

Problem

A **triangle** in an undirected graph is a 3-clique. Show that $TRIANGLE \in P$, where $TRIANGLE = \{ \langle G \rangle \mid G \text{ contains a triangle} \}$.

Step-by-step solution

Step 1 of 1

Class P : P is a class of languages that are decidable in polynomial time on a deterministic single – tape Turing – machine.

Specified language,

- A triangle in an undirected graph is a 3 – clique.
- $TRIANGLE = \{ \langle G \rangle \mid G \text{ contains a triangle} \}$.
- Now we have to show that $TRIANGLE \in P$
- Let A be the Turing machine that decides $TRIANGLE$ in polynomial time
- A can be described as follows:

A ="on input $\langle G \rangle$:"

V denotes set of vertices of the graph G .

E denotes set of edges of the graph G .

1. For $u, v, w \in V$ and $u < v < w$, we enumerate all triples $\langle u, v, w \rangle$.
2. Check whether all three edges $(u, v), (v, w)$ and (w, u) exist in E or not. If exist then accept.
3. Otherwise reject."

• Enumeration of all triple require $O(|V|^3)$ time

• Checking whether all three edges belong to E take $O(|E|)$ time.

• Overall time is $O(|V|^3 |E|)$ which is polynomial in the length of the input

• Therefore $TRIANGLE \in P$

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