})^2$$
$$=(\frac{1-\cos\theta}{\sin\theta})^2$$
$$=\frac{(1-\cos\theta)(1-\cos\theta)}{\sin^2\theta}$$

LHS = RHS

Hence proved.

Q5 Prove the following identities, where the angles involved are acute angles for which the expressions are defined.

$$(ii)\frac{\cos A}{1+\sin A}+\frac{1+\sin A}{\cos A}=2\sec A$$

Answer:

We need to prove-

$$\frac{\cos A}{1+\sin A}+\frac{1+\sin A}{\cos A}=2\sec A$$

taking LHS;

= RHS

Hence proved.

Q5 Prove the following identities, where the angles involved are acute angles for which the expressions are defined.

$$(iii)\frac{\tan\theta}{1-\cot\theta}+\frac{\cot\theta}{1-\tan\theta}=1+\sec\theta\csc\theta$$

[**Hint** : Write the expression in terms of $\sin\theta$ and $\cos\theta$]

Answer:

We need to prove-

$$\frac{\tan\theta}{1-\cot\theta}+\frac{\cot\theta}{1-\tan\theta}=1+\sec\theta\, cosec\theta$$

Taking LHS;

By using the identity $a^3 - b^3 = (a - b)(a^2 + b^2 + ab)$

Hence proved.

Q5 Prove the following identities, where the angles involved are acute angles for which the expressions are defined.

$$(iv)\frac{1+\sec A}{\sec A}=\frac{\sin^2 A}{1-\cos A}$$

[**Hint** : Simplify LHS and RHS separately]

Answer:

We need to prove-

$$\frac{1+\sec A}{\sec A}=\frac{\sin^2 A}{1-\cos A}$$

taking LHS;

$$\Rightarrow \frac{1+\sec A}{\sec A}$$
$$\Rightarrow (1+\frac{1}{\cos A})/\sec A$$
$$\Rightarrow 1+\cos A$$

Taking RHS;

We know that identity $1-\cos^2\theta=\sin^2\theta$

LHS = RHS

Hence proved.

Q5 Prove the following identities, where the angles involved are acute angles for which the expressions are defined. $(v)\frac{\cos A-\sin A+1}{\cos A+\sin A-1}=\csc A+\cot A$, using the identity $\csc^2 A=1+\cot^2 A$

Answer:

We need to prove -

$$\frac{\cos A-\sin A+1}{\cos A+\sin A-1}=cosecA+\cot A$$

Dividing the numerator and denominator by $\sin A$, we get;

Hence Proved.

Q5 Prove the following identities, where the angles involved are acute angles for which the expressions are defined.

$$(vi)\sqrt{\frac{1+\sin A}{1-\sin A}} = \sec A + \tan A$$

Answer:

We need to prove -

$$\sqrt{\frac{1+\sin A}{1-\sin A}} = \sec A + \tan A$$

Taking LHS;

By rationalising the denominator, we get;

Hence proved.

Q5 Prove the following identities, where the angles involved are acute angles for which the expressions are defined. $(vii)\dfrac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta} = \tan\theta$

Answer:

We need to prove -

$$\frac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta} = \tan\theta$$

Taking LHS;

[we know the identity $\cos 2\theta = 2\cos^2\theta - 1 = \cos^2\theta - \sin^2\theta$]

Hence proved.

Q5 Prove the following identities, where the angles involved are acute angles for which the expressions are defined.

$(viii)(\sin A+\csc A)^2+(\cos A+\sec A)^2=7+\tan^2 A+\cot^2 A$

Answer:

Given equation,
$(\sin A+\csc A)^2+(\cos A+\sec A)^2=7+\tan^2 A+\cot^2 A$ ....(i)

Taking LHS;

$(\sin A+\csc A)^2+(\cos A+\sec A)^2$
$\Rightarrow \sin^2 A+\csc^2 A+2+\cos^2 A+\sec^2 A+2$

$\Rightarrow 1+2+2+(1+\cot^2 A)+(1+\tan^2 A)$
[since $\sin^2\theta+\cos^2\theta=1, \csc^2\theta-\cot^2\theta=1, \sec^2\theta-\tan^2\theta=1$]

$7+\csc^2 A+\tan^2 A$
$=RHS$

Hence proved

Q5 Prove the following identities, where the angles involved are acute angles for which the expressions are defined.

$(ix)\ (cosecA-\sin A)(\sec A-\cos A)=\dfrac{1}{\tan A+\cot A}$

[**Hint** : Simplify LHS and RHS separately]

Answer:

We need to prove-

$$(coescA - \sin A)(\sec A - \cos A) = \frac{1}{\tan A + \cot A}$$

Taking LHS;

Taking RHS;

LHS = RHS

Hence proved.

Q5 Prove the following identities, where the angles involved are acute angles for which the expressions are defined.

$$(x)(\frac{1+\tan^2 A}{1+\cot^2 A}) = (\frac{1-\tan A}{1-\cot A})^2 = \tan^2 A$$

Answer:

We need to prove,

$$(\frac{1+\tan^2 A}{1+\cot^2 A}) = (\frac{1-\tan A}{1-\cot A})^2 = \tan^2 A$$

Taking LHS;

$$\Rightarrow \frac{1+\tan^2 A}{1+\cot^2 A} = \frac{\sec^2 A}{\csc^2 A} = \tan^2 A$$

Taking RHS;

LHS = RHS

Hence proved.