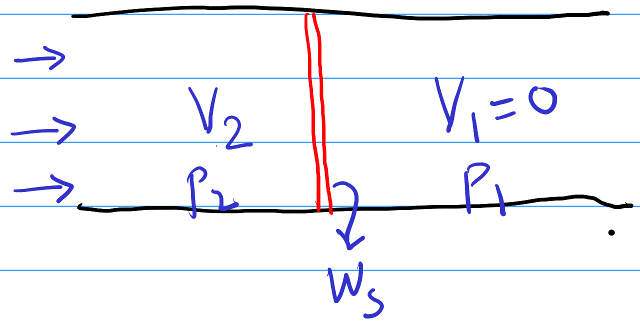
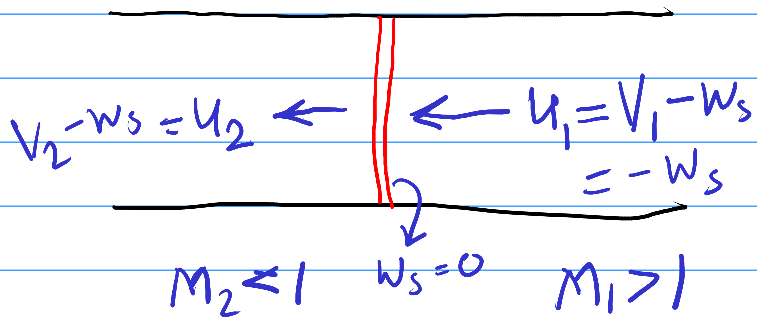
Moving Normal shock



Earth Reference frame



Shock Reference frame

$$V_2 - V_1 = u_2 - u_1 \qquad \frac{u_2}{u_1} = \left(\frac{\rho_1}{\rho_2}\right)$$

$$V_2 = u_2 - u_1$$

$$= u_1\left(\frac{u_2}{u_1} - 1\right)$$

$$= -W_s\left(\frac{u_2}{u_1} - 1\right)$$

$$\boxed{V_2 = W_s\left(1 - \frac{u_2}{u_1}\right)}$$

$$|W_s| = |u_1| = M_1 a_1$$

$$W_s = M_1 a_1$$
$$= a_1 M_1$$

If I know $p_2/p_1 \longrightarrow$ Strength of the normal shock,

$W_s(p_2/p_1, \gamma)$

$V_2(p_2/p_1, \gamma)$

$M_2(p_2/p_1, \gamma)$

$$\boxed{\begin{aligned} W_s &= M_1 a_1 \\ &= a_1 \left[\left(\frac{\gamma+1}{2\gamma}\right)\left(\frac{P_2}{P_1}\right) + \frac{\gamma-1}{2\gamma} \right]^{1/2} \end{aligned}}$$

$1 < \gamma < 5/3 \,(1.67)$

$$V_2 = w_s \left(1 - \frac{u_2}{u_1}\right)$$

$$= w_s \left(1 - \rho_1/\rho_2\right)$$

$$V_2 = a_1 \left(\frac{\gamma+1}{2\gamma} \cdot \frac{P_2}{P_1} + \frac{\gamma-1}{2\gamma} \right)^{1/2} \left[1 - \frac{\frac{\gamma+1}{\gamma-1} + P_2/P_1}{\underbrace{\left(\frac{\gamma+1}{\gamma-1}\right)\left(\frac{P_2}{P_1}\right) + 1}_{\left(1 - \frac{\gamma-1}{\gamma+1}\right)}} \right]$$

If $P_2/P_1 < 1 \rightarrow$ Expansion

$\rightarrow$ Compression

$P_2/P_1 = 1 \rightarrow$ Weakest compression wave.

$$\lim_{P_2/P_1 \to 1} V_2 = 0$$

$$\lim_{P_2/P_1 \to 1} W_s = a_1$$

$\frac{P_2}{P_1} = 1$

Sound wave.

<u>Strong shock</u> :-

$$\lim_{P_2/P_1 \to \infty} V_2 = \infty$$

$$\lim_{P_2/P_1 \to \infty} W_s = \infty$$

As the strength of the shock increases, i.e. $p_2/p_1 \uparrow$, W_s is going to increase.

$\not\longrightarrow$

$\rightarrow$ W_s is always supersonic.

$\rightarrow$

$$\frac{V_2}{a_2} = M_2$$

$$= \frac{a_1}{a_2}\cdot\underbrace{\left(\frac{\gamma+1}{2\gamma}\cdot\frac{p_2}{p_1}+\frac{\gamma-1}{2\gamma}\right)^{1/2}}\left[1-\frac{\left(\frac{\gamma+1}{\gamma-1}\right)+p_2/p_1}{\left(\frac{\gamma+1}{\gamma-1}\right)\left(\frac{p_2}{p_1}\right)+1}\right]$$

$$\frac{a_1}{a_2} \Downarrow \sqrt{\frac{T_1}{T_2}}$$

$$\frac{T_1}{T_2} = \left(\frac{\frac{\gamma+1}{\gamma-1}+p_1/p_2}{\frac{\gamma+1}{\gamma-1}+\frac{p_2}{p_1}}\right)^{1/2}$$

$$\Downarrow \quad \sqrt{\frac{\gamma+1}{2\gamma}\cdot\frac{p_2}{p_1}} \qquad \Downarrow \quad \left[1-\frac{\gamma-1}{\gamma+1}\right]$$

$$^{1/2} = \boxed{\sqrt{\frac{2}{\gamma(\gamma+1)}\cdot\frac{p_2}{p_1}}}$$

$$= \frac{1}{(p_2/p_1)^{1/2}}\left[\frac{\gamma+1/\gamma-1}{1+\underbrace{\left(\frac{\gamma+1}{\gamma-1}\right)\left(\frac{p_1}{p_2}\right)}_{=0}}\right]$$

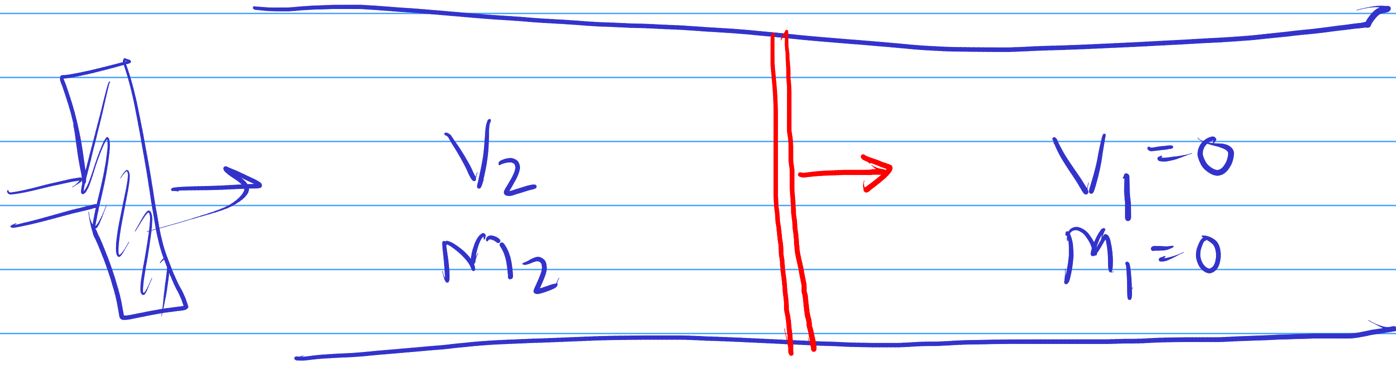
$$= \frac{1}{(p_2/p_1)^{1/2}}\left(\frac{\gamma+1}{\gamma-1}\right)^{1/2}$$

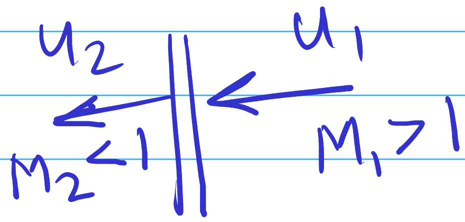
$$\lim_{p_2/p_1 \to \infty} V_2/a_2 = \frac{1}{(p_2/p_1)^{1/2}} \left(\frac{\gamma+1}{\gamma-1}\right)^{1/2} \cdot \sqrt{\frac{2}{\gamma(\gamma+1)}}$$

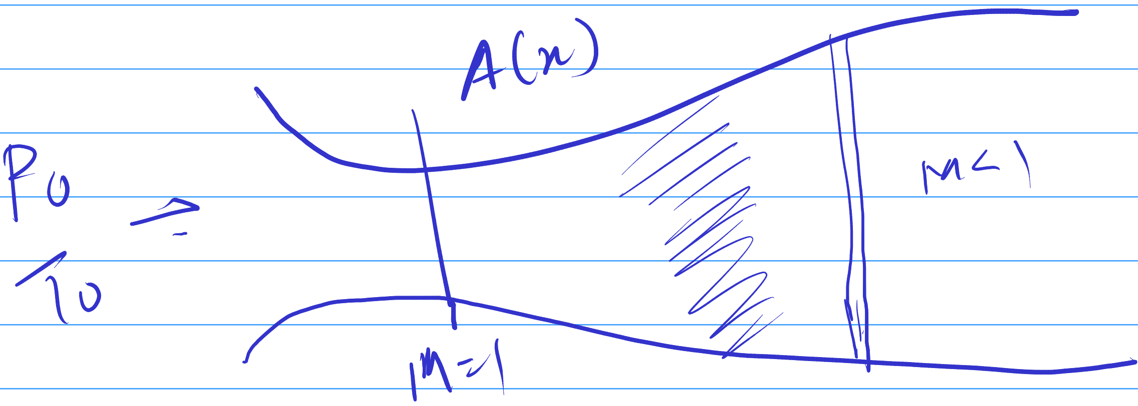
$$\boxed{\lim_{p_2/p_1 \to \infty} M_2 = \sqrt{\frac{2}{\gamma(\gamma-1)}}}$$

For $\gamma = 1.4$,

$M_2 = 1.89$







$$\lim_{\frac{p_2}{p_1} \to \infty} M_2 = \sqrt{\frac{2}{\gamma(\gamma-1)}}$$