Show that the class of context-free languages is closed under the regular operations, union, concatenation, and star.

Step-by-step solution

Step 1 of 3

In order to show that context free language is closed under union operation considers two starts variable S_1 and S_2 for the two different languages L_1 and L_2.

Grammar for union operation is as shown:

$S \rightarrow S_1 \mid S_2$

If both the language belongs to the context free language then union of both the language should belong to context free language.

By the above definition if user generates S_1 and S_2 string or both then in that case union of both the language is generated.

Hence, $L_1 \cup L_2 \in CFL$.

This implies context free language is closed under union operation.

Comments (1)

Step 2 of 3

In order to show that context free language is closed under concatenation operation considers two starts variable S_1 and S_2 for the two different languages L_1 and L_2.

Grammar for union operation is as shown:

$S \rightarrow S_1 S_2$

If both the language belongs to the context free language then concatenation of both the language should belong to context free language.

$\forall L_1, L_2 \in CFL$

$\{w_1 w_2 : w_1 \in L_1 \wedge w_2 \in L_2\} \in CFL$

By the above definition if user generates S_1 string for language L_1 followed by S_2 string of language L_2. Then it concatenation of both the language is generated.

Hence, $\{w_1 w_2 : w_1 \in L_1 \wedge w_2 \in L_2\} \in CFL$

This implies context free language is closed under concatenation operation.

Comments (1)

Step 3 of 3

In order to show that context free language is closed under star operation.

Consider one start variable S_1 for the languages L_1

Grammar for union operation is as shown:

$S \rightarrow S_1 S \mid \in$

If the language belongs to the context free language then star of the language should belong to context free language.

$\forall L_1 \in CFL$

By the above definition, if user generates zero or many string which is the definition of the star.

This implies that context free language is closed under star operation.

Comments (1)