Weak Shock relations.

$$\frac{\tan(\beta-\theta)}{\tan\beta} = \frac{(\gamma-1)\, M_1^2 \sin^2\beta + 2}{(\gamma+1)\, M_1^2 \sin^2\beta}$$

$$\therefore \frac{\gamma+1}{2} \frac{\tan(\beta-\theta)}{\tan(\beta)} = \frac{\frac{\gamma-1}{2} M_1^2 \sin^2\beta + 1}{M_1^2 \sin^2\beta} = \frac{\gamma-1}{2} + \frac{1}{M_1^2 \sin^2\beta}$$

$$\frac{1}{M_1^2 \sin^2\beta} = \frac{\gamma+1}{2} \cdot \frac{\tan(\beta-\theta)}{\tan(\beta)} - \frac{\gamma-1}{2}$$

$$\Rightarrow M_1^2 \sin^2\beta - 1 = \frac{\gamma+1}{2} M_1^2 \frac{\sin\beta \sin\theta}{\cos(\beta-\theta)}$$

$\sin\theta \rightarrow \theta$, $\cos(\beta-\theta) \rightarrow \cos\beta$

What happens if θ have small deflection? $\theta \approx 0$

$$M_1^2 \sin^2\beta - 1 = \left(\frac{\gamma+1}{2} M_1^2 \tan\beta\right) \theta$$

For weak shocks $\beta \approx 0$.
Also assuming high M_1

$$\Rightarrow M_1^2 \beta^2 - 1 \approx \left(\frac{\gamma+1}{2}\right) M_1^2 \beta \theta$$

For large M_1, $M_1^2 \beta^2 - 1 \approx M_1^2 \beta^2$

$$\Rightarrow M_1^2 \beta^2 \approx \frac{\gamma+1}{2} M_1^2 \beta \theta$$

$$\Rightarrow \beta \approx \frac{\gamma+1}{2} \theta$$

$\Rightarrow$ Large M_1 weak shocks with small deflection.

Another more general approx can be,

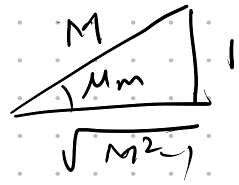
given, $$M_1^2 \sin^2\beta - 1 = \frac{\gamma+1}{2} M_1^2 \frac{\sin\beta \sin\theta}{\cos(\beta-\theta)}$$

$$\Rightarrow M_1^2 \sin^2\beta - 1 \approx \frac{\gamma+1}{2} M_1^2 \cdot \tan\beta \cdot \theta$$

For weak shocks, β is close to Mach angle.

$$\beta \approx \mu_{M_1} = \sin^{-1}\left(\frac{1}{M_1}\right)$$

So $\tan\beta \simeq \tan\mu_{M_1} = \dfrac{1}{\sqrt{M_1^2-1}}$



$$M_1^2\sin^2\beta - 1 \approx \frac{\gamma+1}{2}\frac{M_1^3}{\sqrt{M_1^2-1}}\cdot\theta$$

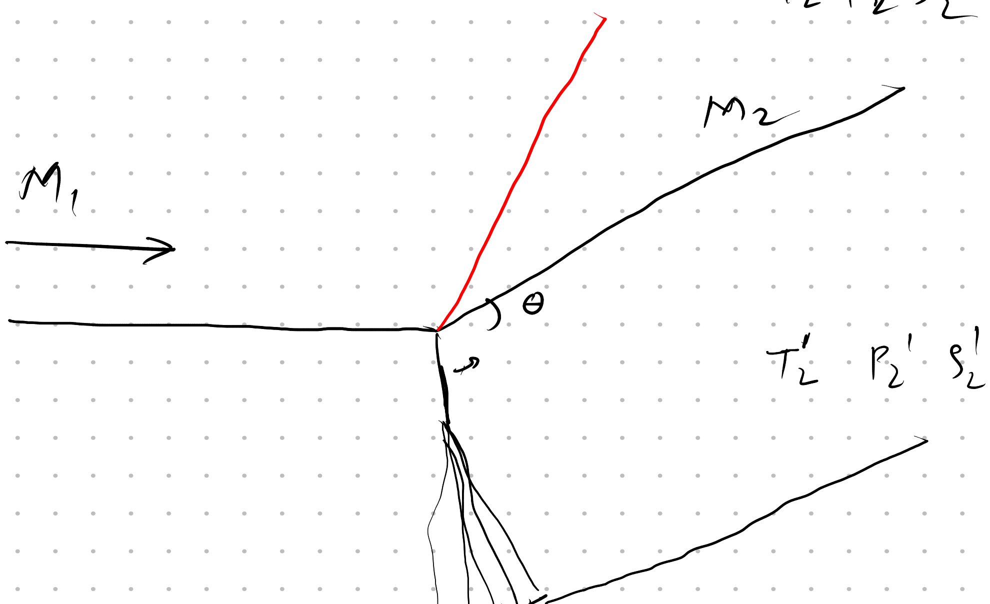
$$\frac{\Delta p}{p_1} = \frac{p_2}{p_1} - 1 = 1 + \frac{2\gamma}{\gamma+1}\left(M_1^2\sin^2\beta - 1\right) - 1$$

$$\frac{\Delta p}{p} \approx \frac{\gamma M_1^2}{\sqrt{M_1^2-1}}\cdot\theta$$

$$\frac{\Delta\rho}{\rho},\ \frac{\Delta T}{T},\ \propto \theta$$

But for entropy,

$$\frac{\Delta s}{R} \propto \theta^3$$

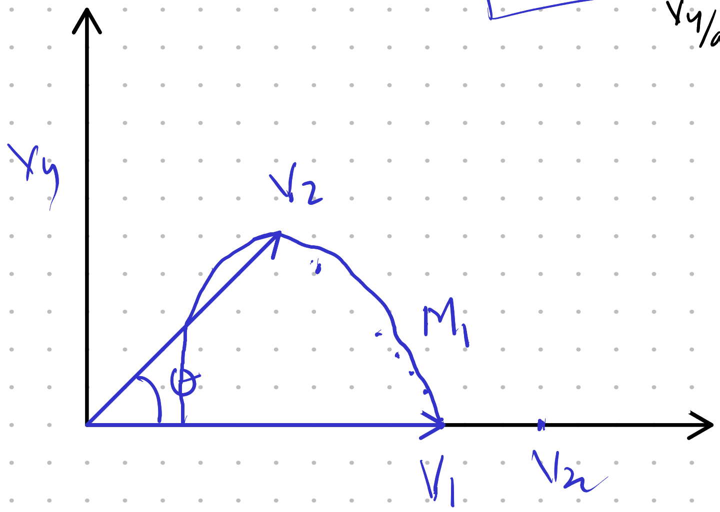


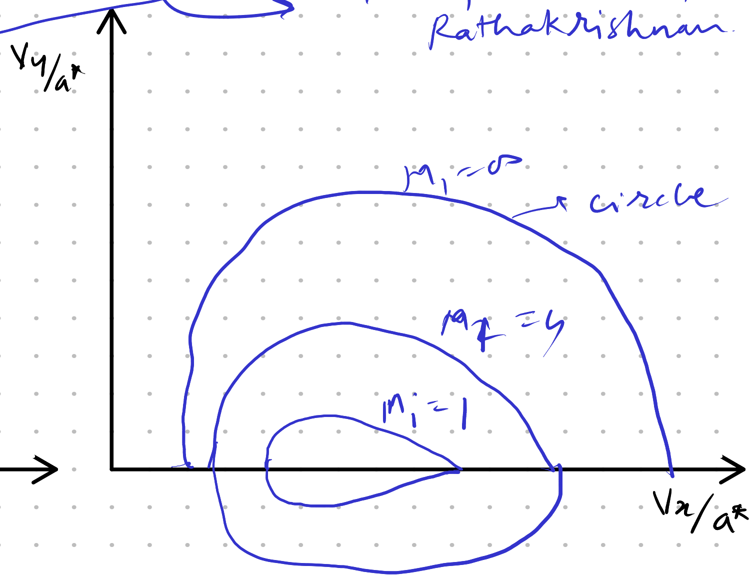
$\sum \theta_n = \theta$

Sec 6.6
Rathakrishnan

Shock Polar

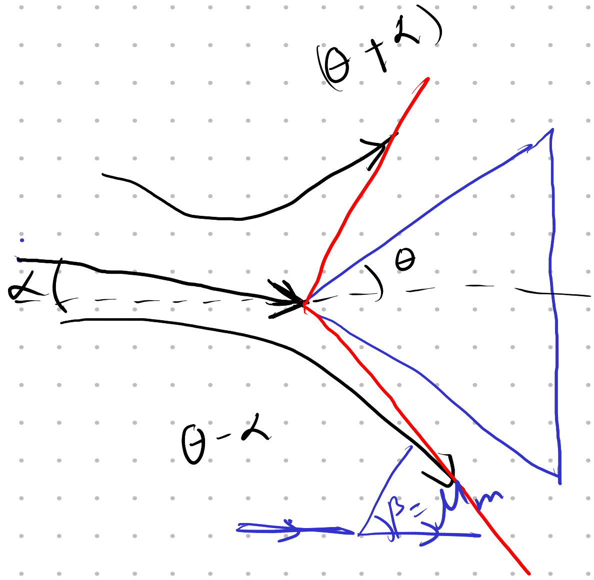
$\boxed{6.4 / \text{Rathakrishnan}}$ → Gasdynamics by Rathakrishnan





Show $\Rightarrow$ $a^*\big|_{\text{Before shock}} = a^*\big|_{\text{After shock}}$

Q :-



Bow shock