The eigenvalue method: instructions

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What you (should) have done so far:

1. By discretising the beam equation

$$\mu\,\ddot{w} + (EI\,w'')'' = q$$

you have obtained the DAE (Differential Algebraic Equation)

$$\underbrace{\begin{bmatrix} M & 0 \\ 0 & 0 \end{bmatrix}}_{M_e} \begin{bmatrix} \ddot{w} \\ \ddot{\mu} \end{bmatrix} + \underbrace{\begin{bmatrix} S & C \\ C^\top & 0 \end{bmatrix}}_{S_e} \begin{bmatrix} w \\ \mu \end{bmatrix} = \begin{bmatrix} f \\ a \end{bmatrix}. \tag{1}$$

2. You have solved the DAE in the static case (w constant, algebraic equation) and you have compared the result in some simple cases with analytic solutions of $(EI\,w'')'' = q$ (found in the literature).

3. You have implemented the Newmark method for solving the dynamic case and generated some movies.

There is another method to solve the dynamic case which is very effective if there are no external forces and moments and only homogeneous boundary conditions ($q = 0$, $f = 0$, $a = 0$). This method is called the eigenvalue method. It is explained in the attached two scripts. The basic idea is to write the general motion as a superposition of standing waves (eigenmodes). The shapes of that waves are called eigenfunctions (eigenvectors). The frequencies are called eigenfrequencies. The amplitudes of the waves are found from initial conditions via Fourier analysis.

Tasks:

1. Read the attached scripts, the analytic one first.

2. Apply the eigenvalue method to the discretisation (1) of the cantilever beam (E, I and mass density constant). Generate some movies of eigenmodes and superpositions.

3. Compare the computed eigenfrequencies ω_j with their analytic counterparts. You will find that only the lowest frequencies almost coincide.

4. You don't need to perform Fourier analysis to compute the amplitudes α_j, β_j. For your experiments just choose some values.

5. Additionally to the cantilever beam you could investigate the simply supported beam (see script). This is not mandatory.