Problem 2.39

We will determine the autocorrelation function of the signal $g(t)$ depicted in Fig. 2.45 by proceeding on a segment-by-segment basis:

1. The maximum value of $R_g(\tau)$ occurs at $\tau = 0$, for then $g(t)$ and $g(t\text{-}\tau)$ overlap exactly, yielding

$$R_g(0) = 3(A^2)(T) = 3A^2T$$

2. For $0 < \tau < (T/2)$, we have the picture depicted in Fig. 1. From this figure, we obtain

$$\begin{aligned} R_g(\tau) &= \int_{-(3T/2)+\tau}^{-T/2}(+A)(+A)dt + \int_{-T/2}^{-(T/2)+\tau}(+A)(\text{-}A)dt \\ &\quad + \int_{-(T/2)+\tau}^{T/2}(+A)(+A)dt + \int_{T/2}^{(T/2)+\tau}(\text{-}A)(+A)dt \\ &\quad + \int_{(T/2)+\tau}^{3T/2}(\text{-}A)(\text{-}A)dt \\ &= A^2(T-\tau) - A^2\tau + A^2(T-\tau) - A^2\tau + A^2(T-\tau) \\ &= A^2(3T-5\tau), \qquad 0 < |\tau| < (T/2) \end{aligned}$$

where the use of $|\tau|$ is invoked in light of the symmetric property of the autocorrelation function.

3. Next, for $(T/2) < \tau < T$, we have the picture depicted in Fig. 2, from which we obtain

$$\begin{aligned} R_g(\tau) &= \int_{-(3T/2)+\tau}^{-T/2}(\text{-}A)(\text{-}A)dt + \int_{-T/2}^{-(T/2)+\tau}(+A)(\text{-}A)dt \\ &\quad + \int_{-(T/2)+\tau}^{T/2}(+A)(+A)dt + \int_{T/2}^{(T/2)+\tau}(\text{-}A)(+A)dt \\ &\quad + \int_{(T/2+\tau)}^{3T/2}(\text{-}A)(\text{-}A)dt \\ &= A^2(T-\tau) - A^2\tau + A^2(T-\tau) - A^2\tau + A^2(T-\tau) \\ &= A^2(3T-5\tau), \qquad \frac{T}{2} < |\tau| < T \end{aligned}$$

4. Next, for $T < \tau < 3T/2$, we have the picture depicted in Fig. 3, from which we obtain

$$\begin{aligned} R_g(\tau) &= \int_{-(3T/2)+\tau}^{T/2}(+A)(\text{-}A)dt + \int_{T/2}^{-(T/2)+\tau}(\text{-}A)(\text{-}A)dt \\ &\quad + \int_{-(T/2)+\tau}^{3T/2}(\text{-}A)(+A)dt \\ &= -A^2(2T-\tau) + A^2(-T+\tau) - A^2(2T-\tau) \\ &= A^2(-5T+3\tau) \qquad \text{for } T < |\tau| < 3T/2 \end{aligned}$$

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5. For $(3T/2) < \tau < 2T$ we have the picture depicted in Fig. 4, from which we obtain

$$R_g(\tau) = \int_{-(3T/2)+\tau}^{T/2} (+A)(-A)dt + \int_{T/2}^{-(T/2)+\tau} (-A)(-A)dt + \int_{-(T/2)+\tau}^{3T/2} (-A)(+A)dt$$

$$= -A^2(2T-\tau) + A^2(-T+\tau) - A^2(2T-\tau)$$

$$= A^2(-5T+3\tau) \qquad \text{for } (3T/2) + |\tau| < 2T$$

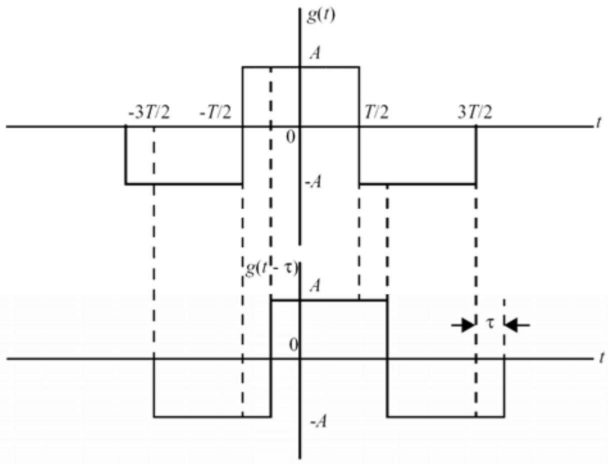

Figure 1: $-(T/2) < \tau < (T/2)$

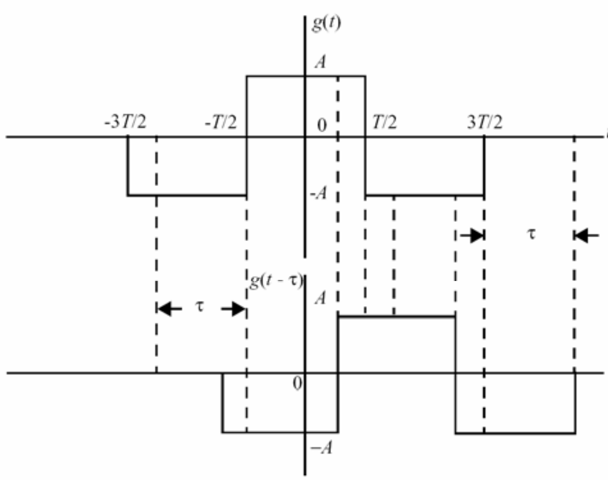

Figure 2: $(T/2) < \tau < (3T/2)$

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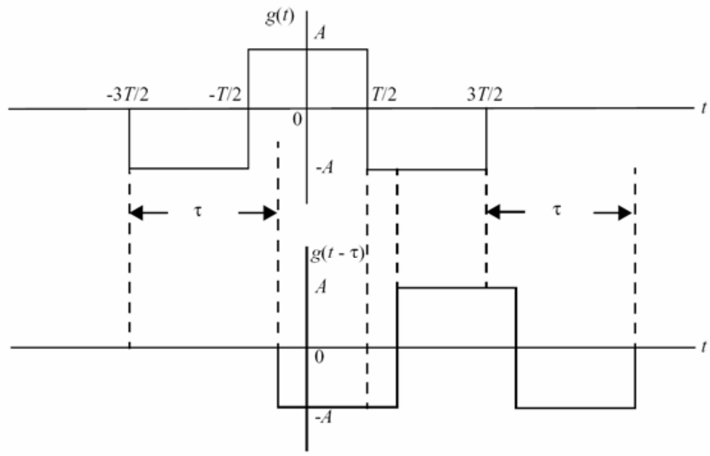

Figure 3: $T < \tau < (3T/2)$

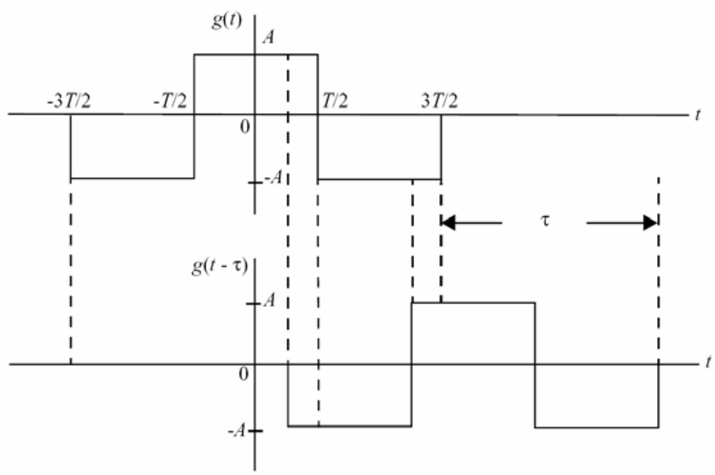

Figure 4: $(3T/2) < \tau < 2T$

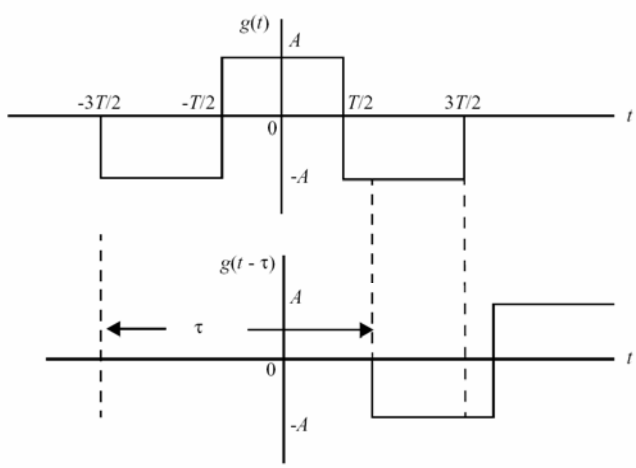

Figure 5: $2T < \tau < 3T$

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Problem 2-39 continued

7. Finally, for $|\tau| > 3T$, we find that $R_g(\tau) = 0$.

Putting all these pieces together, we get the autocorrelation function $R_g(\tau)$ plotted in Fig. 6, which is symmetric about the origin $\tau = 0$.

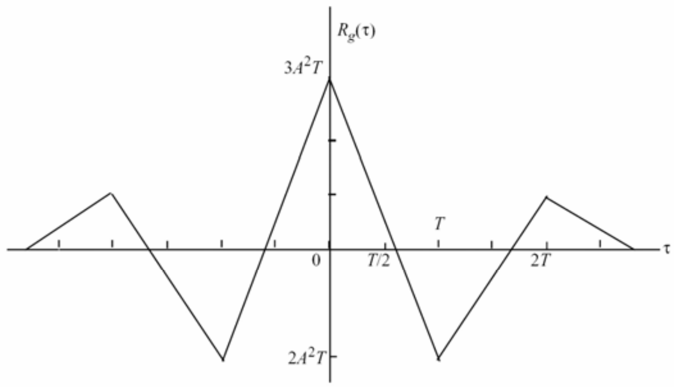

Figure 6: Plot of the autocorrelation function $R_g(t\tau)$