Problem

Each of the following languages is the intersection of two simpler languages. In each part, construct DFAs for the simpler languages, then combine them using the construction discussed in footnote 3 (page 46) to give the state diagram of a DFA for the language given. In all parts, ∑ = {a, b}.

a. {wl w has at least three a's and at least two b's}

A**b.** {wl w has exactly two a's and at least two b's}

c. {wl w has an even number of a's and one or two b's}

A**d.** {wl w has an even number of a's and each a is followed by at least one b}

e. {wl w starts with an a and has at most one b}

f. {wl w has an odd number of a's and ends with a b}

g. {wl w has even length and an odd number of a's}

Step-by-step solution

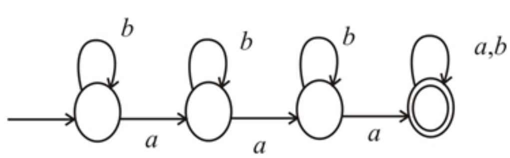
Step 1 of 18

a.

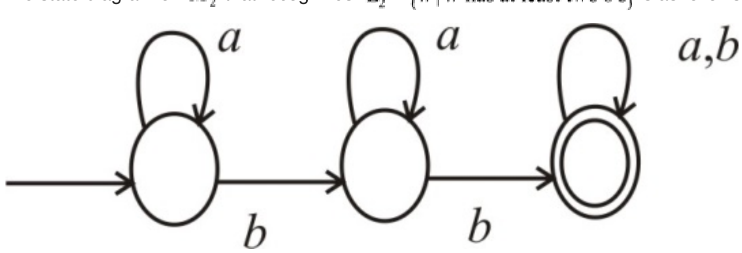
Consider the Language $L = \{w \mid w \text{ has at least three } a\text{'s and at least two } b\text{'s}\}$. The language *L* is the intersection of two simpler languages L_1 and L_2.

Now $L_1 = \{w \mid w \text{ has at least three } a\text{'s}\}$ and $L_2 = \{w \mid w \text{ has at least two } b\text{'s}\}$. Let *M* be the DFA (Deterministic Finite automata) that recognizes *L*. Let M_1 and M_2 be the DFAs that recognizes L_1 and L_2.

The state diagram of M_1 that recognizes $L_1 = \{w \mid w \text{ has at least three } a\text{'s}\}$ is as follows:



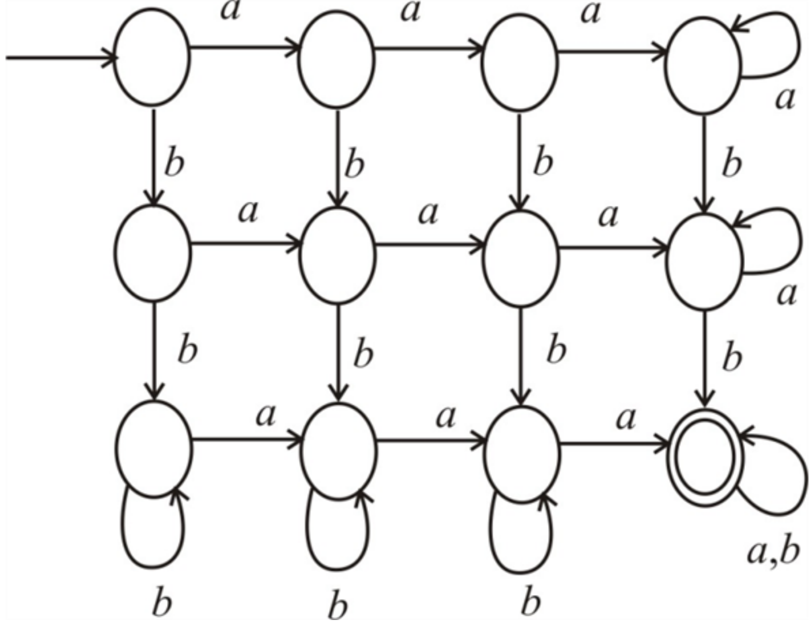
The state diagram of M_2 that recognizes $L_2 = \{w \mid w \text{ has at least two } b\text{'s}\}$ is as follows:



Comments (1)

Step 2 of 18

The machine *M* will accept the input if and only if both M_1 and M_2 accept it. Because, the language *L* is the intersection of the languages L_1 and L_2. Therefore, the state diagram of *M* that recognizes the Language L is as follows:



Comment

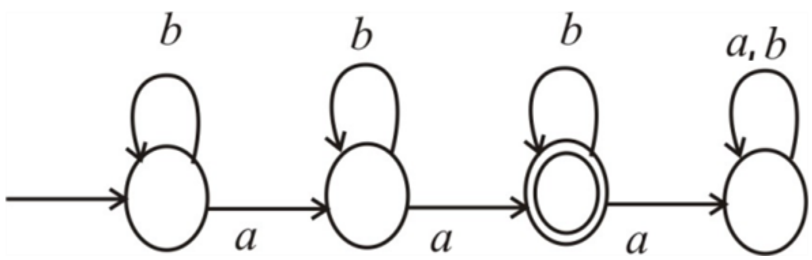
Step 3 of 18

b.

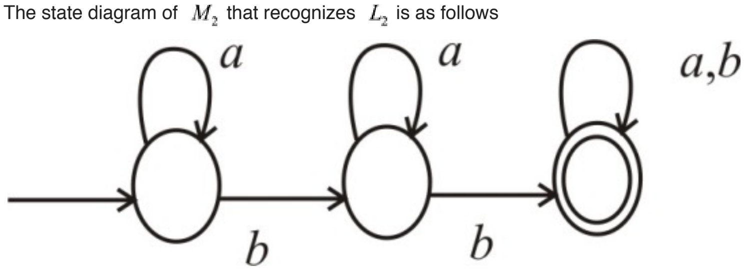
Consider the language $L = \{ w \mid w$ has at exactly two *a*'s and at least two *b*'s$\}$. The language *L* is the intersection of two simpler languages say L_1 and L_2.

Now $L_1 = \{ w \mid w$ has exactly two *a*'s$\}$ and $L_2 = \{ w \mid w$ has at least two *b*'s$\}$. Let *M* be the DFA(Deterministic Finite automata) that recognizes *L* and M_1 and M_2 be the DFAs that recognizes L_1 and L_2.

The state diagram of M_1 that recognizes L_1 is as follows



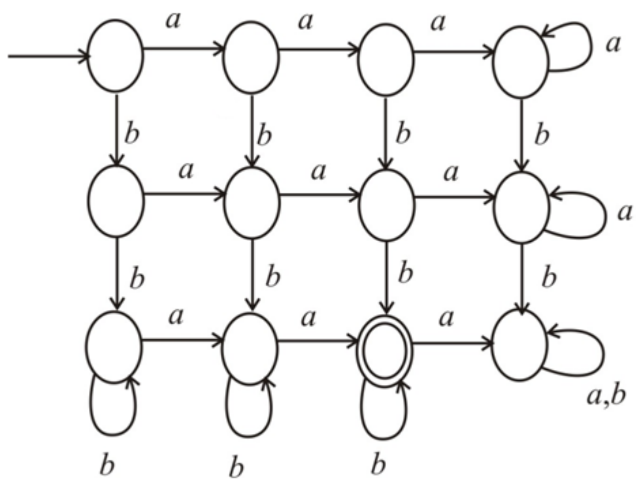
The state diagram of M_2 that recognizes L_2 is as follows



Comments (1)

Step 4 of 18

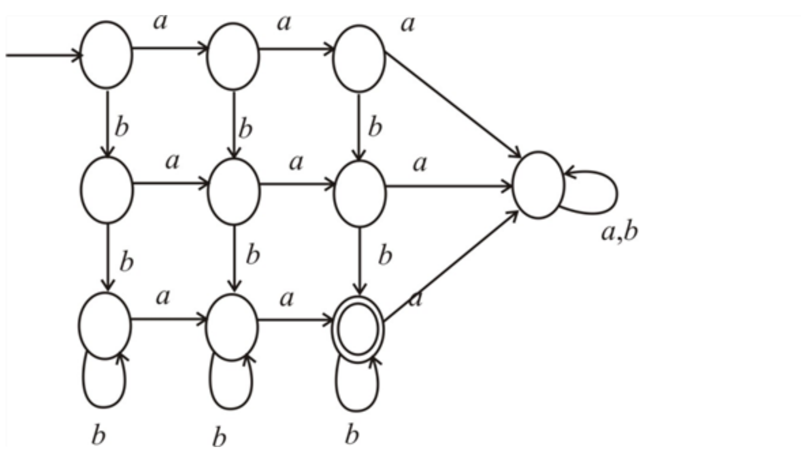
The machine *M* will accept the input if and only if both M_1 and M_2 accept it. Because, the language *L* is the intersection of L_1 and L_2. The state diagram of *M* that recognizes *L* is as follows:



Combine some states to get more simplified form as follows:

Comment

Step 5 of 18



Comments (1)

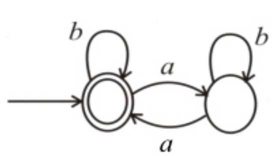
Step 6 of 18

c.

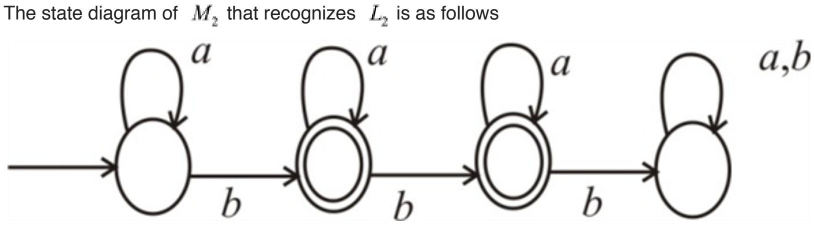
Consider the language $L = \{ w \mid w$ has even number of *a*'s and one or two *b*'s}. The language *L* is the intersection of two simpler languages say L_1 and L_2.

Now $L_1 = \{ w \mid w$ has even number of *a*'s } and $L_2 = \{ w \mid w$ has one or two *b*'s}. Let *M* be the DFA(Deterministic Finite automata) that recognizes *L* and M_1 and M_2 be the DFAs that recognizes L_1 and L_2.

The state diagram of M_1 that recognizes L_1 is as follows



The state diagram of M_2 that recognizes L_2 is as follows

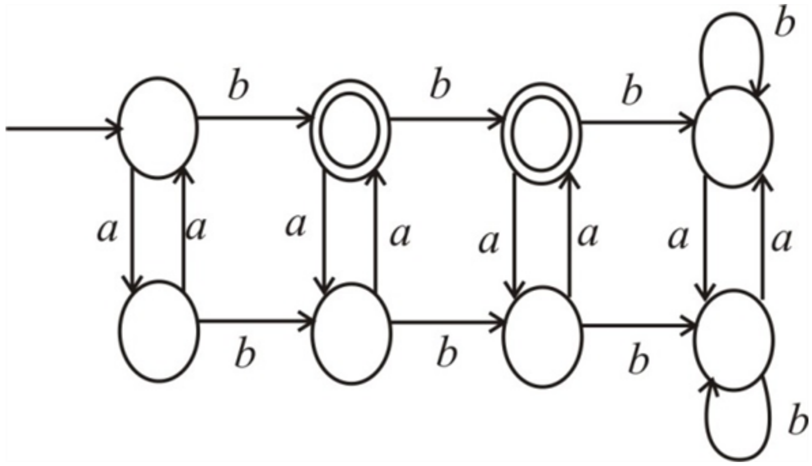


Comment

Step 7 of 18

The machine *M* will accept the input if and only if both M_1 and M_2 accept it. Because language *L* is the intersection of L_1 and L_2.

The state diagram of *M* which recognizes the language *L* is as follows



Comments (3)

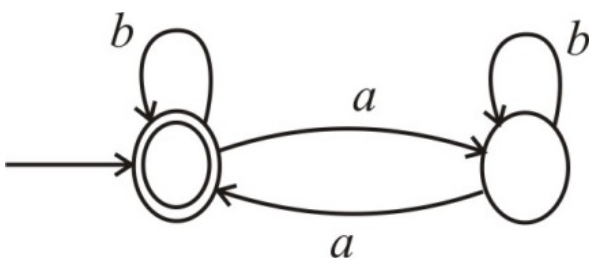
Step 8 of 18

d.

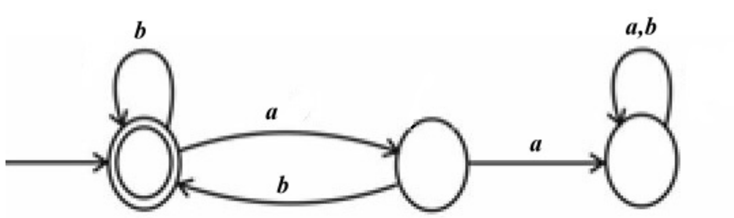
Consider the language $L=\{ w \mid w$ has an even number of *a*'s and each a is a followed by at least on *b*}. The language *L* is the intersection of two simpler languages say L_1 and L_2.

Now L_1={ $w \mid w$ has an even number of *a*'s } and L_2={ $w \mid w$ has each a is a followed by at least on *b* }. Let *M* be the DFA(Deterministic Finite automata) that recognizes *L* and M_1 and M_2 be the DFAs that recognizes L_1 and L_2.

The state diagram of M_1 that recognizes L_1 is as follows where $L_1=\{ w \mid w$ has an even number of *a*'s}.



The state diagram of M_2 that recognizes L_2 is as follows where $L_2=\{ w \mid w$ has each *a* is followed by at least one *b*}.



Comment

Step 9 of 18

The machine *M* will accept the input if and only if both M_1 and M_2 accept it. Because, the language *L* is the intersection of L_1 and L_2.

The state diagram of *M* that recognizes *L* is as follows:

Comment

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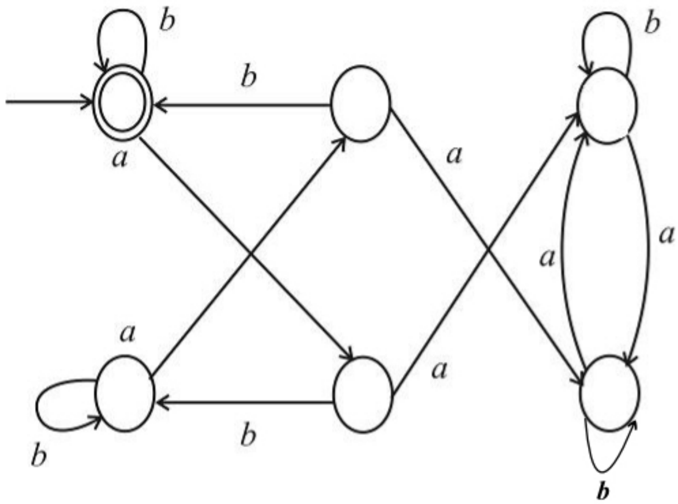
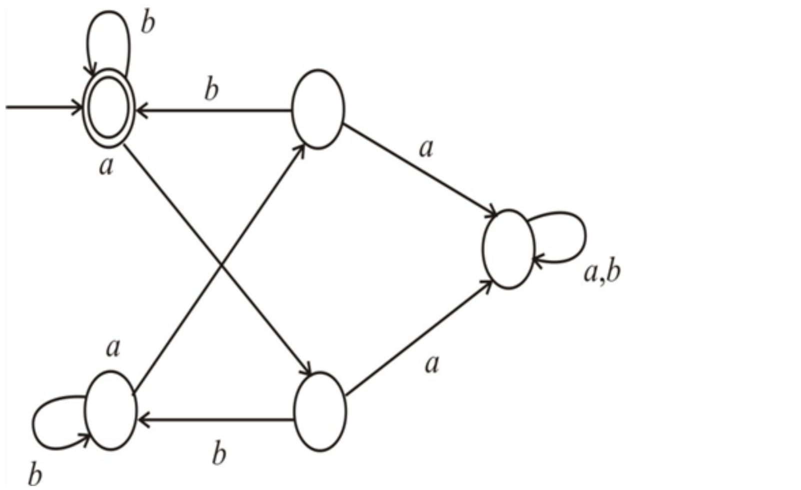


Comments (4)

Step 11 of 18

Combine some states to get more simplified form as follows:



Comments (1)

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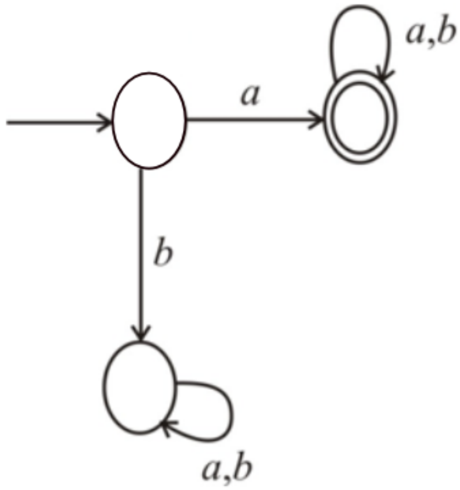
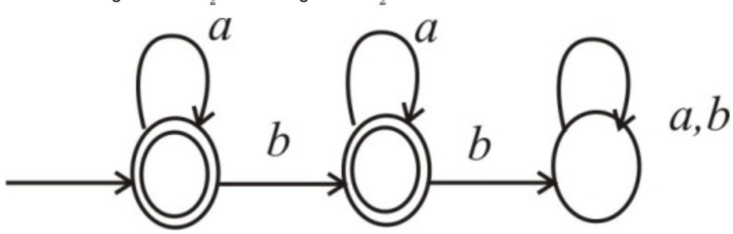
e.

Consider the language $L = \{ w | w$ Start with an *a* and has at most one *b*}. The language *L* is the intersection of two simpler languages say L_1 and L_2. Now $L_1 = \{ w | w$ starts with *a*}and $L_2 = \{ w | w$ has at most one *b*}. Let *M* be the DFA(Deterministic Finite automata) that recognizes *L* and M_1 and M_2 be the DFAs that recognizes L_1 and L_2.

The state diagram of M_1 that recognizes L_1 is as follows:



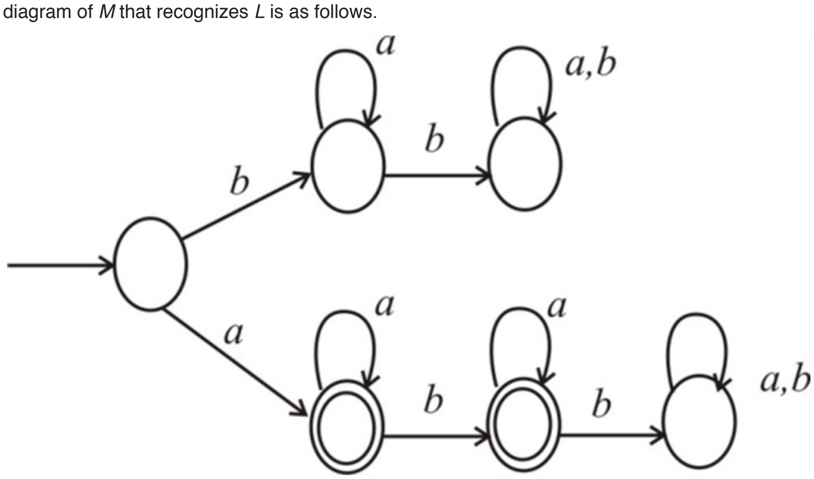
The state diagram of M_2 that recognizes L_2 is as follows.



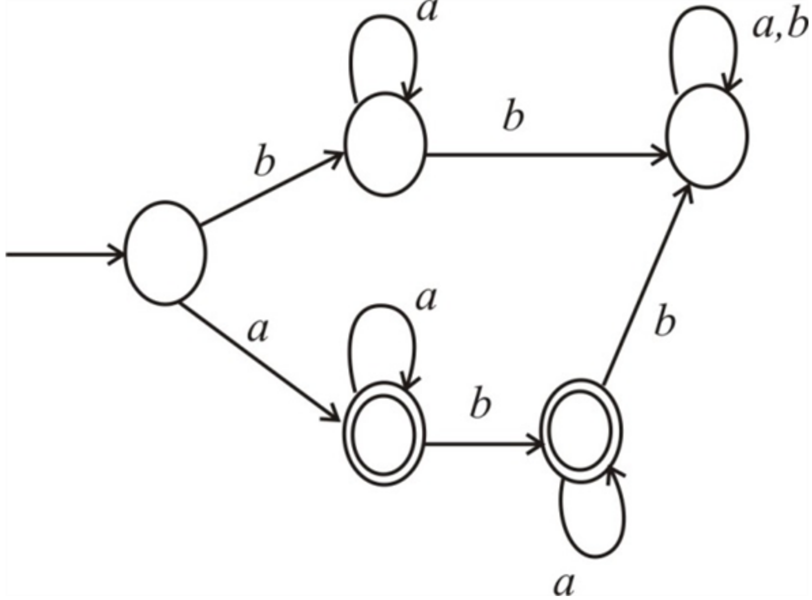
Comment

Step 13 of 18

The machine *M* will accept the input if and only if both M_1 and M_2 accept it. Because, the language *L* is the intersection of L_1 and L_2. The state diagram of *M* that recognizes *L* is as follows.



Combine some states to get more simplified form as follows:



Comments (3)

Step 14 of 18

f.

Consider the language $L = \{ w \mid w$ has an odd number of *a*'s and ends with *b*}. The language *L* is the intersection of two simpler languages say L_1 and L_2. Now $L_1 = \{ w \mid w$ has an odd

Comment

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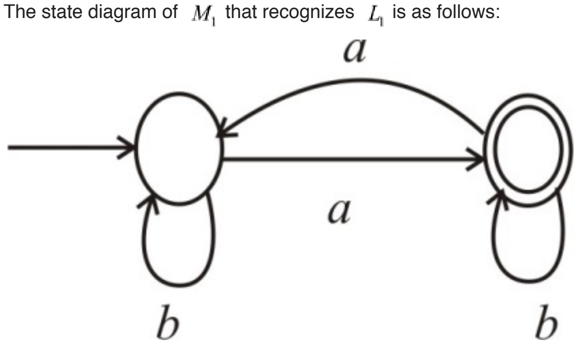
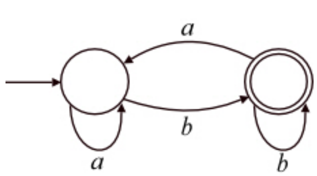
number of *a*'s} and $L_2 = \{ w \mid w$ ends with a *b*}. Let *M* be the DFA(Deterministic Finite automata) that recognizes *L* and M_1 and M_2 be the DFAs that recognizes L_1 and L_2.

The state diagram of M_1 that recognizes L_1 is as follows:



The language L_2 accepts the strings that ends with a symbol *b*. The state diagram of M_2 that recognizes L_2 is as follows:



Comment

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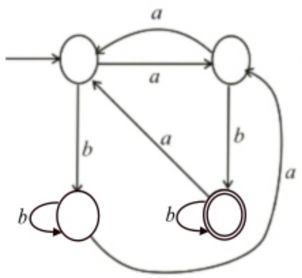
The machine *M* will accept the input if and only if both M_1 and M_2 accept it. Because, the language *L* is the intersection of L_1 and L_2.

The state diagram of *M* that recognizes *L* is as follows:



Comments (9)

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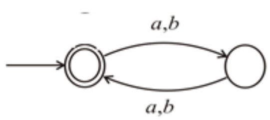
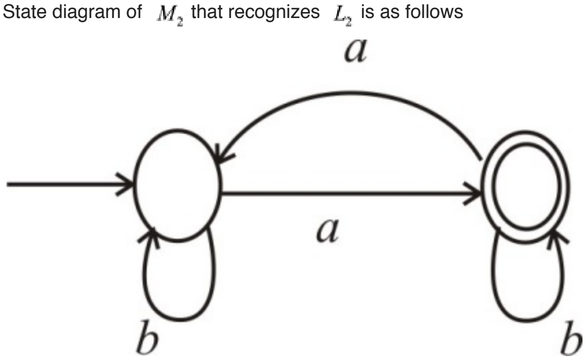
g.

Consider the language $L = \{ w | w$ has even length and an odd number of *a*'s }. The language *L* is the intersection of two simpler languages say L_1 and L_2. Now $L_1 = \{ w | w$ has even length } and $L_2 = \{ w | w$ has odd number of *a*'s }. Let *M* be the DFA(Deterministic Finite automata) that recognizes *L* and M_1 and M_2 be the DFAs that recognizes L_1 and L_2.

State diagram of M_1 that recognizes L_1 is as follows



State diagram of M_2 that recognizes L_2 is as follows

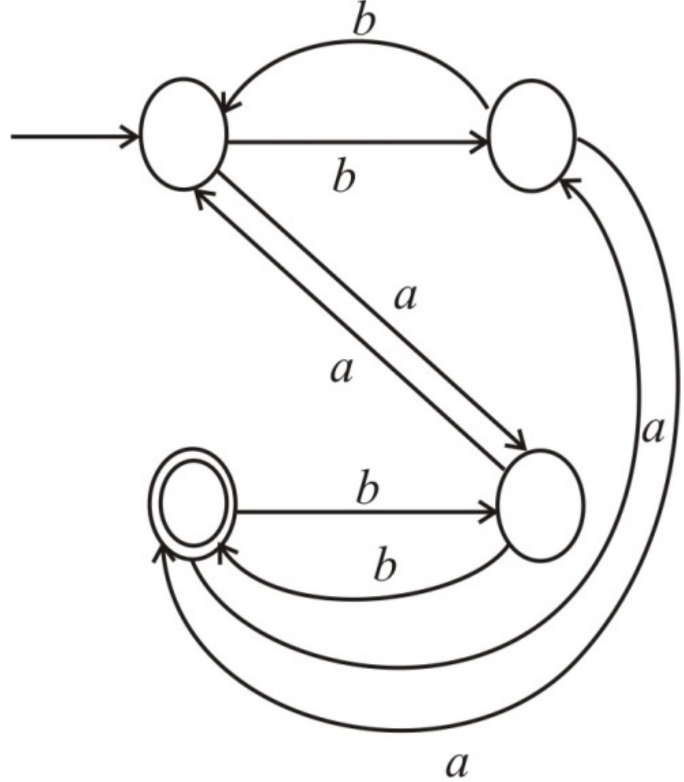


Comment

Step 18 of 18

The machine *M* will accept the input if and only if both M_1 and M_2 accept it. Because, the language *L* is the intersection of L_1 and L_2.

The state diagram of *M* that recognize *L* is as follows:



Comments (2)