Problem

Answer each part TRUE or FALSE.

a. $2n = O(n)$.

b. $n^2 = O(n)$.

A**c.** $n^2 = O(n \log^2 n)$.

A**d.** $n \log n = O(n^2)$.

e. $3^n = 2^{O(n)}$.

f. $2^{2^n} = O(2^{2^n})$.

Step-by-step solution

Step 1 of 7

TRUE (or) FALSE

Big – *O* Notation:

Let *f* and *g* be functions $f, g : N \rightarrow R^{+}$ say that $f(n) = O(g(n))$ if positive integers *c* and n_0 exist such that for every integer $n \geq n_0$ $f(n) \leq c(g(n))$

When $f(n) = O(g(n))$ we say that $g(n)$ is an upper bound for $f(n)$.

Comment

Step 2 of 7

(a)

True.

The statement $2n = O(n)$ is valid, because from the definition of Big-*O* notation it is clear that $f(n) = c(g(n))$.

Comment

Step 3 of 7

(b)

False.

The statement $n^2 = O(n)$ is not valid, because $n^2 = n \cdot n$ which will grow faster than *n*.

That contradicts Big – *O* notation. Thus, $n^2 = O(n)$ is False.

Comment

Step 4 of 7

(c)

False.

The statement $n^2 = O(n \log^2 n)$ is not valid, because factor *n* grows faster than the factor $\log^2 n$. That means $f(n) > g(n)$, which contracts the Big –*O* notation.

Hence $n^2 = O(n \log^2 n)$ is false.

Comment

Step 5 of 7

(d)

True.

The statement $n\log n = O\left(n^2\right)$ is valid, because the factor $\log n$ grows slower than the factor *n.* That means $f\left(n\right) < g\left(n\right)$.From Big-*O* notation $n\log n - O\left(n^2\right)$ is true.

Comments (1)

Step 6 of 7

(e)

True.

The statement $3^n = 2^{O(n)}$ is valid, because $3^n = 2^{n\log 3}$ and $n\log 3 = O\left(n\right)$.

From Big-*O* notation $3^n = 2^{O(n)}$ is true.

Comment

Step 7 of 7

(f)

True.

The statement $2^{2^n} = O\left(2^{2^n}\right)$ is valid, because from Big- *O* notation $f\left(n\right) = O\left(f\left(n\right)\right)$ for any function $f\left(n\right)$. Hence $2^{2^n} = O\left(2^{2^n}\right)$ is true.

Comment