

Problem

Read the informal definition of the finite state transducer given in Exercise 1.24. Prove that no FST can output w^R for every input w if the input and output alphabets are $\{0,1\}$.

Step-by-step solution

Step 1 of 1

Let $w = w_1 w_2 \dots w_n$ then reverse of w can be written as $w^R = w_n \dots w_2 w_1$

We have to prove that no FST(Finite state transducer) can output w^R for every input w over the alphabets $\{0,1\}$

Let us assume that FST T output w^R on input w .

Consider two input strings 00 and 01

(i) On input 00, FST T must produce output 00 because T outputs w^R on input w .

(ii) On input 01, FST T must produce output 10 because T outputs w^R on input w .

In these both case the first input bit is same i.e., 0.

But first output bits differ (0 and 1).

But from actual definition of FST, FST can produce its first output bit before it reads its second input.

Thus FST cannot operate to produce $w^R = 10$ on input $w = 01$.

Thus our assumption that FST T outputs w^R on input w is wrong.

Hence no FST outputs w^R on input w .

[Comment](#)