Problem

Let A and B be two disjoint languages. Say that language C *separates* A and B if $A \subseteq C \text{ and } B \subseteq \overline{C}.$ Describe two disjoint Turing-recognizable languages that aren't separable by any decidable language.

Step-by-step solution

Step 1 of 2

Let A and B are two disjoint languages. Consider that language C, that separates A and B if $A \subseteq C and B \subseteq \overline{C}$. It can be proved as follows:

• It is quite easy to understand this statement. Read the statement carefully things are pretty obvious all you need to understand is $A \subseteq C and B \subseteq \overline{C}$.

• Here, A **is a set of** C **and** B **is a set of Complement of** C. So, it is quite obvious now A and B both are not related to each other anyhow.

• Here, no use of C because even if we use C it won't be able to prove decidability of A on the basis of B and Turing reducibility of A on the basis of B.If it comes to languages those are fully different and belong to different sets then no separators are required. So, here it is worthless to use C as separator.

Comment

Step 2 of 2

Now, consider a Turing machine T that will work as a decider for the language C that separates A and B.**Consider both languages as Regular Expressions that will be decided by** M_1 **and** M_2**.**

1. $S = < M, w >$ Where M is a Turing machine.

2. Now, run $< M_1, W >$ and $< M_2, W >$

3. If M_1 accepts then M rejects and if M_2 accepts M rejects.

• **Remember** M **will always halt in each situation. Where C decides A or B. Now it is pretty easy to understand the situation**.

• Therefore, it can be said that only the first statement of question is enough to prove the concept" **No decidable languages can be used to separate two disjoint Turing recognizable languages**".

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