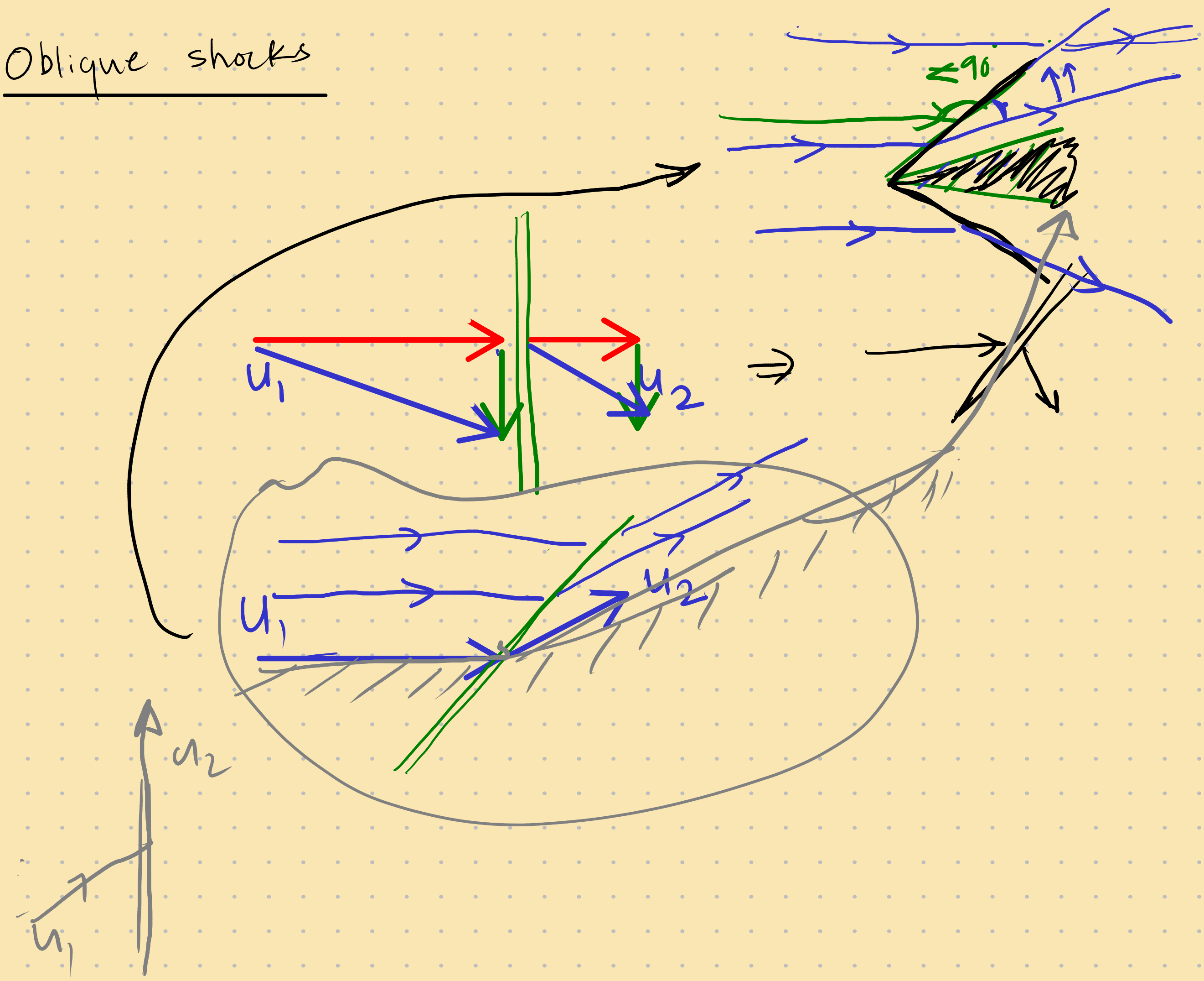
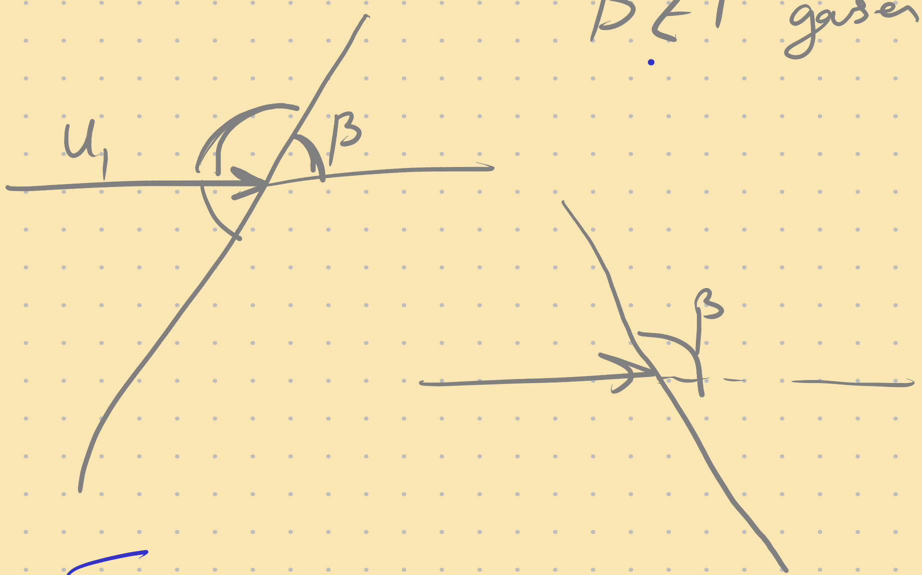
Oblique shocks



BZT gases



$\frac{\Delta s}{R}$

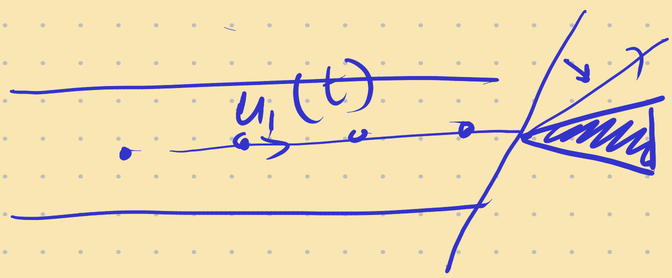
$M_2 = \sqrt{\frac{2}{\gamma(\gamma-1)}}$

$t \to t + \Delta t$

$u_2 \uparrow \Rightarrow T_2 \uparrow$

$\frac{u_2}{\sqrt{T_2}}$

$p_0(t)$

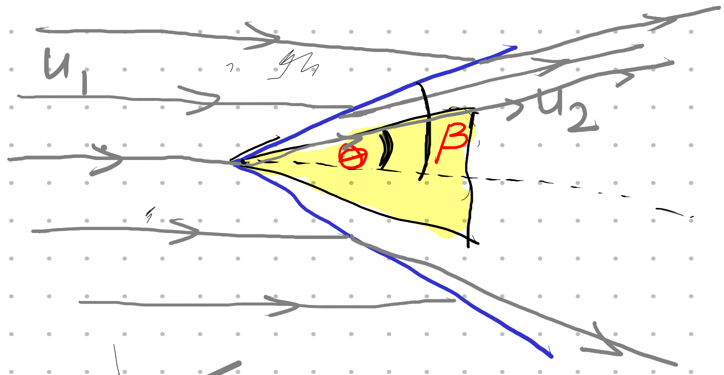


Oblique Shock :-

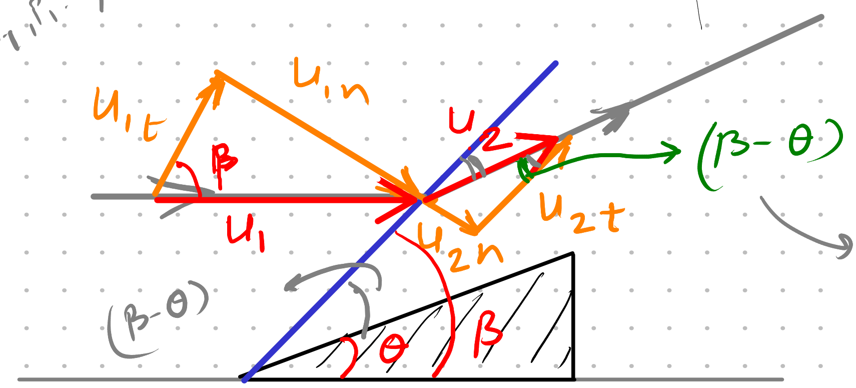
→ 2D

→ θ - Half angle of wedge

β - Half angle of shock



$\beta = f(\theta, M_1, \gamma, P_1, \ldots)$



$\tan^{-1}(\beta-\theta) = \dfrac{u_{2n}}{u_{2t}}$

$\theta = \beta - \tan^{-1}\left(\dfrac{u_{2n}}{u_{2t}}\right)$

$u_{1n} = u_1 \sin\beta \quad , \quad u_{1t} = u_1 \cos\beta$

$u_{2n} = u_2 \sin(\beta-\theta) \quad , \quad u_{2t} = u_2 \cos(\beta-\theta)$

Clearly, there is no change in the tangential velocity.

$$\Rightarrow \quad u_{1t} = u_{2t}$$

The normal component of the velocity encounters a normal shock. Hence normal shock relations apply.

So, the algo is this.

Given M_1 & β,

$M_{1n} = M_1 \sin\beta, \quad M_{1t} = M_1 \cos\beta$

$M_{2n} = M_2 \sin(\beta-\theta) \quad M_{2t} = M_2 \cos(\beta-\theta)$

Clearly, $M_{1t} = M_{2t}$ ✗ $\quad u_{1t} = u_{2t}$ ✓

$$\left.\begin{aligned} M_{2n} &= f_1(M_{1n}) \\ \rho_2/\rho_1 &= f_2(M_{1n}) \\ P_2/P_1 &= f_3(M_{1n}) \\ T_2/T_1 &= f_4(M_{1n}) \end{aligned}\right\} \text{NORMAL SHOCK RELATIONS}$$

$\rightarrow$ $u_{2n} = f_5(M_{2n}, T_2) \Rightarrow u = \sqrt{u_{2n}^2 + u_{2t}^2}$

$\searrow M_2(T_2, u_2)$

$$(M_1, P_1, \rho_1, T_1, \beta) \longrightarrow M_{1n}$$

$$\Big\downarrow \text{Normal shock relations}$$

$$(M_{2n}, P_2, \rho_2, T_2)$$

$$\Big\downarrow$$

$$\theta = \beta - \tan^{-1}\left(\frac{M_{2n}}{M_{2t}}\right), \quad M_2 = \sqrt{M_{2n}^2 + M_{1t}^2}$$

$\rightarrow$ But what if β is not known. Instead the angle of the flow (θ) is known.

i.e. We know $(M_1, \rho_1, P_1, T_1, \theta)$

For this scenario, we calculate,

$$\beta = f(\theta).$$

$\rightarrow$ Turns out, it is easier to find the

the inverse function.

i.e. $\theta = g(\beta) = f^{-1}(\beta)$

So we know,

$$u_{1t} = u_{2t}$$

$$\therefore u_1 \cos\beta = u_2 \cos(\beta - \theta)$$

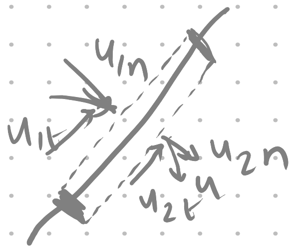
$$\therefore \frac{u_1}{u_2} = \frac{\cos(\beta-\theta)}{\cos(\beta)}$$

$$\frac{u_{1n}/\sin\beta}{u_{2n}/\sin(\beta-\theta)} = \text{''}$$

$$\therefore \frac{u_{1n}}{u_{2n}} = \frac{\cos(\beta-\theta)}{\cos(\beta)} \cdot \frac{\sin(\beta)}{\sin(\beta-\theta)} = \frac{\tan\beta}{\tan(\beta-\theta)} \quad \text{— (A)}$$

From mass conservation,

$$\rho_1 u_{1n} = \rho_2 u_{2n} \quad \text{— (B)}$$



From (A) & (B).

$$\frac{\rho_2}{\rho_1} = \frac{\tan(\beta-\theta)}{\tan(\beta)} \quad \text{— (C)}$$

Now,

$$\frac{\rho_2}{\rho_1} = f_3(M_1\sin\beta) = \frac{(\gamma+1)M_1^2\sin^2\beta}{(\gamma-1)M_1^2\sin^2\beta + 2} \quad \text{— (D)}$$

$$\frac{u_{1n}}{a_1} = \frac{u_1\sin\beta}{a_1} = M_1\sin\beta$$

Using (C) & (D) it can be shown that

$$\tan\theta = 2\cot\beta\left[\frac{M_1^2\sin^2\beta - 1}{2 + M_1^2[\gamma + \cos 2\beta]}\right]$$

This gives us $\theta = g(\beta) = f^{-1}(\beta)$

Now we can either use a plot or use a computer

Q: What is the minimum achievable value of β?

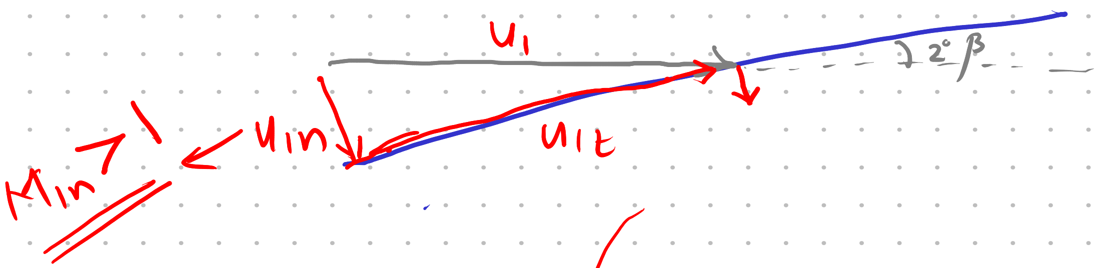
$\rightarrow$ $$\frac{p_2}{p_1} = 1 + \frac{2\gamma}{\gamma+1}(M_1^2\sin^2\beta - 1)$$

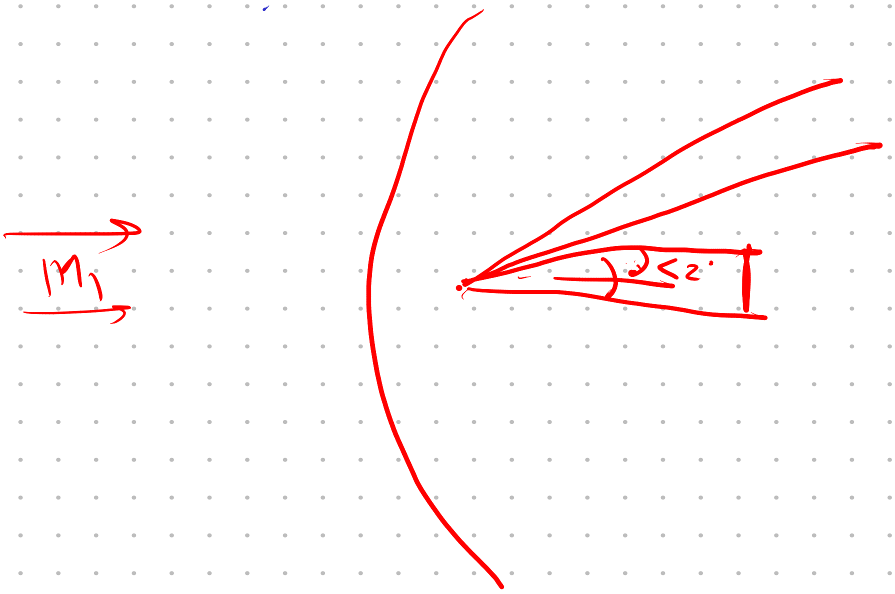
$\sin^{-1}\left(\frac{1}{M_1}\right) < \beta < 90$

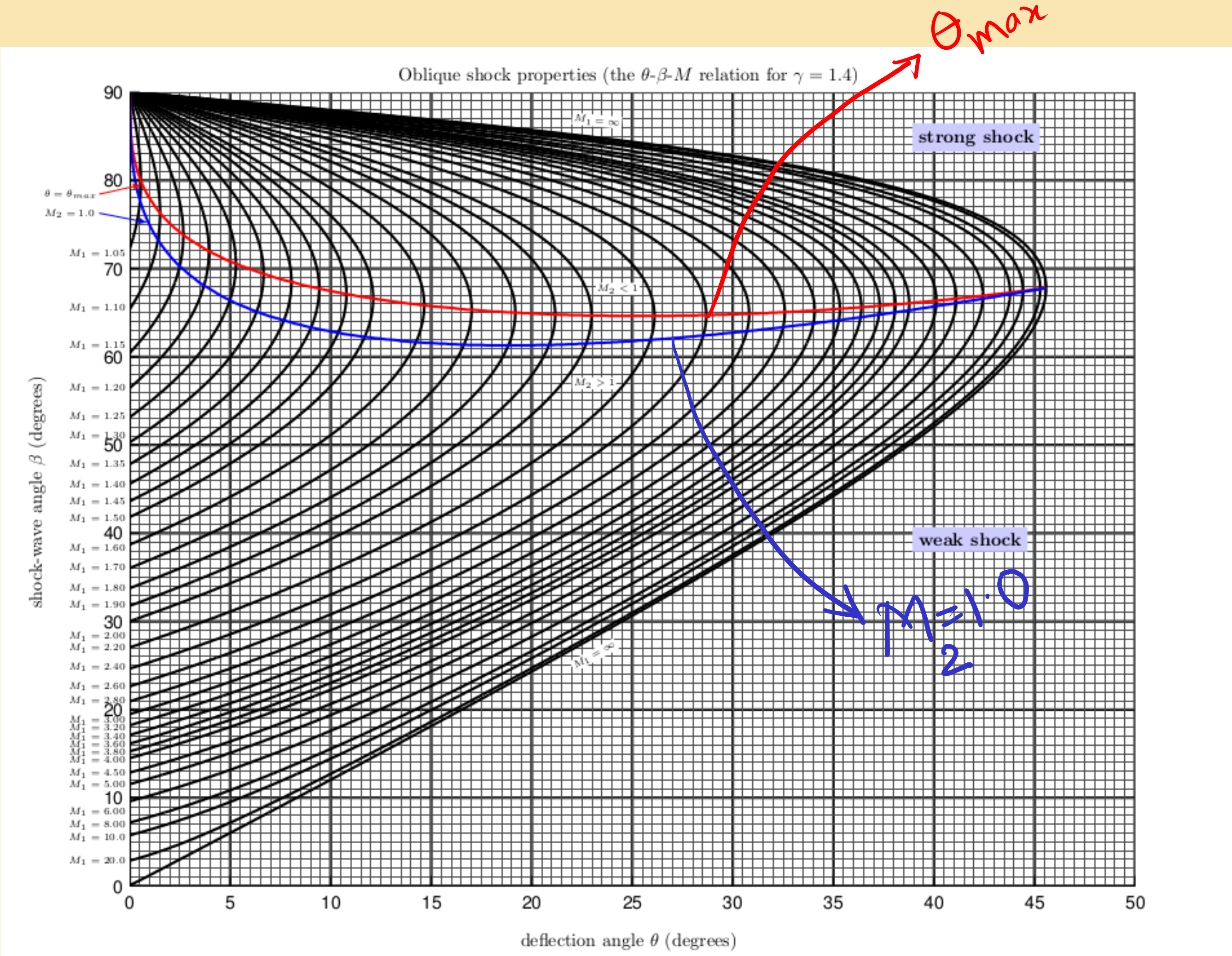
$p_2 > p_1 \Rightarrow M_1^2\sin^2\beta > 1$

$\sin^2(\beta) \geq 1/M_1^2 \Rightarrow \sin\beta \geq 1/M_1$

$\beta \geq \sin^{-1}(1/M_1)$






θ_{max}
Oblique shock properties (the θ-β-M relation for $\gamma = 1.4$)
strong shock
weak shock
$M_1 = \infty$
$M_2 < 1$
$M_2 > 1$
$\theta = \theta_{max}$
$M_2 = 1.0$
$M_2 = 1.0$
shock-wave angle β (degrees)
deflection angle θ (degrees)

Q: What is maximum value of β?

Q:- What is the possible range of θ?

$\rightarrow$

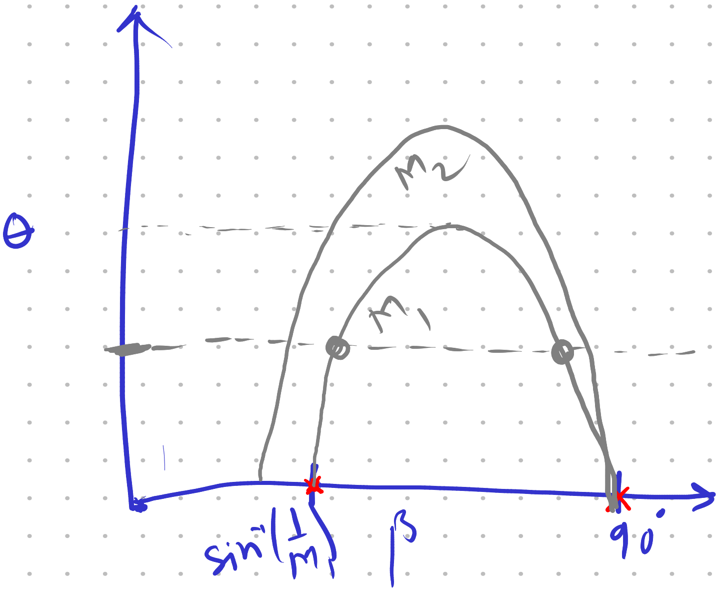
$$\tan(\theta) = \frac{2\cot\beta(M_1^2\sin^2\beta - 1)}{2 + M_1^2(\gamma + \cos 2\beta)}$$

$0 < \theta < 90^\circ$

$\theta \begin{cases} \nearrow \beta_1 \\ \searrow \beta_2 \end{cases}$

Fix γ - gas

M_1 — Upstream Mach num.



M_1, γ

M_2, γ

$M_2 > M_1$