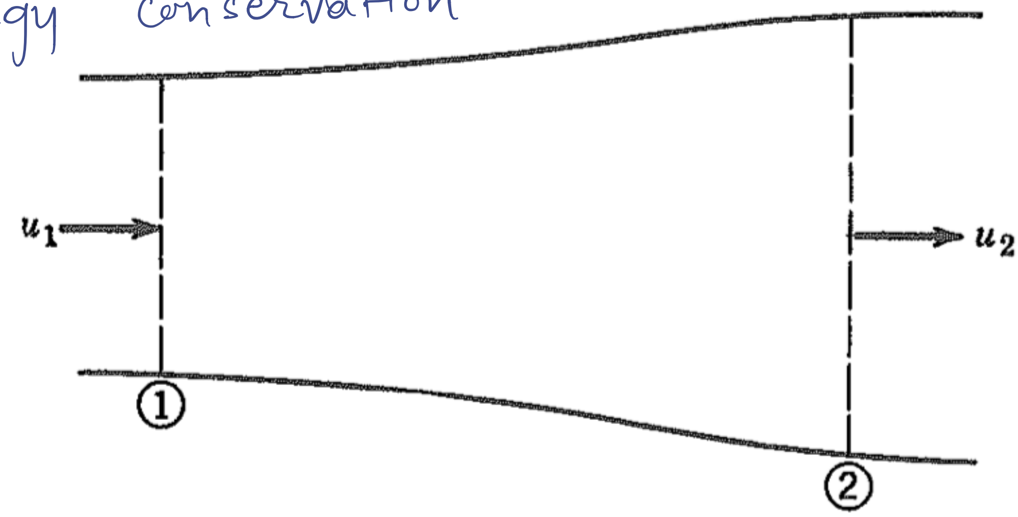
Mass conservation (Liepmann sec. 2.2)

$$\frac{\partial}{\partial t}(\rho A) + \frac{\partial}{\partial x}(\rho u A) = 0$$

$$\frac{\partial}{\partial t}\int_{cv} \rho \, dV + \int_{cs} \rho(\vec{V}\cdot\hat{n})\, dA = 0$$

(Zucker chap2)

Energy conservation



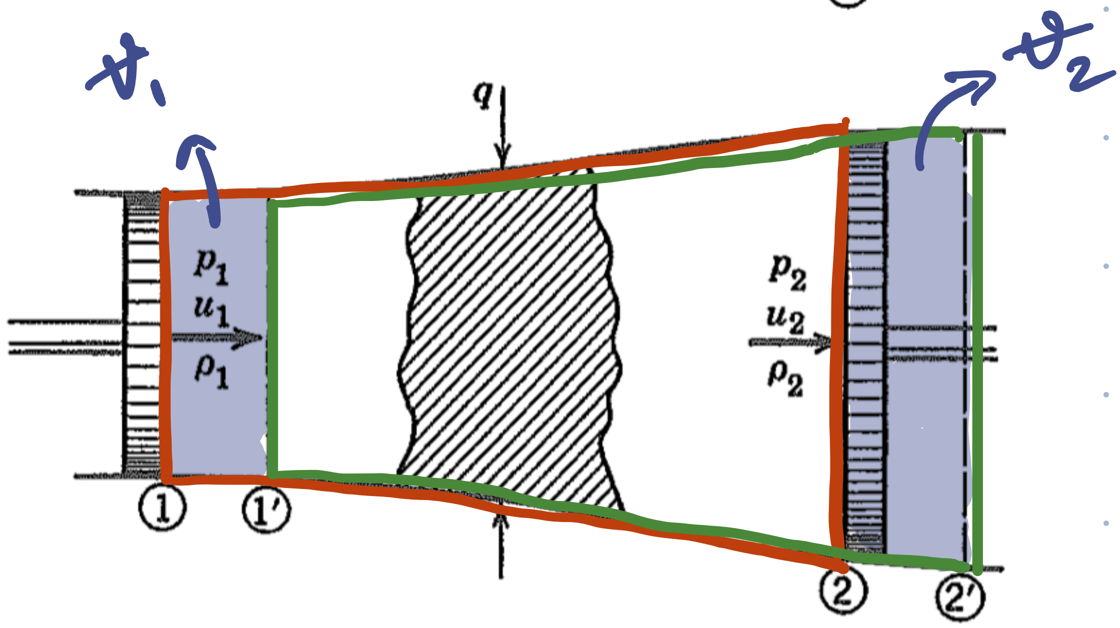


FIG. 2·3 System for calculating energy relations in flow.

Change in internal energy

$$= \left(e_2 + \tfrac{1}{2}u_2^2\right) - \left(e_1 + \tfrac{1}{2}u_1^2\right)$$

By 1^{st} law,

$$\left(e_2 + \tfrac{1}{2}u_2^2\right) - \left(e_1 + \tfrac{1}{2}u_1^2\right) = q + p_1 v_1 - p_2 v_2$$

$h = e + pv$, $\Rightarrow$ $q = h_2 - h_1 + \tfrac{1}{2}u_2^2 - \tfrac{1}{2}u_1^2$

If $q = 0$,

$$h_2 + \tfrac{1}{2}u_2^2 = h_1 + \tfrac{1}{2}u_1^2$$

If equilibrium exists all along the flow,

$$dh + u\,du = 0$$

Thermally perfect gas,

$dh = C_p(T)\,dT$ $\Rightarrow$ $C_p\,dT + u\,du = 0$

Calorically perfect gas,

$\dfrac{dC_p}{dT} = 0 \Rightarrow C_p = 0$

$$C_p T + \tfrac{1}{2}u^2 = \text{const.}$$