

## Problem

In Theorem 3.21, we showed that a language is Turing-recognizable iff some enumerator enumerates it. Why didn't we use the following simpler algorithm for the forward direction of the proof? As before,  $s_1, s_2, \dots$  is a list of all strings in  $\Sigma^*$ .

E = "Ignore the input.

1. Repeat the following for  $i = 1, 2, 3, \dots$

2. RunM on  $s_i$ .

3. If it accepts, print out  $s_i$ ."

### THEOREM 3.21

A language is Turing-recognizable if and only if some enumerator enumerates it.

## Step-by-step solution

### Step 1 of 1

**Theorem:** - A language is Turing – recognizable if and only if some enumerator enumerates it.

The given simpler algorithm for the forward direction of the proof of this theorem is

Say that  $s_1, s_2, \dots$  is a list of all strings in  $\Sigma^*$

If Turing Machine M recognizes a language L, then we can construct following enumerator E for L.

The Enumerator E works as follows:

E = "Ignore the input

1. Repeat the following for  $i = 1, 2, 3, \dots$

2. Run M on  $S_i$ ;

3. If it accepts, print out  $S_i$ ."

### Defects in this proof:

In stage 2 of this algorithm (Run M on  $S_i$ )

If M loops on a certain input  $S_i$  runs forever, E could not check any input after  $S_i$ .

If it occurs, then E might fail to enumerate its language L as required.

Thus this procedure does not give the effect of running M in parallel on all possible input strings.

So, this proof is not suited for forward direction of above theorem.

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