Problem

Give context-free grammars that generate the following languages. In all parts, the alphabet Σ is {0,1}.

A**a.** {wl w contains at least three 1s}

b. {wl w starts and ends with the same symbol}

c. {wl the length of w is odd}

A**d.** {wl the length of w is odd and its middle symbol is a 0}

e. {wl w = w^R, that is, w is a palindrome}

f. The empty set

Step-by-step solution

Step 1 of 6

The alphabet $\sum$ is given by $\{0, 1\}$

(a)

The context free grammar that generates the language

$\{\omega \mid \omega \text{ contains at least three 1s}\}$ is given by

$S \rightarrow P1P1P1P$

$P \rightarrow 0P \mid 1P \mid \varepsilon$

Comments (4)

Step 2 of 6

(b)

The context free grammar that generates the language $\{\omega \mid \omega \text{ starts and ends with the same symbol}\}$ is given by

$S \rightarrow 0P0 \mid 1P1 \mid 0 \mid 1$

$P \rightarrow 0P \mid 1P \mid \varepsilon$

Comments (3)

Step 3 of 6

(c)

The context free grammar that generates the language

$\{\omega \mid \text{the length of } \omega \text{ is odd}\}$ is given by

$S \rightarrow 0 \mid 1 \mid 00S \mid 01S \mid 10S \mid 11S$

$\qquad\qquad (\text{or})$

$S \rightarrow 0 \mid 1 \mid 0S0 \mid 0S1 \mid 1S0 \mid 1S1$

Comments (14)

(d)

The context free grammar that generates the language $\{\omega \mid \text{the length of } \omega \text{ is odd and its middle symbol is a } 0\}$ is given by

$$S \to 0 \mid 0S0 \mid 0S1 \mid 1S0 \mid 1S1$$

Comments (5)

(e)

The context free grammar that generates the language $\{\omega \mid \omega = \omega^R, \text{that is, } \omega \text{ is a palindrome}\}$ is given by

$$S \to 0 \mid 1 \mid 0S0 \mid 1S1 \mid \varepsilon$$

Comments (12)

(f)

The context free grammar that generates the language

$\{\ \}$ is given by

$$S \to S$$

Comments (2)