Problem

Convert the following CFG into an equivalent CFG in Chomsky normal form, using the procedure given in Theorem 2.9.

$$A \rightarrow BAB \mid B \mid \varepsilon$$
$$B \rightarrow 00 \mid \varepsilon$$

THEOREM **2.9**

Any context-free language is generated by a context-free grammar in Chomsky normal form.

Step-by-step solution

Step 1 of 7

Given CFG (Context-free Grammar) is

$A \rightarrow BAB \mid B \mid \varepsilon$
$B \rightarrow 00 \mid \varepsilon$

Now, construct an equivalent CFG (Context-free Grammar) in Chomsky normal form.

Chomsky normal form:

A context i– free grammar s in **Chomsky normal form** if every rule is of the form

$A \rightarrow BC$
$A \rightarrow a$

Here, a is terminal,

A, B and C are variables,

In addition, it permits the rule $S \rightarrow \varepsilon$, here S is the start variable.

convert the given CFG into an equivalent CFG in Chomsky normal form.

Comment

Step 2 of 7

Let's add a new start variable S_0 and the rule $S_0 \rightarrow A$.

Thus the obtained grammar is

$S_0 \rightarrow A$
$A \rightarrow BAB \mid B \mid \varepsilon$
$B \rightarrow 00 \mid \varepsilon$

In the addition of new start variable guarantees that the start variable doesn't occur on the right-hand side of a rule.

Comment

Step 3 of 7

Removing all rules that containing ε.

Removing $A \rightarrow \varepsilon$ and $B \rightarrow \varepsilon$ gives

$S_0 \rightarrow A \mid \varepsilon$
$A \rightarrow BAB \mid BA \mid AB \mid A \mid B \mid BB$
$B \rightarrow 00$

The rule $S_0 \rightarrow \varepsilon$ is accepted since S_0 is the start variable and that is allowed in Chomsky normal form.

Comments (1)

Step 4 of 7

Now remove the unit rules.

Removing $A \to A$ gives

$S_0 \to A \mid \varepsilon$
$A \to BAB \mid BA \mid AB \mid B \mid BB$
$B \to 00$

Removing $S \to B$ gives

$S_0 \to A \mid \varepsilon$
$A \to BAB \mid BA \mid AB \mid 00 \mid BB$
$B \to 00$

Removing $S_0 \to S$ gives

$S_0 \to BAB \mid BA \mid AB \mid 00 \mid BB \mid \varepsilon$
$A \to BAB \mid BA \mid AB \mid 00 \mid BB$
$B \to 00$

Comments (2)

Step 5 of 7

Now replace ill placed terminals 0 by variable U with new

$S_0 \to BAB \mid BA \mid AB \mid UU \mid BB \mid \varepsilon$
$A \to BAB \mid BA \mid AB \mid UU \mid BB$
$B \to UU$
$U \to 0$

Comment

Step 6 of 7

Shorten the right-hand side of rules with only 2 variables each.

To shorten the rules, replace $S_0 \to BAB$ with two rules $S_0 \to BA_1$ and $A_1 \to AB$.

The rule $A \to BAB$ is replaced by the two rules $A \to BA_2$ and $A_2 \to AB$.

After replacing these rules, the final Context-free grammar in Chomsky normal form is $G = (V, \Sigma, R, S_0)$,

Here the set of variables is $V = \{S_0, S, B, U, A_1, A_2\}$,

the start variable is S_0.

The set of terminals is $\Sigma = \{0\}$, and the rules R are given by

Comment

Step 7 of 7

$S_0 \to BA_1 \mid BA \mid SB \mid UU \mid BB \mid \varepsilon$
$A \to BA_1 \mid BA \mid SB \mid UU \mid BB$
$B \to UU$
$U \to 0$
$A_1 \to AB$

This is the final CFG in Chomsky normal form equivalent to the given CFG.

Comments (4)