Problem

Answer each part TRUE or FALSE.

$$\textbf{a. } n = o(2n).$$
$$\textbf{b. } 2n = o(n^2).$$
$$^{A}\textbf{c. } 2^n = o(3^n).$$
$$^{A}\textbf{d. } 1 = o(n).$$
$$\textbf{e. } n = o(\log n).$$
$$\textbf{f. } 1 = o(1/n).$$

Step-by-step solution

Step 1 of 7

TRUE (or) FALSE

Small – *o* Notation:

Let *f* and *g* be functions $f,g:N \to R^{+}$ say that $f(n)=O(g(n))$ if for any real number *c* > 0,a number n_0 exists, where *f(n)* < *c* . *g(n)* for all $n \ge n_0$.

Comment

Step 2 of 7

(a)

False.

The statement $n=o(2n)$ is invalid, because the functions *n* and 2*n* grows equality

That is $f(n)=c\cdot g(n)$.But according to definition $f(n)<c\cdot g(n)$

Therefore $n=o(2n)$ is false

Comments (3)

Step 3 of 7

(b)

True.

The statement $2n=o(n^2)$ is valid, because the functions $n^2=n\cdot n$ which will grow faster than *n.* That is $f(n)<g(n)$.

Therefore from the definition of small – *o* notation, $2n=o(n^2)$ is true.

Comment

Step 4 of 7

(c)

True.

The statement $2^n=o(3^n)$ is valid, because the function 2^n runs shower than the function 3^n .

Then, $2^n < 3^n$.

That is $f(n) < c \cdot g(n)$

Hence from the definition of small – *o* notation, $2^n = o(3^n)$ is true.

Comment

Step 5 of 7

(d)

True.

The statement $1 = o(n)$ is valid, because the function *n* grows faster than a number 1.

Therefore $f(n) < c \cdot g(n)$

By the definition of small – *o* notation, $1 = o(n)$ is true.

Comment

Step 6 of 7

(e)

False.

The statement $n = o(\log n)$ is not valid, because the functions $\log n$ grows slower than the function *n* , which is a contradiction.

Hence $n = o(\log n)$ is false.

Comment

Step 7 of 7

(f)

False.

The statement $1 = o(1/n)$ is not valid, because the function $1/n$ grows slower than 1.

Therefore $f(n) > g(n)$.

Which is a contradiction

Hence $1 = o(1/n)$ is false .

Comment