UDAAN 2025

MATHS

DHA:06

Trigonometry

Q1 Prove the following identity: $\sin^4 A + \cos^4 A = 1 - 2\sin^2 A \cos^2 A$

Q2 Prove the following identity:
$\frac{\sin^3\theta + \cos^3\theta}{\sin\theta + \cos\theta} + \sin\theta\cos\theta = 1$

Q3 If $\cos\theta + \sin\theta = \sqrt{2}\cos\theta$, show that $\cos\theta - \sin\theta = \sqrt{2}\sin\theta$

Q4 If $\tan\theta + \sin\theta = m$ and $\tan\theta - \sin\theta = n$, show that $m^2 - n^2 = 4\sqrt{mn}$

Q5 If x = a $\sin\theta$ and y = b $\tan\theta$, then prove that $\frac{a^2}{x^2} - \frac{b^2}{y^2} = 1$

Q6 If a $\cos\theta + b\sin\theta = m$ and $a\sin\theta - b\cos\theta = n$,
prove that $a^2 + b^2 = m^2 + n^2$.

Q7 If $a\cos\theta - b\sin\theta = c$, prove that $a\sin\theta + b\cos\theta = \pm\sqrt{a^2 + b^2 - c^2}$

Q8 Prove the following identity.
$\frac{\tan\theta + \sec\theta - 1}{\tan\theta - \sec\theta + 1} = \frac{1 + \sin\theta}{\cos\theta}$

Q9 Prove the following identity:
$2\left(\sin^6\theta + \cos^6\theta\right) - 3\left(\sin^4\theta + \cos^4\theta\right) + 1 = 0$

Q10 If $\sin\theta + \cos\theta = p$ and $\sec\theta + cosec\theta = q$, show that $q\left(p^2 - 1\right) = 2p$.

Foundation

Answer Key

Q1 $\sin^4 A+\cos^4 A=1-2\sin^2 A\cos^2 A$

Q2 $\frac{\sin^3\theta+\cos^3\theta}{\sin\theta+\cos\theta}+\sin\theta\cos\theta=1$

Q3 $\cos\theta-\sin\theta=\sqrt{2}\sin\theta$

Q4 $m^2-n^2=4\sqrt{mn}$

Q5 $\frac{a^2}{x^2}-\frac{b^2}{y^2}=1$

Q6 $a^2+b^2=m^2+n^2.$

Q7 $a\sin\theta+b\cos\theta=\pm\sqrt{a^2+b^2-c^2}$

Q8 $\frac{\tan\theta+\sec\theta-1}{\tan\theta-\sec\theta+1}=\frac{1+\sin\theta}{\cos\theta}$

Q9 $2\left(\sin^6\theta+\cos^6\theta\right)-3\left(\sin^4\theta+\cos^4\theta\right)+1$
$=0$

Q10 $q\left(p^2-1\right)=2p$

Hints & Solutions

Q1 Text Solution:

Taking LHS

$= \sin^4 A + \cos^4 A$

$= (\sin^2 A)^2 + \left(\cos^2 A\right) + (\cos^2 A)^2$

$+ 2\sin^2 A\cos^2 A - 2\sin^2 A\cos^2 A$

[Adding and subtracting 2 $\sin^2$A $\cos^2$ A]

$= (\sin^2 A + \cos^2 A)^2 - 2\sin^2 A\cos^2 A = 1$

$- 2\sin^2 A \ \cos^2 A$

Taking RHS

$1 - 2\sin^2 A \ \cos^2 A$

LHS = RHS

Hence Proved.

Video Solution:

Q2 Text Solution:

We have,

Taking LHS

$= \frac{\sin^3\theta + cos^3\theta}{sin\theta + cos\theta} + sin\theta cos\theta$

$= \frac{(\sin\theta + \cos\theta)(\sin^2\theta + \cos^2\theta - \sin\theta\cos\theta)}{\sin\theta + \cos\theta}$

$+ \sin\theta\cos\theta$

$= 1 - \sin\theta\cos\theta + \sin\theta\cos\theta$

$= 1$

$Taking$ RHS

1

$LHS = RHS$

$Hence\ Proved.$

Video Solution:

Q3 Text Solution:

We have,

$\cos\theta + \sin\theta = \sqrt{2}\cos\theta$

$(\cos\theta + \sin\theta)^2 = 2\cos^2\theta$

$\cos^2\theta + \sin^2\theta + 2\cos\theta\sin\theta = 2\cos^2\theta$

$\cos^2\theta - 2\cos\theta\sin\theta = \sin^2\theta$

$\cos^2\theta - 2\cos\theta\sin\theta + \sin^2\theta = 2\sin^2\theta$

$(\cos\theta - \sin\theta)^2 = 2\sin^2\theta$

$\cos\theta - \sin\theta = \sqrt{2}\sin\theta$

Hence Proved.

Video Solution:

Q4 Text Solution:

We have, $m = \tan\theta + \sin\theta$ and, $n = \tan\theta - \sin\theta$.

$\therefore$ LHS $= m^2 - n^2 = (m+n)(m-n)$

$= (\tan\theta + \sin\theta + \tan\theta - \sin\theta)(\tan\theta$

$+ \sin\theta - \tan\theta + \sin\theta)$

$= \left(2\tan\theta\right)\left(2\sin\theta\right) = 4\tan\theta\sin\theta$

$= 4\sqrt{\tan^2\theta\sin^2\theta}$

$= 4\sqrt{\tan^2\theta(1 - \cos^2\theta)}$

$= 4\sqrt{\tan^2\theta - \tan^2\theta\cos^2\theta}$

$= 4\sqrt{\tan^2\theta - \sin^2\theta}$

$= 4\sqrt{(\tan\theta + \sin\theta)(\tan\theta - \sin\theta)}$

$= 4\sqrt{mn} = $ RHS

Hence Proved.

Video Solution:

Q5 Text Solution:

We have, $x = a\sin\theta$ and $y = b\tan\theta$

$\therefore \quad \text{LHS} = \frac{a^2}{x^2} - \frac{b^2}{y^2}$

$= \frac{a^2}{a^2\sin^2\theta} - \frac{b^2}{b^2\tan^2\theta} \qquad \left[\because x = a\sin\theta,\right.$

$\left. y = b\tan\theta\right]$

$= \frac{1}{\sin^2\theta} - \frac{1}{\tan^2\theta}$

$= \operatorname{co sec}^2\theta - \cot^2\theta$

$\left[\because 1 + \cot^2\theta = \operatorname{cosec}^2\theta \therefore \quad \operatorname{cosec}^2\theta - \cot^2\theta = 1\right]$

$= 1 = \text{RHS}$

Hence Proved.

Video Solution:

Q6 Text Solution:

We have,

$m = a\cos\theta + b\sin\theta$ and $n = a\sin\theta$
$- b\cos\theta$

$\therefore$ Taking RHS

$= m^2 + n^2$

$= (a\cos\theta + b\sin\theta)^2 + (a\sin\theta - b\cos\theta)^2$

$= \left(a^2\cos^2\theta + b^2\sin^2\theta + 2ab\cos\theta\sin\theta\right)$

$+ \left(a^2\sin^2\theta + b^2\cos^2\theta - 2ab\sin\theta\cos\theta\right)$

$= a^2\left(\cos^2\theta + \sin^2\theta\right)$

$+ b^2\left(\sin^2\theta + \cos^2\theta\right) = a^2 + b^2$

Taking LHS

$= a^2 + b^2$

LHS = RHS

Hence Proved.

Video Solution:

Q7 Text Solution:

$\Rightarrow \quad a\cos\theta - b\sin\theta = c$

$\Rightarrow (a\cos\theta - b\sin\theta)^2 = c^2$

$\Rightarrow \left(a^2\cos^2\theta + b^2\sin^2\theta - 2ab\sin\theta\cos\theta\right)$

$= c^2$

$\Rightarrow \quad a^2\left(1 - \sin^2\theta\right) + b^2\left(1 - \cos^2\theta\right)$

$- 2ab\sin\theta\cos\theta = c^2$

$\Rightarrow a^2 - a^2\sin^2\theta + b^2 - b^2\cos^2\theta$

$- 2ab\sin\theta\cos\theta = c^2$

$\Rightarrow a^2 + b^2 - a^2\sin^2\theta - b^2\cos^2\theta$

$- 2ab\sin\theta\cos\theta = c^2$

$\Rightarrow a^2 + b^2$

$- \left(a^2\sin^2\theta + b^2\cos^2\theta + 2ab\sin\theta\cos\theta\right)$

$= c^2$

$\Rightarrow \quad (a\sin\theta + b\cos\theta)^2 = a^2 + b^2 - c^2$

$\Rightarrow \quad a\sin\theta + b\cos\theta = \pm\sqrt{a^2 + b^2 - c^2}$

Video Solution:

Q8 Text Solution:

We have,

Taking LHS

$= \frac{\tan\theta + \sec\theta - 1}{\tan\theta - \sec\theta + 1}$

$= \frac{(\tan\theta + \sec\theta) - 1}{(\tan\theta - \sec\theta) + 1}$

$= \frac{(\sec\theta + \tan\theta) - (\sec^2\theta - \tan^2\theta)}{\tan\theta - \sec\theta + 1}$

$\left[\because \quad \sec^2\theta - \tan^2\theta = 1\right]$

$= \frac{(\sec\theta + \tan\theta) - (\sec\theta + \tan\theta)(\sec\theta - \tan\theta)}{\tan\theta - \sec\theta + 1}$

$= \frac{(\sec\theta + \tan\theta)[1 - (\sec\theta - \tan\theta)]}{\tan\theta - \sec\theta + 1}$

$= \frac{(\sec\theta + \tan\theta)(1 - \sec\theta + \tan\theta)}{(\tan\theta - \sec\theta + 1)}$

$= \frac{(\sec\theta + \tan\theta)(\tan\theta - \sec\theta + 1)}{(\tan\theta - \sec\theta + 1)}$

$= \sec\theta + \tan\theta = \frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta} = \frac{1 + \sin\theta}{\cos\theta}$

$= \text{RHS}$

Hence Proved.

Video Solution:

Q9 Text Solution:

We have,

$$\text{Taking LHS } = 2\left(\sin^6\theta + \cos^6\theta\right)$$
$$-3\left(\sin^4\theta + \cos^4\theta\right) + 1$$
$$= 2\left\{\left(\sin^2\theta\right)^3 + \left(\cos^2\theta\right)^3\right\}$$
$$-3\left(\sin^4\theta + \cos^4\theta\right) + 1$$
$$\text{Using } a^3 + b^3 = (a+b)^3$$
$$-3ab\left(a+b\right) \text{ and } a^2 + b^2 = (a+b)^2$$
$$-2ab, \text{ we obtain}$$

$$= 2$$
$$\left\{\left(\sin^2\theta + \cos^2\theta\right)^3\right.$$
$$\left.-3\sin^2\theta\cos^2\theta\left(\sin^2\theta + \cos^2\theta\right)\right\}$$
$$-3\left\{\left(\sin^2\theta + \cos^2\theta\right)^2 - 2\sin^2\theta\cos^2\theta + 1\right\}$$
$$= 2\left(1 - 3\sin^2\theta\cos^2\theta\right)$$
$$-3\left(1 - 2\sin^2\theta\cos^2\theta\right) + 1$$
$$= 2 - 6\sin^2\theta\cos^2\theta - 3 + 6\sin^2\theta\cos^2\theta$$
$$+1 = 0 = \text{ RHS}$$

Hence Proved.

Video Solution:

Q10 Text Solution:

We have, $p = \sin\theta + \cos\theta$ and $q = \sec\theta + \operatorname{cosec}\theta$

$$\text{LHS} = q\left(p^2 - 1\right)$$
$$= \left(\sec\theta + cosec\theta\right)\left\{\left(\sin\theta + \cos\theta\right)^2 - 1\right\}$$
$$= \left(\frac{1}{\cos\theta} + \frac{1}{\sin\theta}\right)\left\{\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta - 1\right\}$$
$$= \left(\frac{\sin\theta + \cos\theta}{\cos\theta\sin\theta}\right)\left(1 + 2\sin\theta\cos\theta - 1\right)$$
$$= \left(\frac{\sin\theta + \cos\theta}{\cos\theta\sin\theta}\right)\left(2\sin\theta\cos\theta\right)$$
$$= 2\left(\sin\theta + \cos\theta\right) = 2p = \text{ RHS}$$

Hence Proved.

Video Solution: