Let Extension

Let extension violates the progress rule. In this section, we will provide a counter example.

$$let\ x': int = Int(1)\ in\ y$$

According to (D-Let) rule, we can make a small step, which is

$$let\ x': int = Int(1)\ in\ y \ \mapsto [x' \to Int(1)]\ y$$

Observed from the above statement, we know that y is neither **VAL** nor can go any further with a small step (no τ' can be the next step of y). Therefore, this example violates the progress theorem.

Rec Extension

Rec extension obeys both the progress rule and the preservation rule. In this section, we will prove this.

- For progress,

What we need to show is, if $rec(t_0; x.y.t_1)(t): \tau$, then either $rec(t_0; x.y.t_1)(t): \tau$ **VAL** or $\exists t'$ such that $rec(t_0; x.y.t_1)(t) \to t'$.

1. t VAL

By typing inversion for int, $t = 0 \vee t = n(n \neq 0)$

If $t = 0$, then $rec(t_0; x.y.t_1)(t) \mapsto t_0$ (D-Rec2), where t_0 is also an int (VAL).

If $t = n(n \neq 0)$, then $rec(t_0; x.y.t_1)(t) \mapsto [x \to n, y \to rec(t_0; x.y.t_1)(n-1)]t_1$ (D-Rec3).

Denote $[x \to n, y \to rec(t_0; x.y.t_1)(n-1)]t_1$ as t', then we have $rec(t_0; x.y.t_1)(t) \mapsto t'$.

2. t is not VAL.

In this case, make a small step according to (D-Rec1)

$$rec(t_0; x.y.t_1)(t) \mapsto rec(t_0; x.y.t_1)(t')$$

Denote $rec(t_0; x.y.t_1)(t')$ as t'', then we have $rec(t_0; x.y.t_1)(t) \to t''$.

- For preservation,

What we need to show is, if $rec(t_0; x.y.t_1)(t): \tau$ and $rec(t_0; x.y.t_1)(t): \tau \mapsto t'$, then $t': \tau$.

1. t is VAL

By typing inversion for int, $t = 0 \vee t = n(n \neq 0)$

If $t = 0$, then $rec(t_0; x.y.t_1)(t) \mapsto t_0$, where $t_0: \tau$, according to D-Rec1.

If $t = n(n \neq 0)$, then we can prove by induction.

Base case: the theorem holds for $t = 0$.

Suppose: the theorem holds for $t = n - 1$.

We examine the case for $t = n$.

According to D-Rec3, we have

$$rec(t_0; x.y.t_1)(t) \mapsto [x \to n, y \to rec(t_0; x.y.t_1)(n-1)]t_1$$

where $rec(t_0; x.y.t_1)(n-1): \tau$. Apply this to T-Rec, then we know $t_1: \tau$. Hence, it holds.

2. t is not VAL

$t \mapsto t''$, then by inductive hypothesis, $rec(t_0; x.y.t_1)(t''): \tau$, according to D-Rec1.