Problem

Prove that NTIME(n) $\subsetneq$ PSPACE.

Step-by-step solution

Step 1 of 1

$\mathbf{NTIME(n)}$ **is strict subset of** $\mathbf{PSPACE(n)}$

At most $t(n)$ tape cells on each branch can be used by any Turing machine that operates in time $t(n)$ on each computation branch. So, it can be stated that $\mathrm{NTIME}(n) \subsetneq \mathrm{NSPACE}(n)$

• Now, consider the **Savitch's theorem** which says that: "Let $f : N \rightarrow R$ be a function, with $f(n) \geq n$ then $\mathrm{NSPACE}(f(n)) \subseteq \mathrm{SPACE}\left((f(n))^2\right)$".

Therefore according to Savitch's theorem $\mathrm{NSPACE}(n) \subseteq \mathrm{SPACE}(n^2)$.

• Now, consider the **space hierarchy theorem** which says that "if g is space-constructible ($1^n \rightarrow 1^{g(n)}$ can be computed in space $O(g(n))$), $f(n) = O(g(n))$ then $\mathrm{SPACE}(f(n)) \subsetneq \mathrm{SPACE}(g(n))$.

Therefore, according to space hierarchy theorem it can be said that $\mathrm{SPACE}(n^2) \subsetneq \mathrm{SPACE}(n^3)$. The result follows because $\mathrm{SPACE}(n^3) \subseteq \mathrm{PSPACE}$.

From the above explanation, it can be said that $\mathrm{NTIME}(n) \subsetneq \mathrm{PSPACE}(n)$.

Comment