Tutorial - 3. (solution).

1.

$z<0$	$z=0$	$z>0$.
$\mu_0, \epsilon_1, \sigma_1 = 0$ (η_1)		$\mu_0, \epsilon_2, \sigma_2 = 0$. ($\eta_2$)
$\lambda_1 = 5$ cm		$\lambda_2 = 3$ cm.

$\vec{\hat{a}_z}$.

$2\pi f = 3\times10^{10}$ rad/s.

$$\beta = \omega\sqrt{\mu\epsilon}$$

$$\Rightarrow \frac{2\pi}{\lambda_1} = \frac{\omega\sqrt{\epsilon_{r1}}}{c} \Rightarrow \frac{2\pi}{5} = \frac{3\times10^{10}\sqrt{\epsilon_{r1}}}{3\times10^{10}}.$$

$$\Rightarrow \sqrt{\epsilon_{r1}} = 2\pi/5$$

similarly $\sqrt{\epsilon_{r2}} = 2\pi/3$.

(a) $$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{\sqrt{\epsilon_{r1}} - \sqrt{\epsilon_{r2}}}{\sqrt{\epsilon_{r1}} + \sqrt{\epsilon_{r2}}}.$$

$$= \frac{2\pi/5 - 2\pi/3}{2\pi/5 + 2\pi/3}$$

$$\Rightarrow \Gamma = -2/8 = -1/4 = -0.25$$

(a) $|\Gamma|^2 = 0.0625$

(b) $1 - |\Gamma|^2 = 0.9375$

(c) $SWR = \frac{1+|\Gamma|}{1-|\Gamma|} = 1.67$.

2. The instantaneous electric field intensity is ($\alpha = 0$ for freespace).

$$E_1(z,t) = \hat{x}\, 2E_{i1} \sin(\beta_1 z) \cdot \sin(\omega t) \quad V/m.$$

$f = 1$ GHz.

phase constant $\beta = \omega\sqrt{\mu_0 \epsilon_0}$

$$= \frac{2\pi \times 10^9}{3 \times 10^8} = \frac{20\pi}{3} \text{ rad/m.}$$

$$E_1(z,t) = \hat{x}\, 2 \cdot \sin\left(\frac{20\pi}{3}z\right) \cdot \sin(2\pi \times 10^9 t) \quad V/m.$$

$$H_1(z,t) = \hat{y}\, \frac{2}{120\pi} \cos\left(\frac{20\pi}{3}z\right) \cdot \cos(2\pi \times 10^9 t) \quad A/m.$$

E and H fields are in time quadrature and orthogonal in their direction in space.

E-field zero at

$$\frac{20\pi}{3} z = n\pi, \quad n = 0, 1, 2, \dots$$

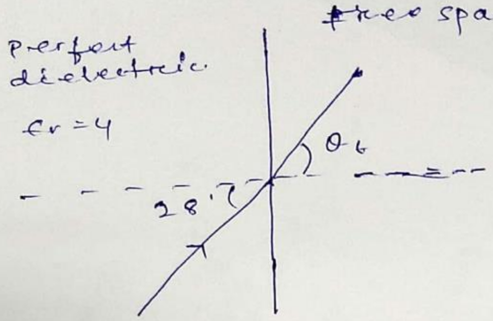
$$\Rightarrow z = \frac{3n}{20}.$$ [from the interface, towards opposite to the perfect conductor].

H-field zero at:

$$\frac{20\pi}{3} z = n\pi/2, \quad n = 0, 1, 2, \dots$$

$$\Rightarrow z = \frac{3n}{40}$$ [from the interface, towards opposite to the perfect conductor].

3.



Snell's law $\quad n_1 \sin\theta_i = n_2 \sin\theta_t$

$\Rightarrow \quad 2\sin 28' = \sin\theta_t$

$\Rightarrow \quad \theta_t = 69.87'$

(a) $$\Gamma_{\parallel} = \frac{\eta_2 \cos\theta_t - \eta_1 \cos\theta_i}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i}$$

$$= \frac{0.344 - 0.441}{0.344 + 0.441} = -0.12$$

$$T_{\parallel} = \frac{2\eta_2 \cos\theta_i}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i}$$

$$= 2.249$$

(b) $\theta_c = 30'$

(c) No Brewster's angle [$\because$ There is no concept of interface in a single medium].

4. $\epsilon_1 = \epsilon_0$, $\epsilon_2 = 1.44\,\epsilon_0$.

$\mu_1 = \mu_2 = \mu_0$

$$\eta_1 = \sqrt{\frac{\mu_1}{\epsilon_1}} = \sqrt{\frac{\mu_0}{\epsilon_0}}$$

$$\eta_2 = \sqrt{\frac{\mu_2}{\epsilon_2}} = \sqrt{\frac{\mu_0}{1.44\epsilon_1}} = \frac{1}{1.2}\sqrt{\frac{\mu_0}{\epsilon_0}} = \frac{\eta_1}{1.2}$$

(a) $$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{\frac{1}{1.2} - 1}{\frac{1}{1.2} + 1} = -0.0909$$

(b) $$T = \frac{2\eta_2}{\eta_1 + \eta_2} = \frac{2\,(1/1.2)}{2.2/1.2} = 0.909$$

(c) Incident power $= \dfrac{|E_0|^2}{2\eta_1} = \dfrac{(1\times10^{-3})^2}{2\eta_1}$

$= 1.327$ nW/m^2

transmitted power $= (0.909)^2 \left(\dfrac{\eta_1}{\eta_2}\right)\dfrac{|E_0|^2}{2\eta_1}$

or $(1-|\Gamma|^2)\dfrac{|E_0|^2}{2\eta_1} = 1.315$ nW/m^2

(d) E-field in media 1

$$E_i + E_r = E_0 e^{-j\beta z} + \Gamma E_0 e^{j\beta z}$$

$$= E_0 e^{-j\beta z}\left[1 + \Gamma e^{2j\beta z}\right]$$

H-field in media 1, $H_i + H_r$

$$= \cancel{+} \frac{E_0}{\eta_1} e^{-j\beta z} - \Gamma \frac{E_0}{\eta_1} e^{j\beta z}$$

$$= \frac{E_0}{\eta_1} e^{-j\beta z}\left[1 - \Gamma e^{+j2\beta z}\right].$$

5. $\lambda_0 = \frac{c}{f} = \frac{3\times10^8}{100\times10^6} = 3m.$

power density in air

$$\frac{|E_0|^2}{2\eta_0} = 10 \Rightarrow E_0 = 86.833 \text{ V/m}.$$

$$\eta_2 = \frac{\eta_0}{\sqrt{\epsilon_r}} = \frac{377}{\sqrt{25}} = 75.4\,\Omega.$$

$$\beta_0 = \frac{2\pi}{\lambda} = 2\pi/3.$$

$$\beta_2 = \beta_0\sqrt{\epsilon_r} = 2\pi/3\,(5) = 10\pi/3.$$

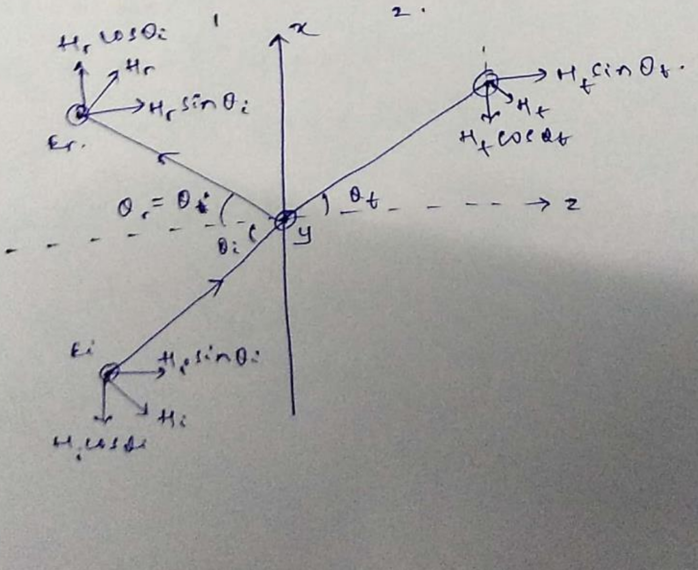
Snell's law:

$$\sin\theta_t = \frac{\sin\theta_i}{\sqrt{\epsilon_r}} = \frac{\sin 45}{\sqrt{25}} = \frac{1}{5\sqrt{2}}$$

$$\theta_t = 8.13, \quad \cos\theta_t = 0.9899$$

$$\Gamma_\perp = \frac{\eta_2\cos\theta_i - \eta_1\cos\theta_t}{\eta_2\cos\theta_i + \eta_1\cos\theta_t} = -0.75$$

$$\tau_\perp = \frac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_i + \eta_1\cos\theta_t} = 0.25$$



$$E_i = E_0 e^{-j(k\cdot r)}\,\hat{a}_y$$

$$= E_0 e^{-j\beta(x\cos\theta_i + z\sin\theta_i)}\,\hat{a}_y$$

$$= E_0 e^{-j\left(\frac{2\pi}{3\sqrt{2}}\right)(x+z)}\,\hat{a}_y$$

$$= \cancel{122.8}\ 86.833\, e^{-j\left(\frac{2\pi}{3\sqrt{2}}\right)(x-z)}$$

$$E_r = -0.75\,(86.833)\, e^{-j(10\pi/3)\left(\frac{x}{5\sqrt{2}} + 0.98\,997\right)}\,\hat{a}_y.$$

$$E_t = 0.25\,(86.833)\, e$$

Electric field at $z = 1m$.

$$E_1 = E_i + E_r$$

$$= \left(86.833\, e^{-j2\pi/3\sqrt{2}} - 65.124\, e^{j2\pi/(3\sqrt{2})}\right) e^{-j2\pi/3\sqrt{2}\,x}.$$

$$|E_1| = \cancel{108.54}\ 151.356\ V/m.$$

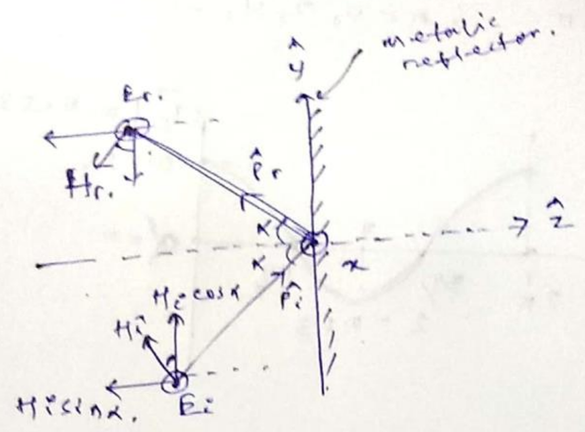
similarly for magnetic field.

6. Brewster's angle. $\theta = \tan^{-1}\sqrt{\frac{\epsilon_0}{81\epsilon_0}}$.

$$\theta_{iB} = 6.34^\circ$$

(a) $\theta_{iB} = \tan^{-1}\sqrt{\frac{81\epsilon_0}{\epsilon_0}} = \tan^{-1}(9) = 83.66.$

7.



$\vec{J}_s = \hat{n} \times \vec{H}_{t_1}$

$\vec{H}_{t_1} = \vec{H}_{i_t} + \vec{H}_{r_t}$

$\vec{H}_i = (\hat{y}\cos\alpha - \hat{z}\sin\alpha)\frac{E_{io}}{\eta} e^{-j\beta(y\sin\alpha + z\cos\alpha)}$ A/m.

$\vec{H}_r = (-\hat{y}\cos\alpha - \hat{z}\sin\alpha)\frac{E_{io}}{\eta} e^{-j\beta(y\sin\alpha - z\cos\alpha)}$ A/m.

$\vec{H}_{t_1} = \hat{y}\cos\alpha \frac{E_{io}}{\eta} e^{-j\beta y\sin\alpha} + (-\hat{y})\cos\alpha\, e^{-j\beta y\sin\alpha}$ A/m.

$\hat{n} = \hat{z}$.

(unit out ward normal to the interface.

$\vec{J}_s = \hat{z} \times \left[\hat{y}\cos\alpha \frac{E_{io}}{\eta} e^{-j\beta\sin\alpha\cdot y}(1-\rho_1)^{-1}\right]$

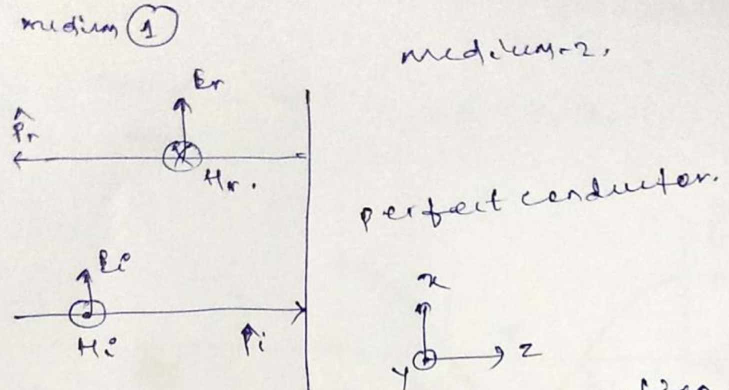
$= 2\hat{x}\frac{E_{io}}{\eta} e^{-j\beta\sin\alpha\cdot y}\cos\alpha$

$= 2\hat{x}\frac{100}{120\pi} e^{-j\frac{2\pi\times100\times10^9}{3\times10^8}\sin\alpha\cdot y} \times \cos\alpha$

$= \hat{x}\, 0.53\cos\alpha \cdot e^{-j2094.4y\sin\alpha}$ A/m.

$|\vec{J}_s| = 0.53\cos\alpha$ A/m.

A11.



Derivation of E-field for question No. 2.

$$\vec{E}_1 = \hat{x}\left(E_{i_1} e^{-j\beta z} + E_{r_1} e^{j\beta z}\right)$$

$$= E_{i_1}\left(e^{-j\beta z} + \Gamma e^{j\beta z}\right)\hat{x}$$

$$= E_{i_1}\left[e^{-j\beta z} + \Gamma e^{j\beta z} + \Gamma e^{-j\beta z} - \Gamma e^{-j\beta z}\right]\hat{x}$$

$$= E_{i_1}\left[e^{-j\beta z}(1+\Gamma) + \Gamma\left(e^{j\beta z} - e^{-j\beta z}\right)\right]\hat{x}$$

$$= E_{i_1}\left[e^{-j\beta z}\cancel{(1+\Gamma)}^{\,0} + \cancel{\Gamma}^{\,-1} 2j\sin(\beta z)\right]\hat{x}$$

$$= -\hat{x} E_{i_1} 2j \sin(\beta z)$$

In time harmonic form.

$$E(z,t) = Re\left\{E_1 e^{j\omega t}\right\}.$$

$$= Re\left[-\hat{x} E_{i_1} 2j \sin(\beta z) \cdot e^{j\omega t}\right]$$

$$= Re\left[-\hat{x} E_{i_1} 2j \sin(\beta z) \cdot \sin(\omega t)\right].$$

$$= \hat{x}\, 2j E_{i_1} \sin(\beta z) \cdot \sin(\omega t).$$