Physics 5B

Assignment 5

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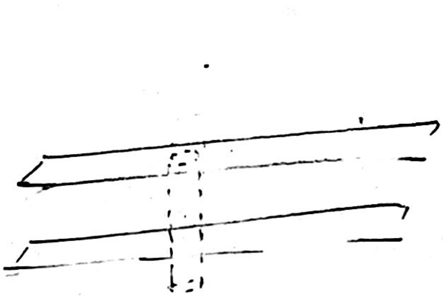
54 Charge Distribution



$$E_1 A = \frac{\sigma_1 A}{\epsilon_0} \qquad E_2 = \frac{\sigma_2}{\epsilon_0}$$

$$V_1 = V_2$$

$$V_{01} = -\int_0^{0.05} E_1 \cdot ds = -E_1(0.05)$$

$$V_{02} = -\int_0^{0.08} E_2 \cdot ds = -E_2(0.08)$$

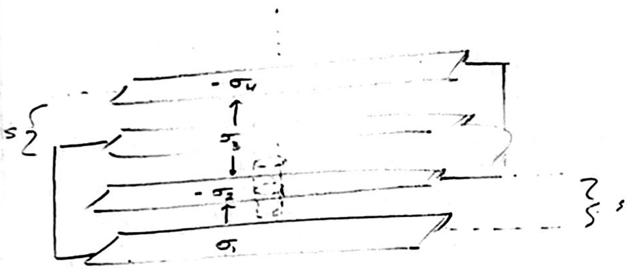
$$E_1(0.05) = E_2(0.08)$$

$$\sigma_1(5) = \sigma_2(8)$$

$$\sigma_1 = \frac{8}{5}\sigma_2 \qquad \sigma_1 + \sigma_2 = \sigma$$

$$\rightarrow \sigma_1 = \frac{8}{13}\sigma \qquad \sigma_2 = \frac{5}{13}\sigma$$

3.57 N Plates



$$\xi_1 = \frac{\sigma_1}{\epsilon_0}, \quad \xi_2 = \frac{-\sigma_2}{\epsilon_0}, \quad \ldots,$$

$$\varphi_{12} = -\int_0^s \xi_1 \, d_s = \frac{\sigma_1}{\epsilon_0} s = \varphi_{\sigma_n \sigma_m}$$

$C = Q\varphi_0$

$\sigma_2, \sigma_3, \sigma_4, \ldots = \pm 2\sigma_1$

$$Q_{total} = \left(\sigma_1 + (N-1)2\sigma_1\right)A \Longrightarrow \sigma_1 = \frac{Q}{(2n-1)A}$$

$$\varphi = \frac{\sigma_1}{\epsilon_0} s = \frac{Q s}{(2n-1)A\epsilon_0}$$

$$Q = C\varphi, \quad C = \frac{Q}{\varphi} = \frac{Q}{\frac{Qs}{(2n-1)A\epsilon_0}} \Longrightarrow \underline{\underline{C = \frac{(2n-1)A\epsilon_0}{s}}}$$

3.61 Spheroid

$$C = \frac{8\pi\epsilon_0 a\epsilon}{\ln\left(\frac{1+\epsilon}{1-\epsilon}\right)}, \quad \epsilon = \sqrt{1 - \frac{b^2}{a^2}}$$

$b \to a$

$\epsilon \ll 1$

$\ln(1+\epsilon) \approx \epsilon$

Capacitance of sphere: $4\pi\epsilon_0 R$

$$\lim_{b \to a} \frac{8\pi\epsilon_0 a\sqrt{1-\frac{b^2}{a^2}}}{\ln\left(\frac{1+\epsilon}{1-\epsilon}\right)} = \frac{8\pi\epsilon_0 a\epsilon}{\epsilon - (-\epsilon)} = \frac{8\pi\epsilon_0 a}{2} = \underline{\underline{4\pi\epsilon_0 a}}$$

Capacitance of Sphere: $4\pi\epsilon_0 a$

Capacitance of Spheroid: $\dfrac{8\pi\epsilon_0 a\epsilon}{\ln(1+\epsilon)-\ln(1-\epsilon)}$

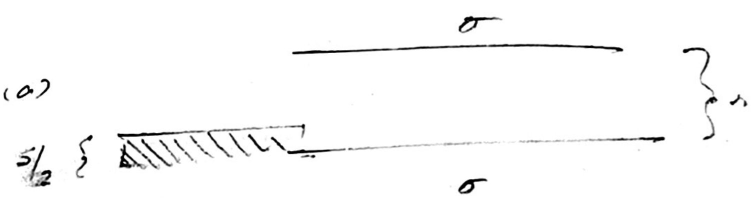
$$\frac{C_{spheroid}}{C_{sphere}} = \frac{\dfrac{8\pi\epsilon_0 a\epsilon}{\ln(1+\epsilon)-\ln(1-\epsilon)}}{4\pi\epsilon_0 a} = \frac{\cancel{8}^{\,2}\cancel{\pi\epsilon_0 a}\,\epsilon}{\ln(1+\epsilon)-\ln(1-\epsilon)}\cdot\frac{1}{\cancel{4\pi\epsilon_0 a}} = \boxed{\frac{2\epsilon}{\ln\left(\frac{1+\epsilon}{1-\epsilon}\right)}}$$

$$\frac{4}{3}\pi a^3 = \frac{4}{3}\pi a b^2 \qquad b \simeq \frac{1}{\sqrt{a}} \Longrightarrow \epsilon = \sqrt{1-\frac{1}{a^3}}$$

$C_{spheroid} > C_{sphere}$

3.71 Conductor in Capacitor

(a)

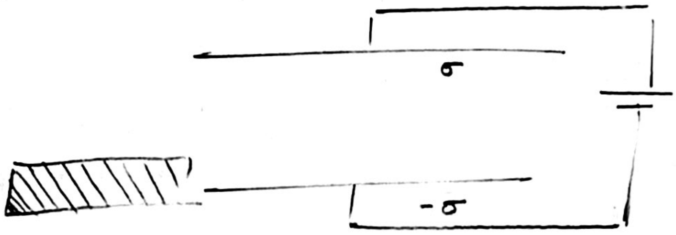


$$C = \epsilon_0\left(\frac{\cancel{A}}{s}\right) = \frac{Q}{\phi_0} = \frac{\sigma \cancel{A}}{\phi_0}$$

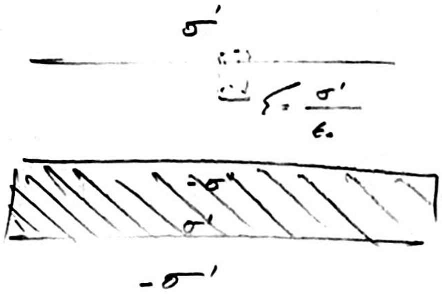
$$\phi_0 = \frac{\sigma s}{\epsilon_0}$$

$$U = KE\phi = \frac{1}{2}Q\phi_0 = \frac{1}{2}\sigma A\left(\frac{\sigma s}{\epsilon_0}\right) = \frac{\sigma^2 A s}{2\epsilon_0}$$

(b)



$$u_1 = \frac{\sigma^2 A s}{2\epsilon_0}$$

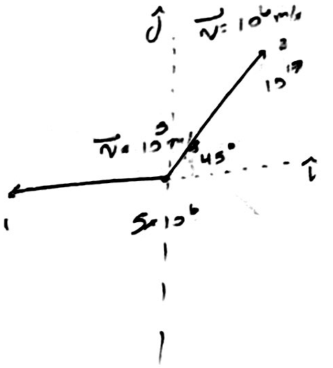


$$\Big\} = \frac{1}{2}\,\frac{\sigma'}{\cancel{\epsilon_0}}\left(\frac{\cancel{s}}{2}\right) = \frac{\sigma \cancel{s}}{\cancel{\epsilon_0}}$$

$$\underline{\sigma' = 2\sigma}$$

$$u_f = \frac{1}{2}\sigma' A\left(\frac{\sigma'}{\epsilon_0}\left(\frac{s}{2}\right)\right) = \frac{\sigma'^2 A s}{4\epsilon_0} = \underline{\frac{\sigma^2 A s}{\epsilon_0}}$$

4.20 Combining the Current Densities



$$\vec{J} = \sum_k n_k q_k \vec{u}_k$$

$J_1 = (5\times 10^{6})(2e)(10)(-\hat{i})$, $J_2 = (10^{17})(-e)(10^{6})\cos(45)\hat{i} + (10^{17})(-1)(10^{6})\sin(45)\hat{j}$

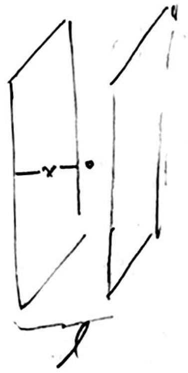
$$= (-10^{23}\cos(45)\hat{i} - 10^{23}\sin(45)\hat{j})e$$

$$= -20(5\times 10^{6})e\,\hat{i} = -1.6\times 10^{3}\,\frac{A}{m^2}\,\hat{i}$$

$\vec{J} = \langle -1.29\times 10^{4}, -1.13\times 10^{4} \rangle$, $|\vec{J}| = 1.71\times 10^{4}\ A/m^2$

2) Current pulse from α particle

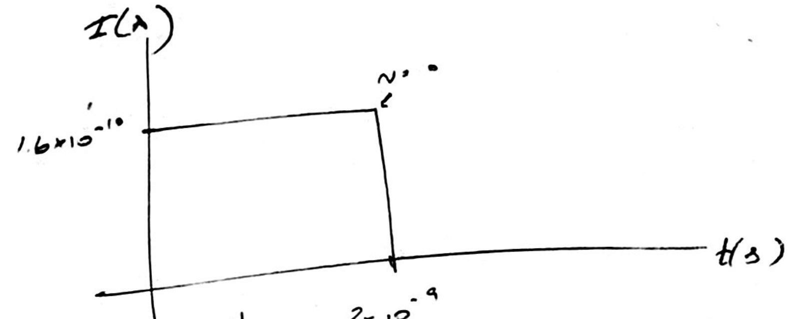
a)

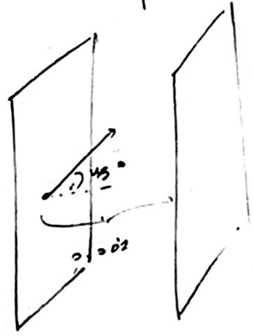


$Q = \dfrac{-2ex}{0.002} \Rightarrow I = \dfrac{-2e}{0.002} \cdot \dfrac{\delta x}{\delta t}$

$= \dfrac{-2e}{0.002} v = 1.6 \times 10^{-10} A$

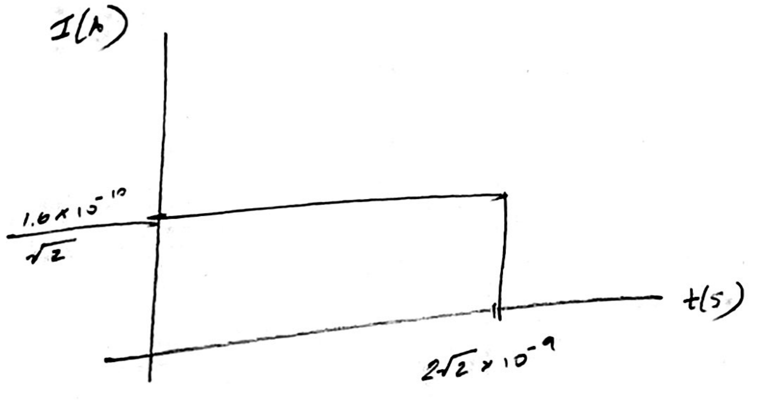
time it takes particle to traverse $0.002m = 2 \times 10^{-9} s$





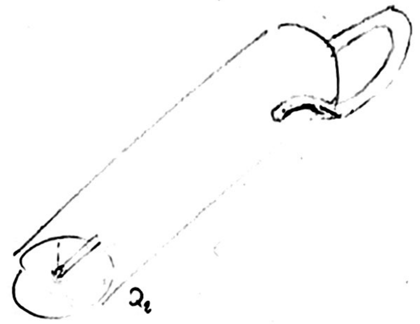
$I = \dfrac{-2e}{0.002} v \cos(45) = \dfrac{1.6 \times 10^{-10}}{\sqrt{2}}$

Distance to travel $= \dfrac{0.002}{\cos(45)} = \sqrt{2}(0.002) = \sqrt{2}\, t$



b)

$Q_1 + Q_2 = -2e$



$$k\int_r^a \frac{Q_1}{r}\,dr = k\int_r^b \frac{Q_2}{r}\,dr \rightarrow Q_1\left[\ln(a)-\ln(r)\right] = Q_2\left[\ln(b)-\ln(r)\right]$$

$$\rightarrow Q_1 \ln(a/r) = Q_2 \ln(b/r)$$

$$Q_1 + Q_2 = -2e$$

$$Q_1 = Q_2\left(\ln(b/r) / \ln(a/r)\right)$$

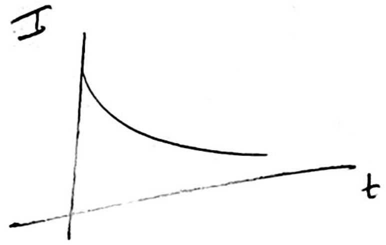
$$Q_1 = -2e\,\frac{\ln(b/r)}{\ln(b/a)}, \quad Q_2 = -2e\,\frac{\ln(r/a)}{\ln(b/a)}$$

$$I = -\frac{dQ_2}{dt} = \frac{2e\ln(r/a)}{\ln(b/a)} \cdot \frac{d}{dt}\Big|_{r=a+vt} \Rightarrow \frac{2e}{\ln(b/a)}\frac{d\left(\ln\left(b/(a+vt)\right)\right)}{dt}$$

distance initial + distance traveled

$$\frac{2e}{\ln(b/a)}\frac{d\left(\cancel{\ln(b)} - \ln(a+vt)\right)}{dt} = -\frac{2e}{\ln(b/a)}\left(\frac{v}{a+vt}\right) = \frac{-2ev}{\ln(b/a)(a+vt)},$$

t on denominator



if 45°, stretch by $\frac{1}{\sqrt{2}}$, time increased by $\sqrt{2}$