## Problem

**a.** Let C be a context-free language and R be a regular language. Prove that the language C ∩ R is context free.

**b.** Let A = {w|w ? {a, b, c}* and w contains equal numbers of a's, b's, and c's}. Use part (a) to show that A is not a CFL.

## Step-by-step solution

**Step 1** of 2

2287-2-18P AID: 824

RID: 944

a) Let $C$ be a context free language and $R$ be a regular language.

Now we have to prove that the language $C \cap R$ is a context free.

In order to prove, let us consider

$P$ be the PDA (Push Down Automata) recognizes $C$, and

$D$ be the DFA (Deterministic Finite Automata) recognizes $R$.

Now we construct a PDA that recognizes $C \cap R$ with the set of states $Q \times Q'$

where

$Q$ be the set of states of $P$ and

$Q'$ is the set of states of $D$

Here $P'$ will do what $P$ does and keep track of the states of D.

The PDA that recognizes $C \cap R$ accepts the string $w$ if and only if it stops a state $q \in F_P \times F_D$

where

$F_P$ is the states of accepts of $P$ and

$F_D$ is the states of accepts of $D$.

So $C \cap R$ is recognized by $P'$.

**Therefore, $C \cap R$ is context free.**

Comment

**Step 2** of 2

b) Given the language is

$$A = \left\{ w \mid w \in \{a,b,c\}^* \text{ and contains equal number of } a\text{'s, } b\text{'s and } c\text{'s} \right\}$$

Now we have to prove $A$ is not a CFL (Context Free Language) .

Let $R$ be the regular language $a^*b^*c^*$.

If $A$ were a CFL (Context Free Language) then $A \cap R$ would be a CFL (using the result proved above in part (a) of this problem).

Hence, inorder to prove that $A$ is not a CFL it is enough to prove that $A \cap R$ is not a CFL.

We have $A \cap R = \left\{ a^n b^n c^n \mid n \geq 0 \right\}$.

We will prove $A \cap R$ is not a CFL by taking a contradiction.

Assume that $A \cap R$ is a CFL.

Using the pumping lemma, which states thet every context-free language has a special calue called *pumping length* such that all longer strings in the language can be "pumped", let $p$ be the pumping length for $A \cap R$.

Consider a string $s = a^p b^p c^p$.

Clearly $s$ is a member of $A \cap R$ and of length at least $p$.

Now we prove that one condition of pumping lemma violated by proving $s$ cannot be pumped.

If we divide $s$ into $uvxyz$ , condition 2 stipulates that either $v$ or $y$ is non-empty.

Now consider one of the two cases, depending on wheather substring $v$ and $y$ contains more than one type of alphabet symbol.

1. If both $v$ and $y$ contain only one type of symbol, $v$ doesn't contain both $a$'s and $b$'s or both $b$'s and $c$'s, and the same holds for $y$. Here the string $uv^2xy^2z$ cannot contain equal number of $a$'s, $b$'s and $c$'s. Therefore it cannot be a member of $A \cap R$ which violates the first condition of the pumping lemma and thus is a contradiction to our hypothsis.

2. If either $v$ or $y$ contain more than one type of symbol $uv^2xy^2z$ may contain equal number of the three alphabet symbols but not in the correct order. Hence it cannot be a member of $A \cap R$ and thus is a contradiction to our hypothsis.

One of the above two case must occur. However, both the cases raised contradiction. This is because of our assumption $A \cap R$ is a CFL.

Hence our assumption is false and $A \cap R$ is not a CFL

**Therefore, $A$ is not a CFL.**

---

Comments (3)