Foundation

UDAAN 2025

MATHS

Trigonometry

DHA : 05

Do Again Very Important DHA

Q1 Express the ratios cosec***B***, cot***B*** in terms of tan***B***.

Q2 Express the ratios cos A, tan A and sec A in terms of sin A.

Q3 Prove that sec A(1 – sin A)(sec A + tan A) = 1.

Q4 Prove that $\frac{\cot A-\cos A}{\cot A+\cos A}=\frac{\operatorname{cosec} A-1}{\operatorname{cosec} A+1}$

Q5 Prove that $\frac{\sin \theta-\cos \theta+1}{\sin \theta+\cos \theta-1}=\frac{1}{\sec \theta-\tan \theta}$, using the identity $\sec^2 \theta=1+\tan^2 \theta$

Q6 Prove the following trigonometric identity: $\frac{\sin^3 \theta+\cos^3 \theta}{\sin \theta+\cos \theta}=1-\sin \theta \cos \theta$

Q7 Prove the following trigonometric identity: $\frac{\cos^2 \theta}{(\cos \theta-\sin \theta)}+\frac{\sin^2 \theta}{(\sin \theta-\cos \theta)}=(\cos \theta+\sin \theta)$.

Q8 Prove the following: $\sqrt{\frac{1+\sin \theta}{1-\sin \theta}}=\frac{1}{\sec \theta-\tan \theta}$

Q9 Prove the following trigonometric identity: $\sqrt{\frac{1-\cos \theta}{1+\cos \theta}}=\frac{1}{cosec\theta+\cot \theta}$

Q10 If $1+\sin^2 \theta=3 \sin \theta \cos \theta$, prove that $\tan \theta=1$ or $\frac{1}{2}$.

Answer Key

Q1 $\cot B=\frac{1}{\tan B}$

$\operatorname{cosec} B=\frac{\sqrt{\tan ^2 B+1}}{\tan B}$

Q2 $\cos \mathrm{A}=\sqrt{1-\sin ^2 \mathrm{~A}}$

$\tan \mathrm{A}=\frac{\sin \mathrm{A}}{\sqrt{1-\sin ^2 \mathrm{~A}}}$ and $\sec \mathrm{A}=\frac{1}{\sqrt{1-\sin ^2 \mathrm{~A}}}$

Q3 **sec A(1 – sin A)(sec A + tan A) = 1**

Q4 $\frac{\cot A-\cos A}{\cot A+\cos A}=\frac{\operatorname{cosec} A-1}{\operatorname{cosec} A+1}$

Q5 $\frac{\sin \theta-\cos \theta+1}{\sin \theta+\cos \theta-1}=\frac{1}{\sec \theta-\tan \theta}$

Q6 $\frac{\sin ^3 \theta+\cos ^3 \theta}{\sin \theta+\cos \theta}=1-\sin \theta \cos \theta$

Q7 $\frac{\cos ^2 \theta}{(\cos \theta-\sin \theta)}+\frac{\sin ^2 \theta}{(\sin \theta-\cos \theta)}=(\cos \theta+\sin \theta)$

Q8 $\sqrt{\frac{1+\sin \theta}{1-\sin \theta}}=\frac{1}{\sec \theta-\tan \theta}$

Q9 $\sqrt{\frac{1-\cos \theta}{1+\cos \theta}}=\frac{1}{\operatorname{cosec} \theta+\cot \theta}$

Q10 $\tan \theta=1$ or $\frac{1}{2}$

Hints & Solutions

Q1 Text Solution:

$\cot B=\frac{1}{\tan B}$ $\quad\left(\because \cot\theta=\frac{1}{\tan\theta}\right)$

$\operatorname{cosec}^2 B=1+\cot^2 B$

$\left(\therefore 1+\cot^2\theta=\operatorname{cosec}^2\theta\right)$

$\operatorname{cosec}^2 B=1+\frac{1}{\tan^2 B}$

$\operatorname{cosec}^2 B=\frac{\tan^2 B+1}{\tan^2 B}$

$\operatorname{cosec} B=\sqrt{\frac{\tan^2 B+1}{\tan^2 B}}$

$\operatorname{cosec} B=\frac{\sqrt{\tan^2 B+1}}{\tan B}$

Video Solution:

Q2 Text Solution:

Since

$\cos^2 \mathrm{~A}+\sin^2 \mathrm{~A}=1$, therefore,

$\cos^2 A=1-\sin^2 A$, i.e., $\cos A=\pm\sqrt{1-\sin^2 A}$

This gives $\quad \cos\mathrm{A}=\sqrt{1-\sin^2 \mathrm{~A}}$

Hence,

$\tan\mathrm{A}=\frac{\sin\mathrm{A}}{\cos\mathrm{A}}=\frac{\sin\mathrm{A}}{\sqrt{1-\sin^2 \mathrm{~A}}}$ and $\sec\mathrm{A}=\frac{1}{\cos\mathrm{A}}$

$=\frac{1}{\sqrt{1-\sin^2 \mathrm{~A}}}$

Video Solution:

Q3 Text Solution:

Taking LHS

$=\sec\mathrm{A}\left(1-\sin\mathrm{A}\right)\left(\sec\mathrm{A}+\tan\mathrm{A}\right)$

$=\left(\frac{1}{\cos A}\right)\left(1-\sin\mathrm{A}\right)\left(\frac{1}{\cos A}+\frac{\sin A}{\cos A}\right)$

$=\frac{(1-\sin\mathrm{A})(1+\sin\mathrm{A})}{\cos^2 \mathrm{~A}}$

$=\frac{1-\sin^2 \mathrm{~A}}{\cos^2 \mathrm{~A}}$

$=\frac{\cos^2 \mathrm{~A}}{\cos^2 \mathrm{~A}}$

$=1--------\left(i\right)$

Taking RHS

$1--------(ii)$

From eq(i) & eq(ii)

LHS = RHS

hence proved.

Video Solution:

Q4 Text Solution:

Taking LHS

$=\frac{\cot A-\cos A}{\cot A+\cos A}$

$=\frac{\frac{\cos A}{\sin A}-\cos A}{\frac{\cos A}{\sin A}+\cos A}$

$=\frac{\cos A\left(\frac{1}{\sin A}-1\right)}{\cos A\left(\frac{1}{\sin A}+1\right)}$

$=\frac{\left(\frac{1}{\sin A}-1\right)}{\left(\frac{1}{\sin A}+1\right)}$

$=\frac{\operatorname{cosec} A-1}{\operatorname{cosec} A+1}-------\left(i\right)$

Taking RHS

$=\frac{\operatorname{cosec} A-1}{\operatorname{cosec} A+1}------\left(ii\right)$

From eq $\left(i\right)$ *& eq* $\left(ii\right)$

LHS = RHS

Hence Proved.

Video Solution:

Q5 Text Solution:

Since we well apply the identity involving sec θ and tan θ, let us first ocnvert the LHS (of the idenitty we need to prove) in terms of sec θ and tan by dividing numerator and denominator by cos θ.

Taking LHS

$$= \frac{\sin\theta - \cos\theta + 1}{\sin\theta + \cos\theta - 1}$$

$$= \frac{\tan\theta - 1 + \sec\theta}{\tan\theta + 1 - \sec\theta}$$

$$= \frac{(\tan\theta + \sec\theta) - 1}{(\tan\theta - \sec\theta) + 1}$$

$$= \frac{\{(\tan\theta + \sec\theta) - 1\}(\tan\theta - \sec\theta)}{\{(\tan\theta - \sec\theta) + 1\}(\tan\theta - \sec\theta)}$$

$$= \frac{(\tan^2\theta - \sec^2\theta) - (\tan\theta - \sec\theta)}{\{\tan\theta - \sec\theta + 1\}(\tan\theta - \sec\theta)}$$

$$= \frac{-1 - \tan\theta + \sec\theta}{(\tan\theta - \sec\theta + 1)(\tan\theta - \sec\theta)}$$

$$= \frac{-1}{\tan\theta - \sec\theta} = \frac{1}{\sec\theta - \tan\theta},$$

which is the RHS of the identity, we are required to prove.

Video Solution:

Q6 Text Solution:

We have

$$\text{LHS} = \frac{\sin^3\theta + \cos^3\theta}{\sin\theta + \cos\theta}$$

$$= \frac{(\sin\theta + \cos\theta)(\sin^2\theta + \cos^2\theta - \sin\theta\cos\theta)}{(\sin\theta + \cos\theta)}$$

$$\left[\because \left(a^3 + b^3\right) = (a + b)\left(a^2 + b^2 - ab\right)\right]$$

$$\left[\because \sin^2\theta + \cos^2\theta\right]$$

$$= \frac{(1 - \sin\theta\cos\theta)(\sin\theta + \cos\theta)}{(\sin\theta + \cos\theta)}$$

Taking RHS

$$= [1 - \sin\theta\cos\theta]$$

$$\therefore \text{LHS} = \text{RHS}$$

Hence Proved

Video Solution:

Q7 Text Solution:

Taking LHS

$$= \frac{\cos^2\theta}{(\cos\theta - \sin\theta)} + \frac{\sin^2\theta}{(\sin\theta - \cos\theta)}$$

$$= \frac{\cos^2\theta}{(\cos\theta - \sin\theta)} - \frac{\sin^2\theta}{(\cos\theta - \sin\theta)}$$

$$= \frac{\cos^2\theta - \sin^2\theta}{(\cos\theta - \sin\theta)} = \frac{(\cos\theta - \sin\theta)(\cos\theta + \sin\theta)}{(\cos\theta - \sin\theta)}$$

$$= (\cos\theta + \sin\theta)$$

$$Taking\ \text{RHS}$$

$$= (\cos\theta + \sin\theta)$$

$$\therefore LHS = RHS$$

$$Hence\ Proved.$$

Video Solution:

Q8 Text Solution:

$$Taking\ LHS$$ (Multiplying and

$$= \sqrt{\frac{(1 + \sin\theta)(1 + \sin\theta)}{(1 - \sin\theta)(1 + \sin\theta)}}$$

dividing by (1 + sinθ))

$$= \sqrt{\frac{(1 + \sin\theta)^2}{1^2 - \sin^2\theta}}$$

$$= \frac{1 + \sin\theta}{\cos\theta}$$

$$= \frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta}$$

$$= \sin\theta + \tan\theta$$

$$\left(\because \sec^2\theta - \tan^2\theta = 1\right)$$

$$= \frac{1}{\sec\theta - \tan\theta} - - - - - eq\left(i\right)$$

$$Taking\ RHS$$

$$== \frac{1}{\sec\theta - \tan\theta} - - - - - eq\left(ii\right)$$

$$From\ eq\left(i\right)\ \&\ eq\left(ii\right)$$

$$LHS = RHS$$

Video Solution:

Q9 **Text Solution:**

$Taking\ LHS$

$= \sqrt{\frac{(1-\cos\theta)}{(1+\cos\theta)}}$

$= \sqrt{\frac{(1-\cos\theta)(1-\cos\theta)}{(1+\cos\theta)(1-\cos\theta)}}$

$= \sqrt{\frac{(1-\cos\theta)^2}{1^2-\cos^2\theta}}$

$= \frac{1-\cos\theta}{\sin\theta}$

$= \frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}$

$= cosec\theta$

$- \cot\theta \quad (\because 1 + \cot^2\theta - cosec^2\theta)$

$= \frac{1}{\cos ec\theta + \cot\theta} - - - \left(i\right)$

$Taking\ RHS$

$= \frac{1}{\cos ec\theta + \cot\theta} - - - \left(ii\right)$

$From\ eq\left(i\right)\ \&\ \left(ii\right)$

$LHS = RHS$

Video Solution:

Q10 **Text Solution:**

Given 1 + $\sin^2\theta \ = \ 3\sin\theta\cos\theta$

Dividing both sides by $\cos^2\theta$, we get

$\frac{1}{\cos^2\theta} + \frac{\sin^2\theta}{\cos^2\theta} = 3\frac{\sin\theta\cos\theta}{\cos^2\theta}$

$\Rightarrow \sec^2\theta + \tan^2\theta = 3\tan\theta$

$\Rightarrow \left(1 + \tan^2\theta\right) + \tan^2\theta = 3\tan\theta$

$\Rightarrow 2\tan^2\theta - 3\tan\theta + 1 = 0$

$\Rightarrow 2\tan^2\theta - 2\tan\theta - \tan\theta + 1 = 0$

$\Rightarrow 2\tan\theta\left(\tan\theta - 1\right) - 1\left(\tan\theta - 1\right) = 0$

$\Rightarrow \left(\tan\theta - 1\right)\left(2\tan\theta - 1\right) = 0 \Rightarrow \tan\theta$

$- 1 = 0 \text{ or } 2\tan\theta - 1 = 0$

$\Rightarrow \tan\theta = 1 \text{ or } \tan\theta = \frac{1}{2}$

$Hence\ Proved.$

Video Solution:

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