Problem

Let A be a Turing-recognizable language consisting of descriptions of Turing machines, $\{\langle M_1\rangle, \langle M_2\rangle, \ldots\}$, where every M_i is a decider. Prove that some decidable language D is not decided by any decider M_i whose description appears in A.

(Hint: You may find it helpful to consider an enumerator for A.)

Step-by-step solution

Step 1 of 2

Consider the Turing-recognizable language *A* which contains the descriptions of all the Turing machines, therefore there must exist an enumerator *E* to enumerate it.

Consider $\langle M_i\rangle$ is the i^{th} output of *E*. Assume $s_1, s_2, s_3 \ldots, s_i$ are the all possible strings of $\{0,1\}^*$. It means $s_1, s_2, s_3 \ldots, s_i$ are made up of combinations of 0's and 1's.

Comment

Step 2 of 2

Consider a decidable language *D* is defined as follows:

For a string S_i,

• If $\langle M_i\rangle$ accepts then S_i does not belongs to the language *D.*

• If $\langle M_i\rangle$ rejects then S_i belongs to the language *D.*

Here, the language *D* is a decidable language and its decider is not present in the list. Therefore, it is proved that there is a decidable language *D* whose decider is not present in *A*.

Comment