## Problem

Show that for any c, some strings x and y exist, where
$$K(xy) > K(x) + K(y) + c.$$

## Step-by-step solution

### Step 1 of 1

**Given:**

Consider that *x* and *y* are strings whereas *c* is constant.

**Proof:**

Consider a Turing machine *TM* having input *w*. Assume that *x* is a binary string in *w*. The minimal description $d(x)$ is the shortest string in *w*. The descriptive complexity of *x* is

$$K(x) = |d(x)|$$

Consider a string *xy* that has been broken in 2 parts *x* and *y* and *c* can be considered as a default value that is used to match left hand side with right hand side.

Now use a Turing machine *T* that breaks its input *w* in description of 2 separate parts $d(x)$ and $d(y)$. The bits of first part and bits of second part are combined together or concatenate.

Once both of the part are obtained than both parts can be run to obtain the string *x* and *y* and can perform concatenate operation to achieve desired output of *xy*.

The length of description of part *xy* is clearly greater than individual parts description of *x* and *y* as some extra part *c* can be seen in the statement. The constant *c* can be termed as a constant for balancing between *xy* and individual parts.

**Conclusion:**

Hence, the Description complexity of *xy* is greater than sum of description of *x, y* and constant *c*.

$$K(xy) = K(x) + K(y) + c$$