UDAAN

2025

Real Numbers

Mathematics | Lecture - 04

By – Ritik Sir

Topics *to be covered*

1. 1 more important Question on HCF and LCM
2. Recalling irrational numbers
3. Proof of irrationality

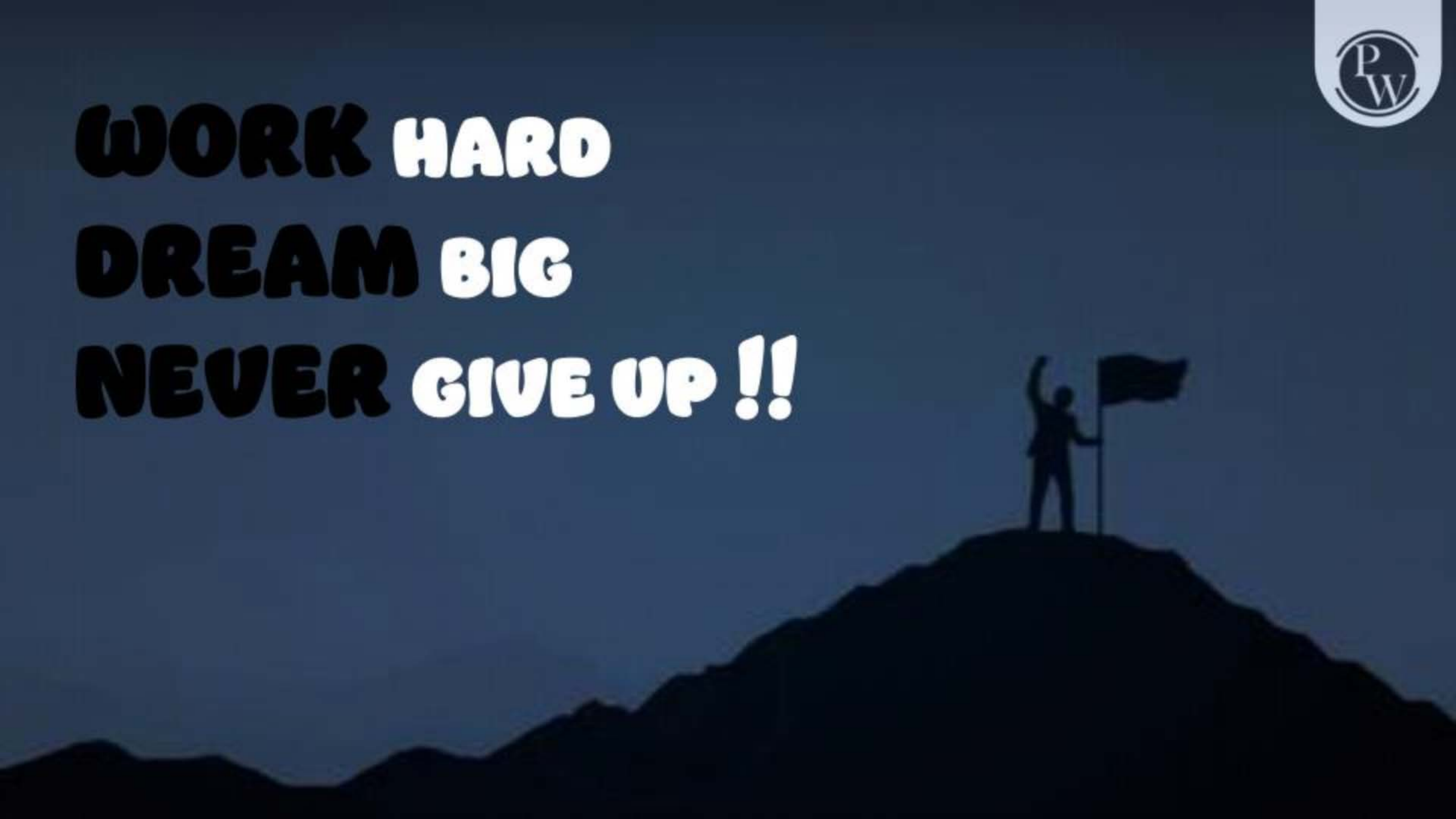
MP Board Result

AIIMS

Sneha → Udaan Batch 2024 (2nd Rank)

Utkarsh → Udaan Batch (yt) (7th Rank)

IIT Bombay


WORK HARD
DREAM BIG
NEVER GIVE UP !!

Topic : HCF and LCM

Revision 'O'

#Q. There are 312, 260 and 156 students in class X, XI and XII respectively. Buses are to he hired to take these students to a picnic. Find the maximum numbers of students who can sit in a bus if each bus takes equal number of students.

(A) 52

(B) 56

(C) 48

(D) 63

Chota → Factor → HCF

HCF (312, 260, 156) = 52

Topic : HCF and LCM

#Q. Find the number of possible pairs if the product of two numbers and HCF are 4500 and 15 respectively.

(A) 1

(B) 2

(C) 3

(D) 4

Product = 4500

HCF = 15

coprime

let the no.s be = $\boxed{15x, 15y}$

Here x and y coprime no.s

$15x \times 15y = 4500$

$x \times y = \frac{\overset{20}{\cancel{\overset{300}{\cancel{4500}}}}}{\cancel{15} \times \cancel{15}}$

$\boxed{x \times y = 20}$

(4, 5), (20, 1)

Topic : HCF and LCM

#Q. The sum of two positive numbers is 240 and their $\boxed{\text{HCF is 15}}$. Find the number of pairs of numbers satisfying the given condition.

Sum = 240

HCF = 15

let the nos be = $15x, 15y$

coprime

$15x + 15y = 240$

$15(x+y) = 240$

$x+y = \frac{240}{15}$

$\boxed{x+y=16}$

$(11,5), (9,7), (15,1), (13,3)$

Ans = 4 pairs

$a = 3 \times 7$ coprimenos

$b = 3 \times 8$

$HCF = 3$

$a \neq 3x$

$b \neq 3y$

Real no.s

Rational no.

N, W, Z

- **Terminating**

  $3.5,\ 2.52,\ 3.9,\ 41.25$

- **Non terminating repeating**

  $3.\overline{2},\ 3.\overline{81},\ 4.92\overline{3}$

Irrational no.

N.T.N.R

3.145297090294565123 ------

Irrational Numbers

TPYE 1

N.T.N.R

TPYE 2

If m is a positive integer which is not a perfect square then $\sqrt{m}$ is irrational.

Square root of 2 = root 2 = $\sqrt{2}$ = $2^{1/2}$

$\sqrt{\text{not a perfect square}}$ = Irrational no.

$(a^m)^n = a^{mn}$

Perfect square :-

1, 4, 9, 16, 25, 36, 49, 64, 81, 100 - - - - - .

$\sqrt{36} = \sqrt{6^2} = (6^2)^{1/2}$

$= 6^{\cancel{2} \times \frac{1}{\cancel{2}}} = 6^1 = \boxed{6}$

Important Points

(i) Sum of a rational and irrational is irrational.

(ii) Difference of a rational and an irrational is irrational.

(iii) Product of a rational and an irrational is irrational.

(iv) Quotient of a rational and an irrational is irrational.

→ R + Irr = Irr

→ R − Irr = Irr

→ R × Irr = Irr

→ $\frac{R}{Irr}$ or $\frac{Irr}{R}$ = Irr

Non-Irr

Q. $3+\sqrt{2}$

R Irr = Irr

→ Irr + Irr

→ Irr - Irr

→ Irr X Irr

→ $\frac{Irr}{Irr}$

Rational / Irrational

$\sqrt{2} \times \sqrt{8} = \sqrt{16} = 4$

I I R

$\sqrt{3} \times \sqrt{7} = \sqrt{21}$

Important Points

(i) Sum of two irrationals need not be an irrational.

(ii) Difference of two irrationals need not be an irrational.

(iii) Product of two irrationals need not be an irrational.

(iv) Quotient of two irrationals need not be an irrational.

Topic : Irrational Number

#Q. Write whether $\frac{2\sqrt{45}+3\sqrt{20}}{2\sqrt{5}}$ on simplification gives an irrational or a rational number.

$$\begin{array}{c|c} 3 & 45 \\ \hline 3 & 15 \\ 5 & 5 \\ & 1 \end{array} \qquad \begin{array}{c|c} 2 & 20 \\ \hline 2 & 10 \\ 5 & 5 \\ & 1 \end{array}$$

$$= \frac{2(3\sqrt{5})+3(2\sqrt{5})}{2\sqrt{5}}$$

$$\sqrt{45} = \sqrt{3\times3\times5} = \boxed{3\sqrt{5}}$$

$$\sqrt{20} = \sqrt{2\times2\times5} = \boxed{2\sqrt{5}}$$

$$= \frac{6\sqrt{5}+6\sqrt{5}}{2\sqrt{5}}$$

$$= \frac{12\cancel{\sqrt{5}}}{2\cancel{\sqrt{5}}} = 6 = \boxed{\text{Rational}}$$

Concept-1

$p = 2c$ $\quad$ $q = 3c$ $\quad$ $p^2 = 2c^2$

$\boxed{\text{2 divides } p}$ $\quad$ $\boxed{\text{3 divides } q}$ $\quad$ $\boxed{\text{2 divides } p^2}$

5 divides p. $\quad$ 7 divides q^2

$\boxed{p = 5c}$ $\quad$ $\boxed{q^2 = 7c}$

$p = 5z$

$p = 5q$

$p = 5x$

Theorem

$\boxed{\text{Concept \#2}}$

Let p be a prime number. If p divides a^2, then p divides a, where a is a positive integer.

$p \to a^2$

$p \to a$

prime no.

positive integer

$2 \to 8^2$

$2 \to 8$

$5 \to 105^2$

$5 \to 105$

Concept #3

Rational No. = $\frac{P}{q}$] integers.

Rational No. = $\frac{P}{q}$] $\boxed{\text{coprime Integers.}}$

$\frac{35}{7} = \frac{5}{1}$

$\frac{42}{6} = \frac{7}{1}$

$\frac{21}{42} = \frac{1}{2}$

Topic : Irrational Number

#Q. Prove that $\boxed{\sqrt{2} \text{ is irrational.}}$

let $\sqrt{2}$ be rational

$\therefore$ $\sqrt{2} = \frac{p}{q}$ [where p and q coprime integers]

p or q ka 1 key alava koi or common factor nahi hoga.

Squaring both sides..

$(\sqrt{2})^2 = \left(\frac{p}{q}\right)^2$

$2 = \frac{p^2}{q^2}$

$\Rightarrow 2q^2 = p^2 \longrightarrow 2q^2 = (2c)^2$

$\Rightarrow$ 2 divides p^2

$\Rightarrow$ $\boxed{2 \text{ divides } p}$ — (1)

let $p = 2c$

$2q^2 = 4c^2$

$q^2 = \frac{4c^2}{2}$

$q^2 = 2c^2$

$\Rightarrow$ 2 divides q^2

$\Rightarrow$ $\boxed{2 \text{ divides } q}$ — (2)

From ① and ②

2 is also a factor of p and q.

Which implies that our assumption was wrong.

$\boxed{\therefore \sqrt{2} \text{ is irrational.}}$

Topic : Irrational Number

#Q. Prove that $\boxed{\sqrt{3} \text{ is irrational.}}$

let $\sqrt{3}$ be rational

$\therefore \sqrt{3} = \frac{p}{q}$ [where p and q are coprime integers]

Squaring...

$(\sqrt{3})^2 = \left(\frac{p}{q}\right)^2$

$3 = \frac{p^2}{q^2}$

$3q^2 = p^2$

$\Rightarrow$ 3 divides p^2

$\Rightarrow$ 3 divides p

let, $\boxed{p = 3c}$

$3q^2 = (3c)^2$

$3q^2 = 9c^2$

$q^2 = 3c^2$

$\Rightarrow$ 3 divides q^2

$\Rightarrow$ 3 divides q

3 is also a factor of p and q.

common

this makes our assumption wrong.

$\therefore \sqrt{3}$ is irrational.

H.W

Q proov $\sqrt{5}$, $\sqrt{7}$ irrational

My lectures

THANK YOU