Problem

Show that the collection of decidable languages is closed under the operation of

A**a.** union.

b. concatenation.

c. star.

d. complementation.

e. intersection.

Step-by-step solution

Step 1 of 6

A language is said to be decidable if and only if some Turing machine decides it. The Turing machine is a decider if all braches halts on all inputs.

Comments (2)

Step 2 of 6

a.

Let L_1 and L_2 be two decidable Languages. M_1 and M_2 be the Turing machines that decides L_1 and L_2 respectively.

There exists a Turing machine M' such that, decides $L_1 \cup L_2$ i.e. $L(M') = L_1 \cup L_2$.

The description of M' is as follows:

$M' =$ on input w:

1. Run M_1 on *w*. if M_1 accepts, then ***accept***.

2. Else Run M_2 on *w*. If M_2 accepts, then ***accept***

3. Else ***reject***

M' Accepts *w* if either M_1 or M_2 accepts it. If both rejects, M' rejects.

Therefore, $L(M') = L_1 \cup L_2$. The decidable languages are closed under union.

Comment

Step 3 of 6

b.

Let L_1 and L_2 be two decidable Languages. M_1 and M_2 be the Turing machines that decides L_1 and L_2 respectively.

There is a Turing machine M' such that, it decides concatenation of L_1 and L_2 i.e., $L(M') = L_1 \circ L_2$.

The description of M' is as follows:

$M' =$ on input *w*:

1. Split *w* into two parts w_1, w_2 such that $w = w_1 w_2$

2. Run M_1 on w_1. If M_1 rejected then ***reject***.

3. Else run M_2 on w_2. If M_2 rejected then ***reject***.

4. Else ***accept***

Try each possible cut of *w*. If first part is accepted by M_1 and the second part is accepted by M_2 then *w* is accepted by M'. Else, *w* does not belong to the concatenation of languages and is rejected.

Therefore, $L(M') = L_1 o L_2$. The decidable languages are closed under concatenation.

Comment

Step **4** of 6

c.

Let *L* be a Turing decidable Language and *M* be the Turing machine that decides *L*.

There is a Turing machine M' such that, it decides star of *L* i.e., *L*(M')=L^*.

The description of M' is as follows:

$M' =$ On input *w*:

1. Split *w* into *n* parts such that

$w = w_1 w_2 ... w_n$ in different ways.

2. Run *M* on w_i for $i = 1, 2, ...n$.

If *M* accepts each of these strings w_i, ***accept***.

3. All cuts have been tried without success then ***reject***.

When *w* is cut into different substrings such that every string is accepted by *M*, then *w* belongs to the star of *L* and thus M' accepts *w* after finite number of steps, else *w* will be rejected. Since, there are finitely many possible cuts of *w*, M' will halt after finitely many steps.

Therefore, $L(M') = L^*$. The decidable languages are closed under star.

Comment

Step **5** of 6

d.

For a Turing decidable language L, Turing machine decides language *M* then the complement is M' on input *w.*

The description of M' is as follows:

$M' =$on input *w*:

1. Accepts if *M* rejects

2. Else ***accept.***

Since M' does the opposite of what ever M does, it decides the complement of L.

Therefore, decidable languages are closed under complementation.

Comments (3)

Step **6** of 6

e.

Let L_1 and L_2 be two Turing decidable Languages. M_1 and M_2 be the Turing machines that decides L_1 and L_2 respectively.

There is a Turing machine M' such that, it decides intersection of L_1 and L_2 i.e., $L(M') = L_1 \cap L_2$.

The description of M' is as follows:

$M' =$on input *w*:

1. Run M_1 on *w.* if M_1 rejects then ***reject***.

2. Else run M_2 on *w.* if M_2 rejects then ***reject***.

3. Else ***accept***.

M' Accepts *w* if both M_1 and M_2 accept it. If either of them rejects then M' rejects *w.*

Therefore, $L(M') = L_1 \cap L_2$. The decidable languages are closed under intersection.

Comments (2)