Practice Midterm Exam (Solutions)

(9:45 AM – 11:00 AM : 75 Minutes)

- This exam will account for 30% of your overall grade.
- There are six (6) questions, worth 75 points in total. Please answer all of them.
- This is a *closed book*, *closed notes* exam. *No cheat sheets* are allowed.
- You are allowed to *use scratch papers* for your calculations.
- You are *not allowed to use your own calculator*. A scientific calculator will be available inside the Respondus Lockdown Browser.

Good Luck!

Question	Parts	Points
1. DFA Construction	(i), (ii)	$10 + 10 = 20$
2. DFA Composition	–	10
3. Regular Expressions	(i), (ii)	$5 + 5 = 10$
4. NFA to DFA	–	15
5. Non-regularity	–	15
6. Context-free Grammar	–	5
Total		75

QUESTION 1. [**20 Points**] **DFA Construction.** Write down a DFA in the 5-tuple form to accept each of the following two regular languages.

Assume that $\Sigma = \{a, b\}$.

Your answers do not need to include DFA diagrams (though you may draw them on your scratch papers if you like).

(a) [**10 Points**] $L = \{w|$ $aaba$ **is a substring of** $w\}$

(b) [**10 Points**] $L = \{w|$ w **does not contain** $ab\}$

SOLUTION.

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 (i) 5-tuple, M = {Q, Sigma, q, F, delta}, where,

     Set of states, Q = { q0, q1, q2, q3, q4 }
     Set of symbols, Sigma = { a, b }
     Start state, q = q0
     Set of accept states, F = { q4 }
     Transition function, delta:

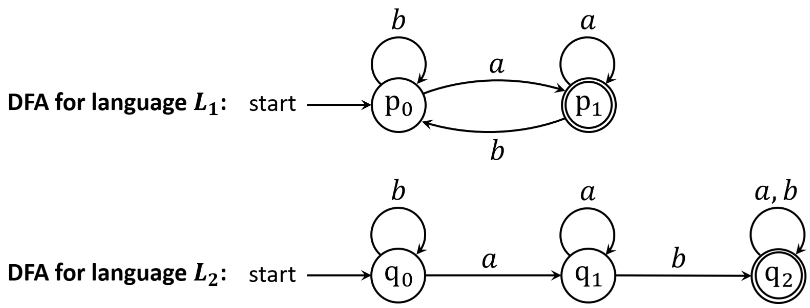
     state    a    b
     q0       q1   q0
     q1       q2   q0
     q2       q2   q3
     q3       q4   q0
     q4       q4   q4

(ii) 5-tuple, M = {Q, Sigma, q, F, delta}, where,

     Set of states, Q = { q0, q1, q2 }
     Set of symbols, Sigma = { a, b }
     Start state, q = q0
     Set of accept states, F = { q0, q2 }
     Transition function, delta:

     state    a    b
     q0       q1   q0
     q1       q1   q2
     q2       q2   q2
```

QUESTION 2. [**10 Points**] **DFA Composition.** Consider the following two DFAs.



Write down a DFA in the 5-tuple form that accepts the language $L_1 \cup L_2$.

Your answer does not need to include the DFA diagram (though you may draw it on your scratch papers if you like).

SOLUTION.

```
5-tuple, M = {Q, Sigma, q, F, delta}, where,

Set of states, Q = { (p0, q0), (p0, q1), (p0, q2), (p1, q0), (p1, q1), (p1, q2) }
Set of symbols, Sigma = { a, b }
Start state, q = (p0, q0)
Set of accept states, F = { (p0, q2), (p1, q0), (p1, q1), (p1, q2) }
Transition function, delta:

state       a          b
(p0, q0)    (p1, q1)   (p0, q0)
(p0, q1)    (p1, q1)   (p0, q2)
(p0, q2)    (p1, q2)   (p0, q2)
(p1, q0)    (p1, q1)   (p0, q0)
(p1, q1)    (p1, q1)   (p0, q2)
(p1, q2)    (p1, q2)   (p0, q2)
```

Question 3. [**10 Points**] **Regular Expressions.** Write down regular expressions for the following two languages.

(a) [**5 Points**] $L = \{w |\ n_a(w) = 2$ **or** $n_b(w) = 2\}$, $\Sigma = \{a, b, c\}$

(b) [**5 Points**] $L = \{w|$ **binary number** w **is not divisible by 8**$\}$, $\Sigma = \{0, 1\}$

Solution.

In the solutions below the letter 'U' represents the union operation.

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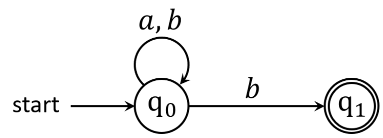
  (i) Let A = a U c
          B = b U c

      Regular expression = B*aB*aB* U A*bA*bA*

 (ii) Let S = Sigma

      Regular expression = 1 U 01 U 1S U S*(SS1 U S1S U 1SS)
```

QUESTION 4. [**15 Points**] **NFA to DFA.** Consider the following NFA.



Convert this NFA into a DFA and write it down in the 5-tuple form.

Your answer does not need to include the DFA diagram (though you may draw it on your scratch papers if you like).

SOLUTION.

```
5-tuple, M = {Q, Sigma, q, F, delta}, where,

Set of states, Q = { q0, (q0, q1) }
Set of symbols, Sigma = { a, b }
Start state, q = q0
Set of accept states, F = { (q0, q1) }
Transition function, delta:

state          a       b
q0             q0      (q0, q1)
(q0, q1)       q0      (q0, q1)
```

Question 5. [15 Points] **Non-regularity.** Use the pumping lemma to prove that the following language is not regular.

$$L = \{w|\ w = a^m b^n,\ 0 \leq m < n\},\quad \Sigma = \{a, b\}$$

Solution.

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Suppose L is regular. Then it must satisfy the pumping property.

Let s be the number of states in any finite state machine that accepts L.

Let w = a^s b^(s + 1) = xyz, where, x = a^p, y = a^q, and z = a^r b^(s + 1),

         such that |xy| <= s, |y| >= 1, and p + q + r = s.

Then xy^(i)z must belong to L for all i >= 0.

Now xyyz = a^p a^q a^q a^r b^(s + 1)
         = a^(p + q + r + q) b^(s + 1)
         = a^(s + q) b^(s + 1).

But since |y| = q >= 1, we have s + q >= s + 1.

So, xyyz = a^(s + q) b^(s + 1) is not in L, which is a contradiction!

Hence, L is not regular.
```

Question 6. [**5 Points**] **Context-free Grammar.** Write down a context-free grammar to accept the following language:

$$L = \{w|\ w = a^{n+1}b^{n+2},\ n \geq 0\},\quad \Sigma = \{a, b\}$$

Solution.

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S -> aSb
S -> abb
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