Problem

Prove that EQ_{DFA} is decidable by testing the two DFAs on all strings up to a certain size. Calculate a size that works.

Step-by-step solution

Step 1 of 1

Given:

Assume that $EQ_{DFA} = \{\langle A, B\rangle \mid A, B \text{ are } DFAs \text{ and } L(A) = L(B)\}$ and suppose M is a Turing machine for determining the decidability of EQ_{DFA}.

Construction of Turing machine M :

• M = "On input $\langle A, B\rangle$, where A and B are DFAs with the same terminal symbols. If symbols are not same then reject.

• Calculate the number of states in the DFA, A and B and stored them in n and m respectively.

• Iterate all strings which comes under Σ till $n.m$

• Now for each and every string w

o Simulate DFA A on the string w

o Simulate DFA B on the string w

o Here Turing Machine M is working as a decider by running on the output provided both DFA A and B. If result of both the simulation comes different the reject the string, otherwise accept it.

• After that all $n.m$ strings then accept it otherwise reject it.

• If M accepts, accept. If M rejects, reject."

Size for working:

The reason behind checking first $n.m$ strings is that if both DFAs do not accept the same language so there is a string w of size $|w|$ and it is less than equals to size of $n.m$ for $A(w) \neq B(w)$.

By using contradiction, suppose that the first string provides a different output of DFAs A and B is w'. The length l of $|w'|$ is greater than $n.m$.

Now the sequence of states for the DFA A is $a_0, a_1, a_2 \ldots\ldots a_l$ and the sequence of states for the DFA B is $b_0, b_1, b_2 \ldots\ldots b_l$ for describing the transitions for w' in DFA A and B.

As the value of l is greater than $n.m$, if the user places above sequences side by side so there is some repetition in pairs like a_i, b_i and a_j, b_j such that $a_i = a_j, b_i = b_j, i < j$ also present.

So user can remove all the sequences remaining a_i, b_i and get a smaller string w''. As the DFA A and B is same over as w' thus our contradiction becomes false.

As the string w'' has length less than equal to $n.m$ such that $A(w'') \neq B(w'')$.

So, checking all the strings up to size $n.m$ is enough.

Conclusion:

Here two Turing Machines are used M is used as decider to find decidability of A and B Turing Machine M is used to find Running condition of Turing machines M. This way A and B are being decided by M, A and B are components of EQ_{DFA} so, EQ_{DFA} is also decidable.

Comments (1)