Problem

Let $D = \{xy | x, y \in \{0,1\}^* \text{ and } |x| = |y| \text{ but } x \neq y\}.$ Show that D is a context-free language.

Step-by-step solution

Step 1 of 2

By the definition of **Context free language,** for showing that the language D is a CFL i.e. context free language, generate a context free grammar CFG G.

Consider the following grammar G:

$S \rightarrow AB|BA$

$A \rightarrow 0|0A0|0A1|1A0 | 1A1$

$B \rightarrow 1|0B0|0B1|1B0 | 1B1$

The given grammar L (G) generates the language in the form $w_1 x w_2 v_1 y v_2$, where $| w_1 |=| w_2 |= k,| v_1 |=| v_2 |=l, x \neq y$ for $\Sigma = (0,1)*$.

Comments (2)

Step 2 of 2

- By the definition, any language which is generated by a context-free grammar is termed as a context-free language.
- The grammar generated above is a Context Free Grammar. The language D can be generated using the above context free grammar G as follows:
- A string is in D iff it can be written as xy with $|x|=|y|$ s.t. for some i, the ith character of x and y are different from one another. The above grammar can be used to obtain the required string by generating the ith characters and filling up with the remaining characters.
- The generated language $w_1 x w_2 v_1 y v_2$ can be subjected to nested induction over k and l with case distinction over pairs (x, y).
- Now, w2 and v1 can exchange symbols because both carry symbols that are independent of the rest of the string.
- Therefore, x and y in their respective half can have the same position, which implies L (G) = L because G doesn't impose any restrictions on its language.

Hence, the given language **D** is **context free language**.

Comment