Problem

Say that a CFG is *minimal* if none of its rules can be removed without changing the language generated. Let MIN_{CFG} =

$$\{\langle G \rangle \mid G \text{ is a minimal CFG}\}.$$

a. Show that MIN_{CFG} is T-recognizable.

b. Show that MIN_{CFG} is undecidable.

Step-by-step solution

Step 1 of 6

a)

A minimal context-free grammar MIN_{CFG} is one in which no rule can be modified without changing the language generated. Now, it can Proved that MIN_{CFG} is T-recognizable.

• The grammar MIN_{CFG} can be converted to an equivalent grammar in Chomsky normal form.

• The generated CFG can be simulated by a Turing machine because for a string of length k there will be a finite number of derivations with $2k-1$ steps.

• This Turing machine U will accept the strings that are in the language.

Comment

Step 2 of 6

The Turing machine is:

U ="On input $\langle G, w \rangle$, where G is a minimal CFG and w is a string:

1. Create an equivalent grammar H in Chomsky normal form from G.

2. Let $n = |w|$.

3. If $n = 0$, then check all the derivations with one step in H, else check all the derivations with $2n-1$ steps.

4. *Accept* if any of the derivations accept w, else *reject.*

Comments (1)

Step 3 of 6

Therefore, **a minimal context-free grammar is T-recognizable.**

Comment

Step 4 of 6

b)

Consider the MIN_{CFG}. Now, it can be shown that MIN_{CFG} is un-decidable. It can be proved by contradiction, that is, by taking an assumption that MIN_{CFG} is decidable. Assume the opposite, which is the grammar MIN_{CFG} is decidable.

Comment

Step 5 of 6

It is known that if a language is decidable/ undecidable if and only if the complement of this language is decidable/ undecidable. Now consider ALL_{CFG}, which is already known as undecidable.

On input $\langle G \rangle$:

• Construction of G' takes place by adding to G a new terminal, together with productions: $G'S \rightarrow A, A \rightarrow \varepsilon, and\ A \rightarrow aA\ for\ each\ a \in \Sigma$

• Output $\langle G', A \rangle$.

Comment

Step 6 of 6

The construction of the grammar G' is takes place by f in such a way that it always show $L(G') = \Sigma^*$. Thus, if $L(G) = \Sigma^*$ then it is not necessary that A is for G'.

• The reason behind it is that every string $l = \Sigma^*$ can already be derived from G, which shows a contradiction.

Hence, from the above explanation it can be said that," MIN_{CFG} **is undecidable**".

Comment