### **Problem**

Suppose that A and B are two oracles. One of them is an oracle for *TQBF*, but you don't know which. Give an algorithm that has access to both A and B, and that is guaranteed to solve *TQBF* in polynomial time.

#### Step-by-step solution

### Step 1 of 3

An Oracle Turing machine is a type of Turing machine having many different tapes and these tapes are called oracle tapes. The different states may be represented by  $q_{states}$ .

A TQBF is True Qualified Boolean formula. To Show TQBF in polynomial problem which has access to turing machines A and B can be solved by using Baker Gill Solovay Theorem. The problem can be broken up in two parts of algorithm:

Comment

## Step 2 of 3

#### For Oracle A:

- 1. Consider TQBF has access to A then A=TQBF.
- 2. In the given problem it can be proved by showing  $P^A = NP^B$ .
- 3. As TQBF is PSPACE problem,
- 4. Hence  $PSPACE \subseteq P^{TQBF}$  and  $PSPACE^{TQBF} \subseteq PSPACE$
- 5. Therefore, combining both result  $P^A = NP^B$ .

Comments (1)

# **Step 3** of 3

#### For Oracle B:

- 1. For oracle B it is required to show that  $P^B \neq NP^B$  .
- 2. For this define a language  $L_{B}$  as:  $L_{B} = \left\{0^{n} \mid w \in \left\{0,1\right\}\right\}$  Here, B(w) = 1
- 3. For any oracle B the defined language  $L_B$  is  $NP^B$ .
- 4. Now the machine's output will be  $\ 1$  if x=0'' with  $|\mathbf{w}|=|x|$
- 5. Here  $TQBF \in NL$  this shows that  $PSPACE \in NL$
- 6. By using hierarchy theorems and Baker Gill which states that  $\ ^{NL}\varsubsetneq PSPACE$  .
- 7. It shows that  $P^B \neq NP^B$

Using both two parts of algorithm result it is sure that  $^{TQBF}$  is in polynomial time for both oracle  $^{A}$  and  $^{B}$ .

Comments (1)