Problem

Define $CYCLE$ = { $\langle G \rangle$ I G is a directed graph that contains a directed cycle}. Show that $CYCLE$ is NL-complete.

Step-by-step solution

Step 1 of 2

<u>NL – completeness:</u>

A language *B* is *NL*- complete if

1. $B \in NL$, and

2. Every *A* in *NL* is log space reducible to *B*.

Given that

$CYCLE =$ { $\langle G \rangle | G$ is a directed graph that contains a directed cycle}.

Now we have to prove that CYCLE is *NL* – complete.

1. $\underline{CYCLE \in NL}$:

We know that

"*NL* is the class of languages that are decidable in logarithmic space on a Non deterministic Turing machine"

Let *N* be the nondeterministic Turing machine that decides $CYCEL$.

The construction of *N* is as follows:

$N =$ "On input $\langle G \rangle$ (*G* is an directed graph):

1. Select a vertex *u* as starting vertex.

2. Select an edge (u,v) from *u*.

3. Run $PATH(u,v)$.

4. Start traversal through (u,v), if we come back to *u* through on edge different that

(u,v), then direct cycle will exist

5. Otherwise, reject."

• Since all vertices and all the edges are enumerated in log space *N* decides CYCLE in logarithmic space.

• Therefore, $CYCLE \in NL$.

Comment

Step 2 of 2

2. $\underline{PATH \leq_L CYCLE}$:

• Now we have to reduce $PATH$ to $CYCLE$.

• For that we have modify the *PATH* problem instance $\langle G,s,t \rangle$ by adding an edge from *t* to *s* in *G*.

• If path exists from *s* to *t* in *G* then direct cycle will exist in modified *G*.

• But some cycles may already be present in *G*.

• So, so solve that problem change *G* so that it contains no cycles.

• A leveled directed graph is non where the nodes are divided into graphs, $A_1, A_2, \ldots A_k$ called levels.

• Only edges from one level to next higher level are permitted.

• G' is the leveled graph of *G* which has two nodes *s* and *t*, and *m* nodes in total.

• Draw an edge from node *i* at each level to node, *j* in the next level if *G* contain an edge from *i* to *j*.

• Also, Draw on edge from node 1 in each level to node *l* in the next level.

• Let s' be the node *s* in the first level and t' be the node *t* in the next level.

• Graph *G* contains a path from *s* to *t* iff G' contain a path from s' to t'.

• If we add an edge from t' to s' in G' then reduction from *PATH* to *CYCLE* will be obtained, this reduction is implemented in log space.

Therefore (1) and (2) $CYCEL$ is $NL-$complete.

Comment