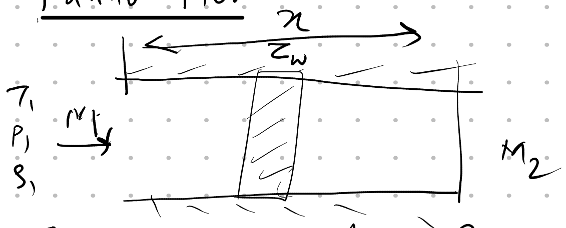
Fanno Flow



Subsonic $M_2 > M_1$

Supersonic $M_2 < M_1$

$$\frac{dT}{T} + (\gamma-1)\, M^2 \cdot \frac{du}{u} = 0$$

$$\frac{dp}{p} + \frac{du}{u}\left[1 + (\gamma-1) M^2\right] = 0$$

$$P = \rho R T$$

$$\frac{d\rho}{\rho} = f(M, u) \qquad \frac{du}{u} = \frac{dM/M}{\left[1 + \frac{\gamma-1}{2} M^2\right]}$$

$$\frac{ds}{R} = \frac{(1-M^2)}{\left[1+\frac{\gamma-1}{2}M^2\right] M}\, dM$$

$$\boxed{M_2 = f(M_1, \gamma, x, \tau_w, D_H)}$$

Fanny friction factor :-

$$f = \frac{\tau_w}{\frac{1}{2}\rho u^2}$$

Darcy's " " $f_D = 4f$

$f_D = f\left(\underline{\underline{Re}}, M, \frac{\epsilon}{D_H}\right)$ — surface roughness; M → weak function

→ Moody charts

$$\frac{-dp}{\rho u^2} - \frac{\tau_w}{\rho u^2} \cdot \frac{4dx}{D_H} = \frac{du}{u}$$

$$-\frac{dp}{p} - \left(\frac{f}{2}\right) \cdot \frac{4dx}{D_H} \cdot \frac{\rho u^2}{p} = du \cdot \frac{(\rho u)}{p}$$

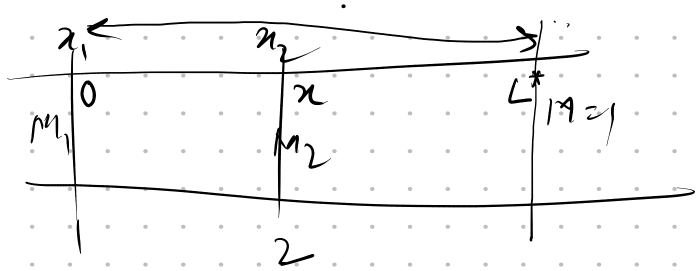
$$\frac{\rho u^2}{p} = \frac{u^2}{RT} = \frac{\gamma u^2}{a^2} = \gamma M^2$$

After a LOT of algebra

$$\boxed{\int_0^{L^*} \frac{4f\,dx}{D_H} = \int_{M_1}^{1} \frac{(1-M^2)}{\left(1+\frac{\gamma-1}{2}M^2\right)} \cdot \left(\frac{2}{\gamma M^2}\right) \frac{dM}{M}} \rightarrow f(M, \gamma) \quad — (A)$$

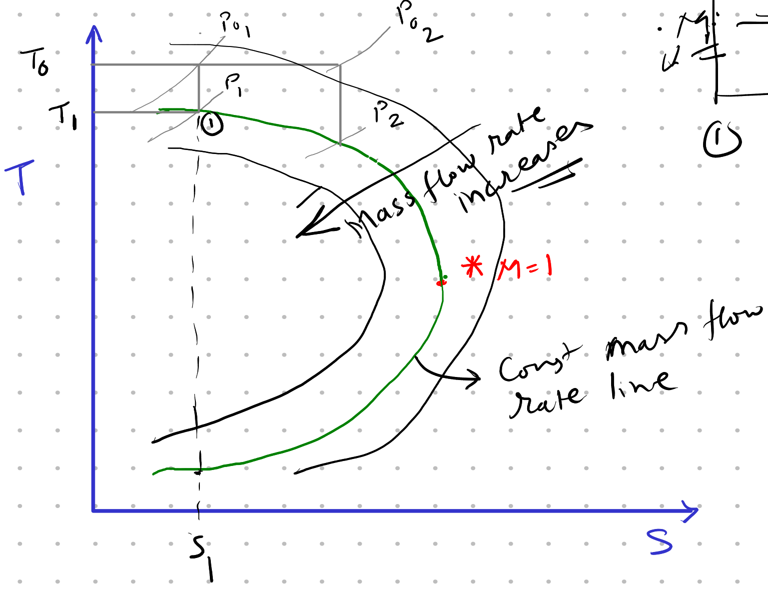
$$\int_{x_1}^{x_2} \frac{4f\,dx}{D_H} = \int_{x_1}^{L^*} \frac{4f\,dx}{D_H} - \int_{x_2}^{L^*} \frac{4f\,dx}{D_H} = \int_{M_1}^{1} (\,)\, dM - \int_{M_2}^{1} (\,)\, dM$$

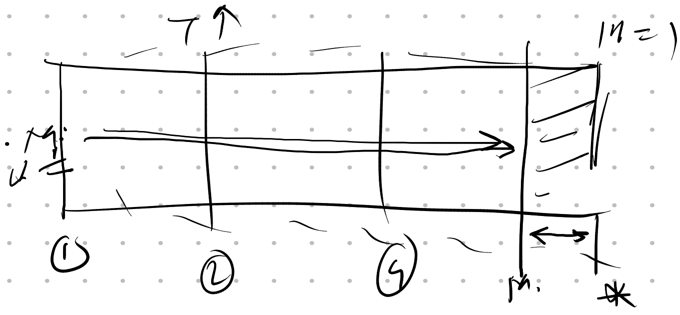
L^* – As the length required to reach $M=1$ from (x_1, M_1)



M	$\frac{4fL^*}{D_H}$	T/T^*	p/p^*	p_0/p_0^*	ρ/ρ^*

Fanno line





T ↓ S ↑

$$h + \frac{u^2}{2} = const \qquad \rho u$$

$$h + \frac{\rho^2 u^2}{2\rho^2} = const$$

$$c_p T + \frac{G^2}{2\rho^2} = const$$