Introduction to Quantum Mechanics

deval deliwala

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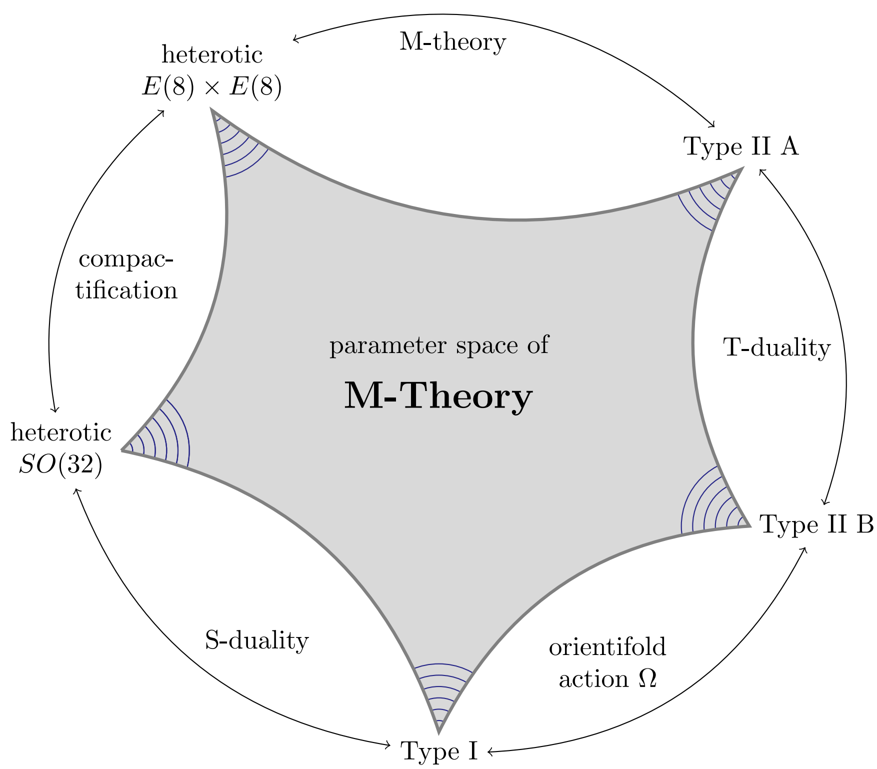
Adapted from *Introduction to Quantum Mechanics* 3rd ed. By David J. Griffiths



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1.1 The Schrödinger Equation

Imagine a particle of mass m, constrained to move along the x axis, subject to move to some specified force $F(x,t)$. The program of *classical mechanics* is to determine the position of the particle at any given time $x(t)$. Once we know that, we can figure out the velocity ($v = \frac{dx}{dt}$), the momentum $p = mv$, the kinetic energy ($T = \frac{1}{2}mv^2$), or any other dynamical variable of interest. To determine $x(t)$, we apply Newton's Second Law: $F = ma$, or more specifically, $F = -\frac{\partial V}{\partial x}$, the derivative of a potential energy function, where $m\frac{\partial^2 x}{\partial t^2} = -\frac{\partial V}{\partial x}$. This together, with initial conditions, determines $x(t)$.

Quantum mechanics approaches this same problem a bit differently. In this case what we're looking for is the particles **wave function**, $\Phi(x,t)$, and we get it by solving the **Schrödinger Equation:**

Schrödinger Equation

$$i\hbar\frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2 \Phi}{\partial x^2} + V\Phi$$

Here i is the square root of -1, and $\hbar$ is Planck's constant - or rather, his *original* constant (h) divided by 2π:

$$\hbar = \frac{h}{2\pi} = 1.054573 \times 10^{-34} Js.$$

The Schrödinger equation plays a role logically analogous to Newton's second law.

1.2 The Statistical Interpretation

But what exactly *is* this wave function, and what does it do for you once you've *got* it? After all a particle by nature is a point, whereas the wave function (as its name suggests) is spread out in space (a function of x, for any given t).

How can such an object represent the state of a *particle*? The answer is provided by Born's **statistical interpretation**, which says that $|\Psi(x,t)|^2$ gives the *probability* of finding the particle at point x, at time t - or, more precisely,

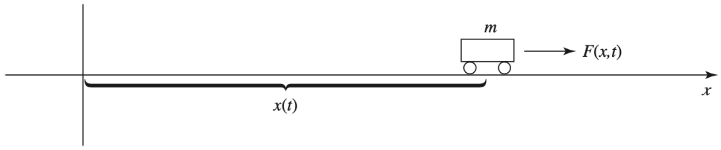

Figure 1: a "particle" constrained to move in one dimension under the influence of a specified force

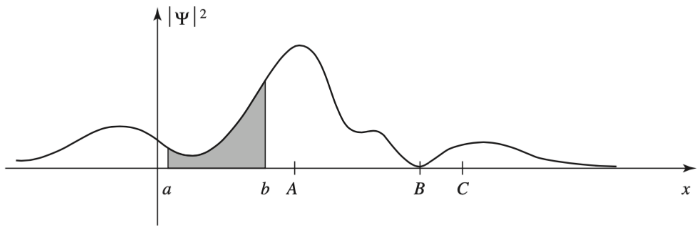

Figure 2: a typical wave function. the shaded area represents the probability of finding the particle between a and b. the particle would be relatively likely to be found near A, and unlikely to be found near B.

Born's Statistical Interpretation

$$\int_a^b |\Psi(x,t)|^2 \, dx = \{\text{probability of finding particle between } a \text{ and } b\text{, at time } t \ \}$$

Probability is the *area* under the graph of $|\Psi|^2$. For the wave function in the figure above, you would be quite likely to find the particle in the vicinity of point A, where $|\Psi|^2$ is large, and relatively unlikely to find it near point B.

The statistical interpretation introduces a kind of **indeterminacy** into quantum mechanics, for even if you know everything, the theory has to tell you about the particle, still you can't predict with certainty the outcome of a simple experiment to predict its position - all quantum mechanics has to offer is *statistical* information about *possible* results. It is natural to wonder whether this indeterminacy is a fact of nature, or a defect in the theory.

Suppose I *do* measure the position of the particle, and I find it to be a point C.

Question

Where was the particle just *before* I made the measurement?

Solution

There are three plausible answers

1. The **realist** position: The *particle was at C*. This certainly seems reasonable, and it is the response Einstein advocated. However, if this is true, quantum mechanics is an *incomplete* theory, since the particle *really was* at C, and yet quantum mechanics was unable to tell us so.

2. The **orthodox** position: The *particle wasn't really anywhere*. It was the act of measurement that forced it to "take a stand" (though how and why it chose the point C we dare not ask). This view is associated with Bohr and his followers. Among physicists it is the most widely accepted position. However, if it is correct, a century worth of debate about the act of measurement has done preciously little to illuminate.

The **agnostic** position: *Refuse to answer*. This is not as silly as it sounds - what sense can there be in making assertions about the status of a particle *before* a measurement. For decades this was the "fall-back" position of most physicists: they'd try to sell you the orthodox answer, but if you were persistent they'd retreat to the agnostic response, and terminate the conversation.