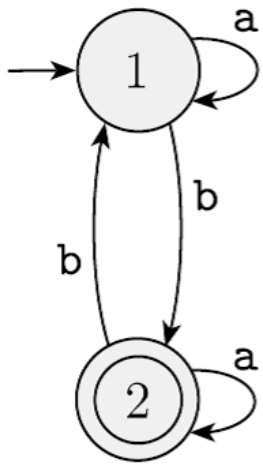
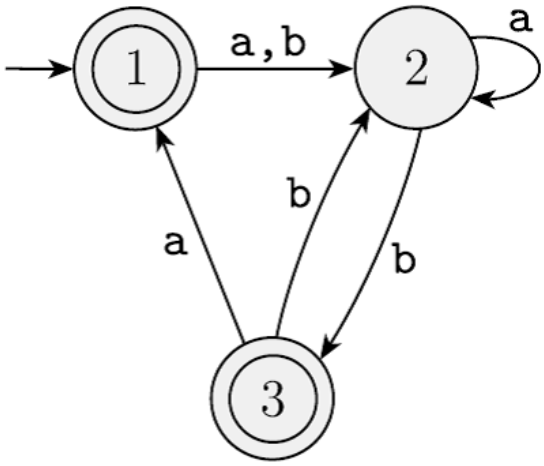
Problem

Use the procedure described in Lemma 1.60 to convert the following finite automata to regular expressions.



(a)



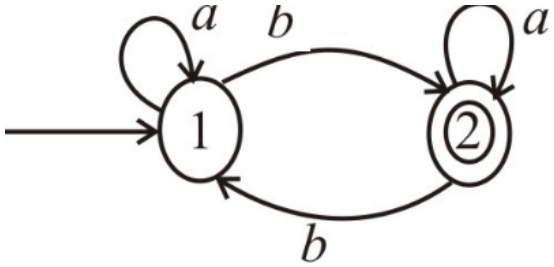
(b)

Step-by-step solution

Step 1 of 14

(a)

Consider the Finite Automata:



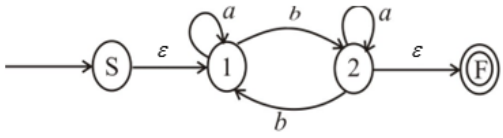
Now convert this finite automaton to a regular expression in the following steps as below:

Comment

Step 2 of 14

Step 1:

Add the start state(S) and new accept state (F) to make the original accept state as non – accepting state as:

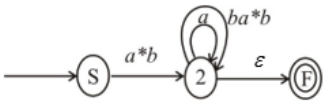


Comment

Step 3 of 14

Step 2:

In the second step eliminate the state(1), no need to add the loop for the state 1 and directly add loop to the state (2) and write the expression by passing state (S) to state (2)

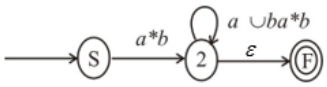


Comment

Step 4 of 14

Step 3:

From the above step one loop is represented a s union with the a as follows:

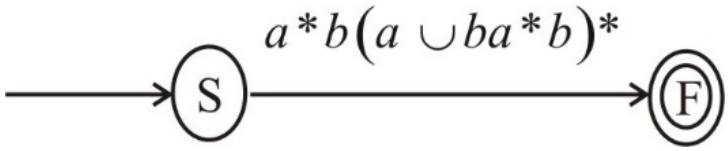


Comment

Step 5 of 14

Step 4:

Now remove the loop over the state (2) and eliminate it and write expression directly from state (S) to State(F)



Comment

Step 6 of 14

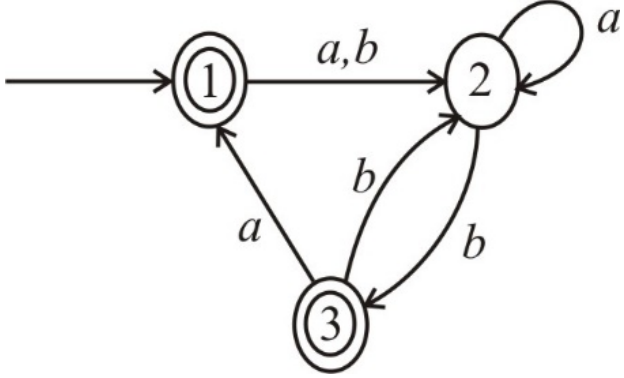
So, the regular expression for the given finite automata is $\boxed{a*b(a \cup ba*b)*}$

Comment

Step 7 of 14

(b)

Consider the second finite automata is



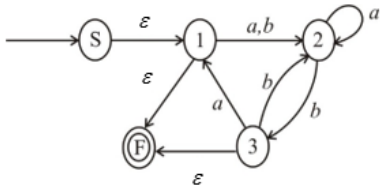
Now convert this finite automaton to a regular expression in the following steps as below:

Comment

Step 8 of 14

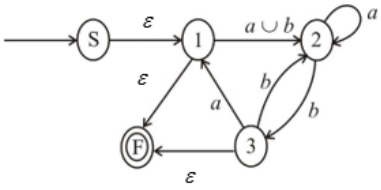
Step 1:

Add new start state (S) and new accept state (F). Make original accept states as non-accepting states then the Finite Automata becomes:



Step 2:

Perform union on the edge from state 1 to state2.

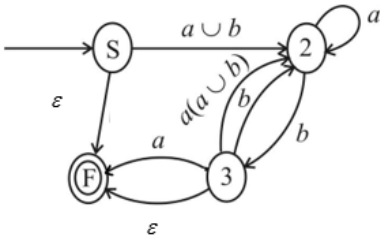


Comment

Step 9 of 14

Step 3:

From the above step 2, there are no unions or loops for the state 1, So eliminate the state 1 as follows:

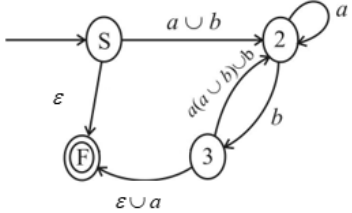


Comments (4)

Step 10 of 14

Step 4:

Perform unions on edges from state 3 to state 2 and from state 3 to the final state, Then the Automata becomes as below:

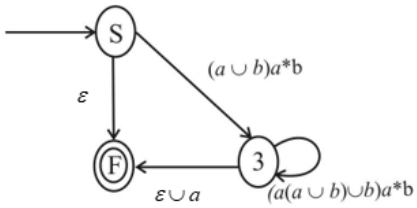


Comment

Step 11 of 14

Step 5:

Here to minimize the automata eliminate 2 and perform union on 3 and write expression for the state (S) to state(3), then apply loop on state (3) with the expression of state(2)

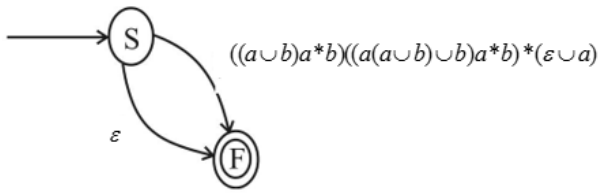


Comment

Step 12 of 14

Step 6:

Eliminate the state 3 and write the expression from state(S) to state (F), because there are no loops and unions.

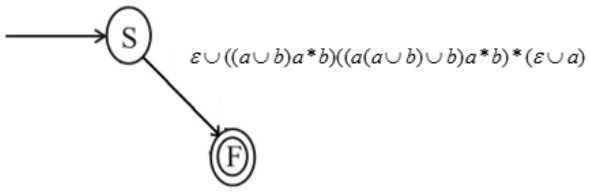


Comment

Step 13 of 14

Step 7:

Perform union on edge from state *S* to state *F*



Comment

Step 14 of 14

So, the regular expression for the given finite automata is

$$\varepsilon \cup ((a \cup b)a^*b)((a(a \cup b) \cup b)a^*b)^*(\varepsilon \cup a)$$

Comment