Problem

Find the error in the following proof that all horses are the same color.

CLAIM: In any set of h horses, all horses are the same color.

PROOF: By induction on h.

Basis: For h = 1. In any set containing just one horse, all horses clearly are the same color.

Induction step: For k ≥ 1, assume that the claim is true for h = k and prove that it is true for h = k+1. Take any set H of k+1 horses. We show that all the horses in this set are the same color. Remove one horse from this set to obtain the set H^1 with just k horses. By the induction hypothesis, all the horses in H^1 are the same color. Now replace the removed horse and remove a different one to obtain the set H^2. By the same argument, all the horses in H^2 are the same color. Therefore, all the horses in H must be the same color, and the proof is complete.

Step-by-step solution

Step 1 of 2

The error has occurred in the last sentence. There are two statements “all horses in H_1 are the same or identical in color” And “all horses in H_2 are identical or the same color” for the choice of H_1 and H_2which has taken.

Comment

Step 2 of 2

The induction is valid only for value greater than 2 but fails for 2.

• It indicates that all the horses present in $H = H_1 \cup H_2$ are identical (that is, the same color) if $H = H_1 \cap H_2$ is not empty which returns always true result except when exactly two horses $(h = 2)$ contained in H. Then exactly one horse out of the two horses will be consist in each H_1 and H_2 .

• Certainly the single horse in H_1 and only the single horse in H_2has its peculiar color. Therefore, it has no meaning to conclude that the horses have the same color.

• So the base case should be for value 2 not for 1. Since the base case is false, the above proof is wrong.

Comments (2)