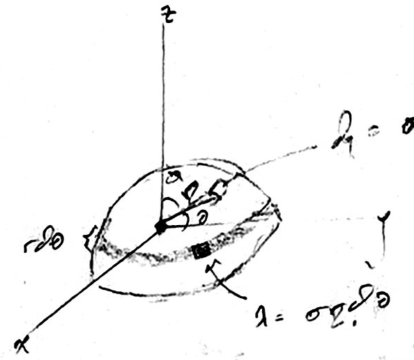
Physics 5B

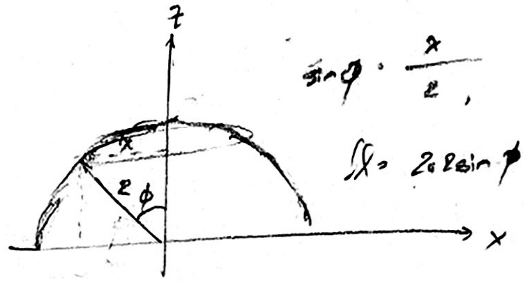
Assignment #2

Deval Deliwala

1.50 field from center of hemisphere

a) hollow hemisphere of charge density σ





$\sin\phi = \frac{x}{R}, \quad x = R\sin\phi$

$dl = 2\pi R\sin\phi$

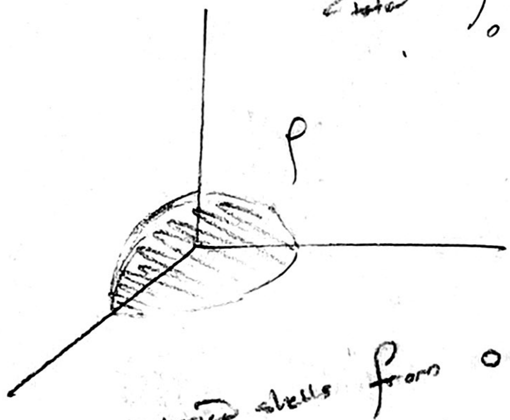
$q_{ring} = 2\pi R^2 \sin\phi\, \sigma$

$$E_{ring} = \frac{1}{4\pi\varepsilon_0} \frac{dq \, R\sin\phi}{(R^2\sin^2\phi + R^2\cos^2\phi)^{3/2}}$$

$$E_{total} = \int_0^{\pi/2} \frac{\sigma}{2\varepsilon_0} \sin\phi\cos\phi \, d\phi = \frac{\sigma}{4\varepsilon_0} \int_0^{\pi/2} \sin 2\phi \, d\phi = \frac{\sigma}{4\varepsilon_0} \frac{-\cos 2\phi}{2} \Big|_0^{\pi/2} = \underline{\underline{\frac{\sigma}{4\varepsilon_0}}}$$

b) E_{shell} of radius R $= \frac{\rho}{4\varepsilon_0} dr$

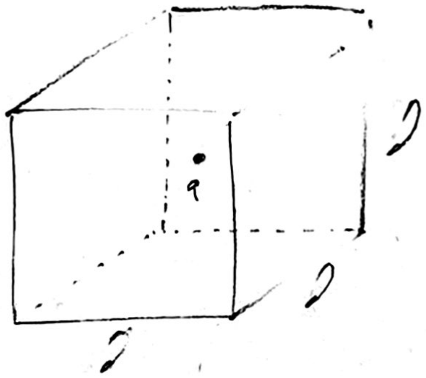
$$E_{total} = \int_0^R \frac{\rho}{4\varepsilon_0} dr = \underline{\underline{\frac{\rho R}{4\varepsilon_0}}}$$



Integrating spherical shells from $0 \to R$

1.56 flux through a cube

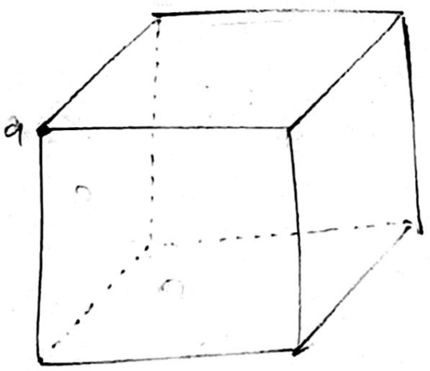
a)



$$\oint \vec{E} \cdot d\vec{a} = \frac{q}{\varepsilon_0} \quad \text{for entire cube}$$

$$\int_{\text{one face}} \vec{E} \cdot d\vec{a} = \underline{\underline{\frac{q}{6\varepsilon_0}}}$$

b)



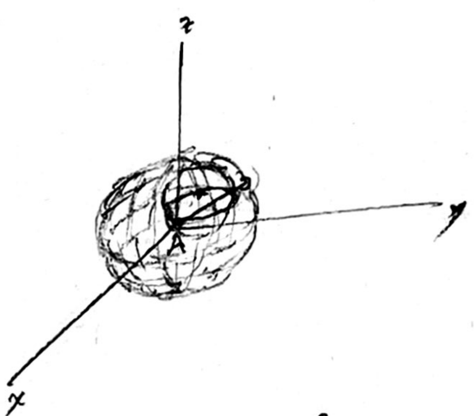
$$\int_{\text{total}} \vec{E} \cdot d\vec{a} = \frac{q}{\varepsilon_0}$$

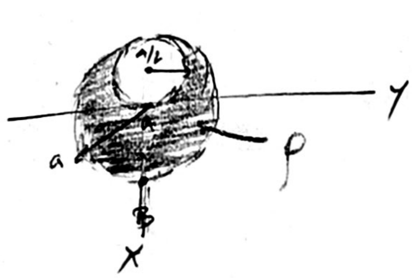
$$\int_{\text{cube}} \vec{E} \cdot d\vec{a} = \frac{1}{8} \frac{q}{\varepsilon_0} \quad \text{(only 1/8 of flux is entering the cube)}$$

$$\Phi_{\text{front}} = \Phi_{\text{left}} = \Phi_{\text{top}} = 0$$

$$\Phi_{\text{bottom}} = \Phi_{\text{right}} = \Phi_{\text{back}} = \frac{1}{3}\left(\frac{1}{8}\frac{q}{\varepsilon_0}\right) = \underline{\underline{\frac{1}{24}\frac{q}{\varepsilon_0}}}$$

1.69 Electric field of carved out sphere





$$E_{\text{sphere}} = \frac{q_{enc}}{\varepsilon_0 (4\pi r^2)} = \underline{\frac{q_{enc}}{4\pi \varepsilon_0 r^2}}$$

$$E_{\text{carved}} = E_{\text{sphere}} - E_{\text{carved sphere}}$$

superposition

Carved portion

$$\oint_A \vec{E}_{carved} \cdot d\vec{a} = \frac{Q_{enc}}{\varepsilon_0} = \frac{\rho\left(\frac{4}{3}\pi\left(\frac{a}{2}\right)^3\right)}{\varepsilon_0}$$

$$\vec{E}\left(\cancel{4\pi} r^2\right) = \frac{\cancel{4}\rho\cancel{\pi}a^3}{24\varepsilon_0} \Longrightarrow \vec{E}_{carved} = \frac{\rho a^3}{24\varepsilon_0 r^2}\hat{r}$$

$$\vec{E}_{sphere} = \frac{\rho\left(\frac{4}{3}\cancel{\pi}a^3\right)}{4\cancel{\pi}\varepsilon_0 r^2}\hat{r} = \frac{4\rho a^3}{\cancel{12}_3 r^2\varepsilon_0} = \frac{\rho a^3}{3r^2\varepsilon_0}\hat{r}$$

$$E_A = E_{sphere} - E_{carved} = -E_{carved} = \frac{-\rho\left(\frac{4}{3}\right)\pi\left(\frac{a}{2}\right)^3}{4\pi\varepsilon_0\left(\frac{a}{2}\right)^2} = \boxed{\frac{a\rho}{6\varepsilon_0}}$$

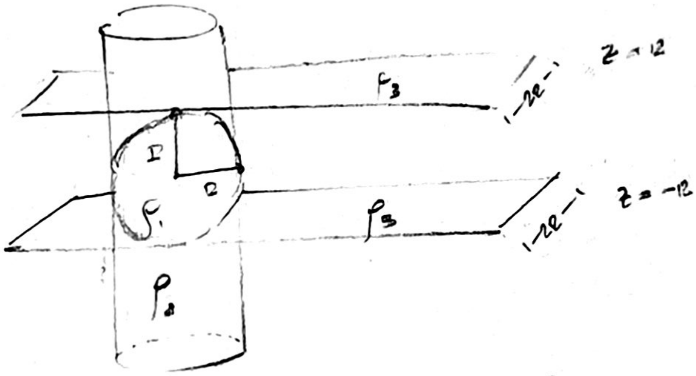
$$E_B = E_{sphere} - E_{carved} = \frac{\left(\frac{4}{3}\right)\cancel{\pi}a^{\cancel{3}}\rho}{4\cancel{\pi}\varepsilon_0\cancel{a^2}} - \frac{\left(\frac{4}{3}\right)\cancel{\pi}\left(\frac{a}{2}\right)^3\rho}{4\cancel{\pi}\varepsilon_0\left(\frac{3a}{2}\right)^2}$$

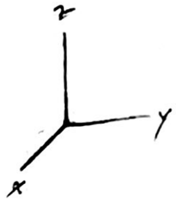
$$= \frac{\left(\frac{4}{3}\right)a\rho}{\cancel{4}\varepsilon_0} - \frac{\left(\frac{4}{3}\right)\left(\frac{a^{\cancel{3}}}{\cancel{8}_2}\right)\rho}{\cancel{4}\varepsilon_0\left(\frac{9a^{\cancel{2}}}{4}\right)}$$

$$= \frac{a\rho}{3\varepsilon_0} - \frac{a\rho}{54\varepsilon_0} = \boxed{\frac{17a\rho}{54\varepsilon_0}}$$

$$\vec{E}_A = \frac{a\rho}{6\varepsilon_0}\hat{k} \qquad \vec{E}_B = \frac{17a\rho}{54\varepsilon_0}\left(-\hat{k}\right)$$

1.74 Zero field in a Sphere





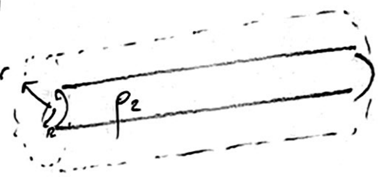
$$E_{sphere} = \frac{\rho(\frac{4}{3}\pi R^3)}{4\pi r^2 \varepsilon_0} = \frac{\rho R^3}{3 r^2 \varepsilon_0}\Bigg|_{r=R} = \frac{\rho_1 r}{3\varepsilon_0}\langle x, y, z\rangle$$

E_{slabs}

$$\oint\!\!\oint_S \vec{E}\cdot\hat{n}\,da = \frac{\pi R^2 \rho_3 l}{\varepsilon_0} \Longrightarrow E_{plane} = \frac{\rho_3 r}{2\varepsilon_0}$$

$$\vec{E}_{slabs} = \frac{\rho_3 r}{\varepsilon_0}\hat{k}.$$

$E_{cylinder}$



$$\int \vec{E}\cdot d\vec{s} = \frac{\rho_2 \pi R^2 \cancel{l}}{\varepsilon_0} = \vec{E}(2\pi r \cancel{l}) = \frac{\rho_2 r}{2\varepsilon_0}\langle x, y, 0\rangle$$

$$\vec{E}_{total} = \frac{\rho_1 r}{3\varepsilon_0}\langle x, y, z\rangle + \frac{\rho_2 r}{\varepsilon_0}\langle x, y, 0\rangle + \frac{\rho_3 r}{\varepsilon_0}\langle 0, 0, z\rangle = \vec{0}$$

$$x:\ \frac{\rho_1 x}{3\cancel{\varepsilon_0}} + \frac{\rho_2 x}{2\cancel{\varepsilon_0}} = 0 \Longrightarrow \rho_1 = \frac{-3\rho_2}{2} \rightarrow \boxed{\rho_1 = \frac{-3\rho_2}{2} = -3\rho_3}$$

$$y:\ \frac{\rho_1 r}{3\varepsilon_0} + \frac{\rho_2 r}{2\varepsilon_0} = 0$$

$$z:\ \frac{\rho_1 x}{3\cancel{\varepsilon_0}} + \frac{\rho_3 x}{\cancel{\varepsilon_0}} = 0 \Longrightarrow \rho_1 = -3\rho_3$$

1.77. Electron Jelly

Protons lie on same diameter w/ same radius due to symmetry

$$E_{\substack{\text{proton}\\ \text{from } e^-}} = \frac{1}{\cancel{4\pi}\varepsilon_0 \atop 2}\left(\frac{-\cancel{2}e\left(\frac{r^3}{a^3}\right)}{\cancel{r^2}}\right) = \frac{-e^- r}{2\varepsilon_0 a^3}$$

$$E_{\substack{\text{proton}\\ \text{from proton}}} = \frac{1}{4\pi\varepsilon_0}\left(\frac{e^-}{(2r)^2}\right) = \frac{e^-}{16\pi r^2\varepsilon_0}$$

$$E_{\substack{\text{proton}\\ \text{total}}} = \frac{e^-}{16\pi r^2\varepsilon_0} - \frac{e^- r}{2\varepsilon_0 a^3} = 0$$

$$\frac{\cancel{e^-}}{16\cancel{\pi} r^2\cancel{\varepsilon_0}} = \frac{\cancel{e^-} r}{2\cancel{\varepsilon_0} a^3\cancel{\pi}} \implies 2a^3 = 16r^3$$

$$a^3 = 8r^3 \implies r^3 = \frac{1}{8}a^3$$

$$r = \frac{1}{2}a$$