Problem Set 2

Physics 5C, UC Berkeley, Spring 2024

Due Monday, 2/5, at 11:59PM

Problem 1

Consider a collection of particles confined to move on a *two*-dimensional plane. (Electrons moving in graphene are an example of such system.) For simplicity, we assume the particles are free to move in both directions and do not interact with each other. The mass of each particles is m and the system is in thermal equilibrium at a temperature T. The Boltzmann constant is denoted as k_B.

(a) The probability distribution of the particles' *speed* is denoted as $\mathcal{F}(v)$, while the x-component of the particles' *velocity*, v_x, has a probability distribution $\phi_x(v_x)$. Similarly, the distribution for v_y is $\phi_y(v_y)$. Assuming v_x and v_y are independent variables, find the relations between $\mathcal{F}(v)$, $\phi_x(v_x)$ and $\phi_y(v_y)$ based on the rotational symmetry.

(b) Derive the probability distribution $\mathcal{F}(v)$ from (a) and the equipartition theorem.

(c) What is the average speed $\langle v \rangle$ of the particles?

Problem 2

In this problem we will derive the pressure of an ideal gas at an equilibrium temperature T from the Maxwell-Boltzmann distribution

$$\mathcal{F}(v) = 4\pi \left(\frac{m}{2\pi\, k_B T} \right)^{3/2} v^2\, e^{-\frac{mv^2}{2k_B T}},$$

where m is the mass of the gas particles and k_B is the Boltzmann constant.

(a) Find the number of particles colliding on the wall per unit time per unit area with an angle θ with respect to the normal of the surface with a speed v. Express your answer in terms of v, θ and n, the number of particles per volume.

(b) Show that the pressure caused by the gas collisions is $P = nk_B T$.

Problem 3

Show that the time dependence of the pressure inside an oven (volume V) containing hot gas (molecular mass m, temperature T) with a small hole of area A is given by

$$P(t) = P(0)e^{-t/\tau},$$

with

$$\tau = \frac{V}{A}\sqrt{\frac{2\pi m}{k_B T}}.$$

Below are selected optional problems. We do not collect your work, but you are encouraged to do as many practice problems as you can.

Problem 4

Calculate the rms speed of Hydrogen (H_2, helium (He) and oxygen (O_2) at room temperature. The atomic masses of H, He and O are 1, 4, and 16 respectively. Compare these speeds with the escape velocity on the surface of (i) the Earth, (ii) the Sun.

Problem 5

A gas effuses into a vacuum through a small hole of area A. The particles that then collimated by passing through a very small of radius a, in a screen a distance d from the first hole. Find the rate at which particles emerge from the second hole. Express your answer in terms of A, a, d, n (the number density of the gas) and $\langle v \rangle$ (the average speed of the gas). Assume that no collisions take place after the gas effuses through the first hole and that $d \gg a$.

Problem 6

An astronaut goes for a space walk and her space suit is pressurized to 1 atm. Unfortunately, a tiny piece of space dust punctures her suit and it develops a small hole of radius 1 μm. What force does she feel due to the effusion gas?

Problem 7

Show that if a gas were allowed to leak through a small hole into an evacuated sphere and the particles condensed where they first hit the surface, they would form a uniform coating.

Problem 8

Suppose we have a cubic room with sides of length ℓ, at temperature T and pressure P, containing N molecules of ideal gas.

(a) Show that the frequency f with which gas molecules strikes a wall lying in the yz-plane is

$$f = \frac{\langle |v_x| \rangle}{2} \frac{P}{k_B T} \ell^2.$$

(b) Show that the equation can then be written as

$$f \simeq \frac{P \ell^2}{\sqrt{4 m k_B T}}$$

where m is the mass of the gas molecule.

(c) Assume the cubic air-filled room has pressure of 1 atm, temperature 20 °C, and has sides of 3 m. Determine f.