UDAAN

2025

Trigonometry

Mathematics | Lecture - 08

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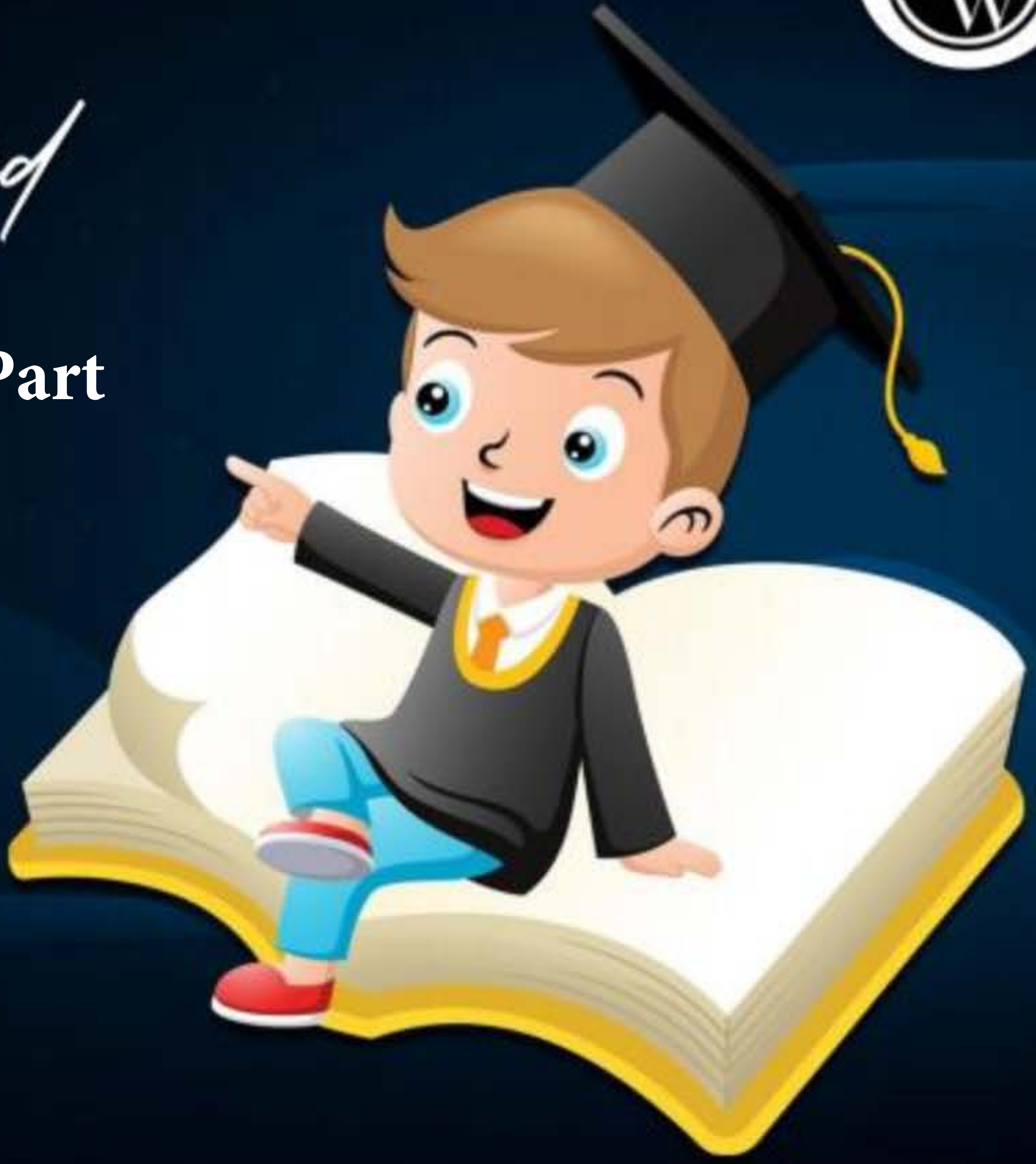
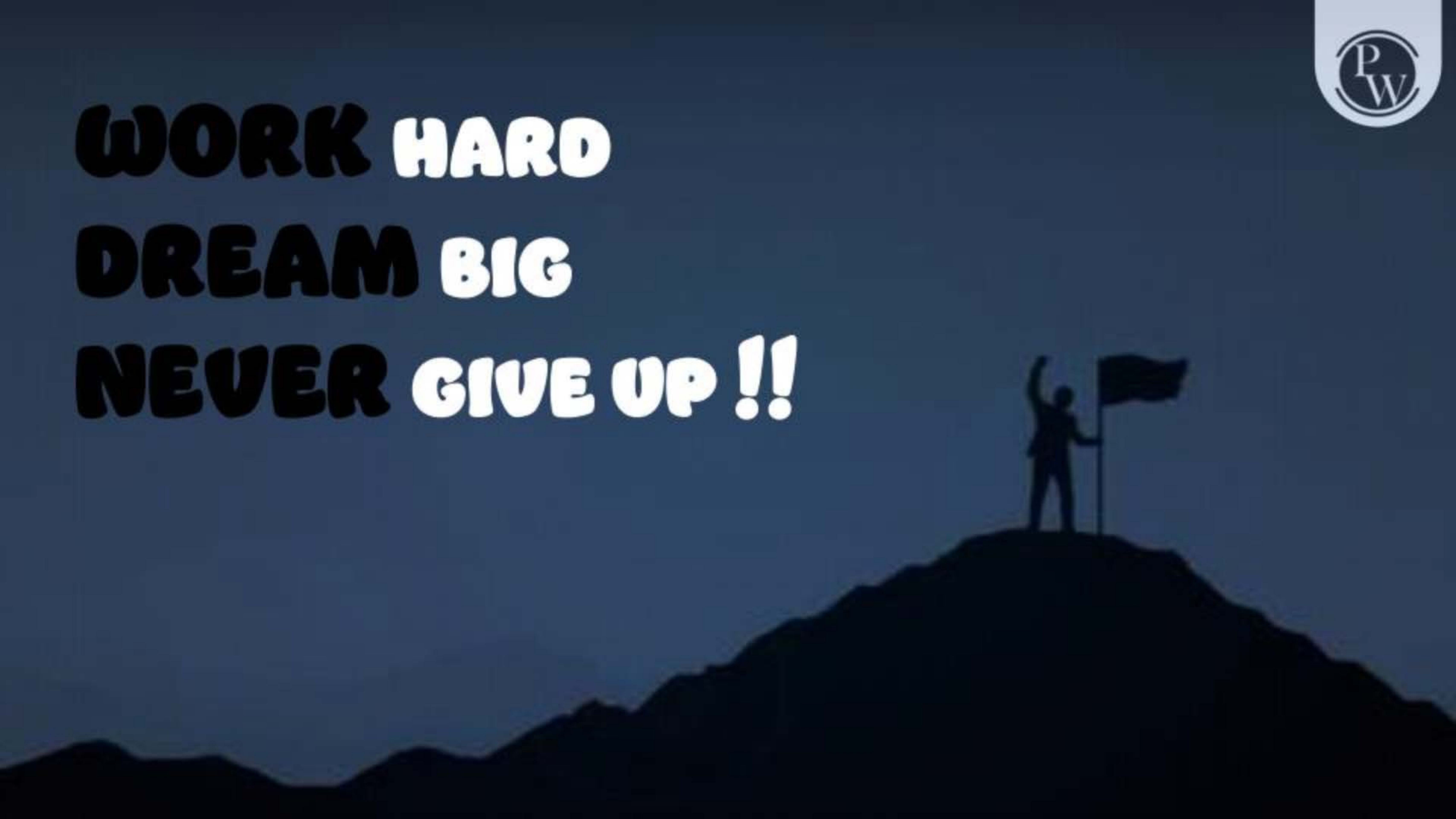
Topics *to be covered*

1. Problems on Trigonometric Identities (Part -2)




WORK HARD
DREAM BIG
NEVER GIVE UP !!

Topic : Trigonometric Identities

#Q. Prove that : $\boxed{\dfrac{(\sin^4\theta+\cos^4\theta)}{1-2\sin^2\theta\cos^2\theta}=1}$

[Board Term - 1, 2015]

$$= \frac{(s^2)^2+(c^2)^2}{1-2s^2c^2}$$

$$= \frac{(s^2+c^2)^2-2s^2c^2}{1-2s^2c^2}$$

$$= \frac{\cancel{1-2s^2c^2}}{\cancel{1-2s^2c^2}}$$

$$= 1$$

$$\boxed{a^2+b^2=(a+b)^2-2ab}$$

$$\boxed{\sin^4\theta+\cos^4\theta=1-2\sin^2\theta\cos^2\theta}$$

Topic : Trigonometric Identities ★★★

$a^6 = (a^2)^3$

#Q. Prove the following identities

$$2(\sin^6\theta + \cos^6\theta) - 3(\sin^4\theta + \cos^4\theta) + 1 = 0$$

$a^3+b^3 = (a+b)(a^2-ab+b^2)$

$= \sin^6\theta + \cos^6\theta$

$= (\sin^2\theta)^3 + (\cos^2\theta)^3$

$= (\sin^2\theta + \cos^2\theta)\left[(\sin^2\theta)^2 - \sin^2\theta\cos^2\theta + (\cos^2\theta)^2\right]$

$= (1)(s^4 + c^4 - s^2c^2)$

$= s^4 + c^4 - s^2c^2$

$= 1 - 2s^2c^2 - s^2c^2$

$= \boxed{1 - 3s^2c^2}$

$= 2(1-3s^2c^2) - 3(1-2s^2c^2) + 1$

$= 2 - \cancel{6s^2c^2} - 3 + \cancel{6s^2c^2} + 1$

$= -1 + 1$

$= \boxed{0}$

$$= S^4 + C^4$$

$$= (S^2)^2 + (C^2)^2$$

$$= (S^2 + C^2)^2 - 2S^2C^2$$

$$= \boxed{1 - 2S^2C^2}$$

$$\sin^6\theta + \cos^6\theta = 1 - 3\sin^2\theta\cos^2\theta$$

$$\sin^4\theta + \cos^4\theta = 1 - 2\sin^2\theta\cos^2\theta$$

$\sec\theta + \tan\theta = p$

$\sec\theta - \tan\theta = ?$

$\sec^2\theta = 1 + \tan^2\theta$

$$\boxed{\sec^2\theta - \tan^2\theta = 1}$$

$(\sec\theta + \tan\theta)(\sec\theta - \tan\theta) = 1$

$p(\sec\theta - \tan\theta) = 1$

$$\boxed{\sec\theta - \tan\theta = \frac{1}{p}}$$

$\operatorname{cosec}\theta - \cot\theta = a$

$\operatorname{cosec}\theta + \cot\theta =$

$\operatorname{cosec}^2\theta - \cot^2\theta = 1$

$(\operatorname{cosec}\theta + \cot\theta)(\operatorname{cosec}\theta - \cot\theta) = 1$

$$\boxed{\operatorname{cosec}\theta + \cot\theta = \frac{1}{a}}$$

Very important

$$\boxed{\sec\theta + \tan\theta = \frac{1}{\sec\theta - \tan\theta}}$$

$$\boxed{\operatorname{cosec}\theta + \cot\theta = \frac{1}{\operatorname{cosec}\theta - \cot\theta}}$$

Topic : Trigonometric Identities

#Q. If $\boxed{\sec\theta + \tan\theta = p}$, obtain the values of $\boxed{\sec\theta, \tan\theta \text{ and } \sin\theta}$ in terms of $\boxed{p}$.

$$\sec\theta + \tan\theta = p \quad -(1)$$

$$\therefore \sec\theta - \tan\theta = \frac{1}{p} \quad -(2)$$

(1) + (2)

$$\sec\theta + \cancel{\tan\theta} = p$$
$$\sec\theta - \cancel{\tan\theta} = \frac{1}{p}$$

$$2\sec\theta = p + \frac{1}{p}$$

$$2\sec\theta = \frac{p^2+1}{p}$$

$$\boxed{\sec\theta = \frac{p^2+1}{2p}}$$

(1) − (2)

$$\cancel{\sec\theta} + \tan\theta = p$$
$$\cancel{\sec\theta} - \tan\theta = \frac{1}{p}$$
$$- \quad + \quad -$$

$$2\tan\theta = p - \frac{1}{p}$$

$$= \frac{p^2-1}{p}$$

$$\boxed{\tan\theta = \frac{p^2-1}{2p}}$$

$$= \frac{\tan\theta}{\sec\theta}$$

$$= \frac{\dfrac{\sin\theta}{\cancel{\cos\theta}}}{\dfrac{1}{\cancel{\cos\theta}}}$$

$$= \sin\theta$$

$$\sin\theta = \frac{\tan\theta}{\sec\theta}$$

$$= \frac{\dfrac{p^2-1}{\cancel{2p}}}{\dfrac{p^2+1}{\cancel{2p}}}$$

$$\boxed{\sin\theta = \frac{p^2-1}{p^2+1}}$$

#Q. If $\boxed{\operatorname{cosec}\theta + \cot\theta = p}$, then prove that $\boxed{\cos\theta = \frac{p^2-1}{p^2+1}}$

$$\frac{\cot\theta}{\operatorname{cosec}\theta} = \cos\theta$$

$$\operatorname{cosec}\theta + \cot\theta = p \quad -(1)$$

$$\therefore \operatorname{cosec}\theta - \cot\theta = \frac{1}{p} \quad -(2)$$

H.W

(1) + (2)

$$\operatorname{cosec}\theta = !$$

(1) − (2)

$$\cot\theta = !$$

Topic : Trigonometric Identities

#Q. Prove the following identity : [NCERT Exemplar]

$$\frac{1}{\text{cosec}A - \cot A} - \frac{1}{\sin A} = \frac{1}{\sin A} - \frac{1}{\text{cosec}A + \cot A}$$

$\sin^2\theta + \cos^2\theta = 1$

$\sin^2\theta - 1 = \cos^2\theta$

M.I

$$= \frac{1}{\frac{1}{S} - \frac{C}{S}} - \frac{1}{S}$$

$$= \frac{1}{\frac{1-C}{S}} - \frac{1}{S}$$

$$= \frac{S}{1-C} - \frac{1}{S}$$

$$= \frac{S^2 - 1(1-C)}{(1-C)S}$$

$$= \frac{S^2 - 1 + C}{(1-C)S}$$

$$= \frac{-C^2 + C}{(1-C)S}$$

$$= \frac{C\cancel{(-C+1)}}{\cancel{(1-C)}(S)} = \boxed{\cot A}$$

R.H.S:

$$= \frac{1}{\sin A} - \frac{1}{\operatorname{cosec} A + \cot A}$$

$$= \frac{1}{s} - \frac{1}{\frac{1}{s} + \frac{c}{s}}$$

$$= \frac{1}{s} - \frac{1}{\frac{1+c}{s}}$$

$$= \frac{1}{s} - \frac{s}{1+c}$$

$$= \frac{1(1+c) - s^2}{s(1+c)}$$

$$= \frac{1+c-s^2}{s(1+c)}$$

$$= \frac{c^2+c}{s(1+c)}$$

$$= \frac{c\cancel{(c+1)}}{s\cancel{(1+c)}}$$

$$= \boxed{\cot A}$$

$\therefore$ L.H.S = R.H.S

$= \cot A$

Topic : Trigonometric Identities

#Q. Prove the following identity : [NCERT Exemplar]

$$\frac{1}{\text{cosec}A - \cot A} - \frac{1}{\sin A} = \frac{1}{\sin A} - \frac{1}{\text{cosec}A + \cot A}$$

$\text{cosec}^2\theta - \cot^2\theta = 1$

$(\text{cosec}\theta - \cot\theta)(\text{cosec}\theta + \cot\theta) = 1$

L.H.S

$= \cancel{\text{cosec}A} + \cot A - \cancel{\text{cosec}A}$

$= \cot A$

R.H.S

$= \text{cosec}A - (\text{cosec}A - \cot A)$

$= \cancel{\text{cosec}A} - \cancel{\text{cosec}A} + \cot A$

$= \cot A$

Topic : Trigonometric Identities

#Q. If $\sin\theta + \cos\theta = \sqrt{2}$, then prove that $\boxed{\tan\theta + \cot\theta = 2.}$ [NCERT]

$$(\sin\theta + \cos\theta)^2 = (\sqrt{2})^2$$

$$\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = 2$$

$$1 + 2\sin\theta\cos\theta = 2$$

$$2\sin\theta\cos\theta = 1$$

$$\sin\theta\cos\theta = \frac{1}{2}$$

$$= \tan\theta + \cot\theta$$

$$= \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$$

$$= \frac{\sin^2\theta + \cos^2\theta}{\cos\theta\sin\theta}$$

$$= \frac{1}{\cos\theta\sin\theta}$$

$$= \frac{1}{\frac{1}{2}} = 2$$

Topic : Trigonometric Identities

#Q. If $\sin\theta + \cos\theta = \sqrt{3}$, then prove that $\tan\theta + \cot\theta = 1$.

$(s+c)^2 = (\sqrt{3})^2$

$s^2 + c^2 + 2sc = 3$

$1 + 2sc = 3$

$2sc = 2$

$sc = 1$

$= \tan\theta + \cot\theta$

$= \frac{s}{c} + \frac{c}{s}$

$= \frac{s^2 + c^2}{cs}$

$= \frac{1}{cs}$

$= \frac{1}{1}$

$= \boxed{1}$

Topic : Trigonometric Identities

Bhaari Sawaal

#Q. If $1 + \sin^2\theta = 3\sin\theta\cos\theta$, prove that $\boxed{\tan\theta = 1 \text{ or } 1/2.}$

[NCERT Exemplar]

Divide both sides By $\cos^2\theta$

$$\frac{1+\sin^2\theta}{\cos^2\theta} = \frac{3\sin\theta\cancel{\cos\theta}}{\cos^{\cancel{2}}\theta}$$

$$\sec^2\theta + \tan^2\theta = 3\tan\theta$$

$$1+\tan^2\theta + \tan^2\theta - 3\tan\theta = 0$$

$$\boxed{2\tan^2\theta - 3\tan\theta + 1 = 0}$$

let, $\tan\theta = x$

$$2x^2 - 3x + 1 = 0$$

$S = -3,\ P = 2$ $\quad -2,\ -1$

$$2x^2 - 2x - 1x + 1 = 0$$

$$2x(x-1) - 1(x-1) = 0$$

$$(x-1)(2x-1) = 0$$

$$x = 1,\ x = \frac{1}{2}$$

$$\tan\theta = 1,\ \tan\theta = \frac{1}{2}$$

Topic : Trigonometric Identities

Bhaari Sawaal

#Q. If $1 + \sin^2\theta = 3 \sin\theta \cos\theta$, prove that $\boxed{\tan\theta = 1 \text{ or } 1/2.}$

[NCERT Exemplar]

M.II

$1+\sin^2\theta - 3\sin\theta\cos\theta = 0$

$\cos^2\theta+\sin^2\theta+\sin^2\theta - 3\sin\theta\cos\theta = 0$

$\boxed{2\sin^2\theta - 3\sin\theta\cos\theta + \cos^2\theta = 0}$

$2S^2 - 2SC - SC + C^2 = 0$

$2S(S-C) - C(S-C) = 0$

$\boxed{(S-C)(2S-C) = 0}$

$S-C=0, \quad 2S-C=0$

$S=C, \quad 2S=C$

$\frac{S}{C}=1, \quad \frac{S}{C}=\frac{1}{2}$

$\tan\theta=1, \quad \tan\theta=\frac{1}{2}$

Homework

Poactice sheet

level-01

level-02

THANK YOU