

Problem

Let $D = \{w \mid w \text{ contains an even number of } a\text{'s and an odd number of } b\text{'s and does not contain the substring } ab\}$. Give a DFA with five states that recognizes D and a regular expression that generates D . (Suggestion: Describe D more simply.)

Step-by-step solution

Step 1 of 4

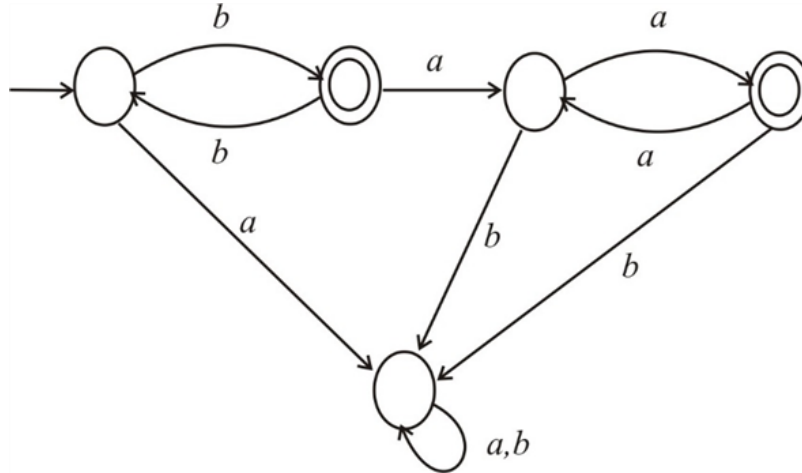
Consider the language $D = \{w \mid w \text{ contains an even number of } a\text{'s and an odd number of } b\text{'s and does not contain the substring } ab\}$.

The language D can be described simply as follows $D = \{w \mid w \text{ contains an odd number of } b\text{'s followed by even number of } a\text{'s}\}$.

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Step 2 of 4

Let M be the DFA with five states that recognizes the language D . The state diagram of M is as follows:



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Step 3 of 4

The language accepts the strings like $\{b, baa, bbbaaaa, \dots\}$. The string b is accepted by the language because, it contains the odd number of b 's (1) followed by even number of a 's (0).

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Step 4 of 4

Now, the language D can be expressed as combination of following two languages D_1 and D_2 .

$D_1 = \{w \mid w \text{ contain odd number of b's}\}$

$D_2 = \{w \mid w \text{ contains even number of a's}\}$

$D = D_1 o D_2$

R_1 be the regular expression that generates D_1

R_2 be the regular expression that generates D_2

R be the regular expression that generates D

$R = R_1 o R_2$

$R_1 = b(bb)^*$

$R_2 = (aa)^*$

$R = b(bb)^* o (aa)^*$

$R = b(bb)^*(aa)^*$

Therefore, the regular expression that generates the language D is $b(bb)^*(aa)^*$.

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