Problem

Show that if PH = PSPACE, then the polynomial time hierarchy has only finitely many distinct levels.

Step-by-step solution

Step 1 of 2

The hierarchy in polynomial-time exists between deterministically accepted languages of classes P **in polynomial time** and deterministically or non-deterministically accepted languages of class PSPACE **in polynomial space**. There exists a relativization, which allowed minimum three levels of the hierarchy. The number of distinct levels, which are used to determine "low" and "high", in the hierarchy of polynomial-time are, depends upon the perishing of the NP class. Now consider the facts of "low" and "high", which is explained below:

• If there exist some i for which $\sum_{\mathrm{i}}^{\mathrm{P}}(Y)\subseteq\sum_{i}^{\mathrm{P}}$ then a set E in NP is known as "low" and if there exists $\sum_{\mathrm{i+1}}^{\mathrm{P}}\subseteq\sum_{i}^{\mathrm{P}}(z)$ for some i, then it is known as "high".

Comment

Step 2 of 2

Now, suppose PH **is defined as the union of the various classes of polynomial time hierarchy**. From the explanation of "low" and "high" (as it is defined above) , it can be shown that "**the hierarchy collapses are the only way of simultaneously existence of high and low for a set of PH.**

• There are two principle results exists. First one is based on the fact "either all or none (that is, every sparse set in PH is high or no one is low. Second one is that "every set of sparse will be extended high or no one sets will be extended high.

• Simply, it can be said that "**the hierarchy collapses in polynomial-time are** the only way for simultaneous existence of high and low behavior".

• A disjoint set can be obtained by combining high sets and low sets. The reason behind it is "the existence of immeasurably many levels extended by the hierarchy in polynomial time and in NP there exist some sets which show neither low nor high.

• **Therefore, the hierarchy in polynomial-size can be extended to only distinct finitely level if a sparse set in** NP**.** Hence, it can be said that **if** $\mathrm{PH{=}PSPACE}$**, then the polynomial time hierarchy consists levels which are distinct and finite**.

Comment