Show that F = {aⁱbʲ | i = k_j for some positive integer k} is not context free.

Step-by-step solution

Step 1 of 3

The language given in the question is as follows:

Language $F = \left\{ a^i b^j \,\middle|\, i = kj \text{ for some positive integer } k \right\}$.

A context free language (CFL) is generated by a context free grammar. **In order to prove that a language is not a CFL, Pumping Lemma is used.**

Comment

Step 2 of 3

Proof that Language L is not a Context Free Language:

Assume that F is CFL. Obtain a contradiction using pumping lemma to prove that the assumed statement is false.

• Let p be the pumping length for F that is guaranteed to exit by pumping lemma.

• Select string $s = a^p b^{2p} \in F$ where $k = 2$ and divide the string s into *uvxyz*.

• According to pumping lemma, v and y in string cannot be empty sets.

• Now, consider these two case, depending on whether substring v and y contain more than one type of alphabet symbol:

1. **Both v and y contain only one type of alphabet symbol:** In this case, both $v \text{ and } y$ does not contain mixed a's and b's. Thus, the string $uv^2 xy^2 z$ cannot contain equal number of a's and b's. Also, a pattern for a's and b's can be obtained that contains a relation between number of a's and b's. So, none of the conditions of lemma violates and thus it does not contradict.

For example:

Consider $uv^i xy^i z$ such that $v = a, y = bb$ and $u = x = z = \phi$. Thus, the strings generated will be $s = abb, aabbbb, aaabbbbbb...$

All the strings s are a member of F. Hence, no contradiction is obtained.

2. **Either of v or y contains more than one type of alphabet symbols**: In this case, both $v \text{ and } y$ contain mixed a's and b's. Thus, the string $uv^2 xy^2 z$ will contain strings with some order of ab followed by some order of ab again. Thus, it produces a wrong order of strings thereby producing a contradiction

For example:

Consider $uv^i xy^i z$ such that $v = ab \text{ and } y = b$ and $u = x = z = \phi$. Thus the strings generated will be $s = abb, ababbb...$

The string s = $ababbb$ is not member of F. This violates our assumption and thus, a contradiction is obtained.

Comments (1)

Step 3 of 3

The second case results in a contradiction. Hence, the assumption that F is context free language is false and therefore, F is not a context free language.

Comments (2)