

## Problem

Show that if  $P = NP$ , then every language  $A \in P$ , except  $A = \emptyset$  and  $A = \Sigma^*$ , is NP-complete.

## Step-by-step solution

### Step 1 of 2

A language B is said to be NP-complete if the following conditions are satisfied:

1.  $B \in NP$
2. Every language L can be polynomial-time reduced to B.

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### Step 2 of 2

Let  $P = NP$  and let  $A \in P$  such that  $A \neq \emptyset$  and  $A \neq \Sigma^*$  so, it is required to prove that for every  $A \in NP$  and  $L \in NP$ ,  $L \leq_p A$ .

Let there exist an arbitrary language L from  $NP=P$ . Hence, the language L has polynomial decider  $X_L$  so, the polynomial reduction f from L to A will be as follows:

When the input string is x:

1. Run  $X_L$  on the string x.
2. If the decider  $X_L$  accepts the string then output  $x_{in}$
3. If the decider  $X_L$  rejects the string then output  $x_{output}$

Thus, there exists a poly-time reduction from L to A, so, A is NP-complete.

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