Problem

A k-***query oracle Turing machine*** is an oracle Turing machine that is permitted to make at most k queries on each input. A k-query oracle Turing machine M with an oracle for A is written $M^{A,k}$. Define $P^{A,k}$ to be the collection of languages that are decidable by polynomial time k-query oracle Turing machines with an oracle for A.

a. Show that $\mathrm{NP} \cup \mathrm{coNP} \subseteq \mathrm{P}^{SAT,1}$.

b. Assume that $\mathrm{NP} \neq \mathrm{coNP}$. Show that $\mathrm{NP} \cup \mathrm{coNP} \subsetneq \mathrm{P}^{SAT,1}$.

Step-by-step solution

Step 1 of 2

In this user need to prove that union of the *NP* and *coNP* can be decided in polynomial time by using the oracle of the *SAT* problem.

This implies $\mathrm{NP} \cup \mathrm{coNP} \subseteq \mathrm{P}^{SAT,1}$

It is being known that the oracle problem *SAT* is basically the *NP*-complete problem. *NP* language is basically encoded in the polynomial time *SAT*.

As, it is known that $\mathrm{L} \subseteq \mathrm{NP}$ and $\mathrm{P}^{A,K} \in \mathrm{NP}$.

This implies *L* and $\mathrm{P}^{A,K} \subseteq \mathrm{NP}$ it can reduced in Poly-time by the oracle problem *SAT*.

So, $\mathrm{NP} \subseteq \mathrm{P}^{SAT}$

Similarly,

As, it is known that $\mathrm{L} \subseteq \mathrm{NP}$ and $\mathrm{P}^{A,K} \in \mathrm{NP}$.

This implies *L* and $\mathrm{P}^{A,K} \subseteq \mathrm{NP}$ it can reduced in Poly-time by the oracle problem *SAT*.

Even, $\neg\mathrm{L}\left(\in \mathrm{NP} \subseteq \mathrm{P}^{SAT}\right)$ can be reduced in Poly-time by the oracle problem *SAT*.

So, $\mathrm{coNP} \subseteq \mathrm{P}^{SAT}$

In case of *NP* "yes" answer is checked by the oracle Turing machine and in case of *coNP* "no" answer is check by the oracle Turing machine in polynomial time.

It is being known that for each and every *NP* complete problem there is *coNP* complete problem.

Suppose, *coNP* and *NP* are equal then in that case the polynomial collapsed to either *NP* or *coNP*. But as it is shown earlier that $\mathrm{NP} \subseteq \mathrm{P}^{SAT}$ and $\mathrm{coNP} \subseteq \mathrm{P}^{SAT}$.

Here union operation is done between *NP* and *coNP* as here, the output is in either "yes" or "no".

So, union is computed in the polynomial time.

Hence, $\mathrm{NP} \cup \mathrm{coNP} \subseteq \mathrm{P}^{SAT,1}$

Comment

Step 2 of 2

In this user need to prove that union of the *NP* and *coNP* can be decided in polynomial time by using the oracle of the *SAT* problem.

This implies $\mathrm{NP} \cup \mathrm{coNP} \not\subset \mathrm{P}^{SAT,1}$

It is being known that the oracle problem *SAT* is basically the *NP*-complete problem. *NP* language is basically encoded in the polynomial time *SAT*.

As, it is known that $\mathrm{L} \subseteq \mathrm{NP}$ and $\mathrm{P}^{A,K} \in \mathrm{NP}$.

This implies *L* and $\mathrm{P}^{A,K} \subseteq \mathrm{NP}$ it can reduced in Poly-time by the oracle problem *SAT*.

So, $\mathrm{NP} \subseteq \mathrm{P}^{SAT}$

Similarly,

As, it is known that $\mathrm{L} \subseteq \mathrm{NP}$ and $\mathrm{P}^{A,K} \in \mathrm{NP}$.

This implies *L* and $P^{A,K} \subseteq NP$ it can reduced in Poly-time by the oracle problem *SAT*.

Even, $\neg L\left(\in NP \subseteq P^{SAT}\right)$ can be reduced in Poly-time by the oracle problem *SAT*.

So, $coNP \subseteq P^{SAT}$

In case of *NP* "yes" answer is checked by the oracle Turing machine and in case of *coNP* "no" answer is check by the oracle Turing machine in polynomial time.

It is being known that for each and every *NP* complete problem there is *coNP* complete problem.

In this *coNP* and *NP* are not equal.

$NP \subseteq P^{SAT}$ and $coNP \subseteq P^{SAT}$.

So, union is not computed in the polynomial time.

Hence, $NP \cup coNP \not\subset P^{SAT,1}$

Comment