

UPDAAN



2025

Trigonometry

Mathematics

Lecture - 02

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Topics

to be covered



- 1 Questions on Trigonometric Ratios
- 2 Trigonometric Ratios for some Specific Angles
- 3 Proof of Trigonometric Ratios for some Specific Angles

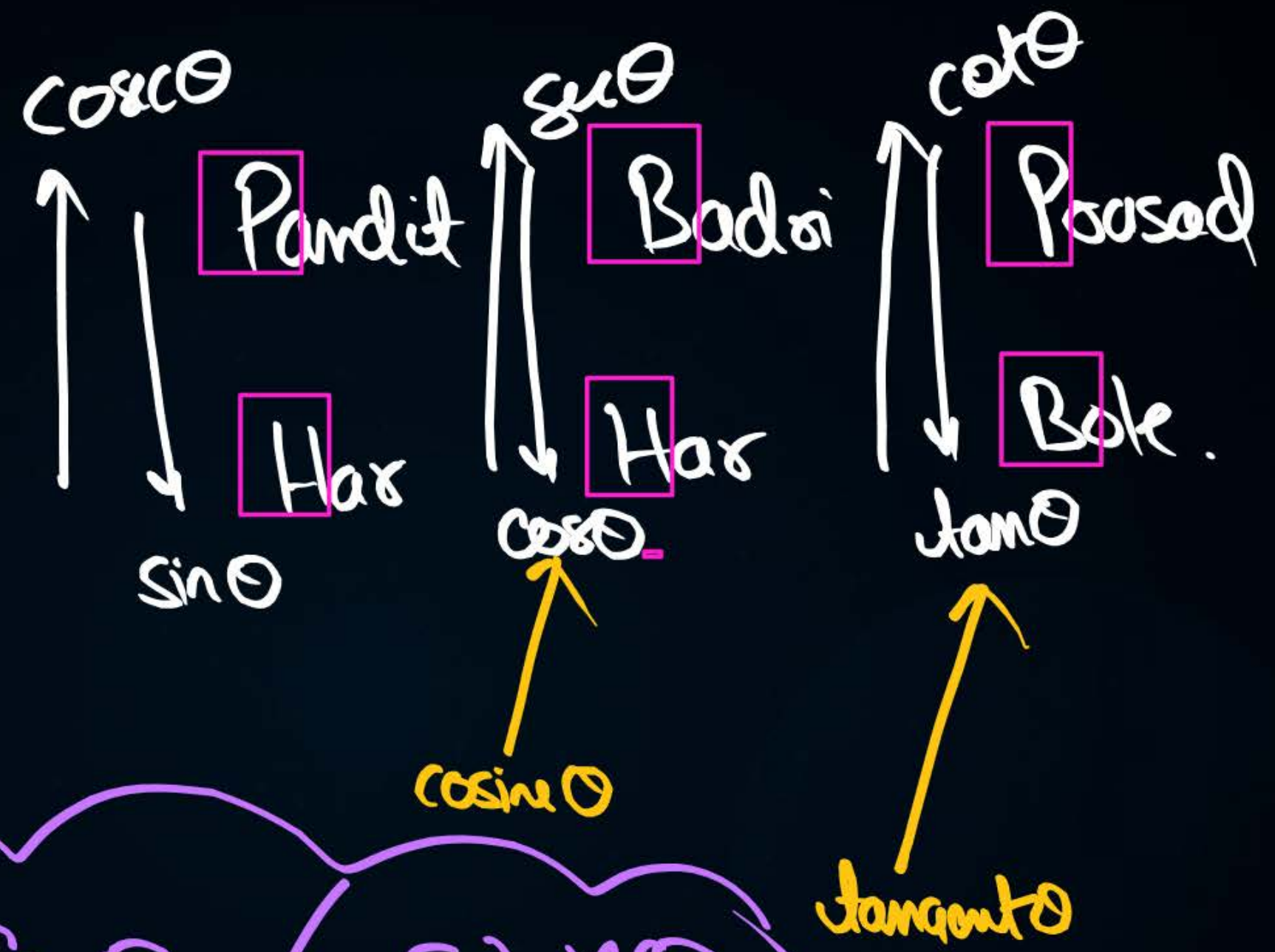
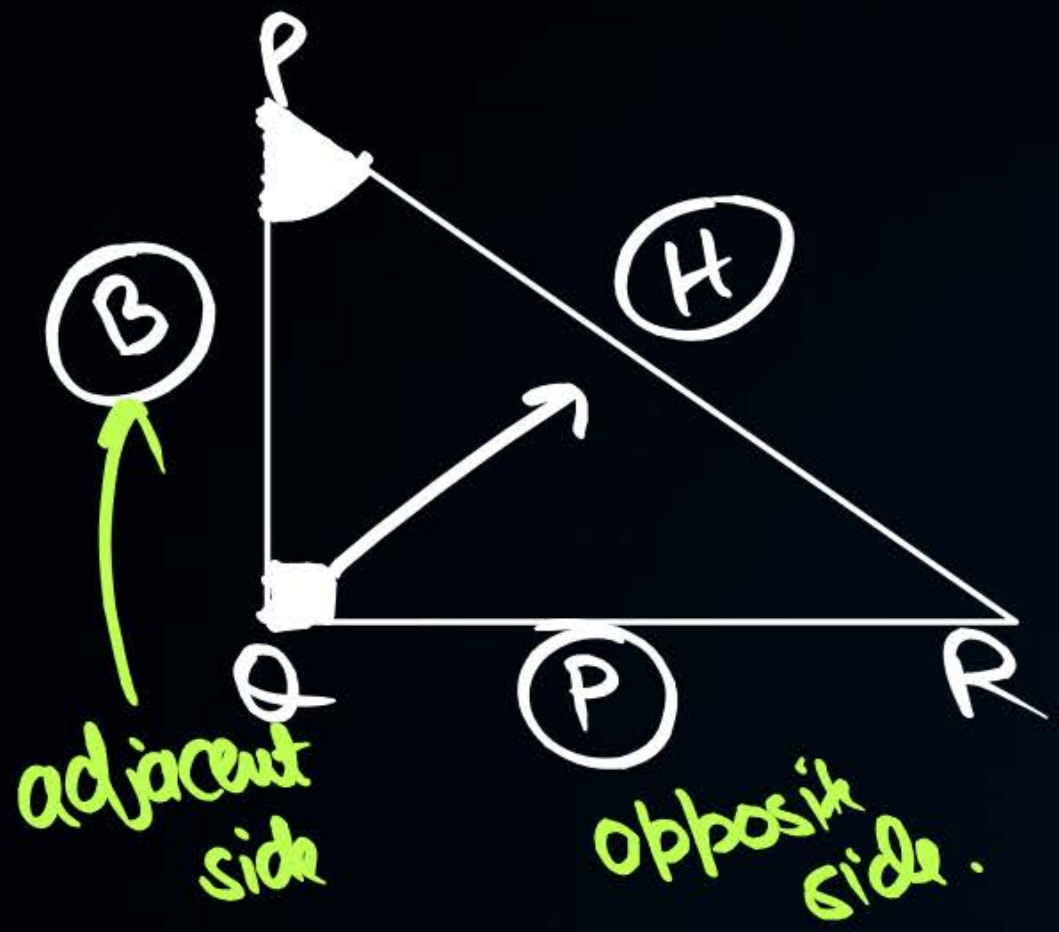




WORK HARD
DREAM BIG
NEVER GIVE UP !!



Janam kabh lena hai or Marna kabh hai vo hum decide nhi kr skte, pr kese jeena hai vo hum decide kr skte hain.



$\sin \theta \neq \sinh \theta$

tangent θ



P	B	P	
H	H	B	H.w

Pyax	Babuq	Pyax		P	B	P
Hamax	Homan	Babus.		H	H	B

Topic : T Ratios



#Q. If $8 \tan A = 15$, find $\sin A - \cos A$.

$$\tan A = \frac{15}{8}$$

$$\frac{P}{B} = \frac{15}{8}$$

$$P = 15x$$

$$B = 8x$$

$$H^2 = P^2 + B^2$$

$$(H)^2 = (15x)^2 + (8x)^2$$
$$= 225x^2 + 64x^2$$

$$H^2 = 289x^2$$

$$H = \pm \sqrt{289x^2}$$

$$H = +17x$$

$$= \sin A - \cos A$$

$$= \frac{P}{H} - \frac{B}{H}$$

$$= \frac{15x}{17x} - \frac{8x}{17x}$$

$$= \frac{7x}{17x}$$

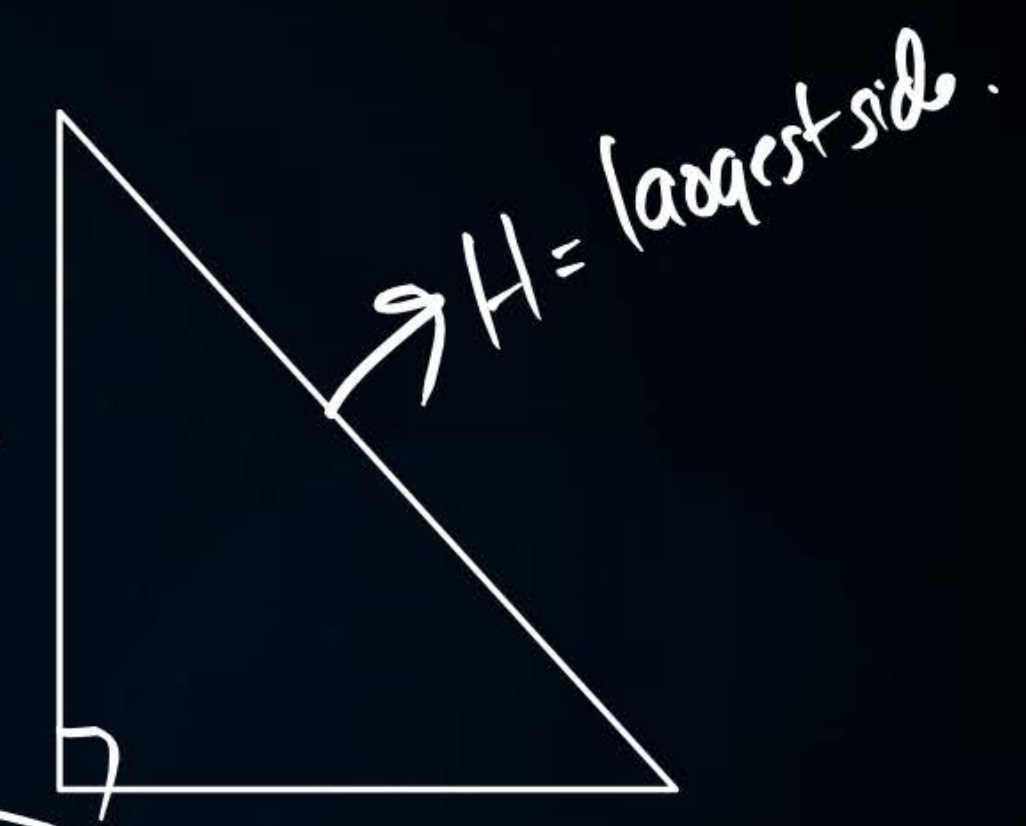
$$= \frac{7}{17}$$

Q $\sin \theta = \frac{7}{3}$ $\neq \frac{P}{H}$ $\begin{matrix} P=7x \\ H=9x \end{matrix}$

- A) True
~~B) False.~~
 C) —
 D) —

Q $\sin \theta > 1$
 A) True
~~B) False~~
 C) —
 D) —

$\sin \theta = \frac{P}{H}$
 $H > P$
 $D > N$



$\frac{1}{2}, \frac{3}{5}, \frac{5}{9}, \frac{4}{5},$
 $0-1$

$H_B = \frac{12}{5}$

- $$\downarrow$$
-
- $\csc A$

Topic : T Ratios



#Q. If $\sin B = \frac{1}{2}$, then $3\cos B - 4\cos^3 B =$

[DPP Question]

$$\sqrt{3} \times \sqrt{3} \times \sqrt{3} = 3\sqrt{3}$$

$$\begin{aligned}\sin^3 B &= (\sin B)^3 \\ &= \sin B \times \sin B \times \sin B\end{aligned}$$

$$\sin B = \frac{1}{2}$$

$$\frac{P}{H} = \frac{1}{2}$$

$$P = 1x$$

$$H = 2x$$

$$H^2 = P^2 + B^2$$

$$(2x)^2 = (1x)^2 + B^2$$

$$4x^2 = 1x^2 + B^2$$

$$3x^2 = B^2$$

$$\pm \sqrt{3}x = B$$

$$\sqrt{3}x = B$$

$$\cos B = \frac{B}{H}$$

$$= \frac{\sqrt{3}x}{2x}$$

$$= \frac{\sqrt{3}}{2}$$

$$= 3\cos B - 4(\cos B)^3$$

$$= 3\left(\frac{\sqrt{3}}{2}\right) - 4\left(\frac{\sqrt{3}}{2}\right)^3$$

$$= \frac{3\sqrt{3}}{2} - 4\left(\frac{3\sqrt{3}}{8}\right)$$

$$= \frac{3\sqrt{3}}{2} - \frac{3\sqrt{3}}{2} = 0$$

$$x^2 = 4$$

$$x = \pm \sqrt{4}$$

$$x = +2, -2$$

Topic : T Ratios



#Q. In a right triangle ABC right angled at B, $\angle ACB = \theta$, AB = 2 cm and BC = 1 cm. Find the value of $\sin^2\theta + \tan^2\theta$. [DPP Type Question]

$$\sin^2\theta + \tan^2\theta = 9$$
$$(\sin\theta)^2 + (\tan\theta)^2 = 9$$

$$(AC)^2 = (AB)^2 + (BC)^2$$

$$(AC)^2 = (2)^2 + (1)^2$$

$$AC^2 = 5$$

$$AC = \pm\sqrt{5}$$

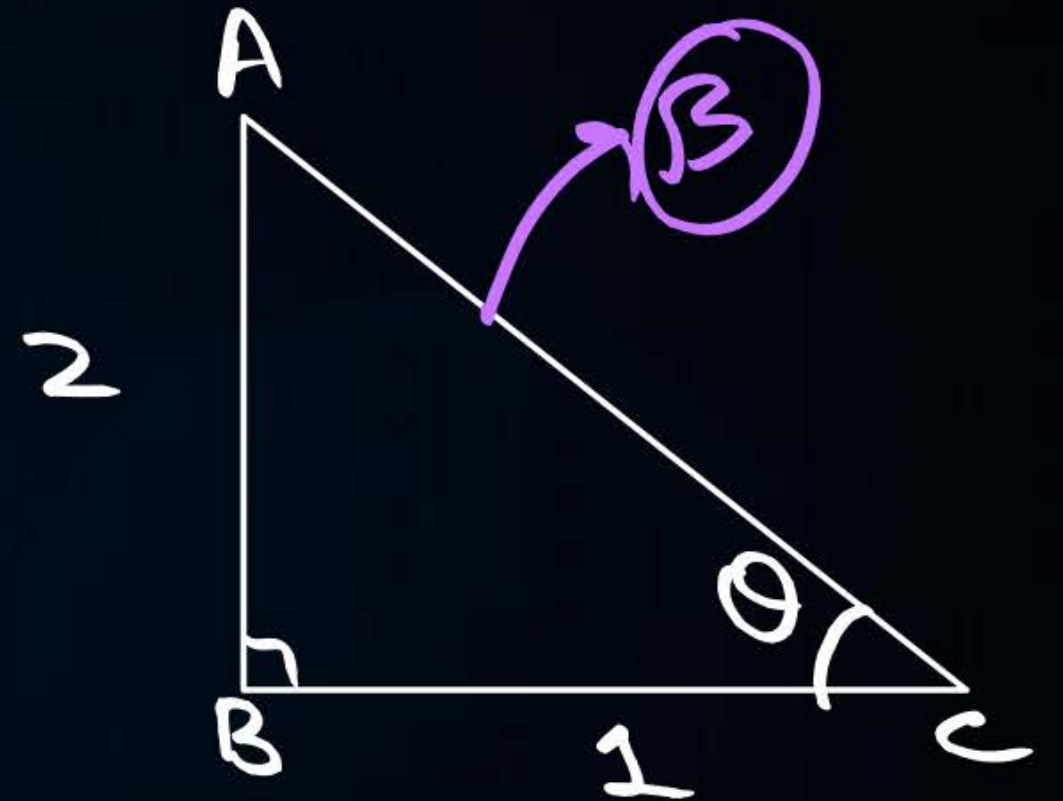
$$AC = \sqrt{5}$$

$$\sin\theta = \frac{P}{H} = \frac{2}{\sqrt{5}}$$

$$\tan\theta = \frac{P}{B} = \frac{2}{1}$$

$$= \left(\frac{2}{\sqrt{5}}\right)^2 + \left(\frac{2}{1}\right)^2$$

$$= \frac{4}{5} + 4$$
$$= \frac{24}{5}$$



Topic : T Ratios



#Q. In $\triangle OPQ$ right angled at P, $OP = 7$ cm, $OQ - PQ = 1$ cm. Determine the values of $\sin Q$ and $\cos Q$. [NCERT]

$$(OQ)^2 = (OP)^2 + (PQ)^2$$

$$(OQ)^2 = 49 + (PQ)^2$$

$$(1 + PQ)^2 = 49 + (PQ)^2$$

$$1^2 + \cancel{PQ^2} + 2(1)(PQ) = 49 + \cancel{PQ^2}$$

$$1 + 2PQ = 49$$

$$2PQ = 48$$

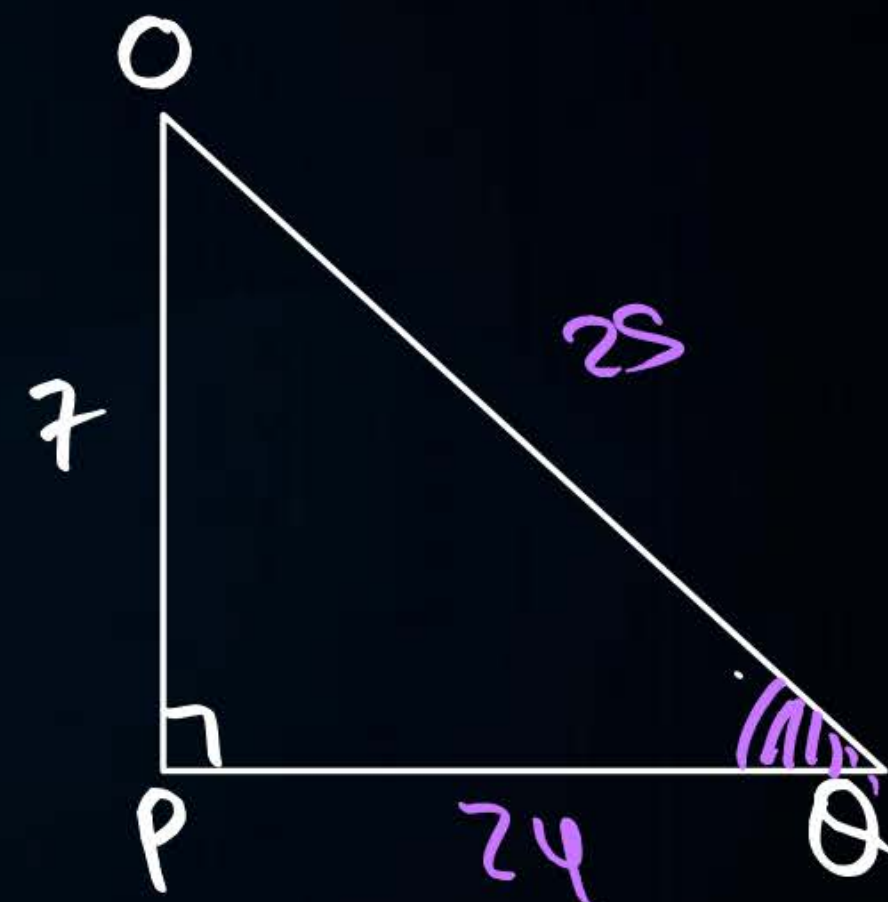
$$PQ = 24$$

$$OQ = 1 + PQ$$

$$OQ = 25$$

$$\sin Q = \frac{P}{H} = \frac{7}{25}$$

$$\cos Q = \frac{B}{H} = \frac{24}{25}$$



Topic : T Ratios



#Q. In $\triangle PQR$, right angled at Q, $PR + QR = 25$ cm and $PQ = 5$ cm. Determine the values of $\sin P$, $\cos P$ and $\tan P$. [NCERT]

$$(PR)^2 = (QR)^2 + (PQ)^2$$

$$(PR)^2 = (QR)^2 + 25$$

$$(25 - QR)^2 = QR^2 + 25$$

$$(25)^2 + \cancel{QR^2} - 2(25)(QR) = \cancel{QR^2} + 25$$

$$625 - 50QR = 25$$

$$625 - 25 = 50QR$$

$$600 = 50QR$$

$$PR = 25 - QR$$

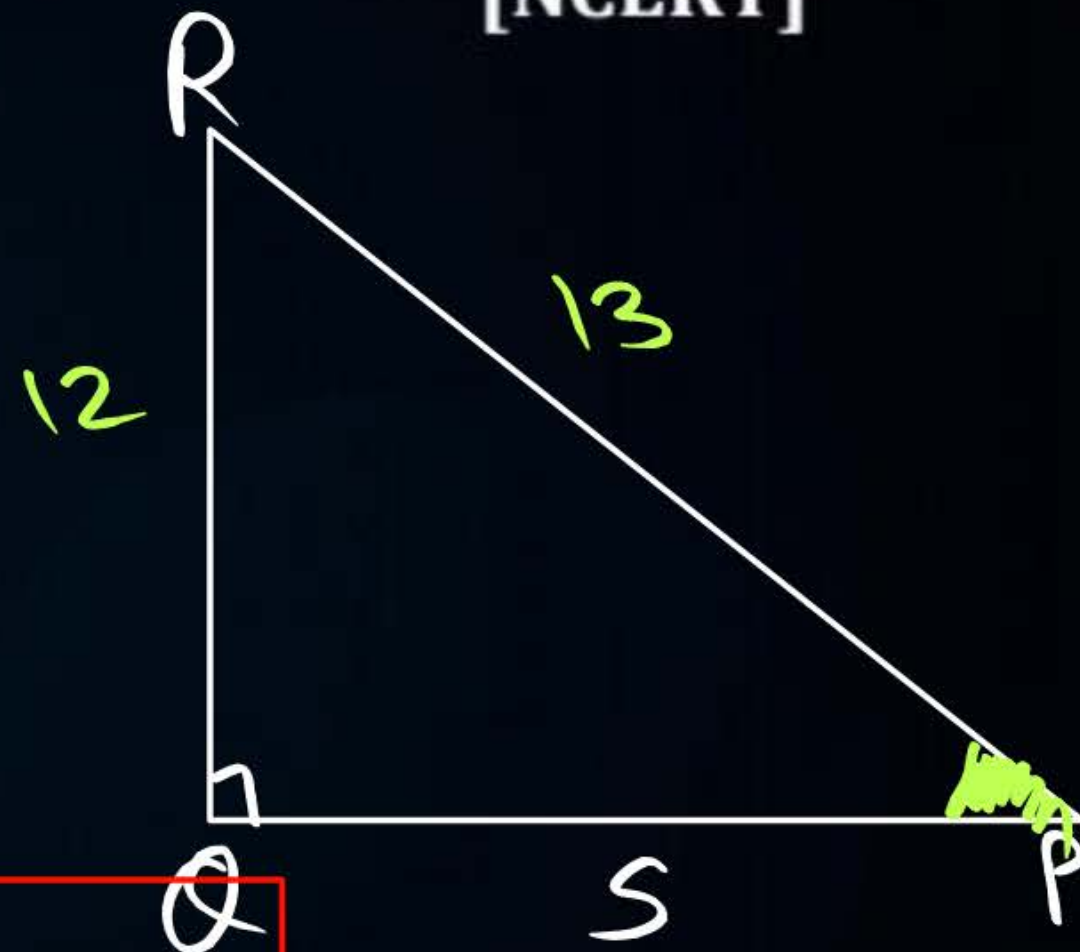
$$PR = 25 - 12$$

$$\star PR = 13$$

$$\frac{600}{50} = QR$$

$$\star 12 = QR$$

$$\begin{aligned} \tan P &= \frac{P}{B} = \frac{12}{5} \\ \sin P &= \frac{P}{H} = \frac{12}{13} \end{aligned} \quad \cos P = \frac{5}{13}$$



Reciprocal.

$$\sin \theta = \frac{2}{3} \quad \cos \theta = \frac{3}{2}$$



$$\frac{P}{H} \sin \theta \longleftrightarrow \cos \theta \quad \frac{H}{P}$$

$$\frac{B}{H} \cos \theta \longleftrightarrow \sec \theta \quad \frac{H}{B}$$

$$\frac{P}{B} \tan \theta \longleftrightarrow \cot \theta \quad \frac{B}{P}$$

$$\textcircled{1} \quad \sin \theta = \frac{1}{\cos \theta} \quad \text{or} \quad \cos \theta = \frac{1}{\sin \theta}$$

$$\textcircled{2} \quad \cos \theta = \frac{1}{\sec \theta} \quad \text{or} \quad \sec \theta = \frac{1}{\cos \theta}$$

$$\textcircled{3} \quad \tan \theta = \frac{1}{\cot \theta} \quad \text{or} \quad \cot \theta = \frac{1}{\tan \theta}$$

$$\textcircled{1} \quad \frac{1}{\cos \theta}$$

$$= \frac{1}{\frac{3}{2}}$$

$$= \frac{1}{\frac{3}{2} \times \frac{2}{2}} = \frac{2}{3}$$

$$= \frac{P}{H}$$

$$= \sin \theta$$

$$\textcircled{2} \quad \frac{2}{\frac{2}{3}} = \frac{2}{1} = 2$$

$$\textcircled{3} \quad \frac{2}{\frac{2}{3}} = \frac{2}{1} = 2$$

Quotient Relation.



$$\textcircled{1} \quad \frac{\sin \theta}{\cos \theta} = \frac{\frac{P}{H}}{\frac{B}{H}} = \frac{P \cancel{H}}{B \cancel{H}} = \frac{P}{B} = \tan \theta$$

$$\textcircled{1} \quad \tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\textcircled{2} \quad \cot \theta = \frac{\cos \theta}{\sin \theta}$$



Topic: Trigonometric Ratios of some specific Angles

$\frac{1}{0} = \text{not defined}$

$\frac{1}{\text{n.d.}} = 0$

$$\tan 0^\circ = \frac{\sin 0^\circ}{\cos 0^\circ}$$

$$\tan 30^\circ = \frac{\sin 30^\circ}{\cos 30^\circ}$$

$$\tan 60^\circ = \frac{\sin 60^\circ}{\cos 60^\circ}$$

T. ratios \ θ	0°	30°	45°	60°	90°
$\sin \theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos \theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan \theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	n.d
$\operatorname{cosec} \theta$	n.d	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
$\sec \theta$	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	n.d
$\cot \theta$	n.d	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0



Topic: Trigonometric Ratios of some specific Angles

T. ratios θ	0°	30°	45°	60°	90°
$\sin \theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos \theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan \theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not defined
$\operatorname{cosec} \theta$	Not defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
$\sec \theta$	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Not defined
$\cot \theta$	Not defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

$$T \frac{\sin i}{\cos i} c$$

$$\frac{\sin \text{Doori}}{\cos \text{Doori}}$$



Topic : T Ratios for some specific angles



#Q. Evaluate each of the following in the simplest form:

$$\sin 60^\circ \cos 30^\circ + \cos 60^\circ \sin 30^\circ$$

[NCERT]

$$= \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} + \frac{1}{2} \times \frac{1}{2}$$

$$= \frac{3}{4} + \frac{1}{4}$$

$$= \frac{4}{4}$$

$$= 1$$



Topic : T Ratios for some specific angles



#Q. Evaluate the following expressions:
 $2 \sin^2 30^\circ \tan 60^\circ - 3 \cos^2 60^\circ \sec^2 30^\circ$

$$= 2 \left(\frac{1}{2} \right)^2 \times \sqrt{3} - 3 \left(\frac{1}{2} \right)^2 \left(\frac{2}{\sqrt{3}} \right)^2$$

$$= 2 \left(\frac{1}{4} \right) \times \sqrt{3} - 3 \left(\frac{1}{4} \right) \left(\frac{4}{3} \right)$$

$$= \frac{\sqrt{3}}{2} - 1$$

$$= \boxed{\frac{\sqrt{3} - 2}{2}}$$

$$\begin{aligned} \tan 60^\circ &= \frac{\sin 60^\circ}{\cos 60^\circ} \\ &= \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} \\ &= \textcircled{\sqrt{3}} \end{aligned}$$

Topic : T Ratios for some specific angles



#Q. Prove that: $\frac{\cos 30^\circ + \sin 60^\circ}{1 + \cos 60^\circ + \sin 30^\circ} = \frac{\sqrt{3}}{2}$

$$\begin{aligned} &= \frac{\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2}}{1 + \frac{1}{2} + \frac{1}{2}} \\ &= \frac{\frac{\sqrt{3} + \sqrt{3}}{2}}{\frac{2 + 1 + 1}{2}} \end{aligned}$$

$$\begin{aligned} &= \frac{\frac{2\sqrt{3}}{2}}{\frac{4}{2}} \\ &= \frac{2\sqrt{3} \times 2}{2 \times 4} \\ &= \frac{\sqrt{3}}{2} \quad \text{H.P.} \end{aligned}$$

Topic : T Ratios for some specific angles



#Q. Evaluate : $\frac{3\tan^2 30^\circ + \tan^2 60^\circ + \operatorname{cosec} 30^\circ - \tan 45^\circ}{\cot^2 45^\circ}$

[Board Term - I, 2016]

$$= \frac{3\left(\frac{1}{\sqrt{3}}\right)^2 + (\sqrt{3})^2 + 2 - 1}{(1)^2}$$

$$= \cancel{3}\left(\frac{1}{\cancel{3}}\right) + 3 + 2 - 1$$

$$= 1 + 3 + 2 - 1$$

$$= \boxed{5}$$

$$\begin{aligned}\tan 30^\circ &= \frac{\sin 30^\circ}{\cos 30^\circ} \\ &= \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} \\ &= \boxed{\frac{1}{\sqrt{3}}}\end{aligned}$$

$$\begin{aligned}\tan 60^\circ &= \frac{\sin 60^\circ}{\cos 60^\circ} \\ &= \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} \\ &= \boxed{\sqrt{3}}\end{aligned}$$

When a 10th Class student solves
Trigonometry problem without
converting
Tan into Sin and Cos :



Topic :



#Q. If $(x-2)\sin^2 30^\circ + (x-3)\tan^2 60^\circ - x\cos^2 45^\circ = 17/4$, find the value of x .

$$(x-2)\left(\frac{1}{2}\right)^2 + (x-3)(\sqrt{3})^2 - x\left(\frac{1}{\sqrt{2}}\right)^2 = \frac{17}{4}$$

$$(x-2)\left(\frac{1}{4}\right) + (x-3)(3) - x\left(\frac{1}{2}\right) = \frac{17}{4}$$

$$\frac{x-2}{4} + \frac{3x-9}{1} - \frac{x}{2} = \frac{17}{4}$$

$$\frac{1(x-2) + 4(3x-9) - 2(x)}{4} = \frac{17}{4}$$

$$\rightarrow x-2+12x-36-2x = \frac{17}{1}$$

$$11x-38=17$$

$$11x=17+38$$

$$11x=55$$

$$x=5$$

Topic :

(L.W) Discussion



#Q. If $\tan \theta = \frac{2x(x+1)}{2x+1}$ find $\sin \theta$ ^{and} $\cos \theta$.

$$(ab)^2 = a^2b^2$$

$$\tan \theta = \frac{P}{B}$$

$$P = 2x(x+1)$$

$$B = 2x+1$$

$$H^2 = (x+2)^2$$

$$H = \pm \sqrt{(x+2)^2}$$

$$H = x+2$$

$$H^2 = P^2 + B^2$$

$$H^2 = [2x(x+1)]^2 + (2x+1)^2$$

$$= 4x^2(x+1)^2 + 4x^2 + 1 + 4x$$

$$= 4x^2(x^2 + 1 + 2x) + 4x^2 + 1 + 4x$$

$$H^2 = 4x^4 + 4x^2 + 8x^3 + 4x^2 + 1 + 4x$$

$$= 4x^4 + 4x^2 + 1 + 8x^3 + 4x^2 + 4x$$

$$H^2 = 4x^4 + 4x^2 + 1 + 8x^3 + 4x^2 + 4x$$

$$= \underbrace{(2x^2)^2}_a + \underbrace{(2x)^2}_b + \underbrace{(1)^2}_c + \underbrace{2(2x^2)(2x)}_{2ab} + \underbrace{2(2x)(1)}_{2bc} + \underbrace{2(2x^2)(1)}_{2ac}$$

$$(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ac$$

$$H^2 = (2x^2 + 2x + 1)^2$$

$$H = \pm \sqrt{(2x^2 + 2x + 1)^2}$$

$$H = 2x^2 + 2x + 1$$



Homework

DPP



THANK
YOU

