

SL Paper 2

Given $f(x) = x^2 - 3x^{-1}$, $x \in \mathbb{R}$, $-5 \leq x \leq 5$, $x \neq 0$,

A football is kicked from a point A ($a, 0$), $0 < a < 10$ on the ground towards a goal to the right of A.

The ball follows a path that can be modelled by **part** of the graph

$$y = -0.021x^2 + 1.245x - 6.01, x \in \mathbb{R}, y \geq 0.$$

x is the horizontal distance of the ball from the origin

y is the height above the ground

Both x and y are measured in metres.

i.a. Write down the equation of the vertical asymptote. [1]

i.b. Find $f'(x)$. [2]

i.c. Using your graphic display calculator or otherwise, write down the coordinates of any point where the graph of $y = f(x)$ has zero gradient. [2]

i.d. Write down all intervals in the given domain for which $f(x)$ is increasing. [3]

ii.a. Using your graphic display calculator or otherwise, find the value of a . [1]

ii.b. Find $\frac{dy}{dx}$. [2]

ii.c.(i) Use your answer to part (b) to calculate the horizontal distance the ball has travelled from A when its height is a maximum. [4]

(ii) Find the maximum vertical height reached by the football.

ii.d. Draw a graph showing the path of the football from the point where it is kicked to the point where it hits the ground again. Use 1 cm to represent 5 m on the horizontal axis and 1 cm to represent 2 m on the vertical scale. [4]

ii.e. The goal posts are 35 m from **the point where the ball is kicked**. [2]

At what height does the ball pass over the goal posts?

Markscheme

i.a. equation of asymptote is $x = 0$ **(A1)**

(Must be an equation.)

[1 mark]

i.b. $f'(x) = 2x + 3x^{-2}$ (or equivalent) **(A1)** for each term **(A1)(A1)**

[2 marks]

i.c. stationary point $(-1.14, 3.93)$ **(G1)(G1)(ft)**

$(-1,4)$ or similar error is awarded **(G0)(G1)(ft)**. Here and also as follow through in part (d) accept exact values $-\left(\frac{3}{2}\right)^{\frac{1}{3}}$ for the x coordinate and $3\left(\frac{3}{2}\right)^{\frac{2}{3}}$ for the y coordinate.

OR $2x + \frac{3}{x^2} = 0$ or equivalent

Correct coordinates as above **(M1)**

Follow through from candidate's $f'(x)$. **(A1)(ft)**

[2 marks]

i.d. In all alternative answers for (d), follow through from candidate's x coordinate in part (c).

Alternative answers include:

$-1.14 \leq x < 0$, $0 < x < 5$ **(A1)(A1)(ft)(A1)**

OR $[-1.14, 0), (0, 5)$

Accept alternative bracket notation for open interval $].$ (Union of these sets is not correct, award **(A2)** if all else is right in this case.)

OR $-1.14 \leq x < 5, x \neq 0$

In all versions 0 **must** be excluded **(A1)**. -1.14 must be the left bound. 5 must be the right bound **(A1)**. For $x \geq -1.14$ or $x > -1.14$ alone, award **(A1)**. For $-1.4 \leq x < 0$ together with $x > 0$ award **(A2)**.

[3 marks]

ii.a. $a = 5.30$ (3sf) (Allow $(5.30, 0)$ but 5.3 receives an **(AP)**). **(A1)**

[1 mark]

ii.b. $\frac{dy}{dx} = -0.042x + 1.245$ **(A1)** for each term. **(A1)(A1)**

[2 marks]

ii.c. Unit penalty (**UP**) is applicable where indicated in the left hand column.

(i) Maximum value when $f'(x) = 0, -0.042x + 1.245 = 0$, **(M1)**

(M1) is for either of the above but at least one must be seen.

$(x = 29.6.)$

Football has travelled $29.6 - 5.30 = 24.3$ m (3sf) horizontally. **(A1)(ft)**

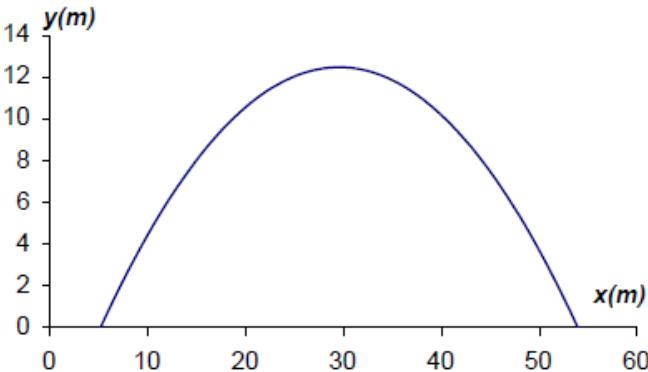
For answer of 24.3 m with no working or for correct subtraction of 5.3 from candidate's x -coordinate at the maximum (if not 29.6), award **(A1)(d)**.

(UP) (ii) Maximum vertical height, $f(29.6) = 12.4$ m **(M1)(A1)(ft)(G2)**

(M1) is for substitution into f of a value seen in part (c)(i). $f(24.3)$ with or without evaluation is awarded **(M1)(A0)**. For any other value without working, award **(G0)**. If lines are seen on the graph in part (d) award **(M1)** and then **(A1)** for candidate's value ± 0.5 (3sf not required.)

[4 marks]

ii.d.(not to scale)



(A1)(A1)(A1)(ft)(A1)(ft)

Award (A1) for labels (units not required) and scale, (A1)(ft) for max(29.6, 12.4), (A1)(ft) for x-intercepts at 5.30 and 53.9, (all coordinates can be within 0.5), (A1) for well-drawn parabola ending at the x-intercepts.

[4 marks]

ii.e.

Unit penalty (UP) is applicable where indicated in the left hand column.

(UP) $f(40.3) = 10.1$ m (3sf).

Follow through from (a). If graph used, award (M1) for lines drawn and (A1) for candidate's value ± 0.5 . (3sf not required). (M1)(A1)(ft)(G2)

[2 marks]

Examiners report

i.a.(i) An attempt at part (a) was seen only rarely. If there was an attempt, it was often not a meaningful equation. If an equation was seen, sometimes it was for y , not x .

i.b. The derivative seemed manageable for many, though with the expected mis-handling of the negative power quite often. Parts (c) and (d) proved problematical. Marking of (d) was lenient and it was reaffirmed that testing of the concept in (d) will be done in a more straightforward context in future, when done at all.

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Nevertheless, a lot of candidates could have earned more marks here if they had persevered. Many gave up on the graph, and elementary marks for scale and labels were lost unnecessarily.

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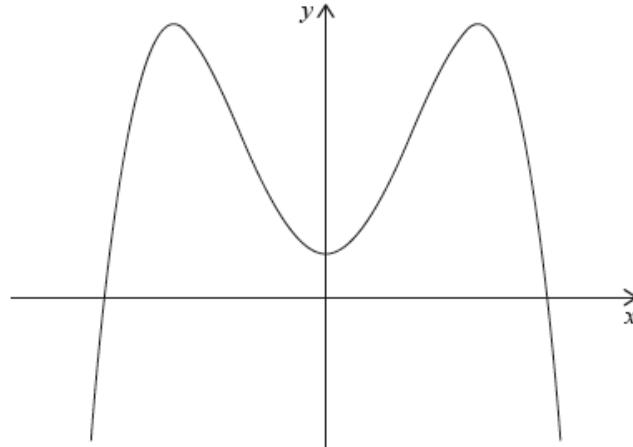
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As this was the first time for the unit penalty, we were lenient about the units left off the labels but this is likely to change in the future.

Consider the function $f(x) = -x^4 + ax^2 + 5$, where a is a constant. Part of the graph of $y = f(x)$ is shown below.



It is known that at the point where $x = 2$ the tangent to the graph of $y = f(x)$ is horizontal.

There are two other points on the graph of $y = f(x)$ at which the tangent is horizontal.

- a. Write down the y -intercept of the graph. [1]
- b. Find $f'(x)$. [2]
- c.i. Show that $a = 8$. [2]
- c.ii. Find $f(2)$. [2]
- d.i. Write down the x -coordinates of these two points; [2]
- d.ii. Write down the intervals where the gradient of the graph of $y = f(x)$ is positive. [2]
- e. Write down the range of $f(x)$. [2]
- f. Write down the number of possible solutions to the equation $f(x) = 5$. [1]
- g. The equation $f(x) = m$, where $m \in \mathbb{R}$, has four solutions. Find the possible values of m . [2]

Markscheme

a. 5 (**A1**)

Note: Accept an answer of $(0, 5)$.

[1 mark]

b. $(f'(x) =) -4x^3 + 2ax$ (**A1**)(**A1**)

Note: Award (**A1**) for $-4x^3$ and (**A1**) for $+2ax$. Award at most (**A1**)(**A0**) if extra terms are seen.

[2 marks]

c.i. $-4 \times 2^3 + 2a \times 2 = 0$ (**M1**)(**M1**)

Note: Award (**M1**) for substitution of $x = 2$ into their derivative, (**M1**) for equating their derivative, written in terms of a , to 0 leading to a correct answer (note, the 8 does not need to be seen).

$a = 8$ (**AG**)

[2 marks]

c.ii. $(f(2) =) -2^4 + 8 \times 2^2 + 5$ (**M1**)

Note: Award (**M1**) for correct substitution of $x = 2$ and $a = 8$ into the formula of the function.

21 (**A1**)(**G2**)

[2 marks]

d.i. $(x =) -2$, $(x =) 0$ **(A1)(A1)**

Note: Award **(A1)** for each correct solution. Award at most **(A0)(A1)(ft)** if answers are given as $(-2, 21)$ and $(0, 5)$ or $(-2, 0)$ and $(0, 0)$.

[2 marks]

d.ii. $x < -2$, $0 < x < 2$ **(A1)(ft)(A1)(ft)**

Note: Award **(A1)(ft)** for $x < -2$, follow through from part (d)(i) provided their value is negative.

Award **(A1)(ft)** for $0 < x < 2$, follow through only from their 0 from part (d)(i); 2 must be the upper limit.

Accept interval notation.

[2 marks]

e. $y \leqslant 21$ **(A1)(ft)(A1)**

Notes: Award **(A1)(ft)** for 21 seen in an interval or an inequality, **(A1)** for “ $y \leqslant$ ”.

Accept interval notation.

Accept $-\infty < y \leqslant 21$ or $f(x) \leqslant 21$.

Follow through from their answer to part (c)(ii). Award at most **(A1)(ft)(A0)** if x is seen instead of y . Do not award the second **(A1)** if a (finite) lower limit is seen.

[2 marks]

f. 3 (solutions) **(A1)**

[1 mark]

g. $5 < m < 21$ or equivalent **(A1)(ft)(A1)**

Note: Award **(A1)(ft)** for 5 and 21 seen in an interval or an inequality, **(A1)** for correct strict inequalities. Follow through from their answers to parts (a) and (c)(ii).

Accept interval notation.

[2 marks]

Examiners report

- a. [N/A]
- b. [N/A]
- c.i. [N/A]
- c.ii. [N/A]
- d.i. [N/A]
- d.ii. [N/A]
- e. [N/A]
- f. [N/A]
- g. [N/A]

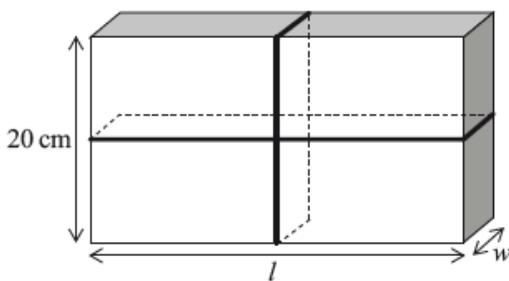
A parcel is in the shape of a rectangular prism, as shown in the diagram. It has a length l cm, width w cm and height of 20 cm.

The total volume of the parcel is 3000 cm³.

- a. Express the volume of the parcel in terms of l and w . [1]

b. Show that $l = \frac{150}{w}$. [2]

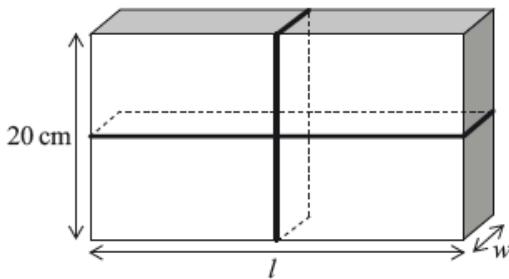
- c. The parcel is tied up using a length of string that fits **exactly** around the parcel, as shown in the following diagram. [2]



Show that the length of string, S cm, required to tie up the parcel can be written as

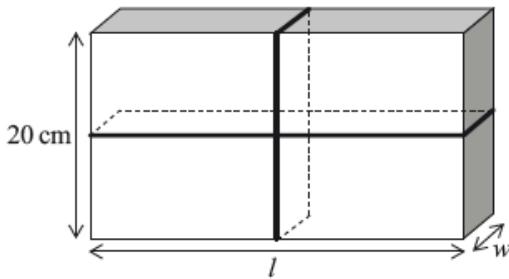
$$S = 40 + 4w + \frac{300}{w}, \quad 0 < w \leq 20.$$

- d. The parcel is tied up using a length of string that fits **exactly** around the parcel, as shown in the following diagram. [2]



Draw the graph of S for $0 < w \leq 20$ and $0 < S \leq 500$, clearly showing the local minimum point. Use a scale of 2 cm to represent 5 units on the horizontal axis w (cm), and a scale of 2 cm to represent 100 units on the vertical axis S (cm).

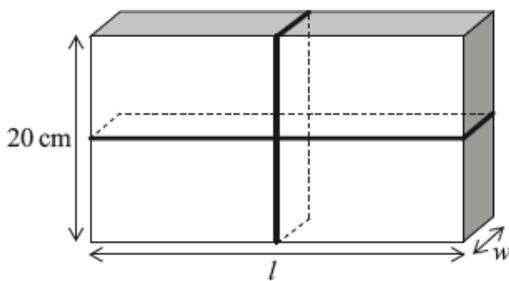
- e. The parcel is tied up using a length of string that fits **exactly** around the parcel, as shown in the following diagram. [3]



Find $\frac{dS}{dw}$.

- f. The parcel is tied up using a length of string that fits **exactly** around the parcel, as shown in the following diagram.

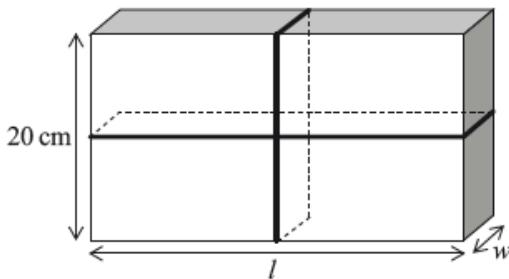
[2]



Find the value of w for which S is a minimum.

- g. The parcel is tied up using a length of string that fits **exactly** around the parcel, as shown in the following diagram.

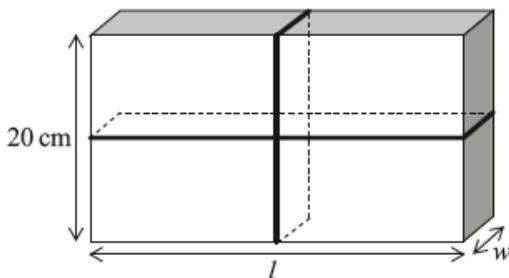
[1]



Write down the value, l , of the parcel for which the length of string is a minimum.

- h. The parcel is tied up using a length of string that fits **exactly** around the parcel, as shown in the following diagram.

[2]



Find the minimum length of string required to tie up the parcel.

Markscheme

a. $20lw$ OR $V = 20lw$ (A1)

[1 mark]

b. $3000 = 20lw$ (M1)

Note: Award (M1) for equating their answer to part (a) to 3000.

$$l = \frac{3000}{20w} \quad (\text{M1})$$

Note: Award (M1) for rearranging equation to make l subject of the formula. The above equation must be seen to award (M1).

OR

$$150 = lw \quad (\text{M1})$$

Note: Award **(M1)** for division by 20 on both sides. The above equation must be seen to award **(M1)**.

$$l = \frac{150}{w} \quad (\text{AG})$$

[2 marks]

c. $S = 2l + 4w + 2(20) \quad (\text{M1})$

Note: Award **(M1)** for setting up a correct expression for S .

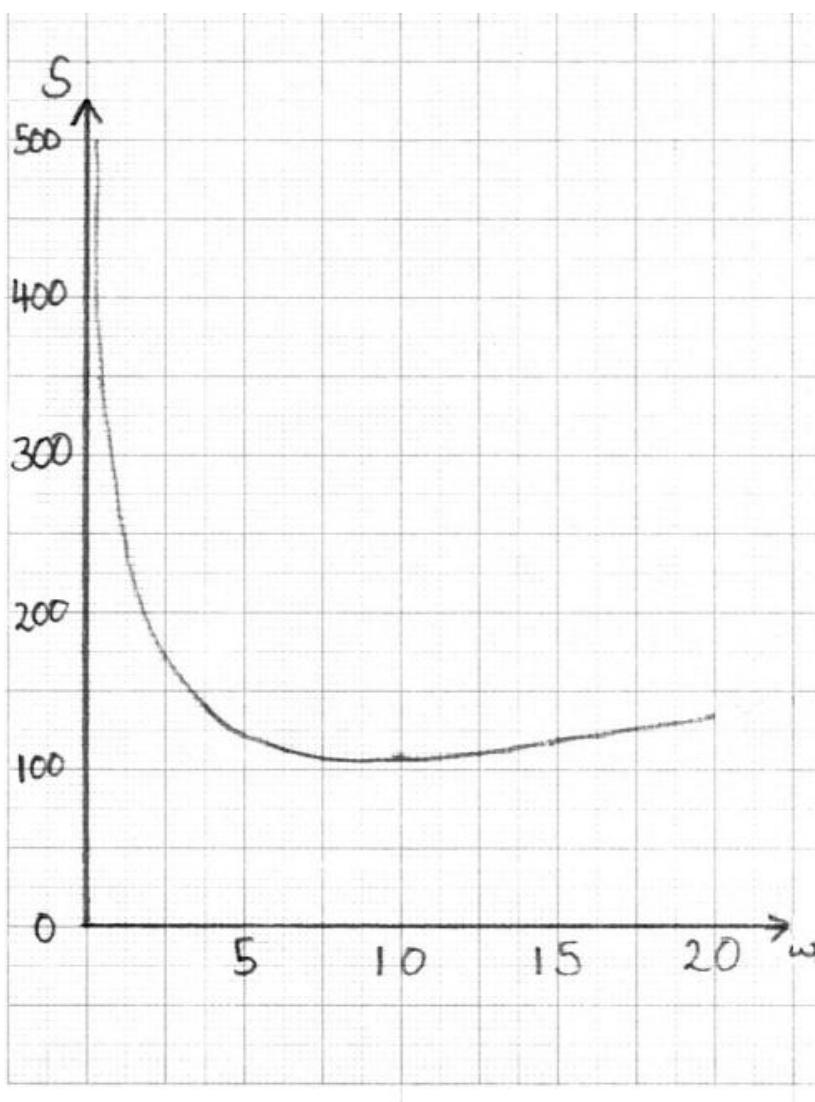
$$2\left(\frac{150}{w}\right) + 4w + 2(20) \quad (\text{M1})$$

Notes: Award **(M1)** for correct substitution into the expression for S . The above expression must be seen to award **(M1)**.

$$= 40 + 4w + \frac{300}{w} \quad (\text{AG})$$

[2 marks]

d.



(A1)(A1)(A1)(A1)

Note: Award (A1) for correct scales, window and labels on axes, (A1) for approximately correct shape, (A1) for minimum point in approximately correct position, (A1) for asymptotic behaviour at $w = 0$.

Axes must be drawn with a ruler and labeled w and S .

For a smooth curve (with approximately correct shape) there should be **one** continuous thin line, no part of which is straight and no (one-to-many) mappings of w .

The S -axis must be an asymptote. The curve must not touch the S -axis nor must the curve approach the asymptote then deviate away later.

[4 marks]

e. $4 - \frac{300}{w^2}$ (A1)(A1)(A1)

Notes: Award (A1) for 4, (A1) for -300 , (A1) for $\frac{1}{w^2}$ or w^{-2} . If extra terms present, award at most (A1)(A1)(A0).

[3 marks]

f. $4 - \frac{300}{w^2} = 0$ OR $\frac{300}{w^2} = 4$ OR $\frac{dS}{dw} = 0$ (M1)

Note: Award (M1) for equating their derivative to zero.

$$w = 8.66 \left(\sqrt{75}, 8.66025\dots \right) \quad (\mathbf{A1})(\mathbf{ft})(\mathbf{G2})$$

Note: Follow through from their answer to part (e).

[2 marks]

g. $17.3 \left(\frac{150}{\sqrt{75}}, 17.3205\dots \right) \quad (\mathbf{A1})(\mathbf{ft})$

Note: Follow through from their answer to part (f).

[1 mark]

h. $40 + 4\sqrt{75} + \frac{300}{\sqrt{75}} \quad (\mathbf{M1})$

Note: Award **(M1)** for substitution of their answer to part (f) into the expression for S .

$$= 110 \text{ (cm)} \left(40 + 40\sqrt{3}, 109.282\dots \right) \quad (\mathbf{A1})(\mathbf{ft})(\mathbf{G2})$$

Note: Do not accept 109.

Follow through from their answers to parts (f) and (g).

[2 marks]

Examiners report

- a. [N/A]
- b. [N/A]
- c. [N/A]
- d. [N/A]
- e. [N/A]
- f. [N/A]
- g. [N/A]
- h. [N/A]

Consider the function $g(x) = x^3 + kx^2 - 15x + 5$.

The tangent to the graph of $y = g(x)$ at $x = 2$ is parallel to the line $y = 21x + 7$.

- a. Find $g'(x)$.

[3]

- b.i. Show that $k = 6$.

[2]

b.ii. Find the equation of the tangent to the graph of $y = g(x)$ at $x = 2$. Give your answer in the form $y = mx + c$. [3]

c. Use your answer to part (a) and the value of k , to find the x -coordinates of the stationary points of the graph of $y = g(x)$. [3]

d.i. Find $g'(-1)$. [2]

d.ii. Hence justify that g is decreasing at $x = -1$. [1]

e. Find the y -coordinate of the local minimum. [2]

Markscheme

a. $3x^2 + 2kx - 15 \quad (\text{A1})(\text{A1})(\text{A1})$

Note: Award **(A1)** for $3x^2$, **(A1)** for $2kx$ and **(A1)** for -15 . Award at most **(A1)(A1)(A0)** if additional terms are seen.

[3 marks]

b.i. $21 = 3(2)^2 + 2k(2) - 15 \quad (\text{M1})(\text{M1})$

Note: Award **(M1)** for equating their derivative to 21. Award **(M1)** for substituting 2 into their derivative. The second **(M1)** should only be awarded if correct working leads to the final answer of $k = 6$.

Substituting in the known value, $k = 6$, invalidates the process; award **(M0)(M0)**.

$k = 6 \quad (\text{AG})$

[2 marks]

b.ii. $g(2) = (2)^3 + (6)(2)^2 - 15(2) + 5 (= 7) \quad (\text{M1})$

Note: Award **(M1)** for substituting 2 into g .

$7 = 21(2) + c \quad (\text{M1})$

Note: Award **(M1)** for correct substitution of 21, 2 and their 7 into gradient intercept form.

OR

$y - 7 = 21(x - 2) \quad (\text{M1})$

Note: Award **(M1)** for correct substitution of 21, 2 and their 7 into gradient point form.

$y = 21x - 35 \quad (\text{A1}) \quad (\text{G2})$

[3 marks]

c. $3x^2 + 12x - 15 = 0$ (or equivalent) **(M1)**

Note: Award **(M1)** for equating their part (a) (with $k = 6$ substituted) to zero.

$x = -5, x = 1$ **(A1)(ft)(A1)(ft)**

Note: Follow through from part (a).

[3 marks]

d.i. $3(-1)^2 + 12(-1) - 15$ **(M1)**

Note: Award **(M1)** for substituting -1 into their derivative, with $k = 6$ substituted. Follow through from part (a).

$= -24$ **(A1)(ft) (G2)**

[2 marks]

d.ii. $g'(-1) < 0$ (therefore g is decreasing when $x = -1$) **(R1)**

[1 marks]

e. $g(1) = (1)^3 + (6)(1)^2 - 15(1) + 5$ **(M1)**

Note: Award **(M1)** for correctly substituting 6 and their 1 into g .

$= -3$ **(A1)(ft) (G2)**

Note: Award, at most, **(M1)(A0)** or **(G1)** if answer is given as a coordinate pair. Follow through from part (c).

[2 marks]

Examiners report

- a. [N/A]
- b.i. [N/A]
- b.ii. [N/A]
- c. [N/A]
- d.i. [N/A]
- d.ii. [N/A]
- e. [N/A]

Consider the function $f(x) = \frac{96}{x^2} + kx$, where k is a constant and $x \neq 0$.

- a. Write down $f'(x)$. [3]
- b. The graph of $y = f(x)$ has a local minimum point at $x = 4$. [2]
- Show that $k = 3$.
- c. The graph of $y = f(x)$ has a local minimum point at $x = 4$. [2]
- Find $f(2)$.
- d. The graph of $y = f(x)$ has a local minimum point at $x = 4$. [2]
- Find $f'(2)$
- e. The graph of $y = f(x)$ has a local minimum point at $x = 4$. [3]
- Find the equation of the normal to the graph of $y = f(x)$ at the point where $x = 2$.
- Give your answer in the form $ax + by + d = 0$ where $a, b, d \in \mathbb{Z}$.
- f. The graph of $y = f(x)$ has a local minimum point at $x = 4$. [4]
- Sketch the graph of $y = f(x)$, for $-5 \leq x \leq 10$ and $-10 \leq y \leq 100$.
- g. The graph of $y = f(x)$ has a local minimum point at $x = 4$. [2]
- Write down the coordinates of the point where the graph of $y = f(x)$ intersects the x -axis.
- h. The graph of $y = f(x)$ has a local minimum point at $x = 4$. [2]
- State the values of x for which $f(x)$ is decreasing.

Markscheme

a. $\frac{-192}{x^3} + k$ (A1)(A1)(A1)

Note: Award (A1) for -192 , (A1) for x^{-3} , (A1) for k (only).

b. at local minimum $f'(x) = 0$ (M1)

Note: Award (M1) for seeing $f'(x) = 0$ (may be implicit in their working).

$$\frac{-192}{4^3} + k = 0 \quad (\text{A1})$$

$$k = 3 \quad (\text{AG})$$

Note: Award (A1) for substituting $x = 4$ in their $f'(x) = 0$, provided it leads to $k = 3$. The conclusion $k = 3$ must be seen for the (A1) to be awarded.

c. $\frac{96}{2^2} + 3(2)$ (M1)

Note: Award (M1) for substituting $x = 2$ and $k = 3$ in $f(x)$.

$$= 30 \quad (\text{A1})(\text{G2})$$

d. $\frac{-192}{2^3} + 3$ (M1)

Note: Award (M1) for substituting $x = 2$ and $k = 3$ in their $f'(x)$.

$$= -21 \quad (\mathbf{A1})(\mathbf{ft})(\mathbf{G2})$$

Note: Follow through from part (a).

e. $y - 30 = \frac{1}{21}(x - 2) \quad (\mathbf{A1})(\mathbf{ft})(\mathbf{M1})$

Notes: Award **(A1)(ft)** for their $\frac{1}{21}$ seen, **(M1)** for the correct substitution of their point and their normal gradient in equation of a line.

Follow through from part (c) and part (d).

OR

$$\text{gradient of normal} = \frac{1}{21} \quad (\mathbf{A1})(\mathbf{ft})$$

$$30 = \frac{1}{21} \times 2 + c \quad (\mathbf{M1})$$

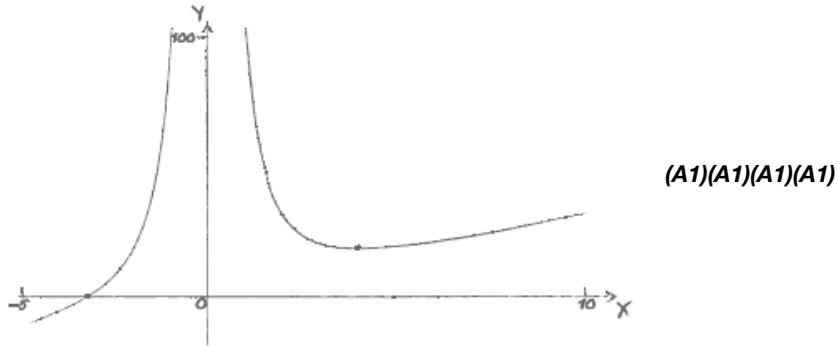
$$c = 29\frac{19}{21}$$

$$y = \frac{1}{21}x + 29\frac{19}{21} \quad (y = 0.0476x + 29.904)$$

$$x - 21y + 628 = 0 \quad (\mathbf{A1})(\mathbf{ft})(\mathbf{G2})$$

Notes: Accept equivalent answers.

f.



Notes: Award **(A1)** for correct window (at least one value, other than zero, labelled on each axis), the axes must also be labelled; **(A1)** for a smooth curve with the correct shape (graph should not touch y -axis and should not curve away from the y -axis), on the given domain; **(A1)** for axis intercept in approximately the correct position (nearer -5 than zero); **(A1)** for local minimum in approximately the correct position (first quadrant, nearer the y -axis than $x = 10$).

If there is no scale, award a maximum of **(A0)(A1)(A0)(A1)** – the final **(A1)** being awarded for the zero and local minimum in approximately correct positions relative to each other.

g. $(-3.17, 0) \quad ((-3.17480 \dots, 0)) \quad (\mathbf{G1})(\mathbf{G1})$

Notes: If parentheses are omitted award **(G0)(G1)(ft)**.

Accept $x = -3.17$, $y = 0$. Award **(G1)** for -3.17 seen.

h. $0 < x \leqslant 4$ or $0 < x < 4 \quad (\mathbf{A1})(\mathbf{A1})$

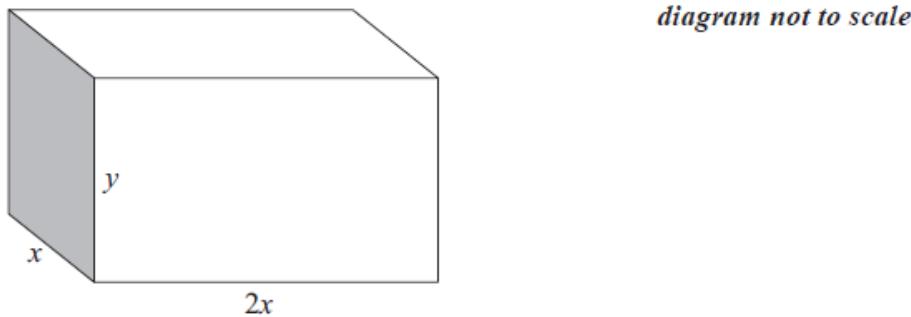
Notes: Award **(A1)** for correct end points of interval, **(A1)** for correct notation (note: lower inequality must be strict).

Award a maximum of **(A1)(A0)** if y or $f(x)$ used in place of x .

Examiners report

- a. Differentiation of terms including negative indices remains a testing process; it will continue to be tested. There was, however, a noticeable improvement in responses compared to previous years. The parameter k was problematic for a number of candidates.
- b. In part (b), the manipulation of the derivative to find the local minimum point caused difficulties for all but the most able; note that a GDC approach is not accepted in such questions and that candidates are expected to be able to apply the theory of the calculus as appropriate. Further, once a parameter is given, candidates are expected to use this value in subsequent parts.
- c. Parts (c) and (d) were accessible and all but the weakest candidates scored well.
- d. Parts (c) and (d) were accessible and all but the weakest candidates scored well.
- e. Part (e) discriminated at the highest level; the gradient of the normal often was not used, the form of the answer not given correctly.
- f. Curve sketching is a skill that most candidates find very difficult; axes must be labelled and some indication of the window must be present; care must be taken with the domain and the range; any asymptotic behaviour must be indicated. It was very rare to see sketches that attained full marks, yet this should be a skill that all can attain. There were many no attempts seen, yet some of these had correct answers to part (g).
- g. Curve sketching is a skill that most candidates find very difficult; axes must be labelled and some indication of the window must be present; care must be taken with the domain and the range; any asymptotic behaviour must be indicated. It was very rare to see sketches that attained full marks, yet this should be a skill that all can attain. There were many no attempts seen, yet some of these had correct answers to part (g).
- h. Part (h) was not well attempted in the main; decreasing (and increasing) functions is a testing concept for the majority.
-

A shipping container is to be made with six rectangular faces, as shown in the diagram.



The dimensions of the container are

length $2x$
width x
height y .

All of the measurements are in metres. The total length of all twelve edges is 48 metres.

- a. Show that $y = 12 - 3x$.

[3]

- b. Show that the volume $V \text{ m}^3$ of the container is given by

[2]

$$V = 24x^2 - 6x^3$$

c. Find $\frac{dV}{dx}$. [2]

d. Find the value of x for which V is a maximum. [3]

e. Find the maximum volume of the container. [2]

f. Find the length and height of the container for which the volume is a maximum. [3]

g. The shipping container is to be painted. One litre of paint covers an area of 15 m^2 . Paint comes in tins containing four litres. [4]

Calculate the number of tins required to paint the shipping container.

Markscheme

a. $4(2x) + 4y + 4x = 48 \quad (\text{M1})$

Note: Award **(M1)** for setting up the equation.

$$12x + 4y = 48 \quad (\text{M1})$$

Note: Award **(M1)** for simplifying (can be implied).

$$y = \frac{48-12x}{4} \quad \text{OR} \quad 3x + y = 12 \quad (\text{A1})$$

$$y = 12 - 3x \quad (\text{AG})$$

Note: The last line must be seen for the **(A1)** to be awarded.

[3 marks]

b. $V = 2x \times x \times (12 - 3x) \quad (\text{M1})(\text{A1})$

Note: Award **(M1)** for substitution into volume equation, **(A1)** for correct substitution.

$$= 24x^2 - 6x^3 \quad (\text{AG})$$

Note: The last line must be seen for the **(A1)** to be awarded.

[2 marks]

c. $\frac{dV}{dx} = 48x - 18x^2 \quad (\text{A1})(\text{A1})$

Note: Award **(A1)** for each correct term.

[2 marks]

d. $48x - 18x^2 = 0 \quad (\text{M1})(\text{M1})$

Note: Award **(M1)** for using their derivative, **(M1)** for equating their answer to part (c) to 0.

OR

(M1) for sketch of $V = 24x^2 - 6x^3$, **(M1)** for the maximum point indicated **(M1)(M1)**

OR

(M1) for sketch of $\frac{dV}{dx} = 48x - 18x^2$, **(M1)** for the positive root indicated **(M1)(M1)**

$$2.67 \left(\frac{24}{9}, \frac{8}{3}, 2.66666\dots \right) \quad (\text{A1})(\text{ft})(\text{G2})$$

Note: Follow through from their part (c).

[3 marks]

e. $V = 24 \times \left(\frac{8}{3}\right)^2 - 6 \times \left(\frac{8}{3}\right)^3$ **(M1)**

Note: Award **(M1)** for substitution of their value from part (d) into volume equation.

$$56.9(\text{m}^3) \left(\frac{512}{9}, 56.8888\dots\right) \quad \mathbf{(A1)(ft)(G2)}$$

Note: Follow through from their answer to part (d).

[2 marks]

f. length = $\frac{16}{3}$ **(A1)(ft)(G1)**

Note: Follow through from their answer to part (d). Accept 5.34 from use of 2.67

$$\text{height} = 12 - 3 \times \left(\frac{8}{3}\right) = 4 \quad \mathbf{(M1)(A1)(ft)(G2)}$$

Notes: Award **(M1)** for substitution of their answer to part (d), **(A1)(ft)** for answer. Accept 3.99 from use of 2.67.

[3 marks]

g. $\text{SA} = 2 \times \frac{16}{3} \times 4 + 2 \times \frac{8}{3} \times 4 + 2 \times \frac{16}{3} \times \frac{8}{3}$ **(M1)**

OR

$$\text{SA} = 4\left(\frac{8}{3}\right)^2 + 6 \times \frac{8}{3} \times 4 \quad \mathbf{(M1)}$$

Note: Award **(M1)** for substitution of their values from parts (d) and (f) into formula for surface area.

$$92.4 (\text{m}^2) (92.4444\dots(\text{m}^2)) \quad \mathbf{(A1)}$$

Note: Accept 92.5 (92.4622...) from use of 3 sf answers.

$$\text{Number of tins} = \frac{92.4444\dots}{15 \times 4} (= 1.54) \quad \mathbf{(M1)}$$

[4 marks]

Note: Award **(M1)** for division of their surface area by 60.

2 tins required **(A1)(ft)**

Note: Follow through from their answers to parts (d) and (f).

Examiners report

- a. Many candidates did not answer this question at all and others did not get past part (c). It was unclear if this was because they could not do the question or they ran out of time.
 - (a) This was very poorly done. Most candidates had no idea what they were supposed to do here. Many tried to find values for x .
- b. Many candidates did not answer this question at all and others did not get past part (c). It was unclear if this was because they could not do the question or they ran out of time.
 - (a) This was very poorly done. Most candidates had no idea what they were supposed to do here. Many tried to find values for x .

(b) Similar comment as for part (a) although more candidates made an attempt at finding the Volume.

c. Many candidates did not answer this question at all and others did not get past part (c). It was unclear if this was because they could not do the question or they ran out of time.

(c) This part was very well done.

d. Many candidates did not answer this question at all and others did not get past part (c). It was unclear if this was because they could not do the question or they ran out of time.

(d) Not many correct answers seen. Many candidates graphed the wrong equation and found 1.333 as their answer.

e. Many candidates did not answer this question at all and others did not get past part (c). It was unclear if this was because they could not do the question or they ran out of time.

(e) Some managed to gain follow through marks for this part.

f. Many candidates did not answer this question at all and others did not get past part (c). It was unclear if this was because they could not do the question or they ran out of time.

(f) Again here follow through marks were gained by those who attempted it.

g. Many candidates did not answer this question at all and others did not get past part (c). It was unclear if this was because they could not do the question or they ran out of time.

(g) Very few correct answers for the surface area were seen. Most candidates thought that there were 4 equal faces $2xy$ and 2 faces xy . Some managed to get follow through marks for the last part if they divided by 60.

Consider the curve $y = x^3 + \frac{3}{2}x^2 - 6x - 2$.

a. (i) Write down the value of y when x is 2.

[3]

(ii) Write down the coordinates of the point where the curve intercepts the y -axis.

b. Sketch the curve for $-4 \leq x \leq 3$ and $-10 \leq y \leq 10$. Indicate clearly the information found in (a).

[4]

c. Find $\frac{dy}{dx}$.

[3]

d. Let L_1 be the tangent to the curve at $x = 2$.

[8]

Let L_2 be a tangent to the curve, parallel to L_1 .

(i) Show that the gradient of L_1 is 12.

(ii) Find the x -coordinate of the point at which L_2 and the curve meet.

(iii) Sketch and label L_1 and L_2 on the diagram drawn in (b).

e. It is known that $\frac{dy}{dx} > 0$ for $x < -2$ and $x > b$ where b is positive.

- (i) Using your graphic display calculator, or otherwise, find the value of b .
- (ii) Describe the behaviour of the curve in the interval $-2 < x < b$.
- (iii) Write down the equation of the tangent to the curve at $x = -2$.

Markscheme

a. (i) $y = 0$ (A1)

(ii) $(0, -2)$ (A1)(A1)

Note: Award (A1)(A0) if brackets missing.

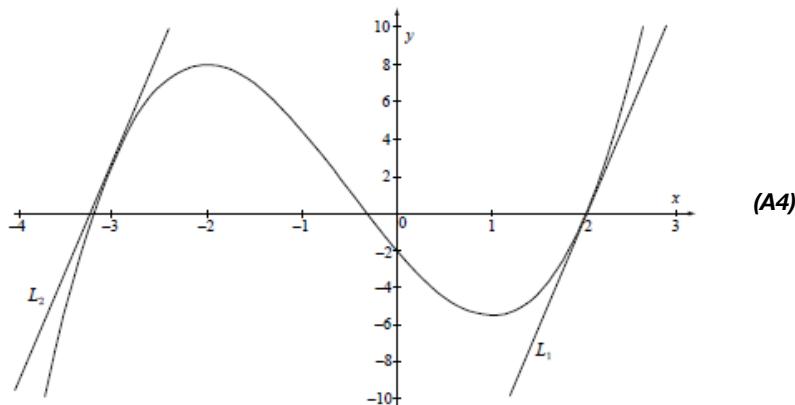
OR

$x = 0, y = -2$ (A1)(A1)

Note: If coordinates reversed award (A0)(A1)(ft). Two coordinates must be given.

[3 marks]

b.



(A4)

Notes: (A1) for appropriate window. Some indication of scale on the x -axis must be present (for example ticks). Labels not required. (A1) for smooth curve and shape, (A1) for maximum and minimum in approximately correct position, (A1) for x and y intercepts found in (a) in approximately correct position.

[4 marks]

c. $\frac{dy}{dx} = 3x^2 + 3x - 6$ (A1)(A1)(A1)

Note: (A1) for each correct term. Award (A1)(A1)(A0) at most if any other term is present.

[3 marks]

d. (i) $3 \times 4 + 3 \times 2 - 6 = 12$ (M1)(A1)(AG)

Note: (M1) for using the derivative and substituting $x = 2$. (A1) for correct (and clear) substitution. The 12 must be seen.

(ii) Gradient of L_2 is 12 (can be implied) (A1)

$$3x^2 + 3x - 6 = 12 \quad (\text{M1})$$

$$x = -3 \quad (\text{A1})(\text{G2})$$

Note: (M1) for equating the derivative to 12 or showing a sketch of the derivative together with a line at $y = 12$ or a table of values showing the 12 in the derivative column.

(iii) (A1) for L_1 correctly drawn at approx the correct point (A1)

(A1) for L_2 correctly drawn at approx the correct point (A1)

(A1) for 2 parallel lines (A1)

Note: If lines are not labelled award at most (A1)(A1)(A0). Do not accept 2 horizontal or 2 vertical parallel lines.

[8 marks]

e. (i) $b = 1$ (G2)

(ii) The curve is decreasing. (A1)

Note: Accept any valid description.

(iii) $y = 8$ (A1)(A1)(G2)

Note: (A1) for “ $y =$ a constant”, (A1) for 8.

[5 marks]

Examiners report

- a. Many candidates managed to gain good marks in this question as they were able to answer the first three parts of the question. Good sketches were drawn with the required information shown on them. Very few candidates did not recognise the notation $\frac{dy}{dx}$ but they showed that they knew how to differentiate as in (d)(i) they found the derivative to show that the gradient of L_1 was 12. Candidates found it difficult to find the other x for which the derivative was 12. However, some could draw both tangents without having found this value of x . In general, tangents were not well drawn. The last part question did act as a discriminating question. However, those candidates that had the function drawn either in their GDC or on paper recognised that at $x = -2$ there was a maximum and so wrote down the correct equation of the tangent at that point.
- b. Many candidates managed to gain good marks in this question as they were able to answer the first three parts of the question. Good sketches were drawn with the required information shown on them. Very few candidates did not recognise the notation $\frac{dy}{dx}$ but they showed that they knew how to differentiate as in (d)(i) they found the derivative to show that the gradient of L_1 was 12. Candidates found it difficult to find the other x for which the derivative was 12. However, some could draw both tangents without having found this value of x . In general, tangents were not well drawn. The last part question did act as a discriminating question. However, those candidates that had the function drawn either in their GDC or on paper recognised that at $x = -2$ there was a maximum and so wrote down the correct equation of the tangent at that point.
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Consider the function $f(x) = \frac{3}{4}x^4 - x^3 - 9x^2 + 20$.

- a. Find $f(-2)$. [2]
- b. Find $f'(x)$. [3]
- c. The graph of the function $f(x)$ has a local minimum at the point where $x = -2$. [5]

Using your answer to part (b), show that there is a second local minimum at $x = 3$.

- d. The graph of the function $f(x)$ has a local minimum at the point where $x = -2$. [4]

Sketch the graph of the function $f(x)$ for $-5 \leq x \leq 5$ and $-40 \leq y \leq 50$. Indicate on your sketch the coordinates of the y -intercept.

- e. The graph of the function $f(x)$ has a local minimum at the point where $x = -2$. [2]

Write down the coordinates of the local maximum.

- f. Let T be the tangent to the graph of the function $f(x)$ at the point $(2, -12)$. [2]

Find the gradient of T .

- g. The line L passes through the point $(2, -12)$ and is perpendicular to T . [5]

L has equation $x + by + c = 0$, where b and $c \in \mathbb{Z}$.

Find

- (i) the gradient of L ;
- (ii) the value of b and the value of c .

Markscheme

a. $\frac{3}{4}(-2)^4 - (-2)^3 - 9(-2)^2 + 20 \quad (\text{M1})$

Note: Award **(M1)** for substituting $x = -2$ in the function.

$= 4 \quad (\text{A1})(\text{G2})$

Note: If the coordinates $(-2, 4)$ are given as the answer award, at most, **(M1)(AO)**. If no working shown award **(G1)**.

If $x = -2$, $y = 4$ seen then award full marks.

[2 marks]

b. $3x^3 - 3x^2 - 18x \quad (\text{A1})(\text{A1})(\text{A1})$

Note: Award **(A1)** for each correct term, award at most **(A1)(A1)(AO)** if extra terms seen.

[3 marks]

c. $f'(3) = 3 \times (3)^3 - 3 \times (3)^2 - 18 \times 3 \quad (\text{M1})$

Note: Award **(M1)** for substitution in their $f'(x)$ of $x = 3$.

$= 0 \quad (\text{A1})$

OR

$3x^3 - 3x^2 - 18x = 0 \quad (\text{M1})$

Note: Award **(M1)** for equating their $f'(x)$ to zero.

$x = 3 \quad (\text{A1})$

$f'(x_1) = 3 \times (x_1)^3 - 3 \times (x_1)^2 - 18 \times x_1 < 0 \text{ where } 0 < x_1 < 3 \quad (\text{M1})$

Note: Award **(M1)** for substituting a value of x_1 in the range $0 < x_1 < 3$ into their f' and showing it is negative (decreasing).

$f'(x_2) = 3 \times (x_2)^3 - 3 \times (x_2)^2 - 18 \times x_2 > 0 \text{ where } x_2 > 3 \quad (\text{M1})$

Note: Award **(M1)** for substituting a value of x_2 in the range $x_2 > 3$ into their f' and showing it is positive (increasing).

OR

With or without a sketch:

Showing $f(x_1) > f(3)$ where $x_1 < 3$ and x_1 is close to 3. (M1)

Showing $f(x_2) > f(3)$ where $x_2 > 3$ and x_2 is close to 3. (M1)

Note: If a sketch of $f(x)$ is drawn in this part of the question and $x = 3$ is identified as a stationary point on the curve, then

(i) award, at most, (M1)(A1)(M1)(M0) if the stationary point has been found;

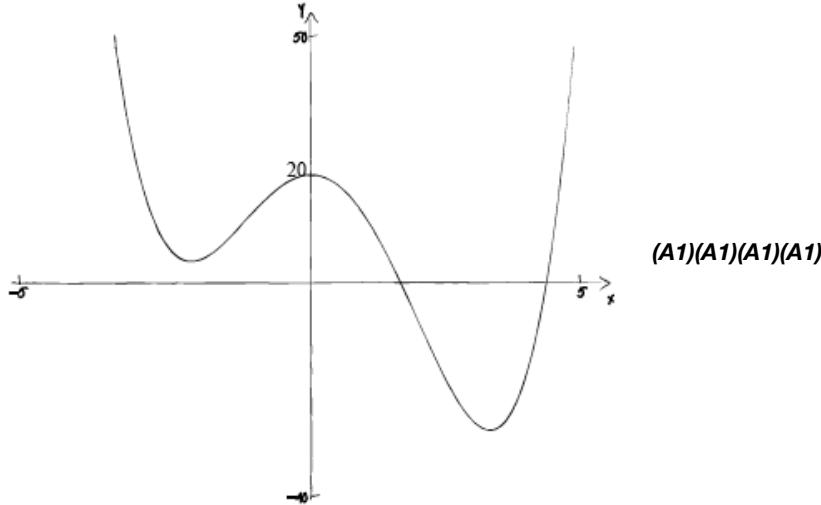
(ii) award, at most, (M0)(A0)(M1)(M0) if the stationary point has not been previously found.

Since the gradients go from negative (decreasing) through zero to positive (increasing) it is a local minimum (R1)(AG)

Note: Only award (R1) if the first two marks have been awarded ie $f'(3)$ has been shown to be equal to 0.

[5 marks]

d.



(A1)(A1)(A1)(A1)

Notes: Award (A1) for labelled axes and indication of scale on both axes.

Award (A1) for smooth curve with correct shape.

Award (A1) for local minima in 2nd and 4th quadrants.

Award (A1) for y intercept (0, 20) seen and labelled. Accept 20 on y-axis.

Do not award the third (A1) mark if there is a turning point on the x-axis.

If the derivative function is sketched then award, at most, (A1)(A0)(A0)(A0).

For a smooth curve (with correct shape) there should be ONE continuous thin line, no part of which is straight and no (one to many) mappings of x .

[4 marks]

e. (0, 20) (G1)(G1)

Note: If parentheses are omitted award (G0)(G1).

OR

$x = 0, y = 20$ (G1)(G1)

Note: If the derivative function is sketched in part (d), award **(G1)(ft)(G1)(ft)** for $(-1.12, 12.2)$.

[2 marks]

f. $f'(2) = 3(2)^3 - 3(2)^2 - 18(2)$ **(M1)**

Notes: Award **(M1)** for substituting $x = 2$ into their $f'(x)$.

$= -24$ **(A1)(ft)(G2)**

[2 marks]

g. (i) Gradient of perpendicular $= \frac{1}{24}$ $(0.0417, 0.041666 \dots)$ **(A1)(ft)(G1)**

Note: Follow through from part (f).

(ii) $y + 12 = \frac{1}{24}(x - 2)$ **(M1)(M1)**

Note: Award **(M1)** for correct substitution of $(2, -12)$, **(M1)** for correct substitution of their perpendicular gradient into equation of line.

OR

$$-12 = \frac{1}{24} \times 2 + d \quad \textbf{(M1)}$$

$$d = -\frac{145}{12}$$

$$y = \frac{1}{24}x - \frac{145}{12} \quad \textbf{(M1)}$$

Note: Award **(M1)** for correct substitution of $(2, -12)$ and gradient into equation of a straight line, **(M1)** for correct substitution of the perpendicular gradient and correct substitution of d into equation of line.

$b = -24, c = -290$ **(A1)(ft)(A1)(ft)(G3)**

Note: Follow through from parts (f) and g(i).

To award **(ft)** marks, b and c must be integers.

Where candidate has used 0.042 from g(i), award **(A1)(ft)** for -288 .

[5 marks]

Examiners report

- a. Surprisingly, a correct method for substituting the value of -2 into the given function led many candidates to a variety of incorrect answers.

This suggests a poor handling of negative signs and/or poor use of the graphic display calculator. Many correct answers were seen in part (b) as candidates seemed to be well-drilled in the process of differentiation. Part (c), however, proved to be quite a discriminator. There were 5 marks for this part of the question and simply showing that $x - 3$ is a turning point was not sufficient for all of these marks. Many simply scored only two marks by substituting $x - 3$ into their answer to part (b). Once they had shown that there was a turning point at $x - 3$, candidates were not

expected to use the second derivative but to show that the function decreases for $x < 3$ and increases for $x > 3$. Part (d) required a sketch which could have either been done on lined paper or on graph paper. The majority of candidates obtained at least two marks here with the most common errors seen being incomplete labelled axes and curves which were far from being smooth. In part (e), many candidates identified the correct coordinates for the two marks available. But for many candidates, this is where responses stopped as, in part (f), connecting the gradient function found in part (b) to the given coordinates proved problematic and only a significant minority of candidates were able to arrive at the required answer of -24 . Indeed, there were many NR (no responses) to this part and the final part of the question. As many candidates found part (f) difficult, even more candidates found getting beyond the gradient of L very difficult indeed. A minority of candidates wrote down the gradient of their perpendicular but then did not seem to know where to proceed from there. Substituting their gradient for b and the coordinates $(2, -12)$ into the equation $x + by + c = 0$ was a popular, but erroneous, method. It was a rare event indeed to see a script with a correct answer for this part of the question.

- b. Surprisingly, a correct method for substituting the value of -2 into the given function led many candidates to a variety of incorrect answers.

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- c. Surprisingly, a correct method for substituting the value of -2 into the given function led many candidates to a variety of incorrect answers.

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- d. Surprisingly, a correct method for substituting the value of -2 into the given function led many candidates to a variety of incorrect answers.

This suggests a poor handling of negative signs and/or poor use of the graphic display calculator. Many correct answers were seen in part (b) as candidates seemed to be well-drilled in the process of differentiation. Part (c), however, proved to be quite a discriminator. There were 5 marks for this part of the question and simply showing that $x - 3$ is a turning point was not sufficient for all of these marks. Many simply scored only two marks by substituting $x - 3$ into their answer to part (b). Once they had shown that there was a turning point at $x - 3$, candidates were not expected to use the second derivative but to show that the function decreases for $x < 3$ and increases for $x > 3$. Part (d) required a sketch which could have either been done on lined paper or on graph paper. The majority of candidates obtained at least two marks here with the most common errors seen being incomplete labelled axes and curves which were far from being smooth. In part (e), many candidates identified the correct coordinates for the two marks available. But for many candidates, this is where responses stopped as, in part (f), connecting the gradient function found in part (b) to the given coordinates proved problematic and only a significant minority of candidates were able to arrive at the required answer of -24 . Indeed, there were many NR (no responses) to this part and the final part of the question. As many candidates found part (f) difficult, even more candidates found getting beyond the gradient of L very difficult indeed. A minority of candidates wrote down the gradient of their perpendicular but then did not seem to know where to proceed from there. Substituting their gradient for b and the coordinates $(2, -12)$ into the equation $x + by + c = 0$ was a popular, but erroneous, method. It was a rare event indeed to see a script with a correct answer for this part of the question.

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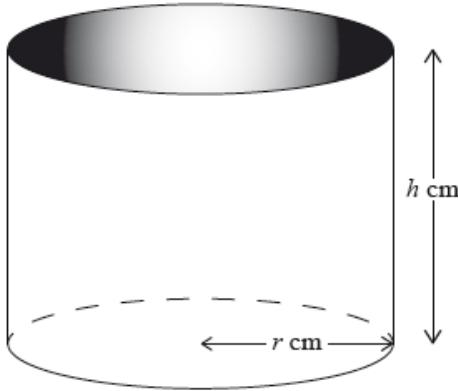
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A water container is made in the shape of a cylinder with internal height h cm and internal base radius r cm.



The water container has no top. The inner surfaces of the container are to be coated with a water-resistant material.

The volume of the water container is 0.5 m^3 .

The water container is designed so that the area to be coated is minimized.

One can of water-resistant material coats a surface area of 2000 cm^2 .

a. Write down a formula for A , the surface area to be coated. [2]

b. Express this volume in cm^3 . [1]

c. Write down, in terms of r and h , an equation for the volume of this water container. [1]

d. Show that $A = \pi r^2 \frac{1\,000\,000}{r}$. [2]

d. Show that $A = \pi r^2 + \frac{1\,000\,000}{r}$. [2]

e. Find $\frac{dA}{dr}$. [3]

f. Using your answer to part (e), find the value of r which minimizes A . [3]

g. Find the value of this minimum area. [2]

h. Find the least number of cans of water-resistant material that will coat the area in part (g). [3]

Markscheme

a. $(A =) \pi r^2 + 2\pi r h \quad (\mathbf{A1})(\mathbf{A1})$

Note: Award **(A1)** for either πr^2 OR $2\pi r h$ seen. Award **(A1)** for two correct terms added together.

[2 marks]

b. 500 000 **(A1)**

Notes: Units **not** required.

[1 mark]

c. $500\ 000 = \pi r^2 h$ **(A1)(ft)**

Notes: Award **(A1)(ft)** for $\pi r^2 h$ equating to their part (b).

Do not accept unless $V = \pi r^2 h$ is explicitly defined as their part (b).

[1 mark]

d. $A = \pi r^2 + 2\pi r \left(\frac{500\ 000}{\pi r^2} \right)$ **(A1)(ft)(M1)**

Note: Award **(A1)(ft)** for their $\frac{500\ 000}{\pi r^2}$ seen.

Award **(M1)** for correctly substituting **only** $\frac{500\ 000}{\pi r^2}$ into a **correct** part (a).

Award **(A1)(ft)(M1)** for rearranging part (c) to $\pi r h = \frac{500\ 000}{r}$ and substituting for $\pi r h$ in expression for A .

$$A = \pi r^2 + \frac{1\ 000\ 000}{r} \quad \textbf{(AG)}$$

Notes: The conclusion, $A = \pi r^2 + \frac{1\ 000\ 000}{r}$, must be consistent with their working seen for the **(A1)** to be awarded.

Accept 10^6 as equivalent to 1 000 000.

[2 marks]

d. $A = \pi r^2 + 2\pi r \left(\frac{500\ 000}{\pi r^2} \right)$ **(A1)(ft)(M1)**

Note: Award **(A1)(ft)** for their $\frac{500\ 000}{\pi r^2}$ seen.

Award **(M1)** for correctly substituting **only** $\frac{500\ 000}{\pi r^2}$ into a **correct** part (a).

Award **(A1)(ft)(M1)** for rearranging part (c) to $\pi r h = \frac{500\ 000}{r}$ and substituting for $\pi r h$ in expression for A .

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Notes: The conclusion, $A = \pi r^2 + \frac{1\ 000\ 000}{r}$, must be consistent with their working seen for the **(A1)** to be awarded.

Accept 10^6 as equivalent to 1 000 000.

[2 marks]

e. $2\pi r - \frac{1\ 000\ 000}{r^2}$ **(A1)(A1)(A1)**

Note: Award **(A1)** for $2\pi r$, **(A1)** for $\frac{1}{r^2}$ or r^{-2} , **(A1)** for $-1\ 000\ 000$.

[3 marks]

f. $2\pi r - \frac{1\ 000\ 000}{r^2} = 0$ **(M1)**

Note: Award **(M1)** for equating their part (e) to zero.

$$r^3 = \frac{1\ 000\ 000}{2\pi} \text{ OR } r = \sqrt[3]{\frac{1\ 000\ 000}{2\pi}} \quad (\text{M1})$$

Note: Award **(M1)** for isolating r .

OR

sketch of derivative function **(M1)**

with its zero indicated **(M1)**

$$(r =) 54.2 \text{ (cm)} (54.1926\dots) \quad (\text{A1})(\text{ft})(\text{G2})$$

[3 marks]

g. $\pi(54.1926\dots)^2 + \frac{1\ 000\ 000}{(54.1926\dots)}$ **(M1)**

Note: Award **(M1)** for correct substitution of their part (f) into the given equation.

$$= 27\ 700 \text{ (cm}^2\text{)} (27\ 679.0\dots) \quad (\text{A1})(\text{ft})(\text{G2})$$

[2 marks]

h. $\frac{27\ 679.0\dots}{2000}$ **(M1)**

Note: Award **(M1)** for dividing their part (g) by 2000.

$$= 13.8395\dots \quad (\text{A1})(\text{ft})$$

Notes: Follow through from part (g).

14 (cans) **(A1)(ft)(G3)**

Notes: Final **(A1)** awarded for rounding up their 13.8395... to the next integer.

[3 marks]

Examiners report

a. [N/A]

- b. [N/A]
 c. [N/A]
 d. [N/A]
 d. [N/A]
 e. [N/A]
 f. [N/A]
 g. [N/A]
 h. [N/A]
-

Consider the function $f(x) = x^3 + \frac{48}{x}$, $x \neq 0$.

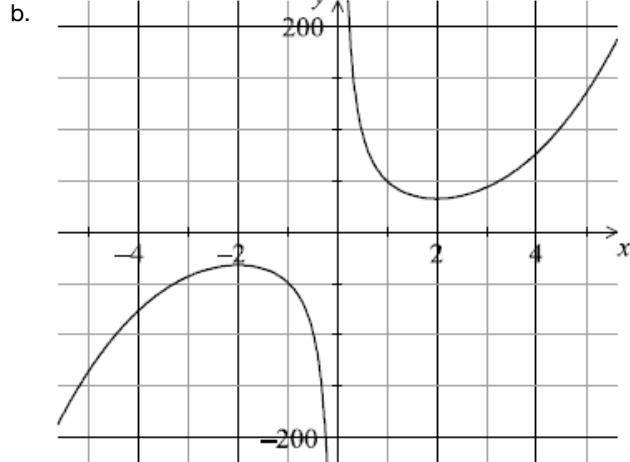
- a. Calculate $f(2)$. [2]
- b. Sketch the graph of the function $y = f(x)$ for $-5 \leq x \leq 5$ and $-200 \leq y \leq 200$. [4]
- c. Find $f'(x)$. [3]
- d. Find $f'(2)$. [2]
- e. Write down the coordinates of the local maximum point on the graph of f . [2]
- f. Find the range of f . [3]
- g. Find the gradient of the tangent to the graph of f at $x = 1$. [2]
- h. There is a second point on the graph of f at which the tangent is parallel to the tangent at $x = 1$. [2]

Find the x -coordinate of this point.

Markscheme

a. $f(2) = 2^3 + \frac{48}{2} \quad (\text{M1})$
 $= 32 \quad (\text{A1})(\text{G2})$

[2 marks]



(A1) for labels and some indication of scale in an appropriate window

(A1) for correct shape of the two unconnected and smooth branches

(A1) for maximum and minimum in approximately correct positions

(A1) for asymptotic behaviour at y -axis (A4)

Notes: Please be rigorous.

The axes need not be drawn with a ruler.

The branches must be smooth: a single continuous line that does not deviate from its proper direction.

The position of the maximum and minimum points must be symmetrical about the origin.

The y -axis must be an asymptote for both branches. Neither branch should touch the axis nor must the curve approach the asymptote then deviate away later.

[4 marks]

c. $f'(x) = 3x^2 - \frac{48}{x^2}$ (A1)(A1)(A1)

Notes: Award (A1) for $3x^2$, (A1) for -48 , (A1) for x^{-2} . Award a maximum of (A1)(A1)(A0) if extra terms seen.

[3 marks]

d. $f'(2) = 3(2)^2 - \frac{48}{(2)^2}$ (M1)

Note: Award (M1) for substitution of $x = 2$ into their derivative.

= 0 (A1)(ft)(G1)

[2 marks]

e. $(-2, -32)$ or $x = -2, y = -32$ (G1)(G1)

Notes: Award (G0)(G0) for $x = -32, y = -2$. Award at most (G0)(G1) if parentheses are omitted.

[2 marks]

f. $\{y \geq 32\} \cup \{y \leq -32\}$ (A1)(A1)(ft)(A1)(ft)

Notes: Award (A1)(ft) $y \geq 32$ or $y > 32$ seen, (A1)(ft) for $y \leq -32$ or $y < -32$, (A1) for weak (non-strict) inequalities used in both of the above.

Accept use of f in place of y . Accept alternative interval notation.

Follow through from their (a) and (e).

If domain is given award (A0)(A0)(A0).

Award (A0)(A1)(ft)(A1)(ft) for $[-200, -32], [32, 200]$.

Award (A0)(A1)(ft)(A1)(ft) for $]-200, -32], [32, 200[$.

[3 marks]

g. $f'(1) = -45$ (M1)(A1)(ft)(G2)

Notes: Award (M1) for $f'(1)$ seen or substitution of $x = 1$ into their derivative. Follow through from their derivative if working is seen.

[2 marks]

h. $x = -1$ (M1)(A1)(ft)(G2)

Notes: Award (M1) for equating their derivative to their -45 or for seeing parallel lines on their graph in the approximately correct position.

[2 marks]

Examiners report

a. As usual and by intention, this question caused the most difficulty in terms of its content; however, for those with a sound grasp of the topic, there were many very successful attempts. Much of the question could have been answered successfully by using the GDC, however, it was also clear that a number of candidates did not connect the question they were attempting with the curve that they had either sketched or were viewing on their GDC. Where there was no alternative to using the calculus, many candidates struggled.

The majority of sketches were drawn sloppily and with little attention to detail. Teachers must impress on their students that a mathematical sketch is designed to illustrate the main points of a curve – the smooth nature by which it changes, any symmetries (reflectional or rotational), positions of turning points, intercepts with axes and the behaviour of a curve as it approaches an asymptote. There must also be some indication of the dimensions used for the “window”.

Differentiation of terms with negative indices remains a testing process for the majority; it will continue to be tested.

It was also evident that some centres do not teach the differential calculus.

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A, a Sketch the graph of $y = 2^x$ for $-2 \leq x \leq 3$. Indicate clearly where the curve intersects the y -axis. [3]

A, b Write down the equation of the asymptote of the graph of $y = 2^x$. [2]

A, c On the same axes sketch the graph of $y = 3 + 2x - x^2$. Indicate clearly where this curve intersects the x and y axes. [3]

A, d Using your graphic display calculator, solve the equation $3 + 2x - x^2 = 2^x$. [2]

A, e Write down the maximum value of the function $f(x) = 3 + 2x - x^2$. [1]

A, f Use Differential Calculus to verify that your answer to (e) is correct. [5]

B, a The curve $y = px^2 + qx - 4$ passes through the point $(2, -10)$. [2]

Use the above information to write down an equation in p and q .

B, b The gradient of the curve $y = px^2 + qx - 4$ at the point $(2, -10)$ is 1. [2]

Find $\frac{dy}{dx}$.

B, c The gradient of the curve $y = px^2 + qx - 4$ at the point $(2, -10)$ is 1. [1]

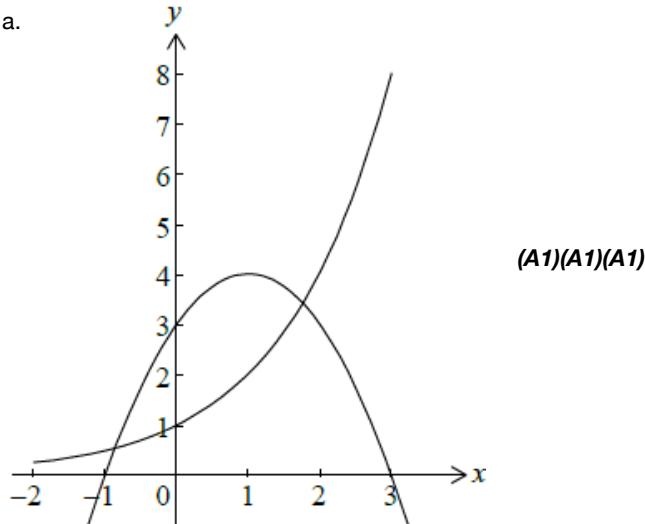
Hence, find a second equation in p and q .

B, d The gradient of the curve $y = px^2 + qx - 4$ at the point $(2, -10)$ is 1. [3]

Solve the equations to find the value of p and of q .

Markscheme

A, a.



Note: Award (A1) for correct domain, (A1) for smooth curve, (A1) for y -intercept clearly indicated.

[3 marks]

A, b $y = 0$ (A1)(A1)

Note: Award **(A1)** for $y = \text{constant}$, **(A1)** for 0.

[2 marks]

A, **Note:** Award **(A1)** for smooth parabola,

(A1) for vertex (maximum) in correct quadrant.

(A1) for all clearly indicated intercepts $x = -1$, $x = 3$ and $y = 3$.

The final mark is to be applied very strictly. **(A1)(A1)(A1)**

[3 marks]

A, $a = -0.857$ $x = 1.77$ **(G1)(G1)**

Note: Award a maximum of **(G1)** if x and y coordinates are both given.

[2 marks]

A, $a = 1$ **(G1)**

Note: Award **(G0)** for (1, 4).

[1 mark]

A, $f'(x) = 2 - 2x$ **(A1)(A1)**

Note: Award **(A1)** for each correct term.

Award at most **(A1)(A0)** if any extra terms seen.

$$2 - 2x = 0 \quad (\text{M1})$$

Note: Award **(M1)** for equating their gradient function to zero.

$$x = 1 \quad (\text{A1})(\text{ft})$$

$$f(1) = 3 + 2(1) - (1)^2 = 4 \quad (\text{A1})$$

Note: The final **(A1)** is for substitution of $x = 1$ into $f(x)$ and subsequent correct answer. Working must be seen for final **(A1)** to be awarded.

[5 marks]

B, $2^2 \times p + 2q - 4 = -10 \quad (\text{M1})$

Note: Award **(M1)** for correct substitution in the equation.

$$4p + 2q = -6 \quad \text{or} \quad 2p + q = -3 \quad (\text{A1})$$

Note: Accept equivalent simplified forms.

[2 marks]

$$B, \frac{dy}{dx} = 2px + q \quad (\text{A1})(\text{A1})$$

Note: Award **(A1)** for each correct term.

Award at most **(A1)(A0)** if any extra terms seen.

[2 marks]

$$B, \frac{dp}{dx} + q = 1 \quad (\text{A1})(\text{ft})$$

[1 mark]

$$B, 4p + 2q = -6$$

$$4p + q = 1 \quad (\text{M1})$$

Note: Award **(M1)** for sensible attempt to solve the equations.

$$p = 2, q = -7 \quad (\text{A1})(\text{A1})(\text{ft})(\text{G3})$$

[3 marks]

Examiners report

A, **J**ndoubtedly, this question caused the most difficulty in terms of its content. Where there was no alternative to using the calculus, the majority of candidates struggled. However, for those with a sound grasp of the topic, there were many very successful attempts.

The most common error was using the incorrect domain.

B, **H**ndoubtedly, this question caused the most difficulty in terms of its content. Where there was no alternative to using the calculus, the majority of candidates struggled. However, for those with a sound grasp of the topic, there were many very successful attempts.

Many had little idea of asymptotes. Others did not write their answer as an equation.

A, **J**ndoubtedly, this question caused the most difficulty in terms of its content. Where there was no alternative to using the calculus, the majority of candidates struggled. However, for those with a sound grasp of the topic, there were many very successful attempts.

The intercepts being inexact or unlabelled was the most frequent cause of loss of marks.

A, **J**ndoubtedly, this question caused the most difficulty in terms of its content. Where there was no alternative to using the calculus, the majority of candidates struggled. However, for those with a sound grasp of the topic, there were many very successful attempts.

Often, only one solution to the equation was given. Elsewhere, a lack of appreciation that the solutions were the x coordinates was a common mistake.

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The maximum is the y coordinate only; again a common misapprehension was the answer “(1, 4)”.

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This was a major discriminator in the paper. Many candidates were unable to follow the analytic approach to finding a maximum point.

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A dog food manufacturer has to cut production costs. She wishes to use as little aluminium as possible in the construction of cylindrical cans. In the following diagram, h represents the height of the can in cm and x , the radius of the base of the can in cm.

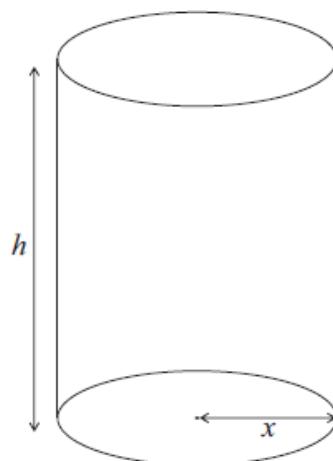


diagram not to scale

The volume of the dog food cans is 600 cm^3 .

a. Show that $h = \frac{600}{\pi x^2}$.

[2]

b.i. Find an expression for the curved surface area of the can, in terms of x . Simplify your answer.

[2]

b.ii.Hence write down an expression for A , the total surface area of the can, in terms of x .

[2]

c. Differentiate A in terms of x .

[3]

d. Find the value of x that makes A a minimum.

[3]

e. Calculate the minimum total surface area of the dog food can.

[2]

Markscheme

a. $600 = \pi x^2 h \quad (\text{M1})(\text{A1})$

$$\frac{600}{\pi x^2} = h \quad (\text{AG})$$

Note: Award **(M1)** for correct substituted formula, **(A1)** for correct substitution. If answer given not shown award at most **(M1)(A0)**.

[2 marks]

b.i. $C = 2\pi x \frac{600}{\pi x^2} \quad (\text{M1})$

$$C = \frac{1200}{x} \text{ (or } 1200x^{-1}\text{)} \quad (\text{A1})$$

Note: Award **(M1)** for correct substitution in formula, **(A1)** for correct simplification.

[??? marks]

b.ii. $A = 2\pi x^2 + 1200x^{-1} \quad (\text{A1})(\text{A1})(\text{ft})$

Note: Award **(A1)** for multiplying the area of the base by two, **(A1)** for adding on their answer to part (b) (i).

For both marks to be awarded answer must be in terms of x .

[??? marks]

c. $\frac{dA}{dx} = 4\pi x - \frac{1200}{x^2} \quad (\text{A1})(\text{ft})(\text{A1})(\text{ft})(\text{A1})(\text{ft})$

Notes: Award **(A1)** for $4\pi x$, **(A1)** for -1200 , **(A1)** for x^{-2} . Award at most **(A2)** if any extra term is written. Follow through from their part (b) (ii).

[??? marks]

d. $4\pi x - \frac{1200}{x^2} = 0 \quad (\text{M1})(\text{M1})$

$$x^3 = \frac{1200}{4\pi} \text{ (or equivalent)}$$

$$x = 4.57 \quad (\text{A1})(\text{ft})(\text{G2})$$

Note: Award **(M1)** for using their derivative, **(M1)** for setting the derivative to zero, **(A1)(ft)** for answer.

Follow through from their derivative.

Last mark is lost if value of x is zero or negative.

[3 marks]

e. $A = 2\pi(4.57)^2 + 1200(4.57)^{-1}$ **(M1)**

$A = 394$ **(A1)(ft)(G2)**

Note: Follow through from their answers to parts (b) (ii) and (d).

[2 marks]

Examiners report

- a. This was the most difficult question for the candidates. It was clear that the vast majority of them had not had exposure to this style of question. Part (a) was well answered by most of the students. In part (b) the correct expression “in terms of x ” for the curve surface area was not frequently seen. In many cases the impression was that they did not know what “in terms of x ” meant as correct equivalent expressions were seen but where the h was also involved. Those candidates that made progress in the question, even with the wrong expression for the total area of the can, A were able to earn follow through marks.
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-

A function is defined by $f(x) = \frac{5}{x^2} + 3x + c, x \neq 0, c \in \mathbb{Z}$.

- a. Write down an expression for $f'(x)$. [4]

- b. Consider the graph of f . The graph of f passes through the point P(1, 4). [2]

Find the value of c .

- c, i. There is a local minimum at the point Q. [4]

Find the coordinates of Q.

- c, ii. There is a local minimum at the point Q. [3]

Find the set of values of x for which the function is decreasing.

- d, i. Let T be the tangent to the graph of f at P. [2]

Show that the gradient of T is -7 .

- d, ii. Let T be the tangent to the graph of f at P. [2]

Find the equation of T .

- e. T intersects the graph again at R. Use your graphic display calculator to find the coordinates of R. [2]

Markscheme

a. $f'(x) = \frac{-10}{x^3} + 3 \quad (\text{A1})(\text{A1})(\text{A1})(\text{A1})$

Note: Award **(A1)** for -10 , **(A1)** for x^3 (or x^{-3}), **(A1)** for 3 , **(A1)** for no other constant term.

[4 marks]

b. $4 = 5 + 3 + c \quad (\text{M1})$

Note: Award **(M1)** for substitution in $f(x)$.

$$c = -4 \quad (\text{A1})(\text{G2})$$

[2 marks]

$$\text{c, i. } f'(x) = 0 \quad (\text{M1})$$

$$0 = \frac{-10}{x^3} + 3 \quad (\text{A1})(\text{ft})$$

$$(1.49, 2.72) \quad (\text{accept } x = 1.49 \text{ or } y = 2.72) \quad (\text{A1})(\text{ft})(\text{A1})(\text{ft})(\text{G3})$$

Notes: If answer is given as (1.5, 2.7) award **(A0)(AP)(A1)**.

Award at most **(M1)(A1)(A1)(A0)** if parentheses not included. **(ft)** from their (a).

If no working shown award **(G2)(G0)** if parentheses are not included.

OR

Award **(M2)** for sketch, **(A1)(ft)(A1)(ft)** for correct coordinates. **(ft)** from their (b). **(M2)(A1)(ft)(A1)(ft)**

Note: Award at most **(M2)(A1)(ft)(A0)** if parentheses not included.

[4 marks]

$$\text{c, ii } 0 < x < 1.49 \quad \text{OR} \quad 0 < x \leq 1.49 \quad (\text{A1})(\text{A1})(\text{ft})(\text{A1})$$

Notes: Award **(A1)** for 0, **(A1)(ft)** for 1.49 and **(A1)** for correct inequality signs.

(ft) from their x value in (c) (i).

[3 marks]

$$\text{d, i For P(1, 4) } f'(1) = -10 + 3 \quad (\text{M1})(\text{A1})$$

$$= -7 \quad (\text{AG})$$

Note: Award **(M1)** for substituting $x = 1$ into their $f'(x)$. **(A1)** for $-10 + 3$.

-7 must be seen for **(A1)** to be awarded.

[2 marks]

$$\text{d, ii } 4 = -7 \times 1 + c \quad 11 = c \quad (\text{A1})$$

$$y = -7x + 11 \quad (\text{A1})$$

[2 marks]

$$\text{e. Point of intersection is R}(-0.5, 14.5) \quad (\text{A1})(\text{ft})(\text{A1})(\text{ft})(\text{G2})(\text{ft})$$

Notes: Award **(A1)** for the x coordinate, **(A1)** for the y coordinate.

Allow **(ft)** from candidate's (d)(ii) equation and their (b) even with no working seen.

Award **(A1)(ft)(A0)** if brackets not included and not previously penalised.

[2 marks]

Examiners report

a. A large number of candidates were not able to answer this question well because of the first term of the function's negative index. Many candidates who did make an attempt lacked understanding of the topic.

Most candidates could score 4 marks.

b. A large number of candidates were not able to answer this question well because of the first term of the function's negative index. Many candidates who did make an attempt lacked understanding of the topic.

A good number of candidates correctly substituted into the original function.

c, iA large number of candidates were not able to answer this question well because of the first term of the function's negative index. Many candidates who did make an attempt lacked understanding of the topic.

Very few managed to answer this part algebraically. Those candidates who were aware that they could read the values from their GDC gained some easy marks.

c, iiA large number of candidates were not able to answer this question well because of the firstterm of the function's negative index. Many candidates who did make an attempt lacked understanding of the topic.

Proved to be difficult for many candidates as increasing/decreasing intervals were not well understood by many.

d, iA large number of candidates were not able to answer this question well because of the first term of the function's negative index. Many candidates who did make an attempt lacked understanding of the topic.

As this part was linked to part (a), candidates who could not find the correct derivative could not show that the gradient of T was -7 . Many candidates did not realize that they had to substitute into the first derivative. For those who did, finding the equation of T was a simple task.

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e. A large number of candidates were not able to answer this question well because of the first term of the function's negative index. Many candidates who did make an attempt lacked understanding of the topic.

For those who had part (d) (ii) correct, this allowed them to score easy marks. For the others, it proved a difficult task because their equation in (d) would not be a tangent.

A function f is given by $f(x) = (2x + 2)(5 - x^2)$.

The graph of the function $g(x) = 5^x + 6x - 6$ intersects the graph of f .

a. Find the **exact** value of each of the zeros of f .

[3]

b.i.Expand the expression for $f(x)$.

[1]

b.ii. Find $f'(x)$.

[3]

c. Use your answer to part (b)(ii) to find the values of x for which f is increasing.

[3]

d. Draw the graph of f for $-3 \leq x \leq 3$ and $-40 \leq y \leq 20$. Use a scale of 2 cm to represent 1 unit on the x -axis and 1 cm to represent 5 units on the y -axis.

[4]

e. Write down the coordinates of the point of intersection.

[2]

Markscheme

a. $-1, \sqrt{5}, -\sqrt{5}$ (A1)(A1)(A1)

Note: Award (A1) for -1 and each exact value seen. Award at most (A1)(A0)(A1) for use of $2.23606\dots$ instead of $\sqrt{5}$.

[3 marks]

b.i. $10x - 2x^3 + 10 - 2x^2$ (A1)

Notes: The expansion may be seen in part (b)(ii).

[1 mark]

b.ii. $10 - 6x^2 - 4x$ (A1)(ft)(A1)(ft)(A1)(ft)

Notes: Follow through from part (b)(i). Award (A1)(ft) for each correct term. Award at most (A1)(ft)(A1)(ft)(A0) if extra terms are seen.

[3 marks]

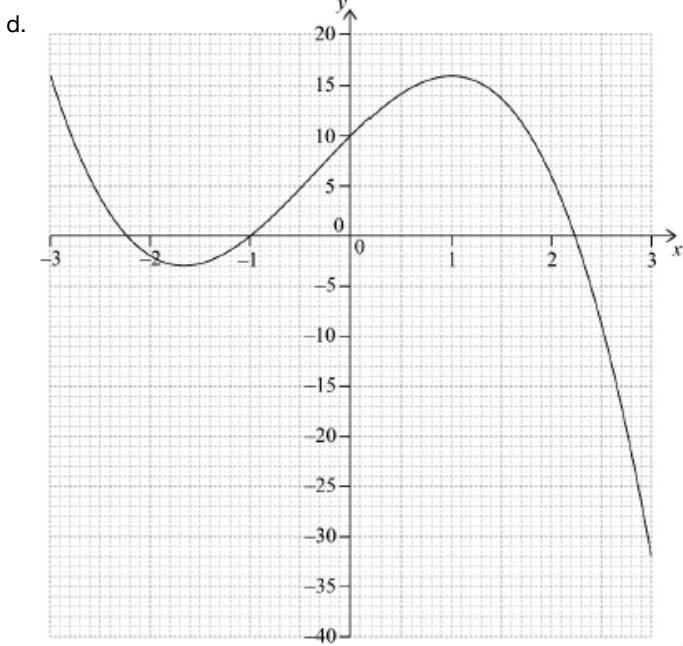
c. $10 - 6x^2 - 4x > 0$ (M1)

Notes: Award (M1) for their $f'(x) > 0$. Accept equality or weak inequality.

$$-1.67 < x < 1 \left(-\frac{5}{3} < x < 1, -1.66666\dots < x < 1 \right) \text{(A1)(ft)(A1)(ft)(G2)}$$

Notes: Award (A1)(ft) for correct endpoints, (A1)(ft) for correct weak or strict inequalities. Follow through from part (b)(ii). Do not award any marks if there is no answer in part (b)(ii).

[3 marks]



(A1)(A1)(ft)(A1)(ft)(A1)

Notes: Award (A1) for correct scale; axes labelled and drawn with a ruler.

Award (A1)(ft) for their correct x -intercepts in approximately correct location.

Award (A1) for correct minimum and maximum points in approximately correct location.

Award (A1) for a smooth continuous curve with approximate correct shape. The curve should be in the given domain.

Follow through from part (a) for the x -intercepts.

[4 marks]

e. $(1.49, 13.9)$ ((1.48702..., 13.8714...)) (G1)(ft)(G1)(ft)

Notes: Award (G1) for 1.49 and (G1) for 13.9 written as a coordinate pair. Award at most (G0)(G1) if parentheses are missing. Accept $x = 1.49$ and $y = 13.9$. Follow through from part (b)(i).

[2 marks]

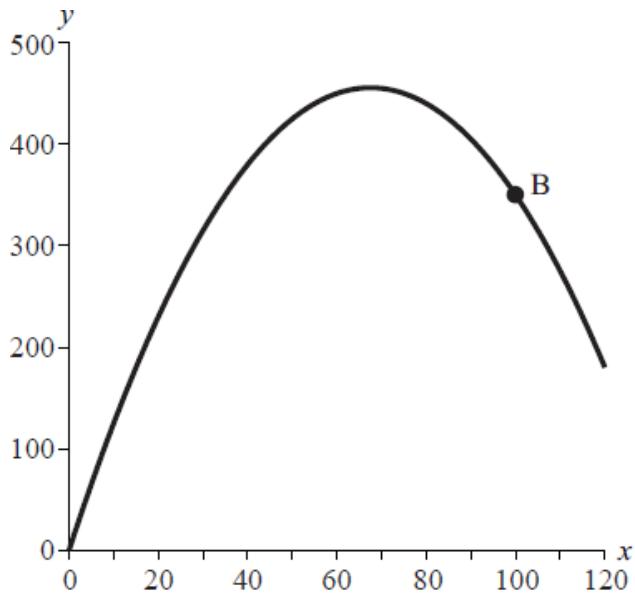
Examiners report

- a. [N/A]
- b.i. [N/A]
- b.ii. [N/A]
- c. [N/A]
- d. [N/A]
- e. [N/A]

The diagram shows an **aerial** view of a bicycle track. The track can be modelled by the quadratic function

$$y = \frac{-x^2}{10} + \frac{27}{2}x, \text{ where } x \geq 0, y \geq 0$$

(x, y) are the coordinates of a point x metres east and y metres north of O , where O is the origin $(0, 0)$. B is a point on the bicycle track with coordinates $(100, 350)$.



- a. The coordinates of point A are (75, 450). Determine whether point A is on the bicycle track. Give a reason for your answer. [3]
- b. Find the derivative of $y = \frac{-x^2}{10} + \frac{27}{2}x$. [2]
- c. Use the answer in part (b) to determine if A (75, 450) is the point furthest north on the track between O and B. Give a reason for your answer. [4]
- d. (i) Write down the midpoint of the line segment OB.
[3]
- (ii) Find the gradient of the line segment OB.
- e. Scott starts from a point C(0,150) . He hikes along a straight road towards the bicycle track, parallel to the line segment OB. [3]
- Find the equation of Scott's road. Express your answer in the form $ax + by = c$, where a, b and $c \in \mathbb{R}$.
- f. Use your graphic display calculator to find the coordinates of the point where Scott first crosses the bicycle track. [2]

Markscheme

a. $y = -\frac{75^2}{10} + \frac{27}{2} \times 75 \quad (\text{M1})$

Note: Award **(M1)** for substitution of 75 in the formula of the function.

$= 450 \quad (\text{A1})$

Yes, point A is on the bike track. **(A1)**

Note: Do not award the final **(A1)** if correct working is not seen.

b. $\frac{dy}{dx} = -\frac{2x}{10} + \frac{27}{2} \left(\frac{dy}{dx} = -0.2x + 13.5 \right) \quad (\text{A1})(\text{A1})$

Notes: Award **(A1)** for each correct term. If extra terms are seen award at most **(A1)(A0)**. Accept equivalent forms.

c. $-\frac{2x}{10} + \frac{27}{2} = 0 \quad (\text{M1})$

Note: Award **(M1)** for equating their derivative from part (b) to zero.

$$x = 67.5 \quad (\text{A1})(\text{ft})$$

Note: Follow through from their derivative from part (b).

$$(\text{Their}) 67.5 \neq 75 \quad (\text{R1})$$

Note: Award **(R1)** for a comparison of their 67.5 with 75. Comparison may be implied (eg 67.5 is the x -coordinate of the furthest north point).

OR

$$\frac{dy}{dx} = -\frac{2 \times (75)}{10} + \frac{27}{2} \quad (\text{M1})$$

Note: Award **(M1)** for substitution of 75 into their derivative from part (b).

$$= -1.5 \quad (\text{A1})(\text{ft})$$

Note: Follow through from their derivative from part (b).

$$(\text{Their}) -1.5 \neq 0 \quad (\text{R1})$$

Note: Award **(R1)** for a comparison of their -1.5 with 0. Comparison may be implied (eg The gradient of the parabola at the furthest north point (vertex) is 0).

Hence A is not the furthest north point. **(A1)(ft)**

Note: Do not award **(R0)(A1)(ft)**. Follow through from their derivative from part (b).

d. (i) $M(50, 175) \quad (\text{A1})$

Note: If parentheses are omitted award **(A0)**. Accept $x = 50, y = 175$.

(ii) $\frac{350-0}{100-0} \quad (\text{M1})$

Note: Award **(M1)** for correct substitution in gradient formula.

$$= 3.5 \left(\frac{350}{100}, \frac{7}{2} \right) \quad (\text{A1})(\text{ft})(\text{G2})$$

Note: Follow through from (d)(i) if midpoint is used to calculate gradient. Award **(G1)(G0)** for answer $3.5x$ without working.

e. $y = 3.5x + 150 \quad (\text{A1})(\text{ft})(\text{A1})(\text{ft})$

Note: Award **(A1)(ft)** for using their gradient from part (d), **(A1)(ft)** for correct equation of line.

$$3.5x - y = -150 \text{ or } 7x - 2y = -300 \text{ (or equivalent)} \quad (\text{A1})(\text{ft})$$

Note: Award **(A1)(ft)** for expressing their equation in the form $ax + by = c$.

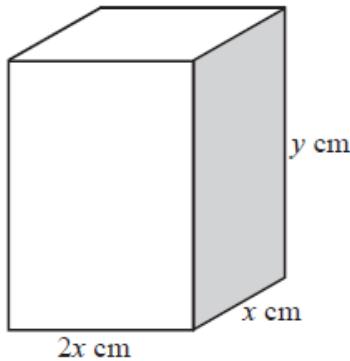
- f. (18.4, 214) (18.3772..., 214.320...) **(A1)(ft)(A1)(ft)(G2)(ft)**

Notes: Follow through from their equation in (e). Coordinates must be positive for follow through marks to be awarded. If parentheses are omitted and not already penalized in (d)(i) award at most **(A0)(A1)(ft)**. If coordinates of the two intersection points are given award **(A0)(A1)(ft)**. Accept $x = 18.4$, $y = 214$.

Examiners report

- a. [N/A]
 - b. [N/A]
 - c. [N/A]
 - d. [N/A]
 - e. [N/A]
 - f. [N/A]
-

A closed rectangular box has a height y cm and width x cm. Its length is twice its width. It has a fixed outer surface area of 300 cm^2 .



i.a. Factorise $3x^2 + 13x - 10$.

[2]

i.b. Solve the equation $3x^2 + 13x - 10 = 0$.

[2]

i.c. Consider a function $f(x) = 3x^2 + 13x - 10$.

[2]

Find the equation of the axis of symmetry on the graph of this function.

i.d. Consider a function $f(x) = 3x^2 + 13x - 10$.

[2]

Calculate the minimum value of this function.

ii.a. Show that $4x^2 + 6xy = 300$.

[2]

ii.b. Find an expression for y in terms of x .

[2]

ii.c. Hence show that the volume V of the box is given by $V = 100x - \frac{4}{3}x^3$.

[2]

ii.d. Find $\frac{dV}{dx}$.

[2]

ii.e.(i) Hence find the value of x and of y required to make the volume of the box a maximum.

[5]

(ii) Calculate the maximum volume.

Markscheme

i.a. $(3x - 2)(x + 5) \quad (\text{A1})(\text{A1})$

[2 marks]

i.b. $(3x - 2)(x + 5) = 0$

$x = \frac{2}{3}$ or $x = -5 \quad (\text{A1})(\text{ft})(\text{A1})(\text{ft})(\text{G2})$

[2 marks]

i.c. $x = \frac{-13}{6} (-2.17) \quad (\text{A1})(\text{A1})(\text{ft})(\text{G2})$

Note: **(A1)** is for $x =$, **(A1)** for value. **(ft)** if value is half way between roots in (b).

[2 marks]

i.d. Minimum $y = 3\left(\frac{-13}{6}\right)^2 + 13\left(\frac{-13}{6}\right) - 10 \quad (\text{M1})$

Note: **(M1)** for substituting their value of x from (c) into $f(x)$.

$= -24.1 \quad (\text{A1})(\text{ft})(\text{G2})$

[2 marks]

ii.a. Area $= 2(2x)x + 2xy + 2(2x)y \quad (\text{M1})(\text{A1})$

Note: **(M1)** for using the correct surface area formula (which can be implied if numbers in the correct place). **(A1)** for using correct numbers.

$300 = 4x^2 + 6xy \quad (\text{AG})$

Note: Final line must be seen or previous **(A1)** mark is lost.

[2 marks]

ii.b. $6xy = 300 - 4x^2 \quad (\text{M1})$

$y = \frac{300-4x^2}{6x} \text{ or } \frac{150-2x^2}{3x} \quad (\text{A1})$

[2 marks]

ii.c. Volume $= x(2x)y \quad (\text{M1})$

$V = 2x^2 \left(\frac{300-4x^2}{6x} \right) \quad (\text{A1})(\text{ft})$

$= 100x - \frac{4}{3}x^3 \quad (\text{AG})$

Note: Final line must be seen or previous **(A1)** mark is lost.

[2 marks]

ii.d. $\frac{dV}{dx} = 100 - \frac{12x^2}{3} \text{ or } 100 - 4x^2 \quad (\text{A1})(\text{A1})$

Note: **(A1)** for each term.

[2 marks]

ii.e. **Unit penalty (UP)** is applicable where indicated in the left hand column

(i) For maximum $\frac{dV}{dx} = 0$ or $100 - 4x^2 = 0$ (M1)

$x = 5$ (A1)(ft)

$y = \frac{300 - 4(5)^2}{6(5)}$ or $\left(\frac{150 - 2(5)^2}{3(5)} \right)$ (M1)

$= \frac{20}{3}$ (A1)(ft)

(UP) (ii) $333\frac{1}{3}$ cm³ (333 cm³)

Note: (ft) from their (e)(i) if working for volume is seen.

[5 marks]

Examiners report

i.a. Most candidates made a good attempt to factorise the expression.

i.b. Many gained both marks here from a correct answer or ft from the previous part.

i.c. Many used the formula correctly. Some forgot to put $x =$.

i.d. Most candidates found this value from their GDC.

ii.a. A good attempt was made to show the correct surface area.

ii.b. Many could rearrange the equation correctly.

ii.c. Although this was not a difficult question it probably looked complicated for the candidates and it was often left out.

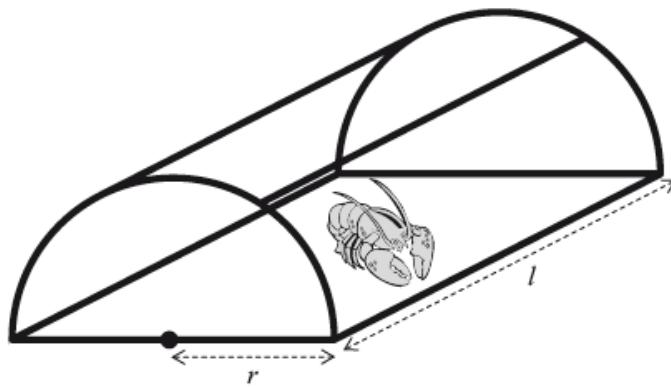
ii.d. Those who reached this length could usually manage the differentiation.

ii.e.(i) Many found the correct value of x but not of y .

(ii) This was well done and again the units were included in most scripts.

A lobster trap is made in the shape of half a cylinder. It is constructed from a steel frame with netting pulled tightly around it. The steel frame consists of a rectangular base, two semicircular ends and two further support rods, as shown in the following diagram.

diagram not to scale



The semicircular ends each have radius r and the support rods each have length l .

Let T be the total length of steel used in the frame of the lobster trap.

- a. Write down an expression for T in terms of r , l and π .

[3]

- b. The volume of the lobster trap is 0.75 m^3 .

[3]

Write down an equation for the volume of the lobster trap in terms of r , l and π .

- c. The volume of the lobster trap is 0.75 m^3 .

[2]

Show that $T = (2\pi + 4)r + \frac{6}{\pi r^2}$.

- d. The volume of the lobster trap is 0.75 m^3 .

[3]

Find $\frac{dT}{dr}$.

- e. The lobster trap is designed so that the length of steel used in its frame is a minimum.

[2]

Show that the value of r for which T is a minimum is 0.719 m , correct to three significant figures.

- f. The lobster trap is designed so that the length of steel used in its frame is a minimum.

[2]

Calculate the value of l for which T is a minimum.

- g. The lobster trap is designed so that the length of steel used in its frame is a minimum.

[2]

Calculate the minimum value of T .

Markscheme

- a. $2\pi r + 4r + 4l \quad (\text{A1})(\text{A1})(\text{A1})$

Notes: Award **(A1)** for $2\pi r$ (" π " must be seen), **(A1)** for $4r$, **(A1)** for $4l$. Accept equivalent forms. Accept $T = 2\pi r + 4r + 4l$. Award a maximum of **(A1)(A1)(AO)** if extra terms are seen.

[3 marks]

- b. $0.75 = \frac{\pi r^2 l}{2} \quad (\text{A1})(\text{A1})(\text{A1})$

Notes: Award **(A1)** for their formula equated to 0.75, **(A1)** for l substituted into volume of cylinder formula, **(A1)** for volume of cylinder formula divided by 2.

If “ π ” not seen in part (a) accept use of 3.14 or greater accuracy. Award a maximum of **(A1)(A1)(AO)** if extra terms are seen.

[3 marks]

$$\begin{aligned} c. \quad T &= 2\pi r + 4r + 4 \left(\frac{1.5}{\pi r^2} \right) \quad (\mathbf{A1})(\mathbf{ft})(\mathbf{A1}) \\ &= (2\pi + 4)r + \frac{6}{\pi r^2} \quad (\mathbf{AG}) \end{aligned}$$

Notes: Award **(A1)(ft)** for correct rearrangement of their volume formula in part (b) seen, award **(A1)** for the correct substituted formula for T . The final line must be seen, with no incorrect working, for this second **(A1)** to be awarded.

[2 marks]

$$d. \quad \frac{dT}{dr} = 2\pi + 4 - \frac{12}{\pi r^3} \quad (\mathbf{A1})(\mathbf{A1})(\mathbf{A1})$$

Note: Award **(A1)** for $2\pi + 4$, **(A1)** for $\frac{-12}{\pi}$, **(A1)** for r^{-3} .

Accept 10.3 (10.2832...) for $2\pi + 4$, accept $-3.82 - 3.81971\dots$ for $\frac{-12}{\pi}$. Award a maximum of **(A1)(A1)(AO)** if extra terms are seen.

[3 marks]

$$e. \quad 2\pi + 4 - \frac{12}{\pi r^3} = 0 \quad \text{OR} \quad \frac{dT}{dr} = 0 \quad (\mathbf{M1})$$

Note: Award **(M1)** for setting their derivative equal to zero.

$$r = 0.718843\dots \quad \text{OR} \quad \sqrt[3]{0.371452\dots} \quad \text{OR} \quad \sqrt[3]{\frac{12}{\pi(2\pi+4)}} \quad \text{OR} \quad \sqrt[3]{\frac{3.81971}{10.2832\dots}} \quad (\mathbf{A1})$$

$$r = 0.719 \text{ (m)} \quad (\mathbf{AG})$$

Note: The rounded and unrounded or formulaic answers must be seen for the final **(A1)** to be awarded. The use of 3.14 gives an unrounded answer of $r = 0.719039\dots$

[2 marks]

$$f. \quad 0.75 = \frac{\pi \times (0.719)^2 l}{2} \quad (\mathbf{M1})$$

Note: Award **(M1)** for substituting 0.719 into their volume formula. Follow through from part (b).

$$l = 0.924 \text{ (m)} \quad (0.923599\dots) \quad (\mathbf{A1})(\mathbf{ft})(\mathbf{G2})$$

[2 marks]

$$g. \quad T = (2\pi + 4) \times 0.719 + \frac{6}{\pi(0.719)^2} \quad (\mathbf{M1})$$

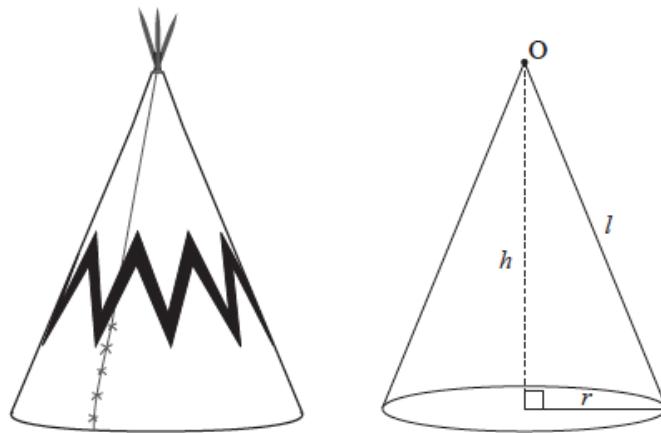
Notes: Award **(M1)** for substituting 0.719 in their expression for T . Accept alternative methods, for example substitution of their l and 0.719 into their part (a) (for which the answer is 11.08961024). Follow through from their answer to part (a).

$$= 11.1 \text{ (m)} \quad (11.0880\dots) \quad (\mathbf{A1})(\mathbf{ft})(\mathbf{G2})$$

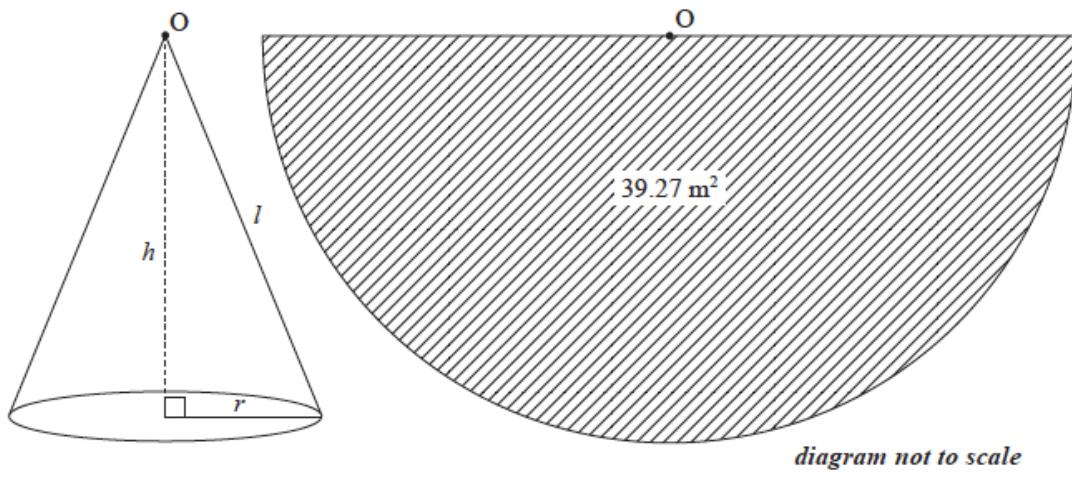
Examiners report

- a. [N/A]
- b. [N/A]
- c. [N/A]
- d. [N/A]
- e. [N/A]
- f. [N/A]
- g. [N/A]

Tepees were traditionally used by nomadic tribes who lived on the Great Plains of North America. They are cone-shaped dwellings and can be modelled as a cone, with vertex O, shown below. The cone has radius, r , height, h , and slant height, l .



A model tepee is displayed at a Great Plains exhibition. The curved surface area of this tepee is covered by a piece of canvas that is 39.27 m^2 , and has the shape of a semicircle, as shown in the following diagram.



- a. Show that the slant height, l , is 5 m, correct to the nearest metre.

[2]

- b. (i) Find the circumference of the base of the cone.

[6]

- (ii) Find the radius, r , of the base.

- (iii) Find the height, h . [1]
- c. A company designs cone-shaped tents to resemble the traditional tepees.
- These cone-shaped tents come in a range of sizes such that the sum of the diameter and the height is equal to **9.33 m**.
- Write down an expression for the height, h , in terms of the radius, r , of these cone-shaped tents.

- d. A company designs cone-shaped tents to resemble the traditional tepees. [1]

These cone-shaped tents come in a range of sizes such that the sum of the diameter and the height is equal to **9.33 m**.

Show that the volume of the tent, V , can be written as

$$V = 3.11\pi r^2 - \frac{2}{3}\pi r^3.$$

- e. A company designs cone-shaped tents to resemble the traditional tepees. [2]
- These cone-shaped tents come in a range of sizes such that the sum of the diameter and the height is equal to **9.33 m**.
- Find $\frac{dV}{dr}$.
- f. A company designs cone-shaped tents to resemble the traditional tepees. [4]

These cone-shaped tents come in a range of sizes such that the sum of the diameter and the height is equal to **9.33 m**.

- (i) Determine the exact value of r for which the volume is a maximum.
(ii) Find the maximum volume.

Markscheme

a. $\frac{\pi l^2}{2} = 39.27 \quad (\text{M1})(\text{A1})$

Note: Award **(M1)** for equating the formula for area of a semicircle to 39.27, award **(A1)** for correct substitution of l into the formula for area of a semicircle.

$l = 5 \text{ (m)} \quad (\text{AG})$

b. (i) $5 \times \pi \quad (\text{M1})$

$= 15.7 \quad (15.7079\dots, 5\pi) \text{ (m)} \quad (\text{A1})(\text{G2})$

(ii) $2\pi r = 15.7079\dots \quad \text{OR} \quad 5\pi r = 39.27 \quad (\text{M1})$

$(r =) 2.5 \text{ (m)} \quad (\text{A1})(\text{ft})(\text{G2})$

Note: Follow through from part (b)(i).

(iii) $(h^2 =) 5^2 - 2.5^2 \quad (\text{M1})$

Notes: Award **(M1)** for correct substitution into Pythagoras' theorem. Follow through from part (b)(ii).

$(h =) 4.33 \quad (4.33012\dots) \text{ (m)} \quad (\text{A1})(\text{ft})(\text{G2})$

c. $9.33 - 2 \times r \quad (\text{A1})$

d. $V = \frac{\pi r^2}{3} \times (9.33 - 2r) \quad (\text{M1})$

Note: Award **(M1)** for correct substitution in the volume formula.

$$V = 3.11\pi r^2 - \frac{2}{3}\pi r^3 \quad (\text{AG})$$

e. $6.22\pi r - 2\pi r^2 \quad (\text{A1})(\text{A1})$

Notes: Award **(A1)** for $6.22\pi r$, **(A1)** for $-2\pi r^2$.

If extra terms present, award at most **(A1)(AO)**.

f. (i) $6.22\pi r - 2\pi r^2 = 0 \quad (\text{M1})$

Note: Award **(M1)** for setting their derivative from part (e) to 0.

$$r = 3.11 \text{ (m)} \quad (\text{A1})(\text{ft})(\text{G2})$$

Notes: Award **(A1)** for identifying 3.11 as the answer.

Follow through from their answer to part (e).

(ii) $\frac{1}{3}\pi(3.11)^3 \quad \text{OR} \quad 3.11\pi(3.11)^2 - \frac{2}{3}\pi(3.11)^3 \quad (\text{M1})$

Note: Award **(M1)** for correct substitution into the correct volume formula.

$$31.5 \text{ (m}^3\text{)}(31.4999\dots) \quad (\text{A1})(\text{ft})(\text{G2})$$

Note: Follow through from their answer to part (f)(i).

Examiners report

- a. [N/A]
- b. [N/A]
- c. [N/A]
- d. [N/A]
- e. [N/A]
- f. [N/A]

The function $f(x)$ is defined by $f(x) = 1.5x + 4 + \frac{6}{x}$, $x \neq 0$.

a. Write down the equation of the vertical asymptote.

[2]

b. Find $f'(x)$.

[3]

c. Find the gradient of the graph of the function at $x = -1$.

[2]

d. Using your answer to part (c), decide whether the function $f(x)$ is increasing or decreasing at $x = -1$. Justify your answer.

[2]

e. Sketch the graph of $f(x)$ for $-10 \leq x \leq 10$ and $-20 \leq y \leq 20$.

[4]

f. P_1 is the local maximum point and P_2 is the local minimum point on the graph of $f(x)$.

[4]

Using your graphic display calculator, write down the coordinates of

(i) P_1 ;

(ii) P_2 .

g. Using your sketch from (e), determine the range of the function $f(x)$ for $-10 \leq x \leq 10$.

[3]

Markscheme

a. $x = 0$ **(A1)(A1)**

Note: Award **(A1)** for $x = \text{constant}$, **(A1)** for 0.

[2 marks]

b. $f'(x) = 1.5 - \frac{6}{x^2}$ **(A1)(A1)(A1)**

Notes: Award **(A1)** for 1.5, **(A1)** for -6 , **(A1)** for x^{-2} . Award **(A1)(A1)(AO)** at most if any other term present.

[3 marks]

c. $1.5 - \frac{6}{(-1)} = \mathbf{(M1)}$

$= -4.5$ **(A1)(ft)(G2)**

Note: Follow through from their derivative function.

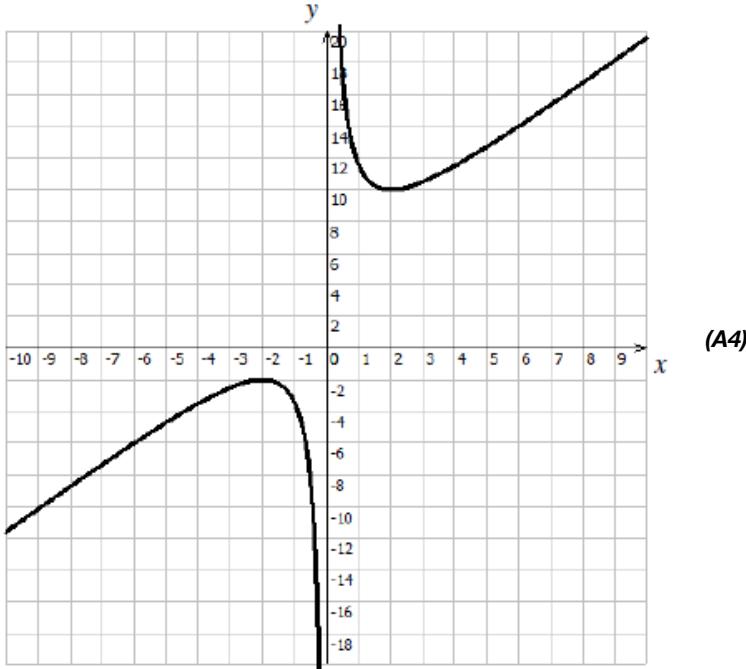
[2 marks]

d. Decreasing, the derivative (gradient or slope) is negative (at $x = -1$) **(A1)(R1)(ft)**

Notes: Do not award **(A1)(R0)**. Follow through from their answer to part (c).

[2 marks]

e.



(A4)

Notes: Award **(A1)** for labels and some indication of scales and an appropriate window.

Award **(A1)** for correct shape of the two unconnected, and smooth branches.

Award **(A1)** for the maximum and minimum points in the approximately correct positions.

Award **(A1)** for correct asymptotic behaviour at $x = 0$.

Notes: Please be rigorous.

The axes need not be drawn with a ruler.

The branches must be smooth and single continuous lines that do not deviate from their proper direction.

The max and min points must be symmetrical about point $(0, 4)$.

The y -axis must be an asymptote for **both** branches.

[4 marks]

f. (i) $(-2, -2)$ or $x = -2, y = -2$ **(G1)(G1)**

(ii) $(2, 10)$ or $x = 2, y = 10$ **(G1)(G1)**

[4 marks]

g. $\{ -2 \geq y \}$ or $\{ y \geq 10 \}$ **(A1)(A1)(ft)(A1)**

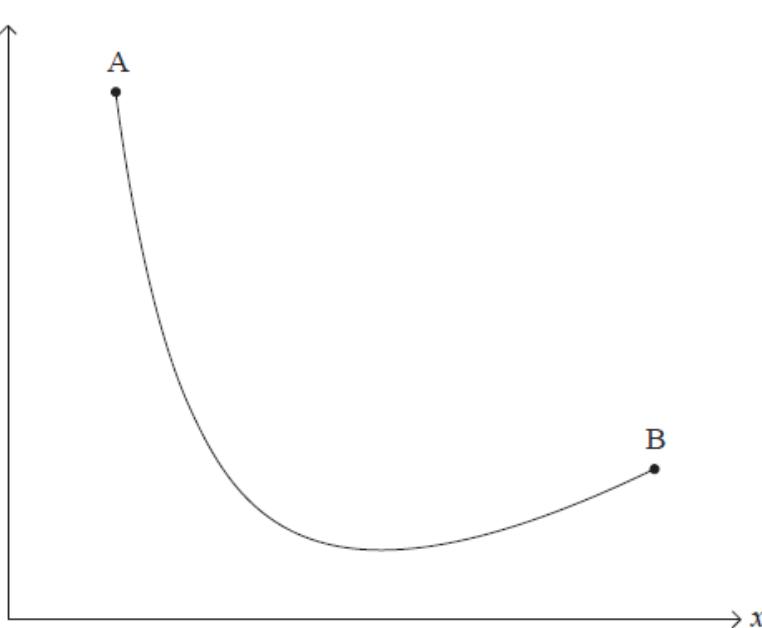
Notes: **(A1)(ft)** for $y > 10$ or $y \geq 10$. **(A1)(ft)** for $y < -2$ or $y \leq -2$. **(A1)** for weak (non-strict) inequalities used in **both** of the above. Follow through from their (e) and (f).

[3 marks]

Examiners report

- a. Part a) was either answered well or poorly.
- b. Most candidates found the first term of the derivative in part b) correctly, but the rest of the terms were incorrect.
- c. The gradient in c) was for the most part correctly calculated, although some candidates substituted incorrectly in $f(x)$ instead of in $f'(x)$.
- d. Part d) had mixed responses.
- e. Lack of labels of the axes, appropriate scale, window, incorrect maximum and minimum and incorrect asymptotic behaviour were the main problems with the sketches in e).
- f. Part f) was also either answered correctly or entirely incorrectly. Some candidates used the trace function on the GDC instead of the min and max functions, and thus acquired coordinates with unacceptable accuracy. Some were unclear that a point of local maximum may be positioned on the coordinate system “below” the point of local minimum, and exchanged the pairs of coordinates of those points in f(i) and f(ii).
- g. Very few candidates were able to identify the range of the function in (g) irrespective of whether or not they had the sketches drawn correctly.

The graph of the function $f(x) = \frac{14}{x} + x - 6$, for $1 \leq x \leq 7$ is given below.



a. Calculate $f(1)$.

[2]

b. Find $f'(x)$.

[3]

c. Use your answer to part (b) to show that the x -coordinate of the local minimum point of the graph of f is 3.7 correct to 2 significant figures.

[3]

d. Find the range of f .

[3]

e. Points A and B lie on the graph of f . The x -coordinates of A and B are 1 and 7 respectively.

[1]

Write down the y -coordinate of B.

f. Points A and B lie on the graph of f . The x -coordinates of A and B are 1 and 7 respectively.

[2]

Find the gradient of the straight line passing through A and B.

g. M is the midpoint of the line segment AB.

[2]

Write down the coordinates of M.

h. L is the tangent to the graph of the function $y = f(x)$, at the point on the graph with the same x -coordinate as M.

[2]

Find the gradient of L.

i. Find the equation of L. Give your answer in the form $y = mx + c$.

[3]

Markscheme

a. $\frac{14}{(1)} + (1) - 6 \quad (\text{M1})$

Note: Award **(M1)** for substituting $x = 1$ into f .

$= 9 \quad (\text{A1})(\text{G2})$

b. $-\frac{14}{x^2} + 1 \quad (\text{A3})$

Note: Award **(A1)** for -14 , **(A1)** for $\frac{14}{x^2}$ or for x^{-2} , **(A1)** for 1 .

Award at most **(A2)** if any extra terms are present.

c. $-\frac{14}{x^2} + 1 = 0$ or $f'(x) = 0$ **(M1)**

Note: Award **(M1)** for equating **their** derivative in part (b) to 0.

$\frac{14}{x^2} = 1$ or $x^2 = 14$ or equivalent **(M1)**

Note: Award **(M1)** for correct rearrangement of their equation.

$x = 3.74165\dots(\sqrt{14})$ **(A1)**

$x = 3.7$ **(AG)**

Notes: Both the unrounded and rounded answers must be seen to award the **(A1)**. This is a “show that” question; appeals to their GDC are not accepted –award a maximum of **(M1)(M0)(AO)**.

Specifically, $-\frac{14}{x^2} + 1 = 0$ followed by $x = 3.74165\dots$, $x = 3.7$ is awarded **(M1)(M0)(AO)**.

d. $1.48 \leq y \leq 9$ **(A1)(A1)(ft)(A1)**

Note: Accept alternative notations, for example $[1.48, 9]$. ($x = \sqrt{14}$ leads to answer $1.48331\dots$)

Note: Award **(A1)** for $1.48331\dots$ seen, accept $1.48378\dots$ from using the given answer $x = 3.7$, **(A1)(ft)** for their 9 from part (a) seen, **(A1)** for the correct notation for their interval (accept $\leq y \leq$ or $\leq f \leq$).

e. 3 **(A1)**

Note: Do not accept a coordinate pair.

f. $\frac{3-9}{7-1}$ **(M1)**

Note: Award **(M1)** for their correct substitution into the gradient formula.

$= -1$ **(A1)(ft)(G2)**

Note: Follow through from their answers to parts (a) and (e).

g. $(4, 6)$ **(A1)(ft)(A1)**

Note: Accept $x = 4$, $y = 6$. Award at most **(A1)(AO)** if parentheses not seen.

If coordinates reversed award **(AO)(A1)(ft)**.

Follow through from their answers to parts (a) and (e).

h. $-\frac{14}{4^2} + 1$ **(M1)**

Note: Award **(M1)** for substitution into their gradient function.

Follow through from their answers to parts (b) and (g).

$$= \frac{1}{8}(0.125) \quad (\mathbf{A1})(\mathbf{ft})(\mathbf{G2})$$

i. $y - 1.5 = \frac{1}{8}(x - 4)$ **(M1)(ft)(M1)**

Note: Award **(M1)** for substituting their (4, 1.5) in any straight line formula,

(M1) for substituting their gradient in any straight line formula.

$$y = \frac{x}{8} + 4 \quad (\mathbf{A1})(\mathbf{ft})(\mathbf{G2})$$

Note: The form of the line has been specified in the question.

Examiners report

- a. Most candidates were able to evaluate the function and find the derivative for $x + 6$ but the term with the negative index was problematic. The few candidates who equated their derivative to zero at the local minimum point progressed well and showed a thorough understanding of the differential calculus. Many did not attain full marks for the range of the function, either confusing this with the statistical concept of range or using the y -coordinate at B. Most were able to find the gradient and midpoint of the straight line passing through A and B. The final parts were also challenging for the majority: many had difficulty finding the gradient of the tangent L, instead using the slope formula for a straight line; the most common error in part (i) was to substitute in the coordinates of midpoint M rather than the point on the curve. Greater insight into the problem would have come from using the given sketch of the curve and annotating it; it seems that many candidates do not link the algebraic nature of the differential calculus with the curve in question.
- b. Most candidates were able to evaluate the function and find the derivative for $x + 6$ but the term with the negative index was problematic. The few candidates who equated their derivative to zero at the local minimum point progressed well and showed a thorough understanding of the differential calculus. Many did not attain full marks for the range of the function, either confusing this with the statistical concept of range or using the y -coordinate at B. Most were able to find the gradient and midpoint of the straight line passing through A and B. The final parts were also challenging for the majority: many had difficulty finding the gradient of the tangent L, instead using the slope formula for a straight line; the most common error in part (i) was to substitute in the coordinates of midpoint M rather than the point on the curve. Greater insight into the problem would have come from using the given sketch of the curve and annotating it; it seems that many candidates do not link the algebraic nature of the differential calculus with the curve in question.
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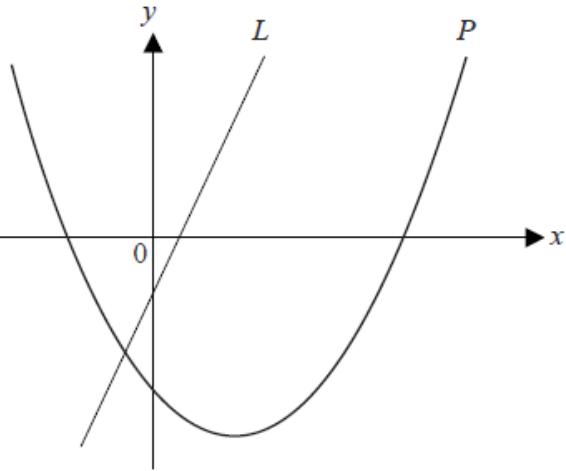
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The diagram below shows the graph of a line L passing through $(1, 1)$ and $(2, 3)$ and the graph P of the function $f(x) = x^2 - 3x - 4$



- a. Find the gradient of the line L .

[2]

- b. Differentiate $f(x)$. [2]
- c. Find the coordinates of the point where the tangent to P is parallel to the line L . [3]
- d. Find the coordinates of the point where the tangent to P is perpendicular to the line L . [4]
- e. Find [3]
- (i) the gradient of the tangent to P at the point with coordinates $(2, -6)$.
 - (ii) the equation of the tangent to P at this point.
- f. State the equation of the axis of symmetry of P . [1]
- g. Find the coordinates of the vertex of P and state the gradient of the curve at this point. [3]

Markscheme

a. for attempt at substituted $\frac{y\text{distance}}{x\text{distance}}$ (M1)

$$\text{gradient} = 2 \quad (\text{A1})(\text{G2})$$

[2 marks]

b. $2x - 3$ (A1)(A1)

(A1) for $2x$, (A1) for -3

[2 marks]

c. for their $2x - 3 =$ their gradient and attempt to solve (M1)

$$x = 2.5 \quad (\text{A1})(\text{ft})$$

$$y = -5.25 \quad (\text{ft}) \text{ from their } x \text{ value} \quad (\text{A1})(\text{ft})(\text{G2})$$

[3 marks]

d. for seeing $\frac{-1}{\text{their}(a)}$ (M1)

$$\text{solving } 2x - 3 = -\frac{1}{2} \text{ (or their value)} \quad (\text{M1})$$

$$x = 1.25 \quad (\text{A1})(\text{ft})(\text{G1})$$

$$y = -6.1875 \quad (\text{A1})(\text{ft})(\text{G1})$$

[4 marks]

e. (i) $2 \times 2 - 3 = 1$ (ft) from (b)) (A1)(ft)(G1)

$$\text{(ii) } y = mx + c \text{ or equivalent method to find } c \Rightarrow -6 = 2 + c \quad (\text{M1})$$

$$y = x - 8 \quad (\text{A1})(\text{ft})(\text{G2})$$

[3 marks]

f. $x = 1.5$ (A1)

[1 mark]

g. for substituting their answer to part (f) into the equation of the parabola $(1.5, -6.25)$ accept $x = 1.5, y = -6.25$ (M1)(A1)(ft)(G2)

gradient is zero (accept $\frac{dy}{dx} = 0$) **(A1)**

[3 marks]

Examiners report

- a. Parts (a) and (b) were very well done. After that, only the stronger candidates were able to cope. The equation of the tangent at the point with coordinates (2, 6) was badly done but some candidates managed to find the equation of the tangent line from their GDC. The equation of the axis of symmetry was reasonably well done although many just wrote down 1.5 instead of $x = 1.5$.

Some forgot to write down that the gradient at the vertex was 0.

- b. Parts (a) and (b) were very well done. After that, only the stronger candidates were able to cope. The equation of the tangent at the point with coordinates (2, 6) was badly done but some candidates managed to find the equation of the tangent line from their GDC. The equation of the axis of symmetry was reasonably well done although many just wrote down 1.5 instead of $x = 1.5$.

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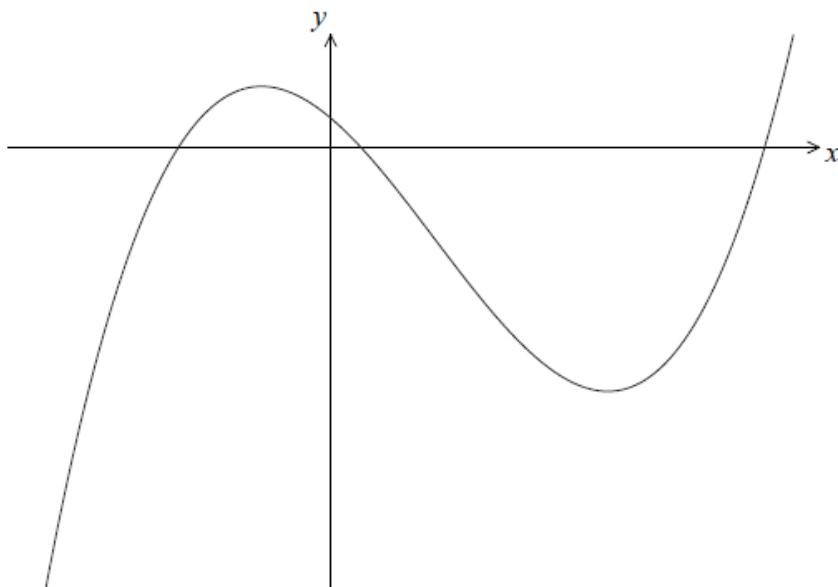
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Some forgot to write down that the gradient at the vertex was 0.

The diagram shows a sketch of the function $f(x) = 4x^3 - 9x^2 - 12x + 3$.

diagram not to scale



- a. Write down the values of x where the graph of $f(x)$ intersects the x -axis.

[3]

- b. Write down $f'(x)$.

[3]

- c. Find the value of the local maximum of $y = f(x)$.

[4]

- d. Let P be the point where the graph of $f(x)$ intersects the y axis.

[1]

Write down the coordinates of P.

- e. Let P be the point where the graph of $f(x)$ intersects the y axis.

[2]

Find the gradient of the curve at P.

- f. The line, L, is the tangent to the graph of $f(x)$ at P.

[2]

Find the equation of L in the form $y = mx + c$.

- g. There is a second point, Q, on the curve at which the tangent to $f(x)$ is parallel to L.

[1]

Write down the gradient of the tangent at Q.

- h. There is a second point, Q, on the curve at which the tangent to $f(x)$ is parallel to L.

[3]

Calculate the x -coordinate of Q.

Markscheme

- a. $-1.10, 0.218, 3.13$ (A1)(A1)(A1)

[3 marks]

- b. $f'(x) = 12x^2 - 18x - 12$ (A1)(A1)(A1)

Note: Award (A1) for each correct term and award maximum of (A1)(A1) if other terms seen.

[3 marks]

c. $f'(x) = 0$ **(M1)**

$x = -0.5, 2$

$x = -0.5$ **(A1)**

Note: If $x = -0.5$ not stated, can be inferred from working below.

$$y = 4(-0.5)^3 - 9(-0.5)^2 - 12(-0.5) + 3 \quad \text{**(M1)**}$$

$$y = 6.25 \quad \text{**(A1)(G3)**}$$

Note: Award **(M1)** for their value of x substituted into $f(x)$.

Award **(M1)(G2)** if sketch shown as method. If coordinate pair given then award **(M1)(A1)(M1)(A0)**. If coordinate pair given with no working award **(G2)**.

[4 marks]

d. $(0, 3)$ **(A1)**

Note: Accept $x = 0, y = 3$.

[1 mark]

e. $f'(0) = -12$ **(M1)(A1)(ft)(G2)**

Note: Award **(M1)** for substituting $x = 0$ into their derivative.

[2 marks]

f. Tangent: $y = -12x + 3$ **(A1)(ft)(A1)(G2)**

Note: Award **(A1)(ft)** for their gradient, **(A1)** for intercept = 3.

Award **(A1)(A0)** if $y =$ not seen.

[2 marks]

g. -12 **(A1)(ft)**

Note: Follow through from their part (e).

[1 mark]

h. $12x^2 - 18x - 12 = -12$ **(M1)**

$$12x^2 - 18x = 0 \quad \text{**(M1)**}$$

$$x = 1.5, 0$$

Note: Award (M1)(G2) for $12x^2 - 18x - 12 = -12$ followed by $x = 1.5$.

Follow through from their part (g).

[3 marks]

Examiners report

- a. This question was either very well done – by the majority – or very poor and incomplete attempts were seen. This would perhaps indicate a lack of preparation in this area of the syllabus from some centres, though it is recognised that the differential calculus is one of the more problematic topics for the candidature.

It was however disappointing to note the number of candidates who do not use the GDC to good effect; in part (a) for example, the zeros were not found accurately due to “trace” being used; this is not a suitable approach – there is a built-in zero finder which should be used. Much of the question was accessible via a GDC approach, a sketch was given that could have been verified on the GDC; this was lost on many.

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Consider the function $f(x) = x^3 - 3x - 24x + 30$.

- a. Write down $f(0)$. [1]
- b. Find $f'(x)$. [3]
- c. Find the gradient of the graph of $f(x)$ at the point where $x = 1$. [2]
- d. (i) Use $f'(x)$ to find the x -coordinate of M and of N.
(ii) Hence or otherwise write down the coordinates of M and of N. [5]
- e. Sketch the graph of $f(x)$ for $-5 \leq x \leq 7$ and $-60 \leq y \leq 60$. Mark clearly M and N on your graph. [4]
- f. Lines L_1 and L_2 are parallel, and they are tangents to the graph of $f(x)$ at points A and B respectively. L_1 has equation $y = 21x + 111$.
(i) Find the x -coordinate of A and of B.
(ii) Find the y -coordinate of B. [6]

Markscheme

a. 30 (A1)

[1 mark]

b. $f'(x) = 3x^2 - 6x - 24$ **(A1)(A1)(A1)**

Note: Award **(A1)** for each term. Award at most **(A1)(A1)** if extra terms present.

[3 marks]

c. $f'(1) = -27$ **(M1)(A1)(ft)(G2)**

Note: Award **(M1)** for substituting $x = 1$ into their derivative.

[2 marks]

d. (i) $f'(x) = 0$

$3x^2 - 6x - 24 = 0$ **(M1)**

$x = 4; x = -2$ **(A1)(ft)(A1)(ft)**

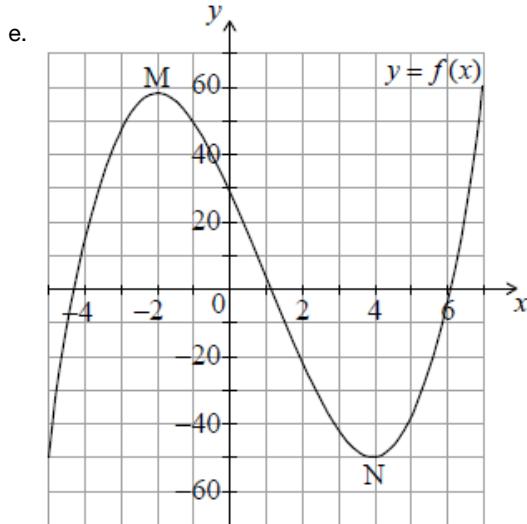
Notes: Award **(M1)** for either $f'(x) = 0$ or $3x^2 - 6x - 24 = 0$ seen. Follow through from their derivative. Do not award the two answer marks if derivative not used.

(ii) M(-2, 58) accept $x = -2, y = 58$ **(A1)(ft)**

N(4, -50) accept $x = 4, y = -50$ **(A1)(ft)**

Note: Follow through from their answer to part (d) (i).

[5 marks]



(A1) for window

(A1) for a smooth curve with the correct shape

(A1) for axes intercepts in approximately the correct positions

(A1) for M and N marked on diagram and in approximately correct position **(A4)**

Note: If window is not indicated award at most **(A0)(A1)(A0)(A1)(ft)**.

[4 marks]

f. (i) $3x^2 - 6x - 24 = 21$ **(M1)**

$3x^2 - 6x - 45 = 0$ **(M1)**

$x = 5$; $x = -3$ **(A1)(ft)(A1)(ft)(G3)**

Note: Follow through from their derivative.

OR

Award **(A1)** for L_1 drawn tangent to the graph of f on their sketch in approximately the correct position ($x = -3$), **(A1)** for a second tangent parallel to their L_1 , **(A1)** for $x = -3$, **(A1)** for $x = 5$. **(A1)(ft)(A1)(ft)(A1)(A1)**

Note: If only $x = -3$ is shown without working award **(G2)**. If both answers are shown irrespective of working award **(G3)**.

(ii) $f(5) = -40$ **(M1)(A1)(ft)(G2)**

Notes: Award **(M1)** for attempting to find the image of their $x = 5$. Award **(A1)** only for $(5, -40)$. Follow through from their x -coordinate of B only if it has been clearly identified in (f) (i).

[6 marks]

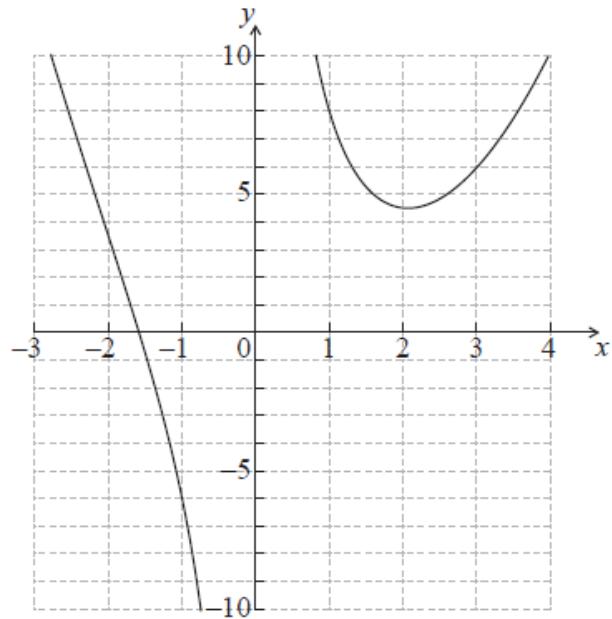
Examiners report

- a. The value of $f(0)$ and the derivative function, $f'(x)$ were well done in parts (a) and (b). In part (c) many candidates found $f(1)$ instead of $f'(1)$. In part (d) many students did not use their $f(x)$ to find the x -coordinates of M and N and instead used their GDC. The sketch was generally well done although some students forgot to label M and N or did not use the specified window. The last part of the question was a clear discriminator. Examiners were pleased to see how this challenging question was solved using alternative methods.
- b. The value of $f(0)$ and the derivative function, $f'(x)$ were well done in parts (a) and (b). In part (c) many candidates found $f(1)$ instead of $f'(1)$. In part (d) many students did not use their $f(x)$ to find the x -coordinates of M and N and instead used their GDC. The sketch was generally well done although some students forgot to label M and N or did not use the specified window. The last part of the question was a clear discriminator. Examiners were pleased to see how this challenging question was solved using alternative methods.
- c. The value of $f(0)$ and the derivative function, $f'(x)$ were well done in parts (a) and (b). In part (c) many candidates found $f(1)$ instead of $f'(1)$. In part (d) many students did not use their $f(x)$ to find the x -coordinates of M and N and instead used their GDC. The sketch was generally well done although some students forgot to label M and N or did not use the specified window. The last part of the question was a clear discriminator. Examiners were pleased to see how this challenging question was solved using alternative methods.
- d. The value of $f(0)$ and the derivative function, $f'(x)$ were well done in parts (a) and (b). In part (c) many candidates found $f(1)$ instead of $f'(1)$. In part (d) many students did not use their $f(x)$ to find the x -coordinates of M and N and instead used their GDC. The sketch was generally well done although some students forgot to label M and N or did not use the specified window. The last part of the question was a clear discriminator. Examiners were pleased to see how this challenging question was solved using alternative methods.
- e. The value of $f(0)$ and the derivative function, $f'(x)$ were well done in parts (a) and (b). In part (c) many candidates found $f(1)$ instead of $f'(1)$. In part (d) many students did not use their $f(x)$ to find the x -coordinates of M and N and instead used their GDC. The sketch was generally well done although some students forgot to label M and N or did not use the specified window. The last part of the question was a clear discriminator. Examiners were pleased to see how this challenging question was solved using alternative methods.

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The diagram shows part of the graph of $f(x) = x^2 - 2x + \frac{9}{x}$, where $x \neq 0$.



a. Write down

[5]

- (i) the equation of the vertical asymptote to the graph of $y = f(x)$;
- (ii) the solution to the equation $f(x) = 0$;
- (iii) the coordinates of the local minimum point.

b. Find $f'(x)$.

[4]

c. Show that $f'(x)$ can be written as $f'(x) = \frac{2x^3 - 2x^2 - 9}{x^2}$.

[2]

d. Find the gradient of the tangent to $y = f(x)$ at the point A(1, 8).

[2]

e. The line, L , passes through the point A and is perpendicular to the tangent at A.

[1]

Write down the gradient of L .

f. The line, L , passes through the point A and is perpendicular to the tangent at A.

[3]

Find the equation of L . Give your answer in the form $y = mx + c$.

g. The line, L , passes through the point A and is perpendicular to the tangent at A.

[2]

L also intersects the graph of $y = f(x)$ at points B and C. Write down the **x-coordinate** of B and of C.

Markscheme

a. (i) $x = 0$ **(A1)(A1)**

Note: Award **(A1)** for $x =$ a constant, **(A1)** for the constant in their equation being 0.

(ii) $-1.58 (-1.58454\dots)$ **(G1)**

Note: Accept -1.6 , do not accept -2 or -1.59 .

(iii) $(2.06, 4.49) (2.06020\dots, 4.49253\dots)$ **(G1)(G1)**

Note: Award at most **(G1)(G0)** if brackets not used. Award **(G0)(G1)(ft)** if coordinates are reversed.

Note: Accept $x = 2.06, y = 4.49$.

Note: Accept 2.1 , do not accept 2.0 or 2 . Accept 4.5 , do not accept 5 or 4.50 .

[5 marks]

b. $f'(x) = 2x - 2 - \frac{9}{x^2}$ **(A1)(A1)(A1)(A1)**

Notes: Award **(A1)** for $2x$, **(A1)** for -2 , **(A1)** for -9 , **(A1)** for x^{-2} . Award a maximum of **(A1)(A1)(A1)(A0)** if there are extra terms present.

[4 marks]

c. $f'(x) = \frac{x^2(2x-2)}{x^2} - \frac{9}{x^2}$ **(M1)**

Note: Award **(M1)** for taking the correct common denominator.

$$= \frac{(2x^3-2x^2)}{x^2} - \frac{9}{x^2} \quad \textbf{(M1)}$$

Note: Award **(M1)** for multiplying brackets or equivalent.

$$= \frac{2x^3-2x^2-9}{x^2} \quad \textbf{(AG)}$$

Note: The final **(M1)** is not awarded if the given answer is not seen.

[2 marks]

d. $f'(1) = \frac{2(1)^3-2(1)-9}{(1)^2}$ **(M1)**

$$= -9 \quad \textbf{(A1)(G2)}$$

Note: Award **(M1)** for substitution into **given** (or their correct) $f'(x)$. There is no follow through for use of their incorrect derivative.

[2 marks]

e. $\frac{1}{9}$ **(A1)(ft)**

Note: Follow through from part (d).

[1 mark]

f. $y - 8 = \frac{1}{9}(x - 1)$ **(M1)(M1)**

Notes: Award **(M1)** for substitution of their gradient from (e), **(M1)** for substitution of given point. Accept all forms of straight line.

$$y = \frac{1}{9}x + \frac{71}{9} (y = 0.111111\dots x + 7.88888\dots) \quad \textbf{(A1)(ft)(G3)}$$

Note: Award the final **(A1)(ft)** for a correctly rearranged formula of **their** straight line in (f). Accept $0.11x$, do not accept $0.1x$. Accept 7.9, do not accept 7.88, do not accept 7.8.

[3 marks]

- g. $-2.50, 3.61 (-2.49545 \dots, 3.60656 \dots)$ **(A1)(ft)(A1)(ft)**

Notes: Follow through from their line L from part (f) even if no working shown. Award at most **(A0)(A1)(ft)** if their correct coordinate pairs given.

Note: Accept -2.5 , do not accept -2.49 . Accept 3.6, do not accept 3.60.

[2 marks]

Examiners report

- a. As usual, the content in this question caused difficulty for many candidates. However, for those with a sound grasp of the topic, there were many very successful attempts. The curve was given so that a comparison could be made to a GDC version and the correct form of the derivative was also given to permit weaker candidates to progress to the latter stages. Unfortunately, some decided to proceed with their own incorrect versions, in which case **very limited follow through accrued**. It should be emphasized to candidates that when an answer is given in this way it should be used in subsequent parts of the question.

As in previous years, much of the question could have been answered successfully by using the GDC. However, it was also clear that a large number of candidates did not attempt either to verify their work with their GDC or to use it in place of an algebraic approach.

Differentiation of terms with negative indices remains a testing process for the majority; it will continue to be tested. Some centres still do not teach the differential calculus.

- b. As usual, the content in this question caused difficulty for many candidates. However, for those with a sound grasp of the topic, there were many very successful attempts. The curve was given so that a comparison could be made to a GDC version and the correct form of the derivative was also given to permit weaker candidates to progress to the latter stages. Unfortunately, some decided to proceed with their own incorrect versions, in which case **very limited follow through accrued**. It should be emphasized to candidates that when an answer is given in this way it should be used in subsequent parts of the question.

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As in previous years, much of the question could have been answered successfully by using the GDC. However, it was also clear that a large number of candidates did not attempt either to verify their work with their GDC or to use it in place of an algebraic approach.

Differentiation of terms with negative indices remains a testing process for the majority; it will continue to be tested. Some centres still do not teach the differential calculus.

Consider the function $g(x) = bx - 3 + \frac{1}{x^2}$, $x \neq 0$.

a. Write down the equation of the vertical asymptote of the graph of $y = g(x)$. [2]

b. Write down $g'(x)$. [3]

c. The line T is the tangent to the graph of $y = g(x)$ at the point where $x = 1$. The gradient of T is 3. [2]

Show that $b = 5$.

d. The line T is the tangent to the graph of $y = g(x)$ at the point where $x = 1$. The gradient of T is 3. [3]

Find the equation of T .

e. Using your graphic display calculator find the coordinates of the point where the graph of $y = g(x)$ intersects the x -axis. [2]

f. (i) Sketch the graph of $y = g(x)$ for $-2 \leq x \leq 5$ and $-15 \leq y \leq 25$, indicating clearly your answer to part (e). [6]

(ii) Draw the line T on your sketch.

g. Using your graphic display calculator find the coordinates of the local minimum point of $y = g(x)$. [2]

h. Write down the interval for which $g(x)$ is increasing in the domain $0 < x < 5$. [2]

Markscheme

a. $x = 0$ **(A1)(A1)**

Notes: Award **(A1)** for x =constant, **(A1)** for 0. Award **(AO)(AO)** if answer is not an equation.

[2 marks]

b. $b - \frac{2}{x^3}$ **(A1)(A1)(A1)**

Note: Award **(A1)** for b , **(A1)** for -2 , **(A1)** for $\frac{1}{x^3}$ (or x^{-3}). Award at most **(A1)(A1)(AO)** if extra terms seen.

[3 marks]

c. $3 = b - \frac{2}{(1)^3}$ **(M1)(M1)**

Note: Award **(M1)** for substituting 1 into their gradient function, **(M1)** for equating their gradient function to 3.

$b = 5$ **(AG)**

Note: Award at most **(M1)(AO)** if final line is not seen or b does not equal 5.

[2 marks]

d. $g(1) = 3$ or $(1, 3)$ (seen or implied from the line below) **(A1)**

$3 = 3 \times 1 + c$ **(M1)**

Note: Award **(M1)** for correct substitution of their point $(1, 3)$ and gradient 3 into equation $y = mx + c$. Follow through from their point of tangency.

$y = 3x$ **(A1)(ft)(G2)**

OR

$$y - 3 = 3(x - 1) \quad (\mathbf{M1})(\mathbf{A1})(\mathbf{ft})(\mathbf{G2})$$

Note: Award **(M1)** for substitution of gradient 3 and their point (1, 3) into $y - y_1 = m(x - x_1)$, **(A1)(ft)** for correct substitutions. Follow through from their point of tangency. Award at most **(A1)(M1)(A0)(ft)** if further incorrect working seen.

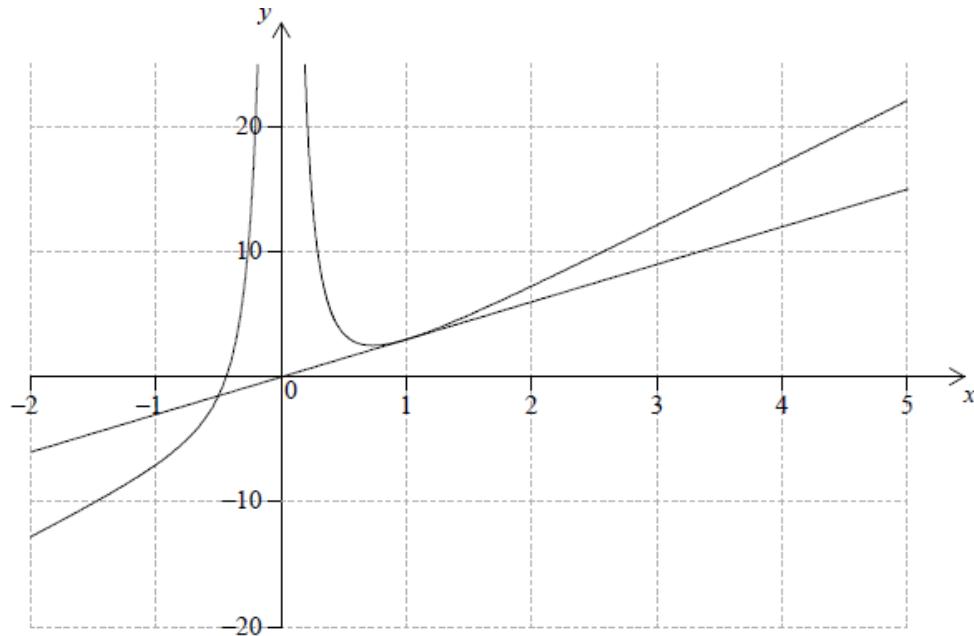
[3 marks]

e. $(-0.439, 0)$ $((-0.438785..., 0))$ **(G1)(G1)**

Notes: If no parentheses award at most **(G1)(G0)**. Accept $x = 0.439, y = 0$.

[2 marks]

f. (i)



Award **(A1)** for labels and some indication of scale in the stated window.

Award **(A1)** for correct general shape (curve must be smooth and must not cross the y-axis)

Award **(A1)(ft)** for x-intercept consistent with their part (e).

Award **(A1)** for local minimum in the first quadrant. **(A1)(A1)(A1)(ft)(A1)**

(ii) Tangent to curve drawn at approximately $x = 1$ **(A1)(A1)**

Note: Award **(A1)** for a line tangent to curve approximately at $x = 1$. Must be a straight line for the mark to be awarded. Award **(A1)(ft)** for line passing through the origin. Follow through from their answer to part (d).

[6 marks]

g. $(0.737, 2.53)$ $((0.736806..., 2.52604...))$ **(G1)(G1)**

Notes: Do not penalize for lack of parentheses if already penalized in (e). Accept $x = 0.737, y = 2.53$.

[2 marks]

h. $0.737 < x < 5$ **OR** $(0.737; 5)$ **(A1)(A1)(ft)**

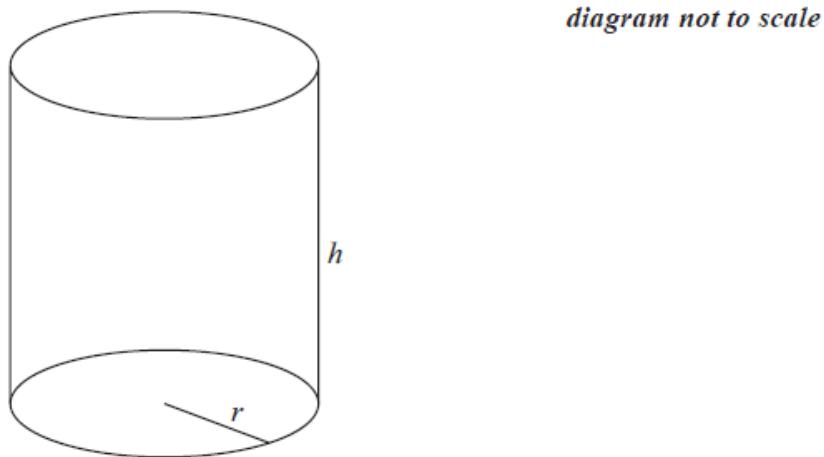
Notes: Award **(A1)** for correct strict or weak inequalities with x seen if the interval is given as inequalities, **(A1)(ft)** for 0.737 and 5 or their value from part (g).

[2 marks]

Examiners report

- a. This question was moderately well answered. The concept of vertical asymptote in part (a) seemed to be problematic for a great number of candidates. In many cases students showed partial understanding of the vertical asymptote but found it difficult to write a correct equation.
- b. This question was moderately well answered. The concept of vertical asymptote in part (a) seemed to be problematic for a great number of candidates. In many cases students showed partial understanding of the vertical asymptote but found it difficult to write a correct equation. Finding the derivative in part (b) proved problematic as well. It seems that the presence of the parameter b in the function may have contributed to this.
- c. This question was moderately well answered. In part (c) a great number of students substituted $b = 5$ in the equation of the function instead of substituting it in the equation of their derivative.
- d. This question was moderately well answered. Very few students used the GDC to find the equation of the tangent at $x = 1$ in part (d).
- e. This question was moderately well answered. Good use of the GDC was seen in part (e), although some students wrote the x -coordinates of the point of intersection and neglected to write the y -coordinate.
- f. This question was moderately well answered. The sketch in part (f) was, for the most part, not well done. Often the axes labels were missing. Very few tangents to the curve at the correct point were seen. Often the intended tangent lines intersected the curve, which shows that candidates either did not know what a tangent is or did not make sense of the sketch.
- g. This question was moderately well answered. Good use of the GDC was shown in part (g) for finding the coordinates of the minimum point.
- h. This question was moderately well answered. Few acceptable answers were given in part (h).

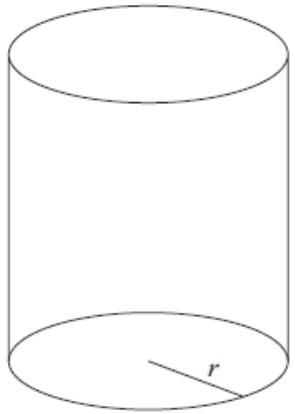
Nadia designs a wastepaper bin made in the shape of an **open** cylinder with a volume of 8000 cm^3 .



Nadia decides to make the radius, r , of the bin 5 cm.

Merryn also designs a cylindrical wastepaper bin with a volume of 8000 cm^3 . She decides to fix the radius of its base so that the **total external surface area** of the bin is minimized.

diagram not to scale



Let the radius of the base of Merryn's wastepaper bin be r , and let its height be h .

a. Calculate

[7]

- (i) the area of the base of the wastepaper bin;
- (ii) the height, h , of Nadia's wastepaper bin;
- (iii) the total **external** surface area of the wastepaper bin.

b. State whether Nadia's design is practical. Give a reason.

[2]

c. Write down an equation in h and r , using the given volume of the bin.

[1]

d. Show that the total external surface area, A , of the bin is $A = \pi r^2 + \frac{16000}{r}$.

[2]

e. Write down $\frac{dA}{dr}$.

[3]

f. (i) Find the value of r that minimizes the total external surface area of the wastepaper bin.

[5]

- (ii) Calculate the value of h corresponding to this value of r .

g. Determine whether Merryn's design is an improvement upon Nadia's. Give a reason.

[2]

Markscheme

a. (i) Area = $\pi(5)^2$ **(M1)**

$$= 78.5 \text{ (cm}^2\text{)} (78.5398\dots) \quad \mathbf{(A1)(G2)}$$

Note: Accept 25π .

(ii) $8000 = 78.5398\dots \times h$ **(M1)**

$$h = 102 \text{ (cm)} (101.859\dots) \quad \mathbf{(A1)(ft)(G2)}$$

Note: Follow through from their answer to part (a)(i).

$$(iii) \text{ Area} = \pi(5)^2 + 2\pi(5)(101.859\dots) \quad \mathbf{(M1)(M1)}$$

Note: Award **(M1)** for their substitution in curved surface area formula, **(M1)** for addition of their two areas.

$$= 3280 \text{ (cm}^2\text{)} (3278.53\dots) \quad (\mathbf{A1})(\mathbf{ft})(\mathbf{G2})$$

Note: Follow through from their answers to parts (a)(i) and (ii).

- b. No, it is too tall/narrow. **(A1)(ft)(R1)**

Note: Follow through from their value for h .

c. $8000 = \pi r^2 h \quad (\mathbf{A1})$

d. $A = \pi r^2 + 2\pi r \left(\frac{8000}{\pi r^2} \right) \quad (\mathbf{A1})(\mathbf{M1})$

Note: Award **(A1)** for correct rearrangement of **their** part (c), **(M1)** for substitution of **their** rearrangement into area formula.

$$= \pi r^2 + \frac{16000}{r} \quad (\mathbf{AG})$$

e. $\frac{dA}{dr} = 2\pi r - 16000r^{-2} \quad (\mathbf{A1})(\mathbf{A1})(\mathbf{A1})$

Note: Award **(A1)** for $2\pi r$, **(A1)** for $-16000r^{-2}$. If an extra term is present award at most **(A1)(A1)(A0)**.

f. (i) $\frac{dA}{dr} = 0 \quad (\mathbf{M1})$

$$2\pi r^3 - 16000 = 0 \quad (\mathbf{M1})$$

$$r = 13.7 \text{ cm } (13.6556\dots) \quad (\mathbf{A1})(\mathbf{ft})$$

Note: Follow through from their part (e).

(ii) $h = \frac{8000}{\pi(13.65\dots)^2} \quad (\mathbf{M1})$

$$= 13.7 \text{ cm } (13.6556\dots) \quad (\mathbf{A1})(\mathbf{ft})$$

Note: Accept 13.6 if 13.7 used.

- g. Yes or No, accompanied by a consistent and sensible reason. **(A1)(R1)**

Note: Award **(A0)(R0)** if no reason is given.

Examiners report

- a. [N/A]
- b. [N/A]
- c. [N/A]
- d. [N/A]
- e. [N/A]
- f. [N/A]
- g. [N/A]

Consider the function $f(x) = 0.5x^2 - \frac{8}{x}$, $x \neq 0$.

- a. Find $f(-2)$. [2]
- b. Find $f'(x)$. [3]
- c. Find the gradient of the graph of f at $x = -2$. [2]
- d. Let T be the tangent to the graph of f at $x = -2$. [2]

Write down the equation of T .

- e. Let T be the tangent to the graph of f at $x = -2$. [4]

Sketch the graph of f for $-5 \leq x \leq 5$ and $-20 \leq y \leq 20$.

- f. Let T be the tangent to the graph of f at $x = -2$. [2]

Draw T on your sketch.

- g. The tangent, T , intersects the graph of f at a second point, P. [2]

Use your graphic display calculator to find the coordinates of P.

Markscheme

a. $0.5 \times (-2)^2 - \frac{8}{-2}$ (**M1**)

Note: Award (**M1**) for substitution of $x = -2$ into the formula of the function.

6 (**A1**)(**G2**)

b. $f'(x) = x + 8x^{-2}$ (**A1**)(**A1**)(**A1**)

Notes: Award (**A1**) for x , (**A1**) for 8, (**A1**) for x^{-2} or $\frac{1}{x^2}$ (each term must have correct sign). Award at most (**A1**)(**A1**)(**AO**) if there are additional terms present or further incorrect simplifications are seen.

c. $f'(-2) = -2 + 8(-2)^{-2}$ (**M1**)

Note: Award (**M1**) for $x = -2$ substituted into their $f'(x)$ from part (b).

$= 0$ (**A1**)(**ft**)(**G2**)

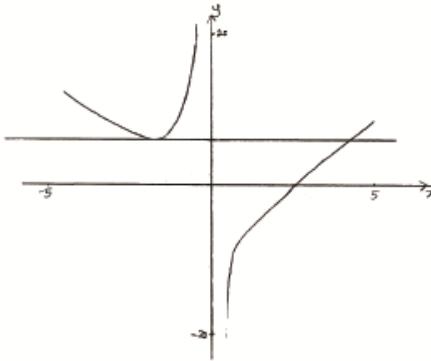
Note: Follow through from their derivative function.

d. $y = 6$ OR $y = 0x + 6$ OR $y - 6 = 0(x + 2)$ (**A1**)(**ft**)(**A1**)(**ft**)(**G2**)

Notes: Award (**A1**)(**ft**) for their gradient from part (c), (**A1**)(**ft**) for their answer from part (a). Answer must be an equation.

Award (**AO**)(**AO**) for $x = 6$.

e.



(A1)(A1)(A1)(A1)

Notes: Award (A1) for labels and some indication of scales in the stated window. The point $(-2, 6)$ correctly labelled, or an x -value and a y -value on their axes in approximately the correct position, are acceptable indication of scales.

Award (A1) for correct general shape (curve must be smooth and must not cross the y -axis).

Award (A1) for x -intercept in approximately the correct position.

Award (A1) for local minimum in the second quadrant.

- f. Tangent to graph drawn approximately at $x = -2$ (A1)(ft)(A1)(ft)

Notes: Award (A1)(ft) for straight line tangent to curve at approximately $x = -2$, with approximately correct gradient. Tangent must be straight for the (A1)(ft) to be awarded.

Award (A1)(ft) for (extended) line passing through approximately their y -intercept from (d). Follow through from their gradient in part (c) and their equation in part (d).

- g. $(4, 6)$ OR $x = 4, y = 6$ (G1)(ft)(G1)(ft)

Notes: Follow through from their tangent from part (d). If brackets are missing then award (G0)(G1)(ft).

If line intersects their graph at more than one point (apart from $(-2, 6)$), follow through from the first point of intersection (to the right of -2).

Award (G0)(G0) for $(-2, 6)$.

Examiners report

- a. [N/A]
- b. [N/A]
- c. [N/A]
- d. [N/A]
- e. [N/A]
- f. [N/A]
- g. [N/A]

Consider the function $f : x \mapsto \frac{kx}{2^x}$.

The cost per person, in euros, when x people are invited to a party can be determined by the function

$$C(x) = x + \frac{100}{x}$$

- i.a. Given that $f(1) = 2$, show that $k = 4$.

[2]

i.b. Write down the values of q and r for the following table.

[2]

x	-1	0	1	2	4	8
$f(x)$	-8	0	2	q	1	r

i.c. As x increases from -1, the graph of $y = f(x)$ reaches a maximum value and then decreases, behaving asymptotically.

[4]

Draw the graph of $y = f(x)$ for $-1 \leq x \leq 8$. Use a scale of 1 cm to represent 1 unit on both axes. The position of the maximum, M , the y -intercept and the asymptotic behaviour should be clearly shown.

i.d. Using your graphic display calculator, find the coordinates of M , the maximum point on the graph of $y = f(x)$.

[2]

i.e. Write down the equation of the horizontal asymptote to the graph of $y = f(x)$.

[2]

i.f. (i) Draw and label the line $y = 1$ on your graph.

[4]

(ii) The equation $f(x) = 1$ has two solutions. One of the solutions is $x = 4$. Use your **graph** to find the other solution.

ii.a. Find $C'(x)$.

[3]

ii.b. Show that the cost per person is a minimum when 10 people are invited to the party.

[2]

ii.c. Calculate the minimum cost per person.

[2]

Markscheme

i.a. $f(1) = \frac{k}{2^1}$ **(M1)**

Note: **(M1)** for substituting $x = 1$ into the formula.

$$\frac{k}{2} = 2 \quad \mathbf{(M1)}$$

Note: **(M1)** for equating to 2.

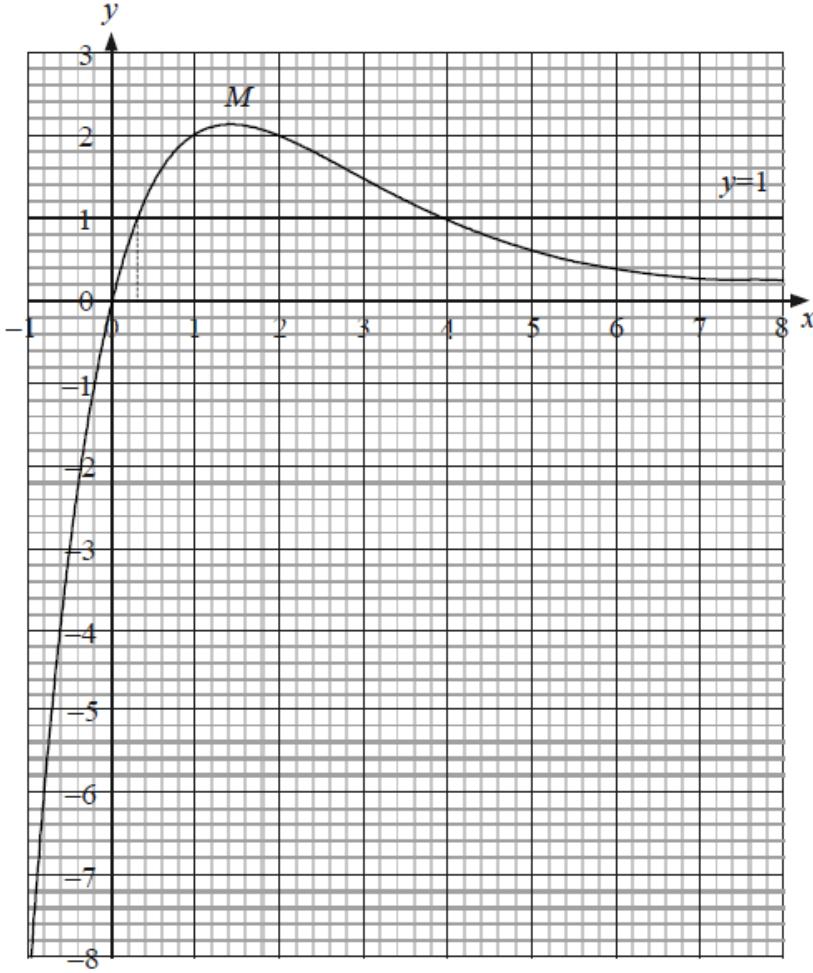
$$k = 4 \quad \mathbf{(AG)}$$

[2 marks]

i.b. $q = 2, r = 0.125$ **(A1)(A1)**

[2 marks]

i.c.



(A4)

Notes: (A1) for scales and labels.

(A1) for accurate smooth curve passing through (0, 0) drawn at least in the given domain.

(A1) for asymptotic behaviour (curve must not go up or cross the x -axis).

(A1) for indicating the position of the maximum point.

[4 marks]

i.d. M (1.44, 2.12) (G1)(G1)

Note: Brackets required, if missing award (G1)(G0). Accept $x = 1.44$ and $y = 2.12$.**[2 marks]**i.e. $y = 0$ (A1)(A1)**Note:** (A1) for ' $y =$ ' provided the right hand side is a constant. (A1) for 0.**[2 marks]**

i.f. (i) See graph (A1)(A1)

Note: (A1) for correct line, (A1) for label.(ii) $x = 0.3$ (ft) from candidate's graph. (A2)(ft)**Notes:** Accept ± 0.1 from their x . For 0.310 award (G1)(G0). For other answers taken from the GDC and not given correct to 3 significant figures award (G0)(AP)(G0) or (G1)(G0) if (AP) already applied.**[4 marks]**

$$\text{ii.a. } C'(x) = 1 - \frac{100}{x^2} \quad (\mathbf{A1})(\mathbf{A1})(\mathbf{A1})$$

Note: (A1) for 1, (A1) for -100 , (A1) for x^2 as denominator or x^{-2} as numerator. Award a maximum of (A2) if an extra term is seen.

[3 marks]

ii.b For studying signs of the derivative at either side of $x = 10$ (M1)

For saying there is a change of sign of the derivative (M1)(AG)

OR

For putting $x = 10$ into C' and getting zero (M1)

For clear sketch of the function or for mentioning that the function changes from decreasing to increasing at $x = 10$ (M1)(AG)

OR

For solving $C'(x) = 0$ and getting 10 (M1)

For clear sketch of the function or for mentioning that the function changes from decreasing to increasing at $x = 10$ (M1)(AG)

Note: For a sketch with a clear indication of the minimum or for a table with values of x at either side of $x = 10$ award (M1)(M0).

[2 marks]

ii.c. $C(10) = 10 + \frac{100}{10}$ (M1)

$C(10) = 20$ (A1)(G2)

[2 marks]

Examiners report

i.a. There were many well drawn graphs using correctly scaled and labelled axes with a good curve drawn. A number of students did not label the maximum point. Although many students showed in their graph the asymptotic behaviour of the curve, they did not know how to describe the asymptote. It was noticed that some students were tracing the curve to find the coordinates of the maximum instead of finding the maximum directly. The intersection between the line $y = 1$ and the curve was not always read from their graph but from their GDC's graph.

i.b. There were many well drawn graphs using correctly scaled and labelled axes with a good curve drawn. A number of students did not label the maximum point. Although many students showed in their graph the asymptotic behaviour of the curve, they did not know how to describe the asymptote. It was noticed that some students were tracing the curve to find the coordinates of the maximum instead of finding the maximum directly. The intersection between the line $y = 1$ and the curve was not always read from their graph but from their GDC's graph.

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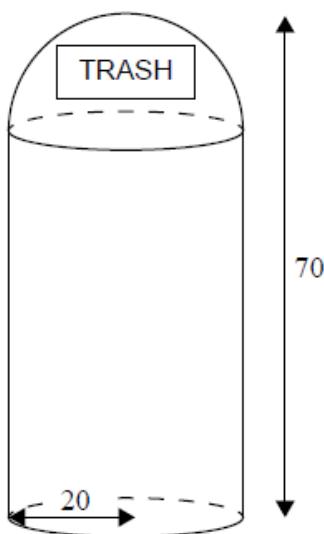
ii.a. Finding the derivative was done at least partially correctly by most of the candidates. However, using it to find the minimum and to justify why it is a minimum was troublesome for the majority of the candidates. Even those who used a graph in their reasoning neglected to mention the change from decreasing to increasing or to supply a sign diagram. Many candidates recovered in the last part of the question when finding the minimum cost.

ii.b. Finding the derivative was done at least partially correctly by most of the candidates. However, using it to find the minimum and to justify why it is a minimum was troublesome for the majority of the candidates. Even those who used a graph in their reasoning neglected to mention the change from decreasing to increasing or to supply a sign diagram. Many candidates recovered in the last part of the question when finding the minimum cost.

ii.c. Finding the derivative was done at least partially correctly by most of the candidates. However, using it to find the minimum and to justify why it is a minimum was troublesome for the majority of the candidates. Even those who used a graph in their reasoning neglected to mention the change from decreasing to increasing or to supply a sign diagram. Many candidates recovered in the last part of the question when finding the minimum cost.

A manufacturer makes trash cans in the form of a cylinder with a hemispherical top. The trash can has a height of 70 cm. The base radius of both the cylinder and the hemispherical top is 20 cm.

diagram not to scale

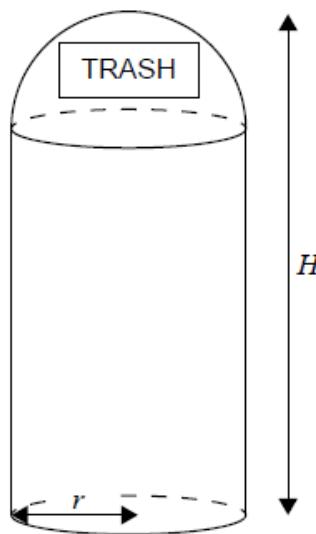


A designer is asked to produce a new trash can.

The new trash can will also be in the form of a cylinder with a hemispherical top.

This trash can will have a height of H cm and a base radius of r cm.

diagram not to scale



There is a design constraint such that $H + 2r = 110$ cm.

The designer has to maximize the volume of the trash can.

a. Write down the height of the cylinder.

[1]

b. Find the total volume of the trash can.

[4]

c. Find the height of the **cylinder**, h , of the new trash can, in terms of r .

[2]

d. Show that the volume, V cm³, of the new trash can is given by

[3]

$$V = 110\pi r^3.$$

e. Using your graphic display calculator, find the value of r which maximizes the value of V .

[2]

f. The designer claims that the new trash can has a capacity that is at least 40% greater than the capacity of the original trash can.

[4]

State whether the designer's claim is correct. Justify your answer.

Markscheme

- a. 50 (cm) **(A1)**

[1 mark]

b. $\pi \times 50 \times 20^2 + \frac{1}{2} \times \frac{4}{3} \times \pi \times 20^3$ **(M1)(M1)(M1)**

Note: Award **(M1)** for their correctly substituted volume of cylinder, **(M1)** for correctly substituted volume of sphere formula, **(M1)** for halving the substituted volume of sphere formula. Award at most **(M1)(M1)(M0)** if there is no addition of the volumes.

$$= 79600 \text{ (cm}^3\text{)} \left(79587.0\dots \text{ (cm}^3\text{)}, \frac{76000}{3}\pi \right) \quad \mathbf{(A1)(ft)(G3)}$$

Note: Follow through from part (a).

[4 marks]

- c. $h = H - r$ (or equivalent) **OR** $H = 110 - 2r$ **(M1)**

Note: Award **(M1)** for writing h in terms of H and r or for writing H in terms of r .

$$(h =) 110 - 3r \quad \mathbf{(A1)(G2)}$$

[2 marks]

d. $(V =) \frac{2}{3}\pi r^3 + \pi r^2 \times (110 - 3r)$ **(M1)(M1)(M1)**

Note: Award **(M1)** for volume of hemisphere, **(M1)** for correct substitution of their h into the volume of a cylinder, **(M1)** for addition of two correctly substituted volumes leading to the given answer. Award at most **(M1)(M1)(M0)** for subsequent working that does not lead to the given answer. Award at most **(M1)(M1)(M0)** for substituting $H = 110 - 2r$ as their h .

$$V = 110\pi r^2 - \frac{7}{3}\pi r^3 \quad \mathbf{(AG)}$$

[3 marks]

- e. $(r =) 31.4 \text{ (cm)} (31.4285\dots \text{ (cm)})$ **(G2)**

OR

$$(\pi)(220r - 7r^2) = 0 \quad \mathbf{(M1)}$$

Note: Award **(M1)** for setting the correct derivative equal to zero.

$$(r =) 31.4 \text{ (cm)} (31.4285\dots \text{ (cm)}) \quad \mathbf{(A1)}$$

[2 marks]

f. $(V =) 110\pi(31.4285\dots)^3 - \frac{7}{3}\pi(31.4285\dots)^3$ **(M1)**

Note: Award **(M1)** for correct substitution of their 31.4285... into the given equation.

$$= 114000 (113781\dots) \quad \mathbf{(A1)(ft)}$$

Note: Follow through from part (e).

$$(\text{increase in capacity } =) \frac{113.781\dots - 79587.0\dots}{79587.0\dots} \times 100 = 43.0 \text{ (\%)} \quad \mathbf{(R1)(ft)}$$

Note: Award **(R1)(ft)** for finding the correct percentage increase from their two volumes.

OR

$$1.4 \times 79587.0\dots = 111421.81\dots \quad \mathbf{(R1)(ft)}$$

Note: Award **(R1)(ft)** for finding the capacity of a trash can 40% larger than the original.

Claim is correct **(A1)(ft)**

Note: Follow through from parts (b), (e) and within part (f). The final **(R1)(A1)(ft)** can be awarded for their correct reason and conclusion. Do not award **(R0)(A1)(ft)**.

[4 marks]

Examiners report

- a. [N/A]
- b. [N/A]
- c. [N/A]
- d. [N/A]
- e. [N/A]
- f. [N/A]

Consider the curve $y = 2x^3 - 9x^2 + 12x + 2$, for $-1 < x < 3$

- a. Sketch the curve for $-1 < x < 3$ and $-2 < y < 12$.

[4]

- b. A teacher asks her students to make some observations about the curve.

[1]

Three students responded.

Nadia said “*The x-intercept of the curve is between –1 and zero*”.

Rick said “*The curve is decreasing when $x < 1$* ”.

Paula said “*The gradient of the curve is less than zero between $x = 1$ and $x = 2$* ”.

State the name of the student who made an **incorrect** observation.

- c. Find the value of y when $x = 1$.

[2]

- d. Find $\frac{dy}{dx}$.

[3]

- e. Show that the stationary points of the curve are at $x = 1$ and $x = 2$.

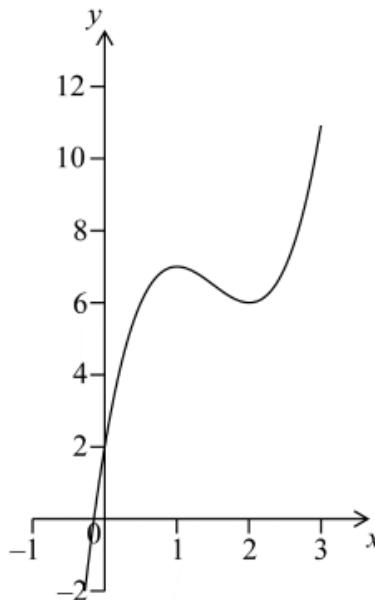
[2]

- f. Given that $y = 2x^3 - 9x^2 + 12x + 2 = k$ has **three** solutions, find the possible values of k .

[3]

Markscheme

a.



(A1)(A1)(A1)(A1)

Note: Award (A1) for correct window (condone a window which is slightly off) and axes labels. An indication of window is necessary. –1 to 3 on the x-axis and –2 to 12 on the y-axis and a graph in that window.

(A1) for correct shape (curve having cubic shape and must be smooth).

(A1) for both stationary points in the 1st quadrant with approximate correct position,

(A1) for intercepts (negative x-intercept and positive y intercept) with approximate correct position.

[4 marks]

b. Rick (A1)

Note: Award (A0) if extra names stated.

[1 mark]c. $2(1)^3 - 9(1)^2 + 12(1) + 2$ (M1)

Note: Award (M1) for correct substitution into equation.

$$= 7 \quad (\text{A1})(\text{G2})$$

[2 marks]d. $6x^2 - 18x + 12$ (A1)(A1)(A1)

Note: Award (A1) for each correct term. Award at most (A1)(A1)(A0) if extra terms seen.

[3 marks]e. $6x^2 - 18x + 12 = 0$ (M1)

Note: Award (M1) for equating their derivative to 0. If the derivative is not explicitly equated to 0, but a subsequent solving of their correct equation is seen, award (M1).

$$6(x - 1)(x - 2) = 0 \quad (\text{or equivalent}) \quad (\text{M1})$$

Note: Award (M1) for correct factorization. The final (M1) is awarded only if answers are clearly stated.

Award (M0)(M0) for substitution of 1 and of 2 in their derivative.

$$x = 1, x = 2 \quad (\text{AG})$$

[2 marks]

- f. $6 < k < 7$ **(A1)(A1)(ft)(A1)**

Note: Award **(A1)** for an inequality with 6, award **(A1)(ft)** for an inequality with 7 from their part (c) provided it is greater than 6, **(A1)** for their correct strict inequalities. Accept $]6, 7[$ or $(6, 7)$.

[3 marks]

Examiners report

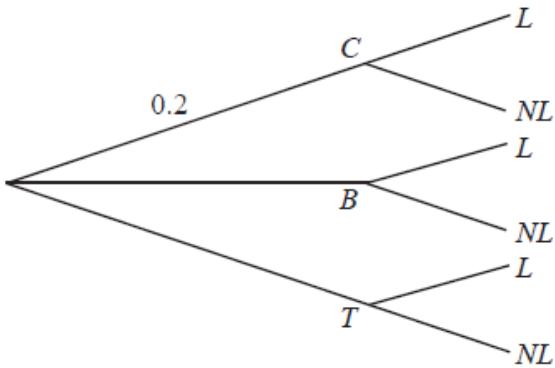
- a. [N/A]
 - b. [N/A]
 - c. [N/A]
 - d. [N/A]
 - e. [N/A]
 - f. [N/A]
-

When Geraldine travels to work she can travel either by car (C), bus (B) or train (T). She travels by car on one day in five. She uses the bus 50 % of the time. The probabilities of her being late (L) when travelling by car, bus or train are 0.05, 0.12 and 0.08 respectively.

*It is **not** necessary to use graph paper for this question.*

- i.a. Copy the tree diagram below and fill in all the probabilities, where NL represents not late, to represent this information.

[5]



- i.b. Find the probability that Geraldine travels by bus and is late.

[1]

- i.c. Find the probability that Geraldine is late.

[3]

- i.d. Find the probability that Geraldine travelled by train, given that she is late.

[3]

- ii.a. Sketch the curve of the function $f(x) = x^3 - 2x^2 + x - 3$ for values of x from -2 to 4 , giving the intercepts with both axes.

[3]

- ii.b. On the same diagram, sketch the line $y = 7 - 2x$ and find the coordinates of the point of intersection of the line with the curve.

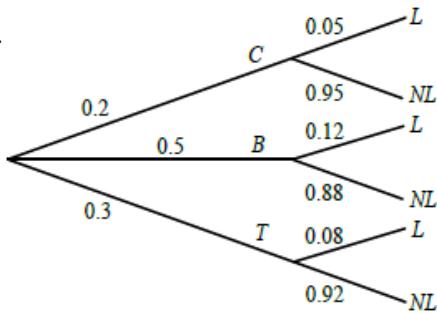
[3]

- ii.c. Find the value of the gradient of the curve where $x = 1.7$.

[2]

Markscheme

i.a.



Award (A1) for 0.5 at B, (A1) for 0.3 at T, then (A1) for each correct pair. Accept fractions or percentages. (A5)

[5 marks]

i.b. 0.06 (accept 0.5×0.12 or 6%) (A1)(ft)

[1 mark]

i.c. for a relevant two-factor product, either $C \times L$ or $T \times L$ (M1)

for summing three two-factor products (M1)

$$(0.2 \times 0.05 + 0.06 + 0.3 \times 0.08)$$

$$0.094 \quad (\text{A1})(\text{ft})(\text{G2})$$

[3 marks]

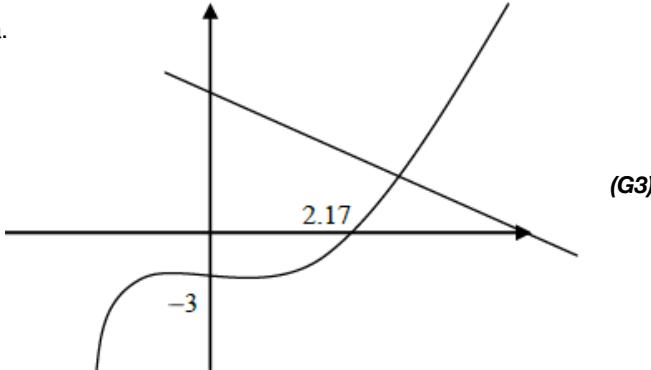
i.d. $\frac{0.3 \times 0.08}{0.094} \quad (\text{M1})(\text{A1})(\text{ft})$

award (M1) for substituted conditional probability formula seen, (A1)(ft) for correct substitution

$$= 0.255 \quad (\text{A1})(\text{ft})(\text{G2})$$

[3 marks]

ii.a.



[3 marks]

ii.b. line drawn with -ve gradient and +ve y-intercept (G1)

$$(2.45, 2.11) \quad (\text{G1})(\text{G1})$$

[3 marks]

ii.c. $f'(1.7) = 3(1.7)^2 - 4(1.7) + 1 \quad (\text{M1})$

award (M1) for substituting in their $f'(x)$

$$2.87 \quad (\text{A1})(\text{G2})$$

[2 marks]

Examiners report

i.a. This should have been an easy first question but, even so, there were some candidates who were unable to fill in the tree diagram correctly let alone evaluate any probabilities. The majority of candidates were confident with answering parts (a), (b) and (c) but the conditional probability question was not well answered with few candidates managing to recognise that it was a conditional type.

i.b. This should have been an easy first question but, even so, there were some candidates who were unable to fill in the tree diagram correctly let alone evaluate any probabilities. The majority of candidates were confident with answering parts (a), (b) and (c) but the conditional probability question was not well answered with few candidates managing to recognise that it was a conditional type.

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i.d. This should have been an easy first question but, even so, there were some candidates who were unable to fill in the tree diagram correctly let alone evaluate any probabilities. The majority of candidates were confident with answering parts (a), (b) and (c) but the conditional probability question was not well answered with few candidates managing to recognise that it was a conditional type.

ii.a. This should have been an easy first question but, even so, there were some candidates who were unable to fill in the tree diagram correctly let alone evaluate any probabilities. The majority of candidates were confident with answering parts (a), (b) and (c) but the conditional probability question was not well answered with few candidates managing to recognise that it was a conditional type.

The curve sketching and straight line were well drawn but not all candidates indicated the intersection points with the axes. In finding the line / curve intersection some candidates did not use the intersection function on the GDC. Few candidates managed the last part. Many just chose two sets of coordinates and used the gradient formula.

ii.b. This should have been an easy first question but, even so, there were some candidates who were unable to fill in the tree diagram correctly let alone evaluate any probabilities. The majority of candidates were confident with answering parts (a), (b) and (c) but the conditional probability question was not well answered with few candidates managing to recognise that it was a conditional type.

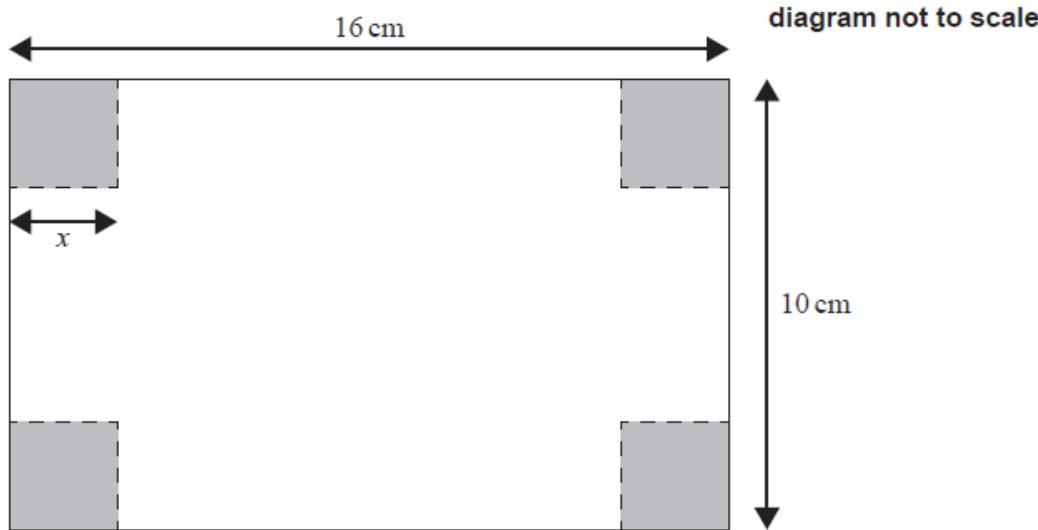
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ii.c. This should have been an easy first question but, even so, there were some candidates who were unable to fill in the tree diagram correctly let alone evaluate any probabilities. The majority of candidates were confident with answering parts (a), (b) and (c) but the conditional probability question was not well answered with few candidates managing to recognise that it was a conditional type.

The curve sketching and straight line were well drawn but not all candidates indicated the intersection points with the axes. In finding the line / curve intersection some candidates did not use the intersection function on the GDC. Few candidates managed the last part. Many just chose two sets of coordinates and used the gradient formula.

- a. Hugo is given a rectangular piece of thin cardboard, 16 cm by 10 cm. He decides to design a tray with it. [2]

He removes from each corner the shaded squares of side x cm, as shown in the following diagram.



The remainder of the cardboard is folded up to form the tray as shown in the following diagram.

diagram not to scale



Write down, in terms of x , the length and the width of the tray.

- b. (i) State whether x can have a value of 5. Give a reason for your answer. [4]

- (ii) Write down the interval for the possible values of x .

- c. Show that the volume, V cm³, of this tray is given by [2]

$$V = 4x^3 - 52x^2 + 160x.$$

- d. Find $\frac{dV}{dx}$. [3]

- e. Using your answer from part (d), find the value of x that maximizes the volume of the tray. [2]

- f. Calculate the maximum volume of the tray. [2]

- g. Sketch the graph of $V = 4x^3 - 51x^2 + 160x$, for the possible values of x found in part (b)(ii), and $0 \leq V \leq 200$. Clearly label the maximum point. [4]

Markscheme

- a. $16 - 2x, 10 - 2x$ (A1)(A1)

- b. (i) no (A1)

(when x is 5) the width of the tray will be zero / there is no short edge to fold / $10 - 2(5) = 0$ (R1)

Note: Do not award (R0)(A1). Award the (R1) for reasonable explanation.

(ii) $0 < x < 5$ **(A1)(A1)**

Note: Award **(A1)** for 0 and 5 seen, **(A1)** for correct strict inequalities (accept alternative notation). Award **(A1)(A0)** for “between 0 and 5” or “from 0 to 5”.

Do not accept a list of integers.

c. $V = (16 - 2x)(10 - 2x)(x)$ **(M1)**

Note: Award **(M1)** for their correct substitution in volume of cuboid formula.

$$= 160x - 32x^2 - 20x^2 + 4x^3 \text{ (or equivalent)} \quad \text{**(M1)**}$$

$$= 160x - 52x^2 + 4x^3 \quad \text{**(AG)**}$$

Note: Award **(M1)** for showing clearly the expansion and for simplifying the expression, and this **must** be seen to award second **(M1)**. The **(AG)** line must be seen for the final **(M1)** to be awarded.

d. $12x^2 - 104x + 160$ (or equivalent) **(A1)(A1)(A1)**

Note: Award **(A1)** for $12x^2$, **(A1)** for $-104x$ and **(A1)** for $+160$. If extra terms are seen award at most **(A1)(A1)(A0)**.

e. $12x^2 - 104x + 160 = 0$ **(M1)**

Note: Award **(M1)** for equating **their** derivative to 0.

$$x = 2 \quad \text{**(A1)(ft)**}$$

Note: Award **(M1)** for a sketch of their derivative in part (d), **(A1)(ft)** for reading the x -intercept from their graph.

Award **(M0)(A0)** for $x = 2$ with no working seen.

Award at most **(M1)(A0)** if the answer is a pair of coordinates.

Award at most **(M1)(A0)** if the answer given is $x = 2$ and $x = \frac{20}{3}$

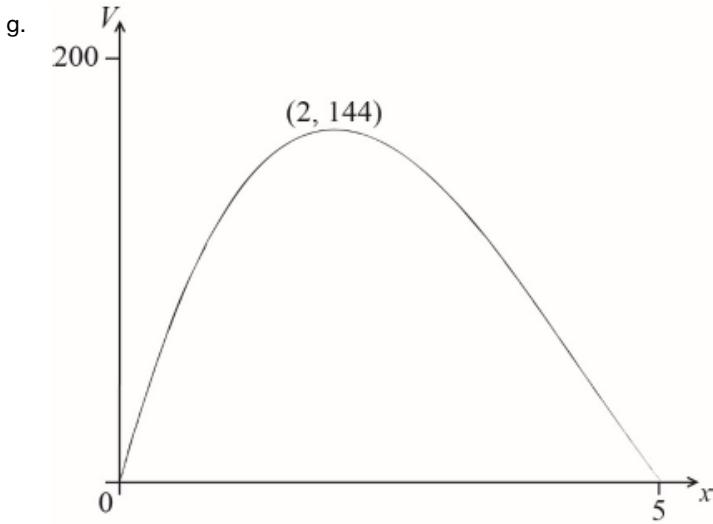
Follow through from their derivative in part (d). Award **(A1)(ft)** only if answer is positive and less than 5.

f. $4(2)^3 - 52(2)^2 + 160(2)$ **(M1)**

Note: Award **(M1)** for correct substitution of their answer to part (e) into volume formula.

$$144 \text{ (cm}^3\text{)} \quad \text{**(A1)(ft)(G2)**}$$

Note: Follow through from part (d).



(A1)(A1)(ft)(A1)(A1)(ft)

Note: Award **(A1)** for correctly labelled axes and window for V , ie $0 \leqslant V \leqslant 200$.

Award **(A1)(ft)** for the correct domain ($0 < x < 5$). Follow through from part (b)(ii) if a different domain is shown on graph.

Award **(A1)** for smooth curve with correct shape.

Award **(A1)(ft)** for **their** maximum point indicated (coordinates, cross or dot etc.) in approximately correct place.

Follow through from parts (e) and (f) only if the maximum on their graph is different from (2, 144).

Examiners report

a. Question 5: Differential calculus

In general, many candidates struggled in some parts. Most candidates who could state the dimensions of the box gave a reasonable justification of why x could not be 5. Very few candidates scored the two marks in part (d)(ii). Either their inequalities were not strict or their limits were incorrect or both. Some candidates stated the range of x as 1, 2, 3, 4. The algebra in part (c) caused problems for a number of candidates. It seemed that there was a lack of understanding of what the question required. Some substituted $x = 2$ in the volume formula. A few candidates wrote the product of the length, width, height, omitting the appropriate brackets. Part (d) was well answered by most candidates. However, its application in the following part was not as good in part (e). In part (e) some candidates left both solutions for x , not appreciating the fact that one was outside the range. Others lost both marks as they did not show that they had used their derivative to part (d) as required by the question. Very few candidates scored full marks for the sketch in part (g). Not following the given instructions about the domain and range let most candidates down in this question.

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c. Question 5: Differential calculus

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d. Question 5: Differential calculus

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f. Question 5: Differential calculus

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g. Question 5: Differential calculus

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a. A function, f , is given by

[2]

$$f(x) = 4 \times 2^{-x} + 1.5x - 5.$$

Calculate $f(0)$

b. Use your graphic display calculator to solve $f(x) = 0$.

[2]

c. Sketch the graph of $y = f(x)$ for $-2 \leq x \leq 6$ and $-4 \leq y \leq 10$, showing the x and y intercepts. Use a scale of 2 cm to represent 2 units on [4] both the horizontal axis, x , and the vertical axis, y .

d. The function f is the derivative of a function g . It is known that $g(1) = 3$.

[4]

i) Calculate $g'(1)$.

ii) Find the equation of the tangent to the graph of $y = g(x)$ at $x = 1$. Give your answer in the form $y = mx + c$.

Markscheme

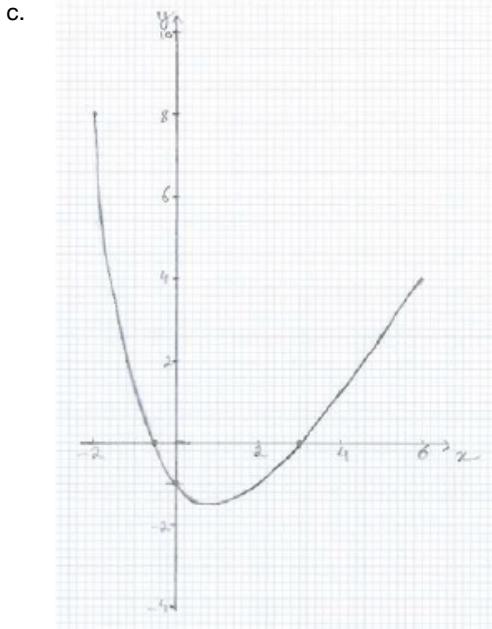
a. $4 \times 2^{-0} + 1.5 \times 0 - 5 \quad (\text{M1})$

Note: Award **(M1)** for substitution of 0 into the expression for $f(x)$.

$= -1 \quad (\text{A1})(\text{G2})$

b. $-0.538 \ (-0.537670\dots)$ and 3 $\quad (\text{A1})(\text{A1})$

Note: Award at most **(AO)(A1)(ft)** if answer is given as pairs of coordinates.



$(\text{A1})(\text{A1})(\text{A1})(\text{ft})(\text{A1})(\text{ft})$

Note: Award **(A1)** for labels and some indication of scale in the correct given window.

Award **(A1)** for smooth curve with correct general shape with $f(-2) > f(6)$ and minimum to the right of the y -axis.

Award **(A1)(ft)** for correct y -intercept (consistent with their part (a)).

Award **(A1)(ft)** for approximately correct x -intercepts (consistent with their part (b), one zero between -1 and 0 , the other between 2.5 and 3.5).

d. i) $g'(1) = f(1) = 4 \times 2^{-1} + 1.5 - 5 \quad (\text{M1})$

Note: Award **(M1)** for substitution of 1 into $f(x)$.

$= -1.5 \quad (\text{A1})(\text{G2})$

ii) $3 = -1.5 \times 1 + c \text{ OR } (y - 3) = -1.5(x - 1) \quad (\text{M1})$

Note: Award **(M1)** for correct substitution of gradient and the point $(1, 3)$ into the equation of a line. Follow through from (d)(i).

$y = -1.5x + 4.5 \quad (\text{A1})(\text{ft})(\text{G2})$

Examiners report

a. Question 6: Functions and Calculus

Many candidates were able to calculate $f(0)$ although some had a problem with 2^0 . Very few were able to find both roots in part (b), even when they represented correctly both zeros in their sketch. Many sketches were reasonably well drawn, presenting a smooth curve with correct points of

intersection. Many lost a mark for not labelling their axes and the given scale was ignored by some, which was not penalized but often resulted in sketches which were difficult to read. The stronger candidates calculated $g'(1)$ correctly, but very few represented the gradient of the function at $x = 1$ correctly.

b. Question 6: Functions and Calculus

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c. Question 6: Functions and Calculus Many candidates were able to calculate $f(0)$ although some had a problem with 2^0 . Very few were able to find both roots in part (b), even when they represented correctly both zeros in their sketch. Many sketches were reasonably well drawn, presenting a smooth curve with correct points of intersection. Many lost a mark for not labelling their axes and the given scale was ignored by some, which was not penalized but often resulted in sketches which were difficult to read. The stronger candidates calculated $g'(1)$ correctly, but very few represented the gradient of the function at $x = 1$ correctly.

d. Question 6: Functions and Calculus

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Consider the function $f(x) = \frac{48}{x} + kx^2 - 58$, where $x > 0$ and k is a constant.

The graph of the function passes through the point with coordinates $(4, 2)$.

P is the minimum point of the graph of $f(x)$.

a. Find the value of k .

[2]

b. Using your value of k , find $f'(x)$.

[3]

c. **Use your answer** to part (b) to show that the minimum value of $f(x)$ is -22 .

[3]

d. Write down the **two** values of x which satisfy $f(x) = 0$.

[2]

e. Sketch the graph of $y = f(x)$ for $0 < x \leq 6$ and $-30 \leq y \leq 60$.

[4]

Clearly indicate the minimum point P and the x-intercepts on your graph.

Markscheme

a. $\frac{48}{4} + k \times 4^2 - 58 = 2$ (M1)

Note: Award (M1) for correct substitution of $x = 4$ and $y = 2$ into the function.

$k = 3$ (A1) (G2)

[2 marks]

b. $\frac{-48}{x^2} + 6x$ (A1)(A1)(A1)(ft) (G3)

Note: Award (A1) for -48 , (A1) for x^{-2} , (A1)(ft) for their $6x$. Follow through from part (a). Award at most (A1)(A1)(AO) if additional terms are seen.

[3 marks]

c. $\frac{-48}{x^2} + 6x = 0$ (M1)

Note: Award (M1) for equating their part (b) to zero.

$x = 2$ (A1)(ft)

Note: Follow through from part (b). Award (M1)(A1) for $\frac{-48}{(2)^2} + 6(2) = 0$ seen.

Award (MO)(AO) for $x = 2$ seen either from a graphical method or without working.

$\frac{48}{2} + 3 \times 2^2 - 58 (= -22)$ (M1)

Note: Award (M1) for substituting their 2 into their function, but only if the final answer is -22 . Substitution of the known result invalidates the process; award (MO)(AO)(MO).

-22 (AG)

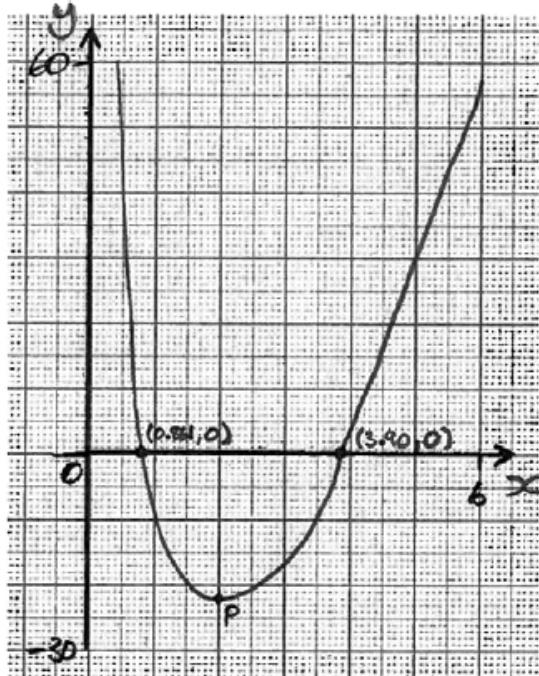
[3 marks]

d. 0.861 ($0.860548\dots$), 3.90 ($3.90307\dots$) (A1)(ft)(A1)(ft) (G2)

Note: Follow through from part (a) but only if the answer is positive. Award at most (A1)(ft)(AO) if answers are given as coordinate pairs or if extra values are seen. The function $f(x)$ only has two x -intercepts within the domain. Do not accept a negative x -intercept.

[2 marks]

e.



(A1)(A1)(ft)(A1)(ft)(A1)(ft)

Note: Award (A1) for correct window. Axes must be labelled.

(A1)(ft) for a smooth curve with correct shape and zeros in approximately correct positions relative to each other.

(A1)(ft) for point P indicated in approximately the correct position. Follow through from their x-coordinate in part (c). (A1)(ft) for two x-intercepts identified on the graph and curve reflecting asymptotic properties.

Examiners report

- a. [N/A]
 - b. [N/A]
 - c. [N/A]
 - d. [N/A]
 - e. [N/A]
-

- a. A distress flare is fired into the air from a ship at sea. The height, h , in metres, of the flare above sea level is modelled by the quadratic function [1]

$$h(t) = -0.2t^2 + 16t + 12, t \geq 0,$$

where t is the time, in seconds, and $t = 0$ at the moment the flare was fired.

Write down the height from which the flare was fired.

- b. Find the height of the flare 15 seconds after it was fired. [2]

- c. The flare fell into the sea k seconds after it was fired. [2]

Find the value of k .

- d. Find $h'(t)$. [2]

- e. i) Show that the flare reached its maximum height 40 seconds after being fired. [3]

ii) Calculate the maximum height reached by the flare.

- f. The nearest coastguard can see the flare when its height is more than 40 metres above sea level. [3]

Determine the total length of time the flare can be seen by the coastguard.

Markscheme

a. $12 \text{ (m)} \quad (\text{A1})$

b. $(h(15) =) - 0.2 \times 15^2 + 16 \times 15 + 12 \quad (\text{M1})$

Note: Award **(M1)** for substitution of 15 in expression for h .

$= 207 \text{ (m)} \quad (\text{A1})(\text{G2})$

c. $h(k) = 0 \quad (\text{M1})$

Note: Award **(M1)** for setting h to zero.

$(k =) 80.7 \text{ (s)} \quad (80.7430) \quad (\text{A1})(\text{G2})$

Note: Award at most **(M1)(AO)** for an answer including $K = -0.743$.

Award **(AO)** for an answer of 80 without working.

d. $h'(t) = -0.4t + 16 \quad (\text{A1})(\text{A1})$

Note: Award **(A1)** for $-0.4t$, **(A1)** for 16. Award at most **(A1)(AO)** if extra terms seen. Do not accept x for t .

e. i) $-0.4t + 16 = 0$ **(M1)**

Note: Award **(M1)** for setting their derivative, from part (d), to zero, provided the correct conclusion is stated and consistent with their $h'(t)$.

OR

$$t = \frac{-16}{2 \times (-0.2)} \quad \text{**(M1)**}$$

Note: Award **(M1)** for correct substitution into axis of symmetry formula, provided the correct conclusion is stated.

$$t = 40 \text{ (s)} \quad \text{**(AG)**}$$

ii) $-0.2 \times 40^2 + 16 \times 40 + 12 \quad \text{**(M1)**}$

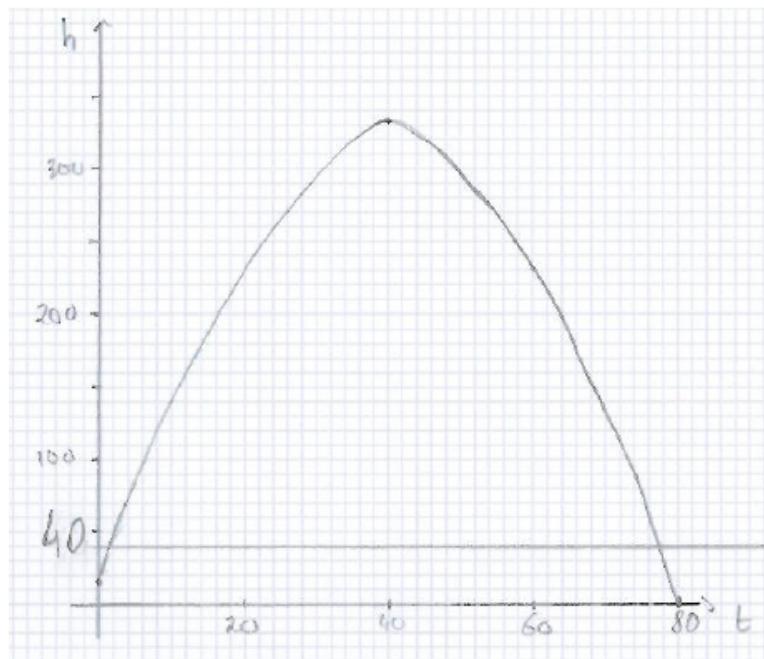
Note: Award **(M1)** for substitution of 40 in expression for h .

$$= 332 \text{ (m)} \quad \text{**(A1)(G2)**}$$

f. $h(t) = 40 \quad \text{**(M1)**}$

Note: Award **(M1)** for setting h to 40. Accept inequality sign.

OR



M1

Note: Award **(M1)** for correct sketch. Indication of scale is not required.

$$78.2 - 1.17 \quad (78.2099\dots - 1.79005\dots) \quad \text{**(A1)**}$$

Note: Award **(A1)** for 1.79 and 78.2 seen.

$$\text{(total time} =) 76.4 \text{ (s)} \quad (76.4198\dots) \quad \text{**(A1)(G2)**}$$

Note: Award **(G1)** if the two endpoints are given as the final answer with no working.

Examiners report

a. Question 3: Quadratic function, problem solving.

Parts (a) (finding the initial height) and (b) (finding the height after 15 seconds), were done very well by the majority of candidates. Many struggled to translate question (c) to find the (positive) zeros of the function, or did not write that down, losing a possible method mark. The derivative in part (d) was no problem for most; only very few used x instead of t . The maximum height reached was calculated correctly by the majority of candidates, but many lost the mark in part (e)(i) as they simply substituted 40 into their derivative or calculated the height at points close to 40. Only a few candidates showed correct method for part (f). Several were still able to obtain 2 marks as a result of “trial and error” of integer values for t . Some candidate seem to have a problem with the notation “ $h(t) = \dots$ ”, where this is interpreted as $h \times t$, resulting in incorrect answers throughout.

b. Question 3: Quadratic function, problem solving.

Parts (a) (finding the initial height) and (b) (finding the height after 15 seconds), were done very well by the majority of candidates. Many struggled to translate question (c) to find the (positive) zeros of the function, or did not write that down, losing a possible method mark. The derivative in part (d) was no problem for most; only very few used x instead of t . The maximum height reached was calculated correctly by the majority of candidates, but many lost the mark in part (e)(i) as they simply substituted 40 into their derivative or calculated the height at points close to 40. Only a few candidates showed correct method for part (f). Several were still able to obtain 2 marks as a result of “trial and error” of integer values for t . Some candidate seem to have a problem with the notation “ $h(t) = \dots$ ”, where this is interpreted as $h \times t$, resulting in incorrect answers throughout.

c. Question 3: Quadratic function, problem solving.

Parts (a) (finding the initial height) and (b) (finding the height after 15 seconds), were done very well by the majority of candidates. Many struggled to translate question (c) to find the (positive) zeros of the function, or did not write that down, losing a possible method mark. The derivative in part (d) was no problem for most; only very few used x instead of t . The maximum height reached was calculated correctly by the majority of candidates, but many lost the mark in part (e)(i) as they simply substituted 40 into their derivative or calculated the height at points close to 40. Only a few candidates showed correct method for part (f). Several were still able to obtain 2 marks as a result of “trial and error” of integer values for t . Some candidate seem to have a problem with the notation “ $h(t) = \dots$ ”, where this is interpreted as $h \times t$, resulting in incorrect answers throughout.

d. Question 3: Quadratic function, problem solving.

Parts (a) (finding the initial height) and (b) (finding the height after 15 seconds), were done very well by the majority of candidates. Many struggled to translate question (c) to find the (positive) zeros of the function, or did not write that down, losing a possible method mark. The derivative in part (d) was no problem for most; only very few used x instead of t . The maximum height reached was calculated correctly by the majority of candidates, but many lost the mark in part (e)(i) as they simply substituted 40 into their derivative or calculated the height at points close to 40. Only a few candidates showed correct method for part (f). Several were still able to obtain 2 marks as a result of “trial and error” of integer values for t . Some candidate seem to have a problem with the notation “ $h(t) = \dots$ ”, where this is interpreted as $h \times t$, resulting in incorrect answers throughout.

e. Question 3: Quadratic function, problem solving.

Parts (a) (finding the initial height) and (b) (finding the height after 15 seconds), were done very well by the majority of candidates. Many struggled to translate question (c) to find the (positive) zeros of the function, or did not write that down, losing a possible method mark. The derivative in part (d) was no problem for most; only very few used x instead of t . The maximum height reached was calculated correctly by the majority of candidates, but many lost the mark in part (e)(i) as they simply substituted 40 into their derivative or calculated the height at points close to 40. Only a few candidates showed correct method for part (f). Several were still able to obtain 2 marks as a result of “trial and error” of integer values for t . Some candidate seem to have a problem with the notation “ $h(t) = \dots$ ”, where this is interpreted as $h \times t$, resulting in incorrect answers throughout.

f. Question 3: Quadratic function, problem solving.

Parts (a) (finding the initial height) and (b) (finding the height after 15 seconds), were done very well by the majority of candidates. Many struggled to translate question (c) to find the (positive) zeros of the function, or did not write that down, losing a possible method mark. The derivative in part (d) was no problem for most; only very few used x instead of t . The maximum height reached was calculated correctly by the majority of candidates, but many lost the mark in part (e)(i) as they simply substituted 40 into their derivative or calculated the height at points close to 40.

Only a few candidates showed correct method for part (f). Several were still able to obtain 2 marks as a result of “trial and error” of integer values for t . Some candidate seem to have a problem with the notation “ $h(t) = \dots$ ”, where this is interpreted as $h \times t$, resulting in incorrect answers throughout.

Consider the function $f(x) = 3x + \frac{12}{x^2}$, $x \neq 0$.

- a. Differentiate $f(x)$ with respect to x . [3]
- b. Calculate $f'(x)$ when $x = 1$. [2]
- c. Use your answer to part (b) to decide whether the function, f , is increasing or decreasing at $x = 1$. Justify your answer. [2]
- d. Solve the equation $f'(x) = 0$. [3]

e, i. The graph of f has a local minimum at point P. Let T be the tangent to the graph of f at P. [2]

Write down the coordinates of P.

e, ii. The graph of f has a local minimum at point P. Let T be the tangent to the graph of f at P. [1]

Write down the gradient of T .

e, iii. The graph of f has a local minimum at point P. Let T be the tangent to the graph of f at P. [2]

Write down the equation of T .

f. Sketch the graph of the function f , for $-3 \leq x \leq 6$ and $-7 \leq y \leq 15$. Indicate clearly the point P and any intercepts of the curve with the axes. [4]

g, i. On your graph draw and label the tangent T . [2]

g, ii. T intersects the graph of f at a second point. Write down the x-coordinate of this point of intersection. [1]

Markscheme

a. $f'(x) = 3 - \frac{24}{x^3}$ (A1)(A1)(A1)

Note: Award (A1) for 3, (A1) for -24 , (A1) for x^3 (or x^{-3}). If extra terms present award at most (A1)(A1)(AO).

[3 marks]

b. $f'(1) = -21$ (M1)(A1)(ft)(G2)

Note: (ft) from their derivative only if working seen.

[2 marks]

c. Derivative (gradient, slope) is negative. Decreasing. (R1)(A1)(ft)

Note: Do not award **(R0)(A1)**.

[2 marks]

d. $3 - \frac{24}{x^3} = 0$ **(M1)**

$$x^3 = 8 \quad \mathbf{(A1)}$$

$$x = 2 \quad \mathbf{(A1)(ft)(G2)}$$

[3 marks]

e, i. $(2, 9)$ (Accept $x = 2, y = 9$) **(A1)(A1)(G2)**

Notes: **(ft)** from their answer in (d).

Award **(A1)(A0)** if brackets not included and not previously penalized.

[2 marks]

e, ii. $y = 9$ **(A1)**

[1 mark]

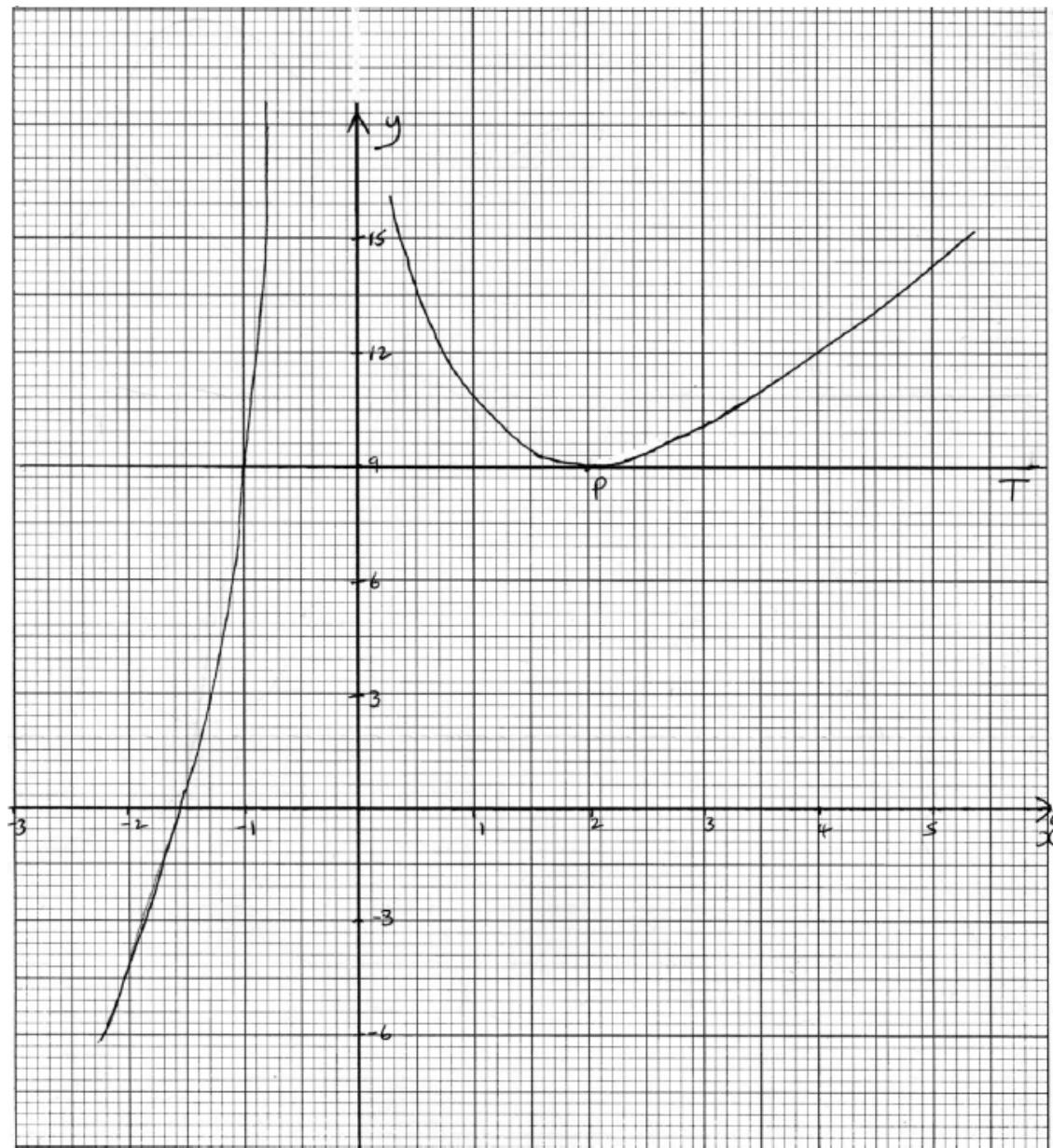
e, iii. $y = 9$ **(A1)(A1)(ft)(G2)**

Notes: Award **(A1)** for $y = \text{constant}$, **(A1)** for 9.

Award **(A1)(ft)** for their value of y in (e)(i).

[2 marks]

f.



(A4)

Notes: Award **(A1)** for labels and some indication of scale in the stated window.

Award **(A1)** for correct general shape (curve must be smooth and must not cross the y -axis).

Award **(A1)** for x -intercept seen in roughly the correct position.

Award **(A1)** for minimum (P).

[4 marks]

g, i. Tangent drawn at P (line must be a tangent and horizontal). **(A1)**

Tangent labeled T. **(A1)**

Note: **(ft)** from their tangent equation only if tangent is drawn and answer is consistent with graph.

[2 marks]

g, ii. $k = -1$ **(G1)(ft)**

Examiners report

- a. Many students did not know the term “differentiate” and did not answer part (a).
 - b. However, the derivative was seen in (b) when finding the gradient at $x = 1$. The negative index of the formula did cause problems for many when finding the derivative. The meaning of the derivative was not clear for a number of students.
 - c. [N/A]
 - d. Part (d) was handled well by some but many substituted $x = 0$ into $f'(x)$.
 - e, i It was clear that most candidates neither knew that the tangent at a minimum is horizontal nor that its gradient is zero.
 - e, ii It was clear that most candidates neither knew that the tangent at a minimum is horizontal nor that its gradient is zero.
 - e, iii It was clear that most candidates neither knew that the tangent at a minimum is horizontal nor that its gradient is zero.
 - f. There were good answers to the sketch though setting out axes and a scale seemed not to have had enough practise.
 - g, i Those who were able to sketch the function were often able to correctly place and label the tangent and also to find the second intersection point with the graph of the function.
 - g, ii Those who were able to sketch the function were often able to correctly place and label the tangent and also to find the second intersection point with the graph of the function.
-

Consider the function $f(x) = -\frac{1}{3}x^3 + \frac{5}{3}x^2 - x - 3$.

- a. Sketch the graph of $y = f(x)$ for $-3 \leq x \leq 6$ and $-10 \leq y \leq 10$ showing clearly the axes intercepts and local maximum and minimum points. Use a [4] scale of 2 cm to represent 1 unit on the x -axis, and a scale of 1 cm to represent 1 unit on the y -axis.
- b. Find the value of $f(-1)$. [2]
- c. Write down the coordinates of the y -intercept of the graph of $f(x)$. [1]
- d. Find $f'(x)$. [3]
- e. Show that $f'(-1) = -\frac{16}{3}$. [1]
- f. Explain what $f'(-1)$ represents. [2]
- g. Find the equation of the tangent to the graph of $f(x)$ at the point where x is -1 . [2]
- h. Sketch the tangent to the graph of $f(x)$ at $x = -1$ on your diagram for (a). [2]

- i. P and Q are points on the curve such that the tangents to the curve at these points are horizontal. The x-coordinate of P is a , and the x-coordinate of Q is b , $b > a$. [2]

Write down the value of

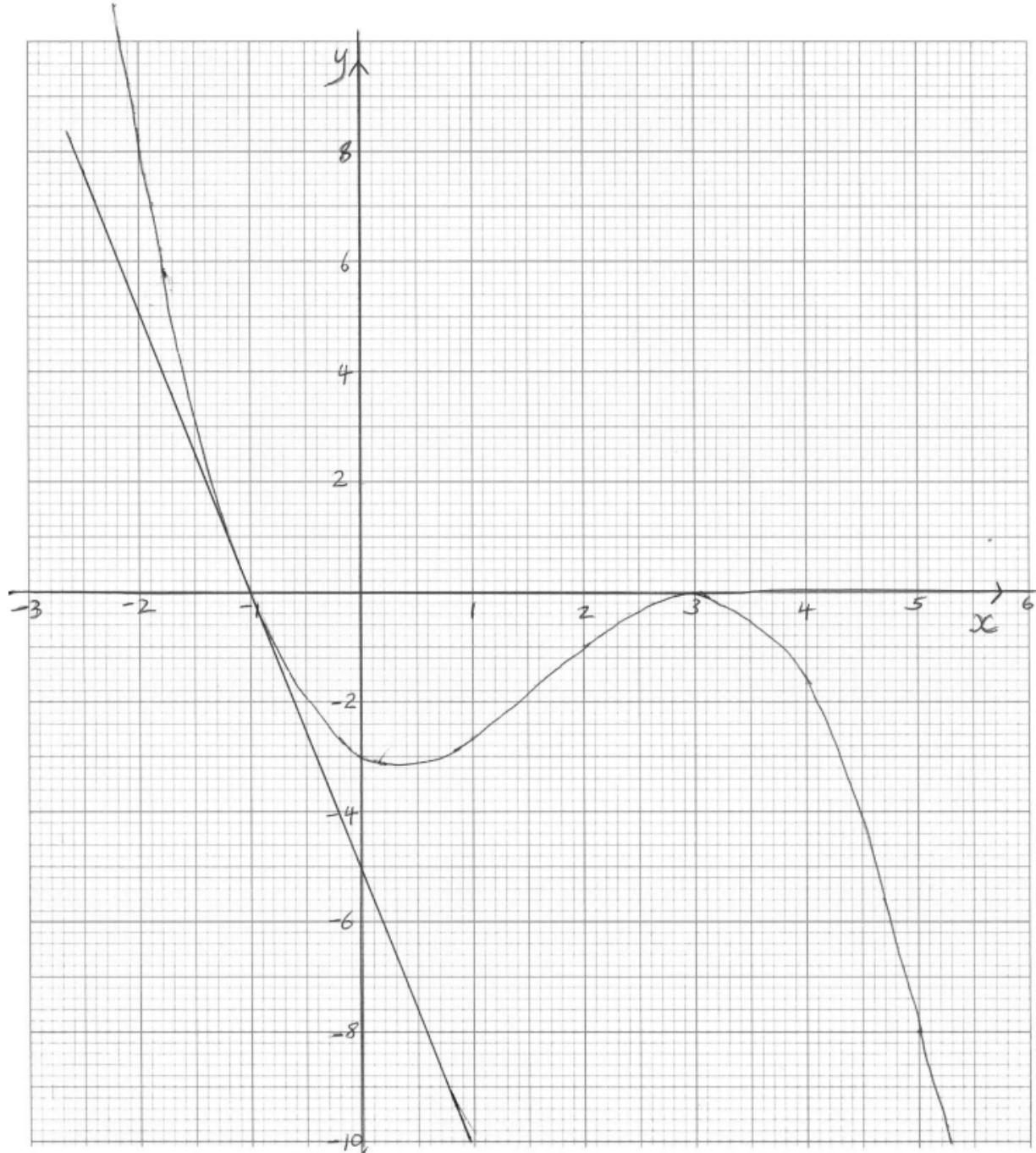
- (i) a ;
(ii) b .

- j. P and Q are points on the curve such that the tangents to the curve at these points are horizontal. The x-coordinate of P is a , and the x-coordinate of Q is b , $b > a$. [1]

Describe the behaviour of $f(x)$ for $a < x < b$.

Markscheme

a.



(A1) for indication of window and labels. (A1) for smooth curve that does not enter the first quadrant, the curve must consist of one line only.

(A1) for x and y intercepts in approximately correct positions (allow ± 0.5).

(A1) for local maximum and minimum in approximately correct position. (minimum should be $0 \leq x \leq 1$ and $-2 \leq y \leq -4$), the y -coordinate of the maximum should be 0 ± 0.5 . **(A4)**

[4 marks]

b. $-\frac{1}{3}(-1)^3 + \frac{5}{3}(-1)^2 - (-1) - 3$ **(M1)**

Note: Award **(M1)** for substitution of -1 into $f(x)$

$= 0$ **(A1)(G2)**

[2 marks]

c. $(0, -3)$ **(A1)**

OR

$x = 0, y = -3$ **(A1)**

Note: Award **(A0)** if brackets are omitted.

[1 mark]

d. $f'(x) = -x^2 + \frac{10}{3}x - 1$ **(A1)(A1)(A1)**

Note: Award **(A1)** for each correct term. Award **(A1)(A1)(A0)** at most if there are extra terms.

[3 marks]

e. $f'(-1) = -(-1)^2 + \frac{10}{3}(-1) - 1$ **(M1)**

$= -\frac{16}{3}$ **(AG)**

Note: Award **(M1)** for substitution of $x = -1$ into correct derivative only. The final answer must be seen.

[1 mark]

f. $f'(-1)$ gives the gradient of the tangent to the curve at the point with $x = -1$. **(A1)(A1)**

Note: Award **(A1)** for “gradient (of curve)”, **(A1)** for “at the point with $x = -1$ ”. Accept “the instantaneous rate of change of y ” or “the (first) derivative”.

[2 marks]

g. $y = -\frac{16}{3}x + c$ **(M1)**

Note: Award **(M1)** for $-\frac{16}{3}$ substituted in equation.

$0 = -\frac{16}{3} \times (-1) + c$

$c = -\frac{16}{3}$

$y = -\frac{16}{3}x - \frac{16}{3}$ **(A1)(G2)**

Note: Accept $y = -5.33x - 5.33$.

OR

$(y - 0) = -\frac{16}{3}(x + 1)$ **(M1)(A1)(G2)**

Note: Award **(M1)** for $-\frac{16}{3}$ substituted in equation, **(A1)** for correct equation. Follow through from their answer to part (b). Accept $y = -5.33(x + 1)$. Accept equivalent equations.

[2 marks]

h. **(A1)(ft)** for a tangent to their curve drawn.

(A1)(ft) for their tangent drawn at the point $x = -1$. **(A1)(ft)(A1)(ft)**

Note: Follow through from their graph. The tangent must be a straight line otherwise award at most **(A0)(A1)**.

[2 marks]

i. (i) $a = \frac{1}{3}$ **(G1)**

(ii) $b = 3$ **(G1)**

Note: If a and b are reversed award **(A0)(A1)**.

[2 marks]

j. $f(x)$ is increasing **(A1)**

[1 mark]

Examiners report

a. This question caused the most difficulty to candidates for two reasons; its content and perhaps lack of time.

Drawing/sketching graphs is perhaps the area of the course that results in the poorest responses. It is also the area of the course that results in the best. It is therefore the area of the course that good teaching can influence the most.

Candidates should:

- Use the correct scale and window. Label the axes.
- Enter the formula into the GDC and use the table function to determine the points to be plotted.
- Refer to the graph on the GDC when drawing the curve.
- Draw a curve rather than line segments; ensure that the curve is smooth.
- Use a pencil rather than a pen so that required changes once further information has been gathered (the turning points, for example) can be made.

b. This question caused the most difficulty to candidates for two reasons; its content and perhaps lack of time.

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In part (b) the answer could have been checked using the table on the GDC.

c. This question caused the most difficulty to candidates for two reasons; its content and perhaps lack of time.

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In part (c) **coordinates** were required.

d. This question caused the most difficulty to candidates for two reasons; its content and perhaps lack of time.

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The responses to part (d) were generally correct.

e. This question caused the most difficulty to candidates for two reasons; its content and perhaps lack of time.

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- Use a pencil rather than a pen so that required changes once further information has been gathered (the turning points, for example) can be made.

The “show that” nature of part (e) meant that the final answer had to be stated.

f. This question caused the most difficulty to candidates for two reasons; its content and perhaps lack of time.

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- Draw a curve rather than line segments; ensure that the curve is smooth.
- Use a pencil rather than a pen so that required changes once further information has been gathered (the turning points, for example) can be made.

The interpretive nature of part (f) was not understood by the majority.

g. This question caused the most difficulty to candidates for two reasons; its content and perhaps lack of time.

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- Draw a curve rather than line segments; ensure that the curve is smooth.
- Use a pencil rather than a pen so that required changes once further information has been gathered (the turning points, for example) can be made.

h. This question caused the most difficulty to candidates for two reasons; its content and perhaps lack of time.

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- Refer to the graph on the GDC when drawing the curve.
- Draw a curve rather than line segments; ensure that the curve is smooth.
- Use a pencil rather than a pen so that required changes once further information has been gathered (the turning points, for example) can be made.

i. This question caused the most difficulty to candidates for two reasons; its content and perhaps lack of time.

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Candidates should:

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- Refer to the graph on the GDC when drawing the curve.
- Draw a curve rather than line segments; ensure that the curve is smooth.
- Use a pencil rather than a pen so that required changes once further information has been gathered (the turning points, for example) can be made.

Parts (i) and (j) had many candidates floundering; there were few good responses to these parts.

j. This question caused the most difficulty to candidates for two reasons; its content and perhaps lack of time.

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