Problem

In the traditional method for cutting a deck of playing cards, the deck is arbitrarily split two parts, which are exchanged before reassembling the deck. In a more complex cut, called Scarne's cut, the deck is broken into three parts and the middle part in placed first in the reassembly. We'll take Scarne's cut as the inspiration for an operation on languages. For a language A, let *CUT*(A) = {yxzl xyz ? A}.

a. Exhibit a language B for which *CUT*(B) ≠ *CUT*(*CUT*(B)).

b. Show that the class of regular languages is closed under *CUT*.

Step-by-step solution

Step 1 of 2

Consider the following language:

$B = \left\{0^n 1^n \mid \{0,1\} \in \Sigma, n \geq 0\right\}$.

The above language B is a language for which $CUT(B) \neq CUT(CUT(B))$. In the above language, if the string w exists in B and user divides this string w in xyz as $0^n 1^m 1^{n-m}$ then $CUT(B) = 1^m 0^n 1^{n-m}$. After again dividing it becomes $CUT(CUT(B)) = 0^n 1^n$.

Hence $CUT(B) \neq CUT(CUT(B))$

Comment

Step 2 of 2

Assume that a regular language A and prove that $CUT(A)$ is also a regular language. Again, let the DFA that accepts A be M_A.

Now construct another DFA M_{CUT} that accepts the language $CUT(A)$ and hence it will be proved that $CUT(A)$ is a regular language.

DFA Construction: Let $M = (Q, \Sigma, \delta, q_0, F)$ recognizes A. Construct $M_{CUT} = (Q_{CUT}, \Sigma, \delta_{CUT}, q_{CUT}, F_{CUT})$ to recognize $CUT(A)$.

i) $Q_{CUT} = Q$. The states of M_{CUT} are same as M.

ii) The new start state of the DFA is q_{CUT}.

iii) $F_{CUT} = F$. Let w be a string in A accepted by M. If user divides this string w in xyz then w' in $CUT(A)$ be yxz which will be accepted by M_{CUT}. Now in both cases the machine enters into a accept state on input of the substring z. Hence the accepting states of both of the machines are same.

iv) Now define transition function of M_{CUT}, δ_{CUT} for $q_{CUT} \in Q_{CUT}$ and $a \in \Sigma$ as follows:

$$\begin{aligned}\delta_{CUT}(q_{CUT}, a) &= \delta(q, a) \text{ where } q \in Q \text{ and } q \notin F \\ &= \delta(q, a) \text{ where } q \in F\end{aligned}$$

Hence, from the above DFA it is clear that accepting states of both of the machines are same. So, the class of regular languages is closed under CUT operation.

Comment