MATHS

Some Application Of Trigonometry

Q1 A wire connects the top of two poles of height 20m and 14m. Find the length of the wire if it makes an angle of 30° with the horizontal.

Q2 If the angle of elevatin of a cloud from a point *h* metres above a lake is α and the angle of depression of its reflection in the lake is β, prove that the height of the cloud is $\frac{h(\tan\beta + \tan\alpha)}{\tan\beta - \tan\alpha}$

Q3 An aeroplane when flying at a height of 4000 m from the ground passes vertically above another aeroplane at an instant when the angles of the elevation of the two planes from the same point on the ground are 60° and 45° respectively. Find the vertical distance between the aeroplanes at that instant.

Q4 From the top of a cliff, the angles of depression of two consecutive kilometer stones due east are found to be 45° and 30° respectively .Find the height of the cliff.

Q5 The height of a building is h units and angle of elevation of the building is α. On moving a distance $\frac{h}{2}$units towards the building, the angle of elevation becomes β. What is the value of $(\cot\alpha - \cot\beta)$?

Q6 The angle of elevation of top of an unfinished pillar at a point 150m from its base is 30°. If angle of elevation at the same point is to be 45°then the pillar has to be raised to a height of how many meters?

Q7 Two poles are x meters apart and the height of one is double that of other. If, from the midpoint of the line joining their feet, an observer find the angular elevation of their tops to be complementary, then find the height (in meters) of shorter pole?

Q8 When the Sun is at 30° ,the shadow of a tower standing on level ground is 40 m longer than when it is at 60° altiude. Determine the tower's height.

Answer Key

Q1 AC = 12 m

Q2 Height of the cloud is $\frac{h(\tan\beta + \tan\alpha)}{\tan\beta - \tan\alpha}$

Q3 $1690.53\mathrm{m}$

Q4 The height of the cliff is $500\left(\sqrt{3}+1\right)m.$

Q5 $\therefore \cot\alpha - \cot\beta = \frac{1}{2}$

Q6 $x = 50\left(3-\sqrt{3}\right)\mathrm{m}.$

Q7 Height of shorter pole is $\frac{x}{2\sqrt{2}}m.$

Q8 $\therefore$ The height of a tower is $20\sqrt{3}\,m.$

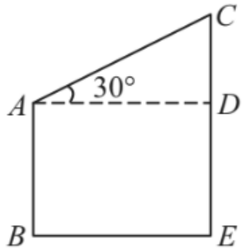
Hints & Solutions

Q1 Text Solution:

AB = 14 m, CE = 20 m

CD = 6 m

In right angled ΔADC



$\sin 30^\circ = \frac{CD}{AC}$

$\frac{1}{2} = \frac{6}{AC}$

AC = 12 m

Video Solution:

Q2 Text Solution:

Let AB be the surface of the lake and let P be a point of observartion such that AP = h metres. Let C be the position of the cloud and C' be its reflection in the lake. Then, CB = C'B. Let PM be perpendicular from P on CB. Then, $\angle CPM = \alpha$ and $\angle MPC' = \beta$.

Let CM = x. Then, CB = CM + MB = CM + PA = x + h.

In ΔCPM, we have

$\tan\alpha = \frac{CM}{PM}$

$\Rightarrow \quad \tan\alpha = \frac{x}{AB}$

$[\because \ PM = AB]$

$\Rightarrow \quad AB = x\cot\alpha$ ... (i)

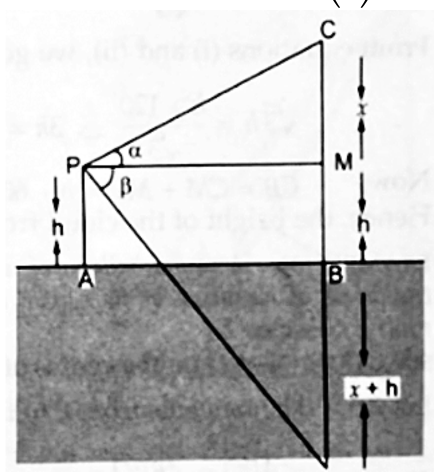
In $\Delta\, PMC'$, we have

$\tan\beta = \frac{\mathrm{C'M}}{\mathrm{PM}}$

$\Rightarrow \quad \tan\beta = \frac{\mathrm{x+2h}}{\mathrm{AB}}$

$\left[\because C'M = \mathrm{C'B + BM = x + h + h}\right]$

$\Rightarrow \quad \mathrm{AB} = \left(\mathrm{x + 2h}\right)\cot\beta$ ... $\left(\mathrm{ii}\right)$



From (i) and (ii), we have

$\Rightarrow \quad x\cot\alpha = (x+2h)\cot\beta$ [On equating the values of AB]

$\Rightarrow \quad x\left(\frac{1}{\tan\alpha} - \frac{1}{\tan\beta}\right) = \frac{2h}{\tan\beta}$

$\Rightarrow \quad x\left(\frac{\tan\beta - \tan\alpha}{\tan\alpha\tan\beta}\right) = \frac{2h}{\tan\beta}$

$\Rightarrow \quad x = \frac{2h\tan\alpha}{\tan\beta - \tan\alpha}$

Hence, the height CB of the cloud is given by

CB = x + h

$\Rightarrow \quad CB = \frac{2h\tan\alpha}{\tan\beta - \tan\alpha} + h$

$\Rightarrow \quad CB = \frac{2h\tan\alpha + h\tan\beta - h\tan\alpha}{\tan\beta - \tan\alpha}$

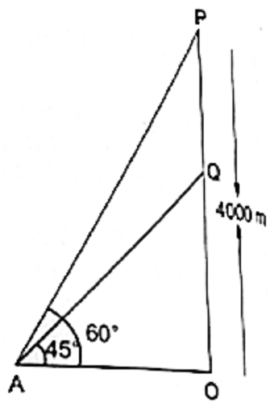
$= \frac{h(\tan\alpha + \tan\beta)}{\tan\beta - \tan\alpha}$

Video Solution:

Q3 Text Solution:

Let P and Q be the positions of two aeroplanes when Q is vertically below P and OP = 4000 m. Let the angles of elevation of P and Q at a point A on the ground be 60° and 45° respectively.

In triangles AOP and AOQ, we have



$\tan 60^\circ = \frac{OP}{OA}$ and $\tan 45^\circ = \frac{OQ}{OA}$

$\Rightarrow \quad \sqrt{3} = \frac{4000}{OA}$ and $1 = \frac{OQ}{OA}$

$\Rightarrow \quad OA = \frac{4000}{\sqrt{3}}$ and $OQ = OA$

$\Rightarrow \quad OQ = \frac{4000}{\sqrt{3}}\text{m}$

$\therefore$ Vertical distance PQ between the aeroplanes is given by

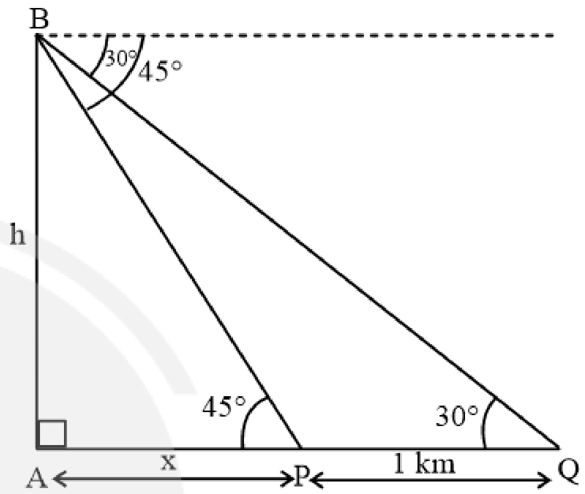
PQ = OP – OQ

$\Rightarrow \quad PQ = \left(4000 - \frac{4000}{\sqrt{3}}\right)\text{m}$

$= 4000\frac{(\sqrt{3}-1)}{\sqrt{3}}\text{m} = 1690.53\text{m}$

Video Solution:

Q4 Text Solution:



Let us suppose the hight of the cliff be h meters and P, Q are two consecutive stones having distance 1 km or 1000 meter between them and AP = x meters

Now, In ΔABP

$\tan 45^\circ = \frac{h}{x} \Rightarrow x = h$

$(\because \tan 45^\circ = 1)\ldots(\text{i})$

and, In ΔABQ

$\tan 30^\circ = \frac{h}{x+1000} \Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{x+1000}$

$\left(\because \tan 30^\circ = \frac{1}{\sqrt{3}}\right)$

$\Rightarrow h + 1000 = \sqrt{3}h$ (From equation (i))

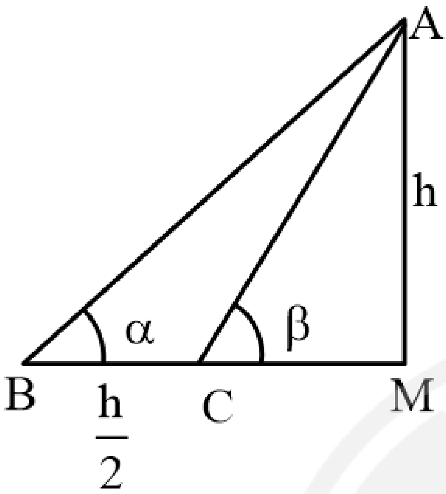
$\Rightarrow h\left(\sqrt{3}-1\right) = 1000$

$\Rightarrow h = \frac{1000}{\sqrt{3}-1}$ or $500\left(\sqrt{3}+1\right)m$

Therefore, the height of the cliff is $500\left(\sqrt{3}+1\right)m$.

Video Solution:

Q5 Text Solution:



$In\ \Delta ACM, \tan\beta = \frac{h}{CM}$

$\Rightarrow\ CM = h\cot\ \beta$ $\quad\left(\because \frac{1}{\tan\beta} = \cot\beta\right)$

...(i)

and in $\Delta\ AMB,$

$\tan\alpha = \frac{h}{\frac{h}{2}+CM} \Rightarrow \frac{h}{2} + CM = h\cot\alpha$

...(ii)

On putting equaion (i) in (ii), we get

$\Rightarrow\ \frac{h}{2} + h\cot\beta = h\cot\alpha$

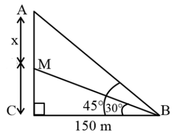
$\Rightarrow h\cot\alpha - h\cot\beta = \frac{h}{2}$

$h(\cot\alpha - \cot\beta) = \frac{h}{2}$

$\therefore \cot\alpha - \cot\beta = \frac{1}{2}$

Video Solution:

Q6 Text Solution:



In $\Delta\ MCB,$

$\tan 30^\circ = \frac{MC}{BC}$

$\Rightarrow MC = 50\sqrt{3}$ meter

In $\Delta\ ACB,$

Let AM = x meters then, $AC = \left(50\sqrt{3} + x\right)\text{m}$

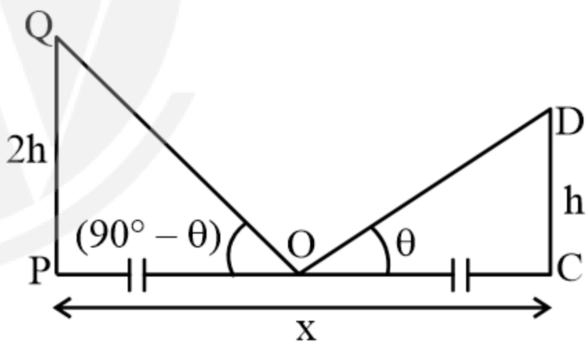
$\therefore \tan 45^\circ = \frac{x+50\sqrt{3}}{150}$

$\Rightarrow x + 50\sqrt{3} = 150 \qquad (\therefore\ tan45^\circ = 1)$

$x = 50\left(3 - \sqrt{3}\right)\text{m}.$

Video Solution:

Q7 Text Solution:



It is given that both poles are x meters apart.

Hence, PC = x m

$PO = OC = \frac{x}{2}$ m

In $\Delta\ OCD$, we have

$\tan\theta = \frac{h}{\frac{x}{2}} = \frac{2h}{x}$ ...(i)

Now, in $\Delta\ OPQ$, we have

$\tan\left(90^\circ - \theta\right) = \frac{PQ}{PO}$

$\cot\theta = \frac{2h}{\frac{x}{2}} = \frac{4h}{x}$ $\left(\because \tan\left(90-\theta\right)\right.$ ...

$\left.= \cot\theta\right)$

(ii)

On multiplying equations (i) and (ii), we get

$\tan\theta \cdot \cot\theta = \frac{2h}{x} \times \frac{4h}{x}$ $\left(\because \tan\theta = \frac{1}{\cot\theta}\right)$

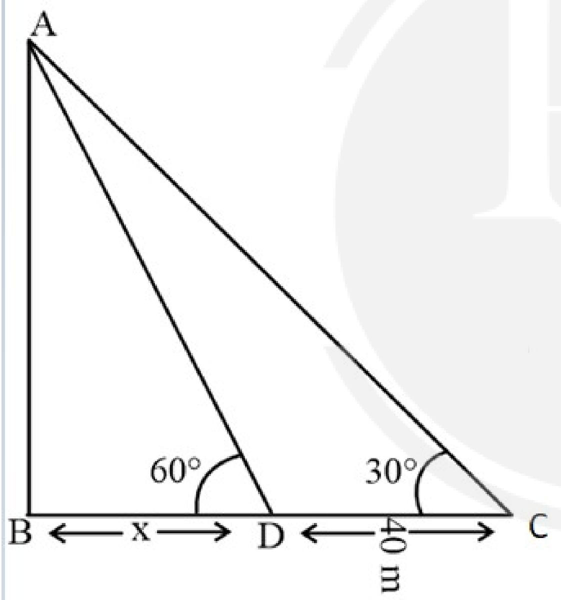
$\Rightarrow x^2 = 8h^2 \Rightarrow h^2 = \frac{x^2}{8}$

$\Rightarrow h = \frac{x}{2\sqrt{2}}$ m

Therefore, height of shorter pole is $\frac{x}{2\sqrt{2}}m$.

Video Solution:

Q8 Text Solution:



Let AB represents the tower and height of tower is h meters. Also let the length of shadow when sun is at 60° altitude.

In $\Delta\, ABD$, tan 60° = $\frac{AB}{BD}$

$\Rightarrow \sqrt{3} = \frac{h}{x}$

$\therefore x = \frac{h}{\sqrt{3}}$ ...(i)

Now in $\Delta\, ABC$, $\tan 30^\circ = \frac{AB}{BC}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{(x+40)}$

$\Rightarrow x + 40 = \sqrt{3}h$

$\therefore \frac{h}{\sqrt{3}} + 40 = \sqrt{3}h$ [from equation (i)]

$\Rightarrow h + 40\sqrt{3} = 3h$

$\Rightarrow 2h = 40\sqrt{3}$

$\therefore h = 20\sqrt{3}$ m

$\therefore$ The height of a tower is $20\sqrt{3}\, m$.

Video Solution: