Physics 5C

Review Lecture

Thermal Expansion of Solids & Fluids

Intuitively thermal expansion occurs because as temperature increases, the constituent particles move faster & thus move further from their average position. Each particle thus requires more space, expanding the material.

It is reasonable to assume that each part of an object expands proportionally due to thermal expansion: the larger the dimension L the larger the change of L. As such, it is reasonable to suggest the following formula.

$$\frac{dL}{dT} = \alpha(T) L$$

where $\alpha(T)$ is the coefficient of linear expansion at temperature T. If the variation ΔT isn't too large, we can approximate α to be constant. The change in linear dimension ΔL is then

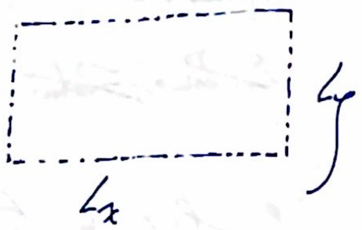
$$\Delta L = \alpha L \Delta T$$

For larger variations in temperature, the temperature dependence of $\alpha(T)$ can't be ignored. We solve the differential equation given by

$$\int_{L_i}^{L_f} \frac{dL}{L} = \int_{T_i}^{T_f} \alpha(T)\, dT$$

Example

The coefficient of area expansion α_A



$$\alpha_A = \frac{1}{A}\frac{dA}{dT}$$

Consider a rectangular tile of dimension L_x + L_y made of material w/ linear expansion coefficient α. Show that when the expansion is small compared to the original dimension, the two expansion coefficients are related by

$$\alpha_A = 2\alpha$$

$$A + dA = (L_x + dL_x)(L_y + dL_y)$$

$$= (L_x + \alpha L_x dT)(L_y + \alpha L_y dT)$$

$$= L_x L_y + 2\alpha L_x L_y dT + \cancel{\alpha^2 L_x L_y dT^2}^{\,0}$$

$$= A + 2\alpha A\, dT$$

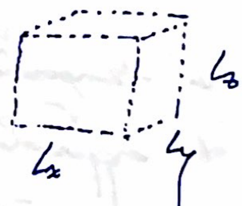
Therefore,

$$dA = 2\alpha A\, dT$$

$$\alpha_A = 2\alpha$$

Exercise #2

The coefficient of volume expansion β



$$\beta = \frac{1}{V}\frac{dV}{dT}$$

Consider a cuboid of dimension L_x L_y L_z made of material w/ coefficient of linear expansion α. The cuboid has a mass density ρ_0 @ an initial temperature T_0. Show that if expansion is small,

$$\beta = 3\alpha$$

for an expansion,

$$V + dV = (L_x + dL_x)(L_y + dL_y)(L_z + dL_z)$$

$$= (L_x + \alpha L_x dT)(L_y + \alpha L_y dT)(L_z + \alpha L_z dT)$$

$$= (L_x L_y + 2\alpha L_x L_y dT)(L_z + \alpha L_z dT)$$

$$= L_x L_y L_z + 3\alpha L_x L_y L_z dT + \cancelto{0}{2\alpha^2 L_x L_y L_z dT^2}$$

$$= V + 3\alpha V dT$$

Therefore,

$$\frac{1}{V}\frac{dV}{dT} = \underline{\beta = 3\alpha.}$$

Show that the mass density of the solid at temperature T is given by

$$\rho(T) = \rho_0 e^{-3\alpha(T-T_0)}$$

where $\alpha(T-T_0)$ is of order one and not small.

Mass density $\sim \frac{1}{V}$

$$\int_{V_0}^{V_f} \frac{1}{V} dV = \int_{T_0}^{T} 3\alpha \, dT$$

$$\Longrightarrow \ln\left(\frac{V_f}{V_0}\right) = 3\alpha(T-T_0)$$

$$\frac{V_f}{V_0} = e^{3\alpha(T-T_0)} \Longrightarrow V_f = V_0 e^{3\alpha(T-T_0)}$$

$$\rho(T) = \frac{m}{V_0 e^{3\alpha(T-T_0)}} = \underline{\rho_0 e^{-3\alpha(T-T_0)}}$$

The Equation of State of Gases & Ideal Gas Law

$$PV = nk_B T \qquad k_B = 1.380649 \times 10^{-23}\ J/K$$

where n is the total # of particles contained & k_B is the Boltzmann Constant. Here, Temperature T is measured in Kelvin.

If you define a variable "mole" such that

$$1 \text{ mole} = N_A \text{ particles}$$

where N_A is the Avogadro constant 6.0223×10^{23} particles/mole. You can then define

$$PV = nRT, \quad n = \# \text{ moles}$$

$$R = k_B N_A = 8.314\ J/\text{mol}\cdot K$$

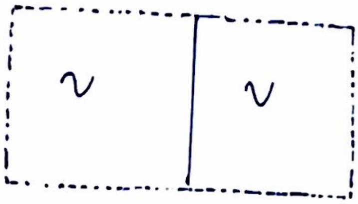
This equation assumes gas molecules do not interact with each other, but models behaviour well when the pressure is low. In reality, gas molecules do interact w/ each other. Due to the vacuum fluctuation of the E&M field, gas molecules are electrically polarized creating a force between molecules that decreases over distance w/ $1/r^6$. A better ideal gas law can then be defined

$$\left(P + a\frac{N^2}{V^2}\right)\left(\frac{V}{N} - b\right) = k_B T$$

known as the van der Waal's equation of state.

ple

A container filled with ideal gas is separated by a movable piston



At $T = 27°C$, the chambers are in equilibrium. If the left is increased to $127°C$, what will be its volume when thermal equilibrium is reached?

Initially both temperatures are $T_0 = 300k$. We then raise the left to $400°C$, $\frac{4}{3}T_0$. Therefore,

$$P_{lf}V_{lf} = \frac{4}{3}P_{li}V_{li} \Longrightarrow P_{lf}V_{lf} = \frac{4}{3}P_{rf}V_{f}$$

$$P_{f}V_{f} = P_{ri}V_{ri}$$

And since the pressures must be the same for equilibrium,

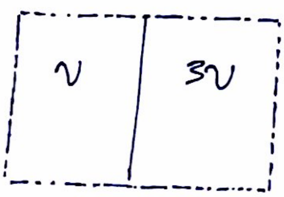
$$V_{l,f} = \frac{4}{3}V_{r,f}$$

Knowing that $V_{l,f} + V_{r,f} = 2V$, we finally get

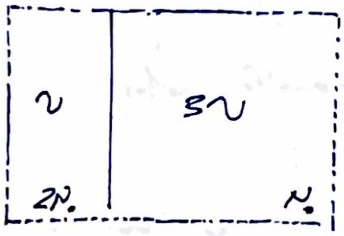
$$V_{l,f} = \frac{8}{7}V.$$

Exercise

Two chambers of volume V & $3V$ are separated by a thin wall.



Initially the left chamber has ~~that~~ $2N_0$ gas molecules, while the right side has N_0 molecules of the same kind. If we open the seal how many particles flow from the left to right side?



Initially,

$$\cancel{P_{l,i} V = P_{r,i}(3V)}$$

$$\cancel{2N_0 k_B T = N_0 k_B T}$$

Once Equilibrium is reached, pressure is the same.

$$\cancel{P_{l,f}} V = n_{l,f} k_B T$$

$$\cancel{3P_{r,f}} V = n_{r,f} k_B T \longrightarrow n_{l,f} = 3n_{r,f} \; ; \; n_{l,f} + 3n_{r,f} = 3N_0$$

$$4P_f V = (3N_0) k_B T \qquad n_{l,f} = \frac{P_f V}{k_B T} = \frac{3N_0}{4}$$

$$P_f = \frac{3N_0 k_B T}{4V}$$

Therefore,

$$\Delta n_l = n_{l,i} - n_{l,f} = 2N_0 - \frac{3N_0}{4} = \frac{5}{4} N_0 \text{ moved from left to right.}$$