Problem

Show that P is closed under union, concatenation, and complement.

Step-by-step solution

Step 1 of 4

The class P is closed under union, concatenation, and complement

Class P: P is the class of languages that are decidable in polynomial time on a deterministic single – tape Turing machine.

That is, $P = \bigcup_k \mathrm{TIME}(n^k)$

Now we have to show that P is closed under union, concatenation and complement.

Comment

Step 2 of 4

Union:

Assume two languages $P_1 \in P$ and $P_2 \in P$

The Turing machine *M* that accepts $P_1 \cup P_2$ works as follows:

$M =$ "On input *w*:

1. Check if $w \in P_1$

2. if not then check if $w \in P_2$

3. Accept *w* if and only if P_1 or P_2 accepts.

4. If both reject then rejects the input *w*".

Since each membership check requires polynomial time the overall time is polynomial.

Comment

Step 3 of 4

Concatenation:

Assume two languages $P_1 \in P$ and $P_2 \in P$.

The Turing machine *M* that accepts $P_1.P_2$ works as follows

$M =$ "On input *w* of length *n:*

1. *w* can be split into two strings in *n* different ways

2. For each split,

(a) Check if the first substring belongs to P_1

(b) check if second substring belongs to P_2

3. If any split succeeds, then accept".

Clearly the overall time is polynomial.

Comments (1)

Step 4 of 4

Complement :

Assume a language $P_1 \in P$

The Turing machine *M* that accepts $\overline{P_1}$ works as follows

$M =$ "On input *w*:

1. Check if $w \in P_1$

2. If $w \in P_1$ then reject.

3. If $w \not\in P_1$, then accept"

Clearly the overall time is polynomial.

Therefore, the class P is closed under union, concatenation, and complement.

Comments (1)