

## Problem

This exercise concerns TM  $M_2$ , whose description and state diagram appear in Example 3.7. In each of the parts, give the sequence of configurations that  $M_2$  enters when started on the indicated input string.

- a. 0.
- Ab. 00.
- c. 000.
- d. 000000.

### EXAMPLE 3.7

Here we describe a Turing machine (TM)  $M_2$  that decides  $A = \{0^{2^n} \mid n \geq 0\}$ , the language consisting of all strings of 0s whose length is a power of 2.

$M_2$  = “On input string  $w$ :

1. Sweep left to right across the tape, crossing off every other 0.
2. If in stage 1 the tape contained a single 0, *accept*.
3. If in stage 1 the tape contained more than a single 0 and the number of 0s was odd, *reject*.
4. Return the head to the left-hand end of the tape.
5. Go to stage 1.”

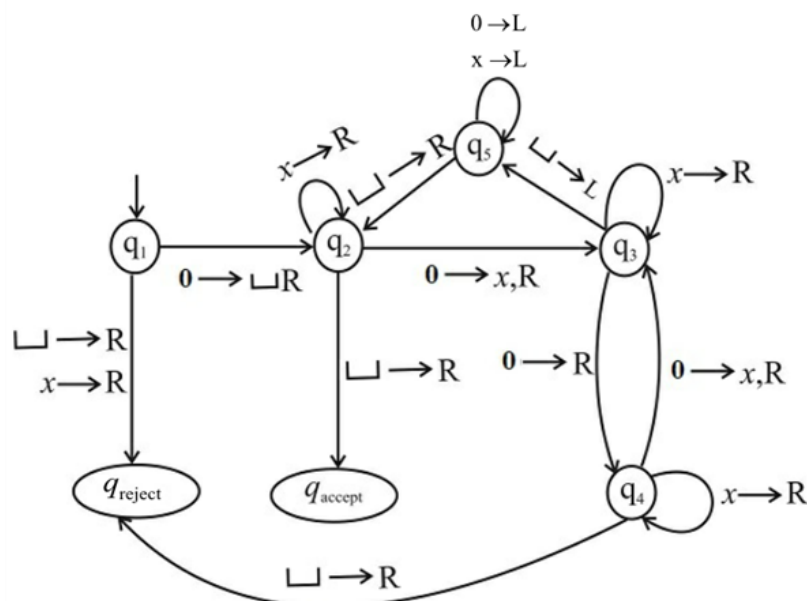
### Step-by-step solution

#### Step 1 of 7

Consider the language  $A = \{0^{2^n} \mid n \geq 0\}$ , consisting of all strings of 0s whose length is a power of 2.

Turing machine  $M_2$  decides a language  $A$ .

The state diagram for  $M_2$  is as follows.



[Comment](#)

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In this state diagram the label  $0 \rightarrow x, R$ , appears on the transition from  $q_4$  to  $q_3$ . This label signifies that, the state  $q_4$  with head reading 0, the machine goes to state  $q_3$ , writes  $x$ , and moves the head to the right. In the similar manner, other transitions also occur.

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#### Step 3 of 7

a. 0

Run the machine  $M_2$  on the input 0. The starting configuration is  $q_1 0$ . The sequence of configurations that the machine enters when started on the input string is as follows:

$q_1 0$	(At $q_1, 0 \rightarrow \sqcup, R$ goes to the $q_2$ )
$\sqcup q_2 \sqcup$	(At $q_2, \sqcup \rightarrow R$ goes to the accept state)
$\sqcup \sqcup q_{\text{accept}}$	

The state  $q_1$  on 0, the machine goes to state  $q_2$ , writes  $\sqcup$  and moves the head to the right. The state  $q_2$  on  $\sqcup$ , the machine goes to state  $q_{\text{accept}}$ , then halts.

As  $M_2$  enters  $q_{\text{accept}}$  state the input 0 is accepted.

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#### Step 4 of 7

b. 00

Run the machine  $M_2$  on the input 00. The starting configuration is  $q_1 00$ . The sequence of configurations that the machine enters when started on the input string is as follows:

$q_1 00$	(At $q_1, 0 \rightarrow \sqcup, R$ goes to the $q_2$ )
$\sqcup q_2 0$	(At $q_2, 0 \rightarrow x, R$ goes to the $q_3$ )
$\sqcup x q_3 \sqcup$	(At $q_3, \sqcup \rightarrow L$ goes to the $q_5$ )
$\sqcup q_5 x \sqcup$	(At $q_5, x \rightarrow L$ goes to the $q_5$ )
$q_5 \sqcup x \sqcup$	(At $q_5, \sqcup \rightarrow R$ goes to the $q_2$ )
$\sqcup q_2 x \sqcup$	(At $q_2, x \rightarrow R$ goes to the accept state)
$\sqcup x q_2 \sqcup$	(At $q_2, \sqcup \rightarrow R$ goes to the accept state)
$\sqcup x \sqcup q_{\text{accept}}$	

The state  $q_1$  on 0, the machine goes to state  $q_2$ , writes  $\sqcup$  and moves the head to the right. The state  $q_2$  on 0, the machine goes to state  $q_3$ , writes  $x$  and moves the head to the right. The state  $q_3$  on  $\sqcup$ , the machine goes to state  $q_5$ , moves the head to the left. The state  $q_5$  on  $x$ , the machine goes to state  $q_5$ , moves the head to the left. The state  $q_5$  on  $\sqcup$ , the machine goes to state  $q_2$ , moves the head to the right. The state  $q_2$  on  $x$ , the machine goes to state  $q_2$  itself, moves the head to the right. The state  $q_2$  on  $\sqcup$ , the machine goes to state  $q_{\text{accept}}$ , then halts.

Finally,  $M_2$  enters  $q_{\text{accept}}$  state. Thus, the input 00 is accepted.

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#### Step 5 of 7

c. 000

Run the machine  $M_2$  on the input 000. The starting configuration is  $q_1 000$ . The sequence of configurations that the machine enters when started on the input string is as follows:

[Comment](#)

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$q_1 000$  (At  $q_1, 0 \rightarrow \sqcup, R$  goes to the  $q_2$ )  
 $\sqcup q_2 00$  (At  $q_2, 0 \rightarrow x, R$  goes to the  $q_3$ )  
 $\sqcup x q_3 0$  (At  $q_3, 0 \rightarrow R$  goes to the  $q_4$ )  
 $\sqcup x 0 q_4 \sqcup$  (At  $q_4, \sqcup \rightarrow R$  goes to the  $q_{\text{reject}}$ )  
 $\sqcup x 0 \sqcup q_{\text{reject}}$

The state  $q_1$  on 0, the machine goes to state  $q_2$ , writes  $\sqcup$  and moves the head to the right. The state  $q_2$  on 0, the machine goes to state  $q_3$ , writes  $x$  and moves the head to the right. The state  $q_3$  on 0, the machine goes to state  $q_4$ , moves the head to the right. The state  $q_4$  on  $\sqcup$ , the machine goes to state  $q_{\text{reject}}$ , moves the head to the right.

Finally,  $M_2$  enters  $q_{\text{reject}}$  state. Thus, input 000 is rejected.

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d. 000000

Run the machine  $M_2$  on the input 000000. The starting configuration is  $q_1 000000$ . The sequence of configurations that the machine enters when started on the input string is as follows:

$q_1 000000$  (At  $q_1, 0 \rightarrow \sqcup$ , R goes to the  $q_2$ )  
 $\sqcup q_2 00000$  (At  $q_2, 0 \rightarrow x$ , R goes to the  $q_3$ )  
 $\sqcup x q_3 0000$  (At  $q_3, 0 \rightarrow R$  goes to the  $q_4$ )  
 $\sqcup x 0 q_4 000$  (At  $q_4, 0 \rightarrow x$ , R goes to the  $q_3$ )  
 $\sqcup x 0 x q_3 00$  (At  $q_3, 0 \rightarrow R$  goes to the  $q_4$ )  
 $\sqcup x 0 x 0 q_4 0$  (At  $q_4, 0 \rightarrow x$ , R goes to the  $q_3$ )  
 $\sqcup x 0 x 0 x q_3 \sqcup$  (At  $q_3, \sqcup \rightarrow L$  goes to the  $q_5$ )  
 $\sqcup x 0 x 0 q_5 x \sqcup$  (At  $q_5, x \rightarrow L$  goes to the  $q_5$ )  
 $\sqcup x 0 x q_5 0 x \sqcup$  (At  $q_5, 0 \rightarrow L$  goes to the  $q_5$ )  
 $\sqcup x 0 q_5 x 0 x \sqcup$  (At  $q_5, x \rightarrow L$  goes to the  $q_5$ )  
 $\sqcup x q_5 0 x 0 x \sqcup$  (At  $q_5, 0 \rightarrow L$  goes to the  $q_5$ )  
 $\sqcup q_5 x 0 x 0 x \sqcup$  (At  $q_5, x \rightarrow L$  goes to the  $q_5$ )  
 $q_5 \sqcup x 0 x 0 x \sqcup$  (At  $q_5, \sqcup \rightarrow R$  goes to the  $q_2$ )  
 $\sqcup q_2 x 0 x 0 x \sqcup$  (At  $q_2, x \rightarrow R$  goes to the  $q_2$ )  
 $\sqcup x q_2 0 x 0 x \sqcup$  (At  $q_2, 0 \rightarrow x$ , R goes to the  $q_3$ )  
 $\sqcup x x q_3 x 0 x \sqcup$  (At  $q_3, x \rightarrow R$  goes to the  $q_3$ )  
 $\sqcup x x x q_3 0 x \sqcup$  (At  $q_3, 0 \rightarrow R$  goes to the  $q_4$ )  
 $\sqcup x x x 0 q_4 x \sqcup$  (At  $q_4, x \rightarrow R$  goes to the  $q_4$ )  
 $\sqcup x x x 0 x q_4 \sqcup$  (At  $q_4, \sqcup \rightarrow R$  goes to the  $q_{\text{reject}}$ )  
 $\sqcup x x x 0 x \sqcup q_{\text{reject}}$

Finally,  $M_2$  enters  $q_{\text{reject}}$  state. Hence, the input 000000 is rejected.

[Comment](#)