Physics 7A, Spring 2014 final

1)

Conservation of Energy

$$\int_\infty^R \vec{F}_g \cdot dr = \frac{1}{2}mv_s^2 - \frac{1}{2}mv_i^2 \qquad -\frac{GM_0 m_r}{r}\Bigg|_\infty^R = \frac{1}{2}mv_s^2 - \frac{1}{2}mv_i^2$$

$$\int \frac{GM_0 m_r}{r^2}\,dr = GM_0 m_r \int \frac{1}{r^2}\,dr =$$

$$= -GM_0 m_r\left(\frac{1}{R} - \cancel{\frac{1}{\infty}}^{\,0}\right) = -\frac{GM_0 m_r}{R} = \frac{1}{2}mv_s^2 - \frac{1}{2}mv_i^2$$

$$\Longrightarrow -\frac{GM_0 \cancel{m_r}}{R} + \frac{1}{2}\cancel{m}v_i^2 = \frac{1}{2}\cancel{m}v_s^2 \Longrightarrow v_s^2 = \frac{-2GM_0}{R} + v_i^2$$

$$\underline{\underline{v_s = \sqrt{\frac{2GM_0}{R} + v_i^2}}}$$

b) $$-\frac{GM_0 m_r}{r}\Bigg|_R^\infty = \frac{1}{2}m_r v_f^2 - \frac{1}{2}m_r v_s^2 \Longrightarrow \frac{GM_0 \cancel{m_r}}{R} + \frac{1}{2}\cancel{m_r}v_s^2 = \frac{1}{2}\cancel{m_r}v_f^2$$

$$\Longrightarrow \frac{2GM_0}{R} + v_s^2 = v_f^2 \Longrightarrow \underline{\underline{v_f = \sqrt{\frac{2GM_0}{R} + v_s^2}}}$$

c) $F\Delta t = m(v_f - v_i)$

$= (m_r - m_f)v_f - m_r v_i$

$(m_r - m_f)v_f = F\Delta t + m_r v_i$

$$v_f = \frac{F\Delta t + m_r v_i}{m_r - m_f}$$

$$-\frac{GM_0(m_r - m_f)}{r}\Bigg|_\infty^R = \frac{1}{2}(m_r - m_f)(v_f^2 - v_i^2)$$

$$\frac{-GM_0\cancel{(m_r - m_f)}}{R} + \frac{1}{2}\cancel{(m_r - m_f)}v_i^2 = \frac{1}{2}\cancel{(m_r - m_f)}v_s^2$$

$$v_s = \sqrt{\frac{-2GM_0}{R} + \left(\frac{F\Delta t + m_r v_i}{m_r - m_f}\right)^2} \Longrightarrow v_f = \sqrt{\frac{2GM_0}{R} - \frac{2GM_0}{R} + \left(\frac{F\Delta t + m_r v_i}{m_r - m_f}\right)^2}$$

$$\underline{\underline{v_f = \frac{F\Delta t + m_r v_i}{m_r - m_f}}}$$

d) $\cancel{F\Delta t = m(v_{s_{new}} - v_s)}$

$\cancel{F\Delta t + m v_s = m v_{s_{new}}}$

$\cancel{\dfrac{F\Delta t + m\sqrt{\frac{-2GM_0}{R} + v_i^2}}{m} = v_{s_{new}}}$

$\cancel{\Rightarrow v_s' = \dfrac{F\Delta t}{m} + \sqrt{\dfrac{-2GM_0}{R} + v_i^2}}$

$F\Delta t = (m_r - m_f)\, v_s' - m_r v_s$

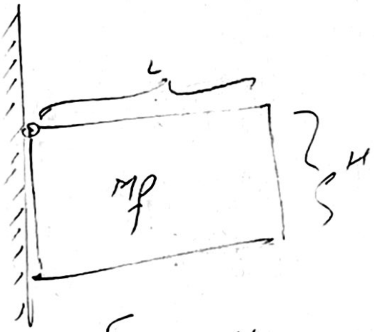
$\Rightarrow v_s = \dfrac{F\Delta t + m_r v_s}{(m_r - m_f)} = \dfrac{F\Delta t + m_r\sqrt{\frac{-2GM_0}{R} + v_i^2}}{(m_r - m_f)}$

$$\int_R^\infty \frac{GM_0(m_r - m_f)}{r^2}\,dr = \frac{1}{2}(m_r - m_f)\, v_f^2 - \frac{1}{2}(m_r - m_f)\, v_s'^2$$

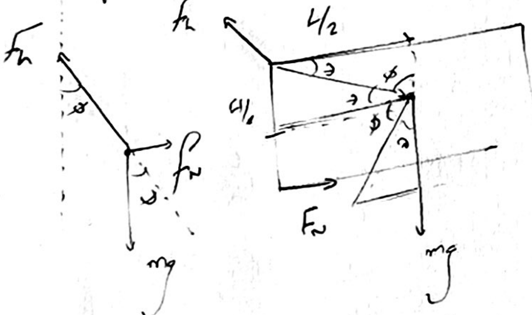
$$\Rightarrow \frac{GM_0\cancel{(m_r - m_f)}}{R} + \frac{1}{2}\cancel{(m_r - m_f)}\, v_s'^2 = \frac{1}{2}\cancel{(m_r - m_f)}\, v_f^2$$

$$\Rightarrow \frac{GM_0}{R} + \frac{1}{2} v_s'^2 = \frac{1}{2} v_f^2 \Rightarrow v_f = \frac{2GM_0}{R} + v_s'^2$$

$$= \frac{2GM_0}{R} + \left(\frac{F\Delta t + m_r\sqrt{\frac{-2GM_0}{R} + v_i^2}}{(m_r - m_f)}\right)^2$$



a)



b) Equations of Motion

$F_y: F_{h_y} - mg = 0 \Longrightarrow F_{h_y} = mg$, vertical

$F_x: f_N - F_{hx} = 0$

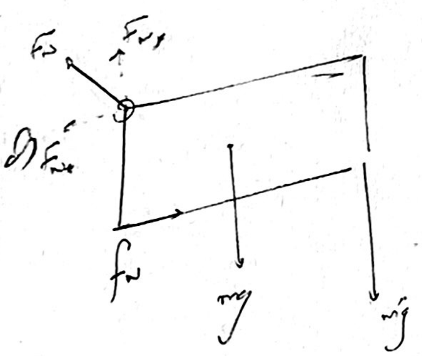
c) $f_N = F_{h_x}$, $\theta = \tan^{-1}\left(\frac{H}{L}\right)$ $\quad r = \sqrt{\frac{L^2}{4} + \frac{H^2}{4}}$

$\tau_h: \tau_N - \tau_g = 0$

$\tau_N = \tau_g = mg\left(\frac{L}{2}\right)$

$F_N(H) = mg\left(\frac{L}{2}\right) \Longrightarrow F_N = \frac{mgL}{2H}$

d)



Equations of motion

$F_{hy} - mg - m'g = 0$

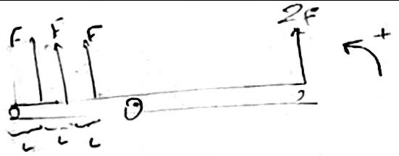
$F_{hx} - f_N = 0$

$\tau: \tau_N - \tau_g - \tau_{g'} = 0 \longrightarrow F_N(H) - mg\left(\frac{L}{2}\right) - m'g(L) = 0$

$F_N = \frac{gL\left(\frac{m}{2} + m'\right)}{H}$

$\frac{\cancel{gL}\left(\frac{m}{2} + m'\right)}{\cancel{H}} = \frac{5m\cancel{gL}}{2\cancel{H}}$

$\frac{m}{2} + m' = \frac{5m}{2} \longrightarrow m' = 2m$

3. 

a) Equilibrium of Motion

$\tau: F(L) + F(2L) + F(3L) = \underline{\underline{6FL}}$

b) $\tau: 2F(4L) - 6FL = I\alpha$

$= 8L - 6FL = I\alpha$

$= 2FL = \tau$ → Counter Clockwise

c) Mg

d) Circumference $= 8L\pi$, Distance for $90° \to \underline{2L\pi}$

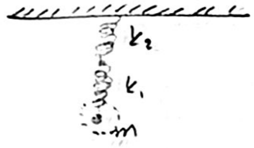
$2F\cancel{L} = \frac{1}{12} ML^2 \alpha, \quad \alpha = \frac{24F}{M}$

$2L\pi = \frac{1}{2}\alpha t^2 = \frac{12F}{M} t^2$

$t^2 = \frac{\cancel{2}L\pi M}{\cancel{12}F} = \frac{ML\pi}{6F}, \quad \underline{\underline{t = \sqrt{\frac{ML\pi}{6F}}}}$

e) $\tau_{middle} = F(L)$

$\int_{\theta_i}^{\theta_f} \tau \, d\theta = \int_0^{-\pi/2} F(L)\, d\theta = F(L)\left(-\frac{\pi}{2}\right) = \underline{\underline{\frac{-FL\pi}{2}}}$



a) Equations of Motion

$f_x : mg - k_1 \Delta x = 0$

$k_1 \Delta x = mg$

$\Delta x_1 = \frac{mg}{k_1}$

$f_x' : mg - k_2 \Delta x = 0$

$k_2 \Delta x = mg$

$\Delta x_2 = \frac{mg}{k_2}$

$\Delta x_{tot} = mg\left(\frac{1}{k_1} + \frac{1}{k_2}\right)$

$D = l_1 + l_2 + mg\left(\frac{1}{k_1} + \frac{1}{k_2}\right)$

b) $0 = m_b v_b - m v_f$

$m v_f = m_b v_b$

$v_f = \frac{m_b}{m} v_b$

c) $J = \Delta p = p_f - p_i = m_b v_b$

d) Equations of Motion

lowest point: $\frac{3}{4} T$

$T = 2\pi\sqrt{\frac{m}{k_{eff}}}$,

$\Delta x = mg\left(\frac{1}{k_1} + \frac{1}{k_2}\right)$

$\Delta x = F\left(\frac{1}{k_1} + \frac{1}{k_2}\right)$

$F = k_{eff} \Delta x = \frac{1}{\left(\frac{1}{k_1} + \frac{1}{k_2}\right)} \Delta x$

$T = 2\pi\sqrt{\frac{m}{\frac{1}{1/k_1 + 1/k_2}}}$

$\frac{3}{4} T = \frac{3}{2}\pi\sqrt{\frac{m}{\frac{1}{1/k_1 + 1/k_2}}}$

e) $v(t) = \frac{d}{dt}\left(A\sin(\omega t)\right) = A\omega\cos(\omega t)$

$v(0) = \frac{m_b}{m} v_b = A\omega\cancel{\cos(\omega t)}$

$A = \frac{m_b v_b}{\omega m}$

$a(t) = -A\omega^2 \sin(\omega t)$

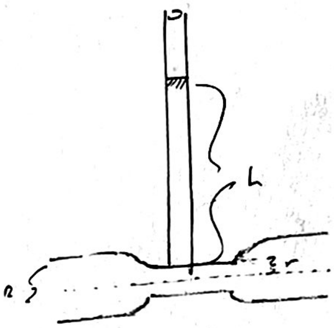
At highest point, $a(t) = -A\omega^2 \cancel{\sin(\omega t)}$

$= -\frac{m_b v_b}{\omega m}\omega^2 = \frac{-m_b v_b \omega}{m}$

$\omega = \sqrt{\frac{k_{eff}}{m}} = \sqrt{\frac{1}{(1/k_1 + 1/k_2)m}}$

$\Longrightarrow a = \frac{-m_b v_b \sqrt{\frac{1}{(1/k_1 + 1/k_2)m}}}{m}$

7.



a) v_1 (m/s)

B (m³/s) $\quad v_1 = \frac{B}{A_1} = \underline{\underline{\frac{B}{\pi R^2}}}$

$A = m^2$

b) $\cancel{\pi R^2}\left(\frac{B}{\cancel{\pi R^2}}\right) = \pi r^2 v$

$\underline{\underline{v = \frac{B}{\pi r^2}}}$

c) $P_1 + \cancel{\rho g h}^{0} + \frac{1}{2}\rho v_1^2 = 1 + \rho g h + 0$

$P_1 + \frac{1}{2}\rho\left(\frac{B^2}{\pi^2 R^4}\right) = 1 + \rho g h$

$h = \underline{\underline{\frac{P_1 - 1 + \frac{1}{2}\rho\left(\frac{B^2}{\pi^2 R^4}\right)}{\rho g}}}$

d) $\underline{\underline{\frac{B}{\pi R^2}, \text{ Continuity}}}$