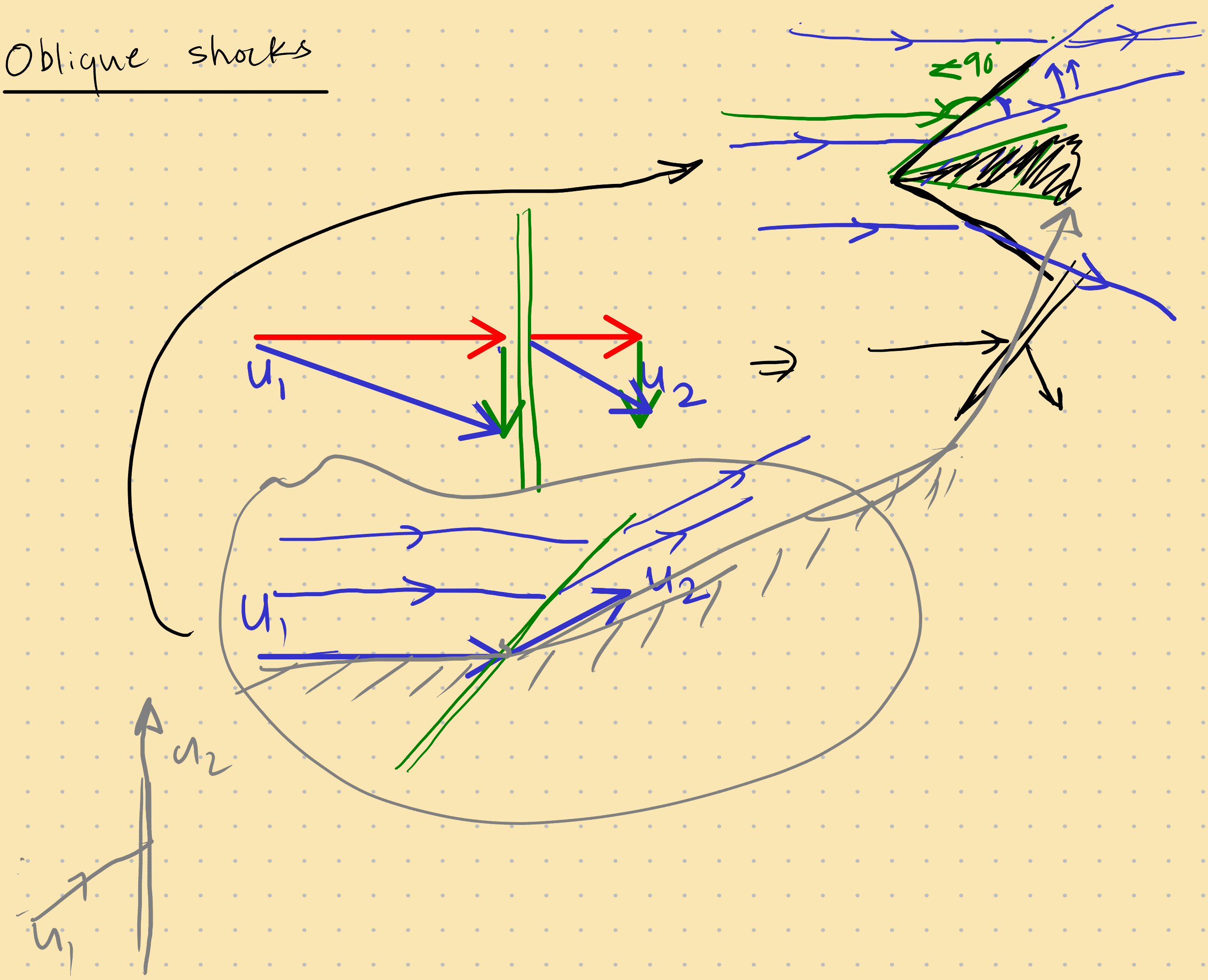
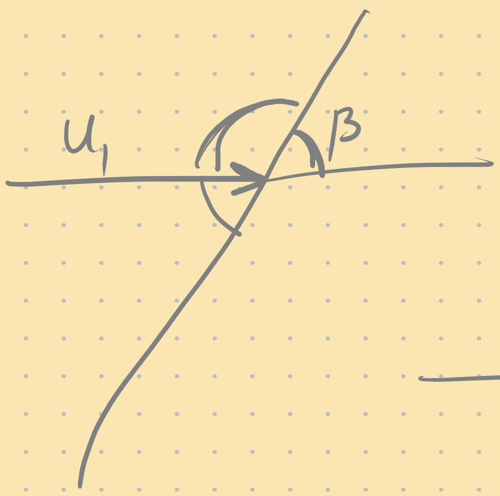
Oblique shocks



BZT gases

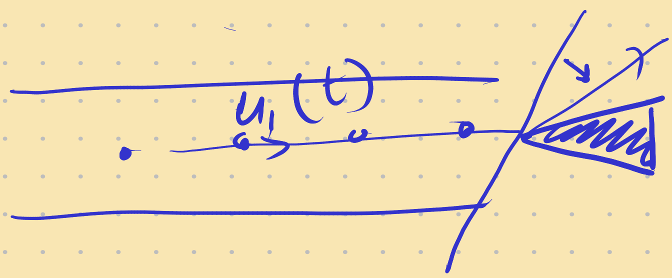


$\frac{\Delta s}{R}$

$M_2 = \sqrt{\frac{2}{\gamma(\gamma-1)}}$

$t \to t+\Delta t$ $\quad u_2 \uparrow \Rightarrow T_2 \uparrow$ $\quad \frac{u_2}{\sqrt{T_2}}$

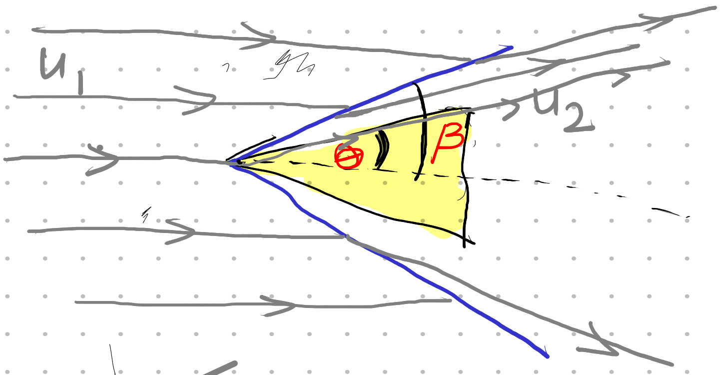
$p_0(t)$



Oblique Shock :-

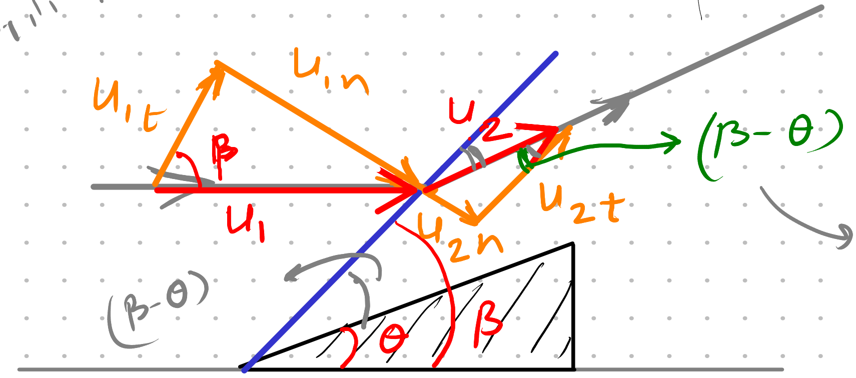
→ 2D

→ θ - Half angle of wedge



β - Half angle of shock

$\beta = f(\theta, M_1, \gamma, P_1, \ldots)$



$\tan^{-1}(\beta-\theta) = \dfrac{u_{2n}}{u_{2t}}$

$\theta = \beta - \tan^{-1}\left(\dfrac{u_{2n}}{u_{2t}}\right)$

$u_{1n} = u_1 \sin\beta \quad , \quad u_{1t} = u_1 \cos\beta$

$u_{2n} = u_2 \sin(\beta-\theta) \quad , \quad u_{2t} = u_2 \cos(\beta-\theta)$

Clearly, there is no change in the tangential velocity.

$$\Rightarrow \quad u_{1t} = u_{2t}$$

The normal component of the velocity encounters a normal shock. Hence normal shock relations apply.

So, the algo is this.

Given M_1 & β,

$M_{1n} = M_1 \sin\beta, \qquad M_{1t} = M_1 \cos\beta$

$M_{2n} = M_2 \sin(\beta-\theta) \qquad M_{2t} = M_2 \cos(\beta-\theta)$

the inverse function.

i.e. $\theta = g(\beta) = f^{-1}(\beta)$

So we know,

$$u_{1t} = u_{2t}$$

$$\therefore u_1 \cos\beta = u_2 \cos(\beta - \theta)$$

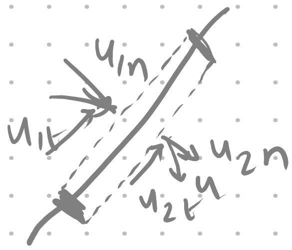
$$\therefore \frac{u_1}{u_2} = \frac{\cos(\beta - \theta)}{\cos(\beta)}$$

$$\frac{u_{1n}/\sin\beta}{u_{2n}/\sin(\beta-\theta)} = \text{''}$$

$$\therefore \frac{u_{1n}}{u_{2n}} = \frac{\cos(\beta-\theta)}{\cos(\beta)} \cdot \frac{\sin(\beta)}{\sin(\beta-\theta)} = \frac{\tan\beta}{\tan(\beta-\theta)} \quad \text{— (A)}$$

From mass conservation,

$$\rho_1 u_{1n} = \rho_2 u_{2n} \quad \text{— (B)}$$



From (A) & (B).

$$\frac{\rho_2}{\rho_1} = \frac{\tan(\beta-\theta)}{\tan(\beta)} \quad \text{— (C)}$$

Now,

$$\frac{\rho_2}{\rho_1} = f_3(M_1 \sin\beta) = \frac{(\gamma+1) M_1^2 \sin^2\beta}{(\gamma-1) M_1^2 \sin^2\beta + 2} \quad \text{— (D)}$$

$$\frac{u_{1n}}{a_1} = \frac{u_1 \sin\beta}{a_1} = M_1 \sin\beta$$

Using (C) & (D) it can be shown that

$$\tan\theta = 2\cot\beta\left[\frac{M_1^2\sin^2\beta - 1}{2 + M_1^2[\gamma + \cos 2\beta]}\right]$$

This gives us $\theta = g(\beta) = f^{-1}(\beta)$

Now we can either use a plot or use a computer

Q.. What is the minimum achievable value of β ?

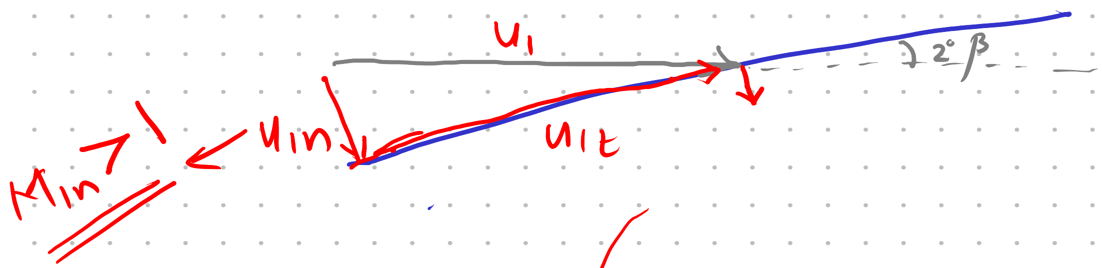
$\rightarrow$ $$\frac{p_2}{p_1} = 1 + \frac{2\gamma}{\gamma+1}(M_1^2\sin^2\beta - 1)$$

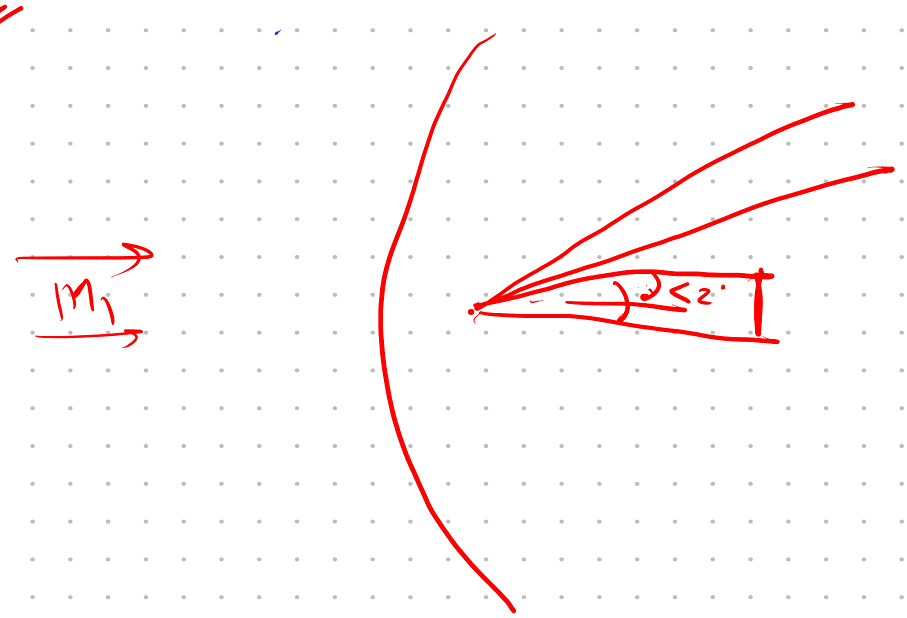
$\sin^{-1}\left(\frac{1}{M_1}\right) < \beta < 90$

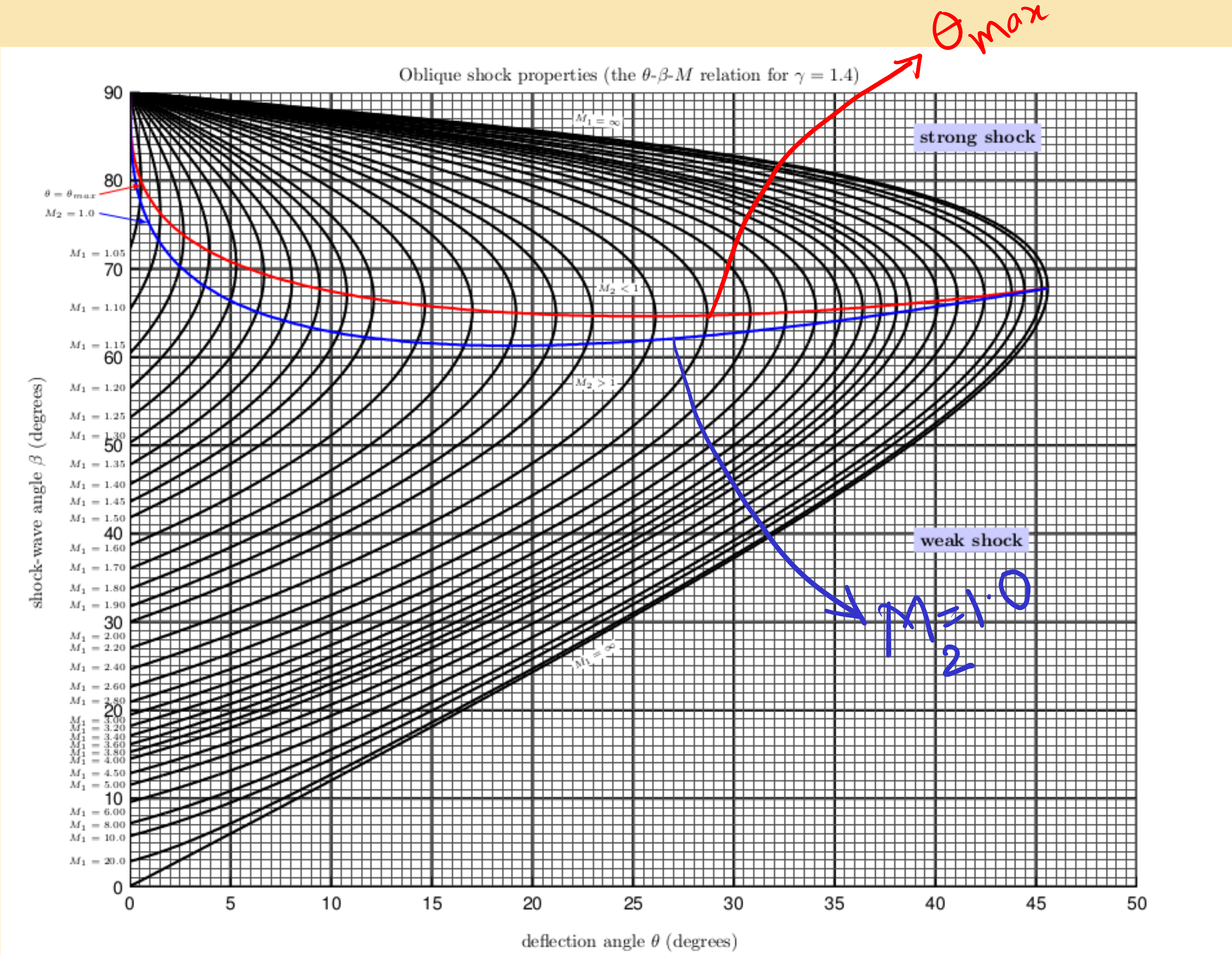
$P_2 > P_1 \Rightarrow M_1^2 \sin^2\beta > 1$

$\sin^2(\beta) \geq 1/M_1^2 \Rightarrow \sin\beta \geq 1/M_1$

$\beta \geq \sin^{-1}(1/M_1)$







Oblique shock properties (the θ-β-M relation for γ = 1.4)
θmax
strong shock
weak shock
$M_2 = 1.0$
θ = θ$_{max}$
$M_2 = 1.0$
$M_1 = \infty$
$M_2 < 1$
$M_2 > 1$
$M_1 = 1.05$
$M_1 = 1.10$
$M_1 = 1.15$
$M_1 = 1.20$
$M_1 = 1.25$
$M_1 = 1.30$
$M_1 = 1.35$
$M_1 = 1.40$
$M_1 = 1.45$
$M_1 = 1.50$
$M_1 = 1.60$
$M_1 = 1.70$
$M_1 = 1.80$
$M_1 = 1.90$
$M_1 = 2.00$
$M_1 = 2.20$
$M_1 = 2.40$
$M_1 = 2.60$
$M_1 = 2.80$
$M_1 = 4.50$
$M_1 = 5.00$
$M_1 = 6.00$
$M_1 = 8.00$
$M_1 = 10.0$
$M_1 = 20.0$
shock-wave angle β (degrees)
deflection angle θ (degrees)

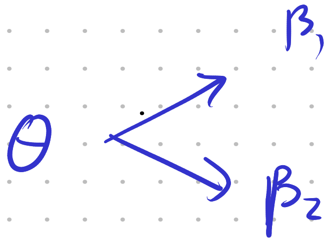
Q:- What is maximum value of β ?

Q:- What is the possible range of θ ?

$\rightarrow$

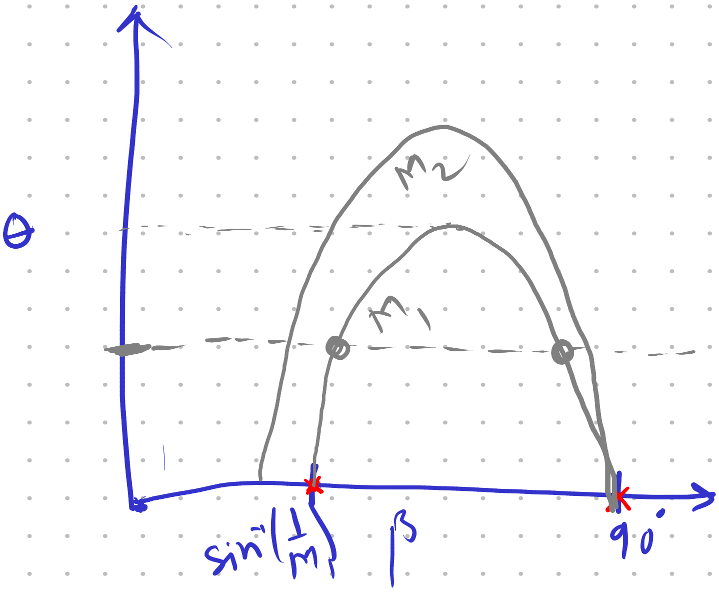
$$\tan(\theta) = \frac{2\cot\beta(M_1^2\sin^2\beta - 1)}{2 + M_1^2(\gamma + \cos 2\beta)}$$

$0 < \theta < 90^\circ$



Fix γ - gas

M_1 — Upstream Mach num.

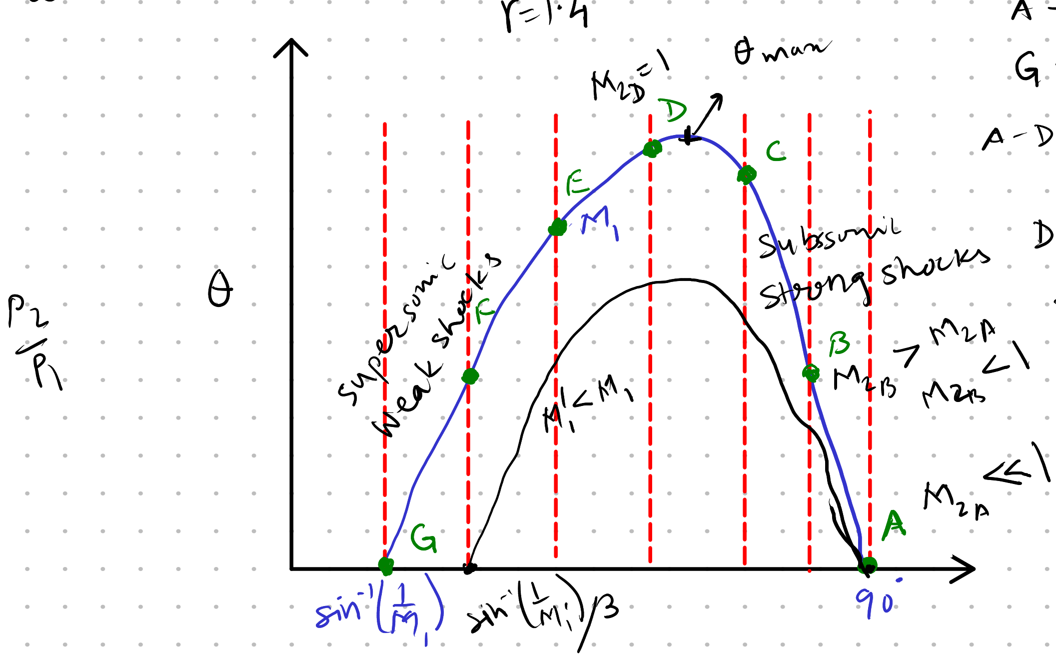


M_1, γ

M_2, γ

$M_2 > M_1$

Q:-



A – Normal shock

G – Mach wave $M_1 = M_2$

A-D – subsonic after the oblique shock

D – $M_{2D} = 1$

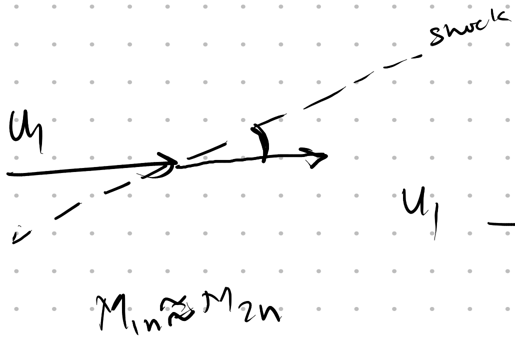
D-G – supersonic flow after the shock

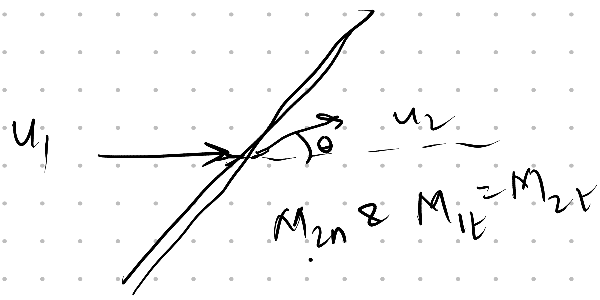
$$\frac{\Delta s}{R} = \left.\frac{s_2 - s_1}{R}\right|_G = 0$$

As $\beta\downarrow$, $\frac{\Delta s}{R}\downarrow$ for a given γ, M_1

As $\beta\downarrow$, $\frac{p_2}{p_1}\downarrow$, $\frac{T_2}{T_1}$, $\frac{\rho_2}{\rho_1}$, $\frac{p_{02}}{p_{01}}$

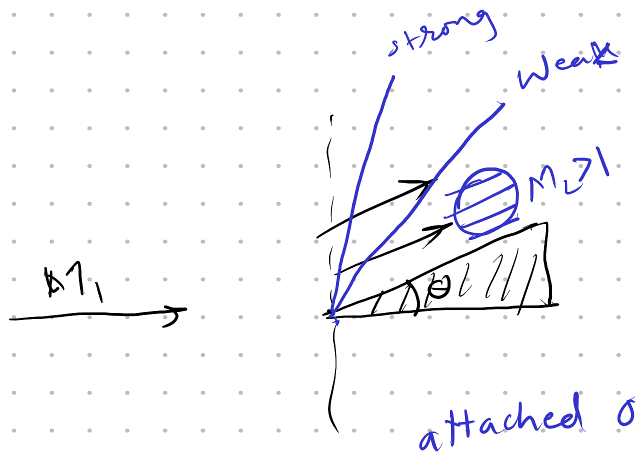
For $M_1' < M_1$,





Q:- So I know that for given θ (flow deflection), I have two possible values of β and corresponding two possible shocks (strong & weak). So which one will occur?

→



→ Both shocks are feasible. But attached oblique strong shocks are unstable and hence very improbable.