Problem

Read the definition of *MIN-FORMULA* in Problem 7.46.

a. Show that *MIN-FORMULA* $\in$ PSPACE.

b. Explain why this argument fails to show that *MIN-FORMULA* $\in$ coNP: If $\phi \notin$ *MIN-FORMULA*, then ϕ has a smaller equivalent formula. An NTM can verify that $\phi \in \overline{\textit{MIN-FORMULA}}$ by guessing that formula.

Problem 7.46

Say that two Boolean formulas are ***equivalent*** if they have the same set of variables and are true on the same set of assignments to those variables (i.e., they describe the same Boolean function). A Boolean formula is ***minimal*** if no shorter Boolean formula is equivalent to it. Let *MIN-FORMULA* be the collection of minimal Boolean formulas. Show that if P = NP, then *MIN-FORMULA* ? P.

Step-by-step solution

Step 1 of 1

a. Consider following algorithm

On Input F :

1. For each string s such that $|s|<|F|$, if s is a valid representation of a formula (this can be easily checked) which is equivalent to F (this can be checked in polynomial space by evaluating both F and s over all possible truth assignments and comparing the results) then reject.

2. If all string has been tried without rejecting, accept.

Correctness of the algorithm should be evident. Only space used by the algorithm is for storing formula F, string s and current assignment of literals which amounts to polynomial space. Hence, $MIN-FORMULA \in PSPACE$.

b. It fails to show $MIN-FORMULA \in coNP$ because it is not known if one can verify equivalence of two Boolean formulae is polynomial time.

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