$$H_{\mathrm{HF_{coupled}}} = \begin{bmatrix}
\frac{\hbar^2(A_{z1z}+A_{z2z}+A_{z1z}+A_{z2z})}{4} & 0 & 0 & 0 & 0 & \frac{\sqrt{2}\hbar^2(A_{z2z}-A_{z2y}+A_{z2x}-A_{z2y})}{8} & \frac{\sqrt{2}\hbar^2(A_{z1z}-A_{z1y}+A_{z1x}-A_{z1y})}{8} & 0 & \frac{\sqrt{2}\hbar^2(A_{z1z}-A_{z1y}+A_{z1x}-A_{z1y})}{8} & 0 & \frac{\sqrt{2}\hbar^2(-A_{z2x}+A_{z2y}+A_{z2x}-A_{z2y})}{8} & \frac{\sqrt{2}\hbar^2(-A_{z1x}+A_{z1y}+A_{z1x}-A_{z1y})}{8} & 0 & 0 & 0 & 0 \\
0 & \frac{\hbar^2(A_{z1z}-A_{z2z}+A_{z1z}-A_{z2z})}{4} & 0 & 0 & \frac{\sqrt{2}\hbar^2(A_{z2x}+A_{z2y}+A_{z2x}+A_{z2y})}{8} & \frac{\sqrt{2}\hbar^2(A_{z1x}-A_{z1y}+A_{z1x}-A_{z1y})}{8} & 0 & \frac{\sqrt{2}\hbar^2(A_{z1x}-A_{z1y}+A_{z1x}-A_{z1y})}{8} & 0 & \frac{\sqrt{2}\hbar^2(-A_{z2x}+A_{z2y}+A_{z2x}-A_{z2y})}{8} & \frac{\sqrt{2}\hbar^2(-A_{z1x}+A_{z1y}+A_{z1x}-A_{z1y})}{8} & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{\hbar^2(-A_{z1z}+A_{z2z}-A_{z1z}+A_{z2z})}{4} & 0 & 0 & 0 & 0 & \frac{\hbar^2(A_{z1z}-A_{z2z}+A_{z1z}-A_{z2z})}{4} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{\hbar^2(A_{z1z}+A_{z2z}+A_{z1z}+A_{z2z})}{4} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{\sqrt{2}\hbar^2(A_{z2x}-A_{z2y}+A_{z2x}-A_{z2y})}{8} & 0 & 0 & 0 & \frac{\sqrt{2}\hbar^2(A_{z2x}+A_{z2y}+A_{z2x}+A_{z2y})}{8} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{\sqrt{2}\hbar^2(A_{z1x}-A_{z1y}+A_{z1x}-A_{z1y})}{8} & 0 & 0 & 0 & 0 & \frac{\sqrt{2}\hbar^2(A_{z1x}+A_{z1y}+A_{z1x}+A_{z1y})}{8} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{\sqrt{2}\hbar^2(A_{z1x}-A_{z1y}+A_{z1x}-A_{z1y})}{8} & 0 & 0 & \frac{\sqrt{2}\hbar^2(A_{z2x}-A_{z2y}+A_{z2x}-A_{z2y})}{8} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{\sqrt{2}\hbar^2(-A_{z2x}+A_{z2y}+A_{z2x}-A_{z2y})}{8} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{\sqrt{2}\hbar^2(-A_{z1x}+A_{z1y}+A_{z1x}-A_{z1y})}{8} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{\sqrt{2}\hbar^2(-A_{z1x}+A_{z1y}+A_{z1x}-A_{z1y})}{8} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{\sqrt{2}\hbar^2(-A_{z2x}+A_{z2y}+A_{z2x}-A_{z2y})}{8} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{\sqrt{2}\hbar^2(-A_{z1x}-A_{z1y}+A_{z1x}+A_{z1y})}{8} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}$$

$$\begin{aligned}
&\left|1,1\right\rangle \otimes \left|+\tfrac{1}{2},+\tfrac{1}{2}\right\rangle \\
&\left|1,0\right\rangle \otimes \left|+\tfrac{1}{2},+\tfrac{1}{2}\right\rangle \\
&\left|0,0\right\rangle \otimes \left|+\tfrac{1}{2},+\tfrac{1}{2}\right\rangle \\
&\left|1,-1\right\rangle \otimes \left|+\tfrac{1}{2},+\tfrac{1}{2}\right\rangle \\
&\left|1,1\right\rangle \otimes \left|+\tfrac{1}{2},-\tfrac{1}{2}\right\rangle \\
&\left|1,0\right\rangle \otimes \left|+\tfrac{1}{2},-\tfrac{1}{2}\right\rangle \\
&\left|0,0\right\rangle \otimes \left|+\tfrac{1}{2},-\tfrac{1}{2}\right\rangle \\
&\left|1,-1\right\rangle \otimes \left|+\tfrac{1}{2},-\tfrac{1}{2}\right\rangle \\
&\left|1,1\right\rangle \otimes \left|-\tfrac{1}{2},+\tfrac{1}{2}\right\rangle \\
&\left|1,0\right\rangle \otimes \left|-\tfrac{1}{2},+\tfrac{1}{2}\right\rangle \\
&\left|0,0\right\rangle \otimes \left|-\tfrac{1}{2},+\tfrac{1}{2}\right\rangle \\
&\left|1,-1\right\rangle \otimes \left|-\tfrac{1}{2},+\tfrac{1}{2}\right\rangle \\
&\left|1,1\right\rangle \otimes \left|-\tfrac{1}{2},-\tfrac{1}{2}\right\rangle \\
&\left|1,0\right\rangle \otimes \left|-\tfrac{1}{2},-\tfrac{1}{2}\right\rangle \\
&\left|0,0\right\rangle \otimes \left|-\tfrac{1}{2},-\tfrac{1}{2}\right\rangle \\
&\left|1,-1\right\rangle \otimes \left|-\tfrac{1}{2},-\tfrac{1}{2}\right\rangle
\end{aligned}$$

$$H_{\mathrm{HF,iso}} = \begin{bmatrix}
\frac{\hbar^2(Aa_1+Aa_2+Ab_1+Ab_2)}{4} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{\hbar^2(Aa_1-Aa_2+Ab_1-Ab_2)}{4} & 0 & 0 & \frac{\hbar^2(Aa_2+Ab_2)}{4\sqrt{2}} & 0 & 0 & \frac{\hbar^2(Aa_1+Ab_1)}{4\sqrt{2}} & -\frac{\hbar^2(Aa_2+Ab_2)}{4\sqrt{2}} & 0 & 0 & -\frac{\hbar^2(Aa_1+Ab_1)}{4\sqrt{2}} & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{\hbar^2(-Aa_1+Aa_2-Ab_1+Ab_2)}{4} & 0 & \frac{\hbar^2(Aa_1+Ab_1)}{4\sqrt{2}} & 0 & 0 & \frac{\hbar^2(Aa_2+Ab_2)}{4\sqrt{2}} & -\frac{\hbar^2(Aa_1+Ab_1)}{4\sqrt{2}} & 0 & 0 & -\frac{\hbar^2(Aa_2+Ab_2)}{4\sqrt{2}} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -\frac{\hbar^2(Aa_1+Aa_2+Ab_1+Ab_2)}{4} & 0 & \frac{\hbar^2(Aa_1+Ab_1)}{4\sqrt{2}} & \frac{\hbar^2(Aa_2+Ab_2)}{4\sqrt{2}} & 0 & 0 & -\frac{\hbar^2(Aa_1+Ab_1)}{4\sqrt{2}} & -\frac{\hbar^2(Aa_2+Ab_2)}{4\sqrt{2}} & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{\hbar^2(Aa_2+Ab_2)}{4\sqrt{2}} & \frac{\hbar^2(Aa_1+Ab_1)}{4\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & \frac{\hbar^2(Aa_1+Aa_2-Ab_1-Ab_2)}{4} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{\hbar^2(Aa_1+Aa_2+Ab_1+Ab_2)}{4\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & \frac{\hbar^2(Aa_1-Aa_2-Ab_1+Ab_2)}{4} & 0 & 0 & \frac{\hbar^2(Aa_2+Ab_2)}{4\sqrt{2}} & 0 & 0 & \frac{\hbar^2(Aa_1+Ab_1)}{4\sqrt{2}} \\
0 & 0 & 0 & \frac{\hbar^2(Aa_2+Ab_2)}{4\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{\hbar^2(-Aa_1+Aa_2+Ab_1-Ab_2)}{4} & 0 & \frac{\hbar^2(Aa_1+Ab_1)}{4\sqrt{2}} & 0 & 0 & \frac{\hbar^2(Aa_2+Ab_2)}{4\sqrt{2}} \\
0 & -\frac{\hbar^2(Aa_1+Ab_1)}{4\sqrt{2}} & -\frac{\hbar^2(Aa_2+Ab_2)}{4\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{\hbar^2(-Aa_1-Aa_2+Ab_1+Ab_2)}{4} & 0 & \frac{\hbar^2(Aa_1+Ab_1)}{4\sqrt{2}} & \frac{\hbar^2(Aa_2+Ab_2)}{4\sqrt{2}} & 0 \\
0 & 0 & 0 & 0 & \frac{\hbar^2(Aa_2+Ab_2)}{4\sqrt{2}} & \frac{\hbar^2(Aa_1+Ab_1)}{4\sqrt{2}} & 0 & 0 & -\frac{\hbar^2(Aa_2+Ab_2)}{4\sqrt{2}} & -\frac{\hbar^2(Aa_1+Ab_1)}{4\sqrt{2}} & 0 & 0 & -\frac{\hbar^2(Aa_1+Aa_2+Ab_1+Ab_2)}{4} & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{\hbar^2(Aa_2+Ab_2)}{4\sqrt{2}} & 0 & 0 & \frac{\hbar^2(Aa_1+Ab_1)}{4\sqrt{2}} & \frac{\hbar^2(Aa_2+Ab_2)}{4\sqrt{2}} & 0 & 0 & \frac{\hbar^2(Aa_1+Ab_1)}{4\sqrt{2}} & 0 & \frac{\hbar^2(-Aa_1+Aa_2-Ab_1+Ab_2)}{4} & 0 \\
0 & 0 & 0 & 0 & \frac{\hbar^2(Aa_1+Ab_1)}{4\sqrt{2}} & 0 & 0 & \frac{\hbar^2(Aa_2+Ab_2)}{4\sqrt{2}} & \frac{\hbar^2(Aa_1+Ab_1)}{4\sqrt{2}} & 0 & 0 & \frac{\hbar^2(Aa_2+Ab_2)}{4\sqrt{2}} & 0 & 0 & \frac{\hbar^2(Aa_1-Aa_2+Ab_1-Ab_2)}{4} & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{\hbar^2(Aa_1+Ab_1)}{4\sqrt{2}} & \frac{\hbar^2(Aa_2+Ab_2)}{4\sqrt{2}} & 0 & 0 & \frac{\hbar^2(Aa_1+Ab_1)}{4\sqrt{2}} & \frac{\hbar^2(Aa_2+Ab_2)}{4\sqrt{2}} & 0 & 0 & 0 & 0 & \frac{\hbar^2(Aa_1+Aa_2+Ab_1+Ab_2)}{4}
\end{bmatrix}$$

$$H_{\mathrm{HF}_{____}} = \begin{bmatrix}
h^2\left(\frac{A_{0000}}{4}+\frac{A_{0033}}{4}+\frac{A_{3300}}{4}+\frac{A_{3333}}{4}\right) & 0 & 0 & 0 & 0 & \frac{h^2(A_{0032}-A_{0023})}{4} & \frac{h^2(A_{0132}-A_{0123})}{4} & 0 & 0 & \frac{h^2(A_{0232}-A_{0223})}{4} & \frac{h^2(A_{0132}-A_{0123})}{4} & 0 & 0 & 0 & 0 & 0 \\
0 & h^2\left(\frac{A_{0000}}{4}-\frac{A_{0033}}{4}+\frac{A_{3300}}{4}-\frac{A_{3333}}{4}\right) & 0 & 0 & \frac{h^2(A_{0032}+A_{0023})}{4} & 0 & 0 & \frac{h^2(A_{0132}-A_{0123})}{4} & \frac{h^2(A_{0232}+A_{0223})}{4} & 0 & 0 & \frac{h^2(A_{0132}-A_{0123})}{4} & 0 & 0 & 0 & 0 \\
0 & 0 & h^2\left(-\frac{A_{0000}}{4}+\frac{A_{0033}}{4}-\frac{A_{3300}}{4}+\frac{A_{3333}}{4}\right) & 0 & \frac{h^2(A_{0132}+A_{0123})}{4} & 0 & 0 & \frac{h^2(A_{0132}-A_{0123})}{4} & \frac{h^2(A_{0232}+A_{0223})}{4} & 0 & 0 & \frac{h^2(A_{0232}-A_{0223})}{4} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & h^2\left(-\frac{A_{0000}}{4}-\frac{A_{0033}}{4}-\frac{A_{3300}}{4}-\frac{A_{3333}}{4}\right) & 0 & \frac{h^2(A_{0132}+A_{0123})}{4} & \frac{h^2(A_{0232}+A_{0223})}{4} & 0 & 0 & \frac{h^2(A_{0132}+A_{0123})}{4} & \frac{h^2(A_{0232}+A_{0223})}{4} & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{h^2(A_{0032}+A_{0023})}{4} & \frac{h^2(A_{0132}+A_{0123})}{4} & 0 & h^2\left(\frac{A_{0000}}{4}+\frac{A_{0033}}{4}-\frac{A_{3300}}{4}-\frac{A_{3333}}{4}\right) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{h^2(A_{0232}-A_{0223})}{4} & \frac{h^2(A_{0132}-A_{0123})}{4} & 0 \\
\frac{h^2(A_{0032}-A_{0023})}{4} & 0 & 0 & \frac{h^2(A_{0132}+A_{0123})}{4} & 0 & h^2\left(\frac{A_{0000}}{4}-\frac{A_{0033}}{4}-\frac{A_{3300}}{4}+\frac{A_{3333}}{4}\right) & 0 & 0 & 0 & 0 & 0 & 0 & \frac{h^2(A_{0232}+A_{0223})}{4} & 0 & 0 & \frac{h^2(A_{0132}-A_{0123})}{4} \\
\frac{h^2(A_{0132}-A_{0123})}{4} & 0 & 0 & \frac{h^2(A_{0232}+A_{0223})}{4} & 0 & 0 & h^2\left(-\frac{A_{0000}}{4}+\frac{A_{0033}}{4}+\frac{A_{3300}}{4}-\frac{A_{3333}}{4}\right) & 0 & 0 & 0 & 0 & 0 & \frac{h^2(A_{0232}+A_{0223})}{4} & 0 & 0 & \frac{h^2(A_{0232}-A_{0223})}{4} \\
0 & \frac{h^2(A_{0132}-A_{0123})}{4} & \frac{h^2(A_{0232}-A_{0223})}{4} & 0 & 0 & 0 & 0 & h^2\left(-\frac{A_{0000}}{4}-\frac{A_{0033}}{4}+\frac{A_{3300}}{4}+\frac{A_{3333}}{4}\right) & 0 & 0 & 0 & 0 & 0 & \frac{h^2(A_{0132}+A_{0123})}{4} & \frac{h^2(A_{0232}+A_{0223})}{4} & 0 \\
0 & \frac{h^2(A_{0232}+A_{0223})}{4} & \frac{h^2(A_{0132}+A_{0123})}{4} & 0 & 0 & 0 & 0 & h^2\left(-\frac{A_{0000}}{4}-\frac{A_{0033}}{4}+\frac{A_{3300}}{4}+\frac{A_{3333}}{4}\right) & 0 & 0 & 0 & 0 & 0 & \frac{h^2(A_{0232}+A_{0223})}{4} & \frac{h^2(A_{0132}-A_{0123})}{4} & 0 \\
\frac{h^2(A_{0232}-A_{0223})}{4} & 0 & 0 & \frac{h^2(A_{0132}+A_{0123})}{4} & 0 & 0 & 0 & 0 & h^2\left(-\frac{A_{0000}}{4}+\frac{A_{0033}}{4}+\frac{A_{3300}}{4}-\frac{A_{3333}}{4}\right) & 0 & 0 & 0 & 0 & 0 & \frac{h^2(A_{0132}+A_{0123})}{4} & \frac{h^2(A_{0232}-A_{0223})}{4} \\
\frac{h^2(A_{0132}-A_{0123})}{4} & 0 & 0 & \frac{h^2(A_{0232}+A_{0223})}{4} & 0 & 0 & 0 & 0 & 0 & h^2\left(\frac{A_{0000}}{4}+\frac{A_{0033}}{4}+\frac{A_{3300}}{4}+\frac{A_{3333}}{4}\right) & 0 & 0 & 0 & 0 & \frac{h^2(A_{0232}+A_{0223})}{4} & \frac{h^2(A_{0132}-A_{0123})}{4} \\
0 & \frac{h^2(A_{0232}-A_{0223})}{4} & \frac{h^2(A_{0132}-A_{0123})}{4} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & h^2\left(\frac{A_{0000}}{4}+\frac{A_{0033}}{4}-\frac{A_{3300}}{4}-\frac{A_{3333}}{4}\right) & 0 & 0 & 0 & 0 & \frac{h^2(A_{0132}+A_{0123})}{4} \\
0 & 0 & 0 & 0 & \frac{h^2(A_{0232}+A_{0223})}{4} & 0 & 0 & 0 & 0 & \frac{h^2(A_{0132}+A_{0123})}{4} & \frac{h^2(A_{0232}+A_{0223})}{4} & 0 & h^2\left(-\frac{A_{0000}}{4}+\frac{A_{0033}}{4}-\frac{A_{3300}}{4}+\frac{A_{3333}}{4}\right) & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{h^2(A_{0132}+A_{0123})}{4} & 0 & 0 & \frac{h^2(A_{0232}+A_{0223})}{4} & \frac{h^2(A_{0132}-A_{0123})}{4} & 0 & 0 & \frac{h^2(A_{0232}+A_{0223})}{4} & 0 & h^2\left(\frac{A_{0000}}{4}-\frac{A_{0033}}{4}+\frac{A_{3300}}{4}-\frac{A_{3333}}{4}\right) & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{h^2(A_{0132}+A_{0123})}{4} & \frac{h^2(A_{0232}+A_{0223})}{4} & 0 & 0 & \frac{h^2(A_{0132}-A_{0123})}{4} & \frac{h^2(A_{0232}+A_{0223})}{4} & 0 & 0 & 0 & h^2\left(\frac{A_{0000}}{4}+\frac{A_{0033}}{4}+\frac{A_{3300}}{4}+\frac{A_{3333}}{4}\right) & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{h^2(A_{0232}+A_{0223})}{4} & \frac{h^2(A_{0132}-A_{0123})}{4} & 0 & 0 & \frac{h^2(A_{0232}-A_{0223})}{4} & \frac{h^2(A_{0132}-A_{0123})}{4} & 0 & 0 & 0 & 0 & h^2\left(\frac{A_{0000}}{4}+\frac{A_{0033}}{4}+\frac{A_{3300}}{4}+\frac{A_{3333}}{4}\right)
\end{bmatrix}$$

$$\begin{bmatrix}
\left|+\frac{1}{2},+\frac{1}{2},+\frac{1}{2},+\frac{1}{2}\right\rangle \\
\left|+\frac{1}{2},+\frac{1}{2},+\frac{1}{2},-\frac{1}{2}\right\rangle \\
\left|+\frac{1}{2},+\frac{1}{2},-\frac{1}{2},+\frac{1}{2}\right\rangle \\
\left|+\frac{1}{2},+\frac{1}{2},-\frac{1}{2},-\frac{1}{2}\right\rangle \\
\left|+\frac{1}{2},-\frac{1}{2},+\frac{1}{2},+\frac{1}{2}\right\rangle \\
\left|+\frac{1}{2},-\frac{1}{2},+\frac{1}{2},-\frac{1}{2}\right\rangle \\
\left|+\frac{1}{2},-\frac{1}{2},-\frac{1}{2},+\frac{1}{2}\right\rangle \\
\left|+\frac{1}{2},-\frac{1}{2},-\frac{1}{2},-\frac{1}{2}\right\rangle \\
\left|-\frac{1}{2},+\frac{1}{2},+\frac{1}{2},+\frac{1}{2}\right\rangle \\
\left|-\frac{1}{2},+\frac{1}{2},+\frac{1}{2},-\frac{1}{2}\right\rangle \\
\left|-\frac{1}{2},+\frac{1}{2},-\frac{1}{2},+\frac{1}{2}\right\rangle \\
\left|-\frac{1}{2},+\frac{1}{2},-\frac{1}{2},-\frac{1}{2}\right\rangle \\
\left|-\frac{1}{2},-\frac{1}{2},+\frac{1}{2},+\frac{1}{2}\right\rangle \\
\left|-\frac{1}{2},-\frac{1}{2},+\frac{1}{2},-\frac{1}{2}\right\rangle \\
\left|-\frac{1}{2},-\frac{1}{2},-\frac{1}{2},+\frac{1}{2}\right\rangle \\
\left|-\frac{1}{2},-\frac{1}{2},-\frac{1}{2},-\frac{1}{2}\right\rangle
\end{bmatrix}$$