Problem

In both parts, provide an analysis of the time complexity of your algorithm.

a. Show that EQ_{DFA} ? P.

b. Say that a language A is ***star-closed*** if A = A*. Give a polynomial time algorithm to test whether a DFA recognizes a star-closed language. (Note that EQ_{NFA} is not known to be in P.)

Step-by-step solution

Step 1 of 6

The given details are as follows:

$EQ_{DFA} \in P$

Where *P* is a class of language and *EQ* represents a deterministic finite automata (*DFA*).

Comment

Step 2 of 6

Consider *E* and Q be two DFA, then a third DFA C can be constructed, which is defined as follows:

$$L(C) = \left(L(E) \cap \overline{L(Q)}\right) \cup \left(L(Q) \cap \overline{L(E)}\right).$$

Here, first the intersection of the language recognized by the *DFA E* and the complement of the language recognized by the *DFA Q* takes place and then its union takes place with the intersection of the language recognized by the *DFA Q* and the complement of the language recognized by the *DFA E*.

• In the process of *DFA* C construction, interchanging in accepting state can be used for complement operation.

• It is already known that, polynomial time can be taken in intersection and union operations. So, it can be said that "the polynomial time required finishing all the construction procedure".

Comment

Step 3 of 6

Algorithm:

L="On input ":

1. **w will be run by L**

2. **If** $L(C) = \varnothing$ **(NULL) or** $L(E) = L(Q)$ **is accepted.**

Else it is rejected.

Comment

Step 4 of 6

Now testing is performed on $L(C)$ and check that where it becomes NULL. In other words, $L(C) = \varnothing$ or $L(E) = L(Q)$.

• Now suppose, k = the number of states of C (polynomial size of E and Q), then it is already known that "the number of step 2 $\leq n$ and every step 2 takes a cost less than n^2. Hence, it can be said that," $L(C) = \varnothing$ **is polynomial**".

• Therefore, **a Turing machine is obtained which decide a polynomial time is used to run** EQ_{DFA}**.** In other words, it can be said that**,** "$EQ_{DFA} \in P$"**.**

• Hence, the time complexity of the algorithm is $O(n^2)$ because both *DFA E* and *Q* take $O(n)$ time so, the total time is $O(n) \times O(n) = O(n^2)$.

Comment

Step **5** of 6

b)

A language A is known as star-closed if $A = A^{*}$. An algorithm can be provided, which is used to **test whether a *DFA* recognizes a star-closed language**. It can be achieved by the following algorithm:

• For a given language A, suppose T be the Turing machine which decides it in polynomial in time.

• Now a Turing machine T' is constructed which is used to decide the star closed of the given language A in polynomial time.

T'="On input ~~":~~

~~1. **s will be run by T**~~

~~2. **If T is accepted, reject. Else it is rejected, accepted**."~~

Comments (1)

Step **6** of 6

Here, T' is used to decide the star of the language A. Since T runs on polynomial time, then T' will also be run on polynomial time.

Therefore, the above describes that "**DFA recognized a star-closed language**".

The input string ***s*** for a language ***A*** can be (ε) or an input for any number of times possible. Let, the input string ***k***-times possible, thus for each input time complexities are different and consider upper bound time $O(1)\times O(2)\times ....\times O(n^{k}) = O(n^{k})$.

So, the total time complexity for *k* times is $O(n^{k})$.

Comment