Problem

A ***2cnf-formula*** is an AND of clauses, where each clause is an OR of at most two literals. Let

Let $2SAT = \{\langle\phi\rangle \mid \phi \text{ is a satisfiable 2cnf-formula}\}$. Show that $2SAT \in P$.

Step-by-step solution

Step 1 of 5

Class – P: P is a class of Languages that are decidable in polynomial time on a deterministic single – tape Turing – machine.

The Language is $2\mathrm{SAT} = \{\langle\phi\rangle \mid \phi \text{ is a satisfiable 2CNF formula}\}$

A $cnf-\text{formula}$ is said to be $2\ cnf$ if all the clauses have two literals.

Now we have to prove that $2SAT \in P$.

• Let ϕ be the $2cnf-\text{formula}$ one variables $x_1, x_2, \ldots, x_n$

• Let us construct the graph G for the give ϕ as follows :

→ The variables and their negations in ϕ are taken as vertices of graph G. That is, $V = \{x_1, \ldots, x_n\} \cup \{\overline{x}_1, \ldots, \overline{x}_n\}$.

→For every clause of the form $\mathrm{A} \vee \mathrm{B}$ in ϕ, add a directed edge from $\overline{\mathrm{A}}$ to B and one from $\overline{\mathrm{B}}$ to A in graph G.

• So by the construction of the graph, it is follows that, if there is an edge from *A* to *B* then there is an edge from $\overline{\mathrm{B}}$ to $\overline{\mathrm{A}}$.

• Now let us suppose that there is a directed path from $x_i\ to\ \overline{x_i}$ and from $\overline{x_i}\ to\ x_i$.

• The existence of a directed path from $x_i\ to\ \overline{x_i}$ is equivalent to saying that $x_i \Rightarrow \overline{x_i}$ and the existence of a directed path from $\overline{x_i}\ to\ x_i$ is equivalent to saying that $\overline{x_i} \Rightarrow x_i$. Together, they implying that $x_i \cong \overline{x_i}$, which is false.

• So if this condition occurs then the formula ϕ has an un-satisfiable clause embedded in it.

• Conversely we will show that, if there is no such pair of paths (one form $x_i\ to\ \overline{x_i}$ and another from $x_i\ to\ \overline{x_i}$) then a satisfying assignment can be found for ϕ, by the following algorithm.

Comment

Step 2 of 5

Step – 1 For each variable x_i, check if there is a path from $x_i\ to\ \overline{x_i}$. If there is such a path, assign $x_i = false$. For all variables V such that there is a path from $\overline{x_i}\ to\ v$, assign v to true and $\overline{v}$ to false.

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Step 3 of 5

Step – 2 for each variable x_i, check if there is a path from $\overline{x_i}\ to\ x_i$. If there is such a path, assign $x_i = true$. For all literal v such that there: a path form $x_i\ to\ v$, assign v to true and $\overline{v}$ to false.

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Step 4 of 5

Step – 3 Propagate all the “true values” down the paths, and the “false values” up the paths.

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Step 5 of 5

- So this algorithm never assigns both true and false values to the same variable.
- The entire algorithm will be executed in polynomial time.
- Therefore $2SAT$ is in P.

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