CSE303: Theory of Computation, Fall 2021 **Date:** 10/01/2021

Homework # 2

(Due: 10/05/2021 **)**

GROUP NUMBER: 33

Group Members		
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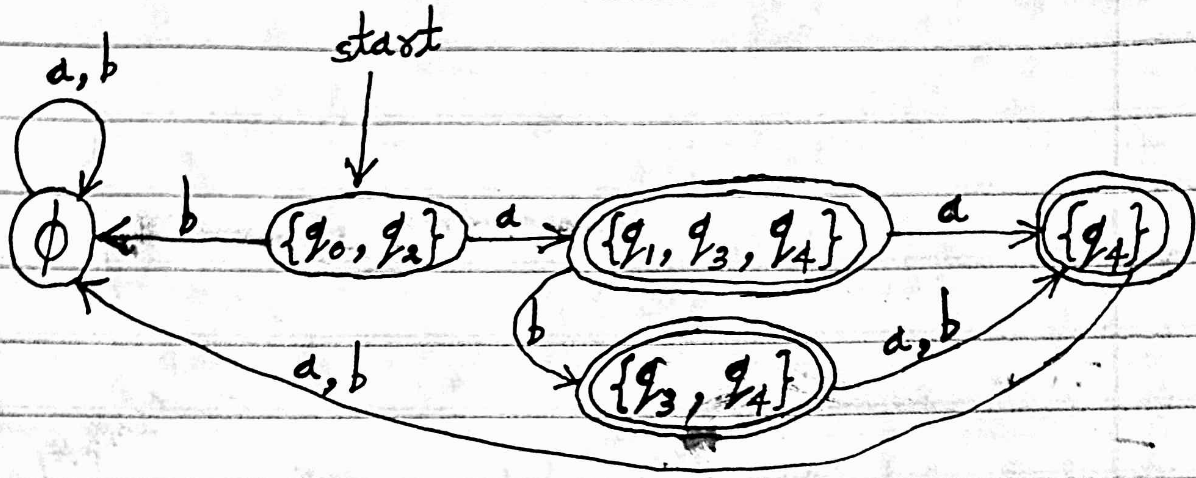
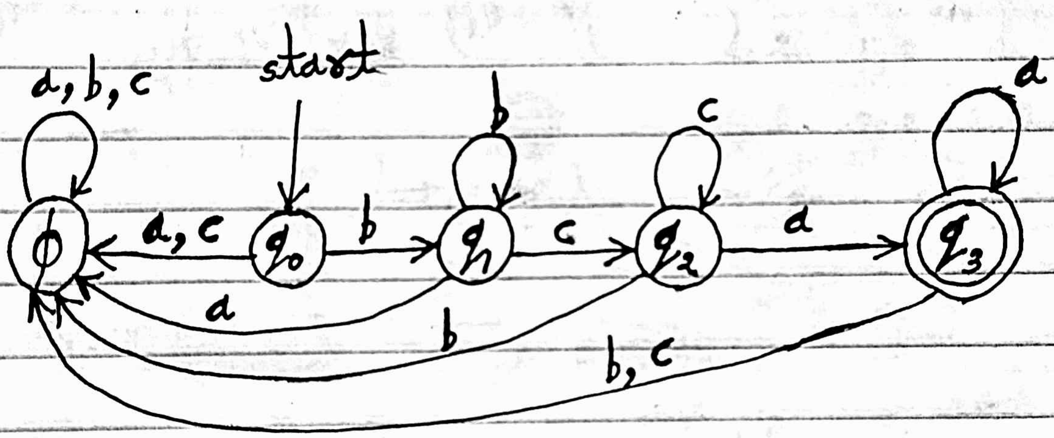
CSE303: Homework #2

Task 1

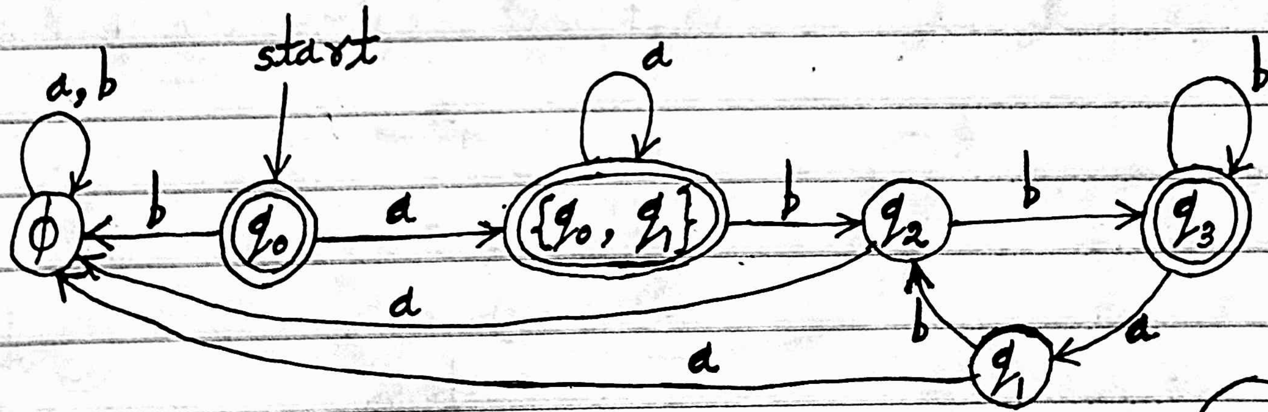
(1) (a)



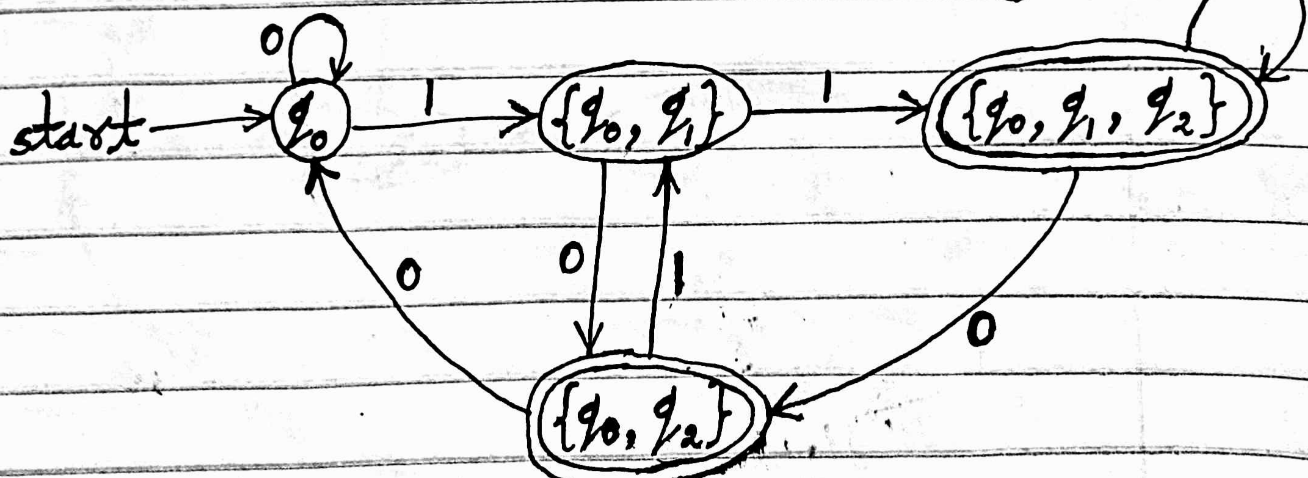
(b)



(c)



(d)



Task 2

(2.) (a) Regular expression is $R = (0 \cup 1(01^*0)^*1)^*$

(b) Regular expression is $R = (a \cup bbb^*(abbb^*))^*$

(c) Regular expression is $R = a \cup aa \cup ab \cup aba \cup abb$

(d) Regular expression is
$R = ((a \cup b)^* \cup ((a \cup b)^2)^* \cup ((a \cup b)^3)^* \cup ((a \cup b)^4)^* \cup ((a \cup b)^5)^*)$

Task 3

(3.) (a) Suppose L is regular. Then it must satisfy pumping property.

Suppose $w = a^s b^s c^{2s}$

Let $w = xyz = \boxed{a^p}\boxed{a^q}\boxed{a^r b^s c^{2s}}$

where $|xy| = p+q \leq s$, $|y| = q \geq 1$ and $p+q+r = s$

Also, xy^iz must belong to L for all $i \geq 0$

But, $xz = a^p a^r b^s c^{2s} = a^{p+r} b^s c^{2s} \notin L$

Reason: $p+r+s = 2s-q < 2s$ i.e., sum of #a's and #b's is not equal to #c's.

Contradiction! Hence L is not regular.

(b) Suppose L is regular. Then it must satisfy pumping property.

Suppose $w = a^n$, where n is the k^{th} Fibonacci number and $n \geq s$ and $F_{k+1} - F_k > s$

Let $w = xyz = \boxed{a^p}\boxed{a^q}\boxed{a^r}$

where $|xy| = p+q \leq s$, $|y| = q \geq 1$ and $p+q+r = n$.
Also, xy^iz must belong to L for all $i \geq 0$
But, xy^2z is not in L.
Reason: $xy^2z = a^p a^{2q} a^r = a^{n+q} \notin L$
Because $n = F_k < n+q \leq F_k + s < F_{k+1}$ so $n+q$ is not a Fibonacci number.
Contradiction! Hence, L is not regular.

(c) Suppose L is regular. Then it must satisfy pumping property.
Suppose $w = a^s b^{s^3}$
Let $w = xyz = \boxed{a^p} \boxed{a^q} \boxed{a^r b^{s^3}}$
where $|xy| = p+q \leq s$, $|y| = q \geq 1$ and $p+q+r = s$
Also, xy^iz must belong to L for all $i \geq 0$
But, xy^2z is not in L
Reason: $xy^2z = a^p a^{2q} a^r b^{s^3} = a^{s+q} b^{s^3} \notin L$
Contradiction! Hence, L is not regular

Task 4
(4) (a) Suppose L is regular.
As regular languages are closed under complementation, $\overline{L}$ must also be regular.
But, $\overline{L} = \{a^n \mid n \text{ is prime}\}$ was proved to be non-regular in the class.
Contradiction! Hence L is not regular.

(b) Suppose L is regular.
As regular languages are closed under complementation, $\bar{L}$ must also be regular.
$L_1 = \{a^m \mid m \text{ is prime}\}$
$L_2 = \{a^m \mid m \text{ is divisible by } 3\}$
$\bar{L} = L_1 \cup L_2$
L_2 was proved to be regular in the class.
As regular languages are closed under intersection and union, L_1 must also be regular.
But, L_1 was proved to be non-regular in the class.
Contradiction! Hence L is not regular.