Physics 5B

Assignment 3

1.61. Potential Energy of a Sphere

(1) $\mathcal{U} = \frac{1}{2}\epsilon_0 \int_V \vec{E}\cdot\vec{E}\, dV$

(2) $\oint \vec{E}\cdot d\vec{a} = \frac{q_{enc}}{\epsilon_0} = \frac{\rho\left(\frac{r}{R}\right)^3}{\epsilon_0}$

$E(4\pi r^2) = \frac{\rho\left(\frac{r}{R}\right)^3}{\epsilon_0} \Longrightarrow E = \frac{\rho r^{\cancel{3}}}{4\pi \cancel{r^2} R^3 \epsilon_0} = \frac{\rho r}{4\pi R^3 \epsilon_0}$

$\vec{E}\cdot\vec{E} = E^2 = \frac{\rho^2 r^2}{4^2\pi^2 R^6 \epsilon_0^2}$

$\mathcal{U} = \frac{1}{2}\epsilon_0 \int \frac{\rho^2 r^2}{4^2\pi^2 R^6 \epsilon_0^2}\, dV = \frac{1}{2}\epsilon_0 \frac{1}{16\pi^2 R^6 \epsilon_0}$

for $r < R$, $q_{enc} = \frac{4}{3}\pi r^3 \rho$

for a small dr, $dq = 4\pi r^2 \rho\, dr$ (charge of shell)

$\mathcal{U} = \frac{1}{4\pi\epsilon_0}\frac{q_{enc}\, dq}{r} = \frac{1}{4\pi\epsilon_0}\left(\frac{\frac{4}{3}\pi r^3 \rho\,(4\pi r^2 \rho\, dr)}{r}\right) = \frac{1}{4\pi\epsilon_0}\frac{16\pi \rho^2 r^4}{3}\, dr$

$Q_{tot} = \frac{4}{3}\pi R^3$

$\mathcal{U}_{total} = \int_0^R \frac{1}{4\pi\epsilon_0}\frac{16\pi^2 \rho^2 r^4}{3}\, dr = \frac{4\pi\rho^2}{3\epsilon_0}\frac{R^5}{5} = \frac{\frac{3}{5}Q^2}{4\pi\epsilon_0 R}$ ✓

Trying Another Method

$$U = \frac{1}{2}\epsilon_0 \int_V \vec{E}\cdot\vec{E}\, dv$$

for $r < R$, $E(4\pi r^2) = \frac{\frac{4}{3}\pi r^3 \rho}{\epsilon_0} \Rightarrow E = \frac{\frac{4}{3}\cancel{\pi} r^{\cancel{3}} \rho}{\cancel{4\pi r^2}\,\epsilon_0} = \frac{r}{3\epsilon_0}\rho$

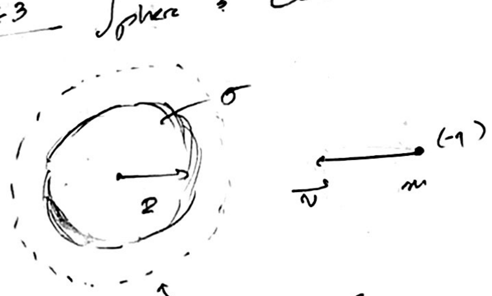
$$E^2 = \frac{r^2}{9\epsilon_0^2}\rho^2$$

$$U = \frac{1}{2}\epsilon_0 \int_V \frac{r^2}{9\epsilon_0^2}\rho^2\, dv = \frac{\rho^2}{18\epsilon_0}\int_0^{2\pi}\int_0^{\pi}\int_0^{R} \rho^2 \rho^2 \sin\phi \, d\rho\, d\phi\, d\theta$$

$$= \frac{\rho^2}{18\epsilon_0}\int_0^{2\pi}\int_0^{\pi}\int_0^{R} \rho^4 \sin\phi \, d\rho\, d\phi\, d\theta = \frac{\rho^2}{18\epsilon_0}\left(\frac{4\pi R^5}{5}\right)$$

This should be 3 to match my first answer. Not sure what I did wrong.

1.43 Sphere & Cones



$q_{enc} = 4\pi R^2\sigma$

a)

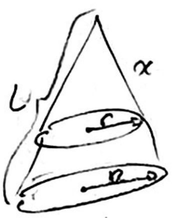
$$W = \frac{1}{\cancel{4\pi}\epsilon_0}\frac{\cancel{4\pi} R^{\cancel{2}}\sigma(-q)}{\cancel{R}} = \left|\frac{-R\sigma q}{\epsilon_0}\right| = K_f - \cancel{K_i}^{\,0} = \frac{1}{2}mv^2$$

$$v = \sqrt{\frac{2R\sigma q}{m\epsilon_0}}$$

b)

$$dQ = 2\pi r \sigma\, \delta x$$

$$dU = \frac{dQ(-q)}{4\pi\varepsilon_0 x} = \frac{-2\pi r \sigma q\, dx}{4\pi\varepsilon_0 x}$$



$$r = R\left(\frac{x}{L}\right)$$

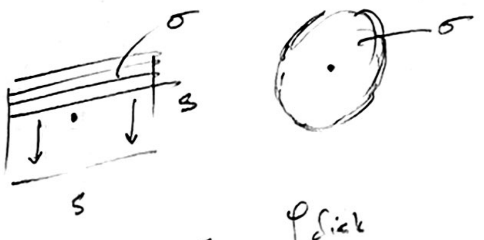
$$\underset{(\text{one of the cones})}{U} = \int_0^L \frac{-\cancel{2}\pi(Rx/L)q\sigma}{\cancel{4}_{2}\pi\varepsilon_0}\cdot\frac{1}{x}\,dx$$

$$-\frac{R}{2L\varepsilon_0}q\sigma\int_0^L dx = \frac{-Rq\sigma}{2\varepsilon_0} \Longrightarrow \text{Total } U = \cancel{2}\left(\frac{-q\sigma R}{\cancel{2}\varepsilon_0}\right) = \frac{-q\sigma R}{\varepsilon_0}$$

$$\frac{-q\sigma R}{\varepsilon_0} = U_f - \cancel{U_i}^{\,0} = \frac{1}{2}mv^2 \Longrightarrow v = \sqrt{\frac{2q\sigma R}{\varepsilon_0 m}}$$

Exact Same!

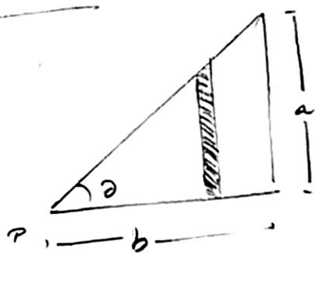
247 ½ square ?. a Disk



$$\varphi_{square} = \varphi_{disk}$$

$$\varphi = \frac{1}{4\pi\varepsilon_0}\int_D \frac{\sigma(\vec r')}{|\vec r - \vec r'|}\,d^3r'$$

Exercise 246



$$\varphi_P = \frac{\sigma b}{4\pi\varepsilon_0}\ln\left(\frac{(1+\sin\theta)}{\cos\theta}\right)$$

$\theta = 45°$

8 triangles

$$V_{P\,(\text{one triangle})} = \frac{\sigma b}{4\pi\epsilon_0}\ln\left[\frac{(1+\sin\theta)}{\cos\theta}\right]\Bigg|_{\theta=45°,\ b=s/2}$$

$$V_{P\,\text{square}} = 8\,V_{P\,(\text{triangle})} = 2\sigma(s/2)\ln\left(\frac{1+1/\sqrt{2}}{1/\sqrt{2}}\right)$$

$$= \frac{s\sigma}{\pi\epsilon_0}\ln\left(\frac{\sqrt{2}+1}{\sqrt{2}}\cdot\frac{2}{\sqrt{2}}\right) = \frac{s\sigma}{\pi\epsilon_0}\ln\left(\frac{2\sqrt{2}+2}{2}\right) = s\sigma\ln(\sqrt{2}+1)$$

$$= \frac{3.525\,s\sigma}{4\pi\epsilon_0}$$

V_{square}

$dq = 2\pi r\sigma\,dr$

$$V_{P\,\text{disk}} = \frac{1}{4\pi\epsilon_0}\int_0^{d/2}\frac{2\pi x\sigma}{x}\,dr = \frac{2\pi\sigma}{4\pi\epsilon_0}\,\frac{d}{2} = \frac{\sigma}{2\epsilon_0}\cdot\frac{d}{2} = \frac{\sigma d}{4\epsilon_0}$$

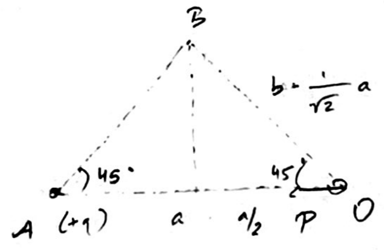
V_{disk}

$V_{\text{disk}} = V_{\text{square}}$

$$\frac{3.525\,s\cancel{\sigma}}{\cancel{4\pi\epsilon_0}} = \frac{\cancel{\sigma}\,d}{\cancel{4\epsilon_0}} \Longrightarrow \frac{3.525\,s}{\pi} = d$$

$$\Longrightarrow \frac{s}{d} = \frac{\pi}{3.525}$$

10.20 Work in a Dipole field

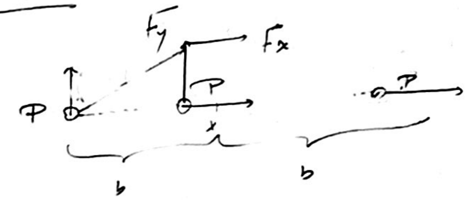


$$* \quad \varphi = \frac{p\cos\theta}{4\pi\epsilon_0 r^2} \Longrightarrow \varphi_a = \frac{p\cos(0)}{4\pi\epsilon_0 a^2} = \underline{\frac{p}{4\pi\epsilon_0 a^2}}$$

$$\varphi_b = \frac{p\cos(45)}{4\pi\epsilon_0 \left(\frac{1}{\sqrt{2}}a\right)^2} = \underline{\frac{\sqrt{2}\,p}{4\pi\epsilon_0 a^2}}$$

$$\text{Work } (+q)\ A \to B = \varphi_b - \varphi_a = \underline{\underline{\frac{\sqrt{2}-1}{4\pi\epsilon_0 a^2}}}$$

10.28



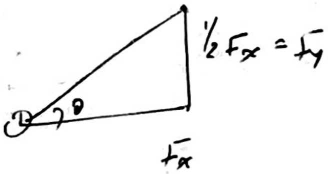
$$\vec{F} = \langle \vec{p}\cdot\nabla E_x,\ \vec{p}\cdot\nabla E_y,\ \cancel{\vec{p}\cdot\nabla E_z} \rangle$$

$$E = \frac{p}{4\pi\epsilon_0 r^3} \Longrightarrow E_{x\ \text{right dipole}} = \frac{2p}{4\pi\epsilon_0 (b-x)^3} =$$

$$E_{y\ \text{left dipole}} = \frac{-p}{4\pi\epsilon_0 (b+x)^3}$$

$$\frac{\partial E_x\, p}{\partial x} = \frac{p}{2\pi\epsilon_0}\frac{\partial}{\partial x}\left((b-x)^{-3}\right) = \frac{p}{2\pi\epsilon_0}\left(-3(b-x)^{-4}(-1)\right) = \left.\frac{6p}{4\pi\epsilon_0 (b-x)^4}\right|_{x=0} = \underline{\underline{\frac{6p}{4\pi\epsilon_0 b^4}}}$$

$$\frac{\partial E_y\, p}{\partial y} = \frac{-p}{4\pi\epsilon_0}\left(\frac{\partial}{\partial y}(b+x)^{-3}\right) = \frac{-p}{4\pi\epsilon_0}\left(-3(b+x)^{-4}\right) = \frac{3p}{4\pi\epsilon_0 (b+\cancel{x}^{\,0})^4} = \underline{\underline{\frac{1}{2} f_x}}$$



$\theta = \tan^{-1}\left(\frac{1}{2}\right)$

$$\vec{F} = \left\langle \frac{6p}{4\pi\epsilon_0 b^4}, \frac{3p}{4\pi\epsilon_0 b^4} \right\rangle$$

$$|\vec{F}| = \sqrt{\frac{36p^2}{(4\pi\epsilon_0 b^4)^2} + \frac{9p^2}{(4\pi\epsilon_0 b^4)^2}} = \sqrt{\frac{45p^2}{(4\pi\epsilon_0 b^4)^2}} = \frac{p}{(4\pi\epsilon_0 b^4)}\sqrt{45}$$

$$\boxed{= \frac{3\sqrt{5}}{4\pi\epsilon_0 b^4}}$$