Problem

Give a counterexample to show that the following construction fails to prove that the class of context-free languages is closed under star. Let A be a CFL that is generated by the CFG G = (V,∑,R, S). Add the new rule S → SS and call the resulting grammar G′. This grammar is supposed to generate A^*.

Step-by-step solution

Step 1 of 2

Example to show the failure of closure of Context free language under Star

Context free language is the language which is generated by CFG or the context free grammar. It is possible to get different types of context free languages from different types of context free grammar.

Given: Here, it is given that *A* is a context free language and the context free grammar which is responsible for its generation is given as $G=(V,\Sigma,R,S)$.

Here it is required to show the failure of the construction in proving the closure of the context free language is closed under star.

Comment

Step 2 of 2

Proof:

This can be done by adding a new rule as $S \rightarrow SS$ and the grammar which will be obtained as resultant will be called as G'. This grammar is expected to generate A^*.

Now, the counter example can be considered by supposing the CFL as A and the grammar corresponding to it along with its production rules as $G=\{\{S\},\{(,)\},\{S \Rightarrow (S), S \Rightarrow \in\},S\}$.

Now, as required and given add the new production rule as $S \rightarrow SS$. As the new rule is being added a new grammar represented as G' will be obtained.

This grammar will generate the language containing the string symbols as $((\ ),(\ ))$. And, thus, this grammar is not capable of generating the star closure of *A* represented by A^*.

Conclusion: Thus, it can be concluded that the new grammar G' fails to prove that the class of context free languages is closed under star.

This condition is true only if the A^* contains empty string else false. So, A^* must contains the empty string.

Comment