

Problem

Let

$\Sigma = \{0, 1, \#\}$. Let $C = \{x\#x^R\#x \mid x \in \{0, 1\}^*\}$. Show that \overline{C} is a CFL.

Step-by-step solution

Step 1 of 4

Suppose $\Sigma = \{0, 1, \#\}$. Now consider a language C , where $C = \{x\#x^R\#x \mid x \in \{0, 1\}^*\}$, then it has to show that \overline{C} is a context free language. This can be achieved by showing the contradiction, that is, the language C is not a context free language.

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Step 2 of 4

Each string $k = x\#x^R\#x \in C$, contains $|x| = |x^R|$, and then $|k|$ is multiple of three. For a contradiction suppose that C is context free. As it is considered as a context free, so it contains a pumping length l . Take $k = 0^{2l}0^l1^l0^{2l} \in C$ with $|k| > l$. Hence, there exists $abcde$ in such a way that:

1. $ab^mcd^me \in A$, for all $m \geq 0$.
2. $|bd| > 0$. It means that either b or d should not be empty string.
3. $|bcd| \leq l$. It means that length of the parts b, c, and d should be at most p.

Now, the following cases, which are given below, show that "the value taken for $abcde$ is independent and always shows a contradiction".

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Step 3 of 4

Consider the different cases which are given below:

Case 1: $|bd|$ is not multiple of three. Then, $k' = ab^2cd^2e \notin C$ since $|k'|$ will no longer be multiple of three.

Case 2: Only 0s from the prefix set of 0s exists in bcd and $|bd| = 3q$ for some q . Then, $ab^2cd^2e = 0^{3l+3q}1^l0^{2l} = 0^{2l+q}0^{l+2q}1^q0^{2l} \notin C$. Since $x = 0^{2l+q}$ and $x^R \neq 1^q0^{2l}$, the resultant third of the string.

Case 3: bcd has only 1s and $|bd| = 3q$ for some q . Then, the resultant third string is given by $ab^2cd^2e = 0^{3l}1^{l+3q}0^{2l} = 0^{2l+q}0^{l+2q}1^q0^{2l} \notin C$.

Case 4: Only 0s from the suffix set of 0s exists in bcd and $|bd| = 3q$ for some q . Then, the resultant third string is given by, $ab^0cd^0e = 0^{3l}1^l0^{2l-3q} = 0^{2l-q}0^{l+q}1^{l-2q}1^{2q}0^{2l-3q} \notin C$, since $x = 0^{2l-q}$ and $x^R \neq 1^{2q}0^{2l-3q}$.

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Step 4 of 4

Now, if $i < l + 2$, $q = 1$ is taken then, $ab^i cd^i e = 0^{3l}1^{l+3(i-1)}0^{2l+(i-1)} = 0^{2l+(i-1)}0^{l-(i-1)}1^{l+2(i-1)} \in C$. Since there are not enough 1s to force then into x and $l + 2$ is first time is guaranteed. Hence, there is a contradiction occurred in each case. Thus C is not a context free language. In other words, it can be said that " \overline{C} is a context free language".