

Category 2 Number Properties

1. If x is an even integer and y is an odd integer, which of the following CANNOT be true?

- (A) x^y is an even integer.
- (B) y^x is an odd integer.
- (C) x is a multiple of y .
- (D) y is a multiple of x .
- (E) xy is an even integer.

2. If x is an even integer, which of the following is an odd integer?

- (A) $3x + 2$
- (B) $7x$
- (C) $8x + 5$
- (D) x^2
- (E) x^3

3. If x is a positive odd integer and y is a negative even integer, which of the following could be a negative odd integer?

- (A) y^x
- (B) xy
- (C) $x - y$
- (D) $x^2 + y$
- (E) $x + y^2$

4. An integer n that is greater than 1 is said to be “prime-saturated” if it has no prime factor greater than or equal to \sqrt{n} . Which of the following integers is prime-saturated?
- (A) 6
 - (B) 35
 - (C) 46
 - (D) 66
 - (E) 75
5. If a, b , and c are three consecutive odd integers such that $10 < a < b < c < 20$ and if b and c are prime numbers, what is the value of $a + b$?
- (A) 24
 - (B) 28
 - (C) 30
 - (D) 32
 - (E) 36
6. If n is a positive integer, which of the following could be a prime number?
- (A) $6n$
 - (B) $6n+1$
 - (C) $6n+2$
 - (D) $6n+3$
 - (E) $6n+4$

7. What is the least common multiple of 3, 4, 5, and 8?

- (A) 480
- (B) 240
- (C) 120
- (D) 105
- (E) 60

8. Which of the following is NOT a factor of 252?

- (A) 2
- (B) 3
- (C) 6
- (D) 7
- (E) 8

9. There are 125 chips on a table. If as many of the chips as possible are to be arranged into an equal number of 3-chip and 4-chip stacks and the remaining chips are to be removed, how many of the chips are to be removed?

- (A) One
- (B) Two
- (C) Five
- (D) Six
- (E) Seven

$$x = 0.9$$

$$y = \frac{1}{0.9}$$

$$z = (0.9)^2$$

10. The values of x , y , and z are shown above. Which of the following gives these numbers in order from least to greatest?

- (A) x, y, z
- (B) x, z, y
- (C) y, z, x
- (D) z, y, x
- (E) z, x, y

11. If x and y are positive integers, which of the following is NOT necessarily an integer?

- (A) $x + y$
- (B) $x - y$
- (C) $\frac{x}{y}$
- (D) xy
- (E) x^y

12. For any number x , \boxed{x} denotes the least non-negative number y such that $x + y$ is an integer. What is the value of $8.4 - \boxed{8.4}$?

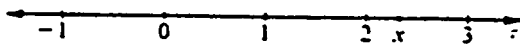
- (A) -0.4
- (B) 0
- (C) 0.6
- (D) 7.8
- (E) 8.0

13. What is the least odd integer, greater than 1, that is both the square of an integer and the cube of an integer?

(A) 9 (B) 27 (C) 81 (D) 243 (E) 729

14. What is the least possible product of 4 different integers, each of which has a value between -5 and 10 , inclusive?

(A) -5040
(B) -3600
(C) -720
(D) -600
(E) -120



15. The number line above shows the position of a point that has coordinate x . Which of the following statements about x must be true?

I. $2 < x < 4$
II. $-x < -3$
III. $0 < 2x - 3 < 1$

(A) I only
(B) II only
(C) III only
(D) I and II
(E) I and III

16. Which of the following must be true?

I. If $a + b < a + c$ then $b < c$.

II. If $a^2b < a^2c$ then $b < c$.

III. If $b^2 < c^2$ then $b < c$.

(A) None

(B) I only

(C) II only

(D) I and II only

(E) I, II, and III

17. If x and y are two consecutive odd integers and $x + y = 2(x - y)^2$, what is the value of $x + y$?

(A) 2

(B) 4

(C) 8

(D) 12

(E) 16

18. Which of the following integers does NOT have a divisor greater than 1 that is the square of an integer?

(A) 75

(B) 42

(C) 32

(D) 25

(E) 12

19. When the integer n is divided by 6, the remainder is 3. Which of the following is *NOT* a multiple of 6?
- (A) $n - 3$
 - (B) $n + 3$
 - (C) $2n$
 - (D) $3n$
 - (E) $4n$
20. If the remainder is 7 when positive integer n is divided by 18, what is the remainder when n is divided by 6?
- (A) 0
 - (B) 1
 - (C) 2
 - (D) 3
 - (E) 4

<High Level Questions>

21. If the two-digit integers M and N are positive and have the same digits, but in reverse order, which of the following CANNOT be the sum of M and N ?
- (A) 181
 - (B) 165
 - (C) 121
 - (D) 99
 - (E) 44

22. If the product of two positive integers is 630, which of the following must be true?
- I. Both integers are even numbers.
 - II. At least one of the integers is a multiple of 3.
 - III. One of the integers is 10.
- (A) I only
(B) II only
(C) III only
(D) I and II
(E) II and III
23. For any numbers a and b , $a \cdot b = a + b - ab$. If $a \cdot b = 0$, which of the following CANNOT be a value of b ?
- (A) 2
(B) 1
(C) 0
(D) -1
(E) $-\frac{3}{2}$
24. When the integer k is divided by 12, the remainder is 3. Which of the following, when divided by 12, will have a remainder of 6?
- I. $2k$
 - II. $6k$
 - III. $4k + 6$
- (A) I only
(B) II only
(C) III only
(D) I and II only
(E) I, II, and III

25. What is the least number of digits (including repetitions) needed to express 10^{100} in decimal notation?
- (A) 4
 - (B) 100
 - (C) 101
 - (D) 1,000
 - (E) 1,001
26. What is the smallest positive integer n for which 324 is a factor of 6^n ?
- (A) 2
 - (B) 3
 - (C) 4
 - (D) 5
 - (E) 6
27. If n is an integer, which of the following CANNOT be a factor of $3n + 4$?
- (A) 4
 - (B) 5
 - (C) 6
 - (D) 7
 - (E) 8
28. If n and k are integers whose product is 400, which of the following statements must be true?
- (A) $n + k > 0$
 - (B) $n \neq k$
 - (C) Either n or k is a multiple of 10.
 - (D) If n is even, then k is odd.
 - (E) If n is odd, then k is even.

29. If a is a positive integer, and if the units' digit of a^2 is 9 and the units' digit of $(a+1)^2$ is

4, what is the units' digit of $(a+2)^2$?

- (A) 1 (B) 3 (C) 5 (D) 7 (E) 9

30. An "Armstrong number" is an n -digit number that is equal to the sum of the n th powers of its individual digits. For example, 153 is an Armstrong number because it has 3 digits and $1^3 + 5^3 + 3^3 = 153$. What is the digit k in the Armstrong number 1, $6k4$?

- (A) 2
(B) 3
(C) 4
(D) 5
(E) 6

31. If the sum of the first n positive integers is S , what is the sum of the first n positive even integers, in terms of S ?

- (A) $\frac{S}{2}$
(B) S
(C) $2S$
(D) $2S + 2$
(E) $4S$

32. The positive integers a, b, c , and d are such that $a > b > c$. If $a + c = b + d$, which of the following CANNOT be true?
- (A) $d > a$
 - (B) $d = b$
 - (C) $d > b$
 - (D) $d > c$
 - (E) $b > d$
33. If $[x]$ is the greatest integer less than or equal to x , what is the value of $[-1.6] + [3.4] + [2.7]$?
- (A) 3
 - (B) 4
 - (C) 5
 - (D) 6
 - (E) 7

STOP



Category 2 Number Properties

1. If x is an even integer and y is an odd integer, which of the following CANNOT be true?

- (A) x^y is an even integer.
- (B) y^x is an odd integer.
- (C) x is a multiple of y .
- (D) y is a multiple of x .
- (E) xy is an even integer.

(A) x^y ,

(B) y^x ,

(C)

(D) “ \times ”, “ \times ” . “ \times ”

##

summary of arithmetic



(D)

2. If x is an even integer, which of the following is an odd integer?

- (A) $3x + 2$
- (B) $7x$
- (C) $8x + 5$
- (D) x^2
- (E) x^3

“ $+$ ”, “ \times ” . x 가

가

$8x + 5 (\quad + \quad)$.



(C)

3. If x is a positive odd integer and y is a negative even integer, which of the following could be a negative odd integer?

- (A) y^x
 (B) xy
 (C) $x - y$
 (D) $x^2 + y$
 (E) $x + y^2$

x 가 , y 가

. (A) . (B) . (C) .

(D) $x^2 < |y|$, . (E) .



(D) .

4. An integer n that is greater than 1 is said to be “prime-saturated” if it has no prime factor greater than or equal to \sqrt{n} . Which of the following integers is prime-saturated?

- (A) 6
 (B) 35
 (C) 46
 (D) 66
 (E) 75

n \sqrt{n} \sqrt{n} prime-saturated
 , prime-saturated
 6 2 3 . 2 $\sqrt{6}$ 3 $\sqrt{6}$ prime-saturated
 saturated . 75 5 $\sqrt{75}$ prime-saturated
 .



(E) .

5. If a, b , and c are three consecutive odd integers such that $10 < a < b < c < 20$ and if b and c are prime numbers, what is the value of $a + b$?

- (A) 24
(B) 28
(C) 30
(D) 32
(E) 36

a, b, c 가 b 가 c $a + b$.
 $10 < a < b < c < 20$. (11,13,15),
 (13,15,17), (15,17,19) b, c 가 (15,17,19) . $a + b = 32$.
 (D) .

6. If n is a positive integer, which of the following could be a prime number?

- (A) $6n$
(B) $6n+1$
(C) $6n+2$
(D) $6n+3$
(E) $6n+4$

2 . (B), (D)가 가 (D)
 $6n+3=3(2n+1)$ 3 .
 (B) .

7. What is the least common multiple of 3, 4, 5, and 8?

- (A) 480
(B) 240
(C) 120
(D) 105
(E) 60

3,4,5,8 (the least common multiple) .
 가 !!!
 (C) .

8. Which of the following is NOT a factor of 252?

- (A) 2 (B) 3 (C) 6 (D) 7 (E) 8

252 가 . !!!

$$2^2 \times 3^2 \times 7 = 252$$

📖 (E) .

9. There are 125 chips on a table. If as many of the chips as possible are to be arranged into an equal number of 3-chip and 4-chip stacks and the remaining chips are to be removed, how many of the chips are to be removed?

- (A) One (B) Two (C) Five (D) Six (E) Seven

125 3 , 4 . 3 4
125

3 . 4 가 7
125 ÷ 7 .

📖 (D) .

$$x = 0.9$$

$$y = \frac{1}{0.9}$$

$$z = (0.9)^2$$

10. The values of x , y , and z are shown above. Which of the following gives these numbers in order from least to greatest?

- (A) x, y, z
(B) x, z, y
(C) y, z, x
(D) z, y, x
(E) z, x, y

$$x = 0.9 = \frac{9}{10}, \quad y = \frac{1}{0.9} = 9, \quad z = (0.9)^2 = \left(\frac{9}{10}\right)^2$$

📖 (E) .

11. If x and y are positive integers, which of the following is NOT necessarily an integer?


- (A) $x + y$
- (B) $x - y$
- (C) $\frac{x}{y}$
- (D) xy
- (E) x^y

 (C) $\frac{x}{y}$ 가 정수일 필요는 없다.

12. For any number x , \boxed{x} denotes the least non-negative number y such that $x + y$ is an integer. What is the value of $8.4 - \boxed{8.4}$?

- (A) -0.4
- (B) 0
- (C) 0.6
- (D) 7.8
- (E) 8.0

\boxed{x} 은 x 가 정수가 되도록 하는 가장 작은 음이 아닌 수 y 를 나타낸다. $8.4 - \boxed{8.4}$ 의 값을 구하시오. $\boxed{8.4}$ 가 0.6, 1.6, ...를 나타낸다.

 (D) 0.6

13. What is the least odd integer, greater than 1, that is both the square of an integer and the cube of an integer?

- (A) 9
- (B) 27
- (C) 81
- (D) 243
- (E) 729

(the square of an integer)

(the cube of an integer)

$$3^6 = 3^2 \times 3^2 \times 3^2 = 3^3 \times 3^3 = 729$$

 (E) 729

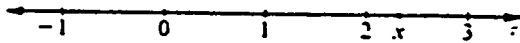
14. What is the least possible product of 4 different integers, each of which has a value between -5 and 10 , inclusive?

- (A) -5040
 (B) -3600
 (C) -720
 (D) -600
 (E) -120

-5 10 4 가
 $-5 \times 8 \times 9 \times 10 = -3,600$



(B) .



15. The number line above shows the position of a point that has coordinate x . Which of the following statements about x must be true?

- I. $2 < x < 4$
 II. $-x < -3$
 III. $0 < 2x - 3 < 1$
 (A) I only
 (B) II only
 (C) III only
 (D) I and II
 (E) I and III

x 가 x x
 2 3 . , $-x < -3 (x > 3)$ $0 < 2x - 3 < 1 (\frac{3}{2} < x < 2)$



(A) .

16. Which of the following must be true?

I. If $a + b < a + c$ then $b < c$.

II. If $a^2b < a^2c$ then $b < c$.

III. If $b^2 < c^2$ then $b < c$.

- (A) None
- (B) I only
- (C) II only
- (D) I and II only
- (E) I, II, and III

가

가



(D)

17. If x and y are two consecutive odd integers and $x + y = 2(x - y)^2$, what is the value of $x + y$?

- (A) 2
- (B) 4
- (C) 8
- (D) 12
- (E) 16

2

 $a - 1, a + 1$

$$x + y = 2(x - y)^2 \Rightarrow 2a = 2(-2)^2$$



(C)

18. Which of the following integers does NOT have a divisor greater than 1 that is the square of an integer?

- (A) 75
- (B) 42
- (C) 32
- (D) 25
- (E) 12

. 75 $25(=5^2)$, 32 $4(=2^2)$

, 25 25 , 12 $4(=2^2)$



(B)

19. When the integer n is divided by 6, the remainder is 3. Which of the following is *NOT* a multiple of 6?

- (A) $n - 3$
 (B) $n + 3$
 (C) $2n$
 (D) $3n$
 (E) $4n$

$$\begin{array}{l} n \div 6 \text{ 가 } 3 \text{ .} \\ n = 6a + 3 \end{array} \qquad \begin{array}{l} n = 6a + 3 \\ n \div 6 \end{array}$$

 (D) .

20. If the remainder is 7 when positive integer n is divided by 18, what is the remainder when n is divided by 6?

- (A) 0 (B) 1 (C) 2 (D) 3 (E) 4

$$\begin{array}{l} n \div 18 \text{ 가 } 7 \text{ .} \\ n = 18a + 7 \end{array} \qquad \begin{array}{l} n \\ 6 \end{array} \qquad \begin{array}{l} n = \frac{18a}{6} + \frac{7}{6} \\ \text{가 } 1 \end{array}$$

 (B) .

21. If the two-digit integers M and N are positive and have the same digits, but in reverse order, which of the following CANNOT be the sum of M and N ?

- (A) 181 (B) 165 (C) 121 (D) 99 (E) 44

$$\begin{array}{l} M \div 10 \text{ 가 } a \text{ , } N \div 10 \text{ 가 } b \text{ .} \\ M = 10a + b, N = 10b + a \Rightarrow M + N = 11(a + b) \end{array}$$

$$\begin{array}{l} M + N \div 11 \text{ 가 } 11 \text{ .} \\ \text{(A) 181 (E) 44} \end{array}$$

 (A) .

22. If the product of two positive integers is 630, which of the following must be true?

- I. Both integers are even numbers.
- II. At least one of the integers is a multiple of 3.
- III. One of the integers is 10.

- (A) I only
- (B) II only
- (C) III only
- (D) I and II
- (E) II and III

$$630 = 2 \times 3^2 \times 5 \times 7$$

2 I III 3×210 가

. II .



(B)

23. For any numbers a and b , $a \cdot b = a + b - ab$. If $a \cdot b = 0$, which of the following CANNOT be a value of b ?

- (A) 2 (B) 1 (C) 0 (D) -1 (E) $-\frac{3}{2}$

$$a \cdot b = a + b - ab = 0 \quad b \quad a - ab = -b \Rightarrow a = \frac{-a}{1-b}$$

$$1 - b \neq 0 \quad b = 1$$



(B)

24. When the integer k is divided by 12, the remainder is 3. Which of the following, when divided by 12, will have a remainder of 6?

- I. $2k$
- II. $6k$
- III. $4k + 6$

- (A) I only (B) II only (C) III only (D) I and II only (E) I, II, and III

$$k = 12Q + 3$$

- I. $2k = 2(12Q + 3) = 24Q + 6$, 가 6 !
 - II. $6k = 6 \times 12Q + 18$, 18 12 가 6 !
 - III. $4k + 6 = 4(12Q + 3) + 6 = 48Q + 18$, 18 12 가 6,
- !



(E)

25. What is the least number of digits (including repetitions) needed to express 10^{100} in decimal notation?

- (A) 4 (B) 100 (C) 101 (D) 1,000 (E) 1,001

$$10^{100} = 10^3 = 1000 \dots 10^{100} = 100 \dots 101 \text{ 가 } 10^1 = 2, 10^2 = 3$$

 (C)

26. What is the smallest positive integer n for which 324 is a factor of 6^n ?

- (A) 2
(B) 3
(C) 4
(D) 5
(E) 6

$$324 = 6^2 \times 3^2, \quad 6^n = 324 \text{ 가 } 6^n \geq 324, \quad n = 4, \quad 6^4 = 1296$$

 (C)

27. If n is an integer, which of the following CANNOT be a factor of $3n + 4$?

- (A) 4
(B) 5
(C) 6
(D) 7
(E) 8

$$3n + 4 = 3(n + 1) + 1, \quad n = 3, \quad 3n + 4 = 13 \text{ factor가 } 3, 6$$

 (C)

28. If n and k are integers whose product is 400, which of the following statements must be true?

- (A) $n + k > 0$
- (B) $n \neq k$
- (C) Either n or k is a multiple of 10.
- (D) If n is even, then k is odd.
- (E) If n is odd, then k is even.

$$nk = 400 = 2^4 5^2$$

- (A) $n = -2, k = -5$ $nk = 400$ $n + k < 0$. False!
- (C) $n = 25, k = 16$ n $k \nmid 10$ \nmid . False!
- (D) $n = 2, k = 200$. False!



(E)

29. If a is a positive integer, and if the units' digit of a^2 is 9 and the units' digit of $(a+1)^2$ is

4, what is the units' digit of $(a+2)^2$?

- (A) 1
- (B) 3
- (C) 5
- (D) 7
- (E) 9

a

$$a^2 = 9, (a+1)^2 = 4, (a+1)^2 = a^2 + 2a + 1 = 4 \quad a^2$$

$$\begin{array}{ccccccc} 9 & 1 & 0 & 2a & 4 & 4 \\ \text{가} & 2a & 4\text{가} & & & \end{array}$$

$$(a+2)^2 = (a+1)^2 + 2a + 3, (a+1)^2 = 4, 2a$$

$$4 \quad 3 \quad 1$$



(A)

30. An “Armstrong number” is an n -digit number that is equal to the sum of the n th powers of its individual digits. For example, 153 is an Armstrong number because it has 3 digits and $1^3 + 5^3 + 3^3 = 153$. What is the digit k in the Armstrong number 1, 6 k 4?


(A) 2 (B) 3 (C) 4 (D) 5 (E) 6

$$1, 6k4 = 1^4 + 6^4 + k^4 + 4^4 = 1553 + k^4, \quad \text{“Armstrong number”}$$

k^4 1553

3 4 . k^4 1 . (A) (E) 4

1 3 .

 (B)

31. If the sum of the first n positive integers is S , what is the sum of the first n positive even integers, in terms of S ?

(A) $\frac{S}{2}$ (B) S (C) $2S$ (D) $2S + 2$ (E) $4S$

$$1 + 2 + 3 + \dots + n = \frac{n(n+1)}{2} = S$$

$$2 + 4 + 6 + \dots + 2n = 2(1 + 2 + 3 + \dots + n) = 2S$$

n S ,

1 n $\frac{n(n+1)}{2} = S$ $n(n+1) = 2S$ 가 .

 (C)

32. The positive integers a, b, c , and d are such that $a > b > c$. If $a + c = b + d$, which of the following CANNOT be true?

(A) $d > a$
 (B) $d = b$
 (C) $d > b$
 (D) $d > c$
 (E) $b > d$

$$a > b > c \quad a + c = b + d, \quad a > b \quad a + c = b + d \text{ 가 } d > a \text{ 가}$$

$$d > a \quad b > c \quad a + c = b + d \text{ 가}$$

.

 (A)

33. If $[x]$ is the greatest integer less than or equal to x , what is the value of $[-1.6] + [3.4] + [2.7]$?
- (A) 3
(B) 4
(C) 5
(D) 6
(E) 7

$[x]$ is the greatest integer less than or equal to x . For example, $[-1.6] = -2$, $[3.4] = 3$, $[2.7] = 2$.

⏏ (A)

< Summary of Arithmetic >

Divisibility Tests for 2, 3, 5, and 10

A number is divisible by

- 2, if its last digit is even – 0, 2, 4, 6, or 8;
- 3, if the sum of its digits is a number divisible by 3
- 5, if its last digit is 0 or 5; and
- 10, if its last digit is 0

Rules for Odds and Evens

<i>odd</i>	+	<i>odd</i>	=	<i>even</i>	<i>odd</i>	−	<i>odd</i>	=	<i>even</i>
<i>even</i>	+	<i>even</i>	=	<i>even</i>	<i>even</i>	−	<i>even</i>	=	<i>even</i>
<i>odd</i>	+	<i>even</i>	=	<i>odd</i>	<i>odd</i>	−	<i>even</i>	=	<i>odd</i>

Factors()

The factors of a number are the positive integers that evenly divide that number.

Ex: 36 has factors 1, 2, 3, 4, 6, 9, 12, 18, 36

$36 = 2^2 \times 3^2$ (exponent 2, 1, 2, 1)

$\Rightarrow (2+1)(2+1) = 9$ (factors 3, 3)

(the greatest common divisor) :

, 가 가 가 .

(the lowest common divisor):

, 가 가 가 .

Ex. : 180, 420 (GCD) (LCD) ?

$180 = 2^2 \cdot 3^2 \cdot 5$, $420 = 2^2 \cdot 3 \cdot 5 \cdot 7$

$GCD = 2^2 \cdot 3 \cdot 5 = 60$, $LCD = 2^2 \cdot 3^2 \cdot 5 \cdot 7 = 1260$

Decimals()

P 1) : 0 가 $(\frac{3}{5}=0.6)$

2) :

$(0.123123123 \dots = 0.\dot{1}2\dot{3})$

P 3) : 0 가

ex) $1/6 = 0.1666 \dots$

4) : 가 2 5

ex.) $\frac{12}{80} = \frac{3}{20} = \frac{3}{2^2 \times 5} = 0.15$