Problem

Prove that the following languages are not regular. You may use the pumping lemma and the closure of the class of regular languages under union, intersection, and complement.

a. $\{0^n 1^m 0^n \mid m, n \geq 0\}$

A b. $\{0^m 1^n \mid m \neq n\}$

c. $\{w \mid w \in \{0,1\}^* \text{ is not a palindrome}\}$ 8

* d. $\{wtw \mid w, t \in \{0,1\}^+\}$

Step-by-step solution

Step 1 of 5

Pumping lemma:

For a regular language *A* with the pumping length *P*, if S is any string of *A* with minimum length of *P*, then the string S can be divided into 3 pieces *x, y* and *z* represented as $S = xyz$ should satisfy the following conditions:

• for each

• $|y| > 0$, and

• $|xy| \leq P$

Comment

Step 2 of 5

a.

Consider the Language $L = \{0^n 1^m 0^n \mid m, n \geq 0\}$ **.**

Assume that *L* is regular language and a string $S = 0^P 1 0^P$. Divide the string into three pieces *x, y* and *z*. So, $S = 0^P 1 0^P = xyz$ where, *P* is the pumping length.

Assume that $x = 0^{P-K}, y = 0^K$ and $z = 10^P$ (where $K > 0$)

Now $xy^0 z = 0^{P-K} \left(0^K\right)^0 1\, 0^P$

$= 0^{P-K} 10^P \notin L \quad [\because y^0 = \epsilon]$

The String $xy^0 z$ does not belong to *L* because *P-K < P.*

So, the assumption that *L* is regular is a contradiction. Thus, by using pumping lemma it is proved that *L* is not regular.

Comments (3)

Step 3 of 5

b.

Consider the Language $L=\{0^m1^n \mid m\neq n\}$ **.**

Assume that *L* is regular language and string $S=0^P1^{P+P!}$. Divide the string into three pieces *x, y* and *z*. So, $S=0^P1^{P+P!}=xyz\in L$ and $|S|\geq P$ where, *P* is the pumping length.

Note: *P!* is divisible by all integers from 1 to *P*, where $P!=P\times(P-1)\times(P-2)\times\ldots\times1.$

Assume that $x=0^a, y=0^b$ and $z=0^c1^{P+P!}$, where $b\geq 1$ and $a+b+c=P$.

Now take string $s'=xy^{i+1}z$, where $i=\dfrac{P!}{b}$.

Then $y^i=0^{P!}$ so $y^{i+1}=0^{b+P!}$, and

So $xyz,=0^{a+b+c+P!}1^{P+P!}$.

That gives $xyz=0^{P+P!}1^{P+P!}\notin L$, a contridiction.

Here, $m=P+P!$, $n=P+P!$ and $m=n$. This is a contradiction to the assumption because $m=n$. Thus, by using pumping lemma it is proved *L* is not regular.

Comment

Step 4 of 5

c.

Consider the Language $L=\{w \mid w\in\{0,1\}^*$ is not a palindrome$\}$ **.**

Assume that *L* is regular language.

The compliment of the language L is $\overline{L}=\{w \mid w\in\{0,1\}^* \textit{ is palindrome}\}$ is also regular.

Assume a string $S=0^P1\,0^P$. Divide the string into three pieces *x, y* and *z*. So, $S=0^P1\,0^P=xyz\in L$ where, *P* is the pumping length.

Assume that $x=0^{P-K}, y=0^K$ and $z=10^P$ where $K>0$

Now $xy^0z=0^{P-K}\left(0^K\right)^0 10^P$

$=0^{P-K}10^P\notin L \quad [\because y^0=\in]$

The String xy^0z is not same from forward and backward direction because *P-K* < *P*.

So, the string xy^0z does not belong to $\overline{L}$. So, the assumption is a contradiction.

Thus, by using pumping lemma it is proved *L* is not regular.

Comment

Step 5 of 5

d.

Consider the Language $L=\{wtw \mid w,t\in\{0,1\}^+\}$ **.**

Assume that *L* is regular language.

Assume a string $S=0^P1\,0^P$. Divide the string into three pieces *x, y* and *z*. So, $S=0^P1\,0^P=xyz\in L$ where, *P* is the pumping length.

Assume that $x=0^{P-K}, y=0^K$ and $z=10^P$ where $K>0$

Now $xy^0z=0^{P-K}\left(0^K\right)^0 10^P$

$=0^{P-K}10^P\notin L \quad [\because y^0=\in]$

The String xy^0z does not belong to *L* because *P-K* < *P* and not of the form *wtw* as in *L*.

So, the assumption is a contradiction. Thus, by using pumping lemma it is proved *L* is not regular.

Comments (3)