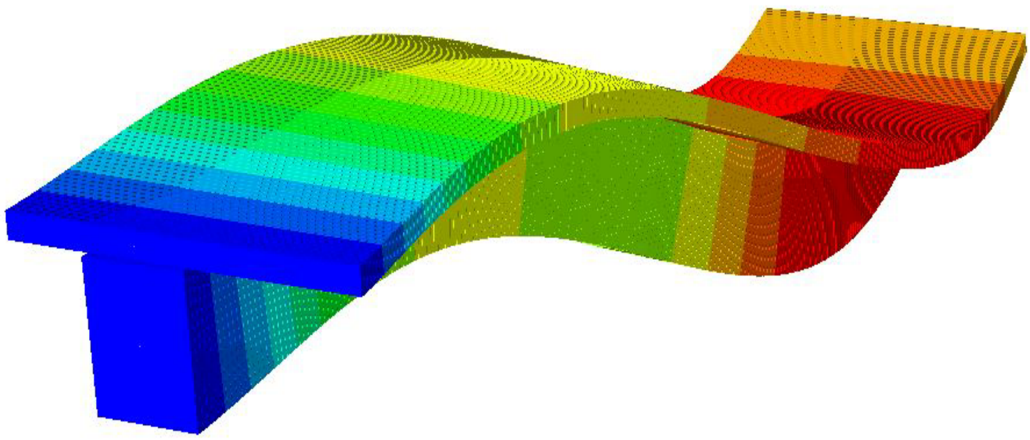
DEGREE PROJECT IN CIVIL ENGINEERING AND URBAN MANAGEMENT,
SECOND CYCLE, 30 CREDITS
STOCKHOLM, SWEDEN 2016

Comparison of ballasted and ballastless bridges for high speed trains

DAVID MATOS SÁNCHEZ

MAŠA NIKOLIĆ



KTH ROYAL INSTITUTE OF TECHNOLOGY
SCHOOL OF ARCHITECTURE AND THE BUILT ENVIRONMENT

Preface

The master thesis presented in this document is the final requisite to finish the Master's program Civil and Architectural Engineering, 120 ECTS, from the Royal Institute of Technology, KTH. The thesis has been carried out at SWECO Civil in Stockholm, Sweden with the supervision from KTH.

We are truly grateful to our supervisors, Dr. *Andreas Andersson* from KTH and Dr. *Ignacio González Silva* from SWECO Civil. Their dedication and guidance has helped us to reach the goal of presenting this thesis and get the possibility to learn more about dynamics in bridge design. This support has been crucial to find us writing this preface as the final step in this period of our education. Furthermore, our appreciation goes to *Joakim Woll*, who has always shown disposition to act as a guide whenever it was needed.

We would like to thank *Thomas Brutar* and *Alexander Ahne* for the opportunity to write the thesis at SWECO Civil and to Professor *Raid Karoumi*, Head of the Division of Structural engineering at KTH, for helping us to choose a topic as interesting as this one.

A special mention goes to *Andreas Fridholm*, SWECO Civil, for the time he invested in commenting and discussing results. In addition, we would like to thank everyone in the bridge group at SWECO Civil, for the warm welcoming and the help provided.

David's personal appreciation goes to his parents and brother who have encouraged him, gave him the possibility to study the Master's degree abroad and stood by his side in the best and worst. Without them, this would not have been possible. Finally, he would like to appreciate the patience shown by his relatives and friends when the times were a bit rougher.

Masa's personal appreciation goes to her parents for always being there when it was needed and to her sister for that unconditional support that only a sister can give.

Stockholm, May 2016

Maša Nikolić

David Matos Sánchez

Abstract

The purpose of the project is to investigate the difference in performance between ballasted and ballastless railway bridges dedicated to high speed trains by taking into account both static and dynamic requirements. The main questions are:

a) whether choosing a ballastless bridge results in a more slender section due to the lower self-weight
b) if the design of bridges for high speed trains is governed by the static or by the dynamic requirements.

The method followed was to first make a complete static design of a ballasted and a ballastless bridge, and then subject them to a 2D dynamic analyses in order to see if the cross section dimensions must be changed. Some of the bridges required a more thorough dynamic analyses, and for these, a 3D model was developed.

The analysed bridge is a simply supported beam with a T section carrying one track. Some variations were also considered, namely a simply supported bridge with a double T section carrying two tracks, as well as a single track bridge in two spans.

It was found that all of the analysed bridges are somewhat more slender for the ballastless alternative, and require a 10 -30% less reinforcement. Simply supported bridges carrying one track are governed by the dynamic requirements; the bridges in two spans are for shorter spans governed by the statics and for longer spans by the dynamics. Bridges in double T fulfilled all the requirements according to the 2D analyses, but were found to be greatly affected by the 3 dimensional effects and failed to satisfy the criteria when these were taken into account.

Finally, the optimal design according to these analyses is a ballastless bridge in a simple T section. If the bridge constructed should carry two tracks, then it should be constructed as two T beams that are not connected to one another in order to avoid the unfavourable 3D effects.

Keywords: High-speed train, concrete bridge, beam bridge, dynamic, 3D dynamic, ballastless

Sammanfattning

Syftet med detta projekt är att undersöka skillnaden mellan ballasterade och ballastfria järnvägsbroar för höghastighetståg genom att beakta både de statiska och de dynamiska kraven. Examensarbetet har gjorts med utgångspunkt i följande frågor:

a) kommer en ballastfri bro att resultera i ett slankare tvärsnitt på grund av den minskade egenvikten
b) huruvida det är de statiska eller de dynamiska kraven som är dimensionerande för broar avsedda för höghastighetståg.

Tillvägagångssättet var att först göra en fullständig statisk dimensionering av en ballasterad och en ballastfri bro, och därefter utföra en dynamisk analys i 2D för att kontrollera om tvärsnittsdimensionerna behöver ändras. En del av broarna krävde en noggrannare dynamisk kontroll och för dessa skapades en 3D modell.

Bron som analyserades är en fritt upplagd balkbro med T tvärsnitt som bär ett spår. Vissa variationer beaktades, nämligen fritt upplagd balk med dubbel T tvärsnitt och dubbelspår, samt enkelspårig bro i två fack.

Resultaten för alla broar som analyserades visade att ballastfria broar kan konstrueras någorlunda slankare, samt att de kräver mycket mindre armering, mellan 10-30%. Balkbroar i ett fack dimensioneras av de dynamiska kraven medan broarna i två fack dimensioneras av statiken för kortare spann och dynamiken för längre spann. Dubbelspårsbroarna uppfyllde alla krav i 2D analysen men påverkades i hög grad av tredimensionella effekter och de dynamiska kraven kunde inte uppfyllas när 3D effekterna beaktades.

Den optimala designen enligt dessa studier vore en ballastfri balkbro med enkel T tvärsnitt. Om bron planeras för dubbelspårig trafik bör T balkarna utformas på så sätt att de inte samverkar sinsemellan för att undvika ogynnsamma 3D effekter.

Nyckelord: Höghastighetståg, betongbro, balkbro, dynamik, 3D dynamik, ballastfritt

Nomenclature

Latin upper case letters

A_c	Concrete area
A_s	Reinforcement area
D	Dynamic train load factor (Static design)
DAF	Dynamic amplification factor
D_c	Fatigue damage in the concrete
D_s	Fatigue damage in the steel
E	Modulus of elasticity
E_{cm}	Concrete Young's modulus
E_d	Combination of actions
E_s	Steel Young's modulus
$G_{k,i}$	Permanent action (Static design)
I	Moment of inertia
L	Theoretical span length
M_{crd}	Bending moment resistance
M_{Ed}	Design bending moment
$M_{Ed,derail}$	Design derailment moment
M_{Rfw}	Derailment moment resistance
N_{Ed}	Design axial force
P	Point force
$Q_{k,i}$	Variable action (Static design)
SLS	Serviceability limit state
T_b	Time span of the bridge
T_{crd}	Torsional resistance
T_{Ed}	Torsional resistance
ULS	Ultimate limit state

V_{crd}	Shear force resistance
V_{Ed}	Design shear force

Latin lower case letters

$b_{ballast}$	Width of the ballast
b_{eff}	Effective width of the upper flange
b_f	Width of the upper flange
c_{min}	Minimum cover
$c_{min,b}$	Minimum cover due to bonding
$c_{min,dur}$	Minimum cover due to durability
d_{max}	Maximum vertical displacement
f_{ck}	Characteristic compressive strength of concrete
f_{ctk}	Characteristic tensile strength of concrete
$f_{ctk,0,05}$	Characteristic axial tensile strength of concrete 0.05 proof-stress
f_{ctm}	Mean axial tensile strength of concrete
f_{yd}	Steel design yield strength
f_{yk}	Steel characteristic yield strength
h	Total height of the beam
h_b	Height of the ballast
h_w	Height of the web
k_1	is the coefficient affecting the fatigue strength (Concrete)
k_3	is the coefficient affecting the fatigue strength (Steel)
t_f	Thickness of the upper flange
t_w	Thickness of the web

Greek letters

α	Ratio between the steel Young's modulus and the effective concrete Young's Modulus
α_{cc}	Coefficient taking account of long term effects on the compressive strength and of unfavourable effects, resulting from the way the load is applied
α_{ct}	Coefficient taking account of long term effects on the tensile strength and of unfavourable effects, resulting from the way the load is applied.
α_{cw}	Coefficient taking account for stress in compression chord, resulting from the way the load is applied
Υ	Reinforced concrete density
γ_C	Safety factor for loading
$\gamma_{G,j}$	Partial factor for permanent action j
$\gamma_{Q,j}$	Partial factor for variable action j
δ_{max}	Maximum vertical deformation
δ_{lim}	Limit for the vertical deformation
ε_{cu2}	Ultimate compressive strain of concrete
σ_c	Concrete stress
$\psi_{0,i}$	Combination factor for variable action i
ω_{flange}	Flange crack width
ω_{max}	Maximum crack width
ω_{web}	Web crack width

Table of Contents

1 Introduction

1.1 Background

The master thesis topic is in close relationship with the ongoing plan of the *Ostlänken* (The East Link) in Sweden. The *Ostlänken* is a high speed railway line that will connect the cities of Järna and Linköping. It will consist of 160 km double track railway allowing speeds up to 320 km/h. The travel time between Stockholm and Linköping will be about 40 minutes shorter with an intermediate stop at Norrköping. Regional trains will stop also in Vagnhärad and Skavsta Airport (Trafikverket[2], 2016).

Figure 1.1 Ostlänken project plan (Trafikverket[1], 2016)

According to preliminary plans, the tracks will be designed with ballastless tracks, instead of the conventional ballasted tracks. This will impact on the mass of the bridges along the line, since ballastless bridges are lighter than conventional ones. Furthermore, the railway is dedicated to passenger and not freight trains, meaning the maximum allowable axle load is decreased from 33 ton/axle to 25 ton/axle. These two factors, that is to say, removing the weight of the ballast and decreasing the axle load, can result in more slender and cost-effective bridges.

In a static design it is likely that the cross-section can be reduced due to smaller loads. However, with increasing speed, the dynamic effects start to have a larger influence on the behaviour of the bridge. The focus of the thesis will therefore be to investigate if a slenderer bridge design can be used without exceeding the dynamic design criteria.

1.2 Aim and scope

The aim of the thesis is to investigate whether bridges dedicated to high speed trains (further denoted HST) can be constructed more slender by removing the weight of the ballast or if this would result in a mass too low to fulfil the dynamic requirements. A static design of a railway bridge will be done for a ballasted as well as a ballastless bridge and then the static design for the both alternatives will be checked against dynamic effects. Finally, a parametric study will be performed in order to check some variations of the original bridge and see the impact on the design cross section.

1.3 Main questions

This thesis will try to answer the following questions:

i. Is it possible to design a ballastless bridge to be more slender than a conventional (ballasted) one when taking into account the dynamic effects of high speed trains?
ii. Is the dimensioning of bridges for high speed trains governed by the static or dynamic requirements?
iii. In what cases is it necessary to carry out a thorough dynamic analyses?

1.4 Method

The method followed will be:

1) *Static design* of a simply supported concrete bridge (span 20-30 m, T section carrying one track), for two cases:
    a. conventional line with 33 ton/axle and ballasted tracks,
    b. high-speed line with 25 ton/axle and no ballast (excluding the dynamic requirements)

   The static design is done in Matlab® for both bridge alternatives.
2) *Dynamic design,* in other words, check that the bridge dimensioned in part 1 fulfils the dynamic requirements.
    a. *2D dynamics:* This step is done by design diagrams developed by A. Andersson and C. Svedholm (Andersson & Svedholm, 2016). These design diagrams are developed for a beam in 2D, which does not cover such effects as torsion and shear lag.
    b. *3D dynamics:* A model of the bridge is developed in Brigade Plus®. The bridge can now be analysed taking into account the 3 dimensional effects as well.
3) *Parametric study* where the following alternatives will be checked:
    a. Composite cross section where the two T sections are connected and carry two tracks
    b. Continuous bridge in two spans

1.5 Limitations

The thesis will consider a simply supported concrete bridge with a constant cross section. The cross section is a double T section, each T carrying one track. Since the T sections are not connected (the reason for this is explained later in the text), the analysed section can be said to work as a simple T section carrying one train track. The bridge is assumed to behave linearly and the concrete is assumed not to have any cracks. No second order effects are considered and the analyses is thus only a linear one. The design of foundations of the bridge is not included, they are instead taken into account by applying correct boundary conditions. The design of the reinforcement is limited to calculating the minimum amount of reinforcement required and checking that it fits into the chosen section.

Some actions due to railway operations are omitted, namely

- Vertical loading for earthworks
- Centrifugal forces
- Aerodynamic effects from passing trains
- Nosing forces (consider forces generated in the wheel-rail contact)

Other loads that are neglected are snow, wind load, water pressure on the bearings and maintenance vehicle.

1.6 Structure of the thesis

The first chapter gives a background to the thesis and the project East link. The purpose and scope of the project are presented and the limitations defined.

The second chapter presents general requirements for railway bridges according to Eurocode and TRVK Bro, both regarding static and dynamic analyses. The relevant limit states are also presented here.

Chapter three deals with the static dimensioning of two bridges, the conventional and the ballastless alternatives. A description of the calculation model is given and the results from the two designs are presented.

In the fourth chapter the design of both bridges continues with the dynamic check. This chapter also starts with an explanation of the calculation model, and further on, results of different checks are presented together with the needed cross-section modifications.

In the fifth chapter, some properties about the bridge are changed and the effect of this is evaluated. First alteration is going from a bridge section carrying one track to a composite section that carries two tracks. The second modification is going from a simply supported beam to a continuous one in this case a continuous bridge in two spans.

Finally, in chapter six, the results from all the analyses are discussed and conclusions are presented. Chapter six ends with a design proposal based on the results and conclusions of this thesis, and suggestions for further research.

1.7 High-Speed railway

The definition given by the UIC (*Union Internationale des Chemins de Fer*) for high speed railway traffic is a train line which allows traffic over 250 km/h in specially designed tracks or 200 km/h in existing lines. This high speed allows these trains to compete with flying traffic in medium distances (UIC [1], 2016). The following Figure 1.2 shows the railway network in Europe.

Figure 1.2 High-speed network in Europe (UIC [2], 2016)

As it can be seen, the countries with most high speed lines are Spain, Italy, France and Germany. Sweden has some upgraded lines in service, and the *Ostlänken* in development.

The increasing demand for these kind of trains is at peak levels right now, as this kind of transportation is competitive, comfortable and safe and allows quick travels within every country in Europe.

Some facts that UIC remarks are:

- The world's first high speed train service was implemented from Tokyo to Osaka on the 1st October 1964.
- There are 29 792 km of high speed lines in the world (1st April 2015)
- There are 3 603 high speed train sets in operation (April 2015)
- The world speed record is 574.8 km/h achieved in France in 2007
- 1 600 million passengers per year are carried by high speed trains in the world
- 80% modal split obtained by high speed trains in relation to air transport when travel time by train is less than 2,5 hours

UIC presents also the advantages of high speed lines

- High capacity
- Environmental respect
- High safety
- Commercial speed
- Total time of travel
- Frequency
- Reliability
- Accessibility
- Price
- Comfort
- "*Freedom*" which they define as the possibility of going to the restaurant, promenade, electronic devices not limited, among others
- Helps contain urban sprawl
- Helps economic development

The prediction for development, which can be extracted from UIC, is that the high speed railway network is going to grow exponentially in the coming years.

The increment and estimated increasing demand for High Speed lines creates a need to study the effects that those trains have on the tracks and structures. When train velocities surpass 200 km/h, the dynamic effects start to play a role in structure behaviour and must be taken into account in the structural design.

Sweden has a huge territory in comparison with the number of inhabitants which makes communications and travel an important issue, and creates a need for traveling long distances in a short time. The existent railway lines are not developed to carry the high speed trains and therefore the need for new railway lines is crucial for the further development and connection of different regions in Sweden.

Sweden has already some upgraded lines that connect the most important cities in the country. However, the *Ostlänken* is the first high-speed line which will be designed specifically for high speed passenger trains. This means it will require some research in order to obtain a more clear understanding of structure behaviour when subjected to trains of high speeds.

2 Requirements according to Eurocode and TRVK Bro

2.1 Static analyses

2.1.1 Loads

Out of actions due to railway operations given in the Eurocode (EN-1991-2, 2003), following loads will be used

- Vertical loads
    - Self-weight of the concrete
    - Self-weight of the ballast (for the conventional bridge)
    - Load model 71 (normal rail traffic on mainline bridges)
    - Load model SW/2 (heavy traffic, i.e. freight trains on the conventional, ballasted bridge)
    - Load model "unloaded train"
    - Derailment actions from rail traffic
- Horizontal forces
    - Actions due to traction and braking

2.1.2 Limit states

Following limit states will be checked

- Ultimate limit state
    - Bending resistance
    - Shear resistance
    - Torsion
    - Fatigue
- Serviceability limit state
    - Stresses
    - Crack control
    - Deflection at mid span
    - Displacement at supports due to change of angle

2.2 Dynamic analyses

2.2.1 Dynamic effects

The deformations of the bridge (for instance deflection) can change once the dynamic effects are taken into account. By accounting for the dynamics one also encounters other problems, such as vertical acceleration of the deck. These phenomenon are due to the following (EN-1991-2, 2003):

- Rapid rate of loading due to the speed of traffic
- The passage of successive loads with uniform spacing leading to resonance
- Variations in wheel loads resulting from track of vehicle imperfections

The factors that are determinant to the dynamic effects are the speed of the vehicles, the span length, the mass of the structure, the natural frequency of the whole structure, the number of axles, the damping of the structure and imperfections which will be explained further on in chapter 4.1.

2.2.2 Dynamic requirements

The following limit states will be checked:

- Vertical displacement
- Vertical acceleration
- Deck twist
- Change of angles at bearings

3 Static analyses

The first design is based on a simplified model, dealing only with static loads and considering any dynamic effects through a dynamic amplification factor. The objective is to find optimal cross sections in the static design. That section will then be analysed taking into account the dynamic effects more carefully.

3.1 Calculation model and method

The cross section of the bridge is a double T-section, which is the design chosen by Sweco for bridges ranging between 20-30 m, see figure 3.1. The two T-sections are, however, not connected for practical reasons (both construction and maintenance are thereby simplified), meaning that each beam acts like a single (and asymmetrical) T-section carrying one track, as shown in Figure 3.1 The lengths 20 and 30 m are considered as “extreme cases” of the span length for which this cross section is used, and because of this, the results for all analyses will be presented for L = 20 m and L = 30 m, even though the code developed can be used to perform cross-section checks for any intermediate length by changing the input values (the code itself can of course be used to analyse any arbitrary length of the bridge, but the designed section is only valid for spans between 20 and 30 m).

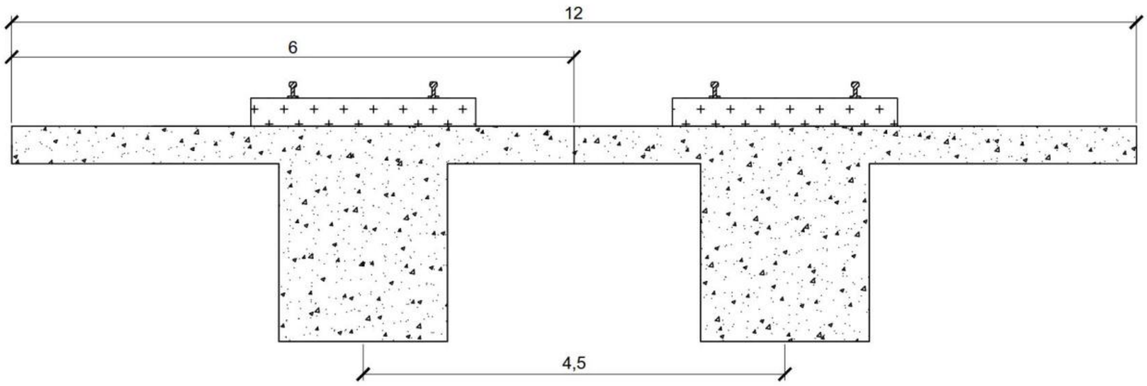


Figure 3.1 Double T-cross-section

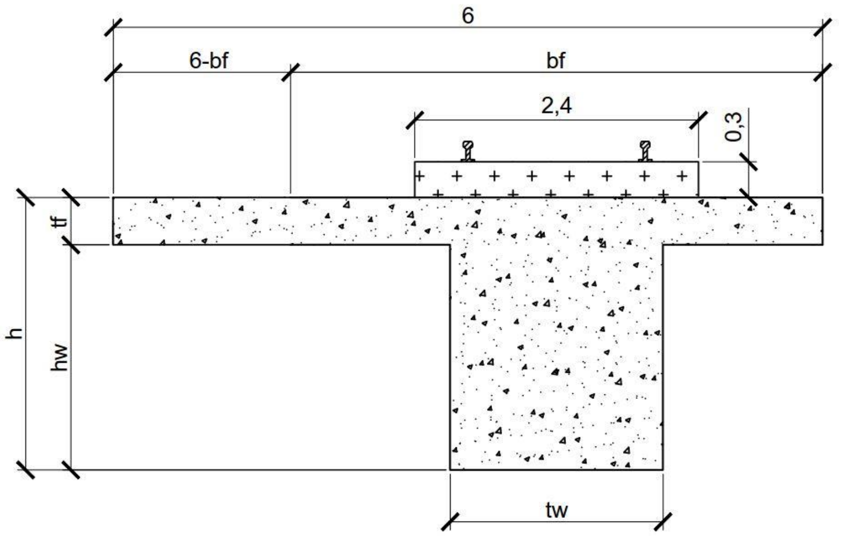


Figure 3.2 Design section

A model of the section in figure 3.2 is analysed in Matlab® in order to check the static design. The bridge is simply supported, with no curvature or initial displacements, and it is analysed as an Euler-Bernoulli 2D beam.

The weight density of the reinforced concrete is assumed to be 25 kN/m^3 and the weight density of the ballast is taken as 20 kN/m^3.

The loads taken as input are the ones shown in the Chapter 2.1.1 and the limit states checked are the ones shown in the chapter 2.1.2.

3.2 Material properties

Both bridges are reinforced concrete bridges. In the Table 3.1 the properties of the concrete are shown according to (EN-1992-1-1, 2004).

Table 3.1 Concrete properties

Concrete properties		
Concrete class		C35
Characteristic compressive strength of concrete	f_{ck} [MPa]	35
Secant modulus of elasticity of concrete	E_{cm} [GPa]	34
Mean value of axial tensile strength of concrete	f_{ctm} [MPa]	3.2
Characteristic axial tensile strength of concrete 0.05 proof-stress	$f_{ctk,0.05}$ [MPa]	2.2
Reinforced concrete density	Υ [kN/m^3]	25
Ultimate compressive strain of concrete	ε_{cu2} [‰]	3.5

The steel properties used for reinforcement are shown in the Table 3.2.

Table 3.2 Steel properties

Steel properties		
Steel class		C35
Characteristic yield strength of steel	f_{yk} [MPa]	500
Elastic modulus of steel	E_s [GPa]	200
Design yield strength of steel	f_{yd} [MPa]	435

The ballast density is taken as 20 kN/m3.

3.3 Loads and load combinations for the static design

The loads and load combinations were presented in chapter 2.1.1. However, in Table 3.3, they are shown in a more detailed way.

Table 3.3 Loads and load combinations

Permanent loads	*Loads considered*	Self-weight
		Ballast
Variable Loads	*Loads considered*	Load Model 71
		Load Model SW/2
		Unloaded train
		Traction/braking forces
		Derailment actions
	Not considered loads	Wind load
		Snow load
		Nosing forces
		Water pressure on bearings
		Aerodynamic effects from passing trains
		Centrifugal forces (Straight bridge)

Permanent loads

The permanent loads are self-weight and ballast. They are separated because the total self-weight will vary depending on whether there is ballast on the bridge or not. Both loads are distributed and calculated as the product of density and area.

Traffic loads

There are three traffic loads for the static design of the bridge. They are LM71, load model SW/2 and the unloaded train as is specified in Eurocode 1 (EN-1991-2, 2003). The load models are analysed by finding sectional forces (moment and shear) they cause and combine these effects with the self-weight of the bridge. The results are then compared with the corresponding resistances of the section.

The first load model to be presented is the LM71 and is defined in the Figure 3.3. Load model 71 represents the static effect of normal rail traffic.

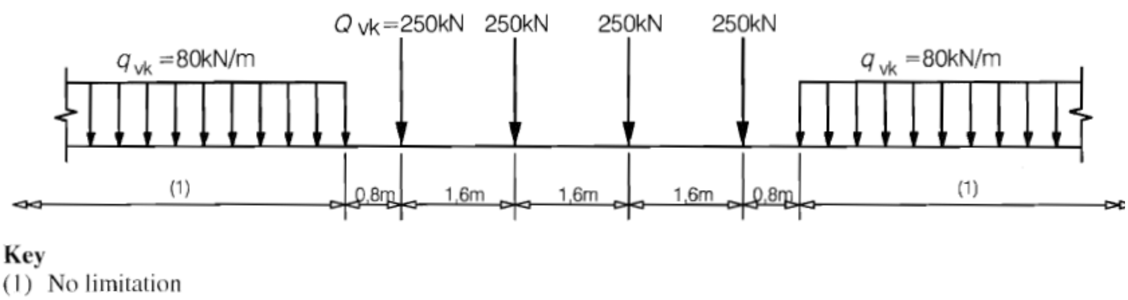


Figure 3.3 Load model LM-71 (EN-1991-2, 2003)

The Eurocode (EN-1991-2, 2003) states:

"(3) P The characteristic values given in Figure 3.3 shall be multiplied by a factor "α", on lines carrying rail traffic which is heavier or lighter than normal rail traffic. When multiplied by the factor "α" the loads are called "classified vertical loads". This factor a shall be one of the following:

0,75 - 0,83 - 0,91 - 1,00 - 1,10 - 1,21 - 1,33 - 1.46"

Next load model is the SW/2 that can be seen in Figure 3.4.

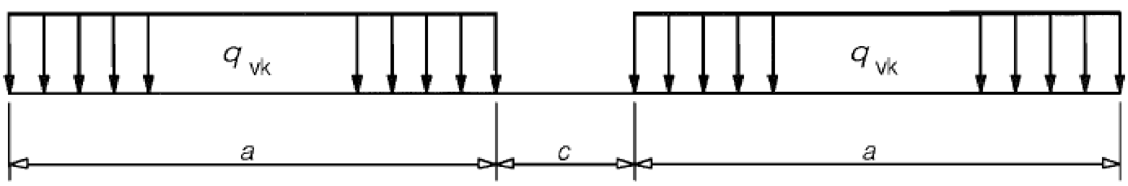


Figure 3.4 Load model SW/2 (EN-1991-2, 2003)

Table 3.4 shows the characteristic values for the vertical loads for the model SW/2 (EN-1991-2, 2003).

Table 3.4 Characteristic values for the load model SW/2 (EN-1991-2, 2003)

Load Model	q_{vk} [kN/m]	a [m]	c [m]
SW/0	133	15,0	5,3
SW/2	150	25,0	7,0

Finally, the unloaded train consists of a vertical uniformly distributed load with a characteristic value of 10.0 kN/m.

Braking/acceleration forces

These actions are acting at the top of the rails in the longitudinal direction of the track. They are uniformly distributed of the corresponding influence length and it is shown in in the following figure, Figure 3.5, as is established in (EN-1991-2, 2003).

Traction force: $Q_{lak} = 33$ [kN/m] $L_{a,b}$ [m] ≤ 1000 [kN]
for Load Models 71,
SW/0, SW/2 and HSLM

Braking force: $Q_{lbk} = 20$ [kN/m] $L_{a,b}$ [m] ≤ 6000 [kN]
for Load Models 71,
SW/0 and Load Model HSLM

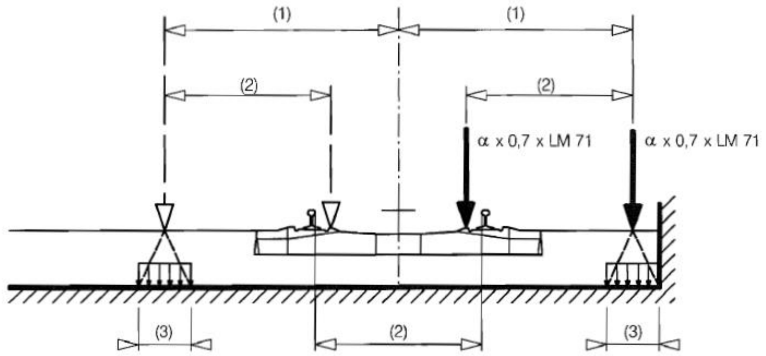
$Q_{lbk} = 35$ [kN/m] $L_{a,b}$ [m]
for Load Model SW/2

Figure 3.5 Traction and braking forces (EN-1991-2, 2003)

Derailment actions

As shown in (EN-1991-2, 2003) 6.7.1 the derailment actions are considered to be accidental design situations. For this case, two different situations are shown:

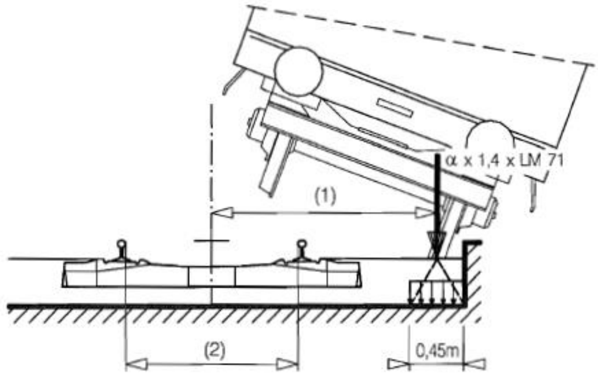
<u>Design situation I</u>: Where the train is still on the bridge, shown in Figure 3.6:



Key
(1) max. 1,5*s* or less if against wall
(2) Track gauge *s*
(3) For ballasted decks the point forces may be assumed to be distributed on a square of side 450mm at the top of the deck.

Figure 3.6 Derailment actions. Design situation 1 (EN-1991-2, 2003)

<u>Design Situation II</u>: where the load is applied at the worst point possible, which is the edge of the bridge as it is shown in Figure 3.7.



Key
(1) Load acting on edge of structure
(2) Track gauge *s*

Figure 3.7 Derailment actions. Design situation 2 (EN-1991-2, 2003)

Load combinations

The load combination in Sweden in the ULS is done according the (EN-1990, 2002)

$$E_d = E\left[\sum_j \gamma_{G,j} G_{k,j} + \sum_i \gamma_{Q,i} \psi_{0,i} Q_{k,i}\right] \qquad (3.1)$$

Or

$$E_d = E\left[\sum_j \xi_j \gamma_{G,j} G_{k,j} + \gamma_{Q,1} Q_{k,1} + \sum_{i>1} \gamma_{Q,i} \psi_{0,i} Q_{k,i}\right] \tag{3.2}$$

From these two equations (3.1)and (3.2) the most unfavourable alternative is used, where ξ = 0.89 according to the Eurocode Swedish amendments (SIS, 2002).

The combinations have been made for the Ultimate Limit State (Stage III) and the Serviceability Limit State (Stage II). The effects of loads are combined in order to achieve the most unfavourable case. A simply supported bridge will have the maximum shear forces at the supports and the maximum bending moments at mid-span. In tables Table 3.5 and Table 3.6 the different load combination factors are shown.

Table 3.5 Load combination factors for ULS.

Load combination factors for ULS			
Load		min	max
Permanent Loads	Self-weight	0.95	1.05
	Ballast	0.8	1.3
Variable loads	Load Model 71	0.7	1.45
	Load Model SW/2	0.7	1.45
	Unloaded train	1	1.45
	Traction/braking forces	0.8	1.45
	Derailment action	0	0.8

Table 3.6 Load combination factors for SLS.

Load combination factors for SLS			
Load		min	max
Permanent Loads	Self-weight	0.95	1.05
	Ballast	0.8	1.3
Variable loads	Load Model 71	0.7	1.45
	Load Model SW/2	0.7	1.45
	Unloaded train	1	1.45
	Traction/braking forces	0.8	1.45
	Derailment action	0.8	1.45

All these calculations are made by a Matlab® code which are presented in Appendix A:

3.4 Structural capacity

3.4.1 Ultimate Limit State (ULS)

In order to fulfil the limit states defined in 2.1.2, the following criteria must be satisfied in ULS, presented in equations (3.3) to (3.8):

$$\frac{M_{Ed}}{M_{Rd}} \leq 1 \tag{3.3}$$

$$\frac{V_{Ed}}{V_{Rd}} \leq 1 \tag{3.4}$$

$$\frac{T_{Ed}}{T_{Rd}} + \frac{V_{Ed}}{V_{Rd}} \leq 1 \tag{3.5}$$

$$\frac{M_{EdDerail}}{M_{Rfw}} \leq 1 \tag{3.6}$$

$$\frac{V_{EdDerail}}{V_{Rfw}} \leq 1 \tag{3.7}$$

$$Cycles\ before\ failure \geq 10^6 \tag{3.8}$$

Since the project deals with a concrete bridge, it is required to check the amount of reinforcement used. This work is limited to obtaining the necessary reinforcement areas for the cross section and making sure the number of bars required can fit into the section without limiting the effective height. No further reinforcement design is done.

Moment resistance

The moment resistance will be checked at the position for the maximum moment, which for a simply supported beam is the mid span, taking into account that there is an acting axial force, and it is calculated using the expression (3.9):

$$M_{Rd} = A_s \sigma_s (d - 0{,}4x) \tag{3.9}$$

Where:
A_s Area of the tensile reinforcement
σ_s Stress in the reinforcement
d Distance between the top of the section and the reinforcement
x Lever arm (distance from the top of the beam to the neutral layer)

The tensile reinforcement is obtained following the procedure outlined in sections 9.2.1 and 7.3 in (EN-1992-2, 2005). The longitudinal reinforcement is taken as the maximum between equations (3.10) to (3.13).

Minimum longitudinal reinforcement (3.10):

$$A_{s,min} = 0.26\frac{f_{ctm}}{f_{yk}}b_t d \tag{3.10}$$

Lower limit of minimum reinforcement (3.11):

$$0.0013 b_t d \tag{3.11}$$

Reinforcement needed to balance the bending moment in the section (3.12)

$$A_s = \frac{M_{Ed}}{f_{yd} * z} \tag{3.12}$$

Where:
d Distance between the top of the section and the reinforcement
z Lever arm = 0.8d

Reinforcement required to resist torsion is presented in (3.13)

$$A_{s,tor} = \frac{T_{Ed}\frac{1}{tan\theta}\cdot u_k}{2A_c * f_{yd}} \tag{3.13}$$

A more detailed explanation of the terms used in the equations can be found in the corresponding Eurocode.

Moment resistance for derailment actions

Moment capacity in the section between the flanges and the web (denoted M_{Rfw}) must be checked for the case of derailment, when the load from the train acts on the flanges instead of straight over the web. In this case, the flange is assumed to work as a cantilever, and the moment resistance (equation (3.14)) comes from the tensile capacity of the concrete and the reinforcement in the top of the beam, which is set as minimum reinforcement (see equation (3.10)).

$$M_{Rfw} = (f_{ctm}t_f f_{yd}A_{s,min}) * (t_f - 2c) \tag{3.14}$$

Shear resistance

It is also necessary check the shear resistance of the section, which is done as prescribed in section 6.2.2 of (EN-1992-2, 2005) for members not requiring shear reinforcement.

$$V_{cRd} = \min(V_{cRd,1}, V_{cRd,2}, V_{cRd,3}) \tag{3.15}$$

Where

$$V_{cRd,1} = \left(C_{Rd}k\left(\frac{100\rho_1}{f_{ck}}\right)^{\frac{1}{3}} + k_1\sigma_{cp}\right)b_w d \tag{3.16}$$

$$V_{cRd,2} = (\upsilon_{min} + k_1\sigma_{cp})b_w d \tag{3.17}$$

$$V_{cRd,3} = 0{,}5\upsilon b_w d f_{cd} \tag{3.18}$$

The Matlab® code checks weather the section given the loads requires additional shear reinforcement, in which case it calculates it according to section 6.2.3 (EN-1992-2, 2005). In case that the shear resistance isn't sufficient, a calculation is made for the shear reinforcement A_{sw} and the total amount of reinforcement A_s is the sum of the longitudinal and shear reinforcement.

Since the section is a T section, it is necessary to check the shear force between the web and the flanges of the section, following equation (3.19) as outlined in section 6.2.4 (EN-1992-2, 2005).

$$V_{Rfw} = kwb_w f_{ctd} \tag{3.19}$$

In the case of derailment, the flange must have enough shear resistance to support the train. This resistance is taken as the resistance between the flanges and the web, as it was estimated that that was the place where the fracture would occur. The resistance is calculated according to (3.19).

Torsional resistance

Torsional resistance is calculated according to equation 6.30 in the (EN-1992-1-1, 2004) and the section is checked for a combination of torsion and shear (Sundquist, 2008). The calculations for the torsional resistance are shown in (3.20):

$$T_{Rd} = 2\nu\alpha_{cw}f_{cd}A_k t_{ef,i}\sin\theta\cos\theta \tag{3.20}$$

Where:
ν Strength reduction factor for concrete cracked (expression 6.6N (EN-1992-1-1, 2004))
A_k Area enclosed by centre lines of connection walls
$t_{ef,i}$ Effective wall thickness
Θ Angle between shear reinforcement and the beam axis

Fatigue

The fatigue resistance is considered sufficient if it allows for the given number of train passages per day during the technical lifetime of the bridge. The number of load cycles before failure is obtained following the procedure in section 6.8.7 of (EN-1992-2, 2005), which is the lambda method, a simplified version and quite conservative.

3.4.2 Serviceability Limit State (SLS)

The limit states defined in 2.1.2 for the SLS must satisfy the following criteria presented in equations (3.21) to (3.25):

$$\frac{\delta_{max}}{\delta_{lim}} \leq 1 \tag{3.21}$$

$$\frac{\omega_{web}}{\omega_{max}} \leq 1 \tag{3.22}$$

$$\frac{\omega_{flange}}{\omega_{max}} \leq 1 \tag{3.23}$$

$$\frac{\sigma_s}{k_3 f_{yk}} \leq 1 \tag{3.24}$$

$$\frac{\sigma_c}{k_1 f_{ck}} \leq 1 \tag{3.25}$$

Vertical displacement

The vertical displacement is one of the most important aspects when checking the SLS. The displacement is calculated with all the quasi-permanent actions on the bridge. The maximum limit displacement,δ_{lim} is calculated as a the L/600, (EN-1992-2, 2005).

Crack control

The crack control will be carried out according to the Eurocode (EN-1992-2, 2005) taking as the limiting width for the cracks 0.30 mm and expecting cracks after 28 days. It will be carried out for both the flange and the web.
Reinforcement to control cracks (3.26):

$$A_{s,crack} = \frac{k_c \cdot k \cdot f_{ct,eff} \cdot A_{ct}}{\sigma_s} \tag{3.26}$$

Stress limitations

The stresses need to be controlled in both steel and concrete according to (EN-1992-2, 2005).

3.5 Validation of the model

To carry out the validation of the Matlab® code, the results are compared to some examples from (JSC, 2012) that deal with concrete, as well as a thesis dealing with fatigue assessment (Olsson & Pettersson, 2010).

The Matlab® code is defined for a T-Cross-section but it can be modified to work with a rectangular cross-section by changing the geometry input. This step is needed to compare the model with the examples in (JSC, 2012). Also the design moments have been taken as input. The comparison between the example and the results of the code are shown in Table 3.7.

Table 3.7 Validation of the Matlab® code with the Bridge Worked Examples compendium

Values to be compared	Worked examples (JSC, 2012)	Matlab® Code
σ_s	$448\ MPa$	$448.5\ MPa$
ε_s	$20.6\frac{mm}{m}$	$20.7\frac{mm}{m}$
M_{Rd}	$0.281\ MNm$	$0.2812\ MNm$
k	1.75	1.745
ρ_1	0.51%	0.513%
σ_{cp}	$0\ MPa$	$0\ MPa$
v_{min}	$0.48\ MPa$	$0.477\ MPa$
$V_{Rd,c}$	$198\ kN$	$197.5\ kN$
$V_{Rd,s}$	$240\ kN$	$239.55\ kN$
$V_{Rd,max}$	$2020\ kN$	$2018\ kN$
n_{s_SLS}	5.9	$5.88\ MPa$
σ_{s_SLS}	$344\ MPa$	$343.6\ MPa$
σ_{c_SLS}	$15.6\ MPa$	$15.612\ MPa$

In this example, only the bending moment capacity, the shear resistance and the cracking can be compared, concluding that these aspects in the model are validated. However, the load combinations, the loads and the deflection requirements are not computed there, which lead to further validation with other examples.

The next verification deals with the fatigue and it has been made comparing the Matlab® code results with a master thesis about fatigue in reinforced concrete according to Eurocode (Olsson & Pettersson, 2010). The results of the comparison are shown in Table 3.8.

Table 3.8 Validation of the Matlab® code with the Master's Thesis about fatigue in RC

Values to be compared	Master Thesis (Olsson & Pettersson, 2010)	Matlab® Code
λ_s	0.892	0.8919
D_s	0.273	0.272
λ_c	0.893	0.87
$E_{cd,min,eq}$	0.164	0.16
$E_{cd,min,eq}$	0.433	0.424
D_c	0.77	0.761

As it can be seen, the results are quite similar which means that the model can be validated for fatigue.

3.6 Results of the static analyses

The results for the static analyses are presented for both bridge alternatives in order to make the comparison easier to read. The ballasted alternative of the bridge will have the following cross section shown in Figure 3.8 with the measures shown in the Table 3.9.

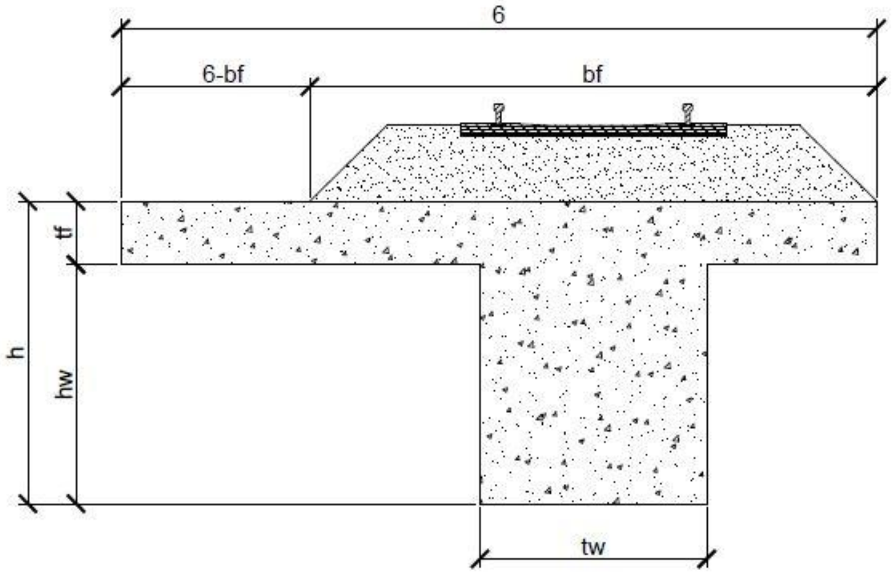


Figure 3.8 T-cross section, ballasted

The ballastless alternative of the bridge will have the following cross section shown in Figure 3.9 with the measures shown in the Table 3.9.

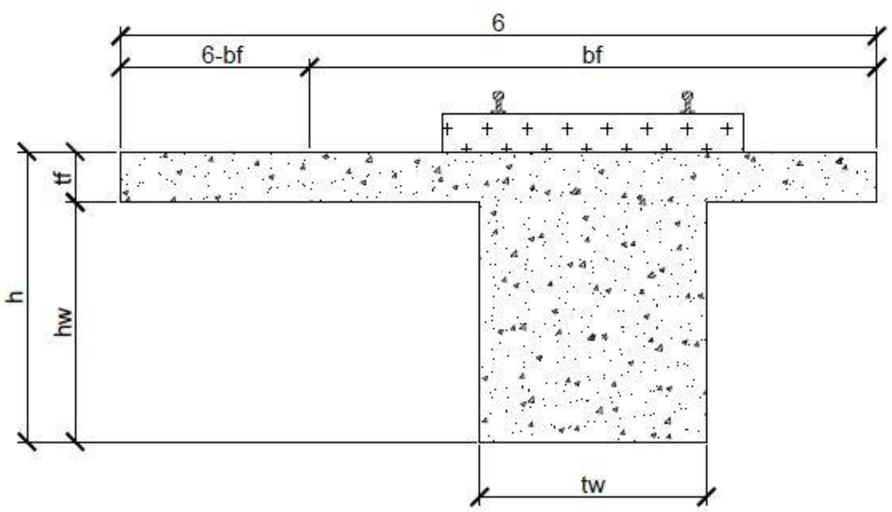


Figure 3.9 T-cross section, ballastless

Table 3.9 Cross-section dimensions for both alternatives

	BALLASTED		BALLASTLESS	
Dimensions	**L = 20**	**L = 30**	**L = 20**	**L = 30**
Flange width, b_f $[m]$	4.5	4.5	4.5	4.5
Flange thickness, t_f $[m]$	0.4	0.4	0.4	0.4
Web height, h_w $[m]$	2.0	2.2	2.0	2.0
Web thickness, t_w $[m]$	1.9	1.8	1.4	1.4
Total height, h $[m]$	2.4	2.6	2.4	2.6
Reinforcement area, A_s	37ϕ32	73ϕ32	27ϕ32	57ϕ32
Shear reinforcement	No	No	No	No

The results obtained from the optimal cross section are shown in Table 3.10 and Table 3.11. The procedure for calculating the different resistances has been described in section 3.4. Note that every section has the minimum shear reinforcement required – Table 3.10 states weather more shear reinforcement than the minimum is required.

Table 3.10 Usage ratios in the ULS

Ultimate Limit State	**BALLASTED**		**BALLASTLESS**	
	L = 20	**L = 30**	**L = 20**	**L = 30**
Bending moment resistance, M_{cRd} [kNm]	31 000	61 000	23 500	44 600
Design bending moment, M_{Ed} [kNm]	23 400	49 200	17 000	35 400
Usage ratio M_{Ed}/M_{cRd}	**0.75**	**0.81**	**0.73**	**0.80**
Usage ratio $M_{EdDerail}/M_{Rfw}$	**1.002**	**1.001**	**0.99**	**0.99**
Shear force resistance, V_{cRd} [kN]	3 500	4 500	3 100	4 000
Design shear force, V_{Ed} [kN]	2 100	3 600	1 500	2 500
Usage ratio V_{Ed}/V_{cRd}	**0.61**	**0.8**	**0.47**	**0.63**
Torsional resistance, T_{cRd} [kNm]	230 600	237 100	189 400	189 400
Design Torsion, T_{Ed} [kNm]	770	770	580	580
Usage ratio T_{Ed}/T_{cRd}	**0.003**	**0.003**	**0.003**	**0.003**
Number of cycles before fatigue failure	10^6	10^6	10^6	10^6
Damage in the concrete D_c	**0.62**	**0.80**	**0.62**	**0.85**

Table 3.11 Usage ratios in the SLS

Serviceability Limit State	BALLASTED		BALLASTLESS	
	L = 20	L = 30	L = 20	L = 30
Long term-Tensile stresses	**0.09**	**0.09**	**0.08**	**0.09**
Short term-Compressive stresses	**0.17**	**0.26**	**0.14**	**0.23**
Maximum crack width [mm]	0.30	0.30	0.30	0.30
Crack width in webs. [mm]	0.1	0.1	0.1	0.1
Crack width in flanges. [mm]	0.1	0.1	0.1	0.1
Ratio of usage in webs	**0.33**	**0.34**	**0.33**	**0.33**
Ratio of usage in flanges	**0.33**	**0.34**	**0.33**	**0.33**
Maximum allowed deflection [mm]	33	50	33	50
Maximum deflection in SLS, [mm]	**3**	**11**	**3**	**12**

The fatigue has been calculated for a 100 cycles/day. This means that there are 200 trains passing daily (100 bridges in each direction i.e. 100 trains per beam) taking as reference the most loaded train lines in Stockholm.

As can be seen in the table, the cross section cannot be significantly reduced by taking away the ballast. The big difference between the conventional and the ballastless option is the amount of reinforcement, and this is where the savings are made.

The dimensioning load case is M_{derail}, when the total load of the train is placed at the end point of the flange and the resistance comes from tensile capacity of the concrete and the minimum reinforcement in the upper part of the beam. The corresponding somewhat low ratios of usage for the moment and shear capacity are due to the following:

- The width of the beam (t_w) cannot be reduced because this would make the load from the train at derailment act even further away from the beam, thereby increasing M_{derail}
- The height of the beam (h_w) cannot be reduced because this limits the effective height (the distance between the top of the beam and the reinforcement) and the capacity of moment and shear are exceeded, since there is not enough room to put the required reinforcement

It is possible to reduce the web of the section somewhat more by making the slab a bit thicker and in this way increase the resistance to derailment action. However, the web cannot be reduced by much, since there must be place for the reinforcement bars. Therefore, the presented section was found to be the optimal one.

4 Dynamic analyses

The first design model to be tested against the dynamic requirements is based on the static design and corrected with the 2D dynamic analyses provided by (Andersson & Svedholm, 2016) that will be explained in this chapter. These checks have limitations because they are based in 2D which leaves out effects from the torsional modes. These will be taken into account later when preforming a 3D dynamic analyses.

Eurocode states that during the static analysis with the LM71 or SW/2 (SW/0 when applicable) the loads must be multiplied by a dynamic factor, φ, and then these results must be compared to the ones obtained from the dynamic analysis, if such is required (EN-1991-2, 2003). In this case the comparison between the results from a dynamics analyses and a static analyses multiplied with a dynamic factor has been omitted, since it is assumed it will not give additional information.

To obtain the torsional modes it is necessary to perform a 3D analysis that is carried out in Brigade Plus® using beam elements.

When a dynamic analysis is required, the Brigade Plus® model is used and the different HSLM trains, (EN-1991-2, 2003), applied on the bridge. The different acceleration and stressed are obtained from the analysis.

4.1 Dynamic characteristics

The most important dynamic characteristics are the natural frequency of the structure and the dynamic amplification factor.

4.1.1 Resonance

The phenomena of resonance is one of the key aspects to consider when dealing with dynamics. Resonance occurs when the structure is excited by an external dynamic force having a frequency close to the natural frequency of the bridge. This phenomenon is crucial for railway bridges where train velocities are above 200 km/m, but considered unlikely to occur under this speed limit. The larges dynamic effects occur at resonance speed (EN-1991-2, 2003) , (Karoumi, 2015), (Chopra, 2001).

4.1.2 Natural frequency

The natural frequencies are a characteristic inherent to the structure and determine the behaviour of the structure and the sensitivity to dynamic loads. They are dependant of the moment of inertia and the mass as the equation to calculate the natural frequencies. Equation (4.1) shows the calculation for the natural frequencies for a simply supported beam:

$$f_n = \frac{n^2 \cdot \pi^2}{2\pi \cdot L^2}\sqrt{\frac{EI}{m}}\ [Hz] \tag{4.1}$$

Where:

n Mode number
L Span length [m]
E Young's modulus [N/m]
I Moment of inertia [m4]
m Mass of the structure [kg/m]

The external loads affect the structure considerably when the external frequencies are close to the natural frequencies of the structure (Chopra, 2001).

With the first natural frequency it is possible to calculate the lowest resonance speed due to external loading as (4.2) (EN-1991-2, 2003):

$$v_{crit} = f_1 d \tag{4.2}$$

Where:

f_1 First mode frequency [Hz]
d Spacing between axles [m]

4.1.3 Dynamic amplification factor

The dynamic amplification factor, DAF, is used to multiply the value of the static effects and obtain a dynamic response. It is calculated as the ratio between the response given by the statics and the response given by the dynamics. It helps to understand the bridge behaviour when subjected to a dynamic loading by doing a very simple calculation, namely finding an absolute ratio between the maximum dynamic response to the maximum static response shown in (4.3), (Chopra, 2001):

$$DAF = \frac{|R_{dyn}|}{|R_{sta}|} \tag{4.3}$$

The Eurocode proposes a dynamic factor for the static calculations (to be applied to LM71) in equation (4.4) (EN-1991-2, 2003) :

$$\phi = \frac{1.44}{\sqrt{L_\phi} - 0.2} + 0.82 \tag{4.4}$$

Where:
L_ϕ Dynamic length [m]

4.2 Factors affecting the dynamic behaviour

4.2.1 Damping

Damping is one of the most important factors that affect the dynamic behaviour of structures. However, it is difficult to estimate because there are many different sources of damping, such as material damping, friction in structural joints, bearings or interaction between different materials; as well as radiation damping from the soil-structure interaction, aerodynamic and hydrodynamic damping. These sources of damping are normally non-linear and modelling them is quite complicated (Chopra, 2001). Normally damping is modelled as proportional to the stiffness- and mass matrices in order to reduce the difficulty of the computations.

Eurocode proposes the following values of damping when designing structures, shown in Table 4.1 from (EN-1991-2, 2003).

Table 4.1 Damping values according to EN 1991-2

Bridge Type	ζ Lower limit of percentage of critical damping [%]	
	Span $L < 20\ m$	Span $L \geq 20\ m$
Steel and composite	$\zeta = 0.5 + 0{,}125\ (20 - \mathrm{L})$	$\zeta = 0.5$
Prestressed concrete	$\zeta = 1.0 + 0{,}07\ (20 - \mathrm{L})$	$\zeta = 1.0$
Filler beam and reinforced concrete	$\zeta = 1.5 + 0{,}07\ (20 - \mathrm{L})$	$\zeta = 1.5$

The value chosen for the critical damping in this thesis is 1.5 % as the span lengths are always equal to or above 20 m and the material used is reinforced concrete.

4.2.2 Mass of the bridge

The maximum dynamic effects occur at resonance peaks, that is when a multiple of the frequency of loading and a natural frequency of the structure coincide. The mass plays a big role in the natural frequency of the structure and underestimating the mass will lead to overestimating the natural frequency and consequently to overestimating the traffic speeds at which resonance occurs (EN-1991-2, 2003).

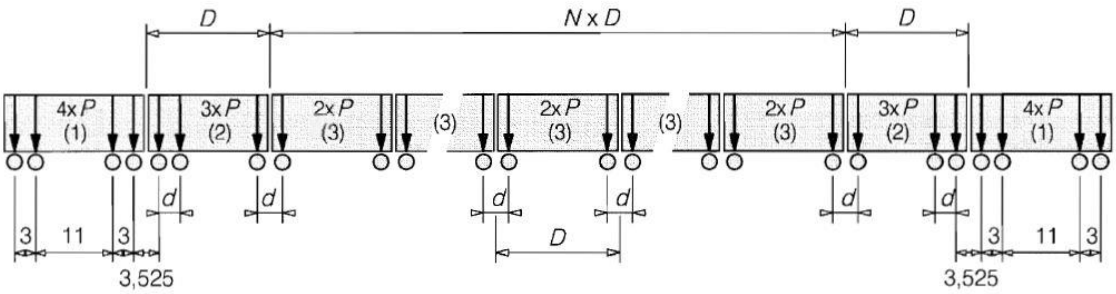
Therefore as (EN-1991-2, 2003), a lower bound for the mass is used to predict the maximum deck accelerations using the minimum likely dry clean density (1700 kg/m^3) and minimum thickness of ballast. An upper bound estimate is used to predict the lowest speeds at which resonance occurs with the maximum saturated density of dirty ballast taken from (EN-1991-1-1, 2002).

4.2.3 Stiffness of the bridge

The stiffness is another important factor influencing the behaviour of a structure. Overestimating the stiffness will overestimate the natural frequency and the speeds at which resonance occurs (EN-1991-2, 2003). This will be taken into account when conclusions need to be drawn from the results.

4.3 Loads

According to Eurocode, (EN-1991-2, 2003), the loads that need to be applied to a bridge in order to perform a dynamic analysis are HSLM-A or HSLM-B and these will be presented in figures Figure 4.1 and Figure 4.2



Key
(1) Power car (leading and trailing power cars identical)
(2) End coach (leading and trailing end coaches identical)
(3) Intermediate coach

Figure 4.1 HSLM-A load model (EN-1991-2, 2003)

Where the different kind of trains HSLM-A are presented in the Table 4.2

Table 4.2 HSLM-A (EN-1991-2, 2003)

Universal Train	Number of intermediate coaches N	Coach length D [m]	Bogie axle spacing d [m]	Point force P [kN]
A1	18	18	2,0	170
A2	17	19	3,5	200
A3	16	20	2,0	180
A4	15	21	3,0	190
A5	14	22	2,0	170
A6	13	23	2,0	180
A7	13	24	2,0	190
A8	12	25	2,5	190
A9	11	26	2,0	210
A10	11	27	2,0	210

The train HSLM-B is presented in Figure 4.3

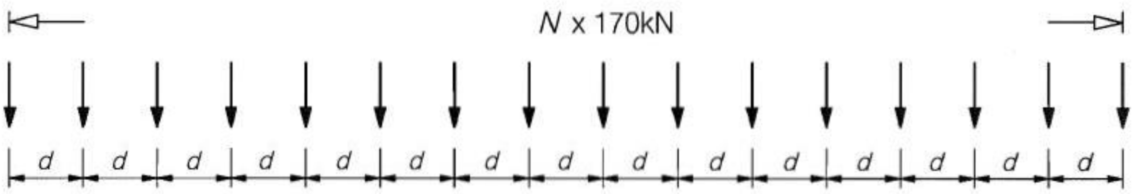


Figure 4.2 HSLM-B Load model (EN-1991-2, 2003)

With a distance, d [m], and a number of point forces N, defined in Figure 4.3.

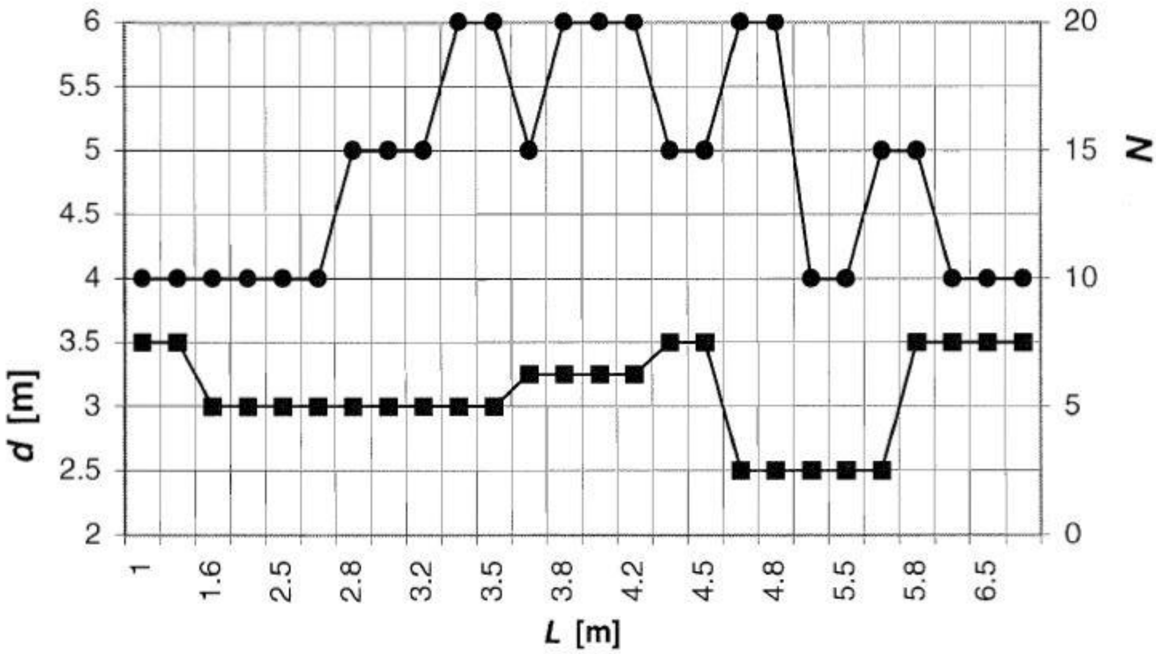


Figure 4.3 HSLM-B input (EN-1991-2, 2003)

The application of every train is shown in Table 4.3 (EN-1991-2, 2003):

Table 4.3 Application of HSLM-A and HSLM-B (EN-1991-2, 2003)

Structural configuration	Span	
	$L < 7$m	$L \geq 7$m
Simply supported span[a]	HSLM-B[b]	HSLM-A[c]
Continuous structure[a] or Complex structure[e]	HSLM-A Trains A1 to A10 inclusive[d]	HSLM-A Trains A1 to A10 inclusive[d]

[a] Valid for bridges with only longitudinal line beam or simple plate behaviour with negligible skew effects on rigid supports.
[b] For simply supported spans with a span of up to 7 m a single critical Universal Train from HSLM-B may be used for the analysis in accordance with 6.4.6.1.1(5).
[c] For simply supported spans with a span of 7 m or greater a single critical Universal Train from HSLM-A may be used for the dynamic analysis in accordance with annex E (Alternatively Universal trains A1 to A10 inclusive may be used).
[d] All Trains A1 to A10 inclusive should be used in the design.
[e] Any structure that does not comply with Note a above. For example a skew structure, bridge with significant torsional behaviour, half through structure with significant floor and main girder vibration modes etc. In addition, for complex structures with significant floor vibration modes (*e.g.* half through or through bridges with shallow floors) HSLM-B should also be applied.

NOTE The National Annex or the individual project may specify additional requirements relating to the application of HSLM-A and HSLM-B to continuous and complex structures.

In the present project, all the studied spans surpass 20 m meaning only HSLM-A trains are required for the design.

4.3.1 Speeds to be considered

According to (EN-1991-2, 2003), the maximum design speed must be multiplied by a safety factor of 1.2 and the trains that will pass will be doing it with series that will start in 40 m/s (144 km/h) to the maximum design speed multiplied by 1.2. The speed steps between them can be chosen freely but it is recommended to reduce them and closely look when the speeds studied are close to a resonance speed. In this case, the maximum speed for the tracks is stated to be 320 km/h, meaning that the maximum calculated speed is 390 km/h. However, in order to save computational time, it has been decided that the analysed speeds will be between 200 km/h and 390 km/h, after checking the resonance speed according to equation (4.2) in order to ensure there is no resonance problems below 200 km/h.

4.4 Dynamic controls

The structural capacity of the bridge is calculated by the limit states, namely maximum vertical deflection and maximum vertical acceleration in the deck, as well as the change of angle at the supports (EN-1990, 2002).

4.4.1 Criteria for traffic safety

Vertical acceleration of the deck

To ensure traffic safety, where a dynamic analysis is necessary, the verification of the maximum deck acceleration due to rail traffic actions is seen as a traffic safety requirement (EN-1990, 2002).

The maximum peak values of a bridge deck acceleration along each track must not exceed 3.5 m/s^2 for ballasted tracks and 5 m/s^2 for slab tracks or other fastened elements (EN-1990, 2002).

The frequencies to be considered for all members supporting the track is the maximum between (EN-1990, 2002):

- 30 Hz;
- 1,5 times the frequency of the first mode of vibration;
- The frequency of the third bending mode

Deck twist

The twist of the deck will be checked for HSLM trains as specified in EN-1990 (EN-1990, 2002). Twisting is checked for a train approaching the bridge, crossing it and exiting. The requirement to be fulfilled is (4.5):

$$t_{max} \leq 1{,}5\ [mm] \qquad (4.5)$$

According to table A2.7 of (EN-1990, 2002) for speeds over 200 km/h.

Vertical displacement

For the vertical deformation, the same limit is taken for the train as it was taken for the static design, (EN-1990, 2002), namely L/600.

Change of angles at bearings

The change of angle depends on the total height of the cross section and the distance from the beam end and the position of the supports (EN-1990, 2002). The maximum allowed change of angle is $2*10^{-3}/h$ radians at the bearings and $4*10^{-3}/h$ at an intermediate support, h being the total height of the cross section (this requirement is specified by TRVK Bro and not by the Eurocode) (TRVK Bro 11, 2011).

4.4.2 Comfort criteria

The passenger comfort depends on the vertical acceleration inside the coach during travel on the approach to, passage and departure from the bridge. The levels of comfort are normally associated to the vertical acceleration of the deck. (EN-1990, 2002), sets those limits in Table 4.4.

Table 4.4 Recommended levels of comfort (EN-1990, 2002)

Level of comfort	Vertical acceleration of the deck (m/ s^2)
Very good	1,0
Good	1,3
Acceptable	2,0

The deflection criteria for passenger comfort must also be checked. In the case of exceptional structures a specific calculation must be carried out, i.e. continuous beams with a wide variation in stiffness (EN-1990, 2002). However, this is not the case here.

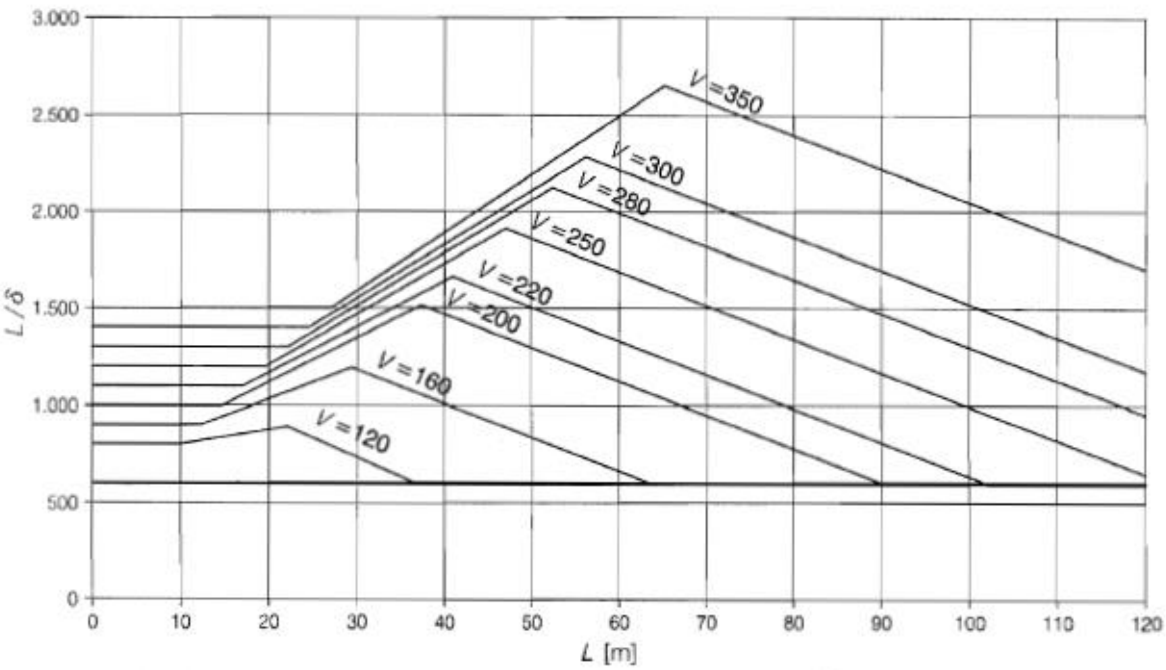


Figure 4.4 Maximum allowed deflection for railway bridges for more than three simply supported spans (EN-1990, 2002)

The comfort criteria cannot be directly checked by the acceleration because the acceleration that the passengers experience is affected by the damping between structure and the track itself, the damping of the train, and the imperfections of the track, among others. According to the Eurocode, the response of the bridge should be multiplied with a dynamic factor which is calculated according to equation (4.6) taken from (EN-1991-2, 2003)

$$t_{max} \leq 1{,}5\ [mm] \tag{4.6}$$

$$Dynamic\ factor = \ 1 + 0.5\ \varphi''$$

For a simply supported bridge with less than 3 spans, the limits are taken from Figure 4.4, in this case taken the speed, V = 350 km/h, as there is not a line for 390 km/h. The graph shows the comfort criteria limits for “*Very good conditions*”, i.e. a = 1 m/s^2, and if one is interested in finding the values for “*Acceptable conditions*”, the deflection must be divided by 2 m/s^2.

4.5 Calculation method

As it has been mentioned before, the cross section of the bridge is a double T-section. The T sections are not connected in such a way that they can be said to work together or affect one another. As in the static analyses, the results will be presented for lengths 20 and 30 m. For the 2D analyses the developed code can easily be used for checking any intermediate length, but the 3D analyses done in Brigade® is only done for these specific lengths. One can however preform the same analyses for any length of the bridge following the procedure outlined in this report.

The bridge section found by the static design is first subjected to a 2D dynamic analyses using diagrams provided by (Andersson & Svedholm, 2016). If the requirements (vertical acceleration and displacement, and the change of angle at supports) are fulfilled, the section is accepted. If not, the dimensions are changed until the mass and stiffness are sufficient to fulfil the requirements while trying to maintain a high usage ratio in the statics.

It is also explained in the Eurocodes (EN-1991-2, 2003), in which cases it is compulsory to perform a dynamic analysis and this is shown in Figure 4.5.

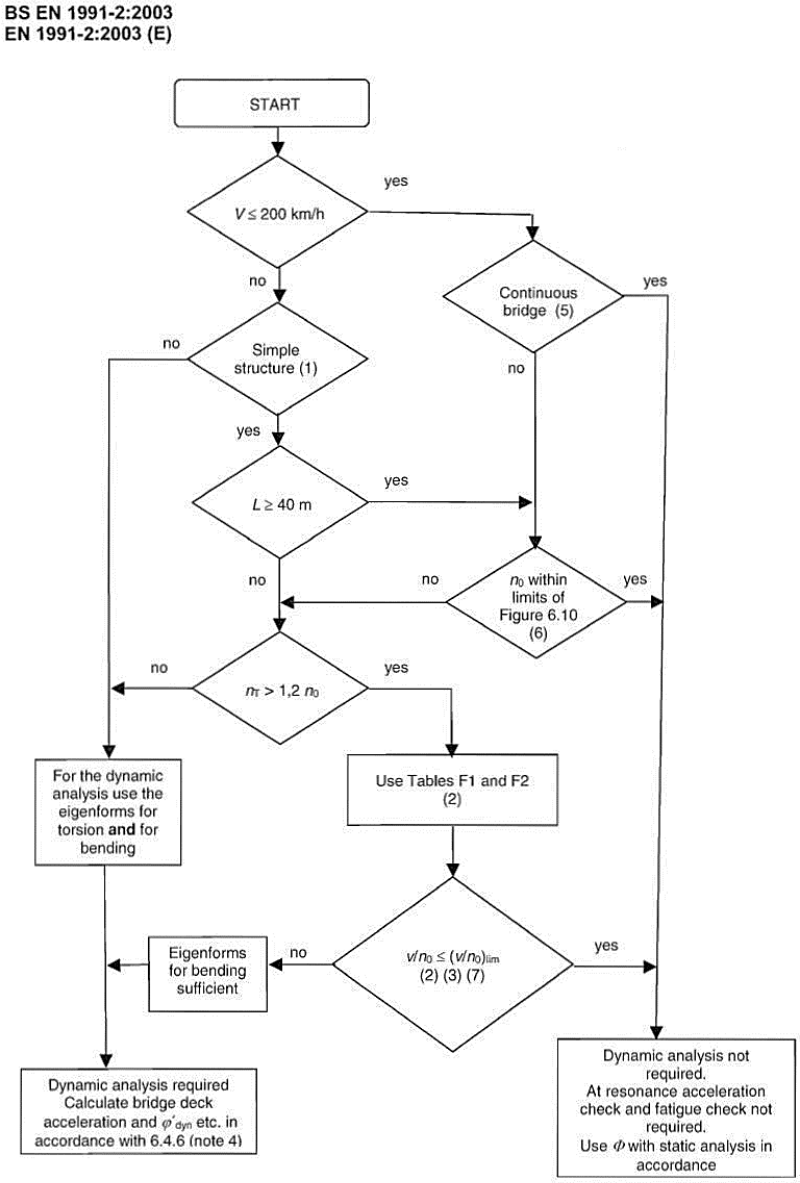


Figure 4.5 Flow chart for determining whether a dynamic analysis is required, (EN-1991-2, 2003)

4.6 Dynamics in 2D

The first step in the dynamic analyses is to check if the bridge has the necessary mass and stiffness to fulfil the dynamic requirements specified in the chapter 2.2.2. This is done by comparing the mass and stiffness of the bridge with the diagram A1 of (Andersson & Svedholm, 2016). Given the mass and the stiffness of the bridge, one can find directly if the bridge fulfils the dynamic requirement. If not, the dimensions are changed until the requirements are met while the degree of utilization in the static calculation is maintained as high as possible.

As mentioned, the dynamic requirements are the vertical acceleration, deflection and the change of angle at the supports. However, for the the analyses in this project, it has been foun that whenever the vertical acceleration requirement is fulfilled, so is the deflection. Therefore, the deflection will not be shown here. Andersson and Svedholm provide diagrams for testing weather the mass of the bridge is sufficient in order to limit vertical acceleration. The mass required for this is denoted m_{erf} and the requirement is here considered fulfilled if (4.7)

$$\frac{m}{m_{erf}} \geq 1 \qquad (4.7)$$

Regarding the angle at the supports, in some cases this requirement is not fulfilled, but this has been accepted, since this requirement is governed by the total height of the section at the support. It is considered possible and probably more effective to construct a beam with a smaller height at the supports then to make the whole section more massive.

4.6.1 Dynamics in 2D - Results

The cross section presented in section 3.6, when tested according to (Andersson & Svedholm, 2016) does not fulfil the requirements for any of the limit states. New sections are found that both satisfy the dynamic requirements according to (Andersson & Svedholm, 2016) as well as the static. In order to see how much the section can be reduced for ballastless compared to conventional bridges, both alternatives are checked and presented below.

As mentioned and explained above, it is in some cases accepted that the requirement regarding the angle change is not fulfilled. In the Table 4.5, the maximum total height, h_T, (between the centre of rotation of the beam to the top of the rail) that fulfils the angle requirement is given.

Table 4.5 Cross-section dimensions for both alternatives (2D dynamics)

	BALLASTED		**BALLASTLESS**	
Dimensions	**L = 20**	**L = 30**	**L = 20**	**L = 30**
Flange width b_f $[m]$	4.5	4.5	4.5	4.5
Flange thickness t_f $[m]$	0.4	0.4	0.4	0.4
Web height h_w $[m]$	2.5	2.5	2.2	2.4
Web thickness t_w $[m]$	1.8	1.8	1.8	1.8
Total height h_T $[m]$	3.4	3.4	3.1	3.2
Max total height allowed [m]	3.4	3.2	3.2	3.1
Reinforcement area A_s	31 φ32	66 φ32	27φ32	55φ32
Shear Reinforcement	No	No	No	No
Mass [ton/m]	21.9	21.9	16.3	17.3
Stiffness [GNm²]	172.4	172.4	126.23	156

Since the dimensions of the cross section are altered in order to fulfil the dynamic requirements, it must be verified that they still work statically. It was also considered of interest to see the usage ratio for the static limit states when the cross section is designed for the dynamic effects. These verifications and usage ratios can be seen in Table 4.6 and Table 4.7.

Table 4.6 Usage ratios in ULS, section optimized for 2D dynamics

Ultimate Limit State		**BALLASTED**		**BALLASTLESS**	
		L = 20	**L = 30**	**L = 20**	**L = 30**
Static	Bending moment resistance, M_{cRd} [kNm]	33 800	64 300	25 900	51 400
	Design bending moment, M_{Ed} [kNm]	24 300	51 000	18 500	40 000
	Usage ratio M_{Ed}/M_{cRd}	**0.72**	**0.79**	**0.72**	**0.78**
	Usage ratio $M_{EdDerail}/M_{Rfw}$	**0.93**	**0.93**	**0.75**	**0.72**
	Shear force resistance, V_{cRd} [kN]	2 300	4 700	3 400	4 300
	Design shear force, V_{Ed} [kN]	3 750	3 800	1 800	3 100
	Usage ratio V_{Ed}/V_{cRd}	**0.62**	**0.82**	**0.53**	**0.72**
	Torsional resistance, T_{cRd} [kNm]	259 400	259 400	237 150	251 800
	Design Torsion, T_{Ed} [kNm]	770	770	580	580
	Usage ratio T_{Ed}/T_{cRd}	**0.003**	**0.003**	**0.0025**	**0.0023**
	Number of cycles before fatigue failure	10^6	10^6	10^6	10^6
	Damage in the concrete D_c	**0.55**	**0.72**	**0.54**	**0.67**
Dynamic	m/m_{erf}	**1.05**	**1.21**	**1.05**	**1.3**
	Eigenfrequency	**10.9**	**4.9**	**10.2**	**4.9**

Table 4.7 Usage ratios in SLS, section optimized for 2D dynamics

Serviceability Limit State	**BALLASTED**		**BALLASTLESS**	
	L = 20	**L = 30**	**L = 20**	**L = 30**
Long term-Tensile stresses. Usage ratio	**0.09**	**0.10**	**0.09**	**0.10**
Short term-Compressive stresses. Usage ratio	**0.15**	**0.25**	**0.15**	**0.23**
Maximum crack width [mm]	0.30	0.30	0.30	0.30
Crack width in webs. [mm]	0.11	0.11	0.11	0.11
Crack width in flanges. [mm]	0.10	0.10	0.11	0.11
Ratio of usage in webs	**0.35**	**0.35**	**0.37**	**0.37**
Ratio of usage in flanges	**0.34**	**0.34**	**0.37**	**0.37**
Maximum allowed deflection [mm]	33	50	33	50
Maximum deflection in SLS, [mm]	**1.8**	**8.3**	**1.9**	**6.9**

As it can be seen from the ratio of usages the dynamic requirements govern the design of the section. The shorter bridges, L = 20 m have significantly higher Eigen frequencies.

4.7 Dynamics in 3D

In order to determine whether further dynamic analyses is required according to the flow chart in Figure 4.5, it is necessary to study the torsional modes, meaning a 3D model needs to be constructed. The model will be built and analysed in the FEM software Brigade Plus®.

Since the bridge is constructed for speeds over 200 km/h, the bridge is a simple structure (a simply supported beam) and shorter than 40 m, the dynamic analyses is needed if (4.8) or (4.9).

$$n_t =< 1.2\, n_0 \tag{4.8}$$

Or

$$v/n_0 \leq (v/n_0)_{lim} \tag{4.9}$$

Where

n_t Frequency for the first torsional mode [Hz]
n_0 Frequency for the first bending mode [Hz]
v/n_0 Ratio between the speed at site and the first bending frequency
$(v/n_0)_{lim}$ Ratio between the resonance speed and the first bending frequency

The values of $(v/n_0)_{lim}$ can be found in tables Table 4.8 and Table 4.9.

Table 4.8 Table F.1 EN 1991-2 (EN-1991-2, 2003)Max value of $(v/n_0)_{lim}$ for simply supported ballasted bridges

Mass m 10^3 kg/m		≥5,0 <7,0	≥7,0 <9,0	≥9,0 <10,0	≥10,0 <13,0	≥13,0 <15,0	≥15,0 <18,0	≥18,0 <20,0	≥20,0 <25,0	≥25,0 <30,0	≥30,0 <40,0	≥40,0 <50,0	≥50,0 -
Span $L \in$ m [a]	ζ %	v/n_0 m	v/n_0 m	v/n_0 m	v/n_0 m	v/n_0 m	v/n_0 m	v/n_0 m	v/n_0 m	v/n_0 m	v/n_0 m	v/n_0 m	v/n_0 m
[5,00,7,50)	2	1,71	1,78	1,88	1,88	1,93	1,93	2,13	2,13	3,08	3,08	3,54	3,59
	4	1,71	1,83	1,93	1,93	2,13	2,24	3,03	3,08	3,38	3,54	4,31	4,31
[7,50,10,0)	2	1,94	2,08	2,64	2,64	2,77	2,77	3,06	5,00	5,14	5,20	5,35	5,42
	4	2,15	2,64	2,77	2,98	4,93	5,00	5,14	5,21	5,35	5,62	6,39	6,53
[10,0,12,5)	1	2,40	2,50	2,50	2,50	2,71	6,15	6,25	6,36	6,36	6,45	6,45	6,57
	2	2,50	2,71	2,71	5,83	6,15	6,25	6,36	6,36	6,45	6,45	7,19	7,29
[12,5,15,0)	1	2,50	2,50	3,58	3,58	5,24	5,24	5,36	5,36	7,86	9,14	9,14	9,14
	2	3,45	5,12	5,24	5,24	5,36	5,36	7,86	8,22	9,53	9,76	10,36	10,48
[15,0,17,5)	1	3,00	5,33	5,33	5,33	6,33	6,33	6,50	6,50	6,50	7,80	7,80	7,80
	2	5,33	5,33	6,33	6,33	6,50	6,50	10,17	10,33	10,33	10,50	10,67	12,40
[17,5,20,0)	1	3,50	6,33	6,33	6,33	6,50	6,50	7,17	7,17	10,67	12,80	12,80	12,80
[20,0,25,0)	1	5,21	5,21	5,42	7,08	7,50	7,50	13,54	13,54	13,96	14,17	14,38	14,38
[25,0,30,0)	1	6,25	6,46	6,46	10,21	10,21	10,21	10,63	10,63	12,75	12,75	12,75	12,75
[30,0,40,0)	1				10,56	18,33	18,33	18,61	18,61	18,89	19,17	19,17	19,17
≥40,0	1				14,73	15,00	15,56	15,56	15,83	18,33	18,33	18,33	18,33

[a] $L \in [a,b)$ means $a \le L < b$

NOTE 1 Table F.1 includes a safety factor of 1.2 on $(v/n_0)_{\text{lim}}$ for acceleration, deflection and strength criteria and a safety factor of 1,0 on the $(v/n_0)_{\text{lim}}$ for fatigue.

NOTE 2 Table F.1 includes an allowance of $(1+\varphi''/2)$ for track irregularities.

Table 4.9 Table F.2 EN 1991-2 (EN-1991-2, 2003) Max value of $(v/n_0)_{lim}$ for simply supported ballastless bridges

Mass m 10^3 kg/m		≥5,0 <7,0	≥7,0 <9,0	≥9,0 <10,0	≥10,0 <13,0	≥13,0 <15,0	≥15,0 <18,0	≥18,0 <20,0	≥20,0 <25,0	≥25,0 <30,0	≥30,0 <40,0	≥40,0 <50,0	≥50,0 -
Span $L \in$ m [a]	ζ %	v/n_0 m	v/n_0 m	v/n_0 m	v/n_0 m	v/n_0 m	v/n_0 m	v/n_0 m	v/n_0 m	v/n_0 m	v/n_0 m	v/n_0 m	v/n_0 m
[5,00,7,50)	2	1,78	1,88	1,93	1,93	2,13	2,13	3,08	3,08	3,44	3,54	3,59	4,13
	4	1,88	1,93	2,13	2,13	3,08	3,13	3,44	3,54	3,59	4,31	4,31	4,31
[7,50,10,0)	2	2,08	2,64	2,78	2,78	3,06	5,07	5,21	5,21	5,28	5,35	6,33	6,33
	4	2,64	2,98	4,86	4,93	5,14	5,21	5,35	5,42	6,32	6,46	6,67	6,67
[10,0,12,5)	1	2,50	2,50	2,71	6,15	6,25	6,36	6,36	6,46	6,46	6,46	7,19	7,19
	2	2,71	5,83	6,15	6,15	6,36	6,46	6,46	6,46	7,19	7,19	7,75	7,75
[12,5,15,0)	1	2,50	3,58	5,24	5,24	5,36	5,36	7,86	8,33	9,14	9,14	9,14	9,14
	2	5,12	5,24	5,36	5,36	7,86	8,22	9,53	9,64	10,36	10,36	10,48	10,48
[15,0,17,5)	1	5,33	5,33	6,33	6,33	6,50	6,50	6,50	7,80	7,80	7,80	7,80	7,80
	2	5,33	6,33	6,50	6,50	10,33	10,33	10,50	10,50	10,67	10,67	12,40	12,40
[17,5,20,0)	1	6,33	6,33	6,50	6,50	7,17	10,67	10,67	12,80	12,80	12,80	12,80	12,80
[20,0,25,0)	1	5,21	7,08	7,50	7,50	13,54	13,75	13,96	14,17	14,38	14,38	14,38	14,38
[25,0,30,0)	1	6,46	10,20	10,42	10,42	10,63	10,63	12,75	12,75	12,75	12,75	12,75	12,75
[30,0,40,0)	1				18,33	18,61	18,89	18,89	19,17	19,17	19,17	19,17	19,17
≥40,0	1				15,00	15,56	15,83	18,33	18,33	18,33	18,33	18,33	18,33

[a] $L \in [a,b)$ means $a \le L < b$

NOTE 1 Table F.2 includes a safety factor of 1.2 on $(v/n_0)_{\text{lim}}$ for acceleration, deflection and strength criteria and a safety factor of 1,0 on the $(v/n_0)_{\text{lim}}$ for fatigue.

NOTE 2 Table F.2 include an allowance of $(1+\varphi''/2)$ for track irregularities.

where:

L is the span length of bridge [m],
m is the mass of bridge [10^3 kg/m],
ζ is the percentage of critical damping in [%],
v is the Maximum Nominal Speed and is generally the Maximum Line Speed at the site. A reduced speed may be used for checking individual Real Trains for their associated Maximum Permitted Vehicle Speed [m/s],
n_0 is the first natural frequency of the span [Hz].
Φ_2 and φ'' are defined in 6.4.5.2 and annex C.

4.7.1 3D Calculation model

The 3D model of the bridge is built in Brigade Plus® using beam elements B31[1] (Dassault Systèmes Simulia Corp, 2012) since they are considered to represent the behaviour of the bridge in a correct way without making the computations too heavy. Beam elements cannot take into account shear lag, but this effect is not considred dimensioning for the section analysed. The trains are applied directly to the rails, meaning no further distribution of loads is applied. In Figure 4.6 the two beam elements (corresponding to the web and the flange of the T section) are presented. The beams have rectangular cross sections and are joined by tie constraints. The same applies to the connection between the lower beam (i.e. the web of the T section) and the reference points RP1 and RP2 (representing the support of the bridge). The rendering of the profiles is shown in Figure 4.7.

Figure 4.6 Beam model of the bridge

[1] B31 is a beam element in 3D that follows the Timoshenko beam theory (Shear flexible) (Dassault Systèmes Simulia Corp, 2012)

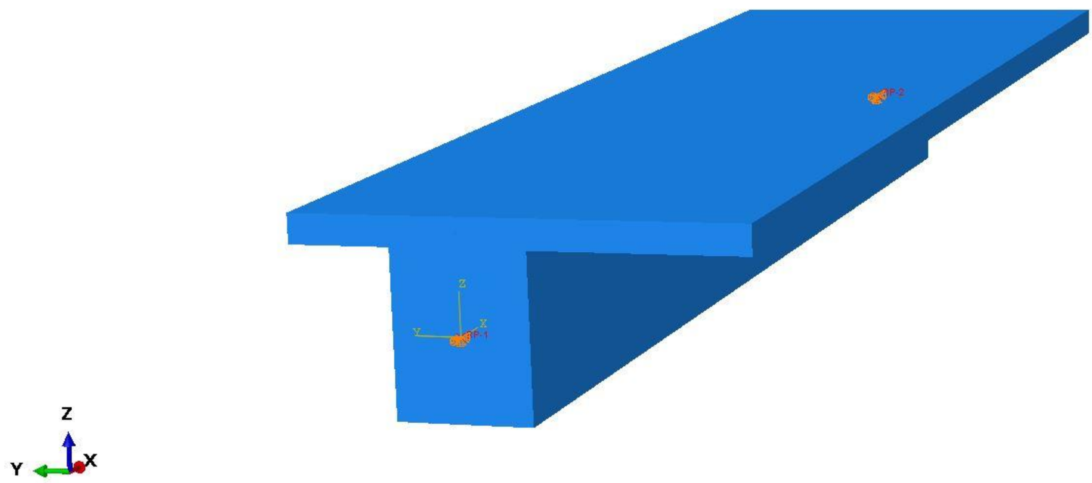


Figure 4.7 Rendered profiles of the model, L=20m

The boundary conditions, as shown in Table 4.10, are applied to RP1 and RP2. U1, U2, U3 are the degrees of freedom in 1, 2, 3 directions in general coordinates, while UR1, UR2, UR3 are the degrees of rotational freedom in 1, 2, 3 direction according to general coordinates of the system as it is shown in Figure 4.7**.**

Table 4.10 Boundary conditions of the 3D model

Point	U1	U2	U3	UR1	UR2	UR3
RP-1	Restrained	Restrained	Restrained	Restrained	Free	Free
RP-2	Free	Restrained	Restrained	Restrained	Free	Free

The selected mesh size for all elements is 0.1.

In order to know if the mesh size is appropriate, a convergence analyses must be performed to insure that the model represents the bridge in an accurate way.

If the preliminary analyses of the 3D model shows that it is necessary to perform a dynamic analyses according to (EN-1991-2, 2003), i.e. that it is not enough to account for the dynamic effects through the DAF, the dynamic analysis is preformed according to the requirements in (EN-1991-2, 2003) by allowing the HSLM-A trains 1 – 10 to pass over the bridge. Trains are predefined in Brigade Plus® software (Scanscot Technology , 2016) but the range of speeds to be analysed as well as the maximal frequency of interest are chosen by the user. In this case, the speeds analysed range between 200 km/h and 390 km/h (using an increment of 5 km/h and a time increment of 0.0013 s) and the maximum frequency is set to 120 Hz.

When an analyses of one train is complete, the maximal vertical displacement and acceleration are found, as well as the node where the maximum acceleration occurs. For this node, the acceleration is plotted against time and speed. This makes it possible to find the speed at which maximum acceleration occurs. Brigade® separates the analysed speeds into two result files in order to optimize the computational time, so both of these files must be checked.

Once the analyses of all train passages are done, a final check is done for an overall maximum acceleration and deflection, taking into account all HSLM-A trains and all speeds.

The twist of the deck is checked as specified in section 4.4.1 by finding the maximum and minimum displacement in the analysed section and thus checking the relative displacement i.e. the twisting.

4.7.2 Validation of the 3D Model

Before any conclusions can be drawn, the model must be verified which is done by comparing the obtained results against the Matlab® model for the static design and the 2D dynamic assessment provided by (Andersson & Svedholm, 2016). The model is considered verified when

- The mass of the model in Brigade® is within 3 % of the mass calculated in the static design
- The maximum deflection due to self-weight is within 5 % of the one calculated in the static design
- The first bending frequency should be similar to the one given by (Andersson & Svedholm, 2016)
- The frequencies for the first three bending modes are similar to the frequencies obtained by an analytical solution.

The reason for different levels of agreement required between the models, is that convergence is easier to reach for the mass than for the displacement. Regarding the frequencies, they should be in the same order of magnitude as the ones calculated by the Matlab® model, but the accepted difference is quite large. This because the Matlab® model uses an analytical solution to obtain the natural frequencies, where the torsional modes are omitted, in other words, the model is considered to be infinitely stiff in torsion. In reality, whenever the beam bends, there is some torsion involved, which the Brigade® model takes into account. In Table 4.11 an example of the validation for the section of 20 m length, is shown. For the validation of all 3D models, please see Appendix C: Validation of 3D models.

Table 4.11 Validation of the 3D model, L = 20

	Matlab® model	Diagrams by Andersson and Svedholm	Brigade® model	Difference [%]
Mass [ton]	324		318	1.9
Displacement [mm]	2.7		2.6	3.7
1:st bending frequency [Hz]	10.9	11	10.8	0.5 (Matlab®) 0.8 (Andersson and Svedholm)
2:nd bending frequency [Hz]	43		38.76	9.9
3:rd bending frequency [Hz]	98		82.7	16

Furthermore, a convergence analyses is performed by reducing the size of the mesh in the FEM model and calculating the displacement at mid –span. Convergence is confirmed if the difference between the displacements calculated using different mesh sizes decreases when the mesh size is reduced. The result of the convergence analyses is shown in Table 4.12. The difference in the table refers to the difference between the obtained deflections when reducing the mesh size.

Table 4.12 Convergence analysis

L = 20 m			L = 30 m		
Mesh Size	Vertical deformation [mm]	Difference	Mesh Size	Vertical deformation [mm]	Difference
0,5	2,59	0,3846154	0,5	11,79	0,084746
0,2	2,6	0	0,2	11,8	0
0,1	2,6	0	0,1	11,8	0

4.7.3 Dynamics in 3D - Results

The tables Table 4.13 and Table 4.14 show the vibration modes obtained from both analytical solution in Matlab® and the numerical solution, i.e. the Brigade® model. The comparison between the torsional and bending frequencies shows that the longer bridge, L = 30 m does not require a 3D dynamic analyses according to equation (4.8). For L = 20 m, however, a 3D dynamic analysis is required according to equation (4.9), in other words, because of the ratio between the train speed and the first bending frequency. These results are consistent with the 2D dynamic analyses where the shorter bridge was found to be more susceptible to dynamic problems.

Table 4.13 Comparison of the first bending and torsional modes for ballastless bridges

Mode	L=20 m		L=30 m	
	2-D Matlab®	Brigade® Model	2-D Matlab®	Brigade® Model
1st Bending mode	10.9 Hz	10.8 Hz	5.26 Hz	5.15 Hz
1st Torsional mode	-	36.84 Hz	-	24.63 Hz
3D dynamic analysis	Needed		Not needed	

Table 4.14 Comparison of the first bending and torsional modes for ballasted bridges

Mode	L=20 m		L=30 m	
	2-D Matlab®	Brigade® Model	2-D Matlab®	Brigade® Model
1st Bending mode	9.76 Hz	9.32 Hz	4.72 Hz	4.39 Hz
1st Torsional mode	-	29.22 Hz	-	19.64 Hz
3D dynamic analysis	Needed		Not needed	

The results obtained from Matlab® and Brigade® are consistent, and the difference is explained by the fact that the elements used in the Brigade® model follow the Timoshenko beam theory, which gives lower frequencies compared to an Euler-Bernoulli beam model used in Matlab®. The following subchapters present mode shapes and corresponding frequencies for the analysed bridges, and where required, the results from a dynamic studies. For more details on the modal analyses, such as the frequencies for the higher modes and the graphic representation of the generalized mass, please see Appendix D: Modal Analyses.

4.7.3.1 Mode shapes L=20 m (Ballastless)

The following figures, Figure 4.8 to Figure 4.11, will show the different modes that have been considered relevant to show. Note that even though the beam elements take torsion into account when performing the calculations, the display options do not to show this in a correct way. Therefore, the torsional modes appear peculiar, but when the values and directions of the displacements are checked, one finds that it is indeed torsion represented.

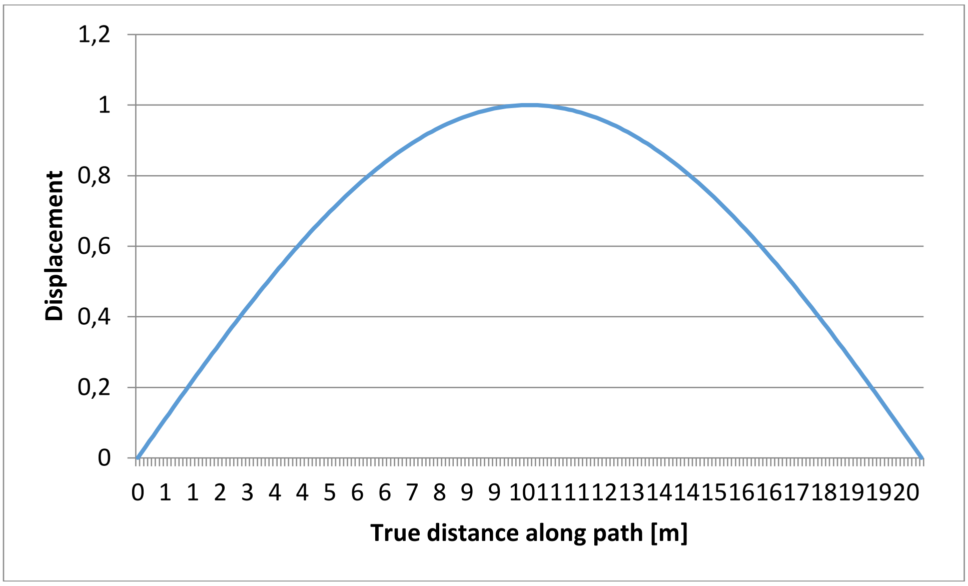


Figure 4.8 First bending mode L=20 m, f =10.8 Hz

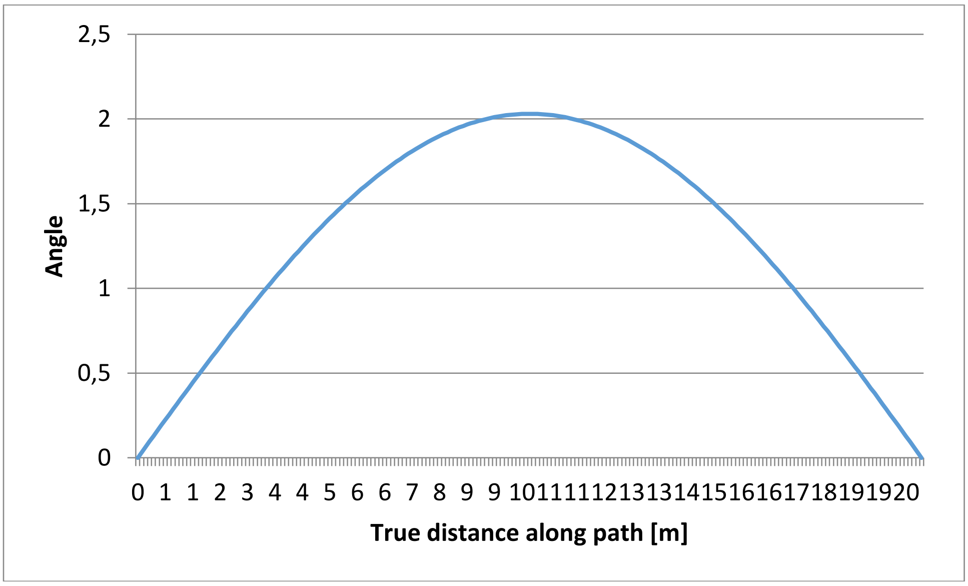


Figure 4.9 First torsional mode L=20 m, f =36.94 Hz

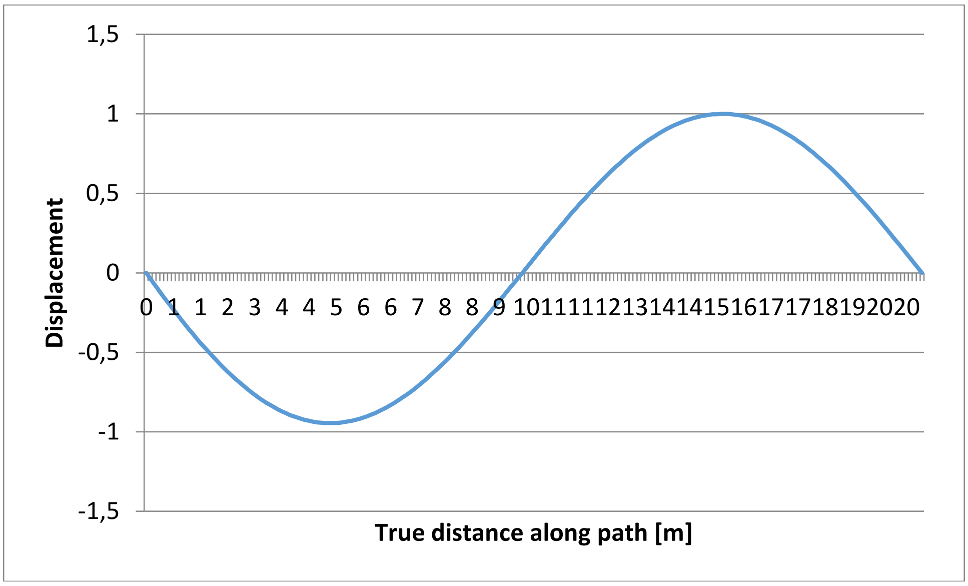


Figure 4.10 Second bending mode L=20 m, f =38,76 Hz

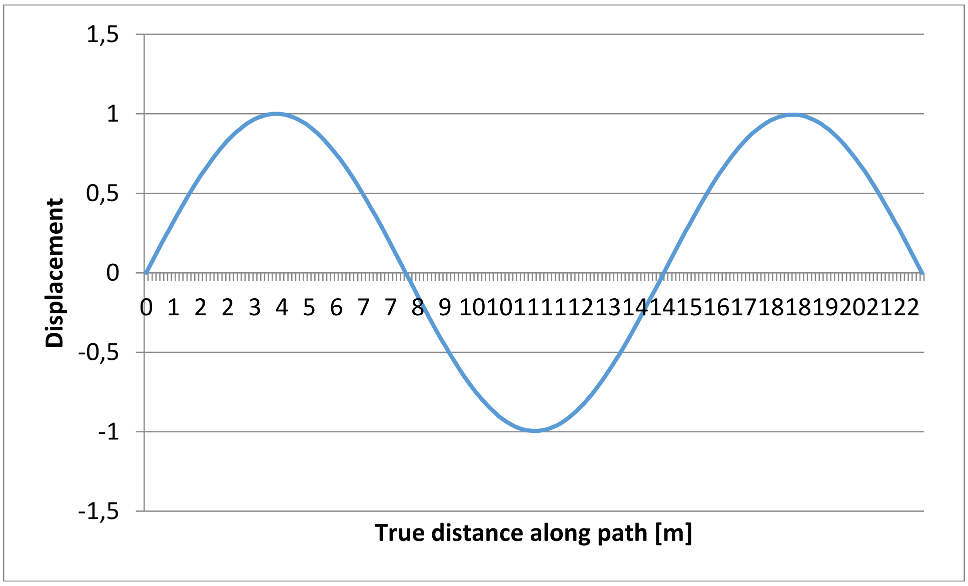


Figure 4.11 Third bending mode L=20 m, f =82,67 Hz

4.7.3.2 Dynamic analysis of the bridge L=20 m (Ballastless)

The output field used to perform the calculations is shown in Figure 4.12. The centre part is analysed since this is the region that will be most excited by the trains. Due to resources limitations, i.e. CPU time and disk space, the calculations have been reduced to this region.

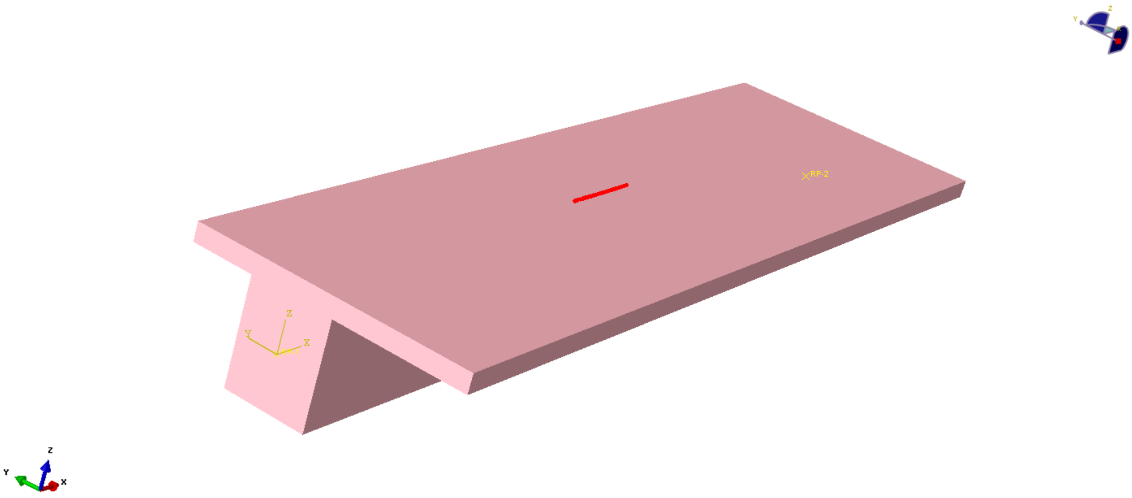


Figure 4.12 Dynamic output node region

Table 4.15 shows, for each of the HSLM trains, maximum acceleration (A3), maximum and minimum vertical displacement of the analyzed segment ($U3_{max}$ and $U3_{min}$, respectively). The difference between the maximum and minimum displacement is used to verify the twist of the deck is below the limit, as specified in (4.5)(Deck twist). In order to check the level of comfort, the maximum displacement must be multiplied by a dynamic factor as explained in section 4.4.2 (U3 *dyn. fac.)

Table 4.15 Dynamic analysis checks, L=20 m Ballastless

Train HSLM	Worst Speed [km/h]	A3 [m/s2]	A3*dyn.fac. [m/s2]	U3,max [mm]	U3*dyn.fac.	U3,min [mm]	Comfort	Max A3	Max U3	Max Torsion
A1	350	3,01	3,1906	1,27	1,3462	1,26	Good	OK	OK	OK
A2	370	2,483	2,63198	1,294	1,37164	1,276	Acceptable	OK	OK	OK
A3	**390**	**4,673**	4,95338	**1,66**	1,7596	**1,637**	**Acceptable**	**OK**	**OK**	**OK**
A4	390	2,2267	2,360302	1,031	1,09286	1,013	Good	OK	OK	OK
A5	390	1,947	2,06382	0,966	1,02396	0,948	Very Good	OK	OK	OK
A6	390	1,791	1,89846	1,023	1,08438	1,055	Good	OK	OK	OK
A7	390	1,63	1,7278	1,08	1,1448	1,06	Good	OK	OK	OK
A8	390	1,56	1,6536	1,057	1,12042	1,038	Good	OK	OK	OK
A9	335	1,089	1,15434	1,27	1,3462	1,25	Good	OK	OK	OK
A10	350	2,54	2,6924	1,337	1,41722	1,32	Acceptable	OK	OK	OK
ALL trains	390	4,673	4,95338	1,66	2,324	1,637	Acceptable	OK	OK	OK

It is clear from the table that the dynamic requirements regarding vertical acceleration, deflection and torsion are fulfilled. The comfort criteria is fulfilled as well, and in the worst case the comfort is at an acceptable level.

Figure 4.13 shows the envelope maximum and minimum accelerations over speed ranging between 320-390 km/h. The diagram shows acceleration over speed for the HSLM_A03 train. Similar diagrams are found for all other HSLM trains but these are not limiting and thus not shown here. Figure 4.13 shows which speed gives the highest acceleration, and for this speed acceleration is plotted against time in Figure 4.14 for HSLM_A03 and a speed of 390 km/h. The remaining graphs for the HSLM-A trains are not limiting and the interest must focus on the train giving the worst effects, as it is the limiting one. The limiting speeds are between the maximum allowed speed and the design speed.

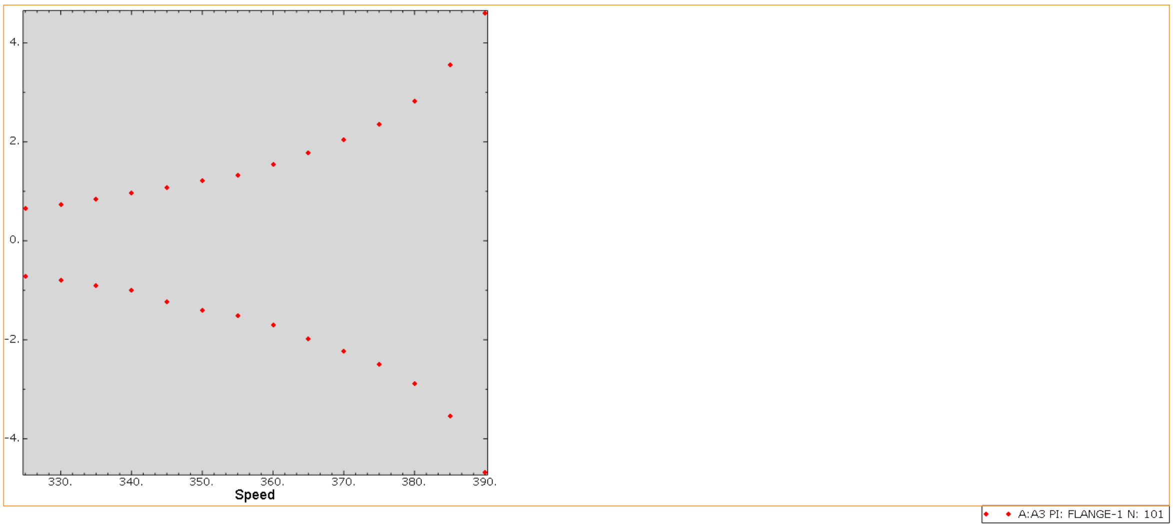


Figure 4.13 Max/min vertical acceleration for HSLM-A3, L=20 m

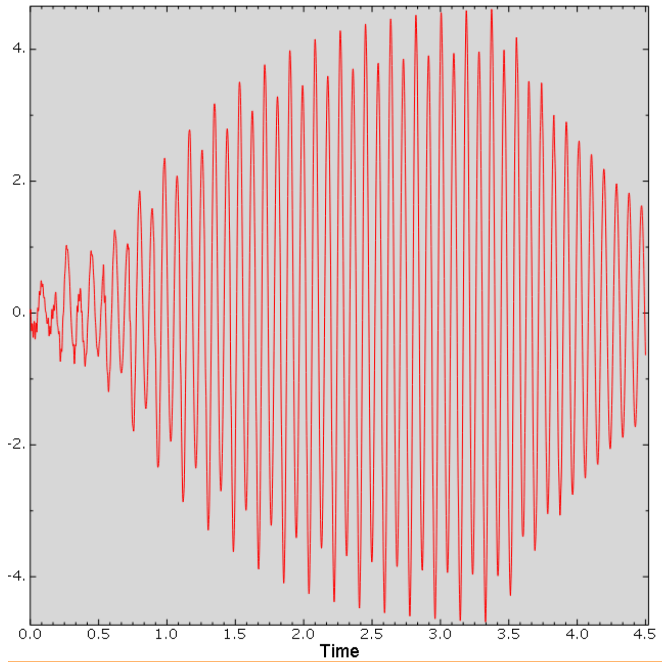


Figure 4.14 Vertical acceleration due to HSLM-A3 at 390 km/h, L=20 m

4.7.3.3 Mode shapes L=30 m (Ballastless)

Figures Figure 4.15 to Figure 4.18, will show the different modes that have been considered relevant to show, as it was presented in the section 4.7.3.1.

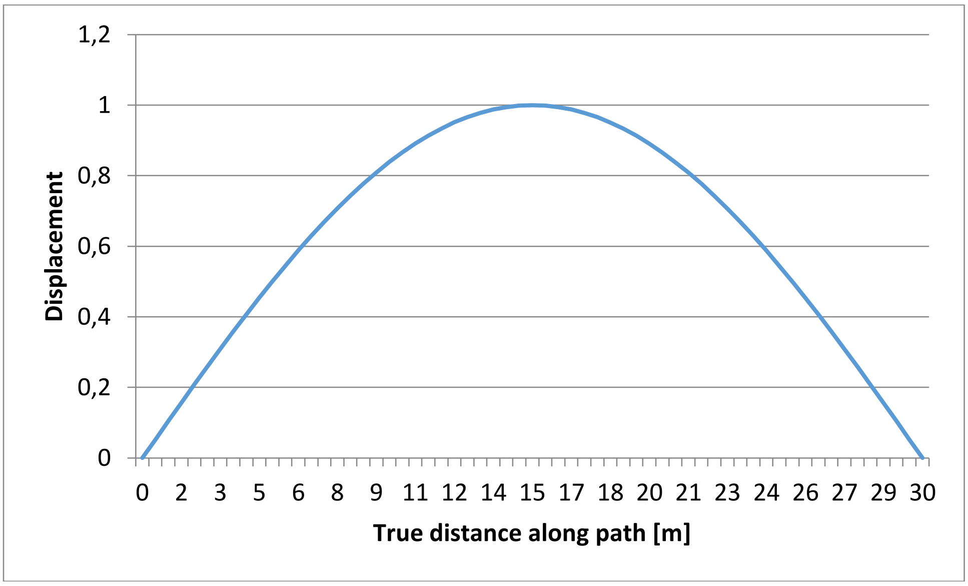


Figure 4.15 First bending mode L=30 m, f =5.045 Hz

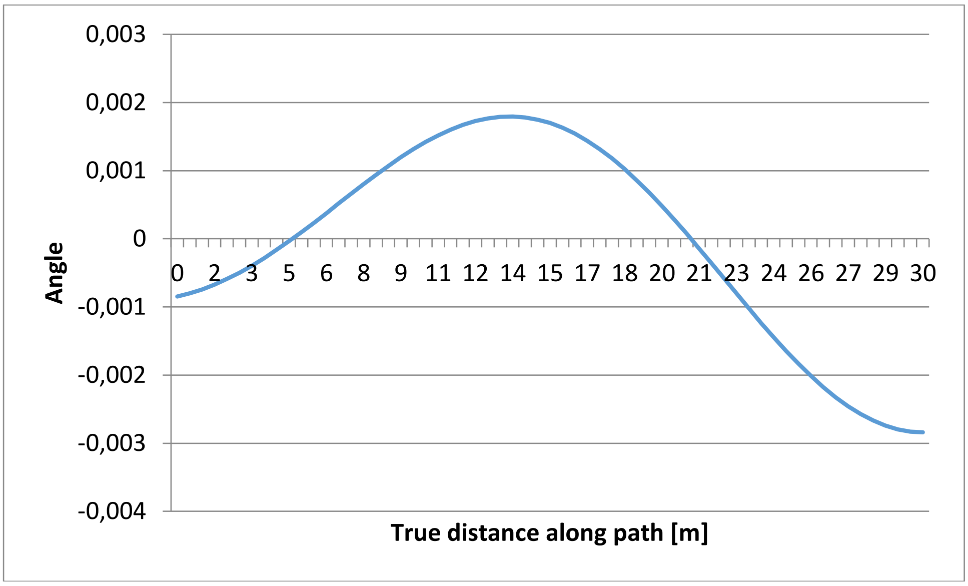


Figure 4.16 First torsional mode L=30 m, f =24.63 Hz

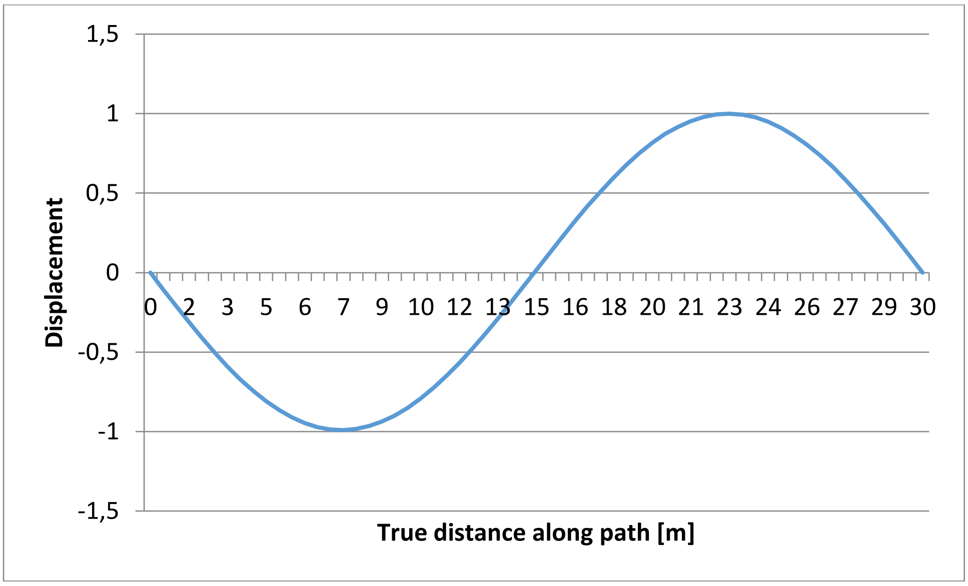


Figure 4.17 Second bending mode L=30 m, f =19,273 Hz

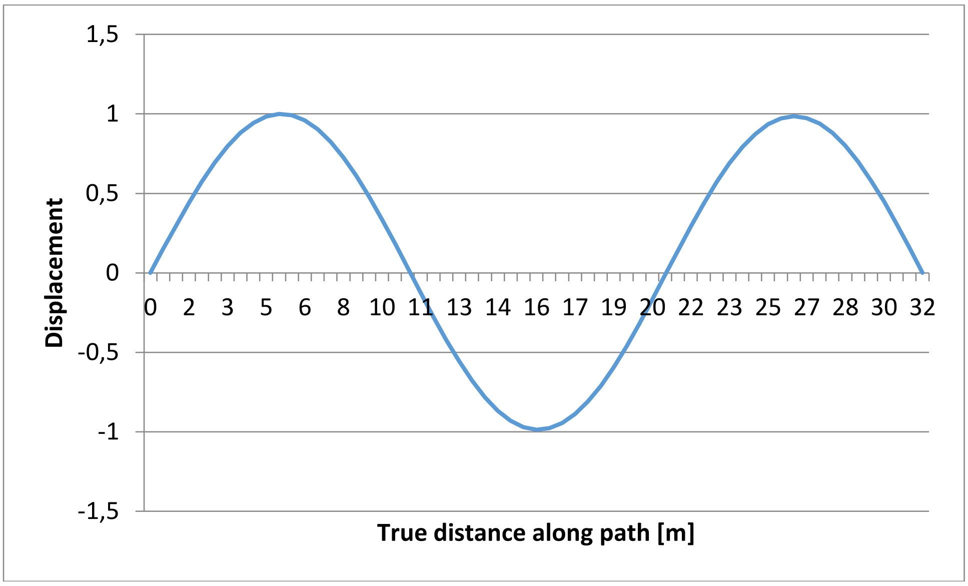


Figure 4.18 Third bending mode L=30 m, f =41,513 Hz

As mentioned earlier, no further dynamic assessment is required for this length.

4.7.3.4 Mode shapes L=20 m (Ballasted)

The following figures, Figure 4.19 to Figure 4.22, will show the different modes that have been considered relevant to show.

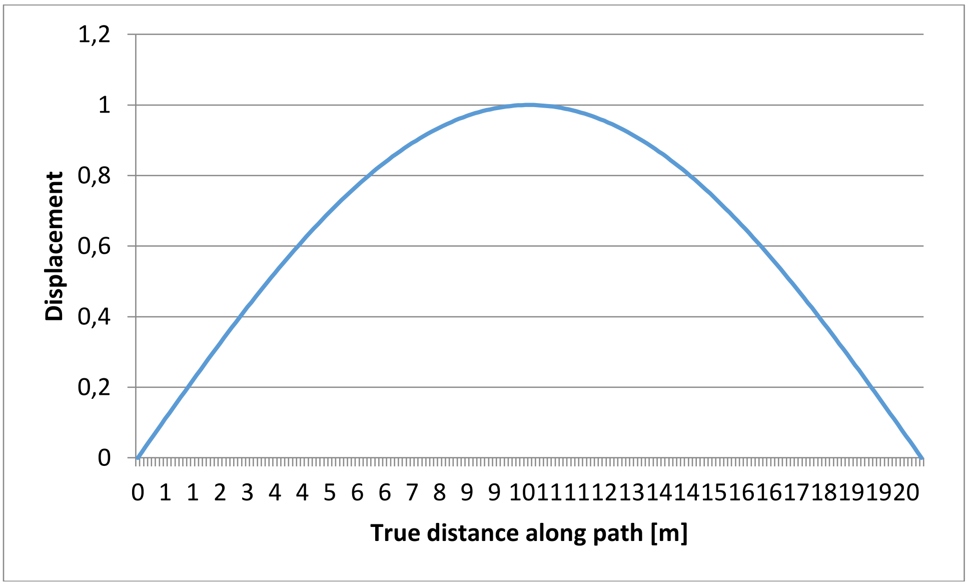


Figure 4.19 First bending mode L=20 m, Ballasted, f =9,32 Hz

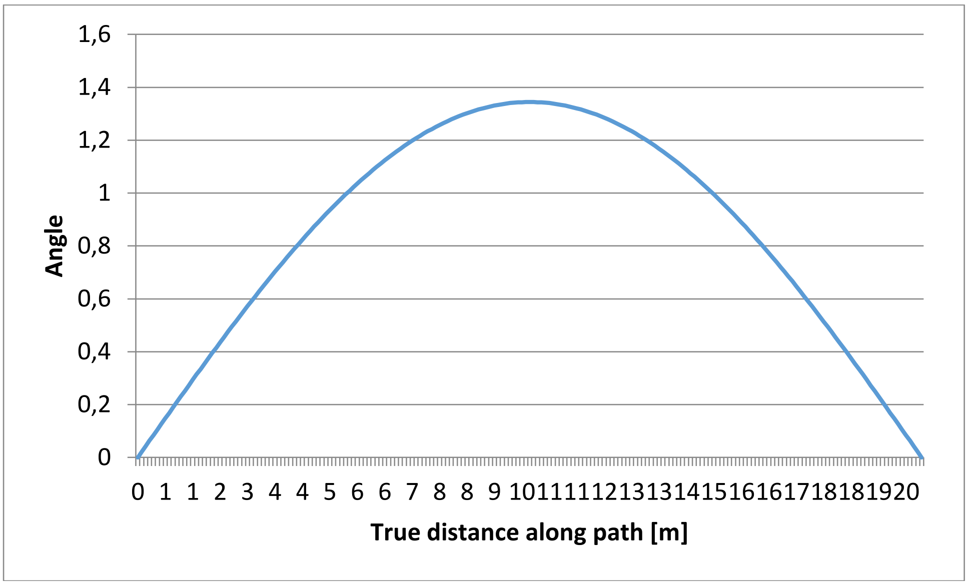


Figure 4.20 First torsional mode L=20 m, Ballasted, f =29,22 Hz

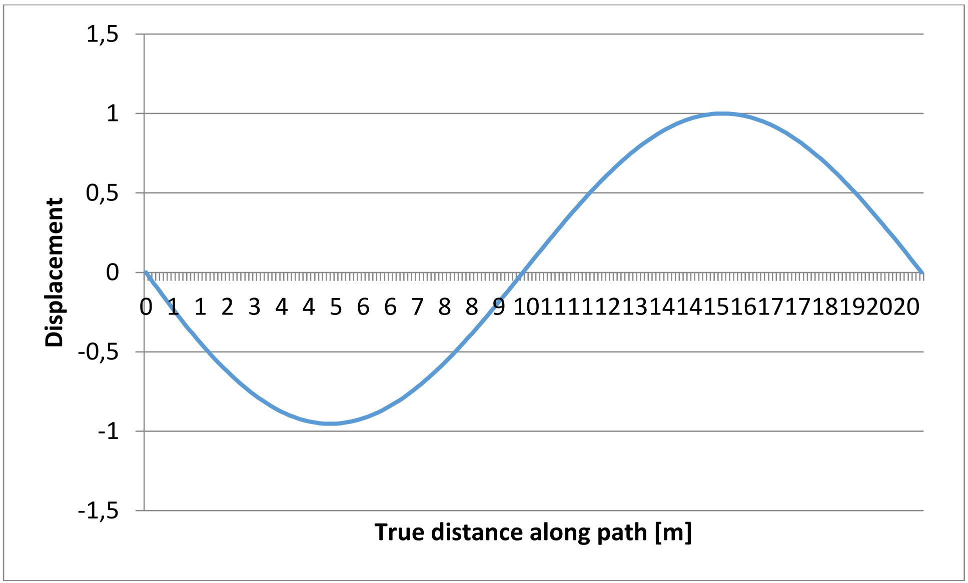


Figure 4.21 Second bending mode L=20 m, Ballasted, f =33,43 Hz

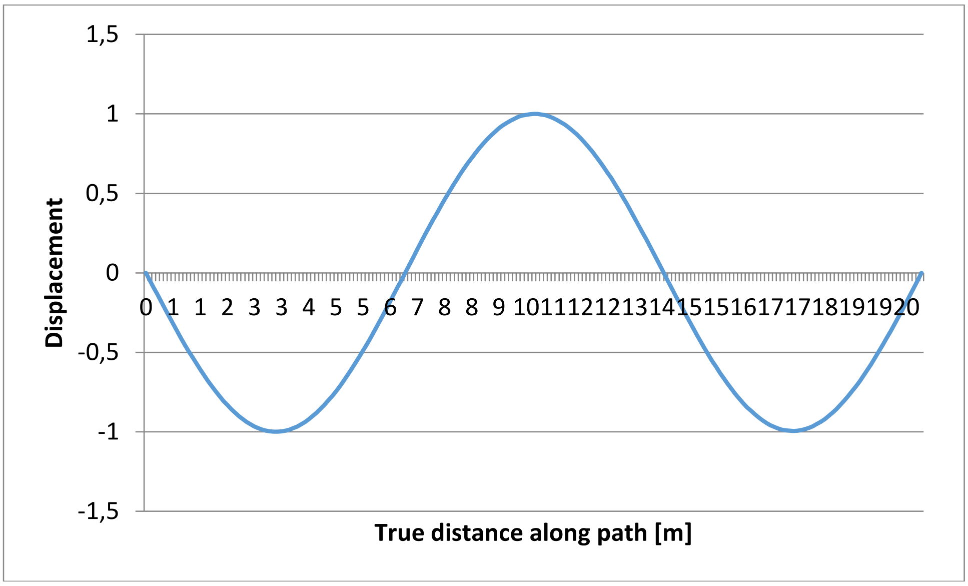


Figure 4.22 Third bending mode L=20 m, Ballasted f =70,5 Hz

4.7.3.5 Dynamic analysis of the bridge L=20 m (Ballasted)

The output field used to perform the calculations is the same as it was used for the ballastless version of the bridge. The centre part is analysed since this is the region that will be most excited by the trains. Due to resources limitations, i.e. CPU time and disk space, the calculations have been reduced to this region. The mass of the ballast has been taken into account by introducing it to the model as a non-structural mass. The results from the dynamic analysis are shown in Table 4.16.

Table 4.16 Dynamic analysis checks, L=20 m Ballasted

Train HSLM	Worst Speed [km/h]	A3 [m/s2]	A3*dyn.fac. [m/s2]	U3,max [mm]	U3*dyn.fac.	U3,min	Comfort	Max A3	Max U3	Deck Twist
A1	365	1,06	1,1236	0,996	1,05576	0,97	Good	OK	OK	OK
A2	325	1,395	1,4787	1,248	1,32288	1,229	Good	OK	OK	OK
A3	**335**	**3,5**	3,75028	**1,65**	1,749	**1,62**	**Acceptable**	**NOT OK**	**OK**	**OK**
A4	355	2,86	3,0316	1,49	1,5794	1,47	Acceptable	OK	OK	OK
A5	370	3,16	3,3496	1,54	1,6324	1,52	Acceptable	OK	OK	OK
A6	385	3,069	3,25314	1,52	1,6112	1,494	Acceptable	OK	OK	OK
A7	390	1,698	1,79988	1,081	1,14586	1,061	Good	OK	OK	OK
A8	380	1,31	1,3886	1,06	1,1236	1,04	Good	OK	OK	OK
A9	360	1,395	1,4787	1,195	1,2667	1,172	Good	OK	OK	OK
A10	365	1,282	1,35892	1,195	1,2667	1,172	Good	OK	OK	OK
ALL trains	335	3,5	3,75028	1,65	1,749	1,62	Acceptable	OK	OK	OK

It is clear from the table that the dynamic requirements regarding vertical acceleration, deflection and torsion are fulfilled for all cases except for HSLM_A3 train, where the acceleration after being multiplied with the dynamic factor is too high. The comfort criterion are fulfilled, with the worst case being only "acceptable" level of comfort.

Figure 4.23 shows the envelope maximum and minimum accelerations over speed ranging between 320-390 km/h. The diagram shows acceleration over speed for the HSLM_A03 train. This diagram shows which speed gives the most acceleration, and for this speed acceleration is plotted against time in Figure 4.24 for HSLM_A3 and a speed of 335 km/h.

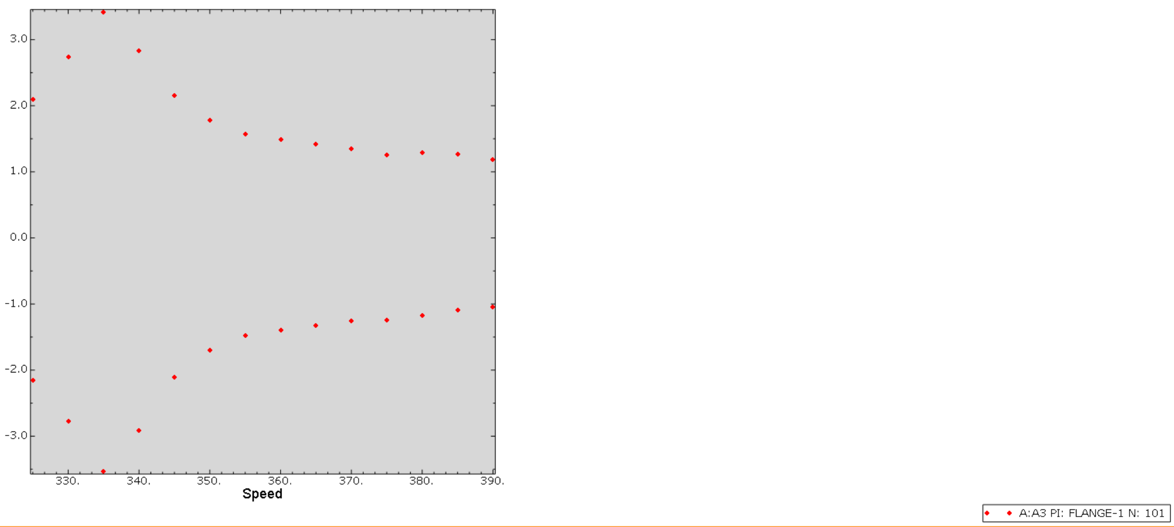


Figure 4.23 Max/min vertical acceleration for HSLM-A3, L=20 m

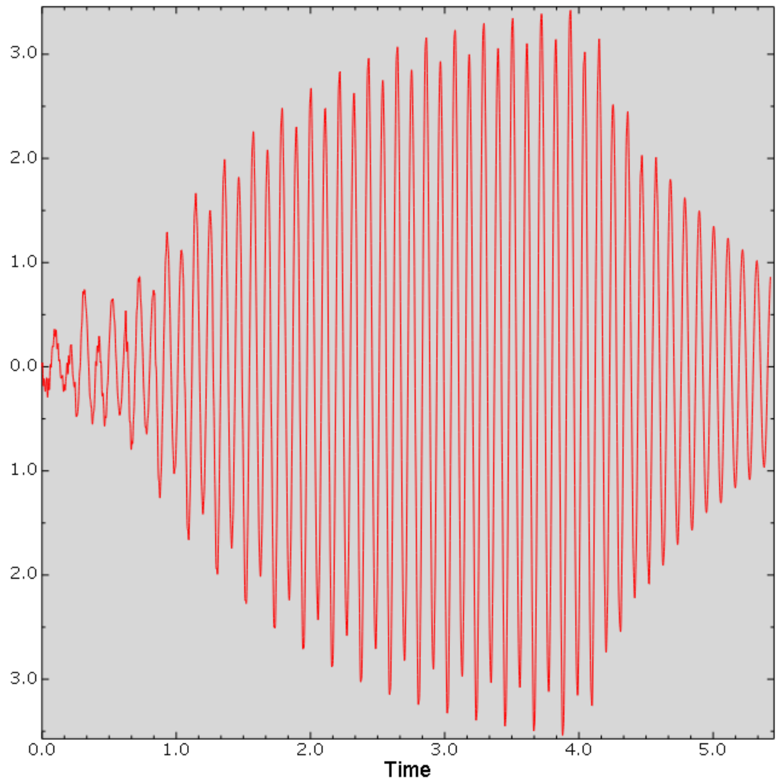


Figure 4.24 Vertical acceleration at node 101 due to HSLM-A3 at 335 km/h, L=20 m

4.7.3.6 Mode shapes L=30 m (Ballasted)

Figures Figure 4.25 to Figure 4.28, will show the different modes that have been considered relevant to show, as it was presented in the section 4.7.3.1.

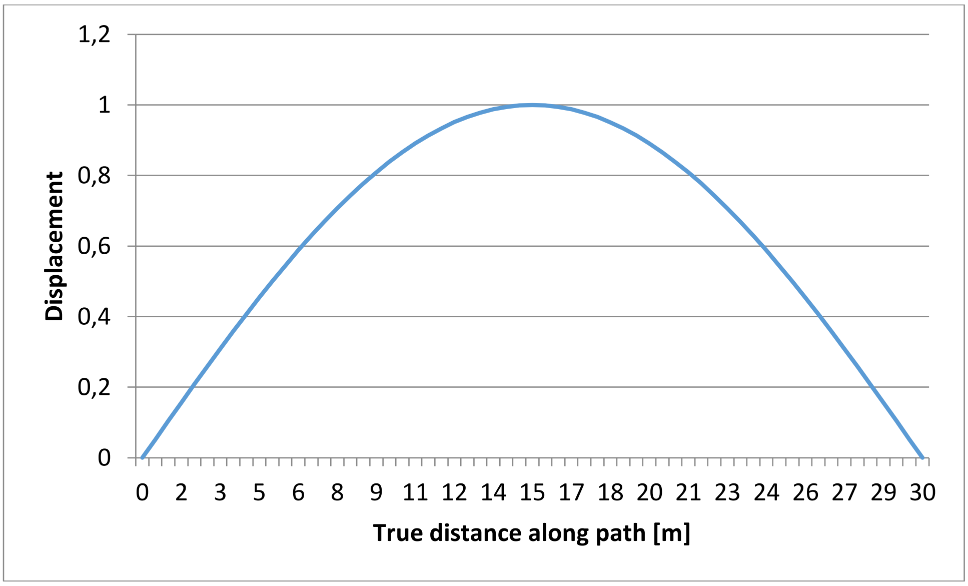


Figure 4.25 First bending mode L=30 m, Ballasted, f =4,39 Hz

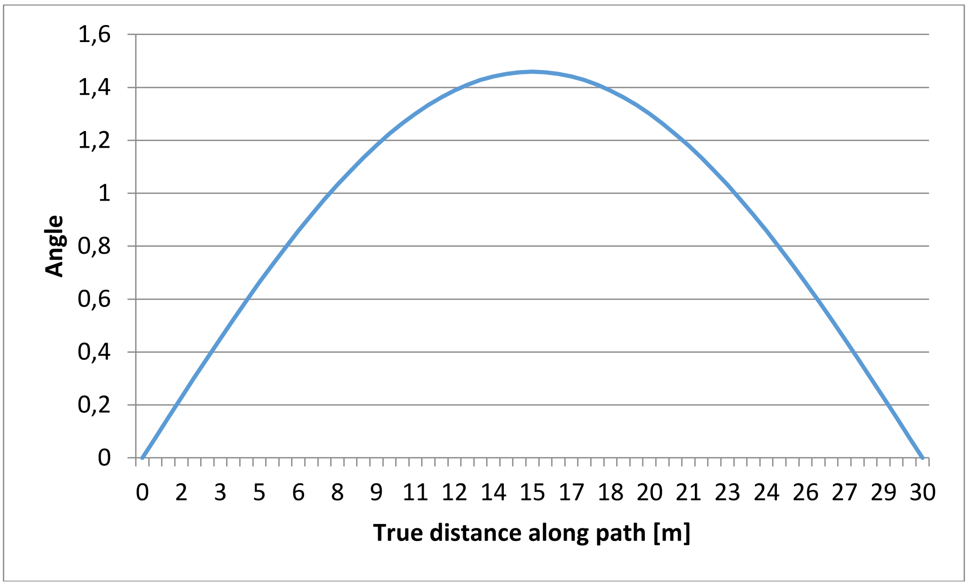


Figure 4.26 First torsional mode L=30 m, Ballasted, f =19,64 Hz

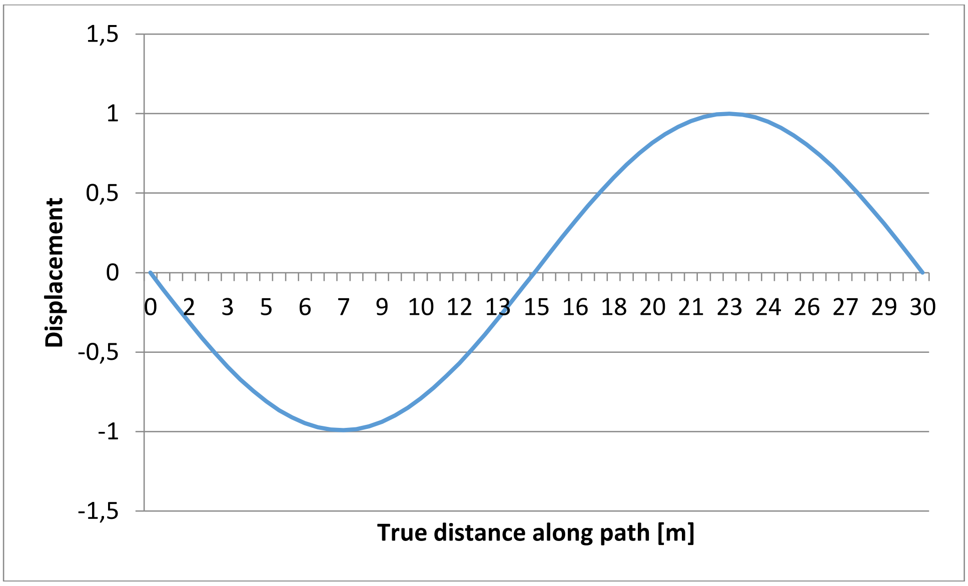


Figure 4.27 Second bending mode L=30 m, Ballasted, f =16,73 Hz

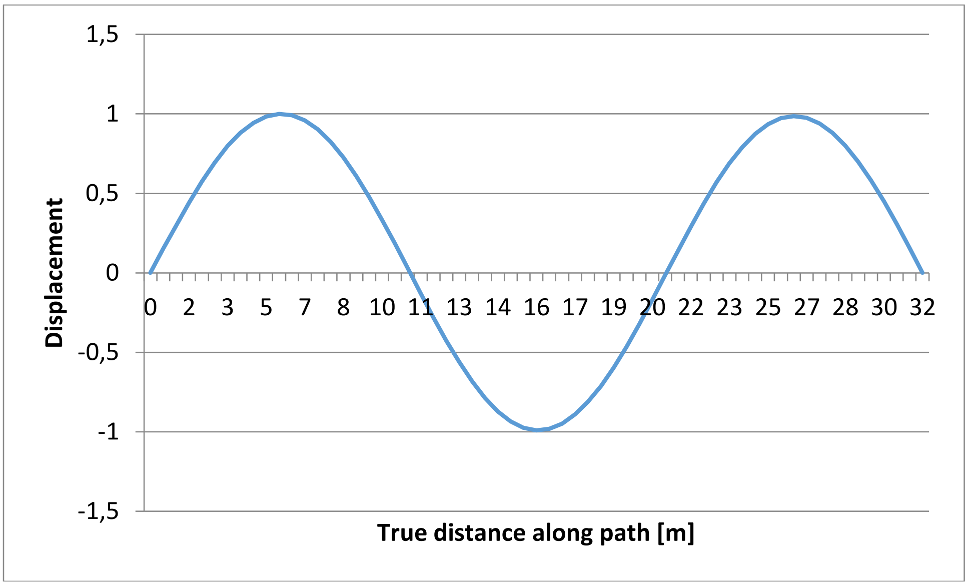


Figure 4.28 Third bending mode L=30 m, Ballasted, f =35,876 Hz

5 Parametric analyses

The following chapters will analyse the behaviour of the bridge when modifying some of its parameters, such as implementing a composite cross section and changing from a simply supported to a continuous bridge.

5.1 Composite T section

As it has been mentioned, these kind of sections are normally avoided due to more complicated construction and maintenance. However, it has been considered interesting to understand the behaviour of a composite section because it is expected that it will be more influenced by 3D effects and will allow for a comparison between a dynamic analyses in 2 and 3 dimensions. The analysed section is the one shown in Figure 5.1. For this part of the parametric study, only the ballastless alternative is analysed.

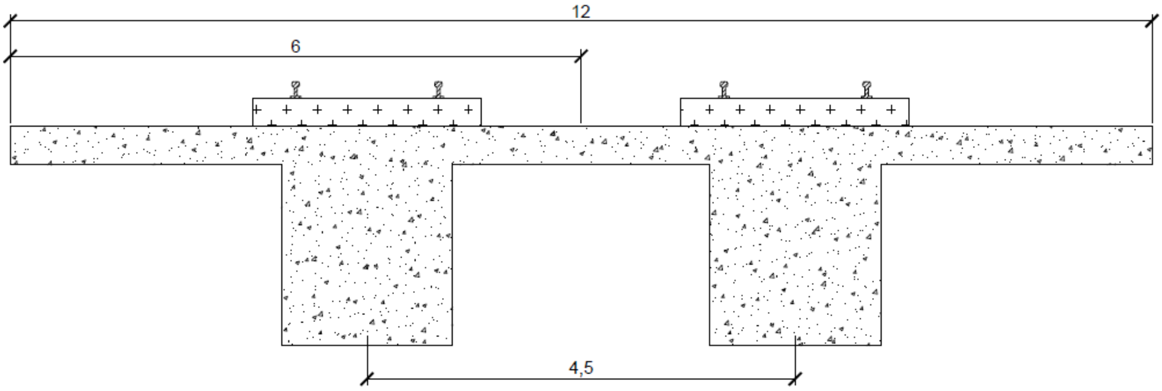


Figure 5.1 Composite section

5.1.1 Calculation method

For these sections, a slightly different design approach has been used. It was assumed that this type of section will have larger problems with dynamics and that the dynamic requirements will be dimensioning. This because the dynamic assessment is performed by having trains only on one track, meaning the loading is assymetric which will result in more severe problems for torsion. Therefore, instead of making a complete static analyses and then checking the optimized section for dynamic, it was decided to analyse the section from chapter 3.6 and check whether it fulfilled the static and dynamic requirements.

5.1.2 Static analyses

The composite section is not optimized for statics since the hypothesis was that the dynamics will be dimensioning. It is however necessary to check that a composite section fulfils the static requirements as well, and interesting to see the ratio of utilization for different limit states.
The static calculations are quite simplified since they are not thought to be dimensioning. The following changes are made in the calculations:

- In order to take into account the composite action of the bridge, the code is modified in order to take into account the increased torsional moment, due to the weight of the second T-section. The resistance stays the same.
- The shear force is also checked in the connection between the two T sections, at the connection between the web and the flange.

5.1.2.1 Results statics

In Table 5.1 the dimensions for the composite bridge are shown.

Table 5.1 Dimensions composite bridge

	BALLASTLESS	
Dimensions	**L = 20**	**L = 30**
Flange width, b_f $[m]$	4.5	4.5
Flange thickness, t_f $[m]$	0.4	0.4
Web height, h_w $[m]$	2.2	2.4
Web thickness, t_w $[m]$	1.8	1.8
Total height, h $[m]$	3.072	3.27
Reinforcement area, A_s	27 φ 32	55 φ 32
Shear reinforcement	No	No

The results obtained from the cross section chosen and optimised are shown in Table 3.10 and Table 3.11. The procedure for calculating the different resistances has been described in section 3.4. Note that every section has the minimum shear reinforcement required – Table 3.10 states weather more shear reinforcement than the minimum is required.

Table 5.2 Usage ratios in ULS, composite bridge

Ultimate Limit State	BALLASTLESS	
	L = 20	**L = 30**
Bending moment resistance, M_{cRd} [kNm]	25 900	51 400
Design bending moment, M_{Ed} [kNm]	18 500	39 900
Usage ratio M_{Ed}/M_{cRd}	**0.72**	**0.78**
Usage ratio $M_{EdDerail}/M_{Rfw}$	**0.75**	**0.72**
Shear force resistance, V_{cRd} [kN]	3 400	4 200
Design shear force, V_{Ed} [kN]	1 800	3 100
Usage ratio V_{Ed}/V_{cRd}	**0.54**	**0.74**
Torsional resistance, T_{cRd} [kNm]	237 150	251 970
Design Torsion, T_{Ed} [kNm]	3 393	3 436
Usage ratio T_{Ed}/T_{cRd}	**0.0143**	**0.0136**
Number of cycles before fatigue failure	10^6	10^6
Damage in the concrete D_c	**0.54**	**0.67**

Table 5.3 Usage ratio in SLS composite bridge

Serviceability Limit State	BALLASTLESS	
	L = 20	**L = 30**
Long term-Tensile stresses. Usage ratio	**0.1**	**0.1**
Short term-Compressive stresses. Usage ratio	**0.15**	**0.34**
Maximum crack width [mm]	0.3	0.3
Crack width in webs. [mm]	0.11	0.11
Crack width in flanges. [mm]	0.11	0.11
Ratio of usage in webs	**0.37**	**0.37**
Ratio of usage in flanges	**0.37**	**0.37**
Maximum allowed deflection [mm]	33	50
Maximum deflection in SLS, [mm]	**1.8**	**6.9**

5.1.3 Dynamics

5.1.3.1 2D dynamics

As mentioned, the composite section is not optimized, thus in the 2D dynamic analyses it has the same dimensions as for the static (see Table 5.1). The 2D dynamic properties are presented in Table 5.4

Table 5.4 2D dynamics composite bridge

Dynamics 2D	BALLASTLESS	
	L = 20	L = 30
Mass [ton/m]	32.6	34.5
Stiffness [GNm²]	303	374
m/m_{erf}	3.90	2.62
Eigenfrequency	12	5.7

5.1.3.2 3D dynamics

The model that has been built in Brigade Plus® is formed by shell elements S4R[2] for the plate (flanges) while the webs are designed with beam elements B31. They are joined using tie constrains, descriptions of which can be found in the Abaqus® manual, as Brigade Plus® is based on the former (Dassault Systèmes Simulia Corp, 2012). The Figure 5.2 shows the model as it is and the rendered profiles are in Figure 5.3.

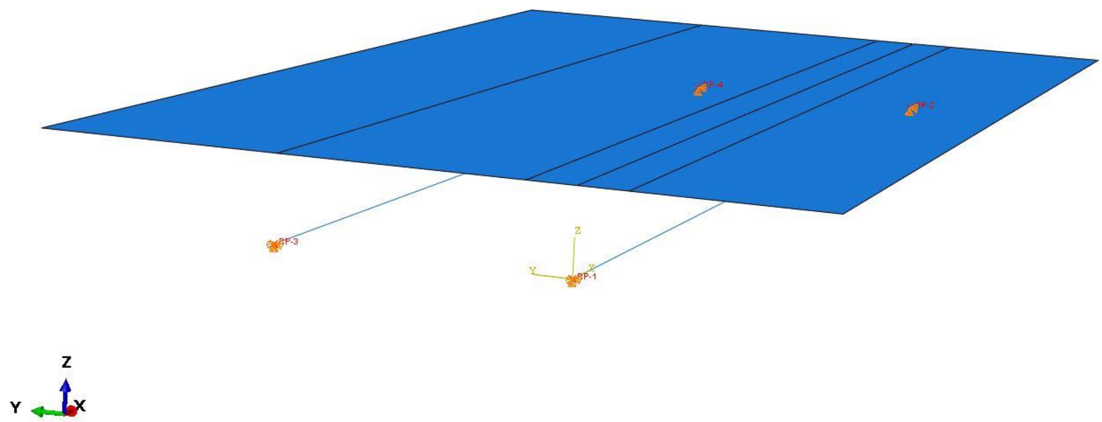


Figure 5.2 Composite section model, L=20 m

[2] A 4 node doubly curved thin or thick shell, reduced ingration, hourglass control, finite membrane strains, (Dassault Systèmes Simulia Corp, 2012)

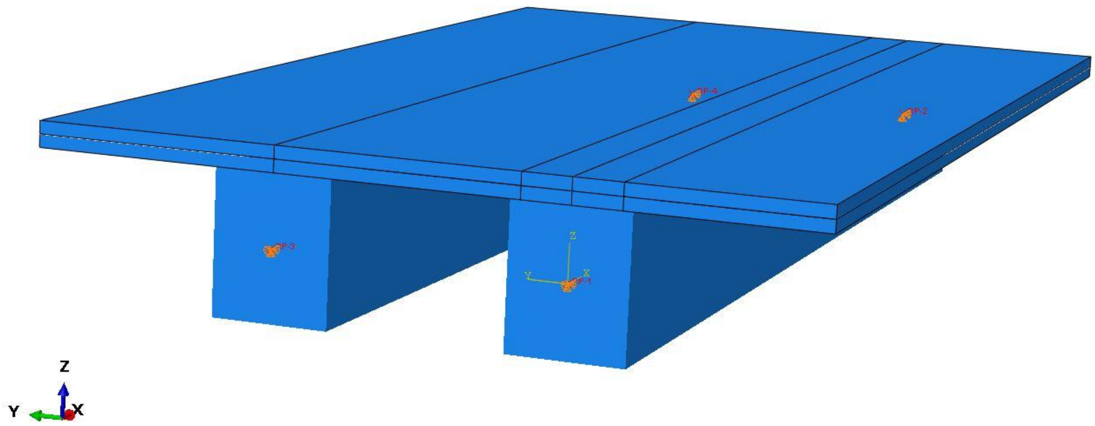


Figure 5.3 Rendered preview for the composite section model, L=20 m

The boundary conditions are applied in the reference points which are connected to the web beams by using tie constraints. The boundary conditions are shown in Table 5.5. The supports are not fixed for rotation, but since neither of the supporting beams can move vertically, this restrains the model from spinning. It is a very conservative way of modelling since it gives the supports a low torsional stiffness.

Table 5.5 Boundary conditions 3D model, composite bridge

Point	U1	U2	U3	UR1	UR2	UR3
RP-1	Restrained	Restrained	Restrained	Free	Free	Free
RP-2	Free	Restrained	Restrained	Free	Free	Free
RP-3	Restrained	Restrained	Restrained	Free	Free	Free
RP-4	Free	Restrained	Restrained	Free	Free	Free

The mesh size is chosen as 0.5 as a compromise between the computational time, RAM and precision needed for the dynamic analysis. The mesh is shown in Figure 5.4. A convergence analyses is of course performed to insure the mesh is sufficient to provide a good accuracy.

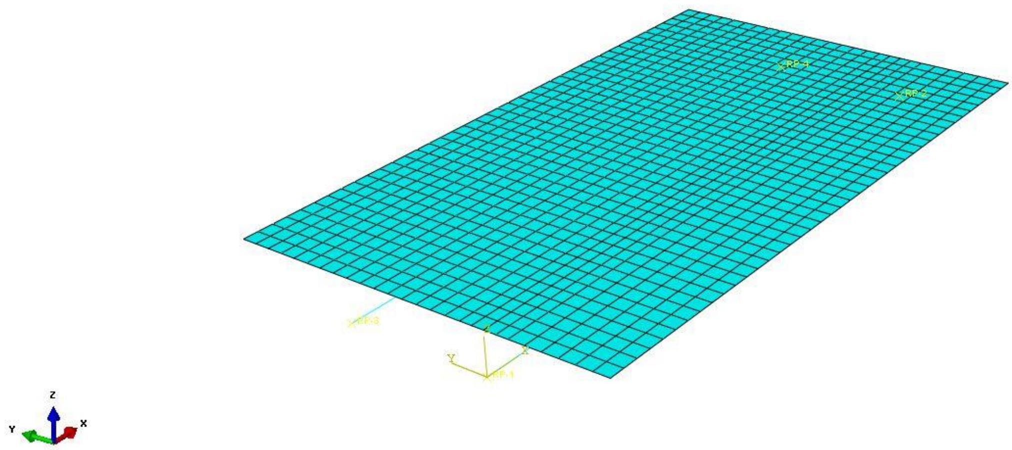


Figure 5.4 Mesh size for the composite section model, size=0.5

5.1.3.2.1 Validation of the model

To validate the model a convergence analysis is required. This will be performed as it was done for the simple T-sections.

The convergence analysis is performed over the vertical displacement of the model when subjected only to self weight, at mid-span. The value of displacement obtained from Brigade Plus® is compared to an analytical calculation of the displacement. It is considered that this value is easy to reach and make it converge, but it is the ultimate check to validate the model and it is shown in Table 5.6. The difference in the table refers to the difference between the obtained deflections when reducing the mesh size.

Table 5.6 Convergence analysis composite bridge, L=20 m and L=30 m

L=20 m			L=30 m		
Mesh Size	Vertical deformation [mm]	Difference	Mesh Size	Vertical deformation [mm]	Difference
1	2,69	0,2230483	1	11,64	0,0858369
0,5	2,684	0,3353204	0,5	11,65	0
0,1	2,675	0,0373832	0,1	11,65	0

Due to the expected calculation time and the precision for this beam model, the mesh size has been chosen to be 0.5. The difference in time invested for a mesh of 0.5 and one with 0.1 is exponential and this will affect the time invested in the dynamic analysis.

5.1.3.2.2 Modal analysis

When the sections are studied in order to know if the dynamic analysis is needed, torsional mode obtained in the same way as it is obtained in the single T-section, it is determined that both bridges need a dynamic analysis. The Table 5.7 shows the comparison of frequencies. The difference between the hand calculations and the model is attributed to the shear lag that appears in the flanges reducing the effective mass.

Table 5.7 Comparison of the first bending and torsional modes, composite section

Mode	L=20 m		L=30 m	
	2-D Matlab®	Brigade® Model	2-D Matlab®	Brigade® Model
1st Bending mode	12 Hz	10.2 Hz	5.77 Hz	5.12 Hz
1st Torsional mode	-	8.4 Hz	-	5.53 Hz
3D dynamic Analysis	Needed		Needed	

As it can be concluded the dynamic analysis is required according to Figure 4.5 (EN-1991-2, 2003) due to the torsional modes.

5.1.3.2.2.1 20 metres bridge

The most interesting modes that are considered are presented in figures, Figure 5.5 to Figure 5.8.

Figure 5.5 First bending mode, composite section L=20 m, f =10.2 Hz

Figure 5.6 First torsional mode, composite section L=20 m, f =8.4 Hz

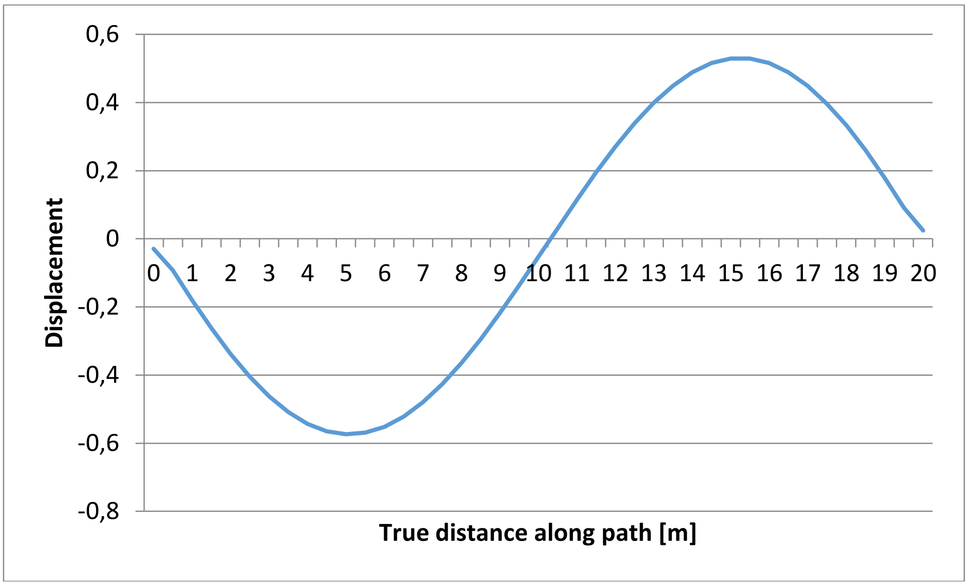


Figure 5.7 Second bending mode, composite section L=20 m, f =43.2 Hz

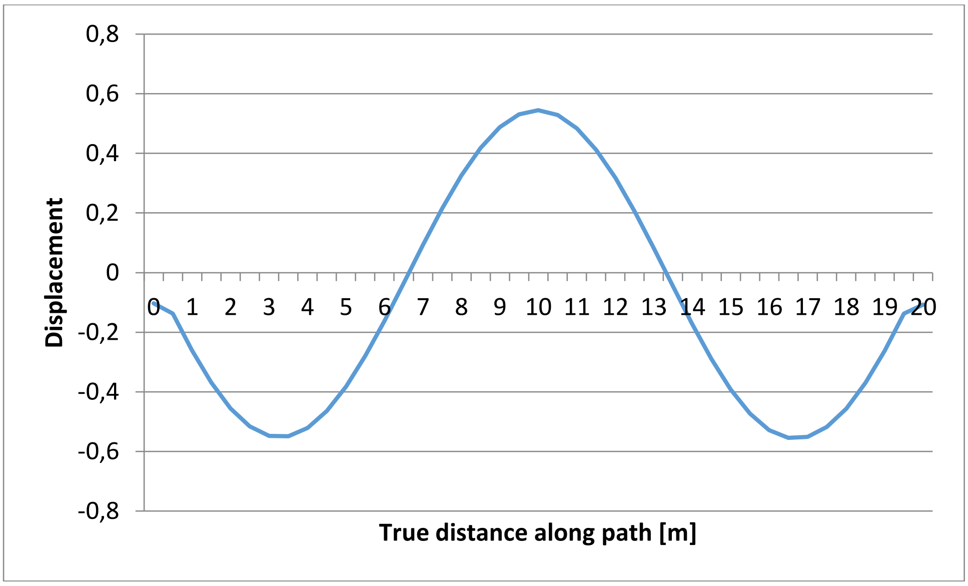


Figure 5.8 Third bending mode, composite section L=20 m, f =98.1 Hz

5.1.3.2.2.2 30 metres bridge

The most interesting modes that are considered are presented in figures, Figure 5.9 to Figure 5.12.

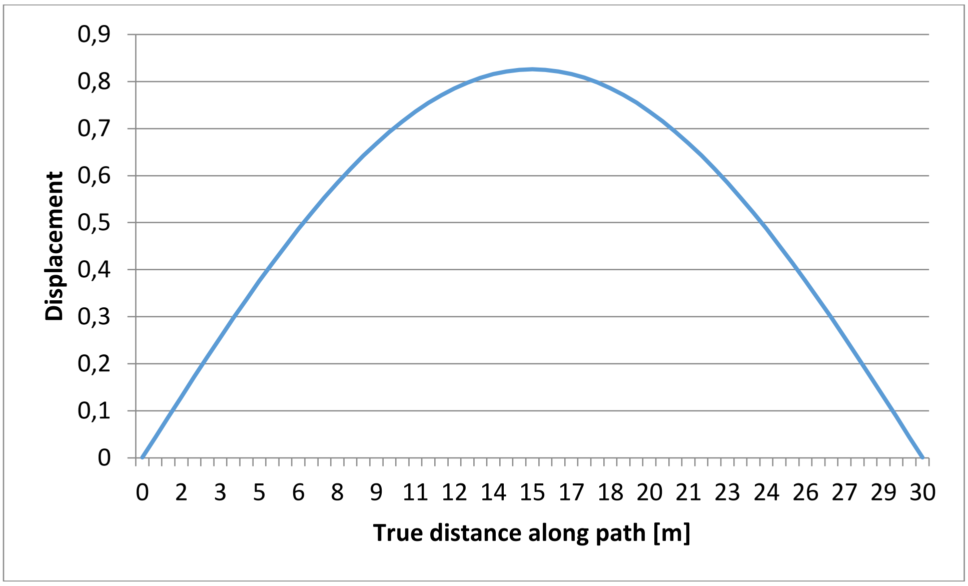


Figure 5.9 First bending mode, composite section L=30 m, f =5.12 Hz

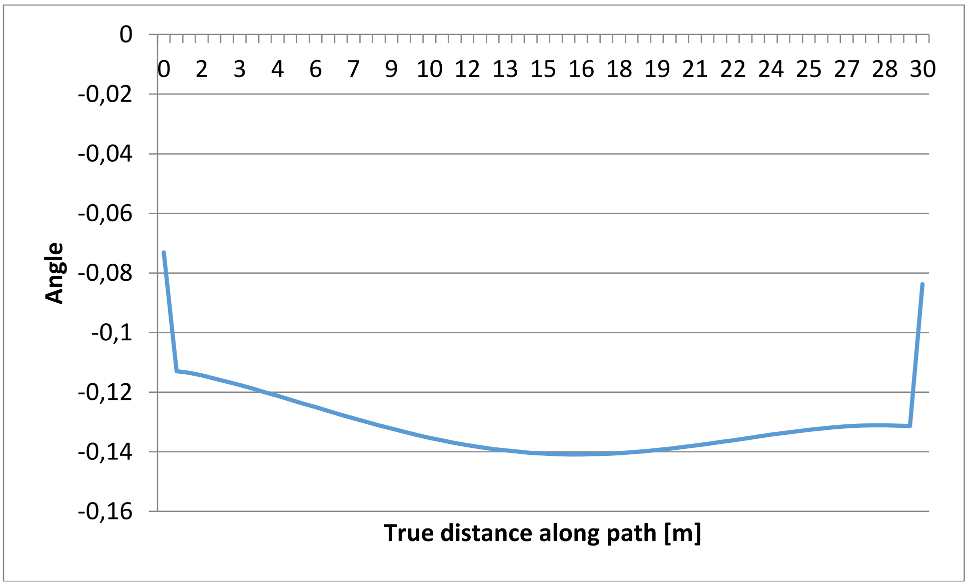


Figure 5.10 First torsional mode, composite section L=30 m, f =5.53 Hz

Figure 5.11 Second bending mode, composite section L=20 m, f =23.8 Hz

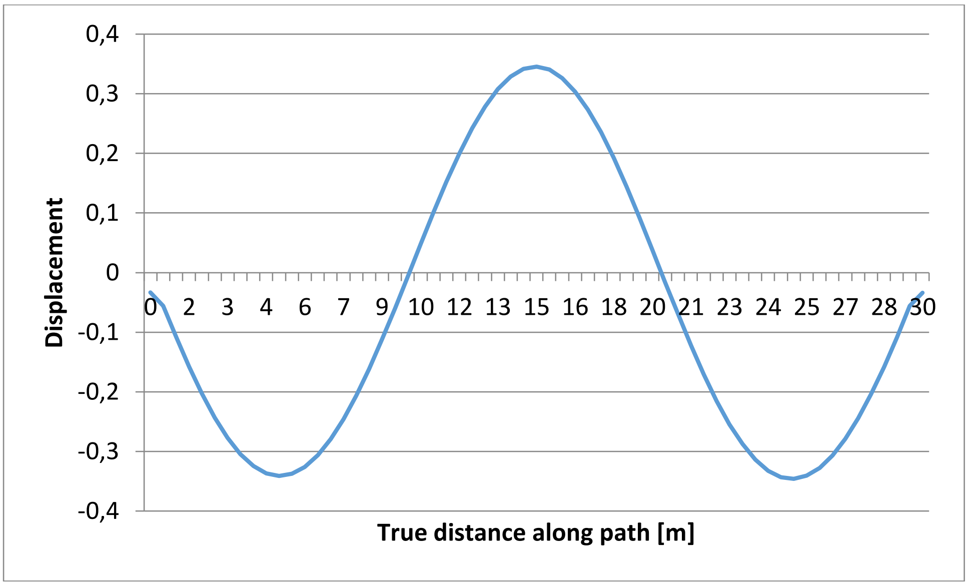


Figure 5.12 Third bending mode, composite section L=30 m, f =41.53 Hz

5.1.3.2.3 Dynamic analysis

In order to minimize the computational time for the analyses as well as, the size of the result files, only certain points of interest will be analysed. These points are the midpoint of the bridge (over one track) together with 1.5 m towards the supports, as shown in Figure 5.13.

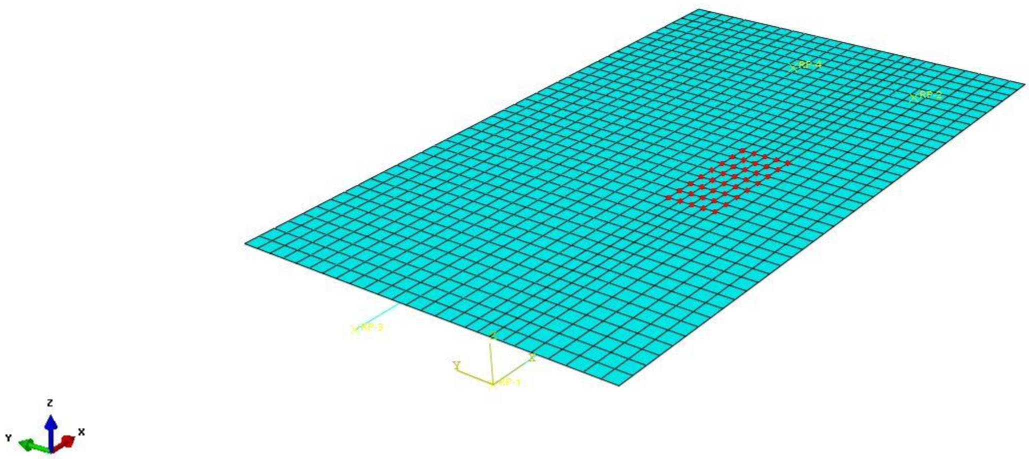


Figure 5.13 Dynamic output node region

These nodes are interesting because they are the ones that will be excited by the bending and torsional modes and the ones affecting the passenger comfort. The length of the analysed segment is the minimal possible that still allows for torsional analyses. The analysed region is the one directly below the rail, since it is not interesting to know what happens at the ends of the slab – the accelerations and deformations here do not affect the passenger comfort. Also, the ends of the slab are stiffer in reality than in the model, because of the edge beams that have been omitted in this analyses.

5.1.3.2.3.1 20 metres bridge

The results from the dynamic analysis are shown in Table 5.8

Table 5.8 Dynamic analysis checks, composite section, L=20 m

Train HSLM	Worst Speed [km/h]	A3 [m/s2]	A3*dyn.fac. [m/s2]	U3 [mm]	U3*dyn.fac.	U3	Comfort	Max A3	Max U3	Deck twist
A1	365	6,98	7,3988	1,004	1,06424	0,7526	Good	NOT OK	OK	OK
A2	345	6,65	7,049	1,078	1,14268	0,83	Good	NOT OK	OK	OK
A3	350	7	7,42	1,227	1,30062	0,938	Good	NOT OK	OK	OK
A4	355	7,5	7,95	1,17	1,2402	0,9	Good	NOT OK	OK	OK
A5	390	6,75	7,155	1,028	1,08968	0,66	Good	NOT OK	OK	OK
A6	335	6,58	6,9748	1,072	1,13632	0,699	Good	NOT OK	OK	OK
A7	365	6,76	7,1656	1,114	1,18084	0,738	Good	NOT OK	OK	OK
A8	390	5,69	6,0314	1,03	1,0918	0,722	Good	NOT OK	OK	OK
A9	**355**	**7,55**	8,003	**1,089**	**1,15434**	**0,825**	**Good**	**NOT OK**	**OK**	**OK**
A10	390	7,47	7,9182	1,12	1,1872	0,86	Good	NOT OK	OK	OK
ALL trains	355	7,55	8,003	1,227	1,30062	0,938	Good	NOT OK	OK	OK

The table shows that the dynamic requirements regarding vertical acceleration, are not fulfilled for any train. The cross section selected for the analysis is the one that according to the 2D dynamic analyses fulfilled the dynamic requirements. Due to effects that are not being taken into account, such as torsion and shear lag, when the 3D is performed, the bridge reaction is much higher and over the limits.

Figure 5.14 shows the envelope maximum and minimum accelerations over speed ranging between 320-390 km/h. The diagram shows acceleration over speed for the HSLM_A09 train at node 112 because it is the one experiencing the highest acceleration for this train. Similar diagrams are found for all other HSLM trains and all of them above the limit speed. This diagram shows which speed gives the most acceleration, and for this speed acceleration is plotted against time in Figure 5.15 for HSLM_A09 and a speed of 355 km/h.

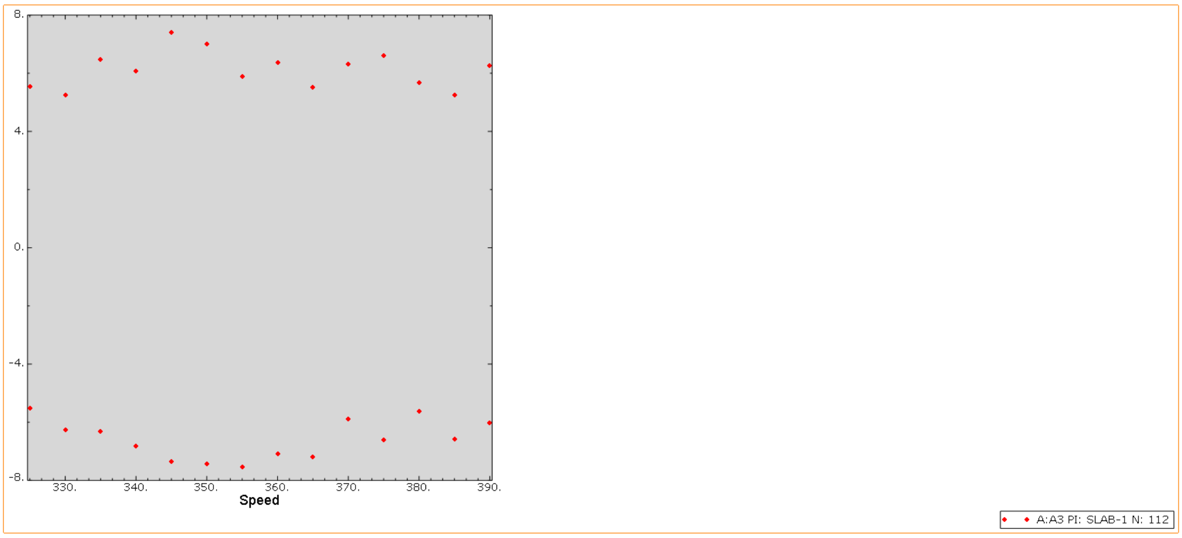


Figure 5.14 Max/min vertical acceleration at Node 112 for HSLM-A9, L=20 m

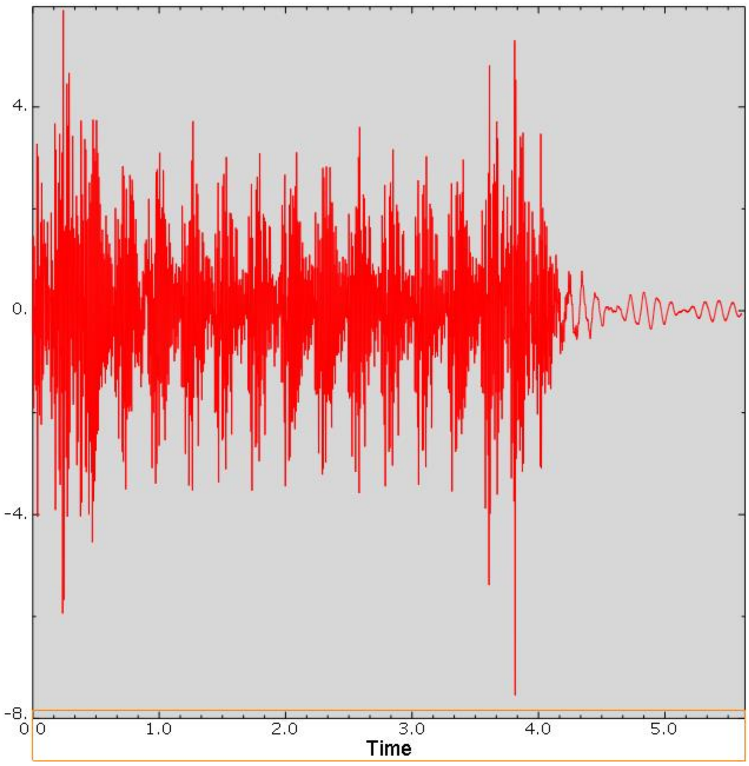


Figure 5.15 Vertical acceleration at node 112 due to HSLM-A9 at 355 km/h, L=20 m

5.1.3.2.3.2 30 metres bridge

The same controls are studied for the 30 m bridge and presented in Table 5.9.

Table 5.9 Dynamic analysis checks, composite section, L=30 m

Train HSLM	Worst Speed [km/h]	A3 [m/s2]	U3*Dyn Fac [m/s2]	U3 [mm]	U3*Dyn Fac	U3	Comfort	Max A3	Max U3	Max Torsion
A1	330	4,9	5,194	3,768	3,99408	3,295	NOT OK	OK	OK	OK
A2	365	5,96	6,3176	3,43	3,6358	2,767	NOT OK	NOT OK	OK	OK
A3	375	4,633	4,91098	2,689	2,85034	2,168	NOT OK	OK	OK	OK
A4	380	4,728	5,01168	3,073	3,25738	2,556	NOT OK	OK	OK	OK
A5	375	4,375	4,6375	2,988	3,16728	2,484	NOT OK	OK	OK	OK
A6	375	4,63	4,9078	2,939	3,11534	2,4	NOT OK	OK	OK	OK
A7	380	5,089	5,39434	2,938	3,11428	2,392	NOT OK	NOT OK	OK	OK
A8	365	6,107	6,47342	2,982	3,16092	2,42	NOT OK	NOT OK	OK	OK
A9	**325**	**6,69**	7,0914	**3,3207**	**3,519942**	**2,609**	**NOT OK**	**NOT OK**	**OK**	**OK**
A10	375	2,256	2,39136	3,279	3,47574	2,655	NOT OK	OK	OK	OK
ALL trains	325	6,69	7,0914	3,768	3,99408	2,609	NOT OK	NOT OK	OK	OK

As well as for the composite section of 20 m, there are some values that are over the limit, even though they should not be according to the 2D analyses. The reasons for this are explained in the previous subchapter.

Figure 5.14 shows the envelope maximum and minimum accelerations over speed ranging between 320-390 km/h. The diagram shows acceleration over speed for the HSLM_A09 train at node 43 because it is the one experiencing the highest acceleration for this train. Acceleration plotted over time can be found in Figure 5.15 for HSLM_A09 and a speed of 325 km/h.

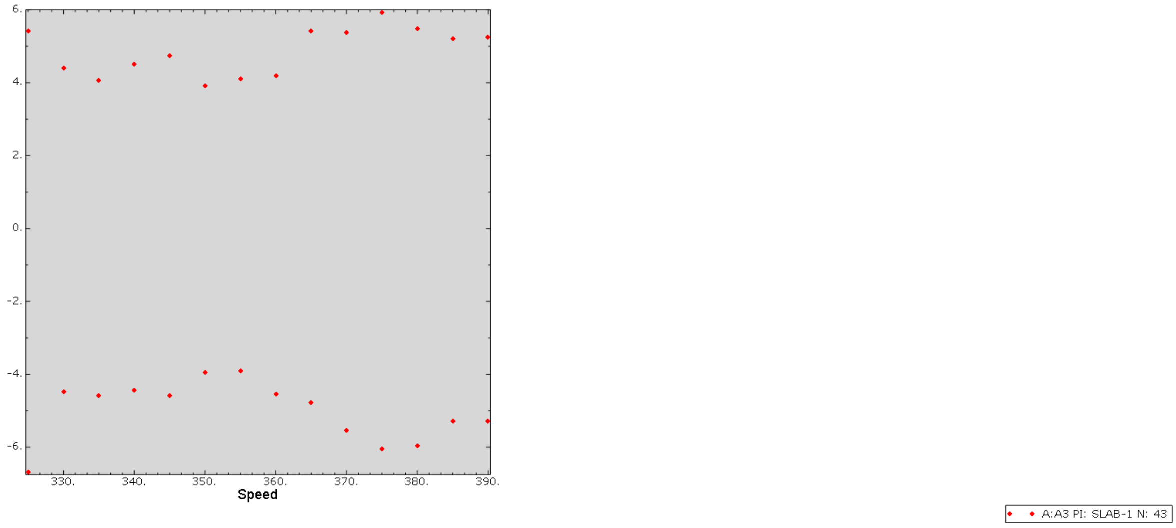


Figure 5.16 Max/min vertical acceleration at Node 43 for HSLM-A9, L=20 m

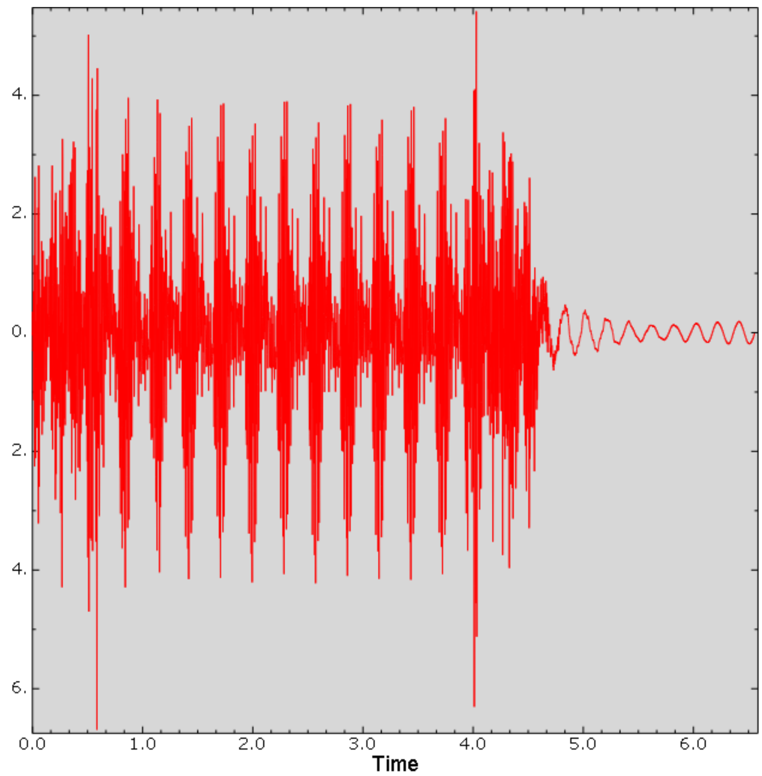


Figure 5.17 Vertical acceleration node 43 due to HSLM-A9 at 330 km/h. L= 30 m

5.2 Continuous bridge in two spans

The continuous bridge is the second kind of further analysis carried out. The behaviour of the continuous beam is interesting to study since it is common to have bridges in several spans.

The analysed cross section is the same as described in the beginning, namely an asymmetric single T section.

5.2.1 Calculation method

First of all, and as it was done for the simple bridge in the previous chapters, the static analysis is carried out by the Matlab® code and the 2D dynamics according to the report by (Andersson & Svedholm, 2016). Finally, a 3D model is created for the selected cases where a more detailed dynamic analyses is necessary.

The statics will be done for a continuous bridge with spans ranging between 20 and 30 m and both ballasted and ballastless tracks are analysed. The results will, as previously, be presented for cases L = 20 m and L = 30 m. In other words, the code can be used to analyse bridges of total length from 40 to 60 m, given the two spans are equally long.

When an optimal design is obtained from the static analyses, it will be checked against the dynamic requirements in 2D according to design diagrams from (Andersson & Svedholm, 2016). Should the cross section fail to fulfil the requirements, the dimensions will be changed until it does.

5.2.2 Static analyses

5.2.2.1 Structural capacity

The calculations for designing the section statically are described in section 3.4. However, for a continuous beam some additional calculations must be done and some procedures altered as it will be presented in the following subchapters.

5.2.2.1.1 Ultimate Limit State (ULS)

Moment resistance

For a continuous bridge in two spans one must check the moment resistance at two critical points, namely the position where the moment has its maximum positive and maximum negative values. These positions are found by calculating the sectional forces along the whole beam and extracting the maximum and minimum values.

When applying the traffic loads, specifically LM71 and SW/2, the critical load distribution is not trivial, and it is therefore necessary to create an influence line in order to find the worst position for the load.

Due to the negative moment over the support, in the case of a continuous bridge, it is necessary to have tensile reinforcement in the top part of the flange over the mid support. The required reinforcement area is calculated as it was a tensile reinforcement as in section 3.4.1.

Shear resistance

The shear resistance of the section is checked in the same way as for the simply supported beam. The only difference is the position where the maximum shear force occurs. The Matlab® code automatically finds the dimensioning section and the whole beam is dimensioned according to the sectional forces that occur at this position.

Other actions

Torsion, fatigue, derailment action and normal forces due to acceleration and braking of the train are calculated in the same manner as for the simply supported beam.

5.2.2.1.2 Serviceability Limit State (SLS)

The SLS requirements are not affected by the number of spans, and no alterations had to be made. The dimensioning section is not necessarily the same as for the simply supported bridge, but since the Matlab® code automatically preforms the dimensioning for the worst section, it works for the continuous as well as for the simply supported beam.

5.2.2.1.3 Results static analyses

The results of the static analyses and optimization and the measures in Table 5.10.

Table 5.10 Cross-section dimensions for a continuous beam, static design

	BALLASTED		**BALLASTLESS**	
Dimensions	**L = 20**	**L = 30**	**L = 20**	**L = 30**
Flange width, b_f $[m]$	4.5	4.5	4.5	4.5
Flange thickness, t_f $[m]$	0.4	0.4	0.4	0.4
Web height, h_w $[m]$	2.0	2.4	2.0	2.2
Web thickness, t_w $[m]$	1.9	1.8	1.4	1.4
Total height, h $[m]$	2.4	2.8	2.4	2.8
Reinforcement area, A_s	23ϕ32	46ϕ32	17ϕ32	35ϕ32
Reinforcement area over middle support, A_{ss}	23ϕ32	46ϕ32	17ϕ32	35ϕ32
Shear reinforcement	Yes	Yes	No	Yes
Mass [ton/m]	20	21.5	13.4	13.97
Stiffness [GNm^2]	103.68	156	86.19	108.33

The procedure for calculating the different resistances has been described in section 3.4. The usage ratios for different limit states are presented in Table 5.11 and Table 5.12.

Table 5.11 Usage ratios in the ULS, continuous bridge

Ultimate Limit State	**BALLASTED**		**BALLASTLESS**	
	L = 20	**L = 30**	**L = 20**	**L = 30**
Bending moment resistance, M_{cRd} [kNm]	21 600	45 800	16 400	33 000
Design bending moment, M_{Ed} [kNm]	15 100	34 600	10 800	24 400
Usage ratio M_{Ed}/M_{cRd}	**0.70**	**0.76**	**0.66**	**0.74**
Usage ratio $M_{Ed,neg}/M_{cRd}$	**071**	**0.77**	**0.67**	**0.75**
Usage ratio $M_{EdDerail}/M_{Rfw}$	**1.0**	**0.95**	**0.99**	**0.95**
Shear force resistance, V_{cRd} [kN]	6 600	10 200	3 200	7 200
Design shear force, V_{Ed} [kN]	3 700	5 700	2 600	4 000
Usage ratio V_{Ed}/V_{cRd}	**0.55**	**0.56**	**0.83**	**0.56**
Torsional resistance, T_{cRd} [kNm]	230 560	251 970	189 390	200 920
Design Torsion, T_{Ed} [kNm]	772	772	580	580
Usage ratio T_{Ed}/T_{cRd}	**0.0033**	**0.003**	**0.0031**	**0.0029**
Number of cycles before fatigue failure	10^6	10^6	10^6	10^6
Damage in the concrete D_c	**0.41**	**0.51**	**0.31**	**0.42**

Table 5.12 Usage ratios in the SLS, continuous bridge

Serviceability Limit State	**BALLASTED**		**BALLASTLESS**	
	L = 20	**L = 30**	**L = 20**	**L = 30**
Long term-Tensile stresses. Usage ratio	**0.11**	**0.15**	**0.09**	**0.14**
Short term-Compressive stresses. Usage ratio	**0.19**	**0.30**	**0.14**	**0.25**
Maximum crack width [mm]	0.30	0.30	0.30	0.30
Crack width in webs. [mm]	0.14	0.17	0.11	0.16
Crack width in flanges. [mm]	0.14	0.17	0.11	0.16
Ratio of usage in webs	**0.47**	**0.56**	**0.36**	**0.55**
Ratio of usage in flanges	**0.47**	**0.56**	**0.36**	**0.55**
Maximum allowed deflection [mm]	33	50	33	50
Maximum deflection in SLS, [mm]	**5.8**	**17**	**5.2**	**18.6**

As Table 5.10 shows, there is some reduction in the cross section dimensions for the ballastless alternative and a significant reduction in the amount of reinforcement needed. This is consistent with the results from the analyses of the simply supported bridge.

5.2.3 Dynamics

5.2.3.1 2D dynamics

Table 5.13 Cross section dimensions optimized for 2D dynamics

	BALLASTED		BALLASTLESS	
Dimensions	**L = 20**	**L = 30**	**L = 20**	**L = 30**
Flange width, b_f $[m]$	4.5	4.5	4.5	4.5
Flange thickness, t_f $[m]$	0.4	0.4	0.4	0.4
Web height, h_w $[m]$	2.0	2.4	2.0	2.2
Web thickness, t_w $[m]$	1.9	1.8	1.4	1.4
Total height, h $[m]$	2.4	2.8	2.4	2.8
Reinforcement area, A_s	23ϕ32	46ϕ32	17ϕ32	35ϕ32
Reinforcement area over middle support, A_{ss}	23ϕ32	46ϕ32	17ϕ25	35ϕ32
Shear reinforcement	Yes	Yes	No	Yes
Mass [ton/m]	20	21.5	13.4	13.97
Stiffness [GNm^2]	103.68	156	86.19	108.33
m/m_{erf}	**1.26**	**1.00**	**1.88**	**1.1**
Eigenfrequency	**8.9**	**4.7**	**10**	**4.9**

The results obtained from the statics and the 2D analysis are quite revealing. A section optimized for the static analyses fulfils the dynamic requirements according to the 2D analyses. In other words, a continuous bridge with the same span length as a simply supported one is less susceptible to dynamic effects and thus governed by the static design. Only exception is the 30 span ballasted bridge where the highest usage ratio is m/m_{erf} meaning is dimensioned by the dynamic requirements.

Further, it can be seen that, as in the case of a simply supported bridge, the cross section is not significantly reduced by removal of the ballast, and the savings are made mostly in saving the reinforcement.

The dimensioning load is again the case of derailment. As for the simply supported beam, the capacity for derailment can be increased by making the slab thicker. However, that would mean adding to the self-weight and at the same time reducing the space for the tensile reinforcement.

The cross sections have similar dimensions as the corresponding span lengths for the simply supported bridge, and require the similar amount of reinforcement, which is considered reasonable.

5.2.3.2 3D dynamics

Since the bridge is no longer a “*simple structure*” according to Figure 4.5 it is necessary to perform a more thorough dynamic analyses. In order to do this a 3D model is built in Brigade Plus®, which can be seen in Figure 5.18.

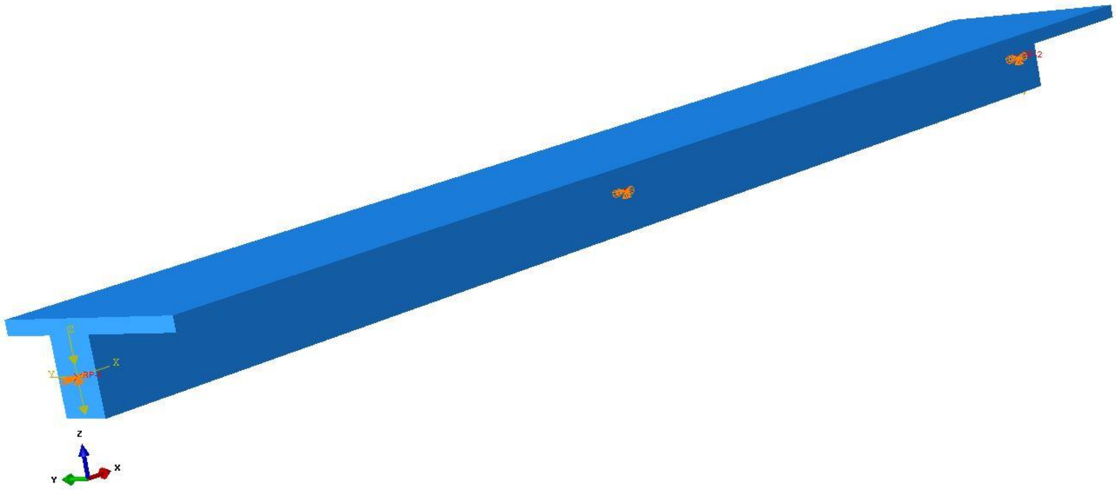


Figure 5.18 Continuous bridge model, L=20 m

The model consists of two beam elements because they are thought to represent the behaviour of the beam in the optimal way. The bridge in two spans of 20 m each is constructed as a 40 m long beam with boundary conditions at ends and at the middle as presented in Table 5.14

Table 5.14 Boundary Conditions, continuous bridge

Point	U1	U2	U3	UR1	UR2	UR3
RP-1	Restrained	Restrained	Restrained	Restrained	Free	Free
RP-2	Free	Restrained	Restrained	Restrained	Free	Free
Midpoint	Free	Restrained	Restrained	Restrained	Free	Free

In order to save computational time, only a segment in the middle of one of the spans is fully analysed.

5.2.3.2.1 Validation of the model

As for the simply supported bridge, the Brigade® model must be verified before any analyses can be done. The validation is performed in the same way as described in section 4.7.2. Table 5.15 shows the comparison between the models in Matlab® and Brigade® for the 2-span ballastless bridge of 20 m.

Table 5.15 Validation of the 3D model, continuous bridge, L = 20

	Matlab® model	Diagrams by Andersson and Svedholm	Brigade® model	Difference [%]
Mass [ton]	530		520	1.9
Displacement [mm]	13.0		13.5	3.7
1:st bending frequency [Hz]	10.02	10.0	9.93	1 (Matlab®) 0 (Andersson & Svedholm)
2:nd bending frequency [Hz]	40.06		37.7	5.9
3:rd bending frequency [Hz	90.14		78.12	12

5.2.3.2.2 Mode shapes and dynamic analysis for the L=20 m ballastless bridge

The first bending modes and first torsional mode are shown in figures, Figure 5.19 to Figure 5.22.

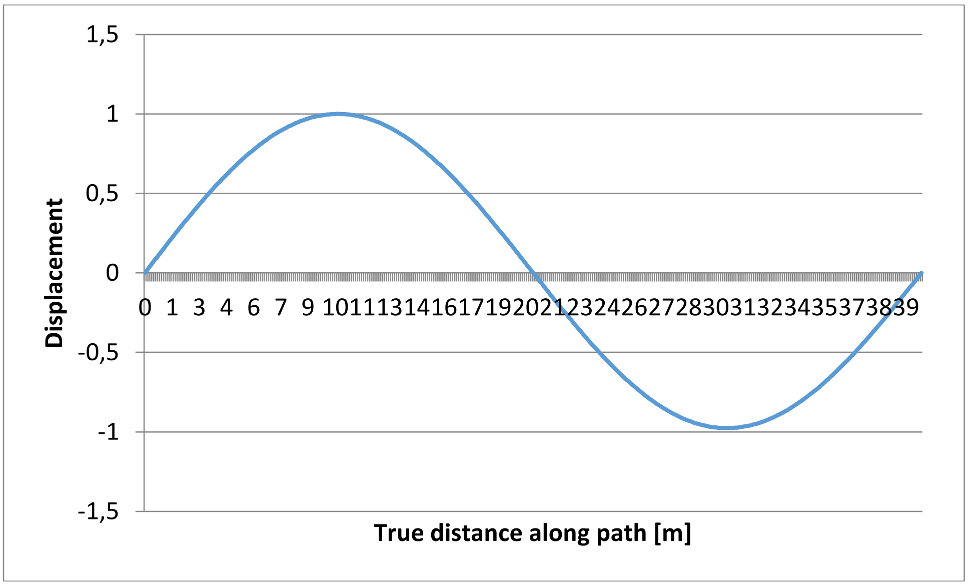


Figure 5.19 First bending mode for the continuous bridge L=20 m ballastless, f = 9.93 Hz

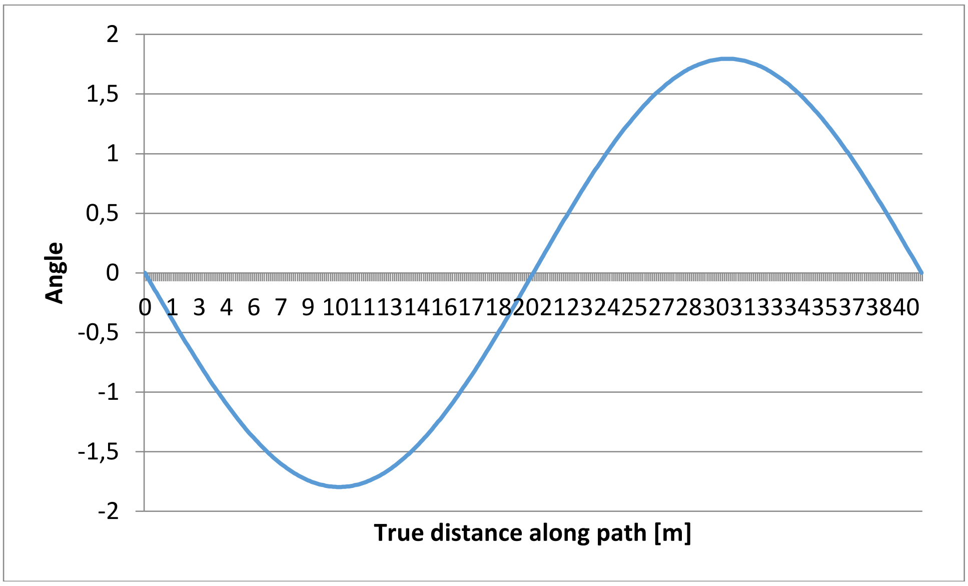


Figure 5.20 First torsional mode for the continuous bridge, L=20 m ballastless, f = 32,37 Hz

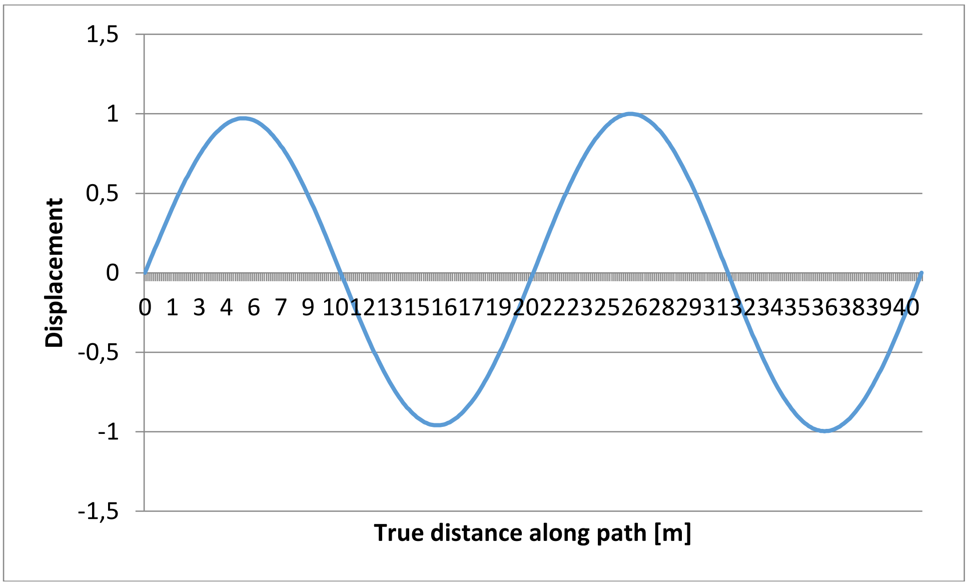


Figure 5.21 Second bending mode for the continuous bridge L=20 m ballastless f = 37.72 Hz

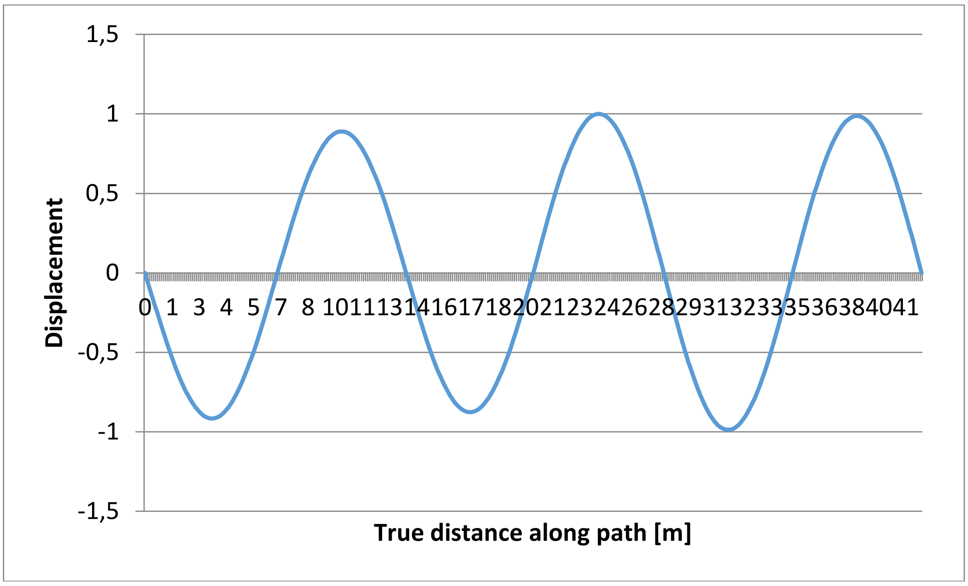


Figure 5.22 Third bending mode for the continuous bridge L=20 m ballastless f = 78,12 Hz

Table 5.16 shows the dynamic analysis performed.

Table 5.16 Dynamic analysis, continuous bridge, L=20 m, ballastless

Train HSLM	Worst Speed [km/h]	A3 [m/s2]	U3 *dyn. fac. [m/s2]	U3 max[mm]	U3 *dyn. fac.	U3 min [mm]	Comfort	Max A3	Max U3	Deck twist
A1	325	2,17	2,3002	1,05	1,47	0,94	Acceptable	OK	OK	OK
A2	335	1,23	1,3038	0,81	1,134	0,24	Good	OK	OK	OK
A3	360	1,71	1,8126	0,839	1,1746	0,758	Good	OK	OK	OK
A4	380	1,56	1,6536	1,04	1,456	0,95	Acceptable	OK	OK	OK
A5	**390**	**2,634**	2,79204	**1,24**	**1,736**	**1,13**	**Acceptable**	**OK**	**OK**	**OK**
A6	390	1,95	2,067	0,79	1,106	0,74	Good	OK	OK	OK
A7	390	1,55	1,643	0,85	1,19	0,76	Good	OK	OK	OK
A8	390	1,43	1,5158	0,92	1,288	0,85	Good	OK	OK	OK
A9	350	1,52	1,6112	1,11	1,555	1,027	Acceptable	OK	OK	OK
A10	325	1,84	1,9504	1,17	1,638	1,07	Acceptable	OK	OK	OK
ALL trains		2,634	2,79204	1,24	1,736	1,13	Acceptable	OK	OK	OK

As it can been seen, the worst train is the HSLM-5 at a speed of 390 km/h, which accelerates to approximately half of the limit according to the regulations. The dynamic requirements regarding vertical acceleration, deflection and torsion are fulfilled, as is the comfort criteria.

What seems to be the limiting check for this bridge is the ULS in the static design. Figure 5.23 shows the envelope maximum and minimum accelerations over speed ranging between 320-390 km/h. The diagram shows acceleration over speed for the HSLM_A05 train at node 83 because it is the one experiencing the highest acceleration for this train. This diagram shows which speed gives the most acceleration, and for this speed acceleration is plotted against time in Figure 5.24 for HSLM_A05 and a speed of 390 km/h.

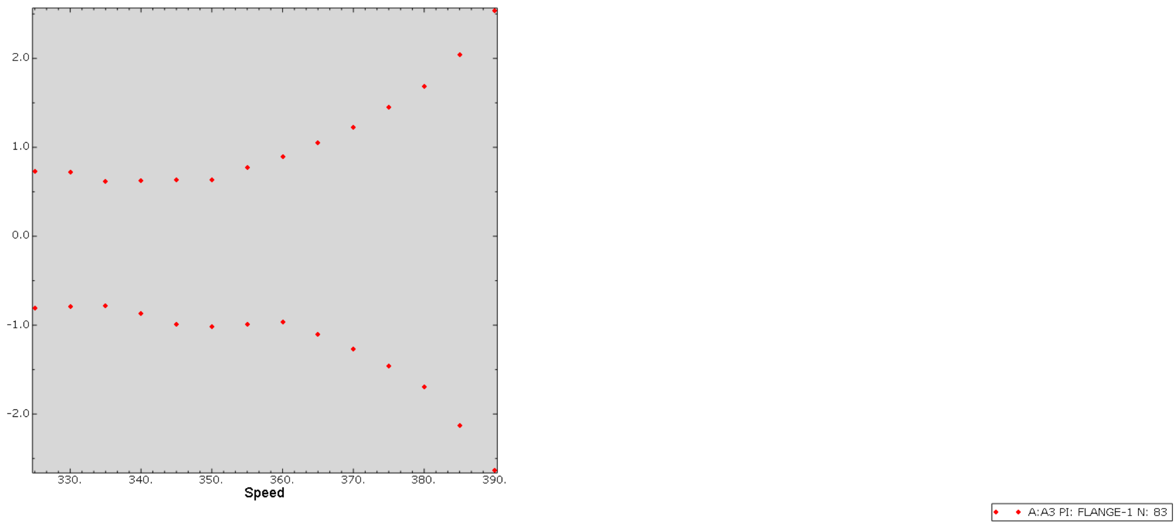


Figure 5.23 Max/min vertical acceleration at Node 83 for HSLM-A5, L=20 m

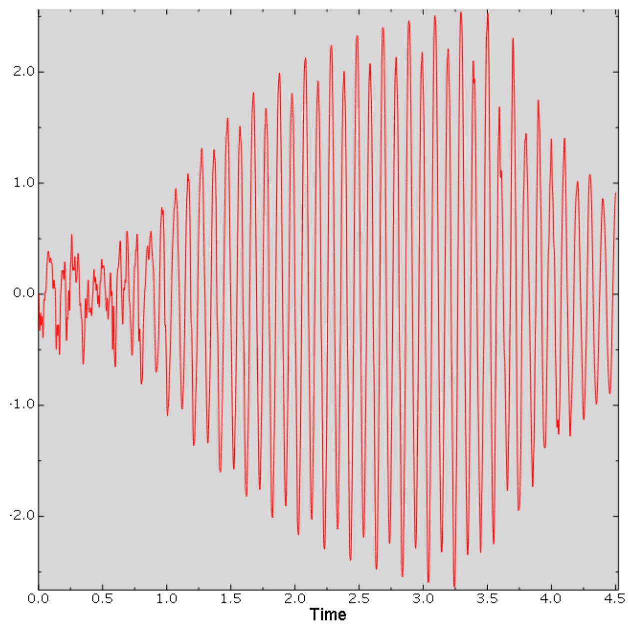


Figure 5.24 Vertical acceleration due to HSLM-A5 at 390 km/h, L=20 m

5.2.3.2.3 Mode shapes and dynamic analysis for the L=20 m ballasted bridge

The first bending modes and first torsional mode are shown in Figure 5.25 to Figure 5.28.

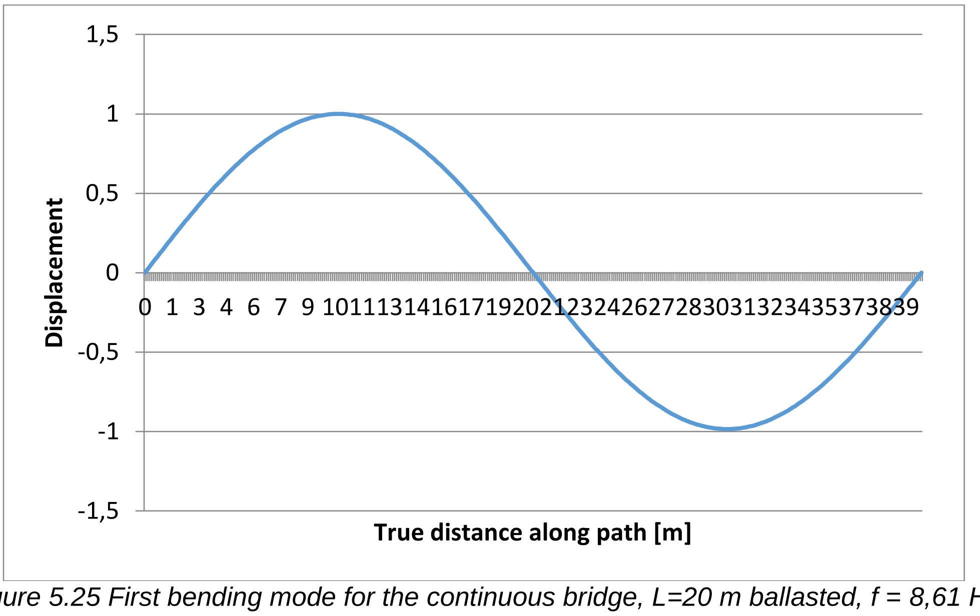


Figure 5.25 First bending mode for the continuous bridge, L=20 m ballasted, f = 8,61 Hz

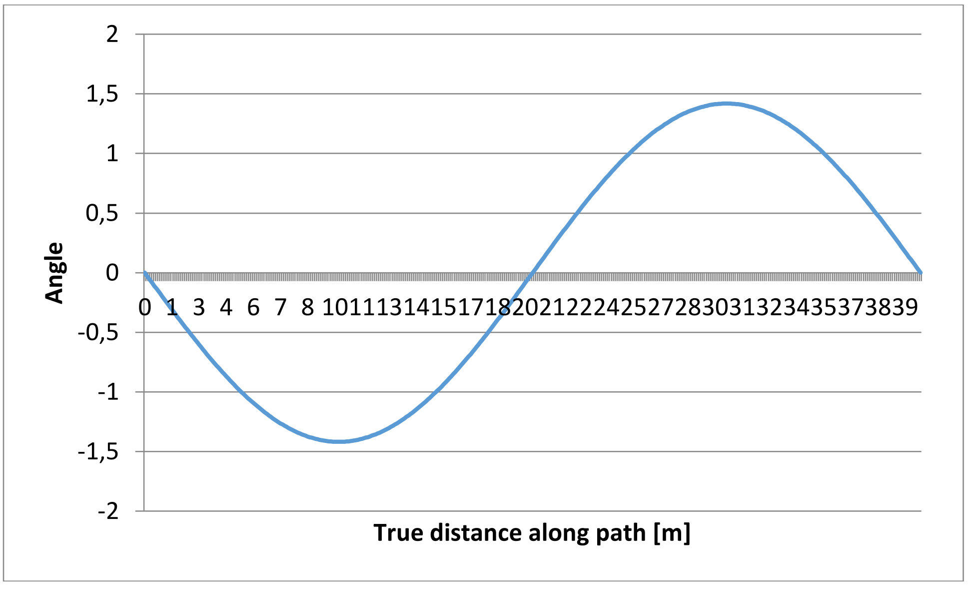


Figure 5.26 First torsional mode for the continuous bridge, L=20 m ballasted, f = 28,4 Hz

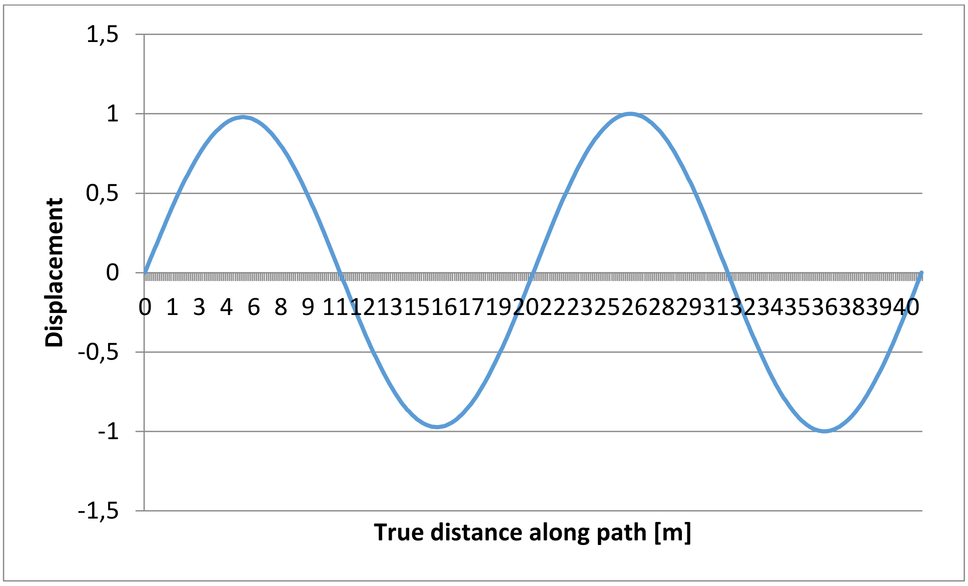


Figure 5.27 Second bending mode for the continuous bridge, L=20 m ballasted, f = 32,5 Hz

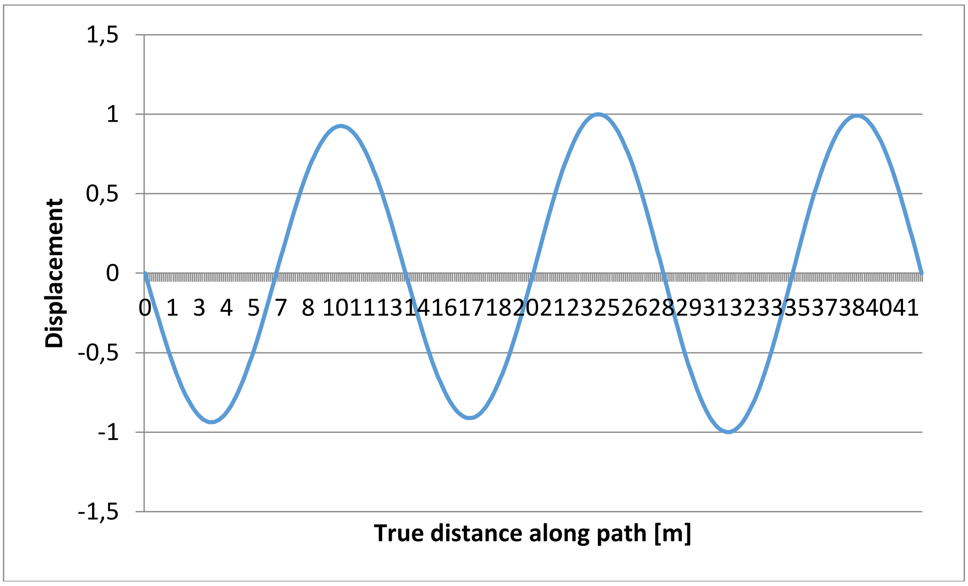


Figure 5.28 Third bending mode for the continuous bridge, L=20 m ballasted, f = 66,78 Hz

The dynamic analysis performed is presented in Table 5.17 below.

Table 5.17 Dynamic analyses, continuous bridge, ballasted, L = 20 m

Train HSLM	Worst Speed [km/h]	A3 [m/s2]	U3 * Dyn Fac [mm]	U3 [mm]	U3 * Dyn Fac [mm]	U3	Comfort	Max A3	Max U3	Max Torsion
A1	350	0,71	0,7526	0,59	0,826	0,57	Very Good	OK	OK	OK
A2	370	0,96	1,0176	0,64	0,896	0,6	Very Good	OK	OK	OK
A3	350	0,78	0,8268	0,63	0,882	0,6	Very Good	OK	OK	OK
A4	325	1,06	1,1236	0,88	1,232	0,84	Very Good	OK	OK	OK
A5	340	1,83	1,9398	1,11	1,554	1,05	Very Good	OK	OK	OK
A6	355	2,78	2,9468	1,23	1,722	1,17	Good	OK	OK	OK
A7	**370**	**3,11**	3,2966	**1,23**	**1,722**	**1,17**	**Good**	**OK**	**OK**	**OK**
A8	385	2,14	2,2684	1,01	1,414	0,97	Very Good	OK	OK	OK
A9	390	1,09	1,1554	0,8	1,12	0,76	Very Good	OK	OK	OK
A10	365	0,91	0,9646	0,86	1,204	0,81	Very Good	OK	OK	OK
ALL trains	370	3,11	3,2966	1,23	1,722	1,17	Good	OK	-	OK

As the table shows, the critical train is HSLM-7 giving the maximum acceleration which is still below the permitted limit. It can thus be concluded that the bridge fulfils the dynamic requirements. The following figures show which speed results in the maximum acceleration Figure 5.29 and the acceleration against time for this speed Figure 5.30.

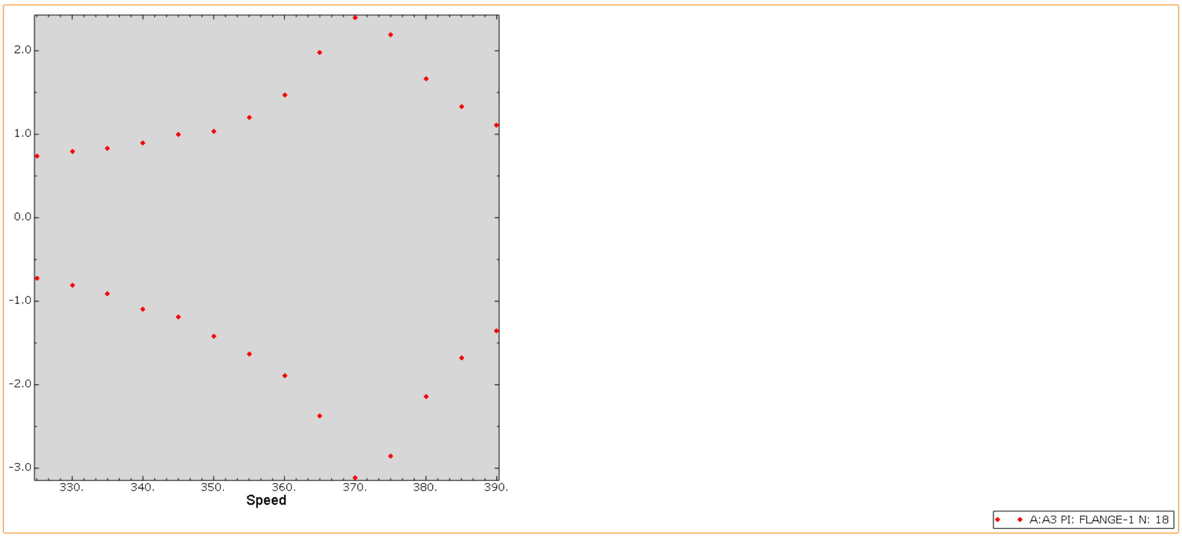


Figure 5.29 Max/min vertical acceleration for HSLM-A7, L=20 m

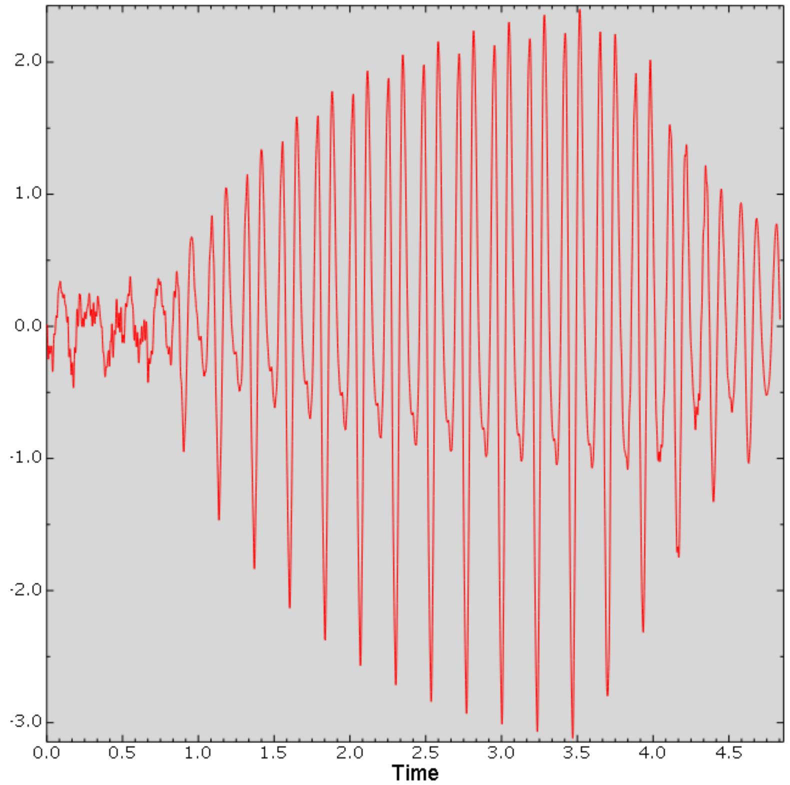


Figure 5.30 Vertical acceleration due to HSLM-A7 at 370 km/h, L=20 m

5.2.3.2.4 Mode shapes and dynamic analysis for the L=30 m ballastless bridge

The first bending modes and first torsional mode are shown in figures, Figure 5.31 to Figure 5.34.

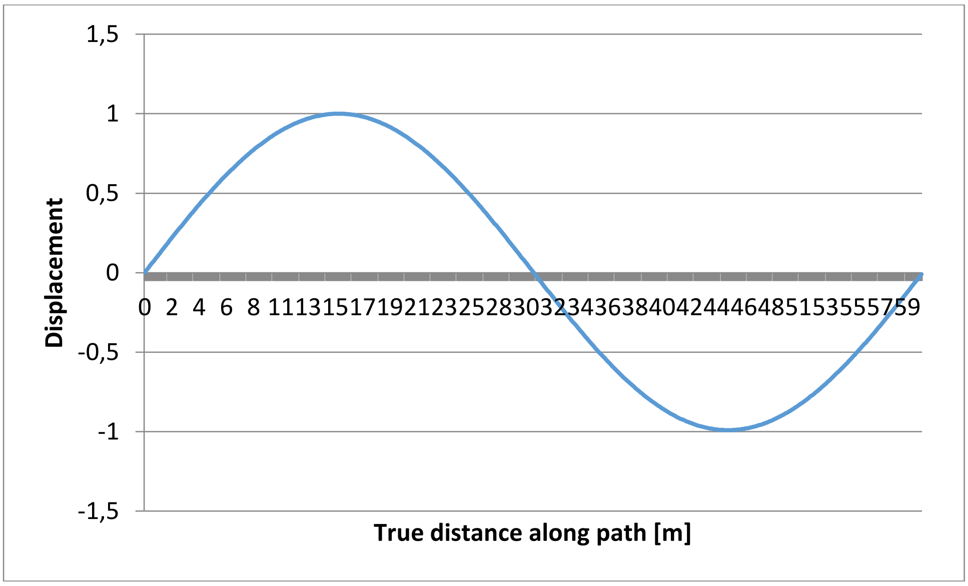


Figure 5.31 First bending mode for the continuous bridge, L=30 m and ballastless, f = 4,86 Hz

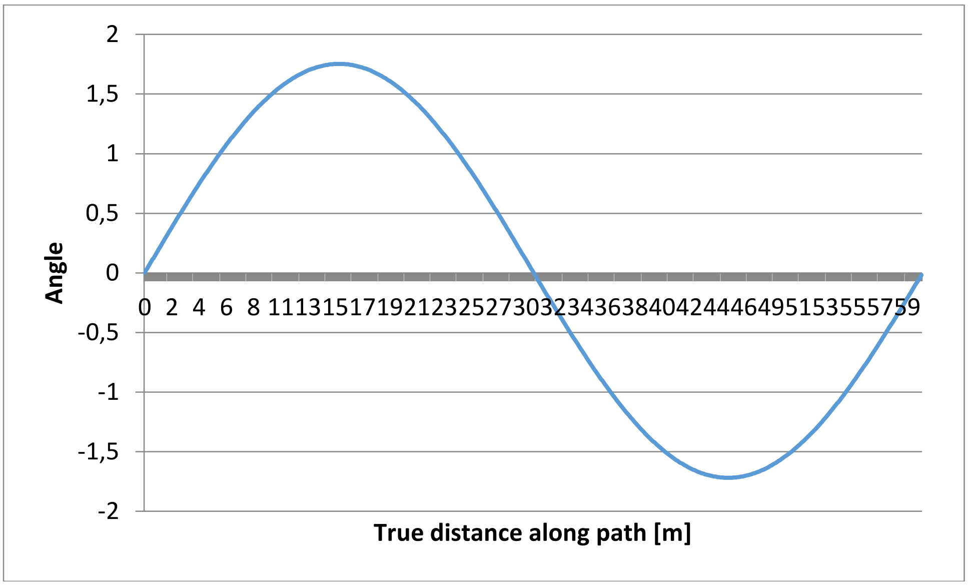


Figure 5.32 First torsional mode for the continuous bridge, L=30 m and ballastless, f = 22,5 Hz

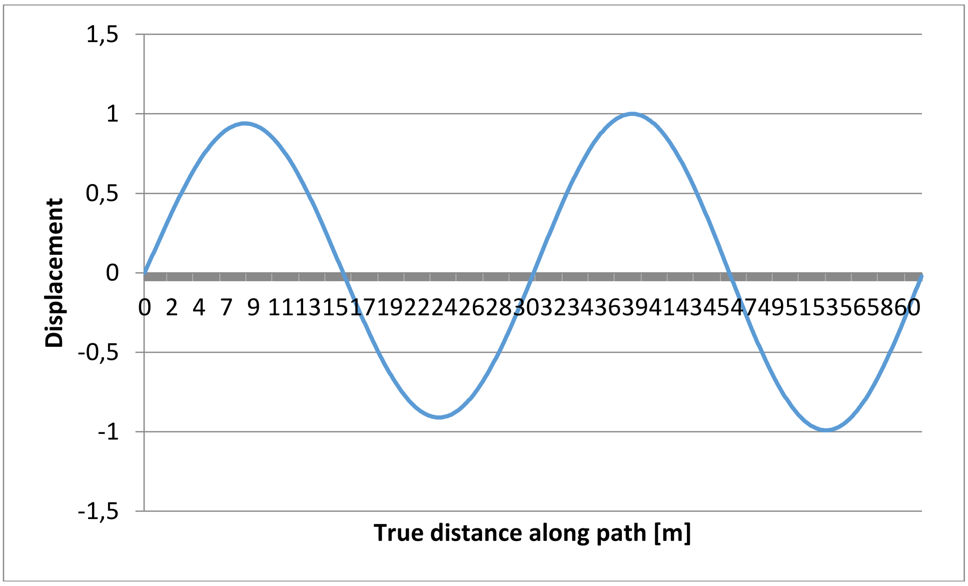


Figure 5.33 Second bending mode for the continuous bridge, L=30 m and ballastless, f = 19,02 Hz

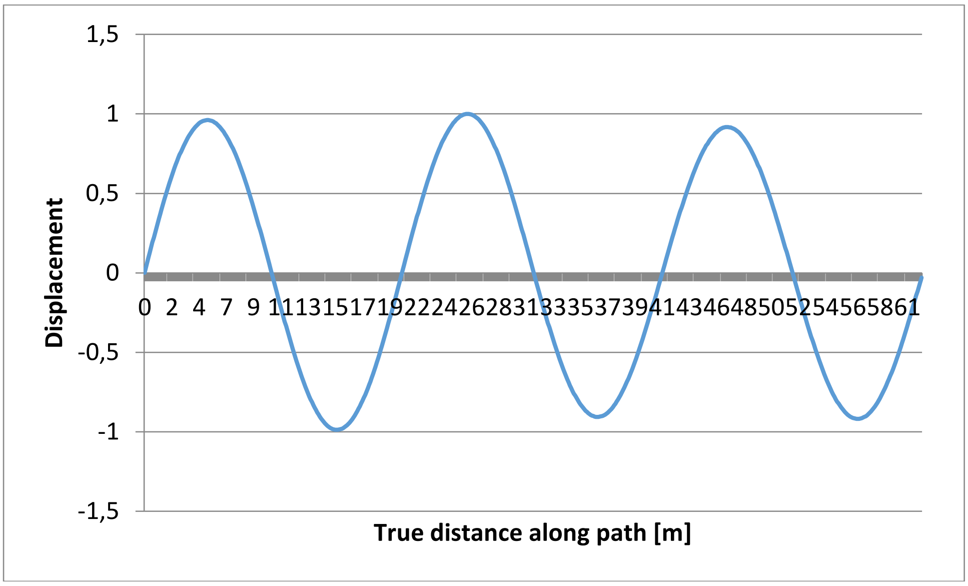


Figure 5.34 Third bending mode for the continuous bridge, L=30 m and ballastless, f = 40,03 Hz

The results for the required dynamic evaluation is presented in Table 5.18.

Table 5.18 Dynamic evaluation, continuous bridge, ballastless, L = 30 m

Train HSLM	Worst Speed [km/h]	A3 [m/s2]	U3* dyn.fac. [mm]	U3 [mm]	U3* dyn.fac.	U3	Comfort	Max A3	Max U3	Max Torsion
A1	325	2,53	2,6818	4,1	5,371	4,05	NOT OK	OK	OK	OK
A2	330	2,73	2,8938	4,62	6,052	4,52	NOT OK	OK	OK	OK
A3	330	1,27	1,3462	3,07	4,0217	2,99	NOT OK	OK	OK	OK
A4	370	2,43	2,5758	4,3	5,633	4,15	NOT OK	OK	OK	OK
A5	385	3,97	4,2082	6,088	7,9752	5,18	NOT OK	OK	OK	OK
A6	**390**	**4,03**	4,2718	**6,23**	**8,1613**	**6,02**	**NOT OK**	**OK**	**OK**	**OK**
A7	390	2,82	2,9892	4,01	5,2531	3,9	NOT OK	OK	OK	OK
A8	390	2,3	2,438	3,48	4,558	3,34	NOT OK	OK	OK	OK
A9	390	2,05	2,173	3,58	4,689	3,49	NOT OK	OK	OK	OK
A10	390	2,08	2,2048	3,58	4,689	3,49	NOT OK	OK	OK	OK
ALL trains		4,03	4,2718	6,23	8,722		NOT OK	OK	OK	OK

The data shows that the main criteria is fulfilled for every train. However, the comfort criteria is not achieved for the mentioned speeds. The worst train found is the HSLM-A6 at the highest speed, v = 390 km/h as it is shown in Figure 5.35. A plot at that mentioned speed with the worst vibrations and time is in Figure 5.36.

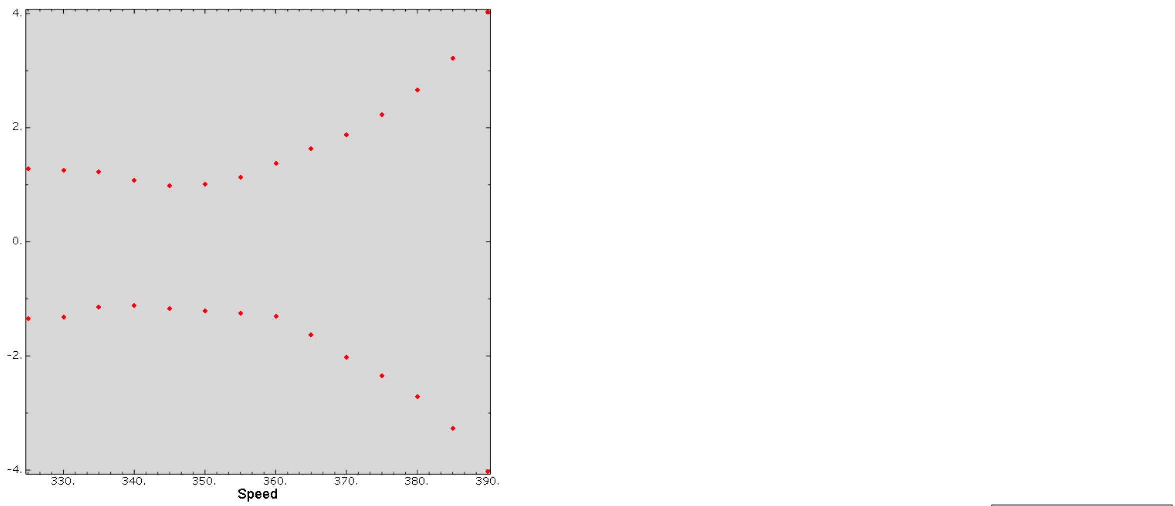


Figure 5.35 Max/min vertical acceleration for HSLM-A6, L=30 m

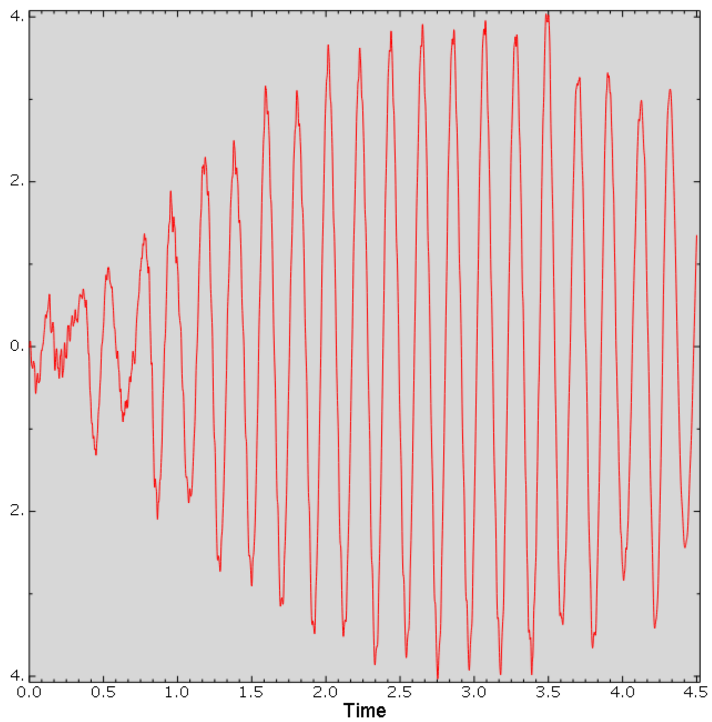


Figure 5.36 Vertical acceleration at node 31 due to HSLM-A6 at 390 km/h, L=30 m

5.2.3.2.5 Mode shapes and dynamic analysis for the L=30 m ballasted bridge

The first bending modes and first torsional mode are shown in figures, Figure 5.37 to Figure 5.40.

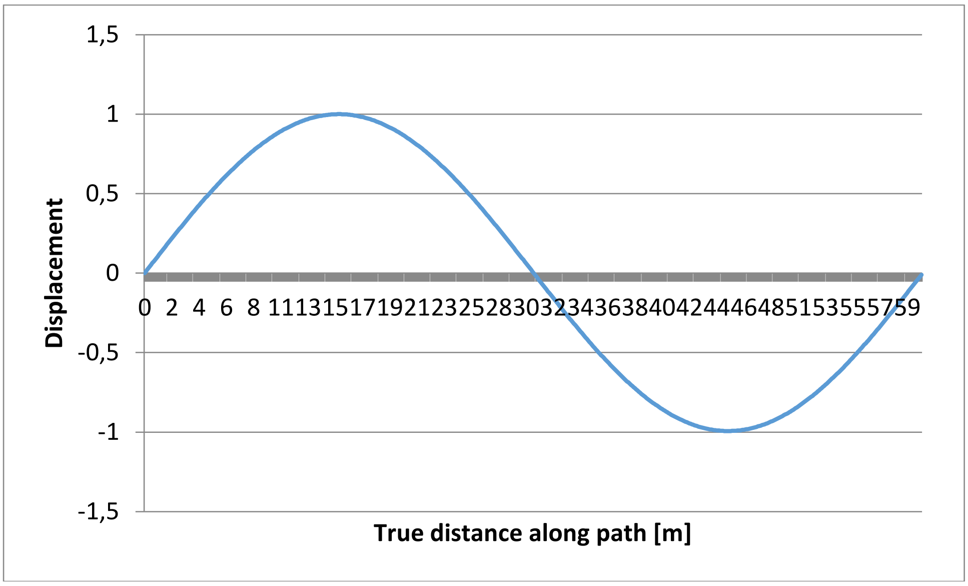


Figure 5.37 First bending mode for the continuous bridge, L=30 m and ballasted, f = 4,67 Hz

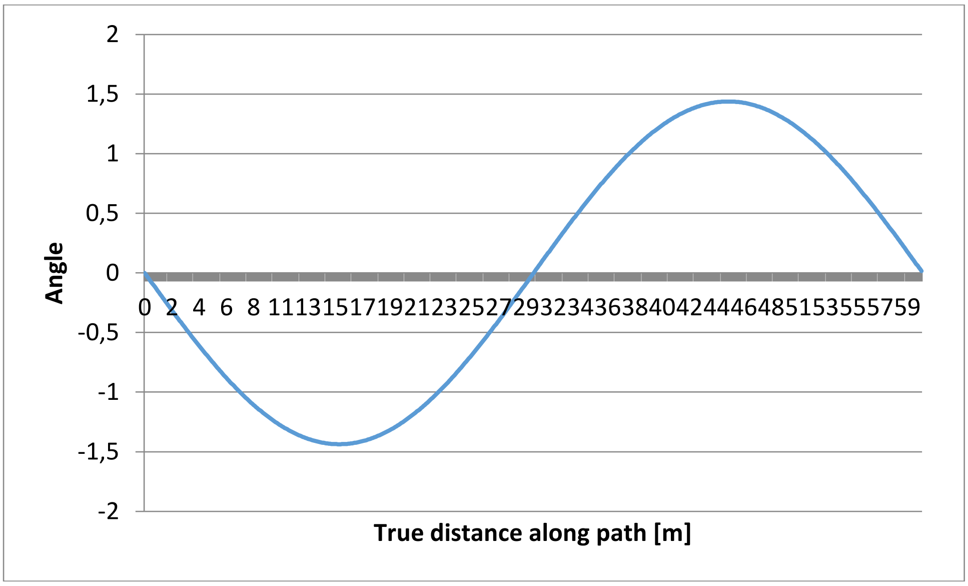


Figure 5.38 First torsional mode for the continuous bridge, L=30 m and ballasted, f = 21,08 Hz

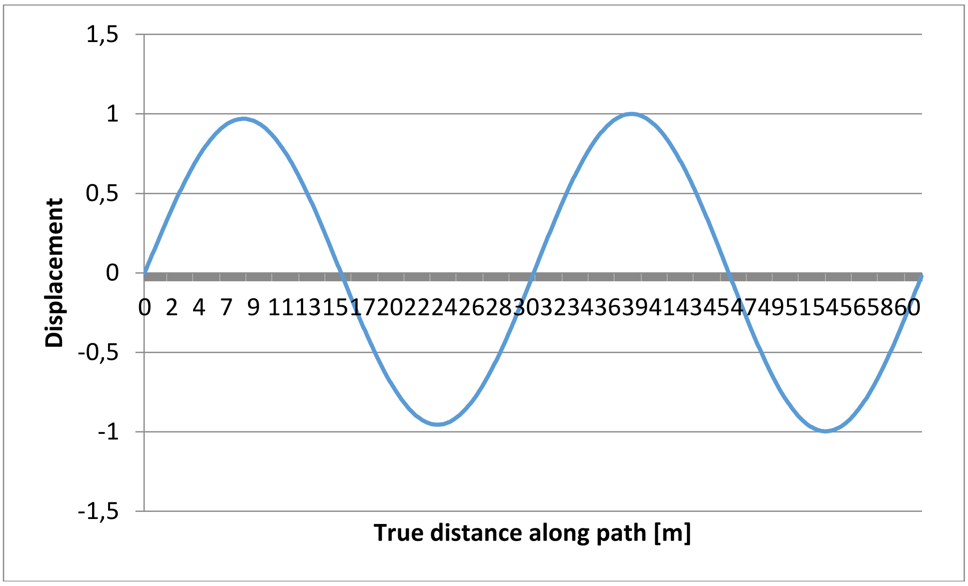


Figure 5.39 Second bending mode for the continuous bridge, L=30 m and ballasted, f = 18,034 Hz

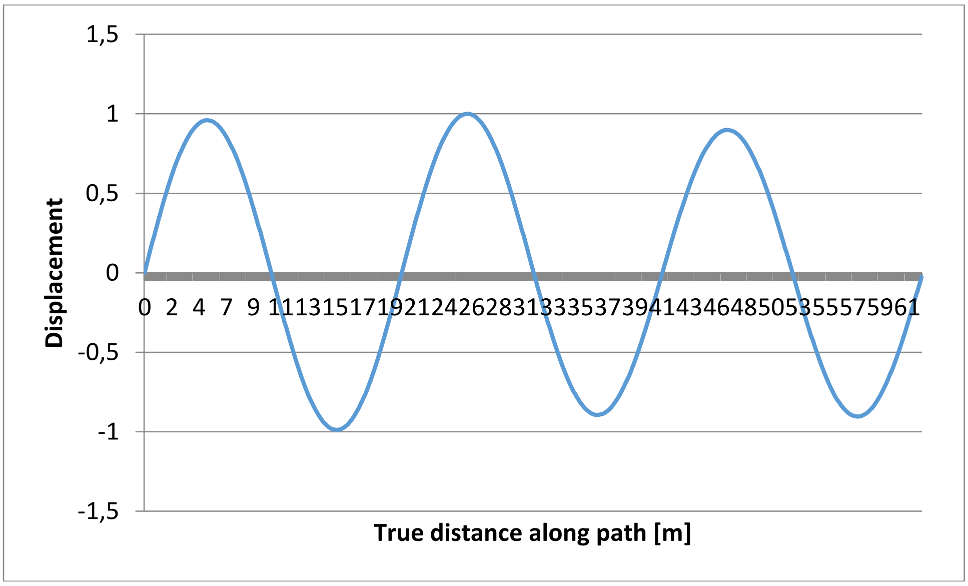


Figure 5.40 Third bending mode for the continuous bridge, L=30 m and ballastless, f = 37,82 Hz

The dynamic evaluation is shown in Table 5.19.

Table 5.19 Dynamic evaluation, continuous bridge, ballasted, L = 30 m

Train HSLM	**Worst Speed [km/h]**	**A3 [m/s2]**	**A3 * dyn.fac [m/s2]**	**U3,max [mm]**	**U3 * dyn.fac.**	**U3,min**	**Comfort**	**Max A3**	**Max U3**	**Max Torsion**
A1	295	2,25	2,385	4,016	5,6224	3,89	NOT OK	OK	OK	OK
A2	310	1,59	1,6854	3,17	4,438	3,1	NOT OK	OK	OK	OK
A3	250	1,04	1,1024	2,13	2,982	2,09	NOT OK	OK	OK	OK
A4	350	1,39	1,4734	2,93	4,102	2,8	NOT OK	OK	OK	OK
A5	360	2,3	2,438	4,11	5,754	3,92	NOT OK	OK	OK	OK
A6	375	3,214	3,40684	5,26	7,364	5,02	NOT OK	OK	OK	OK
A7	**390**	**3,85**	4,081	**6,14**	**8,6**	**5,86**	**NOT OK**	**NOT OK**	**OK**	**OK**
A8	390	2,3	2,438	3,74	5,236	3,64	NOT OK	OK	OK	OK
A9	390	1,77	1,8762	2,93	4,102	2,86	NOT OK	OK	OK	OK
A10	380	1,39	1,4734	2,39	3,346	2,35	NOT OK	OK	OK	OK
ALL trains	390	3,85	4,081	6,14	8,596	5,86	NOT OK	NOT OK	OK	OK

The dynamic requirements regarding deflection and torsion are fulfilled. However, they are in one case not fulfilled for vertical acceleration, nor for comfort in any of the cases.

The worst case, where the train HSLM-A7 passes at the speed 390 km/h is shown in figures, Figure 5.41 and Figure 5.42. As it can be seen in the first figure, the section does not fulfil the criteria for the HSLM-A7 at speeds over 385 km/h, this mean that they are in the part that is above the maximum allowed speed for the track and meaning that the case that it would happen is rarely occurring.

The comfort criteria is not fulfilled for any case, meaning this cross section does not fulfil the requierments.

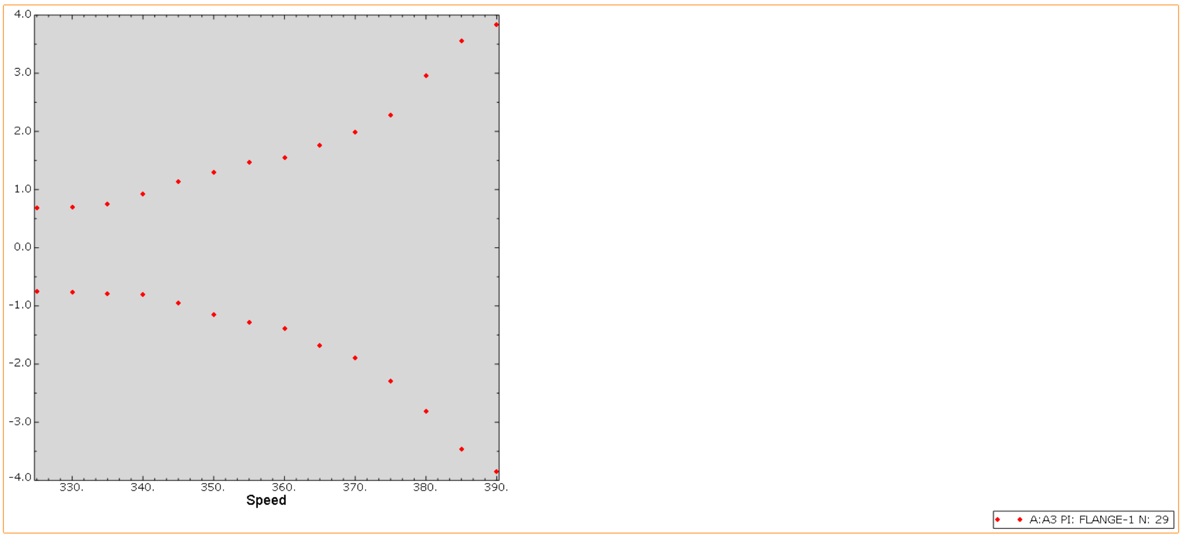


Figure 5.41 Max/min vertical acceleration for HSLM-A7, L=30 m

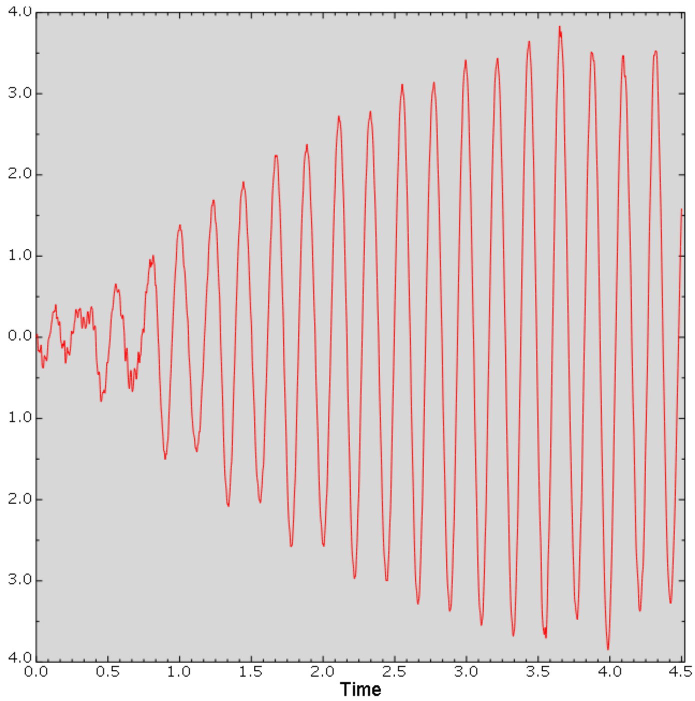


Figure 5.42 Vertical acceleration due to HSLM-A7 at 390 km/h, L=30 m

6 Discussion and conclusions

6.1 Discussion

6.1.1 Simply supported bridge, T section

The cross section cannot be significantly reduced by removing the weight of the ballast. The quantity of the reinforcement can however be reduced.

The optimal cross section from the static analyses does not fulfil the dynamic requirements for L = 20 m neither for the conventional nor the ballastless option, meaning that bridges of this length are limited by the dynamic requirements. Similar to the static design, the optimal section from the dynamic point of view is not reduced by removing the ballast, although it requires fewer reinforcement bars.

Regarding the L = 30 bridge, only the ballasted bridge needed a slight increase of section dimensions in order to fulfil the dynamic requirements. It is thus concluded that this section is to a higher degree governed by the static requirements.

As a conclusion, removing the ballast does not result in a reduced section, but some saving can be made by reducing the amount of reinforcement steel needed to fulfil the requirements. However, one should keep in mind that the dynamic requirements for a ballasted track are more strict (maximum allowed acceleration are 3,5 and 5 m/s^2 for ballasted and ballastless tracks, respectively; ballasted tracks are dimensioned for heavier traffic), meaning that a given cross section has a higher capacity if ballasted, provided it has the necessary reinforcement. In the end, it is the constructor’s decision to choose between a more resistant cross section and one requiring less reinforcement steel.

6.1.2 Composite bridge

The alternative where the two T sections are connected has been dimensioned in such a way that it fulfils the dynamic requirements in 2D analyses. But when the 3D effects were taken into account the accelerations were found to be too high and the section could not be accepted. Since the analysed section had the same dimensions as the one in simple T which fulfilled the requirements, it can be concluded that this way of constructing the bridge requires a heavier cross section. An alternative is to model the cross section with different boundary conditions and restrain it from torsion which would increase its stiffness. This would decrease its susceptibility to 3D effects.

6.1.3 Continuous bridge

In the same way as for the simply supported, a bridge in two spans cannot be designed much slimmer by removing the ballast, but the ballastless alternative saves the amount of reinforcement needed.

All the tested alternatives were optimised for the statics and when subjected to the dynamic analyses in 2 dimensions, they fulfilled the criteria, with the L = 30 m bridges somewhat closer to the limit values. Consequently, in the 3D analyses, it was found that these bridges do not fulfil the comfort criterion, and the ballasted alternative encounters problems with the acceleration as well.

6.1.4 Difference 2D and 3D analyses

An interesting observation was made when performing cross section checks for the composite bridge in 2 and 3 dimensions. A cross section that fulfilled the dynamic requirements by a large margin when checked in 2 dimensions, was, in the 3D analyses, found to be unsatisfactory due to the acceleration being too high.

The design diagrams used for the 2D analyses (Andersson & Svedholm, 2016) use the mass m and the stiffness EI as input data and calculate whether the bridge fulfils the dynamic requirements or not. In this case, the inertia I for a composite bridge was simply calculated using Steiner's theorem for the given cross section. The comparison with a 3 dimensional analyses shows that this way of calculating the stiffness is much too simplified. Clearly, the whole section is not contributing to resist the acceleration demands and in order to analyse a section like this it is crucial to estimate the effective inertia correctly, otherwise one is at risk of overestimating the stiffness.

Further, the composite section was modelled with very conservative boundary conditions. If the stiffness to torsion would be increased, the results between 2D and 3D analyses would be more similar.

6.2 Conclusions

(i) Simply supported, as well as bridges in two spans, will have somewhat smaller dimensions if the weight of ballast is removed, and more importantly they will require less reinforcement.

(ii) For simply supported bridges, the design is governed by the dynamic requirements. Shorter bridges require more thorough dynamic analyses due the enhanced the risk of resonance as a result of their lower mass.

(iii) Bridges in two spans are slightly stiffer than simply supported ones, and for shorter spans, they are less susceptible to dynamic effects. The longer bridges however fail to fulfil the criteria when subjected to a 3D analyses.

(iv) Bridges with a composite cross section (two T sections carrying one track each) are sensitive to 3D effects. If not restrained against torsion, they will have very unfavourable dynamic behaviour. Important to evaluate the effective mass and inertia correctly.

6.3 Proposed design

Due to the previous conclusions, the selected design would be a ballastless bridge, due to the fact that the amount of material used is the least. It is also recommended to avoid designing the composite cross sections, as it would result in the increase of dynamic and static problems and a need for a bigger cross-section. If the bridge needs to carry two tracks, it is better to construct it in such a way that the T sections are not connected.

6.4 Further research

The following ideas are proposed as further research on the topic:

- Check variable cross sections.
- Dynamics for a multi-span continuous bridge with more than 2 spans. The dynamics seem to demand less when the number of spans are increased.
- Change the boundary conditions for the simple supported bridges, making them more restrictive or actually modelling the real foundations, which will affect the damping and the behaviour of the bridge.
- Model the middle support as a column for the continuous bridges in order to obtain a better idea of how much the support is going to affect the structure.
- Model the composite bridge with more torsional stiffness in order to obtain a more favourable behaviour of the bridge.
- Study the difference between the effective inertia and the basic inertia used in the 3D to do a more accurate 2D analysis.

References

Andersson, A., & Svedholm, C. (2016). *Designdiagram för förenklad dynamisk kontroll av järnvägsbroar*. Stockholm: KTH.

Beeby, A. W., & Narayanan, R. S. (2009). *Designers Guide to Eurocode 2: Design of Concrete Structures*. London: Thomas Telford Publishing.

Chopra, A. K. (2001). *Dynamic of Structures - Theory and Applications to Earthquake Engineering*. Prentice Hall.

Dassault Systèmes Simulia Corp. (2012). *Abaqus 6.12 User's Manual.* Providence, RI, USA.

EN-1990. (2002). *Eurocode: Basis of structural design. EN-1990*. Brussels: European Committee for Standardization, CEN.

EN-1991-1-1. (2002). *Eurocode 1: Actions on structures. General Actions - Part 1-1: Densities self-weight, imposed loads for buildings. EN-1991-1-1*. Brussels: European Comitee for Standarization, CEN.

EN-1991-2. (2003). *Eurocode 1: Actions on structures - Part 2: Traffic loads on bridges. EN-1991-2*. Brussels: European Committee for Standardization, CEN.

EN-1992-1-1. (2004). *Eurocode 2: Design of Concrete Structures - Part 1-1. General rules and rules for buildings. EN-1992-1-1*. Brussels: European Comitee For Standardization, CEN.

EN-1992-2. (2005). *Eurocode 2: Design of Concrete Structures - Part 2: Concrete Bridges - Design and detailing rules. EN-1992-2*. Brussels: European Comitee For Standardization, CEN.

JSC, E. C. (2012). *Bridge Design to Eurocodes. Worked Examples*. Luxembourg: European Commision Joint Research Centre.

Karoumi, R. (2015, Autumn semester). Lectures on Structural Dynamics. Stockholm, Sweden: KTH.

Olsson, K., & Pettersson, J. (2010). *Fatigue Assessment Methods for Reinforced Concrete Bridges in Eurocode*. Göteborg: Chalmers University of Technology, Department of Civil and Environmental Engineering. Division of Structural Engineering. Concrete Structures.

Scanscot Technology . (2016, March). *Brigade Plus*. Retrieved from http://www.scanscot.com

SIS. (2002, February). *Eurokod: Grundläggande dimensioneringsregler SS-EN 1990* . Stockholm: SIS Förlag AB. Retrieved from sis.se.

Sundquist, H. (2008). *Torsion of concrete beams. Report 119 TRITA-BKN*. Stockholm: Department of Structural Design and Bridges, KTH.

Trafikverket[1]. (2016, January). Retrieved from Trafikverket.se: http://www.trafikverket.se/nara-dig/Sodermanland/projekt-i-sodermanlands-lan/Ostlanken/Om-projektet-Ostlanken-/

Trafikverket[2]. (2016, January). *Trafikverket.se*. Retrieved from http://www.trafikverket.se/nara-dig/Sodermanland/projekt-i-sodermanlands-lan/Ostlanken/Om-projektet-Ostlanken-/

TRVK Bro 11. (2011). *Trafikverkets tekniska krav Bro*. Stockholm: Trafikverket.

UIC [1]. (2016, February). *UIC*. Retrieved from uic.org: http://www.uic.org/highspeed

UIC [2]. (2016, February). *uic*. Retrieved from uic.org: http://www.uic.org/IMG/jpg/carte_europe_2013.jpg

UIC-1. (2016, February). *UIC, The world*. Retrieved from UIC: http://www.uic.org/High-Speed-principles-and

Appendices

A. Appendix A: Matlab® codes, static design

In this appendix, all the Matlab® functions will be presented in alphabetic order. All of them follow the flow chart presented in Figure 8.1.

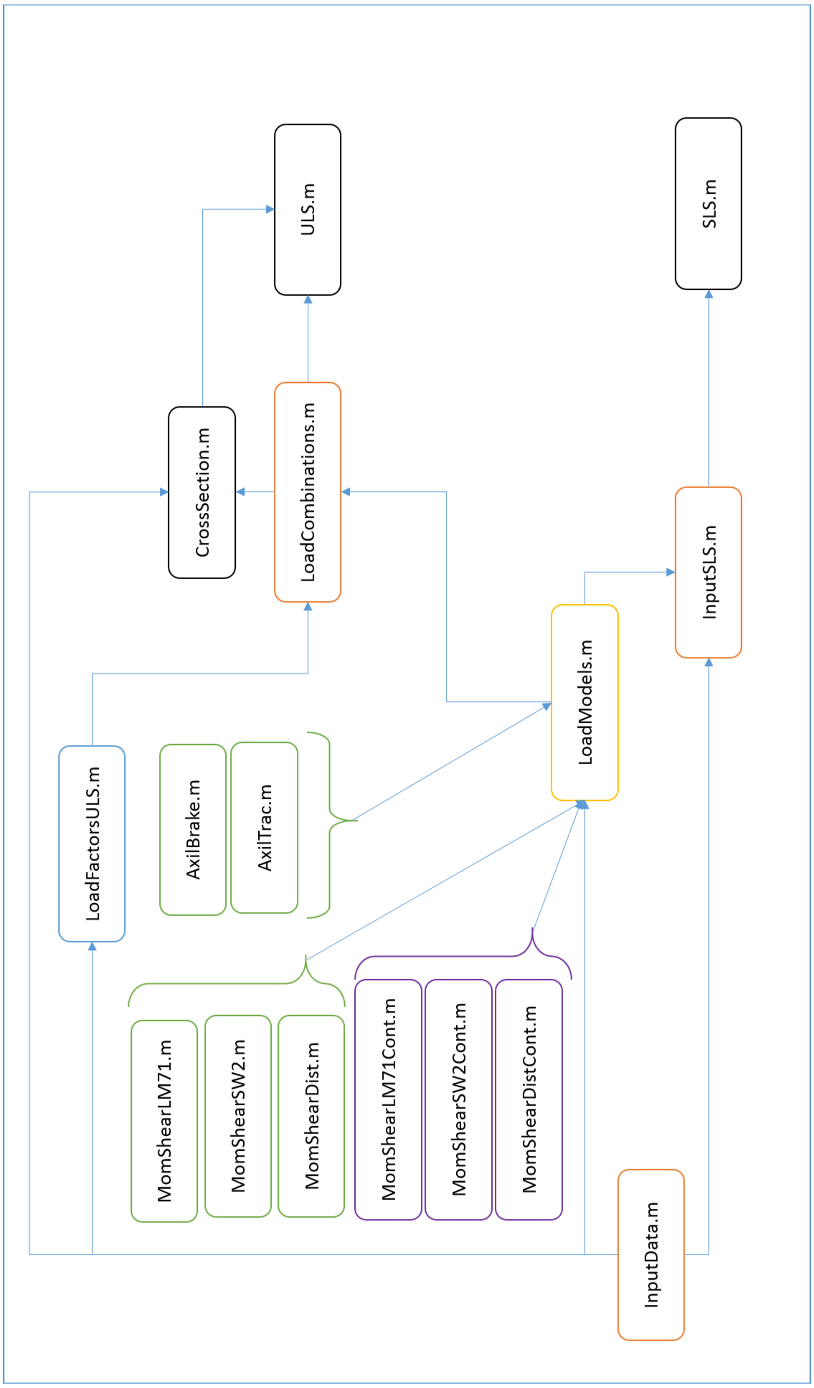


Figure 8.1 Flow chart of the static Matlab® codes

Axil Brake

```
%AxilBrake

%Effect of horizontal loading due to traction
%Function [N] = AxilBrake(Q,Lab,x)
%Indata: Q = Axial force due to breaking [N/m]
%         L = Span length [m]
%         x = Position of the section force [m]
%         Lab = Influence length [m]
%Output: N = Axial force at x [N]

function [N] = AxilBrake(Q,Lab,x)
N = min(Q*Lab,6000E3);
end
```

Axil Traction

```
%AxilTrac

%Effect of horizontal loading due to traction
%Function [N] = AxilTrac(Q,Lab,x)
%Indata: Q = Axial force due to acc/break [N/m]
%         L = Span length [m]
%         x = Position of the section force [m]
%         Lab = Influence length [m]
%Output: N = Axial force at x [N]

function [N] = AxilTrac(Q,Lab,x)
N = min(Q*Lab,1000E3);
end
```

Cross Section

```
%Cross section check for a T-Beam. Checks if the section can be
calculated
%as a rectangular section
%Input Data Function
function [NA] = CrossSection()
[bf,tf,hw,tw,h,bef,l_0,fyk,fyd,fck,alfa_cc,alfa_ct,bballast,gammaC,gammaD
,T_b,fcd,Es,Ecm,fctm,fctk_0_05,fctd,c,d,L,alfa,ep_cu3,Aconcrete,qconcrete
,qballast,Con,Train,freq,ep_uk,t,fcm,gammaF_Fat,gammas_Fat,sigma_rsk,Main
tenance,phi,phi_neg,SEC,SPAN,Ltot]=InputData();
[Mmid,Msup_as,Med,Med_min,Vmid_as,Vsup,Ved,Ted,Ned,DIFFx,Vderail,Mderail,
sigmamax,sigmamin,dyn_fat]=LoadCombinations(bf,tf,hw,tw,h,bef,fyk,fyd,fck
,alfa_cc,alfa_ct,gammaC,fcd,Es,Ecm,fctm,c,d,L,alfa,ep_cu3,bballast,Aconcr
ete,qconcrete,qballast,Con,Train);
disp(['____________']);
disp(['T-beam']);

q=(1-sqrt(1-Med/(0.45*bef*d^2*fcd)));
NA=q*d;                                       %NA computed for a Tbeam in ULS

disp(['____________']);
```

```
disp(['The Neutral axis is placed (from the top) NA=', num2str(NA),'
m']);

if NA/tf <=1
    disp('The beam can be computed as a rectangular section');
    disp(['The Neutral axis is placed (from the top) NA=',
num2str((NA*100)/tf),' %']);
 else
    disp('%%%%%%%%%%%%%%%%%%%%WARNING%%%%%%%%%%%%%%%%%%%%%%%%%%%%');
    disp('%%%%%%%%%%%%%%%_________________%%%%%%%%%%%%%%%%%%%');
    disp('NOT OK! Change your input or choose another method to
calculate');
    disp(['The Neutral axis is placed (from the top) NA=',
num2str((NA*100)/tf),' %']);
    disp('%%%%%%%%%%%%%%%%%%%%WARNING%%%%%%%%%%%%%%%%%%%%%%%%%%%%');
    disp('%%%%%%%%%%%%%%%_________________%%%%%%%%%%%%%%%%%%%');
end
```

Input Data.

```
% File with the indata

%
function[bf,tf,hw,tw,h,bef,l_0,fyk,fyd,fck,alfa_cc,alfa_ct,bballast,gamma
C,gammaD,T_b,fcd,Es,Ecm,fctm,fctk_0_05,fctd,c,d,As,L,alfa,ep_cu3,Aconcret
e,qconcrete,qballast,Con]=InputData
% bf,tf: width and thickness of the upper flange [m]
% hw,tw: height and thickness of the web [m]
% h: total height of the beam [m]
% fyk: Characteristic steel tensile strength of the bars [Pa]
% fyd: Design steel tensile strength of the bars [Pa] Taken from EN 1992-
1-1 Annex C
% fck: Characteristic concrete compresive strength of the concrete [Pa]
% alfa_cc: Recommended value 0.85
% alfa_ct: Recommended value 1.0
% gammaC: Safety factor
% gammaD: Safety factor
% T_b: Life span [Years]
% fcd: Design concrete compresive strength of the concrete [Pa]
% Es: Steel Young Modulus [Pa]
% Ecm: Concrete Young Modulus [Pa]
% fctm: Mean concrete tensile strength [Pa] %table 3.1 En 1992-1-1
Depends on fck
% c: Minimum cover [m]
% c_minb: Minimum cover due to bonding [mm](table 4.2 EN1992-1-1 Agg size
<32mm)
% c_mindur: Minimum cover due to durability [mm]
% d: Distance from the top to the reinforcement [m]
% As: Area reinforcement[m2]
% L: Span length [m]
% alfa: Ratio between Young Modulus
% ep_cu3: Strain Value extracted from table if fck<=50 MPa [mm/m]
% b: Assumed width of the ballast at the top (below rails)
% bballast:Mean width of the ballast
% hballast: Assumed height of the ballast
% Aconcrete: Area of the concrete section (TOTAL) [m2]
% rho_concrete = concrete density [N/m3]
```

```
% rho_ballast: ballast density [N/m3]
% qconcrete: Self weight load of the concrete [N/m]
% qballast: Weight load of the ballast [N/m]
% b_slab: Width of the concrete slab
% h_slab: Height of the concrete slab
% Con: Set to 1 for the Conventional line (Ballast) and set to 0 for the
HST (Unballasted)
% Train: Variable train load chosen: if 0, no train is on. if 1 only LM71
passes. If 2 SW2 train passes.
function[bf,tf,hw,tw,h,bef,l_0,fyk,fyd,fck,alfa_cc,alfa_ct,bballast,gamma
C,gammaD,T_b,fcd,Es,Ecm,fctm,fctk_0_05,fctd,c,d,L,alfa,ep_cu3,Aconcrete,q
concrete,qballast,Con,Train,freq,ep_uk,t,fcm,gammaF_Fat,gammas_Fat,sigma_
rsk,Maintenance,phi,phi_neg,SEC,SPAN,Ltot]=InputData

Cas = 1;  % for checking different configurations of the bridge
          % where Case = 1 and Case = 2 are the extreme cases pf 20 and
30
          % and case 3 is used to check intermediate lengths
%Geometric parameters
if Cas==1
 bf=4.5;  %Case 1
 tf=0.4;  %Case 1
 hw=2.4;  %Case 1
 tw=1.8;  %Case 1
 L=30; %Always 30m
 elseif Cas==2
 bf=4.5; %Case 2
 tf=0.4; %Case 2
 hw=2.2;   %Case 2
 tw=1.8;   %Case 2
 L=20; %Always 20m
 elseif Cas==3
 bf=4.5; %Case 3
 tf=0.4; %Case 3
 hw=2.2;   %Case 3
 tw=1.8 ;   %Case 3
 L=25; %Variable max 30m min 20m
end

%Variables
SPAN = 2; % Span = 1 single span, Span = 2 2-span bridge
Con=1; %Choose between 1(ballasted) or 0 (no ballast)
SEC=1; %SEC=1 single cross section/SEC=2 Double cross section
Train=1; %If 0, no train is on. if 1 only LM71 passes. If 2 SW2 train
passes.
freq=100; % number of trains per day
h=hw+tf;
Ltot = SPAN*L;

%Steel Class input
Es=200e9;
fyk=500e6;
fyd=435e6;
phi = 32E-3;    % Diameter of reinforcement bars
phi_neg = 32E-3; % Diameter of reinforcment bars for negative moment
%Concrete input
Concrete=35; %Choose between C30 or C35.
```

```
if Concrete==30
fck=30e6;
Ecm=33e9;
fctm=2.9e6;
fctk_0_05=2e6;
elseif Concrete==35
fck=35e6;
Ecm=34e9;
fctm=3.2e6;
fctk_0_05=2.2e6;
else
    disp '%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%';
    disp '%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%';
    disp 'The concrete you are using needs to be added manually';
    disp '%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%';
    disp '%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%';
end
rho_concrete = 25e3;% Concrete density
alfa_cc=0.85;
alfa_ct=1;
gammaC=1.5;
fcd=alfa_cc*fck/gammaC;
fctd=alfa_ct*fctk_0_05/gammaC;
fcm = fck+8E6;
ep_uk=5;            % Percentage Characteristic strain at full force
ep_cu3=3.5;
t = 28;             % Age of concrete [days]
%Other parameters
alfa=Es/Ecm;
b = 2.4;            % Assumed width of the ballast at the top (below
rails)
bballast=(bf+b)/2; % Mean width of the ballast
hballast = 0.6;    % Assumed height of the ballast
rho_ballast = 20e3;% Density of the ballast
gammaD=1;
T_b=120;
c_minb= 25;
c_mindur= 30; %Table 4.4N (Exposure class [XC4 top flange]4.1)
(Structural class [S4]table 4.3N)
c1=max([c_minb c_mindur 10]); %[mm]
c=c1/1e3;%[m]
d=h-c;
%Fatigue parameters
gammaF_Fat=1;        % Recommended value SS-EN 1992-1-1; 6.8.4(1)
gammas_Fat=1.15;     % Partial coefficient for steel subjected to
fatigue.Recommended value SS-EN 1992-1-1; 2.4.2.4
sigma_rsk=162.5e6;  % [Pa] Straight and bended bars wöhler curves SS-EN
1992-1-1. Figure 2. table 6.3N
%Loads
Aconcrete=bf*tf+hw*tw;
b_slab = 2.4;    % Width of the concrete slab
h_slab = 0.3;    % Height of the concrete slab
Aslabconcrete=b_slab*h_slab;
if Con==1
 Aslabconcrete=0;
elseif Con==0
 Aslabconcrete=Aslabconcrete;
end
qconcrete=(Aconcrete+Aslabconcrete)*rho_concrete;
```

```
qballast=hballast*bballast*rho_ballast;

Maintenance=1; %Maintenance=1 Carefully maintained track Maintenance=2
Normalm maintenance.
%For a T beam Calculations for bef when assuming that it can be
calculated as a rectangular cross section
l_0=0.7*L; %Simply supported Figure 5.2 1992-1-1
bi=(bf-tw)/2;
befi=0.2*bi+0.1*l_0;
if befi<=0.2*l_0
 befi=0.2*bi+0.1*l_0;
else
befi=0.2*l_0;
end

if befi<=bi
befi=befi;
else
befi=bi;
end

bef=2*(befi)+tw; %[m]
```

Input SLS

```
% Calculation of the load combinations for SLS
% Function
%
function[Ned_SLS]=InputSLS(bf,tf,hw,tw,h,bef,fyk,fyd,fck,alfa_cc,alfa_ct,
gammaC,fcd,Es,Ecm,fctm,c,d,As,L,alfa,ep_cu3,bballast,Aconcrete,qconcrete,
qballast,Con,Train))
% Output:
% Ned_SLS: Design mxial force for qusipermanente loads [N]
% ymax: maximum deflection
% Med_SLS: Design bending moment for qusipermanente loads [N]

function[Ned_SLS,ymax,Med_SLS]=InputSLS(Nbra,Ntr)
%Get initial data
[bf,tf,hw,tw,h,bef,l_0,fyk,fyd,fck,alfa_cc,alfa_ct,bballast,gammaC,gammaD
,T_b,fcd,Es,Ecm,fctm,fctk_0_05,fctd,c,d,L,alfa,ep_cu3,Aconcrete,qconcrete
,qballast,Con,Train,freq,ep_uk,t,fcm,gammaF_Fat,gammas_Fat,sigma_rsk,Main
tenance,phi,phi_neg,SEC,SPAN,Ltot]=InputData();
[Mconcrete,Vconcrete,Mballast,Vballast,Munloaded,Vunloaded,MSW2,VSW2,MLM7
1,VLM71,Nbra,Mbra,Vbra,Ntr,Mtr,Vtr,Tt,Tballast,Tsw,Tderail,yconcrete,ybal
last,yunloaded,ySW2,yLM71,Vderail,Mderail]=LoadModels(qconcrete,L,qballas
t,Train);

%Get the load factors

fbkacc_SLS=[0.8 1];
%Get axial force Nbkacc
Nbkacc=[max(Nbra) max(Ntr)];
[s1,n]=max(Nbkacc);

if (n==1)
  lfbra=fbkacc_SLS(2);
```

```
  lftr=fbkacc_SLS(1);
elseif (n==2)
  lfbra=fbkacc_SLS(1);
  lftr=fbkacc_SLS(2);
end
Ned_SLS=lfbra*(max(Nbra))+lftr*(max(Ntr));
%%%Get deflection at midspan (max deflection)
if Train==1
 flm71_SLS=1;
else
 flm71_SLS=0;
end

if Train==2
 fsw2_SLS=1;
else
 fsw2_SLS=0;
end
if Con==1
  fballast=1;
  fballast_M=1.2;
elseif Con==0
  fballast=0;
  fballast_M=0;
end
%yconcrete=yconcrete
disp(['The fballast is = ',num2str(fballast),' fsw2_SLS = ',
num2str(fsw2_SLS),' flm71_SLS = ', num2str(flm71_SLS)]);
disp(['The yballast is = ',num2str(yballast),' ySW2 = ', num2str(ySW2),'
yLM71 = ', num2str(yLM71),' yconcrete = ', num2str(yconcrete)]);
ymax=ySW2*fsw2_SLS+yLM71*flm71_SLS;
if SPAN==1
Med_SLS=max(Mconcrete)+max(Mballast)*fballast_M;
elseif SPAN==2
Mconcreteabs=abs(Mconcrete);
Mballastabs=abs(Mballast);
Med_SLS=max(Mconcreteabs)+max(Mballastabs)*fballast_M;
end
```

Load Combinations

```
%Calculation of the load combinations for:
%-----Shear force
%-----Bending moment
%-----Axial force
%-----Torsional moment
%In the ULS

%Function
%%function[Mmid,Msup_as,Med,Vmid_as,Vsup,Ved,Ted,Ned]=LoadCombinations()
%Output
%
%Mmid: Moment at mid span (Max moment) [Nm]
%Msup_as: Moment associated to the max shear force [Nm]
%Med: Design moment [Nm]
%Vmid_as: Shear force associated to the max moment [N]
%Vsup: Max Shear force (Placed at the support) [N]
%Ved: Design Shear force [N]
```

```
%Ted: Design Torsional moment [Nm]
%Ned: Design Axial force [N]


function[Mmid,Msup_as,Med,Med_min,Vmid_as,Vsup,Ved,Ted,Ned,DIFFx,Vderail,
Mderail,sigmamax,sigmamin,dyn_fat]=LoadCombinations(bf,tf,hw,tw,h,bef,fyk
,fyd,fck,alfa_cc,alfa_ct,gammaC,fcd,Es,Ecm,fctm,c,d,L,alfa,ep_cu3,bballas
t,Aconcrete,qconcrete,qballast,Con,Train)
%Get initial data
[bf,tf,hw,tw,h,bef,l_0,fyk,fyd,fck,alfa_cc,alfa_ct,bballast,gammaC,gammaD
,T_b,fcd,Es,Ecm,fctm,fctk_0_05,fctd,c,d,L,alfa,ep_cu3,Aconcrete,qconcrete
,qballast,Con,Train,freq,ep_uk,t,fcm,gammaF_Fat,gammas_Fat,sigma_rsk,Main
tenance,phi,phi_neg,SEC,SPAN,Ltot]=InputData();
%Get the load factors
[fsw,flm71,fsw2,fhslm,fet,fballast,fbkacc,fderail]=LoadFactorsULS(Con,Tra
in);
%Get the individual bending and torsional moments, as the axial and shear
forces
[Mconcrete,Vconcrete,Mballast,Vballast,Munloaded,Vunloaded,MSW2,VSW2,MLM7
1,VLM71,Nbra,Mbra,Vbra,Ntr,Mtr,Vtr,Tt,Tballast,Tsw,Tderail,yconcrete,ybal
last,yunloaded,ySW2,yLM71,Vderail,Mderail,deltaVSW2,deltaHSW2,deltaVLM71,
deltaHLM71,Mconcretemax,Mconcretemin,Mconcreteposmax,Mconcreteposmin,Vcon
cretemax,Vconcreteposmax,Mballastmax,Mballastmin,Mballastposmax,Mballastp
osmin,Vballastmax,Vballastposmax,Munloadedmax,Munloadedmin,Vunloadedmax,V
unloadedposmax,MSW2max,MSW2min,MSW2posmax,MSW2posmin,VSW2max,VSW2posmax,M
LM71max,MLM71min,MLM71posmax,MLM71posmin,VLM71max,VLM71posmax]=LoadModels
(qconcrete,L,qballast);
Ldyn=L;
%According to the level of maintenance chosen in InputData
dyn_fat2=1.44/(sqrt(Ldyn)-.2)+0.82; %Dynamic factor.SS-EN 1991-2. 6.4.5.2
if dyn_fat2<=1
 dyn_fat2=1;
elseif dyn_fat2>=1.67
 dyn_fat2=1.67;
else
 dyn_fat2=dyn_fat2;
end

dyn_fat3=2.16/(sqrt(Ldyn)-.2)+0.73; %Dynamic factor.SS-EN 1991-2. 6.4.5.2
if dyn_fat3<=1
 dyn_fat3=1;
elseif dyn_fat3>=2
 dyn_fat3=2;
else
 dyn_fat3=dyn_fat3;
end
%Dynamic factor
if Maintenance==1
 dyn_fat=dyn_fat2;
 else
 dyn_fat=dyn_fat3;
end
Vbkacc = max([max(Vbra(1)) max(Vtr(1))]);
Mbkacc = max([max(Mbra(1)) max(Mtr(1))]);

disp(['The Mbkacc max is = ',num2str(Mbkacc/1e3),' kNm at position:
midspan ']);
```

```
disp(['The Vbkacc max is = ',num2str(Vbkacc/1e3),' kN at position :
Supports ']);
%Dynamic factor for statics
D=1+4/(8+L);
%At Supports
%Shear Force and associated moment
if SPAN == 1
    Vperm=abs([Vconcrete(1) Vballast(1)]);      % Permanent loads
(Selfweight and ballast [in such a case])
    Vvar=abs([D*VLM71(1) D*VSW2(1) Vbkacc]);    % Variable loads (LM71,
SW2,braking/acceleration)
    Mperm=abs([Mconcrete(1) Mballast(1)]);      % Permanent loads
(Selfweight and ballast [in such a case])
    Mvar=abs([D*MLM71(1) D*MSW2(1) Mbkacc]);    % Variable loads (LM71,
SW2,braking/acceleration)
elseif SPAN == 2
    Vperm = ([Vconcretemax Vballastmax]);
    Vvar = ([D*VLM71max D*VSW2max Vbkacc]);
    Mperm = ([Mconcrete(Vconcreteposmax) Mballast(Vballastposmax)]);
    Mvar = ([D*MLM71(VLM71posmax) D*MSW2(VSW2posmax) Mbkacc]);
end
[s1,n]=max(Vvar);
if n==1
  lflm71=flm71(2);
  lfsw2=fsw2(1);
  lfbkacc=fbkacc(1);
  lfhslm=fhslm(1);
elseif n==2
  lflm71=flm71(1);
  lfsw2=fsw2(2);
  lfbkacc=fbkacc(1);
  lfhslm=fhslm(1);
else%if n==3
  lflm71=flm71(1);
  lfsw2=fsw2(1);
  lfbkacc=fbkacc(2);
  lfhslm=fhslm(1);
end

%PermanentLoading

if Vperm(1)>=0
  lfsw=fsw(2);
  else
  lfsw=fsw(1);
end

if Vperm(2)>=0
  lfballast=fballast(2);
  else
  lfballast=fballast(1);
end

Vsup=lfsw*Vperm(1)+lfballast*Vperm(2)+lflm71*Vvar(1)+lfsw2*Vvar(2)+lfbkac
c*Vvar(3);%+lfhslm*Vvar(4);
Msup_as=lfsw*Mperm(1)+lfballast*Mperm(2)+lflm71*Mvar(1)+lfsw2*Mvar(2)+lfb
kacc*Mvar(3);%+lfhslm*Mvar(4);
```

```
%Bending moment and associated Shear Force
if SPAN == 1
    %At Midspan
    Vperm=[Vconcrete(Mconcreteposmax) Vballast(Mballastposmax)];
%Permanent loads (Selfweight and ballast [in such a case])
    Vvar=[D*VLM71(MLM71posmax) D*VSW2(MSW2posmax) Vbkacc]; %Variable
loads (LM71, SW2,braking/acceleration)
    Mperm=[Mconcrete(Mconcreteposmax)
Mballast(Mballastposmax)];%Permanent loads (Selfweight and ballast [in
such a case])
    Mvar=abs([D*MLM71(MLM71posmax) D*MSW2(MSW2posmax) Mbkacc]);%Variable
loads (LM71, SW2,braking/acceleration,)
    % Assign partial coeffcients variable loads
    [s1,n]=max(Mvar);
    if n==1
        lflm71=flm71(2);
        lfsw2=fsw2(1);
        lfbkacc=fbkacc(1);
        lfhslm=fhslm(1);
    elseif n==2
        lflm71=flm71(1);
        lfsw2=fsw2(2);
        lfbkacc=fbkacc(1);
        lfhslm=fhslm(1);
    else
        lflm71=flm71(1);
        lfsw2=fsw2(1);
        lfbkacc=fbkacc(2);
        lfhslm=fhslm(1);
    end
    %Permanent loading
     if Mperm(1)>=0
        lfsw=fsw(2);
    else
        lfsw=fsw(1);
    end
    if Mperm(2)>=0
        lfballast=fballast(2);
    else
        lfballast=fballast(1);
    end
  % Load combination
disp(['lfsw = ',num2str(lfsw),' lfballast = ', num2str(lfballast),'
lflm71 = ', num2str(lflm71),' lfsw2 = ', num2str(lfsw2),' lfbkacc = ',
num2str(lfbkacc)]);
Mmid=lfsw*Mperm(1)+lfballast*Mperm(2)+lflm71*Mvar(1)+lfsw2*Mvar(2)+lfbkac
c*Mvar(3);
Vmid_as=lfsw*Vperm(1)+lfballast*Vperm(2)+lflm71*Vvar(1)+lfsw2*Vvar(2)+lfb
kacc*Vvar(3);
elseif SPAN == 2
    % Maximum moments and associated shear for field moment
    POSmax = [Mconcreteposmax MLM71posmax MSW2posmax];
    Mpos = zeros (3);
    Vpos_as = zeros (3);
    for i = 1:3
        Mperm = ([Mconcrete(POSmax(i)) Mballast(POSmax(i))]);
        Mvar = ([D*MLM71(POSmax(i)) D*MSW2(POSmax(i)) Mbkacc]);
        Vperm = ([Vconcrete(POSmax(i)) Vballast(POSmax(i))]);
        Vvar = ([D*VLM71(POSmax(i)) D*VSW2(POSmax(i)) Vbkacc]);
```

```
    %Variable loading
        [s1,n]=max(Mvar);
    if n==1
        lflm71=flm71(2);
        lfsw2=fsw2(1);
        lfbkacc=fbkacc(1);
        lfhslm=fhslm(1);
    elseif n==2
        lflm71=flm71(1);
        lfsw2=fsw2(2);
        lfbkacc=fbkacc(1);
        lfhslm=fhslm(1);
    else
         lflm71=flm71(1);
        lfsw2=fsw2(1);
        lfbkacc=fbkacc(2);
        lfhslm=fhslm(1);
    end
    %Permanent loading
    if Mperm(1)>=0
        lfsw=fsw(2);
    else
        lfsw=fsw(1);
    end
    if Mperm(2)>=0
        lfballast=fballast(2);
    else
        lfballast=fballast(1);
    end
  % Load combination

Mpos(i)=lfsw*Mperm(1)+lfballast*Mperm(2)+lflm71*Mvar(1)+lfsw2*Mvar(2)+lfb
kacc*Mvar(3);

Vpos_as(i)=lfsw*Vperm(1)+lfballast*Vperm(2)+lflm71*Vvar(1)+lfsw2*Vvar(2)+
lfbkacc*Vvar(3);
  end

Mmid = max (Mpos);
[s2,n]=max (Mpos);
Vmid_as = Vpos_as(n);
Position=n
end
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%                 Minimum moments                %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%Moments

if SPAN == 1
    Med_min = 0;
elseif SPAN == 2
    Mpermmin = ([Mconcrete(Mconcreteposmin) Mballast(Mconcreteposmin)]);
    Mvarmin = ([D*MLM71(Mconcreteposmin) D*MSW2(Mconcreteposmin)
Mbkacc]);
    Vpermmin = ([Vconcrete(Mconcreteposmin) Vballast(Mconcreteposmin)]);
    Vvarmin = ([D*VLM71(Mconcreteposmin) D*VSW2(Mconcreteposmin)
Vbkacc]);
    [s1,n]=min(Mvarmin);
```

```
    if n==1
        lflm71=flm71(2);
        lfsw2=fsw2(1);
        lfbkacc=fbkacc(1);
        lfhslm=fhslm(1);
    elseif n==2
        lflm71=flm71(1);
        lfsw2=fsw2(2);
        lfbkacc=fbkacc(1);
        lfhslm=fhslm(1);
    else
        lflm71=flm71(1);
        lfsw2=fsw2(1);
        lfbkacc=fbkacc(2);
        lfhslm=fhslm(1);
    end
    % Assign partial coefficents permanent loads
    if Mpermmin(1)<=0
        lfsw=fsw(2);
    else
        lfsw=fsw(1);
    end
    if Mpermmin(2)<=0
        lfballast=fballast(2);
    else
        lfballast=fballast(1);
    end
    %Load combination
    Med_min =
lfsw*Mpermmin(1)+lfballast*Mpermmin(2)+lflm71*Mvarmin(1)+lfsw2*Mvarmin(2)
+lfbkacc*Mvarmin(3);
    Ved_min =
lfsw*Vpermmin(1)+lfballast*Vpermmin(2)+lflm71*Vvarmin(1)+lfsw2*Vvarmin(2)
+lfbkacc*Vvarmin(3)
end

%Axial force Ned (Due to horizontal forces)
Nbkacc=[max(Nbra) max(Ntr)];
[s1,n]=max(Nbkacc);
%DIFFx=max([abs(Nbra(1)-Nbra(3)) abs(Ntr(1)-Ntr(3))]);
DIFFx=max(abs(Nbra(1)-Ntr(1)));
if n==1
  lfbra=fbkacc(2);
  lftr=fbkacc(1);
elseif n==2
  lfbra=fbkacc(1);
  lftr=fbkacc(2);
end
Ned=lfbra*(max(Nbra))+lftr*(max(Ntr));
%Torsional moment Ted
Tperm=[Tsw Tballast];       % Permanent loads (Selfweight and ballast [in
such a case])
Tvar=[Tt Tderail]           % Variable loads (LM71,
SW2,braking/acceleration,)
[s1,n]=max(Tvar)

if n==1
  if Train==1
```

```
      lftrain=flm71(2);
   elseif Train==2
     lftrain=fsw2(2);
   else
     lftrain=0;
   end
     lfde=fderail(1);
elseif n==2
   if Train==1
     lftrain=flm71(1);
   elseif Train==2
      lftrain=fsw2(1);
   else
      lftrain=0;
   end
     lfde=fderail(2);
end

if Tperm(1)>=0
  lfsw=fsw(2);
  else
  lfsw=fsw(1);
end

if Tperm(2)>=0
  lfballast=fballast(2);
  else
  lfballast=fballast(1);
end

Ted=lfsw*Tperm(1)+lfballast*Tperm(2)+lftrain*Tvar(1)+lfde*Tvar(2);
%Design moments and forces
Med=max(abs([Msup_as Mmid])); %N
Ved=max(abs([Vmid_as Vsup])); %N
Med_min = Med_min; %N
Ted=Ted; %N
Ned=Ned; %N

% Stress range for fatigue resistance from LM71
y=h-(h^2*tw+tf^2*(bf-tw))/(2*(bf*tf+hw*tw));
I=(tw*y^3+bf*(h-y)^3-(bf-tw)*(h-y-tf)^3)/3;
if SPAN == 1
    sigmaperm = Nbra/Aconcrete+(Mperm(1)*-y)/I;      % Naviers formula
    sigmavar = Nbra/Aconcrete+(Mvar(1)*-y)/I;        % Naviers formula
    sigmavarmin = 0;                                 % Lowest stress after
the train has passed, ie zero
elseif SPAN == 2
    sigmaperm = Nbra/Aconcrete+(-Mconcretemin*-y)/I;% Naviers formula
    Mvar = Mvar(n);
    sigmavarmin = Nbra/Aconcrete+(Mvar(1)*-y)/I;     % Naviers formula
    sigmavar = Nbra/Aconcrete+(-Mvarmin(1)*-y)/I;    % Lowest stress after
the train has passed
end
sigmapermmax = max(abs((sigmaperm)));
sigmavarmax = max(abs((sigmavar)));
sigmavarmin = max(abs((sigmavarmin)));
% Calc of fatigue strength for equvalent stresses
```

```
t = 28;                                    % Age of concrete [days]
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%ASSUMED
s = 0.2;                                   % Coeff depending on cement class
given in EN 1992-1-1 (3.1.2)
betacc = exp(s*(1-sqrt(28/t)));            % Coeff depending on the age of
concrete
k1 = 0.85;                                 % Given by EN 1992-2-2005
fcm = fck+8E6;
fcmt = betacc*fcm;
if (t>=3) && (t<28)
     fck = fcmt-8E6;
elseif (t>=28)
    fck = fck;
else
     disp 'Age of concrete t should not be under 3 days'
end
fcdfat = k1*betacc*fcd*(1-(fck/1E6)/250);        % fatigue strength [Pa
lambdac0 = min (0.94+(0.2*sigmapermmax)/fcdfat,1);
lambdac1 = 0.75;                                 % Simply sdupported,
compressed zone (no prestressing)
Vol = 25*freq*10*365/2;                          % Volume of traffic (25
tonnes*number of trains*10 vagons) (Tonnes/year/track)
lambdac23 = 1+(1/8)*log10(Vol/25E6)+(1/8)*log10(T_b/100);
a = 1;                                            % Should be a ratio
(LM71 on one track)/(LM71 on two tracks) but only one track/cross section
n = 0.12;
if a > 0.8
     lambdac4 = 1;
elseif a <= 0.84
     lambdac4 = 1+(1/8)*log10(n);
end

lambdac = lambdac0*lambdac1*lambdac23*lambdac4;
disp(['lambdac0 = ',num2str(lambdac0),' lambdac1 = ', num2str(lambdac1),'
lambdac23 = ', num2str(lambdac23),' lambdac4 = ', num2str(lambdac4)]);
sigmamax = sigmapermmax+lambdac*(sigmavarmax*dyn_fat-sigmapermmax);   %
EN 1992-2 Annex NN eq NN 113. EC uses LM71, here max variable load
sigmamin = sigmapermmax-lambdac*(sigmapermmax-sigmavarmin*dyn_fat);
```

Load Factors ULS

```
%%ULS combination load factors
%function[fsw,flm71,fsw2,fhslm,fet,fballast,fbkacc]=LoadFactorsULS(Con)
%Each factor is going to be a vector where the min and max value are
shown.
%Output:
%fsw: Selft weight factors
%flm71: Train Load LM71 factors
%fsw2: Train Load SW2 factors
%fhslm: Train Load HSLM factors
%fet: Train Load for Empty train
%fballast: Ballast  factors
%fbkacc: Braking and acceleration factors
%fderail: Derailment factor
%Input:
```

```
%%Con: Set to 1 for the Conventional line (Ballast) and set to 0 for the
HST (Unballasted)
%Train: Variable train load chosen: if 0, no train is on. if 1 only LM71
passes. If 2 SW2 train passes.
function[fsw,flm71,fsw2,fhslm,fet,fballast,fbkacc,fderail]=LoadFactorsULS
(Con,Train)
%Get initial data
[bf,tf,hw,tw,h,bef,l_0,fyk,fyd,fck,alfa_cc,alfa_ct,bballast,gammaC,gammaD
,T_b,fcd,Es,Ecm,fctm,fctk_0_05,fctd,c,d,L,alfa,ep_cu3,Aconcrete,qconcrete
,qballast,Con,Train,freq,ep_uk,t,fcm,gammaF_Fat,gammas_Fat,sigma_rsk,Main
tenance,phi,phi_neg,SEC,SPAN,Ltot]=InputData();
%Data for function
fsw=[0.95 1.05];

%Traffic loads depending on varibale Train
if Train==1
 flm71=[0.7 1.45];
else
 flm71=[0 0];
end

if Train==2
 fsw2=[0.7 1.45];
else
 fsw2=[0 0];
end
%%%%%Other variable loads
fhslm=[1 1.45];
fet=[1 1.45];
%Ballast load depending on Con variable
if Con==1
  fballast=[0.8 1.3];
elseif Con==0
  fballast=[0 0];
end
fbkacc=[0.8 1.45];
fderail=[0 .8];
%Check values again EN1990 A.2 Trains
```

Load Models

```
% Mconcrete: Section moment along the beam due to selfweight of the
concrete
% Vconcrete: Shear force along the beam due to selfweight of the concrete
% yconcrete: Deflection due to concrete
% Mballast: Section moment along the beam due to selfweight of the
ballast
% Vballast: Shear force along the beam due to selfweight of the ballast
% yballast: Deflection due to ballast
% Munloaded: Section moment along the bem due to unloaded train
% Vunloaded: Shear force along the beam due to the unloaded train
% yunloaded: Deflection due to the unloaded train
% MSW2: Section moment along the beam due to SW2 train
% VSW2: Shear force along the beam due to SW2
% ySW2: Deflection due to SW2
% deltaVSW2: vertical displacment at support due to SW2
% deltaHSW2: horizontal displacment at support due to SW2
% MLM71: Section moment along the beam due to LM71
```

```
% VLM71: Shear force along the beam due to LM71
% yLM71: Deflection due to LM71
% deltaVLM71: vertical displacment at support due to LM71
% deltaHLM71: horizontal displacment at support due to LM71
% Mbra: Section moment along the beam due to braking
% Vbra: Shear force along the beam due to braking
% Mtr: Section moment along the beam due to traction
% Vtr: Shear force along the beam due to traction
% Mderail: Section moment along the beam due to derailment
% Vderail: Shear force along the beam due to derailment
function[Mconcrete,Vconcrete,Mballast,Vballast,Munloaded,Vunloaded,MSW2,V
SW2,MLM71,VLM71,Nbra,Mbra,Vbra,Ntr,Mtr,Vtr,Tt,Tballast,Tsw,Tderail,yconcr
ete,yballast,yunloaded,ySW2,yLM71,Vderail,Mderail,deltaVSW2,deltaHSW2,del
taVLM71,deltaHLM71,Mconcretemax,Mconcretemin,Mconcreteposmax,Mconcretepos
min,Vconcretemax,Vconcreteposmax,Mballastmax,Mballastmin,Mballastposmax,M
ballastposmin,Vballastmax,Vballastposmax,Munloadedmax,Munloadedmin,Vunloa
dedmax,Vunloadedposmax,MSW2max,MSW2min,MSW2posmax,MSW2posmin,VSW2max,VSW2
posmax,MLM71max,MLM71min,MLM71posmax,MLM71posmin,VLM71max,VLM71posmax]=Lo
adModels(qconcrete,L,qballast,Train)
[bf,tf,hw,tw,h,bef,l_0,fyk,fyd,fck,alfa_cc,alfa_ct,bballast,gammaC,gammaD
,T_b,fcd,Es,Ecm,fctm,fctk_0_05,fctd,c,d,L,alfa,ep_cu3,Aconcrete,qconcrete
,qballast,Con,Train,freq,ep_uk,t,fcm,gammaF_Fat,gammas_Fat,sigma_rsk,Main
tenance,phi,phi_neg,SEC,SPAN,Ltot]=InputData();
n = 200;
Ln = L/n;
E=Ecm;
y=h-(h^2*tw+tf^2*(bf-tw))/(2*(bf*tf+hw*tw));
I=(tw*y^3+bf*(h-y)^3-(bf-tw)*(h-y-tf)^3)/3;
delta = 0.3;                                % Part sticking out behind the
support

%%%%%%%%%%%%%%%%%%%%%%%%%% Selfweight concrete %%%%%%%%%%%%%%%%%

Mconcrete = zeros (1,L+1);  % Bending moment along the beam [kNm]
Vconcrete = zeros (1,L+1);  % Shear force along the beam [kN]
if SPAN == 1
    for i = 0:L
        [M,V] = MomShearDist(qconcrete,L,i);
        Mconcrete(i+1)= M;
        Vconcrete(i+1) = V;
    end
    [Mconcretemax,Mconcreteposmax] = max(Mconcrete);
    [Mconcretemin,Mconcreteposmin] = min(Mconcrete);
    Mconcretexmax = (Mconcreteposmax-1);          % Position on the beam
for max pos moment
    Mconcretexmin = (Mconcreteposmin-1);          % Position on the beam
for max neg moment
    Vconcreteabs = abs(Vconcrete);
    [Vconcretemax,Vconcreteposmax] = max(Vconcreteabs);
    Vconcretexmax = (Vconcreteposmax-1);          % Position on the beam
for max pos shear
    disp(['The Mconcretemax is = ',num2str(Mconcretemax/1e3),' kNm at x =
', num2str(Mconcretexmax)]);
    disp(['The Mconcretemin is = ',num2str(Mconcretemin/1e3),' kNm at  x
= ', num2str(Mconcretexmin)]);
    disp(['The Vconcretemax is = ',num2str(Vconcretemax/1e3),' kN at
position = ', num2str(Vconcretexmax)]);
    % Deflection at midspan
```

```
    yconcrete = ((qconcrete*(L/2))/(24*E*I))*(L^3-2*L*(L/2)^2+(L/2)^3)
    %figure
    %plot (Mconcrete)
elseif SPAN == 2
        for i = 1:(Ltot/Ln+1)
        [M,V] = MomShearDistCont(qconcrete,L,Ltot,(i-1)*Ln);
        Mconcrete(i+1)= M;
        Vconcrete(i+1) = V;
        end
    [Mconcretemax,Mconcreteposmax] = max(Mconcrete);
    [Mconcretemin,Mconcreteposmin] = min(Mconcrete);
    Mconcretexmax = ((Mconcreteposmax-2)*Ln);          % Position on the
beam for max pos moment
    Mconcretexmin = ((Mconcreteposmin-2)*Ln);          % Position on the
beam for max neg moment
    Vconcreteabs = abs(Vconcrete);
    [Vconcretemax,Vconcreteposmax] = max(Vconcreteabs);
     Vconcretexmax = ((Vconcreteposmax-2)*Ln);          % Position on the
beam for max pos shear
    disp(['The Mconcretemax is = ',num2str(Mconcretemax/1e3),' kNm at x =
', num2str(Mconcretexmax)]);
    disp(['The Mconcretemin is = ',num2str(Mconcretemin/1e3),' kNm at  x
= ', num2str(Mconcretexmin)]);
    disp(['The Vconcretemax is = ',num2str(Vconcretemax/1e3),' kN at
position = ', num2str(Vconcretexmax)]);
    %Deflection at midspan
    yconcrete = (0.54*qconcrete*L^4)/(100*E*I);
    %figure
    %plot (Mconcrete)
end

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% Selfweight ballast %%%%%%%%%%%%%%%%%%%%%%

Mballast = zeros (1,L+1);   % Bending moment along the beam [kNm]
Vballast = zeros (1,L+1);   % Shear force along the beam [kN]
if SPAN == 1
    for i = 0:L
        [M,V] = MomShearDist(qballast,L,i);
        Mballast(i+1)= M;
        Vballast(i+1) = V;
    end
        [Mballastmax,Mballastposmax] = max(Mballast);
        [Mballastmin,Mballastposmin] = min(Mballast);
        Mballastxmax = (Mballastposmax-1);           % Position on the
beam for max pos moment
        Mballastxmin = (Mballastposmin-1);           % Position on the
beam for max neg moment
        Vballastabs = abs(Vballast);
        [Vballastmax,Vballastposmax] = max(Vballastabs);
        Vballastxmax = (Vballastposmax-1);           % Position on the
beam for max pos shear
        disp(['The Mballastmax is = ',num2str(Mballastmax/1e3),' kNm at x
= ', num2str(Mballastxmax)]);
        disp(['The Mballastmin is = ',num2str(Mballastmin/1e3),' kNm at
x = ', num2str(Mballastxmin)]);
        disp(['The Vballastmax is = ',num2str(Vballastmax/1e3),' kN at
position = ', num2str(Vballastxmax)]);
    % Deflection at midspan
```

```
    yballast = ((qballast*(L/2))/(24*E*I))*(L^3-2*L*(L/2)^2+(L/2)^3);
elseif SPAN == 2
        for i = 1:(Ltot/Ln+1)
        [M,V] = MomShearDistCont(qballast,L,Ltot,(i-1)*Ln);
        Mballast(i+1)= M;
        Vballast(i+1) = V;
        end
        [Mballastmax,Mballastposmax] = max(Mballast);
        [Mballastmin,Mballastposmin] = min(Mballast);
        Mballastxmax = ((Mballastposmax-2)*Ln);          % Position on
the beam for max pos moment
        Mballastxmin = ((Mballastposmin-2)*Ln);          % Position on
the beam for max neg moment
        Vballastabs = abs(Vballast);
        [Vballastmax,Vballastposmax] = max(Vballastabs);
        Vballastxmax = ((Vballastposmax-2)*Ln);          % Position on
the beam for max pos shear
        disp(['The Mballastmax is = ',num2str(Mballastmax/1e3),' kNm at x
= ', num2str(Mballastxmax)]);
        disp(['The Mballastmin is = ',num2str(Mballastmin/1e3),' kNm at
x = ', num2str(Mballastxmin)]);
        disp(['The Vballastmax is = ',num2str(Vballastmax/1e3),' kN at
position = ', num2str(Vballastxmax)]);
        yballast = (0.54*qballast*L^4)/(100*E*I);
end

%%%%%%%%%%%%%%%%%%% Load model "Unloaded train" EN 1992 - 2003 6.3.4
%%%%%%%%%%%%%

qunloaded = 10e3;                 % Load from selfweight of ballast
Munloaded = zeros (1,L+1);        % Bending moment along the beam [kNm]
Vunloaded = zeros (1,L+1);        % Shear force along the beam [kN]

if SPAN == 1
    for i = 1:L
        [M,V] = MomShearDist(qunloaded,L,i);
        Munloaded(i+1)= M;
        Vunloaded(i+1) = V;
    end
        [Munloadedmax,Munloadedposmax] = max(Munloaded);
        [Munloadedmin,Munloadedposmin] = min(Munloaded);
        Munloadedxmax = (Munloadedposmax-1);          % Position on the
beam for max pos moment
        Munloadedxmin = (Munloadedposmin-1);          % Position on the
beam for max neg moment
        [Vunloadedmax,Vunloadedposmax] = max(Vunloaded);       % Max
positiv shear force
        [Vunloadedmin,Vunloadedposmin] = min(Vunloaded);       % Max
negative shear force
        Vunloadedxmax = (Vunloadedposmax-1);          % Position on the
beam for max pos shear
        Vunloadedxmin = (Vunloadedposmin-1);          % Position on the
beam for max neg shear
        disp(['The Munloadedmax is = ',num2str(Munloadedmax/1e3),' kNm at
x = ', num2str(Munloadedxmax)]);
        disp(['The Munloadedmin is = ',num2str(Munloadedmin/1e3),' kNm at
x = ', num2str(Munloadedxmin)]);
```

```
        disp(['The Vunloadedmax is = ',num2str(Vunloadedmax/1e3),' kN at
position = ', num2str(Vunloadedxmax)]);
        disp(['The Vunloadedmin is = ',num2str(Vunloadedmin/1e3),' kN at
position = ', num2str(Vunloadedxmin)]);
    % Deflection at midspan
    yunloaded = ((qunloaded*(L/2))/(24*E*I))*(L^3-2*L*(L/2)^2+(L/2)^3);
elseif SPAN == 2
        for i = 1:(Ltot/Ln+1)
        [M,V] = MomShearDistCont(qunloaded,L,Ltot,(i-1)*Ln);
        Munloaded(i+1)= M;
        Vunloaded(i+1) = V;
        end
        [Munloadedmax,Munloadedposmax] = max(Munloaded);
        [Munloadedmin,Munloadedposmin] = min(Munloaded);
        Munloadedxmax = ((Munloadedposmax-2)*Ln);          % Position on
the beam for max pos moment
        Munloadedxmin = ((Munloadedposmin-2)*Ln);          % Position on
the beam for max neg moment
        [Vunloadedmax,Vunloadedposmax] = max(Vunloaded);      % Max
positiv shear force
        [Vunloadedmin,Vunloadedposmin] = min(Vunloaded);      % Max
negative shear force
        Vunloadedxmax = ((Vunloadedposmax-2)*Ln);          % Position on
the beam for max pos shear
        Vunloadedxmin = ((Vunloadedposmin-2)*Ln);          % Position on
the beam for max neg shear
        disp(['The Munloadedmax is = ',num2str(Munloadedmax/1e3),' kNm at
x = ', num2str(Munloadedxmax)]);
        disp(['The Munloadedmin is = ',num2str(Munloadedmin/1e3),' kNm at
x = ', num2str(Munloadedxmin)]);
        disp(['The Vunloadedmax is = ',num2str(Vunloadedmax/1e3),' kN at
position = ', num2str(Vunloadedxmax)]);
        disp(['The Vunloadedmin is = ',num2str(Vunloadedmin/1e3),' kN at
position = ', num2str(Vunloadedxmin)]);
        yunloaded = (0.54*qunloaded*L^4)/(100*E*I);
end

%%%%%%%%%%%%%%%%%%%% Load model SW/2 EN 1992 - 2003 6.3.3 %%%%%%%%%%%%%%


qSW2 = 155E3;              % Load from
MSW2 = zeros (1,L+1);      % Bending moment along the beam [kNm]
VSW2 = zeros (1,L+1);      % Shear force along the beam [kN]

if SPAN == 1
    c = 7;                     % Unloaded length of the span
    a = (L/2)-c/2;             % Loaded length of the span
    for i = 1:(L/Ln+1)
        [M,V] = MomShearSW2(a,qSW2,L,(i-1)*Ln);
        MSW2(i)= M;
        VSW2(i) = V;
    end

        [MSW2max,MSW2posmax] = max(MSW2);       % [Max positiv moment ie
at midspan, position in vector]
        [MSW2min,MSW2posmin] = min(MSW2);       % [Max negative moment ie
over the mid support, position in the vector]
```

```
        MSW2xmax = (MSW2posmax-1)*Ln;          % Position on the beam for
max pos moment
        MSW2xmin = (MSW2posmin-1)*Ln;          % Position on the beam for
max neg moment
        [VSW2max,VSW2posmax] = max(VSW2);      % Max positiv shear force
        [VSW2min,VSW2posmin] = min(VSW2);      % Max negative shear force
        VSW2xmax = (VSW2posmax-1)*Ln;          % Position on the beam for
max pos shear
        VSW2xmin = (VSW2posmin-1)*Ln;          % Position on the beam for
max neg shear
        disp(['The MSW2max is = ',num2str(MSW2max/1e3),' kNm at x = ',
num2str(MSW2xmax)]);
        disp(['The MSW2min is = ',num2str(MSW2min/1e3),' kNm at  x = ',
num2str(MSW2xmin)]);
        disp(['The VSW2max is = ',num2str(VSW2max/1e3),' kN at position =
', num2str(VSW2xmax)]);
        disp(['The VSW2min is = ',num2str(VSW2min/1e3),' kN at position =
', num2str(VSW2xmin)]);
    % Deflection at midspan
    yX=(qSW2*a^2)/(24*L*E*I)*(L-L/2)*(-2*(L/2)^2+(4*L*L/2)-(L/2)^2);
%Deflection of a distributed load
    ySW2=2*yX %Superposition principle
    % Change of angle
    phiA = ((qSW2*L^3)/(6*E*I*L))*((a/L)^2*(1-a/(2*L))^2);
    phiB = -((qSW2*L^3)/(6*E*I))*(-0.5*(a/L)^2*(1-0.5*(a/L)^2));
    phiSW2 = phiA+phiB;
    h = hw+tf;
    deltaVSW2 = delta*sind(phiSW2);      % Vertical displacment
    deltaHSW2 = h*sind(phiSW2);          % Horizontal displacment
elseif SPAN == 2
        c = 7;                           % Unloaded part
        a = L-c/2;                       % Loaded part
        for i = 1:(Ltot/Ln+1)
        [M,V,MB] = MomShearSW2Cont(a,c,qSW2,L,(i-1)*Ln);
        MSW2(i)= M;
        VSW2(i) = V;
        MB = MB;
        end
        % figure
        % plot (MSW2);
        % figure
        % plot (VSW2);
        [MSW2max,MSW2posmax] = max(MSW2);      % [Max positiv moment ie
at midspan, position in vector]
        [MSW2min,MSW2posmin] = min(MSW2);      % [Max negative moment ie
over the mid support, position in the vector]
        MSW2xmax = (MSW2posmax-1)*Ln;          % Position on the beam for
max pos moment
        MSW2xmin = (MSW2posmin-1)*Ln;          % Position on the beam for
max neg moment
        [VSW2max,VSW2posmax] = max(VSW2);      % Max positiv shear force
        [VSW2min,VSW2posmin] = min(VSW2);      % Max negative shear force
        VSW2xmax = (VSW2posmax-1)*Ln;          % Position on the beam for
max pos shear
        VSW2xmin = (VSW2posmin-1)*Ln;          % Position on the beam for
max neg shear
        disp(['The MSW2max is = ',num2str(MSW2max/1e3),' kNm at x = ',
num2str(MSW2xmax)]);
```

```
        disp(['The MSW2min is = ',num2str(MSW2min/1e3),' kNm at  x = ',
num2str(MSW2xmin)]);
        disp(['The VSW2max is = ',num2str(VSW2max/1e3),' kN at position =
', num2str(VSW2xmax)]);
        disp(['The VSW2min is = ',num2str(VSW2min/1e3),' kN at position =
', num2str(VSW2xmin)]);
        % Change of angle at end supports
        phi1 = ((qSW2*a^2)/(6*E*I))*(L-a/2)^2;
        phi2 = (-MB*L)/(6*E*I);
        phiSW2 = phi1+phi2;
        h = hw+tf;
        deltaVSW2 = delta*sind(phiSW2);         % Vertical displacment at
support
        deltaHSW2 = h*sind(phiSW2);             % Horizontal displacment
at support
        % Deflection at max field moment
        ySW2 =sind(phiSW2)*MSW2xmax;    % Maximum displacment
end

%%%%%%%%%%%%%%%%%%%% Load model 71 EN 1992 - 2003 6.3.2 %%%%%%%%%%%%%%

c = 6.4;                             % Unloaded length of the span
% Classifying factor
if Con==0
 alpha = 1.0;                        % HST are just for passengers
elseif Con==1
 alpha = 1.33;                       % Conventional trains need to be multiplied
for freight
end
qLM71 = alpha*80E3;                  % Distr load
QLM71 = alpha*250E3;                 % Point load
MLM71 = zeros (1,L+1);               % Bending moment along the beam [kNm]
VLM71 = zeros (1,L+1);               % Shear force along the beam [kN]

if SPAN == 1
    a = (L/2)-c/2;                       % Loaded length of the span
    for i = 1:(L/Ln+1)
        [M,V] = MomShearLM71(a,qLM71,QLM71,L,(i-1)*Ln);
        MLM71(i)= M;
        VLM71(i) = V;
    end

        [MLM71max,MLM71posmax] = max(MLM71)        % [Max positiv moment ie
at midspan, position in vector]
        [MLM71min,MLM71posmin] = min(MLM71);       % [Max negative moment
ie over the mid support, position in the vector]
        MLM71xmax = (MLM71posmax-1)*Ln;            % Position on the beam
for max pos moment
        MLM71xmin = (MLM71posmin-1)*Ln;            % Position on the beam
for max neg moment
        [VLM71max,VLM71posmax] = max(VLM71);       % Max positiv shear
force
        [VLM71min,VLM71posmin] = min(VLM71);       % Max negative shear
force
        VLM71xmax = (VLM71posmax-1)*Ln;            % Position on the beam
for max pos shear
        VLM71xmin = (VLM71posmin-1)*Ln;            % Position on the beam
for max neg shear
```

```
        disp(['The MLM71max is = ',num2str(MLM71max/1e3),' kNm at x = ',
num2str(MLM71xmax)]);
        disp(['The MLM71min is = ',num2str(MLM71min/1e3),' kNm at  x = ',
num2str(MLM71xmin)]);
        disp(['The VLM71max is = ',num2str(VLM71max/1e3),' kN at position
= ', num2str(VLM71xmax)]);
        disp(['The VLM71min is = ',num2str(VLM71min/1e3),' kN at position
= ', num2str(VLM71xmin)]);
        % figure
        % plot (MLM71);
        % figure
        % plot (VLM71);

    % Deflection at midspan

    % Due to point loads
    yp = 0;
    for i = (L/2-2.4):1.6:(L/2+2.4)
        y = ((QLM71*(L-i))/(48*E*I))*(3*L^2-4*(L-i)^2);
        yp = yp+y;
    end
    %Due to distr loads
    yX2=(qLM71*a^2)/(24*L*E*I)*(L-L/2)*(-2*(L/2)^2+(4*L*L/2)-(L/2)^2);
    yq=2*yX2;
    yLM71 = yp+yq;

% Change of angle at support
    % Due to point loads
    phipoint = 0;
    for i = (L/2-2.4):1.6:(L/2+2.4)
        phi = ((QLM71*L^2*(L-i))/(6*E*I))*(1-(L-i)^2/L^2);
        phipoint = phipoint+phi;
    end
    % Due to distr loads
    phidistrA = ((qLM71*L^3)/(6*E*I))*((a/L)^2*(1-a/(2*L))^2);
    phidistrB = -((qLM71*L^3)/(6*E*I))*(-0.5*(a/L)^2*(1-0.5*(a/L)^2));
    % Tot change of angle
    phiLM71 = phipoint+phidistrA+phidistrB;
    % Displacment at supports
    deltaVLM71 = delta*sind(phiLM71);      % Vertical displacment
    deltaHLM71 = h*sind(phiLM71);          % Horizontal displacment
elseif SPAN == 2
    e = (3*L)/8;
    a = e-c/2;
        for i = 1:(Ltot/Ln+1)
        [M,V,MB] = MomShearLM71Cont(a,qLM71,QLM71,L,(i-1)*Ln);
        MLM71(i)= M;
        VLM71(i) = V;
        MB = MB;
        end
        MB = MB;
        [MLM71max,MLM71posmax] = max(MLM71);       % [Max positiv moment
ie at midspan, position in vector]
        [MLM71min,MLM71posmin] = min(MLM71);       % [Max negative moment
ie over the mid support, position in the vector]
        MLM71xmax = (MLM71posmax-1)*Ln;            % Position on the beam
for max pos moment
```

```
        MLM71xmin = (MLM71posmin-1)*Ln;           % Position on the beam
for max neg moment
        [VLM71max,VLM71posmax] = max(VLM71);      % Max positiv shear
force
        [VLM71min,VLM71posmin] = min(VLM71);      % Max negative shear
force
        VLM71xmax = (VLM71posmax-1)*Ln;           % Position on the beam
for max pos shear
        VLM71xmin = (VLM71posmin-1)*Ln;           % Position on the beam
for max neg shear
        disp(['The MLM71max is = ',num2str(MLM71max/1e3),' kNm at x = ',
num2str(MLM71xmax)]);
        disp(['The MLM71min is = ',num2str(MLM71min/1e3),' kNm at  x = ',
num2str(MLM71xmin)]);
        disp(['The VLM71max is = ',num2str(VLM71max/1e3),' kN at position
= ', num2str(VLM71xmax)]);
        disp(['The VLM71min is = ',num2str(VLM71min/1e3),' kN at position
= ', num2str(VLM71xmin)]);

        % Change of angle at end supports
        b = L-a-c;
        phidistA = ((qLM71*a^2)/(6*E*I*L))*(L-a/2)^2;
        phidistB = ((qLM71*b^2)/(12*E*I*L))*(L^2-b^2/2);
        phipoint = 0;
        for A = (a+0.8):1.6:(a+5.6)
            B = L-A;
            phipoint = phipoint +(QLM71*B*(L^2-B^2))/(6*E*I*L);
        end
        phipoint = phipoint;
        phimoment = (-MB*L)/(6*E*I);
        phiLM71 = phidistA+phidistB+phipoint+phimoment;

        % Displacment at supports
        deltaVLM71 = delta*sind(phiLM71);      % Vertical displacment
        deltaHLM71 = h*sind(phiLM71);          % Horizontal displacment

        % Deflection at point of maximum field moment
        yLM71 = phiLM71*MLM71xmax;
        % figure
        % plot (VLM71);
        % figure
        % plot (MLM71);
end

%%%%%%%%%%%%%%%%%%%% Action due to traction and breaking EN 1991 - 2003
6.5.3 %%%%%%%%%%%%%%%

% Traction
Ntr = zeros (1,L+1);
Qtr = 33E3;
Lab = 0.7*L;

for i = 1:(L/Ln+1)
    [N] = AxilTrac(Qtr,Lab,(i-1)*Ln);
    Ntr(i)= N;
end
Ntrabs = abs(Ntr);
[Ntrmax,Ntrpos] = max(Ntrabs);
```

```
disp(['The Ntrmax is = ',num2str(Ntrmax/1e3),' kN at position = ',
num2str(Ntrpos)]);
if Con==1
  hballast=0.6;
elseif Con==0
  hballast=0;
end

hrail=0.172; %Chosen to be a UIC60 [m]
Ftr=max(Ntrabs);
Mrestr=Ftr*(h+hballast+hrail); %Resultant moment due to
braking/accelerating [Nm] related to the height (Eccentricity)
Mtr=zeros(1,L+1);
Vtr=zeros(1,L+1);

for i=1:L+1
  Vtr(i)=Mrestr/L;
  Mtr(i)=Mrestr*((i-1)/(L-1));
end
% Breaking
Nbra = zeros (1,L);
Qbra71 = 20E3;
QbraSW2 = 35E3;
if (Train == 1)
    Q = Qbra71;
elseif (Train == 2);
    Q = QbraSW2;
else
    Q = 0;
end

for i = 1:(L/Ln+1)
    [N] = AxilBrake(Q,Lab,(i-1)*Ln);
    Nbra(i)= N;
end
Nbraabs = abs(Nbra);
[Nbramax,Nbrapos] = max(Nbraabs);
disp(['The Nbramax is = ',num2str(Nbramax/1e3),' kN at position = ',
num2str(Nbrapos)]);
if Con==1
  hballast=0.6;
elseif Con==0
  hballast=0;
end

hrail=0.172; %Chosen to be a UIC60 [m]
Fbra=max(Nbraabs);
Mres=Fbra*(h+hballast+hrail); %Resultant moment due to
braking/accelerating [Nm] related to the height (Eccentricity)
Mbra=zeros(1,L+1);
Vbra=zeros(1,L+1);

for i=1:L+1
  Vbra(i)=Mres/L;
  Mbra(i)=Mres*((i-1)/(L-1));
end

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%Torsion%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

```
twidth=1.435;        % Track width in Sweden
r1=bf+twidth/2;      % Outermost rail
r2=bf-twidth/2;      % Innermost rail
if Train==1          % LM1
  Q=alpha*250E3;     % LM1
elseif Train==2
  Q=330E3;           % SW2
else
  Q=0;               % No train
end
e= twidth/18;        % Eccentricity.
rI1=bf/2-0.45/2;     % One side
rI2=rI1-twidth;      % One side
rII=bf/2-0.45/2;     % One side
if Train==1          % LM1
  Q=alpha*250E3;     % LM1
elseif Train==2
  Q=330E3;           % SW2
else
  Q=0;               % No train
end

if SEC==1
 Tt=Q*e;             % [Nm] One T alone
 Tsw=0;              % [Nm] One T alone
 Tballast=0;         % [Nm] One T alone
 Tderail1=(Q/2*rI1+Q/2*rI2)*0.7;
 Tderail2=Q*rII*1.4;
elseif SEC==2
 Tt=Q*(e+4.5);
 Tsw=qconcrete*bf;       % Torsional moment due to Selfweight of the other
part of the beam (Second T)[Nm]
 Tballast=qballast*4.5; % Torsional moment due to ballast[Nm]
 Tderail1=(Q/2*(rI1+4.5)+Q/2*(rI2+4.5))*0.7;
 Tderail2=Q*(rII+4.5)*1.4;
end

Tderail = max(Tderail1, Tderail2);    % Torsional moment due to derailment
[Nm]
%Derailment (Shear + Moment)
Vderail = Q*1.4;                      % Shear between the web and the
flange due to derailment
Mderail = Q*(6-bf+(bf-tw)/2)*1.4;     % Moment between the web and the
flange due to derailment
```

Mom Shear Dist

```
%MomShearDist
%Function [M,V] = MomShearDist(q,L,x)
%Indata: q = Distributed load [N/m]
%         L = Span length [m]
%         x = Position of the section force [m]
%Output: M = Bending moment at x [Nm]
%         V = Shear force at x [N]

function [M,V] = MomShearDist(q,L,x);
M = ((q*L^2)/2)*(x/L-x^2/L^2);
```

```
V = q*L*(0.5-x/L);
end
```

Mom Shear DistCont

```
%MomShearDistCont
%Function [M,V] = MomShearDist(q,L,x)
%Indata: q = Distributed load [N/m]
%        L = Span length [m]
%        Ltot = Total length [m]
%        x = Position of the section force [m]
%Output: M = Bending moment at x [Nm]
%        V = Shear force at x [N]

function [M,V] = MomShearDistCont(q,L,Ltot,x);
if (x>=0) && (x<=Ltot/2)
    M = (q*x/8)*(3*L-4*x);
    V = q/8*(3*L-8*x);
elseif (x>Ltot/2) && (x<=Ltot)
    M = q/8*(2*L-x)*(4*x-5*L);
    V = q/8*(13*L-8*x);
end
```

Mom Shear LM71

```
%MomShearLM71

%Effect of vertical loading due to normal rail traffic
%Function [M,V] = MomShearLM71(a,c,q,L,x)
%Indata: q = Distributed load [N/m]
%        L = Span length [m]
%        x = Position of the section force [m]
%        a = loaded length [m]
%        c = unloaded length [m]
%Output: M = Bending moment at x [Nm]
%        V = Shear force at x [N]

function [M,V] = MomShearLM71(a,q,Q,L,x);

% Section forces from the distributed load 0<x<a
if (x >= 0) && (x <= a)
    Ma = (q*x^2)/2-q*a*x*(1-a/(2*L));
    Va = q*x-q*a*(1-a/(2*L));
elseif (x>a) && (x<=L)
    Ma = -(q*a^2*(L-x))/(2*L);
    Va = -(q*a^2)/(2*L);
end

% Section forces from the distributed load (L-a)<x<L
if (x >= 0) && (x <= (L-a))
    Mb = -(q*a^2*x)/(2*L);
    Vb = -(q*a^2)/(2*L);
elseif (x > (L-a)) && (x<= L)
    Mb = -((q*a^2*x)/(2*L)-(q*(x-L+a)^2)/2);
```

```
    Vb = -((q*a^2)/(2*L)-q*(x-L+a));
end

 Ra1 = Q*(L-a-0.8)/L;
if (x >= 0) && (x < (a+0.8))
    Mc1 = Ra1*x;
    Vc1 = Ra1;
elseif (x >= (a+0.8)) && (x<=L);
    Mc1 = (Ra1*x-Q*(x-a-0.8));
    Vc1 = Ra1-Q;
end

 Ra2 = Q*(L-a-2.4)/L;
if (x >= 0) && (x < (a+2.4))
    Mc2 = Ra2*x;
    Vc2 = Ra2;
elseif (x >= (a+2.4)) && (x<=L);
    Mc2 = (Ra2*x-Q*(x-a-2.4));
    Vc2 = Ra2-Q;
end

Ra3 = Q*(L-a-4)/L;
if (x >= 0) && (x < (a+4))
    Mc3 = Ra3*x;
    Vc3 = Ra3;
elseif (x >= (a+4)) && (x<=L);
    Mc3 = (Ra3*x-Q*(x-a-4));
    Vc3 = Ra3-Q;
end

Ra4 = Q*(L-a-5.6)/L;
if (x >= 0) && (x < (a+5.6))
    Mc4 = Ra4*x;
    Vc4 = Ra4;
elseif (x >= (a+5.6)) && (x<=L);
    Mc4 = (Ra4*x-Q*(x-a-5.6));
    Vc4 = Ra4-Q;
end

Mc = -(Mc1+Mc2+Mc3+Mc4);
Vc = Vc1+Vc2+Vc3+Vc4;

M = -(Ma+Mb+Mc);
V = -(Va+Vb+Vc);

end
```

Mom Shear LM71 Cont

```
%Effect of vertical loading due to normal rail traffic
%Function [M,V] = MomShearLM71(a,c,q,L,x)
%Indata: q = Distributed load [N/m]
%        L = Span length [m]
%        x = Position of the section force [m]
%        a = loaded length [m]
%        c = unloaded length [m]
```

```
%Output: M = Bending moment at x [Nm]
%        V = Shear force at x [N]


function [M,V,MB] = MomShearLM71Cont(a,q,Q,L,x);
 a1 = a+0.8;
 a2 = a1+1.6;
 a3 = a2+1.6;
 a4 = a3+1.6;
 b = L-(a4+0.8);
 MB = -(1/(2*L))*(q*a^2/(4*L)*(L^2-a^2/2)+q*b^2/(2*L)*(L-
b/2)^2+q*L^3/8+Q/(2*L)*(a1*(L^2-a1^2)+a2*(L^2-a2^2)+a3*(L^2-
a3^2)+a4*(L^2-a4^2)));
 RA = (MB+q*a*(L-a/2)+Q*((L-a1)+(L-a2)+(L-a3)+(L-a4))+q*b^2/2)/L;
 RB1 = (q*a+q*b+4*Q-RA);
 RB2 = (1/L)*((q*L^2)/2-MB);
 RB = RB1+RB2;
 RC = (1/L)*(MB+(q*L^2)/2);

if (x >= 0) && (x < a)
    M = (RA*x-.5*q*x^2);
    V = (RA-q*x);
elseif (x >= (a)) && (x<a1);
    M = (RA*x-q*a*(x-a/2));
    V = (RA-q*a);
elseif (x >= (a1)) && (x<a2);
    M = (RA*x-q*a*(x-a/2)-Q*(x-a1));
    V = (RA-q*a-Q);
elseif (x >= (a2)) && (x<a3);
    M = (RA*x-q*a*(x-a/2)-Q*(x-a1)-Q*(x-a2));
    V = (RA-q*a-2*Q);
elseif (x >= (a3)) && (x<a4);
    M = (RA*x-q*a*(x-a/2)-Q*(x-a1)-Q*(x-a2)-Q*(x-a3));
    V = (RA-q*a-3*Q);
elseif (x >= (a4)) && (x<(L-b));
    M = (RA*x-q*a*(x-a/2)-Q*(x-a1)-Q*(x-a2)-Q*(x-a3)-Q*(x-a4));
    V = (RA-q*a-4*Q);
elseif (x >= (L-b)) && (x<(L));
    M = (RA*x-q*a*(x-a/2)-Q*(x-a1)-Q*(x-a2)-Q*(x-a3)-Q*(x-a4)-.5*q*(x-
L+b)^2);
    V = (RA-q*a-4*Q-q*(x-L+b));
elseif (x >= (L)) && (x<=2*L);
    M = (MB+RB2*(x-L)-0.5*q*(x-L)^2);
    V = RB2-q*(x-L);
end
```

Mom Shear SW2

```
%Effect of vertical loading due to heavy rail traffic
%Function [M,V] = MomShearSW2(a,c,q,L,x)
%Indata: q = Distributed load [N/m]
%        L = Span length [m]
%        x = Position of the section force [m]
%        a = loaded length [m]
%        c = unloaded length [m]
%Output: M = Bending moment at x [Nm]
```

```
%         V = Shear force at x [N]

function [M,V] = MomShearSW2(a,q,L,x);
if (x>=0) && (x <= a)
    Ma = (q*x^2)/2-q*a*x*(1-a/(2*L));
    Va = q*x-q*a*(1-a/(2*L));
elseif (x>a) && (x<=L)
    Ma = -(q*a^2*(L-x))/(2*L);
    Va = -(q*a^2)/(2*L);
end

if (x>=0) && (x<= (L-a))
    Mb = -(q*a^2*x)/(2*L);
    Vb = -(q*a^2)/(2*L);
elseif (x > (L-a)) && (x<= L)
    Mb = -((q*a^2*x)/(2*L)-(q*(x-L+a)^2)/2);
    Vb = -((q*a^2)/(2*L)-q*(x-L+a));
end

M = -(Ma+Mb);
V = -(Va+Vb);

end
```

Mom Shear SW2 Cont

```
%Effect of vertical loading due to heavy rail traffic
%Function [M,V] = MomShearSW2(a,c,q,L,x)
%Indata: q = Distributed load [N/m]
%         L = Span length [m]
%         x = Position of the section force [m]
%         a = loaded length [m]
%         c = unloaded length [m]
%Output: M = Bending moment at x [Nm]
%         V = Shear force at x [N]

function [M,V,MB] = MomShearSW2Cont(a,c,q,L,x);
 MB = -q*a^2/(4*L^2)*(L^2-(a^2)/2);
 RA = (MB+q*a*(L-a/2))/L;
 RB = 2*(q*a-RA);
 RC = RA;
if (x >= 0) && (x < a)
    M = (RA*x-.5*q*x^2);
    V = (RA-q*x);
elseif (x >= (a)) && (x<L);
    M = (RA*x-q*a*(x-a/2));
    V = (RA-q*a);
elseif (x >= (L)) && (x<L+c/2);
    M = (MB+RB/2*(x-L));
    V = RB/2;
elseif (x >= (L+c/2)) && (x<=2*L);
    M = (MB+RB/2*(x-L)-0.5*q*(x-L-c/2)^2);
    V = (RB/2-q*(x-L-c/2));
end

end
```

SLS

```
clc
close all
clear all

%%Calculations in Stage II (SLS)
%%The present script checks if the parameters chosen will fit the design
criteria
%%for the SLS

%Input Data
Function[bf,tf,hw,tw,h,bef,l_0,fyk,fyd,fck,alfa_cc,alfa_ct,bballast,gamma
C,gammaD,T_b,fcd,Es,Ecm,fctm,fctk_0_05,fctd,c,d,L,alfa,ep_cu3,Aconcrete,q
concrete,qballast,Con,Train,freq,ep_uk,t,fcm,gammaF_Fat,gammas_Fat,sigma_
rsk,Maintenance,phi,phi_neg,SEC,SPAN,Ltot]=InputData();
[bf,tf,hw,tw,h,bef,l_0,fyk,fyd,fck,alfa_cc,alfa_ct,bballast,gammaC,gammaD
,T_b,fcd,Es,Ecm,fctm,fctk_0_05,fctd,c,d,L,alfa,ep_cu3,Aconcrete,qconcrete
,qballast,Con,Train,freq,ep_uk,t,fcm,gammaF_Fat,gammas_Fat,sigma_rsk,Main
tenance,phi,phi_neg,SEC,SPAN,Ltot]= InputData();
%InputSLS.m
[Mconcrete,Vconcrete,Mballast,Vballast,Munloaded,Vunloaded,MSW2,VSW2,MLM7
1,VLM71,Nbra,Mbra,Vbra,Ntr,Mtr,Vtr,Tt,Tballast,Tsw,Tderail,yconcrete,ybal
last,yunloaded,ySW2,yLM71,Vderail,Mderail,deltaVSW2,deltaHSW2,deltaVLM71,
deltaHLM71,Mconcretemax,Mconcretemin,Mconcreteposmax,Mconcreteposmin,Vcon
cretemax,Vconcreteposmax,Mballastmax,Mballastmin,Mballastposmax,Mballastp
osmin,Vballastmax,Vballastposmax,Munloadedmax,Munloadedmin,Vunloadedmax,V
unloadedposmax,MSW2max,MSW2min,MSW2posmax,MSW2posmin,VSW2max,VSW2posmax,M
LM71max,MLM71min,MLM71posmax,MLM71posmin,VLM71max,VLM71posmax]=LoadModels
(qconcrete,L,qballast,Train);
[Ned_SLS,ymax,Med_SLS]=InputSLS(Nbra,Ntr);
[Mmid,Msup_as,Med,Vmid_as,Vsup,Ved,Ted,Ned,DIFFx,Vderail,Mderail,sigmamax
,sigmamin,dyn_fat]=LoadCombinations(bf,tf,hw,tw,h,bef,fyk,fyd,fck,alfa_cc
,alfa_ct,gammaC,fcd,Es,Ecm,fctm,c,d,L,alfa,ep_cu3,bballast,Aconcrete,qcon
crete,qballast,Con,Train);
[NA] = CrossSection();
%-----Stress limitations
%-----Crack control
%-----Angle at the supports/Deflection at the supports
%-----Deflection control


% Longitudinal reinforcement
z = 0.8*d;                                          % Lever arm for
tension in the reinforcement
Asl1 = Med/(fyd*z);
Asmin = min(0.26*(fctm/fyk)*tw*d, 0.0013*tw*d);     % Min long
reinforcment EN 1992-1-1 9.2.1 [m2]
fav = 0.459*fck;                                    % Assuming parabolic-
rec stress-stran curve, partial coefficent = 1. and alpha = 0.8
uk = 2*tf+2*hw+2*bf;
teta = 25;                                          % Angle between
concrete compression strut and the beam axis guessed
Asltor = (Ted*(1/tand(teta)*uk))/(2*Aconcrete*fyd); % Longitudinal
reinforcement requiered for torsion EN 1992-2 6.3.2
Ast = max([Asmin,Asl1,Asltor]);
```

```
Asmax = 0.04*Aconcrete;
Asl = min(Ast,Asmax);
n = Asl/(pi*(phi/2)^2);
if tw/0.07 <= n                                     % Min spacing
between bars 10 cm
    disp 'Longitudinal reinforcement fits into chosen section'
else
    disp ' %%%%%%%%%%%%%%%%%%%%%%%%%'
    disp ' WARNING!'
    disp 'Requiered longitudinal reinforcement doesnt fit into chosen
section'
    disp '%%%%%%%%%%%%%%%%%%%%%%%%%%'
end
%Shear reinforcement
alpha = 90;                          % Angle between shear reinforcment
and the axis of the beam guessed
slmax = 0.75*d*(1+1/tand(alpha))
s = slmax;
rhowmin = (0.08*sqrt(fck))/fyk;
rhow = rhowmin;
s = 0.1;                             % Spacing of the stirrups
z = 0.9*d;
fywd = fyd;
teta = 25;                           % Angle between concrete compression
strut and the beam axis guessed
cotteta = 1/tand(teta);
    if cotteta <1
    disp ' WARNING cotangent teta below limit, check angle teta'
      elseif cotteta > 2.5
    disp ' WARNING cotangent teta above limit, check angle teta'
    end
hred = 0.5*d;
Asw = (s*Ved)/(hred*fywd*cotteta);
% Total amount of reinforcementwkw
%n = 37;
%Asl = n*pi*(phi^2/4);
%disp 'Number of bars chosen manually'
As = Asl+Asw

Ned=Ned_SLS;
Med=Med_SLS;
disp(['____________']);
disp(['The design value for the bending moment is Med_SLS=',
num2str(Med_SLS/1e3),'kNm']);
disp(['____________']);
disp(['Serviceability Limit State']);
disp(['____________']);
disp(['Stress Limitations for SLS']);
k1=0.6; %Recommended
k3=0.8; %Recommended
b=bef; %m Section width befective because it is considered to be a
rectangular cross section. Check CrossSection.m
disp(['____________']);
disp(['Long term-Tensile']);
nl=15; %Recommended
disp(['Long term n=', num2str(nl)]);
M=Med;%Max Moment
xl=(nl*Asl)/b*(sqrt(1+(2*b*d/(nl*Asl)))-1);
sigma_s=M/(Asl*(d-xl/3)); %stress in the Steel [Pa]
```

```
disp (Asl)
while sigma_s > k3*fyk
    Asl =Asl+pi*(phi^2/4);
    xl=(nl*Asl)/b*(sqrt(1+(2*b*d/(nl*Asl)))-1);
    sigma_s=M/(Asl*(d-xl/3));
end
disp(['The ratio of usage for stresses is =',
num2str(sigma_s/(k3*fyk))]);
disp (Asl)
disp(['____________']);
disp(['Short Term Compressive']);

ns=Es/Ecm; %Short term
disp(['Short term n=', num2str(ns)]);
xs=(ns*As)/b*(sqrt(1+(2*b*d/(ns*As)))-1);
sigma_c=(2*M)/(b*xs*((d-xs/3))); %stress arisen in concrete [Pa]

if sigma_c <k1*fck
    disp('The stress fulfill the requirements for concrete tension in the
SLS ');
    disp(['The ratio of usage for stresses is =',
num2str(sigma_c/(k1*fck))]);
    else
    disp('%%%%%%%%%%%%%%%%%%%%WARNING%%%%%%%%%%%%%%%%%%%%%%%%');
    disp('%%%%%%%%%%%%%%%_________________%%%%%%%%%%%%%%%%%%%');
    disp('NOT OK! The stresses do not fulfill criteria. Change your
input');
    disp('%%%%%%%%%%%%%%%%%%%%WARNING%%%%%%%%%%%%%%%%%%%%%%%%');
    disp('%%%%%%%%%%%%%%%_________________%%%%%%%%%%%%%%%%%%%');
end


disp('____________');
disp('Control of Cracking for SLS');
disp('____________');
if sigma_c>-fctm
disp(['Section assumed to be uncracked (EN1992-1-1,7,1(2))']);
end
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%DEPENDING ON
SECTION%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
hceff = min([2.5*(hw+tf)-d,(h-xs)/3,h/2]);
Act=b*hceff;
%Two conditions for k value depending on b and h
kb=zeros(2,1);
if bef<=0.3
kb(1)=1;
elseif bef>=0.8
kb(1)=0.65;
else
kb(1)=-(0.35*(bef-0.3))/0.5+1;
end
%Now for h
if h<=0.3
kb(2)=1;
elseif h>=0.8
kb(2)=0.65;
else
kb(2)=-(0.35*(h-0.3))/0.5+1;
```

```
end
k=min(kb); %Size effects
disp('                  ');
disp('Bending in webs');
%kc for pure tension is Kc=1 but it is not our case

sigma_s_kc=Ned/b/h; %mean stress of the concrete on the section
%Parameter h_prime
if h<1
h_prime=h;
else
h_prime=1;
end
%Parameter k1: Effects of axial forces on stress distribution
if Ned<0
k1=1.5;
else
k1=2*h_prime/(3*h);
end
%kc becomes
fct_eff=fctm; %MPa Cracks are expected after 28 days. Otherwise
fct_eff=fctm(t)
kc=0.4*(1-(sigma_s_kc/(k1*h/h_prime*fct_eff))); %Stress distribution and
of the change of lever arm when cracking occurs
w_max=0.3E-3; %According to Table 7.101N EN1992-2 for quasipermanent
combination of actions
sigma_s_crack=fyk; %Pa
As_crackweb=(kc*k*fct_eff*Act)/sigma_s_crack;
disp('The minimum area to fulfill the requirements for concrete cracking
in the SLS ');
disp(['As_crack in web = ', num2str((As_crackweb*1e4)), ' cm2']);

%Check crack width
kt=0.6;         %For short term actions kt=0.6 and for long kt=0.4 worst
case is chosen
hc_eff=min([2.5*(h-d) (h-xs)/3 h/2]);
Ac_eff=b*hc_eff; %En 1992-1-1 7.3.2
Rho_eff=Asl/Ac_eff;

Diff_ten_strain=max([(sigma_s-kt*fct_eff/Rho_eff*(1+alfa*Rho_eff))/Es
0.6*sigma_s/Es]); %Difference of the mean tensile strains

k1=0.8;         % Good adherence Passive reinforcement
k2=0.5;         % Bending
k3=3.4;         % Value given
k4=0.425;       % Value given
diam=phi;       % m diameter of the bars
c=0.03;         % Cover Taken from other
sr_max=k3*c+k1*k2*k4*diam/Rho_eff;
wkw=sr_max*Diff_ten_strain;
disp('                  ');
disp(['The crack width for webs is =', num2str(wkw), ' m']);
disp (Asl)
while wkw > w_max
    Asl =Asl+pi*(phi^2/4);
    Rho_eff=Asl/Ac_eff;
    Diff_ten_strain=max([(sigma_s-kt*fct_eff/Rho_eff*(1+alfa*Rho_eff))/Es
0.6*sigma_s/Es]); %Difference of the mean tensile strains
```

```
    sr_max=k3*c+k1*k2*k4*diam/Rho_eff;
    wkw=sr_max*Diff_ten_strain;
end
disp(['The ratio of usage for cracks in web is =', num2str(wkw/w_max)]);
disp (Asl)

%FLANGES
disp('                  ');
disp('Bending in flanges');
fct_eff=fctm;                           % Pa Cracks are expected after 28
days. Otherwise fct_eff=fctm(t)
Fcr=fct_eff/(tf*bf);                    % N Tensile force in the flanges
kc=max([(0.9*(Fcr/Act/fct_eff)) 0.5]);  % Stress distribution and of the
change of lever arm when cracking occurs
w_max=0.3E-3;                           % According to Table 7.101N
EN1992-2 for quasipermanent combination of actions
sigma_s_crack=fyk;                      % Pa
As_crackfl=(kc*k*fct_eff*Act)/sigma_s_crack;
disp('The minimum area to fulfill the requirements for concrete cracking
in the SLS ');
disp(['As_crack in flanges = ', num2str(As_crackfl*1e4),' cm2']);
As_crack=As_crackfl+As_crackweb;

%Check crack width
kt=0.6;                                 % For short term actions kt=0.6
and for long kt=0.4 so worst case is chosen
hc_eff=min([2.5*(h-d) (h-xs)/3 h/2]);
Ac_eff=b*hc_eff;                        % En1992-1-1 7.3.2
Rho_eff=Asl/Ac_eff;
Diff_ten_strain=max([(sigma_s-kt*fct_eff/Rho_eff*(1+alfa*Rho_eff))/Es
0.6*sigma_s/Es]); %Difference of the mean tensile strains

k1=0.8;                                 % Good adherence Passive
reinforcement
k2=0.5;                                 % Bending if it was pure tension
k2=1
k3=3.4; %Value given
k4=0.425; %Value given
diam=phi;%0.02; %m diameter of the bars.
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%Assumed
sr_max=k3*c+k1*k2*k4*diam/Rho_eff;
wkf=sr_max*Diff_ten_strain;
disp('                  ');
disp(['The crack width for flanges is =', num2str(wkf), ' m']);
disp 'Asl'
disp (Asl)
while wkf > w_max
    Asl =Asl+pi*(phi^2/4);
    Rho_eff=Asl/Ac_eff;
    Diff_ten_strain=max([(sigma_s-kt*fct_eff/Rho_eff*(1+alfa*Rho_eff))/Es
0.6*sigma_s/Es]); %Difference of the mean tensile strains
    sr_max=k3*c+k1*k2*k4*diam/Rho_eff;
    wkf=sr_max*Diff_ten_strain;
end
disp(['The ratio of usage for cracks in web is =', num2str(wkf/w_max)]);
disp 'Asl'
disp (Asl)
```

```
%%%%%%%%%%%%%%%%%%% Deflection check %%%%%%%%%%%%%%%%%%%%%%%%%
disp(['____________']);
disp(['Deflection control for SLS']);
disp(['____________']);

disp(['____________']);
disp(['Deflection at supports']);
disp(['____________']);

if Train == 1
    deltaV = deltaVLM71;
    deltaH = deltaHLM71;
elseif Train == 2
    deltaV = deltaVSW2;
    deltaH = deltaHSW2;
end
deltaVlim = 2E-3;       % Max vertical displ at supports for STH > 160
km/h EN 1991-2 6.5.4.5.2 (3)
deltaHlim = 10E-3;      % Max horozontal displ at supports for STH > 160
km/h EN 1991-2 6.5.4.5.2 (2)

if (deltaV <= deltaVlim) && (deltaH <= deltaHlim)
    disp 'Displacement at supports OK'
else
    disp('%%%%%%%%%%%%%%%%%%%%WARNING%%%%%%%%%%%%%%%%%%%%%%%%');
    disp('%%%%%%%%%%%%%%%_________________%%%%%%%%%%%%%%%%%%%');
        disp ' Displacement at supports NOT OK!!!'
    disp('%%%%%%%%%%%%%%%%%%%%WARNING%%%%%%%%%%%%%%%%%%%%%%%%');
    disp('%%%%%%%%%%%%%%%_________________%%%%%%%%%%%%%%%%%%%');
end

disp(['____________']);
disp(['Deflection at midspan']);
disp(['____________']);
Lmm = L*1e3;                % Span length [mm]
Ac = Aconcrete*1e6;         % Cross section area [m^2]
fck_MPa = fck/1e6;          % Compressive strength [MPa]
dlim = Lmm/(600);           % Deflection limit check [mm]

disp(['The maximum vertical deflection allowed,dlim, is=',
num2str(dlim/1e3), ' m']);

dmax=ymax*1e3;
disp('                  ');
disp(['The maximum deflection,dmax is=', num2str(dmax/1e3), ' m']);
disp(['The usage ratio for the deflection is=', num2str(dmax/dlim)]);

if dmax <= dlim
    disp('The maximum vertical deflection is verified in the SLS ');
else
    disp('%%%%%%%%%%%%%%%%%%%%WARNING%%%%%%%%%%%%%%%%%%%%%%%%');
    disp('%%%%%%%%%%%%%%%_________________%%%%%%%%%%%%%%%%%%%');
    disp('NOT OK! The maximum vertical deflection  does not fulfill
criteria. Change your input');
    disp('%%%%%%%%%%%%%%%%%%%%WARNING%%%%%%%%%%%%%%%%%%%%%%%%');
    disp('%%%%%%%%%%%%%%%_________________%%%%%%%%%%%%%%%%%%%');
end
```

```
%%%% Check to see if the required reinforcement area fits into section
%%%%%%%%%%%%
As = Asl+Asw;
n = Asl/(pi*(phi/2)^2);
nrow = tw/0.07;              % Max number of bars in a row using 10 cm
spacing
if n <= nrow
    disp 'The bars fit in one row'
    disp 'The number of bars in one rows is n = ', num2str(n),' bars and
allowed in row is ntot ='; num2str(nrow);
elseif n > nrow
    d = d-0.07;              % New effective height using 7 cm spacing
between rows of reinforcement bars
    z = 0.8*d;               % Lever arm for tension in the reinforcement
    Asl1 = Med/(fyd*z);
    As2 = max([Asmin,Asl1,Asltor,Asl]);
    As3 = min(As2,Asmax);
    n = Asl/(pi*(phi/2)^2);
if n <= 2*nrow
    disp ' Long reinforcement fits into two rows';
    disp 'The number of bars in two rows is n = ', num2str(n),' bars and
allowed in 2 rows is ntot ='; num2str(2*nrow);
elseif n > 2*nrow
    d = d-0.07;              % New effective height using 7 cm spacing
between rows of reinforcement bars
    z = 0.8*d;               % Lever arm for tension in the reinforcement
    Asl1 = Med/(fyd*z);
    As2 = max([Asmin,Asl1,Asltor,Asl]);
    As3 = min(As2,Asmax);
    n = Asl/(pi*(phi/2)^2);
    if n <=3*nrow
        disp ' Long reinforcement fits into three rows';
    else
    disp 'Long reinforcement does not fit into section. Change input.';
    disp 'The number of bars in one rows is n = ', num2str(n),' bars and
allowed in 2 rows is ntot ='; num2str(2*nrow);
end
end
end

% Condition to see if reinforcement bars fit into the section
if n <= nrow | n <= 2*nrow | n <=3*nrow
    bars = 1;
else
    bars = 0;
end

disp(['______________________________________________________']);
disp(['_______________________SUMMARY_________________________']);
disp(['______________________________________________________']);
if  dmax <= dlim && wkw<=w_max && wkf<=w_max&& sigma_c <k1*fck && sigma_s
<k3*fyk && bars > 0
    disp('The SLS resistance checks for the cross section are OK!');
else
    disp('%%%%%%%%%%%%%%%%%%%%WARNING%%%%%%%%%%%%%%%%%%%%%%%%%');
    disp('%%%%%%%%%%%%%%%_________________%%%%%%%%%%%%%%%%%%%');
    disp('NOT OK! Check the file again. Change your input');
```

```
    disp('%%%%%%%%%%%%%%%%%%%%WARNING%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%');
    disp('%%%%%%%%%%%%%%%_________________%%%%%%%%%%%%%%%%%%%');

end
    disp(['_____________________________________________________']);

    disp(['_____________________________________________________']);

disp(['  dmax/dlim % wkw/w_max % wkf<=w_max % sigma_c/(k1*fck) %
sigma_s/(k3*fyk)']);
disp(['__________________________________________________________________
________']);
disp([' %  ', num2str(dmax/dlim),' % ', num2str(wkw/w_max),'   % ',
num2str(wkf/w_max),'   %     ', num2str(sigma_c/(k1*fck)),'        %  ',
num2str(sigma_s/(k3*fyk))]);
disp (['Number of reinforcement bars is ' num2str(n), ' phi '
num2str(phi*1000)]);


disp (['wkw = ' num2str(wkw*1000), ' Wkf = ' num2str(wkf*1000)]);
disp (['dmax = ' num2str(dmax)]);
```

ULS.

```
clc
close all
clear all

%%Calculations in Stage III (ULS)
%%The present script checks if the parameters chosen will fit the design
criteria
%%for the ULS
%-------Bending Moment
%-------Shear force
%-------Fatigue resistance
%-------Torsion
%Input Data Function
[bf,tf,hw,tw,h,bef,l_0,fyk,fyd,fck,alfa_cc,alfa_ct,bballast,gammaC,gammaD
,T_b,fcd,Es,Ecm,fctm,fctk_0_05,fctd,c,d,L,alfa,ep_cu3,Aconcrete,qconcrete
,qballast,Con,Train,freq,ep_uk,t,fcm,gammaF_Fat,gammas_Fat,sigma_rsk,Main
tenance,phi,phi_neg,SEC,SPAN,Ltot]=InputData
%Load combinations input
[Mmid,Msup_as,Med,Med_min,Vmid_as,Vsup,Ved,Ted,Ned,DIFFx,Vderail,Mderail,
sigmamax,sigmamin,dyn_fat]=LoadCombinations(bf,tf,hw,tw,h,bef,fyk,fyd,fck
,alfa_cc,alfa_ct,gammaC,fcd,Es,Ecm,fctm,c,d,L,alfa,ep_cu3,bballast,Aconcr
ete,qconcrete,qballast,Con,Train);
[NA] = CrossSection();
% Longitudinal reinforcement
z = 0.8*d;                                                   % Lever arm for
tension in the reinforcement
Asl1 = Med/(fyd*z);
Asmin = min(0.26*(fctm/fyk)*tw*d, 0.0013*tw*d);              % Min long
reinforcement EN 1992-1-1 9.2.1 [m2]
fav = 0.459*fck;                                             % Assuming
parabolic-rec stress-stran curve, partial coefficent = 1. and alpha = 0.8
uk = 2*tf+2*hw+2*bf;
```

```
teta = 25;                                                  % Angle between
concrete compression strut and the beam axis guessed
Asltor = (Ted*(1/tand(teta)*uk))/(2*Aconcrete*fyd);         % Longitudinal
reinforcement requiered for torsion EN 1992-2 6.3.2
Ast = max([Asmin,Asl1,Asltor]);
Asmax = 0.04*Aconcrete;
Asl = min(Ast,Asmax);
n = Asl/(pi*(phi/2)^2);
nrow = tw/0.07                                               % Max number of
bars in a row using 10 cm spacing

if n <= nrow
    disp 'The bars fit in one row'
    disp 'The number of bars in one rows is n = ', num2str(n),' bars and
allowed in row is ntot ='; num2str(nrow);
elseif n > nrow
    d = d-0.07;                                             % New effective
height using 7 cm spacing between rows of reinforcement bars
    z = 0.8*d;                                              % Lever arm for
tension in the reinforcement
    Asl1 = Med/(fyd*z);
    Ast = max([Asmin,Asl1,Asltor]);
    Asl = min(Ast,Asmax);
    n = Asl/(pi*(phi/2)^2);
if n <= 2*nrow
    disp ' Long reinforcement fits into two rows';
    disp 'The number of bars in two rows is n = ', num2str(n),' bars and
allowed in 2 rows is ntot ='; num2str(2*nrow);
elseif n > 2*nrow
    d = d-0.07;                                             % New effective
height using 7 cm spacing between rows of reinforcement bars
    z = 0.8*d;                                              % Lever arm for
tension in the reinforcement
    Asl1 = Med/(fyd*z);
    Ast = max([Asmin,Asl1,Asltor]);
    Asl = min(Ast,Asmax);
    n = Asl/(pi*(phi/2)^2)
    if n <=3*nrow
        disp ' Long reinforcement fits into three rows';
    else
    disp '
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%';
    disp '
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%';
    disp 'Long reinforcement does not fit into section. Change input.';
    disp '
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%';
    disp '
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%';
    disp 'The number of bars is n = ', num2str(n),' bars and allowed in 2
rows is ntot ='; num2str(2*nrow);
end
end
end
```

```
% Condition to see if reinforcement bars fit into max 3 rows in the web
if n <= nrow | n <= 2*nrow | n <=3*nrow
    bars = 1;
else
    bars = 0;
end

%Reinforcement over the middle support (In case of negative moment and
continuous bridge)

if SPAN == 1
  Asnb=0;
  n_neg=0
  disp 'The bridge is simply supported, no negative reinforcement
needed';
  disp 'The number of bars is n = ', num2str(n);
 elseif SPAN == 2
  zn = 0.8*d;                         % Lever arm [m]
  Asbn = -Med_min/(fyd*zn);           % Reinforcement over the support
  Asminbn = min(0.26*(fctm/fyk)*bf*d, 0.0013*bf*d);
  nrow_neg = bf/0.1;
  n_neg=Asbn/(pi*(phi_neg/2)^2);
  lreinf=1/4*L;
  disp 'The continuous bridge needs negative reinforcement over the
supports';

disp(['____________________________________________________________________
']);

disp(['____________________________________________________________________
']);
    if n_neg <= nrow_neg
      disp 'The bars fit in one row'
      disp 'The number of bars in one rows is n = ', num2str(n),' bars
and allowed in row is ntot ='; num2str(nrow);
    elseif n > nrow
     disp 'The number of bars in one rows is n = ', num2str(n),' bars and
allowed in row is ntot ='; num2str(nrow);
     disp 'The number of bars does not fit in one row!';
    end
end
%Shear reinforcement
alpha = 90;                           % Angle between shear reinforcment
and the axis of the beam assumed
slmax = 0.75*d*(1+1/tand(alpha));
s = slmax;
rhowmin = (0.08*sqrt(fck))/fyk;
rhow = rhowmin;
s = 0.1;                              % Spacing of the stirrups
z = 0.9*d;
fywd = fyd;
teta = 25;                            % Angle between concrete compression
strut and the beam axis assumed
cotteta = 1/tand(teta);
    if cotteta <1
    disp ' WARNING cotangent teta below limit, check angle teta'
      elseif cotteta > 2.5
```

```
    disp ' WARNING cotangent teta above limit, check angle teta'
    end
hred = 0.5*d;
Asw = (s*Ved)/(hred*fywd*cotteta);
% Total amount pf reinforcement
As = Asl+Asw;

%%%%%%%%%%%%% Shear resistance %%%%%%%%%%%%%%%%%%%%%
disp(['_____________']);
disp(['Shear Force']);

% Shear resistance members not requiring shear reinforcment

bwm = bef*1e3;                                          % smallest width of
the cross section in tension [mm]
dmm=d*1e3;                                              % d in mm
Crd = 0.18/gammaC;                                     % Given in EN
k = min(1+sqrt(200/(dmm)),2);
pho1 = min(Asl*1e6/(bwm*dmm),0.02);
k1 = 0.15;                                              % Given EN
Nedp=0;                                                 % N Prestressing
sigmacp = min(Nedp/(Aconcrete*1e6),0.2*fcd/1e6);
vmin = 0.035*k^(3/2)*fck^(1/2);
v = 0.6*(1-(fck/1e6)/250);

Vcrd1 = (Crd*k*(100*pho1*fck/1e6)^(1/3)+k1*sigmacp)*bwm*dmm;
Vcrd2 = (vmin+k1*sigmacp)*bwm*d;
Vcrd3 = 0.5*bwm*dmm*v*(fcd/1e6);

Vcrd = min (Vcrd3, max (Vcrd1, Vcrd2));

Vcrd = Vcrd/gammaD;

disp(['_____________']);
disp(['The design value for the Shear Force is Ved=',
num2str(Ved/10^3),'kN']);
disp(['If Ved/Vcrd<=1']);
if Ved/Vcrd <=1
    disp('The shear resistance of the cross section is OK!');
    disp(['The ratio of usage for the shear force is =',
num2str(Ved/Vcrd)]);
    else
    disp('%%%%%%%%%%%%%%%%%%%%WARNING%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%');
    disp('%%%%%%%%%%%%%%%_________________%%%%%%%%%%%%%%%%%%%');
    disp('NOT OK! The shear resistance does not fulfill criteria');
    disp('Change your input or Shear reinforcement required');
    disp('%%%%%%%%%%%%%%%%%%%%WARNING%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%');
    disp('%%%%%%%%%%%%%%%_________________%%%%%%%%%%%%%%%%%%%');

    % Shear resistance for members requiring shear reinforcment

    VRds = (Asw/s)*z*fywd*cotteta;

    if (sigmacp >= 0) && (sigmacp <= 0.25*fcd)
        alphacw = (1+sigmacp/fcd);                          % coeff account
for stress in compression chord
      elseif (sigmacp > 0.25*fcd) && (sigmacp <= 0.5*fcd)
```

```
          alphacw = 1.25;
        elseif (sigmacp > 0.5*fcd) && (sigmacp <= fcd)
           alphacw = 2.5*(1-sigmacp/fcd);
    end

    if fck <= 60E6
      ny1 = 0.6;
    elseif fck > 60E6
      ny1 = min(0.9-fck/200E6,0.5);
     end

    VRdmax = alphacw*tw*z*ny1*fck*(1-fck/250e6)/(cotteta+tand(teta));

    VRd = min(VRds,VRdmax);

    if Ved/VRd <= 1
        disp('The shear resistance of the cross section is OK!');
        disp(['The ratio of usage for the shear force with shear
reinforcment is =', num2str(Ved/VRd)]);
        disp(['The shear reinforcement needed is Asw= ',
num2str(Asw*1e4), ' cm2']);
        disp ' Change the value of Asw in SLS '
    else
        disp('%%%%%%%%%%%%%%%%%%%%WARNING%%%%%%%%%%%%%%%%%%%%%%%%');
        disp('%%%%%%%%%%%%%%%_________________%%%%%%%%%%%%%%%%%%%');
        disp('NOT OK! The shear resistance does not fulfill criteria with
Shear Reinforcement');
        disp ('Change your input');
        disp('%%%%%%%%%%%%%%%%%%%%WARNING%%%%%%%%%%%%%%%%%%%%%%%%');
        disp('%%%%%%%%%%%%%%%_________________%%%%%%%%%%%%%%%%%%%');
    end
end

if Ved/Vcrd <=1
 VcrdT=Vcrd;
else
 VcrdT=VRd;
end
%%%%%%%%%%%%%% Shear between web flanges and the web %%%%%%%%%%%%%%%%%%

disp(['____________']);
disp(['Shear resistance between web the flanges']);

DIFFpos=3;                    % Assummed value of the deepness [m]
ved=DIFFx/(tf*DIFFpos);
kwbw=0.4;                     % Recommended value from EC1992-1-1 6.2.4
tetaI=30;                     % Angle between the longitudinal direction of
the beam and the reinforcement Assumed
sinSigmaf=sind(tetaI);
cosSigmaf=cosd(tetaI);
v=0.6*(1-(fck/250e6));        % Reduction coefficient for the shear
resistance of cracked concrete. 1992-1-1 eq 6.6N
VRfw = kwbw*fctd;             % Shear resistance between the flenges and
the web
if ved<=(VRfw)
    disp('No need to check the shear resistance between web flanges and
flanges');
```

```
    else
    disp('NOT OK! Checking needed');
    if ved<=v*fcd*sinSigmaf*cosSigmaf
    disp('The shear resistance of the cross section is OK!');
    disp(['The ratio of usage for the shear force with shear reinforcment
is =', num2str(Ved/VRd)]);
    else
        disp('%%%%%%%%%%%%%%%%%%%%WARNING%%%%%%%%%%%%%%%%%%%%%%%%');
        disp('%%%%%%%%%%%%%%%_________________%%%%%%%%%%%%%%%%%%%');
        disp('NOT OK! The shear resistance in the flanges does not
fulfill criteria');
        disp ('Change your input');
        disp('%%%%%%%%%%%%%%%%%%%%WARNING%%%%%%%%%%%%%%%%%%%%%%%%');
        disp('%%%%%%%%%%%%%%%_________________%%%%%%%%%%%%%%%%%%%');
    end
end

% Shear between flanges and the web due to derailment

if VRfw >= Vderail
    disp ('Shear force due to derailment is OK!!');
else
  disp('%%%%%%%%%%%%%%%%%%%%WARNING%%%%%%%%%%%%%%%%%%%%%%%%');
  disp('%%%%%%%%%%%%%%%_________________%%%%%%%%%%%%%%%%%%%');
  disp ('Not enough shear resistance between the flanges and the web due
to derailment action');
  disp ('Change your input');
  disp('%%%%%%%%%%%%%%%%%%%%WARNING%%%%%%%%%%%%%%%%%%%%%%%%');
  disp('%%%%%%%%%%%%%%%_________________%%%%%%%%%%%%%%%%%%%');
end

%%%%%%%% Shear resistance in the slab between the T sections %%%%%%%%%
    disp(['____________']);
    disp(['Shear Force between the T sections']);
    bwm = (4.5-tw)*1e3;   % smallest width of the cross section in
tension [mm]
    dmm=(tf-c)*1e3;          %d in mm assuming the minimum reinforcement
in the flenge at the distance c from the bottom of the flenge
    Crd = 0.18/gammaC ;      % Given in EN
    k = min(1+sqrt(200/(dmm)),2);
    pho1 = min(Asmin*1e6/(bwm*dmm),0.02);
    k1 = 0.15;                                          % Given EN
    Nedp=0;                                             % N Prestressing
    sigmacp = min(Nedp/((tf*1)*1e6),0.2*fcd/1e6);       % tf*1 =
Aconcrete per meter in the slab
    vmin = 0.035*k^(3/2)*fck^(1/2);
    v = 0.6*(1-(fck/1e6)/250);

    Vcrd1_slab = (Crd*k*(100*pho1*fck/1e6)^(1/3)+k1*sigmacp)*bwm*dmm;
    Vcrd2_slab = (vmin+k1*sigmacp)*bwm*d;
    Vcrd3_slab = 0.5*bwm*dmm*v*(fcd/1e6);

    Vcrd_slab = min (Vcrd3_slab, max (Vcrd1_slab, Vcrd2_slab));

    Vcrd_slab = Vcrd_slab/gammaD;
    if SEC == 1
        Ved_slab = 0;
    elseif SEC ==2
```

```
        Ved_slab = Ved;
    end
    disp(['The design value for the Shear Force between the T sections is 
Ved_slab=', num2str(Ved_slab/10^3),'kN']);
    disp(['If Ved_slab/Vcrd_slab<=1']);
    if Ved_slab/Vcrd_slab <=1
        disp('The shear resistance of the cross section is OK!');
        disp(['The ratio of usage for the shear force is =', 
num2str(Ved_slab/Vcrd_slab)]);
    else
        disp('%%%%%%%%%%%%%%%%%%%%WARNING%%%%%%%%%%%%%%%%%%%%%%%%');
        disp('%%%%%%%%%%%%%%%_________________%%%%%%%%%%%%%%%%%%%');
        disp('NOT OK! The shear resistance does not fulfill criteria');
        disp('Change your input or Shear reinforcement required in the 
slab');
        disp('%%%%%%%%%%%%%%%%%%%%WARNING%%%%%%%%%%%%%%%%%%%%%%%%');
        disp('%%%%%%%%%%%%%%%_________________%%%%%%%%%%%%%%%%%%%');
    end


 %%%%% Total reinforcement area %%%%%%%%%%%%%%%%%%%%

As = Asl+Asw;

%%%%%%%%%%%%%%% Moment resistance %%%%%%%%%%%%%%%%%
%Code
disp(['____________']);
disp(['Bending Moment']);
disp(['____________']);
disp(['Concrete']);
disp(['Concrete class fck=', num2str(fck/1e6), 'MPa']);
M=Med+Ned*0.1;%Nm
b=bef; %m Section width befective because it is considered to be a 
rectangular cross section. Check CrossSection.m
lamda=0.8; %Because fck<=50MPa
nu=1; %Because fck<=50MPa
alfaIII= 0.8; %Recommended
betaIII= 0.4; %Recommended
Ratio_xd=1/(2*betaIII)-sqrt((1/(4*betaIII^2)-
(M/(alfaIII*betaIII*fcd*b*d^2))));
x=Ratio_xd*d; %m Lever arm
disp(['____________']);
disp(['Reinforcement']);
disp(['Steel Class Selected is B']);
k=1.08; %ft/fy
ep_ud=.9*ep_uk*10; %Permile mm/m Design strain at full force
ep_s=ep_cu3/1000*(d-x)/x; %Elastic strain of concrete


if ep_s<=fyd/Es
Sigma_s=Es*ep_s;
else
Sigma_s=fyd+(k-1)*fyd*(ep_s-fyd/Es)/(ep_uk/100-fyd/Es);
end
Mcrd=As*Sigma_s*(d-.4*x);

%Design resistance Check (Bending Moment)
```

```
disp(['____________']);
disp(['The design value for the bending moment is Med=',
num2str(Med/1e3),'kNm']);
disp(['If Med/Mcrd<=1']);

if Med/Mcrd <=1
    disp('The moment resistance of the cross section is OK!');
    disp(['The ratio of usage for the bending moment is =',
num2str(Med/Mcrd)]);
    else
    disp('%%%%%%%%%%%%%%%%%%%%WARNING%%%%%%%%%%%%%%%%%%%%%%%%');
    disp('%%%%%%%%%%%%%%%_________________%%%%%%%%%%%%%%%%%%%');
    disp('NOT OK! The moment resistance does not fulfill criteria. Change
your input');
    disp(['The ratio of usage for the bending moment is =',
num2str(Med/Mcrd)]);
    disp('%%%%%%%%%%%%%%%%%%%%WARNING%%%%%%%%%%%%%%%%%%%%%%%%');
    disp('%%%%%%%%%%%%%%%_________________%%%%%%%%%%%%%%%%%%%');
end
if SPAN==1
Mcrdmin=1;
Med_min=0;
elseif SPAN==2
Mcrdmin=Asbn*Sigma_s*(d-.4*x);
if -Med_min/Mcrdmin <=1
    disp('The moment resistance of the cross section is OK!');
    disp(['The ratio of usage for the bending moment is =',
num2str(Med/Mcrd)]);
    else
    disp('%%%%%%%%%%%%%%%%%%%%WARNING%%%%%%%%%%%%%%%%%%%%%%%%');
    disp('%%%%%%%%%%%%%%%_________________%%%%%%%%%%%%%%%%%%%');
    disp('NOT OK! The moment resistance does not fulfill criteria. Change
your input');
    disp(['The ratio of usage for the bending moment is =',
num2str(Med/Mcrd)]);
    disp('%%%%%%%%%%%%%%%%%%%%WARNING%%%%%%%%%%%%%%%%%%%%%%%%');
    disp('%%%%%%%%%%%%%%%_________________%%%%%%%%%%%%%%%%%%%');
end
end
% Moment between flanges and the web due to derailment

Ft = fctm*1*tf+fyd*Asmin;
MRfw = Ft*(tf-2*c);

if MRfw >= Mderail
    disp ('Moment due to derailment is OK!!');
else
  disp('%%%%%%%%%%%%%%%%%%%%WARNING%%%%%%%%%%%%%%%%%%%%%%%%');
  disp('%%%%%%%%%%%%%%%_________________%%%%%%%%%%%%%%%%%%%');
  disp 'Not enough moment resistance between the flanges and the web due
to derailment action'
  disp ('Change your input');
  disp('%%%%%%%%%%%%%%%%%%%%WARNING%%%%%%%%%%%%%%%%%%%%%%%%');
  disp('%%%%%%%%%%%%%%%_________________%%%%%%%%%%%%%%%%%%%');
end

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%Fatigue
Control%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

```
disp(['____________']);
disp(['Fatigue control']);
disp(['____________']);
disp(['Fatigue for the concrete']);

s = 0.2;                                       % Coeff depending on cement class
given in EN 1992-1-1 (3.1.2)
gammaC_fat=1.5;                                % Design partial factor taking
material uncertainties into account En1992-1-1:2005 2.4.2.4 (1)
fcdft=fck/gammaC_fat;                         %Design compressive concrete strength
in MPa EN 1992-1-1:2005 3.1.3 table 3.1
betacc = exp(s*(1-sqrt(28/t)));                % Coeff depending on the age of
concrete En 1992-2:2005 6.8.7 (1)
k1 = 0.85;                                     % Given by EN 1992-2-2005
fcmt = betacc*fcm;
if (t>=3) && (t<28)
    fck = fcmt-8E6;
elseif (t>=28)
    fck = fck;
else
    disp 'Age of concrete t should not be under 3 days'
end
fcdfat = k1*betacc*fcdft*(1-(fck/1E6)/250);      % fatigue strength [Pa]
Ecdmax = sigmamax/fcdfat;          % Max compressive stress
Ecdmin = sigmamin/fcdfat;                      % Min compressive stress
R = Ecdmin/Ecdmax;                             % Stress ratio
%N = 10^((14*(1-Ecdmax))/sqrt(1-R));            % Num of cycles before failure

N=1e6;
Dc=Ecdmax+log10(N)/14*sqrt(1-R);
if Dc<=1
 disp('The fatigue of the concrete the cross section is OK for the life
span!');
 disp(['The total damage inflicted on the concrete is =', num2str(Dc)]);
else
        disp('%%%%%%%%%%%%%%%%%%%%WARNING%%%%%%%%%%%%%%%%%%%%%%%%%');
        disp('%%%%%%%%%%%%%%%_________________%%%%%%%%%%%%%%%%%%%');
        disp('NOT OK! The damage inflicted does not fullfil criteria');
        disp ('Change your input');
        disp(['The total damage inflicted on the concrete is =',
num2str(Dc)]);
        disp('%%%%%%%%%%%%%%%%%%%%WARNING%%%%%%%%%%%%%%%%%%%%%%%%%');
        disp('%%%%%%%%%%%%%%%_________________%%%%%%%%%%%%%%%%%%%');
end
%%%%%%%%%%%%%%Fatigue for the reinforcement En 1992-1-1 6.8.5
disp(['____________']);
disp(['Fatigue for the reinforcement']);

k2_fat=9; %SS-EN 1992-1-1 Table 6.3N
lamda_1=0.65; %Factor that takes into account the influence line EN1992-2
Figure NN.2 or Annex F of EN 1991-2 for Traffic mix
Vol = 25*freq*10*365/2% Volume of traffic (25 tonnes*number of trains*10
vagons) (Tonnes/year/track)
lamda_2=(Vol/25e6)^(1/k2_fat); %Factor that takes into account the
traffic volume. SS_EN1992-2 Eq NN.109
lamda_3=(T_b/100)^(1/k2_fat); %Factor that takes into account the life
span. SS-EN1992-2 Eq NN.110
n_fat=0.12; %SS-EN1992-2 Eq NN.111 Recommended
```

```
s1=sigmamax/(sigmamax+sigmamax);
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
Check again
s2=s1;
lamda_4=1;%(n_fat+(1-n_fat)*s1^k2_fat+(1-n_fat)*s1^k2_fat)^(1/k2_fat);
%Factor that takes into account the simultaneous loading of the tracks.
SS-EN1992-2 Eq NN.111
sigma_sEc=sigmamax-sigmamin;
lamda_s=lamda_1*lamda_2*lamda_3*lamda_4; %Lamda factor taking into
account the traffic volume, life span and stress range
sigma_seq=sigma_sEc*dyn_fat*lamda_s; %[Pa] Equivalent Stress range SS-EN
1992-2

if gammaF_Fat*sigma_seq<=sigma_rsk/gammas_Fat
   disp('The fatigue of the concrete the cross section is OK for the life
span!');
   disp(['The ratio of usage for fatigue in the reinforcement is =',
num2str(gammaF_Fat*sigma_seq/(sigma_rsk/gammas_Fat))]);
else
   disp('%%%%%%%%%%%%%%%%%%%%WARNING%%%%%%%%%%%%%%%%%%%%%%%%');
   disp('%%%%%%%%%%%%%%%_________________%%%%%%%%%%%%%%%%%%%');
   disp('NOT OK! The fatigue failure for the reinforcement will be
produced before the end of the life span');
   disp ('Change your input');
   disp('%%%%%%%%%%%%%%%%%%%%WARNING%%%%%%%%%%%%%%%%%%%%%%%%');
   disp('%%%%%%%%%%%%%%%_________________%%%%%%%%%%%%%%%%%%%');
end



%%%%%%%%%%%%%%%%%%%%%%%%% Torsion
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%Torsional resistance member subjected to torsion + shear
disp(['____________']);
disp(['Torsional resistance (Torsion + Shear']);
TEd = Ted;
bw=bef;
ny = 0.6*(1-(fck/1e6)/250);          % fck is changed into in MPa
Ak = Aconcrete;                      % Area inclosed by centre lines of
connection walls En 1992-1-1 fig 6.1
tefi = bw;                           % Effective wall thickness
teta=45;                             % Angle between shear reinforcement and
the axes of the beam
%alphacw needs to be calculated just in case above conditions are safe
and shear reinforcement is not fulfilled
if (sigmacp >= 0) && (sigmacp <= 0.25*fcd)
        alphacw = (1+sigmacp/fcd);                      % coeff account
for stress in compression chord
      elseif (sigmacp > 0.25*fcd) && (sigmacp <= 0.5*fcd)
        alphacw = 1.25;
      elseif (sigmacp > 0.5*fcd) && (sigmacp <= fcd)
         alphacw = 2.5*(1-sigmacp/fcd);
end
TRd = 2*ny*alphacw*fcd*Ak*tefi*sin(teta)*cos(teta); %Computations
according to EC1992 Torsion

%Computations according to Hakan Sundquist. Torsion in concrete beams
if TEd/TRd+Ved/VcrdT <= 1
```

```
    disp('The torsional resistance of the cross section is OK!');
    disp(['The ratio of usage for the torsion an shear force is =',
num2str(TEd/TRd+Ved/VcrdT)]);
else
    disp('%%%%%%%%%%%%%%%%%%%%WARNING%%%%%%%%%%%%%%%%%%%%%%%%%');
    disp('%%%%%%%%%%%%%%%_________________%%%%%%%%%%%%%%%%%%%');
    disp('NOT OK! The torsional resistance does not fulfil criteria.
Change your input');
    disp('%%%%%%%%%%%%%%%%%%%%WARNING%%%%%%%%%%%%%%%%%%%%%%%%%');
    disp('%%%%%%%%%%%%%%%_________________%%%%%%%%%%%%%%%%%%%');
    disp(['The ratio of usage for the torsion an shear force is =',
num2str(TEd/TRd+Ved/VcrdT)]);
end

%%%%%%%%%%%% Resistance check summary %%%%%%%%%%%%%%%%%%%%%

disp(['___________________________________________________________________
_____']);
disp(['*******************************************************************
*****']);
disp(['           Resistance checks summary']);
disp(['*******************************************************************
*****']);
disp(['*******************************************************************
*****']);
if TEd/TRd+Ved/VcrdT <= 1 && Ved/VcrdT <=1 && Med/Mcrd <=1 && Dc<=1 &&
VRfw >= Vderail && MRfw >= Mderail && bars >0 && NA<=tf && Mderail/MRfw
<= 1
    disp('The Resistance checks for the cross section are OK!');
else

disp(['___________________________________________________________________
']);

disp(['___________________________________________________________________
']);

disp('%%%%%%%%%%%%%%%%%%%%WARNING%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
');

disp('%%%%%%%%%%%%%%%_________________%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
');
    disp('NOT OK! Check the file again. Change your input');

disp('%%%%%%%%%%%%%%%%%%%%WARNING%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
');

disp('%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
');
end

disp(['___________________________________________________________________
____']);

disp(['Fatigue %   Med/Mcrd % Med_min/Mcrdmin % Ved/VcrdT %
TEd/TRd+Ved/VcrdT %  Mderail/MRfw']);
disp(['___________________________________________________________________
____']);
```

```
disp(['%', num2str(Dc),'    %    ', num2str(Med/Mcrd),'  %   ', num2str(-
Med_min/Mcrdmin), '%    ', num2str(Ved/VcrdT),'     %    ',
num2str(TEd/TRd+Ved/VcrdT) '      %         ' num2str(Mderail/MRfw)]);
disp (['Number of reinforcement bars is ' num2str(n), ' phi '
num2str(phi*1000)]);
if SPAN == 2

disp('%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%');
disp(['_________________________________________________________________
']);
disp '                      Negative reinforcement
';
disp(['_________________________________________________________________
']);
  disp 'The continuous bridge needs negative reinforcement over the
supports';

disp(['_________________________________________________________________
']);

disp(['_________________________________________________________________
']);
    if n_neg <= nrow_neg
      disp 'The bars fit in one row'
      disp (['Number of reinforcement bars is ' num2str(n_neg), ' phi '
num2str(phi_neg*1000)]);
    else
     disp 'The number of bars does not fit in one row!!!';
     disp (['Number of reinforcement bars is ' num2str(n_neg), ' phi '
num2str(phi_neg*1000)]);
    end
end

disp (['Med = ' num2str(Med/1000), ' Mcrd = ' num2str(Mcrd/1000)]);
disp (['Ved = ' num2str(Ved/1000), ' VcrdT = ' num2str(VcrdT/1000)]);
disp (['TEd = ' num2str(TEd/1000), ' TRd = ' num2str(TRd/1000)]);
disp (['TEd/TRd = ' num2str(TEd/TRd)]);
```

B. Appendix B: Matlab® codes, dynamic design

In this appendix the dynamic evaluation is shown. The Matlab® codes will be presented first and then other kind of script used in Brigade Plus®. The Figure 8.2 shows the flowchart for the Matlab® functions.

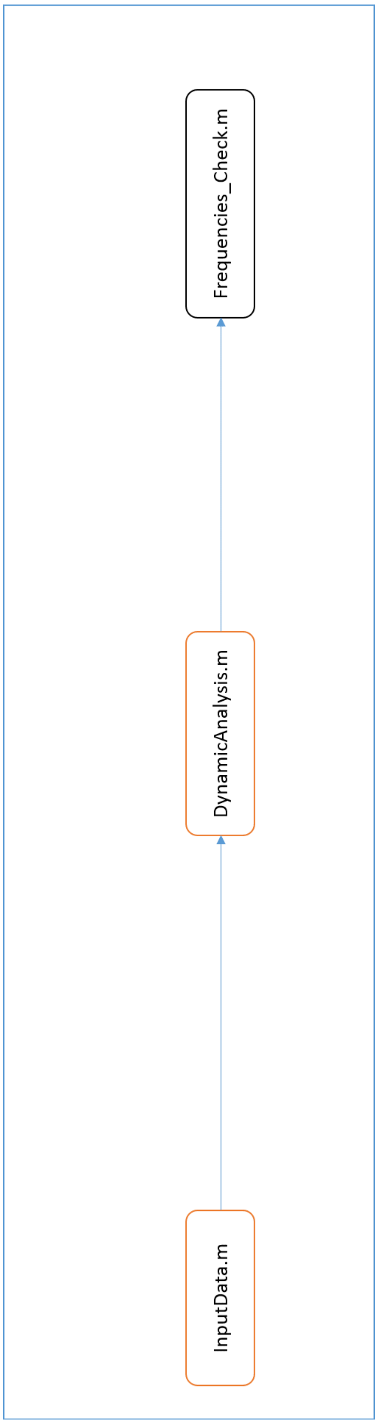


Figure 8.2 Flow chart of the dynamic Matlab® functions.

Dynamic Evaluation.

```
% function [m,mL,EI,EIx] = DynamicEvaluation ()
% Output
% m: Total mass of the bridge [Kg]
% mL: Mass per meter of the bridge [Kg/m]
% EI: Product of the Inertia and young modulus
% EIx: Product of the Inertia and young modulus with respect to x

function [m,mL,EI,EIx] = DynamicEvaluation ();
disp(['____________']);
disp(['Dynamic evaluation. Mass and Inertia of the model']);
disp(['____________']);

%Mass Evaluation of the model
%Linear mass and moment of inertia for the dynamic calculations
[bf,tf,hw,tw,h,bef,l_0,fyk,fyd,fck,alfa_cc,alfa_ct,bballast,gammaC,gammaD
,T_b,fcd,Es,Ecm,fctm,fctk_0_05,fctd,c,d,L,alfa,ep_cu3,Aconcrete,qconcrete
,qballast,Con,Train,freq,ep_uk,t,fcm,gammaF_Fat,gammas_Fat,sigma_rsk,Main
tenance,phi,phi_neg,SEC,SPAN,Ltot]=InputData();

disp(['The total length of the bridge is L = ', num2str(L),' m']);
disp(['____________']);

%Linear Mass
Aedge=0.2*(0.3+tf); % [m2] Area of the edge beam
Aslab=2.4*0.3; %[m2] Area of the slab track (Width of the slab
track)*(depth)*number
if Con == 1
  qballast = qballast;
  Aslab = 0;
  elseif Con == 0
  qballast = 0;
  Aslab = Aslab;
  end
Aextra=0;%Aedge+Aslab; %[m2] Area total of extra elements on the cross
section
Aflanges_extra=(6-bf)*tf;
mballast = qballast/9.81;
Atotal=Aconcrete+Aextra+Aflanges_extra;
dconcrete=25e3/9.81; %[kg/m3] Reinforced concrete density
mconcrete=dconcrete*Atotal; %[kg/m] Mass of the concrete
mUIC60=2*60;    %[kg/m] UIC60 rail mass
m=1*(mconcrete+mUIC60+mballast); %[kg/m]Total mass of the bridge per
length unit

if SEC == 1
  m=m;
  elseif SEC==2
  m=2*m;
end

mL=m*Ltot; %[kg]Total mass of the bridge

disp(['____________']);
disp(['The total mass of the bridge is mL = ', num2str(mL),' kg']);
```

```
disp(['_____________']);
disp(['The total mass of the bridge is m = ', num2str(m/1e3),' Ton/m']);

%Moment of Inertia for the dynamic calculations
if SEC == 1
    y=h-(h^2*tw+tf^2*(bf-tw))/(2*(bf*tf+hw*tw)); %[m] Center of gravity
of the section
    I=(tw*y^3+bf*(h-y)^3-(bf-tw)*(h-y-tf)^3)/3; %[m4] Moment of inertia
of the single section
    xcg = (6*tf*(6/2)+hw*tw*(bf/2))/(6*tf+hw*tw);
    Ix = (hw*tw^3)/12+hw*tw*(xcg-bf/2)^2+(tf*6^3)/12+tf*6*(6/2-xcg)^2;
  elseif SEC == 2
    %Cross section X
    %Find X
    xw=0; %Chosen as 0
    xf=0.75; %Hand calculation
    Aw=hw*tw;
    Af=tf*12;
    xtot=(xw*Aw+xf*Af)/(Aw+Af);
    disp(['The position of x is placed x=', num2str(xtot),' m from the
center of the web']);
    difftoWeb=0.75+xtot;
    disp(['The position of the flange is placed x=', num2str(difftoWeb),'
m from the center of gravity']);
    %Find Y
    yw=hw/2;
    yf=hw+tf/2;
    Aw=hw*tw;
    Af=tf*12;
    ytot=(yw*Aw+yf*Af)/(Aw+Af);
    I = 2*((tw*hw^3)/12+tw*hw*(yw-ytot)^2)+(tf^3)+tf*12*(yf-ytot)^2;
    disp(['The position of y is placed y=', num2str(ytot),' m from the
bottom of the web']);
    disp(['The moment of inertia of the beam is I = ', num2str(I),'
m4']);
    disp(['_____________']);
    Ix = (tf*12^3)/12+tf*12*(12/2)^2+2*((hw*tw^3)/12+hw*tw*(bf/2)^2);
    end

factor=1; % Reduction factor for the E modulus of the beam in dynamics
Ebeam=Ecm*factor;
disp(['The Youngs Modulus of the beam is E = ', num2str(Ebeam/1e9),'
GPa']);
disp(['_____________']);
EI= Ebeam*I;
EIx = Ebeam*Ix;
disp(['The EI = ', num2str(EI/1e9),' GNm2']);
disp(['_____________']);
%Total height of the beam+rails
hrail=0.172;
tslab=0.3;
h_tot=hw+tf+tslab+hrail;
disp(['The total height is h = ', num2str(h_tot),' m']);
disp(['_____________']);
```

Frequencies Check.

```
clc
clear all
close all

% Input data
[bf,tf,hw,tw,h,bef,l_0,fyk,fyd,fck,alfa_cc,alfa_ct,bballast,gammaC,gammaD
,T_b,fcd,Es,Ecm,fctm,fctk_0_05,fctd,c,d,L,alfa,ep_cu3,Aconcrete,qconcrete
,qballast,Con,Train,freq,ep_uk,t,fcm,gammaF_Fat,gammas_Fat,sigma_rsk,Main
tenance,phi,phi_neg,SEC,SPAN,Ltot]=InputData();

[m,mL,EI] = DynamicEvaluation();
vlim = 320/3.6;
disp '*************************************************'
disp '*                 Eigenfrequencies              *'
disp '*************************************************'

% Eigenfrequencies
n0 = 7;
n = zeros(1,n0);
for i = 1:n0
    n(i) = i;
end

if SPAN==1
w = (n.^2)*sqrt(EI/m)*(pi^2/L^2);
f = w./(2*pi)
elseif SPAN==2
f = n.^2*pi/(2*L^2)*sqrt(EI/m)
end

% Critical speed
disp '*************************************************'
disp '*                Resonance speed                *'
disp '*************************************************'
lambda = 20;
vcr = f(1)*lambda*3.6;
disp (['The critical resonance speed is ', num2str(vcr), ' km/h']);
if SPAN==1
% Check for dynamic evaluation
nt = 36.84;         % From Brigade
n0 = 9.93;%f(1);

if Con == 0
  %Additional checks Table F.2 En 1991-2:2003
  %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
  v_1=[5.21 7.08 7.5 7.5 13.54 13.75 13.96 14.17 14.38 14.38 14.38
14.38];
  v_2=[6.46 10.2 10.42 10.48 10.63 10.63 12.75 12.75 12.75 12.75 12.75
12.75];
elseif Con == 1
  %Additional checks Table F.1 En 1991-2:2003
  %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
  v_1=[5.21 5.21 5.42 7.08 7.5 7.5 13.54 13.54 13.96 14.17 14.38 14.38];
  v_2=[6.25 6.46 6.46 10.21 10.21 10.21 10.63 10.63 12.75 12.75 12.75
12.75];
```

```
end

if (L<=20) && (L<25)
    v_n=v_1;
elseif (L>=25) && (L<=30)
    v_n=v_2;
end

if (m>=5000) && (m<7000)
    v_n0= v_n(1);
elseif (m>=7000) && (m<9000)
    v_n0= v_n(2);
elseif (m>=9000) && (m<10000)
    v_n0= v_n(3);
elseif (m>=10000) && (m<13000)
    v_n0= v_n(4);
elseif (m>=13000) && (m<15000)
    v_n0= v_n(5);
elseif (m>=15000) && (m<18000)
    v_n0= v_n(6);
elseif (m>=18000) && (m<20000)
    v_n0= v_n(7);
elseif (m>=20000) && (m<25000)
    v_n0= v_n(8);
elseif (m>=25000) && (m<30000)
    v_n0= v_n(9);
elseif (m>=30000) && (m<40000)
    v_n0= v_n(10);
elseif (m>=40000) && (m<50000)
    v_n0= v_n(11);
elseif (m>50000)
    v_n0= v_n(12);
end


if (L<=20) && (L<25)
    v_n=v_1;
elseif (L>=25) && (L<=30)
    v_n=v_2;
end

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
disp '_____________'

disp '*************************************************'
disp 'Check for dynamic evaluation. Figure 6.9 EN1991-2'
disp '*************************************************'
if nt > 1.2*n0
    disp 'Use Tables F1 and F2 in Eurocode'
    if v_n0 <= vlim/f(1)
    disp 'No need to check dynamics'
    disp '_____________'
    else
    disp '!!!!!!!!!!!!!!!!!!!!!!!!!!'
    disp 'Dynamic analyses required.'
    disp '!!!!!!!!!!!!!!!!!!!!!!!!!!'
    end
else
```

```
    disp '!!!!!!!!!!!!!!!!!!!!!!!!!!!'
    disp 'Dynamic analyses required.'
    disp '!!!!!!!!!!!!!!!!!!!!!!!!!!!'
    end

elseif SPAN==2

disp '_____________'

disp '**************************************************'
disp 'Check for dynamic evaluation. Figure 6.9 EN1991-2'
disp '**************************************************'


    disp '!!!!!!!!!!!!!!!!!!!!!!!!!!!'
    disp 'Dynamic analyses required.'
    disp '!!!!!!!!!!!!!!!!!!!!!!!!!!!'
end

%% Dynamic factor 1+phi for real trains EN 1991-2-2003 %%

Lf = L;
v = vlim;
K = v/(2*Lf*n0);
if K < 0.76
    phi_prim = K/(1-K+K^4);
elseif K >= 0.76
    phiprim = 1.325;
end

if v <= 22
    alpha = v/22;
elseif v > 22
    alpha = 1;
end
phi_bis = (alpha/100)*(56*exp(-(Lf/10)^2)+50*((Lf*n0)/80-1)*exp(-
(Lf/20)^2));
if phi_bis >= 0
    phi_bis = phi_bis;
elseif phi_bis < 0
    phi_bis = 0;
end
phi_prim=0;
dynfac = phi_prim+0.5*phi_bis;

disp ([' The static load shall be multiplied by 1+',num2str(dynfac),' for
carefully maintained track'])
```

C. Appendix C: Validation of 3D models

3D models constructed in Brigade Plus® are before any analyses verified against the Matlab® model. The criteria for the model to be considered is explained in section 4.7.2 and briefly repeated here. The model is considered verified when

- The mass of the model in Brigade® is within 3 % of the mass calculated in the static design
- The maximum deflection due to self-weight is within 5 % of the one calculated in the static design
- The first bending frequency should be similar to the one given by (Andersson & Svedholm, 2016)
- The frequencies for the first three bending modes are similar to the frequencies obtained by an analytical solution.

C.1 Simply supported bridge

Table 8.1 Simply supported bridge, L = 20 m

	Matlab® model	Diagrams by Andersson and Svedholm	Brigade® model	Difference [%]
Mass [ton]	324		318	1.85
Displacement [mm]	2.7		2.6	3.85
1:st bending frequency [Hz]	11.0	11	10.8	1.8 (Matlab®) 1.8 (Andersson & Svedholm)
2:nd bending frequency [Hz]	43.4		38.8	10.6
3:rd bending frequency [Hz]	98.6		82.8	16

Table 8.2 Simply supported bridge L = 30 m

	Matlab® model	Diagrams by Andersson and Svedholm	Brigade® model	Difference [%]
Mass [ton]	513		504	1.75
Displacement [mm]	11.6		11.7	0.8
1:st bending frequency [Hz]	5.26	5.3	5.0	4.1 (Matlab®) 4.8 (Andersson & Svedholm)
2:nd bending frequency [Hz]	21.1		19.3	8.5
3:rd bending frequency [Hz]	47.4		41.5	12.4

C.2 Composite section

Table 8.3 Double T section L = 20 m

	Matlab® model	Diagrams by Andersson and Svedholm	Brigade® model	Difference [%]
Mass [ton]	648		636	1.8
Displacement [mm]	2.7		2.7	0
1:st bending frequency [Hz]	12	12	10.2	17 (Matlab®) 17 (Andersson & Svedholm)
2:nd bending frequency [Hz]	48		38	20.8
3:rd bending frequency [Hz]	108		80	26

Table 8.4 Double T section L = 30 m

	Matlab® model	Diagrams by Andersson and Svedholm	Brigade® model	Difference [%]
Mass [ton]	1027		1008	1.85
Displacement [mm]	11.6		11.6	0
1:st bending frequency [Hz]	5.8	5.8	5.1	12 (Matlab®) 12 (Andersson & Svedholm)
2:nd bending frequency [Hz]	23.1	-	23.8	3
3:rd bending frequency [Hz]	52.0	-	41.6	20

C.3 Continuous bridge

Table 8.5 Continuous bridge L = 20 m

	Matlab® model	Diagrams by Andersson and Svedholm	Brigade® model	Difference [%]
Mass [ton]	530		520	1.9
Displacement [mm]	13		13.5	3.7
1:st bending frequency [Hz]	10.0	10	9.9	1 (Matlab®) 1 (Andersson & Svedholm)
2:nd bending frequency [Hz]	40.1		37.7	5.9
3:rd bending frequency [Hz]	90.1		78.12	12

Table 8.6 Continuous bridge L = 30 m

	Matlab® model	Diagrams by Andersson and Svedholm	Brigade® model	Difference [%]
Mass [ton]	838		822	1.9
Displacement [mm]	5.7		5.5	3.5
1:st bending frequency [Hz]	4.9	4.9	4.9	0 (Matlab®) 0 (Andersson & Svedholm)
2:nd bending frequency [Hz]	19.4		19.02	1.9
3:rd bending frequency [Hz]	43.7		40.03	8.4

D. Appendix D: Modal Analyses

When reading this appendix, it is important to take into account that the rendered visualisation of the modes is not performed well in Brigade Plus®. However, the results that the software present are well represented in the analysis.

D.1 Modal analysis of the bridge L=20 m (Ballastless)

The results for the 10 first modes for the simple section bridge and length L=20 m are presented in Figure 8.3 and Table 8.7.

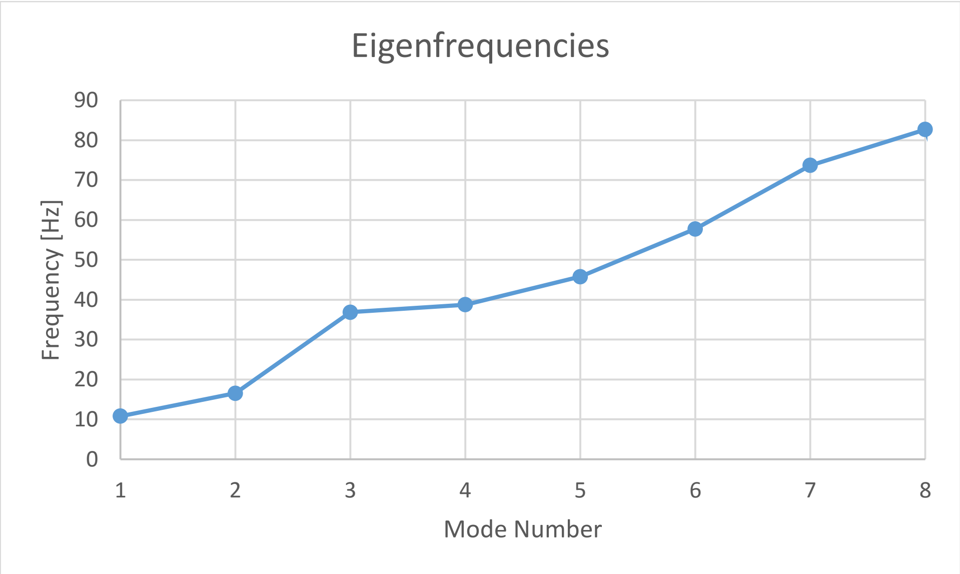


Figure 8.3 Eigenfrequencies, simple T-section. L=20 m

Table 8.7 Eigenfrequencies, simple T-section. L=20 m

Mode	Frequency (Hz)
1	10,793
2	16,531
3	36,844
4	38,755
5	45,784
6	57,751
7	73,688
8	82,672

The effective mass for those mentioned modes is presented in Figure 8.4 and Table 8.8.

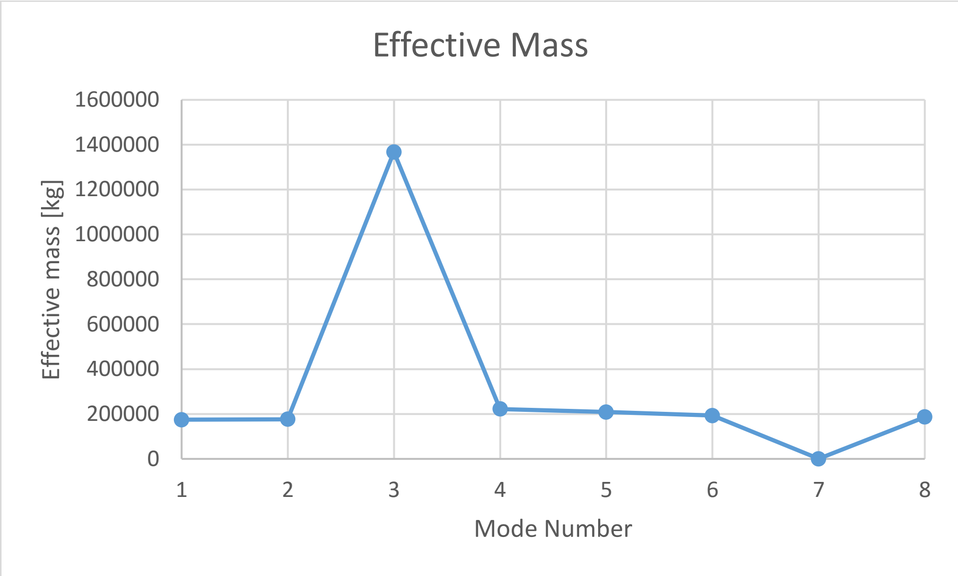


Figure 8.4 Effective mass, simple T-section. L=20 m

Table 8.8 Effective mass, simple T-section. L=20 m

Mode	Effective Mass (kg)
1	174122
2	176327
3	1367570
4	222196
5	208559
6	192870
7	1,32025
8	186082

The rendered mode shapes are shown in Figure 8.5 to Figure 8.8.

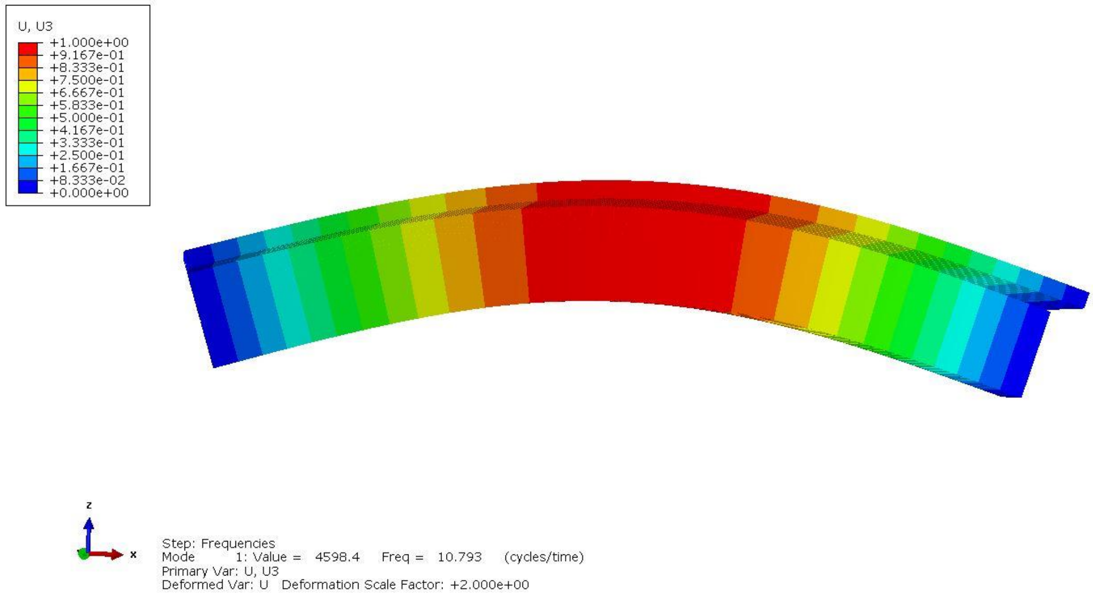


Figure 8.5 First bending mode L=20 m, f =10.8 Hz

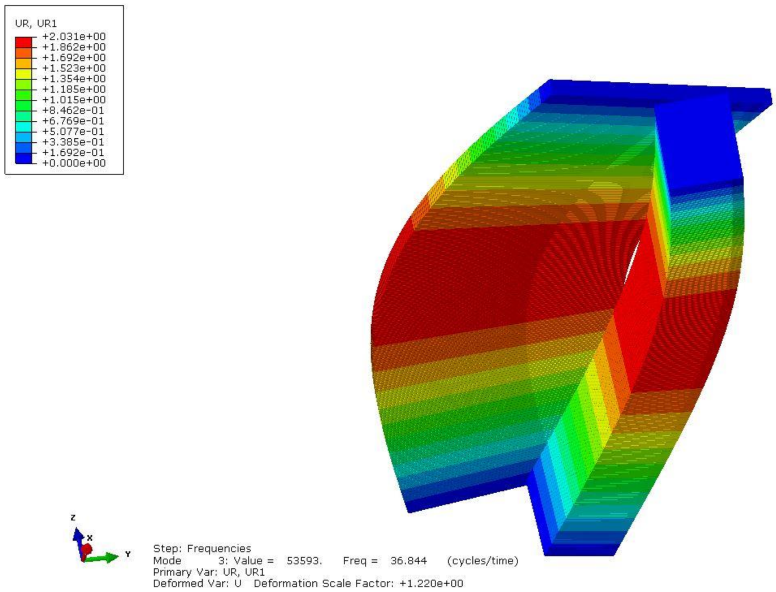


Figure 8.6 First torsional mode L=20 m, f =36.94 Hz

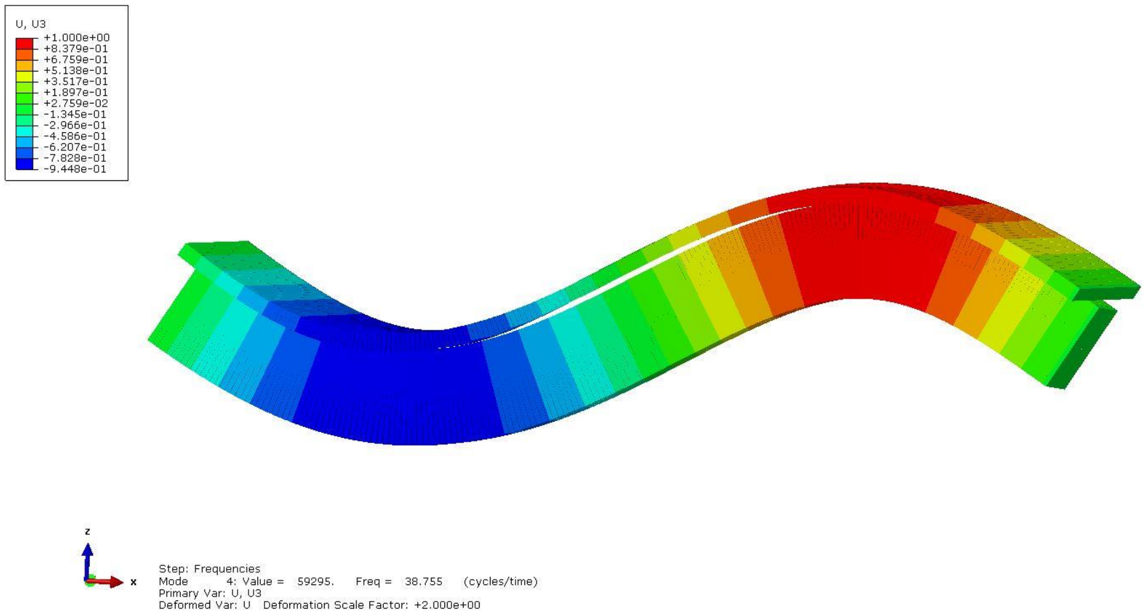


Figure 8.7 Second bending mode L=20 m, f =38,76 Hz

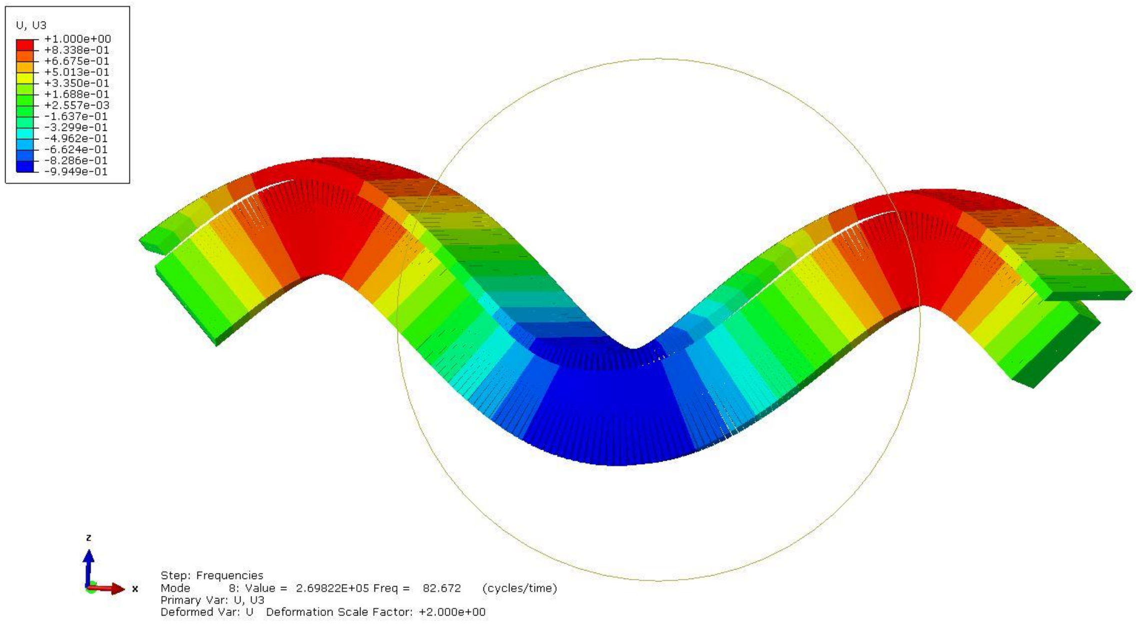


Figure 8.8 Third bending mode L=20 m, f =82,67 Hz

D.2 Modal analysis of the bridge L=30 m (Ballastless)

The results for the 10 first modes for the simple section bridge and length L=20 m are presented in Figure 8.9 and Table 8.9.

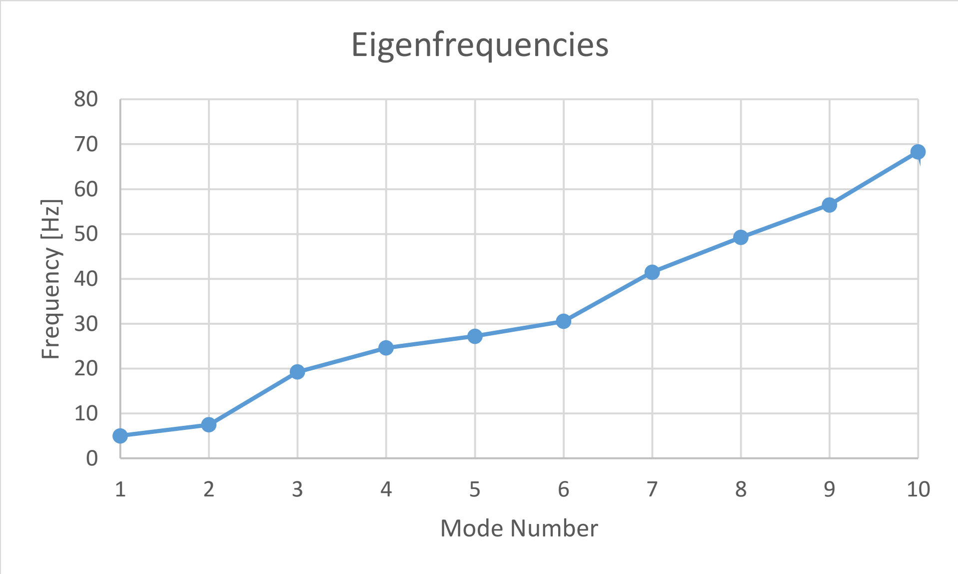


Figure 8.9 Eigenfrequencies for the simple T-section. L=30 m

Table 8.9 Eigenfrequencies, simple T-section. L=30 m

Mode	Frequency (Hz)
1	5,045
2	7,491
3	19,273
4	24,63
5	27,21
6	30,556
7	41,513
8	49,248
9	56,496
10	68,317

The effective mass for those mentioned modes is presented in Figure 8.10 and Table 8.10.

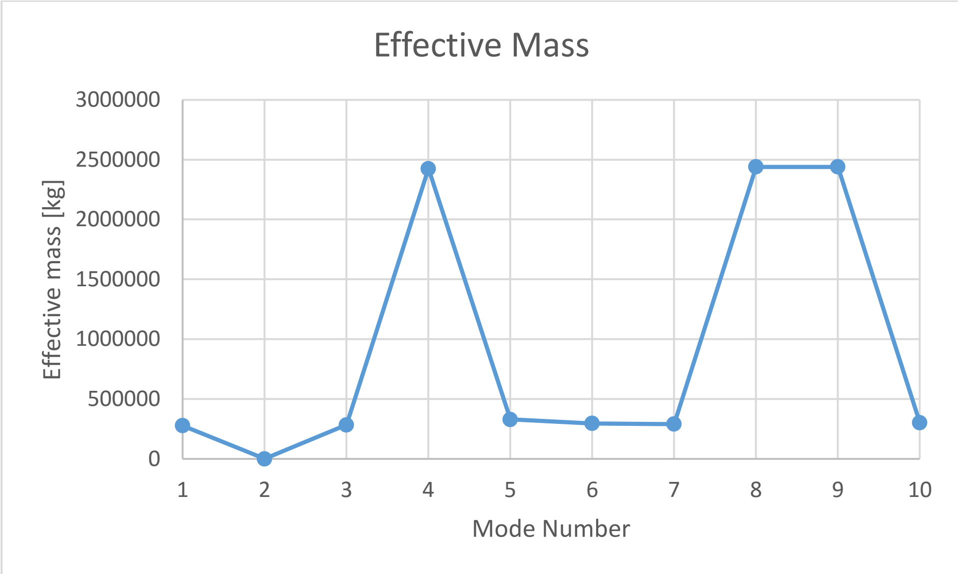


Figure 8.10 Effective mass, simple T-section. L=30 m

Table 8.10 Effective mass, simple T-section. L=30 m

Mode	Effective Mass (kg)
1	277340
2	2,7869
3	284280
4	2425000
5	329507
6	296000
7	290850
8	2440000
9	2440000
10	301020

The rendered mode shapes are shown in Figure 8.11 to Figure 8.14.

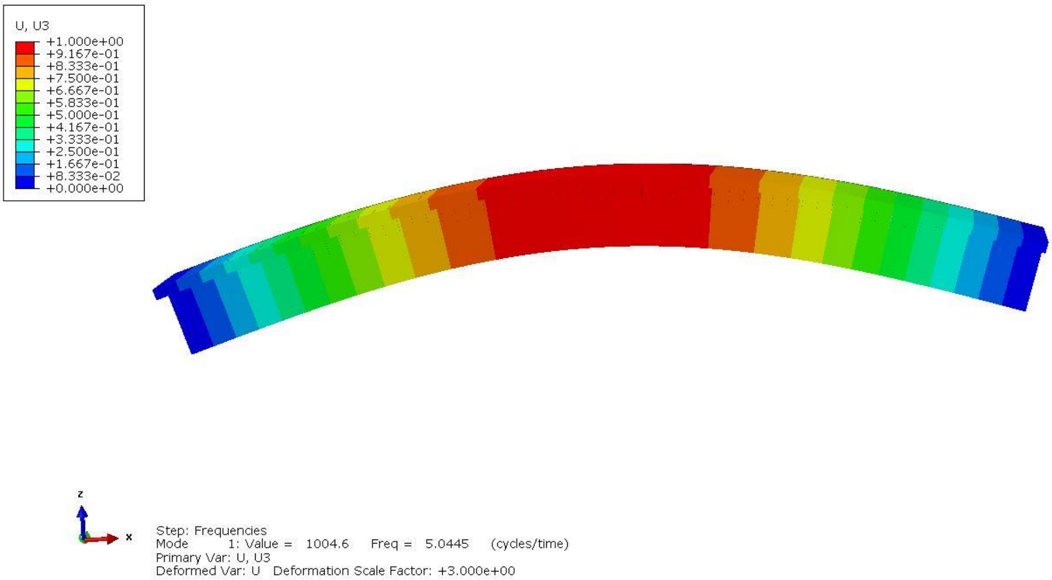


Figure 8.11 First bending mode L=30 m, f =5.045 Hz

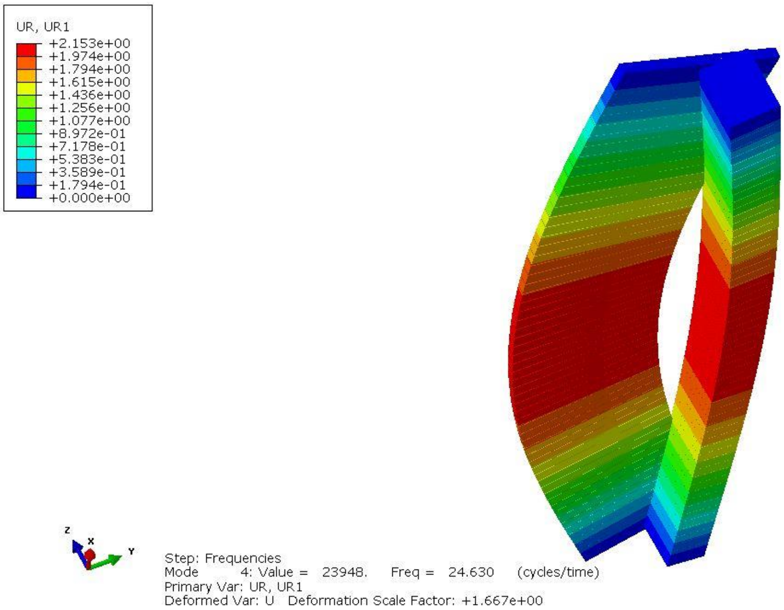


Figure 8.12 First torsional mode L=30 m, f =24.63 Hz

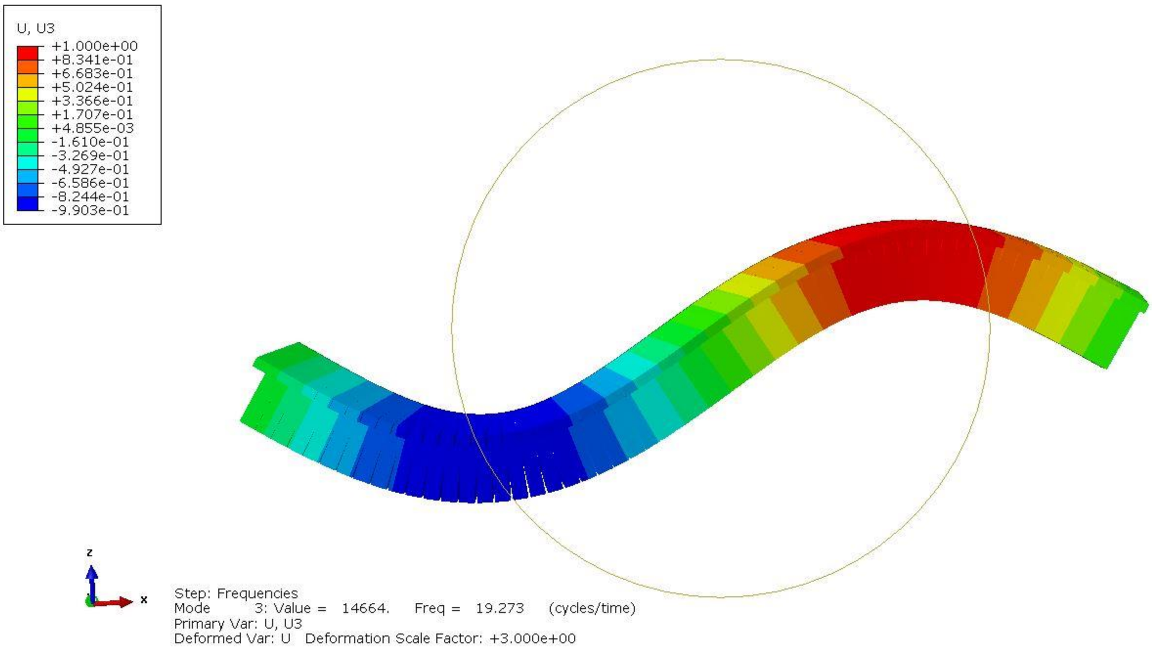


Figure 8.13 Second bending mode L=30 m, f =19,273 Hz

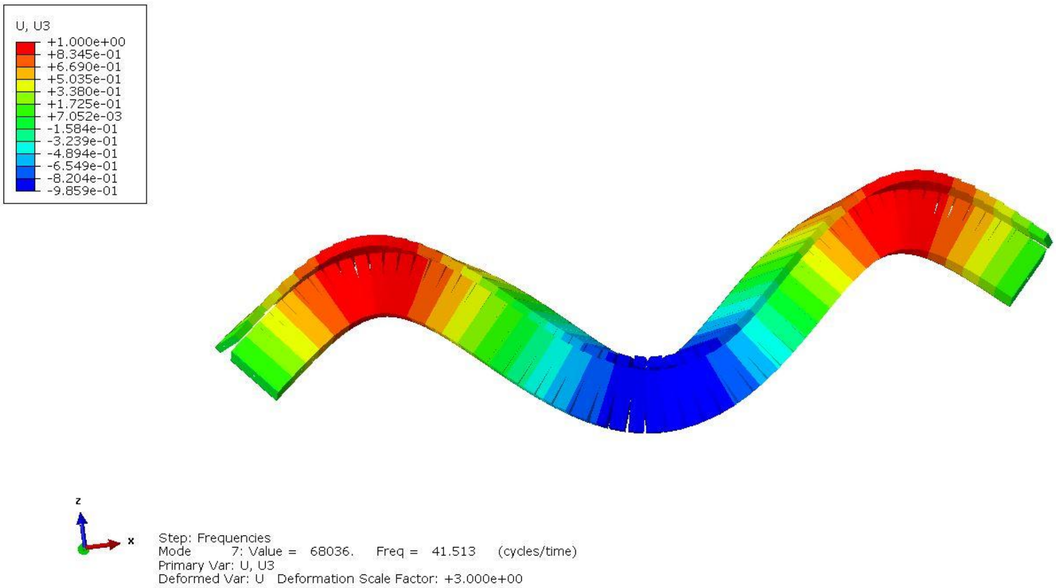


Figure 8.14 Third bending mode L=30 m, f =41,513 Hz

D.3 Modal analysis of the bridge L=20 m (Ballasted)

The results for the 10 first modes for the simple section bridge and length L=20 m are presented in Figure 8.15 and Table 8.11.

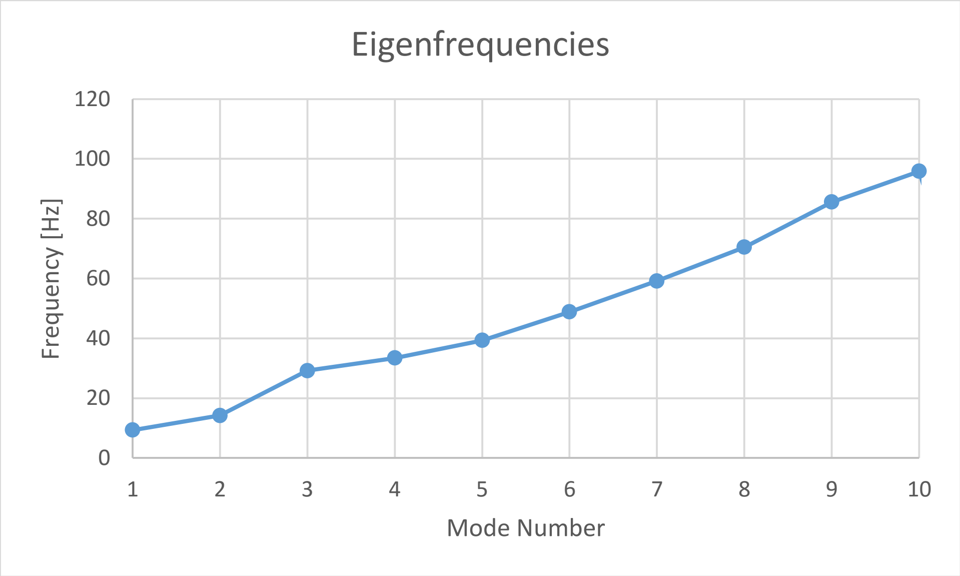


Figure 8.15 Eigenfrequencies, simple T-section. L=20 m

Table 8.11 Eigenfrequencies, simple T-section. L=20 m

Mode	Frequency (Hz)
1	9,3168
2	14,196
3	29,221
4	33,43
5	39,321
6	48,819
7	59,161
8	70,494
9	85,563
10	95,868

The effective mass for those mentioned modes is presented in Figure 8.16 and Table 8.12.

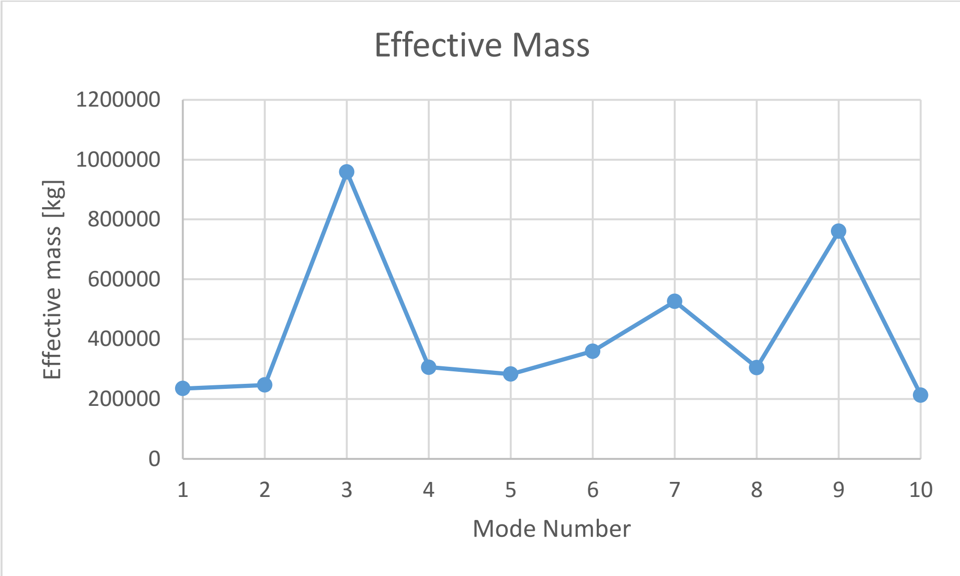


Figure 8.16 Effective mass, simple T-section. L=20 m

Table 8.12 Effective mass simple T-section. L=20 m

Mode	EffectiveMass (kg)
1	235000
2	246600
3	958800
4	306400
5	283217
6	359800
7	526000
8	305000
9	760800
10	212847

The rendered mode shapes are shown in Figure 8.17 to Figure 8.20.

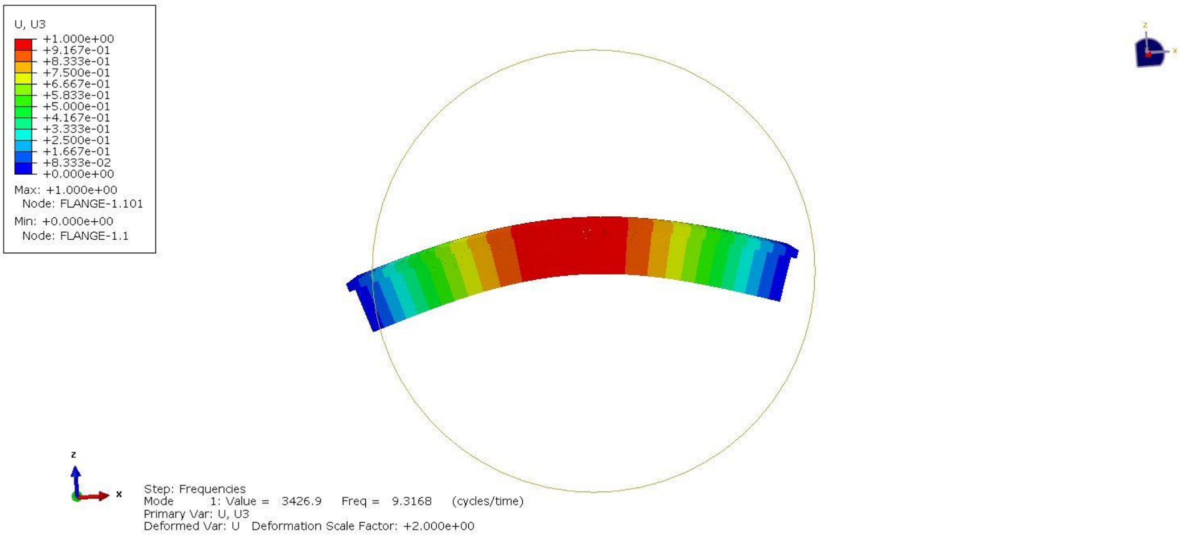


Figure 8.17 First bending mode L=20 m, Ballasted, f =9,32 Hz

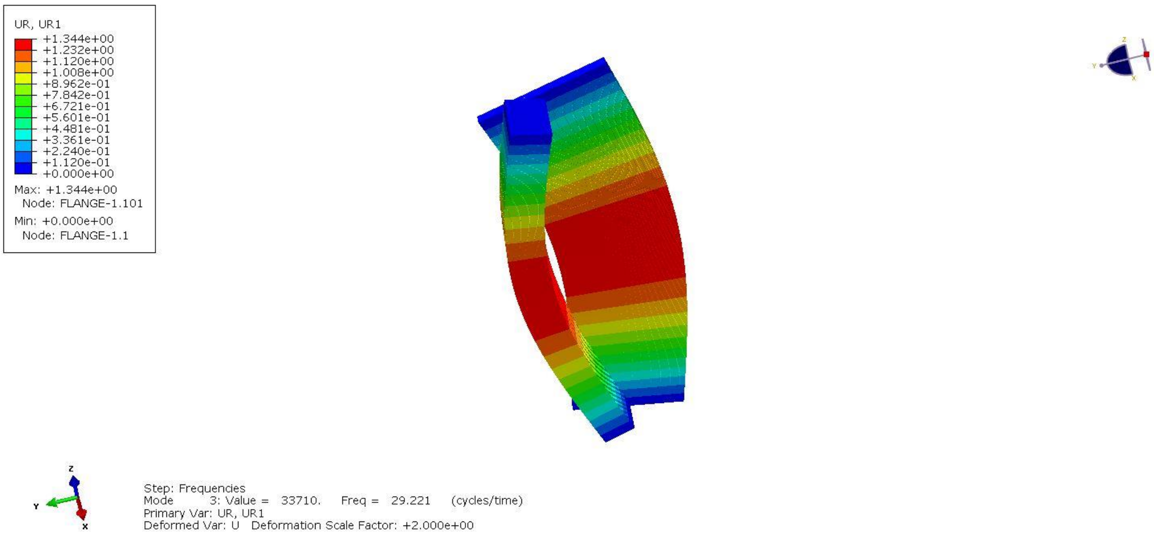


Figure 8.18 First torsional mode L=20 m, Ballasted, f =29,22 Hz

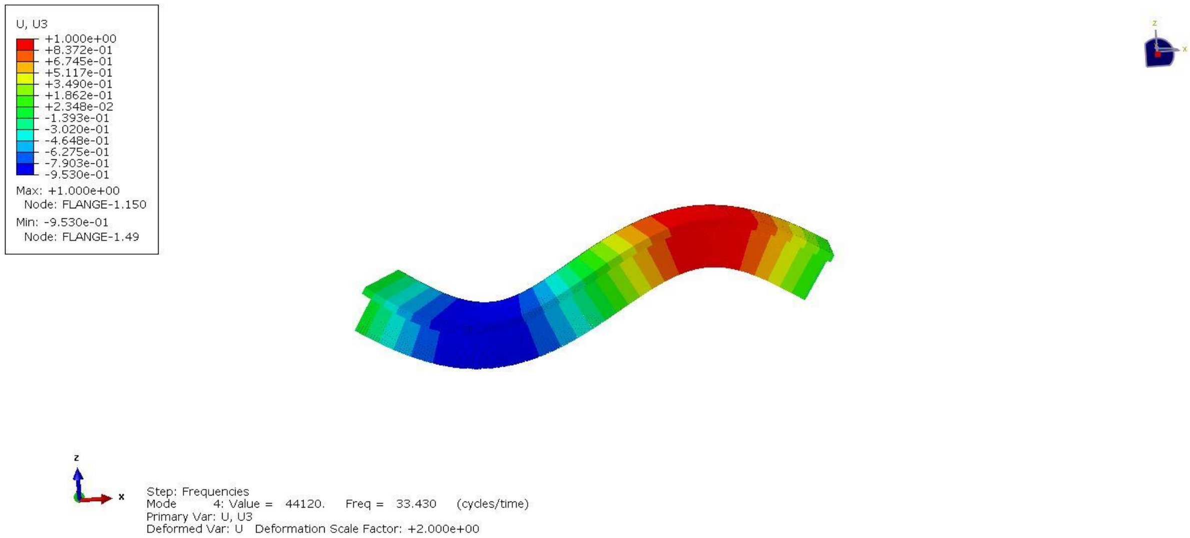


Figure 8.19 Second bending mode L=20 m, Ballasted, f =33,43 Hz

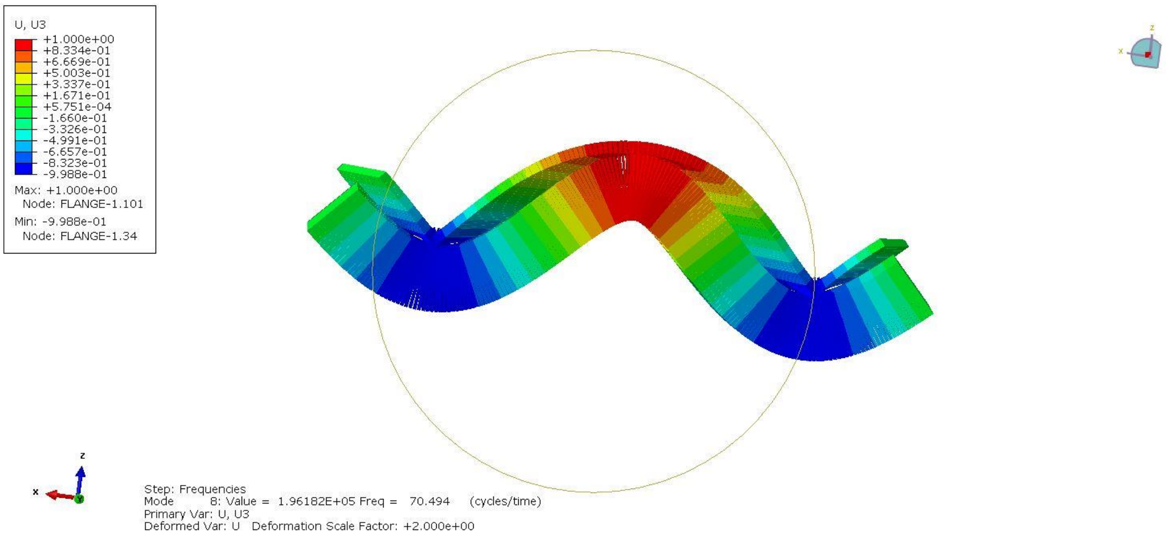


Figure 8.20 Third bending mode L=20 m, Ballasted f =70,5 Hz

D.4 Modal analysis of the bridge L=30 m (Ballasted)

The results for the 10 first modes for the simple section bridge and length L=30 m are presented in Figure 8.21 and Table 8.13.

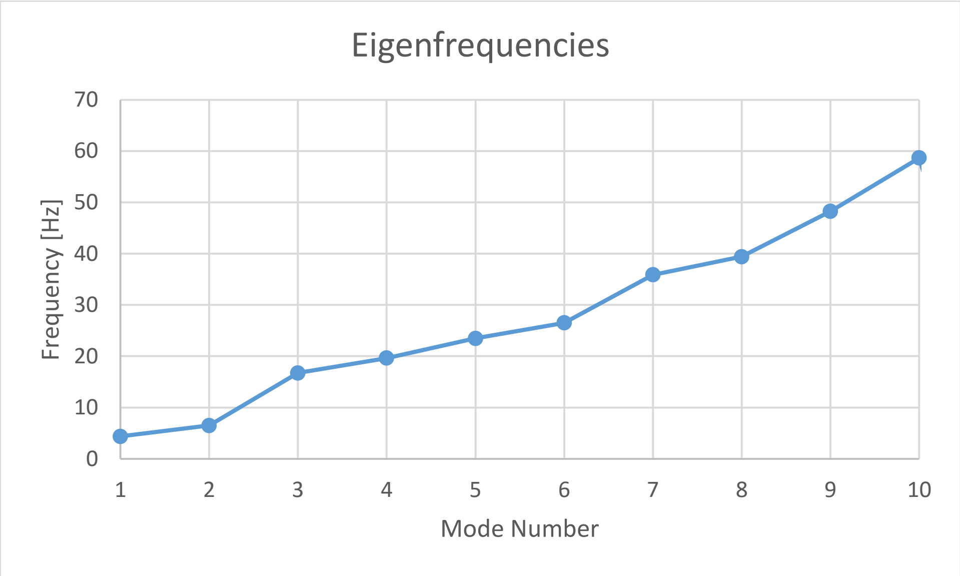


Figure 8.21 Eigenfrequencies for the simple T-section. L=30 m

Table 8.13 Eigenfrequencies for the simple T-section. L=30 3m

Mode	Frequency (Hz)
1	4,385
2	6,4991
3	16,73
4	19,64
5	23,491
6	26,504
7	35,876
8	39,393
9	48,246
10	58,676

The generalised mass for those mentioned modes is presented in Figure 8.22 and Table 8.14.

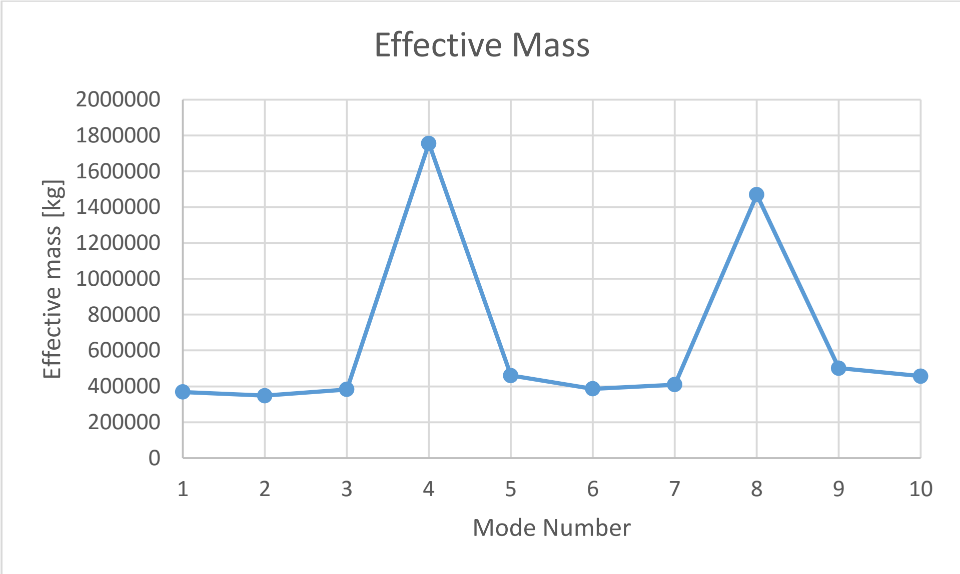


Figure 8.22 Effective mass simple T-section. L=30 m

Table 8.14 Effective mass simple T-section. L=30 m

Mode	EffectiveMass (kg)
1	368190
2	347650
3	383000
4	1756000
5	460400
6	386600
7	409218
8	1469000
9	501700
10	457000

The rendered mode shapes are shown in Figure 8.23 to Figure 8.26.

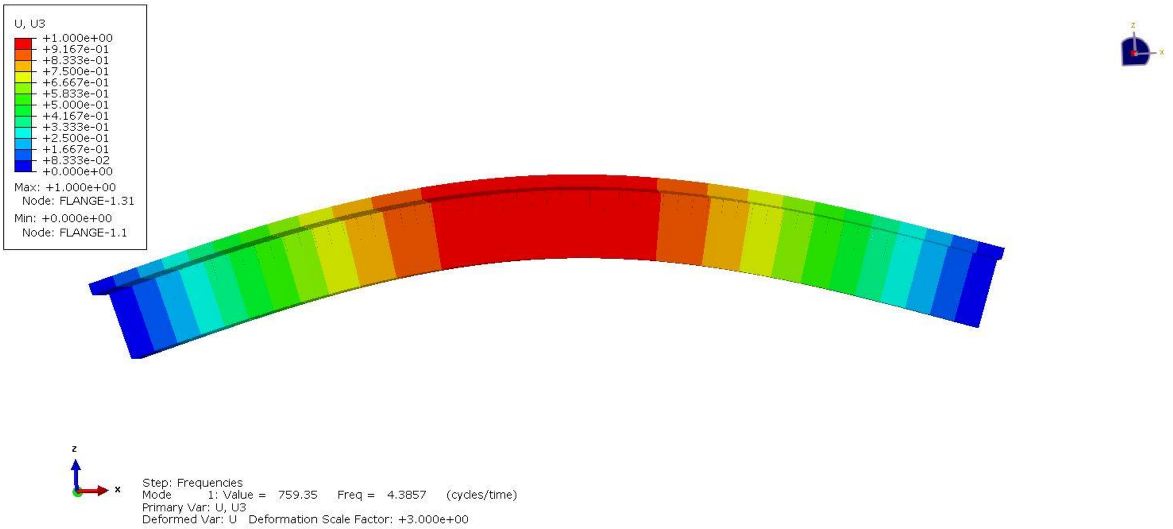


Figure 8.23 First bending mode L=30 m, Ballasted, f =4,39 Hz

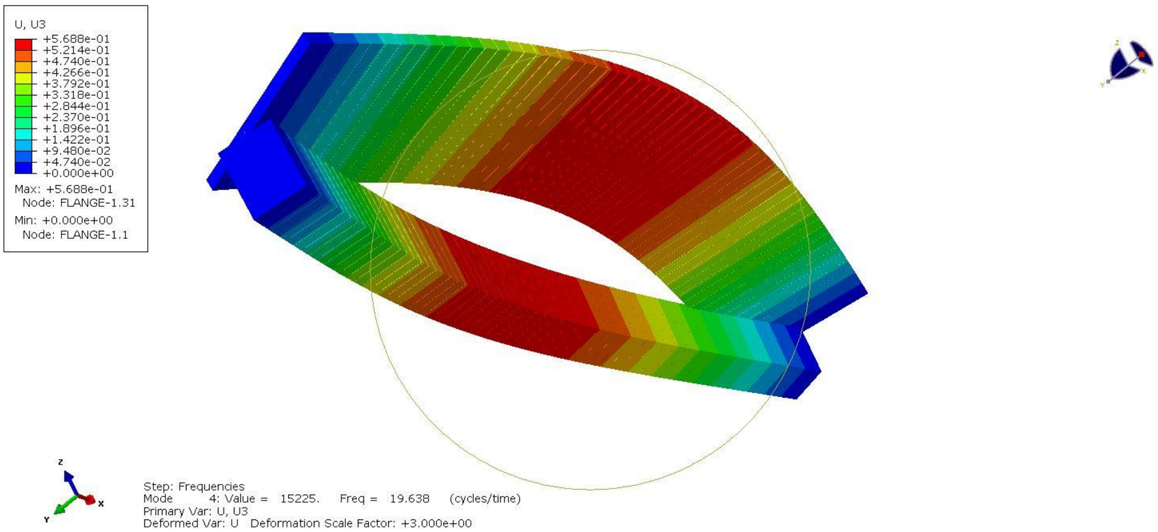


Figure 8.24 First torsional mode L=30 m, Ballasted, f =19,64 Hz

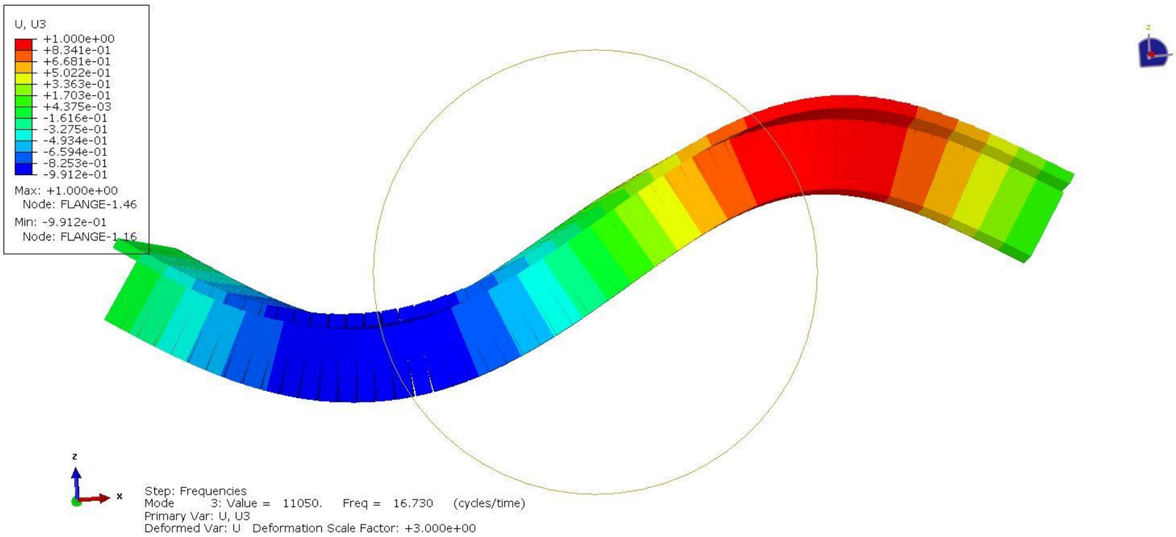


Figure 8.25 Second bending mode L=30 m, Ballasted, f =16,73 Hz

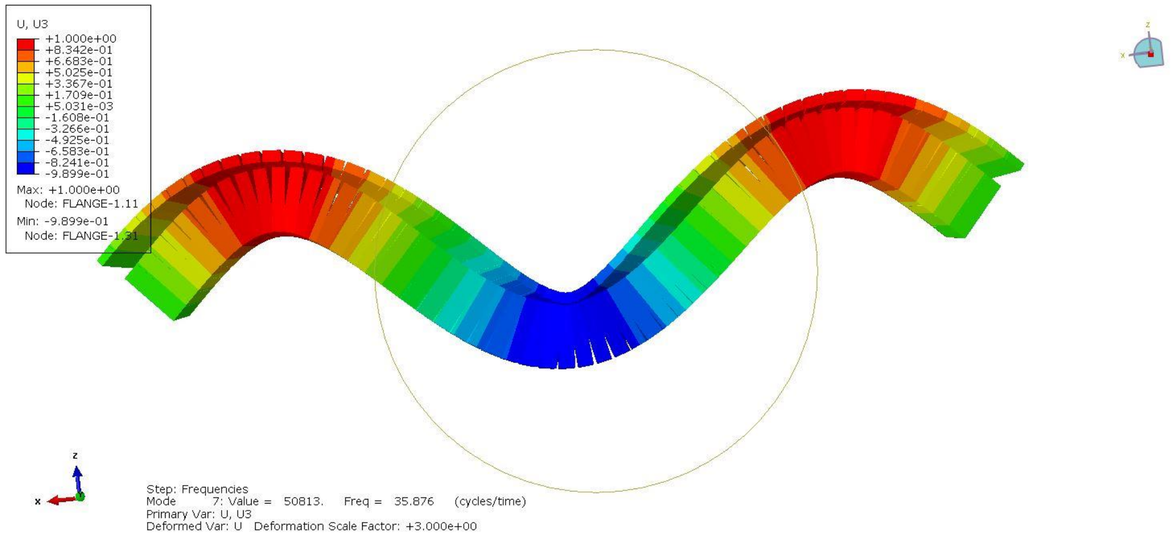


Figure 8.26 Third bending mode L=30 m, Ballasted, f =35,876 Hz

D.5 Modal analysis composite bridge L=20 m

The progression of the modes is presented in Figure 8.27 and Table 8.15.

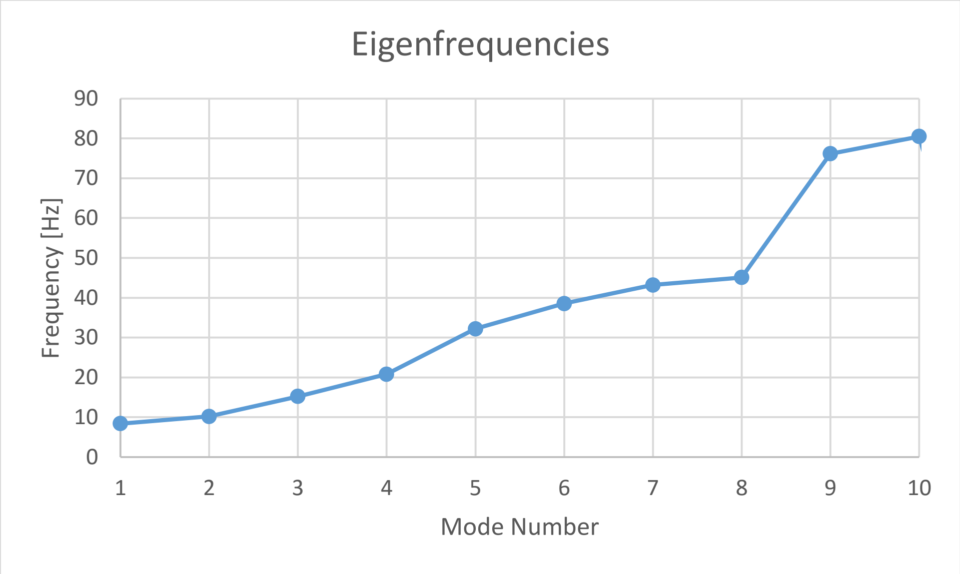


Figure 8.27 Eigen frequencies, composite bridge L = 20 m

Table 8.15 Eigen frequencies, composite bridge L = 20 m

Mode	Frequency (Hz)
1	8,39
2	10,2
3	15,208
4	20,804
5	32,21
6	38,539
7	43,187
8	45,081
9	76,17
10	80,483

The generalised mass for those modes is presented in Figure 8.28 and Table 8.16 .

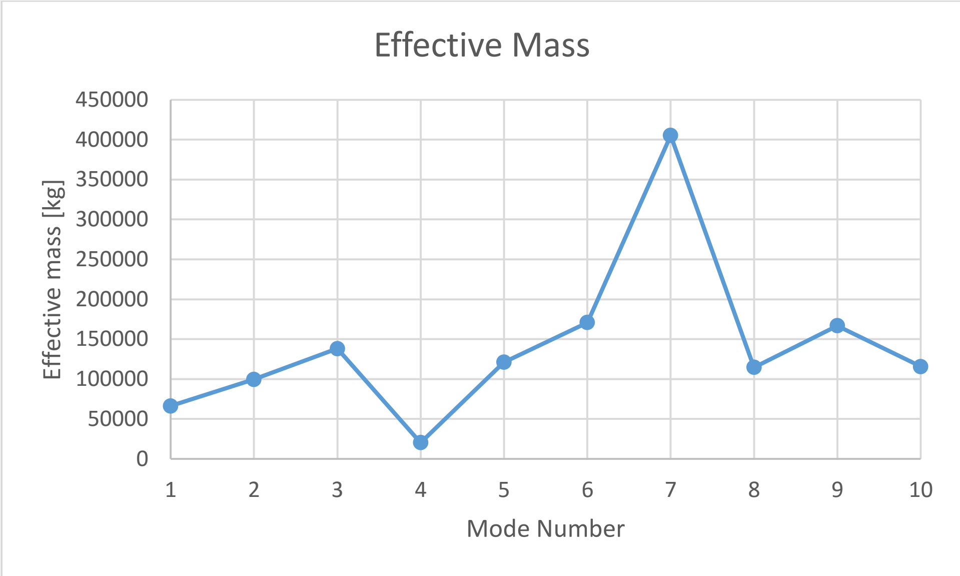


Figure 8.28 Effective mass composite bridge L = 20

Table 8.16 Effective mass composite bridge, L = 20

Mode	EffectiveMass (kg)
1	66305
2	99591
3	138183
4	20190
5	121109
6	170831
7	405291
8	114526
9	166750
10	115580

The rendered mode shapes are shown in Figure 8.29 to Figure 8.32.

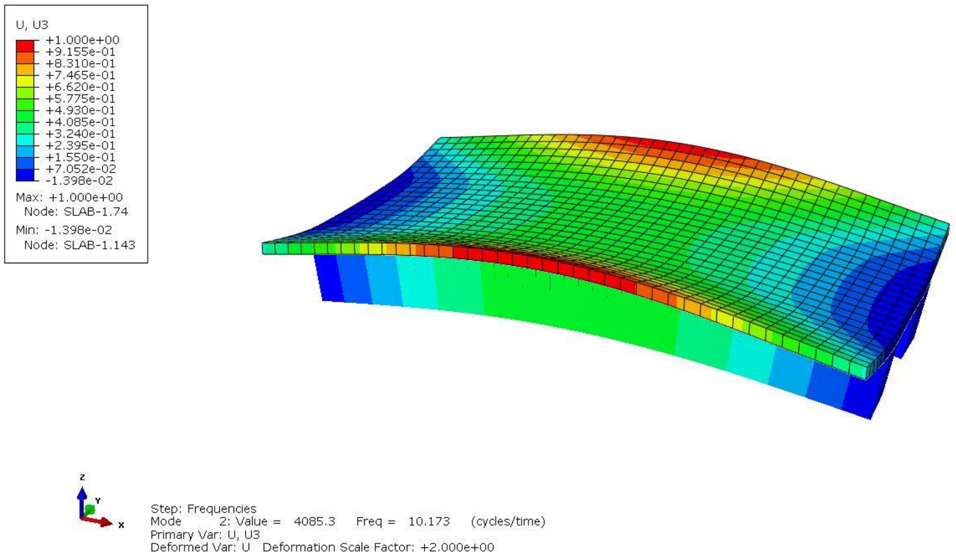


Figure 8.29 First bending mode, composite section L=20 m, f =10.2 Hz

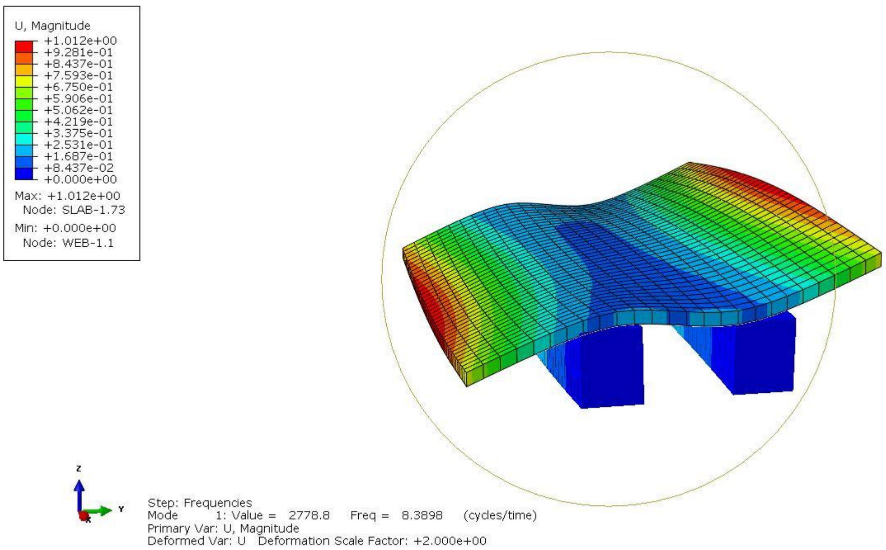


Figure 8.30 First torsional mode, composite section L=20 m, f =8.4 Hz

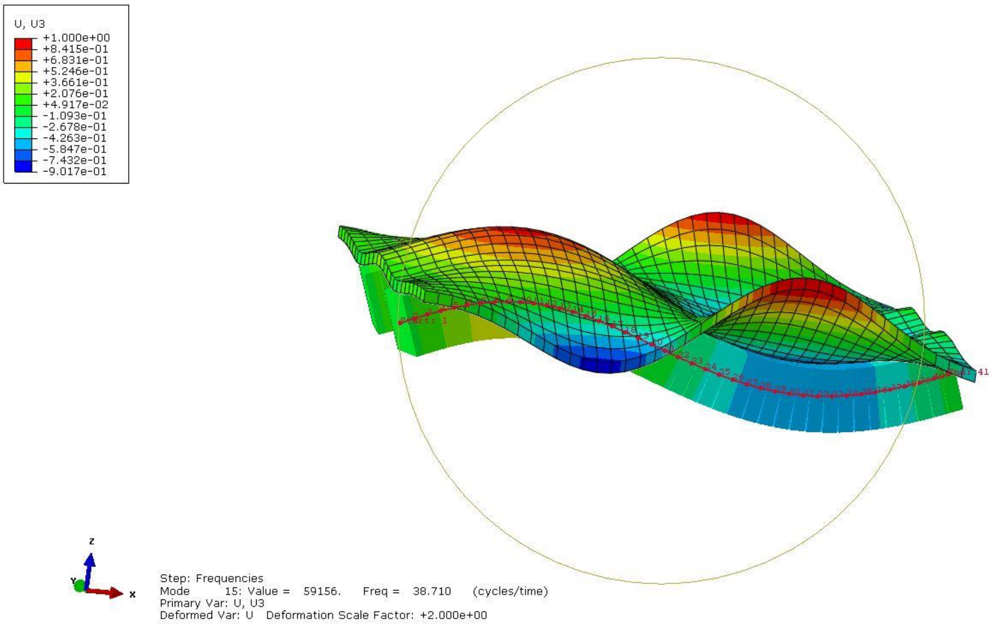


Figure 8.31 Second bending mode, composite section L=20 m, f =43.2 Hz

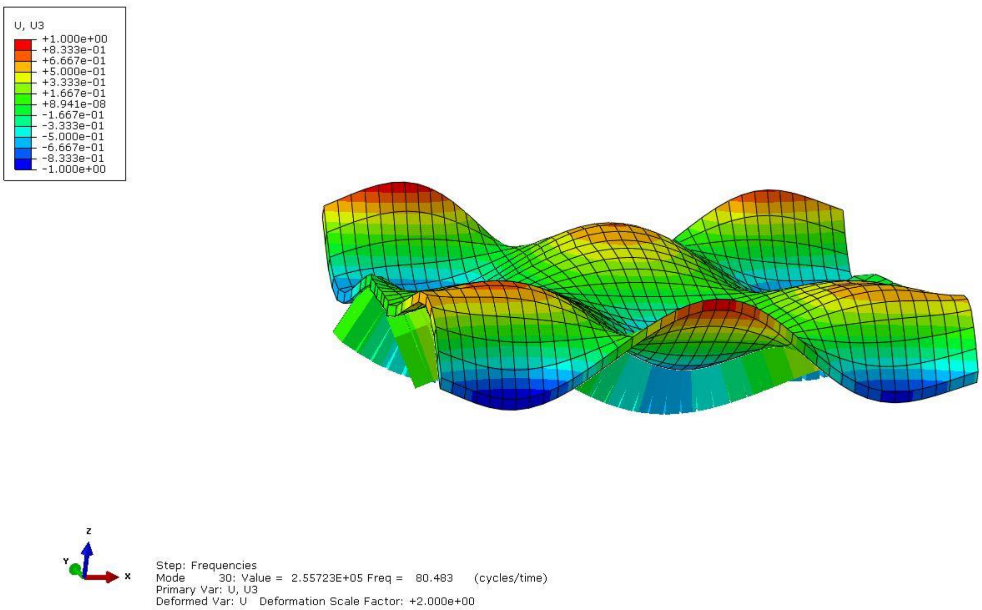


Figure 8.32 Third bending mode, composite section L=20 m, f =98.1 Hz

D.6 Modal analysis composite bridge L=30 m

The progression of the modes is presented in Figure 8.33 and Table 8.17.

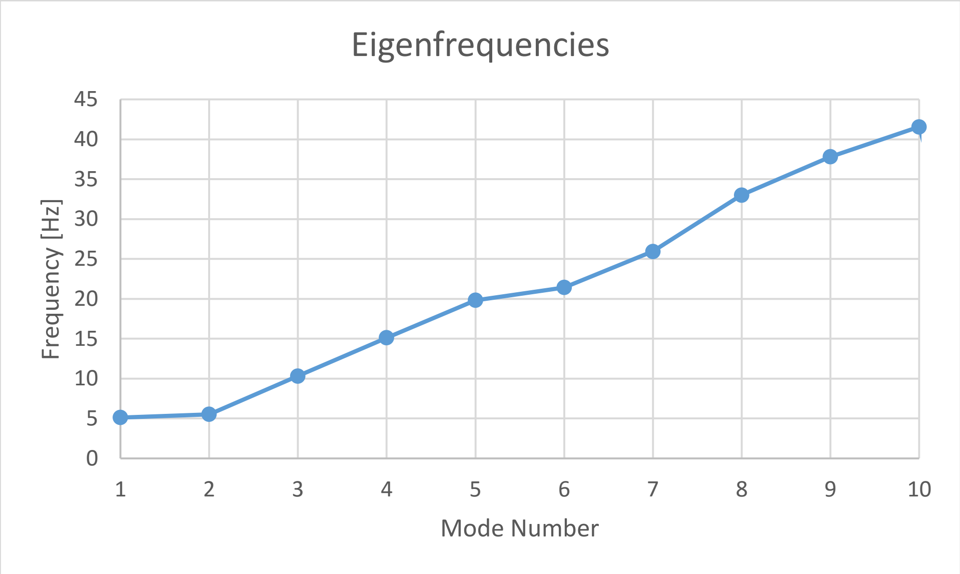


Figure 8.33 Eigen frequencies, composite bridge, L = 30 m

Table 8.17 Eigen frequencies, composite bridge, L = 30 m

Mode	Frequency (Hz)
1	5,1231
2	5,5325
3	10,332
4	15,114
5	19,821
6	21,416
7	25,935
8	32,998
9	37,812
10	41,565

The generalised mass of these modes is shown in Figure 8.34 and Table 8.18.

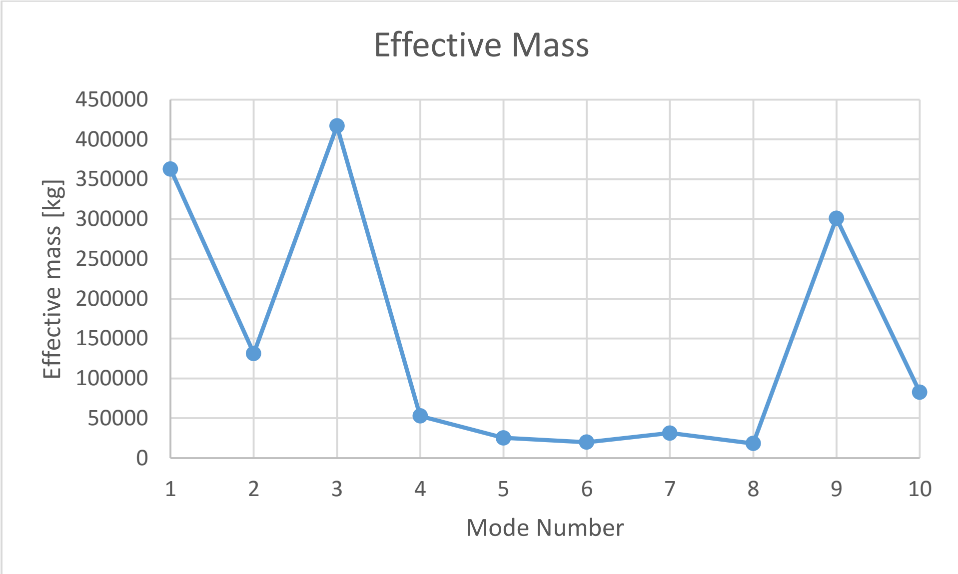


Figure 8.34 Effective mass composite bridge, L = 30

Table 8.18 Effective mass composite bridge, L = 30

Mode	Effective Mass (kg)
1	362704
2	131304
3	416828
4	52784
5	25268
6	19925
7	31213
8	18336
9	300990
10	82396

The rendered mode shapes are shown in Figure 8.35 to Figure 8.38.

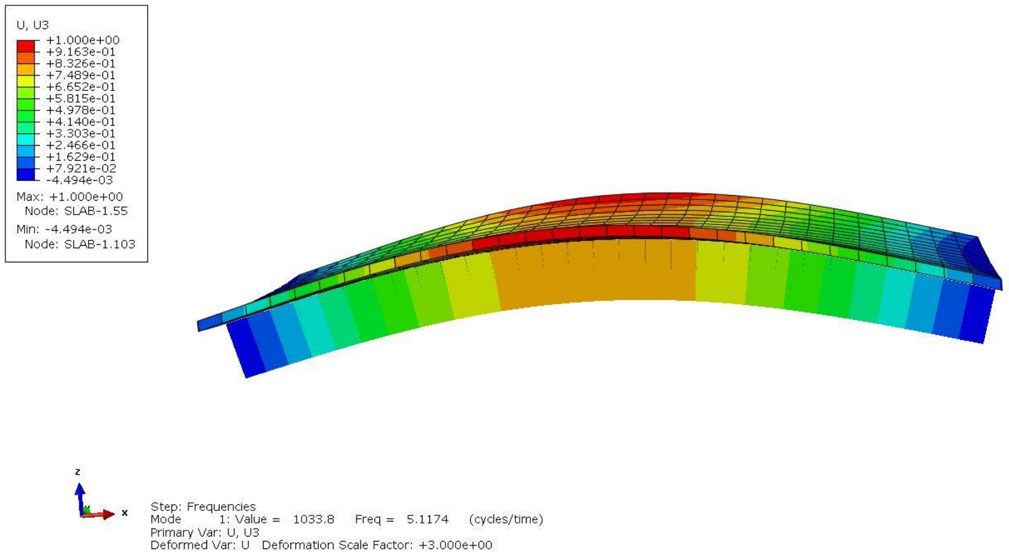


Figure 8.35 First bending mode, composite section L=30 m, f =5.12 Hz

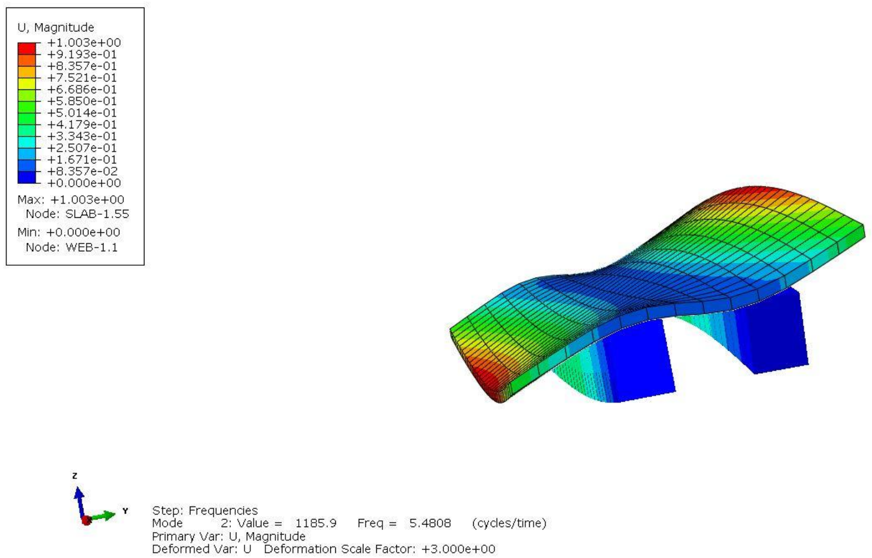


Figure 8.36 First torsional mode, composite section L=30 m, f =5.53 Hz

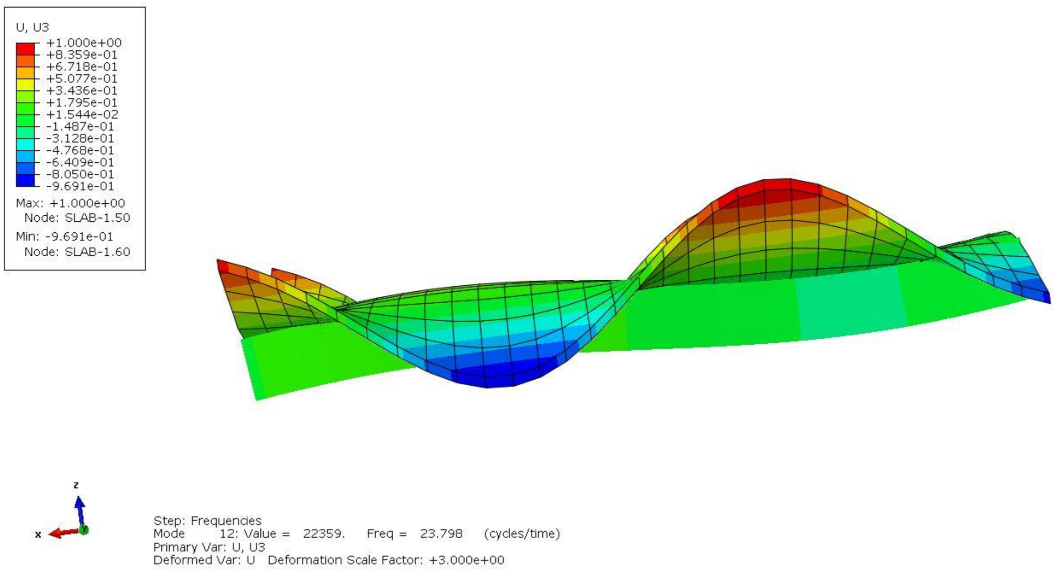


Figure 8.37 Second bending mode, composite section L=20 m, f =23.8 Hz

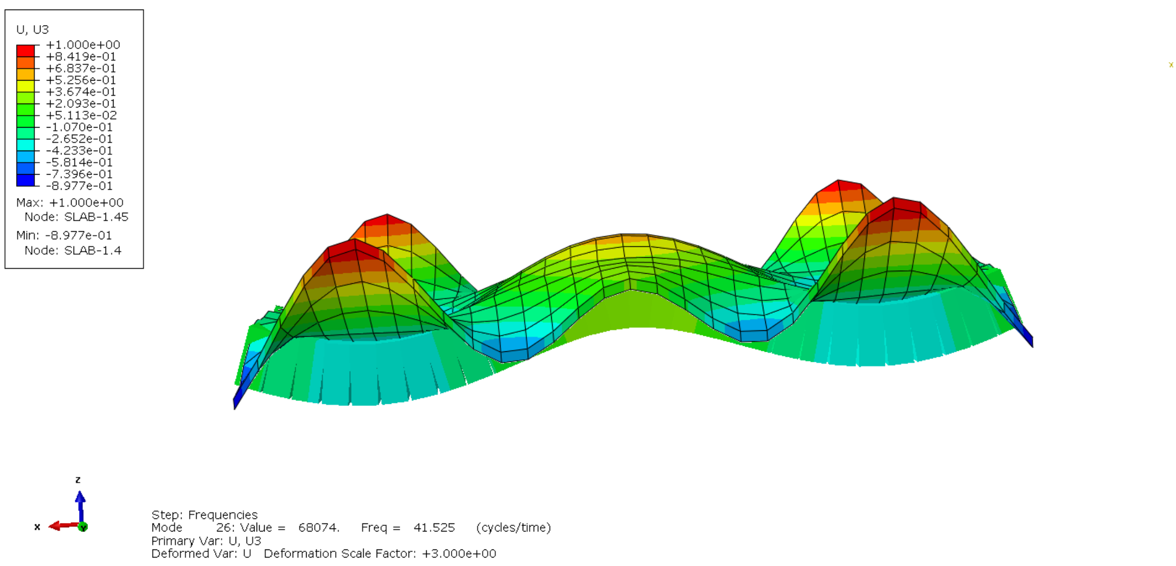


Figure 8.38 Third bending mode, composite section L=30 m, f =41.53 Hz

D.7 Modal analysis continuous bridge L=20 m ballastless

The results of the first 10 modes are presented in Table 8.19 and Figure 8.39.

Table 8.19 Eigen frequencies, continuous bridge L=20 m ballastless

Mode	Frequency (Hz)
1	9,9
2	14,9
3	17,2
4	22,2
5	25,3
6	32,4
7	37,7
8	45,4
9	68,6
10	78,1

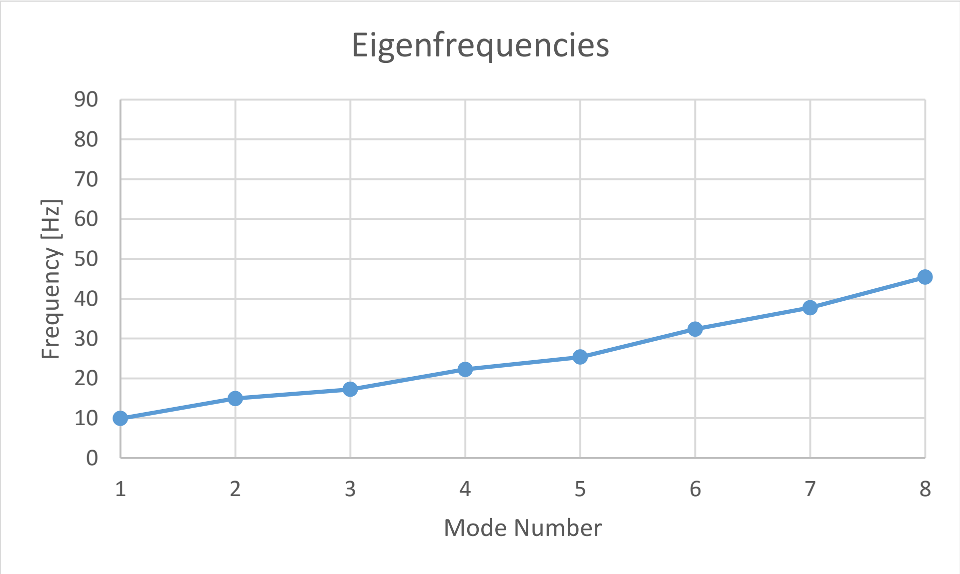


Figure 8.39 Eigen frequencies, continuous bridge L=20 m ballastless

The effective mass for those mentioned modes is presented in Figure 8.40 and Table 8.20.

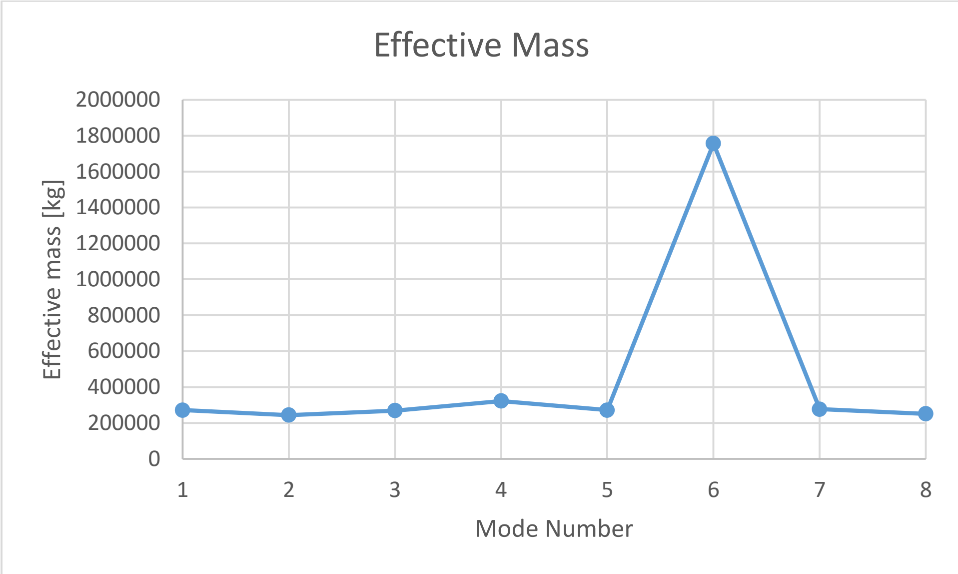


Figure 8.40 Effective mass continuous bridge, L=20 m ballastless

Table 8.20 Effective mass continuous bridge, L=20 m ballastless

Mode	Effective Mass (kg)
1	271197
2	243430
3	268700
4	321980
5	271580
6	1756700
7	276200
8	250370
9	420930
10	272800

The rendered mode shapes are shown in Figure 8.41 to Figure 8.44.

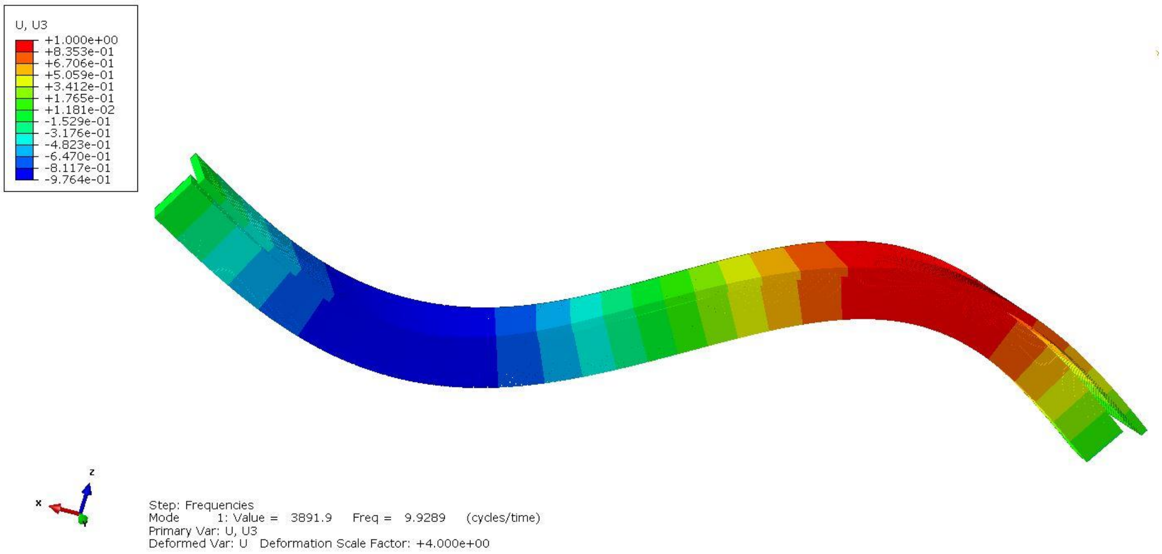


Figure 8.41 First bending mode for the continuous bridge L=20 m ballastless, f = 9.93 Hz

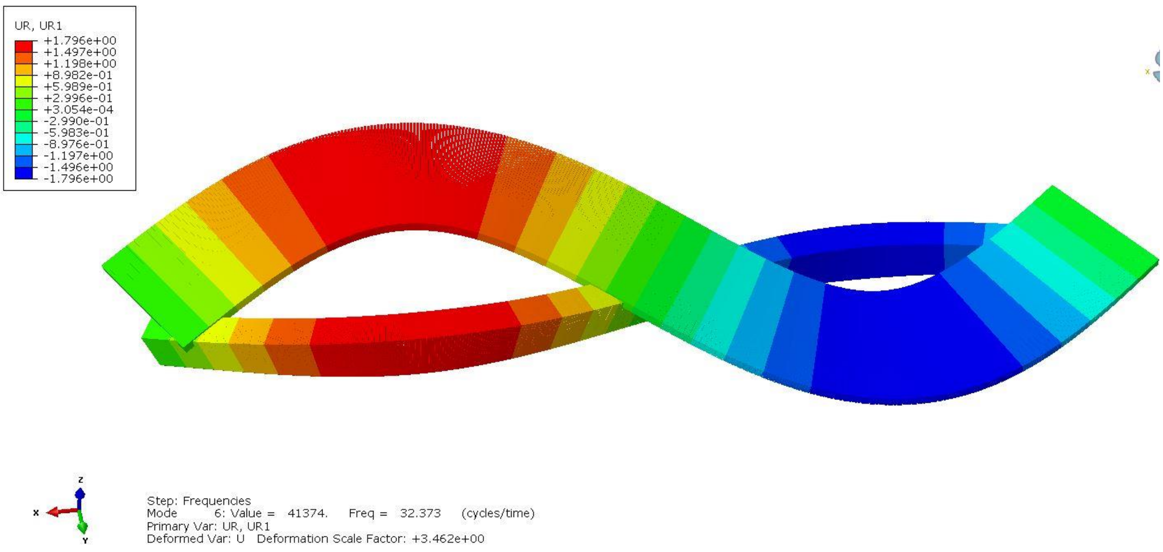


Figure 8.42 First torsional mode for the continuous bridge, L=20 m ballastless, f = 32,37 Hz

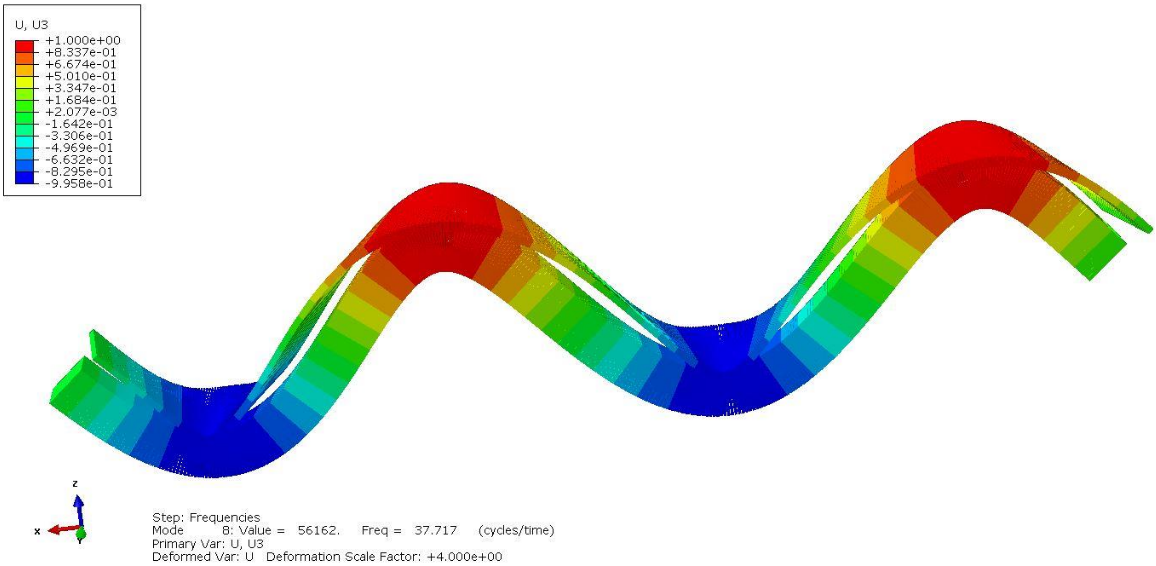


Figure 8.43 Second bending mode for the continuous bridge L=20 m ballastless f = 37.72 Hz

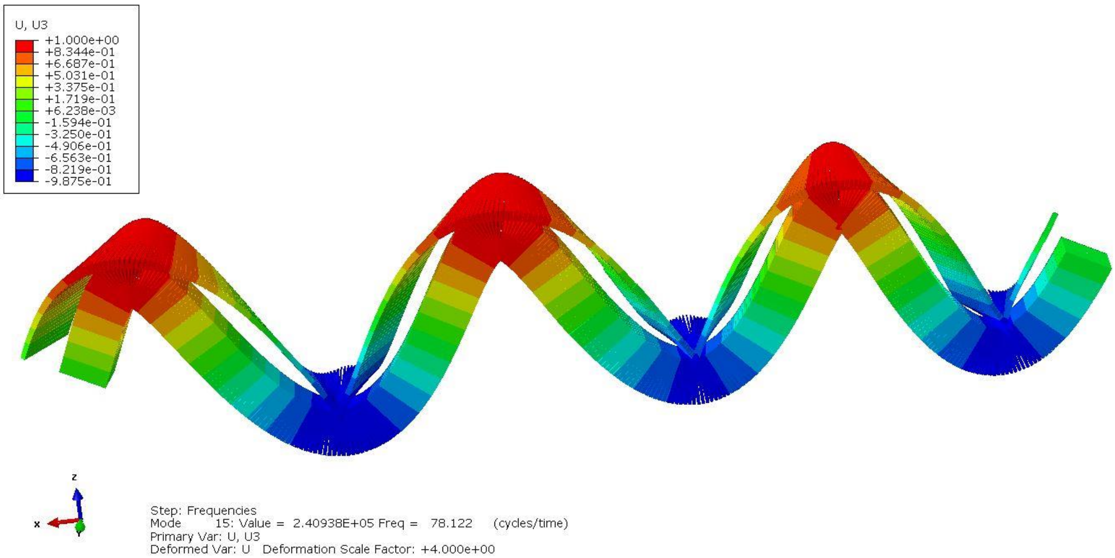


Figure 8.44 Third bending mode for the continuous bridge L=20 m ballastless f = 78,12 Hz

D.8 Modal analysis continuous bridge L=20 m ballasted

The results of the first 10 modes are presented in Table 8.21 and Figure 8.45

Table 8.21 Eigenfrequencies, continuous bridge, L=20 m ballasted

Mode	Frequency (Hz)
1	8,6
2	13,0
3	14,3
4	19,2
5	21,0
6	28,4
7	32,5
8	39,1
9	53,7
10	66,8

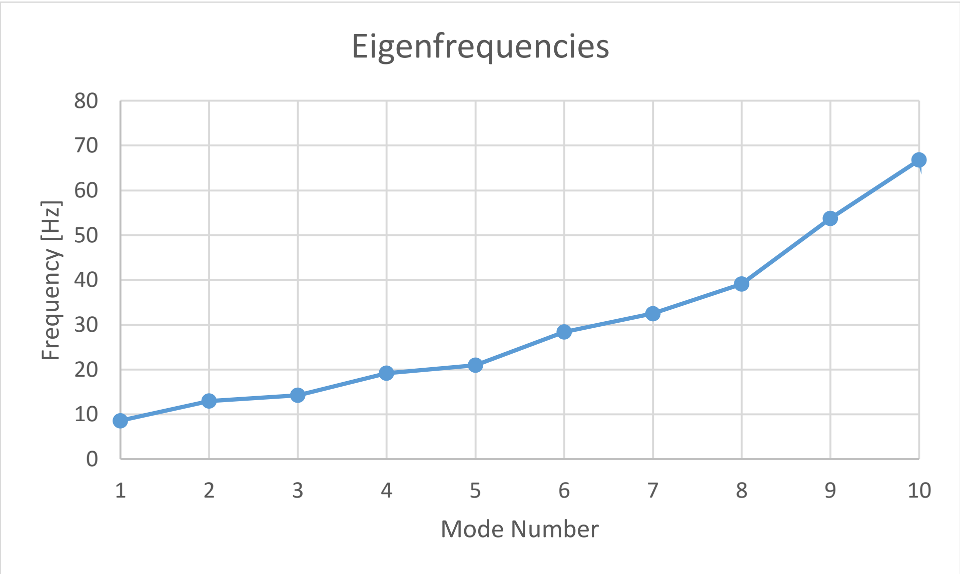


Figure 8.45 Eigenfrequencies, continuous bridge L=20 m ballasted

The effective mass for those mentioned modes is presented in Figure 8.46 and Table 8.22.

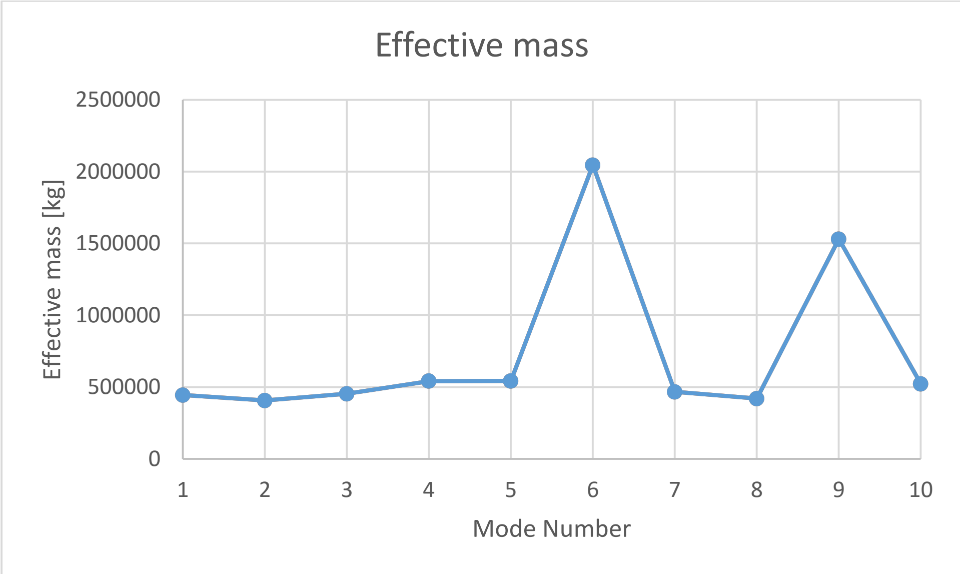


Figure 8.46 Effective mass continuous bridge, L=20 m ballasted

Mode	EffectiveMass (kg)
1	443250
2	405880
3	453000
4	541100
5	541639
6	2045700
7	466349
8	419700
9	1530000
10	522130

Table 8.22 Effective mass continuous bridge. L=20 m ballasted

The rendered mode shapes are shown in Figure 8.47 to Figure 8.50.

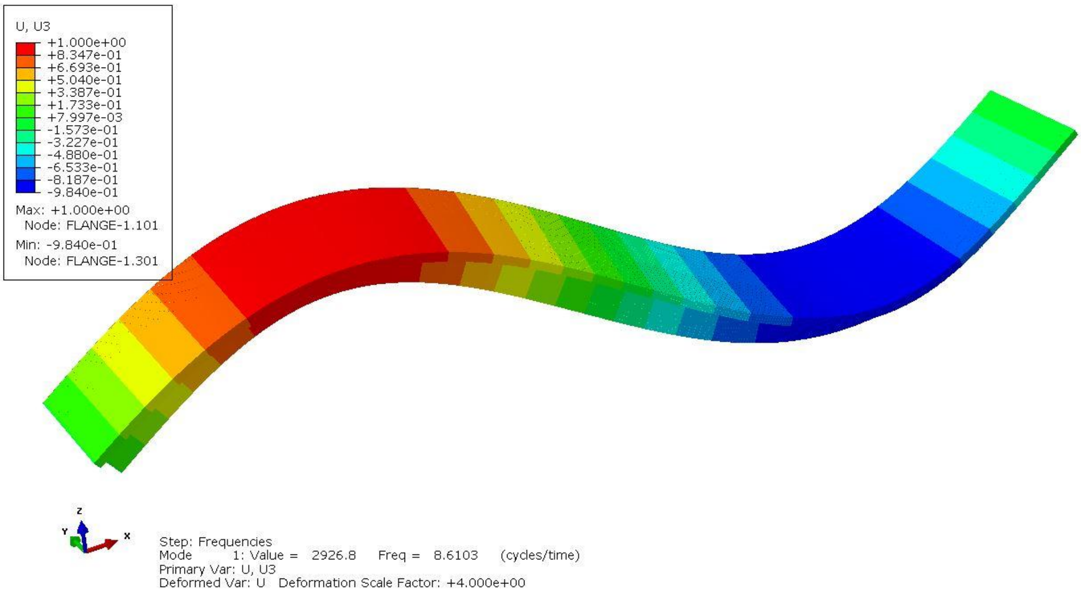


Figure 8.47 First bending mode for the continuous bridge, L=20 m ballasted, f = 8,61 Hz

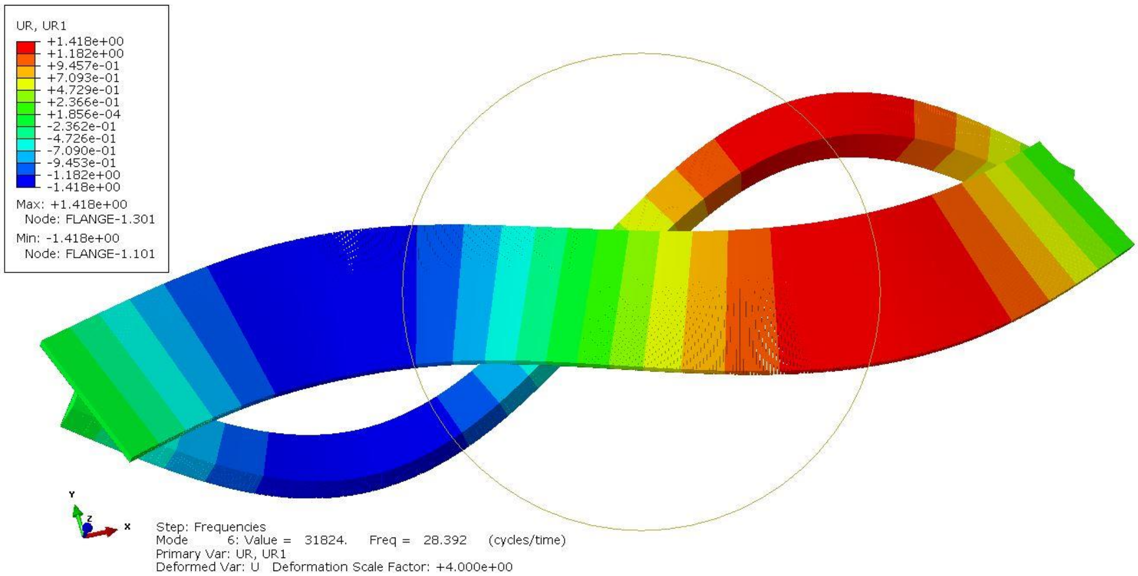


Figure 8.48 First torsional mode for the continuous bridge, L=20 m ballasted, f = 28,4 Hz

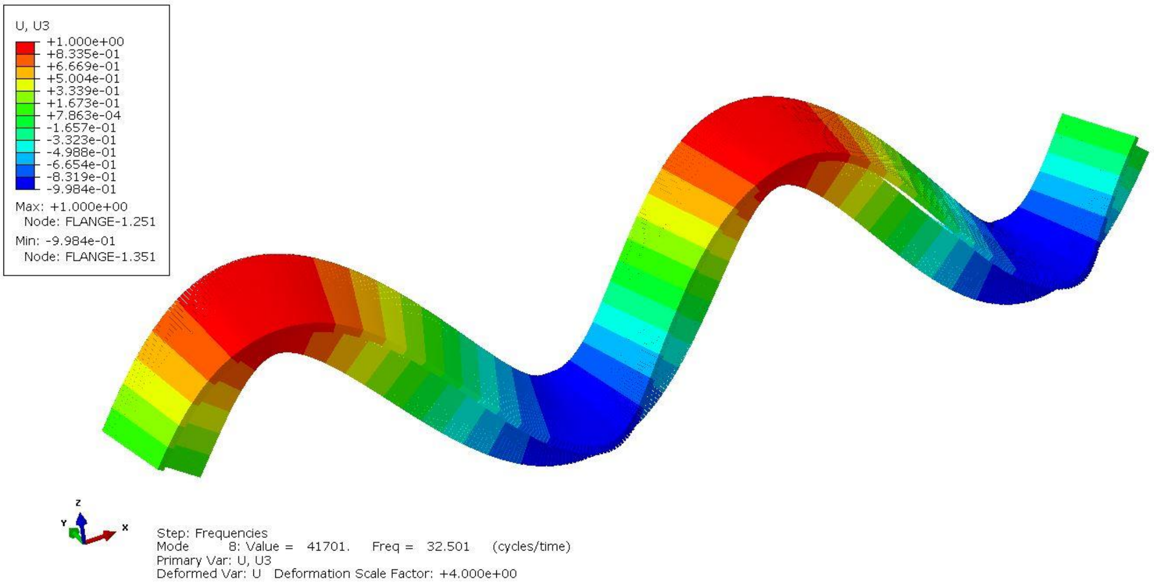


Figure 8.49 Second bending mode for the continuous bridge, L=20 m ballasted, f = 32,5 Hz

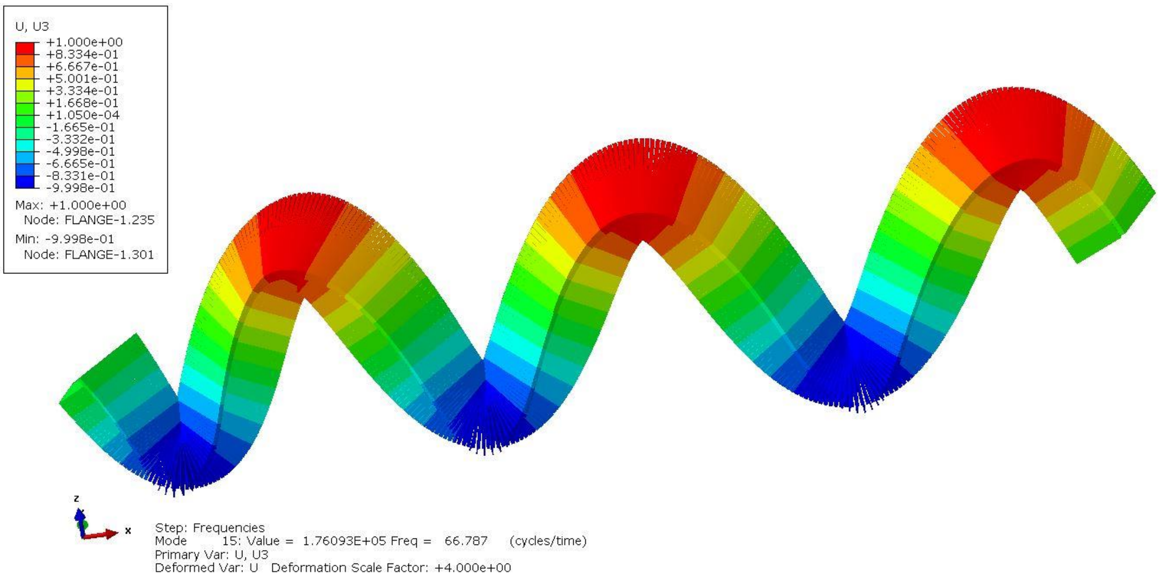


Figure 8.50 Third bending mode for the continuous bridge, L=20 m ballasted, f = 66,78 Hz

D.9 Modal analysis continuous bridge L=30 m ballastless

The results of the first 10 modes are presented in Table 8.23 and *Figure 8.51*.

Table 8.23 Eigenfrequencies, continuous bridge L=30 m ballastless

Mode	Frequency (Hz)
1	4,8636
2	7,4583
3	11,751
4	14,938
5	19,022
6	22,502
7	23,296
8	29,051
9	34,55
10	40,026

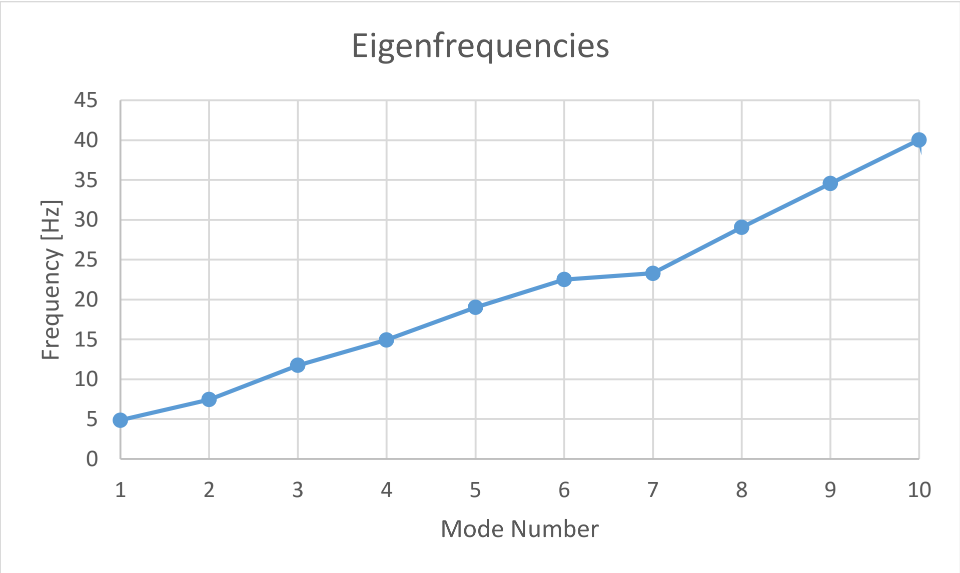


Figure 8.51 Eigenfrequencies, continuous bridge L=30 m and ballastless

The effective mass for those mentioned modes is presented in Figure 8.52 and Table 8.24.

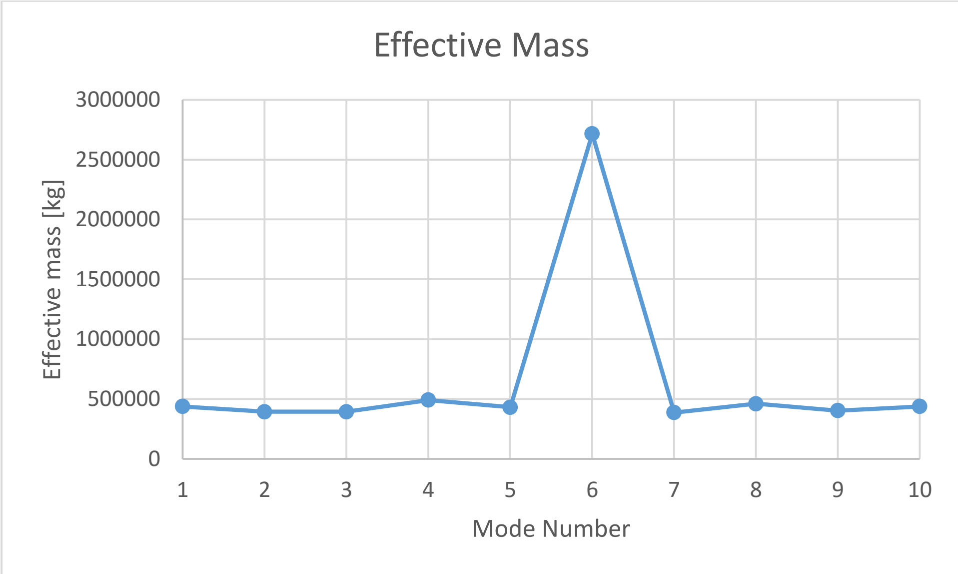


Figure 8.52 Effective mass, continuous bridge L=30 m ballastless

Table 8.24 Effective mass continuous bridge, L=30 m ballastless

Mode	EffectiveMass (kg)
1	437470
2	393000
3	393350
4	490840
5	430680
6	2715000
7	387000
8	460800
9	402400
10	437500

The rendered mode shapes are shown in Figure 8.53 to Figure 8.56.

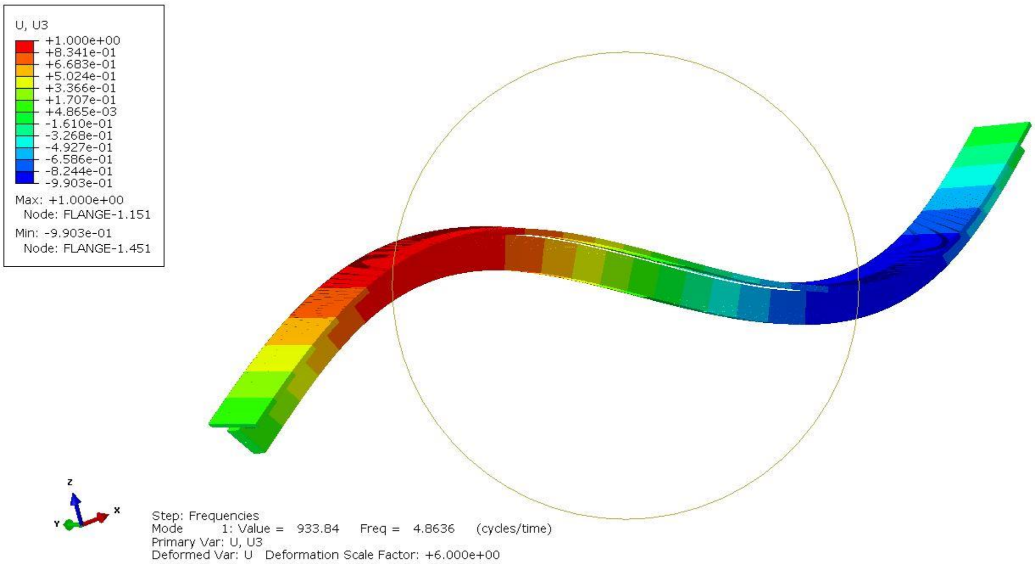


Figure 8.53 First bending mode for the continuous bridge, L=30 m and ballastless, f = 4,86 Hz

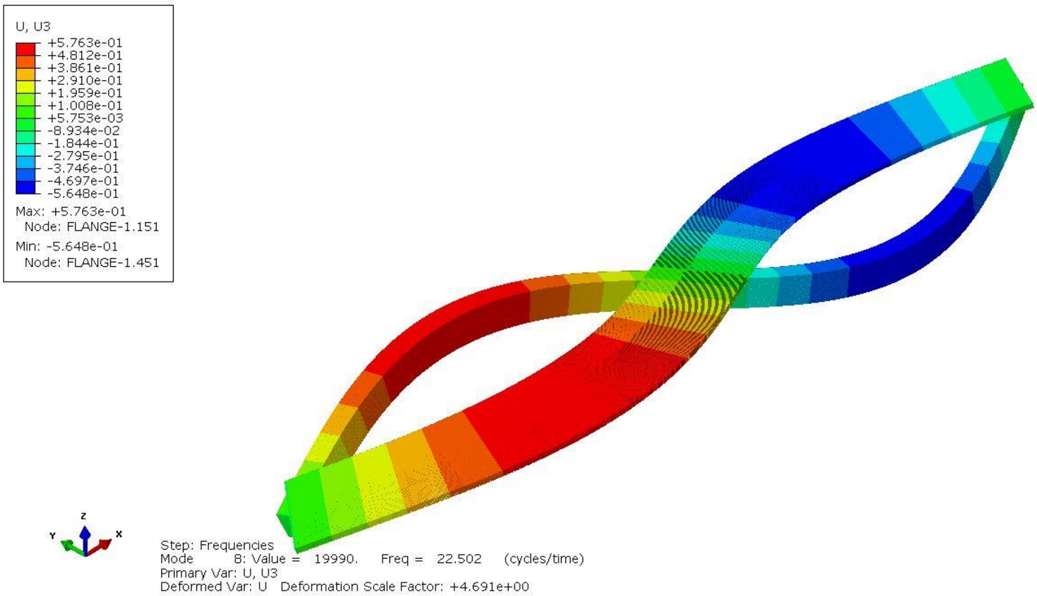


Figure 8.54 First torsional mode for the continuous bridge, L=30 m and ballastless, f = 22,5 Hz

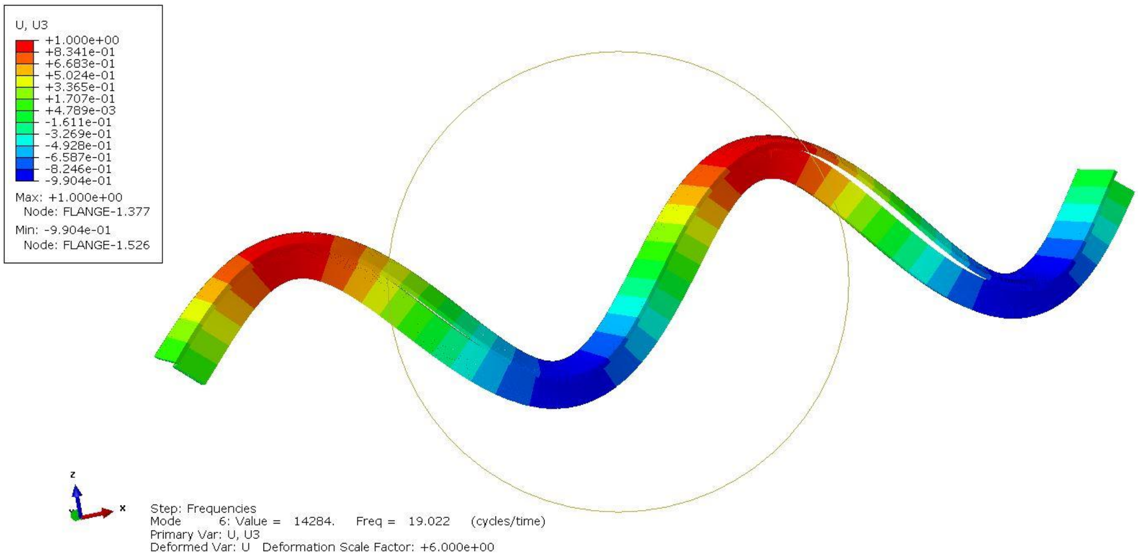


Figure 8.55 Second bending mode for the continuous bridge, L=30 m and ballastless, f = 19,02 Hz

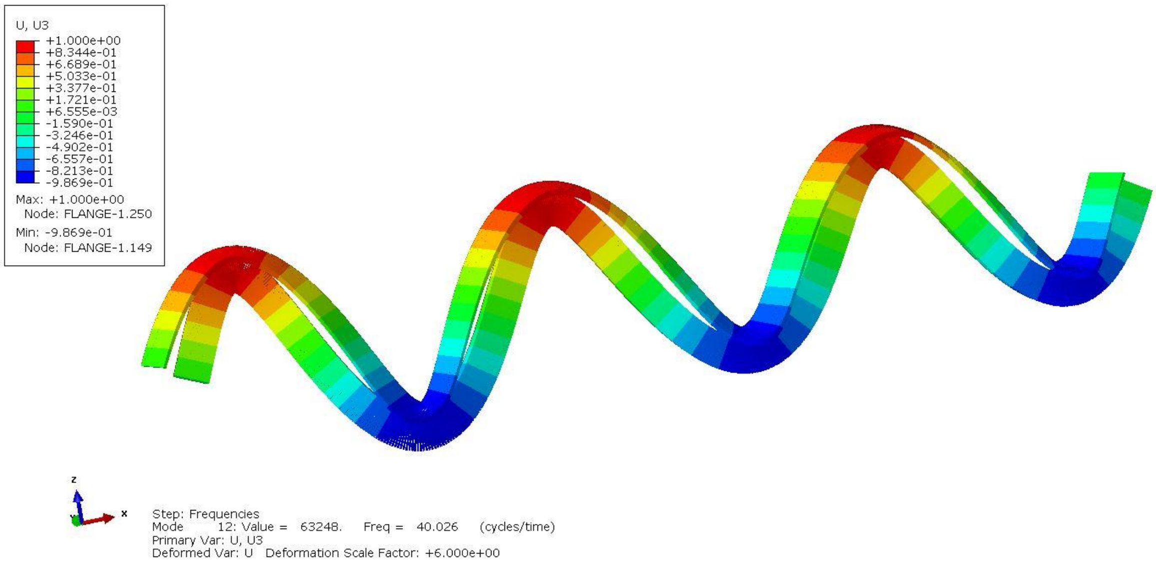


Figure 8.56 Third bending mode for the continuous bridge, L=30 m and ballastless, f = 40,03 Hz

D.10 Modal analysis continuous bridge L=30 m ballasted

The results of the first 10 modes are presented in Table 8.25 and Figure 8.57.

Table 8.25 Eigen frequencies, continuous bridge, L=30 m ballasted

Mode	Frequency (Hz)
1	4,667
2	6,7145
3	7,14
4	10,068
5	13,502
6	18,034
7	21,078
8	22,109
9	29,62
10	37,814

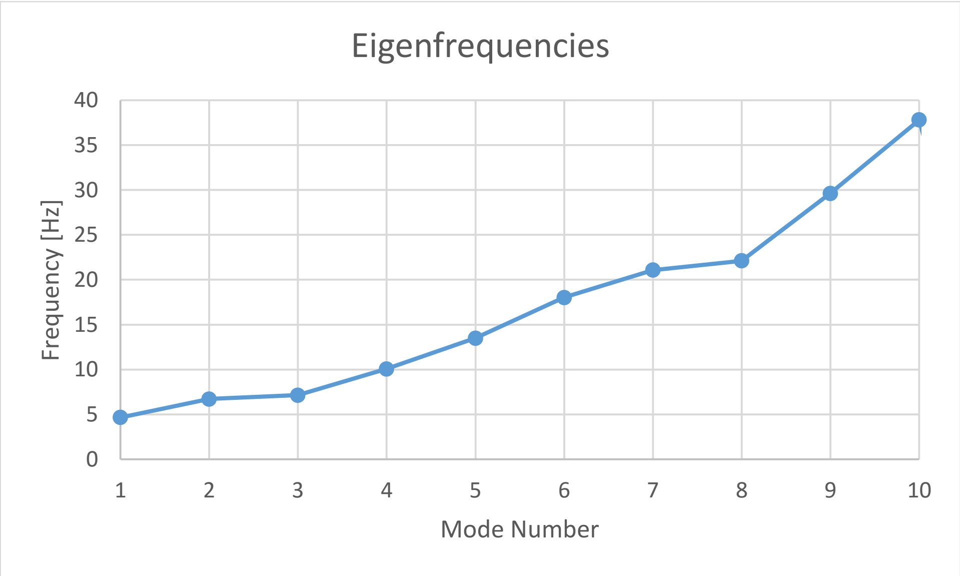


Figure 8.57 Eigenfrequencies, continuous bridge L=30 m and ballasted

The effective mass for mentioned modes is presented in Figure 8.58 and Table 8.26.

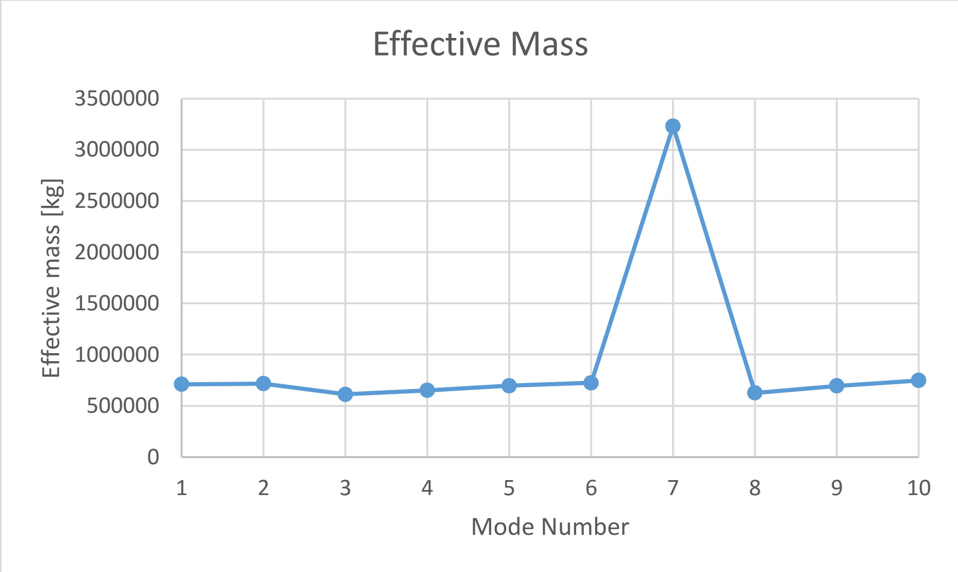


Figure 8.58 Effective mass continuous bridge, L=30 m ballasted

Table 8.26 Effective mass continuous bridge, L=30 m ballasted

Mode	EffectiveMass (kg)
1	710872
2	715760
3	611980
4	650890
5	696000
6	724000
7	3232180
8	626000
9	695000
10	747770

The rendered mode shapes are shown in Figure 8.59 to Figure 8.62.

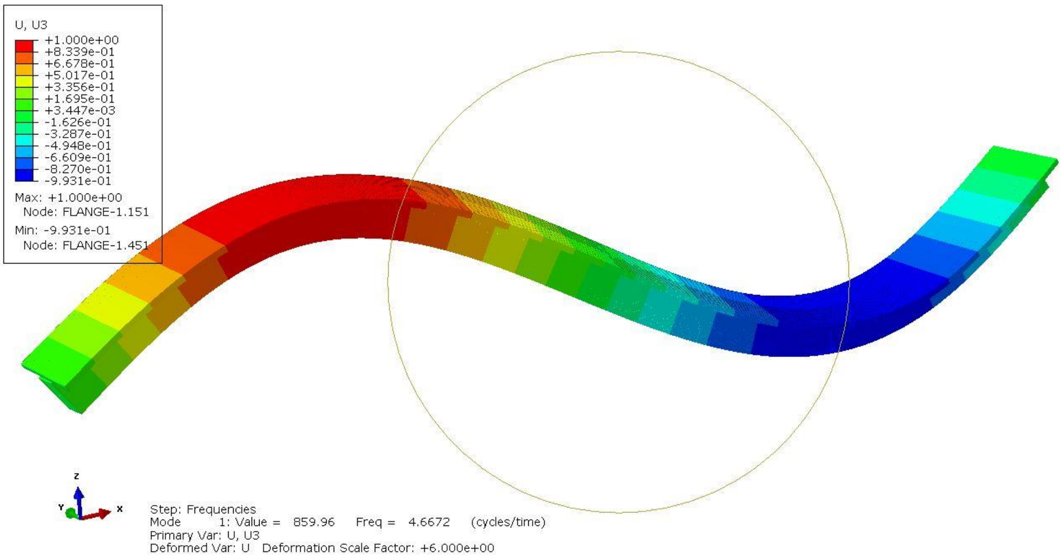


Figure 8.59 First bending mode for the continuous bridge, L=30 m and ballasted, f = 4,67 Hz

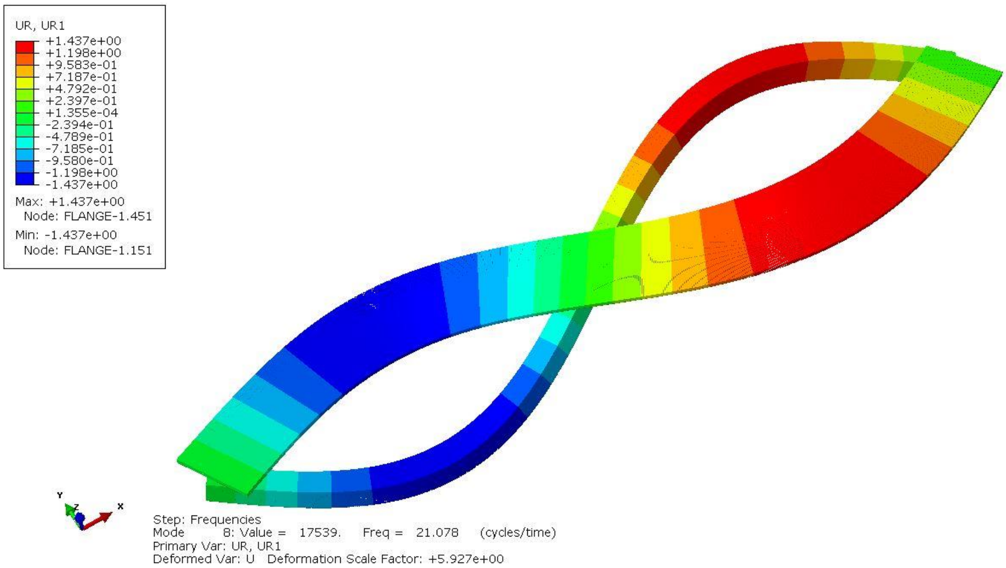


Figure 8.60 First torsional mode for the continuous bridge, L=30 m and ballasted, f = 21,08 Hz

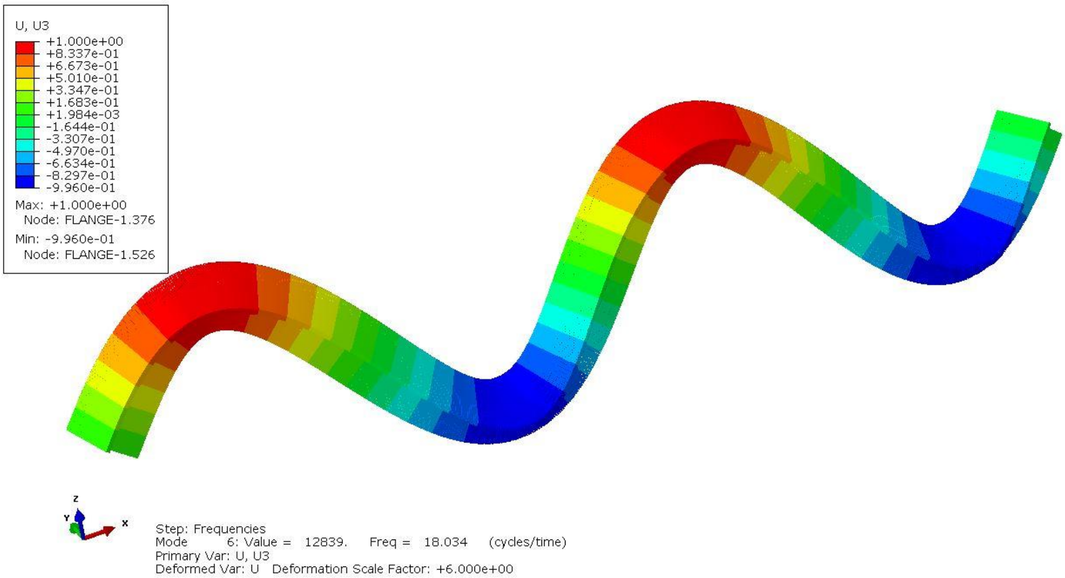


Figure 8.61 Second bending mode for the continuous bridge, L=30 m and ballasted, f = 18,034 Hz

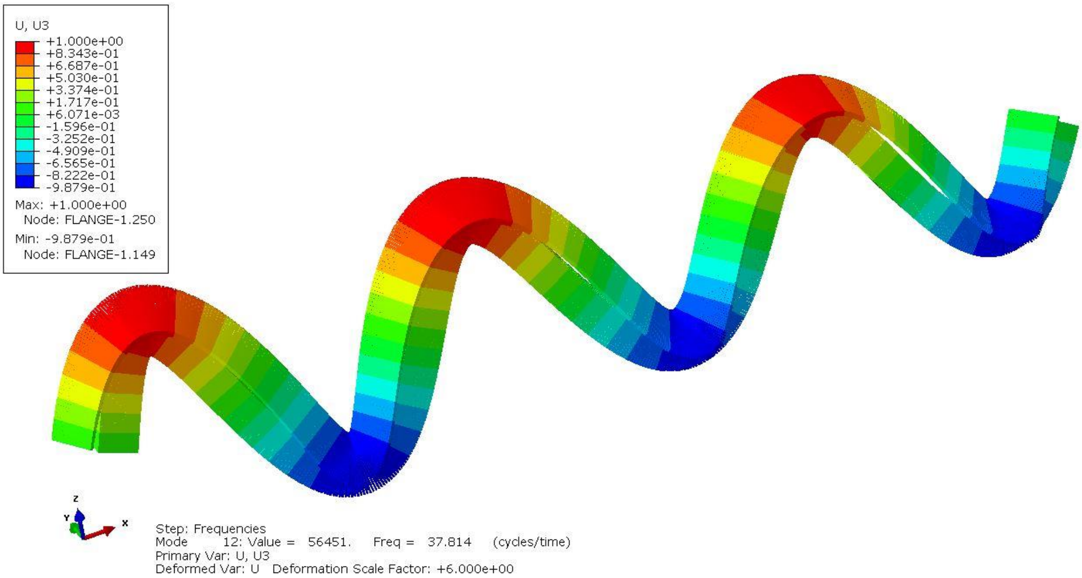


Figure 8.62 Third bending mode for the continuous bridge, L=30 m and ballastless, f = 37,82 Hz

List of figures

List of tables