Problem

Show that for any function $f : \mathcal{N} \longrightarrow \mathcal{R}^{+}$, where $f(n) \geq n$, the space complexity class SPACE(f(n)) is the same whether you define the class by using the single tape TM model or the two-tape read-only input TM model.

Step-by-step solution

Step 1 of 3

$\mathrm{SPACE}(f(n))$: Let $f : N \to R^{+}$ be a function. The space completely class $\mathrm{SPACE}(f(n))$ is defined as follows:

$$\mathrm{SPACE}(f(n)) = \left\{ \begin{array}{l} L \mid L \text{ is a language decided by an } \mathrm{O}(f(n)) \text{ space} \\ \text{deterministic Turning machine} \end{array} \right\}$$

Now we have to prove that, class $\mathrm{SPACE}(f(n))$ is same whether we define z by using the single tape *TM* or two tape read-only input *TM* model.

Comment

Step 2 of 3

To prove this, we have to do the following two things

(i) We have to simulate single – tape *TM* on two tape read – only *TM* and

(ii) We have to simulate two tape read only *TM* on single – tape *TM*.

(i) **<u>Simulation of single – tape *TM* on two tape read – only *TM:*</u>**

1. First we have to scan the read only tape.

2. Contents of the read only tape are copied to the work tape.

3. Remainder of the contents is simulated by treating the read I write work – tape as the input tape of a single – tape *TM*.

• Clearly only $\log n$ space is used to copy the contents.

• Since $f(n) \geq n$, we can write all of the input on are work – tape

In this way we simulated single-tape *TM* on two tape read only *TM*.

Comment

Step 3 of 3

(ii) **<u>Simulation two tape read only *TM* on single – tape *TM:*</u>**

• There is no participation from single tape TM; we are working over the first input symbols, using remainder of the tape as our work area.

• For n symbols, add n amount of space. By increasing space we add constant to n.

So from (i) and (ii) simulations we proved that class $SPACE(f(n))$ is same whether we defined it by using the single tape *TM* or two – tape read _ only input *TM*.

Comment