

Problem

Give a formal definition of an enumerator. Consider it to be a type of two-tape Turing machine that uses its second tape as the printer. Include a definition of the enumerated language.

Step-by-step solution

Step 1 of 3

An enumerator is Turing machine(TM) variant with as attached printer. The enumerator uses the printer as output device to print strings. Every time Turing machine wants to add a string to the list, it sends the string to the printer.

Formal definition of an enumerator:

An Enumerator is a 7-tuple $\langle Q, \Sigma, \Gamma, \delta, q_0, q_{\text{print}}, q_{\text{halt}} \rangle$

Where Q, Σ, Γ are all finite sets and

- (i) Q is the set of states
- (ii) Γ is the work tape alphabet,
- (iii) Σ is the output/print tape alphabet
- (iv) $\delta: Q \times \Gamma \rightarrow Q \times \Gamma \times \{L, R\} \times \Sigma_{\epsilon}$ is the transition function,
- (v) $q_0 \in Q$ is the start state
- (vi) $q_{\text{print}} \in Q$ is the print state, and
- (vii) $q_{\text{halt}} \in Q$ is the halt state, where $q_{\text{print}} \neq q_{\text{halt}}$

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Step 2 of 3

Computation of an enumerator:

The computation of an enumerator E is defined as an ordinary TM, except for following three points.

1. It has two tapes,
 - First tape is work tape and
 - Second tape is print tape

Both are initially blank.

2. At each step, the machine may write a symbol from Σ on the output tape, or nothing, as determined by δ .

For example,

If $\delta(q_{\text{prev}}, a) = (q_{\text{next}}, b, L, c)$, it means that in state q_{prev} , reading a , enumerator E enters state q_{next} , writes b on the work tape moves the work tape head left and writes c on the print tape, and moves the print tape head to the right if $c \neq \epsilon$.

3. Whenever state q_{print} is entered, the output tape is reset to blank and the head returns to the left hand end and the output is printed.

When q_{halt} is entered the machine will halt.

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Step 3 of 3

Enumerated language:

The language enumerated by Enumerator is the collection of all the strings that Enumerator eventually prints out.

Enumerator may generate the strings of the language in any order, possibly with repetitions.

$L(E)$: be the language enumerated by enumerator E

Thus $L(E)$ is the language that contains the strings which are printed out.

$$L(E) = \{w \in \Sigma^* \mid w \text{ appears on the work tape if } q_{\text{print}} \text{ is entered}\}$$

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