UDAAN 2024

– FOR CLASS 10th STUDENTS

Mathematics

Trigonometry

Practice Sheet Discussion

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Today's Targets

1. Practice Sheet Discussion

$\sin^2\theta + \cos^2\theta = 1$

$\rightarrow \sin^2\theta = 1 - \cos^2\theta$

$\rightarrow \cos^2\theta = 1 - \sin^2\theta$

$\sec^2\theta = 1 + \tan^2\theta$

$\rightarrow \sec^2\theta - \tan^2\theta = 1$

$\rightarrow \sec^2\theta - 1 = \tan^2\theta$

Level - 01

$\operatorname{cosec}^2\theta - \cot^2\theta = 1$

$\rightarrow \operatorname{cosec}^2\theta = 1 + \cot^2\theta$

$\rightarrow \operatorname{cosec}^2\theta - 1 = \cot^2\theta$

"OR"

$\sin\theta \leftrightarrow \cos\theta$

$\sec\theta \leftrightarrow \csc\theta$

$\tan\theta \leftrightarrow \cot\theta$

"OR"

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$\cot\theta = \frac{\cos\theta}{\sin\theta}$

Question-01

Prove the following Trigonometric Identities:

$(1 - \cos^2 A)\operatorname{cosec}^2 A = 1$

$= \sin^2\theta \times \frac{1}{\sin^2\theta}$

$= 1$

$=$ RHS

$=$ H.P

Question-02

Prove the following Trigonometric Identities:

$\tan^2\theta\cos^2\theta = 1 - \cos^2\theta$

$= \frac{\sin^2\theta}{\cos^2\theta} \times \cos^2\theta$

$= \sin^2\theta$

$= 1 - \cos^2\theta$

$= R.H.S$

$= H.P$

Question-03

Prove the following Trigonometric Identities:

$\operatorname{cosec}\theta\sqrt{1-\cos^2\theta} = 1$

→ L.H.S

$= \operatorname{cosec}\theta\sqrt{\sin^2\theta}$

$= \operatorname{cosec}\theta \times \sin\theta$

$= \frac{1}{\sin\theta} \times \sin\theta$

$= 1$

$=$ R.H.S

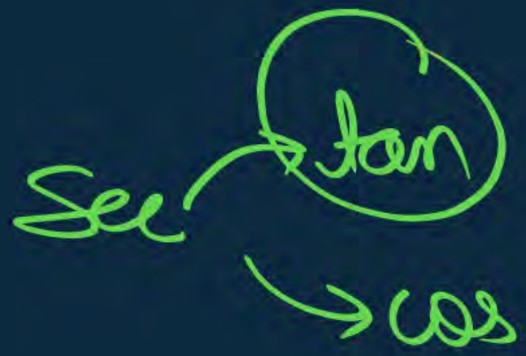
Question-04

Prove the following Trigonometric Identities:

$(\sec^2\theta - 1)(\operatorname{cosec}^2\theta - 1) = 1$

$\sec^2\theta = 1 + \tan^2\theta$

$\operatorname{cosec}^2\theta = 1 + \cot^2\theta$



$= \tan^2\theta \times \cot^2\theta$

$= \frac{1}{\cot^2\theta} \times \cot^2\theta$

$= 1$

Question-05

Prove the following Trigonometric Identities:

$$\frac{\tan\theta}{1} + \frac{1}{\tan\theta} = \sec\theta\,\mathrm{cosec}\theta$$

M-1

$$= \frac{\tan\theta}{1} + \frac{1}{\tan\theta}$$

$$= \frac{\tan^2\theta + 1}{\tan\theta}$$

$$= \frac{\sec^2\theta}{\tan\theta}$$

$$= \frac{\sec^2\theta / 1}{\sin\theta/\cos\theta}$$

$$= \frac{\cos\theta \times \sec^2\theta}{\sin\theta}$$

$$= \frac{\frac{1}{\sec\theta} \times \sec^2\theta}{\frac{1}{\cos\theta}}$$

$$= \sec\theta\cos\theta$$

Question-06

Prove the following Trigonometric Identities:

$$\frac{\cos\theta}{1-\sin\theta} = \frac{1+\sin\theta}{\cos\theta} = \sec\theta + \tan\theta$$

$\sin^2\theta + \cos^2\theta = 1$

$$= \frac{\cos\theta}{(1-\sin\theta)} \times \frac{(1+\sin\theta)}{(1+\sin\theta)} \quad (a-b)(a+b) = a^2-b^2$$

$$= \frac{\cos\theta(1+\sin\theta)}{1^2-\sin^2\theta}$$

$$= \frac{\cos\theta(1+\sin\theta)}{\cos^2\theta}$$

$$\frac{1+\sin\theta}{\cos\theta}$$

$$= \frac{1}{C} + \frac{S}{C}$$

$$= \sec\theta + \tan\theta$$

Question-07

Prove the following Trigonometric Identities:

$$\sin^2 A + \frac{1}{1 + \tan^2 A} = 1$$

$= \sin^2 A + \frac{1}{\sec^2 A}$ ⟶ $\sec^2\theta = 1 + \tan^2\theta$

$= \sin^2 A + \cos^2 A$ ⟶ $\frac{1}{\sec\theta} = \cos\theta$

$= 1$

Question-08

Prove the following Trigonometric Identities:

$$\sqrt{\frac{1-\cos\theta}{1+\cos\theta}} = \operatorname{cosec}\theta - \cot\theta$$

$$= \sqrt{\frac{1-\cos\theta}{1+\cos\theta}} \times \sqrt{\frac{1+\cos\theta}{1-\cos\theta}}$$

$$= \frac{\sqrt{(1-\cos\theta)} \times \sqrt{(1-\cos\theta)}}{\sqrt{(1+\cos\theta)} \times \sqrt{(1-\cos\theta)}}$$

$$= \frac{\sqrt{(1-\cos\theta)^2}}{\sqrt{1^2-\cos^2\theta}}$$

$$= \frac{1-\cos\theta}{\sqrt{\sin^2\theta}}$$

$$= \frac{1-\cos\theta}{\sin\theta}$$

$$= \frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}$$

R.H.S

Question-09

Prove the following Trigonometric Identities:

$$\frac{1-\cos\theta}{\sin\theta} = \frac{\sin\theta}{1+\cos\theta}$$

L.H.S

$$= \frac{1-\cos\theta}{\sin\theta} \times \frac{(1+\cos\theta)}{(1+\cos\theta)}$$

$$= \frac{1-\cos^2\theta}{\sin\theta(1+\cos\theta)}$$

$$= \frac{\sin^2\theta}{\sin\theta(1+\cos\theta)} = \frac{\sin\theta}{1+\cos\theta}$$

Question-10

Prove the following Trigonometric Identities:

$$\frac{1-\sin\theta}{1+\sin\theta} = (\sec\theta - \tan\theta)^2$$

$$= \frac{1-\sin\theta}{1+\sin\theta} \times \frac{1-\sin\theta}{1-\sin\theta}$$

$$= \frac{(1-\sin\theta)^2}{1-\sin^2\theta}$$

$$= \frac{(1-\sin\theta)^2}{\cos^2\theta}$$

$$= \left(\frac{1-\sin\theta}{\cos\theta}\right)^2$$

$$= \left(\frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta}\right)^2$$

$$= \boxed{(\sec\theta - \tan\theta)^2}$$

$$\frac{a^2}{b^2} = \left(\frac{a}{b}\right)^2$$

Question-11

Prove the following Trigonometric Identities:

$$\frac{(1+\cot^2\theta)\tan\theta}{\sec^2\theta} = \cot\theta$$

$$= \frac{\operatorname{cosec}^2\theta \times \tan\theta}{\sec^2\theta}$$

$$= \frac{\frac{1}{\sin^2\theta} \times \frac{\sin\theta}{\cos\theta}}{\frac{1}{\cos^2\theta}}$$

$$= \frac{\sin\theta \times \cos^2\theta}{\sin^2\theta \times \cos\theta}$$

$$= \frac{\cos\theta}{\sin\theta}$$

$$= \cot\theta$$

Question-12

Prove the following Trigonometric Identities:

$\tan^2\theta - \sin^2\theta = \tan^2\theta\sin^2\theta$

$= \frac{\sin^2\theta}{\cos^2\theta} - \frac{\sin^2\theta}{1}$

$= \sin^2\theta\left(\frac{1}{\cos^2\theta} - 1\right)$

$= \sin^2\theta\left(\frac{1-\cos^2\theta}{\cos^2\theta}\right)$

$= \frac{\sin^2\theta\sin^2\theta}{\cos^2\theta}$

$= \tan^2\theta \times \sin^2\theta$

Question-13

Prove the following Trigonometric Identities:

$(\sec\theta + \cos\theta)(\sec\theta - \cos\theta) = \tan^2\theta + \sin^2\theta$

$= \sec^2\theta - \cos^2\theta$

$= 1 + \tan^2\theta - (1 - \sin^2\theta)$

$= 1 + \tan^2\theta - 1 + \sin^2\theta$

$= \tan^2\theta + \sin^2\theta$

Using identities

$\sec^2\theta = 1 + \tan^2\theta$

$\sin^2\theta + \cos^2\theta = 1$

Question-14

Prove the following Trigonometric Identities:

$\sec A(1-\sin A)(\sec A+\tan A)=1$

$$=\left(\frac{1}{\cos A}\right)\left(1-\sin A\right)\left(\frac{1}{\cos A}+\frac{\sin A}{\cos A}\right)$$

$$=\left(\frac{1}{\cos A}\right)\left(1-\sin A\right)\left(\frac{1+\sin A}{\cos A}\right)$$

$$=\frac{1-\sin^2 A}{\cos^2 A}=\frac{\cos^2 A}{\cos^2 A}=\boxed{1}$$

Question-15

Prove the following Trigonometric Identities:

$(\text{cosec}A - \sin A)(\sec A - \cos A)(\tan A + \cot A) = 1$

$$= \left(\frac{1}{S} - S\right)\left(\frac{1}{C} - C\right)\left(\frac{S}{C} + \frac{C}{S}\right)$$

$$= \left(\frac{1-S^2}{S}\right)\left(\frac{1-C^2}{C}\right)\left(\frac{S^2+C^2}{SC}\right)$$

$$= \frac{C^2}{S} \times \frac{S^2}{C} \times \frac{1}{CS}$$

$$= \frac{C^2S^2}{C^2S^2} = 1$$

Question-16

Prove the following Trigonometric Identities:

$\sin^2 A \cot^2 A + \cos^2 A \tan^2 A = 1$

$= \sin^2 A \times \frac{\cos^2 A}{\sin^2 A} + \cos^2 A \frac{\sin^2 A}{\cos^2 A}$

$= \cos^2 A + \sin^2 A$

$= 1$

Question-17

Prove the following Trigonometric Identities:

$$\cot\theta - \tan\theta = \frac{2\cos^2\theta - 1}{\sin\theta\cos\theta}$$

$$= \frac{\cos\theta}{\sin\theta} - \frac{\sin\theta}{\cos\theta}$$

$$= \frac{\cos^2\theta - \sin^2\theta}{\sin\theta\cos\theta}$$

$$= \frac{\cos^2\theta - (1 - \cos^2\theta)}{\sin\theta\cos\theta}$$

$$= \frac{\cos^2\theta - 1 + \cos^2\theta}{\sin\theta\cos\theta}$$

$$= \boxed{\frac{2\cos^2\theta - 1}{\sin\theta\cos\theta}}$$

Question-18

Prove the following Trigonometric Identities:

$$\frac{\cos^2\theta}{\sin\theta} - \operatorname{cosec}\theta + \sin\theta = 0$$

$$= \frac{\cos^2\theta}{\sin\theta} - \frac{1}{\sin\theta} + \frac{\sin\theta}{1}$$

$$= \frac{\cos^2\theta - 1 + \sin^2\theta}{\sin\theta}$$

$$= \frac{1-1}{\sin\theta} = \frac{0}{\sin\theta} = 0$$

Question-19

Prove the following Trigonometric Identities:

$$\frac{1}{1+\sin A}+\frac{1}{1-\sin A}=2\sec^2 A$$

$$= \frac{(1-s)+(1+s)}{(1+s)(1-s)}$$

$$= \frac{1-s+1+s}{1-s^2}$$

$$= \frac{2}{c^2} = 2\sec^2\theta$$

Q.20 [NCERT]

Prove the following Trigonometric Identities:

$$\frac{1+\sin\theta}{\cos\theta}+\frac{\cos\theta}{1+\sin\theta}=2\sec\theta$$

$$= \frac{(1+\sin\theta)^2+\cos^2\theta}{\cos\theta(1+\sin\theta)}$$

$$= \frac{1^2+\sin^2\theta+2\sin\theta+\cos^2\theta}{\cos\theta(1+\sin\theta)} \quad (1)$$

$$= \frac{2+2\sin\theta}{\cos\theta(1+\sin\theta)}$$

$$= \frac{2(1+\sin\theta)}{\cos\theta(1+\sin\theta)}$$

$$= 2\sec\theta$$

Question-21

Prove the following Trigonometric Identities:

$$\frac{(1+\sin\theta)^2+(1-\sin\theta)^2}{2\cos^2\theta}=\frac{1+\sin^2\theta}{1-\sin^2\theta}$$

$$=\frac{1^2+s^2+2s+1^2+s^2-2s}{2(1-s^2)}$$

$$=\frac{2+2s^2}{2(1-s^2)}$$

$$=\frac{2(1+s^2)}{2(1-s^2)}$$

$$\frac{1+s^2}{1-s^2}$$

Q.22 [NCERT]

$$(a-b)^2 = (b-a)^2$$

Prove the following Trigonometric Identities:

$$\frac{1+\tan^2\theta}{1+\cot^2\theta} = \left(\frac{1-\tan\theta}{1-\cot\theta}\right)^2 = \tan^2\theta$$

L.H.S

$$= \frac{\sec^2\theta}{\operatorname{cosec}^2\theta}$$

$$= \frac{\frac{1}{\cos^2\theta}}{\frac{1}{\sin^2\theta}}$$

$$= \frac{\sin^2\theta}{\cos^2\theta} = \tan^2\theta$$

R.H.S

$$= \frac{(1-\tan\theta)^2}{\left(1-\frac{1}{\tan\theta}\right)^2}$$

$$= \frac{(1-\tan\theta)^2}{\left(\frac{\tan\theta-1}{\tan\theta}\right)^2}$$

$$= \frac{(1-\tan\theta)^2}{1}$$

$$= \frac{(1-\tan\theta)^2}{\frac{(\tan\theta-1)^2}{\tan^2\theta}}$$

$$= \frac{\tan^2\theta(1-\tan\theta)^2}{(\tan\theta-1)^2}$$

$$= \tan^2\theta$$

H.P

Question-24

Prove the following Trigonometric Identities:

$$\frac{(1+\tan^2\theta)\cot\theta}{\operatorname{cosec}^2\theta} = \tan\theta$$

$$= \frac{\sec^2\theta \times \cot\theta}{\operatorname{cosec}^2\theta}$$

$$= \frac{\frac{1}{C^2} \times \frac{C}{S}}{\frac{1}{S^2}}$$

$$= \frac{S^{\cancel{2}}\,C}{C^{\cancel{2}}\,\cancel{S}}$$

$$= \frac{S}{C}$$

$$= \tan\theta$$

Question-25

Prove the following Trigonometric Identities:

$$\frac{1+\cos A}{\sin^2 A} = \frac{1}{1-\cos A}$$

$$= \frac{1+\cos A}{1-\cos^2 A}$$

$1^2 - \cos^2 A = a^2 - b^2$

$= (a+b)(a-b)$

$$= \frac{1+\cos A}{(1+\cos A)(1-\cos A)}$$

$$= \frac{1}{1-\cos A}$$

Question-26

Prove the following Trigonometric Identities:

$$\frac{\sec A - \tan A}{\sec A + \tan A} = \frac{\cos^2 A}{(1+\sin A)^2}$$

$$= \frac{\frac{1}{C} - \frac{S}{C}}{\frac{1}{C} + \frac{S}{C}}$$

$$= \frac{\frac{1-S}{C}}{\frac{1+S}{C}}$$

$$= \frac{1-S}{1+S} \times \frac{(1+S)}{(1+S)}$$

$$= \frac{1-S^2}{(1+S)^2}$$

$$= \boxed{\frac{\cos^2 A}{(1+\sin A)^2}}$$

Q.23 [NCERT]

Prove the following Trigonometric Identities:

$$\frac{1+\sec\theta}{\sec\theta}=\frac{\sin^2\theta}{1-\cos\theta}$$

$= \dfrac{1+\frac{1}{c}}{\frac{1}{c}}$

$= \dfrac{\frac{c+1}{c}}{\frac{1}{c}}$

$c+1$

$= 1+\cos\theta \times \dfrac{1-\cos\theta}{1-\cos\theta}$

$= \dfrac{1^2-\cos^2\theta}{1-\cos\theta}$

$= \dfrac{\sin^2\theta}{1-\cos\theta}$

Question-27

Prove the following Trigonometric Identities:

$$\frac{1+\cos A}{\sin A}=\frac{\sin A}{1-\cos A}$$

FTW

Question-28

Prove the following Trigonometric Identities:

$$\sqrt{\frac{1+\sin A}{1-\sin A}} = \sec A + \tan A$$

HW

Question-29

Prove the following Trigonometric Identities:

$$\sqrt{\frac{1-\cos A}{1+\cos A}} + \sqrt{\frac{1+\cos A}{1-\cos A}} = 2\operatorname{cosec} A$$

$$= \frac{\sqrt{1-\cos A}}{\sqrt{1+\cos A}} \times \frac{\sqrt{1-\cos A}}{\sqrt{1-\cos A}}$$

$$= \frac{\sqrt{(1-\cos A)^2}}{\sqrt{1-\cos^2 A}}$$

$$= \frac{1-\cos A}{\sqrt{\sin^2 A}} = \frac{1-\cos A}{\sin A}$$

$$\frac{1+\cos A}{\sin A}$$

$$= \frac{1-\cos A}{\sin A} + \frac{1+\cos A}{\sin A}$$

$$= \frac{1-\cancel{\cos A}+1+\cancel{\cos A}}{\sin A}$$

$$= \frac{2}{\sin A} = 2\operatorname{cosec} A$$

Question-30

Prove the following Trigonometric Identities:

$$(\sec A - \tan A)^2 = \frac{1 - \sin A}{1 + \sin A}$$

$$= \left(\frac{1}{c} - \frac{s}{c}\right)^2$$

$$= \left(\frac{1-s}{c}\right)^2$$

$$= \frac{(1-s)^2}{c^2}$$

$$= \frac{(1-s)^2}{(1-s^2)}$$

$$= \frac{(1-s)(1-s)}{(1-s)(1+s)}$$

$$= \frac{1 - \sin A}{1 + \sin A}$$

Question-31

Prove the following Trigonometric Identities:

$$\frac{1}{\sec A - 1} + \frac{1}{\sec A + 1} = 2\operatorname{cosec} A \cot A$$

$\frac{1}{S} \times \frac{C}{S}$

$$= \frac{1}{\frac{1}{C} - 1} + \frac{1}{\frac{1}{C} + 1}$$

$$= \frac{1}{\frac{1-C}{C}} + \frac{1}{\frac{1+C}{C}}$$

$$= \frac{C}{1-C} + \frac{C}{1+C}$$

$$= \frac{C(1+C) + C(1-C)}{(1-C)(1+C)}$$

$$= \frac{C + C^2 + C - C^2}{1 - C^2}$$

$$= \frac{2C}{S^2}$$

$$= \frac{2 \times C}{S \times S}$$

$2\operatorname{cosec}\theta\cot\theta$

Question-32

Prove the following Trigonometric Identities:

$$\frac{\tan A + \tan B}{\cot A + \cot B} = \tan A \tan B$$

$$= \frac{\tan A + \tan B}{\frac{1}{\tan A} + \frac{1}{\tan B}}$$

$$= \frac{\tan A + \tan B}{\frac{\tan B + \tan A}{\tan A \tan B}}$$

$$\rightarrow \frac{\tan A \tan B (\tan A + \tan B)}{(\tan B + \tan A)}$$

$$= \tan A \tan B$$

Question-33

Prove the following Trigonometric Identities:

$\tan^2 A\sec^2 B - \sec^2 A\tan^2 B = \tan^2 A - \tan^2 B$

$= \tan^2 A(1+\tan^2 B) - (1+\tan^2 A)\tan^2 B$

$= \tan^2 A + \tan^2 A\tan^2 B - \tan^2 B - \tan^2 B\tan^2 A$

$= \tan^2 A - \tan^2 B$

Level - 02

$$\sec\theta - \tan\theta = \frac{1}{\sec\theta + \tan\theta}$$

$$\sec\theta + \tan\theta = \frac{1}{\sec\theta - \tan\theta}$$

$$\operatorname{cosec}\theta - \cot\theta = \frac{1}{\operatorname{cosec}\theta + \cot\theta}$$

$$\operatorname{cosec}\theta + \cot\theta = \frac{1}{\operatorname{cosec}\theta - \cot\theta}$$

Question-01

Prove the following trigonometric Identities :

$$\frac{1+\cos\theta+\sin\theta}{1+\cos\theta-\sin\theta}=\frac{1+\sin\theta}{\cos\theta}$$

$\sec\theta+\tan\theta$

$$\frac{1+\cos\theta+\sin\theta}{\cos\theta}$$

$$=\frac{1+\cos\theta+\sin\theta}{1+\cos\theta-\sin\theta}$$

$$=\frac{\sec\theta+1+\tan\theta}{\sec\theta+1-\tan\theta}$$

$$=\frac{\sec\theta+\tan\theta+1}{\sec\theta+1-\tan\theta}$$

$$=\frac{\frac{1}{\sec\theta-\tan\theta}+\frac{1}{1}}{\sec\theta+1-\tan\theta}$$

$$=\frac{\frac{+1+\sec\theta-\tan\theta}{\sec\theta-\tan\theta}}{\frac{\sec\theta+1-\tan\theta}{1}}$$

$$= \frac{1}{\sec\theta \tan\theta}$$

$$= \sec\theta + \tan\theta$$

$$= \frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta}$$

$$= \frac{1+\sin\theta}{\cos\theta}$$

Q.02 [CBSE 2001, NCERT]

Prove the following trigonometric Identities :

$$\frac{\sin\theta - \cos\theta + 1}{\sin\theta + \cos\theta - 1} = \frac{1}{\sec\theta - \tan\theta}$$

H.W

Q.03 **[NCERT Exemplar]**

Prove the following trigonometric Identities :

$(\sin\theta + \cos\theta)(\tan\theta + \cot\theta) = \sec\theta + \operatorname{cosec}\theta$

$= (s+c)\left(\frac{s}{c} + \frac{c}{s}\right)$

$= (s+c)\left(\frac{s^2+c^2}{sc}\right)$ (1)

$= \frac{s+c}{sc}$

$= \frac{s}{sc} + \frac{c}{sc}$

$\sec\theta + \operatorname{cosec}\theta$

Q.04 [CBSE 2005]

Prove the following trigonometric Identities :

$$\frac{1}{\sec A + \tan A} - \frac{1}{\cos A} = \frac{1}{\cos A} - \frac{1}{\sec A - \tan A}$$

L.H.S

$= (\sec A - \tan A) - \sec A$

$= -\tan A$

R.H.S

$= \sec A - (\sec A + \tan A)$

$= \sec A - \sec A - \tan A$

$= -\tan A$

H.P

Q.05 **[NCERT Exemplar]**

Prove the following trigonometric Identities :

$$\frac{\tan A}{1+\sec A}-\frac{\tan A}{1-\sec A}=2\operatorname{cosec} A$$

$\sin^2\theta = 1-\cos^2\theta$

$-\sin^2\theta = -1+\cos^2\theta$

$$= \tan A\left(\frac{1}{1+\sec A}-\frac{1}{1-\sec A}\right)$$

$$= \frac{\sin A}{\cos A}\left(\frac{1}{1+\frac{1}{c}}-\frac{1}{1-\frac{1}{c}}\right)$$

$$= \frac{s}{c}\left(\frac{1}{\frac{c+1}{c}}-\frac{1}{\frac{c-1}{c}}\right)$$

$$= \frac{s}{c}\left(\frac{c}{c+1}-\frac{c}{c-1}\right)$$

$$= \frac{s}{c}\left[\frac{(c-1)-(c+1)}{(c+1)(c-1)}\right]$$

$$= \frac{s}{c}\left[\frac{c-1-c-1}{c^2-1}\right]$$

$$= \frac{s}{c}\times\frac{-2}{c^2-1}$$

$$\frac{s}{c}\times\frac{+2\times c}{+s^2}$$

$$= \operatorname{cosec} A$$

Q.06 [CBSE 2010]

If $\text{cosec}\theta = 2x$ and $\cot\theta = \frac{2}{x}$, find the value of $2\left(x^2 - \frac{1}{x^2}\right)$

$\text{cosec}^2\theta = (2x)^2$ $\quad$ $\cot^2\theta = \left(\frac{2}{x}\right)^2$

$\text{cosec}^2\theta = 4x^2$ $\quad$ $\cot^2\theta = \frac{4}{x^2}$

$\text{cosec}^2\theta - \cot^2\theta = 4x^2 - \frac{4}{x^2}$

$1 = 4\left(a^2 - \frac{1}{a^2}\right)$

$\frac{1}{2} = 2\left(x^2 - \frac{1}{a^2}\right)$

Ans//

Question-07

If $\operatorname{cosec}\theta + \cot\theta = m$ and $\operatorname{cosec}\theta - \cot\theta = n$, prove that $mn = 1$.

$= (\operatorname{cosec}\theta + \cot\theta)(\operatorname{cosec}\theta - \cot\theta)$

$= \operatorname{cosec}^2\theta - \cot^2\theta$

$= 1$

Question-08

$a^2+b^2=(a+b)^2-2ab$

Prove the following trigonometric Identities :

$$\frac{\tan^3\theta}{1+\tan^2\theta}+\frac{\cot^3\theta}{1+\cot^2\theta}=\sec\theta\,\text{cosec}\theta-2\sin\theta\cos\theta$$

$$= \frac{\tan^3\theta}{\sec^2\theta}+\frac{\cot^3\theta}{\text{cosec}^2\theta}$$

$$= \frac{\sin^3\theta}{\cos^3\theta}\times\frac{\cos^2\theta}{1}+\frac{\cos^3\theta}{\sin^3\theta}\times\frac{\sin^2\theta}{1}$$

$$= \frac{\sin^3\theta}{\cos\theta}+\frac{\cos^3\theta}{\sin\theta}$$

$$= \frac{\sin^4\theta+\cos^4\theta}{\cos\theta\sin\theta}$$

$$= \frac{(\sin^2\theta)^2+(\cos^2\theta)^2}{\cos\theta\sin\theta}$$

$$= \frac{(\sin^2\theta+\cos^2\theta)^2-2\sin^2\theta\cos^2\theta}{\cos\theta\sin\theta}$$

$$= \frac{1-2s^2c^2}{cs}$$

Hw

Question-09

If $x = a\cos^3\theta, y = b\sin^3\theta$, prove that $\left(\frac{x}{a}\right)^{2/3} + \left(\frac{y}{b}\right)^{2/3} = 1$.

$$= \left(\frac{a\cos^3\theta}{a}\right)^{2/3} + \left(\frac{b\sin^3\theta}{b}\right)^{2/3}$$

$$= (\cos^3\theta)^{2/3} + (\sin^3\theta)^{2/3}$$

$$= (\cos\theta)^{3\times\frac{2}{3}} + (\sin\theta)^{3\times\frac{2}{3}}$$

$$= \cos^2\theta + \sin^2\theta$$

$$= 1$$

$$= (a^3)^2 = a^{3\times2} = a^6$$

$$= (a^3)^{2/3} = a^{3\times\frac{2}{3}} = a^2$$

Question-10

If $a\cos\theta + b\sin\theta = m$ and $a\sin\theta - b\cos\theta = n$ prove that $a^2 + b^2 = m^2 + n^2$.

$$m^2+n^2 = (a\cos\theta + b\sin\theta)^2 + (a\sin\theta - b\cos\theta)^2$$

$$= a^2\cos^2\theta + b^2\sin^2\theta + 2ab\sin\theta\cos\theta + a^2\sin^2\theta + b^2\cos^2\theta - 2ab\sin\theta\cos\theta$$

$$= a^2(\cos^2\theta+\sin^2\theta) + b^2(\sin^2\theta+\cos^2\theta)$$

$$m^2+n^2 = a^2+b^2$$

Question-11

If $\cos A + \cos^2 A = 1$, prove that $\sin^2 A + \sin^4 A = 1$.

Atpata question

- $\cos A = 1 - \cos^2 A$

$\cos A = \sin^2 A$

$(\cos A)^2 = (\sin^2 A)^2$

$\cos^2 A = \sin^4 A$

$= \cos A + \cos^2 A$

$= 1$

Question-12

Prove the following trigonometric Identities:

$$\left(\tan\theta + \frac{1}{\cos\theta}\right)^2 + \left(\tan\theta - \frac{1}{\cos\theta}\right)^2 = 2\left(\frac{1+\sin^2\theta}{1-\sin^2\theta}\right)$$

$$= \left(\frac{S}{C} + \frac{1}{C}\right)^2 + \left(\frac{S}{C} - \frac{1}{C}\right)^2$$

$$= \frac{(S+1)^2}{C^2} + \frac{(S-1)^2}{C^2}$$

$$= \frac{(S+1)^2 + (S-1)^2}{C^2}$$

$$= \frac{S^2+1+2S + S^2+1-2S}{1-S^2}$$

$$= \frac{2S^2+2}{1-S^2}$$

$$= \frac{2(S^2+1)}{(1-S^2)}$$

Question-13

Prove the following trigonometric Identities :

$(\sec A + \tan A - 1)(\sec A - \tan A + 1) = 2\tan A$

$$= \left(\frac{1}{c} + \frac{s}{c} - \frac{1}{1}\right)\left(\frac{1}{c} - \frac{s}{c} + \frac{1}{1}\right)$$

$$= \left(\frac{1+s-c}{c}\right)\left(\frac{1-s+c}{c}\right)$$

$$= \frac{1-s+c+s-s^2+sc-c+sc-c^2}{c^2}$$

$$= \frac{1+2sc-s^2-c^2}{c^2}$$

$$= \frac{1+2sc-(s^2+c^2)}{c^2}$$

$$= \frac{1+2sc-1}{c^2}$$

$$= \frac{2sc}{c^2}$$

$$= 2\tan A$$

$$= (1+S-C)(1-S+C)$$

$$= [(1)+(S-C)][(1)-(S-C)]$$

$$= a^2-b^2 = (1)^2-(S-C)^2$$

$$= 1-(S^2+C^2-2SC)$$

$$= 1-(1-2SC)$$

$$= 1-1+2SC$$

$$= \boxed{2SC}$$

Question-14

Prove the following trigonometric Identities :

$(1 + \cot A - \operatorname{cosec} A)(1 + \tan A + \sec A) = 2$

H.W

Question-15

Prove the following trigonometric Identities :

$$\frac{\cos A\operatorname{cosec} A - \sin A\sec A}{\cos A + \sin A} = \operatorname{cosec} A - \sec A$$

$$= \frac{C \times \frac{1}{S} - S \times \frac{1}{C}}{C+S}$$

$$= \frac{\frac{C}{S} - \frac{S}{C}}{\frac{C+S}{1}}$$

$$= \frac{\frac{C^2-S^2}{CS}}{\frac{C+S}{1}}$$

$$= \frac{(C^2-S^2)(1)}{(CS)(C+S)}$$

$$= \frac{(C+S)(C-S)}{(CS)(C+S)}$$

$$= \frac{C-S}{CS}$$

$$= \frac{C}{CS} - \frac{S}{CS}$$

$$= \operatorname{cosec} A - \sec A$$

Question-16

Prove the following trigonometric Identities :

$$\frac{\tan A}{(1+\tan^2 A)^2}+\frac{\cot A}{(1+\cot^2 A)^2}=\sin A\cos A$$

$$= \frac{\tan A}{\sec^4 A}+\frac{\cot A}{\operatorname{cosec}^4 A}$$

$$= \frac{\frac{S}{C}}{\frac{1}{C^4}}+\frac{\frac{C}{S}}{\frac{1}{S^4}}$$

$$= \frac{C^4 S}{C}+\frac{S^4 C}{S}$$

$$= C^3S + S^3C$$

$$= CS(C^2+S^2)$$

$$= \boxed{CS}$$

Q.17 [CBSE 2008]

Prove the following trigonometric Identities :

$$(1 + \cot A + \tan A)(\sin A - \cos A) = \frac{\sec A}{\operatorname{cosec}^2 A} - \frac{\operatorname{cosec} A}{\sec^2 A} = \sin A \tan A - \cot A \cos A$$

H·W

Question-18

If $\tan\theta + \cot\theta = 2$, find the value of $\tan^2\theta + \cot^2\theta$.

$$\Rightarrow (\tan\theta + \cot\theta)^2 = (2)^2$$

$$= \tan^2\theta + \cot^2\theta + 2\tan\theta\cot\theta = 4$$

$$= \tan^2\theta + \cot^2\theta + 2 = 4$$


THANK
YOU