Problem

Let ∑ = {0,1}.

a. Let $A = \{0^k u 0^k \mid k \geq 1 \text{ and } u \in \Sigma^*\}$. Show that A is regular.

b. Let $B = \{0^k 1 u 0^k \mid k \geq 1 \text{ and } u \in \Sigma^*\}$. Show that B is not regular.

Step-by-step solution

Step 1 of 6

Pumping Lemma: There is an integer i for any language L which is regular in such a way that X belongs to L with $|X| \geq i$, There exits $p, q, r \in \sum *$, such that $X = pqr$, and(a) $|pq| \leq i$ (b) $|q| > 0$ (c) $\forall n \geq 0 : pqr \, \varepsilon \, L$

Comment

Step 2 of 6

(a)

Consider the following details which is as follows:

Given language $A = \left\{0^k u \, 0^k \,\middle|\, k \geq 1 \text{and } u \in \Sigma^*\right\}$ and $\Sigma = \{0,1\}$

• In order to prove whether the given language is regular, the principle of pumping lemma is applied.

• Now, applying pumping lemma of regular language on the language A and let p be the minimum pumping length.

• Consider a string $\omega = 0^k u 0^k$ where ω is in language A of length p where p= $p = (2k + |u|)$

• Now, if ω is divided in xyz and it is known that $|xy| \leq p$ for $x = 0^k, y = u, z = 0^k$.

Comments (3)

Step 3 of 6

Case 1:

• If $u = \xi$

• Then language must be $0^k 0^k \in A$

$$\begin{aligned} &\omega = xy^i z \\ &let\, i = 0 \\ &then\, \omega = 0^k \xi 0^k \\ &\qquad = 0^k 0^k \in A \end{aligned}$$

Hence, for i=0 and ω is in specified language.

Comment

Case 2:

$\omega = xyz$

$then\, x = z = 0^k$

$(|x|+|y|+|z|) <= p$

$k+|y|+k <= p$

$then |y| <= p-2k$

• Let y=1 then the language will be:

$0^k 1 0^k \in A$

Hence, for y=1 and ω is in specified language.

Hence for any y belonging to $\sum *$. So ω is in specified language.

Comment

Case 3:

$\omega = xy^{p-2k} z$

$\omega = 0^k y^{p-2k} 0^k$

So, the y^{p-2k} always belongs to $\sum *$

• So, from the pumping lemma it is proved that the language is regular as ω always belongs to specified language.

Comments (2)

(b)

Consider the following details which is as follows:

Given language $\mathrm{B} = \left\{ 0^k 1 u\, 0^k \,\middle|\, k \geq 1 \text{ and } u \in \Sigma^* \right\}$ and $\Sigma = \{0,1\}$

• In order to prove whether the given language is regular, the principle of pumping lemma is applied.

• Now, applying pumping lemma of regular language on the language B and let p be the minimum pumping length.

• Consider a string $\omega = 0^2 1 u 0^2$ where ω is in language B of length *p* for k=2.

• Now, if ω is divided in xyz and it is known that $|xy| \leq p$ for x=0, y= 0 and $z = 1u0^2$

• Apply condition 1 in pumping lemma that is $xy^i z \in \mathrm{B}$ for $i \geq 0$.

• Assume that i=2, so x=0 $y^2 = 00$ z=1 $u0^2$

• Now $xy^i z$ is $0001u0^2$ and can be written as $0^3 1u0^2$

• It is clear that $0^3 1u0^2$ does not belong to language *B* because the value of k is not same for string $0^k 1 u\, 0^k$ | k≥1.

• So { $xy^i z = 0^3 1 u 0^2$ for i=2} $\notin$ *B*.

Hence, it is proved that *B* is a not a regular language.

Comments (2)