

Natural Language Processing and Machine Translation

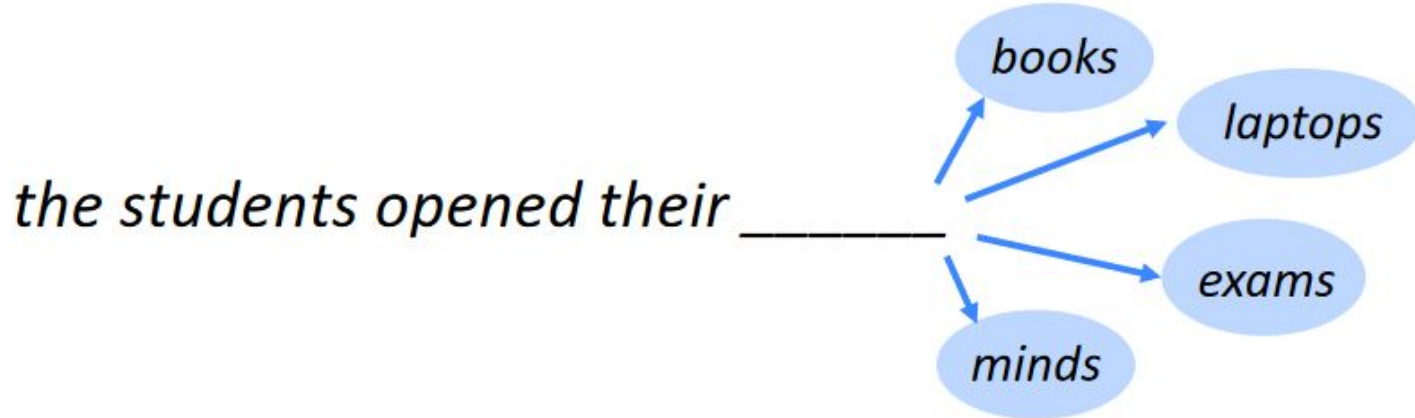
Language Models

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Introduction

- Use of various statistical and probabilistic techniques to determine the probability of a given sequence of words occurring in a sentence
- Analyze bodies of text data to provide a base for word predictions



<https://medium.com/@antonio.lopardo/the-basics-of-language-modeling-1c8832f21079>

N-gram

The cow jumps over the moon

Unigram/ 1-gram

The
cow
jumps
over
the
moon

Bigram/2-gram

The cow
cow jumps
jumps over
over the
the moon

3-gram

The cow jumps
cow jumps over
jumps over the
over the moon

4-gram

The cow jumps over
cow jumps over the
jumps over the moon

If X = Num of words in a given sentence K , the number of n -grams for sentence K would be:

$$Ngrams_K = X - (N - 1)$$

N-gram Language Models

Its water is so transparent that

$P(\text{the} | \text{its water is so transparent that})$.

One approach to calculate this using frequency approach

$$P(\text{the} | \text{its water is so transparent that}) = \frac{C(\text{its water is so transparent that the})}{C(\text{its water is so transparent that})}$$

Will this give us a good estimate in all possible scenarios ??

N-gram Language Models

Another way to do this is using **chain rule of probability**

$$p(w_1 \dots w_n) = p(w_1) \cdot p(w_2 | w_1) \cdot p(w_3 | w_1 w_2) \cdot p(w_4 | w_1 w_2 w_3) \dots p(w_n | w_1 \dots w_{n-1})$$

But this is again computationally expensive

We make this more simpler with an assumption:

- We approximate the context of the word w_k by looking at the last word of the context.
(**Markov Assumption**)

Eg. for bigram

$$p(w) = \prod_{i=1}^{k+1} p(w_i | w_{i-1})$$

N-gram language models

<s> I am a human </s>
<s> I am not a stone </s>
<s> I live in Lahore </s>

$$P(I|<S>) = C(<s>|I) / C(<s>) = 3/3 = 1$$

$$P(am|I) = C(I|am) / C(I) = 2/3$$

$$P(a|am) = C(am|a) / C(a) = 1/2$$

$$P(human|a) = C(a|human) / C(a) = 1/2$$

$$P(</s>|human) = C(human|</s>) / C(human) = 1$$

$$P(not|am) = C(am|not) / C(am) = 1/2$$

$$P(a|not) = C(not|a) / C(not) = 1$$

$$P(stone|a) = C(a|stone) / C(a) = 1/2$$

$$P(</s>|stone) = C(stone|</s>) / C(stone) = 1$$

$$P(live|I) = C(I|live) / C(I) = 1/3$$

$$P(in|live) = C(live|in) / C(live) = 1$$

$$P(Lahore|in) = C(in|Lahore) / C(in) = 1$$

$$P(</s>|Lahore) = C(Lahore|</s>) / C(Lahore) = 1$$

P(I am a human)

$$\begin{aligned} &= P(I|<s>) P(am|I) P(a|am) P(human|a) P(</s>|human) \\ &= 1 * 2/3 * 1/2 * 1/2 * 1 \\ &= 1/6 \end{aligned}$$

P(I am human)

$$\begin{aligned} &= P(I|<s>) P(am|I) P(human|am) P(</s>|human) \\ &= 1 * 2/3 * 0 * 1 \\ &= 0 \Rightarrow \text{Does this seem correct?} \end{aligned}$$

Laplace Smoothing

<s> I am a human </s>
<s> I am not a stone </s>
<s> I live in Lahore </s>

$$P(I|<S>) = C(<s>|I) / C(<s>) = 3/3 = 1$$

$$P(am|I) = C(I|am) / C(I) = 2/3$$

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$$P(human|a) = C(a|human) / C(a) = 1/2$$

$$P(</s>|human) = C(human|</s>) / C(human) = 1$$

$$P(not|am) = C(am|not) / C(am) = 1/2$$

$$P(a|not) = C(not|a) / C(not) = 1$$

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$$P(in|live) = C(live|in) / C(live) = 1$$

$$P(Lahore|in) = C(in|Lahore) / C(in) = 1$$

$$P(</s>|Lahore) = C(Lahore|</s>) / C(Lahore) = 1$$

The solution to the problem of unseen N-grams is to re-distribute some of the probability mass from the observed frequencies to unseen N-grams. This is a general problem in probabilistic modeling called **smoothing**.

$$P_{\text{Laplace}}(w_i) = \frac{c_i + 1}{N + V}$$

Using laplace smoothing (Vocab = 11)

P(I am human)

$$\begin{aligned} &= P(I|<s>) P(am|I) P(human|am) P(</s>|human) \\ &= (3+1)/(3+11) * (2+1)/(3+11) * (0+1)/(2+11) * (1+1)/(1+11) \\ &= 4/14 * 3/14 * 1/13 * 2/12 \\ &= 0.00078 \end{aligned}$$

Good Turing Discounting

- Re-estimate the amount of probability mass to assign N-gram with zero or low counts by looking at the number of N-grams with higher counts
- Use the count of things which are seen once to help estimates the count of things never seen.
- Let N_c be number of N-grams that occur c times
 - For bigrams, N_0 , is the number of bigrams of count 0, N_1 , is the number of bigrams with count 1, etc
- Revised count

$$c^* = (c + 1) \frac{N_{c+1}}{N_c}$$

Kneser Ney Smoothing

- Let the count assigned to each unigram be the number of different words that it follows. Define:

$$N_{1+}(\bullet w_i) = |\{w_{i-1} : c(w_{i-1}w_i) > 0\}|$$

$$N_{1+}(\bullet \bullet) = \sum_{w_i} N_{1+}(\bullet w_i)$$

- Let lower-order distribution be:

$$p_{KN}(w_i) = \frac{N_{1+}(\bullet w_i)}{N_{1+}(\bullet \bullet)}$$

- Put it all together:

$$p_{KN}(w_i | w_{i-n+1}^{i-1}) = \frac{\max\{c(w_{i-n+1}^i) - \delta, 0\}}{\sum_{w_i} c(w_{i-n+1}^i)} + \frac{\delta}{\sum_{w_i} c(w_{i-n+1}^i)} N_{1+}(w_{i-n+1}^{i-1} \bullet) p_{KN}(w_i | w_{i-n+2}^{i-1})$$

Evaluating Language Models

2 types of evaluation

- Extrinsic evaluation
- Intrinsic evaluation

Extrinsic Evaluation

- Model metrics compared with respect to applications implemented in

Intrinsic Evaluation

- Single model evaluation
 - Perplexity

Perplexity

I always order burger with

A good language models with add words like fries, drinks or sausage to the above sentence while a bad language model could add completely random words

fries
drinks
sausage

....

....

burgers
with burgers

A better model of text is the one that assigns a higher probability to the words that actually occurs.

Perplexity is the probability of test set normalized by the number of words

$$\begin{aligned} PP(W) &= P(w_1 w_2 \dots w_N)^{-\frac{1}{N}} \\ &= \sqrt[N]{\frac{1}{P(w_1 w_2 \dots w_N)}} \end{aligned}$$

Perplexity as average branching factor

How hard is the task of recognizing digits '0', '1', '2', '3', '4', '5', '6', '7', '8', '9' ?

There were people in the room.

Assuming that the above space can be filled with any digit from 0-9 and probability of all these digits are equally likely

$$P(\text{any digit}) = 1/10$$

$$PP(\text{any digit}) = (1/10)^{-1}$$

$$= 10$$

Lower the perplexity, better the model

Perplexity as average branching factor

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Lower the perplexity, better the model

Backoff

- Non linear method
- Estimate for an n-gram is allowed to back off through progressively shorter histories
- Trigram version

$$\hat{P}(w_i | w_{i-2}w_{i-1}) = \begin{cases} P(w_i | w_{i-2}w_{i-1}), & \text{if } C(w_{i-2}w_{i-1}w_i) > 0 \\ \alpha_1 P(w_i | w_{i-1}), & \text{if } C(w_{i-2}w_{i-1}w_i) = 0 \\ & \text{and } C(w_{i-1}w_i) > 0 \\ \alpha_2 P(w_i), & \text{otherwise.} \end{cases}$$

Word Representation

How do we represent the meaning of a word?

⇒ One common NLP solution: Use a taxonomic resource(eg. **WordNet**), a thesaurus containing a list of synonyms set and hypernyms set (“is a” relationships)

E.g., synonyms set containing “good”:

```
noun: good
noun: good, goodness
noun: good, goodness
noun: commodity, trade_good, good
adj: good
adj (sat): full, good
adj: good
adj (sat): estimable, good, honorable, respectable
adj (sat): beneficial, good
adj (sat): good
adj (sat): good, just, upright
...
adverb: well, good
adverb: thoroughly, soundly, good
```

E.g., hypernyms of “panda”

```
[Synset('procyonid.n.01'),
Synset('carnivore.n.01'),
Synset('placental.n.01'),
Synset('mammal.n.01'),
Synset('vertebrate.n.01'),
Synset('chordate.n.01'),
Synset('animal.n.01'),
Synset('organism.n.01'),
Synset('living_thing.n.01'),
Synset('whole.n.02'),
Synset('object.n.01'),
Synset('physical_entity.n.01'),
Synset('entity.n.01')]
```

Word Representation

Problems with such discrete representation

- Missing context and nuances
 - Eg. “There is a stone in river **bank**”, “The **bank** should be closed by now”.
- Missing new meanings of words
 - Eg. wicked, badass, wizard, ninja
 - Impossible to keep up-to-date forever
- Requires human labor to maintain
- Cannot compute accurate word similarity

Word Similarity

- Naively, words can be represented by one-hot vectors
 - motel = [0 0 1 0 0 0 0 0 0 0]
 - hotel = [0 0 0 0 0 0 0 0 1 0 0]
- Dot products of these two vectors = 0 (orthogonal)
- Thus, there is no natural notion of similarity for one-hot vectors
- Solution?
 - Rely on WordNet to get similarity?
 - Incompleteness, inconsistency, difficult to maintain
 - **Actual Solution: Learn to encode similarity in the vectors**

Word Similarity

- How to encode similarity?
 - Distributional semantics: **A word's meaning is given by the words that frequently appear close-by**
 - “You shall know a word by the company it keeps” (J.R Firth 1957)
 - One of the most successful ideas of modern NLP
 - When a word w appears in a text, its context is the set of words that appear nearby (within a fixed size window)

...government debt problems turning into **banking** crises as happened in 2009...
...saying that Europe needs unified **banking** regulation to replace the hodgepodge...
...India has just given its **banking** system a shot in the arm...

These **context words** will represent **banking**

Word Vectors

expect =

$$\begin{pmatrix} 0.286 \\ 0.792 \\ -0.177 \\ -0.107 \\ 0.109 \\ -0.542 \\ 0.349 \\ 0.271 \\ 0.487 \end{pmatrix}$$

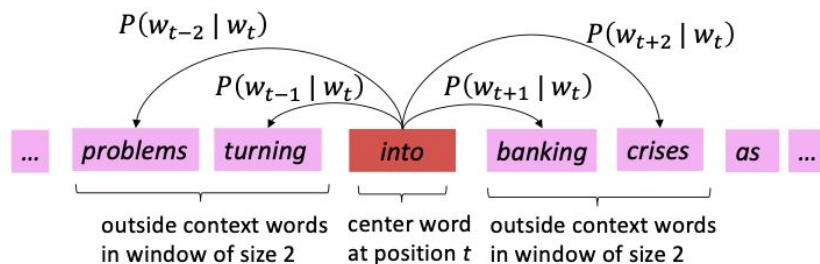


Suggested Readings

1. <https://web.stanford.edu/~jurafsky/slp3/6.pdf> (vector semantics and embeddings)
2. [Efficient Estimation of Word Representations in Vector Space](#) (original word2vec paper)

Word2Vec is an initial framework for learning word vectors

- We got large corpus of text
- Go through each position t in the text, which has a **center word c** , and **context (“outside”) words o**
- Calculate the **probability** of o given c (or vice versa)
- **Gradient descent** to maximize this probability
- Example windows for $P(w_{t+j} | w_t)$



Efficient Estimation of Word Representations in Vector Space, Mikolov et al., 2013, <https://arxiv.org/pdf/1301.3781.pdf>

Word2Vec: Objective Function

For each position $t = 1, \dots, T$, predict context words within a window of fixed size m , given center word w_j , the likelihood is

$$L(\theta) = \prod_{t=1}^T \prod_{\substack{-m \leq j \leq m \\ j \neq 0}} \log P(w_{t+j} | w_t; \theta)$$

The objective function $J(\theta)$ is the average negative log likelihood:

$$J(\theta) = -\frac{1}{T} \sum_{t=1}^T \sum_{\substack{-m \leq j \leq m \\ j \neq 0}} \log P(w_{t+j} | w_t; \theta)$$

Two flavors of Word2Vec algorithm

- **CBOW (Continuous Bag of words)**
 - Uses the context words to predict current word
- **Skip-gram**
 - Use the current word to predict its context

The probability of a predicted word occurring given a center word:

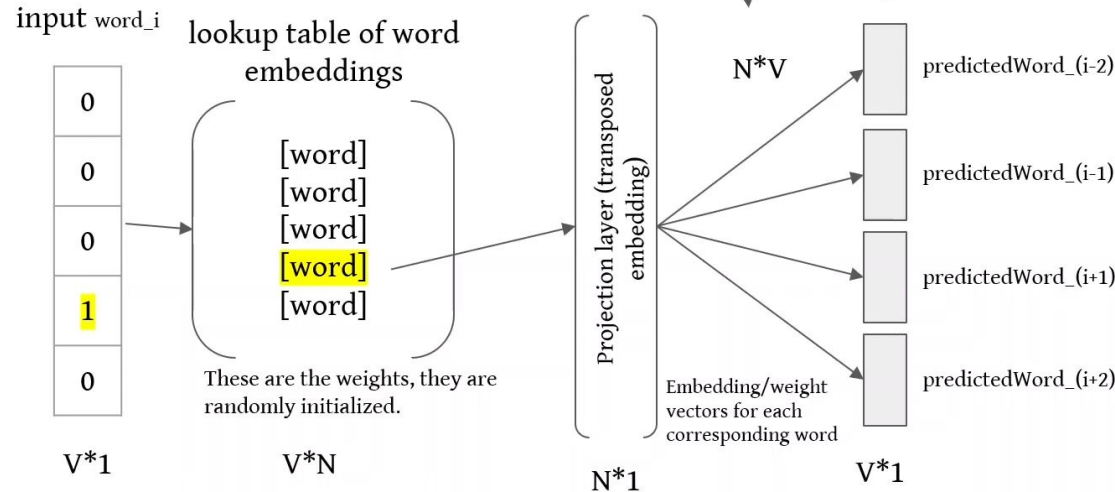
$$P(\text{predictedWord}_n | \text{centerWord}) = \frac{e^{\text{dot}(\text{predictedWord}_n, \text{centerWord})}}{\sum_i e^{\text{dot}(\text{predictedWord}_i, \text{centerWord})}}$$

A closer look...

One hot vector in:

Activation function:

$$\text{softmax}(\text{predictedWord}_n) = \frac{e^{\text{predictedWord}_n}}{\sum_i e^{\text{predictedWord}_i}}$$



The softmax activation normalizes the outputs as a probability distribution. This means a percentage is associated with each predicted word.

Backprop from here

V: # of words in the corpus, N: # of values in our vectors

The weight vector is actually what becomes your word embedding!

Word2Vec

```
[12] from gensim.models import Word2Vec
import numpy as np
```

```
[13] sentences=[['this','is','a','sentence','about','school'],
                ['school','has','students','and','teachers'],
                ['the','students','learn'],
                ['the','teachers','make','money']]
```

```
[28] model=Word2Vec(sentences, min_count=1, window=3)
```

```
▶ first=model['students']
target=model['learn']
second=model['teachers']

print("First: ", np.linalg.norm(target-first))
print("Second: ", np.linalg.norm(target-second))
```

```
↳ First: 0.04220174
Second: 0.03774856
```

```
▶ model.most_similar('school')
```

```
↳ [('has', 0.17184367775917053),
    ('the', 0.15330740809440613),
    ('this', 0.12881389260292053),
    ('students', 0.08287020027637482),
    ('a', 0.06733693182468414),
    ('sentence', 0.05654553323984146),
    ('and', 0.008189082145690918),
    ('money', -0.0024816691875457764),
    ('teachers', -0.029205819591879845),
    ('learn', -0.06788718700408936)]
```

sg parameter in Word2Vec():

sg → 0 (CBOW)

sg → 1 (Skipgrams)

Word2Vec : Skip-gram vs CBOW

- **CBOW** (Predict center word given outside words) and **Skip-gram** (Predict context ("outside") words given center word) uses the **same training procedure**.
- **CBOW** is much simpler, this implies a **much faster convergence** for CBOW than for Skip-gram, in the original paper, CBOW took hours to train, Skip-gram 3 days.
- CBOW learns better **syntactic relationships** between words while Skip-gram is better in capturing better **semantic relationships**. For the word 'cat':
 - CBOW would retrieve as closest vectors morphologically like plurals, i.e. '**cats**'
 - Skip-gram would consider morphologically different words (but semantically relevant) like '**dog**' **much closer to 'cat'** in comparison.
- Because Skip-gram rely on single word input, it is **less sensitive to overfit frequent words** (and it's also the reason of the better performances of Skip-gram in capturing semantic relationships).

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Word2Vec : Summary

- **Word2Vec** is the the first framework to encode word vectors using nearby words
 - Comes in 2 variants : **Skip-gram** and **CBOW**
 - Skip gram seems better but takes longer time to train
 - Amazingly effective to capture word similarity
- The current approach is inefficient given the huge computational cost in the lower term. We can revisit this using **negative sampling** instead

$$\frac{\exp(\mathbf{u}_o^\top \mathbf{v}_c)}{\sum_{w=1}^V \exp(\mathbf{u}_w^\top \mathbf{v}_c)} \leftarrow \text{Huge cost!}$$

Negative Sampling

- Instead of using all vocabularies, we can just pick some “negative” samples
- Steps:
 - We draw k random negative samples
 - We maximize the probability of real outside word appearing and minimize the probability of random words appearing around the center word.

$$J_{\text{neg-sample}}(\mathbf{v}_c, o, \mathbf{U}) = -\log(\sigma(\mathbf{u}_o^T \mathbf{v}_c)) - \sum_{k=1}^K \log(\sigma(-\mathbf{u}_k^T \mathbf{v}_c))$$

- Since we are not normalizing, we use sigmoid instead to turn the dot product to probabilities.
- Negative sampling technique is a widely used technique in deep learning field.

Limitations of Word2Vec

- Only looks at local words
 - Does not utilize global co occurrence statistics
 - **Possible solution:** Use co occurrence counts (**GloVe**)
- Does not work well with OOV tokens
 - **Possible solution:** Use character based or sub-words based embeddings (eg. FastText)
- Unsure whether contextual information was fully captured
 - **Possible solution:**
 - Pass these trained embeddings through some RNN/LSTM and get the resulting encodings as embeddings (ELMo)
 - Provide some prediction task, so that the model can capture better context (BERT)

Co-occurrence matrix

- 2 options
 - Window
 - Full document
- **Window:** Similar to Word2Vec, use window around each word -> capture some syntactic and semantic information, (Use window-based for word embedding)
- **Document:** similar to Latent Semantic Analysis

Co-occurrence matrix

- Example : Window length 1 (common: 5-10)
- Example corpus:
 - *"I like deep learning."*
 - *"I like NLP."*
 - *"I enjoy flying."*

| counts | I | like | enjoy | deep | learning | NLP | flying | . |
|----------|---|------|-------|------|----------|-----|--------|---|
| I | 0 | 2 | 1 | 0 | 0 | 0 | 0 | 0 |
| like | 2 | 0 | 0 | 1 | 0 | 1 | 0 | 0 |
| enjoy | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 0 |
| deep | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 |
| learning | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 |
| NLP | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 |
| flying | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 1 |
| . | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 0 |

Also has its own problem !!

Global Vectors for word representation

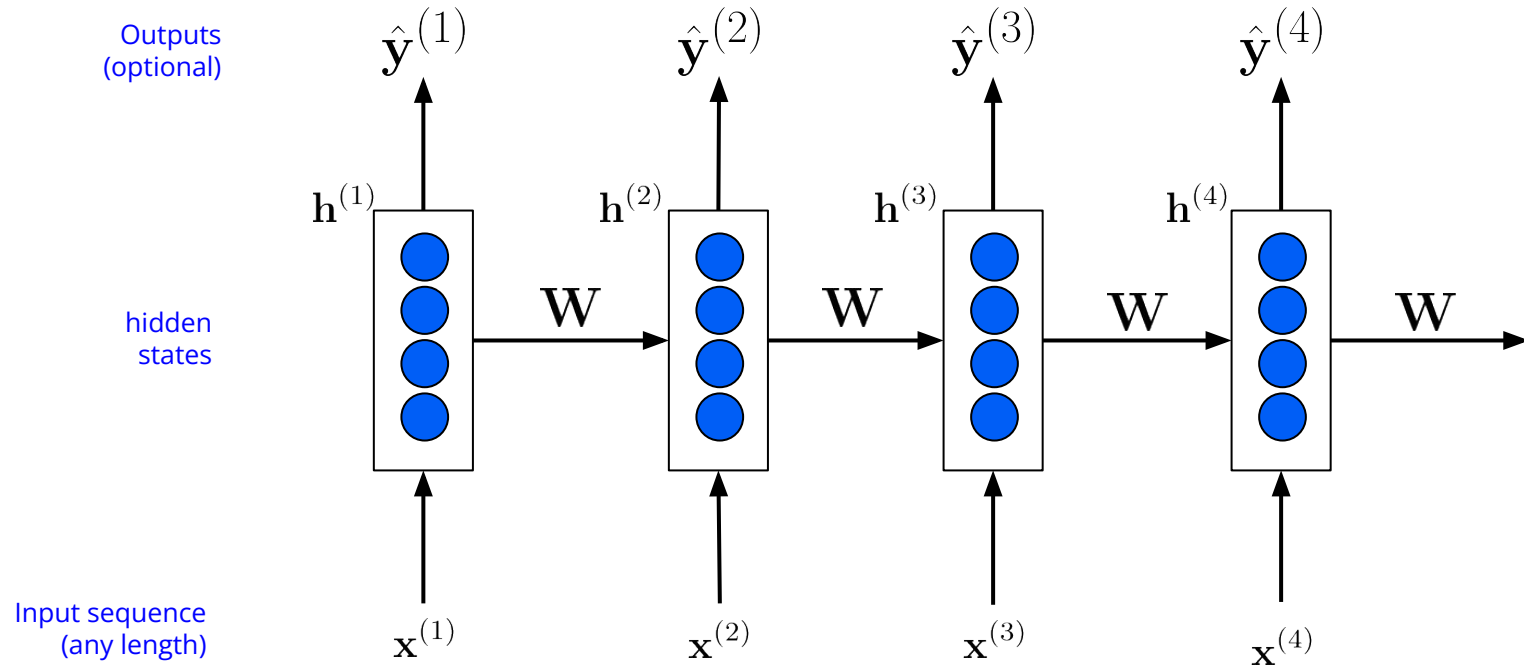
- Paper : <https://aclanthology.org/D14-1162.pdf>
- Mathematical explanation and derivation :
<https://towardsdatascience.com/light-on-math-ml-intuitive-guide-to-understanding-glove-em-beddings-b13b4f19c010>

Recurrent Neural Network

- **Suggested Readings**

- [N-gram Language Models](#) (textbook chapter)
- [The Unreasonable Effectiveness of Recurrent Neural Networks](#) (blog post overview about RNN)
- [Sequence Modeling: Recurrent and Recursive Neural Nets](#) (Sections 10.1 and 10.2)
- [On Chomsky and the Two Cultures of Statistical Learning](#) (some cool stuffs about LM)

Recurrent Neural Network



Recurrent Neural Network

output distribution

$$\hat{\mathbf{y}}^{(t)} = \text{softmax}(\mathbf{U}\mathbf{h}^{(t)} + \mathbf{b}_2) \in \mathbb{R}^{|V|}$$

hidden states

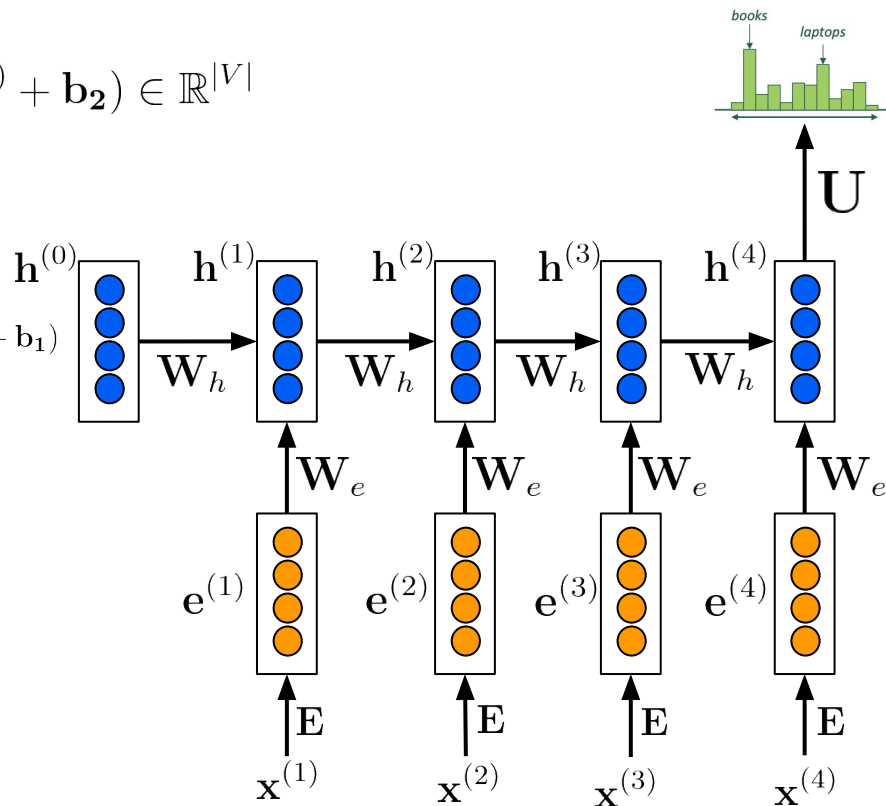
$$\mathbf{h}^{(t)} = f(\mathbf{W}_h \mathbf{h}^{(t-1)} + \mathbf{W}_e \mathbf{e}^{(t)} + \mathbf{b}_1)$$

words embeddings

$$\mathbf{e}^{(t)} = \mathbf{E}\mathbf{x}^{(t)}$$

words/one-hot vectors

$$\mathbf{x}^{(t)} \in \mathbb{R}^{|V|}$$



- Can process any length input
- Can use information from many steps back
- Model size does not increase because W is shared

Remaining problems:

- Recurrent computation is **slow** because it's sequential
- In reality, **difficult to access information from many steps back** (more on this later in the course)

Recurrent Neural Network

Training a RNN LM

- Get a **big corpus of text** which is a sequence of words
- Feed into RNN-LM; compute output distribution for **every step t**
 - i.e., predict probability dist of every word, given words so far
- **Loss function** on step t is **cross-entropy** between predicted probability distribution , and the true next word (one hot for):

$$J^{(t)}(\theta) = CE(\mathbf{y}^t, \hat{\mathbf{y}}^{(t)}) = - \sum_{w \in V} \mathbf{y}_w^{(t)} \log \hat{\mathbf{y}}_w^{(t)} = - \log \hat{\mathbf{y}}_{\mathbf{x}_{t+1}}^{(t)}$$

- Average this to get overall loss for the entire training set

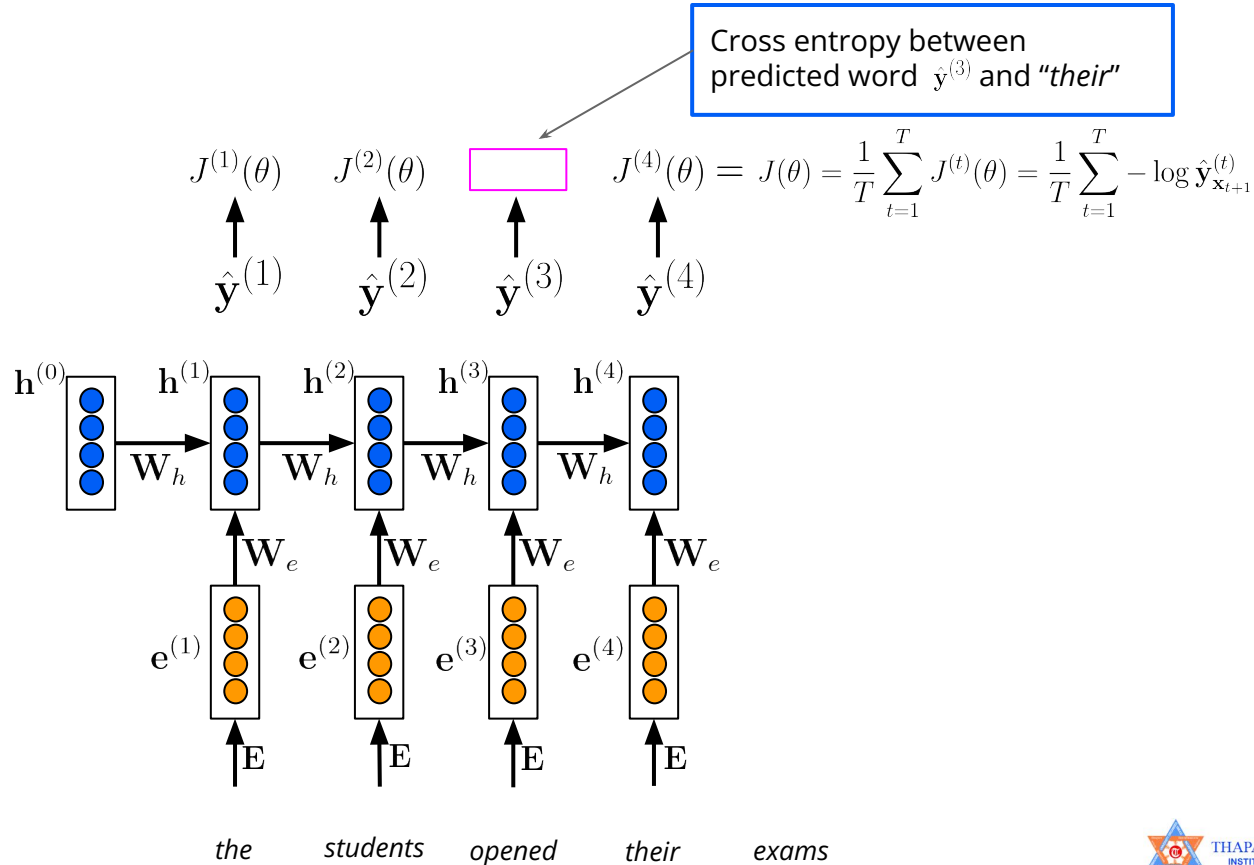
$$J(\theta) = \frac{1}{T} \sum_{t=1}^T J^{(t)}(\theta) = \frac{1}{T} \sum_{t=1}^T - \log \hat{\mathbf{y}}_{\mathbf{x}_{t+1}}^{(t)}$$

Recurrent Neural Network

Training a RNN LM

Loss

Predicted prob dists.



Corpus

Training a RNN LM

- Better to perform **stochastic gradient descent** instead to save computational time
 - Use batch of sentences, instead of the whole corpus
- The derivative w.r.t the repeated weight matrix is simply the sum of all gradients of each time step

$$\frac{\partial J^{(t)}}{\partial \mathbf{W}_h} = \sum_{i=1}^t \frac{\partial J^{(t)}}{\partial \mathbf{W}_h} \Big|_{(i)}$$

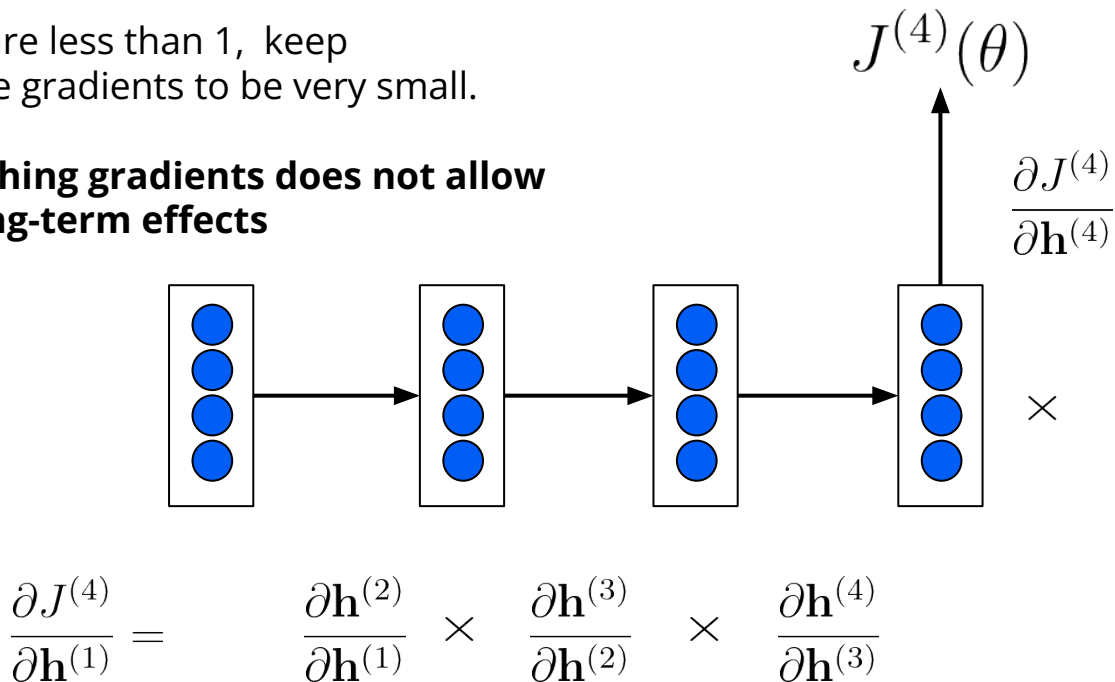
This is also known as “**backpropagation through time**”

Recurrent Neural Network

Vanishing Gradients

Imagine if all these gradients are less than 1, keep multiplying them will cause the gradients to be very small.

The real problem is that **vanishing gradients does not allow the network to learn any long-term effects**



Effect of vanishing gradients

- **LM task:** “*The writer of the books _____*” (possible words: is, are)
- **Correct answer:** *The writer of the books is planning a sequel*
- **Syntactic** recency: The writer of the book is (correct)
- **Sequential** recency: The writer of the books are (incorrect)
- Due to vanishing gradient, RNN-LMs are better at learning from sequential recency than syntactic recency, so they make this type of error more often [[Linzen et al. 2016](#)]

Exploding Gradient

- If the gradient becomes too big, the SGD update can easily overshoot:

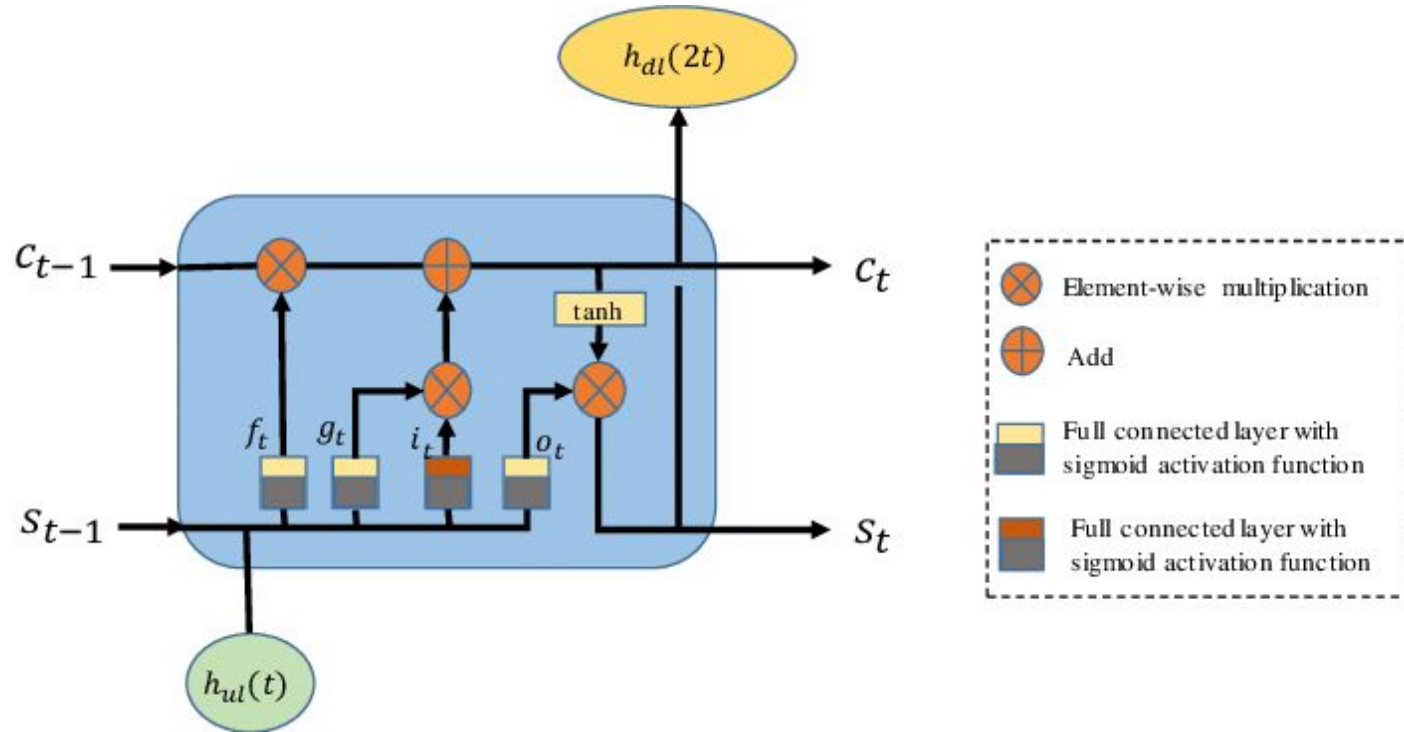
$$\theta^{\text{new}} = \theta^{\text{old}} - \alpha \nabla_{\theta} J(\theta)$$

- This can cause bad updates: we take too large a step and reach a weird and bad parameter configuration (with large loss)
- In the worst case, this will result in **Inf** or **NaN** in your network
 - a. (then you have to restart training from an earlier checkpoint)

Solving vanishing gradient

- As hinted earlier, vanishing gradients cause the model **inability to learn long-term relationships**
- Instead of trying to fix vanishing gradients which is difficult, can we try to **preserve long-term relationships better?**
- How about a RNN with a separate **memory?**

Long Short Term Memory (LSTM)



Long Short Term Memory (LSTM)

A type of RNN proposed by Hochreiter and Schmidhuber in 1997 as a solution to the vanishing gradients problem. Everyone cites that paper but really a crucial part of the modern LSTM is from Gerseta.(2000) :-)

Forget gate: controls what is kept vs. forgotten, from previous cell state

Input gate: controls what parts of the new cell contents are written to cell

Output gate: controls what parts of cell are output to hidden state

New cell content: this is the new content to be written to the cell

Cell state: erase ("forget") some content from last cell state, write ("input") some new cell state

Hidden state: read ("output") some content from the cell

$$\mathbf{f}^{(t)} = \sigma(\mathbf{W}_f \mathbf{h}^{(t-1)} + \mathbf{U}_f \mathbf{x}^{(t)} + \mathbf{b}_f)$$

$$\mathbf{i}^{(t)} = \sigma(\mathbf{W}_i \mathbf{h}^{(t-1)} + \mathbf{U}_i \mathbf{x}^{(t)} + \mathbf{b}_i)$$

$$\mathbf{o}^{(t)} = \sigma(\mathbf{W}_o \mathbf{h}^{(t-1)} + \mathbf{U}_o \mathbf{x}^{(t)} + \mathbf{b}_o)$$

$$\tilde{\mathbf{c}}^{(t)} = \tanh(\mathbf{W}_c \mathbf{h}^{(t-1)} + \mathbf{U}_c \mathbf{x}^{(t)} + \mathbf{b}_c)$$

$$\mathbf{c}^{(t)} = \mathbf{f}^{(t)} \circ \mathbf{c}^{(t-1)} + \mathbf{i}^{(t)} \circ \tilde{\mathbf{c}}^{(t)}$$

$$\mathbf{h}^{(t)} = \mathbf{o}^{(t)} \circ \tanh(\mathbf{c}^{(t)})$$

All these vectors are of same length

Gates are applied using element-wise (Hadamard product)

Long Short Term Memory (LSTM)

- In **2013-2015**, LSTMs started achieving **state-of-the-art** results
 - Successful tasks: handwriting recognition, speech recognition, machine translation, parsing, and image captioning, as well as language models
 - LSTMs became the dominanch approach for most NLP tasks

Gated Recurrent Unit (GRU)

Paper: <https://arxiv.org/pdf/1406.1078v3.pdf>

