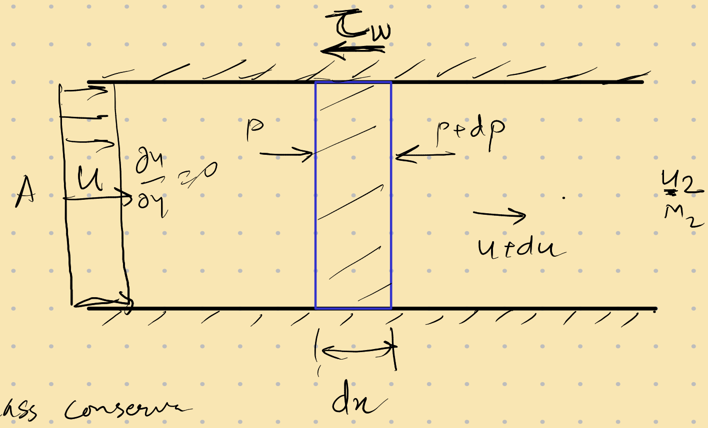
Non Isentropic flow

Isentropic flow
→ No viscosity.
→ No heat addition



1D
steady
Adiabatic
$\frac{dA}{dx} = 0$

Mass conserve

$$\rho_1 u_1 A = \rho_2 u_2 A$$

$$\rho u A = \text{const.}$$

$$\frac{d\rho}{\rho} + \frac{du}{u} + \frac{dA}{A} = 0 \quad \text{— (1)}$$

Energy conservation

$$h + \frac{u^2}{2} = \text{const.}$$

$$h_0 = \text{const.} \quad \text{— (2)}$$

Momentum conservation

$$pA - (p+dp)A - \tau_w(\underbrace{\Gamma_w}_{\text{perimeter}} dx) = (\rho u A)(u+du) - (\rho u A)u$$

Hydraulic diameter $D_H = \frac{4 A_w}{\Gamma_w}$ $\qquad \frac{4 \times \pi r^2}{2\pi r} = 2r$

$$-dp - \tau_w \frac{4dx}{D_H} = \rho u du$$

Dividing by ρu^2,

$$\frac{-dp}{\rho u^2} - \frac{\tau_w}{\rho u^2} \cdot \frac{4dx}{D_H} = \frac{du}{u} \quad \text{— (3)}$$

Derivations from Rathakrishnan or Zucker.

Energy eq^n :- $\quad \frac{dT}{T} + (\gamma-1) M^2 \frac{du}{u} = 0 \quad$ — (4)

Using mass & energy :- $\quad \frac{dp}{p} = -\frac{du}{u}\left[1 + (\gamma-1)M^2\right] \quad$ — (5)

But, $\quad M = \frac{u}{a} = \frac{u}{\sqrt{\gamma R T}}$

$$\frac{dM}{M} = \frac{du}{u} - \frac{1}{2}\frac{dT}{T}$$

$$\frac{dM}{M} = \frac{du}{u}\left[1 + \frac{\gamma-1}{2}M^2\right] \quad \text{— (6)}$$

Similarly $\quad ds = c_p \frac{dT}{T} - R \frac{dp}{p}$

$c_p \to \frac{\gamma R}{\gamma - 1}$, $\frac{dT}{T} \to$ from eq (4), $\frac{dp}{p} \to$ eq (5)

$$\frac{ds}{R} = (1 - M^2) \frac{du}{u}$$

$$\frac{ds}{R} = \frac{(1 - M^2)}{\left[1 + \frac{\gamma - 1}{2} M^2\right] M} dM$$

Subsonic $\quad M < 1 \Rightarrow (1 - M^2) > 0 \Rightarrow dM > 0$

Supersonic $\quad M > 1 \Rightarrow (1 - M^2) < 0 \Rightarrow dM < 0$

Sonic $\quad M = 1 \Rightarrow ds = 0$