Problem

Use the results of Exercise 2.16 to give another proof that every regular language is context free, by showing how to convert a regular expression directly to an equivalent context-free grammar.

Step-by-step solution

Step 1 of 3

Consider the following data:

From Exercise 2.16, it is clear that Context free language is closed under union operation, concatenation operation and star operation.

Comment

Step 2 of 3

The definition of the regular expressions it can be inferred and known that regular expression consist of series of operations such as union, concatenation and star.

Statement: Every regular language is context free language

Proof by using induction:

Assume that *E* is a regular expression and there exists a CFG *G* such that $L(E)=L(G)$. The proof will be carried out by different number of operators in an expression *E*.

Step-1:

If $Operation(E)=0$ then E is either $\phi, \in or\ a$ in $\sum$.

Step-2:

- If $E=\phi$: $G=(\{S\},\{\ \},P,S)$. Here $P=\{\ \}$.
- If $E=\in$: $G=(\{S\},\{\ \},P,S)$. Here $P=\{S\rightarrow\in\}$.
- If $E=a$: $G=(\{S\},\{a\},P,S)$. Here $P=\{S\rightarrow a\}$.

For every regular expression consisting of $\phi, \in or\ a$ the CFG can be written. So, every regular expression is context free.

Thus, every regular language is context free language.

Comment

Step 3 of 3

The context free languages are superset of regular languages. The rules for converting the regular languages consisting of union, concatenation and Kleene closure are as follows:

- If the regular expression E consists of E_1 and E_2 such that the concatenation $E=E_1E_2$, then the production can be easily expressed as $S\rightarrow E_1E_2$.
- If the regular expression E consists of E_1 and E_2 such that the union $E=E_1 \mid E_2$, then the production can be easily expressed as $S\rightarrow E_1 \mid E_2$.
- If the regular expression E consists of E_1 such that the Kleene closure $E=E_1^*$, then the production can be easily expressed as $E_1^*\rightarrow EE_1^* \mid \in$.

Comment