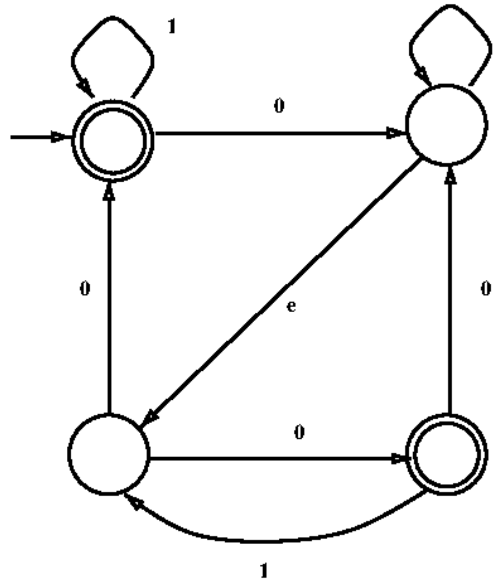
Homework 6 — Due: Wednesday, October 12, 2022

Please submit your work on Brightspace, in PDF format only.

1. Find a regular expression equivalent to the following NFA:



2. Prove in two different ways that the class of regular languages is closed under reversal:
   (a) Give a proof in terms of regular expressions.
   (b) Give a proof in terms of NFAs.

3. For any language L, let

$$\mathrm{MIN}(L) = \{x \in L \mid \text{no proper prefix of } x \text{ belongs to } L\}.$$

   Prove that if L is regular, then so is $\mathrm{MIN}(L)$.

4. For any language $L \subseteq \Sigma^*$, define the binary relation R_L on Σ^* as follows:

$$R_L = \{(x, y) \mid \forall w.\ xw \in L \leftrightarrow yw \in L\}.$$

   (a) Prove that R_L is an equivalence relation.

(b) Suppose $L = L(M)$ for some DFA M. Prove that $(x, y) \in R_L$ if and only if M reaches the same state q when input x is read starting from the start state, as it does when input y is read starting from the start state. Conclude that if L is regular, then R_L has a finite number of equivalence classes.

(c) Prove that, if L is a language for which R_L has a finite number of equivalence classes, then there exists a DFA M, having exactly those equivalence classes as its states, such that $L(M) = L$.

(This proof requires that you define the start state, set of accept states, and transition function for M, and argue that $L(M) = L$. The argument should involve the idea that the state reached by M after reading x starting from the start state is the R_L-equivalence class of x.)