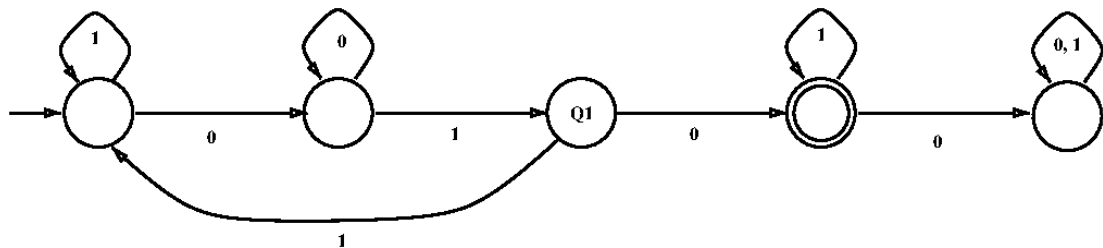


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Homework 2 — Due: Tuesday, September 20, 2022

Please submit your work on Brightspace, in PDF format only.

1. Prove that if  $A$ ,  $B$ , and  $C$  are any three at-most-countable sets, not necessarily infinite or disjoint, then their union is also at most countable.
2. Prove by induction: Every partial order on a nonempty finite set has at least one minimal element. (The induction will be on the number  $n$  of elements in the finite set.) Is the same statement true for nonempty infinite sets? Justify your answer.
3. Describe in English the language recognized by the following finite automaton:



4. Construct a finite automaton that recognizes the following language: The set of strings over the alphabet  $\{1, 2, 3\}$  in which the sum of all symbols is divisible by 5.
5. Use a product automaton construction to construct a finite automaton that recognizes the following language: The set of strings over the alphabet  $\{0, 1\}$  that either begin with 010 or end with 101.
6. Let  $M$  be a (deterministic) finite automaton. Under exactly what circumstances is  $\epsilon$  in  $L(M)$ ? Prove your answer.