Problem

The pumping lemma says that every regular language has a pumping length p, such that every string in the language can be pumped if it has length p or more. If p is a pumping length for language A, so is any length p' ≥ p. The ***minimum pumping length*** for A is the smallest p that is a pumping length for A. For example, if A = 01^*, the minimum pumping length is 2. The reason is that the string s = 0 is in A and has length 1 yet s cannot be pumped; but any string in A of length 2 or more contains a 1 and hence can be pumped by dividing it so that x = 0, y = 1, and z is the rest. For each of the following languages, give the minimum pumping length and justify your answer.

a. 0001^*

b. 0^*1^*

c. $001 \cup 0^*1^*$

d. $0^*1^+0^+1^* \cup 10^*1$

e. $(01)^*$

f. ε

g. $1^*01^*01^*$

h. $10(11^*0)^*0$

i. 1011

j. Σ^*

Step-by-step solution

Step 1 of 10

a.

The pumping length could not be 3 as 000 being in the language it cannot be pumped. Consider the string length be 4 or more and divide in xyz as x being 000, y being the first 1 and z being everything after then it satisfies every condition of pumping lemma.

Hence the minimum pumping length is 4.

Comment

Step 2 of 10

b.

String ε is in the language but it could not be pumped so the pumping length could not be 0. According to pumping lemma's three condition arises which are as follows:

- If user divide the string in xyz as x is ε
- y is first symbol $(0|1)$
- z being the everything after then it holds.

Hence the minimum pumping length is 1.

Comment

Step 3 of 10

c.

The string 001 is in the language but if it is generated by **001** then it cannot be pumped. If string s is larger than 3 and in the language, then it is generated by 0^*1^*. Dividing the string according to Pumping Lemma's condition into a string xyz where x is ε, y be the first symbol and z be the remaining, user can pump the string.

Hence the minimum pumping length is 4.

Comment

Step 4 of 10

d.

The string 11 is in the language but it cannot be pumped so the length is not 2. Let s be the string in the language of length at least 3. If s is generated by $0^*1^+0^+1^*$ it can be divided in xyz as x is ε, y is the first string and z is everything else so it can be pumped.

Again if s is generated by 10^*1 then also user can write it as xyz where x is 1, y is 0, and z is the remainder so it could be pumped.

Hence the minimum pumping length is 3.

Comment

Step 5 of 10

e.

Let s be a string in the language.

Now s could be ε but it cannot be pumped so the length is not 0. Next s could be 01 which if divide in xyz as x is empty string ε, y is 01, and z is everything after then it satisfies the three conditions of pumping lemma.

Pumping length is not 1 because since there is no string of length 1 in the language.

Hence the minimum pumping length is 2.

Comment

Step 6 of 10

f.

Let s be a string in the language then s is ε and according to pumping lemma it cannot be pumped. As per the pumping lemma, pumping length should greater then equal to 1.

Hence the minimum pumping length is 0.

Comments (1)

Step 7 of 10

g.

The minimum pumping length of the language could not be 2 as 00 being in the language it could not be pumped. Let s be the string of length at least 3 in the language so minimally s could be 100 or 010 or 001.

Now dividing s in xyz in all the three cases user get, for 100 x is ε, y is 1, and z is the remainder, for 010 x is 0, y is 1, and z is the remainder and for 001 x is 00, y is 1, and z is the remainder. All satisfies Pumping Lemma's three conditions.

Hence the minimum pumping length is 3.

Comment

Step 8 of 10

h.

The minimum length string in the given language is 100 but it cannot be pumped. Next minimum length string is 10100. If divide it according to the pumping lemma in xyz then x be 10, y be 10, and z be the rest of the string then y can be pumped.

Hence the minimum pumping length is 4.

Comment

Step 9 of 10

i.

If s be a string in the language then s is 1011. If set p = 4, then claim s is pumpable (which it is not, as it is the only string in the language). This should be 5.

Hence then the minimum pumping length is 5.

Comment

Step 10 of 10

j.

Say s be a string in the language Σ^*. According to pumping lemma if divide s in xyz then x be the empty string, y is $(\varepsilon \mid 0 \mid 1)$ and z is empty string. Now ε could not be pumped.

Hence the minimum pumping length is 1.

Comment