The Shape Matching Element Method for Meshless Animation of NURBs Models

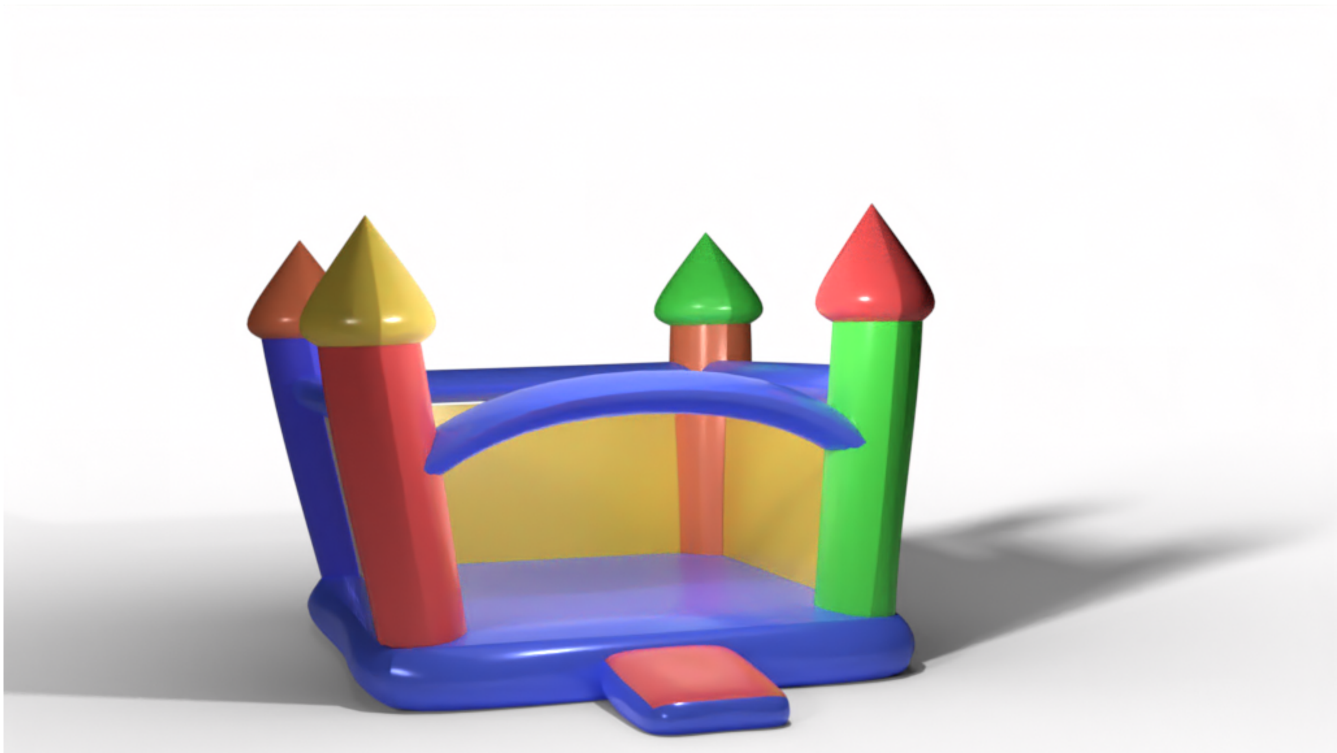

Figure 1: tmp

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1 INTRODUCTION

The consumption of geometric surface models by physics-based animation algorithms is fraught with difficulty. For volumetric objects, this process often involves identifying and discretizing the interior of the modelled object, typically either as a tetrahedral or hexahedral mesh.This procedure is both expensive and difficult, especially if the surface model is constructed from higher-order boundary representations, or if the volumetric discretization is required to be conforming or feature aligned. Removing explicit volumetric discretization from the physics-based-animation pipeline can avoid these difficulties and also provide a more unified modelling and simulation experience.

NURBS (Non-uniform Rational B Splines) are a popular higher-order modelling primitive which are used for computer-aided design (CAD), computational fabrication and computer animation. NURBS primitives were the first geometric representation used for physics-based animation, yet, despite over three decades of research, animation of NURBS objects remains a challenge.

Isogeometric Analysis (IGA) is a physics simulation methodology that uses the control variables of the NURBS model as the degrees-of-freedom (DOFS) of the simulation itself. Unfortunately IGA approaches for volumetric objects still require background volumetric structures, typically regular grids that make satisfaction of boundary conditions difficult (which makes collision resolution difficult) or more complicated cut-cell grids which introduce non-trivial root finding problems into the mix. Crucially, these simulation schemes typically assume models arise from engineering applications and meet tight geometric criteria such as that the mesh is manufacturable. These inputs are much cleaner than those produced by a typical modeller.

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We present the first truly meshless (no volumetric discretization generated) algorithm for direct, nonlinear elastodynamic simulation of NURBs models. Our nonlinear elastodynamic simulation scheme requires only a boundary description of the object (we do not strictly require a solid model) and approriate physical parameters. Because we explicitly use the NURBS boundary representation in the simulation its is straightforward to handle Dirichilet, Neumann boundary conditions and to apply contact resolution. Crucially because we broadly target animation not necessarily simulation for engineering or manufacturing we don't require that models satisfy the rigourous geometric requirements common for these applications

Our approach is an extension of the recently developed Virtual Element Method (VEM) for solving PDEs on domains tiled with arbitrary polygons. We establish a connection between VEM and the well-known shape matching simulation algorithm which enables us to derive equations of motion for an arbitrary NURBS model using Lagrangian Mechanics. Importantly, we show how to replace volumetric data structures for integration with ray casting approaches which enables our meshless approach to isogeometric elastodynamic physically-based animation of volumetric structures.

2 RELATED WORK

Geometric modeling is a necessary precursor to physics-based animation, however differing geometric representations for modelling and simulation often require time-consuming, complex geometry processing pipelines. For instance, the popular tetrahedral finite element approach for simulating solid elastodynamics requires robust algorithms for converting input surface geometry into a volumetric tetrahedral mesh. This is a difficult problem and while significant progress has been [Sifakis and Barbic 2012] made, even the most robust volumetric methods [Hu et al. 2018] can be time consuming, fail to maintain correspondence between the input model and output simulation mesh, and don't work directly on curved surface representations such as NURBS.

For many physically-based animation tasks, it would be desirable to bypass volumetric meshing entirely and directly simulate the geometric model. An ideal approach would avoid meshing of any kind (no volumetric meshes or cutcells), support continuum mechanics type constitutive models and energies that have become standard in physics-based animation pipelines, be compatible with a wide range of time integration schemes and ensure that simulation output can be edited in the same modelling software used to create the input (important for post-processing). Finally, our method should put only moderate constraints on input model quality to facilitate ease-of-use.

Isogeometric Analysis [Cottrell et al. 2009] endeavors to perform simulation directly on the NURBS output from Computer-Aided Design (CAD) software. Initial attempts used NURBS surfaces to represent the mid-surface of thin objects [Terzopoulos and Qin 1994]. Initial attempts at volumetric simulation relied on volumetric NURBS [Aigner et al. 2009] but were limited to a narrow class of geometries. Finite volume methods are applicable to more general geometries [Heinrich et al. 2012; Sevilla et al. 2008] but require a volumetric mesh be generated. Modern approaches are constructed around the extended finite element method which enriches the standard finite element basis with discontinuous basis functions to enable improved boundary handling [Haasemann et al. 2011; Hafner et al. 2019; Legrain 2013; Safdari et al. 2015, 2016]. These methods typically start with an easy-to-generate structured volumetric mesh (tetrahedral or hexahedral), "cutting" the NURBS geomtric model against it to enable boundary handling (such a mesh is called a *cutcell* mesh). Like volumetric meshing, this cutting operation can be difficult and our ideal method would avoid it if possible. Some cutcell algorithms assume engineering/manufacturing quality input, which puts tight requirements on input models quality [Hafner et al. 2019]. Finally these approaches require additional mechanisms to ensure that simulation results lie inside the shape space of the input model's primitives, increasing complexity.

Shape Matching is a meshless approach to physics-based animation [Diziol et al. 2011; Müller et al. 2005] and geometry processing [Bouaziz et al. 2012] built around shape registration. The algorithm has been extended from volumetric triangle mesh input to cloth [Stumpp et al. 2008] to particles [Müller and Chentanez 2011] and even to visual geometry for video games [Müller et al. 2016]. Shape Matching is fast [Rivers and James 2007; Steinemann et al. 2008] and meshless, but state-of-the-art methods require additional modeling input to position simulation primitives [?] or limit simulation primitives to be collections of convex polytopes [Müller et al. 2016]. Finally, Shape Matching is tightly coupled to the position-based dynamics [Müller et al. 2007] time integration methodology. While incredibly performant, this approach is incompatible with the constitutive models that are popular for physics-based animation as well as other time integration schemes. The popular Projective Dynamics algorithm [Bouaziz et al. 2014] enables a more flexible Shape Matching implementation but is still limited to a subset of constitutive models for elastic solids.

To alleviate these restrictions we can to turn to other meshless methods popular in computer graphics and engineering [Faure et al. 2011; Gilles et al. 2011; Liu et al. 1995; Martin et al. 2010; Müller et al. 2004]. These support more advanced constitutive models and integration schemes but often require background integration meshes and lose the direct connection with modelling geometry. Like cutcell-based, Isogeometric Analysis approaches, additional constraints must be added to ensure that simulation output can be represented by the input model. Given this, we conclude that there is no existing algorithm that meets our desiderata for success.

Our algorithm takes the Shape Matching approach as inspiration, but rather than follow the PBD formalism, we interpret Shape Matching as a Virtual Element Method (VEM) [Beirão da Veiga et al. 2014; Veiga et al. 2012]. Virtual Elements are an extension of mimetic finite differences [Brezzi et al. 2005; Lipnikov et al. 2014] to weak-form variational problems. VEM relaxed the mesh generation requirements of finite element method by enabling the solution of partial differential equations on domains partitioned with arbitrary polytopes [De Goes et al. 2020]. The solution inside each polytope is approximated using a polynomial function of a specified order. VEM typically assumes that the boundary of the the problem domain is described by a piecewise linear complex which makes its standard formulation incompatible with our NURBS based input geometry.

Contributions

In this paper we develop a new Shape Matching-based, Virtual Element Method which is directly compatible with NURBS input geometry rather than piecewise linear surfaces. Furthermore, we improve the expressivity of the VEM basis by using a shape blending approach inspired by algorithms for skinning [Jacobson et al. 2014]. Our method is entirely meshless (requiring no volumetric meshes or cutcells) and is compatible with standard constitutive models and time integrators. It guarantees that simulation output is directly consumable by the input modeling software and can ingest models which include large gaps, intersections and disconnected primitives without additional user input. These features mean that our algorithm is the first truly meshless approach for the simulation of NURBS models for physics-based animation.

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Dump of some equations

$$\mathbf{M}(\mathbf{X}) = (X_0 - \bar{X}_0, X_1 - \bar{X}_1)$$

$$\bar{\mathbf{x}} = \frac{1}{n}\sum_i^n x_i$$

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