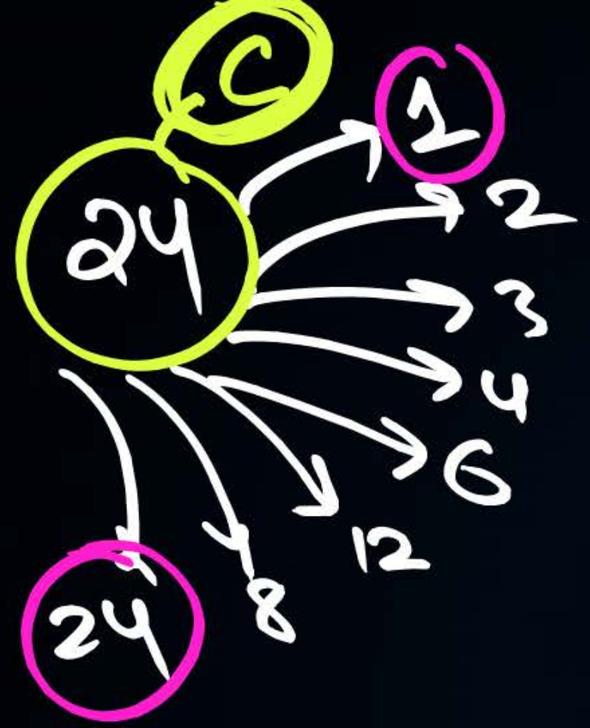
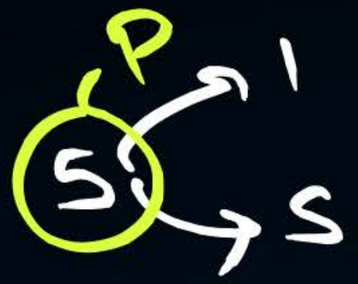
Backlog (Revision) Marathon

Real no's

- Factors
- Prime Factors
- Fundamental theorem of Arithmetic.
- HCF and LCM.
- Word problems.
- proof of irrationality.

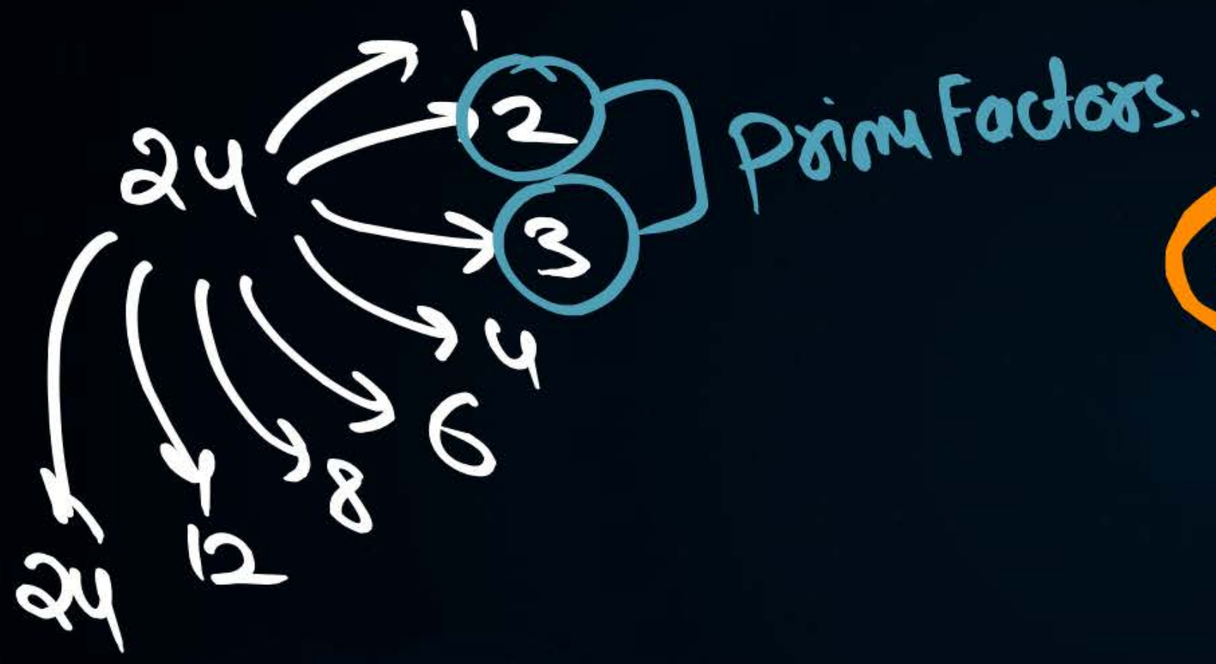




Nahi prime, nahi composite.

- Prime no. → only two Factors.
- Composite no. → More than 2 Factors

Prime Factorisation.



$84 = 2 \times 2 \times 3 \times 7$

$84 = 2^2 \times 3^1 \times 7^1$

2	84
2	42
3	2^1
7	7
	1

Q 480 → PF

2	480
2	240
2	120
2	60
2	30
3	15
5	5
	1

$480 = 2^5 \times 3^1 \times 5^1$

Theorem 1

(Fundamental Theorem of Arithmetic):
Every composite number can be expressed (factorised) as a product of primes, and this factorisation is unique, apart from the order in which the prime factor's occur.

The prime factorisation of a natural number is unique, except for the order of its factors.

Composite no. = product of primes.

unique.

Q Check whether 12^n can end with the digit 0.
n (natural no.)

$12^n = 12^1 = 12$

$= 12^2 = 144$

$= 12^3 = 1728$

2	12
2	6
3	3
	1

$12^n = (2^2 \times 3^1)^n$

$= (2^2)^n \times (3^1)^n$

$= \boxed{2^{2n} \times 3^n}$

$(a \times b)^n = a^n \times b^n$

clearly 12^n only contains 2 and 3 as prime factors, $\therefore 12^n$ cannot end with the digit '0'.

koi bhi no. kay last main agar '0' hai, toh usse "PF" main '2 or 5' hoga hi hoga.

Solution :

If the number 4^n, for any natural number n, were to end with the digit zero, then it would be divisible by 5 and 2.

That is, the prime factorization of 4^n would contain the prime 5 and 2. This is not possible because $4^n = (2)^{2n}$; So, the only prime in the factorization of 4^n is 2. So, the uniqueness of the Fundamental Theorem of Arithmetic guarantees that there are no other primes in the factorization of 4^n. So, there is no natural number n for which 4^n ends with the digit zero.

HCF & LCM

HCF → Highest Common Factor.

LCM → Least common multiple.

$8 = 8, 16, 24, 32, 40, 48$

$12 = 12, 24, 36, 48, 60$

$LCM(8,12) = 24$

$HCF(8,12) = 4$

8: 1, 2, 4, 8

12: 1, 2, 3, 4, 6, 12

Topic : HCF and LCM

#Q. Find the HCF and LCM of 30, 72 and 432 using the prime factorization method.

$30 = 2^1 \times 3^1 \times 5^1$

$72 = 2^3 \times 3^2 \times 5^0$

$432 = 2^4 \times 3^3 \times 5^0$

2	30
3	15
5	5
	1

2	72
2	36
2	18
3	9
3	3
	1

2	432
2	216
2	108
2	54
3	27
3	9
3	3
	1

$HCF = 2^1 \times 3^1 \times 5^0 = \boxed{6}$

$LCM = 2^4 \times 3^3 \times 5^1 = 16 \times 27 \times 5$

$= \boxed{2160}$

$43 \rightarrow R = 1$

7

$43 - 1 \rightarrow \boxed{R = 0}$

$$\begin{array}{r} 6 \\ 7 \overline{)\,43} \\ 42 \\ \hline 1 \end{array}$$

Topic : HCF and LCM

#Q. Find the largest number that divides 2053 and 967 and leaves a remainder of 5 and 7 respectively.

2053 → R = 5

967 → R = 7

2048 → R = 0

960 → R = 0

HCF(2048, 960)

2	2048
2	1024
2	512
2	256
2	128
2	64
2	32
2	16
2	8
2	4
	2
	1

2	960
2	480
2	240
2	120
2	60
2	30
3	15
5	5
	1

$2048 = 2^{11} \times 3^{0} \times 5^{0}$

$960 = 2^{6} \times 3^{1} \times 5^{1}$

$HCF = 2^{6} \times 3^{0} \times 5^{0}$

$= 64$

Relation b/w HCF and LCM for two positive integers

natural nos.

For any two positive integers a and b

HCF (a, b) × LCM (a, b) = a × b

$$\boxed{HCF(a,b) \times LCM(a,b) = a \times b}$$

Topic : HCF and LCM

#Q. If HCF(336, 54) = 6, find LCM(336, 54).

M.I → P.F

HCF × LCM = Product of two Nos

$$\boxed{6 \times LCM = 336 \times 54}$$

$$LCM = \frac{\cancel{336}^{56} \times 54}{\cancel{6}_{1}}$$

$$LCM = 56 \times 54$$

$$= \boxed{3024}$$

[redacted] Given that HCF (2520, 6600) = 40 and LCM (2520, 6600) = 252 k, then the value of k is

(a) 1650 (b) 1600 (c) 165 (d) 1625

$HCF(2520, 6600) = 40$, $LCM(2520, 6600) = 252k$

$HCF \times LCM = \text{product}$

$40 \times 252k = 2520 \times 6600$

$k = \frac{\cancel{2520} \times \cancel{6600}^{1650}}{\cancel{40} \times \cancel{252}}$

$k = 1650$

[redacted]

If p and q are two distinct prime numbers, then their HCF is

(a) 2 (b) 0 (c) either 1 or 2 (d) 1

[redacted]

alg-alg

1 2 → 2

1 5 → 5

1 3 → 3

1 11 → 11

[redacted] If p and q are two distinct prime numbers, then LCM (p, q) is

(a) 1 (b) p (c) q (d) pq

$P \rightarrow$ prime

$q \rightarrow$ prime

$LCM(P, q) =$

$2 \mid 2$ $5 \mid 5$

$2 = 2^1 \times 5^0$

$5 = 5^1 \times 2^0$

$LCM = 2^1 \times 5^1 = 10$

[redacted] The LCM of the smallest two digit composite number and the smallest composite number is

(a) 12 (b) 20 (c) 4 (d) 44

Smallest two digit Composite no = 10

" Composite no = 4

2	4, 10
2	2, 5
5	1, 5
	1, 1

20

If the LCM of two numbers is 3600, then which of the following numbers cannot be their HCF?

(a) 600 (b) 500 (c) 400 (d) 150

HCF is always a Factor of LCM.

LCM is always a Multiple of HCF.

a, b → LCM = 3600

HCF = 600 ✓, 500 ✗, 400 ✓, 150 ✓

natural no.

[redacted] If two positive integers a and b are written as $a = p^3 q^4$ and $b = p^2 q^3$, where p and q are prime numbers, such that $HCF(a, b) = p^m q^n$ and $LCM(a, b) = p^r q^s$, then $(m + n)(r + s)$ equal to

(a) 15 (b) 30 (c) 35 (d) 72

Ans (c)

$a = p^3 q^4 \rightarrow HCF = p^2 q^3$

$b = p^2 q^3 \rightarrow LCM = p^3 q^4$

$HCF(a,b) = p^m q^n$

$LCM(a,b) = p^r q^s$

$m = 2$

$n = 3$

$r = 3$

$s = 4$

$(m+n)(r+s) = ?$

$(2+3)(3+4) = 35$ Ans

[redacted] $\Rightarrow x = 10 \times 165 = 1650$

If $a = 2^2 \times 3^x$, $b = 2^2 \times 3 \times 5$, $c = 2^2 \times 3 \times 7$, and LCM $(a, b, c) = 3780$, then $x =$

(a) 0 (b) 1 (c) 2 (d) 3

[redacted]

1-1.w

12

Factors: 1, 2, 3, 4, 6, 12

chota

multiples: 12, 24, 36, 48 - - - - - -

Bada.

Points to be noted!!

Word Problems

- ✓ Read the questions carefully, very carefully.

Bada → Multiple → LCM

- ✓ Abh ye judge kro ki aapka answer given data sey bada ya chota aayega.

Chota → Factor → HCF

- ✓ HCF of students students hi aayega.

8 pens
12 pens

Topic : HCF and LCM

given data

#Q. The traffic lights at three different road crossings change after every 48 seconds, 72 seconds and 108 seconds respectively. If they all change simultaneously at 8 a.m. then at what time will they again change simultaneously.

8 a.m

Ans: 432 seconds.

I 48 sec.

II 72 ''.

III 108 ''.

LCM

2	48, 72, 108
2	24, 36, 54
3	12, 18, 27
3	4, 6, 9
2	4, 2, 3
2	2, 1, 3
3	1, 1, 3

Topic : HCF and LCM

#Q. A fruit vendor has 990 apples and 945 oranges. He packs them into baskets. Each basket contains only one of the two fruits but in equal number. Find the number of fruits to be put in each basket in order to have minimum number of baskets. [Board-Term – I, 2016]

990 Apples.

945 oranges.

HCF (990, 945) = 45 Fruits

Topic : HCF and LCM

#Q. Three sets of Science, History and Drawing books have to be stacked in such a way that all the books are stored subject wise and the height of each stack is the same. The number of Science books is 192, the number of History books is 480 and the number of Drawing books is 672. Assuming that the books are of the same thickness, determine the number of stacks of Science, History and Drawing books.

Sc = 192

H = 480

D = 672

HCF = 96

$Sc = \frac{192}{96} = 2$ stacks.

$H = \frac{480}{96} = 5$ stacks.

$D = \frac{672}{96} = 7$ stacks

Topic : HCF and LCM

#Q. Four bells ring at an interval of 4, 7, 12 and 14 seconds respectively. If the four bells begin to ring at 12 O'clock when will this next ring together and how often will they do so in the next 14 minutes.

H.W

Prime and Co-prime Numbers

2 prime no. hamesha co prime.

2 nos

→ HCF = 1

→ only one common factor i.e., 1.

Ex: 1 → 2, 1 → 5 → Co-prime.

9, 16

9 → 1, 3, 9

16 → 1, 2, 4, 8, 16

Theorem

Concept-1 #1

Let p be a prime number. If p divides a^2, then p divides a, where a is a positive integer.

natural no.

$p \to a^2$

$p \to a$ +ve integer

Prime

$b^2 \to b$

$5 \to b$

$5 \to 5^2 (25)$

$5 \to 5$

Rational no:

- $\frac{p}{q}$
- "p and q" → integer → coprime integer
- $q \neq 0$

#2

H3

$\boxed{p = 3c}$ → 3 divides p.

$\boxed{p^2 = 3c}$ → 3 divides p^2

Q 7 divides p^2.

$p^2 = 7c$ $p^2 = 7x$

$p^2 = 7a$

Ulta

Q 5 divides q.

$q = 5 \times 0$

$\boxed{q = 5c}$

$q = 5x$ $q = 5z$

$q = 5y$

Topic : Irrational Number

#Q. Prove that $\sqrt{3}$ is irrational.

let $\sqrt{3}$ be rational.

$\therefore \sqrt{3} = \frac{p}{q}$ ["p and q" coprime integers]

p or q ka 1 key alava koi or common factor nahi hoga

$(\sqrt{3})^2 = \left(\frac{p}{q}\right)^2$

$3 = \frac{p^2}{q^2}$

$3q^2 = p^2$

$p^2 = 3q^2$

$\Rightarrow$ 3 divides p^2.

$\Rightarrow$ 3 divides p. (1)

let,

$p = 3c$

$(3c)^2 = 3q^2$

$9c^2 = 3q^2$

$3\frac{9c^2}{3} = q^2$

$q^2 = 3c^2$

$\Rightarrow$ 3 divides q^2

$\Rightarrow$ 3 divides q. (2)

$\Rightarrow$ 3 is a common factor of p and q.
which means p and q are not coprime.

This makes our assumption wrong.

$\boxed{\therefore \sqrt{3} \text{ is irrational.}}$

Topic : Irrational Number

#Q. Given that $\sqrt{5}$ is irrational, prove that $2\sqrt{5} - 3$ is an irrational number.

[CBSE SQP - 2020]

let $2\sqrt{5}-3$ be rational.

$\therefore 2\sqrt{5}-3 = \frac{p}{q}$ ['p' and 'q' coprime integers]

$2\sqrt{5} = \frac{p}{q} + \frac{3}{1}$

$2\sqrt{5} = \frac{p+3q}{q}$

$\boxed{\sqrt{5} = \frac{p+3q}{2q}}$

Rational

Irr

this is not possible

$\therefore 2\sqrt{5}-3$ is irrational.

$I \times I$

$I + I$

$I - I$

$\rightarrow I$

$\boxed{\dfrac{I}{I} = \text{Rational no.}}$

Topic : Irrational Number

#Q. Prove that $7\sqrt{5}$ is an irrational number. [NCERT]

H.W

Polynomials:

- → what?
- → types.
- → zeroes.
- → zeroes on graph.
- → Relationship b/w zeroes and coefficients of a Quadratic poly.

[redacted]

MULTIPLE CHOICE QUESTIONS (MCQs)

EXAMPLE 1 *Which of the following is a polynomial?*

(a) $2x^2 + \frac{3}{x} - 5$

(b) $-3x^2 + \sqrt{2x} + 4$

[redacted] $\sqrt{2}x^3 + \sqrt{3}x^2 + \sqrt{5}x - 3$

(d) $\frac{5}{x^3} + 2x^2 - 3x + \frac{1}{7}$

$5x^{-3}$

Ans. (c)

Types of polynomials

- On the basis of degree
- distinct variables
- On the basis of no. of terms

Variable ki highest power.

Q $2x^2 - 3x^3 + 4x + 5$

$d = 3$

no. of terms = 4

On the Basis of Degree

$\cancel{4x^4} - 3x^3 + 2x^2 - \cancel{2x^4}$
$\cancel{-2x^4} + x$
$\cancel{-4x^4}$

cubic

1. ***Linear Polynomial:*** A polynomial of degree 1 is called a linear polynomial.

d=1

2. ***Quadratic Polynomial:*** A polynomial of degree 2 is called a Quadratic polynomial.

d=2

3. ***Cubic Polynomial:*** A polynomial of degree 3 is called a Cubic polynomial.

d=3

Quadratic

4. ***Biquadratic Polynomial:*** A polynomial of degree 4 is called a Biquadratic polynomial.

d=4

Constant Polynomial

(non-zero)

$2x^0, 3x^0, \frac{22}{7}x^0, -3x^0, \pi x^0$

degree

Zero Polynomial

$0x^0, 0x, 0x^2, 0x^{1000}, 0x^{10000}, 0x^{\square}$

$\boxed{\text{degree} = \text{not defined.}}$

Number of terms in a polynomials

1. **Monomial:** A polynomial containing one term is called a monomial ('Mono' means 'one'.)
   Example : 9, –14, 6x, $-8x^2$, $5x^3$, $2x^4$, etc. are all monomials.

2. **Binomials:** A polynomial containing two non zero terms is called a binomial. ('Bi' means 'two'.)
   Example : $(9 + 4x)$, $(x - 3x^2)$, $(8 + x^3)$, $(-x^4 + 7)$ are all binomials.

3. **Trinomials:** A polynomial containing three non zero terms is called a trinomial. ('Tri' means 'Three'.)
   Example : $(x^2 + 2x - 3)$, $(2x^3 + 5x^2 - 4)$, $(-7x^4 + 5x^2 + 6)$, $(5x^6 - 3x^4 + x)$ are all trinomials.

Q Monomial of $d = 5$.

$x^5, -5x^5, -4x^5, -\frac{2}{7}x^5 \ldots\ldots$

Topic : Basics

numerical coefficient

#Q. Write

(i) The coefficient of x^3 in $x + 3x^2 - 5x^3 + x^4$ $= -5$

(ii) The coefficient of x in $\sqrt{3} - 2\sqrt{2}x + 6x^2$ $= -2\sqrt{2}$

(iii) The coefficient of x^2 in $3x - 3 + x3$ $\rightarrow 3x - 3 - x^3 + 0x^2 \rightarrow 0$

[redacted]

Value of a polynomial:

$P(x) = x+3$

$P\left(\frac{22}{7}\right) = \frac{22}{7} + 3 = \frac{22+21}{7} = \frac{43}{7}$

$x=2$ $\quad P(2) = 5$

$P(-3) = 0$

$P(4) = 7$

$P(-100) = -97$

Zero of a polynomial.

↳ Variable ki aisan value

(jo poly = 0)

-3 is the zero of $(x+3)$

Q. $q(x) = x - 5$

$\boxed{\text{zero} = 5}$

Linear polynomial

$\boxed{d = 1}$

only one zero

Q. $r(x) = x$

$\boxed{\text{zero} = 0}$

Q. $g(x) = 2x + 3$

$2x + 3 = 0$

$2x = -3$

$\boxed{x = -\frac{3}{2}}$

$\boxed{\text{zero} = -\frac{3}{2}}$

Q $p(x) = x^2 - 4$

$\boxed{\text{zero} = 2, -2}$

Q $q(x) = x^2$

$\boxed{\text{zero} = 0}$

Quadratic polynomial.

$\boxed{d = 2}$

Maximum '2'
or
atmost '2'

Q $d = 3$

no. of zeroes = $\boxed{\text{Maximum } 3}$

Some Important Observations

(i) A constant polynomial does not have any zero

(ii) Every linear polynomial has one and only one zero.

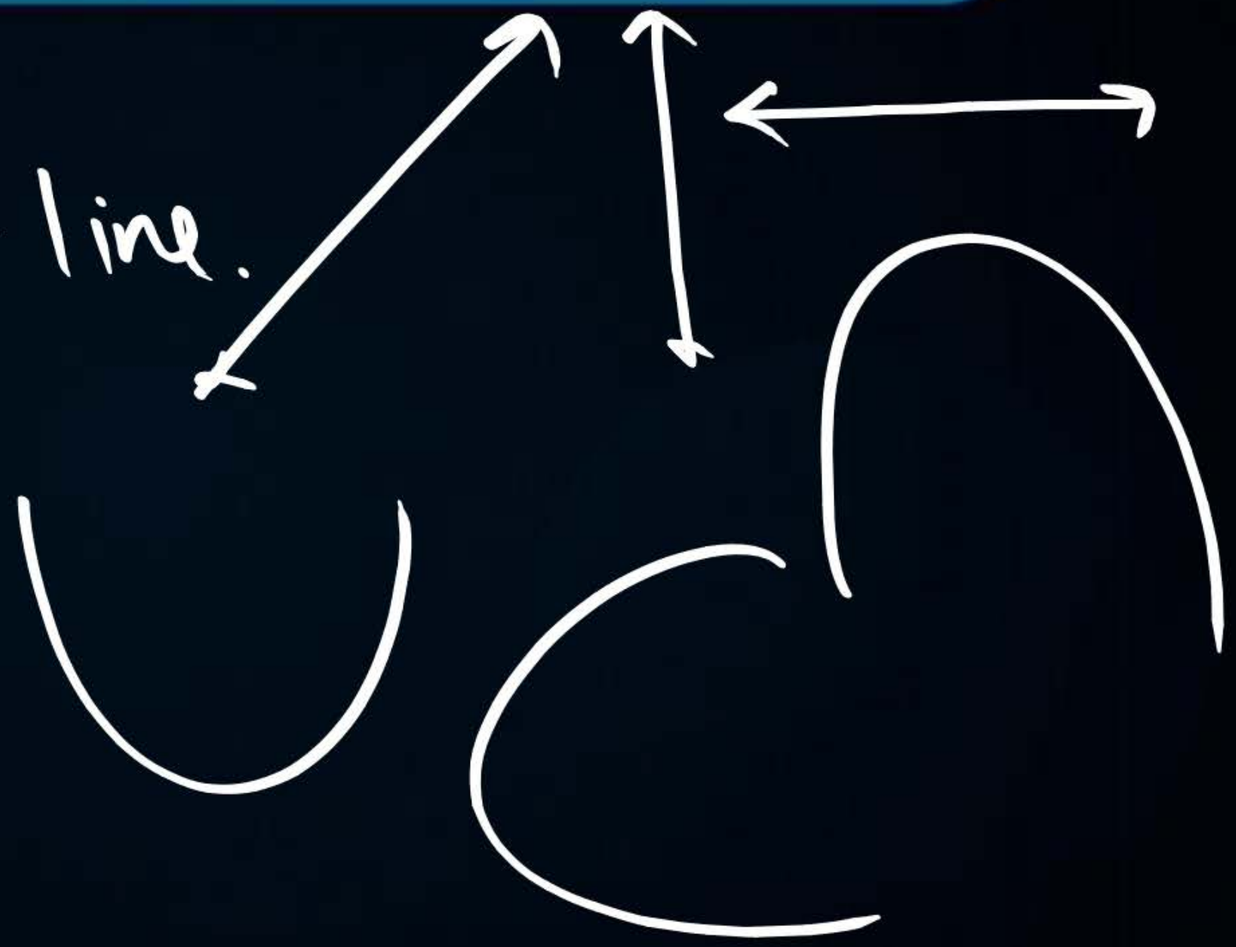
(iii) 0 may or may not be the zero of a given polynomial

(iv) Number of zero of a polynomial cannot exceed its degree.

Geometrical meaning of zeroes of a polynomial

Linear → straight line.

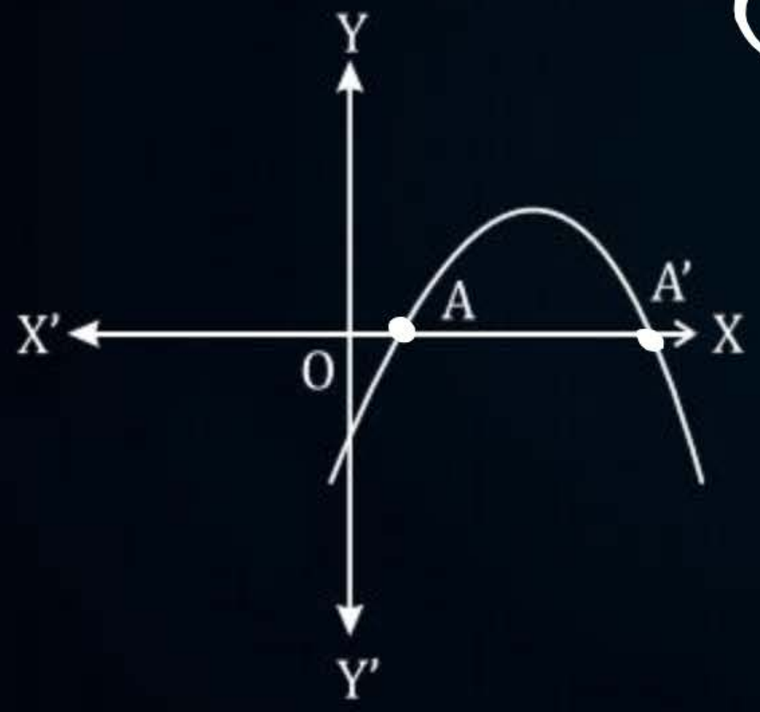
Quadratic → parabola

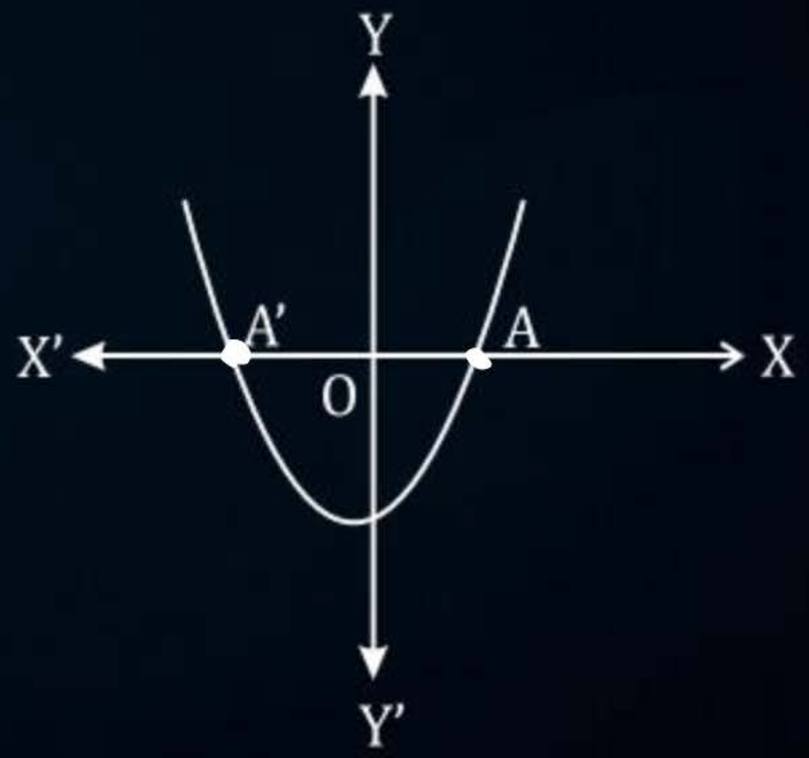


Case : (i)

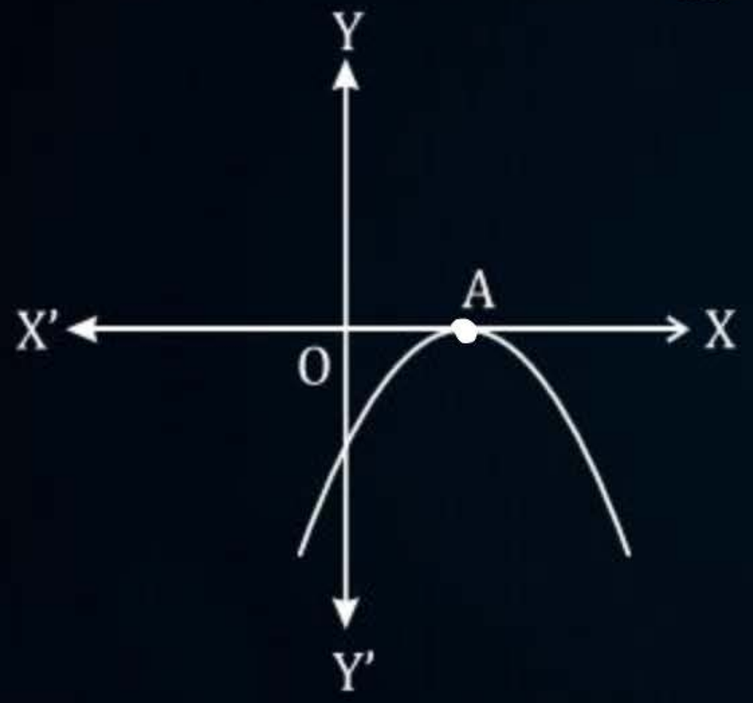
Here, the graph cuts x-axis at two distinct points A and A′.

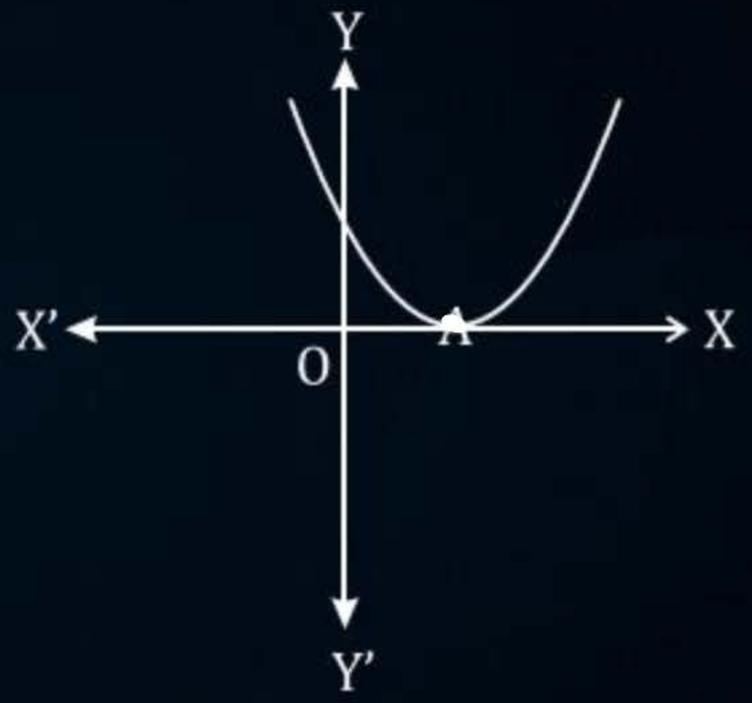
2





Case : (ii)





Case : (iii)

Here, the graph is either completely above the x-axis or completely below the x-axis. So, it does not cut the x-axis at any point.

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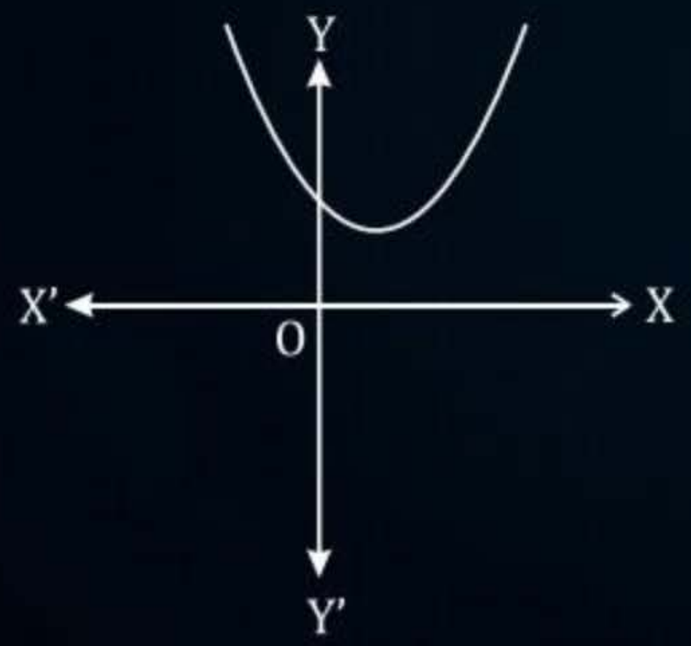
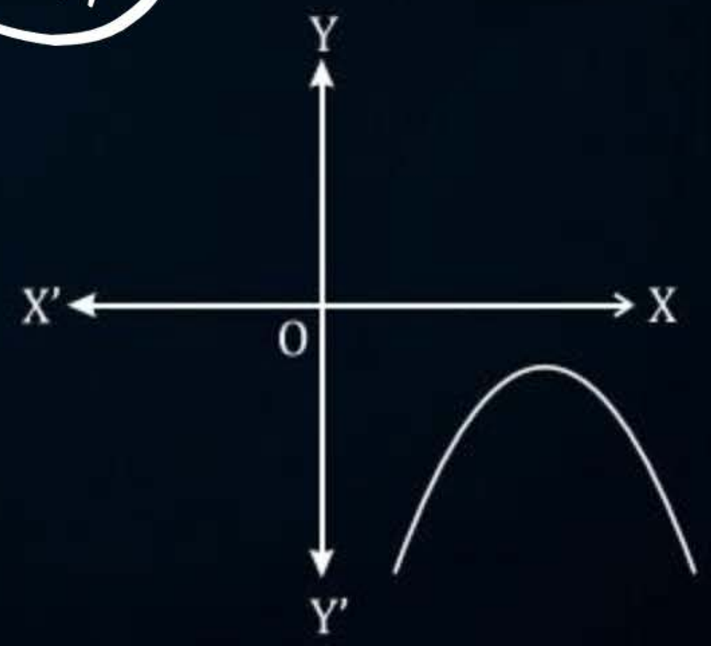
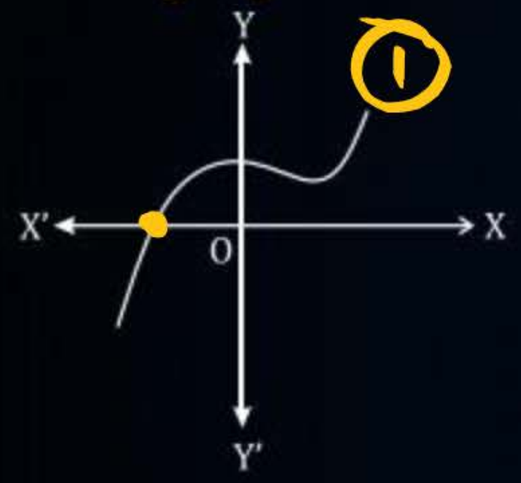
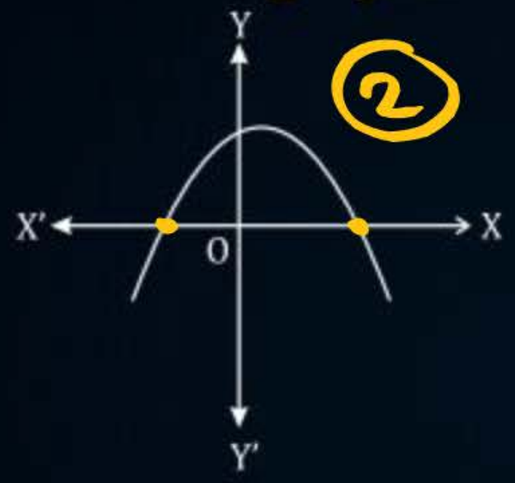
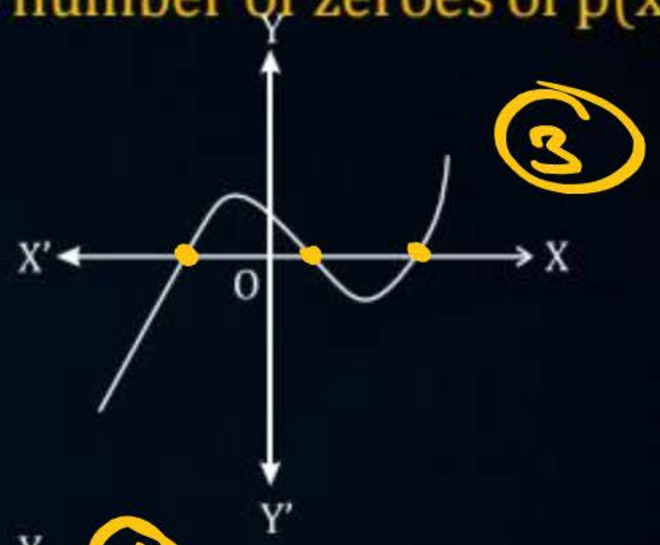


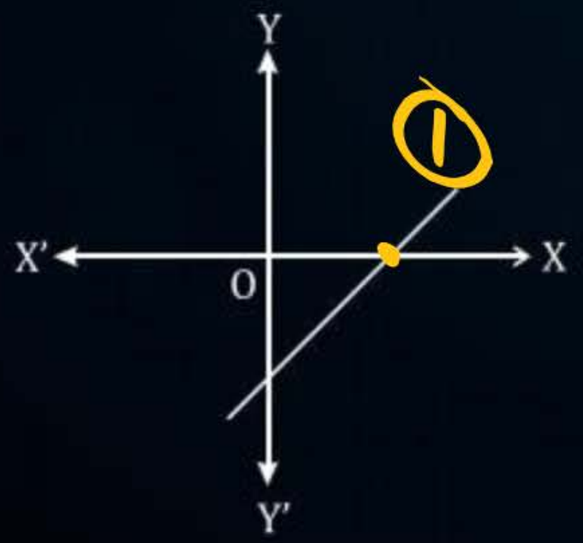


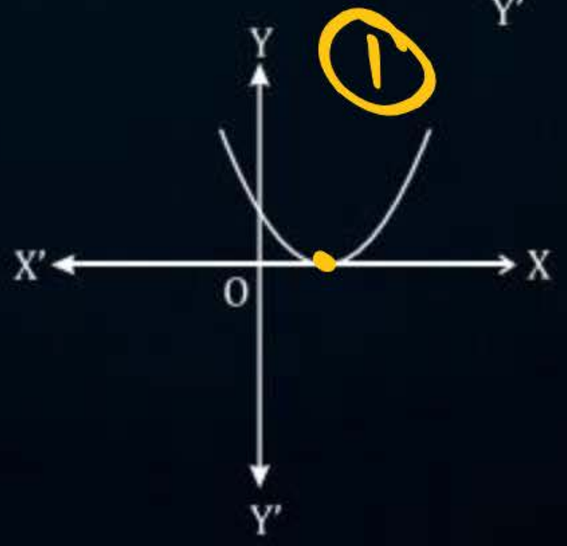
#Q. Look at the graphs given below. Each is the graph of y = p(x), where p(x) is a polynomial. For each of the graph, find the number of zeroes of p(x).

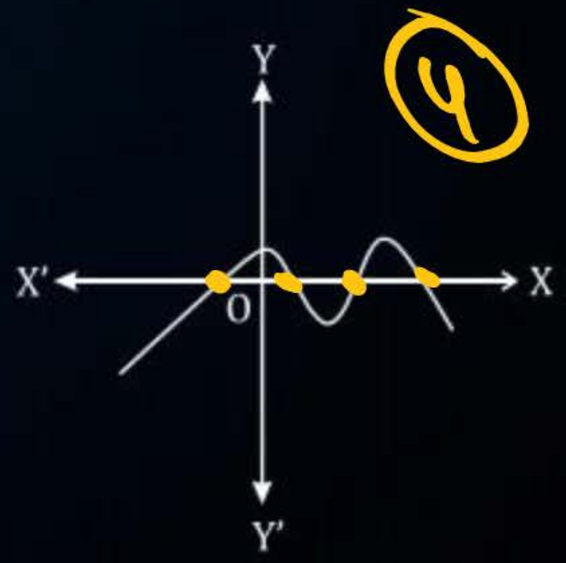












Quadratic polynomial.

ax^2+bx+c

$a, b, c \in R$
$a \neq 0$

Q $2x^2-3x+2$

$a=2,\ b=-3,\ c=2$

Q $4x^2-3x+0$

$\boxed{a=4,\ b=-3,\ c=0}$

Q $2x^2-3+0x$

$\boxed{a=2,\ b=0,\ c=-3}$

Topic : Zero of a Polynomial

#Q. If one of the zeroes of the quadratic polynomial $(k-1)x^2 + kx + 1$ is -3, then the value of k is

(A) $\frac{4}{3}$

(B) $\frac{-4}{3}$

(C) $\frac{2}{3}$

(D) $\frac{-2}{3}$

$(k-1)x^2+kx+1$

ax^2+bx+c

$a = k-1$
$b = k$
$c = 1$

$(k-1)(-3)^2+k(-3)+1=0$

$(k-1)(9)-3k+1=0$

$9k-9-3k+1=0$

$6k-8=0$

$6k=8$

$k=\frac{4}{3}$

Middle term splitting for finding zeroes of quadratic polynomial

$3x^2+10x+3$

ax^2+bx+c

$a=3,\ b=10,\ c=3$

(i) $3x^2 + 10x + 3$

$3x^2+10x+3=0$

$P=9$

$S=10$

$9, 1$

$3x^2+9x+1x+3=0$

$3x(x+3)+1(x+3)=0$

$(x+3)(3x+1)=0$

$x+3=0,\ 3x+1=0$

$x=-3,\ x=-\frac{1}{3}$

α β

$\alpha+\beta=-\frac{b}{a}$

$(-3)+\left(-\frac{1}{3}\right)=-\frac{10}{3}$

$-\frac{10}{3}=-\frac{10}{3}$

$\alpha\beta=\frac{c}{a}$

$(-3)\left(-\frac{1}{3}\right)=\frac{3}{3}$

$1=1$

Q. $7x-6-2x^2$

ax^2+bx+c

$a=-2,\ b=7,\ c=-6$

$-2x^2+7x-6=0$

$P=12$

$S=7$

$3, 4$

$-2x^2+3x+4x-6=0$

$x(-2x+3)+2(2x-3)=0$

$x(-2x+3)-2(-2x+3)=0$

$(-2x+3)(x-2)=0$

$x=\frac{3}{2},\ x=2$

α β

$\alpha+\beta=\frac{-b}{a}$

$\frac{3}{2}+2=\frac{-7}{-2}$

$\frac{7}{2}=\frac{7}{2}$

$\alpha\beta=\frac{c}{a}$

$\frac{3}{2}\times 2=\frac{-6}{-2}$

$3=3$

Middle term splitting for finding zeroes of quadratic polynomial

$7x^2 - 19x - 6$

$P = -42$
$S = -19$

$-21, 2$

2	42
3	21
7	7
	1

$7x^2 - 21x + 2x - 6$

$7x(x-3) + 2(x-3)$

$(x-3)(7x+2) = 0$

$x = 3, -\frac{2}{7}$

α β

Relationship between the zeros and coefficients of a Quadratic Polynomial

ax^2+bx+c

$a, b, c \in R$

$a \neq 0$

α = alpha.

β = Beta.

$\alpha + \beta = -\frac{b}{a}$

(Sum of zeroes)

$\alpha\beta$

(Product of zeroes) $= \frac{c}{a}$

Topic : Relationship b/w zeroes and coefficients of quadratic polynomial

#Q. If α and β are the zeroes of a polynomial $x^2 - 4\sqrt{3}x + 3$, then find the value of $\alpha + \beta - \alpha\beta$.

[Board Term – I, 2015]

$x^2 - 4\sqrt{3}x + 3$

$ax^2 + bx + c$

$a = 1,\ b = -4\sqrt{3},\ c = 3$

$\alpha + \beta = -\frac{b}{a}$

$\alpha + \beta = \frac{-(-4\sqrt{3})}{1}$

$\alpha + \beta = 4\sqrt{3}$

$\alpha\beta = \frac{c}{a}$

$\alpha\beta = \frac{3}{1}$

$\alpha + \beta - \alpha\beta$

$= 4\sqrt{3} - 3$

Topic : Relationship b/w zeroes and coefficients of quadratic polynomial

#Q. P and q are the zeroes of the polynomial $4y^2 - 4y + 1$. What is the value of $\frac{1}{p} + \frac{1}{q} + pq$?

[CBSE, Q.B., 2021 - 22]

(A) $-\frac{15}{4}$

(B) $-\frac{3}{4}$

(C) $\frac{5}{4}$

(D) $\frac{17}{4}$

$= \frac{1}{p} + \frac{1}{q} + pq$

$= \frac{q+p}{pq} + pq$

$= \frac{1}{\frac{1}{4}} + \frac{1}{4}$

$= \frac{4}{1} + \frac{1}{4} = \frac{16+1}{4} = 17/4$

$4y^2 - 4y + 1$

$ay^2 + by + c$

$a = 4,\ b = -4,\ c = 1$

$\alpha + \beta = -\frac{b}{a}$

$p + q = -\frac{(-4)}{4}$

$p + q = 1$

$\alpha\beta = \frac{c}{a}$

$pq = \frac{1}{4}$

EXAMPLE 14 *If zeroes of the quadratic polynomial* $f(x) = (k^2 + 4)x^2 + 7x + 4k$ *are reciprocal of each other, then the value (s) of k is (are)*

(a) 1 (b) –1 (c) 2 (d) –2

$\alpha, \frac{1}{\alpha}$

$(k^2+4)x^2+7x+4k$

$a = k^2+4,\ b=7,\ c=4k$

$\alpha+\beta = \frac{-b}{a}$ | $\alpha\beta = \frac{c}{a}$

$\alpha+\beta = -\frac{7}{k^2+4}$ | $\alpha\beta = \frac{4k}{k^2+4}$

$\alpha\beta = \frac{4k}{k^2+4}$

$\alpha\left(\frac{1}{\alpha}\right) = \frac{4k}{k^2+4}$

$1 = \frac{4k}{k^2+4}$

$k^2+4 = 4k$

$k^2-4k+4=0$

$k^2-2k-2k+4=0$

$k(k-2)-2(k-2)=0$

$(k-2)(k-2)=0$

$k=2,2$

Q 2, 3

$S = 5$
$P = 6$

Formula: $k[x^2 - Sx + P]$

k: non-zero constant.

S: sum of zeroes

P: product of zeroes.

$$\boxed{k[x^2 - 5x + 6]}$$

$1[x^2 - 5x + 6]$

$2[x^2 - 5x + 6]$

$-3[x^2 - 5x + 6]$

Q $\sqrt{2}, 3$ → Polynomial = ?

$S = \sqrt{2} + 3$

$P = 3\sqrt{2}$

$k[x^2 - S x + P]$

→ $k[x^2 - (\sqrt{2}+3)x + 3\sqrt{2}]$

let $k = 1$

$x^2 - (\sqrt{2}+3)x + 3\sqrt{2}$

Topic : Relationship b/w zeroes and coefficients of quadratic polynomial

#Q. The number of polynomials having zeroes as –2 and 5 is

[NCERT Exemplar]

(A) 1

(B) 2

(C) 3

(D) More than 3

Topic : Relationship b/w zeroes and coefficients of quadratic polynomial

#Q. A quadratic polynomial, the product and sum of whose zeroes are 5 and 8 respectively is [CBSE, Board Term – I, 2021]

(A) $k[x^2 - 8x + 5]$

(B) $k[x^2 + 8x + 5]$

(C) $k[x^2 - 5x + 8]$

(D) $k[x^2 + 5x + 8]$

$P = 5$
$S = 8$

$k[x^2 - Sx + P]$

$k[x^2 - 8x + 5]$

Topic : Relationship b/w zeroes and coefficients of quadratic polynomial

#Q. Find a quadratic polynomial where zeroes are $5 - 3\sqrt{2}$ and $5 + 3\sqrt{2}$.

[CBSE SQP, 2020 -21]

$S = 5 - \cancel{3\sqrt{2}} + 5 + \cancel{3\sqrt{2}}$

$S = 10$

$P = (5-3\sqrt{2})(5+3\sqrt{2})$

$(a-b)(a+b) = a^2 - b^2$

$P = (5)^2 - (3\sqrt{2})^2$

$= 25 - 18 = 7$

$S = 5, P = 7$

$= k[x^2 - Sx + P]$

$k = 1$

$x^2 - 10x + 7$

Topic : Relationship between zeroes and coefficients of a quadratic polynomial

#Q. Quadratic polynomial $2x^2 - 3x + 1$ has zeroes as α and β. Now form a quadratic polynomial whose zeroes are 3α and 3β. [Board Term – 2, 2015]

$2x^2 - 3x + 1$

$a = 2,\ b = -3,\ c = 1$

$\alpha + \beta = -\frac{b}{a}$

$\alpha + \beta = -\frac{(-3)}{2}$

$\alpha + \beta = \frac{3}{2}$

$\alpha\beta = \frac{c}{a}$

$\alpha\beta = \frac{1}{2}$

$S = 3\alpha + 3\beta$

$S = 3(\alpha + \beta)$

$S = 3\left(\frac{3}{2}\right) = 9/2$

$P = (3\alpha)(3\beta)$

$P = 9\alpha\beta$

$P = 9\left(\frac{1}{2}\right) = \frac{9}{2}$

$S = 9/2$

$P = 9/2$

$k[x^2 - Sx + P]$

$= k\left[x^2 - \frac{9}{2}x + \frac{9}{2}\right]$

$k = 2$

$\boxed{2x^2 - 9x + 9}$

Topic : Relationship b/w zeroes and coefficients of quadratic polynomial

#Q. If α and β are the zeroes of the quadratic polynomial $f(x) = x^2 - x - 2$, find a polynomial whose zeroes are $2\alpha + 1$ and $2\beta + 1$.

$S = 2\alpha + 1 + 2\beta + 1$

$= 2\alpha + 2\beta + 2$

$= 2(\alpha + \beta) + 2$

$= 2(1) + 2$

$\boxed{S = 4}$

$P = (2\alpha + 1)(2\beta + 1)$

$P = 2\alpha(2\beta + 1) + 1(2\beta + 1)$

$P = 4\alpha\beta + 2\alpha + 2\beta + 1$

$P = 4\alpha\beta + 2(\alpha + \beta) + 1$

$P = 4(-2) + 2(1) + 1$

$P = -8 + 2 + 1$

$\boxed{P = -5}$

$x^2 - x - 2$

$\boxed{a = 1,\ b = -1,\ c = -2}$

$\alpha + \beta = -\frac{b}{a}$

$\alpha + \beta = -\frac{(-1)}{1}$

$\boxed{\alpha + \beta = 1}$

$\alpha\beta = \frac{c}{a}$

$\boxed{\alpha\beta = -\frac{2}{1}}$

$S = 4$

$P = -5$

$k[x^2 - 4x + 5]$

$k = 1$

$x^2 - 4x + 8$

Topic : Relationship b/w zeroes and coefficients of quadratic polynomial

#Q. If α and β are the zeroes of the quadratic polynomial $f(x) = 2x^2 - 5x + 7$, find a polynomial whose zeroes are $2\alpha + 3\beta$ and $3\alpha + 2\beta$.

H.W

Hint:

$$a^2 + b^2 = (a+b)^2 - 2ab$$

$$\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$$

Topic : Relationship between zeroes and coefficients of a quadratic polynomial

#Q. If the zeroes of the polynomial $x^2 + px + q$ are double in values to the zeroes of $2x^2 - 5x - 3$, find the value of p and q. [2012]

$x^2+px+q \to 2\alpha, 2\beta$

$a=1,\ b=p,\ c=q$

$2\alpha+2\beta=-\frac{b}{a}$

$2(\alpha+\beta)=-\frac{p}{1}$

$\cancel{2}\left(\frac{5}{\cancel{2}}\right)=-p$

$-5=p$

$2\alpha\times2\beta=\frac{c}{a}$

$4\alpha\beta=\frac{q}{1}$

$\overset{2}{\cancel{4}}\left(\frac{-3}{\cancel{2}}\right)=q$

$-6=q$

$\alpha, \beta \leftarrow 2x^2-5x-3$

$a=2,\ b=-5,\ c=-3$

$\alpha+\beta=-\frac{b}{a}$

$\alpha+\beta=-\frac{(-5)}{2}$

$\alpha+\beta=\frac{5}{2}$

$\alpha\beta=\frac{c}{a}$

$\alpha\beta=-\frac{3}{2}$

Pair of Linear Equations.

- General Form.
- Meaning of solution.
- Graphical method of solving.
- Substituition method.
- Elimination method.
- No. of solutions.

2 cheeze

Topic : Pair of Linear Equation in Two Variable

$$x + y = 1$$
$$2x + 3y = 5$$

$a_1 = 1$ $a_2 = 2$
$b_1 = 1$ $b_2 = 3$
$c_1 = -1$ $c_2 = -5$

$$a_1x + b_1y + c_1 = 0$$
$$a_2x + b_2y + c_2 = 0$$

solution
↓
Variables (x, y)
dono
↓
Jo eqn ko
satisfy
↓
L.H.S = R.H.S

How many solutions?

unique (only one solution)

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

Infinite many solution

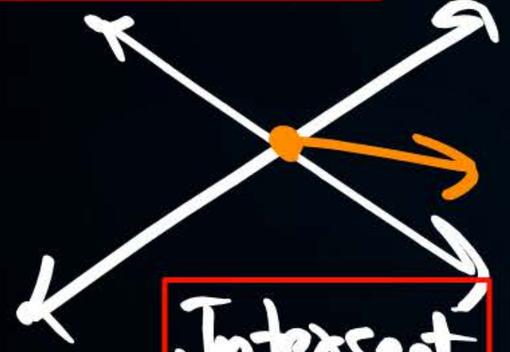
$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

No solution

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

Consistent

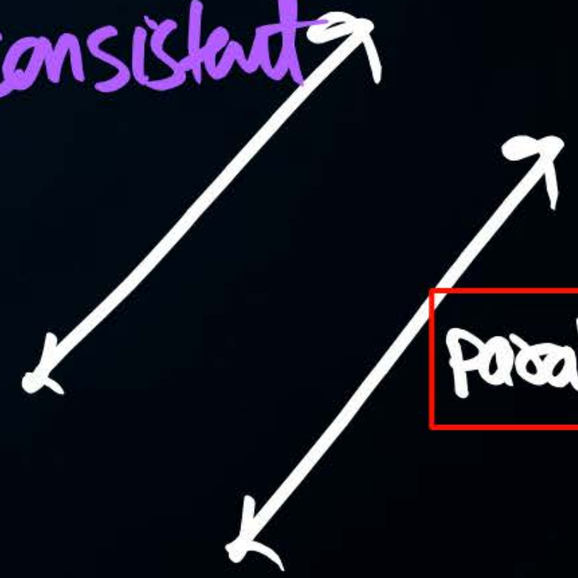
Coordinates → solution.



Intersect

inconsistent

Coincident



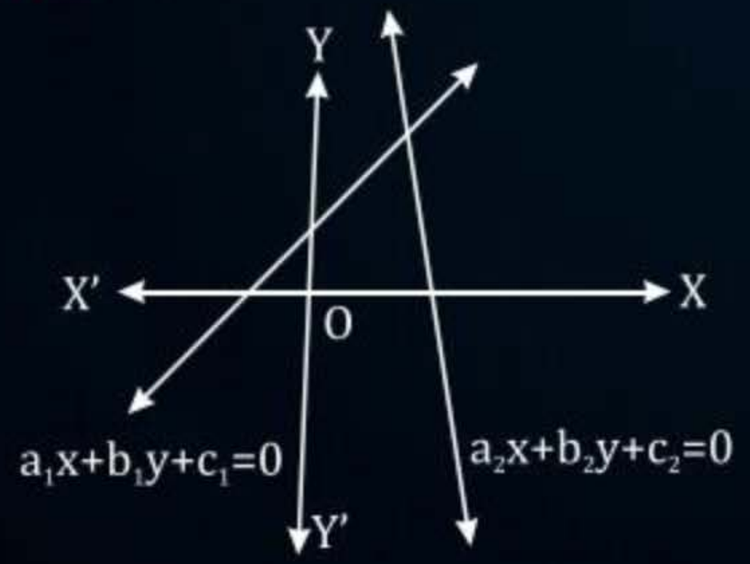
Parallel

Topic : Graphical Representation of Linear Equations

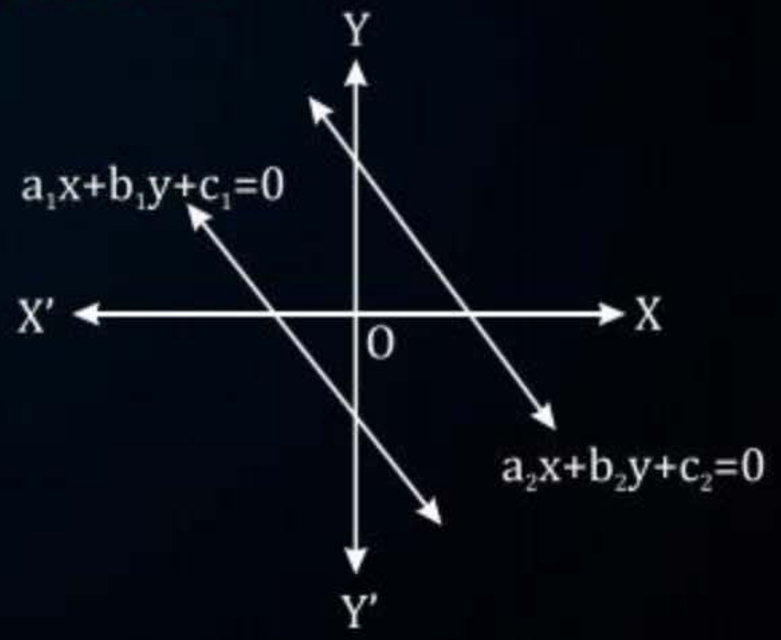
(i) The two lines intersect at one point.

(ii) The two lines are parallel i.e. they do not intersect however far they are extended.

Intersecting lines



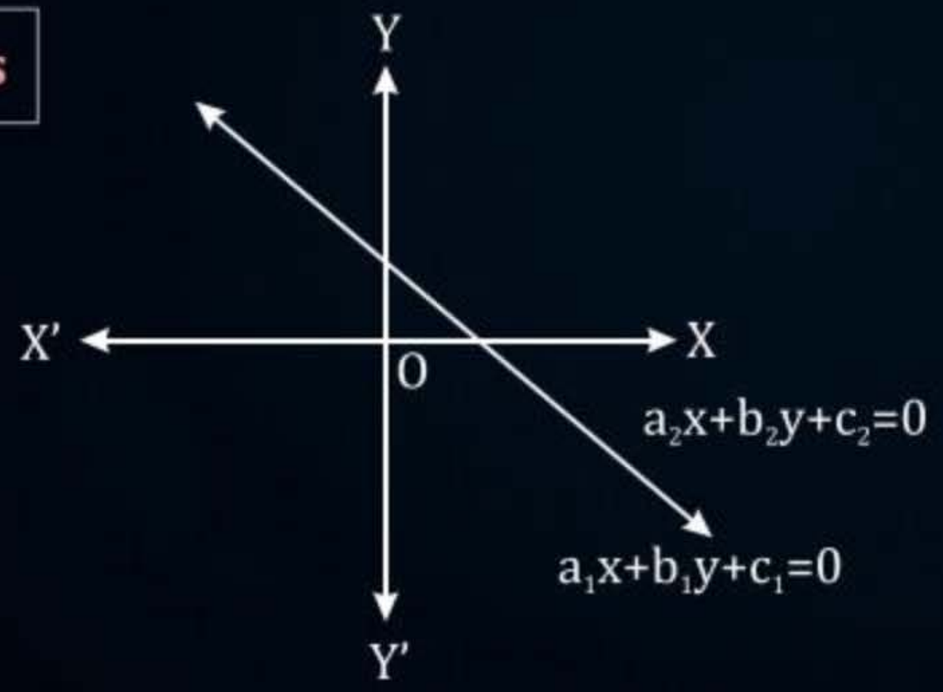
Parallel lines



Topic : Graphical Representation of Linear Equations

(iii) The two lines are coincident lines i.e. one line overlaps the other line.

Coincident lines



Topic : Graphical Method of Solving Simultaneous Linear Equation

#Q. Solve graphically the system of equations:

$$x + y = 3$$
$$3x - 2y = 4$$

$a_1x + b_1y + c_1 = 0$

$a_2x + b_2y + c_2 = 0$

$a_1 = 1 \quad b_1 = 1 \quad c_1 = -3$

$a_2 = 3 \quad b_2 = -2 \quad c_2 = -4$

$\frac{1}{3} \quad \frac{1}{-2} \quad -\frac{3}{-4}$

$\frac{1}{3} \neq -\frac{1}{2} \quad \frac{3}{4}$

$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

→ Intersecting

→ unique solution

→ Consistent.

$x+y=3$

x	0	3	1
y	3	0	2

$3x-2y=4$

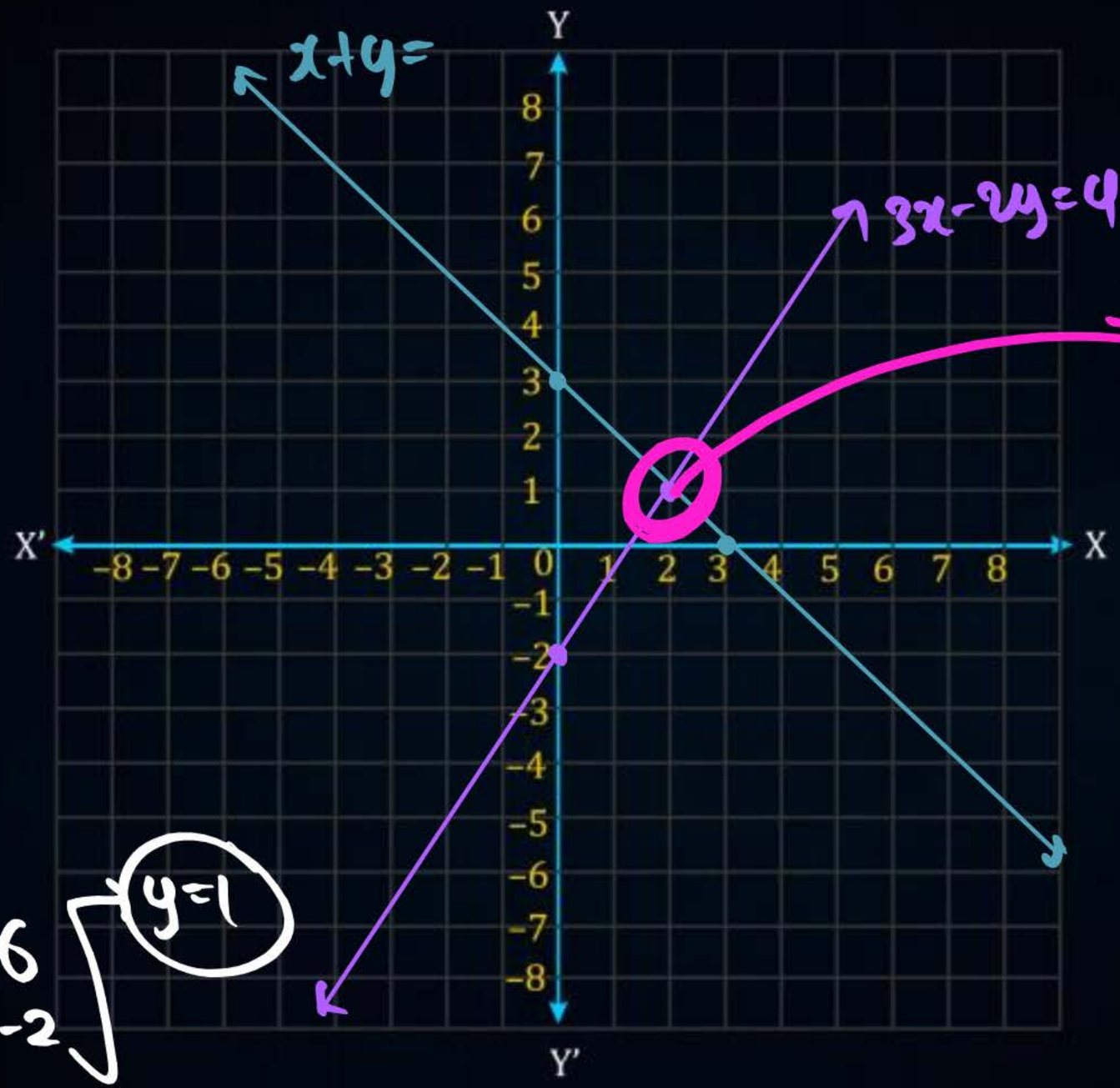
x	0	2		
y	-2	1		

$6-2y=4$

$-2y=4-6$

$-2y=-2$

$y=1$



$(2,1)$ Ans

Topic : Graphical Method of Solving Simultaneous Linear Equation

#Q. If a pair of linear equations is consistent then the lines will be :

(A) Parallel

(B) Always coincident

(C) Intersecting or coincident

(D) Always intersecting

solution hai

unique → Intersecting

Infinite → Coincident

Topic : Graphical Method of Solving Simultaneous Linear Equation

#Q. Draw the graphs of $2x + y = 6$ and $2x - y + 2 = 0$. Shade the region bounded by these lines and x-axis. Find the area of the shaded region. [CBSE 2002]

$2x+y=6$

x	0	3
y	6	0

$2x-y+2=0$

x	0	-1
y	2	0

$0-y+2=0$

$-y=-2$

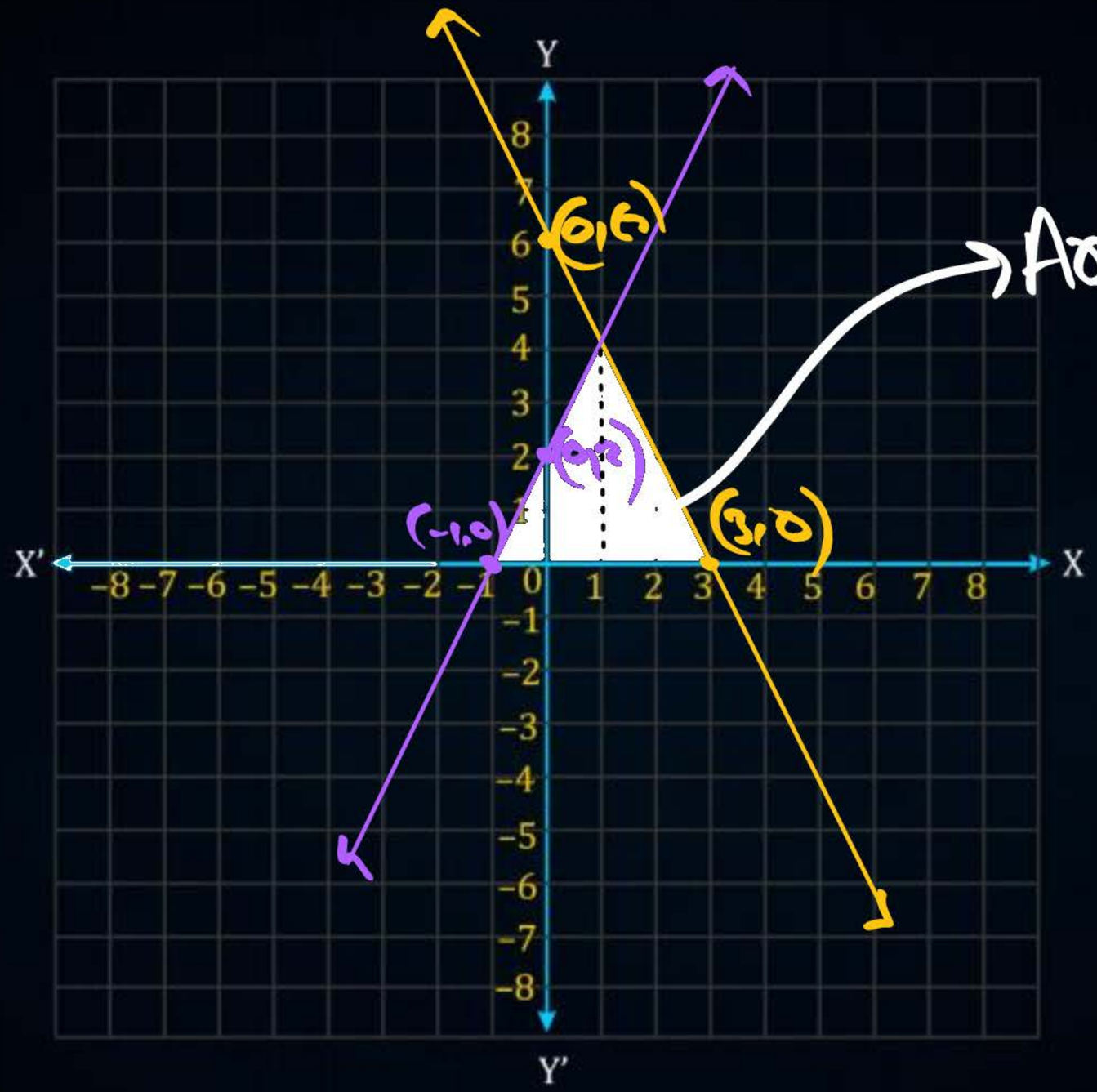
$y=2$

$2x-0+2=0$

$2x+2=0$

$2x=-2$

$x=-1$



$$\text{Area} = \frac{1}{2} \times b \times h$$

$$= \frac{1}{2} \times 4 \times 4$$

$$= 8 \text{ sq. units}$$

Topic : Substitution Method

#Q. Solving the following systems of equations by using the method of substitution:

(1) $3x - 5y = -1$; (2) $x - y = -1$

[redacted]

$3x - 5y = -1$

$3x = -1 + 5y$

$x = \frac{-1+5y}{3}$ (3)

Put (3) in (2)

$x - y = -1$

$\frac{-1+5y}{3} - y = -1$

$\frac{-1+5y-3y}{3} = -1$

$-1 + 5y - 3y = -3$

$2y = -3 + 1$

$2y = -2$

$y = -1$

Put (4) in (3)

$x = \frac{-1+5(-1)}{3} = -2$

Topic : Substitution Method

#Q. Solve by Substitution Method :

$$\frac{x}{3}+\frac{y}{4}=11 \rightarrow \frac{4x+3y}{12}=11 \rightarrow \boxed{4x+3y=132} \quad (1)$$

$$\frac{5x}{6}-\frac{y}{3}=-7 \rightarrow \frac{5x-2y}{6}=-7 \rightarrow \boxed{5x-2y=-42} \quad (2)$$

$5x-2y=-42$

$5x=-42+2y$

$$\boxed{x=\frac{-42+2y}{5}} \quad (3)$$

Put (x) in (1)

$$4\left(\frac{-42+2y}{5}\right)+\frac{3y}{1}=132$$

$$\frac{-168+8y}{5}+\frac{3y}{1}=132$$

$$\frac{-168+8y+15y}{5}=132$$

$23y-168=660$

$23y=660+168$

$23y=828$

$y=\frac{828}{23}=36$

Topic : Elimination Method

#Q. Solve :

(i) $3x + 2y = 11$ $\times 2$

$2x + 3y = 4$ $\times 3$

$6x + 4y = 22$

$6x + 9y = 12$

$-5y = 10$

$\boxed{y = -2}$

$3x + 2y = 11$

$3x + 2(-2) = 11$

$3x - 4 = 11$

$3x = 15$

$\boxed{x = 5}$

3	3, 2
2	1, 2
	1, 1

LCM = 6

Topic : Elimination Method

#Q. Solve:

$$\frac{x}{10}+\frac{y}{5}+1=15 \rightarrow \frac{x+2y}{10}=14 \rightarrow \boxed{x+2y=140} \text{ — (1)} \times 2$$

$$\frac{x}{8}+\frac{y}{6}=15 \rightarrow \frac{3x+4y}{24}=15 \rightarrow \boxed{3x+4y=360} \text{ — (2)}$$

$$2x+\cancel{4y}=280$$

$$3x+\cancel{4y}=360$$

$$\ominus \quad \ominus \quad \ominus \quad \ominus$$

$$-x=-80$$

$$\boxed{x=80}$$

$$y=30$$

Topic : Miscellaneous Problems

#Q. Solve:

$37x + 41y = 70$ — (1)

$41x + 37y = 86$ — (2)

(1) + (2)

$$37x + 41y = 70$$
$$41x + 37y = 86$$
$$78x + 78y = 156$$
$$78[x+y] = 156$$
$$x+y = \frac{156}{78}$$
$$\boxed{x+y=2}$$

(1) − (2)

$$37x + 41y = 70$$
$$\ominus\ 41x \ominus\ 37y = \ominus\ 86$$
$$-4x + 4y = -16$$
$$-4(x-y) = -16$$
$$x-y = \frac{-16}{-4}$$
$$\boxed{x-y=4}$$

$$x + \cancel{y} = 2$$
$$x - \cancel{y} = 4$$

$$2x = 6$$

$$x = 3$$

Topic : Condition of Solvability of Equations

#Q. Find the value of k for which the given system has a unique solution:

[NCERT]

(i) $x - ky = 2$
$3x + 2y = -5$

(ii) $2x + ky = 1$
$5x - 7y = 5$

(i) $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

$\frac{1}{3} \neq \frac{-k}{2}$

$-\frac{2}{3} \neq k$

All Real nos except $-\frac{2}{3}$.

(ii) $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

$\frac{2}{5} \neq \frac{k}{-7}$

$\frac{-14}{5} \neq k$

All Real nos except $-14/5$.

Topic : Condition of Solvability of Equations

#Q. Find the value of k for which the given system has infinitely many solutions.

(i) $5x + 2y = k$

$10x + 4y = 3$

[NCERT]

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\boxed{\frac{5}{10} = \frac{2}{4} = \frac{-k}{-3}}$$

$$\frac{2}{4} = \frac{-k}{-3}$$

$$\frac{1}{2} = \frac{k}{3}$$

$$\frac{3}{2} = k$$

Topic : Condition of Solvability of Equations

#Q. For what value of k will the following system of linear equations has no solution?

[NCERT, CBSE 2000]

$3x + y = 1$

$(2k - 1)x + (k - 1)y = 2k + 1$

No solution

$$\boxed{\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}}$$

$$\frac{3}{2k-1} = \frac{1}{k-1} \neq \frac{-1}{-2k-1}$$

$$\frac{3}{2k-1} = \frac{1}{k-1}$$

$$3(k-1) = 1(2k-1)$$

$$3k-3 = 2k-1$$

$$3k-2k = -1+3$$

$$\boxed{k = 2}$$

check

$$\frac{3}{2(2)-1} \quad \frac{1}{2-1} \quad \frac{-1}{-2(2)-1}$$

$$\frac{3}{3} \quad \frac{1}{1} \quad \frac{-1}{-5}$$

$$1 = 1 \neq \frac{1}{5}$$

Topic : Condition of Solvability of Equations

#Q. For what value of k, will the system of equations [CBSE 2001]

$x + 2y = 5$

$3x + ky - 15 = 0.$

has no solution?

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

$$\frac{1}{3} = \frac{2}{k} \neq \frac{-5}{-15}$$

$$\frac{1}{3} = \frac{2}{k} \neq \frac{1}{3}$$

$$\frac{1}{3} = \frac{2}{k}$$

$$k = 6$$

No value of k will give no solution.

Topic : Condition of Solvability of Equations

#Q. Find the value(s) of k for which the system of equations

$kx - y = 2$

$6x - 2y = 3$ has

(i) A unique solution

(ii) No solution

Is there a value of k for which the system has infinitely many solution?

H.W

12. 8 chairs and 5 tables cost ₹ 10,500, while 5 chairs and 3 tables cost ₹ 6,450. The cost of each chair will be

(a) ₹ 750 (b) ₹ 600 (c) ₹ 850 (d) ₹ 900

let cost of one chair $= x$

" " " " table $= y$

$$8x + 5y = 10{,}500$$
$$5x + 3y = 6450$$

Topic : Problems Based on Numbers

#Q. The sum of the digits of a two digit number is 8 and the difference between the number and that formed by reversing the digits is 18. Find the number.

$x+y=8$ — (1)

$(10y+x)-(10x+y)=18$

$10y+x-10x-y=18$

$9y-9x=18$ — (2)

$9(y-x)=18$

$y-x=2$

unit's digit $= x$

ten's digit $= y$

(Number) two digit no. $= 10y+x$

Reversed no. (interchange) $= 10x+y$

25. Aruna has only ₹ 1 and ₹ 2 coins with her. If the total number of coins that she has is 50 and the amount of money with her is ₹ 75, then the number of ₹ 1 and ₹ 2 coins are, respectively

(a) 35 and 15 (b) 35 and 20 (c) 15 and 35 (d) 25 and 25

let no. of 1₹ coins = x.

" " " 2₹ " = y

$x + y = 50$

$1x + 2y = 75$

Total

1₹ → 2 coins = (1×2)

1₹ → 4 coins = (1×4)

1₹ → 10 coins = (1×10)

1₹ → 'x' coins = (1x)

2₹ → 2 coins = (2×2)

2₹ → 50 coins = (2×50)

2₹ → 'y' coins = (2×y)

Taxi

Fixed charge

x

distance.
(charge per km)

y

50km

Total money

$= x + 50y$

Topic : Miscellaneous Problems

#Q. The taxi charges in a city comprise of a fixed charge together with the charge for the distance covered. For a journey of 10 km the charge paid is Rs. 75 and for a journey of 15 km the charge paid is Rs. 110. What will a person have to pay for travelling a distance of 25 km?

[NCERT, CBSE 2000]

Fixed charge = x

charge per km = y

For 10 km

$x + 10y = 75$ — (1)

For 15 km

$x + 15y = 110$ — (2)

Topic : Miscellaneous Problems

#Q. A part of monthly hostel charges in a college are fixed and the remaining depend on the number of days one has taken food in the mess. When a student A takes food for 20 days, he has to pay Rs 1000 as hostel charges whereas a student B, who takes food for 26 days, pays Rs 1180 as hostel charges. Find the fixed charge and the cost of food per day.

[NCERT, CBSE 2000]

Hostel charges

Fixed x

cost of food per day. y

20 days → 1000 Rs

$x + 20y = 1000$ — (1)

26 days → 1180 Rs

$x + 26y = 1180$ — (2)

Topic : Miscellaneous Problems

#Q. A lending library has a fixed charge for the first three days and an additional charge for each day thereafter. Shristi paid ₹ 27 for a book kept for seven days, while Rekha paid ₹ 21 for the book, she kept for five days. Find the fixed charge and the additional charge paid by them.

[Board Term-I, 2015]

library

Fixed charge (3days) → x

charge per day after 3 days. → y

10din

3d 7din

$x + 7y$

12din

3 9

$x + 9y$

Topic : Miscellaneous Problems

#Q. Raghav scored 70 marks in a test, getting 4 marks for each right answer and losing 1 mark for each wrong answer. Had 5 marks been awarded for each correct answer and 2 marks been deducted for each wrong answer, then Raghav would have scored 80 marks. How many questions were there in the test? [Board Term – I, 2015]

Hw

H.W

CASE BASED

Dhoni is a former Indian cricketer and captain of the Indian national cricket team. He is known for his powerful and innovative batting, and he has scored many boundaries throughout his career. He has a particular penchant for hitting sixes, and he is one of the most successful finishers in limited- overs cricket. Dhoni has hit a total of 359 fours and 209 sixes in One Day Internationals (ODIs), and 78 fours and 16 sixes in Twenty20 Internationals (T20Is). Some of his most memorable boundary shots include the helicopter shot, which he invented himself, and his signature flick shot over the leg side.

In on of the matches he scored 98 runs only in boundaries.
Some of the boundaries were fours (4 runs each) and
some of the boundaries were sixes (6 runs each).
If the number of sixes made was two less than that of fours
answer the following questions.

Topic : Case Based

#Q. (i) What is the algebraic representation of the above problem?

Hw

Topic : Case Based

#Q. (ii) How many fours did he make?

OR

How many sixes did he make?

H.W

Topic : Case Based

#Q. (iii) What are total number of boundaries scored by Dhoni?

H.W

There is an artist, who doesn't need to understand Math...

There is an entrepreneur, who doesn't care about History or English literature...

There is a musician, whose Chemistry marks won't matter...

There's an athlete... whose physical fitness is more important than Physics...

TRIGONOMETRY

- T. Ratios
- T. Ratios For specific angles.
- Identities.

$\text{Sine}\,\theta = \frac{P}{H}$ $(\sin\theta)$

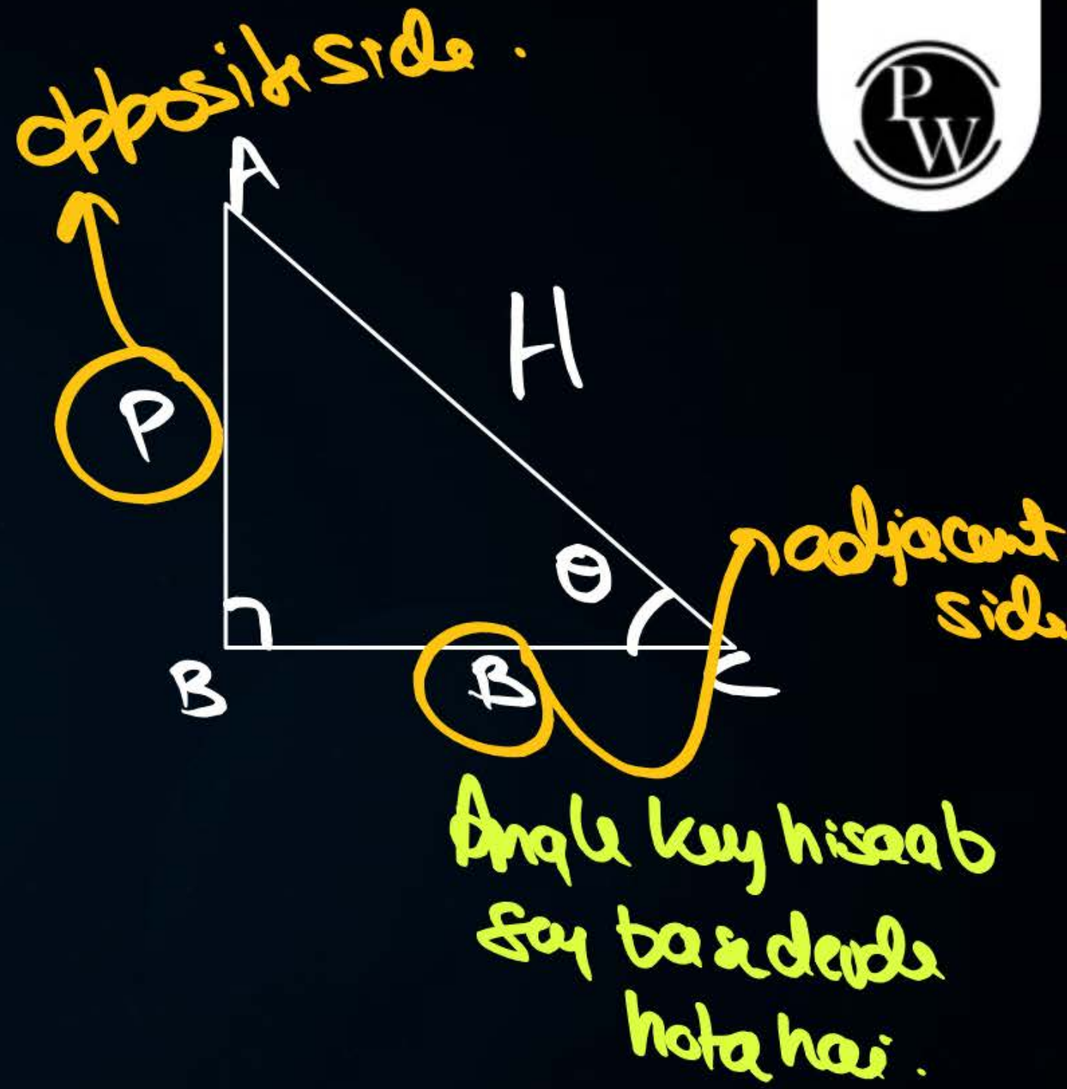
$\text{Cosecant}\,\theta = \frac{H}{P}$ $(\operatorname{cosec}\theta)$

$\text{Cosine}\,\theta = \frac{B}{H}$ $(\cos\theta)$

$\text{Secant}\,\theta = \frac{H}{B}$ $(\sec\theta)$

$\text{tangent}\,\theta = \frac{P}{B}$ $(\tan\theta)$

$\text{Cotangent}\,\theta = \frac{B}{P}$ $(\cot\theta)$

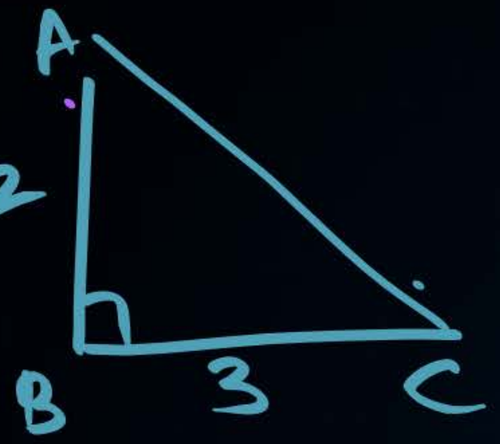


Pizza burger Pyar

hain Homaar Babua

super ⭐ hain

Q



$$\sin C = \frac{P}{H} = \frac{AB}{AC} = \frac{2}{\sqrt{13}}$$

$$\sin A = \frac{P}{H} = \frac{BC}{AC} = \frac{3}{\sqrt{13}}$$

$$(AC)^2 = (AB)^2 + (BC)^2$$

$$AC^2 = 2^2 + 3^2$$

$$= 4 + 9$$

$$AC^2 = 13$$

$$\boxed{AC = \pm\sqrt{13}}$$

$$AC = +\sqrt{13}$$

Q $\tan A = \frac{P}{B} = \frac{BC}{AB}$

$$= \frac{3}{2}$$

Q $\tan C - \cos C$

$$\frac{AB}{BC} - \frac{BC}{AC}$$

$$= \frac{2}{3} - \frac{3}{\sqrt{13}}$$

$$= \boxed{\frac{2\sqrt{13} - 9}{3\sqrt{13}}} \times \frac{\sqrt{13}}{\sqrt{13}}$$

Topic : T Ratios

#Q. If $\cos B = \frac{1}{3}$, find the other five trigonometric ratios.

$\cos B = \frac{1}{3} = \frac{B}{H}$

$B = 1x$
$H = 3x$

$H^2 = P^2 + B^2$

$(3x)^2 = P^2 + (1x)^2$

$9x^2 = P^2 + 1x^2$

$9x^2 - 1x^2 = P^2$

$8x^2 = P^2$

$\pm\sqrt{8x^2} = P$

$+\sqrt{8}\sqrt{x^2} = P$

$2\sqrt{2}x = P$

$\sin B = \frac{P}{H} = \frac{2\sqrt{2}x}{3x} = \frac{2\sqrt{2}}{3}$

$\tan B =$

$\sec B = \frac{H}{B} = \frac{3x}{1x} = \frac{3}{1}$

$\operatorname{cosec} B =$

$\cot B = \frac{B}{P} = \frac{1x}{2\sqrt{2}x} = \frac{1}{2\sqrt{2}}$

Topic : T Ratios

#Q. If in a triangle ABC right angled at B, AB = 6 units and BC = 8 units, then find the value of sin A · cos C + cos A · sin C. **[Board Term – I, 2016]**

$\sin A = \frac{8}{10}$

$\cos C = \frac{8}{10}$

$\cos A = \frac{6}{10}$

$\sin C = \frac{6}{10}$

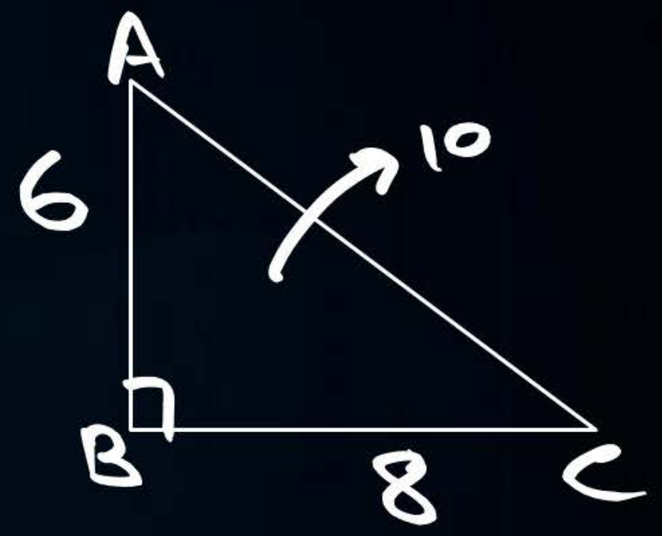
$= \frac{8}{10} \times \frac{8}{10} + \frac{6}{10} \times \frac{6}{10}$

$= \frac{64}{100} + \frac{36}{100}$

$= \frac{64+36}{100}$

$= \frac{100}{100}$

$= \boxed{1}$



$H^2 = P^2 + B^2$

$H = \sqrt{P^2 + B^2}$

$\sin A = \frac{5}{2}$ not possible

$x = \text{dia}$

$\sin A = \frac{P}{H}$

$H = 2x$

$P = 5x$

Topic : T Ratios ★★★★★

#Q. In ΔPQR, right angled at Q, PR + QR = 25 cm and PQ = 5 cm. Determine the values of sin P, cos P and tan P. [NCERT]

$PR = 25 - QR$

$PR = 13$

$\sin P = \frac{P}{H} = \frac{QR}{PR}$

$\cos P = \frac{B}{H} = \frac{PQ}{PR}$

$\tan P = \frac{P}{B} = \frac{QR}{PQ}$

Hw

$PR^2 = PQ^2 + QR^2$

$PR^2 = 25 + QR^2$

$(25 - QR)^2 = 25 + QR^2$

$(25)^2 + (QR)^2 - 2(25)(QR) = 25 + QR^2$

$625 + \cancel{QR^2} - 50QR = 25 + \cancel{QR^2}$

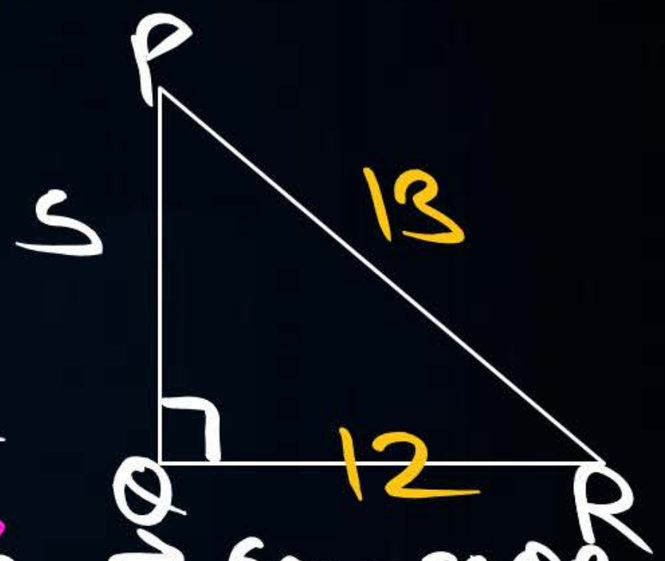
$625 - 50QR = 25$

$625 - 25 = 50QR$

$600 = 50QR$

$\frac{600}{50} = QR$

$\boxed{12 = QR}$



Reciprocal.

$\frac{P}{H} \longleftrightarrow \frac{H}{P}$

$\sin\theta \longleftrightarrow \operatorname{cosec}\theta$

$\frac{P}{B} \longleftrightarrow \frac{B}{P}$

$\tan\theta \longleftrightarrow \cot\theta$

$\frac{B}{H} \longleftrightarrow \frac{H}{B}$

$\cos\theta \longleftrightarrow \sec\theta$

Quotient Relation.

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\cot\theta = \frac{\cos\theta}{\sin\theta}$$

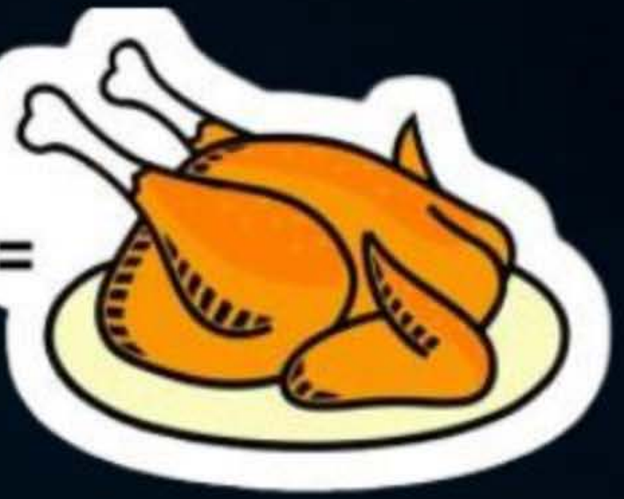
Q. $\sin\theta = \frac{1}{2}$

$\cos\theta = \frac{\sqrt{3}}{2}$

$$\tan\theta = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}}$$

$$\cot\theta = \frac{\sqrt{3}}{1}$$

$$\frac{\text{Sin Doori}}{\text{Cos Doori}} =$$



Topic: Trigonometric Ratios of some specific Angles

$\frac{1}{0} = n.d$

$\frac{1}{n.d} = 0$

T. ratios \ θ	0°	30°	45°	60°	90°
sin θ	0	1/2	1/√2	√3/2	1
cos θ	1	√3/2	1/√2	1/2	0
tanθ	0	1/√3	1	√3	n.d.
cosec θ	n.d	2/1	√2/1	2/√3	1
sec θ	1	2/√3	√2/1	2/1	n.d
cot θ	n.d	√3/1	1	1/√3	0

Topic: Trigonometric Ratios of some specific Angles

T. ratios \ θ	0°	30°	45°	60°	90°
sin θ	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos θ	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tanθ	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not defined
cosec θ	Not defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
sec θ	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Not defined
cot θ	Not defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

Topic : T Rations for some specific angles

#Q. Evaluate the following expressions:

$2\sin^2 30° \tan 60° - 3\cos^2 60° \sec^2 30°$

$$2\left(\frac{1}{2}\right)^2(\sqrt{3}) - 3\left(\frac{1}{2}\right)^2\left(\frac{2}{\sqrt{3}}\right)^2$$

$$\cancel{2}\left(\frac{1}{\cancel{4}_2}\right)\sqrt{3} - \cancel{3}\left(\frac{1}{\cancel{4}}\right)\left(\frac{\cancel{4}}{\cancel{3}}\right)$$

$$= \boxed{\frac{\sqrt{3}}{2} - 1}$$

$\sin^2 30° = (\sin 30°)^2$

$= (\sin 30° \times \sin 30°)$

Topic : T Rations for some specific angles

#Q. Evaluate : $\frac{3\tan^2 30° + \tan^2 60° + \text{cosec}30° - \tan 45°}{\cot^2 45°}$ [Board Term – I, 2016]

$$= \frac{3\left(\frac{1}{\sqrt{3}}\right)^2 + (\sqrt{3})^2 + 2 - 1}{(1)^2}$$

$$= \frac{\cancel{3}\left(\frac{1}{\cancel{3}}\right) + 3 + 1}{1}$$

$$= 1 + 3 + 1 = 5$$

Topic : T Rations for some specific angles

#Q. [redacted] $\dfrac{\cos 30^\circ+\sin 60^\circ}{1+\cos 60^\circ+\sin 30^\circ} = ?$

$$= \frac{\frac{\sqrt{3}}{2}+\frac{\sqrt{3}}{2}}{1+\frac{1}{2}+\frac{1}{2}}$$

$$= \frac{\frac{\sqrt{3}+\sqrt{3}}{\cancel{2}}}{\frac{2+1+1}{\cancel{2}}} = \frac{\cancel{2}\sqrt{3}}{\cancel{4}\,2}$$

$$= \boxed{\frac{\sqrt{3}}{2}}$$

Q. $\sin\theta = \frac{1}{2}$

$\sin\theta = \sin 30^\circ$

on comparison.

$\boxed{\theta = 30^\circ}$

Q. $\cos 2\theta = \frac{\sqrt{3}}{2}$

$\theta = ?$

$\cos 2\theta = \cos 30^\circ$

On comparison -

$2\theta = 30^\circ$

$\theta = \frac{30}{2}$

$\boxed{\theta = 15^\circ}$

Topic : T Rations for some specific angles

#Q. Find the value of θ in each of the following:

(i) $2\sin 2\theta = \sqrt{3}$

$\sin 2\theta = \frac{\sqrt{3}}{2}$

$\sin 2\theta = \sin 60^\circ$

$2\theta = 60^\circ$

$\theta = 30$

(ii) $2\cos 3\theta = 1$

$\cos 3\theta = \frac{1}{2}$

$\cos 3\theta = \frac{1}{2}$

$\cos 3\theta = \cos 60^\circ$

$3\theta = 60^\circ$

$\theta = 20$

Topic : T Rations for some specific angles

#Q. If $\sin(A + B) = 1$ and $\sin(A - B) = 1/2$, $0 \le A + B \le 90°$ and $A > B$, then find A and B.

[Board Term – I, 2016]

$\sin(A+B) = 1$

$\sin(A+B) = \sin 90°$

on comp.

$A+B=90°$ — (1)

$\sin(A-B) = \frac{1}{2}$

$\sin(A-B) = \sin 30°$

on comp.

$A-B=30$ — (2)

$A-B=30$

$60-B=30$

$30=B$

$$\begin{array}{r} A+\cancel{B}=90 \\ (+)\quad A-\cancel{B}=30 \\ \hline 2A=120 \end{array}$$

$2A=120$

$A=60°$

Topic : T Rations for some specific angles

#Q. If $\sqrt{3}\tan\theta = 1$, then find the value of $\sin^2\theta - \cos^2\theta$.

$\sqrt{3}\tan\theta = 1$

$\tan\theta = \frac{1}{\sqrt{3}} = \frac{P}{B}$

M.I

M.II

$\tan\theta = \frac{1}{\sqrt{3}}$

$\tan\theta = \tan 30^\circ$

$\theta = 30^\circ$

$= \sin^2 30^\circ - \cos^2 30^\circ$

$= \left(\frac{1}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2$

$\frac{1}{4} - \frac{3}{4}$

$= \frac{2}{4}$

$= \frac{1}{2}$

Topic : T Rations for some specific angles

#Q. In figure, lengths of sides BC and AB are respectively

(A) 12 cm, $3\sqrt{3}$ cm

(B) 3 cm, $3\sqrt{3}$ cm

(C) 12 cm, $6\sqrt{3}$ cm

(D) 18 cm, $9\sqrt{3}$ cm

$\frac{P}{H} = \sin 30$

$\frac{BC}{6} = \frac{1}{2}$

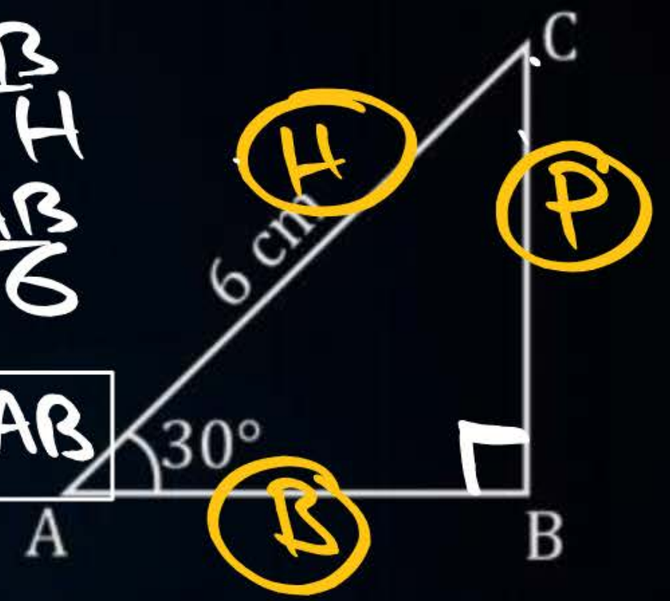
$BC = \frac{6}{2}$

$BC = 3cm$

$\cos 30 = \frac{B}{H}$

$\frac{\sqrt{3}}{2} = \frac{AB}{6}$

$3\sqrt{3}cm = AB$



Reciprocal Relation:

$\sin\theta \leftrightarrow \operatorname{cosec}\theta$

$\cos\theta \leftrightarrow \sec\theta$.

$\tan\theta \leftrightarrow \cot\theta$.

Quotient Relation:

$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$

$\cot\theta = \dfrac{\cos\theta}{\sin\theta}$

1 or Square Vali

$\boxed{\sin^2\theta + \cos^2\theta = 1}$

$\sin^2\theta = 1 - \cos^2\theta$.

$\cos^2\theta = 1 - \sin^2\theta$

$\boxed{\sec^2\theta = 1 + \tan^2\theta}$

$\sec^2\theta - \tan^2\theta = 1$

$\sec^2\theta - 1 = \tan^2\theta$

$\boxed{\operatorname{cosec}^2\theta = 1 + \cot^2\theta}$.

$\operatorname{cosec}^2\theta - \cot^2\theta = 1$

$\operatorname{cosec}^2\theta - 1 = \cot^2\theta$.

Topic : Trigonometric Identities

#Q. Prove the following identity :

$$\frac{1}{1+\sin\theta}+\frac{1}{1-\sin\theta}=2\sec^2\theta$$

$$=\frac{1(1-\sin\theta)+1(1+\sin\theta)}{(1+\sin\theta)(1-\sin\theta)}$$

$(a+b)(a-b)=a^2-b^2$

$$=\frac{1-\cancel{\sin\theta}+1+\cancel{\sin\theta}}{(1)^2-(\sin^2\theta)}$$

$$=\frac{2}{1-\sin^2\theta}=\frac{2}{\cos^2\theta}=\frac{2\times1}{\cos^2\theta}=\boxed{2\sec^2\theta}$$

Topic : Trigonometric Identities

#Q. Prove the following identity :

$$\frac{\cos\theta}{1-\sin\theta}+\frac{\cos\theta}{1+\sin\theta}=2\sec\theta$$

$$=\frac{\cos\theta(1+\sin\theta)+\cos\theta(1-\sin\theta)}{(1-\sin\theta)(1+\sin\theta)}$$

$$=\frac{\cos\theta+\cancel{\cos\theta\sin\theta}+\cos\theta-\cancel{\cos\theta\sin\theta}}{1^2-\sin^2\theta}$$

$$=\frac{2\cos\theta}{\cos^2\theta}=\frac{2\cancel{\cos\theta}}{\cos\theta\times\cancel{\cos\theta}}=\frac{2}{\cos\theta}=\frac{2\times1}{\cos\theta}=\boxed{2\sec\theta}$$

Topic : Trigonometric Identities

Prove the following identity : [NCERT Exemplar]

$$\frac{\sin\theta}{1+\cos\theta}+\frac{1+\cos\theta}{\sin\theta}=2\operatorname{cosec}\theta$$

$$=\frac{\sin^2\theta+(1+\cos\theta)^2}{(1+\cos\theta)(\sin\theta)}$$

$$=\frac{\sin^2\theta+1+\cos^2\theta+2\cos\theta}{(1+\cos\theta)\sin\theta}$$

$$=\frac{1+1+2\cos\theta}{(1+\cos\theta)\sin\theta}$$

$$=\frac{2+2\cos\theta}{(1+\cos\theta)\sin\theta}$$

$$=\frac{2\cancel{(1+\cos\theta)}}{\cancel{(1+\cos\theta)}\sin\theta}$$

$$=\frac{2}{\sin\theta}$$

$$=\boxed{2\operatorname{cosec}\theta}$$

Topic : Trigonometric Identities

#Q. Prove the following identity :

$$\frac{\sin\theta}{1-\cos\theta} = \operatorname{cosec}\theta + \cot\theta$$

$$= \frac{\sin\theta}{1-\cos\theta} \times \frac{1+\cos\theta}{1+\cos\theta}$$

$$= \frac{\sin\theta(1+\cos\theta)}{1^2-\cos^2\theta}$$

$$= \frac{\cancel{\sin\theta}(1+\cos\theta)}{\sin^{\cancel{2}}\theta}$$

$$= \frac{1+\cos\theta}{\sin\theta}$$

$$= \frac{1}{\sin\theta} + \frac{\cos\theta}{\sin\theta}$$

$$= \boxed{\operatorname{cosec}\theta + \cot\theta}$$

Topic : Trigonometric Identities

#Q. Prove the following identity :

$$\frac{1-\sin\theta}{1+\sin\theta} = (\sec\theta - \tan\theta)^2$$

$$= \frac{1-\sin\theta}{1+\sin\theta} \times \frac{1-\sin\theta}{1-\sin\theta}$$

$$= \frac{(1-\sin\theta)^2}{1^2-\sin^2\theta}$$

$$= \frac{(1-\sin\theta)^2}{\cos^2\theta}$$

$$\left(\frac{1-\sin\theta}{\cos\theta}\right)^2$$

$$= \left(\frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta}\right)^2$$

$$= \boxed{(\sec\theta - \tan\theta)^2}$$

$$\frac{a^2}{b^2} = \left(\frac{a}{b}\right)^2$$

Topic : Trigonometric Identities

#Q. Prove the following identity :

$(\text{cosec}\theta - \sin\theta)(\sec\theta - \cos\theta)(\tan\theta + \cot\theta) = 1$

$$= \left(\frac{1}{s} - \frac{s}{1}\right)\left(\frac{1}{c} - \frac{c}{1}\right)\left(\frac{s}{c} + \frac{c}{s}\right)$$

$$= \left(\frac{1-s^2}{s}\right)\left(\frac{1-c^2}{c}\right)\left(\frac{s^2+c^2}{cs}\right)$$

$$= \frac{c^2}{s} \times \frac{s^2}{c} \times \frac{1}{cs}$$

$$= \frac{\cancel{c^2}\,\cancel{s^2}}{\cancel{s^2}\,\cancel{c^2}} = \boxed{1}$$

Topic : Trigonometric Identities

#ImpQ. Prove the following identity : [NCERT Exemplar]

$$\frac{\tan\theta - \cot\theta}{\sin\theta\cos\theta} = \sec^2\theta - \operatorname{cosec}^2\theta = \tan^2\theta - \cot^2\theta$$

$$= \frac{\frac{s}{c} - \frac{c}{s}}{\frac{sc}{1}}$$

$$= \frac{\frac{s^2 - c^2}{cs}}{\frac{sc}{1}}$$

$$= \frac{s^2 - c^2}{s^2c^2}$$

$$= \frac{s^2}{s^2c^2} - \frac{c^2}{s^2c^2}$$

$$= \boxed{\sec^2\theta - \operatorname{cosec}^2\theta}$$

$$= (1 + \tan^2\theta) - (1 + \cot^2\theta)$$

$$= 1 + \tan^2\theta - 1 - \cot^2\theta = \boxed{\tan^2\theta - \cot^2\theta}$$

Topic : Trigonometric Identities

$a^4 = (a^2)^2$

#Q. Prove the following identity :

$\sec^4\theta - \sec^2\theta = \tan^4\theta + \tan^2\theta$

[NCERT Exemplar]

$\boxed{\sec^2\theta = 1 + \tan^2\theta}$

$= (\sec^2\theta)^2 - \sec^2\theta$

$= (1+\tan^2\theta)^2 - (1+\tan^2\theta)$

$= \cancel{1} + \tan^4\theta + 2\tan^2\theta - \cancel{1} - \tan^2\theta$

$= \boxed{\tan^4\theta + \tan^2\theta}$

Topic : Trigonometric Identities

#Q. Prove the following identity : [NCERT, CSBE 2000]

$(\sin\theta + \operatorname{cosec}\theta)^2 + (\cos\theta + \sec\theta)^2 = 7 + \tan^2\theta + \cot^2\theta$

$(\sin\theta)^2 + (\operatorname{cosec}\theta)^2 + 2\boxed{\sin\theta\operatorname{cosec}\theta}$
$+ (\cos\theta)^2 + (\sec\theta)^2 + 2\boxed{\cos\theta\sec\theta}$

$\sin^2\theta + \operatorname{cosec}^2\theta + 2\cancel{\sin\theta} \times \frac{1}{\cancel{\sin\theta}} + \cos^2\theta + \sec^2\theta + 2\cancel{\cos\theta} \times \frac{1}{\cancel{\cos\theta}}$

$= \sin^2\theta + 1 + \cot^2\theta + 2 + \cos^2\theta + 1 + \tan^2\theta + 2$

$= \boxed{7 + \tan^2\theta + \cot^2\theta}$

Topic : Trigonometric Identities

#Q. Prove the following identity : [CBSE 2008]

$$(1 + \cot\theta - \operatorname{cosec}\theta)(1 + \tan\theta + \sec\theta) = 2$$

$$= \left(\frac{1}{1} + \frac{c}{s} - \frac{1}{s}\right)\left(\frac{1}{1} + \frac{s}{c} + \frac{1}{c}\right)$$

$$= \left(\frac{s + c - 1}{s}\right)\left(\frac{c + s + 1}{c}\right)$$

$$= \frac{sc + s^2 + \cancel{s} + c^2 + cs + \cancel{c} - \cancel{c} - \cancel{s} - 1}{sc}$$

$$= \frac{2sc + s^2 + c^2 - 1}{sc}$$

$$= \frac{2sc + \cancel{1} - \cancel{1}}{sc}$$

$$= \frac{2\cancel{sc}}{\cancel{sc}}$$

$$= \boxed{2}$$

Topic : Trigonometric Identities

$$a^3+b^3=(a+b)(a^2-ab+b^2)$$
$$a^3-b^3=(a-b)(a^2+ab+b^2)$$

#Q. Prove the following identity :

$$\frac{\cos^2\theta}{1-\tan\theta}+\frac{\sin^3\theta}{\sin\theta-\cos\theta}=1+\sin\theta\cos\theta$$

$$=\frac{c^2}{1-\frac{s}{c}}+\frac{s^3}{s-c}$$

$$=\frac{\frac{c^2}{1}}{\frac{c-s}{c}}+\frac{s^3}{s-c}$$

$$=\boxed{\frac{c^3}{c-s}+\frac{s^3}{s-c}}$$

$$=\frac{c^3}{c-s}+\frac{s^3}{s-c}$$

$$=\frac{c^3}{c-s}+\frac{s^3}{-(c-s)}$$

$$=\frac{c^3}{c-s}-\frac{s^3}{c-s}$$

$$=\frac{c^3-s^3}{c-s}$$

$$=\frac{\cancel{(c-s)}(c^2+s^2+cs)}{\cancel{(c-s)}}$$

$$=\boxed{1+cs}$$

Topic : Trigonometric Identities

#Q. Prove the following identity : **[NCERT Exemplar]**

$$\frac{(1+\cot A+\tan A)(\sin A-\cos A)}{\sec^3 A-\operatorname{cosec}^3 A} = \sin^2 A\cos^2 A$$

$$= \frac{\left(1+\frac{c}{s}+\frac{s}{c}\right)\left(\frac{s-c}{1}\right)}{\left(\frac{1}{c^3}-\frac{1}{s^3}\right)}$$

$$= \frac{\left(\frac{sc+c^2+s^2}{sc}\right)\left(\frac{s-c}{1}\right)}{\frac{s^3-c^3}{c^3s^3}}$$

$$= \frac{(sc+c^2+s^2)(s-c)(c^3s^3)}{(sc)(s^3-c^3)}$$

$$= \frac{\cancel{(sc+c^2+s^2)}\,\cancel{(s-c)}\,(c^{\cancel{3}}s^{\cancel{3}})}{(\cancel{sc})\,\cancel{(s-c)}\,\cancel{(s^2+c^2+sc)}}$$

$$= \boxed{c^2s^2}$$

Topic : Trigonometric Identities

#Q. If $\sin\theta + \cos\theta = \sqrt{2}$, then prove that $\tan\theta + \cot\theta = 2$. [NCERT]

$(s+c)^2 = (\sqrt{2})^2$

$s^2 + c^2 + 2sc = 2$

$1 + 2sc = 2$

$2sc = 2 - 1$

$2sc = 1$

$sc = \frac{1}{2}$

$= \frac{s}{c} + \frac{c}{s}$

$= \frac{s^2 + c^2}{cs}$

$= \frac{1}{cs}$

$= \frac{1}{\frac{1}{2}}$

$= 2$

Topic : Trigonometric Identities

#Q. If $\sin\theta + \cos\theta = \sqrt{3}$, then prove that $\tan\theta + \cot\theta = 1$.

$(s+c)^2 = (\sqrt{3})^2$

$s^2 + c^2 + 2sc = 3$

$1 + 2sc = 3$

$2sc = 3 - 1$

$2sc = 2$

$sc = 1$

$= \frac{s}{c} + \frac{c}{s}$

$= \frac{s^2 + c^2}{cs}$

$= \frac{1}{cs}$

$= \frac{1}{1}$

$= 1$

#Q. $\dfrac{\tan^3\theta}{1+\tan^2\theta}+\dfrac{\cot^3\theta}{1+\cot^2\theta}=\sec\theta\,\mathrm{cosec}\,\theta-2\sin\theta\cos\theta$

$$=\frac{\frac{S^3}{C^3}}{1+\frac{S^2}{C^2}}+\frac{\frac{C^3}{S^3}}{1+\frac{C^2}{S^2}}$$

$$=\frac{\frac{S^3}{C^3}}{\frac{C^2+S^2}{C^2}}+\frac{\frac{C^3}{S^3}}{\frac{S^2+C^2}{S^2}}$$

$$=\frac{\frac{S^3}{C^3}}{\frac{1}{C^2}}+\frac{\frac{C^3}{S^3}}{\frac{1}{S^2}}$$

$$=\frac{S^3\times\cancel{C^2}}{C^{\cancel{3}}}+\frac{C^3\times\cancel{S^2}}{S^{\cancel{3}}}$$

$$=\boxed{\frac{S^3}{C}+\frac{C^3}{S}}$$

$$a^2+b^2=(a+b)^2-2ab$$

$$= \frac{s^3}{c}+\frac{c^3}{s}$$

$$= \frac{s^4+c^4}{cs}$$

$$= \frac{(s^2)^2+(c^2)^2}{cs}$$

$$= \frac{(s^2+c^2)^2-2s^2c^2}{cs}$$

$$= \frac{1-2s^2c^2}{cs}$$

$$= \frac{1}{cs}-\frac{2\cancel{s^2}c^{\cancel{2}}}{\cancel{c}\cancel{s}}$$

$$= \boxed{\sec\theta\operatorname{cosec}\theta-2\sin\theta\cos\theta}$$

#Q. $\left(\tan\theta + \frac{1}{\cos\theta}\right)^2 + \left(\tan\theta - \frac{1}{\cos\theta}\right)^2 = 2\left(\frac{1+\sin^2\theta}{1-\sin^2\theta}\right)$

H.W

#Q. $(1 + \cot A + \tan A)(\sin A - \cos A) = \dfrac{\sec A}{\operatorname{cosec}^2 A} - \dfrac{\operatorname{cosec} A}{\sec^2 A} = \sin A \tan A - \cot A \cos A$

[CBSE 2018]

H.W

Thankyou