Problem

Let A = { $\langle R \rangle \mid R$ is a regular expression describing a language containing at least one string w that has 111 as a substring (i.e., w = x111y for some x and y)}. Show that A is decidable.

Step-by-step solution

Step 1 of 3

Consider the language,

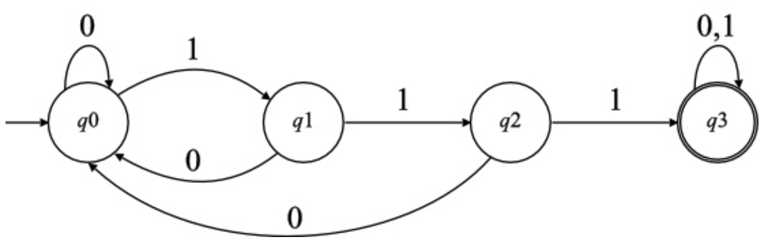
$$A = \left\{ \begin{array}{l} \langle R \rangle \mid R \text{ is a regular expression describing a language which contain} \\ \text{at least one string } w \text{ containing } 111 \text{ as its substring} \end{array} \right\}$$

The decidability of the language *A* is proved as follows:

• Define a language *S* such that $S = \{w \in \Sigma^* \mid w \text{ consists } 111 \text{ as a substring}\}$.

• The regular expression (RE) for the language *S* is $(0 \cup 1)^*111(0 \cup 1)^*$.

• The DFA D_S for the language *S* is shown below:



Comment

Step 2 of 3

• Now think about some RE *R* on input alphabet Σ.

• If $S \cap L(R) \neq \phi$, then *R* produces a string containing 111 as a substring. Thus, $\langle R \rangle \in A$.

• Similarly, if $S \cap L(R) \neq \phi$ then R produces a string that does not contain 111. Thus, $\langle R \rangle$ does not belongs to *A*.

• Since $L(R)$ is described by regular language, $L(R)$ is a regular language. Both *S* and $L(R)$ are regular languages.

• $S \cap L(R)$ is regular because, regular languages are closed under intersection. Thus, $S \cap L(R)$ has some DFA $D_{S \cap L(R)}$.

• Theorem 4.4 shows that $E_{DFA} = \{\langle K \rangle \mid K \text{ is a DFA with } L(K) \neq \phi\}$ is decidable. Thus, there exists a Turing Machine *TM* which determines E_{DFA}.

• Relate TM *T* to $D_{S \cap L(R)}$ to determine if $L(R) \cap S \neq \phi$.

Comments (2)

Step 3 of 3

Summarization of the above discussion contributes the subsequent Turing machine *M* to decide *A*:

M = "On input $\langle R \rangle$, where R is a regular expression:

• Transform *R* into a DFA D_R by means of the algorithm in the proof of Kleene's Theorem.

• Build a DFA $D_{S \cap L(R)}$ for the language $S \cap L(R)$ from the DFAs D_S and D_R.

• Run TM *T* that decides E_{DFA} on input $\langle D_{S \cap L(R)} \rangle$.

• If T accepts, reject. If T rejects, accept.

The Turing machine T decides A . Therefore, the language A is decidable.

Comment