Problem

Show that NP is closed under union and concatenation.

Step-by-step solution

Step 1 of 3

The class NP is closed under union and concatenation

***NP* – class:**

NP is a class of languages that are decidable in nondeterministic polynomial time on a non – deterministic Turing machine.

Comment

Step 2 of 3

Union:

• Let *A* and *B* be languages are decided by $NP-$machines T_A and T_B.

• Now we want to show that, there is a non – deterministic poly time decider $T_{A\cup B}$ that decides union of *A* and *B.*

• The construction of $T_{A\cup B}$ is as follows:

$T_{A\cup B}$ =" On input *S*:

1. Run T_A on *S*. If T_A accepts *S,* then accept.

2. Else run T_B on *S*. If T_B accepts *S,* then accept.

3. Else reject"

As the new *TM* $T_{A\cup B}$ calls T_A and T_B each once, it runs on $O(T_A+T_B)$,

as both are *NP* is $T_{A\cup B}$.

Comment

Step 3 of 3

Concatenation:

• Let *A* and *B* be languages are decided by $NP-$machines T_A and T_B.

• Now we want to show that, there is a non – deterministic poly time decider $T_{A\cup B}$ that decides concatenation of *A* and *B.*

• The construction of $T_{A\circ B}$ is as follows:

$T_{A\circ B}$ = "On input S:

1. Split S into S_1, S_2 such that $S=S_1S_2$.

2. Run the NP machine T_A on S_1. If T_A is rejected, then reject.

3. Else run T_B on S_2. If T_B is rejected, then reject.

4. Else accept.

The time taken by step 1 is $O(n)$ in a two tape Turing Machine. Thus, T is a poly-time non-deterministic decider for $A\circ B$.

Comments (1)