

### Problem

$$A \not\leq_T B \text{ and } B \not\leq_T A.$$

Prove that there exist two languages  $A$  and  $B$  that are Turing-incomparable—that is, where

### Step-by-step solution

#### Step 1 of 1

**Given:**

In this problem two languages  $A$  and  $B$  are present.

**Proof:**

Two languages  $A$  and  $B$  are Turing incomparable if the language is not Turing comparable with language  $B$  and it's vice versa.

Languages Incomparability can be proved as follows:

- In the Previous question there was one more language  $J$  was given for proving Turing reducibility of 2 languages  $A$  and  $B$ .
- If it comes to previous situation there was one more language  $J$  exists and it is being used to decide the Turing reducibility of two languages  $A$  and  $B$ .
- But if it comes to discuss about current situation then no other language or can say oracle support with Turing machine or language is given to detect Turing Comparability.
- Here no Turing machine is given that will work as an oracle Turing machine to find decidability and Turing reducibility.
- So none language from  $A$  and  $B$  can be replaced by any intermediate language and further no oracle Turing machine can be produced.
- For producing any Turing machine at least one intermediate language is required.
- So both languages can be compared through intermediate languages and Turing reducibility can be found.
- Oracle Turing machine set is countable that is why it is required to find the Turing reducibility and on the other hand set of all languages are uncountable.
- Most important thing is still missing that Turing reducibility cannot be established until decidability is proved as Turing reducibility is generalized concept of decidability.
- If discuss about further concept that how to find Turing comparability then machine must be added up with oracle support.

**Conclusion:**

No solution or method is defined here to prove the decidability of 2 languages so languages  $A$  and  $B$  are not Turing reducible.

Hence it is clear that language  $A$  and  $B$  is Turing incomparable.

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