Problem

Show that the following problem is NP-complete. You are given a set of states Q = {q_0, q_1, . . . , q_l} and a collection of pairs {(s_1, r_1), . . . , (s_k, r_k)} where the s_i are distinct strings over Σ = {0, 1}, and the r_i are (not necessarily distinct) members of Q. Determine whether a DFA M = (Q,Σ, ?, q_0, F) exists where ?(q_0, s_i) = r_i for each i. Here, ?(q, s) is the state that M enters after reading s, starting at state q.

(Note that F is irrelevant here.)

Step-by-step solution

Step 1 of 2

Correct DFA which satisfy C constraints and in polynomial time $\prod$ can be guessed by Non Deterministic Turing Machine iff such DFA available or exist.

For showing that problem is NP complete reduce it to polynomial time.

Consider the formula $F = \wedge_{j=1}^{m} R_j$ where $R_j = \left(s_j \vee t_j \vee u_j\right)$ and construction some constraints C and $\prod$.

- $C = \{c_T, c_F, c_1, c_2\}$ are states.
- Creating pair (ε, c_F) in $\prod$ for enforcing c_F as starting state.
- Every variable s belongs to F will create the pairs $(s\overline{s}, c_T)$ and $(\overline{s}s, c_T)$.
- Every clause $R_j{}'$ in formula F will have pair in $\prod$ that is $(s\#_s, c_1)$ and $(\overline{s}\#_s, c_2)$ that enforces that when reading s and $\overline{s}$, DFA must be in different state.
- Choose any s in F. Now for $\forall$ variable t create other three points in $\prod$: $(s\overline{s}t, c_T)$, $(s\#_s t, c_1)$, $(\overline{s}\#_s t, c_2)$.

Comment

Step 2 of 2

F is satisfiable iff there is some DFA that satisfy C and R. Reduction is taking some polynomial time therefore given problem is NP-complete.

Comment