Problem

If A is any language, let $A_{\frac{1}{3}-\frac{1}{3}}$ be the set of all strings in A with their middle thirds removed so that

$$A_{\frac{1}{3}-\frac{1}{3}} = \{xz \mid \text{ for some } y,\ |x| = |y| = |z| \text{ and } xyz \in A\}.$$

Show that if A is regular, then $A_{\frac{1}{3}-\frac{1}{3}}$ is not necessarily regular.

Step-by-step solution

Step 1 of 2

Constraints:

A is any language and $A_{\frac{1}{3}-\frac{1}{3}} = \{x \mid \text{ for some } y, |x| = |y| = |z| \text{ and } xyz \in A\}$ is the set of all strings in A with their middle thirds removed.

- Let $A = \{a^*\#b^*\}$ is regular language
- We know that $\{a^*b^*\}$ is a regular language.
- Also we know that "Regular languages are closed under intersection"
- Now $A_{\frac{1}{3}-\frac{1}{3}} \cap \{a^*b^*\} = \{a^n b^n \mid n \geq 0\}$
- Clearly $\{a^n b^n \mid n \geq 0\}$ is not regular, because if p is the pumping length and

$S = xyz = aabb$ is $p = 2$ Here $x = a$ $y = a$ $z = bb$, obtain $xy^2z = aaabb$

Comment

Step 2 of 2

<u>**Pumping lemma**</u> : If A is a regular language, then there is a pumping length *p* where, if s is any string in A of length at least P, then s may be divided into three pieces, $s = xyz$, satisfying following conditions

(i) For each $i \geq 0, xy^iz \in A$

(ii) $|y| > 0$ and

(iii) $|xy| \leq p$

So according to pumping lemma $xy^2z = a^3b^2 \notin \{a^*b^*\}$

Hence $\{0^*1^*\}$ is not regular.

As regular languages are closed under intersection and $\{0^*1^*\}$ is not regular, $A_{\frac{1}{3}-\frac{1}{3}}$ is not regular. If A is regular, then $A_{\frac{1}{3}-\frac{1}{3}}$ is not necessarily regular is proved.

Comment