Homework # 3

(Due: 11/12/2021)

Group Number: 33

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CSE303: Homework #3

Task 1

(1) (a) $S \to aScc \mid A$

$A \to bAccc \mid \epsilon$

(b) $S \to XY \mid UV$

$X \to AD \mid DB$

$Y \to cY \mid \epsilon$

$U \to aU \mid \epsilon$

$V \to BE \mid EC$

$D \to aDb \mid \epsilon$

$E \to bEc \mid \epsilon$

$A \to aA \mid a$

$B \to bB \mid b$

$C \to cC \mid c$

(c) $S \to aSd \mid aPc \mid bQd \mid R$

$P \to aPc \mid bRc \mid \epsilon$

$Q \to bQd \mid bRc \mid \epsilon$

$C \to bRc \mid \epsilon$

(d) $S \to AB$

$A \to aAa \mid bAb \mid cB$

$B \to aB \mid bB \mid cB \mid \epsilon$

(e) $S \rightarrow AB$

$A \rightarrow aABa \mid bABb \mid cB$

$B \rightarrow aB \mid bB \mid cB \mid \epsilon$

Task 2

(2) (a)

$S \to bS \mid aA$

$A \to bA \mid aB$

$B \to aB \mid bB \mid \epsilon$

(b)

$S \to aA \mid bC$

$A \to aC \mid bB$

$B \to aB \mid bB \mid \epsilon$

$C \to aC \mid bC$

(c)

$S \to aA \mid bB \mid cC$

$A \to aA \mid bD \mid cD \mid \epsilon$

$B \to aE \mid bB \mid cE \mid \epsilon$

$C \to aF \mid bF \mid cC \mid \epsilon$

$D \to aA \mid bD \mid cD$

$E \to aE \mid bB \mid cE$

$F \to aF \mid bF \mid cC$

(d)

$S \to aA \mid bC$

$A \to aB \mid bD \mid \epsilon$

$B \to aS \mid bE$

$C \to aG \mid bC$

$D \to aH \mid bD \mid \epsilon$

$E \to aF \mid bE$

$F \to aG \mid bF \mid \epsilon$

$G \to aH \mid bG \mid \epsilon$

$H \to aF \mid bH \mid \epsilon$

(e) $S \to 0A \mid 1B$

$A \to 0A \mid 1B \mid \epsilon$

$B \to 0C \mid 1D$

$C \to 0E \mid 1F$

$D \to 0G \mid 1A$

$E \to 0B \mid 1C$

$F \to 0D \mid 1E$

$G \to 0F \mid 1G$

Task 3

(3) (a) $S \to 0S \mid 1A1S \mid \epsilon$

$A \to 0B0A \mid \epsilon$

$B \to 1B \mid \epsilon$

(b) $S \to \epsilon \mid A \mid AbBC$

$A \to aA \mid a$

$B \to bB \mid b$

$C \to abBC \mid \epsilon$

(c) $S \to a \mid aa \mid ab \mid aba \mid abb$

(d) $S \to ABCCCC$

$A \to BBBBBBA \mid \epsilon$

$B \to a \mid b$

$C \to B \mid \epsilon$

Task 4

(4.) (d) Suppose L is CFL. Then it must satisfy pumping property. Suppose $s = a^p b^{2p} a^p$

Let $s = uvxyz$ where $|vxy| \leq p$ and $|vy| \geq 1$

Then $uv^i xy^i z$ must belong to L for all $i \geq 0$

We will show that $uxz \notin L$ for all possible cases.

Two cases:

Case 1: vxy consists of exactly 1 symbol. (a's or b's)

Subcase 1: vxy consists only of a's.

Let $s = uvxyz = a^p b^{2p} a^p$

uxz is not in L

Reason: $uxz = a^{p-(|v|+|y|)} b^{2p} a^p \notin L$ as $(|v|+|y|) > 0$

uxz has fewer a's than b's.

Subcase 2: vxy consists only of b's

Let $s = uvxyz = a^p b^{2p} a^p$

uxz is not in L

Reason: $uxz = a^p b^{2p-(|v|+|y|)} a^p \notin L$ as $(|v|+|y|) > 0$

uxz has fewer b's than a's.

<u>Case 2</u>: vxy consists of exactly 2 symbols. (ab's or ba's)

Subcase 1: vxy consists of ab's.

Let $s = uvxyz = a^p b^{2p} c^p$

uxz is not in L

Reason: $uxz = a^{k_1} b^{k_2} a^p \notin L$

where $k_1 + k_2 = 3p - (|v| + |y|) < 3p$ as $(|v| + |y|) > 0$

Either uxz has fewer b's than a's (when $k_1 = p$)

or uxz has fewer a's before b's than after b's.

Subcase 2: vxy consists of ba's

Similar to subcase 1

Subcase 3: vxy consists of aba's

This subcase is impossible as $|vxy| \geq 2p+2 > p$

Contradiction! Hence L is not CFL.

(b) Suppose L is CFL. Then it must satisfy pumping property. Suppose $w = a^r b^s c^t$ and $p = 2s$ be the pumping length.

Let $w = uvxyz = \underbrace{a^{r-1}}_{u}\,\underbrace{a}_{v}\,\underbrace{b^s}_{x}\,\underbrace{\epsilon}_{y}\,\underbrace{c^t}_{z}$ $\quad t > r > 0$, $t > s > 0$

where $|vy| = 1 \geq 1$

and $|vxy| = s+1 \leq p = 2s$

Then $uv^i xy^i z$ must belong to L for all $i \geq 0$

Choosing $i = t - r + 2$, we get

$$uv^i x y^i z = a^{r-1} a^{t-r+2} b^s c^t$$

$$= a^{t+1} b^s c^t \notin L$$

Reason: $uv^i x y^i z$ has fewer c's than a's

Contradiction! Hence L is not CFL.

(c) Suppose L is CFL. Then, it must satisfy pumping property. Suppose $s = a^{p!}$

Let $s = uvxyz$ where $|vxy| \leqslant p$ and $|vy| \geqslant 1$

Then $uv^i xy^i z$ must belong to L for all $i \geqslant 0$

But $uv^2 xy^2 z \notin L$

Reason: Let $|vy| = k$. Then, $1 \leqslant k \leqslant p$

$uv^2 xy^2 z = a^{p!+k} \notin L$

Because, as $k \in [1, p]$

$p! < p! + k \leqslant p! + p < p! + p!p = p!(1+p) = (p+1)!$

Contradiction! Hence, L is not CFL.