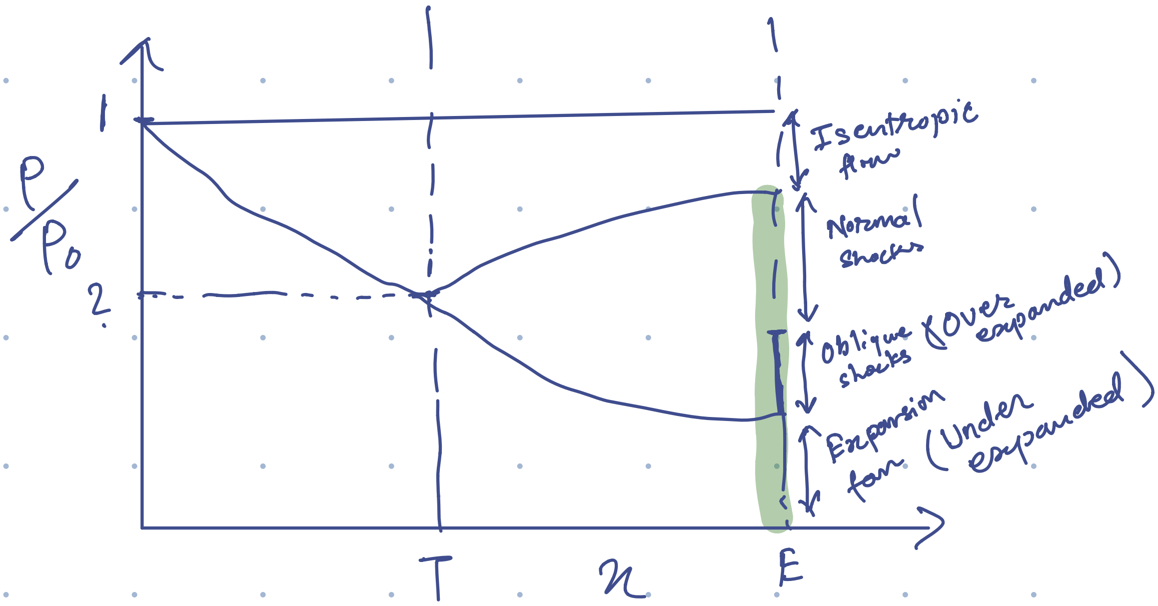
Please make sure, you have isentropic table with you tomorrow. (M P/P_0 T/T_0 ρ/ρ_0 A/A^*)

Choked flow

Q:- What is P^*/P_r or P^*/P_0 for the choked flow?

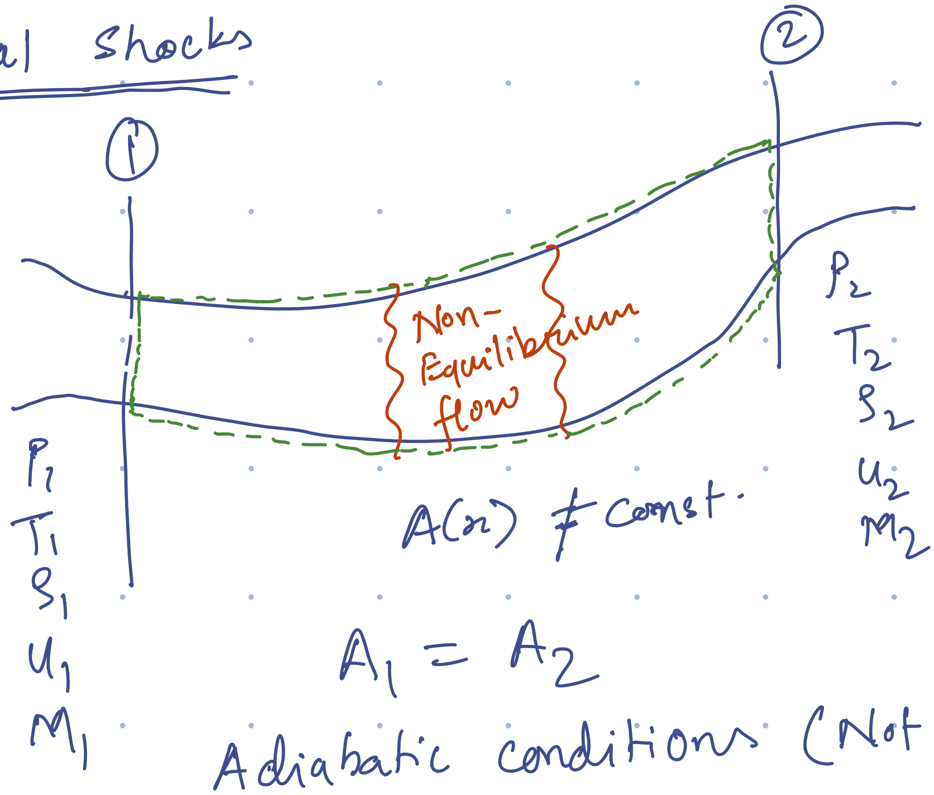


From isentropic tables,

$$P/P_0 \big|_{M=1} \approx 0.528 \Bigg\} \text{ Choked flow}$$

$\dot{m}$ = const. (choked flow)

Normal shocks



$$A_1 = A_2$$

Adiabatic conditions (Not Isentropic)

Perfect gas.

Conservation laws still apply

$$\rightarrow \quad A_1 \rho_1 u_1 = A_2 \rho_2 u_2 \implies \rho_1 u_2 = \rho_2 u_2 \longrightarrow (1)$$

Momentum $\quad P_1 + \rho_1 u_1^2 = P_2 + \rho_2 u_2^2 \longrightarrow (2)$

Energy

$$h_{o_1} = h_{o_2}$$

$$C_p T_{o_1} = C_p T_{o_2}$$

$$C_p T_1 + \frac{1}{2} u_1^2 = C_p T_2 + \frac{1}{2} u_2^2 \longrightarrow (3)$$

$$M_1 \longrightarrow M_2 \longrightarrow \begin{matrix} P_2/P_1 \\ T_2/T_1 \\ \rho_2/\rho_1 \end{matrix}$$

$$\frac{T_2}{T_1} = \frac{T_{01}/T_1}{T_{02}/T_2} = \frac{1+\frac{\gamma-1}{2}M_1^2}{1+\frac{\gamma-1}{2}M_2^2} \longrightarrow (4)$$

from momentum cons. (2),

$$P_1 + \rho_1 u_1^2 = P_2 + \rho_2 u_2^2$$

$$P_1\left(1+\frac{\rho_1}{P_1}\cdot u_1^2\right) = P_2\left(1+\frac{\rho_2}{P_2}u_2^2\right)$$

$$\therefore \frac{P_2}{P_1} = \frac{1+\frac{u_1^2}{RT_1}}{1+\frac{u_2^2}{RT_2}} = \frac{1+\gamma M_1^2}{1+\gamma M_2^2} \longrightarrow (5)$$

$$\frac{\rho_2}{\rho_1} = \left(\frac{P_2}{P_1}\right)\left(\frac{T_1}{T_2}\right) = f(M_1, M_2) \longrightarrow (6)$$

$$P, \rho, T, M, u \xrightarrow[\text{Stagnation}]{\text{Isentropically}} P_0, \rho_0, T_0, \; M_0 = 0, \; u_0 = 0$$

$$\Big\downarrow \text{ Isentropically, Critical Condition}$$

$u^* = a^*$

$M^* = 1$

p^*, ρ^*, T^*

From energy conservation eq^{n}s. (3),

$$c_p T_1 + \frac{u_1^2}{2} = c_p T_2 + \frac{u_2^2}{2}.$$

Since $c_p = \frac{\gamma R}{\gamma - 1}$, and $a^2 = \gamma R T$

$$\frac{a_1^2}{\gamma-1} + \frac{u_1^2}{2} = \frac{a_2^2}{\gamma-1} + \frac{u_2^2}{2} \quad \text{——— (7)}$$

Now, going to critical condition,

$$C_p T_1 + \frac{1}{2} u_1^2 = C_p T^* + \frac{1}{2} u^{*2}$$

But $u^* = a^*$

$$\therefore \frac{a_1^2}{\gamma-1} + \frac{u_1^2}{2} = \frac{a^{*2}}{\gamma-1} + \frac{a^{*2}}{2} = \frac{(\gamma+1)}{2(\gamma-1)} a^{*2} \quad \text{——— (8)}$$

$$\begin{tikzcd} a_1 \ (1) \arrow[r, "\text{Adiabatic}"] \arrow[d] & (2) \ a_2 \arrow[d] \\ T_1^*,\ a_1^*,\ p_1^* & T_2^*,\ a_2^*,\ p_2^* \end{tikzcd}$$

$a_1^* = a_2^*$ $\because$ flow between (1) & (2) is adiabatic

$$\frac{a_1^2}{r-1} + \frac{u_1^2}{2} = \frac{(r+1)}{2(r-1)} a^{*2} \quad \text{--- } (8) \quad \times \rho_1$$

Using (7) & (8),

$$\frac{a_2^2}{r-1} + \frac{u_2^2}{2} = \frac{(r+1)}{2(r-1)} a^{*2} \quad \text{--- } (9) \quad \times \rho_2$$

(8) $\times \rho_1$ — (9) $\times \rho_2$ gives

$$\underbrace{\left(\frac{\rho_1 a_1^2}{r-1} - \frac{\rho_2 a_2^2}{r-1}\right)}_{\alpha} + \underbrace{\frac{\left(\rho_1 u_1^2 - \rho_2 u_2^2\right)}{2}}_{\beta} = \frac{r+1}{2(r-1)} \cdot a^{*2} (\rho_1 - \rho_2) \quad \text{--- } (10)$$

(α) $\rho_1 a_1^2 - \rho_2 a_2^2$

$= \rho_1 rRT_1 - \rho_2 rRT_2$

$= r(P_1 - P_2)$

$= r(\rho_2 u_2^2 - \rho_1 u_1^2)$ (Using momentum eq^n)

$$= r\left((\rho_1 u_1)u_2 - (\rho_2 u_2)u_1\right)$$ (Using mass $\rho_1 u_1 = \rho_2 u_2$)

$$= r u_1 u_2 (\rho_1 - \rho_2)$$

for β

$$\frac{\rho_1 u_1^2 - \rho_2 u_2^2}{2} = \frac{(\rho_2 u_2)u_1 - (\rho_1 u_1)u_2}{2}$$

$$= \frac{u_1 u_2 (\rho_2 - \rho_1)}{2}$$

$$= \frac{-u_1 u_2 (\rho_1 - \rho_2)}{2}$$

from (10), substituting α & β,

$$\frac{r u_1 u_2}{r-1}(\rho_1 - \rho_2) - u_1 u_2 \frac{(\rho_1 - \rho_2)}{2}$$

$$= \frac{r+1}{2(r-1)} \cdot a^{*2}(\rho_1 - \rho_2)$$

$$u_1 u_2 \left(\frac{\gamma}{\gamma-1} - \frac{1}{2} \right) = \frac{\gamma+1}{2(\gamma-1)} a^{*2}$$

$$\boxed{u_1 u_2 = a^{*2}}$$

Prandtl's relations.

$$M_1^* = \frac{u_1}{a^*}, \quad M_2^* = \frac{u_2}{a^*}.$$

Then $M_1^* M_2^* = 1$