Problem

Show that the problem of determining whether a CFG generates all strings in 1^* is decidable. In other words, show that

$$\{\langle G\rangle \mid G \text{ is a } \mathsf{CFG} \text{ over } \{0,1\} \text{ and } 1^* \subseteq L(G)\}$$

is a decidable language.

Step-by-step solution

Step 1 of 1

Given:

$S = \{\langle G\rangle \mid G\}$ is a context free grammar over the string $\{0,1\}$ and $1^* \subseteq L(G)$

User need to proof that string 1* is generated by CFG and it is decidable.

Proof:

First the assumption is made that grammar G is in the CNF. If it is not present in the CNF then Turing machine should be use to make the grammar in CNF.

Suppose there is total n variable in the grammar G and the grammar there is one pumping lemma constant in grammar. Assume pumping lemma constant is s and the value of this pumping lemma constant is assumed to be 2^{n-1}.

It is prove that if the value of k is greater than or equal to s. If the grammar G can generate 1^k then it would be capable of generating 1^{k+s}.

Grammar G is capable of generating string which is in the form 1^m for $0 \le m < s! + s$ then G is use for generating Σ^*.

Turing machine is use for verifying that whether the string is accepted or not. If the string is recognized then Turing machine accepts that particular string otherwise Turing machine reject that particular string.

Construction:

If the Grammar G is use for generating 1^k then the grammar G should also generates $1^{k+p!}$

With the help of the lemma 1^k is divided into the multiple string named a, b, c, d and e in such a manner that $abcde = 1^k$. $|bcd| \le s, |bd| > 0$, $ab^i cd^i e \in L(G)$ for all the value of i which is greater than or equal to zero.

Suppose, $h = |bd|$ in such a way that $1 \le h \le s$.

With the help of lemma $1^{k+ih} \in L(G)$ for the integer i greater than or equal to zero.

Hence, $1^{k+ih} \in L(G)$ it implies grammar G generates 1^{k+ih}.

Conclusion:

Turing Machine is working as a decider for CFG and deciding whether the string is accepted or not by using the predefined rules mentioned above.

Hence, it can be said that the Turing machine S is decidable.

Comment