## Problem

Describe the error in the following “proof” that $0^*1^*$ is not a regular language. (An error must exist because $0^*1^*$ *is* regular.) The proof is by contradiction. Assume that $0^*1^*$ is regular. Let p be the pumping length for $0^*1^*$ given by the pumping lemma. Choose s to be the string $0^p1^p$. You know that s is a member of $0^*1^*$, but Example 1.73 shows that s cannot be pumped. Thus you have a contradiction. So $0^*1^*$ is not regular.

## Step-by-step solution

**Step 1** of 4

The pumping lemma is used as a negative test to prove that the given language is non-regular. The language that violates the any of the three conditions of the pumping lemma is classified as non-regular.

Since the pumping lemma starts by assuming that the given language is regular, the belongingness of the string, which is used as a counter example, is tested only for the given language and not for all the languages that accepts the string.

Comment

**Step 2** of 4

The proof given in the example 1.73 (refer to the textbook example 1.73) is for a different language. The counter examples given to prove the language as non-regular does not hold true for the language given in the question as shown below:

Let A be the language $\{0^*1^*\}$ given in the question.

Let B be the language $\{0^n1^n \mid n>=0\}$ given in the example 1.73.

As per the example 1.73, s is $0^p1^p$ and hence as per the pumping lemma, s should be split into three pieces as s = xyz, where for any i>=0, the string $xy^iz$ is in B. The cases are as follows:

• The string y contains all 0s: The string xyyz will result in more number of 0s than the number of 1s which does not belong to the language B but the string belongs to language A and hence, the pumping lemma is not violated.

• The same reason holds true for the case when the string y contains all 1s. The string xyyz will results in more number of 1s which is again accept by the given language A.

Comment

**Step 3** of 4

Since the above conditions are satisfied, the given language cannot be classified as non-regular by pumping lemma.

Comment

**Step 4** of 4

Therefore, the error in the given proof is that a string which is used as a counter example to prove a certain language as non-regular does not signifies that all the languages that accept that string are considered as non-regular.

The above problems arise when the pumping lemma is used on a language which is regular since violating a condition of the pumping lemma indicates that the language is non-regular but the vice versa is not always true.

Comment