CSE 303 : ToC. 115060128
HW12.

Problem 1:-

Sipser Ex 4.2.

we have from Thm 4.5

$$EQ_{DFA} = \{ \langle A,B \rangle \mid A \,\&\, B \text{ are DFAs } \& \; L(A) = L(B) \}.$$

Let T be a TM which decides language L given below.

$$L = \{ \langle M, R \rangle \mid M \text{ is a DFA } \& \; R \text{ is a regular exp.} \}$$

we will give a definition of T.

We have already proved that ~~a~~ any reg. exp can be converted to an eq. DFA.
Let the eq. DFA for R be R_{DFA}.

So we have def of T as follows:

T = "On input $\langle M, R \rangle$ where M is a DFA & R is a reg ex.

1) Convert R to its eq. DFA R_{DFA}.

2) Let ~~a~~ S be a TM $= \{ \langle M, R_{DFA} \rangle \mid M \,\&\, R_{DFA}$ are DFA's & $L(M) = L(R_{DFA}) \}$.

3) Acc to Thm 4.5, ~~a~~ S is a decider.

4) If S accepts, accept language L.

5) If S rejects, reject language L.

Note $L(M) = L(R) = L(R_{DFA})$.

Thus this problem is expressed in terms of language L & it is decidable.

Problem 2:-

Sipser Ex 4.4.

Let $A\varepsilon_{CFG} = \{ \langle G \rangle \mid G \text{ is a CFG that generates } \varepsilon \}$.

Let T be a TM that decides $A\varepsilon_{CFG}$. We will give the def. of T.

T = " On input $\langle G \rangle$"

1). Convert G into G' its equivalent Chomsky Normal Form.

2) $L(G) = L(G')$ we have proved this. Now ~~...~~ if G' has the rule $S \to \varepsilon$ then G' generates ε. Only using this rule ε can be generated. Else it cannot.

3) If G' has the rule $S \to \varepsilon$ then T accepts.

4) Else it rejects.

Note all ~~this term~~ these steps are decidable. Hence, this problem is decidable.

Problem 3:-

Sipser Ex 4.11

This language is decidable.

We will construct a TM T on this, ~~for~~ The Steps for it ~~will be~~ are shown below.

T = " On input <M> "

1) Read <M> & create an equivalent CFG G.

2) Convert G to its Chomsky Normal Form G'

3) For a rule $A \to BC$ create a directed graph with A, B, C as vertex & $A \to B$ edge & $A \to C$ edge. Do this for all rules of grammar.

4) Do a Breadth first search in this graph. ~~If it~~ To check if it consists of a cycle.

5) A cycle implies there exist a derivation $A \overset{*}{\Rightarrow} uAv$.

6) If there is a cycle accept.

7) Else reject.

Thus it is decidable.

Note if there is a cycle $\Rightarrow \exists$ a rule $A \overset{*}{\Rightarrow} uAv$.
Thus we can get uA^iv $i \geq 1$ all strings in this. Hence $L(M)$ is infinite.

Problem 4:

Sipser Ex 4.21

$S = \{ \langle M \rangle \mid M$ is a DFA that accepts w^R whenever it accepts $w \}$.

We will give the def of S.

S = " On input $\langle M \rangle$ "

1) Construct a DFA N which accepts reverse of strings accepted by M.

i.e $L(N) = L(M^R)$

2) Check <M, N> is accepted by EQ_{DFA}

3) If M is accepted by EQ_{DFA} then accept.

4) Else reject.

Note that we have already constructed N which accepts the reverse of strings accepted by M. ~~E~~ First we can create N N_{NFA} an NFA

Initial state of N_{NFA} is accepted state of M.
If there are multiple accepted states of M use ε. transition

Reverse all the ~~E~~ transitions.
Put initial state of M as accepted by N_{NFA}.

Convert NFA to N a DFA.

Thus S is decidable.