Problem

Next

Say that a variable A in CFL G is ***usable*** if it appears in some derivation of some string w ? G. Given a CFG G and a variable A, consider the problem of testing whether A is usable. Formulate this problem as a language and show that it is decidable.

Step-by-step solution

Step 1 of 3

Consider the variable *A* in CFG *G* is usable if it appears in some derivation of some string $w \in L(G)$.

Proof:

Let's consider the problem to find whether *A* is usable and formulate this problem as a language and show that it is decidable. The language is:

$USABLE_{CFG} = \{\langle G, A\rangle : G \text{ is a CFG}, A \text{ usable for } G\}$.

Comment

Step 2 of 3

For CFG *G*, *A* variable is usable if the derivation of: $s \Rightarrow *xAy \text{ of } G$ such that $L(x), L(A) \text{ and } L(y)$ are all nonempty, is exist.

Decidability of language can be proved with the help of some steps. There are two further steps required to find decidability of the language. Now, according to the question:

• Consider, a usable CFG is A_{CFG} that generates decidable language.

• If usable A_{CFG} is decidable then language will be decidable as well.

Comment

Step 3 of 3

Construction:

Now construct and use the approach to find the decidability of A_{CFG} and to find decidability of the language.

Consider a decider *M* for A_{CFG} and $L(x), L(A) \text{ and } L(y)$ to decide *M* and also assume a Turing Machine *R* that will decide, whether output provided by M_1 can be accepted or not.

$B = "on\ input < M >, where\ M\ is\ a\ CFG"$, *M* is another CFG that is working as a decider for A_{CFG}, Now

• Run Turing Machine *R* on input $<M,M_1>$, where M_1 is a CFG that generates $\in^*$, and working as decider for *M1*.

• If R acceptable then accepts, else reject.

Conclusion:

Here, it is quite obvious M_1 is working as a decider for *M* and *M* is working as a decider for A_{CFG} so A_{CFG} is decidable and then language produced by A_{CFG} will also be accepted by Turing machine so it will also be decidable.

Comment