Problem

Let $B = \{\langle M_1 \rangle, \langle M_2 \rangle, \ldots\}$ be a Turing-recognizable language consisting of TM descriptions. Show that there is a decidable language C consisting of TM descriptions such that every machine described in B has an equivalent machine in C and vice versa.

Step-by-step solution

Step 1 of 2

In order to solve this problem, we need to know the definition of enumerator and some theorems

Enumerator:-

An enumerator is a Turing machine that consist a work tape and the output tape. It outputs the strings by using the work tape without accepting any input.

Also we use the following theorem

Theorem 1:

"A language is Turing – decidable if and only if some enumerator enumerates the strings of this language in lexicographic order"

Comment

Step 2 of 2

Consider the language $B = \{\langle M_1 \rangle, \langle M_2 \rangle \ldots\}$.

B is a Turing recognizable language.

C is a language consisting of Turing machines descriptions.

Consider *E* be the enumerator for the Turing recognizable language *B.*

Construct an enumerator E_o which output the strings of *C* in lexicographic order.

From the above Theorem1, *C* is decidable.

Enumerator E_o simulates *E.*

When *E* gives the i^{th} TM $\langle M_i \rangle$ as output, then enumerator E_o pads M_i by adding sufficiently many extra useless states to obtain a new TM M_i' where the length of $\langle M_i' \rangle$ is greater than the length of $\langle M_{i-1}' \rangle$. Then E outputs $\langle M_i' \rangle$.

Thus simulation occurs in both directions.

Therefore, E_o and *E* are equivalent.

Comments (1)