Foundation

Udaan 2025

Maths

Real Numbers

DHA - 03

Q 1 Prove that $3\sqrt{2}$ is irrational

Q 2 Prove that $2-3\sqrt{5}$ is an irrational number.

Q 3 Prove that $\sqrt{3}+\sqrt{5}$ is an irrational number.

Q 4 Prove that $\sqrt{p}+\sqrt{q}$ is irrational, where p, q are primes.

Q 5 How many rational number would be there between $\sqrt{3}$ and $\sqrt{5}$.

(A) 2
(B) 1
(C) 0
(D) Infinite

Q 6 The given number 1.3456386794........ is

(A) Rational
(B) Irrational
(C) Integer
(D) Prime

Q 7 State whether true or false, "Sum of any two irrational number is always irrational."

Answer Key

Q1	proving question
Q2	proving question
Q3	proving question
Q4	(proving question)
Q5	D
Q6	B
Q7	false

Hints & Solutions

Q 1 Text Solution:

Let us assume that $3\sqrt{2}$ is rational.

That is, we can find co-prime p and q (q ≠ 0) such that $3\sqrt{2} = \frac{p}{q}$

Rearranging, we get $\sqrt{2} = \frac{p}{3q}$

Since 3, p and q are integers, $\frac{p}{3q}$ is rational, and so $\sqrt{2}$ is rational.

But this contradicts the fact that $\sqrt{2}$ is irrational.

So, we conclude that $3\sqrt{2}$ is irrational.

Video Solution:

Q 2 Text Solution:

$$\begin{aligned}
&Given\ 2-3\sqrt{5}\\
&let\ us\ assume\ that\ 2-3\sqrt{5}\ is\ rational.\\
&we\ can\ express\ 2-3\sqrt{5}\ =\frac{p}{q} where\ p,q\ are\ co\\
&-primes\\
&\Rightarrow 3\sqrt{5}=2-\frac{p}{q}=\frac{2q-p}{q}\\
&\Rightarrow \sqrt{5}=\frac{2q-p}{3q}\\
&RHS\ =\ \frac{2q-p}{3q} is\ rational\\
&LHS=\sqrt{5} is\ irrational\\
&This\ leads\ to\ a\ contradiction\\
&Our\ assumption\ is\ wrong\\
&So,\ 2-3\sqrt{5} is\ irrational.
\end{aligned}$$

Video Solution:

Q 3 Text Solution:

$$\begin{aligned}
&let\ \sqrt{3}+\sqrt{5}\ is\ a\ rational\ number\\
&So,\ \sqrt{3}+\sqrt{5}=\frac{p}{q},\ p,q\ are\ co-primes\\
&Squaring\ on\ both\ sides,\ we\ get\\
&\left(\sqrt{3}+\sqrt{5}\right)^2=\left(\frac{p}{q}\right)^2 \Rightarrow 3+5+2\sqrt{15}=\frac{p^2}{q^2}\\
&\Rightarrow 2\sqrt{15}=\frac{p^2}{q^2}-8\\
&\Rightarrow \sqrt{15}=\frac{p^2-8q^2}{2q^2}\\
&RHS\ =\ Rational\\
&LHS\ =\ Irrational\\
&It\ leads\ to\ a\ contradiction\\
&Our\ assumption\ is\ wrong.\\
&So,\ \sqrt{3}+\sqrt{5}\ is\ irrational.
\end{aligned}$$

Video Solution:

Q 4 Text Solution:

Let us suppose that $\sqrt{p}+\sqrt{q}$ is rational.

let $\sqrt{p}+\sqrt{q}=\frac{a}{b}$, where $\frac{a}{b}$ is rational, a,b are co-primes and $b\neq 0$

On squaring both sides, we get

$$\begin{aligned}
&\left(\sqrt{p}+\sqrt{q}\right)^2=\left(\frac{a}{b}\right)^2 \Rightarrow p+q+2\sqrt{pq}=\frac{a^2}{b^2}\\
&\Rightarrow 2\sqrt{pq}=\frac{a^2}{b^2}-p-q\\
&\Rightarrow \sqrt{pq}=\frac{a^2-2pb^2-2qb^2}{2b^2}\\
&RHS\ =\ Rational\\
&LHS\ =\ Irrational\\
&It\ leads\ to\ a\ contradiction\\
&Our\ assumption\ is\ wrong.\\
&So,\ \sqrt{p}+\sqrt{q} is\ irrational.
\end{aligned}$$

Video Solution:

Q 5 Text Solution:

There exists infinite rational numbers between any two real numbers.

Video Solution:

Q 6 Text Solution:

Given number is non-terminating non-recurring decimal.

So, it is an irrational number

Video Solution:

Q 7 Text Solution:

$$\begin{aligned}
&Let\ \sqrt{2}\ and\ \sqrt{3}\ are\ two\ irrational\ numbers\\
&The\ sum\ of\ \sqrt{2}\ and\ \sqrt{3}\ is\ irrational,\\
&but\ if\ \sqrt{2}\ and\ -\sqrt{2}\ are\ the\ two\ irrationals.\\
&The\ sum\ of\ \sqrt{2}\ and\ -\sqrt{2}=\sqrt{2}\ +\left(-\sqrt{2}\right)=0\\
&which\ is\ rational\\
&So,\ The\ sum\ of\ two\ irrationals\ is\ always\ not\ a\\
&n\ irrational\ number.
\end{aligned}$$

Video Solution: