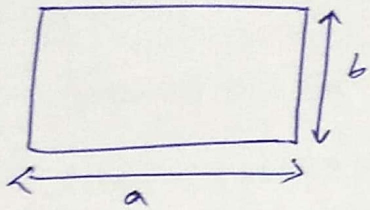
① ⓐ



for WR-90 waveguide

$a = 2.286$ cm.

$b = 1.016$ cm.

cut-off frequency $f_c = \frac{1}{2\sqrt{\mu\epsilon}} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$.

Dominant mode in WR-90 (Rectangular waveguide) is TE_{10}.

$m = 1,\ n = 0$

$$f_c = \frac{1}{2\sqrt{\mu\epsilon}}\left(\frac{1}{a}\right) = \frac{1}{2} \times \frac{3\times10^8}{3} \times \frac{100}{2.286}$$

$$= 2.187\,GHz$$

ⓑ Phase velocity

$$v_p = v_0 \Big/ \sqrt{1 - \left(\frac{f_c}{f}\right)^2}$$

$$\sqrt{1 - \left(\frac{f_c}{f}\right)^2} = \sqrt{1 - \left(\frac{2.187}{3}\right)^2}$$

$$= 0.6845$$

group velocity

$$\cancel{v_g = v_0 / \sqrt{1 - f'}}$$

$$v_g = v_0 \sqrt{1 - (f_c/f)^2}$$

$$v_0 = \frac{1}{\sqrt{\mu\epsilon}} = \frac{3\times10^8}{3} = 1\times10^8\ m$$

$$v_p = \frac{1\times10^8}{0.6845} = 1.46\times10^8\ m/s.$$

$$v_g = 0.68\times10^8\ m/s.$$

ⓒ
$$\lambda_g = \frac{\lambda}{\sqrt{1 - \left(\frac{f_c}{f}\right)^2}}$$

$$\lambda = \frac{3\times10^{10}}{3\times10^9\times3} = \frac{10}{3}.$$

$$\lambda_g = \frac{10}{3} \times \frac{1}{0.6845} = 4.869\ cm.$$

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(d) $Z_{TE_{mn}} = \dfrac{\eta}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$ $\eta = \sqrt{\dfrac{\mu}{\varepsilon}} = \dfrac{120\pi}{3} = 40\pi$

$Z_{TE_{10}} = \dfrac{40\pi}{0.6845} = 58.43\pi\ \Omega$

(e)

TE_{10}: $f_c = 2.187$ GHz

TE_{01}: $f_c = 4.921$ GHz.

TE_{11}/TM_{11}: $f_c = 5.385$ GHz.

TE_{20}: $f_c = 4.374$ GHz.

TE_{02}: $f_c = 9.842$ GHz.

TE_{21}/TM_{21}: $f_c = 6.584$ GHz.

TE_{12}/TM_{12}: $f_c = 10.08$ GHz.

TE_{22}/TM_{22}: $f_c = 10.77$ GHz.

TE_{30}: $f_c = 6.4761$

TE_{03}: $f_c = 14.763$ GHz.

$f = 1$ GHz.

Modes	TE_{10}	TE_{20}	TE_{01}	TE_{11}/TM_{11}	TE_{30}	TE_{21}/TM_{21}	— — —
f_c (GHz)	2.18	4.37	4.92	5.38	6.47	6.58	— — —

$f = 3$ GHz. (between 2.18 and 4.37)

$f = 6$ GHz. (between 5.38 and 6.47)

If $f = 1$ GHz $\rightarrow$ no propagation.

$f = 3$ GHz $\rightarrow$ TE_{10} mode.

$f = 6$ GHz $\rightarrow$ TE_{10}, TE_{20}, TE_{01}, TE_{11}, TM_{11} modes.

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2 TE_{10} mode.

$f = 4\,GHz$

$a = 5cm = 0.05m$.

$b = 2.5cm = 0.025m$.

$\sigma_d = 2\times10^{-6}$ mho/m.

$B_1 = j1000$

$$f_c = \frac{c}{2a}$$

$$= \frac{3\times10^8}{2\times0.05} = 3\times10^9 = 3GHz.$$

$$\sqrt{1-\left(\frac{f_c}{f}\right)^2} = \sqrt{1-\left(\frac{3}{4}\right)^2}$$

$$= 0.6614.$$

$$H_z = B\cos\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right)e^{-j\bar{\beta}z}.$$

for TE_{10} mode $\quad H_z = B\cos\left(\frac{\pi}{a}x\right)e^{-j\bar{\beta}z}$

$E_z = 0$

$E_x = 0$

$H_y = 0$

$$E_y = \frac{j\omega\mu}{h^2}\frac{\partial}{\partial x}H_z.$$

$$E_y = -\frac{j\omega\mu}{(\pi/a)}B\sin\left(\frac{\pi}{a}x\right)e^{-j\bar{\beta}z}$$

$$H_x = -\frac{\gamma}{h^2}\frac{\partial}{\partial z}H_z$$

$$H_x = \frac{j\bar{\beta}}{(\pi/a)}B\sin\left(\frac{\pi}{a}x\right)e^{-j\bar{\beta}z}$$

$$\text{Power} = \frac{1}{2}Re\left\{\int \bar{E}\times\bar{H}^*\cdot ds\right\}$$

$$\bar{E}\times\bar{H}^* = \left(-\frac{j\omega\mu}{(\pi/a)}B\sin\left(\frac{\pi}{a}x\right)e^{-j\bar{\beta}z}\hat{y}\right)\times\left(\frac{-j\bar{\beta}}{\pi/a}B\sin\left(\frac{\pi}{a}x\right)e^{j\bar{\beta}z}\hat{x}\right)$$

$$= \frac{\omega\mu\bar{\beta}}{(\pi/a)^2}B^2\sin^2\left(\frac{\pi}{a}x\right)\hat{z}$$

$$\int_0^a\int_0^b \frac{\omega\mu\bar{\beta}}{(\pi/a)^2}B^2\sin\left(\frac{\pi}{a}x\right)dx\,dy = \left(\frac{1}{2}ab\right)\frac{\omega\mu\bar{\beta}}{\left(\frac{\pi}{a}\right)^2}B^2$$

$$= \frac{1}{2}a^3b\frac{\omega\mu\bar{\beta}}{\pi^2}B^2$$

$$\text{Power} = \frac{1}{4} a^3 b B^2 \left(\frac{\omega \mu \bar{\beta}}{\pi^2}\right)$$

$$\frac{\omega \mu}{\pi/a} B = 1000 \Rightarrow \frac{2\pi f \mu a}{\pi} B = 1000$$

$$\Rightarrow B = \frac{1000}{2 f \mu a} = \frac{1000}{2 \times 4 \times 10^9 \times 4\pi \times 10^{-7} \times 0.05}$$

$$= \frac{100}{16\pi} = 1.99$$

$$\sqrt{1 - \left(\frac{f_c}{f}\right)^2} = 0.6614$$

$$V_p = \frac{c}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}} = \frac{3 \times 10^8}{0.6614} = 4.53 \times 10^8 \text{ m/s.}$$

$$V_g = c\sqrt{1-\left(\frac{f_c}{f}\right)^2} = 3 \times 10^8 \times 0.6614 = 1.9842 \times 10^8 \text{ m/s.}$$

$$\bar{\beta} = \beta\sqrt{1-\left(\frac{f_c}{f}\right)^2} = \omega\sqrt{\mu\varepsilon}\sqrt{1-\left(\frac{f_c}{f}\right)^2}$$

$$= \frac{2\pi \times 4 \times 10^9}{3 \times 10^8} \times 0.6614$$

$$= 55.4 \text{ rad/m.}$$

$$\text{Power} = \frac{1}{4} \times \frac{(0.05)^3 (0.025)(1.99)^2 \times 2\pi \times 4 \times 10^9 \times 4\pi \times 10^{-7} \times 55.4}{\pi^2}$$

$$= \frac{21.65}{4\pi^2} = 548.4 \text{ mW}$$

$$\alpha_d = \frac{\omega \mu \sigma}{2\bar{\beta}} = \frac{2\pi \times 4 \times 10^9 \times 4\pi \times 10^{-7} \times 2 \times 10^{-6}}{2 \times 55.4}$$

$$\alpha_d = 5.7 \times 10^{-4} \text{ NP/m.}$$

Power after 10m

$$P = P_o e^{-2\alpha z}$$

$$= 548.4\times10^{-3}\ e^{-2\times5.7\times10^{-4}\times10}.$$

$$= 542.1\ mW.$$

$$H_z = 1.99\cos(20\pi x)\ e^{-5.4\times10^{-4}z}\cos(8\pi\times10^{9}t - 55.4z)$$

$$E_y = -j1000\sin(20\pi x)\ e^{-5.4\times10^{-4}z}\cos(8\pi\times10^{9}t - 55.4z)$$

$$= 1000\sin(20\pi x)\ e^{-5.4\times10^{-4}z}\cos(8\pi\times10^{9}t - 55.4z - 90^\circ)$$

$$H_x = 1.75\sin(20\pi x)\ e^{-j5.4\times10^{-4}z}\cos(8\pi\times10^{9}t - 55.4z + 90^\circ)$$

3)

$$\frac{\lambda_g}{2} = 2\times10^{-2} \Rightarrow \lambda_g = 4cm$$
$$= 0.04m.$$

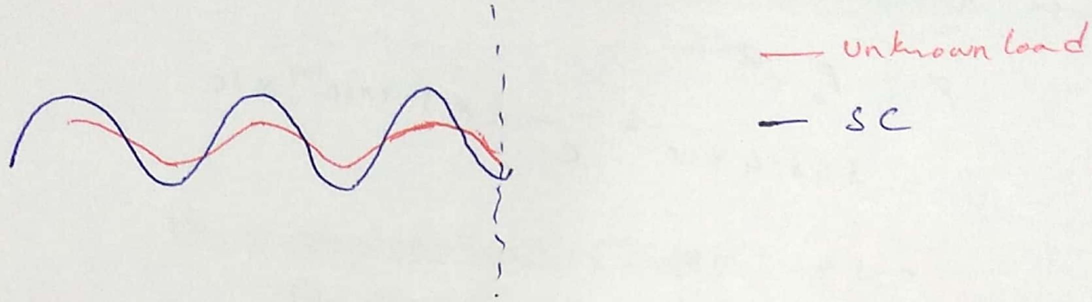
$$f_c = \frac{c}{2a} = \frac{3\times10^{8}\times100}{2.286\times2} = 6.56\ GHz.$$

$$\lambda_c = 2a = 4.572\ cm.$$
$$= 0.045\ m.$$

$$\frac{1}{\lambda^2} = \frac{1}{\lambda_c^2} + \frac{1}{\lambda_g^2} \Rightarrow \frac{1}{\lambda^2} = \left(\frac{1}{4.572}\right)^2 + \left(\frac{1}{4}\right)^2$$

$$\Rightarrow \lambda = 3.01\ cm.$$

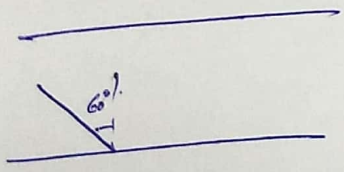
$$f = \frac{3\times10^{8}\times100}{3.01} = 9.965\ GHz.$$



$$Z_0 = \frac{\eta}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}} = \frac{377}{\sqrt{1-\left(\frac{6.56}{9.965}\right)^2}} = 506.2\ \Omega.$$

unknown load $= 0.5 \times 506.2 = 253.1\ \Omega$.

(4.)



(a) $\theta = 60°$

$$V_p = \frac{c}{\sin\theta} = \frac{3\times10^8}{\sin 60°} = 3.46\times10^8\ m/s.$$

$$V_g = c\sin\theta = 3\times10^8 \times \sin 60° = 2.59\times10^8\ m/s.$$

(b) TM_0 mode is TEM mode so cutoff frequency is zero.

The Complete Smith Chart

Black Magic Design