2023

Full Marks : 75

Time : 3 hours

Answer from **both** the Groups as directed.

The figures in the right-hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP—A

(Compulsory)

1. Answer *all* the questions: 1×5

(*a*) Suppose the set A has m elements and the set B has n elements. Then the number of different relations from A to B is

(*i*) 2^{m+n}

(*ii*) $m+n$

(*iii*) mn

(*iv*) 2^{mn}

(*b*) In the poset $P=\{2,3,6,12,24,36\}$ with divisibility relation, the minimal elements are

(*i*) 2, 3

(*ii*) 2, 6

(*iii*) 3, 6

(*iv*) 3, 12

(*c*) Let the truth values of p and q be true (T). Then

(*i*) $p \wedge q = F$

(*ii*) $(p \rightarrow q) = F$

(*iii*) $p \vee q = F$

(*iv*) $(p \wedge q) = F$

(*d*) Let $A=\{1,2,3\}$ and consider the relation $R=\{(1,1),(2,2),(3,3),(1,2),(2,3),(1,3)\}$. Then R is

(*i*) Reflexive and transitive but not symmetric

(*ii*) Reflexive but not transitive

(*iii*) Symmetric and transitive

(*iv*) Reflexive and symmetric

(*e*) Let $R=\{(2,a),(4,a),(4,b)\}$ be a relation from set $A=\{1,2,3,4\}$ to set $B=\{a,b,c\}$. Then domain of R is

(*i*) $\{a,c\}$

(*ii*) $\{1,2,4\}$

(*iii*) $\{2,4\}$

(*iv*) $\{a,b\}$

2. Define equivalence and partial order relations on a non-empty set A. 5

3. Let S be a non-empty set and $P(S)$ be its power set. Then show that the partially ordered set $(P(S), \leq)$ is a lattice where $A \leq B \Rightarrow A$ is a subset of B. 5

GROUP—B

Answer any *four* questions: 15 × 4

4. Let $A=\{2,3,4,6,8,12,24,48\}$ and '$\leq$' be the partial order relation of divisibility. Let $B=\{4,6,12\}$ be a subset of A. Then find

   (*a*) Hasse diagram of the poset $(A, \leq)$.

   (*b*) All upper bounds of B

   (*c*) All lower bounds of B

   (*d*) The least upper bound of B

   (*e*) The greatest lower bound of B

5. Show that each of the following is a tautology:

   (*a*) $[p \wedge (p \to q)] \to q$

   (*b*) $[(p \to q) \wedge (q \to r)] \to (p \to r)$

6. Show that

   (*a*) $p \to (q \to r) = (p \wedge q) \to r$

   (*b*) $(p \leftrightarrow q) = (p \to q) \wedge (q \to p)$

7. (*a*) Define relation and function on a non-empty set.

   (*b*) Write difference between relations and functions by giving an example.

8. Define partition of a set. State and prove fundamental theorem on equivalence relation.

9. (*a*) Define tautology and contradiction.

   (*b*) Show that

   $\sim(p \wedge q) \to (\sim p \vee (\sim p \vee q)) \equiv \sim p \vee q$

   without constructing truth table.