### **Problem**

Problem 8.13 showed that  $A_{LBA}$  is PSPACE-complete.

- a. Do we know whether A<sub>LBA</sub> ? NL? Explain your answer.
- **b.** Do we know whether A<sub>LBA</sub> ? P? Explain your answer.

# Step-by-step solution

### Step 1 of 3

It is being given and it is being proved in the previous chapter that  $A_{\textit{LBA}} \in \textit{PSPACE}$ 

*PSPACE* is basically the class of the language, whether the language belongs to deterministic finite automata or non-deterministic finite automata it is decidable in the polynomial time Turing machine.

- A language can be said *PSPACE* complete if that particular language belongs to *PSPACE*.
- For each and every language PSPACE hardness is satisfied.

Comment

#### **Step 2** of 3

Here, user needs to prove that  $A_{LBA} \in NL$ , but as it is being given that  $NP \in NPSPACE$  is PSPACE- complete.

With the help of space hierarchy theorem it can be proved that  $A_{\mathit{LBA}} \in \mathit{NL}$  .

It is being known by the Corollary of the Savitch's theorem that *PSPACE=NSPACE*, it implies deterministic space complexity and non-deterministic polynomial space complexity are basically the same.

Space complexity of *PSPACE* is  $O(\log n)$  whereas space complexity of the NSPACE is  $O(n^k)$  for each and every value of  $k_i$ .

So, this implies that language is basically accepted by NSPACE but it is not accepted by NL.

But, it is being known by the space hierarchy theorem that the language is accepted by PSPACE and also language is accepted by NL.

This implies that  $NL \in PSPACE$ 

As, it is being given that  $A_{LBA} \in PSPACE$ 

Hence, it is proved that  $A_{LBA} \in NL$ 

Comment

## Step 3 of 3

Here, user needs to prove that  $A_{LBA} \in P$ , but as it is being given that  $NL \in PSPACE$  is PSPACE- complete.

With the help of space hierarchy theorem it can be proved that  $A_{\mathit{LBA}} \in P$  .

It is being known by the Corollary of the Savitch's theorem that *PSPACE=NSPACE*, it implies deterministic space complexity and non-deterministic polynomial space complexity are basically the same.

Poly-time Turing machine is not able to consume space greater than poly-space.

It implies  $P \in PSPACE$  and  $NP \in NPSPACE$ 

As, it is being given that  $A_{LBA} \in PSPACE$ 

Hence, it is proved that  $A_{\mathit{LBA}} \in P$ 

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Comment