Problem

Let $\Sigma = \{0,1\}$. Let C_1 be the language of all strings that contain a 1 in their middle third. Let C_2 be the language of all strings that contain two 1s in their middle third.

So $C_1 = \{xyz \mid x, z \in \Sigma^* \text{ and } y \in \Sigma^* 1 \Sigma^*, \text{ where } |x| = |z| \geq |y|\}$

and $C_2 = \{xyz \mid x, z \in \Sigma^* \text{ and } y \in \Sigma^* 1 \Sigma^* 1 \Sigma^*, \text{ where } |x| = |z| \geq |y|\}$.

a. Show that C_1 is a CFL.

b. Show that C_2 is not a CFL.

Step-by-step solution

Step 1 of 7

A language L is said to be **context-free** if there exist some integer $q \geq 1$ (it is also known as pumping length) in such a way that all the string S in L which is equal or longer than q symbols or $|S| \geq q$.

It can be written as $S = abcde$ with substring $a, b, c, d \text{ and } e$ such that

1. $|bcd| \leq q$
2. $|bd| \geq 1$, and
3. $ab^x cd^x e$ is in L for all $x \geq 0$.

Comment

Step 2 of 7

a.

Consider the language C_1 which is given below:

$C_1 = \{xyz \mid x, z \in \Sigma^* \text{ and } y \in \Sigma^* 1 \Sigma^*, where |x| = |z| \geq y\}$, where $\Sigma = \{0,1\}$

Now, a Push down automata needs to construct which help in determining the language C_1. The PDA M that recognizes C_1 is $(Q, \Sigma\frac{1}{3}, \Gamma, \delta, q_0, F)$, where:

$Q = \{q_0, q_1, q_2\}$
$\Sigma = \{0,1\}$
$\Gamma = \{x\}$
$F = \{q_2\}$

Comment

Step 3 of 7

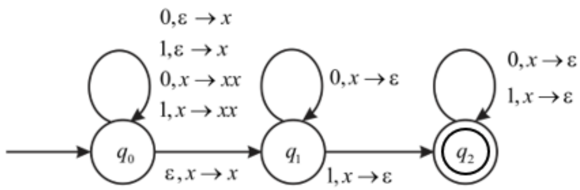
The transition functions δ of the represented in a tabular format:

Input:	0		1		ε	
Stack:	x	ε	x	ε	x	ε
q_0	$\{(q_0, xx)\}$	$\{(q_0, x)\}$	$\{(q_0, xx)\}$	$\{(q_0, x)\}$	$\{(q_1, x)\}$	
q_1	$\{(q_1, \varepsilon)\}$		$\{(q_2, \varepsilon)\}$			
q_2	$\{(q_2, \varepsilon)\}$		$\{(q_2, \varepsilon)\}$			

Comment

Step 4 of 7

The state diagram for the PDA M is given below:



Hence from the above explanation it can be said that there exists a Turing machine which accepts the given language.

Hence, C_1 is a context free language.

Comments (4)

Step 5 of 7

b.

Now consider the language C_2 that accepts all the string that contains two 1's in their middle of the string. The language C_2 can be defined as:

$$C_2 = \left\{xyz \mid x,z \in \Sigma^* \text{ and } y \in \Sigma^* 1 \Sigma^* 1 \Sigma^*, \text{where} |x| = |z| \geq y\right\}$$

Now, using a pumping lemma to show Language C_2 is not CFL.

Let as assume that C_2 is CFL and obtain a contradiction.

Let pumping length of the pumping lemma is *p.*

Let select a string $S = 0^{p+2}10^p10^{p+2}$ of given language.

Comment

Step 6 of 7

Let, divide S into five pieces $S = uvxyz$, it must satisfy the conditions according to the pumping lemma,

1. For each $i \geq 0, uv^ixy^iz \in C_2$,
2. $|vy| > 0$, and
3. $|vxy| \leq p$

Where

$u = 0^{p+2}$

$v = 1$

$x = 0^p$

$y = 1$

$z = 0^{p+2}$

Comments (1)

Step 7 of 7

Let $i = 1$

After pumping, string becomes $S = 0^{p+2}10^p10^{p+2}$.

According to the pumping lemma third condition, new string $|10^p1| \leq p$ becomes fails, that means, length of the *vxz* is greater than the pumping length *p*, i.e. $|vxy| \leq p$.

Hence, C_2 is not context free language.