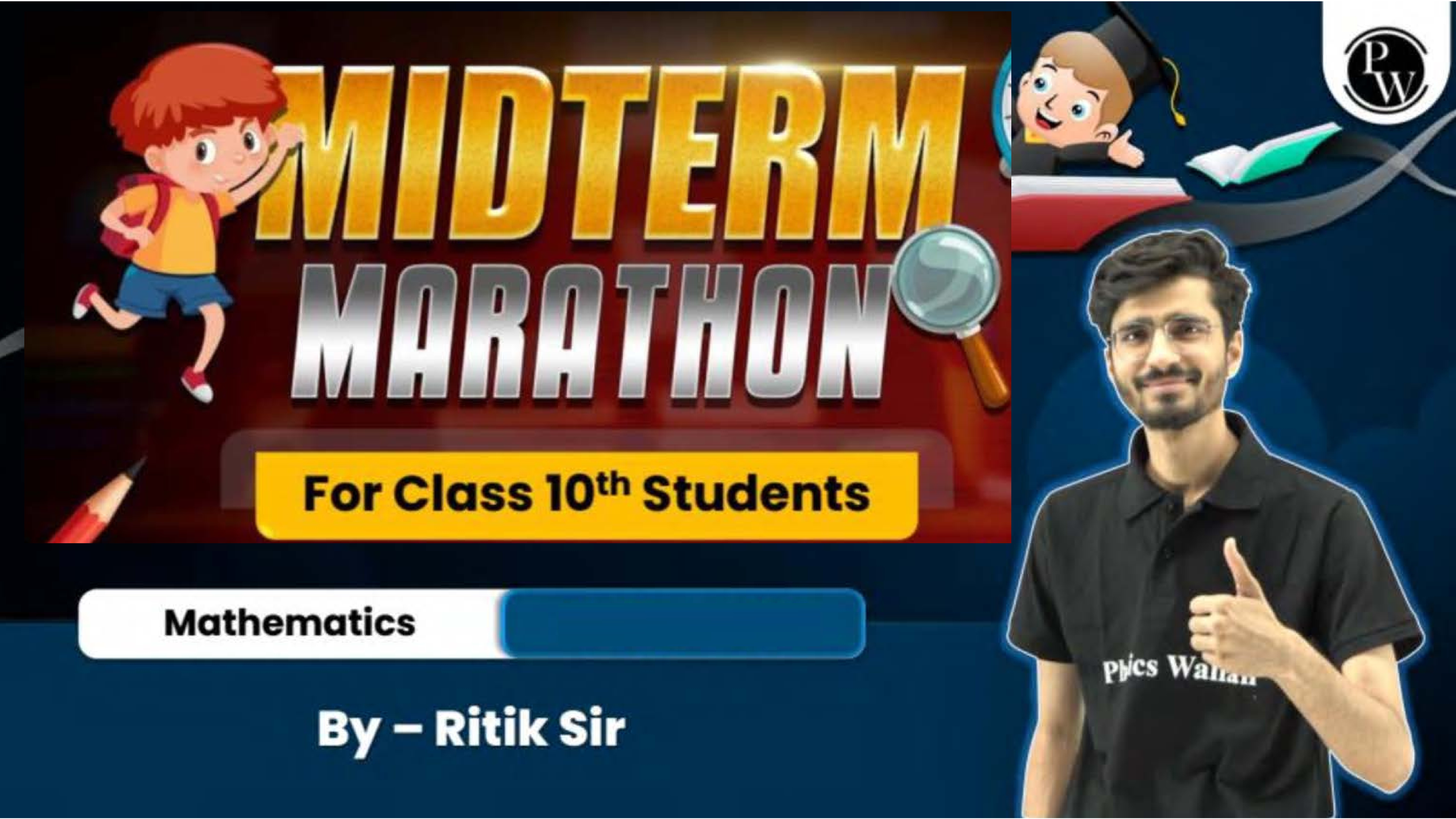

MIDTERM
MARATHON
For Class 10th Students
Mathematics
By – Ritik Sir

Topics *to be covered*

1. Complete Mid Term Syllabus
2.
3.
4.

Next 12 hours main you will be Legends

Are you Ready?

16 → 1, 2, 4, 8, 16

Composit.

33 → 1, 33, 3, 11

Comp.

5 → 1, 5

Poime no.

Poime no → only two Factors.

Composit → More than two Factoos.

Q 1 → 1 (only one Factor)

Nahi 'p' nahi Composite.

$$\boxed{720 = 2^4 \times 3^2 \times 5^1}$$

comp.

Prime no.

2	720
2	360
2	180
2	90
3	45
3	15
5	5
	1

THEOREM

(Fundamental Theorem of Arithmetic)

Every composite number can be expressed (factorised) as a product of primes, and this factorization is unique except for the order in which the prime factors occur.

Composite no. = product of primes.

unique.

QUESTION [NCERT, CBSE 2023]

Check whether 6^n can end with the digit 0 for any natural number n.

6^n

$n=1,\ 6^1=\underline{6}$

$n=2,\ 6^2=3\underline{6}$

$n=3,\ 6^3=21\underline{6}$

$n=4,\ \ ,\ \ ,\ \ ,\ \ \underline{6}$

$n=5,\ \ ,\ \ ,\ \ ,\ \ \underline{6}$

$n=6,\ \ ,\ \ ,\ \ ,\ \ \underline{6}$

$6^n=(2\times 3)^n$

$$\boxed{6^n = 2^n \times 3^n}$$

Since 6^n cannot have 5 as a factor,

$\therefore$ 6^n will never end with the digit '0'.

[By F.T.O.A]

QUESTION [NCERT EXEMPLAR]

Explain why $3 \times 5 \times 7 + 7$ is a composite number.

$= 3 \times 5 \times 7 + 7$

$= 7[15+1]$

$= 7 \times 16$

1

Clearly $(3 \times 5 \times 7 + 7)$ has 7, 16, 1 and the no. itself as factors so it is a composite no.

12 → 1, 2, 3, 4, 6, 12

8 → 1, 2, 4, 8

$HCF(12, 8) = 4.$

$LCM(8, 12) = 24$

$12 = 12, 24, 36, 48 \ldots\ldots$

$8 = 8, 16, 24, 32, 40, 48 \ldots\ldots$

QUESTION

Find the H.C.F and L.C.M of 480 and 720 using the Prime factorisation method.

OR

The H.C.F of 85 and 238 is expressible in the form 85m -238. Find the value of m.

$480 = 2^5 \times 5^1 \times 3^1$

$720 = 2^4 \times 5^1 \times 3^2$

$HCF = 2^4 \times 5^1 \times 3^1 = \boxed{240}$

$LCM = 2^5 \times 3^2 \times 5^1 = \boxed{1440}$

2	480
2	240
2	120
2	60
2	30
5	15
3	3
	1

2	720
2	360
2	180
2	90
3	45
5	15
3	3
	1

QUESTION

Find the H.C.F and L.C.M of 480 and 720 using the Prime factorisation method.

OR

The H.C.F of 85 and 238 is expressible in the form 85m -238. Find the value of m.

$$\begin{array}{r|l} 5 & 85 \\ \hline 17 & 17 \\ & 1 \end{array} \qquad \begin{array}{r|l} 2 & 238 \\ \hline 17 & 119 \\ 7 & 7 \\ & 1 \end{array}$$

$85 = 5^1 \times 17^1 \times 2^0 \times 7^0$

$238 = 2^1 \times 17^1 \times 7^1 \times 5^0$

$HCF = 2^0 \times 17^1 \times 7^0 \times 5^0$

$\boxed{HCF = 17}$

$17 = 85m - 238$

$17 + 238 = 85m$

$255 = 85m$

$\frac{255}{85} = m$

$3 = m$

QUESTION

alg-alg

If p and q are two distinct prime numbers, then their HCF is

A) 2

B) 0

C) either 1 or 2

D) 1

2 → 1, 2

5 → 1, 5

QUESTION

If p and q are two distinct prime numbers, then LCM (p, q) is

A 1

B p

C q

D pq

$2 = 2, 4, 6, 8, 10, 12 \ldots$

$3 = 3, 6, 9, 12 \ldots$

6

QUESTION

Let p be a prime number. The sum of its factors is

(A) p

(B) 1

(C) p + 1

(D) p − 1

$p \to 1$, $p \to p$

$1+p$

QUESTION

If 3 is the least prime factor of m and 5 is the least prime factor of n, then the least prime factor of (m +n) is

(A) 11

(B) 2

(C) 3

(D) 5

m ← 3

n ← 5

odd

m+n ← ?

even

odd + odd = even

8 → 1, 2, 4, 8

2 — Prim. Factor.

Even no → least prime Factor 2

QUESTION

The LCM of the smallest two digit composite number and the smallest composite number is

A) 12

B) 20

C) 4

D) 44

QUESTION

The HCF of smallest prime number and the smallest composite number is

A 2

B 4

C 6

D 8

QUESTION

The smallest number divisible by all natural numbers between 1 and 10 (both inclusive) is

LCM.

(A) 2020

(B) 2520

(C) 1010

(D) 5040

2 | 1, 2, 3, 4, 5, 6, 7, 8, 9, 10
2 | 1, 3, 2, 5, 3, 7, 4, 9, 5
3 | 1, 3, 1, 5, 3, 7, 2, 9, 5
2 | 1, 1, 5, 1, 7, 2, 3, 5
5 | 1, 1, 5, 1, 7, 1, 3, 5
3 | 1, 1, 7, 1, 1, 1
7 |

QUESTION [CBSE 2023]

Find the greatest number which divides 85 and 72 leaving remainders 1 and 2 respectively.

$85 \longrightarrow R = 1$

$72 \longrightarrow R = 2$

$HCF(84, 70) = \text{Ans}$

$84 \longrightarrow R = 0$

$70 \longrightarrow R = 0$

$\boxed{\text{For two +ve Integers}}$ $\to -\infty \ldots -2, -1, 0, 1, 2, 3 \ldots \infty$

a, b

$$\boxed{HCF(a,b) \times L.C.M(a,b) = a \times b}$$

QUESTION

Two numbers are in the ratio 2 : 3 and their LCM is 180. What is the HCF of these numbers?

1^{st} no. $= 2x$.

2^{nd} no. $= 3x$.

$LCM(2x, 3x) = 180$.

$HCF(2x, 3x) = ?$

$2x = 2^1 \times x^1 \times 3^0$

$3x = 3^1 \times x^1 \times 2^0$

$HCF = 2^0 \times 3^0 \times x^1 = x$

$\boxed{HCF \times LCM = Product}$

$HCF \times 180 = 2x \times 3x$

$\cancel{x} \times 180 = 2x \times 3\cancel{x}$

$180 = 6x$

$\boxed{30 = x}$

Ans

Points to be noted

$8 \rightarrow 1, 2, 4, 8$

$8 = 8, 16, 24, 32 \ldots$

- Read the questions carefully, very carefully.

- Abh ye judge kro ki aapka answer given data sey bada ya chota aayega.

M → LCM

F → HCF

- HCF of students students hi aayega.

12 p
14 p
} HCF = 0 case

QUESTION

Three bells ring at intervals of 6, 12 and 18 minutes. If all the three bells rang at 6 AM, when will they ring together again?

$B_1 \rightarrow 6\text{min}$

$B_2 \rightarrow 12\text{min}$

$B_3 \rightarrow 18\text{min}$

6AM

6:06, 6:12, 6:18.....

6:12, 6:24, 6:36......

6:18, 6:36........

$\boxed{\text{LCM} = 36\text{min}}$

$\boxed{\text{Ans: } 6:36\text{min}}$

2	6, 12, 18
3	3, 6, 9
3	1, 2, 3
2	1, 2, 1
	1, 1, 1

QUESTION [NCERT EXEMPLAR]

On a morning walk, three persons step off together and their steps measure 40cm, 42 cm and 45 cm respectively. What is the minimum distance each should walk so that each can cover the same distance and complete steps?

1 → 40cm

2 → 42cm

3 → 45cm.

$\boxed{LCM = 2520 cm}$

2	40, 42, 45
2	20, 21, 45
5	10, 21, 45
	2, 21, 9

QUESTION

There are 312, 260 and 156 students in class X, XI and XII respectively. Buses are to he hired to take these students to a picnic. Find the maximum numbers of students who can sit in a bus if each bus takes equal number of students.

HCF

(A) 52

(B) 56

(C) 48

(D) 63

QUESTION

If the LCM of two numbers is 3600, then which of the following numbers cannot be their HCF?

(A) 600

(B) 500

(C) 400

(D) 150

LCM = 3600

a b

HCF is always a factor of LCM.

QUESTION

If two positive integers a and b are written as $a = p^3 q^4$ and $b = p^2 q^3$, where p and q are prime numbers, such that HCF (a, b) = $p^m q^n$ and LCM (a, b) = $p^r q^s$, then $(m + n)(r + s)$ equal to

(A) 15

(B) 30

(C) 35

(D) 72

$a = p^3 q^4$

$b = p^2 q^3$

$$\boxed{\begin{aligned} HCF &= p^2 \times q^3 \rightarrow p^m q^n \\ LCM &= p^3 q^4 \rightarrow p^r q^s \end{aligned}}$$

$= (m+n)(r+s)$

$= (2+3)(3+4)$

$= 35$

QUESTION

If HCF $(x, 8) = 4$ LCM $(x, 8) = 24$, then x is

(A) 8

(B) 10

(C) 12

(D) 14

HCF × LCM = poodnct

$4 \times 24 = x \times 8$

$12 = x$

A seminar is being conducted by an educational organisation, where the participants will be educators of different subjects. The numbers of participants in Hindi, English and Mathematics are 60, 84 and 108 respectively.

$H \rightarrow 60 \, p'$

$E \rightarrow 84 \, p'$

$M \rightarrow 108 \, p'$

QUESTION

(i) In each room the same number of participants are to be seated and all of them being in the same subject, hence the maximum number of participants that can be accomodated in each room is

HCF : 12 'p'

(A) 14

(B) 12

(C) 16

(D) 18

QUESTION

(ii) The minimum number of rooms required during the event, is

(A) 21

(B) 9

(C) 7

(D) 5

$$H \rightarrow \frac{60\text{ 'p'}}{12} = \boxed{5}\text{ Rooms.}$$

$$E = \frac{84\text{ 'p'}}{12} = \boxed{7}\text{ Rooms}$$

$$M = \frac{108\text{ 'p'}}{12} = \boxed{9}\text{ Rooms}$$

$\boxed{\text{Coprime Nos.}}$ → (ak hi common factor hoga, vo hoga 1)

2 nos → HCF = 1

→ 9, 16

$9 \to 1, 3, 9$

$16 \to 1, 2, 4, 8, 16$

→ 2, 5

$2 \to 1, 2$

$5 \to 1, 5$

→ 8, 27

→ 3, 5

→ 15, 17

QUESTION

Prove that $\sqrt{3}$ is an irrational number.

let, $\sqrt{3}$ be rational.

$\therefore \sqrt{3} = \frac{p}{q}$ ['p' and 'q' are coprime integers]

('1' key alawa koi or factor nahi hoga)

$\boxed{S.B.S}$

$(\sqrt{3})^2 = \left(\frac{p}{q}\right)^2$

$3 = \frac{p^2}{q^2}$

$3q^2 = p^2$

$\Rightarrow$ 3 divides p^2

$\Rightarrow$ $\boxed{\text{3 divides } p.}$ — (1)

$3c = p$

$3q^2 = (3c)^2$

$3q^2 = 9c^2$

$q^2 = 3c^2$

$\Rightarrow$ 3 divides q^2

$\Rightarrow$ $\boxed{\text{3 divides } q.}$ — (2)

From (1) and (2)

3 is a common factor of p and q.

This makes our assumption wrong.

$\therefore \sqrt{3}$ is irrational.

QUESTION

Prove that $3 + 2\sqrt{5}$ is irrational

Let, $3+2\sqrt{5}$ rational.

$\therefore 3+2\sqrt{5} = \frac{p}{q}$ ['p' and 'q' are integers]

$2\sqrt{5} = \frac{p}{q} - 3$

$2\sqrt{5} = \frac{p-3q}{q}$

$$\boxed{\sqrt{5} = \frac{p-3q}{2q}} \to R$$

I

This is not possible!

$\therefore$ our assumption was wrong.

Hence, $3+2\sqrt{5}$ is irrational.

$p, q \in Z$

$\frac{p}{q} \to R$

$\frac{2p}{q} \to R$

$\frac{2p-3q}{q} \to R$

DIRECTION: In the question number 19 and 20, a statement of Assertion (A) is followed by a statement of Reason (R).

H.W

Choose the correct option

A) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A)

B) Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of assertion (A)

C) Assertion (A) is true but reason (R) is false.

D) Assertion (A) is false but reason (R) is true.

QUESTION

Assertion (A): HCF of any two consecutive even natural numbers is always 2.
Reason (R): Even natural numbers are divisible by 2.

H.W

QUESTION

The sum of two positive numbers is 240 and their HCF is 15. Find the number of pairts of numbers satisfying the given condition.

QUESTION

Find the number of possible pairs of the product of two numbers and HCF are 4500 and 15 respectively.

$\boxed{\text{Polynomial.}}$ → Special Algebraic expression

→ Variable ki power Whole no.

$\{0, 1, 2, 3, 4 \ldots\}$

$\rightarrow 2x^{1/2} + 2x^2 - 3x - 5x^3$ ✗

$\rightarrow \sqrt{2}x - \sqrt{3}x^2 + 2x^3$ ✓

$\rightarrow \frac{1}{x} = x^{-1}$ ✗

$\rightarrow \frac{x^2}{x^3} = x^{2-3} = x^{-1}$ ✗

$\rightarrow \sqrt{2x^2} + 2x = \sqrt{2}\sqrt{x^2} + 2x = \boxed{\sqrt{2}x + 2x}$ ✓

Types of polynomials

On the basis of degree

On the basis of no. of terms

$\rightarrow 2x^3+4x^4-5x$

$D=4$

$\rightarrow \cancel{2x^2}-\cancel{2x^2}+4x+3x+9x$

$=16x$

$D=1$

Q $2x^2-2x+5x^3-6x$

$2x^2+5x^3-8x$

$T=3$

$D=3$

Number of terms in a polynomials

1. **Monomial:** A polynomial containing one term is called a monomial ('Mono' means 'one'.)
   Example : $9, -14, 6x, -8x^2, 5x^3, 2x^4$, etc. are all monomials.

2. **Binomials:** A polynomial containing two non zero terms is called a binomial. ('Bi' means 'two'.)
   Example : $(9 + 4x)$, $(x - 3x^2)$, $(8 + x^3)$, $(-x^4 + 7)$ are all binomials.

3. **Trinomials:** A polynomial containing three non zero terms is called a trinomial. ('Tri' means 'Three'.)
   Example : $(x^2 + 2x - 3)$, $(2x^3 + 5x^2 - 4)$, $(-7x^4 + 5x^2 + 6)$, $(5x^6 - 3x^4 + x)$ are all trinomials.

On the Basis of Degree

1. *Linear Polynomial:* A polynomial of degree 1 is called a linear polynomial.

2. *Quadratic Polynomial:* A polynomial of degree 2 is called a Quadratic polynomial.

3. *Cubic Polynomial:* A polynomial of degree 3 is called a Cubic polynomial.

Q Monomial, $d=3$.

$x^3, -x^3, -2x^3, 4x^4-x^3-4x^4$

Q Binomial, $d=2$

$x^2+2, 4x^2-x$

Constant Polynomial

non-zero

$2x^0, 3x^0, 4x^0, -5x^0, \frac{1}{2}x^0, 1000x^0$

$d = 0$

Zero Polynomial

'0'

$0x^0$

$d=0$

$0x^1$

$0x^2$

$0x^{100}$

$0x^{10000}$

d = not defined

$P(x) = 2x+3 \to$ zero $= -\frac{3}{2}$

Value of a polynomial :

$P(1) = 2(1)+3 = \boxed{5}$

$P(2) = 2(2)+3 = \boxed{7}$

$P(100) = 2(100)+3 = \boxed{203}$

$P(-3/2) = \cancel{2}\left(\frac{-3}{\cancel{2}}\right)+3 = -3+3 = \boxed{0}$

Zero of a poly:

Variable ki value

Poly = 0

Q $g(x) = x+2$

$\text{zero} = -2$

Q $g(x) = x$

$\text{zero} = 0$

Q $r(x) = 4x-3$

$4x-3=0$

$4x=3$

$x = 3/4$

$\text{zero} = 3/4$

Q $g(x) = x^2-4$

$\text{zero} = 2, -2$

Q $g(x) = x^2$

$\text{zero} = 0$

Some Important Observations

(i) A constant polynomial does not have any zero

(ii) Every linear polynomial has one and only one zero.

(iii) 0 may or may not be the zero of a given polynomial

(iv) Number of zero of a polynomial cannot exceed its degree.

Poly:

$D=3 \longrightarrow$ Max. 3

$D=2 \longrightarrow$ Max. 2

$D=1 \longrightarrow$ 1

$D=4 \longrightarrow$ Max. 4

Geometrical meaning of zeroes of a Linear polynomial

Example : p(x) = 2x + 3 Linear

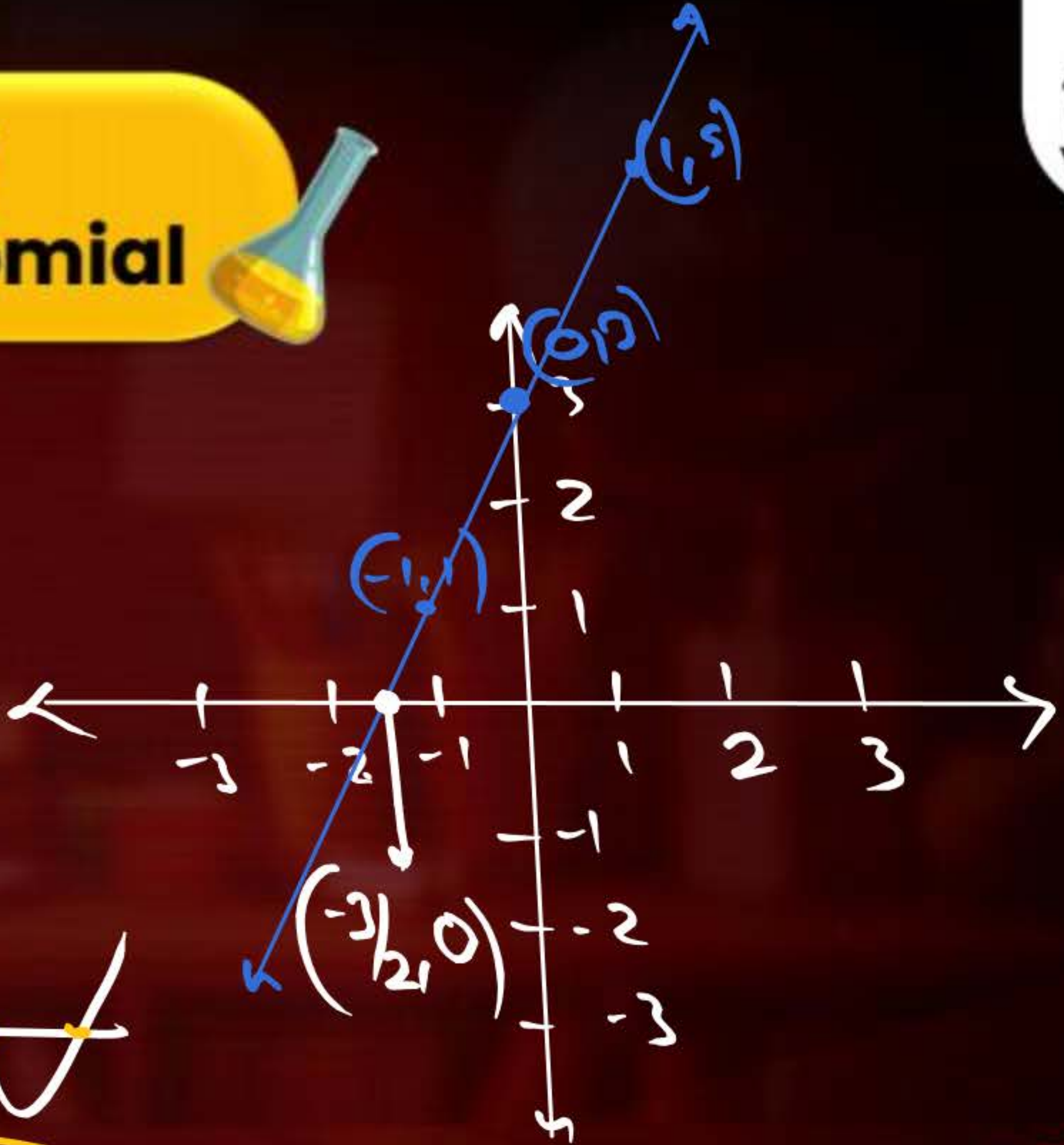
$p(x) = 2x+3$

$\text{zero} = -3/2$

$y = 2x+3$

x	0	1	-1
y	3	5	1

zeroes = 5



Geometrical meaning of zeroes of a polynomial

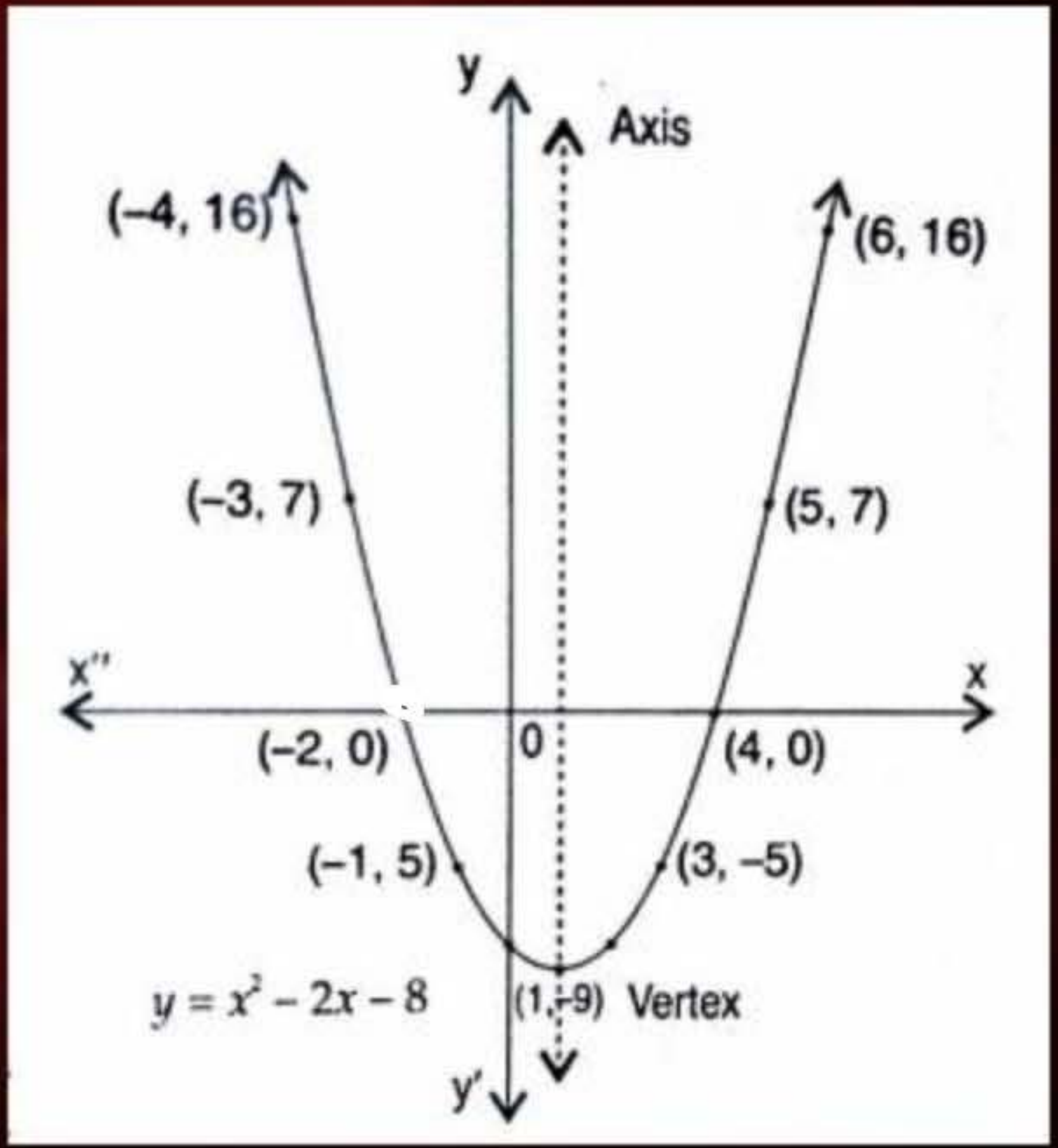
[redacted]

f(x)

x	−4	−3	−2	−1	0	1	2	3	4	5	6
$y = \boxed{x^2 - 2x - 8}$	16	7	0	−5	−8	−9	−8	−5	0	7	16

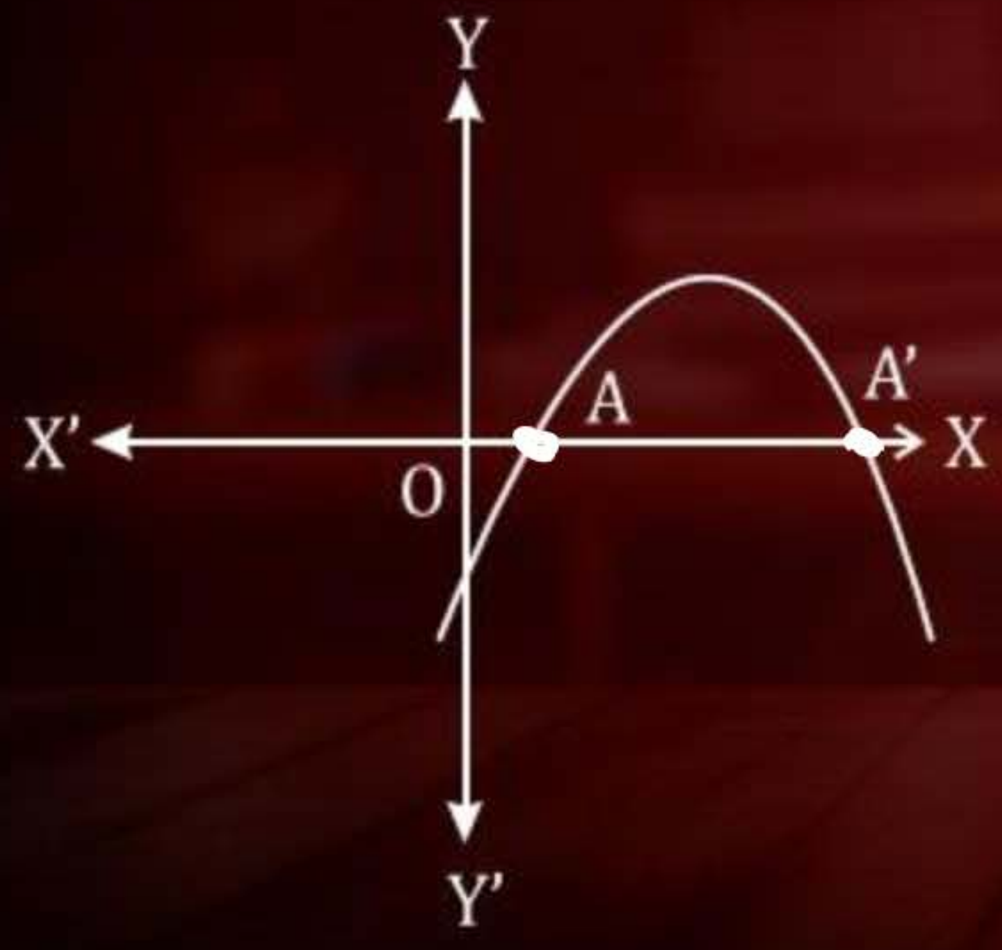
f(x)

−2, 4



Case : (i)

Here, the graph cuts x-axis at two distinct points A and A′.



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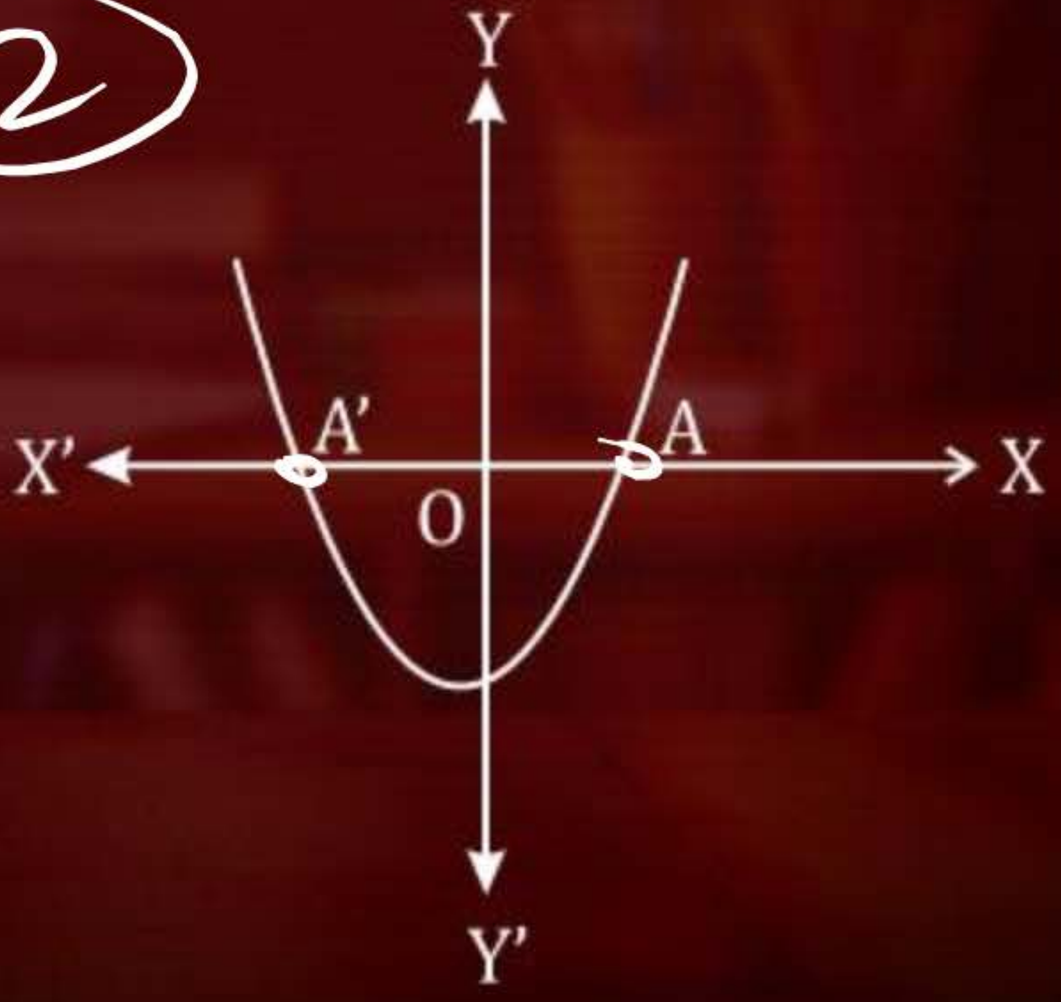
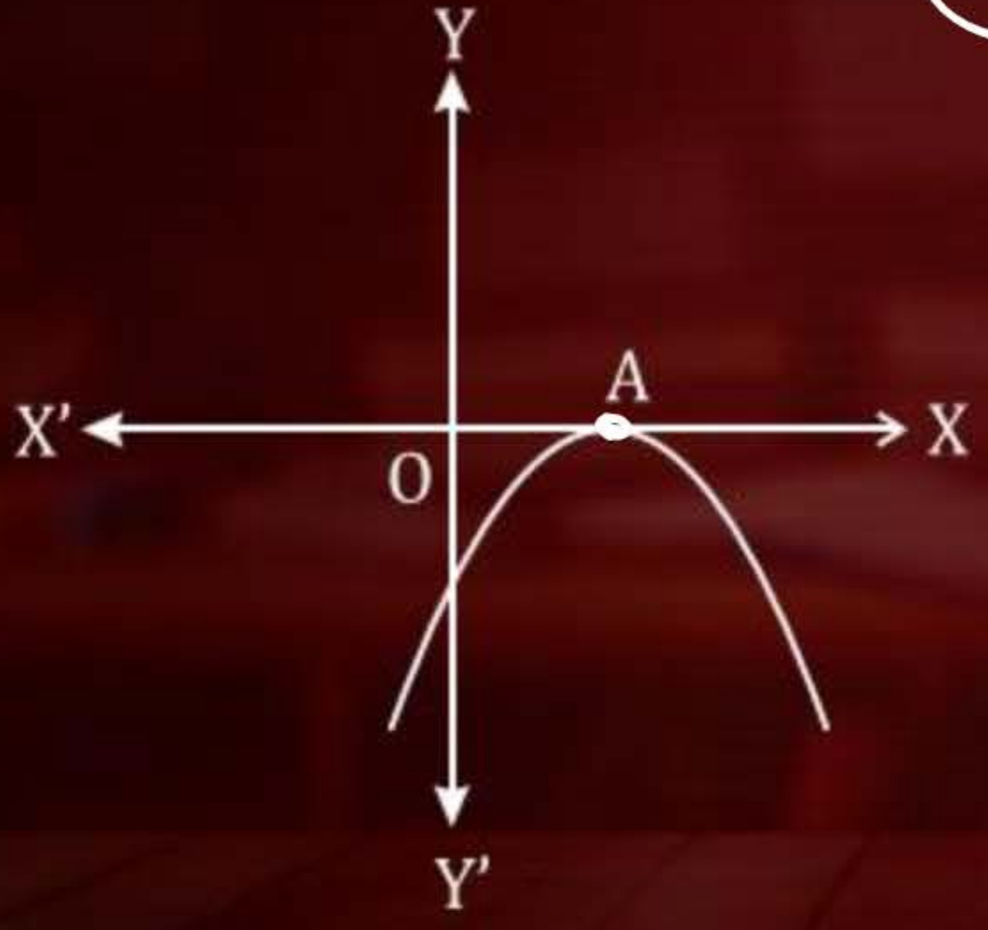
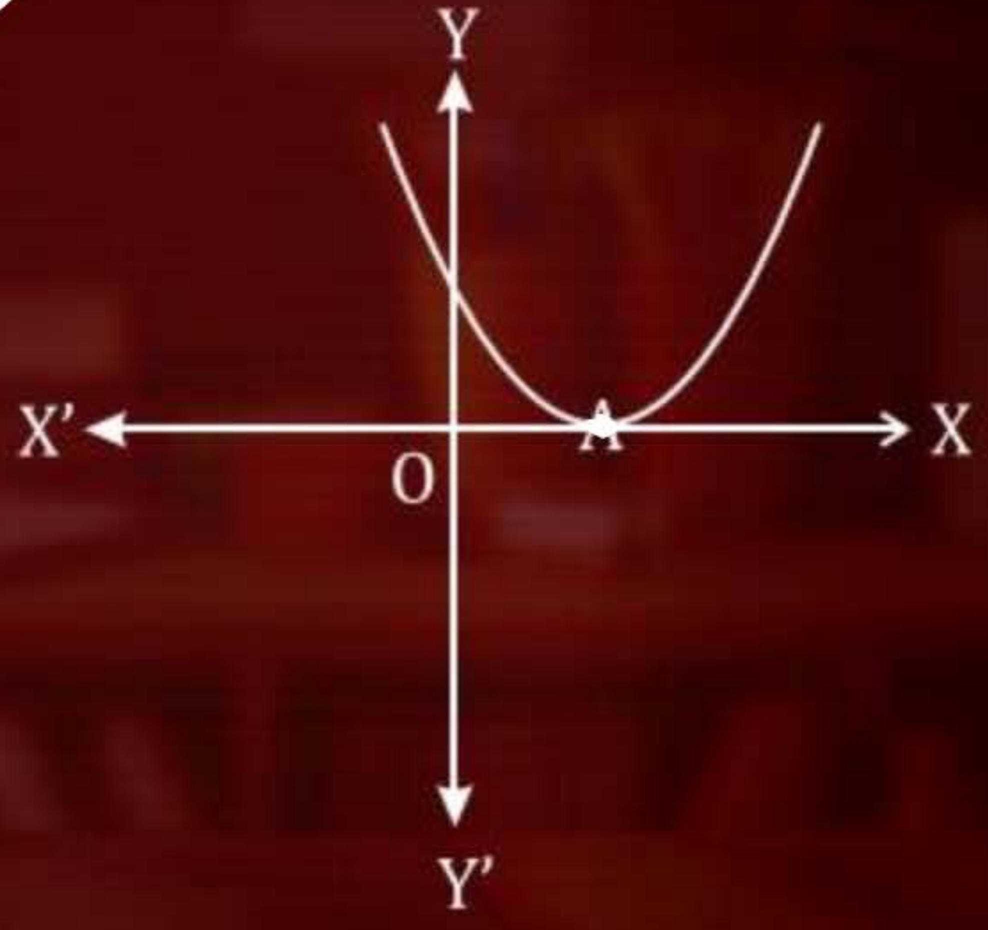


Case : (ii)

(1)





Case : (iii)

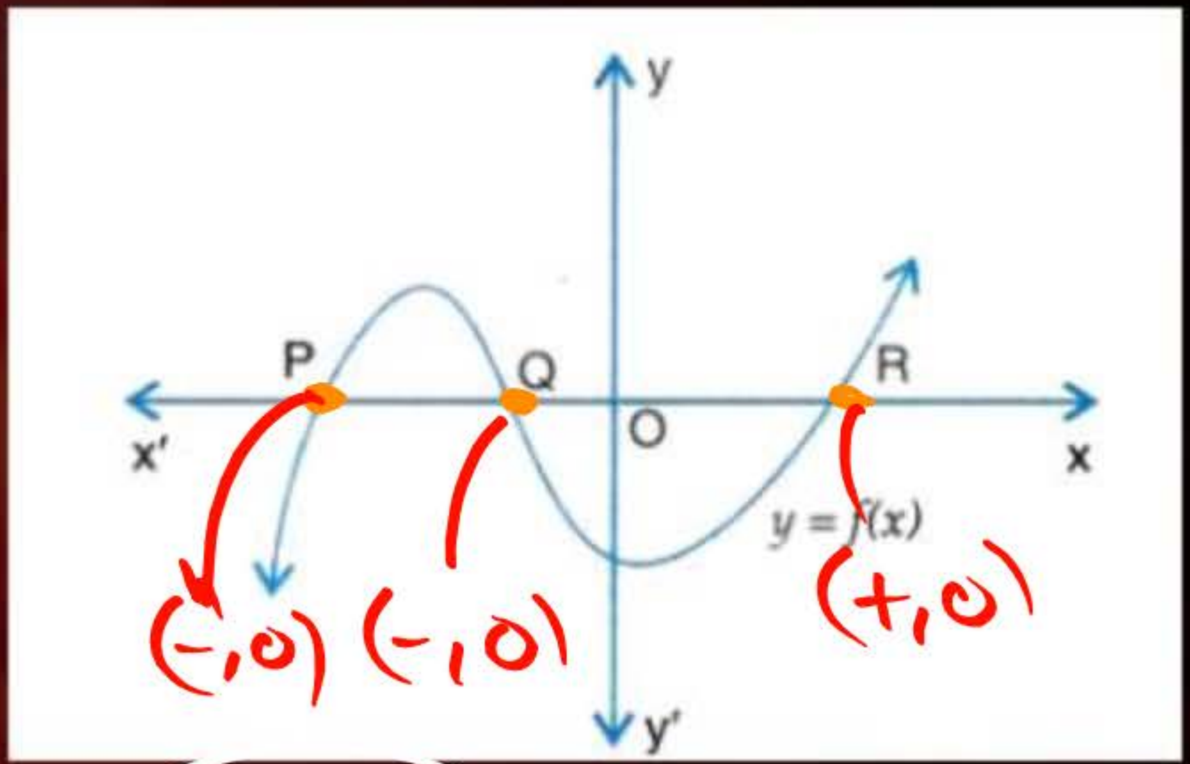
Here, the graph is either completely above the x-axis or completely below the x-axis. So, it does not cut the x-axis at any point.

0

QUESTION

The graph of a polynomial f(x) is shown in fig. The number of zeroes of f(x) is

3



$-,-,+ \rightarrow +$

QUESTION

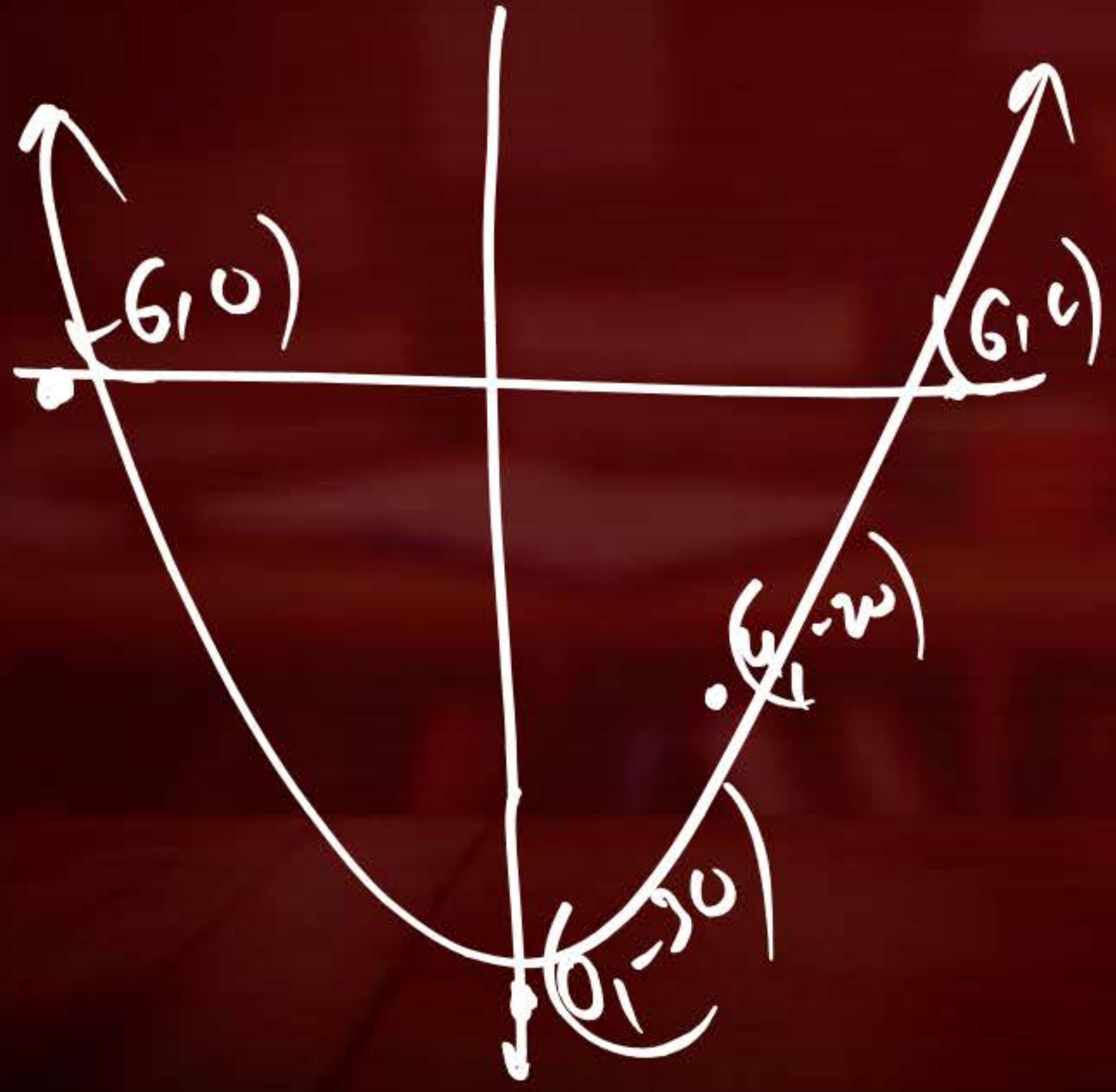
The graph of a quadratic polynomial p(x) passes through the point (–6, 0), (0, –30), (4, –20) and (6, 0). The zeroes of the polynomial are

(A) –6, 0

(B) 4, 6

(C) –30, –20

(D) –6, 6



QUESTION

If one zero of the quadratic polynomial $kx^2 + 3x + k$ is 2, then the value of k is

(A) 5/6

(B) –5/6

(C) 6/5

(D) –6/5

$P(x) = kx^2 + 3x + k \rightarrow 2$

$\rightarrow ?$

$P(2) = 0$

$k(2)^2 + 3(2) + k = 0$

$4k + 6 + k = 0$

$5k = -6$

$k = -6/5$

QUESTION

Zeroes?

$$f(x) = \boxed{x^2 - \frac{11}{6}x - \frac{5}{3}.}$$

| 60
|---

$$\frac{x^2}{1} - \frac{11x}{6} - \frac{5}{3} = 0$$

$$\frac{6x^2 - 11x - 10}{6} = 0$$

$$\boxed{6x^2 - 11x - 10 = 0}$$

$P = -60,\ S = -11$

$-15, 4$

$$6x^2 - 11x - 10 = 0$$

$$\underbrace{6x^2 - 15x}\ \underbrace{+4x - 10} = 0$$

$$3x(2x-5) + 2(2x-5) = 0$$

$$(2x-5)(3x+2) = 0$$

$2x - 5 = 0,\ 3x + 2 = 0$

$x = 5/2,\ x = -2/3$

$\alpha \quad \beta$

ax^2+bx+c

$a \neq 0$

$a, b, c \in R$

(1) $2x^2-3x+2$

$$\boxed{\begin{aligned} a&=2\\ b&=-3\\ c&=2 \end{aligned}}$$

(2) x^2-2+0x

$$\boxed{\begin{aligned} a&=1\\ b&=0\\ c&=-2 \end{aligned}}$$

(3) $-3x^2$

$$\boxed{\begin{aligned} a&=-3\\ b&=0\\ c&=0 \end{aligned}}$$

ax^2+bx+c — α = alpha, β = beta.

$$\boxed{\begin{array}{l} \alpha+\beta = -b/a \\ \text{(sum of zeroes)} \\ \alpha\beta = \dfrac{c}{a} \\ \text{(Product of zeroes.)} \end{array}}$$

Q 2, −3

$S = -1$
$P = -6$

Sum
Product

$k\left[x^2 - (\text{sum})x + \text{product}\right]$

non zero constant

$k\left[x^2 - (-1)x + (-6)\right]$

$k\left[x^2 + x - 6\right]$

$k = 1$
$k = 2$
$k = 3$
$k = a$

$x^2 + x - 6$
$2x^2 + 2x - 12$
$3x^2 + 3x - 18$

QUESTION [NCERT Exemplar]

Find the zeros of the polynomial $x^2+\frac{1}{6}x-2$, and verify the relation between the coefficients and zeros of the polynomial.

$$\frac{x^2}{1}+\frac{1}{6}x-\frac{2}{1}=0$$

$$\frac{6x^2+x-12}{6}=0$$

ax^2+bx+c

$a=6$

$b=1$

$c=-12$

$$\boxed{6x^2+x-12=0}$$

$P=-72,\ S=1$

$9,-8$

$6x^2+x-12=0$

$\underbrace{6x^2+9x}\underbrace{-8x-12}=0$

$3x(2x+3)-4(2x+3)=0$

$(2x+3)(3x-4)=0$

$x=-3/2 \rightarrow \alpha$

$x=4/3 \rightarrow \beta$

$\alpha+\beta=\frac{-b}{a}$

$-\frac{3}{2}+\frac{4}{3}=-\frac{1}{6}$

$\frac{-9+8}{6}=\frac{-1}{6}$

$\frac{-1}{6}=\frac{-1}{6}$

$\alpha\beta=\frac{c}{a}$

$\left(\frac{-3}{2}\right)\left(\frac{4}{3}\right)=\frac{-12}{6}$

$\frac{-12}{6}=\frac{-12}{6}$

QUESTION [CBSE 2003]

Which of the following is a quadratic polynomial having zeroes –2/3 and 2/3

(A) $4x^2 - 9$

(B) $\frac{4}{9}(9x^2 + 4)$

(C) $x^2 + \frac{9}{4}$

(D) $5(9x^2 - 4)$

$S = \frac{-2}{3} + \frac{2}{3} = 0$

$P = \frac{-2}{3} \times \frac{2}{3} = \frac{-4}{9}$

$k\left[x^2 - 0x - \frac{4}{9}\right]$

$k\left(x^2 - \frac{4}{9}\right)$

$k = 9$

$9\left[x^2 - \frac{4}{9}\right]$

$9x^2 - 4$

QUESTION

A quadratic polynomial having zeroes $-\sqrt{\frac{5}{2}}$ and $\sqrt{\frac{5}{2}}$ is

(A) $x^2 - 5\sqrt{2}x + 1$ ✗

(B) $8x^2 - 20$ ✓

(C) $15x^2 - 6$

(D) $x^2 - 2\sqrt{5}x - 1$ ✗

$S = 0$

$P = -\sqrt{\frac{5}{2}} \times \sqrt{\frac{5}{2}} = -\frac{5}{2}$

$k\left[x^2 - 0x - \frac{5}{2}\right]$

$\boxed{k\left[x^2 - \frac{5}{2}\right]}$

$k = 2$

$2x^2 - 5$

QUESTION

If $\boxed{(a-2)x^2 + 3x - 5}$ is a quadratic $\boxed{\text{polynomial}}$, then

$ax^2 + bx + c$

A a can take any real value

B a can take any non-zero value

C $a \neq 2$ $\quad a - 2 \neq 0 \quad a \neq 0$

D $a = 2$ $\quad a \neq 2$

QUESTION [CBSE 2003]

The number of polynomials having zeros –3 and 5 is

(A) 1

(B) 2

(C) 3

(D) More than 3

QUESTION

The zeros of the quadratic polynomial $f(x) = x^2 + 99x + 127$ are

(A) both positive

(B) both negative

(C) one positive and one negative

(D) both equal

$\alpha+\beta = -\frac{b}{a}$

$\boxed{\alpha+\beta = -99}$

$\alpha\beta = \frac{c}{a}$

$\boxed{\alpha\beta = 127}$

QUESTION

If α and β are the zeroes of the polynomial $f(x) = px2 - 2x + 3p$ and $\alpha + \beta = \alpha\beta$, then the value of p is

(A) $-2/3$

(B) $2/3$

(C) $1/3$

(D) $-1/3$

$\alpha+\beta=\alpha\beta$

$\frac{2}{p}=3$

$\frac{2}{3}=p$

$px^2-2x+3p$

$\alpha+\beta=-\frac{b}{a}$

$\alpha+\beta=-\frac{(-2)}{p}$

$\alpha+\beta=\frac{2}{p}$

$\alpha\beta=\frac{c}{a}$

$\alpha\beta=\frac{3p}{p}$

$\alpha\beta=3$

QUESTION

If α and β are the zeroes of the quadratic polynomial $p(x) = x^2 - ax - b$, then the value of $\alpha^2 + \beta^2$ is

ax^2+bx+c

$(a+b)^2 = a^2+b^2+2ab$

$a^2+b^2 = (a+b)^2-2ab$

$\alpha+\beta = -\frac{b}{a}$ | $\alpha\beta = \frac{c}{a}$

$\alpha+\beta = -\frac{(-a)}{1}$ | $\boxed{\alpha\beta = -\frac{b}{1}}$

$\boxed{\alpha+\beta = a}$

(A) $a^2 - 2b$

(B) $a^2 + 2b$

(C) $b^2 - 2a$

(D) $b^2 + 2a$

$\alpha^2+\beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$

$= (a)^2 - 2(-b)$

$= a^2+2b$

QUESTION

If zeroes of the quadratic polynomial $f(x) = (k^2 + 4)x^2 + 7x + 4k$ are reciprocal of each other, then the value (s) of k is (are)

(A) 1

(B) –1

(C) 2

(D) –2

let $\alpha, \frac{1}{\alpha}$

$a = k^2+4$
$b = 7$
$c = 4k$

$\alpha+\beta = \frac{-b}{a}$ | $\alpha\beta = \frac{c}{a}$

$\alpha+\beta = \frac{-7}{k^2+4}$ | $\alpha\beta = \frac{4k}{k^2+4}$

$\cancel{\alpha} \times \frac{1}{\cancel{\alpha}} = \frac{4k}{k^2+4}$

$k^2+4 = 4k$

$\boxed{k^2-4k+4=0}$

$k^2-2k-2k+4=0$

$k(k-2)-2(k-2)=0$

$(k-2)(k-2)=0$

$k=2,2$

QUESTION

If α and β are zeroes of a polynomial $6x^2 - 5x + 1$ then form a quadratic polynomial whose zeroes are α^2 and β^2.

$$\alpha+\beta = \frac{-b}{a} \qquad \alpha\beta = \frac{c}{a}$$

$$\alpha+\beta = \frac{-(-5)}{6} \qquad \boxed{\alpha\beta = \frac{1}{6}}$$

$$\boxed{\alpha+\beta = 5/6}$$

$$\text{Sum} = \alpha^2+\beta^2$$
$$= (\alpha+\beta)^2 - 2\alpha\beta$$
$$= (5/6)^2 - 2(1/6)$$
$$= \frac{25}{36} - \frac{2}{6}$$
$$= \frac{25-12}{36} = \boxed{13/36}$$

$$\text{Product} = \alpha^2 \times \beta^2$$
$$= (\alpha\beta)^2$$
$$= \left(\frac{1}{6}\right)^2$$
$$= \boxed{\frac{1}{36}}$$

α^2, β^2

$S = 13/36 \quad = k\left[x^2 - Sx + P\right]$

$P = 1/36 \quad = k\left[x^2 - \frac{13}{36}x + \frac{1}{36}\right]$

$k = 36$

$= \boxed{36x^2 - 13x + 1}$ //

QUESTION

If α and β are the zeroes of the quadratic polynomial $f(x) = 2x^2 - 5x + 7$, find a polynomial whose zeros are $2\alpha + 3\beta$ and $3\alpha + 2\beta$.

QUESTION

The quadratic polynomial whose zeroes are reciprocal of the zeroes of quadratic polynomial $ax^2 + bx + c$, $a \neq 0$, $c \neq 0$, are given by

(A) $k(cx^2 + ax + b)$

(B) $k(cx^2 + bx + a)$

(C) $k(cx^2 - bx + a)$

(D) $k(cx^2 + bx - a)$

$\boxed{?}$ → $1/\alpha$, $1/\beta$

ax^2+bx+c → α, β

$$\alpha+\beta = \frac{-b}{a} \quad \Big| \quad \alpha\beta = \frac{c}{a}$$

$$\text{Sum} = \frac{1}{\alpha}+\frac{1}{\beta}$$

$$S = \frac{\beta+\alpha}{\alpha\beta}$$

$$S = \frac{-b/\cancel{a}}{c/\cancel{a}} = \boxed{-b/c}$$

$$\text{Product} = \frac{1}{\alpha}\times\frac{1}{\beta}$$

$$= \frac{1}{\alpha\beta}$$

$$= \frac{1/1}{c/a}$$

$$= \boxed{a/c}$$

$$k[x^2 - Sx + P]$$

$$k\left[x^2 + \frac{b}{c}x + \frac{a}{c}\right]$$

$$cx^2 + bx + a$$

QUESTION

If α, β are the zeros of polynomial $f(x) = x^2 - p(x + 2) - c$, then $(\alpha + 1)(\beta + 1) =$

(A) ~~c - 1~~

(B) ~~1 - c~~

(C) ~~c~~

(D) ~~1 + c~~

$x^2 - px - 2p - c$

$a = 1$

$b = -p$

$c = -2p - c$

$\alpha + \beta = \frac{-b}{a}$

$\alpha + \beta = \frac{-(-p)}{1}$

$\alpha + \beta = p$

$\alpha\beta = \frac{c}{a}$

$\alpha\beta = -2p - c$

$= \alpha\beta + \alpha + \beta + 1$

$= -2p - c + p + 1$

$= -c - p + 1$

QUESTION

If α and β are the zeros of the polynomial $f(x) = x^2 - 5x + k$ such that $\alpha - \beta = 1$, find the value of k.

H.W

S.B.S

QUESTION

If α and β are the zeros of the quadratic polynomial $f(x) = kx^2 + 4x + 4$ such that $\alpha^2 + \beta^2 = 24$, find the value of k.

Ch-3

$x+y=1$ — linear eqn
2 variable.

Infinite

$(100,-99)$
$(5,-4)$
$(-4,5)$

solution
↓
variable
ki
value
↓
eqn = satisfy

General Form
↓
$$\boxed{ax+by+c=0}$$

Q $-2x+3y=5 \to -2x+3y-5=0$

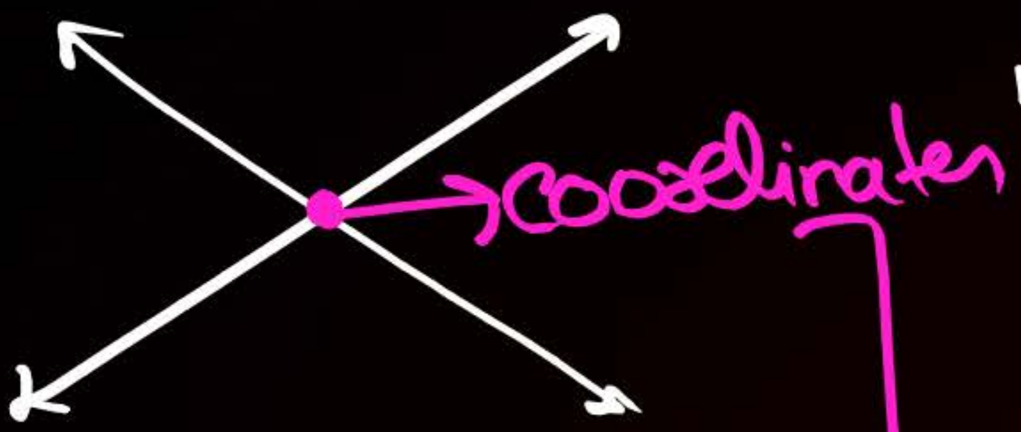
$a=-2$ $c=-5$
$b=3$

$2x+4y+9=0$

$3x-5y+3=0$

$a_1x+b_1y+c_1=0$

$a_2 \quad b_2y+c_2=0$

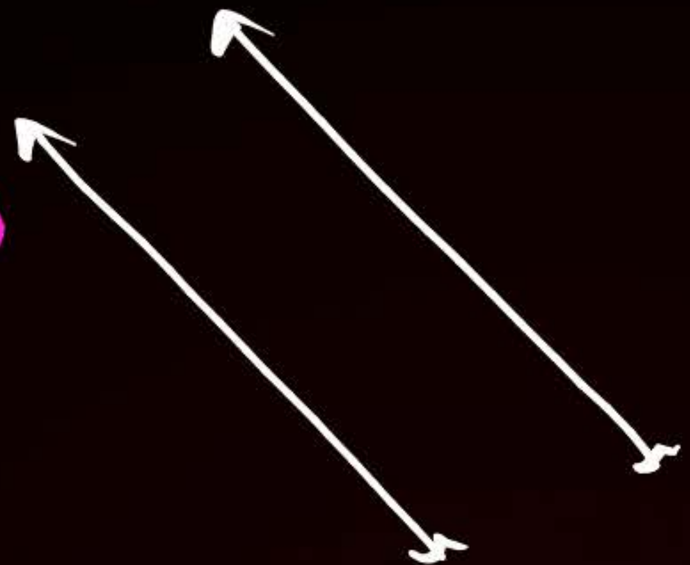


→ Intersecting

→ unique solution.

→ $\boxed{\frac{a_1}{a_2} \neq \frac{b_1}{b_2}}$

→ Consistent system.



→ parallel lines

→ No solution.

→ $\boxed{\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}}$

→ Inconsistent.

→ coincident.

→ Infinite

→ $\boxed{\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}}$

→ Consistent system.

→ dependent

QUESTION

If a pair of equations is consistent, then the lines representing them are

solution hai

I

u

(A) parallel

(B) intersecting or coincident

(C) always coincident

(D) always intersecting

QUESTION

The pair of linear equations $2x = 5y + 6$ and $15y = 6x - 18$ represents two lines which are:

$\boxed{2x - 5y - 6 = 0}$ $\boxed{-6x + 15y + 18 = 0}$

$a_1 = 2$ $a_2 = -6$

$b_1 = -5$ $b_2 = 15$

$c_1 = -6$ $c_2 = 18$

A intersecting

B parallel

C coincident

D either intersecting or parallel

$\frac{2}{-6}$ $\frac{-5}{15}$ $-\frac{6}{18}$

$-\frac{1}{3} = -\frac{1}{3} = -\frac{1}{3}$

QUESTION

A pair of equations $ax + 2y = 9$ and $3x + by = 18$ represent parallel lines, where a, b are integers, if

$a_1 = a$ $\quad$ $a_2 = 3$

$b_1 = 2$ $\quad$ $b_2 = b$

$c_1 = -9$ $\quad$ $c_2 = -18$

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

$$\frac{a}{3} = \frac{2}{b} \neq \frac{-9}{-18}$$

$ab = 6$

(A) $a = b$

(B) $3a = 2b$

(C) $2a = 3b$

(D) $ab = 6$

QUESTION

If the pair of equations $3x - y + 8 = 0$ and $6x - ry + 16 = 0$ represent coincident lines, then the value of r is

$\frac{3}{6} = \frac{1}{r} = \frac{8}{16}$

$\frac{1}{2} = \frac{1}{r}$

$r = 2$

(A) $-\frac{1}{2}$

(B) $\frac{1}{2}$

(C) 2

(D) -2

QUESTION

The pair of equations $\boxed{x = a \text{ and } y = b}$ graphically represents lines which are:

$x=2$ $y=-3$

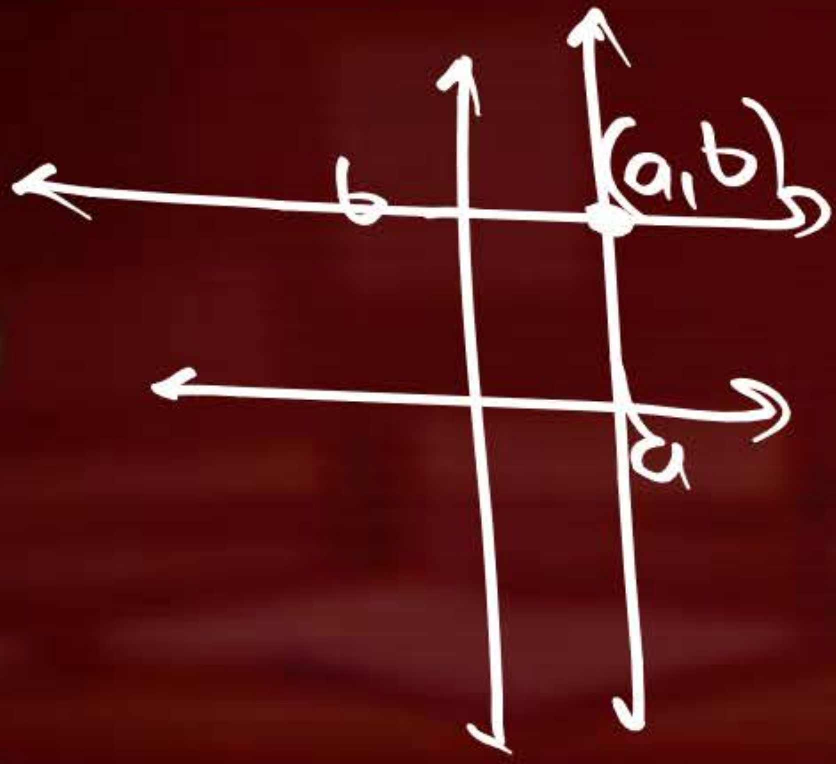
(A) parallel

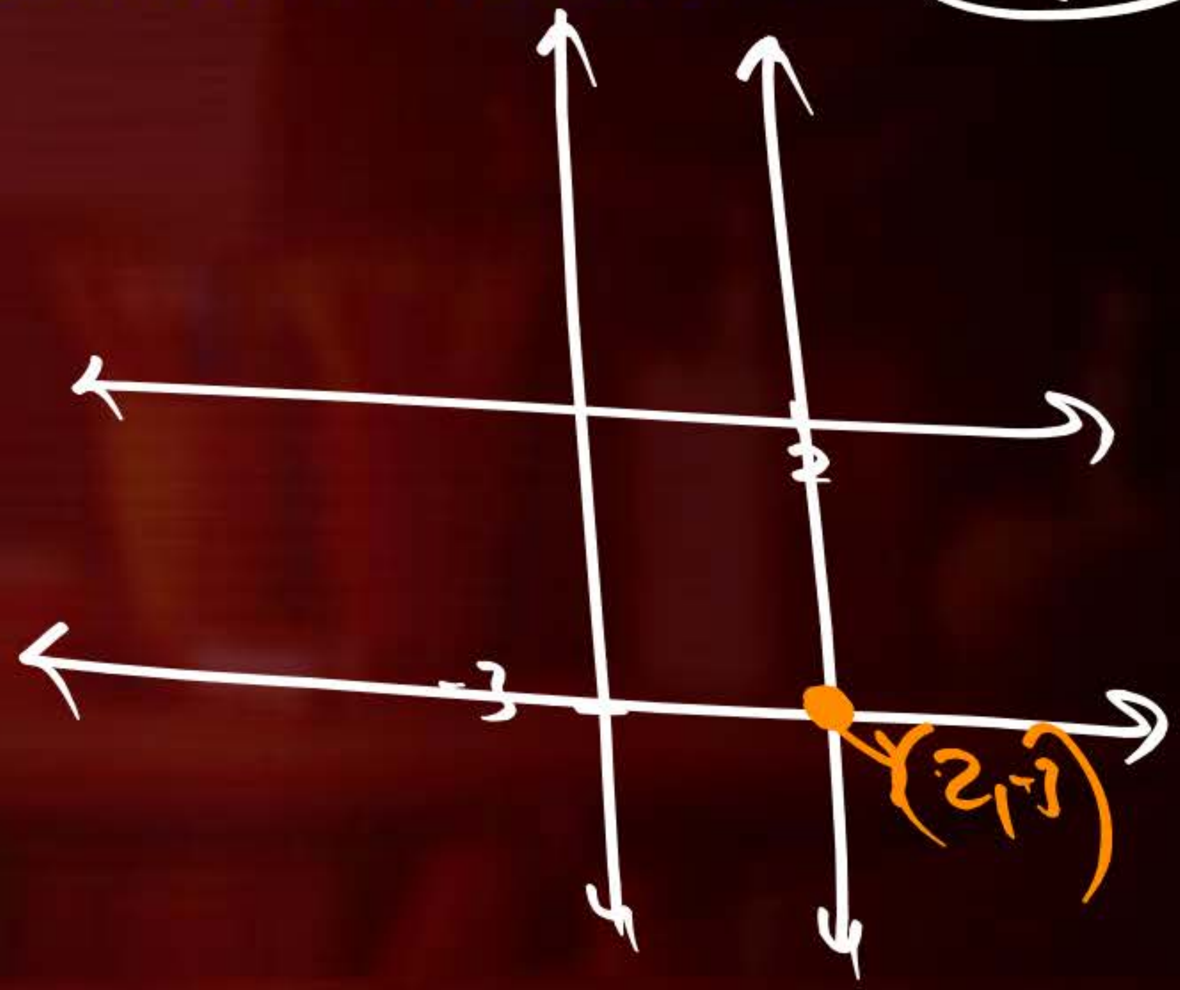
(B) intersecting at (b, a)

(C) coincident

(D) intersecting at (a. b)

$x \rightarrow$ galti





QUESTION

One equation of a pair of dependent linear equations is $-5x + 7y - 2 = 0$, the second equation can be

[CBSE 2011]

(A) $10x + 14y + 4 = 0$ ✗

(B) $-10x - 14y + 4 = 0$ ✗

(C) $-10x + 14y + 4 = 0$ ✗

(D) $10x - 14y = -4$ ✓

$-5x + 7y - 2 = 0$ infinite.

$\boxed{}$

QUESTION

Solve graphically the system of equations :

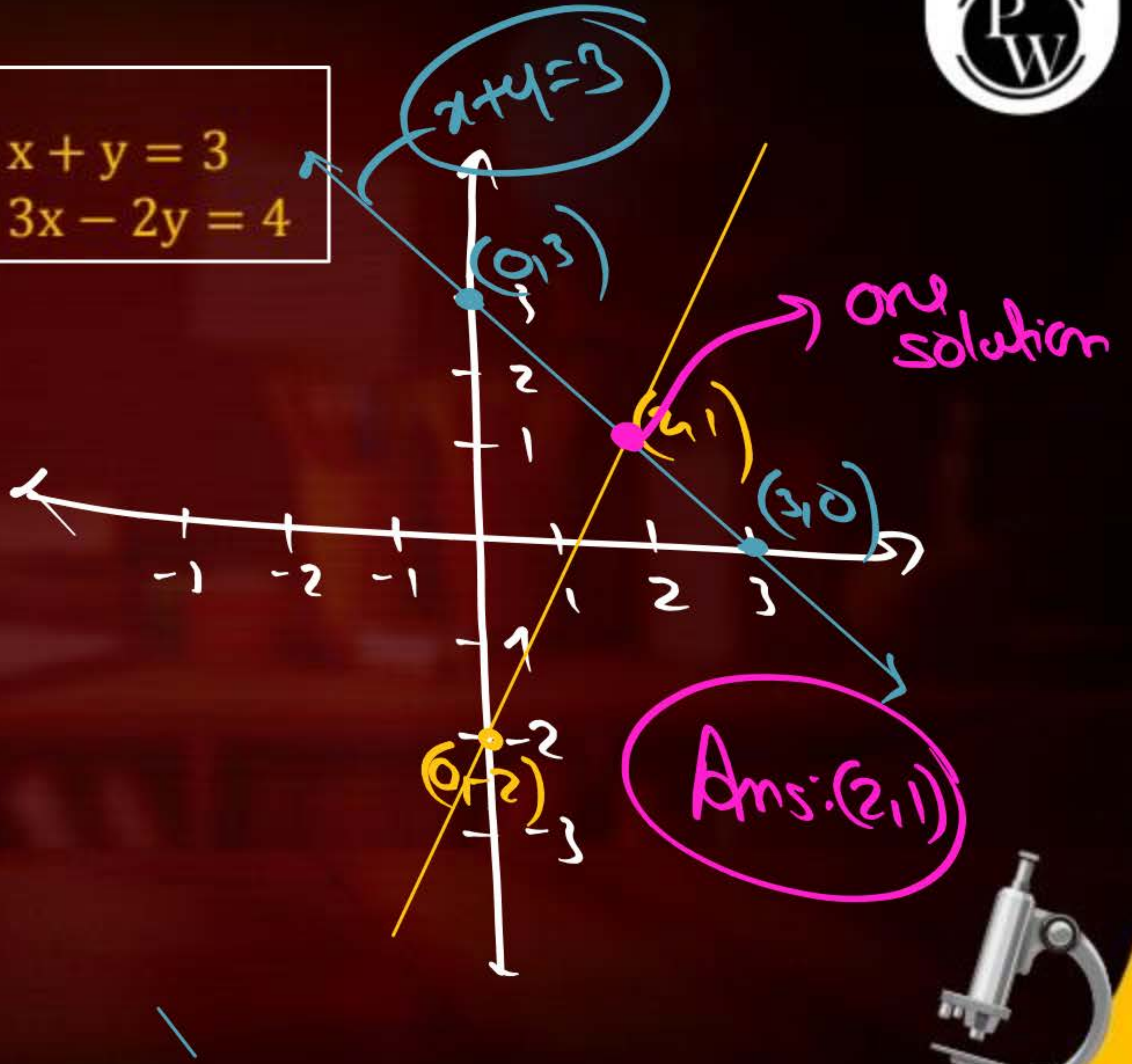
$$x + y = 3$$
$$3x - 2y = 4$$

$x+y=3$

x	0	3
y	3	0

$3x-2y=4$

x	0	$\cancel{4/3}$	2
y	-2	$\cancel{0}$	1



QUESTION

graphically that the system of equations : $\boxed{\begin{aligned} 2x + 4y &= 10 \\ 3x + 6y &= 12 \end{aligned}}$ has no solution.

Parallel

QUESTION

Solve graphically that the system of equations : $\begin{matrix} 3x - y = 2 \\ 9x - 3y = 6 \end{matrix}$ has no infinitely many solution.

QUESTION

Draw the graphs of $2x + y = 6$ and $2x - y + 2 = 0$. Shade the region bounded by these lines and x-axis. Find the area of the shaded region. [CBSE 2002]

$2x + y = 6$

x	0	3
y	6	0

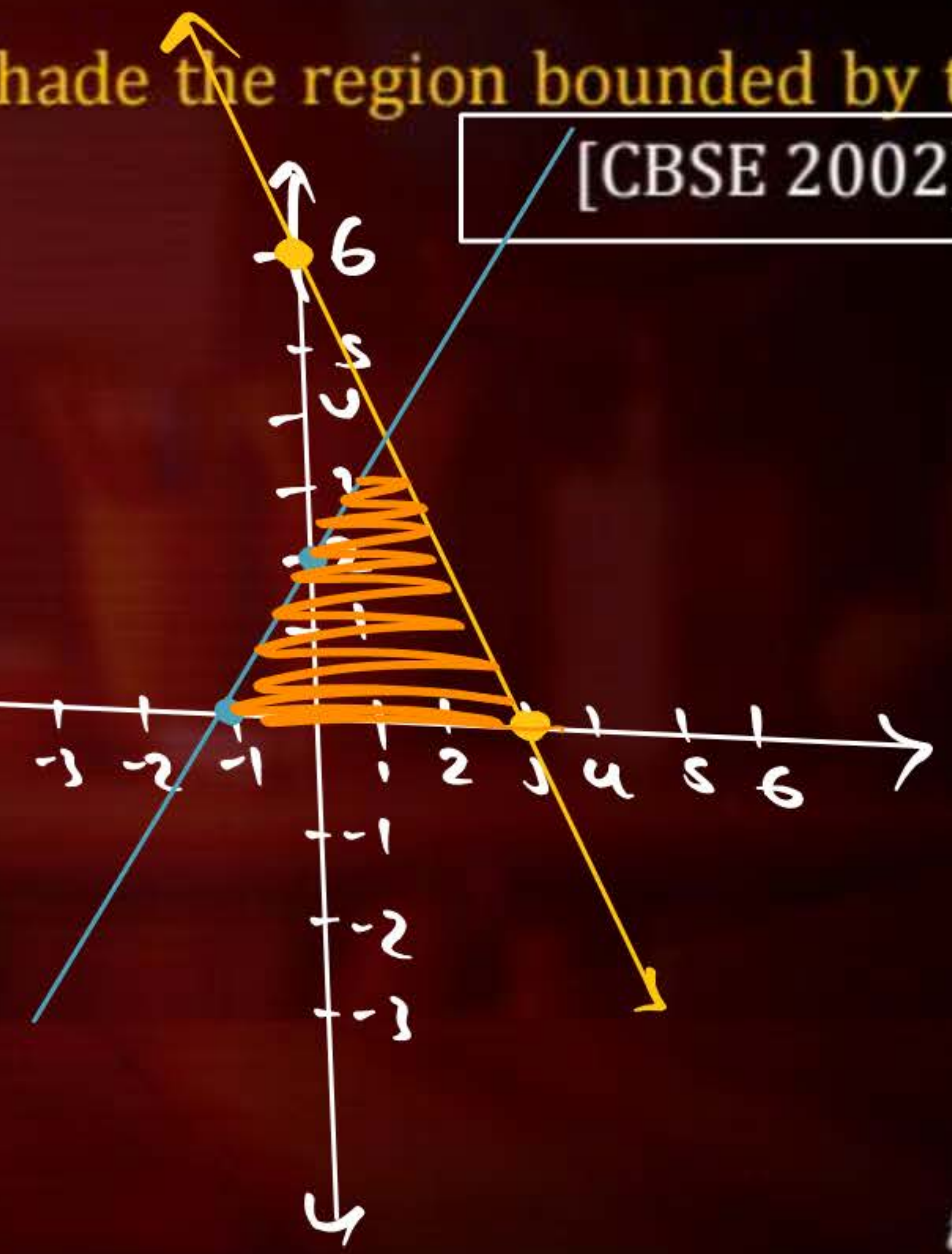
$2x - y + 2 = 0$

x	0	-1
y	2	0

$A = \frac{1}{2} \times b \times h$

4

4



QUESTION

Solve [redacted]

$2x + 3y = 9$ —(1)

$3x + 4y = 5$ —(2)

$$2x+3y=9$$

$$2x = 9-3y$$

$$\boxed{x = \frac{9-3y}{2}} \quad \text{—(3)}$$

Put (3) in (2)

$$3x+4y=5$$

$$3\left(\frac{9-3y}{2}\right)+4y=5$$

$$\frac{27-9y}{2}+\frac{4y}{1}=5$$

$$\frac{27-9y+8y}{2}=5$$

$$27-y=10$$

$$27-10=y$$

$$17=y$$

Put y in (3)

$$x=\frac{9-3(17)}{2}$$

$$x=-21$$

QUESTION

Solve the following system of equations by using the method of elimination by equating the coefficients :

$$\frac{x}{10}+\frac{y}{5}+1=15, \frac{x}{8}+\frac{y}{6}=15$$

$\frac{x}{10}+\frac{y}{5}=14$

$\frac{x+2y}{10}=14$

$\boxed{x+2y=140}$ (1)

$\frac{x}{8}+\frac{y}{6}=15$

$\frac{3x+4y}{24}=15$

$\boxed{3x+4y=360}$ (2)

$3\times(x+2y=140)$

$3x+4y=360$

$3x+6y=420$

$-\quad 3x+4y=360$

$\quad - \quad - \quad -$

$2y=60$

$y=30$

$x+60=140$

$x=80$

$$\cancel{x} + y = -2$$

$$+ \quad -\cancel{x} - 2y = 3$$

$$-y = 1$$

$$y = -1$$

QUESTION

$21x + 47y = 110$
$47x + 21y = 162$

$$21x + 47y = 110$$
$$47x + 21y = 162$$
$$68x + 68y = 272$$
$$68(x+y) = 272$$
$$\boxed{x+y=4} \quad (1)$$

$$21x + 47y = 110$$
$$47x + 21y = 162$$
$$-26x + 26y = -52$$
$$26(-x+y) = -52$$
$$\boxed{-x+y=-2}$$

$$\cancel{x} + y = 4$$
$$-\cancel{x} + y = -2$$
$$2y = 2$$
$$y = 1$$

$$x+y=4$$
$$x+1=4$$
$$x=3$$

QUESTION

$$99x + 101y = 499$$
$$101x + 99y = 501$$

Same

QUESTION

Foreach of the following systems of equations determine the value of k for which the given system of equations has a unique solution:

$2x + 3y - 5 = 0$

$kx - 6y - 8 = 0$

0, Rakh sakte ho.

$\boxed{\frac{a_1}{a_2} \neq \frac{b_1}{b_2}}$

$\frac{2}{k} \neq \frac{3}{-6}$

$\frac{-6 \times 2}{3} \neq k$

$-4 \neq k$

Ans: All Real no.s, except -4

QUESTION

For each of the following systems of equations determine the value of k for which the given system has $\boxed{\text{no solution:}}$

$3x - 4y + 7 = 0$

$kx + 3y - 5 = 0$

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

$$\boxed{\frac{3}{k} = \frac{-4}{3} \neq \frac{7}{-5}}$$

$$\frac{9}{-4} = k$$

$1x + 2y - 3 = 0$

$kx + 4y - 6 = 0$

$$\frac{1}{k} = \frac{2}{4} \neq \frac{-3}{-6}$$

$$k = 2$$

QUESTION

For what value of k, will the following system of equations has infinitely many solutions?

$2x + 3y = 4$

$(k + 2)x + 6y = 3k + 2$

$$\frac{2}{k+2} = \frac{3}{6} = \frac{-4}{-3k-2}$$

$$\frac{2}{k+2} = \cancel{\frac{3}{6}}\frac{1}{2}$$

$4 = k+2$

$2 = k$

QUESTION

[redacted]

One says, "Give me a hundred, friend! I shall then become twice as rich as you." The other replies, "If you give me ten, I shall be six times as rich as you." Tell me what is the amount of their respective capital? [NCERT]

QUESTION

37 pens and 53 pencils together cost ₹320, while 53 pens and 37 pencils together cost ₹400. Find the cost of a pen and that of a pencil.

let, cost of a pen = x Rs

cost of a pencil = y Rs

$37x + 53y = 320$

$53x + 37y = 400$

$+$

$-$

1 'p' = x Rs

37 'p' = 37x Rs

QUESTION

$92 = 9 \times 10 + 2 \times 1$

$89 = 8 \times 10 + 9 \times 1$

$98 = 9 \times 10 + 8 \times 1$

$yx = 10y + x$

BASED ON BASIC CONCEPTS (BASIC)

In a two digit number, the unit's digit is twice the ten's digit. If 27 is added to the number, the digits interchange their places. Find the number.

$x = 2y$ — (1)

$27 + 10y + x = 10x + y$

$-9x + 9y = 27$

$9(-x + y) = 27$

$-x + y = 3$ — (2)

Units' digit $= x$

Ten's digit $= y$

Two digit no./No. $= 10y + x$

Reversed no./Interchanged $= 10x + y$

QUESTION

The sum of the digits of a two digit number is 8 and the difference between the number and that formed by reversing the digits is 18. Find the number.

$\boxed{x+y=8}$ — (1)

$\boxed{(10y+x)-(10x+y)=18}$ — (2)

Units' digit $= x$

Ten's digit $= y$

No. $= 10y+x$

R. no. $= 10x+y$

QUESTION

The sum of a two digit number and the number formed by interchanging the digit is 132. If 12 is added to the number, the new number becomes 5 times the sum of the digits. Find the number.

[CBSE 2002C]

$$\boxed{10y+x+10x+y=132} \quad -(1)$$

$$\boxed{12+10y+x=5(x+y)} \quad -(2)$$

new no.

x

y

$10y+x$

$10x+y$

QUESTION

The sum of a two-digit number and the number obtained by reversing the order of its digits is 165. If the digits differ by 3, find the number. [CBSE 2002]

QUESTION

BASED ON BASIC CONCEPTS (BASIC)

A fraction becomes 4/5, if 1 is added to both numerator and denominator. If, however, 5 is subtracted from both numerator and denominator, the fraction becomes 1/2. What is the fraction?

QUESTION

The denominator of a fraction is 4 more than twice the numerator. When both the numerator and denominator are decreased by 6, then the denominator becomes 12 times the numerator. Determine the fraction. [CBSE 2001C]

$D = 4 + 2(N)$

$\boxed{y = 4 + 2x}$ — ①

$D = 12(N)$

$\boxed{y - 6 = 12(x - 6)}$ — ②

$N = x - 6$ $\quad N = x$

$D = y - 6$ $\quad D = y$

$F = \frac{x-6}{y-6}$ $\quad F = x/y$

QUESTION

BASED ON BASIC CONCEPTS (BASIC)

If twice the son's age in years is added to the father's age, the sum is 70. But if twice the father's age is added to the son's age, the sum is 95. Find the ages of father and son.

QUESTION

BASED ON LOTS

Two years ago, a father was five times as old as his son. Two years later, his age will be 8 more than three times the age of the son. Find the present ages of father and son.

[CBSE 2004]

Father

	past	Pr.	Fu.
Father	$5x-2$	x	$x+2$
Son	$y-2$	y	$y+2$

$$\boxed{x-2=5(y-2)} \quad (1)$$

$$\boxed{x+2=8+3(y+2)} \quad (2)$$

$$x-2=5y-10$$

$$\boxed{x-5y=-8} \quad (1)$$

$$x+2=8+3y+6$$

$$\boxed{x-3y=12} \quad (2)$$

QUESTION

BASED ON LOTS

Father's age is three times the sum of ages of his two children. After 5 years his age will be twice the sum of ages of two children. Find the age of father.

[CBSE 2003, 2019]

$\boxed{x = 3(y)}$ — (1)

$\boxed{x+5 = 2(y+10)}$ — (2)

	Pa. Pr.	Fy
Father	x	$x+5$
sum of ages of bacha	y	$y+5+5$

Taxi

Fixed charge

x

charge per km

y

For 10km

$x + 10y$

For 15km

$x + 15y$

QUESTION

BASED ON BASIC CONCEPTS (BASIC)

The taxi charges in a city comprise of a fixed charge together with the charge for the distance covered. For a journey of 10 km the charge paid is ₹75 and for a journey of 15 km the charge paid is ₹110. What will a person have to pay for travelling a distance of 25 km? [NCERT, CBSE 2000]

Fixed charge = 'x' Rs

charge per km = 'y' Rs

For 10 km

$\boxed{x + 10y = 75}$ — (1)

15 km.

$\boxed{x + 15y = 110}$ — (2)

Ans: $x + 25y$

QUESTION

A part of monthly hostel charges in a college are fixed and the remaining depend on the number of days one has taken food in the mess. When a student A takes food for 20 days, he has to pay ₹1000 as hostel charges whereas a student B, who takes food for 26 days, pays ₹1180 as hostel charges. Find the fixed charge and the cost of food per day.

[NCERT, CBSE 2000]

x

y

A → 20 days.

$$\boxed{x + 20y = 1000} \quad (1)$$

B → 26 days.

$$\boxed{x + 26y = 1180} \quad (2)$$

QUESTION

A shopkeeper gives books on rent for reading. She takes a fixed charge for the first two days, and an additional charge for each day thereafter. Latika paid ₹22 for a book kept for 6 days, while Anand paid ₹16 for the book kept for four days. Find the fixed charges and charge for each extraday. [NCERT Exemplar]

Fixed charge for '2' days = x.

charge per day thereafter = y

Latika → 6 days

2 ↙ ↘ 4

$\boxed{x + 4y = 22}$

Anand → 4 days

2 ↙ ↘ 2

$\boxed{x + 2y = 16}$

QUESTION

The ratio of incomes of two persons is 9 : 7 and the ratio of their expenditures is 4: 3. If each of them saves ₹200 per month, find their monthly incomes. [NCERT]

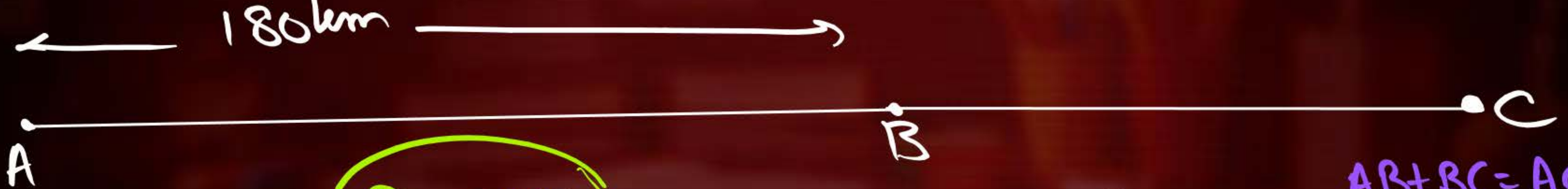
QUESTION

The area of a rectangle gets reduced by 9 square units if its length is reduced by 5 units and the breadth is increased by 3 units. If we increase the length by 3 units and breadth by 2 units, the area is increased by 67 square units. Find the length and breadth of the rectangle. [NCERT]

QUESTION

$D = S \times T$, $S = \frac{D}{T}$, $T = \frac{D}{S}$

Places A and B are 180 km apart on a highway. One car starts from A and another from B at the same time. If the car travels in the same direction at different speeds, they meet in 9 hours. If they travel towards each other with the same speeds as before, they meet in an hour. What are the speeds of the two cars?



$D = S \times T$

$S = x$
$D = AC$
$T = 9$

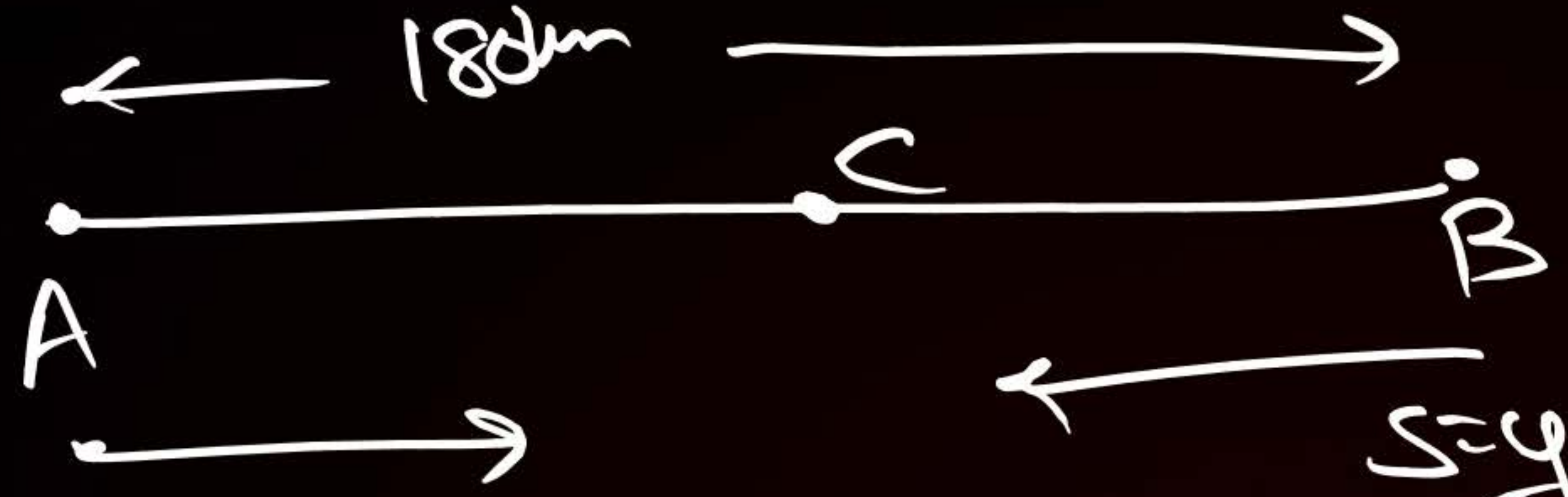
$AC = 9x$

$S = y$
$D = BC$
$T = 9$

$BC = 9y$

$AB + BC = AC$

$\boxed{180 + 9y = 9x}$ (1)



$S = x$

$D = AC$

$T = 1$

$AC = x$

$S = y$

$D = BC$

$T = 1$

$BC = y$

$AC + CB = AB$

$\boxed{x + y = 180}$ (2)

QUESTION

Yash scored 40 marks in a test, getting 3 marks for each right answer and losing 1 mark for each wrong answer. Had 4 marks been awanded for each correct answer and 2 marks been deducted for each incorrect answer, then Yash would have scored 50 marks. How many questions were there in the test? [NCERT]

H.W

Hint: no. of right answers $= x$
" " wrong " $= y$

Quadratic Equation

Ch-4.

$ax^2+bx+c=0$ → always '2' Roots

$a \neq 0$

$\boxed{a, b, c \in R}$

Roots → variable ki value jo eqn ko satisfy.

QUESTION [redacted]

The coefficient x^0 the quadratic equation $x(x-1) - 5 = 0$ is:

constant term

$$\boxed{x^2 - x - 5 = 0}$$

$-5x^0$

(A) 5

(B) −5

(C) 1

(D) 1/2

QUESTION

~~Factorise~~ : Roots

(i) $\boxed{x^2 + 5\sqrt{3}x + 12} = 0$

(ii) $x^2 + 3\sqrt{3}x - 30$

(i) $x^2 + 5\sqrt{3}x + 12 = 0$

$P = 12, \; S = 5\sqrt{3}$

$4\sqrt{3} + \sqrt{3} = 5\sqrt{3}$

$x \qquad x$

$x^2 + 5\sqrt{3}x + 12 = 0$

$x^2 + 4\sqrt{3}x + \sqrt{3}x + 12 = 0$

$x(x + 4\sqrt{3}) + \sqrt{3}(x + 4\sqrt{3}) = 0$

$(x + 4\sqrt{3})(x + \sqrt{3}) = 0$

$x = -4\sqrt{3}$

$x = -\sqrt{3}$

(iii) $x^2 + 3\sqrt{3}x - 30 = 0$

$P = -30$, $S = 3\sqrt{3}$

$5\sqrt{3}$, $-2\sqrt{3}$

H.W

$5\sqrt{3} + 2\sqrt{3}$

QUESTION

Factorise: $4\sqrt{3}x^2 + 5x - 2\sqrt{3}$

QUESTION [CBSE Term - II, 2015]

Solve the quadratic equation $\boxed{(x-1)^2 - 5(x-1) - 6 = 0}$

$$x^2 + 1 - 2x - 5x + 5 - 6 = 0$$

$$x^2 - 7x + \cancel{x} - \cancel{1} = 0$$

$$x^2 - 7x = 0$$

$$x(x-7) = 0$$

$$x = 0, x = 7$$

QUESTION [CBSE Term - II, 2015, 2017]

Find the positive roots of $\sqrt{3x^2+6}=9$.

Ans: 5

S.B.S

$$\left(\sqrt{3x^2+6}\right)^2=(9)^2$$

$$3x^2+6=81$$

$$3x^2=75$$

$$x^2=25$$

$$x=\pm 5$$

QUESTION [CBSE 2002]

If one root of the quadratic equation $2x^2 + kx - 6 = 0$ is 2, find the value of k. Also, find the other root.

$2(2)^2 + k(2) - 6 = 0$

$8 + 2k - 6 = 0$

$2 + 2k = 0$

$2k = -2$

$k = -1$

$2x^2 - x - 6 = 0$

M.I → Factorise.

M.II

$\alpha\beta = \frac{c}{a}$

$2(\beta) = \frac{-6}{2}$

$\beta = -3/2$

QUESTION [NCERT]

$$\frac{1}{x+4}-\frac{1}{x-7}=\frac{11}{30}, x \neq 4,7$$

(A) 1,2

(B) −1, −2

(C) −1, 2

(D) 1, −2

QUESTION [CBSE 2005]

Solve the following quadratic equations by factorization method:

$$\boxed{\frac{1}{a+b+x}=\frac{1}{a}+\frac{1}{b}+\frac{1}{x},\ a+b\neq 0}$$

$$\frac{1}{a+b+x}-\frac{1}{x}=\frac{1}{a}+\frac{1}{b}$$

$$\frac{x-(a+b+x)}{(a+b+x)x}=\frac{b+a}{ab}$$

$$\frac{\cancel{x}-a-b-\cancel{x}}{ax+bx+x^2}=\frac{a+b}{ab}$$

$$\frac{-a-b}{ax+bx+x^2}=\frac{a+b}{ab}$$

$a+b\neq 0$

$$\frac{-\cancel{(a+b)}}{ax+bx+x^2}=\frac{\cancel{a+b}}{ab}$$

$$\frac{-1}{ax+bx+x^2}=\frac{1}{ab}$$

$$-ab=ax+bx+x^2$$

$$\boxed{x^2+ax+bx+ab=0}$$

$$x(x+a)+b(x+a)=0$$

$$(x+a)(x+b)=0$$

$$x=-a$$

$$x=-b$$

$ax^2+bx+c \rightarrow$ 2 Roots

$$\boxed{D = b^2 - 4ac}$$

Discriminant

$$x = \frac{-b \pm \sqrt{D}}{2a}$$

(I) Real and distinct $D > 0$

(II) Real and equal. $D = 0$

(III) No Real roots. $D < 0$

Q $4x^2 - 4x + 4 = 0$

Nature of roots?

$\boxed{a = 4,\ b = -4,\ c = 4}$

$D = b^2 - 4ac$

$D = (-4)^2 - 4(4)(4)$

$D = 16 - 64$

$\boxed{D = -48}$

No real roots.

Q $2x^2 - 3x + 1 = 0$

Nature = ?

$D = b^2 - 4ac$

$D = (-3)^2 - 4(2)(1)$

$9 - 8$

$D = 1 \rightarrow D > 0 \rightarrow$ Real & Distint

Q $x^2 + 4 - 4x = 0$

N = ?

$D = b^2 - 4ac$

$D = (-4)^2 - 4(1)(4)$

$16 - 16 = 0$

$D = 0$

Real & equal.

QUESTION [CBSE 2016, 2019]

If $x = 2/3$ and $x = -3$ are the roots of the equation $ax^2 + 7x + b = 0$, find the values of a and b.

QUESTION [CBSE 2014]

$$3\left(\frac{7x+1}{5x-3}\right)-4\left(\frac{5x-3}{7x+1}\right)=11; x\neq\frac{3}{5},-\frac{1}{7}$$

$$\frac{21x+3}{5x-3}-\frac{(20x-12)}{7x+1}=11$$

$$\frac{(7x+1)(21x+3)-(20x-12)(5x-3)}{(5x-3)(7x+1)}=11$$

$$\frac{147x^2+21x+21x+3-100x^2+60x+60x-36}{35x^2+5x-21x-3}=11$$

$$\frac{47x^2+162x-33}{35x^2-16x-3}=11$$

$$47x^2+162x-\cancel{33}=385x^2-176x-\cancel{33}$$

$$47x^2-385x^2+162x+176x=0$$

$$-338x^2+338x=0$$

$$(-338x)(x-1)=0$$

$$x=0,\ x=1$$

QUESTION [CBSE 2013, 17]

Find the Value of k for which the given equation has real and equal roots:

$D=0$

$(k-12)x^2 + 2(k-12)x + 2 = 0$

$a = k-12$

$b = 2(k-12)$

$c = 2$

$D = 0$

$b^2 - 4ac = 0$

$[2(k-12)]^2 - 4(k-12)(2) = 0$

$4(k-12)^2 - 8(k-12) = 0$

$4[k-12][k-12-2] = 0$

$(k-12)(k-14) = 0$

$k = 12, k = 14$

QUESTION [CBSE 2017]

If the *equation* $(1+m^2)x^2 + 2mcx + (c^2 - a^2) = 0$ has equal roots, prove that $c^2 = a^2(1+m^2)$.

$D=0$

$D=0$

$b^2 - 4ac = 0$

$(2mc)^2 - 4(1+m^2)(c^2 - a^2) = 0$

$4m^2c^2 - 4[c^2 - a^2 + m^2c^2 - m^2a^2] = 0$

$\cancel{4m^2c^2} - 4c^2 + 4a^2 - \cancel{4m^2c^2} + 4m^2a^2 = 0$

$-4c^2 + 4a^2 + 4m^2a^2 = 0$

$-4(c^2 - a^2 - m^2a^2) = 0$

$c^2 - a^2 - m^2a^2 = 0$

$c^2 = a^2 + m^2a^2$

$\boxed{c^2 = a^2(1+m^2)}$ //

QUESTION [CBSE 2017]

If the roots of the equation $(a^2 + b^2)x^2 - 2(ac + bd)x + (c^2 + d^2) = 0$ are equal, prove that $\frac{a}{b} = \frac{c}{d}$.

H.W

$D = 0$

QUESTION [CBSE 2014]

Find the values of p for which the quadratic equation

$(2p+1)x^2-(7p+2)x+(7p-3)=0$ has equal roots.

Also, find these roots.

QUESTION

The sum of two numbers is 18 and the sum of their reciprocals is 9/40. Find the numbers.

Let the nos be x and $18-x$.

$$\boxed{\frac{1}{x}+\frac{1}{18-x}=\frac{9}{40}}$$

$$\frac{1(18-x)+1(x)}{x(18-x)}=\frac{9}{40}$$

18

x	$18-x$
8	18-8
10	18-10

QUESTION [NCERT, CBSE 2014]

Find two consecutive odd positive integers, sum of whose squares is 290.

Let, x and $x+2$ be two odd consecutive nos.

$$x^2+(x+2)^2=290$$

QUESTION [CBSE 2016]

A two-digit number is four times the sum and three times the product of its digits. Find the number.

QUESTION

$S=\frac{D}{T}$ $T=\frac{D}{S}$ $D=S\times T$

A train travels a distance of 300 km at constant speed. If the speed of the train is increased by 5 km an hour, the journey would have taken 2 hours less. Find the original speed of the train.

let speed $= x$.

case-I

$S = x$

$D = 300$

$T = T$

$T = \frac{300}{x}$

case II

$S' = x+5$

$D = 300$

$T' = T-2$

$T-2 = \frac{300}{x+5}$

$\frac{300}{x} - 2 = \frac{300}{x+5}$

$\boxed{\frac{300}{x} - \frac{300}{x+5} = 2}$

$300\left[\frac{1}{x} - \frac{1}{x+5}\right] = 2$

$$\left[\frac{\cancel{x}+5-\cancel{x}}{x(x+5)}\right]=\frac{1}{150}$$

$$\frac{5}{x^2+5x}=\frac{1}{150}$$

$$750=x^2+5x$$

$$\boxed{0=x^2+5x-750}$$

Quadratic

M.T.S

QUESTION

$T = \frac{D}{S}$

A fast train takes 3 hours less than a slow train for a journey of 600 km. If the speed of the slow train is 10km/hr less than that of the fast train, find the speeds of the two trains.

Speed → Fast train = x

→ slow train = $x-10$

$T = \frac{600}{x-10}$

$T-3 = \frac{600}{x}$

$\frac{600}{x-10} - 3 = \frac{600}{x}$

	I (S.T)	II (F.T)
Speed	$x-10$	x
distance	600	600
time	T	$T-3$

QUESTION

$T = \frac{D}{S}$

In a flight of 600 km, a aircraft was slowed down due to bad weather. Its average speed for the trip was reduced by 200 km/hr and the time of flight increased by 30 minutes. Find the duration of flight.

Let actual speed = x km/hr.

60 min = 1 hr

30 min = $\frac{1}{2}$ hr

x = speed

	I	II
Speed	x	$x-200$
Distance	600	600
time	T	$T+\frac{1}{2}$

$T = \frac{600}{x}$

$T + \frac{1}{2} = \frac{600}{x-200}$

$\frac{600}{x} + \frac{1}{2} = \frac{600}{x-200}$

QUESTION [CBSE 2023]

A train travels at a certain average speed for a distance of 54 km and then travels a distance of 63 km at an average speed of 6 km/hr more than the first speed. If it takes 3 hours to complete the journey, what was its first average speed?

H.W

QUESTION [CBSE 2014]

The area of a rectangular plot is 528 m^2. The length of the plot (in metres) is one metre more then twice its breadth. Find the length and the breadth of the plot.

$l = x$

$b = y$

$l = 1 + 2b$

$x = 1 + 2y$

$A = 528$

$xy = 528$

$(1+2y)y = 528$

Speed of boat / your speed = x km/h

Speed of stream / current = y km/h

Upstream → water key against. $x-y$

Downstream → water key sath. $x+y$

QUESTION [CBSE 2017]

The speed of a boat in still water is 15 km/hr. It can go 30 km upstream and return downstream to the original point in 4 hours 30 minutes. Find the speed of the stream.

Speed of boat = 15 km/hr

Let, speed of stream = x km/hr

$S_{up.} = 15 - x$

$S_{do.} = 15 + x$

$= 4h\ 30min$

$= 4h + 30min$

$= 4h + \frac{30}{60}h$

$= 4 + \frac{1}{2}$

$= \frac{9}{2}h$

$\underline{\underline{\text{Up}}}$

S	$15-x$	$15+x$
D	30	30
T	t_1	t_2

$\underline{\underline{\text{Down}}}$

$T = \frac{D}{S}$

$$\boxed{t_1 = \frac{30}{15-x}}$$

$$\boxed{t_2 = \frac{30}{15+x}}$$

$$\boxed{t_1 + t_2 = \frac{9}{2}}$$

$$\frac{30}{15-x} + \frac{30}{15+x} = \frac{9}{2}$$

QUESTION

The product of Ramu's age (in years) five years ago with his age (in years) 9 years later is 15. Find Ramu's present age.

QUESTION [CBSE 2006C]

Seven years ago Varun's age was five times the square of Swati's age. Three years hence Swati's age will be two fifth of Varun's age. Find their present ages.

QUESTION

The hypotenuse of right-angled triangle is 6 metres more than twice the shortest side. If the third side is 2 metres less than the hypotenuse, find the sides of the triangle.

QUESTION [NCERT, CBSE 2008, 2013]

Sum of the areas of two squares is 640 m^2. If the difference of their perimeters is 64 m, find the sides of the two squares.

QUESTION

If $1 + \sqrt{2}$ is a root of a quadratic equation with rational coefficients, write its other root.

$ax^2 + bx + c$

Rational no.

Conjugate

$\alpha = 1 + \sqrt{2}$

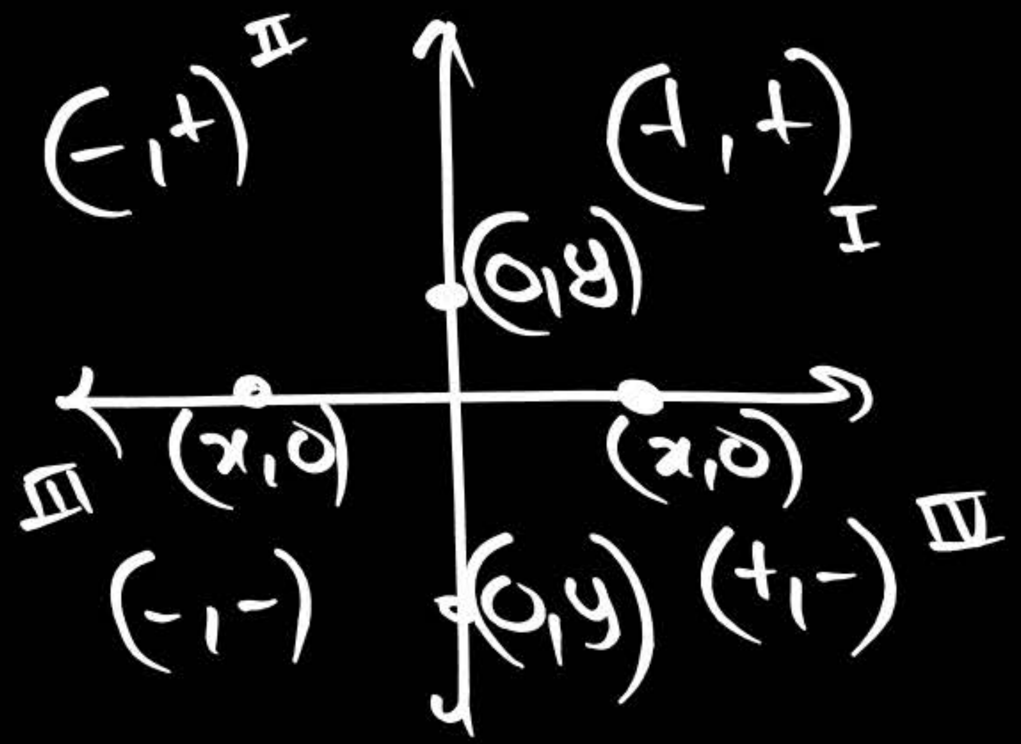
$\beta = 1 - \sqrt{2}$

QUESTION [NCERT, CBSE 2008, 14, 19]

In a class test, the sum of Shefali's marks in Mathematics and English is 30. Had she got 2 marks more in Mathematics and 3 marks less in English, the product of her marks would have been 210. Find her marks in two subjects.

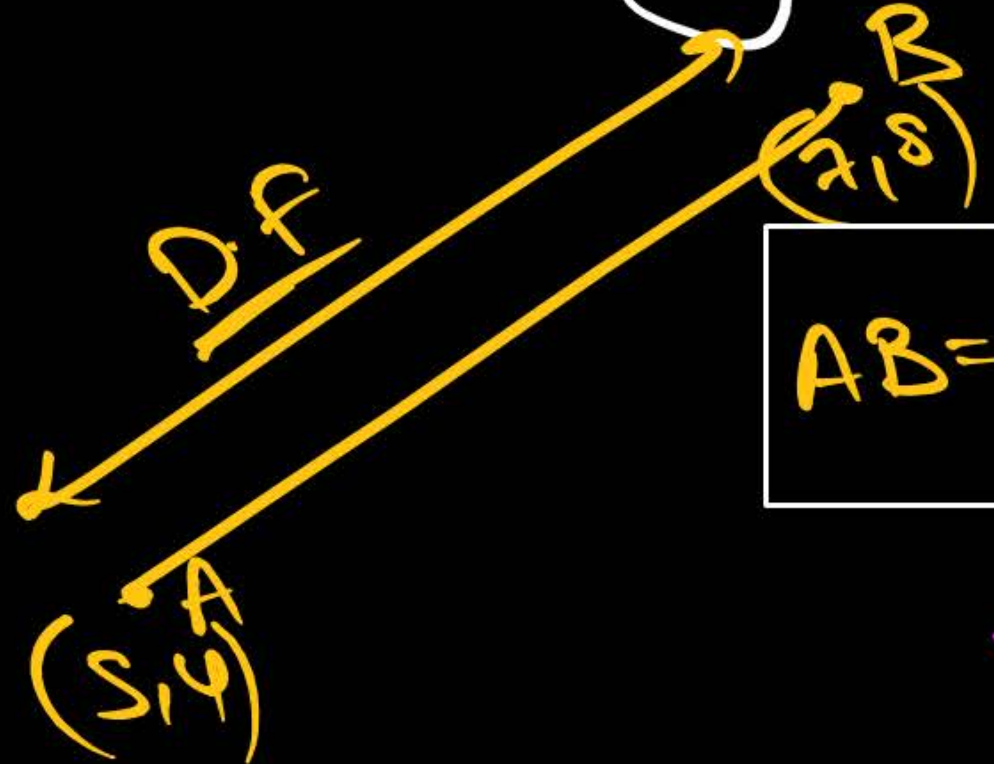
H.W

Coordinate geometry



(x, y)

absassa ordinate.



$$\boxed{AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}}$$

$$= \sqrt{(5-7)^2 + (4-8)^2}$$

$$= \sqrt{4 + 16}$$

$$= \sqrt{20}$$

$$= 2\sqrt{5} \text{ units}$$

Q $P(-4,-5)$ $\quad\quad\quad$ $Q(0,5)$

$$PQ = \sqrt{(-4-0)^2 + (-5-5)^2}$$

$$= \sqrt{16+100}$$

$$= \sqrt{116} \text{ units}$$

$$PQ = \sqrt{(0-(-4))^2 + (5-(-5))^2}$$

$$= \sqrt{16+100}$$

$$= \sqrt{116} \text{ units}$$

QUESTION

Find the value of x, if the distance between the points (x, –1) and (3, 2) is 5

$(x, -1)$ P ———— S ———— Q $(3, 2)$

Formula:

$$PQ = \sqrt{(3-x)^2 + (2 - -1)^2}$$

$$S = \sqrt{9 + x^2 - 6x + 9}$$

S.B.S

$$25 = x^2 - 6x + 18$$

$$\boxed{0 = x^2 - 6x - 7}$$

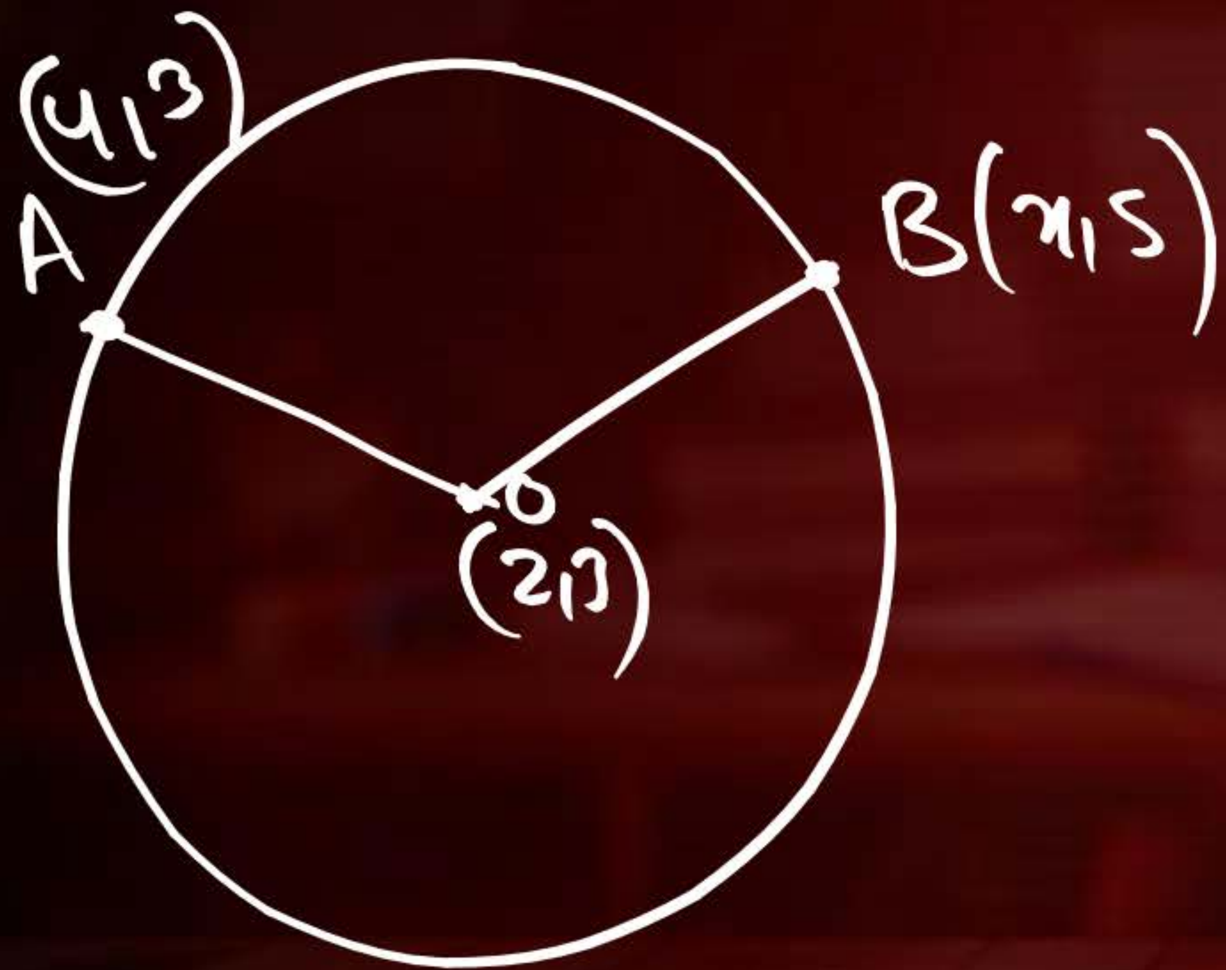
$P = -7,\ S = -6$

$-7, 1$

$x = 7, -1$

QUESTION [CBSE 2009]

If the points A (4, 3) and B (x, 5) are on the circle with centre O(2, 3), find the value of x.



$OA = OB$

$$\sqrt{(2-4)^2+(3-3)^2} = \sqrt{(2-x)^2+(3-5)^2}$$

S.B.S

$$\cancel{4} = 4 + x^2 - 4x + \cancel{4}$$

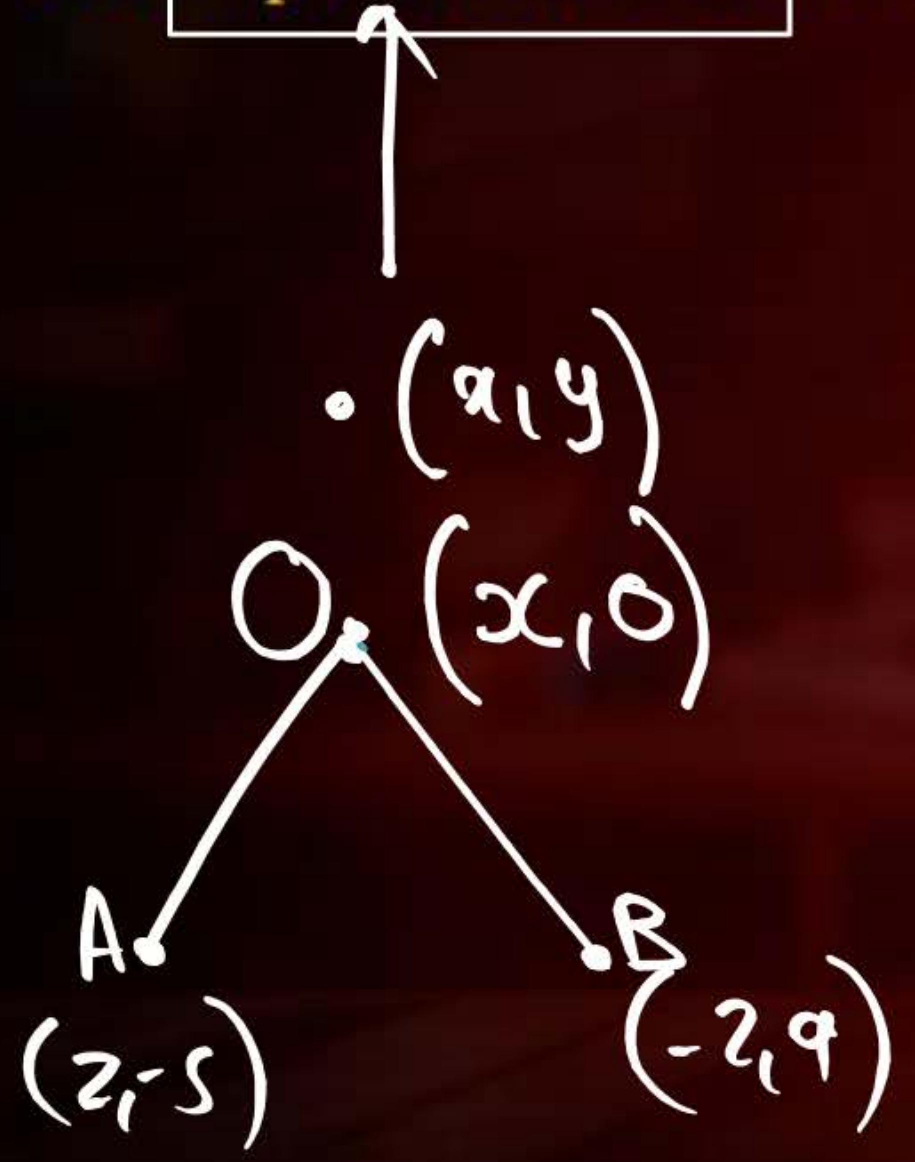
$$\boxed{0 = x^2 - 4x + 4}$$

$x = 2, 2$

QUESTION [NCERT, CBSE 2009, 2017]

Find a point on x-axis which is equidistant from A (2, –5) and B (–2, 9).

equal distance



$$OA = OB$$

$$\boxed{OA^2 = OB^2}$$

$$(x-2)^2 + (0 - -5)^2 = (x - -2)^2 + (0 - 9)^2$$

$$\cancel{x^2} + \cancel{4} - 4x + 25 = \cancel{x^2} + 4x + \cancel{4} + 81$$

$$-4x + 25 = 4x + 81$$

$$-56 = 8x$$

$$-7 = x$$

Ans $(-7, 0)$

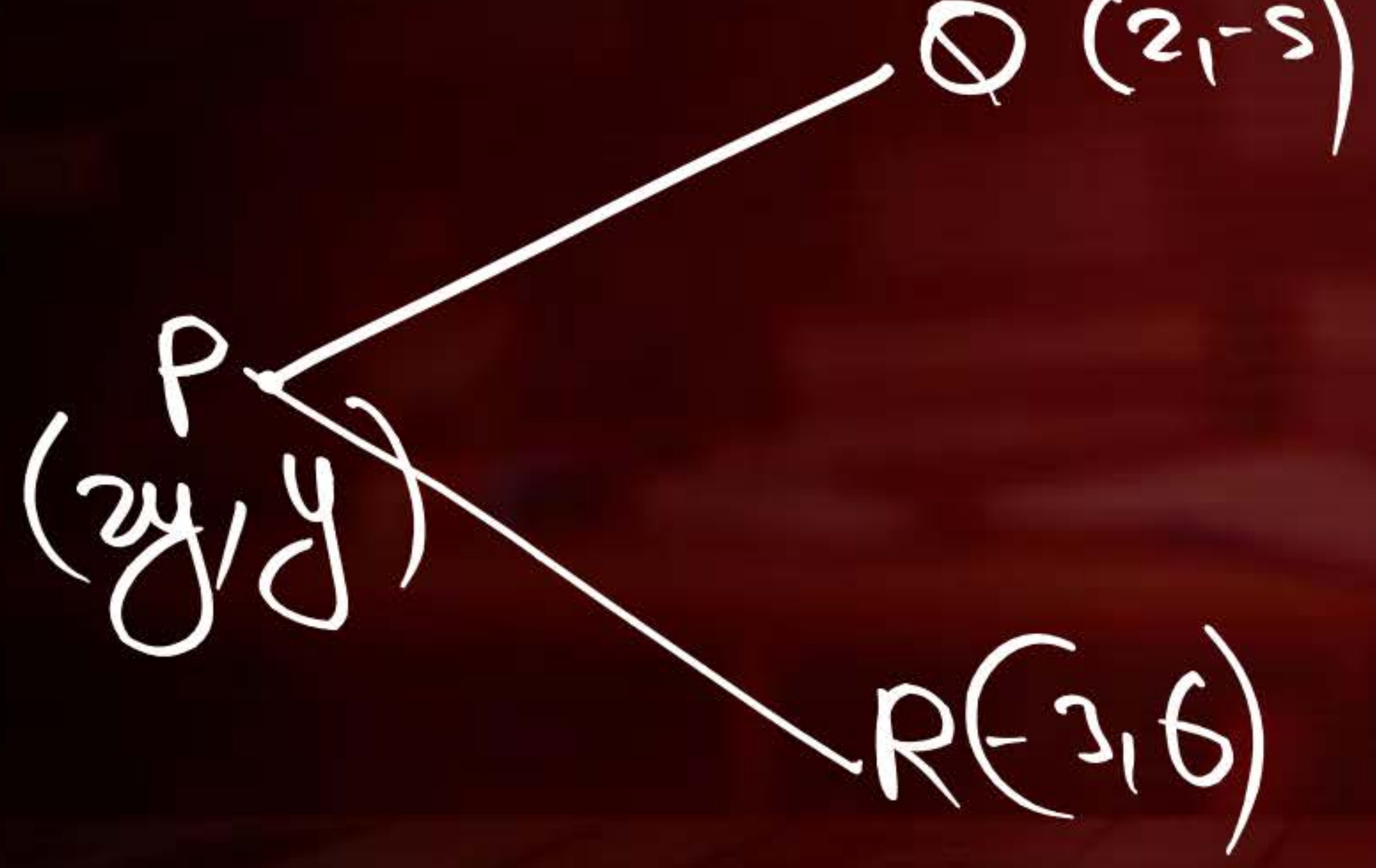
QUESTION [NCERT, CBSE 2017]

Find a point on the y-axis which is equidistant from the point A (6, 5) and B(−4, 3).

H.W

QUESTION [CBSE 2010, 2016]

The x-coordinate of a point P is twice its y-coordinate. If P is equidistant from Q (2, –5) and R (–3, 6), then find the coordinates of P.



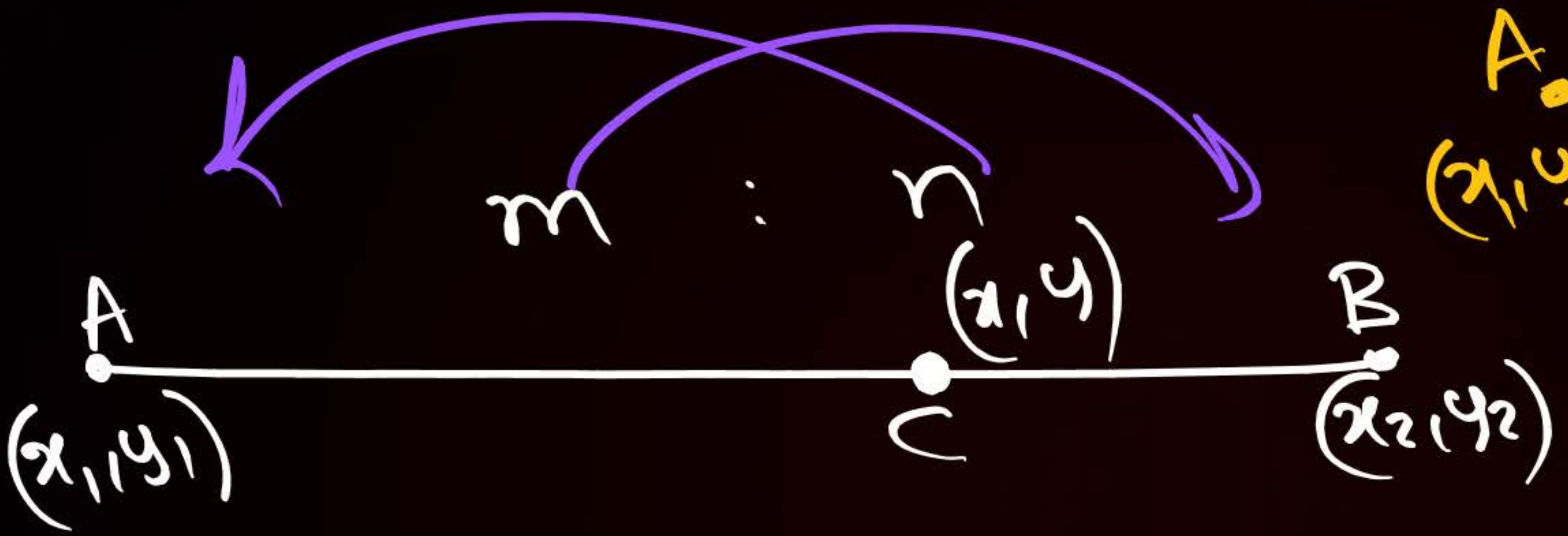
$$PQ = PR$$

$$PQ^2 = PR^2$$

$$(2y-2)^2 + (y+5)^2 = (2y+3)^2 + (y-6)^2$$

$$4y^2 + 4 - 8y + y^2 + 25 + 10y = 4y^2 + 9 + 12y + y^2 + 36 - 12y$$

Section Formulae



$$x = \frac{mx_2 + nx_1}{m+n}$$

$$y = \frac{my_2 + ny_1}{m+n}$$

Mid Point Formula

$$x = \frac{x_2 + x_1}{2}$$

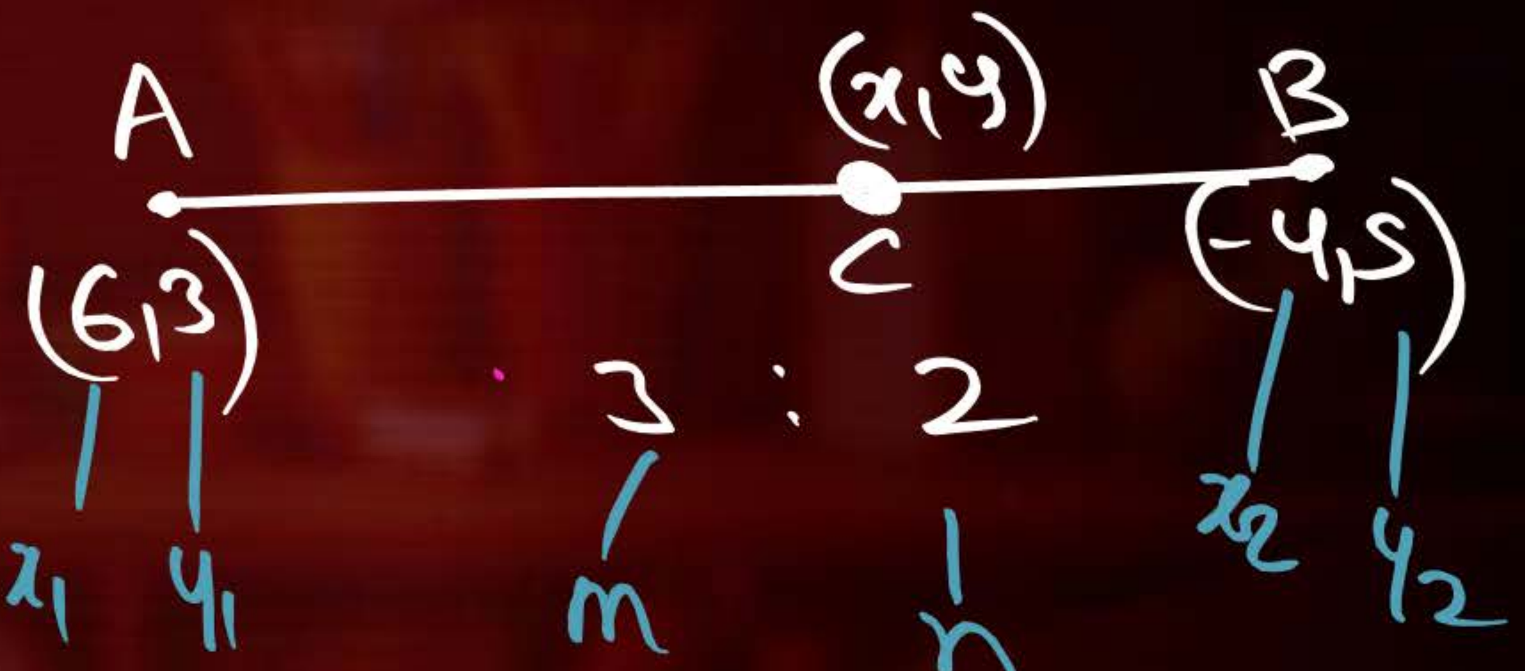
$$y = \frac{y_2 + y_1}{2}$$

QUESTION

diagram Banao

Find the coordinates of the point which divides the line segment joining the points (6, 3) and (–4, 5) in the ratio 3 : 2 internally.

$$x = \frac{mx_2 + nx_1}{m+n} \qquad y = \frac{my_2 + ny_1}{m+n}$$



$$x = \frac{-12+12}{5} \qquad y = \frac{15+6}{5}$$

$$x = 0 \qquad y = 21/5$$

Ans: $(0, 21/5)$

QUESTION [CBSE 2017]

Find the coordinates of points which trisect the line segment joining (1, –2) and (–3, 4).

trisect → 3 equal parts

A (1, -2) —— P —— B (-3, 4)

1 : 2

$P_x = \frac{-3+2}{3} = \boxed{-\frac{1}{3}}$

$P_y = \frac{4+-4}{3} = \boxed{0}$

A (1, -2) —— P —— Q —— B (-3, 4)

1 : 1 : 1

A (1, -2) —— Q —— B (-3, 4)

2 : 1

$Q_x = \frac{-6+1}{3} = \boxed{\frac{-5}{3}}$

$Q_y = \frac{8-2}{3} = \boxed{2}$

QUESTION [CBSE 2019]

In what ratio does the x-axis divide the line segment joining the points (2, –3) & (5, 6)?

[redacted]

let, the ratio be $k:1$

$$y = \frac{my_2 + ny_1}{m+n}$$

$$0 = \frac{6k-3}{k+1}$$

$$0 = 6k - 3$$

$$3 = 6k$$

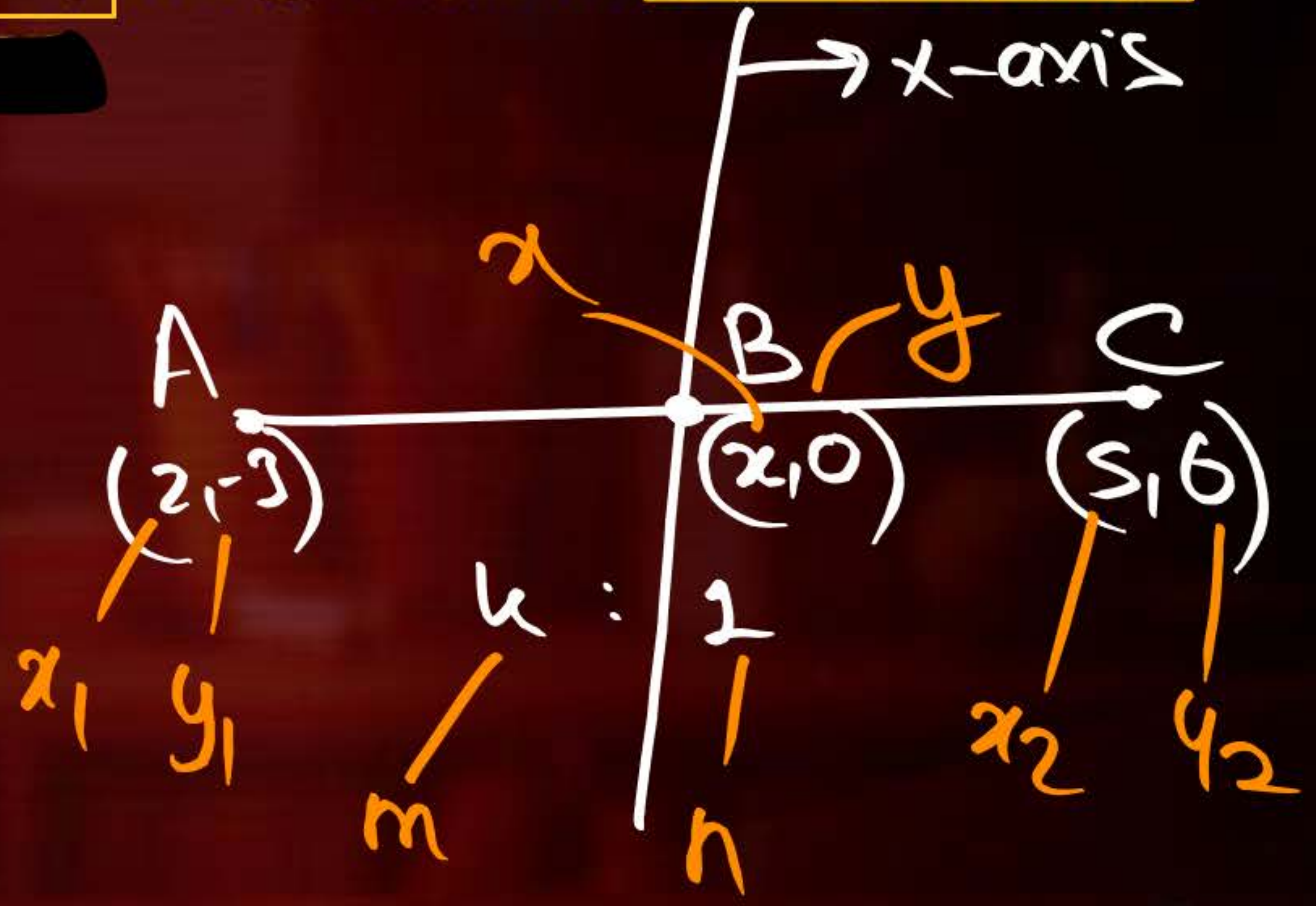
$$\frac{1}{2} = k$$

$\frac{1}{2} : 1$

Ans $1:2$

$$x = \frac{mx_2 + nx_1}{m+n}$$

$$x = \frac{5k+2}{k+1}$$



QUESTION [CBSE 2010]

Show that A(6, 4), B(5, 2) and C(7, –2) are the vertices of an isosceles triangle. Also, find the length of the median through A.

H.W

QUESTION [CBSE 2020]

If the point C (–1, 2) divides internally the line segment joining A (2, 5) and B in ratio 3 : 4, find the coordinates of B.

QUESTION [CBSE 2016]

Find the ratio in which the point $(-3, p)$ divides the line segment joining the points $(-5, -4)$ and $(-2, 3)$. Hence, find the value of p.

QUESTION [NCERT EXEMPLAR, CBSE 2016]

The lines segment joining the points A(3, 2) and B(5, 1) is divided at the point P in the ratio 1: 2 and it lies on the line 3x – 18y + k = 0 Find the value of k.

$x = \frac{5+6}{3}$ $y = \frac{1+4}{3}$

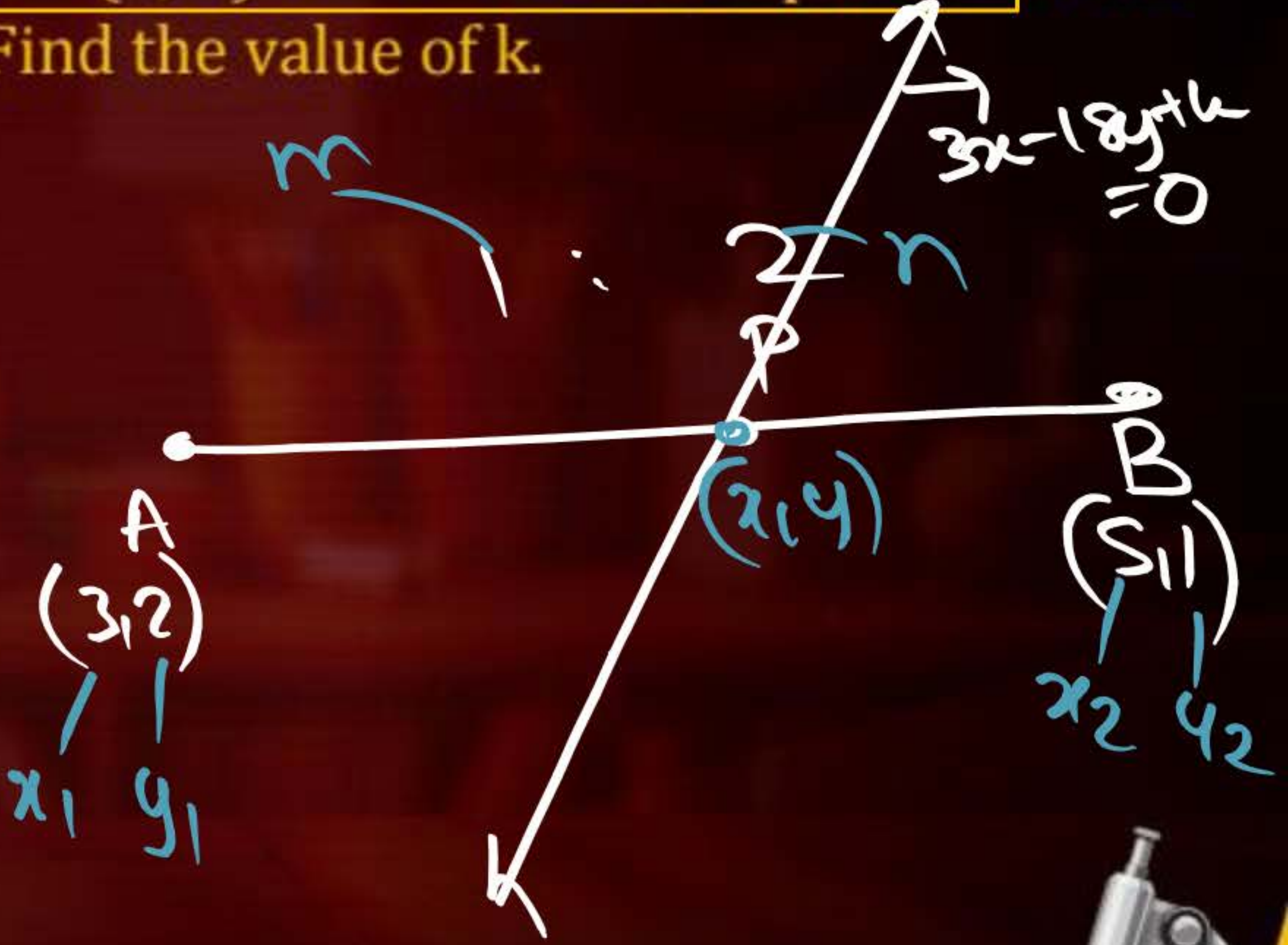
$\boxed{x = \frac{11}{3}}$ $\boxed{y = \frac{5}{3}}$

$3x - 18y + k = 0$

$\cancel{3}\left(\frac{11}{\cancel{3}}\right) - \cancel{18}^{6}\left(\frac{5}{\cancel{3}}\right) + k = 0$

$-19 + k = 0$

$k = 19$



QUESTION [CBSE 2008]

Determine the ratio in which the line $3x + y - 9 = 0$ divides the segment joining the points (1, 3) and (2, 7).

$$\boxed{x = \frac{2k+1}{k+1}}, \quad \boxed{y = \frac{7k+3}{k+1}}$$

$$3x + y - 9 = 0$$

$$3\left(\frac{2k+1}{k+1}\right) + \left(\frac{7k+3}{k+1}\right) = 9$$

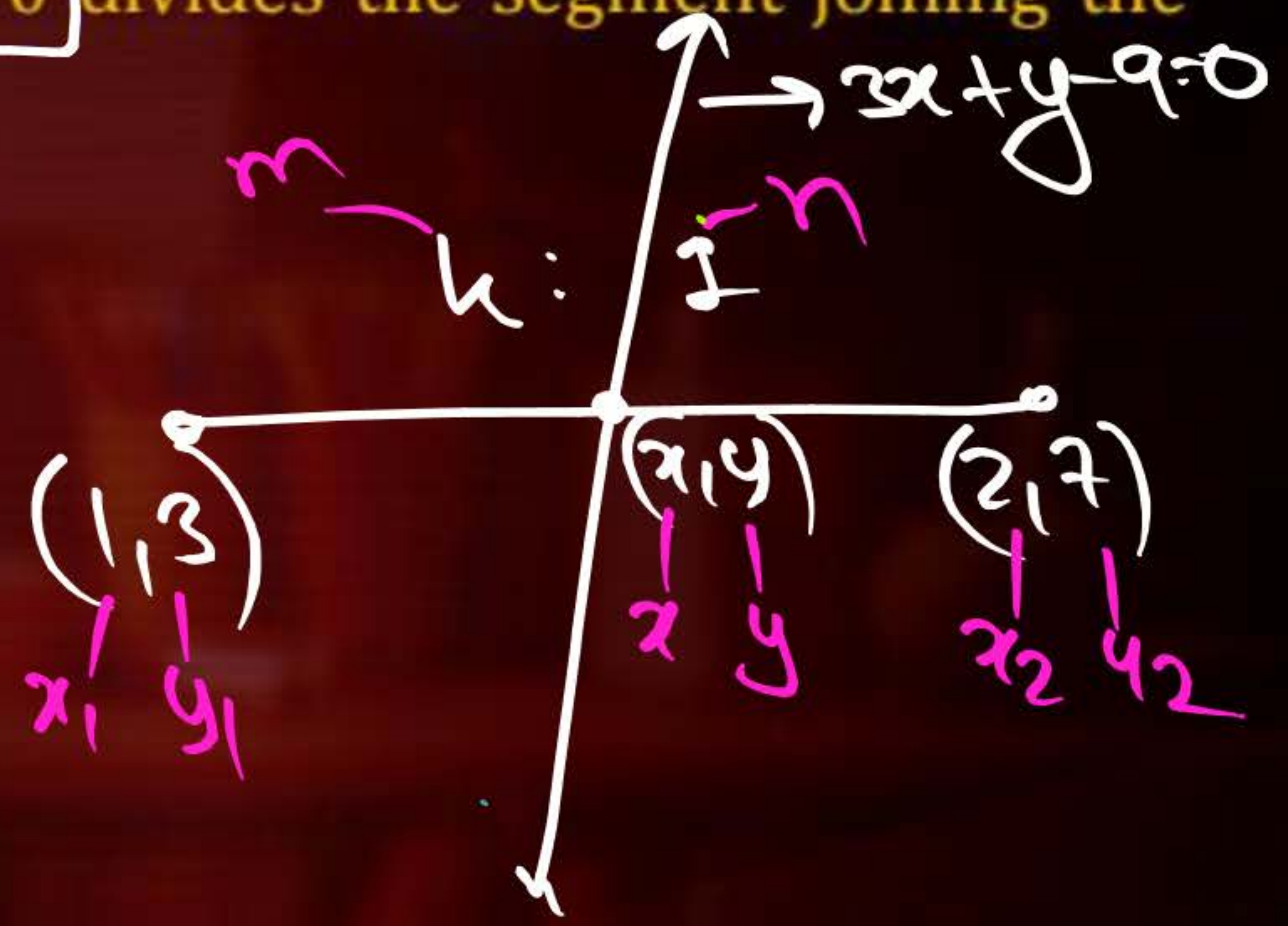
$$\frac{6k+3+7k+3}{k+1} = 9$$

$$13k + 6 = 9k + 9$$

$$4k = 3$$

$$\boxed{k = 3/4}$$

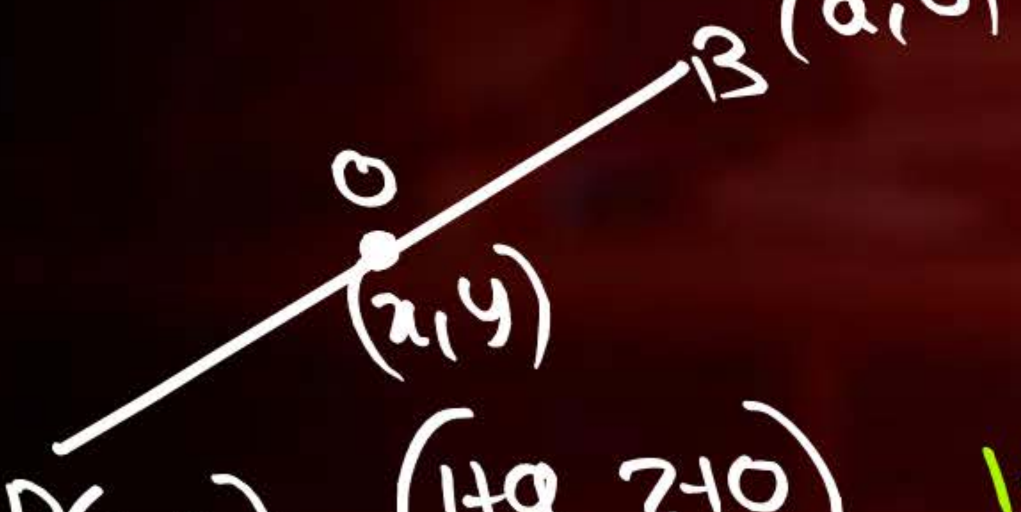
Ans: 3 : 4



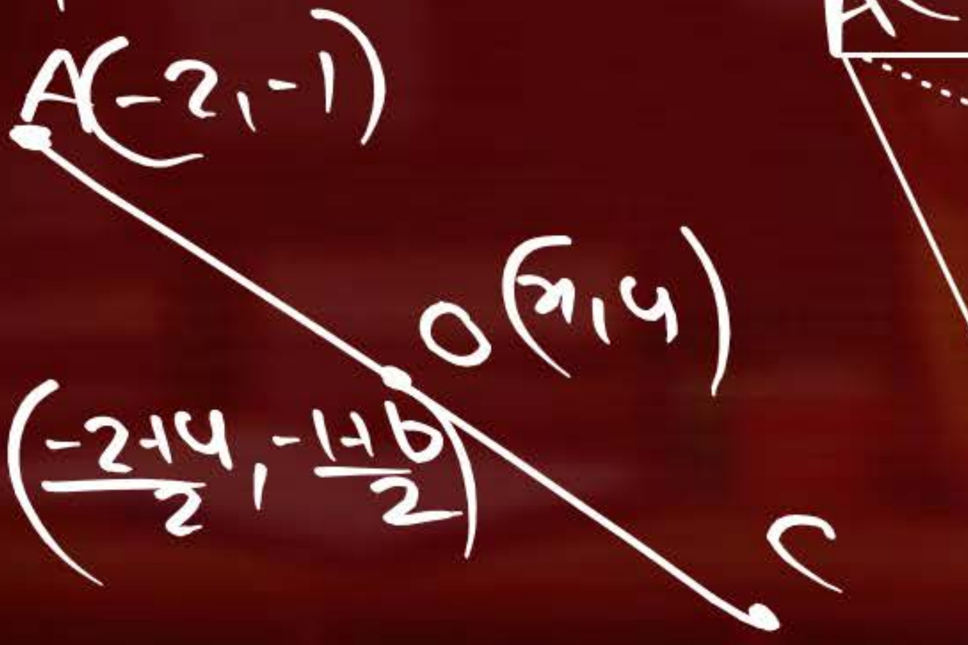
QUESTION [CBSE 2006 C, 2018]

If A(−2, −1), B(a, 0), C(4, b) and D(1, 2) are the vertices of a parallelogram, find the values of a and b.

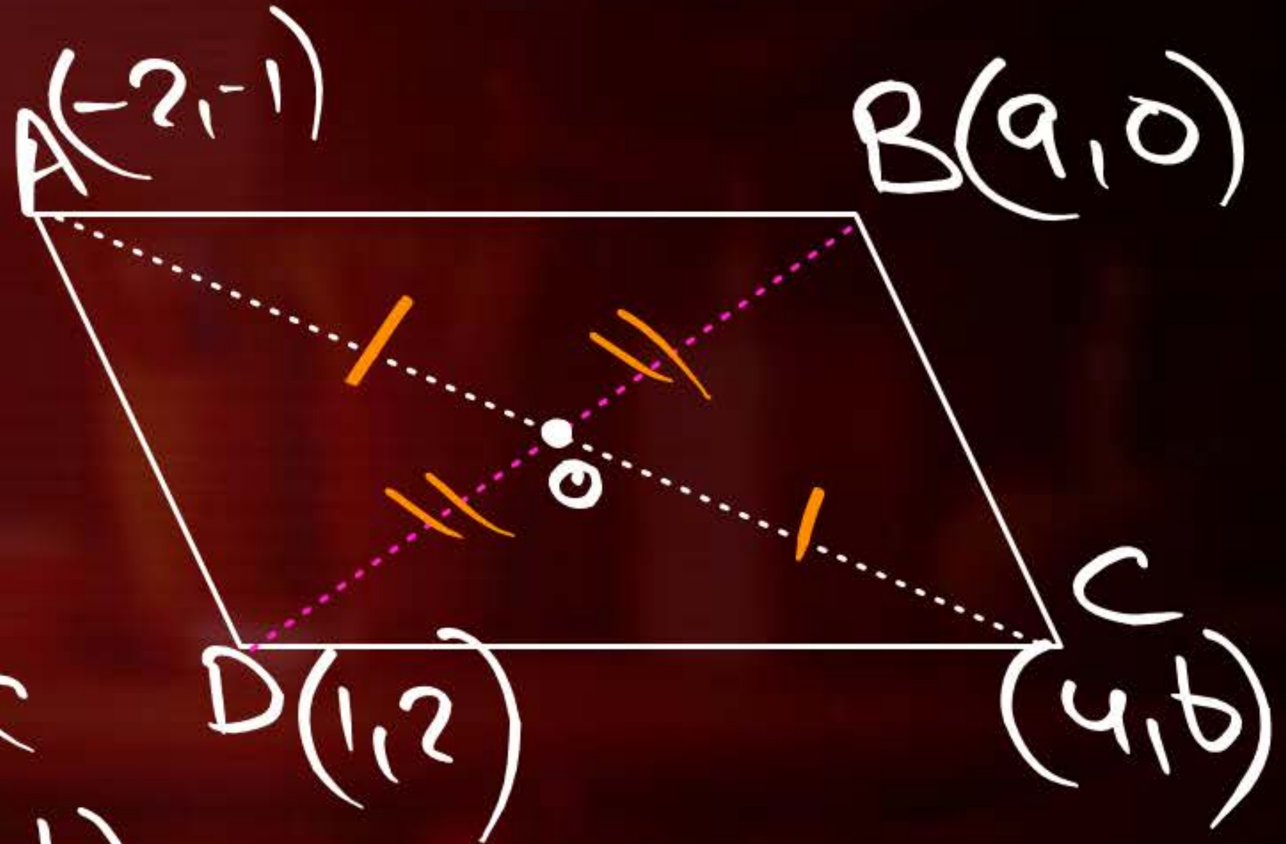
diagonals of a llgm bisect each other.



$\left(\frac{1+a}{2}, \frac{2+0}{2}\right)$



$\left(\frac{-2+4}{2}, \frac{-1+b}{2}\right)$



$\frac{1+a}{\cancel{2}} = \frac{-2+4}{\cancel{2}}$

$a = 1$

$\frac{2+0}{\cancel{2}} = \frac{-1+b}{\cancel{2}}$

$3 = b$

QUESTION [NCERT, CBSE 2008, 2009]

If A and B are two points having coordinates (–2, –2) and (2, 4) respectively, find the coordinates of P such that $AP = \frac{3}{7} AB$.

H.W

A.P

Sequence

1, 4, 9, 16, 25 . . . , 36 . . . . (natural no. ka square)

Sequence hai

$$2 \xrightarrow{+3} 5 \xrightarrow{+3} 8 \xrightarrow{+3} 11 \xrightarrow{+3} 14 \xrightarrow{+3} 17 \xrightarrow{+3} 20 \; . . . . .$$

$a_7 = 7^{th}$ term (7^{th} position par jo term hai)

$$\underset{a=a_1}{5}, \underset{a_2}{15}, \underset{a_3}{25}, \underset{a_4}{35}, \underset{a_5}{45} \; . . . . .$$

$a_7 = 65$

$a_{100} = 100^{th}$ term

$a_n = n^{th}$ term (general term)

n^{th} position ka term hai

koi bhi term nikal sakte hain

$\overset{+2}{\curvearrowright}$ $\overset{+2}{\curvearrowright}$

2, 4, 6, 8, 10 . . . .

$a = 2$
$d = 2$

First term = a

common difference = d   2

a, a+d, a+d+d, a+d+d+d . . . .

a, a+1d, a+2d, a+3d, a+4d, a+5d . . .

a_1 a_2 a_3 a_4 a_5 a_6

$a_7 = a + 6d$

$a_8 = a + 7d$

$a_{10} = a + 9d$

$\vdots$

$a_{100} = a + 99d$

$a_n = a + (n-1)d$

QUESTION

The next term of the A.P. $\sqrt{18}, \sqrt{50}, \sqrt{98}, \ldots$ is a_4

(A) $\sqrt{146}$

(B) $\sqrt{128}$

(C) $\sqrt{162}$

(D) $\sqrt{200}$

$$a = \sqrt{18} = \sqrt{3\times3\times2}$$

$$\boxed{a = 3\sqrt{2}}$$

$$d = \sqrt{50} - \sqrt{18}$$

$$= \sqrt{5\times5\times2} - 3\sqrt{2}$$

$$= 5\sqrt{2} - 3\sqrt{2}$$

$$\boxed{d = 2\sqrt{2}}$$

$$a_4 = a + 3d$$

$$= 3\sqrt{2} + 3(2\sqrt{2})$$

$$= 3\sqrt{2} + 6\sqrt{2}$$

$$= 9\sqrt{2}$$

$$= \sqrt{9\times9\times2}$$

$$= \sqrt{162}$$

QUESTION

The $\boxed{12^{th} \text{ term of an AP}}$ whose first $\boxed{\text{two terms are } -3 \text{ and } 4}$ is

(A) 67

(B) 74

(C) 60

(D) 81

$a_{12} = a + 11d$

$= -3 + 11(7)$

$= -3 + 77$

$= 74$

$-3, 4 \ldots$

$\boxed{a = -3}$

$d = a_2 - a_1$

$d = 4 - (-3)$

$\boxed{d = 7}$

QUESTION

If the common difference of an AP is 10, then $a_{25} - a_{21}$ is equal to

$d = 10$

(A) 14

(B) 20

(C) 28

(D) 35

$$= a + 24d - (a + 20d)$$
$$= a + 24d - a - 20d$$
$$= 4d$$
$$= 4(10)$$
$$= 40$$

QUESTION

If p −1, p + 1 and 2p + 3 are in A.P., then the value of p is.

(A) −2

(B) 4

(C) 0

(D) 2

$$p-1,\ p+1,\ 2p+3 \ldots$$

$a_1 \quad a_2 \quad a_3$

$$a_2 - a_1 = a_3 - a_2$$

$$(p+1)-(p-1) = (2p+3)-(p+1)$$

$$\cancel{p}+1-\cancel{p}+1 = 2p+3-p-1$$

$$2 = p+2$$

QUESTION

If the n^{th} term of an AP is $3n + 7$, then its common difference is

[CBSE 2023]

$$a_n = 3n+7$$

(A) 7

(B) 3

(C) 3n

(D) 1

$$a_1 = 3(1)+7 = 10$$
$$a_2 = 3(2)+7 = 13$$
$$a_3 = 3(3)+7 = 16$$

19

22

QUESTION

The 10^{th} term of an A.P. is 52 and 16^{th} term is 82. Find the 32^{nd} term and the general term.

$a_{10} = 52$, $a_{16} = 82$

$a + 9d = 52$, $a + 15d = 82$

$\cancel{a} + 9d = 52$

$\cancel{a} + 15d = 82$

$- \quad - \quad -$

$-6d = -30$

$d = 5$

$a + 9d = 52$

$a + 45 = 52$

$a = 7$

To Find: $a_{32} = a + 31d$

$a_n = a + (n-1)d$

$a_{32} = a + 31d$

$= 7 + 31(5)$

$\boxed{a_{32} = 162}$

$a_n = a + (n-1)d$

$= 7 + (n-1)5$

$= 7 + 5n - 5 = \boxed{2 + 5n}$

QUESTION

The sum of 5^{th} and 9^{th} terms of an A.P. is 72 and the sum of 7^{th} and 12^{th} terms is 97. Find the A.P.

[CBSE 2009]

$\boxed{a_5 + a_9 = 72}$ — (1)

$a + 4d + a + 8d = 72$

$2a + 12d = 72$ — (1)

$\boxed{a_7 + a_{12} = 97}$ — (2)

$a + 6d + a + 11d = 97$

$2a + 17d = 97$ — (2)

$a_1, a_2, a_3 \ldots$

$\boxed{a,\ a+d,\ a+2d,\ a+3d \ldots}$

end → A.P ulta kardo

QUESTION

The 7^{th} term from the end of the A.P. 7, 11, 15, ..., 107, is

(A) 79

(B) 83

(C) 81

(D) 87

7, 11, 15 .... 99, 103, 107

107, 103, 99 .... 15, 11, 7.

$a_7 = a + 6d$

$= 107 + 6(-4)$

$= 107 - 24$

$= 83$

QUESTION

If five times the fifth term of an A.P. is equal to 8 times its eighth term, show that its 13^{th} term is zero.

QUESTION

Determine the general term of an A.P. whose 7^{th} term is -1 and 16^{th} term 17.

QUESTION

How many numbers of two digits are divisible by 7 ?

10

$\boxed{14, 21, 28 \ldots\ldots\ldots\ldots 98}$

99

Ans 13

This is an A.P.

$a = 14$
$d = 7$

let, $a_n = 98$

$a + (n-1)d = 98$

$14 + (n-1)7 = 98$

$(n-1)7 = 98 - 14$

$(n-1)7 = 84$

$(n-1) = \frac{84}{7}$

$(n-1) = 12$

$n = 13$

$a_n = 98$

$a_{13} = 98$

Last term.

QUESTION

Find the number of integers between $\boxed{50 \text{ and } 500}$ which are divisible by 7.

50

500

56, 63, 70 . . . . 497.

$a = 56$

$d = 7$

let $a_n = 497$

QUESTION

Which term of the sequence –1, 3, 7, 11,... is 95 ?

Let, $a_n = 95$

$a+(n-1)d = 95$

$-1+(n-1)4 = 95$

$(n-1)4 = 96$

$n-1 = 24$

$n = 25$

$a_n = 95$

$a_{25} = 95$

Total terms = 25

QUESTION

Find the middle term of the A.P. 6, 13, 20, ..., 216.

[CBSE 2015]

let, $a_n = 216$

$a + (n-1)d = 216$

$6 + (n-1)7 = 216$

$(n-1)7 = 210$

$n - 1 = 30$

$n = 31$

$a_{31} = 216$

total terms = 31

$M.T = \left(\frac{n+1}{2}\right)^{th}$

$\boxed{M.T = 16^{th}}$

$a_{16} = a + 15d$

$= 6 + 15(7)$

$a_{16} = 111$ Ans.

odd → 1 middle term

even → 2 middle terms.

QUESTION

Find the middle term(s) of the A.P. 7, 13, 19,..., 241.

H.W

$n = 100$

$\left(\frac{n}{2}\right)^{th}, \left(\frac{n}{2}+1\right)^{th}$

$50^{th}, 51^{st}$

a_{50}, a_{51}

QUESTION

How many terms are there in the sequence 3, 6, 9, 12,...,111 ?

QUESTION

Which term of the arithmetic progression 5, 15, 25,... will be 130 more than its 31^{st} general term?

[CBSE 2006C, 2017]

QUESTION

Which term of the A.P. 65, 61, 57, 53 is the first negative term?

Let a_n be negative.

$\boxed{a_n < 0}$

$a + (n-1)d < 0$

$65 + (n-1) - 4 < 0$

$(n-1) - 4 < -65$

$(n-1) > \frac{\cancel{-}65}{\cancel{-}4}$

$n - 1 > \frac{65}{4}$

$n > \frac{65}{4} + 1$

$n > \frac{69}{4}$

$n > 17.25$

$n = 18$

a_{18} is your first negative term

$$\boxed{a_n = a + (n-1)d}$$

$$\boxed{a_p = a + (p-1)d}$$

p^{th} term

$$\boxed{a_m = a + (m-1)d}$$

m^{th} term

$$\boxed{a_{mn} = a + (mn-1)d}$$

mn^{th} term.

QUESTION

If the p^{th} term of an A.P. is q and the q^{th} term is p, prove that its n^{th} term is $(p + q - n)$.

[CBSE 2008, 2017, 2023]

$a_p = q \quad a_q = p$

$a + (p-1)d = q$ — (1)

$a + (q-1)d = p$ — (2)

$(p-1)d - (q-1)d = q - p$

$d[p \cancel{-1} - q \cancel{+1}] = q - p$

$d(p-q) = q-p$

$d = \frac{q-p}{p-q}$

$d = \frac{-1\cancel{(p-q)}}{\cancel{p-q}}$

$d = -1$

$a + (p-1)d = q$

$a + (p-1)(-1) = q$

$a - p + 1 = q$

$a = q + p - 1$

$a_n = a + (n-1)d$

$= q + p - 1 + (n-1)(-1)$

$= q + p \cancel{-1} - n \cancel{+1}$

$a_n = q + p - n$

$a_n = p + q - n$

$$S_n = \frac{n}{2}\left[2a+(n-1)d\right]$$

$$S_n = \frac{n}{2}\left[a+a_n\right]$$ l = last term.

Q $1+2+3+4+5 \ldots\ldots n.$
(sum of 'n' natural no's)

$$S_n = \frac{n}{2}\left[1+n\right]$$

$$\boxed{S_n = \frac{n(n+1)}{2}}$$

Q $1+2+3+4+5+6 \ldots\ldots\ldots\ldots +100.$
(sum of 100 natural no)

$a=1$
$d=1$

$$S_{100} = \frac{100}{2}\left[1+100\right]$$

$= 50 \times 101$

$= \boxed{5050}$

QUESTION

The sum of the first n odd natural numbers is

(A) $2n$

(B) $2n + 1$

(C) n^2

(D) $n^2 - 1$

QUESTION

The sum of first n even natural numbers is

(A) $2n$

(B) n^2

(C) $n^2 + n$

(D) $n^2 - 1$

QUESTION

If nth term of an A.P. is $(2n + 1)$, then the sum of first n terms of the A.P. is

(A) $n(n - 2)$

(B) $n(n + 2)$

(C) $n(n + 1)$

(D) $n(n - 1)$

$a_n = 2n+1$

$a_1 = 3$

$a_2 = 5$

$a_3 = 7$

$3, 5, 7, 9 \ldots\ldots$

$a = 3$
$d = 2$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$= \frac{n}{2}[6 + (n-1)2]$

$= \frac{n}{2}[6 + 2n - 2]$

$= \frac{n}{2}[4 + 2n] = 2n + n^2$

QUESTION

John wants to participate in a 200 m race. He can currently run that distance in 51 seconds and with each day of practice it takes him 2 seconds less. He wants to do it in 31 seconds.

Which of the following are in A.P. for the given situation?

Fig. 5.2

$51, 49, 47 \ldots\ldots 31.$

$a_1 \quad a_2$

(A) 51, 55, 59, ……

(B) – 51, – 53, – 55, ….

(C) 51, 49, 47, ….

(D) 51, 53, 55, ……

QUESTION

John wants to participate in a 200 m race. He can currently run that distance in 51 seconds and with each day of practice it takes him 2 seconds less. He wants to do it in 31 seconds.

The minimum number of days he needs to practice to achieve the goal is

Fig. 5.2

(A) 10

(B) 12

(C) 11

(D) 9

$51, 49, 47 \ldots\ldots 31$

$a=1$

$d=-2$

let,

$a_n = 31$

$a+(n-1)d = 31$

$(n-1)-2 = -20$

$(n-1) = 10$

$n = 11$

QUESTION

Rishi wants to buy a car and plans to take loan from a bank to buy the car. He pays his total loan of ₹1,180,000 by paying every month starting with the first instalment of ₹10,000. If he increases the instalment by ₹1000 every month answer the following:

(i) The amount paid by Rishi in 30th instalment, is

$\boxed{1,180,000}$

Ans: 930

10000, 11000, 12000 - - - - - -

a_1 a_2 a_3

(A) ₹ 39,000

(B) ₹ 35,000

(C) ₹ 37,000

(D) ₹ 36,000

QUESTION

Rishi wants to buy a car and plans to take loan from a bank to buy the car. He pays his total loan of ₹1,180,000 by paying every month starting with the first instalment of ₹10,000. If he increases the instalment by ₹1000 every month answer the following:

(ii) The amount paid by Rishi in 30 instalments, is

S_{30}

A. ₹ 370,000

B. ₹ 735,000

C. ₹ 753,000

D. ₹ 750,000

QUESTION

Rishi wants to buy a car and plans to take loan from a bank to buy the car. He pays his total loan of ₹1,180,000 by paying every month starting with the first instalment of ₹10,000. If he increases the instalment by ₹1000 every month answer the following:

(iii) After $\boxed{\text{paying } 30^{th} \text{ instalment}}$ the amount $\boxed{\text{still to be paid is}}$

$a_1 + a_2 + a_3 \cdots\cdots a_{30} + \cdots\cdots\cdots$

Total money - S_{30}

(A) ₹ 455,000

(B) ₹ 490,000

(C) ₹ 445,000

(D) ₹ 540,000

QUESTION

Rishi wants to buy a car and plans to take loan from a bank to buy the car. He pays his total loan of ₹1,180,000 by paying every month starting with the first instalment of ₹10,000. If he increases the instalment by ₹1000 every month answer the following:

(iv) If the loan is to be repaid in 40 instalments, then amount paid in the last instalment is

a_{40}

A) ₹ 49,000

B) ₹ 39,000

C) ₹ 59,000

D) ₹ 94,000

QUESTION

Rishi wants to buy a car and plans to take loan from a bank to buy the car. He pays his total loan of ₹1,180,000 by paying every month starting with the first instalment of ₹10,000. If he increases the instalment by ₹1000 every month answer the following:

(v) The ratio of the first instalment to the last instalment is

$\frac{a_1}{a_{40}}$

(A) 1 : 49

(B) 10 : 49

(C) 10 : 39

(D) 39 : 10

QUESTION

If S_n, the sum of first n terms of an A.P., is given by $S_n = 3n^2 + 5n$, then find its n^{th} term.

[CBSE 2009]

$S_n = 3n^2 + 5n$

$n = 1, \quad S_1 = 3(1)^2 + 5(1)$

$= 3 + 5$

$\boxed{S_1 = 8}$

$a_1 = a = 8$

$n = 2, \quad S_2 = 3(2)^2 + 5(2)$

$= 12 + 10$

$\boxed{S_2 = x}$

$\cancel{a_2 = 22}$

$a_1 + a_2 = 22$

$8 + a_2 = 22$

$a_2 = 14$

$d = a_2 - a_1$

$= 14 - 8$

$d = 6$

$a_n = a + (n-1)d$

$a_n = 8 + (n-1)6$

$= 8 + 6n - 6$

$a_n = 2 + 6n$

QUESTION

How many terms of the series 54, 51, 48,... be taken so that their sum is 513? Explain the double answer.

[CBSE 2005]

let, $\boxed{S_n = 513}$

$\frac{n}{2}[2a+(n-1)d] = 513$

$n[2(54)+(n-1)-3] = 513 \times 2$

$n[108-3n+3] = 1026$

$n(111-3n) = 1026$

$111n - 3n^2 = 1026$

$0 = 3n^2 - 111n + 1026$

$0 = 3(n^2 - 37n + 342)$

$\boxed{n^2 - 37n + 342 = 0}$

$P = 342, S = -37$

$n^2 - 37n + 342 = 0$

$P = 342, \quad S = -37$

$-19, -18$

$n^2 - 37n + 342 = 0$

$n^2 - 19n - 18n + 342 = 0$

$n(n-19) - 18(n-19) = 0$

$(n-19)(n-18) = 0$

$n = 19, n = 18$

$S_n = S_{13}$

$S_{18} = S_{13}$

$S_{19} = S_{13}$

$a_1 + a_2 + a_3 \cdots\cdots + a_{18} = S_{13}$

$\underbrace{a_1 + a_2 + a_3 + \cdots\cdots a_{18}}_{S_{13}} + \underbrace{a_{19}}_{0} = S_{13}$

$a_{19} = a + 18d$

$= 54 + 18(-3)$

$a_{19} = 0$

QUESTION

Solve the equation: $\boxed{1 + 4 + 7 + 10 + \cdots + x = 287}$ [NCERT EXEMPLAR]

a_n

HW

$a=1, d=3$

let $S_n = 287$

$a_n = x$

$S_n = \frac{n}{2}[a + a_n]$

$287 = \frac{n}{2}[1 + x]$

$574 = n(1+x)$

$574 = n(1+3n-2)$

$= n(3n-1)$

$\boxed{0 = 3n^2 - n - 574}$

$a_n = x$

$a+(n-1)d = x$

$1+(n-1)3 = x$

$1+3n-3 = x$

$3n-2 = x$

Ex:

$S_{40} = 287$

$a_{40} = x$

QUESTION

Find the sum of all natural numbers between 250 and 1000 which are exactly divisible by 3 .

H.W

QUESTION

If the m^{th} term of an A.P. is $\frac{1}{n}$ and the n^{th} term is $\frac{1}{m}$, show that the sum of mn terms is $\frac{1}{2}(mn + 1)$.

[CBSE 2015, 2017]

QUESTION

If the ratio of the sums of first n terms of two A.P.s is $\frac{5n+3}{7n+27}$, then the ratio of their 4^{th} terms is -

I: a, d, a_n, S_n, n

II: a', d', a_n', S_n', n

$$\boxed{\frac{S_n}{S_n'} = \frac{5n+3}{7n+27}}$$

$$\frac{\frac{n}{2}\left[2a+(n-1)d\right]}{\frac{n}{2}\left[2a'+(n-1)d'\right]} = \frac{5n+3}{7n+27}$$

$$\boxed{\frac{2a+(n-1)d}{2a'+(n-1)d'} = \frac{5n+3}{7n+27}}$$

$$\frac{a_4}{a_4'} = \frac{a+3d}{a'+3d'}$$

$n=7$

$$\frac{2a+6d}{2a'+6d'} = \frac{5(7)+3}{7(7)+27}$$

$$\frac{a+3d}{a'+3d'} = \frac{38}{76}$$

$$\frac{a_4}{a_4'} = \frac{38}{76} = \frac{1}{2}$$ Ans//

$\boxed{\text{Triangles.}}$

Khana kha lo.....

QUESTION

Ms. Sheel visited a store near her house and found that the glass jars are arranged one above the other in a specific pattern.

QUESTION

On the top layer there are 3 jars. In the next layer there are 6 jars. In the 3^{rd} layer from the top there are 9 jars and so on till the 8^{th} layer.

On the basis of the above situation answer the following questions.

(i) Write an A.P whose terms represent the number of jars in different layers starting from top. Also, find the common difference.

QUESTION

On the top layer there are 3 jars. In the next layer there are 6 jars. In the 3^{rd} layer from the top there are 9 jars and so on till the 8^{th} layer.

On the basis of the above situation answer the following questions.

(ii) Is it possible to arrange 34 jars in a layer if this pattern is continued? Justify your answer.

CONCEPT OF SIMILARITY

In earlier classes, we have learnt about congruent figures. Two geometric figures having the same shape and size are known as congruent figures. Note that congruent figures are alike in every respect. In this chapter, we shall study about similarity of geometric figures. Geometric figures having the same shape but different sizes are known as similar figures. Two congruent figures are always similar but similar figures need not be congruent as discussed in the following illustrations.

Congruent → shape, size] same.

Similar → shape → same
size → alg-alg bhi ho sakte hai.

ILLUSTRATION 1

Any two line segments are always similar but they need not be congruent. They are congruent, if their lengths are equal.

Similar line segments

ILLUSTRATION 2

Any two circles are similar but not necessarily congruent. They are congruent if their radii are equal.

Similar circles

ILLUSTRATION 3

(i) Any two squares are similar

Similar Squares

ILLUSTRATION 3

(ii) Any two equilateral triangles are similar.

Similar Squares

DEFINITION

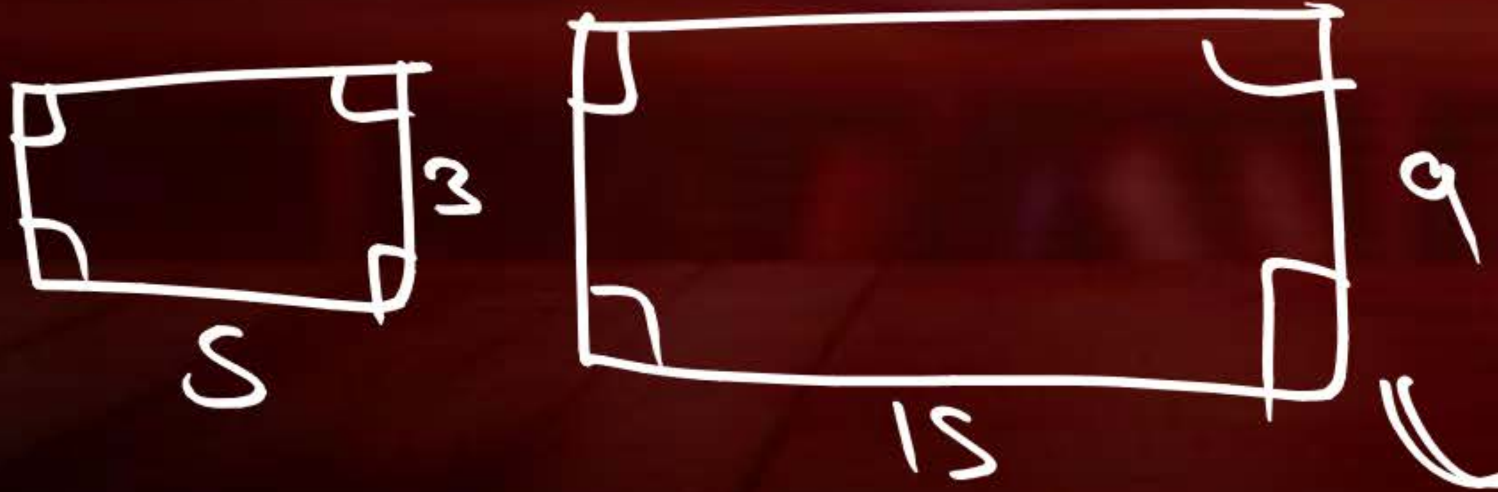
other than triangles

Two polygons are said to be similar to each other, if

(i) Their corresponding angles are equal, and

(ii) The lengths of their corresponding sides are proportional.

$\frac{1}{3} = \frac{1}{3}$



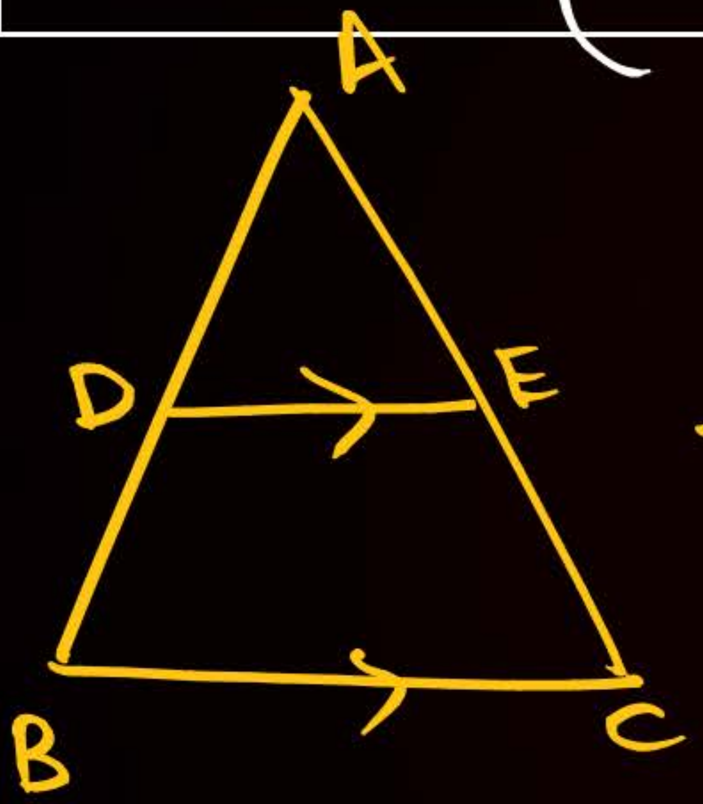
QUESTION (NCERT EXAMPLAR)

Is the following statement true? Why?

"Two quadrilaterals are similar, if their corresponding angles are equal."

False!

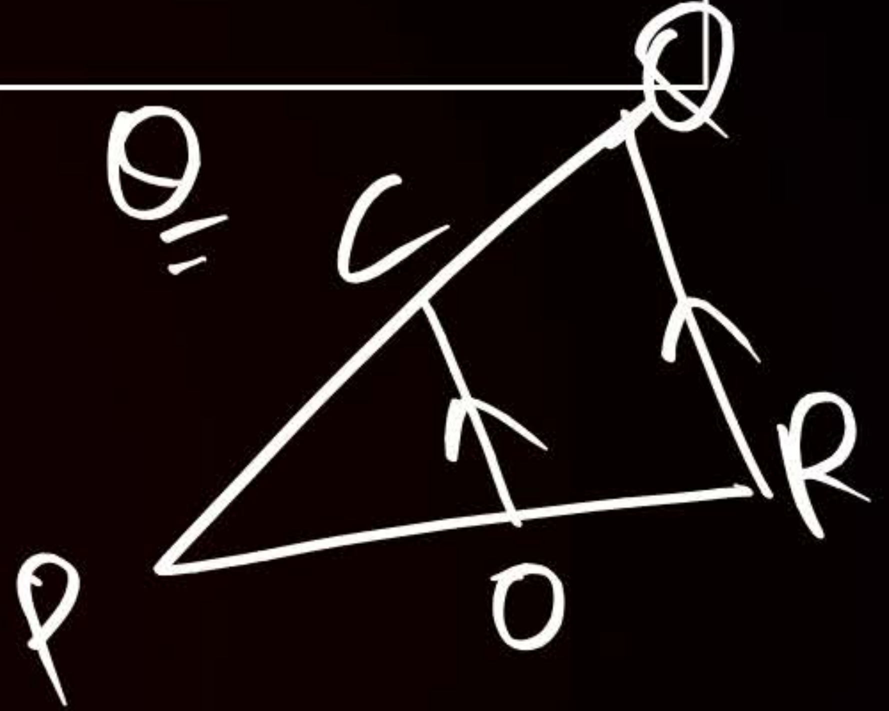
B.P.T (Basic Proportionality Theorem)
(Thale's theorem)



If, $DE \parallel BC$

then, $\dfrac{AD}{DB} = \dfrac{AE}{EC}$

Q:

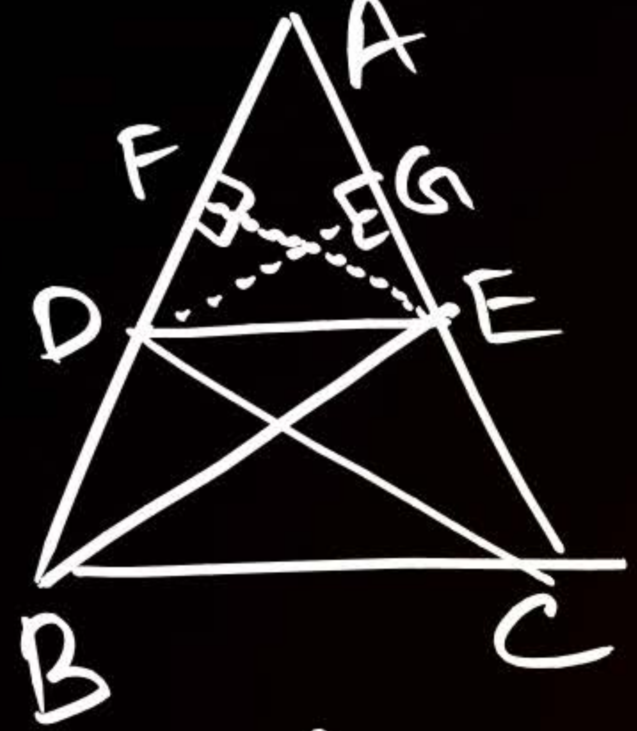


By B.P.T ...

$\dfrac{OP}{OR} = \dfrac{CP}{CQ}$

THEOREM 1

(Basic Proportionality Theorem (BPT) or Thales Theorem) If a line is drawn parallel to one side of a triangle intersecting the other two sides, then it divides the two sides in the same ratio.

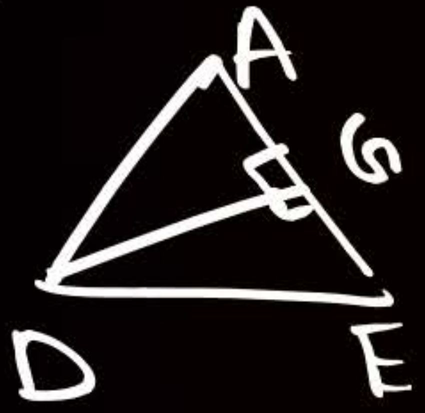


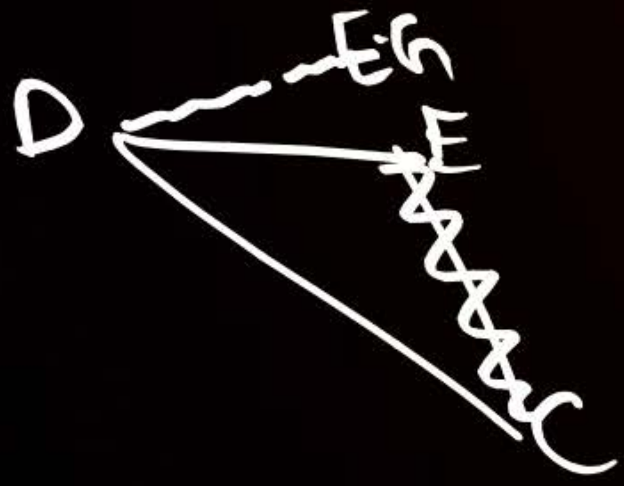
$$A.\text{of } \Delta ADE = \frac{1}{2} \times AE \times DG \quad \text{—} (1)$$

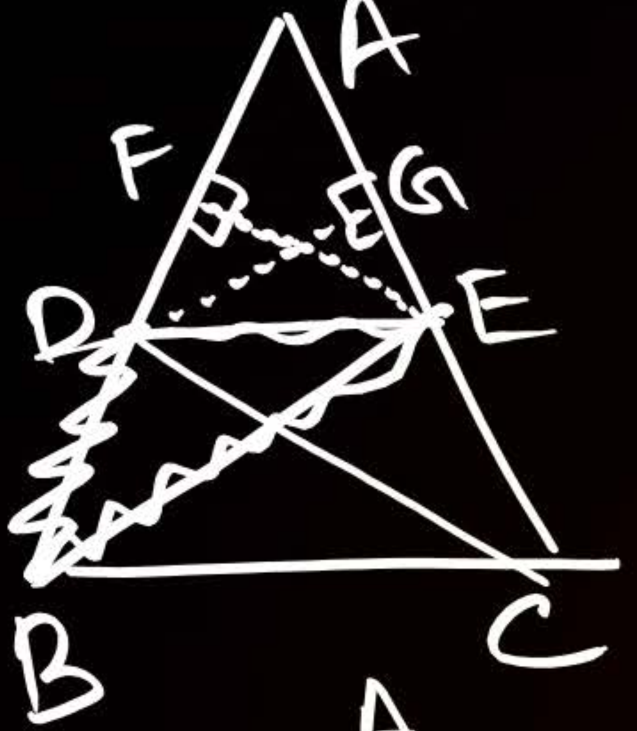
$$\text{Area of } \Delta DEC = \frac{1}{2} \times EC \times DG \quad \text{—} (2)$$

$$(1) \div (2)$$

$$\boxed{\frac{A.\Delta ADE}{A.\Delta DEC} = \frac{AE}{EC}} \quad \text{—} (3)$$

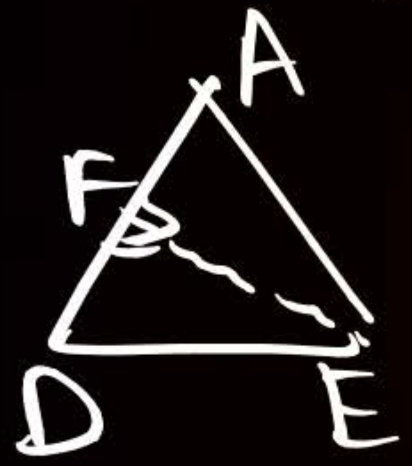






$$A.\text{ of } \triangle ADE = \frac{1}{2} \times AD \times EF \quad \text{— (4)}$$

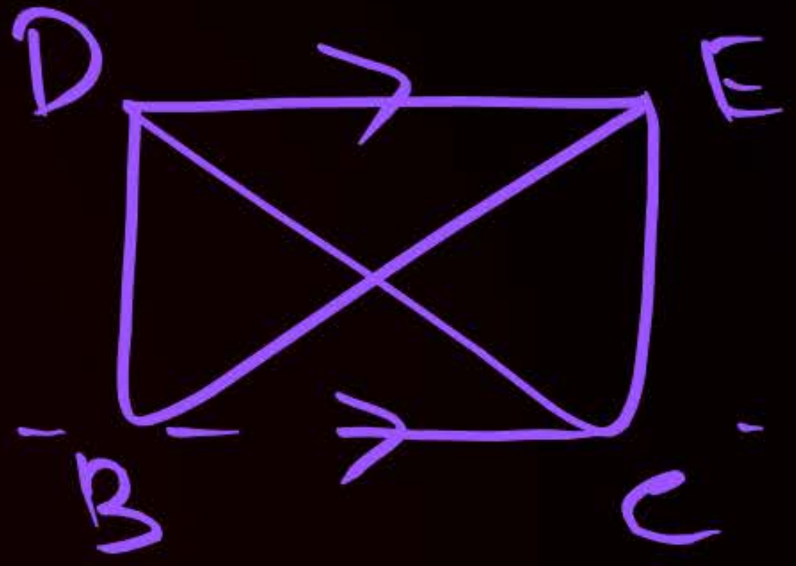
$$A.\text{ of } \triangle BDE = \frac{1}{2} \times DB \times EF \quad \text{— (5)}$$



$$(4) \div (5)$$

$$\boxed{\frac{A.\triangle ADE}{A.\triangle BDE} = \frac{AD}{DB}} \quad \text{— (6)}$$

$$\boxed{\frac{A.\triangle ADE}{A\triangle DEC} = \frac{AE}{EC}} \quad \text{— (8)}$$



$$A.\triangle BDE = A.\triangle DEC$$

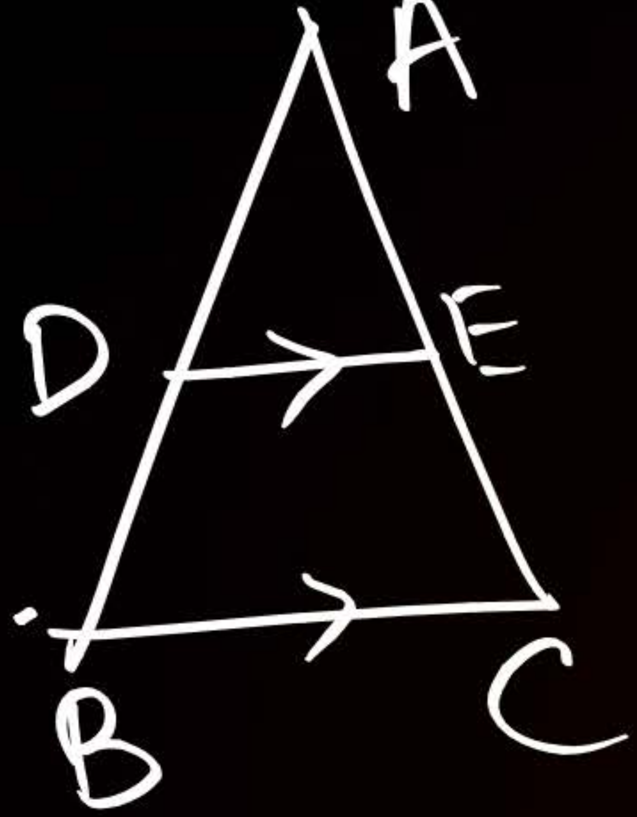
they lie b/w same parallels and same base.

0
60

$$\frac{A \cdot \Delta ADE}{A \cdot \Delta BDE} = \frac{A \cdot \Delta ADE}{A \cdot \Delta DEC}$$

$$\frac{AD}{DB} = \frac{AE}{EC}$$

H.P.



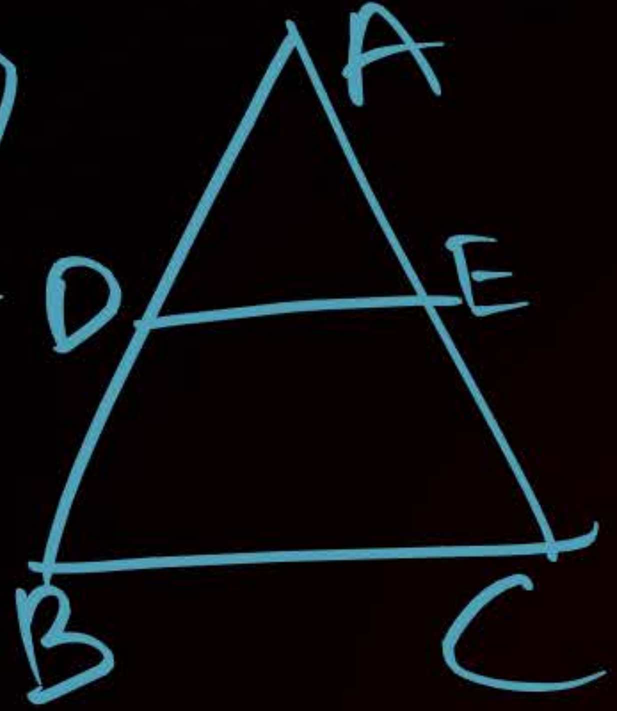
1) $\dfrac{AD}{DB} = \dfrac{AE}{EC}$ → By. B.P.T

2) $\dfrac{AD}{AB} = \dfrac{AE}{AC}$

3) $\dfrac{DB}{AB} = \dfrac{EC}{AC}$

→ corollary

Q



G: $DE \parallel BC$

top: $\dfrac{AD}{AB} = \dfrac{AE}{AC}$

By B.P.T. --

$$\frac{AD}{DB} = \frac{AE}{EC}$$

$$\frac{DB}{AD} = \frac{EC}{AE}$$

$$\frac{DB}{AD} + 1 = \frac{EC}{AE} + 1$$

$$\frac{DB + AD}{AD} = \frac{EC + AE}{AE}$$

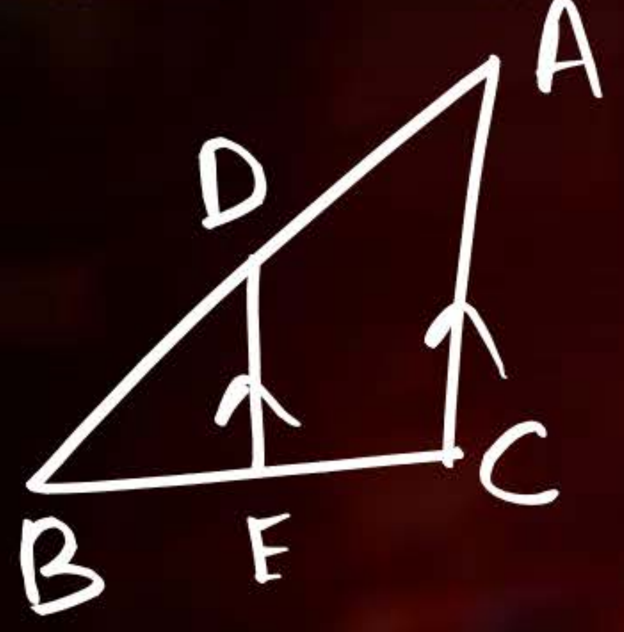
$$\frac{AB}{AD} = \frac{AC}{AE}$$

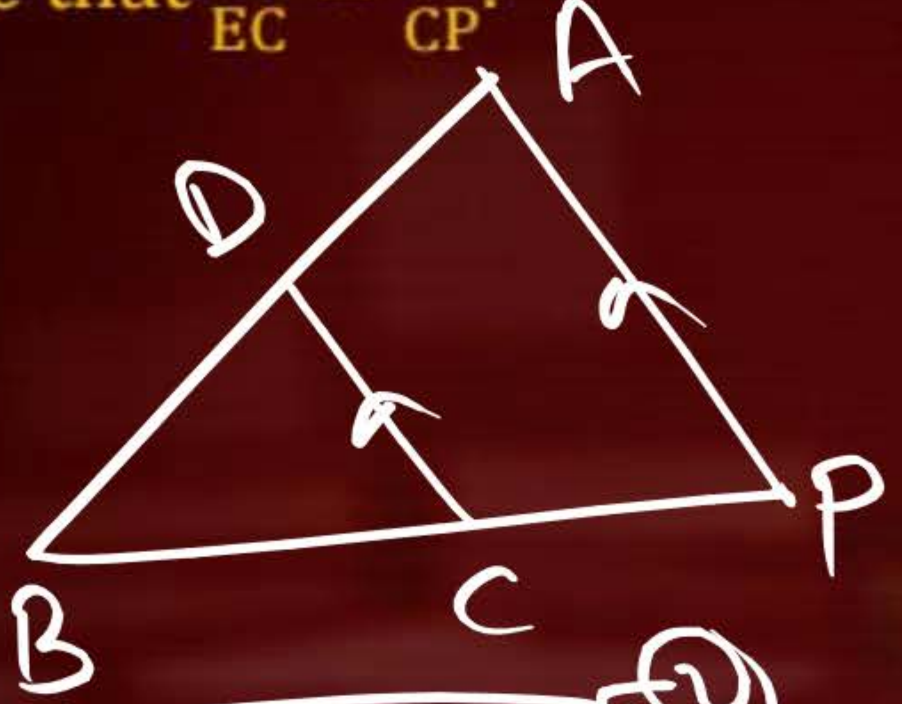
$$\frac{AD}{AB} = \frac{AE}{AC}$$

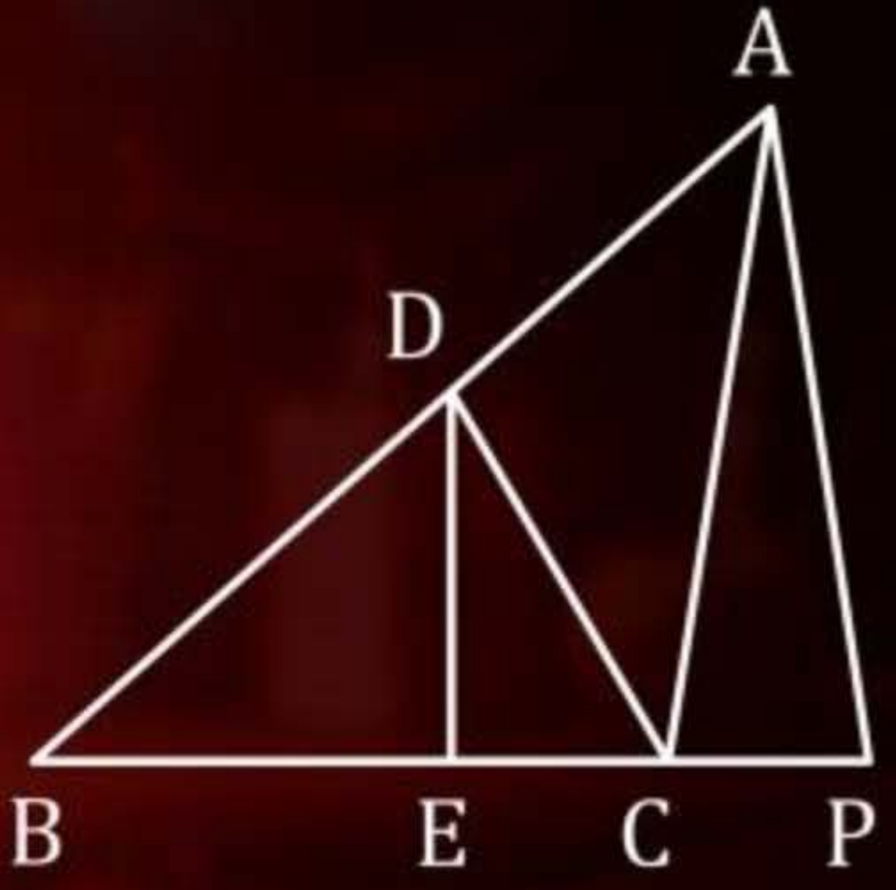
H.P.

QUESTION [CBSE 2005, 07, 20]

$DE \parallel AC$ and $DC \parallel AP$. Prove that $\frac{BE}{EC} = \frac{BC}{CP}$.







By B.P.T. ...

$$\boxed{\frac{BE}{EC} = \frac{BD}{DA}} \quad (1)$$

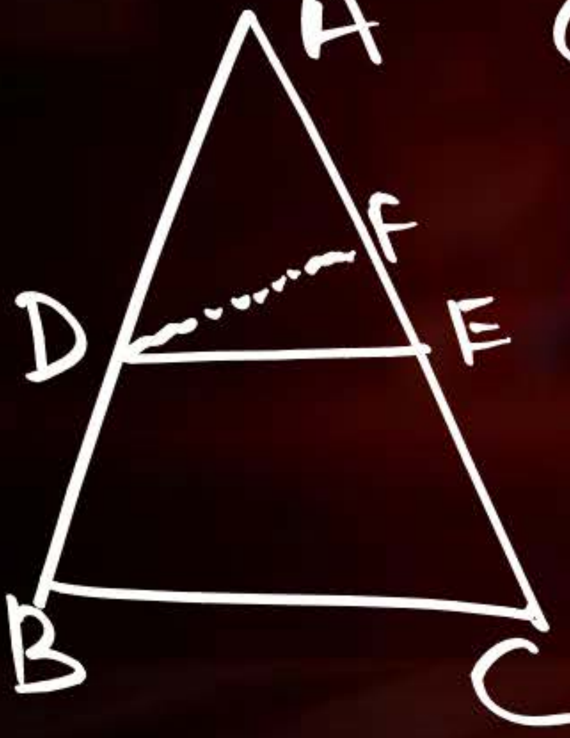
$$\boxed{\frac{BC}{CP} = \frac{BD}{DA}} \quad (2)$$

From (1) and (2)

$$\frac{BE}{EC} = \frac{BC}{CP}$$

THEOREM 2

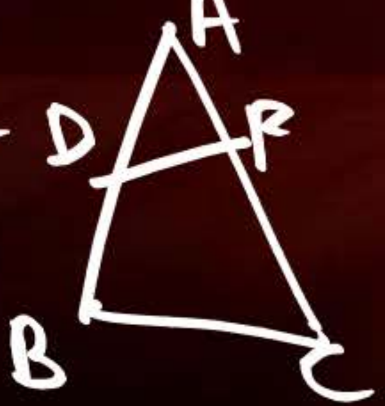
(Converse of Basic Proportionality Theorem (BPT)) If a line divides any two sides of a triangle in the same ratio, then the line must be parallel to the third side.



G: $\boxed{\frac{AD}{DB} = \frac{AE}{EC}}$ — (1)

To p: DE || BC

Const: let DF || BC.



Proof: DF || BC

$\boxed{\frac{AD}{DB} = \frac{AF}{FC}}$ — (2)

From (1) and (2)

$\frac{AE}{EC} = \frac{AF}{FC}$

$\frac{AE}{EC} + 1 = \frac{AF}{FC} + 1$

$\frac{AE+EC}{EC} = \frac{AF+FC}{FC}$

$\frac{\cancel{AC}}{EC} = \frac{\cancel{AC}}{FC}$

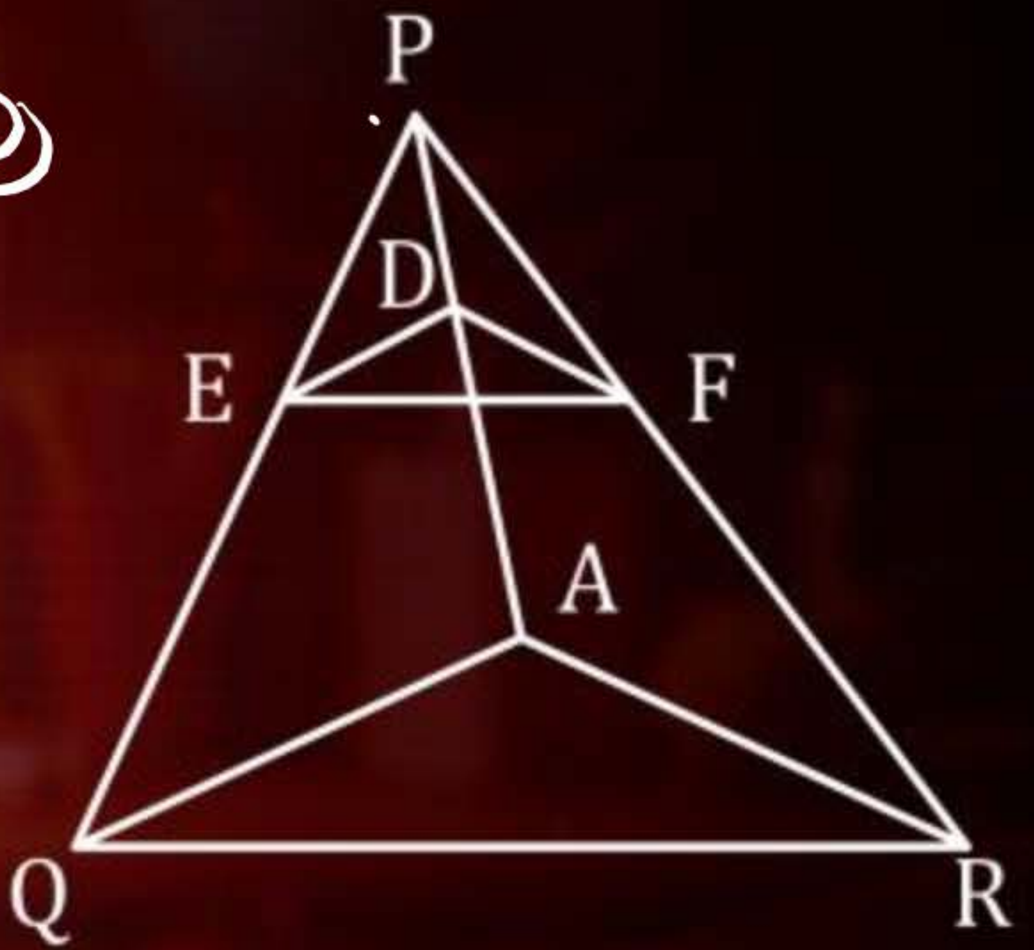
FC = EC

this is only possible if E = F.

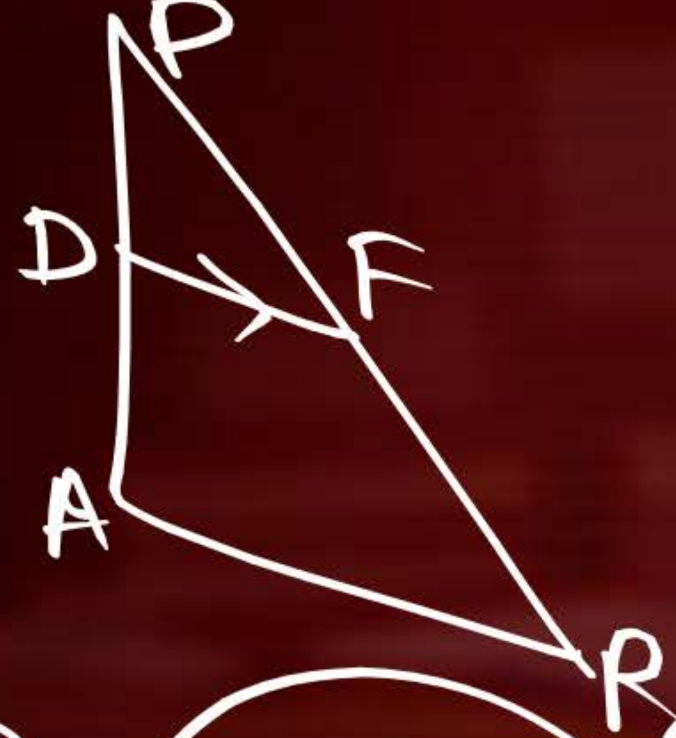
∴ DE || BC

QUESTION [NCERT, CBSE 2008, 2014]

In fig., if DE || AQ and DF || AR. Prove that EF || QR.



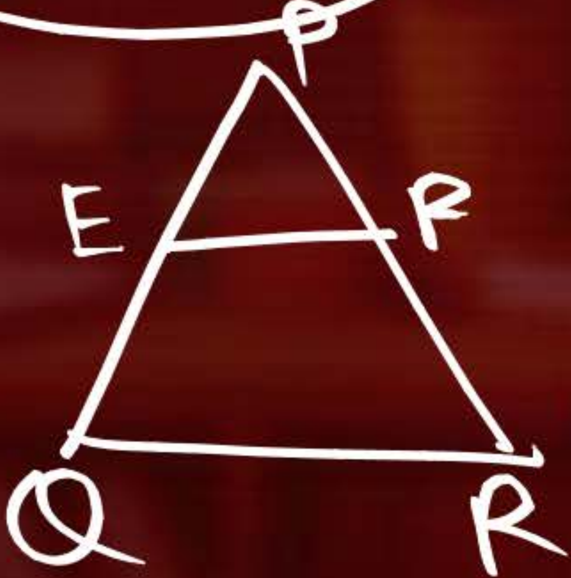
$\frac{PE}{EQ} = \frac{PD}{DA}$ — (1)



$\frac{PD}{DA} = \frac{PF}{FR}$ — (2)

From (1) and (2)

$\frac{PE}{EQ} = \frac{PF}{FR}$



∴ By C.B.P.T

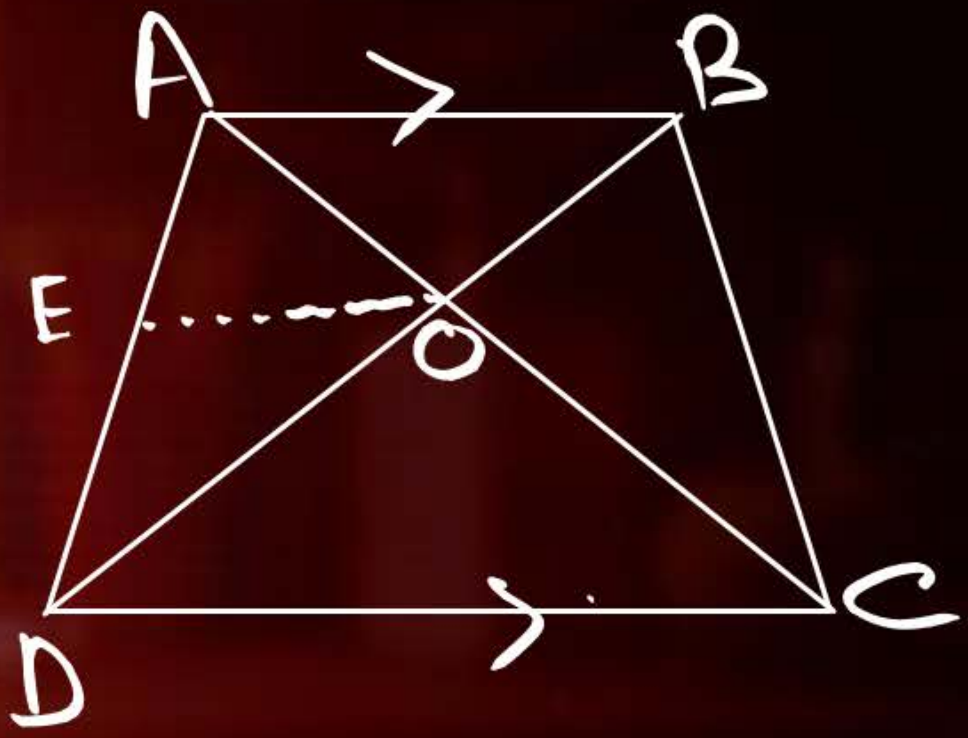
EF || QR

QUESTION [CBSE 2010, 2023]

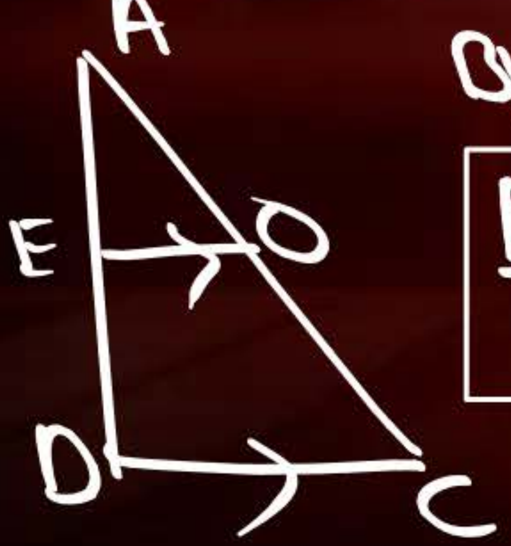
ABCD is a trapezium in which AB||DC and its diagonals intersect each other at O. Using basic proportionality theorem, prove that $\frac{AO}{BO} = \frac{CO}{DO}$.

G: AB || DC

To p: $\frac{AO}{BO} = \frac{CO}{DO}$



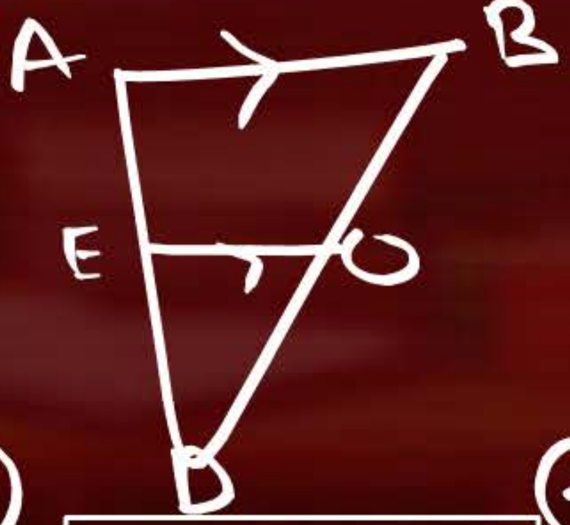
Proof: EO || DC



By B.P.T. .... — (1)

$$\boxed{\frac{AE}{ED} = \frac{AO}{OC}}$$

Also, EO || AB



— (2)

$$\boxed{\frac{AE}{ED} = \frac{BO}{DO}}$$

$$\frac{AO}{OC} = \frac{OB}{OD}$$

$$\frac{AO}{OB} = \frac{OC}{OD}$$

QUESTION [NCERT]

The diagonals of a quadrilateral ABCD intersect each other at O such that $\frac{AO}{BO} = \frac{CO}{DO}$. Using Basic proportionality Theorem and its converse, prove that ABCD is a trapezium.

To p: $AB \parallel DC$

G: $\frac{AO}{BO} = \frac{CO}{DO} \rightarrow \boxed{\frac{AO}{OC} = \frac{OB}{OD}}$ — (1)

Proof: $EO \parallel DC$ — (2)

$\boxed{\frac{AE}{ED} = \frac{AO}{OC}}$

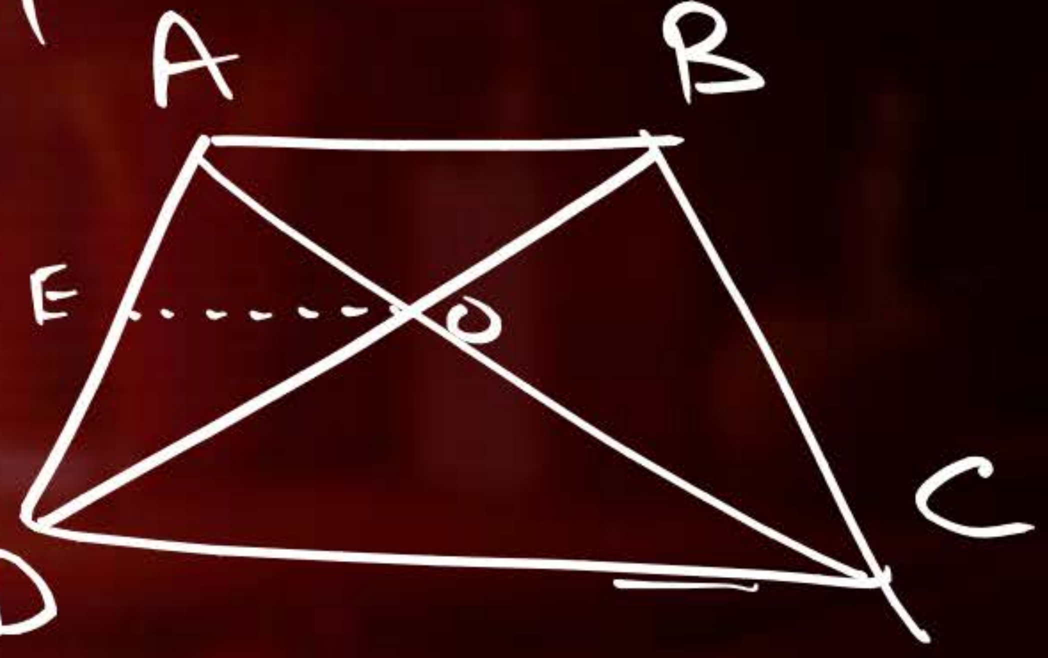
$\frac{AE}{ED} = \frac{OB}{OD}$

By C.B.P.T

$EO \parallel AB$

$\therefore$

$DC \parallel AB$



QUESTION [NCERT, NCERT EXEMPLAR, CBSE 2020]

In Fig., $\frac{PS}{SQ} = \frac{PT}{TR}$ and $\angle PST = \angle PRQ$. Prove that $\triangle$ PQR is an isoscels triangle.

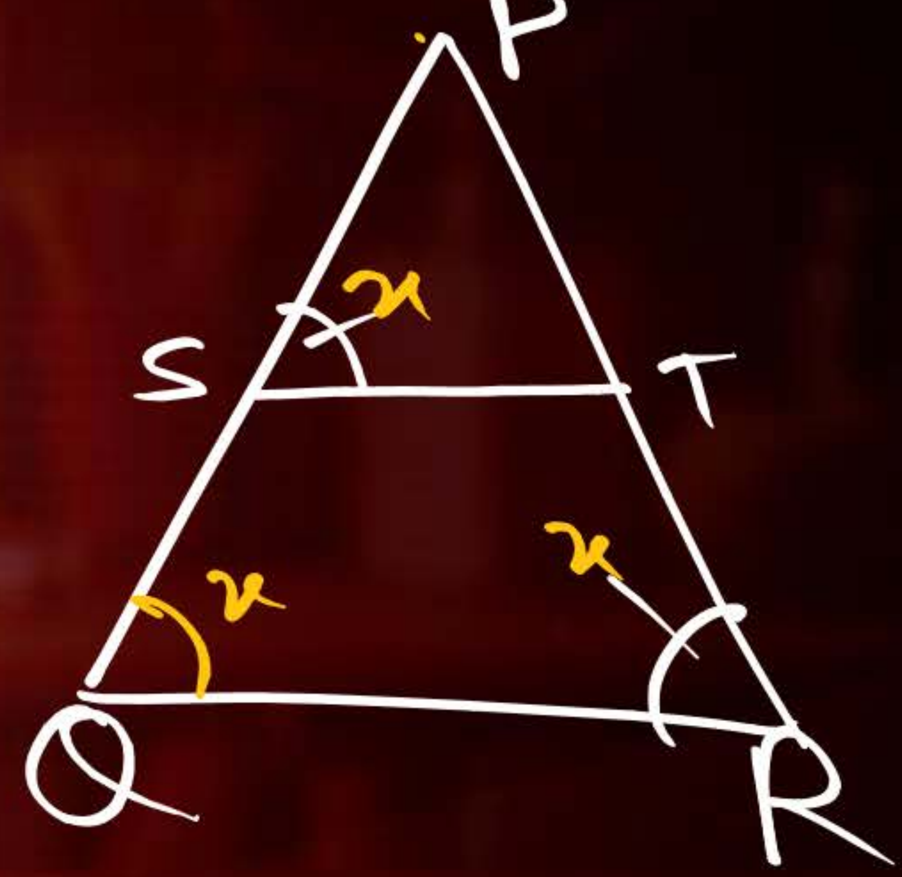
By C.BPT

$\boxed{ST \parallel QR}$

$\angle PST = \angle PQR = x$ [corres. $\angle$'s]

$\therefore \angle PQR = \angle PRQ$

$\therefore \Delta PQR$ is isoscales.



If $\Delta ABC \sim \Delta DEF$

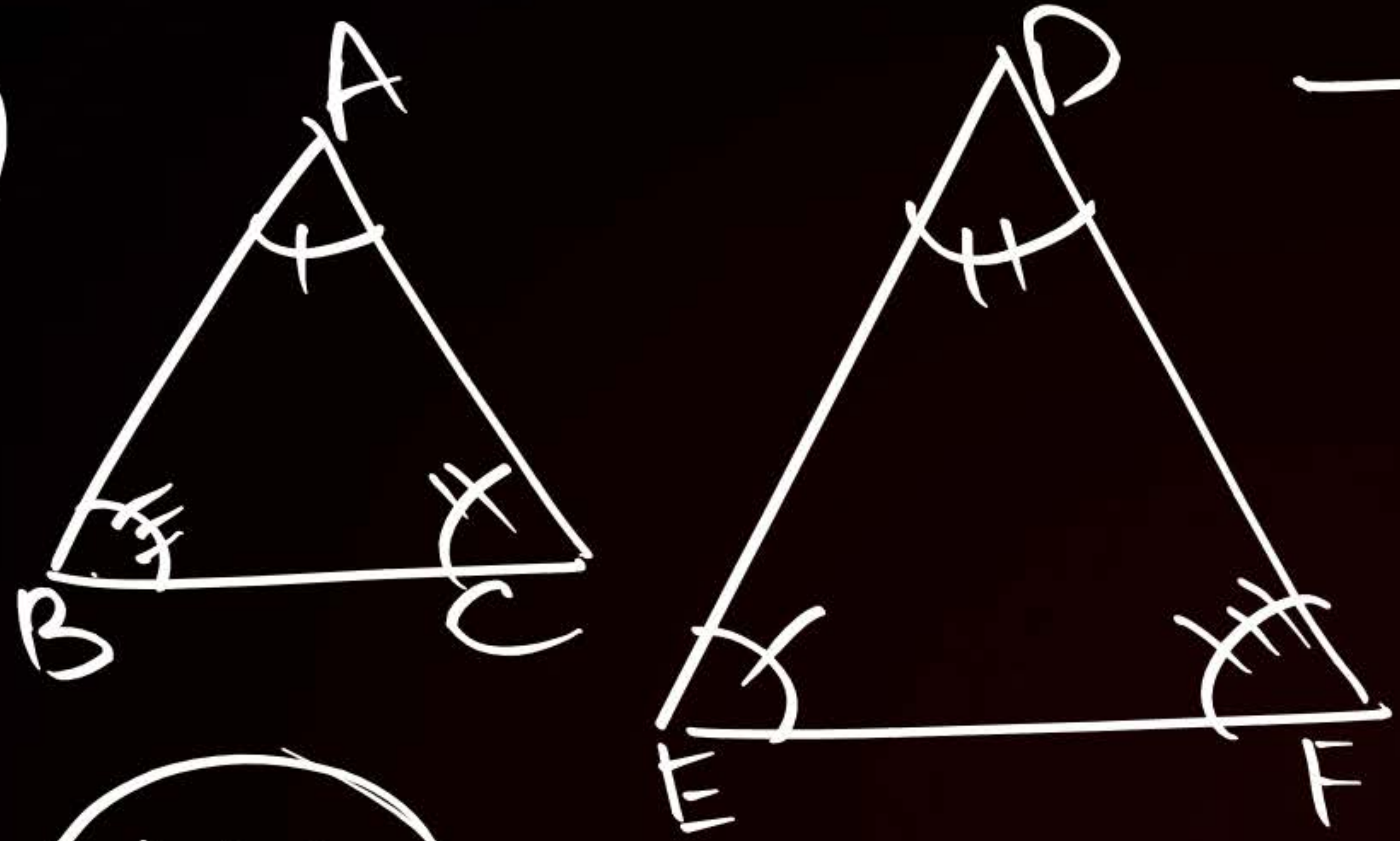
Then, $\dfrac{AB}{DE} = \dfrac{BC}{EF} = \dfrac{AC}{DF}$

$\angle A = \angle D, \angle B = \angle E, \angle C = \angle F$

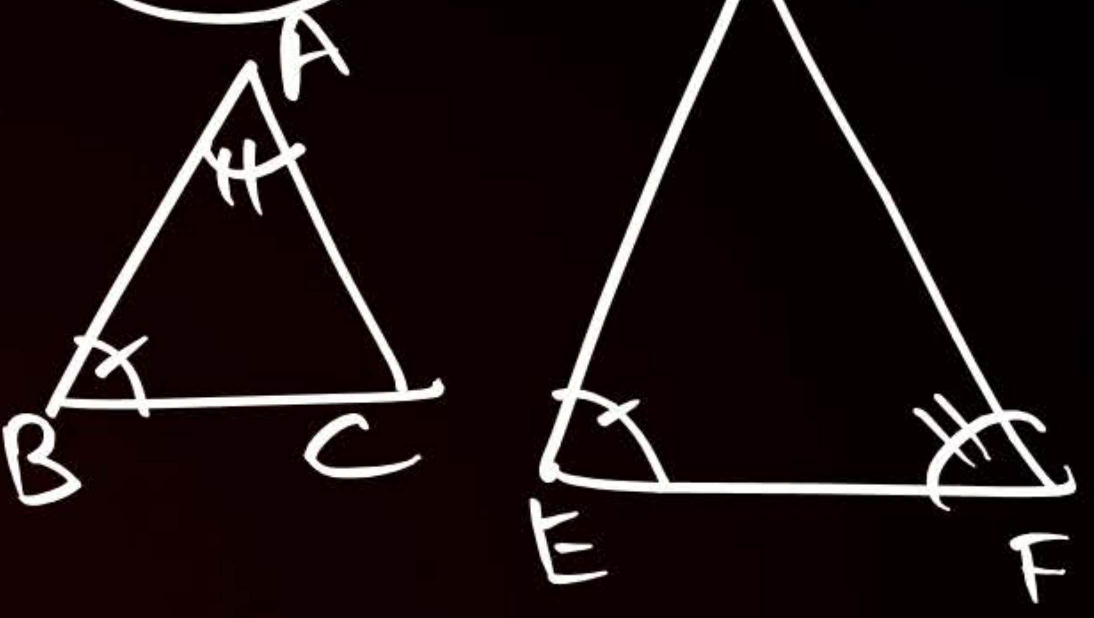
Q $\Delta PQR \sim \Delta ABC$

$\dfrac{QR}{BC} = \dfrac{PR}{AC} = \dfrac{PQ}{AB}$

1



→ AA



AAA

$\boxed{\Delta ACB \sim \Delta EDF}$

By CPST

$$\frac{AC}{ED} = \frac{CB}{DF} = \frac{AB}{EF}$$

By AA

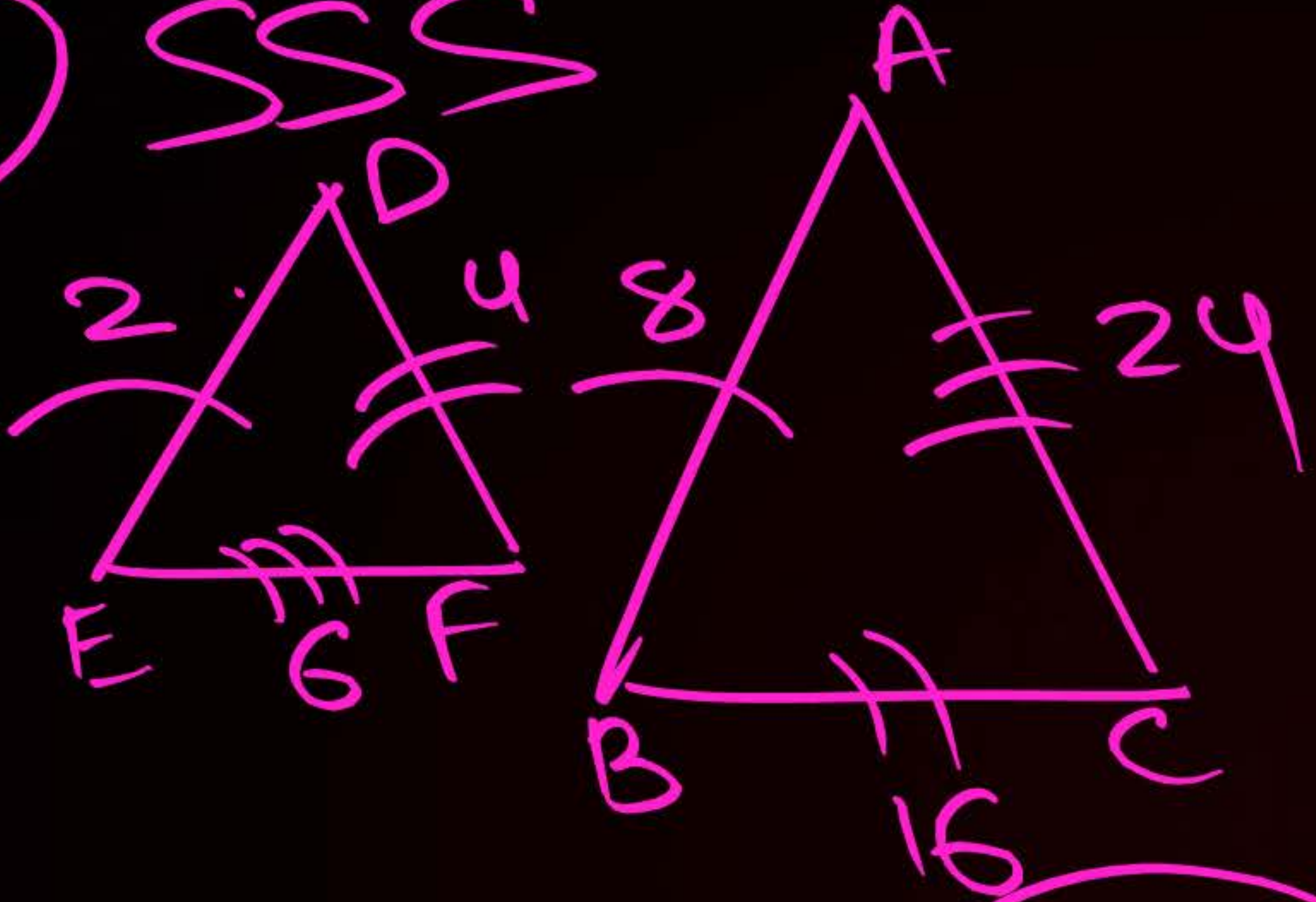
$\boxed{\Delta CAB \sim \Delta DFE}$

By CPST...

$$\frac{CA}{DF} = \frac{AB}{FE} = \frac{CB}{DE}$$

$$\angle C = \angle D$$

(3) SSS



$$\frac{DE}{AB} = \frac{DF}{BC} = \frac{EF}{AC}$$

$$\frac{2}{8} = \frac{4}{10} = \frac{6}{24}$$

By SSS

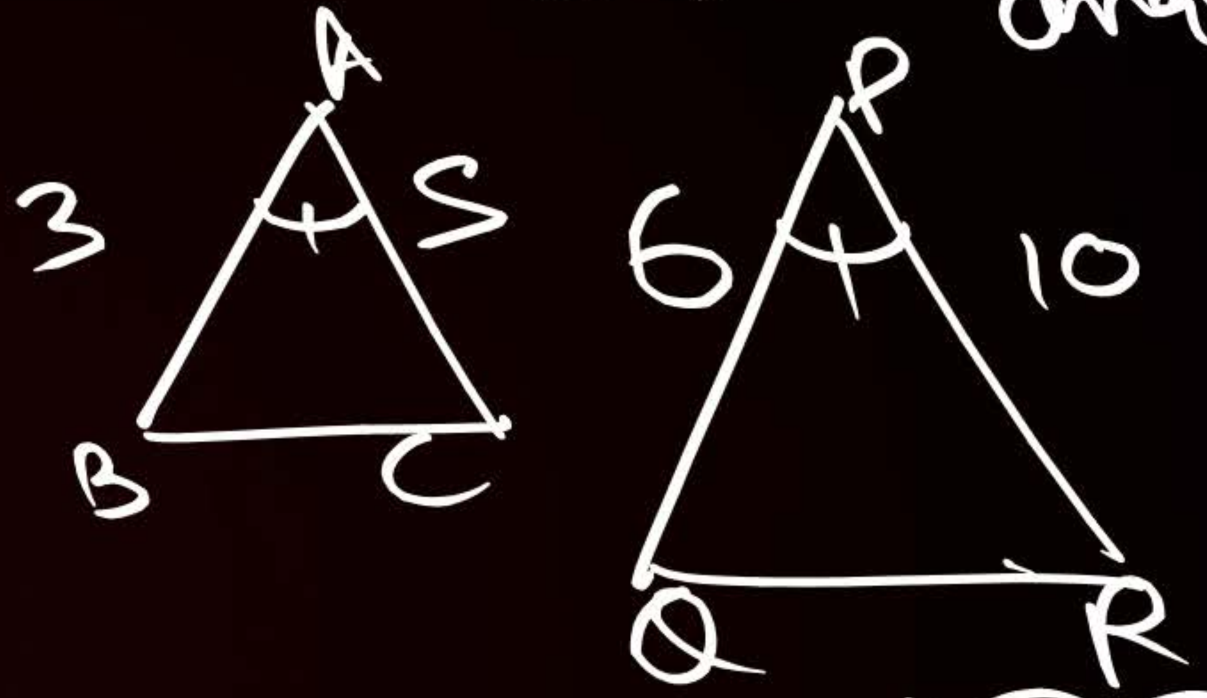
$$\boxed{\Delta DEF \sim \Delta BAC}$$

By CPST - - -

$\angle D = \angle B$, $\angle E = \angle A$, $\angle F = \angle C$

(4) SAS

included angle.



$$\frac{BA}{QP} = \frac{AC}{PR}$$

$$3/6 = 5/10$$

By SAS -

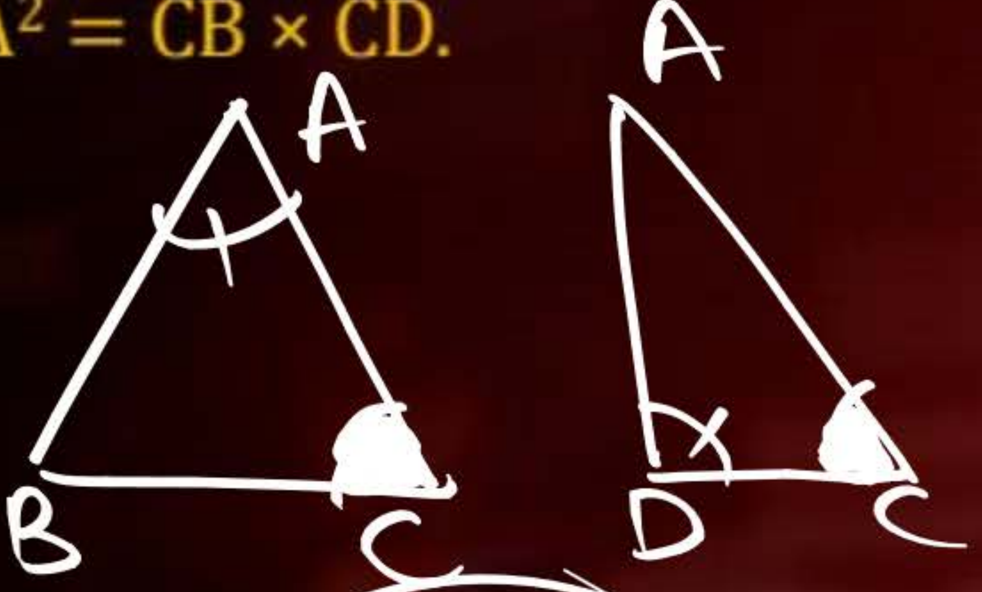
$\Delta BCA \sim \Delta QRP$

QUESTION [NCERT EXEMPLAR]

It is given that $\triangle ABC \sim \triangle EDF$ such that $AB = 5$ cm, $AC = 7$ cm, $DF = 15$ cm and $DE = 12$ cm, find the lengths of the remaining sides of the triangles.

QUESTION [NCERT, CBSE 2004, 2023]

D is a point on the side BC of $\triangle ABC$ such that $\angle ADC = \angle BAC$. Prove that $\frac{CA}{CD} = \frac{CB}{CA}$ or, $CA^2 = CB \times CD$.



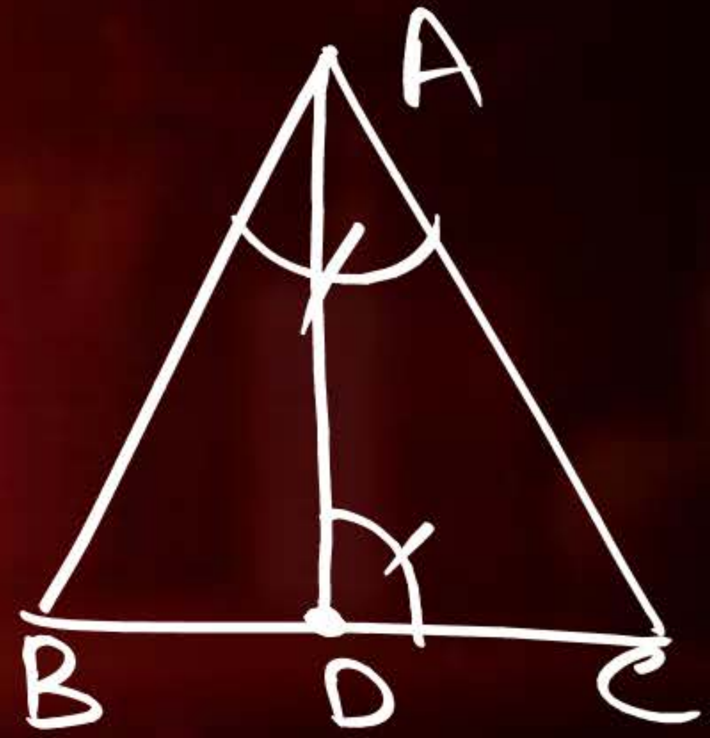
By AA

$$\boxed{\Delta ABC \sim \Delta DAC}$$

By CPST...

$$\frac{AB}{DA} = \frac{BC}{AC} = \frac{AC}{DC}$$

$$BC \cdot DC = AC^2$$



QUESTION

In Fig., QA and PB are perpendiculars to AB. If $AO = 10$ cm, $BO = 6$ cm and $PB = 9$ cm. Find AQ.

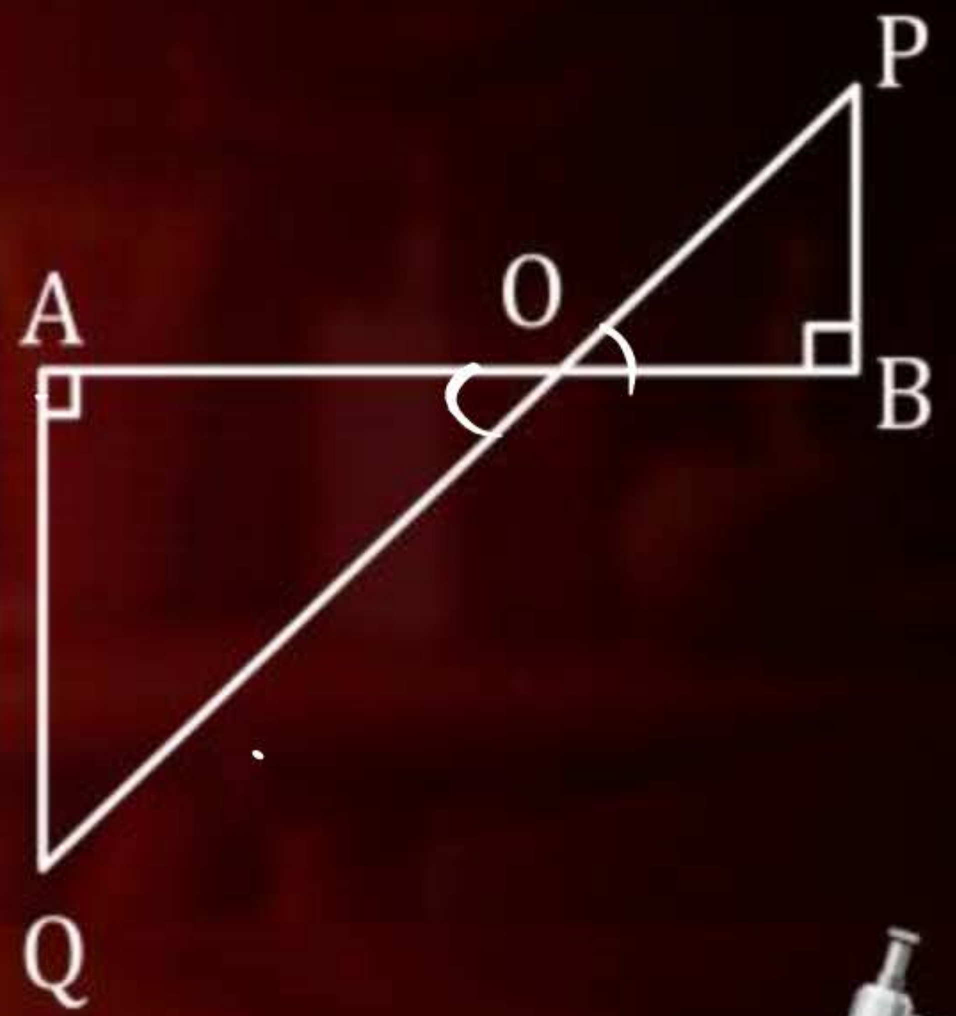
$\boxed{\Delta AOQ \sim \Delta BOP}$ (AA)

CPST...

$$\boxed{\frac{AO}{BO} = \frac{OQ}{OP} = \frac{AQ}{BP}}$$

$$\frac{10}{6} = x = \frac{AQ}{9}$$

15 = AQ cm



QUESTION [CBSE 2023]

In Fig, CD and GH are respectively the medians of $\triangle ABC$ and $\triangle EFG$. If $\triangle ABC \sim \triangle FEG$, prove that

(i) $\triangle ADC \sim \triangle FHG$

G, $\Delta ABC \sim \Delta FEG$

$$\frac{AB}{FE} = \frac{BC}{EG} = \frac{AC}{FG}$$

$\angle A = \angle F$ ✓

$\angle B = \angle E$ ✗

$\angle C = \angle G$ ✗

(i) In ΔADC & ΔFHG

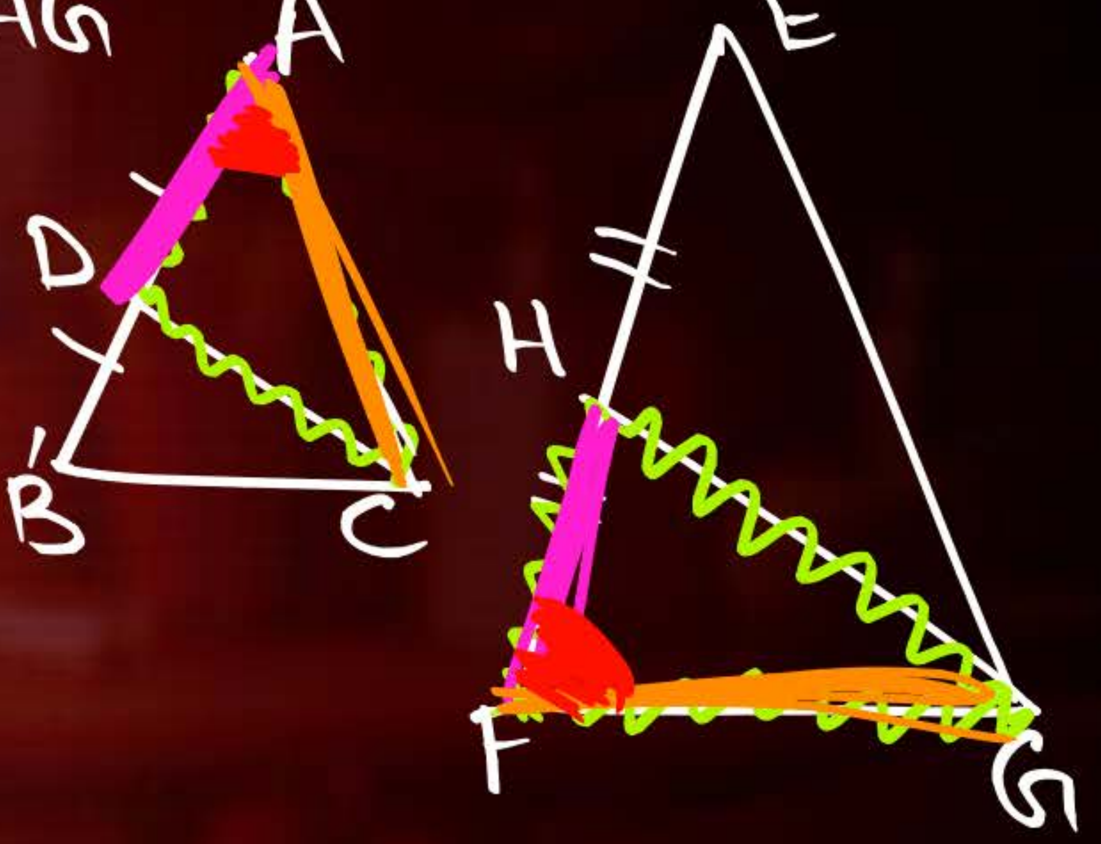
$$\boxed{\angle A = \angle F}$$

$$\frac{AB}{FE} = \frac{AC}{FG}$$

$$\boxed{\frac{\cancel{2}AD}{\cancel{2}FH} = \frac{AC}{FG}}$$

By SAS

$\Delta ADC \sim \Delta FHG$



QUESTION

The perimeters of two similar triangles ABC and PQR are respectively 36 cm and 24 cm . If $PQ = 10$ cm, find AB.

QUESTION [NCERT EXEMPLER]

If in two triangles ABC and PQR, then $\frac{AB}{QR} = \frac{BC}{PR} = \frac{CA}{PQ}$.

(A) $\triangle PQR \sim \triangle CAB$

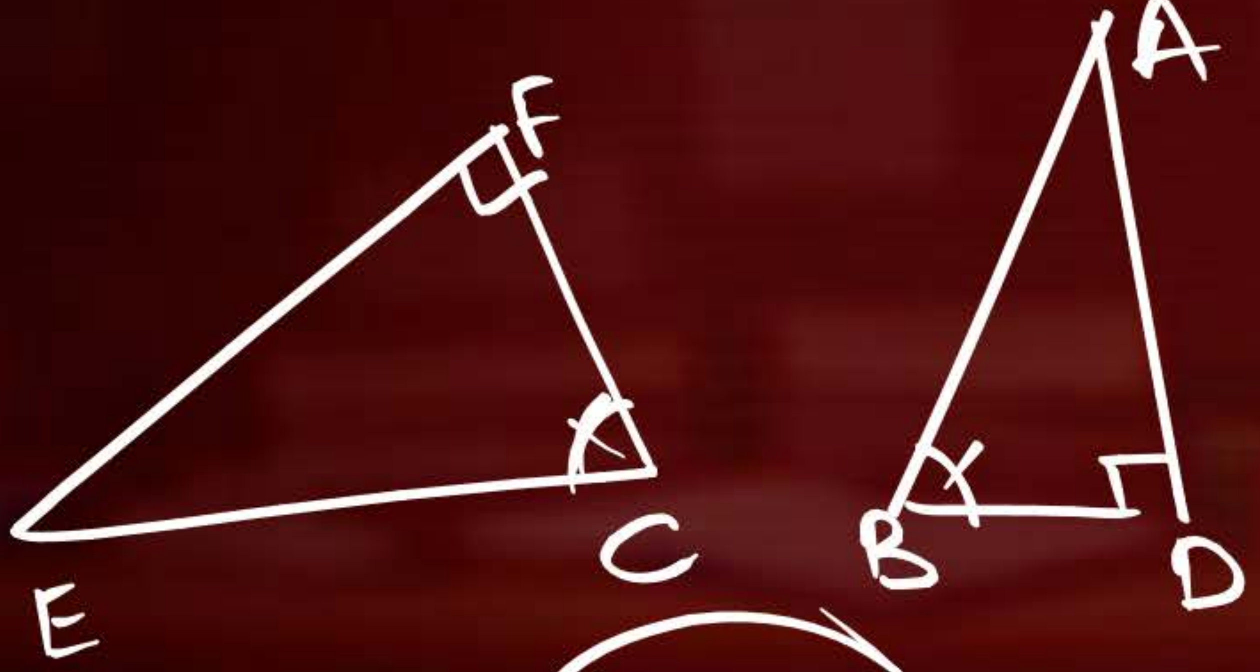
(B) $\triangle PQR \sim \triangle ABC$

(C) $\triangle CBA \sim \triangle PQR$

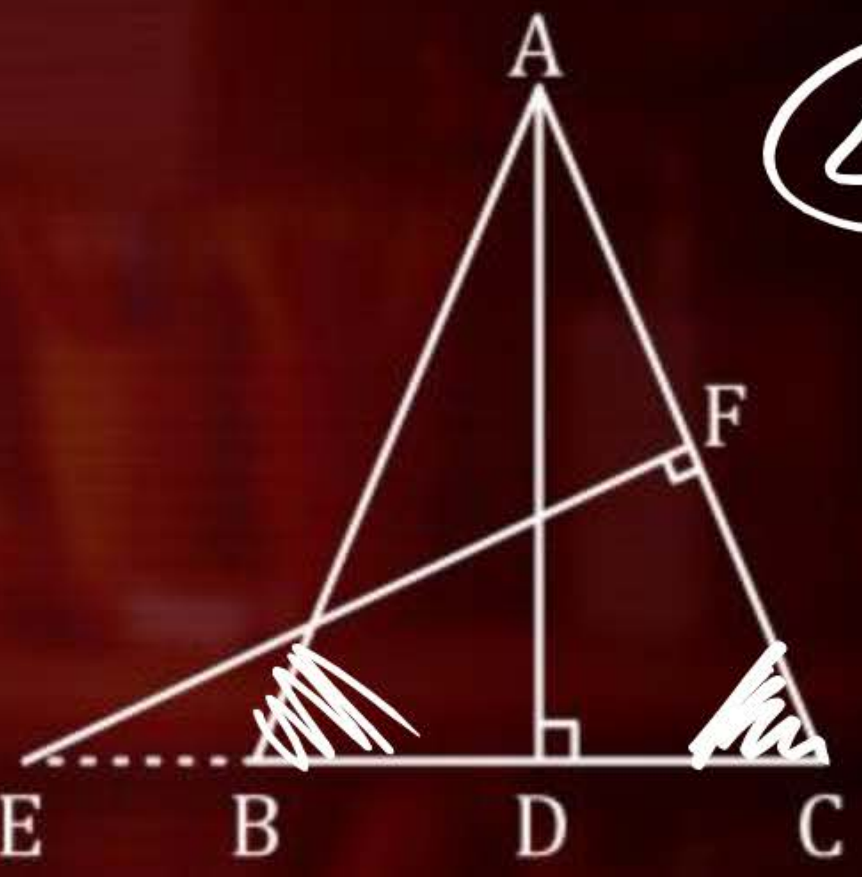
(D) $\triangle BCA \sim \triangle PQR$

QUESTION [NCERT, CBSE 2011]

In fig. E is a point on side CB produced of an isosceles triangle ABC with AB = AC. If AD ⊥ BC and EF ⊥ AC, prove that $\boxed{\triangle ABD \sim \triangle ECF.}$



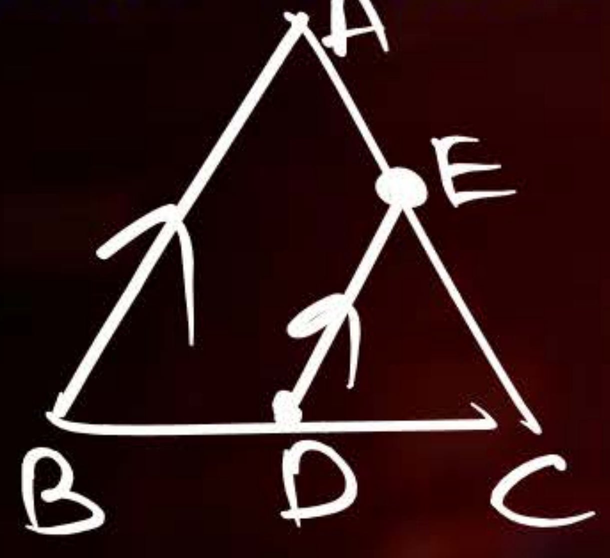
By AA

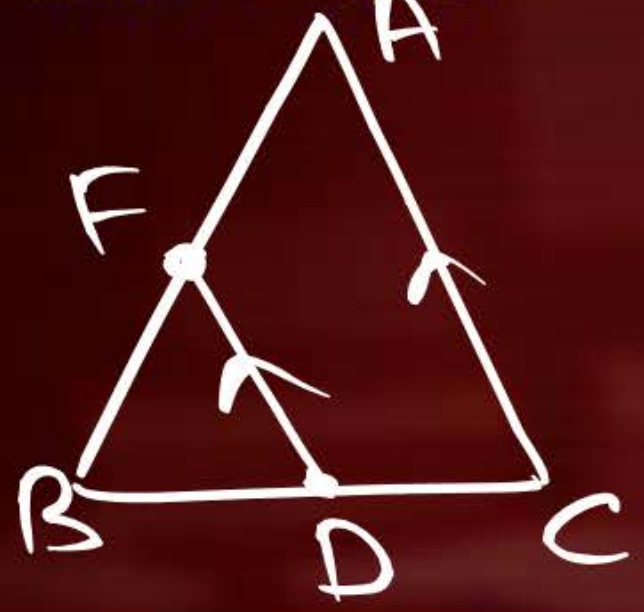


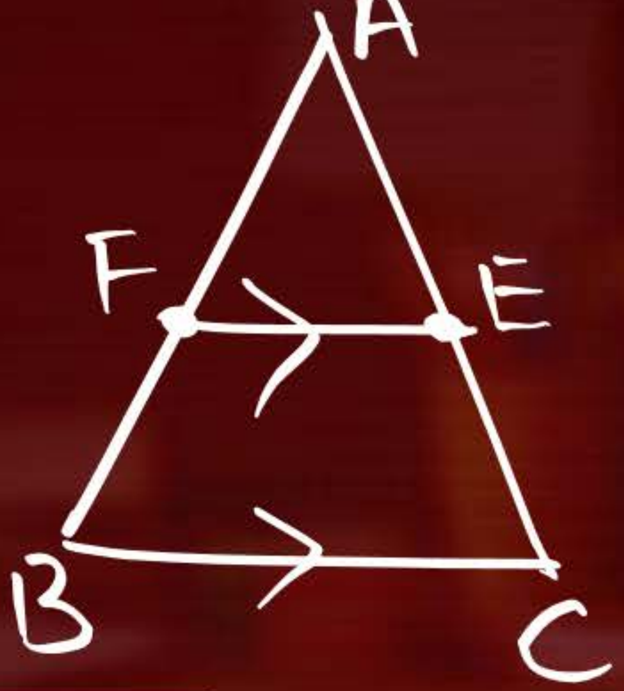
$\angle C = \angle B$

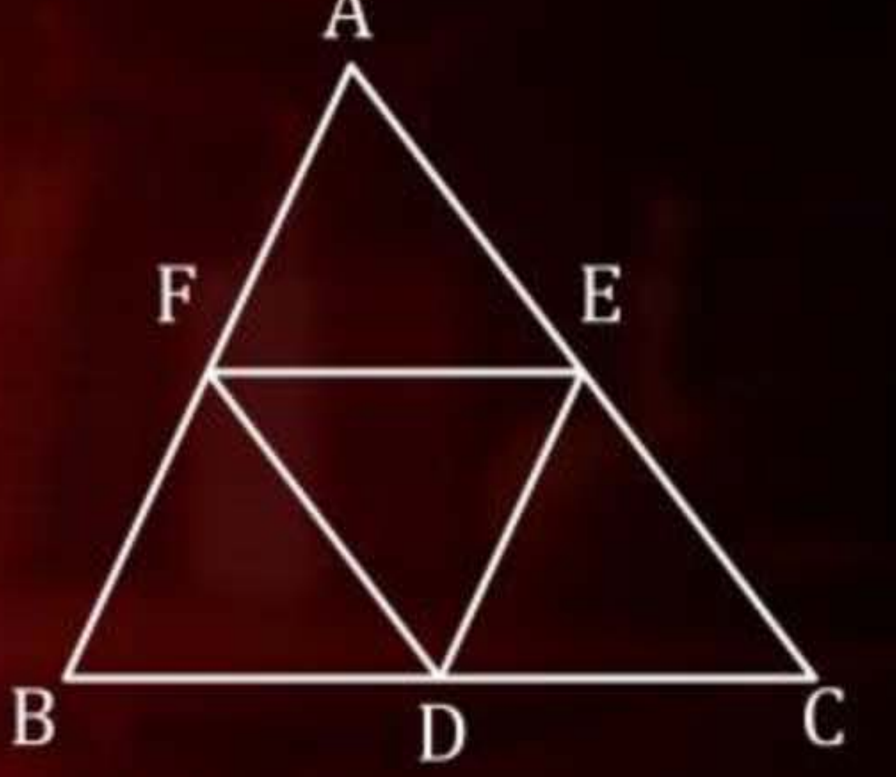
QUESTION [NCERT, CBSE 2011]

In $\triangle ABC$, D, E and F are midpoints of BC, CA and AB respectively. Prove that $\triangle FBD \sim \triangle DEF$ and $\triangle DEF \sim \triangle ABC$





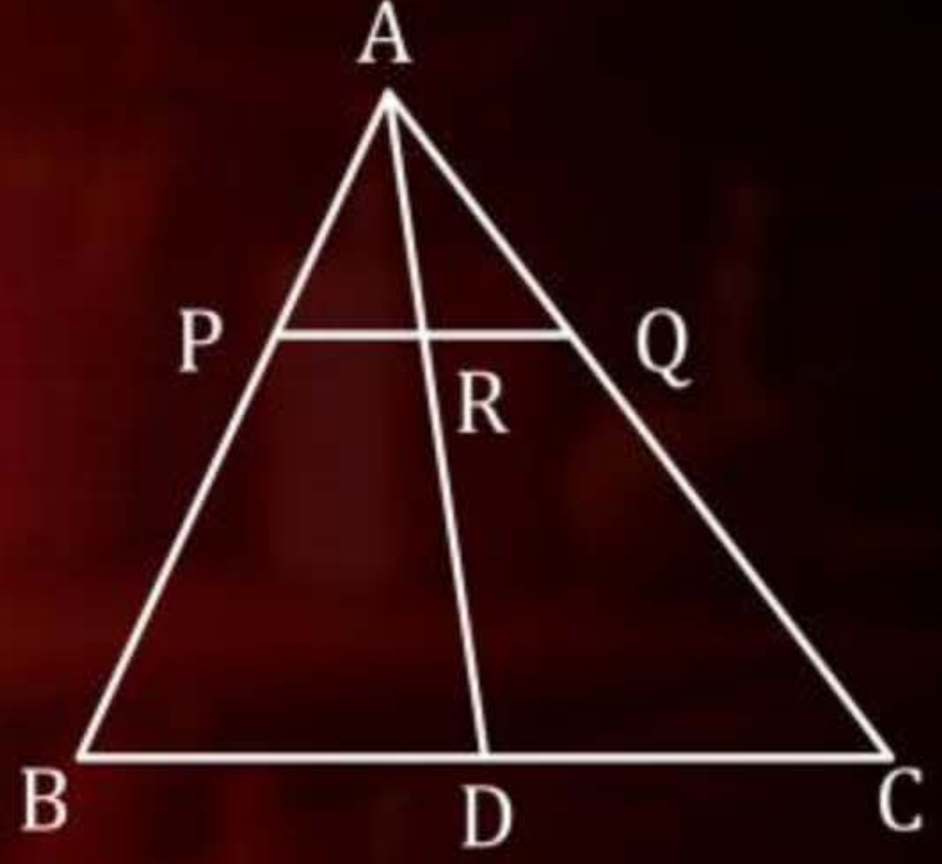




$\boxed{\text{Mid}\cdot\text{point theorem}}$

QUESTION [NCERT, CBSE 2011]

Prove that the median AD drawn from A on BC bisects PQ .



Topic : Important Points

If two triangles are equiangular/similar, the ratio of the corresponding side is same as the ratio of the:

- Corresponding, medians.
- Corresponding angle bisector segments.
- Corresponding altitude.

TRIGONOMETRY

REVISION

$\frac{P}{H}$ = sine θ (sin θ)

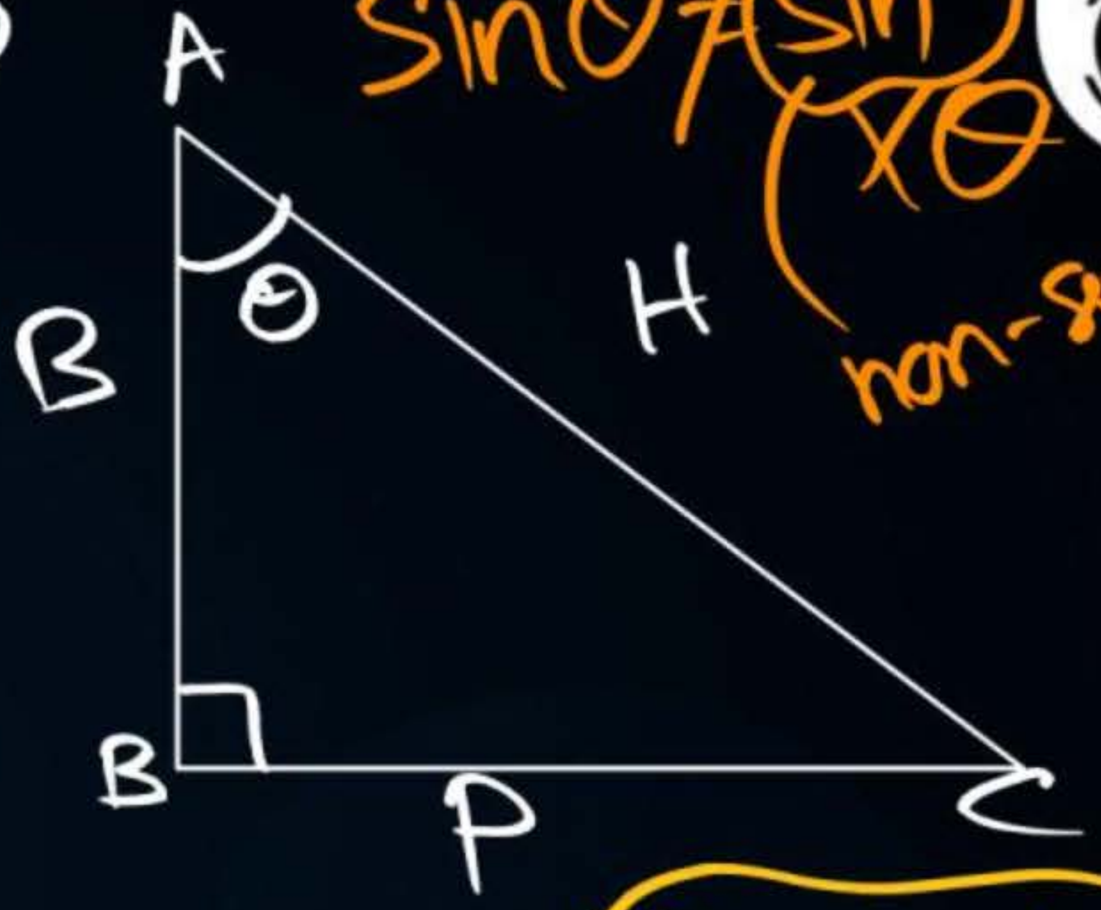
$\frac{H}{P}$ = cosecant θ (cosec θ)

$\frac{B}{H}$ = cosine θ (cos θ)

$\frac{H}{B}$ = secant θ (sec θ)

$\frac{P}{B}$ = tangent θ (tan θ)

$\frac{B}{P}$ = cotangent θ (cot θ)



$\sin\theta \neq \sin \times \theta$

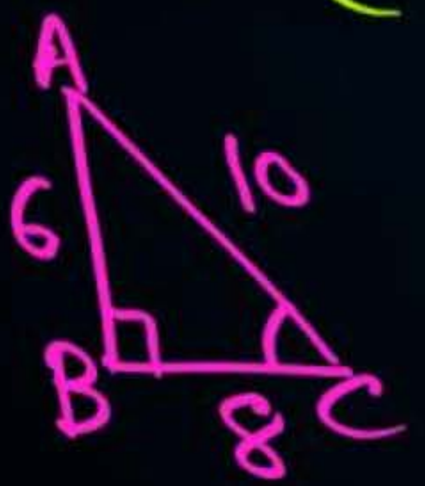
non-sense.

θ = theta

Perpendicular = opposite side

Base = adjacent side.

Q



$\sin C = \frac{6}{10} = \frac{3}{5}$

$\tan A = \frac{8}{6} = \frac{4}{3}$

Topic : T Ratios

#Q. If $\cos B = \frac{1}{3}$, find the other five trigonometric ratios.

of angle B.

$\cos B = \frac{1}{3}$

$\frac{B}{H} = \frac{1}{3}$

$B = 1x$

$H = 3x$

$H^2 = P^2 + B^2$

$(3x)^2 = P^2 + (1x)^2$

$9x^2 = P^2 + 1x^2$

$8x^2 = P^2$

$\pm\sqrt{8x^2} = P$

$\sqrt{2\times2\times2\times x\times x} = P$

$\boxed{2\sqrt{2}x = P}$

$\sin B = \frac{P}{H} = \frac{2\sqrt{2}x}{3x} = \frac{2\sqrt{2}}{3}$

$\tan B = \frac{P}{B} = \frac{2\sqrt{2}x}{1x} = 2\sqrt{2}$

$\sec B = H/B = 3/1$

$\cot B = B/P = 1/2\sqrt{2}$

$\operatorname{cosec} B = H/P = 3/2\sqrt{2}$

$\frac{P}{B}$ $\tan\theta \longleftrightarrow \cot\theta$ B/P

$\frac{P}{H}$ $\sin\theta \longleftrightarrow \operatorname{cosec}\theta$ H/P

$\frac{B}{H}$ $\cos\theta \longleftrightarrow \sec\theta$ H/B

$\sin\theta = \frac{2}{3}$

$\boxed{\operatorname{cosec}\theta = 3/2}$

$\sin\theta = \frac{1}{\operatorname{cosec}\theta}$

$\operatorname{cosec}\theta = \frac{1}{\sin\theta}$

$\tan\theta = \frac{1}{\cot\theta}$

$\cot\theta = \frac{1}{\tan\theta}$

$\cos\theta = \frac{1}{\sec\theta}$

$\sec\theta = 1/\cos\theta$

Q. [redacted] If $4\tan\theta = 3$, then $\left(\dfrac{4\sin\theta - \cos\theta}{4\sin\theta + \cos\theta}\right)$ is equal to

(a) $\frac{2}{3}$ (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) $\frac{3}{4}$

$4\tan\theta = 3$

$\tan\theta = 3/4$

$\frac{P}{B} = \frac{3}{4}$

$P = 3x$

$B = 4x$

$H = 5x.$

$\sin\theta = \frac{P}{H} = \frac{3}{5}$

$\cos\theta = \frac{B}{H} = \frac{4}{5}$

$$\frac{4\times\frac{3}{5} - \frac{4}{5}}{4\times\frac{3}{5} + \frac{4}{5}} = \frac{\frac{12}{5} - \frac{4}{5}}{\frac{12}{5} + \frac{4}{5}} = \frac{\frac{8}{\cancel{5}}}{\frac{16}{\cancel{5}}} = \frac{1}{2}$$

Method : 2

$$\frac{\dfrac{4\sin\theta - \cos\theta}{\cos\theta}}{\dfrac{4\sin\theta + \cos\theta}{\cos\theta}}$$

$$= \frac{4\tan\theta - 1}{4\tan\theta + 1}$$

Topic : T Ratios

#Q. If in a triangle ABC right angled at B, AB = 6 units and BC = 8 units, then find the value of $\sin A \cdot \cos C + \cos A \cdot \sin C$. [Board Term – I, 2016]

$$\sin A = \frac{P}{H} = \frac{BC}{AC} = \frac{8}{10} = \boxed{\frac{4}{5}}$$

$$\cos C = \frac{B}{H} = \frac{BC}{AC} = \frac{8}{10} = \boxed{\frac{4}{5}}$$

$$\cos A = \frac{B}{H} = \frac{AB}{AC} = \frac{6}{10} = \boxed{\frac{3}{5}}$$

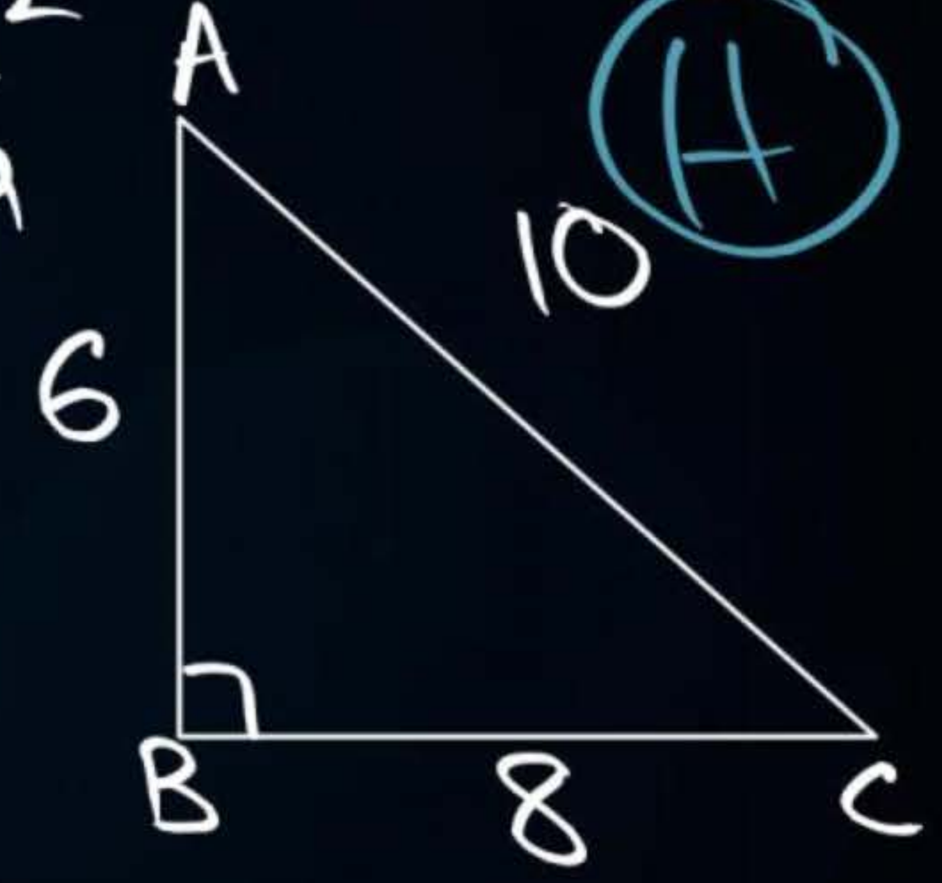
$$\sin C = \frac{P}{H} = \frac{AB}{AC} = \frac{6}{10} = \boxed{\frac{3}{5}}$$

$$AC^2 = AB^2 + BC^2$$

$$AC^2 = 36 + 64$$

$$AC = \pm\sqrt{100}$$

$$\boxed{AC = 10\,cm}$$



$$\frac{4}{5} \times \frac{4}{5} + \frac{3}{5} \times \frac{3}{5}$$

$$\frac{16}{25} + \frac{9}{25} = \frac{25}{25} = 1 \text{ Ans.}$$

Topic : T Rations for some specific angles

#Q. In a ΔABC, if $\angle B = 90°$, $BC = 5$ cm, $\boxed{AC - AB = 1 \text{ cm.}}$ Then the value of $\boxed{\dfrac{1 + \sin C}{1 + \cos C}}$ is

(A) $\dfrac{18}{25}$

(B) $\dfrac{36}{31}$

(C) $\dfrac{25}{18}$

(D) $\dfrac{31}{36}$

$\boxed{AC - AB = 1}$ $\quad AC = 1 + AB$

By P.T

$AC^2 = AB^2 + BC^2$

$\boxed{AC^2 = AB^2 + 25}$

$(1+AB)^2 = AB^2 + 25$

$1^2 + \cancel{AB^2} + 2AB = \cancel{AB^2} + 25$

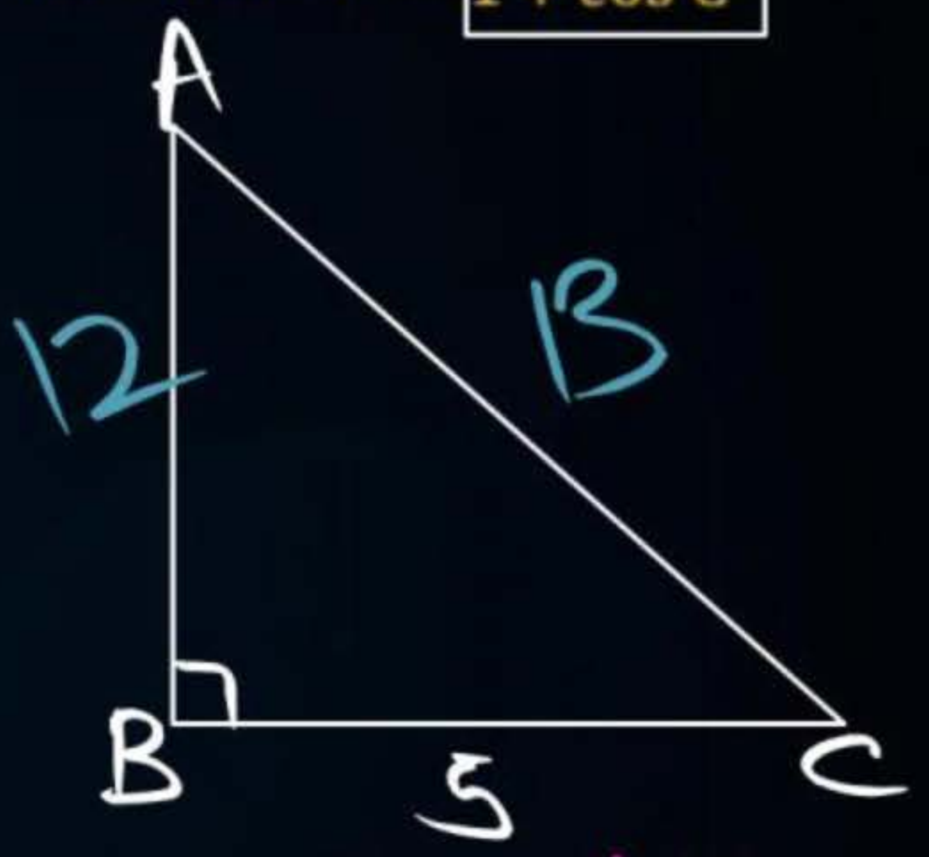
$1 + 2AB = 25$

$2AB = 24$

$AB = 12$

$AC = 1 + 12$

$= 13$



$\sin C = \dfrac{12}{13}$

$\cos C = 5/13$

$\dfrac{1 + \frac{12}{13}}{1 + \frac{5}{13}} = \dfrac{\frac{25}{\cancel{13}}}{\frac{18}{\cancel{13}}}$

Reciprocal Identity

$\sin\theta \longleftrightarrow \operatorname{cosec}\theta$

$\cos\theta \longleftrightarrow \sec\theta$

$\tan\theta \longleftrightarrow \cot\theta$

Quotient Relation.

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\cot\theta = \frac{\cos\theta}{\sin\theta}$$

$$\tan 30^\circ = \frac{\sin 30^\circ}{\cos 30^\circ}$$

$$\tan 60^\circ = \frac{\sin 60^\circ}{\cos 60^\circ}$$

Topic: Trigonometric Ratios of some specific Angles

$\frac{1}{0} = nd$

$\frac{1}{nd} = 0$

θ / T. ratios	0°	30°	45°	60°	90°
sin θ	0	$1/2$	$1/\sqrt{2}$	$\sqrt{3}/2$	1
cos θ	1	$\sqrt{3}/2$	$1/\sqrt{2}$	$1/2$	0
tanθ	0	$1/\sqrt{3}$	1	$\sqrt{3}$	n.d
cosec θ	n.d	2	$\sqrt{2}$	$2/\sqrt{3}$	1
sec θ	1	$2/\sqrt{3}$	$\sqrt{2}$	2	n.d
cot θ	n.d	$\sqrt{3}$	1	$1/\sqrt{3}$	0

Find the value of x if

$$2\,\text{cosec}^2 30 + x \sin^2 60 - \frac{3}{4}\tan^2 30 = 10$$

$$2(2)^2 + x\left(\frac{\sqrt{3}}{2}\right)^2 - \frac{3}{4}\left(\frac{1}{\sqrt{3}}\right)^2 = 10$$

$$8 + \frac{3x}{4} - \frac{\cancel{3}}{4} \times \frac{1}{\cancel{3}}$$

$$8 + \frac{3x}{4} - \frac{1}{4} = 10$$

$$\frac{3x}{4} - \frac{1}{4} = 2$$

$$\frac{3x-1}{4} = 2$$

$$3x - 1 = 8$$

$$3x = 9$$

$$x = 3$$

Q $\sin\theta = \frac{1}{2}$

$\theta = ?$

$\sin\theta = \sin 30^\circ$

"on comp"

$\boxed{\theta = 30^\circ}$

Q $\sin 2\theta = \frac{1}{2}$

$\sin 2\theta = \sin 30^\circ$

"on comp"

$2\theta = 30$

$\theta = 15$

Q $\cos 3\theta = \frac{1}{\sqrt{2}}$

$\cos 3\theta = \cos 45^\circ$

$3\theta = 45^\circ$

$\theta = 15$

Q $A = 30^\circ$, $\sin(A+B) = \frac{1}{2}$

$\sin(A+B) = \sin 30^\circ$

$A+B = 30^\circ$

$30+B = 30^\circ$

$\boxed{B = 0}$

Q. [redacted] If $\sin\alpha = \frac{1}{2}$ and $\cos\beta = \frac{1}{2}$, then the value of $(\alpha + \beta)$ is

(a) $0°$ (b) $30°$ (c) $60°$ (d) $90°$

$\sin\alpha = \frac{1}{2}$ $\cos\beta = \frac{1}{2}$

$\sin\alpha = \sin 30°$ $\cos\beta = \cos 60°$

$\alpha = 30°$ $\beta = 60°$

#Q. If A and B are acute angles such that $\sin(A - B) = 0$ and $2\cos(A + B) - 1 = 0$, then A =

$\sin(A-B)=0$

$\sin(A-B)=\sin 0^\circ$

"oncomp"

$\boxed{A-B=0}$ (1)

$2\cos(A+B)-1=0$

$2\cos(A+B)=1$

$\cos(A+B)=\frac{1}{2}$

$\cos(A+B)=\cos 60^\circ$

$\boxed{A+B=60^\circ}$ (2)

$$\begin{array}{r} A-\cancel{B}=0 \\ +\ A+\cancel{B}=60 \\ \hline 2A=60 \end{array}$$

$A=30$

Topic : T Rations for some specific angles

#Q. In figure, lengths of sides BC and AB are respectively

(A) 12 cm, $3\sqrt{3}$ cm

(B) 3 cm, $3\sqrt{3}$ cm

(C) 12 cm, $6\sqrt{3}$ cm

(D) 18 cm, $9\sqrt{3}$ cm

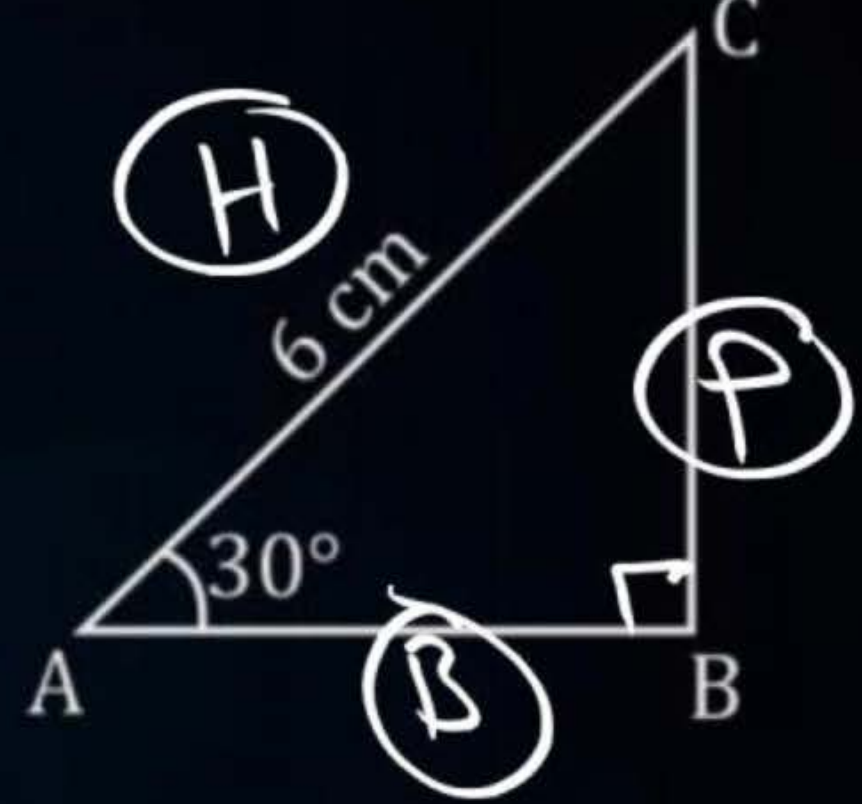
$\sin 30^\circ = \frac{P}{H}$ $\cos 30^\circ = \frac{B}{H}$

$\sin 30^\circ = \frac{BC}{6}$ $\cos 30^\circ = \frac{AB}{AC}$

$\frac{1}{2} = \frac{BC}{6}$ $\frac{\sqrt{3}}{2} = \frac{AB}{6}$

$\boxed{3\text{cm} = BC}$ $\frac{6\sqrt{3}}{2} = AB$

$\boxed{3\sqrt{3}\text{cm} = AB}$



Reciprocal Identity

$\sin\theta \longleftrightarrow \operatorname{cosec}\theta$

$\cos\theta \longleftrightarrow \sec\theta$

$\tan\theta \longleftrightarrow \cot\theta$

Quotient Relation

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\cot\theta = \frac{\cos\theta}{\sin\theta}$$

$$\boxed{\operatorname{cosec}^2\theta = 1+\cot^2\theta}$$

$$\operatorname{cosec}^2\theta - 1 = \cot^2\theta$$

$$\operatorname{cosec}^2\theta - \cot^2\theta = 1$$

(1) or Square vali

$$\boxed{\sin^2\theta + \cos^2\theta = 1}$$

$$\sin^2\theta = 1-\cos^2\theta$$

$$\cos^2\theta = 1-\sin^2\theta$$

$$\boxed{\sec^2\theta = 1+\tan^2\theta}$$

$$\sec^2\theta - 1 = \tan^2\theta$$

$$\sec^2\theta - \tan^2\theta = 1$$

Topic : Trigonometric Identities

#Q. Express the ratios cos A, tan A and sec A in terms of sin A.

$\cos A \rightarrow \sin A$

$\cos^2 A + \sin^2 A = 1$

$\cos^2 A = 1 - \sin^2 A$

$\cos A = \pm\sqrt{1 - \sin^2 A}$

$$\boxed{\cos A = \sqrt{1 - \sin^2 A}}$$

$\tan A \rightarrow \sin A$

$\tan A = \frac{\sin A}{\cos A}$

$$\boxed{\tan A = \frac{\sin A}{\sqrt{1 - \sin^2 A}}}$$

$\sec A \rightarrow \sin A$

$\sec \rightarrow \cos \rightarrow \sin A$

$\sec \rightarrow \tan \rightarrow \sin A$

$\sec A = \frac{1}{\cos A}$

$$\boxed{\sec A = \frac{1}{\sqrt{1 - \sin^2 A}}}$$

[redacted]

EXAMP[redacted] 2 *The value of* $(1+\tan^2\theta)(1+\sin\theta)(1-\sin\theta)(1+\cos\theta)(1-\cos\theta)(1+\cot^2\theta)$ is

(a) 1 (b) –1 (c) 0 (d) –2

[redacted]

$$= \sec^2\theta\,(1-\sin^2\theta)(1-\cos^2\theta)(\operatorname{cosec}^2\theta)$$

$$= \sec^2\theta \times \cos^2\theta \times \sin^2\theta \times \operatorname{cosec}^2\theta$$

$$= \frac{1}{\cancel{\cos^2\theta}} \times \cancel{\cos^2\theta} \times \cancel{\sin^2\theta} \times \frac{1}{\cancel{\sin^2\theta}}$$

$$= 1$$

$$(1+\sin\theta)(1-\sin\theta)$$

$$= (1)^2 - (\sin\theta)^2$$

$$= 1-\sin^2\theta$$

Topic : Trigonometric Identities

#Q. Prove the following identity :

$$\frac{1}{1+\sin\theta}+\frac{1}{1-\sin\theta}=2\sec^2\theta$$

L.H.S

$$=\frac{1}{1+\sin\theta}+\frac{1}{1-\sin\theta}$$

$$=\frac{1(1-\sin\theta)+1(1+\sin\theta)}{(1+\sin\theta)(1-\sin\theta)}$$

$$=\frac{1-\cancel{\sin\theta}+1+\cancel{\sin\theta}}{(1)^2-(\sin\theta)^2}$$

$$=\frac{2}{1-\sin^2\theta}$$

$$=\frac{2}{\cos^2\theta}$$

$$=2\times\frac{1}{\cos^2\theta}$$

$$=\boxed{2\sec^2\theta}=\text{R.H.S.}$$

H.P.

Topic : Trigonometric Identities

#ImpQ. Prove the following identity: $\boxed{\text{cosec}^2\theta + \sec^2\theta = \text{cosec}^2\theta \sec^2\theta}$

[CBSE 2001]

$$= \frac{1}{s^2} + \frac{1}{c^2}$$

$$= \frac{c^2 + s^2}{s^2 c^2}$$

$$= \frac{1}{s^2 c^2}$$

$$= \frac{1}{s^2} \times \frac{1}{c^2} = \boxed{\text{cosec}^2\theta \times \sec^2\theta}$$

Topic : Trigonometric Identities

#Q. Prove the following identity :

$$\frac{\sin\theta}{1-\cos\theta} = \operatorname{cosec}\theta + \cot\theta$$

$$= \left(\frac{\sin\theta}{1-\cos\theta}\right) \times \left(\frac{1+\cos\theta}{1+\cos\theta}\right)$$

$$= \frac{\sin\theta(1+\cos\theta)}{1-\cos^2\theta}$$

$$= \frac{\cancel{\sin\theta}(1+\cos\theta)}{\sin^{\cancel{2}}\theta}$$

$$= \frac{1+\cos\theta}{\sin\theta}$$

$$= \frac{1}{\sin\theta} + \frac{\cos\theta}{\sin\theta}$$

$$= \boxed{\operatorname{cosec}\theta + \cot\theta}$$

Topic : Trigonometric Identities

#Q. Prove the following identity :

$$\frac{\tan\theta + \sin\theta}{\tan\theta - \sin\theta} = \frac{\sec\theta + 1}{\sec\theta - 1}$$

$$= \frac{\frac{\sin\theta}{\cos\theta} + \frac{\sin\theta}{1}}{\frac{\sin\theta}{\cos\theta} - \frac{\sin\theta}{1}}$$

$$= \frac{\cancel{\sin\theta}\left(\frac{1}{\cos\theta} + 1\right)}{\cancel{\sin\theta}\left(\frac{1}{\cos\theta} - 1\right)}$$

$$= \frac{\sec\theta + 1}{\sec\theta - 1}$$

Topic : Trigonometric Identities

#Q. Prove the following identity :

$$\cot\theta - \tan\theta = \frac{2\cos^2\theta - 1}{\sin\theta\cos\theta}$$

$$= \frac{c}{s} - \frac{s}{c}$$

$$= \frac{\cos^2\theta - \sin^2\theta}{\sin\theta\cos\theta}$$

$$= \frac{\cos^2\theta - (1-\cos^2\theta)}{\sin\theta\cos\theta}$$

$$= \frac{2\cos^2\theta - 1}{\sin\theta\cos\theta}$$

Prove that: $\sec^2\theta - \left[\frac{\sin^2\theta - 2\sin^4\theta}{2\cos^4\theta - \cos^2\theta}\right] = 1$

$$= \sec^2\theta - \frac{\sin^2\theta\left(1-2\sin^2\theta\right)}{\cos^2\theta\left(2\cos^2\theta-1\right)}$$

$$= \sec^2\theta - \tan^2\theta\left(\frac{1-2\sin^2\theta}{2\cos^2\theta-1}\right)$$

$$= \sec^2\theta - \tan^2\theta\left[\frac{1-2(1-\cos^2\theta)}{2\cos^2\theta-1}\right]$$

$$= \sec^2\theta - \tan^2\theta\left[\frac{1-2+2\cos^2\theta}{2\cos^2\theta-1}\right]$$

$$= \sec^2\theta - \frac{\tan^2\theta\,\cancel{(2c^2-1)}}{\cancel{(2c^2-1)}}$$

$$= \sec^2\theta - \tan^2\theta$$

$$= 1$$

Topic : Trigonometric Identities

#Q. Prove the following identity :

$$\sqrt{\frac{1-\sin\theta}{1+\sin\theta}} = \sec\theta - \tan\theta$$

$$= \sqrt{\left(\frac{1-\sin\theta}{1+\sin\theta}\right) \times \left(\frac{1-\sin\theta}{1-\sin\theta}\right)}$$

$$= \sqrt{\frac{(1-\sin\theta)^2}{1^2-\sin^2\theta}}$$

$$= \sqrt{\frac{(1-\sin\theta)^2}{\cos^2\theta}}$$

$$\frac{a^2}{b^2} = \left(\frac{a}{b}\right)^2$$

$$= \cancel{\sqrt{\left(\frac{1-\sin\theta}{\cos\theta}\right)^{2}}}$$

$$= \frac{1-\sin\theta}{\cos\theta}$$

$$= \frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta}$$

$$= \sec\theta - \tan\theta$$

Topic : Trigonometric Identities

#Q. Prove the following identity :

$$\frac{1-\sin\theta}{1+\sin\theta} = (\sec\theta - \tan\theta)^2$$

$$= \left(\frac{1-\sin\theta}{1+\sin\theta}\right) \times \left(\frac{1-\sin\theta}{1-\sin\theta}\right)$$

$$= \frac{(1-\sin\theta)^2}{1-\sin^2\theta}$$

$$= \frac{(1-\sin\theta)^2}{\cos^2\theta} = \left(\frac{1-\sin\theta}{\cos\theta}\right)^2 = \left(\frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta}\right)^2 = \boxed{(\sec\theta - \tan\theta)^2}$$

Topic : Trigonometric Identities

#Q. Prove the following identity :

$$\frac{\cos\theta}{1-\sin\theta}+\frac{\cos\theta}{1+\sin\theta}=2\sec\theta$$

$$=\frac{\cos\theta(1+\sin\theta)+\cos\theta(1-\sin\theta)}{(1-\sin\theta)(1+\sin\theta)}$$

$$=\frac{c+\cancel{cs}+c-\cancel{cs}}{1-\sin^2\theta}$$

$$=\frac{2\cancel{c}}{c^{\cancel{2}}}=\frac{2}{c}=\boxed{2\sec\theta}$$

Topic : Trigonometric Identities

#Q. Prove the following identity : [NCERT CBSE 2000C]

$$(\text{cosec}\theta - \cot\theta)^2 = \frac{1-\cos\theta}{1+\cos\theta}$$

$$= \left(\frac{1}{S} - \frac{C}{S}\right)^2$$

$$= \left(\frac{1-C}{S}\right)^2$$

$$= \frac{(1-C)^2}{S^2}$$

$$= \frac{(1-C)^2}{(1^2-C^2)}$$

$$= \frac{\cancel{(1-C)}(1-C)}{\cancel{(1-C)}(1+C)}$$

$$= \boxed{\frac{1-C}{1+C}}$$

[redacted]

Q. [redacted] $1+\dfrac{\cot^2\alpha}{1+\operatorname{cosec}\alpha}=\operatorname{cosec}\alpha$

[redacted]

$$= 1+\frac{\frac{c^2}{s^2}}{\frac{1}{1}+\frac{1}{s}}$$

$$= 1+\frac{\frac{c^2}{s^2}}{\frac{s+1}{s}}$$

$$= 1+\frac{\cancel{s}c^2}{s^{\cancel{2}}(s+1)}$$

$$= 1+\frac{c^2}{s(s+1)}$$

$$= \frac{1}{1}+\frac{c^2}{s^2+s}$$

$$= \frac{s^2+s+c^2}{s^2+s}$$

$$= \frac{1+s}{s^2+s}$$

$$= \frac{\cancel{(1+s)}}{s(\cancel{s+1})}$$

$$= \frac{1}{s}$$

$$= \operatorname{cosec}\alpha$$

Topic : Trigonometric Identities

#Q. Prove the following identity :

$$(\text{cosec}\theta - \sin\theta)(\sec\theta - \cos\theta)(\tan\theta + \cot\theta) = 1$$

$$= \left(\frac{1}{S} - \frac{S}{1}\right)\left(\frac{1}{C} - \frac{C}{1}\right)\left(\frac{S}{C} + \frac{C}{S}\right)$$

$$= \left(\frac{1 - S^2}{S}\right)\left(\frac{1 - C^2}{C}\right)\left(\frac{S^2 + C^2}{CS}\right)$$

$$= \frac{C^2}{S} \times \frac{S^2}{C} \times \frac{1}{SC}$$

$$= \frac{\cancel{C^2} \times \cancel{S^2}}{\cancel{S^2}\cancel{C^2}} = 1$$

$$\frac{1}{a \times b \times a \times b}$$

Topic : Trigonometric Identities

#ImpQ. Prove the following identity : [NCERT, CBSE 2000]

$$\frac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta} = \tan\theta$$

$$= \frac{\sin\theta(1-2\sin^2\theta)}{\cos\theta(2\cos^2\theta-1)}$$

$$= \tan\theta\left(\frac{1-2\sin^2\theta}{2\cos^2\theta-1}\right)$$

$$= \tan\theta\left(\frac{1-2(1-c^2)}{2c^2-1}\right)$$

$$= \tan\theta\left(\frac{1-2+2c^2}{2c^2-1}\right)$$

$$= \tan\theta\left(\frac{\cancel{2c^2-1}}{\cancel{2c^2-1}}\right)$$

$$= \tan\theta$$

Topic : Trigonometric Identities

#Q. Prove the following identity : [NCERT, CBSE 2000]

$(\sin\theta + \mathrm{cosec}\theta)^2 + (\cos\theta + \sec\theta)^2 = 7 + \tan^2\theta + \cot^2\theta$

$$= \sin^2\theta + \mathrm{cosec}^2\theta + 2\sin\theta\,\mathrm{cosec}\theta$$
$$+ \cos^2\theta + \sec^2\theta + 2\cos\theta\sec\theta$$

$$= 1 + 2 + 2 + \mathrm{cosec}^2\theta + \sec^2\theta$$

$$= 5 + 1 + \cot^2\theta + 1 + \tan^2\theta$$

$$= \boxed{7 + \tan^2\theta + \cot^2\theta}$$

Topic : Trigonometric Identities

#Q. Prove the following identity :

$(\sin\theta - \sec\theta)^2 + (\cos\theta - \operatorname{cosec}\theta)^2 = (1 - \sec\theta \operatorname{cosec}\theta)^2$

$= \left(s - \frac{1}{c}\right)^2 + \left(c - \frac{1}{s}\right)^2$

$= s^2 + \frac{1}{c^2} - \frac{2s}{c} + c^2 + \frac{1}{s^2} - \frac{2c}{s}$

$= s^2 + c^2 + \frac{1}{c^2} + \frac{1}{s^2} - \frac{2s}{c} - \frac{2c}{s}$

$= 1 + \frac{s^2 + c^2}{s^2c^2} - 2\left[\frac{s}{c} + \frac{c}{s}\right]$

$\Rightarrow \boxed{1 + \frac{1}{s^2c^2} - \frac{2}{cs}}$

$= 1 + \operatorname{cosec}^2\theta \sec^2\theta - 2\sec\theta \operatorname{cosec}\theta$

$= \boxed{(1 - \sec\theta \operatorname{cosec}\theta)^2}$

Q. Prove that:

$$\frac{1}{1+\sin^2\theta}+\frac{1}{1+\cos^2\theta}+\frac{1}{1+\sec^2\theta}+\frac{1}{1+\operatorname{cosec}^2\theta}=2$$

$$= \frac{1}{1+S^2}+\frac{1}{1+C^2}+\frac{1}{1+\frac{1}{C^2}}+\frac{1}{1+\frac{1}{S^2}}$$

$$= \frac{1}{1+S^2}+\frac{1}{1+C^2}+\frac{1/1}{C^2+1/C^2}+\frac{1/1}{S^2+1/S^2}$$

$$= \frac{1}{1+S^2}+\frac{1}{1+C^2}+\frac{C^2}{1+C^2}+\frac{S^2}{1+S^2}$$

$$= \frac{\cancel{1+S^2}}{\cancel{1+S^2}}+\frac{\cancel{1+C^2}}{\cancel{1+C^2}} = 1+1 = 2$$

$$\frac{1}{2}+\frac{3}{4}+\frac{5}{2}+\frac{9}{4}$$

$$\frac{1+5}{2}+\frac{3+9}{4}$$

Topic : Trigonometric Identities

#Q. Prove the following identity : **[CBSE 2008]**

$(1 + \cot\theta - \operatorname{cosec}\theta)(1 + \tan\theta + \sec\theta) = 2$

$$= \left(\frac{1}{1} + \frac{c}{s} - \frac{1}{s}\right)\left(\frac{1}{1} + \frac{s}{c} + \frac{1}{c}\right)$$

$$= \left(\frac{s+c-1}{s}\right)\left(\frac{c+s+1}{c}\right)$$

$$= \frac{sc + s^2 + \cancel{s} + c^2 + cs + \cancel{c} - \cancel{c} - \cancel{s} - 1}{sc}$$

$$= \frac{2sc + s^2 + c^2 - 1}{sc}$$

$$= \frac{2sc + \cancel{1} - \cancel{1}}{sc}$$

$$= \frac{2\cancel{sc}}{\cancel{sc}}$$

$$= 2$$

$$a^3+b^3=(a+b)(a^2+b^2-ab)$$

$$a^3-b^3=(a-b)(a^2+b^2+ab)$$

$$\boxed{(a+b)^2=a^2+b^2+2ab}$$

$$\boxed{a^2+b^2=(a+b)^2-2ab}$$

Q $\sin^4\theta+\cos^4\theta$

$$(\sin^2\theta)^2+(\cos^2\theta)^2=(\sin^2\theta+\cos^2\theta)^2-2\sin^2\theta\cos^2\theta$$

$$=\boxed{1-2\sin^2\theta\cos^2\theta}$$

Topic : Trigonometric Identities

#Q. Prove the following identity : **[NCERT Exemplar]**

$(\sin^4\theta - \cos^4\theta + 1)\operatorname{cosec}^2\theta = 2$

$$= \left[(s^2)^2 - (c^2)^2 + 1\right]\frac{1}{s^2}$$

$$= \left[(s^2 - c^2)(s^2 + c^2) + 1\right]\frac{1}{s^2}$$

$$= \left[s^2 - c^2 + 1\right]\frac{1}{s^2}$$

$$= (s^2 + s^2)\frac{1}{s^2} = 2\cancel{s^2} \times \frac{1}{\cancel{s^2}} = 2$$

Topic : Trigonometric Identities

#Q. Prove the following identity :

$$\frac{\sin^3\theta+\cos^3\theta}{\sin\theta+\cos\theta}+\sin\theta\cos\theta=1$$

$$a^3+b^3=(a+b)(a^2+b^2-ab)$$

$$=\frac{\cancel{(\sin\theta+\cos\theta)}(\sin^2\theta+\cos^2\theta-\sin\theta\cos\theta)}{\cancel{(\sin\theta+\cos\theta)}}+\sin\theta\cos\theta$$

$$=1-\cancel{sc}+\cancel{sc}$$

$$=1$$

Topic : Trigonometric Identities

#Q. Prove the following identity : [NCERT Exemplar]

$\sin^4 A + \cos^4 A = 1 - 2\sin^2 A \cos^2 A$

$= (s^2)^2 + (c^2)^2$

$= (s^2 + c^2)^2 - 2s^2c^2$

$= \boxed{1 - 2s^2c^2}$

Topic : Trigonometric Identities

#Q. Prove the following identity :

$$\frac{\cos A}{1-\tan A}+\frac{\sin A}{1-\cot A}=\cos A+\sin A$$

$$=\frac{c}{1-\frac{s}{c}}+\frac{s}{1-\frac{c}{s}}$$

$$=\frac{c/1}{c-s/c}+\frac{s/1}{s-c/s}$$

$$=\frac{c^2}{c-s}+\frac{s^2}{s-c}$$

$$=\frac{c^2}{c-s}+\frac{s^2}{-(c-s)}$$

$$=\frac{c^2}{c-s}-\frac{s^2}{c-s}$$

$$=\frac{c^2-s^2}{c-s}$$

$$=\frac{\cancel{(c-s)}(c+s)}{\cancel{(c-s)}}=c+s$$

Topic : Trigonometric Identities

#Q. Prove the following identity :

$$\frac{\cos^2\theta}{1-\tan\theta}+\frac{\sin^3\theta}{\sin\theta-\cos\theta}=1+\sin\theta\cos\theta$$

$$=\frac{c^2}{1-\frac{s}{c}}+\frac{s^3}{s-c}$$

$$=\frac{c^2/1}{c-s/c}+\frac{s^3}{s-c}$$

$$=\frac{c^3}{c-s}+\frac{s^3}{s-c}$$

$$=\frac{c^3}{c-s}-\frac{s^3}{c-s}$$

$$=\frac{c^3-s^3}{c-s}$$

$$=\frac{\cancel{(c-s)}(c^2+s^2+sc)}{\cancel{(c-s)}}$$

$$=\boxed{1+sc}$$

Topic : Trigonometric Identities

#Q. Prove the following identity :

$$\frac{\tan A}{1-\cot A}+\frac{\cot A}{1-\tan A}=1+\tan A+\cot A=1+\sec A\,\mathrm{cosec}A$$

$$=\frac{t}{1-\frac{1}{t}}+\frac{\frac{1}{t}}{1-t}$$

$$=\frac{t/1}{t-1/t}+\frac{1/t}{1-t}$$

$$=\frac{t^2}{t-1}+\frac{1/t}{1-t}$$

$$=\frac{t^2}{t-1}-\frac{1/t}{t-1}$$

$$=\frac{t^2-\frac{1}{t}}{t-1}$$

$$=\frac{t^3-1^3}{t(t-1)}$$

$$=\frac{\cancel{(t-1)}(t^2+1+t)}{t\cancel{(t-1)}}$$

$$=\frac{t^2+1+t}{t}$$

$$=\frac{t^{\cancel{2}}}{\cancel{t}}+\frac{1}{t}+\frac{t}{t}$$

$$=\boxed{\tan A+\cot A+1}$$

$$=\frac{s}{c}+\frac{c}{s}+\frac{1}{1}$$

$$=\frac{s^2+c^2+cs}{cs}$$

$$= \frac{1+CS}{CS}$$

$$= \frac{1}{CS} + \cancel{\frac{CS}{CS}}$$

$$= \boxed{\sec A \operatorname{cosec} A + 1}$$

$$\frac{1}{CS} = \frac{1}{C} \times \frac{1}{S} = \sec A \times \operatorname{cosec} A$$

Topic : Trigonometric Identities

#Q. Prove the following identity : [NCERT Exemplar]

$$\frac{(1+\cot A+\tan A)(\sin A-\cos A)}{\sec^3 A-\operatorname{cosec}^3 A}=\sin^2 A\cos^2 A$$

$$=\frac{\left(1+\frac{c}{s}+\frac{s}{c}\right)\left(\frac{s-c}{1}\right)}{\left(\frac{1}{c^3}-\frac{1}{s^3}\right)}$$

$$=\frac{\left(\frac{sc+c^2+s^2}{sc}\right)\left(\frac{s-c}{1}\right)}{\frac{s^3-c^3}{c^3s^3}}$$

$$=\frac{(sc+c^2+s^2)(s-c)(c^{\cancel{3}\,2}s^{\cancel{3}\,2})}{(s^3-c^3)(\cancel{sc})}$$

$$=\frac{\cancel{(sc+c^2+s^2)}\cancel{(s-c)}(s^2c^2)}{\cancel{(s-c)}\cancel{(s^2+c^2+sc)}}$$

$$=s^2c^2$$

Topic : Trigonometric Identities

#Q. If $\sin\theta + \cos\theta = p$ and $\sec\theta + \operatorname{cosec}\theta = q$, show that $q(p^2 - 1) = 2p$.

[NCERT Exemplar]

L.H.S

$= q(p^2-1)$

$= [\sec\theta + \operatorname{cosec}\theta][(\sin\theta+\cos\theta)^2 - 1]$

$= \left(\frac{1}{c} + \frac{1}{s}\right)(s^2 + c^2 + 2sc - 1)$

$= \left(\frac{s+c}{cs}\right)(\cancel{1} + 2sc - \cancel{1})$

$= \frac{(s+c)}{\cancel{(sc)}}(2\cancel{sc})$

$= 2(s+c)$

$= 2(p)$

$2(\sin\theta + \cos\theta)$

HP

2. If $\tan\alpha + \cot\alpha = 2$, then $\boxed{\tan^{2020}\alpha + \cot^{2020}\alpha} =$

(a) 0 (b) 2 (c) 2020 (d) 2^{2020}

$\tan\alpha + \cot\alpha = 2$

$\tan\alpha + \dfrac{1}{\tan\alpha} = 2$

$\dfrac{\tan^2\alpha + 1}{\tan\alpha} = 2$

$\tan^2\alpha + 1 = 2\tan\alpha$

$\boxed{\tan^2\alpha - 2\tan\alpha + 1 = 0}$

let, $\tan\alpha = x$

$x^2 - 2x + 1 = 0$

$x = 1, 1$

$\tan\alpha = 1$

$\tan\alpha = \tan 45^\circ$

$\alpha = 45^\circ$

$= \tan^{2020}\alpha + \cot^{2020}\alpha$

$= (\tan\alpha)^{2020} + (\cot\alpha)^{2020}$

$= (1)^{2020} + (1)^{2020}$

$= 1 + 1$

$= 2$

Topic : Trigonometric Identities

#Q. If $\boxed{\sin\theta + \cos\theta = \sqrt{3}}$, then prove that $\boxed{\tan\theta + \cot\theta = 1.}$

$$s + c = \sqrt{3}$$

S.B.S

$$(s+c)^2 = (\sqrt{3})^2$$

$$s^2 + c^2 + 2sc = 3$$

$$1 + 2sc = 3$$

$$2sc = 2$$

$$sc = \frac{2}{2}$$

$$\boxed{sc = 1}$$

$$= \frac{s}{c} + \frac{c}{s}$$

$$= \frac{s^2 + c^2}{cs}$$

$$= \frac{1}{cs}$$

$$= \frac{1}{1}$$

$$= 1$$

Topic : Trigonometric Identities

#Q. If θ is an acute angle and $\boxed{\tan\theta + \cot\theta = 2,}$ then the value of $\boxed{\sin^3\theta + \cos^3\theta}$ is

[CBSE, Board Term - I, 2021]

(A) 1

(B) $\frac{1}{2}$

(C) $\frac{\sqrt{2}}{2}$

(D) $\sqrt{2}$

$\frac{s}{c} + \frac{c}{s} = 2$

$\frac{s^2 + c^2}{cs} = 2$

$\frac{1}{cs} = 2$

$\boxed{\frac{1}{2} = cs}$

$= (s+c)(s^2 + c^2 - sc)$

$= (s+c)(1 - sc)$

$= (s+c)\left(1 - \frac{1}{2}\right)$

$= \frac{1}{2}(s+c)$

$(s+c)^2 = s^2 + c^2 + 2sc$

$(s+c)^2 = 1 + \cancel{2}\left(\frac{1}{\cancel{2}}\right)$

$(s+c)^2 = 2$

$s+c = \pm\sqrt{2}$

$\boxed{s+c = \sqrt{2}}$

Topic : Trigonometric Identities

#Q. If $\cos\theta + \sin\theta = \sqrt{2}\cos\theta$, show that $\cos\theta - \sin\theta = \sqrt{2}\sin\theta$

[CBSE 2002 C]

Hw

Topic : Trigonometric Identities

#Q. If $\tan\theta + \sin\theta = m$, $\tan\theta - \sin\theta = n$, show that $m^2 - n^2 = 4\sqrt{mn}$.

H.W

Topic : Trigonometric Identities

#Q. Prove that $(1 - \sin\theta + \cos\theta)^2 = 2(1 + \cos\theta)(1 - \sin\theta)$

$$= \left((1-\sin\theta)+(\cos\theta)\right)^2$$

$$= (1-\sin\theta)^2 + (\cos\theta)^2 + 2(1-\sin\theta)(\cos\theta)$$

$$= 1+\sin^2\theta - 2\sin\theta + \cos^2\theta + 2\cos\theta - 2\sin\theta\cos\theta$$

$$= 2 - 2\sin\theta + 2\cos\theta - 2\sin\theta\cos\theta$$

$$= 2(1-\sin\theta) + 2\cos\theta(1-\sin\theta)$$

$$= (1-\sin\theta)(2+2\cos\theta) = 2(1-\sin\theta)(1+\cos\theta)$$

Topic : Trigonometric Identities

#Q. If $a\cos\theta + b\sin = m$ and $a\sin\theta - b\cos\theta = n$, prove that $a^2 + b^2 = m^2 + n^2$.

H.W

Topic : Trigonometric Identities

#Q. Prove that: $\boxed{(\sec\theta - \operatorname{cosec}\theta)(1 + \tan\theta + \cot\theta) = \sec\theta\tan\theta - \operatorname{cosec}\theta\cot\theta.}$

$$= \left(\frac{1}{c} - \frac{1}{s}\right)\left(\frac{1}{1} + \frac{s}{c} + \frac{c}{s}\right)$$

$$= \left(\frac{s-c}{cs}\right)\left(\frac{cs+s^2+c^2}{cs}\right)$$

$$= \frac{s^3-c^3}{c^2s^2}$$

$$= \frac{s^{\cancel{3}}}{c^2\cancel{s^2}} - \frac{c^{\cancel{3}}}{\cancel{c^2}s^2} = \frac{s}{c^2} - \frac{c}{s^2} = \frac{s\times 1}{c\times c} - \frac{c\times 1}{s\times s}$$

#Q. $\left(\tan\theta + \frac{1}{\cos\theta}\right)^2 + \left(\tan\theta - \frac{1}{\cos\theta}\right)^2 = 2\left(\frac{1+\sin^2\theta}{1-\sin^2\theta}\right)$

H.W

[redacted]

#Q. $\left(\tan\theta + \frac{1}{\cos\theta}\right)^2 + \left(\tan\theta - \frac{1}{\cos\theta}\right)^2 = 2\left(\frac{1+\sin^2\theta}{1-\sin^2\theta}\right)$

H.W

[redacted]

#Q. $\dfrac{\tan A}{(1+\tan^2 A)^2} + \dfrac{\cot A}{(1+\cot^2 A)^2} = \sin A \cos A$

Hw

[redacted]

#Q. $(1+\cot A+\tan A)(\sin A-\cos A)=\dfrac{\sec A}{\operatorname{cosec}^2 A}-\dfrac{\operatorname{cosec} A}{\sec^2 A}=\sin A\tan A-\cot A\cos A$

[CBSE 2018]

H.W

Work Hard!
Dream Big!!
Never Give up!!!


THANK
YOU