6. The Arbitrage Pricing Theory (APT) and Multifactor Models of Risk and Return

FRM Part 1: Foundations of Risk Management

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1 Introduction

Recall, the CAPM is a *single factor* model that describes an asset's expected rate of return as a linear function of the market's risk premium.

- i.e. this model only considers *market risk*

In this chapter, we're now dealing with what's known as the *Asset Pricing Theory (APT)* models which seeks to explain the relationship between expected returns and risk but it considers more than just market risk.

- expected return is now a linear function of *multiple* factors
- APT doesn't specify what these factors are, it's up to the analyst to decide on what these factors should be

One thing to note is the term *arbitrage* in APT doesn't have the classical meaning of the word in this context. Here is simply means a model that measures expected return relative to multiple risk factor.

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2 The Arbitrage Pricing Theory (APT)

Unlike CAPM, APT doesn't assume that investors hold efficient portfolios and it doesn't assume investors are risk averse.

The following are the primary assumptions underlying the APT model:

1. asset returns can be explained by systematic factors that affect all securities
2. diversification can eliminate idiosyncratic risk
3. no arbitrage opportunities exist among well-diversified portfolios (if they did exist, they would be exploited anyways)
4. Market participants are seeking to maximize their profits.
5. Markets are frictionless (i.e., no barriers due to transaction costs, taxes, or lack of access to short selling).

APT return on a security is given by,

$$R_i = E(R_i) + \beta_{i1}[I_1 - E(I_1)] + \cdots + \beta_{ik}[I_k - E(I_k)] \tag{1}$$

where

- R_i = the return on a security i
- $E(R_i)$ = the expected rate of return on security i
- $I_k - E(I_k)$ = the difference between the observed value of the k^{th} factor and the expected value of the k^{th} factor

- β_{ik} = the coefficient measuring the effect of changes on the rate of return for security i by the k^{th} factor

In English, this means the model helps explain how much each factor impacts our realized rate of return for a given security.

Again, β measures the systemic risk of a given factor and how it affects our returns.

The premise of APT is that investors can create a zero-β portfolio with zero net investment. In other words, the core of the model is to find a combination of granular risk factors that more closely predict the return of a financial asset. Once we've picked the factors, we can then compute them and hedge by selecting securities to offset the systemic risk we don't want exposure to.

The absence of arbitrage opportunities requires that the expected return on all well-diversified portfolios to satisfy,

$$E(R_P) = E(R_F) + \beta_{P1}[E(I_1) - E(R_F)] + \cdots + \beta_{Pk}[E(I_k) - E(R_F)] \tag{2}$$

where

- $E(R_P)$ = expected return on a well-diversified portfolio
- $E(R_Z)$ = the expected return on a zero-β portfolio
- $E(I_k) - E(R_F)$ = the risk premium associated with the k^{th} factor

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3 Different Types of Factor Models

3.1 Macroeconomic Factor Models

These models seek to explain returns using macroeconomic variables.

Chen, Roll, and Ross gave some macroeconomic factors that help explain the returns from stocks offered on the NYSE:

- spread between long-term and short-term interest rates (yield curve)
- expected and unexpected inflation
- industrial production
- spread between high-risk and low-risk corporate bond yields

Later, the following factors were considered:

- confidence risk
- time horizon risk
- real business activity (business cycle risk)
- market risk

3.2 Fundamental Factor Models

Fundamental factor models have factors that attributes of a company or an industry.

The three-factor model proposed by Fama and French extend the CAPM by including the following factors:

1. Small minus big (SMB) - this is the difference between the returns from small sticks and those from large stocks
2. High minus low (HML) - the difference between the returns on stocks with high book-to-market values and those with low book-to-market values

The model proposed by Fama and French is given by

$$E(R_i) = R_F + \beta_{i,M} RP_M + \epsilon_i \tag{3}$$

Extension of this model include the following factors:

- Robust minus weak (RMW) - difference between companies with robust (high) and weak (low) operation profitability
- Conservative minus aggressive (CMA) - the difference between companies that invest aggressively vs those that do so conservatively

The *momentum factor (MOM)* is the difference between stocks that have a risk in value over the past month vs. those that have fallen in value in the past month.

3.3 Statistical Factor Models

These models use historical and cross-sectional data on stock returns. We can use PCA to help determine the most important uncorrelated factors and afterwards, we'll need to determine the meaning of each.