CSE 303 ToC 115060128
HW 10

Problem 3.

$S \to LA$
$A \to BR$
$B \to LA \mid BB \mid LR$
$L \to ($
$R \to)$

Let $w_1 = (())()$

We will write $D(i,l) = X$ if $D(i,l,X)$ is true and ϕ otherwise. for $X \in V$.

i.e $X \in \{S, A, B, L, R\}$ we will check if $D(i,l)$ contains X. If yes then $D(i,l,X) =$ true otherwise False
We make the matrix $D(i,l)$ below.

length l ↓ / i →	1 (	2 (	3)	4)	5 (	6)
1	L					
2	-	L				
3	-	B	R			
4	S, B	A		R		
5	-				L	
6	B				B	R

Now we convert it to $D(i,l,X)$ form

S T = True , F = False.

D(i, j, S)

	(	(	)	)	(	)
1	F					
2	F	F				
3	F	F	F			
4	T	F	F	F		
5	F	F	F	F	F	
6	F	F	F	F	F	F

D(i, j, A)

	(	(	)	)	(	)
1	F					
2	F	F				
3	F	F	F			
4	F	T	F	F		
5	F	F	F	F	F	
6	F	F	F	F	F	F

D(i, j, B)

	(	(	)	)	(	)
1	F					
2	F	F				
3	F	T	F			
4	T	F	F	F		
5	F	F	F	F	F	
6	T	F	F	F	T	F

D(i, d, L)

	(	(	)	)	(	)
1	T					
2	F	T				
3	F	F	F			
4	F	F	F	F		
5	F	F	F	F	T	
6	F	F	F	F	F	F

D(i, d, R)

	(	(	)	)	(	)
1	F					
2	F	F				
3	F	F	T			
4	F	F	F	T		
5	F	F	F	F	F	
6	F	F	F	F	F	T

Since D(1, 6, S) = F ; w_1 is not in L.

ii) $w_2 = ())(()$.

We again will construct D(i, d) and then convert it to D(i, d, X)

length
↓

i →	(	)	)	(	(	)
1	L					
2	B	R				
3	A	-	R			
4	-	-	-	L		
5	-	-	-	-	L	
6	-	-	-	-	B	R

D(i, J, S)

	(	)	)	(	(	)
1	F					
2	F	F				
3	F	F	F			
4	F	F	F	F		
5	F	F	F	F	F	
6	F	F	F	F	F	F

D(i, J, A)

	(	)	)	(	(	)
1	F					
2	F	F				
3	T	F	F			
4	F	F	F	F		
5	F	F	F	F	F	
6	F	F	F	F	F	F

D(i, 1, B)

	(	(	)	(	(	)
1	F					
2	T	F				
3	F	F	F			
4	F	F	F	F		
5	F	F	F	F	F	
6	F	F	F	F	T	F

D(i, J, L)

	(	(	)	(	(	)
1	T					
2	F	F				
3	F	F	F			
4	F	F	F	F		
5	F	F	F	F	F	
6	F	F	F	F	F	F

D(i, J, R)

	(	)	)	(	(	)
1	F					
2	F	T				
3	F	F	T			
4	F	F	F	F		
5	F	F	F	F	F	
6	F	F	F	F	F	T

since $D(1,6,5)$ is false w_2 is not in L.

Problem 2:-

Let us assume that $L = \{0^n 1^m \mid m \leq n^2\}$ is context free.

Thus we can apply the Pumping lemma.
Let p be the pumping length.

Take $S = 0^p 1^{p^2}$ Note $n = p$, $m = p^2$ & $p^2 \leq p^2$.

So by PL, we get $S = uvxyz$ s.t.

$|vy| > 0$, $|vxy| \leq p$ & $uv^i xy^i z \in L$ $i \geq 0$.

We will deal with vxy depending on cases.

Case i) either v or y contain both 0 & 1.

Say v contains both 0 & 1.
Then $S_2 = uvvxyyz$ will be of the form
$00\ldots11\ldots00\ldots11\ldots11$
Because vv will be $00\ldots11\ldots00\ldots11$, it will have 0 followed by 1, which is not allowed in L

Thus $S_2 \notin L \Rightarrow$ Contradiction

Case ii) vxy contains only 0.
Say $|vxy| = d$. Note $d \neq 0$
In this case $S_0 = uv^0xy^0z = uxz$ will have the form $0^{p-d}1^{p^2}$
Here $n = p-d$, $m = p^2$ & $m > n^2$
$\Rightarrow S_0$ cannot be in L. $\Rightarrow$ contradiction

Case iii) vxy contains only 1
Say $|vxy| = d$ note $d \neq 0$.
In this case, $S_2 = uv^2xy^2z = 0^{p}1^{p^2+d}$.
Here $n = p$, $m = p^2+d$ & $m > n^2$.
$\Rightarrow S_2$ cannot be in L $\Rightarrow$ Contradiction

Case iv) v contains 0, y contains 1.
$v = 0^a$, $y = 1^b$
Here if $a \neq b$, then $S_0 = uxz$ will be of the form $0^{p-a}1^{p^2-b}$
Since $a \neq b$, $p^2 - b > p - a \Rightarrow S_0 \notin L$.
which leads to contradiction

if $a < b$, then $S_2 = uv^2xy^2z$ will be of the form $0^{p+a}1^{p^2+b}$
Since $a < b$, $p^2 + b > (p+a)^2 \Rightarrow S_2 \notin L$.

which leads to contradiction.

Thus in each case we get a contradiction.

$\Rightarrow L = \{0^n 1^m \mid m \leq n^2\}$ is not CF.

Problem 1:-

We will construct a PDA for the same.

Assume $\beta/\alpha = k$.

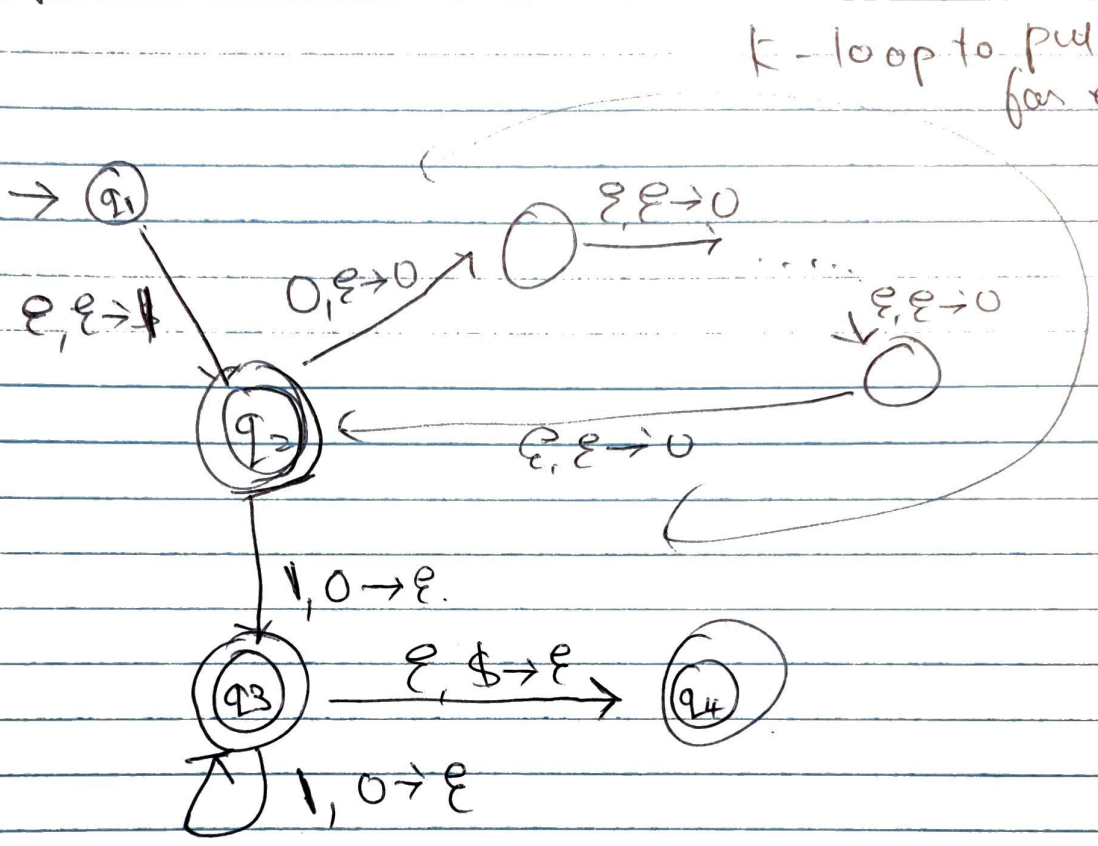
Idea:- for every 0 we read, we input k 0's in the stack. Thus at the end of 0's we will ~~get~~ have $k \cdot n$ 0's in stack. Now we will start popping 1's as we see them.

If we reach the end of the input & stack is not empty, it means $m < kn \Rightarrow$ accept.

if at the end of input stack is empty $\Rightarrow m = kn$ $\Rightarrow$ accept.

However, if before end of input if stack is empty $\Rightarrow$ $m > kn \rightarrow$ not accept.

PDA


k-loop to put k 0's for every 0 input
q1
q2
q3
q4
ε, ε → #
0, ε → 0
ε, ε → 0
ε, ε → 0
ε, ε → 0
1, 0 → ε
ε, $ → ε
1, 0 → ε