

Problem

Consider the problem of determining whether a two-tape Turing machine ever writes a nonblank symbol on its second tape during the course of its computation on any input string. Formulate this problem as a language and show that it is undecidable.

Step-by-step solution

Step 1 of 1

Undecidability of the Turing machine problems

In the problem it is given that a single tape Turing machine writes a blank symbol over a non-blank symbol or not in the time of computation on any string which is provided as input. For showing the language generated by formulating the problem is undecidable following proof has been written:

$$K = \left\{ \langle M, w \rangle \mid \begin{array}{l} M \text{ is a Turing machine for input } w, \\ \text{Writes a non blank symbol on second tape.} \end{array} \right\}$$

Now, define a two tape Turing machine N_1 . This Turing machine takes a pair of M and w or $\langle M, w \rangle$ as input. Here, M is a Turing machine and w is a string which is passed as input to Turing machine. Turing machine N_1 is stimulating the Turing machine M , by using the first tape of machine M on input w .

Turing machine N_1 will ignore the second tape of the Turing machine M till end of the string does not come. If the Turing machine M accepts the input string w then the Turing machine N_1 will write a non-blank symbol on its second tape. If the simulation is halt then the Turing machine has stopped working.

Now assume T decides K . Then, user create T' for deciding A_{TM} . When the input $\langle M, w \rangle$ is passed to the Turing machine, firstly, it simulates the Turing machine N_1 and after that it will run on the Turing machine T on N_1 . The language K is accepted when T accept it otherwise, vice versa.

So, Turing machine T' is accepted if and only if when T accepts. T accepts the $\langle M_1, \langle M, w \rangle \rangle$ if and only if M_1 has written on second tape and M accepts w . So, ATM is decided by T' . But, A_{TM} cannot contain any decider. Here, in the above problem, for deciding K , any T is not present so the language K is undecidable.

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