Lecture 1: The Quantum Regime

The Limits of Theories: As physical theories are constructed to account for patterns that emerge from observation and experiment, it is not surprising that such theories are limited by the data that inspired them, and thus are subject to failure when extended into new regimes. We are familiar with historical examples. Newton observed a world that was Galilean invariant – velocities added, observers in different Galilean frames agree on their measurements of space and time separations of events, thus $x_1 - x_2 = x'_1 - x'_2$ and $t_1 - t_2 = t'_1 - t'_2$, so that events simultaneous in one observer's frame are also simultaneous in another's. Yet these properties were aspects of a theory constructed to account for measurements for which the dynamics were governed by the condition

$$\frac{v^2}{c^2} << 1.$$

Einstein's special theory of relativity did not replace classical mechanics, but rather incorporated it: the invariant separation among events changed to $c^2(t_1 - t_2)^2 - (x_1 - x_2)^2 =$ constant, but in the limit where all velocities are small, the classical results are all recovered – valid apart from corrections of order v^2/c^2. That is, classical mechanics is an *effective theory*, fully consistent with special relativity provided measurement are restricted to the effective theory's range of validity, $v << c$. If this condition is fulfilled, one can use the simpler classical theory, with confidence that the predictions we make will be valid with small errors controlled by v^2/c^2.

> Classical mechanics is contained in special relativity:
>
> The former is an effective theory equivalent to the latter in applications where $v \ll c$

One expects in physics to discover the simplest theories – the most "effective" theories – first. We discovered classical mechanics first because it is the theory of falling apples, planetary motion, and sailing ships. Only when we started probing higher velocities were the limitations of the theory recognized – motivating the creation of an extended theory, special relativity.

Effective theory is one of the most powerful and pervasive concepts in modern physics. We in fact will be able to see some of its uses in problems we do. Physics can often be viewed as a tower of effective theories, with each successive layer more complete and more predictive (valid over a wider range of parameters). When we analyze as experiment, we use the effective theory lowest in this "tower" that is adequate for our needs. This theory will be simpler, the physics we are studying will be accurately encoded in a smaller set of parameters. If you are dealing with a non-relativistic mechanics problem, you can of course chose to use special relativity in your analysis – but you will need to work harder and you will learn no new information. Weinberg put it nicely

> You can use any degrees of freedom you want, but if you pick the wrong ones, you will be sorry!

Once you have completed you analysis, the information you have obtain can be "ported up" to the more general effective theories that reside above: this is the process of "matching" one effective theory to another. Nothing is lost by using the simplest appropriate effective theory.

In subatomic physics today the last experimentally validated member of this tower is the Standard Model (SM) – decades of experiments have established its validity, in some cases to precisions exceeding a part in a billion. But we also know that there is something more – massive neutrinos, dark matter, and dark energy provided rather direct evidence of this. 'There is enormous effort underway to learn more about such phenomena, so that we will have a bit more guidance from experiment about the next effective theory in the tower. This theory will not replace the SM, but rather incorporate it in a generalization that accounts for the new phenomena recently discovered.)

Just as special relativity emerged when experiments began to probe higher velocities, quantum mechanics emerged when experiments started to probe atomic scales. The classical mechanics in use was not only nonrelativistic, but also deterministic. Knowledge of the initial conditions (e.g., particle positions and velocities) and interactions among the particles allow one to compute the future evolution of the system. In principle this can be done to arbitrary accuracy, as the theory places no limits on the precision with which those initial conditions can be determined, or the classical equations solved.

This aspect of classical mechanics again reflects the limited range of the data informing the theory, e.g., macroscopic apples falling meters/sec. A second scale that further restricts the applicability of classical physics and thus the boundary beyond which a more general, quantum description must be used, is defined in terms of Planck's reduced constant $\hbar$,

$$\Delta E\ \Delta t >> \hbar = 6.58 \times 10^{-16}\ \text{eV s}$$

$$\Delta E\ \Delta x >> \hbar c = 197.3\ \text{eV nm}$$

If one were to ask, is quantum mechanics relevant to a squash ball confined to a squash court, the second expression above tells us only if we are interested in changes of the squash ball's energy of about one part in 10^{70}. You are welcome to calculate the future trajectory of a squash ball using quantum mechanics – you might need about 10^{70} states in your calculation – but any attempt to do this will convince you that Weinberg provided good advice. The mistakes we make in treating squash ball dynamics using deterministic classical mechanics are extraordinarily small. Newton gave us the right effective theory for this purpose.

But $\hbar$ tells us where the determinism of the classical theory will fail us, and from the numbers above, the failures will begin with atomic physics, and continue as we probe the nuclear and particle scales. The hydrogen atom has the size (Bohr radius) a_0 of about an angstrom, or $\sim$ 0.053 nm – and its electron is bound by E_b $\sim$13.6 eV. How these parameters relate to the expressions above we will determine in this class, but we observe $E_b\, a_0 \sim 1$. The product is certainly not large on the $\Delta E \Delta x$ scale defined above.

Why did QM emerge when it did? It is the usual answer: Because experiment started telling us our prevailing theories were not up to the task of understanding the emerging subatomic world. It is helpful to look back to those early times to recognize what an interesting but confusing time it was.

1. In the early 1800s photoabsorption lines in the solar spectrum – a signature of the discrete transitions between atomic levels – were observed, but there was no theory context for their interpretation. By the middle of the century specific spectral lines were understood to be associated with specific elements, and lines seen in the laboratory were correlated with some seen in the solar spectrum. Late in the century work of Balmer and Rydberg revealed the regularity of the hydrogen spectrum, with $1/\lambda$, where λ is the wavelength, related to integer differences in quantities $1/n_i^2$, where n_i is an integer.

   Why were lines unexpected? Classically accelerating charges radiate, but the spectra produced are continuous. Thus even if someone brilliant in the 19th century had managed to come up with a quasi-modern description of atoms, she would have been hard pressed to explain why electrons are confined to orbits of definite energy, the origin of the discrete lines.

2. The first necessary steps in understanding spectra came with J. J. Thompson's discovery of the electron in 1897, followed by Rutherford's discovery through alpha-particle scattering of a dense nuclear core in atoms. Rutherford correctly concluded that the nuclear mass was a multiple of the hydrogen (proton) mass. In 1911 Rutherford proposed that the atom consisted on a central positive charge surrounded by orbiting negatively charged electrons. As discussed below, the conceptual difficulties presented by the instability of such a system in classical physics – accelerated charges radiate – led to Bohr's early quantum mechanical theory of the atom. About a decade later – 1926 – a much more complete and self-consistent theory of wave mechanics emerged when Schroedinger introduced his equation, the focus of much of this class.

3. Concurrent with these discoveries radioactivities associated with nuclear decays were studied by Roentgen, Becquerel, and Marie and Pierre Curie. These included x-rays, β rays (energetic electrons produced in the weak process of β decay), and nuclear fission via α (the He nucleus) emission.

4. A correct theoretical interpretation of either the structure of atoms or the radiation coming from atoms would have nearly impossible at the turn of the last century, as some of the particles participating in these reactions had not even been discovered. In 1916 Chadwick, then a student, studied the continuous spectrum of electrons omitted in β decay. Rather than being pleased that a spectrum (not lines) was observed, he was unhappy because, if the radioactive decay released some definite energy, a continuous spectrum would seem to contradict energy conservation. To avoid this, Chadwick speculated that some unobserved radiation was also coming out (preserving energy conservation). In 1930 Pauli proposed that this radiation was a new, spin-1/2, light elementary particle he called the neutron, but we call the neutrino. In 1932 Chadwick discovered the "real" neutron, of nearly the same mass as the proton, which quickly resolved enormous confusion over the varying masses, charges, and angular momentum/statistics of nuclei. It is remarkable that we have had a

basic understanding of the constituents of the atom – the neutron, proton, and electron – for less than 100 years.

5. Quantum mechanics is the theory that grew out of our need to understand atoms – their structure, stability, and radiation – as well as other phenomena we will discuss this semester. It plus special relativity were the two revolutions that rocked physics early in the last century.

6. Further, the need to reconcile special relativity and quantum mechanics was also recognized in the 1920s. One year after Schroedinger introduced his wave mechanics, Dirac proposed a relativistic equation for the electron, the Dirac equation. In 1933, in an extraordinary step, Fermi combined the new particles into a remarkably modern theory of β decay

$$n \rightarrow p + e^- + \bar{\nu}_e$$

His paper was rejected from Physical Review for being too speculative. His theory involved the spontaneous production of new particles – the electron and neutrino are not constituents of a nucleus, but instead are produced spontaneously from the vacuum. Fermi's guess for the form of the interaction mediating beta decay was based on analogies with the Coulomb interaction of electromagnetism, though Fermi somehow recognized that there should be no electric field – the interaction occurred between all four particles at a point. He later incorporated into his theory aspects of special relativity – charges viewed in a moving frame produce currents. Four years later Gamow and Teller argued that a second interaction contributed to beta decay, involving the spins of the particles, and to account for experiment this second interaction must be of comparable strength to Fermi's interaction. Remarkably, by this point an effective theory equivalent at low energies to the SM with its vector and axial interactions was being formulated — including the capacity to account for phenomena like parity violation that would not be discovered for another 20 years. This quantum mechanics, relativity, and particle production by fields were being cobbled together in these early times. The SM, a field theory, was formulated in the 1960s, treating electromagnetism and the weak interaction as aspects of one theory, with the final step in validating the basic structure of the SM coming with the recent discovery of the Higgs, in 2012.

The Utility of Quantum Mechanics: QM emerged from studies of physics at the atomic and nuclear scales, and remains the effective theory of choice for an enormous range of phenomena in

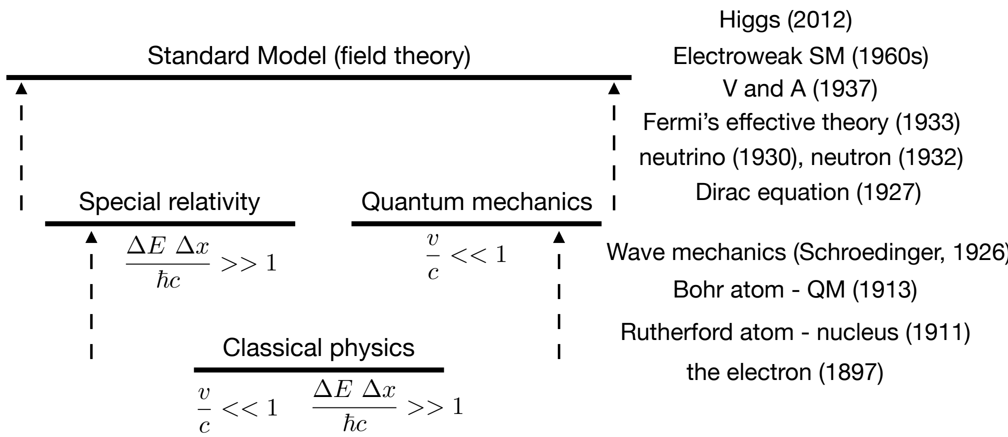

Figure 1: Summary of the developments leading up to and beyond wave mechanics and the Schroedinger equation.

materials and condensed matter, atomic physics, and nuclear physics. Such systems are typically nonrelativistic. In an atom, while the interior 1s orbital does become increasingly relativistic with increasing nuclear charge (scaling as Z^2), still the Coulomb 1s energy is less than a tenth the electron rest mass, provided $Z < 60$. The quasi-particles of nuclear physics – bound states of quarks and gluons we call nucleons – typically have $\frac{v^2}{c^2} \sim \frac{1}{100}$ throughout the table of nuclei. (The lack of variation is because, unlike the Coulomb interaction, the strong interaction is repulsive at short range, so nucleons keep their distance from one another, whether they are in deuterium or in uranium.) Yet these systems are far, far from the classical deterministic limit: a wave description and all the associated interference effects are essential to the physics. Consequently, QM s is the effective theory of choice.

Second, there is a huge "buzz" surrounding QM today, sometimes termed the *second quantum revolution.* The first quantum revolution was acknowledged through a series of Nobel Prizes over the last thirty years recognizing the development of tools for *manipulating atoms* and other quantum matter, including the laser, the maser, quantum electronics, atom traps, optical tweezers, laser cooling, ultra-fast laser pulses, optical frequency combs, and atom interferometers. The first revolution allowed physicists and others to build computers and other devices that were based on classical concepts like the bit – information stored as a series of 0s or 1s – while achieving new milestones in speed and storage because devices could be packed ever more densely on silicon chips.

The second revolution – quantum information and computation computation – envisions new devices that employ quantum mechanics directly in the manipulation and processing of information. If one envisions the two possibilities encoded in a bit as a point either at the north or south poles of a unit sphere, its quantum mechanical analog – a qubit consisting of two interfering states carrying arbitrary phases – covers the entire surface of that sphere, vastly increasing the information that is stored and potentially read out on interrogation. So it is a great time, as a student, to learn QM. Berkeley Physics offers you some excellent quantum information courses, and there are multi-disciplinary centers on campus and at LBL that provided research opportunities for advanced students. This course can be fun just because QM can stretch your mind – the rules of the subatomic world contradict so many of those of our macroscopic one – but it also can prepare you for future steps, should you catch the quantum information/computing bug.