MA122: Assignment 3

Marks (20)

25/12/2020

Instructions:

1. Grading will be done using script files which will compile and run your assignments automatically. All the programs should be compilable and executable from the command prompt.
2. Since extensive use of scripting will be done for evaluation, your program should output only the values being asked in the question. **Do not put `cout` statements asking for user input or similar things**.
3. Use default precision for `cout` unless mentioned otherwise.
4. You are free to discuss and also take help from internet. However, do not copy/paste entire programs from internet or from a friend.

Q1

We know that Taylor's series is

$$f(x_0 + \Delta x) = f(x_0) + \Delta x f'(x_0) + \frac{(\Delta x)^2}{2!} f''(x_0) + \dots$$

If we ignore the higher order terms then a linear approximation can be written as

$$f(x_0 + \Delta x) \approx f(x_0) + \Delta x f'(x_0)$$

which gives,

$$f'_{FD}(x_0) \approx \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x}$$

This is called as the finite difference approximation of a derivative. This is extensively used in scientific computing to approximate derivatives of functions. Clearly, as $\Delta x \to 0$, right hand side tends to the analytical derivative $f'(x_0)$. We want to investigate the accuracy of this approximation as we reduce Δx. Consider $f'(x_0)$ for $f(x) = \sin x$ at $x_0 = \pi/4$ and $\Delta x = 10^{-\alpha},\ \alpha = [1, \dots, 13]$. Error of finite difference (FD) approximation can be defined as $E_{FD} = \|1 - \frac{f'_{FD}(x_0)}{f'(x_0)}\|$. Write a program that takes no input and outputs α and E_{FD} values for all α's. One error per line. Output should look like:

```
1   0.0516242
2   0.00501662
.
.
13  .........
```

Note-1: Does error reduce as α increases? If not, why not?

Note-2: What happens if I declare all the variables as `float` instead of a `double`?

Q2. Write a program to calculate *exact* $n!$ $\forall$ $2 < n \leq 20$. Input of the program will be n and the output should be $n!$. **Note**: Can you calculate exact $n!$ for $n = 21$ and above?

Q3. Write a program to give an *estimate* for $n!$ $\forall$ $n > 20$. Input of the program is n, and the output should be a single number (an estimate for $n!$).

Note: Play with the program and find out the maximum n upto which it works.