Problem

Show that single-tape TMs that cannot write on the portion of the tape containing the input string recognize only regular languages.

Step-by-step solution

Step 1 of 3

Let M be the single – tape Turing machine that cannot write on the portion of the tape containing the input string.

$M = (Q, \Sigma, \tau, q_0, q_{accept}, q_{reject})$

M works on an input string *x* as follows.

Here we consider two events.

(i) **Out event:**

In out event, the tape head moves from input portion to non – input portion, i.e., the portion of the tape on the right of the $(x)^{th}$ cell.

(ii) **In event:**

In In-event tape head moves from non – input portion to input portion.

Comment

Step 2 of 3

Consider the state q_x for Turing machine $M = (Q, \Sigma, \tau, q_0, q_{accept}, q_{reject})$ when it first enters the non – input portion (i.e., after it's first *out event*)

• In case M never enters the non – input portion.

(a) If M accepts *x,* assign $q_x = q_{accept}$

(b) If M does not accept *x,* assign $q_x = q_{reject}$

For any $q \in Q$, define a characteristic function f_x such that

$f_x(q) = q'$

That implies

If *M* is in the state *q* and about to perform an "*in event*", the next "*out event*" will change M in state q'

• In case M never enters the non – input portion again,

(a) If M enters the accept state inside the input portion, assign $f_x(q) = q_{accept}$

(b) If M does not enter the accept state, assign $f_x(q) = q_{reject}$

For two strings *x* and *y*,

If $q_x = q_y$ for all *q*, $f_x(q) = f_y(q)$, then *x* and *y* are indistinguishable by M. That is, M accepts *xz* if and only if M accepts *yz.*

Comments (1)

Step 3 of 3

As there are finite choice of q_x and f_x $\left(\text{Precisely } |\text{Q}|^{|Q|+1} \text{ such choices}\right)$, the number of indistinguishable strings are finite.

"Myhill – Nerode theorem" is used to prove whether the language is regular or not.

Statement:

A language L over alphabet Σ is regular if and only if the set of equivalent classes of I_L is finite.

I_L is the relation on Σ^* such that for two strings *x* and *y* of Σ^*

$x\, I_L\, y \Leftrightarrow \{z \mid xz \in L\} = \{z \mid yz \in L\}$

That is $x I_L y$ if and only if they are indistinguishable with respect to L

So, by Myhill – Nerode theorem the language recognized by M is regular.

Comments (1)