

Proofs

→ Proofs exists beyond mathematics

(concept): method to ascertaining the truth

"Verification of a proposition by chain of logical deductions from a set of axioms."

↳ Math concept

Proposition: statement that is either true or false

Ex: $2+3=5$ (true)

$\forall n \in \mathbb{N}, \text{ such that } n^2+n+41 \in \mathbb{P}$ (false)

"for all n in Natural numbers, such that
 n^2+n+41 is a Prime number"

↪ Predicate: proposition whose truth depends on the value of a variable

In this case, not truth for $n \geq 40$.

$\nexists (a, b, c, d) \in \mathbb{N}^4$ such that $a^4 + b^4 + c^4 = d^4$ (false)

" $a^4 + b^4 + c^4 = d^4$ has no positive solution"

→ Conjectured by Euler in 1769 and disproved by Elkies in 1988.

'The regions in any map can be colored in four colors so that adjacent regions have different colors' (?)

↳ Four-color theorem

↳ Is "Proved" in 1977 by computer processing of thousands of cases. Not math enough.

$\forall n \in \mathbb{Z}, n \geq 2, n \neq 0 \Rightarrow \exists p, q \in \mathbb{P}, p+q=n$ (?)???

"For every even integer $n \geq 2$, there exist prime numbers p and q where $p+q=n$ "

→ Goldbach's Conjecture, unsolved since 1742

$\forall n \in \mathbb{Z}, n \geq 2 \Rightarrow n^2 \geq 4$

⇒ Implication $p \Rightarrow q$ is true if $P = \text{false}$

or q is true.

false implying
anything is true

P	Q	$P \Rightarrow Q$
T	T	T
T	F	F
F	T	T
F	F	T

"If pigs could fly, I would be King" (true!)

pigs flying \rightarrow I'm King, is true since
pigs flying is false. *so go outside*

Axiom: A proposition that is "assumed" to be true.

Ex: If $a=b$ and $b=c$, Then $a=c$

In Euclidian Geometry: Given a line L and a point P not on L , there is exactly one line through P parallel to L

In Spherical Geometry: Given a line L and a point P not on L , there is NO line through P parallel to L .

In Hyperbolic Geometry: Given a line L and a point P not on L , there is an infinite number of lines P parallel to L .

There are 2 main properties of axioms. They should be:

- Consistent: no proposition can prove both true and false.
- Complete: it can be used to prove every proposition is either true or false.

"If you want consistency, there will be facts
that you will never be able to prove"

Kurt Gödel

If you work hard enough, you can do
anything, right? right?

Nope.