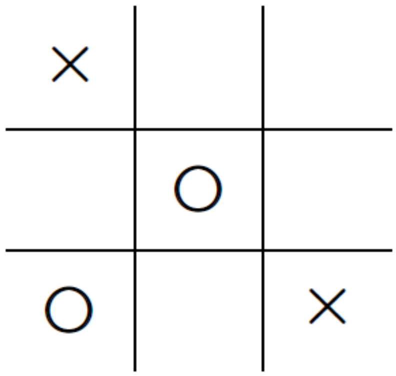
Problem

Consider the following position in the standard tic-tac-toe game.



Let's say that it is the ×-player's turn to move next. Describe a winning strategy for this player. (Recall that a winning strategy isn't merely the best move to make in the current position. It also includes all the responses that this player must make in order to win, however the opponent moves.)

Step-by-step solution

Step 1 of 3

Tic- tac – toe game:

Game Constraints:

• This game is played by two players X and O, who take turns marking the spaces in a 3×3 grid.

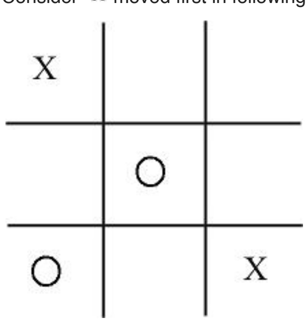
• Any X or O player goes first.

• **Winning the game:** who will first mark three places of vertical, horizontal or diagonal rows.

Comment

Step 2 of 3

Consider X moved first in following problem.



Winning strategy for the game:

The winning strategy of the game corresponds to quantified statement because games are closely related to quantifiers. These quantified statements help in understanding the complexity of the game.

Comment

Step 3 of 3

Consider the quantified Boolean formula in prenex normal form, $\phi = \exists x_1 \forall x_2 \exists x_3 ... Qx_k[\psi]$. Here, Q is either $\forall$ or $\exists$. Two players called *X* and *O* selects the gr in the tic-tac-toe in their respective turn. Here, $x_1, x_2, x_3, \ldots$ are the grids in the tic-tac-toe. At this position of the ga e, three grids decide the winner.

The player *X* selects the grids that are bound to $\exists$ quantifier and player *O* selects the grids that are bound to $\forall$ quantifier. In this problem, player *X* starts the game. So, the formula starts with $\exists$ quantifier. The player *X* will win the game if ψ is TRUE.

The formula is as follows:

$$\phi = \exists x_1 \forall x_2 \exists x_3 [(x_1 \vee x_2) \wedge (x_2 \vee x_3) \wedge (\overline{x_2} \vee \overline{x_3})]$$

In the above formula, x_1 denotes the grid selected by player *X*, x_2 denotes the grid selected by player *O* and x_3 denotes the grid selected by player *X*.

The player *X* always win the game if the third grid in the first row is selected (Consider it as 1 i.e., $x_1 = 1$)) and then selecting x_3 to be the negation of whatever player *O* selects.

The formula will become true, if the third grid in the first row is selected by player *X* then player *O* selects either third grid in the second row or the second grid in the first row or the first grid in the second row or the second grid in the third row. The player *X* can win the game if the second grid in the first row or the third grid in the second row is selected. Depending on the player *O*'s move, player *X* will select any of these grids and wins the game.

Comment