Problem

Let $C = \{ww^{\mathcal{R}} \mid w \in \{0,1\}^*\}.$ Prove that C is not a DCFL. (Hint: Suppose that when some DPDA P is started in state q with symbol x on the top of its stack, P never pops its stack below x, no matter what input string P reads from that point on. In that case, the contents of P's stack at that point cannot affect its subsequent behavior, so P's subsequent behavior can depend only on q and x.)

Step-by-step solution

Step 1 of 2

A deterministic context-free language (DCFL) is defined as a language for which there is a DPDA that accepts the same language. Consider the language which is given below:

Language $L_0 = \left\{ww^R \mid w \in \{0,1\}^*\right\}$

The language L_0 which is given above is not deterministic context free language because any pushed down automata (PDA), the use of non-determinism is essential to "guess" the midpoint.

Comment

Step 2 of 2

Consider the table which is given below:

Current State	Input	Stack top	String pushed	Current State	Description
q0	0	z0	0z0	q1	
q0	1	z0	1z0	q1	0. Have to push on this one
q1	0	0	00	q1	…or this one
q1	0	1	01	q1	1a: Suppose not at midpoint have to push on this one.
q1	0	0	$\in$	q1	1b: Suppose at the midpoint
q1	1	1	11	q1	2a: Suppose not at midpoint have to push on this one.
q1	1	0	10	q1	2b:Assume at the midpoint
q1	1	1	$\in$	q1	3. Matched around midpoint
q1	$\in$	z0	z0	q2	

The above table explained the ∂ function of **pushed down automata (PDA), which is designed to recognize L_0.**

• The given PDA is described by the structure:

$$P_{L_0} = \left(\{q_0\}, \{0,1\}, \{0,1,z_0\}, \partial, q_0, z_0, \{q_0, q_2\}\right)$$

The given PDA starts by stacking 0 or 1, depending on which one comes first. The comment 1a and 1b explain the non-deterministic selection of assuming not being at a midpoint, and being a midpoint, respectively. For 2a and 2b similar logic is followed as well.

• Therefore, from the above explanation it can be said that **the language L_0, which is given by $\mathbf{L_0 = \left\{ww^R \mid w \in \{0,1\}^*\right\}}$, is not a Deterministic Context Free Language (DCFL).**

Comment