

## Problem

If we disallow  $\epsilon$ -rules in CFGs, we can simplify the  $DK$ -test. In the simplified test, we only need to check that each of  $DK$ 's accept states has a single rule. Prove that a CFG without  $\epsilon$ -rules passes the simplified  $DK$ -test iff it is a DCFG.

## Step-by-step solution

### Step 1 of 1

#### Given:

Consider a context free grammar  $C$  which does not have  $\epsilon$  rule. Apply  $DK$ -test on  $C$  to check whether it is a deterministic CFG or not.

#### DK Test:

This test is used to check whether CFG is deterministic CFG or not. Consider a CFG  $C$ , create  $DK$  an associated DFA. Check whether CFG is deterministic by checking the accept state of  $DK$ . Every accept states must have:

- Only 1 completed rule.
- There should be no dotted rule that is mean dot should not be immediately after the terminal.

#### Proof:

Assume a contrary that  $C$  is not a deterministic CFG to show the failure of  $DK$ -test. Consider a valid string  $ahb$  which has  $h$  as unforced handle. It may possible that some other valid string  $ahb'$  has another handle  $\hat{h} \neq h$ . Therefore  $ahb'$  can be rewrite as  $\hat{a}\hat{h}\hat{b}$  where  $b'$  is a terminal.

- When  $ah = \hat{a}\hat{h}$ , this lead to change the reduce rule because  $h$  and  $\hat{h}$  both are two different handle. Hence,  $ah$  take the  $DK$  to state which has two completed rule which does not satisfy the  $DK$ -test.
- When  $ah \neq \hat{a}\hat{h}$ , the one extends the other. Assume that proper prefix of  $\hat{a}\hat{h}$  is  $ah$ . There are same arguments with the interchanged string and in place of  $b'$  use  $b$ .

Assume that  $w$  is an accepting state in which  $DK$  enters on  $ah$  input. The state  $w$  must be an accepting states because  $h$  is the handle of  $ahb$ . As  $w$  is the accepting state, so the transition arrow must terminate at  $w$  also  $\hat{a}\hat{h}$  take  $DK$  to accepting state by  $w$ .

The transition must label with  $b'$  because  $b' \in \Sigma^+$  also  $C$  does not have null rule. Hence  $w$  have dotted rule that is, dot exist immediately after terminal symbol which does not satisfy the  $DK$ -test.

#### Conclusion:

The  $DK$  test fails; therefore contradiction occurs that  $C$  is not a deterministic CFG. Hence CFG is a DCFG.

---

[Comment](#)