

mt2113(PROBABILITY)

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1 COMBINATORIAL ANALYSIS

The Basic Principle of Counting

Suppose that two experiments are to be performed. Then if experiment 1 can result in any one of m possible outcomes and if for each outcome of experiment 1, there are n possible outcomes of experiment 2 together there are mn possible outcomes of two experiments.

Proof

$$\left[\begin{array}{cccc|c} 1,1 & 1,2 & \cdots & 1,n & \\ 2,1 & 2,2 & \cdots & 2,n & \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ m,1 & m,2 & \cdots & m,n & \end{array} \right]$$

for each outcome i.e. (i,j) if experiment 1 result in its i^{th} position outcome and experiment 2 then result in its j^{th} possible outcome. Hence the set of possible outcomes consists of m rows each containing n elements.

Generalized Basic Principle of Counting

If r experiments that are to be performed are such that the first one may result in any of n_1 possible outcomes and if for each of these n_1 possible outcomes, there are n_2 possible outcomes of the second experiments and if for each of the possible outcomes of the first two experiments, there are n_3 possible outcomes of the third experiments and if \dots then there is a total of $n_1.n_2.\dots n_r$ possible outcomes of the r experiments.

Example

How many different 7-place license plates are possible if the first 3 places are to be occupied by letters and the final 4 by numbers?

Sol Generalized version of the basic principle is $26*26*26*10*10*10*10= 175760000$ outcomes

Abstract

So the **Generalized Principle of Counting**

Suppose r experiments are to be performed and

- For Experiments 1 we have n_1 possible outcomes
- For each outcome of Experiment $i \rightarrow$ there are n_{i+1} outcomes for Experiment $i+1$
- Total number of possible outcomes is

$$\sum_{i=1}^r n_i = n_1 * n_2 * \dots * n_r \quad (1)$$

2 PERMUTATIONS

2.0.1 Defination:

A permutation of n objects is an ordered sequence of those n objects.

How many different ordered arrangements of the letters a,b, and c?

Let there be an object. $n(n-1)(n-2)\dots 3.2.1 = n!$

different permutations of n objects

2.0.2 Counting

Let P_n be the number of permutations for n objects. Then

$$P_n = n! = n * (n-1) \dots * 2 = \sum_{j=1}^n j \quad (2)$$

In the same there are

$$\frac{n!}{n_1!n_2!\dots n_r!} \quad (3)$$

different permutations of n objects of which n_1 are alike n_2 are alike $\dots n_r$ are alike

3 Combinations

3.0.1 Defination

A combination of p objects among n objects is a non-ordered subset of p objects.

Property: Two combinations only differ according to the nature of their objects

$$\binom{n}{p} = \frac{n!}{k!(n-p)!} \quad (4)$$

Proof of Counting

Combination when order is relevant
The number of possibilities is

$$n * (n - 1) \dots * (n - p + 1) = \frac{n!}{(n - p)!} \quad (5)$$

3.0.2 Combinations when order is irrelevant:

When divided by permutations of p objects the number of possibilities is, if we select r items from n items, then the number of different groups is given by

$$\frac{n * (n - 1) \dots * (n - p - 1)}{p!} = \frac{n!}{p!(n - p)!} = \binom{n}{p} \binom{n}{r} = \binom{n}{n - r} = \frac{n!}{(n - r)!r!} \quad (6)$$

3.0.3 A Combinatorial Identity

$$\binom{n}{r} = \binom{n - 1}{r - 1} + \binom{n - 1}{r} \quad (7)$$

Proof:

$$\binom{n - 1}{r - 1} + \binom{n - 1}{r} = \frac{(n - 1)!}{(r - 1)!(n - r)!} + \frac{(n - 1)!}{r!(n - 1 - r)!} = \frac{(n - 1)!r}{r!(n - r)!} + \frac{(n - 1)!(n - r)}{r!(n - r)!} \quad (8)$$

$$= \frac{(n - 1)!(r + n - r)}{r!(n - r)!} = \frac{(n - 1)!n}{r!(n - r)!} = \frac{n!}{r!(n - r)!} = \binom{n}{r} \quad (9)$$

4 BINOMIAL THEOREM

$$(x + y)^n = \sum_{k=0}^n \binom{n}{k} x^k y^{n-k} \quad (10)$$

Multinomial Coefficient

It is used in combinatorics and is an extension of the binomial coefficient. It is used to find permutations when you have repeating values or duplicate items.

Formula

$$\binom{n}{k_1, k_2, \dots, k_r} = \frac{n!}{k_1!k_2! \dots k_{r-1}!k_r!} \quad (11)$$

The multinomial coefficient formula gives an expansion of $(k_1 + k_2 + \dots + k_n)$ where k_i are non negative integer

4.0.1 Division of n objects into r groups with size n1...nr

- n objects and r groups
- n_j objects in group j and

$$\sum_{j=1}^r n_j = n$$

Set of notation:

$$\binom{n}{n_1, \dots, n_r} = \frac{n!}{\prod_{j=1}^r (n_j!)}$$

Counting: Division of n objects into r groups with size $n_1, \dots, n_r =$

$$\binom{n}{n_1, \dots, n_r}$$

5 Multinomial Theorem

let

- $x_1, x_2, \dots, x_r \in R$
- $n \geq 1$

$$(x_1 + x_2 + \dots + x_r)^n = \sum_{n_1, \dots, n_r \in A_{n,r}} * \binom{n}{n_1, n_2, \dots, n_r} x_1^{n_1} x_2^{n_2} \dots x_r^{n_r} \quad (12)$$

The numbers

$$\binom{n}{n_1, n_2, \dots, n_r}$$

are known as **multinomial coefficient**

Permutation and Combinations

- No. 1 point is fuck off