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CSE 303: Theory of Computation
HW3

Problem 1:-
Given three at-most countable sets A, B, C.
Let us assume the elements of A, B and C are as follows:-
$a_1, a_2, a_3, \ldots \in A$.
$b_1, b_2, b_3, \ldots \in B$
$c_1, c_2, c_3, \ldots \in C$.

There may be common elements or not. We need to prove that $A \cup B \cup C$ is also at-most finite.
~~If there are common elements als~~
It will suffice to ~~pr~~ show a ~~countin~~. method to count every member of $A \cup B \cup C$.
We will start with below.

1 a_1	4 a_2	7 a_3	10 a_4	....
2 b_1	5 b_2	8 b_3	11 b_4	....
3 c_1	6 c_2	9 c_3	12 c_4	....

This is how we can count each and every element of $X = A \cup B \cup C$. If there is an element which is appearing again then we skip that element & move to the next.
For eg: if $a_1 = b_2$ then $4 \leftrightarrow a_2$ & $5 \leftrightarrow c_2$
Since $b_2 = a_1$ is already counted.

~~If~~ This counting method works if any of A, B, C is infinite since the members of X will also be infinite.

Thus, for three at-most countable sets, their union is also at most countable.

Problem 2:-

Let $P(n)$ be the property that a partial order on a set of n elements has atleast one minimal element.

We prove this by Induction.

Basis:- $P(1)$: Given a partial order on a set of a single element, ~~that set~~ Say R.
$A = \{a\}$.
$R = \{(a,a)\}$ Since R is reflexive.
This element is the only element in R thus is the minimal element.

Induction Hypothesis:- Let k be a fixed but arbitary no. Let $P(k)$. ~~be the set of k element~~ partial order on a set of k elements has minimal element be true.

~~Let the min~~
Induction :- TPT $P(k+1)$ holds.

~~Let x be any element of A. Now for the partial order on $A - \{x\}$ there is a minimal element.~~
~~Say y.~~
~~Now if $y < x$ then y is the minimal element for A.~~
~~ESP~~

~~Fake~~ ~~th~~

Let $A = \{a_1, a_2, \dots, a_k, a_{k+1}\}$.

Let $B = A - \{a_{k+1}\}$. Then the Partial order R on B will have a minimal element.
Let the minimal element be x for B.

Thus $\forall y \in B$, if $(y, x) \in R \Rightarrow y = x$.
[Definition of minimal element]

Case i) $(a_i, a_{k+1}) \in R$ for some $i \in \{\cancel{0}, 1, 2, \dots, k\}$.

If $a_i = x$ then $(x, a_{k+1}) \in R \Rightarrow x$ is minimal element of R over A.

If $a_i \neq x$ then $(x, a_i) \in R \because a_i \in B$ & x is minimal element of R over B

Since $(x, a_i) \in R$ & $(a_i, a_{k+1}) \in R \Rightarrow (x, a_{k+1}) \in R$
$\Rightarrow x$ is minimal element of R over A. by transitivity.

Case ii) $(a_i, a_{k+1}) \in R \Rightarrow a_i = a_{k+1}$
i.e there is no such $a_i \in B$.

In that case, a_{k+1} is an minimal element of R over A

In either case $\exists$ atleast one minimal element of R over A.
Induction: Thus $P(k) \rightarrow P(k+1)$.

Thus it holds for all non-empty finite sets.

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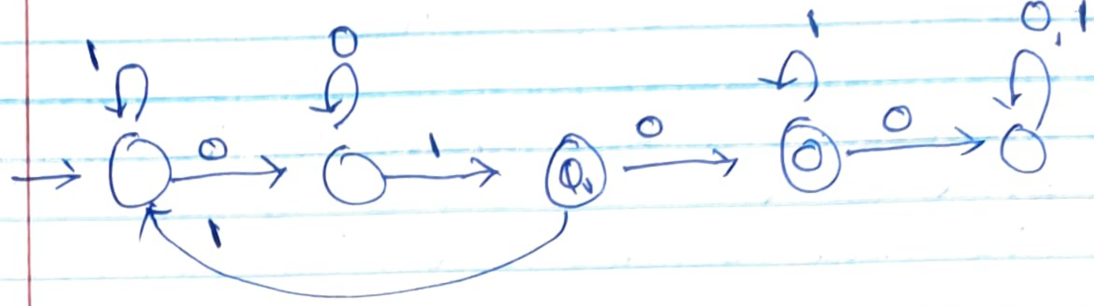
Problem 3:-



~~The tan~~

Let A = Any no. (≥0) of 1's followed by any no (≥1) of 0's, followed by 1

B = a 0 followed by any no. of 1's

~~C = 1 followed by~~ any

Then language recognised is

Starts with A then either B or 1 followed by A.

This is because when in Q_1 by getting an input of 1, we go back to initial state.

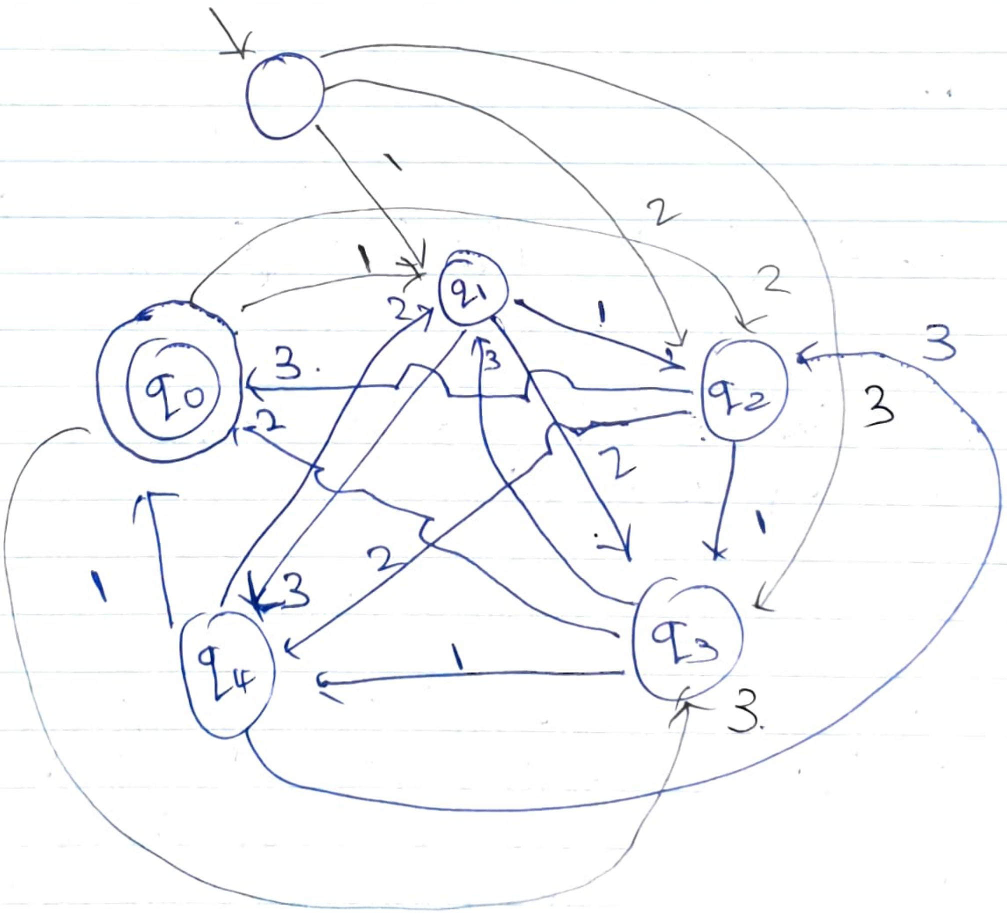
~~Then language recognised will be~~

~~Starts with A then either B or (C followed by A)~~

Problem 4:-

Assuming that for ~~ϵ~~ no input, it is not an accepted state since the sum of all symbols is not defined for no input.

Let q_i denote the state where i denotes the ~~~~ sum of all symbols modulo 5.



This is a finite automaton that ~~accepts~~ recognises the set of strings over the alphabet $\{0, 1, 2, 3\}$ in which sum of all symbols is divisible by 5.

Problem 6:-

$\in \in L(M)$ iff the initial state is an ~~accep~~ acceptable state.

Proof:- Let $\in \in L(M)$.
So our FA is in its initial state. Since after giving no input, it will be in its initial state. Since ~~E~~ Since no input is a ~~ac~~ language accepted by $M \Rightarrow$ initial state is final state.

Part II:- Initial state is acceptable state.
When FA is in its initial state, it is its its final state without any input i.e $\in$ as input $\Rightarrow \in \in L(M)$

Thus $\in \in L(M)$ iff initial state is an acceptable state

Problem 2:
Part II:-
For infinite sets, this property may not hold good.

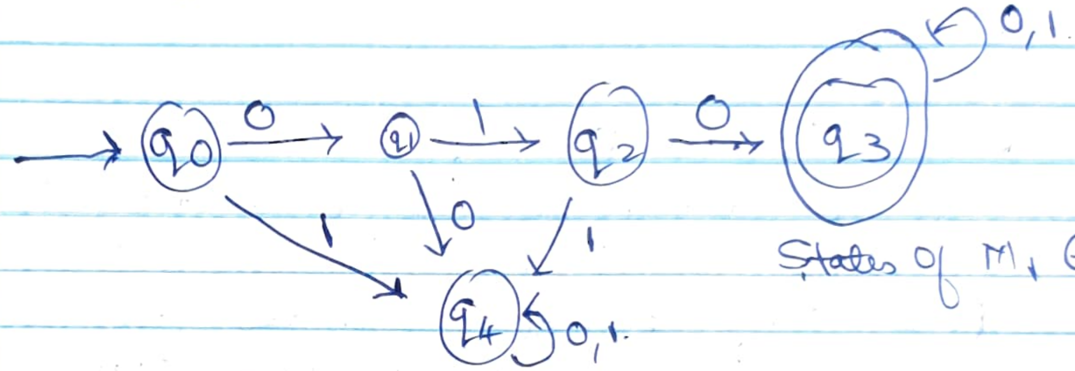
Eg: $A = \{-1, -2, -3, \dots\}$.

$R = \{(m, n) \in A \times A.\ m \leq n\}$

Then there is no minimal element since it extends all the way to infinity.

Problem 5:
We will construct two FA ~~we~~ one which accepts strings beginning with 010 & 2nd which accepts strings ~~with~~ ending in 101.

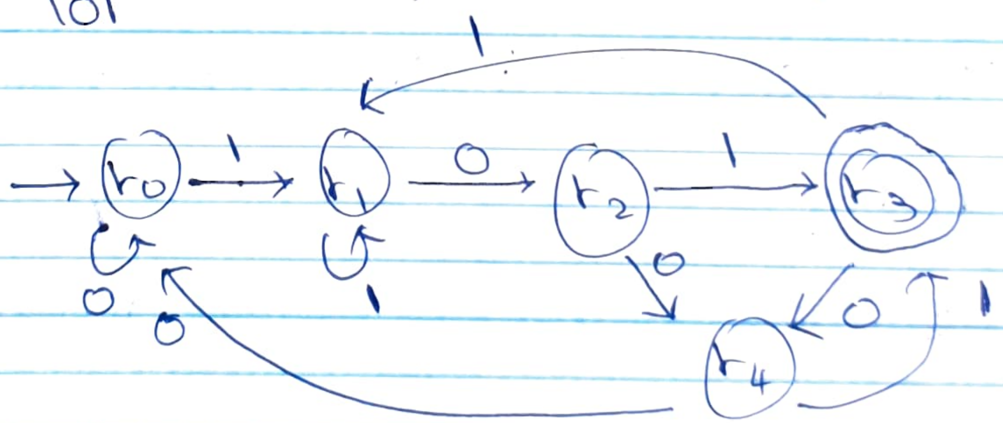
M1: FA accepting strings over $\Sigma = \{0,1\}$ that begin with 010.



States of $M_1 \in Q$.

The initial state of this is q_0, final/acceptable state is q_3. q_4 is dead state.

M2: FA accepting strings over $\Sigma = \{0,1\}$ that end with 101



The initial state is r_0, final/acceptable state is r_3

States of $M_2 \in R$.

We will define a new automata $M = M_1 \cup M_2$ with States given as follows:

$P = \{(q, r) . q \in Q \wedge r \in R\}$

Initial State of M is (q_0, r_0).

Accepting States of $M = \{(q_1, r_1) . q_1$ is acceptable for $M_1 \vee r_1$ is acceptable for $M_2\}$.

M is defined over same $\Sigma = \{0, 1\}$.

Transition function δ is given by

$$\delta((q, r)), \sigma) = (\delta_1(q, \sigma), \delta_2(r, \sigma))$$

where $\sigma \in \{0, 1\}$ is a symbol.
$q \in Q$ & $r \in R$.

δ_1 is transition func. of M_1
δ_2 ——— " ——— " M_2.