Problem

Refer to Problem 1.41 for the definition of the perfect shuffle operation. Show that the class of context-free languages is not closed under perfect shuffle.

Problem 1.41

For languages A and B, let the ***perfect shuffle*** of A and B be the language

$$\{w|\ w = a_1b_1\cdots a_kb_k,\ \text{where } a_1\cdots a_k \in A \text{ and } b_1\cdots b_k \in B,\ \text{each } a_i, b_i \in \Sigma\}.$$

Show that the class of regular languages is closed under perfect shuffle.

Step-by-step solution

Step 1 of 1

Context free language is the language which is generated by CFG or the context free grammar. It is possible to get different types of context free languages from different types of context free grammar.

Consider the languages A and B as given below:

Language *A* $\{0^k1^k|k \geq 0\}$

Language *B* $\{a^kb^{3k}|k \geq 0\}$

After the perfect shuffle, other language (*C*) obtained will be $\{(0a)^k(0b)^k(1b)^{2k} \mid k >= 0\}$

It is clear that languages *A* and *B* are context free languages but language *C* is not a context-free language. It can be proved by contradiction follow:

- Suppose that the language defined for the shuffle of *A* and *B* is context free and let *p* be the length of its pumping lemma and *s* is string $(0a)^p(0b)^p(1b)^{2p}$.
- As string is longer as compared with pumping lemma and string is a part of language *C*, divide $s = uvxyz$.
- In language *C*, string is equal to one-fourth part of 1s and one-eighth part of *a*'s. For uv^2xy^2z, to have the same property, it should contain *a*'s and 1's. But it is not possible because both are separated by another symbol 2*p*. Also $|vxy| \leq p$.

Thus, it can be concluded and deduced that context free languages or CFL's are not closed under perfect shuffle.

Comment