Problem

Consider the language $F = \{\mathrm{a}^i\mathrm{b}^j\mathrm{c}^k \mid i, j, k \geq 0 \text{ and if } i = 1 \text{ then } j = k\}$.

a. Show that F is not regular.

b. Show that F acts like a regular language in the pumping lemma. In other words, give a pumping length p and demonstrate that F satisfies the three conditions of the pumping lemma for this value of p.

c. Explain why parts (a) and (b) do not contradict the pumping lemma.

Step-by-step solution

Step 1 of 4

Consider the language:

$F = \left\{a^i b^j c^k \mid i, j, k \geq 0 \text{ and if } i = 1 \text{ then } j = k\right\}$

The language F is the union of three disjoint languages.

$F_0 = \left\{b^j c^k \mid j, k \geq 0\right\}, F_1 \left\{ab^j c^j \mid j \geq 0\right\}, F_2 = \left\{a^i b^j c^k \mid i \geq j, k \geq 0\right\}$

Clearly F_0 and F_2 are regular languages.

The class of regular languages is closed under union and complement.

Thus $\overline{F_0 \cup F_2}$ is also regular.

We have $F_1 = F - \left(F_0 \cup F_2\right)$

$= F \cap \overline{F_0 \cup F_2}$

Since the class of regular languages is closed under intersection if *F* is regular, then so is F_1.

Comments (1)

Step 2 of 4

a.

Use pumping lemma to show that F_1 is not regular and hence neither is F.

Assume that F_1 is regular language.

Let *P* be the pumping length given by pumping lemma.

Consider a string $S = ab^P c^P \in F_1$

Clearly $|S| > P$.

By using the pumping lemma, *S* can be divided into three pieces.

i.e., $S = ab^P c^P = uvw$ such that $|uv| \leq P, |v| > 0$ and $uv^i w \in F_1 \quad \forall i \geq 0$.

Take $u = a \quad v = b^P \quad w = c^P$

$uv^0 w = a\left(b^P\right)^0 c^P \quad \therefore (i = 0)$

$= ac^P \notin F_1$

The string *w* consist of '*c*' s. In this case, string $a\left(b^{P}\right)^{0}c^{P}$ has more *c*s than '*b*'s and so is not a member of F_1, violating the condition of pumping lemma.

This is contradiction. The previous assumption that F_1 is regular is wrong. Thus F_1 is not regular.

Therefore, *F* is also not a regular language.

Comments (1)

Step 3 of 4

b.

Let pumping length $P=2$

- Show that every string $S\in F$ of length at least *P* can be divided into three pieces $S=uvw$ such that, $|uv|\le P,|v|>0$ and $uv^{l}w\in F\ \forall l\ge 0$.
- Consider a string $a^{i}b^{j}c^{k}\in F$ of length at least 2.
- Choose *x* to be the empty string.
- If $i\ne 2$, then choose *y* to be the first symbol in $a^{i}b^{j}c^{k}$.
- If $i=2$, then choose $y=aa$

Clearly these chosen x,y satisfies the three conditions of pumping lemma.

Comment

Step 4 of 4

c.

part (a) and part(b) do not contradict the pumping lemma because the pumping lemma just says if a language is regular then there is a pumping length for that language. But, if the language satisfies the pumping lemma, the language may not be regular.

Comment