

6/15/20

Math MTRV 5

1. Given - $\triangle ABC$ where $DE \parallel BC$

To prove $\rightarrow \frac{AD}{DB} = \frac{AE}{EC}$

Construction \rightarrow Join BE and CD
 Draw $DM \perp AC$ and $EN \perp AB$

Proof \rightarrow

$$\text{ar}(\triangle ADE) = \frac{1}{2} \cdot B \cdot H$$

$$= \frac{1}{2} \cdot AD \cdot EN \quad \text{--- (i)}$$

$$\text{ar}(\triangle BDE) = \frac{1}{2} \cdot DB \cdot EN \quad \text{--- (ii)}$$

$$\text{ar}(\triangle ADE) = \frac{1}{2} \cdot AE \cdot DM \quad \text{--- (iii)}$$

$$\text{ar}(\triangle DEC) = \frac{1}{2} \cdot EC \cdot DM \quad \text{--- (iv)}$$

Divide (i) and (ii)

$$\text{ar}(\triangle ADE) = \frac{1}{2} \cdot AD \cdot EN$$

$$\text{ar}(\triangle BDE) = \frac{1}{2} \cdot DB \cdot EN$$

$$\therefore \frac{\text{ar}(\triangle ADE)}{\text{ar}(\triangle BDE)} = \frac{AD}{DB} \quad \text{--- (A)}$$

$$\text{Divide (iii) by (iv)} \rightarrow \frac{\text{ar}(\triangle ADE)}{\text{ar}(\triangle DEC)} = \frac{\frac{1}{2} \cdot AE \cdot DM}{\frac{1}{2} \cdot EC \cdot DM}$$

$$\frac{A_1(\triangle ADE)}{A_1(\triangle DEC)} = \frac{AE}{EC} \quad \text{--- (D)}$$

Now, $\triangle ADE$ and $\triangle DEC$ are on same base DE between same parallels BC and DE .

$$A_1(\triangle ADE) = A_1(\triangle DEC)$$

Hence,

$$\frac{A_1(\triangle ADE)}{A_1(\triangle BDE)} = \frac{A_1(\triangle ADE)}{A_1(\triangle DEC)}$$

$$\therefore \frac{AD}{DB} = \frac{AE}{EC} \quad (\text{from A & B})$$

Hence proved.

2. Given: $\triangle ABC$ right angled at B .

$$\text{To prove: } AC^2 = AB^2 + BC^2$$

Construction: Draw $BD \perp AC$

Proof: Since $BD \perp AC$

using theorem 6.7: If a perpendicular is drawn from vertex of right angle of the \triangle to the hypotenuse then triangles on B.S. of the \triangle are similar to whole \triangle and each other.

$\triangle ABC \rightarrow \rightarrow$ P.T.O

$$\triangle ADB \sim \triangle ABC$$

Since, sides of similar triangles are in the same ratio,

$$\Rightarrow \frac{AD}{AB} = \frac{AB}{AC}$$

$$\Rightarrow AD \cdot AC = AB^2 \quad \text{--- (i)}$$

$$\triangle BDC \sim \triangle ABC$$

Since, sides of sim Δ are in same ratio.

$$\Rightarrow \frac{CD}{BC} = \frac{BC}{AC}$$

$$\Rightarrow CD \cdot AC = BC^2 \quad \text{--- (ii)}$$

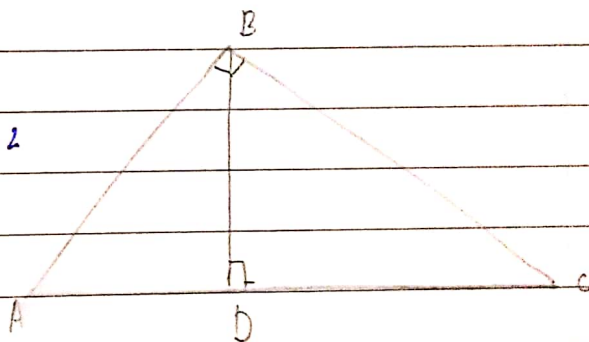
Adding (i) and (ii),

$$AD \cdot AC + CD \cdot AC = AB^2 + BC^2$$

$$AC(AD + CD) = AB^2 + BC^2$$

$$AC \cdot AC = AB^2 + BC^2$$

$$AC^2 = AB^2 + BC^2$$



Hence proved.