Quasi - One dimensional conservation eq^ns.

Mass conservation → Thermodynamics laws

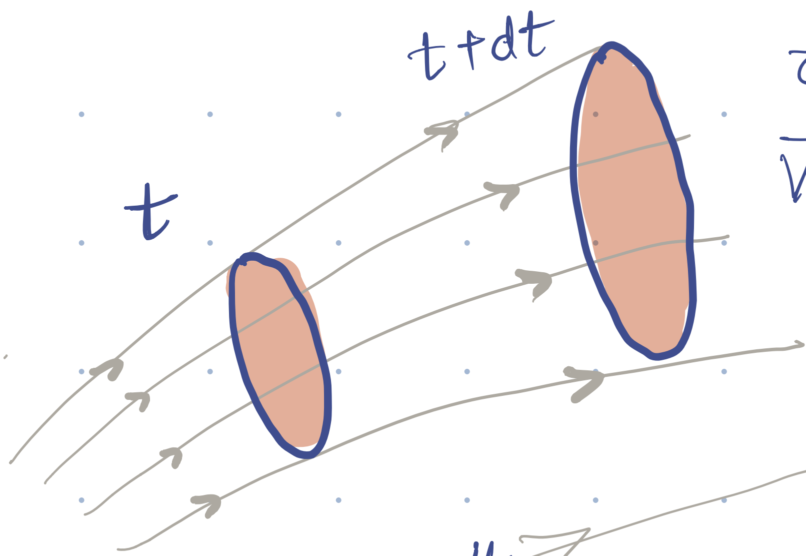
Momentum " → Newton's 2^{nd} law.

Energy "

There are two standard approaches.

1. Lagrangian (Control mass)
2. Euler (Control volume)

① Control mass →



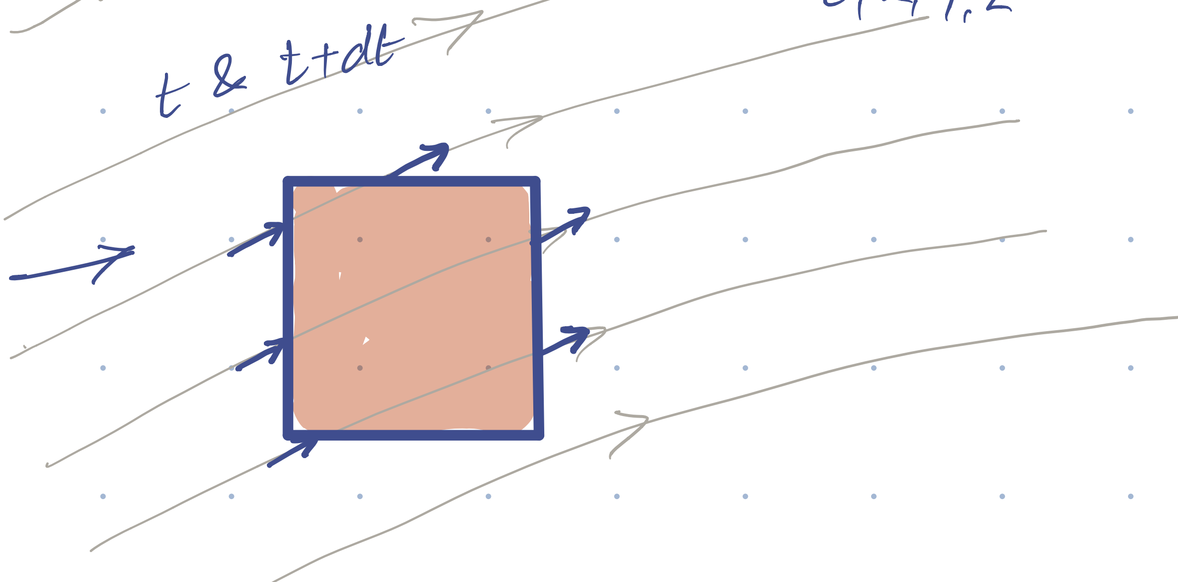
$\tau, X(t), Y(t), Z(t)$

$\vec{V} = \frac{\partial X}{\partial \tau}\hat{i} + \frac{\partial Y}{\partial t}\hat{j} + \frac{\partial Z}{\partial t}\hat{k}$

$\tau = t$

t, x, y, z

② Control volume →



Reynold's transport theorem

Lagrangian equation $\xrightarrow{RTT}$ Eulerean equation

$$\left(\frac{DN}{Dt}\right) \frac{dN}{dt} = \frac{\partial N}{\partial t} + \underbrace{\vec{V} \cdot \vec{\nabla} N}_{\text{Convection}}$$

$$\frac{d}{dt} \int_{\Omega(t)} f \, dV = \int_{\Omega(t)} \frac{\partial f}{\partial t} \, dV + \underbrace{\int_{\partial\Omega(t)} (\vec{V} \cdot \vec{n}) f \, dA}_{\text{Convection}}$$

Material OR Substantial Derivative

Assumptions:-

→ Quasi-one dimensional flow

→ $|V_x| \gg |V_y|$, $|V_x| \gg |V_z|$

→ Duct / Nozzle flows qualify

→ 1D Vs. Quasi-1D (Area variation) $A(x)$

→ Unsteady

→ No body forces (gravity / electromagnetism)

→ Inviscid flow (No shear forces)

→ No heat transfer (Adiabatic)

There are essentially, three types of treatments for the derivation of the conservation laws.

① Liepmann & Roshko (Sec 2.1 to 2.7)

→ Mostly physics based.

② Zucker & Biblarz (Chap 2 & 3)

→ Control volume. Detailed explanation.

→ Integral form as well as differential form.

③ Emmanuel (Chapter 3)

→ Mostly mathematical arguments.

→ Only differential form