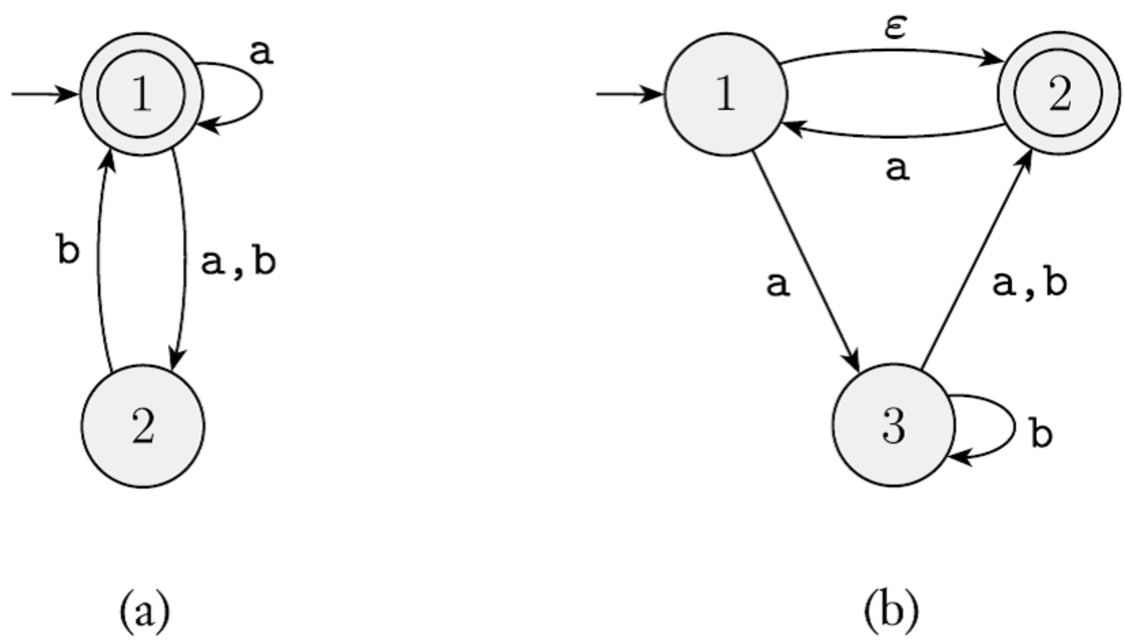
Problem

Use the construction given in Theorem 1.39 to convert the following two nondeterministic finite automata to equivalent deterministic finite automata.

THEOREM **1.39**

Every nondeterministic finite automaton has an equivalent deterministic finite automaton.

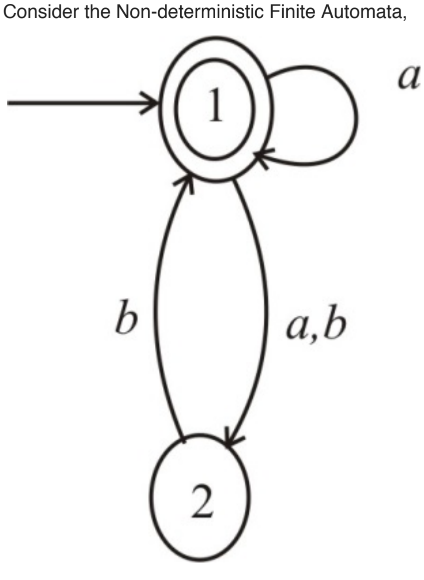


Step-by-step solution

Step 1 of 2

a.

Consider the Non-deterministic Finite Automata,



By using Theorem 1.39, "*For every non-deterministic finite automata, there is an equivalent Deterministic finite automation*".

Constructing equivalent DFA for the given NFA:

1. $Q^{1} = P(Q)$ where Q^{1} is the subset of all sets of *Q*.

So, $Q^{1} = \{\phi,\{1\},\{2\},\{1,2\}\}$

2. For an element *R* in Q^{1} and *a* in set of alphabets Σ, Calculate $\delta^{1}(R,a)=\{q\in Q \mid q\in\delta(r,a)\ \textit{for some}\ r\in R\}$. Here, δ^{1} performs the transition on *r* for some value of *a*.

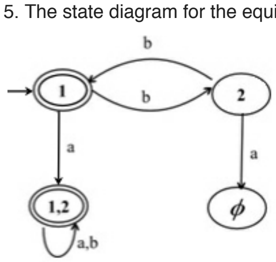
$$\begin{aligned}\delta^{1}(\phi,a)&=\delta(\phi,a)\\&=\phi\\\delta^{1}(\phi,b)&=\delta^{1}(\phi,b)\\&=\phi\\\delta^{1}(\{1\},a)&=\delta(1,a)\\&=\{1,2\}\\\delta^{1}(\{1\},b)&=\delta(1,b)\\&=\{2\}\\\delta^{1}(\{2\},a)&=\delta(2,a)\\&=\phi\\\delta^{1}(\{2\},b)&=\delta(2,b)\\&=\{1\}\\\delta^{1}(\{1,2\},a)&=\delta(\{1,2\},a)\\&=\delta(1,a)\cup\delta(2,a)\\&=\{1,2\}\cup\phi\\&=\{1,2\}\\\delta^{1}(\{1,2\},b)&=\delta(\{1,2\},b)\\&=\delta(1,b)\cup\delta(2,b)\\&=\{2\}\cup\{1\}\\&=\{1,2\}\end{aligned}$$

3. $q_0' = \{q_0\}$ where q_0 is the start state in NFA.

Here, q_0'={1}.

4. $F' = \{R\in Q' \mid R \text{ contains an accept state of NFA}\}$. The machine M accepts the possible states where the NFA is present in the accept state.

5. The state diagram for the equivalent DFA is as follows:

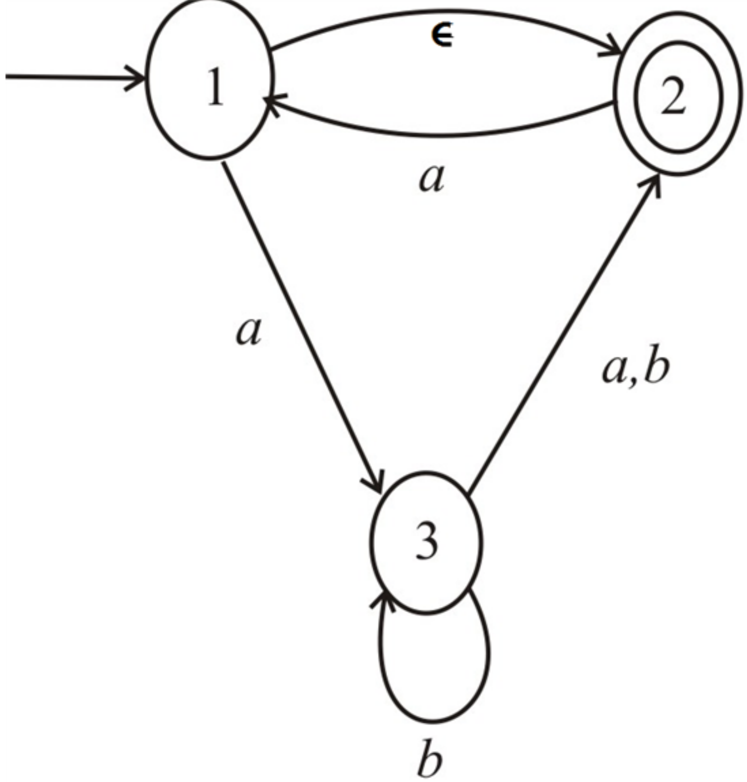


Comments (2)

Step **2** of 2

b.

Consider the Non-deterministic Finite Automata,



By using Theorem 1.39, "*For every non-deterministic finite automata, there is an equivalent Deterministic finite automation*".

Constructing equivalent DFA for the given NFA:

The initial state of DFA is 1 let x= (Q_x, Σ, δ_x, q_0, F_x).

1. $Q^1 = P(Q)$ where Q^1 is the subset of all sets of *Q*.

So, $Q^1 = \{\phi, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}\}$

2. Considering $\in$ notations for each $R \subseteq Q$.

$E(R) = \{q \mid q \text{ can be reached from } R \text{ by traveling along or more } \in \text{ arrows}\}$

The collection of states reached from R by moving along the $\in$notations is,

$$
\begin{aligned}
E(\phi) &= \phi \\
E(\{1\}) &= \{1,2\} \\
E(\{2\}) &= \{2\} \\
E(\{3\}) &= \{3\} \\
E(\{1,2\}) &= \{1,2\} \\
E(\{1,3\}) &= \{1,2,3\} \\
E(\{2,3\}) &= \{2,3\} \\
E(\{1,2,3\}) &= \{1,2,3\}
\end{aligned}
$$

3. Calculate $\delta^1(R,a) = \{q \in Q \mid q \in \delta(r,a) \text{ for some } r \in R\}$. Here, δ^1 performs the transition on *r* for some value of *a*.

$$\delta'(\phi, a) = \phi$$
$$\delta'(\phi, b) = \phi$$

$$
\begin{aligned}
\delta'(\{1\}, a) &= E(\delta(1,a)) \\
&= E(\{3\}) \\
&= \{3\}
\end{aligned}
$$

$$
\begin{aligned}
\delta'(\{1\}, b) &= E(\delta(1,b)) \\
&= E(\phi) \\
&= \phi
\end{aligned}
$$

$$
\begin{aligned}
\delta'(\{2\}, a) &= E(\delta(1,a)) \\
&= E(\{1\}) \\
&= \{1,2\}
\end{aligned}
$$

$$
\begin{aligned}
\delta'(\{2\}, b) &= E(\delta(2,b)) \\
&= E(\phi) \\
&= \phi
\end{aligned}
$$

$$\begin{aligned}\delta'(\{3\},a) &= E(\delta(3,a))\\ &= E(\{2\})\\ &= \{2\}\end{aligned}$$

$$\begin{aligned}\delta'(\{3\},b) &= E(\delta(3,b))\\ &= E(\{2,3\})\\ &= \{2,3\}\end{aligned}$$

$$\begin{aligned}\delta'(\{1,2\},a) &= E(\delta(1,a)) \cup E(\delta(2,a))\\ &= E(\{3\}) \cup E(\{1\})\\ &= \{3\} \cup \{1,2\}\\ &= \{1,2,3\}\end{aligned}$$

$$\begin{aligned}\delta'(\{1,2\},b) &= E(\delta(1,b)) \cup E(\delta(2,b))\\ &= E(\{\phi\}) \cup E(\{\phi\})\\ &= \phi \cup \phi\\ &= \phi\end{aligned}$$

$$\begin{aligned}\delta'(\{1,3\},a) &= E(\delta(1,a)) \cup E(\delta(3,a))\\ &= E(\{3\}) \cup E(\{2\})\\ &= \{3\} \cup \{2\}\\ &= \{3\} \cup \{2\}\\ &= \{2,3\}\end{aligned}$$

$$\begin{aligned}\delta'(\{1,3\},b) &= E(\delta(1,b)) \cup E(\delta(3,b))\\ &= E(\{\phi\}) \cup E(\{2,3\})\\ &= \phi \cup \{2,3\}\\ &= \{2,3\}\end{aligned}$$

$$\begin{aligned}\delta'(\{2,3\},a) &= E(\delta(2,a)) \cup E(\delta(3,a))\\ &= E(\{1\}) \cup E(\{2\})\\ &= \{1,2\} \cup \{2\}\\ &= \{1,2\}\end{aligned}$$

$$\begin{aligned}\delta'(\{2,3\},b) &= E(\delta(2,b)) \cup E(\delta(3,b))\\ &= E(\phi) \cup E(\{2,3\})\\ &= \phi \cup \{2,3\}\\ &= \{2,3\}\end{aligned}$$

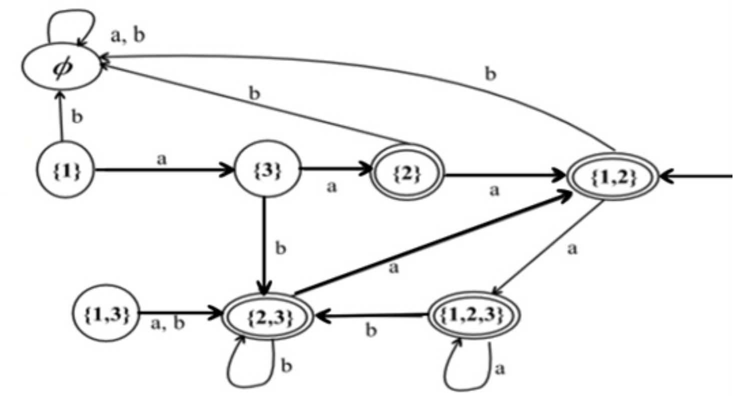
$$\begin{aligned}\delta'(\{1,2,3\},a) &= E(\delta(1,a)) \cup E(\delta(2,a)) \cup E(\delta(3,a))\\ &= E(\{3\}) \cup E(\{1\}) \cup E(\{2\})\\ &= \{3\} \cup \{1,2\} \cup \{2\}\\ &= \{1,2,3\}\end{aligned}$$

$$\begin{aligned}\delta'(\{1,2,3\},b) &= E(\delta(1,b)) \cup E(\delta(2,b)) \cup E(\delta(3,b))\\ &= E(\phi) \cup E(\phi) \cup E(\{2,3\})\\ &= \phi \cup \phi \cup \{2,3\}\\ &= \{2,3\}\end{aligned}$$

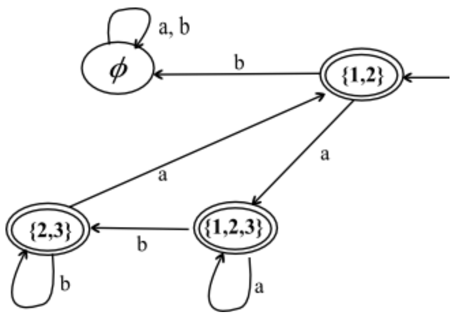
4. Changing q_0' to $E(q_0)$ the start state becomes,

$$\begin{aligned}q_0^1 &= E(q_0)\\ q_0^1 &= E(\{1\})\\ q_0^1 &= \{1,2\}\end{aligned}$$

5. The state diagram for the equivalent DFA is as follows:



Simplifying the machine by eliminating no arrow points. Here {1},{2},{1,3} and {3} do not contain any incoming arrows. Thus, the simplified machine is:



Comments (7)