Problem

Let CFG G be the following grammar.

$$S \rightarrow \mathtt{a}S\mathtt{b} \mid \mathtt{b}Y \mid Y\mathtt{a}$$
$$Y \rightarrow \mathtt{b}Y \mid \mathtt{a}Y \mid \varepsilon$$

Give a simple description of L(G) in English. Use that description to give a CFG for $\overline{L(G)},$ the complement of L(G).

Step-by-step solution

Step 1 of 2

Consider the context free grammar (CFG) G is as follows:

$S \rightarrow aSb \mid bY \mid Ya$

$Y \rightarrow bY \mid aY \mid \varepsilon$

Language *L* (*G*) for the G is as follows:

Consider the productions in the grammar

$S \rightarrow aSb$

$S \rightarrow bY$

$S \rightarrow Ya$

$Y \rightarrow bY$

$Y \rightarrow aY$

$Y \rightarrow \varepsilon$

Case 1:

Consider production $S \rightarrow Ya$ to derive the language.

Substitute Y with production $Y \rightarrow \varepsilon$ then

$S \rightarrow \in a$

$S \rightarrow a$

Case 2:

Consider production $S \rightarrow bY$ to derive the language.

Substitute Y with production $Y \rightarrow \varepsilon$ then

$S \rightarrow \in b$

$S \rightarrow b$

Case 3:

Consider production $S \rightarrow aSb$ to derive the language

Substitute S with production $S \rightarrow bY$ then

$S \rightarrow abYb$

Substitute Y with production $Y \rightarrow bY$ then

$S \rightarrow abbYb$

Substitute Y with production $Y \rightarrow \varepsilon$ then

$S \rightarrow abb \in b$

$S \rightarrow abbb$

Case 4:

Consider production $S \rightarrow bY$ to derive the language.

Substitute Y with production $Y \rightarrow bY$ then

$S \rightarrow bbY$

Substitute Y with production $Y \rightarrow \varepsilon$ then

$S \rightarrow bb \in$
$S \rightarrow bb$

Therefore form the Case 1, Case 2, Case 3 and Case 4 the language obtained is as follows:

$L(G) = \{a, b, abbb, bb\ldots\}$

Using the grammar *G*, many more strings can be generated.

Comment

Step 2 of 2

Description of the *L* (*G*) is as follows:

The grammar G generates a language *L*(*G*) consists of the strings which are described as follows:

• Strings with consecutive number of ***a***'s with a length ranging from 1 to infinity.

• Strings with consecutive number of ***b*'s** with a length ranging from 1 to infinity.

• String with start symbol ***a*** followed by number of ***b*'s**.

• Strings with start symbol ***b*** followed by number of ***a*'s.**

• Strings with ***a*** as start symbol and ***b*** as end symbol.

• Strings with ***b*** as start symbol and ***a*** as end symbol.

• Strings that contains the same start and end symbols. For example, $aba,\ bab$ etc.

From the above description as *L*(*G*) is generating all the possible combination of *a*'s and *b*'s except $a^i b^i \ where\ i \geq 0$. The *L*(*G*) does not produce strings like $\in,\ ab,\ aabb,\ aaabbb\ \ldots$

The complements of $L(G)$ i.e. $\overline{L(G)} = \{\in,\ ab,\ aabb,\ aaabbb\ \ldots\}$

The grammar for $\overline{L(G)}$ is $a^i b^i \ where\ i \geq 0$.

Therefore, the CFG G' for $\overline{L(G)}$ is as follows:

$S \rightarrow aSb \mid \in$

Comment