

Show that  $EQ_{CFG}$  is co-Turing-recognizable.

### Step-by-step solution

#### Step 1 of 1

Remember that  $EQ_{CFG}$  is co-Turing-recognizable language if and only if its complement  $\overline{EQ_{CFG}}$  is a Turing-recognizable language.

Now,  $\overline{EQ_{CFG}} = A \cup B$ , where

$A = \{ w \mid w \text{ does not have the form } \langle G1, G2 \rangle \text{ for some CFGs } G1 \text{ and } G2 \}$ ,

$B = \{ \langle G1, G2 \rangle \mid G1 \text{ and } G2 \text{ are CFGs and } L(G1) \neq L(G2) \}$ .

- A contains strings that defy the syntax for encoding  $\langle G1, G2 \rangle$  is simple to accept.
- The set B is realized in the following way:
- Transform the  $G1, G2$  to Chomsky Normal Form (CNF).
- Then begin numbering strings in  $\Sigma^*$  lexicographically, where  $\Sigma$  is the group of terminals for  $G1, G2$ .
- For every string  $w$  numbered, check if it is produced by  $G1$  and by  $G2$ .
- If the 2 Context Free Grammars or both of them cannot produce  $w$ , then TM goes on to generate the next string in the lexicographic order.
- Else, precisely one of the CFGs produces the string, and the TM accepts.
- Therefore, B is a Turing recognizable language.
- It is already proved that Turing-recognizable languages are closed under union, so  $EQ_{CFG}$  is Turing-recognizable language.
- Let the list of strings that are listed in lexicographic order are as follows:

$s_1, s_2, s_3 \dots$  over the input  $\Sigma^*$ .

Now  $EQ_{CFG}$  is realized by the following TM  $M$ :

$M =$  "On input  $\langle G1, G2 \rangle$ , where  $G1, G2$  are Context Free Grammars:

1. Examine if  $G1, G2$  are valid Context Free Grammars. If atleast 1 does not, accept.
2. Transform  $G1, G2$  into corresponding Context Free Grammars  $G'_1, G'_2$ . both into CNF.
3. Replicate the below step 4 for  $j = 1, 2, 3 \dots$
4. Examine if  $G'_1$  and  $G'_2$  produce  $s_j$  if precisely one of them does, accept."

**Hence it is proved that  $EQ_{CFG}$  is co-Turing-recognizable language.**

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