Problem

This problem investigates ***resolution***, a method for proving the unsatisfiability of cnf-formulas. Let $\phi = C_1 \wedge C_2 \wedge \cdots \wedge C_m$ be a formula in cnf, where the C_i are its clauses. Let $\mathcal{C} = \{C_i | \ C_i$ is a clause of $\phi\}$. In a *resolution step*, we take two clauses C_a and C_b in $\mathcal{C}$, which both have some variable x occurring positively in one of the clauses and negatively in the other. Thus, $C_a = (x \vee y_1 \vee y_2 \vee \cdots \vee y_k)$ and $C_b = (\overline{x} \vee z_1 \vee z_2 \vee \cdots \vee z_l)$, where the y_i and z_i are literals. We form the new clause $(y_1 \vee y_2 \vee \cdots \vee y_k \ \vee \ z_1 \vee z_2 \vee \cdots \vee z_l)$ and remove repeated literals. Add this new clause to C. Repeat the resolution steps until no additional clauses can be obtained. If the empty clause ( ) is in C, then declare ? unsatisfiable.

Say that resolution is ***sound*** if it never declares satisfiable formulas to be unsatisfiable. Say that resolution is ***complete*** if all unsatisfiable formulas are declared to be unsatisfiable.

a. Show that resolution is sound and complete.

b. Use part (a) to show that *2SAT* ? P.

Step-by-step solution

Step 1 of 4

a)

The **Resolution** is defined as "it is proof procedure which uses **refutation**". In other words, "it can be defined as a formula which is used to proof that a formula is unsatisfiable by deriving $\perp$ from the formula". Resolution shows both of the behavior, that is, complete and sound.

- The **Resolution** is said to be **sound** if satisfiable formula can never be declared unsatisfiable by it.
- The **Resolution** is said to be **complete** if all the unsatisfiable formulas are declared to be unsatisfiable.

Comment

Step 2 of 4

Now, consider **the lemma** which said that" if an application of resolution rule produces a clause H (which is given as $H = H_1 \setminus \{L\} \cup C_2 \setminus \{\neg L\}$) under a valuation Q, then from the conjunction of the hypothesis of the rule, $H_1 \wedge H_2$, is false under Q".

- The above lemma can be proved by assuming H is false under Q, but $H_1 \wedge H_2$ gives true value under Q. It is already given that H_1 shows a disjunction behavior, then one of its literals must be possess true under Q.
- All the literals other that L, then it is also exist in H and therefore H show true behavior, that shows a contradiction. Now, if L is taken as a literal then $\neg L$ will be false under Q.
- It is already known that under Q, H_2 is true, then it must consists a literals other than $\neg L$ that shows true behavior under Q. But this show that it is also exists in H and therefore H shows a true nature under Q, that is a contradiction.

Thus, it can be said that $H_1 \wedge H_2$ must be false under Q. Now by using the concept of **induction**, it can be said that a **satisfiable formulas can never be declared as unsatisfiable.** Therefore, it can be said that **Resolution is sound.**

Comment

Step 3 of 4

The **complete** property of **Resolution** can be proved by using the concept of induction. The induction will be applied on the excess number of literals.

- The excess number of literals in a clause is explained to be the number of literals, except in the clause. That is,

  $excess(H) = \{0 \, if \, |H| < 2 \, and \, |H| - 1 \, if \, |H| \geq 2$

- The number of excess literals in a clause set is just the sum of excess literals in every clause, that is,

  $excess(H) = \sum_{i=1}^{n} excess(H_i)$

Thus, an induction concept will be applied on the above. Therefore, it can be said that **Resolution is complete**.

Comment

Step **4** of 4

b)

Consider **the decidability** of $2SAT$. It is known that $2SAT$ is **polynomial time decidable**. To proof this problem, efficiently a path searches in graphs can be used. A depth search or breadth search algorithm can be used in path search of graphs.

- If there exists an edge between (a,b) then there must be exists a clause similar to $(\neg a, b)$ and also there exists a path between $(\neg b, \neg a)$.
- Now, consider a $2-CNF$ formula β. This formula is **unsatisfiable if and only if** there exists a variable q in such a way that: "In graph, there exists a path from q to $\neg q$ and also a path from $\neg q$ to q".
- Now consider there exists a path $q..\neg q$ and $\neg q..q$ fro a given variable q, but there exists also an assignment δ fro which $\delta(q) = T$ and similarly, $\delta(q) = F$.

Hence, from the above **explanation, it can be said that** $2SAT \in P$.

Comment