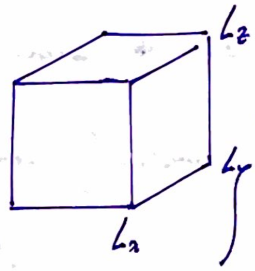
Physics 734a

Review Lecture

Rayleigh-Jeans formula & Stefan-Boltzmann



volume L^3
temperature T

Cubic-Blackbody: walls absorb & perfectly emit all incident radiation maintaining thermal equilibrium between contained radiation & the walls characteristic of T

$$\frac{P}{A} = \sigma T^4 \qquad \frac{\text{power } (P)}{\text{unit surface area } A}$$

$$\sigma \sim 5.67 \, 10^{-8} \, W/m^2 K^4$$

Rayleigh & Jeans attempted to derive this formula from first principles by summing over the E&M fields in the box. This requires one to calculate the # of standing E&M modes in a volume L^3.

E&M waves satisfy Laplaces Equation

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2} + k^2 \psi = 0$$

where $k = k_x^2 + k_y^2 + k_z^2$. This separates the equation into a product of sin in the x, y, z directions all of which vanish at the boundaries 0 & L.

The soln are:

$$\psi[n_x, n_y, n_z] = N \sin\left(\frac{\pi n_x}{L} x\right) \sin\left(\frac{\pi n_y}{L} y\right) \sin\left(\frac{\pi n_z}{L} z\right)$$

$$k^2 = \frac{\pi^2}{L^2}\left(n_x^2 + n_y^2 + n_z^2\right)$$

where (n_x, n_y, n_z) are positive integers. We wish to count how many modes (n_x, n_y, n_z) there are. Switching to spherical coordinates, integrating over k, taking into account that $k_i/(\pi/L) = n_i$

$$N(k)\, dk = \frac{1}{8} \cdot 2 \cdot \frac{4\pi k^2}{(\pi/L)^3} dk = \frac{V k^2}{\pi^2} \qquad V = L^3$$

The $1/8$ factor comes from the portion of the sphere $n_i > 0$

The 2 factor comes from supporting both $\vec{E}$ & $\vec{B}$ wave projections

Implementing a classical Boltzmann distribution, calculating the avg. energy per cavity mode,

$$\bar{\varepsilon} = \frac{\int_0^\infty \varepsilon e^{-\varepsilon/k_B T} d\varepsilon}{\int_0^\infty e^{-\varepsilon/k_B T} d\varepsilon} = k_B T$$

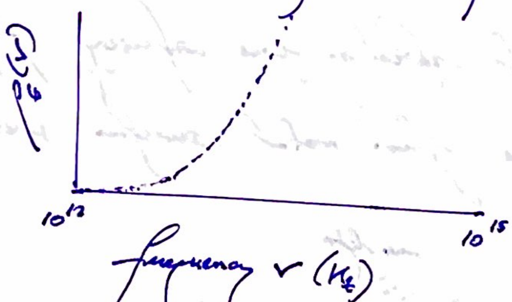
Adding the result to the expression for the # of standing wave modes, to get the Energy Density,

$$\frac{\varepsilon}{V} = k_B T \int \frac{k^2}{\pi^2} dk = k_B T \int \frac{8\pi \nu^2 d\nu}{c^3} = k_B T \int \frac{8\pi}{\lambda^4} d\lambda$$

since $k = \frac{2\pi\nu}{c} = \frac{2\pi}{\lambda}$.

This result neither reproduces observation nor the Stefan Boltzmann Law. The calculation is not self consistent, as the integral diverges for large k/ν, small λ.

Rayleigh Jeans Result.



In 1900, Planck revised this by replacing the classical Boltzmann integral over energy-weights by a discrete sum corresponding to energy quantized as $\mathcal{E} = nh\nu$, $n = 0, 1, 2, \ldots$ where h is a new physical constant. This modifies the calculation into

$$\bar{\mathcal{E}} = \frac{\sum_{n=0}^{\infty} nh\nu e^{-nh\nu/k_BT}}{\sum_{n=0}^{\infty} e^{-nh\nu/k_BT}} = -\frac{1}{\sum_{n=0}^{\infty} e^{-nh\nu/k_BT}} \frac{d}{d\left(\frac{1}{k_BT}\right)} \sum_{n=0}^{\infty} e^{-nh\nu/k_BT}$$

A bit of algebra yields

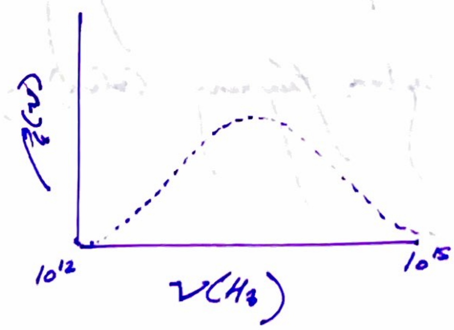
$$\bar{\mathcal{E}} = \frac{h\nu}{e^{h\nu/k_BT} - 1}$$

which yields

$$\frac{\mathcal{E}}{V} = \int_0^{\infty} \rho_E(\nu)\, d\nu$$

$$\rho_E(\nu) = \frac{8\pi h}{c^3} \frac{\nu^3}{e^{h\nu/k_BT} - 1} \Longrightarrow \begin{cases} k_BT \dfrac{8\pi\nu^2}{c^3} & \dfrac{h\nu}{k_BT} \ll 1 \\ \dfrac{8\pi h}{c^3} e^{-h\nu/k_BT} & \dfrac{h\nu}{k_BT} \gg 1 \end{cases}$$

Max Planck Result.

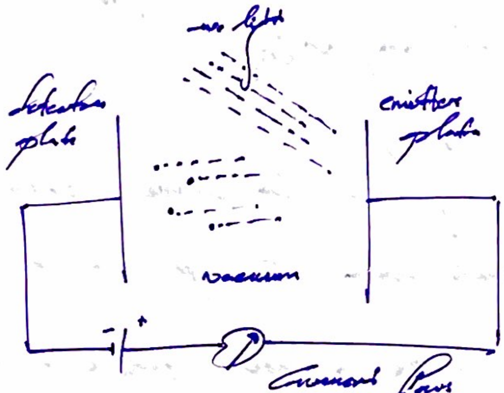


Recovers RJ for small frequencies ν

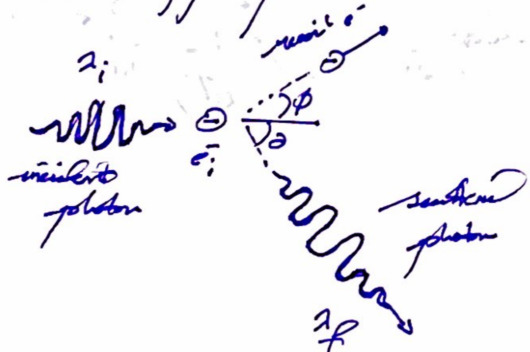
Photoelectric Effect.

At the same time black body radiation was confusing physicists, experiments examine the emission of electrons from metal surfaces when shined w/ UV light.

photoelectric effect experimental setup



Electrons get shot off from the emitter plate via Compton Scattering



$$\lambda_f - \lambda_i = \Delta\lambda = \frac{h}{m_0 c}(1-\cos\theta)$$

Results from experiment:

1. the ~~#~~ but not the Energy of photoelectrons depends on light intensity
2. photoelectrons appear as soon as light is turned on, even if at a low intensity
3. photoelectron energy depends on the frequency of light

   higher frequency (blue) → high energy e^-

These results were unexpected in the classical picture of light as a wave. If light was a wave the light's energy from the wave would be absorbed across the plate, around e^-. When the threshold for e^- emission is reached an e^- would be emitted w/ same energy as if the incident light had a different frequency.

Einstein in 1909 proposed the notion of wave particle duality to solve this issue. He argued that the photoelectric effect observations were because light was absorbed in packets of quanta of energy $h\nu$. Energy conservation then yields

$$h\nu = KE_{e^-} + h\nu_0$$

where $h\nu_0$ is the energy required to knock an e^- from the metal, the work function. If $\nu < \nu_0$, no emission is produced. This solves all 5 results from the experiment

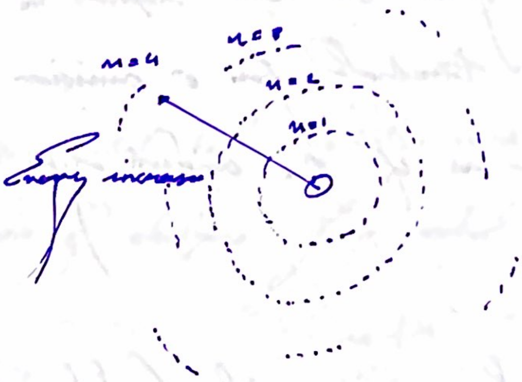
de Broglie & the Bohr Atom

By the early 1900s many experiments had been done observing the absorption & emission of visible & other frequencies of light from simple atoms

As the binding energy of hydrogen is $-13.6\,eV/n_i^2$, where the principle quantum # takes on integer values $n_i = 1, 2, 3, \ldots$. The emission lines correspond to energies

$$E_{n_f} - E_{n_i} = -13.6\,eV\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right), \quad n_f < n_i .$$

The principle quantum # describes how close an e^- is from the nucleus.



Various "series" had been identified

$$\text{Balmer (visible)}: \quad \Delta E = -13.6\,\text{eV}\left(\frac{1}{2^2} - \frac{1}{n_i^2}\right)$$

$$\text{Lyman (UV)}: \quad \Delta E = -13.6\,\text{eV}\left(\frac{1}{1^2} - \frac{1}{n_i^2}\right)$$

$$\text{Paschen (IR)}: \quad \Delta E = -13.6\,\text{eV}\left(\frac{1}{3^2} - \frac{1}{n_i^2}\right)$$

Rutherford proposed the model of the atom as e^- bound & orbiting a nucleus. Bohr related this idea of quantisation in 1913. He formed a classical model of e in circular orbits around the nucleus, where

$$|\vec{r} \times \vec{p}| = mvr = \frac{nh}{2\pi} = n\hbar$$

where $\hbar$ is planck's constant. This model reproduces the results above.

The model gets two important things right.

1. Atomic systems can exist only in certain stationary or quantised states, each characterised by a definite energy
2. Transitions between e^- states can occur via absorption or emission of radiation of energy $\Delta E = h\nu$, in agreement of Einstein & Planck.

Einstein & Planck had treated E&M waves as particles, de Broglie suggested that massive particles may also act as waves — that is quantum mechanics.

For a photon:

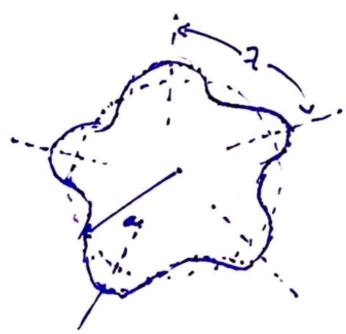
$$p_\gamma = \frac{h\nu}{c} = \frac{h}{\lambda} \quad \text{momentum}$$

→ perhaps a massive particle satisfies the same relationship

$$p_e = m_e v = \frac{h}{\lambda} \implies \lambda = \frac{h}{m_e v} \quad \text{a particle's de Broglie wavelength}$$

If one calculates the circumference of an atom & the de Broglie wavelength of an e^- moving at $v/c \sim 0.01$ one finds $\lambda \sim 2$ Å, about the same.

De Broglie accounted for the Bohr model of the atom by supposing the e^- orbits in hydrogen correspond to an integral number of de Broglie wavelengths.



Atomic orbitals correspond to an integral # of de Broglie wavelengths.