

Problem

Answer each part for the following context-free grammar G .

$$\begin{aligned} R &\rightarrow XRX \mid S \\ S &\rightarrow aTb \mid bTa \\ T &\rightarrow XTX \mid X \mid \varepsilon \\ X &\rightarrow a \mid b \end{aligned}$$

- | | |
|----------------------------------------------|-----------------------------------------------------|
| a. What are the variables of G ? | i. True or False: $T \xRightarrow{*} T$. |
| b. What are the terminals of G ? | j. True or False: $XXX \xRightarrow{*} aba$. |
| c. Which is the start variable of G ? | k. True or False: $X \xRightarrow{*} aba$. |
| d. Give three strings in $L(G)$. | l. True or False: $T \xRightarrow{*} XX$. |
| e. Give three strings <i>not</i> in $L(G)$. | m. True or False: $T \xRightarrow{*} XXX$. |
| f. True or False: $T \Rightarrow aba$. | n. True or False: $S \xRightarrow{*} \varepsilon$. |
| g. True or False: $T \xRightarrow{*} aba$. | o. Give a description in English of $L(G)$. |
| h. True or False: $T \Rightarrow T$. | |

Step-by-step solution

Step 1 of 16

Given the context – free grammar G is

$$\begin{aligned} R &\rightarrow XRX \mid S \\ S &\rightarrow aTb \mid bTa \\ T &\rightarrow XTX \mid X \mid \varepsilon \\ X &\rightarrow a \mid b \end{aligned}$$

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a. The variables of G are R, S, T and X

Variables are the non-terminal symbols that appear in the rules of the grammar.

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b. The terminals of G are a, b .

Terminals are the terminal symbols that appear in the rules of the grammar.

[Comments \(2\)](#)

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c. The start variable of G is R .

$$R \rightarrow XRX \mid S$$

Start variable is a variable usually occurs on the left – hand side of the topmost rule.

[Comments \(3\)](#)

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d. Case1: Consider the rule $R \rightarrow S$

Substitute S with rule $S \rightarrow aTb$

$$R \rightarrow aTb$$

Substitute T with rule $T \rightarrow \varepsilon$.

$$R \rightarrow a \in b$$

$$R \rightarrow ab$$

Case2: Consider the rule $R \rightarrow S$

Substitute S with rule $S \rightarrow bTa$

$$R \rightarrow bTa$$

Substitute T with rule $T \rightarrow \varepsilon$.

$$R \rightarrow b \in a$$

$$R \rightarrow ba$$

Case3: Consider the rule $R \rightarrow S$

Substitute S with rule $S \rightarrow aTb$

$$R \rightarrow aTb$$

Substitute T with rule $T \rightarrow X$.

$$R \rightarrow aXb$$

Substitute X with rule $X \rightarrow a$.

$$R \rightarrow aab$$

Therefore, the 3 strings in $L(G)$ are ab , ba and aab .

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e. The three strings not in $L(G)$ are aba , b and ε . Since these strings cannot be derived from the given grammar G .

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f. False

$T \Rightarrow aba$: the string cannot be derived using G .

[Comments \(1\)](#)

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g. True

$T \stackrel{*}{\Rightarrow} aba$, the string can be derived using G .

[Comments \(16\)](#)

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h. False

$T \Rightarrow T$, the string cannot be derived using G .

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i. True

$T \stackrel{*}{\Rightarrow} T$, the string can be derived using G .

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j. True

$XXX \stackrel{*}{\Rightarrow} aba$, the string can be derived using G .

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k. False

$X \stackrel{*}{\Rightarrow} aba$, the string cannot be derived using G .

[Comments \(2\)](#)

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l. True

$T \stackrel{*}{\Rightarrow} XX$, the string can be derived using G .

[Comments \(2\)](#)

Step 14 of 16

m. True

$T \stackrel{*}{\Rightarrow} XXX$, the string can be derived using G .

[Comments \(1\)](#)

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n. False

$S \xRightarrow{*} \varepsilon$, the string cannot be derived using G .

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o. $L(G)$ consists of all strings that are not palindromes and are formed over terminal symbols a and b .

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