Problem

Prove that $TQBF \notin \text{SPACE}(n^{1/3})$.

Step-by-step solution

Step 1 of 1

The **space hierarchy theorem** says "if g is a space-constructible ($1^n \to 1^{g(n)}$ can be computed in space $O(g(n))$, $f(n) = o(g(n))$, then $\mathbf{SPACE}(f(n)) \subsetneq \mathbf{SPACE}(g(n))$". So, from the space hierarchy theorem it can be said that there exists a language L, which is **solvable in linear space but it cannot be solved by sub-linear space**.

• Since, $\mathbf{TQBF}$ is space complete then L can be reduced to $\mathbf{TQBF}$ in log space. Therefore, if $\mathbf{TQBF} \in \mathbf{SPACE}\left(\mathbf{n}^{1/3}\right)$, then $L \in \mathbf{SPACE}\left(\left(\mathbf{n}^{c}\right)^{1/3} + \log(\mathbf{n})\right)$.

• Now suppose, if $0.33 < 1/c$, **there exists a contradiction**.

Thus, from the above result it can be said that $TQBF \notin SPACE\left(n^{1/3}\right)$.

Comment