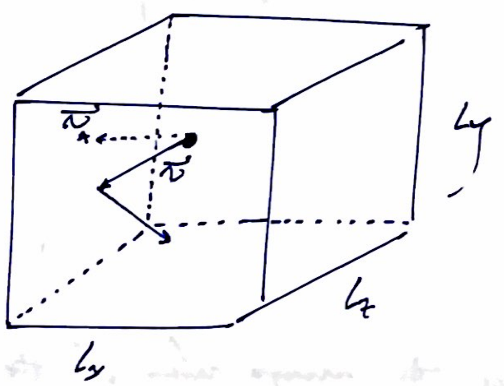
Physics 5C
Review Lecture

The Kinetic Theory of Gases

1. Temperature & microscopic kinetic energy

In this section we will concretely define the relationship between temperature, a macroscopic physical quantity and the kinetic energy of microscopic particles. We will also see the role of the Boltzmann constant as fundamental constant that relates microscopic to macroscopic physics.

Consider a closed container of dimension L_x, L_y, L_z filled w/ gas. The pressure on the container's walls is due to collision of particles on the surface.



Consider a particle colliding on the left wall of the container. Assuming the collision is elastic, the particle remains in the same velocity in the y & z direction after the collision, but the x component is flipped.

$\vec{v}_{xi} \longrightarrow -\vec{v}_{xi}$ ith particle.

The magnitude of the change in momentum is then

$$|\Delta \vec{P}_i| = 2 m v_{xi}$$

After colliding w/ the left wall the particle moves towards the right wall and bounces back again. The time it takes to travel from the left to right wall is

$$\Delta t = L_x / v_{xi}$$

This particle thus strikes w/ the left wall every $2\Delta t$.

The average force exerted on the ith particle by the left wall is then,

$$F_{avg,i} = \frac{|\Delta \vec{p}_i|}{2\Delta t} = \frac{m v_{x_i}^2}{L_x}$$

By newton's third law, the force on the wall by the ith particle is then also

$$F_{avg,\ \text{ith on wall}} = \frac{m v_{x_i}^2}{L_x}$$

Adding up contribution from N particles,

$$F_{avg} = \frac{m}{L_x} \sum_{i=1}^{N} v_{x_i}^2$$

Average force on the wall by all N particles.

Because there are so many particles F_{avg} is the same for any time interval, therefore,

$$F = F_{avg} = \frac{m}{L_x} \sum_{i=1}^{N} v_{x,i}^2$$

The average value of $v_{x_i}^2$ is defined as

$$\langle v_x^2 \rangle = \frac{1}{N} \sum_{i=1}^{N} v_{xi}^2$$

$\langle \ldots \rangle$ denotes the average value of the N particles of the quantity inside the brackets.

Therefore, the force on the left wall is

$$F = \frac{m N \langle v_x^2 \rangle}{L_x}$$

The pressure on the left wall is then given by

$$P = \frac{F}{L_y L_z} = \frac{m N \langle v_x^2 \rangle}{L_x L_y L_z} = \frac{m N \langle v_x^2 \rangle}{V}$$

V volume of the container.

Although we focus on the left wall, the discussion is general & applies to all directions. Since $v_i^2 = v_{xi}^2 + v_{yi}^2 + v_{zi}^2$ the same relation holds for all average values $\langle v^2 \rangle$

$$\langle v^2 \rangle = \langle v_x^2 \rangle + \langle v_y^2 \rangle + \langle v_z^2 \rangle$$

Even though the container has different dimensions, the particles hit all across the walls when traveling and so

$$\langle v_x^2 \rangle = \langle v_y^2 \rangle = \langle v_z^2 \rangle = \frac{1}{3}\langle v^2 \rangle$$

We can thus express pressure on the walls as

$$P = \frac{mN\langle v^2 \rangle}{3V} = \frac{2N\langle k \rangle}{3V}$$

$$\langle k \rangle = \frac{1}{2} m \langle v^2 \rangle$$

We see that we connect the pressure of the gas, a macroscopic property, with the average kinetic energy of the microscopic ~~constituents~~. Furthermore, if we write the ideal gas law as

$$P = \frac{N k_B T}{V}$$

we see that

$$P = \frac{2N\langle k \rangle}{3V} = \frac{N k_B T}{V} \implies \langle k \rangle = \frac{3}{2} k_B T$$

We now explicitly see that the higher kinetic energy particles have, the higher the temperature). The Boltzmann constant plays the role of the proportional constant between macroscopic & microscopic physics.

The equation

$$\langle K \rangle = \frac{3}{2} k_B T$$

is actually a special case of the equipartition theorem, which states that if the energy of a degree of freedom has a quadratic form

$$\frac{1}{2} m v^2, \quad \frac{1}{2} I \omega^2, \quad \frac{1}{2} k x^2, \ \ldots$$

and if the temperature is high enough to treat the allowed energy levels to be continuous, then for each degree of freedom, the average energy is

$$\langle E \rangle = \frac{1}{2} k_B T$$

And since the particles have 3 degrees of freedom v_x, v_y, v_z we get an an additional factor of 3.