Problem

Let $A \subseteq 1^*$ be any unary language. Show that if A is NP-complete, then P = NP. (Hint: Consider a polynomial time reduction f from *SAT* to A. For a formula ?, let $?_{0100}$ be the reduced formula where variables x_1, x_2, x_3, and x_4 in ? are set to the values 0, 1, 0, and 0, respectively. What happens when you apply f to all of these exponentially many reduced formulas?)

Step-by-step solution

Step 1 of 2

A language is defined as a **unary language** if it is subset of 1^* (at most n strings of length $\le n$). **A unary language is** NP **-complete then** $\mathrm{P{=}NP}$. The given statement can be shown by the following way:

Suppose $\mathrm{U} \subset 1^*$ be a unary language and let that $\mathrm{SAT} \le \mathrm{U}$ through reduction R. The function φ, which is defined as $\varphi(\mathrm{x}_1, x_2, \ldots, x_n)$ is an instance of SAT.

Here, SAT **can be defined by the following algorithm:**

$\mathrm{SAT}(\varphi(\mathrm{A}))$

If $(|\varphi| == 1)$ return φ *//trivial case True or False*

If $(A(f(\varphi))! = \text{undefined})$ return $A(f(\varphi))$

$A(f(\varphi)) = \mathrm{SAT}(\varphi(\mathrm{T}, x_2, \ldots, x_n)) || \mathrm{SAT}(\varphi(\mathrm{F}, x_2, \ldots, x_n))$

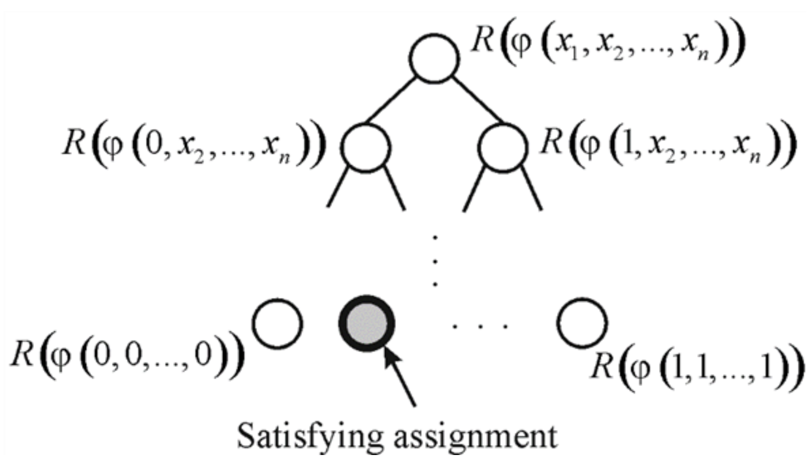
return $A(f(\varphi))$

The above algorithm explains how the self-reduction and reduction applied on the function φ (which is further described below).

Comment

Step 2 of 2

Consider the figure which is given below:



The above figure **explain self-reduction tree for** φ. Now, by the input of length $m = |\varphi(\mathrm{x}_1, x_2, \ldots, x_n)|$, R produces a string of length $\le p(m)$.

- Here $\mathrm{R's}$ different outputs are colors. One color for the string which is not in 1^* and at most $p(m)$ for other colors .
- Hence, puzzle solution $\Rightarrow$ can solve SAT in $(\mathrm{p}(\mathrm{m}) + 1, \mathrm{n} + 1) = \mathrm{poly}(\mathrm{m})$ time!.

Hence from the above explanation it can be said that "**A unary language is** NP **-complete then** $\mathrm{P{=}NP}$."

Comment