Problem

Define *UPATH* to be the counterpart of *PATH* for undirected graphs. Show that $\overline{BIPARTITE} \leq_L UPATH$. (Note: In fact, we can prove $UPATH \in L$, and therefore $BIPARTITE \in L$, but the algorithm [62] is too difficult to present here.)

Step-by-step solution

Step 1 of 2

An assumption $UPATH \in L$ can be used to show $BIPART \in L$. **Now,** As it is known that, all paths which contain a length of two in G are used to acquire the graph of G^2. To show the bipartite condition, all graphs have to show a certain criteria. One of the criteria is that all the graphs should consist a cycle of odd length and also an edge between p and q or an edge (p,q). By considering the above explanation, G can be said as bipartite iff $\forall$ edges (p,q) of G, there is no connection between p and q by the path taken in G^2. Consider the algorithm which is given below shows how a logarithmic space can be used to solve $BIPART$.

//algorithm

On input G

For every edges (p,q) **in** G

// check for the existence of path.

"Check if there is a path exists from p **to** q **in** G^2**"**.

Now, the logarithmic space can be used to implement whole the procedure discussed above and **this logarithmic space can be achieved by checking if there are edges between** (u,r) and (r,q).

Comment

Step 2 of 2

Now, $UPATH \in L$ **can be shown by taking an assumption** $BIPART \in L$. The condition of bipartite can also be checked by the color of the vertices (which is generated by any algorithm). Now, consider the graph algorithm (which is used to generate the graph) which is given below:

For all vertex q of G

"**Two copy of** q_1, q_2 **is made in** H**"**.

For all edge (q,r) of G

"**Put the edges** (q_1,r_2) **and** (q_2,r_1) **in** H".

By using the facts which is discussed above, H will be **bipartite** only if every vertices of q_1 and q_2 are colored red and blue respectively. It gives that there exists no monochromatic edge. So, a cycle of odd length cannot exist. Suppose, a moment (G,m,n) of $BIPART$ is given.

- Now, by connecting the edges (m_1,n_1) and (m_2,n_2), the graph H_1 and H_2 can be obtained respectively. If there exists a path from m to n in G, then minimum one of the graphs H_1 and H_2 **must consists a cycle of odd length**.
- However, if n and m are not joined in G, then both H_1 and H_2 will show bipartite behavior. In H_1, m_1 and n_1 will be in different component.
- Since, bipartite behavior is shown by both of these components, it exists also after adding the edge (m_1,n_1) and acquiring H_1, the graph remains bipartite**. In the same way,** H_2 **must be bipartite.**

Therefore, it can be said that $UPATH \in L$ **and therefore,** $BIPART \in L$ **or In other words "** $\overline{BIPARTITE} \leq_L UPATH$ ".

Comment