Forward Kinematics

To calculate the configuration of the robot based on new wheel configuration ϕ_1:

$$\Delta\phi = \phi_1 - \phi_0 \quad \text{where } \phi = \begin{bmatrix} \phi_L \\ \phi_R \end{bmatrix}$$

Next, the twist generated would be:

$$V = \begin{bmatrix} \dot{\theta} \\ \dot{x} \\ \dot{y} \end{bmatrix} = \underbrace{\begin{pmatrix} \Delta\theta \\ \Delta x \\ \Delta y \end{pmatrix}}_{\text{Assume unit time}} = \frac{r}{2}\begin{bmatrix} (\Delta\phi_R - \Delta\phi_L)/d \\ (\Delta\phi_R + \Delta\phi_L) \\ 0 \end{bmatrix}$$

where r is the radius of the wheel and d is half the track-width.

To integrate this twist to a transformation matrix:

$$T_{bb'} = T_{bs} T_{ss'} T_{s'b'}$$

where $T_{bb'}$ is transformation to new configuration in the body frame. T_{bs} is the inverse of T_{sb}.

$$T_{sb} = \begin{bmatrix} 1 & 0 & \Delta y/\Delta\theta \\ 0 & 1 & -\Delta x/\Delta\theta \\ 0 & 0 & 1 \end{bmatrix}$$

$$T_{ss'} = \begin{bmatrix} \cos(\Delta\theta) & -\sin(\Delta\theta) & 0 \\ \sin(\Delta\theta) & \cos(\Delta\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$T_{s'b'} = T_{sb}$$

Lastly, transforming $T_{bb'}$ in the world frame, one can get $T_{wb'} = T_{wb} T_{bb'}$.

Inverse Kinematics

Assuming no slip, given a twist V, where $V = \begin{bmatrix} \Delta\theta \\ \Delta x \\ \Delta y \end{bmatrix}$

$$\dot{\phi} = \begin{bmatrix} \dot{\phi}_L \\ \dot{\phi}_R \end{bmatrix} = \frac{1}{r}\begin{bmatrix} -d\Delta\theta + \Delta x \\ d\Delta\theta + \Delta x \end{bmatrix}$$

where $\dot{\phi}$ is the new wheel velocity.

* $\Delta y = 0$ as no slip is assumed.