Problem

Use the construction in the proof of Theorem 1.45 to give the state diagrams of NFAs recognizing the union of the languages described in

a. Exercises 1.6a and 1.6b.

b. Exercises 1.6c and 1.6f.

THEOREM **1.45**

The class of regular languages is closed under the union operation.

Step-by-step solution

Step 1 of 9

a)

Consider the languages,

$L_1 = \{ w \mid w$ begins with 1 and ends with *a* 0 $\}$ and

$L_2 = \{ w \mid w$ contains at least three 1s$\}$ on $\Sigma = \{0,1\}$

M_1 be the NFA that recognizes L_1 and

M_2 be the NFA that recognizes L_2.

Comment

Step 2 of 9

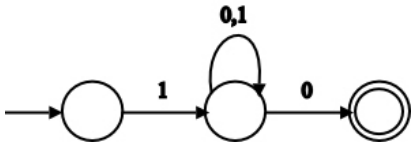
Let $L = L_1 \cup L_2$

Now *M* be the NFA that recognizes *L*.

- $L_1 = \{ w \mid w$ begins with *a* 1 and ends with *a* 0$\}$

$L_1 = 1(0,1)^* 0$

The state diagram of M_1 that recognizes L_1 is



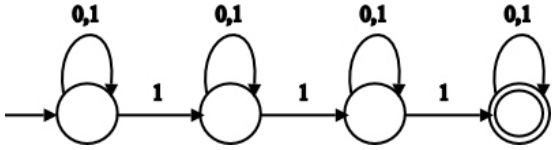
Comments (5)

Step 3 of 9

- $L_2 = \{ w \mid w$ contains at least three 1s$\}$

$L_2 = (0,1)^* 1(0,1)^* 1(0,1)^* 1(0,1)^*$

The state diagram of M_2 that recognizes L_2 is

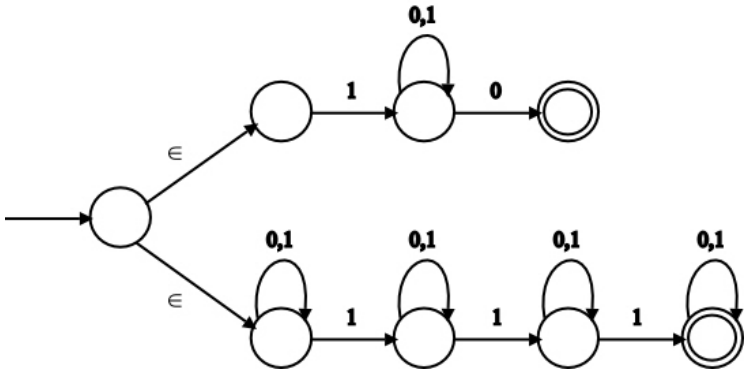


Comments (3)

Step 4 of 9

Now *L* is union of L_1 and L_2.

So the state diagram of *M* that recognizes *L* is described as follows.



Comments (1)

Step 5 of 9

(b) Languages are

$L_1 = \{w \mid w \text{ contains the substring } 0101 \text{ i.e, } w = x0101y \text{ for some } x \text{ and } y\} \text{on} \sum = \{0,1\}$

$L_1 = \{w \mid w \text{ doesn't contain the substring } 110\}$ on $\Sigma = \{0,1\}$

M_1 be NFA that recognizes L_1 and

M_2 be the NFA that recognizes L_2

Comment

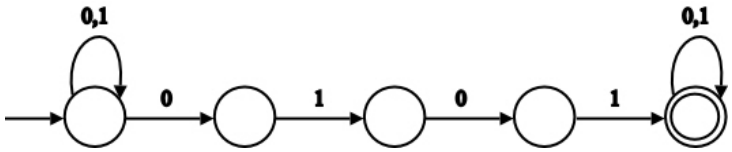
Step 6 of 9

Let $L = L_1 \cup L_2$

Now *M* be the NFA that recognizes *L*.

- $L_1 = \{w \mid w \text{ contains the substring } 0101\}$

The state diagram of M_1 that recognizes L_1 is

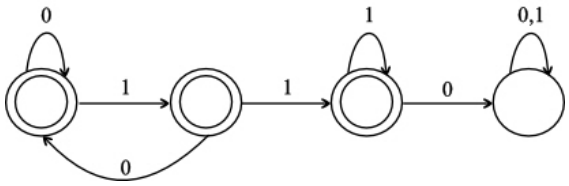


Comments (3)

Step 7 of 9

- $L_2 = \{w \mid w \text{ doesn't contain the substring } 110\}$

First we give the state diagram of the machine which recognize the language machine which recognize the language



Comments (4)

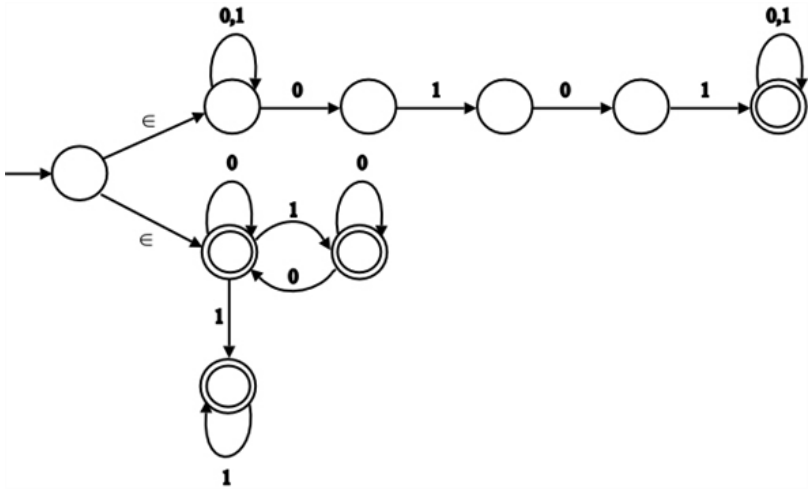
Step 8 of 9

Now *L* is the union of L_1 and L_2

So the state diagram of *M* that recognizes *L* is described as follows

Comment

Step 9 of 9



Comments (3)