Problem

Use the pumping lemma to show that the following languages are not regular.

^{A}**a.** $A_1 = \{0^n1^n2^n \mid n \geq 0\}$

b. $A_2 = \{www \mid w \in \{a, b\}^*\}$

^{A}**c.** $A_3 = \{a^{2^n} \mid n \geq 0\}$ (Here, a^{2^n} means a string of 2^n a's.)

Step-by-step solution

Step 1 of 4

Pumping Lemma:

If A is regular language, there is a number p (the pumping length) where *S* is any string in A of length at least *P*, then *S* may be divided into three pieces, $S = xyz$, satisfying the following conditions.

1. For each $i \geq 0, xy^iz \in A$

2. $|y| > 0$, and

3. $|xy| \leq p$

Comment

Step 2 of 4

(a)

Consider the language, $A_1 = \{0^n1^n2^n \mid n \geq 0\}$.

Assume A_1 is a regular language.

Let p be the pumping length given by the pumping lemma consider a string $S = 0^p1^p2^p \in A_1$

$|S| > P$ so, by pumping lemma, take $S = 0^p1^p2^p = xyz$ such that $|xy| \leq p, |y| > 0$ consider the following 2 possibilities:

Let 001122 be the string that belongs to A_1. $S = 0^p1^p2^p = 001122$. The pumping length of the string is 2. To satisfy the conditions of the pumping lemma, x = 0, y = 0, z = 1122.

$$\begin{aligned} S &= 001122 \\ &= \underset{x}{\underline{0}}\ \underset{y}{\underline{0}}\ \underset{z}{\underline{1122}} \end{aligned}$$

Pump the middle part such that $xy^iz\ (i \geq 0)$. For *i*=2, the *y* becomes 00. The string after pumping is 0001122.

$$\begin{aligned} S &= (0)\ (0)^i\ (1122) \\ &= \underset{x}{\underline{0}}\ \underset{y}{\underline{00}}\ \underset{z}{\underline{1122}} \quad [when\ i = 2] \end{aligned}$$

The string $0001122 \notin A_1$ because the string that is accepted by the language should have equal number of 0's, 1's and 2's. It is a contradiction. So, the pumping lemma is violated.

Therefore, A_1 is not a regular language.

Comment

Step 3 of 4

(b)

Consider the language, $A_2 = \{www \mid w \in \{a,b\}^*\}$.

Assume A_2 is a regular language.

Let p be the pumping length given by the pumping lemma.

Consider a string $S = a^p ba^p ba^p b \in A_2$.

By pumping lemma, this string can be divided into three pieces xyz such that $|xy| \le p, |y| > 0$ and $xy^i z \in A_2 \forall i \ge 0$

So $S = a^p ba^p ba^p b = xyz$.

Let *aabaabaab* be the string that belongs to A_2. The pumping length of the string is 2. To satisfy the conditions of the pumping lemma, x = *a*, y = *a*, z = *baabaab*.

$$\begin{aligned} S &= aabaabaab \\ &= \underset{x}{\underline{a}}\ \underset{y}{\underline{a}}\ \underset{z}{\underline{baabaab}} \end{aligned}$$

Pump the middle part such that $xy^i z\ (i \ge 0)$. For *i*=2, the *y* becomes *aa*. The string after pumping is *aaabaabaab*.

$$\begin{aligned} S &= (a)\ (a)^i\ (baabaab) \\ &= \underset{x}{\underline{a}}\ \underset{y}{\underline{aa}}\ \underset{z}{\underline{baabaab}} \quad [when\ i = 2] \end{aligned}$$

The string $aaabaabaab \notin A_2$.It is a contradiction. So, the pumping lemma is violated.

Therefore, A_2 is not a regular language.

Comments (8)

Step 4 of 4

(c)

Consider the language, $A_3 = \left\{a^{2^n} \mid n \ge 0\right\}$ (Here, a^{2^n} means a string of 2^n a's).

Assume that A_3 is regular language.

Let p be the pumping length given by pumping lemma consider a string $S = a^{2^p} \in A_3$. And $|S| > p$

By pumping lemma, this string can be divided into three pieces xyz such that $|xy| \le p, |y| > 0$ and $xy^i z \in A_2 \forall i \ge 0$

Let *aaaa* be the string that belongs to A_3. The pumping length of the string is 2. To satisfy the conditions of the pumping lemma, x = *a*, y = *a*, z = *aa*.

$$\begin{aligned} S &= aaaa \\ &= \underset{x}{\underline{a}}\ \underset{y}{\underline{a}}\ \underset{z}{\underline{aa}} \end{aligned}$$

Pump the middle part such that $xy^i z\ (i \ge 0)$. For *i*=2, the *y* becomes *aa*. The string after pumping is *aaaaa*.

$$\begin{aligned} S &= (a)\ (a)^i\ (aa) \\ &= \underset{x}{\underline{a}}\ \underset{y}{\underline{aa}}\ \underset{z}{\underline{aa}} \quad [when\ i = 2] \end{aligned}$$

The string $aaaaa \notin A_3$.It is a contradiction. So, the pumping lemma is violated.

Therefore, A_3 is not a regular language.

Comments (9)