Let Σ = {0,1} and let B be the collection of strings that contain at least one 1 in their second half. In other words,

$$B = \{uv \mid u \in \Sigma^*, v \in \Sigma^* 1 \Sigma^* \text{ and } |u| \geq |v|\}.$$

a. Give a PDA that recognizes B.

b. Give a CFG that generates B.

Step-by-step solution

Step 1 of 5

Consider the following information:

The alphabet set $\Sigma = \{0,1\}$.

$B = \{uv \mid u \in \Sigma^*, v \in \Sigma^* 1 \Sigma^* \text{ and } |u| \geq |v|\}$.

Comment

Step 2 of 5

a.

Construction of PDA:

1. The PDA for this language works by first reading and pushing the string u onto the stack. The string u consists of either 0's or 1's.

2. After pushing the string u into stack is completed, the machine has all the letters of u on the stack. Now, match the letters in u with the letters in v.

3. Remember that the length of u is greater than or equal to v.

4. If the condition that number of 1's in the input is at least 1 in second half, then accept the entire string.

Problem:

• But here a problem takes place in finding the middle of the string uv.

• That is the position at which u ends and v starts.

• The problem can be solved by using **Non deterministic push down automata**.

• The PDA will be designed such that the empty string $\$$ can be accepted by the PDA.

Construction:

The PDA M that recognizes B is $(Q, \Sigma \frac{1}{2}, \Gamma, \delta, q_0, F)$, where:

$Q = \{q_0, q_1, q_2, q_3\}$,
$\Sigma = \{0,1\}$,
$\Gamma = \{x\}$,

$F = \{q_3\}$

Transition function δ of the represented in a tabular format:

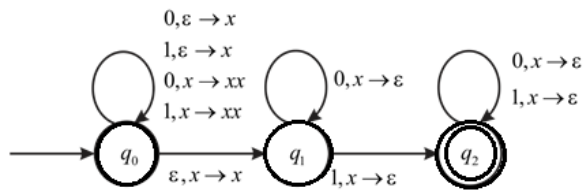| Input:	0		1		ε	
Stack:	x	ε	x	ε	x	ε
q_0	$\{(q_0, xx)\}$	$\{(q_0, x)\}$	$\{(q_0, xx)\}$	$\{(q_0, x)\}$	$\{(q_1, x)\}$	
q_1	$\{(q_1, \varepsilon)\}$		$\{(q_2, \varepsilon)\}$			
q_2	$\{(q_2, \varepsilon)\}$		$\{(q_2, \varepsilon)\}$			

Comment

Step 3 of 5

The state diagram for the PDA M is given below:

Step 4 of 5

The diagram shows three states q_0, q_1, q_2 with transitions:

q_0 self-loop:
$0, \varepsilon \to x$
$1, \varepsilon \to x$
$0, x \to xx$
$1, x \to xx$

q_1 self-loop: $0, x \to \varepsilon$

q_2 self-loop:
$0, x \to \varepsilon$
$1, x \to \varepsilon$

$q_0 \to q_1$: $\varepsilon, x \to x$

$q_1 \to q_2$: $1, x \to \varepsilon$

Step 5 of 5

b.

The CFG for the language B is as follows:

$S \to UV$
$U \to AB$
$V \to A1A \mid A1B \mid A1U \mid B1U \mid U1U$
$A \to 00^* \mid \in$
$B \to 11^* \mid \in$

Explanation:

• The string S is concatenation of U and V.

• The string U may consist of any number of 0's and 1's.

• The string V may consist of atleast one 1 or only one 1.

• The string A may consists of atleast one zero or more than one zeros.

• The string B may consist of atleast one 1 or more than one 1's.