Physics 5B (Spring 2023): Homework #1

Please submit on online before 6:00 pm on Monday, January 30, 2022.

Review of gradient and divergence

Problem 1 Find the gradients of the following functions:
(a) $f(x, y, z) = x^2 + y^3 + z^4$.
(b) $f(x, y, z) = x^2 y^3 z^4$.

Problem 2 The height of a certain hill (in feet) is given by
$h(x, y) = 10(2xy - 3x^2 - 4y^2 - 18x + 28y + 12)$,
where y is the distance (in miles) north, x the distance east of the Campanile.
(a) Where is the top of the hill located?
(b) How high is the hill?
(c) How steep is the slope (in feet per mile) at a point 1 mile north and one mile east of the Campanile? In what direction is the slope steepest, at that point?

Problem 3 Calculate the divergence of the following vector functions:
(a) $\boldsymbol{v} = x^2\hat{\boldsymbol{x}} + 3xz^2\hat{\boldsymbol{y}} - 2xz\hat{\boldsymbol{z}}$
(b) $\boldsymbol{v} = xy\hat{\boldsymbol{x}} + 2yz\hat{\boldsymbol{y}} - 3zx\hat{\boldsymbol{z}}$

Divergence theorem

Problem 4 Test the divergence theorem for the function $\boldsymbol{v} = xy\hat{\boldsymbol{x}} + 2yz\hat{\boldsymbol{y}} - 3xz\hat{\boldsymbol{z}}$, taking the surface to be a cube of dimension a.

Flux

Problem 5 (1.56 in PM)

Coulomb's Law

Problem 6 (1.38) in PM