UDAAN 2024

– FOR CLASS 10th STUDENTS

Lecture No.- 03

- Subject Name- Mathematics
- Chapter Name- Surface Area and Volume

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Today's Targets

1. Important Questions (Part - 2)

Name of Solid	Volume	Total Surface Area	Lateral Surface Area
Cube	$V = a^3$	$TSA = 6a^2$	$LSA = 4a^2$
Cuboid	$V = l \times b \times h$	$TSA = 2(lb + bh + hl)$	$LSA = 2h(l + b)$
Cylinder	$V = \pi r^2 h$	$TSA = 2\pi r(h + r)$	$CSA = 2\pi rh$
Hollow Cylinder ($R > r$)	$V = \pi(R^2 - r^2)h$	$TSA = 2\pi(R + r)(h + R - r)$	$2\pi(R + r)$
Cone	$V = \frac{1}{3}\pi r^2 h$	$TSA = \pi r(l + r)$	$CSA = \pi rl$
Sphere	$V = \frac{4}{3}\pi r^3$	$TSA = 4\pi r^2$	$CSA = 4\pi r^2$
Hemisphere	$V = \frac{2}{3}\pi r^3$	$TSA = 3\pi r^2$	$CSA = 2\pi r^2$

Topic : surface area of combination of solids

#Q. A pen stand made of wood is in the shape of a cuboid with four conical depressions to hold pens. The dimensions of the cuboid are 15 cm by 10 cm by 3.5 cm. The radius of each of the depressions is 0.5 cm and the depth is 1.4 cm. Find the volume of wood in the entire stand (see fig.).

$l = 15\text{cm}$
$b = 10\text{cm}$
$h = 3.5\text{cm}$
→ cuboid

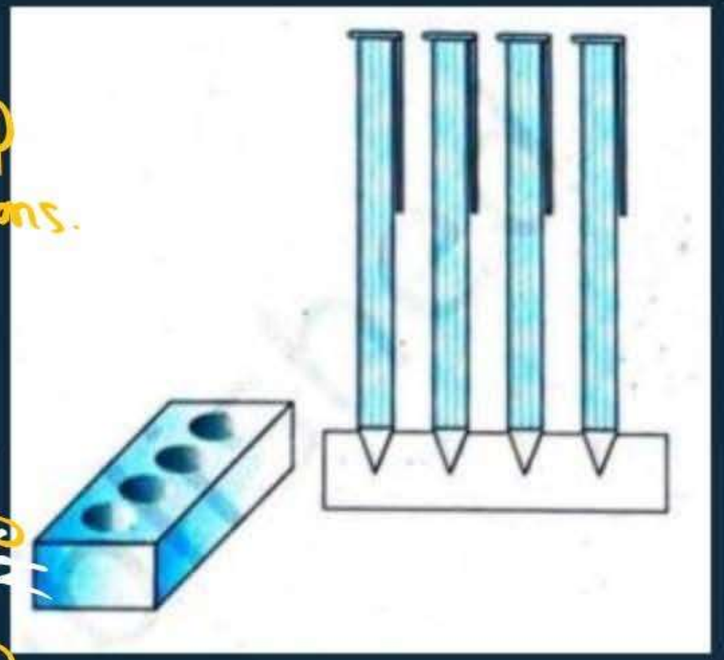
Cone . . .
$r = 0.5\text{cm}$
$h = 1.4\text{cm}$

Volume of wood
= V. of cuboid − V. of 4 conical depressions.

$= lbh - 4\left(\frac{1}{3}\pi r^2 h\right)$

$= 15 \times 10 \times \frac{35}{10} - \frac{1}{3} \times \frac{22}{7} \times \frac{1}{2} \times \frac{1}{2} \times \frac{14}{10}$

$= 525 - \frac{22}{3 \times 7 \times 10}$



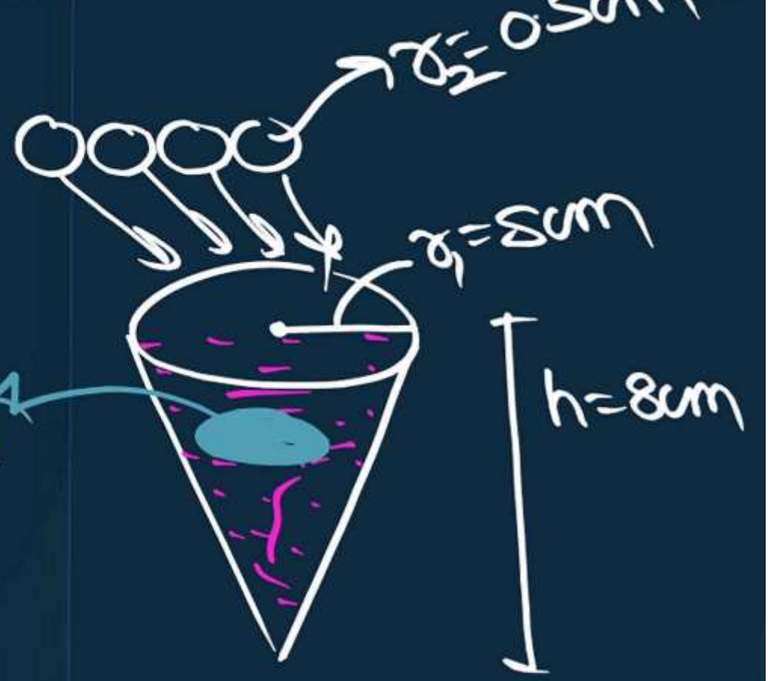
$= 525 - \frac{22}{15}$

$= 525 - 1.46$

$= \boxed{523.54\ cm^3}$ (approx)

Topic : surface area of combination of solids

#Q. A vessel is in the form of an inverted cone. Its height is 8 cm and the radius of its top, which is open, is 5 cm. It is filled with water up to the brim. When lead shots, each of which is a sphere of radius 0.5 cm are dropped into the vessel, one-fourth of the water flows out. Find the number of lead shots dropped in the vessel.



1 lead shot's (v) $= \frac{4}{3}\pi r^3$

let 'n' no. of lead shots $= n \times \frac{4}{3}\pi r^3$

$\frac{1}{4}$ volume of water in the cone $= n \times \frac{4}{3}\pi r_2^3$

$\frac{1}{4} \times \frac{1}{3} \times \pi r_1^2 h = n \times \frac{4}{3} \times \pi r_2^3$

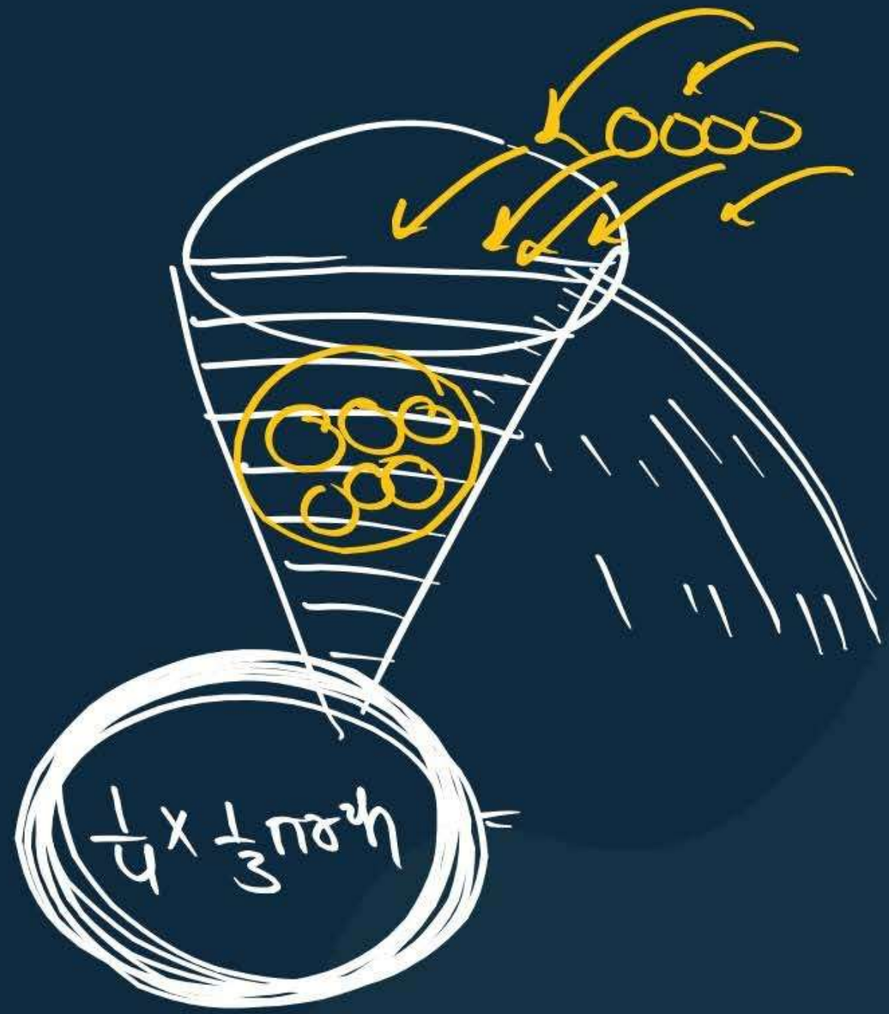
$$\frac{1}{4} \times r_1^2 \times h = n \times 4 \times r_2^3$$

$$\frac{1}{4} \times 8 \times 8 \times 8 = n \times 4 \times \frac{2}{10} \times \frac{2}{10} \times \frac{2}{10}$$

$$2 = n \times 4 \times \frac{1}{10} \times \frac{1}{10} \times \frac{2}{10}$$

$$\frac{50}{200} \quad \frac{1000 \times 2}{4 \times 8} = n$$

$$100 = n$$



#Q. A cylindrical glass tube with radius 10 cm has water upto a height of 9 cm. A metal cube of 8 cm edge is immersed completely. By how much the water level will rise in the glass tube ?

[CBSE Term-II, 2015]

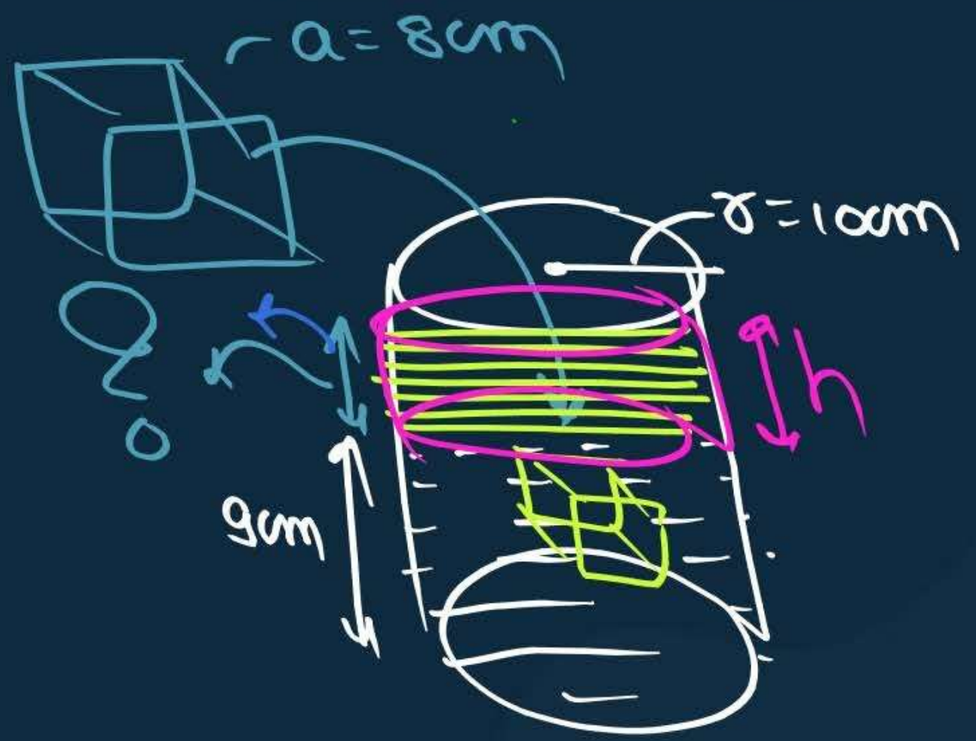
Volume of water displaced = volume of cube.

$\pi r^2 h = a^3$

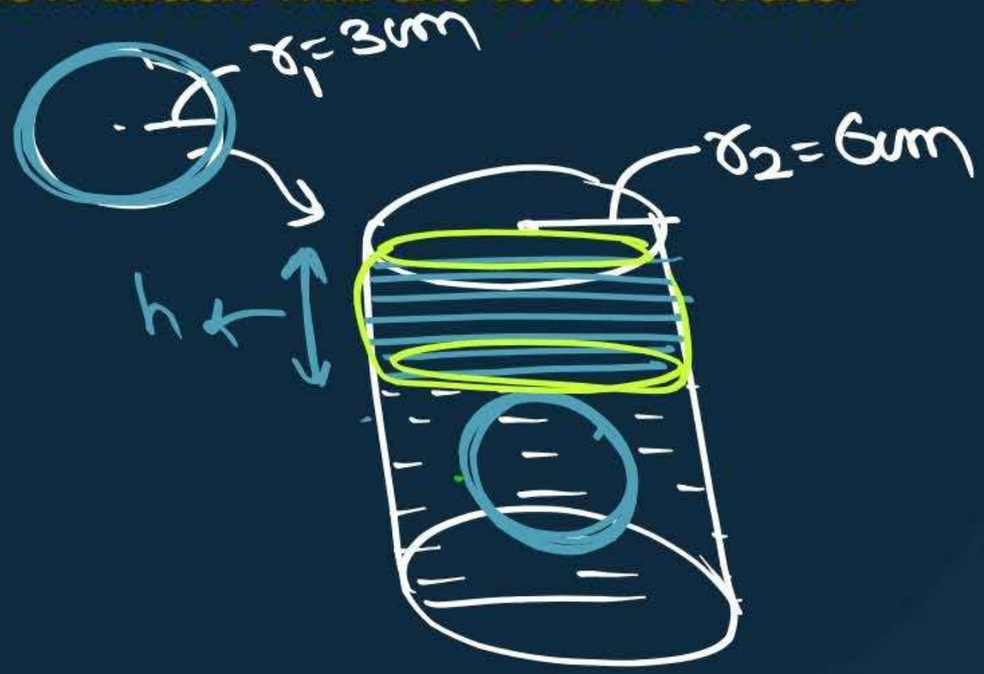
$\frac{22}{7} \times 10 \times 10 \times h = 8 \times 8 \times 8$

$h = \frac{8 \times 8 \times 8 \times 7}{22 \times 10 \times 10}$

$h = 1.629$ cm



#Q. A sphere of diameter 6 cm is dropped in a right circular cylindrical vessel partly filled with water. The diameter of the cylindrical vessel is 12 cm. If the sphere is completely submerged in water, by how much will the level of water rise in the cylindrical vessel?



Volume of water displaced = volume of sphere

$$\pi r_2^2 h = \frac{4}{3}\pi r_1^3$$

$$6 \times 6 \times h = \frac{4}{3} \times 3 \times 3 \times 3$$

$$4h = 4$$

$$h = 1 \text{ cm}$$

#Q. A solid iron pole consists of a cylinder of height 220 cm and base diameter 24 cm, which is surmounted by another cylinder of height 60 cm and radius 8 cm. Find the mass of the pole, given that 1 cm^3 of iron has approximately 8g mass. (Use $\pi = 3.14$)

Sir kaya upar

$1\ cm^3$ → volume

$1\ cm^3 = 8g$

$2\ cm^3 = (2 \times 8)g$

$\vdots$

$10\ cm^3 = (10 \times 8)g$.

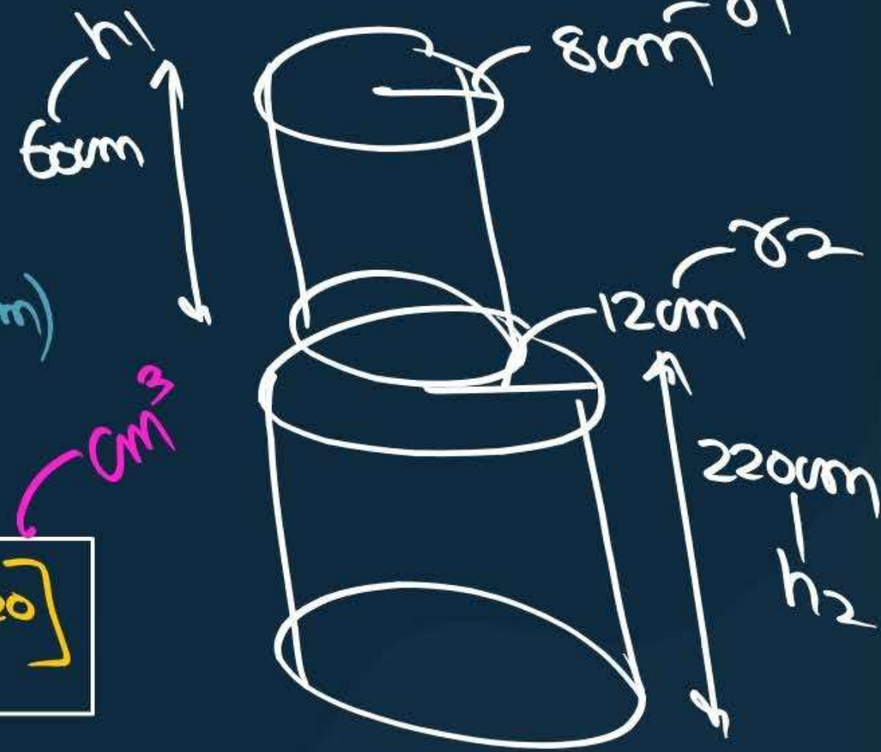
Volume of iron pole

= V. of cylinder ($r = 8cm$) + V. of cylinder ($r = 12cm$)

$= \pi r_1^2 h_1 + \pi r_2^2 h_2$ cm^3

$= \pi [8 \times 8 \times 60 + 12 \times 12 \times 220]$

Final Ans: 892262.4g

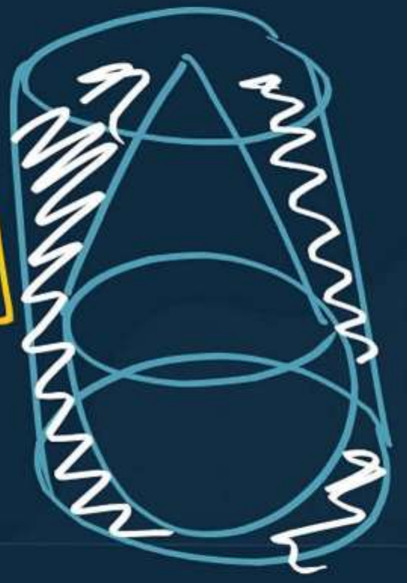


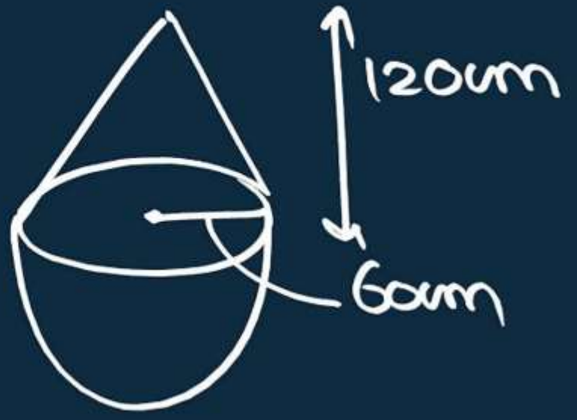
Topic : surface area of combination of solids

#Q. A solid consisting of a right circular cone of height 120 cm and radius 60 cm standing on a hemisphere of radius 60 cm is placed upright in a right circular cylinder full of water such that it touches the bottom. Find the volume of water left in the cylinder, if the radius of the cylinder is 60 cm and its height is 180 cm.

Volume left in the cylinder

= V.of cylinder - [V.of cone + V.of hemi.s]





Ans = 113428.57 cm³

Topic : surface area of combination of solids

#Q. A spherical glass vessel has a cylindrical neck 8 cm long, 2 cm in diameter; the diameter of the spherical part is 8.5 cm. By measuring the amount of water it holds, a child finds its volume to be 345 cm^3. Check whether she is correct, taking the above as the inside measurements, and $\pi = 3.14$.

H.W

#Q. Two cubes have their volumes in the ratio 1 : 27. Find the ratio of their surface area.

[CBSE OD Comptt. Set-I, II, III 2018]

$\frac{V_1}{V_2} = \frac{1}{27}$ → $\frac{a_1^3}{a_2^3} = \frac{1}{27}$

$\left(\frac{a_1}{a_2}\right)^3 = \frac{1}{27}$

$\frac{a_1}{a_2} = \sqrt[3]{\frac{1}{27}}$

$\boxed{\frac{a_1}{a_2} = \frac{1}{3}}$

$\frac{S_1}{S_2} = ?$

$S_1 = 6a_1^2$

$S_2 = 6a_2^2$

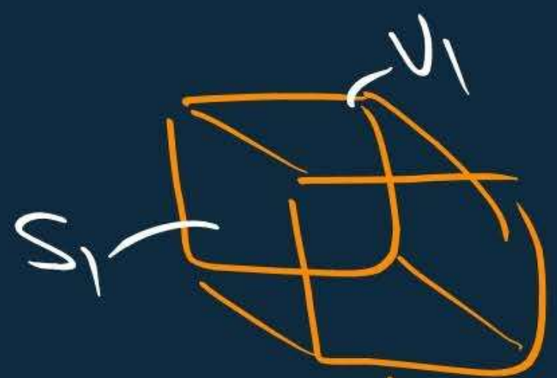
$\frac{S_1}{S_2} = \frac{6a_1^2}{6a_2^2}$

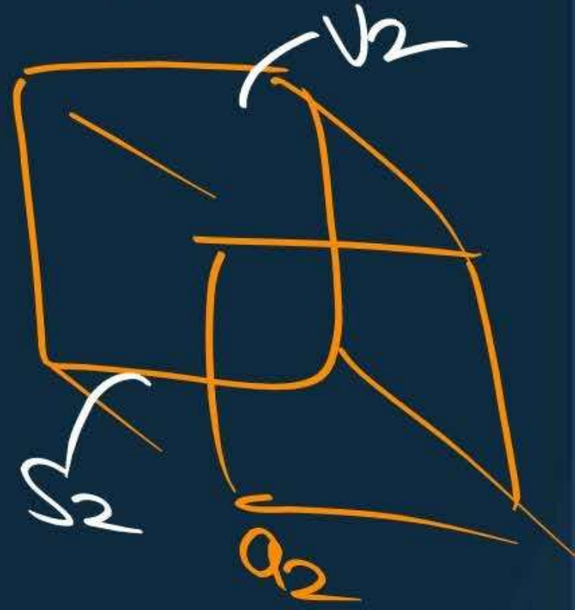
$= \frac{a_1^2}{a_2^2}$

$\frac{S_1}{S_2} = \left(\frac{a_1}{a_2}\right)^2$

$\frac{S_1}{S_2} = \left(\frac{1}{3}\right)^2$

$S_1 : S_2 = 1 : 9$





#Q. A solid is in the shape of a cone mounted on a hemisphere of same base radius. If the curved surface areas of the hemispherical part and the conical part are equal, then find the ratio of the radius and the height of the conical part.

To Find = r/h

C.S.A of hemisphere = C.S.A of Cone

$2\pi r^2 = \pi r l$

$2 \times \pi \times r \times r = \pi \times r \times l$

$2r = l$

$2r = \sqrt{h^2 + r^2}$

$(2r)^2 = \left(\sqrt{h^2 + r^2}\right)^2$

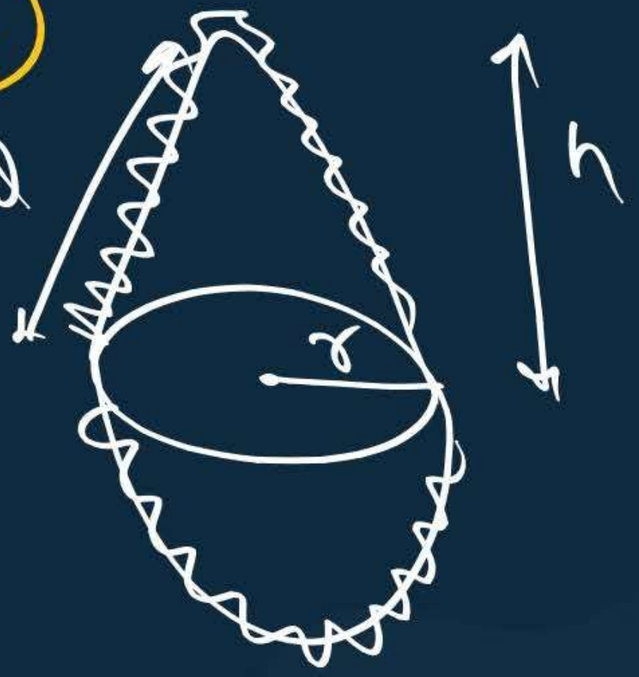
$4r^2 = h^2 + r^2$

$3r^2 = h^2$

$\frac{r^2}{h^2} = \frac{1}{3}$

$\left(\frac{r}{h}\right)^2 = 3^{-1}$

$\frac{r}{h} = \frac{1}{\sqrt{3}}$



#Q. The sum of the radius of base and height of a solid right circular cylinder is 37 cm. If the total surface area of the solid cylinder is 1628 s q. cm, find the volume of the cylinder. (Use $\pi = 22/7$).

$r + h = 37\text{cm}$

$T.S.A = 1628\ cm^2$

Volume of cylinder $= \pi r^2 h$

$2\pi r(r+h) = 1628$

$2 \times \frac{22}{7} \times r \times 37 = 1628$

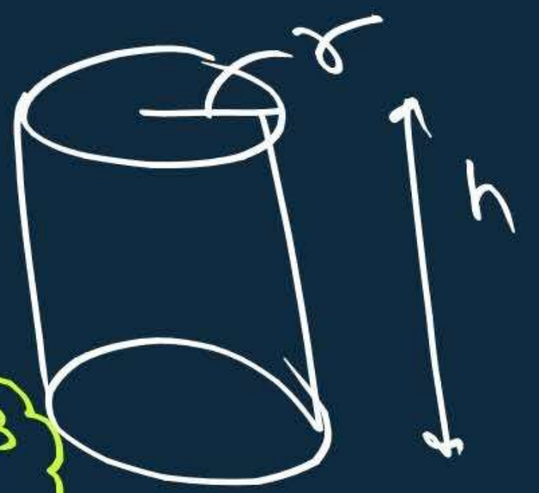
$r = \frac{1628 \times 7}{2 \times 22 \times 37}$

$r = 7\text{cm}$

$r + h = 37$

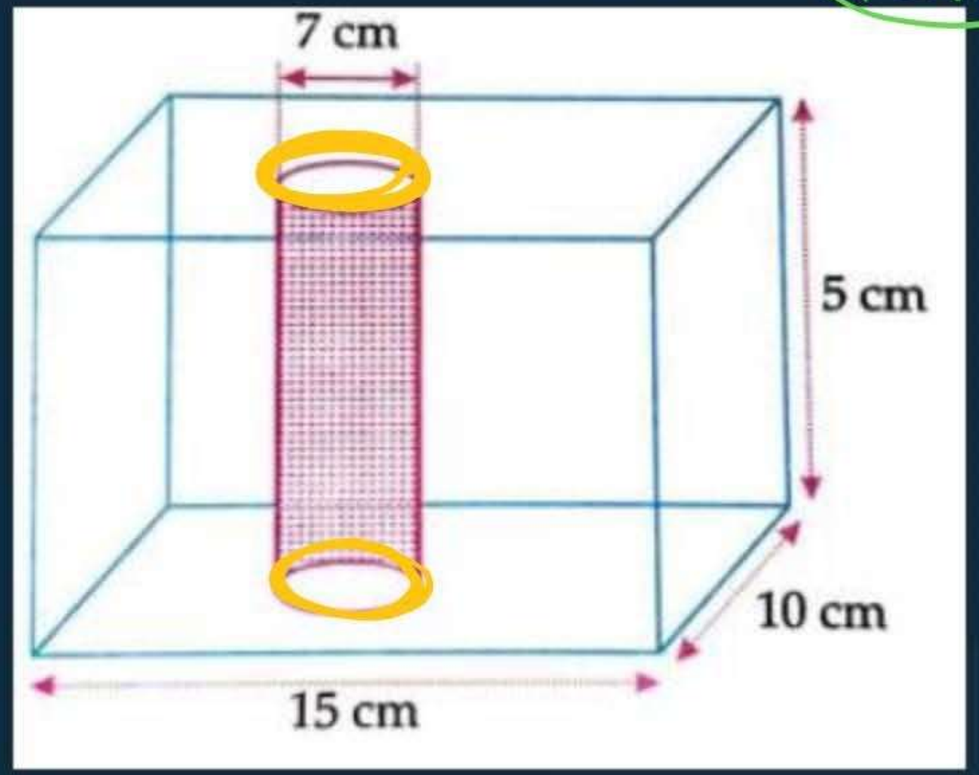
$h = 30\text{cm}$

Volume $= 4620\ cm^3$



Topic : surface area of combination of solids

#Q. In fig. from a cuboidal solid metallic block of dimension 15 cm × 10 cm × 5 cm, a cylindrical hole of diameter 7 cm is drilled out. Find the surface area of the remaining block. (Use $\pi = 22/7$)



T.S.A of Remaining block = T.S.A of cuboid $- \pi r^2 - \pi r^2$ + C.S.A of cylinder

$= 2(lb + bh + hl) - 2\pi r^2 + 2\pi rh$

$= 2(150 + 50 + 75) - 2\pi r(r - h)$

$= 2(275) - 2 \times \frac{22}{7} \times \frac{7}{2}\left(\frac{7}{2} - 5\right)$

$= 550 \cdot 22\left(-\frac{3}{2}\right)$

$= 550 + 33$

$= 583 \text{ cm}^2$

#Q. Two cones of equal height have their radii in the ratio 3 : 2. The ratio of their volumes will be equal to

A 3 : 2

B 9 : 4

C 27 : 8

D 81 : 16

$\frac{r_1}{r_2} = \frac{3}{2}$

$\frac{V_1}{V_2} = \frac{\frac{1}{3}\pi r_1^2 h}{\frac{1}{3}\pi r_2^2 h}$

$\frac{V_1}{V_2} = \frac{r_1^2}{r_2^2}$

$\frac{V_1}{V_2} = \left(\frac{r_1}{r_2}\right)^2$

$= \frac{9}{4}$

Topic : surface area of combination of solids

#Q. Two cones have their heights in the ratio 1 : 3 and radii in the ratio 3 : 1. What is the ratio of their volumes? **[CBSE 2020]**

Topic : surface area of combination of solids

#Q. A cone and a cylinder have the same radii but the height of the cone is 3 times that of the cylinder. Find the ratio of their volumes. **[CBSE 2020]**

H.W

#Q. The volume of a right circular cylinder with the height equal to the radius is $25\frac{1}{7}\ cm^3$. Find the height of the cylinder. [CBSE 2020]

$h = r$

$V = 25\frac{1}{7}\ cm^3$

$V = \frac{176}{7}\ cm^3$

$\pi r^2 h = \frac{176}{7}$

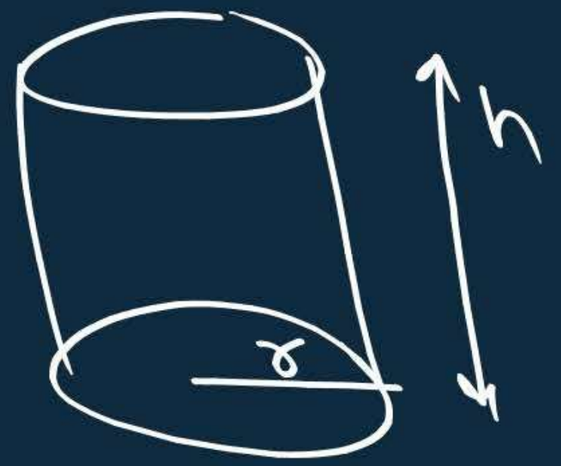
$\pi h^3 = \frac{176}{7}$

$h^3 = \frac{176}{7 \times \pi}$

$h^3 = \frac{176 \times 7}{7 \times 22}$

$h^3 = 8$

$h = 2\ cm$



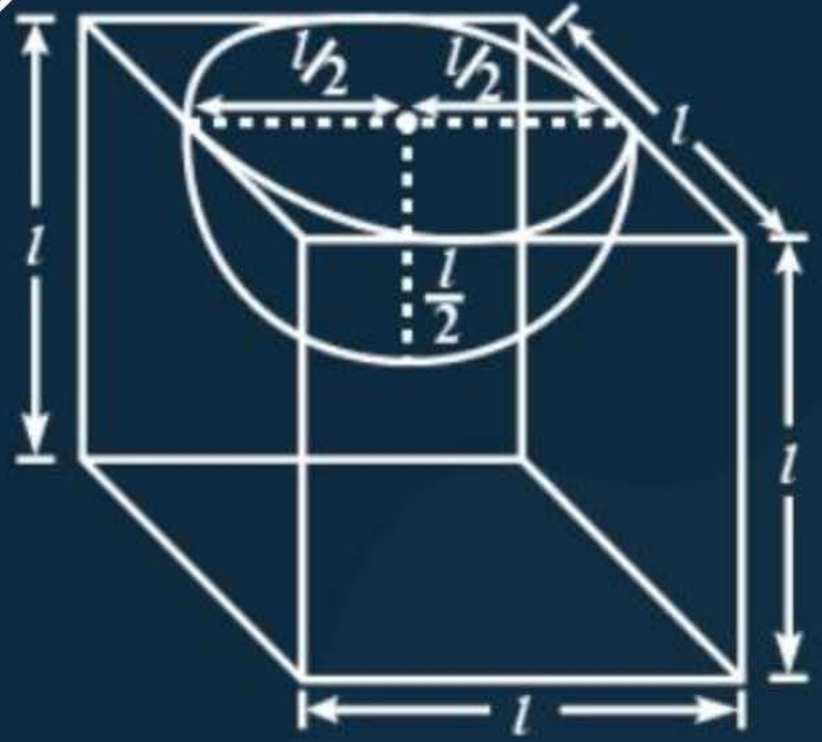
Topic : surface area of combination of solids

#Q. A hemispherical depression is cut out form one face of the cubical wooden block such that the diameter of the hemisphere is equal to the edge of the cube. Determine the surface area of the remaining solid.

[NCERT, CBSE 2010, 2014]

H.W

Ans: $\frac{l^2}{4}(24+\pi)$ sq. units.



#Q. A toy is in the form of hemisphere surmounted by a right circular cone of the same radius as that of the hemisphere. If the radius of the base of the cone is 21 cm and its volume is 2/3 of the volume of the hemisphere, calculate the height of the cone and surface area of the toy. (Use π 22/7)

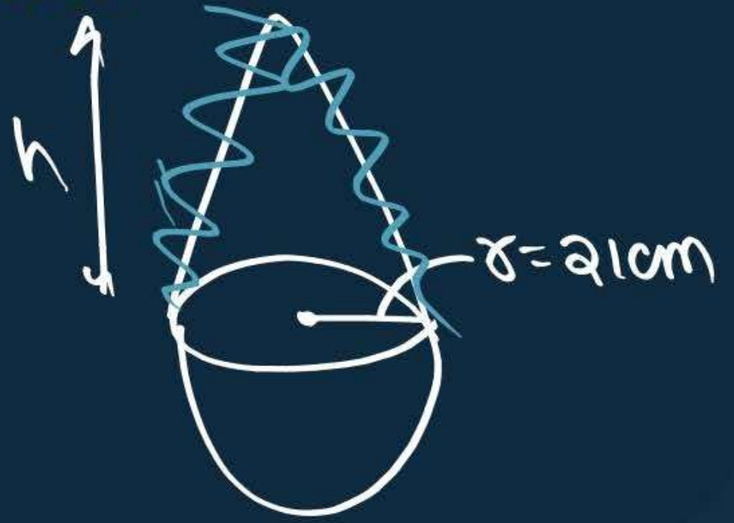
V. of cone = $\frac{2}{3}$ V. of hemisphere

$\frac{1}{3}\pi r^2 h = \frac{2}{3} \times \frac{2}{3} \times \pi r^3$

$\frac{1}{3} \times h = \frac{4}{9} \times r$

$h = \frac{4}{9} \times 21 \times \frac{7}{22} \times 3$

$h = 28$ cm



Surface area of toy = $\pi rl + 2\pi r^2$

Topic : surface area of combination of solids

#Q. A juice seller was serving his customers using glass. The inner diameter of the cylindrical glass was 5 cm, but the bottom of the glass had a hemispherical raised portion which reduced the capacity of the glass. If the height of the glass was 10 cm, find what the apparent capacity of the glass was and what the actual capacity was (Use $\pi = 3.14$) [NCERT, CBSE 2009]

Hw

Ans

apparent capacity = 196.25 cm^3

Actual capacity = 163.55 cm^3


THANK
YOU