Problem

Show that any function with n inputs can be computed by a branching program that has O(2^n) nodes.

Step-by-step solution

Step 1 of 4

A program is defined as a branching program if "**a directed graph, which also shows an acyclic** property, where labels of all the given nodes are maintained by the variables. These variables will not be used for two output nodes which are labeled 1 or 0. Here, all the nodes **whose labels are maintained by the variables** are also known as **query nodes**. Every query nodes consists of two edges which is outgoing from itself. In which one output node is labeled as 1 and another one is labeled as 0.

Comment

Step 2 of 4

Now, consider the **majority function** $\left(majority_n : \{0,1\}^n \rightarrow \{0,1\}\right)$, which is defined as:

$$\begin{aligned} majority_n\left(x_1, x_2, \ldots, x_n\right) &= 0 \text{ if } \sum x_i < n/2; \\ &= 1 \text{ if } \sum x_i \geq n/2. \end{aligned}$$

As it is known "the computation of the **majority function** can be done by using **a branching program** of $O\left(2^n\right)$nodes"

Comment

Step 3 of 4

Again, consider the function **parity**; it is known that "**a branching program with** $O(n)$ **gates may be used to compute the n-input parity function".**

• It can be achieved by **building a binary tree of gates which is used to calculate function XOR**, **where the function XOR is used as equivalent to the parity function. The implementation of each XOR gate can be done by using two AND's, two NOT's and one OR gates.**

• It is known that each **AND's, OR gates takes two nodes as an input and produces a single output w**hich again considered as an input of another gate because a binary tree of gates is considered**.**

• Finally, the total number of nodes, which are used in computation, is an order of 2^n or $O\left(2^n\right)$.

Comment

Step 4 of 4

As discussed above, the **majority** and the **function parity, which** takes n-inputs, can be obtained by a branching program, which consist $O\left(2^n\right)$ nodes. Therefore, **as the definition of branching program says (which is explained above), it can be said that any function, that consists n-inputs, can be computed by using a branching program which consist $O\left(2^n\right)$ nodes.**

Comments (1)