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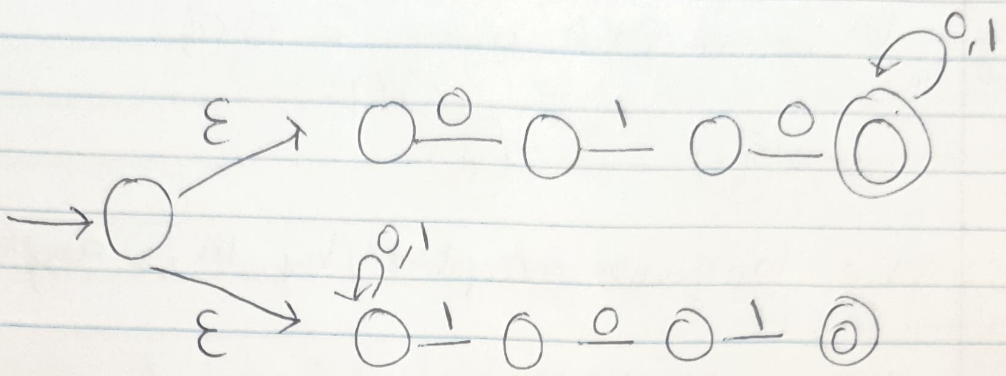
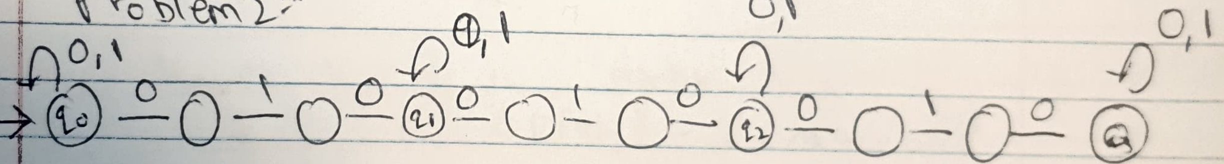
CSE 303 ToC
HW 4

Problem 1:



This is a NFA that accepts the set of binary strings beginning with 010 or ending with 110.

Problem 2:



Since the NFA has to be s.t it has to accept atleast three occurrences of 010, we need to have comb. like

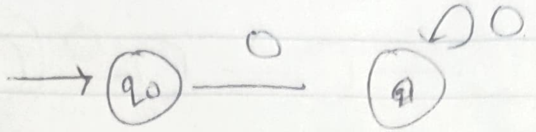
$\{0,1\}^m \; 010 \; \{0,1\}^n \; 010, \{0,1\}^k \; 0,10 \; \{0,1\}^l$

$m, n, k, l \geq 0$.

So we need self loops at q_0, q_1 q_2 because of the pairs of $\{0,1\}$ if at all.

Problem 5:-

M is a DFA over $\Sigma = \{0\}$



The language accepted by M is any no. of 0's (≥ 1)

Take $m = 1, n = 1$.

So $L(M) = \{ 0 . 0^k \mid k \geq 0 \}$

i.e $L(M)$ is any non zero nos of 0's which M accepts since after one 0 it goes from q_0 to q_1. And then it will stay in q_1 for any input.

Problem 4:-

We define M using M_1, M_2

$(Q, \Sigma, \delta, q_0, F)$ are given as

$Q = Q_1 \cup Q_2$ where Q_1, Q_2 are sets for M_1, M_2.

Σ is the same alphabet over M_1, M_2 are defined

q_0 is initial State of M_1

F is accepted States of M_2

δ_1 & δ_2 are function for M_1 & M_2

②

$\delta(w,a) = \delta_1(w,a)$ if w ~~&~~ is string accepted by M_1 & $a \in M_2$ state.

$= \delta_2(w,a)$ if w is string accepted by M_2 & $a \in M_1$ state.

This is how M is defined.

Let L be the language given below

$L = \{ w \mid w = a_1 b_1 \ldots a_k b_k$ where $a_1 \ldots a_k \in L(M_1)$
$b_1, \ldots b_k \in L(M_2) \}$

Proof that M accepts L:

☆ we have $w = a_1 b_1 \ldots a_k b_k$ where $a_1 \ldots a_k \in L(M_1)$
& $b_1 \ldots b_k \in L(M_2)$

Thus $\exists$ states $c_0, \ldots c_{k+1}$ s.t. $c_0 = q_0$ (initial state of a)

& $\delta_1(c_i, a_i) = c_{i+1}$ for $i = 0, 1, \ldots k-1$

& $\exists$ states $d_0, \ldots, d_{k+1}$ s.t $d_0 = r_0$ (initial state of b)

& $\delta_2(d_i, b_i) = b_{i+1}$ for $i = 0, 1, \ldots k-1$

~~Then~~ Take the states $c_0, d_0, c_1 d_1, \ldots c_{k+1} d_{k+1}$ for the ~~...~~ DFA M.

Thus we have,

$\delta(c_0, q_1) = c_1$ & so on & finally we will reach the accepted states of ~~q~~ M_2. which is also the accepted state of M.

Thus M accepts L.

Problem 3:-

we take states corresponding to n, m mod 5.

for n mod 5 we will have 0, 1, 2, 3, 4 values
m mod 5 " " " "

Also, Since $q \equiv mn \bmod 5$, we have multiplication table for $m \times n \pmod 5$.

↓m / n→	0	1	2	3	4
0	0	0	0	0	0
1	0	1	2	3	4
2	0	2	4	1	3
3	0	3	1	4	2
4	0	4	3	2	1

So we have values of q mod 5 as per (m, n) values

		(m,n)								
q_0	0	(0,0)	(0,1)	(0,2)	(0,3)	(0,4)	(1,0)	(2,0)	(3,0)	(4,0)
q_1	1	(1,1)	(3,2)	(2,3)	(4,4)					
q_2	2	(2,1)	(1,2)	(4,3)	(3,4)					
q_3	3	~~(3,3)~~	(3,1)	(1,3)	(4,2)	(2,4)				
q_4	4	~~(4,4)~~	(4,1)	(1,4)	(2,2)	(3,3)