Midterm Exam #1

INSTRUCTIONS:

- Put your NAME and SBU ID # on this exam booklet in the space provided.
- This is a CLOSED-BOOK exam, which TERMINATES AT 2:35PM (80 minutes). NO ELECTRONIC DEVICES, including calculators, may be used during the exam.
- Please place ALL ANSWERS IN THIS BOOKLET, on the sheet where the corresponding question is printed.
- THINK BEFORE YOU WRITE. A partial solution can get you partial credit, but too much extraneous information can prevent me from finding your correct solution.
- SOME QUESTIONS ARE HARDER THAN OTHERS, and you might not have time to answer all questions completely. LOOK OVER ALL THE QUESTIONS BEFORE STARTING, and work first on those that will get you the most credit fastest. Use the number of points listed for each question as a guide.

Question:	1	2	3	4	5	6	7	8	9	Total
Points:	10	9	5	5	5	5	5	15	10	69
Score:	10	7	3	5	5	5	5	8	[illegible]	[illegible]

9.57

Note: Point values have been assigned so that you should expect to be answering roughly one point per minute.

NAME and SBU ID#: Deven Diwakar 115060128

1. Give complete (with justifications) Fitch-style proofs for the following, using *only* the rules listed on the last pages of this exam.

   (a) (5 points)

   1. $\neg F \rightarrow G$
   2. $F \rightarrow H$

   3. | F
   4. | H $\rightarrow$E 2, 3
   5. | G∨H ∨I 4.
   6. | ¬F
   7. | G $\rightarrow$E 1, 6.
   8. | G∨H ∨I 7
   9. G∨H LEM 3-5, 6-8.

   $G \vee H$

   (b) (5 points)

   1. $\exists x\ Dx$
   2. $\forall x\ (x = p \leftrightarrow Dx)$

   3. | Dc
   ~~4. c=p ↔E~~
   4 | c=p ⇔ Dc. ∀E 2
   5. | c=p ⇔E 4.
   6. | Dp =E 3, 5
   7. Dp ∃E 1, 3-6.

   Dp

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2. Let R be the binary relation on natural numbers defined as follows:

$$R = \{(m, n) \in \mathcal{N} \times \mathcal{N} \mid m < n\}.$$

(a) (2 points) Is R reflexive? Why or why not?

No. Let $a \in N$. Then we check if $(a,a) \in R$. Since $a<a$ is not True $(a,a) \notin R$. So R is not reflexive.

✓ 2

(b) (2 points) Is R symmetric? Why or why not?

No. Let $(m,n) \in R$. $m,n \in N$. Then $m<n$. Since $n<m$ is not true. [$\because m<n$] $(n,m) \notin R$. So R is not symmetric.

✓ 2

(c) (2 points) Is R transitive? Why or why not?

Yes. Let (a,b) $(b,c) \in R$ $a,b,c \in N$. Then $a<b$ & $b<c$. Thus $a<b<c \Rightarrow a<c \Rightarrow (a,c) \in R$. Thus R is transitive.

✓ 2

(d) (3 points) Give a simple description of the reflexive, transitive closure of R.

R^* reflexive transitive closure of $R = \{(a,b) \mid \exists$ a sequence $a_0, a_1, \ldots, a_n$ $a_i \in N$, $i = 0, 1 \ldots n$, s.t $a_0 = a$, $a_n = b$ & $(a_i, a_{i+1}) \in R$ $i = 0, 1, \ldots n\}$. Reflexive transitive closure of R is R^* which here is R itself. ?

1

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3. (5 points) Does the following statement hold for sets? Explain why or why not.

$$\forall A.\ \exists B.\ \forall X.\ X \in B \leftrightarrow X \notin A$$

Yes it holds. set B is essentially the complement of set A.
Here A, B are sets. X is element. It says that
$X \in B$ if and only if it doesnt belong to A. Thus
B set is the complement of A. This assumes we have
a universal set so that we can take complement.
[i.e a set of all elements.]

Careful!

does not exist!

3

4. (5 points) Write a regular expression R over $\{a, b\}$, such that a string w is in $L(R)$ if and only if w has no more than three a's.

The regular exp R is given by

$R = b^* \cup \{b^*ab^*\} \cup \{b^*ab^*ab^*\}$
$\cup \{b^*ab^*ab^*ab^*\}$

No set brackets in reg exps

5

5. (5 points) For a nondeterministic finite automaton N, under exactly what conditions is the empty string in $L(N)$?

Let q_0 be the initial state of N. We have $E(\{q_0\})$ is set of all states reachable from q_0 through 0 or more ε transitions. Empty string is in $L(N)$ iff at least one state of $E(\{q_0\})$ is in F.

5

6. (5 points) Prove that every infinite language has a subset that is a regular language.

Any infinite language if it has a single element then that single element is a regular language. Any finite subset of infinite language is a regular language.
~~Else Let A = {aaa...}~~

5

7. (5 points) Does the following equivalence hold for regular expressions? Give a proof or counterexample to justify your answer.

$$(R \cup S) \circ (T \cup V) = (R \circ T) \cup (S \circ V)$$

This is False. Take $L(R) = \{r\}$, $L(S) = \{s\}$, $L(T) = \phi$, $L(V) = \{v\}$. Then string rv is present in LHS since LHS $= (R \cup S) \circ (\phi \cup V) = (R \cup S) \circ V$. But not in RHS since RHS $= (R \circ \phi) \cup (S \circ V) = \phi \cup (S \circ V) = S \circ V$. Anything in r can't appear in RHS since ϕ eliminates it.

5'

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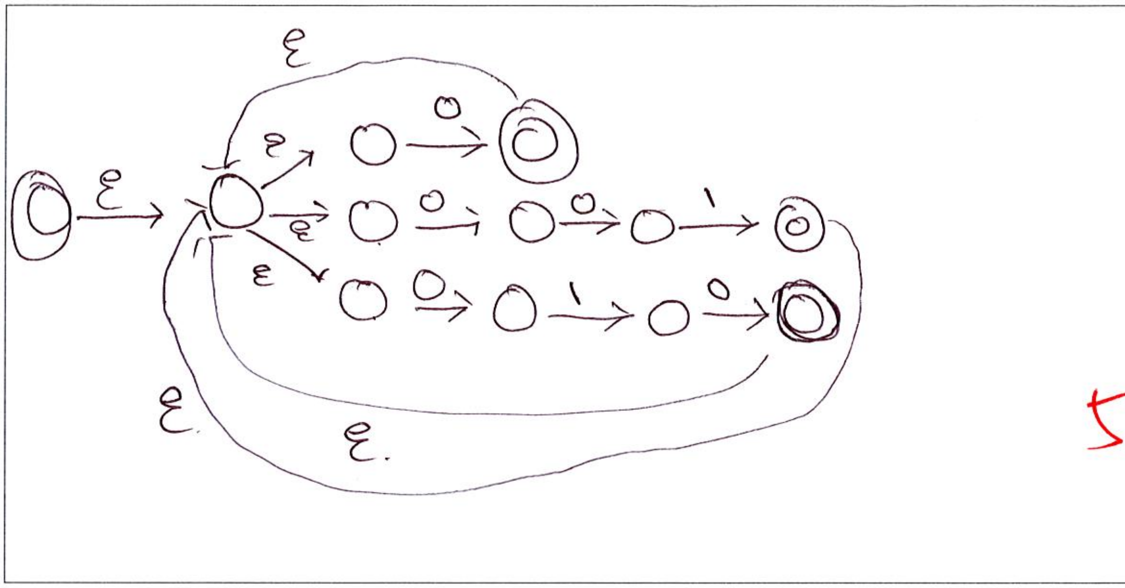
8. (a) (5 points) Give an NFA recognizing the language $(0 \cup 001 \cup 010)^*$ (a state diagram is sufficient). For the second part of the question it will be helpful if you use as few states as possible (it can be done with four states).



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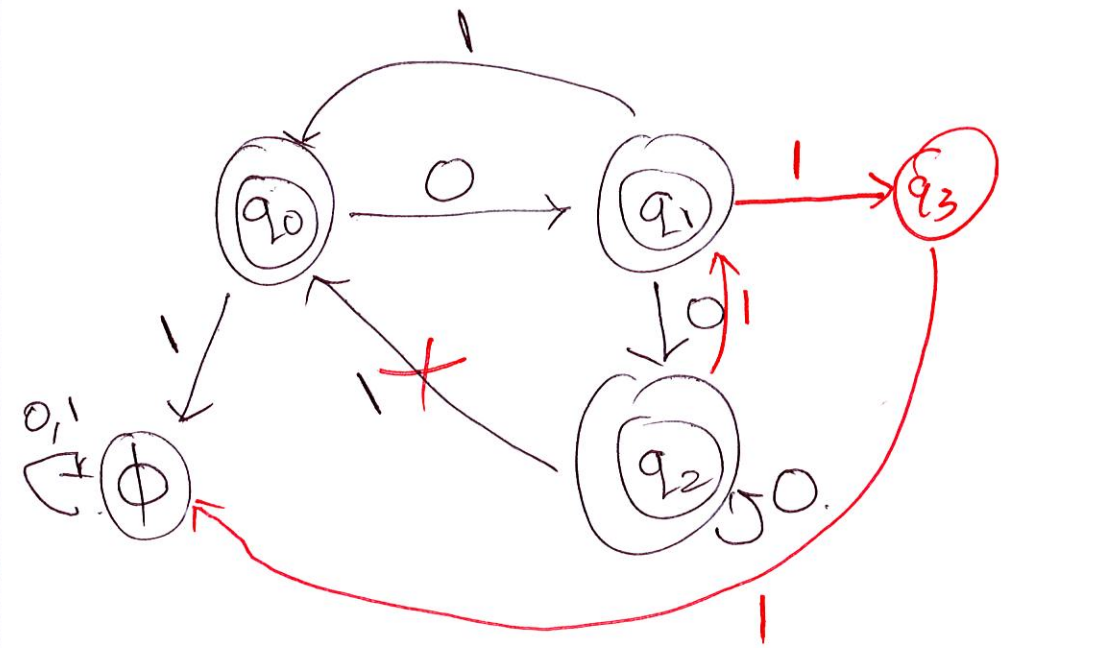
(b) (10 points) Convert this NFA to an equivalent DFA (again, a state diagram is sufficient). Give only the portion of the DFA that is reachable from the start state.

You missed one state.



3'

9. (10 points) Prove that every NFA can be converted to an equivalent one with a single accept state.

(**Note:** "Prove" means not just to describe an idea but also to give an explicit formal construction and to use the definition of acceptance to show that the construction works.)

Let M be a NFA. $M = (Q, \Sigma, \delta, q_0, F)$

Assume M has multiple accept states. Construct another NFA N from N s.t it has an additional state.

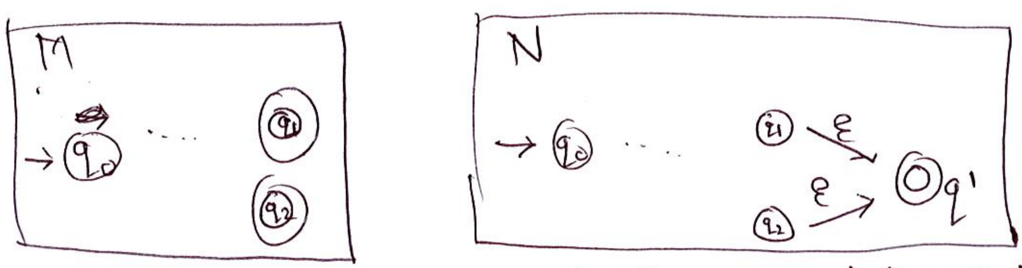
$N = (Q', \Sigma, \delta', q_0, F')$ ~~set~~ Let q' be a state $\in Q'$.

$Q' = Q \cup \{q'\}$. $F' = \{q'\}$.

$\delta'(r, a) = \delta(r, a)$ if r is not accept state of M.

$= q'$ if r is accept state of M & $a = \varepsilon$.

$\delta'(q', a) = q'$ if $\delta(r, a)$ = accept state & r is accept state.



Let ~~Q~~ $w \in \Sigma^*$ w accepted by M. Then at the end of w we go to an accept state of M. Say q_1. ~~Same~~

For N with w we will reach q_1 & from q_1 we can reach q' through ε. Thus N accepts w. $\because$ q' is accept state.

Let $w \in \Sigma^*$ w accepted by N. Since q' is only reached by accept states of M through ε transitions, w reaches q' in N

$\Rightarrow$ w reaches accept states of M. $\Rightarrow$ M accepts w

Thus M accepts $w \Leftrightarrow$ N accepts w.

$\Rightarrow$ M & N are equivalent

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(scratch paper)

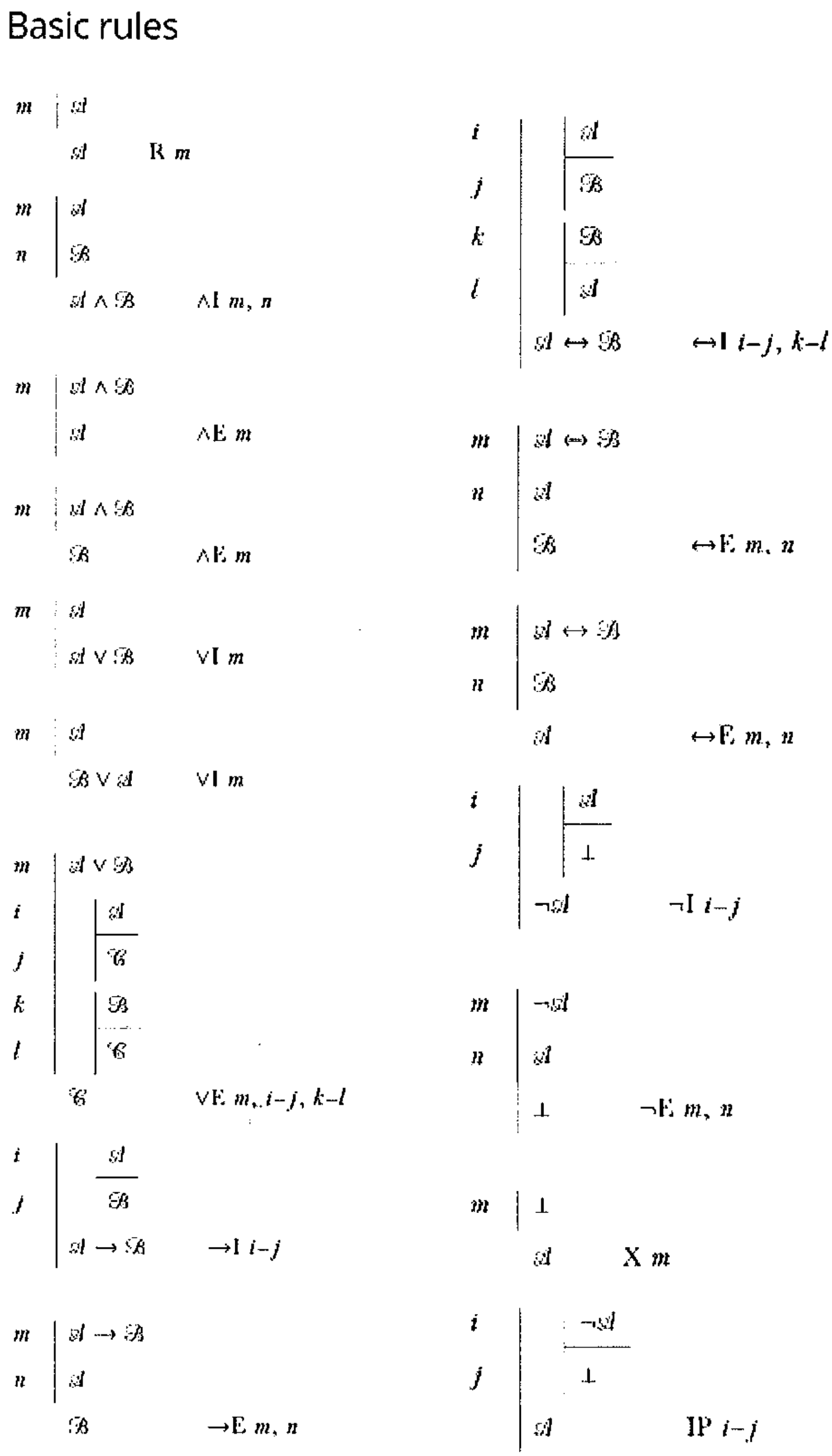

Figure 1: Rules for Fitch-Style Proofs

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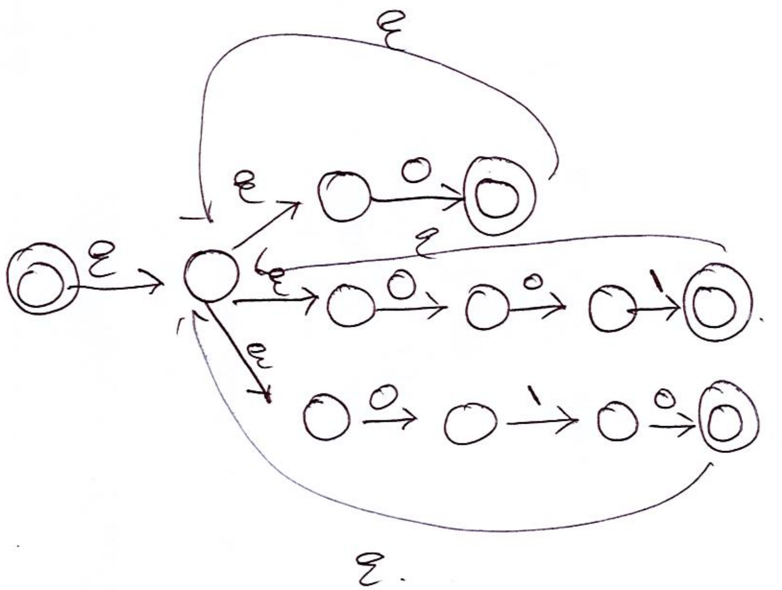
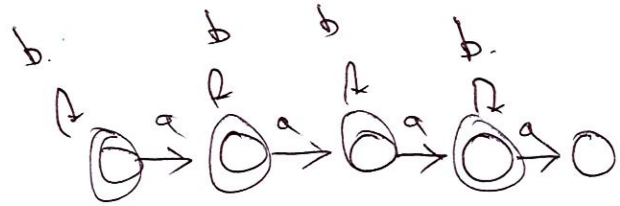
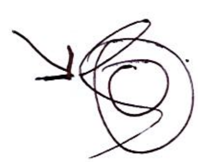
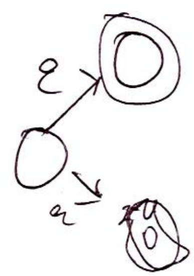




$b^* \cup \{b^*ab^*\} \cup \{b^*ab^*ab^*\} \cup$

$\{b^*ab^*ab^*ab^*\}$





$(r \cup s)\ v.$

$r\ v.$

$R = \{r\}\quad (r \cup s)\ o(t \cup v)$

$S = \{s\}$

$\cancel{T = \{t\}}\ T = \phi\quad rt$

$V = \{v\}$

$s \circ v.\quad sv$

$rt\ sv$

$T = \phi$

$$\begin{array}{l|ll} m & \mathscr{A}(\ldots c \ldots c \ldots) & \\ & \forall x \mathscr{A}(\ldots x \ldots x \ldots) & \forall\mathrm{I}\ m \end{array}$$

$$\begin{array}{l|ll} m & \forall x \mathscr{A}(\ldots x \ldots x \ldots) & \\ & \mathscr{A}(\ldots c \ldots c \ldots) & \forall\mathrm{E}\ m \end{array}$$

$$\begin{array}{l|ll} m & \mathscr{A}(\ldots c \ldots c \ldots) & \\ & \exists x \mathscr{A}(\ldots x \ldots c \ldots) & \exists\mathrm{I}\ m \end{array}$$

$$\begin{array}{l|ll} m & \exists x \mathscr{A}(\ldots x \ldots x \ldots) & \\ i & \quad \mathscr{A}(\ldots c \ldots c \ldots) & \\ j & \quad \mathscr{B} & \\ & \mathscr{B} & \exists\mathrm{E}\ m,\ i\text{–}j \end{array}$$

$$\begin{array}{l|ll} & c = c & =\mathrm{I} \end{array}$$

$$\begin{array}{l|ll} m & a = b & \\ n & \mathscr{A}(\ldots a \ldots a \ldots) & \\ & \mathscr{A}(\ldots b \ldots a \ldots) & =\mathrm{E}\ m,\ n \end{array}$$

$$\begin{array}{l|ll} m & a = b & \\ n & \mathscr{A}(\ldots b \ldots b \ldots) & \\ & \mathscr{A}(\ldots a \ldots b \ldots) & =\mathrm{E}\ m,\ n \end{array}$$

Derived rules

$$\begin{array}{l|ll} m & \mathscr{A} \vee \mathscr{B} & \\ n & \neg\mathscr{A} & \\ & \mathscr{B} & \mathrm{DS}\ m,\ n \end{array}$$

$$\begin{array}{l|ll} m & \mathscr{A} \vee \mathscr{B} & \\ n & \neg\mathscr{B} & \\ & \mathscr{A} & \mathrm{DS}\ m,\ n \end{array}$$

$$\begin{array}{l|ll} m & \mathscr{A} \to \mathscr{B} & \\ n & \neg\mathscr{B} & \\ & \neg\mathscr{A} & \mathrm{MT}\ m,\ n \end{array}$$

$$\begin{array}{l|ll} m & \neg\neg\mathscr{A} & \\ & \mathscr{A} & \mathrm{DNE}\ m \end{array}$$

$$\begin{array}{l|ll} i & \quad \mathscr{A} & \\ j & \quad \mathscr{B} & \\ k & \quad \neg\mathscr{A} & \\ l & \quad \mathscr{B} & \\ & \mathscr{B} & \mathrm{LEM}\ i\text{–}j,\ k\text{–}l \end{array}$$

Figure 2: Rules for Fitch-Style Proofs (cont'd.)

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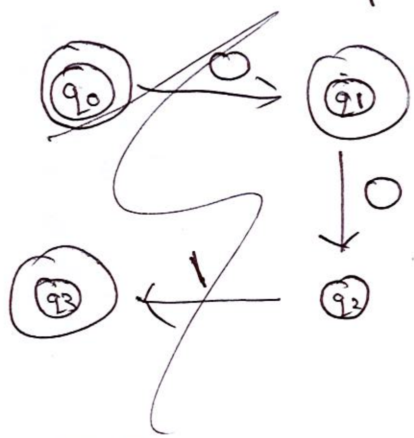
$\partial_0 q_1 = (\phi \cup 1 \cup \phi) q_0 \cup q_1 = 1q_0 \cup q_1 = q_2.$ $q_0 = (0 \cup 001 \cup 010)^*$

$\partial_1 q_1 = 1 q_0 \cup \phi = 1 q_0.$

$\partial_0 q_0 = \overset{R_1}{(\varepsilon \cup 01 \cup 10)}\ \overset{R_2}{q_0} = q_1$

$\partial_1 q_0 = \phi.$



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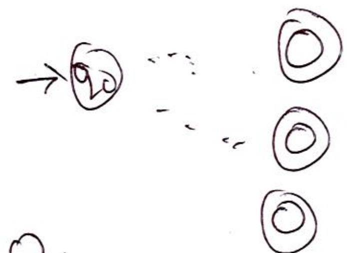
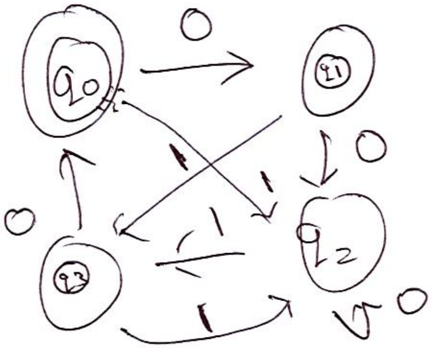


001010.

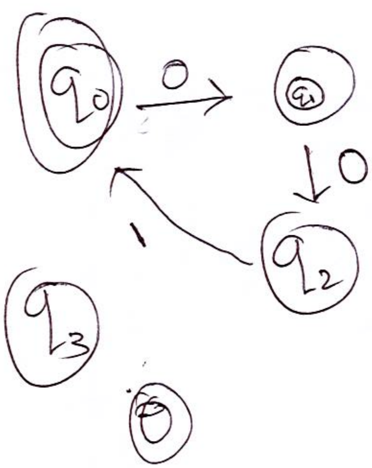
$\partial_a R_1 R_2 = (\partial_a R_1) R_2 \cup \partial_a R_2 \quad \varepsilon \in L(R_1)$

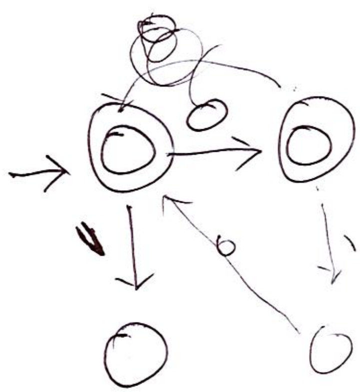
$= (\partial_a R_1) R_2$

1, 2, 3, 4, 5, 6, 7, 8, 9



0, 001, 010.





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