

Aptitude Assignment 2

1. What quantity of water should be added to the milk water mixture so that the milk water ratio changes from 2:3 to 4:11. The quantity of milk in the mixture is 40 litres?

Solution:

Quantity of milk = 40 litters

Milk-water ratio = 2:3

The initial milk-water ratio of 2:3 implies that the total mixture can be divided into 2 parts milk and 3 parts waters. Let's calculate the initial quantities of milk and water.

$$\begin{aligned}\text{Milk quantity} &= (2 / (2+3)) * \text{Total quantity} \\ &= (2 / 5) * 40 \text{ litters} \\ &= 16 \text{ litters}\end{aligned}$$

$$\begin{aligned}\text{Water quantity} &= (3 / (2+3)) * \text{Total quantity} \\ &= (3 / 5) * 40 \text{ litters} \\ &= 24 \text{ litters}\end{aligned}$$

Now, we want to change the milk-water ratio to 4:11. This new ratio implies that the total mixture can be divided into 4 parts milk and 11 parts waters.

Let's assume we need to add 'x' litters of water to achieve the desired ratio.

$$\begin{aligned}\text{New milk quantity} &= 4 * (\text{Total quantity} + x) / (4+11) \\ &= 4 * (40 + x) / 15\end{aligned}$$

$$\begin{aligned}\text{New water quantity} &= 11 * (\text{Total quantity} + x) / (4+11) \\ &= 11 * (40 + x) / 15\end{aligned}$$

According to the problem, the new milk quantity should be equal to the initial milk quantity, i.e.,

$$\begin{aligned}\text{New milk quantity} &= \text{Initial milk quantity} \\ 4 * (40 + x) / 15 &= 16\end{aligned}$$

Solving this equation for 'x', we can find the amount of water to be added.

$$\begin{aligned}4 * (40 + x) &= 16 * 15 \\ 160 + 4x &= 240 \\ 4x &= 240 - 160 \\ 4x &= 80 \\ x &= 80 / 4 \\ x &= 20\end{aligned}$$

Therefore, 20 liters of water should be added to the milk-water mixture to change the milk-water ratio from 2:3 to 4:11.

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2. Linear equation $2x+3y=0$ meets the x & y-axis at the point

Solution:

To find the points where the equation $2x + 3y = 0$ intersects the x-axis and y-axis, we can set one of the variables to 0 and solve for the other variable.

When the equation intersects the x-axis, $y = 0$. Let's substitute $y = 0$ into the equation:

$$2x + 3(0) = 0$$

$$2x = 0$$

$$x = 0$$

So, the point where the equation intersects the x-axis is $(0, 0)$.

When the equation intersects the y-axis, $x = 0$. Let's substitute $x = 0$ into the equation:

$$2(0) + 3y = 0$$

$$3y = 0$$

$$y = 0$$

So, the point where the equation intersects the y-axis is also $(0, 0)$.

Therefore, the equation $2x + 3y = 0$ intersects both the x-axis and y-axis at the point $(0, 0)$.

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3. a & b are positive integers such that $a^2-b^2=19$. Find a & b?

Solution:

Now, let's consider the factors of 19: 1 and 19.

Since a and b are positive integers, we can set up the following equations:

$$a + b = 19 \text{ ---- (Equation 1)}$$

$$a - b = 1 \text{ ---- (Equation 2)}$$

Solving Equation 1 and Equation 2 simultaneously will give us the values of a and b.

Adding Equation 1 and Equation 2:

$$(a + b) + (a - b) = 19 + 1$$

$$2a = 20$$

$$a = 20 / 2$$

$$a = 10$$

Substituting the value of an into Equation 2:

$$10 - b = 1$$

$$b = 10 - 1$$

$$b = 9$$

Therefore, the values of a and b that satisfy the equation $a^2 - b^2 = 19$ are:

$$a = 10$$

$$b = 9$$

4. Find $a^3 + b^3 + c^3 + 3abc$, where $a + b + c = 5$ & , $a^2 + b^2 + c^2 = 10$?

Solutions:

To find the value of $a^3 + b^3 + c^3 + 3abc$, we can use the identity:

$$(a + b + c)(a^2 + b^2 + c^2 - ab - ac - bc) = a^3 + b^3 + c^3 - 3abc$$

Given:

$$a + b + c = 5$$

$$a^2 + b^2 + c^2 = 10$$

Let's substitute these values into the identity:

$$(5)(10 - ab - ac - bc) = a^3 + b^3 + c^3 - 3abc$$

We can rewrite the equation as:

$$50 - 5(ab + ac + bc) = a^3 + b^3 + c^3 - 3abc$$

Now, we need to find the value of $ab + ac + bc$.

To find $ab + ac + bc$, we can square the equation $a + b + c = 5$ and the equation $a^2 + b^2 + c^2 = 10$, and then subtract the second equation from the first:

$$(a + b + c)^2 - (a^2 + b^2 + c^2) = (5)^2 - 10$$

$$a^2 + b^2 + c^2 + 2(ab + ac + bc) - a^2 - b^2 - c^2 = 25 - 10$$

$$2(ab + ac + bc) = 15$$

$$ab + ac + bc = 15/2$$

Substituting this value back into the equation, we get:

$$50 - 5(15/2) = a^3 + b^3 + c^3 - 3abc$$

$$50 - 75/2 = a^3 + b^3 + c^3 - 3abc$$

$$(100 - 75)/2 = a^3 + b^3 + c^3 - 3abc$$

$$25/2 = a^3 + b^3 + c^3 - 3abc$$

Therefore, the value of $a^3 + b^3 + c^3 + 3abc$ is $25/2$.

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5. Sum of two, two-digit numbers is a perfect square. The digits of the first two-digit number are two consecutive positive integers; also, when the digits of the first number are reversed, the second number is formed. Find these numbers & the square root of their sum

Solution:

Let's solve this problem step by step.

Let's assume the first two-digit number is represented as " $10a + b$," where 'a' and 'b' are the two consecutive positive integers.

When the digits of the first number are reversed, the second number is formed, which means the second two-digit number is " $10b + a$."

Given that the sum of these two numbers is a perfect square, we can write the equation:

$$(10a + b) + (10b + a) = k^2$$

Simplifying the equation, we have:

$$11a + 11b = k^2$$

$$11(a + b) = k^2$$

Since 11 is a prime number, the sum $(a + b)$ must also be a multiple of 11. This means 'a' and 'b' are either both multiples of 11 or both not multiples of 11.

Let's consider both cases:

Case 1: 'a' and 'b' are multiples of 11:

The only consecutive positive integers that are multiples of 11 are 11 and 22. Therefore, 'a' must be 11, and 'b' must be 22.

So the first two-digit number is $110 + 22 = 132$, and the second two-digit number is $220 + 11 = 231$.

The sum of these numbers is $132 + 231 = 363$, which is a perfect square. The square root of 363 is approximately 19.07.

Case 2: 'a' and 'b' are not multiples of 11:

Let's consider the consecutive positive integers 1 and 2. In this case, 'a' is 1, and 'b' is 2.

So the first two-digit number is $10 + 2 = 12$, and the second two-digit number is $20 + 1 = 21$.

The sum of these numbers is $12 + 21 = 33$, which is a perfect square. The square root of 33 is approximately 5.74.

Therefore, we have two possibilities:

1. The first two-digit numbers are 132 and 231, and the square root of their sum is approximately 19.07.
2. The first two-digit numbers are 12 and 21, and the square root of their sum is approximately 5.74.