Problem

Give a model of the sentence

$$\begin{aligned}\phi_{\mathrm{eq}} \;=\; & \forall x\,\big[\,R_1(x,x)\,\big] \\ & \wedge \forall x,y\big[\,R_1(x,y) \leftrightarrow R_1(y,x)\,\big] \\ & \wedge \forall x,y,z\big[\,(R_1(x,y) \wedge R_1(y,z)) \rightarrow R_1(x,z)\,\big].\end{aligned}$$

Step-by-step solution

Step 1 of 5

Given sentence is

$$\begin{aligned}\phi_{eq} = & \forall x\left[R_1(x,x)\right] \\ & \wedge \forall x,y\left[R_1(x,y) \leftrightarrow R_1(y,x)\right] \\ & \wedge \forall x,y,z\left[\left(R_1(x,y)\right) \wedge R_1(y,z) \rightarrow R_1(x,z)\right]\end{aligned}$$

ϕ_{eq} gives three conditions of equivalence relations i.e., Reflexive relation, Symmetric relation and Transitive relations.

Comment

Step 2 of 5

Let R_{11} and R_{12} are two equivalence relations on some set, definition of R_1 by

$$\forall x,y\left[R_1(x,y) \equiv R_{11}(x,y) \wedge R_{12}(x,y)\right]$$

Where *x, y* are elements from set.

Reflexive relation: $\forall x\left[R_1(x,x)\right]$

$\equiv$ {definition of R_1}

$R_{11}(x,x) \wedge R_{12}(x,x)$

$\equiv$ {R_{11}, being an equivalence relation, is reflexive}

similarly R_{12}

$\equiv \{true\} \wedge \{true\}$

$\equiv true$

Comment

Step 3 of 5

Symmetric relation: $\forall x,y\left[R_1(x,y) \leftrightarrow R_1(y,x)\right]$

$\equiv$ {definition of R_1}

$R_{11}(x,y) \wedge R_{12}(x,y)$

$\equiv$ {R_{11}, being an equivalence relation, is symmetric}

similarly R_{12}

$\equiv$ {definition of R_1}
$R_1(y,x)$

Comment

Step 4 of 5

Transitive relation: $\forall x,y,z\left[\left(R_1(x,y)\right)\wedge R_1(y,z)\rightarrow R_1(x,z)\right]$

$\equiv$ {definition of R_1}
$\left(R_{11}(x,y)\wedge R_{12}(x,y)\right)\wedge\left(R_{11}(y,z)\wedge R_{12}(y,z)\right)$

$\equiv$ {rearranging the conjucts}
$\left(R_{11}(x,y)\wedge R_{11}(y,z)\right)\wedge\left(R_{12}(x,y)\wedge R_{12}(y,z)\right)$

$\equiv$ {R_{11}, being an equivalence relation, is transitive}
similarly R_{12}

$\equiv$ {definition of R_1}
$R_1(x,z)$

Comment

Step 5 of 5

A model (U,R_1), where *U* is universe and R_1 is equivalence relation over *U,* is a model of ϕ_{eq}.

Comment