Problem

Let Σ_2 be the same as in Problem 1.33. Consider each row to be a binary number and let

$$D = \{w \in \Sigma_2^* | \text{ the top row of } w \text{ is a larger number than is the bottom row}\}.$$

For example, $\begin{bmatrix}0\\0\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix}\begin{bmatrix}1\\1\end{bmatrix}\begin{bmatrix}0\\0\end{bmatrix} \in D$, but $\begin{bmatrix}0\\0\end{bmatrix}\begin{bmatrix}0\\1\end{bmatrix}\begin{bmatrix}1\\1\end{bmatrix}\begin{bmatrix}0\\0\end{bmatrix} \notin D$. Show that D is regular.

Step-by-step solution

Step 1 of 4

Given language is

$D = \left\{ w \in \Sigma_2^* \mid \text{ the top row of } w \text{ is the larger number than is the bottom row} \right\}$

Over the alphabet $\Sigma_2 = \left\{ \begin{bmatrix}0\\0\end{bmatrix}, \begin{bmatrix}0\\1\end{bmatrix}, \begin{bmatrix}1\\0\end{bmatrix}, \begin{bmatrix}1\\1\end{bmatrix} \right\}$

Language for given expression L = $\left\{ \varepsilon, \begin{bmatrix}1\\0\end{bmatrix}, \begin{bmatrix}1\\0\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix}, \begin{bmatrix}1\\0\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix}, ... \right\}$

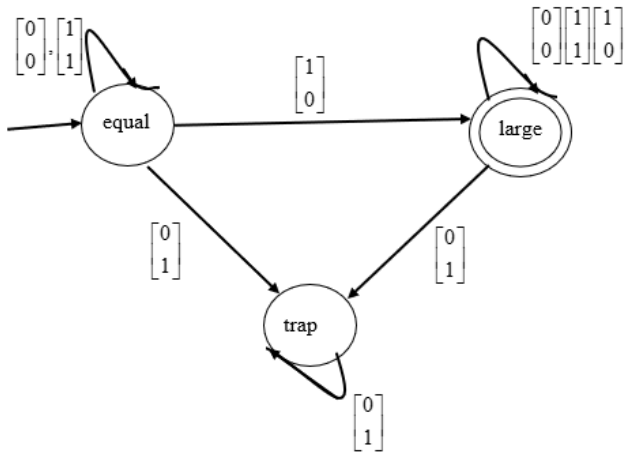
Here each row is binary number.

Comment

Step 2 of 4

Let **M** be the DFA, over the input alphabet $\Sigma_2 = \left\{ \begin{bmatrix}0\\0\end{bmatrix}, \begin{bmatrix}0\\1\end{bmatrix}, \begin{bmatrix}1\\0\end{bmatrix}, \begin{bmatrix}1\\1\end{bmatrix} \right\}$.

The state transition diagram of **M** is as follows:



Comments (1)

Step 3 of 4

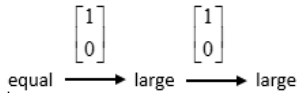
We must prove that *D* is a regular language.

A language is said to be regular if it recognizes by a DFA.

Let take string form language D, $w = \left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \right\}$

Initial state of the above DFA is ‘equal’

Parse string



Here ‘large’ is final state , the string is accepted by the DFA.

Comment

Step 4 of 4

Thus, language of given D is accepted by the given DFA.

we defined a DFA to recognize the language D.

Therefore, D is a regular language.

Comment