Problem

Give a counterexample to show that the following construction fails to prove Theorem 1.49, the closure of the class of regular languages under the star operation.7

Let $N_1 = (Q_1, \Sigma, \delta_1, q_1, F_1)$ recognize A_1. Construct $N = (Q_1, \Sigma, \delta, q_1, F)$ as follows. N is supposed to recognize A_1^*.

a. The states of N are the states of N_1.

b. The start state of N is the same as the start state of N_1.

c. $F = \{q_1\} \cup F_1$.
The accept states F are the old accept states plus its start state.

d. Define δ so that for any $q \in Q_1$ and any $a \in \Sigma_\varepsilon$,

$$\delta(q,a) = \begin{cases} \delta_1(q,a) & q \notin F_1 \text{ or } a \neq \varepsilon \\ \delta_1(q,a) \cup \{q_1\} & q \in F_1 \text{ and } a = \varepsilon. \end{cases}$$

(Suggestion: Show this construction graphically, as in Figure 1.50.)

Figure 1.50

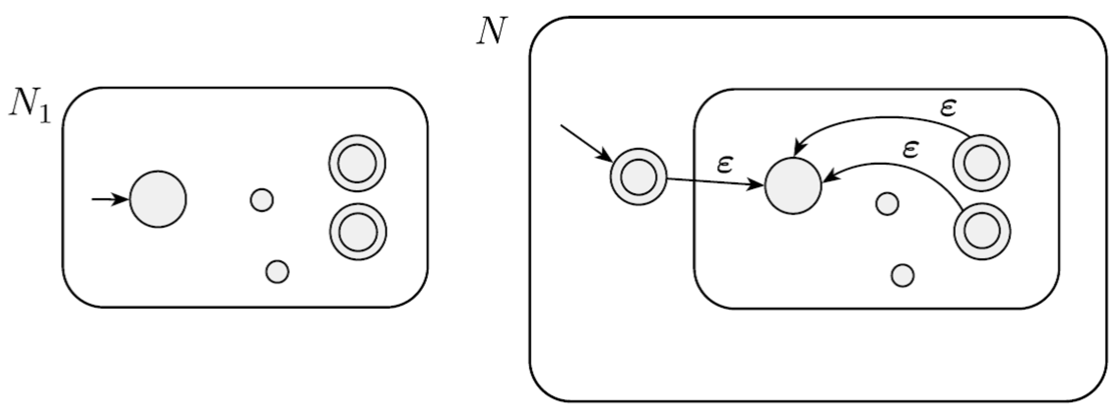


FIGURE 1.50
Construction of N to recognize A^*

Step-by-step solution

Step 1 of 1

Theorem 1.49 states that "*The class of regular languages closed under the star operation*".

Consider the data,

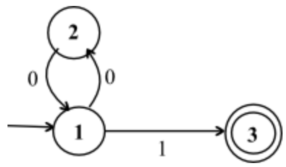
A language A is recognized by the automata $N_1 = (Q_1, \Sigma, \delta_1, q_1, F_1)$.

Assume N is the Non-deterministic finite automata which recognize the language A_1^*.

Example:

Assume a language $A_1 = \{(00)^* 1\}$.

The finite state automata N_1, which recognizes the language A_1 is as follows:



The following procedure is used to construct the finite state automata *N*, which recognizes the language A_1^*:

a. States of *N* are the states of N_1.

States of N_1 are {1,2,3}.

So, States of *N* are {1,2,3}.

b. The start state of *N* is same as start state of N_1.

Start state of N_1 is {1}.

So, start state of *N* is {1}.

c. $F = \{q_1\} \cup F_1$. The accept states for F are the accept states of F_1 including the start state. So, the accept state for F are **1** and **3.**

d. Define the transition δ for any $q \in Q_1$ and any $a \in \Sigma_\epsilon$ by using the following transition:

$$\delta(q,a) = \begin{Bmatrix} \delta^1(q,a) & q \notin F_1 \text{ and } a \neq \epsilon \\ \delta^1(q,a) \cup \{q_1\} & q \in F_1 \text{ and } a = \epsilon \end{Bmatrix}$$

$$\begin{aligned} \delta(1,\epsilon) &= \delta_1(1,\epsilon) \cup \{1\} \\ &= \phi \cup \{1\} \\ &= \{1\} \end{aligned}$$

$$\begin{aligned} \delta(3,\epsilon) &= \delta_1(3,\epsilon) \cup \{1\} \\ &= \phi \cup \{1\} \\ &= \{1\} \end{aligned}$$

$$\begin{aligned} \delta(1,0) &= \delta_1(1,0) \\ &= 2 \end{aligned}$$

$$\begin{aligned} \delta(1,1) &= \delta_1(1,1) \\ &= 3 \end{aligned}$$

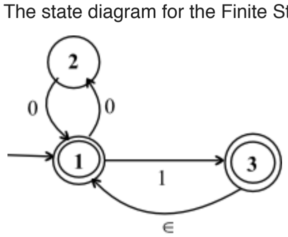
$$\begin{aligned} \delta(2,0) &= \delta_1(2,0) \\ &= 1 \end{aligned}$$

$$\begin{aligned} \delta(2,1) &= \delta_1(2,1) \\ &= \phi \end{aligned}$$

$$\begin{aligned} \delta(3,0) &= \delta_1(3,0) \\ &= \phi \end{aligned}$$

$$\begin{aligned} \delta(3,1) &= \delta_1(3,1) \\ &= \phi \end{aligned}$$

The state diagram for the Finite State automata is as follows:



• The above finite automata adds the start state to the set of accept states, which adds some other undesired strings and ϵ to the recognized language .

• A new start state which is also an accept state is not added to the automata. Thus, the new state is not added to the automata and leads to different automata from original automata.

Comment