

Problem

Let $T = \{(i, j, k) \mid i, j, k \in \mathcal{N}\}$. Show that T is countable.

Step-by-step solution

Step 1 of 5

Countability: A set is countable if a set is either finite or has the same cardinality as the set of positive integers.

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Step 2 of 5

Given $T = \{(i, j, k) \mid i, j, k \in \mathcal{N}\}$

- The goal is to prove that T is a countable set.
- First, let's define a set $P = \{(i, j, k) \in T \mid i+j+k = s\}$ for each triple $\langle i, j, k \rangle$ where $i, j, k \in \mathcal{N}$, let $i+j+k$ be the sum s of the triplet.
- Now, for each number $s \in \mathcal{N}$,
- There are finitely many triples that has sum equal to s .
- Enumerating the triples with sum zero, then triples with sum equal to 1, then sum equal to 2 and so on.
- The previous step will follow all the triples in T .
- Hence, set P is finite for every $s \in \mathcal{N}$.

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Step 3 of 5

Now, since P is finite and according to the given definition, it is countable too, therefore the set $P' = \{(i', j', k') \in T \mid i'+j'+k' = s'\}$ is also countable.

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Step 4 of 5

Therefore, any set P_i where $i \in \mathcal{N}$, the union $T = \bigcup_{i \in \mathcal{N}} P_i$ is also countable since, a countable union of a number of finite sets is countable.

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Step 5 of 5

Hence, it is proved that T is countable.

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