Fanno Flow

Subsonic $\quad M_2 > M_1$

Supersonic $\quad M_2 < M_1$

$$\frac{dT}{T} + (\gamma-1)\, M^2 \cdot \frac{du}{u} = 0$$

$$\frac{dP}{P} + \frac{du}{u}\left[1 + (\gamma-1)M^2\right] = 0$$

$$P = \rho RT$$

$$\frac{d\rho}{\rho} = f(M, u)$$

$$\frac{du}{u} = \frac{dM/M}{\left[1 + \frac{\gamma-1}{2}M^2\right]}$$

$$\frac{ds}{R} = \frac{(1-M^2)}{\left[1 + \frac{\gamma-1}{2}M^2\right]M}\, dM$$

$$\boxed{M_2 = f(M_1, \gamma, x, \tau_w, D_H)}$$

Fanny friction factor :-

$$f = \frac{\tau_w}{\frac{1}{2}\rho u^2}$$

Darcy's " "

$$f_D = 4f$$

$$f_D = f\left(\underline{Re}, M, \frac{\epsilon}{D_H}\right) \quad \leftarrow \text{surface roughness}$$

$\qquad\qquad\qquad\qquad\qquad \searrow$ weak function

$\to$ Moody charts

$$-\frac{dp}{\rho u^2} - \left[\frac{\tau_w}{\rho u^2}\right]\cdot \frac{4\,dx}{D_H} = \frac{du}{u}$$

$$-\frac{dP}{P} - \left(\frac{f}{2}\right)\cdot \frac{4\,dx}{D_H}\cdot\frac{\rho u^2}{P} = du\cdot\frac{(\rho u)}{P}$$
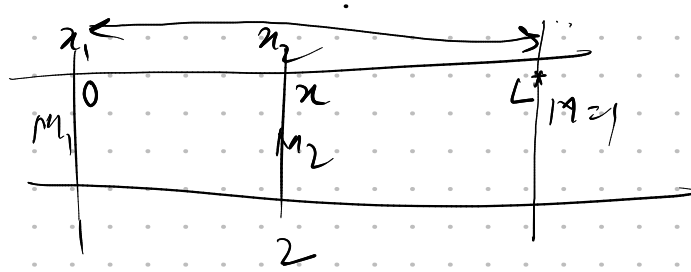
$$\frac{\rho u^2}{P} = \frac{u^2}{RT} = \frac{\gamma\, u^2}{a^2} = \gamma M^2$$

After a **LOT** of algebra

$$\int_0^{L^*} \frac{4f\,dx}{D_H} = \int_{M_1}^{1} \frac{(1-M^2)}{\left(1 + \frac{\gamma-1}{2}M^2\right)}\cdot\left(\frac{2}{\gamma M^2}\right)\frac{dM}{M} \;\longrightarrow\; f(M,\gamma)$$

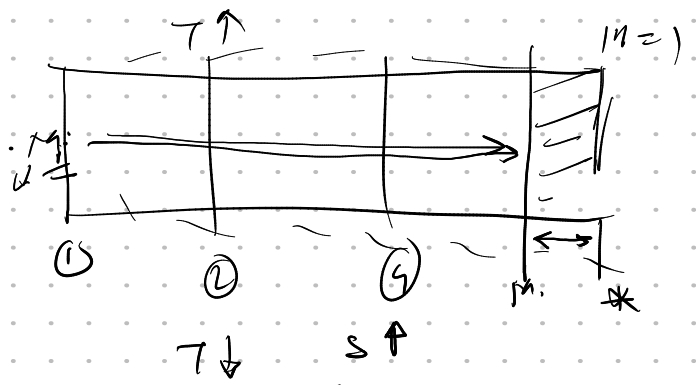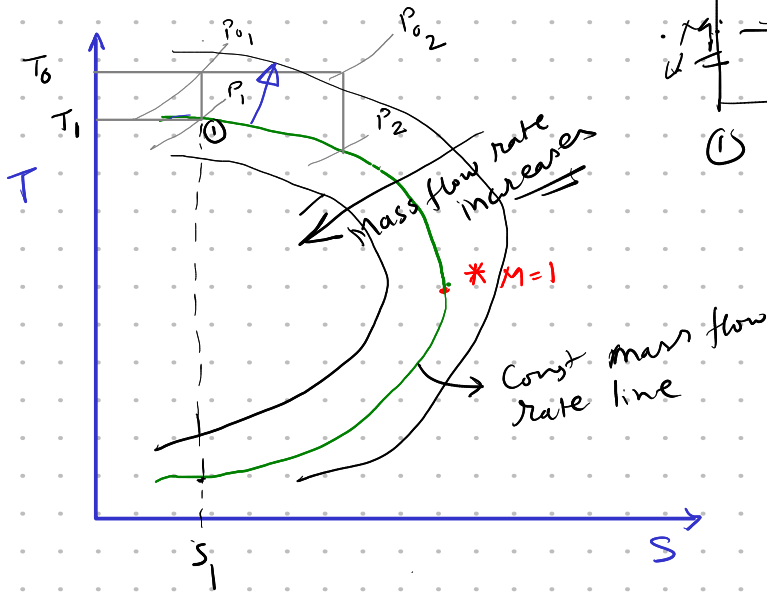$$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad -Ⓐ$$

$$\int_{x_1}^{x_2} \frac{4f\,dx}{D_H} = \int_{x_1}^{L^*} \frac{4f\,dx}{D_H} - \int_{x_2}^{L^*} \frac{4f\,dx}{D_H} = \int_{M_1}^{1} (\bigcirc)\,dM - \int_{M_2}^{1} (\,\cdot\,)\,dM$$

L^* — As the length required to reach $M=1$ from (x_1, M_1)

$$M \qquad \frac{4fL^*}{D_H} \qquad T/T^* \qquad P/p^* \qquad P_0/P_0^* \qquad \rho/\rho^*$$
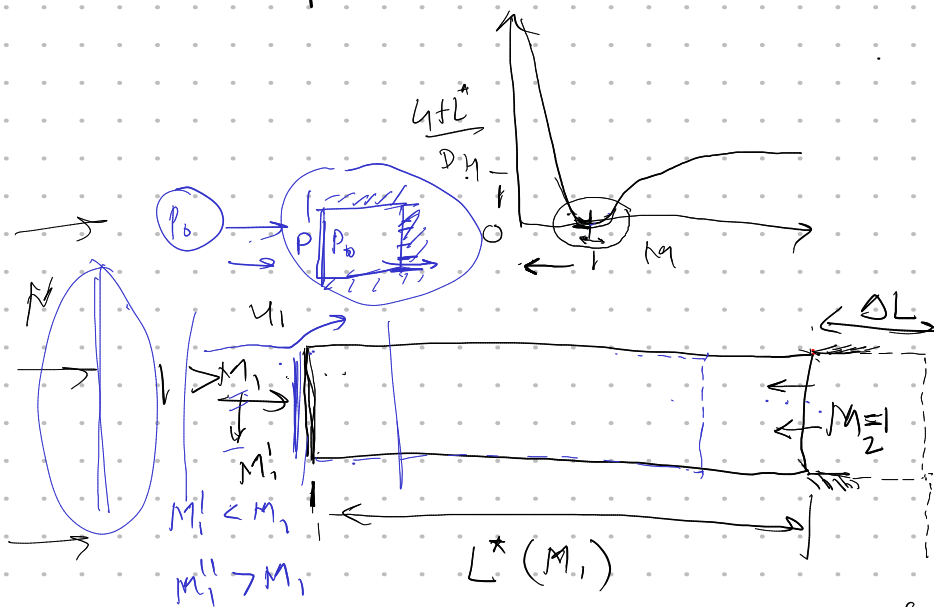
Fanno line

T_0
T_1
P_{0_1} P_{0_2}
P_1 P_2
T
①
mass flow rate increases
* $M=1$
Const mass flow rate line
S_1 S

$T \uparrow$ $M=1$
M
u
① ② ⑤ M_2 *
$T \downarrow$ $S \uparrow$

$$h + \frac{w^2}{2} = const \qquad g_u$$

$$h + \frac{g_u^2 v^2}{2 g^2} = const$$

$$k + C_p T + \frac{G^2}{2 g^2} = const$$

$\uparrow \Rightarrow \uparrow$
v S

$$\frac{4 f \, dx}{D_H} = \frac{(1-M^2)}{1 + \frac{\gamma-1}{2} M^2} \left(\frac{2}{\gamma M^2}\right) \frac{dM}{M}$$
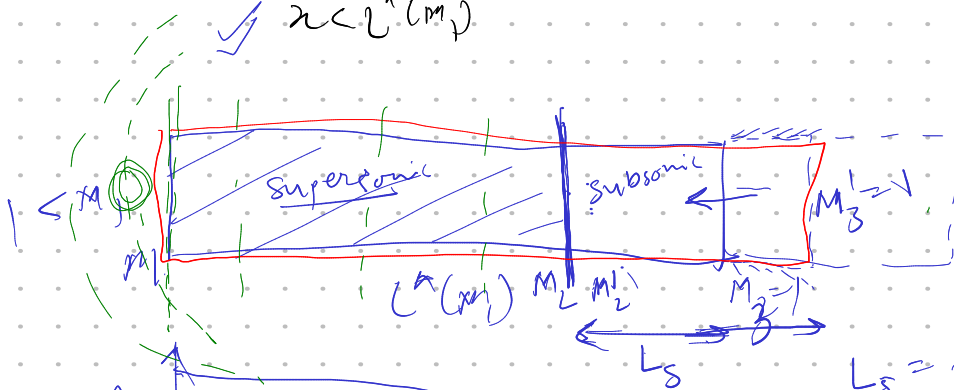
$\frac{4 f L^*}{D_H}$

P_0 P P_b
u_1
M
M_1
M_1'
$M_1' < M_1$
$M_1'' > M_1$

$L^*(M_1)$

ΔL
$M_2 = 1$ $M_2' = ?$

$$f(M_1, f, D_H) = L^*$$
$$f(M_1', f, D_H) = (L^* + \Delta L)$$

$x < L^*(M_1)$

$1 < M$
supersonic subsonic $M_3' = 1$
$L^*(M_1)$ M_2 M_2' $M_3 = 1$
L_S
M
x
$$L_S = f(M_2', f, D_H)$$
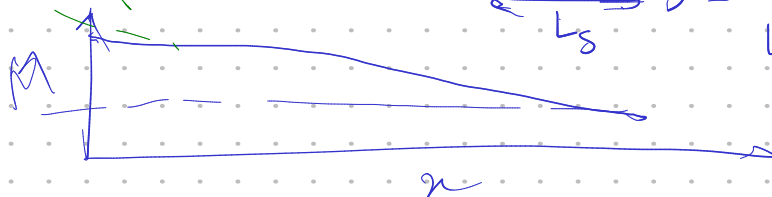
Tables

1. Isentropic flow $\gamma = 1.4$
2. Normal shock   ,,
3. Oblique shock   ,,
4. Fanno flow   ,,

To standardise, I will send a PDF over the weekend.

Numericals

1. C-D nozzle
2. Normal shock
3. Moving NS
4. Reflected moving NS
5. Oblique shock
6. Oblique shock reflection
7. Oblique shock interaction
8. Fanno Flow

→ Infinite number of problems can be set up by combining two or more elements from above.