CSE 303 115060128.
HW 7.

Problem 1:-
Let us assume $L = \{ww^R \mid w \in \{0,1\}^*\}$ is a regular language

Thus, by using Pumping lemma we get p as pumping length

Take $w = 0^p 1^p$
So $w^R = 1^p 0^p$
& $ww^R = 0^p 1^p 1^p 0^p = 0^p 1^{2p} 0^p$
Note $ww^R \in L$ since it is "palindrome"
Let $a = ww^R = 0^p 1^{2p} 0^p$

Using $\exists$ elimination we get x, y & z s.t.
- $a = xyz$
- $xy^iz \in L \;\; \forall i \geq 0$
- $|y| > 0$
- $|xy| \leq p$

~~Since $|y| > 0$ & $|xy| \leq p \Rightarrow x$ is of the form 0^{p-k} where $k > 0$ but $k \leq p$.~~

~~Assume~~

~~Thus $xz = 0^{p-k}$~~

Please Turn Over

~~Assume~~

~~z will be of the form $0^b 1^{2p} 0^p$ where $b > 0$~~

~~$\therefore$ xz will be $0^{p-k+b} 1^{2p} 0^p$~~

Since $xyz = 0^p 1^{2p} 0^p$ & y is of the form 0^k
$k > 0$
then xz must be of the form $0^{p-k} 1^{2p} 0^p$

Clearly $xz \notin L$ since $p-k \neq p$. But it must $\in L$
$\therefore$ We have a contradiction.

$\Rightarrow$ L is not regular

Problem 2:
a). L = Set of binary strings having equal no of 0's & 1's.

This language is not regular.

We will prove it using Proof of Contradiction method.

Assume L is regular. Thus we get p through pumping lemma.

Take $w = 0^p 1^p$.

Using $\exists$ elimination, we get x, y, z s.t.
- $w = xyz$
- $xy^iz \in L \quad \forall i \geq 0$
- $|y| > 0$
- $|xy| \leq p$.

Since $|y| > 0 \Rightarrow y$ is of the form 0^k
$0 < k \leq p$.

Thus $xz = 0^{p-k}1^{p}$.

However $xz = 0^{p-k}1^{p}$ does not belong to L.

But it must belong to L by pumping lemma.

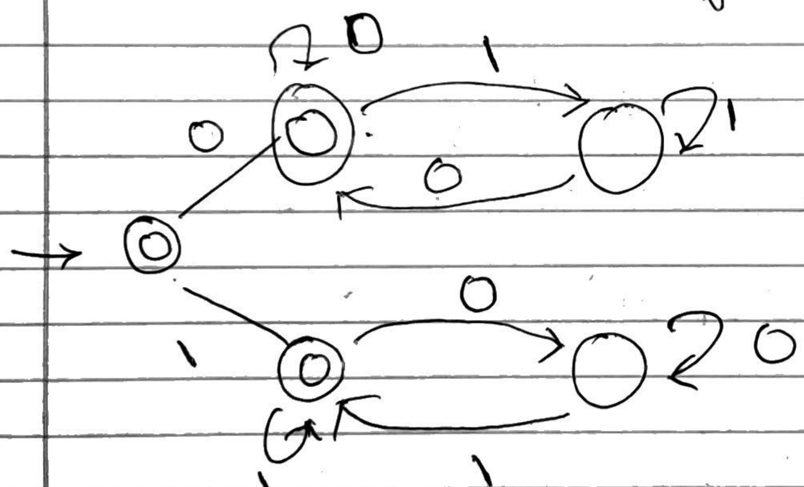
∴ We have a contradiction

⇒ L is not regular

b) L = Set of strings having equal no of 01's & 10's

⇒ This language is regular.

This is because the following DFA M recognises it.



Because we have a DFA recognising L.
~~We have~~ Thus L is regular.

Problem 3:-

a) Since L is accepted by DFA M $\Rightarrow$ L is regular

~~Let $w \in L$ s.t $|w| \geq S$.~~

~~At~~ Let $w \in L$ s.t $w = xyz$ with $|y| \geq S$.

Choose states $r_0, r_1, \ldots, r_{|w|}$ s.t $r_0 = q_0$. $r_{|w|} \in F$ & $r_{i+1} = \delta(r_i, w_{i+1})$ for $0 \leq i \leq |w|$.
Such states exist $\because$ M accepts w.

Let after accepting x, we are in state r_j
Let after accepting xy, ... r_k.

~~We have~~ Let $y = y_1 y_2 \ldots y_{|y|}$.
Since ~~after~~ $|y| \geq S$, there exists states $t_0, t_1, \ldots, t_{|y|}$ s.t $t_0 = r_j$, $t_{|y|} = r_k$ &

$t_{i+1} = \delta(t_i, y_i)$ for $0 \leq i \leq |y|$

where y_i is i^{th} input alphabet from left in y.

Since $|y| \geq S$ we have $t_0, t_1, \ldots, t_{|y|}$ to be atleast $S+1$ in no.
But DFA M only has S states.
Thus by Pigeon Hole Principle, two states are same say $t_a = t_b$. $a < b$.
$b - a < S$.

Let $u = y_1 y_2 \ldots y_a$.
$v = y_{a+1} \ldots y_b$.
$w = y_{b+1} \ldots y_{|y|}$.

Thus $y = uvw$.
Since $b - a > 0 \Rightarrow v \neq \varepsilon$.

For any given $i \geq 0$ the seq of states

$$r_0, r_1, \ldots, r_j = t_0, t_1, \ldots, \underbrace{t_a \ldots t_b}_{i \text{ times}} \ldots t_{|y|} = r_k \ldots r_{|w|}$$

~~is accepted by~~ exist which correspond to ~~the~~ transitions of the ~~sta~~ DFA after taking input of xuv^iwz which M accepts.

$\Rightarrow$ xuv^iwz, $i \geq 0 \in L$.

This proves the stronger ~~fro~~ form of pumping lemma.

b) $L = \{ww^Rx \mid w \in \{0,1\}^+, x \in \{0,1\}^*\}$

$L = \{ww^Rz \mid w \in \{0,1\}^+, z \in \{0,1\}^*\}$

Assume L is regular $\Rightarrow \exists$ a DFA M with S states which recognises L.

Take string $c = 0^s 1^s \Rightarrow c^R = 1^s 0^s$
So we have $a = cc^R z = 0^s 1^{2s} 0^s z$
Take $a = xyz$ where $x = c$, $y = c^R$

$|y| = 2s \geq s$.

~~Take z at.~~

Thus we can apply stronger form of Pumping lemma & $\exists u, v, w$ s.t.

$y = uvw$; $v \neq \varepsilon$

& $\forall b$, $b = xuv^iwz$, $i \geq 0 \in L$.

we have $y \equiv c^R = 1^s 0^s$.

Since $y = uvw$; $v \neq \varepsilon$ we have.

$v = 1^m 0^n$ where either m or n is nonzero.
else $v = \varepsilon$ which is not possible

Consider the string $b = xuwz$ if $s, s-m$ & $s-n$ have diff parity
$= 0^s 1^s 1^{s-m} 0^{s-n} z$.
Since either once of m, n is non zero,
we cant express b as $ww^R z$ for any $w \in \{0,1\}^+$.

Even if we can try to make it in the form ww^R we have control over z ~~& by choosing z~~.

If $s, s-m$ or $s, s-n$ have same parity.

Take $b = xuv^3wz$.
Then we will get $s-2m$ or $s-2n$ 1's or 0's which wont belong to L.

Thus L is not regular.