UDAAN 2024

– FOR CLASS 10th STUDENTS

Lecture No.- 05

- **Subject Name- Mathematics**

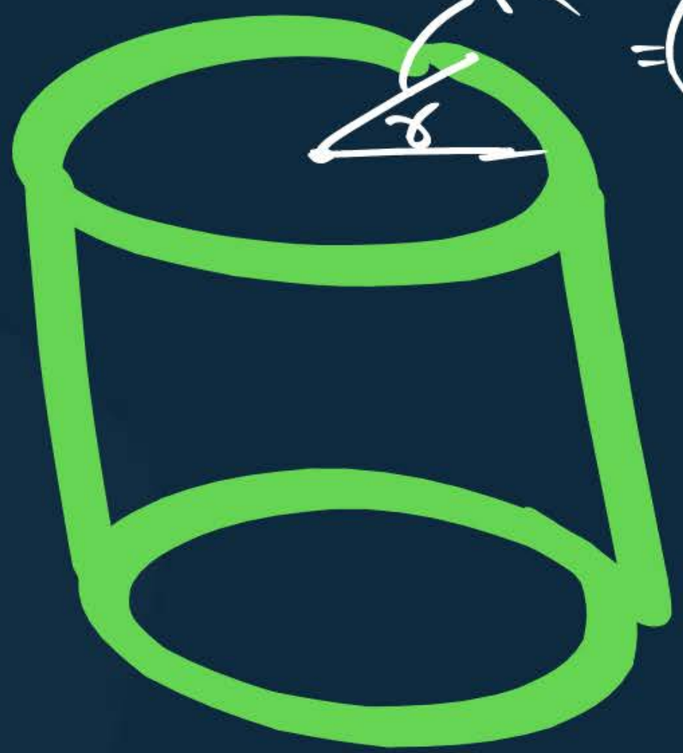
- **Chapter Name- Surface Area and Volume**

By- RITIK SIR

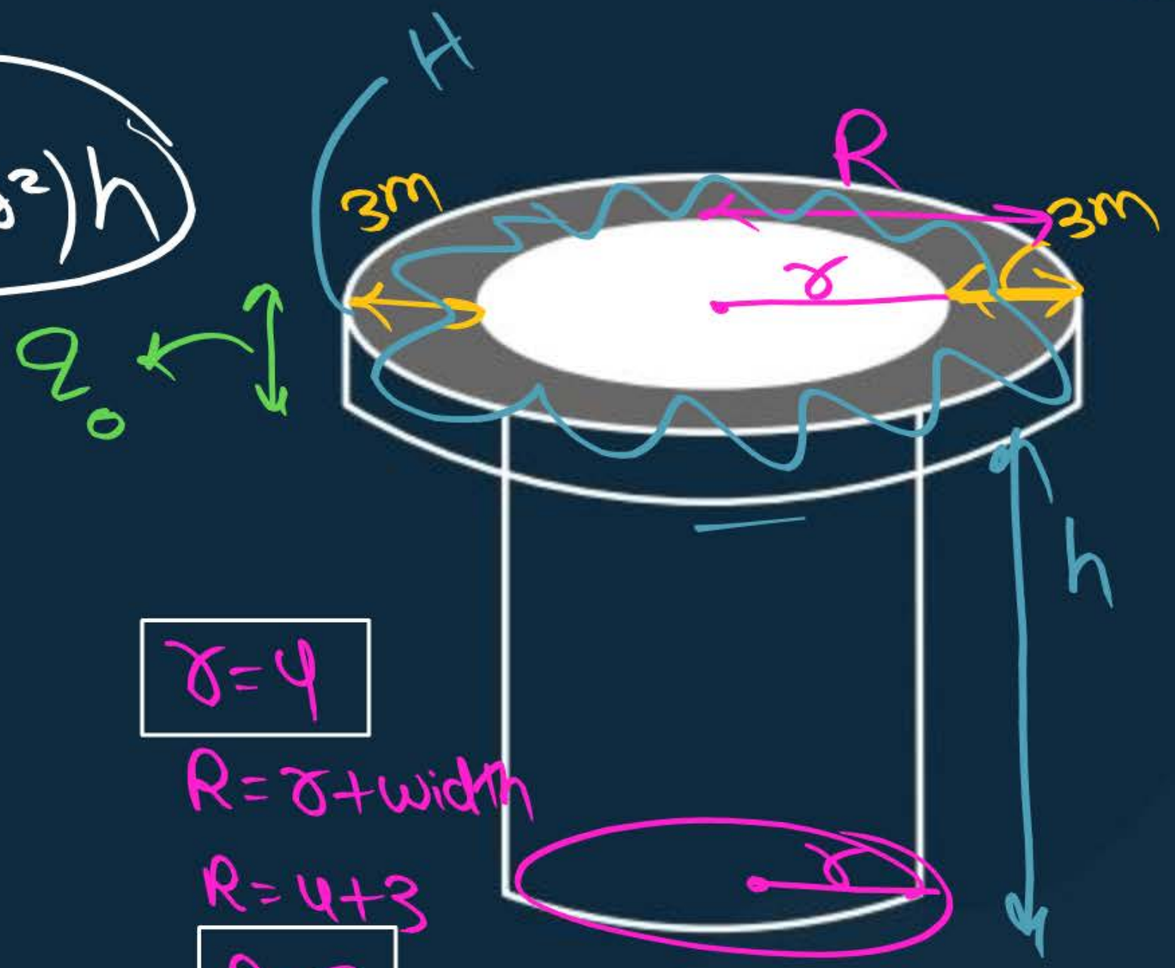
Topic to be Covered

Topic Conversion of solids

Topic Frustum of A Right Circular Cone



$= \pi(R^2 - r^2)h$



$r = 4$

$R = r + \text{width}$

$R = 4 + 3$

$R = 7$

Topic : Conversion of solids

#Q. A well with inner radius 4 m is dug 14 m deep. Earth taken out of it has been spread evenly all around a width of 3 m it to form an embankment. Find the height of the embankment. [CBSE 2016]

Volume of earth dug out = volume of embankment

$\pi r^2 h = \pi(R^2 - r^2)H$

$4 \times 4 \times 14 = (7^2 - 4^2)H$

$4 \times 4 \times 14 = 33H$

$\frac{4 \times 4 \times 14}{33} = H$

$6.78m = H$

∴ Height of embankment = 6.78m.

$1\,m^3 = 1000\,l$

$1000\,cm^3 = 1\,l$

Volume → m^3, cm^3, litres

$\frac{185}{1000}\,m^3 = 185\,l$

$0.185\,m^3 = 185\,l$

Q $185\,l = ?\,m^3$

$1\,m^3 = 1000\,l$

$\frac{1\,m^3}{1000} = 1\,l$

khaali horaha hai

#Q. A hemispherical tank full of water is emptied by a pipe at the rate of $3\frac{4}{7}$ litres per second. How much time will it take to make the tank half- empty, if the tank is 3 m in diameter?

[CBSE 2016]

$1 l = \frac{2s}{7} l$

Volume of water in the tank

$= \frac{2}{3}\pi r^3$

$= \frac{2}{3} \times \frac{22}{7} \times \frac{3}{2} \times \frac{3}{2} \times \frac{3}{2}$

$= \frac{99}{14} m^3$

Half volume of water $= \left(\frac{99}{14} \times \frac{1}{2}\right) m^3$

$= \boxed{\frac{99}{28} m^3}$

$1 m^3 = 1000 \text{ l}$

$\boxed{\frac{99}{28} m^3 = \left(\frac{99}{28} \times 1000\right) \text{ l}}$

$1 \text{ s} = \frac{25}{7} \text{ l}$

$\frac{7}{25} \text{ s} = 1 \text{ l}$

$\left(\frac{99 \times 1000}{28} \times \frac{7}{25}\right) \text{s} = \left(\frac{99 \times 1000}{28}\right) \text{ l}$

990 seconds $\frac{99000}{28}$ l

$1m = 60s$

$\frac{1}{60}m = 1s$

$\frac{33}{60 \times 2}m = 990s$

$\frac{33}{2}m = 990s$

16.5m -- 990

$1s = \frac{25}{7}l$

$\frac{25}{7}s = 1l$

$\left(\frac{99 \times 11}{28 \times 33}\right)s = \frac{99 \times 1}{28}l$

Volume $= \frac{2}{3}\pi r^3$

$1m^3 = 1000l$

$\frac{99}{28}m^3 = \frac{99000}{28}l$

$\frac{1}{2} \times \frac{2}{3}\pi r^3$

$\frac{99}{28}m^3$

Name of Solid	Volume	Total Surface Area	Lateral Surface Area
Cube	$V = a^3$	$TSA = 6a^2$	$LSA = 4a^2$
Cuboid	$V = l \times b \times h$	$TSA = 2(lb + bh + hl)$	$LSA = 2h(l + b)$
Cylinder	$V = \pi r^2 h$	$TSA = 2\pi r(h + r)$	$CSA = 2\pi rh$
Hollow Cylinder ($R > r$)	$V = \pi(R^2 - r^2)h$	$TSA = 2\pi(R + r)(h + R - r)$	$2\pi(R + r)$
Cone	$V = \frac{1}{3}\pi r^2 h$	$TSA = \pi r(l + r)$	$CSA = \pi rl$
Sphere	$V = \frac{4}{3}\pi r^3$	$TSA = 4\pi r^2$	$CSA = 4\pi r^2$
Hemisphere	$V = \frac{2}{3}\pi r^3$	$TSA = 3\pi r^2$	$CSA = 2\pi r^2$

Topic : Conversion of solids

#Q. Water in a canal, 30 dm wide and 12 dm deep is flowing with velocity of 10km/hr. How much area will it irrigate in 30 minutes, if 8 cm of standing water is required for irrigation? **[CBSE 2014]**

$1 dm = \frac{1}{10} m$

$30 dm = \frac{30}{10} m$

$30 dm = 3 m$

$1 dm = \frac{1}{10} m$

$12 dm = \frac{12}{10} m$

$12 dm = 1.2 m$

10 km = 1 hr

pani length

10000 m = 1 hr

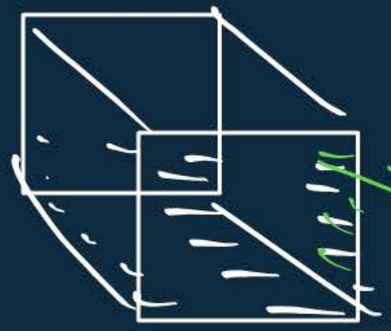
5000 m = $\frac{1}{2}$ hr

$\frac{1}{2}$ hr

5000 m

$1 dm = \frac{1}{10} m$

example



b = 3m

h = 1.2m

l = ?

h = 8 cm

Area

Volume of water flown through the canal = V. of water in the field

$$= \text{lxbxh} = \text{Area} \times \text{height}$$

$$5000 \times 3 \times \frac{12}{10} = A \times \frac{8}{100}$$

$$100 \times \frac{250 \quad 3}{5000 \times 3 \times 12} = A$$

$$\frac{10 \times 8}{2}$$

225000 m^2 = A

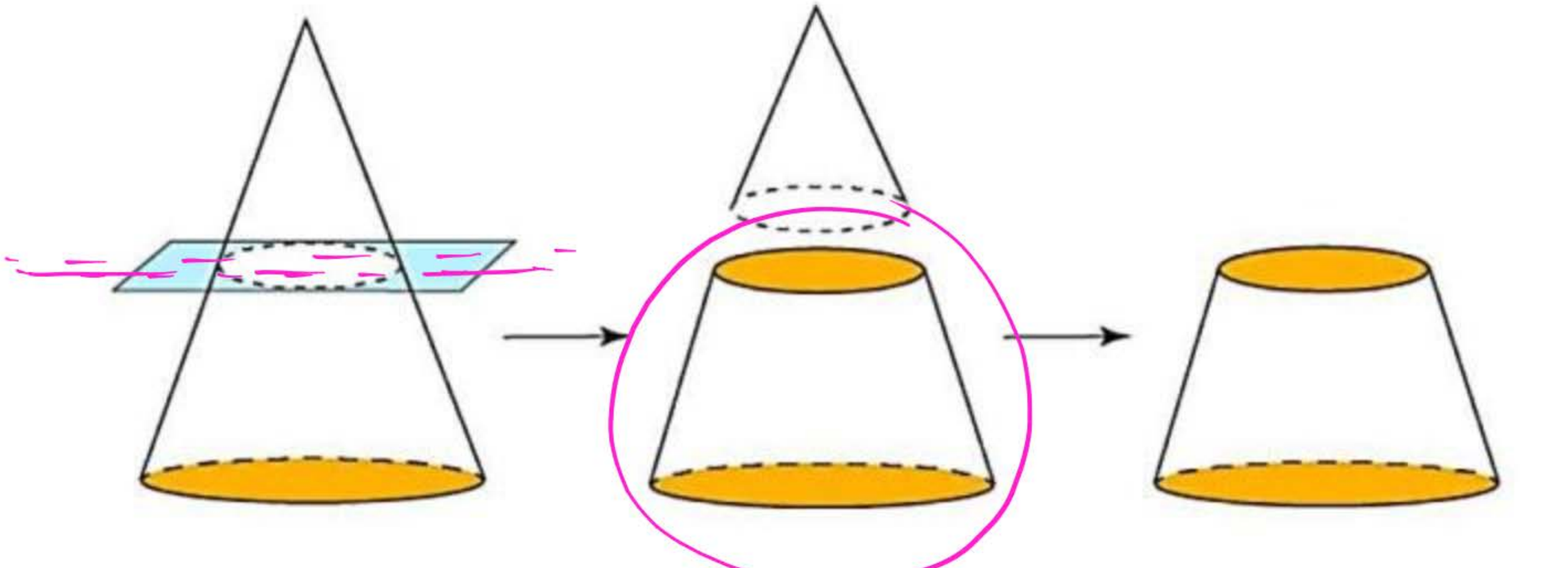
Topic : Conversion of solids

#Q. A cistern, internally measuring 150 cm × 120 cm × 110 cm has 129600 cm^3 of water in it. Porous bricks are placed in the water until the cistern is full to the brim. Each brick absorbs one seventeenth of its own volume of water. How many bricks can be put in without the water overflowing, each brick being 22.5 cm × 7.5 cm × 6.5 cm?

[NCERT Exemplar]

H.W

Frustum of A Cone

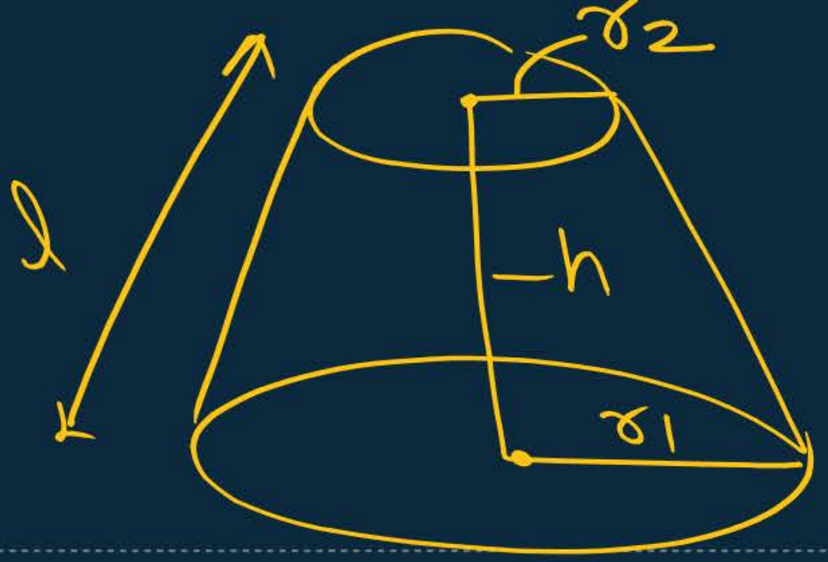


Slant height =

Volume =

C.S.A of frustum =

T.S.A of frustum =

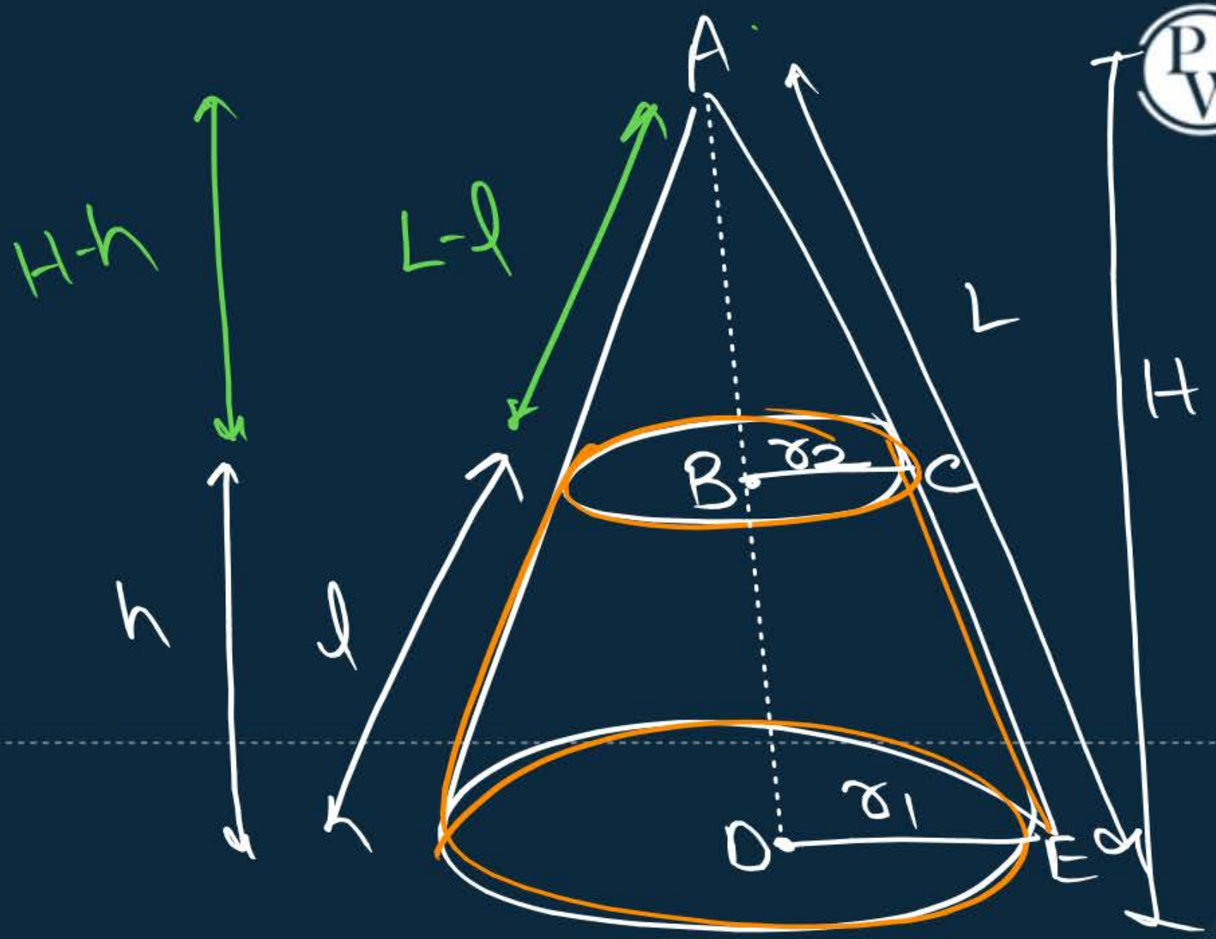


$\triangle ABC \sim \triangle ADE$

By CPST - - -

$$\frac{AB}{AD} = \frac{BC}{DE} = \frac{AC}{AE}$$

$$\frac{H-h}{H} = \frac{r_2}{r_1} = \frac{L-l}{L}$$



$$C.S.A = \pi r_1 L - \pi r_2 (L-l)$$

$$= \pi r_1 L - \pi r_2 L + \pi r_2 l$$

$$= \pi L (r_1 - r_2) + \pi r_2 l$$

$$= \pi \frac{r_1 l}{r_1 - r_2} \times (r_1 - r_2) + \pi r_2 l$$

C.S.A of Frustum $= \pi r_1 l + \pi r_2 l$

$= \pi l (r_1 + r_2)$

$$\frac{r_2}{r_1} = \frac{L-l}{L}$$

$$L r_2 = r_1 (L-l)$$

$$L r_2 = r_1 L - r_1 l$$

$$r_1 l = r_1 L - L r_2$$

$$r_1 l = L (r_1 - r_2)$$

$$\frac{r_1 l}{r_1 - r_2} = L$$

T.S.A = upper vala + niche vala + C.S.A

$= \pi r_2^2 + \pi r_1^2 + \pi(r_1+r_2)l$

Slant height $(l) = h^2 + (r_1 - r_2)^2$

Volume = Volume of bigcone - Vol smaller cone

$$= \frac{1}{3}\pi r_1^2 H - \frac{1}{3}\pi r_2^2 (H-h)$$

$$= \frac{1}{3}\pi r_1^2 H - \frac{1}{3}\pi r_2^2 H + \frac{1}{3}\pi r_2^2 h$$

$$= \frac{1}{3}\pi H (r_1^2 - r_2^2) + \frac{1}{3}\pi r_2^2 h$$

$$= \frac{1}{3}\pi \frac{hr_1}{r_1 - r_2} \times (r_1 - r_2)(r_1 + r_2) + \frac{1}{3}\pi r_2^2 h$$

$$= \frac{1}{3}\pi h r_1 (r_1 + r_2) + \frac{1}{3}\pi r_2^2 h$$

$$= \frac{1}{3}\pi h \left[r_1 (r_1 + r_2) + r_2^2 \right]$$

$$= \frac{1}{3}\pi h \left[r_1^2 + r_2^2 + r_1 r_2 \right]$$

$$\frac{H-h}{H} = \frac{r_2}{r_1}$$

$$Hr_1 - hr_1 = Hr_2$$

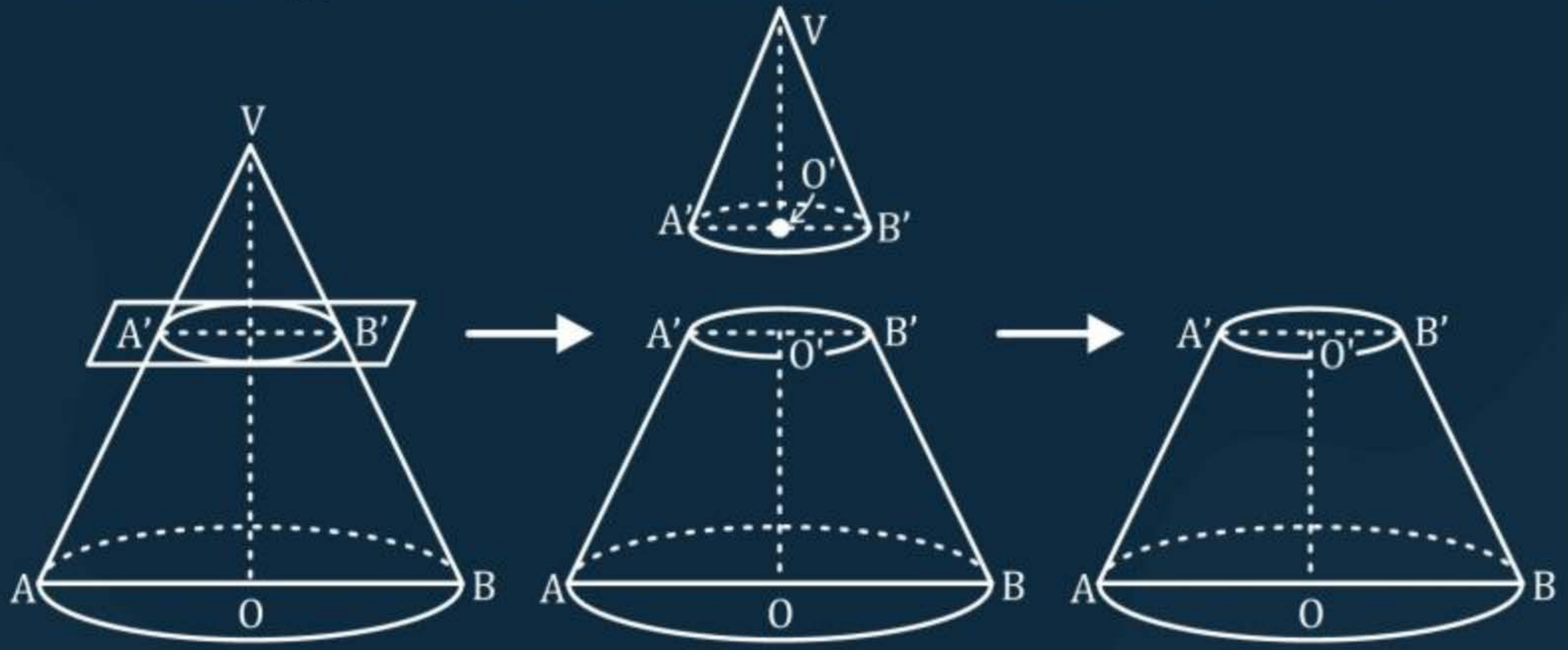
$$Hr_1 - Hr_2 = hr_1$$

$$H(r_1 - r_2) = hr_1$$

$$H = \frac{hr_1}{r_1 - r_2}$$

Frustum of A Right Circular Cone

Frustum: If a right circular cone is cut off by a plane parallel to its base, then the portion of the cone between the cutting plane and the base of the cone is called a frustum of the cone.



Topic : Frustum of A Right Circular Cone

frustum

#Q. If the radii of the circular ends of a conical bucket which is 45 cm high, are 28 cm and 7 cm, find the capacity of the bucket (use $\pi = 22/7$).

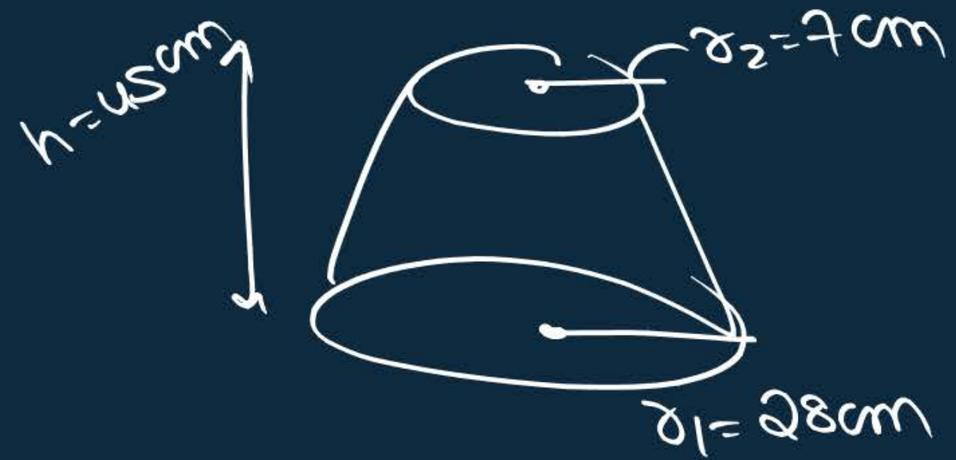
[CBE 2004, 2005]

$$V = \frac{1}{3}\pi h\left(r_1^2 + r_2^2 + r_1 r_2\right)$$

$$= \frac{1}{3} \times \frac{22}{7} \times 45 \left[28^2 + 7^2 + 28 \times 7\right]$$

$$= \frac{22 \times 15}{7}\left[784 + 49 + 196\right]$$

$$= \frac{330}{7}\left[1039\right] \text{ cm}^3 \longrightarrow 48510 \text{ cm}^3$$



Topic : Frustum of A Right Circular Cone

#Q. The perimeter of the ends of a frustum are 48 cm and 36 cm. If the height of the frustum be 11 cm, find its volume.

Perimeter of upper end $= 2\pi r_2$

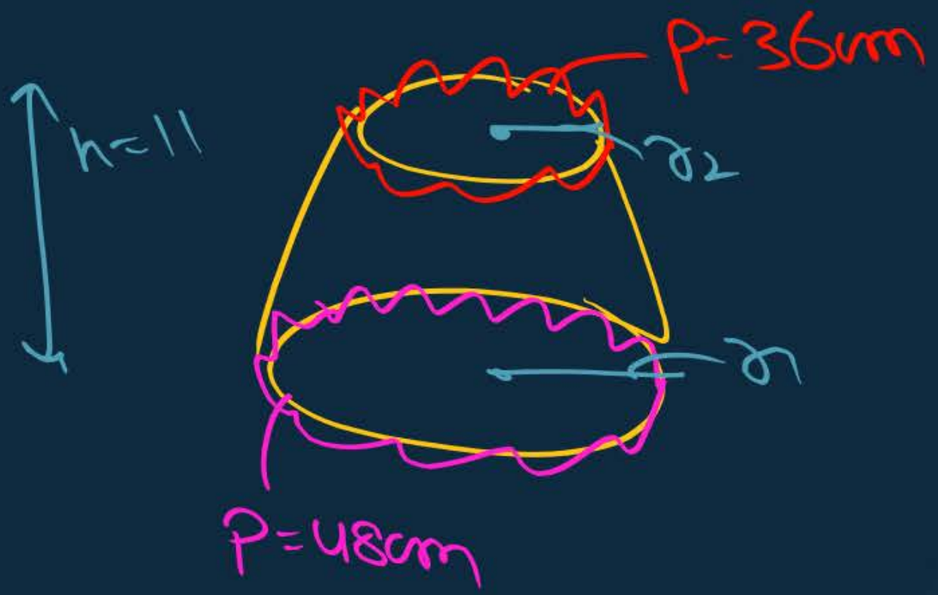
$36 = 2 \times \pi \times r_2$

$\frac{18}{\pi} = r_2$

" " lower end $= 2\pi r_1$

$48 = 2\pi r_1$

$\frac{24}{\pi} = r_1$



$$V = \frac{1}{3}\pi h\left(r_1^2 + r_2^2 + r_1 r_2\right)$$

$$= \frac{1}{3} \times \pi \times 11\left[\left(\frac{24}{\pi}\right)^2 + \left(\frac{18}{\pi}\right)^2 + \left(\frac{24}{\pi} \times \frac{18}{\pi}\right)\right]$$

$$= \frac{11}{3}\pi\left[\frac{576}{\pi^2} + \frac{324}{\pi^2} + \frac{432}{\pi^2}\right]$$

$$= \frac{11}{3} \times \pi\left[\frac{1332}{\pi^2}\right]$$

$$= \frac{11}{3} \times \frac{7}{22} \times 1332$$

$$V = 1554\ cm^3$$

#Q. A bucket is in the form of a frustum of a cone and hold 28.490 litres of water. The radii of the top and bottom are 28 cm and 21 cm respectively. Find the height of the bucket. **[CBSE 2012, 2014]**

$V = 28.490\ l$

$r_1 = 28\ cm$

$r_2 = 21\ cm$

$h = ?$

$$V = \frac{1}{3}\pi h\left(r_1^2 + r_2^2 + r_1 r_2\right)$$

$1000\ cm^3 = 1\ l$

$28.490 \times 1000\ cm^3 = 28.490\ l$

$28490\ cm^3 = 28.490\ l$

#Q. A container, open front the top, made up of a metal sheet is in the form of a frustum of a cone of height 16 cm with radii of its lower and upper ends as 8 cm and 20 cm respectively. Find the cost of milk which can completely fill the container at the rate of ₹15 per litre and the cost of metal sheet used, if the cost ₹5 per 100 cm^2. (use $\pi = 3.14$) **[CBSE 2008, 2014, 2016]**

Topic : Frustum of A Right Circular Cone

#Q. The height of a cone is 30 cm. A small cone is cut off at the top by a plane parallel to the base. If its volume be $\frac{1}{27}$ of the volume of the given cone, at what height above the base is the section mode? [CBSE 2016, 2017]

Tay Rasna

Topic : Frustum of A Right Circular Cone

#Q. The height of a cone is 10 cm. The cone is divided into two parts using a plane parallel to its base at the middle of its height. Find the ratio of the volumes of two parts.

[CBSE 2017]

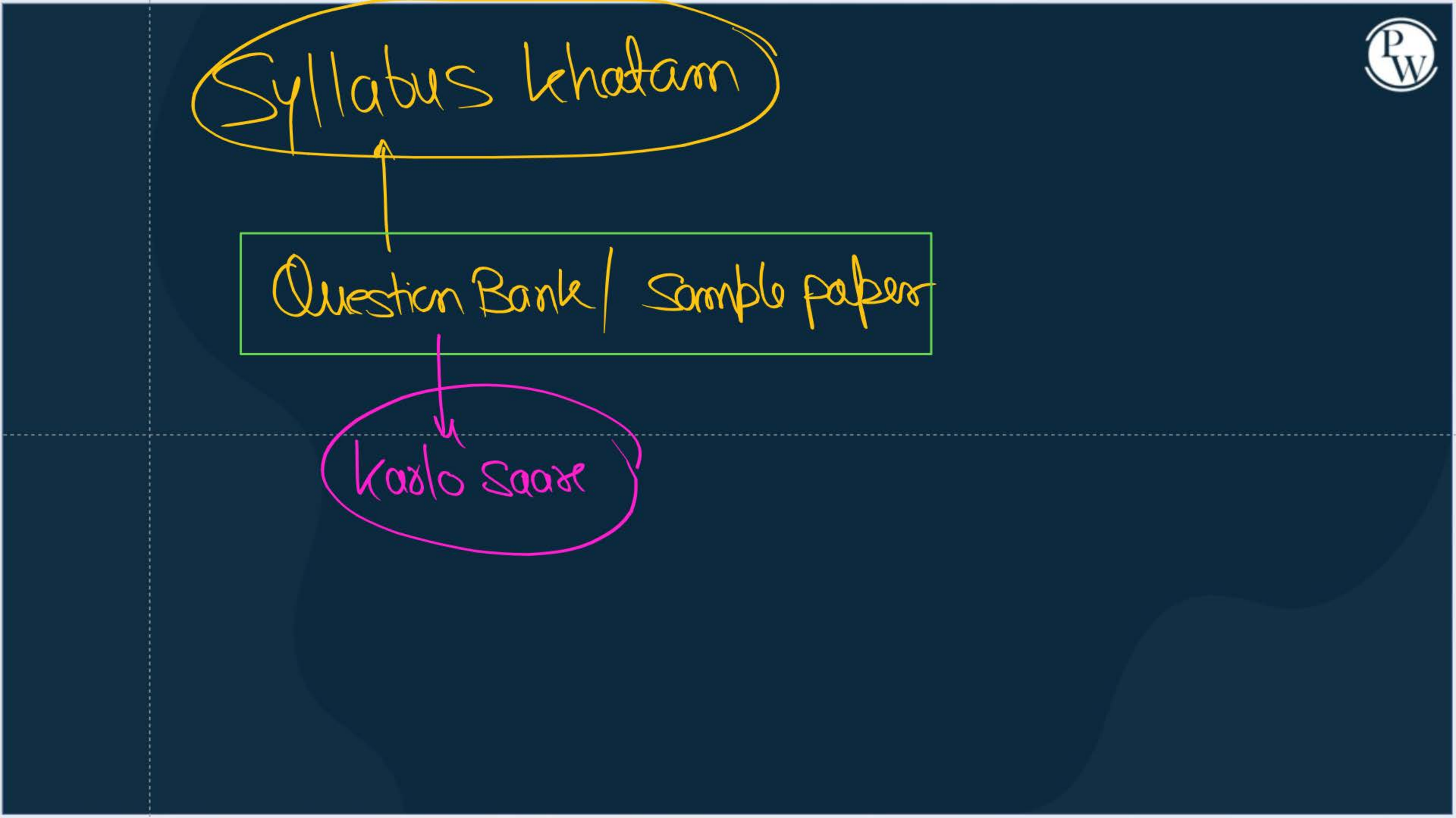
Toy Karna

$$l^2 = h^2 + (r_1 - r_2)^2$$

$$V = \frac{1}{3}\pi h\left(r_1^2 + r_2^2 + r_1 r_2\right)$$

$$CSA : \pi (r_1 + r_2) l$$

$$TSA = \pi r_1^2 + \pi r_2^2 + \pi (r_1 + r_2) l$$


Syllabus khatam
Question Bank / Sample papers
Karlo Saare


THANK
YOU