Problem

Let

$$T = \{\langle M \rangle |\ M \text{ is a TM that accepts } w^{\mathcal{R}} \text{ whenever it accepts } w\}.$$

Show that T is undecidable.

Step-by-step solution

Step 1 of 4

Consider the problem statement provided in the textbook.

Comment

Step 2 of 4

Let $T = \{ <M>| M \text{ is a TM that accepts } w^R \text{ whenever it accepts } w \}$.

- It is already known that $L = \{(w, M): w \textit{ is accepted} \text{ by } M\}$ is undecidable.
- Assume that T is decidable, then there must exist a TM by which T can be decided. Let's say P is the Turing Machine that decides T.

Comment

Step 3 of 4

For any input (w, M), M' can be constructed as follows:

If $w = w^R$, simulate M on w. The Σ is the alphabet set of M and let $a, b \notin M$.

Let $\Sigma \cup \{a,b\}$ be the alphabet set of M'. Then for input ab, M' will reject all the other strings except ab.

Now, simulate M on w.

- If M accepts w, M' rejects.
- If M rejects w, M' accepts.

Claim: P accepts M' iff M accepts w.

Proof: If P accepts M'. Since, M' rejects all the other strings which include ba also, then M' rejects ab which implies M accepts w.

If w is accepted by M, then M' rejects ab. Since, M' rejects all the other strings, M' is accepted by P.

Now, construct a TM, Q for L. Construct M' on input (w, M) and run P on it. Q accepts iff P accepts.

This contradict the fact that L is undecidable.

Comment

Step 4 of 4

Therefore, T is undecidable. Hence Proved.

Comments (2)