Problem

Let

$$C = \{\langle G, x\rangle |\ G \text{ is a CFG } x \text{ is a substring of some } y \in L(G)\}.$$

Show that C is decidable. (Hint: An elegant solution to this problem uses the decider for E_{CFG}.)

Step-by-step solution

Step 1 of 3

Suppose $C=\{\langle G,x\rangle \mid G \text{ is a CFG } x \text{ is a substring of some } y \in L(G)\}$. Now consider the following proof which shows that C is decidable.

A Turing machine M is constructed in such a way that decides C as follows:

• A DFA *A* is constructed in such a way that it is used to recognize that language of the regular expression $\Sigma^* \circ \{x\} \circ \Sigma^*$ (every string with *x* as their substring).

• A DFA *F* is constructed which is used for the context free language $L(G) \cap L(A)$.

• Perform simulation on the Turing machine that decides E_{CFG} on $L(F)$. If it accept, reject, otherwise accept.

Comments (1)

Step 2 of 3

It is already know that a language is also a context free language if it is an intersection of a regular language and a context free language. Therefore, *F* will be CFG.

• Furthermore, $L(A)$ is a language of every strings with *x* as their substring and it is described above that it is also a regular language.

• So, **if *G* produces some string *y* with *x* as its substring, the intersection,** $L(F)$ **, should be non-empty.**

Comment

Step 3 of 3

Now it can be concluded from the above that ***C* is decidable.**

Comment