Problem

Let B be the language of all palindromes over {0,1} containing equal numbers of 0s and 1s. Show that B is not context free.

Step-by-step solution

Step 1 of 3

Let B be the language of all palindromes over $\{0, 1\}$ containing the equal numbers of 0's and 1's . To prove B is not a context free language by taking a contradiction.
Assume that B is a context free language.

Since B is a context free language, then by pumping lemma, there is a number p (the pumping length) where, if s is any string in B of length at least p , then s may be divided into five pieces $s = uvxyz$ satisfying the conditions

1. for each $i \geq 0, uv^i xy^i z \in A$,
2. $|vy| > 0$, and
3. $|vxy| \leq p$.

Comment

Step 2 of 3

Case 1:

Now select a string $s = 0^n 1^{2n} 0^n$.

Clearly s is a member of B of length at least p.

Assume the value of n=2 for the string *s*.

$s = 0^2 1^4 0^2$

$s = 00111100$

The string s can be divided into *uvxyz* as follows:

$$\underbrace{00}_{u}\ \underbrace{11}_{v}\ \underbrace{1}_{x}\ \underbrace{10}_{y}\ \underbrace{0}_{z}$$

Now apply the first condition of the pumping lemma.

$$\text{for each } i \geq 0, uv^i xy^i z \in A$$

For i = 2:

$$\underbrace{00}_{u}\left(\underbrace{11}_{v}\right)^2\ \underbrace{1}_{x}\left(\underbrace{10}_{y}\right)^2\ \underbrace{0}_{z}$$

$$\underbrace{00}_{u}\left(\underbrace{1111}_{v}\right)\ \underbrace{1}_{x}\left(\underbrace{1010}_{y}\right)\ \underbrace{0}_{z}$$

Assume the obtained string 00 1111 1 1010 0 as s'.

The obtained string s' is not a palindrome after applying the first condition of pumping lemma and $s' \notin B$. In the pumped string, the number of 0's and 1's is not equal.

So, the language B is not following the condition1 of the pumping lemma.

Comment

Step 3 of 3

Case 2:

The same string is selected as $s = 0^n 1^{2n} 0^n$

Clearly s is a member of B of length at least p.

Assume the value of n=2 for the string s.

$s=0^2 1^4 0^2$

s=00111100

The string s can be divided into *uvxyz* as follows:

$$\frac{0}{u}\ \frac{01}{v}\ \frac{11}{x}\ \frac{1}{y}\ \frac{00}{z}$$

Now apply the first condition of the pumping lemma.

$$\text{for each } i \geq 0, uv^i xy^i z \in A$$

For i=2:

$$\frac{0}{u}\left(\frac{01}{v}\right)^2\ \frac{11}{x}\left(\frac{1}{y}\right)^2\ \frac{00}{z}$$

$$\frac{0}{u}\left(\frac{0101}{v}\right)\ \frac{11}{x}\left(\frac{11}{y}\right)\ \frac{00}{z}$$

Assume the obtained string 0 0101 11 11 00 as s'.

The obtained string s' is not a palindrome after applying the first condition of pumping lemma and $s' \notin B$. In the pumped string, the number of 0's and 1's is not equal.

Hence, the assumption B is a context free language is wrong.

Therefore, by the two cases it can be proved that B is not a context free language.

Comments (3)