## Problem

Let $\phi_{\text{eq}}$ be defined as in Problem 6.10. Give a model of the sentence

$$\begin{aligned}\phi_{\text{lt}} \;=\;& \phi_{\text{eq}}\\ &\wedge \forall x,y \left[\, R_1(x,y) \to \neg R_2(x,y) \,\right]\\ &\wedge \forall x,y \left[\, \neg R_1(x,y) \to (R_2(x,y) \oplus R_2(y,x)) \,\right]\\ &\wedge \forall x,y,z \left[\, (R_2(x,y) \wedge R_2(y,z)) \to R_2(x,z) \,\right]\\ &\wedge \forall x\, \exists y \left[\, R_2(x,y) \,\right].\end{aligned}$$

Give a model of the sentence

$$\begin{aligned}\phi_{\text{eq}} \;=\;& \forall x \left[\, R_1(x,x) \,\right]\\ &\wedge \forall x,y \left[\, R_1(x,y) \leftrightarrow R_1(y,x) \,\right]\\ &\wedge \forall x,y,z \left[\, (R_1(x,y) \wedge R_1(y,z)) \to R_1(x,z) \,\right].\end{aligned}$$

## Step-by-step solution

**Step 1** of 2

Consider the following sentence that is provided in the problem 6.10 in the textbook,

$$\begin{aligned}\phi_{eq} &= \forall x\,[R_1(x,x)]\\ &\wedge \forall x,y\,[R_1(x,y) \leftrightarrow R_1(y,x)]\\ &\wedge \forall x,y,z\,[(R_1(x,y) \wedge R_1(y,z)) \to R_1(x,z)]\end{aligned}$$

The above statement explains about the conditions of the equivalence relation. A model $(A, R_1)$, where *A* is any universe and $R_1$ is the equivalence relation over the elements of *A*. The line 1 describes that for all *x*, *x* is equal to itself. The line 2 describes that for all *x* and *y*, if and only if *x* is equal to *y* then *y* is equal to *x*. The line 3 describes that for all *x, y*, and *z*, if *x* is equal to *y* and *y* is equal to *z* then *x* is equal to *z.*

Comment

**Step 2** of 2

Consider the following sentence,

$$\begin{aligned}\phi_{lt} &= \phi_{eq}\\ &\wedge \forall x,y\,[R_1(x,y) \to \neg R_2(x,y)]\\ &\wedge \forall x,y\,[\neg R_1(x,y) \to (R_2(x,y) \oplus R_2(y,x))]\\ &\wedge \forall x,y,z\,[(R_2(x,y) \wedge R_2(y,z)) \to R_2(x,z)]\\ &\wedge \forall x \exists y\,[R_2(x,y)]\end{aligned}$$

The above statement explains about the conditions of the equivalence relation and less than relation. A model $(A, R_1, R_2)$, where *A* is any universe, $R_1$ is the equivalence relation over the elements of *A* and $R_2$ is the less than relation over *A.*

• The line 1 describes the conditions of the equivalence relation.

• The line 2 describes that for all $x$ and $y$, if $x$ is equal to $y$ then it is tends to complement of less than relation. This means if $x$ is equal to $y$ then it can be said that $x$ is not less than $y$.

• The line 3 describes that for all $x$ and $y$, if $x$ is not equal to $y$ then either $x$ less than $y$ or $y$ less than $x$ is tru

• The line 4 describes that for all $x$, $y$ and $z$, if $x$ is less than $y$ and $y$ is less than $z$ then $x$ is less than $z$.

• The line 5 describes that for all $x$ there exist $y$, $x$ is less than $y$.

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