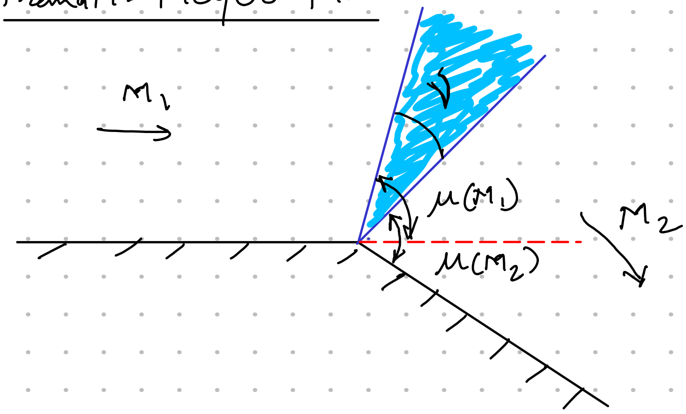
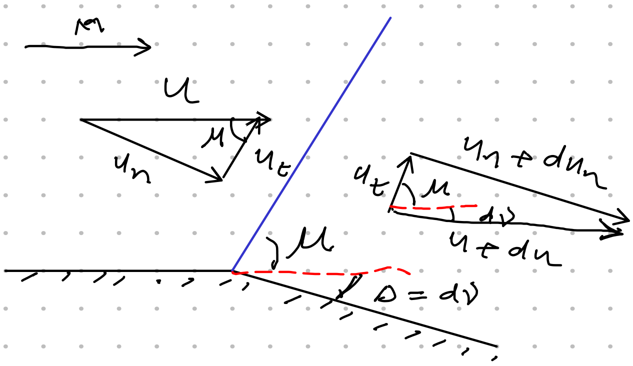
Prandtl – Meyer flow





single expansion wave.

$$u_{t_1} = u_{t_2}$$

$$u \cos\mu = (u + du)\cos(\mu + d\nu)$$

After expansion and

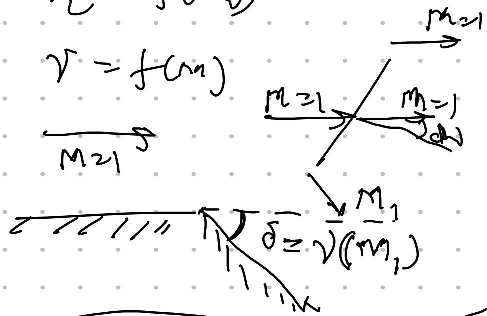
$\cos(d\nu) \approx 1, \quad \sin(d\nu) \approx d\nu$

$$\nu_2 - \nu_1 = f(M_2) - f(M_1)$$

$$\nu_2 - \underbrace{\nu_{ref}}_{0} = f(M_2) - \underbrace{f(M_{ref} = 1)}_{0}$$

$$\nu_2 = f(M_2)$$

$$\nu = f(M)$$



$\nu(M_2)$

$$\Rightarrow \frac{du}{u} = \tan(\mu)\, d\nu$$

$$\Downarrow$$

$$\mu = \sin^{-1}\left(\frac{1}{M}\right) \Rightarrow \tan\mu = \frac{1}{\sqrt{1 - M^2}}$$

$$\Rightarrow \frac{du}{u} = \frac{1}{\sqrt{1 - M^2}}\, d\nu \quad \text{— (A)}$$

Since $u = Ma = M\sqrt{\gamma R T}$

$$\frac{du}{u} = \frac{dM}{M} + \frac{1}{2}\frac{dT}{T} \quad \text{— (B)}$$

Since $T_0 = \text{const} := T\left(1 + \frac{\gamma - 1}{2} M^2\right) = \text{const}$

$\Downarrow$ Taking derivative.

$$\frac{dT}{T} + \frac{(\gamma - 1) M\, dM}{1 + \frac{\gamma - 1}{2} M^2} = 0 \quad \text{— (C)}$$

Using (A), (B), (C),

$$d\nu = \frac{\sqrt{M^2 - 1}}{\left(1 + \frac{\gamma - 1}{2} M^2\right)} \frac{dM}{M} \quad \text{— (D)}$$

$$\int_{\nu_1}^{\nu_2} d\nu = \int_{M_1}^{M_2} (\quad)\, dM$$

$$\nu_2 - \nu_1 = \underline{f(M_2)} - f(M_1)$$

$$\nu_2 - \nu_{ref} = f(M_2) - f(\underline{M_{ref}})$$

$M_{ref} = 1$

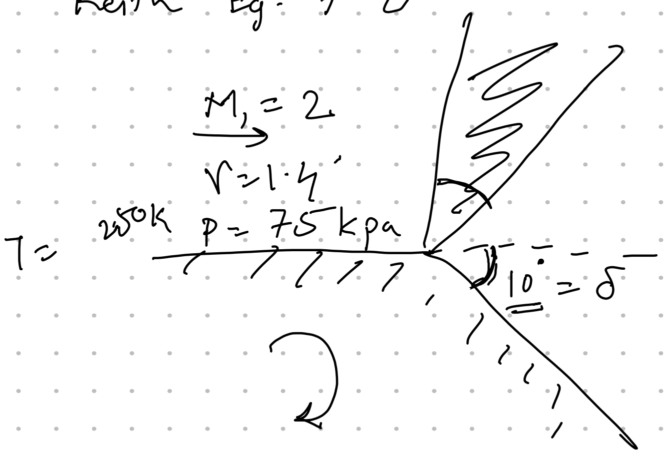
$f(M=1) = 0$

$$\boxed{\nu_2 = f(M_2)}$$

$$\boxed{\nu = f(M)}$$

Keith Chapter 7

Keith Eg. 7.2



$M_2 = ?$

$p_2 =$

$T_2 =$

$\rho_2 =$

$$\boxed{\nu_2 - \nu_1 = \delta}$$

$\nu($

Given $M_1 = 2$, from isentropic tables, $\nu_1 = \nu(M_1) = 26.37^\circ$

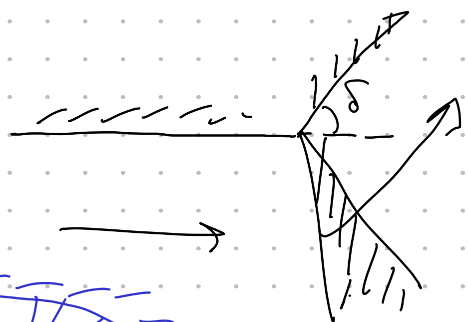
$$-\delta = \nu_1 - \nu_2 \implies \nu_2 = \nu_1 + \delta = 26.37 + 10$$

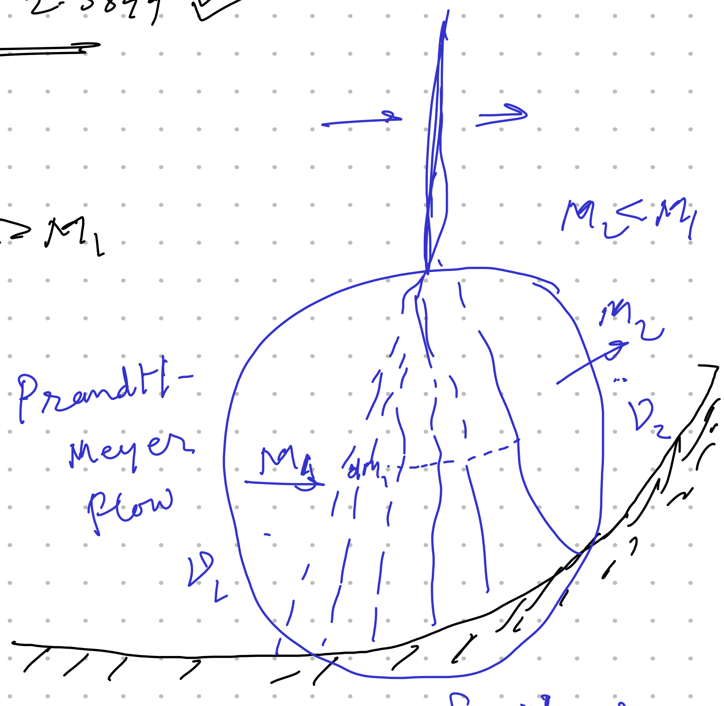
$\nu(M_1)$ $\quad$ $\nu(M_2)$

$$= 36.37^\circ$$

from tables, $\underline{M_2 = 2.3847}$ ✓

$\nu(M_2) > \nu(M_1)$ if $M_2 > M_1$





Since $M_2 < M_1 \implies \nu_2 < \nu_1$. $\delta = \nu_1 - \nu_2$

Q ⇒ What is the upperlimit of ν ?

$\Rightarrow \lim_{M \to \infty} f(M) = \lim_{M \to \infty} \nu = 134°$