Problem

Define pad as in Problem 9.13.

a. Prove that for every A and natural number k, A ? P iff pad(A, n^k) ? P.

b. Prove that P ≠ SPACE(n).

Step-by-step solution

Step 1 of 2

For any language A and function $f : N \rightarrow N$, the language $pad(A, f)$ is defined as:

$(\mathrm{A}, f) = \{pad(s, f(m)) \mid \text{where } s \in \mathrm{A} \text{ and m is the length of } s\}$.

(a) Suppose A be any language and $k \in N$. If $\mathrm{A} \in P$, then $pad(\mathrm{A}, n^k) \in P$ because it can be determined whether $w \in pad(\mathrm{A}, n^k)$ by writing w as $s\#$ where s does not contain the # symbol then it is tested whether $|w| = |s|^k$; and finally it is tested that whether $s \in A$.the implementation of the first test in polynomial time is straight forward. The second test runs in time $\mathrm{poly}(|\mathrm{s}|)$ because $|s| \le |w|$, the test runs in time $\mathrm{poly}(|w|)$ and hence is in polynomial in time. If $pad(A, n^k) \in P$, then $A \in P$ because it can be determined whether $w \in A$ by padding w with # symbols until it has length $|w|^k$ and then test, whether the result is in $pad(A, n^k)$. The above explanation shows that $A \in P$ holds if and only if $pad(A, n^k) \in P$.

Comment

Step 2 of 2

(b) Assume that $\mathrm{P{=}SPACE}(n)$. Suppose A be a language in $\mathrm{SPACE}(n^2)$ but it will not exist in $\mathrm{SPACE}(n)$ as the space hierarchy theorem says. The language $pad(A, n^2) \in \mathrm{SPACE}(n)$ because it has enough space to run the $O(n^2)$ space algorithm for A, using that is linier in the padded language. Because of the assumption, $pad(A, n^2) \in P$, hence $A \in \mathrm{P}$ as discussed above. Hence $\mathrm{A} \in \mathrm{SPACE}(n)$ by taking assumption once again. But that is a contradiction. Hence, It can be said that $\mathrm{P} \neq \mathrm{SPACE}(n)$.

Comment