Problem

Let BPL be the collection of languages that are decided by probabilistic log space Turing machines with error probability 1/3 . Prove that BPL $\subseteq$ P.

Step-by-step solution

Step 1 of 1

Suppose $\mathbf{BPL}$ be the collection of languages which are judged by **probabilistic log space** TM (Turing Machine) with an error probability of $1/3$. Now, suppose L be a $\mathbf{BPL}$ language and **the machine** M **is required** as the definition of $\mathbf{BPL}$ says.

- On input x of length n, suppose the number of configuration of $M(.,x)$ is defined as C. A $C \times C$ matrix is constructed in such a way that $P[c_1, c_2] = 1/3$ if c_2 is reachable from c_1 in a single step, and $P[c_1, c_2] = 0$ otherwise.
- For all t, $P^t[c_1, c_2]$ is defined as the **probability of approaching configuration** c_2 **from configuration** c_1 in t number of steps. Here, P^t is defined as the matrix obtained by multiplying P with itself t times.
- The accepting probability of $M(.,x)$ can be **computed by** computing all powers of P till the running time of $M(.,x)$ and **decide** if $x \in L$.
- The exact calculation can be performed at this time: **each probability is an integer multiple of** $1/3^{p(n)}$. **So, the polynomial number of digits can be used to represent it**.

Hence, it can be said that $\mathbf{BPL} \subseteq \mathbf{P}$.

Comment