Problem

Give an example of an NL-complete context-free language.

Step-by-step solution

Step 1 of 3

The statement of Ginsburg theorem state that, "Suppose G is used to generate a language $L(G)$ where G is defined as a reducible context free grammar. Then $|L(G)\cap\{0,1\}^n|$ will show a polynomial behavior in n if and only if for each non-terminal $z, l(z)$ and $r(z)$ are commutative".

Comment

Step 2 of 3

Now, suppose $M\subseteq\{0,1\}^*$ be a language which **is context free and exponential** in size. The above context free language shows a $NP-complete$ behavior.

• To perform this, suppose, D is defined as **a reduced-free grammar** for M. Here, M_n will not be in polynomial of n because M consists an **exponential size**.

• Now from the above given theorem, it can be said that there exists a nonterminal and for each non-terminal $Z, l(Z)$ and $r(Z)$ are **commutative**.

Comment

Step 3 of 3

Consider, $l(Z)$ (where, $a_1, a_2 \in l(Z)$) is not showing the commutative property and $a_1, a_2 \neq a_2, a_1$. Hence, there exists a position k such that $(a_1, a_2)_k = 0$ and $(a_2, a_1)_k = 1$.

• As from the above explanation, there exists $(b_1, b_2)\in\{0,1\}^*$ in such a way that $Z\succ^* a_1Zb_1$ and $Z\succ^* a_2Zb_2$. An arbitrary $1s$ and $0s$ can be generated at the position $k\cdot|a_1a_2|+i$ for any k, by applying either $Z\succ^* a_1a_2Zb_2b_1$ or $Z\succ^* a_2a_1Zb_1b_2$, k times.

• Now, to reach Z from M_0, user can use $M_0\succ^* aZb$. Again, to acquire a word in $\{0,1\}^*$ for some $x, y, w\subseteq\{0,1\}^*$, $Z\succ^* w$ can be used.

• Hence from the above discussion, $I:=\{|x|+k\cdot|a_1a_2|+i : 0\le k\le n-1\}$ is of size n can be obtained if N is set as $N:=\{|x|+|y|+|w|+n(|a_1a_2|+|b_1b_2|)\}$ and also I can be shattered by M_N.

• All the calculations (which are done above) take a time in $O(n)$ and N is linier in n and it is already known that $S-SAT$ is $NP-hard$ and $S-SAT\in NP$. Hence, it shows $NP-complete$ behavior.

Hence the above explanation shows, the context free language $M\subseteq\{0,1\}^*$ is $NP-complete$.

Comment