Problem

Call graphs *G* and *H* ***isomorphic*** if the nodes of *G* may be reordered so that it is identical to *H*. Let *ISO* = { $\langle G, H \rangle$ I G and H are isomorphic graphs}. Show that *ISO* ? NP.

Step-by-step solution

Step 1 of 4

Class *NP* :

NP is a class of languages that are nondeterministic polynomial time on a non – deterministic single – tape Turing Machine.

From the definition 7.19 *NP* is the class of languages that have polynomial time verifies

Consider the given expression:

ISO $=\{\langle G,H\rangle \mid\ G \text{ and } H \text{ are isomorphic graphs}\}$

• If the nodes of *G* may be reordered so that it is identical to *H* then Graphs *G* and *H* are said to be isomorphic.

• Now it must be proved that $ISO \in NP$

• Let $G=(V_G,E_G)$ and $H=(V_H,E_H)$ be the two graphs

• Let $V_G=\{u_1,u_2,..u_m\},V_H=\{v_1,v_2,...v_n\}$ be the sets of vertices of *G* and *H.*

Comment

Step 2 of 4

Isomorphism:

An isomorphism is defined by a mapping $f:V_G \rightarrow V_H$ with the property that it is a one – to –one correspondence. That means it is both one – to – one and onto.

• This one – to – one correspondence is possible only if $m=n$ and for all $u,v \in V_G$ we have $(v,v)\in E_G$ if and only if $(f(u),f(v))\in E_H$.

• Thus, the correspondence takes edges into edges and non – edges into non – edges.

• A mapping *f* can be represented. By a sequence $S=(S_1,S_2,...S_m)$ of indices with the property that $f(u_i)=v_{s_i}$, that is i^{th} point of *G* is mapped into the S_i^{th} point of *H.*

• This sequence *S* can be taken as certificate.

Comment

Step 3 of 4

Now *N* is the non – deterministic Turing machine (NTM) that decides *ISO* in polynomial time.

$N =$ "On input $(\langle G,H\rangle,S)$:

Where *G* and *H* are graph as defined above *S* is the certificate.

1. Check whether *G* and *H* have same number of points.

2. If *G* and *H* have same number of points then checks that for each pair i,j

$$\Rightarrow \left(v_{S_i}, v_{S_j}\right) \in E_U \quad ....(1)$$

$$\Rightarrow \left(u_i, u_j\right) \in E_U \quad ....(2)$$

$$From\ (1)\ and\ (2)$$

i. E_U can be derived from the above mapping procedure, $f\left(u_i\right) = v_{s_i} = E_U$

ii. have $S_i \neq S_j$ and that $\left(u_i, u_j\right) \in E_U$ if and only if $\left(v_{S_i}, v_{S_j}\right) \in E_U$

iii. If the above condition satisfies, then “accept”.

3. Otherwise “reject”.

Comment

Step 4 of 4

All these checking can be done in time $O\left(m^2\right)$, so in time polynomial in the description of $\left(G, H\right)$. **Therefore** $ISO \in NP$**.**

Comment