Problem

Give a circuit that computes the parity function on three input variables and show how it computes on input 011.

Step-by-step solution

Step 1 of 7

The *n*-input **parity function** is the Boolean function f : $\{0,1\}^n \rightarrow \{0,1\}$ gives an output 1 if an odd number of 1's appear in the input variable and otherwise gives 0. Here f is defined as:

$f(x) = x_1 \oplus x_2 \oplus x_3 \oplus ... \oplus x_n$, where $\oplus$ denotes an EX-OR gate operation.

• In this parity functional circuit, a number of gates are used. Therefore, first of all, the working of gates will be discussed here to understand the process performed in that circuit.

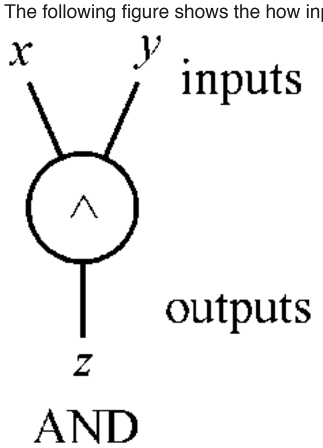
Comment

Step 2 of 7

The Truth table for **and** operator is as follow:

Input		Output z
x	y	
0	0	0
1	0	0
0	1	0
1	1	1

The following figure shows the how input and output can be taken in the above table (for **and** operator):



Comment

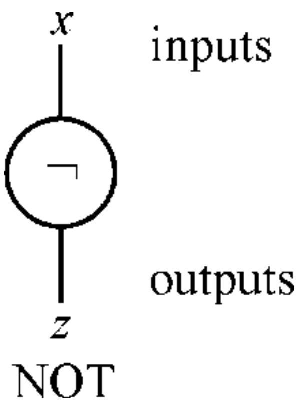
Step 3 of 7

The Truth table for **not** operator is as follow:

Input(x)	Output(z)

0	1
1	0

The following figure shows the how input and output can be taken in the above table (for **not** operator):



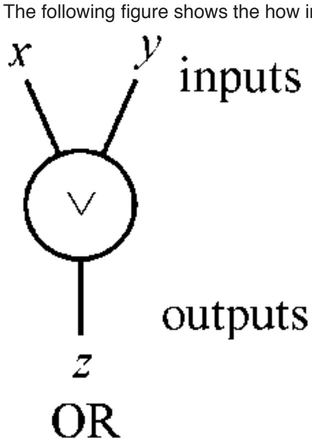
Comment

Step 4 of 7

The Truth table for **or** operator is as follow:

Input		Output z
x	y	
0	0	0
0	1	1
1	0	1
1	1	1

The following figure shows the how input and output can be taken in the above table (for **or** operator):



Comment

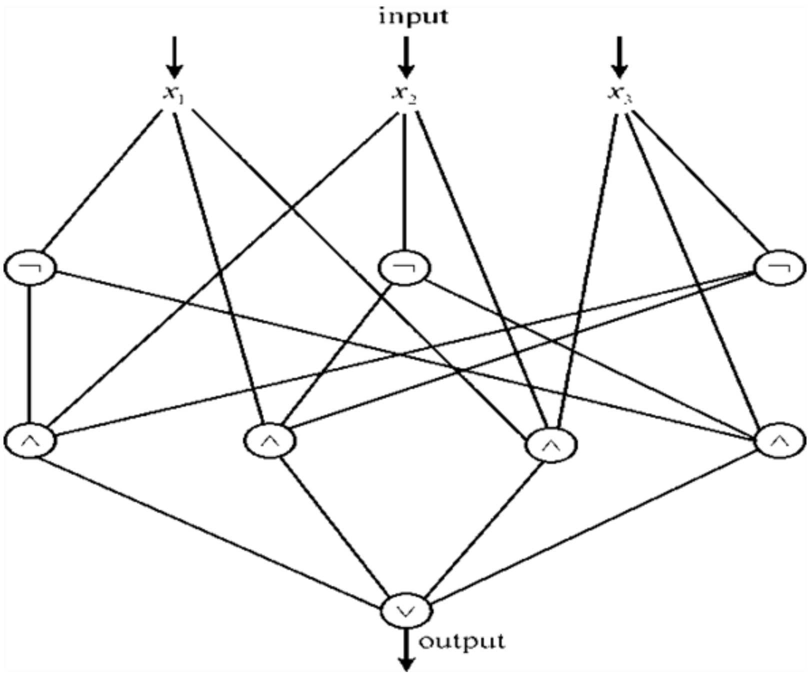
Step 5 of 7

From the definition of **parity function**, the n-input parity functional circuit can be obtained by applying an EX-OR gate operation on the n- inputs.

• For three input variables, the parity function may be defined as:

$$
\begin{aligned}
f(x) &= x_1 \oplus x_2 \oplus x_3 \\
&= \left[(x_1 \cdot \overline{x}_2) + (\overline{x}_1 \cdot x_2) \right] \oplus x_3 \\
&- \left[x_1 \cdot \overline{x}_2 \cdot \overline{x}_3 + \overline{x}_1 \cdot x_2 \cdot \overline{x}_3 + x_1 \cdot x_2 \cdot \overline{x}_3 + \overline{x}_1 \cdot \overline{x}_2 \cdot x_3 \right]
\end{aligned}
$$

Consider the figure which is given below:



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Comment

Step 6 of 7

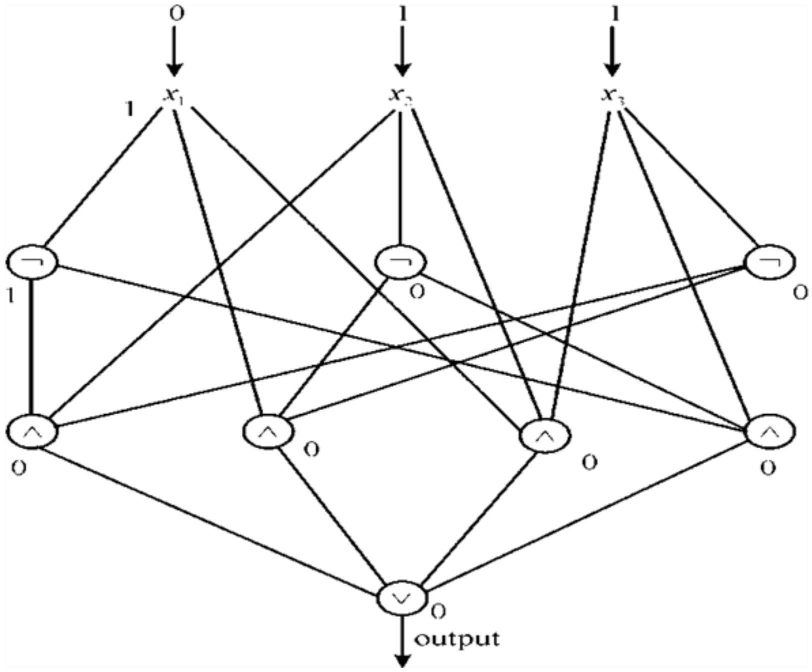
Above figure shows the functional circuit on three input variable. It shows how three variables are taken as an input, and how an EX-OR gate operation is applied on these inputs.

• An EX-OR gate consists of an AND, NOT and OR gate operations.

Comment

Step 7 of 7

Consider the figure which is given below:



Above figure shows, the computation performed by three input parity functional circuit when 001 is taken as an input. After taking an input , an EX-OR operation is applied as discussed above.

• The output given by the parity functional circuit is 0 because even number of 1's are present in the input.

• which follow the definition of parity funtional circuit(that is, it gives an output 1 if an odd number of 1's appear in the input variable and otherwise gives 0).

Comment