Problem

Show that the set of incompressible strings is undecidable.

Step-by-step solution

Step 1 of 2

Incompressible strings:

Let w_i be a string. If w_i doesn't have any description shorter than itself then w_i is incompressible.

Now we have to show that set of incompressible strings is un-decidable.

Let *A* be the set of incompressible strings and assume the contradiction *A* is decidable.

We construct a machine *M* which enumerates *A*.

Enumeration: $f : A \to N$ such that $f(w1) = 1, f(w2) = 2, f(w3) = 3...$ where first, second, and third shortest strings are respectively *w*1, *w*2, & *w*3.

Since *A* reaches infinite there is a string $w_i \in A$.

Comment

Step 2 of 2

Define a Turing machine *T* which computes w_i incompressible string of length *n*

$T ="$ on input *n*

1. Returns the first string w_i that *M* enumerates of length *n*.

2. If $K(\langle T, n \rangle) = c + \log(n)$. For any constant *c*

Then we find *n* such that

$|w_i| = n > c + \log(n)$

The string w_i is shorter description on $\langle M', f(w_i) \rangle$. Where M' is a machine, $f(w_i)$ is input and output as w_i.

Run machine M' each string in lexicographic order from and output the same from *M.*

It contradicts that w_i is compressible. Therefore our assumption that "*A* is decidable" is wrong. So for *A* set of incompressible strings *A* is un-decidable.

Comment