Problem

Ramsey's theorem. Let G be a graph. A ***clique*** in G is a subgraph in which every two nodes are connected by an edge. An ***anti-clique***, also called an ***independent set***, is a subgraph in which every two nodes are not connected by an edge. Show that every graph with n nodes contains either a clique or an anti-clique with at least 1/2 $\log^2$ n nodes.

Step-by-step solution

Step 1 of 2

Consider the graph *G*. A clique is a subgraph of *G* in which every pair of vertices are connected. An anti-clique is a subgraph in which in which every pair of vertices are not connected.

Comment

Step 2 of 2

In order to show that every graph with *n* vertices contains either clique or anti-clique with at least $\frac{1}{2}\log_2 n$ vertices, create two piles *A* and *B* to store the vertices of a graph. Here, the pile *A* contains the vertices of a clique whereas the pile *B* contains the vertices of an anti-clique.

Procedure to identify a clique or an anti-clique is as follows:

• Take each vertex *v* of the graph *G*.

• If the degree of the vertex is greater than one half of the remaining vertices then add the vertex to pile *A*. Otherwise, add the vertex to the pile *B*.

• Discard all vertices to which *v* is not connected if it was added to the pile *A*.

• Discard all vertices to which *v* is connected if it was added to the pile *B*.

• Continue this procedure until no vertices left.

Consider the whole procedure as a step. For each step, at most half of the vertices are discarded. Thus, at least $\log_2 n$ steps occur before completion of the process. Each step adds a vertex to one of the piles. Thus, one of the piles contains at least $\frac{1}{2}\log_2 n$ vertices.

Therefore, it is proved that that every graph with *n* vertices contains either clique or anti-clique with at least $\frac{1}{2}\log_2 n$ vertices.

Comment