Problem

We defined the *CUT* of language A to be *CUT*(A) = {yxzl xyz ? A}. Show that the class of CFLs is not closed under *CUT*.

Step-by-step solution

Step 1 of 4

A CFL language is closed under few operations. If L_1 and L_2 are two CFL languages then the following language will also be context free:

1. Cyclic shift of CFL
2. Union of both is CFL
3. The reversal of L_1
4. Concatenation of both languages
5. Kleen star of L_1
6. Image of L_1 under homomorphism
7. Image inverse of L_1 under inverse homomorphism

Comment

Step 2 of 4

Consider the language $CUT(A)$ is defined as $CUT(A) = \{yxz \mid xyz \in A\}$.

Consider $L_1 = x,\ L_2 = y,\ L_3 = z$.

If the CFL $CUT(A)$ is closed under above operation then all conditions must be true. For showing it is not closed it will be sufficient to prove that one condition does not hold.

Comment

Step 3 of 4

Cyclic shift of CFL:

Consider $L_1 \circ L_2$ exist in CFL, then it there must be Cyclic shift exist in the definition of language that is $L_2 \circ L_1$ must exist.

Consider a string $s = xyz$ in the language $CUT(A)$. It can be formed from the concatenation of the languages X and Y and Z, such that $x \in X$ and $y \in Y$ and $z \in Z$.

As per the definition of language $CUT(A)$, if $yxz \in A \text{ then } xyz \in A$. For given definition it will be closed if cyclic shift is also CFL. That is, if $xy \in A$ then $yx \in A$. But given condition is not true as definition of $CUT(A)$.

Comments (1)

Step 4 of 4

The Cyclic shift of CFL which is first condition does not follow. Hence, given CFL cannot be closed under $CUT(A) = \{yxz \mid xyz \in A\}$.

Comment