Problem

Is the statement $\exists x \, \forall y \, [\, x+y=y \,]$

Step-by-step solution

Step 1 of 1

Theory of *N* :-

If *N* is a model, theory of *N*, written $Th(N)$, be the collection of true sentences in the language of that model.

• Given sentence is $\phi_1 = \exists x \forall y [x + y = y]$ and the given theory of model is $Th(N,+)$.

The statement ϕ_1 is true in model $(N,+)$.Because for $N = \{0,1,2,3...\}$ and for $x = 0$ the statement $0 + y = y$ is true.

So the statement $\exists x \forall y [x + y = y]$ is a member of $Th(N,+)$

• Given statement is $\phi_2 = \exists x \forall y [x + y = x]$

This statement ϕ_2 is false in the model $(N,+)$

Because for any *x* the statement $x + y = x$ is false, for all y (except if $y = 0$).

So the statement ϕ_2 is false in the model $(N,+)$

Hence the statement $\exists x \forall y [x + y = x]$ is not a member of $Th(N,+)$

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