mt2113(PROBABILITY)

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1 COMBINATORIAL ANALYSIS

The Basic Principle of Counting

Suppose that two experiments are to be performed. Then if experiment 1 can result in any one of m possible outcomes and if for each outcome of experiment 1, there are n possible outcomes of experiment 2 together there are mn possible outcomes of two experiments.

Proof

$$\left[\begin{array}{cccc:c} 1,1 & 1,2 & \cdots & 1,n & \\ 2,1 & 2,2 & \cdots & 2,n & \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ m,1 & m,2 & \cdots & m,n & \end{array}\right]$$

for each outcome i.e. (i,j) if experiment 1 result in its i^{th} position outcome and experiment 2 then result in its j^{th} possible outcome. Hence the set of possible outcomes consists of m rows each containing n elements.

Generalized Basic Principle of Counting

If r experiments that are to be performed are such that the first one may result in any of n_1 possible outcomes and if for each of these n_1 possible outcomes, there are n_2 possible outcomes of the second experiments and if for each of the possible outcomes of the first two experiments, there are n_3 possible outcomes of the third experiments and if $\ldots$ then there is a total of $n_1.n_2.\ldots.n_r$ possible outcomes of the r experiments.

Example

How many different 7-place license plates are possible if the first 3 places are to be occupied by letters and the final 4 by numbers?
Sol Generalized version of the basic principle is 26*26*26*10*10*10*10= 175760000 outcomes

Abstract

So the **Generalized Principle of Counting**
Suppose r experiments are to be performed and

- For Experiments 1 we have n_1 possible outcomes
- For each outcome of Experiment i $\rightarrow$ there are n_{i+1} outcomes for Experiment i+1
- Total number of possible outcomes is

$$\sum_{i=1}^{r} n_i = n_1 * n_2 * \ldots * n_r \tag{1}$$

2 PERMUTATIONS

2.0.1 Defination:

A permutation of n objects is an ordered sequence of those n objects.
How many different ordered arrangements of the letters a,b, and c?
Let there be an object. n(n-1)(n-2)...3.2.1= n!
different permutations of n objects

2.0.2 Counting

Let P_n be the number of permutations for n objects. Then

$$P_n = n! = n * (n-1) \ldots * 2 = \sum_{j=1}^{n} j \tag{2}$$

In the same there are

$$\frac{n!}{n_1! n_2! \ldots n_r!} \tag{3}$$

different permutations of n objects of which n_1 are alike n_2 are alike $\ldots n_r$ are alike

3 Combinations

3.0.1 Defination

A combination of p objects among n objects is a non-ordered subset of p objects.
Property: Two combinations only differ according to the nature of their objects

$$\binom{n}{p} = \frac{n!}{k!(n-p)!} \tag{4}$$

Proof of Counting

Combination when order is relevant
The number of possibilities is

$$n * (n-1) \ldots * (n-p+1) = \frac{n!}{(n-p)!} \tag{5}$$

3.0.2 Combinations when order is irrelevant:

When divided by permutations of p objects the number of possibilities is, if we select r items from n items, then the number of different groups is given by

$$\frac{n * (n-1) \ldots * (n-p-1)}{p!} = \frac{n!}{p!(n-p)!} = \binom{n}{p}\binom{n}{r} = \binom{n}{n-r} = \frac{n!}{(n-r)!r!} \tag{6}$$

3.0.3 A Combinatorial Identity

$$\binom{n}{r} = \binom{n-1}{r-1} + \binom{n-1}{r} \tag{7}$$

Proof:

$$\binom{n-1}{r-1}+\binom{n-1}{r} = \frac{(n-1)!}{(r-1)!(n-r)!}+\frac{(n-1)!}{r!(n-1-r)!} = \frac{(n-1)!r}{r!(n-r!)}+\frac{(n-1)!(n-r)}{r!(n-r)!} \tag{8}$$

$$= \frac{(n-1)!(r+n-r)}{r!(n-r)!} = \frac{(n-1)!n}{r!(n-r)!} = \frac{n!}{r!(n-r)!} = \binom{n}{r} \tag{9}$$

4 BINOMIAL THEOREM

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^k y^{n-k} \tag{10}$$

Multinomial Coeffiecent

It is used in combinatorics and is an extension of the binomial coefficient. It is used to find permutations when you have repeating values or duplicate items.
Formula

$$\binom{n}{k_1, k_2, ..., kr-1} = \frac{n!}{k_1!k_2!\ldots k_{r-1}!k_r!} \tag{11}$$

The multinomial coefficient formula gives an expansion of $(k_1 + kk_2 + \ldots + k_n)$ where k_i are non negative integer

4.0.1 Division of n objects into r groups with size n1...nr

- n objects and r groups
- n_j objects in group j and

$$\sum_{j=1}^{r} n_j = n$$

Set of notation:

$$\binom{n}{n_1, \ldots, n_r} = \frac{n!}{\prod_{j=1}^{r}(n_j!)}$$

Counting: Division of n objects into r groups with size $n-1, \ldots, n_r =$

$$\binom{n}{n_1, \ldots, n_r}$$

$$(x_1 + x_2 + \ldots + x_r)^n = \sum_{n_1, \ldots, n_r \in A_{n,r}} * \binom{n}{n_1, n_2, \ldots n_r} x_1^{n_1} x_2^{n_2} \ldots x_r^{n_r} \tag{12}$$

The numbers

$$\binom{n}{n_1, n_2, \ldots, n_r}$$

are known as **multinomial coefficent**

Multinomial Coefficent

Abstract

A set of n distinct items is to be divided into r distinct group of respective size $n_1, n_2, n_3, \ldots, n_r$ where

$$\sum_{i=1}^{r} n_i = n \tag{13}$$

there are $\binom{n}{n_1}$ possible choices for the first group for each group choice of the first group there are $\binom{n-n_1}{n_2}$ possible choice for the second group for each choice of the first two group there are $\binom{n-n_1-n_2}{n_3}$ possible choices for the third group and so on.

The Generalized Version of the Basic counting Principle are

$$\binom{n}{n_1}, \binom{n-n_1}{n_2} \ldots, \binom{n-n_1-n_2-\ldots-n_{r-1}}{n_r} = \frac{n!}{(n-n_1!n_1!)} \frac{(n-n_[1])!}{(n-n_1!n_1!)n_2!} \ldots \frac{(n-n_1-n_2-\ldots-n_{r-1}}{0!n_r!} \tag{14}$$