Problem

Use the pumping lemma to show that the following languages are not context free.

a. $\{0^n 1^n 0^n 1^n \mid n \geq 0\}$

A**b.** $\{0^n \# 0^{2n} \# 0^{3n} \mid n \geq 0\}$

A**c.** $\{w\#t \mid w \text{ is a substring of } t, \text{ where } w, t \in \{a, b\}^*\}$

d. $\{t_1 \# t_2 \# \cdots \# t_k \mid k \geq 2, \text{ each } t_i \in \{a, b\}^*, \text{ and } t_i = t_j \text{ for some } i \neq j\}$

Step-by-step solution

Step 1 of 5

a) Consider the language $B = \{0^n 1^n 0^n 1^n \mid n \geq 0\}$.

Let p be the pumping length of B given by the pumping lemma.

To show that B is not a CFL, it is enough to show that a string $s = 0^p 1^p 0^p 1^p$ cannot be pumped.

Consider s is of the form $uvxyz$.

• If both v and y contain at most one type of alphabet symbol, the string will be of the form uv^2xy^2z runs of 0's and 1's of unequal length. Hence the string s cannot be a member of B.

• If either v or y contains more than one type of alphabet symbol, the string will be of the form uv^2xy^2z which does not contain the symbols in correct order. Hence the string s cannot be a member of B.

Since the string s cannot be pumped without violating the pumping lemma condition, B is not a CFL (context-free language).

Comment

Step 2 of 5

b) Consider the language $B = \{0^n \# 0^{2n} \# 0^{3n} \mid n \geq 0\}$.

Let p be the pumping length of B given by the pumping lemma.

To show that B is not a CFL, it enough to show that a string $s = 0^p \# 0^{2p} \# 0^{3p}$ cannot be pumped.

Consider s is of the form $uvxyz$.

Neither v nor y can contains $\#$, otherwise uv^2xy^2z contains more than two $\#$s. If the string s is divided into three segments by $\#$'s at least one of the segments $0^p, 0^{2p}$ and 0^{3p} is not contained within either v or y.

Because the length ratio of the segments is not maintained as 1:2:3, xv^2wy^2z is not in B.

Hence the string s cannot be a member of B.

Since the string s cannot be pumped without violating the pumping lemma condition, B is not a CFL (context-free language).

Comment

Step 3 of 5

c) Consider the language $B = \{w\#t \mid w \text{ is a substring of } t, \text{ where } w, t \in \{a, b\}^*\}$.

Let p be the pumping length of B given by the pumping lemma.

To show that B is not a CFL, it enough to show that a string $s = a^p b^p \# a^p b^p$ cannot be pumped.

Consider s is of the form $uvxyz$.

• Neither v nor y can contair $\#$, otherwise uv^0xy^0z does not contain $\#$. Hence the string s cannot be a member of B.

• If both v and y are nonempty and occur on the left-hand side of $\#$, the string uv^2xy^2z is longer on the left-hand side of $\#$. Hence the string s cannot be a member of B.

• Similarly, if both v and y are nonempty and occure on the right-hand side of $\#$, the string uv^0xy^0z is longer on the rifht hand side of $\#$. Hence the string s cannot be a member of B.

• If only one of v and y is nonempty we can treat them as if both occurred on the same side of $\#$. Hence the string s cannot be a member of B.

• In the remaining case if both v and y are nonempty and include the $\#$, then by the third pumping lemma condition $|vxy| \leq p$, we have v consists of b's and y consists of a's. Hence uv^2xy^2z contains more b's on the left-hand side of the $\#$. Hence the string s cannot be a member of B.

Since the string s cannot be pumped without violating the pumping lemma condition, B is not a CFL (context-free language).

Comments (3)

Step 4 of 5

d) Consider the language

$$B = \left\{ t_1 \# t_2 \# \cdots \# t_k \mid k \geq 2,\ t_i \in \{a,b\}^*, \text{ and } t_i = t_j \text{ for some } i \neq j \right\}.$$

Let p be the pumping length of B given by the pumping lemma.

The t_i can be equal to t_j for different *i* and *j* values. Hence, the same terms will be appeared in the string *s* separated by *#*.

For example, if the *k* value is 2 the string *s* can be *ab#ab.* The string *s* has the same term *ab* for different *k* values separated by *#*. The language generates the strings that contains same terms comprised of *a, b* separated by *#*. The strings that can be generated from the language *B* are *ab#ab, b#b#b, aba#aba#aba#aba, …* etc.

To show that B is not a CFL, it enough to show that a string $s = a^pb^p\#a^pb^p$ cannot be pumped.

Consider s is of the form $uvxyz$.

• Neither v nor y can contains $\#$, otherwise uv^0xy^0z does not contain $\#$. Hence the string s cannot be a member of B.

• If only one of v and y is nonempty we can treat them as if both occurred on the same side of $\#$. Hence the string s cannot be a member of B.

• If both v and y are nonempty and occur on the left-hand side of $\#$, the string uv^2xy^2z is longer on the left-hand side of $\#$. Hence the string s cannot be a member of B.

• If both v and y are nonempty and occur on the right-hand side of $\#$, the string uv^0xy^0z is longer on the rifht hand side of $\#$. Hence the string s cannot be a member of B.

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Comment

Step 5 of 5

In the remaining case if both v and y are nonempty and include the $\#$, then by the third pumping lemma condition $|vxy| \leq p$, we have v consists of b's and y consists of a's. Hence uv^2xy^2z contains more b's on the left-hand side of the $\#$. Hence the string s cannot be a member of B.

Since the string s cannot be pumped without violating the pumping lemma condition, B is not a CFL (context-free language).

Comment