Problem

Next

Show that A_{TM} is not mapping reducible to E_{TM}. In other words, show that no computable function reduces A_{TM} to E_{TM}. (Hint: Use a proof by contradiction, and facts you already know about A_{TM} and E_{TM}.)

Step-by-step solution

Step 1 of 3

Refer theorem 5.2 in the textbook. It states that E_{TM} is undecidable. It is known that A_{TM} is Turing recognizable but not co-Turing recognizable.

Comment

Step 2 of 3

The complement of E_{TM} is denoted with $\overline{E_{TM}}$. The TM for $\overline{E_{TM}}$ is as follows:

M="On input <M>,
1. For l = 1, 2, 3, ...
   a. Run M on all strings of length l for l steps.
   b. If any string is accepted then accept.
2. Reject if no string is accepted."

There exists a TM that recognizes $\overline{E_{TM}}$. Thus, E_{TM} is co-Turing recognizable.

Comment

Step 3 of 3

Assume that A_{TM} is mapping reducible to E_{TM}. Thus, $\overline{A_{TM}}$ is mapping reducible to $\overline{E_{TM}}$. $\overline{A_{TM}}$ is not Turing recognizable but $\overline{E_{TM}}$ is Turing recognizable which is a contradiction to the theorem 5.28. This a contradiction to the earlier assumption.

Therefore, A_{TM} is not mapping reducible to E_{TM}.

Comment