An Analysis of Vibrating Euler-Bernoulli Beams

using Finite Element, Newmark and Eigenvalue Methods

Sergi Andreu, Dylan Everingham,
Carsten van de Kamp, Sebastian Myrbäck

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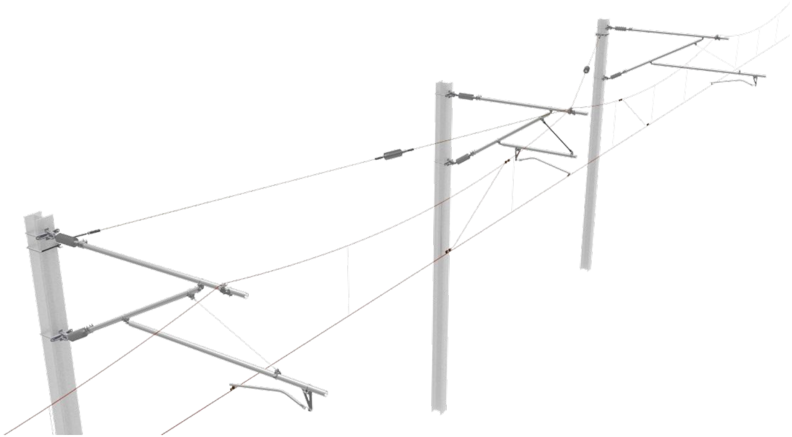
Outline

- Relevance of beam modeling in modern engineering
- Model and numerical methods
  - FEM for static case
  - Newmark and Eigenvalue methods for dynamic case
- Results
- Conclusions

Relevance

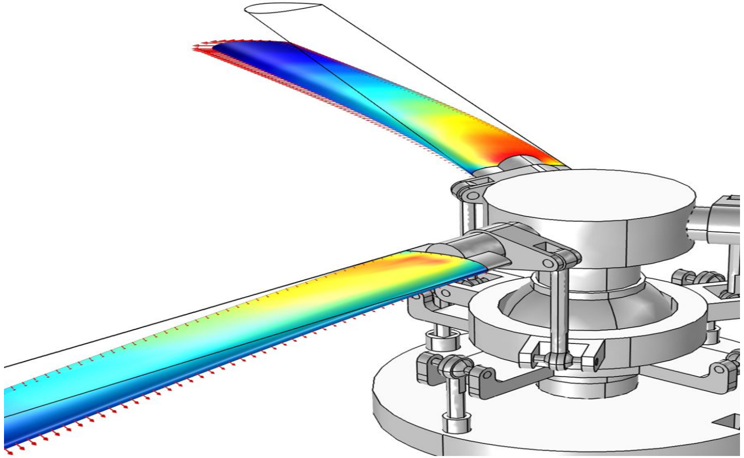
Cantilever construction for power cables

Kummler+Matter - Fixed point cantilever pipe



Helicopter rotor blades

Vince D - Helicopter COMSOL Multiphysics Mechanics Structure Finite Element Method

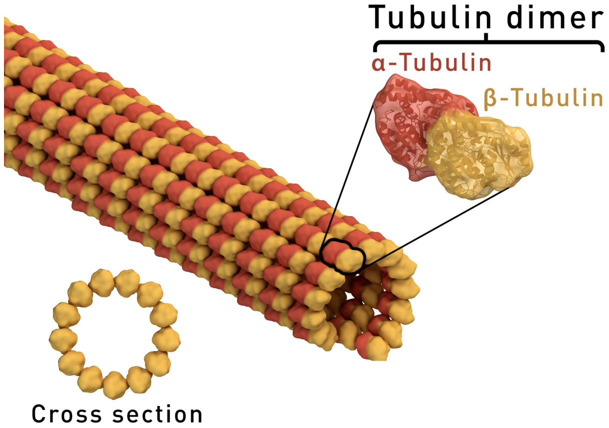


Relevance, continued

Podvis Viaduct Kičevo, North Macedonia

Microtubules in cell cytoskeletons

Y. Shi, W. Guo, and C. Ru - Relevance of Timoshenko-beam model to microtubules of low shear modulus



Model

Euler-Bernoulli beam theory

Assumptions:

- Small, smooth deformations
- Negligible shear/torsion

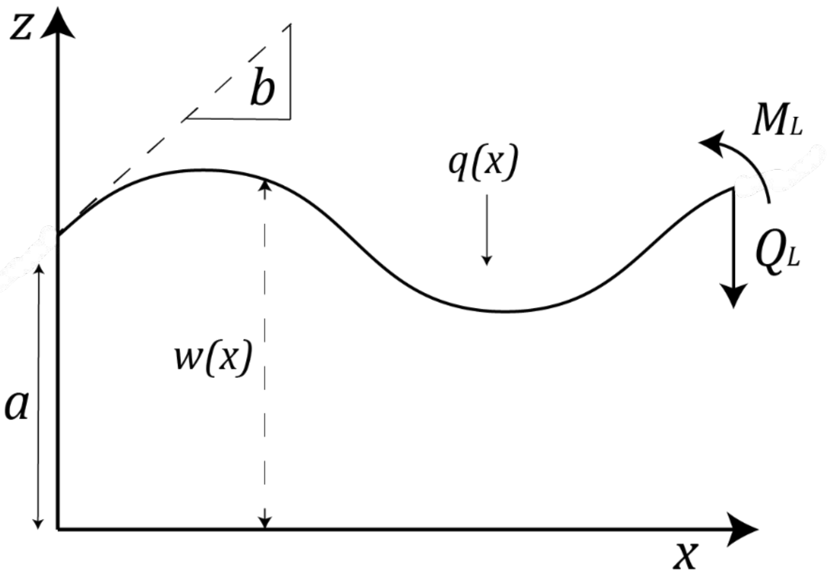
Consider 1D deformations in a beam of length L

Distinction cantilever and two-sided support

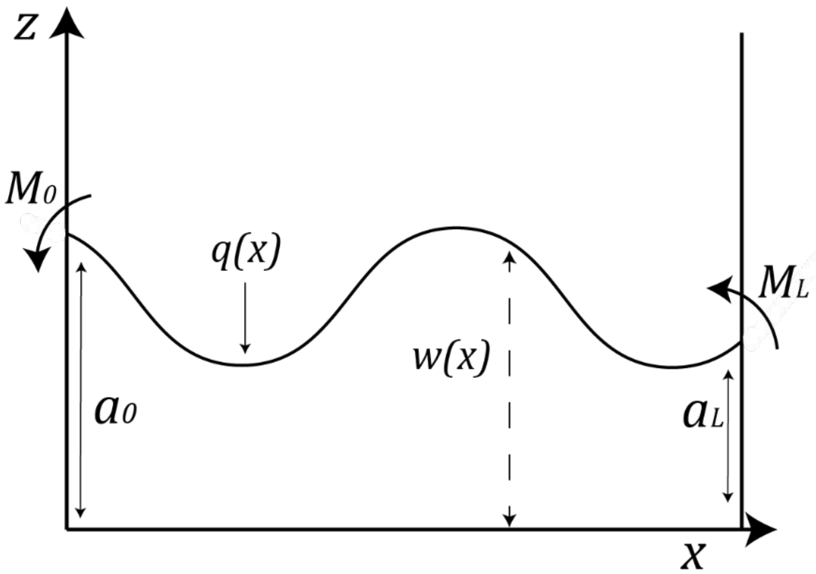
M - moment at boundary;
Q - load at boundary;
$q(x)$ - load distribution over beam

$w(x)$ - beam deflection

Cantilever



Two-sided support



Model and governing equation

Euler-Bernoulli beam theory

Assumptions:

- Small, smooth deformations
- Negligible shear/torsion

$$\mu \frac{\partial^2 w}{\partial t^2} + EI \frac{\partial^4 w}{\partial x^4} = q$$

μ - mass per unit length
E - Young's modulus, measure of stiffness
I - moment of inertia

Solving the Euler-Bernoulli beam equation

Static case: $\frac{\partial^2 w}{\partial t^2} = 0$

- Analytical solutions
- Finite element method

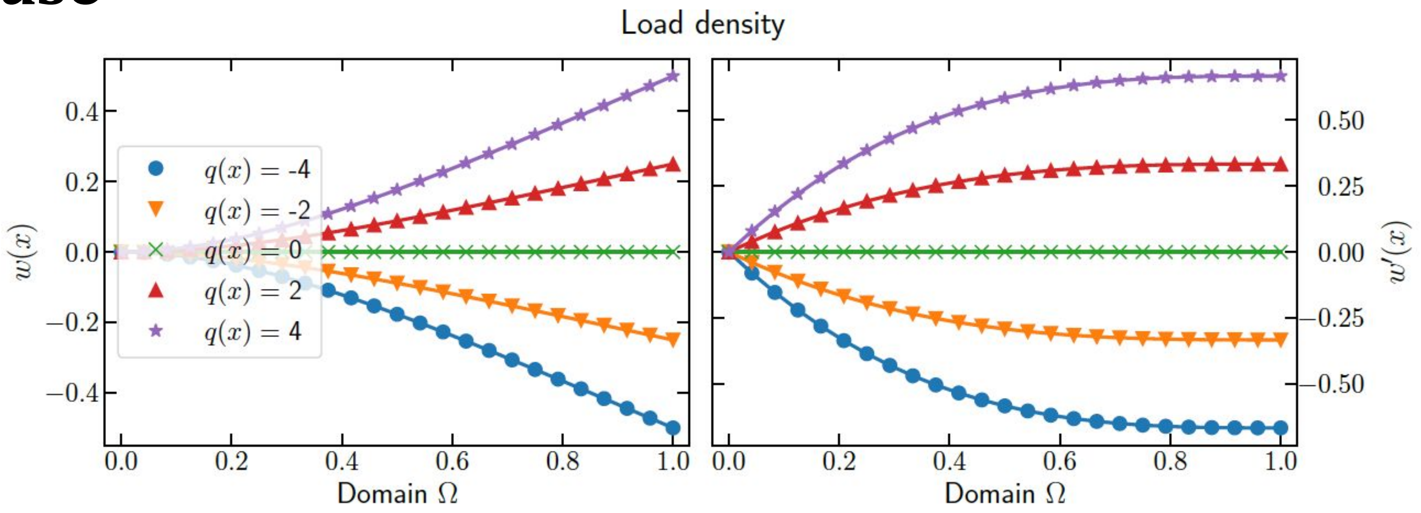
Dynamic case:

- Newmark method
- Eigenvalue method

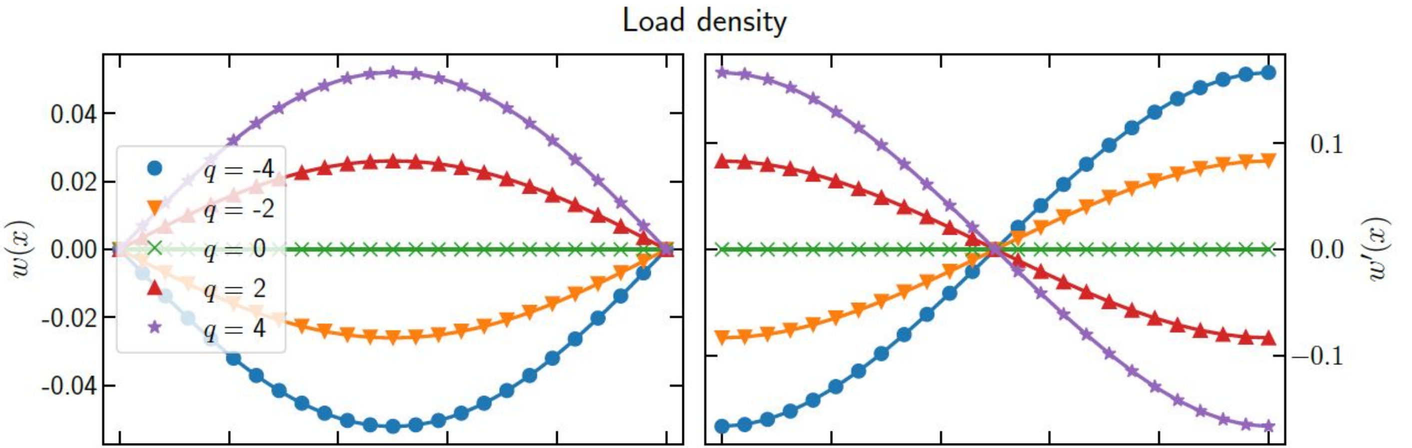
For details, see report

Results: **static case**

Cantilever beam:

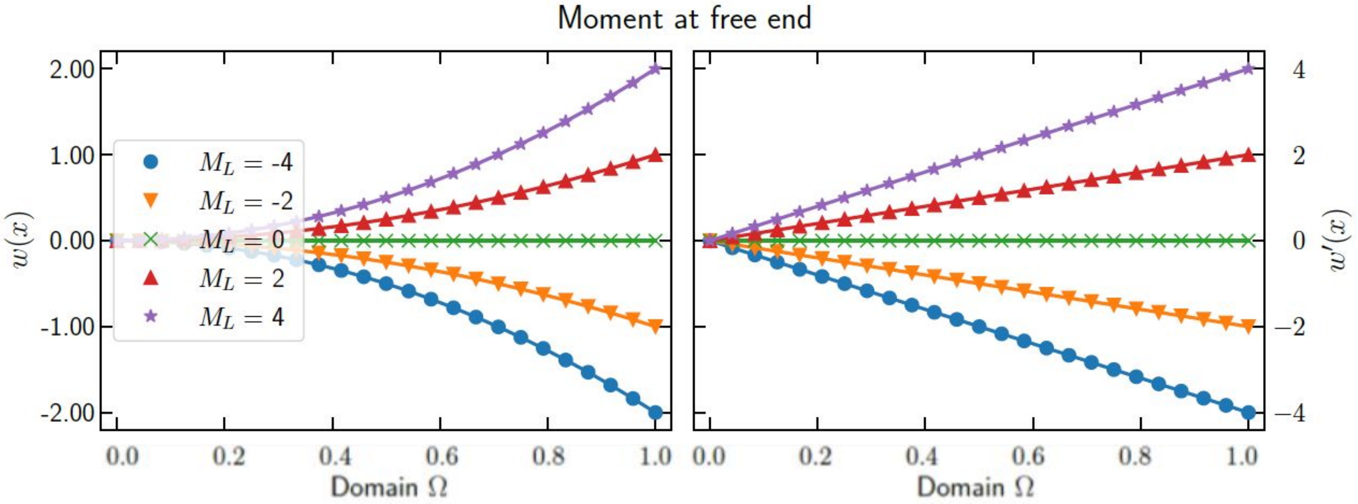


Beam supported at both ends:

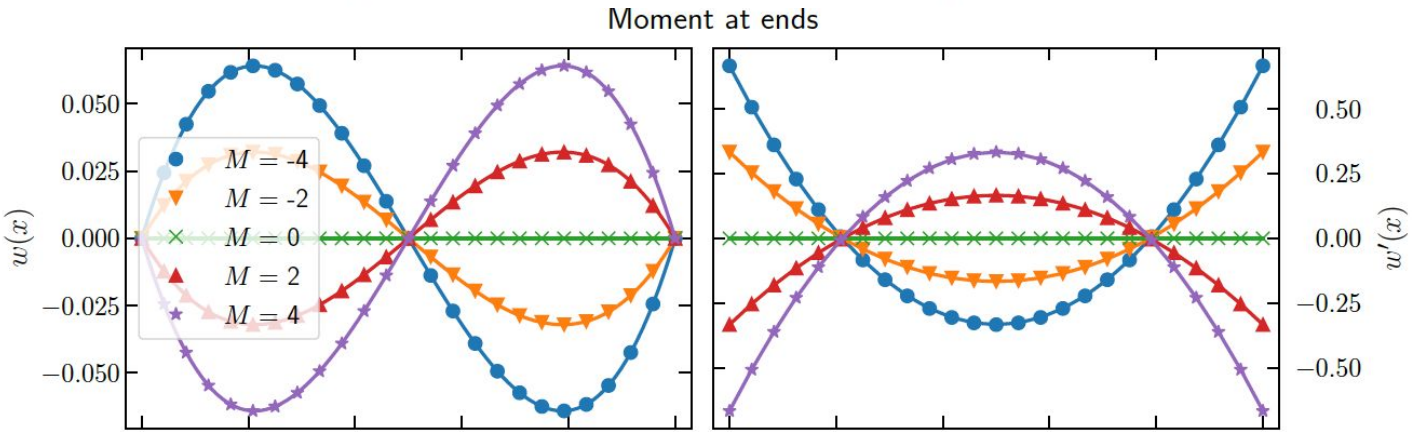


Results: **static case**

Cantilever beam:



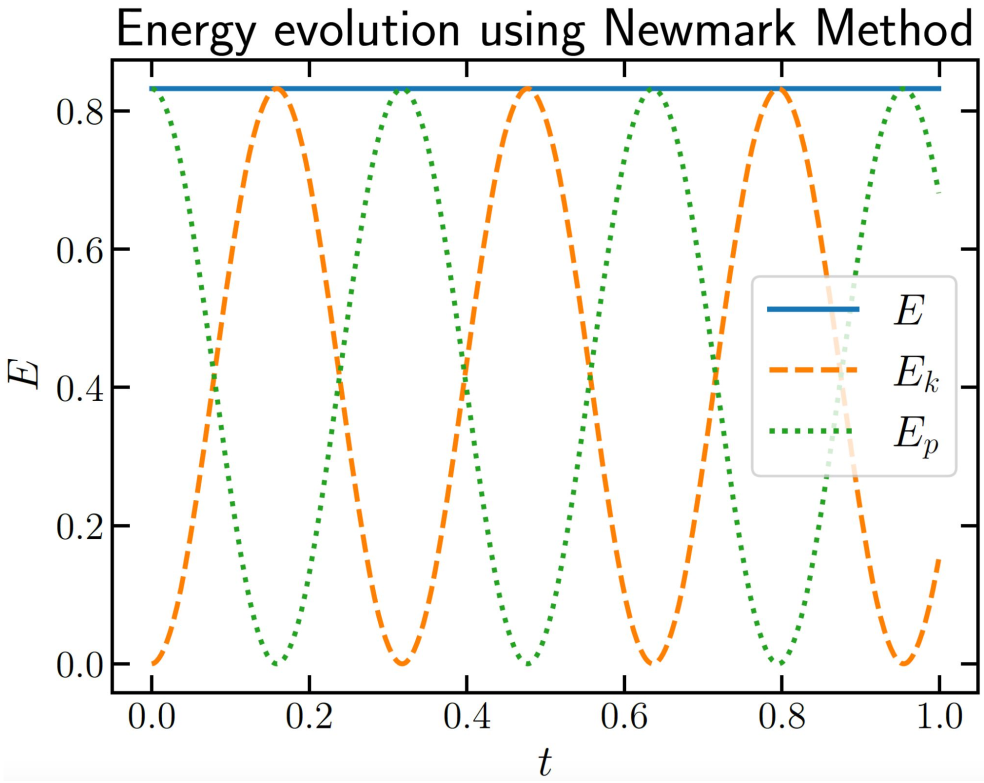
Beam supported at both ends:



Results: **dynamic case**

Newmark method:

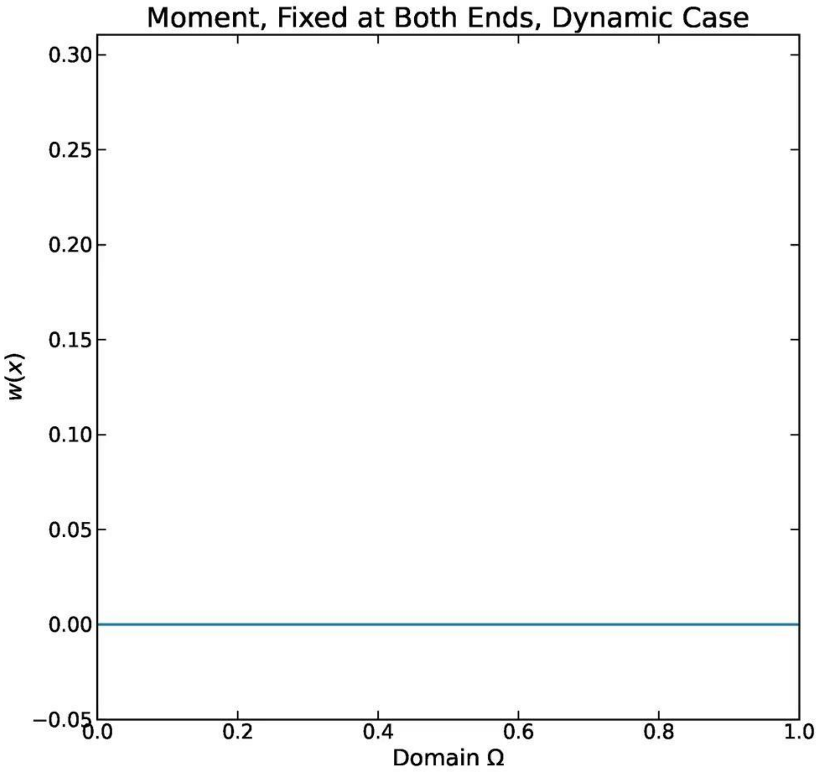
We look at the energy of the system.



Results: **dynamic case**

Newmark method:

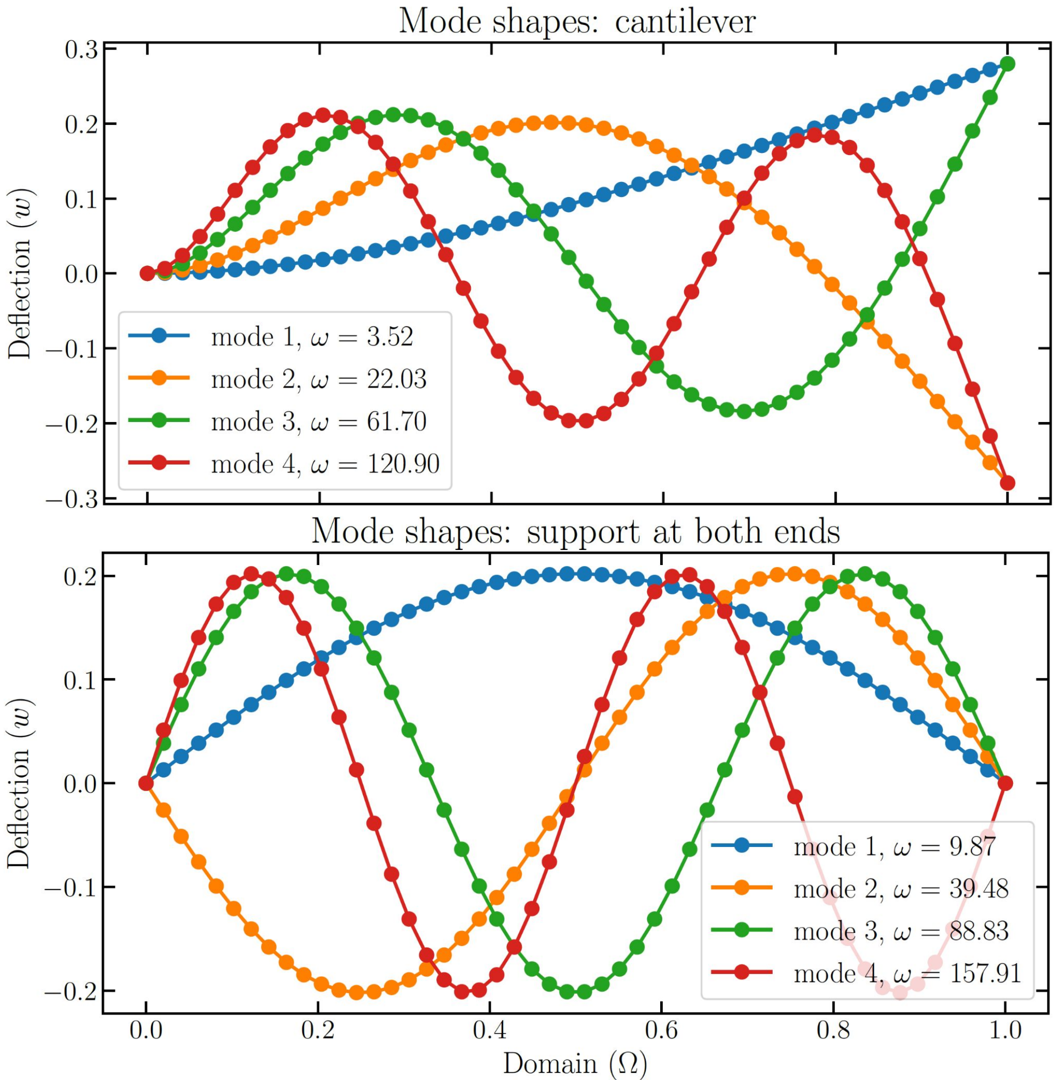
Here's an animation.



Results: **dynamic case**

Eigenvalue method:

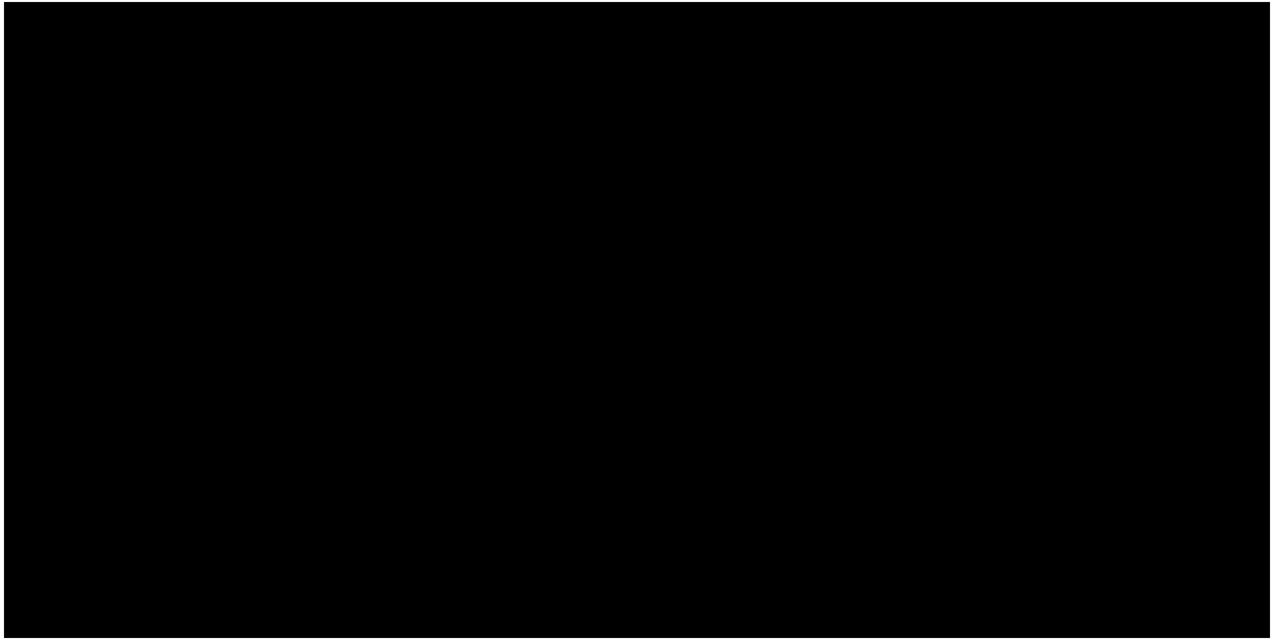
We look at some of the vibration modes, that constitute the numerical solution.



Results: **dynamic case**

Eigenvalue method:

Here's an animation of the cantilever case.



Conclusions

- Finite element results of the static case follow the analytical solutions very well.
- Newmark method
  - in the dynamic case preserves energy, and simulates realistic beam behavior.
  - can simulate cases of more general boundary conditions.
- Eigenvalue method
  - vibration modes correspond well to theoretical expectations, and simulates realistic beam behavior.
  - can only be used to model a very simple base case of boundary conditions.
  - gives more insight into beam vibrations and eigenfrequencies