Assignment 1:

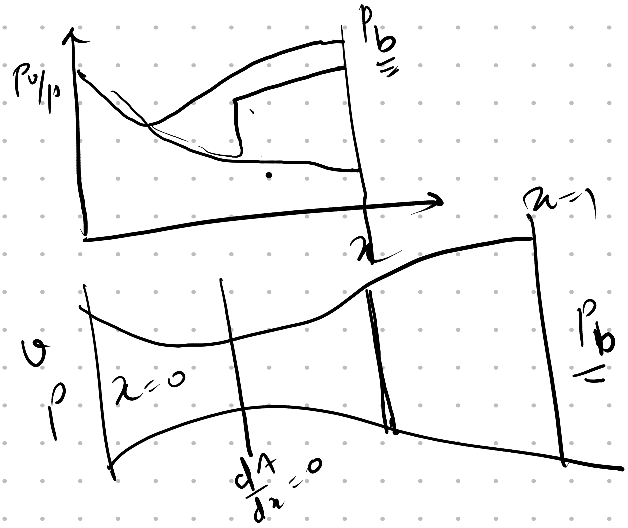
The aim of the assignment is to plot the p0/p variation along x for a CD nozzle.
Write a code (in the language of choice) which has two functions.

1. First is an area function whose input is x and output is A(x). You can take the area function as a quadratic of the form a*x^2 + b*x + c, x = [0, 1].

2. Second function has the input
   a. Velocity at x=0
   b. Pressure at x=0
   c. Back pressure pb

   and it should output
   a. Plot of p0/p vs. x
   b. Value of pressure at x=1
   c. (s(x=0) - s(x=1))/R

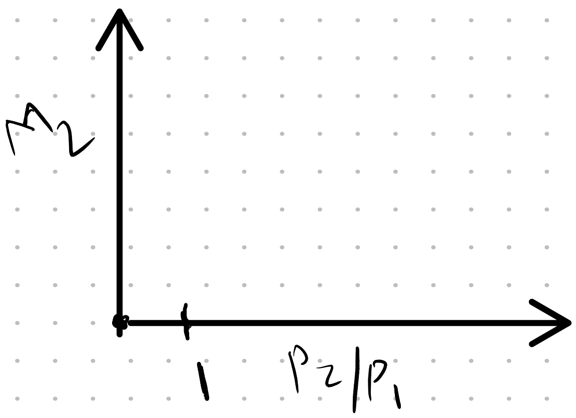
Code should work for any values of a, b, c.



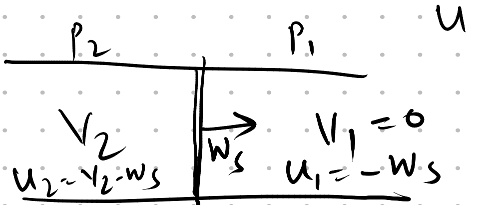
Review :-

		$p_2/p_1 = 1$	$p_2/p_1 \to \infty$
1.	What is the range of V_2?	0	∞
2.	What is the range of W_s?	a_1	∞
3.	—— " —— M_2?	0	$\sqrt{\frac{2}{\gamma(\gamma-1)}}$

4. How will I solve a problem if I am in a reference frame where $V_1 \neq 0$?



$u_1 = V_1 - W_s$



$M_2 \to$ Induced Mach number

Moving normal shock.

We know that

$$V_2 = w_s\left(1 - \frac{u_2}{u_1}\right)$$

$$= w_s\left(1 - \frac{\rho_1}{\rho_2}\right)$$

$$= a_1 M_1 \left(1 - \frac{1}{\rho_2/\rho_1}\right)$$

$$= a_1 \left(\frac{\gamma+1}{2\gamma}\frac{p_2}{p_1} + \frac{\gamma-1}{2\gamma}\right)^{1/2}\left(1 - \frac{\left(\frac{\gamma+1}{\gamma-1}\right) + \frac{p_2}{p_1}}{\left(\frac{\gamma+1}{\gamma-1}\right)\left(\frac{p_2}{p_1}\right) + 1}\right)$$

Now, $M_2 = \dfrac{V_2}{a_2} = \left(\dfrac{V_2}{a_1}\right)\left(\dfrac{a_1}{a_2}\right)$

$$\therefore M_2 = \left(\left(\frac{\gamma+1}{2\gamma}\right)\left(\frac{p_2}{p_1}\right) + \frac{\gamma-1}{2\gamma}\right)^{1/2}\left[1 - \frac{\frac{\gamma+1}{\gamma-1} + \frac{p_2}{p_1}}{\left(\frac{\gamma+1}{\gamma-1}\right)\left(\frac{p_2}{p_1}\right) + 1}\right]\underbrace{\left[\frac{\frac{\gamma+1}{\gamma-1} + p_1/p_2}{\frac{\gamma+1}{\gamma-1} + \frac{p_2}{p_1}}\right]^{1/2}}_{\text{This comes from } \frac{a_1}{a_2} = \sqrt{\frac{T_1}{T_2}}}$$

For weak shock, as $\frac{p_2}{p_1} \to 1$,

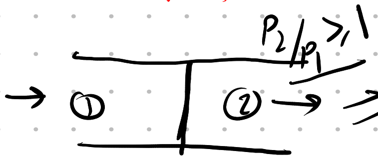
$$w_s \to a_1 \quad \& \quad V_2 \to 0 \quad \& \quad M_2 \to 0$$

For strong shock, as $\frac{p_2}{p_1} \to \infty$,

$$w_s \to \infty \quad \& \quad V_2 \to \infty \quad \& \quad M_2 = \sqrt{\frac{2}{\gamma(\gamma-1)}}$$

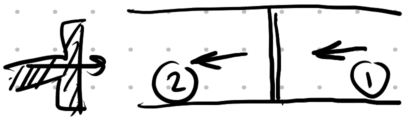
Now, let's see what happens to P_{02}/P_{01} ?
(In earth ref. frame)

$$\frac{P_{02}}{P_{01}} = \frac{P_{02}}{P_1} = \frac{P_{02}}{P_2}\cdot\frac{P_2}{P_1}$$



$\Rightarrow \boxed{\dfrac{P_{02}}{P_{01}} \leq 1}$

$$\therefore \frac{P_{02}}{P_{01}} = \left(1+\frac{\gamma-1}{2}M_2^2\right)^{\gamma/\gamma-1}\cdot\frac{P_2}{P_1}$$



$\Rightarrow \dfrac{P_{02}}{P_{01}} \geq 1$

$M_2 = \dfrac{u_2}{a_2}$ or $\dfrac{V_2}{a_2}$

This means that $\dfrac{P_{02}}{P_{01}} \geq 1$ (Always!!)

$$\frac{T_{02}}{T_{01}} = \frac{T_{02}}{T_1} = \frac{T_2+\frac{V_2^2}{2C_p}}{T_1} = \frac{T_2}{T_1}+\frac{V_2^2\,(\gamma-1)}{2\gamma R T_1}$$

$$= \frac{T_2}{T_1} + \left(\frac{V_2}{a_1}\right)^2\cdot\frac{\gamma-1}{2}$$

$$\therefore \frac{T_{02}}{T_{01}} = \frac{\frac{\gamma+1}{\gamma-1}+\frac{P_2}{P_1}}{\frac{\gamma+1}{\gamma-1}+\frac{P_1}{P_2}} + \left(\frac{\gamma-1}{2}\right)\left[\left(\frac{\gamma+1}{2\gamma}\frac{P_2}{P_1}+\frac{\gamma-1}{2\gamma}\right)\left(1-\frac{\frac{\gamma+1}{\gamma-1}+\frac{P_2}{P_1}}{\left(\frac{\gamma+1}{\gamma-1}\right)\left(\frac{P_2}{P_1}\right)+1}\right)^2\right]$$

This is not equal to 1, because we are in earth fixed ref. In the shock ref. frame, this will be 1.

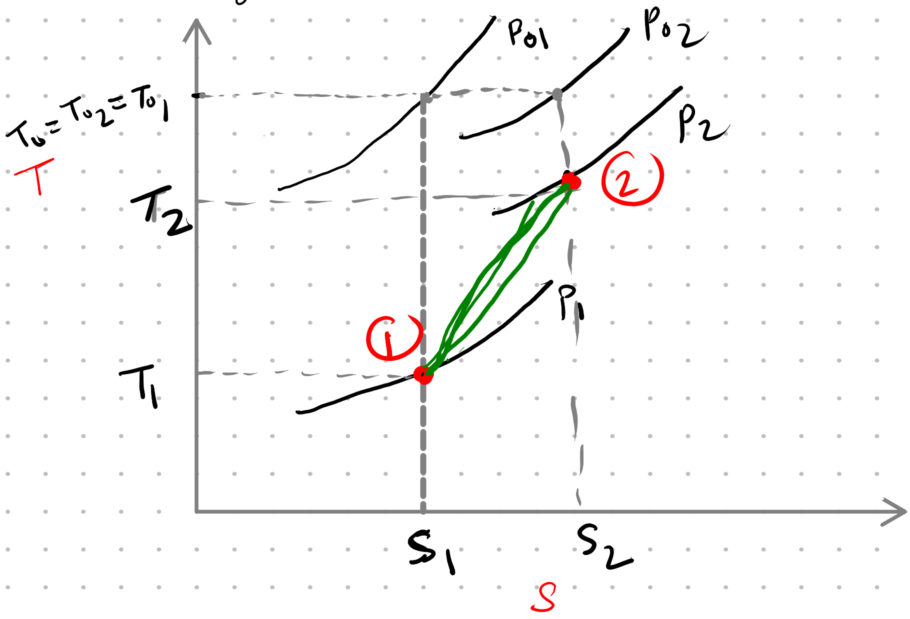
May be it is a good exercise to prove that $\frac{T_{02}}{T_{01}} = 1$ in the shock reference frame.

Entropy change

$$\Delta s = s_2 - s_1 = C_p \ln T_2/T_1 - R \ln P_2/P_1$$

$$\frac{\Delta s}{R} = \ln \frac{(T_2/T_1)^{\gamma/\gamma-1}}{P_2/P_1}$$

T-s diagram of a stationary normal Shock.



$$\frac{P_{02}}{P_{01}} \leq 1$$

Similarly draw the p-v diagram for a normal shock.
The process is conceptually similar.

Hugoniot equation (Sec. 5.5 Rathakrishnan)

→ We ask ourselves, is it possible to get a relationship between only the thermodynamic quantities across a shock?

→ Can we get a relationship, independent of the M_1 & M_2 also (if possible) any assumptions on the nature of the gas.

Mass conservation,
$$\rho_1 u_1 = \rho_2 u_2 \quad — ①$$
$$p_1 + \rho_1 u_1^2 = p_2 + \rho_2 u_2^2 \quad — ②$$

Using ① & ②,
$$p_1 + \rho_1 u_1^2 = p_2 + \rho_2 \left(\frac{\rho_1 u_1}{\rho_2}\right)^2$$
$$\Rightarrow \quad u_1^2 = \frac{p_2 - p_1}{\rho_2 - \rho_1}\left(\frac{\rho_2}{\rho_1}\right). \quad \text{Similarly,} \quad u_2^2 = \frac{p_2 - p_1}{\rho_2 - \rho_1}\left(\frac{\rho_1}{\rho_2}\right)$$

$$h_{0_1} = h_{0_2}$$
$$\therefore \underbrace{e_1 + p_1/\rho_1}_{h_1} + \frac{u_1^2}{2} = e_2 + p_2/\rho_2 + \frac{u_2^2}{2}$$

$$e_2 - e_1 = \frac{p_1 + p_2}{2}\left(\frac{1}{\rho_1} - \frac{1}{\rho_2}\right)$$

$e(p, \rho)$ $\quad p_2 = f(p_1, v_1, v_2)$

$$e_2 - e_1 = \frac{P_1 + P_2}{2}(v_1 - v_2)$$

$$\Delta e = -P_{av}\Delta v$$

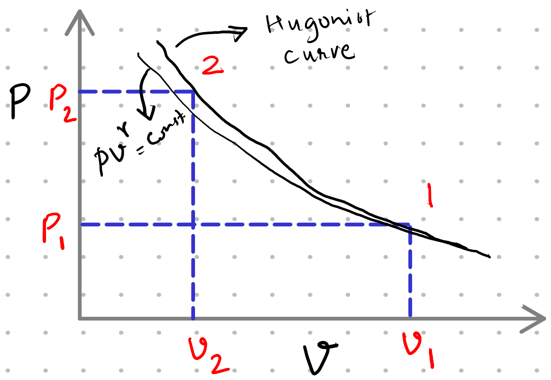
→ Now, let us ask if this equation is valid for a moving normal shock?

→ We note that it is also valid for real gases, chemically reacting flows etc.

→ We know that $e = e(p, v)$ is valid for any equilibrium thermodynamic state

∴ Hugoniot equation can be written as

$$P_2 = f(P_1, v_1, v_2)$$



Hugoniot curve represents all possible (p_2, v_2) that can be attained by shocks of different strengths from (p_1, v_1).

So how do we use this plot?

→ What determines the strength of a shock for a given upstream condition of (P_1, v_1)?

$x-t$ diagram representation of a moving normal shock.