# Homework #3

**( Due: Nov 12 )**

**Task 1. [ 25 Points ] Construct CFGs**

Construct CFG for each of the following languages.

(a) [ **5 Points** ] $L = \{a^i b^j c^k |\ i \geq 0,\ j \geq 0,\ k = 2i + 3j\}$, $\Sigma = \{a, b, c\}$

(b) [ **5 Points** ] $L = \{a^i b^j c^k |\ i \neq j$ **or** $j \neq k\}$, $\Sigma = \{a, b, c\}$

(c) [ **5 Points** ] $L = \{a^i b^j c^k d^l |\ i + j = k + l\}$, $\Sigma = \{a, b, c, d\}$

(d) [ **5 Points** ] $L = \{ucv |\ u^R \in \{a, b\}^*$ **is a substring of** $v\}$, $\Sigma = \{a, b, c\}$

(e) [ **5 Points** ] $L = \{ucv |\ u^R \in \{a, b\}^*$ **is a subsequence of** $v\}$, $\Sigma = \{a, b, c\}$

**Task 2. [ 25 Points ] Construct CFGs from DFAs**

You constructed a DFA for each of the following languages in HW1. This task asks you to convert each of those DFAs to an equivalent CFG. Assume that $\Sigma = \{a, b\}$ unless specified otherwise.

(a) [ **5 Points** ] $L = \{w |\ n_a(w) \geq 2\}$

(b) [ **5 Points** ] $L = \{w |\ w$ **starts with** $ab\}$

(c) [ **5 Points** ] $L = \{w |\ w$ **starts and ends with the same symbol**$\}$ for $\Sigma = \{a, b, c\}$

(d) [ **5 Points** ] $L = \{w |\ n_a(w) \bmod 3 = 1$ **or** $w$ **contains** $ba\}$

(e) [ **5 Points** ] $L = \{w |$ **binary number** $w$ **is divisible by** $7\}$ for $\Sigma = \{0, 1\}$

**Task 3. [ 20 Points ] Regular expressions to CFGs**

For each of the following regular expressions construct a CFG to accept the language it represents.

(a) [ **5 Points** ] $(0 \cup 1(01^*0)^*1)^*$

(b) [ **5 Points** ] $\epsilon \cup a^+ \cup a^+bb^+(abb^+)^*$

(c) [ **5 Points** ] $a(\epsilon \cup a \cup b(a \cup b \cup \epsilon))$

(d) [ **5 Points** ] $((a \cup b)^6)^*(a \cup b)(a \cup b \cup \epsilon)^4$

**Task 4. [ 30 Points ] Non-CFLs**

Use the pumping lemma to show that the following languages are not context-free.

(*a*) [ **10 Points** ] $L = \{a^n b^{2n} a^n | \ n \geq 0\}$, $\Sigma = \{a, b\}$

(*b*) [ **10 Points** ] $L = \{a^i b^j c^k | \ k > i > 0, \ k > j > 0\}$, $\Sigma = \{a, b, c\}$

(*c*) [ **10 Points** ] $L = \{a^{n!} | \ \ n \geq 0, \ n! = 1 \times 2 \times \ldots \times n, \ 0! = 1\}$, $\Sigma = \{a\}$