Problem

Let *DOUBLE-SAT* = { $\langle\phi\rangle \mid \phi$ has at least two satisfying assignments}. Show that *DOUBLE-SAT* is NP-complete.

Step-by-step solution

Step 1 of 3

***NP* –complete:** A language *B* is *NP* – complete if it satisfies two conditions.

1.*B* is in *NP* and

2. Every *A* in *NP* is polynomial time reducible to *B*.

Comment

Step 2 of 3

1. $DOUBLE-SAT \in NP$

NTM *N* can decide double – *SAT* as follows:

$N =$" on input a Boolean formula ϕ:

1. Non-deterministically guess two Boolean assignments t_1 and t_2 which are different from each other.

2. If both t_1 and t_2 satisfies ϕ then accept

3. Otherwise, reject.

Thus, we construct a $\text{NTM } N$ to decide Double – *SAT*.

Hence, $DOUBLE-SAT \in NP$.

Comments (1)

Step 3 of 3

2. $SAT \leq_p DOUBLE-SAT$

• The function *f* which maps an instance ϕ of *SAT* to an instance ϕ' of $DOUBLE-SAT$ work as follows:

$\phi' = \phi \wedge (x_1 \vee x_2)$

• Where x_1 and x_2 are new variables. They do not occur in ϕ.

• This reduction is certainly polynomial time.

• If ϕ is unsatifiable, certainly ϕ' is also unsatisfiable. Because we have only conducted an additional term.

• But if ϕ has some satisfying assignments t, then ϕ' has at least three satisfying assignments, corresponding to the 3 different ways of extending *t* to the new variables x_1 and x_2.

Thus, the $DUBLE-SAT$ is *NP* - complete.

Comment