Problem

Let the ***rotational closure*** of language A be $RC(A) = \{yx|\ xy \in A\}$.

a. Show that for any language A, we have *RC*(A) = *RC*(*RC*(A)).

b. Show that the class of regular languages is closed under rotational closure.

Step-by-step solution

Step 1 of 4

a) The Rotational Closure of a language A is defined as $RC(A)=\{yx|xy \in A\}$. Now for any language A a string $\omega \in A$ also $\omega \in \Sigma^*$, and therefore $\varepsilon \in A$.

It is known that $\omega\varepsilon = \varepsilon\omega = \omega$. Here for any language $A = \{\omega\varepsilon\}$, $RC(\omega\varepsilon) = \varepsilon\omega = \omega$ and $RC(RC(\omega\varepsilon)) = RC(\varepsilon\omega) = \omega\varepsilon = \omega$.

Therefore, $\boxed{RC(A) = RC(RC(A))}$

Comments (1)

Step 2 of 4

b) Proof: Assume the language A is being defined with the help of regular expression E. Now structural induction on the size of regular expression E is to be proved.

Now, it is to be shown that $A(E) = A(RC(A))$, which implies the language $RC(A)$ is the reverse language of the specified language A.

Basis: If E is $\varepsilon, \varnothing$, or a for some symbol a, then $RC(E) = E$. That is, $RC(\varepsilon) = \varepsilon$, $RC(\varnothing) = \varnothing$ and $RC(a) = a$.

Induction: Three cases arise depending upon the specified expressions E which are as follows:

- $E = E_1 + E_2$. Then $RC(E) = RC(E_1) + RC(E_2)$.

The relational closure of any two languages is obtained with the computation and after that taking the union of the specified two languages.

Comment

Step 3 of 4

- $E = E_1E_2$. Then $RC(E) = RC(E_2)RC(E_1)$.

Here the expression is rotated with respect to two languages and language itself

For an example let if, $L(E_1) = \{01, 111\}$ and $A(E_2) = \{00, 10\}$, then,

$$A(E_1E_2) = \{0100, 0110, 11100, 11110\}, \text{ then we can say}$$
$$RC(A(E_1E_2)) = \{0010, 0110, 00111, 01111\}, \text{ where } \varepsilon \in A.$$

Again, $RC(A(E_1)) = \{10, 111\}$

$$RC(A(E_2)) = \{00, 01\}$$
$$A(RC(E_2)RC(E_1)) = \{0010, 00111, 0110, 01111\}$$

and therefore, $RC(A(E)) = A(RC(E))$.

Comment

Step 4 of 4

- $E = E_1^*$. Then $RC(E) = (RC(E_1))^*$. Here the justification is any string ω in $A(E)$ can be written as $\omega_1\omega_2\cdots\omega_n$ where $\omega_i \in L(E)$.

But $RC(\omega) = RC(\omega_n)RC(\omega_{n-1})\cdots RC(\omega_1)$ where each $RC(\omega_i) \in A(RC(E))$ and hence $RC(\omega)$ is in $L((RC(E_1))^*)$.

Therefore it is proved that the set of regular languages are closed under rotational closure.

Comment