Problem

Show that A_{NFA} is NL-complete.

Step-by-step solution

Step 1 of 1

Consider

$A_{NFA} = \{\langle M, w\rangle \mid M \text{ is } NFA \text{ and } M \text{ accepts } w\}$ and
$\text{PATH} = \{\langle G, s, t\rangle \mid G \text{ is any directed graph also } G \text{ has path from } s \text{ to } t\}$

Firstly it is required to show A_{NFA} is NL.

Use a verifier /certificate definition of NL, similar to verifier /certificate of NP. A_{NFA} is in NL if there is some deterministic type Turing machine V such that V uses $O(\log n)$ space and $\langle M, w\rangle$ in A_{NFA} iff there is any certificate c such that V accepts $\langle\langle M, w\rangle, c\rangle$. The certificate will be a path in the underlying graph of NFA M, and the verifier operates as given below:

• The states in the path denoted by $(p_0, \ldots\ldots, p_n)$.

• Check that the $p_0 = q_0$.

• For each $i \in \{1, \ldots\ldots, n\}$, check that $\delta(p_{i-1}, w_i) = p_i$.

• Check that p_n is accepting state.

• If any checks fail, then reject. Otherwise accept.

This algorithm checks every condition which is required by the definition of "M accept w", if it has M accept w iff V accept $\langle\langle M, w\rangle, (p_0, \ldots, p_n)\rangle$ for any path $(p_0, \ldots, p_n)$. Therefore it is concluded that A_{NFA} is in NL.

Now it is required to prove that $PATH \le_M^L A_{NFA}$. Since Path is $NL-complete$, this shows that A_{NFA} is $NL-complete$.

The reduction of mapping $\langle G, s, t\rangle \to \langle M, \varepsilon\rangle$, where M is NFA whose graph is G with $\in$ labels on each edge, where s is initial state, and t is its accept state. If G has path from s to t, then there is few sequence of the vertices $(v_1, v_2, v_3, \ldots., v_k)$ so that

- $v_1 = s$
- $(v_i, v_{i+1}) \in E(G)$ for each $i \in \{1, 2, 3 \ldots., k-1\}$
- $v_k = t$

Thus, $(v_1, v_2, v_3, \ldots, v_k)$ is sequence of the state in M such that

- v_1 is initial state,
- $\delta(v_i \in) = v_{i+1}$ for each $i \in \{1, 2, 3, \ldots, k-1\}$
- v_k is the terminal state,

Three conditions which are necessary to conclude that M accepts ε. Conversely, if G has not any path from point s to point t, then the path will violate one of three given conditions for having the path, so any state sequence will violate any one of three conditions to accept. Therefore $PATH \le_M^L A_{NFA}$ (and thus A_{NFA} is $NL-complete$).

Comment