Problem

Read the definition of a 2DFA (two-headed finite automaton) given in Problem 5.26. Prove that P contains a language that is not recognizable by a 2DFA.

Step-by-step solution

Step 1 of 2

Suppose $L = \{p \mid \text{either } p = 0x \text{ for some } x \in B_{TM}, or\ p = 1y \text{ for some } y \notin B_{TM}\}$.

• It can be designed a Turing machine S and S may be defined as:

S: On input, write 0 followed by $\langle M, p \rangle$ in the tapes and halts. Then it is easy to check that:

$\langle M, p \rangle \in B_{TM} \Leftrightarrow output\ of\ Q \in L$

Thus, a mapping reduction of B_{TM} to L or $B_{TM} \leq_m L$ can be obtained .

• Now, a Turing machine(TM) R can be formed, which shows the functionality $B_{TM} \leq_m \overline{L}$. The Turing machine(TM) R can be defined as:

R: On input, write 1 followed by $\langle M, p \rangle$ in the tapes and halts. Then it is easy to check that:

$\langle M, p \rangle \in \overline{B}_{TM} \Leftrightarrow output\ of\ R \in L$

• Similarly,

$\langle M, p \rangle \in B_{TM} \Leftrightarrow output\ of\ R \in \overline{L}$

Thus, a mapping reduction of B_{TM} to $\overline{L}$ can be obtained .

Comment

Step 2 of 2

Since, $B_{TM} \leq_m L$ and $\overline{B}_{TM} \leq_m \overline{L}$. This show that $\overline{L}$ is non Turing recognizable because B_{TM} is **non Turing recognizable**. Similarly, since $B_{TM} \leq_m \overline{L}$ and $\overline{B}_{TM} \leq_m L$. So, this allows that L is non Turing recognizable. **Therefore, the above explanation shows that “ P contains a language which is not recognizable by a 2DFA”**.

Comment