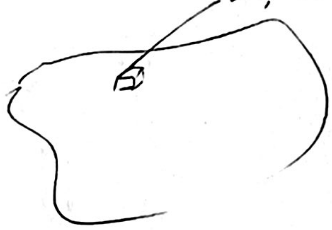
$$\nabla \cdot \vec{\mathcal{E}} = \frac{\rho}{\varepsilon_0}$$

Calculating $\vec{\mathcal{E}}$

① Coulomb's Law + Lorentz $\longrightarrow$ $\vec{F} = \dfrac{k q_1 q_2}{r^2} = q_1 \vec{\mathcal{E}}$

$$d\vec{F} = \frac{k\, dq}{r^2}\,\hat{r}$$

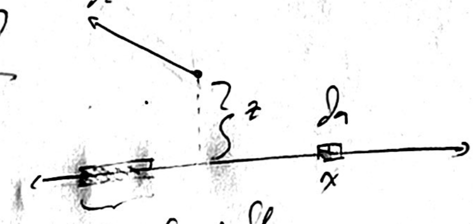


$dv,\ dq$

$$\mathcal{E} = \iiint \frac{\rho}{r^2}\,\hat{r}\,dV$$

Infinite Line of Charge

1) Coulomb's Method



$d\vec{\mathcal{E}}$, z, dq, x

$dl,\ dq = \lambda\, dl$

$d\vec{\mathcal{E}}$, θ, θ, z, x, dq

$$d\mathcal{E}_z = |d\vec{\mathcal{E}}|\cos\theta$$

$$= |d\vec{\mathcal{E}}|\,\frac{z}{\sqrt{x^2+z^2}}$$

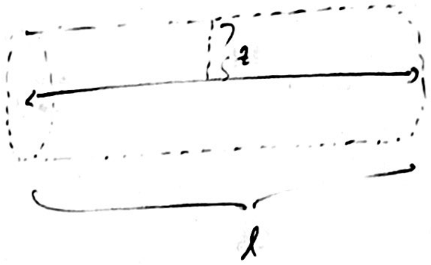
$$= \frac{k\,dq}{x^2+z^2}\cdot\frac{z}{\sqrt{x^2+z^2}}$$

$$= k(\lambda\,dx)\cdot\frac{z}{(x^2+z^2)^{3/2}}$$

$$\mathcal{E}_z = \int_{-\infty}^{\infty} \frac{k z \lambda}{(x^2+z^2)^{3/2}}\,dx = \frac{2k\lambda}{z}$$

2.

Gauss's Law



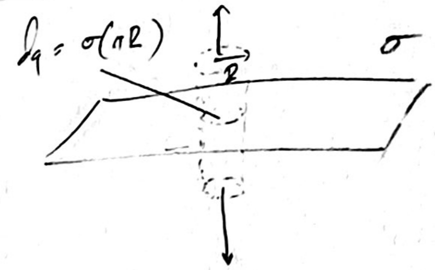
$$\oiint_S \vec{E} \cdot dS = \frac{Q_{enc}}{\varepsilon_0}$$

$$|\vec{E}|(2\pi z \cancel{l}) = \frac{\lambda \cancel{l}}{\varepsilon_0}$$

$$E = \frac{\lambda}{2\pi z \varepsilon_0}$$

Infinite Plane of Charge

1. Gauss's Law



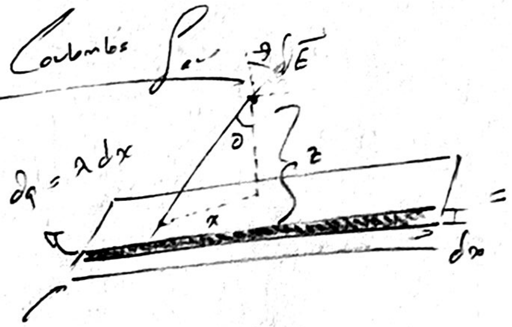
$$\oiint_S \vec{E} \cdot \hat{n}\, d\sigma = \frac{Q_{enc}}{\varepsilon_0}$$

$$= E(z)(2\pi \cancel{R^2}) = \frac{\pi \cancel{R^2} \sigma}{\varepsilon_0}$$

$$\longrightarrow E(z) = \frac{\sigma}{2\varepsilon_0}$$

$$E_{wire} = \frac{\lambda}{2\pi\varepsilon_0 r} \quad \sqrt{x^2+z^2}$$

2. Coulombs Law

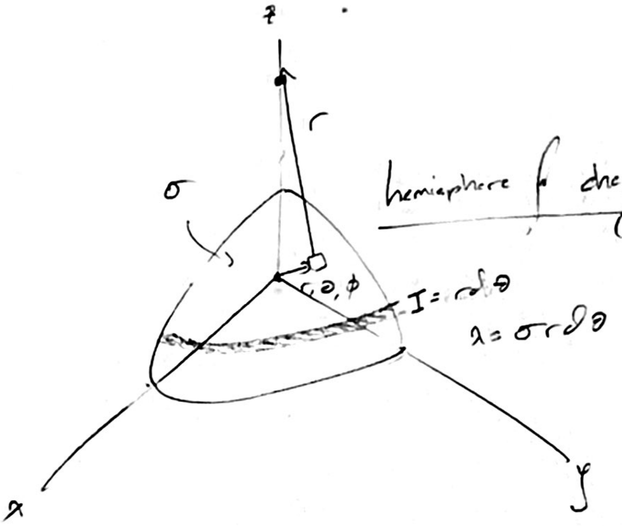


$$dE_z = |dE| \cos\theta$$

$$= \frac{\lambda}{2\pi\varepsilon_0 \sqrt{x^2+z^2}} \cdot \frac{z}{\sqrt{x^2+z^2}}$$

$$= \frac{\sigma\, dx}{2\pi\varepsilon_0} \cdot \frac{z}{x^2+z^2}$$

$$E_z = \int_{-\infty}^{\infty} \frac{\sigma z}{2\pi\varepsilon_0 (x^2+z^2)}\, dx$$



hemisphere of charge $\quad E = \frac{k\rho\, dA}{r^2}$ ← ugly $\int_L$

Oscillating Charge in a Ring

$$U(x) = \frac{kq\lambda}{R^2}$$

$$dU(x) = \frac{kq(\lambda\, dl)}{R} = \frac{kq\lambda}{R}\, dl, \quad U_{total} = \int_0^{2\pi} \frac{kq\lambda}{R}\, dl = \frac{2\pi kq\lambda}{R}$$

$$dU(x+dr) = \frac{kq(\lambda\, dl)}{(R^2 + dr^2 - 2R\,dr\cos\theta)^{1/2}} = \frac{kq\lambda}{(R^2 + dr^2 - 2R\,dr\cos\theta)^{1/2}}\, dl$$

$$U(x+dr) = \int_0^{2\pi} \frac{kq\lambda}{(R^2 + dr^2 - 2R\,dr\cos\theta)^{1/2}}\, dl \qquad dl = R\,d\theta$$

$$\vec{F} = -\frac{dU}{dx} = -\frac{d}{dx}\int_0^{2\pi} \frac{kq\lambda\,(R\,d\theta)}{\sqrt{R^2 + dr^2 - 2R\,dr\cos\theta}} \qquad \text{Given } \frac{1}{\sqrt{1+\varepsilon}} = 1 - \frac{1}{2}\varepsilon + \frac{3}{4}\varepsilon^2 + \dots$$

$$= -\frac{d}{dx}\int_0^{2\pi} \left\{1 - \frac{1}{2}\left(\frac{r^2}{R^2} - \frac{2r}{R}\cos\theta\right) + \frac{3}{8}\left(-\frac{2r}{R}\cos\theta\right)^2 + \dots\right\} d\theta$$

$$\boxed{\omega = \frac{q\lambda}{4\pi\varepsilon_0 m R}}$$

$$= -\frac{d}{dr}\left\{\frac{q\lambda}{2\varepsilon_0} + \frac{q\lambda r^2}{8\varepsilon_0 R^2}\right\} = -\frac{q\lambda}{4\varepsilon_0 R^2}\, r = m\ddot{r}$$

↑ k

$$f = \frac{1}{2\pi}\sqrt{\frac{m}{\frac{q\lambda}{4\varepsilon_0 R^2}}}$$