Problem

Let ? be a 3cnf-formula. An ≠***-assignment*** to the variables of ? is one where each clause contains two literals with unequal truth values. In other words, an ≠ -assignment satisfies ? without assigning three true literals in any clause.

a. Show that the negation of any ≠ -assignment to ? is also an ≠ -assignment.

b. Let ≠ *SAT* be the collection of 3cnf-formulas that have an ≠ -assignment. Show that we obtain a polynomial time reduction from *3SAT* to ≠ *SAT* by replacing each clause c_i

$$(y_1 \vee y_2 \vee y_3)$$

with the two clauses

$$(y_1 \vee y_2 \vee z_i) \quad \text{and} \quad (\overline{z_i} \vee y_3 \vee b),$$

where z_i is a new variable for each clause ci, and b is a single additional new variable.

c. Conclude that ≠ *SAT* is NP-complete.

Step-by-step solution

Step 1 of 4

Assumption: is a 3CNF formula.

ϕ is a clause that contains two literals with unequal truth values of an $\neq -$ assignment to the variable.

CNF is Conjunctive Normal Form. It has the following rules:

• A literal is Boolean variable or negated Boolean variable in the form.

• Boolean formula is in CNF called a CNF formula.

• If all clauses have three literals, then it is called 3cnf formula.

• Clause contains several literals connected with $\vee$s and $\wedge$s.

• Each clause has at least one satisfied literal and one unsatisfied literal in $\neq -$ assignment to ϕ.

• The negation of an $\neq -$assignment conserve this property.

• Hence, negation of any $\neq -$to ϕ is also an $\neq -$assignment.

Comment

Step 2 of 4

b)

To show: The formula ϕ is mapped to ϕ' then ϕ is satisfiable if ϕ' has an $\neq -$ assignment.

Assumption: $\neq$SAT is the collection of 3cnf formulas that have an $\neq -$assignment.

• Obtain a polynomial time reduction from 3SAT to $\neq -$SAT by replacing each close c_i of the form $(y_1 \vee y_2 \vee y_3)$ with the two clause $(y_1 \vee y_2 \vee z_i)$ and $(\overline{z}_i \vee y_3 \vee b)$

Where

• z_i is a new variable for each clause c_i

• b is a single additional new variable.

It is known that $SAT = \{(\phi) \mid \phi \text{ is a boolean formula}\}$

• Let ϕ and ϕ' are 3 CNF formulas of input and reduction on input ϕ and ϕ' as output.

• We must prove $\phi \in 3-SAT \Leftrightarrow \phi^{'} \in \neq SAT$ therefore the reduction is correct.

Suppose that $\phi \in 3-SAT$

• ϕ is satisfiable.

• 1 is true and 0 is false. We get an $\neq$- assignment to $\phi^{'}$ by extending a satisfying assignment to ϕ in such a way that we assign 1 to $\phi^{'} \in k \neq SAT$.

• Else if both literals y_1 and y_2 are clauses c_i are unsatisfied, else we assign 0 to z_i.

• Finally, we assign 0 to b.

• Extended assignment satisfies $\phi^{'}$ and it is an $\neq -$assignment to $\phi^{'}$.

Therefore, $\boxed{\phi^{'} \in \neq SAT}$

Comment

Step 3 of 4

Suppose that $\phi^{'} \in k \neq SAT$

• $\neq$ has an $\neq$- assignment

Satisfying assignment to ϕ as follows:

• From part(a) we obtain $\neq -$assignment assigns 0 to b, otherwise simply negate the assignment.

• This $\neq -$assignment cannot assign 0 to all y_1, y_2 and y_3 as doing so would force one of the two clauses, $(y_1 \vee y_2 \vee z_i)$ and $(\overline{z}_i \vee y_3 \vee b)$, to have all 0's

• Hence restricting this assignment to the variables of ϕ yields a satisfying assignment to ϕ.

Therefore, 3SAT is polynomial time reducible to $\neq$SAT

Comment

Step 4 of 4

(c)

NP-COMPLETE: A language B is NP-complete if it satisfies two conditions

• B is in NP

• Every A in NP is polynomial time reducible to B.

$\neq SAT \in NP$, as it is easy to verify whether an assignment is an $\neq -$ assignment.

Part(b) also proved $3SAT \leq_p \neq SAT$, $\neq$SAT is NP-complete.

Comment