## Problem

Consider the following two-person version of the language *PUZZLE* that was described in Problem 7.28. Each player starts with an ordered stack of puzzle cards. The players take turns placing the cards in order in the box and may choose which side faces up. Player I wins if all hole positions are blocked in the final stack, and Player II wins if some hole position remains unblocked. Show that the problem of determining which player has a winning strategy for a given starting configuration of the cards is PSPACE-complete.

## Step-by-step solution

**Step 1** of 4

Consider the two-people version of the language $PUZZLE$. In this, every player uses an ordered puzzle card stack to start.

• The cards are placed in each turn in order, by the player, in the box and can select which part faces up. Player $I$ wins if every hole place are closed in the resultant stack and player $II$ wins if some position of the holes remains in unblocked state.

• Now, it can be shows that “the problem of determining which player has a winning strategy for a given starting configuration of the cards is $PSPACE$-complete”.

Comment

**Step 2** of 4

Consider the formula $\varphi$, which is defined as:

$$\varphi = \exists p_1 \forall p_2 \exists p_3 \left[ (p_1 \vee p_2) \wedge (p_2 \vee p_3) \wedge (\overline{p_2} \vee \overline{p_3}) \right]$$

In the above given formula for $\varphi$, player $I$ picks the value of $p_1$ and then player $II$ picks the value of $p_2$ and finally player $I$ picks the value of $p_3$.

• Now, assign true with the value 1 and false with the value 0. Suppose that player $I$ picks $p_1$=1, then the player $II$ picks $p_2$=0, and finally player $I$ picks $p_3$=1.

• Now by using these values in the sub-formula which is described above as

$$(p_1 \vee p_2) \wedge (p_2 \vee p_3) \wedge (\overline{p_2} \vee \overline{p_3})$$

is1, therefore the player $I$ has won the game. Then, it can be said that “player $I$ has a strategy of winning for the given puzzle”.

Comment

**Step 3** of 4

Now, a slightly change in formula, which is given above, will change the strategy of winning of the player.

• Consider the modified formula:

$$(p_1 \vee p_2) \wedge (p_2 \vee p_3) \wedge (p_2 \vee \overline{p_3})$$

The above formula is for winning strategy for the player $II$.

Comment

**Step 4** of 4

Consider the $PUZZLE$, which is described above, is $PSPACE-complete$. It is because, it is same as $TQBF$. The $PUZZLE$ problem is $PSPACE-complete$ can be proved by using the fact of $PUZZLE = TQBF$.

• Consider the formula $\varphi = \exists p_1 \forall p_2 \exists p_3 ... [\psi]$ became TRUE when setting of $p_1$ exists in such a way that, for every setting of $p_2$, a setting of $p_3$ exist is such a way and so on. Where, $\psi$ sows TRUE value under the setting of the taken variable.

• The same process will be applied for the winning strategy for the player $II$.

• If the formula $\varphi$ has of the form $\forall p_1, p_2, p_3 \exists p_4, p_5 \forall p_6 [\psi]$, player $I$ would make the first three move in e $PUZZLE$, to assign values to $p_1, p_2 \; and \; p_3$.

• Then, player $II$ makes two moves to assign $p_4 \; and \; p_5$. Finally, the player $I$ assigned a value $p_6$.

**Therefore, $\varphi \in TQBF$ exactly when $\varphi \in PUZZLE$. Hence it can be said that "the problem of determining which player has a winning strategy for a given starting configuration of the cards is $PSPACE$ -complete".**

Comment