

## Problem

A **useless state** in a pushdown automaton is never entered on any input string. Consider the problem of determining whether a pushdown automaton has any useless states. Formulate this problem as a language and show that it is decidable.

## Step-by-step solution

### Step 1 of 4

Useless states in automata are the **states whose removal from the automata does not make any difference or put any impact on the language accepted by the automaton.**

**Given:** A pushdown automaton.

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### Step 2 of 4

Here, it is required to test whether the machine has useless states or not and whether the problem of testing is decidable or not.

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### Step 3 of 4

- Let  $P$  be the set of all the strings accepted by a pushdown automaton.
- Let the language  $L = \{x \in P \mid x \text{ contains a useless state}\}$ .
- To show that language  $L$  is decidable, construct a Turing machine which accepts strings in language  $L$ .
- In reference to this book, consider that the question of whether a PDA has an empty language is decidable.
- It can reduce the question of whether a given state  $n$  is useless to this question by making  $n$  the only accept state and then determine whether the resulting push down automata has an empty language.
- If it does then,  $n$  is a useless state.

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### Step 4 of 4

**Hence, our Turing machine successfully decides whether there is any useless state, by performing this test for each and every state in order.**

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