Problem 2.15

Derive the relationship of Eq. (2.142) between the two cross-correlation factors $R_{xy}(\tau)$ and $R_{yx}(\tau)$.

Solution

By definition

$$R_{yx}(\tau) = \int_{-\infty}^{\infty} y(t) x^{*}(t-\tau) dt$$

Complex conjugate both sides of the equation:

$$R_{yx}^{*}(\tau) = \int_{-\infty}^{\infty} x(t-\tau)y^{*}(t)dt$$

Next, replace τ with $-\tau$:

$$R_{yx}^*(-\tau) = \int_{-\infty}^{\infty} x(t+\tau)y^*(t)dt$$

Finally, replace $t + \tau$ with t, which is equivalent to replacing t with $t - \tau$; we therefore (since dt remains unchanged)

$$R_{yx}^*(-\tau) = \int_{-\infty}^{\infty} x(t)y^*(t-\tau)dt = R_{xy}(\tau)$$