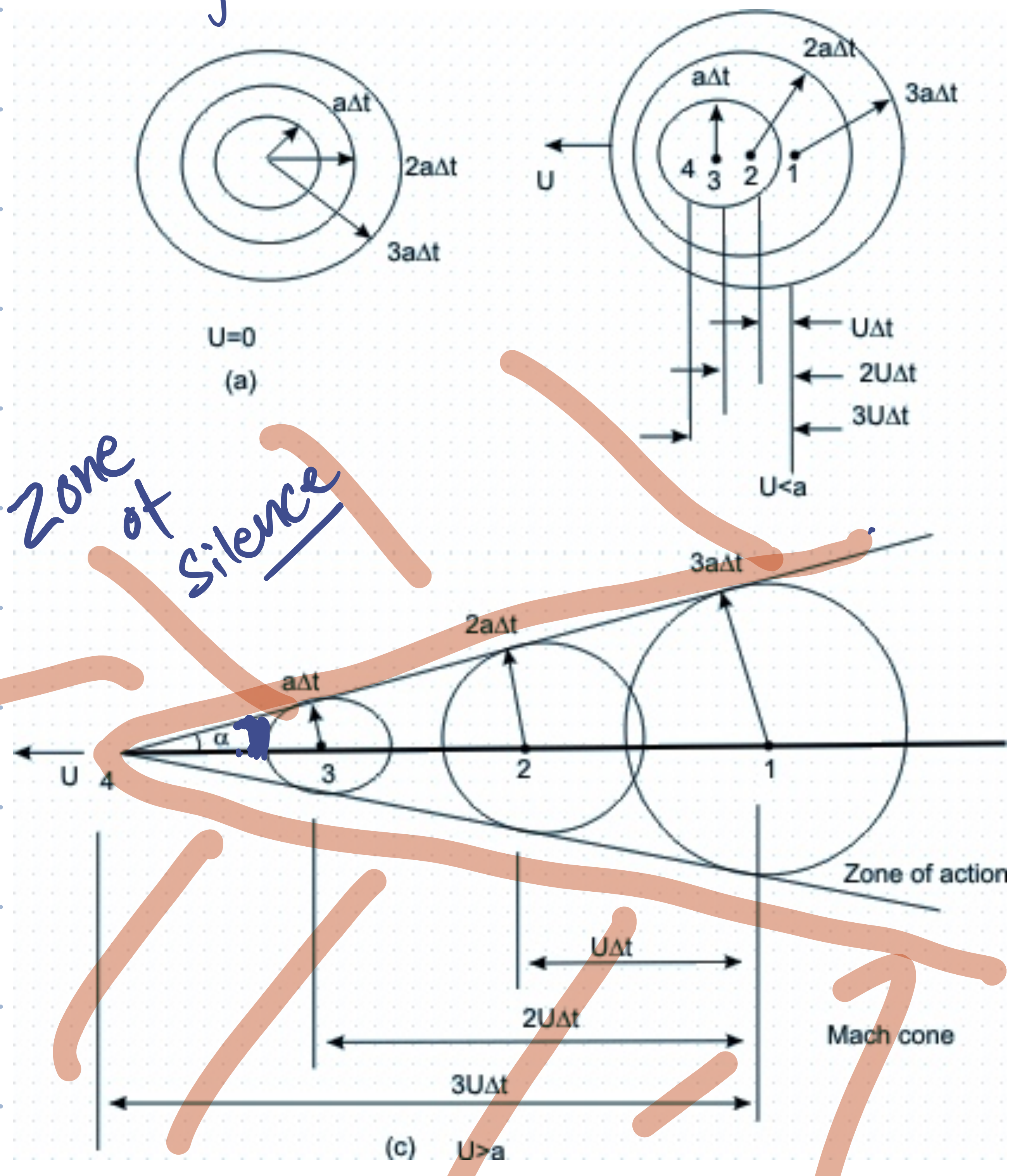
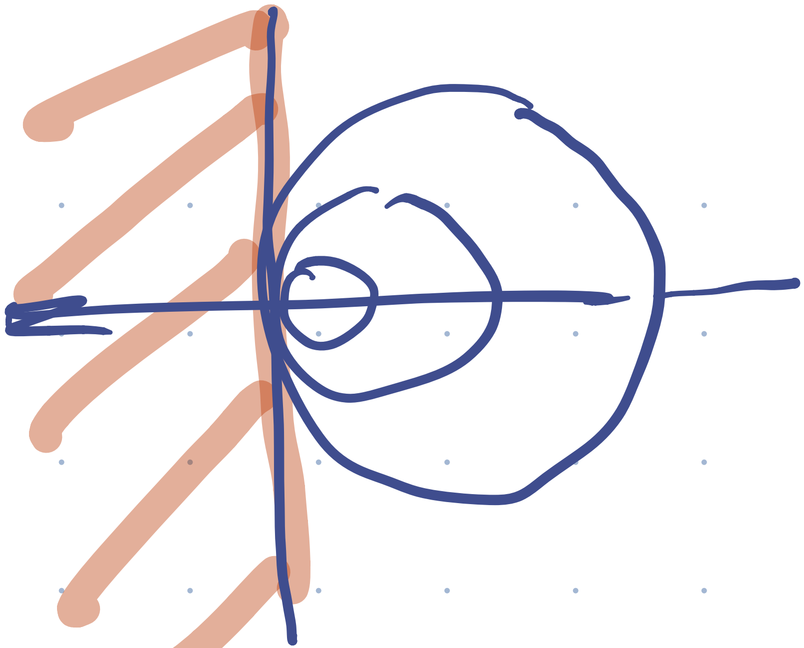
Mach angle. (μ)

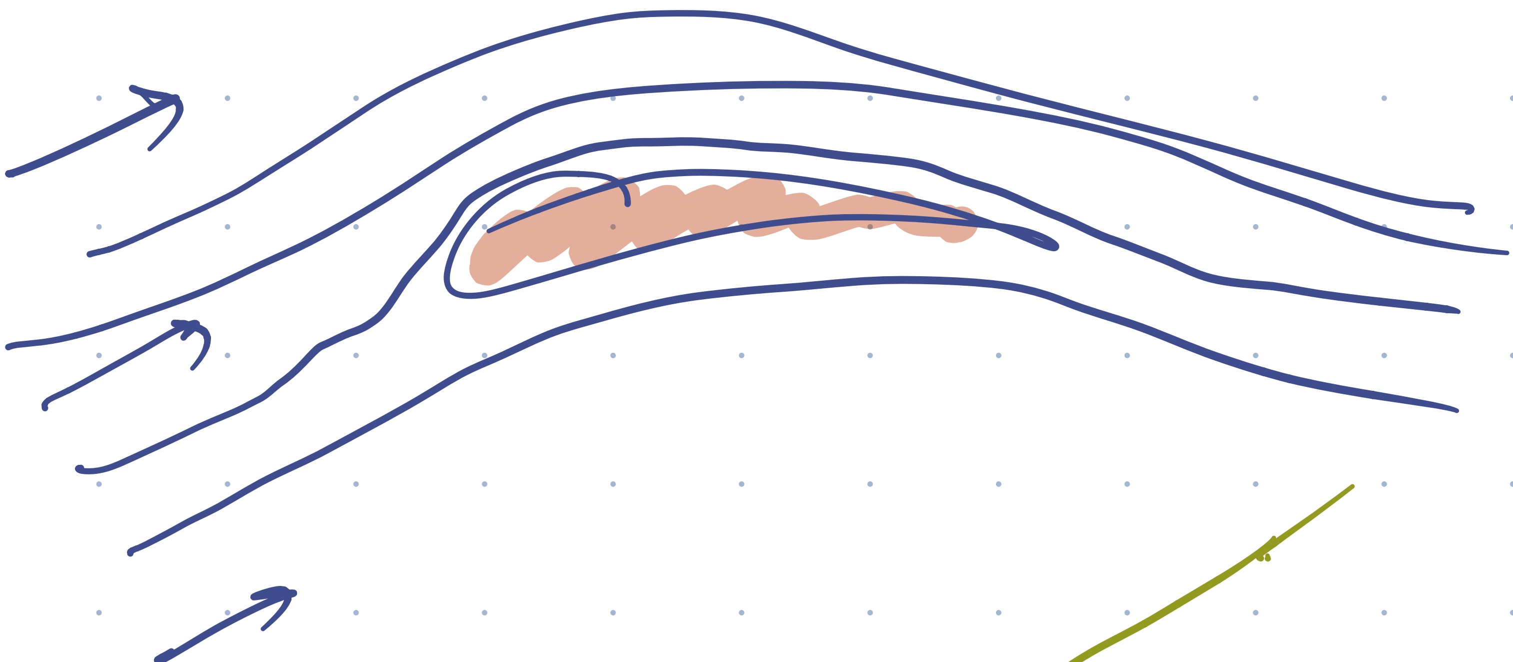


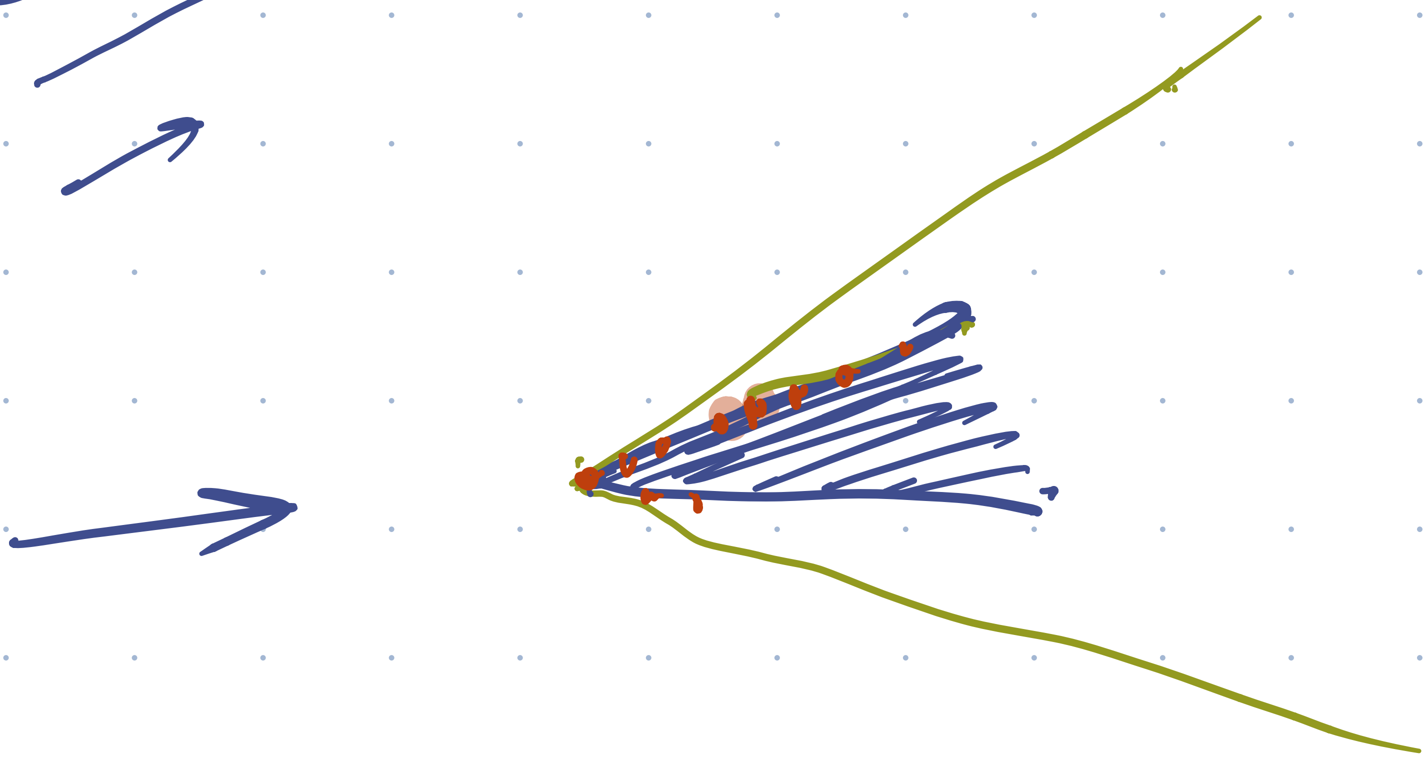
Source: NPTEL

Mach Cone $\quad \mu = \sin^{-1} \frac{1}{M}$

M=1





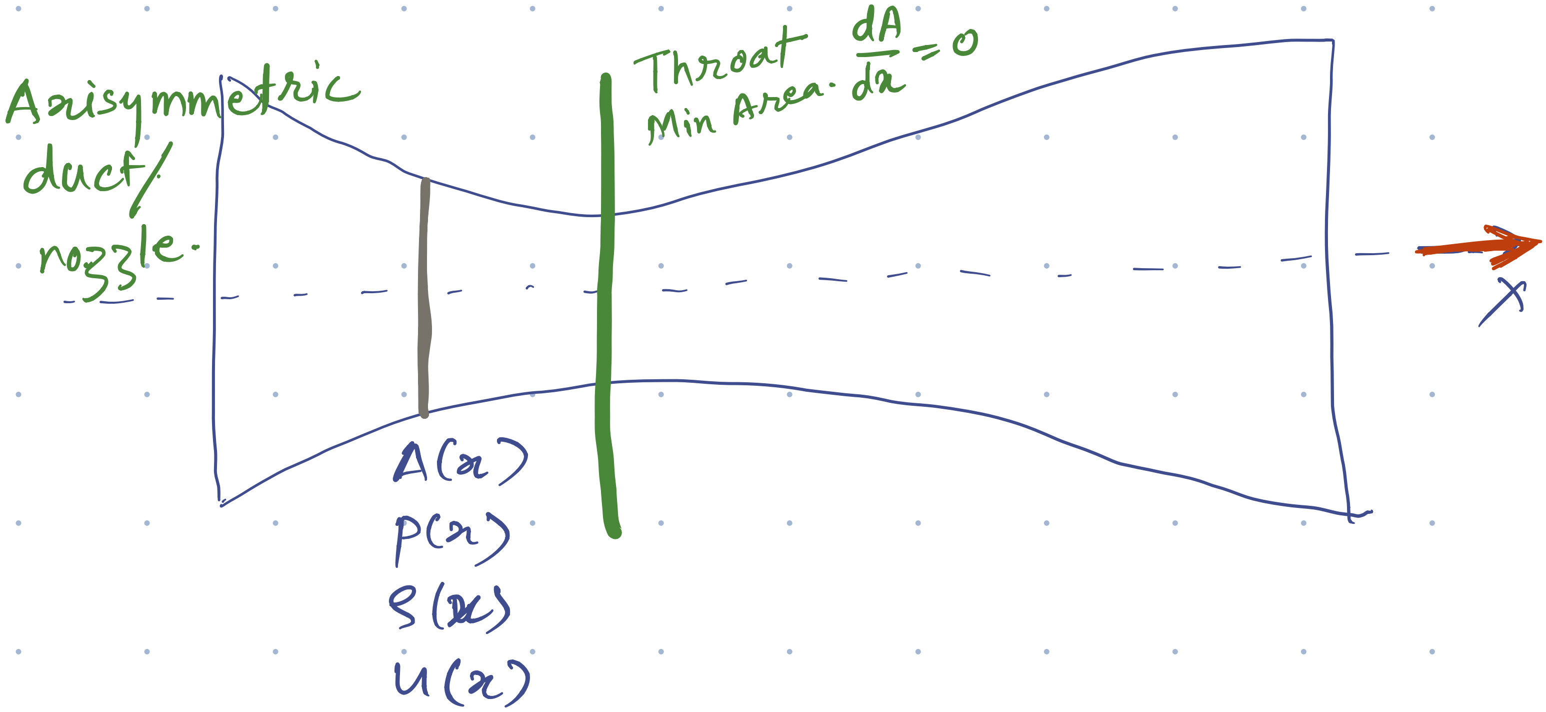


Isentropic Flow of a perfect gas

- Adiabatic $dq = 0$
- Equillibrium flow

$\Big\}$ Hence Isentropic

- No entropy generation $ds = 0$
- $pv = RT$ (perfect gas)
- steady flow; hence $\left(\frac{\partial(\cdot)}{\partial t} = 0\right)$



Conservation of mass

$$\rho A u = \dot{m} = \text{constant}.$$

$$\therefore \quad \frac{d\rho}{\rho} + \frac{dA}{A} + \frac{du}{u} = 0 \qquad \text{--- (1)}$$

Momentum conservation

$$\cancel{\frac{\partial u}{\partial t}}^{\,0} + u\frac{\partial u}{\partial x} = \frac{-1}{\rho}\frac{\partial p}{\partial x}$$

$$\therefore \frac{dp}{dx} + \rho u \frac{du}{dx} = 0 \qquad \text{--- } (2)$$

Note :- If ρ = const. ; then this will lead to the Bernaulli's eq^n :- $p + \rho\frac{u^2}{2} = \text{const.}$

Energy conservation

As we have seen already

$$h_0 = h_0'$$

As there is equillibrium through out the direct;

$$dh_0 = 0 \Rightarrow dh + u\,du = 0$$

But we know, $dh = T\,ds + v\,dp$.

$$\therefore T\,ds + v\,dp + u\,du = 0$$

As $ds = 0$ (Isentropic process),

$$\frac{dp}{\rho} + u\,du = 0$$

$$\therefore \quad \frac{dp}{dx} + \rho u \frac{du}{dx} = 0 \qquad \text{—— (3)}$$

(3) is same as (2). Hence redundant.

So, combining (1) & (2)

$$dp + \rho u^2 \left(-\frac{d\rho}{\rho} - \frac{dA}{A} \right) = 0$$

Defining $\left.\frac{\partial p}{\partial \rho}\right|_s = a^2$,

We have,

$$dp + \rho u^2 \left(-\frac{dp}{\rho a^2} - \frac{dA}{A} \right) = 0$$

$$\therefore \quad dp(1 - M^2) = \rho u^2 \frac{dA}{A}$$

where $M = u/a$ = Mach number.

$$\therefore \quad \frac{dp}{dA} = \left(\frac{\rho u^2}{1 - M^2} \right) \frac{1}{A}.$$

Since, $a^2 = \gamma p/\rho$, we have

$$\boxed{\frac{dp}{dA} = \left(\frac{\gamma M^2}{1-M^2}\right)\frac{p}{A}}$$

→ we note that the sign of RHS depends on $\left(\frac{1}{1-M^2}\right)$.

→ RHS not defined for $M=1$

For $M<1$,

$$\frac{dp}{dA} > 0$$

This means as $A\uparrow \Rightarrow p\uparrow \Rightarrow u\downarrow$

OR

$A\downarrow \Rightarrow p\downarrow \Rightarrow u\uparrow$

For $M>1$,

$$\frac{dp}{dA} < 0$$

$\therefore$ $A\uparrow \Rightarrow p\downarrow \Rightarrow u\uparrow$

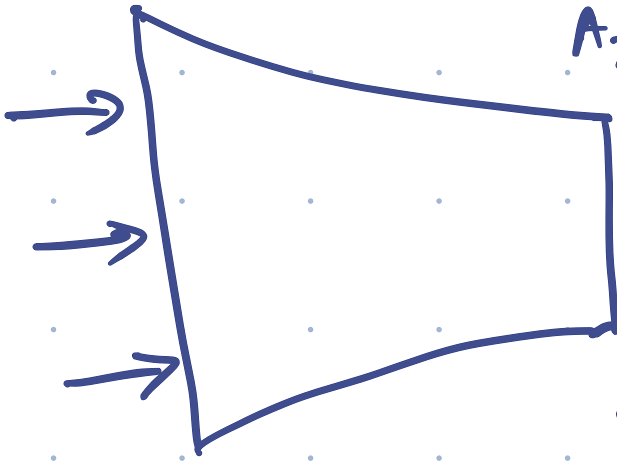
OR

$A\downarrow \Rightarrow p\uparrow \Rightarrow u\downarrow$

Converging duct

$A_1 = 50\, cm^2$

$A_2 = 40\, cm^2$



1

2

$T_1 = 300K$

$P_1 = 100\, kPa$

$V_1 = 100\, m/s$

$\rho_1 = P_1 / R T_1$

$M_2 = ?$

$P_2 = ?$

$T_2 = ?$

$T_{o_1} = T_{o_2}$

$P_{o_1} = P_{o_2}$

$\Downarrow$

$\rho_{o_1} = \rho_{o_2}$

$\rho_1 A_1 V_1 = \rho_2 A_2 V_2$

$\Downarrow$

T_o

$$\dot{m} = \rho A v$$

$$= \rho A \frac{v}{a} \cdot a$$

$$= \rho A M \cdot \sqrt{\gamma R T} \quad \text{— (1)}$$

Also, $$\frac{T_0}{T} = 1 + \frac{\gamma-1}{2} M^2 \quad \text{— (2)}$$

$$P_0/p = \left(1 + \frac{\gamma-1}{2} M^2\right)^{\gamma/\gamma-1} = \left(\frac{T_0}{T}\right)^{\gamma/\gamma-1}$$

$$\frac{\rho_0}{\rho} = \left(\frac{T_0}{T}\right)^{1/\gamma-1} = \left(1 + \frac{\gamma-1}{2} M^2\right)^{1/\gamma-1}$$

Using (1) & (2),

$$\dot{m} = \rho A M \sqrt{\gamma R} \cdot \left(\frac{\left(1 + \frac{\gamma-1}{2} M^2\right)^{1/2}}{\sqrt{T_0}}\right)^{-1}$$

$$\dot m = \frac{p_0}{\sqrt{RT_0}} A \sqrt{\gamma M}\left(1+\frac{\gamma-1}{2}M^2\right)^{-\frac{(\gamma+1)}{2(\gamma-1)}}$$

$$= \frac{p_0}{\sqrt{RT_0}} A \cdot F(\gamma, M)$$

$$\dot m\big|_1 = \dot m\big|_2$$

$$\boxed{\frac{A_1}{A_2} = \frac{F(\gamma, M_2)}{F(\gamma, M_1)}} \quad (3)$$

$M_2 =$ solving (3)

$\hookrightarrow T_2 = T_0 \Big/ \left(1+\frac{\gamma-1}{2}M_2^2\right)$

$= f(p_0, M_2)$

$p_2 = (\ldots)$

$\rho_2 = f(\rho_0, m_2)$