

Problem

Let $G = (V, \Sigma, R, S)$ be the following grammar. $V = \{S, T, U\}$; $\Sigma = \{0, \#\}$; and R is the set of rules:

$$\begin{aligned} S &\rightarrow TT \mid U \\ T &\rightarrow 0T \mid T0 \mid \# \\ U &\rightarrow 0U00 \mid \# \end{aligned}$$

- Describe $L(G)$ in English.
- Prove that $L(G)$ is not regular.

Step-by-step solution

Step 1 of 4

Let $G = (V, \Sigma, R, S)$ be the grammar. $V = \{S, T, U\}$; $\Sigma = \{0, \#\}$. R is the set of rules.

$$\begin{aligned} S &\rightarrow TT \mid U \\ T &\rightarrow 0T \mid T0 \mid \# \\ U &\rightarrow 0U00 \mid \# \end{aligned}$$

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Step 2 of 4

Description of $L(G)$ in English is given as

$L(G)$ is the set of all strings containing 0's and #'s of the form(s)

- Strings containing exactly two #'s and any number of 0's.
- Strings containing exactly one # and the number of 0's on the left of # are half the number of 0's to the right.

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Step 3 of 4

To prove the language $L(G)$ is not regular, consider by contradiction $L(G)$ is regular.

Let $A = L(G) \cap 0^* \# 0^*$.

From our consideration $L(G)$ is regular. So A is regular.

Since A is regular, using pumping lemma let p be the pumping length of the regular language.

Consider the string $w = 0^p \# 0^{2p}$.

Clearly length of the string $w \in A$ is greater than p . That is $|w| > p$.

So, by pumping lemma we have $w = xyz$ such that

$|xy| \leq p$, $y \neq \epsilon$ and $xy^i z \in L(G)$ for all $i \geq 0$.

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Step 4 of 4

Since A is a regular language, consider the possible ways of cutting the string w .

- If x contains the character $\#$, then y will be on the right side of $\#$. Pumping y down, increases the number of 0 's on the left such that number of 0 's on the left of $\#$ are not equal to half the number of 0 's to the right.

- If y contains the character $\#$, then pumping y down, increases the number of $\#$'s in the string.

- If z contains the character $\#$, then y will be on the left side of $\#$. Pumping y down, decreases the number of 0 's on the right such that number of 0 's on the left of $\#$ are not equal to half the number of 0 's to the right.

The resulting string of all the above cases does not belong to A . So, we can say that A does not satisfy pumping lemma. Hence A is not a regular language.

This is a contradiction to our statement that $L(G)$ regular language.

Therefore, $L(G)$ is not a regular language.

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