Problem

Show that *TQBF* restricted to formulas where the part following the quantifiers is in conjunctive normal form is still PSPACE-complete.

Step-by-step solution

Step 1 of 3

TQBF: *TQBF* problem is to determine whether a fully quantified Boolean formula is true or false.

$TQBF =$ { $\langle\phi\rangle \mid \phi$ is a true fully quantified Boolean formula}

Show that *TQBF* restricted to formulas where the part following quantifiers is in conjunctive normal form (cnf) is $PSPACE$ - complete.

That is $cnf - TQBF = \left\{\vec{Q}\phi \in TQBF \mid \phi \text{ is in } cnf\right\}$ is *PSPACE*- complete.

Comment

Step 2 of 3

PSPACE- complete:

A language *B* is PSPACE- complete if it satisfies two conditions:

1. *B* is in *PSPACE,* and

2. Every *A* in *PSPACE* is polynomial time reducible to *B*.

If *B* merely satisfies condition 2, we say that it is *PSPACE*-hard.

1. $\underline{cnf - TQBF \in PSPACE}$: we know that $TQBF \in PSPACE$

As a subset of *TQBF* characterized by a simple syntactic test, $cnf - TQBF$ is obviously still in *PSPACE.*

2. $\underline{cnf - TQBF \in PSPACE - \text{hard}}$:

• We show $PSPACE - \text{hardness}$ by proving $TQBF \leq_p cnf - TQBF$

• Given a *TQBF* instance $\vec{Q}\phi$ where $\vec{Q}$ is a sequence of quantifiers and ϕ is a Boolean formula, we construct in polynomial time an equivalent $cnf - TQBF$ instance $\vec{Q}\vec{E}\psi$ where $\vec{E}$ is a sequence of existential quantifiers concerning the fresh proposition in ψ but not in ϕ.

• Here we use the technique for transforming the *SAT* instance ϕ in to an equi – satisfiable *CSAT* instance ψ.

• $\phi F \phi$ if and only if there exist an extension π' of π that make ψ true.

• This construction establishes that

$\pi \models \phi$ that is $\pi \models \vec{\mathrm{E}}\psi$ and hence $\models \vec{Q}\phi$ iff $\models \vec{Q}\,\vec{E}\psi$.

Comment

Step 3 of 3

From (1) and (2) $cnf - TQBF$ is $PSPACE$ complete.

Thus, it is proved that the $TQBF$ restricted to formulas where the part following the quantifiers is conjunctive normal from, is still $PSPACE -$ complete.

Comment