Problem

Let

$$PAL_{\text{DFA}} = \{\langle M\rangle \mid M \text{ is a DFA that accepts some palindrome}\}.$$

Show that PAL_{DFA} is decidable. (Hint: Theorems about CFLs are helpful here.)

Step-by-step solution

Step 1 of 4

Assume that $PAL_{DFA} = \{\langle M\rangle \mid M \text{ is a } DFA \text{ that accepts some palindrome}\}$. If a Turing machine can be presented for the given DFA that runs finitely and halts, then the PAL_{DFA} is decidable.

Comment

Step 2 of 4

Construct a decider *D* for PAL_{DFA} and a Turing machine K that can decides E_{CFG} :

D = "On input $\langle M\rangle$,

1. A PDA *P* is constructed as: $L(P) = \{w \mid w \text{ is a palindrome}\}$

2. A PDA *P'* is constructed so that $L(P') = L(P) \cap L(M)$

3. Now P' is converted into an equivalent CFG *G.*

4. Check if $L(G)$ is empty using Theorem 4.8 over Turing Machine K.

5. If $L(G)$ is empty then reject, otherwise, accept.

Comment

Step 3 of 4

For Turing machine K:

• Both steps 1 and 2 can be done in finite steps.

• Step 3 also takes finite steps to convert P' into its equivalent CFG.

• In step 4, the decider *K* checks whether the language $L(G)$ is empty or not. It can also be done in a finite step.

Comment

Step 4 of 4

Since D takes finite steps for any input, it means that it is a decider. Hence, PAL_{DFA} is decidable.

Comment