Problem

Use the result of Problem 6.21 to give a function f that is computable with an oracle for A_{TM}, where for each n, f(n) is an incompressible string of length n.

Step-by-step solution

Step 1 of 2

Compute descriptive complexity of strings $\mathrm{K}(x)$ with an oracle for A_{TM}.

• For the given string *x*, start testing all the strings '*S*' up to the length $|x|+c$

Where $c =$ length of *TM* (Turing machine) that halts immediately upon starting.

• All the strings up to the length $|x|+c$ are potential description of *x*.

• If *S* is well formed as $\langle M,w\rangle$ from all binary strings in lexicographic order, then we simulate *M* with input *w* and see if it halts with *x* on the tape.

• Here we do not know whether *M* will halt on input *w* or not.

• An oracle for A_{TM} can determine this.

$A_{TM} = \{\langle M,w\rangle \mid M \text{ is a } TM \text{ and } M \text{ accepts } w\}$

• An oracle for A_{TM} will take $\langle M,w\rangle$ as input and determine whether *M* accepts *w* or not.

• If *M* doesn't halt we move on to the next string *S*, and so on.

• After that we will find lexicographically first string *S* among them.

• In this way shortest string will be determined and it is represented as minimal description $d(x)$.

• From $d(x)$, we find $\mathrm{K}(x)$ as

$\mathrm{K}(x) = |d(x)|$

• By this procedure, we will calculate $\mathrm{K}(x)$ with an oracle for A_{TM}.

Comment

Step 2 of 2

From the above procedure, $K(x)$ is the descriptive complexity of strings and computed with an oracle A_{TM}.

Now, we have to know the definition of incompressible strings.

Incompressible strings:

Let *x* be a string. If *x* doesn't have any description shorter than itself then *x* is incompressible.

Let *f* be the function that is computable with an oracle for A_{TM}, for each *n*, calculate *f*(*n*) i.e., incompressible length of string *n.*

Create an oracle TM (Turing machine) that does the following :

1. For a given number *n*, begin

2. Enumerate all strings *x* of length *n.*

3. For each *x*, calculate $\mathrm{K}(x)$ by using oracle for A_{TM}.

4. As soon as we find $x = f(n)$ with $\mathrm{K}(x) >= |x|$ output *x.*

[$K(x) >= |x|$ means getting a string which is greater than its description.]

5. Halt.

Comment