Homework #1

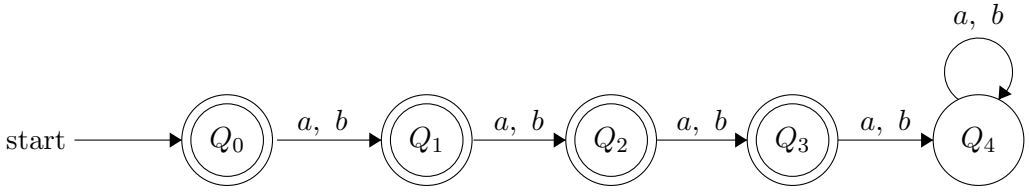
(Due: Sep 23)

Task 1. [100 Points] Construct DFAs

Construct a DFA to accept each of the following regular languages. Assume that $\Sigma = \{a, b\}$ unless specified otherwise.

(*a*) **[5 Points]** $L = \{w| \ |w| \leq 3\}$

DFA Diagram:



5-Tuple: $M = (Q, \Sigma, \delta, q_0, F)$, where,

Set of states is	$Q = \{Q_0, Q_1, Q_2, Q_3, Q_4\}$
Set of symbols is	$\Sigma = \{a, b\}$
Start state is	$q_0 = Q_0$
Set of accept states is	$F = \{Q_0, Q_1, Q_2, Q_3\}$
Transition function is	

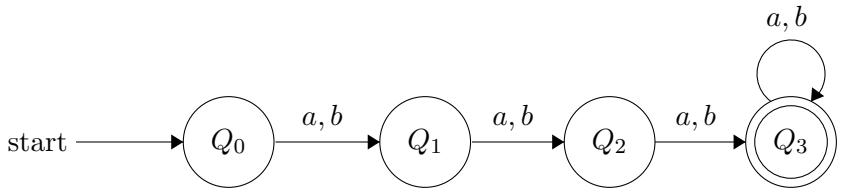
δ:

	a	b
Q_0	Q_1	Q_1
Q_1	Q_2	Q_2
Q_2	Q_3	Q_3
Q_3	Q_4	Q_4
Q_4	Q_4	Q_4

Regular Expression: $\epsilon \cup \Sigma \cup \Sigma^2 \cup \Sigma^3$

(*b*) **[5 Points]** $L = \{w| \ |w| \geq 3\}$

DFA Diagram:



5-Tuple: $M = (Q, \Sigma, \delta, q_0, F)$, where,

Set of states is	$Q = \{Q_0, Q_1, Q_2, Q_3\}$
Set of symbols is	$\Sigma = \{a, b\}$
Start state is	$q_0 = Q_0$
Set of accept states is	$F = \{Q_3\}$
Transition function is	

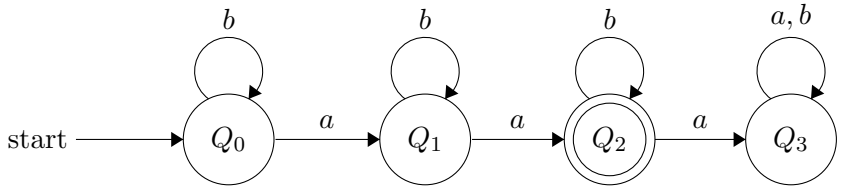
δ:

	a	b
Q_0	Q_1	Q_1
Q_1	Q_2	Q_2
Q_2	Q_3	Q_3
Q_3	Q_3	Q_3

Regular Expression: $\Sigma^3\Sigma^*$

(*c*) [**5 Points**] $L = \{w |\ n_a(w) = 2\}$

DFA Diagram:



5-Tuple: $M = (Q, \Sigma, \delta, q_0, F)$, where,

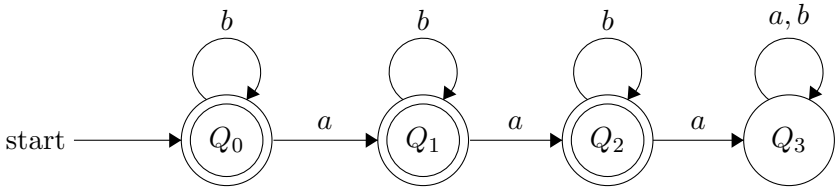
Set of states is	$Q = \{Q_0, Q_1, Q_2, Q_3\}$
Set of symbols is	$\Sigma = \{a, b\}$
Start state is	$q_0 = Q_0$
Set of accept states is	$F = \{Q_2\}$
Transition function is	

δ:	a	b
Q_0	Q_1	Q_0
Q_1	Q_2	Q_1
Q_2	Q_3	Q_2
Q_3	Q_3	Q_3

Regular Expression: $b^*ab^*ab^*$

(*d*) [**5 Points**] $L = \{w|\ n_a(w) \leq 2\}$

DFA Diagram:



5-Tuple: $M = (Q, \Sigma, \delta, q_0, F)$, where,

Set of states is $Q = \{Q_0, Q_1, Q_2, Q_3\}$

Set of symbols is $\Sigma = \{a, b\}$

Start state is $q_0 = Q_0$

Set of accept states is $F = \{Q_0, Q_1, Q_2\}$

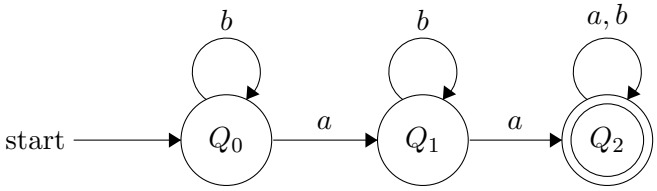
Transition function is

δ:	a	b
Q_0	Q_1	Q_0
Q_1	Q_2	Q_1
Q_2	Q_3	Q_2
Q_3	Q_3	Q_3

Regular Expression: $b^* \cup b^*ab^* \cup b^*ab^*ab^*$

(*e*) [**5 Points**] $L = \{w|\ n_a(w) \geq 2\}$

DFA Diagram:



5-Tuple: $M = (Q, \Sigma, \delta, q_0, F)$, where,

Set of states is	$Q = \{Q_0, Q_1, Q_2\}$
Set of symbols is	$\Sigma = \{a, b\}$
Start state is	$q_0 = Q_0$
Set of accept states is	$F = \{Q_2\}$
Transition function is	δ

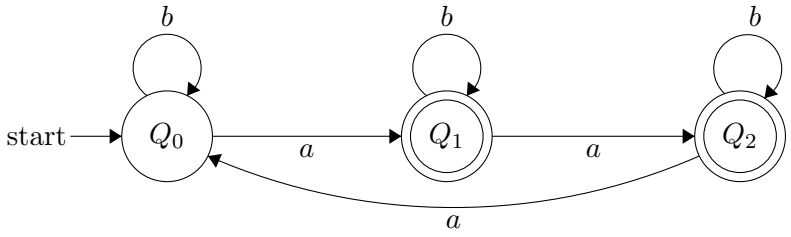
δ:

	a	b
Q_0	Q_1	Q_0
Q_1	Q_2	Q_1
Q_2	Q_2	Q_2

Regular Expression: $b^*ab^*a\Sigma^*$

(*f*) [**5 Points**] $L = \{w|\ n_a(w) \bmod 3 = 1\}$

DFA Diagram:



5-Tuple: $M = (Q, \Sigma, \delta, q_0, F)$, where,

Set of states is	$Q = \{Q_0, Q_1, Q_2\}$
Set of symbols is	$\Sigma = \{a, b\}$
Start state is	$q_0 = Q_0$
Set of accept states is	$F = \{Q_1\}$
Transition function is	δ

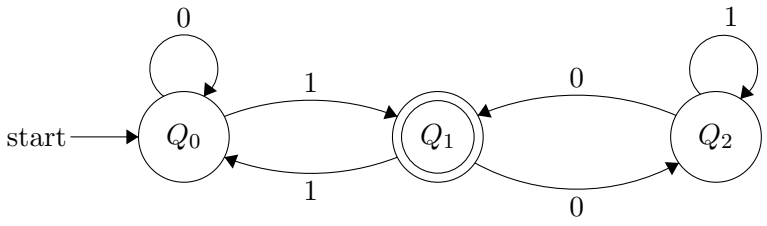
δ:

	a	b
Q_0	Q_1	Q_0
Q_1	Q_2	Q_1
Q_2	Q_0	Q_2

Regular Expression: $b^*ab^*(ab^*)^{3*}$

(*g*) [**5 Points**] $L = \{w|$ **binary number** $w \bmod 3 = 1\}$ for $\Sigma = \{0, 1\}$

DFA Diagram:



5-Tuple: $M = (Q, \Sigma, \delta, q_0, F)$, where,

Set of states is	$Q = \{Q_0, Q_1, Q_2\}$
Set of symbols is	$\Sigma = \{0, 1\}$
Start state is	$q_0 = Q_0$
Set of accept states is	$F = \{Q_1\}$
Transition function is	δ

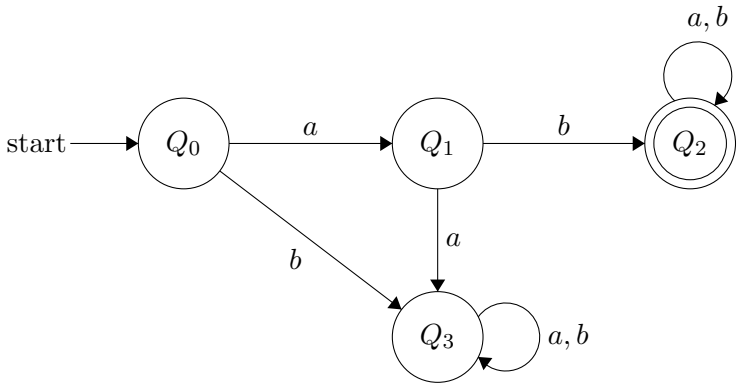
δ:

	0	1
Q_0	Q_0	Q_1
Q_1	Q_2	Q_0
Q_2	Q_1	Q_2

Regular Expression: 0*1(10*1 ∪ 01*0)*

(*h*) [**5 Points**] $L = \{w|\ w$ **starts with** $ab\}$

DFA Diagram:



5-Tuple: $M = (Q, \Sigma, \delta, q_0, F)$, where,

Set of states is	$Q = \{Q_0, Q_1, Q_2, Q_3\}$
Set of symbols is	$\Sigma = \{0, 1\}$
Start state is	$q_0 = Q_0$
Set of accept states is	$F = \{Q_2\}$
Transition function is	δ

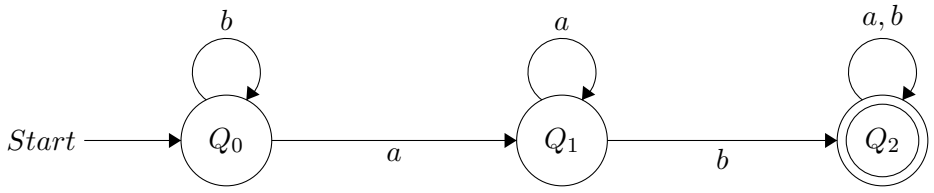
δ:

	0	1
Q_0	Q_1	Q_3
Q_1	Q_3	Q_2
Q_2	Q_2	Q_2
Q_3	Q_3	Q_3

Regular Expression: $ab\Sigma^*$

(i) [**5 Points**] $L = \{w |\ w$ **contains** $ab\}$

DFA Diagram:



5-Tuple: $M = (Q, \Sigma, \delta, q_0, F)$, where,

Set of states is	$Q = \{Q_0, Q_1, Q_2\}$
Set of symbols is	$\Sigma = \{a, b\}$
Start state is	$q_0 = Q_0$
Set of accept states is	$F = \{Q_2\}$
Transition function is	

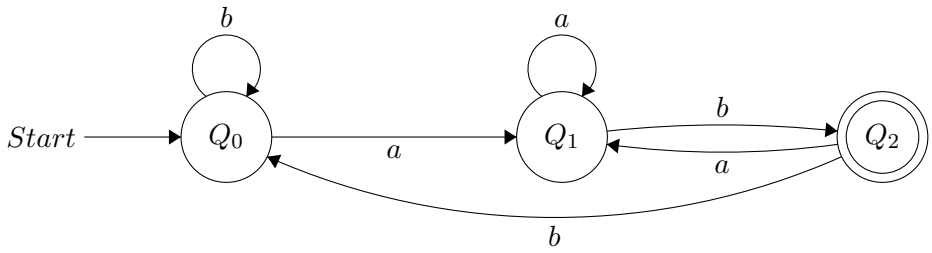
δ:

	a	b
Q_0	Q_1	Q_0
Q_1	Q_1	Q_2
Q_2	Q_2	Q_2

Regular Expression: $\Sigma^*ab\Sigma^*$

(*j*) [**5 Points**] $L = \{w |\ w$ **ends with** $ab\}$

DFA Diagram:



5-Tuple: $M = (Q, \Sigma, \delta, q_0, F)$, where,

Set of states is	$Q = \{Q_0, Q_1, Q_2\}$
Set of symbols is	$\Sigma = \{a, b\}$
Start state is	$q_0 = Q_0$
Set of accept states is	$F = \{Q_2\}$
Transition function is	

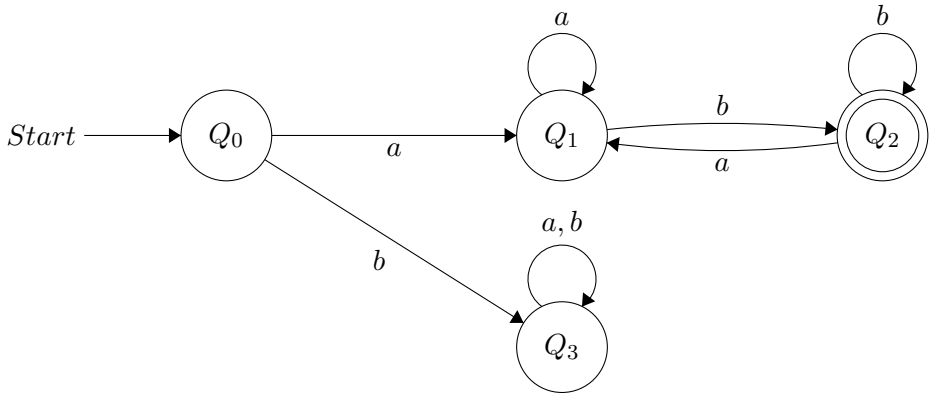
δ:

	a	b
Q_0	Q_1	Q_0
Q_1	Q_1	Q_2
Q_2	Q_1	Q_0

Regular Expression: $\Sigma^* ab$

(*k*) [**5 Points**] $L = \{w |\ w$ **starts with** a **and ends with** $b\}$

DFA Diagram:



5-Tuple: $M = (Q, \Sigma, \delta, q_0, F)$, where,

Set of states is	$Q = \{Q_0, Q_1, Q_2, Q_3\}$
Set of symbols is	$\Sigma = \{a, b\}$
Start state is	$q_0 = Q_0$
Set of accept states is	$F = \{Q_2\}$
Transition function is	

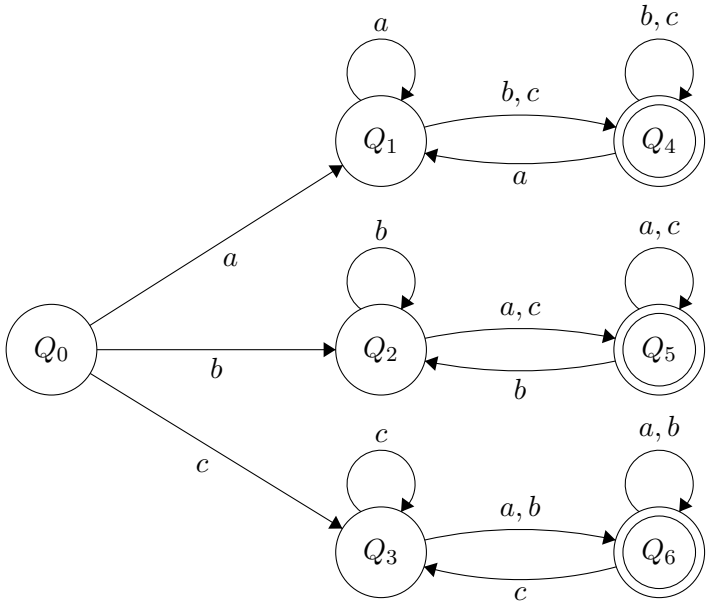
δ:

	a	b
Q_0	Q_1	Q_3
Q_1	Q_1	Q_2
Q_2	Q_1	Q_2
Q_3	Q_3	Q_3

Regular Expression: $a\Sigma^*b$

(l) [**5 Points**] $L = \{w |\ w$ **starts and ends with different symbols**$\}$ for $\Sigma = \{a, b, c\}$

DFA Diagram:



5-Tuple: $M = (Q, \Sigma, \delta, q_0, F)$, where,

Set of states is	$Q = \{Q_0, Q_1, Q_2, Q_3, Q_4, Q_5, Q_6\}$
Set of symbols is	$\Sigma = \{a, b, c\}$
Start state is	$q_0 = Q_0$
Set of accept states is	$F = \{Q_4, Q_5, Q_6\}$
Transition function is	

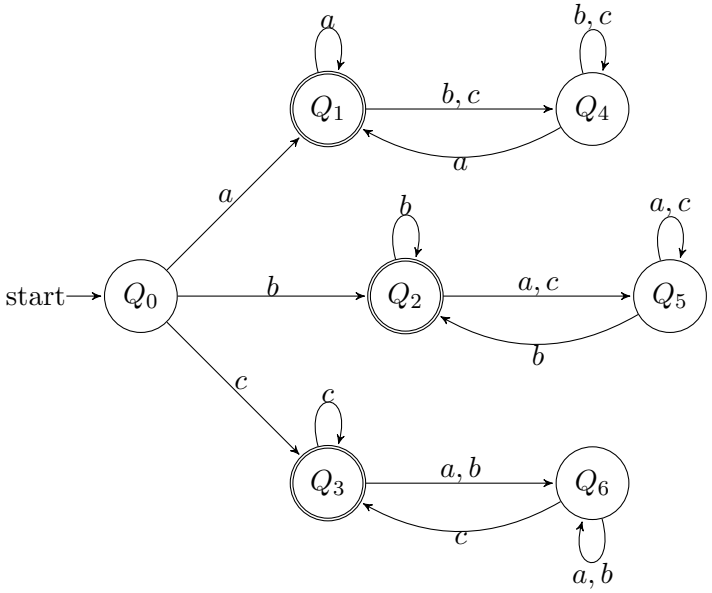
δ:

	a	b	c
Q_0	Q_1	Q_2	Q_3
Q_1	Q_1	Q_4	Q_4
Q_2	Q_5	Q_2	Q_5
Q_3	Q_6	Q_6	Q_3
Q_4	Q_1	Q_4	Q_4
Q_5	Q_5	Q_2	Q_5
Q_6	Q_6	Q_6	Q_3

Regular Expression: $\{a\Sigma^*\{b, c\}\} \cup \{b\Sigma^*\{a, c\}\} \cup \{c\Sigma^*\{a, b\}\}$

(*m*) [**5 Points**] $L = \{w|\ w$ **starts and ends with the same symbol**$\}$ for $\Sigma = \{a, b, c\}$

DFA Diagram:



5-Tuple: $M = (Q, \Sigma, \delta, q_0, F)$, where,

Set of states is	$Q = \{Q_0, Q_1, Q_2, Q_3, Q_4, Q_5, Q_6\}$
Set of symbols is	$\Sigma = \{a, b, c\}$
Start state is	$q_0 = Q_0$
Set of accept states is	$F = \{Q_1, Q_2, Q_3\}$
Transition function is	

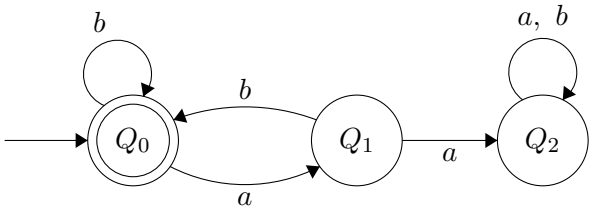
δ:

	a	b	c
Q_0	Q_1	Q_2	Q_3
Q_1	Q_1	Q_4	Q_4
Q_2	Q_5	Q_2	Q_5
Q_3	Q_6	Q_6	Q_3
Q_4	Q_1	Q_4	Q_4
Q_5	Q_5	Q_2	Q_5
Q_6	Q_6	Q_6	Q_3

Regular Expression: $\mathrm{a}(\Sigma^* a)^* \cup \mathrm{b}(\Sigma^* b)^* \cup \mathrm{c}(\Sigma^* c)^*$

(*n*) **[5 Points]** $L = \{w|$ **every** a **in** w **is followed by a** $b\}$

DFA Diagram:



5-Tuple: $M = (Q, \Sigma, \delta, q_0, F)$, where,

Set of states is	$Q = \{Q_0, Q_1, Q_2\}$
Set of symbols is	$\Sigma = \{a, b\}$
Start state is	$q_0 = Q_0$
Set of accept states is	$F = \{Q_0\}$
Transition function is	

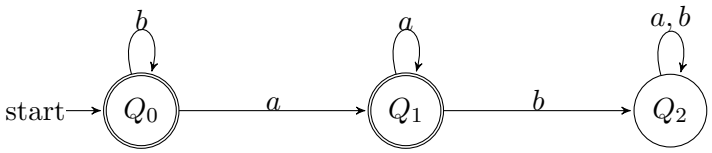
δ:

	a	b
Q_0	Q_1	Q_0
Q_1	Q_2	Q_0
Q_2	Q_2	Q_2

Regular Expression: $(b^*ab^+)^*$

(o) [**5 Points**] $L = \{w|$ **no** a **in** w **is followed by a** $b\}$

DFA Diagram:



5-Tuple: $M = (Q, \Sigma, \delta, q_0, F)$, where,

Set of states is	$Q = \{Q_0, Q_1, Q_2\}$
Set of symbols is	$\Sigma = \{a, b\}$
Start state is	$q_0 = Q_0$
Set of accept states is	$F = \{Q_0, Q_1\}$
Transition function is	

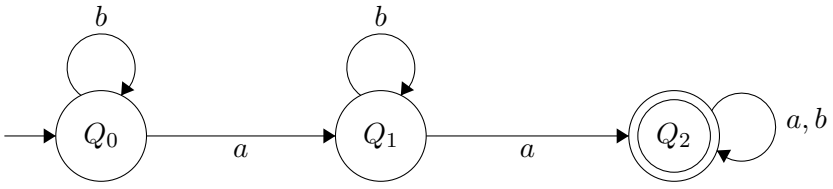
δ:

	a	b
Q_0	Q_1	Q_0
Q_1	Q_1	Q_2
Q_2	Q_2	Q_2

Regular Expression: b^*a^*

(p) [**5 Points**] $L = \{w|\ w$ **contains** $(a(a \cup b)^*a)^+\}$

DFA Diagram:



5-Tuple: $M = (Q, \Sigma, \delta, q_0, F)$, where,

Set of states is	$Q = \{Q_0, Q_1, Q_2, Q_3\}$
Set of symbols is	$\Sigma = \{a, b\}$
Start state is	$q_0 = Q_0$
Set of accept states is	$F = \{Q_2\}$
Transition function is	

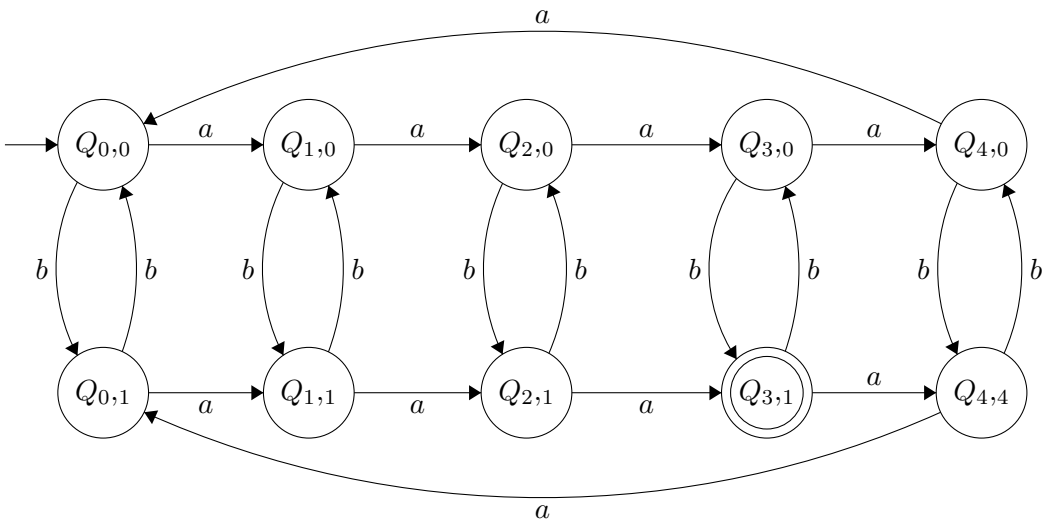
δ:

	a	b
Q_0	Q_1	Q_0
Q_1	Q_2	Q_1
Q_2	Q_2	Q_2

Regular Expression: $b^*ab^*a\Sigma^*$

(*q*) [**5 Points**] $L = \{w |\ n_a(w) \bmod 5 = 3$ **and** $n_b(w) \bmod 2 = 1\}$

DFA Diagram:



5-Tuple: $M = (Q, \Sigma, \delta, q_0, F)$, where,

Set of states is	$Q = \{Q_{0,0}, Q_{1,0}, Q_{2,0}, Q_{3,0}, Q_{4,0}, Q_{0,1}, Q_{1,1}, Q_{2,1}, Q_{3,1}, Q_{4,1}\}$
Set of symbols is	$\Sigma = \{a, b\}$
Start state is	$q_0 = Q_{0,0}$
Set of accept states is	$F = \{Q_{3,1}\}$
Transition function is	

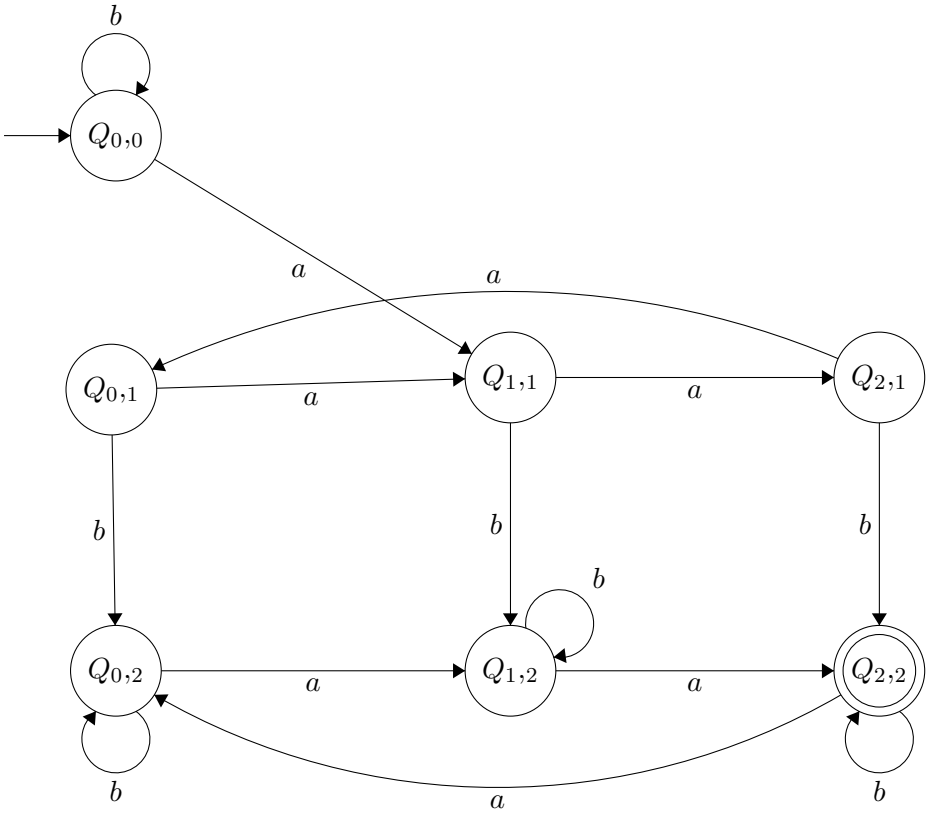
δ:

	a	b
$Q_{0,0}$	$Q_{1,0}$	$Q_{0,1}$
$Q_{1,0}$	$Q_{2,0}$	$Q_{1,1}$
$Q_{2,0}$	$Q_{3,0}$	$Q_{2,1}$
$Q_{3,0}$	$Q_{4,0}$	$Q_{3,1}$
$Q_{4,0}$	$Q_{0,0}$	$Q_{4,1}$
$Q_{0,1}$	$Q_{1,1}$	$Q_{0,0}$
$Q_{1,1}$	$Q_{2,1}$	$Q_{1,0}$
$Q_{2,1}$	$Q_{3,1}$	$Q_{2,0}$
$Q_{3,1}$	$Q_{4,1}$	$Q_{3,0}$
$Q_{4,1}$	$Q_{0,1}$	$Q_{4,0}$

Regular Expression:

(*r*) [**5 Points**] $L = \{w |\ n_a(w) \bmod 3 = 2$ **and** w **contains** $ab\}$

DFA Diagram:



5-Tuple: $M = (Q, \Sigma, \delta, q_0, F)$, where,

Set of states is $Q = \{Q_{0,0}\,, Q_{0,1}\,, Q_{1,1}\,, Q_{2,1}\,, Q_{0,2}\,, Q_{1,2}\,, Q_{2,2}\,\}$

Set of symbols is $\Sigma = \{a, b\}$

Start state is $q_0 = Q_{0,0}$

Set of accept states is $F = \{Q_{2,2}\,\}$

Transition function is

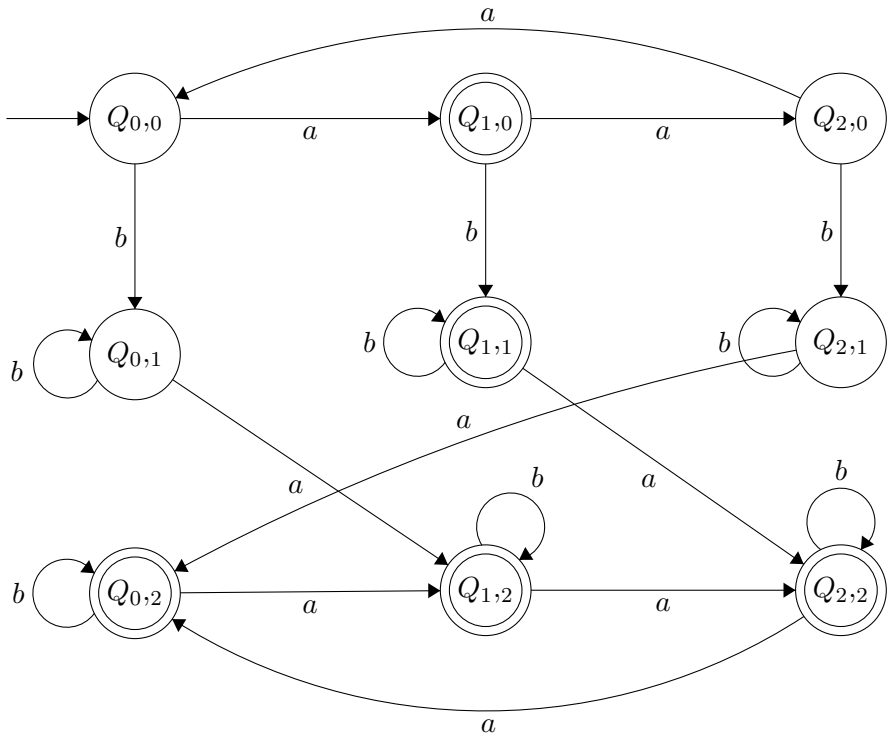
δ:

	a	b
$Q_{0,0}$	$Q_{1,1}$	$Q_{0,0}$
$Q_{0,1}$	$Q_{1,1}$	$Q_{0,2}$
$Q_{1,1}$	$Q_{2,1}$	$Q_{1,2}$
$Q_{2,1}$	$Q_{0,1}$	$Q_{2,2}$
$Q_{0,2}$	$Q_{1,2}$	$Q_{0,2}$
$Q_{1,2}$	$Q_{2,2}$	$Q_{1,2}$
$Q_{2,2}$	$Q_{0,2}$	$Q_{2,2}$

Regular Expression:

(s) [**5 Points**] $L = \{w |\ n_a(w) \bmod 3 = 1$ **or** w **contains** $ba\}$

DFA Diagram:



5-Tuple: $M = (Q, \Sigma, \delta, q_0, F)$, where,

Set of states is $Q = \{Q_{0,0}\,, Q_{1,0}\,, Q_{2,0}\,, Q_{0,1}\,, Q_{1,1}\,, Q_{2,1}\,, Q_{0,2}\,, Q_{1,2}\,, Q_{2,2}\,\}$

Set of symbols is $\Sigma = \{a, b\}$

Start state is $q_0 = Q_{0,0}$

Set of accept states is $F = \{Q_{1,0}\,, Q_{1,1}\,, Q_{0,2}\,, Q_{1,2}\,, Q_{2,2}\,\}$

Transition function is

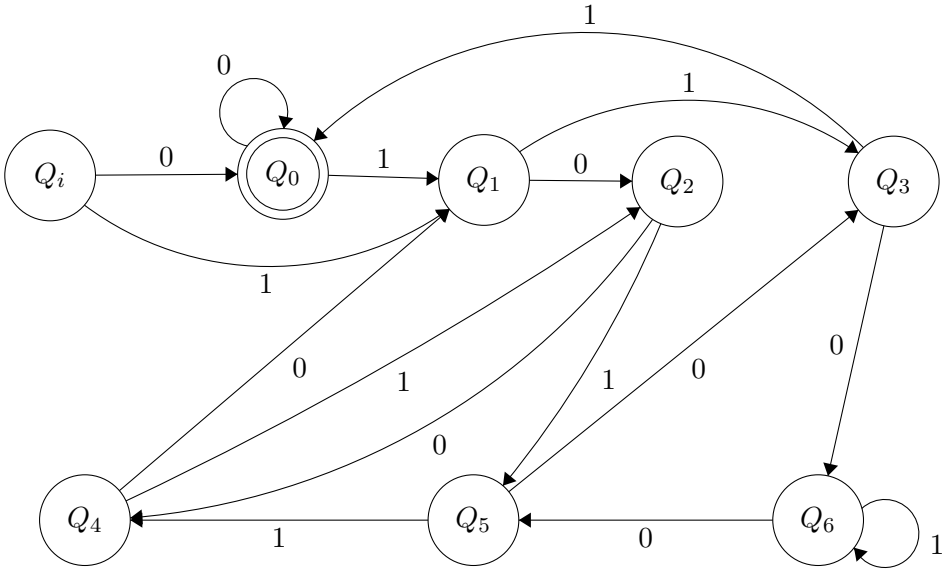
δ:

	a	b
$Q_{0,0}$	$Q_{1,0}$	$Q_{0,1}$
$Q_{1,0}$	$Q_{2,0}$	$Q_{1,1}$
$Q_{2,0}$	$Q_{0,0}$	$Q_{2,1}$
$Q_{0,1}$	$Q_{1,2}$	$Q_{0,1}$
$Q_{1,1}$	$Q_{2,2}$	$Q_{1,1}$
$Q_{2,1}$	$Q_{0,2}$	$Q_{2,1}$
$Q_{0,2}$	$Q_{1,2}$	$Q_{0,2}$
$Q_{1,2}$	$Q_{2,2}$	$Q_{1,2}$
$Q_{2,2}$	$Q_{0,2}$	$Q_{2,2}$

Regular Expression: $(b^*ab^*ab^*ab^*ab^*)^*b^*ab^*(b^*ab^*ab^*ab^*ab^*)^* \cup \Sigma^*ba\Sigma^*$

(t) [**5 Points**] $L = \{w|$ **binary number** w **is divisible by** $7\}$ for $\Sigma = \{0, 1\}$

DFA Diagram:



5-Tuple: $M = (Q, \Sigma, \delta, q_0, F)$, where,

Set of states is $Q = \{Q_i, Q_0, Q_1, Q_2, Q_3, Q_4, Q_5, Q_6\}$

Set of symbols is $\Sigma = \{0, 1\}$

Start state is $q_0 = Q_i$

Set of accept states is $F = \{Q_0\}$

Transition function is

δ:

	0	1
Q_i	Q_0	Q_1
Q_0	Q_0	Q_1
Q_1	Q_2	Q_3
Q_2	Q_4	Q_5
Q_3	Q_6	Q_0
Q_4	Q_1	Q_2
Q_5	Q_3	Q_4
Q_6	Q_5	Q_6