Problem

Let C_{CFG} = $\{\langle G, k\rangle \mid$ G is a CFG and L(G) contains exactly k strings where k ≥ 0 or k = 1}. Show that C_{CFG} is decidable.

Step-by-step solution

Step 1 of 1

Decidability

Consider a decider *M* which is used to check whether language of CFG is finite or infinite. Use another decider that is Turing machine *W* which shows that is C_{CFG} decidable.

1. *W* = "on input $\langle G,k\rangle$" where *G* is CFG and *k* is string

2. Check $L(G)$ is infinite using decider *M.*

• If $L(G)$ is infinite and $k=\infty$, it is accepted

• If $L(G)$ is infinite and $k\neq\infty$, it is rejected

• If $L(G)$ is finite and $k=\infty$, it is rejected

• If $L(G)$ is finite and $k\neq\infty$, continue

3. Calculate the pumping length *l* for grammar *G.*

4. Set $count=0$

5. Use *for* loop $i=0$ to l

• Use *for* loop to get all strings *S* whose length equal to *i*

• If *S* can be generated by *G* then make an increment in *count.*

6. Check value of *count* is equal to *k* then it is accept, otherwise reject.

Explanation:

• The Step 2 checks whether $L(G)$ is infinite or not. After step 2 there is grammar whose language which has finite set. In order to prove C_{CFG} is decidable there is only need to prove that the size of language is *k.*

• To do so use loop to find the all the possible string can be generated by grammar G. The grammar is finite therefore the length of string cannot be more than pumping length *l.*

• Make an increment in variable count if the string can be generated by grammar *G.*

• In the last step check value of *count* is equal to *k.*

• Now, it has finite number of steps therefore it can easily check.

Thus *W* is decider, therefore C_{CFG} is also decidable language.