Problem

Let $\Gamma = \{0, 1, \sqcup\}$ as follows. For each value of k, consider all k-state TMs that halt when started with a blank tape. Let *BB*(k) be the maximum number of 1s that remain on the tape among all of these machines. Show that *BB* is not a computable function.

Step-by-step solution

Step 1 of 1

Given: A tape of alphabets Γ for all the Turing machines is given. Γ Is composed of string of three terminals which is as follows: $\Gamma = \{0,1,\sqcup\}$

For every value of k all k states, of Turing machine halts, when the Turing machine starts from the blank tape. Maximum number of 1s is $BB(k)$ which remains in tape.

Proof:

By using contradiction, assume that the busy beaver function BB is computable. If function BB is computable then there exists a Turing machine F for computing it. Now without loss of generality a Turing machine F on input 1^n, and the Turing machine F halts with $1^{BB(n)}$ for each value of n.

Now build a Turing machine M, and this Turing machine halts when it will start from a blank tape based on F.

Construction of Turing machine:

Here M is a Turing machine which halts when starts from blank tape.

Step1: Now the Turing machine M writes n number of 1s on the tape.

Step2: Turing machine M doubles the number of 1s on the tape.

Step 3: Now M executes the Turing machine F on the input 1^{2n}.

Hence Turing machine M will halts with $BB(2n)$ number of 1s if it is starts from the blank tape.

For implementing the Turing machine M, at most n numbers of states are required for the step 1 of the Turing machine and c numbers of states are required for step 2 and 3, c is a constant.

Conclusion:

By definition, $BB(n+c)$ is the maximum number of1s on which Turing machine with states $(n+c)$ will halt is at least the number of 1s on which the Turing machine M halts. Which means $BB(n+c) \geq BB(2n)$ and this relationship will hold for all values of n.

Therefore, $BB(k)$ is strictly increasing function so $BB(n+c) < BB(2n)$. It proves wrong to our contradiction.

Hence, it is clear that $BB(k)$ is not a computable function.

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