Problem 4.17

Consider first the message signal

$$m_1(t) = \begin{cases} a_1 t + a_0, & t \geq 0 \\ 0, & t = 0 \end{cases}$$

applied to a frequency modulator. The signal produced by this modulator is defined by

$$\begin{aligned} s_1(t) &= A_c \cos\left[2\pi f_c t + 2\pi k_f \int_0^t m_1(\tau) d\tau\right] \\ &= A_c \cos\left[2\pi f_c t + 2\pi k_f \int_0^t (a_1 t + a_0) d\tau\right] \\ &= A_c \cos\left[2\pi f_c t + 2\pi k_f \left(\frac{1}{2} a_1 t^2 + a_0 t + C\right)\right], \qquad t \geq 0 \qquad (1) \end{aligned}$$

where C is the constant of integration.

Consider next the message signal

$$m_2(t) = \begin{cases} b_2 t^2 + b_1 t + b_0 & t \geq 0 \\ 0, & t = 0 \end{cases}$$

applied to a phase modulator. The signal produced by this second modulator is defined by

$$\begin{aligned} s_2(t) &= A_c \cos[2\pi f_c t + k_p m_2(t)] \\ &= A_c \cos[2\pi f_c t + k_p (b_2 t^2 + b_1 t + b_0)], \qquad t \geq 0 \qquad (2) \end{aligned}$$

For the FM signal $s_1(t)$ of Eq. (1) and the PM signal of Eq. (2) to be exactly equal for $t \geq 0$, we require that the following conditions be satisfied:

(i) $\pi k_f a_1 = k_p b_2$

(ii) $2\pi k_f a_0 = k_p b_1$

(iii) $2\pi k_f C = k_p b_0$