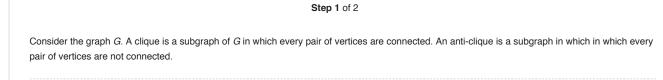
## **Problem**

Ramsey's theorem. Let G be a graph. A *clique* in G is a subgraph in which every two nodes are connected by an edge. An *anti-clique*, also called an *independent set*, is a subgraph in which every two nodes are not connected by an edge. Show that every graph with n nodes contains either a clique or an anti-clique with at least 1/2 log<sup>2</sup> n nodes.

# Step-by-step solution



#### Comment

## Step 2 of 2

In order to show that every graph with n vertices contains either clique or anti-clique with at least  $\frac{1}{2}\log_2 n$  vertices, create two piles A and B to store the vertices of a graph. Here, the pile A contains the vertices of a clique whereas the pile B contains the vertices of an anti-clique. Procedure to identify a clique or an anti-clique is as follows:

- Take each vertex v of the graph G.
- If the degree of the vertex is greater than one half of the remaining vertices then add the vertex to pile A. Otherwise, add the vertex to the pile B.
- Discard all vertices to which v is not connected if it was added to the pile A.
- Discard all vertices to which v is connected if it was added to the pile B.
- Continue this procedure until no vertices left.

Consider the whole procedure as a step. For each step, at most half of the vertices are discarded. Thus, at least  $\log_2 n$  steps occur before completion of the process. Each step adds a vertex to one of the piles. Thus, one of the piles contains at least  $\frac{1}{2}\log_2 n$  vertices.

Therefore, it is proved that that every graph with n vertices contains either clique or anti-clique with at least  $\frac{1}{2}\log_2 n$  vertices.

# Comment