Problem

Show that any infinite subset of MIN_{TM} is not Turing-recognizable.

Step-by-step solution

Step 1 of 3

Definition of MIN $_{TM}$ =

If *M* is a Turing machine, then we say that the length of the description $\langle M \rangle$ of *M* is the number of symbols in the string description *M*.

Say that *M* is minimal if there is no Turing machine equivalent to *M* that has a shorter description. $\boxed{\mathrm{MIN}_{\mathrm{TM}} = \{\langle M \rangle \mid \text{M is a minimal TM}\}}$

Comment

Step 2 of 3

Now we have to prove that any infinite subset of $\mathrm{MIN}_{\mathrm{TM}}$ is not Turing – recognizable.

We will prove by taking contradiction.

We assume that there exists *A*, an infinite subset of $\mathrm{MIN}_{\mathrm{TM}}$, such that *A* is Turing – recognizable.

We know that

"A language is Turing – recognizable if and only if some enumerator enumerates it".

So, Let *E* be the enumerator that enumerates *A* .

By using this *E*, we construct another *TM* (Turing – machine) *N* as follows.

$\mathrm{N} =$"On input *w*:

1. from recursion theorem. Own description $\langle c \rangle$ is obtained

2. Run the Enumerator *E* until a machine *P* is obtained with a longer

description than that of *N*.

3. Simulate *P* on input *w*".

Comment

Step 3 of 3

As we know that $\mathrm{MIN}_{\mathrm{TM}}$ is infinite *A* is infinite subset of $\mathrm{MIN}_{\mathrm{TM}}$.

1. When *A* is infinite, *E*'s list must contain a *TM* with longer description than $\mathrm{N's}$ description. So obviously *N* terminates with some *TM* P which is longer than *N*. Then *N* simulates *P* and so is equivalent to *P*.

2. It also notify that N is shorter than *P*. So *P* cannot be minimal. But *P* appears on the list that is produced by *E*.

3. *E's* list must contain a *TM* with longer description than *N*'s description.

From above three conditions we have a contradiction.

Thus our assumption that *A* is Turing – recognizable is wrong.

Therefore *A* is not Turing – recognizable.

Thus an infinite subset of $\mathrm{MIN}_{\mathrm{TM}}$ is not Turing – recognizable.

Comment