Problem

A *ladder* is a sequence of strings $s_1, s_2, \ldots, s_k$, wherein every string differs from the preceding one by exactly one character. For example, the following is a ladder of English words, starting with "head" and ending with "free":

$$\text{head, hear, near, fear, bear, beer, deer, deed, feed, feet, fret, free.}$$

Let $LADDER_{\mathsf{DFA}} = \{\langle M, s, t\rangle |\ M$ is a DFA and $L(M)$ contains a ladder of strings, starting with s and ending with $t\}$. Show that $LADDER_{\mathsf{DFA}}$ is in PSPACE.

Step-by-step solution

Step 1 of 3

$LADDER_{DFA} \in PSPACE$ $LADDER_{DFA}$= {(M, s, t) I M is a DFA and L(M) contains a ladder of strings, starting with s and ending with t}

The ladder is a sequence of strings where every string differs from preceding one in exactly one character.

To prove: $LADDER_{DFA}$ is in PSPACE

It is known that PSPACE=NPSPACE so, if it is shown that $LADDER_{DFA} \in NPSPACE$ then it implies that $LADDER_{DFA} \in PSPACE$

It is known that NPSPACE is non-deterministic Turing Machine that clears the class of languages which can be decided in polynomial space.

Comment

Step 2 of 3

Follow the following steps:

• If $|s| \neq |t|$, then reject.

• Otherwise, the graph G of size vertices are indexed by strings in $2^{|s|}$ and there is a directed edge from w_1 to w_2 vary in precisely one character and $w_1, w_2 \in L(M)$

• $(M, s, t) \in LADDER_{DFA}$ if and only if there is a path from s to t in G.

• It can be checked in NPSPACE by guessing the path and at each step storing only the name of the current vertex.

• To guess the path at the vertex w_1 non deterministically select a new vertex w_2 that differs from w_1 in precisely one character and verify that M accepts w_2.

• The machine M always halts by keeping a counter and incrementing it with each guess and rejecting when the counter hits $|\Sigma|^{|s|}$

Comment

Step 3 of 3

Thus, the non-deterministic Turing machine is constructed to decide $LADDER_{DFA}$

Therefore, $LADDER_{DFA} \in NPSPACE$ so, $LADDER_{DFA} \in PSPACE$

Comment