

### Problem

- a. Show that if  $M$  is a DFA that recognizes language  $B$ , swapping the accept and nonaccept states in  $M$  yields a new DFA recognizing the complement of  $B$ . Conclude that the class of regular languages is closed under complement.
- b. Show by giving an example that if  $M$  is an NFA that recognizes language  $C$ , swapping the accept and nonaccept states in  $M$  doesn't necessarily yield a new NFA that recognizes the complement of  $C$ . Is the class of languages recognized by NFAs closed under complement? Explain your answer.

### Step-by-step solution

#### Step 1 of 4

(a)

$M$  is a DFA that recognizes the regular language  $B$

Let  $M'$  be the new DFA that has swapped accept and non accept states in  $M$ .

- Consider  $M'$  accepts a string  $x$ .
- Run  $M'$  on  $x$  then  $M'$  surely enters into accept state.
- The machines  $M, M'$  have swapped accept and non-accept states.
- So if we run  $M$  on  $x$ , then  $M$  will end in a non-accept state.
- Thus if  $x$  is accepted by  $M'$  then  $x$  does not accepted by  $M$ .
- Similarly if  $x$  is accepted by  $M$  then  $x$  does not accepted by  $M'$ .
- One can say that if  $x \in B$  then  $x \notin \bar{B}$  (complement of  $B$ ) and vice – versa
- So  $M'$  will accept the strings that are not accepted by  $M$ .
- Therefore  $M'$  recognizes the languages which are complement of  $B$ .
- As  $M$  recognizes a regular language  $B$ , there exists  $M'$  which recognizes complement of  $B$  which is also regular.
- Hence class of regular languages closed under complement.

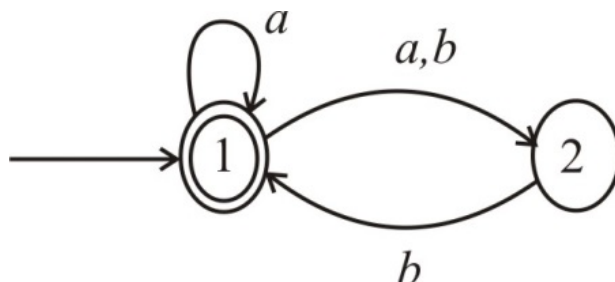
[Comment](#)

#### Step 2 of 4

(b)

Let  $M$  be a NFA that recognizes a languages  $C$ .

The state diagram of  $M$  is as follows

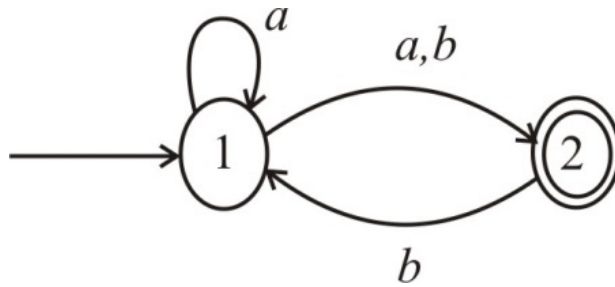


[Comments \(5\)](#)

Step 3 of 4

Clearly the string  $a$  is accepted by  $M$ .

If we swap accept and non accept states of  $M$  then we will get the following  $M'$



---

[Comment](#)

Step 4 of 4

Clearly the string  $a$  is accepted by  $M'$ .

This shown that swapping accept and non - accept states of NFA doesn't necessarily yield a new NFA recognizing the complement of the original one.

Consider the facts,

"The class of languages recognized by NFAs is precisely the class of languages recognizes by DFAs" ..... (a)

"The class of languages recognized by DFAs is closed under complement" ..... (b)

Therefore from the facts (a) and (b)

Class of languages recognize by NFAs closed under complement.

---

[Comment](#)