

### Problem

Give state diagrams of DFAs recognizing the following languages. In all parts, the alphabet is  $\{0,1\}$ .

- a.  $\{w \mid w \text{ begins with a 1 and ends with a 0}\}$
- b.  $\{w \mid w \text{ contains at least three 1s}\}$
- c.  $\{w \mid w \text{ contains the substring 0101 (i.e., } w = x0101y \text{ for some } x \text{ and } y)\}$
- d.  $\{w \mid w \text{ has length at least 3 and its third symbol is a 0}\}$
- e.  $\{w \mid w \text{ starts with 0 and has odd length, or starts with 1 and has even length}\}$
- f.  $\{w \mid w \text{ doesn't contain the substring 110}\}$
- g.  $\{w \mid \text{the length of } w \text{ is at most 5}\}$
- h.  $\{w \mid w \text{ is any string except 11 and 111}\}$
- i.  $\{w \mid \text{every odd position of } w \text{ is a 1}\}$
- j.  $\{w \mid w \text{ contains at least two 0s and at most one 1}\}$
- k.  $\{\epsilon, 0\}$
- l.  $\{w \mid w \text{ contains an even number of 0s, or contains exactly two 1s}\}$
- m. The empty set
- n. All strings except the empty string

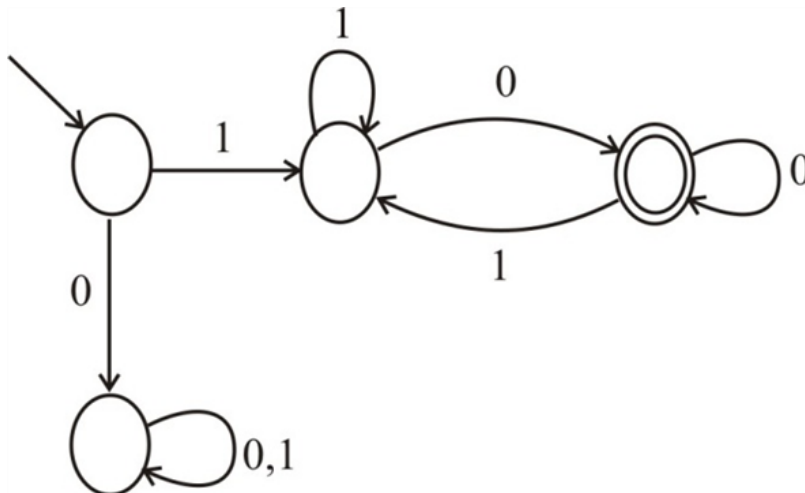
### Step-by-step solution

Step 1 of 14

(a)

Language  $L = \{w \mid w \text{ begins with a 1 and ends with a 0}\}$

State diagram of DFA that recognizes  $L$  is given by:



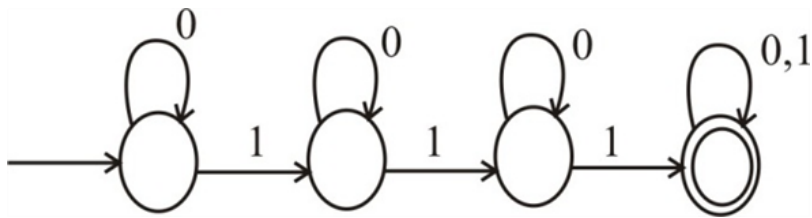
[Comments \(7\)](#)

Step 2 of 14

(b)

Language  $L = \{ w \mid w \text{ contains at least three 1s} \}$

State diagram of DFA that recognizes  $L$  is given by:



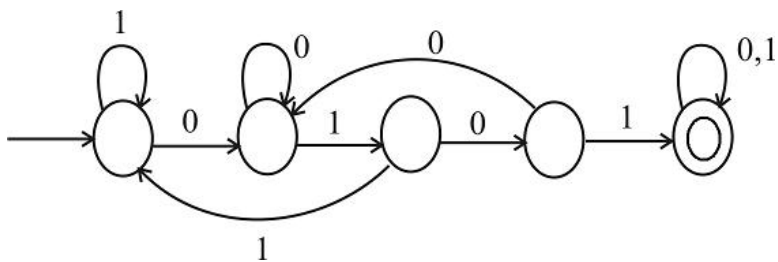
[Comments \(2\)](#)

Step 3 of 14

(c)

Language  $L = \{ w \mid w \text{ contains the substring } 0101, \text{ i.e. } w = x0101y \text{ for some } x \text{ and } y \}$

State diagram of DFA that recognizes  $L$  is given by:



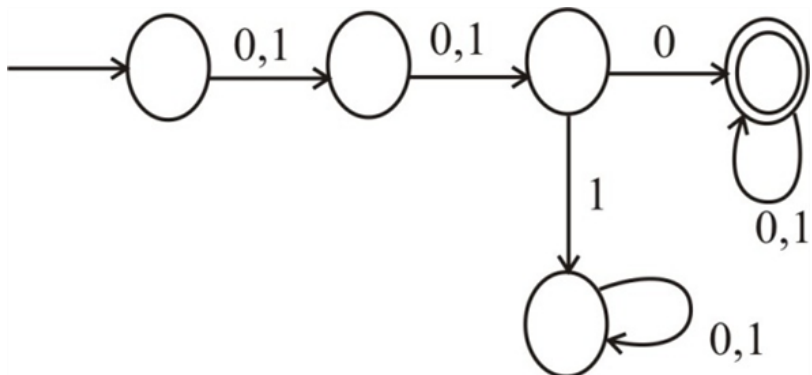
[Comment](#)

Step 4 of 14

(d)

Language  $L = \{ w \mid w \text{ has length at least 3 and its third symbol is a 0} \}$

State diagram of DFA that recognizes  $L$  is given by:



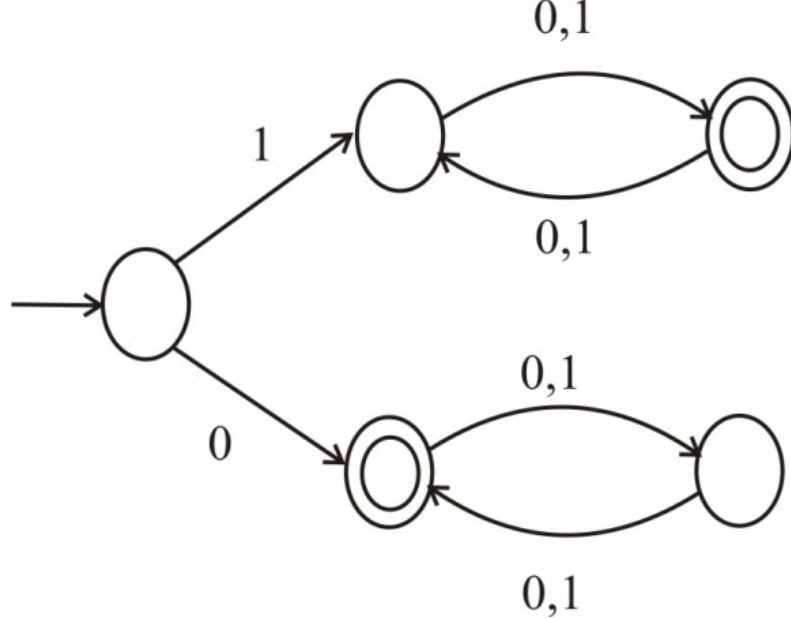
[Comment](#)

Step 5 of 14

(e)

Language  $L = \{ w \mid w \text{ starts with 0 and has odd length, or starts with 1 and has even length} \}$

State diagram of DFA that recognizes  $L$  is given by:



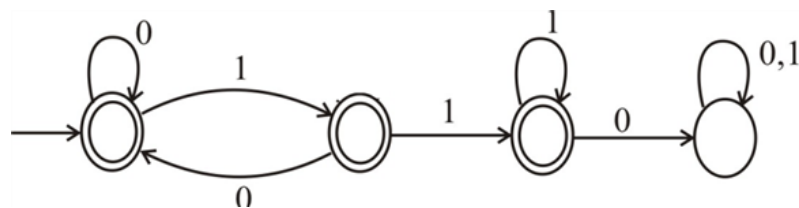
[Comments \(3\)](#)

Step 6 of 14

(f)

Language  $L = \{ w \mid w \text{ doesn't contain the substring } 110 \}$

State diagram of DFA that recognizes  $L$  is given by:



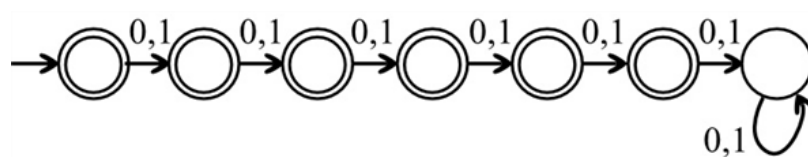
[Comment](#)

Step 7 of 14

(g)

Language  $L = \{ w \mid \text{length of } w \text{ is at most } 5 \}$

State diagram of DFA that recognizes  $L$  is given by:



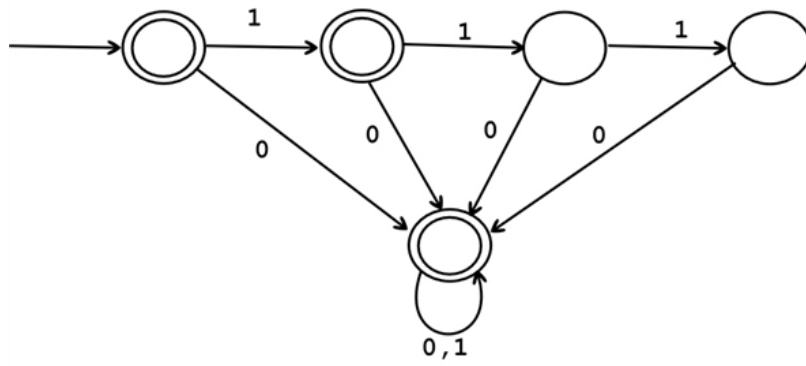
[Comments \(2\)](#)

Step 8 of 14

(h)

Language  $L = \{ w \mid w \text{ is any string except } 11 \text{ and } 111 \}$

State diagram of DFA that recognizes  $L$  is given by:



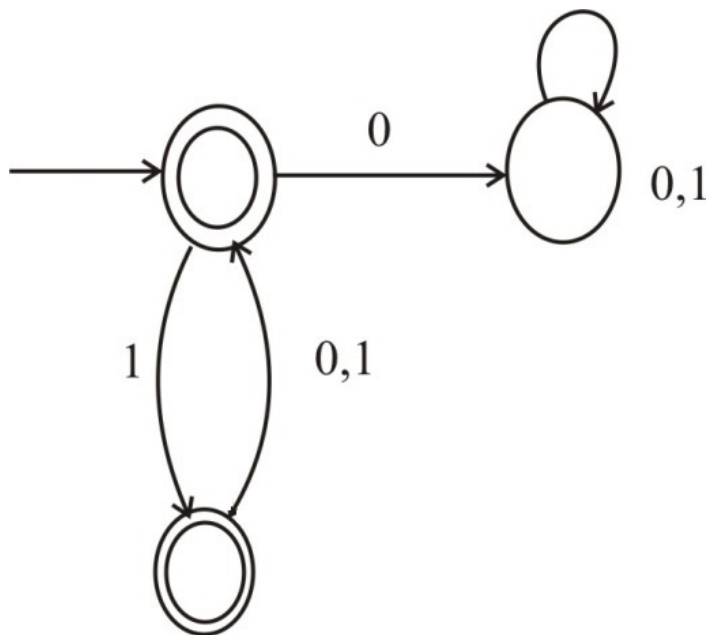
[Comments \(11\)](#)

#### Step 9 of 14

(i)

Language  $L = \{ w \mid w \text{ every odd position of } w \text{ is a } 1 \}$

**State diagram of DFA that recognizes  $L$  is given by:**



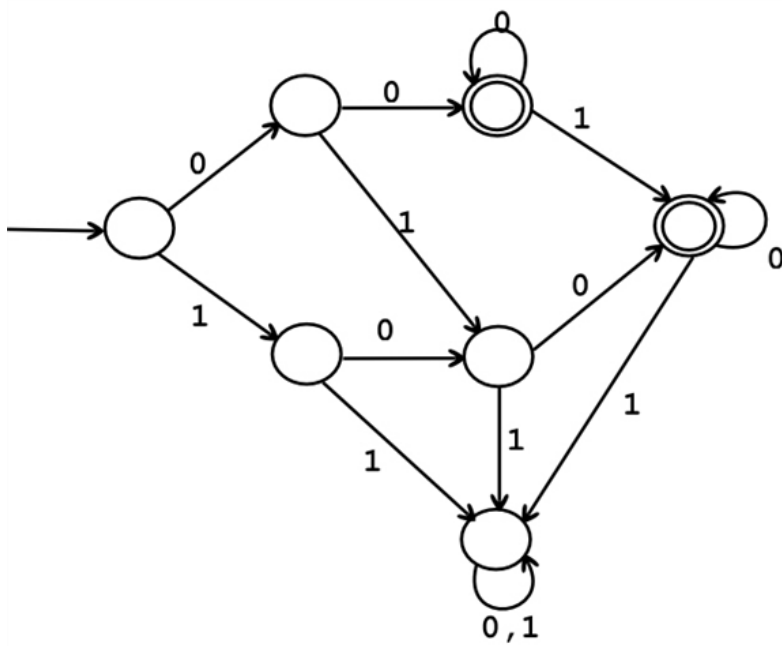
[Comments \(3\)](#)

#### Step 10 of 14

(j)

Language  $L = \{ w \mid w \text{ contain at least two 0s and at most one 1} \}$

**State diagram of DFA that recognizes  $L$  is given by:**



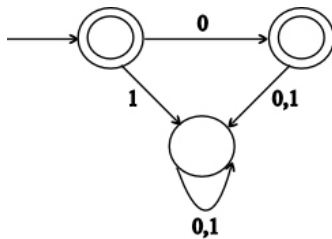
[Comment](#)

Step 11 of 14

(k)

Language  $L = \{\epsilon, 0\}$

State diagram of DFA that recognizes  $L$  is given by:



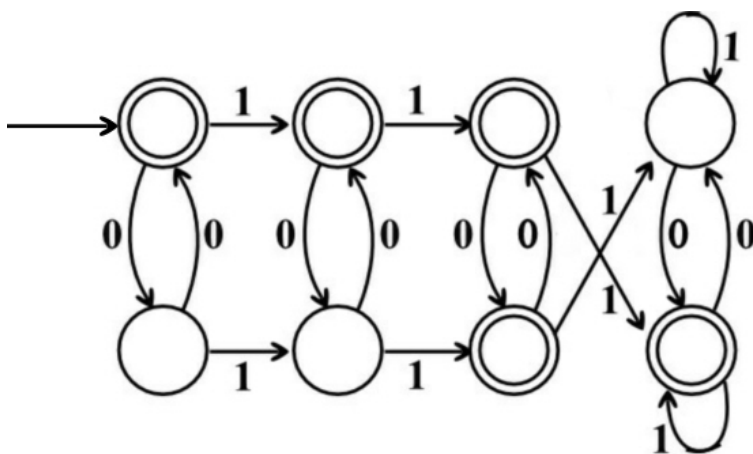
[Comment](#)

Step 12 of 14

(l)

Language  $L = \{ w \mid w \text{ contains an even number of 0s, or contains exactly two 1s} \}$ .

State diagram of DFA that recognizes  $L$  is given by:



The language  $L$  accepts the strings that contain even number of 0s or contains exactly two 1s. This language accepts the string if any one of the two conditions is satisfied.

Consider the string 11, the string 11 contains zero number of 0s which is even. In this case, the first condition is accepted. Thus, the language  $L$  accepts the string 11.

[Comments \(10\)](#)

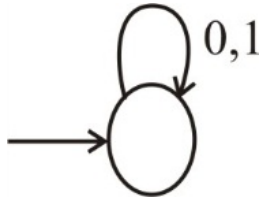
**Step 13 of 14**

(m)

Language  $L = \{\text{The empty set}\}$

The empty set does not contain the null string. The language  $L$  does not accept any string even the null string.

**State diagram of DFA that recognizes  $L$  is given by:**



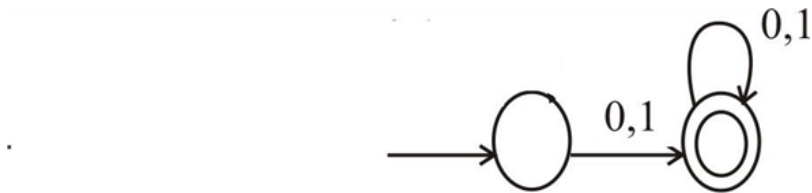
[Comments \(2\)](#)

**Step 14 of 14**

(n)

Language  $L = \{\text{all strings except the empty string}\}$

**State diagram of DFA that recognizes  $L$  is given by:**



[Comment](#)