Topics

to be covered

Questions on

① Sum of n terms of an AP

$a_n = a + (n-1)d$

$S_n = \dfrac{n}{2}\left[2a + (n-1)d\right]$

$S_n = \dfrac{n}{2}\left[a + a_n\right]$

$l = $ last term.

#Q. The sum of first six terms of an arithmetic progression is 42. The ratio of its 10th term to its 30th term is $1:3$. Calculate the first and the thirteen term of the A.P.

[CBSE 2009]

$$S_6 = 42$$

$$\frac{a_{10}}{a_{30}} = \frac{1}{3}$$

$$\frac{a+9d}{a+29d} = \frac{1}{3}$$

$$3(a+9d) = 1(a+29d)$$

$$3a + 27d = a + 29d$$

$$2a - 2d = 0$$

$$\boxed{a - d = 0} \text{ ②}$$

$$\underline{a = d}$$

To Find : a, a_{13}

Put $a = d$ in eq. ①

$$14 = 2d + 5d$$

$$14 = 7d$$

$$\underline{2 = d}$$

$$\Rightarrow) \; \underline{a = 2}$$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_6 = \frac{6}{2}[2a + 5d]$$

$$42 = 3[2a + 5d]$$

$$\boxed{14 = 2a + 5d} \text{ ①}$$

② $a + 12d$
$2 + 12(2)$

Ans, ㉖

#Q. Find the sum of all three-digit natural numbers, which are divisible by 7.

[CBSE 2006C]

~~100~~ $105, 112, 119, 126 \cdots \cdots \cdots 994.$ ~~999~~

$$a = 105, d = 7$$

$$S_n = \frac{n}{2}[a + 994]$$

$$S_{128} = \frac{128}{2}[105 + 994]$$

let $a_n = 994$

$$a + (n-1)d = 994$$

$$105 + (n-1)7 = 994$$

$$(n-1)7 = 889$$

$$n-1 = \frac{889}{7} = 127$$

$$n-1 = 127$$

$$\boxed{n = 128}$$

$$a_n = 994$$

$$a_{128} = 994$$

total terms = 128

$$S_{128} = 70336 \quad \text{Ans.}$$

#Q. Find the sum of all natural numbers between 250 and 1000 which are exactly divisible by 3.

$$252, 255, 258 \cdots \cdots \cdots 999.$$

$$a = 252, \ d = 3$$

let,

$$a_n = 999$$

$$a + (n-1)d = 999$$

$$252 + (n-1)3 = 999$$

$$(n-1)3 = 747$$

$$(n-1) = \frac{747}{3}$$

$$n - 1 = 249$$

$$\begin{cases} n = 250 \\ a_n = 999 \\ \boxed{a_{250} = 999} \\ \therefore \ \text{total terms} = 250 \\ S_n = \frac{n}{2}[a+l] \\ S_{250} = \frac{250}{2}[252+999] = \boxed{156375} \end{cases}$$

#Q. How many terms of the series 54, 51, 48, ... Be taken so that their sum is 513? Explain the double answer.

let $S_n = 513$

$$\frac{n}{2}\left[2a+(n-1)d\right] = 513$$

$$n\left[2(54)+(n-1)\cdot-3\right] = 513\times2$$

$$n\left[108-3n+3\right] = 1026$$

$$n\left[111-3n\right] = 1026$$

$$111n-3n^2 = 1026$$

$$0 = 3n^2 - 111n + 1026$$

$$0 = 3(n^2 - 37n + 342)$$

$$\boxed{n^2 - 37n + 342 = 0}$$

$$S = -37, \quad P = 342$$

$$\boxed{-18, -19}$$

$$n^2 - 37n + 342 = 0$$
$$n^2 - 18n - 19n + 342 = 0$$
$$n(n-18) - 19(n-18) = 0$$

$$(n-18)(n-19) = 0$$
$$n = 18, \quad n = 19$$

$$S_{18} = 513$$
$$S_{19} = 513$$

$$a_1 + a_2 + a_3 - - - - - - - - a_{18}.$$

$$a_1 + a_2 + a_3 - - - - - - - - a_{18} + a_{19}.$$

513

513

0

$$a_{19} = a + 18d = 0$$

Both the answers are possible as $a_{19} = 0$.

$S_{25} = 200$

$S_{36} = 200$?

A) Ph

B) Pnh

$$a_1 + - - - - - - - - - + a_{25} \Big/ = 200$$

$$a_1 + a_2 - - - - - - - + a_{25} + a_{26} - - - - - - - - - + a_{36} = 200$$

$0 \| $

$-10, -8, -6, -4, -2, 0, 2, 4, 6, 8, 10$

Question

How many terms of the AP $20, 19\frac{1}{3}, 18\frac{2}{3}, \ldots$

must be taken so that their sum is 300? Explain the

double answer.

H.W

#Q. In an A.P., if $S_5 + S_7 = 167$ and $S_{10} = 235$, then find the A.P., where S_n denotes the sum of first n terms.
[CBSE Board, Term – 2, 2015]

$$S_5 + S_7 = 167$$

$$S_{10} = 235$$

$$\frac{5}{2}[2a+4d] + \frac{7}{2}[2a+6d] = 167$$

$$5a + 10d + 7a + 21d = 167$$

$$\boxed{12a + 31d = 167} \text{—①}$$

$$S_{10} = 235$$

$$\frac{10}{2}[2a+9d] = 235$$

$$10a + 45d = 235$$

$$\boxed{2a + 9d = 47} \text{—②}$$

Ans

$$a, a+d, a+2d \cdots\cdots$$

#Q. Let there be an A.P. with first term 'a', common difference 'd'. If a_n denotes its nth terms and S_n the sum of first n terms, find.

(i) n and S_n, if $a = 5$, $d = 3$ and $a_n = 50$.

$$S_n = \frac{n}{2}[a + a_n]$$

$$a_n = a + (n-1)d$$

$$50 = 5 + (n-1)3$$

$$S_n = \frac{n}{2}[5 + 50]$$

$$45 = (n-1)3$$

$$S_n = \frac{n}{2}[55]$$

$$15 = n-1$$

$$\boxed{16 = n}$$

$$S_{16} = 8(55)$$

$$\boxed{S_{16} = 440}$$

#Q. Let there be an A.P. with first term 'a', common difference 'd'. If a_n denotes its nth terms and S_n the sum of first n terms, find.

(ii) n and a, if $a_n = 4$, $d = 2$ and $S_n = -14$.

$$S_n = \frac{n}{2}[a + a_n]$$

$$-14 = \frac{n}{2}[a + 4]$$

$$-28 = n[a + 4]$$

$$-28 = n[6 - 2n + 4]$$

$$-28 = n[10 - 2n]$$

$$-28 = 10n - 2n^2$$

$$a_n = a + (n-1)d$$
$$4 = a + (n-1)2$$
$$4 = a + 2n - 2$$

$$\boxed{6 - 2n = a}$$

$$2n^2 - 10n - 28 = 0$$

$$\boxed{n^2 - 5n - 14 = 0}$$

$$S = 5, \quad P = -14$$

$$\text{7, -2}$$

$$n = 7$$
$$n = -2 \quad \times$$

$$6 - 2(7) = a$$

$$-8 = a$$

#Q. Let there be an A.P. with first term 'a', common difference 'd '. If a_n denotes its nth terms and S_n the sum of first n terms, find.

(iii) n and a_n, if $a = 2, d = 8$ and $S_n = 90$.

$$a_n = a + (n-1)d$$
$$a_n = 2 + (n-1)8$$
$$a_n = 2 + 8n - 8$$
$$a_n = 8n - 6$$

$$a_n = 8(5) - 6$$
$$a_n = 34$$

$$S_n = \frac{n}{2}[a + a_n]$$
$$90 = \frac{n}{2}[2 + a_n]$$
$$180 = n[2 + a_n]$$
$$180 = n[2 + 8n - 6]$$
$$180 = n[8n - 4]$$
$$180 = 8n^2 - 4n$$

$$0 = 8n^2 - 4n - 180$$

$$\boxed{0 = 2n^2 - n - 45}$$

$$P = -90, \ S = -1$$

$$\bigcirc -10, 9$$

$$0 = 2n^2 - 10n + 9n - 45$$
$$0 = 2n(n-5) + 9(n-5)$$
$$0 = (2n+9)(n-5)$$
$$n = -9/2 \quad \boxed{n = 5}$$

#Q. In an A.P. the first term is 2, the last term is 29 and the sum of the terms is 155. Find the common difference of the A.P.

[CBSE 2010]

$$a = 2$$

last term = 29, let $a_n = 29$.

Sum of terms = 155, $S_n = 155$

$$d = ?$$

$$a_n = a + (n-1)d$$

$$29 = 2 + (n-1)d$$

$$27 = (n-1)d$$

$$27 = 9d$$

$$3 = d$$

$$S_n = \frac{n}{2}[a + a_n]$$

$$155 = \frac{n}{2}[2 + 29]$$

$$310 = n(31)$$

$$10 = n$$

#Q. Solve : $1 + 4 + 7 + 10 + ... + x = 287$ ← a_n

(NCERT Exemplar (CBSE 2020) (CBT-2)

this is an A.P, $a = 1$, $d = 3$.

let, $a_n = x$ $S_n = 287$

$a_n = a + (n-1)d$

$x = 1 + (n-1)3$

$x = 1 + 3n - 3$

$x = 3n - 2$

$x = 3(14) - 2$

$x = 40$ Ans.

$S_n = \frac{n}{2}[a + a_n]$

$287 = \frac{n}{2}[1 + x]$

$574 = n[1 + x]$

$574 = n[1 + 3n - 2]$

$574 = n[3n - 1]$

$3n^2 - n = 574$

$3n^2 - n - 574 = 0$

$D = b^2 - 4ac$

$D = (-1)^2 - 4(3)(-574)$

$D = 1 + 6888$

$D = 6889$

$$n = \frac{-b \pm \sqrt{D}}{2a}$$

$$n = \frac{-(-1) \pm \sqrt{6889}}{2(3)}$$

$$n = \frac{1 \pm \sqrt{6889}}{6}$$

$$n = \frac{1 \pm 83}{6}$$

$$n = \frac{1+83}{6}, \frac{1-83}{6} \times$$

$$\boxed{n = 14}$$

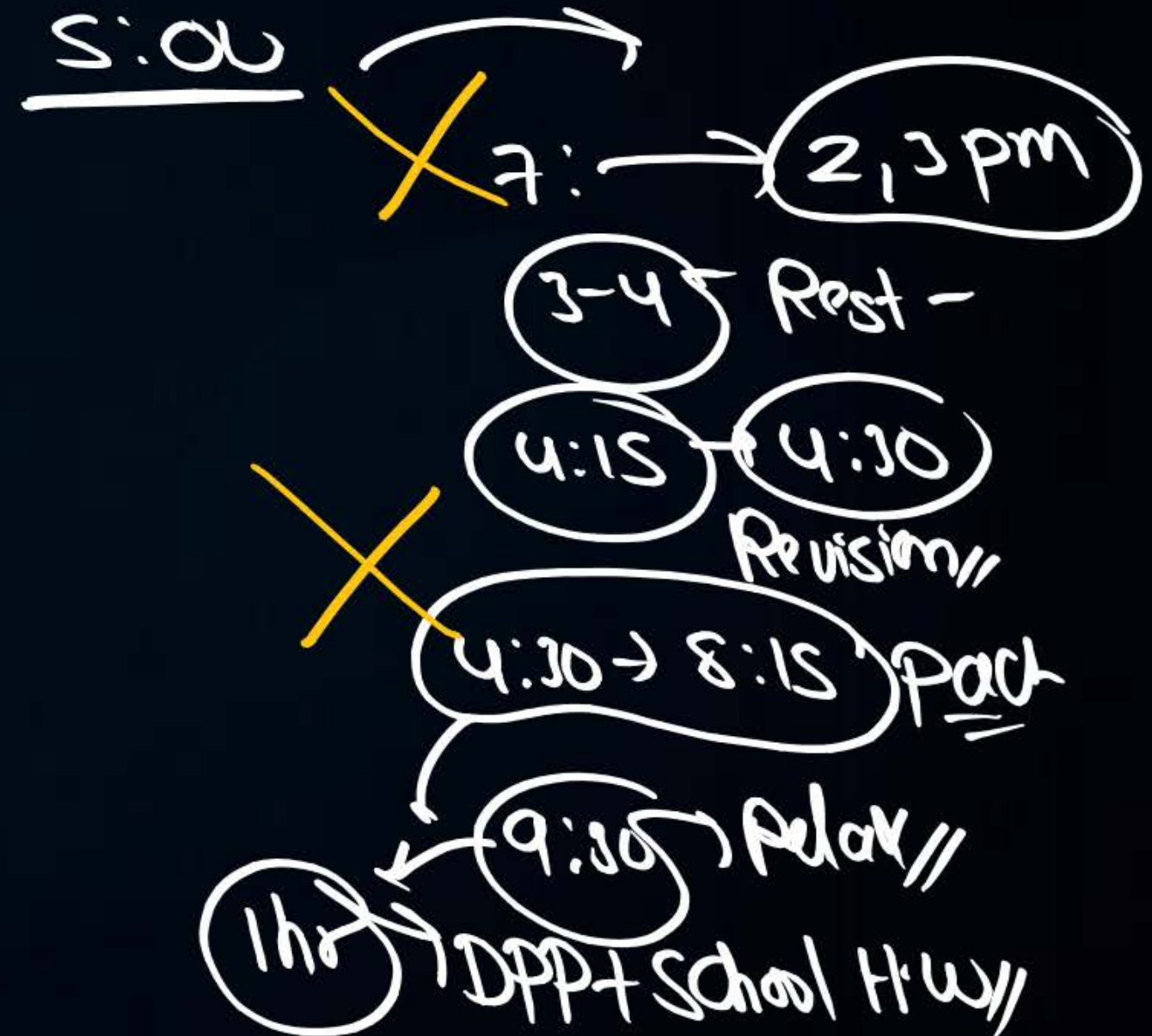$\sqrt{6400}$ $\sqrt{8100}$ $\sqrt{10000}$

$80 \longleftrightarrow 90$ 100

$81, 82, 83 \cdots 89$

$\cdots \cdots \textcircled{1} \cdots 4 \quad 9$

#Q. In an A.P., the sum of first ten terms is -150 and the sum of its next ten terms is -550. Find the A.P.

[CBSE 2010]

#Q. In an A.P., the sum of first ten terms is −150 and the sum of its next ten terms is −550. Find the A.P.

[CBSE 2010]

$$S_{10} = -150 \quad \text{(1)}$$

$$a_{11} + a_{12} + a_{13} + \cdots + a_{20} = -550$$

$$a_1 + a_2 + a_3 \cdots \cdots a_{10} + a_{11} + a_{12} \cdots \cdots a_{20}$$

$$S_{20} - S_{10} = \text{Sum of next 10 terms}$$

$$S_{20} - S_{10} = -550$$

$$S_{20} - (-150) = -550$$

$$S_{20} + 150 = -550$$

$$S_{20} = -550 - 150$$

$$S_{20} = -700 \quad \text{(2)}$$

$$S_{10} = -150$$

$$\frac{10}{2}\left[2a + 9d\right] = -150$$

$$10a + 45d = -150$$

$$\boxed{2a + 9d = -30}$$

$$S_{20} = -700$$

$$\frac{20}{2}\left[2a + 19d\right] = -700$$

$$20a + 190d = -700$$

$$\boxed{2a + 19d = -70}$$

#Q. An A.P., consists of 37 terms. The sum of the three middle most terms is 225 and the sum of the last three terms is 429. Find the A.P.

H.w

DPP

THANK YOU