Problem

Let *HALF-CLIQUE* = { $\langle G \rangle$ | G is an undirected graph having a complete subgraph with at least m/2 nodes, where m is the number of nodes in G}. Show that *HALF-CLIQUE* is NP-complete.

Step-by-step solution

Step 1 of 2

Clique is an undirected graph where every two nodes connected by an edge.

***NP* - complete:**

A language *B* is *NP* – complete if by an edge it satisfies two conditions.

1. *B* is in *NP*

2. Every *A* in *NP* is polynomial time reducible to *B*.

Comment

Step 2 of 2

1. $HALF-CLIQUE \in NP$**:**

Let *N* be the nondeterministic polynomial time (*NTM)* that decides $HALF-CLIQUE$ in polynomial time.

N can be described as follows:

$N =$"on input graph $\langle G \rangle$:

1. Non-deterministically choose at least $n/2$ nodes

2. Verify whether $n/2$ nodes form a clique

3. If they form a clique then accept.

4. Otherwise, reject".

Therefore, $HALF-CLIQUE \in NP$

2. $CLIQUE \leq_p HALF-CLIQUE$:

A reduction from $CLIQUE$ to $HALF-CLIQUE$ as follows:

On input $\langle G, k \rangle$, where *G* is a graph on *n* verifies and *k* is an integer:

1. If $k = n/2$ then output $\langle G \rangle$.

2. If $k < n/2$, then construct a new graph G' by adding a complete graph with $n-2k$ vertices and connecting them to all vertices in *G*, and output $\langle G' \rangle$.

3. If $k > n/2$, then construct a new graph G'' by adding $2k-n$ isolated vertices to *G*, and output $\langle G'' \rangle$.

$When\, k = n/2$:It is clear that $\langle G, n/2 \rangle \in CLIQUE$ if and only if $\langle G \rangle \in HALF-CLIQUE$.

When $k < n/2$:If *G* has a *k*-clique, then G' has a clique of size

$k + (n-2k) = (2n-2k)/2$.

Therefore, $\langle G' \rangle \in HALF-CLIQUE$ as G' is a graph with $2n-2k$ vertices.

Conversely,if $\langle G' \rangle \in HALF-CLIQUE$, that is G' has a clique of size $n-k = k + (n-2k)$, then at most $n-2k$ of the clique come from the $n-2k$ new vertices. Therefore the reaming at least *k* vertices form a clique in *G*.

Hence, $\langle G, k \rangle \in CLIQUE$

When $k > \frac{n}{k}$: if *G* has a $k-$clique, then G'' has a clique size $k = \frac{2k}{2}$, and

Therefore, $\langle G' \rangle \in HALF-CLIQUE$ as G'' is a graph with $n+2k-n=2k$ vertices.

Conversely, if $\langle G'' \rangle \in HALF-CLIQUE$, that is if G'' a clique of size has *k*, then the clique does not contain any of the new vertices as they are isolated.

Thus, the clique is a *k* – clique of G, and hence $\langle G,k \rangle \in CLIQUE$.

Therefore, the $HALF-CLIQUE$ **is *NP*- complete.**

Comment