

## Problem

Consider the problem of determining whether a Turing machine  $M$  on an input  $w$  ever attempts to move its head left at any point during its computation on  $w$ . Formulate this problem as a language and show that it is decidable.

## Step-by-step solution

### Step 1 of 2

Consider a problem of determining whether a Turing machine  $M$  on input  $w$  ever attempts to move its head left at any point during its computation on  $w$ . The language that describes the problem is,

$$L = \{ \langle M, w \rangle \mid M \text{ moves its head left on input } w \}$$

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### Step 2 of 2

Construct a Turing machine  $A$  that decides the problem.

$A =$  "On input  $\langle M, w \rangle$ :

1. Run the machine for  $|Q|+|w|+1$  steps.
2. If the Turing machine  $M$ 's head moved to the left then accept. Otherwise, reject."

Here,  $|Q|$  represents the number of states and  $|w|$  represents the length of the input string. The problem is said to be decidable, if the Turing machine  $M$  moves its head left on input  $w$  within the first  $|Q|+|w|+1$  steps. The problem is decidable because, there exists a Turing machine for it.

**Therefore, the language  $L$  is decidable.**

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