

## Problem

Let  $G$  be a CFG in Chomsky normal form that contains  $b$  variables. Show that if  $G$  generates some string with a derivation having at least  $2^b$  steps,  $L(G)$  is infinite.

## Step-by-step solution

### Step 1 of 3

#### CFG:

- A finite set of grammar rules is known as **CFG (context free grammar)**. It consisting of that is quadruple  $(N, T, P, S)$ .
- Where, set of non-terminal symbol represented by  $N$ .
- $T$  is group of terminal  $N \cap T = \text{NULL}$ .
- $P$  is group of rule,  $P: N \rightarrow (N \cup T)^*$ .
- $S$  is start symbol.

[Comment](#)

### Step 2 of 3

#### CNF:

- In **CNF (Chomsky normal form)** we have a restriction on the length of RHS, which is a CFG (context free grammar) is in Chomsky normal form if the productions are in the following terms:

$$U \rightarrow u$$

$$U \rightarrow VW$$

- Where  $U, V$  and  $W$  are non-terminals and  $u$  is a terminal.

[Comment](#)

### Step 3 of 3

- At most two terminal can generate in every deviation.
- In any parse string with use of  $G$ .
- An internal node can have at most two children.
- Parse tree with height  $k$  has at most  $2^k - 1$  internal node.
- If several string generated by  $G$  with a derivation taking at least  $2^b$  steps.
- At least  $2^b$  inner node takes parse tree of that string.
- At least consuming  $b+1$  height in Parse tree.
- It exists a path from root to leaf containing  $b+1$  variable.
- In this one variable occurring twice.

Hence, user can use the technique in proof of pumping lemma to construct infinity many string which are all in  $L(G)$ .

[Comments \(1\)](#)

