Use python libraries such as numpy to solve the following problems.

1) Compute the determinant of the following matrix.

$$\begin{vmatrix} 1 & 3 & 2 \\ 4 & 1 & 3 \\ 2 & 5 & 2 \end{vmatrix}.$$

2) Find the eigenvalues and corresponding eigenvectors to matrix A, where

$$A = \begin{bmatrix} 4 & 2 & 3 \\ -1 & 1 & -3 \\ 2 & 4 & 9 \end{bmatrix}.$$

3) Find the inverse of the matrix B, where B =

$$\begin{bmatrix} 2 & 2 & 0 \\ 0 & 0 & 1 \\ 4 & 2 & 0 \end{bmatrix}.$$

4) Use matrix multiplication to verify that B x B = I, where I is the identity matrix.

5) Compute the matrix given by $AB + 2A^2 - 3B$

6) From your answer in question 2, find the matrix Q and D such that $A = QDQ^{-1}$

7) Solve the following system of linear equations

$$\begin{cases} 2x - 4y + 9z = -38 \\ 4x - 3y + 8z = -26 \\ -2x + 4y - 2z = 17 \end{cases}$$

8) Find the dimensions and transposes of matrices C and D, where

```
  [[0 1]
C= [2 3].       D = [[0 1 2 3]
   [4 5]]             [4 5 6 7]]
```

9) Verify that $(CD)^T = D^T C^T$

10) Reshape the following matrix into a 6 x 2 matrix.

```
[ [ 1 2 4 ]
  [ 3 4 6 ]
  [ 7 8 5 ]
  [ 9 2 1 ] ]
```

11) This is an advanced question : not required for the course.

Download the following dataset. “iris.data.csv”. Details of the dataset are found here: https://archive.ics.uci.edu/ml/datasets/iris

We are going to use the data from columns 1-4, namely sepal length, sepal width, petal length, petal width. Use pandas and numpy here.

A) Scale/Normalise each column by subtracting its mean and dividing by its standard deviation.
B) Find the covariance matrix explaining the covariances between each pair of elements of a random vector. (This should be a 4 x 4 matrix with diagonal values close to 1)
C) Find the Eigenvalues and eigenvector of the covariance matrix
D) Divide each eigenvalue by the sum of eigenvalues to obtain percentage of explained variance per principal component. What percentage of the data is explained by the first 2 principal components?
E) Project the data onto the first 2 principal components (a.k.a. the 2 eigenvectors with the largest eigenvalues)
F) Using matplotlib, seaboard. Scatter plot the projected data and compare it with a scatter plot of any 2 variables of the original data. Notice any differences?