Problem

In a directed graph, the ***indegree*** of a node is the number of incoming edges and the ***outdegree*** is the number of outgoing edges. Show that the following problem is NP-complete. Given an undirected graph G and a designated subset C of G's nodes, is it possible to convert G to a directed graph by assigning directions to each of its edges so that every node in C has indegree 0 or outdegree 0, and every other node in G has indegree at least 1?

Step-by-step solution

Step 1 of 3

In a directed graph, the numbers of incoming edges are known as **in-degree** of a node and the numbers of edges, which are **out-degree,** are known as outgoing edges.

• Now, consider an undirected graph G and a subset C that consists of $G's$ node. The conversion of G as a directed graph may be possible by assigning directions to each of its edges.

• This conversion is done in such a way that each node in C consists an in-degree 0 or out-degree 0 and all other nodes in G consists an in-degree of at-least 1. It can also be proved that the above problem is $NP-complete$.

Comment

Step 2 of 3

First of all, consider $C=\varnothing$, then a graph is directed in such a way that all nodes consists a minimum degree of 1 ($DA1$). It can be achieved by using the concept of directed Hamiltonian cycle (DHC) and how the reduction takes place between DHC to $DA1$.

• Suppose the undirected graph G is given by $G=(V,E)$, where V denotes the nodes and E denotes the edges. Now, a graph G' is created (which is directed) as $G'=(V,E')$, where $(u,v),(v,u)\in E'$ if $(u,v)\in E$.

• If the new graph consists a directed Hamiltonian cycle, that conclude that the real graph G must consists a Hamiltonian cycle and vice versa.

Comment

Step 3 of 3

Now, the directed Hamiltonian cycle can be reduced to the directed Hamiltonian path. Suppose a graph $G'=(V',E')$ is constructed from the given di-graph $G=(V,E)$. It is constructed by picking any vertex $v\in V$ and then break this in two vertices, which is given by v_0 and v_1.

• Suppose $V'=(V-v)\cup\{v_0,v_1\}$. Now, add an edge $(v_0,u)\in E'$, for every edge $(v,u)\in E$ and add an edge $(u,v_1)\in E'$, for every edge $(v,u)\in E$. Now, every other edges in E is copied into E'.

• Hence, it is claimed that a Hamiltonian path exists that goes from v_0 to v_1 iff G consists a directed Hamiltonian graph.

• Now, from the above discussion, G' consists a Hamiltonian path from v_0 to v_1. Then, there exist an order of E' that initialized from v_0 and terminated at v_1 and each is visited once.

• From the way of construction E' takes place, it consists a path in G which is initialized at v and terminated at v in such a way that it goes through each nodes $u\neq v$ perfectly once and v exactly visited twice. Such a path in G is known as its Hamiltonian cycle.

Therefore, from the discussion it is concluded that "the problem given above is $NP-complete$ ".

Comment