Problem

Use Theorem 0.25 to derive a formula for calculating the size of the monthly payment for amortgage in terms of the principal P, the interest rate I, and the number of payments t. Assume that after t payments have been made, the loan amount is reduced to 0. Use the formula to calculate the dollar amount of each monthly payment for a 30-year mortgage with 360 monthly payments on an initial loan amount of $100,000 with a 5% annual interest rate.

Step-by-step solution

Step 1 of 3

Given formula related to loan is

$$P_t = PM^t - Y\left(\frac{M^t - 1}{M - 1}\right)$$

where,

P_t is the amount of loan outstanding after the t^{th} month.

P is the principal (original loan amount).

Y is the monthly payment.

t is the number of months in which loan is repaid.

I is the yearly interest rate.

M is the monthly multiplier $(M = 1 + I/12)$.

Now we have to derive the formula for calculating the size of the monthly payments for a mortgage in terms of the principal P, interest rate I, and the number of payments t.

Comment

Step 2 of 3

In order to derive the formula we have to get Y (monthly payment) on left hand side and remaining terms to right hand side.

$$P_t = PM^t - Y\left(\frac{M^t - 1}{M - 1}\right)$$

$$Y\left(\frac{M^t - 1}{M - 1}\right) = PM^t - P_t$$

$$Y = \left(\frac{M - 1}{M^t - 1}\right)\left(PM^t - P_t\right)$$

The formula required for calculation is

$$Y = \left(\frac{M - 1}{M^t - 1}\right)\left(PM^t - P_t\right)$$

Comment

Step 3 of 3

Given data is

$P_t = \$0$

$P = \$100,000$

$t = 360$ months

$I = 5\% = \dfrac{5}{100} = 0.05$

$M = 1 + I$

$= 1 + \dfrac{0.05}{12} \quad = 1.0042(\text{approx})$

We get

$$Y = \left(\frac{M-1}{M^t - 1}\right)\left(PM^t - P_t\right)$$
$$= \left(\frac{1.00417 - 1}{1.00417^{360} - 1}\right)\left(100000 \times 1.00417^{360} - 0\right)$$
$$= \left(\frac{0.00417}{3.47309}\right)(100000 \times 4.47309)$$
$$= 0.0012 \times 447309$$
$$\approx 536.7708$$

Therefore, the monthly payment is $536.78.

Comment