UDAAN 2024

– FOR CLASS 10th STUDENTS

Lecture No.- 06

- Subject Name- Mathematics
- Chapter Name- Surface Area and Volume

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Topic to be Covered

Topic

More Questions on Frustum of a Right Circular Cone

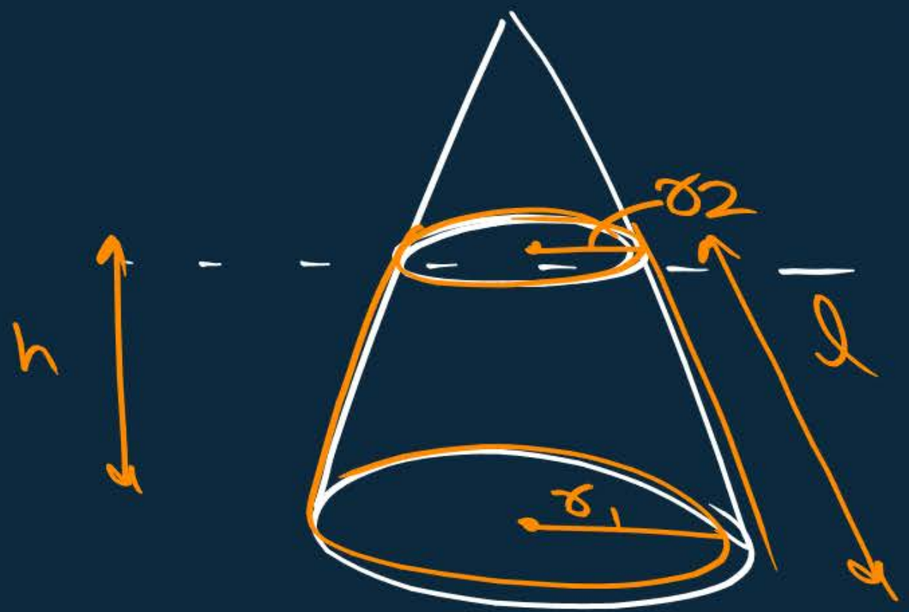
Frustum of a cone.

$$V = \frac{1}{3}\pi h\left[r_1^2 + r_2^2 + r_1 r_2\right]$$

$$C.S.A = \pi(r_1 + r_2)l$$

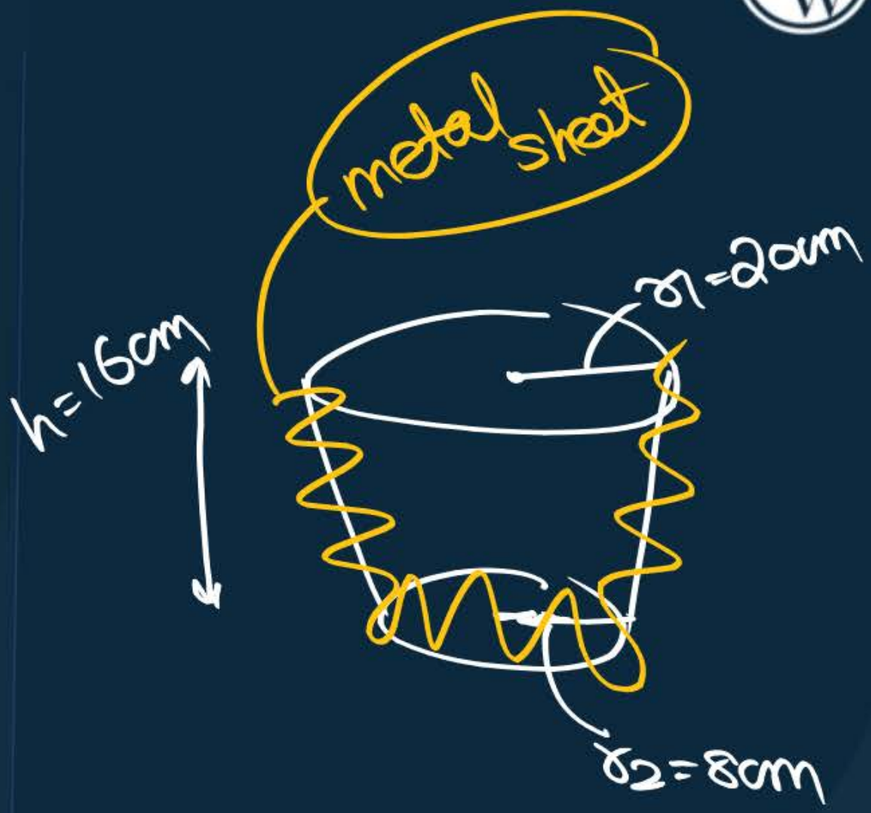
$$T.S.A = \pi r_2^2 + \pi r_1^2 + \pi(r_1 + r_2)l$$

$$l = \sqrt{(r_1 - r_2)^2 + h^2}$$



Topic : Frustum of A Right Circular Cone

#Q. A container, open from the top, made up of a metal sheet is in the form of a frustum of a cone of height 16 cm with radii of its lower and upper ends as 8 cm and 20 cm respectively. Find the cost of milk which can completely fill the container at the rate of ₹15 per litre and the cost of metal sheet used, if the cost ₹ 5 per 100 cm^2. (use $\pi = 3.14$) **[CBSE 2008, 2014, 2016]**



Rate = Rs 15/l

1 l = 15 Rs

Capacity of the container = V. of Foustum

$$= \frac{1}{3}\pi h\left(r_1^2 + r_2^2 + r_1 r_2\right)$$

$$= \frac{1}{3} \times \frac{314}{100} \times 16 \left[(20)^2 + (8)^2 + (20)(8)\right]$$

$$= \frac{1256}{75}\left[400 + 64 + 160\right]$$

$$V = \frac{1256}{\cancel{25}\ 25} \times \cancel{624}^{208}$$

$$V = \frac{261248 \times 4}{25 \times 4}$$

$$\boxed{V = 10449.92\ cm^3}$$

$1000\ cm^3 = 1\ l$

$1\ cm^3 = \frac{1}{1000}\ l$

$10449.92\ cm^3 = \frac{10449.92}{1000}\ l$

$= 10.449921$

$= 10.45\ l$

$1\ Rs = 1\ l$

Cost of milk $10.45 \times 1 Rs = 10.45\ l$

$= 1567.5\ Rs$

Area of metal sheet = $\pi r_2^2 + \pi(r_1+r_2)l$

$= \pi\left[r_2^2 + (r_1+r_2)l\right]$

$= \frac{314}{100}\left[64 + (24+16)20\right]$

$= \frac{314}{100}\left[64 + 800\right]$

$= \frac{314}{100} \times 624$

$= \boxed{195936\ cm^2}$

$l = \sqrt{h^2 + (r_1 - r_2)^2}$

$l = \sqrt{(16)^2 + (20-8)^2}$

$= \sqrt{256 + 144}$

$= \sqrt{400}$

$l = 20cm$

4 2413
195936
9 7968 S
0

Rs S = 100 cm^2

$\frac{S}{100}$ Rs = 1 cm^2

$\frac{S}{100} \times 1959.36$ Rs = 1959.36 cm^2

97.96 Rs → 1959.36 cm^2

Cost of metal sheet

#Q. The height of a cone is 10 cm. The cone is divided into two parts using a plane parallel to its base at the middle of its height. Find the ratio of the volumes of two parts. [CBSE 2017]

$$\frac{V_1}{V_2} = \frac{\frac{1}{3}\pi r_2^2 h}{\frac{1}{3}\pi H(r_1^2 + r_2^2 + r_1 r_2)}$$

$$= \frac{r_2^2 h}{H(r_1^2 + r_2^2 + r_1 r_2)}$$

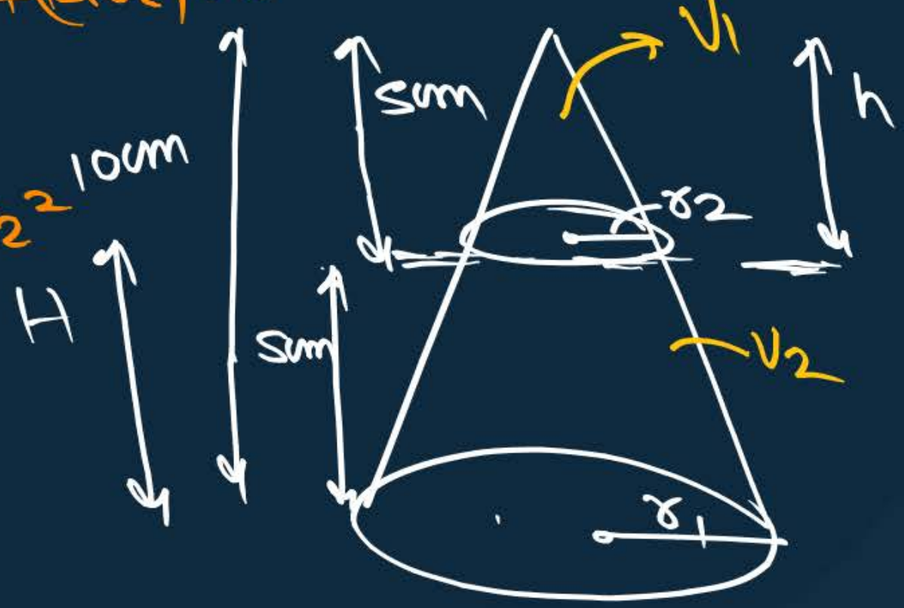
$$\frac{V_1}{V_2} = \frac{r_2^2}{r_1^2 + r_2^2 + r_1 r_2}$$

$$\frac{V_1}{V_2} = \frac{r_2^2}{(2r_2)^2 + (r_2)^2 + (2r_2)r_2}$$

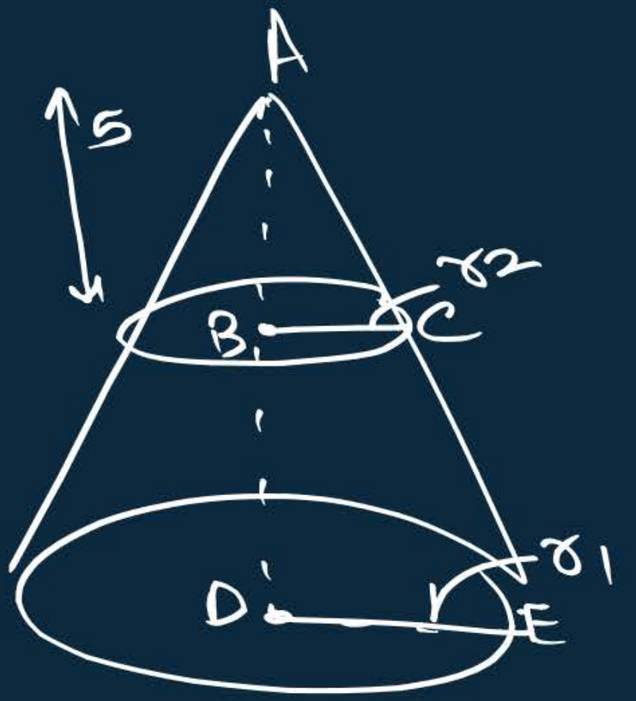
$$\frac{V_1}{V_2} = \frac{r_2^2}{4r_2^2 + r_2^2 + 2r_2^2}$$

$$= \frac{r_2^2}{7r_2^2}$$

$$\frac{V_1}{V_2} = \frac{1}{7}$$ - Ans



$\triangle ABC \sim \triangle ADE$



10

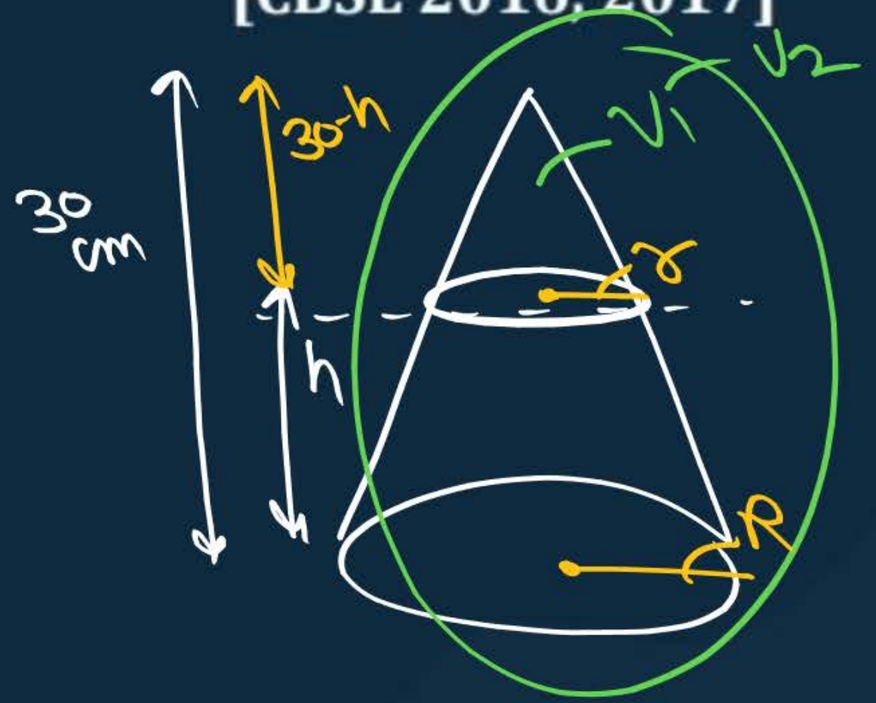
$$\frac{AB}{AD} = \frac{BC}{DE} = \frac{AC}{AE}$$

$$\frac{5}{10} = \frac{r_2}{r_1}$$

$$\frac{5}{10} = \frac{r_2}{r_1}$$

$$\boxed{r_1 = 2r_2}$$

#Q. The height of a cone is 30 cm. A small cone is cut off at the top by a plane parallel to the base. If its volume be $\frac{1}{27}$ of the volume of the given cone, at what height above the base is the section mode? [CBSE 2016, 2017]



$V_1 = \frac{1}{27} V_2$

$\frac{1}{3}\pi r^2 (30-h) = \frac{1}{27} \times \frac{1}{3} \times \pi R^2 (30)$

$r^2 (30-h) = \frac{10}{9} R^2$

$\frac{r^2}{R^2}\left[30-h\right] = \frac{10}{9}$

$$\left(\frac{r}{R}\right)^2(30-h)=\frac{10}{9}$$

$$\left(\frac{30-h}{30}\right)^2(30-h)=\frac{10}{9}$$

$$\frac{(30-h)^3}{30^2}=\frac{10}{9}$$

$$(30-h)^3=\frac{10}{9}\times 900^{100}$$

$$(30-h)^3=1000$$

$$30-h=\sqrt[3]{1000}$$

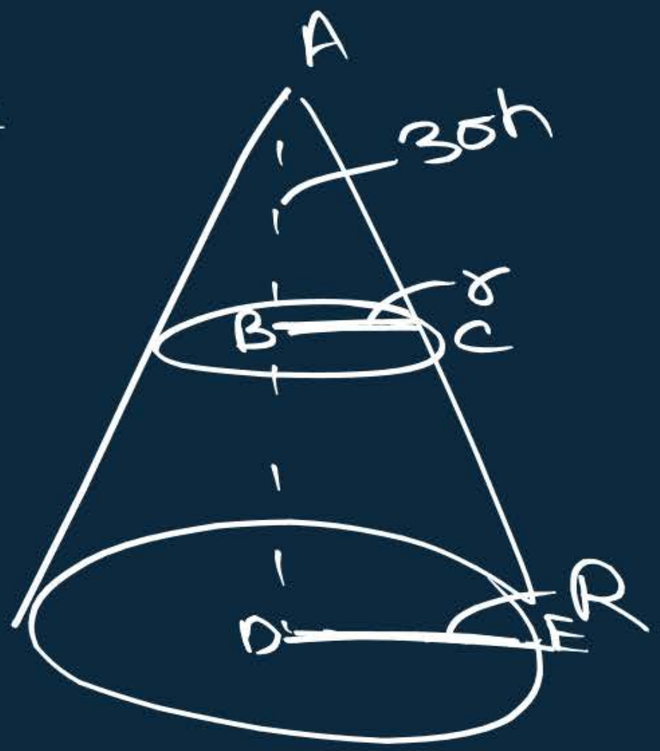
$$30-h=10$$

$h=20\text{cm}$

$\triangle ABC \sim \triangle ADE$

$$\frac{AB}{AD}=\frac{BC}{DE}$$

$$\boxed{\frac{30-h}{30}=\frac{r}{R}}$$



Topic : frustum of a cone

#Q. A metallic right circular cone 20 cm high and whose vertical angle is 60° is cut into two parts at the middle of its height by a plane parallel to its base. If the frustum so obtained be drawn into a wire of diameter 1/16 cm, find the length of the wire.

V. of Frustum = V. of cylindrical wire

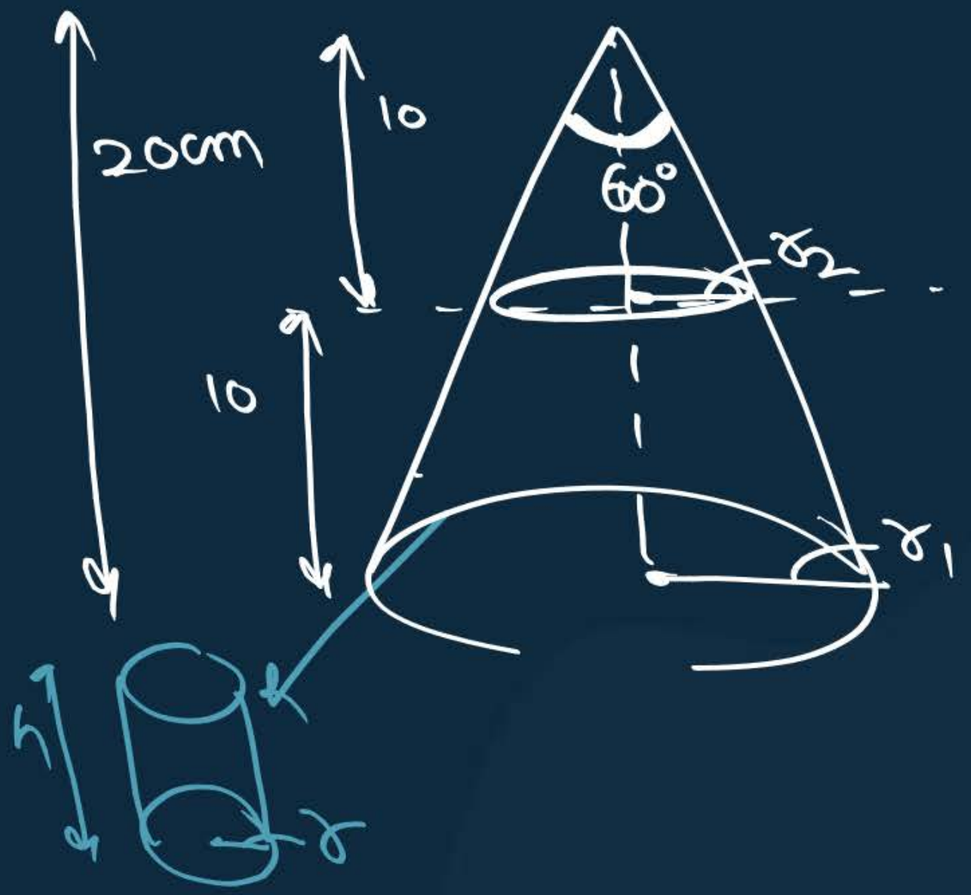
$\frac{1}{3}\pi 10\left[r_1^2 + r_2^2 + r_1 r_2\right] = \pi r^2 h$

diameter of cylinder $= \frac{1}{16}$ cm

$r = \frac{D}{2}$

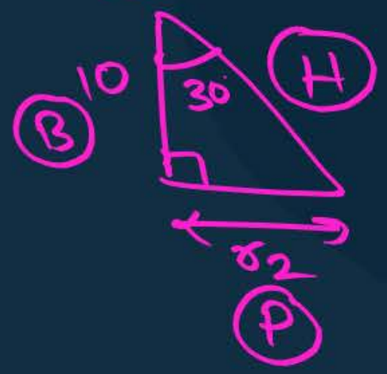
$r = \frac{1/16}{2}$

$r = \frac{1}{32}$ cm



$$\frac{10}{3}\pi\left[r_1^2+r_2^2+r_1r_2\right]=\pi\left(\frac{1}{32}\times\frac{1}{32}\right)\times h$$

ΔABC ...

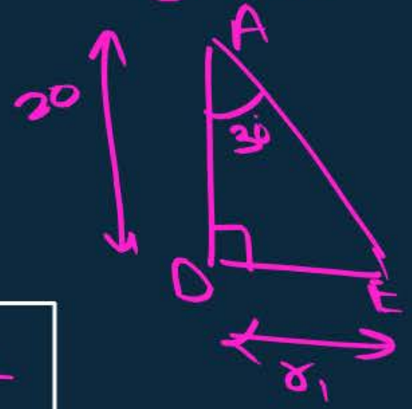


$$\tan 30^\circ=\frac{P}{B}$$

$$\frac{1}{\sqrt{3}}=\frac{r_2}{10}$$

$$\frac{10}{\sqrt{3}}=r_2$$

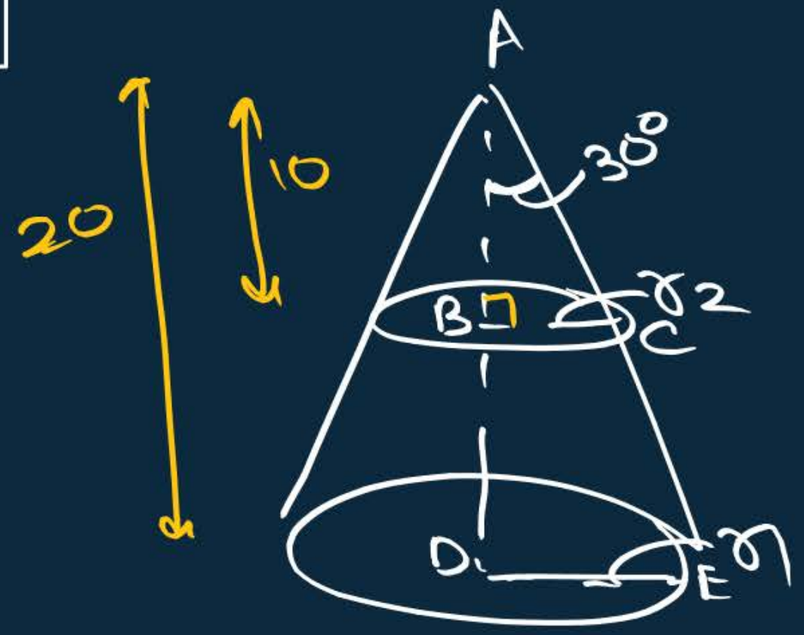
ΔADE



$$\tan 30^\circ=\frac{r_1}{20}$$

$$\frac{1}{\sqrt{3}}=\frac{r_1}{20}$$

$$\frac{20}{\sqrt{3}}=r_1$$



$$\frac{1}{3}\left[\left(\frac{20}{\sqrt{3}}\right)^2+\left(\frac{10}{\sqrt{3}}\right)^2+\left(\frac{20}{\sqrt{3}}\times\frac{10}{\sqrt{3}}\right)\right] = \frac{1}{32}\times\frac{1}{32}\times h$$

$$\frac{1}{3}\left[\frac{400}{3}+\frac{100}{3}+\frac{200}{3}\right]\times 32\times 32 = h$$

$$\frac{7000}{9}\times 1024 = h \text{ cm}$$


THANK
YOU