CSE 303

Intro to Theory of Computation
Homework: 1

Problem 1:-

a) 1) $J \to \neg J \therefore \neg J$ Chapter 15. ExC

1	$J \to \neg J$	
2.	$\quad J$	
3.	$\quad \neg J$	$\to$E 1, 2
4.	$\quad \bot$	$\neg$E 2, 3
5.	$\neg J$	$\neg$I 2-4

5) $(C \land D) \lor E \therefore E \lor D$

1.	$(C \land D) \lor E$	
2.	$\quad E$	
3.	$\quad E \lor D$	$\lor$I 2
4.	$\quad C \land D$	
5.	$\quad D$	$\land$E 4
6.	$\quad E \lor D$	$\lor$I 5
7.	$E \lor D$	$\lor$E 1, 2-3, 4-6.

Chapter 17, ExB.

1. $C \to (E \land G), \neg C \to G \vdash G$

1	$C \to (E \land G)$	
2.	$\neg C \to G$	
3.	$\quad C$	
4.	$\quad E \land G$	$\to$E 1, 3
5.	$\quad G$	$\land$E 4
6.	$\quad \neg C$	
7.	$\quad G$	$\to$E 2, 6
8.	G	LEM 3-5, 6-7.

2) $M \land (\neg N \to \neg M) \vdash (N \land M) \lor \neg M$

1.	$M \land (\neg N \to \neg M)$	
2.	M	$\land$E 1
3.	$\neg N \to \neg M$	$\land$E 1
4.	$\quad \neg N$	
5.	$\quad \neg M$	$\to$E 3, 4
6.	$\quad \bot$	$\neg$E 2, 5
7.	$\neg(\neg N)$	$\neg$I 4-6
8	N	DNE 7
9	$N \land M$	$\land$I 2, 8
10	$(N \land M) \lor \neg M$	$\lor$I 9

b) Chapter 32, Exercise E

1. $\vdash \forall x Fx \lor \neg \forall x Fx$

1	$\quad \forall x Fx$	
2.	$\quad \forall x Fx \lor \neg \forall x Fx$	$\lor$I 1
3.	$\quad \neg \forall x Fx$	
4.	$\quad \forall x Fx \lor \neg \forall x Fx$	$\lor$I 3
5.	$\forall x Fx \lor \neg \forall x Fx$	LEM 1-2, 3-4.

5. $\forall x \forall y Gxy \vdash \exists x Gxx$

1.	$\forall x \forall y Gxy$	
2.	$\forall y Gcy$	$\forall$E 1
3.	Gcc	$\forall$E 2
4.	$\exists x Gxx$	$\exists$I 3

Chapter 34, Exercise A

1.8. $m = n \lor n = 0,\ Fn \vdash Fm \lor Fo$

1.	$m = n \lor n = 0$	
2.	Fn	
3.	$\quad m = n$	
4.	$\quad Fm$	$=$E 2, 3
5.	$\quad Fm \lor Fo$	$\lor$I 4
6.	$\quad n = 0$	
7.	$\quad Fo$	$=$E 2, 6
8.	$\quad Fm \lor Fo$	$\lor$I 7
9.	$Fm \lor Fo$	$\lor$E 1, 3-5, 6-8

3. $\forall x\, x = m,\ Rma \vdash \exists x Rxx$

1.	$\forall x\ x = m$	
2.	Rma	
3.	$a = m$	$\forall$E 1
4.	Rmm	$=$E 2, 3
5.	$\exists x Rxx$	$\exists$I 4

Problem 2:-
Proof of PMI as English para.

We wish to show that "$n^2 + n$ is even" for all nat nos n using the Principle of Math. Induction. Let $P(n)$ denote "$n^2 + n$ is even", we first check the basis case i.e $P(0)$. $P(0) = 0^2 + 0 = 0$ is even. Thus $P(0)$ holds. Let k be a arbitary but fixed no and assume that $P(k)$ holds. Thus $k^2 + k$ is even. We intend to prove that given $P(k)$ holds, $P(k+1)$ also holds. We have $\cancel{P(k+1)} = (k+1)^2 + (k+1) = k^2 + 2k + 1 + k + 1 = k^2 + k + 2k + 2 = k^2 + k + 2(k+1)$. We have from $P(k)$, $k^2 + k$ is even and $2(k+1)$ is also even since it is a multiple of 2. It follows that $(k^2+k) + 2(k+1)$ is even since sum of two even nos is even. Thus $P(k+1)$ also holds. Thus by Principle of Mathematical Induction, we can say that $n^2 + n$ is always even for all natural nos n.

Proof of PMI in Fitch-Style

Let $P(n)$ be the statement. $\forall n.\ n^2+n$ is even

$\cancel{\forall}$

$P(0)$: $0^2+0 = 0$ is even	Basis
$\quad$ $P(k)$: k^2+k is even	Induction Hypothesis
$\quad$ $k^2+k+2(k+1)$ is even	
$\quad$ $k^2+2k+1+(k+1)$ is even	
$\quad$ $(k+1)^2+(k+1)$ is even	
$\quad$ $P(k+1)$ is even	Induction Step
$P(k) \rightarrow P(k+1)$	$\rightarrow I$
$\forall n.\ P(n) \rightarrow P(n+1)$	$\forall I$
$\forall n.\ P(n)$	

Problem 3 :-

In the problem, the RHS should be $(A-B) \cup (A-C)$.

We have $A \cap B \equiv \{x.\ x \in A \wedge x \in B\}$

$A - B \equiv \{x \in A,\ x \notin B\}$

We introduce if $x \in A$ Ax & if $x \notin A$ $\neg Ax$

$\cancel{\forall x. x \in \{A - B \wedge C\}}$

$\cancel{\forall x.\ x \in A \wedge x \notin \{B \wedge C\}}$

$\cancel{\forall x.\ x \in A \wedge x \notin B \cup}$

1.	$\forall x.\ x \in A \wedge x \notin \{B \cap C\}$	
2.	$Ac \wedge \neg(Bc \wedge Cc)$	$\forall E$ 1
3.	Ac	$\wedge E$ 2
4.	$\neg(Bc \wedge Cc)$	$\wedge E$ 2
5.	$\neg Bc \vee \neg Cc$	DeM 4.
6.	$\neg Bc$	
7.	$Ac \wedge \neg Bc$	$\wedge I$ 3, 6
8.	$(Ac \wedge \neg Bc) \vee (Ac \wedge \neg Cc)$	$\vee I$ 7
9	$\forall x \{x \in A.\ x \notin B\} \cup \{x \in A;\ x \notin C\}$	$\forall I$ 7.
10	$\neg Cc$	
11.	$\cancel{Ac \wedge \neg Cc}$ $Ac \wedge \neg Cc$	$\wedge I$ 3, 10.
12	$(Ac \wedge \neg Bc) \vee (Ac \wedge \neg Cc)$	$\vee I$ 11
13.	$\forall x \{x \in A.\ x \notin B\} \cup \{x \in A;\ x \notin C\}$.	$\forall I$. 12.
14	$\forall x \{x \in A.\ x \notin B\} \cup \{x \in A\ x \notin C\}$	$\vee E$ 5, 6-9, 10-13

So we have proved that.
$$\forall x \in A - (B \cap C),\ x \in (A - B) \cup (A - C).$$

~~the~~ the converse will go along similar lines & we can prove that
$$\forall x \in (A - B) \cup (A - C),\ x \in A - (B \cap C).$$

Thus $A - (B \cap C) = (A - B) \cup (A - C)$.