Problem

Prove that an oracle C exists for which $NP^C \neq coNP^C$

Step-by-step solution

Step 1 of 2

It is known that an oracle A exists such that $P^A \neq NP^A$ and $L_A \notin P^A$. For a given oracle A, L_A may be defined as:

$L_A = \{w : |w| = |x| \text{ for some } x \in A\}$

Then, it is clear that L_A is in NP^A.So, finally $L_A \notin coNP^A$ will have to proof (that is, same as $\overline{L}_A \notin NP^A$).

• For this purpose, first of all A will be constructed and $M_1, M_2, \ldots$ is now a list of all nondeterministic polytime oracle TMs, instead of all deterministic polytime oracle TMs.

Comment

Step 2 of 2

For Stage i **,** choose n and run M_i in input 1^n. It is respond NO to the query if M_i queries a string y whose status has not yet been determined.

• If there does not exist any computation and also, if M_i does not accept 1^n under these conditions then M_i is forced to make a mistake. It is performed because A can be maintained permanently that contains no string of length n. Then, $1^n \in \overline{L}_A$, but M_i does not accept 1^n.

If, on the other hand, there is an accepting computation for M_i with input 1^n, then it may be noted that this accepting computation can only query polynomially many strings y.

• Hence, there is a string x of length n which this computation does not query. We specify that this string x is in A and no other string of length n is in A.

• M_i Still has the same accepting computation on input $1^n \notin \overline{L}_A$. So makes a mistake in this case also. So, it may be concluded from the above explanation is that, an oracle C exists for which $NP^c \neq coNP^c$.

Comment