

Problem

Let

$$A_{\text{TM}}' = \{ \langle M, w \rangle \mid M \text{ is an oracle TM and } M^{A_{\text{TM}}} \text{ accepts } w \}.$$

is undecidable relative to A_{TM} .

Step-by-step solution

Step 1 of 1

Given that

$$A_{\text{TM}}' = \{ \langle M, w \rangle \mid M \text{ is an oracle TM (turing machine) and } M^{A_{\text{TM}}} \text{ accepts } w \}$$

We have to show that A_{TM}' is undecidable relative to A_{TM} .

Take a contradiction of A_{TM}' is decidable relative to A_{TM} .

Hence there exists an oracle TM T with oracle access to A_{TM} which decides A_{TM}' .

Now we construct another oracle TM N as follows:

$N =$ "on input

1. Run $T^{A_{\text{TM}}}$ on input $\langle \rangle$

2. If T accepts, *reject*.

3. Else if T rejects, *accept*."

• So $N^{A_{\text{TM}}}$ accepts if and only if M rejects.

• When the input of N is $\langle N \rangle$, we have $N^{A_{\text{TM}}}$ accepts $\langle N \rangle$ if and only if N rejects $\langle N \rangle$.

This is a contradiction to our hypothesis, that A_{TM}' is decidable relative to A_{TM} is wrong. Hence, A_{TM}' is undecidable relative to A_{TM} .

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