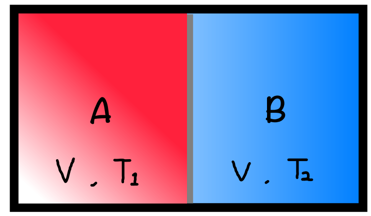
Problem Set 1

Physics 5C, UC Berkeley, Spring 2024

Due Monday, 1/29, at 11:59PM

Problem 1

As shown in the right figure, a thermally insulated container have two chambers of equal volume V. The two chambers respectively contain N_1 and N_2 of ideal gas molecules of the same type, with absolute temperatures of T_1 and T_2 respectively.



(a) What is the ratio of the two chambers' pressures, P_1/P_2?

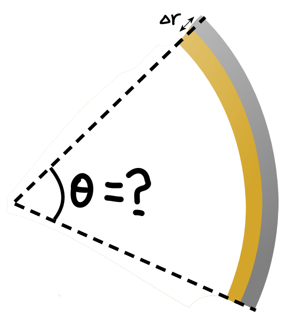
(b) Suppose we remove the divider between the two chambers, what is the final temperature of the mixed gas?

(c) Continuing from part (b), what is the pressure of the mixed gas?

(d) Find the number of gas molecules that flow from chamber A to chamber B.

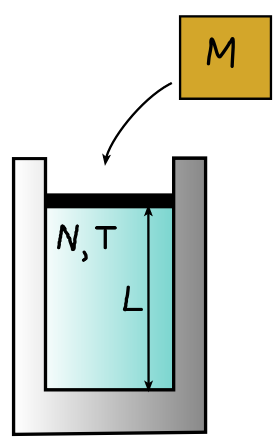
Problem 2

A bimetallic metal strip consist of two ribbons of metals with linear expansion coefficient α_1 and α_2 respectively, with $\alpha_2 > \alpha_1$. The two metals are bonded together. At the initial temperature T_0, both metals have the same initial length L_0. The distance between the centers of the two strips is Δr, where $\Delta r \ll L_0$ so that the thermal expansion along that dimension is negligible compared with the expansion of the length. When we raise the temperature by an amount ΔT, the bimetallic strip bends due to the difference in expansion coefficient. Find the angle of the curvature θ in terms of L_0, α_1, α_2, Δr and ΔT.



Problem 3

A container filled with N particles of ideal gas is originally in equilibrium with temperature T, and the frictionless movable piston is at a height L. The cross-sectional area of the container is constant. We now place a block of mass M on the piston and release it from rest. In below, we assume the temperature of the gas is maintained at T, and the variation of the piston's height δh is much smaller than L so that we can expand our equations to the order of $\delta h/L$.



(a) After releasing the block, the piston oscillates. What is the amplitude of the oscillation?

(b) What is the frequency of the oscillation?

Below are selected optional problems. We do not collect your work, but you are encouraged to do as many practice problems as you can.

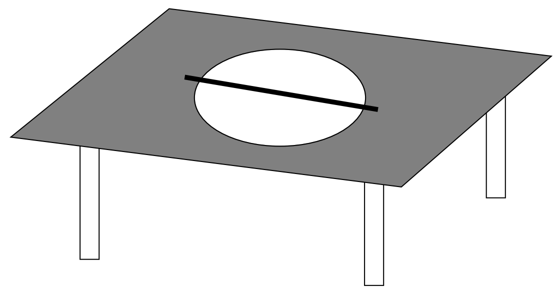
Problem 4

(a) Assuming that you are nearly all water, how many water molecules are there in your body?

(b) How many drops of water are there in all the oceans of the world? The mass of the world's ocean is about 10^{21} kg. And you can assume the radius of a droplet is about 1 mm.

Problem 5

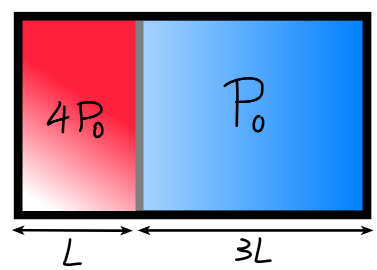
A rod made out of aluminum (linear expansion coefficient $\alpha_{\mathrm{Al}} = 25\times10^{-6}\,^\circ\mathrm{C}^{-1}$) has a length $L_0 = 100$ cm at $100\,^\circ$C. It rests on top of a circular hole in a steel plate (linear expansion coefficient $\alpha_{\mathrm{Fe}} = 12\times10^{-6}\,^\circ\mathrm{C}$) that forms the top of a table. This hole has a radius $R = 49.7$ cm at $20\,^\circ$C. The center of the rod lies at the center of the hole. The temperature of the system is then changed to T. Is there a T below which the rod will fall through the hole? Neglect the thickness of the rod, and the friction between the rod and the plate.



(Physics 7B, UC Berkeley, Spring 2014)

Problem 6

A cuboid chamber is divided by a movable piston. The left chamber initially has a pressure $4P_0$ and a width L. The right chamber has a pressure P_0 and a width $3L$. The two chambers have the same temperature maintained throughout the process. What will be the final width of the left chamber when equilibrium is reached?

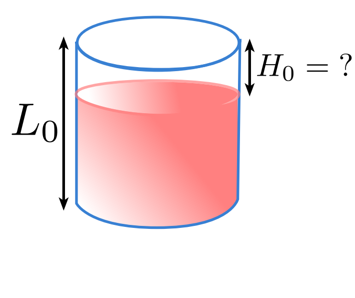


Problem 7

Two rods made of material with linear expansion coefficient α_1 and $\alpha_2 = 1.2\alpha_1$ have a length difference ΔL at temperature T_0. As we heat up the rods, the length difference between the two rods remains the same. Assuming the expansion coefficient is temperature-independent and $\alpha\,\Delta T \ll 1$ where ΔT is the temperature variation, find the lengths of the two rods at T_0. Express your answer in terms of ΔL.

Problem 8

Wine bottles are never completely filled as some headspace is needed due to wine's large coefficient of expansion. In this problem we model this by considering a cylindrical glass container of inner radius R_0 and height L_0, both measured at a reference temperature T_0. Suppose we want to leave some headspace such that the alcohol will not fully fill up the headspace until a critical temperature T_c, how much headspace H_0 (the length between the top surface of the alcohol and the top surface of the glass container) do we need at temperature T_0? Express your answer in terms of L_0, coefficient of *volume* expansion for alcohol and glass, β_{a} and β_{g} respectively, the reference temperature T_0, and the critical temperature T_c.



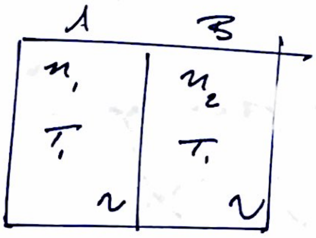
Problem Set 7

[illegible]

Physics 5c

Problem 1

a)



ideal gas

for A,

$$P_1 V = n_1 k_B T_1 \implies P_1 = \frac{n_1 k_B T_1}{V}$$

for B,

$$P_2 V = n_2 k_B T_2 \implies P_2 = \frac{n_2 k_B T_2}{V}$$

Therefore, $\underline{\underline{P_1/P_2 = \dfrac{n_1 T_1}{n_2 T_2}}}$.

b) Energy must be conserved so

$$n_1 \cancel{k_B} T_1 + n_2 \cancel{k_B} T_2 = (n_1 + n_2) \cancel{k_B} T_F$$

$$\underline{\underline{T_F = \frac{n_1 T_1 + n_2 T_2}{(n_1 + n_2)}}}$$

c)

$$P_f (2V) = \cancel{(n_1 + n_2)} k_B \left(\frac{n_1 T_1 + n_2 T_2}{\cancel{(n_1 + n_2)}} \right)$$

$$\underline{\underline{P_f = \frac{n_1 T_1 + n_2 T_2}{2V} k_B}}$$

d) After thermal equilibrium is reached, pressure is the same,

$$\begin{cases} P_f V = n_{1f} k_B T_f \\ P_f V = n_{2f} k_B T_f \end{cases} \longrightarrow n_{1f} = n_{2f} = \frac{P_f V}{k_B T_f}$$

Therefore, the change in molecules from A to B is given by

$$\Delta n_1 = n_1 - n_{1f} = n_1 - \frac{P_f V}{k_B T_f}$$

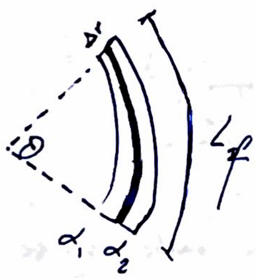
$$= n_1 - \left(\frac{n_1 T_1 + n_2 T_2}{2 T_f}\right)$$

$$= n_1 - \frac{1}{2}\cancel{(n_1 T_1 + n_2 T_2)}\left(\frac{n_1 + n_2}{\cancel{n_1 T_1 + n_2 T_2}}\right)$$

$$= n_1 - \frac{1}{2}(n_1 + n_2)$$

$$= \underline{\underline{\frac{1}{2} n_1 + n_2}} \text{ gas molecules } A \longrightarrow B$$

Problem 2



for α_1,

$$L_0 + dL_1 = L_0 + \alpha_1 L_0 \Delta T$$
$$= L_0(1 + \alpha_1 \Delta T)$$
$$dL_1 = \alpha_1 L_0 \Delta T$$

for α_2,

$$L_0 + dL_2 = L_0 + \alpha_2 L_0 \Delta T$$
$$= L_0(1 + \alpha_2 \Delta T)$$
$$dL_2 = \alpha_2 L_0 \Delta T > dL_1$$

find θ such that arc length between α_1 & α_2 differs by $2\pi r\left(\frac{\theta}{360}\right)$ vs. $2\pi(r+\Delta r)\left(\frac{\theta}{360}\right)$?

$$L_{1f} = L_0 + \alpha_1 L_0 \Delta T = \cancel{2}\pi r\left(\frac{\theta}{\cancel{360}\,180}\right) \Longrightarrow \theta = \frac{180(L_0 + \alpha_1 L_0 \Delta T)}{\pi r}$$

$$L_{2f} = L_0 + \alpha_2 L_0 \Delta T = \cancel{2}\pi(r+\Delta r)\left(\frac{\theta}{\cancel{360}\,180}\right) \Longrightarrow \theta = \frac{180(L_0 + \alpha_2 L_0 \Delta T)}{(r+\Delta r)\pi}$$

$$\frac{L_0 + \alpha_1 L_0 \Delta T}{r} = \frac{L_0 + \alpha_2 L_0 \Delta T}{r + \Delta r}$$

$$\frac{r + \Delta r}{r} = \frac{L_0 + \alpha_2 L_0 \Delta T}{L_0 + \alpha_1 L_0 \Delta T} \Longrightarrow \frac{\Delta r}{r} = \frac{L_0 + \alpha_2 L_0 \Delta T}{L_0 + \alpha_1 L_0 \Delta T} \underline{\underline{-1}}$$

~~Therefore,~~

$$\cancel{r = \frac{(L_0 + \alpha_1 L_0 \Delta T)\Delta r}{L_0 + \alpha_2 L_0 \Delta T}} \quad \text{forgot } \underline{\underline{-1}} \Longrightarrow \theta = \cancel{\frac{180(L_0 + \alpha_1 L_0 \Delta T)}{\pi\left(\frac{(L_0 + \alpha_1 L_0 \Delta T)\Delta r}{(L_0 + \alpha_2 L_0 \Delta T)}\right)}}$$

Simplifying,

$$\partial = \cancel{\frac{\dfrac{180(L_0 + \alpha_1 L_0 \Delta T)}{\pi (L_0 + \alpha_1 L_0 \Delta T)\Delta r}}{L_0 + \alpha_2 L_0 \Delta T}} = \cancel{\frac{180(L_0 + \alpha_2 L_0 \Delta T)}{\pi \Delta r}}$$

doesn't make sense, there's no α_1.

$$\frac{\Delta r}{r} = \frac{\cancel{L_0} + \alpha_2 L_0 \Delta T - \cancel{L_0} - \alpha_1 L_0 \Delta T}{L_0 + \alpha_1 L_0 \Delta T} = \frac{L_0 \Delta T(\alpha_2 - \alpha_1)}{L_0 + \alpha_1 L_0 \Delta T}$$

$$\frac{r}{\cancel{\Delta r}} = \cancel{\frac{L_0 \Delta T(\alpha_2 - \alpha_1)\Delta r}{L_0 + \alpha_1 L_0 \Delta T}} \qquad \frac{r}{\cancel{\Delta r}} = \frac{(L_0 + \alpha_1 L_0 \Delta T)\cancel{\Delta r}}{L_0 \Delta T(\alpha_2 - \alpha_1)}$$

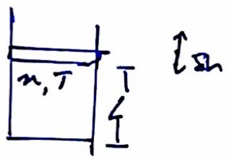
Therefore,

$$\partial = \cancel{\frac{180(L_0 + \alpha_1 L_0 \Delta T)}{\pi\left(\dfrac{L_0 \Delta T(\alpha_2 - \alpha_1)\Delta r}{L_0 + \alpha_1 L_0 \Delta T}\right)}} \qquad \partial = \frac{180(L_0 + \alpha_1 L_0 \Delta T)}{\pi\left(\dfrac{(L_0 + \alpha_1 L_0 \Delta T)\Delta r}{L_0 \Delta T(\alpha_2 - \alpha_1)}\right)}$$

$$\partial = \frac{180}{\pi}\left(\cancel{L_0 + \alpha_1 L_0 \Delta T} \cdot \frac{L_0 \Delta T(\alpha_2 - \alpha_1)}{\cancel{(L_0 + \alpha_1 L_0 \Delta T)}\Delta r}\right) = \frac{180}{\pi \Delta r}\left(L_0 \Delta T(\alpha_2 - \alpha_1)\right)$$

$$\partial = \frac{180}{\pi}\left(\frac{L_0 \Delta T(\alpha_2 - \alpha_1)}{\Delta r}\right)$$ in degrees.

Problem 3



force summation:

$$\sum F_y = P(L)A - Mg = m\frac{d^2h}{dt^2}$$

$\Delta T = 0$, cross-sectional area constant.

At Equilibrium,

$$P_0(AL_0) = nk_BT = P(AL) \qquad P_0A = Mg,$$

$$P_0L_0 = PL$$

$$P_0 = \frac{Mg}{A} = \frac{PL}{L_0} \qquad P_0 = \frac{nk_BT}{AL_0} = \frac{Mg}{A}$$

Therefore,

$$L_0 = \frac{nk_BT}{Mg}. \qquad \text{Oscillation } L_0 \pm \delta h.$$

$$P_0 = \frac{Mg}{A} \Longrightarrow \sum F_y = PA - P_0A = m\frac{d^2h}{dt^2}$$

$$= A(P - P_0) = m\frac{d^2h}{dt^2}$$

The pressure as a function δh is then.

$$P(\delta h) = \frac{P_0AL_0}{AL_0 + A\delta h} \approx \cancel{\frac{P_0AL_0}{V_0}}\left(1 - \frac{A\delta}{V_0}\right) \qquad V_0 = AL_0$$

$$\approx P_0\left(1 - \frac{A\delta}{AL_0}\right)$$

Therefore, we get

$$\sum F_y = A\left(P_0\left(\cancel{1} - \frac{A\delta h}{AL_0}\right) - \cancel{P_0}\right) = \frac{md^2(\delta h)}{dt^2}$$

$$= -A\left(\frac{P_0\cancel{A}\,\delta h}{\cancel{A}L_0}\right) = \frac{-Mg\,\delta h}{\left(\frac{nk_BT}{Mg}\right)} = \frac{md^2\delta h}{dt^2}$$

Simplifying,

$$\frac{-M g\, \delta h}{\left(\frac{n k_B T}{M g}\right)} = M \frac{d^2(\delta h)}{dt^2}$$

$$= -\frac{M^2 g^2\, \delta h}{n k_B T} = \frac{M\, d^2(\delta h)}{dt^2} \Longrightarrow -\left(\frac{g^2}{n k_B T}\right) \delta h = \frac{d^2(\delta h)}{dt^2}$$

The final differential equation:

$$\frac{d^2(\delta h)}{dt^2} + \left(\frac{g^2}{n k_B T}\right) \delta h = 0$$

Therefore, the angular frequency of the block M:

(b) $$\omega = \frac{g}{\sqrt{n k_B T}} \Longrightarrow f = \frac{1}{2\pi}\omega$$

The solution of the differential equation

$$\delta h(t) = C_1 \cos(\omega t) + C_2 \sin(\omega t)$$

(a) $$A = \sqrt{C_1^2 + C_2^2} = L - \frac{n k_B T}{M g}$$

$$\delta h(0) = C_1 = L - L_0 = L - \frac{n k_B T}{M g}$$

$$\delta h'(0) = \omega C_2 = v_0 \Longrightarrow C_2 = 0$$

released from rest.

~~This makes no sense.~~

$$\delta h(t) = L \cos\left(\frac{g}{\sqrt{n k_B T}} t\right)$$

$$\delta h(t) = \left(L - \frac{n k_B T}{M g}\right)\left(\frac{g}{\sqrt{n k_B T}} t\right)$$