Problem

Review the way that we define sets to be the same size in Definition 4.12 (page 203). Show that “is the same size” is an equivalence relation.

DEFINITION 4.12

Assume that we have sets A and B and a function f from A to B. Say that f is ***one-to-one*** if it never maps two different elements to the same place—that is, if $f(a) \neq f(b)$ whenever $a \neq b$. Say that f is ***onto*** if it hits every element of B—that is, if for every $b \in B$ there is an $a \in A$ such that $f(a) = b$. Say that A and B are the ***same size*** if there is a one-to-one, onto function $f\colon A \longrightarrow B$. A function that is both one-to-one and onto is called a ***correspondence***. In a correspondence, every element of A maps to a unique element of B and each element of B has a unique element of A mapping to it. A correspondence is simply a way of pairing the elements of A with the elements of B.

Step-by-step solution

Step 1 of 3

Given:

A function $f: A \to B$ in which A and B are two sets. The function $f: A \to B$ is **one to one function** as it never maps two different elements of same set with one element of another set.

The function $f: A \to B$ is **onto function** as each and every element of the set B is hit by the element of set A.

As the function $f: A \to B$ is one to one and onto at the same time then it means the set A and B has the same cardinality. If the cardinality of these two sets is same so these sets are of **same size or equinumerous**.

If the function $f: A \to B$ is one to one and onto at the same time it means the function $f: A \to B$ is **correspondence function** also. **Correspondence function** is also known as **bijective function**.

If the function $f: A \to B$ is one to one and onto or bijective function, then sets A and B are of **same size**.

Comment

Step 2 of 3

Proof:

Equivalence relation: A relation is known as equivalence in nature if it is reflexive, transitive and symmetric.

Same size relation is equivalence relation if and only if it is symmetric, reflexive and transitive.

• **For reflexivity:** If the user checks the identity function on the set A then this identity function is a bijection from A to A.

Hence the **same size** relation is reflexive relation.

• **For symmetry: if the function** $f: A \to B$ is a bijective function then it means the inverse function f^{-1} is also bijective function from the set B to set A.

Hence if $A \sim B$ then $B \sim A$, so **the same size** relation is symmetric relation also.

• **For transitivty: Assume that** $A \sim B$ and $B \sim C$. Then the function $f: A \to B$ is bijective function from A to B and the function $g: B \to C$ from B to C.

Therefore the composition of two bijective functions f and g is also a bijective function from A to C.

Hence the **same size** relation is transitive relation as $A \sim C$.

Comment

Step 3 of 3

Conclusion:

Hence the **same size** relation is reflexive, symmetric and transitive in nature so the **same size** relation is equivalence relation.

Comment