

Problem

Let $A/B = \{w \mid wx \in A \text{ for some } x \in B\}$. Show that if A is regular and B is any language, then A/B is regular.

Step-by-step solution

Step 1 of 2

Given language is

$$A/B = \{w \mid wx \in A \text{ for some } x \in B\}$$

Here, A is a regular language and B is any language.

Now, the objective is to prove that A/B is regular.

Since A is a regular language, some DFA will recognize the language A .

Let $M = (Q, \Sigma, \delta, q_0, F)$ be the DFA that recognizes A .

Here, Q is the set of states.

Σ is set of alphabets = of the alphabets for A and B .

δ is the transition function.

q_0 is the start state.

F is the set of final states.

[Comment](#)

Step 2 of 2

To prove A/B is a regular language, construct a DFA that recognizes the language A/B .

Let $M' = (Q', \Sigma', \delta', q'_0, F')$ be the DFA that recognizes A/B .

- $Q' = \text{set of states} = Q$
- $\Sigma' = \text{set of alphabets} = \Sigma$
- $\delta' = \text{transition function} = \delta$
- $q'_0 = \text{start state} = q_0$
- $F' = \{q \in Q \mid \exists x \in B \text{ such that } M \text{ goes from } q \text{ to some state in } F \text{ on reading } x\}$

Thus, a DFA M' to recognize the language A/B has been constructed.

Hence A/B is a regular language.

[Comment](#)