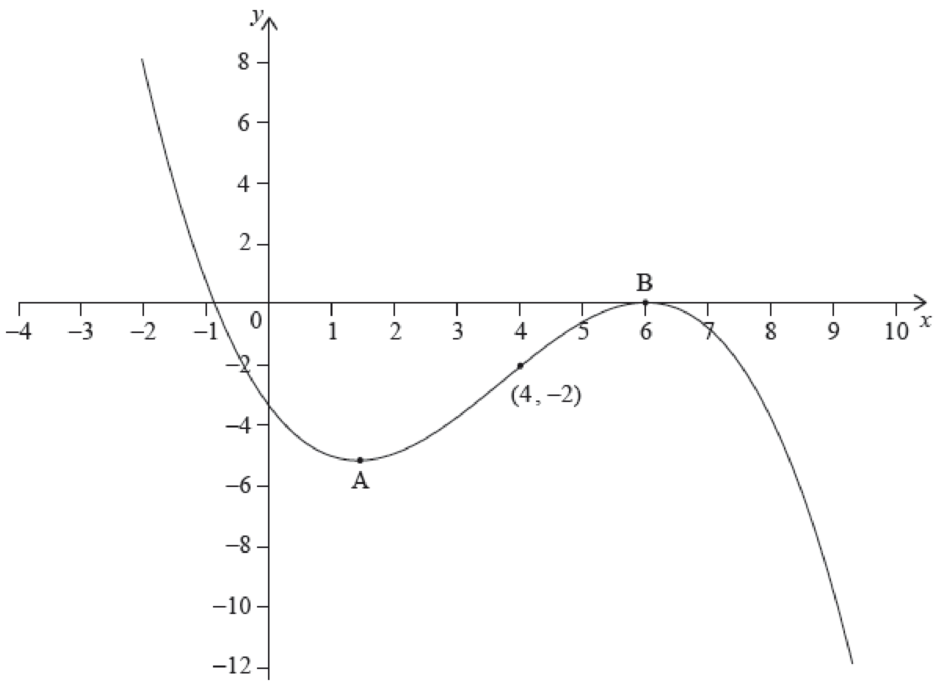
SL Paper 1

The following diagram shows the graph of f', the derivative of f.



The graph of f' has a local minimum at A, a local maximum at B and passes through $(4,\ -2)$.

The point $\mathrm{P}(4,\ 3)$ lies on the graph of the function, f.

a.i. Write down the gradient of the curve of f at P. [1]

a.ii. Find the equation of the normal to the curve of f at P. [3]

b. Determine the concavity of the graph of f when $4 < x < 5$ **and** justify your answer. [2]

Markscheme

a.i. -2 ***A1*** ***N1***

[1 mark]

a.ii. gradient of normal $= \frac{1}{2}$ ***(A1)***

attempt to substitute their normal gradient and coordinates of P (in any order) ***(M1)***

eg $y - 4 = \frac{1}{2}(x - 3),\ 3 = \frac{1}{2}(4) + b,\ b = 1$

$y - 3 = \frac{1}{2}(x - 4),\ y = \frac{1}{2}x + 1,\ x - 2y + 2 = 0$ ***A1*** ***N3***

[3 marks]

b. correct answer **and** valid reasoning ***A2*** ***N2***

answer: *eg* graph of f is concave up, concavity is positive (between $4 < x < 5$)

reason: *eg* slope of f' is positive, f' is increasing, $f'' > 0$,

sign chart (must clearly be for f'' and show A and B)

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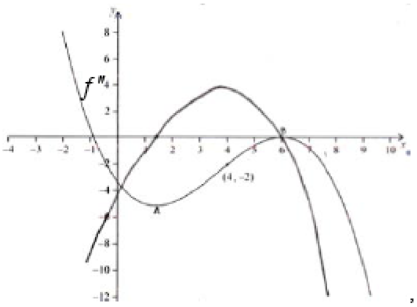

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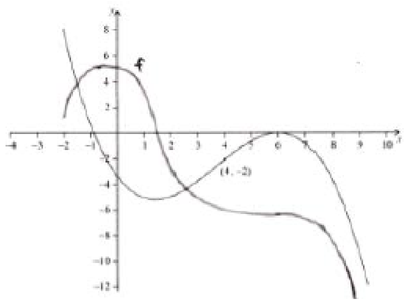


Note: The reason given must refer to a specific function/graph. Referring to “the graph” or “it” is not sufficient.

[2 marks]

Examiners report

a.i. [N/A]
a.ii. [N/A]
b. [N/A]

The values of the functions f and g and their derivatives for $x = 1$ and $x = 8$ are shown in the following table.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	2	4	9	−3
8	4	−3	2	5

Let $h(x) = f(x)g(x)$.

a. Find $h(1)$. [2]

b. Find $h'(8)$. [3]

Markscheme

a. expressing $h(1)$ as a product of $f(1)$ and $g(1)$ ***(A1)***

eg $f(1) \times g(1),\ 2(9)$

$h(1) = 18$ ***A1*** ***N2***

[2 marks]

b. attempt to use product rule (do **not** accept $h' = f' \times g'$) ***(M1)***

eg $h' = fg' + gf',\ h'(8) = f'(8)g(8) + g'(8)f(8)$

correct substitution of values into product rule ***(A1)***

eg $h'(8) = 4(5) + 2(-3),\ -6 + 20$

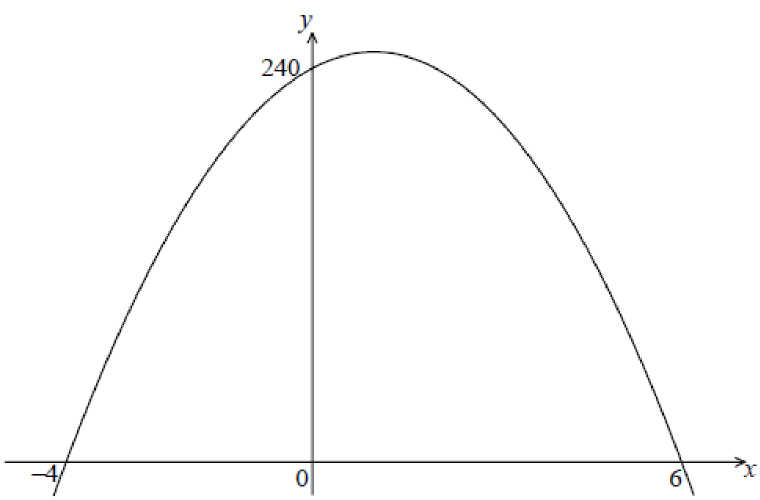
$h'(8) = 14$ ***A1 N2***

[3 marks]

Examiners report

a. [N/A]
b. [N/A]

The following diagram shows part of the graph of a quadratic function f.



The x-intercepts are at $(-4, 0)$ and $(6, 0)$, and the y-intercept is at $(0, 240)$.

a. Write down $f(x)$ in the form $f(x) = -10(x-p)(x-q)$. [2]

b. Find another expression for $f(x)$ in the form $f(x) = -10(x-h)^2 + k$. [4]

c. Show that $f(x)$ can also be written in the form $f(x) = 240 + 20x - 10x^2$. [2]

d(i) And (ii). A particle moves along a straight line so that its velocity, $v\ \mathrm{ms}^{-1}$, at time t seconds is given by $v = 240 + 20t - 10t^2$, for $0 \le t \le 6$. [7]

(i) Find the value of t when the speed of the particle is greatest.

(ii) Find the acceleration of the particle when its speed is zero.

Markscheme

a. $f(x) = -10(x+4)(x-6)$ ***A1A1*** ***N2***

[2 marks]

b. **METHOD 1**

attempting to find the x-coordinate of maximum point ***(M1)***

e.g. averaging the x-intercepts, sketch, $y' = 0$, axis of symmetry

attempting to find the y-coordinate of maximum point ***(M1)***

e.g. $k = -10(1+4)(1-6)$

$f(x) = -10(x-1)^2 + 250$ ***A1A1*** ***N4***

METHOD 2

attempt to expand $f(x)$ ***(M1)***

e.g. $-10(x^2 - 2x - 24)$

attempt to complete the square ***(M1)***

e.g. $-10((x-1)^2 - 1 - 24)$

$f(x) = -10(x-1)^2 + 250$ ***A1A1*** ***N4***

[4 marks]

c. attempt to simplify ***(M1)***

e.g. distributive property, $-10(x-1)(x-1) + 250$

correct simplification ***A1***

e.g. $-10(x^2 - 6x + 4x - 24)$, $-10(x^2 - 2x + 1) + 250$

$f(x) = 240 + 20x - 10x^2$ ***AG*** ***N0***

[2 marks]

d(i) and (ii). (i) valid approach ***(M1)***

e.g. vertex of parabola, $v'(t) = 0$

$t = 1$ ***A1*** ***N2***

(ii) recognizing $a(t) = v'(t)$ ***(M1)***

$a(t) = 20 - 20t$ ***A1A1***

speed is zero $\Rightarrow t = 6$ ***(A1)***

$a(6) = -100$ (ms^{-2}) ***A1*** ***N3***

[7 marks]

Examiners report

a. Parts (a) and (c) of this question were very well done by most candidates.

b. In part (b), many candidates attempted to use the method of completing the square, but were unsuccessful dealing with the coefficient of -10. Candidates who recognized that the x-coordinate of the vertex was 1, then substituted this value into the function from part (a), were generally able to earn full marks here.

c. Parts (a) and (c) of this question were very well done by most candidates.

d(i) and (ii). In part (d), it was clear that many candidates were not familiar with the relationship between velocity and acceleration, and did not understand how those concepts were related to the graph which was given. A large number of candidates used time $t = 1$ in part b(ii), rather than $t = 6$. To find the acceleration, some candidates tried to integrate the velocity function, rather than taking the derivative of velocity. Still others found the derivative in part b(i), but did not realize they needed to use it in part b(ii), as well.

Let $g(x) = 2x \sin x$.

a. Find $g'(x)$. [4]

b. Find the gradient of the graph of g at $x = \pi$. [3]

Markscheme

a. evidence of choosing the product rule ***(M1)***

e.g. $uv' + vu'$

correct derivatives $\cos x$, 2 ***(A1)(A1)***

$g'(x) = 2x \cos x + 2 \sin x$ ***A1*** ***N4***

[4 marks]

b. attempt to substitute into gradient function ***(M1)***

e.g. $g'(\pi)$

correct substitution ***(A1)***

e.g. $2\pi \cos \pi + 2 \sin \pi$

gradient $= -2\pi$ ***A1*** ***N2***

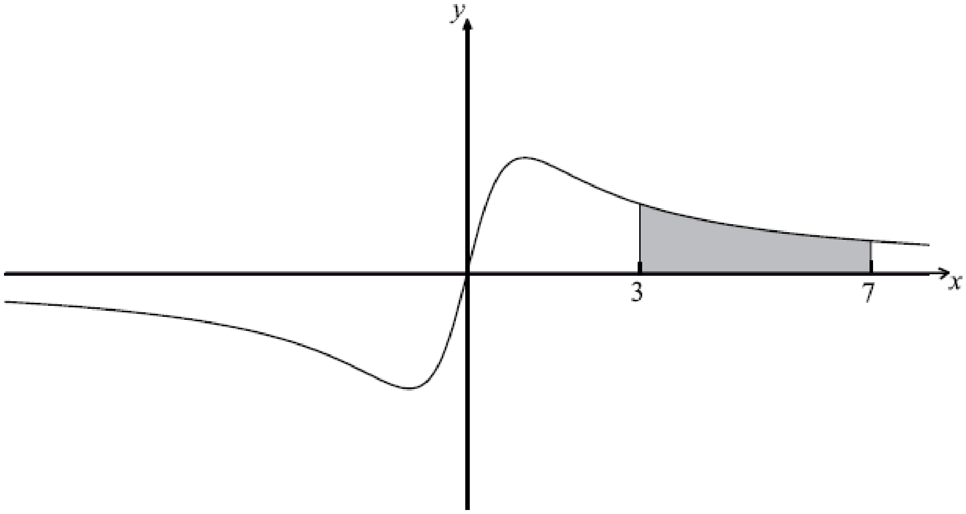
[3 marks]

Examiners report

a. Most candidates answered part (a) correctly, using the product rule to find the derivative, and earned full marks here. There were some who did not know to use the product rule, and of course did not find the correct derivative.

b. In part (b), many candidates substituted correctly into their derivatives, but then used incorrect values for $\sin x$ and $\cos x$, leading to the wrong gradient in their final answers.

Let $f(x) = \frac{ax}{x^2+1}$, $-8 \le x \le 8$, $a \in \mathbb{R}$ .The graph of f is shown below.



The region between $x = 3$ and $x = 7$ is shaded.

a. Show that $f(-x) = -f(x)$. [2]

b. Given that $f''(x) = \frac{2ax(x^2-3)}{(x^2+1)^3}$, find the coordinates of all points of inflexion. [7]

c. It is given that $\int f(x)\mathrm{d}x = \frac{a}{2}\ln(x^2+1) + C$. [7]

(i) Find the area of the shaded region, giving your answer in the form $p\ln q$.

(ii) Find the value of $\int_4^8 2f(x-1)\mathrm{d}x$.

Markscheme

a. **METHOD 1**

evidence of substituting $-x$ for x ***(M1)***

$f(-x) = \frac{a(-x)}{(-x)^2+1}$ ***A1***

$f(-x) = \frac{-ax}{x^2+1}$ $(= -f(x))$ ***AG*** ***N0***

METHOD 2

$y = -f(x)$ is reflection of $y = f(x)$ in x axis

and $y = f(-x)$ is reflection of $y = f(x)$ in y axis ***(M1)***

sketch showing these are the same ***A1***

$f(-x) = \frac{-ax}{x^2+1}$ $(= -f(x))$ ***AG*** ***N0***

[2 marks]

b. evidence of appropriate approach ***(M1)***

e.g. $f''(x) = 0$

to set the numerator equal to 0 ***(A1)***

e.g. $2ax(x^2-3) = 0$; $(x^2-3) = 0$

$(0, 0)$, $\left(\sqrt{3}, \frac{a\sqrt{3}}{4}\right)$, $\left(-\sqrt{3}, -\frac{a\sqrt{3}}{4}\right)$ (accept $x = 0$, $y = 0$ etc) ***A1A1A1A1A1*** ***N5***

[7 marks]

c. (i) correct expression ***A2***

e.g. $\left[\frac{a}{2}\ln(x^2+1)\right]_3^7$, $\frac{a}{2}\ln 50 - \frac{a}{2}\ln 10$, $\frac{a}{2}(\ln 50 - \ln 10)$

area = $\frac{a}{2}\ln 5$ ***A1A1*** ***N2***

(ii) **METHOD 1**

recognizing the shift that does not change the area ***(M1)***

e.g. $\int_4^8 f(x-1)\mathrm{d}x = \int_3^7 f(x)\mathrm{d}x$, $\frac{a}{2}\ln 5$

recognizing that the factor of 2 doubles the area ***(M1)***

e.g. $\int_4^8 2f(x-1)\mathrm{d}x = 2\int_4^8 f(x-1)\mathrm{d}x \left(= 2\int_3^7 f(x)\mathrm{d}x\right)$

$\int_4^8 2f(x-1)\mathrm{d}x = a\ln 5$ (i.e. $2\times$ **their** answer to (c)(i)) ***A1*** ***N3***

METHOD 2

changing variable

let $w = x - 1$, so $\frac{\mathrm{d}w}{\mathrm{d}x} = 1$

$2\int f(w)\mathrm{d}w = \frac{2a}{2}\ln(w^2+1) + c$ ***(M1)***

substituting correct limits

e.g. $\left[a\ln\left[(x-1)^2+1\right]\right]_4^8$, $\left[a\ln(w^2+1)\right]_3^7$, $a\ln 50 - a\ln 10$ ***(M1)***

$\int_4^8 2f(x-1)\mathrm{d}x = a\ln 5$ ***A1*** ***N3***

[7 marks]

Examiners report

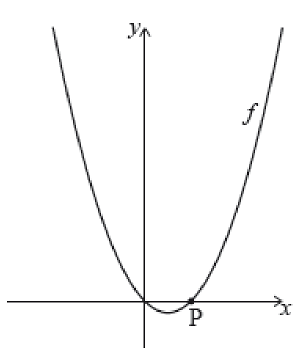
a. Part (a) was achieved by some candidates, although brackets around the $-x$ were commonly neglected. Some attempted to show the relationship by substituting a specific value for x . This earned no marks as a general argument is required.

b. Although many recognized the requirement to set the second derivative to zero in (b), a majority neglected to give their answers as ordered pairs, only writing the x-coordinates. Some did not consider the negative root.

c. For those who found a correct expression in (c)(i), many finished by calculating $\ln 50 - \ln 10 = \ln 40$. Few recognized that the translation did not change the area, although some factored the 2 from the integrand, appreciating that the area is double that in (c)(i).

Let $f(x) = x^2 - x$, for $x \in \mathbb{R}$. The following diagram shows part of the graph of f.

diagram not to scale

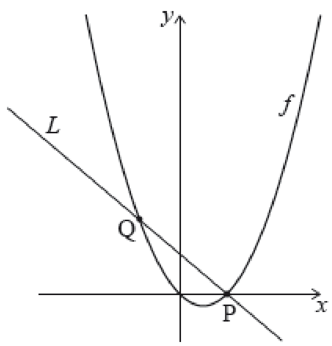


The graph of f crosses the x-axis at the origin and at the point $\mathrm{P}(1,\ 0)$.

The line *L* is the normal to the graph of *f* at P.

The line L intersects the graph of f at another point Q, as shown in the following diagram.

diagram not to scale



a. Show that $f'(1) = 1$. [3]

b. Find the equation of L in the form $y = ax + b$. [3]

c. Find the x-coordinate of Q. [4]

d. Find the area of the region enclosed by the graph of f and the line L. [6]

Markscheme

a. $f'(x) = 2x - 1$ ***A1A1***

correct substitution ***A1***

eg $2(1) - 1,\ 2 - 1$

$f'(1) = 1$ ***AG*** ***N0***

[3 marks]

b. correct approach to find the gradient of the normal ***(A1)***

eg $\frac{-1}{f'(1)},\ m_1 m_2 = -1,\ \text{slope} = -1$

attempt to substitute correct normal gradient and coordinates into equation of a line ***(M1)***

eg $y - 0 = -1(x - 1),\ 0 = -1 + b,\ b = 1,\ L = -x + 1$

$y = -x + 1$ ***A1*** ***N2***

[3 marks]

c. equating expressions ***(M1)***

eg $f(x) = L,\ -x + 1 = x^2 - x$

correct working (must involve combining terms) ***(A1)***

eg $x^2 - 1 = 0,\ x^2 = 1,\ x = 1$

$x = -1 \quad (\text{accept } Q(-1,\ 2))$ ***A2*** ***N3***

[4 marks]

d. valid approach ***(M1)***

eg $\int L - f,\ \int_{-1}^{1} (1 - x^2) \mathrm{d}x$, splitting area into triangles and integrals

correct integration ***(A1)(A1)***

eg $\left[x - \frac{x^3}{3}\right]_{-1}^{1}$, $-\frac{x^3}{3} - \frac{x^2}{2} + \frac{x^2}{2} + x$

substituting **their** limits into **their** integrated function and subtracting (in any order) ***(M1)***

eg $1 - \frac{1}{3} - \left(-1 - \frac{-1}{3}\right)$

Note: Award ***M0*** for substituting into original or differentiated function.

area $= \frac{4}{3}$ ***A2 N3***

[6 marks]

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]
d. [N/A]

Let $f(x) = \frac{\cos x}{\sin x}$, for $\sin x \neq 0$.

In the following table, $f'\left(\frac{\pi}{2}\right) = p$ and $f''\left(\frac{\pi}{2}\right) = q$. The table also gives approximate values of $f'(x)$ and $f''(x)$ near $x = \frac{\pi}{2}$.

x	$\frac{\pi}{2} - 0.1$	$\frac{\pi}{2}$	$\frac{\pi}{2} + 0.1$
$f'(x)$	-1.01	p	-1.01
$f''(x)$	0.203	q	-0.203

a. Use the quotient rule to show that $f'(x) = \frac{-1}{\sin^2 x}$. [5]

b. Find $f''(x)$. [3]

c. Find the value of p and of q. [3]

d. Use information from the table to explain why there is a point of inflexion on the graph of f where $x = \frac{\pi}{2}$. [2]

Markscheme

a. $\frac{\mathrm{d}}{\mathrm{d}x}\sin x = \cos x$, $\frac{\mathrm{d}}{\mathrm{d}x}\cos x = -\sin x$ (seen anywhere) ***(A1)(A1)***

evidence of using the quotient rule ***M1***

correct substitution ***A1***

e.g. $\frac{\sin x(-\sin x) - \cos x(\cos x)}{\sin^2 x}$, $\frac{-\sin^2 x - \cos^2 x}{\sin^2 x}$

$f'(x) = \frac{-(\sin^2 x + \cos^2 x)}{\sin^2 x}$ ***A1***

$f'(x) = \frac{-1}{\sin^2 x}$ ***AG*** ***N0***

[5 marks]

b. **METHOD 1**

appropriate approach ***(M1)***

e.g. $f'(x) = -(\sin x)^{-2}$

$f''(x) = 2(\sin^{-3} x)(\cos x)$ $\left(= \frac{2\cos x}{\sin^3 x}\right)$ ***A1A1*** ***N3***

Note: Award ***A1*** for $2\sin^{-3} x$, ***A1*** for $\cos x$.

METHOD 2

derivative of $\sin^2 x = 2 \sin x \cos x$ (seen anywhere) ***A1***

evidence of choosing quotient rule ***(M1)***

e.g. $u = -1$, $v = \sin^2 x$, $f'' = \frac{\sin^2 x \times 0 - (-1) 2 \sin x \cos x}{(\sin^2 x)^2}$

$f''(x) = \frac{2 \sin x \cos x}{(\sin^2 x)^2}$ $\left(= \frac{2\cos x}{\sin^3 x}\right)$ ***A1*** ***N3***

[3 marks]

c. evidence of substituting $\frac{\pi}{2}$ ***M1***

e.g. $\frac{-1}{\sin^2 \frac{\pi}{2}}$, $\frac{2\cos\frac{\pi}{2}}{\sin^3\frac{\pi}{2}}$

$p = -1$, $q = 0$ ***A1A1*** ***N1N1***

[3 marks]

d. second derivative is zero, second derivative changes sign ***R1R1*** ***N2***

[2 marks]

Examiners report

a. Many candidates comfortably applied the quotient rule, although some did not completely show that the Pythagorean identity achieves the numerator of the answer given. Whether changing to $-(\sin x)^{-2}$, or applying the quotient rule a second time, most candidates neglected the chain rule in finding the second derivative.

b. Whether changing to $-(\sin x)^{-2}$, or applying the quotient rule a second time, most candidates neglected the chain rule in finding the second derivative.

c. Those who knew the trigonometric ratios at $\frac{\pi}{2}$ typically found the values of p and of q, sometimes in follow-through from an incorrect $f''(x)$.

d. Few candidates gave two reasons from the table that supported the existence of a point of inflexion. Most stated that the second derivative is zero and neglected to consider the sign change to the left and right of q. Some discussed a change of concavity, but without supporting this statement by referencing the change of sign in $f''(x)$, so no marks were earned.

Consider $f(x) = x^2 + \frac{p}{x}$, $x \neq 0$, where p is a constant.

a. Find $f'(x)$. [2]

b. There is a minimum value of $f(x)$ when $x = -2$. Find the value of p . [4]

Markscheme

a. $f'(x) = 2x - \frac{p}{x^2}$ ***A1A1*** ***N2***

Note: Award ***A1*** for $2x$, ***A1*** for $-\frac{p}{x^2}$.

[2 marks]

b. evidence of equating derivative to 0 (seen anywhere) ***(M1)***

evidence of finding $f'(-2)$ (seen anywhere) ***(M1)***

correct equation ***A1***

e.g. $-4 - \frac{p}{4} = 0$, $-16 - p = 0$

$p = -16$ ***A1*** ***N3***

[4 marks]

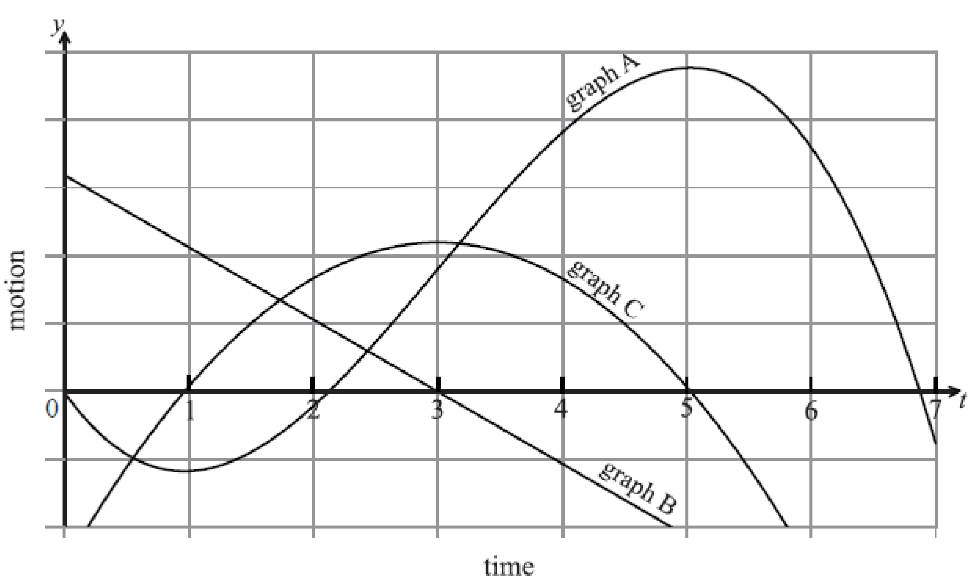
Examiners report

a. Candidates did well on (a).

b. For (b), a great number of candidates substituted into the function instead of into the derivative.

The derivate of x^2 was calculated without difficulties, but there were numerous problems regarding the derivative of $\frac{p}{x}$. There were several candidates who considered both *p* and *x* as variables; some tried to use the quotient rule and had difficulties, others used negative exponents and were not successful.

The following diagram shows the graphs of the **displacement**, **velocity** and **acceleration** of a moving object as functions of time, *t*.



a. Complete the following table by noting which graph A, B or C corresponds to each function. [4]

Function	Graph
displacement	
acceleration	

b. Write down the value of *t* when the velocity is greatest. [2]

Markscheme

a.

Function	Graph
displacement	A
acceleration	B

A2A2 N4

[4 marks]

b. $t = 3$ *A2 N2*

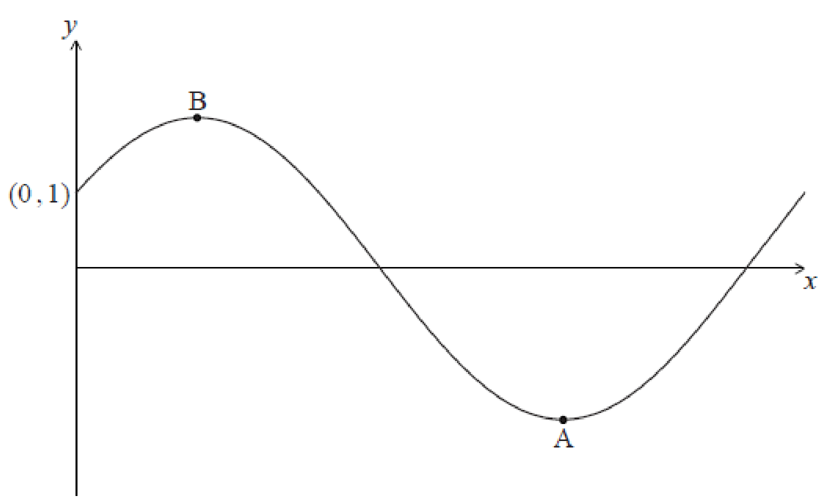
[2 marks]

Examiners report

a. Many candidates answered this question completely and correctly, showing a good understanding of the graphical relationship between displacement, velocity and acceleration. When done incorrectly, many answered with the displacement as graph B and acceleration as graph C.

b. Many candidates found the value of *t* which gave a maximum in the remaining graph, and were awarded follow through marks.

Let $f(x) = \cos x + \sqrt{3}\sin x$, $0 \le x \le 2\pi$. The following diagram shows the graph of f .



The y-intercept is at (0, 1) , there is a minimum point at A (p, q) and a maximum point at B.

a. Find $f'(x)$. [2]

b(i) and (ii). Hence [10]

(i) show that $q = -2$;

(ii) verify that A is a minimum point.

c. Find the maximum value of $f(x)$. [3]

d. The function $f(x)$ can be written in the form $r\cos(x - a)$. [2]

Write down the value of r and of a .

Markscheme

a. $f'(x) = -\sin x + \sqrt{3}\cos x$ ***A1A1*** ***N2***

[2 marks]

b(i) and (ii). (i) at A, $f'(x) = 0$ ***R1***

correct working ***A1***

e.g. $\sin x = \sqrt{3}\cos x$

$\tan x = \sqrt{3}$ ***A1***

$x = \frac{\pi}{3}, \frac{4\pi}{3}$ ***A1***

attempt to substitute **their** x into $f(x)$ ***M1***

e.g. $\cos\left(\frac{4\pi}{3}\right) + \sqrt{3}\sin\left(\frac{4\pi}{3}\right)$

correct substitution ***A1***

e.g. $-\frac{1}{2} + \sqrt{3}\left(-\frac{\sqrt{3}}{2}\right)$

correct working that clearly leads to -2 ***A1***

e.g. $-\frac{1}{2} - \frac{3}{2}$

$q = -2$ ***AG*** ***N0***

(ii) correct calculations to find $f'(x)$ either side of $x = \frac{4\pi}{3}$ ***A1A1***

e.g. $f'(\pi) = 0 - \sqrt{3}$, $f'(2\pi) = 0 + \sqrt{3}$

$f'(x)$ changes sign from negative to positive ***R1***

so A is a minimum ***AG*** ***N0***

[10 marks]

c. max when $x = \frac{\pi}{3}$ ***R1***

correctly substituting $x = \frac{\pi}{3}$ into $f(x)$ ***A1***

e.g. $\frac{1}{2} + \sqrt{3}\left(\frac{\sqrt{3}}{2}\right)$

max value is 2 ***A1*** ***N1***

[3 marks]

d. $r = 2$, $a = \frac{\pi}{3}$ ***A1A1*** ***N2***

[2 marks]

Examiners report

[N/A]

a(i) and (ii). [N/A]
b. [N/A]
c. [N/A]
d. [N/A]

Consider a function f. The line L_1 with equation $y = 3x + 1$ is a tangent to the graph of f when $x = 2$

Let $g(x) = f(x^2 + 1)$ and P be the point on the graph of g where $x = 1$.

a.i. Write down $f'(2)$. [2]

a.ii. Find $f(2)$. [2]

b. Show that the graph of *g* has a gradient of 6 at P. [5]

c. Let L_2 be the tangent to the graph of *g* at P. L_1 intersects L_2 at the point Q. [7]

Find the y-coordinate of Q.

Markscheme

a.i. recognize that $f'(x)$ is the gradient of the tangent at x ***(M1)***

eg $f'(x) = m$

$f'(2) = 3$ (accept *m* = 3) ***A1 N2***

[2 marks]

a.ii. recognize that $f(2) = y(2)$ ***(M1)***

eg $f(2) = 3 \times 2 + 1$

$f(2) = 7$ ***A1 N2***

[2 marks]

b. recognize that the gradient of the graph of *g* is $g'(x)$ ***(M1)***

choosing chain rule to find $g'(x)$ ***(M1)***

eg $\frac{\mathrm{d}y}{\mathrm{d}u} \times \frac{\mathrm{d}u}{\mathrm{d}x},\ u = x^2 + 1,\ u' = 2x$

$g'(x) = f'(x^2 + 1) \times 2x$ ***A2***

$g'(1) = 3 \times 2$ ***A1***

$g'(1) = 6$ ***AG N0***

[5 marks]

c. at Q, $L_1 = L_2$ (seen anywhere) ***(M1)***

recognize that the gradient of L_2 is *g*'(1) (seen anywhere) ***(M1)***
eg *m* = 6

finding *g*(1) (seen anywhere) ***(A1)***
eg $g(1) = f(2),\ g(1) = 7$

attempt to substitute gradient and/or coordinates into equation of a straight line ***M1***
eg $y - g(1) = 6(x-1)$, $y - 1 = g'(1)(x-7)$, $7 = 6(1) + \mathrm{b}$

correct equation for L_2

eg $y - 7 = 6(x-1)$, $y = 6x + 1$ ***A1***

correct working to find Q ***(A1)***
eg same *y*-intercept, $3x = 0$

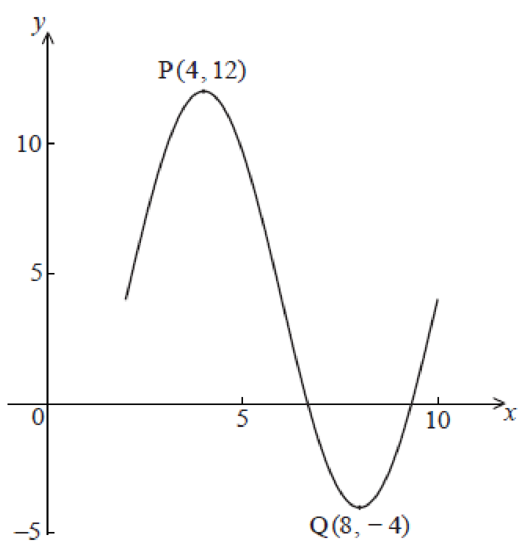
$y = 1$ ***A1 N2***

[7 marks]

Examiners report

a.i. [N/A]
a.ii. [N/A]
b. [N/A]
c. [N/A]

The following diagram shows the graph of $f(x) = a\sin(b(x-c)) + d$, for $2 \le x \le 10$.



There is a maximum point at P(4, 12) and a minimum point at Q(8, −4).

a(i), (ii) and (iii). Use the graph to write down the value of [3]

(i) a;

(ii) c;

(iii) d.

b. Show that $b = \frac{\pi}{4}$. [2]

c. Find $f'(x)$. [3]

d. At a point R, the gradient is -2π. Find the *x*-coordinate of R. [6]

Markscheme

a(i), (ii) and (iii). (i) $a = 8$ ***A1*** ***N1***

(ii) $c = 2$ ***A1*** ***N1***

(iii) $d = 4$ ***A1*** ***N1***

[3 marks]

b. **METHOD 1**

recognizing that period $= 8$ ***(A1)***

correct working ***A1***

e.g. $8 = \frac{2\pi}{b}$, $b = \frac{2\pi}{8}$

$b = \frac{\pi}{4}$ ***AG*** ***N0***

METHOD 2

attempt to substitute ***M1***

e.g. $12 = 8\sin(b(4-2)) + 4$

correct working ***A1***

e.g. $\sin 2b = 1$

$b = \frac{\pi}{4}$ ***AG*** ***N0***

[2 marks]

c. evidence of attempt to differentiate or choosing chain rule ***(M1)***

e.g. $\cos\frac{\pi}{4}(x-2)$, $\frac{\pi}{4} \times 8$

$f'(x) = 2\pi\cos\left(\frac{\pi}{4}(x-2)\right)$ (accept $2\pi\cos\frac{\pi}{4}(x-2)$) ***A2*** ***N3***

[3 marks]

d. recognizing that gradient is $f'(x)$ ***(M1)***

e.g. $f'(x) = m$

correct equation ***A1***

e.g. $-2\pi = 2\pi\cos\left(\frac{\pi}{4}(x-2)\right)$, $-1 = \cos\left(\frac{\pi}{4}(x-2)\right)$

correct working ***(A1)***

e.g. $\cos^{-1}(-1) = \frac{\pi}{4}(x-2)$

using $\cos^{-1}(-1) = \pi$ (seen anywhere) ***(A1)***

e.g. $\pi = \frac{\pi}{4}(x-2)$

simplifying ***(A1)***

e.g. $4 = (x-2)$

$x = 6$ ***A1*** ***N4***

[6 marks]

Examiners report

a(i), (ii) and (iii). Part (a) of this question proved challenging for most candidates.

b. Although a good number of candidates recognized that the period was 8 in part (b), there were some who did not seem to realize that this period could be found using the given coordinates of the maximum and minimum points.

c. In part (c), not many candidates found the correct derivative using the chain rule.

d. For part (d), a good number of candidates correctly set their expression equal to -2π , but errors in their previous values kept most from correctly solving the equation. Most candidates who had the correct equation were able to gain full marks here.

Given that $f(x) = \frac{1}{x}$, answer the following.

a. Find the first four derivatives of $f(x)$. [4]

b. Write an expression for $f^{(n)}(x)$ in terms of *x* and *n* . [3]

Markscheme

a. $f'(x) = -x^{-2}$ (or $-\frac{1}{x^2}$) ***A1 N1***

$f''(x) = 2x^{-3}$ (or $\frac{2}{x^3}$) ***A1 N1***

$f'''(x) = -6x^{-4}$ (or $-\frac{6}{x^4}$) ***A1 N1***

$f^{(4)}(x) = 24x^{-5}$ (or $\frac{24}{x^5}$) ***A1 N1***

[4 marks]

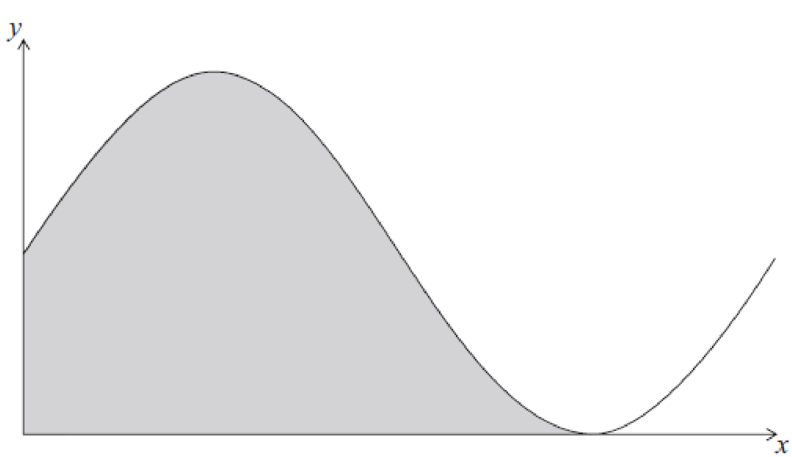
b. $f^{(n)}(x) = \frac{(-1)^n n!}{x^{n+1}}$ or $(-1)^n n!(x^{-(n+1)})$ ***A1A1A1 N3***

[3 marks]

Examiners report

a. [N/A]
b. [N/A]

Let $f(x) = 6 + 6\sin x$. Part of the graph of *f* is shown below.



The shaded region is enclosed by the curve of *f* , the *x*-axis, and the *y*-axis.

a(i) and (ii). Solve for $0 \le x < 2\pi$ [5]

(i) $6 + 6\sin x = 6$;

(ii) $6 + 6\sin x = 0$.

b. Write down the exact value of the x-intercept of f , for $0 \le x < 2\pi$. [1]

c. The area of the shaded region is k . Find the value of k , giving your answer in terms of π . [6]

d. Let $g(x) = 6 + 6\sin\left(x - \frac{\pi}{2}\right)$. The graph of f is transformed to the graph of g. [2]

Give a full geometric description of this transformation.

e. Let $g(x) = 6 + 6\sin\left(x - \frac{\pi}{2}\right)$. The graph of f is transformed to the graph of g. [3]

Given that $\int_p^{p+\frac{3\pi}{2}} g(x)\mathrm{d}x = k$ and $0 \le p < 2\pi$, write down the two values of p.

Markscheme

a(i) and (ii). (i) $\sin x = 0$ ***A1***

$x = 0$, $x = \pi$ ***A1A1*** ***N2***

(ii) $\sin x = -1$ ***A1***

$x = \frac{3\pi}{2}$ ***A1*** ***N1***

[5 marks]

b. $\frac{3\pi}{2}$ ***A1*** ***N1***

[1 mark]

c. evidence of using anti-differentiation ***(M1)***

e.g. $\int_0^{\frac{3\pi}{2}} (6 + 6\sin x)\mathrm{d}x$

correct integral $6x - 6\cos x$ (seen anywhere) ***A1A1***

correct substitution ***(A1)***

e.g. $6\left(\frac{3\pi}{2}\right) - 6\cos\left(\frac{3\pi}{2}\right) - (-6\cos 0)$, $9\pi - 0 + 6$

$k = 9\pi + 6$ ***A1A1*** ***N3***

[6 marks]

d. translation of $\begin{pmatrix} \frac{\pi}{2} \\ 0 \end{pmatrix}$ ***A1A1*** ***N2***

[2 marks]

e. recognizing that the area under g is the same as the shaded region in f ***(M1)***

$p = \frac{\pi}{2}$, $p = 0$ ***A1A1*** ***N3***

[3 marks]

Examiners report

a(i) and (ii). Many candidates again had difficulty finding the common angles in the trigonometric equations. In part (a), some did not show sufficient working in solving the equations. Others obtained a single solution in (a)(i) and did not find another. Some candidates worked in degrees; the majority worked in radians.

b. While some candidates appeared to use their understanding of the graph of the original function to find the x-intercept in part (b), most used their working from part (a)(ii) sometimes with follow-through on an incorrect answer.

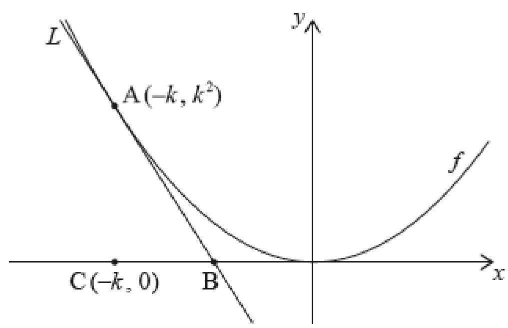
c. Most candidates recognized the need for integration in part (c) but far fewer were able to see the solution through correctly to the end. Some did not show the full substitution of the limits, having incorrectly assumed that evaluating the integral at 0 would be 0; without this working, the mark for evaluating at the limits could not be earned. Again, many candidates had trouble working with the common trigonometric values.

d. While there was an issue in the wording of the question with the given domains, this did not appear to bother candidates in part (d). This part was often well completed with candidates using a variety of language to describe the horizontal translation to the right by $\frac{\pi}{2}$.

e. Most candidates who attempted part (e) realized that the integral was equal to the value that they had found in part (c), but a majority tried to integrate the function g without success. Some candidates used sketches to find one or both values for p. The problem in the wording of the question did not appear to have been noticed by candidates in this part either.

Let $f(x) = x^2$. The following diagram shows part of the graph of f.

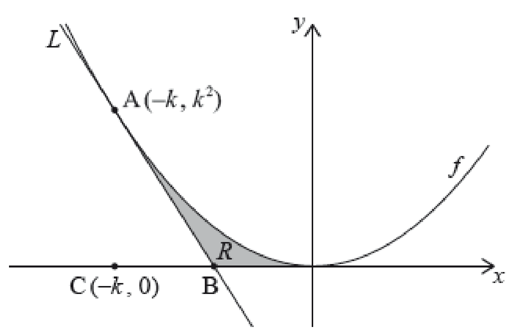
diagram not to scale



The line L is the tangent to the graph of f at the point $\mathrm{A}(-k,\ k^2)$, and intersects the x-axis at point B. The point C is $(-k,\ 0)$.

The region R is enclosed by L, the graph of f, and the x-axis. This is shown in the following diagram.

diagram not to scale



a.i. Write down $f'(x)$. [1]

a.ii. Find the gradient of L. [2]

b. Show that the x-coordinate of B is $-\frac{k}{2}$. [5]

c. Find the area of triangle ABC, giving your answer in terms of k. [2]

d. Given that the area of triangle ABC is p times the area of R, find the value of p. [7]

Markscheme

a.i. $f'(x) = 2x$ ***A1 N1***

[1 mark]

a.ii. attempt to substitute $x = -k$ into their derivative ***(M1)***

gradient of L is $-2k$ ***A1 N2***

[2 marks]

b. **METHOD 1**

attempt to substitute coordinates of A and their gradient into equation of a line ***(M1)***

eg $k^2 = -2k(-k) + b$

correct equation of L in any form ***(A1)***

eg $y - k^2 = -2k(x + k),\ y = -2kx - k^2$

valid approach ***(M1)***

eg $y = 0$

correct substitution into L equation ***A1***

eg $-k^2 = -2kx - 2k^2,\ 0 = -2kx - k^2$

correct working ***A1***

eg $2kx = -k^2$

$x = -\frac{k}{2}$ ***AG N0***

METHOD 2

valid approach ***(M1)***

eg $\text{gradient} = \frac{y_2 - y_1}{x_2 - x_1},\ -2k = \frac{\text{rise}}{\text{run}}$

recognizing $y = 0$ at B ***(A1)***

attempt to substitute coordinates of A and B into slope formula ***(M1)***

eg $\frac{k^2 - 0}{-k - x},\ \frac{-k^2}{x + k}$

correct equation ***A1***

eg $\frac{k^2 - 0}{-k - x} = -2k,\ \frac{-k^2}{x + k} = -2k,\ -k^2 = -2k(x + k)$

correct working ***A1***

eg $2kx = -k^2$

$x = -\frac{k}{2}$ ***AG N0***

[5 marks]

c. valid approach to find area of triangle ***(M1)***

eg $\frac{1}{2}(k^2)\left(\frac{k}{2}\right)$

area of $\text{ABC} = \frac{k^3}{4}$ ***A1 N2***

[2 marks]

d. **METHOD 1 ($\int f - \text{triangle}$)**

valid approach to find area from $-k$ to 0 ***(M1)*** *eg* $\int_{-k}^{0} x^2 \mathrm{d}x,\ \int_0^{-k} f$

correct integration (seen anywhere, even if ***M0*** awarded) ***A1***

eg $\frac{x^3}{3},\ \left[\frac{1}{3}x^3\right]_{-k}^{0}$

substituting **their** limits into **their** integrated function and subtracting ***(M1)***

eg $0 - \frac{(-k)^3}{3}$, area from $-k$ to 0 is $\frac{k^3}{3}$

Note: Award ***M0*** for substituting into original or differentiated function.

attempt to find area of R ***(M1)***

eg $\int_{-k}^{0} f(x)\mathrm{d}x -$ triangle

correct working for R ***(A1)***

eg $\frac{k^3}{3} - \frac{k^3}{4},\ R = \frac{k^3}{12}$

correct substitution into triangle $= pR$ ***(A1)***

eg $\frac{k^3}{4} = p\left(\frac{k^3}{3} - \frac{k^3}{4}\right),\ \frac{k^3}{4} = p\left(\frac{k^3}{12}\right)$

$p = 3$ ***A1 N2***

METHOD 2 ($\int (f - L)$)

valid approach to find area of R ***(M1)***

eg $\int_{-k}^{-\frac{k}{2}} x^2 - (-2kx - k^2)\mathrm{d}x + \int_{-\frac{k}{2}}^{0} x^2\mathrm{d}x,\ \int_{-k}^{-\frac{k}{2}} (f - L) + \int_{-\frac{k}{2}}^{0} f$

correct integration (seen anywhere, even if ***M0*** awarded) ***A2***

eg $\frac{x^3}{3} + kx^2 + k^2x,\ \left[\frac{x^3}{3} + kx^2 + k^2x\right]_{-k}^{-\frac{k}{2}} + \left[\frac{x^3}{3}\right]_{-\frac{k}{2}}^{0}$

substituting **their** limits into **their** integrated function and subtracting ***(M1)***

eg $\left(\frac{\left(-\frac{k}{2}\right)^3}{3} + k\left(-\frac{k}{2}\right)^2 + k^2\left(-\frac{k}{2}\right)\right) - \left(\frac{(-k)^3}{3} + k(-k)^2 + k^2(-k)\right) + (0) - \left(\frac{\left(-\frac{k}{2}\right)^3}{3}\right)$

Note: Award ***M0*** for substituting into original or differentiated function.

correct working for R ***(A1)***

eg $\frac{k^3}{24} + \frac{k^3}{24},\ -\frac{k^3}{24} + \frac{k^3}{4} - \frac{k^3}{2} + \frac{k^3}{3} - k^3 + k^3 + \frac{k^3}{24},\ R = \frac{k^3}{12}$

correct substitution into triangle $= pR$ ***(A1)***

eg $\frac{k^3}{4} = p\left(\frac{k^3}{24} + \frac{k^3}{24}\right),\ \frac{k^3}{4} = p\left(\frac{k^3}{12}\right)$

$p = 3$ ***A1 N2***

[7 marks]

Examiners report

a.i. [N/A]
a.ii. [N/A]
b. [N/A]
c. [N/A]
d. [N/A]

Let $f(x) = \sqrt{4x+5}$, for $x \geqslant -1.25$.

Consider another function g. Let R be a point on the graph of g. The x-coordinate of R is 1. The equation of the tangent to the graph at R is

$y = 3x + 6$.

a. Find $f'(1)$. [4]

b. Write down $g'(1)$. [2]

c. Find $g(1)$. [2]

d. Let $h(x) = f(x) \times g(x)$. Find the equation of the tangent to the graph of h at the point where $x = 1$. [7]

Markscheme

a. choosing chain rule ***(M1)***

eg $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}u} \times \frac{\mathrm{d}u}{\mathrm{d}x},\ u = 4x + 5,\ u' = 4$

correct derivative of f ***A2***

eg $\frac{1}{2}(4x+5)^{-\frac{1}{2}} \times 4,\ f'(x) = \frac{2}{\sqrt{4x+5}}$

$f'(1) = \frac{2}{3}$ ***A1 N2***

[4 marks]

b. recognize that $g'(x)$ is the gradient of the tangent ***(M1)***

eg $g'(x) = m$

$g'(1) = 3$ ***A1 N2***

[2 marks]

c. recognize that R is on the tangent ***(M1)***

eg $g(1) = 3 \times 1 + 6$, sketch

$g(1) = 9$ ***A1 N2***

[2 marks]

d. $f(1) = \sqrt{4+5}\ (= 3)$ (seen anywhere) ***A1***

$h(1) = 3 \times 9\ (= 27)$ (seen anywhere) ***A1***

choosing product rule to find $h'(x)$ ***(M1)***

eg $uv' + u'v$

correct substitution to find $h'(1)$ ***(A1)***

eg $f(1) \times g'(1) + f'(1) \times g(1)$

$h'(1) = 3 \times 3 + \frac{2}{3} \times 9\ (= 15)$ ***A1***

EITHER

attempt to substitute coordinates (in any order) into the equation of a straight line ***(M1)***

eg $y - 27 = h'(1)(x-1),\ y - 1 = 15(x - 27)$

$y - 27 = 15(x - 1)$ ***A1 N2***

OR

attempt to substitute coordinates (in any order) to find the y-intercept ***(M1)***

eg $27 = 15 \times 1 + b,\ 1 = 15 \times 27 + b$

$y = 15x + 12$ ***A1*** ***N2***

[7 marks]

Examiners report

a. Part a) was relatively well answered – the obvious errors seen were not using the chain rule correctly and simple fraction calculations being wrong.

b. In parts b) and c) it seemed that the students did not have a good conceptual understanding of what was actually happening in this question. There was lack of understanding of tangents, gradients and their relationship to the original function, g. A working sketch may have been beneficial but few were seen and many did a lot more work than required.

c. In parts b) and c) it seemed that the students did not have a good conceptual understanding of what was actually happening in this question. There was lack of understanding of tangents, gradients and their relationship to the original function, g. A working sketch may have been beneficial but few were seen and many did a lot more work than required.

d. In part d) although candidates recognized $h(x)$ as a product and may have correctly found $h(1)$, they did not necessarily use the product rule to find $h'(x)$, instead incorrectly using $h'(x) = f'(x) \times g'(x)$. It was rare for a candidate to get as far as finding the equation of a straight line but those who did usually gained full marks.

Let $f(x) = \cos x$.

Let $g(x) = x^k$, where $k \in \mathbb{Z}^+$.

Let $k = 21$ and $h(x) = \left(f^{(19)}(x) \times g^{(19)}(x)\right)$.

a. (i) Find the first four derivatives of $f(x)$. [4]

(ii) Find $f^{(19)}(x)$.

b. (i) Find the first three derivatives of $g(x)$. [5]

(ii) Given that $g^{(19)}(x) = \frac{k!}{(k-p)!}(x^{k-19})$, find p.

c. (i) Find $h'(x)$. [7]

(ii) Hence, show that $h'(\pi) = \frac{-21!}{2}\pi^2$.

Markscheme

a. (i) $f'(x) = -\sin x,\ f''(x) = -\cos x,\ f^{(3)}(x) = \sin x,\ f^{(4)}(x) = \cos x$ ***A2*** ***N2***

(ii) valid approach ***(M1)***

eg recognizing that 19 is one less than a multiple of 4, $f^{(19)}(x) = f^{(3)}(x)$

$f^{(19)}(x) = \sin x$ ***A1*** ***N2***

[4 marks]

b. (i) $g'(x) = kx^{k-1}$

$g''(x) = k(k-1)x^{k-2},\ g^{(3)}(x) = k(k-1)(k-2)x^{k-3}$ ***A1A1*** ***N2***

(ii) **METHOD 1**

correct working that leads to the correct answer, involving the correct expression for the 19th derivative ***A2***

eg $k(k-1)(k-2)\dots(k-18) \times \frac{(k-19)!}{(k-19)!},\ {}_kP_{19}$

$p = 19$ (accept $\frac{k!}{(k-19)!}x^{k-19}$) ***A1*** ***N1***

METHOD 2

correct working involving recognizing patterns in coefficients of first three derivatives (may be seen in part (b)(i)) leading to a general rule for 19th coefficient ***A2***

eg $g'' = 2!\begin{pmatrix} k \\ 2 \end{pmatrix},\ k(k-1)(k-2) = \frac{k!}{(k-3)!},\ g^{(3)}(x) = {}_kP_3(x^{k-3})$

$g^{(19)}(x) = 19!\begin{pmatrix} k \\ 19 \end{pmatrix},\ 19! \times \frac{k!}{(k-19)! \times 19!},\ {}_kP_{19}$

$p = 19$ (accept $\frac{k!}{(k-19)!}x^{k-19}$) ***A1*** ***N1***

[5 marks]

c. (i) valid approach using product rule ***(M1)***

eg $uv' + vu',\ f^{(19)}g^{(20)} + f^{(20)}g^{(19)}$

correct 20th derivatives (must be seen in product rule) ***(A1)(A1)***

eg $g^{(20)}(x) = \frac{21!}{(21-20)!}x,\ f^{(20)}(x) = \cos x$

$h'(x) = \sin x(21!x) + \cos x\left(\frac{21!}{2}x^2\right)$ $\left(\text{accept } \sin x\left(\frac{21!}{1!}x\right) + \cos x\left(\frac{21!}{2!}x^2\right)\right)$ ***A1*** ***N3***

(ii) substituting $x = \pi$ (seen anywhere) ***(A1)***

eg $f^{(19)}(\pi)g^{(20)}(\pi) + f^{(20)}(\pi)g^{(19)}(\pi),\ \sin\pi\frac{21!}{1!}\pi + \cos\pi\frac{21!}{2!}\pi^2$

evidence of one correct value for $\sin\pi$ or $\cos\pi$ (seen anywhere) ***(A1)***

eg $\sin\pi = 0,\ \cos\pi = -1$

evidence of correct values substituted into $h'(\pi)$ ***A1***

eg $21!(\pi)\left(0 - \frac{\pi}{2!}\right),\ 21!(\pi)\left(-\frac{\pi}{2}\right),\ 0 + (-1)\frac{21!}{2}\pi^2$

Note: If candidates write only the first line followed by the answer, award ***A1A0A0***.

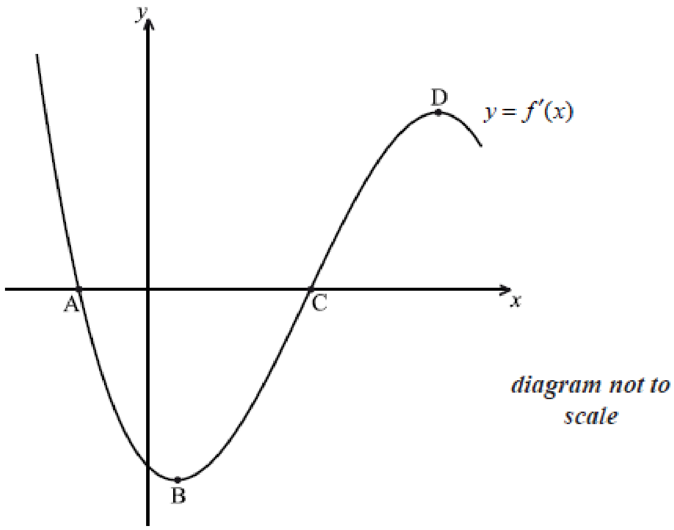
$\frac{-21!}{2}\pi^2$ ***AG*** ***N0***

[7 marks]

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]

The diagram shows part of the graph of $y = f'(x)$. The x-intercepts are at points A and C. There is a minimum at B, and a maximum at D.



a(i) and (ii). (i) Write down the value of $f'(x)$ at C. [3]

(ii) **Hence**, show that C corresponds to a minimum on the graph of f, i.e. it has the same x-coordinate.

b. Which of the points A, B, D corresponds to a maximum on the graph of f? [1]

c. Show that B corresponds to a point of inflexion on the graph of f. [3]

Markscheme

a(i) and (ii). (i) $f'(x) = 0$ ***A1 N1***

(ii) **METHOD 1**

$f'(x) < 0$ to the left of C, $f'(x) > 0$ to the right of C ***R1R1 N2***

METHOD 2

$f''(x) > 0$ ***R2 N2***

[3 marks]

b. A ***A1 N1***

[1 mark]

c. **METHOD 1**

$f''(x) = 0$ ***R2***

discussion of sign change of $f''(x)$ ***R1***

e.g. $f''(x) < 0$ to the left of B and $f''(x) > 0$ to the right of B; $f''(x)$ changes sign either side of B

B is a point of inflexion ***AG N0***

METHOD 2

B is a minimum on the graph of the derivative f' ***R2***

discussion of sign change of $f''(x)$ ***R1***

e.g. $f''(x) < 0$ to the left of B and $f''(x) > 0$ to the right of B; $f''(x)$ changes sign either side of B

B is a point of inflexion ***AG N0***

[3 marks]

Examiners report

a(i) and (ii). The variation in successful and unsuccessful responses to this question was remarkable. Many candidates did not even attempt it. Candidates could often determine from the graph, the minimum and maximum values of the original function, but few could correctly use the graph to analyse and justify these results. Responses indicated that some candidates did not realize that they were looking at the graph of f' and not the graph of f .

b. The variation in successful and unsuccessful responses to this question was remarkable. Many candidates did not even attempt it. Candidates could often determine from the graph, the minimum and maximum values of the original function, but few could correctly use the graph to analyse and justify these results. Responses indicated that some candidates did not realize that they were looking at the graph of f' and not the graph of f .

c. In part (c), many candidates once more failed to respect the command term "show" and often provided an incomplete answer. Candidates should be encouraged to refer to the number of marks available for a particular part when deciding how much information should be given.

Let $f(x) = \mathrm{e}^{6x}$.

a. Write down $f'(x)$. [1]

b(i) and (ii). The tangent to the graph of f at the point $\mathrm{P}(0, b)$ has gradient m . [4]

(i) Show that $m = 6$.

(ii) Find b .

c. Hence, write down the equation of this tangent. [1]

Markscheme

a. $f'(x) = 6\mathrm{e}^{6x}$ ***A1*** ***N1***

[1 mark]

b(i) and (ii). (i) evidence of valid approach ***(M1)***

e.g. $f'(0)$, $6\mathrm{e}^{6\times 0}$

correct manipulation ***A1***

e.g. $6\mathrm{e}^0$, 6×1

$m = 6$ ***AG*** ***N0***

(ii) evidence of finding $f(0)$ ***(M1)***

e.g. $y = \mathrm{e}^{6(0)}$

$b = 1$ ***A1*** ***N2***

[4 marks]

c. $y = 6x + 1$ ***A1*** ***N1***

[1 mark]

Examiners report

a. On the whole, candidates handled this question quite well with most candidates correctly applying the chain rule to an exponential function and successfully finding the equation of the tangent line.

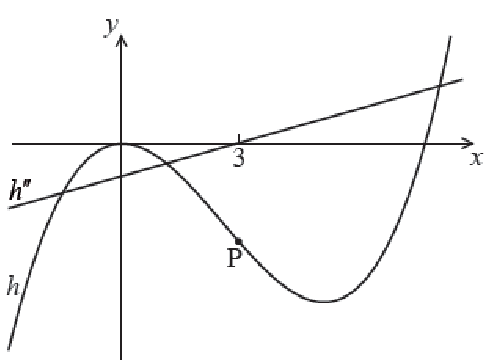
b(i) and (ii). On the whole, candidates handled this question quite well with most candidates correctly applying the chain rule to an exponential function and successfully finding the equation of the tangent line. Some candidates lost a mark in (b)(i) for not showing sufficient working leading to the given answer.

c. On the whole, candidates handled this question quite well.

Consider the functions $f(x)$, $g(x)$ and $h(x)$. The following table gives some values associated with these functions.

x	2	3
$f(x)$	2	3
$g(x)$	–14	–18
$f'(x)$	1	1
$g'(x)$	–5	–3
$h''(x)$	–6	0

The following diagram shows parts of the graphs of h and h'' .



There is a point of inflexion on the graph of h at P, when $x = 3$.

Given that $h(x) = f(x) \times g(x)$,

a. Write down the value of $g(3)$, of $f'(3)$, and of $h''(2)$. [3]

b. Explain why P is a point of inflexion. [2]

c. find the y-coordinate of P. [2]

d. find the equation of the normal to the graph of h at P. [7]

Markscheme

a. $g(3) = -18$, $f'(3) = 1$, $h''(2) = -6$ ***A1A1A1*** ***N3***

[3 marks]

b. $h''(3) = 0$ ***(A1)***

valid reasoning ***R1***

eg h'' changes sign at $x = 3$, change in concavity of h at $x = 3$

so P is a point of inflexion ***AG*** ***N0***

[2 marks]

c. writing $h(3)$ as a product of $f(3)$ and $g(3)$ ***A1***

eg $f(3) \times g(3)$, $3 \times (-18)$

$h(3) = -54$ ***A1 N1***

[2 marks]

d. recognizing need to find derivative of h ***(R1)***

eg h' , $h'(3)$

attempt to use the product rule (do **not** accept $h' = f' \times g'$) ***(M1)***

eg $h' = fg' + gf'$, $h'(3) = f(3) \times g'(3) + g(3) \times f'(3)$

correct substitution ***(A1)***

eg $h'(3) = 3(-3) + (-18) \times 1$

$h'(3) = -27$ ***A1***

attempt to find the gradient of the normal ***(M1)***

eg $-\frac{1}{m}$, $-\frac{1}{27}x$

attempt to substitute **their** coordinates and **their** normal gradient into the equation of a line ***(M1)***

eg $-54 = \frac{1}{27}(3) + b$, $0 = \frac{1}{27}(3) + b$, $y + 54 = 27(x - 3)$, $y - 54 = \frac{1}{27}(x + 3)$

correct equation in any form ***A1*** ***N4***

eg $y + 54 = \frac{1}{27}(x - 3)$, $y = \frac{1}{27}x - 54\frac{1}{9}$

[7 marks]

Examiners report

a. Nearly all candidates who attempted to answer parts (a) and (c) did so correctly, as these questions simply required them to understand the notation being used and to read the values from the given table.

b. In part (b), the majority of candidates earned one mark for stating that $h''(x) = 0$ at point P. As this is not enough to determine a point of inflexion, very few candidates earned full marks on this question.

c. Nearly all candidates who attempted to answer parts (a) and (c) did so correctly, as these questions simply required them to understand the notation being used and to read the values from the given table.

d. Part (d) proved to be quite challenging for even the strongest candidates, as almost none of them used the product rule to find $h'(3)$. The most common error was to say $h'(3) = f'(3) \times g'(3)$. Despite this error, many candidates were able to earn further method marks for their work in finding the equation of the normal. There were also a small number of candidates who were able to find the equation for $h'(x)$, and from that $h''(x)$. These candidates were often successful in earning full marks, although this method was quite time-consuming.

---

a. Find $\int \frac{1}{2x+3} \mathrm{d}x$. [2]

b. Given that $\int_0^3 \frac{1}{2x+3} \mathrm{d}x = \ln \sqrt{P}$, find the value of *P*. [4]

Markscheme

a. $\int \frac{1}{2x+3} \mathrm{d}x = \frac{1}{2}\ln(2x+3) + C$ (accept $\frac{1}{2}\ln|(2x+3)| + C$) ***A1A1*** ***N2***

[2 marks]

b. $\int_0^3 \frac{1}{2x+3} \mathrm{d}x = \left[\frac{1}{2}\ln(2x+3)\right]_0^3$

evidence of substitution of limits ***(M1)***

e.g. $\frac{1}{2}\ln 9 - \frac{1}{2}\ln 3$

evidence of correctly using $\ln a - \ln b = \ln \frac{a}{b}$ (seen anywhere) ***(A1)***

e.g. $\frac{1}{2}\ln 3$

evidence of correctly using $a \ln b = \ln b^a$ (seen anywhere) ***(A1)***

e.g. $\ln \sqrt{\frac{9}{3}}$

$P = 3$ (accept $\ln \sqrt{3}$) ***A1*** ***N2***

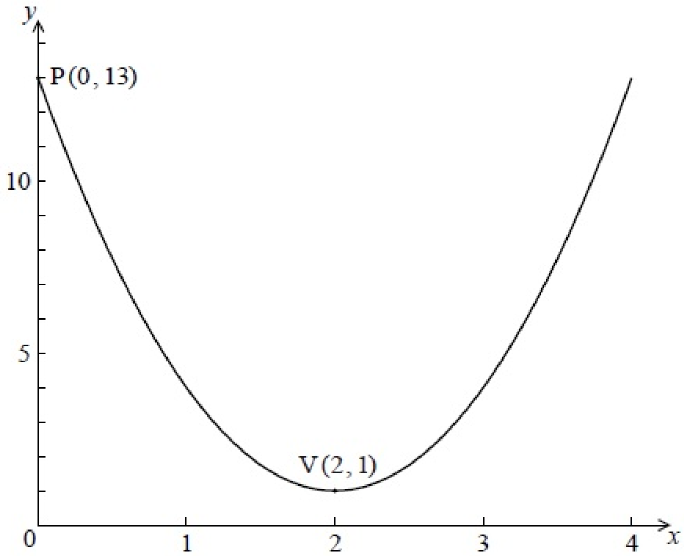
[4 marks]

Examiners report

a. Many candidates were unable to correctly integrate but did recognize that the integral involved the natural log function; they most often missed the factor $\frac{1}{2}$ or replaced it with 2.

b. Part (b) proved difficult as many were unable to use the basic rules of logarithms.

The following diagram shows the graph of a quadratic function f, for $0 \le x \le 4$.



The graph passes through the point P(0, 13) , and its vertex is the point V(2, 1) .

a(i) and (ii). The function can be written in the form $f(x) = a(x-h)^2 + k$. [4]

(i) Write down the value of h and of k .

(ii) Show that $a = 3$.

b. Find $f(x)$, giving your answer in the form $Ax^2 + Bx + C$. [3]

c. Calculate the area enclosed by the graph of f , the x-axis, and the lines $x = 2$ and $x = 4$. [8]

Markscheme

a(i) and (ii). (i) $h = 2$, $k = 1$ ***A1A1*** ***N2***

(ii) attempt to substitute coordinates of any point (except the vertex) on the graph into f ***M1***

e.g. $13 = a(0-2)^2 + 1$

working towards solution ***A1***

e.g. $13 = 4a + 1$

$a = 3$ ***AG*** ***N0***

[4 marks]

b. attempting to expand **their** binomial ***(M1)***

e.g. $f(x) = 3(x^2 - 2 \times 2x + 4) + 1$, $(x-2)^2 = x^2 - 4x + 4$

correct working ***(A1)***

e.g. $f(x) = 3x^2 - 12x + 12 + 1$

$f(x) = 3x^2 - 12x + 13$ (accept $A = 3$, $B = -12$, $C = 13$) ***A1*** ***N2***

[3 marks]

c. **METHOD 1**

integral expression ***(A1)***

e.g. $\int_2^4 (3x^2 - 12x + 13)$, $\int f \mathrm{d}x$

Area $= [x^3 - 6x^2 + 13x]_2^4$ ***A1A1A1***

Note: Award ***A1*** for x^3 , ***A1*** for $-6x^2$, ***A1*** for $13x$.

correct substitution of **correct** limits into **their** expression ***A1A1***

e.g. $(4^3 - 6 \times 4^2 + 13 \times 4) - (2^3 - 6 \times 2^2 + 13 \times 2)$, $64 - 96 + 52 - (8 - 24 + 26)$

Note: Award ***A1*** for substituting 4, ***A1*** for substituting 2.

correct working ***(A1)***

e.g. $64 - 96 + 52 - 8 + 24 - 26, 20 - 10$

$\text{Area} = 10$ ***A1*** ***N3***

[8 marks]

METHOD 2

integral expression ***(A1)***

e.g. $\int_2^4 (3(x-2)^2 + 1)$, $\int f\mathrm{d}x$

$\text{Area} = [(x-2)^3 + x]_2^4$ ***A2A1***

Note: Award ***A2*** for $(x-2)^3$, ***A1*** for x .

correct substitution of **correct** limits into **their** expression ***A1A1***

e.g. $(4-2)^3 + 4 - [(2-2)^3 + 2]$, $2^3 + 4 - (0^3 + 2)$, $2^3 + 4 - 2$

Note: Award ***A1*** for substituting 4, ***A1*** for substituting 2.

correct working ***(A1)***

e.g. $8 + 4 - 2$

$\text{Area} = 10$ ***A1*** ***N3***

[8 marks]

METHOD 3

recognizing area from 0 to 2 is same as area from 2 to 4 ***(R1)***

e.g. sketch, $\int_2^4 f = \int_0^2 f$

integral expression ***(A1)***

e.g. $\int_0^2 (3x^2 - 12x + 13)$, $\int f\mathrm{d}x$

$\text{Area} = [x^3 - 6x^2 + 13x]_0^2$ ***A1A1A1***

Note: Award ***A1*** for x^3 , ***A1*** for $-6x^2$, ***A1*** for $13x$.

correct substitution of **correct** limits into **their** expression ***A1(A1)***

e.g. $(2^3 - 6 \times 2^2 + 13 \times 2) - (0^3 - 6 \times 0^2 + 13 \times 0)$, $8 - 24 + 26$

Note: Award ***A1*** for substituting 2, ***(A1)*** for substituting 0.

$\text{Area} = 10$ ***A1*** ***N3***

[8 marks]

Examiners report

a(i) and (ii). In part (a), nearly all the candidates recognized that h and k were the coordinates of the vertex of the parabola, and most were able to successfully show that $a = 3$. Unfortunately, a few candidates did not understand the "show that" command, and simply verified that $a = 3$ would work, rather than showing how to find $a = 3$.

b. In part (b), most candidates were able to find $f(x)$ in the required form. For a few candidates, algebraic errors kept them from finding the correct function, even though they started with correct values for a, h and k.

c. In part (c), nearly all candidates knew that they needed to integrate to find the area, but errors in integration, and algebraic and arithmetic errors prevented many from finding the correct area.

In this question s represents displacement in metres and t represents time in seconds.

The velocity v m s^{-1} of a moving body is given by $v = 40 - at$ where a is a non-zero constant.

Trains approaching a station start to slow down when they pass a point P. As a train slows down, its velocity is given by $v = 40 - at$, where $t = 0$ at P. The station is 500 m from P.

a. (i) If $s = 100$ when $t = 0$, find an expression for s in terms of a and t. [6]

(ii) If $s = 0$ when $t = 0$, write down an expression for s in terms of a and t.

b. A train M slows down so that it comes to a stop at the station. [6]

(i) Find the time it takes train M to come to a stop, giving your answer in terms of a.

(ii) Hence show that $a = \frac{8}{5}$.

c. For a different train N, the value of a is 4. [5]

Show that this train will stop **before** it reaches the station.

Markscheme

a. **Note**: In this question, do not penalize absence of units.

(i) $s = \int (40 - at)\mathrm{d}t$ ***(M1)***

$s = 40t - \frac{1}{2}at^2 + c$ ***(A1)(A1)***

substituting $s = 100$ when $t = 0$ ($c = 100$) ***(M1)***

$s = 40t - \frac{1}{2}at^2 + 100$ ***A1 N5***

(ii) $s = 40t - \frac{1}{2}at^2$ ***A1 N1***

[6 marks]

b. (i) stops at station, so $v = 0$ ***(M1)***

$t = \frac{40}{a}$ (seconds) ***A1 N2***

(ii) evidence of choosing formula for s from (a) (ii) ***(M1)***

substituting $t = \frac{40}{a}$ ***(M1)*** e.g. $40 \times \frac{40}{a} - \frac{1}{2}a \times \frac{40^2}{a^2}$

setting up equation ***M1***

e.g. $500 = s$, $500 = 40 \times \frac{40}{a} - \frac{1}{2}a \times \frac{40^2}{a^2}$, $500 = \frac{1600}{a} - \frac{800}{a}$

evidence of simplification to an expression which obviously leads to $a = \frac{8}{5}$ ***A1***

e.g. $500a = 800$, $5 = \frac{8}{a}$, $1000a = 3200 - 1600$

$a = \frac{8}{5}$ ***AG*** ***N0***

[6 marks]

c. **METHOD 1**

$v = 40 - 4t$, stops when $v = 0$

$40 - 4t = 0$ ***(A1)***

$t = 10$ ***A1***

substituting into expression for s ***M1***

$s = 40 \times 10 - \frac{1}{2} \times 4 \times 10^2$

$s = 200$ ***A1***

since $200 < 500$ (allow ***FT*** on their s, if $s < 500$) ***R1***

train stops before the station ***AG*** ***N0***

METHOD 2

from (b) $t = \frac{40}{4} = 10$ ***A2***

substituting into expression for s

e.g. $s = 40 \times 10 - \frac{1}{2} \times 4 \times 10^2$ ***M1***

$s = 200$ ***A1***

since $200 < 500$ ***R1***

train stops before the station ***AG*** ***N0***

METHOD 3

a is deceleration ***A2***

$4 > \frac{8}{5}$ ***A1***

so stops in shorter time ***(A1)***

so less distance travelled ***R1***

so stops before station ***AG*** ***N0***

[5 marks]

Examiners report

a. Part (a) proved accessible for most.

b. Part (b), simple as it is, proved elusive as many candidates did not make the connection that $v = 0$ when the train stops. Instead, many attempted to find the value of t using $a = \frac{8}{5}$.

c. Few were successful in part (c).

a. Find $\int x\mathrm{e}^{x^2-1}\mathrm{d}x$. [4]

b. Find $f(x)$, given that $f'(x) = x\mathrm{e}^{x^2-1}$ and $f(-1) = 3$. [3]

Markscheme

a. valid approach to set up integration by substitution/inspection ***(M1)***

eg $u = x^2 - 1,\ \mathrm{d}u = 2x,\ \int 2x\mathrm{e}^{x^2-1}\mathrm{d}x$

correct expression ***(A1)***

eg $\frac{1}{2}\int 2x\mathrm{e}^{x^2-1}\mathrm{d}x,\ \frac{1}{2}\int \mathrm{e}^u\mathrm{d}u$

$\frac{1}{2}\mathrm{e}^{x^2-1} + c$ ***A2*** ***N4***

Notes: Award ***A1*** if missing "$+c$".

[4 marks]

b. substituting $x = -1$ into **their** answer from (a) ***(M1)***

eg $\frac{1}{2}\mathrm{e}^0,\ \frac{1}{2}\mathrm{e}^{1-1} = 3$

correct working ***(A1)***

eg $\frac{1}{2} + c = 3,\ c = 2.5$

$f(x) = \frac{1}{2}\mathrm{e}^{x^2-1} + 2.5$ ***A1*** ***N2***

[3 marks]

Examiners report

a. [N/A]
b. [N/A]

Let $f(x) = kx^4$. The point $\mathrm{P}(1,\ k)$ lies on the curve of f. At P, the normal to the curve is parallel to $y = -\frac{1}{8}x$. Find the value of k.

Markscheme

gradient of tangent $= 8$ (seen anywhere) ***(A1)***

$f'(x) = 4kx^3$ (seen anywhere) ***A1***

recognizing the gradient of the tangent is the derivative ***(M1)***

setting the derivative equal to 8 ***(A1)***

e.g. $4kx^3 = 8$, $kx^3 = 2$

substituting $x = 1$ (seen anywhere) ***(M1)***

$k = 2$ ***A1*** ***N4***

[6 marks]

Examiners report

Candidates‟ success with this question was mixed. Those who understood the relationship between the derivative and the gradient of the normal line were not bothered by the lack of structure in the question, solving clearly with only a few steps, earning full marks. Those who were unclear often either gained a few marks for finding the derivative and substituting $x = 1$, or no marks for working that did not employ the derivative. Misunderstandings included simply finding the equation of the tangent or normal line, setting the derivative equal to the gradient of the normal, and equating the function with the normal or tangent line equation. Among the candidates who demonstrated greater understanding, more used the gradient of the normal (the equation $-\frac{1}{4}k = -\frac{1}{8}$) than the gradient of the tangent ($4k = 8$) ; this led to more algebraic errors in obtaining the final answer of $k = 2$. A number of unsuccessful candidates wrote down a lot of irrelevant mathematics with no plan in mind and earned no marks.

The graph of the function $y = f(x)$ passes through the point $\left(\frac{3}{2}, 4\right)$. The gradient function of *f* is given as $f'(x) = \sin(2x - 3)$. Find $f(x)$.

Markscheme

evidence of integration

e.g. $f(x) = \int \sin(2x - 3)\mathrm{d}x$ ***(M1)***

$= -\frac{1}{2}\cos(2x - 3) + C$ ***A1A1***

substituting initial condition into **their** expression (even if *C* is missing) ***M1***

e.g. $4 = -\frac{1}{2}\cos 0 + C$

$C = 4.5$ ***(A1)***

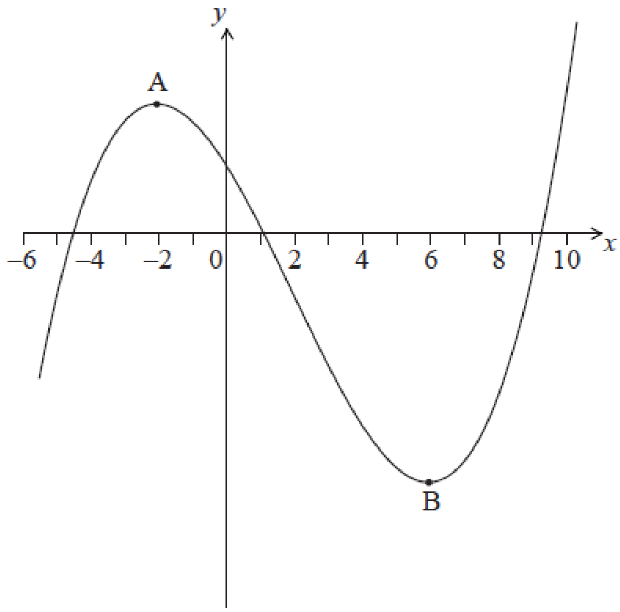
$f(x) = -\frac{1}{2}\cos(2x - 3) + 4.5$ ***A1*** ***N5***

[6 marks]

Examiners report

While most candidates realized they needed to integrate in this question, many did so unsuccessfully. Many did not account for the coefficient of *x*, and failed to multiply by $\frac{1}{2}$. Some of the candidates who substituted the initial condition into their integral were not able to solve for "*c*", either because of arithmetic errors or because they did not know the correct value for $\cos 0$.

The following diagram shows part of the graph of $y = f(x)$.



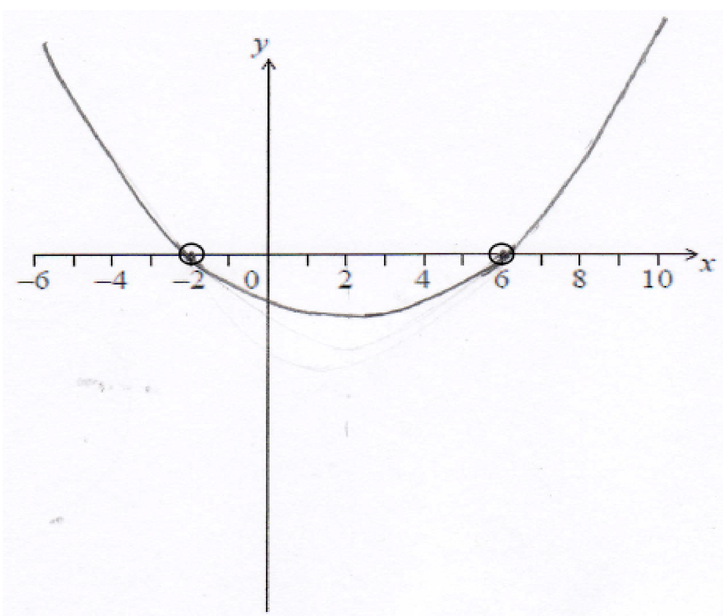
The graph has a local maximum at A, where $x=-2$, and a local minimum at B, where $x=6$.

a. On the following axes, sketch the graph of $y=f'(x)$. [4]

b. Write down the following in order from least to greatest: $f(0),\ f'(6),\ f''(-2)$. [2]

Markscheme

a.



A1A1A1A1 ***N4***

Note: Award ***A1*** for *x*-intercept in circle at -2, ***A1*** for *x*-intercept in circle at 6.

Award ***A1*** for approximately correct shape.

Only if this ***A1*** is awarded, award ***A1*** for a negative *y*-intercept.

[4 marks]

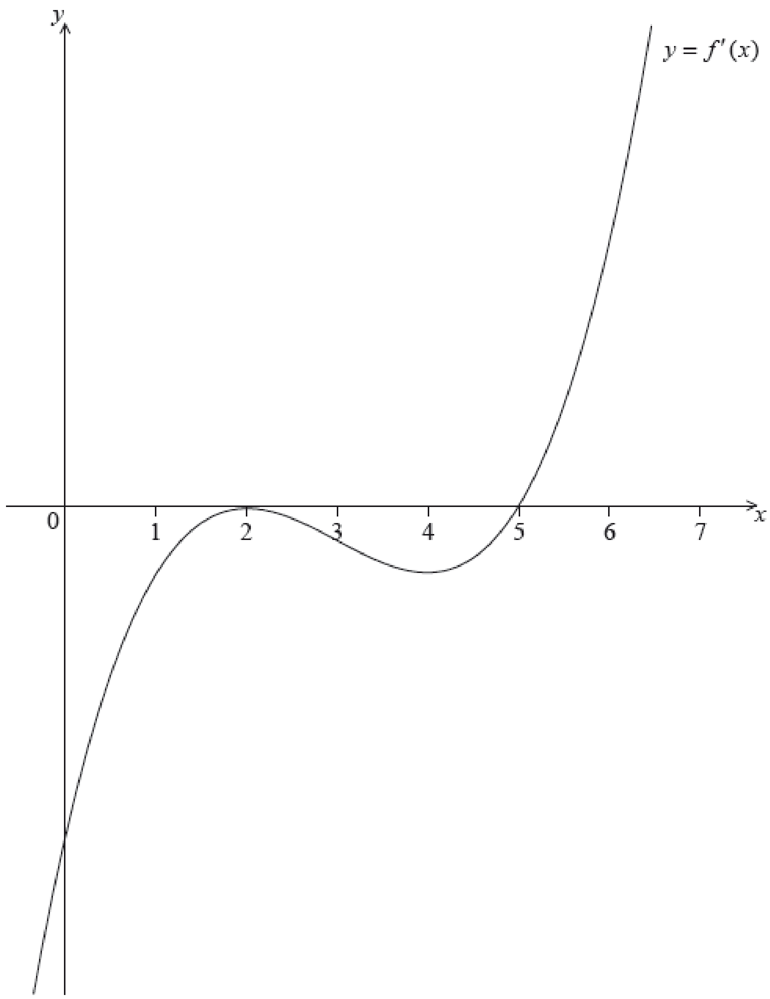
b. $f''(-2),\ f'(6),\ f(0)$ ***A2*** ***N2***

[2 marks]

Examiners report

a. [N/A]
b. [N/A]

Let $y = f(x)$, for $-0.5 \leq$ x ≤ 6.5. The following diagram shows the graph of f', the derivative of f.

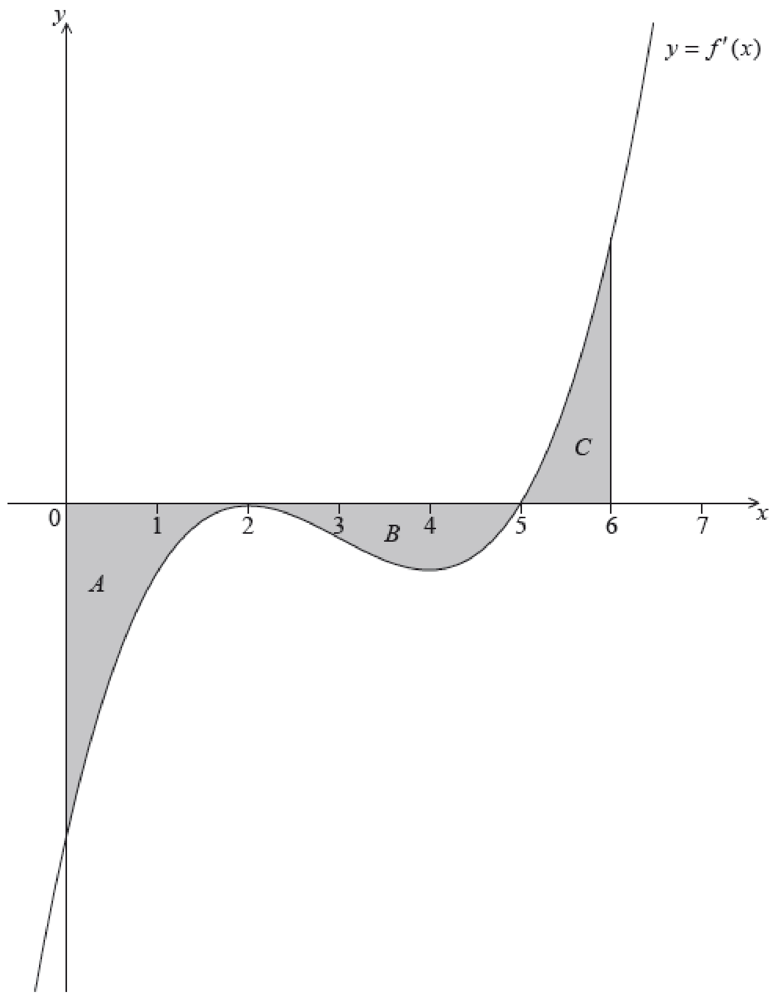


The graph of f' has a local maximum when $x = 2$, a local minimum when $x = 4$, and it crosses the x-axis at the point $(5,\ 0)$.

a. Explain why the graph of f has a local minimum when $x = 5$. [2]

b. Find the set of values of x for which the graph of f is concave down. [2]

c. The following diagram shows the shaded regions A, B and C. [5]

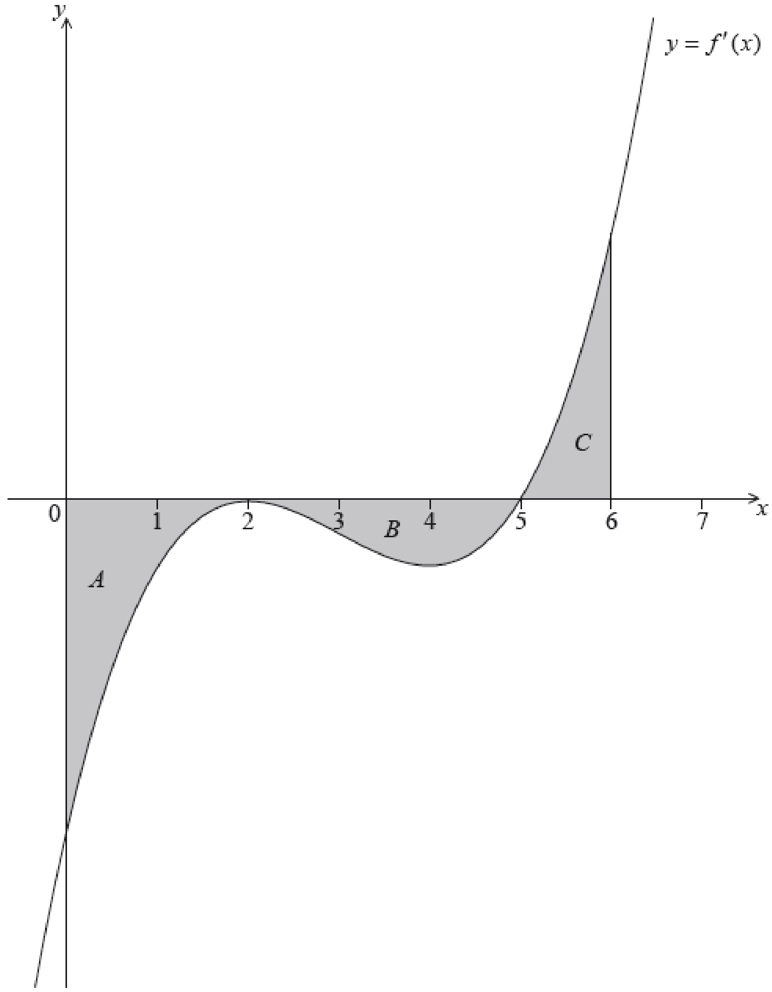


The regions are enclosed by the graph of f', the x-axis, the y-axis, and the line $x = 6$.

The area of region A is 12, the area of region B is 6.75 and the area of region C is 6.75.

Given that $f(0) = 14$, find $f(6)$.

d. The following diagram shows the shaded regions A, B and C. [6]



The regions are enclosed by the graph of f', the *x*-axis, the *y*-axis, and the line $x = 6$.

The area of region A is 12, the area of region B is 6.75 and the area of region C is 6.75.

Let $g(x) = (f(x))^2$. Given that $f'(6) = 16$, find the equation of the tangent to the graph of g at the point where $x = 6$.

Markscheme

a. **METHOD 1**

$f'(5) = 0$ ***(A1)***

valid reasoning including reference to the graph of f' ***R1***

eg f' changes sign from negative to positive at $x = 5$, labelled sign chart for f'

so f has a local minimum at $x = 5$ ***AG*** ***N0***

Note: It must be clear that any description is referring to the graph of f', simply giving the conditions for a minimum without relating them to f' does not gain the ***R1***.

METHOD 2

$f'(5) = 0$ ***A1***

valid reasoning referring to second derivative ***R1***

eg $f''(5) > 0$

so f has a local minimum at $x = 5$ ***AG*** ***N0***

[2 marks]

b. attempt to find relevant interval ***(M1)*** *eg* f' is decreasing, gradient of f' is negative, $f'' < 0$

$2 < x < 4$ (accept "between 2 and 4") ***A1*** ***N2***

Notes: If no other working shown, award ***M1A0*** for incorrect inequalities such as $2 \leq x \leq 4$, or "from 2 to 4"

[2 marks]

c. **METHOD 1 (one integral)**

correct application of Fundamental Theorem of Calculus ***(A1)***

eg $\int_0^6 f'(x)\mathrm{d}x = f(6) - f(0),\ f(6) = 14 + \int_0^6 f'(x)\mathrm{d}x$

attempt to link definite integral with areas ***(M1)***

eg $\int_0^6 f'(x)\mathrm{d}x = -12 - 6.75 + 6.75,\ \int_0^6 f'(x)\mathrm{d}x = \text{Area } A + \text{Area } B + \text{ Area } C$

correct value for $\int_0^6 f'(x)\mathrm{d}x$ ***(A1)***

eg $\int_0^6 f'(x)\mathrm{d}x = -12$

correct working ***A1***

eg $f(6) - 14 = -12,\ f(6) = -12 + f(0)$

$f(6) = 2$ ***A1*** ***N3***

METHOD 2 (more than one integral)

correct application of Fundamental Theorem of Calculus ***(A1)***

eg $\int_0^2 f'(x)\mathrm{d}x = f(2) - f(0),\ f(2) = 14 + \int_0^2 f'(x)$

attempt to link definite integrals with areas ***(M1)***

eg $\int_0^2 f'(x)\mathrm{d}x = 12,\ \int_2^5 f'(x)\mathrm{d}x = -6.75,\ \int_0^6 f'(x) = 0$

correct values for integrals ***(A1)***

eg $\int_0^2 f'(x)\mathrm{d}x = -12,\ \int_5^2 f'(x)\mathrm{d}x = 6.75,\ f(6) - f(2) = 0$

one correct intermediate value ***A1***

eg $f(2) = 2,\ f(5) = -4.75$

$f(6) = 2$ ***A1*** ***N3***

[5 marks]

d. correct calculation of $g(6)$ (seen anywhere) ***A1***

eg $2^2,\ g(6) = 4$

choosing chain rule or product rule ***(M1)***

eg $g'(f(x))\, f'(x),\ \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}u} \times \frac{\mathrm{d}u}{\mathrm{d}x},\ f(x)f'(x) + f'(x)f(x)$

correct derivative ***(A1)***

eg $g'(x) = 2f(x)f'(x),\ f(x)f'(x) + f'(x)f(x)$

correct calculation of $g'(6)$ (seen anywhere) ***A1***

eg $2(2)(16),\ g'(6) = 64$

attempt to substitute **their** values of $g'(6)$ and $g(6)$ (in any order) into equation of a line ***(M1)***

eg $2^2 = (2 \times 2 \times 16)6 + b,\ y - 6 = 64(x - 4)$

correct equation in any form ***A1*** ***N2***

eg $y - 4 = 64(x - 6),\ y = 64x - 380$

[6 marks]

[Total 15 marks]

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]
d. [N/A]

Let $\int_1^5 3f(x)\mathrm{d}x = 12$.

a. Show that $\int_5^1 f(x)\mathrm{d}x = -4$. [2]

b. Find the value of $\int_1^2 (x + f(x))\mathrm{d}x + \int_2^5 (x + f(x))\mathrm{d}x$. [5]

Markscheme

a. evidence of factorising 3/division by 3 ***A1***

e.g. $\int_1^5 3f(x)\mathrm{d}x = 3\int_1^5 f(x)\mathrm{d}x$, $\frac{12}{3}$, $\int_1^5 \frac{3f(x)\mathrm{d}x}{3}$ (do not accept 4 as this is show that)

evidence of stating that reversing the limits changes the sign ***A1***

e.g. $\int_5^1 f(x)\mathrm{d}x = -\int_1^5 f(x)\mathrm{d}x$

$\int_5^1 f(x)\mathrm{d}x = -4$ ***AG*** ***N0***

[2 marks]

b. evidence of correctly combining the integrals (seen anywhere) ***(A1)***

e.g. $I = \int_1^2 (x + f(x))\mathrm{d}x + \int_2^5 (x + f(x))\mathrm{d}x = \int_1^5 (x + f(x))\mathrm{d}x$

evidence of correctly splitting the integrals (seen anywhere) ***(A1)***

e.g. $I = \int_1^5 x\mathrm{d}x + \int_1^5 f(x)\mathrm{d}x$

$\int x\mathrm{d}x = \frac{x^2}{2}$ (seen anywhere) ***A1***

$\int_1^5 x\mathrm{d}x = \left[\frac{x^2}{2}\right]_1^5 = \frac{25}{2} - \frac{1}{2} \left(= \frac{24}{2}, 12\right)$ ***A1***

$I = 16$ ***A1*** ***N3***

[5 marks]

Examiners report

a. This question was very poorly done. Very few candidates provided proper justification for part (a), a common error being to write $\int_1^5 f(x)\mathrm{d}x = f(5) - f(1)$. What was being looked for was that $\int_1^5 3f(x)\mathrm{d}x = 3\int_1^5 f(x)\mathrm{d}x$ and $\int_5^1 f(x)\mathrm{d}x = -\int_1^5 f(x)\mathrm{d}x$.

b. Part (b) had similar problems with neither the combining of limits nor the splitting of integrals being done very often. A common error was to treat $f(x)$ as 1 in order to make $\int_1^5 f(x)\mathrm{d}x = 4$ and then write $\int_1^5 (x + f(x))\mathrm{d}x = [x + 1]_1^5$.

A function f has its derivative given by $f'(x) = 3x^2 - 2kx - 9$, where k is a constant.

a. Find $f''(x)$. [2]

b. The graph of f has a point of inflexion when $x = 1$. [3]

Show that $k = 3$.

c. Find $f'(-2)$. [2]

d. Find the equation of the tangent to the curve of f at $(-2,\ 1)$, giving your answer in the form $y = ax + b$. [4]

e. Given that $f'(-1) = 0$, explain why the graph of f has a local maximum when $x = -1$. [3]

Markscheme

a. $f''(x) = 6x - 2k$ ***A1A1*** ***N2***

[2 marks]

b. substituting $x = 1$ into f'' ***(M1)***

eg $f''(1),\ 6(1) - 2k$

recognizing $f''(x) = 0$ (seen anywhere) ***M1***

correct equation ***A1***

eg $6 - 2k = 0$

$k = 3$ ***AG*** ***N0***

[3 marks]

c. correct substitution into $f'(x)$ ***(A1)***

eg $3(-2)^2 - 6(-2) - 9$

$f'(-2) = 15$ ***A1*** ***N2***

[2 marks]

d. recognizing gradient value (may be seen in equation) ***M1***

eg $a = 15,\ y = 15x + b$

attempt to substitute $(-2,\ 1)$ into equation of a straight line ***M1***

eg $1 = 15(-2) + b,\ (y - 1) = m(x + 2),\ (y + 2) = 15(x - 1)$

correct working ***(A1)***

eg $31 = b,\ y = 15x + 30 + 1$

$y = 15x + 31$ ***A1*** ***N2***

[4 marks]

e. **METHOD 1** (2^{nd} derivative)

recognizing $f'' < 0$ (seen anywhere) ***R1***

substituting $x = -1$ into f'' ***(M1)***

eg $f''(-1),\ 6(-1) - 6$

$f''(-1) = -12$ ***A1***

therefore the graph of f has a local maximum when $x = -1$ ***AG*** ***N0***

METHOD 2 (1^{st} derivative)

recognizing change of sign of $f'(x)$ (seen anywhere) ***R1*** *eg* *sign chart* 

correct value of f' for $-1 < x < 3$ ***A1***

eg $f'(0) = -9$

correct value of f' for x value to the left of -1 ***A1***

eg $f'(-2) = 15$

therefore the graph of f has a local maximum when $x = -1$ ***AG*** ***N0***

[3 marks]

Total [14 marks]

Examiners report

a. Well answered and candidates coped well with k in the expression.

b. Mostly answered well with the common error being to substitute into f' instead of f''.

c. A straightforward question that was typically answered correctly.

d. Some candidates recalculated the gradient, not realising this had already been found in part c). Many understood they were finding a linear equation but were hampered by arithmetic errors.

e. Using change of sign of the first derivative was the most common approach used with a sign chart or written explanation. However, few candidates then supported their approach by calculating suitable values for $f'(x)$. This was necessary because the question already identified a local maximum, hence candidates needed to explain why this was so. Some candidates did not mention the ‘first derivative’ just that ‘it’ was increasing/decreasing. Few candidates used the more efficient second derivative test.

Consider $f(x) = x^2 \sin x$.

a. Find $f'(x)$. [4]

b. Find the gradient of the curve of f at $x = \frac{\pi}{2}$. [3]

Markscheme

a. evidence of choosing product rule ***(M1)***

eg $uv' + vu'$

correct derivatives (must be seen in the product rule) $\cos x$, $2x$ ***(A1)(A1)***

$f'(x) = x^2 \cos x + 2x \sin x$ ***A1 N4***

[4 marks]

b. substituting $\frac{\pi}{2}$ into **their** $f'(x)$ ***(M1)***

eg $f'\left(\frac{\pi}{2}\right)$, $\left(\frac{\pi}{2}\right)^2 \cos\left(\frac{\pi}{2}\right) + 2\left(\frac{\pi}{2}\right)\sin\left(\frac{\pi}{2}\right)$

correct values for **both** $\sin \frac{\pi}{2}$ and $\cos \frac{\pi}{2}$ seen in $f'(x)$ ***(A1)*** *eg* $0+2\left(\frac{\pi}{2}\right)\times 1$

$f'\left(\frac{\pi}{2}\right)=\pi$ ***A1 N2***

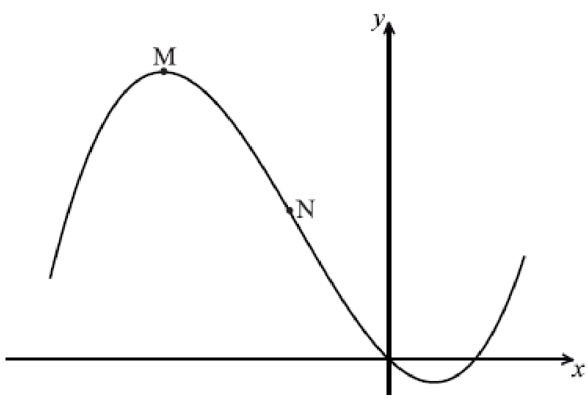
[3 marks]

Examiners report

a. Many candidates correctly applied the product rule for the derivative, although a common error was to answer $f'(x)=2x\cos x$.

b. Candidates generally understood that the gradient of the curve uses the derivative, although in some cases the substitution was made in the original function. Some candidates did not know the values of sine and cosine at $\frac{\pi}{2}$.

Consider $f(x)=\frac{1}{3}x^3+2x^2-5x$. Part of the graph of *f* is shown below. There is a maximum point at M, and a point of inflexion at N.



a. Find $f'(x)$. [3]

b. Find the *x*-coordinate of M. [4]

c. Find the *x*-coordinate of N. [3]

d. The line L is the tangent to the curve of *f* at $(3, 12)$. Find the equation of L in the form $y=ax+b$. [4]

Markscheme

a. $f'(x)=x^2+4x-5$ ***A1A1A1 N3***

[3 marks]

b. evidence of attempting to solve $f'(x)=0$ ***(M1)***

evidence of correct working ***A1***

e.g. $(x+5)(x-1)$, $\frac{-4\pm\sqrt{16+20}}{2}$, sketch

$x=-5, x=1$ ***(A1)***

so $x=-5$ ***A1 N2***

[4 marks]

c. **METHOD 1**

$f''(x) = 2x + 4$ (may be seen later) ***A1***

evidence of setting second derivative = 0 ***(M1)***

e.g. $2x + 4 = 0$

$x = -2$ ***A1*** ***N2***

METHOD 2

evidence of use of symmetry ***(M1)***

e.g. midpoint of max/min, reference to shape of cubic

correct calculation ***A1***

e.g. $\frac{-5+1}{2}$

$x = -2$ ***A1*** ***N2***

[3 marks]

d. attempting to find the value of the derivative when $x = 3$ ***(M1)***

$f'(3) = 16$ ***A1***

valid approach to finding the equation of a line ***M1***

e.g. $y - 12 = 16(x - 3)$, $12 = 16 \times 3 + b$

$y = 16x - 36$ ***A1*** ***N2***

[4 marks]

Examiners report

a. This question was very well done with most candidates showing their work in an orderly manner. There were a number of candidates, however, who were a bit sloppy in indicating when a function was being equated to zero and they “solved” an expression rather than an equation.

b. This question was very well done with most candidates showing their work in an orderly manner. There were a number of candidates, however, who were a bit sloppy in indicating when a function was being equated to zero and they “solved” an expression rather than an equation. Many candidates went through first and second derivative tests to verify that the point they found was a maximum or an inflexion point; this was unnecessary since the graph was given. Many also found the *y*-coordinate which was unnecessary and used up valuable time on the exam.

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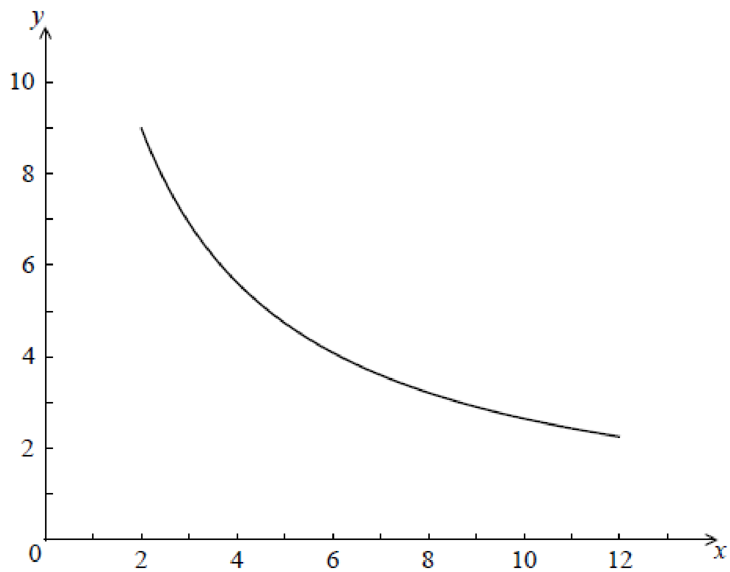
Let $f(x) = \frac{1}{4}x^2 + 2$. The line L is the tangent to the curve of f at (4, 6) .

Let $g(x) = \frac{90}{3x+4}$, for $2 \le x \le 12$. The following diagram shows the graph of g .



a. Find the equation of L . [4]

b. Find the area of the region enclosed by the curve of g , the x-axis, and the lines $x = 2$ and $x = 12$. Give your answer in the form $a \ln b$, where $a, b \in \mathbb{Z}$. [6]

c. The graph of g is reflected in the x-axis to give the graph of h . The area of the region enclosed by the lines L , $x = 2$, $x = 12$ and the x-axis is 120 120 cm^2 . [3]

Find the area enclosed by the lines L , $x = 2$, $x = 12$ and the graph of h .

Markscheme

a. finding $f'(x) = \frac{1}{2}x$ ***A1***

attempt to find $f'(4)$ ***(M1)***

correct value $f'(4) = 2$ ***A1***

correct equation in any form ***A1*** ***N2***

e.g. $y - 6 = 2(x - 4)$, $y = 2x - 2$

[4 marks]

b. area $= \int_2^{12} \frac{90}{3x+4} \mathrm{d}x$

correct integral ***A1A1***

e.g. $30 \ln(3x + 4)$

substituting limits and subtracting ***(M1)***

e.g. $30 \ln(3 \times 12 + 4) - 30 \ln(3 \times 2 + 4)$, $30 \ln 40 - 30 \ln 10$

correct working ***(A1)***

e.g. $30(\ln 40 - \ln 10)$

correct application of $\ln b - \ln a$ ***(A1)*** e.g. $30\ln\frac{40}{10}$

area $= 30\ln 4$ ***A1*** ***N4***

[6 marks]

c. valid approach ***(M1)***

e.g. sketch, area h = area g , 120 + **their** answer from (b)

area $= 120 + 30\ln 4$ ***A2*** ***N3***

[3 marks]

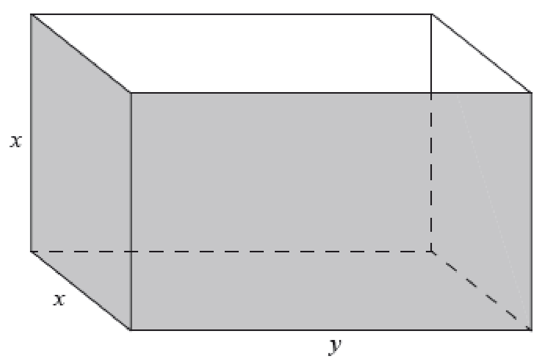
Examiners report

a. While most candidates answered part (a) correctly, finding the equation of the tangent, there were some who did not consider the value of their derivative when $x = 4$.

b. In part (b), most candidates knew that they needed to integrate to find the area, but errors in integration, and misapplication of the rules of logarithms kept many from finding the correct area.

c. In part (c), it was clear that a significant number of candidates understood the idea of the reflected function, and some recognized that the integral was the negative of the integral from part (b), but only a few recognized the relationship between the areas. Many thought the area between h and the x-axis was 120.

Fred makes an open metal container in the shape of a cuboid, as shown in the following diagram.



The container has height x m, width x m and length y m. The volume is $36\ \mathrm{m}^3$.

Let $A(x)$ be the outside surface area of the container.

a. Show that $A(x) = \frac{108}{x} + 2x^2$. [4]

b. Find $A'(x)$. [2]

c. Given that the outside surface area is a minimum, find the height of the container. [5]

d. Fred paints the outside of the container. A tin of paint covers a surface area of $10\ \mathrm{m}^2$ and costs $20. Find the total cost of the tins needed to paint the container. [5]

Markscheme

a. correct substitution into the formula for volume ***A1***

eg $36 = y \times x \times x$

valid approach to eliminate y (may be seen in formula/substitution) ***M1***

eg $y = \frac{36}{x^2},\ xy = \frac{36}{x}$

correct expression for surface area ***A1***

eg $xy + xy + xy + x^2 + x^2,\ \text{area} = 3xy + 2x^2$

correct expression in terms of x only ***A1***

eg $3x\left(\frac{36}{x^2}\right) + 2x^2,\ x^2 + x^2 + \frac{36}{x} + \frac{36}{x} + \frac{36}{x},\ 2x^2 + 3\left(\frac{36}{x}\right)$

$A(x) = \frac{108}{x} + 2x^2$ ***AG*** ***N0***

[4 marks]

b. $A'(x) = -\frac{108}{x^2} + 4x,\ 4x - 108x^{-2}$ ***A1A1*** ***N2***

Note: Award ***A1*** for each term.

[2 marks]

c. recognizing that minimum is when $A'(x) = 0$ ***(M1)***

correct equation ***(A1)***

eg $-\frac{108}{x^2} + 4x = 0,\ 4x = \frac{108}{x^2}$

correct simplification ***(A1)***

eg $-108 + 4x^3 = 0,\ 4x^3 = 108$

correct working ***(A1)***

eg $x^3 = 27$

$\text{height} = 3\ (\text{m})\ (\text{accept } x = 3)$ ***A1*** ***N2***

[5 marks]

d. attempt to find area using **their** height ***(M1)***

eg $\frac{108}{3} + 2(3)^2,\ 9 + 9 + 12 + 12 + 12$

minimum surface area $= 54\ \text{m}^2$ (may be seen in part (c)) ***A1***

attempt to find the number of tins ***(M1)***

eg $\frac{54}{10},\ 5.4$

6 (tins) ***(A1)***

$120 ***A1*** ***N3***

[5 marks]

Examiners report

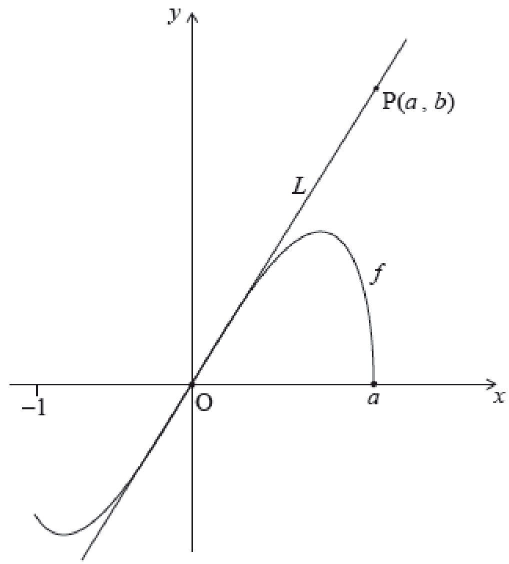
a. Many candidates answered part (a) of this question correctly, though some seemed to be working backwards from the given expression for area, which is not the intention of a "show that" question.

b. In part (b), while many candidates found the correct derivative, some did so using cumbersome methods such as the quotient rule, rather than using the simpler power rule.

c. It was disappointing to see the number of candidates who did not recognize that the derivative they had just found in part (b) would have to be equal to zero in order for the surface area to be a minimum. For the candidates who did set their derivative equal to zero, most were able to find the correct height.

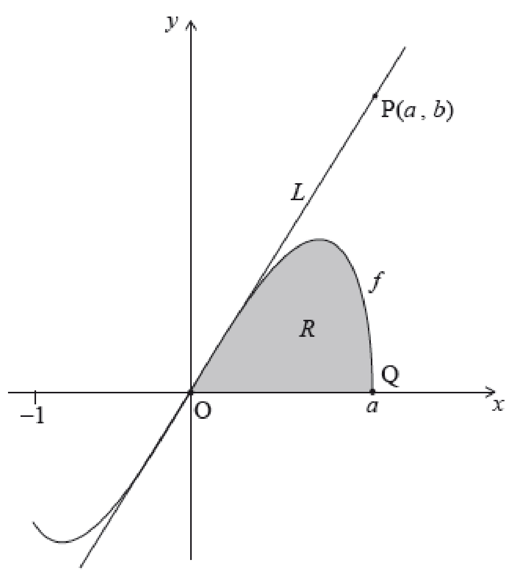
d. In part (d) of this question, there were some arithmetic errors which kept candidates from finding the correct area. The most common error here, by far, was not considering that the number of tins purchased must be an integer.

The following diagram shows the graph of $f(x) = 2x\sqrt{a^2 - x^2}$, for $-1 \leqslant x \leqslant a$, where $a > 1$.



The line L is the tangent to the graph of f at the origin, O. The point $\mathrm{P}(a,\ b)$ lies on L.

The point $\mathrm{Q}(a,\ 0)$ lies on the graph of f. Let R be the region enclosed by the graph of f and the x-axis. This information is shown in the following diagram.



Let A_R be the area of the region R.

a. (i) Given that $f'(x) = \frac{2a^2 - 4x^2}{\sqrt{a^2 - x^2}}$, for $-1 \leqslant x < a$, find the equation of L. [6]

(ii) Hence or otherwise, find an expression for b in terms of a.

b. Show that $A_R = \frac{2}{3}a^3$. [6]

c. Let A_T be the area of the triangle OPQ. Given that $A_T = kA_R$, find the value of k. [4]

Markscheme

a. (i) recognizing the need to find the gradient when $x = 0$ (seen anywhere) ***R1***

eg $f'(0)$

correct substitution ***(A1)***

$f'(0) = \frac{2a^2-4(0)}{\sqrt{a^2-0}}$

$f'(0) = 2a$ ***(A1)***

correct equation with gradient $2a$ (do not accept equations of the form $L = 2ax$) ***A1*** ***N3***

eg $y = 2ax,\ y - b = 2a(x - a),\ y = 2ax - 2a^2 + b$

(ii) **METHOD 1**

attempt to substitute $x = a$ into **their** equation of L ***(M1)***

eg $y = 2a \times a$

$b = 2a^2$ ***A1*** ***N2***

METHOD 2

equating gradients ***(M1)***

eg $\frac{b}{a} = 2a$

$b = 2a^2$ ***A1*** ***N2***

[6 marks]

b. **METHOD 1**

recognizing that area $= \int_0^a f(x)\mathrm{d}x$ (seen anywhere) ***R1***

valid approach using substitution or inspection ***(M1)***

eg $\int 2x\sqrt{u}\mathrm{d}x,\ u = a^2 - x^2,\ \mathrm{d}u = -2x\mathrm{d}x,\ \frac{2}{3}(a^2 - x^2)^{\frac{3}{2}}$

correct working ***(A1)***

eg $\int 2x\sqrt{a^2 - x^2}\mathrm{d}x = \int -\sqrt{u}\mathrm{d}u$

$\int -\sqrt{u}\mathrm{d}u = -\frac{u^{\frac{3}{2}}}{\frac{3}{2}}$ ***(A1)***

$\int f(x)\mathrm{d}x = -\frac{2}{3}(a^2 - x^2)^{\frac{3}{2}} + c$ ***(A1)***

substituting limits and subtracting ***A1***

eg $A_R = -\frac{2}{3}(a^2 - a^2)^{\frac{3}{2}} + \frac{2}{3}(a^2 - 0)^{\frac{3}{2}},\ \frac{2}{3}(a^2)^{\frac{3}{2}}$

$A_R = \frac{2}{3}a^3$ ***AG*** ***N0***

METHOD 2

recognizing that area $= \int_0^a f(x)\mathrm{d}x$ (seen anywhere) ***R1***

valid approach using substitution or inspection ***(M1)***

eg $\int 2x\sqrt{u}\mathrm{d}x,\ u = a^2 - x^2,\ \mathrm{d}u = -2x\mathrm{d}x,\ \frac{2}{3}(a^2 - x^2)^{\frac{3}{2}}$

correct working ***(A1)***

eg $\int 2x\sqrt{a^2 - x^2}\mathrm{d}x = \int -\sqrt{u}\mathrm{d}u$

$\int -\sqrt{u}\mathrm{d}u = -\frac{u^{\frac{3}{2}}}{\frac{3}{2}}$ ***(A1)***

new limits for *u* (even if integration is incorrect) ***(A1)***

eg $u = 0 \text{ and } u = a^2,\ \int_0^{a^2} u^{\frac{1}{2}}\mathrm{d}u,\ \left[-\frac{2}{3}u^{\frac{3}{2}}\right]_{a^2}^{0}$

substituting limits and subtracting ***A1***

eg $A_R = -\left(0 - \frac{2}{3}a^3\right),\ \frac{2}{3}(a^2)^{\frac{3}{2}}$

$A_R = \frac{2}{3}a^3$ ***AG*** ***N0***

[6 marks]

c. **METHOD 1**

valid approach to find area of triangle ***(M1)***

eg $\frac{1}{2}(\mathrm{OQ})(\mathrm{PQ}),\ \frac{1}{2}ab$

correct substitution into formula for A_T (seen anywhere) ***(A1)***

eg $A_T = \frac{1}{2} \times a \times 2a^2,\ a^3$

valid attempt to find k (must be in terms of a) ***(M1)***

eg $a^3 = k\frac{2}{3}a^3,\ k = \frac{a^3}{\frac{2}{3}a^3}$

$k = \frac{3}{2}$ ***A1*** ***N2***

METHOD 2

valid approach to find area of triangle ***(M1)***

eg $\int_0^a (2ax)\mathrm{d}x$

correct working ***(A1)***

eg $[ax^2]_0^a,\ a^3$

valid attempt to find k (must be in terms of a) ***(M1)***

eg $a^3 = k\frac{2}{3}a^3,\ k = \frac{a^3}{\frac{2}{3}a^3}$

$k = \frac{3}{2}$ ***A1*** ***N2***

[4 marks]

Examiners report

a. As is typically the case with question 10, this proved to be quite a challenging question for many candidates. In part (a), while many candidates seemed to recognize that there was some relationship between the given derivative and the gradient of the tangent line, most did not substitute zero for the x-value, and were unable to find the correct gradient of the line.

b. In part (b), nearly every candidate understood that the area was equal to the integral of f from 0 to a, very few were able to integrate correctly using either substitution or inspection. Many candidates did not even attempt to integrate, stopping after writing the integral expression.

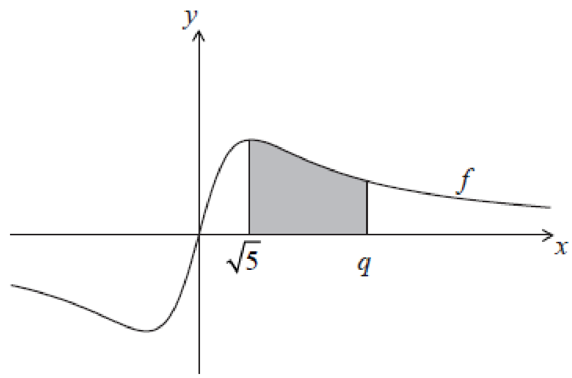
c. In part (c), most candidates started with a correct expression for the area of the triangle such as $\frac{ab}{2}$. However, very few were able to substitute their expression for b from part (a)(ii), and therefore did not find a value for k.

Let $f(x)=\frac{2x}{x^2+5}$.

a. Use the quotient rule to show that $f'(x)=\frac{10-2x^2}{(x^2+5)^2}$. [4]

b. Find $\int \frac{2x}{x^2+5}\mathrm{d}x$. [4]

c. The following diagram shows part of the graph of f. [7]



The shaded region is enclosed by the graph of f, the x-axis, and the lines $x=\sqrt{5}$ and $x=q$. This region has an area of $\ln 7$. Find the value of q.

Markscheme

a. derivative of $2x$ is 2 (must be seen in quotient rule) ***(A1)***

derivative of x^2+5 is $2x$ (must be seen in quotient rule) ***(A1)***

correct substitution into quotient rule ***A1***

eg $\frac{(x^2+5)(2)-(2x)(2x)}{(x^2+5)^2},\ \frac{2(x^2+5)-4x^2}{(x^2+5)^2}$

correct working which clearly leads to given answer ***A1***

eg $\frac{2x^2+10-4x^2}{(x^2+5)^2},\ \frac{2x^2+10-4x^2}{x^4+10x^2+25}$

$f'(x)=\frac{10-2x^2}{(x^2+5)^2}$ ***AG N0***

[4 marks]

b. valid approach using substitution or inspection ***(M1)***

eg $u=x^2+5,\ \mathrm{d}u=2x\mathrm{d}x,\ \frac{1}{2}\ln(x^2+5)$

$\int \frac{2x}{x^2+5}\mathrm{d}x=\int \frac{1}{u}\mathrm{d}u$ ***(A1)***

$\int \frac{1}{u}\mathrm{d}u=\ln u+c$ ***(A1)***

$\ln(x^2+5)+c$ ***A1 N4***

[4 marks]

c. correct expression for area ***(A1)***

eg $\left[\ln\left(x^2+5\right)\right]_{\sqrt{5}}^{q},\ \int_{\sqrt{5}}^{q}\frac{2x}{x^2+5}\mathrm{d}x$

substituting limits into **their** integrated function and subtracting (in either order) ***(M1)***

eg $\ln(q^2+5)-\ln\left(\sqrt{5}^2+5\right)$

correct working ***(A1)***

eg $\ln\left(q^2+5\right)-\ln 10,\ \ln\frac{q^2+5}{10}$

equating **their** expression to $\ln 7$ (seen anywhere) ***(M1)***

eg $\ln\left(q^2+5\right)-\ln 10=\ln 7,\ \ln\frac{q^2+5}{10}=\ln 7,\ \ln(q^2+5)=\ln 7+\ln 10$

correct equation without logs ***(A1)***

eg $\frac{q^2+5}{10}=7,\ q^2+5=70$

$q^2=65$ ***(A1)***

$q=\sqrt{65}$ ***A1 N3***

Note: Award ***A0*** for $q = \pm\sqrt{65}$.

[7 marks]

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]

Let $\int_{\pi}^{a} \cos 2x \mathrm{d}x = \frac{1}{2}$, where $\pi < a < 2\pi$. Find the value of a.

Markscheme

correct integration (ignore absence of limits and "$+C$")   ***(A1)***

eg $\frac{\sin(2x)}{2}, \ \int_{\pi}^{a} \cos 2x = \left[\frac{1}{2}\sin(2x)\right]_{\pi}^{a}$

substituting limits into **their** integrated function and subtracting (in any order)   ***(M1)***

eg $\frac{1}{2}\sin(2a) - \frac{1}{2}\sin(2\pi), \ \sin(2\pi) - \sin(2a)$

$\sin(2\pi) = 0$ ***(A1)***

setting **their** result from an integrated function equal to $\frac{1}{2}$ ***M1***

eg $\frac{1}{2}\sin 2a = \frac{1}{2}, \ \sin(2a) = 1$

recognizing $\sin^{-1} 1 = \frac{\pi}{2}$ ***(A1)***

eg $2a = \frac{\pi}{2}, \ a = \frac{\pi}{4}$

correct value   ***(A1)***

eg $\frac{\pi}{2} + 2\pi, \ 2a = \frac{5\pi}{2}, \ a = \frac{\pi}{4} + \pi$

$a = \frac{5\pi}{4}$ ***A1***   ***N3***

[7 marks]

Examiners report

[N/A]

A rocket moving in a straight line has velocity v km s^{-1} and displacement s km at time t seconds. The velocity v is given by $v(t) = 6\mathrm{e}^{2t} + t$.

When $t = 0$, $s = 10$.

Find an expression for the displacement of the rocket in terms of t .

Markscheme

evidence of anti-differentiation   ***(M1)***

eg $\int (6\mathrm{e}^{2t} + t)$

$s = 3\mathrm{e}^{2t} + \frac{t^2}{2} + C$ ***A2A1***

Note: Award ***A2*** for $3\mathrm{e}^{2t}$, ***A1*** for $\frac{t^2}{2}$.

attempt to substitute (0, 10) into **their** integrated expression (even if C is missing) ***(M1)***

correct working ***(A1)***

eg $10 = 3 + C$, $C = 7$

$s = 3e^{2t} + \frac{t^2}{2} + 7$ ***A1*** ***N6***

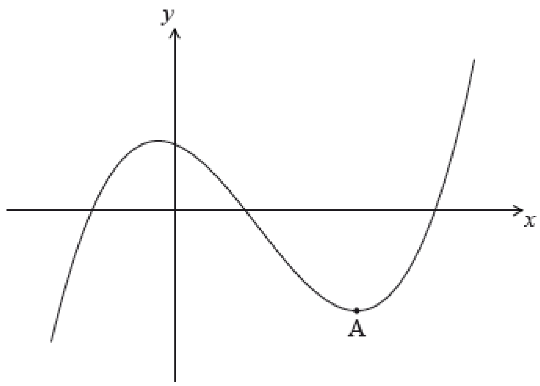
Note: Exception to the ***FT*** rule. If working shown, allow full ***FT*** on incorrect integration which must involve a power of e.

[7 marks]

Examiners report

A good number of candidates earned full marks on this question, and many others were able to earn at least half of the available marks. Most candidates knew to integrate, but there were quite a few who tried to find the derivative instead. Many candidates integrated the term $6e^{2t}$ incorrectly, but most were able to earn some further method marks for substituting into their integrated function. The majority of candidates who substituted (0, 10) into their integrated function knew that $e^0 = 1$.

The following diagram shows the graph of a function f. There is a local minimum point at A, where $x > 0$.



The derivative of f is given by $f'(x) = 3x^2 - 8x - 3$.

a. Find the x-coordinate of A. [5]

b. The y-intercept of the graph is at $(0, 6)$. Find an expression for $f(x)$. [6]

The graph of a function g is obtained by reflecting the graph of f in the y-axis, followed by a translation of $\begin{pmatrix} m \\ n \end{pmatrix}$.

c. Find the x-coordinate of the local minimum point on the graph of g. [3]

Markscheme

a. recognizing that the local minimum occurs when $f'(x) = 0$ ***(M1)***

valid attempt to solve $3x^2 - 8x - 3 = 0$ ***(M1)***

eg factorization, formula

correct working ***A1***

$(3x + 1)(x - 3),\ x = \frac{8 \pm \sqrt{64+36}}{6}$

$x = 3$ ***A2*** ***N3***

Note: Award ***A1*** if both values $x = \frac{-1}{3},\ x = 3$ are given.

[5 marks]

b. valid approach ***(M1)***

$f(x) = \int f'(x)\mathrm{d}x$

$f(x) = x^3 - 4x^2 - 3x + c$ (do not penalize for missing "$+c$") ***A1A1A1***

$c = 6$ ***(A1)***

$f(x) = x^3 - 4x^2 - 3x + 6$ ***A1 N6***

[6 marks]

c. applying reflection ***(A1)***

eg $f(-x)$

recognizing that the minimum is the image of A ***(M1)***

eg $x = -3$

correct expression for x ***A1 N3***

eg $-3 + m,\ \begin{pmatrix} -3+m \\ -12+n \end{pmatrix},\ (m-3,\ n-12)$

[3 marks]

Total [14 marks]

Examiners report

a. The majority of candidates approached part (a) correctly, and most recognized that only one solution was possible within the given domain.

b. Nearly all candidates answered part (b) correctly, earning all the available marks for integrating the polynomial and solving for C.

c. Part (c) proved to be much more difficult for candidates, who either did not know how to apply the transformations correctly, or who engaged in lengthy and unnecessary manipulations of the function, rather than simply finding the image of the local minimum point A.

Let $f(x) = 1 + \mathrm{e}^{-x}$ and $g(x) = 2x + b$, for $x \in \mathbb{R}$, where b is a constant.

a. Find $(g \circ f)(x)$. [2]

b. Given that $\lim\limits_{x \to +\infty} (g \circ f)(x) = -3$, find the value of b. [4]

Markscheme

a. attempt to form composite ***(M1)***

eg $g(1 + \mathrm{e}^{-x})$

correct function ***A1 N2***

eg $(g \circ f)(x) = 2 + b + 2\mathrm{e}^{-x},\ 2(1 + \mathrm{e}^{-x}) + b$

[2 marks]

b. evidence of $\lim\limits_{x \to \infty}(2 + b + 2\mathrm{e}^{-x}) = 2 + b + \lim\limits_{x \to \infty}(2\mathrm{e}^{-x})$ ***(M1)***

eg $2 + b + 2\mathrm{e}^{-\infty}$, graph with horizontal asymptote when $x \to \infty$

Note: Award ***M0*** if candidate clearly has incorrect limit, such as $x \to 0,\ \mathrm{e}^{\infty},\ 2\mathrm{e}^{0}$.

evidence that $\mathrm{e}^{-x} \to 0$ (seen anywhere) ***(A1)***

eg $\lim\limits_{x \to \infty}(\mathrm{e}^{-x}) = 0,\ 1 + \mathrm{e}^{-x} \to 1,\ 2(1) + b = -3,\ \mathrm{e}^{\text{large negative number}} \to 0$, graph of $y = \mathrm{e}^{-x}$ or $y = 2\mathrm{e}^{-x}$ with asymptote $y = 0$, graph of composite function with asymptote $y = -3$

correct working ***(A1)***

eg $2 + b = -3$

$b = -5$ ***A1 N2***

[4 marks]

Examiners report

a. [N/A]
b. [N/A]

Let $f'(x) = \frac{3x^2}{(x^3+1)^5}$. Given that $f(0) = 1$, find $f(x)$.

Markscheme

valid approach ***(M1)***

eg $\int f' \mathrm{d}x,\ \int \frac{3x^2}{(x^3+1)^5} \mathrm{d}x$

correct integration by substitution/inspection ***A2***

eg $f(x) = -\frac{1}{4}(x^3 + 1)^{-4} + c,\ \frac{-1}{4(x^3+1)^4}$

correct substitution into **their** integrated function (must include *c*) ***M1***

eg $1 = \frac{-1}{4(0^3+1)^4} + c,\ -\frac{1}{4} + c = 1$

Note: Award ***M0*** if candidates substitute into f' or f''.

$c = \frac{5}{4}$ ***(A1)***

$f(x) = -\frac{1}{4}(x^3 + 1)^{-4} + \frac{5}{4}\ \left(= \frac{-1}{4(x^3+1)^4} + \frac{5}{4},\ \frac{5(x^3+1)^4 - 1}{4(x^3+1)^4}\right)$ ***A1 N4***

[6 marks]

Examiners report

[N/A]

A function *f* has its first derivative given by $f'(x) = (x-3)^3$.

a. Find the second derivative. [2]

b. Find $f'(3)$ and $f''(3)$. [1]

c. The point P on the graph of *f* has *x*-coordinate 3. Explain why P is not a point of inflexion. [2]

Markscheme

a. **METHOD 1**

$f''(x) = 3(x-3)^2$ ***A2 N2***

METHOD 2

attempt to expand $(x-3)^3$ ***(M1)***

e.g. $f'(x) = x^3 - 9x^2 + 27x - 27$

$f''(x) = 3x^2 - 18x + 27$ ***A1 N2***

[2 marks]

b. $f'(3) = 0$, $f''(3) = 0$ ***A1 N1***

[1 mark]

c. **METHOD 1**

f'' does not change sign at P ***R1***

evidence for this ***R1 N0***

METHOD 2

f' changes sign at P so P is a maximum/minimum (i.e. not inflexion) ***R1***

evidence for this ***R1 N0***

METHOD 3

finding $f(x) = \frac{1}{4}(x-3)^4 + c$ and sketching this function ***R1***

indicating minimum at $x = 3$ ***R1 N0***

[2 marks]

Examiners report

a. Many candidates completed parts (a) and (b) successfully.

b. Many candidates completed parts (a) and (b) successfully.

c. A rare few earned any marks in part (c) - most justifying the point of inflexion with the zero answers in part (b), not thinking that there is more to consider.

Let $f(x) = \frac{6x}{x+1}$, for $x > 0$.

a. Find $f'(x)$. [5]

b. Let $g(x) = \ln\left(\frac{6x}{x+1}\right)$, for $x > 0$. [4]

Show that $g'(x) = \frac{1}{x(x+1)}$.

c. Let $h(x) = \frac{1}{x(x+1)}$. The area enclosed by the graph of h , the x-axis and the lines $x = \frac{1}{5}$ and $x = k$ is $\ln 4$. Given that $k > \frac{1}{5}$, find the value of k . [7]

Markscheme

a. **METHOD 1**

evidence of choosing quotient rule ***(M1)***

e.g. $\frac{u'v-uv'}{v^2}$

evidence of correct differentiation (must be seen in quotient rule) ***(A1)(A1)***

e.g. $\frac{\mathrm{d}}{\mathrm{d}x}(6x) = 6$, $\frac{\mathrm{d}}{\mathrm{d}x}(x+1) = 1$

correct substitution into quotient rule ***A1***

e.g. $\frac{(x+1)6-6x}{(x+1)^2}$, $\frac{6x+6-6x}{(x+1)^2}$

$f'(x) = \frac{6}{(x+1)^2}$ ***A1 N4***

[5 marks]

METHOD 2

evidence of choosing product rule ***(M1)***

e.g. $6x(x+1)^{-1}$, $uv' + vu'$

evidence of correct differentiation (must be seen in product rule) ***(A1)(A1)***

e.g. $\frac{\mathrm{d}}{\mathrm{d}x}(6x) = 6$, $\frac{\mathrm{d}}{\mathrm{d}x}(x+1)^{-1} = -1(x+1)^{-2} \times 1$

correct working ***A1***

e.g. $6x \times -(x+1)^{-2} + (x+1)^{-1} \times 6$, $\frac{-6x+6(x+1)}{(x+1)^2}$

$f'(x) = \frac{6}{(x+1)^2}$ ***A1 N4***

[5 marks]

b. **METHOD 1**

evidence of choosing chain rule ***(M1)***

e.g. formula, $\frac{1}{\left(\frac{6x}{x+1}\right)} \times \left(\frac{6x}{x+1}\right)$

correct reciprocal of $\frac{1}{\left(\frac{6x}{x+1}\right)}$ is $\frac{x+1}{6x}$ (seen anywhere) ***A1***

correct substitution into chain rule ***A1***

e.g. $\frac{1}{\left(\frac{6x}{x+1}\right)} \times \frac{6}{(x+1)^2}$, $\left(\frac{6}{(x+1)^2}\right)\left(\frac{x+1}{6x}\right)$

working that clearly leads to the answer ***A1***

e.g. $\left(\frac{6}{(x+1)}\right)\left(\frac{1}{6x}\right)$, $\left(\frac{1}{(x+1)^2}\right)\left(\frac{x+1}{x}\right)$, $\frac{6(x+1)}{6x(x+1)^2}$

$g'(x) = \frac{1}{x(x+1)}$ ***AG N0***

[4 marks]

METHOD 2

attempt to subtract logs ***(M1)***

e.g. $\ln a - \ln b$, $\ln 6x - \ln(x+1)$

correct derivatives (must be seen in correct expression) ***A1A1***

e.g. $\frac{6}{6x} - \frac{1}{x+1}$, $\frac{1}{x} - \frac{1}{x+1}$

working that clearly leads to the answer ***A1***

e.g. $\frac{x+1-x}{x(x+1)}$, $\frac{6x+6-6x}{6x(x+1)}$, $\frac{6(x+1-x)}{6x(x+1)}$

$g'(x) = \frac{1}{x(x+1)}$ ***AG N0***

[4 marks]

c. valid method using integral of $h(x)$ (accept missing/incorrect limits or missing $\mathrm{d}x$) ***(M1)***

e.g. area $= \int_{\frac{1}{5}}^{k} h(x)\mathrm{d}x$, $\int \left(\frac{1}{x(x+1)}\right)$

recognizing that integral of derivative will give original function ***(R1)***

e.g. $\int \left(\frac{1}{x(x+1)}\right)\mathrm{d}x = \ln\left(\frac{6x}{x+1}\right)$

correct substitution and subtraction ***A1***

e.g. $\ln\left(\frac{6k}{k+1}\right) - \ln\left(\frac{6\times\frac{1}{5}}{\frac{1}{5}+1}\right)$, $\ln\left(\frac{6k}{k+1}\right) - \ln(1)$

setting **their** expression equal to $\ln 4$ ***(M1)***

e.g. $\ln\left(\frac{6k}{k+1}\right) - \ln(1) = \ln 4$, $\ln\left(\frac{6k}{k+1}\right) = \ln 4$, $\int_{\frac{1}{5}}^{k} h(x)\mathrm{d}x = \ln 4$

correct equation without logs ***A1***

e.g. $\frac{6k}{k+1} = 4$, $6k = 4(k+1)$

correct working ***(A1)***

e.g. $6k = 4k + 4$, $2k = 4$

$k = 2$ ***A1 N4***

[7 marks]

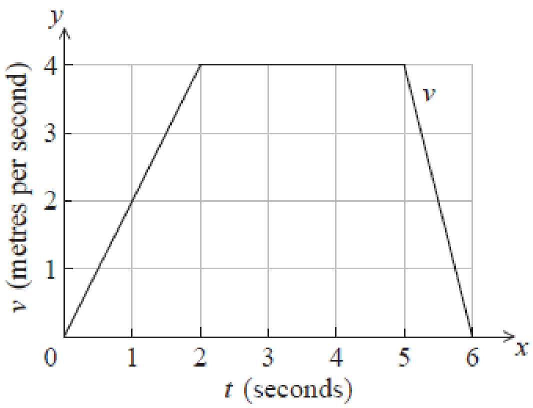
Examiners report

a. In part (a), most candidates recognized the need to apply the quotient rule to find the derivative, and many were successful in earning full marks here.

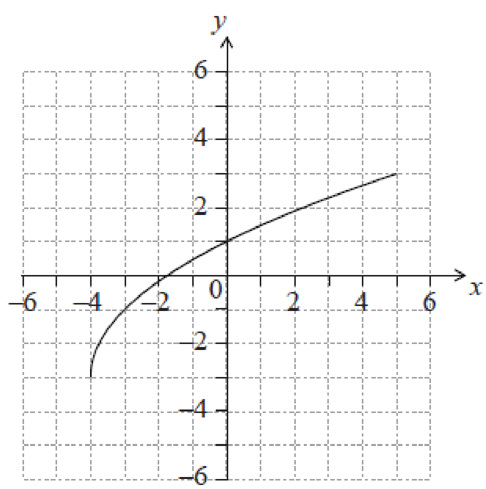
b. In part (b), many candidates struggled with the chain rule, or did not realize the chain rule was necessary to find the derivative. Again, some candidates attempted to work backward from the given answer, which is not allowed in a "show that" question. A few clever candidates simplified the situation by applying properties of logarithms before finding their derivative.

c. For part (c), many candidates recognized the need to integrate the function, and that their integral would equal $\ln 4$. However, many did not recognize that the integral of h is g . Those candidates who made this link between the parts (b) and (c) often carried on correctly to find the value of k , with a few candidates having errors in working with logarithms.

A toy car travels with velocity v ms^{-1} for six seconds. This is shown in the graph below.



The following diagram shows the graph of $y = f(x)$, for $-4 \leq x \leq 5$.



a. Write down the car's velocity at $t = 3$. [1]

a(i).Write down the value of $f(-3)$; [1]

b. Find the car's acceleration at $t = 1.5$. [2]

c. Find the total distance travelled. [3]

Markscheme

a. $4\ (\text{ms}^{-1})$ ***A1 N1***

[1 mark]

a(i). $f(-3) = -1$ ***A1 N1***

[1 mark]

b. recognizing that acceleration is the gradient ***M1***

e.g. $a(1.5) = \frac{4-0}{2-0}$

$a = 2\ (\text{ms}^{-2})$ ***A1 N1***

[2 marks]

c. recognizing area under curve ***M1***

e.g. trapezium, triangles, integration

correct substitution ***A1***

e.g. $\frac{1}{2}(3+6)4$, $\int_0^6 |v(t)|\mathrm{d}t$

distance 18 (m) ***A1*** ***N2***

[3 marks]

Examiners report

a. [N/A]
a(i). [N/A]
b. [N/A]
c. [N/A]

Let L_x be a family of lines with equation given by $r = \begin{pmatrix} x \\ \frac{2}{x} \end{pmatrix} + t\begin{pmatrix} x^2 \\ -2 \end{pmatrix}$, where $x > 0$.

a. Write down the equation of L_1. [2]

b. A line L_a crosses the y-axis at a point P. [6]

Show that P has coordinates $\left(0,\ \frac{4}{a}\right)$.

c. The line L_a crosses the x-axis at $\mathrm{Q}(2a,\ 0)$. Let $d = \mathrm{PQ}^2$. [2]

Show that $d = 4a^2 + \frac{16}{a^2}$.

d. There is a minimum value for d. Find the value of a that gives this minimum value. [7]

Markscheme

a. attempt to substitute $x = 1$ ***(M1)***

eg $\boldsymbol{r} = \begin{pmatrix} 1 \\ \frac{2}{1} \end{pmatrix} + t\begin{pmatrix} 1^2 \\ -2 \end{pmatrix}$, $L_1 = \begin{pmatrix} 1 \\ 2 \end{pmatrix} + t\begin{pmatrix} 1 \\ -2 \end{pmatrix}$

correct equation (vector or Cartesian, but do not accept "L_1")

eg $\boldsymbol{r} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} + t\begin{pmatrix} 1 \\ -2 \end{pmatrix}$, $y = -2x + 4$ (must be an equation) ***A1*** ***N2***

[2 marks]

b. appropriate approach ***(M1)***

eg $\begin{pmatrix} 0 \\ y \end{pmatrix} = \begin{pmatrix} a \\ \frac{2}{a} \end{pmatrix} + t\begin{pmatrix} a^2 \\ -2 \end{pmatrix}$

correct equation for x-coordinate ***A1***

eg $0 = a + ta^2$

$t = \frac{-1}{a}$ ***A1***

substituting **their** parameter to find y ***(M1)***

eg $y=\frac{2}{a}-2\left(\frac{-1}{a}\right),\ \begin{pmatrix} a \\ \frac{2}{a} \end{pmatrix}-\frac{1}{a}\begin{pmatrix} a^2 \\ -2 \end{pmatrix}$

correct working ***A1***

eg $y=\frac{2}{a}+\frac{2}{a},\ \begin{pmatrix} a \\ \frac{2}{a} \end{pmatrix}-\begin{pmatrix} a \\ -\frac{2}{a} \end{pmatrix}$

finding correct expression for y ***A1***

eg $y=\frac{4}{a},\ \begin{pmatrix} 0 \\ \frac{4}{a} \end{pmatrix} \mathrm{P}\left(0,\ \frac{4}{a}\right)$ ***AG*** ***N0***

[6 marks]

c. valid approach ***M1***

eg distance formula, Pythagorean Theorem, $\overrightarrow{\mathrm{PQ}}=\begin{pmatrix} 2a \\ -\frac{4}{a} \end{pmatrix}$

correct simplification ***A1***

eg $(2a)^2+\left(\frac{4}{a}\right)^2$

$d=4a^2+\frac{16}{a^2}$ ***AG*** ***N0***

[2 marks]

d. recognizing need to find derivative ***(M1)***

eg $d',\ d'(a)$

correct derivative ***A2***

eg $8a-\frac{32}{a^3},\ 8x-\frac{32}{x^3}$

setting **their** derivative equal to 0 ***(M1)***

eg $8a-\frac{32}{a^3}=0$

correct working ***(A1)***

eg $8a=\frac{32}{a^3},\ 8a^4-32=0$

working towards solution ***(A1)***

eg $a^4=4,\ a^2=2,\ a=\pm\sqrt{2}$

$a=\sqrt[4]{4}\quad(a=\sqrt{2})\quad(\text{do not accept }\pm\sqrt{2})$ ***A1*** ***N3***

[7 marks]

Total [17 marks]

Examiners report

a. In part (a), most candidates correctly substituted 1 for x, although many of them did not earn full marks for their work here, as they wrote their vector equation using $L_1=$, not understanding that L_1 is the name of the line, and not a vector.

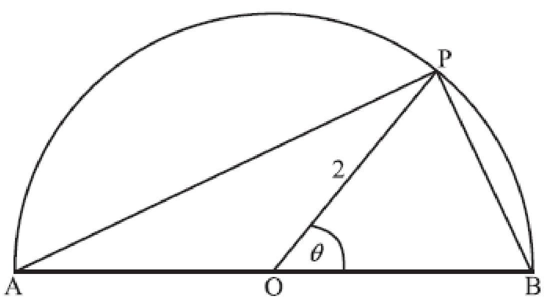
b. Very few candidates answered parts (b) and (c) correctly, often working backwards from the given answer, which is not appropriate in "show that" questions. In these types of questions, candidates are required to clearly show their working and reasoning, which will hopefully lead them to the given answer.

c. Very few candidates answered parts (b) and (c) correctly, often working backwards from the given answer, which is not appropriate in "show that" questions. In these types of questions, candidates are required to clearly show their working and reasoning, which will hopefully lead them to the given answer.

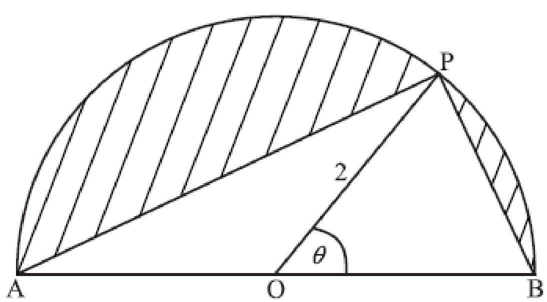
d. Fortunately, a good number of candidates recognized the need to find the derivative of the given expression for d in part (d) of the question, and so were able to earn at least some of the available marks in the final part.

The following diagram shows a semicircle centre O, diameter [AB], with radius 2.

Let P be a point on the circumference, with $\hat{\text{POB}} = \theta$ radians.



Let S be the total area of the two segments shaded in the diagram below.



a. Find the area of the triangle OPB, in terms of θ . [2]

b. Explain why the area of triangle OPA is the same as the area triangle OPB. [3]

c. Show that $S = 2(\pi - 2\sin\theta)$. [3]

d. Find the value of θ when S is a local minimum, justifying that it is a minimum. [8]

e. Find a value of θ for which S has its greatest value. [2]

Markscheme

a. evidence of using area of a triangle ***(M1)***

e.g. $A = \frac{1}{2} \times 2 \times 2 \times \sin\theta$

$A = 2\sin\theta$ ***A1*** ***N2***

[2 marks]

b. **METHOD 1**

$\mathrm{P\widehat{O}A} = \pi - \theta$ ***(A1)***

area $\Delta\mathrm{OPA} = \frac{1}{2} 2 \times 2 \times \sin(\pi - \theta)\ (= 2\sin(\pi - \theta))$ ***A1***

since $\sin(\pi - \theta) = \sin\theta$ ***R1***

then both triangles have the same area ***AG N0***

METHOD 2

triangle OPA has the same height and the same base as triangle OPB ***R3***

then both triangles have the same area ***AG N0***

[3 marks]

c. area semicircle $= \frac{1}{2} \times \pi(2)^2\ (= 2\pi)$ ***A1***

area $\Delta\mathrm{APB} = 2\sin\theta + 2\sin\theta\ (= 4\sin\theta)$ ***A1***

$S =$ area of semicircle $-$ area $\Delta\mathrm{APB}$ $(= 2\pi - 4\sin\theta)$ ***M1***

$S = 2(\pi - 2\sin\theta)$ ***AG N0***

[3 marks]

d. **METHOD 1**

attempt to differentiate ***(M1)***

e.g. $\frac{\mathrm{d}S}{\mathrm{d}\theta} = -4\cos\theta$

setting derivative equal to 0 ***(M1)***

correct equation ***A1***

e.g. $-4\cos\theta = 0$, $\cos\theta = 0$, $4\cos\theta = 0$

$\theta = \frac{\pi}{2}$ ***A1 N3***

EITHER

evidence of using second derivative ***(M1)***

$S''(\theta) = 4\sin\theta$ ***A1***

$S''\left(\frac{\pi}{2}\right) = 4$ ***A1***

it is a minimum because $S''\left(\frac{\pi}{2}\right) > 0$ ***R1 N0***

OR

evidence of using first derivative ***(M1)***

for $\theta < \frac{\pi}{2}, S'(\theta) < 0$ (may use diagram) ***A1***

for $\theta > \frac{\pi}{2}, S'(\theta) > 0$ (may use diagram) ***A1***

it is a minimum since the derivative goes from negative to positive ***R1 N0***

METHOD 2

$2\pi - 4\sin\theta$ is minimum when $4\sin\theta$ is a maximum ***R3***

$4\sin\theta$ is a maximum when $\sin\theta = 1$ ***(A2)***

$\theta = \frac{\pi}{2}$ ***A3 N3***

[8 marks]

e. S is greatest when $4\sin\theta$ is smallest (or equivalent) ***(R1)***

$\theta = 0$ (or π) ***A1 N2***

[2 marks]

Examiners report

a. Most candidates could obtain the area of triangle OPB as equal to $2\sin\theta$, though 2θ was given quite often as the area.

b. A minority recognized the equality of the sines of supplementary angles and the term complementary was frequently used instead of supplementary. Only a handful of candidates used the simple equal base and altitude argument.

c. Many candidates seemed to see why $S = 2(\pi - 2\sin\theta)$ but the arguments presented for showing why this result was true were not very convincing in many cases. Explicit evidence of why the area of the semicircle was 2π was often missing as was an explanation for $2(2\sin\theta)$ and for subtraction.

d. Only a small number of candidates recognized the fact *S* would be minimum when sin was maximum, leading to a simple non-calculus solution. Those who chose the calculus route often had difficulty finding the derivative of *S*, failing in a significant number of cases to recognize that the derivative of a constant is 0, and also going through painstaking application of the product rule to find the simple derivative. When it came to justify a minimum, there was evidence in some cases of using some form of valid test, but explanation of the test being used was generally poor.

e. Candidates who answered part (d) correctly generally did well in part (e) as well, though answers outside the domain of θ were frequently seen.

Consider a function $f(x)$ such that $\int_1^6 f(x)\mathrm{d}x = 8$.

a. Find $\int_1^6 2f(x)\mathrm{d}x$. [2]

b. Find $\int_1^6 \left(f(x)+2\right)\mathrm{d}x$. [4]

Markscheme

a. appropriate approach ***(M1)***

$eg \quad 2\int f(x),\ 2(8)$

$\int_1^6 2f(x)\mathrm{d}x = 16$ ***A1 N2***

[2 marks]

b. appropriate approach ***(M1)***

$eg \quad \int f(x) + \int 2,\ 8 + \int 2$

$\int 2\mathrm{d}x = 2x$ (seen anywhere) ***(A1)***

substituting limits into **their** integrated function and subtracting (in any order) ***(M1)***

$eg \quad 2(6) - 2(1),\ 8 + 12 - 2$

$\int_1^6 \left(f(x)+2\right)\mathrm{d}x = 18$ ***A1 N3***

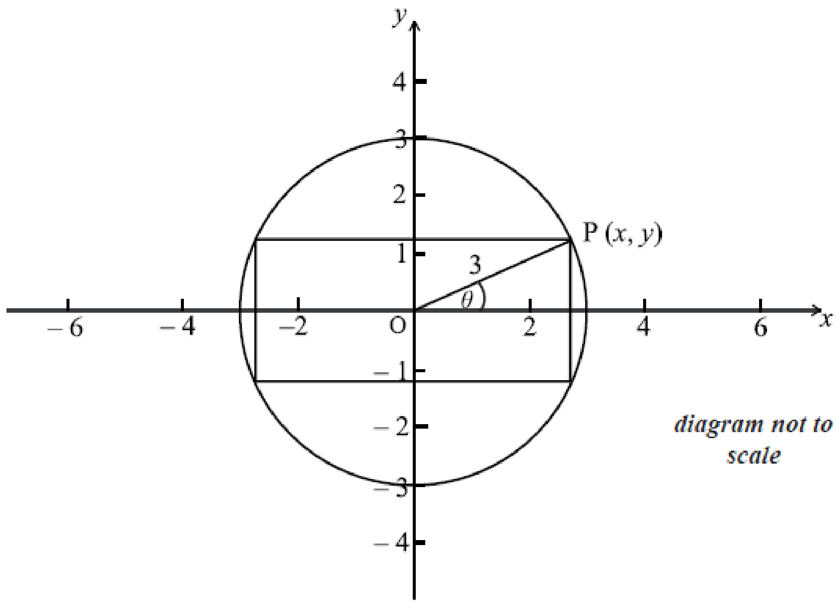
[4 marks]

Examiners report

a. [N/A]
[N/A]

b.

A rectangle is inscribed in a circle of radius 3 cm and centre O, as shown below.



The point P(x , y) is a vertex of the rectangle and also lies on the circle. The angle between (OP) and the x-axis is θ radians, where $0 \le \theta \le \frac{\pi}{2}$.

a. Write down an expression in terms of θ for [2]

(i) x ;

(ii) y .

b. Let the area of the rectangle be A. [3]

Show that $A = 18 \sin 2\theta$.

c. (i) Find $\frac{\mathrm{d}A}{\mathrm{d}\theta}$. [8]

(ii) Hence, find the exact value of θ which maximizes the area of the rectangle.

(iii) Use the second derivative to justify that this value of θ does give a maximum.

Markscheme

a. (i) $x = 3\cos\theta$ ***A1 N1***

(ii) $y = 3\sin\theta$ ***A1 N1***

[2 marks]

b. finding area ***(M1)***

e.g. $A = 2x \times 2y$, $A = 8 \times \frac{1}{2}bh$

substituting ***A1***

e.g. $A = 4 \times 3\sin\theta \times 3\cos\theta$, $8 \times \frac{1}{2} \times 3\cos\theta \times 3\sin\theta$

$A = 18(2\sin\theta\cos\theta)$ ***A1***

$A = 18\sin 2\theta$ ***AG N0***

[3 marks]

c. (i) $\frac{\mathrm{d}A}{\mathrm{d}\theta} = 36\cos 2\theta$ ***A2 N2***

(ii) for setting derivative equal to 0 ***(M1)***

e.g. $36\cos 2\theta = 0$, $\frac{\mathrm{d}A}{\mathrm{d}\theta} = 0$

$2\theta = \frac{\pi}{2}$ ***(A1)***

$\theta = \frac{\pi}{4}$ ***A1 N2***

(iii) valid reason (seen anywhere) ***R1***

e.g. at $\frac{\pi}{4}$, $\frac{\mathrm{d}^2A}{\mathrm{d}\theta^2} < 0$; maximum when $f''(x) < 0$

finding second derivative $\frac{\mathrm{d}^2A}{\mathrm{d}\theta^2} = -72\sin 2\theta$ ***A1***

evidence of substituting $\frac{\pi}{4}$ ***M1***

e.g. $-72\sin\left(2 \times \frac{\pi}{4}\right)$, $-72\sin\left(\frac{\pi}{2}\right)$, -72

$\theta = \frac{\pi}{4}$ produces the maximum area ***AG N0***

[8 marks]

Examiners report

a. Candidates familiar with the circular nature of sine and cosine found part (a) accessible. However, a good number of candidates left this part blank, which suggests that there was difficulty interpreting the meaning of the *x* and *y* in the diagram.

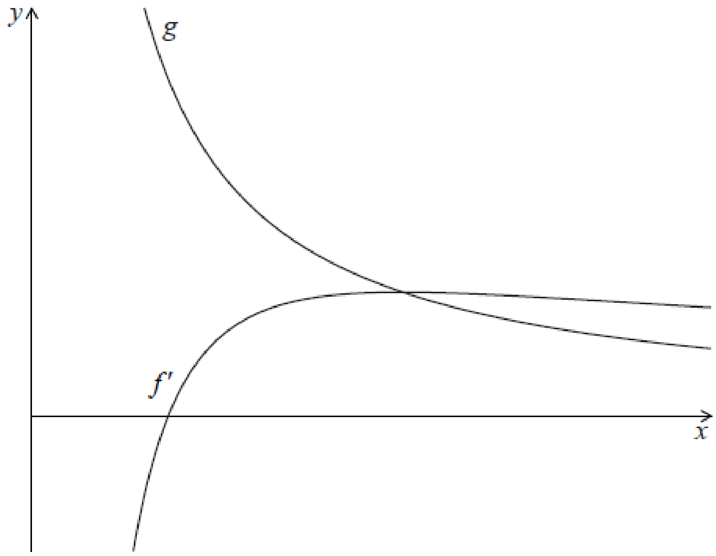
b. Those with answers from (a) could begin part (b), but many worked backwards and thus earned no marks. In a "show that" question, a solution cannot begin with the answer given. The area of the rectangle could be found by using $2x \times 2y$, or by using the eight small triangles, but it was essential that the substitution of the double-angle formula was shown before writing the given answer.

c. As the area function was given in part (b), many candidates correctly found the derivative in (c) and knew to set this derivative to zero for a maximum value. Many gave answers in degrees, however, despite the given domain in radians.

Although some candidates found the second derivative function correctly, few stated that the second derivative must be negative at a maximum value. Simply calculating a negative value is not sufficient for a justification.

Let $f(x) = \frac{(\ln x)^2}{2}$, for $x > 0$.

Let $g(x) = \frac{1}{x}$. The following diagram shows parts of the graphs of f' and g.



The graph of f' has an x-intercept at $x = p$.

a. Show that $f'(x) = \frac{\ln x}{x}$. [2]

b. There is a minimum on the graph of f. Find the x-coordinate of this minimum. [3]

c. Write down the value of p. [2]

d. The graph of g intersects the graph of f' when $x = q$. [3]

Find the value of q.

e. The graph of g intersects the graph of f' when $x = q$. [5]

Let R be the region enclosed by the graph of f', the graph of g and the line $x = p$.
Show that the area of R is $\frac{1}{2}$.

Markscheme

a. **METHOD 1**

correct use of chain rule ***A1A1***
eg $\frac{2\ln x}{2} \times \frac{1}{x},\ \frac{2\ln x}{2x}$

Note: Award ***A1*** for $\frac{2\ln x}{2x}$, ***A1*** for $\times\frac{1}{x}$.

$f'(x) = \frac{\ln x}{x}$ ***AG*** ***N0***
[2 marks]
METHOD 2
correct substitution into quotient rule, with derivatives seen ***A1***
eg $\frac{2\times 2\ln x\times\frac{1}{x}-0\times(\ln x)^2}{4}$
correct working ***A1***
eg $\frac{4\ln x\times\frac{1}{x}}{4}$
$f'(x) = \frac{\ln x}{x}$ ***AG*** ***N0***
[2 marks]

b. setting derivative $= 0$ ***(M1)***
eg $f'(x) = 0,\ \frac{\ln x}{x} = 0$
correct working ***(A1)***
eg $\ln x = 0,\ x = \mathrm{e}^0$
$x = 1$ ***A1*** ***N2***
[3 marks]

c. intercept when $f'(x) = 0$ ***(M1)***

$p = 1$ ***A1 N2***

[2 marks]

d. equating functions ***(M1)***

eg $f' = g,\ \frac{\ln x}{x} = \frac{1}{x}$

correct working ***(A1)***

eg $\ln x = 1$

$q = \mathrm{e}$ (accept $x = \mathrm{e}$) ***A1 N2***

[3 marks]

e. evidence of integrating and subtracting functions (in any order, seen anywhere) ***(M1)***

eg $\int_q^e \left(\frac{1}{x} - \frac{\ln x}{x}\right) \mathrm{d}x,\ \int f' - g$

correct integration $\ln x - \frac{(\ln x)^2}{2}$ ***A2***

substituting limits into **their** integrated function and subtracting (in any order) ***(M1)***

eg $(\ln \mathrm{e} - \ln 1) - \left(\frac{(\ln \mathrm{e})^2}{2} - \frac{(\ln 1)^2}{2}\right)$

Note: Do not award ***M1*** if the integrated function has only one term.

correct working ***A1***

eg $(1 - 0) - \left(\frac{1}{2} - 0\right),\ 1 - \frac{1}{2}$

area $= \frac{1}{2}$ ***AG N0***

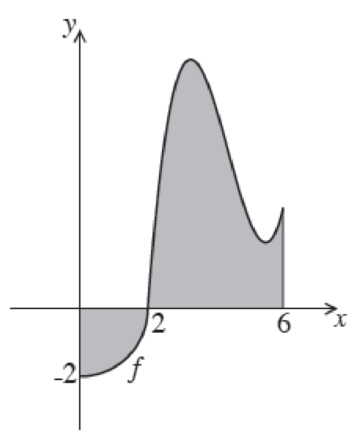
Notes: Candidates may work with two separate integrals, and only combine them at the end. Award marks in line with the markscheme.

[5 marks]

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]
d. [N/A]
e. [N/A]

The following is the graph of a function f, for $0 \leq x \leq 6$.



The first part of the graph is a quarter circle of radius 2 with centre at the origin.

. (a) Find $\int_0^2 f(x)\mathrm{d}x$. [7]

(b) The shaded region is enclosed by the graph of f , the x-axis, the y-axis and the line $x=6$. The area of this region is 3π .

Find $\int_2^6 f(x)\mathrm{d}x$.

a. Find $\int_0^2 f(x)\mathrm{d}x$. [4]

b. The shaded region is enclosed by the graph of f , the x-axis, the y-axis and the line $x=6$. The area of this region is 3π . [3]

Find $\int_2^6 f(x)\mathrm{d}x$.

Markscheme

. (a) attempt to find quarter circle area ***(M1)***

eg $\frac{1}{4}(4\pi)$, $\frac{\pi r^2}{4}$, $\int_0^2 \sqrt{4-x^2 \mathrm{d}x}$

area of region $=\pi$ ***(A1)***

$\int_0^2 f(x)\mathrm{d}x=-\pi$ ***A2 N3***

[4 marks]

(b) attempted set up with both regions ***(M1)***

eg shaded area $-$ quarter circle , $3\pi-\pi$, $3\pi-\int_0^2 f=\int_2^6 f$

$\int_2^6 f(x)\mathrm{d}x=2\pi$ ***A2 N2***

[3 marks]

Total [7 marks]

a. attempt to find quarter circle area ***(M1)***

eg $\frac{1}{4}(4\pi)$, $\frac{\pi r^2}{4}$, $\int_0^2 \sqrt{4-x^2 \mathrm{d}x}$

area of region $=\pi$ ***(A1)***

$\int_0^2 f(x)\mathrm{d}x=-\pi$ ***A2 N3***

[4 marks]

b. attempted set up with both regions ***(M1)***

eg shaded area $-$ quarter circle , $3\pi-\pi$, $3\pi-\int_0^2 f=\int_2^6 f$

$\int_2^6 f(x)\mathrm{d}x=2\pi$ ***A2 N2***

[3 marks]

Total [7 marks]

Examiners report

. There was a minor error on the diagram, where the point on the y-axis was labelled 2 (to indicate the length of the radius), rather than -2. Examiners were instructed to notify the IB assessment centre of any candidates adversely affected. Candidate scripts did not indicate any adverse effect.

While most candidates were able to correctly find the area of the quarter circle in part (a), very few considered that the value of the definite integral is negative for the part of the function below the x-axis. In part (b), most went on to earn full marks by subtracting the area of the quarter circle from 3π.

Candidates who did not understand the connection between area and the value of the integral often tried to find a function to integrate. These candidates were not successful using this method.

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Candidates who did not understand the connection between area and the value of the integral often tried to find a function to integrate. These candidates were not successful using this method.

Consider $f(x) = \ln(x^4 + 1)$.

The second derivative is given by $f''(x) = \frac{4x^2(3-x^4)}{(x^4+1)^2}$.

The equation $f''(x) = 0$ has only three solutions, when $x = 0$, $\pm\sqrt[4]{3}$ $(\pm 1.316\ldots)$.

a. Find the value of $f(0)$. [2]

b. Find the set of values of x for which f is increasing. [5]

c. (i) Find $f''(1)$. [5]

(ii) **Hence**, show that there is no point of inflexion on the graph of f at $x = 0$.

d. There is a point of inflexion on the graph of f at $x = \sqrt[4]{3}$ $(x = 1.316\ldots)$. [3]

Sketch the graph of f , for $x \geq 0$.

Markscheme

a. substitute 0 into f ***(M1)***

eg $\ln(0+1)$, $\ln 1$

$f(0) = 0$ ***A1 N2***

[2 marks]

b. $f'(x) = \frac{1}{x^4+1} \times 4x^3$ (seen anywhere) ***A1A1***

Note: Award ***A1*** for $\frac{1}{x^4+1}$ and ***A1*** for $4x^3$.

recognizing f increasing where $f'(x) > 0$ (seen anywhere) ***R1***

eg $f'(x) > 0$, diagram of signs

attempt to solve $f'(x) > 0$ ***(M1)***

eg $4x^3 = 0$, $x^3 > 0$

f increasing for $x > 0$ (accept $x \geq 0$) ***A1*** ***N1***

[5 marks]

c. (i) substituting $x = 1$ into f'' ***(A1)***

eg $\frac{4(3-1)}{(1+1)^2}$, $\frac{4\times2}{4}$

$f''(1) = 2$ ***A1*** ***N2***

(ii) valid interpretation of point of inflexion (seen anywhere) ***R1***

eg no change of sign in $f''(x)$, no change in concavity,

f' increasing both sides of zero

attempt to find $f''(x)$ for $x < 0$ ***(M1)***

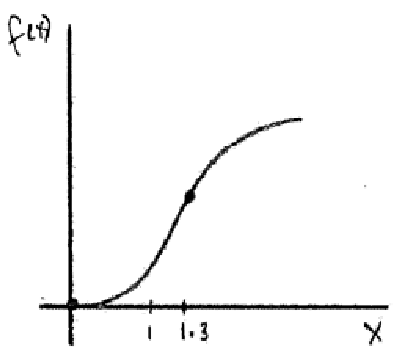
eg $f''(-1)$, $\frac{4(-1)^2(3-(-1)^4)}{((-1)^4+1)^2}$, diagram of signs

correct working leading to positive value ***A1***

eg $f''(-1) = 2$, discussing signs of numerator **and** denominator

there is no point of inflexion at $x = 0$ ***AG*** ***N0***

[5 marks]

d. 

A1A1A1 ***N3***

Notes: Award ***A1*** for shape concave up left of POI and concave down right of POI.

Only if this ***A1*** is awarded, then award the following:

A1 for curve through (0, 0) , ***A1*** for increasing throughout.

Sketch need not be drawn to scale. Only essential features need to be clear.

[3 marks]

Examiners report

a. Many candidates left their answer to part (a) as $\ln 1$. While this shows an understanding for substituting a value into a function, it leaves an unfinished answer that should be expressed as an integer.

b. Candidates who attempted to consider where f is increasing generally understood the derivative is needed. However, a number of candidates did not apply the chain rule, which commonly led to answers such as "increasing for all x". Many set their derivative equal to zero, while neglecting to indicate in their working that $f'(x) > 0$ for an increasing function. Some created a diagram of signs, which provides appropriate evidence as long as it is clear that the signs represent f'.

c. Finding $f''(1)$ proved no challenge, however, using this value to **show that** no point of inflexion exists proved elusive for many. Some candidates recognized the signs must not change in the second derivative. Few candidates presented evidence in the form of a calculation, which follows from the "hence" command of the question. In this case, a sign diagram without numerical evidence was not sufficient.

d. Few candidates created a correct graph from the information given or found in the question. This included the point (0, 0), the fact that the function is always increasing for $x > 0$, the concavity at $x = 1$ and the change in concavity at the given point of inflexion. Many incorrect attempts showed a graph concave down to the right of $x = 0$, changing to concave up.

The acceleration, a ms^{-2}, of a particle at time t seconds is given by $a = 2t + \cos t$.

a. Find the acceleration of the particle at $t = 0$. [2]

b. Find the velocity, v, at time t, given that the initial velocity of the particle is ms^{-1} . [5]

c. Find $\int_0^3 v\mathrm{d}t$, giving your answer in the form $p - q\cos 3$. [7]

d. What information does the answer to part (c) give about the motion of the particle? [2]

Markscheme

a. substituting $t = 0$ *(M1)*

e.g. $a(0) = 0 + \cos 0$

$a(0) = 1$ ***A1*** ***N2***

[2 marks]

b. evidence of integrating the acceleration function *(M1)*

e.g. $\int (2t + \cos t)\mathrm{d}t$

correct expression $t^2 + \sin t + c$ ***A1A1***

Note: If "$+c$" is omitted, award no further marks.

evidence of substituting (2,0) into indefinite integral *(M1)*

e.g. $2 = 0 + \sin 0 + c$, $c = 2$

$v(t) = t^2 + \sin t + 2$ ***A1*** ***N3***

[5 marks]

c. $\int (t^2 + \sin t + 2)\mathrm{d}t = \frac{t^3}{3} - \cos t + 2t$ ***A1A1A1***

Note: Award ***A1*** for each correct term.

evidence of using $v(3) - v(0)$ ***(M1)***

correct substitution ***A1***

e.g. $(9 - \cos 3 + 6) - (0 - \cos 0 + 0)$, $(15 - \cos 3) - (-1)$

$16 - \cos 3$ (accept $p = 16$, $q = -1$) ***A1A1*** ***N3***

[7 marks]

d. reference to motion, reference to first 3 seconds ***R1R1*** ***N2***

e.g. displacement in 3 seconds, distance travelled in 3 seconds

[2 marks]

Examiners report

a. Parts (a) and (b) of this question were generally well done.

b. Parts (a) and (b) of this question were generally well done.

c. Problems arose in part (c) with many candidates not substituting $s(3) - s(0)$ correctly, leading to only a partially correct final answer. There were also a notable few who were not aware that $\cos 0 = 1$ in both parts (a) and (c).

d. There were a variety of interesting answers about the motion of the particle, few being able to give both parts of the answer correctly.

Let $f : x \mapsto \sin^3 x$.

a. (i) Write down the range of the function f . [5]

(ii) Consider $f(x) = 1$, $0 \le x \le 2\pi$. Write down the number of solutions to this equation. Justify your answer.

b. Find $f'(x)$, giving your answer in the form $a\sin^p x\cos^q x$ where $a, p, q \in \mathbb{Z}$. [2]

c. Let $g(x) = \sqrt{3}\sin x(\cos x)^{\frac{1}{2}}$ for $0 \le x \le \frac{\pi}{2}$. Find the volume generated when the curve of g is revolved through 2π about the x-axis. [7]

Markscheme

a. (i) range of f is $[-1, 1]$, $(-1 \le f(x) \le 1)$ ***A2*** ***N2***

(ii) $\sin^3 x \Rightarrow 1 \Rightarrow \sin x = 1$ ***A1***

justification for one solution on $[0, 2\pi]$ ***R1***

e.g. $x = \frac{\pi}{2}$, unit circle, sketch of $\sin x$

1 solution (seen anywhere) ***A1*** ***N1***

[5 marks]

b. $f'(x) = 3\sin^2 x \cos x$ ***A2*** ***N2***

[2 marks]

c. using $V = \int_a^b \pi y^2 \mathrm{d}x$ ***(M1)***

$V = \int_0^{\frac{\pi}{2}} \pi(\sqrt{3}\sin x \cos^{\frac{1}{2}} x)^2 \mathrm{d}x$ ***(A1)***

$= \pi \int_0^{\frac{\pi}{2}} 3\sin^2 x \cos x \mathrm{d}x$ ***A1***

$V = \pi\left[\sin^3 x\right]_0^{\frac{\pi}{2}} \left(= \pi\left(\sin^3\left(\frac{\pi}{2}\right) - \sin^3 0\right)\right)$ ***A2***

evidence of using $\sin\frac{\pi}{2} = 1$ and $\sin 0 = 0$ ***(A1)***

e.g. $\pi(1-0)$

$V = \pi$ ***A1*** ***N1***

[7 marks]

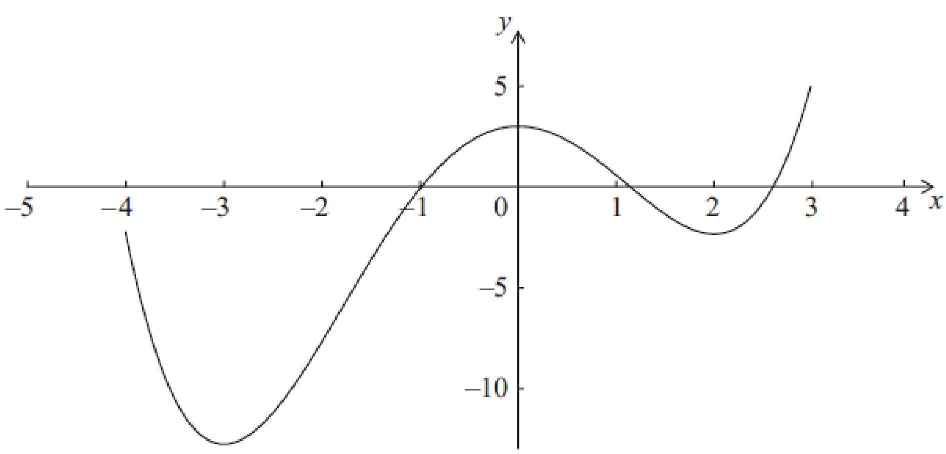
Examiners report

a. This question was not done well by most candidates. No more than one-third of them could correctly give the range of $f(x) = \sin^3 x$ and few could provide adequate justification for there being exactly one solution to $f(x) = 1$ in the interval $[0, 2\pi]$.

b. This question was not done well by most candidates.

c. This question was not done well by most candidates. No more than one-third of them could correctly give the range of $f(x) = \sin^3 x$ and few could provide adequate justification for there being exactly one solution to $f(x) = 1$ in the interval $[0, 2\pi]$. Finding the derivative of this function also presented major problems, thus making part (c) of the question much more difficult. In spite of the formula for volume of revolution being given in the Information Booklet, fewer than half of the candidates could correctly put the necessary function and limits into $\pi\int_a^b y^2 \mathrm{d}x$ and fewer still could square $\sqrt{3}\sin x \cos^{\frac{1}{2}} x$ correctly. From those who did square correctly, the correct antiderivative was not often recognized. All manner of antiderivatives were suggested instead.

A function *f* is defined for $-4 \le x \le 3$. The graph of *f* is given below.



The graph has a local maximum when $x = 0$, and local minima when $x = -3$, $x = 2$.

a. Write down the x-intercepts of the graph of the **derivative** function, f' . [2]

b. Write down all values of x for which $f'(x)$ is positive. [2]

c. At point D on the graph of f, the x-coordinate is -0.5. Explain why $f''(x) < 0$ at D. [2]

Markscheme

a. x-intercepts at -3, 0, 2 ***A2 N2***

[2 marks]

b. $-3 < x < 0$, $2 < x < 3$ ***A1A1 N2***

[2 marks]

c. correct reasoning ***R2***

e.g. the graph of f is **concave-down** (accept convex), the first derivative is decreasing

therefore the second derivative is negative ***AG***

[2 marks]

Examiners report

a. Candidates had mixed success with parts (a) and (b). Weaker candidates either incorrectly used the x-intercepts of f or left this question blank. Some wrote down only two of the three values in part (a). Candidates who answered part (a) correctly often had trouble writing the set of values in part (b); difficulties included poor notation and incorrectly including the endpoints. Other candidates listed individual x-values here rather than a range of values.

b. Candidates had mixed success with parts (a) and (b). Weaker candidates either incorrectly used the x-intercepts of f or left this question blank. Some wrote down only two of the three values in part (a). Candidates who answered part (a) correctly often had trouble writing the set of values in part (b); difficulties included poor notation and incorrectly including the endpoints. Other candidates listed individual x-values here rather than a range of values.

c. Many candidates had difficulty explaining why the second derivative is negative in part (c). A number claimed that since the point D was "close" to a maximum value, the second derivative must be negative; this incorrect appeal to the second derivative test indicates a lack of understanding of how the test works and the relative concept of closeness. Some candidates claimed D was a point of inflexion, again demonstrating poor understanding of the second derivative. Among candidates who answered part (c) correctly, some stated that f was concave down while others gave well-formed arguments for why the first derivative was decreasing. A few candidates provided nicely sketched graphs of f' and f'' and used them in their explanations.

The graph of a function h passes through the point $\left(\frac{\pi}{12}, 5\right)$.

Given that $h'(x) = 4\cos 2x$, find $h(x)$.

Markscheme

evidence of anti-differentiation ***(M1)***

eg $\int h'(x), \int 4\cos 2x\mathrm{d}x$

correct integration ***(A2)***

eg $h(x) = 2\sin 2x + c, \frac{4\sin 2x}{2}$

attempt to substitute $\left(\frac{\pi}{12}, 5\right)$ into their equation ***(M1)***

eg $2\sin\left(2 \times \frac{\pi}{12}\right) + c = 5,\ 2\sin\left(\frac{\pi}{6}\right) = 5$

correct working ***(A1)***

eg $2\left(\frac{1}{2}\right) + c = 5,\ c = 4$

$h(x) = 2\sin 2x + 4$ ***A1*** ***N5***

[6 marks]

Examiners report

[N/A]

Let $f(x) = \mathrm{e}^{-3x}$ and $g(x) = \sin\left(x - \frac{\pi}{3}\right)$.

a. Write down [2]

(i) $f'(x)$;

(ii) $g'(x)$.

b. Let $h(x) = \mathrm{e}^{-3x}\sin\left(x - \frac{\pi}{3}\right)$. Find the exact value of $h'\left(\frac{\pi}{3}\right)$. [4]

Markscheme

a. (i) $-3\mathrm{e}^{-3x}$ ***A1*** ***N1***

(ii) $\cos\left(x - \frac{\pi}{3}\right)$ ***A1*** ***N1***

[4 marks]

b. evidence of choosing product rule ***(M1)***

e.g. $uv' + vu'$

correct expression ***A1***

e.g. $-3\mathrm{e}^{-3x}\sin\left(x - \frac{\pi}{3}\right) + \mathrm{e}^{-3x}\cos\left(x - \frac{\pi}{3}\right)$

complete correct substitution of $x = \frac{\pi}{3}$ ***(A1)***

e.g. $-3\mathrm{e}^{-3\frac{\pi}{3}}\sin\left(\frac{\pi}{3} - \frac{\pi}{3}\right) + \mathrm{e}^{-3\frac{\pi}{3}}\cos\left(\frac{\pi}{3} - \frac{\pi}{3}\right)$□□□□□□□□□

$h'\left(\frac{\pi}{3}\right) = \mathrm{e}^{-\pi}$ ***A1*** ***N3***

[4 marks]

Examiners report

a. A good number of candidates found the correct derivative expressions in (a). Many applied the product rule, although with mixed success.

b. Often the substitution of $\frac{\pi}{3}$ was incomplete or not done at all.

The velocity v ms^{-1} of a particle at time t seconds, is given by $v = 2t + \cos 2t$, for $0 \le t \le 2$.

a. Write down the velocity of the particle when $t = 0$. [1]

b(i) and (ii). When $t = k$, the acceleration is zero. [8]

(i) Show that $k = \frac{\pi}{4}$.

(ii) Find the exact velocity when $t = \frac{\pi}{4}$.

c. When $t < \frac{\pi}{4}$, $\frac{\mathrm{d}v}{\mathrm{d}t} > 0$ and when $t > \frac{\pi}{4}$, $\frac{\mathrm{d}v}{\mathrm{d}t} > 0$. [4]

Sketch a graph of v against t .

d(i) and (ii). Let d be the distance travelled by the particle for $0 \le t \le 1$. [3]

(i) Write down an expression for d .

(ii) Represent d on your sketch.

Markscheme

a. $v = 1$ ***A1*** ***N1***

[1 mark]

b(i) and (ii). (i) $\frac{\mathrm{d}}{\mathrm{d}t}(2t) = 2$ ***A1***

$\frac{\mathrm{d}}{\mathrm{d}t}(\cos 2t) = -2\sin 2t$ ***A1A1***

Note: Award ***A1*** for coefficient 2 and ***A1*** for $-\sin 2t$.

evidence of considering acceleration = 0 ***(M1)***

e.g. $\frac{\mathrm{d}v}{\mathrm{d}t} = 0$, $2 - 2\sin 2t = 0$

correct manipulation ***A1***

e.g. $\sin 2k = 1$, $\sin 2t = 1$

$2k = \frac{\pi}{2}$ (accept $2t = \frac{\pi}{2}$) ***A1***

$k = \frac{\pi}{4}$ ***AG*** ***N0***

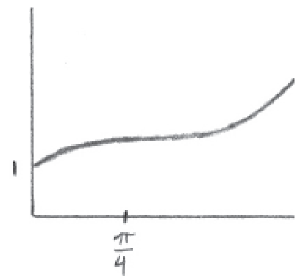
(ii) attempt to substitute $t = \frac{\pi}{4}$ into v ***(M1)***

e.g. $2\left(\frac{\pi}{4}\right) + \cos\left(\frac{2\pi}{4}\right)$

$v = \frac{\pi}{2}$ ***A1*** ***N2***

[8 marks]

c.



A1A1A2 ***N4***

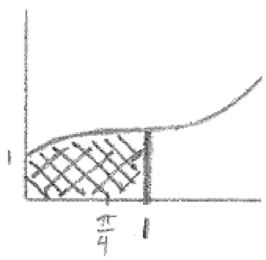
Notes: Award ***A1*** for y-intercept at $(0, 1)$, ***A1*** for curve having zero gradient at $t = \frac{\pi}{4}$, ***A2*** for shape that is concave down to the left of $\frac{\pi}{4}$ and concave up to the right of $\frac{\pi}{4}$. If a correct curve is drawn without indicating $t = \frac{\pi}{4}$, do not award the second ***A1*** for the zero gradient, but award the final ***A2*** if appropriate. Sketch need not be drawn to scale. Only essential features need to be clear.

[4 marks]

d(i) and (ii). (i) correct expression ***A2***

e.g. $\int_0^1 (2t + \cos 2t)\mathrm{d}t$, $\left[t^2 + \frac{\sin 2t}{2}\right]_0^1$, $1 + \frac{\sin 2}{2}$, $\int_0^1 v\mathrm{d}t$

(ii)



A1

Note: The line at $t = 1$ needs to be clearly after $t = \frac{\pi}{4}$.

[3 marks]

Examiners report

a. Many candidates gave a correct initial velocity, although a substantial number of candidates answered that $0 + \cos 0 = 0$.

b(i) and (ii). For (b), students commonly applied the chain rule correctly to achieve the derivative, and many recognized that the acceleration must be zero. Occasionally a student would use a double-angle identity on the velocity function before differentiating. This is not incorrect, but it usually caused problems when trying to show $k = \frac{\pi}{4}$. At times students would reach the equation $\sin 2k = 1$ and then substitute the $\frac{\pi}{4}$, which does not satisfy the “show that” instruction.

c. The challenge in this question is sketching the graph using the information achieved and provided. This requires students to make graphical interpretations, and as typical in section B, to link the early parts of the question with later parts. Part (a) provides the y-intercept, and part (b) gives a point with a horizontal tangent. Plotting these points first was a helpful strategy. Few understood either the notation or the concept that the function had to be increasing on either side of the $\frac{\pi}{4}$, with most thinking that the point was either a max or min. It was the astute student who recognized that the derivatives being positive on either side of $\frac{\pi}{4}$ creates a point of inflexion.

Additionally, important points should be labelled in a sketch. Indicating the $\frac{\pi}{4}$ on the x-axis is a requirement of a clear graph. Although students were not penalized for not labelling the $\frac{\pi}{2}$ on the y-axis, there should be a recognition that the point is higher than the y-intercept.

d(i) and (ii). While some candidates recognized that the distance is the area under the velocity graph, surprisingly few included neither the limits of integration in their expression, nor the "d*t*". Most unnecessarily attempted to integrate the function, often giving an answer with "+C", and only earned marks if the limits were included with their result. Few recognized that a shaded area is an adequate representation of distance on the sketch, with most fruitlessly attempting to graph a new curve.

In this question, you are given that $\cos \frac{\pi}{3} = \frac{1}{2}$, and $\sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}$.

The displacement of an object from a fixed point, O is given by $s(t) = t - \sin 2t$ for $0 \le t \le \pi$.

a. Find $s'(t)$. [3]

b. In this interval, there are only two values of *t* for which the object is not moving. One value is $t = \frac{\pi}{6}$. [4]

Find the other value.

c. Show that $s'(t) > 0$ between these two values of *t* . [3]

d. Find the distance travelled between these two values of *t* . [5]

Markscheme

a. $s'(t) = 1 - 2\cos 2t$ ***A1A2*** ***N3***

Note: Award ***A1*** for 1, ***A2*** for $-2\cos 2t$.

[3 marks]

b. evidence of valid approach ***(M1)***

e.g. setting $s'(t) = 0$

correct working ***A1***

e.g. $2\cos 2t = 1$, $\cos 2t = \frac{1}{2}$

$2t = \frac{\pi}{3}, \frac{5\pi}{3}, \ldots$ ***(A1)***

$t = \frac{5\pi}{6}$ ***A1*** ***N3***

[4 marks]

c. evidence of valid approach ***(M1)***

e.g. choosing a value in the interval $\frac{\pi}{6} < t < \frac{5\pi}{6}$

correct substitution ***A1***

e.g. $s'\left(\frac{\pi}{2}\right) = 1 - 2\cos \pi$

$s'\left(\frac{\pi}{2}\right) = 3$ ***A1***

$s'(t) > 0$ ***AG*** ***N0***

[3 marks]

d. evidence of approach using *s* or integral of s' ***(M1)*** e.g. $\int s'(t)\mathrm{d}t$; $s\left(\frac{5\pi}{6}\right)$, $s\left(\frac{\pi}{6}\right)$; $[t-\sin 2t]_{\frac{\pi}{6}}^{\frac{5\pi}{6}}$

substituting values and subtracting ***(M1)***

e.g. $s\left(\frac{5\pi}{6}\right)-s\left(\frac{\pi}{6}\right)$, $\left(\frac{\pi}{6}-\frac{\sqrt{3}}{2}\right)-\left(\frac{5\pi}{6}-\left(-\frac{\sqrt{3}}{2}\right)\right)$

correct substitution ***A1***

e.g. $\frac{5\pi}{6}-\sin\frac{5\pi}{3}-\left[\frac{\pi}{6}-\sin\frac{\pi}{3}\right]$, $\left(\frac{5\pi}{6}-\left(-\frac{\sqrt{3}}{2}\right)\right)-\left(\frac{\pi}{6}-\frac{\sqrt{3}}{2}\right)$

distance is $\frac{2\pi}{3}+\sqrt{3}$ ***A1A1*** ***N3***

Note: Award ***A1*** for $\frac{2\pi}{3}$, ***A1*** for $\sqrt{3}$.

[5 marks]

Examiners report

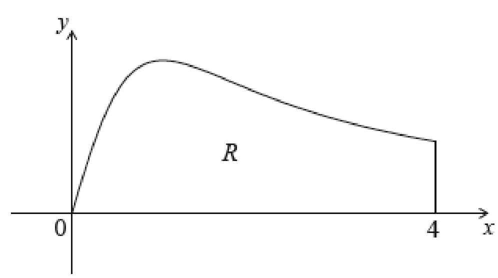
a. The derivative in part (a) was reasonably well done, but errors here often caused trouble in later parts. Candidates occasionally attempted to use the double angle identity for $\sin 2t$ before differentiating, but they rarely were successful in then applying the product rule.

b. In part (b), most candidates understood that they needed to set their derivative equal to zero, but fewer were able to take the next step to solve the resulting double angle equation. Again, some candidates over-complicated the equation by using the double angle identity. Few ended up with the correct answer $\frac{5\pi}{6}$.

c. In part (c), many candidates knew they needed to test a value between $\pi/6$ and their value from part (b), but fewer were able to successfully complete that calculation. Some candidates simply tested their boundary values while others unsuccessfully attempted to make use of the second derivative.

d. Although many candidates did not attempt part (d), those who did often demonstrated a good understanding of how to use the displacement function s or the integral of their derivative from part (a). Candidates who had made an error in part (b) often could not finish, as $\sin(2t)$ could not be evaluated at their value without a calculator. Of those who had successfully found the other boundary of $5\pi/6$, a common error was giving the incorrect sign of the value of $\sin(5\pi/3)$. Again, this part was a good discriminator between the grade 6 and 7 candidates.

The following diagram shows the graph of $f(x)=\frac{x}{x^2+1}$, for $0\le x\le 4$, and the line $x=4$.



Let R be the region enclosed by the graph of f , the x-axis and the line $x=4$.

Find the area of R.

Markscheme

substitution of limits or function ***(A1)***

eg $A = \int_0^4 f(x),\ \int \frac{x}{x^2+1}\mathrm{d}x$

correct integration by substitution/inspection ***A2***

$\frac{1}{2}\ln(x^2+1)$

substituting limits into **their** integrated function and subtracting (in any order) ***(M1)***

eg $\frac{1}{2}\left(\ln(4^2+1) - \ln(0^2+1)\right)$

correct working ***A1***

eg $\frac{1}{2}\left(\ln(4^2+1) - \ln(0^2+1)\right),\ \frac{1}{2}(\ln(17) - \ln(1)),\ \frac{1}{2}\ln 17 - 0$

$A = \frac{1}{2}\ln(17)$ ***A1*** ***N3***

Note: Exception to ***FT*** rule. Allow full ***FT*** on incorrect integration involving a $\ln$ function.

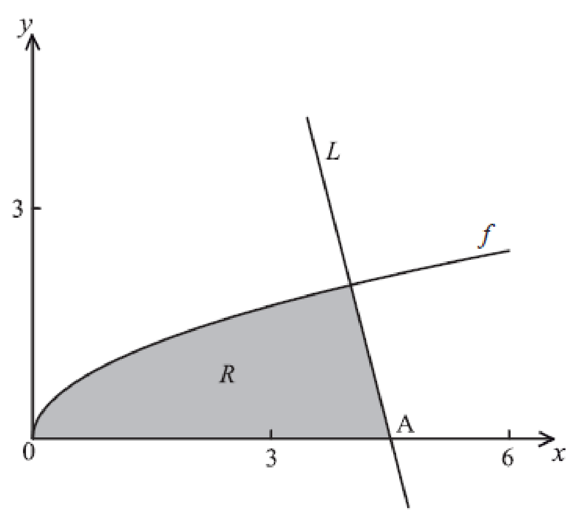
[6 marks]

Examiners report

Very few candidates earned full marks in this question. While most candidates knew to integrate, many seemed unfamiliar with integrating using substitution or inspection. This topic is part of the syllabus, but it did not occur to many candidates to use a substitution method. A large number of them tried to integrate the individual terms in the numerator and denominator as though this were a polynomial function. While there were some candidates who knew the integral would involve a natural log function and substituted 4 and 0 into their function, many ended up with undefined values such as or did not know what to do with expressions containing $\ln 1$.

Let $f(x) = \sqrt{x}$. Line L is the normal to the graph of f at the point (4, 2) .

In the diagram below, the shaded region R is bounded by the x-axis, the graph of f and the line L .



a. Show that the equation of L is $y = -4x + 18$. [4]

b. Point A is the x-intercept of L . Find the x-coordinate of A. [2]

c. Find an expression for the area of R . [3]

d. The region R is rotated 360° about the x-axis. Find the volume of the solid formed, giving your answer in terms of π . [8]

Markscheme

a. finding derivative ***(A1)***

e.g. $f'(x) = \frac{1}{2}x^{\frac{1}{2}}, \frac{1}{2\sqrt{x}}$

correct value of derivative or its negative reciprocal (seen anywhere) ***A1***

e.g. $\frac{1}{2\sqrt{4}}$, $\frac{1}{4}$

gradient of normal = $\frac{1}{\text{gradient of tangent}}$ (seen anywhere) ***A1***

e.g. $-\frac{1}{f'(4)} = -4$, $-2\sqrt{x}$

substituting into equation of line (for normal) ***M1***

e.g. $y - 2 = -4(x - 4)$

$y = -4x + 18$ ***AG*** ***N0***

[4 marks]

b. recognition that $y = 0$ at A ***(M1)***

e.g. $-4x + 18 = 0$

$x = \frac{18}{4} \left(= \frac{9}{2}\right)$ ***A1*** ***N2***

[2 marks]

c. splitting into two appropriate parts (areas and/or integrals) ***(M1)***

correct expression for area of R ***A2*** ***N3***

e.g. area of R = $\int_0^4 \sqrt{x}\mathrm{d}x + \int_4^{4.5} (-4x + 18)\mathrm{d}x$, $\int_0^4 \sqrt{x}\mathrm{d}x + \frac{1}{2} \times 0.5 \times 2$ (triangle)

Note: Award ***A1*** if dx is missing.

[3 marks]

d. correct expression for the volume from $x = 0$ to $x = 4$ ***(A1)***

e.g. $V = \int_0^4 \pi \left[f(x)^2\right]\mathrm{d}x$, $\int_0^4 \pi\sqrt{x}^2\mathrm{d}x$, $\int_0^4 \pi x\mathrm{d}x$

$V = \left[\frac{1}{2}\pi x^2\right]_0^4$ ***A1***

$V = \pi\left(\frac{1}{2} \times 16 - \frac{1}{2} \times 0\right)$ ***(A1)***

$V = 8\pi$ ***A1***

finding the volume from $x = 4$ to $x = 4.5$

EITHER

recognizing a cone ***(M1)***

e.g. $V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi(2)^2 \times \frac{1}{2}$ ***(A1)***

$= \frac{2\pi}{3}$ ***A1***

total volume is $8\pi+\frac{2}{3}\pi\left(=\frac{26}{3}\pi\right)$ ***A1*** ***N4***

OR

$V=\pi\int_4^{4.5}(-4x+18)^2\mathrm{d}x$ ***(M1)***

$=\int_4^{4.5}\pi(16x^2-144x+324)\mathrm{d}x$

$=\pi\left[\frac{16}{3}x^3-72x^2+324x\right]_4^{4.5}$ ***A1***

$=\frac{2\pi}{3}$ ***A1***

total volume is $8\pi+\frac{2}{3}\pi\left(=\frac{26}{3}\pi\right)$ ***A1*** ***N4***

[8 marks]

Examiners report

a. Parts (a) and (b) were well done by most candidates.

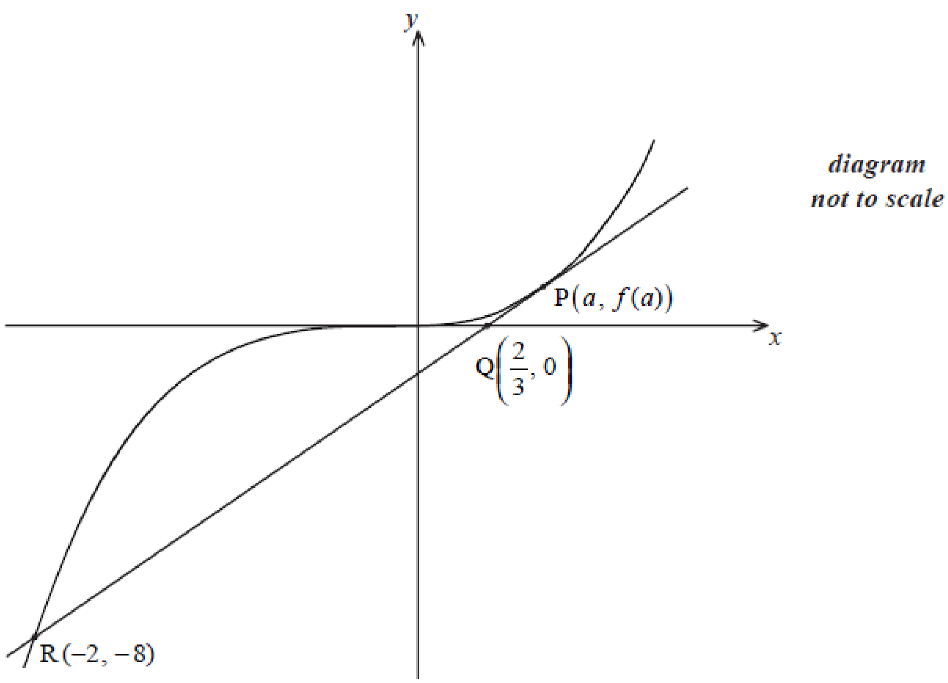
b. Parts (a) and (b) were well done by most candidates.

c. While quite a few candidates understood that both functions must be used to find the area in part (c), very few were actually able to write a correct expression for this area and this was due to candidates not knowing that they needed to integrate from 0 to 4 and then from 4 to 4.5.

d. On part (d), some candidates were able to earn follow through marks by setting up a volume expression, but most of these expressions were incorrect. If they did not get the expression for the area correct, there was little chance for them to get part (d) correct.

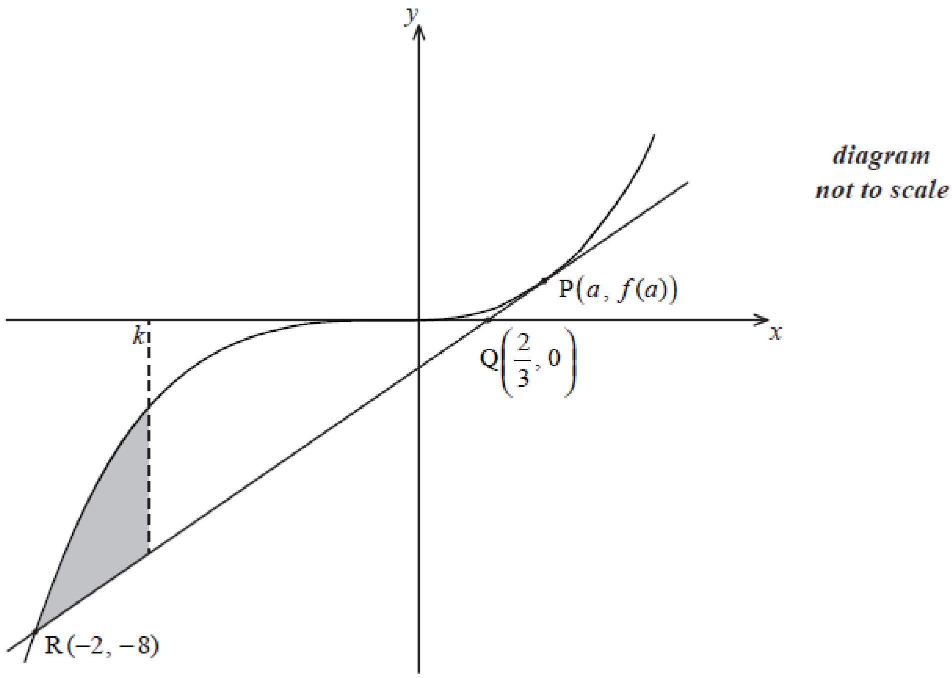
For those candidates who used their expression in part (c) for (d), there was a surprising amount of them who incorrectly applied distributive law of the exponent with respect to the addition or subtraction.

Let $f(x)=x^3$. The following diagram shows part of the graph of f.



The point $P(a, f(a))$, where $a > 0$, lies on the graph of f. The tangent at P crosses the x-axis at the point $Q\left(\frac{2}{3}, 0\right)$. This tangent intersects the graph of f at the point $R(-2, -8)$.

The equation of the tangent at P is $y = 3x - 2$. Let T be the region enclosed by the graph of f, the tangent [PR] and the line $x = k$, between $x = -2$ and $x = k$ where $-2 < k < 1$. This is shown in the diagram below.



a(i), (ii) and (iii). (i) Show that the gradient of [PQ] is $\frac{a^3}{a-\frac{2}{3}}$. [7]

(ii) Find $f'(a)$.

(iii) Hence show that $a = 1$.

b. Given that the area of T is $2k + 4$, show that k satisfies the equation $k^4 - 6k^2 + 8 = 0$. [9]

Markscheme

a(i), (ii) and (iii). (i) substitute into gradient $= \frac{y_1 - y_2}{x_1 - x_2}$ ***(M1)***

e.g. $\frac{f(a)-0}{a-\frac{2}{3}}$

substituting $f(a) = a^3$

e.g. $\frac{a^3-0}{a-\frac{2}{3}}$ ***A1***

gradient $\frac{a^3}{a-\frac{2}{3}}$ ***AG N0***

(ii) correct answer ***A1 N1***

e.g. $3a^2$, $f'(a) = 3$, $f'(a) = \frac{a^3}{a-\frac{2}{3}}$

(iii) **METHOD 1**

evidence of approach ***(M1)***

e.g. $f'(a) = $ gradient, $3a^2 = \frac{a^3}{a-\frac{2}{3}}$

simplify ***A1***

e.g. $3a^2\left(a-\frac{2}{3}\right)=a^3$

rearrange ***A1***

e.g. $3a^3-2a^2=a^3$

evidence of solving ***A1***

e.g. $2a^3-2a^2=2a^2(a-1)=0$

$a=1$ ***AG N0***

METHOD 2

gradient RQ $=\frac{-8}{-2-\frac{2}{3}}$ ***A1***

simplify ***A1***

e.g. $\frac{-8}{-\frac{8}{3}}, 3$

evidence of approach ***(M1)***

e.g. $f'(a)=\text{gradient}$, $3a^2=\frac{-8}{-2-\frac{2}{3}}$, $\frac{a^3}{a-\frac{2}{3}}=3$

simplify ***A1***

e.g. $3a^2=3$, $a^2=1$

$a=1$ ***AG N0***

[7 marks]

b. approach to find area of *T* involving subtraction and integrals ***(M1)***

e.g. $\int f-(3x-2)\mathrm{d}x$, $\int_{-2}^{k}(3x-2)-\int_{-2}^{k}x^3$, $\int(x^3-3x+2)$

correct integration with correct signs ***A1A1A1***

e.g. $\frac{1}{4}x^4-\frac{3}{2}x^2+2x$, $\frac{3}{2}x^2-2x-\frac{1}{4}x^4$

correct limits -2 and *k* (seen anywhere) ***A1***

e.g. $\int_{-2}^{k}(x^3-3x+2)\mathrm{d}x$, $\left[\frac{1}{4}x^4-\frac{3}{2}x^2+2x\right]_{-2}^{k}$

attempt to substitute *k* and -2 ***(M1)***

correct substitution into **their** integral if 2 or more terms ***A1***

e.g. $\left(\frac{1}{4}k^4-\frac{3}{2}k^2+2k\right)-(4-6-4)$

setting **their** integral expression equal to $2k+4$ (seen anywhere) ***(M1)***

simplifying ***A1***

e.g. $\frac{1}{4}k^4-\frac{3}{2}k^2+2=0$

$k^4-6k^2+8=0$ ***AG N0***

[9 marks]

Examiners report

a(i) and (ii). Part (a) seemed to be well-understood by many candidates, and most were able to earn at least partial marks here. Part (ai) was a "show that" question, and unfortunately there were some candidates who did not show how they arrived at the given expression.

b. In part (b), the concept seemed to be well-understood. Most candidates saw the necessity of using definite integrals and subtracting the two functions, and the integration was generally done correctly. However, there were a number of algebraic and arithmetic errors which prevented candidates from correctly showing the desired final result.

A quadratic function f can be written in the form $f(x) = a(x-p)(x-3)$. The graph of f has axis of symmetry $x = 2.5$ and y-intercept at $(0,\ -6)$

a. Find the value of p. [3]

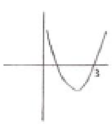
b. Find the value of a. [3]

c. The line $y = kx - 5$ is a tangent to the curve of f. Find the values of k. [8]

Markscheme

a. **METHOD 1 (using *x*-intercept)**

determining that 3 is an x-intercept ***(M1)***

eg $x - 3 = 0,$ 

valid approach ***(M1)***

eg $3 - 2.5,\ \frac{p+3}{2} = 2.5$

$p = 2$ ***A1 N2***

METHOD 2 (expanding *f* (*x*))

correct expansion (accept absence of a) ***(A1)***

eg $ax^2 - a(3+p)x + 3ap,\ x^2 - (3+p)x + 3p$

valid approach involving equation of axis of symmetry ***(M1)***

eg $\frac{-b}{2a} = 2.5,\ \frac{a(3+p)}{2a} = \frac{5}{2},\ \frac{3+p}{2} = \frac{5}{2}$

$p = 2$ ***A1 N2***

METHOD 3 (using derivative)

correct derivative (accept absence of a) ***(A1)***

eg $a(2x-3-p),\ 2x-3-p$

valid approach ***(M1)***

eg $f'(2.5) = 0$

$p = 2$ ***A1 N2***

[3 marks]

b. attempt to substitute $(0,\ -6)$ ***(M1)***

eg $-6 = a(0-2)(0-3),\ 0 = a(-8)(-9),\ a(0)^2 - 5a(0) + 6a = -6$

correct working ***(A1)***

eg $-6 = 6a$

$a = -1$ ***A1*** ***N2***

[3 marks]

c. **METHOD 1 (using discriminant)**

recognizing tangent intersects curve once ***(M1)***

recognizing one solution when discriminant = 0 ***M1***

attempt to set up equation ***(M1)***

eg $g = f,\ kx - 5 = -x^2 + 5x - 6$

rearranging their equation to equal zero ***(M1)***

eg $x^2 - 5x + kx + 1 = 0$

correct discriminant (if seen explicitly, not just in quadratic formula) ***A1***

eg $(k-5)^2 - 4,\ 25 - 10k + k^2 - 4$

correct working ***(A1)***

eg $k - 5 = \pm 2,\ (k-3)(k-7) = 0,\ \frac{10 \pm \sqrt{100 - 4 \times 21}}{2}$

$k = 3,\ 7$ ***A1A1*** ***N0***

METHOD 2 (using derivatives)

attempt to set up equation ***(M1)***

eg $g = f,\ kx - 5 = -x^2 + 5x - 6$

recognizing derivative/slope are equal ***(M1)***

eg $f' = m_T,\ f' = k$

correct derivative of f ***(A1)***

eg $-2x + 5$

attempt to set up equation in terms of either x or k ***M1***

eg $(-2x+5)x - 5 = -x^2 + 5x - 6,\ k\left(\frac{5-k}{2}\right) - 5 = -\left(\frac{5-k}{2}\right)^2 + 5\left(\frac{5-k}{2}\right) - 6$

rearranging their equation to equal zero ***(M1)***

eg $x^2 - 1 = 0,\ k^2 - 10k + 21 = 0$

correct working ***(A1)***

eg $x = \pm 1,\ (k-3)(k-7) = 0,\ \frac{10 \pm \sqrt{100 - 4 \times 21}}{2}$

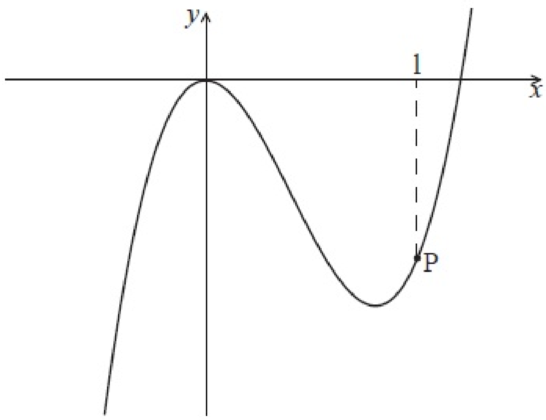
$k = 3,\ 7$ ***A1A1*** ***N0***

[8 marks]

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]

Part of the graph of $f(x) = ax^3 - 6x^2$ is shown below.



The point P lies on the graph of f . At P, $x = 1$.

a. Find $f'(x)$. [2]

b. The graph of f has a gradient of 3 at the point P. Find the value of a . [4]

Markscheme

a. $f'(x) = 3ax^2 - 12x$ ***A1A1*** ***N2***

Note: Award ***A1*** for each correct term.

[2 marks]

b. setting their derivative equal to 3 (seen anywhere) ***A1***

e.g. $f'(x) = 3$

attempt to substitute $x = 1$ into $f'(x)$ ***(M1)***

e.g. $3a(1)^2 - 12(1)$

correct substitution into $f'(x)$ ***(A1)***

e.g. $3a - 12$, $3a = 15$

$a = 5$ ***A1*** ***N2***

[4 marks]

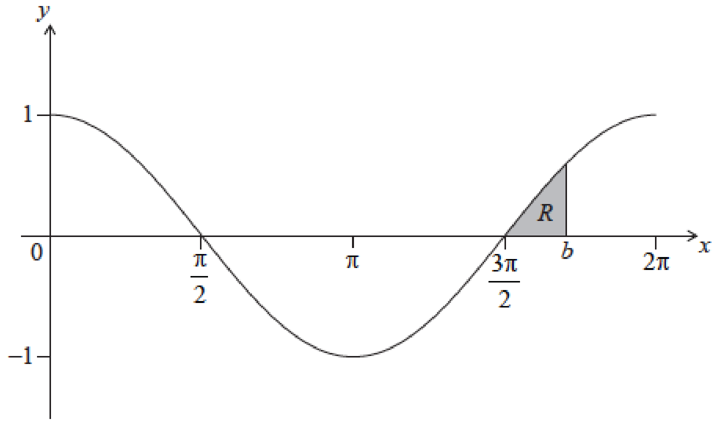
Examiners report

a. A majority of candidates answered part (a) correctly, and a good number earned full marks on both parts of this question. In part (b), some common errors included setting the derivative equal to zero, or substituting 3 for x in their derivative. There were also a few candidates who incorrectly tried to work with $f(x)$, rather than $f'(x)$, in part (b).

b. A majority of candidates answered part (a) correctly, and a good number earned full marks on both parts of this question. In part (b), some common errors included setting the derivative equal to zero, or substituting 3 for x in their derivative. There were also a few candidates who incorrectly tried to work with $f(x)$, rather than $f'(x)$, in part (b).

Let $f(x) = \cos x$, for $0 \leq x \leq 2\pi$. The following diagram shows the graph of f.

There are x-intercepts at $x = \frac{\pi}{2},\ \frac{3\pi}{2}$.



The shaded region R is enclosed by the graph of f, the line $x = b$, where $b > \frac{3\pi}{2}$, and the x-axis. The area of R is $\left(1 - \frac{\sqrt{3}}{2}\right)$. Find the value of b.

Markscheme

attempt to set up integral (accept missing or incorrect limits and missing $\mathrm{d}x$) ***M1***

eg $\int_{\frac{3\pi}{2}}^{b} \cos x \mathrm{d}x,\ \int_{a}^{b} \cos x \mathrm{d}x,\ \int_{\frac{3\pi}{2}}^{b} f \mathrm{d}x,\ \int \cos x$

correct integration (accept missing or incorrect limits) ***(A1)***

eg $[\sin x]_{\frac{3\pi}{2}}^{b},\ \sin x$

substituting correct limits into **their** integrated function and subtracting (in any order) ***(M1)***

eg $\sin b - \sin\left(\frac{3\pi}{2}\right),\ \sin\left(\frac{3\pi}{2}\right) - \sin b$

$\sin\left(\frac{3\pi}{2}\right) = -1$ (seen anywhere) ***(A1)***

setting **their** result from an integrated function equal to $\left(1 - \frac{\sqrt{3}}{2}\right)$ ***M1***

eg $\sin b = -\frac{\sqrt{3}}{2}$

evaluating $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3}$ or $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) = -\frac{\pi}{3}$ ***(A1)***

eg $b = \frac{\pi}{3},\ -60^{\circ}$

identifying correct value ***(A1)***

eg $2\pi - \frac{\pi}{3},\ 360 - 60$

$b = \frac{5\pi}{3}$ ***A1*** ***N3***

[8 marks]

Examiners report

Most candidates recognised that a definite integral was required and many were able to set up a correct equation. Incorrect integration leading to $-\sin x$ was quite common and poor notation was frequently seen. Some candidates appeared to guess their value from the graph, showing little supporting work.

Let $f(x) = px^3 + px^2 + qx$.

a. Find $f'(x)$. [2]

b. Given that $f'(x) \geqslant 0$, show that $p^2 \leqslant 3pq$. [5]

Markscheme

a. $f'(x) = 3px^2 + 2px + q$ ***A2*** ***N2***

Note: Award ***A1*** if only 1 error.

[2 marks]

b. evidence of discriminant (must be seen explicitly, not in quadratic formula) ***(M1)***

eg $b^2 - 4ac$

correct substitution into discriminant (may be seen in inequality) ***A1***

eg $(2p)^2 - 4 \times 3p \times q,\ 4p^2 - 12pq$

$f'(x) \geqslant 0$ then f' has two equal roots or no roots ***(R1)***

recognizing discriminant less or equal than zero ***R1***

eg $\Delta \leqslant 0,\ 4p^2 - 12pq \leqslant 0$

correct working that clearly leads to the required answer ***A1***

eg $p^2 - 3pq \leqslant 0,\ 4p^2 \leqslant 12pq$

$p^2 \leqslant 3pq$ ***AG*** ***N0***

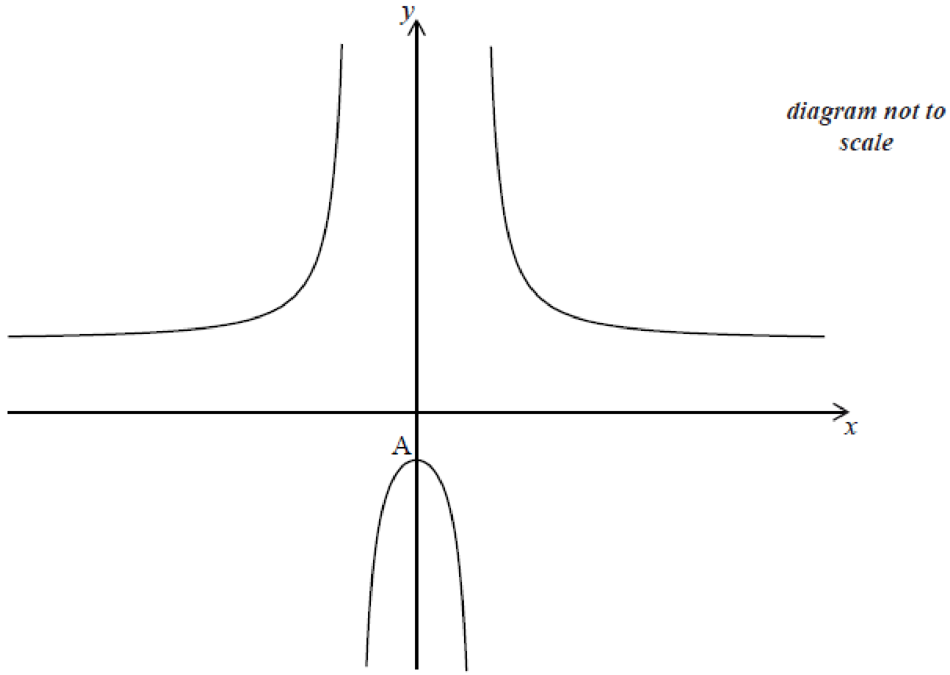
[5 marks]

Examiners report

a. [N/A]

b.

Let $f(x) = 3 + \frac{20}{x^2-4}$, for $x \neq \pm 2$. The graph of *f* is given below.



The y-intercept is at the point A.

a. (i) Find the coordinates of A. [7]

(ii) Show that $f'(x) = 0$ at A.

b. The second derivative $f''(x) = \frac{40(3x^2+4)}{(x^2-4)^3}$. Use this to [6]

(i) justify that the graph of f has a local maximum at A;

(ii) explain why the graph of f does **not** have a point of inflexion.

c. Describe the behaviour of the graph of f for large $|x|$. [1]

d. Write down the range of f . [2]

Markscheme

a. (i) coordinates of A are $(0,\ -2)$ ***A1A1*** ***N2***

(ii) derivative of $x^2 - 4 = 2x$ (seen anywhere) ***(A1)***

evidence of correct approach ***(M1)***

e.g. quotient rule, chain rule

finding $f'(x)$ ***A2***

e.g. $f'(x) = 20 \times (-1) \times (x^2 - 4)^{-2} \times (2x)$, $\frac{(x^2-4)(0)-(20)(2x)}{(x^2-4)^2}$

substituting $x = 0$ into $f'(x)$ (do not accept solving $f'(x) = 0$) ***M1***

at A $f'(x) = 0$ ***AG*** ***N0***

[7 marks]

b. (i) reference to $f'(x) = 0$ (seen anywhere) ***(R1)***

reference to $f''(0)$ is negative (seen anywhere) ***R1***

evidence of substituting $x = 0$ into $f''(x)$ ***M1*** finding $f''(0) = \frac{40 \times 4}{(-4)^3}$ $\left(= -\frac{5}{2}\right)$ ***A1***

then the graph must have a local maximum ***AG***

(ii) reference to $f''(x) = 0$ at point of inflexion ***(R1)***

recognizing that the second derivative is never 0 ***A1*** ***N2***

e.g. $40(3x^2 + 4) \neq 0$, $3x^2 + 4 \neq 0$, $x^2 \neq -\frac{4}{3}$, the numerator is always positive

Note: Do not accept the use of the first derivative in part (b).

[6 marks]

c. correct (informal) statement, including reference to approaching $y = 3$ ***A1*** ***N1***

e.g. getting closer to the line $y = 3$, horizontal asymptote at $y = 3$

[1 mark]

d. **correct** inequalities, $y \leq -2$, $y > 3$, ***FT*** from (a)(i) and (c) ***A1A1*** ***N2***

[2 marks]

Examiners report

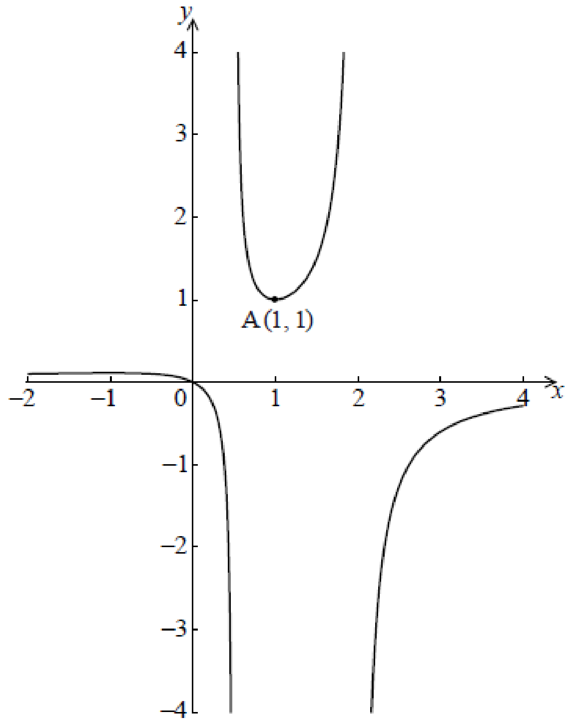
a. Almost all candidates earned the first two marks in part (a) (i), although fewer were able to apply the quotient rule correctly.

b. Many candidates were able to state how the second derivative can be used to identify maximum and inflection points, but fewer were actually able to demonstrate this with the given function. For example, in (b)(ii) candidates often simply said "the second derivative cannot equal 0" but did not justify or explain why this was true.

c. Not too many candidates could do part (c) correctly.

d. In (d) even those who knew what the range was had difficulty expressing the inequalities correctly.

Let $f(x) = \frac{x}{-2x^2+5x-2}$ for $-2 \leq x \leq 4$, $x \neq \frac{1}{2}$, $x \neq 2$. The graph of f is given below.



The graph of f has a local minimum at A(1, 1) and a local maximum at B.

a. Use the quotient rule to show that $f'(x) = \frac{2x^2-2}{(-2x^2+5x-2)^2}$. [6]

b. Hence find the coordinates of B. [7]

c. Given that the line $y = k$ does not meet the graph of f , find the possible values of k . [3]

Markscheme

a. correct derivatives **applied** in quotient rule ***(A1)A1A1***

$1, -4x+5$

Note: Award ***(A1)*** for 1, ***A1*** for $-4x$ and ***A1*** for 5, **only** if it is clear candidates are using the quotient rule.

correct substitution into quotient rule ***A1***

e.g. $\frac{1\times(-2x^2+5x-2)-x(-4x+5)}{(-2x^2+5x-2)^2}$, $\frac{-2x^2+5x-2-x(-4x+5)}{(-2x^2+5x-2)^2}$

correct working ***(A1)***

e.g. $\frac{-2x^2+5x-2-(-4x^2+5x)}{(-2x^2+5x-2)^2}$

expression clearly leading to the answer ***A1***

e.g. $\frac{-2x^2+5x-2+4x^2-5x}{(-2x^2+5x-2)^2}$

$f'(x) = \frac{2x^2-2}{(-2x^2+5x-2)^2}$ ***AG N0***

[6 marks]

b. evidence of attempting to solve $f'(x) = 0$ ***(M1)***

e.g. $2x^2 - 2 = 0$

evidence of correct working ***A1***

e.g. $x^2 = 1, \frac{\pm\sqrt{16}}{4}, 2(x-1)(x+1)$

correct solution to quadratic *(A1)*

e.g. $x = \pm 1$

correct x-coordinate $x = -1$ (may be seen in coordinate form $\left(-1, \frac{1}{9}\right)$) ***A1 N2***

attempt to substitute -1 into f (do not accept any other value) ***(M1)***

e.g. $f(-1) = \frac{-1}{-2\times(-1)^2+5\times(-1)-2}$

correct working

e.g. $\frac{-1}{-2-5-2}$ ***A1***

correct y-coordinate $y = \frac{1}{9}$ (may be seen in coordinate form $\left(-1, \frac{1}{9}\right)$) ***A1 N2***

[7 marks]

c. recognizing values between max and min ***(R1)***

$\frac{1}{9} < k < 1$ ***A2 N3***

[3 marks]

Examiners report

a. While most candidates answered part (a) correctly, there were some who did not show quite enough work for a "show that" question. A very small number of candidates did not follow the instruction to use the quotient rule.

b. In part (b), most candidates knew that they needed to solve the equation $f'(x) = 0$, and many were successful in answering this question correctly. However, some candidates failed to find both values of x, or made other algebraic errors in their solutions. One common error was to find only one solution for $x^2 = 1$; another was to work with the denominator equal to zero, rather than the numerator.

c. In part (c), a significant number of candidates seemed to think that the line $y = k$ was a vertical line, and attempted to find the vertical asymptotes. Others tried looking for a horizontal asymptote. Fortunately, there were still a good number of intuitive candidates who recognized the link with the graph and with part (b), and realized that the horizontal line must pass through the space between the given local minimum and the local maximum they had found in part (b).

Let $f'(x) = 6x^2 - 5$. Given that $f(2) = -3$, find $f(x)$.

Markscheme

evidence of antidifferentiation ***(M1)***

eg $f = \int f'$

correct integration (accept absence of C) ***(A1)(A1)***

$f(x) = \frac{6x^3}{3} - 5x + C,\ 2x^3 - 5x$

attempt to substitute $(2,\ -3)$ into **their** integrated expression (must have C) ***M1***

eg $2(2)^3 - 5(2) + C = -3,\ 16 - 10 + C = -3$

Note: Award ***M0*** if substituted into original or differentiated function.

correct working to find C ***(A1)***

eg $16 - 10 + C = -3,\ 6 + C = -3,\ C = -9$

$f(x) = 2x^3 - 5x - 9$ ***A1*** ***N4***

[6 marks]

Examiners report

[N/A]

Consider $f(x) = \log k(6x - 3x^2)$, for $0 < x < 2$, where $k > 0$.

The equation $f(x) = 2$ has exactly one solution. Find the value of k.

Markscheme

METHOD 1 – using discriminant

correct equation without logs ***(A1)***

eg $6x - 3x^2 = k^2$

valid approach ***(M1)***

eg $-3x^2 + 6x - k^2 = 0,\ 3x^2 - 6x + k^2 = 0$

recognizing discriminant must be zero (seen anywhere) ***M1***

eg $\Delta = 0$

correct discriminant ***(A1)***

eg $6^2 - 4(-3)(-k^2),\ 36 - 12k^2 = 0$

correct working ***(A1)***

eg $12k^2 = 36,\ k^2 = 3$

$k = \sqrt{3}$ ***A2*** ***N2***

METHOD 2 – completing the square

correct equation without logs ***(A1)***

eg $6x - 3x^2 = k^2$

valid approach to complete the square ***(M1)***

eg $3(x^2 - 2x + 1) = -k^2 + 3,\ x^2 - 2x + 1 - 1 + \frac{k^2}{3} = 0$

correct working ***(A1)***

eg $3(x-1)^2 = -k^2 + 3,\ (x-1)^2 - 1 + \frac{k^2}{3} = 0$

recognizing conditions for one solution ***M1***

eg $(x-1)^2 = 0,\ -1 + \frac{k^2}{3} = 0$

correct working ***(A1)***

eg $\frac{k^2}{3} = 1,\ k^2 = 3$

$k = \sqrt{3}$ ***A2*** ***N2***

[7 marks]

Examiners report

[N/A]

Let $f'(x) = 3x^2 + 2$. Given that $f(2) = 5$, find $f(x)$.

Markscheme

evidence of anti-differentiation ***(M1)***

e.g. $\int f'(x)$, $\int (3x^2 + 2)\mathrm{d}x$

$f(x) = x^3 + 2x + c$ (seen anywhere, including the answer) ***A1A1***

attempt to substitute (2, 5) ***(M1)***

e.g. $f(2) = (2)^3 + 2(2)$, $5 = 8 + 4 + c$

finding the value of c ***(A1)***

e.g. $5 = 12 + c$, $c = -7$

$f(x) = x^3 + 2x - 7$ ***A1*** ***N5***

[6 marks]

Examiners report

This question, which required candidates to integrate a simple polynomial and then substitute an initial condition to solve for "c", was very well done. Nearly all candidates who attempted this question were able to earn full marks. The very few mistakes that were seen involved arithmetic errors when solving for "c", or failing to write the final answer as the equation of the function.

Let $f(x) = \int \frac{12}{2x-5}\mathrm{d}x$, $x > \frac{5}{2}$. The graph of f passes through (4, 0) .

Find $f(x)$.

Markscheme

attempt to integrate which involves ln ***(M1)***

eg $\ln(2x-5)$, $12\ln 2x - 5$, $\ln 2x$

correct expression (accept absence of C)

eg $12\ln(2x-5)\frac{1}{2} + C$, $6\ln(2x-5)$ ***A2***

attempt to substitute (4,0) into **their** integrated ***f*** ***(M1)***

eg $0 = 6\ln(2\times 4 - 5)$, $0 = 6\ln(8-5) + C$

$C = -6\ln 3$ ***(A1)***

$f(x) = 6\ln(2x-5) - 6\ln 3 \left(= 6\ln\left(\frac{2x-5}{3}\right)\right)$ (accept $6\ln(2x-5) - \ln 3^6$) ***A1*** ***N5***

Note: Exception to the ***FT*** rule. Allow full ***FT*** on incorrect integration which must involve ln.

[6 marks]

Examiners report

While some candidates correctly integrated the function, many missed the division by 2 and answered $12\ln(2x-5)$. Other common incorrect responses included $\frac{12x}{x^2-5x}$ and $-122(x-5)^{-2}$. Finding the constant of integration also proved elusive for many. Some either did not remember the $+C$ or did not try to find its value, while others misunderstood the boundary condition and attempted to calculate the definite integral from 0 to 4.

Let $f'(x) = \sin^3(2x)\cos(2x)$. Find $f(x)$, given that $f\left(\frac{\pi}{4}\right) = 1$.

Markscheme

evidence of integration ***(M1)***

eg $\int f'(x)\mathrm{d}x$

correct integration (accept missing C) ***(A2)***

eg $\frac{1}{2} \times \frac{\sin^4(2x)}{4}$, $\frac{1}{8}\sin^4(2x) + C$

substituting initial condition into their integrated expression (must have $+C$) ***M1***

eg $1 = \frac{1}{8}\sin^4\left(\frac{\pi}{2}\right) + C$

Note: Award ***M0*** if they substitute into the original or differentiated function.

recognizing $\sin\left(\frac{\pi}{2}\right) = 1$ ***(A1)***

eg $1 = \frac{1}{8}(1)^4 + C$

$C = \frac{7}{8}$ ***(A1)***

$f(x) = \frac{1}{8}\sin^4(2x) + \frac{7}{8}$ ***A1 N5***

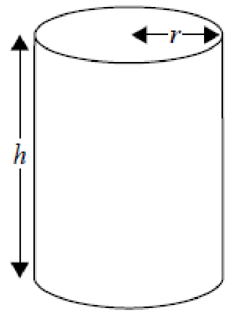
[7 marks]

Examiners report

[N/A]

A closed cylindrical can with radius r centimetres and height h centimetres has a volume of 20π cm^3.

diagram not to scale



The material for the base and top of the can costs 10 cents per cm^2 and the material for the curved side costs 8 cents per cm^2. The total cost of the material, in cents, is *C*.

a. Express *h* in terms of *r*. [2]

b. Show that $C = 20\pi r^2 + \frac{320\pi}{r}$. [4]

c. Given that there is a minimum value for *C*, find this minimum value in terms of π. [9]

Markscheme

a. correct equation for volume ***(A1)***

eg $\pi r^2 h = 20\pi$

$h = \frac{20}{r^2}$ ***A1 N2***

[2 marks]

b. attempt to find formula for cost of parts ***(M1)***

eg 10 × two circles, 8 × curved side

correct expression for cost of two circles in terms of *r* (seen anywhere) ***A1***
eg $2\pi r^2 \times 10$

correct expression for cost of curved side (seen anywhere) ***(A1)***
eg $2\pi r \times h \times 8$

correct expression for cost of curved side in terms of *r* ***A1***
eg $8 \times 2\pi r \times \frac{20}{r^2}, \ \frac{320\pi}{r^2}$

$C = 20\pi r^2 + \frac{320\pi}{r}$ ***AG N0***

[4 marks]

c. recognize $C' = 0$ at minimum ***(R1)***

eg $C' = 0, \ \frac{\mathrm{d}C}{\mathrm{d}r} = 0$

correct differentiation (may be seen in equation)

$C' = 40\pi r - \frac{320\pi}{r^2}$ ***A1A1***

correct equation ***A1***
eg $40\pi r - \frac{320\pi}{r^2} = 0, \ 40\pi r \frac{320\pi}{r^2}$

correct working *(A1)*
eg $40r^3 = 320,\ r^3 = 8$

r = 2 (m) ***A1***

attempt to substitute **their** value of r into C
eg $20\pi \times 4 + 320 \times \frac{\pi}{2}$ ***(M1)***

correct working
eg $80\pi + 160\pi$ ***(A1)***

240π (cents) ***A1 N3***

Note: Do not accept 753.6, 753.98 or 754, even if 240π is seen.

[9 marks]

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]

---

a. Find $\int \frac{e^x}{1+e^x} dx$. [3]

b. Find $\int \sin 3x \cos 3x dx$. [4]

Markscheme

a. attempt to use substitution or inspection ***M1***

e.g. $u = 1 + e^x$ so $\frac{du}{dx} = e^x$

correct working ***A1***

e.g. $\int \frac{du}{u} = \ln u$

$\ln(1 + e^x) + C$ ***A1 N3***

[3 marks]

b. **METHOD 1**

attempt to use substitution or inspection ***M1***

e.g. let $u = \sin 3x$

$\frac{du}{dx} = 3\cos 3x$ ***A1***

$\frac{1}{3}\int u du = \frac{1}{3} \times \frac{u^2}{2} + C$ ***A1***

$\int \sin 3x \cos 3x dx = \frac{\sin^2 3x}{6} + C$ ***A1 N2***

METHOD 2

attempt to use substitution or inspection ***M1***

e.g. let $u = \cos 3x$

$\frac{du}{dx} = -3\sin 3x$ ***A1***

$-\frac{1}{3}\int u du = -\frac{1}{3} \times \frac{u^2}{2} + C$ ***A1***

$\int \sin 3x \cos 3x dx = \frac{\cos^2 3x}{6} + C$ ***A1 N2***

METHOD 3

recognizing double angle ***M1***

correct working ***A1***

e.g. $\frac{1}{2}\sin 6x$

$\int \sin 6x\mathrm{d}x = \frac{-\cos 6x}{6} + C$ ***A1***

$\int \frac{1}{2}\sin 6x\mathrm{d}x = -\frac{\cos 6x}{12} + C$ ***A1*** ***N2***

[4 marks]

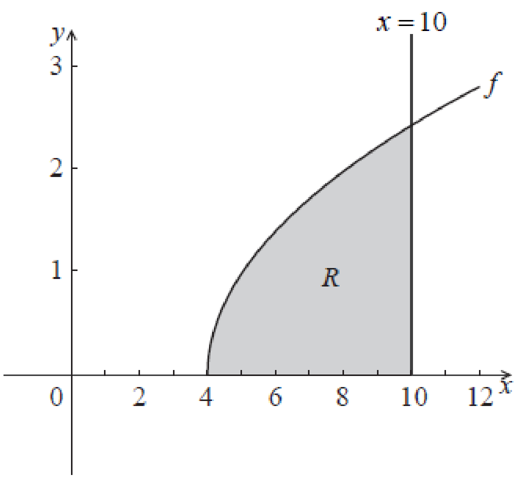
Examiners report

a. [N/A]
b. [N/A]

---

a. Find $\int_4^{10} (x-4)\mathrm{d}x$. [4]

b. Part of the graph of $f(x) = \sqrt{x-4}$, for $x \geq 4$, is shown below. The shaded region R is enclosed by the graph of f , the line $x = 10$, and the x-axis. [3]



The region R is rotated 360° about the x-axis. Find the volume of the solid formed.

Markscheme

a. correct integration ***A1A1***

e.g. $\frac{x^2}{2} - 4x$, $\left[\frac{x^2}{2} - 4x\right]_4^{10}$, $\frac{(x-4)^2}{2}$

Notes: In the first 2 examples, award ***A1*** for each correct term.

In the third example, award ***A1*** for $\frac{1}{2}$ and ***A1*** for $(x-4)^2$.

substituting limits into **their** integrated function and subtracting (in any order) ***(M1)***

e.g. $\left(\frac{10^2}{2} - 4(10)\right) - \left(\frac{4^2}{2} - 4(4)\right)$, $10 - (-8)$, $\frac{1}{2}(6^2 - 0)$

$\int_4^{10} (x-4)\mathrm{d}x = 18$ ***A1*** ***N2***

b. attempt to substitute either limits or the function into volume formula ***(M1)***

e.g. $\pi \int_4^{10} f^2\mathrm{d}x$, $\int_a^b \left(\sqrt{x-4}\right)^2$, $\pi \int_4^{10} \sqrt{x-4}$

Note: Do not penalise for missing π or dx.

correct substitution (accept absence of dx and π) ***(A1)***

e.g. $\pi\int_4^{10}\left(\sqrt{x-4}\right)^2$, $\pi\int_4^{10}(x-4)\mathrm{d}x$, $\int_4^{10}(x-4)\mathrm{d}x$

volume = 18π ***A1*** ***N2***

[3 marks]

Examiners report

a. Many candidates answered both parts of this question correctly. In part (b), a large number of successful candidates did not seem to notice the link between parts (a) and (b), and duplicated the work they had already done in part (a). Also in part (b), a good number of candidates squared $(x-4)$ in their integral, rather than squaring $\sqrt{x-4}$, which of course prevented them from noting the link between the two parts and obtaining the correct answer.

b. Many candidates answered both parts of this question correctly. In part (b), a large number of successful candidates did not seem to notice the link between parts (a) and (b), and duplicated the work they had already done in part (a). Also in part (b), a good number of candidates squared $(x-4)$ in their integral, rather than squaring $\sqrt{x-4}$, which of course prevented them from noting the link between the two parts and obtaining the correct answer.

A particle moves along a straight line so that its velocity, v ms^{-1}at time t seconds is given by $v = 6\mathrm{e}^{3t}+4$. When $t=0$, the displacement, s, of the particle is 7 metres. Find an expression for s in terms of t.

Markscheme

evidence of anti-differentiation ***(M1)***

e.g. $s=\int(6\mathrm{e}^{3x}+4)\mathrm{d}x$

$s=2\mathrm{e}^{3t}+4t+C$ ***A2A1***

substituting $t=0$, ***(M1)***

$7=2+C$ ***A1***

$C=5$

$s=2\mathrm{e}^{3t}+4t+5$ ***A1*** ***N3***

[7 marks]

Examiners report

There were a number of completely correct solutions to this question. However, there were many who did not know the relationship between velocity and position. Many students differentiated rather than integrated and those who did integrate often had difficulty with the term involving e. Many who integrated correctly neglected the C or made $C=7$.

Consider the function f with second derivative $f''(x) = 3x - 1$. The graph of f has a minimum point at A(2, 4) and a maximum point at B $\left(-\frac{4}{3}, \frac{358}{27}\right)$.

a. Use the second derivative to justify that B is a maximum. [3]

b. Given that $f'(x) = \frac{3}{2}x^2 - x + p$, show that $p = -4$. [4]

c. Find $f(x)$. [7]

Markscheme

a. substituting into the second derivative ***M1***

e.g. $3 \times \left(-\frac{4}{3}\right) - 1$

$f''\left(-\frac{4}{3}\right) = -5$ ***A1***

since the second derivative is negative, B is a maximum ***R1*** ***N0***

[3 marks]

b. setting $f'(x)$ equal to zero ***(M1)***

evidence of substituting $x = 2$ (or $x = -\frac{4}{3}$) ***(M1)***

e.g. $f'(2)$

correct substitution ***A1***

e.g. $\frac{3}{2}(2)^2 - 2 + p$, $\frac{3}{2}\left(-\frac{4}{3}\right)^2 - \left(-\frac{4}{3}\right) + p$

correct simplification

e.g. $6 - 2 + p = 0$, $\frac{8}{3} + \frac{4}{3} + p = 0$, $4 + p = 0$ ***A1***

$p = -4$ ***AG*** ***N0***

[4 marks]

c. evidence of integration ***(M1)***

$f(x) = \frac{1}{2}x^3 - \frac{1}{2}x^2 - 4x + c$ ***A1A1A1***

substituting (2, 4) or $\left(-\frac{4}{3}, \frac{358}{27}\right)$ into **their** expression ***(M1)***

correct equation ***A1***

e.g. $\frac{1}{2} \times 2^3 - \frac{1}{2} \times 2^2 - 4 \times 2 + c = 4$, $\frac{1}{2} \times 8 - \frac{1}{2} \times 4 - 4 \times 2 + c = 4$, $4 - 2 - 8 + c = 4$

$f(x) = \frac{1}{2}x^3 - \frac{1}{2}x^2 - 4x + 10$ ***A1*** ***N4***

[7 marks]

Examiners report

a. Many candidates were successful with this question. In part (a), some candidates found $f''\left(-\frac{4}{3}\right)$ and were unclear how to conclude, but most demonstrated a good understanding of the second derivative test.

b. A large percentage of candidates were successful in showing that $p = -4$ but there were still some who worked backwards from the answer. Others did not use the given information and worked from the second derivative, integrated, and then realized that p was the constant of integration. Candidates who evaluated the derivative at $x = 2$ but set the result equal to 4 clearly did not understand the concept being assessed. Few candidates used the point B with fractional coordinates.

c. Candidates often did well on the first part of (c), knowing to integrate and successfully finding some or all terms. Some had trouble with the fractions or made careless errors with the signs; others did not use the value of $p = -4$ and so could not find the third term when integrating. It was very common for candidates to either forget the constant of integration or to leave it in without finding its value.

Let $f'(x) = \frac{6-2x}{6x-x^2}$, for $0 < x < 6$.

The graph of f has a maximum point at P.

The y-coordinate of P is $\ln 27$.

a. Find the x-coordinate of P. [3]

b. Find $f(x)$, expressing your answer as a single logarithm. [8]

c. The graph of f is transformed by a vertical stretch with scale factor $\frac{1}{\ln 3}$. The image of P under this transformation has coordinates $(a,\ b)$. [[N/A]]

Find the value of a and of b, where $a,\ b \in \mathbb{N}$.

Markscheme

a. recognizing $f'(x) = 0$ ***(M1)***

correct working ***(A1)***

eg $6 - 2x = 0$

$x = 3$ ***A1 N2***

[3 marks]

b. evidence of integration ***(M1)***

eg $\int f',\ \int \frac{6-2x}{6x-x^2}\mathrm{d}x$

using substitution ***(A1)***

eg $\int \frac{1}{u}\mathrm{d}u$ where $u = 6x - x^2$

correct integral ***A1***

eg $\ln(u) + c,\ \ln(6x - x^2)$

substituting $(3,\ \ln 27)$ into **their** integrated expression (must have c) ***(M1)***

eg $\ln(6 \times 3 - 3^2) + c = \ln 27,\ \ln(18 - 9) + \ln k = \ln 27$

correct working ***(A1)***

eg $c = \ln 27 - \ln 9$

EITHER

$c = \ln 3$ ***(A1)***

attempt to substitute **their** value of c into $f(x)$ ***(M1)***

eg $f(x) = \ln(6x - x^2) + \ln 3$ ***A1*** ***N4***

OR

attempt to substitute **their** value of c into $f(x)$ ***(M1)***

eg $f(x) = \ln(6x - x^2) + \ln 27 - \ln 9$

correct use of a log law ***(A1)***

eg $f(x) = \ln(6x - x^2) + \ln\left(\frac{27}{9}\right),\ f(x) = \ln\left(27(6x - x^2)\right) - \ln 9$

$f(x) = \ln\left(3(6x - x^2)\right)$ ***A1*** ***N4***

[8 marks]

c. $a = 3$ ***A1*** ***N1***

correct working ***A1***

eg $\frac{\ln 27}{\ln 3}$

correct use of log law ***(A1)***

eg $\frac{3\ln 3}{\ln 3},\ \log_3 27$

$b = 3$ ***A1*** ***N2***

[4 marks]

Examiners report

a. Part a) was well answered.

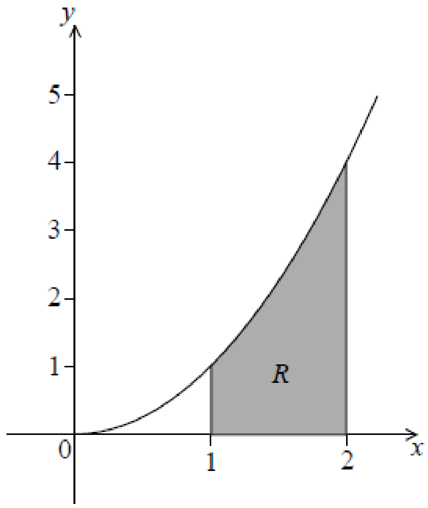
b. In part b) most candidates realised that integration was required but fewer recognised the need to use integration by substitution. Quite a number of candidates who integrated correctly omitted finding the constant of integration.

c. In part c) many candidates showed good understanding of transformations and could apply them correctly, however, correct use of the laws of logarithms was challenging for many. In particular, a common error was $\frac{\ln 27}{\ln 3} = \ln 9$.

Let $f(x) = x^2$.

a. Find $\int_1^2 \left(f(x)\right)^2 \mathrm{d}x$. [4]

b. The following diagram shows part of the graph of f. [2]



The shaded region R is enclosed by the graph of f, the x-axis and the lines $x = 1$ and $x = 2$.
Find the volume of the solid formed when R is revolved 360° about the x-axis.

Markscheme

a. substituting for $(f(x))^2$ (may be seen in integral) ***A1***

eg $(x^2)^2$, x^4

correct integration, $\int x^4 \mathrm{d}x = \frac{1}{5}x^5$ ***(A1)***

substituting limits into **their integrated** function and subtracting (in any order) ***(M1)***

eg $\frac{2^5}{5} - \frac{1}{5}$, $\frac{1}{5}(1-4)$

$\int_1^2 (f(x))^2 \mathrm{d}x = \frac{31}{5} (= 6.2)$ ***A1*** ***N2***

[4 marks]

b. attempt to substitute limits or function into formula involving f^2 ***(M1)***

eg $\int_1^2 (f(x))^2 \mathrm{d}x$, $\pi \int x^4 \mathrm{d}x$

$\frac{31}{5}\pi \ (= 6.2\pi)$ ***A1*** ***N2***

[2 marks]

Examiners report

a. [N/A]
b. [N/A]

Given that $\int_0^5 \frac{2}{2x+5} \mathrm{d}x = \ln k$, find the value of k .

Markscheme

correct integration, $2 \times \frac{1}{2}\ln(2x+5)$ ***A1A1***

Note: Award ***A1*** for $2 \times \frac{1}{2} (= 1)$ and ***A1*** for $\ln(2x+5)$.

evidence of substituting limits into integrated function and subtracting ***(M1)***

e.g. $\ln(2 \times 5 + 5) - \ln(2 \times 0 + 5)$

correct substitution ***A1***

e.g. $\ln 15 - \ln 5$

correct working ***(A1)***

e.g. $\ln \frac{15}{5}$, $\ln 3$

$k = 3$ ***A1*** ***N3***

[6 marks]

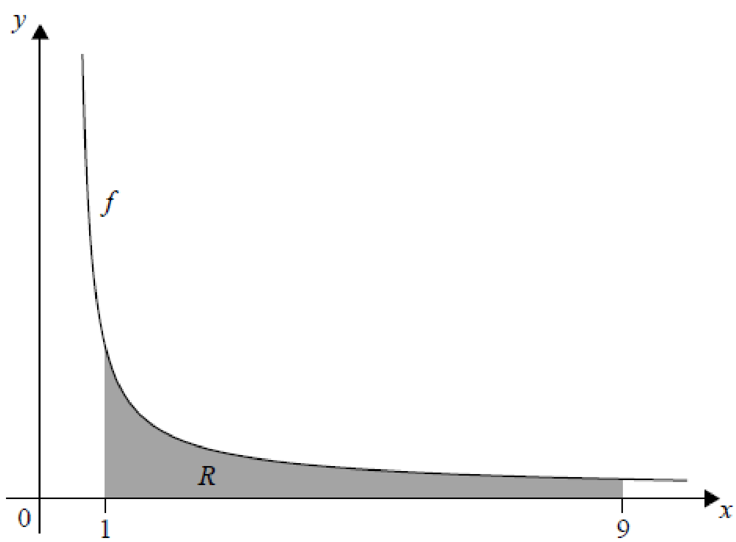
Examiners report

Knowing that the answer to this integration led to a natural logarithm function helped many candidates make progress on this more challenging question, although some candidates simply substituted the limits straight away without integrating. Although some candidates incorrectly simplified $\ln 15 - \ln 5$ as $\ln 10$ or $\frac{\ln 15}{\ln 5} = \ln 3$, a pleasing number applied the logarithm property correctly. Some candidates had difficulty with missing brackets which typically led to $\ln 0$ in their answer.

Let $f(x) = \frac{1}{\sqrt{2x-1}}$, for $x > \frac{1}{2}$.

a. Find $\int (f(x))^2 \mathrm{d}x$. [3]

b. Part of the graph of *f* is shown in the following diagram. [4]



The shaded region *R* is enclosed by the graph of *f*, the *x*-axis, and the lines *x* = 1 and *x* = 9 . Find the volume of the solid formed when *R* is revolved 360° about the *x*-axis.

Markscheme

a. correct working ***(A1)***

eg $\int \frac{1}{2x-1} \mathrm{d}x$, $\int (2x-1)^{-1}$, $\frac{1}{2x-1}$, $\int \left(\frac{1}{\sqrt{u}}\right)^2 \frac{\mathrm{d}u}{2}$

$\int (f(x))^2 \mathrm{d}x = \frac{1}{2}\ln(2x-1) + c$ ***A2 N3***

Note: Award ***A1*** for $\frac{1}{2}\ln(2x-1)$.

[3 marks]

b. attempt to substitute either limits or the function into formula involving f^2 (accept absence of π / dx) ***(M1)***

eg $\int_1^9 y^2\mathrm{d}x,\ \pi\int\left(\frac{1}{\sqrt{2x-1}}\right)^2\mathrm{d}x,\ \left[\frac{1}{2}\ln(2x-1)\right]_1^9$

substituting limits into **their** integral and subtracting (in any order) ***(M1)***

eg $\frac{\pi}{2}(\ln(17)-\ln(1))\ ,\ \pi\left(0-\frac{1}{2}\ln(2\times 9-1)\right)$

correct working involving calculating a log value or using log law ***(A1)***

eg $\ln(1)=0,\ \ln\left(\frac{17}{1}\right)$

$\frac{\pi}{2}\ln 17\ \ \left(\text{accept } \pi\ln\sqrt{17}\right)$ ***A1 N3***

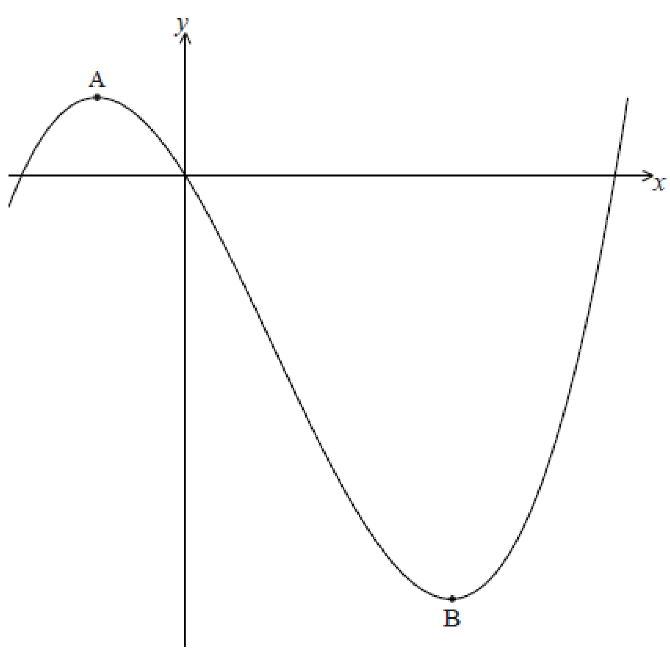
Note: Full ***FT*** may be awarded as normal, from their incorrect answer in part (a), however, do not award the final two ***A*** marks unless they involve logarithms.

[4 marks]

Examiners report

a. [N/A]
b. [N/A]

Let $f(x)=\frac{1}{2}x^3-x^2-3x$. Part of the graph of f is shown below.



There is a maximum point at A and a minimum point at B$(3,-9)$.

a. Find the coordinates of A. [8]

b(i), (ii) and (iii). Write down the coordinates of [6]

(i) the image of B after reflection in the y-axis;

(ii) the image of B after translation by the vector $\begin{pmatrix}-2\\5\end{pmatrix}$;

(iii) the image of B after reflection in the x-axis followed by a horizontal stretch with scale factor $\frac{1}{2}$.

Markscheme

a. $f(x) = x^2 - 2x - 3$ ***A1A1A1***

evidence of solving $f'(x) = 0$ ***(M1)***

e.g. $x^2 - 2x - 3 = 0$

evidence of correct working ***A1***

e.g. $(x+1)(x-3)$, $\frac{2 \pm \sqrt{16}}{2}$

$x = -1$ (ignore $x = 3$) ***(A1)***

evidence of substituting **their** negative x-value into $f(x)$ ***(M1)***

e.g. $\frac{1}{3}(-1)^3 - (-1)^2 - 3(-1)$, $-\frac{1}{3} - 1 + 3$

$y = \frac{5}{3}$ ***A1***

coordinates are $\left(-1, \frac{5}{3}\right)$ ***N3***

[8 marks]

b(i), (ii) and (iii). (i) $(-3, 9)$ ***A1*** ***N1***

(ii) $(1, -4)$ ***A1A1*** ***N2***

(iii) reflection gives (3, 9) ***(A1)***

stretch gives $\left(\frac{3}{2}, 9\right)$ ***A1A1*** ***N3***

[6 marks]

Examiners report

a. A majority of candidates answered part (a) completely.

b(i), (ii) and (iii). Candidates were generally successful in finding images after single transformations in part (b). Common incorrect answers for (biii) included $\left(\frac{3}{2}, \frac{9}{2}\right)$, (6, 9) and (6, 18) , demonstrating difficulty with images from horizontal stretches.

The graph of $y = \sqrt{x}$ between $x = 0$ and $x = a$ is rotated 360° about the x-axis. The volume of the solid formed is 32π . Find the value of a.

Markscheme

attempt to substitute into formula $V = \int \pi y^2 \mathrm{d}x$ ***(M1)***

integral expression ***A1***

e.g. $\pi \int_0^a (\sqrt{x})^2 \mathrm{d}x$, $\pi \int x$

correct integration ***(A1)***

e.g. $\int x \mathrm{d}x = \frac{1}{2}x^2$

correct substitution $V = \pi \left[\frac{1}{2}a^2\right]$ ***(A1)***

equating **their** expression to 32π ***M1*** e.g. $\pi\left[\frac{1}{2}a^2\right]=32\pi$

$a^2=64$

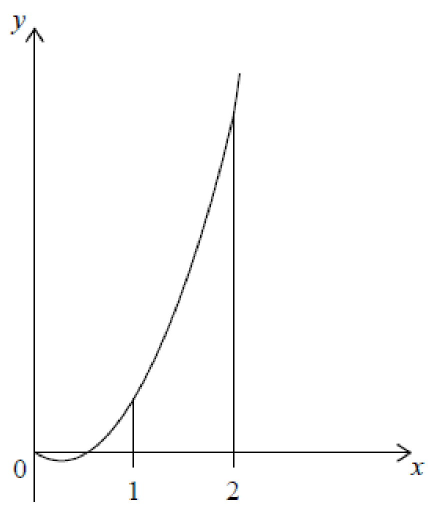
$a=8$ ***A2*** ***N2***

[7 marks]

Examiners report

Despite the “reverse” nature of this question, many candidates performed well with the integration. Some forgot to square the function, while others did not discard the negative value of a. Some attempted to equate 32π to the formula for volume of a sphere, which suggests this topic was not fully covered in some centres.

Let $f(x)=6x^2-3x$. The graph of f is shown in the following diagram.



a. Find $\int\left(6x^2-3x\right)\mathrm{d}x$. [2]

b. Find the area of the region enclosed by the graph of f, the *x*-axis and the lines *x* = 1 and *x* = 2 . [4]

Markscheme

a. $2x^3-\frac{3x^2}{2}+c$ $\left(\text{accept } \frac{6x^3}{3}-\frac{3x^2}{2}+c\right)$ ***A1A1 N2***

Notes: Award ***A1A0*** for both correct terms if +*c* is omitted.
Award ***A1A0*** for one correct term *eg* $2x^3+c$.
Award ***A1A0*** if both terms are correct, but candidate attempts further working to solve for *c*.

[2 marks]

b. substitution of limits or function ***(A1)***

eg $\int_1^2 f(x)\,\mathrm{d}x,\ \left[2x^3-\frac{3x^2}{2}\right]_1^2$

substituting limits into their integrated function and subtracting ***(M1)***

eg $\frac{6\times2^3}{3}-\frac{3\times2^2}{2}-\left(\frac{6\times1^3}{3}+\frac{3\times1^2}{2}\right)$

Note: Award ***M0*** if substituted into original function.

correct working *(A1)*

eg $\frac{6\times8}{3}-\frac{3\times4}{2}-\frac{6\times1}{3}+\frac{3\times1}{2},\ (16-6)-\left(2-\frac{3}{2}\right)$

$\frac{19}{2}$ ***A1 N3***

[4 marks]

Examiners report

a. [N/A]
b. [N/A]

The following table shows the probability distribution of a discrete random variable A, in terms of an angle θ.

a	1	2
$\mathrm{P}(A=a)$	$\cos\theta$	$2\cos 2\theta$

a. Show that $\cos\theta=\frac{3}{4}$. [6]

b. Given that $\tan\theta>0$, find $\tan\theta$. [3]

c. Let $y=\frac{1}{\cos x}$, for $0<x<\frac{\pi}{2}$. The graph of $y$between $x=\theta$ and $x=\frac{\pi}{4}$ is rotated 360° about the x-axis. Find the volume of the solid formed. [6]

Markscheme

a. evidence of summing to 1 ***(M1)***

eg $\sum p=1$

correct equation ***A1***

eg $\cos\theta+2\cos 2\theta=1$

correct equation in $\cos\theta$ ***A1***

eg $\cos\theta+2(2\cos^2\theta-1)=1,\ 4\cos^2\theta+\cos\theta-3=0$

evidence of valid approach to solve quadratic ***(M1)***

eg factorizing equation set equal to 0, $\frac{-1\pm\sqrt{1-4\times4\times(-3)}}{8}$

correct working, clearly leading to required answer ***A1***

eg $(4\cos\theta-3)(\cos\theta+1),\ \frac{-1\pm7}{8}$

correct reason for rejecting $\cos\theta\neq-1$ ***R1***

eg $\cos\theta$ is a probability (value must lie between 0 and 1), $\cos\theta>0$

Note: Award ***R0*** for $\cos\theta\neq-1$ without a reason.

$\cos\theta=\frac{3}{4}$ ***AG N0***

b. valid approach ***(M1)***

eg sketch of right triangle with sides 3 and 4, $\sin^2x+\cos^2x=1$

correct working

(A1)

eg missing side $= \sqrt{7}$, $\frac{\frac{\sqrt{7}}{4}}{\frac{3}{4}}$

$\tan\theta = \frac{\sqrt{7}}{3}$ ***A1 N2***

[3 marks]

c. attempt to substitute either limits or the function into formula involving f^2 ***(M1)***

eg $\pi\int_\theta^{\frac{\pi}{4}} f^2$, $\int\left(\frac{1}{\cos x}\right)^2$

correct substitution of both limits and function ***(A1)***

eg $\pi\int_\theta^{\frac{\pi}{4}}\left(\frac{1}{\cos x}\right)^2 \mathrm{d}x$

correct integration ***(A1)***

eg $\tan x$

substituting **their** limits into **their** integrated function and subtracting ***(M1)***

eg $\tan\frac{\pi}{4} - \tan\theta$

Note: Award ***M0*** if they substitute into original or differentiated function.

$\tan\frac{\pi}{4} = 1$ ***(A1)***

eg $1 - \tan\theta$

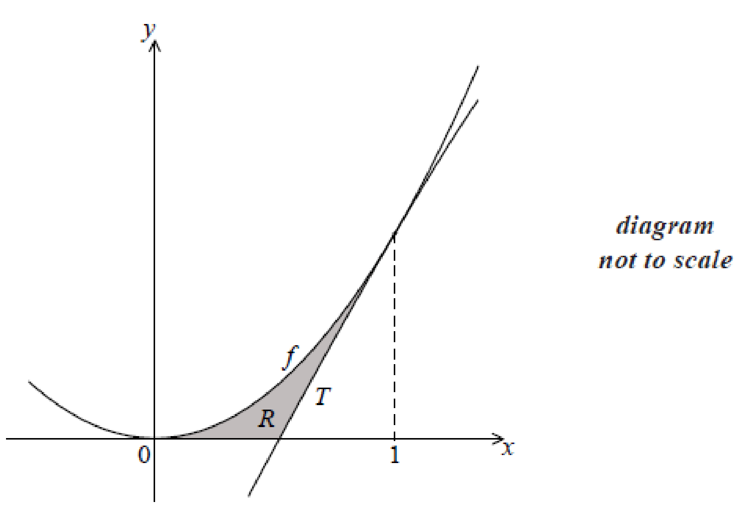
$V = \pi - \frac{\pi\sqrt{7}}{3}$ ***A1 N3***

[6 marks]

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]

The following diagram shows part of the graph of the function $f(x) = 2x^2$.



The line T is the tangent to the graph of f at $x = 1$.

a. Show that the equation of T is $y = 4x - 2$. [5]

b. Find the x-intercept of T . [2]

c(i) and (ii). The shaded region R is enclosed by the graph of f , the line T , and the x-axis. [9]

(i) Write down an expression for the area of R .

(ii) Find the area of R .

Markscheme

a. $f(1) = 2$ ***(A1)***

$f'(x) = 4x$ ***A1***

evidence of finding the gradient of f at $x = 1$ ***M1***

e.g. substituting $x = 1$ into $f'(x)$

finding gradient of f at $x = 1$ ***A1***

e.g. $f'(1) = 4$

evidence of finding equation of the line ***M1***

e.g. $y - 2 = 4(x - 1)$, $2 = 4(1) + b$

$y = 4x - 2$ ***AG*** ***N0***

[5 marks]

b. appropriate approach ***(M1)***

e.g. $4x - 2 = 0$

$x = \frac{1}{2}$ ***A1*** ***N2***

[2 marks]

c(i) and (ii). (i) bottom limit $x = 0$ (seen anywhere) ***(A1)***

approach involving subtraction of integrals/areas ***(M1)***

e.g. $\int f(x) -$ area of triangle , $\int f - \int l$

correct expression ***A2*** ***N4***

e.g. $\int_0^1 2x^2 \mathrm{d}x - \int_{0.5}^1 (4x - 2)\mathrm{d}x$, $\int_0^1 f(x)\mathrm{d}x - \frac{1}{2}$, $\int_0^{0.5} 2x^2 \mathrm{d}x + \int_{0.5}^1 (f(x) - (4x - 2))\mathrm{d}x$

(ii) **METHOD 1 (using only integrals)**

correct integration ***(A1)(A1)(A1)***

$\int 2x^2 \mathrm{d}x = \frac{2x^3}{3}$, $\int (4x - 2)\mathrm{d}x = 2x^2 - 2x$

substitution of limits ***(M1)***

e.g. $\frac{1}{12} + \frac{2}{3} - 2 + 2 - \left(\frac{1}{12} - \frac{1}{2} + 1\right)$

area = $\frac{1}{6}$ ***A1*** ***N4***

METHOD 2 (using integral and triangle)

area of triangle = $\frac{1}{2}$ ***(A1)***

correct integration ***(A1)***

$\int 2x^2\mathrm{d}x = \frac{2x^3}{3}$

substitution of limits ***(M1)***

e.g. $\frac{2}{3}(1)^3 - \frac{2}{3}(0)^3$, $\frac{2}{3} - 0$

correct simplification ***(A1)***

e.g. $\frac{2}{3} - \frac{1}{2}$

area = $\frac{1}{6}$ ***A1*** ***N4***

[9 marks]

Examiners report

a. The majority of candidates seemed to know what was meant by the tangent to the graph in part (a), but there were many who did not fully show their work, which is of course necessary on a "show that" question. While many candidates knew they needed to find the derivative of f, some failed to substitute the given value of x in order to find the gradient of the tangent.

b. Part (b), finding the x-intercept, was answered correctly by nearly every candidate.

c(i) and (ii). In part (c), most candidates struggled with writing an expression for the area of R . Many tried to use the difference of the two functions over the entire interval 0–1, not noticing that the area from 0–0.5 only required the use of function f . Many of these candidates were able to earn follow-through marks in the second part of (c) for their correct integration. There were a few candidates who successfully found the area under the line as the area of a triangle.

Let $g(x) = \frac{\ln x}{x^2}$, for $x > 0$.

a. Use the quotient rule to show that $g'(x) = \frac{1-2\ln x}{x^3}$. [4]

b. The graph of g has a maximum point at A. Find the x-coordinate of A. [3]

Markscheme

a. $\frac{\mathrm{d}}{\mathrm{d}x}\ln x = \frac{1}{x}$, $\frac{\mathrm{d}}{\mathrm{d}x}x^2 = 2x$ (seen anywhere) ***A1A1***

attempt to substitute into the quotient rule (do **not** accept product rule) ***M1***

e.g. $\frac{x^2\left(\frac{1}{x}\right)-2x\ln x}{x^4}$

correct manipulation that clearly leads to result ***A1***

e.g. $\frac{x-2x\ln x}{x^4}$, $\frac{x(1-2\ln x)}{x^4}$, $\frac{x}{x^4}$, $\frac{2x\ln x}{x^4}$

$g'(x) = \frac{1-2\ln x}{x^3}$ ***AG*** ***N0***

[4 marks]

b. evidence of setting the derivative equal to zero ***(M1)***

e.g. $g'(x) = 0$, $1 - 2\ln x = 0$

$\ln x = \frac{1}{2}$ ***A1***

$x = \mathrm{e}^{\frac{1}{2}}$ ***A1 N2***

[3 marks]

Examiners report

a. Many candidates clearly knew their quotient rule, although a common error was to simplify $2x \ln x$ as $2 \ln x^2$ and then "cancel" the exponents.

b. For (b), those who knew to set the derivative to zero typically went on find the correct *x*-coordinate, which must be in terms of e, as this is the calculator-free paper. Occasionally, students would take $\frac{1-2\ln x}{x^3} = 0$ and attempt to solve from $1 - 2\ln x = x^3$.

Consider *f*(*x*), *g*(*x*) and *h*(*x*), for x∈$\mathbb{R}$ where *h*(*x*) = $(f \circ g)$(*x*).

Given that *g*(3) = 7 , *g′*(3) = 4 and *f′*(7) = −5 , find the gradient of the normal to the curve of *h* at *x* = 3.

Markscheme

recognizing the need to find *h′* ***(M1)***

recognizing the need to find *h′*(3) (seen anywhere) ***(M1)***

evidence of choosing chain rule ***(M1)***

eg $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}u} \times \frac{\mathrm{d}u}{\mathrm{d}x},\ f'\left(g\left(3\right)\right) \times g'\left(3\right),\ f'\left(g\right) \times g'$

correct working ***(A1)***

eg $f'\left(7\right) \times 4,\ -5 \times 4$

$h'\left(3\right) = -20$ ***(A1)***

evidence of taking **their** negative reciprocal for normal ***(M1)***

eg $-\frac{1}{h'(3)},\ m_1 m_2 = -1$

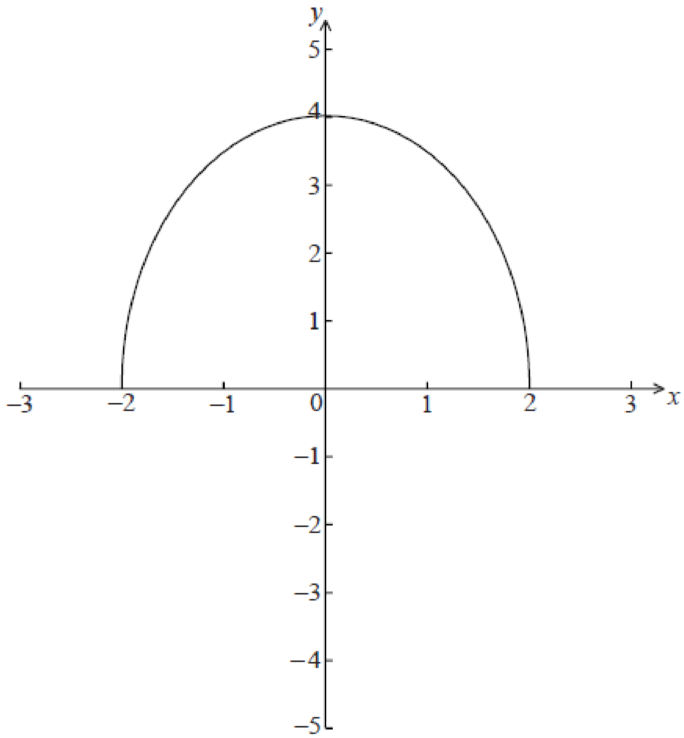
gradient of normal is $\frac{1}{20}$ ***A1 N4***

[7 marks]

Examiners report

[N/A]

The graph of $f(x) = \sqrt{16 - 4x^2}$, for $-2 \le x \le 2$, is shown below.



The region enclosed by the curve of *f* and the *x*-axis is rotated 360° about the *x*-axis.

Find the volume of the solid formed.

Markscheme

attempt to set up integral expression ***M1***

e.g. $\pi\int\sqrt{16-4x^2}^2\mathrm{d}x$, $2\pi\int_0^2(16-4x^2)$, $\int\sqrt{16-4x^2}^2\mathrm{d}x$

$\int 16\mathrm{d}x = 16x$, $\int 4x^2\mathrm{d}x = \frac{4x^3}{3}$ (seen anywhere) ***A1A1***

evidence of substituting limits into the integrand ***(M1)***

e.g. $\left(32-\frac{32}{3}\right)-\left(-32+\frac{32}{3}\right)$, $64-\frac{64}{3}$

volume $=\frac{128\pi}{3}$ ***A2*** ***N3***

[6 marks]

Examiners report

Many candidates correctly integrated using $f(x)$, although some neglected to square the function and mired themselves in awkward integration attempts. Upon substituting the limits, many were unable to carry the calculation to completion. Occasionally the π was neglected in a final answer. Weaker candidates considered the solid formed to be a sphere and did not use integration.

Let $f(x)=\mathrm{e}^{2x}$. The line L is the tangent to the curve of f at $(1,\ \mathrm{e}^2)$.
Find the equation of L in the form $y=ax+b$.

Markscheme

recognising need to differentiate (seen anywhere) ***R1***

eg $f',\ 2e^{2x}$

attempt to find the gradient when $x = 1$ ***(M1)***

eg $f'(1)$

$f'(1) = 2e^2$ ***(A1)***

attempt to substitute coordinates (in any order) into equation of a straight line ***(M1)***

eg $y - e^2 = 2e^2(x - 1),\ e^2 = 2e^2(1) + b$

correct working ***(A1)***

eg $y - e^2 = 2e^2x - 2e^2,\ b = -e^2$

$y = 2e^2x - e^2$ ***A1 N3***

[6 marks]

Examiners report

[N/A]

Let $f(x) = e^x \cos x$. Find the gradient of the normal to the curve of *f* at $x = \pi$.

Markscheme

evidence of choosing the product rule ***(M1)***

$f'(x) = e^x \times (-\sin x) + \cos x \times e^x\ (= e^x \cos x - e^x \sin x)$ ***A1A1***

substituting π ***(M1)***

e.g. $f'(\pi) = e^\pi \cos \pi - e^\pi \sin \pi$, $e^\pi(-1 - 0)$, $-e^\pi$

taking negative reciprocal ***(M1)***

e.g. $-\frac{1}{f'(\pi)}$

gradient is $\frac{1}{e^\pi}$ ***A1 N3***

[6 marks]

Examiners report

Candidates familiar with the product rule easily found the correct derivative function. Many substituted π to find the tangent gradient, but surprisingly few candidates correctly considered that the gradient of the normal is the negative reciprocal of this answer.

Let $h(x) = \frac{6x}{\cos x}$. Find $h'(0)$.

Markscheme

METHOD 1 (quotient)

derivative of numerator is 6 ***(A1)***

derivative of denominator is $-\sin x$ ***(A1)***

attempt to substitute into quotient rule ***(M1)***

correct substitution ***A1***

e.g. $\frac{(\cos x)(6)-(6x)(-\sin x)}{(\cos x)^2}$

substituting $x=0$ ***(A1)***

e.g. $\frac{(\cos 0)(6)-(6\times 0)(-\sin 0)}{(\cos 0)^2}$

$h'(0)=6$ ***A1*** ***N2***

METHOD 2 (product)

$h(x)=6x\times(\cos x)^{-1}$

derivative of $6x$ is 6 ***(A1)***

derivative of $(\cos x)^{-1}$ is $(-(\cos x)^{-2}(-\sin x))$ ***(A1)***

attempt to substitute into product rule **(M1)**

correct substitution ***A1***

e.g. $(6x)(-(\cos x)^{-2}(-\sin x))+(6)(\cos x)^{-1}$

substituting $x=0$ ***(A1)***

e.g. $(6\times 0)(-(\cos 0)^{-2}(-\sin 0))+(6)(\cos 0)^{-1}$

$h'(0)=6$ ***A1*** ***N2***

[6 marks]

Examiners report

The majority of candidates were successful in using the quotient rule, and were able to earn most of the marks for this question. However, there were a large number of candidates who substituted correctly into the quotient rule, but then went on to make mistakes in simplifying this expression. These algebraic errors kept the candidates from earning the final mark for the correct answer. A few candidates tried to use the product rule to find the derivative, but they were generally not as successful as those who used the quotient rule. It was pleasing to note that most candidates did know the correct values for the sine and cosine of zero.

A function $f(x)$ has derivative $f'(x) = 3x^2 + 18x$. The graph of f has an x-intercept at $x = -1$.

a. Find $f(x)$. [6]

b. The graph of f has a point of inflexion at $x = p$. Find p. [4]

c. Find the values of x for which the graph of f is concave-down. [3]

Markscheme

a. evidence of integration ***(M1)***

eg $\int f'(x)$

correct integration (accept absence of *C*) ***(A1)(A1)***

eg $x^3 + \frac{18}{2}x^2 + C,\ x^3 + 9x^2$

attempt to substitute *x* = −1 into **their** *f* = 0 (must have *C*) ***M1***

eg $(-1)^3 + 9(-1)^2 + C = 0,\ -1 + 9 + C = 0$

Note: Award ***M0*** if they substitute into original or differentiated function.

correct working ***(A1)***

eg $8 + C = 0,\ \ C = -8$

$f(x) = x^3 + 9x^2 - 8$ ***A1 N5***

[6 marks]

b. **METHOD 1** (using 2nd derivative)

recognizing that *f*" = 0 (seen anywhere) ***M1***

correct expression for *f*" ***(A1)***

eg 6*x* + 18, 6*p* + 18

correct working ***(A1)***

6*p* + 18 = 0

p = −3 ***A1 N3***

METHOD 1 (using 1st derivative)

recognizing the vertex of *f′* is needed ***(M2)***

eg $-\frac{b}{2a}$ (must be clear this is for *f′*)

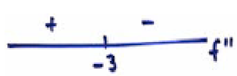
correct substitution ***(A1)***

eg $\frac{-18}{2\times 3}$

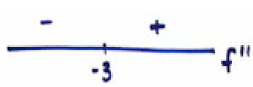
p = −3 ***A1 N3***

[4 marks]

c. valid attempt to use *f*"(*x*) to determine concavity ***(M1)***

eg *f*"(*x*) < 0, *f*"(−2), *f*"(−4), 6*x* + 18 ≤ 0 

correct working ***(A1)***

eg 6*x* + 18 < 0, *f*"(−2) = 6, *f*"(−4) = −6 

f concave down for *x* < −3 (do not accept *x* ≤ −3) ***A1 N2***

[3 marks]

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]

Let $f(x) = \sin x + \frac{1}{2}x^2 - 2x$, for $0 \le x \le \pi$.

Let g be a quadratic function such that $g(0) = 5$. The line $x = 2$ is the axis of symmetry of the graph of g .

The function g can be expressed in the form $g(x) = a(x - h)^2 + 3$.

a. Find $f'(x)$. [3]

b. Find $g(4)$. [3]

c. (i) Write down the value of h . [4]

(ii) Find the value of a .

d. Find the value of x for which the tangent to the graph of f is parallel to the tangent to the graph of g . [6]

Markscheme

a. $f'(x) = \cos x + x - 2$ ***A1A1A1*** ***N3***

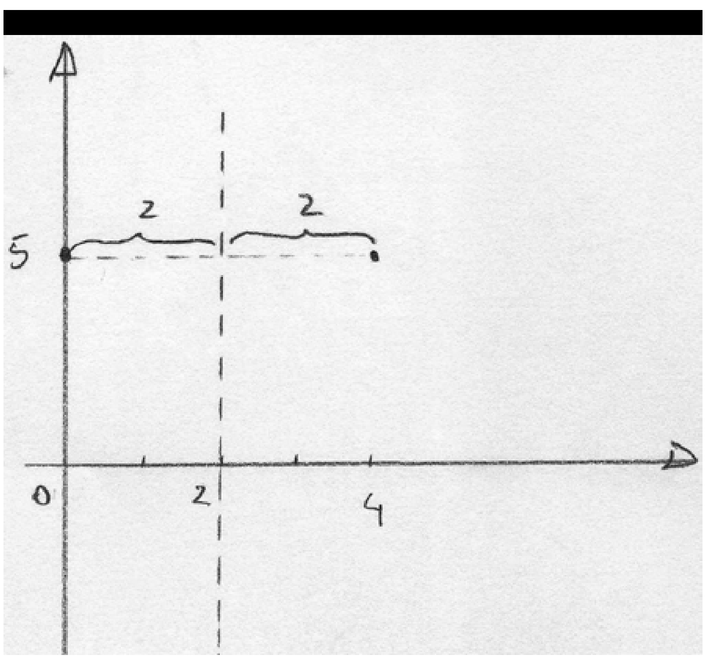
Note: Award ***A1*** for each term.

[3 marks]

b. recognizing $g(0) = 5$ gives the point (0, 5) ***(R1)***

recognize symmetry ***(M1)***

eg vertex, sketch



$g(4) = 5$ ***A1*** ***N3***

[3 marks]

c. (i) $h = 2$ ***A1 N1***

(ii) substituting into $g(x) = a(x - 2)^2 + 3$ (not the vertex) ***(M1)***

eg $5 = a(0 - 2)^2 + 3$, $5 = a(4 - 2)^2 + 3$

working towards solution ***(A1)***

eg $5 = 4a + 3$, $4a = 2$

$a = \frac{1}{2}$ ***A1*** ***N2***

[4 marks]

d. $g(x) = \frac{1}{2}(x-2)^2 + 3 = \frac{1}{2}x^2 - 2x + 5$

correct derivative of g ***A1A1***

eg $2 \times \frac{1}{2}(x-2)$, $x - 2$

evidence of equating both derivatives ***(M1)***

eg $f' = g'$

correct equation ***(A1)***

eg $\cos x + x - 2 = x - 2$

working towards a solution ***(A1)***

eg $\cos x = 0$, combining like terms

$x = \frac{\pi}{2}$ ***A1*** ***N0***

Note: Do not award final ***A1*** if additional values are given.

[6 marks]

Examiners report

a. In part (a), most candidates were able to correctly find the derivative of the function.

b. In part (b), many candidates did not understand the significance of the axis of symmetry and the known point (0, 5), and so were unable to find $g(4)$ using symmetry. A few used more complicated manipulations of the function, but many algebraic errors were seen.

c. In part (c), a large number of candidates were able to simply write down the correct value of h, as intended by the command term in this question. A few candidates wrote down the incorrect negative value. Most candidates attempted to substitute the x and y values of the known point correctly into the function, but again many arithmetic and algebraic errors kept them from finding the correct value for a.

d. Part (d) required the candidates to find the derivative of g, and to equate that to their answer from part (a). Although many candidates were able to simplify their equation to $\cos x = 0$, many did not know how to solve for x at this point. Candidates who had made errors in parts (a) and/or (c) were still able to earn follow-through marks in part (d).