Problem

A k-***head pushdown automaton*** (k-PDA) is a deterministic pushdown automaton with k read-only, two-way input heads and a read/write stack. Define the class PDA_k = {A| A is recognized by a k-PDA}. Show that $\mathrm{P} = \bigcup_k \mathrm{PDA}_k.$ (Hint:Recall that P equals alternating log space.)

Step-by-step solution

Step 1 of 3

A deterministic pushed down automation, which consists k read-only, a read write stack **and two ways input heads**, can be defined as a k**-head pushed-down automation** ($k-\mathrm{PDA}$). Consider the **class** PDA_k, which is defined as:

$\mathrm{PDA}_k = \{A \mid A\, is\, recognized\, by\, a\, k-\mathrm{PDA}\}$

Now, by using the above given facts, $\mathrm{P}=\bigcup_k \mathrm{PDA}_k$ has to be proved. It can be achieved by using, $ASPACE(s(n)) = TIME\left(2^{O(s(n))}\right)$, which shows that P is alternating log-space.

Comment

Step 2 of 3

Now, a machine M, with deterministic time $2^{O(s(n))}$, is constructed. It is used to simulate an alternating space $O(s(n))$ machine S. If an input q is given to the simulator M, a graph is constructed by the simulator for the computation of S on q.

- The nodes are configured for S on q which use maximum $ls(n)$ space, where l is defined as the constant factor approximation for S.
- Edges are drawn from a configuration to those other configuration which can be generated in a single move of S.
- After the construction of graph, M iteratively scans it and marks those configurations which are accepting.
- Initially the acceptance configuration is marked. After that all the universal branching is marked is all of its children marked as an accepting state. Machine S continued marking and scanning until no additional nodes are marked on scan.

Comment

Step 3 of 3

As it is given that, $s(n) \geq \log n$, the configuration's number of S on q is $2^{O(s(n))}$. Hence the configuration graph's size is given by $2^{O(s(n))}$ and its construction may be done in $2^{O(s(n))}$ time.

- It takes roughly the same time to **scan the graph once**. Here, the **number of scans** is **equal to the maximum number of nodes in the graph**. Hence, the total time used is $2^{O(s(n))}$.
- Now, from the above discussion it can be said that $ASPACE(s(n)) = TIME\left(2^{O(s(n))}\right)$. It also shows, P **is alternating log-space.**

Hence, for $\mathrm{PDA}_k = \{A \mid A\, is\, recognized\, by\, a\, k-\mathrm{PDA}\}$, it can be said that $\mathrm{P}=\bigcup_k \mathrm{PDA}_k$.

Comment