Problem

For any language A, let *SUFFIX*(A) = {vl uv ∑ A for some string u}. Show that the class of context-free languages is closed under the *SUFFIX* operation.

Step-by-step solution

Step 1 of 2

For any language *A*, it's suffix is defined as, $SUFFIX(A) = \{v \mid uv \in A \text{ for some string u}\}$. In order to prove that the CFLs are closed under *SUFFIX* operation, the push down automata (PDA) can be constructed or context free grammar (CFG) can be written for *SUFFIX* operation.

Comment

Step 2 of 2

To prove the context free languages closed under context free languages, take a context free language *A*. There exists a PDA and CFG for the language *A* since it is context free. Construct the PDA for *SUFFIX* operation of *A*. Let the PDA for the language *A* be *P*. The PDA for *SUFFIX*(*A*) be *M*. Following is the procedure to construct a PDA *M*.

• Create a copy of the PDA *P* and name it as *Q*. The PDA *Q* has the same transitions as *P* as it is a replica of *P*. The PDAs *P* and *Q* combined to form the PDA *M*.

• Modify the input part of transition in *Q* to ε without changing the stack symbol. If the input transition has $0,1 \to \varepsilon$, modify it to $\varepsilon,1 \to \varepsilon$. The input in the transition $0,1 \to \varepsilon$ is 0 and it is changed $\varepsilon,1 \to \varepsilon$ where the stack symbol ε is unchanged. In this step, just change the input part of each transition irrespective of the stack symbol.

• For each state in PDA *Q*, add a new transition $\varepsilon,\varepsilon \to \varepsilon$ to the corresponding state in PDA *P*. This means, for the input ε and stack symbol ε, the top of the stack will be ε. This step simply connects two PDAs.

• The start state of PDA *Q* should be the start state of the whole PDA *M*. Thus, the PDA *M* is the combination of two PDAs *Q* and *P*.

The PDA *M* simply ignores the alphabet of *u* and starts functioning when it identifies the first alphabet of *v* from which the second part of the PDA *M* (i.e., *P*) accepts the substring *v* (i.e., suffix). Thus, all the suffixes of the string belong to language *A* will be accepted by the PDA *M*.

Therefore, the CFLs are closed under *SUFFIX* operation.

Comment