Problem

Describe two different Turing machines, M and N, where M outputs $\langle N \rangle$, when started on any input.

Step-by-step solution

Step 1 of 2

Given: There are two different Turing machines *M* and *N.*

We have to prove that, it started with any input *w*, *M* outputs $\langle N \rangle$ and *N* outputs $\langle M \rangle$

Here we need to know the recursion theorem.

Recursion theorem: Let T be a Turing machine that computes a function $t : \Sigma^* \times \Sigma^* \to \Sigma^*$. There is a Turing machine R that computes a function $r : \Sigma^* \to \Sigma^*$, where for every w,

$r(w) = t(\langle R \rangle, w)$.

To make a Turing machine that can get its own description and computes with it. Machine T in the statement receives the description of machine as extra input. The recursion theorem produces a new machine R, which operates exactly T, does but R's description filled automatically.

Comment

Step 2 of 2

By using Recursion theorem, remember that $r(w)$ is a Turing machine that prints *w* on its tape, and halts.

The Turing machine *M* is as follows:

$M =$" on input *w*:

1. No need to note the input

2. Obtain $\langle M \rangle$ by using the recursion theorem.

3. Compute $r(\langle M \rangle)$ where r the computable function is write

$r(\langle M \rangle)$ on the tape.

4. Halt."

• When *M* runs, it writes $N = \langle r(\langle M \rangle) \rangle$ in its tape.

• When *N* runs, it writes $\langle M \rangle$ in its tape automatically.

$\langle N \rangle$ is a member of the turing machine M

$\langle M \rangle$ is a member of the turing machine N

Therefore *M* outputs $\langle N \rangle$ and *N* outputs $\langle M \rangle$.

Comment