

### Problem

Give informal descriptions and state diagrams of pushdown automata for the languages in Exercise 2.4.

### Step-by-step solution

#### Step 1 of 10

**Given:**

$L = \{w \mid w\},$  which contains minimum three 1's

Context free grammar of the above language is as shown:

$S \rightarrow R1R1R1R$

$R \rightarrow 0R \mid 1R \mid \epsilon$

**Informal Description:**

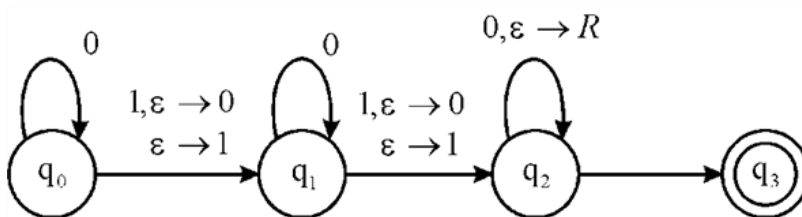
In the above grammar there is no need of stack. So, in the above grammar only the input is being read and that input is accepted only when there is an atleast three 1's. If atleast three 1's are present in any string then push down automata will accept that grammar otherwise it will reject.

**Pushdown Automata:**

Pushdown automata of the informal description is as shown:

[Comment](#)

#### Step 2 of 10



[Comments \(6\)](#)

#### Step 3 of 10

In the above pushdown automata, the state  $q_0$  is the initial state and  $q_3$  is the final state. Transition is done from  $q_0$  to  $q_3$  and string is accepted by the push down automata if there is three 1's otherwise string will not be accepted.

[Comment](#)

#### Step 4 of 10

**Given:**

$L = \{w \mid w\},$  whose starting and ending symbol are same

Context free grammar of the above language is as shown:

$$S \rightarrow 0A0 | 1A1 | 0 | 1$$

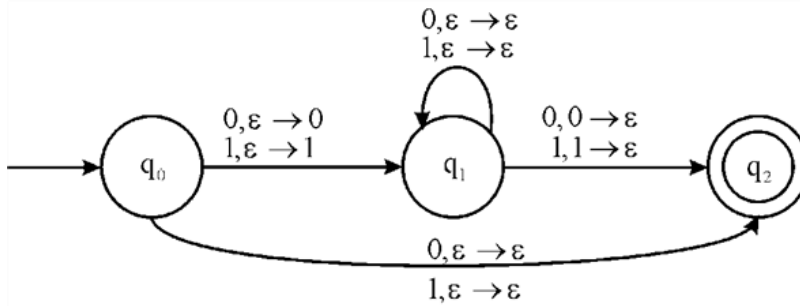
$$A \rightarrow 0A | 1A | \epsilon$$

**Informal Description:**

- In the above grammar user made an assumption that there is only one symbol in the string which can be accepted without the use of the stack.
- If there is more than one symbol in the string then all the symbol are sequentially put into the stack.
- Every time, the user identifies whether it is the last symbol which is to be read. Suppose it is the last symbol which is to be read and this symbol matches with the symbol which is present on the stack and if all the input has been retrieved and no input is being left then the user accepts the input otherwise pushdown automata will reject that input.

**Pushdown Automata:**

Pushdown automata of the informal description is as shown:



In the above push down automata, the state  $q_0$  is the starting state and  $q_2$  is the final state. In the first step user push the element into the push down automata after user pop the symbol from the stack and check whether it match with the final state or not. If it matches with the final stage then string is accepted.

[Comments \(6\)](#)

**Step 5 of 10**

**Given:**  $q_0$

Context free grammar of the above language is as shown:

$$S \rightarrow 0A | 1A$$

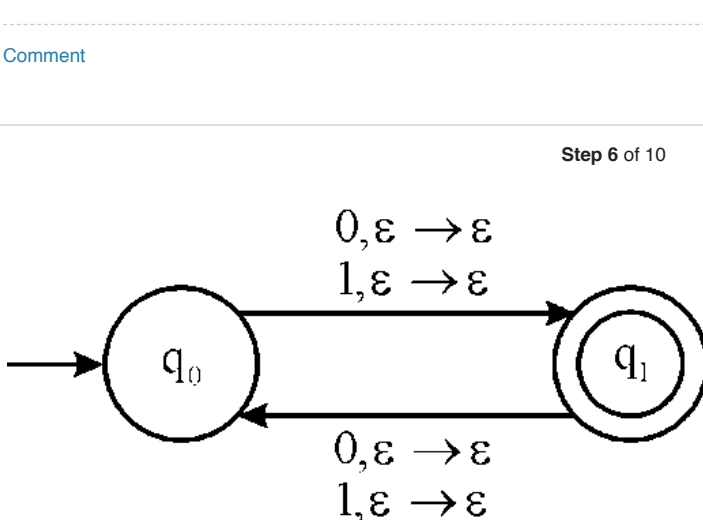
$$A \rightarrow 0S | 1S | \epsilon$$

**Informal Description:**

In the above grammar there is no need of stack. So, in the above grammar only the input is being read and that input is accepted only when there is an odd length. If the first symbol is being read or other symbol is being read then it is the final symbol read.

**Pushdown Automata:**

Pushdown automata of the informal description is as shown:



In the above pushdown automata, the state  $q_0$  is the initial stage and  $q_1$  is the final stage.

[Comment](#)

**Given:**

$$L = \{w \mid \text{the length of } w \text{ is odd and its middle symbol is } 0\}$$

Context free grammar of the above language is as shown:

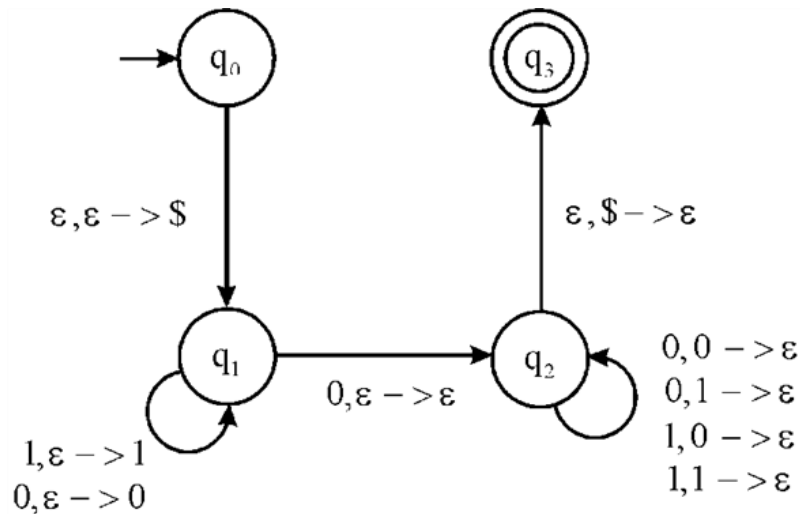
$$S \rightarrow 0S0 \mid 0S1 \mid 1S0 \mid 1S1 \mid 0$$

**Informal Description:**

- In the above grammar input is scanned by the push down automata and symbol is placed into the stack. There is some point in the push down automata where it guesses the input which is placed in the middle.
- It checks whether the symbol which is present in the middle contains 0 or not. If 0 is found as a middle element then all the string are scan sequentially and every time character is pop from the stack which has been scanned.
- If the stack after popping is empty and there is no input which is to be read by the stack then it checks whether 0 is the middle symbol in the string. If it is then the string is accepted otherwise the pushdown automata will reject the string.

**Pushdown Automata:**

Pushdown automata of the informal description is as shown:



In the above push down automata, the state  $q_0$  is the initial state and  $q_3$  is the final state. In each step it checks whether the middle symbol is 0. If the stack is empty and middle symbol is 0 then the string is accepted by the push down automata otherwise pushdown automata will reject the input.

[Comments \(1\)](#)

**Given:**

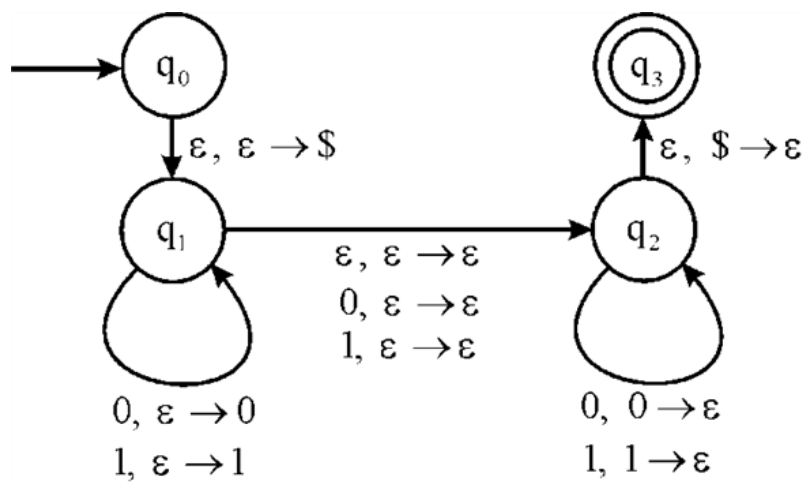
$$L = \{w \mid w = w^R\}, \text{ and } w \text{ is the palindrome.}$$

**Informal Description:**

- In the above grammar first half of the palindrome is pushed into the stack and after that underministically middle is being guessed. After that popping of the element is done from the second half of the palindrome.
- The first half of the input and the second half of the input should match with each other.
- If it matches then stack is made empty and after that push down automata accepts that particular string. If it does not matches then pushdown automata then reject that particular string.

**Pushdown Automata:**

Pushdown automata of the informal description is as shown:



- In the above push down automata  $q_0$  is the starting stage and  $q_3$  is the ending stage. In the stage  $q_1$  first half is pushed into the stack excluding the middle symbol if the palindrome is odd.
- If the string is not odd then user guesses where the middle point of the input is non-deterministically.
- After determining the middle part of the input user goes to state  $q_2$  without the consumption of any input if the string is found to be even.
- In the state  $q_2$  popping of the symbol is done from the stack and it is being checked whether it matches with the input state in the input in the beginning.
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[Comments \(2\)](#)

#### Step 9 of 10

If the first half and second half of the input is matched, then the palindrome is accepted by the push down automata otherwise palindrome is rejected by the push down automata.

[Comment](#)

#### Step 10 of 10

##### Given:

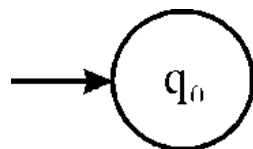
$L$  is an empty set which implies  $S \rightarrow S$

##### Informal Description:

In the above grammar as the language accept only an empty set so it implies it does not terminate any derivation. If non of the derivation is terminated then no any string is accepted by the CFG which also includes an empty string.

##### Pushdown Automata:

Pushdown automata of the informal description is as shown:



[Comments \(1\)](#)

