Problem

Show that the Post Correspondence Problem is decidable over the unary alphabet ∑ = {1}.

Step-by-step solution

Step 1 of 1

Decidability of PCP over unary alphabet

Post Correspondence Problem is basically concerned with manipulation of string and used to find match. Conceptually this concept is quite obvious by its statement itself as there is only 1 alphabet in the Post Correspondence Problem. PCP is decidable over the unary alphabet as follows:

Consider a Turing Machine M that runs on input $< P >$

1. Check for some i, a_i is equals to b_i then accept the string.

2. Check if $a_i > b_i$ or $a_j < b_j$ if there exist some i and j then accepts the string otherwise reject it.

For the first stage, Turing machine M verifies for a domino which is forming a match. In the second stage Turing machine M is looking for two dominos which are forming a match. If the Turing machine finds such pair then it builds a math by extracting $\left(b_j - a_j\right)$ copies of the i^{th} dominos. After extracting copies the Turing machine puts them together with $\left(a_i - b_i\right)$ copies of the j^{th} dominos.

This construction has $a_i\left(b_j - a_j\right) + a_j\left(a_i - b_i\right) = a_i b_j - a_j b_i$

1’s on the top and $b_i\left(b_j - a_j\right) + b_j\left(a_i - b_i\right) = a_j b_i - a_j b_i$

1’s on bottom. If any stage of Turing machine is not accepted then problem instance includes dominos with all upper parts having either more or less 1’s than lower parts.

In this case, any match is not present hence Turing machine M rejects.

Comment