Problem

Let *MULT* = {a#b#cl a, b, c are binary natural numbers and a × b = c}. Show that *MULT* ? L.

Step-by-step solution

Step 1 of 2

The class L: L is the class of languages that are decidable in logarithmic space on a deterministic truing machine.

That is, $L = SPACE(\log n)$

Given that,

$MULT =${ $a\#b\#c\,|$ where a,b,c are binary natural numbers and $a \times b = c$}.

Comment

Step 2 of 2

We have to show that $MULT \in L$.

That means, we have to construct a deterministic Turing machine (DTM) that decides $MULT$ in logarithmic space.

Let M be the *DTM* that decides *MULT* in logarithmic space.

The construction of M is as follows:

$M =$"On input $a\#b\#c$:

1. If either of the three strings is not a binary number as defined above then reject
2. Initialize a binary counter *i* pointing to the O.
3. Initialize *a* binary counter *j* to $\max(0, i+1-\text{length of } 1)$
4. Initialize a binary counter *k* to $i-j$
5. Now take a binary counter $x \leftarrow x + a[j]*b[k]$
6. Repeat steps 3 to 5 $\min(i, n-1)$ times.
7. Repeat steps 2 to 6 $2n-1$ times and calculate $x = \text{floor}(x/2)$
8. If any discrepancy arises between the calculate next bit of *c* and then reject.
9. If the multiplication ended with no errors then accept.

Thus, clearly *M* runs in log space and decides $MULT$.

So, $MULT \in L$.

Comment