Problem

Prove that if A $\leq_L$ B and B is in NC, then A is in NC.

Step-by-step solution

Step 1 of 2

If $A \leq_L B$ **and** B **is in** NC **then it can be proved that** A **is also in** NC. This can be achieved by using the fact of circuit evaluation. In other word, this can be achieved by showing that" **the problem of circuit evaluation is** P **complete".**

For a circuit C and input string w, the value of C on w can be written as $C(w)$. **Suppose**

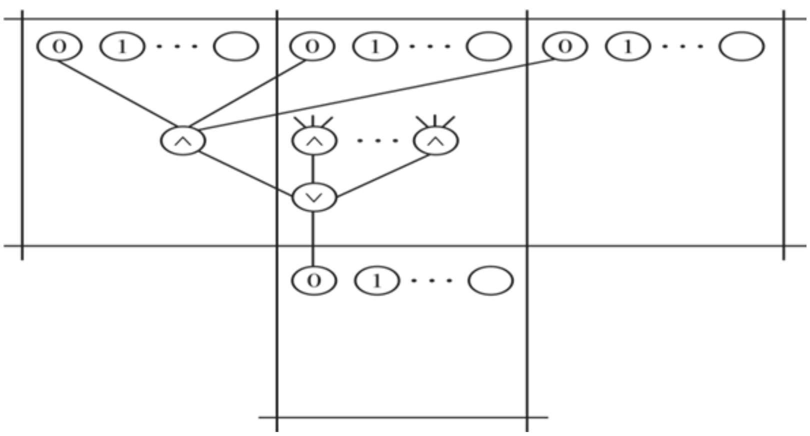
$CIRCUIT-VALUE = \{\langle C, x\rangle \mid C \, is \, a \, Boolean \, circuit \, and \, C(x) = 1\}$

Comment

Step 2 of 2

Consider the given theorem, which says that "**suppose** $t: M \rightarrow M$ **be a function, where** $t(m) \geq m$. **If** $W \in TIME(t(m))$**, then the complexity of the circuit** A **is given by** $O(t^2(m))$

• Now, consider the figure which is given below:



• The above figure shows the construction of the theorem, which is discussed above. It shows the way of **reduction of a language** W **(which is in** P**)** to $CIRCUIT-VALUE$.

• On input w, the production of a circuit takes place by the reduction. The process reduction simulates the Turing machine for W in polynomial time. The w itself can be taken as an input to the circuit.

• A $\log \text{space}$ is used to carried out the reduction because **the circuit produced by it contains a repetitive and a simple structure.**

Hence, **it shows that "** $CIRCUIT-VALUE$ **is** P**-complete" and the circuit produced by it has a repetitive structure.** Therefore it can be said that "**If** $A \leq_L B$ **and** B **is in** NC **then** A **is also in** NC."

Comment