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Chapter · January 2009

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In: Railway Transportation…
Editor: Nicholas P. Scott, pp. 1-55
ISBN: 978-1-60692-863-9

Chapter 10

TRACK STIFFNESS CONSIDERATIONS FOR HIGH SPEED RAILWAY LINES

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Abstract

This chapter discusses the importance of the vertical track stiffness as a means to guide railway track bed design for high speed railway lines and inform decisions regarding track maintenance and renewal. To this end, a rational approach to substructure design is described which it is hoped will further an understanding of the process of appropriate track design and enable the adaptation of existing design procedures to provide a realistic design for the conditions at hand. Despite adopting suitable design and construction standards, however, a number of factors may still result in the track stiffness varying along the track. Sometimes the stiffness variation may be very large within a short distance. These are likely to cause variations in the wheel/rail interaction force and will have a detrimental effect on track degradation increasing wear, fatigue of track components and track settlement due to permanent deformation of the ballast and the underlying substructure. This chapter discusses these issues and describes the use of numerical models which may be used to; assess the influence of track stiffness variations on the wheel/rail contact force, and; investigate the use of possible countermeasures. Techniques which have been developed to measure track stiffness, including the novel approach developed by one of the authors are described, and the

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possible uses of such measurements to make appropriate and timely railway maintenance and renewal decisions are discussed. Devices from different countries are described in relation to the evolution from static to rolling devices capable of effecting measurements at train speeds. Such techniques have undergone a process of continual evolution in recent years and several are now ready to be developed from research tools to those capable of being used practically by commercial organisations.

Introduction

Vertical railway track stiffness and its spatial variation are regarded as important measures of the structural condition of the railway track and essential considerations for the proper design of the track and the correct diagnosis of track maintenance problems 0.

Theoretically there is an optimal track stiffness to which a railway line should be designed, constructed and maintained. Below this optimum, excessive track displacements occur and above it unacceptable track deterioration may take place. Railway track which is too stiff can cause load concentrations as the train load is distributed over fewer supports and this in turn can lead to increased ballast attrition and create variations in track stiffness and therefore differential settlement (Brandl, 2004b; Selig and Waters, 1994). Differential settlement can result in increased train induced dynamic forces which in turn worsen track geometry, thus accelerating the deterioration of the entire track structure. Track which is not stiff enough, however, may lead to excessive rates of settlement and various types of subgrade related failure.

Spatial changes in track stiffness cause variations in train/track interaction forces which can also give rise to the degradation of the track superstructure and substructure. For the former this includes rail and sleeper fatigue, and for the latter, track settlement as a result of permanent deformation of the ballast (Jönsson and Stichel, 2007). The rate of degradation of track components and the rate of track settlement depends on the severity of the stiffness variation and as the track geometry starts to deteriorate, the variations of the train/track interaction forces increase, speeding up the rate of track deterioration. Furthermore, track stiffness irregularities induce vibrations in both the train and in the track. In many cases the stiffness variation is of a random nature along the track. Long wave variations will induce low-frequency random oscillations of the train, causing reduced ride comfort for passengers, and track vibrations may induce disturbances in nearby buildings. At abrupt changes of track stiffness, for example at turnouts and at transitions from ballast to slab track, transient and high-frequency vibrations will be induced in the track. This may result in localised track deterioration such fatigue of the rails and sleepers, cracks, wear and plastic deformation in rails as well as hanging sleepers.

Consequently, it is important to adopt a rational approach to the design of the railway track support system so that the vertical track stiffness and its spatial variation are within an acceptable range of values. It is also necessary to be able to measure track stiffness using techniques and devices which give accurate, repetitive and reproducible results to enable the performance of existing railway track to be quantified so that appropriate decisions regarding track maintenance may be made.

Definitions

Vertical track stiffness (k) can be defined in a number of ways and in its simplest form is the ratio between the track load (F) and track deflection (z) as a function of time (t) as follows:

$$k(t) = \frac{F(t)}{z(t)} \tag{1}$$

Commonly the stiffness of different components of the track structure, such as the rail pad and subgrade, is nonlinear. Further, the sleepers may also have voids beneath them, which lead to large deflections with low loads as indicated in Figure 1. To take into account these factors other definitions of track stiffness may be used. For example, to eliminate the effect of voiding the secant stiffness may be used 0 as follows:

$$k_{x-ykN} = \frac{F_y - F_x}{z_y - z_x} \tag{2}$$

Where F_x and z_x are the seating load and resulting deflection respectively.
Alternatively the tangent stiffness can also be used:

$$k_{tng,y} = \left| \frac{dF(t)}{dz(t)} \right|_y \tag{3}$$

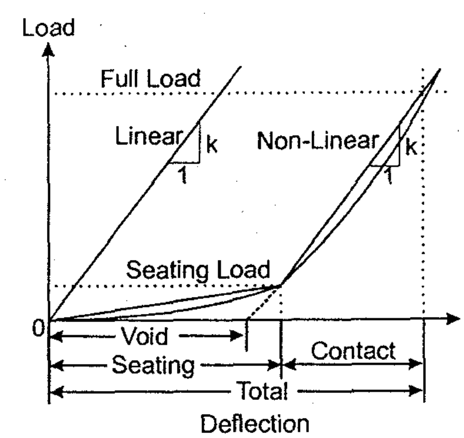

Figure 1. Load – deflection diagram illustrating voids and non-linearities 0.

Increasingly however, devices fitted to moving vehicles (rolling devices) are being used to enable the measurement of track stiffness at normal traffic speeds (several such devices are described below). The track stiffness of such dynamically loaded track also varies with excitation frequency (f) and so in these cases it is also necessary to define stiffness as a function of frequency. Figure 2 shows the measured vertical track stiffness of a section of track in Sweden as a function of frequency. The rail was pre-loaded with a static load of 90 kN and then slowly loaded up to 150 kN with a superimposed dynamic load of amplitude of 10 kN. The resulting force-deflection curve (Figure 2a) is non-linear and shows a hysteresis, which indicates a damping factor.

To facilitate the analysis of dynamic track stiffness using Fourier transforms and associated transfer functions, it is necessary to assume that the stiffness is linear about a certain reference preload. This presumption is approximately valid for a limited portion of the force-deflection diagram. The transfer function between force and displacement is called receptance (α) or dynamic flexibility, Equation 4. The receptance is a complex-valued quantity and is often displayed with magnitude and phase. Receptance is the inverse of the dynamic stiffness and is used in preference to it as most systems studied are force driven and so resonance phenomena can be interpreted as large deflections 0.

$$\alpha(f) = \frac{z(f)}{F(f)} \tag{4}$$

For example, consider the right hand part of Figure 2, which for the dynamically loaded rail describe above, indicates a resonance around 5 - 8 Hz due to soft subgrade (clay). The track is stiffer (lower receptance) for higher frequencies (up to 50 Hz).

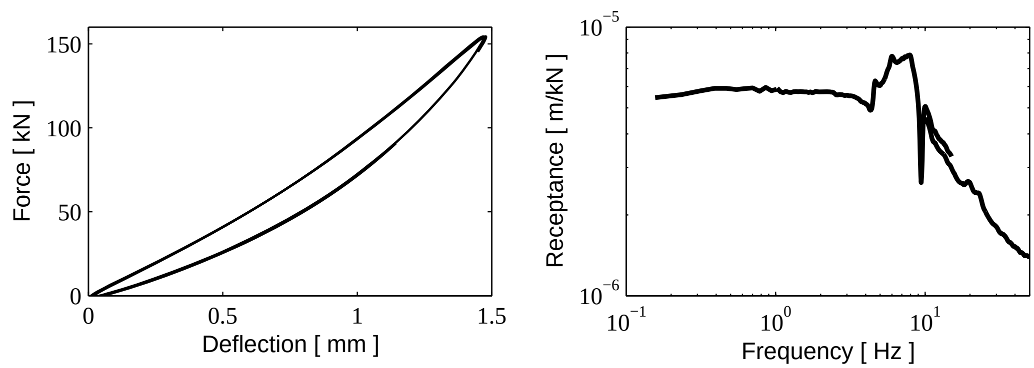

Figure 2 a). Vertical force-deflection diagram of track with quasi-static excitation (measured on rail), b). Magnitude of vertical track receptance with subgrade (measured on rail), F_{stat} = 90 kN, F_{dyn} = 10 kN. 0.

The track modulus, u, is defined as the applied force per unit length of rail per unit deflection (δ) and is sometimes used instead of track stiffness:

$$u = \frac{q}{\delta} \tag{5}$$

where q is the vertical foundation supporting force per unit length. Using the theory of a beam on elastic foundation, a relationship between track modulus and track stiffness can be found 0 as follows:

$$u = \frac{k^{4/3}}{(64EI)^{1/3}} \tag{6}$$

The difference between u and k is that k includes the effect of the rail bending stiffness EI, whereas u is related only to the remainder of the superstructure (i.e. fasteners and sleepers) and the substructure (ballast, subballast and subgrade).

Optimal Track Stiffness

A number of studies have been undertaken to try to define optimal values of track stiffness and its spatial variation. For example, Selig and Li 0 suggest that a track modulus of u = 28 MPa may be considered a minimum to ensure a consistently good track performance under traffic loading. This equals (Eq. (6)) to a stiffness of k = 55 kN/mm (one rail) with rail of type UIC 60. Based on studies of the Spanish high-speed line between Madrid and Seville, Lopez Pita et al. 0 propose that an optimal stiffness value for high speed lines is about 70 – 80 kN/mm (one rail) on a maintenance costs and energy consumption (running resistance) basis. In the EUROBOLT project (EUROBOLT, 2000; Huille, and Hunt, 2000) an analytical model of track geometry deterioration was used to investigate theoretical optimal track stiffness values. For high speed lines this was found to be 200 kN/mm when measured on the sleeper and under the defined conditions of track operation given in Table 1. An optimum track stiffness value of 160 kN/mm was found for non-high speed main lines which carry freight in addition to passenger traffic. Research from the same project suggested that variations in the stiffness of the subgrade should be limited to less than 10% of the mean value (EUROBOLT, 2000). Research continues, however, to quantify acceptable levels of overall track stiffness variation.

Research has also been carried out to determine permissible rail deflections under a given load. For example, Brandl (2004b) and Woldringh (2001) suggest that the following elastic deflections, δ, are acceptable under a passing wheel load of approximately 200 kN:

1.0 mm $\leq \delta \leq$ 2.2 mm for train speeds $\leq$ 160 km/h
1.5 mm $\leq \delta \leq$ 2.0 mm for train speeds > 160 km/h

Table 1. Optimal track parameters (after EUROBOLT, 2000)

Attribute	Value	Comments
Geometry of wheel	1 in 10,000 wheels giving 250 kN impact force or better	
Rail section	UIC 60	
Rail pad stiffness	80 kN/mm	
Sleeper spacing	0.6 m	
Ballast – depth	0.3 m	
Sleeper stiffness	200 kN/mm	0.5 mm rail deflection at 20 tonne axle load
Subgrade stiffness variation	10% or better than mean value	

The Rational Design Process

Introduction

The track stiffness is influenced to a great extent by the upper part of the track substructure. The majority of this consists of subgrade material , which therefore, may be considered to have the greatest influence on track stiffness in comparison to any other factor (Selig and Waters, 1994). Consequently, the railway track structure should be designed in an appropriate manner to maintain, for a predetermined period, an appropriate and uniform track stiffness by withstanding the combined effects of traffic and climate to the extent that the subgrade is adequately protected and that railway vehicle operating costs, safety and comfort of passenger are kept within acceptable limits.

There are a number of design procedures which have been developed by different agencies for such systems. As many of these are based, in part at least, on an empirically founded methodology extrapolating them for the design of high speed lines may not be appropriate (Burrow et al., 2007a).

The trend towards using faster trains and heavier axle loads than those originally considered in many existing design procedures, in conjunction with the need to minimise whole life costs, requires a rational approach to design which is based on a thorough understanding of the track system and its influence on subgrade behaviour.

A rational approach combines two main processes. In the first the stresses, strains and deflections induced by train loading in the component layers of the substructure are determined (Ullidtz, 2002). The second process consists of using experimental methods to determine allowable stresses, strains and deflections in the various materials which constitute the track substructure. To formulate the design, the induced stresses, strains and deflections in each of the component layers are compared with the allowable determined from experimentation.

To determine the induced stresses, strains and deflections the applied traffic loading should be accurately represented and the component layers of the substructure should be characterized to meet the requirements of a theoretical model of the track system. Such a traffic / substructure model allows the stresses, strains and deflections throughout the track system to be predicted.

In the second process, it is necessary to identify the most important parameters in the component layers which cause the track support system to deteriorate over time. By setting limits on these parameters, through serviceability requirements for example, the allowable stresses, strains and deflections may be established.

Traffic Characterization

The effects of repeated traffic loads and climate reduce the performance of the track over time with a consequent lessening of its ability to carry traffic at design loads and speeds. The performance of the track at any particular time can be related to its condition at that time. The design period is the number of years from the time the track is opened for first use until a terminal condition, however defined, is reached.

Track Condition

The condition of the track may be described by its functional and structural condition. The functional condition relates to the ability of track to serve the rail user, whilst the structural condition concerns the track's ability to carry load and protect the subgrade. Under repeated loading, the track moves laterally and vertically causing deviations in line and level from the desired geometry. As these deviations are generally irregular, ride quality decreases and consequently the loads to which the track is subjected increase, causing increased geometry deterioration. Changes in the ride quality, or functional condition, can be measured by a number of parameters, the most common however concern changes in the vertical and horizontal profile of the railway track. Track geometry may change over time due to a number of factors which are related to both the functional and structural property of the track system. For example, line and level deviations over time can be caused solely by poor ballast alignment, or they may be associated with structural problem related to the subgrade. Consequently, in order to identify whether the problem is functional or structural (or both) in nature and make appropriate decisions regarding maintenance it is necessary also to measure the track deflection or stiffness (Huille and Hunt, 2000).

Design Period

The choice of the optimum design period should be evaluated using a life cycle cost analysis to investigate various design options over a fixed analysis period. For example, one strategy could be to design the track substructure to last for the whole of the analysis period with maintenance carried out periodically to adjust the line and level. An alternative strategy could be adopted whereby several design periods are included within the analysis period with various components being replaced at the end of each design period. The most efficient solution should be determined by evaluating total life cycle costs including user, maintenance and renewal costs.

Design Loading

The track structure is subject to repeated vertical, lateral and longitudinal forces induced by traffic and the climate. These forces, transferred through the track superstructure, determine the dynamic loading environment that must be supported by the substructure (Selig and Waters, 1994). The forces applied to the track by moving vehicles are larger than the nominal static weights of the trains in question due to dynamic forces induced by variable track, vehicle characteristics and operating conditions. Research suggests that dynamic forces can be considered to result from vehicle speed effects and irregularities in the track or vehicle wheels. The latter are likely to increase over time as the track deteriorates.

The dynamic forces may be determined from field measurements, appropriate models of the vehicle track system or empirically founded formulae. These typically are of the following form:

$$P_d = K_d P_s \tag{7}$$

Where P_d = dynamic force, K_d = dynamic impact factor and P_s = static train load.

A useful review of many commonly used impact factors is given by Stewart and O'Rourke (1988).

Eisenman (1977 and Esveld, 2001) proposes a probabilistic approach, based on an empirical study of actual stresses, which takes into account both vehicle speed and track condition. From the study, Eisenman suggests that stresses induced in the rail are normally distributed (Figure 3) and the mean value is independent of the operating speed, V, but is a function of the track condition, φ. For speeds greater than 60 km / h, dynamic forces however were found to be a function of both the vehicle speed and track condition. These two cases can be represented by the following equations:

$$k_d = 1 + t\varphi \qquad \text{when } V < 60 \text{ km/h} \tag{8}$$

$$k_d = 1 + t\varphi\left(1 + \frac{V - 60}{140}\right) \qquad \text{when } 60 \leq V \leq 200 \text{ km/h} \tag{9}$$

Where φ, the track condition, has a value of 0.1 for track in very good condition, 0.2 for track in good condition and 0.3 for track in poor condition. t is an integer which takes a value between 1 and 3 depending on the risk associated with the design (see below).

The probability of occurrence of a particular stress or load, P is given by

$$f(p) = \left(\frac{1}{\sigma\sqrt{2\pi}}\right)e^{\frac{(p-\mu)^2}{2\sigma^2}} \tag{10}$$

Where μ is the mean value of stress occurrence and σ is the standard deviation given by

$$\sigma = \mu\varphi \qquad \text{when V} < 60 \text{ km/h} \tag{11}$$

$$\sigma = \mu\varphi\left(1 + \frac{V - 60}{140}\right) \qquad \text{when } 60 \leq V \leq 200 \text{ km/h} \tag{12}$$

Eisenman suggests that the design dynamic stress selected should be based on the application to which the design process will be applied. For example, Eisenman recommends that for a design with safety critical considerations, such as that associated with calculating rail stresses and fastenings, the design dynamic stress should be equal to the mean value, μ, plus 3 standard deviations (i.e. t = 3). As the dynamic loads in Eisenman's model are normally distributed about the mean, the dynamic load calculated with a value of t = 3 represents the maximum of all possible loads occurring within 3 standard deviations of the mean i.e. 99.7 % of all possible loads are likely to be less than this value (Figure 3). For calculations of the lateral load and those in the ballast bed, a value of t = 2 is suggested by Eisenman. Where the design concerns less safety critical work, such as the foundations then t = 1 is appropriate. As an example, consider a foundation design problem (t =1) for a track with a design speed of 200 km / h and assuming the track condition will never fall below what is considered to be a good condition ($\varphi = 0.2$), then using equation 3 the dynamic amplification factor, K, is 1.40 or an increment of 40 % should be added to the mean (static) load.

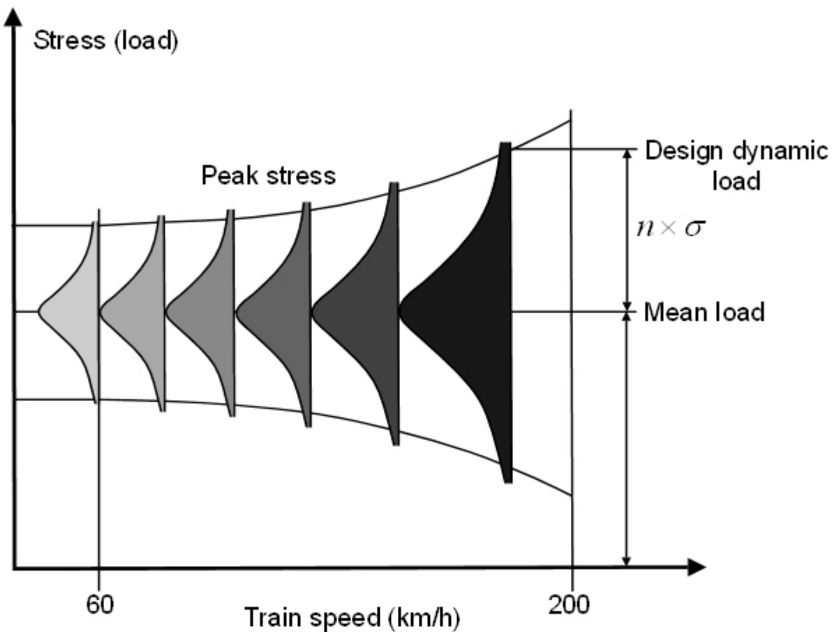

Figure 3. Train induced stresses as a function of train speed (after Esveld, 2001).

Equations 3 has been modified for use by the German railway authorities (Brandl, 2001) as follows:

$$k = 1 + t\varphi\left(1 + \frac{0.5(V - 60)}{190}\right) \qquad \text{for passenger trains when } 60 \le V \le 300 \text{ km/h} \qquad (13)$$

$$k = 1 + t\varphi\left(1 + \frac{0.5(V - 60)}{80}\right) \qquad \text{for freight trains when } 60 \le V \le 140 \text{ km/h} \qquad (14)$$

and $\varphi = 0.15$ for high speed lines and other main lines, $\varphi = 0.20$ for secondary lines and $\varphi = 0.25$ for other tracks.

It is usual to consider the two axles of a leading bogie and the two of a trailing bogie as a single load repetition in order to calculate the stresses and deformations in the subgrade (Grabe and Clayton, 2003; Li and Selig, 1998a; Stewart and O'Rourke, 1988). This four axle loading regime has been shown, from field measurements, to be equivalent to a single load pulse at depths below approximately 0.6 m (Stewart and O'Rourke, 1988).

For design purposes it is convenient to use a single loading configuration. However, as the loads applied to the track over its design period are likely to vary in magnitude, it is necessary to covert the number of applications of all loads to an equivalent number of repetitions of the design load. The equivalent number is that number of loads which will cause the same amount of track damage and can be calculated using equations for track damage which are typically of the form (Esveld, 2001 and Li and Selig, 1998):

$$D = CP_d^{\,c} N^b \tag{15}$$

Where D is damage, C is a constant for vehicles travelling at a particular speed, P_d is the dynamic load, N is the number of load repetitions and c and b are constant for particular track constructions. Then the equivalent number of repetitions, N_e, of the design load, P_{dd}, to achieve the same amount of damage as caused by N_i repetitions of any load P_{di}, can be calculated as follows:

$$D = KP_{dd}^{\,c} N_e^{\,b} = KP_{di}^{\,c} N_i^{\,b} \tag{16}$$

Whence

$$N_e = \left(\frac{P_{di}}{P_{dd}} \right)^{\frac{c}{b}} N_i \tag{17}$$

Where a four axle loading configuration is used, the above calculations involve the superimposition of loads, or the use of modelling to determine the stresses induced by the loading configuration at the depth of interest. This is usually the sub-ballast / subgrade interface.

To quantify the spectra of likely dynamic loads a probabilistic approach may be used. Stewart and O'Rourke (1988) describe a method which considers an unequal load distribution on the four axles. In their method they assume that 3 of the axles carry equal loads whilst the fourth one carries a variable load. The magnitude and probability of occurrence of the latter are determined from field data describing the distribution of all dynamic loads.

Modelling the Track Support System

Analytical models of the track superstructure and substructure are used to determine the effect of traffic loads on the stresses, strains and deformations in the system. These can then be compared with allowable stresses, strains and deformations of the various components in the track support system to formulate a design.

With the advent of the personal computer design procedures have incorporated models based on layered elastic theory, the finite element (FE), the finite difference, boundary element and distinct element methods. These model individual components of the superstructure and substructure and are able to consider non-linear characteristics including plastic, viscous and viscoelastic deformations and strain rates which are non-linear functions of the stress level. Whilst each technique may have advantages for specific applications, arguably the FE method offers the widest and most robust range of computational capabilities (Schwartz, 2004). Further information on the types of models available and their relative merits may be found elsewhere (see for example Blair and Chan, 2006; Selig and Waters, 1994).

Track Foundation Properties Required for Structural Analysis

Analytical models require each layer of the substructure to be characterised in terms of elastic parameters. Usually two parameters, the resilient modulus and Poisson's ratio, are used. The resilient modulus, defined as the quotient of the deviator stress by the resilient strain in the direction of the major principal stress, can be determined directly from laboratory tests, or from an analysis of the response of measured in situ parameters. Poisson's ratio is usually estimated.

Granular Materials

Under repeated loading conditions, the behaviour of granular materials is nonlinear and stress-dependent (Gomez Correia, 2004; Selig and Waters, 1994). Initially, for each cycle of loading some plastic strain occurs whilst the magnitude of the plastic strain decreases as the number of loading cycles increases. Eventually, and if the stresses levels are moderate, after a number of loading cycles the resilient strain becomes constant and the material behaves elastically (see below).

The stress level is the primary factor affecting the resilient modulus which has been shown to increase appreciably with increasing confining stress and slightly with increasing repeated deviator stress, provided that shear failure is not approached.

Four main types of non-linear models are widely used to describe non-linear behaviour. These are the K- θ, the modified K- θ, the Boyce, and the orthotropic Boyce models (Gomez Correia, 2004; Lekarp et al., 2000).

As modelling of resilient behaviour is complex, the simple, and widely used, K- θ model was proposed in the 1960's to describe the results of cyclic load triaxial tests carried out with a constant confining pressure (Gomez Correia, 2004; Brown, 1996; Brown and Pell, 1967). In this model the resilient modulus, E_r, is given by

$$E_r = K_1 \theta_1^{k_2} \tag{18}$$

Where K_1 and K_2 are material constants determined from experimentation and $\theta_1 = 3p'$ where p' is the mean normal effective stress. Poisson's ratio, μ, is constant and usually taken as 0.3. The inaccuracy of this model in computing stress conditions in a granular layer or sand subgrade is widely recognised (Gomes Correia, 2004; Brown, 1996). However, it can usefully be used to model a granular layer when effects in the layers above or below are required. Gomes Correia et al. (1999) also note that the accuracy of the model could be improved if a stress dependent Poisson's ratio is used.

Burrow et al. (2007a) used the K- θ model to represent ballast and sub-ballast material behaviour in a finite element model of an existing section of railway track. In their study they determined the parameters k_1 and k_2 using a procedure based on matching closely actual deflections in the field obtained with the falling weight deflectometer (FWD) device with deflections obtained from the model (Figure 4). The study found that for the ballast material $k_1 = 175$ and $k_2 = 0.5$ (with $\mu = 0.2$) and for highly contaminated ballast slurry $k_1 = 35$ and $k_2 = 0.5$ (with $\mu = 0.49$).

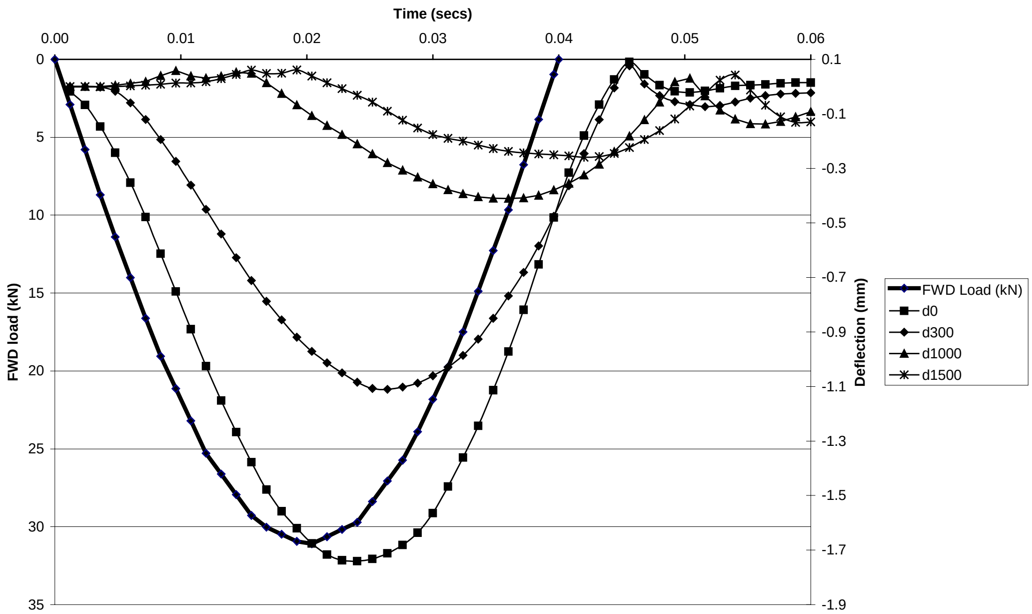

Figure 4. FWD load and sensor deflection time histories.

A modified version of the K- θ model was proposed by Uzan et al. (1992) in which the resilient modulus is given as a function of both the mean stress p and the deviator stress q.

$$E_r = K_1 \theta_1^{k_2} \theta_2^{k_3} \tag{19}$$

Where $\theta_2 = \dfrac{q}{p_a}$ and the deviator stress, q, = σ_1-σ_3, p_a is a reference stress (such as atmospheric pressure). K_1, K_2 and K_3 are material properties obtained from experimentation.

Although the modified model takes into account the effect of the shear stress (q), Poisson's ratio is also assumed to be constant and consequently its use may lead to the inaccuracies as described above.

The Boyce (1980) non-linear elastic model was developed using cyclic triaxial test apparatus that was able to vary the confining pressure. Consequently, it is able to take into account the effect of stress paths. The model is expressed in terms of the bulk modulus, K, and the shear modulus, G, as follows:

$$K = \frac{\left(\dfrac{p}{p_a}\right)^{1-n}}{\dfrac{1}{k_a} - \dfrac{\beta}{k_a}\left(\dfrac{q}{p}\right)^2} \tag{20}$$

$$G = G_a\left(\frac{p}{p_a}\right)^{1-n} \tag{21}$$

where $\beta = G_a \dfrac{(1-n)k_a}{6G_a}$

K_a, G_a and n are constants.

With this model using elasticity theory, Young's modulus, E, and Poisson's ratio, ν, may be expressed as follows:

$$E = \frac{9G_a\left(\dfrac{p}{p_a}\right)^{1-n}}{3+\left(\dfrac{G_a}{K_a}\right)\left[1-\beta\left(\dfrac{q}{p}\right)^2\right]} \tag{22}$$

$$\nu = \frac{\dfrac{3}{2}-\left(\dfrac{G_a}{k_a}\right)\left[1-\beta\left(\dfrac{q}{p}\right)^2\right]}{3+\left(\dfrac{G_a}{K_a}\right)\left[1-\beta\left(\dfrac{q}{p}\right)^2\right]} \tag{23}$$

The Boyce model represents the behaviour of materials with reasonable accuracy as it can describe the effects of the mean normal stress p and also of the stress ratio q/p (Gomez Correia, 2004).

However, it has been found that the Boyce model does not accurately predict values of the shear strain, ε_q for low values of q/p, leading to unrealistic values of k_a, G_a and n. Consequently, Hornych et al. (1998) proposed an orthotropic version of the model by introducing a parameter γ, to scale the principal stress σ_1. Using their model the resilient volumetric (ε_v) and shear (ε_q) strains can be represented by the following (Gomes Correia, 2004):

$$\varepsilon_v = \frac{p^{*n}}{p_a^{\,n-1}}\left[\frac{\gamma+2}{3K_a}+\frac{n-1}{18G_a}(\gamma+2)\left(\frac{q}{p^*}^{*}\right)^2+\frac{\gamma-1}{3G_a}\left(\frac{q}{p^*}^{*}\right)\right] \tag{24}$$

$$\varepsilon_q = \frac{2p^{*n}}{3p_a^{\,n-1}}\left[\frac{\gamma-1}{3K_a}+\frac{n-1}{18G_a}(\gamma-1)\left(\frac{q}{p^*}^{*}\right)^2+\frac{2\gamma+1}{6G_a}\left(\frac{q}{p^*}^{*}\right)\right] \tag{25}$$

and

$$p^* = \frac{\gamma\sigma_1 + 2\sigma_3}{3}$$

$$q^* = \gamma\sigma_1 - \sigma_3$$

Pappin et al. (1992) demonstrated that the resilient response modelled for dry granular material is applicable to saturated and partially saturated conditions, provided that the principle of effective stress is observed (Brown, 1996). Gomes Correia (2004) describes a number of models developed using such an approach. However, in practice the determination of the effective stress state in a granular layer may not be straightforward.

As mentioned above, the accuracy of the constitutive relationships may be improved if stress dependent Poisson's ratios are used and for repeated load triaxial compression tests indicate that Poisson's ratio is strongly correlated to the ratio of principal stresses (σ_1/σ_3).

Fine-Grained Soils

In the case of fine-grained soils, the deviator stress has a primary influence on the resilient modulus which decreases non-linearly with increasing applied deviator stress, when all other factors are kept constant, and therefore constitutive models are primarily established between the resilient modulus and the deviator stress (Fleming et al., 2003). Li and Selig (1994) categorize these models into five different types as follows:

Bilinear model

The bilinear model was proposed by Thompson and Robnett (1976) and takes the following form:

$$E_r = K_1 + K_2 q \qquad \text{when } q < q_i \tag{26}$$

$$E_r = K_3 + K_4 q \qquad \text{when } q > q_i \tag{27}$$

where q_i is the deviator stress at which the gradient of the resilient modulus changes and K_1, K_2, K_3 and K_4 are constants which depend on the soil type and its physical state.

Power models

Mossazadeh and Witczak (1981) formulated a power model to represent the behaviour of three fine-grained soils as follows:

$$E_r = Kq^n \tag{28}$$

Where k and n are constants which depend on soil type and physical state.

Brown et al. (1975) suggested an effective stress version of the model to represent the behaviour of silty clays, in which the resilient modulus is also a function of, p'_0, is the initial (geostatic) mean effective normal compressive stress. Their modified model is given by:

$$E_r = K\left(\frac{p'_0}{q}\right)^n \tag{29}$$

In the work reported by Burrow et al. (2007a) mentioned earlier which investigated a means of calibrating a FE model by using field deflections obtained from an FWD, Brown's model was used to represent the performance of a silty clay subgrade. Using the FE model the parameters K and n in Equation 29 were determined for various subgrade layers beneath the ballast and are given in Table 2 below.

Table 2. Back analysed parameters

Subgrade 1 (Firm CLAY)	Base modulus, K (MPa)	123
	Exponent, *n*	1.1
	Poisson's ratio, ν	0.49
	Layer thickness (m)	0.50
Subgrade 2 (Soft to Firm CLAY)	Base modulus, K (MPa)	30*
	Exponent, *n*	1.1
	Poisson's ratio, ν	0.49
	Layer thickness (m)	1.50
Subgrade 3 (Very soft to soft CLAY)	Base modulus, K (MPa)	6*
	Exponent, *n*	1.1
	Poisson's ratio, ν	0.49
	Layer thickness (m)	0.25

Semilog model

The semilog model was suggested by Fredlund et al. (1977) for a moraine glacial till as follows:

$$M_r = 10^{(k-nq)} \tag{30}$$

Raymond et al. (1979) used a similar model to successfully represent the behaviour of Leda clay.

Hyperbolic model

Drumm et al. (1990) proposed the hyperbolic model as a match for data for fine-grained Tennessee soils and it takes the following form:

$$E_r = \frac{K + nq}{q} \tag{31}$$

Octahedral model

The Octahedral model in which the resilient modulus is a function of the octahedral normal and shear stresses, σ_{OCT} and τ_{OCT} respectively, was proposed by Shackel (1973):

$$E_r = k\frac{\sigma_{OCT}^n}{\tau_{OCT}^m} \tag{32}$$

Li and Selig (1994) carried out an investigation of the first four models described above, which involved determining how well each model fitted to data available in the literature. The Octahedral model was not considered as it was regarded as being too difficult to apply. From their analysis they found that all of the models may be considered to fit the data considered by choosing suitable parameters (such as k and n). However, they found that the bilinear model gave the best representation, followed by the power, semilog and hyperbolic models respectively.

Composites

As described above, large amount of research effort has focused on determining the properties of the individual constituent layers of the track substructure in isolation. However, the interaction of these layers is also important. For example, in the case of a conventional granular trackbed layer which is supported by a softer subgrade, the deflections will be partially controlled by the load spreading ability of the granular material which controls the level of stress transmitted to the subgrade. However, the reaction of subgrade to the stress transmitted will influence the amount of load spreading that can occur. Thus the layer interaction affects the stress distribution which in turn affects the total elastic and plastic strains that are developed within each layer and hence their response to those stresses and vice versa (Fleming et al., 2003).

In many routine highway design methods an attempt is made to take into account the non-linear behaviour of granular materials values by using empirical relationships between the modulus of the layer in question and that of the underlying one (Gomez Correia, 2004; Loizos, 2004). Such relationships are typically of the form:

$$E_n = kE_{n+1} \tag{33}$$

Where E_{n+1} is the resilient modulus of the underlying layer and k is typically between 2 and 4.

Whilst these types of relationships are simple and may be easily incorporated in any railway track design procedure they do not take into account the influence of the water content and quality of the constituent material (Gomez Correia, 2004; Loizos, 2004). Loizos (2004) describes research to provide more accurate relationships, to those described by equation 33 above, which take into account the modulii and thicknesses of the layers above and below those in question.

Field Work and Laboratory Testing

For the models described above appropriate procedures should used to determine the various model parameters. Most models of the type described above have been developed using cyclic load triaxial test apparatus (Gomes Correia, 2004). However, under the passage of a moving wheel load an element of material in the track substructure is subject to a complex regime consisting of vertical, horizontal and shear stresses (see Figure 5). This regime is more closely represented under laboratory conditions using the Hollow Cylinder Apparatus (HCA) which, unlike a conventional triaxial apparatus, allows the normal and shear stresses to be controlled to simulate more closely the in-situ regime. Whilst permanent deformations determined using conventional techniques, when compared to using the HCA, may be significantly underestimated (Gräbe and Clayton, 2003), research by Chan (1990) demonstrated that resilient strains are unaffected by the phenomenon and that the principal planes of strain remain coincident with those of stress. The work demonstrates that the relatively simple resilient modulus models derived from triaxial tests, as described above, rather than more complex apparatus can be used for structural design (Brown, 1996).

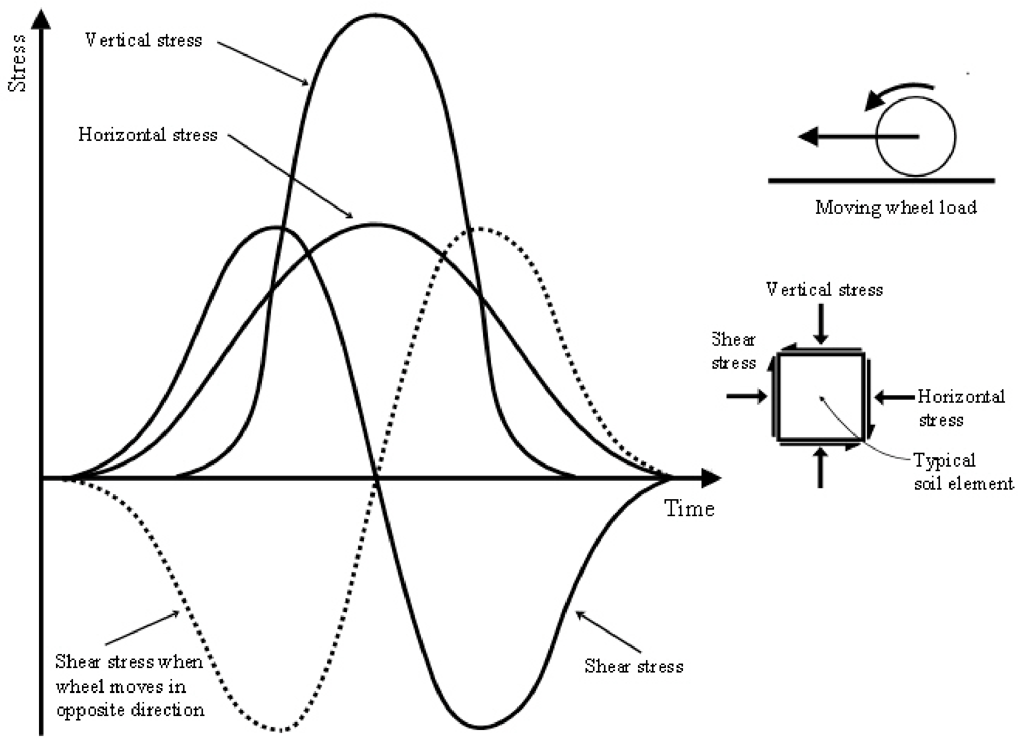

Figure 5. Stresses induced under a moving wheel load (after Chan and Brown, 1994).

Laboratory testing of small elements, however, raises questions concerning the preparation of reliable specimens and whether they are representative of in-situ conditions (Brown, 1996). Consequently, field testing, though more expensive than laboratory testing, plays an important part in the design process as it allows material properties to be determined under representative conditions. To this end, dynamic deflection tests such as single point load tests, deflection basin tests or multiple axle vehicle load tests can be used. An example of a methodology which may be employed to determine the resilient moduli of layers beneath a railway track using FWD device together with a dynamic FE model of the device has been mentioned above and useful summaries of appropriate tests may be found in Brough et al. (2003) and Selig and Waters (1994).

Design Criteria

Two considerations should be considered in design, appropriate track stiffness and the protection of the subgrade. As mentioned previously, it is important to ensure that the stiffness of the track is within acceptable limits. The ability of the track substructure to deform elastically (i.e. in a resilient manner) facilitates the transfer of train induced loads from the wheel, via the rail to sleepers through the ballast, sub-ballast and finally into the subgrade. The entire system should possess a stiffness which limits rail displacements on the one hand, but is not so stiff to cause load concentrations to occur. The former can cause damage to subgrade, whilst the latter can damage the track superstructure. In conventional railway track, the optimum design of such a system should involve a gradual decrease in stiffness from the ballast, through the sub-ballast to the subgrade. The achievable and uniformity of stiffness of each layer and thus of the overall system is a function of material properties, the thickness of the various layers and the quality of construction.

The design criteria related to track stiffness, k, can be expressed as follows:

$$k_{all} \leq k_p \leq k_{aul} \tag{34}$$

Where k_{all} and k_{aul} are lower and upper acceptable limits of track stiffness respectively.

Additionally, it is important to ensure that the track substructure is appropriately designed as it becomes progressively weakened through the cumulative effect of traffic induced repetitions of stresses and strains. As ballast lends itself to maintenance the main objective of the design procedure is to protect the subgrade. In the subgrade, the primary modes of traffic-induced deterioration are subgrade attrition by the ballast, progressive shear failure, massive shear failure and an excessive rate of settlement through the accumulation of plastic strain (Selig and Waters, 1994). These modes are mostly associated with fine grained soils. Subgrade attrition occurs as a result of relative movement of ballast and subgrade at the ballast-subgrade interface. The usual method of preventing its occurrence is to place a layer of sand of appropriate thickness directly between the ballast and subgrade.

Progressive shear failure occurs where cyclic stresses in the subgrade are high enough to cause it to be sheared and remoulded and overstressed soil is squeezed sideways from beneath the track and upwards to cause a form of bearing capacity failure. It is less of a problem with coarse grained materials which possess high values of internal friction such that the increase in shear strength associated with applied normal stress exceeds the increase in associated shear stress.

Massive shear failure can occur due to the weight from the train, track superstructure and unbalanced portions of the substructure. However, as progressive failure usually occurs at stress levels below that causing massive failure it governs performance and therefore design. Massive shear failure is only likely to be problematic when, for example heavy rainfall or flooding, cause the subgrade to have an unusually high water content.

An excessive rate of settlement through plastic deformations may cause a ballast pocket to form as a result of the vertical component of progressive shear deformation, deformations caused by progressive compaction, or consolidation of the subgrade layer (Li and Selig, 1996; Selig and Waters, 1994).

Consequently, the design problem as far as the subgrade is concerned, provided a sufficiently thick sand blanket is included in the construction, can be regarded as identifying, and putting limiting values on, the stresses, strains and deflections that are the major causes of an of excessive rate of settlement and progressive shear failure.

Li and Selig (1998a) suggest that subgrade performance is influenced by (i) vertical strain at top of subgrade and (ii) vertical plastic deformation. Both of these factors are controlled by levels of shear stress (or deviator stress). In the case of (i) the shear stress of interest is that at the top of the subgrade, whilst for deformation the shear stresses throughout the subgrade are important. Accordingly, the design objective is to ensure that the cumulative effects of the repetitions of shear stresses throughout the subgrade do not cause excessive progressive shear failure or excessive rates of settlement to occur before the end of the design period.

The design criteria for the subgrade can be expressed as follows (Li and Selig, 1998a):

$$\varepsilon_p \leq \varepsilon_{pa} \text{ (to prevent progressive shear failure)} \tag{35}$$

And

$$\rho \leq \rho_a \text{ (to prevent excessive plastic deformation)} \tag{36}$$

Noting that

$$\rho = \int_o^D \varepsilon_p ds \tag{37}$$

Where ε_p and ε_{pa} are the actual total cumulative and allowable plastic strains, ρ and ρ_a are the actual and allowable plastic deformations, respectively, at the subgrade surface over the design period. D is the depth of the subgrade.

Material Performance under Repeated Loading

In an analytical design process measures of material performance are used to determine appropriate limits to stresses, strains or deflections. These, in conjunction with stresses, strains and deflections predicted to occur over the lifetime of the railway track are used to formulate the design. Ideally, such measures should be determined under conditions which closely match the in-situ regime. If tests are conducted in the laboratory then these should enable the effect of the rotation of principle stresses to be taken into account. Whilst this effect has been shown to be insignificant in determining resilient properties (see above), permanent deformations may be greatly underestimated when principal stress rotation is ignored. Gräbe and Clayton (2003), for example, found that the axial plastic strain may be underestimated by as much as between 1.6 and 3.2 times depending on the clay content of the material.

Granular Materials

For granular materials, permanent deformation is a function of both the cyclic deviator stress and the confining pressure and it has been shown to increase with the logarithm of the number of cycles of applied stress. Accordingly, performance models for granular materials take the following form:

$$\varepsilon_c = a\left(\frac{1}{N}\right)^b \tag{38}$$

Where ε_c is the permissible vertical compressive strain at the point of interest (usually a layer interface) for *N* load repetitions; *a* and *b* are coefficients determined from experimentation.

Similarly, for ballast the vertical compressive strain is also often expressed as a function of the permanent strain, ε_1, after the first loading cycle and number of cycles, N as follows:

$$\varepsilon_c = \varepsilon_1(1 + CLogN) \tag{39}$$

where C is a material constant and is typically between 0.2 and 0.4 (Selig and Waters, 1994).

Selig and Waters (1994) report laboratory and field trials undertaken by the Office for Research and Experiments (ORE), of the International Union of Railways to characterise ballast settlement. The ORE's results suggested that the settlement is a function of the initial porosity of the sample, n, the deviator stress and the number of load cycles as follows:

$$\varepsilon_c = 0.082(100n - 88.2)(\sigma 1 - \sigma 3)^2(1 + 0.2\log N) \tag{40}$$

Paute et al. (1993) suggest a model to predict permanent axial strain, ε_p in granular materials as a function of *N* as follows:

$$\varepsilon_p = \varepsilon_p(100) + \varepsilon_p^*(N) \tag{41}$$

And

$$\varepsilon_p^*(N) = A\left[1 - \left(\frac{N}{100}\right)^{-\beta}\right] \tag{42}$$

Where $\varepsilon_p(100)$ = accumulated permanent axial strain during the first 100 loading cycles
$\varepsilon^*{}_p$(N) = additional permanent axial strain for N > 100
A, B = regression parameters.

Further analysis of this model by Lekarp et al. (1996) suggested that the model is generally successful in predicting permanent strain.

A limiting deviator stress, known as the shakedown limit, has been found to exist below which the accumulation of plastic strain is stable and above which the accumulation increases rapidly. For example, Brown (1996) describes data from the literature which demonstrate that insignificant plastic strains develop if the peak repeated stress ratio is always less than 70% of that required to cause static failure. However, research continues to define the boundary between stable and unstable behaviour (Werkmeister, 2001).

Fine-Grained Materials

For fine-grained materials permanent deformation has been shown to be a function of the number of loading cycles, soil stress history and drainage conditions. In addition, as with granular materials, it is recognised that a critical level of repeated deviator stress, known as the threshold stress, exists above which the rate of accumulation of deformation increases rapidly. At deviator stress levels below the threshold stress deformation has been found to increase with the logarithm of the number of cycles whilst at deviator stress levels above the threshold the rate of accumulation of deformation increases exponentially. This behaviour has been found to be related to the material stress history and water content, and thus shear strength (Fleming et al., 2003). Brown (1996) suggested that the quotient of the applied shear stress by the soil's shear strength is the principal factor influencing permanent deformation. From experiments on an over-consolidated silty clay, Brown found that the threshold stress was at a deviator stress 1.3 times the value of static yield over the range of initial effective stresses studied.

A number of models have been developed to predict cumulative plastic strain under repeated loading. A commonly used one is a power model of the form (Li and Selig, 1996; Monismith et al., 1975):

$$\varepsilon_p = AN^b \tag{43}$$

Where ε_p is the percentage cumulative plastic strain, N is the number of repeated load applications; and A and b are two parameters related to the stress state and material properties.

In order to take into account soil physical state and type Li and Selig (1996) proposed a modified version of Equation 43 as follows:

$$\varepsilon_p = a\left(\frac{\sigma_d}{\sigma_s}\right)^m N^b \tag{44}$$

Where a, m and b are material parameters, σ_d is the deviator stress and σ_s is the soil static strength.

Track Stiffness Variability

Despite careful design and construction procedures a railway track superstructure is seldom built on a homogeneous substructure leading to significant changes of track stiffness within short distances. These may be due to the presence of pile decks, embankments, bridges, and transition zones between ballasted and slab track. Also, the track stiffness can change very quickly at switches and turnouts, especially at crossings (frogs), insulation joints and where there are hanging sleepers. An example of a 3 km transition zone between an embankment and a bridge is shown in Figure 6. In the upper part of the figure the stiffness variation along the track is shown, whilst the lower part shows the associated settlement over a two year period. The change of stiffness between the embankment and bridge is approximately four fold, from about 40 kN/mm (at km 11.40) to 160 kN/mm (at km 11.45). From Figure 6, also it can be seen that large variations in track stiffness correlate with large track settlements at 9.4 km (due to a bridge) and at 11.4 km and 11.7 km (due to an embankment and light-weight fill respectively).

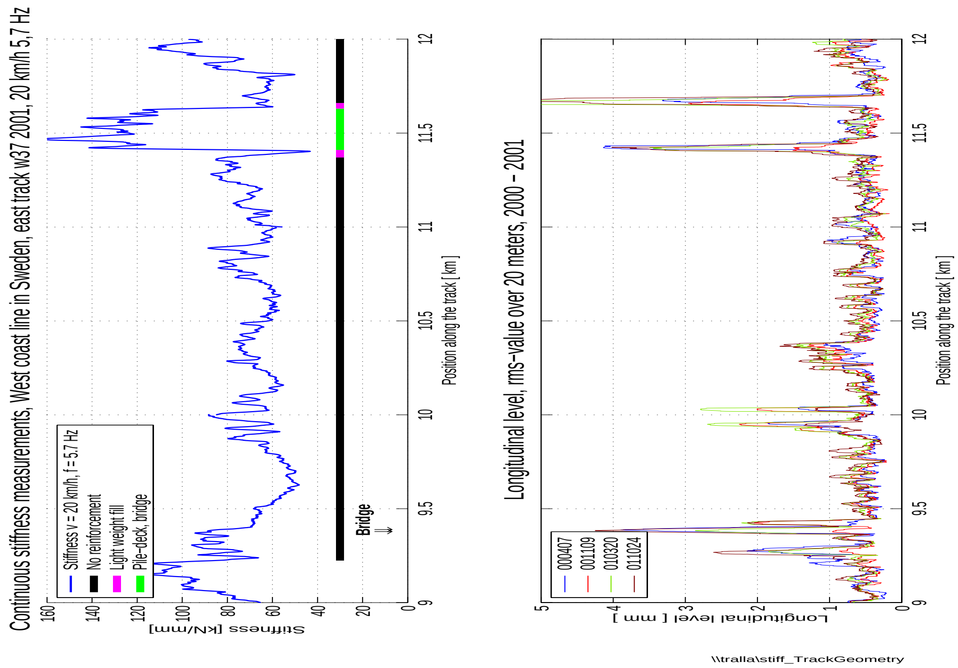

Figure 6. Upper figure: track stiffness (axle load divided by track deflection) along railway track as measured by the Banverket track stiffness measurement trolley. Lower figure: four measurements over a two year period of the longitudinal level of the track (positive downwards, meaning that a large peak in the curve indicates a local settlement of the track) (Berggren, 2007).

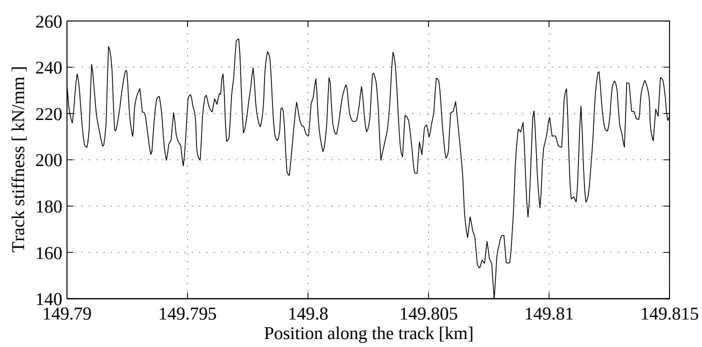

Figure 7. Local stiffness variation along railway track. Stiffness variation due to sleeper passages can be seen, as well as a sharp reduction in of track stiffness for three sleepers at 149.807 km. Measured by the Banverket track stiffness measurement car (Berggren, 2007).

Figure 7 shows the track stiffness variation along a 25m section of a track. It can be seen from the figure that the track stiffness varies with a (spatial) frequency corresponding to the sleeper distance (that here is approximately 0.65m). i.e. the track is stiffer when measured above a sleeper than between two sleepers. It is also interesting to note that the track stiffness is much lower in the vicinity of the three sleepers near km 149.807. This is most probably due to a lack of ballast support at the location (i.e. the sleepers are hanging). A possible cause is considered to be the presence of an insulated joint which has induced irregularities in the wheel/rail contact force causing increased loading on the sleepers and associated vibrations leading to deterioration of the ballast bed below the sleepers. Thus, the associated track deterioration is believed to have started at a point of discontinuous track stiffness at the rail joint.

Previous research

A great deal of research has been conducted on track stiffness variation, it causes and possible means of mitigation. The following section presents a short review of some pertinent research focusing on the following aspects:

- Random track stiffness variation
- The effect of rail joints
- Switches and turnouts
- Reducing sources of track stiffness variation

Random Track Stiffness

As discussed previously, track stiffness varies in a random manner spatially along the railway track. To investigate contributing factors, Mahmoud and Eltawil (1992) modelled the track as a beam resting on elastic supports and found that the response of the beam is highly dependent upon the modulus of subgrade reaction (*i.e*., on track stiffness). Naprstek and Fryba (1995) carried out similar analyses using a beam resting on a Winkler foundation; the stiffness of which was a random function of the length coordinate. Oscarsson (2001, 2002 and 2003) investigated the influence of stochastic properties of the track structure and to obtain sufficient statistical information from the track structures, full-scale in-field measurements and laboratory measurements were carried out. The rail pad stiffness, the ballast stiffness, the dynamic ballast-subgrade mass (a discretized equivalent mass), and the spacing between sleepers were all assumed to be random variables. The influence of scatter on the maximum contact force between the rail and the wheel, the maximum magnitude of the vertical wheelset acceleration, and the maximum sleeper displacement were studied and quantified in terms of probabilities and standard deviations.

Andersen and Nielsen (2003) used a simple track structure with randomly varying support stiffness. In their analysis the vertical support stiffness was assumed to be a stochastic homogeneous field consisting of small random variations around a deterministic mean value. Response spectra were obtained and compared with from numerical solutions obtained from finite element simulations. Wu and Thompson (2000) treated the sleeper spacing and ballast stiffness as random variables and explored their effects on the rail vibration. It was shown that the pinned-pinned resonance phenomenon may be suppressed by the random sleeper spacing, but a randomly varying foundation had no significant effect on the average noise generated by the track.

The effects of varying geometry and foundation stiffness are particularly significant when a train moves onto a bridge abutment. To minimise the rate of track settlement growth Hunt

(1997) suggested that in the vicinity of bridge abutments the track should have carefully prepared variations in foundation stiffness. Li and Davis (2005) state that remedies intended to strengthen the subgrade between the bridge should be designed to produce consistent and acceptable track stiffness between the bridge and the approach in order to be effective.

Nordborg (1998) found that in comparison with surface roughnesses the track support irregularities may be a significant excitation mechanism up to 100 Hz and vibration levels increase with train speed.

Rail Joints

The vertical bending stiffness of a rail joint is generally much lower than that of the rail and so a passing wheel generates larger deflections in the joint region than elsewhere, leading to increased wheel forces and accelerated track deterioration. In order to investigate the effects of joints on track stiffness variation, Koro *et al*. (2004) used a discretely supported Timoshenko beam and finite elements to predict the impulsive wheel-track contact force excited by the wheel passage on the rail joint. Different train speeds and gap sizes at the joint were simulated in the study. They found that rail joints contribute greatly to track deterioration, the settlement of ballast and the failure of other track components. The research was continued by Suzuki *et al*. (2005) who investigated measures to reduce ballast settlement, finding that in many cases soft rail pads could be effective. Rolling contact fatigue and plastic deformations at insulated rail joints were the subject of research carried out by Kabo *et al*. (2006).

Switches and Turnouts

A switch contains several irregularities both in stiffness and in inertia. For example, the bending stiffness of the switch rail differs from that of the stock rail, the lengths and distances between sleepers vary, the crossing (the frog) is stiffer in bending and has a larger mass than the surrounding rails. Andersson and Dahlberg (1998, 2000) investigated, by use of a numerical model, the load impact at the crossing nose when a wheel moves (at the frog) from the wing rail to the nose. It was found that the severity of the load impact depends on variations of track stiffness, mass distribution, and geometric irregularities at the crossing. Zarembski *et al.* (2001) and Zarembski and Palse (2003) performed theoretical formulation, analytical studies, and field tests and concluded that the impact load at a crossing could be virtually eliminated by an appropriate transition.

Zhu (2006) studied the effect of varying stiffness under the switch rail of a high-speed turnout. Results from his work showed that the elasticity under the switch rail could improve effectively the vertical wheel-rail interaction dynamics when the train passes from the stock rail to the switch rail. Kassa (2007) used mathematical models to simulate the dynamic train-turnout interaction and compared the results with field measurements.

Reducing Sources of Track Stiffness Variation

Elastomeric products, such as rail-pads, under-sleeper pads (USP), and sub-ballast mats (SBM), and geogrid (or geotextile) reinforcements, can be used to construct a suitable transition zone with the desired variation of stiffness and geometry. Track settlement in the transition zone has been studied numerically by Guiyu *et al.* (2004), and the influence of tensile-reinforcements on track settlement was investigated by Monley and Wu (1993). Full-scale simulation of geogrid reinforcement for railway ballast was performed by Brown *et al*. (2007) using a specially developed test rig. In Johansson *et al.* (2006) the influence of under-sleeper pads on dynamic train–track interaction was investigated. They used two numerical models, valid for different frequency intervals, to study wheel/rail contact forces, rail bending moments, rail vibrations (displacements, velocities, and accelerations), sleeper vibrations, and loads on sleepers. Frequency-dependent material properties of rail pads, USP, and ballast/substructure were modelled using viscoelastic spring-dampers that were calibrated with respect to measured data. It was found that USP influence dynamic train/track interaction mainly in the frequency range 0 – 250 Hz. Loy (2006) investigated the use of under-sleeper pads to reduce the variation in the static rail deflection in turnouts. He found that that by putting sleeper pads of specific stiffness in different sections of a turnout, the track stiffness could be adjusted so that vertical rail deflections could be smoothed effectively.

In *Anon.* (2006) various track transition designs were reviewed and analysed and a number of techniques were proposed to improve track performance by providing a transition to smooth the stiffness interface between dissimilar track types.

Modelling Dynamic Interaction between Train and Track

This section describes some of the recent work carried out at in the Mechanical Engineering department at the University of Linköping, Sweden to investigate variations in track stiffness using a model of the dynamic interaction of trains and track (Witt, 2008; Dahlberg, 2006; Lundqvist, 2005; Lundqvist and Dahlberg, 2005).

Three studies are described, one to investigate optimum values of track stiffness in a transition between a soft and stiff section of track; another examines the use of under sleeper pads to achieve optimal stiffness for a track whose stiffness is randomly varying. The third considers hanging sleepers. For the three cases the objective was to minimise the dynamic component of the contact force between the wheel and rail.

The model

Train and Track Model

The model consists of a finite element track model comprised of 3-D fully integrated solid elements as shown in Figure 8 with the material properties and other parameters as given in Table 4. The track model is composed of one rail (symmetry with respect to the centre line of the track is assumed), rail pads, sleepers, under sleeper pads, and the ballast/substructure bed. The rail was modelled as a standard UIC60 rail, and the rail pads as an elastic material of defined stiffness.

A moving wheelset is applied to the track model to simulate the load from one axle of a train. The wheelset is modelled as a rigid body moving at speed, v, with a constant load representing the weight of the car body. Inertia from the un-sprung mass (i.e. from the wheel and the axle) is taken into account by including the wheel mass and half of the axle mass. To avoid wave reflections at the boundaries of the limited model, non-reflecting boundary conditions have been used. These prevent artificial stress wave reflections generated at the boundaries from being reflected back into the model and adversely affecting the analysis. The non-reflecting boundary conditions absorb the shear and pressure waves so that no reflections will occur at the boundaries, but still allow bending waves in the rail to be reflected.

Table 4. Components and properties of materials and used in the model

Component	Property	Value
Vehicle		
Wheel and ½ axle	Weight	7.358 kN
Wheel	Speed	90 m/s
Car body	Weight	100 kN
Superstructure		
Rail	Type	UIC 60
Rail pads	Stiffness	300 kN/mm
Sleeper	Mass	125 kg (1/2 sleeper)
	Spacing	0.6 m
Substructure (Ballast + subgrade)	Modulus of elasticity	100 MPa (stiff section)
		30 MPa (soft section)
	Poisson's ratio	0.1
	Density	2, 500 kg/m^3
	Depth	1 m

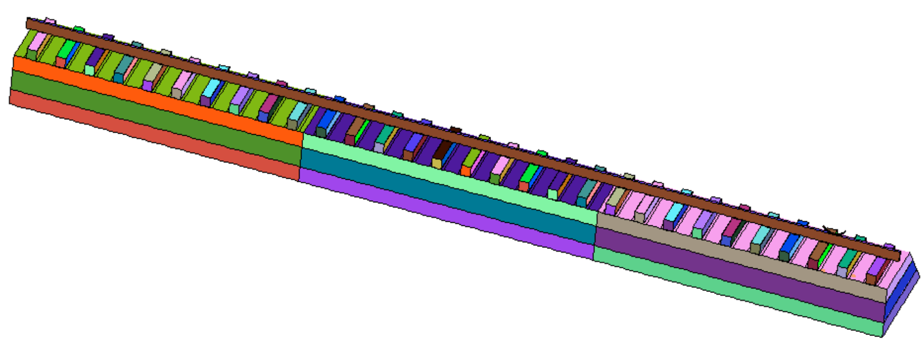

Figure 8. Train/track model consisting of rigid wheel, rail, rail pads, rigid sleepers, under sleeper pads below the ten central sleepers (not shown in the figure), and ballast/substructure. Symmetry with respect to the centre line of the track is assumed.

Train/Track Interaction

In the FE-program used in the study the contact force between two contacting bodies of the structure (for example between wheel and rail or between sleeper/USP and ballast) was calculated using a penalty method described by Belytschko *et al.* (2000). In the penalty algorithm, one of the contact surfaces is defined as the master surface and the other as a slave surface. If contact is obtained between a slave node and the master surface, the slave node attempts to penetrate the master surface. However, as the slave nodes are constrained to slide on the master surface after contact (they must remain on the master surface), the penalty algorithm will introduce normal interface springs between the penetrating nodes and the contact surface. The spring stiffness matrix (from the interface springs) is then assembled into the global stiffness matrix. The stiffness of the interface spring is the minimum of the master segment stiffness and the slave node stiffness. The magnitude of the interface force is thus proportional to the amount of penetration. If there is no contact (slave node does not penetrate), nothing is done. With this contact algorithm, it is possible to simulate loss of contact and recovered contact between wheel and rail and between sleeper/USP and ballast bed.

Optimisation

The finite element model was built-up using the pre-processor TrueGrid (Truegrid, 2001) and the train/track interaction problem was solved using the commercially available finite element software LS-DYNA (Hallquist, 2006). The advantage of the software is that it automatically makes the time step small so that high-frequency variations are well represented. Optimisation was achieved using the associated software optimisation package LS-OP (Stander, 1999) and is described in Lundqvist (2005).

Optimal Ballast / Subgrade Stiffness

In order to investigate an optimal stiffness of the ballast / subgrade in the transition zone the model spanned a length of 45 sleepers. This size of model was selected so that boundary effects would not disturb the track responses investigated in the central portion of 25 sleepers. In the model the substructure (ballast, sub-ballast and subgrade) was modelled as a continuum with elastic material properties. Longitudinally, the ballast bed was divided into several different sections. Two sections at the ends of the model, spanning 15 sleepers were used to represent stiff and soft sections of the track respectively (see Table 4). The central part of the model consisted of five shorter sections, each of three sleeper spans. The stiffnesses of the short sections were varied to determine an optimum combination for the two cases when the load is travelling from stiff to soft track and from soft to stiff track, respectively.

The optimal stiffness of the transition zone determined from the studied, for both travelling directions, can be seen in Figure 9. When moving from stiff to soft track, the ideal stiffness change in the transition zone was found to be smooth in the beginning and at the end of the zone, with a more rapid stiffness change in the central part of the zone, see Figure 9(a). The optimal modulii shown in Figure 9 (from left to right) are 100, 93, 82, 71, 45, 38, and 30 MPa respectively.

For the study of the wheel load moving from soft to stiff track, the ideal change in stiffness of the transition zone was found to be almost linear as shown in Figure 9(b), with optimal values of stiffness of the five sections of 30, 40, 50, 60, 70, 80, and 100 MPa respectively.

The wheel/rail contact force (travelling from stiff to soft track) is shown in Figure 10. A rapid decrease in the contact force is noted when the wheel enters the soft region. This implies a downwards motion of the wheel, and when this downward motion comes to an end, there is a large increase of the contact force. It can be seen that the large amplitude in the contact force that is obtained when there is no transition zone has almost disappeared after the optimisation. Only small variations of the contact force occur at every small change of stiffness in the transition zone. Going from soft to stiff track is worse than going from stiff to soft and the wheel/rail contact force variation is then larger than the variation shown in Figure 10 (see Lundqvist, 2005).

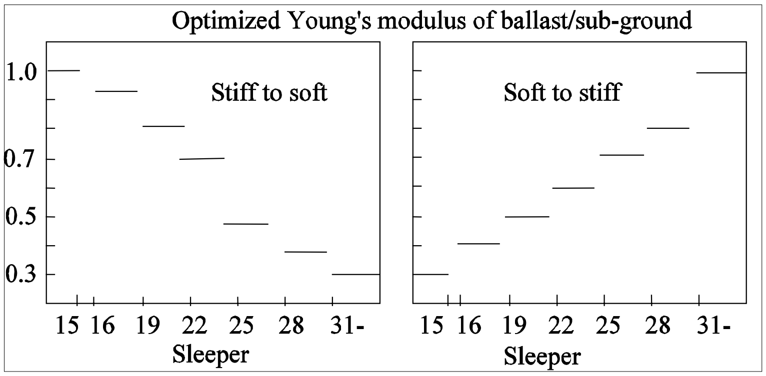

Figure 9. Optimal stiffness of the transition zone for the two cases (a) going from stiff to soft track (left figure, Young's modulus *E* going from 100 to 30 MPa), and (b) going from soft to stiff track (right figure).

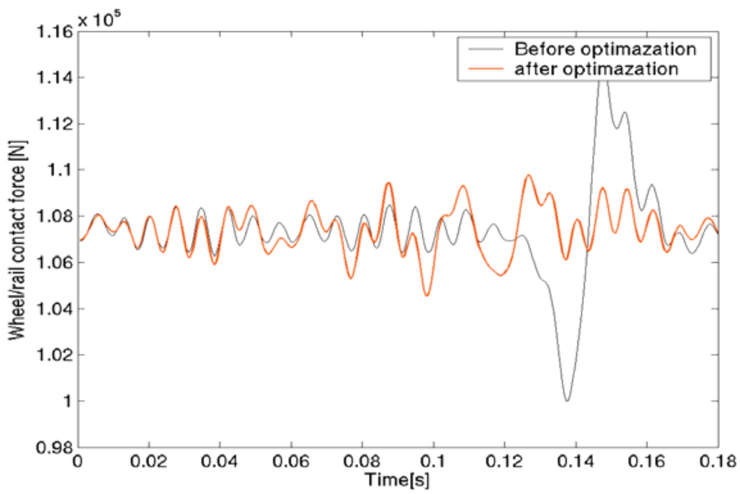

Figure 10. The wheel/rail contact force before and after track stiffness optimisation. Train (wheelset) travelling from stiff to soft track.

The study also demonstrated that if the transition zone is optimised for one direction of travel, then the results are almost as good for trains running in the opposite direction. Consequently, this suggests that if the transition zone is optimised for a particular direction of travel, then the transition when travelling in the opposite direction is almost as smooth as if the transition zone had been optimised for that direction.

Under-sleeper pad stiffness

For the study of the effect of under-sleeper pad stiffnesses on overall track stiffness variation (as represented by the variation in wheel / rail contact force) a numerical model consisting of 30 sleepers was used. The model contained three sections each of 10 sleepers. The sleepers in the central section had 20 mm thick under-sleeper pads. The two other sections at either end of the model, consisted of a soft and stiff section respectively (see Table 4). Under-sleeper pads were placed under each of the 10 sleepers in the stiff section and their stiffness was varied in order to achieve as smooth a transition as possible between the soft and stiff part of the track.

In order to keep the number of optimisation variables low, the same stiffness of the USP was given to two adjacent sleepers, so that the number of optimisation variables was five. The shear modulus G of the USP material was selected as the parameter to be optimized, and its lower limit was set to $G = 10$ MPa.

From the study, the optimal values of the shear modulus of the USP material are shown in Figure 11 where it may be seen that the first two USP (on the stiff part of the track) should have a relatively low stiffness. Perhaps surprisingly, the modelling shows that the stiffness of the USPs under sleepers three and four should be considerably greater than for the first two sleepers, whilst those under sleeper five to eight should be similar in value but less than the previous two USPs. The last two sleepers should have the stiffest pads of all.

The wheel/rail contact force is shown in Figure 12 both with and without the presence of USPs. A large irregularity can be seen at time t = 0.12 s, where the track stiffness changes from soft to stiff. Having optimal values of the USP stiffness this irregularity is almost completely eliminated, as shown by the second curve in Figure 12.

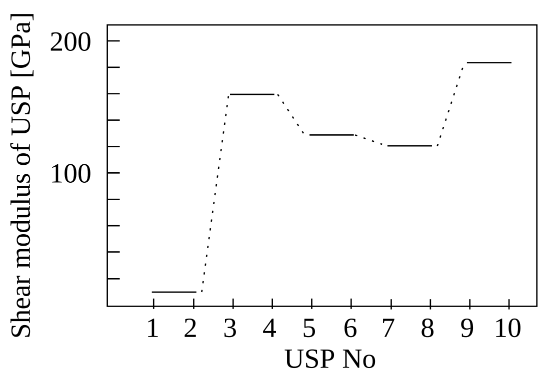

Figure 11. Optimised values of shear modulus of USP material.

In order to investigate the robustness of the optimal solution shown in Figure 11, the model was run with two other distributions of USP stiffness. The two other stiffness distributions investigated (without optimisation) had the following stiffnesses respectively; 10, 100, 125, 150, and 175 GPa, and; 10, 150, 150, 150, 150 GPa. It was found that these two stiffness distributions gave almost the same result as the optimised distribution in Figure 11. This suggests that as long as the two first under-sleeper pads are very soft (by one order of magnitude), the stiffness of the following eight pads does not influence the result very much and therefore it is practical to use USPs to reduce effectively the stiffness variation, for example, at the transition from a “soft” embankment to at “stiff” concrete construction such as a bridge.

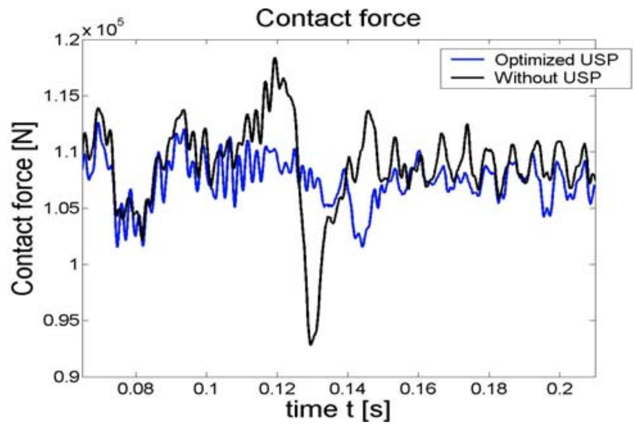

Figure 12. Wheel/rail contact force for track without USP and with five optimised stiffnesses of USP. Transition from soft to stiff track occurs approximately at time t = 0.12 s.

Hanging Sleepers

Hanging, or voided, sleepers occur because of differential track settlement, often due to variations in ballast stiffness. This results in some sleepers not being fully supported to the extent that contact with the ballast bed may be lost. Where hanging sleepers occur the vertical track stiffness becomes very low and this in turn causes high train / track interaction forces which may result in an increased rate of track settlement. One of the tasks of the sleepers is to distribute train induced loads to the ballast. However, in sections of the track with hanging sleepers, the sleepers adjacent to those which are hanging carry an increased load. This in turn may overstress the ballast and cause differential settlement to occur. Thus accelerating track settlement further.

To investigate this phenomena one fully supported sleeper between two hanging ones was modelled and results were obtained when no USP, a USP of medium stiffness and a stiff USP, respectively, were placed under the hanging sleeper (Witt, 2008). The stiffness of the medium and high stiffness USPs were 3000kN/mm and 400 kN/mm respectively. Hanging sleepers were modelled by reducing the Young’s modulus of the ballast from 100 MPa to 0.1 MPa under the hanging sleepers. The results from the analysis are shown as a wheel / rail contact force diagram in Figure 13. The first hanging sleeper is reached after 0.132 s, the supported sleeper after 0.139s and the second hanging sleeper after 0.146 s.

In Figure 13 it may be seen that the contact force decreases in front of the first hanging sleeper for all 3 cases, to 87, 85 and 89 kN without, with stiff and with medium stiffness USPs respectively. Generally the contact force with stiff USPs is lower throughout and reduces faster than with no USPs or USPs with medium stiffness. With USPs of medium stiffness, the contact force oscillates with higher amplitudes with a longer time period after the wheel has passed the sleepers. This demonstrates that USPs can be used to effectively reduce the effects of hanging sleepers, in particular the wheel rail contact forces, however the choice of stiffness of the USP is very important.

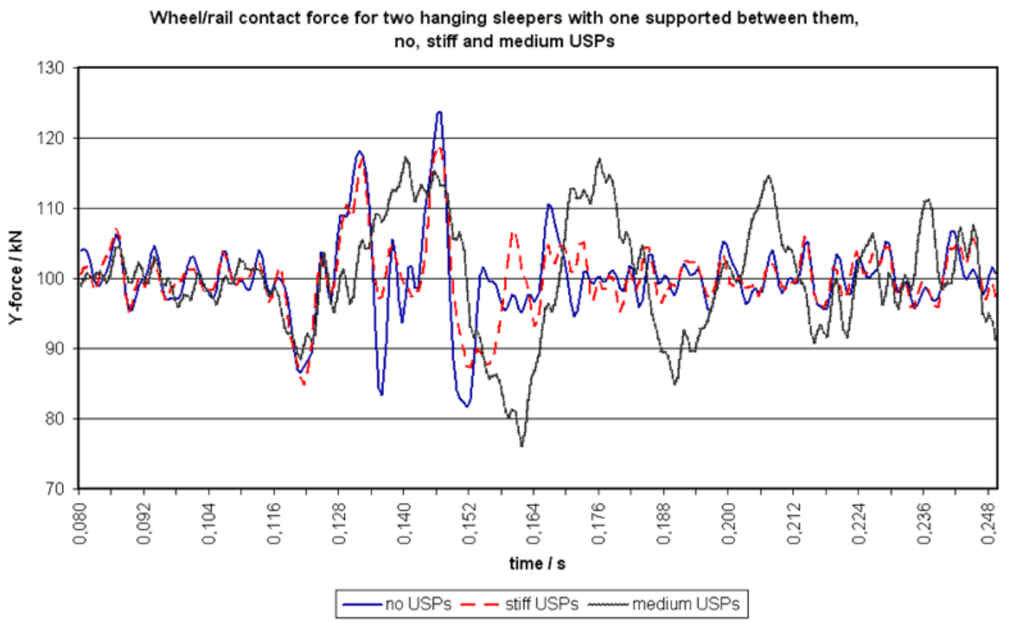

Figure 13. Vertical wheel/rail contact force with one supported sleeper between two hanging ones.

Track Stiffness Measurement

The measurement and assessment of the vertical track stiffness is an important tool for determining the structural causes of poorly performing sites (24, 76, 20, 22). Berggren 0, for example, shows how soft soil properties can be estimated from dynamic stiffness measurements. Many structural problems first manifest themselves as irregularities in track geometry. However, their cause is often related to the structural performance of the substructure and so they cannot be diagnosed correctly by track geometry measurements alone and require the track stiffness to be measured in addition 0, 0, 0, 0, 0.

Sussmann et al. 0 and Brough et al. relate different kind of track problems with track stiffness and recommend possible solutions for each problem (see Table 5). They suggest that the cause of unacceptably low values of track stiffness are mostly likely to be an indication of a weak subgrade as the properties of the subgrade most influence the value of track stiffness. In some cases the weakness may be attributed to poor drainage. However by the time the problem has fully manifested itself, the redesign and reconstruction of the track substructure may be required to reduce the traffic induced stresses in the subgrade to acceptable levels. Alternatively, methods of soil reinforcement to increase the strength and stiffness of the subgrade can be used.

In some cases low track stiffness values can also result from fouled or dirty ballast that prevents adequate support for track loading. This problem occurs especially when the ballast and subgrade deteriorate in the presence of water due to train induced repeated loading. The resulting migration of fines into the ballast and subsequent formation of wet spots, can lead to a reduction in stiffness of the track support system and hence loss of track geometry. Whilst replacing, or cleaning, the ballast may improve the problem in the short term, longer term solutions may require; the improvement of the drainage; the use of appropriate materials at the subgrade/ballast interface to reduce pumping and the migration of fines, and; in particularly problematic cases redesign and reconstruction.

For the problem of a variable track stiffness Sussmann et al. () suggest that potential solutions include the design of rail seat pad stiffness, the appropriate design of the substructure and the use of under ballast mats as discussed above.

Sometimes very large deflections may occur under load indicating the presence of hanging sleepers or loose rail seat fasteners. These types of problems can usually be fixed by repairing fasteners, the appropriate tamping or stone blowing of the ballast, and undercutting when the void is due to fouled ballast that deforms easily under a load (Sussmann et al., 0)

Table 5. Relation between stiffness and track problem and recommended maintenance 0.

Parameter	Problem	Maintenance / Rehabilitation
Low track stiffness	Poor or weak subsoil or fouled ballast	Substructure design, Stabilize subgrade
Variable track stiffness	Variable track support (stiffness or modulus of subgrade)	Matching rail seat pads, substructure design, ballast mats
Voided sleepers	Fouled ballast, local settlement, poor fastener condition	Inspect fasteners, tamp, stoneblow, undercut

As mentioned previously an additional consideration, although one which occurs less often, are problems associated with high stiffness which can lead to faster deterioration of the track and its components due to higher dynamic loads. In such cases several methods may be used to reduce the stiffness including the installation of soft pads and resiliently mounted sleepers 0, 0. Soft pads, however, may have an undesired side effect of increased noise radiation.

Noise Radiation

The noise radiated by the track noise in general is a function, amongst other things of the receptance of the rail and particularly the stiffness the rail pad between the rail and the sleeper (Jones and Thompson 0). For noise, only frequencies in the audible band are of interest (i.e. 20 – 20000 Hz). The upper limit as seen from the track is 2000 – 5000 Hz 0, 0. There is, obviously noise at higher frequencies, for example wheel and braking noise, but the track (rail) is not the dominant. Whilst soft pads may reduce track stiffness effectively they can increase noise emanating from the rail as their uses causes the rail effectively to become uncoupled from the sleeper. This minimizes noise from the sleeper but enables the rail to vibrate more freely so that waves can travel over a greater distance and the noise from the rail is increased. Conversely, with stiff pads the contribution from the rail is reduced but that from the sleeper is increased (Jones and Thompson state 0).

Ground borne vibration problems are associated with subgrade soils of low stiffness and /or a clear resonance as these tend to propagate vibrations more effectively. Smekal et al. 0 give an example of how such potentially problematic sites may be identified. In their work they show how low frequency soil properties can be identified with the help of stiffness measurements at different frequencies.

Measuring Track Stiffness

There are a number of methods which may be used to measure the vertical track stiffness. Perhaps the most important distinction is between those that are static whilst the measurements are made and measure stiffness at discrete intervals, and those that measure stiffness continuously whilst moving along the track (i.e. rolling measurement). Currently, standstill measurements are more widely used, often for research purposes, whereas rolling measurement techniques, which have been developed comparatively recently, are designed to operate at normal traffic speeds and facilitate track inspection without the need to close the track.

As mentioned previously, track stiffness is a function of frequency and it is necessary to select an appropriate device to measure track stiffness depending on the frequency of interest. The static and low frequency dynamics of the track are mostly related to geotechnical and geodynamical issues. Measurements of track stiffness at these frequencies may be very useful for investigations related to the bearing capacity of the subgrade, ground borne vibrations and some soft-soil related problems. High frequencies relate to problems associated with noise and train-track impact forces.

Static Measurement

Amongst the number of static devices available for measuring track stiffness three are described here. These are by the use of simple instrumentation, the impact hammer, the falling weight deflectometer (FWD) and track loading vehicles.

A very simple method of measuring track stiffness can be achieved by instrumenting any number of sleepers, and or rails, with displacement transducers or accelerometers and measuring the response during the passage of a train. The associated stiffness can then be calculated for that track section if the axle load is known. For improved accuracy, where dynamic loads are taken into consideration, the load from the train can also be measured with the help of strain gauges on the rail web, or on the sleeper. The typical results from such a measurement are load – deflection diagrams, where the stiffness can be identified with any of the above definitions of stiffness.

The impact hammer is a hand held device which is used to hit the rail or sleeper [62, 25]. The hammer head is equipped with a force transducer to measure the impulse, and an accelerometer is attached to the rail head or the sleeper. The transfer function between the impulse force of the hammer and acceleration of the rail is calculated (often also double integrated to get receptance instead of acceleration). A frequency interval of 50 – 1500 Hz can often be covered using the hammer depending on the material used to make the top of the hammer head. Rubber gives lower frequencies than a metal top for example. Since frequencies below 50 Hz are not recorded, impact hammer tests are most suited for problems associated with noise, vibration and wheel-rail contact forces, though not at lower frequencies.

The FWD is a device which is most often used to measure the stiffness of the track structure excluding the rails (Burrow et al., 2007) (See Figure 13). The standard FWD device consists of a mass that is dropped from a known height onto rubber buffers mounted on a footplate. The resulting impact is measured by a load cell on the centre of the plate and velocity transducers are used to determine surface velocity at various distances, *d,* from the footplate (see Figure 14). The velocities are integrated to give vertical displacements. For railway tracks in the UK, the device is designed to apply a 125 kN load to a sleeper, disconnected from the rails, via a 1.1 m long loading beam shaped to distribute the load to both ends of the sleeper. This loading system is considered to produce a load pulse which is similar to that applied by a single axle of a train travelling at high speed (Sharpe and Collop, 1998). The magnitude of the applied load is measured in the centre of the loading beam and the velocity transducers are positioned on the loaded sleeper and on the ballast at various distances from the centre of the beam (see Figure 14).

Figure 13. Falling weight deflectometer.

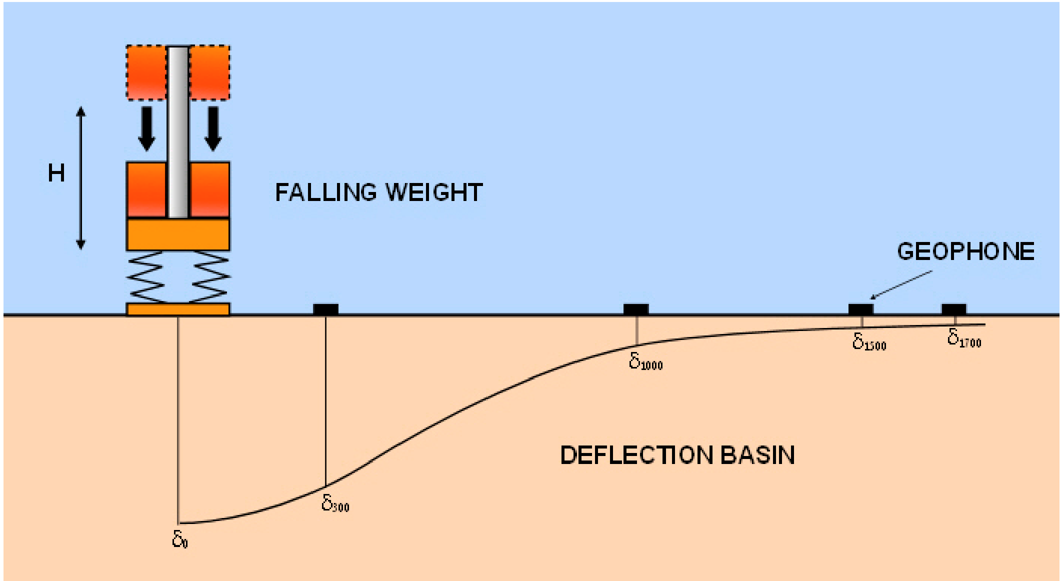

Figure 14. Schematic of the falling weight deflectometer.

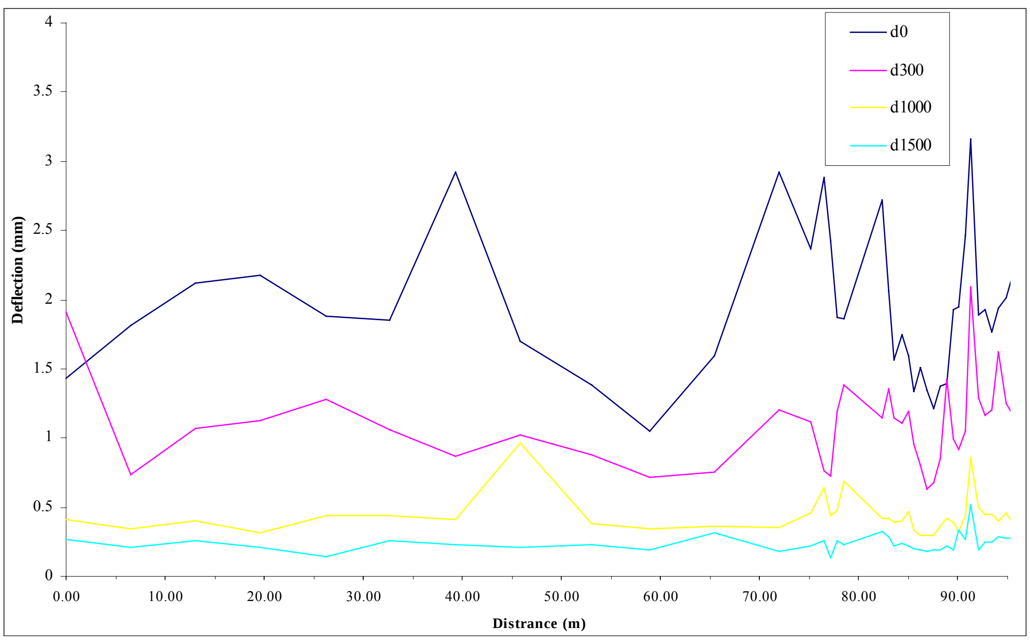

Figure 15. FWD deflections.

The track stiffness is then calculated from the measured load and deflections from some of the geophones, depending on the application. For example, if the stiffness of the ballast and sub-ballast layers is to be established the track stiffness is often determined from the load and deflections measured immediately under the load and from those measured by geophones 1000 mm from the load as follows:

$$k = \frac{62.5}{(d_0 - d_{1000})} \text{ kN/mm/sleeper end} \tag{45}$$

Typical outputs from the FWD are given in Figures 15and 16. Figure 15 shows the resulting deflections as a function of distance along the track for geophones positioned 0, 300, 1000 and 1500 mm from the impact load, whilst Figure 16 shows the track stiffness determine using Equation 45.

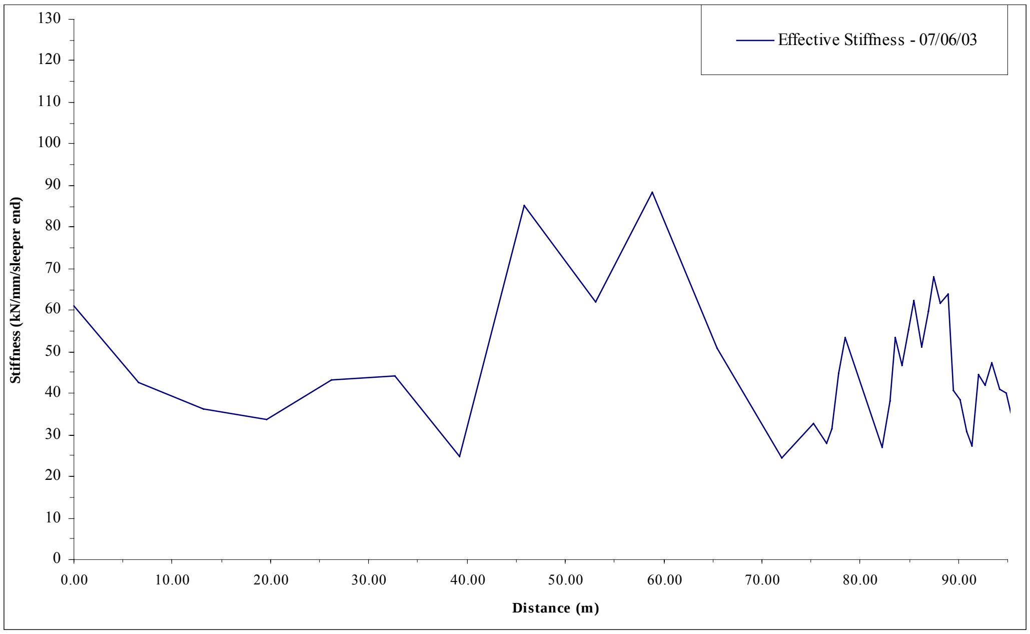

Figure 16. FWD track stiffness.

TLV

A Track Loading Vehicle (TLV) uses its own weight to load the track with the help of hydraulic jacks. Usually the rail heads are loaded, but the sleeper can be loaded also with the rails decoupled. Depending on equipment different loads can be applied.

A number of railway infrastructure companies own and operate TLVs. These include the Swedish TLV which has a weight of 49 tons and can load each rail statically up to 150 kN and excite dynamically up to 200 Hz (see Figure 17). It can also measure lateral track stability / stiffness 0. The Transportation Technology Center, Inc. (TTCI) in USA (which can also effect rolling measurements) 0, 0, the old DECAROTOR in USA 0, the South African BSSM 0 and a modified tamper used by Esveld 0.

The main advantage of a standstill TLV compared to rolling measurements is that the measurements can be much more thorough in terms of variation of preload, dynamic load and frequency range. However, the process is more time consuming and requires the railway track to be closed to other vehicles.

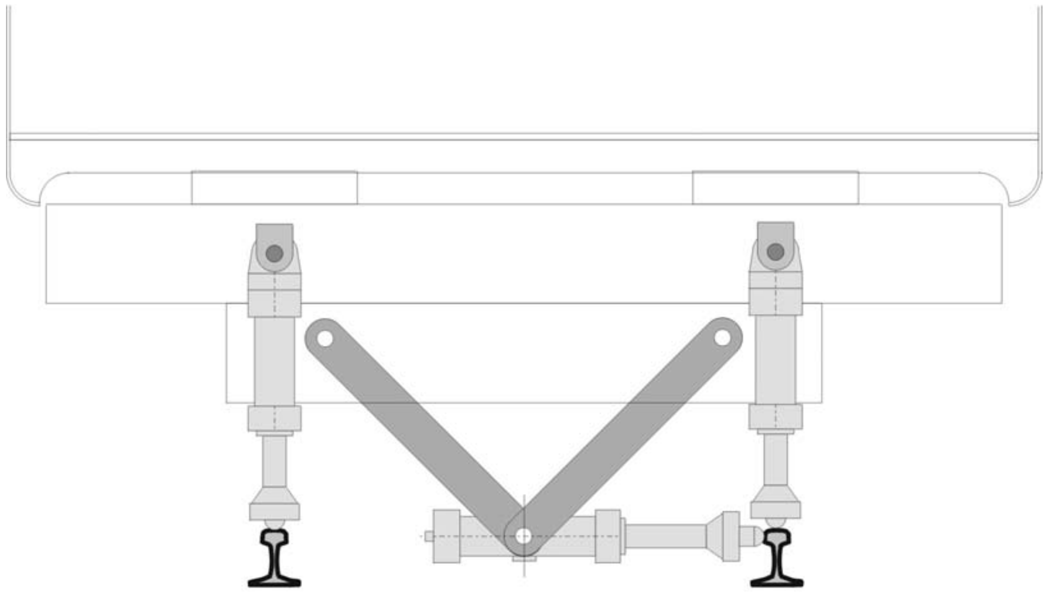

Figure 13. The vertical and lateral hydraulic actuators of the Swedish TLV (Berggrenn, date).

Rolling Measurement

If standstill measurements have been used mainly for research purposes, rolling measurements have the potential to be used on a more regular basis for maintenance purposes. There are several different measurement principles for measuring vertical track stiffness along the track. See for example, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0. Most methods measure the displacement under one or two axles caused by the weight on the axles and the track flexibility. With knowledge of the static axle loads, the track stiffness can be calculated. In case of a two-axle system, the axle loads are different and the lightest loaded axle is used to remove the effect of track irregularities on the stiffness measurement.

Measurement Principles

Due to the variety of methods of measuring track stiffness using a moving load the vertical track stiffness measured by each device, for the same section of track, are unlikely to match for the reasons described below:

- **Static preload:** The static preloads applied to the wheelset by the devices are different and are likely to result in different stiffness values being recorded for the same section of track.
- **Excitation frequency / speed:** Equipment that use a static running wheelset to load the track will (as experienced by the track) excite the track with a range of frequencies which are a function of the speed of the vehicle. As the measuring speed increases, so will the frequency content. Since the dynamic track stiffness is not constant with frequency, the stiffness determined is likely to differ (see Equation 3).

- **Spatial resolution:** The different measurement techniques may have different spatial resolution.
- **Model dependency:** The devices measure the deflection of the rail different distances away from the wheelset. Where the deflection is not measured directly under the wheelset a model for the rail bending has to be used in order to calculate the rail deflection under the wheelset. These models are approximations of reality and can introduce uncertainty and related errors.
- **Degree of influence from track irregularities:** Track geometry irregularities, especially those associated with the vertical alignment, can influence the stiffness measurements since the displacement transducers used in the equipment in most cases measure a combination of deflection due to track flexibility and displacement due to track geometry irregularities. Wheel out-of-roundness and wheel flats introduce similar disturbances.

Devices

A number of organisations have developed rolling devices to measure track stiffness. Some of these are summarised below (Berggrenn,).

People's Republic of China 0

The China Academy of Railway Sciences was one of the first organisations to develop a track stiffness vehicle for continuous track stiffness measurements 0. Their system, which travel at speeds of up to 60 km/h uses two track geometry chord measurement systems with different loading applied to each of the measurement axles (Figure 18). The light-weighted car is used to reduce the effect of track geometry irregularities on the stiffness measurement of the heavy-weighted car. The load of the heavy-weighted car can be varied between 80 and 250 kN enabling the nonlinear characteristics of the same section of track to be investigated by repeating measurements with different loads.

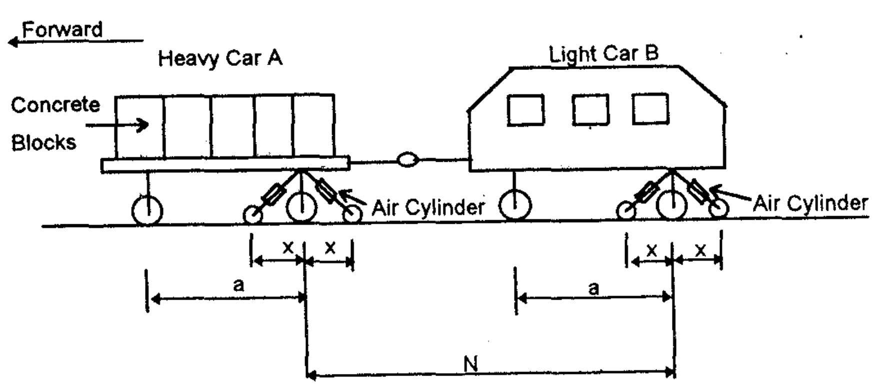

Figure 14. Principle of Chinese track stiffness measurements 0.

The axle load of the lighter car is 40 kN and has been found enough to eliminate the influences voided sleepers.

On railway tracks with a speed limit 160 km/h, a track stiffness of 65 – 100 kN/mm (one rail) obtained using the device has been found to be optimal.

TU Delft, Netherlands

TU Delft's High Speed Deflectograph (HSD) makes use of laser doppler sensors attached to a moving railway vehicle, travelling at speeds of up to 130 km/h, to measure the rail bending velocity 0, 0, 0, 0, 0. An inertial unit (3-axle gyro and accelerometer) is used as an input to a servo system to control the laser position so that the laser rays are perpendicular to the rail.

At the moment the system for railways exists as a concept although a fully working prototype for roads has been developed 0.

The HSD has a number of advantages over other rolling devices including:

a) the effect of track geometry irregularities on the measurement of track stiffness is much less than when displacement transducers are used, although the effect of hanging sleepers still contributes to the rail bending velocity;

b) the rail bending velocity increases with train speed and as a result higher trains speeds are likely to produce more accurate results.

TTCI, USA 0, 0, 0

TTCI's track loading vehicle (TLV) has been developed to measure both lateral and vertical stiffness at standstill and while moving at speeds of up to 16 km/h. For rolling vertical stiffness measurements two railway wagons are used, the TLV for static measurements coupled with an empty car.

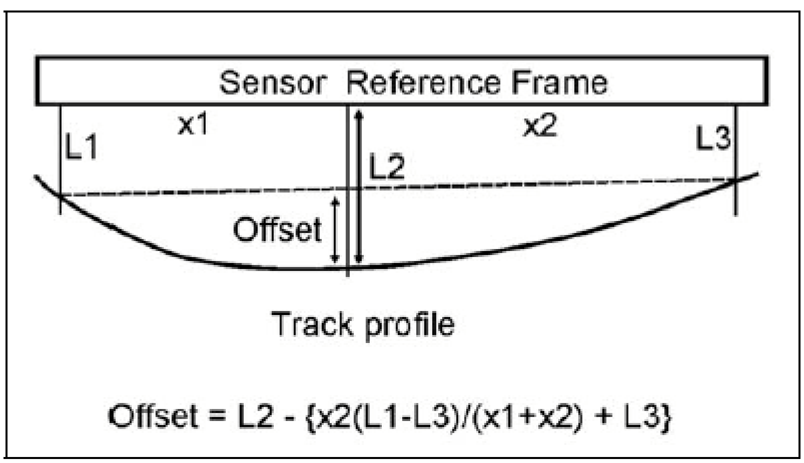

Figure 19: Rail bending deflection measurement with lasers, yielding chord values 0.

The TLV has a fifth wheelset (loaded bogie) mounted underneath the vehicle centre, that can be loaded hydraulically (both vertically and laterally) with vertical loads between 4 – 267 kN. A load of 178 kN is applied to the test axle of the static TLV. If two separate runs are used to differentiate the supports between the ballast and the subsoil, a light test axle load of 44 kN is used for the second run. The deflection is measured with the help of laser sensors, yielding a chord measurement of rail bending deflection (Figure).

Measurements are also made under the empty car, which is also equipped with a centre loaded bogie with pneumatic actuators capable of a nominal load of 9 kN.

Typical results from the TLV are shown in Figure . In the upper part of Figure 20 total deflection is shown for two cases: TLV load of 44 kN and 178 kN. In the middle figure the difference of these two deflections are shown. This should indicate subsoil stiffness, since the 178 kN case measure the whole stiffness and the 44 kN case measure (approximately) the response of rail, pad, sleeper and ballast. The lower figure is a transformation (with help of the theory of beam on elastic foundation) from contact deflection (middle figure) into modulus of the track.

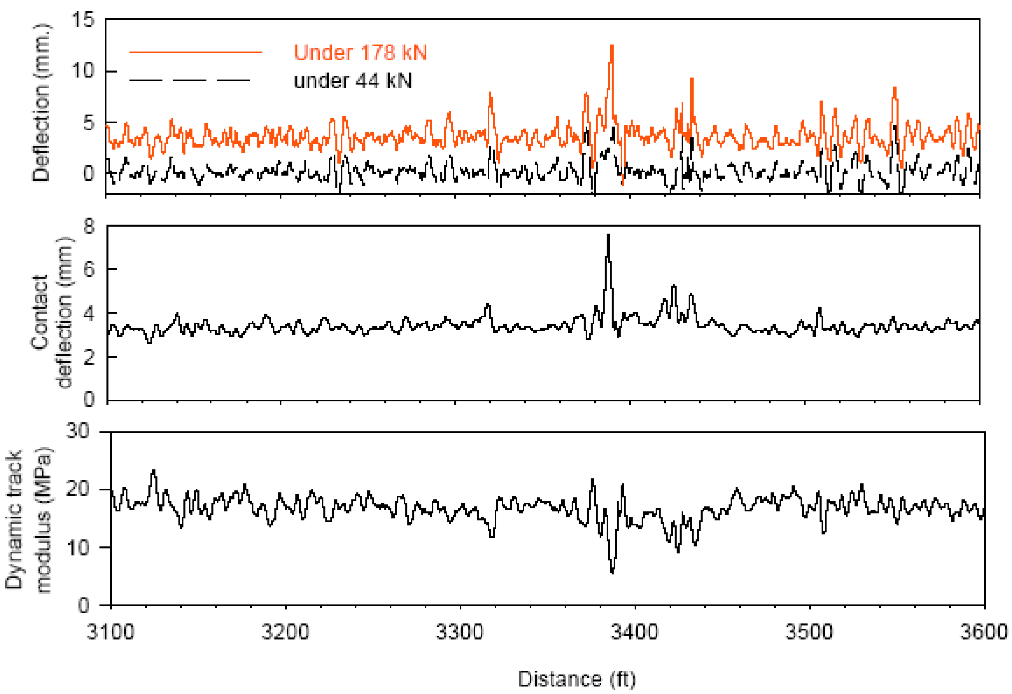

Figure 20. Example of results from TTCI stiffness measurements 0.

Banverket, Sweden

The Swedish vehicle, known as the Rolling Stiffness Measurement Vehicle (RSMV), is a modified two-axle freight wagon. The vehicle dynamically excites the track using two oscillating masses above one of the ordinary wheel axles as shown in Figure and 22. Track stiffness is calculated from the measured force and acceleration 0.

Figure 21. The measurement equipment in the RSMV (vertically moving masses above measuring axle, contained in steel cages) 0.

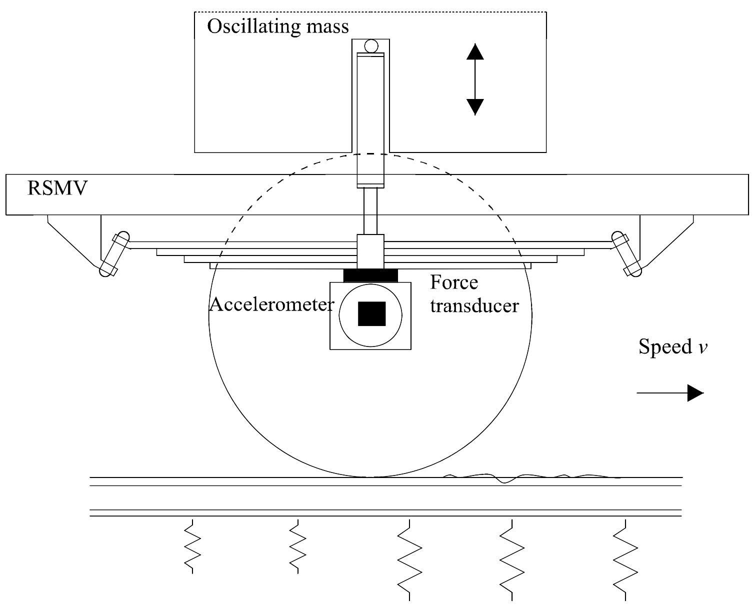

Figure 22. Measurement principle (one side only) of RSMV 0.

The RSMV can measure the dynamic stiffness at frequencies of up to 50 Hz with a static axle load of 180 kN (or more) and a maximum dynamic axle load amplitude of 60 kN. Measurements at higher speeds (up to 60 km/h) with up to 3 simultaneous sinusoidal excitation frequencies or more detailed investigations at lower speeds (below 10 km/h) with noise excitation can be performed.

The measurement principle used by the RSMV has shown very good repeatability. Figure 23 shows the results from a test carried out on 800 metres of track whose stiffness was measured on 6 separate occasions with the same speed and excitation frequency. Also the depth of the clay layer is plotted showing clear correlation with expected substructure behaviour.

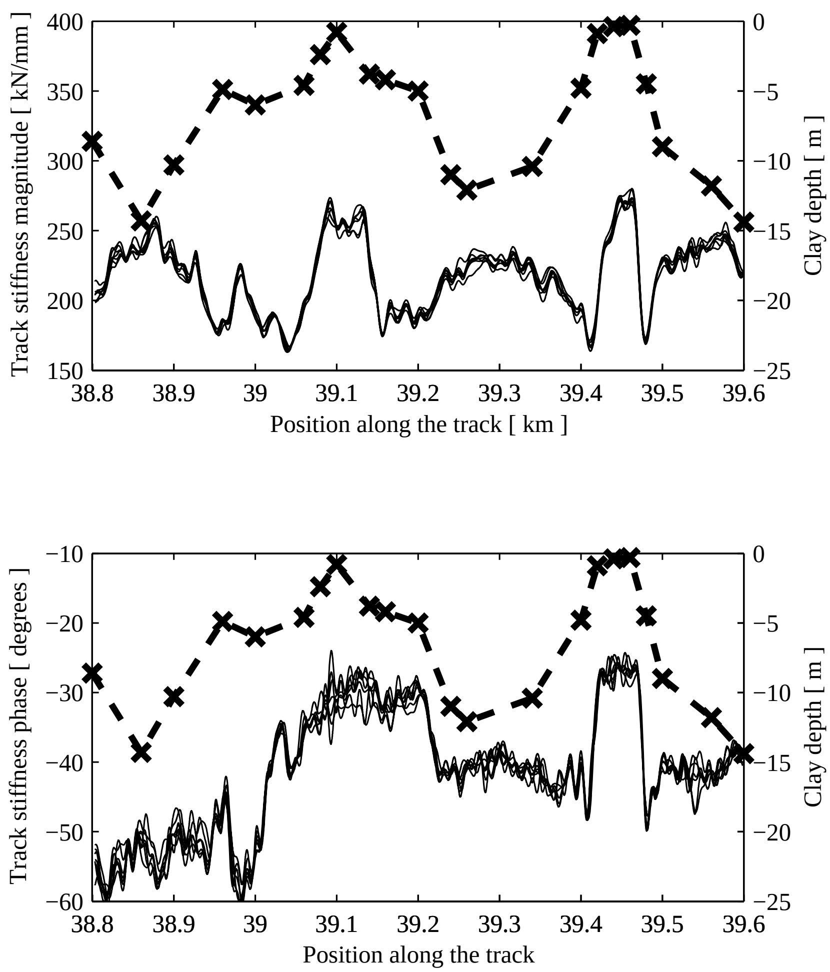

Figure 23. Repeatability test of stiffness magnitude (total) and phase – six measurements on the same track with a speed of 40 km/h and excitation at 11.4 Hz and dynamic load of 2x20 kN (solid lines). Standard deviation: 3.3 kN/mm, 1.3 degrees. Depth of clay layer is also indicated in the figure (x-marked dashed line) 0.

Czech Republic

The Czech stiffness measurement equipment, called the SKMT, has been developed by Czech Technical University of Prague and the Commercial Railway Research Ltd (KZV) 0, 0. It is essentially a chord-based deflection measurement system that measures the deflection from two different loads (two runs are required over the track). The chord measurements are taken on an adapted tamping machine fitted with trusses fitted on five special axles. The angles of rotation between adjacent trusses are measured with Linear Variable Displacement Transducers (LVDTs) and converted to track deflection. The vertical force is applied to both rails where the original lifting mechanism of the tamping machine was located. Two measuring runs are made on the same track, one without additional loading and the other with a static vertical load of 80 kN and the theory of abeam on an elastic foundation is used to obtain a stiffness value.

University of Nebraska, USA 0, 0, 0, 0, 0, 0, 0, 0

The University of Nebraska at Lincoln (UNL) in the USA has developed a chord-based deflection measurement system to effect track stiffness measurements as shown in Figure 24. The technique uses line-lasers to measure relative rail deflection between the bogie and the rail 0, 0, 0, 0, 0, 0, 0, 0.

The measurement principle is shown in Figures 25 and 26. The relative deflection is measured using two lasers and a camera that measures the distance, *d*, between the two lines and as the sensor moves with respect to the rail surface, the distance between the laser lines changes. The Winkler model is used to relate the measured deflections to track modulus/stiffness.

Figure 24. Measurement vehicles 0.

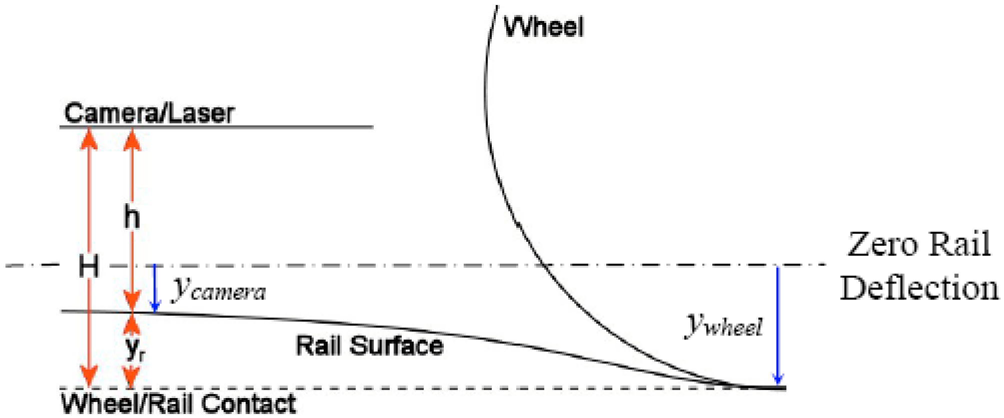

Figure 25. Rail deflection / Sensor measurement of UNL-stiffness equipment 0.

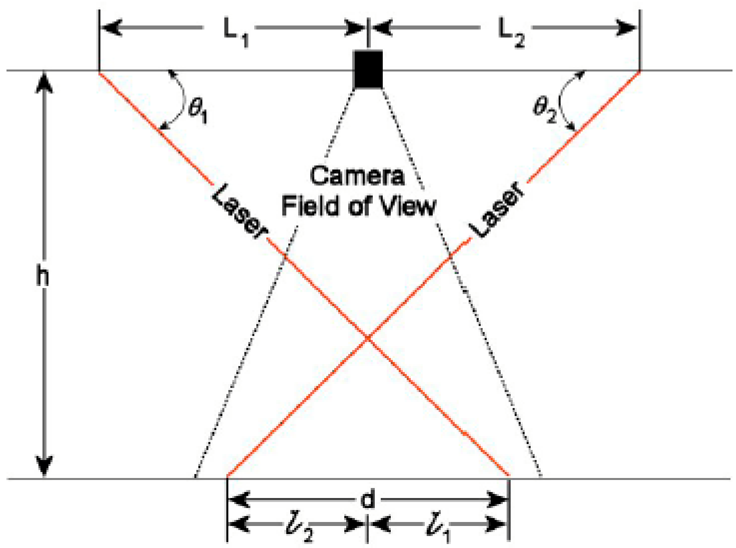

Figure 26. Sensor geometry of UNL stiffness equipment 0.

France, SNCF / CETE 0

The French Portancemeter is a stiffness monitoring tool designed by CETE-Normandie Centre for road structures and is currently being adapted for use on railways (32).

The device applies a dynamic load to the track via a vibrating wheel suspended by a spring and a damper. For road testing 10 kN of static weight, 0.5 mm of theoretical amplitude at 35 Hz is used. For rail testing these characteristics will be changed with increased loads (both static and dynamic) and the capability to alter the frequency.

Switzerland SBB

Swiss railways, Schweizerische Bundesbahnen (SBB), has developed a device (see Figure) which is similar to the Chinese and TTCI equipment and uses two geometry measurement systems (13, 73).

Figure 5. Einsenkungsmesswagen (EMW) 0.

Lateral Track Stiffness Measurement

Lateral track stiffness is associated with track stability (resistance against track buckling) and whilst it has not been discussed in this chapter in any detail, it is an important parameter. Both the Swedish TLV TTCI’s TLV are capable of measuring it.

Conclusion

The vertical track stiffness is an important measure of the structural performance of the railway track and is useful for helping to determine why railway track may be performing poorly. New track, especially that built for high speed lines, must be designed and constructed to appropriate standards so that the track stiffness is within an acceptable range of values. If the stiffness is lower than the acceptable range of values, excessive track displacements can occur and on the other hand unacceptable track deterioration may take place when the stiffness higher than ideal. This chapter described an analytical approach to railway substructure design which has the potential to cater for changes in traffic conditions and materials. A number of design procedures are readily available in the literature and it is hoped that this chapter will help the reader to select and modify, where appropriate, a suitable procedure to give a realistic design for a given set of conditions. Further the issue of variable track stiffness was addressed. It was shown that variations in track stiffness induce a variation of the wheel/rail contact force which will contribute to track structure deterioration and differential track settlement. It may also give rise to unsupported sleepers. The track degradation increases the rate of track deterioration leading to further track quality related problems. Modelling techniques were described and used to investigate techniques which can be used to alleviate track stiffness variations. It was demonstrated that a transition zone between track sections of different stiffness can be created to obtain a smooth transition between the two sections. The optimal stiffness variation in the transition zone depends on the travelling direction, but it is not very sensitive to it. Also, under sleeper pads with non-optimised stiffnesses can significantly reduce the wheel/rail contact force variation. The optimal transition zone can be built by using elastomeric products, such as under sleeper pads and/or sub ballast mats, to construct a tailor-made transition zone with desired stiffness variation and geometry. A thorough understanding of the physical mechanisms causing track deterioration, and understanding of the relationship between the track design parameters and the long-term track maintenance requirement would therefore imply that an optimised (or at least an improved) ballasted track could be constructed. The measurement of the vertical track stiffness and the correct understanding of how measurement procedures calculate stiffness are important so that the correct diagnosis of the causes of track geometry problems can be diagnosed. To this end chapter concluded with a comparison of state of the art techniques which may be used to measure track stiffness. Methods which measure the vertical track stiffness discretely and those which effect its measurement continuously were compared and contrasted.

Acknowledgements

Track modelling and calculations to study changes in track stiffness were carried out by Andreas Lundqvist (2005), Rikard Larsson (Lundquist *et al*. (2006) and Stephen Witt (2008) at Linköping University, Sweden.

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