Problem

Let X = { $\langle M, w \rangle |$ _ M is a single-tape TM that never modifies the portion of the tape that contains the input w}. Is X decidable? Prove your answer.

Step-by-step solution

Step 1 of 1

X is un-decidable which can be proof by reducibility. Reduce *X* from $D_{TM} = \{< M, w > | \text{ Turing machine } M \text{ accepts } w\}$.

Assume *W* be a Turing machine which decides *X*. Use Turing machine *W* to create Turing machine *S* which decides D_{TM}.

1. TM *S*: on input $< M, w >$

2. TM M_{new}: on input $< M, w >$

3. The right end of the input mark with symbol $\$ \notin \Gamma_M$.

4. Copy the string which comes after $. This parts of input denotes as w'

5. Simulate M on w'.

6. If M accepts, then write all strings written in first cell of input tape, **accept**

7. Else **reject.**

8. TM *W*: on input $< M_{new}, w >$

9. When *W* accepts, then **accept**, else **reject**.

When M does not accept w, then M_{new} will not go to left of $\$$. The Turing machine M_{new} writes something on the original input when M accept w.

It is shown above that M_{new} modifies whenever *M* accepts the input *w*. Hence, D_{TM} has a contradiction.

Comments (3)