Problem

Let X be the set $\{1, 2, 3, 4, 5\}$ and Y be the set $\{6, 7, 8, 9, 10\}$. We describe the functions $f: X \to Y$ and $g: X \to Y$ in the following tables. Answer each part and give a reason for each negative answer.

n	f(n)
1	6
2	7
3	6
4	7
5	6

n	g(n)
1	10
2	9
3	8
4	7
5	6

Aa. Is f one-to-one?

b. Is f onto?

c. Is f a correspondence?

Ad. Is g one-to-one?

e. Is g onto?

f. Is g a correspondence?

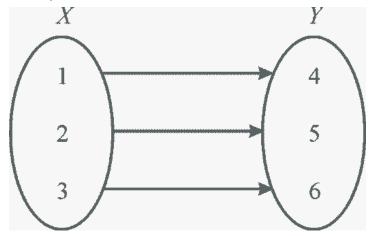
Step-by-step solution

Step 1 of 7

Given that $X=\{1,2,3,4,5\}$ and $Y=\{6,7,8,9,10\}$.

a. **One-to-one Function:** A function $f: X \to Y$ is said to be one to one if and only if when for each and every member of X there exist a unique matching member Y. It is also known as injective function.

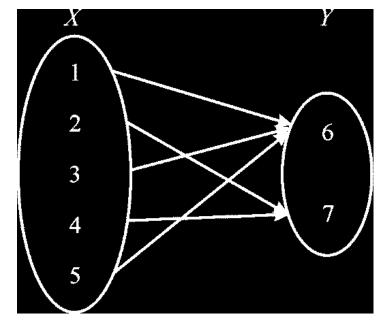
For example:



Here in the above diagram, for every member of X there exists a unique matching member Y. Thus, the above function is one-to-one.

Now, consider the first table given in the textbook draw the diagram for showing the matching between the members.

Consider the following diagram:



Here, in the above diagram for every member of X there exists a matching member Y = f(X).

But in this case:

$$f(1) = f(3) = f(5) = 6$$

 $f(2) = f(4) = 7$

So the function f is not a one-to-one function.

Comment

Step 2 of 7

b. Onto function:

A function $f: X \to Y$ is said to be onto if and only if when for each and every member of $y \in Y$ there exist a matching member $x \in X$ with the property that y = f(x) where $x \in X, y \in Y$.

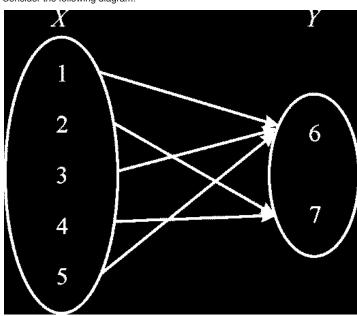
In other words it can be described as follows:

The size of f(x) should be equal to size of the set Y.

It is also known as surjective function.

Now, consider the first table given in the textbook draw the diagram for showing the matching between the members.

Consider the following diagram:



Here, in the above for each and every member of $y \in Y$ there does not exist a matching member $x \in X$ with the property that y = f(x) where $x \in X, y \in Y$

$$f(1) = f(3) = f(5) = 6$$

$$f(2) = f(4) = 7$$

and the elements 8, 9 and 10 in set Y are left out without any elements in $x \in X$ such that y = f(x)

So the function f is not an onto function.

Comments (6)

Step 3 of 7

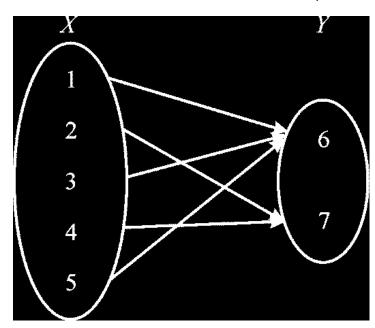
c. Correspondence: A function $f: X \to Y$ is said to be one to one correspondences when this function is one to one and onto at the same time. It is also known as bijective function.

Now, consider the first table given in the textbook draw the diagram for showing the correspondence between them.

Consider the following diagram:

Comment

Step 4 of 7



It is clear from the above diagram that the function $f: X \to Y$ is not a one-to-one and not onto. So the function $f: X \to Y$ is not one-to-one and not onto function.

So, the function f is not a correspondence.

Comment

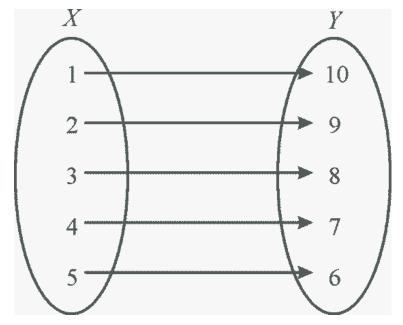
Step 5 of 7

d. **One-to-one Function:** A function $g: X \to Y$ is said to be one-to-one if and only if when for each and every member of X there exist a unique matching member Y.

It is also known as injective function.

Now, consider the first table given in the textbook draw the diagram for showing the matching between the members.

Consider the following diagram:



Here in the above diagram, for every member of X there exists a unique matching member Y = f(X).

So, the function g is a one-to-one function.

Comment

Step 6 of 7

e. Onto function:

A function $f: X \to Y$ is said to be onto if and only if when for each and every member of $y \in Y$ there exist a matching member $x \in X$ with the property that y = f(x) where $x \in X$, $y \in Y$.

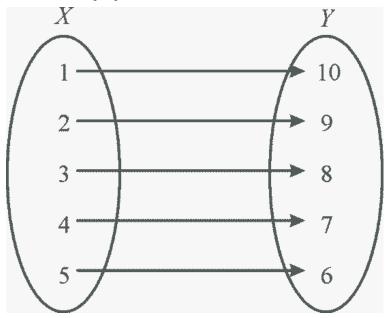
In other words it can be described as follows:

The size of f(x) should be equal to size of the set Y.

It is also known as surjective function.

Now, consider the first table given in the textbook draw the diagram for showing the matching between the members.

Consider the following diagram:



Here in the above diagram, for every member of X, there exists a matching member Y with the property y = f(x).

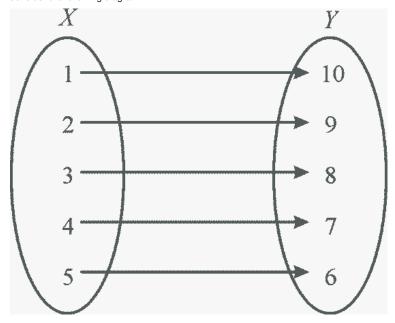
So the function g is a onto function

Step 7 of 7

Correspondence: A function $g: X \to Y$ is said to be one to one correspondences when this function is one to one and onto at the same time. It is also known as bijective function.

Now, consider the first table given in the textbook draw the diagram for showing the correspondence between them.

Consider the following diagram:



It is clear from the above diagram that the function $g: X \to Y$ is one to one and onto at the same time.

So the function g is a correspondence.

Comment