Problem

Let *MAX-CLIQUE* = $\{\langle G, k\rangle \mid \text{a largest clique in } G \text{ is of size exactly } k\}.$ Use the result of Problem 7.47 to show that *MAX-CLIQUE* is DP-complete.

Step-by-step solution

Step 1 of 4

Consider the difference hierarchy D_iP, which is **defined** recursively as

- $D_1P = NP \textit{ and}$
- $D_iP = \{A \mid A = B \cap \overline{C} \text{ for } B \text{ in } NP \text{ and } C \text{ in } D_{i-1}P\}$

Now consider the statement which is given below:

$Z = \{\langle G_1, k_1, G_2, k_2\rangle \mid G_1 \text{ has a } k_1-\text{clique and } G_2 \text{ doesn't have a } k_2 - clique\}$

Comment

Step 2 of 4

Here, it is already known that Z **will be in** DP **and every language in** DP **is polynomial time reducible to** Z**.**

Comment

Step 3 of 4

Now, consider the NP-completeness behavior of $MAX-CLIQUE$. To prove the given statement, first a $3-SAT$ problem will be reduced to $MAX-CLIQUE$.

• Particularly, a m clause and n variables, a $3-CNF$ formula F will be generated. First for every clause d of F, every assignment assigned to a variable c will be created as a node.

$F = (x_1 \vee x_2 \vee \overline{x_4}) \wedge (\overline{x_3} \vee x_4) \wedge (\overline{x_2} \vee \overline{x_3}) \wedge \ldots$

Which show that there exist no edges between any two nodes of the same clauses.

• So it can be said that, the maximum clique size, that it shows, is k. It is well known that if a graph consist k-clique, then this graph will definitely acquire one node per clause d.

• Also, reduction which is taken will be in polynomial time. So, the produced graph shows the quadratic size of the graph.

• In other word it can be said that, it will take $F(O(k))$ nodes which consists $O(k^2)$ edges. **Therefore, it can be said that,** $MAX-CLIQUE$ **is** NP**-complete.**

Comment

Step 4 of 4

Therefore, from the above discussion and from the definition of DP, **every language in** DP **is polynomial time reducible to** Z **and** DP **is also in** NP**. Also,** $MAX-CLIQUE$ **is** NP**-complete. Hence it can be said that,** $MAX-CLIQUE$ **is** DP **complete.**

Comment