Problem

Is the following formula satisfiable?

$$(x \vee y) \wedge (x \vee \overline{y}) \wedge (\overline{x} \vee y) \wedge (\overline{x} \vee \overline{y})$$

Step-by-step solution

Step 1 of 3

The formula $(x \vee y) \wedge (x \vee \overline{y}) \wedge (\overline{x} \vee y) \wedge (\overline{x} \vee y) \wedge (\overline{x} \vee \overline{y})$ **is not satisfiable**.

Explanation:

• A Boolean formula is satisfiable if some assignment of 0s and 1s to the variables makes the formula evaluate to 1.

• We know that

$0 \wedge 0 = 0$ $\quad 0 \vee 0 = 0$ $\quad \overline{0} = 1$
$0 \wedge 1 = 0$ $\quad 0 \vee 1 = 1$ $\quad \overline{1} = 0$
$1 \wedge 0 = 0$ $\quad 1 \vee 0 = 1$
$1 \wedge 1 = 1.$ $\quad 1 \vee 1 = 1$

Comment

Step 2 of 3

Specified Boolean formula is

$$(x \vee y) \wedge (x \vee \overline{y}) \wedge (\overline{x} \vee y) \wedge (\overline{x} \vee y) \wedge (\overline{x} \vee \overline{y})$$

Here consider *x* and *y* are variables.

Case 1:

Assign $x = 0$ and $y = 1$

Then $(0 \vee 1) \wedge (0 \vee \overline{1}) \wedge (\overline{0} \vee 1) \wedge (\overline{0} \vee 1) \wedge (\overline{0} \vee \overline{1})$

$= 1 \wedge (0 \vee 0) \wedge (1 \vee 1) \wedge (1 \vee 0)$
$= 1 \wedge 0 \wedge 1 \wedge 1$
$= 0 \wedge 1$
$= 0$

Comment

Step 3 of 3

Case 2:

Assign $x = 1$ and $y = 0$

Then $(1 \vee 0) \wedge (1 \vee \overline{0}) \wedge (\overline{1} \vee 0) \wedge (\overline{1} \vee 0) \wedge (\overline{1} \vee \overline{0})$

$= 1 \wedge (1 \vee 1) \wedge (0 \vee 0) \wedge (0 \vee 1)$
$= 1 \wedge 1 \wedge 0 \wedge 1$
$= 1 \wedge 0$
$= 0$

From case 1 and case 2 of the Boolean values for *x* and *y*, the formula always evaluated to 0, but we know that if some assignment of 0s and 1s to the variables makes the formula evaluate to 1 then a Boolean formula is satisfiable.

So, the formula $(x \vee y) \wedge (x \vee \overline{y}) \wedge (\overline{x} \vee y) \wedge (\overline{x} \vee y) \wedge (\overline{x} \vee \overline{y})$ is not satisfiable.

Comment