Problem

Show that both conditions in Problem 5.28 are necessary for proving that P is undecidable.

Problem 5.28

Rice's theorem. Let P be any nontrivial property of the language of a Turing machine. Prove that the problem of determining whether a given Turing machine's language has property P is undecidable.

In more formal terms, let P be a language consisting of Turing machine descriptions where P fulfills two conditions. First, P is nontrivial—it contains some, but not all, TM descriptions. Second, P is a property of the TM's language whenever $L(M_1) = L(M_2)$, we have $\langle M_1 \rangle \in P$ iff $\langle M_2 \rangle \in P$. Here, M_1 and M_2 are any TMs. Prove that P is an undecidable language.

Step-by-step solution

Step 1 of 1

Condition to show that some language un-decidable

First, consider P be the language $\{\langle M \rangle | \ M \text{ is a Turing Machine with 5 states}\}$. P is non-trivial, and so it satisfies the second condition of Rice's Theorem but P can be easily decided by checking the number of states of the input Turing Machine.

Second, consider P be the empty set. Then, it does not contain any Turing Machine and so, it satisfies the first condition of Rice's Theorem, but P can be decided by a Turing Machine that always rejects. Hence, both the properties are required for proving P un-decidable.

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