UDAAN

2025

Trigonometry

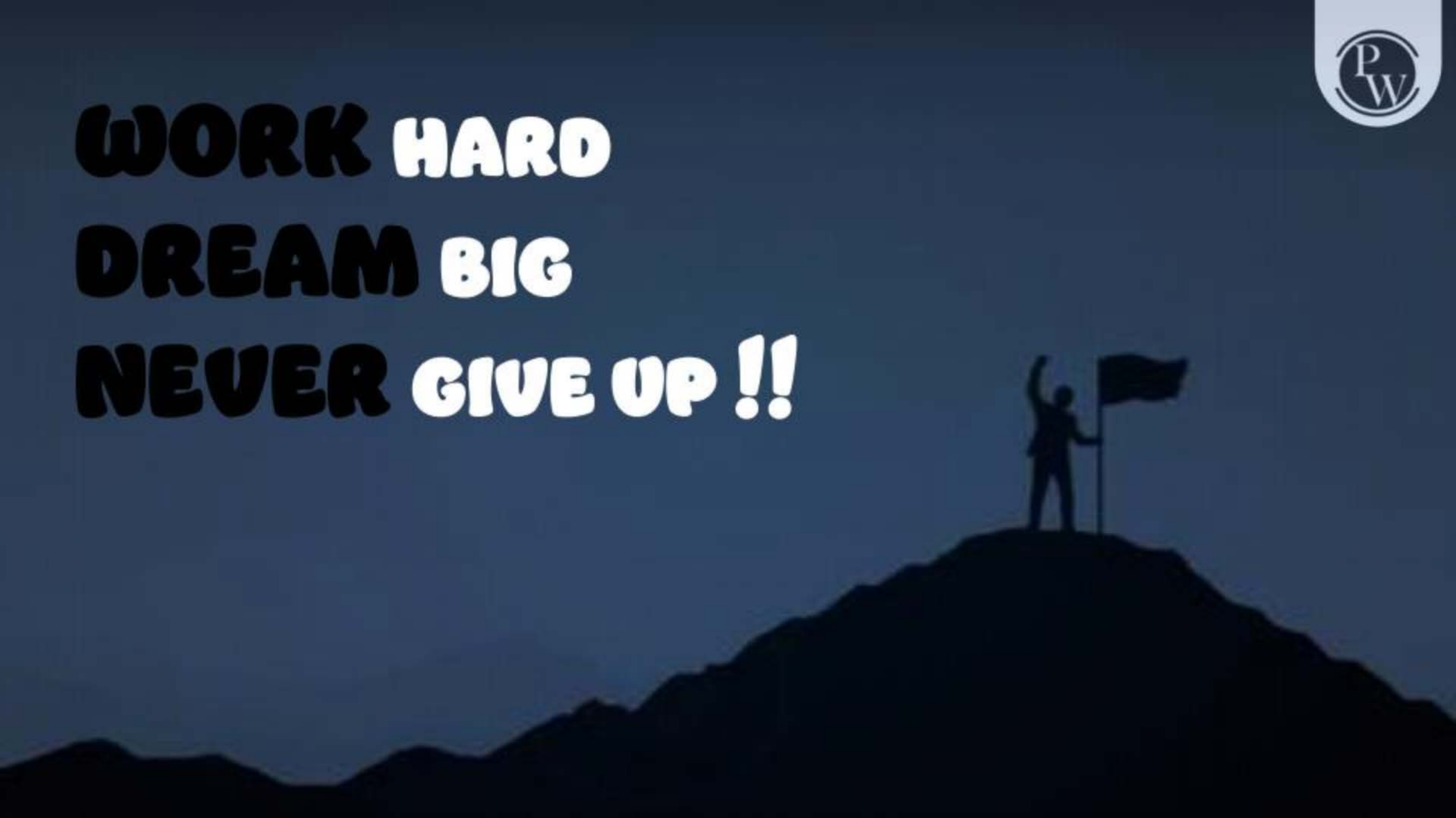
Mathematics | Lecture - 06

By – Ritik Sir

Topics *to be covered*

1. Trigonometric Identities

(Part -3)


WORK HARD
DREAM BIG
NEVER GIVE UP !!

Topic : Trigonometric Identities

Q. Prove the following identity :

$$\tan\theta - \cot\theta = \frac{2\sin^2\theta - 1}{\sin\theta\cos\theta}$$

$$= \frac{\sin\theta}{\cos\theta} - \frac{\cos\theta}{\sin\theta}$$

$$= \frac{(\sin\theta)^2 - (\cos\theta)^2}{\cos\theta\sin\theta}$$

$$= \frac{\sin^2\theta - \cos^2\theta}{\cos\theta\sin\theta}$$

$$= \frac{\sin^2\theta - (1 - \sin^2\theta)}{\cos\theta\sin\theta}$$

$$= \frac{\sin^2\theta - 1 + \sin^2\theta}{\cos\theta\sin\theta}$$

$$= \boxed{\frac{2\sin^2\theta - 1}{\cos\theta\sin\theta}}$$

Topic : Trigonometric Identities

Q. Prove the following identity : **[NCERT Exemplar]**

$$\frac{\sin\theta}{1+\cos\theta}+\frac{1+\cos\theta}{\sin\theta}=2\operatorname{cosec}\theta$$

$$= \frac{\sin^2\theta+(1+\cos\theta)^2}{(1+\cos\theta)(\sin\theta)}$$

$$= \frac{\sin^2\theta+1^2+\cos^2\theta+2\cos\theta}{(1+\cos\theta)\sin\theta}$$

$$= \frac{1+1+2\cos\theta}{(1+\cos\theta)\sin\theta}$$

$$= \frac{2+2\cos\theta}{(1+\cos\theta)\sin\theta}$$

$$= \frac{2(1+\cos\theta)}{(1+\cos\theta)\sin\theta}$$

$$= \frac{2\times 1}{\sin\theta}$$

$$= 2\operatorname{cosec}\theta$$

Topic : Trigonometric Identities

Q. Prove the following identity : **[NCERT]**

$$\frac{1+\sin\theta}{\cos\theta}+\frac{\cos\theta}{1+\sin\theta}=2\sec\theta$$

$$= \frac{(1+s)^2+c^2}{c(1+s)}$$

$$= \frac{1+s^2+2s+c^2}{c(1+s)}$$

$$= \frac{1+1+2s}{c(1+s)}$$

$$= \frac{2+2s}{c(1+s)}$$

$$= \frac{2(1+s)}{c(1+s)}$$

$$= \frac{2}{c}$$

$$= \boxed{2\sec\theta}$$

Topic : Trigonometric Identities

$(a-b)(a+b) = a^2 - b^2$

#Q. Prove the following identity :

$$\frac{\sin A - \sin B}{\cos A + \cos B} + \frac{\cos A - \cos B}{\sin A + \sin B} = 0$$

$$\frac{(\sin A - \sin B)(\sin A + \sin B) + (\cos A - \cos B)(\cos A + \cos B)}{(\cos A + \cos B)(\sin A + \sin B)}$$

$$\frac{(\sin^2 A - \sin^2 B) + (\cos^2 A - \cos^2 B)}{(\cos A + \cos B)(\sin A + \sin B)}$$

$$= \frac{\sin^2 A + \cos^2 A - \sin^2 B - \cos^2 B}{D}$$

$$= \frac{1 - (\sin^2 B + \cos^2 B)}{D}$$

$$= \frac{1 - 1}{D}$$

$$= \frac{0}{D} = \boxed{0}$$

Topic : Trigonometric Identities

#Q. Prove the following identity :

$(\text{cosec}\theta - \sin\theta)(\sec\theta - \cos\theta)(\tan\theta + \cot\theta) = 1$

$$= \left(\frac{1}{\sin\theta} - \frac{\sin\theta}{1}\right)\left(\frac{1}{\cos\theta} - \cos\theta\right)\left(\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}\right)$$

$$= \left(\frac{1-\sin^2\theta}{\sin\theta}\right)\left(\frac{1-\cos^2\theta}{\cos\theta}\right)\left(\frac{\sin^2\theta+\cos^2\theta}{\cos\theta\sin\theta}\right)$$

$$= \frac{\cos^2\theta}{\sin\theta} \times \frac{\sin^2\theta}{\cos\theta} \times \frac{1}{\cos\theta\sin\theta}$$

$$= \frac{\cancel{\cos\theta} \times \cancel{\sin\theta}}{\cancel{\sin\theta} \times \cancel{\cos\theta}} = \boxed{1}$$

Topic : Trigonometric Identities

#Q. Prove the following identity :

$(1 + \cot\theta - \operatorname{cosec}\theta)(1 + \tan\theta + \sec\theta) = 2$

[CBSE 2008]

$$= \left(1 + \frac{\cos\theta}{\sin\theta} - \frac{1}{\sin\theta}\right)\left(1 + \frac{\sin\theta}{\cos\theta} + \frac{1}{\cos\theta}\right)$$

$$= \left(\frac{\sin\theta + \cos\theta - 1}{\sin\theta}\right)\left(\frac{\cos\theta + \sin\theta + 1}{\cos\theta}\right)$$

$$= \frac{\sin\theta\cos\theta + \sin^2\theta + \sin\theta - \cos^2\theta + \cos\theta\sin\theta + \cos\theta - \cos\theta - \sin\theta + 1}{\sin\theta\cos\theta}$$

$$= \frac{2\sin\theta\cos\theta}{\sin\theta\cos\theta} = \boxed{2}$$

Topic : Trigonometric Identities

#ImpQ. Prove the following identity : **[NCERT, CBSE 2000]**

$$\frac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta} = \tan\theta$$

$$\frac{\sin\theta(1-2\sin^2\theta)}{\cos\theta(2\cos^2\theta-1)}$$

$$\frac{\tan\theta\,[1-2(1-\cos^2\theta)]}{2\cos^2\theta-1}$$

$$= \frac{\tan\theta\,[1-2+2\cos^2\theta]}{(2\cos^2\theta-1)}$$

$$= \frac{\tan\theta\,(2\cos^2\theta-1)}{(2\cos^2\theta-1)}$$

$$= \boxed{\tan\theta}$$

Topic : Trigonometric Identities

ImpQ. Prove the following identity : **[NCERT Exemplar]**

$$\frac{\tan\theta - \cot\theta}{\sin\theta\cos\theta} = \sec^2\theta - \operatorname{cosec}^2\theta = \tan^2\theta - \cot^2\theta$$

$$= \frac{\frac{\sin\theta}{\cos\theta} - \frac{\cos\theta}{\sin\theta}}{\sin\theta\cos\theta}$$

$$= \frac{\frac{\sin^2\theta - \cos^2\theta}{\cos\theta\sin\theta}}{\sin\theta\cos\theta}$$

$$= \frac{\sin^2\theta - \cos^2\theta}{\sin^2\theta\cos^2\theta}$$

$$= \frac{\sin^2\theta}{\sin^2\theta\cos^2\theta} - \frac{\cos^2\theta}{\sin^2\theta\cos^2\theta}$$

$$= \frac{1}{\cos^2\theta} - \frac{1}{\sin^2\theta}$$

$$= \sec^2\theta - \operatorname{cosec}^2\theta$$

$$= (1+\tan^2\theta) - (1+\cot^2\theta)$$

$$= 1 + \tan^2\theta - 1 - \cot^2\theta$$

$$= \tan^2\theta - \cot^2\theta$$

Topic : Trigonometric Identities

$a^4 = (a^2)^2$

#Q. Prove the following identity :

$\sec^4\theta - \sec^2\theta = \tan^4\theta + \tan^2\theta$

[NCERT Exemplar]

$= (\sec^2\theta)^2 - \sec^2\theta$

$= (1+\tan^2\theta)^2 - (1+\tan^2\theta)$

$= (1)^2 + (\tan^2\theta)^2 + 2(1)(\tan^2\theta) - 1 - \tan^2\theta$

$= 1 + \tan^4\theta + 2\tan^2\theta - 1 - \tan^2\theta$

$= \boxed{\tan^4\theta + \tan^2\theta}$

M.II

$\sec^2\theta = 1 + \tan^2\theta$

$\sec^4\theta - \sec^2\theta$

$\sec^2\theta(\sec^2\theta - 1)$

$(1+\tan^2\theta)(\tan^2\theta)$

$\tan^2\theta + \tan^4\theta$

Topic : Trigonometric Identities

#Q. Prove the following identity : [CBSE 2000]

$$2\sec^2\theta - \sec^4\theta - 2\operatorname{cosec}^2\theta + \operatorname{cosec}^4\theta = \cot^4\theta - \tan^4\theta$$

$= 2\sec^2\theta - (\sec^2\theta)^2 - 2\operatorname{cosec}^2\theta + (\operatorname{cosec}^2\theta)^2$

$= 2(1+\tan^2\theta) - (1+\tan^2\theta)^2 - 2(1+\cot^2\theta) + (1+\cot^2\theta)^2$

$= 2 + 2\tan^2\theta - (1 + \tan^4\theta + 2\tan^2\theta) - 2 - 2\cot^2\theta + (1 + \cot^4\theta + 2\cot^2\theta)$

$= 2 + 2\tan^2\theta - 1 - \tan^4\theta - 2\tan^2\theta - 2 - 2\cot^2\theta + 1 + \cot^4\theta + 2\cot^2\theta$

$= \boxed{\cot^4\theta - \tan^4\theta}$

Topic : Trigonometric Identities

#Q. Prove the following identity : **[CBSE 2000]**

$$2\sec^2\theta - \sec^4\theta - 2\operatorname{cosec}^2\theta + \operatorname{cosec}^4\theta = \cot^4\theta - \tan^4\theta$$

$= \sec^2\theta(2 - \sec^2\theta) + \operatorname{cosec}^2\theta(-2 + \operatorname{cosec}^2\theta)$

$= (1+\tan^2\theta)\left[2 - (1+\tan^2\theta)\right] + (1+\cot^2\theta)\left[-2 + (1+\cot^2\theta)\right]$

$= (1+\tan^2\theta)(2 - 1 - \tan^2\theta) + (1+\cot^2\theta)(-2 + 1 + \cot^2\theta)$

$= (1+\tan^2\theta)(1 - \tan^2\theta) + (1+\cot^2\theta)(\cot^2\theta - 1)$

Topic : Trigonometric Identities

#Q. Prove the following identity : **[NCERT, CSBE 2000]**

$(\sin\theta + \operatorname{cosec}\theta)^2 + (\cos\theta + \sec\theta)^2 = 7 + \tan^2\theta + \cot^2\theta$

$= \sin^2\theta + \operatorname{cosec}^2\theta + 2\sin\theta\operatorname{cosec}\theta + \cos^2\theta + \sec^2\theta + 2\cos\theta\sec\theta$

$= 1 + \operatorname{cosec}^2\theta + 2\sin\theta\operatorname{cosec}\theta + \sec^2\theta + 2\cos\theta\sec\theta$

$= 1 + 1 + \cot^2\theta + 2\sin\theta\operatorname{cosec}\theta + 1 + \tan^2\theta + 2\cos\theta\sec\theta$

$= 3 + \cot^2\theta + \tan^2\theta + 2\sin\theta\operatorname{cosec}\theta + 2\cos\theta\sec\theta$

$= 3 + \cot^2\theta + \tan^2\theta + 2 + 2 = \boxed{7 + \tan^2\theta + \cot^2\theta}$

Topic : Trigonometric Identities

#Q. Prove the following identity : **[CBSE 2000C]**

$$(\sin\theta + \sec\theta)^2 + (\cos\theta + \operatorname{cosec}\theta)^2 = (1 + \sec\theta \operatorname{cosec}\theta)^2$$

$$= \left(\sin\theta + \frac{1}{\cos\theta}\right)^2 + \left(\cos\theta + \frac{1}{\sin\theta}\right)^2$$

$$= \sin^2\theta + \frac{1}{\cos^2\theta} + \frac{2\sin\theta}{\cos\theta} + \cos^2\theta + \frac{1}{\sin^2\theta} + \frac{2\cos\theta}{\sin\theta}$$

$$= 1 + \frac{1}{\cos^2\theta} + \frac{1}{\sin^2\theta} + \frac{2\sin\theta}{\cos\theta} + \frac{2\cos\theta}{\sin\theta}$$

$$= 1 + \frac{\sin^2\theta + \cos^2\theta}{\cos^2\theta \sin^2\theta} + \frac{2\sin^2\theta + 2\cos^2\theta}{\cos\theta \sin\theta}$$

$$= 1 + \frac{1}{\sin^2\theta\cos^2\theta} + \frac{2(\sin^2\theta+\cos^2\theta)}{\sin\theta\cos\theta}$$

$$= 1 + \frac{1\times 1}{\sin^2\theta\cos^2\theta} + \frac{2\times 1\times 1}{\sin\theta\cos\theta}$$

$$= \boxed{1 + \operatorname{cosec}^2\theta\sec^2\theta + 2\operatorname{cosec}\theta\sec\theta}$$

$$= (1 + \sec\theta\operatorname{cosec}\theta)^2$$

Topic : Trigonometric Identities

#Q. Prove the following identity :

$$(\sin\theta - \sec\theta)^2 + (\cos\theta - \operatorname{cosec}\theta)^2 = (1 - \sec\theta \operatorname{cosec}\theta)^2$$

H.W

Topic : Trigonometric Identities

#Q. Prove the following identity :

$$\frac{(1+\sin\theta)^2+(1-\sin\theta)^2}{\cos^2\theta} = 2\left(\frac{1+\sin^2\theta}{1-\sin^2\theta}\right)$$

$$= \frac{(1+\sin\theta)^2+(1-\sin\theta)^2}{1-\sin^2\theta}$$

$$= \frac{1+\sin^2\theta+2\sin\theta+1+\sin^2\theta-2\sin\theta}{1-\sin^2\theta}$$

$$= \frac{2+2\sin^2\theta}{1-\sin^2\theta} = \boxed{\frac{2(1+\sin^2\theta)}{1-\sin^2\theta}}$$

Homework

① class 'O' Repeat !

② DPP !

+ Aaj i class → practice sheet

THANK YOU