Problem

$$\Sigma_3 = \left\{ \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \ldots, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\}.$$

Σ_3 contains all size 3 columns of 0s and 1s. A string of symbols in Σ_3 gives three rows of 0s and 1s. Consider each row to be a binary number and let

$$B = \{w \in \Sigma_3^* | \text{ the bottom row of } w \text{ is the sum of the top two rows}\}.$$

For example,

$$\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \in B, \quad \text{but} \quad \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \notin B.$$

Show that B is regular. (Hint: Working with $B^{\mathcal{R}}$ is easier. You may assume the result claimed in Problem 1.31.)

Step-by-step solution

Step 1 of 3

Consider the data

- $\Sigma_3 = \left\{ \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \ldots \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\}$
- A string of symbols in Σ_3 gives 3 rows of $0s$ and $1s$.
- Each row to be a binary number
- $B = \{w \in \Sigma_3^* \text{ the bottom row of w is the same of the top two row} \}$ is the language over Σ_3.

Comment

Step 2 of 3

Already know that "regular languages are closed under reversal".
Then, if prove that B^R is regular, then automatically B is regular and vice−versa.
So, first have to prove that B^R is regular.
A language is said to be regular if some automaton recognizes it.

Comment

Step 3 of 3

Let M be the automaton that recognizes B^R.

- M has 2 states.

(i) c_0, which denotes that the string that we have read so for leads to a carry 0.

(ii) c_1, that stands for carry 1.

Now $M = (Q, \Sigma, \delta, q_0, F)$

Where $Q = \{c_0, c_1\}$

= set of states

$$\Sigma = \left\{ \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \ldots \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\}$$

= set of alphabets

$q_0 = c_0$

= start state

$F = \{c_0\}$

= set of final states.

δ is given as:

- $\delta(c_0, a) = c_0$ if $a = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$ or $\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$
- $\delta(c_0, a) = c_1$ if $a = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$
- $\delta(c_1, a) = c_1$ if $a = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ or $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$
- $\delta(c_1, a) = c_0$ if $a = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

All other arrows go to trap state. Then, the defined a automation M to recognize B^R

Therefore B^R is a regular language. As B^R is regular, B is also a regular language.

Comment