

### Problem

Consider the function  $pad: \Sigma^* \times \mathcal{N} \longrightarrow \Sigma^* \#^*$  define the language  $pad(A, f)$  as

$$pad(A, f) = \{pad(s, f(m)) \mid \text{where } s \in A \text{ and } m \text{ is the length of } s\}.$$

Prove that if  $A \in \text{TIME}(n^6)$ , then  $pad(A, n^2) \in \text{TIME}(n^3)$ .

### Step-by-step solution

#### Step 1 of 1

For any function  $f: N \rightarrow N$  and language,  $pad(A, f)$  is defined as:

$$pad(A, f) = \{pad(s, f(m)) \mid \text{where } s \in A \text{ and } m \text{ is the length of } s\}.$$

If  $A \in \text{TIME}(n^6)$  is given then, it is supposed that M be a machine that decide A in time  $n^6$ .

• Now a machine M' can be considered for  $pad(A, n^2)$  that on input x, check if x is of the format  $pad(w, |w|^2)$  for some string  $w \in \Sigma^*$ . Input x will be rejected if it will not matched. Otherwise, simulate M on w.

• The running time of machine M' is  $O(|x|^3) + O(|w|^6) = O(|x|^3)$ .

Hence, it can be said that  $pad(A, n^2) \in \text{TIME}(n^3)$ .

---

[Comment](#)