Problem

Let $S = \{\langle M\rangle \mid M \text{ is a TM and } L(M) = \{\langle M\rangle\}\}.$ is Turing-recognizable.

Step-by-step solution

Step 1 of 1

S and $\overline{S}$ are not recognizable can be proof by following method:

Proof:

This can be proved by reduction that is by reducing $\overline{A_{TM}}$ to S and $\overline{A_{TM}}$ to $\overline{S}$. At first it is necessary to show that $A_{TM} \leq_m S$.

Consider a function *f* having input $\langle M, w\rangle$ and output M' which works as follow on input *x*.

1. When $x == \langle M'\rangle$

ACCEPT

2. When $x == 0$ then execute M on input *w*.

If *w* is accept the **ACCEPT** else **REJECT**

3. **REJECT.**

It is a many-one reduction from $\overline{A_{TM}}$ to S. When $\langle M, w\rangle \in A_{TM}$ then it shows that *w* is accepted by *M*. According to M' definition it can be states that $L(M) = \{\langle M'\rangle, 0\} \neq \{\langle M'\rangle\}$. When $\langle M, w\rangle \notin A_{TM}$ then it shows that *w* is not accepted by *M*. As 0 is not acceptable by M' therefore $L(M') = \{\langle M'\rangle\}$

Now show $\overline{A_{TM}}$ to $\overline{S}$. Consider another function *f* having input $\langle M, w\rangle$ and output M'' which works as follow on input *x*.

1. When $x == \langle M''\rangle$

2. Execute M on input *w*. If w is accept the **ACCEPT** else **REJECT**

3. **REJECT.**

It is also a many-one reduction from $\overline{A_{TM}}$ to $\overline{S}$. When $\langle M, w\rangle \notin A_{TM}$ then it shows that *w* is not accepted by *M*. According to M'' definition it can state that M'' is not acceptable by M'' therefore $M'' \notin S$. When $\langle M, w\rangle \in A_{TM}$ then it shows that *w* is accepted by *M* then according to M'' definition M'' is acceptable by M''.

Comment