Problem

For each part, give a relation that satisfies the condition.

a. Reflexive and symmetric but not trai tive

b. Reflexive and transitive but not symmetric

c. Symmetric and transitive but not reflexive

Step-by-step solution

Step 1 of 4

A binary relation R is

Reflexive if for every x, xRx

Symmetric if for every x and y, xRy implies yRx

Transitive if for every x, y, and z, xRy and yRz implies x

Comments (1)

Step 2 of 4

(a)

Re◊exive and symmetric but not transitive

xRy : x, y are people and they share at least one biological parent.

• This relation is clearly re◊exive, since everyone has their own parents for parents.

• This relation is symmetric, if person x shares a parent with person y, then person y shares that same parent with person x.

• However, this relation is not transitive. Assume a person m who shares exactly one parent (a mother) with person g and exactly one parent with person(s) (a father). g's father is not s's father, and g's mother is not s's mother. Hence, while gRm and mRs are true, gRs is false.

Comment

Step 3 of 4

(b)

Re◊exive and transitive but not symmetric

xRy : $x, y \in$ N and $x - y \leq 0$.

• This relation is Re◊exive, because x-$x = 0$.

• This is transitive, because if xRy then $x \geq y$ and if yRz then $y \geq z$, thus xRx because $x \geq y \geq z$.

• However, this relation is not symmetric, because $5 - 3 \geq 0$, but $3 - 5 < 0$.

Comments (1)

Step 4 of 4

(c)

Symmetric and transitive but not re◊exive

xRy : $x, y \in$ Z and $i * j > 0$

• This relation is Symmetric, because multiplication is symmetric.

• It is transitive, as well, since if xRy then neither x nor y is zero and if yRz , then neither y nor z is zero. Thus xRz because neither x nor z is zero.

• However, this relation is not symmetric, because $0 * 0 = 0$