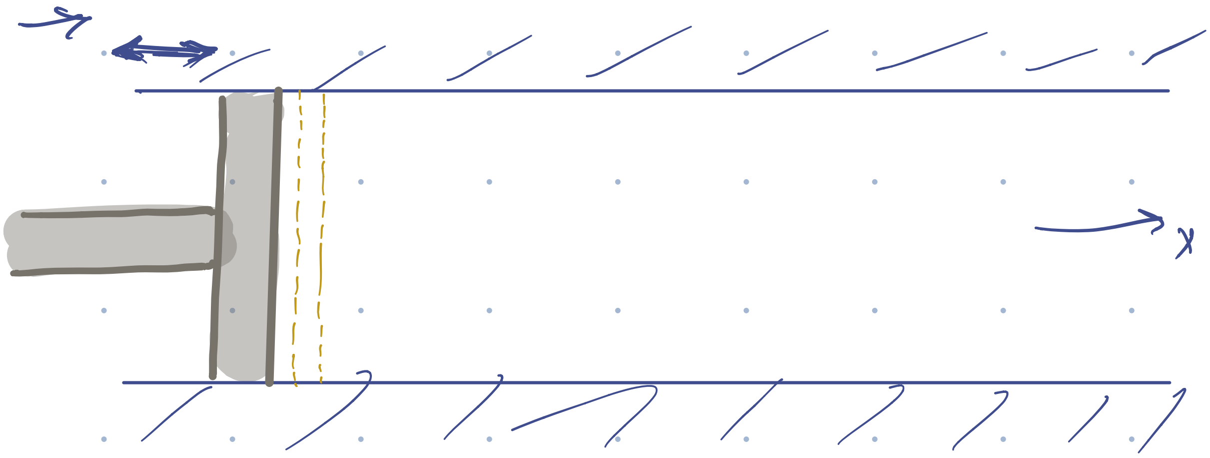
Note on speed of sound.

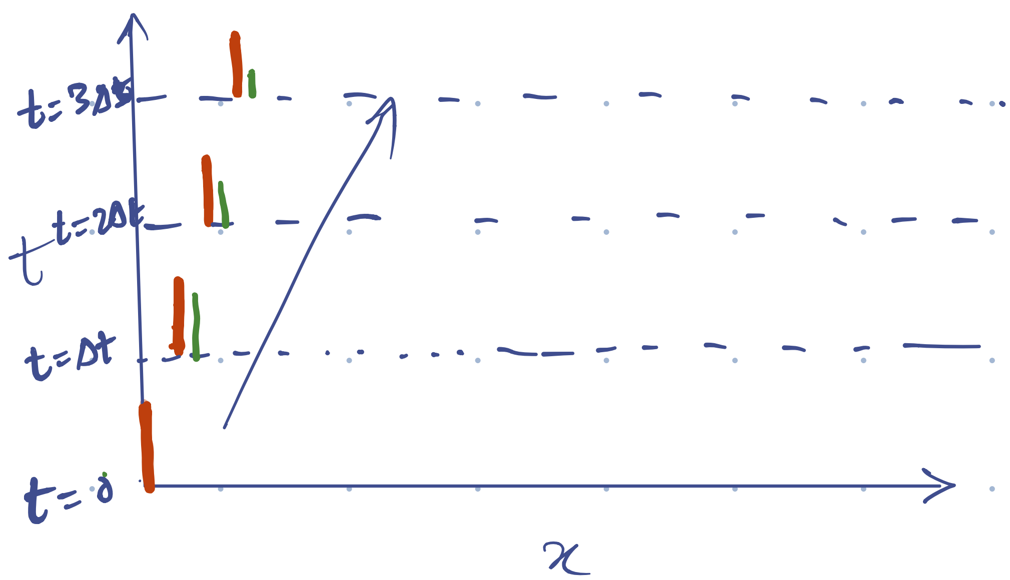
→ Sound is a pressure wave of very low magnitude

$\Delta p = 10^{-9}$ atm – threshold of hearing

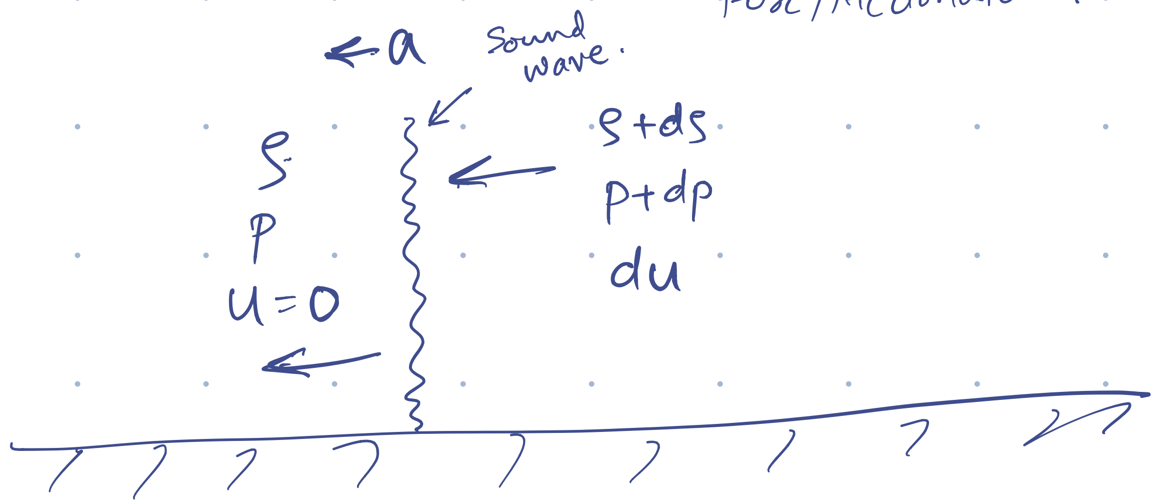
$\Delta p = 10^{-3}$ atm – Pain in ears.

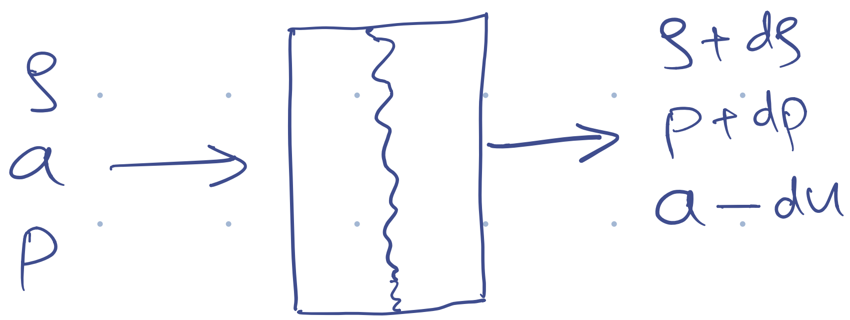
→ It is important because this is the speed at which "information" / "signals" travel in fluids.





Derivation of speed of sound (Fluid Mechanics, Fox/Mcdonald 12.2)





Continuity equation

$$\rho a A - \left[(\rho+d\rho)(a-du)A\right] = 0$$

$$\cancel{\rho a A} - \cancel{\rho a A} + \rho A\,du - aA\,d\rho + \cancel{d\rho\,du\,A}^{\,0} = 0$$

$$\therefore \quad du = \frac{a}{\rho}\,d\rho \qquad \text{—— (1)}$$

Momentum equation

$$F_s = pA - (p+dp)A = -A\,dp$$

$$\therefore \; -A\,dp = (a-du)\left\{(\rho+d\rho)(a-du)A\right\} - a(\rho a A)$$

$$\therefore -A\,dp = (a-du)\left\{ \rho a A - \rho\,du\,A + a\,d\rho\,A \right\} - a(\rho a A)$$

$$= -\rho a A\,du$$

$$\therefore du = \frac{1}{\rho a}\,dp \qquad \text{--- } ②$$

From ① & ②,

$$\frac{a}{\rho}\,d\rho = \frac{1}{\rho a}\,dp$$

$$\therefore a^2 = \frac{dp}{d\rho} = \left.\frac{\partial p}{\partial \rho}\right|_s$$

→ We are considering very small perturbations which are happening very fast.
Hence, no time for heat transfer ($dq = 0$)
Since infinitesimal pressure change, hence reversible.

Hence we can consider the process isentropic.

$$\therefore a^2 = \left.\frac{\partial p}{\partial \rho}\right|_s$$

For incompressible flow, clearly

$$a \to \infty$$

a is large for solids as compared to gases. But the signal attenuates much faster. In gases, the signal travels slow, but farther.

Another way of deriving the expression for speed of sound can be found in George Emmanuel (sec. 3.5).

This is a more rigorous way, and is called as the acoustic approximation.

As for perfect gas in isentropic flow, we know $P/\rho^\gamma = \text{const.}$ & $P = \rho RT$,

$$\therefore \quad a = \sqrt{\gamma RT}$$

Static Values

T_1
P_1
s_1
h_1
u_1

$\xrightarrow{\text{Isentropic}}$ process

$\Delta s = 0$

$\therefore s_1 = s_{o_1}$

T_{o_1}
P_{o_1}
s_{o_1}
h_{o_1}
$u_{o_1} = 0$

Total Values

We know that energy is conserved.

$$\therefore \quad h_1 + \frac{1}{2}u_1^2 = h_{o_1} + \frac{1}{2}u_{o_1}^2$$

Under perfect gas assumption,

$$h_1 = C_p T_1 \quad , \quad h_{o_1} = C_p T_{o_1} \quad \text{where } C_p = \frac{\gamma R}{\gamma - 1}$$

$$\therefore \quad C_p T_1 + \frac{1}{2}u_1^2 = C_p T_{o_1}$$

$$\therefore C_p(T_{o_1} - T_1) = \frac{1}{2}u_1^2$$

$$\therefore \left(\frac{T_{o_1}}{T_1} - 1\right) = \frac{1}{2}u_1^2 \left(\frac{1}{C_p}\right) \cdot \frac{1}{T_1}$$

$$= \frac{1}{2}u_1^2 \cdot \frac{\gamma - 1}{\gamma R} \cdot \frac{1}{T_1}$$

$$= \frac{\gamma - 1}{2} \cdot \frac{u_1^2}{\gamma R T_1}$$

We know that $a_1^2 = \gamma R T_1$.

$$\frac{T_{o_1}}{T_1} - 1 = \frac{\gamma - 1}{2} \cdot \frac{u_1^2}{a_1^2}$$

$$\therefore \boxed{\frac{T_{o_1}}{T_1} = 1 + \frac{\gamma - 1}{2} M_1^2} \quad \text{where } M_1 = \frac{u_1}{a_1}$$

= Mach number at 1

Similarly, $\frac{P_{o_1}}{P_1}$, $\frac{s_{o_1}}{s_1}$ can be calculated using isentropic relations.