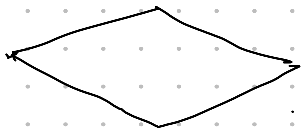
Shock Expansion theory

weak oblique shock
Expansion fans $\Bigg\}$ $\xrightarrow{\text{Simple}}$ geometries $\qquad C_L, C_D$

$\alpha = 0$

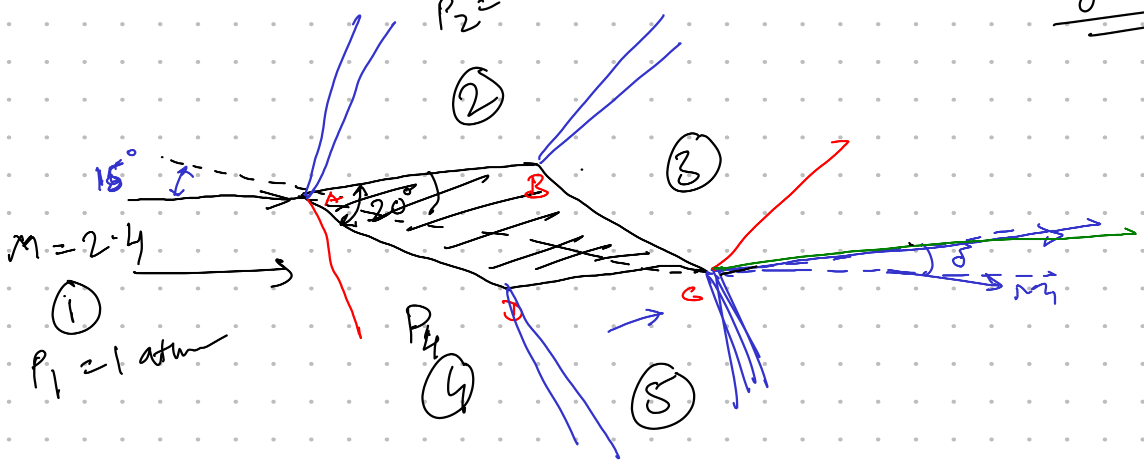
$M > 1$



$D \neq 0 \Rightarrow$ Wave drag

$L = 0$

Ratha
Eg: 6.10



$M_1 = 2.4$

$\nu(M_1) =$

$\nu_1(2.4) = 36.747$

$\nu_2 = \nu_1 + 5° = 41.747 \Rightarrow M_2 = 2.62$

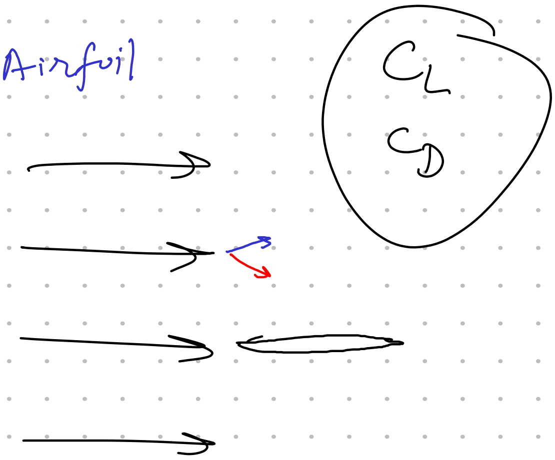
$P_1, M_1 \longrightarrow P_{o_1} = P_{o_2} \xrightarrow{M_2} P_2$

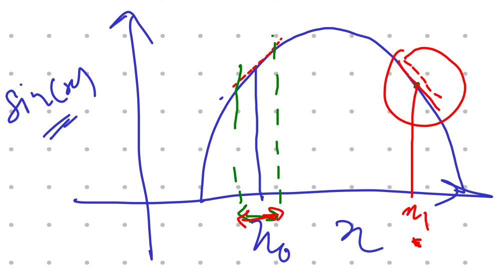
Thin Airfoil theory

Nonlinear eqns of flow
M, M, E conservtn $\Rightarrow$ Linearisation

$\Downarrow$

Thin Airfoil





$$C_p(x) = \frac{p(x) - p_\infty}{q_\infty}$$

Coeff. of pressure

$$= \frac{p(x) - p_1}{q_1}$$

Now $q_1 = \frac{1}{2}\rho_1 v_1^2 = \frac{1}{2} \frac{p_1 \gamma}{\underbrace{R T_1 \gamma}_{a_1^2}} \cdot v_1^2 = \frac{1}{2} p_1 \gamma \cdot M_1^2$

$$C_p(x) = \left(\frac{p(x) - p_1}{p_1}\right)\left(\frac{2}{\gamma M_1^2}\right)$$

Consider

$$\frac{\Delta p}{p_1} = \frac{p(x) - p_1}{p_1} = \frac{p}{p_1} - 1$$

$$= 1 + \frac{2\gamma}{\gamma+1}\left[M_1^2 \sin^2\beta - 1\right] - 1$$

$$= \frac{2\gamma}{\gamma+1}\left[M_1^2 \sin^2\beta - 1\right] \quad \text{——} \quad (1)$$

From Oblique shock relations,

$$\frac{1}{M_1^2 \sin^2\beta} = \left(\frac{\gamma+1}{2}\right)\frac{\tan(\beta-\theta)}{\tan\beta} - \frac{\gamma-1}{2}$$

$$M_1^2 \sin^2\beta - 1 = \frac{\gamma+1}{2} M_1^2 \frac{\sin\beta \sin\theta}{\cos(\beta-\theta)}$$

If weak oblique shock, θ is small,

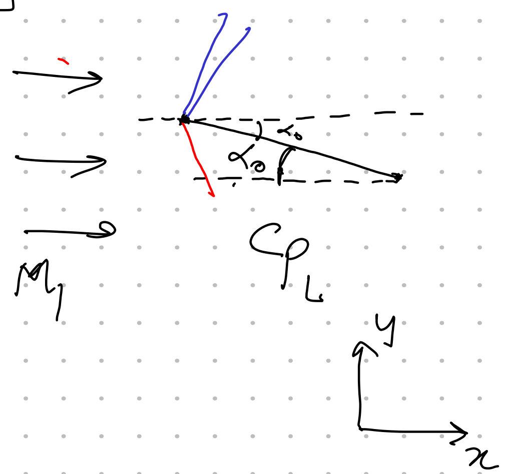
$$\tan\beta \approx \tan\mu = \frac{1}{\sqrt{M^2-1}}$$

$$LHS \approx \frac{\gamma+1}{2} \cdot M_1^2 \cdot \tan\mu \cdot \theta \approx \frac{\gamma+1}{2} \cdot M_1^2 \cdot \frac{1}{\sqrt{M_1^2-1}} \cdot \theta \quad \text{--- (2)}$$

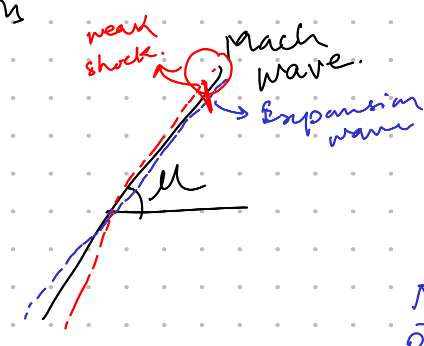
From (1) & (2),

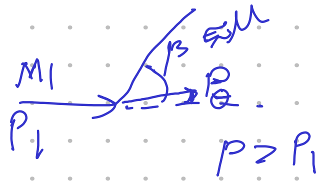
$$\frac{p-p_1}{p_1} \approx \frac{2\gamma}{\gamma+1}\left[\frac{\gamma+1}{2} \cdot M_1^2 \cdot \frac{1}{\sqrt{M_1^2-1}} \cdot \theta\right]$$

$$\approx \frac{\gamma M_1^2}{\sqrt{M_1^2-1}} \cdot \theta$$



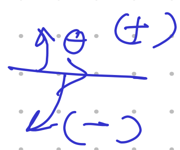
→ Small perturbations
– Linear model.





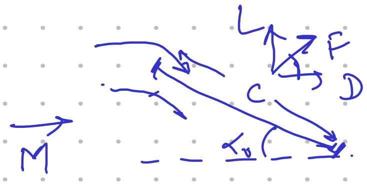
$p > p_1$

$$C_p = \frac{p-p_1}{q_1} \approx \frac{2\theta}{\sqrt{M_1^2-1}}$$



O.S $\quad p > p_1 \quad C_p > 0$

E.F $\quad p < p_1 \quad C_p < 0$



$$F = (P_L - P_U)\cdot c\,(1)$$

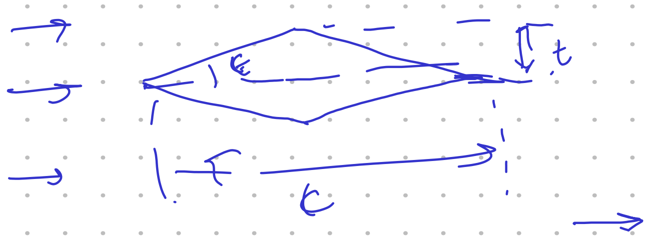
Rathakrishnan 6.14 / 6.15

$$C_{p_U} = \frac{-2\alpha_0}{\sqrt{M^2-1}}$$

$$C_{p_L} = \frac{2\alpha_0}{\sqrt{M^2-1}}$$

$$C_L = \frac{4\alpha_0}{\sqrt{M^2-1}}$$

$$C_D = \frac{4\alpha_0^2}{\sqrt{M^2-1}}$$



$$C_D = \frac{4\varepsilon^2}{\sqrt{M^2-1}} = \frac{4}{\sqrt{M^2-1}}\left(\frac{t}{c}\right)^2$$