Projectile Motion with Varying Air Resistance

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Abstract

In this paper, we first investigate and visualize the gravitational potential generated by a point mass. Then we consider the effects of varying strengths of air resistance on a massive object undergoing projectile motion.

1 Gravitational Potential

For a point mass with mass M, the resulting gravitational potential has the form

$$\phi(r) = -\frac{GM}{r}. \tag{1}$$

In rectangular coordinates, this can be written

$$\phi(x, y, z) = -\frac{GM}{\sqrt{x^2 + y^2 + z^2}}. \tag{2}$$

See Fig. 1 for a plot of the gravitational potential generated by a point mass at the origin.

2 Air Resistance

Air resistance is a force that acts on projectiles opposite to their direction of motion. Newton's second law for a projectile in air resistance is

$$m\frac{\mathrm{d}^2}{\mathrm{d}t^2}x(t) = -mg - \alpha\frac{\mathrm{d}}{\mathrm{d}t}x(t) \tag{3}$$

where α is a constant denoting the strength of air resistance. A sample trajectory for a particular value of α is shown in Fig. 2. There have been experiments about resistance [1].

References

[1] S. Triqueneaux, J. Butterworth, J. Goupy, C. Ribas, D. Schmoranzer, E. Collin and A. Fefferman, "Very low resistance Al/Cu joints for use at cryogenic temperatures," [arXiv:2009.02201 [physics.ins-det]].

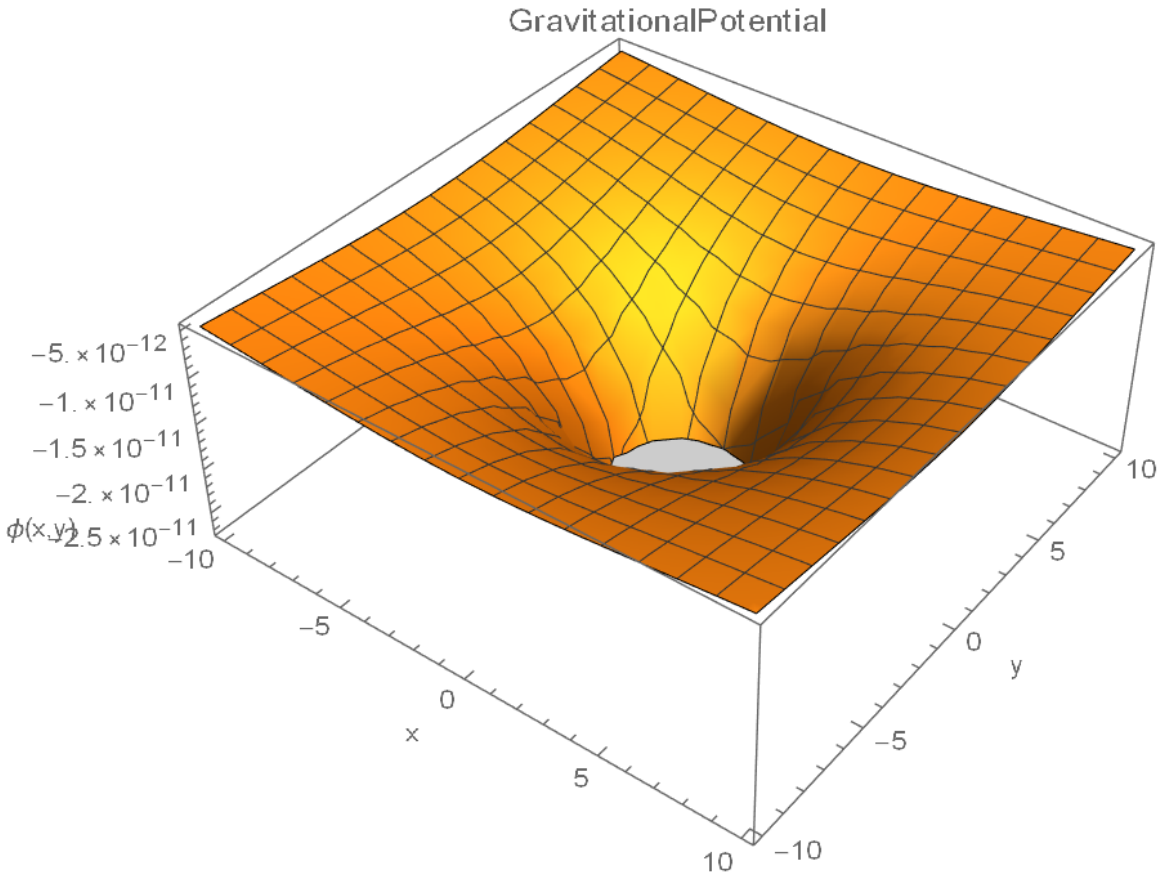

Figure 1: The gravitational potential for a point mass at the origin.

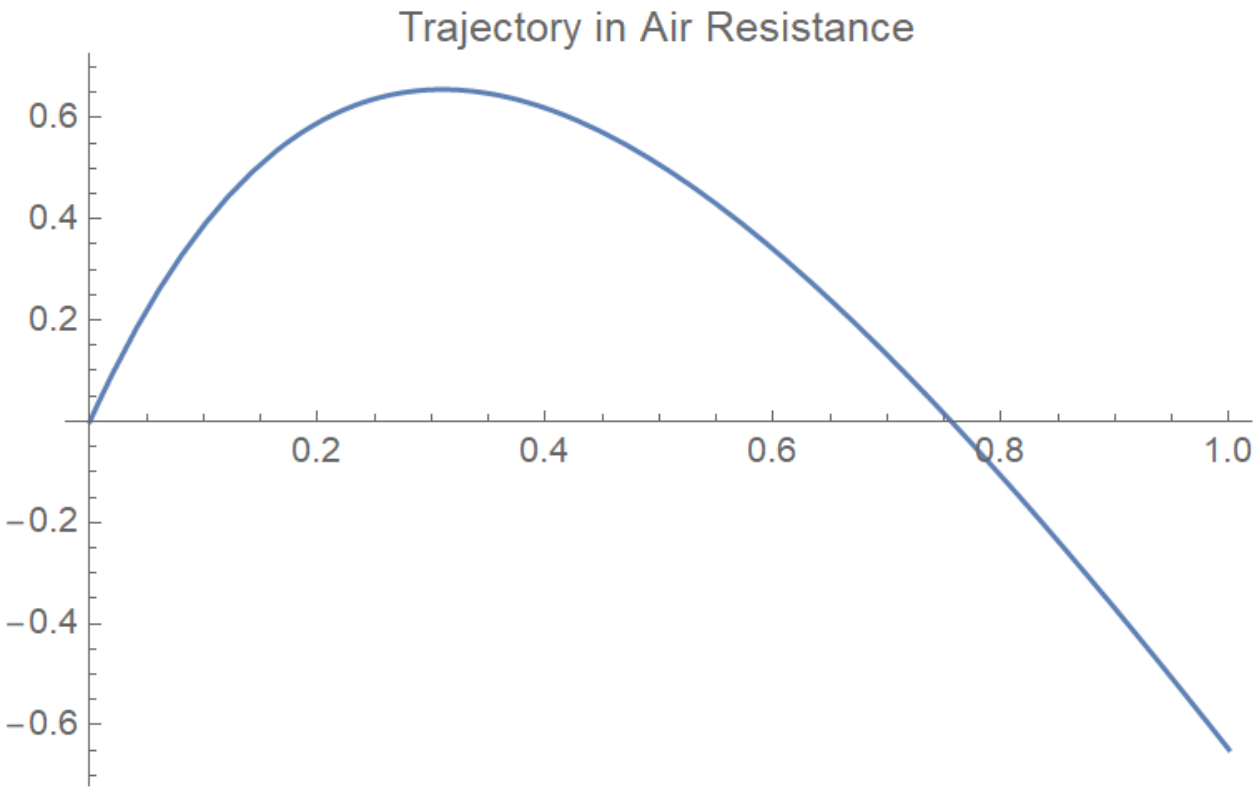

Figure 2: The trajectory of a projectile in air resistance.