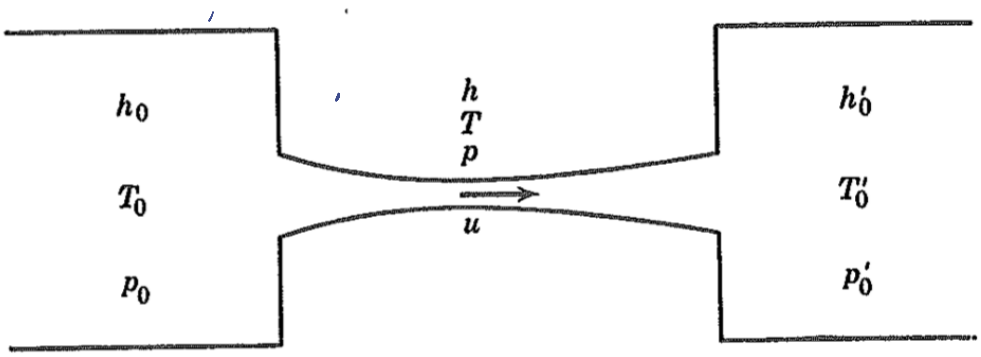
Reservoir conditions (Liepmann Sec. 2.4)



For adiabatic flow, $h_0 = h + \frac{1}{2}u^2 = h_0'$

FIG. 2·4 Flow between two reservoirs.

$$h + \frac{1}{2}u^2 = h_0$$ → Stagnation enthalpy OR Reservoir " OR Total "

Q: What is relationship between h_0 & h_0'?

Adiabatic ($h_0 = h_0'$)

Isentropic ← Adiabatic

- Equilibrium → Isentropic
  - Perfect gas (Calorically perfect gas)
    $p = \rho RT$
    $p\vartheta = RT$
  - Real gas
- Non-equilibrium
  - Perfect gas
    $h_0 = h_0'$
    $T_0 = T_0'$

$e = C_v T \quad , \quad \dfrac{dC_v}{dT} = 0$

$h = C_p T \quad , \quad \dfrac{dC_p}{dT} = 0$

$$\left.\begin{array}{l} h_0 = h_0' \quad \Rightarrow \quad \underline{T_0 = T_0'} \\ \underline{s_0 = s_0'} \end{array}\right\}$$

Isentropic process

$$de = T\,ds - p\,dv$$

$$h = e + pv \Rightarrow dh = de + p\,dv + v\,dp$$
$$= T\,ds + v\,dp$$

If perfect gas, $dh = C_p\,dT$

Then $dh = C_p\,dT = T\,ds + v\,dp$

$$C_p \frac{dT}{T} = ds + R\frac{dp}{p}$$

$$\boxed{\underbrace{s_0' - s_0}_{0} = R \ln \frac{p_0}{p_0'} + \underbrace{C_p \ln \frac{T_0'}{T_0}}_{0}}$$

To make RHS $= 0$, we have to have

for equilibrium perfect gas

$h_0 = h_0'$, $T_0 = T_0'$, $p_0 = p_0'$, $s_0 = s_0'$

For non-equilibrium process

$$s_0' - s_0 \geq 0$$

$$R \ln \frac{p_0}{p_0'} + c_p \ln \frac{T_0'}{T_0} \geq 0$$

Since I know, $T_0 = T_0'$

$$R \ln \frac{p_0}{p_0'} \geq 0$$

$$\Rightarrow \boxed{p_0 \geq p_0'}$$

$$T_0 = T_0', \quad h_0 = h_0'$$

Q:- Show that $P_0/P_0' \geq 1$.

Q: Show that $S_0' - S_0 = S' - S$

Momentum conservation (Section 3.3
(George Emmanuel)

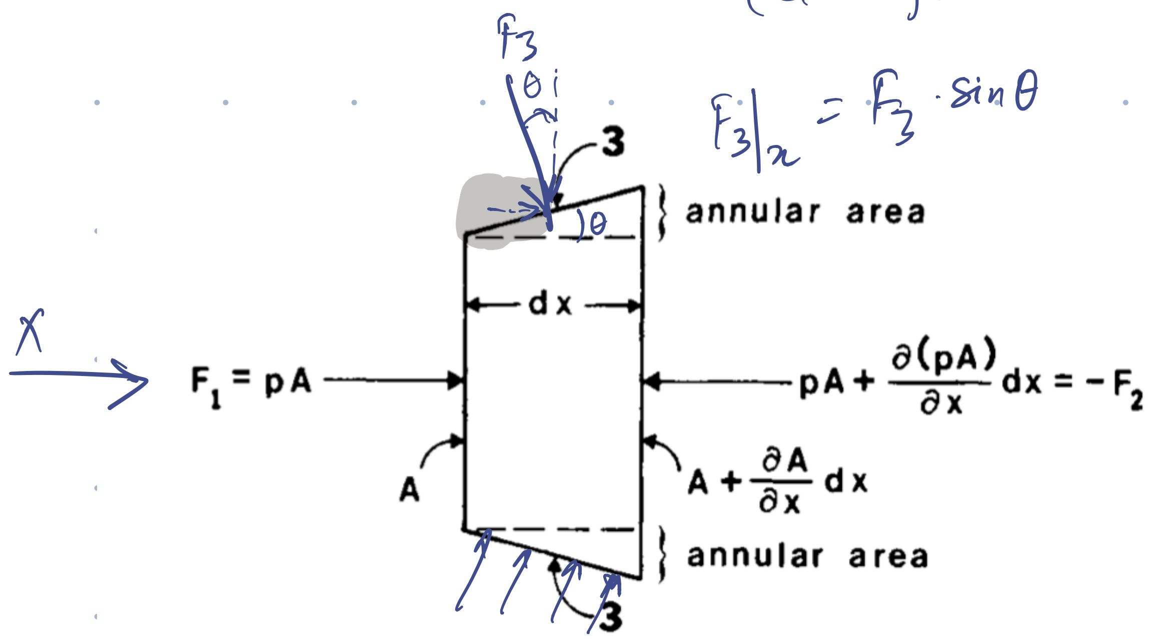


Fig. 3.3 Forces on the differential element of Fig. 3.2.

→ 1D annular duct

Newton's 2nd law.

$$F = \frac{d(mu)}{dt}$$

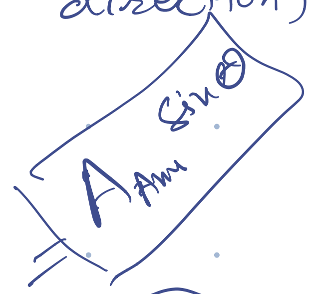
In Lagrangian frame,

$$F = \frac{D(mu)}{Dt}$$

where

$$\frac{D(\cdot)}{Dt} = \frac{\partial(\cdot)}{\partial t} + u\frac{\partial(\cdot)}{\partial x} \quad \text{(For 1D)}$$

LHS :- $F = F_1 + F_2 + F_3$ (Force in X direction)



$$F_1 = pA$$

$$F_2 = -\left[(pA) + \frac{\partial (pA)}{\partial x}\, dx\right]$$

$$F_3 = \left\{\frac{\left[p + \left(p + \frac{\partial p}{\partial x}\, dx\right)\right]}{2}\right\}\left(\left(A + \frac{\partial A}{\partial x}\, dx\right) - A\right)$$

$$= \left(p + \frac{1}{2}\frac{\partial p}{\partial x}\, dx\right)\left(\frac{\partial A}{\partial x}\, dx\right) \approx p\frac{\partial A}{\partial x}\cdot dx$$

$$\therefore F = -\frac{\partial p}{\partial x}\, A\, dx$$

RHS :-
$$\frac{D(mu)}{Dt} = \frac{D(\rho A\, dx\, u)}{Dt}$$

$$= u \cdot \underbrace{\frac{D(\rho A\, dx)}{Dt}}_{\text{Conservation of mass}} + \rho A\, dx \cdot \frac{Du}{Dt}$$

$$= \rho A\, dx\left(\frac{\partial y}{\partial t} + u\frac{\partial u}{\partial x}\right)$$

Since, LHS = RHS,

$$\therefore \quad -\frac{\partial p}{\partial x} \cdot A\,dx = \rho A\,dx \left(\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} \right)$$

$$\therefore \quad \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} = -\frac{1}{\rho} \frac{\partial p}{\partial x}$$

OR

$$\frac{Du}{Dt} = -\frac{1}{\rho} \frac{\partial p}{\partial x}$$

→ Please read conservation of mass and energy from sec. 3.3 (George Emmanuel) using this total derivative approach.