Problem

Show that the class of DCFLs is not closed under the following operations:

a. Union

b. Intersection

c. Concatenation

d. Star

e. Reversal

Step-by-step solution

Step 1 of 5

a) Suppose $L_1 = \{a^i b^j c^k : i,j,k \geq 0\, and\, i \neq j\}$ and $L_2 = \{a^i b^j c^k : i,j,k \geq 0\, and\, i \neq k\}$ are both deterministic context free languages **(DCFL's).** However, $L_1 \cup L_2$ which is defined as $L_1 \cup L_2 = \{a^i b^j c^k : i,j,k \geq 0\, and\, (\, i \neq k\, or\, i \neq j)\}$ is not comes under DCFL's which can be seen as follows.

• Suppose $L_1 \cup L_2$ is a DCFL. T hen its complement is also a DCFL as discussed in theorem. $(L_1 \cup L_2)' \cap \{a\}*\{b\}*\{c\}* = \{a^i b^j c^k : i,j,k \geq 0\, and\, (\, i = k = j)\}$ is a (D)CFL. This shows a **contradiction**.

• Hence, it can be said that" **Deterministic Context Free Language's (DCFL's) is not closed under union"**.

Comments (1)

Step 2 of 5

b) Non-closure under intersection follows from the non-closure under union (as discussed above) and closure under complement: $K \cup L = (K' \cap L')'$.

• Thus, if the family of DCFLs would be closed under intersection, it follows with the help of closure under complement that it would also be closed under union. This shows a **contradiction.**

• Hence, it can be said that" **Deterministic Context Free Language's (DCFL's) is not closed under intersection"**.

Comment

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c) Suppose $L_1 = \{a^i b^j c^k : i,j,k \geq 0\, and\, i \neq j\}$ and $L_2 = \{a^i b^j c^k : i,j,k \geq 0\, and\, i \neq k\}$ be the languages. Now, suppose $L_3 = \{d\} L_1 \cup L_2$,where d is the symbol different from a,b,c. It is simple to see that $L_3\, and\, \{d\}^*$ are **DCFL.** Now, it has intended to show that $\{d\}^* L_3$ is not a DCFL.

• Consider $\{d\}^* L_3 \cap \{d\}\{a\}^*\{b\}^*\{c\}^* = \{d\} L_1 \cup \{d\} L_2$. If $\{d\}^* L_3$ would be in DCFL then also $\{d\} L_1 \cup \{d\} L_2 = \{d\}(L_1 \cup L_2)$ is a DCFL.

• Now, it is fairly easy to see that for all words w and all languages K that consist K are a DCFL whenever $\{w\} k$ is DCFL. Consequently, since $L_1 \cup L_2$ is not a DCFL, also $\{d\}(L_1 \cup L_2)$ is not a DCFL. This shows a **contradiction**.

• Hence, it can be said that" **Deterministic Context Free Language's (DCFL's) is not closed under concatenation"**.

Comment

Step 4 of 5

d) Suppose L_4 is defined as $L_4 = \{d\} \cup \{d\} L_1 \cup L_2$ where L_1 and L_2 is defined same as the above discussion.

• Consider $L_4^* \cap \{d\}^* \left(\{a\}^* \{b\}^* \{c\}^* - \{\wedge\}\right) = \{d\} L_1 \cup \{d\} L_2$. If L_4^* is DCFL then also $\{d\} L_1 \cup \{d\} L_2$ should be in DCFL, but here it is not followed in this case.

• Hence, it can be said that" **Deterministic Context Free Language's (DCFL's) is not closed under star (*)"**.

Comment

Step 5 of 5

e) Non-closure under difference follows from the closure under complement and the non-closure under intersection as discussed in part (b):
$K \cap L = K - L^{'}$, **thus if the family of DCFL's would be closed under difference.**

• The above terms follows with the help of closure under complements that would also be closed under intersection. That shows a contradiction.

• Hence, it can be said that" **Deterministic Context Free Language's (DCFL's) is not closed under reversal"**.

Comment