5B - Introductory Electromagnetism, Waves, and Optics

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Contents

1 Maxwell's Equations

$$\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0}$$

$$\nabla \times \vec{B} = \mu_0 \vec{J} + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \cdot \vec{B} = 0$$

The goal of this course will be to understand Maxwell's Equations, and the unison between the electric and magnetic field.

Lorent'z Force eq

$$\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$$

Rules

1. Charge in nature is quantized in units of e
2. Charge is conserved
3. Charge has 2 types: $\pm$

$\vec{E}$ and $\vec{B}$ are *vector fields*. This means $\vec{E}$ is a function of every point in space: $\vec{E}(x, y, z)$

$$\vec{E}(r) = E_x(x, y, z)\hat{i} + E_y(x, y, z)\hat{j} + E_z(x, y, z)\hat{k}$$

∇ is a vector operator:

$$\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial x}\hat{k}$$

Vector Operations

$$\vec{A}\alpha \rightarrow \vec{B}$$
$$\vec{A} \cdot \vec{B} \rightarrow \alpha$$
$$\vec{A} \times \vec{B} \rightarrow \vec{C}$$

Consider a scalar field: $\varphi(\vec{r})$

$$\begin{aligned}
d\varphi &= \varphi(r + dr) - \varphi(r) \\
d\varphi &= \frac{\partial\varphi(x, y, z)}{\partial x}dx + \frac{\partial\varphi(x, y, z)}{\partial y}dy + \frac{\partial\varphi(x, y, z)}{\partial z}dz \\
d\vec{r} &= \hat{i}dx + \hat{y}dy + \hat{z}dz
\end{aligned}$$

$$d\varphi = \nabla\varphi(\vec{r}) \cdot d\vec{r}$$

$\nabla\varphi =$ "gradient of φ. " Also $\nabla \cdot \vec{E} = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z}$. This is known as the **divergence** of φ.

The **curl** of φ is equal to

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ P & Q & R \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \end{vmatrix}$$

$$\rho = \text{charge density} = \frac{\text{number of particles} q}{dV} = nq$$

where $n = \frac{\text{number of particles}}{dv}$

$$\vec{J} = \text{current density}$$

2 Statics

Equations of Electrostatics

$$\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0}$$
$$\nabla \times \vec{E} = 0$$

Equations of Magnetostatics

$$\nabla \cdot \vec{B} = 0$$
$$\nabla \times \vec{B} = \mu_0 \vec{J}$$

2.1 Flux

"Flux" = Flow

Consider a fluid flow with a velocity vector field $\vec{v}(\vec{r})i$, flowing into a small aperture defined by $\hat{n}\, d\vec{a}$, where $\hat{n}$ is the unit normal vector to the aperture, and $d\vec{a}$ is the area.

$$d\Phi = \vec{v} \cdot d\vec{a}$$

Relating to the Electric field,

$$d\Phi_E = \vec{E}(\vec{r}) \cdot d\vec{a}$$
$$\int_S d\Phi_E = \Phi_E = \int_S \vec{E}(\vec{r}) \cdot d\vec{a}$$

Green's, Gauss's, Divergence Theorem

$$\int_S \vec{E} \cdot d\vec{a} = \iiint_S \nabla \cdot \vec{E}(\vec{r})\, d^3 r$$