Problem

Let M_1 and M_2 be DFAs that have k_1 and k_2 states, respectively, and then let U = $L(M_1)$ [$L(M_2)$.

a. Show that if $U \neq \emptyset$, then U contains some string s, where $|s| < \max(k_1, k_2)$.

b. Show that if $U \neq \Sigma^*$, then U excludes some string s, where $|s| < k_1 k_2$.

Step-by-step solution

Step 1 of 4

a)

Suppose M_1 and M_2 are taken as DFAs which consists k_1 and k_2 states respectively. Now, consider $U = L(M_1) \cup L(M_2)$. So, it can be shown that "if $U \neq \phi$, then some string s exists in U, where $|s| < \max(k_1, k_2)$".

Comment

Step 2 of 4

To prove the above statement first assume that the DFAs M_1 and M_2, which are taken above, consists strings of unequal length and also different from each other.

- Now consider a start state, for the given grammar U, q is taken. Then, the production of M_1 and M_2 with a start variable q is given by:
  $q \rightarrow sM_1 \mid sM_2$
- The string consists of unequal length and also different from each other and the start variable are initiated for the grammar.
- Then, from the property of union, it will take the longest string from the taken two strings and it will add the un-matched string or variable.

Therefore, the final string which is taken should be less than the any string taken. Thus it can be said that **"if $U \neq \phi$, then some string s exists in U, where $|s| < \max(k_1, k_2)$".**

Comment

Step 3 of 4

b)

Suppose M_1 and M_2 are taken as a DFAs which consists k_1 and k_2 states respectively. Now, consider $U = L(M_1) \cup L(M_2)$. So, it can be shown that" if $U \neq \Sigma^*$, then some string s can be excluded from U, where $|s| < k_1 k_2$".

Comment

To prove the above statement first assume that the DFAs M_1 and M_2, which are taken above, consists strings of unequal length and also different from each other.

- Now consider $U = L(M_1) \cup L(M_2)$, which says that U consists the string of M_1 and M_2. The string which is taken may be longer in length than M_1 and M_2.
- Here, concatenation operation is applied between the initial string k_1 and k_2, then the length it consists may be longer than the original length of the string.
- Some string also exists in this final concatenated string, which is already taken. Therefore, these strings will be excluded in the final string taken.

Hence from the above discussion it can be said that "**if** $U \neq \Sigma^*$**, then some string** s **can be excluded from** U **, where** $|s| < k_1k_2$**".**

Comment