Problem

Prove that if NEXPTIME ≠ EXPTIME, then P ≠ NP. You may find the function pad, defined in Problem 9.13, to be helpful.

Step-by-step solution

Step 1 of 1

If $\mathbf{EXPTIME} \neq \mathbf{NEXPTIME}$ then $\mathbf{P} \neq \mathbf{NP}$ can be proved by taking its contra positive . If $\mathbf{P} = \mathbf{NP}$ is assumed then $\mathbf{EXPTIME} = \mathbf{NEXPTIME}$ will have to proof.

• Suppose $L \in \mathrm{NTIME}\left(2^{2^c}\right)$ then the following language $L_{\text{pad}} = \left\{ <x, 1^{2|x|^c} >: x \in L \right\}$ is in NP (in fact in $\mathrm{NTIME}(n)$). This process of adding a string of symbols to each string in the language is called **padding**.

• Hence, if $\mathbf{P} = \mathbf{NP}$ then L_{pad} is in P but if L_{pad} is in P then L is in $\mathbf{EXPTIME}$.

• To conclude whether an input x is in L, it just pads the input and decides whether it is in L_{pad}. This can be achieved by using the polynomial-time machine for L_{pad}.

Therefore, it can be said that if $\mathbf{P} = \mathbf{NP}$ is assumed then $\mathbf{EXPTIME} = \mathbf{NEXPTIME}$. Thus, from the above explanation it may also be concluded that if $\mathbf{EXPTIME} \neq \mathbf{NEXPTIME}$ then $\mathbf{P} \neq \mathbf{NP}$.

Comment