Homework #4

(Due: Dec 4)

Task 1. [30 Points] Convert from CFG to CNF

Convert the CFG for each of the following languages to CNF. You can use the CFGs directly from HW3 solutions (posted on Blackboard) without deriving them again for this task. Please show the transformation step by step (as shown in the class).

(*a*) [**10 Points**] $L = \{a^i b^j c^k | \ i \geq 0, \ j \geq 0, \ k = 2i + 3j\}$, $\Sigma = \{a, b, c\}$

(*b*) [**10 Points**] $L = \{a^i b^j c^k | \ i \neq j$ **or** $j \neq k\}$, $\Sigma = \{a, b, c\}$

(*c*) [**10 Points**] $L = \{a^i b^j c^k d^l | \ i + j = k + l\}$, $\Sigma = \{a, b, c, d\}$

Task 2. [40 Points] Construct TMs

Construct a TM for solving each of the following problems. Write down the TM in the 6-tuple format and draw the state transition diagram showing how it works.

(*a*) [**10 Points**] Check if a string $\in \{a, b\}^*$ (given on the input tape) is a palindrome.

(*b*) [**10 Points**] Increment a binary number (given on the input tape) by 1.

(*c*) [**10 Points**] Decrement a binary number (given on the input tape) by 1 (do nothing if the number is 0).

(*d*) [**10 Points**] Check if a string of parentheses ($\in \{(,)\}^*$) is balanced.

Task 3. [30 Points] Compose TMs

Construct a TM for solving each of the following problems. Give a high level step-by-step description of the algorithm your TM represents and draw a state trnsition diagram showing how it works. You can use TMs you have already constructed under Task 2 (or even Task 3) and/or saw in the class as blackboxes (i.e., subroutines) for solving subproblems.

(*a*) [**10 Points**] Add two binary numbers given on the input tape.

(*b*) [**10 Points**] Compute the n-th Fibonacci number (in binary) given n as a binary number on the input tape.

(*c*) [**10 Points**] Multiply two binary numbers given on the input tape.