Problem

An ***all***-NFA M is a 5-tuple $(Q, \Sigma, \delta, q_0, F)$ if *every* possible state that M could be in after reading input x is a state from F. Note, in contrast, that an ordinary NFA accepts a string if *some* state among these possible states is an accept state. Prove that all-NFAs recognize the class of regular languages.

Step-by-step solution

Step 1 of 5

All NFA *M* is a 5-tuple (Q,Σ,δ,q_0,F) that accepts $x \in \Sigma*$ if every possible state that M could be in after reading input *x* is a state from F.

There are two steps to prove that all – NFAs recognize the class of regular languages.

Step 1: Every regular language is recognized by some all – NFA

Step 2: Every all – NFA recognizes a regular language.

Comment

Step 2 of 5

Proof for the Step 1 as follows:

Clearly every DFA can be viewed as an all – NFA.

• Let *L* be the any regular language.

• Let $M=(Q,\Sigma,\delta,q_0,F)$ be the DFA that recognizes *L*.

• Clearly for each input string *x*,

There is exactly one possible state $q \in Q$ that M could be in after reading *x*.

• Hence, if M is viewed as an all-NFA, then it accept *x* if and only if $q \in F$, which happens if and only if $x \in L$.

• Therefore, when M is viewed as an all – NFA, it also recognizes the language L.

Thus, every regular language is recognized by some all – NFA.

Comment

Step 3 of 5

Proof for the Step 2 as follows:

• Let $N=(Q,\Sigma,\delta,q_0,F)$ be an all – NFA.

• Let *A* be the language recognized by N.

Now, prove that A is regular.

• Constructing a DFA $M=(Q',\Sigma,\delta',q_0',F)$ that recognizes *A*.

$M=(Q',\Sigma,\delta',q_0',F)$

• Where $Q'=P(Q)$, where $P(Q)$ is the set of subset of Q

• For $R \in Q'$ and $a \in \Sigma$

$$\delta'(R,a)=\bigcup_{r\in R}E(\delta(r,a))$$

• For any subset $S\subset Q, E(S)$ is the set of all sates $q\in Q$ that can reached from *S* by travelling along $\in$ arrows including the members of *S* themselves.

$$q_0'=E(\{q_0\})$$

$$F'=\{R\in Q' \mid R\subset F\}$$

Comment

Step 4 of 5

Now, consider an input string $x\in\Sigma*$.

• Let *R* be the set of states that the all NFA *N* could be in after reading *x.*

• Then *x* is fed to the DFA *M* defined above, *M* will end at state *R.*

• By the definition of all-NFA , have $x\in A\Leftrightarrow R\subset F$.

• By the definition of *M*, *M* accepts *x* if and only if $R\in F'$, which is equivalent to $R\subset F$.

Hence M recognize A.

Comment

Step 5 of 5

Therefore, every all- NFA recognizes a regular language.

From step 1 and step 2 all- NFAs recognizes the class of regular language.

Comment