Problem

Let B be any language over the alphabet Σ. Prove that $B = B^{+}$ iff $BB \subseteq B$.

Step-by-step solution

Step 1 of 3

Let B be any language over the alphabet $\sum$.

To Prove: $B = B^{+}$ iff $BB \subseteq B$

The requirement is to prove both the directions of iff.

Comment

Step 2 of 3

One direction:

Assume: $B = B^{+}$ ......(1)

To show: $BB \subseteq B$

Since, for every language $BB \subseteq B^{+}$ ......(2)

By substituting (1) in (2), it can be obtained that $BB \subseteq B$

Hence, it has been proved that: $BB \subseteq B$ iff $B = B^{+}$

Comment

Step 3 of 3

Other direction:

Assume: $BB \subseteq B$

To prove: $B = B^{+}$

It is known that for every language $BB \subseteq B^{+}$ ......(3).

Let *w* be a string of elements, such that $w \in B^{+}$ then $w \in B$.

• If $w \in B^{+}$ then the string *w* can be split into elements $x_1, x_2, x_3 .... x_k$ such that $w = x_1, x_2 ...... x_k$ for $x_i \in B$ and $k \geq 1$. Since, $x_1, x_2 \in B$ and it is assumed that $BB \in B$.

• Similarly, it holds for the element x_3, such that $x_3 \in B$ and $BB \subseteq B$ .Thus, $x_1, x_2, x_3 \in B$.

• If this procedure is continued, then $x_1, x_2, x_3 .... x_k \in B$ that is $w \in B$.

• It means that, if all the elements of the string ($x_1, x_2 .... x_k$) belong to B, then the strin $w \in B$.

Thus, $B^{+} \subseteq B$ ....(4)

From, (3) and (4) it can be concluded that $B = B^{+}$.

Hence, it has been proved for both the directions that, for any language B, $B = B^{+}$ iff $BB \subseteq B$.

Comments (1)