

Problem

Prove that $\text{TIME}(2^n) \subsetneq \text{TIME}(2^{2n})$.

Step-by-step solution

Step 1 of 1

Proving $\text{TIME}(2^n) \subsetneq \text{TIME}(2^{2n})$

The containment $\text{TIME}(2^n) \subseteq \text{TIME}(2^{2n})$ holds because $2^n \leq 2^{2n}$. Now, consider the **time hierarchy theorem** which says that, "if f, g are time-constructible functions and $f(n) \log f(n) = O(g(n))$, then $\text{DTIME}(f(n)) \subsetneq \text{DTIME}(g(n))$ ".

Thus, the above containment is proper by virtue of the time hierarchy theorem as discussed.

- A Turing machine can write the number 1 followed by $2n$ 0s in $O(2^{2n})$ time. So, the function 2^{2n} is time constructible
- Hence, the time hierarchy theorem guarantees that a language A exists that can be decided in $O(2^{2n})$ time but not in $o(2^{2n}/\log 2^{2n}) = o(2^{2n}/2n)$ time.
- Therefore, $A \in \text{TIME}(2^{2n})$ but $A \notin \text{TIME}(2^n)$.

From the above explanation it can be said that $\text{TIME}(2^n) \subsetneq \text{TIME}(2^{2n})$.

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