Problem

The construction inTheorem1.54 shows that every GNFA is equivalent to a GNFA with only two states. We can show that an opposite phenomenon occurs for DFAs. Prove that for every k > 1, a language $A_k \subseteq \{0,1\}^*$ exists that is recognized by a DFA with k states but not by one with only k – 1 states.

Step-by-step solution

Step 1 of 1

Given:

It is being given that if there is two states then GNFA is correspondingly equal to the GNFA. But, here user need to prove that in DFA subsequently opposite phenomenon occurs. It implies user need to prove that no DFA are equivalent to a DFA with lesser states.

Proof:

Assume A_k be the set of words of length at least $k-1$. Therefore it can be said that A_k has at least k equivalence classes of words length $0,1,2,\ldots,k-2,\text{and } k-1$ or more. So it is clear from this that A_k requires a DFA with k states.

For any DFA fewer than k states, by Pigeon Hole Principle, two of the k strings cause the machine to loop in same state results in a rejection from the DFA.

Consider $A_k = \{0,1\}^* 0^{k-1} 0^*$ for $k > 1$.

Now a DFA with exactly k states can recognize the language A_k. Starting from the start state there is a state in the DFA for each 0 it had read after the last 1.

After $k-1$ 0's it arrives at an accepting state whose further transitions are self-loops. Now based on the language let say A_k be the set consisting of the strings $10, 100, \ldots, 10^{k-1}$. If the DFA consist of fewer than k states then by the Pigeon Hole Principle two these strings cause a loop to a single state. Hence the machine fails to accept the strings.

Conclusion:

Hence, it can be said that no DFA's are equivalent to a DFA with lesser states.

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