Problem

Show that for any language A, a language B exists, where $A \leq_T B$ and $B \not\leq_T A$.

Step-by-step solution

Step 1 of 3

Given that

A and *B* are two Languages.

We have to show that

$A \leq_T B$ and $B \not\leq_T A$.

That means

• *A* is Turing reducible to *B*.

But *B* is not Turing reducible to *A*.

Let $A = E_{TM}$ (Empty Turing machine)

$= \{\langle M \rangle | \text{ M is a TM and L}(\text{M}) = \phi\}$

And $B = A_{TM}$ (Oracle Turing machine)

$= \{\langle M, w \rangle, \text{ machine } M \text{ accepts } w\}$

Comment

Step 2 of 3

(i) $A \leq_T B$:

To show $A \leq_T B$, we have to show that $E_{TM} \leq_T A_{TM}$.

We know that E_{TM} is decidable relative to A_{TM}.

Since $E_{TM} \leq_T A_{TM}$, then *A* is decidable relative to *B* then $A \leq_T B$.

Comments (1)

Step 3 of 3

(ii) $B \not\leq_T A$

To shown $B \not\leq_T A$ we have to show that

$A_{TM} \not\leq_T E_{TM}$.

Let us assume the contradiction $A_{TM} \leq_T E_{TM}$.

By mapping reducibility rules for any two languages $x \leq_m y \Leftrightarrow \overline{x} \leq_m \overline{y}$

Language x is mapping reducible to language y, written $x \leq_m y$, if there is a computable function $f : \Sigma^* \rightarrow \Sigma^*$, where for every *w, f* called reduction of *x* to *y.*

$w \in x \Leftrightarrow f(w) \in y$

According to given mapping reducibility we have to derive $A_{TM} \leq_T E_{TM} \Leftrightarrow \overline{A_{TM}} \leq_m \overline{E}_{TM}$

However $\overline{E}_{TM}$ is Turing recognizable, but $\overline{A}_{TM}$ is not Turing recognizable

Accor ¨ ıg mapping reducibility rules

$x \le_m y$ and *y* is Turing – recognizable then *x* must be Turing recognizable.

Therefore this gives a contradiction.

Therefore our assumption that $A_{TM} \le_T E_{TM}$ is wrong.

Hence $A_{TM} \not\le_T E_{TM}$ i.e., $B \not\le_T A$.

From (i) and (ii) we have showed that there exist two languages *A* and *B* such that

$A \le_T B$ But $B \not\le_T A$.

Comments (1)