Problem

Let $f : \mathcal{N} \longrightarrow \mathcal{N}$ be any function where f(n) = o(n log n). Show that TIME(f(n)) contains only the regular languages.

Step-by-step solution

Step 1 of 4

$\text{Time} - \text{Compexity class } TIME(t(n))$:

Let $t : N \to R^{+}$ be a function. Define the time complexity class, $TIME(t(n))$, to be collection of all Languages that are decidable by an $O(t(n))$ time Turing Machine.

Small - O – notation:

Let f and g be functions $f, g : N \to R^{+}$. Say that $f(n) = O(g(n))$

If $\lim_{x \to \infty} \frac{f(n)}{g(n)} = 0$

In other words, $f(n) = 0(g(n))$ means that, form any real number $c > 0$, a number does not exists, where $f(n) < c(n)$ for all $n \geq n_0$.

Given that $f : N \to N$ be any function where $f(n) = 0(n \log n)$

• Now we have to show that $TIME(f(n))$ contains only regular languages.

• Suppose that $f(n) = 0(n \log n)$ and M is a $S(\geq 2)$ state one – tape deterministic Turing machine accepting a set L within time $f(n)$.

• Let $g(n)$ be defined by

$$g(n) = \begin{cases} \dfrac{n \log n}{f(n)}, & n \geq 2 \\ 1, & n = 0,1 \end{cases}$$

• Then we have $\lim_{x \to \infty} g(n) = \infty$ and we can select a value c such that

$$3\frac{n^{(\log s)/g(n)^{\frac{1}{2}}+1} - 1}{S - 1} + 1 \leq n - 2 - \frac{n}{g(n)^{\frac{1}{2}}} + C\frac{g(n)^{\frac{1}{2}}}{\log n}$$

For all $n \geq 2$.

• For this c, we show that the length of any crossing sequence of M for any input x in L with $|x| \to 2$ is at most c.

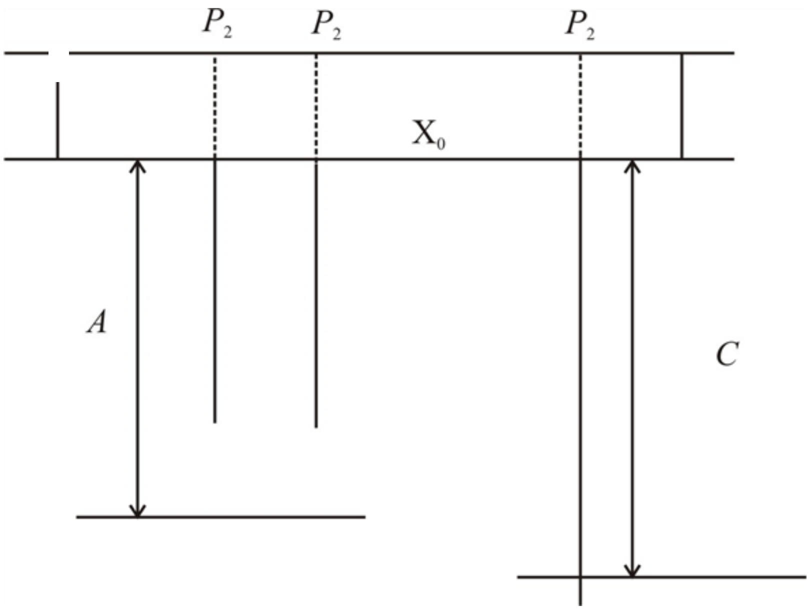
• Form this, it follows, that we can design a finite automaton that accepts L .

• Suppose that there is an x in L with $|x| \geq 2$ such that M generates a crossing sequence of length larger than c in accepting x .

• Let x_0 be the shortest such x, n_0 be its length, and P_1 be the position of one of such long crossing sequences.

Comment

Step 2 of 4



Figure

Comment

Step 3 of 4

In this figure, $A = \frac{\log n_0}{g(n_0)^{\frac{1}{2}}}$

- Suppose that x_0 was given to M.
- Let h be the number of positions in x_0 (excluding both ends)

$(\log n_0)/\left(g(n_0)^{\frac{1}{2}}\right)$

- Then we have

$$\frac{n\log n_0}{g(n_0)} = f(n_0) > c + (n_0 - 2 - h)\frac{\log n_0}{g(n_0)^{\frac{1}{2}}},$$

And hence

$$h \;>\; n_0 - 2 - \frac{n_0}{g(n_0)^{\frac{1}{2}}} + c\frac{g(n_0)^{\frac{1}{2}}}{\log n_0}$$

$$\geq \quad 3\frac{n_0^{(\log s)/g(n_0)^{\frac{1}{2}}+1}}{S-1} + 1$$

$$= \quad 3\frac{S^{(\log n_0)/g(n_0)^{\frac{1}{2}}+1}}{S-1} + 1.$$

- Moreover, there are at most $= \frac{S^{(\log n_0)/g(n_0)^{\frac{1}{2}}+1} - 1}{S-1}$ crossing sequences of lengths smaller then $(\log n_0)/g(n_0)^{\frac{1}{2}}$
- Hence, at least four positions in x_0 have an identical crossing sequence.
- At least two of them are different from P_1 and are on the some side of P_1.
- Let P_2, P_3 be these positions (see figure 1).
- Let $x_0^{\,1}$ is the word obtained form x_0 by deleting the sub word between P_2 *and* P_3.
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Comment

Step 4 of 4

Then, M accepts $x_0{}^1$, generating a crossing sequence of length larger than c for $x_0{}^1$ and $2 \le |x_0{}^1| < |x_0|$.

• This contradicts the selection of x_0.

Comment