UDAAN 2025

Trigonometry

Mathematics | Lecture - 07

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Topics

to be covered

1. Problems on Trigonometric Identities (Part -1)


WORK HARD
DREAM BIG
NEVER GIVE UP !!

Topic : Trigonometric Identities

#Q. Prove the following identity :

$$\frac{\cos A}{1-\tan A}+\frac{\sin^2 A}{\sin A-\cos A}=\sin A+\cos A$$

$$= \frac{c}{1-\frac{s}{c}}+\frac{s^2}{s-c}$$

$$= \frac{c}{\left[\frac{c-s}{c}\right]}+\frac{s^2}{s-c}$$

$$= \frac{c^2}{c-s}+\frac{s^2}{s-c}$$

$$= \frac{c^2}{c-s}+\frac{s^2}{-(-s+c)}$$

$$= \frac{c^2}{c-s}-\frac{s^2}{c-s}$$

$$= \frac{c^2-s^2}{c-s}$$

$$= \frac{(c+s)\cancel{(c-s)}}{\cancel{(c-s)}} = \boxed{\cos A+\sin A}$$

Topic : Trigonometric Identities

#Q. Prove the following identity :

$$\frac{\cos A}{1-\tan A}+\frac{\sin A}{1-\cot A}=\cos A+\sin A$$

$$=\frac{c}{\frac{1}{1}-\frac{s}{c}}+\frac{s}{\frac{1}{1}-\frac{c}{s}}$$

$$=\frac{\frac{c}{1}}{\frac{c-s}{c}}+\frac{\frac{s}{1}}{\frac{s-c}{s}}$$

$$=\frac{c^2}{c-s}+\frac{s^2}{s-c}$$

$$=\frac{c^2}{c-s}+\frac{s^2}{-(-s+c)}$$

$$=\frac{c^2}{c-s}-\frac{s^2}{c-s}$$

$$=\frac{c^2-s^2}{c-s}$$

$$=\frac{(c+s)\cancel{(c-s)}}{\cancel{(c-s)}}$$

$$=\boxed{\cos A+\sin A}$$

Topic : Trigonometric Identities

$a^3-b^3=(a-b)(a^2+b^2+ab)$
$a^3+b^3=(a+b)(a^2+b^2-ab)$

#Q. Prove the following identity :

$$\frac{\cos^2\theta}{1-\tan\theta}+\frac{\sin^3\theta}{\sin\theta-\cos\theta}=1+\sin\theta\cos\theta$$

$$=\frac{c^2}{1-\frac{s}{c}}+\frac{s^3}{s-c}$$

$$=\frac{\frac{c^2}{1}}{\frac{c-s}{c}}+\frac{s^3}{s-c}$$

$$=\frac{c^3}{c-s}+\frac{s^3}{s-c}$$

$$=\frac{c^3}{c-s}-\frac{s^3}{c-s}$$

$$=\frac{c^3-s^3}{c-s}$$

$$=\frac{\cancel{(c-s)}(c^2+s^2+cs)}{\cancel{(c-s)}}$$

$$=\boxed{1+\sin\theta\cos\theta}$$

Topic : Trigonometric Identities

$a^3-b^3=(a-b)(a^2+ab+b^2)$

#Q. Prove the following identity :

$$\frac{\tan A}{1-\cot A}+\frac{\cot A}{1-\tan A}=1+\tan A+\cot A=1+\sec A\ \text{cosec}A$$

$$=\frac{t}{\frac{1}{1}-\frac{1}{t}}+\frac{\frac{1}{t}}{1-t}$$

$$=\frac{\frac{t}{1}}{\frac{t-1}{t}}+\frac{\frac{1}{t}}{1-t}$$

$$=\frac{t^2}{t-1}+\frac{\frac{1}{t}}{1-t}$$

$$=\frac{t^2}{t-1}-\frac{\frac{1}{t}}{t-1}$$

$$=\frac{\frac{t^2}{1}-\frac{1}{t}}{t-1}$$

$$=\frac{\frac{t^3-1}{t}}{\frac{t-1}{1}}$$

$$=\frac{t^3-1^3}{t(t-1)}$$

$$=\frac{\cancel{(t-1)}(t^2+1^2+t)}{t\cancel{(t-1)}}$$

$$=\frac{t^2+1+t}{t}$$

$$=\frac{t^{\cancel{2}}}{\cancel{t}}+\frac{1}{t}+\frac{t}{t}$$

$$=\boxed{\tan A+\cot A+1}$$

$$\tan A + \cot A + 1 = 1 + \sec A \operatorname{cosec} A$$

$$= \frac{S}{C} + \frac{C}{S} + \frac{1}{1}$$

$$= \frac{S^2 + C^2 + CS}{CS}$$

$$= \frac{1 + CS}{CS}$$

$$= \frac{1}{CS} + \frac{\cancel{CS}}{\cancel{CS}} = \frac{1 \times 1}{CS} + 1 = \boxed{\sec A \operatorname{cosec} A + 1}$$

Topic : Trigonometric Identities

#Q. Prove the following identity :

$$\frac{\tan A}{1-\cot A}+\frac{\cot A}{1-\tan A}=1+\tan A+\cot A=1+\sec A\,\mathrm{cosec}A$$

$$=\frac{\frac{s}{c}}{1-\frac{c}{s}}+\frac{\frac{c}{s}}{1-\frac{s}{c}}$$

$$=\frac{\frac{s}{c}}{\frac{s-c}{s}}+\frac{\frac{c}{s}}{\frac{c-s}{c}}$$

$$=\frac{\frac{s^2}{c}}{s-c}+\frac{\frac{c^2}{s}}{c-s}$$

$$=\frac{\frac{s^2}{c}}{s-c}-\frac{\frac{c^2}{s}}{s-c}$$

$$=\frac{\frac{s^2}{c}-\frac{c^2}{s}}{s-c}$$

$$=\frac{\frac{s^3-c^3}{cs}}{\frac{s-c}{1}}$$

$$=\frac{s^3-c^3}{cs(s-c)}$$

$$=\frac{\cancel{(s-c)}(s^2+c^2+sc)}{cs\cancel{(s-c)}}$$

$$=\frac{s^2+c^2+sc}{cs}$$

$$=\frac{s^{\cancel{2}}}{c\cancel{s}}+\frac{c^{\cancel{2}}}{\cancel{c}s}+\frac{\cancel{sc}}{\cancel{cs}}$$

$$=\boxed{\tan A+\cot A+1}$$

Topic : Trigonometric Identities

$$a^3+b^3=(a+b)(a^2+b^2-ab)$$

#Q. Prove the following identity :

$$\frac{\sin^3\theta+\cos^3\theta}{\sin\theta+\cos\theta}+\sin\theta\cos\theta=1$$

$$\frac{\cancel{(\sin\theta+\cos\theta)}(\sin^2\theta+\cos^2\theta-\sin\theta\cos\theta)}{\cancel{(\sin\theta+\cos\theta)}}+\sin\theta\cos\theta$$

$$=1-\cancel{\sin\theta\cos\theta}+\cancel{\sin\theta\cos\theta}$$

$$=\boxed{1}$$

Topic : Trigonometric Identities

#Q. Prove the following identity : [NCERT Exemplar]

$$\frac{(1+\cot A+\tan A)(\sin A-\cos A)}{\sec^3 A-\operatorname{cosec}^3 A}=\sin^2 A\cos^2 A$$

$$=\frac{\left(\frac{1}{1}+\frac{c}{s}+\frac{s}{c}\right)\left(s-c\right)}{\frac{1}{c^3}-\frac{1}{s^3}}$$

$$=\frac{\left(\frac{sc+c^2+s^2}{sc}\right)\left(s-c\right)}{\frac{s^3-c^3}{c^3s^3}}$$

$$=\frac{\left(\frac{sc+1}{sc}\right)\left(\frac{s-c}{1}\right)}{\frac{s^3-c^3}{c^3s^3}}$$

$$=\frac{\frac{(sc+1)(s-c)}{sc}}{\frac{s^3-c^3}{c^3s^3}}$$

$$=\frac{(sc+1)(s-c)(c^3s^3)}{(sc)(s^3-c^3)}$$

$$=\frac{(sc+1)(s-c)(c^3s^3)}{(sc)(s-c)(s^2+c^2+sc)}$$

$$=\frac{\cancel{(sc+1)}\cancel{(s-c)}(c^3s^3)}{(sc)\cancel{(s-c)}\cancel{(1+sc)}}$$

$$=\frac{s^{\cancel{3}}c^{\cancel{3}}}{\cancel{s}\cancel{c}}$$

$$=\boxed{s^2c^2}$$

Topic : Trigonometric Identities

#Q. Prove the following identity : [CBSE 2008]

$\tan^2\theta + \cot^2\theta + 2 = \sec^2\theta \operatorname{cosec}^2\theta$

$= \sec^2\theta - 1 + \operatorname{cosec}^2\theta - 1 + 2$

$= \sec^2\theta + \operatorname{cosec}^2\theta \cancel{-2} \cancel{+2}$

$= \sec^2\theta + \operatorname{cosec}^2\theta$

$= \dfrac{1}{\cos^2\theta} + \dfrac{1}{\sin^2\theta}$

$= \dfrac{\sin^2\theta + \cos^2\theta}{\cos^2\theta \sin^2\theta}$

$= \dfrac{1 \times 1}{\sin^2\theta \times \cos^2\theta}$

$= \boxed{\operatorname{cosec}^2\theta \times \sec^2\theta}$

Topic : Trigonometric Identities

#Q. The value of $\frac{\sin\theta\tan\theta}{1-\cos\theta} + \tan^2\theta - \sec^2\theta$ is

(A) $\sin\theta\cos\theta$

(B) $\sec\theta$

(C) $\tan\theta$

(D) $\operatorname{cosec}\theta$

$\sec^2\theta = 1 + \tan^2\theta$

$\boxed{-1 = \tan^2\theta - \sec^2\theta}$

$$= \frac{s\left(\frac{s}{c}\right)}{1-c} - \frac{1}{1}$$

$$= \frac{\frac{s^2}{c} - 1(1-c)}{1-c}$$

$$= \frac{\frac{s^2}{c} - \frac{1}{1} + \frac{c}{1}}{1-c}$$

$$= \frac{\frac{s^2 - c + c^2}{c}}{\frac{1-c}{1}}$$

$$= \frac{\frac{1-c}{c}}{\frac{1-c}{1}}$$

$$= \frac{\cancel{(1-c)}}{c\cancel{(1-c)}} = \frac{1}{c} = \sec\theta$$

Topic : Trigonometric Identities

#Q. $\cos^4 x - \sin^4 x =$ $= (c^2)^2 - (s^2)^2$

$(a)^2 - (b)^2 = (a+b)(a-b)$

(A) $2\sin^2 x - 1$

(B) $-1 + 2\cos^2 x$

(C) $\sin^2 x - \cos^2 x$

(D) 1

$= (c^2 + s^2)(c^2 - s^2)$

$= \boxed{c^2 - s^2} \rightarrow 1 - s^2 - s^2 = \boxed{1 - 2s^2}$

$= c^2 - (1 - c^2)$

$= c^2 - 1 + c^2$

$= \boxed{2c^2 - 1}$

Topic : Trigonometric Identities

#Q. Prove the following identity :

$(\sin^4\theta - \cos^4\theta + 1)\text{cosec}^2\theta = 2$

$= \left[(s^2)^2 - (c^2)^2 + 1\right]\text{cosec}^2\theta$

$= \left[(s^2+c^2)(s^2-c^2)+1\right]\text{cosec}^2\theta$

$= (s^2 - c^2 + 1)\text{cosec}^2\theta$

$= (s^2+s^2)\text{cosec}^2\theta$

$= 2\cancel{s^2} \times \frac{1}{\cancel{s^2}} = \boxed{2}$

Topic : Trigonometric Identities

#Q. $\frac{\sin^4\theta - \cos^4\theta}{1-\sin^2\theta}$ = how much?

(A) $1 - \cot^2\theta$

(B) $1 - \tan^2\theta$

(C) $\tan^2\theta - 1$

(D) $\cos^2\theta - 1$

$$= \frac{(s^2)^2 - (c^2)^2}{1-s^2}$$

$$= \frac{(s^2+c^2)(s^2-c^2)}{1-s^2}$$

$$= \frac{s^2-c^2}{1-s^2}$$

$$= \frac{s^2-c^2}{c^2} = \frac{s^2}{c^2} - \frac{c^2}{c^2} = \boxed{\tan^2\theta - 1}$$

$$(a+b)^2 = a^2 + b^2 + 2ab$$

$$\boxed{(a+b)^2 - 2ab = a^2 + b^2}$$

Topic : Trigonometric Identities

#Q. Prove the following identity : [NCERT Exemplar]

$\sin^4 A + \cos^4 A = 1 - 2\sin^2 A \cos^2 A$

$= (s^2)^2 + (c^2)^2$

$= (s^2 + c^2)^2 - 2s^2c^2$

$= \boxed{1 - 2s^2c^2}$

$a^2 + b^2 = (a+b)^2 - 2ab$

Topic : Trigonometric Identities

#Q. $\frac{1+\tan^2 A}{1+\cot^2 A}$ is equal to :

(A) $\sec^2 A$

(B) -1

(C) $\cot^2 A$

(D) $\tan^2 A$

$$= \frac{\sec^2 A}{\operatorname{cosec}^2 A}$$

$$= \frac{\frac{1}{\cos^2 A}}{\frac{1}{\sin^2 A}}$$

$$= \frac{\sin^2 A}{\cos^2 A}$$

$$= \tan^2 A$$

Topic : Trigonometric Identities

#Q. Prove the following identity : [NCERT Exemplar]

$$\frac{\tan\theta + \sec\theta - 1}{\tan\theta - \sec\theta + 1} = \frac{1+\sin\theta}{\cos\theta}$$

$\sec^2\theta = 1 + \tan^2\theta$

$\boxed{\sec^2\theta - \tan^2\theta = 1}$

$$= \frac{\tan\theta + \sec\theta - (\sec^2\theta - \tan^2\theta)}{\tan\theta - \sec\theta + 1}$$

$$= \frac{(\sec\theta + \tan\theta) - [(\sec\theta - \tan\theta)(\sec\theta + \tan\theta)]}{t - s + 1}$$

$$= \frac{(\sec\theta + \tan\theta)[1 - (\sec\theta - \tan\theta)]}{t - s + 1}$$

$$\frac{(\sec\theta + \tan\theta)\cancel{(1 - \sec\theta + \tan\theta)}}{\cancel{(\tan\theta - \sec\theta + 1)}}$$

$= \sec\theta + \tan\theta$

$= \frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta} = \frac{1+\sin\theta}{\cos\theta}$

Topic : Trigonometric Identities

#Q. Prove the following identity : [NCERT Exemplar]

$$\frac{\cot A + \operatorname{cosec} A - 1}{\cot A - \operatorname{cosec} A + 1} = \frac{1 + \cos A}{\sin A}$$

$$\boxed{\operatorname{cosec}^2\theta = 1 + \cot^2\theta}$$

$$= \frac{\cot A + \operatorname{cosec} A - (\operatorname{cosec}^2 A - \cot^2 A)}{\cot A - \operatorname{cosec} A + 1}$$

$$= \frac{(\cot A + \operatorname{cosec} A) - [(\operatorname{cosec} A + \cot A)(\operatorname{cosec} A - \cot A)]}{\cot A - \operatorname{cosec} A + 1}$$

$$= \frac{(\cot A + \operatorname{cosec} A)[1 - (\operatorname{cosec} A - \cot A)]}{\cot A - \operatorname{cosec} A + 1}$$

$$\Rightarrow \frac{(C + Co)\cancel{(1 - Co + C)}}{\cancel{(\cot A - \operatorname{cosec} A + 1)}}$$

$$= \cot A + \operatorname{cosec} A$$

$$= \frac{\cos A}{\sin A} + \frac{1}{\sin A}$$

$$= \frac{\cos A + 1}{\sin A}$$

Topic : Trigonometric Identities

#Q. Prove the following identity : [NCERT Exemplar]

$$\frac{\sin\theta - \cos\theta + 1}{\sin\theta + \cos\theta - 1} = \frac{1}{\sec\theta - \tan\theta}$$

H.W

$$= \frac{\dfrac{\sin\theta - \cos\theta + 1}{\cos\theta}}{\dfrac{\sin\theta + \cos\theta - 1}{\cos\theta}}$$

$$= \boxed{\frac{\tan\theta - 1 + \sec\theta}{\tan\theta + 1 - \sec\theta}}$$

Topic : Trigonometric Identities

#Q. Prove the following identities:

$$\frac{\cos A}{1-\sin A}+\frac{\sin A}{1-\cos A}+\frac{1}{1}=\frac{\sin A\cos A}{(1-\sin A)(1-\cos A)}$$

$$=\frac{c(1-c)+s(1-s)+1(1-s)(1-c)}{(1-s)(1-c)}$$

$$=\frac{\cancel{c}-c^2+\cancel{s}-s^2+1-\cancel{c}-\cancel{s}+sc}{(1-s)(1-c)}$$

$$=\frac{-c^2-s^2+1+sc}{(1-s)(1-c)}$$

$$=\frac{\cancel{-(c^2+s^2)}\cancel{+1}+sc}{(1-s)(1-c)}$$

$$=\boxed{\frac{sc}{(1-s)(1-c)}}$$

Topic : Trigonometric Identities

#Q. $\cot^2 A\left(\frac{\sec A - 1}{1+\sin A}\right) + \sec^2 A\left(\frac{\sin A - 1}{1+\sec A}\right) = 0$

$$= \frac{\cot^2 A(\sec A - 1)\overset{(\sec A+1)}{(1+\sec A)} + \sec^2 A(\sin A - 1)\overset{(\sin A+1)}{(1+\sin A)}}{(1+\sin A)(1+\sec A)}$$

$$= \frac{\cot^2 A(\sec^2 A - 1) + \sec^2 A(\sin^2 A - 1)}{(1+\sin A)(1+\sec A)}$$

$\sin^2 A + \cos^2 A = 1$

$\sin^2 A - 1 = -\cos^2 A$

$$= \frac{\cot^2 A\tan^2 A + \sec^2 A(-\cos^2 A)}{(1+\sin A)(1+\sec A)} = \frac{1-1}{D} = \frac{0}{D} = 0$$

Homework

DPP

THANK YOU