Problem

Define A_{LBA} = { $\langle M, w\rangle$ I M is an LBA that accepts input w}. Show that A_{LBA} is PSPACE-complete.

Step-by-step solution

Step 1 of 4

$\underline{PSPACE-\text{complete}:}$ A language *B* is *PSPACE* – complete if it satisfies two conditions.

1. *B* is in *PSPACE*, and

2. every *A* in *PAPACE* is polynomial time reducible to *B*.

If *B* satisfies condition 2, we say that *B* is *PSPACE*- hard

Comment

Step 2 of 4

Given language is

$A_{LBA} = \{\langle M, w\rangle\} M \text{ is an } LBA \text{ that accepts input } w\}$

A linear bound automation (LBA) is one – tape, one – head *NTM*.

We need to show that A_{LBA} is $PSPACE-$complete.

That means A_{LBA} has to satisfy the 2 conditions of *PSPACE*-complete.

Comment

Step 3 of 4

(i) $\underline{A_{LBA} \in PSPACE:}$

To show $A_{LBA} \in PSPACE$, we need to construct a deterministic Turing machine that decides A_{LBA} in polynomial space.

Let *T* be the Turing machine (TM) that decides A_{LBA} in polynomial space.

T can be constructed as follows.

$T =$" on input $\langle M, w\rangle$

Where *M* is a $TM(Q, \Sigma, \Gamma, \delta, q_0, q_{accept}, q_{reject})$ and $w \in \Sigma^*$

1. Take a step count *S* and initialize *S* to O.

2. While $S < |Q| \cdot |w| \cdot |\Gamma|^{|w|}$

(i) Simulate *M* on *w*

(ii) If *M* runs out of $|w|-$bounded tape then reject.

(iii) accept if *M* could in this step.

(iv) increment *S*.

3. Reject."

This machine *T* runs in polynomial space since the extra counter assumes values at most exponential in the length of the input word.

Thus we constructed a *TM T* to decide A_{LBA} in polynomial space.

Therefore $A_{LBA} \subset PSPACE$

Comment

Step 4 of 4

(ii) $\underline{A_{LBA} \text{ is } PSPACE-\text{hard}}:$

Let $L \in PSPACE$

Let M be TM that decides L in space at most n^k

clearly $L \leq_P A_{LBA}$ by giving a reduction that maps w to $\left\langle M, wL_1^{|w|^{k-1}} \right\rangle$.

Thus every L in $PSPACE$ is polynomial time reducible to A_{LBA}.

Hence A_{LBA} is $PSPACE$-hard.

From (i) and (ii) A_{LBA} is $PSPACE-$ complete

Comments (2)