Problem

Show that NL is closed under the operations union, concatenation, and star.

Step-by-step solution

Step 1 of 4

Class _NL(Non-deterministic Logarithmic space):_

_NL_ is the class of languages that are decidable in logarithmic space on a non deterministic Turing machine. i.e., $NL = NSPACE\left(\log n\right)$

let L_1 and L_2 be the languages that are decided by _NL_ - machines M_1 and M_2.

Now we want to show that

· There is a nondeterministic decides $M_\cup$ such that $L\left(M_\cup\right) = L_1 \cup L_2$

· There is a nondeterministic decider $M_\cap$ such that $L\left(M_\cap\right) = L_1 \cap L_2$.

· There is a nondeterministic decider $M*$ such that $L\left(M_*\right) = L_1^*$

Now these 3 machines $M_\cup, M_\cap, M_*$ are for 3 different operations.

Comment

Step 2 of 4

(i)

$M_\cup =$ "on input w:

1. Run M_1 on w, If M_1 accepted then accept

2. Else run M_2 on w, if M_2 accepted then accept

3. Else reject"

The machine $M_\cup$ will accept the input w if either M_1 or M_2 accept w.

If both M_1 and M_2 reject the input w then $M_\cup$ will reject w.

Comment

Step 3 of 4

(ii) **Intersection:**

$M_\cap =$ "on input w:

1. Run M_1 on w, if M_1 rejected then reject.

2. Else run M_2 on w, if M_2 rejected then reject.

3. Else accept.

The machine $M_\cap$ will accept the input w if both M_1 and M_2 reject w then $M_\cap$ will reject w. If any one of M_1 and M_2 reject w then $M_\cap$ will reject w.

Comment

Step 4 of 4

(iii) **Star:**

Machine M_* has more complex algorithm as follows.

$M_* =$ "on input w :

1. P_1 and P_2 be two input positions. P_1 is initialized to 0 and P_2 is initialized to the position immediately preceding the first input symbol.

2. If there is no input symbol after P_2 then accept w.

3. Move P_2 forward to a non-deterministically selected input position.

4. Simulate M_1 non-deterministically on the substring of w from position following P_1 to the position P_2.

5. If M_1 is entered into accept state then copy P_2 to P_1 and go to stage 2

So M_* will decide L_1^*,

In this way **NL is closed under union intersection and star**.

Comment