**Problem**

Give context-free grammars generating the following languages.

[A] **a.** The set of strings over the alphabet $\{\mathtt{a},\mathtt{b}\}$ with more $\mathtt{a}$'s than $\mathtt{b}$'s

**b.** The complement of the language $\{\mathtt{a}^n\mathtt{b}^n \mid n \geq 0\}$

[A] **c.** $\{w\#x \mid w^{\mathcal{R}}$ is a substring of $x$ for $w, x \in \{0,1\}^*\}$

**d.** $\{x_1\#x_2\#\cdots\#x_k \mid k \geq 1$, each $x_i \in \{\mathtt{a},\mathtt{b}\}^*$, and for some $i$ and $j$, $x_i = x_j^{\mathcal{R}}\}$

**Step-by-step solution**

**Step 1** of 2

Given language is

"The set of strings over the alphabet $\{a,b\}$ with more $a$'s than $b$'s"

The context – free grammar generating the given language is

$S \to Aa \mid BS \mid SBA$
$A \to Aa \mid \in$
$B \to \in \mid BB \mid bBa \mid aBb$

In the above grammar $S$ will generate all strings with as many $a$'s as $b$'s. $R$ Forces an extra $a$ which gives the required strings of the language.

Given language is

"The compliment of the language $\{a^n b^n : n \geq 0\}$"

Let $L$ be the language that is a compliment of given language. $L$ can be obtained as $L = \{a^n b^m : n \neq m\} \cup \{(a \cup b)^* ba (a \cup b)^*\}$

Let us conside

$L_1 = \{a^n b^m : n \neq m\}$

$L_2 = \{(a \cup b)^* ba (a \cup b)^*\}$

The context – free grammar generating the language $L_1$ is

$S_1 \to aS_1b \mid T \mid U$
$T \to aT \mid a$
$U \to Ub \mid a$

The context – free grammar generating the language $L_2$ is

$S_2 \to RbaR$
$R \to RR \mid a \mid b \mid \varepsilon$

Therefore, the required context – free grammar generating the language $L$ is given by

$L = L_1 \cup L_2$
$S \to S_1 \mid S_2$
$S_1 \to aS_1b \mid T \mid U$
$S_2 \to RbaR$
$T \to aT \mid a$
$U \to Ub \mid a$
$R \to RR \mid a \mid b \mid \varepsilon$

Given language is

$\{w\#x : w^R$ is a substring of $x$ for $w, x \in \{0,1\}^*\}$

The context – free grammar generating the given language is

$R \to SX$
$S \to 0S0 \mid 1S1 \mid \# X$
$X \to XX \mid 1 \mid 0 \mid \varepsilon$

The nonterminal $S$ ends only with $\# X$ , $S$ must generate a string whose beginning and end are mirror images. Since $X$ generates $(0 \cup 1)^*$, the symbol $S$ generates all strings of the form $w\#(0 \cup 1)^* w^R$. The above grammar generates all the substrings of $x$ for $w, x \in \{0,1\}^*$.

Comments (4)

**Step 2** of 2

Given language is

$\{x_1 \# x_2 \# \cdots \# x_k \mid k \geq 1, \text{ each } x_i \in \{a,b\}^*, \text{ and for some } i \text{ and } j, x_i = x_j{}^R\}$

The context – free grammar generating the given language is

$R \to S \mid J\#S\#J \mid J\#S \mid S\#J$
$S \to aSa \mid bSb \mid \# \mid \# J \#$
$J \to aJ \mid bJ \mid \# J \mid \varepsilon$

The strings in the language contain matching pair of strings with at least one # between them. Before, after and between the matching pairs there can be any number of strings of $a$'s and $b$'s separated by #. Because the strings can be of any length, the strech of the strings of $a$'s, $b$'s and #'s can be of any length. The symbol $S$ generates a matching pair, with strings of $a$'s, $b$'s and #'s optionally inserted in the middle. The symbol $J$ generates strings of $a$'s, $b$'s and #'s. The string generated by $J$ may start or end with $a$ or $b$ , so rules for $M$ and $S$ must ensure that the symbol $J$ is always separated properly from the two matching strings.

Comments (1)