Problem

Next

A ***two-dimensional finite automaton*** (2DIM-DFA) is defined as follows. The input is an m × n rectangle, for any m, n ≥ 2. The squares along the boundary of the rectangle contain the symbol # and the internal squares contain symbols over the input alphabet Σ. The transition function $\delta\colon Q \times (\Sigma \cup \{\#\}) \longrightarrow Q \times \{\mathrm{L}, \mathrm{R}, \mathrm{U}, \mathrm{D}\}$ indicates the next state and the new head position (Left, Right, Up, Down). The machine accepts when it enters one of the designated accept states. It rejects if it tries to move off the input rectangle or if it never halts. Two such machines are equivalent if they accept the same rectangles. Consider the problem of determining whether two of these machines are equivalent. Formulate this problem as a language and show that it is undecidable.

Step-by-step solution

Step 1 of 2

To check **the un-decidability** of $E_{2DIM-DFA}$, first of all, it will have to $m-reduce$ A_{TM} to $E_{2DIM-DFA}$ by using a mapping which takes $\langle M,w\rangle$ **to** $\langle B\rangle$. It means that if w **is accepted by** M **then** $L(B)$ **is desired to be non-empty and when** w **is not accepted by** M **then** $L(B)$ **is desired to be empty.**

Comment

Step 2 of 2

This can be achieved by making $L(B)$ be the set of accepting, all the histories of M on w Here, B can be used as a rectangle (to represent the computation history of $C_1, C_2, \ldots, C_k$) with C_1 in the first row and C_2 in the next row and so on.

• For a given input rectangle, checking is performed. Here, B checks that the initial configuration of M on w is in first row or not and the last row is an accepting or final configuration .

• It also checked that each rows is followed from a previous row by the given rules of M .

• Now, if w is accepted by M , then there exists an accepting configuration history of M on w and $L(B)$ is not equals to $\varnothing$ or $L(B) \neq \varnothing$.

• In the same way, $L(B) = \varnothing$ and there is no accepting configuration history of M on w, when w is not accepted by M . Thus, $m-reduce$ A_{TM} to $\overline{E}_{2DIM-DFA}$ and $E_{2DIM-DFA}$ is un-decidable.

So, It follows that $EQ_{2DIM-DFA}$ is un-decidable.

Comment