Problem

Show that the collection of Turing-recognizable languages is closed under the operation of

A**a.** union.

b. concatenation.

c. star.

d. intersection.

e. homomorphism.

Step-by-step solution

Step 1 of 8

• Suppose, X and Y be two Turing recognizable languages that have Turing machines M_X and M_Y respectively.

• Now the union of these languages is denoted by L_{XY} and the Turing machine recognizing this language is $M_{XY.}$

"On input *w:*"

1. Run X and Y alternately on *w* step by step. If either accepts, *accept*. If both halt and reject, *reject.*

Suppose s be a word from L_{XY}. M_{XY} works for an input string s as shown:

• It executes M_X and M_Y on s individually.

• If at least any one of M_X orM_Y accepts s then M_{XY} also accepts after a finite number of steps and reach to its accepting state.

• If both M_X and M_Y reject and either of them do so by looping then M_{XY} will loop.

Comment

Step 2 of 8

Hence, it can be said that collection of Turing recognizable languages is closed under union operation.

Comment

Step 3 of 8

• Suppose, X and Y be two Turing recognizable languages that have Turing machines M_X and M_Y respectively. • Now the concatenation of these languages is denoted by L_{XY} and the Turing machine recognizing this language is $M_{XY.}$

Let s be a word from L_{XY}. M_{XY} works for an input string s as shown:

• It divides each string of XY into s_1 and s_2 non-deterministically.

• It runs s_1 to M_X. If M_X halts and rejects, *reject*.

• If runs s_2 to M_Y. If M_Y accepts, *accepts*. If M_Y halts and rejects, *reject*.

Comments (2)

Step 4 of 8

Hence, it can be said that collection of Turing recognizable languages is closed under concatenation operation.

Comment

Step 5 of 8

• Suppose, X be a Turing recognizable language and the Turing machine is M_X.

• Now X^* is the language obtained from star operation on X.

The Turing machine for this language be M_{X^*}.

M_{X^*} works as follows:

• For an input string s of X, it non-deterministically divided the string into $s_1, s_2 \ldots s_n$.

• For each of those divided parts M_{X^*} runs, Suppose, M_{X^*} all divided parts then s is accepted by M_{X^*} else s is rejected by M_{X^*}.

Comment

Step 6 of 8

Hence, it can be said that collection of Turing recognizable languages is closed under star operation.

Comment

Step 7 of 8

• Suppose, X, Y be two Turing recognizable languages that have Turing machines M_X and M_Y respectively.

• Now the intersection of these languages is denoted by L_{XY} and the Turing machine recognizing this language is M_{XY}.

For an input string s from L_{XY}, M_{XY} works for as shown:

• Turing machine M_X runs on s. If it accepts s then M_Y runs on s. Else s is rejected.

• Suppose, M_Y accepts s then it is accepted by the Turing machine otherwise s is rejected.

Hence, it can be said that collection of Turing recognizable languages is closed under intersection operation.

Comments (5)

Step 8 of 8

• Suppose, X a Turing recognizable language that have Turing machine M_X.

• To recognize $h(X)$ the other Turing machine M_Y is simulated in such a way that:

On input s, it will consider all strings w such that $h(w) = s$.

• The *TM* M_X will execute on input w by going through all strings in w.

If $h(w) = s$ start executing M_X on input w, using merging to interleave with other executions on M_X. *Accept* if any executions accept.

• M_Y will accept s it any of those executions of M_X accepts s. Else s will be rejected.

Hence, it can be said that collection of Turing recognizable languages is closed under homomorphism operation.

Comment