UDAAN 2025

Trigonometry

Mathematics | Lecture - 09

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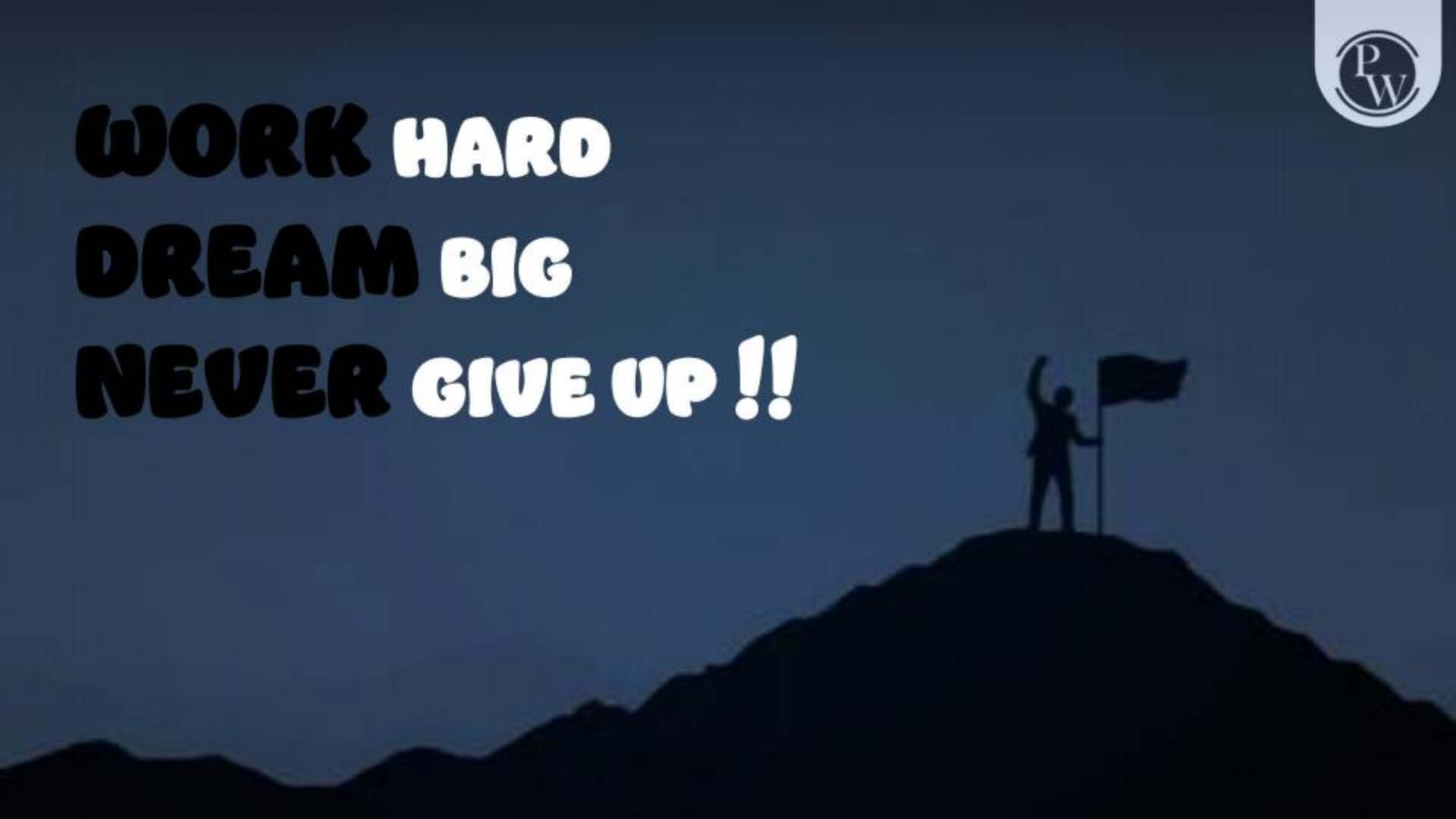
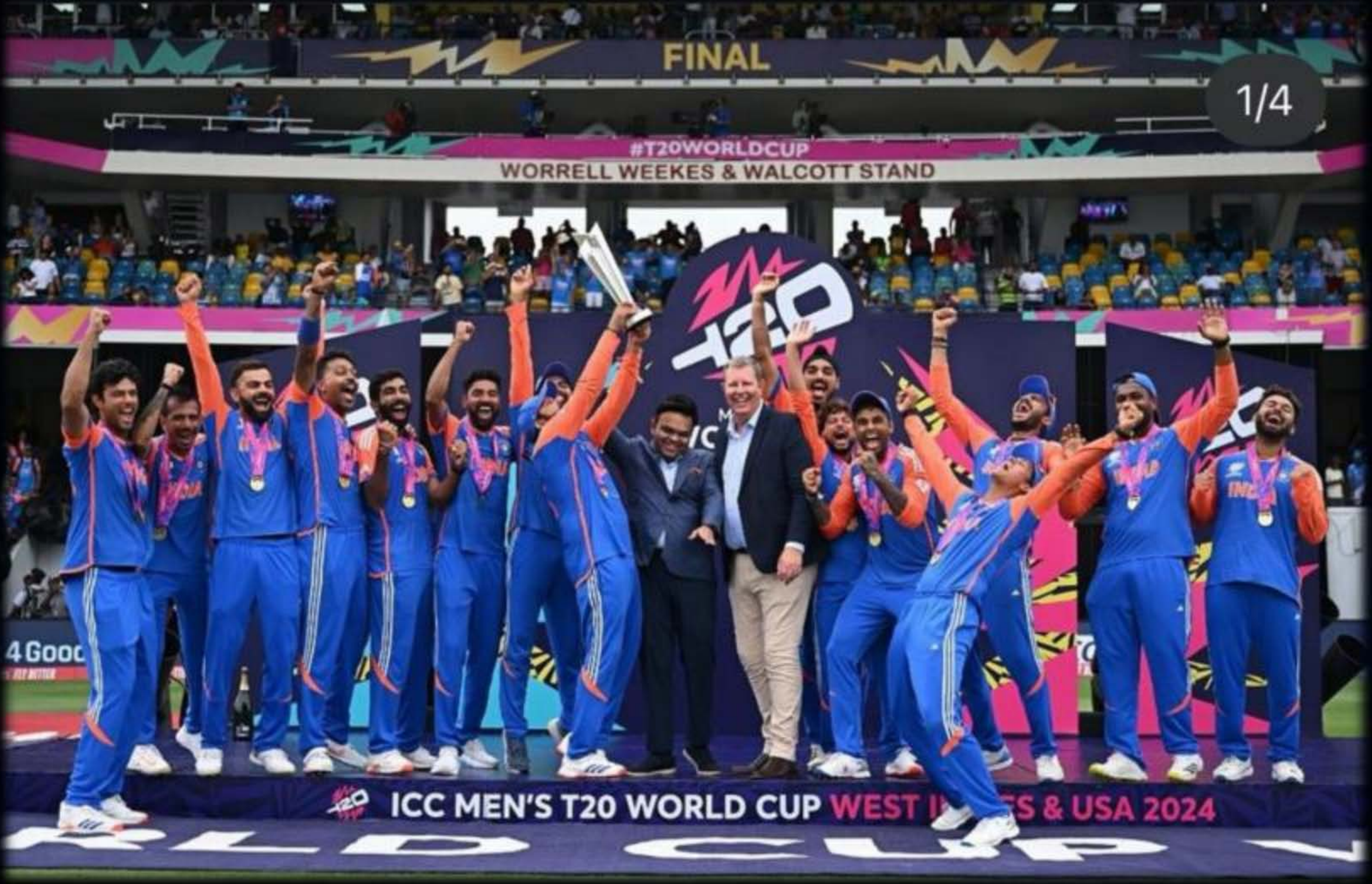
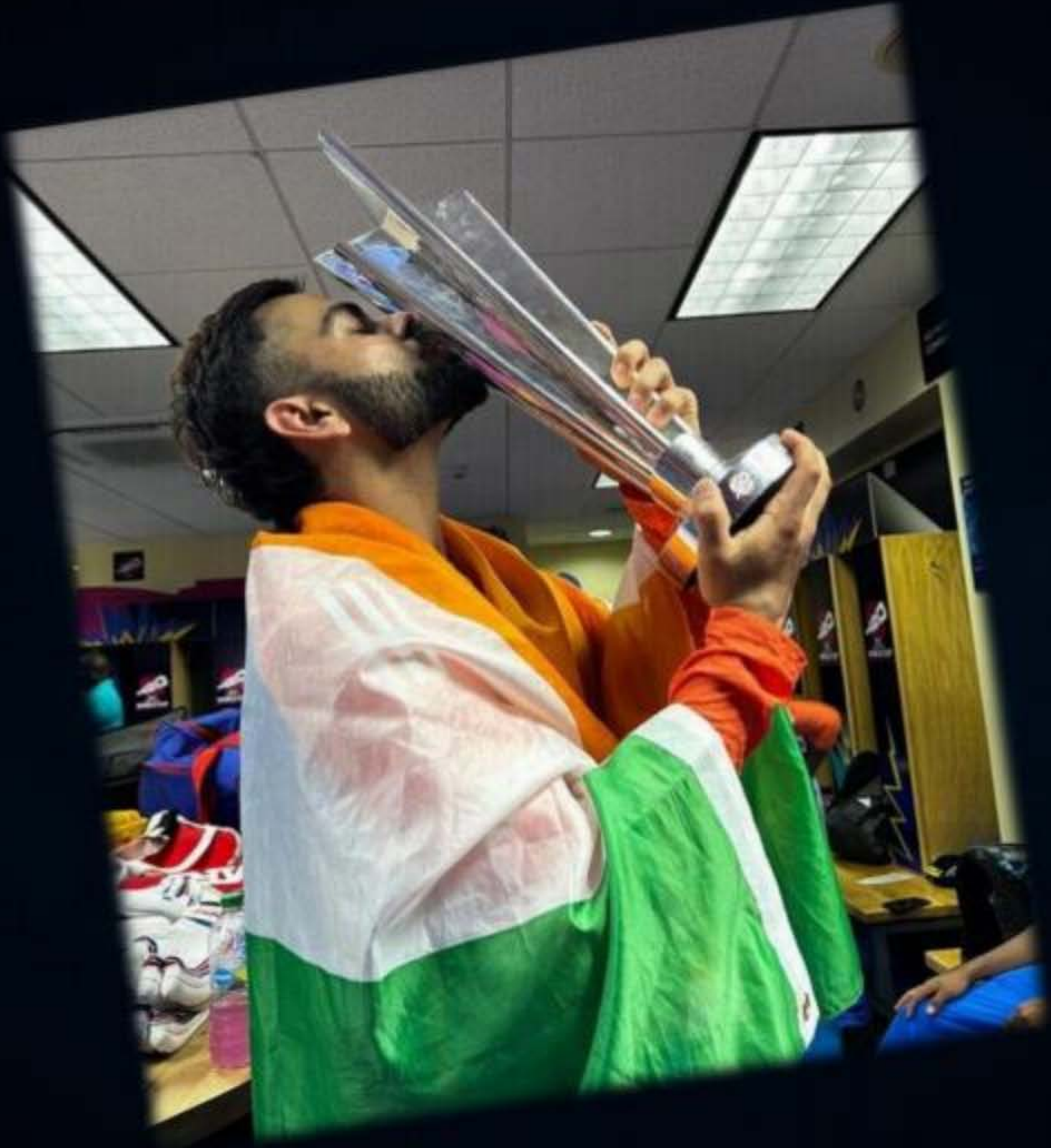
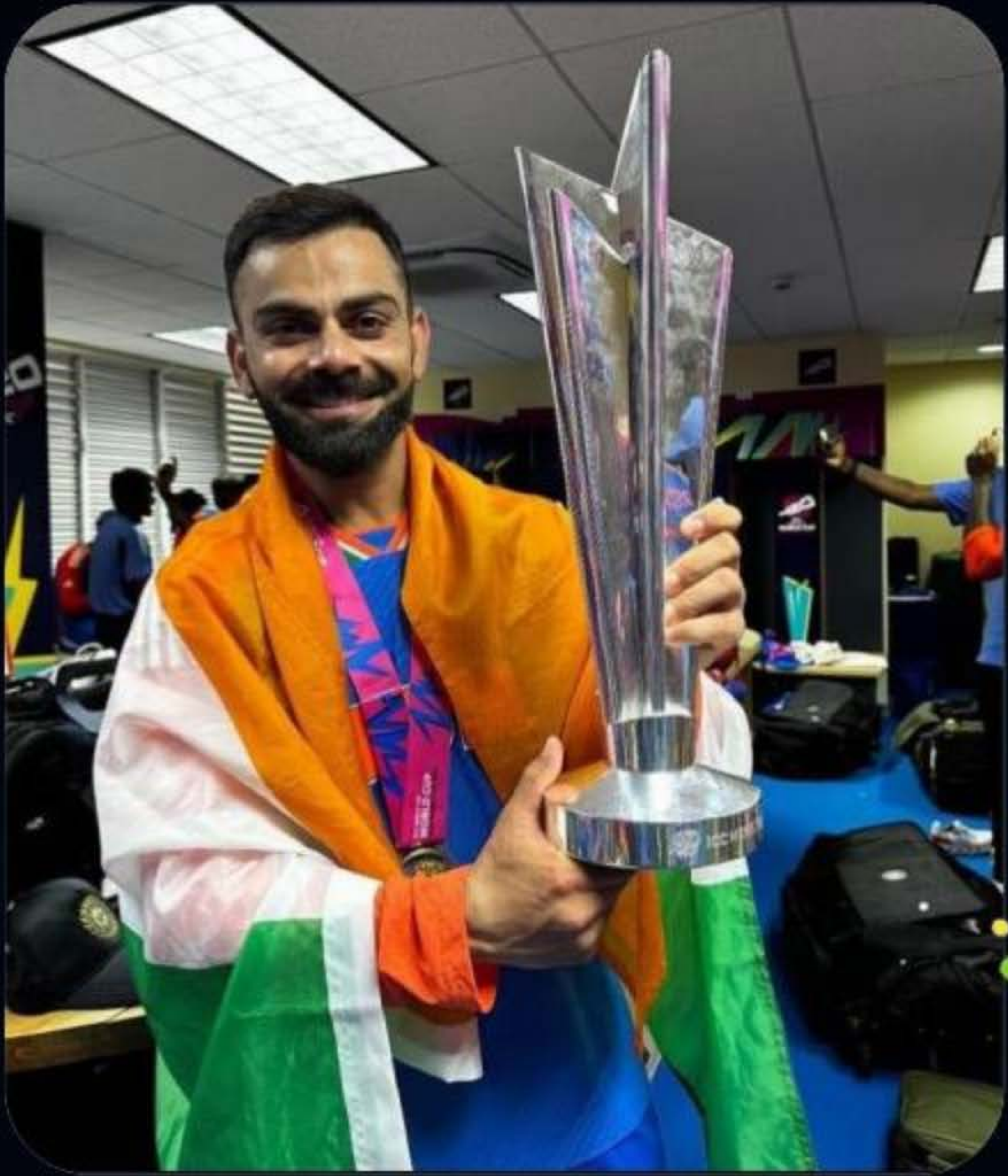
Topics *to be covered*

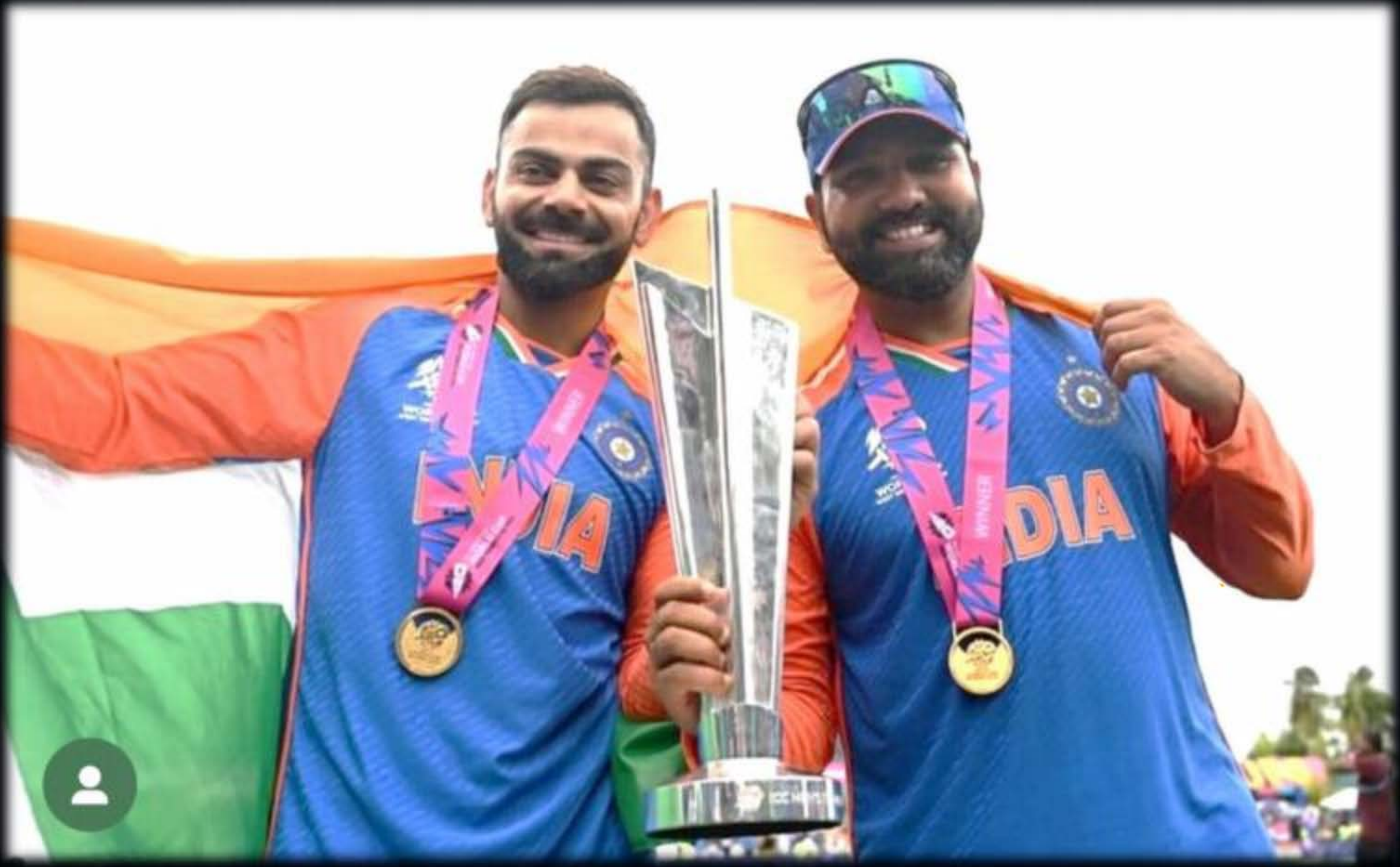
1. Problems on Trigonometric Identities (Part - 3)


WORK HARD
DREAM BIG
NEVER GIVE UP !!


FINAL
#T20WORLDCUP
WORRELL WEEKES & WALCOTT STAND
T20
ICC MEN'S T20 WORLD CUP WEST I ES & USA 2024



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Topic : Trigonometric Identities

#Q. If $\boxed{a \cot\theta + b \operatorname{cosec}\theta = p}$ and $\boxed{b \cot\theta + a \operatorname{cosec}\theta = q}$, then $\boxed{p^2 - q^2}$.

(A) $a^2 - b^2$

(B) $b^2 - a^2$

(C) $a^2 + b^2$

(D) $b - a$

$= p^2 - q^2$

$= (a\cot\theta + b\operatorname{cosec}\theta)^2 - (b\cot\theta + a\operatorname{cosec}\theta)^2$

$= a^2\cot^2\theta + b^2\operatorname{cosec}^2\theta + \cancel{2ab\cot\theta\operatorname{cosec}\theta}$

$- b^2\cot^2\theta - a^2\operatorname{cosec}^2\theta - \cancel{2ab\cot\theta\operatorname{cosec}\theta}$

$= a^2\cot^2\theta + b^2\operatorname{cosec}^2\theta - b^2\cot^2\theta - a^2\operatorname{cosec}^2\theta$

$= a^2(\cot^2\theta - \operatorname{cosec}^2\theta) + b^2(\operatorname{cosec}^2\theta - \cot^2\theta)$

$= a^2(-1) + b^2(1)$

Topic : Trigonometric Identities

#Q. If $\tan\theta + \sin\theta = m$, $\tan\theta - \sin\theta = n$, show that $m^2 - n^2 = 4\sqrt{mn}$.

L.H.S

$$m^2 - n^2$$

$$= (\tan\theta + \sin\theta)^2 - (\tan\theta - \sin\theta)^2$$

$$= (t^2 + s^2 + 2ts) - (t^2 + s^2 - 2ts)$$

$$= \cancel{t^2} + \cancel{s^2} + 2ts - \cancel{t^2} - \cancel{s^2} + 2ts$$

$$= \boxed{4ts}$$

R.H.S

$$= 4\sqrt{mn}$$

$$= 4\sqrt{(t+s)(t-s)}$$

$$= 4\sqrt{t^2 - s^2}$$

$$= 4\sqrt{\frac{s^2}{c^2} - \frac{s^2}{1}}$$

$$= 4\sqrt{s^2\left(\frac{1}{c^2} - 1\right)}$$

$$= 4\sqrt{\frac{s^2(1-c^2)}{c^2}}$$

$$= 4\sqrt{\frac{s^2 \cdot s^2}{c^2}}$$

$$= 4\sqrt{\left(\frac{s \cdot s}{c}\right)^{\cancel{2}}}$$

$$= 4\frac{s}{c} \times s$$

$$= \boxed{4ts}$$

Topic : Trigonometric Identities

#Q. If $\operatorname{cosec}\theta - \cot\theta = \sqrt{2}\cot\theta$, then prove that $\boxed{\operatorname{cosec}\theta + \cot\theta} = \boxed{\sqrt{2}\operatorname{cosec}\theta}$.

[Board Term – 1, 2015]

$$(\operatorname{cosec}\theta - \cot\theta)^2 = (\sqrt{2}\cot\theta)^2$$

$$\operatorname{cosec}^2\theta + \cot^2\theta - 2\operatorname{cosec}\theta\cot\theta = 2\cot^2\theta$$

$$\operatorname{cosec}^2\theta = \cot^2\theta + 2\operatorname{cosec}\theta\cot\theta$$

add $\operatorname{cosec}^2\theta$ both sides.

$$\operatorname{cosec}^2\theta + \operatorname{cosec}^2\theta = \operatorname{cosec}^2\theta + \cot^2\theta + 2\operatorname{cosec}\theta\cot\theta$$

$$2\operatorname{cosec}^2\theta = (\operatorname{cosec}\theta + \cot\theta)^2$$

$$\sqrt{2\operatorname{cosec}^2\theta} = \operatorname{cosec}\theta + \cot\theta$$

$$\boxed{\sqrt{2}\operatorname{cosec}\theta = \operatorname{cosec}\theta + \cot\theta}$$

H.P.//

Topic : Trigonometric Identities

$(a-b)^2 = (b-a)^2$

#Q. If $\cos\theta + \sin\theta = \sqrt{2}\cos\theta$, show that $\cos\theta - \sin\theta = \sqrt{2}\sin\theta$

[CBSE 2002 C]

$(c+s)^2 = (\sqrt{2}c)^2$

$c^2 + s^2 + 2cs = 2c^2$

$s^2 = 2c^2 - c^2 - 2cs$

$s^2 = c^2 - 2cs$

$s^2 + s^2 = s^2 + c^2 - 2cs$

$2s^2 = (s-c)^2$

$(c-s)^2 = 2s^2$

$c - s = \sqrt{2s^2}$

$\boxed{c - s = \sqrt{2}s}$

Topic : Trigonometric Identities

#Q. Prove that $(1 - \sin\theta + \cos\theta)^2 = 2(1 + \cos\theta)(1 - \sin\theta)$

$$= (1-\sin\theta)^2 + (\cos\theta)^2 + 2(1-\sin\theta)(\cos\theta)$$

$$= 1 + S^2 - 2S + C^2 + 2C - 2SC$$

$$= 2 - 2S + 2C - 2SC$$

$$= 2[1 - S + C - SC]$$

$$= 2[(1-S) + C(1-S)]$$

$$= 2[(1-S)(1+C)]$$

$$= \boxed{2(1-S)(1+C)}$$

Topic : Trigonometric Identities

#Q. If $\boxed{\operatorname{cosec}\theta - \sin\theta = m}$ and $\boxed{\sec\theta - \cos\theta = n,}$ prove that $\boxed{(m^2n)^{2/3} + (nm^2)^{2/3} = 1}$.

$$\frac{1}{s} - \frac{s}{1} = m$$

$$\frac{1-s^2}{s} = m$$

$$\boxed{\frac{c^2}{s} = m}$$

$$n = ?$$

$$\frac{1}{c} - \frac{c}{1} = n$$

$$\frac{1-c^2}{c} = n$$

$$\boxed{\frac{s^2}{c} = n}$$

$$= \left[\left(\frac{c^2}{s}\right)^2 \times \frac{s^2}{c}\right]^{2/3} + \left[\left(\frac{s^2}{c}\right)^2 \times \left(\frac{c^2}{s}\right)\right]^{2/3}$$

$$= \left[\frac{c^4}{\cancel{s^2}} \times \frac{\cancel{s^2}}{c}\right]^{2/3} + \left[\frac{s^4}{\cancel{c^2}} \times \frac{\cancel{c^2}}{s}\right]^{2/3}$$

$$= (c^3)^{2/3} + (s^3)^{2/3}$$

$$= c^2 + s^2$$

$$= 1$$

#Q. Prove that: $(\sec\theta - \operatorname{cosec}\theta)(1 + \tan\theta + \cot\theta) = \sec\theta\tan\theta - \operatorname{cosec}\theta\cot\theta$.

$$= \left(\frac{1}{C} - \frac{1}{S}\right)\left(\frac{1}{1} + \frac{S}{C} + \frac{C}{S}\right)$$

$$= \left(\frac{S-C}{CS}\right)\left(\frac{CS + S^2 + C^2}{CS}\right)$$

$$= \frac{S^3 - C^3}{C^2S^2}$$

$$= (a-b)(a^2+b^2+ab)$$

$$= a^3 - b^3$$

$$= \frac{S^{\cancel{3}}}{C^2\cancel{S^2}} - \frac{C^{\cancel{3}}}{\cancel{C^2}S^2}$$

$$= \boxed{\frac{S}{C^2} - \frac{C}{S^2}}$$

$$= \frac{S\times 1}{C\times C} - \frac{C\times 1}{S\times S}$$

$$= \tan\theta \times \sec\theta - \cot\theta \operatorname{cosec}\theta$$

Topic : Practice Sheet Level - 02

#Q. If $\boxed{a\cos\theta - b\sin\theta = c}$, prove that $\boxed{a\sin\theta + b\cos\theta} = \sqrt{a^2 + b^2 - c^2}$

$\Rightarrow (a\cos\theta - b\sin\theta)^2 = (c)^2$

$\Rightarrow a^2\cos^2\theta + b^2\sin^2\theta - 2ab\cos\theta\sin\theta = c^2$

$\Rightarrow a^2(1-\sin^2\theta) + b^2(1-\cos^2\theta) - 2ab\cos\theta\sin\theta = c^2$

$\Rightarrow a^2 - a^2\sin^2\theta + b^2 - b^2\cos^2\theta - 2ab\cos\theta\sin\theta = c^2$

$\Rightarrow a^2 + b^2 - c^2 = a^2\sin^2\theta + b^2\cos^2\theta + 2ab\cos\theta\sin\theta$

$a^2 + b^2 - c^2 = (a\sin\theta + b\cos\theta)^2$

$\boxed{\sqrt{a^2+b^2-c^2} = a\sin\theta + b\cos\theta}$

Topic : Practice Sheet Level - 02

#Q. If $\boxed{\operatorname{cosec}\theta = 2x}$ and $\boxed{\cot\theta = \frac{2}{x}}$, find the value of $\boxed{2\left(x^2 - \frac{1}{x^2}\right)}$.

[CBSE 2010]

$\operatorname{cosec}^2\theta = 4x^2$ — (1) $\quad \cot^2\theta = \frac{4}{x^2}$ — (2)

(1) − (2)

$\operatorname{cosec}^2\theta - \cot^2\theta = 4x^2 - \frac{4}{x^2}$

$1 = 4\left(x^2 - \frac{1}{x^2}\right)$

$\boxed{\frac{1}{2} = 2\left(x^2 - \frac{1}{x^2}\right)}$

Topic : Practice Sheet Level - 02

#Q. $\dfrac{\tan^3\theta}{1+\tan^2\theta} + \dfrac{\cot^3\theta}{1+\cot^2\theta} = \sec\theta \operatorname{cosec}\theta - 2\sin\theta\cos\theta$

$$\frac{\tan^3\theta}{\sec^2\theta} + \frac{\cot^3\theta}{\operatorname{cosec}^2\theta}$$

$$\frac{s^3}{\cancel{c^3}} \times \frac{\cancel{c^2}}{1} + \frac{c^3}{\cancel{s^3}} \times \frac{\cancel{s^2}}{1}$$

$$= \frac{s^3}{c} + \frac{c^3}{s}$$

$$= \boxed{\frac{s^4+c^4}{cs}}$$

$$s^4+c^4 = (s^2)^2 + (c^2)^2$$

$$= (s^2+c^2)^2 - 2s^2c^2$$

$$= \boxed{1-2s^2c^2}$$

$$= \frac{1-2s^2c^2}{cs}$$

$$= \frac{1}{cs} - \frac{2\cancel{s^2}\cancel{c^2}}{\cancel{sc}}$$

$$= \boxed{\sec\theta \operatorname{cosec}\theta - 2\sin\theta\cos\theta}$$

Topic : Practice Sheet Level - 02

#Q. $\dfrac{\tan A}{(1+\tan^2 A)^2}+\dfrac{\cot A}{(1+\cot^2 A)^2}=\sin A\cos A$

$$= \frac{\tan A}{(\sec^2 A)^2}+\frac{\cot A}{(\operatorname{cosec}^2 A)^2}$$

$$= \frac{\tan A}{\sec^4 A}+\frac{\cot A}{\operatorname{cosec}^4 A}$$

$$= \frac{s}{\cancel{c}}\times\frac{c^{\cancel{4}}}{1}+\frac{c}{\cancel{s}}\times\frac{s^{\cancel{4}}}{1}$$

$$= sc^3+cs^3$$

$$= sc(c^2+s^2)=\boxed{sc}$$

Topic : Trigonometric Identities

#Q. If θ is an acute angle and $\tan\theta + \cot\theta = 2$, find the value of $\tan^7\theta + \cot^7\theta$.

$$\frac{t}{1} + \frac{1}{t} = 2$$

$$\frac{t^2+1}{t} = 2$$

$$t^2+1 = 2t$$

$$\boxed{\tan^2\theta - 2\tan\theta + 1 = 0}$$

$\tan\theta = x$

$x^2 - 2x + 1 = 0$

$x^2 - 1x - 1x + 1 = 0$

$x(x-1) - 1(x-1) = 0$

$(x-1)(x-1) = 0$

$x = 1, 1$

$\tan\theta = 1$

$\tan\theta = \tan 45$

$\theta = 45$

THANK YOU