Problem

A ***useless state*** in a Turing machine is one that is never entered on any input string. Consider the problem of determining whether a Turing machine has any useless states. Formulate this problem as a language and show that it is undecidable.

Step-by-step solution

Step 1 of 1

Undecidability of the Turing machine problem:

• The given problem is defined as the following language:

$USELESS_{TM} = \{\langle T, q\rangle \mid q \text{ is a useless state in TM T}\}$.

• Show that $USELESS_{TM}$ is undecidable by reducing E_{TM} to $USELESS_{TM}$,where $E_{TM} = \{\langle T1\rangle \mid T1 \text{ is a TM and } L(T1) = \varnothing\}$.

• Using the Theorem 5.2. it is already proved that E_{TM} is undecidable

• Suppose that $USELESS_{TM}$ is decidable and that *TM R* decides it.

• Note that for any Turing machine M with accept state q_{accept}, q_{accept} is useless if and only if $L(T1) = \varnothing$.

• Accordingly, since *TM* R solves $USELESS_{TM}$, *R* can be used to check if q_{accept} is a useless state to decide E_{TM} .

Specifically, below is a *TM S* that decides E_{TM} by using the decider R for $USELESS_{TM}$ as a subroutine:

S = “On input $\langle T\rangle$, where *M* is a *TM*:

1. Run TM R on input $\langle T, q_{accept}\rangle$, where q_{accept} is the accept state of *T*.

2. If *R* accepts, accept. If *R* rejects, reject.”

However, since it is known E_{TM} is undecidable and there cannot be a *TM* that decides $USELESS_{TM}$.

Hence it is proved, that the given problem is undecidable.

Comments (5)