single_systems

December 18, 2023

[2]:
```python
# Vectors & Matrices in Python

from numpy import array

ket0 = array([1, 0])
ket1 = array([0, 1])

ket0 / 2 + ket1 / 2 # Taking average of ket0 and ket1
```

[2]:
```
array([0.5, 0.5])
```

[3]:
```python
# Matrices & Operations

M1 = array([[1, 1], [0, 0]])
M2 = array([[1, 1], [1, 0]])

M1 / 2 + M2 / 2
```

[3]:
```
array([[1. , 1. ],
       [0.5, 0. ]])
```

[4]:
```python
# Matrix Multiplication using matmul

from numpy import matmul

display(matmul(M1, ket1))
display(matmul(M1, M2))
display(matmul(M2, M1))
```

```
array([1, 0])

array([[2, 1],
       [0, 0]])

array([[1, 1],
       [1, 1]])
```

0.0.1 States, Measurements and Operations

Defining and Displaying State Vectors Qiskits `Statevector` class provides functionality for defining and manipulating quantum state vectors.

[5]:
```
from qiskit.quantum_info import Statevector
from numpy import sqrt

u = Statevector([1/sqrt(2), 1/sqrt(2)])
v = Statevector([(1 + 2.0j)/3, -2/3])
w = Statevector([1/3, 2/3])

print("State vectors u, v, and w have been defined")
```

```
State vectors u, v, and w have been defined
```

The `Statevector` class provides a `draw` method for displaying state vectors including `latex` and `text` options for different visualizations

[6]:
```
display(u.draw("latex"))
display(v.draw("text"))
```

$$\frac{\sqrt{2}}{2}|0\rangle + \frac{\sqrt{2}}{2}|1\rangle$$

```
[ 0.33333333+0.66666667j,-0.66666667+0.j        ]
```

The `Statevector` class also includes the `is_valid` method, which checks to see if a given vector is a valid quantum state vector (it has a euclidean norm $= 1$)

[7]:
```
display(u.is_valid())
display(w.is_valid())
```

```
True
```

```
False
```

Simulating measurements using the `measure` method from the `Statevector` class

[8]:
```
v = Statevector([(1+2.0j)/3, -2/3])
v.draw("latex")
```

[8]:

$$(\frac{1}{3} + \frac{2i}{3})|0\rangle - \frac{2}{3}|1\rangle$$

Running the `measure` method stimulates a standard basis measurement. It returns the result of that measurement, plus the new quantum state of our system after that measurement

[9]:
```
v.measure()
```

[9]:
```
('1',
 Statevector([ 0.+0.j, -1.+0.j],
```

```
            dims=(2,)))
```

Measurements are probabilistic, so the same method can return different results. If the measurement yields 0 the quantum state vector after the measurements takes place to be

$$\frac{1+2i}{\sqrt{5}}|0\rangle$$

(rather than $|0\rangle$).

If the measurement yields 1, the quantum state becomes

$$-|1\rangle$$

(rather than $|1\rangle$).

In both cases the alternatives are equivalent – they are said to *differ* by a *global phase* because one is equal to the other multiplied by a complex number on the unit circle.

As an aside, `Statevector` will throw an error if the `measure` method is applied to an invalid quantum state vector.

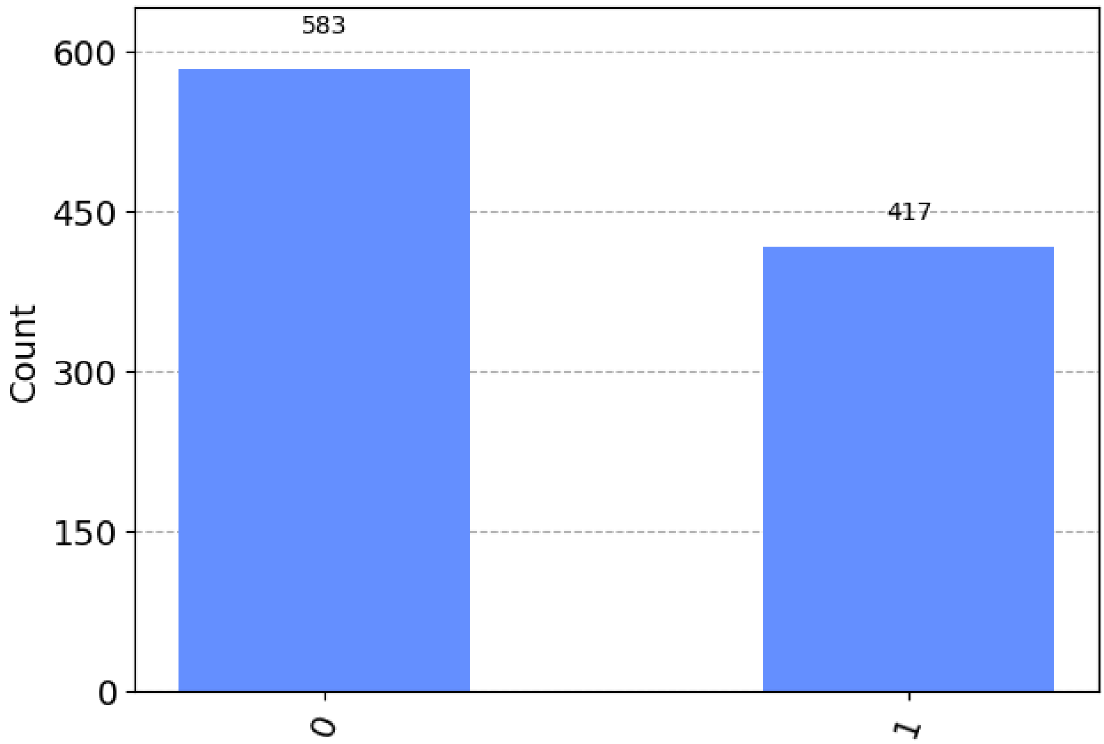
`Statevector` also comes with a `sample_counts` method that allows for the simulation of any # of measurements on the system. We can use `plot histogram` to visualize the results.

[10]:
```
from qiskit.visualization import plot_histogram

statistics = v.sample_counts(1000)
display(statistics)
plot_histogram(statistics)
```

```
{'0': 583, '1': 417}
```

[10]:



0.0.2 Peforming Operations with `Operator` and `Statevector`

Unitary operations can be defined and performed on state vectors in Qiskit using the `Operator` class

[11]:
```
from qiskit.quantum_info import Operator

X = Operator([[0, 1], [1, 0]])
Y = Operator([[0, -1.0j], [1.0j, 0]])
Z = Operator([[1, 0], [0, -1]])
H = Operator([[1 / sqrt(2), 1 / sqrt(2)], [1 / sqrt(2), -1 / sqrt(2)]])
S = Operator([[1, 0], [0, 1.0j]])
T = Operator([[1, 0], [0, (1 + 1.0j) / sqrt(2)]])

v = Statevector([1, 0])

v = v.evolve(H)
v = v.evolve(T)
v = v.evolve(H)
v = v.evolve(T)
v = v.evolve(Z)

v.draw("latex")
```

[11]:

$$(0.8535533906 + 0.3535533906i)|0\rangle + (-0.3535533906 + 0.1464466094i)|1\rangle$$

Looking ahead towards Quantum Circuits

We can define Quantum Circuits (which right now will simply be a sequence of unitary operations performed on a single qubit)

[12]:
```
from qiskit import QuantumCircuit

circuit = QuantumCircuit(1)

circuit.h(0)
circuit.t(0)
circuit.h(0)
circuit.t(0)
circuit.z(0)

circuit.draw()
```

[12]:
```
q:  H   T   H   T   Z
```

The operations are applied sequentially.

Let us first initialize a starting quantum state vector and evolve that state according to the above sequence of operations.

[13]:
```
ket0 = Statevector([1, 0])
v = ket0.evolve(circuit)
v.draw("latex")
```

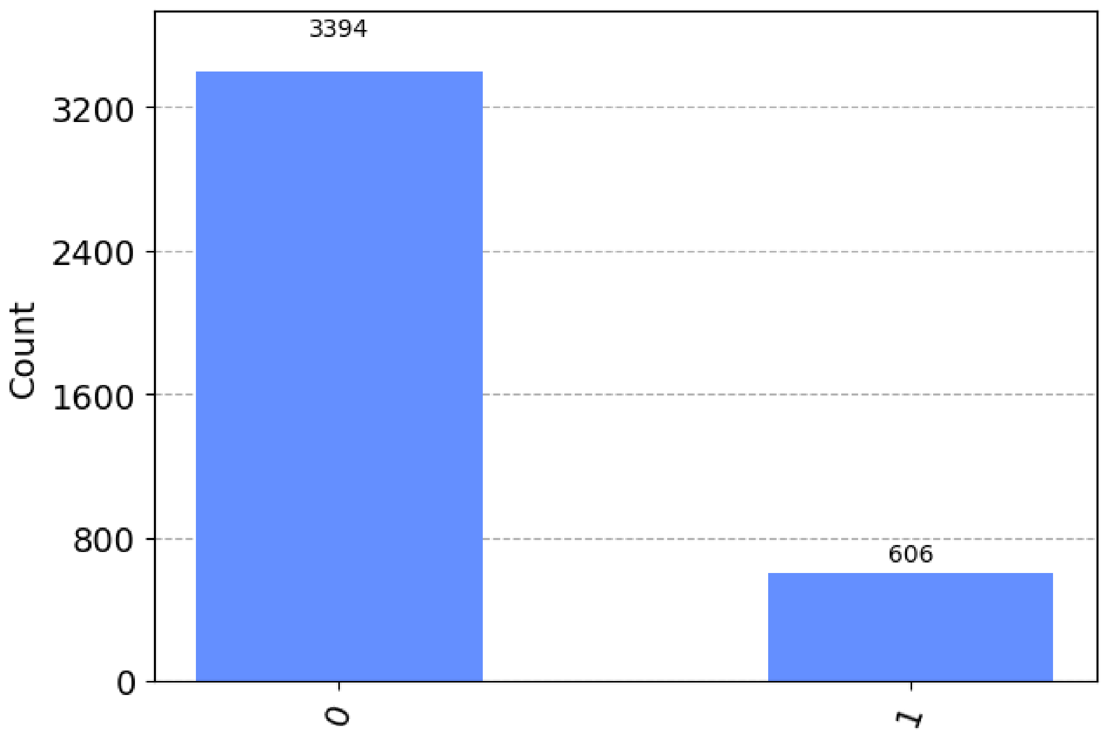
[13]:

$$(0.8535533906 + 0.3535533906i)|0\rangle + (-0.3535533906 + 0.1464466094i)|1\rangle$$

Finally let us simulate the result of running this experiment (i.e. preparing the state $|0\rangle$, applying the sequence of operations represented by the circuit and measuring) 4000 times.

[14]:
```
statistics = v.sample_counts(4000)
plot_histogram(statistics)
```

[14]:


3394
606
3200
2400
1600
800
0
Count
0
1