Problem

Let C_n = {xl x is a binary number that is a multiple of n}. Show that for each n ≥ 1, the language C_n is regular

Step-by-step solution

Step 1 of 3

Given language is

$$C_n = \{x \mid x \text{ is a binary number that is a multiple of } n\}$$

• A language is regular if it is recognized by a DFA.

• So construct a DFA to keep track of the remainder of the input when divided by *n*.

Comment

Step 2 of 3

Construction of DFA is as follows:

• Assume the value of n as 3.

• To determine whether a binary number is a multiple of 3, it is necessary to find its remainder modulo 3.

• If it ends up with remainder zero then accept.

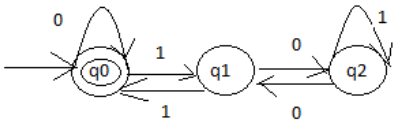
• Otherwise reject.

• Every time read a digit, the preceding string is shifted left one position thereby doubling its value x.

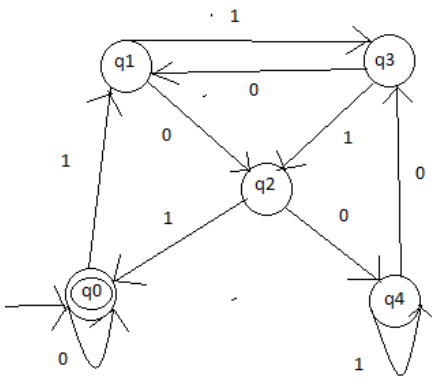
• If the current digit is 0, them the new value is $2x \pmod 3$.

• If the current digit is 1, them the new value is $2x+1 \pmod 3$.

• The DFA for the above example is as follows:



• Similarly for n=5 the DFA is as follows:



Comment

Step 3 of 3

The other case can be proved similarly.

For example:

$C_6 = C_3 0$

$C_{12} = C_6 00$

$C_{15} = C_3 \cap C_5$

....

Thus $M = \left(\{q_0, q_1, ...q_n\}, \{0,1\}, \delta, q_0, \{q_0\}\right)$

Since *M* recognizes language C_n. Hence, it is proved that C_n is regular.

Comment