UDAAN 2025

Polynomials

Mathematics | **Lecture - 02**

By – Ritik Sir

Topics *to be covered*

1. General Forms of Polynomial
2. Value of a Polynomial
3. Zeroes of a Polynomial
4. Geometrical meaning of zeroes of a polynomial
5. Middle term splitting Method


WORK HARD
DREAM BIG
NEVER GIVE UP !!


75% of students are good at Math
I belong to the rest 14%

$\boxed{\text{Linear polynomial}}$

$2x+1$

$-2x$

$3x$

General Form of Polynomials

Linear polynomial.

$2x-1$

$-2x+2$

$-3x+0$

$\frac{5}{2}x+0$

$\boxed{ax+b}$

$a, b \in R$

belongs to

$a \neq 0$

Quadratic polynomial.

$2x^2+3x+2$

$-3x^2+5x+0$

$-5x^2+2+0x$

$-\frac{5}{2}x^2+0x+0$

$\boxed{ax^2+bx+c}$

$a, b, c \in R$

$a \neq 0$

$\boxed{\text{Cubic polynomial.}}$

$2x^3 - 2x^2 + 4x + 9$

$\boxed{ax^3 + bx^2 + cx + d}$

$-3x^3 + 4x^2 + 9x$

$4x^3 + 5x^2$

$-9x^3$

$-2x^2+9x+2$ ax^2+bx+c

$a = -2$

$b = 9$

$c = 2$

$\underline{\underline{Q}}$ $-9x^2-2x+0$ ax^2+bx+c

$a = -9$

$b = -2$

$c = 0$

$a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + a_{n-3} x^{n-3} \dots\dots\dots$

$\dots\dots\dots a_3 x^3 + a_2 x^2 + a_1 x^1$

$+ a_0 x^0.$

Polynomial = degree = n

$3x^3 + 4x^2 - 5x + 2x^0$ — $d = 3$

$3x^0$

$a_3 x^3 + a_2 x^2 + a_1 x^1 + a_0 x^0.$

$a_3 = 3,\ a_2 = 4,\ a_1 = -5,\ a_0 = 2$

Put $n = 2$

$a_2 x^2 + a_1 x^1 + a_0 x^0$

Polynomials in one variables

An expression of the form $a_0 + a_1x + a_2x^2 + \ldots\ldots + a_{n-1}x^{n-1} + a_nx^n$ where, $a_0, a_1, a_2, \ldots\ldots a_{n-1}$, an are real numbers, $a_n \neq 0$ and n is a non-negative integer, is called a polynomial in x of degree n.

Here, a_0 is called the constant term of the given polynomial and $a_1, a_2, a_3, \ldots\ldots a_{n-1}, a_n$ are called the coefficients of $x, x^2, x^3, \ldots\ldots x^{n-1}$ and x^n respectively.

Topic : Basics

#Q. (i) Give an example of a monomial of degree 5.

(ii) Give an example of a binomial of degree 8.

(iii) Give an example of a trinomial of degree 4.

(iv) Give an example of a monomial of degree 0.

$5x^5$, $-4x^5$, x^5,

x^8+2, x^8+x^7, $-x^8+5x^2$

x^4+3x^2+x

5, 7, -5, $\frac{5}{2}$, 100

x^0 x^0 x^0

Topic : Basics

#Q. Write

(i) The coefficient of x^3 in $x + 3x^2 - 5x^3 + x^4$ → -5

(ii) The coefficient of x in $\sqrt{3} - 2\sqrt{2}x + 6x^2$ → $-2\sqrt{2}$

(iii) The coefficient of x^2 in $3x - 3 + x3$ → 0

correct → $3x-3-x^3+0x^2$

(iv) The constant term in $\frac{\pi}{2}x^2 + 7x - \frac{2}{5}\pi$

Ans//

Value of a polynomial

The value of a polynomial p(x) at x = α is obtained by putting x = α in p(x) and it is denoted by p(α).

$P(x), g(x), q(x), r(x)$ - - - - .

$P(100) = 99$

$P(x) = x - 1$

$x = 2$ $\quad \boxed{P(2) = 1}$

Variable

Zeroes of a Polynomial

Let p(x) be a polynomial. If p(α) = 0 then we say that α is a zero of the polynomial p(x).

$P(x) = 2x - 5$

$P(2) = -1$

$P(-2) = -9$

$P(-50) = -105$

$P\left(-\frac{3}{2}\right) = -8$

$P\left(\frac{5}{2}\right) = 0$

Zeroes/Zero of a polynomial

Variable ki aisi value jo poly = 0

Q $P(x) = x$

$P(0) = 0$

$\boxed{\text{Zero} = 0}$

Q $P(x) = x+3$

$x = -3$

$P(-3) = 0$

Zero $= -3$

Topic : Zero of a Polynomial

#Q. Find a zero of the polynomial

(i) $p(x) = x - 3$ $\rightarrow$ 3

(ii) $q(x) = 3x + 2$

(i) $p(x) = x - 3$

$x - 3 = 0$

$x = 3$

(ii) $q(x) = 3x + 2$

$3x + 2 = 0$

$3x = -2$

$x = -\frac{2}{3}$

$q\left(-\frac{2}{3}\right) = \cancel{3}\left(-\frac{2}{\cancel{3}}\right) + 2$

$= 0$

Topic : Zero of a Polynomial

#Q. Find a $\boxed{\text{zero of the polynomial } p(x) = ax + b}$, $a \neq 0$ and a, b are real numbers,

$$p(x) = ax + b$$

$$ax + b = 0$$

$$ax = -b$$

$$x = -\frac{b}{a}$$

$-\frac{b}{a}$ is the zero of the polynomial.

Q $P(x) = x^2 - 4$ $\rightarrow$ '2' zeroes

$P(2) = (2)^2 - 4 = \boxed{0}$

$P(-2) = (-2)^2 - 4 = 4 - 4 = \boxed{0}$

2   -2

Q $p(x) = x^2$

Zero = 0

★ Quadratic polynomial $\xrightarrow{d=2}$ Maximum (atmost) = 2 zeroes.

★ Cubic " $\xrightarrow{d=3}$ Maximum (atmost) = 3 zeroes.

★ Quartic " $\xrightarrow{d=4}$ Max → 4

Some Important Observations

(i) A constant polynomial does not have any zero

(ii) Every linear polynomial has one and only one zero.

(iii) 0 may or may not be the zero of a given polynomial

(iv) Number of zero of a polynomial cannot exceed its degree.

$p(x) = x-3$

$p(x) = x$

#Q. If 2 and 0 are the zeroes of the polynomial $f(x) = 2x^3 - 5x^2 + ax + b$ then find the values of a and b.

$$f(x) = 2x^3 - 5x^2 + ax + b \quad \begin{matrix} 2 \\ 0 \end{matrix} = x$$

$$\boxed{f(2) = 0}$$

$$2(2)^3 - 5(2)^2 + a(2) + b = 0$$

$$16 - 20 + 2a + b = 0$$

$$-4 + 2a + b = 0$$

$$-4 + 2a + 0 = 0$$

$$-4 + 2a = 0$$

$$2a = 4$$

$$a = 2$$

$$\boxed{f(0) = 0}$$

$$2(0)^3 - 5(0)^2 + a(0) + b = 0$$

$$0 - 0 + 0 + b = 0$$

$$b = 0$$

Last class wid pp abh bann Jaegi.

Geometrical meaning of a zero of a Linear Polynomial

Example : $y = 2x + 3$

$F(x) = 2x+3$

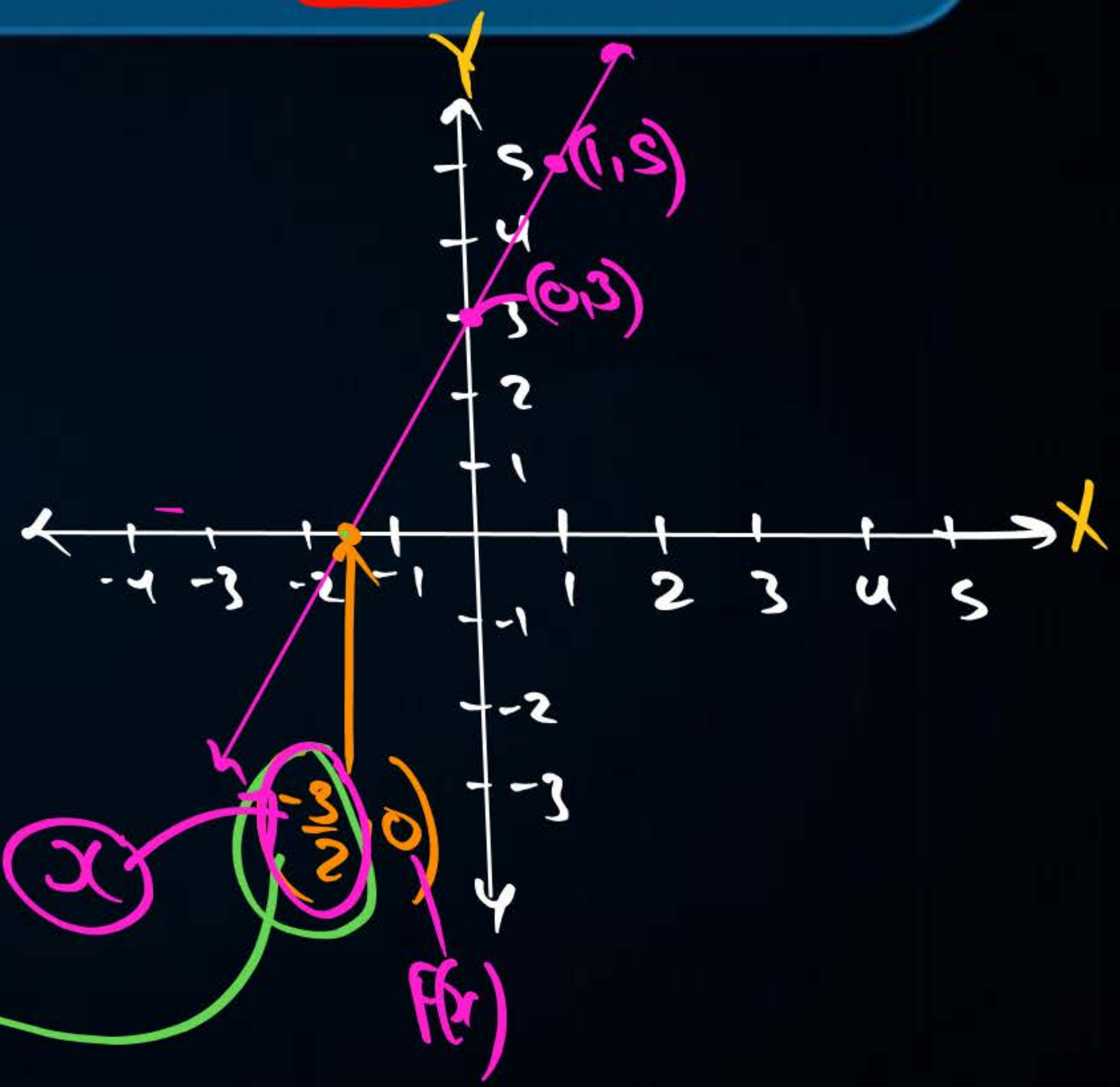
x	0	1
y	3	5

$F(x)$

$2x+3=0$

$\boxed{a = -\frac{3}{2}}$

$-\frac{3}{2}$ is the zero.



Geometrical meaning of zeroes of a polynomial

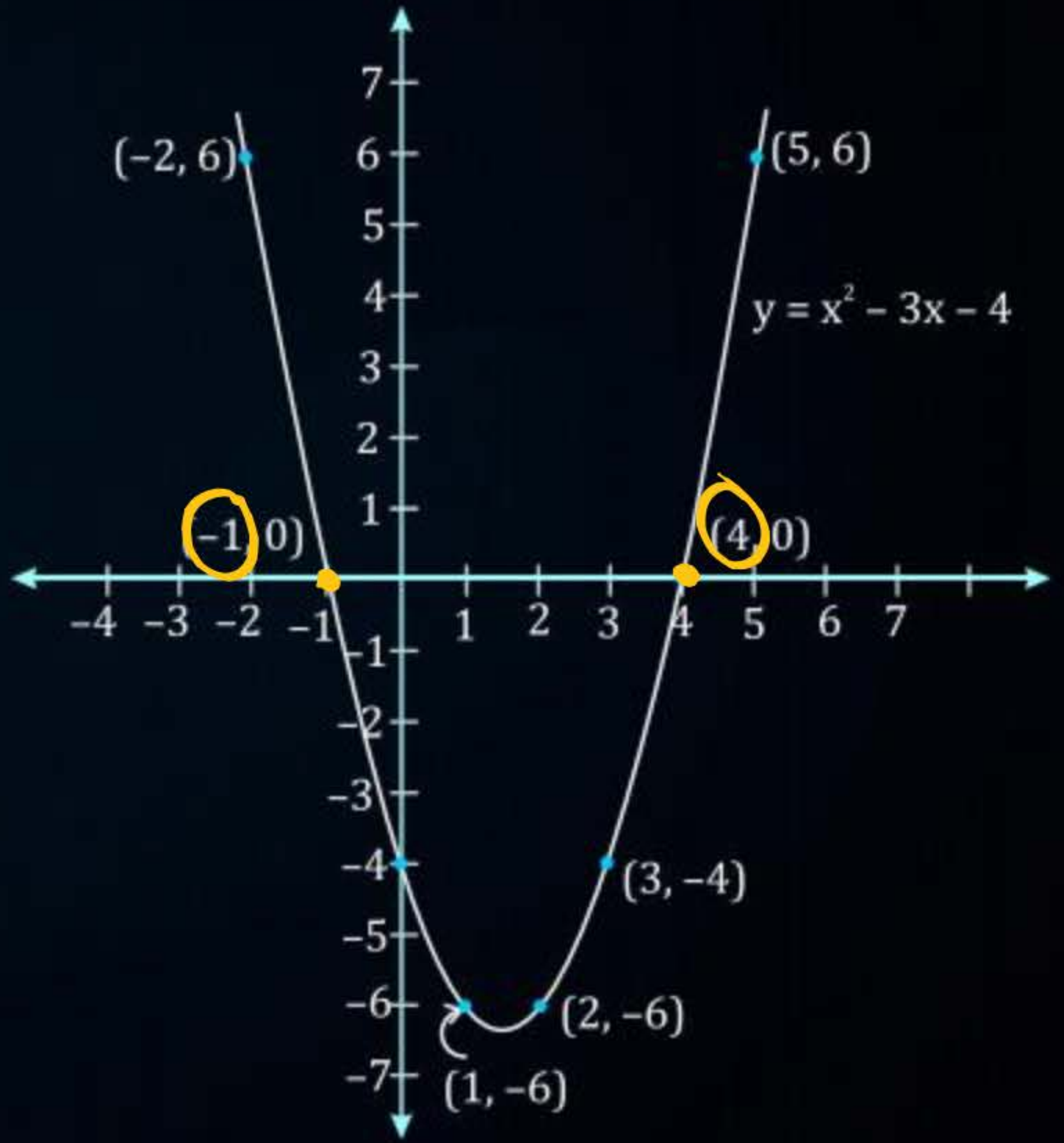
Quadratic

Parabola

zeroes

x	-2	-1	0	1	2	3	4	5
$y = x^2 - 3x - 4$	6	0	−4	−6	−6	−4	0	6

p(x)

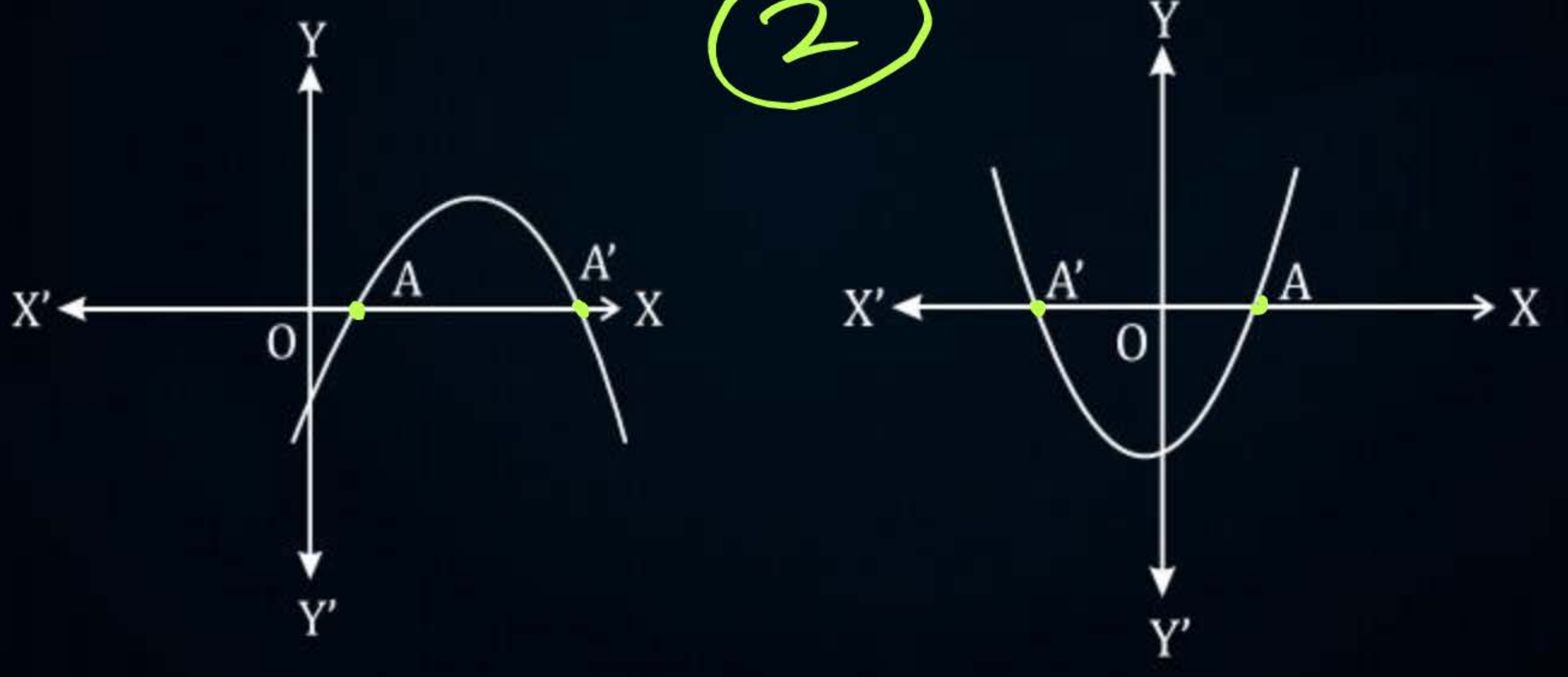


Case : (i)

ax^2+bx+c

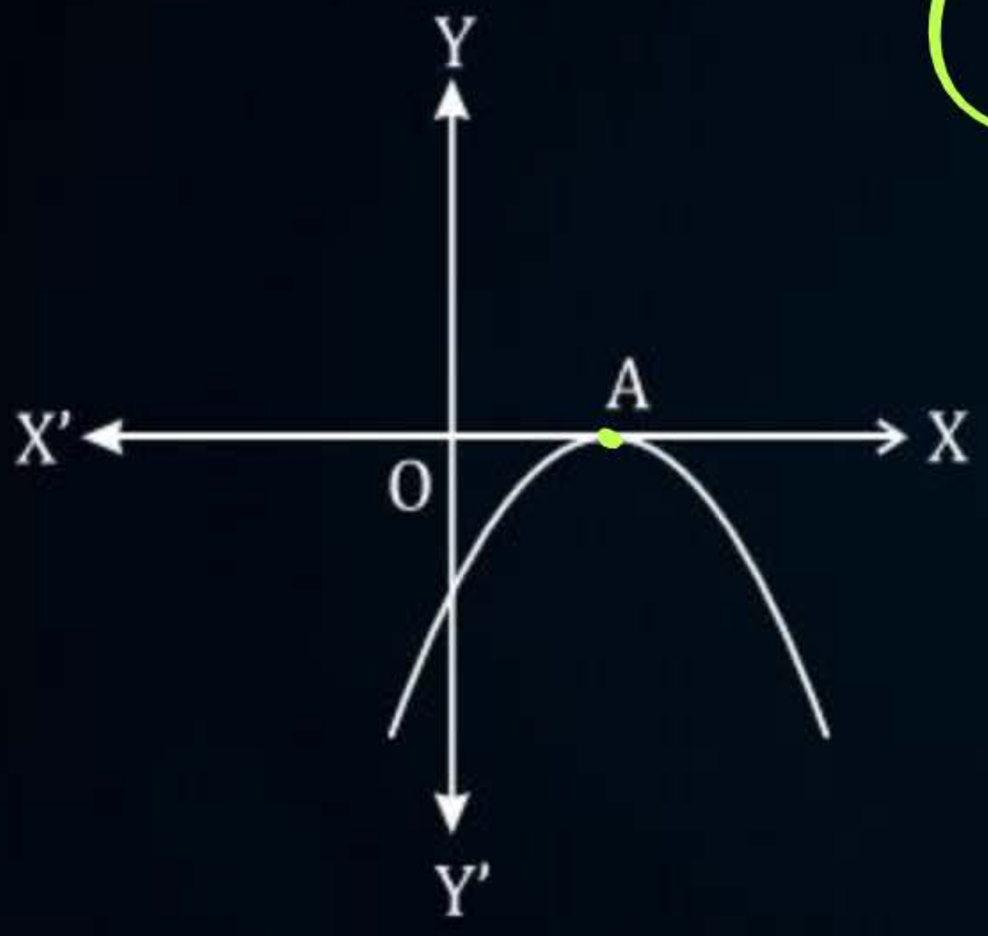
Here, the graph cuts x-axis at two distinct points A and A′.

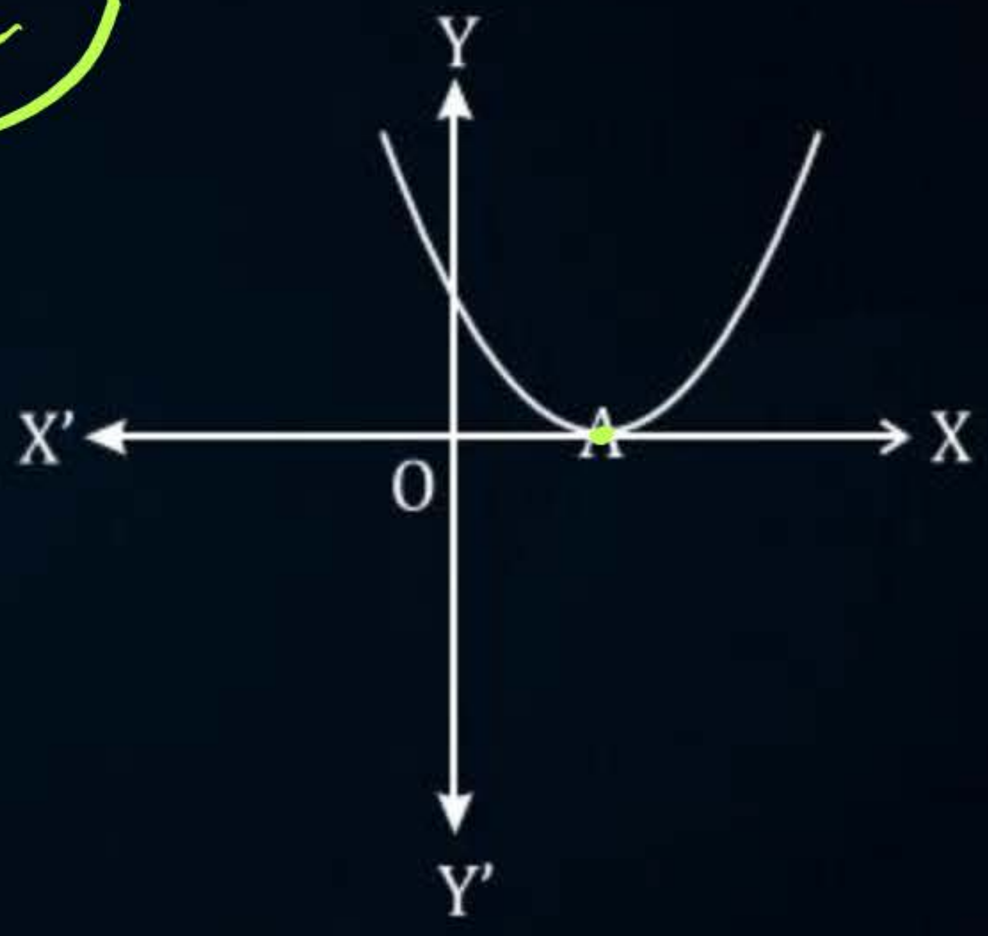
2



Case : (ii)

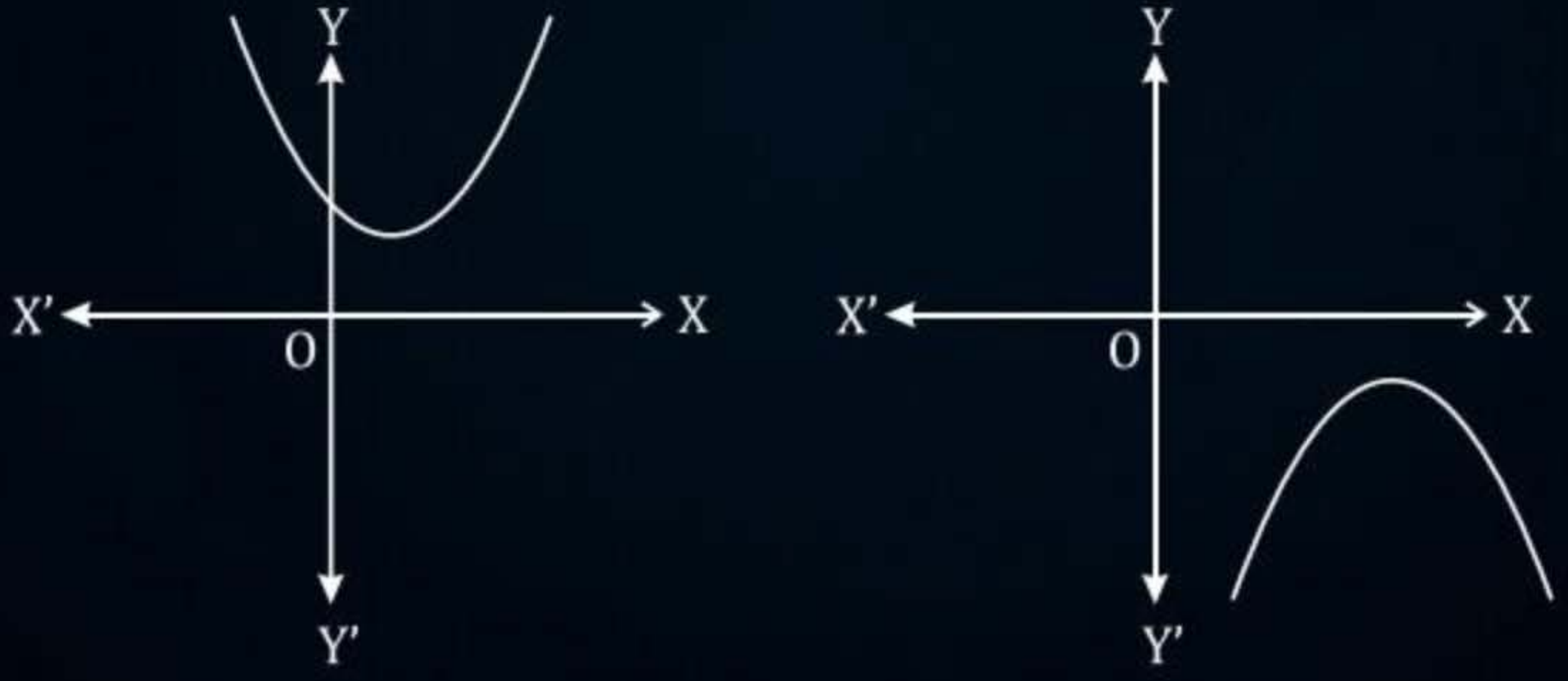
1

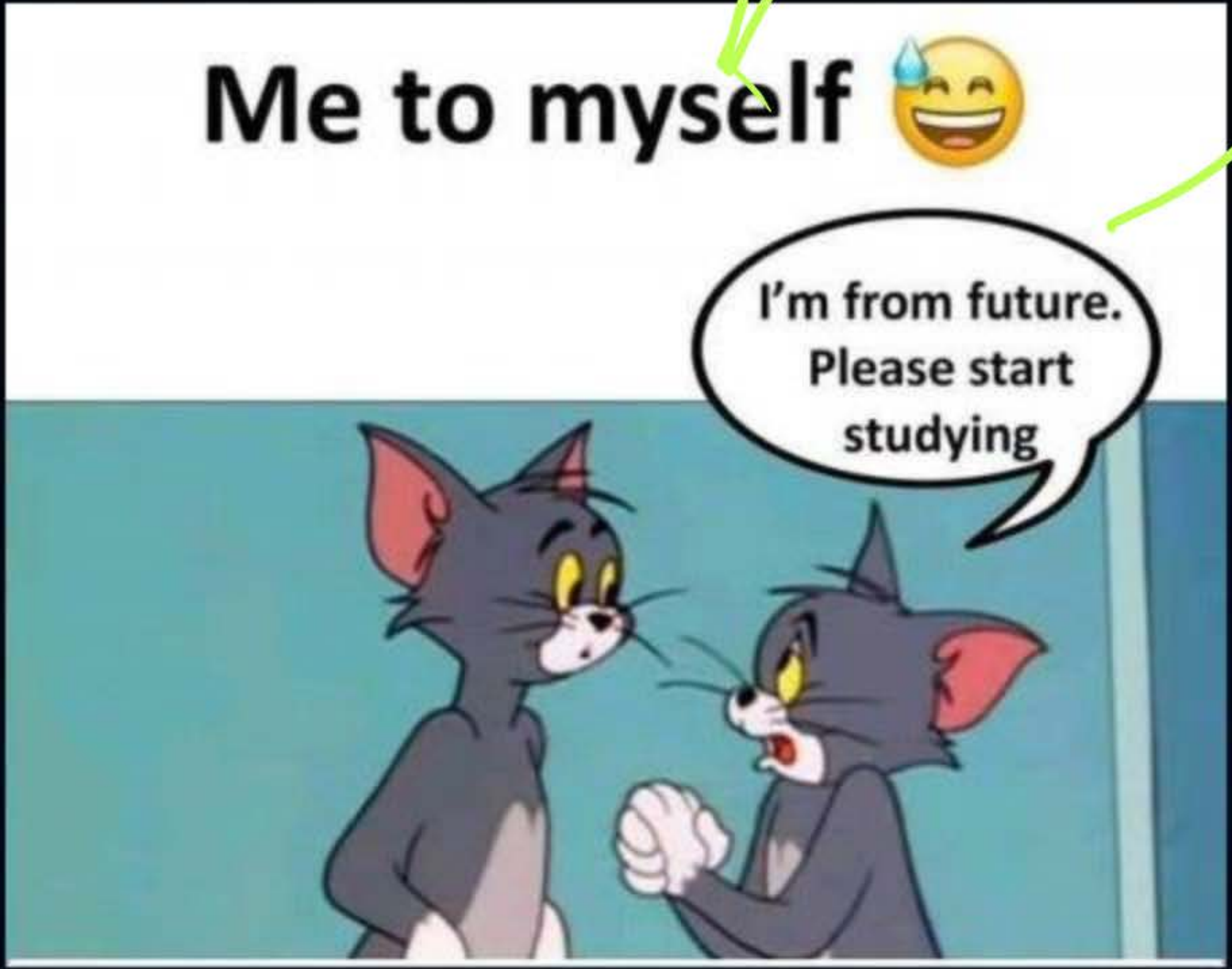




Case : (iii)

Here, the graph is either completely above the x-axis or completely below the x-axis. So, it does not cut the x-axis at any point. 0

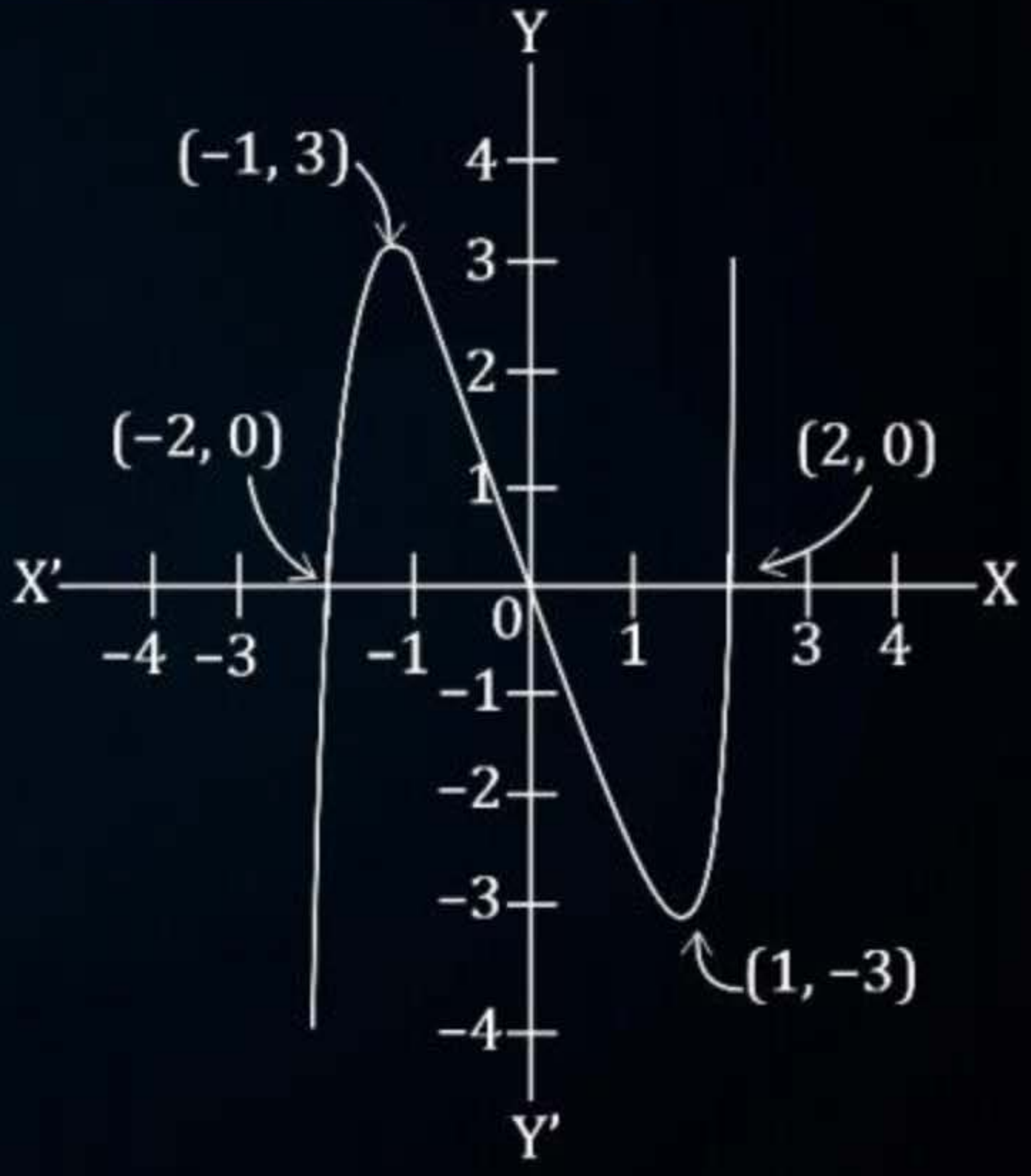



Me to myself
I'm from future. Please start studying

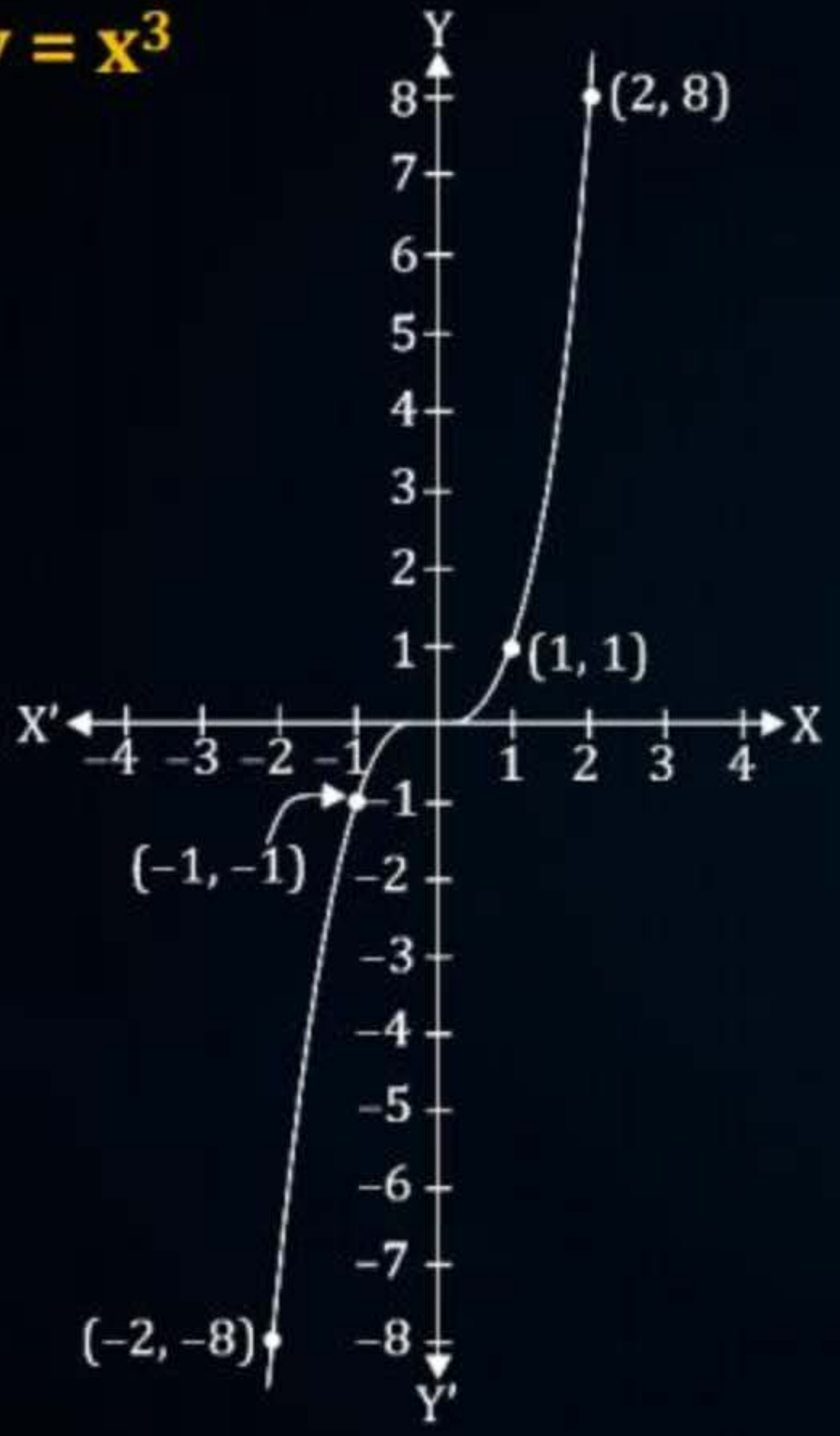
Geometrical meaning of zeroes of a polynomial

Cubic

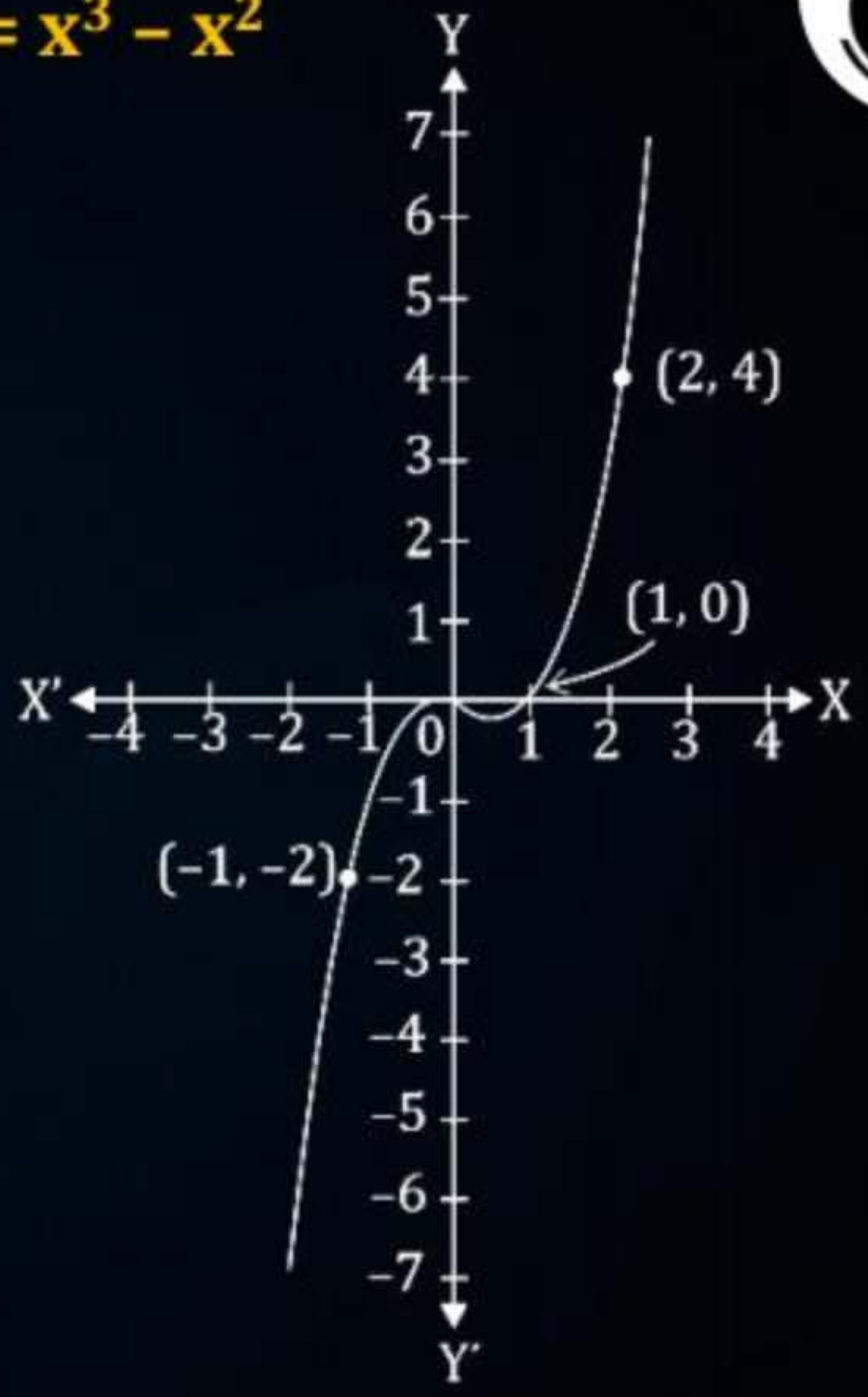
x	−2	−1	0	1	2
$y = x^3 - 4x$	0	3	0	−3	0



(i) $y = x^3$



(ii) $y = x^3 - x^2$

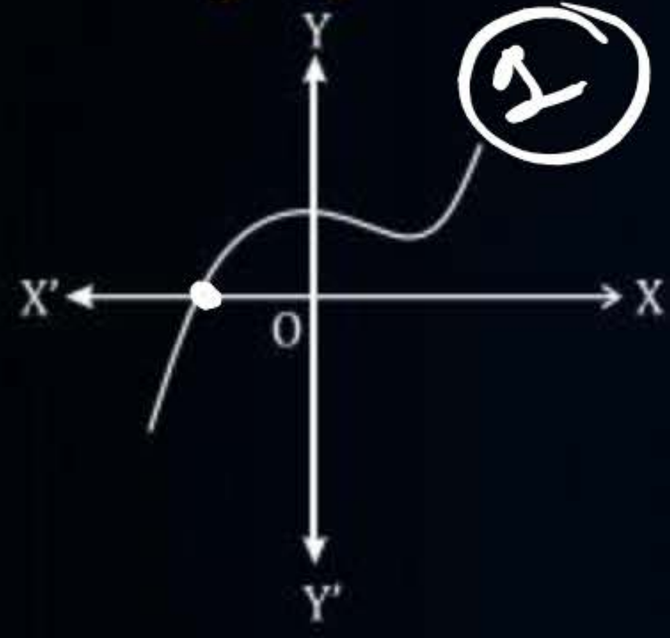


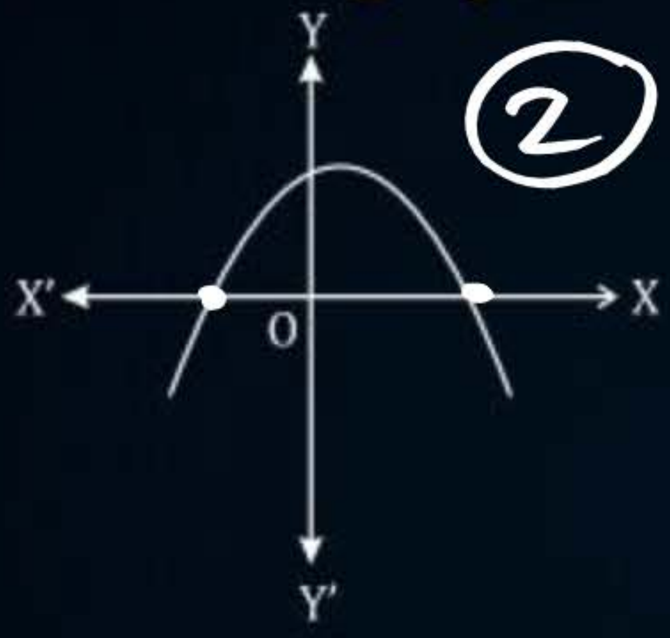
Remark

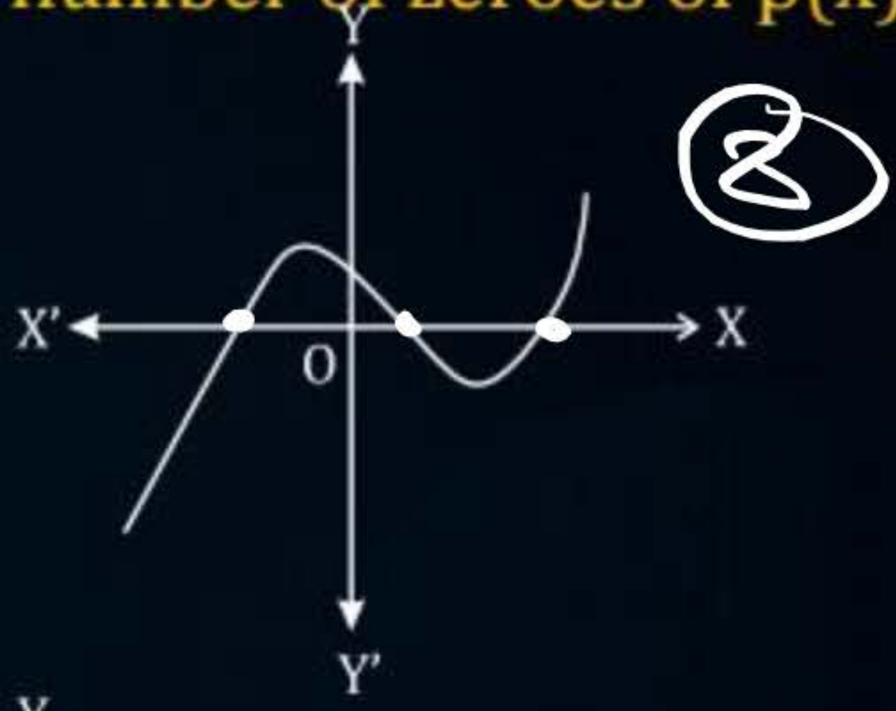
In general, given a polynomial $p(x)$ of degree n, the graph of $y = p(x)$ intersects the x-axis at atmost n points. Therefore, a polynomial $p(x)$ of degree n has at most n zeroes.

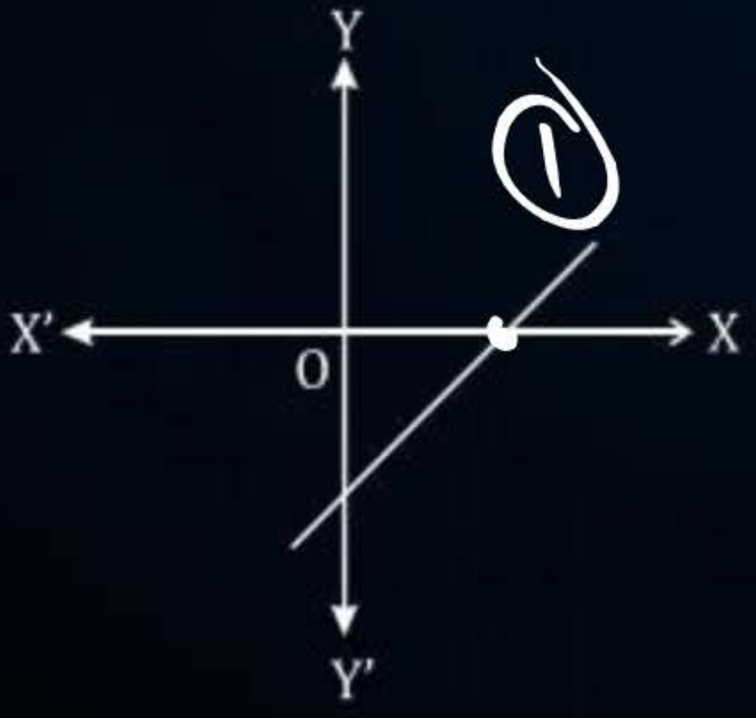
Topic : Zero of a Polynomial

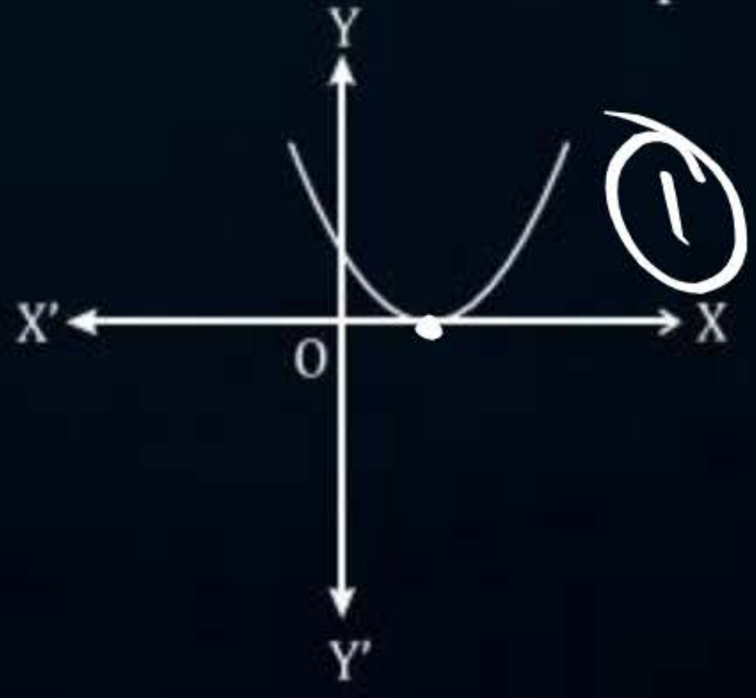
#Q. Look at the graphs given below. Each is the graph of $y = p(x)$, where $p(x)$ is a polynomial. For each of the graph, find the number of zeroes of $p(x)$.

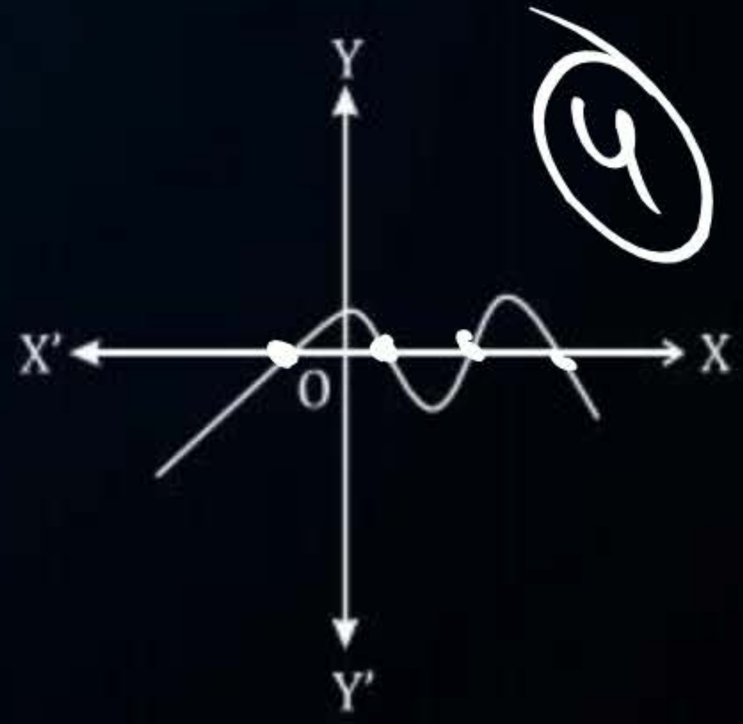












Topic : Zero of a Polynomial

#Q. If one of the zeroes of the quadratic polynomial $(k-1)x^2 + kx + 1$ is -3, then the value of k is

(A) $\frac{4}{3}$

H.W

(B) $\frac{-4}{3}$

(C) $\frac{2}{3}$

(D) $\frac{-2}{3}$

Topic : Zero of a Polynomial

DLU

#Q. Are the following statements 'True' or 'False'? Justify your answer.

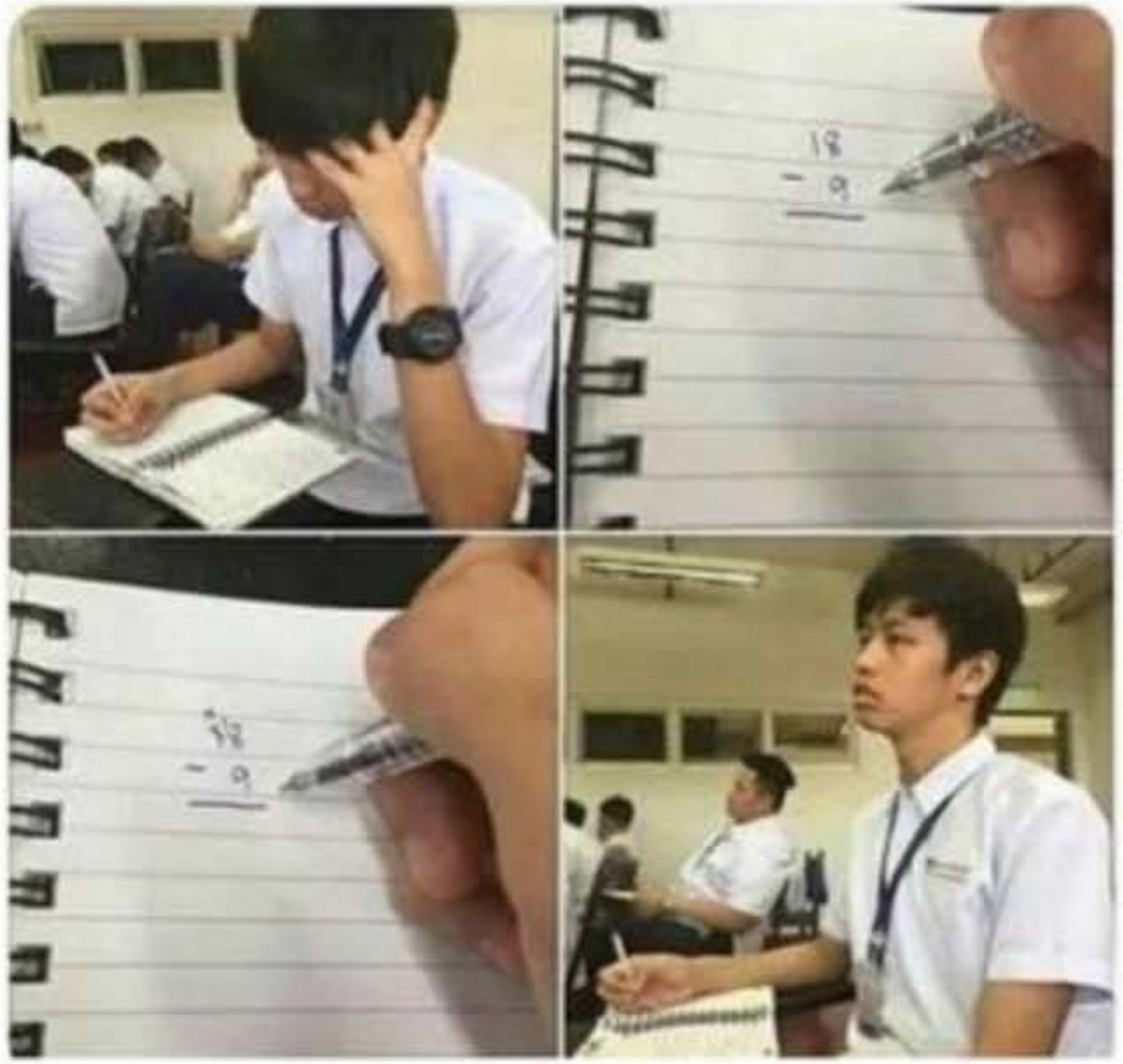
(i) If the graph of a polynomial intersects the X-axis at only one point, it cannot be a quadratic polynomial.

(ii) If the graph of a polynomial intersects the X-axis at exactly two points, it need not be a quadratic polynomial.

HW

How good you are in mathematics ?

Me :



Homework

DPP - Last class.


THANK
YOU