Problem

This exercise concerns TM M_1, whose description and state diagram appear in Example 3.9. In each of the parts, give the sequence of configurations that M_1 enters when started on the indicated input string.

A**a.** 11.

b. 1#1.

c. 1##1.

d. 10#11.

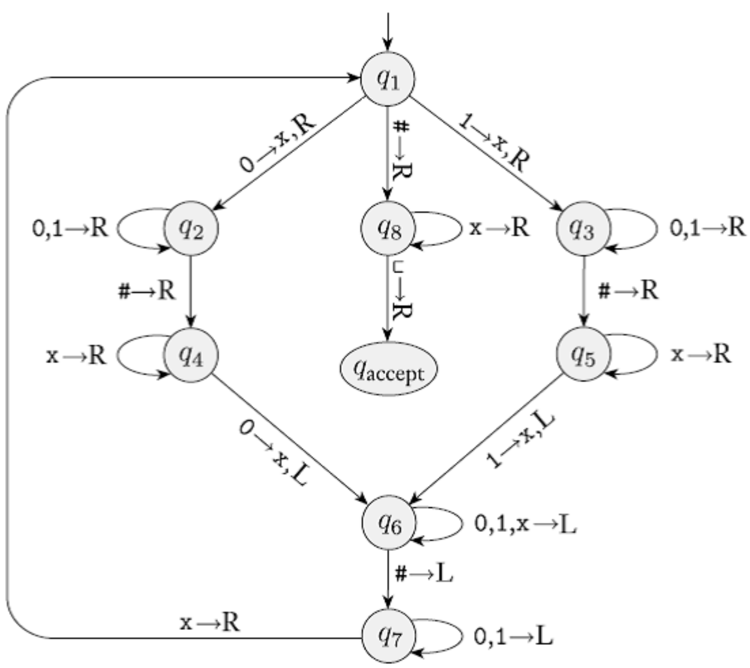
e. 10#10.

Example 3.9

EXAMPLE 3.9

The following is a formal description of $M_1 = (Q, \Sigma, \Gamma, \delta, q_1, q_{\text{accept}}, q_{\text{reject}})$, the Turing machine that we informally described (page 167) for deciding the language $B = \{w\#w \mid w \in \{0,1\}^*\}$.

- $Q = \{q_1, \ldots, q_8, q_{\text{accept}}, q_{\text{reject}}\}$,
- $\Sigma = \{0,1,\#\}$, and $\Gamma = \{0,1,\#,x,\sqcup\}$.
- We describe δ with a state diagram (see the following figure).
- The start, accept, and reject states are q_1, q_{accept}, and q_{reject}, respectively.



Step-by-step solution

Step 1 of 6

Consider the language $B = \{w\#w \mid w \in \{0,1\}^*\}$. The Turing machine M_1 that decides the language B.

In the state diagram, reject state has not shown for simplicity. If the state does not have an outgoing transition for any symbol, then it moves to the reject state q_{reject}.

In the state diagram, the label $0 \rightarrow x, R$ appears on the transition from q_1 to q_2. This label signifies that, the state q_1 with head reading 0, the machine goes to state q_2, writes *x*, and moves the head to the right.

Comment

Step 2 of 6

a.

Consider the input string 11. The sequence of configurations that M_1 enters are as follows:

$\rightarrow q_1 11$
$\rightarrow \mathrm{x} q_3 1$
$\rightarrow \mathrm{x} 1 q_3 \sqcup$ $\quad$ [$\because$ q_3 is not reading $\sqcup$, so it enters to reject state]
$\rightarrow \mathrm{x} 1 \sqcup q_{\text{reject}}$

Finally, M_1 enters the q_{reject} state. Hence input 11 is rejected.

Comment

Step 3 of 6

b.

Consider the input string 1#1. The sequence of configurations that M_1 enters are as follows:

$\rightarrow q_1 1\#1$
$\rightarrow \mathrm{x} q_3 \#1$
$\rightarrow \mathrm{x}\# q_5 1$
$\rightarrow \mathrm{x} q_6 \#\mathrm{x}$
$\rightarrow q_7 \mathrm{x}\#\mathrm{x}$
$\rightarrow \mathrm{x} q_1 \#\mathrm{x}$
$\rightarrow \mathrm{x}\# q_8 \mathrm{x}$
$\rightarrow \mathrm{x}\#\mathrm{x} q_8 \sqcup$
$\rightarrow \mathrm{x}\#\mathrm{x} \sqcup q_{\text{accept}}$

Finally, M_1 enters q_{accept} state. Thus, the input 1#1 is accepted.

Comment

Step 4 of 6

c.

Consider the input string 1##1. The sequence of configurations that M_1 enters are as follows:

$\rightarrow q_1 1\#\#1$
$\rightarrow \mathrm{x} q_3 \#\#1$
$\rightarrow \mathrm{x}\# q_5 \#1$ $\quad$ [$\because$ q_5 is not reading #, so it enters to reject state]
$\rightarrow \mathrm{x}\#\# q_{\text{reject}} 1$

Finally, M_1 enters q_{reject} state. Thus, the input 1##1 is rejected.

Comment

Step 5 of 6

d.

Consider the input string 10#11. The sequence of configurations that M_1 enters are as follows:

$\rightarrow q_1 10\#11$
$\rightarrow \mathrm{x}q_3 0\#11$
$\rightarrow \mathrm{x}0q_3\#11$
$\rightarrow \mathrm{x}0\#q_5 11$
$\rightarrow \mathrm{x}0q_6\#\mathrm{x}1$
$\rightarrow \mathrm{x}q_7 0\#\mathrm{x}1$
$\rightarrow q_7 \mathrm{x}0\#\mathrm{x}1$
$\rightarrow \mathrm{x}q_1 0\#\mathrm{x}1$
$\rightarrow \mathrm{xx}q_2\#\mathrm{x}1$
$\rightarrow \mathrm{xx}\#q_4 \mathrm{x}1$
$\rightarrow \mathrm{xx}\#\mathrm{x}q_4 1$ [$\because q_4$ is not reading 1, so it enters to reject state]
$\rightarrow \mathrm{xx}\#\mathrm{x}1q_{\mathrm{reject}}$

Finally, M_1 enters q_{reject} state. Thus, the input 10#11 is rejected.

Comment

Step 6 of 6

e.

Consider the input string 10#10. The sequence of configurations that M_1 enters are as follows:

$\rightarrow q_1 10\#10$

$\rightarrow \mathrm{x} q_3 0\#10$

$\rightarrow \mathrm{x}0 q_3 \#10$

$\rightarrow \mathrm{x}0\# q_5 10$

$\rightarrow \mathrm{x}0 q_6 \#\mathrm{x}0$

$\rightarrow \mathrm{x} q_7 0\#\mathrm{x}0$

$\rightarrow q_7 \mathrm{x}0\#\mathrm{x}0$

$\rightarrow \mathrm{x} q_1 0\#\mathrm{x}0$

$\rightarrow \mathrm{xx} q_2 \#\mathrm{x}0$

$\rightarrow \mathrm{xx}\# q_4 \mathrm{x}0$

$\rightarrow \mathrm{xx}\#\mathrm{x} q_4 0$

$\rightarrow \mathrm{xx}\# q_6 \mathrm{xx}$

$\rightarrow \mathrm{xx} q_6 \#\mathrm{xx}$

$\rightarrow \mathrm{x} q_7 \mathrm{x}\#\mathrm{xx}$

$\rightarrow \mathrm{xx} q_1 \#\mathrm{xx}$

$\rightarrow \mathrm{xx}\# q_8 \mathrm{xx}$

$\rightarrow \mathrm{xx}\#\mathrm{x} q_8 \mathrm{x}$

$\rightarrow \mathrm{xx}\#\mathrm{xx} q_8 \sqcup$

$\rightarrow \mathrm{xx}\#\mathrm{xx}\sqcup q_{\mathrm{accept}}$

Finally, M_1 enters q_{accept} state. Thus, the input 10#10 is accepted.

Comment