Problem

For a cnf-formula ? with m variables and c clauses, show that you can construct in polynomial time an NFA with $O_{(cm)}$ states that accepts all nonsatisfying assignments, represented as Boolean strings of length m. Conclude that P ≠ NP implies that NFAs cannot be minimized in polynomial time.

Step-by-step solution

Step 1 of 2

Consider a cnf-formula $\varnothing$, which consist c clauses and m variables. A NFA, D is constructed in such a way that, on input $\varnothing$, one of the c clauses is picked deterministically. Also, it reads an input of length m and checks it for acceptance. In other words, it accepts it, if the clause is not satisfied by it and otherwise reject. In addition, all the inputs are accepted by this NFA D which is also not equal to m.

- There is a need of $O(m)$ states for every clause. So that, D consist $O(cm)$ states. Therefore, it is obvious that a polynomial time can be used to compute D.
- Now, minimum one of the clause is not satisfied for any non-satisfying assignment q. So in this case q is accepted by D.
- Now, by using the inverse of the above statement, if q is accepted by D then some clause are not satisfied.

Hence, from the above discussion it can be said that, **every non-satisfying assignments of $\varnothing$ is accepted by D.**

Comment

Step 2 of 2

Now, suppose a polynomial time can be used for minimizing NFA problem. Consider a polynomial time algorithm with a cnf-formula $\varnothing$, which consist c clauses and m variables.

- Now, the construction of a NFA D takes place in such a way that it accepts every non-satisfying assignments of $\varnothing$.
- It can be noticed from the above statement that **every binary string will be accepted by D,** if and only if $\varnothing$ **is not satisfiable**.
- Now, a new NFA D' is generated by running the NFA minimizing algorithm. If D' consists only a **single state**, which accepts every binary strings then **reject** $\varnothing$, otherwise $\varnothing$ is **accepted**.

So, **an algorithm in polynomial time for $3SAT$ can be obtained**. Hence, it can be said that $P = NP$. In other way $P \neq NP$ **implies that NFA cannot be determined in polynomial time.**

Comment