Option Pricing with Deep Learning

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Predicting

- Options are financial derivatives that gives the right to buy (call) or sell (put) a security at or before a certain date.
- The value of a European option (right to buy/sell at a certain date, and not before) can be modeled with the commonly-used Black-Scholes formula [1]:

$$C = S \cdot \boldsymbol{\Phi}(d_1) - Xe^{-rT} \cdot \boldsymbol{\Phi}(d_2)$$

- This model makes many assumptions (especially of *volatility*) and often mismatches empirical findings.
- We use deep learning to construct models that try to price options using historical data.
- With MLP and LSTM architectures to we are able to significantly outperform the Black-Scholes model.

Data

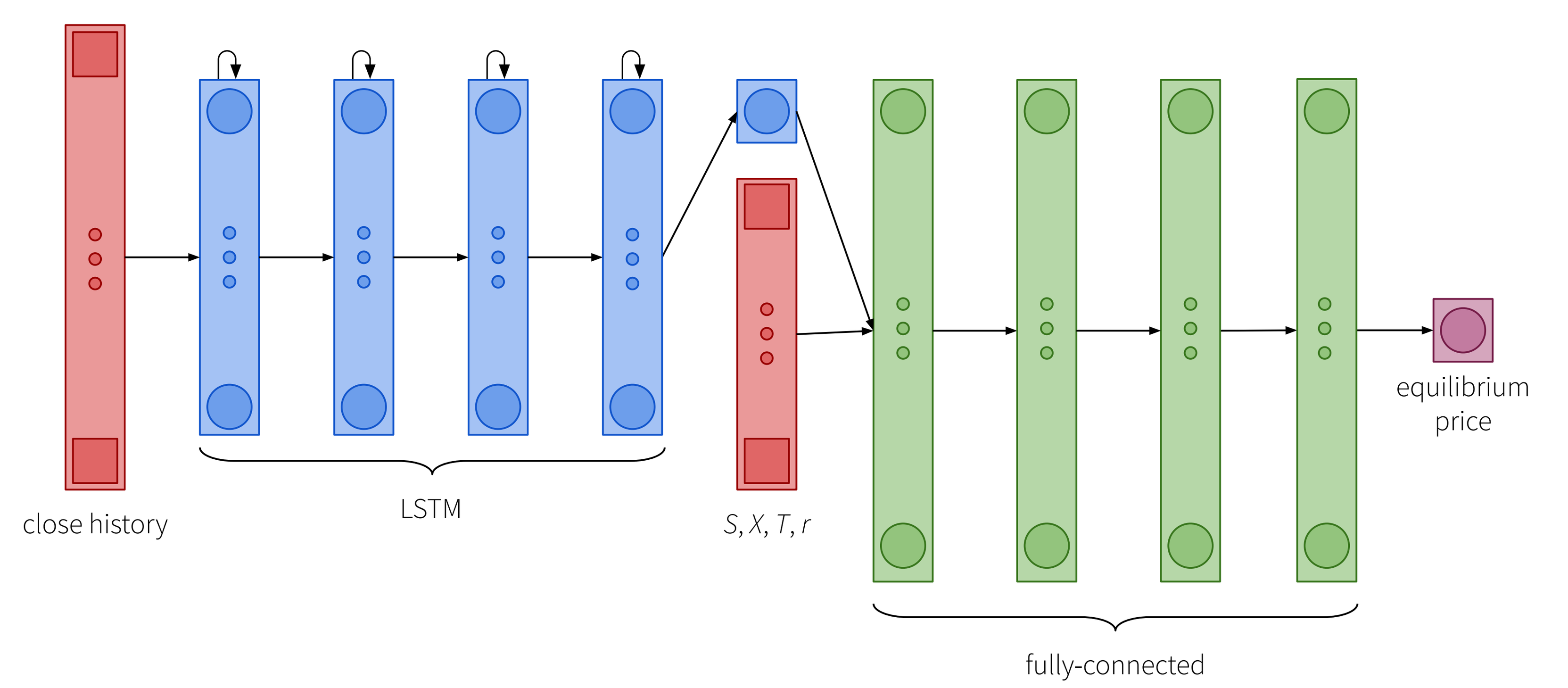
- We obtained 20 years of options and security prices for the S&P 500 from Wharton Research Data Services, for a total of 12,268,772 training examples of roughly half calls and half puts.
- The datasets provides us with contract terms [2]—namely call/put, strike price, date, etc., as well as corresponding securities prices [3].
- We also obtained daily treasury yields from the US Treasury Resource Center.
- Our options price dataset contains bid and ask prices for each contract, which we averaged for equilibrium price for MLP1 and LSTM, or used as labels directly for MLP2.

Features

- Out of 5 features, we had 4 raw features (days until expiry, strike price, risk-free rate, and underlying security price) and 1 derived feature, volatility.
- From the bid and ask prices, we also computed the equilibrium price which we use as an alternative label.
- We used a 98-1-1 train/dev/test split.

Models

- For our problem, we explore three network architectures: two multilayer perceptrons and one LSTM.
- MLP1
  - 4-layer NN with three 400-neuron hidden layers and one output layer with one neuron
  - Uses 20-day historical volatility as an additional input
  - Uses equilibrium price as the label
- MLP2
  - Same architecture as MLP1, but uses multitask learning to predict both bid and ask prices.
- LSTM
  - Feeds 20 timesteps of closing history through LSTM layers, and merges LSTM output with rest of features into fully-connected layers to predict equilibrium price



Results

- All three models outperform Black-Scholes, with MLP2 performing the best.

	Model	train-MSE	MSE	Bias	AAPE	MAPE	PE5	PE10	PE20
Call	BS	322.95	321.37	-0.05	78.79	4.81	50.52	59.33	67.43
	MLP1	23.71	24.00	0.01	24.49	2.12	61.04	68.39	74.33
	MLP2	7.70	15.21	0.09	23.45	1.73	63.03	70.10	75.54
	LSTM	30.61	30.97	0.13	26.58	2.33	58.94	66.35	72.42
Put	BS	543.48	533.25	97.37	68.00	97.46	12.87	18.22	23.58
	MLP1	15.65	15.66	5.03	43.73	18.48	30.46	40.51	51.13
	MLP2	2.03	8.84	3.85	39.59	14.32	33.74	44.25	55.01
	LSTM	22.81	23.15	6.01	48.32	26.05	27.45	36.24	46.17

Discussion

- Our models performs significantly better than the Black-Scholes model, with MLP2 performing the best.
  - Multitask learning is shown to be very effective in predicting bid/ask prices because they are similar tasks
- Our LSTM did not perform as well as either MLP.
  - We believe that we can do better on this task by performing a more thorough hyperparameter search (e.g. number of timesteps)
- Instead of making assumptions about financial mechanics as in the Black-Scholes model, our deep learning approach learns only from historical data, and seems to be a very promising way to forecast options prices.

Future Work

- If given more time, we would like to isolate characteristics such as time until expiry, etc. and conduct deeper error analyses to determine if our models perform differently on contracts with different characteristics.
- Given more time to train models, we would like to refine our LSTM model by increasing the number of timesteps so that we may better predict volatility.
- We would also like to train models on the reverse problem of finding the volatility implied by a given option price.
- Additionally, our findings can be applied toward pricing *exotic options*, e.g. binary or Asian options.

References

1. F. Black and M. Scholes. The pricing of options and corporate liabilities. *Journal of Political Economy*, 81(3):637–654, 1973.
2. Wharton Research Data Services. Optionmetrics - option prices. wrds.wharton.upenn.edu, 2018. Accessed: 2019-10-27.
3. Wharton Research Data Services. Optionmetrics - securities prices. wrds.wharton.upenn.edu, 2018. Accessed: 2019-10-27.