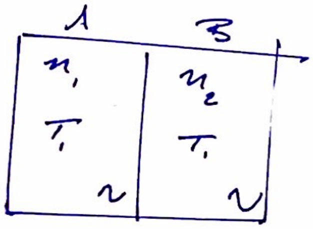
Problem Set 1

Physics 5C

Problem 1

a)



ideal gas

for A,

$$P_1 V = n_1 k_B T_1 \Longrightarrow P_1 = \frac{n_1 k_B T_1}{V}$$

for B,

$$P_2 V = n_2 k_B T_2 \Longrightarrow P_2 = \frac{n_2 k_B T_2}{V}$$

Therefore, $\underline{\underline{P_1/P_2 = \frac{n_1 T_1}{n_2 T_2}}}$.

b) Energy must be conserved so

$$n_1 \cancel{k_B} T_1 + n_2 \cancel{k_B} T_2 = (n_1 + n_2) \cancel{k_B} T_F$$

$$\underline{\underline{T_F = \frac{n_1 T_1 + n_2 T_2}{(n_1 + n_2)}}}$$

c)

$$P_f (2V) = \cancel{(n_1 + n_2)} k_B \left(\frac{n_1 T_1 + n_2 T_2}{\cancel{(n_1 + n_2)}} \right)$$

$$\underline{\underline{P_f = \frac{n_1 T_1 + n_2 T_2}{2V} k_B}}$$

B) After thermal equilibrium is reached, pressure is the same,

$$\begin{cases} P_f V = n_{1f} k_B T_f \\ P_f V = n_{2f} k_B T_f \end{cases} \longrightarrow n_{1f} = n_{2f} = \frac{P_f V}{k_B T_f}$$

Therefore, the change in molecules from A to B is given by

$$\Delta n_1 = n_1 - n_{1f} = n_1 - \frac{P_f V}{k_B T_f}$$

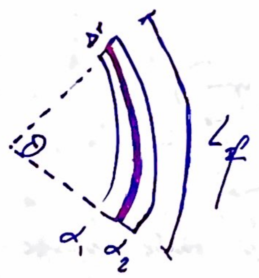
$$= n_1 - \left(\frac{n_1 T_1 + n_2 T_2}{2 T_f} \right)$$

$$= n_1 - \frac{1}{2}\cancel{(n_1 T_1 + n_2 T_2)} \left(\frac{n_1 + n_2}{\cancel{n_1 T_1 + n_2 T_2}} \right)$$

$$= n_1 - \frac{1}{2}(n_1 + n_2)$$

$$= \underline{\underline{\frac{1}{2} n_1 + n_2}} \text{ gas molecules } A \longrightarrow B$$

Problem 2



for α_1,

$$L_0 + dL_1 = L_0 + \alpha_1 L_0 \Delta T$$
$$= L_0(1 + \alpha_1 \Delta T)$$
$$dL_1 = \alpha_1 L_0 \Delta T$$

for α_2,

$$L_0 + dL_2 = L_0 + \alpha_2 L_0 \Delta T$$
$$= L_0(1 + \alpha_2 \Delta T)$$
$$dL_2 = \alpha_2 L_0 \Delta T > dL_1$$

find θ such that arc length between α_1 & α_2 differs by $2\pi r\left(\frac{\theta}{360}\right)$ vs. $2\pi(r+\Delta r)\left(\frac{\theta}{360}\right)$? ✓

$$L_{1f} = L_0 + \alpha_1 L_0 \Delta T = \cancel{2}\pi r\left(\frac{\theta}{\cancel{360}\ 180}\right) \Longrightarrow \theta = \frac{180(L_0 + \alpha_1 L_0 \Delta T)}{\pi r}$$

$$L_{2f} = L_0 + \alpha_2 L_0 \Delta T = \cancel{2\pi}(r+\Delta r)\left(\frac{\theta}{\cancel{360}\ 180}\right) \Longrightarrow \theta = \frac{180(L_0 + \alpha_2 L_0 \Delta T)}{(r+\Delta r)\pi}$$

$$\frac{L_0 + \alpha_1 L_0 \Delta T}{r} = \frac{L_0 + \alpha_2 L_0 \Delta T}{r + \Delta r}$$

$$\frac{r + \Delta r}{r} = \frac{L_0 + \alpha_2 L_0 \Delta T}{L_0 + \alpha_1 L_0 \Delta T} \Longrightarrow \frac{\Delta r}{r} = \frac{L_0 + \alpha_2 L_0 \Delta T}{L_0 + \alpha_1 L_0 \Delta T} \underline{\underline{-1}}$$

~~Therefore,~~ forgot $\underline{\underline{-1}}$

$$\cancel{r = \frac{(L_0 + \alpha_1 L_0 \Delta T)\Delta r}{L_0 + \alpha_2 L_0 \Delta T} \Longrightarrow \theta = \frac{180(L_0 + \alpha_1 L_0 \Delta T)}{\pi\left(\frac{(L_0 + \alpha_1 L_0 \Delta T)\Delta r}{(L_0 + \alpha_2 L_0 \Delta T)}\right)}}$$

Simplifying,

$$\theta = \frac{\cancel{180\left(L_0 + \alpha_1 L_0 \Delta T\right)}}{\cfrac{\cancel{\pi\left(L_0 + \alpha_1 L_0 \Delta T\right)\Delta r}}{L_0 + \alpha_2 L_0 \Delta T}} = \cancel{\frac{180\left(L_0 + \alpha_2 L_0 \Delta T\right)}{\pi \Delta r}}$$

Doesn't make sense, there's no α_1.

$$\frac{\Delta r}{r} = \frac{\cancel{L_0} + \alpha_2 L_0 \Delta T - \cancel{L_0} - \alpha_1 L_0 \Delta T}{L_0 + \alpha_1 L_0 \Delta T} = \frac{L_0 \Delta T\left(\alpha_2 - \alpha_1\right)}{L_0 + \alpha_1 L_0 \Delta T}$$

$$\cancel{\frac{r}{\Delta r} = \frac{L_0 \Delta T\left(\alpha_2 - \alpha_1\right)\Delta r}{L_0 + \alpha_1 L_0 \Delta T}}$$

$$\frac{r}{\cancel{\Delta r}} = \frac{\left(L_0 + \alpha_1 L_0 \Delta T\right)\cancel{\Delta r}}{L_0 \Delta T\left(\alpha_2 - \alpha_1\right)}$$

Therefore,

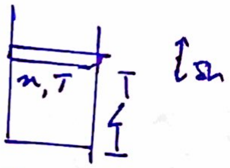
$$\cancel{\theta = \frac{180\left(L_0 + \alpha_1 L_0 \Delta T\right)}{\pi\left(\cfrac{L_0 \Delta T\left(\alpha_2 - \alpha_1\right)\Delta r}{L_0 + \alpha_1 L_0 \Delta T}\right)}}$$

$$\theta = \frac{180\left(L_0 + \alpha_1 L_0 \Delta T\right)}{\pi\left(\cfrac{\left(L_0 + \alpha_1 L_0 \Delta T\right)\Delta r}{L_0 \Delta T\left(\alpha_2 - \alpha_1\right)}\right)}$$

$$\theta = \frac{180}{\pi}\left(\cancel{L_0 + \alpha_1 L_0 \Delta T} \cdot \frac{L_0 \Delta T\left(\alpha_2 - \alpha_1\right)}{\cancel{\left(L_0 + \alpha_1 L_0 \Delta T\right)}\Delta r}\right) = \underline{\underline{\frac{180}{\pi \Delta r}\left(L_0 \Delta T\left(\alpha_2 - \alpha_1\right)\right)}}$$

$$\underline{\underline{\theta = \frac{180}{\pi}\left(\frac{L_0 \Delta T\left(\alpha_2 - \alpha_1\right)}{\Delta r}\right) \text{ in degrees.}}}$$

Problem 3



$\Delta T = 0$, cross-sectional area constant.

force summation:

$$\sum F_y = P(L)A - Mg = m\frac{d^2h}{dt^2}$$

At Equilibrium

$$P_0(AL_0) = nk_BT = P(AL) \qquad P_0A = Mg$$

$$P_0L_0 = PL$$

$$P_0 = \frac{Mg}{A} = \frac{PL}{L_0}. \qquad P_0 = \frac{nk_BT}{AL_0} = \frac{Mg}{A}$$

Therefore,

$$L_0 = \frac{nk_BT}{Mg}. \qquad \text{Oscillation } L_0 \pm \delta h.$$

$$P_0 = \frac{Mg}{A} \Longrightarrow \sum F_y = PA - P_0A = m\frac{d^2h}{dt^2}$$

$$= A(P - P_0) = m\frac{d^2h}{dt^2}$$

The pressure as a function δh is then.

$$P(\delta h) = \frac{P_0AL_0}{AL_0 + A\delta h} \approx \cancel{\frac{P_0AL_0}{V_0}}\left(1 - \frac{A\delta}{V_0}\right)$$

$$\approx P_0\left(1 - \frac{A\delta}{AL_0}\right) \qquad V_0 = AL_0$$

Therefore, we get

$$\sum F_y = A\left(P_0\left(\cancel{1} - \frac{A\delta h}{AL_0}\right) - \cancel{P_0}\right) = m\frac{d^2(\delta h)}{dt^2}$$

$$= -A\left(\frac{P_0\cancel{A}\delta h}{\cancel{A}L_0}\right) = \frac{-Mg\,\delta h}{\left(\frac{nk_BT}{Mg}\right)} = m\frac{d^2\delta h}{dt^2}$$

Simplifying,

$$\frac{-Mg\,\delta h}{\left(\frac{nk_BT}{Mg}\right)} = M\frac{d^2(\delta h)}{dt^2}$$

$$= -\frac{\cancel{M}^2 g^2\,\delta h}{nk_B T} = \frac{\cancel{M}\,d^2(\delta h)}{dt^2} \implies -\left(\frac{g^2}{nk_BT}\right)\delta h = \frac{d^2(\delta h)}{dt^2}$$

The final differential equation:

$$\frac{d^2(\delta h)}{dt^2} + \left(\frac{g^2}{nk_BT}\right)\delta h = 0$$

Therefore, the angular frequency of the block M:

(b) $$\underline{\underline{\omega = \frac{g}{\sqrt{nk_BT}}}} \implies f = \frac{1}{2\pi}\omega$$

The solution of the differential equation

$$\delta h(t) = C_1\cos(\omega t) + C_2\sin(\omega t)$$

(a) $A = \sqrt{C_1^2 + C_2^2} = L - \frac{nk_BT}{Mg}$

$$\delta h(0) = C_1 = L - L_0 = L - \frac{nk_BT}{Mg}$$

$$\delta h'(0) = \omega C_2 = \underset{N_0}{} \implies C_2 = 0$$

released from rest.

~~This makes no sense.~~

$$\cancel{\delta h(t) = L\cos\left(\frac{g}{\sqrt{nk_BT}}t\right)}$$

$$\boxed{\delta h(t) = \left(L - \frac{nk_BT}{Mg}\right)\left(\frac{g}{\sqrt{nk_BT}}t\right)}$$