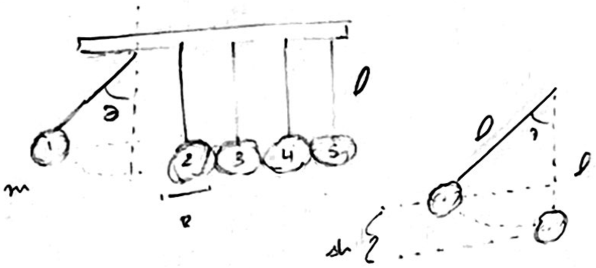
Dennis Delaney   Physics 7A 2017 Section 2

1)



a)

Conservation of Energy

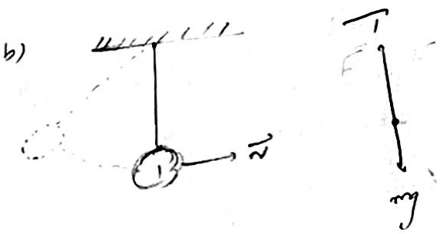
$\Delta h = l - l\cos\theta = l(1-\cos\theta)$

$mgl(1-\cos\theta) = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{P_1}{m}\right)^2$

$mgl(1-\cos\theta) = \frac{1}{2}\cancel{m}\frac{P_1^2}{m^{\cancel{2}}} = \frac{1}{2}\frac{P_1^2}{m}$

$(1-\cos\theta) = \frac{P_1^2}{2m^2gl} \Longrightarrow \cos\theta = 1 - \frac{P_1^2}{2m^2gl} \Longrightarrow \theta = \cos^{-1}\left(1 - \frac{P_1^2}{2m^2gl}\right)$

b)



Equations of Motion

$\Sigma F_y: T - mg = \frac{mv^2}{l} \Longrightarrow T = \frac{\cancel{m}\left(\frac{P_1^2}{m_1\cancel{^2}}\right)}{l} + mg$

$T = \frac{P_1^2}{m_1 l} + mg$

c) Acts as a harmonic oscillator

$\omega_0 = \sqrt{\frac{g}{l}}$ Time to swing down: $\frac{\tau}{4}$

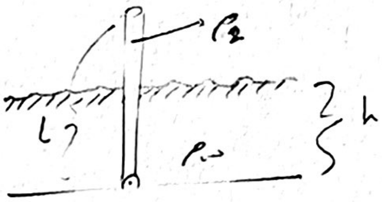
$\tau = 2\pi\sqrt{\frac{g}{l}}$ $\Longrightarrow \frac{\pi}{2}\sqrt{\frac{g}{l}}$

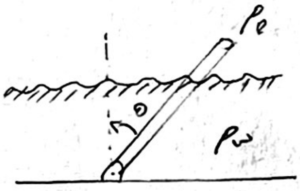
Energy & Momentum Conservation

d) $\vec{v}_5 = \vec{v}_1 = \frac{P_1}{m}$

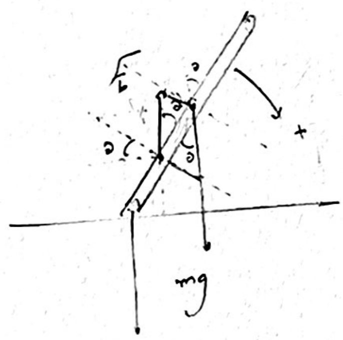
Conservation of Momentum $\Longrightarrow \frac{P_1}{m} - \frac{1}{4}\frac{P_1}{m} = \frac{3}{4}\frac{P_1}{m}$

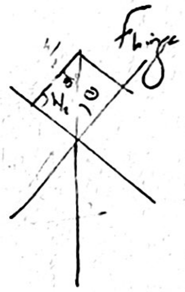
e) $v_1 = \frac{P_1}{m} \Longrightarrow$





a)





b) $\sum \tau_h: F_{hinge}(0) - F_b\left(\frac{h}{2}\right)\tan\theta + mg\left(\frac{L}{2}\right)\sin\theta = 0$

$\Rightarrow F_b\left(\frac{h}{2}\right)\tan\theta = mg\left(\frac{L}{2}\right)\sin\theta$

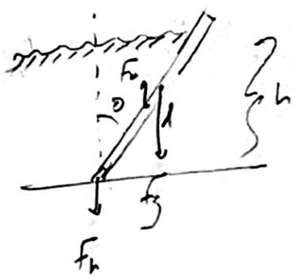
$\Rightarrow \left(\rho_w \cancel{g} \frac{\cancel{A}h}{\cos\theta}\right)\left(\frac{h}{\cancel{2}}\right)\tan\theta = (\rho_R \cancel{g} \cancel{A}L)\left(\frac{L}{\cancel{2}}\right)\sin\theta$

$h\rho_w \frac{h}{\cos\theta}\tan\theta = \rho_R L^2 \sin\theta$

$\cos^2\theta = \frac{h^2}{L^2}\left(\frac{\rho_w}{\rho_R}\right), \quad \theta \to 0 \Rightarrow \frac{h^2}{L^2}\left(\frac{\rho_w}{\rho_R}\right) = 1$

$$\rho_{R_{max}} = \frac{h^2}{L^2}\rho_w$$

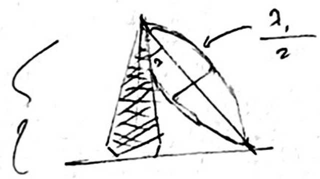
$\theta = \cos^{-1}\left(\frac{h}{L}\sqrt{\frac{\rho_w}{\rho_R}}\right)$

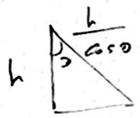


$\sum F_y: F_B - F_h - F_g = 0$

$$F_h = F_B - F_g = \frac{\rho_w g A h}{\cos\theta} - \rho_R g AL$$

a)





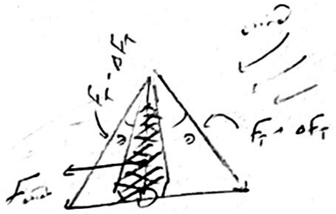
$$f_1 = \frac{v}{\lambda_1}$$

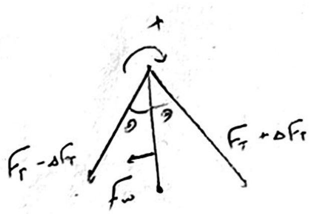
$$\lambda_1 = \frac{2h}{\cos\theta} \Rightarrow f_1 = \frac{v}{2h}\cos\theta$$

$$v = \sqrt{F_T/\mu}$$

$$\underline{\underline{f_1 = \frac{\sqrt{F_T/\mu}}{2h}\cos\theta}}$$

b)





Equation of Motion

$$\sum \tau = (F_T + \Delta F_T)h\sin\theta - (F_T - \Delta F_T)h\sin\theta - F_w = 0$$

$$h\sin\theta\left((\cancel{F_T} + \Delta F_T) - (\cancel{F_T} - \Delta F_T)\right) = F_w$$

$$h\sin\theta\,(2\Delta F_T) = F_w$$

$$2\Delta F_T = \frac{F_w}{h\sin\theta} \Rightarrow \underline{\underline{\Delta F_T = \frac{F_w}{2h\sin\theta}}}$$

c) $$\underline{\underline{f_{beat} = |f_1 - f_2| = \frac{\cos\theta}{2h}\left(\sqrt{\frac{F_T + \Delta F_T}{\mu}} - \sqrt{\frac{F_T - \Delta F_T}{\mu}}\right)}}$$

d) Doppler shift: $f_{obs} = f_0 \dfrac{1}{1 + \frac{u}{v}}$ ← stationary bird

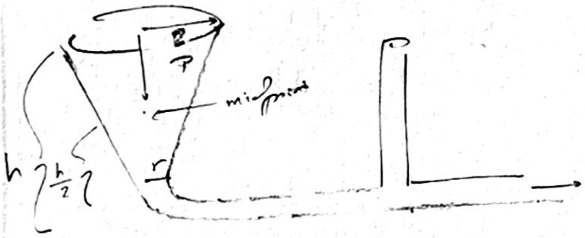
$$\underline{\underline{\lambda_{obs} = \frac{v}{f_{obs}} = \frac{v + u}{f_0}}}$$

e) $f_{obs}' = f_{obs}\left(1 + \dfrac{u' - u}{v}\right)$ ← moving at bird

↑ moving bird

$$= f_0 \frac{1}{1 + \frac{u}{v}}\left(1 + \frac{u' - u}{v}\right)$$

4.



a) Bernoulli's Equation

$$\cancel{\left(P + \rho g h + \frac{1}{2}\rho v_{top}^2\right) = \left(P + \rho g 0 + \frac{1}{2}\rho v_{horizontal}^2\right)}$$

Continuity Equation

$$A_{top} v_{top} = A_h v_h \implies v_h = v_t\left(\frac{A_t}{A_h}\right) = v_t\left(\frac{\pi R^2}{\pi r^2}\right) = v_t\left(\frac{R}{r}\right)^2$$

$$\implies \cancel{\rho} g h + \frac{1}{2}\cancel{\rho} v_t^2 = \frac{1}{2}\cancel{\rho}\left(v_t\left(\frac{R}{r}\right)^2\right)^2$$

$$2gh + v_t^2 = v_t^2 \frac{R^4}{r^4}$$

$$R^4 = \frac{r^4(2gh + v_t^2)}{v_t^2} = \frac{2r^4 gh}{v_t^2}$$

$$R = \sqrt[4]{\frac{2r^4 gh}{v_t^2} + 1} = \underline{\underline{r\sqrt{\frac{2gh}{v_t^2} + 1}}}$$

c) $P_t + \rho g h + \frac{1}{2}\rho v_{top}^2 = P_m + \rho g\left(\frac{h}{2}\right) + \frac{1}{2}\rho v_{mid}^2$

$= P_m + \rho g \frac{h}{2} + \frac{1}{2}\rho(16 v_t^2)$

$= P_m + \rho g \frac{h}{2} + 8\rho v_t^2$

$$P_t + \frac{\rho g h}{2} - \frac{15}{2}\rho v_{top}^2 = P_m$$

$$P_m > P_t$$

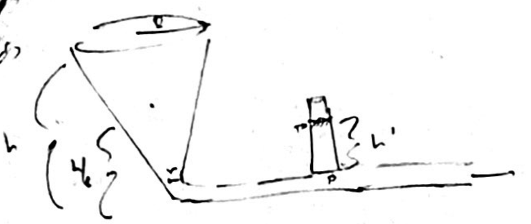
b) $A_1 v_t = A_2 v_m$

$$v_m = v_t\left(\frac{\pi R^2}{\pi r_{mid}^2}\right) = v_t\left(\frac{R^2}{r_{mid}^2}\right)$$

$$r_{mid} = \frac{R + r}{2} \implies v_m = v_t\left(\frac{R^2}{\left(\frac{R+r}{2}\right)^2}\right)$$

Since $R >> r$, $v_m = v_t\left(\frac{4R^2}{R^2}\right) = 4v_t$

$$\underline{\underline{v_m = 4v_t}}$$

d) 

e) Continuity Equation is unchanged by viscosity

$A_t v_t = A_h v_h$

$\Rightarrow v_h = v_t\left(\frac{D^2}{r^2}\right)$

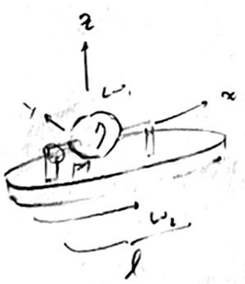
Bernoulli's Equation

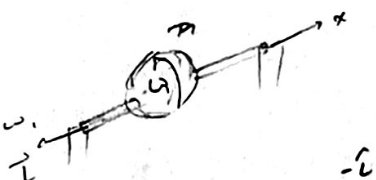
$P_i + \rho g(0) + \frac{1}{2}\rho v^2 = P + \rho g h' + \frac{1}{2}\rho v^2$

$A_i v_i = A_f v_f$ — velocities equivalent

$\Rightarrow P_i = (1\text{ atm}) + \rho g h'$

$h' = \frac{P - 1}{\rho g}$

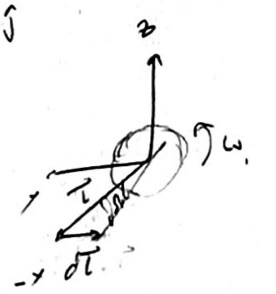
5. 

a) $\omega_2 \ll \omega_1$ 

$\vec{\tau} = \frac{d\vec{L}}{dt}$, $L = I_{sphere}\,\omega_1\,(-\hat{i}) = -\frac{2}{5} m r^2 \omega_1 \hat{i}$

$dL = L\sin(d\Omega)$, $d\Omega \ll 1$ $\Rightarrow dL = L\,d\Omega$

$\frac{dL}{dt} = L\frac{d\Omega}{dt} \Rightarrow \tau = L\frac{d\Omega}{dt}(-\hat{j}) = L(\omega_2)(-\hat{j})$



$\tau = L(\omega_2)(-\hat{j}) = -I_{sphere}\,\omega_1\omega_2\,\hat{j}$

$= -\frac{2}{5} m r^2 \omega_1 \omega_2 \hat{j}$

b) $\vec{L} = I\vec{\omega} = \frac{2}{5} m r^2 \left(\omega_1(-\hat{i}) + \omega_2(\hat{k})\right)$

c) $|\tau| = \left|\frac{dL}{dt}\right| = \frac{2}{5} m r^2 \omega_1 \omega_2$ Torque is the same since $\frac{dL}{dt}$ is the same, L_z isn't changing

d) Equation of Rotation: $\tau = F(l) = \frac{2}{5} m r^2 \omega_1 \omega_2$

$\Rightarrow F = \frac{2}{5l} m r^2 \omega_1 \omega_2$