Problem

Let A = L(G_1) where G_1 is defined in Problem 2.55. Show that A is not a DCFL.

(Hint: Assume that A is a DCFL and consider its DPDA P. Modify P so that its input alphabet is {a, b, c}. When it first enters an accept state, it pretends that c's are b's in the input from that point on. What language would the modified P accept?)

Problem 2.55

Let G_1 be the following grammar that we introduced in Example 2.45. Use the *DK*-test to show that G_1 is not a DCFG.

$$\begin{aligned} R &\rightarrow S \mid T \\ S &\rightarrow \mathtt{a}S\mathtt{b} \mid \mathtt{ab} \\ T &\rightarrow \mathtt{a}T\mathtt{bb} \mid \mathtt{abb} \end{aligned}$$

Step-by-step solution

Step 1 of 3

On the contrary suppose *A* is a deterministic context free language. Consider *p* as the pumping length of *A*, such that *p* length string of *A* will satisfy the pumping lemma. Consider '*m*' as string of *A* with $m = 0^{2p}0^p1^p0^{2p}$. In order to satisfy the assumption, there are following ways so that '*w*' can be written as $uvxyz$, where $|vy| \geq 1$ and $|vxy| \leq p$, uv^ixy^iz string lies in *A* for any value of *i*.

Comment

Step 2 of 3

On the basis of above condition there are only 3 cases which are as follow:

Case 1:

In string uv^ixy^iz, vy only have 0s which are getting from the last 0^{2p} of *m*. Assume that '*i* 'is any number which satisfy the condition $7p > |vy| \times (i+1) \geq 6p$. Then, may be the length of uv^ixy^iz is not become the multiple of 3, or may be the string is in the form of wtw' specified that $|w| = |t| = |w'|$ having all zero's in w' and not all zero's in w $(\text{Means}, w^R \neq w')$.

Case 2:

In string uv^ixy^iz, vy does not have 0s in last 0^{2p} of *m*. Then, may be the length of uv^2xy^2z is not become the multiple of 3, or may be the string is in the form of wtw' specified that $|w| = |t| = |w'|$ having all zero's in w and not all zero's in w' $(\text{Means}, w^R \neq w')$.

Comment

Step 3 of 3

Case 3:

In string uv^ixy^iz, vy have some 0s which are getting from the last 0^{2p} of *m*. In this case $|vxy| \leq p$, therefore vxy must be a substring 1^p0^p. Then, may be the length of uv^2xy^2z is not become the multiple of 3, or may be the string is in the form of wtw' specified that $|w| = |t| = |w'|$ having all zero's in w and not all zero's in w' $(\text{Means}, w^R \neq w')$.

By all these case it is observes that *m* cannot satisfy the conditions of the assumption. Therefore, a contradiction occurs. Hence, it can be state that *A* is not a deterministic context free language.

Comment