Problem

In the fixed-point version of the recursion theorem (Theorem 6.8), let the transformation t be a function that interchanges the states q_{accept} and q_{reject} in Turing machine descriptions. Give an example of a fixed point for t.

THEOREM 6.8

Let $t\colon \Sigma^* \longrightarrow \Sigma^*$ be a computable function. Then there is a Turing machine F for which $t(\langle F\rangle)$ describes a Turing machine equivalent to F. Here we'll assume that if a string isn't a proper Turing machine encoding, it describes a Turing machine that always rejects immediately.

In this theorem, t plays the role of the transformation, and F is the fixed point.

Step-by-step solution

Step 1 of 4

Fixed point of a function is value that is not changed by application of the function. We need to consider functions are computable transformations of Turing machine descriptions. Turing machine behavior is unchanged by transformations that theorem called fixed-point version of recursion theorem.

Fixed – point version of the recursion theorem:- Let $t:\Sigma^* \to \Sigma^*$ be a computable function then there is a Turing machine *F* for which $t(\langle F\rangle)$ describes a Turing machine equivalent to *F*. If a string is not proper Turing machine encoding, then it describes Turing machine rejects immediately.

t : is a role of transformation

F: fixed point

Comment

Step 2 of 4

Given that,

The transformation *t* is a function that interchanges the states q_{accept} and q_{reject} in Turing machine descriptions.

Now we have to define the fixed point for *t*.

Comment

Step 3 of 4

Let $\langle M\rangle$ be a fixed point machine.

(i) *M* Halts on input *w*

- If *M* halts on any input *w*, then clearly *M* cannot be a fixed point since q_{accept} and q_{reject} are replacing.
- The Language recognized by $\langle M\rangle$ and the Language recognized by $t\langle M\rangle$ must differ in *w*.
- So *M* does not halt on any input, instead of that it rejects then only *M* accepts as fixed point.

(ii)

- Suppose *M* loops infinitely on all inputs.
- Then $(t\langle M\rangle)$ must also loop infinitely, because it is the same machine but with accept and reject states swapped.
- Therefore *M* is a fixed – point.

Hence any machine which loops infinitely on all inputs is a fixed – point since it rejects.

Comment

Step 4 of 4

<u>Example:</u> A machine which can never leave the start state is an example of fixed point.

Start with *n*-state Turing machine start with blank tape but the productivity is zero.

Machine *M* on blank input

- Get description for $\langle M \rangle$ from recursion theorem for *n* states of $\langle M \rangle$
- Compute Productivity *p*(*n*)
- Add 1 to obtain *p*(*n*) + 1
- Once end of end of tape print *p*(*n*) + 1
- But *M* is an *n*-state Turing machine that outputs more than the productivity function. It is a contradiction.

Comment