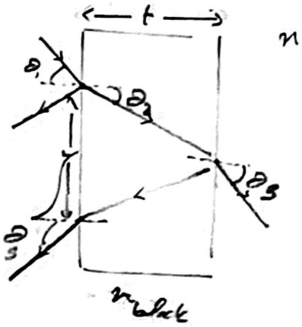
Prelab #0 Physics 5CL.

Donal [illegible]

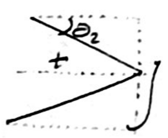
1. Basic Geometry



let $x \equiv \sin\theta_1$

$z \equiv \sin\theta_2$

a)



$$\sin\theta_2 = z = \frac{y/2}{\sqrt{\frac{y^2}{4} + t^2}} = \frac{y}{2\sqrt{\left(\frac{y}{2}\right)^2 + t^2}}$$

b) $x \equiv \sin\theta_1$

$$\sigma_x = \left|\frac{dx}{d\theta_1}\right| \sigma_{\theta_1} = \cos\theta_1\, \sigma_{\theta_1}$$

$$\sigma_z = \sqrt{\left(\frac{\partial z}{\partial y}\sigma_y\right)^2 + \left(\frac{\partial z}{\partial t}\sigma_t\right)^2}$$

$$\frac{\partial z}{\partial y} = \frac{t^2}{2\left(\frac{y^2}{4} + t^2\right)^{3/2}}$$

$$\frac{\partial z}{\partial t} = \frac{-yt}{2\left(t^2 + \frac{y^2}{4}\right)^{3/2}}$$

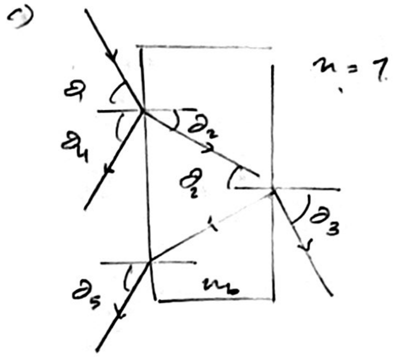
Therefore,

$$\sigma_z = \sqrt{\left(\frac{\sigma_y t^2}{2\left(\frac{y^2}{4} + t^2\right)^{3/2}}\right)^2 - \left(\frac{\sigma_t y t}{2\left(t^2 + \frac{y^2}{4}\right)^{3/2}}\right)^2}$$

$$= \sqrt{\frac{\sigma_y^2 t^4 - \sigma_t^2 y^2 t^2}{2\left(t^2 + \frac{y^2}{4}\right)^{3/2}}} = \frac{1}{\sqrt{2}\left(t^2 + \frac{y^2}{4}\right)^{3/4}}\sqrt{t^2\left(\sigma_y^2 t^2 - \sigma_t^2 y^2\right)}$$

$$\sigma_z = \frac{1}{\sqrt{2}\left(t^2 + \frac{y^2}{4}\right)^{3/4}}\sqrt{t^2\left(\sigma_y^2 t^2 - \sigma_t^2 y^2\right)}$$

a)



$\theta_1 = \theta_4$ by law of reflection

By Snell's Law,

(1)
$$\left.\begin{aligned} \sin\theta_1 &= n_{block}\sin\theta_2 \\ n_{block}\sin\theta_2 &= \sin\theta_3 \end{aligned}\right\} \Longrightarrow \theta_1 = \theta_3$$

By symmetry, $\theta_3 = \theta_5$ $\therefore$ $\theta_1 = \theta_3 = \theta_4 = \theta_5$.

b)

(1)
$$x = n_{block}\, z \Longrightarrow n_{block} = \frac{x}{z} = \frac{\sin\theta_1}{\dfrac{y}{2\sqrt{\left(\frac{y}{2}\right)^2 + t^2}}} = \underline{\underline{\frac{2\sin\theta_1\sqrt{\left(\frac{y}{2}\right)^2 + t^2}}{y}}}$$

c) $n_{block} = \dfrac{x}{z}$, $\qquad \dfrac{\partial n}{\partial x} = \dfrac{1}{z}$

$$\frac{\partial n}{\partial z} = -\frac{x}{z^2}$$

$$\sigma_{n_{block}} = \sqrt{\left(\frac{1}{z}\sigma_x\right)^2 + \left(\frac{x}{z^2}\sigma_z\right)^2} = \sqrt{\frac{\sigma_x^2}{z^2} + \frac{x^2\sigma_z^2}{z^2}}$$

$$\underline{\underline{\sigma_{n_{block}} = \frac{1}{z}\sqrt{\sigma_x^2 + x^2\sigma_z^2}}}$$

2 Least Squares Hypothesis $y = -x + b$

$$\chi^2 = \sum w_i(-y_i + x_i - b)^2 = \sum \left(\frac{-y_i + x_i - b}{\sqrt{\alpha_{y_i}^2 + \alpha_{x_i}^2}}\right)^2$$

To find the b that minimizes χ^2

$$\frac{\partial \chi^2}{\partial b} = \frac{\partial}{\partial b}\left[\sum_i^n \left(\frac{-y_i + x_i - b}{\alpha_{y_i\,\mathrm{equiv}}}\right)^2\right] = \sum_i^n \frac{2}{\alpha_{y_i\,\mathrm{equiv}}}(y_i + x_i - b)(0 + 0 - 1)$$

$$= \sum -\frac{2}{\alpha}(y_i + x_i - b) = 0$$

Therefore,

$$\sum_i^n \frac{y_i}{\alpha_{y_i\,\mathrm{equiv}}} + \sum_i^n \frac{x_i}{\alpha_{y_i\,\mathrm{equiv}}} - \sum_i^n \frac{b}{\alpha_{y_i\,\mathrm{equiv}}} = 0$$

$$\boxed{b = \frac{\sum_i^n \frac{y_i}{\alpha_{y_i\,\mathrm{equiv}}} + \sum_i^n \frac{x_i}{\alpha_{y_i\,\mathrm{equiv}}}}{\sum_i^n \frac{1}{\alpha_{y_i\,\mathrm{equiv}}}}} \quad \text{to minimize } \chi^2$$

b) $$\alpha_b = \sqrt{\left(\frac{\partial b}{\partial y_i}\alpha_{y_i}\right)^2 + \left(\frac{\partial b}{\partial x_i}\alpha_{x_i}\right)^2}$$

$$\frac{\partial b}{\partial y_i} = \frac{1}{\sum_i^n \frac{1}{\alpha_{y\,\mathrm{equiv}}}} \sum_i^n \frac{1}{\alpha_{y\,\mathrm{equiv}}} = 1, \qquad \frac{\partial b}{\partial x_i} = \frac{1}{\sum \frac{1}{\alpha_{y\,\mathrm{equiv}}}} \sum \frac{1}{\alpha_{y\,\mathrm{equiv}}} = 1$$

$$\boxed{\alpha_b = \sqrt{\alpha_{y_i}^2 + \alpha_{x_i}^2} = \alpha_{y,\,\mathrm{equiv},\,i}}$$