Problem

Let $\Sigma = \{a, b\}$. For each $k \geq 1$, let D_k be the language consisting of all strings that have at least one a among the last k symbols. Thus $D_k = \Sigma^* \mathrm{a}(\Sigma \cup \varepsilon)^{k-1}$. Describe a DFA with at most k+1 states that recognizesD_k in terms of both a state diagram and a formal description.

Step-by-step solution

Step 1 of 2

The Language $D_k = \Sigma^* \mathrm{a}(\Sigma \cup \varepsilon)^{k-1}$ for each $k \geq 1$, over the alphabet $\Sigma = \{\mathrm{a}, \mathrm{b}\}$. Here, D_k is the language consisting of all strings that have at least one a among the last k symbols. The DFA (deterministic finite automata) that recognizes the language D_k.

The construction of M where $M = (Q_k, \Sigma, \delta_k, q_0, F)$ is as follows:

- $Q_k = \{q_0, q_1, q_2, \ldots q_k\}$ = set of states
- $\Sigma = \{a, b\}$ = set of alphabets
- $q_0 = \{q_0\}$ = start state.
- $F = \{q_1, q_2 \ldots q_k\}$ = set of final states
- $\delta_k =$ Transition function is given as follows

$$\delta_k(q, l) = \begin{cases} q_1 & i = 0 \wedge l = a \\ q_0 & i = 0 \wedge l = b \\ q_1 & i \neq 0 \wedge l = a \\ q_{(i+1) \bmod k} & i \neq 0 \wedge l = b \end{cases}$$

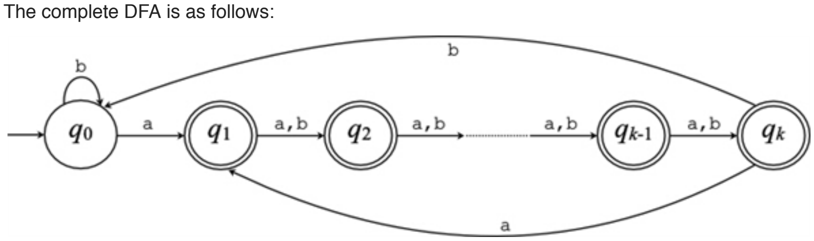
Comment

Step 2 of 2

In other words,

- M is in state q_0 if it has not seen and a within the past k letters then it will reject.
- It is in state q_i if it saw a from i letters ago.

The complete DFA is as follows:



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