

### Problem

Let

$B_n = \{a^k \mid k \text{ is a multiple of } n\}$ . Show that for each  $n \geq 1$ , the language  $B_n$  is regular.

### Step-by-step solution

#### Step 1 of 5

Consider that:

$$B_n = \{a^k \mid k \text{ is a multiple of } n\}$$

In order to prove that the given expression is regular, the value of  $n$  is chosen as greater than or equal to 1.

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#### Step 2 of 5

Suppose,  $k = ni$ , where  $i$  is any positive integer. In starting, suppose value of  $i$  is chosen to be 1.

When  $i = 1$  and  $n = 1$

$$\begin{aligned} B_1 &= \{a^k\} \\ &= \{a^{ni}\} \\ &= \{a^{1 \times 1}\} \\ &= \{a\} \end{aligned}$$

Then, the String comes out to be  $\{a\}$

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#### Step 3 of 5

Increase the value of  $n$  keeping the value of  $i$  equal to 1. When  $i = 1$  and  $n = 2$

$$\begin{aligned} B_2 &= \{a^k\} \\ &= \{a^{ni}\} \\ &= \{a^{2 \times 1}\} \\ &= \{aa\} \end{aligned}$$

Then, the String comes out to be  $\{aa\}$

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#### Step 4 of 5

Increase the value of  $n$  keeping the value of  $i$  equal to 1. When  $i = 1$  and  $n = 3$

$$\begin{aligned} B_3 &= \{a^k\} \\ &= \{a^m\} \\ &= \{a^{3 \times 1}\} \\ &= \{aaa\} \end{aligned}$$

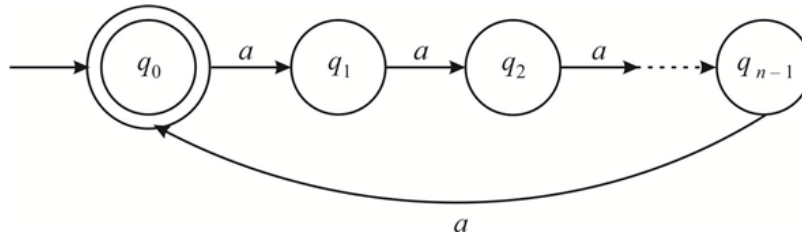
Then, the String comes out to be  $\{aaa\}$  and so on.

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#### Step 5 of 5

Finite automaton of the regular expression is as shown:



In the above finite automaton,  $q_0$  is the initial and final state and  $q_1, q_2, q_3$  and  $q_{n-1}$  are the subsequent states.

The language  $B_n$  is the regular language. According to the closure property of the regular expression, it is clearly seen that the specific expression is a regular expression when the value of  $n$  is greater than and equal to 1.

Closure property includes various operations such as union, intersection, set complement, set reversal, set difference and many more. Assume that  $B_n$  is regular.

- Union of  $B_1$  and  $B_2$  results in the third string and it is also a regular expression.
- Similarly, if user applies any property of closure, then the result is the regular expression.

**Hence, it is proved that the above expression is the regular expression.**

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