

## Problem

Prove that the class of decidable languages is not closed under homomorphism.

## Step-by-step solution

### Step 1 of 1

#### Given:

Consider a decidable language  $K$  and a homomorphism  $m$  to show that  $m(K)$  is un-decidable.

#### Proof:

$$K = \left\{ uv \mid u \in \{0,1\}^*, v \in \{a,b\}^*, u = \langle W, w \rangle, \right. \\ \left. \begin{array}{l} \text{and } v \text{ encodes an integer } n \\ \text{using turing machine } W \text{ having input } w \text{ get halts in } n \text{ steps} \end{array} \right\}$$

- Assume that
- As  $K$  is decidable so simulate Turing machine  $W$  on input  $w$  for  $n$  steps.
- Consider homomorphism  $m(0)=0$ ,  $m(1)=1$  and  $m(a)=m(b)=\varepsilon$
- $m(K) = \text{HALT}$  which is un-decidable.

Hence, Decidable language  $K$  is not closed under homomorphism  $m$ .

---

[Comment](#)