Problem

Consider the problem of determining whether a single-tape Turing machine ever writes a blank symbol over a nonblank symbol during the course of its computation on any input string. Formulate this problem as a language and show that it is undecidable.

Step-by-step solution

Step 1 of 3

Formulating the given problem as a language:

$$L=\left\{\langle M,w\rangle \middle| \begin{array}{l} M \text{ is a single tape Turing machine which writes a blank symbol} \\ \text{on non blank symbol while computing any input string} \end{array}\right\}$$

Comment

Step 2 of 3

Proving that the given problem is undecidable:

By using contradiction, assume that the language L is decidable. Suppose that N is a decider for proving the decidability of the language L. A Turing machine *N* can be constructed as:

N = "On Input $\langle M,s\rangle$

• Construct a Turing machine A' now:

a. A' writes # (a non-blank symbol) if *M* writes a blank symbol

b. Whenever A' reads #, use the transitions specified by the blank symbols.

c. A' Writes # on the tape before accepting and overwrites it with a blank symbol.

• Output of A'' will be input for decider N. If $N(\langle M',s\rangle)$ accepts, accept, otherwise reject.

Comments (2)

Step 3 of 3

Now, the conclusion can be made that a blank symbol is written by A' only when A' takes the input s. That is, N is a decider for A_{TM} which is a contradiction. Hence, **the given problem is undecidable.**

Comment