~~Physics~~ 5B

Assignment 6

Daniel Delivale

4.22 Transatlantic Cable

(a) $\rho_{copper} = 3 \cdot 10^{-8}\ \Omega m$

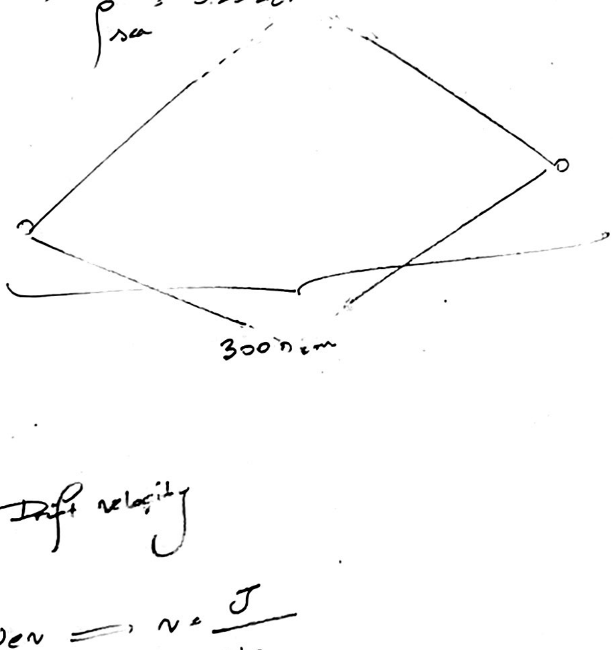
$$R = \rho \frac{L}{A} = (3 \cdot 10^{-8})\left(\frac{3 \times 10^{6}\ m}{7\pi(0.365 \times 10^{-4})^2}\right) = \underline{3.07 \times 10^{4}\ \Omega}$$

Testing different method

$$R_{1\ cable} = 3 \cdot 10^{-8}\left(\frac{3 \times 10^{6}}{\pi(0.365 \times 10^{-4})^2}\right) = 2.15 \times 10^{5}\ \Omega$$

$$\frac{1}{R_T} = \frac{7}{R_{cable}} \Longrightarrow R_T = \frac{R_{cable}}{7} = \underline{3.07 \times 10^{4}\ \Omega}$$

(b) $\rho_{sea} = 0.25\ \Omega m$



$$R_{cone} = \frac{\rho L}{A} = \frac{0.25(1.5 \times 10^{6})}{4.72 \times 10^{9}} = 7.9 \times 10^{-5}$$

$$2R_{cone} = 1.58 \times 10^{-4}\ \Omega \ll 3.07 \times 10^{4}$$

negligible

4.25 Drift velocity

$$J = Nev \Longrightarrow v = \frac{J}{Ne}$$

$$J = \frac{V/R}{A} = \frac{V}{A} \cdot \frac{A}{\rho L} = \frac{V}{\rho L}$$

$$v = \frac{V}{\rho L N e} = \frac{12V}{0.25(2)(2.3 \times 10^{26})(1.6 \times 10^{-19})} = \underline{2.5 \times 10^{-7}\ m/s}$$

4.33 — Conductor Extremum

100Å	Ag
200Å	Sn
100Å	Ag

Solving 4.5

$Ag_{thick} = 100Å$ $\qquad \sigma_{Ag} = 7\sigma_{Sn}$

$Sn_{thick} = 200Å$ $\qquad Sn_{th} = 2Ag_{th}$

Parallel σ

$$R = \frac{L}{\sigma A} \Longrightarrow \frac{1}{R_{\parallel}} = \frac{\sigma_{\parallel} A}{\cancel{L}} = \frac{\sigma_{Ag} A_s}{\cancel{L}} + \frac{\sigma_{Sn} A_{tin}}{\cancel{L}}$$

$$= \sigma_{\parallel} = \frac{\sigma_{Ag} A_s}{A} + \frac{\sigma_{Sn} A_{tin}}{A}$$

$$= \sigma_{Ag} \frac{Ag_{thi}}{th} + \sigma_{Sn} \frac{Sn_{th}}{th}$$

$$= \frac{1}{3}\sigma_{Ag} + \frac{2}{3}\sigma_{Sn} = \frac{7.2}{3} + \frac{2}{3} = \frac{46}{15}$$

Series σ

$$R = \frac{L}{\sigma A} \Longrightarrow \frac{th}{\sigma_{\perp} \cancel{A}} = \frac{A_s}{\sigma_{Ag} \cancel{A}} + \frac{A_{tin}}{\sigma_{S} \cancel{A}} \Longrightarrow \frac{1}{\sigma_{\perp}} = \frac{A_s}{\sigma_{Ag} A_s} + \frac{A}{\sigma_s A_{tin}}$$

$$\frac{1}{\sigma_{\perp}} = \frac{1}{3\sigma_{Ag}} + \frac{2}{3\sigma_{Sn}}$$

$$\sigma_{\perp} = \frac{108}{77} \Longrightarrow \frac{\sigma_{\perp}}{\sigma_{\parallel}} = \underline{0.457}$$

$$\frac{\sigma_\perp}{\sigma_\parallel} = \frac{1}{-c^2\left(\frac{n+1}{n-2}\right) + c\left(\frac{n+1}{n-2}\right) + 1}$$

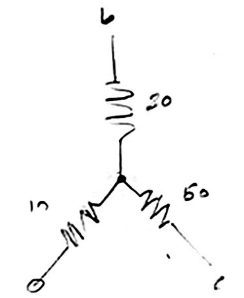
for $c[0, 1]$, $-c^2\left(\frac{n+1}{n-2}\right) + c\left(\frac{n+1}{n-2}\right) + 1 \leq 1$

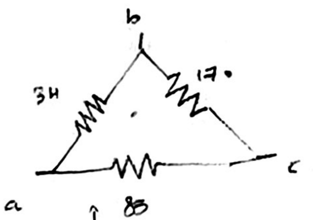
Thus, $\sigma_\perp \leq \sigma_\parallel$ for $c[0,1]$

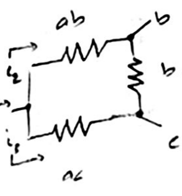
a) Minimum occurs at $c = 1/2$, minimum must exist by MVT.

b) Maximum @ $c = 1$, max must occur for some $n[0, \infty)$.

4.31 Equivalent Boxes







$$Z_{ab} = \frac{34(170 + 85)}{34 + 170 + 85} = 30\ \Omega$$

$Z_{ab} = 30\ \Omega$

$$Z_{bc} = \frac{170(34 + 85)}{34 + 170 + 85} = 70\ \Omega$$

$Z_{bc} = 70\ \Omega$

$$Z_{ac} = \frac{85(170 + 34)}{34 + 170 + 85} = 60\ \Omega$$

$Z_{ac} = 60\ \Omega$

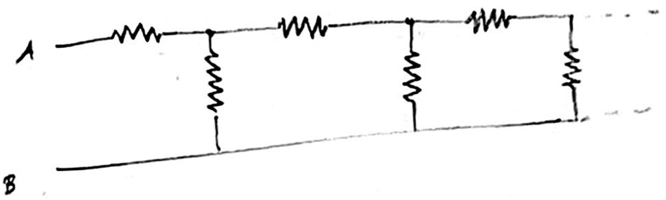
There is no other possibility since solving the Resistance Eqs w/ (3 unknowns) yields only 3 other resistance values.

The boxes are identical since Resistance Ratios (20 : 50, 34 : 85)... are identical

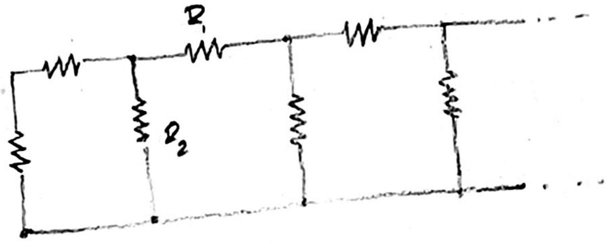
→ current is identical

→ voltage is identical

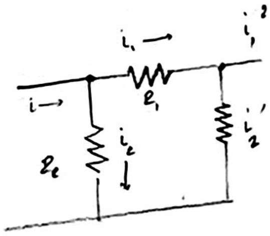
4.36 Attenuator Chain



$$Z_{eq} = R$$



$$R = R_1 + \frac{R_2 R}{R_2 + R} \implies R = \frac{R_1 \pm \sqrt{R_1^2 + 4R_1 R_2}}{2}$$



$i = i_1 + i_2$

$v_2 = i_2 R_2$, $\quad v_1 = i_1 R_1$

$i_1 = i_1' + i_2'$

$v_2' = i_2' R_2$, $\quad v_1' = i_1' R_1$

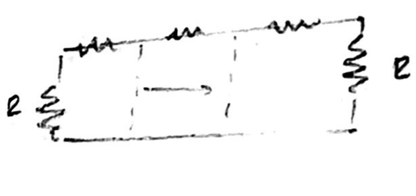
$$\begin{aligned} i &= i_1' + i_2' + i_2 \\ &= i_1'' + i_2'' + i_2' + i_2 \\ &= i_1'' + i_2''' + i_2'' + i_2' + i_2 \\ &= i^{n!} + i_2^{n!} + i_2^{(n-1)!} + i_2^{(n-2)!} \end{aligned}$$

$$\frac{v}{v'} = \frac{i_1}{i} = \frac{i_1'}{i_1} = \frac{i_1''}{i_1'} = \cdots$$

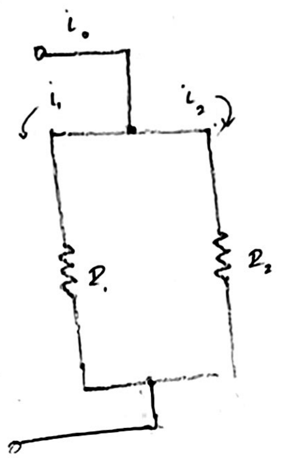
Since $i_1'' < i_1' < i_1$, the resulting voltage decreases in a geometric series

$$\frac{v}{v'} = \frac{1}{2} = \frac{i_1}{i} = \frac{R_2}{R_2 + R}, \quad R = R_2 \implies \underline{\underline{R_2 = 2R_1}}$$

To terminate, add R at the end



7.40 Power Desipation



$$P = i_1^2 R_1 + i_2^2 R_2$$

$$= i_1^2 R_1 + (i_0 - i_1)^2 R_2 = i_1^2 R_1 + (i_0^2 - 2 i_0 i_1 + i_1^2) R_2$$

$$= i_1^2 (R_1 + R_2) - 2 i_0 i_1 R_2 + i_0^2 R_2$$

$$\frac{\partial P}{\partial I_1} = 2 i_1 (R_1 + R_2) - 2 i_0 R_2 + 0 = 0$$

$$\cancel{2} i_1 (R_1 + R_2) = \cancel{2} i_0 R_2$$

$$i_1 = \frac{i_0 R_2}{R_1 + R_2}$$

Ohmic law

$$v_1 = i_1 R_1 \qquad v_2 = i_2 R_2$$

$$i_1 R_1 = i_2 R_2 \Longrightarrow i_1 R_1 = (i_0 - i_1) R_2$$

$$i_1 R_1 + i_1 R_2 = i_0 R_2$$

$$i_1 (R_1 + R_2) = i_0 R_2$$

$$i_1 = \frac{i_0 R_2}{R_1 + R_2}$$

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