Problem

Let $INFINITE_{DFA} = \{\langle A\rangle \mid A \text{ is a DFA and } L(A)$ is an infinite language$\}$. Show that $INFINITE_{DFA}$ is decidable.

Step-by-step solution

Step 1 of 1

Decidability of Infinite $_{DFA}$ can be proved as follows:

For deciding $INFINITE_{DFA}$ construct a Turing machine $T1$.

T1= "On input $\langle A\rangle$ *, where A is* Deterministic Finite Automata.

1. Assume the value of *n* as the number of states of A.
2. Build Deterministic Finite Automata *M1* that accepts all the strings of length *n* or more.
3. Build a Deterministic Finite Automata *M2* such that $L(M2) = L(A) \cap L(M1)$
4. Test $L(M2) = \phi$ using the E_{DFA} decider *T* from the theorem 4.4.
5. If *T* accepts reject; if *T* rejects, accept."

The above algorithm functions because a Deterministic Finite Automata that accepts infinitely many strings must accept randomly long strings.

Hence, the algorithm recognizes such Deterministic Finite Automata.

Contrariwise, if the algorithm recognizes a Deterministic Finite Automata, the Deterministic Finite Automata recognizes certain strings of length *n* or more where *n* is the number of states of the DFA.

This string may be pumped in the method of the pumping lemma in favor of the regular languages to find several recognized strings.

Since Turing machine $T1$ decides $INFINITE_{DFA}$ so $INFINITE_{DFA}$ is decidable.

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