Problem

Give regular expressions generating the languages of Exercise 1.6.

Exercise 1.6.

Give state diagrams of DFAs recognizing the following languages. In all parts, the alphabet is {0,1}.

a. {wl w begins with a 1 and ends with a 0}

b. {wl w contains at least three 1s}

c. {wl w contains the substring 0101 (i.e., w = x0101y for some x and y)}

d. {wl w has length at least 3 and its third symbol is a 0}

e. {wl w starts with 0 and has odd length, or starts with 1 and has even length}

f. {wl w doesn't contain the substring 110}

g. {wl the length of w is at most 5}

h. {wl w is any string except 11 and 111}

i. {wl every odd position of w is a 1}

j. {wl w contains at least two 0s and at most one 1}

k. { ε , 0}

l. {wl w contains an even number of 0s, or contains exactly two 1s}

m. The empty set

n. All strings except the empty string

Step-by-step solution

Step 1 of 14

In the regular expressions, '*' indicates that the preceding regular expression may appear zero or more times and '+' indicates that the preceding regular expression may appear one or more times.

a.

Consider the language $L=\{w \mid w \text{ begins with a 1 and ends with a 0}\}$ over the alphabet $\Sigma=\{0,1\}$. Let *R* be the regular expression that generates the language *L*. The regular expression is as follows:

$$\begin{aligned} R &= 1\Sigma^*0 \\ &= 1(0+1)^*0 \end{aligned}$$

The strings accepted by the regular expression are $10,100,110,1010,1100,10100,\ldots$

Therefore, the regular expression is $1(0+1)^*0$ **.**

Comments (1)

Step 2 of 14

b.

Consider the language $L=\{w \mid w \text{ contains at least three 1s}\}$ over the alphabet $\Sigma=\{0,1\}$. Let *R* be the regular expression that generates the language *L*. The regular expression is as follows:

$$\begin{aligned} R &= \Sigma^*1\Sigma^*1\Sigma^*1\Sigma^* \\ &= (0+1)^*1(0+1)^*1(0+1)^*1(0+1)^* \end{aligned}$$

The strings accepted by the regular expression are $111,010101,01101,00001111,\ldots$

Therefore, the regular expression is $(0+1)^*1(0+1)^*1(0+1)^*1(0+1)^*$ **.**

Comments (4)

Step 3 of 14

c.

Consider the language $L = \{w \mid w \text{ contains the substring } 0101\}$ over the alphabet $\Sigma = \{0,1\}$. Let *R* be the regular expression that generates the language *L*. The regular expression is as follows:

$$R = \Sigma^* 0101 \Sigma^*$$
$$= (0+1)^* 0101 (0+1)^*$$

The strings accepted by the regular expression are $0101, 001011, 101011, 1101010, \ldots$

Therefore, the regular expression is $(0+1)^* 0101 (0+1)^*$ **.**

Comments (1)

Step 4 of 14

d.

Consider the language $L = \{w \mid w \text{ has length at least 3 and its third symbol is a } 0\}$ over the alphabet $\Sigma = \{0,1\}$. Let *R* be the regular expression that generates the language *L*. The regular expression is as follows:

$$R = \Sigma\Sigma 0 \Sigma^*$$
$$= (0+1)(0+1)0(0+1)^*$$

The strings accepted by the regular expression are $000, 1101, \ldots$

Therefore, the regular expression is $(0+1)(0+1)0(0+1)^*$ **.**

Comment

Step 5 of 14

e.

Consider the language,

$L = \{w \mid w \text{ starts with 0 and has odd length, or start with 1 and has even length}\}$ over the alphabet $\Sigma = \{0,1\}$. Let *R* be the regular expression that generates the language *L*. The regular expression is as follows:

$$R = 0(\Sigma\Sigma)^* + 1\Sigma(\Sigma\Sigma)^*$$
$$= 0((0+1)(0+1))^* + 1(0+1)((0+1)(0+1))^*$$

The strings accepted by the regular expression are $0, 011, 010, 00101, 10, 11, 1001, \ldots$

Therefore, the regular expression is $0((0+1)(0+1))^* + 1(0+1)((0+1)(0+1))^*$ **.**

Comment

Step 6 of 14

f.

Consider the language $L = \{w \mid w \text{ doesn't contain the substring } 110\}$ over the alphabet $\Sigma = \{0,1\}$. Let *R* be the regular expression that generates the language *L*. The regular expression is as follows:

$$R = 0^*\left(10^+\right)^*1^*$$

The strings accepted by the regular expression are $010, 011, 0101, \ldots$

Therefore, the regular expression is $0^*\left(10^+\right)^*1^*$ **.**

Comments (4)

Step 7 of 14

g.

Consider the language $L=\{w \mid \text{the length of } w \text{ is at most } 5\}$ over the alphabet $\Sigma=\{0,1\}$. Let *R* be the regular expression that generates the language *L*. The regular expression is as follows:

$$\begin{aligned}R&=\varepsilon+\Sigma+\Sigma\Sigma+\Sigma\Sigma\Sigma+\Sigma\Sigma\Sigma\Sigma+\Sigma\Sigma\Sigma\Sigma\Sigma\\&=\varepsilon+(0+1)+(0+1)(0+1)+(0+1)(0+1)(0+1)+(0+1)(0+1)(0+1)(0+1)+(0+1)(0+1)(0+1)(0+1)(0+1)\\&=\varepsilon+(0+1)+(0+1)^2+(0+1)^3+(0+1)^4+(0+1)^5\end{aligned}$$

The strings accepted by the regular expression are $\varepsilon,0,01,101,1010,00000,\ldots$ The empty string is of length 0. The language accepts the strings of length from 0 to 5.

Therefore, the regular expression is $\varepsilon+(0+1)+(0+1)^2+(0+1)^3+(0+1)^4+(0+1)^5$ **.**

Comments (1)

Step 8 of 14

h.

Consider the language $L=\{w \mid w \text{ is any string except } 11 \text{ and } 111\}$ over the alphabet $\Sigma=\{0,1\}$. Let *R* be the regular expression that generates the language *L*. The regular expression is as follows:

$$\begin{aligned}R&=\varepsilon+\Sigma+0\Sigma+10+0\Sigma\Sigma+10\Sigma+110+\Sigma^3\Sigma^+\\&=\varepsilon+(0+1)+0(0+1)+10+0(0+1)(0+1)+10(0+1)+110+(0+1)^3(0+1)^+\end{aligned}$$

The strings accepted by the regular expression are $\varepsilon,101,110,1010,\ldots$

Therefore, the regular expression is,

$\varepsilon+(0+1)+0(0+1)+10+0(0+1)(0+1)+10(0+1)+110+(0+1)^3(0+1)^+$ **.**

Comments (4)

Step 9 of 14

i.

Consider the language $L=\{w \mid \text{every odd position of } w \text{ is a } 1\}$ over the alphabet $\Sigma=\{0,1\}$. Let *R* be the regular expression that generates the language *L*. The regular expression is as follows:

$$\begin{aligned}R&=(1\Sigma)^*(\varepsilon+1)\\&=(1(0+1))^*(\varepsilon+1)\end{aligned}$$

The strings accepted by the regular expression are $\varepsilon,101,111,1010,\ldots$

Therefore, the regular expression is $(1(0+1))^*(\varepsilon+1)$ **.**

Comments (3)

Step 10 of 14

j.

Consider the language $L=\{w \mid w \text{ contains at least two 0s and at most one } 1\}$ over the alphabet $\Sigma=\{0,1\}$. Let *R* be the regular expression that generates the language *L*. The regular expression is as follows:

$$R=00^*00^*(\varepsilon+1)+00^*(\varepsilon+1)00^*+(\varepsilon+1)00^*00^*$$

The strings accepted by the regular expression are $001,010,100,\ldots$ In the first part of the regular expression $00^*00^*(\varepsilon+1)$, there are two mandatory zeros and at most one 1. The optional 1 may appear at the start or middle or at the end. There are three parts in the regular expression to accept such strings.

Therefore, the regular expression is $00^*00^*(\varepsilon+1)+00^*(\varepsilon+1)00^*+(\varepsilon+1)00^*00^*$ **.**

Comments (1)

Step 11 of 14

k.

Consider the language $L = \{\varepsilon, 0\}$ over the alphabet $\Sigma = \{0,1\}$. Let *R* be the regular expression that generates the language *L*. The regular expression is as follows:

$R = 0 + \varepsilon$

Therefore, the regular expression is $0 + \varepsilon$ **.**

Comment

Step 12 of 14

l.

Consider the language,

$L = \{w \mid w \text{ contains an even number of 0s, or contains exactly two 1s}\}$ over the alphabet $\Sigma = \{0,1\}$. Let *R* be the regular expression that generates the language *L*. The regular expression is as follows:

$R = 1^*(01^*01^*)^* + 0^*10^*10^*$

The strings accepted by the regular expression are $\varepsilon, 00, 11, 0101, 010100, \ldots$

Therefore, the regular expression is $1^*(01^*01^*)^* + 0^*10^*10^*$ **.**

Comments (3)

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m.

Consider the language $L = \text{The empty set}$. Let *R* be the regular expression that generates the language *L*. The regular expression is as follows:

$R = \phi$

Therefore, the regular expression is ϕ **.**

Comment

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n.

Consider the language *L* accepts all the strings except the empty string. Let *R* be the regular expression that generates the language *L*. The regular expression is as follows:

$$R = \Sigma^+$$
$$= (0+1)^+$$

The language accepts all the strings except ε.

Therefore, the regular expression is $(0+1)^+$ **.**

Comment