Problem

Let $E_{\mathsf{TM}} = \{\langle M\rangle \mid M \text{ is a } \mathsf{TM} \text{ and } L(M) = \emptyset\}$. Show that $\overline{E_{\mathsf{TM}}}$, the complement of E_{TM}, is Turing-recognizable.

Step-by-step solution

Step 1 of 1

Re-cognition of the language by Turing machine

Given: In this a language E_{TM} is given. Show that the complement of E_{TM} which is written as $\overline{E_{TM}}$ is recognized by Turing machine M.

Proof: Assume that $t_1, t_2, t_3.....$ is a list of strings which are present in Σ^*. For proving $\overline{E_{TM}}$ is Turing recognizable, user has to determine whether any of the string from $t_1, t_2, t_3.....$ is accepted by the Turing machine M or not.

If the Turing machine M accepts at least one string t_i from the list $L(M) \neq \phi$ so the Turing machine M belongs to $\overline{E_{TM}}$.

If the Turing machine M does not accept any string then $L(M) = \phi$ so the Turing machine M does not belong to $\overline{E_{TM}}$.

List of all strings cannot sequentially execute on the Turing machine M as if the Turing machine M can accept the string $\langle M\rangle \in \overline{E_{TM}}$ but it is looping in string t_1. Since the Turing machine M accept the string t_2 from the list so $\langle M\rangle \in \overline{E_{TM}}$.

If list of all the strings is executed on M in a sequential manner then user never extract the past of first string.

So for avoiding this problem related to sequential execution and recognizing $\overline{E_{TM}}$ construct a Turing machine.

Construction of Turing machine M:

Turing machine S= on input $\langle M\rangle$ here M is a Turing machine.

1. Repeat the following methods for $i = 1, 2, 3, ...$
2. Execute each and every string from the list $t_1, t_2, t_3.....$ on the Turing machine M.
3. If any string is accepted then accept it
4. Otherwise reject it.

Conclusion:

Hence the string is accepted by the Turing machine so the complement of E_{TM} which is written as $\overline{E_{TM}}$ is Turing recognizable.

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