

2022

Full Marks : 75

Time : 3 hours

Answer from both the Groups as directed

*The figures in the right-hand margin indicate marks**Candidates are required to answer in their own words as far as practicable*

## GROUP – A

Answer all questions

## Unit – I

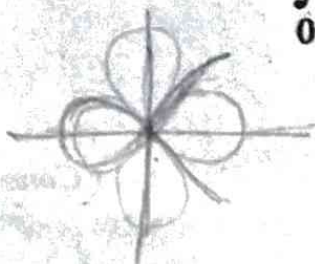
1. Objective type questions :

1 × 5

(i) The value of the integral

$$\int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\cos x} + \sqrt{\sin x}} dx \text{ is}$$

$$2I = \frac{\pi}{4}$$



$$\int_0^{\pi/2}$$

( Turn Over )

( 2 )

- (a)  $\frac{\pi}{2}$   
(b)  $\frac{\pi}{4}$   
(c)  $\frac{\pi}{8}$   
(d) 0
- (ii) If  $y = x^4$  then its 4th derivative ( $y_4$ ) is  
(a) 24  
(b)  $12x^3$   
(c)  $24x$   
(d)  $4x$
- (iii) The value of  $\lim_{x \rightarrow \infty} \frac{\sin x}{x}$  is  
(a) 0  
(b) 1  
(c) -1  
(d) 2

1st Sem-M (MN-1A)

(Continued)

( 3 )

- (iv) Which of the following functions  $f: \mathbb{R} \rightarrow \mathbb{R}$  is one-one?  
(a)  $f(x) = x^3$   
(b)  $f(x) = x^4$   
(c)  $f(x) = x^6$   
(d)  $f(x) = \sin x$
- (v) The area of the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  is  
(a)  $\pi ab$   
(b)  $\pi a^2 b$   
(c)  $4\pi ab$   
(d)  $16\pi ab$

Unit - II

( Short answer type questions )

2. If  $I_n = \int_0^{\pi/4} \tan^n x \, dx$  then show that  
 $I_n + I_{n-2} = \frac{1}{n-1}$ .

1st Sem-M (MN-1A)

( Turn Over )



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( 4 )

3. Trace the curve  $x^3 + y^3 = 3axy$ . 5

GROUP - B

( Long Answer Type Questions )

Answer any four questions : 15 × 4

4. (a) Find the entire length of the astroid  $x^{2/3} + y^{2/3} = a^{2/3}$ .

- (b) Find the asymptotes of the curve  $4x^3 - 3xy^2 - y^3 + 2x^2 - xy - y^2 - 1 = 0$ .

5. (a) Show that the area bounded by the curves  $y^2 = 4ax$  and  $x^2 = 4ay$  is  $\frac{16}{3} a^2$  square units.

- (b) Find 'C' of the Lagrange's mean value theorem, if  $f(x) = x^2 - 3x - 1$ ,

$x \in \left[ -\frac{11}{7}, \frac{13}{7} \right]$



( 5 )

6. (a) If is given that the Rolle's Theorem holds good for the function

$f(x) = x^3 + ax^2 + bx, x \in [1, 2]$

at the point  $x = \frac{4}{3}$ . Then find the value of pair  $(a, b)$ .

- (b) Evaluate  $\int \int e^{2x+3y} dx dy$  over the triangle bounded by the lines  $x = 0, y = 0, x + y = 1$ .

7. (a) Evaluate the integral  $\int_0^{\pi/2} \sin^n x dx$ .

- (b) Show that the total length of the cardioid  $r = a(1 - \cos\theta)$  is  $8a$ .

8. (a) If  $y = (\sin^{-1}x)^2$  then show that

(i)  $(1 - x^2)y_2 - xy_1 - 2 = 0$ .

(ii)  $(1 - x^2)y_{n+2} - (2n+1)xy_{n+1} - n^2y_n = 0$



where  $y_n$  denotes the  $n$ th differential co-efficient of  $y$ .

- (b) Find the volume of the solid generated by the revolution of  $y^2 = \frac{x^3}{a-x}$  about its asymptote.

9. (a) Evaluate the following limits :

(i)  $\lim_{x \rightarrow 0} \left( \frac{\tan x}{x} \right)^{1/x^2}$

(ii)  $\lim_{x \rightarrow 0} \frac{(1+x)^{1/x} - e}{x}$

(b) Let  $f(x) = x \left( \frac{e^{\frac{1}{x}} - e^{-\frac{1}{x}}}{e^{\frac{1}{x}} + e^{-\frac{1}{x}}} \right)$ ,  $x \neq 0$ ;  $f(0) = 0$ .

show that  $f$  is continuous but not differentiable at  $x = 0$ .

