Problem

Consider the problem of determining whether a two-tape Turing machine ever writes a nonblank symbol on its second tape when it is run on input w. Formulate this problem as a language and show that it is undecidable.

Step-by-step solution

Step 1 of 2

Consider the problem of determining whether a two-tape Turing machine ever writes a nonblank symbol on its second tape when it is run on input *w*.

Comment

Step 2 of 2

The language that denotes the problem is,

$L = \{< M, w >| M$ is a two-tape Turing machine writes a nonblank symbol on second tape when it is run on input $w\}$

In order to check the decidability, show that Construct a TM A_{TM} reduces to *L*. Use proof by contradiction method to prove decidability of the language. Assume TM *T* decides *L*. Construct a TM *B* that uses *T* to decide A_{TM}.

$B =$ "On input $< M, w >$:

1. Use M to construct the two-tape Turing machine S.
   S="On input x:
   1. Simulate M using the first tape on input x.
   2. If M is accepted then write a nonblank symbol on the second tape."
2. Run T on $< S, w >$ to check whether S on input w writes a nonblank symbol on second tape.
3. If T accepts, M accepts w then accept. Otherewise, reject."

Thus, *T* decides A_{TM} which is undecidable. Therefore, L is undecidable.

Comment