Problem

If A $\leq_m$ B and B is a regular language, does that imply that A is a regular language? Why or why not?

Step-by-step solution

Step 1 of 1

No, *A* is not a regular language.

• Assume that the languages *A* is defined as follows:

$A = \{ a^n b^n \mid n \geq 0 \}$ *and* $B = \{b\}$, over the input $\Sigma = \{a, b\}$.

• Specify the function $f : \Sigma^* \rightarrow \Sigma^*$ in the following way:

$$f(w) = \begin{cases} b & \text{if } w \in A, \\ a & \text{if } w \notin A. \end{cases}$$

• Notice that if *A* is a context-free language, then it is Turing-decidable.

• Therefore, f is a computable function.

• Besides, $w \in A$ if and only if $f(w) = b$, which is true if and only if $f(w) \in B$.

Hence it is proved that language A is not-regular, but language B is a regular language, because it is finite.

Comment