Problem

For each

$$m > 1 \text{ let } \mathcal{Z}_m = \{0, 1, 2, \ldots, m-1\}, \text{ and let } \mathcal{F}_m = (\mathcal{Z}_m, +, \times)$$

is decidable.

Step-by-step solution

Step 1 of 1

Given that

- $z_m = \{0,1,2,\ldots,m-1\}$ for $m > 1$ is a universe.
- $F_m = (z_m, +, \times)$ be the model whose universe is z_m.
- $+$ and $\times$ are relations over modulo *m*.

Now we have to show that $Th(F_m)$ is decidable.

Clearly z_m is finite.

So we can simply enumerate all the possible values into the formula and see if the formula is true.

If it is true then it is belong to $Th(F_m)$

If it is false then it does not belong to $Th(F_m)$

In particular, the following recursive procedure would work:

- Let $\exists x_i$ such that $\phi_i(x_1, x_2, \ldots x_i)$ be the formula
- Substitute $x_i = 0,1,\ldots,m-1$ into the formula.
- If the formula $\phi_i(x_1, x_2, \ldots x_i)$ is true for any value of x_i then the original formula is true
- That means, if we have $\forall x_i$ instead of $\exists x_i$ then the original formula will be true.

$\rightarrow$ A theory of a model is said to be decidable if we say that the formulae which are true are belong to that model and which are false that does not belong to that model.

So by the above recursive procedure we say that $Th(F_m)$ is decidable.

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