Problem

Describe the error in the following fallacious “proof” that P ≠ NP. Assume that P = NP and obtain a contradiction. If P = NP, then *SAT* ? P and so for some k, *SAT* ? TIME(n^k). Because every language in NP is polynomial time reducible to *SAT*, you have

$$\mathrm{NP} \subseteq \mathrm{TIME}(n^k). \text{ Therefore, } \mathrm{P} \subseteq \mathrm{TIME}(n^k).$$

Step-by-step solution

Step 1 of 2

Consider the following deceptive “proof “that $\mathrm{P} \neq \mathrm{NP}$.

1. Suppose that $\mathrm{P}=\mathrm{NP}$ and obtain a contradiction.

2. If $\mathrm{P}=\mathrm{NP}$ has given, then for some $k \in \mathrm{N}$ and $\mathrm{SAT} \in \mathrm{P}$, $\mathbf{SAT} \in \mathbf{TIME}\left(\mathbf{n}^{\mathbf{k}}\right)$

3. As every language in NP is polynomial time reducible to SAT ,then $\mathbf{NP} \subseteq \mathbf{TIME}\left(\mathbf{n}^{\mathbf{k}}\right)$

4. Due to the assumption $\mathrm{P}=\mathrm{NP}$ which is taken above, $\mathbf{P} \subseteq \mathbf{TIME}\left(\mathbf{n}^{\mathbf{k}}\right)$.

5. Then $\mathbf{TIME}\left(\mathbf{n}^{\mathbf{k}}\right) \subsetneq \mathbf{TIME}\left(\mathbf{n}^{\mathbf{k+1}}\right)$ is a contradiction with $\mathbf{P} \subseteq \mathbf{TIME}\left(\mathbf{n}^{\mathbf{k}}\right)$.

Comment

Step 2 of 2

The above steps are used to proof of $\mathrm{P} \neq \mathrm{NP}$, but it consists some error. Now, consider the **step 3**, **here the time needed for calculating the reduction is not taken into account. So, to make this proof error free, the time needed for calculating the reduction must be taken in to account**.

Comments (1)