

Problem

Consider the problem of determining whether a Turing machine M on an input w ever attempts to move its head left when its head is on the left-most tape cell. Formulate this problem as a language and show that it is undecidable.

Step-by-step solution

Step 1 of 2

Consider the problem of determining whether a Turing machine M on an input w ever attempts to move its head left when its head is on the left-most tape cell. This problem is formulated as a language:

$$L = \{ \langle M, w \rangle \mid M \text{ attempts to move its head left when its head is on the leftmost tape cell} \}$$

[Comment](#)

Step 2 of 2

Assume that the language L is decidable and \hat{M} be a TM that decides the language L . Construct a TM, A that decides the halting problem.

$A =$ "on input $\langle M, w \rangle$:

1. Construct a TM, A' , from A . The TM A' moves w one tape cell to the right and marks the leftmost cell with #.
2. Run the TM A' on $\langle M, w \rangle$.
3. If A' encounters # then A' moves to the right side and simulates M reaching the leftmost tape cell.
4. If M halts and accepts on w then A' simulates to move its head left when its head is on the leftmost tape cell."

Now, TM A runs \hat{M} on the input $\langle A, w \rangle$. If \hat{M} accepts, A accepts. Otherwise, A rejects. It is assumed that \hat{M} be a TM that decides the language L . If M halts and accepts on w , then only A' moves its head left when its head is on the left-most tape cell. If A decides the halting problem, then halting problem is decidable. Thus, the halting problem is undecidable. It is a contradiction.

Therefore, the language L is undecidable.

[Comments \(1\)](#)