

Problem

Say that two Boolean formulas are **equivalent** if they have the same set of variables and are true on the same set of assignments to those variables (i.e., they describe the same Boolean function). A Boolean formula is **minimal** if no shorter Boolean formula is equivalent to it. Let *MIN-FORMULA* be the collection of minimal Boolean formulas. Show that if $P = NP$, then *MIN-FORMULA* \in P .

Step-by-step solution

Step 1 of 2

The two **Booleans** formulas are said to be **equivalent** if it consists the variables of the same set and the assignments of those variable on the same set become true. The definition of the minimal of the Boolean formula is given by "if no other shorter formula (Boolean) is equivalent to it".

- Consider a collection of minimal Boolean formulas, whose name is assigned as *MIN-FORMULA*. Therefore, the language defined for this *MIN-FORMULA* can be given by:

$$MIN-FORMULA = \{ \langle \phi \rangle \mid \phi \text{ is minimal, in the sense that if } \phi \equiv \phi', \text{ then } |\phi| \leq |\phi'| \}$$

Now, consider the language *NON-EQUIV* which consists $\langle \phi, \phi' \rangle$ in such a way that ϕ and ϕ' are not equivalent.

- Now, it can be proved that *NON-EQUIV* is in NP . To achieve this, consider a truth assignment is taken which is named as c . Now V is taken in such a way that it is used to check $t(\phi) \neq t(\phi')$.
- So, *NON-EQUIV* is in P if NP and P are equivalent and also shows the closed property under complementation. Hence, it can be said that *NON-EQUIV* is in P .

[Comments \(1\)](#)

Step 2 of 2

Now, consider the *NOT-MIN-FORMULA*. Suppose a truth assignment is taken which is named as c , that defines a formula ϕ' .

- Now V is taken in such a way that it is used to check two things: $|\phi'| < |\phi|$ and $\phi' \equiv \phi$. Here, first condition gives the polynomial verifiability of second condition.

Now, from the above discussion **that describe the closeness property** (which say that, P is closed under complementation) **of P and equality of NP and P , it can be said that the language *MIN-FORMULA* is in P , if $P = NP$.**

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