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Technical Note

PARAMETRIC STUDY ON BEHAVIOUR OF BOX-GIRDER BRIDGES USING FINITE ELEMENT METHOD

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ABSTRACT

A detailed study of box girder bridge cross-sections namely Rectangular, Trapezoidal and Circular has been carried out in the present investigation. Commercially available software SAP-2000 has been used to carryout linear Analysis of these box girders. Three dimensional 4-noded shell elements have been employed for discretization of domain and to analyze the complex behavior of different box-girders. The linear analysis has been carried out for the Dead Load (Self Weight) and Live Load of Indian Road Congress Class 70R loading, for zero eccentricity as well as maximum eccentricity at mid-span. The paper presents a parametric study for deflections, longitudinal and transverse bending stresses and shear lag for these cross-sections. The Finite element computational model has been validated by comparing few obtained results with the published literature. Effectiveness of four noded shell elements for analysis of box girder bridges is also presented. It is found that the rectangular section is superior to other two sections. It can be stated that the obtained results will provide guidance to the bridge designers.

Keywords: Box girder bridge; longitudinal bending stress; shear lag; transverse bending stress; finite element method, SAP

1. INTRODUCTION

Box girders, have gained wide acceptance in freeway and bridge systems due to their structural efficiency, better stability, serviceability, economy of construction and pleasing aesthetics. Analysis and design of box-girder bridges are very complex because of its three-dimensional behaviors consisting of torsion, distortion and bending in longitudinal and transverse directions. The longitudinal bending stress distribution in wide flange girders is distributed non-uniformly throughout the width. It remains maximum at the edge and reduces towards the centre, and usually cannot be obtained accurately from elementary beam theory.

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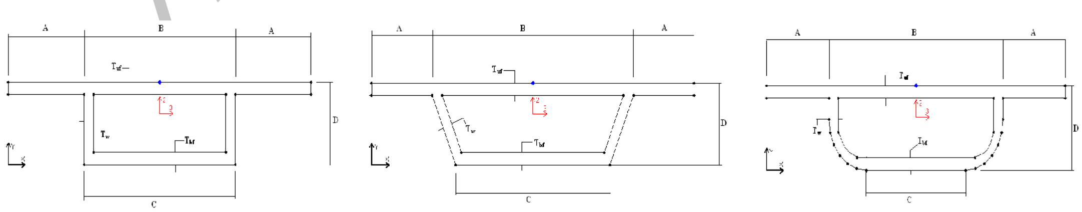
Available research work on the analyses of single-cell and multicell box girders can be classified into two main categories; namely investigations using folded plate or shell elements [1–3], and investigations using box beam elements [4-10]. The latter can be further classified into two categories; namely investigations using box beam elements considering both flexural and torsional behaviors of box girders [4], and investigations using box beam elements considering distortional behavior as well as both flexural and torsional behaviors [5–10]. The problem of deflections, longitudinal and transverse bending stress and shear lag in single cell box girder has been studied by many researchers in the past [11-17]. But, most of the investigations have been limited to rectangular and trapezoidal cross-sectional box girders. Reissner [11] developed a variational procedure based on the principle of minimum potential energy, Luo and Li [12-13] developed polynomial curves from the closed form solutions like Reissner [11]. Yapping et al. [14] gave initial value solution of static equilibrium differential equations of thin walled box beams, considering both shear lag and shear deformation. Upadyay and Kalayanaraman [15] presented a simplified analysis of FRP box girders. With the development of modern tools of analysis, such as finite element method and finite strip method, now it is possible to analyse the box girders by these discrete approaches where in, all the structural actions, both in the longitudinal and transverse direction and also interaction between them, can be considered together. Recently Mishra [16] has studied the behavior of box girder bridges using Finite Element Method and found it suitable and effective to analyse the box sections. In the present paper a detailed study of three box cross-sections (Rectangular, Trapezoidal and Circular) has been carried out using finite element code SAP-2000. The effect of, increase in depth of rectangular Box Girder on its behavior has been also presented. Linear analysis of the behavior of these box girders as per Indian Road Congress provisions [17-18] has been studied and detailed results are obtained and discussed. The validity and accuracy of the present work has been also accessed by comparing the results with published literature.

2. PROBLEM DEFINITION

In the present work three different cross-sections namely Rectangular Trapezoidal and Circular of the box girder bridges are analyzed. The details of the cross-sections are given in Figure 1 and Table 1.



(a) (b) (c)

T_{uf}, T_{bf} and T_w are 0.3 m and same for all cross-sections. All dimensions are in meter

Figure 1. (a) Rectangular, (b) Trapezoidal and (c) Circular cross-section of box girder bridge

Table 1. Geometries of bridges used in parametric study

Depth	Rectangular			Trapezoidal			Circular		
D	A	B	C	A	B	C	A	B	C
2.0	2.4	4.8	4.8	1.8	6.0	4.6	2.0	5.6	3.2
2.4	2.4	4.8	4.8	1.8	6.0	4.7	2.2	5.2	2.8
2.8	2.4	4.8	4.8	1.7	6.2	4.6	2.0	5.6	2.6

The analysis of the bridge was done taking into consideration same width of the bridge deck with same area of cross-section and a span of 20.0 m overall for all types of cross-sections. A constant thickness of 0.3 m was assumed throughout the bridge cross-section with varying depth parameter. Linear analysis was performed for dead load (Self Weight) and I.R.C live load Class 70R loading for all the three cross-sections of the bridges using SAP. Deflection and Stresses were calculated and a comparison of the three bridges has been done for various cross-sections. Results from published literature are used to substantiate the analytical modeling and value of maximum deflection is compared with simple beam theory value of self-weight.

3. FINITE ELEMENT MODELING AND ELEMENT DESCRIPTION

SAP is a commercially available, general-purpose finite element-modeling package for numerically solving a wide variety of civil engineering problems. These problems include static/dynamic analysis (both linear and non-linear), heat transfer and fluid problems. The program employs the matrix displacement method of analysis based on finite element idealization. To discretize the bridge cross-section shell element has been employed (see Figure 2). In order to deduce the theory of the thin elastic shell from the three dimensional elasticity simplifying assumptions, known as Love’s first approximation, are used. The shell element has both bending and membrane capabilities. Both in-plane and normal loads are permitted. The element has six degrees of freedom at each node: translations in the nodal x, y, and z directions and rotations about the nodal x, y, and z-axes. Stress stiffening and large deflection capabilities are included. The geometry, node locations, and the coordinate system for this element are shown in Figure 2. The element is defined by four nodes, thicknesses and the orthotropic material properties. Orthotropic material directions correspond to the element coordinate directions. The element x-axis may be rotated by an angle theta (in degrees). The thickness is assumed to vary smoothly over the area of the element, with the thickness input at the node. In the bridge modeling it is assumed that the element has a constant thickness. Figure 3 shows typical discretized meshes of the all three cross-sections taken for study.

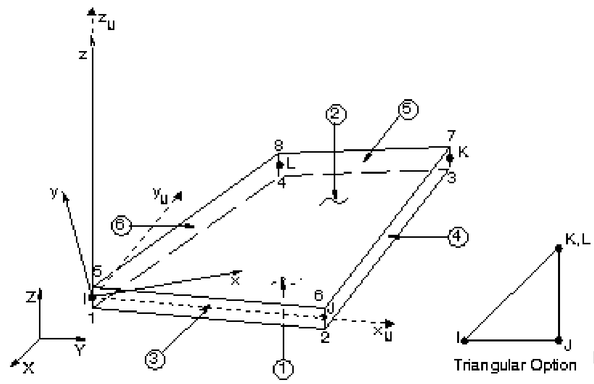
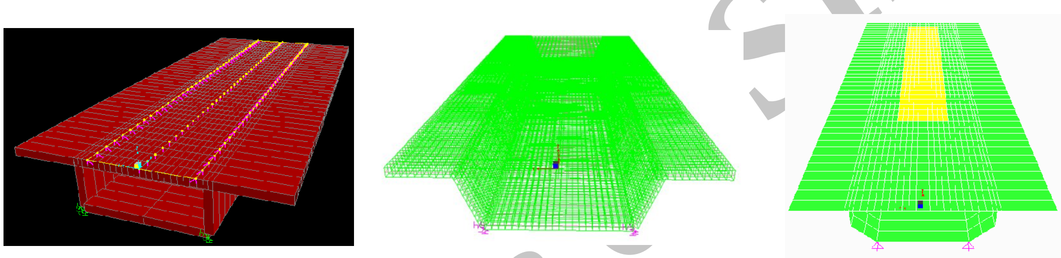


Figure 2. Description of the finite element shell element used in FEA



(a) (b) **(c)**

Figure 3. (a) Rectangular, (b) Trapezoidal and (c) Circular cross-section bridge model in SAP

4. LOADING PLACEMENT

4.1 Longitudinal Placement

I.R.C Class 70R live load [17] were first applied on a simply supported girder, with a span equal to that of the bridge prototype, to determine the case which produced maximum moment at mid span. As an illustration Figure 4 has shown the placement of 70R wheeled loading for 20.0m span bridge. It was found that the critical moment was generated for Class 70R wheeled vehicle loading; therefore, parametric study was done by placing Class 70R wheeled vehicle loading. Subsequently three loading cases were considered for each bridge prototype, central and eccentric IRC Class 70R loading, and bridge dead load.

4.2 Transverse Placement

The transverse load placement conditions are shown in the following Figure 5(a) and (b). To find critical effect on bridge deck, train of IRC Class 70R wheeled loads was placed symmetrically with respect to the central line of bridge deck as well as at 1.2m eccentricity from one end.

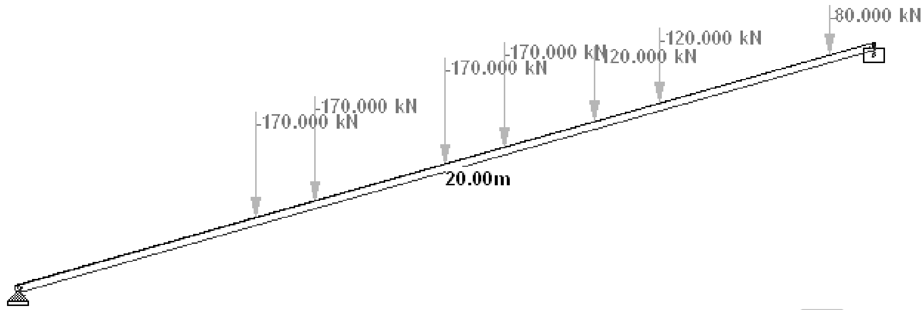
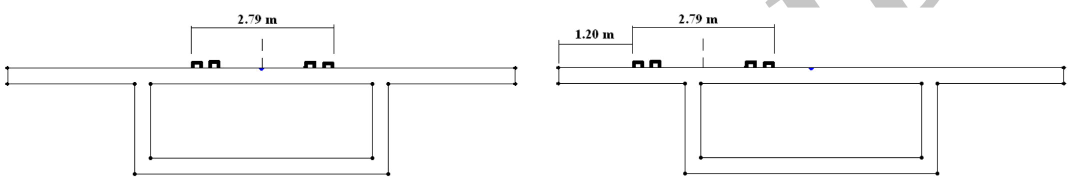


Figure 4. Placement of class 70R wheeled vehicle on 20m span for maximum moment at mid span



(a) (b)

Figure 5. Transverse placement of IRC class 70R vehicle on two lane bridge deck (a) Symmetrically, (b) Eccentrically

5. VALIDATION OF THE FINITE ELEMENT MODEL

To validate the developed finite element model of bridge deck in SAP-2000 a numerical example from the literature has been taken. In the recent past Yapping Wu (2003) has given an initial value solution of the static equilibrium differential equation of thin walled box beam, considering both shear lag and shear deformation. Figure 6 shows the cross section of the simply supported box beam bridge model used for the validation study. It is subjected to two equal concreted load (P=2x800N) at the two webs of mid span, this model was calculated by Guo and Fang [19], who followed Reissner's method to analyze the shear lag without considering shear lag deformation. For the example cross-section, Span length=800mm, Modulus of elasticity (E)=2.842GPa and Modulus of rigidity (G)=1.015GPa are taken.

For the above span length which is 800 mm, the numerical results for the internal forces on hinge point, quarter point and mid span sections of the beam are presented in Table 1. The midspan displacement results are compared with the literature results in Table 2. It can be seen that the deflection obtained using elementary beam theory are most crude as compared to the deflections obtained using other methods. It is clear from Table 2 and 3 that the developed model predicts reasonably accurate results.

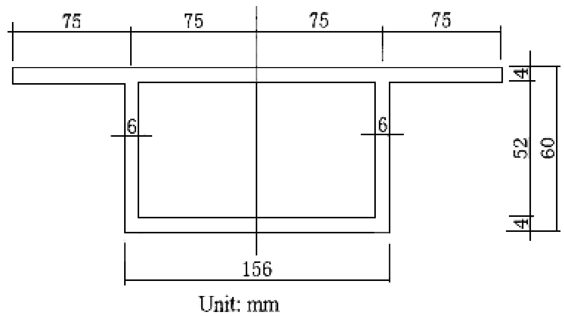


Figure 6. Cross-section of simply supported box-beam

Table 2. Internal forces of simply supported box beam subjected to concentrated force

Result obtained	Internal force	Hinge	L/4	Midspan
SAP	Shear force Q (kN)	0.80	0.80	0.80
	Bending moment M(kN-m)	0.00	0.16	0.32
Initial Value Method	Shear force Q (kN)	0.80	0.80	0.80
	Bending moment M (kN-m)	0.00	0.16	0.32

Table 3. Mid span deflection of simply supported box beam subjected to concentrated force

Quantity	Model test	Elementary beam theory	SAP-2000	Guo and Fang (1983)	Initial value method
Mid Span Deflection (mm)	4.95	4.09	4.92	4.37	4.91
% Error	-----	17.37	0.61	11.7	0.81

6. PARAMETRIC STUDY

In the parametric study three cross-sections namely Rectangular, Trapezoidal and Circular of the box girder bridges are analyzed (for details see Figure 1 and Table 1). The results obtained for various cross-sections of the bridge are compared for different loading configurations Dead load (self weight) and Live load (I.R.C Class 70R) for maximum eccentricity are considered at mid-span. The loads are placed in accordance with IRC:6-2000, Standard Specifications and Code of Practice for road and bridge, section II-Loads and Stresses in the analysis.

6.1 Rectangular Box Girder

Linear Analysis of the rectangular box girder bridge was done for the three depths (2.0, 2.4 & 2.8 m) and a graphical comparison of all three depths was studied in terms of vertical deflection, longitudinal bending stress, transverse bending stress, shear lag and shear flow along the span and across the cross sections in top flange, bottom flange and in the web for dead load and IRC Class 70R live load (centrally and eccentrically). Due to the limitation of space the results for only dead load are presented here. Figure 7 to 14 shows variation of vertical deflection, longitudinal bending stress and shear stress along the span and across the cross sections in top flange, bottom flange and in the web for dead load only.

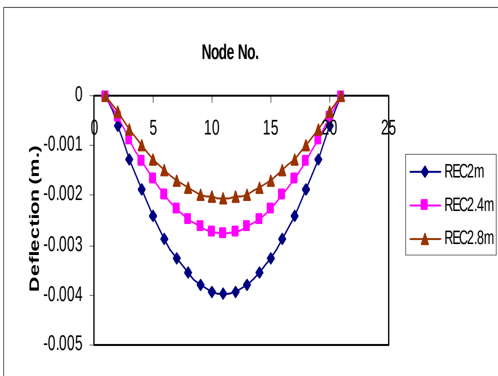


Figure 7. Deflection of top flange in rectangular box girder deck

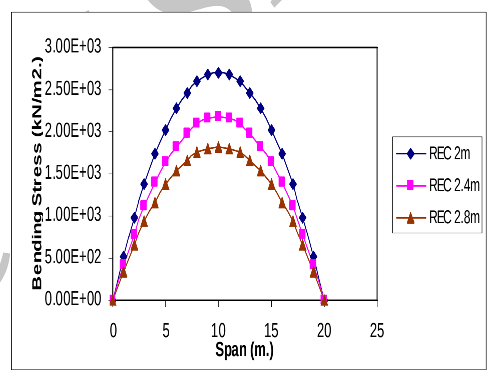


Figure 8. Longitudinal bending stress in bottom flange along the span in rectangular box girder

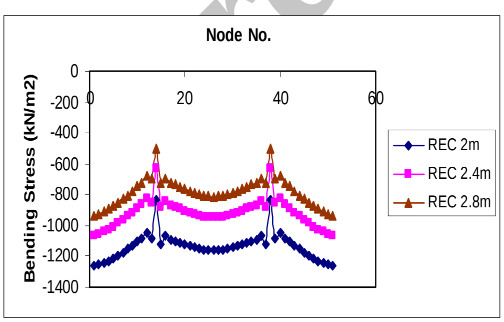


Figure 9. Longitudinal bending stress distribution in top flange across the cross section at 4m in rectangular box girder

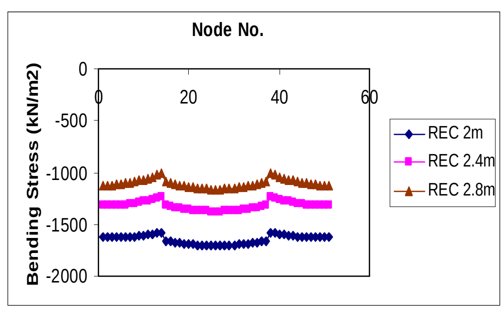


Figure 10. Longitudinal bending stress distribution in top flange across the cross section at mid span in rectangular box girder

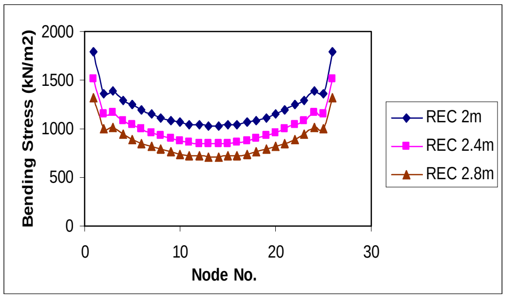


Figure 11. Longitudinal bending stress distribution in bottom flange across the cross section at 4m in rectangular box girder

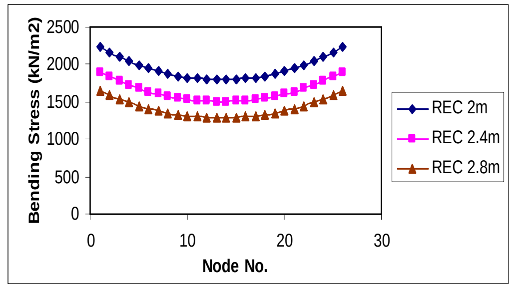


Figure 12. Longitudinal bending stress distribution in bottom flange across the cross section at mid span in rectangular box girder

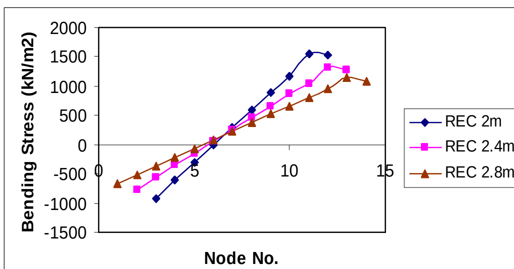


Figure 13. Longitudinal bending stress distribution in the web across the depth at 4m in rectangular box girder

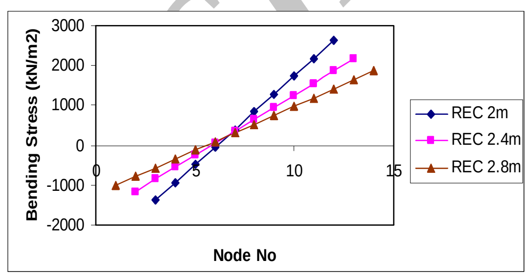


Figure 14. Longitudinal bending stress distribution in the web across the depth at mid span in rectangular box girder

After seeing Figures 7 to 14, it can be understood that with increase in depth of box girder the deflection decreases, but in lower proportion with the same increment of depth. Similarly with increasing the depth, the bending stress distribution along the span also decreases, but the reduction is not in same proportion with the increment in depth. It can also be seen that as the width to depth ratio increases the shear lag coefficient also decreases at the web (1.06, 1.04 & 1.01), but at the flange increases (0.94, 0.96 & 0.98) for dead load case. For centrally placed live load case with increase the width to depth ratio the shear lag coefficient also increases at the web (0.67, 0.74 & 0.82) but at the flange it decreases (1.97, 1.50 & 1.26). In eccentrically placed live load case at the webs with increase in the width to depth ratio the shear lag coefficient also increases (1.01, 1.03 & 1.06) and at the flange it decreases (0.98, 0.96 & 0.94). It is also found that the transverse stress distribution is same for all depth but it is more at the junction of flange and web. The shear stress distribution across the section shows that, the value at the web flange junction is very high. As the depth increases, the shear stress reduces, but the reduction is not proportional to the increase in depth. In live load case for symmetric loading it is zero at the centre of the flange, but in eccentrically placed loading, its distribution at the centre of the flange changes in magnitude.

Table 4 and 5 present the comparison of the results of deflection and stresses for rectangular box girder of different depths for live load placed centrally and eccentrically. It is clear from these Tables that the deflection decreases with increase in depth for both load cases.

Table 4. Comparison of rectangular box section for live load (Centrally placed)

Depth	**Deflection (mm)**	**S_{11} (kN/m^2)**	**S_{22} (kN/m^2)**
2.0m	2.601	Max=7948 Min= 8576	Max=2551 Min=2612
2.4m	2.021	Max=7765 Min= 8519	Max=2473 Min= 2561
2.8m	1.707	Max=7013 Min= 8516	Max=2083 Min= 2082

Table 5. Comparison of rectangular box section for live load (eccentrically placed)

Depth	**Deflection (mm)**	**S_{11} (kN/m^2)**	**S_{22} (kN/m^2)**
2.0 m	2.27	Max=9078.35 Min=10503.46	Max=3537.53 Min=6497.82
2.4 m	1.669	Max=8777.58 Min=10498.20	Max=3408.40 Min=5117.71
2.8 m	1.337	Max=8370.24 Min=10468.64	Max=3297.19 Min=4948.54

6.2 Comparison of Different Cross-Sections

Linear Analysis of the rectangular, trapezoidal and circular cross section box girder bridge was done for the three depths (2.0, 2.4 & 2.8m) and comparison of all three depths was done in terms of vertical deflection, longitudinal bending stress, transverse bending stress, shear stress and shear lag along the span and across the cross sections in top flange, bottom flange and in the web for dead load and IRC Class 70R live load (centrally and eccentrically). Figure 15 to 20 shows variation of vertical deflection, longitudinal bending stress and shear stress across the cross sections in top flange, bottom flange and in the web for only dead load. From these figures following points can be envisaged

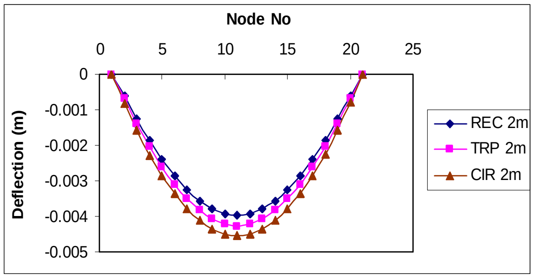


Figure 15. Top flange deflection in different box girders having 2 m depth

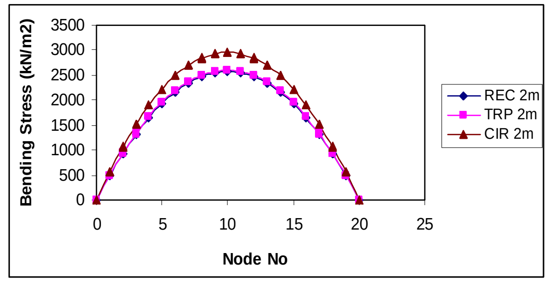
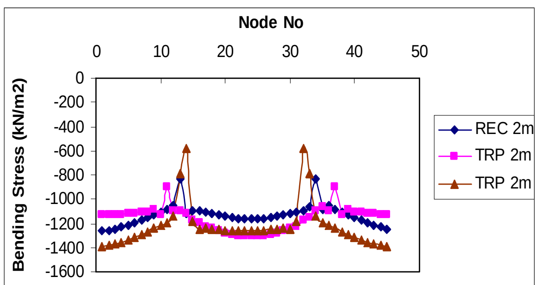
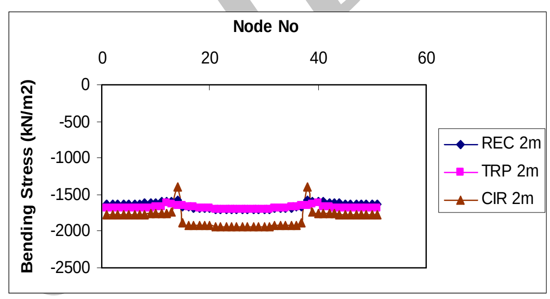


Figure 16. Longitudinal bending stress in bottom flange along span in different box girders having 2 m depth

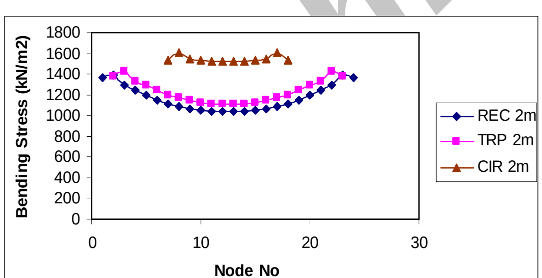


(a) At 4m

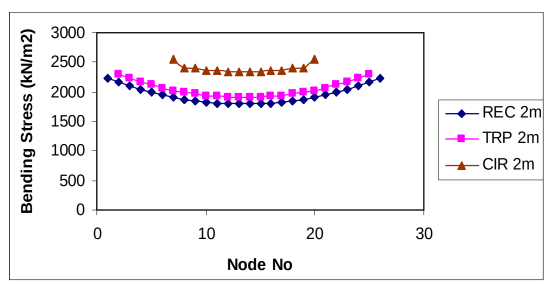


(b) At mid span

Figure 17. Longitudinal bending stress distribution in top flange across the cross section at (a) 4m and (b) mid span in different box girders having 2 m depth

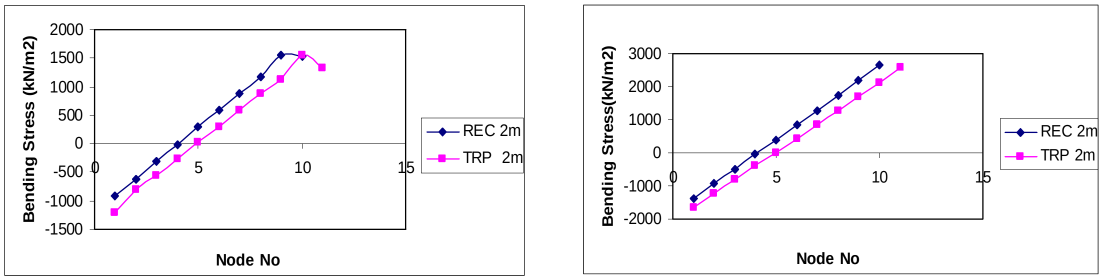


(a) At 4m



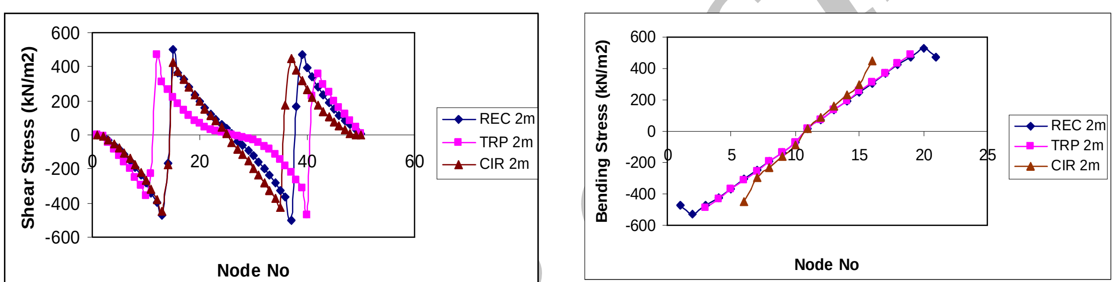
(b) At mid span

Figure 18. Longitudinal bending stress distribution in bottom flange across the cross section at (a) 4m and (b) mid span in different box girders having 2 m depth



(a) At 4m (b) at mid span

Figure 19. Longitudinal bending stress distribution across the depth in the web at (a) 4m and (b) mid span in different box girders having 2 m depth



(a) Top flange (b) Bottom flange

Figure 20. Shear stress distribution across the cross-section at 4m in (a) Top flange (b) Bottom flange in different box girders having 2 m depth

2m Depth Section:

- The longitudinal bending stress along the span is 15% more in circular box girder for the same cross sectional area, compared to trapezoidal and rectangular. In all case bending stress is least in rectangular box girder.
- From the longitudinal bending stress distribution in top flange it can be concluded, that in case of dead load the shear lag effect is respectively 8.4% higher at webs and 4.70% less at flange in circular section compare to rectangular box section. Similar in the case of live load circular box section is having 2.7% lesser shear lag coefficient at webs and 4.16% higher at flange. Shear lag coefficient at the mid span section is nearly 1.0 in rectangular and trapezoidal box section where as in circular box section its value is 1.19 at web for dead load case.
- From the longitudinal bending stress distribution in bottom flange, it can be concluded that, the shear lag effect is respectively 11.49% higher at webs and 12.06% less at flange in circular section compared to rectangular box section in dead load case.

2.4m Depth Section:

- The longitudinal bending stress along the span is 24% more in circular box girder for the same cross sectional area, compared to trapezoidal and rectangular. In all case bending stress is least in rectangular box girder.
- From the longitudinal bending stress distribution in top flange, it can be concluded, that in case of dead load the shear lag coefficient is 11.6% higher at the webs and 7.77% lesser at the flange in circular section compared to rectangular box section. Similarly in the case of live load circular box section is having 8% lesser shear lag coefficient at the webs and 13.81% higher at the flange. Shear lag coefficient at the mid span section is nearly 1.0 in rectangular and trapezoidal box section where as in circular box section its value is 1.56 at web and 0.73 at the flange for dead load case.
- From the longitudinal bending stress distribution in bottom flange, it can be concluded that, the shear lag effect is respectively 12.79% higher at webs and 13.55% less at flange in circular section compared to rectangular box section in dead load case.

2.8m Depth Section:

- The longitudinal bending stress along the span is 37% more in circular box girder for the same cross sectional area, compared to trapezoidal and rectangular. In all case bending stress is least in rectangular box girder.
- From the longitudinal bending stress distribution in top flange, it can be concluded, that in case of dead load the shear lag coefficient is 11.9% higher at the webs and 8% lesser at the flange in circular section compare to rectangular box section. Similar in the case of live load circular box section is having 24% lesser shear lag coefficient at the webs and 7.24% higher at the flange. The distribution of longitudinal bending stress at mid span section is uniform in all section for dead load case.
- From the longitudinal bending stress distribution in bottom flange, it is concluded that, the shear lag effect is respectively 14.11% higher at webs and 15.0% less at flange in circular section compare to rectangular box section in dead load case.

Among rectangular, trapezoidal and circular cross section box girders for all depths, the deflection is highest in circular girder under the dead load and live load (centrally & eccentrically) cases and least in rectangular girder. Therefore it can be concluded that the rectangular section is the stiffest section among these three sections. For all girders the shear stress distribution across the section shows that, the value at the web flange junction is very high as compared to the values at other locations. Moreover, the distribution of shear stress across the section is symmetric for dead load as well as live load and its value in all sections is almost equal and zero at the middle of top flange. Further more the transverse stress distribution is least in rectangular cross-section but in circular cross-section its value is maximum for all load cases. Comparison of deflection and stress development in different shapes of the girders for live load placed centrally and eccentrically are presented in Tables 6 and 7. The deflection of circular section is highest for both loading cases. The magnitudes of the stresses are also of higher magnitude in circular section as compared to the other two sections.

Table 6. Comparison of 2.0m depth three cross sectional shape for live load (Centrally placed)

Parameter	Rectangular	Circular	Trapezoidal
Deflection (mm.)	2.602	3.559	2.996
S_{11} (kN/m^2)	Max. = 7948	Max. = 12322	Max. = 8990
	Min. = 8576	Min. = 13701	Min. = 10547
S_{22} (kN/m^2)	Max. = 2551	Max. = 3155	Max. = 2738
	Min. = 2612	Min. = 3526	Min. = 3159

Table 7. Comparison of 2.0m depth three cross sectional shape for live load (Eccentrically placed)

Parameter	Rectangular	Circular	Trapezoidal
Deflection (mm.)	2.27	2.985	2.601
S_{11} (kN/m^2)	Max. = 9078.35	Max. = 16759.65	Max. = 9990.21
	Min. = 10503.46	Min. = 17419.84	Min. = 11419.19
S_{22} (kN/m^2)	Max. = 3537.53	Max. = 5155.73	Max. = 4896.55
	Min. = 6497.82	Min. = 5526.85	Min. = 6381.68

7. CONCLUSIONS

In this paper, results of linear analysis of three box girder bridge cross-sections namely Rectangular, Trapezoidal and Circular of varying depths have been presented. The results presented highlight, the effects of depth of cross-section and the cross-sectional shape on the behaviour in terms of development of deflection and stresses in different box girders. This detailed study carried out using SAP software has clearly brought out the effectiveness of 4-nodded shell elements for analysis of box girder-bridges. The FEM modeling has been validated by (*Numerical Example*) published literature based on initial value solution of the static equilibrium differential equation of thin walled box beams. It can be concluded from the present study that the simple beam theory is a crude approximation for analysis of box sections. It can also be believed that the results presented in this paper will be of valuable guidance to the designers.

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