Problem

We generally believe that *PATH* is not NP-complete. Explain the reason behind this belief. Show that proving *PATH* is not NP-complete would prove P ≠ NP

Step-by-step solution

Step 1 of 2

***NP* – complete:**

A language *B* is *NP* – complete if it satisfies two conditions

1. *B* is in *NP* and

2. Every *A* is *NP* is polynomial time reducible to *B*.

Comment

Step 2 of 2

PATH $=\{\langle G,s,t\rangle \mid G \text{ is a directed graph that has a directed path from s to t}\}$

1. **PATH is not *NP* – complete:**

◊ Let us assume that PATH would be *NP* – complete.

◊ From the definition of *NP* – completeness,

For all $L \in NP, L$ is polynomial time reducible to PATH.

◊ But this again implies that for all *L* in NP, L is in *P*. Thus $P = NP$ which we believe that it is not true.

Hence, PATH is not *NP* – complete.

2. Proving that PATH is not *NP* – complete implies that $NP \neq P$:

◊ Showing this by contraposition.

◊ Assume that $P = NP$ and then show that PATH is *NP* – complete.

◊ So assume $P = NP$.

◊ “If $P = NP$ then every language $A \in P$ is *NP* – complete”.So, PATH is *NP* – complete.

Thus, if PATH is not *NP* –complete, then $NP \neq P$.

Comment