Problem

a. Let A be an infinite regular language. Prove that A can be split into two infinite disjoint regular subsets.

b. Let B and D be two languages. Write $B \Subset D$ if $B \subseteq D$ and D

Step-by-step solution

Step 1 of 2

a.

The proof that an infinite regular language, say A, can be split into two infinite disjoint regular subsets is as follows:

• Let there be a string, say ‘s’, such that $s \in A$ and s = xyz, where x, y and, z, represent the sub-strings of the string s.

• Since s belongs to the language A, and the language A is regular, xy^iz must belong to A, where $i \geq 0$. (As per the condition 1 of pumping lemma).

• Let A_1 be a language such that $A_1 = \{xy^{2i}z, \text{where } i \geq 0\}$.

• Since all the strings of the form xy^iz belong to A, the strings of the form $xy^{2i}z$ must also belong to A.

• Hence, the language A_1 is a subset of the language A, i.e:

$A_1 \subset A$

• The strings of the language A_1 can be represented by the following regular expression:

$x(yy)^*z$

Hence, the language A_1 is a regular language

• Since in the expression, $A_1 = \{xy^{2i}z, \text{where } i \geq 0\}$, there is no upper limit for the value of i, the language A_1 is infinite.

• Since the regular languages are closed under the operation of complement, the language $\overline{A_1}$ is a regular language.

• Let A_2 be a language such that, $A_2 = \overline{A_1} \cap A$.

• Since the regular languages are closed under the operation of intersection, the language A_2 is a regular language.

• Since the languages, A_1 and A are infinite, the language A2 is also infinite

• Clearly A_2 and A_1 are two disjoint sets.

• Also, $A = A_1 \cup A_2$

Thus, the language A can be split into two infinite disjoint regular subsets.

Hence, proved.

Comment

Step 2 of 2

b.

The steps required to prove the given statement are as follows:

• Divide the regular language D into two regular disjoint subsets and let one of those subsets be B.

• Let the other subset be A, such that A = D – B.

• Since D contains infinitely many strings that are not in B, A also contains infinitely many strings that are not in B.

• Further divide the language A into two disjoint subsets, A_1 and A_2, such that A_2 contains infinitely many strings that are not in A_1 and vice versa.

• Since A contains infinitely many strings that are not present in B, A_1 also contains infinitely many strings that are not in B.

• Create a set C such that $C = A_1 \cup B$.

• Since A_1 contains infinitely many strings that are not in B, C also contains infinitely many strings that are not in B.

• Clearly, B is a subset of C.

• Hence, the following statement is true:

$B \Subset C$

• Since A_2 contains infinitely many strings that are not in A_1, D contains infinitely many strings that are not present in A_1.

• Since D contains infinitely many strings that are not present in A_1, D contains infinitely many strings that are not present in C.

• Hence, the following statement is true:

$C \Subset D$.

• Since $B \Subset C$ and $C \Subset D$, the following statement is true:

$B \Subset C \Subset D$

Hence, proved.

Comment