Problem

Let EQ_{REX} = { $\langle R, S\rangle$ | R and S are equivalent regular expressions}. Show that EQ_{REX} ? PSPACE.

Step-by-step solution

Step 1 of 2

Language is $EQ_{REX} = \{(R,S) \mid R \text{ and S are equivalent regular expression}\}$

To show: $EQ_{REX} \in PSPACE$

PSPACE: PSPACE is deterministic Turing machine that contains the class of languages that are decidable in polynomial space on a deterministic Turing machine that is:

$$PSPACE = \bigcup_k SPACE(n^k)$$

For any language A, it is known that:

$$\begin{aligned} &\overline{A} \in NPSPACE \\ &\Rightarrow \overline{A} \in PSPACE \\ &\Rightarrow A \in PSPACE \end{aligned}$$

Thus, if it is shown that $\overline{EQ_{REX}} \in PSPACE$ then that implies that $EQ_{REX} \in PSPACE$

It is known that NPSPACE is non-deterministic Turing machine that contains the class of languages which are decidable in polynomial space.

Comment

Step 2 of 2

Let M be the non-deterministic Turing machine that decides $\overline{EQ_{REX}}$ in a polynomial space as follows:

M= “On input (R, S) where R, S are equivalent regular expressions.” the following points are followed:

- Construct non-deterministic finite automata $N_x = (Q_x, \Sigma, \delta_x, q_x, A_x)$ such that $L(N_x) = L(X)$ for $X \in \{R,S\}$.
- Let $m_X = \{q_X\}$.
- Repeat $2^{\max}\ X \in \{R,S\}^{|Q_x|}$ times.
- If $m_k \cap A_s = \phi \Leftrightarrow m_s \cap A_s \neq \phi, accept$.
- Pick any $a \in \Sigma$ and change m_x to $\bigcup_{q \in m_x} \delta_x(q,a)$ for $X \in \{R,S\}$.
- Reject

Hence, non-deterministic Turing machine M decides $\overline{EQ_{REX}}$ in polynomial space

Therefore, $\overline{EQ_{REX}} \in NPSPACE$ and hence $\overline{EQ_{REX}} \in PSPACE$

Comment