UDAAN

2025

Polynomials

Mathematics | Lecture - 05

By – Ritik Sir

Topics *to be covered*

1. Most Important Questions


WORK HARD
DREAM BIG
NEVER GIVE UP !!

Recap

→ Zeroes

→ ax^2+bx+c — α, β

★

$$\alpha+\beta=-\frac{b}{a}$$

$$\alpha\beta=\frac{c}{a}$$

Q. $2x^2-5x+3$

$\alpha+\beta=-\frac{b}{a}$ | $\alpha\beta=\frac{c}{a}$

$\alpha+\beta=-\frac{(-5)}{2}$ | $\alpha\beta=\frac{3}{2}$

$\alpha+\beta=\frac{5}{2}$

Q11 2, -5

$\alpha+\beta=-\frac{b}{a}$ | $\alpha\beta=\frac{c}{a}$

$2+(-5)=-\frac{b}{a}$ | $(2)(-5)=\frac{c}{a}$

$\frac{-3}{1}=\frac{-b}{a}$ | $\frac{-10}{1}=\frac{c}{a}$

$-b=-3k$

$b=3k$

$a=1k \quad c=-10k$

$b=3k$

ax^2+bx+c

$1kx^2+3kx-10k$

$k[x^2+3x-10]$

non-zero constant

Concept:

$\frac{a}{b} = \frac{2}{3}$

$a = 2$

$b = 3$

Poeeti = 15y.

Habia = 25y.

$P = 3c$

$k = 5c$

$\frac{P}{k} = \frac{3}{5} \times \frac{\square}{\square}$

$P = 3(5)$ $k = 5(5)$

Quadratic polynomial → Sum of zeroes.
→ Product of zeroes.

★ $k[x^2 - (\text{Sum})x + \text{product}]$

Q 3,7

$k[x^2 - 10x + 21]$ — non-zero constant

$k = 1$

$x^2 - 10x + 21$

$= k[x^2 - (S)x + (P)]$

$= k[x^2 - 7x + 10]$

$k = 1$

$x^2 - 7x + 10$

Topic : Relationship between zeroes and coefficients of a quadratic polynomial

#Q. The number of polynomial having zeroes as –2 and 5 is [NCERT Exemplar]

A 1

B 2

C 3

D More than 3

Topic : Relationship b/w zeroes and coefficients of quadratic polynomial

#Q. Find a quadratic polynomial where zeroes are $5 - 3\sqrt{2}$ and $5 + 3\sqrt{2}$.

[CBSE SQP, 2020 -21]

Sum $= 5 - 3\sqrt{2} + 5 + 3\sqrt{2}$

$S = 10$

Product $= (5 - 3\sqrt{2})(5 + 3\sqrt{2})$

$= 5(5 + 3\sqrt{2}) - 3\sqrt{2}(5 + 3\sqrt{2})$

$= 25 + 15\sqrt{2} - 15\sqrt{2} - 18$

$P = 7$

$= k[x^2 - Sx + P]$

$k = 1$

$= x^2 - 10x + 7$

Topic : Relationship between zeroes and coefficients of a quadratic polynomial

#Q. Quadratic polynomial $2x^2 - 3x + 1$ has zeroes as α and β. Now form a quadratic polynomial whose zeroes are 3α and 3β.

[Board Term - 2, 2015]

$2x^2 - 3x + 1$ → α, β

$3\alpha, 3\beta$ → poly = ?

$\alpha + \beta = \frac{-b}{a}$ | $\alpha\beta = \frac{c}{a}$

$\alpha + \beta = -\frac{(-3)}{2}$ | $\alpha\beta = \frac{1}{2}$

$\alpha + \beta = \frac{3}{2}$

Sum $= 3\alpha + 3\beta$
$= 3(\alpha + \beta) = 3 \times \frac{3}{2} = \frac{9}{2}$

Prod. $= 3\alpha \times 3\beta = 9\alpha\beta$
$= 9 \times \frac{1}{2}$
$= \frac{9}{2}$

$= k[x^2 - Sx + P]$

$= k\left[x^2 - \frac{9}{2}x + \frac{9}{2}\right]$

$$= k\left[x^2 - \frac{9}{2}x + \frac{9}{2}\right]$$

$k = 2$

$$= 2\left[x^2 - \frac{9}{2}x + \frac{9}{2}\right]$$

$$= \boxed{2x^2 - 9x + 9}$$

Topic : Relationship b/w zeroes and coefficients of quadratic polynomial

#Q. If α and β are the zeroes of the quadratic polynomial $f(x) = x^2 - x - 2$, find a polynomial whose zeroes are $2\alpha + 1$ and $2\beta + 1$.

ax^2+bx+c

x^2-x-2 → α, $-\beta$

$2\alpha+1,\ 2\beta+1$

$S = 2\alpha+1+2\beta+1$

$= 2\alpha+2\beta+2$

$S = 2(\alpha+\beta)+2$

$S = 2(1)+2$

$S = 4$

$P = (2\alpha+1)(2\beta+1)$

$= 2\alpha(2\beta+1)+1(2\beta+1)$

$= 4\alpha\beta+2\alpha+2\beta+1$

$P = 4\alpha\beta+2(\alpha+\beta)+1$

$P = 4(-2)+2(1)+1$

$P = -S$

$\alpha+\beta = -\frac{b}{a}$ | $\alpha\beta = \frac{c}{a}$

$\alpha+\beta = -\frac{(-1)}{1}$ | $\alpha\beta = -\frac{2}{1}$

$\alpha+\beta = 1$ | $\alpha\beta = -2$

$S = 4, \quad P = -5$

$= k[x^2 - \overbrace{S}^{\text{sum}}x + \overbrace{P}^{\text{product}}]$

$k = 1$

$x^2 - 4x - 5$

Topic : Relationship between zeroes and coefficients of a quadratic polynomial

#Q. If α and β are zeroes of the quadratic equation $x^2 - 7x + 10$, Find the quadratic polynomial whose zeroes are α^2 and β^2.

$x^2 - 7x + 10$: α, β

α^2, β^2

$\alpha^2 + \beta^2 \neq (\alpha+\beta)^2$

$\alpha + \beta = -\frac{b}{a}$

$\alpha + \beta = -\frac{(-7)}{1}$

$\alpha + \beta = 7$

$\alpha\beta = \frac{c}{a}$

$\alpha\beta = 10$

$S = \alpha^2 + \beta^2$

$S = (\alpha+\beta)^2 - 2\alpha\beta$

$S = (7)^2 - 2(10)$

$= 49 - 20$

$S = 29$

$P = \alpha^2\beta^2$

$P = (\alpha\beta)^2$

$P = (10)^2$

$P = 100$

$S = 29, \; P = 100$

$k[x^2 - Sx + P]$

$k = 1$

$x^2 - 29x + 100$

$$(\alpha+\beta)^2 = \alpha^2+\beta^2+2\alpha\beta$$

$$(\alpha+\beta)^2 - 2\alpha\beta = \alpha^2+\beta^2$$

Topic : Relationship b/w zeroes and coefficients of quadratic polynomial

#Q. If α and β are the zeroes of the quadratic polynomial $f(x) = 2x^2 - 5x + 7$, find a polynomial whose zeroes are $2\alpha + 3\beta$ and $3\alpha + 2\beta$.

$2\alpha+3\beta, 3\alpha+2\beta$

$2x^2-5x+7$ → α, β

$\alpha+\beta=\frac{5}{2}, \alpha\beta=\frac{7}{2}$

$S = 2\alpha+3\beta+3\alpha+2\beta$

$= 5\alpha+5\beta$

$S = 5(\alpha+\beta)$

$\text{Sum} = 5\times\frac{5}{2}$

$\text{Sum} = \frac{25}{2}$

$P = (2\alpha+3\beta)(3\alpha+2\beta)$

$= 2\alpha(3\alpha+2\beta)+3\beta(3\alpha+2\beta)$

$= 6\alpha^2+4\alpha\beta+9\beta\alpha+6\beta^2$

$= 6\alpha^2+6\beta^2+13\alpha\beta$

$6(\alpha^2+\beta^2)+13\alpha\beta$

$6[(\alpha+\beta)^2-2\alpha\beta]+13\alpha\beta$

$6\left[\left(\frac{5}{2}\right)^2-2\times\frac{7}{2}\right]+13\left(\frac{7}{2}\right)$

$$= 6\left[\frac{25}{4} - \frac{7}{1}\right] + \frac{91}{2}$$

$$= \frac{3}{\cancel{6}}\cancel{6}\left[\frac{25-28}{\frac{\cancel{4}}{2}}\right] + \frac{91}{2}$$

$$= \frac{3}{2}[-3] + \frac{91}{2}$$

$$= \frac{-9}{2} + \frac{91}{2}$$

$$= \frac{-9+91}{2} = \frac{82}{2} = \boxed{41}$$

$$S = \frac{25}{2}, P = 41$$

$$= h[x^2 - Sx + P]$$

$$= h\left[x^2 - \frac{25}{2}x + 41\right]$$

$h = 2$

$$= 2\left[x^2 - \frac{25}{2}x + 41\right]$$

$$= 2x^2 - 25x + 82$$

Topic : Relationship between zeroes and coefficients of a quadratic polynomial

#Q. If α, β are the zeroes of the polynomial $f(x) = x^2 - p(x+1) - c$ such that $(\alpha+1)(\beta+1) = 0$ then c =

A 1

B 0

C -1

D 2

$(\alpha+1)(\beta+1) = 0$

$\alpha(\beta+1) + 1(\beta+1) = 0$

$\alpha\beta + \alpha + \beta + 1 = 0$

$-p - c + p + 1 = 0$

$-c + 1 = 0$

$-c = -1$

$c = 1$

$x^2 - p(x+1) - c$ — α, β

$x^2 - px - p - c$

$ax^2 + bx + c$

$a = 1,\ b = -p,\ c = -p - c$

$\alpha + \beta = -\frac{b}{a}$

$\alpha + \beta = -\frac{(-p)}{1}$

$\alpha + \beta = p$

$\alpha\beta = \frac{c}{a}$

$\alpha\beta = -p - c$

Assertion and Reason

Direction: *In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Mark the correct choice as.*

(a) Both Assertion (A) and Reason (R) are true, and Reason (R) is the correct explanation of Assertion (A).

(b) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of Assertion (A).

(c) Assertion (A) is true, but Reason (R) is false.

(d) Assertion (A) is false, but Reason (R) is true.

Topic : Relationship between zeroes and coefficients of a quadratic polynomial

#Q. **Assertion (A):** $(2-\sqrt{3})$ is one zero of the quadratic polynomial then other zero will be $(2+\sqrt{3})$.

Reason (R): Irrational zeroes always occurs in pairs.

True

True

Statements

$ax^2+bx+c \rightarrow 2-\sqrt{3}$, $2+\sqrt{3}$

Rationalise:

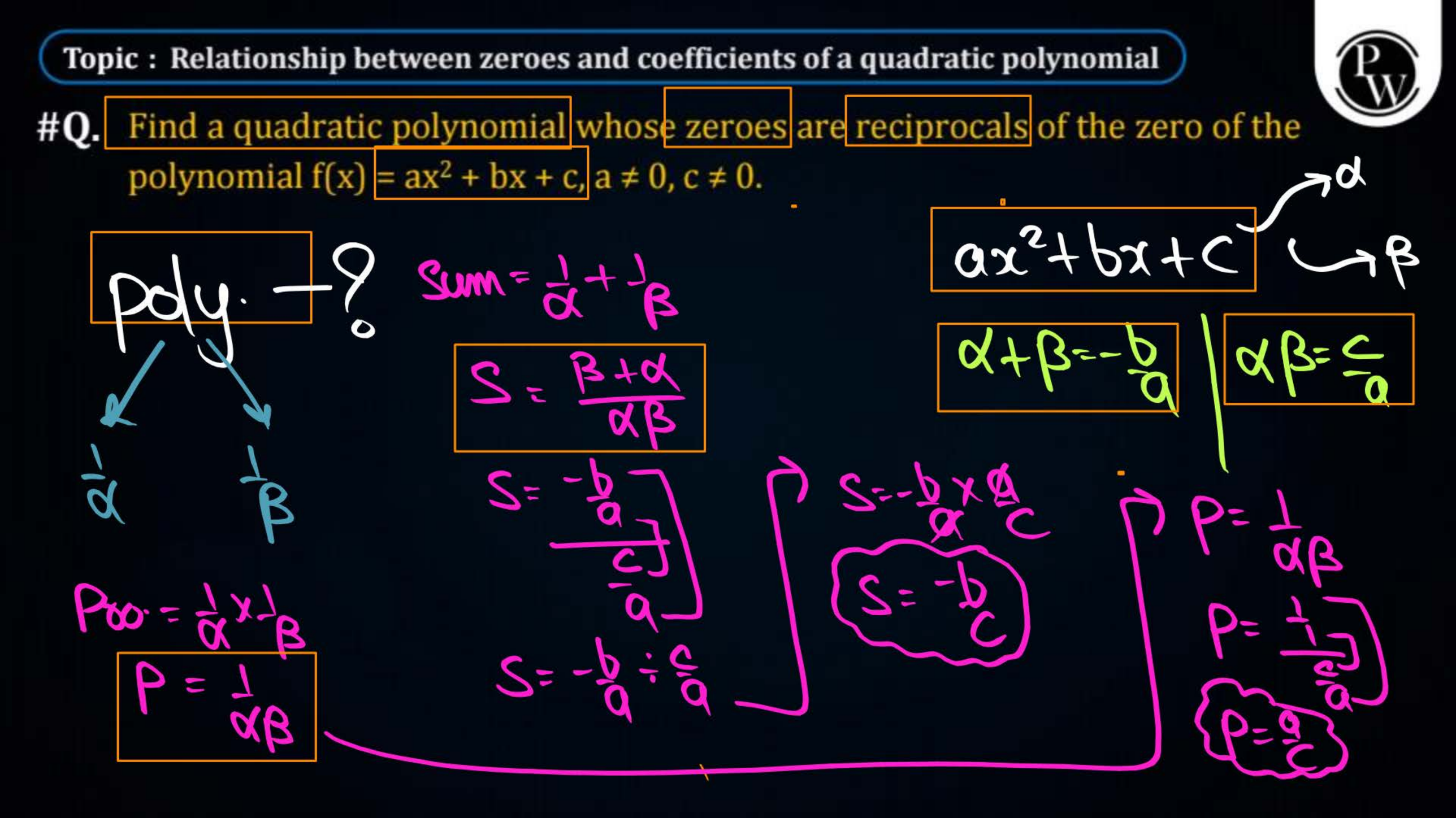
$\frac{1}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}}$

$(2+\sqrt{3}) \longleftrightarrow (2-\sqrt{3})$

Conjugate

Topic : Relationship between zeroes and coefficients of a quadratic polynomial

#Q. Find a quadratic polynomial whose zeroes are reciprocals of the zero of the polynomial $f(x) = ax^2 + bx + c$, $a \neq 0$, $c \neq 0$.



$S = -\frac{b}{c}$, $P = \frac{a}{c}$

$= k\left[x^2 - (\text{Sum})x + \text{Product}\right]$

$= k\left[x^2 - \left(-\frac{b}{c}\right)x + \frac{a}{c}\right]$

$= k\left[x^2 + \frac{b}{c}x + \frac{a}{c}\right]$

$k = c$

$= c\left[x^2 + \frac{bx}{c} + \frac{a}{c}\right]$

$= \boxed{cx^2 + bx + a}$

Topic : Relationship between zeroes and coefficients of a quadratic polynomial

#Q. If the zeroes of the polynomial $x^2 + px + q$ are double in values to the zeroes of $2x^2 - 5x - 3$, find the value of p and q. [2012]

x^2+px+q → 2α, 2β

$2x^2-5x-3$ → α, β

Sum $= -\frac{b}{a}$

$2\alpha+2\beta = -\frac{p}{1}$

$2(\alpha+\beta) = -p$

$2\left(\frac{5}{2}\right) = -p$

$p = -5$

Product $= \frac{c}{a}$

$(2\alpha)(2\beta) = q$

$4\alpha\beta = q$

$2 \times 4 \times \left(\frac{-3}{2}\right) = q$

$q = -6$

$\alpha+\beta = -\frac{b}{a}$

$\alpha+\beta = -\frac{(-5)}{2}$

$\alpha+\beta = \frac{5}{2}$

$\alpha\beta = \frac{c}{a}$

$\alpha\beta = -\frac{3}{2}$

Topic : Relationship between zeroes and coefficients of a quadratic polynomial

#Q. If α and β are the zeroes of the polynomial $x^2 + 4x + 3$, form the polynomial whose zeroes are $1 + \frac{\beta}{\alpha}$ and $1 + \frac{\alpha}{\beta}$.

If you always do what you've always done, you'll always get what you've always got.

Homework

H.W → Class Questions Repeat.

THANK YOU