Problem

Show that if A $\leq_T$ B and B $\leq_T$ C, then A $\leq_T$ C.

Step-by-step solution

Step 1 of 1

Definition of $A \leq_T B$:

Language *A* is Turing reducible to Language *B*, written as $A \leq_T B$, if *A* is decidable relative to *B*. That is the oracle for language *B* decides Language *A*.

Given that

- $A \leq_T B$

That is, Let the oracle M_1^B for Language *B* decides the Language *A*. and

- $B \leq_T C$

That is, Let the oracle M_2^C for Language *C* decides the Language *B*.

We have to prove that

- $A \leq_T C$

That is, there exists an oracle M_3^C for Language *C* which decides the Language *A*.

That means, machine M_3 have to simulate machine M_1.

We will explain this simulation in detail as follows

- Let M_1 queries an oracle about some String *x*.
- M_3 does not have an oracle for *B*. M_3
- So M_3 does not perform the test whether $x \in B$ or not directly.
- Thus M_3 first simulates M_2 on input *x* and get the result.
- Then M_3 provides that answers to M_1.
- But the queries of machine M_2 are directly answered by M_3. Because M_3 and M_2 use same oracle *C*.

In this way the oracle for Language *C* decides Language *A*. That is $A \leq_T C$.

Thus If $A \leq_T B$ and $B \leq_T C$ then $A \leq_T C$.

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