

UPDAAN

2025

Quadratic Equation

Mathematics

Lecture - 04

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Topics

to be covered



Completing the Square.

✓ Nature of roots of a quadratic equation
(Continued)

✓ Badhiya Questions





WORK HARD
DREAM BIG
NEVER GIVE UP !!



Topic : Miscellaneous



#Q. Which of the following equations has the sum of its roots as 3?

A $2x^2 - 3x + 6 = 0$

B $-x^2 + 3x - 3 = 0$

C $\sqrt{2}x^2 - \frac{3}{\sqrt{2}}x + 1 = 0$

D $3x^2 - 3x + 3 = 0$

$$\alpha + \beta = -\frac{b}{a}$$

sum of roots

$$-\frac{b}{a} = -\frac{-3}{2}$$

$$-\frac{b}{a} = 1$$

$$-\frac{b}{a} = -\frac{-\frac{3}{\sqrt{2}}}{\sqrt{2}} = \frac{3}{2}$$

Topic : Miscellaneous



#Q. If one root of the equation $(k-1)x^2 - 10x + 3 = 0$ is the reciprocal of the other, then find the value of k . [CBSE SQP, 2020]

Let roots be α and $\frac{1}{\alpha}$.

$$ax^2 + bx + c = 0$$

$$\begin{aligned} a &= k-1 \\ b &= -10 \\ c &= 3 \end{aligned}$$

Product of roots = $\frac{c}{a}$

$$\cancel{\alpha} \times \frac{1}{\cancel{\alpha}} = \frac{c}{a}$$

$$1 = \frac{c}{a}$$

$$a = c$$

$$\begin{aligned} k-1 &= 3 \\ k &= 4 \end{aligned}$$



$D > 0$
 $D = +ve$  Real and distinct:

$D = 0$  Real and equal.

$D < 0$
 $D = -ve$  no real roots.

Topic : Nature of Roots



#Q. Find the values of k for which the given equation has real and equal roots:

$$2x^2 - 10x + k = 0$$

[CBSE 2002 C]

$$D=0$$

$$ax^2 + bx + c = 0$$

$$a=2, b=-10, c=k$$

$$D = b^2 - 4ac$$

$$= (-10)^2 - 4(2)(k)$$

$$D = 100 - 8k$$

$$0 = 100 - 8k$$

$$8k = 100$$

$$k = \frac{100}{8} = \frac{25}{2}$$

Topic : Nature of Roots



#Q. Find the values of k for which the following equation has equal roots:

$$(k-12)x^2 + 2(k-12)x + 2 = 0$$

$$a = k-12, b = 2(k-12), c = 2$$

$$D = 0$$

$$D = b^2 - 4ac$$

$$0 = [2(k-12)]^2 - 4(k-12)(2)$$

$$0 = 4(k-12)^2 - 8(k-12)$$

$$0 = 4(k^2 + 144 - 24k) - 8k + 96$$

$$0 = 4k^2 + 576 - 96k - 8k + 96$$

$$4k^2 - 104k + 672 = 0$$

$$4(k^2 - 26k + 168) = 0$$

$$k^2 - 26k + 168 = 0$$

$$P = 168, S = -26$$

$$-14, -12$$



$$k^2 - 26k + 168 = 0$$

$$k^2 - 14k - 12k + 168 = 0$$

$$k[k - 14] - 12[k - 14] = 0$$

$$(k - 14)(k - 12) = 0$$

✓ $k = 14$
 $k = 12$ ✗ neglect

Topic : Nature of Roots



#Q. If -4 is a roots of the quadratic equation $x^2 + px - 4 = 0$ and the quadratic equation $x^2 + px + k = 0$ has equal roots, find the value of k.

$$x^2 + px - 4 = 0$$

$$(-4)^2 + p(-4) - 4 = 0$$

$$16 - 4p - 4 = 0$$

$$12 = 4p$$

$$3 = p$$

$$x^2 + px + k = 0$$

$$x^2 + 3x + k = 0$$

equal roots.

$$D = 0$$

$$D = b^2 - 4ac$$

$$D = (3)^2 - 4(1)(k)$$

$$0 = 9 - 4k$$

$$4k = 9$$

$$k = 9/4$$

Topic : Nature of Roots



#Q. If -5 is a root of the quadratic equation $2x^2 + px - 15 = 0$ and the quadratic equation $p(x^2 + x) + k = 0$ has equal roots, find the value of k .

[CBSE 2022, 2009]

$$2x^2 + px - 15 = 0$$

$$2(-5)^2 + p(-5) - 15 = 0$$

$$\text{So } -5p - 15 = 0$$

$$35 - 5p = 0$$

$$35 = 5p$$

$$7 = p$$

$$p(x^2 + x) + k = 0$$

$$px^2 + px + k = 0$$

$$7x^2 + 7x + k = 0$$

$$\downarrow$$
$$D = 0$$

$$D = b^2 - 4ac$$

$$0 = (7)^2 - 4(7)(k)$$

$$0 = 49 - 28k$$

$$28k = 49$$

$$k = 49/28$$

$$k = 7/4$$

Topic : Nature of Roots



#Q. Prove that the equation $x^2(a^2 + b^2) + 2x(ac + bd) + (c^2 + d^2) = 0$ has no real roots, if $ad \neq bc$

$$x^2(a^2 + b^2) + 2x(ac + bd) + c^2 + d^2 = 0$$

$$a = a^2 + b^2, \quad b = 2(ac + bd), \quad c = c^2 + d^2$$

$$D = b^2 - 4ac$$

$$D = [2(ac + bd)]^2 - 4(a^2 + b^2)(c^2 + d^2)$$

$$= 4(ac + bd)^2 - 4(a^2c^2 + a^2d^2 + b^2c^2 + b^2d^2)$$

$$= 4(a^2c^2 + b^2d^2 + 2acbd - a^2c^2 - a^2d^2 - b^2c^2 - b^2d^2)$$

$$= \cancel{4a^2c^2} + \cancel{4b^2d^2} + 8acbd - \cancel{4a^2c^2} - \cancel{4a^2d^2} - \cancel{4b^2c^2} - \cancel{4b^2d^2}$$

$$D = -4a^2d^2 - 4b^2c^2 + 8acbd.$$

$$D = -4(a^2d^2 + b^2c^2 - 2adbc)$$

$$D = -4(ad - bc)^2$$

$$= 4(ad - bc)^2$$

always positive.

$$D = -4(ad - bc)^2$$

$$D = (-)(+)$$

$$D = -ve$$

no real roots

$ad - bc$ \rightarrow -
 \rightarrow +
 \rightarrow 0 ~~X~~ (since $ad \neq bc$)

Topic : Nature of Roots



#Q. If the roots of the equation $(a^2 + b^2)x^2 - 2(ac + bd)x + (c^2 + d^2) = 0$ are equal, prove that $\frac{a}{b} = \frac{c}{d}$.

$$D=0$$

$$D = b^2 - 4ac$$

$$D = -4(ad - bc)^2$$

$$0 = -4(ad - bc)^2$$

$$0 = (ad - bc)^2$$

From P.O

$$ad - bc = \pm \sqrt{0}$$

$$ad - bc = 0$$

$$ad = bc$$

$$\frac{a}{b} = \frac{c}{d}$$

H.P//

Topic : Nature of Roots



#Q. If the roots of the equation $(b - c)x^2 + (c - a)x + (a - b) = 0$ are equal, then prove that $2b = a + c$. [HOTS, CBSE 2002 C]

H.W

Hint: $(a+b+c)^2$
 $= a^2 + b^2 + c^2$
 $+ 2ab + 2bc$
 $+ 2ca$

A cartoon illustration of a student with orange hair wearing a black graduation cap and gown, standing on a purple book. Behind them is a green and blue globe.

Topic : Solution of a Quadratic Equation By Completing the Square

→ Deleted from CBSE

Topic : Completing the Square



#Q. Solve the quadratic equation $9x^2 - 15x - 6 = 0$ by the method of completing the square. [NCERT]

$$\cancel{9}x^2 - \cancel{15}x - 6 = 0$$

$$x^2 - \frac{5}{3}x - \frac{2}{3} = 0$$

$$x^2 - \frac{5}{3}x + \frac{25}{36} - \frac{25}{36} - \frac{2}{3} = 0$$

$$\left(x - \frac{5}{6}\right)^2 = \frac{25}{36} + \frac{2}{3}$$

$$\left(x - \frac{5}{6}\right)^2 = \frac{25 + 24}{36}$$

$$\left(x - \frac{5}{6}\right)^2 = \frac{49}{36}$$

$$x - \frac{5}{6} = \pm \sqrt{\frac{49}{36}}$$

$$x - \frac{5}{6} = \pm \frac{7}{6}$$

$$x = \frac{7}{6} + \frac{5}{6} = \frac{12}{6} = \textcircled{2}$$

$$x = -\frac{7}{6} + \frac{5}{6} = -\frac{2}{6} = \textcircled{-\frac{1}{3}}$$

Topic : Completing the Square



#Q. Solve the equation $2x^2 - 5x + 3 = 0$ by the method of completing square.

[NCERT]

$$\frac{2x^2}{2} - \frac{5x}{2} + \frac{3}{2} = 0$$

$$x^2 - \frac{5}{2}x + \frac{3}{2} = 0$$

$$x^2 - \frac{5}{2}x + \frac{25}{16} - \frac{25}{16} + \frac{3}{2} = 0$$

$$\left(x - \frac{5}{4}\right)^2 = \frac{25}{16} - \frac{3}{2}$$

$$\left(x - \frac{5}{4}\right)^2 = \frac{25 - 24}{16}$$

$$\left(x - \frac{5}{4}\right)^2 = \frac{1}{16}$$

$$x - \frac{5}{4} = \pm \sqrt{\frac{1}{16}}$$

$$x - \frac{5}{4} = \pm \frac{1}{4}$$

$$x = \frac{1}{4} + \frac{5}{4} = \frac{3}{2}$$

$$x = -\frac{1}{4} + \frac{5}{4} = 1$$

$$\left(\frac{5}{2}\right)^2 = \frac{25}{4}$$
$$\frac{25}{4} \times \frac{1}{4} = \frac{25}{16}$$

Topic : Completing the Square



#Q. By using the method of completing the square, show that the equation $4x^2 + 3x + 5 = 0$ has no real roots [NCERT]

$$4x^2 + 3x + 5 = 0$$

$$x^2 + \frac{3x}{4} + \frac{5}{4} = 0$$

$$x^2 + \frac{3x}{4} + \frac{9}{64} - \frac{9}{64} + \frac{5}{4} = 0$$

$$\left(x + \frac{3}{8}\right)^2 = \frac{9}{64} - \frac{5}{4}$$

$$\left(x + \frac{3}{8}\right)^2 = \frac{9 - 80}{64}$$

$$\left(x + \frac{3}{8}\right)^2 = -\frac{71}{64}$$

$$\left(x + \frac{3}{8}\right) = \pm \sqrt{-71/64}$$

this eqn will have no real roots.

Proof of Shreedharacharya Formula



$$ax^2 + bx + c = 0$$

$$\overline{a} \quad \overline{a} \quad \overline{a} \quad \overline{a}$$

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$

$$\left(x^2 + \frac{b}{a}x + \frac{b^2}{4a^2}\right) - \frac{b^2}{4a^2} + \frac{c}{a} = 0$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2}{4a^2} - \frac{c}{a}$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$

$$\left(x + \frac{b}{2a}\right) = \pm \sqrt{\frac{b^2 - 4ac}{4a^2}}$$

$$x + \frac{b}{2a} = \pm \frac{\sqrt{b^2 - 4ac}}{\sqrt{4a^2}}$$

$$x + \frac{b}{2a} = \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$x + \frac{b}{2a} = \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad D//$$

Topic : From the question bank



#Q. Find the values of p so that the equation $x^2 + 4px + p^2 - p + 2 = 0$ has equal roots.

H.w

Topic : From the question bank



#Q. Given, solve for x :

$$\left(\frac{2x}{x-5}\right)^2 + 5\left(\frac{2x}{x-5}\right) - 24 = 0, x \neq 5$$

H.w



Homework



DPP



THANK
YOU

