Problem

Recall that string x is a ***prefix*** of string y if a string z exists where xz = y, and that x is a ***proper prefix*** of y if in addition x ≠ y. In each of the following parts, we define an operation on a language A. Show that the class of regular languages is closed under that operation.

A**a.** $NOPREFIX(A) = \{w \in A |\text{ no proper prefix of } w \text{ is a member of } A\}$.

b. $NOEXTEND(A) = \{w \in A |\ w \text{ is not the proper prefix of any string in } A\}$.

Step-by-step solution

Step 1 of 3

Consider the following information:

• String *x* is a prefix of string *y* if a string *z* exists such that *xz=y*.

• String *x* is a proper prefix of *y* if *xz=y* and $x \neq y$.

• The language *A* is regular language. Assume $M = (Q, \Sigma, \delta, q_0, F)$ be a DFA recognizing *A*.

Comment

Step 2 of 3

a.

$NOPREFIX(A) = \{w \in A \mid no\ proper\ prefix\ of\ w\ is\ a\ member\ of\ A\}$

1. Let $M = (Q, \Sigma, \delta, q_0, F)$ be a DFA recognizing *A*.

2. Initially, find all words that have a proper prefix in *A*. The language *L* is represented as $L = \{w \in \Sigma^* : x \in A\ and\ z \in \Sigma^*\ such\ that\ xz = y\}$.

3. Now, construct the NFA $M^1 = (Q^1, \Sigma, \delta^1, q_0^{\ 1}, F^1)$ for all its components such that:

• $Q^1 = Q \cup \{q_f\}$ and $q_f \notin Q$

• For $q \in Q^1$ and $a \in \Sigma$ define $\delta^1(q,a) = \begin{Bmatrix} \delta(r,a) & if\ r \notin F \\ \phi & if\ r \in F \end{Bmatrix}$

• $q_0^{\ 1} = q_0$

• $F^1 = q_f$

Proof:

• If *w* is a string in Language *L*, there is a string *y* in *A*. Here, *x* is a proper prefix of *y* such that *xz=y* and *x* is non-empty.

• If *w* is taken as input of M^1, the computation on *x* ends at an accepting state in *M* and some computation on *z* ends at state q_f.

• So *w* is accepted by M^1, which means that there is a computation that ends at q_f.

• From the construction of M^1, the computation arrives at one of the accepting states in *M* before it reaches q_f.

• If we conclude that String *x* is a proper prefix of *y*, *M* on input *x* ends in one of its accepting states. So, *w* is a member of *L*, and *x* is in A.

• As, *NOPREFIX*(*A*) is defined as $A \cap \overline{L}$ and class of regular languages are closed under intersection and complement, *NOPREFIX*(*A*) is also regular.

Comments (2)

b.

$NOEXTEND(A) = \{w \in A \mid w \text{ is not proper prefix of any string in } A\}$

- Let $M = (Q, \Sigma, \delta, q_0, F)$ be a DFA recognizing *A*.
- Assume that the DFA for language M accepts that only the strings reaching the final state but not those strings that are added to reach a final state again.
- So, the strings exactly ending in final states are accepted.
- For a state $q \in F$, check whether there is a path from $q \in Q$ to any state in *F* (or a cycle involving *q*) using Depth First Search.
- Let $F^1 \subseteq F$ be the set of all the states from which there is no such path.
- Now, changing the set of final states *F* to F^1 gives a DFA for *NOEXTEND* (*A*).
- Thus, *NOEXTEND(A)* is also regular.

Comment