Problem

Let $\Sigma = \{0,1\}$. Let $WW_k = \{ww|\ w \in \Sigma^*$ and w is of length k}.

a. Show that for each k, no DFA can recognize WW_k with fewer than 2^k states.

b. Describe a much smaller NFA for $\overline{WW_k}$, the complement of WW_k.

Step-by-step solution

Step 1 of 2

Consider the following language:

$WW_k = \{ww \mid w \in \Sigma^* \text{ and } w \text{ is of length } k\}$.

Therefore a string in WW_k language is at least of length k. Now consider two different k- bit strings $x = x_1 \ldots x_k$ and $y = y_1 \ldots y_k$, i be some location such that $x_i \neq y_i$.

Hence one of the strings contains a 1 at the i^{th} position where other contains a 0. Assume that $z = 0^{i-1}$.

Then z distinguishes x and y as exactly one of xz and yz has the k^{th} bit from the end as 1.

Since there exists, total 2^k binary strings of length k which can be mutually distinguish by the argument mentioned above, so any DFA for the given language has at least 2^k states.

Comment

Step 2 of 2

The language $\overline{WW_k}$ can be described as follows:

$\overline{WW_k} = \{ww \mid w \in \Sigma^* \text{ and } |w| < k\}$.

Therefore the user can build non deterministic finite automata with exactly $k+1$ states. This NFA can recognize the language $\overline{WW_k}$.

Assume that the non-deterministic finite automata consist of $Q = \{0,1,\ldots,k\}$ with the names of the state's corresponding to how many of the last k bits the NFA has seen.

Define $\delta(0,0) = 0, \delta(0,1) = 1 \text{ and } \delta(i-1,0|1) = i \text{ for } 2 \leq \text{i} \leq \text{k}$. We set $q_0 = 0$ and $F = \{k\}$

The machine starts from the initial state which is state 0, as it will traverse 1 initial state which is state 0, as it will traverse 1 it understand that it is the k^{th} bit from the finishing state or end state and proceed to state 1.

As it has reached on the state k, it accepts the string if and only if the string contains exactly $k-1$ bits. It checks for $k-1$ bits as it starts traversing from 0^{th} index.

Comments (5)