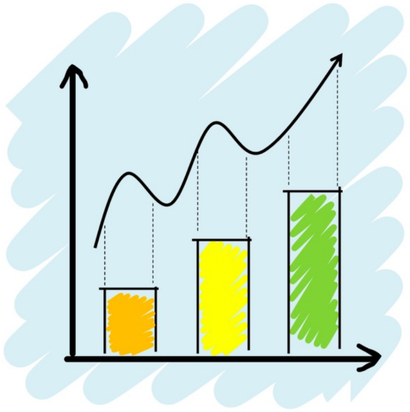


Statistics

Permutation and Combination

Permutation

- In mathematics, permutation relates to the act of arranging all the members of a set into some sequence or order
- Different arrangements of a given number of elements taken one by one, or some, or all at a time
- E.g.
  - if we have two elements A and B, then there are two possible arrangements, AB and BA

AB ≠ BA

order matters

$$\boxed{{}^{n}_{r}P = \frac{n!}{(n-r)!}}$$

$4! = 4 \times 3 \times 2 \times 1$

$\boxed{4! = 4 \times 3!}$

Permutation Example

- How many words can be formed by using 3 letters from the word "DELHI" ?

$n = 5$

$r = 3$

$${}^{n}P_{r} = \frac{n!}{(n-r)!} = \frac{5!}{(5-3)!} = \frac{5 \times 4 \times 3 \times \cancel{2!}}{\cancel{2!}}$$

arrangements = 60

Combination

- The combination is a way of selecting items from a collection, such that (unlike permutations) the order of selection does not matter
- Different selections of a given number of elements taken one by one, or some, or all at a time
- E.g.
  - if we have two elements A and B, then there is only one way select two items, we select both of them

$$ {}^{n}_{r}C = \frac{n!}{r!\,(n-r)!} $$

AB = BA

$$ {}^{n}C_{r} = \frac{n!}{r!\,(n-r)!} $$

Combination Example

- In how many ways, can we select a team of 4 students from a given choice of 15 ?

total students $(n) = 15$

selection $(r) = 4$

$$\text{teams} = \frac{n!}{r!(n-r)!} = \frac{15!}{4!\,11!} = 1365$$

Permutation vs Combination

arrangment

selection

Permutation	Combination
Arranging people, digits, numbers, alphabets, letters, and colours	Selection of menu, food, clothes, subjects, team
Picking a team captain, pitcher and shortstop from a group	Picking three team members from a group
Picking two favourite colours, in order, from a colour brochure	Picking two colours from a colour brochure
Picking first, second and third place winners	Picking three winners

$$\frac{n!}{(n-r)!}$$

$$\frac{n!}{r!\,(n-r)!}$$

Probability

Probability

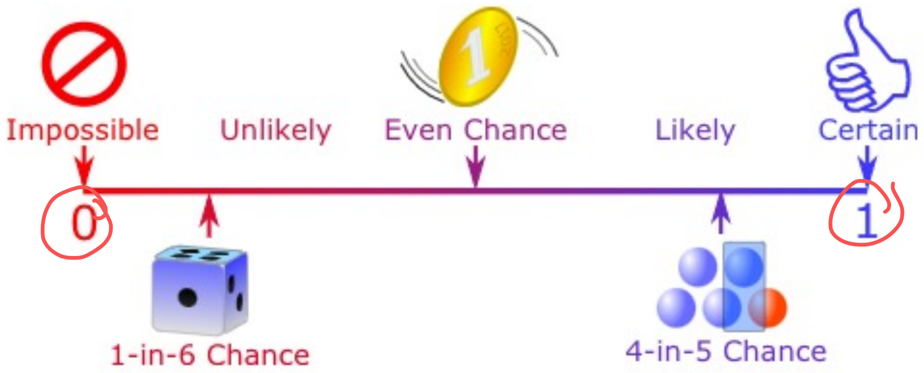
- Probability means possibility
- It is a branch of mathematics that deals with calculating the likelihood of a given event's occurrence
- Expressed as a number between 1 and 0
  - An event with a probability of 1 can be considered a certainty
  - An event with a probability of 0 can be considered a uncertainty
- Probability has been introduced in Maths to predict how likely events are to happen

probability

= number (0-1)

= percentage(%)

= decimal (n/a)



Example

- For example,
  - when we toss a coin, either we get Head OR Tail, only two possible outcomes are possible
    - (H, T) → possible outcomes

$P(H) = \frac{1}{2} = 0.5$

  - But if we toss two coins in the air, there could be three possibilities of events to occur, such as both the coins show heads or both show tails or one shows heads and one tail,
    - i.e.(H, H), (H, T),(T, T)

Definitions

- **Sample Space**
  - The set of all the possible outcomes to occur in any trial
  - E.g.:
    - Tossing a coin, Sample Space (S) = {H,T}
    - Rolling a die, Sample Space (S) = {1,2,3,4,5,6}
- **Sample Point**
  - It is one of the possible results
  - E.g.
    - In a deck of Cards:
      - 4 of hearts is a sample point
      - The queen of clubs is a sample point
- **Experiment or Trial**
  - A series of actions where the outcomes are always uncertain
  - E.g.:
    - The tossing of a coin
    - Selecting a card from a deck of cards
    - throwing a dice

Definitions

- **Event**
  - It is a single outcome of an experiment
  - E.g.
    - Getting a Heads while tossing a coin is an event
- **Outcome**
  - Possible result of a trial/experiment
  - E.g.
    - T (tail) is a possible outcome when a coin is tossed
- **Complimentary event**
  - The non-happening events
  - The complement of an event A is the event, not A (or A')
  - E.g.
    - Standard 52-card deck, A = Draw a heart, then A' = Don't draw a heart

Definitions

- Impossible Event
  - The event cannot happen
  - E.g.
    - In tossing a coin, impossible to get both head and tail at the same time

Probability Formula

- The ratio of number of favorable outcomes to the number of total outcomes is defined as probability of occurrence of any event

$$P(E) = \frac{Number\ of\ favourable\ outcomes}{Total\ number\ of\ outcomes}$$

Probability Example 1

- A coin is tossed one time. What is the probability that it will Head?

sample space = {H, T}

$$P(H) = \frac{\text{favorable outcome}}{\text{total outcome}}$$

$$= 1/2 = 0.5$$

Probability Example 2

- A coin is tossed two times. What is the probability of getting
  - Two heads
  - Two tails
  - One head
  - No head
  - At least one head
  - At most one head

1. $P(2H) = 1/4$: {HH}
2. $P(2T) = 1/4$: {TT}
3. $P(H) = 2/4$ {HT, TH}
4. $P(0H) = 1/4$ {TT}
5. $P(1+H) = 3/4$ {HT, TH, HH}
6. $P(0,1H) = 3/4$ {TT, TH, HT}

sample space =

{HH, HT, TH, TT}

Probability Example 3

- A coin is tossed three times. What is the probability of getting
  - Two heads
  - Two tails
  - One head
  - No head
  - At least one head
  - At most one head

Probability Example 4

- A dice is rolled one time. What is the probability of getting
  - 5
  - Number greater than 3
  - One even number
  - One prime number

Probability Example 5

- A dice is rolled two times. What is the probability of getting
  - 6 on first time
  - Sum if 10 or more
  - Difference in numbers is equal to 3
  - Both are the even numbers
  - Both are 5

Probability Example 6

- A card is drawn from a pack of cards. What is the probability of getting
  - Red color
  - Ace of black
  - Jack of red
  - Any king
  - Dimond and Jack
  - Spade

Probability formula

- $P(A`) = 1 - P(A)$
- $P(A \cup B) = P(A) + P(B) – P(A \cap B)$
- $P(A \cap B) = P(A) . P(B)$
- $P(A . A`) = 0$
- $P(A` . B) = P(B) – P(A.B)$
- $P(A . B`) = P(A) – P(A.B)$
- $P(A + B) = P(A` . B) + P(A . B`) + P(A . B)$

Conditional Probability

Independent Events

- Events can be independent, meaning each event is **not affected** by any other events
- Example:
  - Tossing a coin
  - Each toss of a coin is a perfect isolated thing
  - What it did in the past will not affect the current toss
  - The chance is simply 1-in-2, or 50%, just like ANY toss of the coin
  - So each toss is an **Independent Event**

Dependent Events

- Events can be independent, meaning each event is **affected** by any other events
- Example
  - Consider there are 2 blue and 3 red marbles are in a bag
  - What are the chances of getting a blue marble?
  - What are the chances of getting a blue marble again?

Conditional Probability

- The likelihood of an event occurring, assuming a different one has already happened
- The formula is

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Example 1

- What are the chances of drawing 2 blue marbles from a bag of 2 blue and 3 red marbles?

Example 2

- Drawing 2 Kings from a Deck

Baye’s Rule

- Bayes' theorem, named after 18th-century British mathematician Thomas Bayes, is a mathematical formula for determining conditional probability
- It provides a way to revise existing predictions or theories (update probabilities) given new or additional evidence
- Bayes' theorem is also called Bayes' Rule or Bayes' Law and is the foundation of the field of Bayesian statistics

$$P(A|B) = \frac{P(A \bigcap B)}{P(B)} = \frac{P(A) \cdot P(B|A)}{P(B)}$$

where:

$P(A) =$ The probability of A occurring

$P(B) =$ The probability of B occurring

$P(A|B) =$ The probability of A given B

$P(B|A) =$ The probability of B given A

$P\left(A \bigcap B\right)) =$ The probability of both A and B occurring

How does it work ?

- Below is a data set of weather and corresponding target variable ‘Play’ (suggesting possibilities of playing). We need to classify whether players will play or not based on weather condition.

Weather	Play
Sunny	No
Overcast	Yes
Rainy	Yes
Sunny	Yes
Sunny	Yes
Overcast	Yes
Rainy	No
Rainy	No
Sunny	Yes
Rainy	Yes
Sunny	No
Overcast	Yes
Overcast	Yes
Rainy	No

How does it work ?

- Step 1: Convert the data set into a frequency table

Frequency Table		
Weather	No	Yes
Overcast		4
Rainy	3	2
Sunny	2	3
Grand Total	5	9

How does it work ?

- Step 2: Create Likelihood table by finding the probabilities like Overcast probability = 0.29 and probability of playing is 0.64

Likelihood table

Weather	No	Yes		
Overcast		4	=4/14	0.29
Rainy	3	2	=5/14	0.36
Sunny	2	3	=5/14	0.36
All	5	9		
	=5/14	=9/14		
	0.36	0.64		

How does it work ?

- **Problem:** Players will play if weather is sunny. Is this statement is correct?
- We can solve it using above discussed method of posterior probability.

$$P(Yes \mid Sunny) = P(Sunny \mid Yes) * P(Yes) / P (Sunny)$$

- Here we have

$P (Sunny \mid Yes) = 3/9 = 0.33$
$P(Sunny) = 5/14 = 0.36$
$P(Yes)= 9/14 = 0.64$

- Which means, $P (Yes \mid Sunny) = 0.33 * 0.64 / 0.36 = 0.60$, which has higher probability.