CSE303: ToC 115060128.
HW9

Problem 1:-

Given CGF convert it to Chomsky Normal Form

$S \to aSd \mid A \mid B$
$A \to aAc \mid C$
$B \to bBd \mid C$
$C \to bCc \mid \varepsilon$

Following the 4 steps:-

Step 1:- Introducing $S_0 \to S$

$S_0 \to S$
$S \to aSd \mid A \mid B$
$A \to aAc \mid C$
$B \to bBd \mid C$
$C \to bCc \mid \varepsilon$

Step 2: Removing $C \to \varepsilon$

$S_0 \to S$
$S \to aSd \mid A \mid B$
$A \to aAc \mid C \mid \varepsilon$
$B \to bBd \mid C \mid \varepsilon$
$C \to bCc \mid bc$

Removing $B \to \varepsilon$

$S_0 \to S$
$S \to aSd \mid A \mid B \mid \varepsilon$
$A \to aAc \mid C \mid \varepsilon$
$B \to bBd \mid C \mid bd$
$C \to bCc \mid bc$.

Removing $A \to \varepsilon$.

$S_0 \to S$
$S \to aSd \mid A \mid B \mid \varepsilon$
$A \to aAc \mid C \mid ac$
$B \to bBd \mid C \mid bd$
$C \to bCc \mid bc$

Step3: Removing unit rules $A \to C$ & $B \to C$.

$S_0 \to S$
$S \to aSd \mid A \mid B \mid \varepsilon$.
~~$A \to aAc \mid bBd \mid$~~
$A \to aAc \mid bCc \mid bc \mid ac$.
$B \to bBd \mid bCc \mid bc \mid bd$
$C \to bCc \mid bc$.

Removing rules $S \to A$ & $S \to B$

$S_0 \to S$
$S \to aSd \mid aAc \mid bCc \mid bc \mid ac \mid bBd \mid bd \mid \varepsilon$
$A \to aAc \mid bCc \mid bc \mid a$
$B \to bBd \mid bCc \mid bc \mid bd$
$C \to bCc \mid bc$.

Step4): Converting remaining rules into the proper form.

We will ~~tel~~ have $A_1 \to a$, $B_1 \to b$, $C_1 \to c$ & $D \to d$ as rules for terminals

Thus

$S_0 \to S$

$S \to A_1U_1 \mid A_1U_2 \mid B_1U_3 \mid B_1C_1 \mid A_1C_1 \mid B_1U_4 \mid B_1D \mid \varepsilon$

$A \to A_1U_2 \mid B_1U_3 \mid B_1C_1 \mid A_1C_1$

$B \to B_1U_4 \mid B_1U_3 \mid B_1C_1 \mid B_1D$

$C \to B_1U_3 \mid B_1C_1$

$D \to d$

$U_1 \to SD$

$A_1 \to a$

$C_1 \to c$

$U_2 \to AC_1$

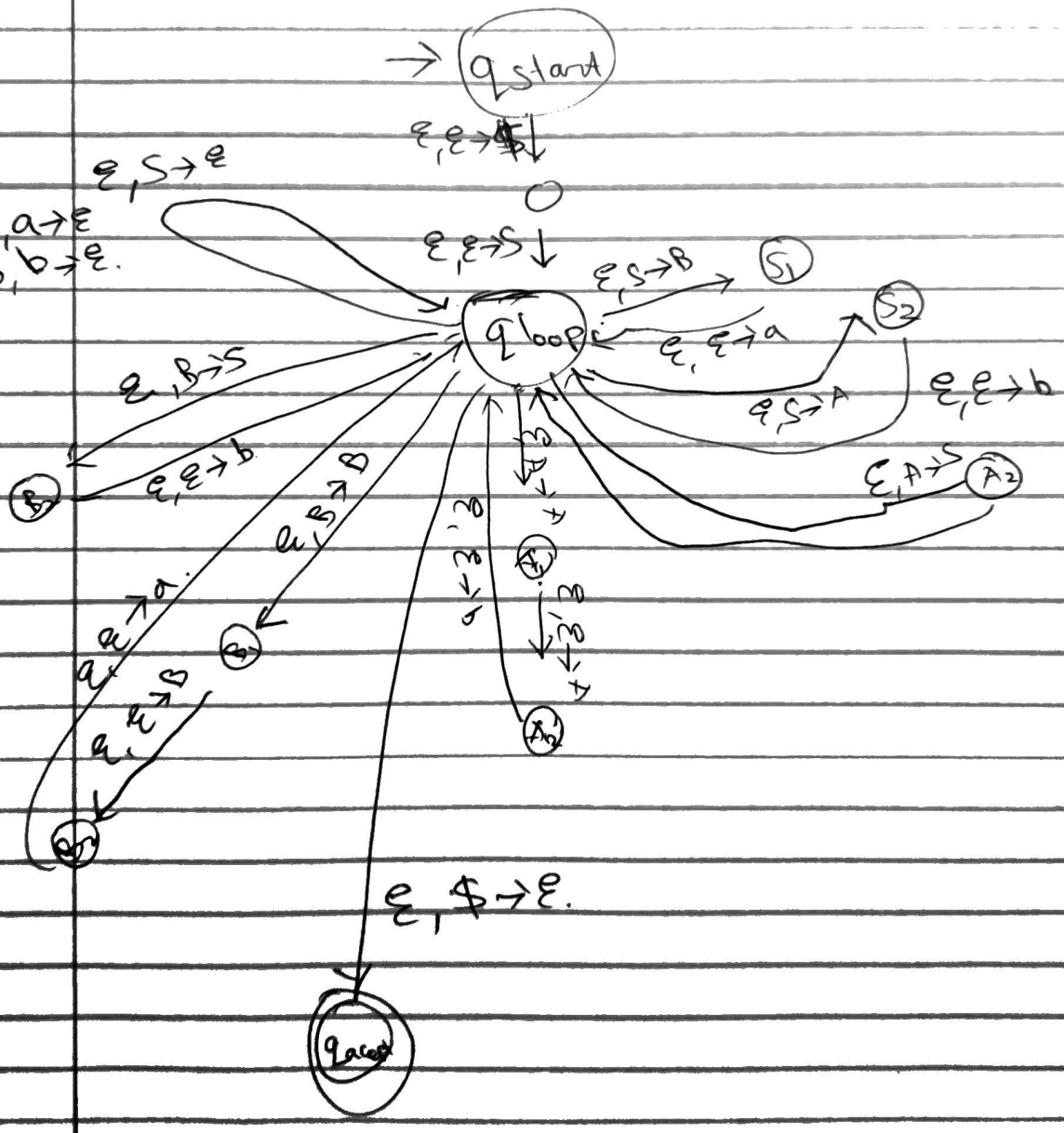
$U_3 \to CC_1$

$B_1 \to b$

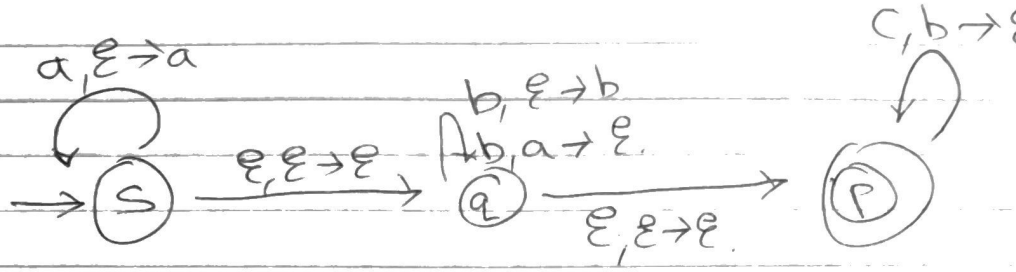
$U_4 \to BD$

Problem 2:-

$S \to \varepsilon \mid aB \mid bA$
$A \to aS \mid bAA$
$B \to bS \mid aBB$



Problem 3:-



Input	Transition / Push	Transition // Pop
a	$S \xrightarrow{\varepsilon \to a} S$	$q \xrightarrow{b\,a \to \varepsilon} q$
b	$q \xrightarrow{\varepsilon \to b} q$	$p \xrightarrow{c,\, b \to \varepsilon} p$
c		

$A_{sp} \to A_{sq} A_{qp}$
$A_{sq} \to a A_{sq} b \mid \varepsilon$
$A_{qp} \to b A_{qp} c \mid \varepsilon$

Problem 4:-

In Chomsky Normal Form we have rules

i) $A \to BC$ B, C are not start symbol
ii) $A \to a$
iii) $S \to \varepsilon$

~~For~~ Consider a string w of length $n (n > 0)$
Note here for using rules ii) & iii) we dont increase the length of the string

Only using (i) the length of string increases.

Also, we don't have S in B & C so, ~~if~~ $S \to \varepsilon$ cannot be used. So only rules (i) & (ii) have to be used

For ~~to~~ achieving length of n, we need to have $A \to BC$ rule applied exactly $n-1$ times since, any greater, we ~~can't get~~ will get string $>$ length n. anyless we will get string $< n$.

So rule (i) has to be used $n-1$ times.

Also to have terminals, we need to use rule ii n times. ~~So~~ Since each application replaces a variable with terminal.

So total steps are $n + n-1 = 2n-1$

Thus we will require $2n-1$ steps for generating w of length n.