Problem

Recall that you may consider circuits that output strings over {0,1} by designating several output gates.

$$\text{Let } add_n : \{0,1\}^{2n} \longrightarrow \{0,1\}^{n+1}$$ take two n bit binary integers and produce the n+1 bit sum. Show that you can compute the add_n function with O(n) size circuits.

Step-by-step solution

Step 1 of 1

Consider the following *add* functions:

$$add_n : \{0,1\}^{2n} \to \{0,1\}^{n+1}$$

This function basically takes two *n* bit binary integer and thus it produces *n*+1 bit sum.

User need to compute the complexity of the *add* function.

Speed of the circuits depends upon the gate which has been chosen. If the complexity is more than at that time, speed should be increased.

So, there is the trade-off between the space and time complexity. It implies that if it is taking less time than speed is more or vice versa.

In case of binary adder, input is two bit binary integer. Suppose there are *n* digits then it is represented with the help of *n* line. Summation of the two *n* bit binary number is equal to *n*+1 bit binary number.

Sum generated by the two *n* bit binary integer is *n*+1 bit long which can be represented in the form of $O(n)$.

As $n + 1$, $n + 2 \ldots$ $n + n$ are represented in form of order of *n*.

Hence, it is proved that complexity of the add_n function is $O(n)$.

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