EE 308: Communication Systems
Instructor: Prof. Shalabh Gupta
DAMP info : Follow Book → Simon Haykin Communication Systems
Grading tough,
Surprise Quizzes

16.7.18

<u>Reference</u>

1) Simon Haykin "Communication Systems"
2) Proakin J. G & Salehi M.
"Communication System Engineering"

<u>Grading</u>

Surprise Quizzes (5 or 6) : 30%
M.S : 25%
Final E.S : 45%

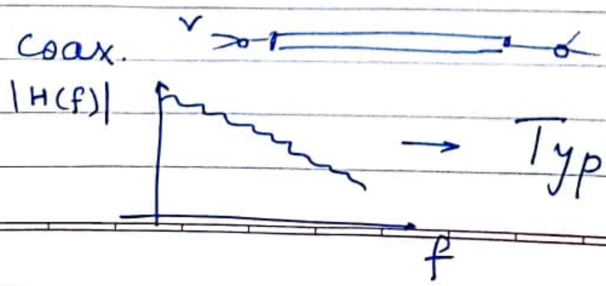
Makeup Classes : Sat : 11-12 July 21, 28
Aug 4

17.7.18 <u>Lecture 2</u>

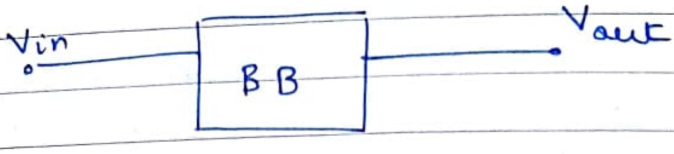
- <u>Fourier Transforms</u>

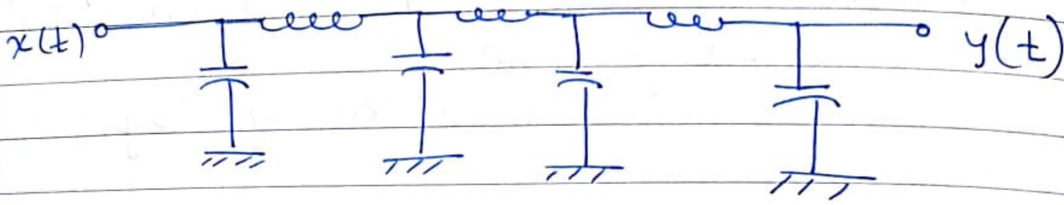
- Why freq. content is imp. : Channels are freq. dep.

<u>ex</u> Coax.



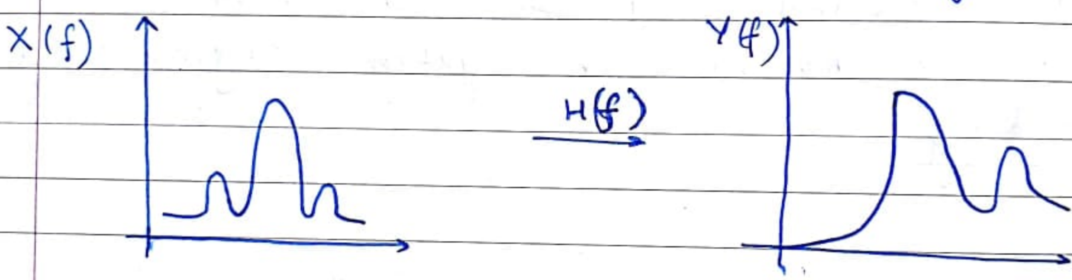
→ Typical





$$x(t) \xrightarrow[\text{comp. math}]{\text{convolution}} y(t) \quad : \text{Inconvenient}$$

$$Y(f) = H(f) \cdot X(f) \quad : \text{Easy}$$



→ Fourier Transforms

$$x(t) \xrightarrow{\text{F.T}} X(f)$$

$$X(f) = \int_{-\infty}^{\infty} x(t) \underbrace{e^{-j2\pi ft}}_{\substack{\text{complex exp.} \\ \text{(sine wave)}}} dt \longrightarrow \cos(2\pi ft) + j\sin(2$$

$$x(t) = \int_{-\infty}^{\infty} X(f) e^{j2\pi ft} df$$

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→ Alternate (EE 210 type) definition

Let, $x_1(\omega) = X\left(\frac{\omega}{2\pi}\right) = \boxed{\int x(t)\, e^{-j\omega t}\, dt}$ (from $X(f)$)

$$x(t) = \int X(f)\, e^{j2\pi f t}\, df$$

$$x(t) = \int X\left(\frac{\omega}{2\pi}\right) e^{j\omega t}\, d\left(\frac{\omega}{2\pi}\right) = \boxed{\frac{1}{2\pi}\int X_1(\omega)\, e^{j\omega t}\, dt}$$

→ Yet Another Symmetric Defn.

$$x(t) \longrightarrow X_2(\omega)$$

$$\boxed{X_2(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt}$$

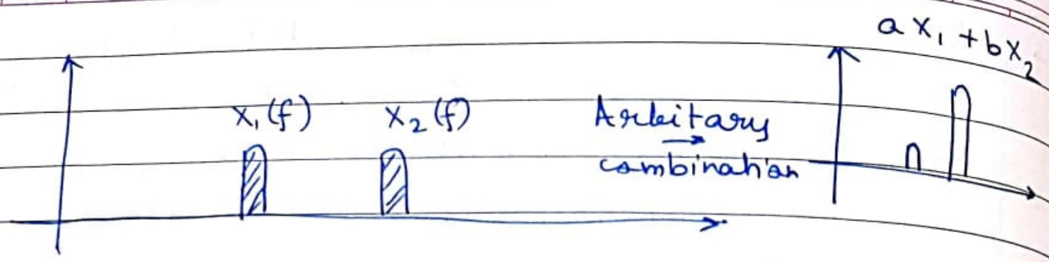
$$\boxed{x(t) = \frac{1}{\sqrt{2\pi}} \int X_2(\omega)\, e^{j\omega t}\, d\omega}$$

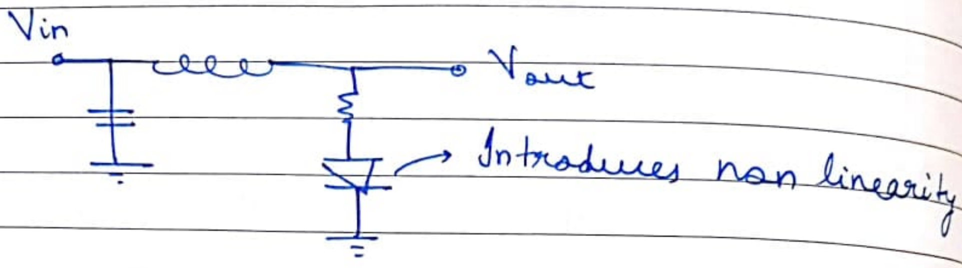
→ Properties of Fourier Transform

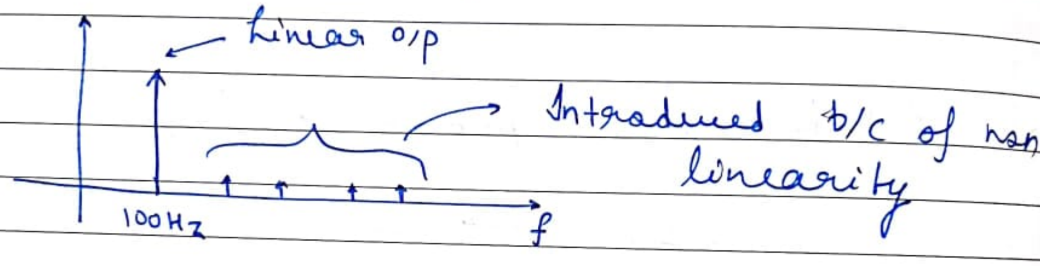
1) Linearity : $x_1(t), x_2(t)$ | a, b
$X_1(f), X_2(f)$

$$a x_1(t) + b x_2(t) \rightleftharpoons a X_1(f) + b X_2(f)$$

→ Implication : No new frequencies are created in the system when we superpose 2 waves







2) <u>Time</u> <u>Shift</u> (may come in the form of ϕ delay)

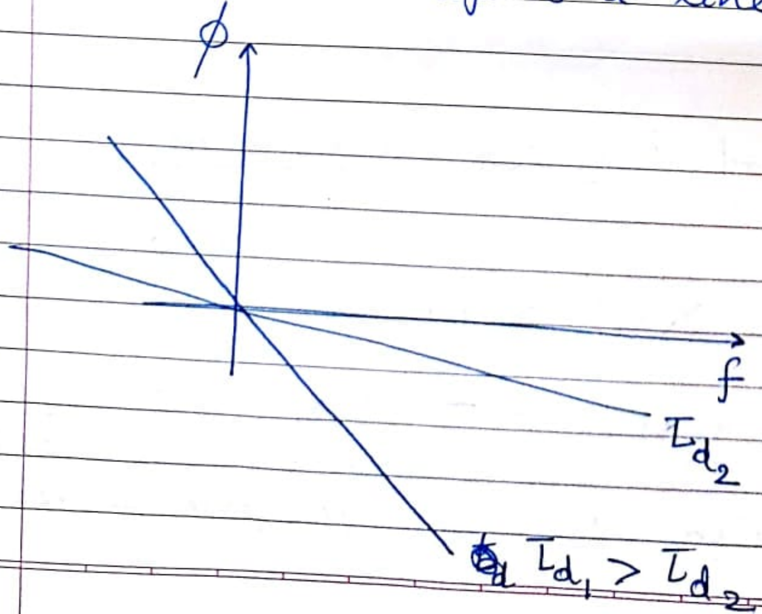
Same, No distortion

$x(t) \rightarrow X(f)$

$x(t-\tau) \rightarrow \underbrace{X(f) \cdot e^{-j2\pi f\tau}}$

↳ f-domain is scaled, but o/p signal still the same.

→ A fixed time shift is a linear freq. dep. ϕ shift



→ If phase shift is not linear with frequency (Due to ↓ quality filter or Channel char.) then there will be distortion.

$$X(f) \xrightarrow{\tau_d} X(f)\, e^{-j2\pi f(\tau_{d_i})}$$

distortion different for different freq.

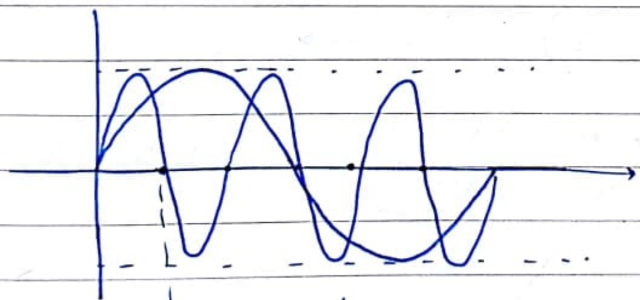
ex

$$x(t) = \cos(2\pi f_0 t) + \cos(2\pi 3 f_0 t)$$

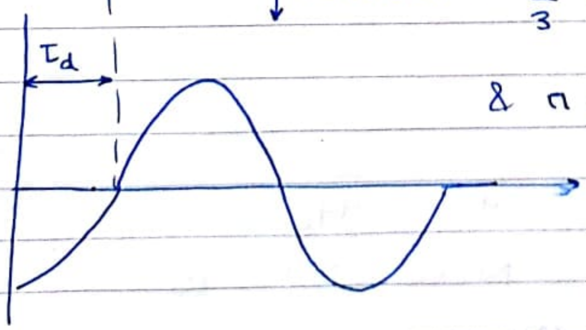
↓ Channel

Linear Phase Channel : $\cos\left(2\pi f_0 t + \frac{\pi}{3}\right) + \cos\left(2\pi 3 f_0 t + \pi\right)$

ex

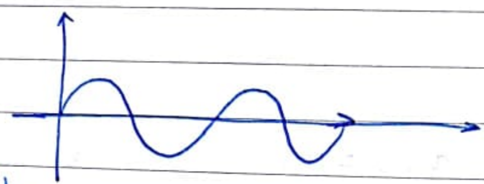


↓ Channel : $\frac{\pi}{3}$ rad phase shift for f_0 & π rad phase shift for $3f_0$



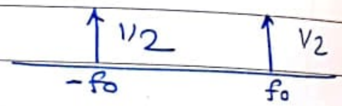
3) <u>Scaling</u>:

$$x(at) \xrightarrow{F} \frac{X\left(\frac{f}{a}\right)}{|a|}$$



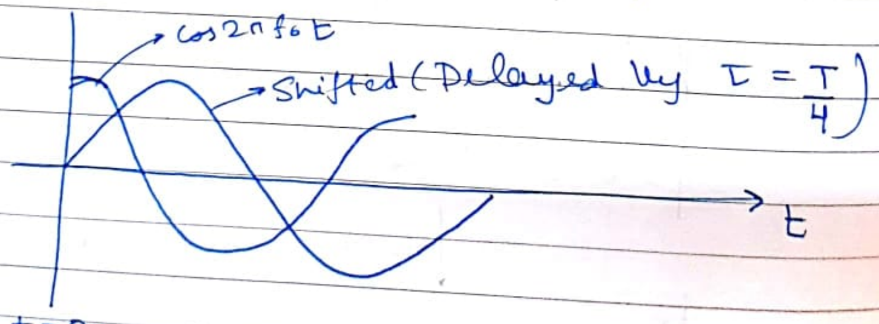
<u>Recap: Time Shift</u>

→ $x(t) = \cos(2\pi f_0 t)$ →



What does a time delay look like in Fourier Domain?

$$x(t-\tau) \longrightarrow X(f)e^{-j2\pi f\tau}$$



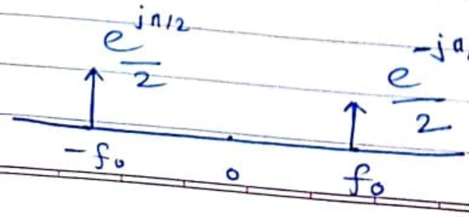
$$\tau = \frac{T}{4} = \frac{1}{4f_0}$$

$$-j \times 2\pi f_0 \times \frac{1}{4f_0}$$

→ Now the New FT is

if $f = f_0$: $e^{-j\pi/2}$

if $f = -f_0$ $e^{j\pi/2}$



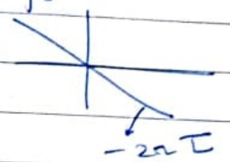
$$\phi(f) = \cancel{-2\pi f T} \; \frac{\phi}{2\pi} \times T$$

$$\cancel{\phi(f) = -Tf\,\phi}$$

$$\frac{\phi}{f} = -j\left[2\pi T\right]$$

some fixed time shift

slope



→ How does Scaling Change f-domain

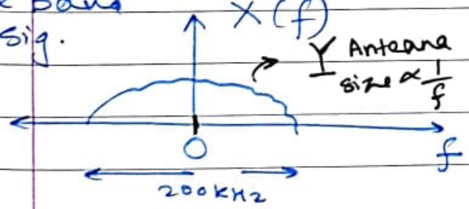
$$x(at) \xrightarrow{\mathcal{F}} \frac{1}{|a|} X\left(\frac{f}{a}\right)$$

$$\cos(2\pi f_0 (at)) \rightarrow \frac{1}{|a|}\left[\delta\left(\frac{f}{a}+f_0\right) + \delta\left(\frac{f}{a}-f_0\right)\right]$$

4) Frequency Shifting/Modulation/Freq. Translation/upconversion

$$x(t)\cdot e^{j2\pi f_0 t} \xrightarrow{F.T} X(f-f_0)$$

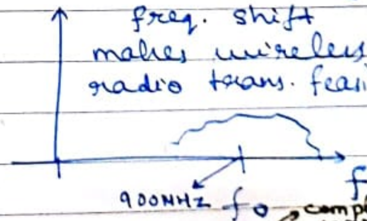
Base Band Sig.



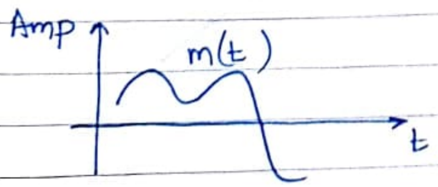
times $e^{j2\pi f t}$ (time domain mult)

Pass Band sig.

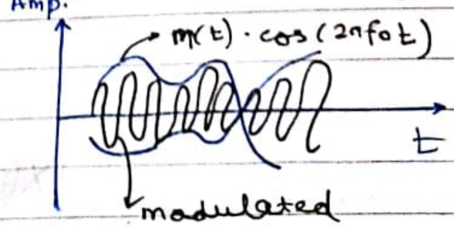
freq. shift makes wireless radio trans. feasible



→ What does times $e^{j2\pi f_0 t} = \underbrace{\cos 2\pi f_0 t}_{\text{carrier}} + \underbrace{j \sin 2\pi f_0 t}_{\text{Ditch this}}$



times $\cos 2\pi f_0 t$



Some hints on complex carriers

$$s_p(t) = \left[m_1(t) \times \cos 2\pi f_0 t \mp m_2(t) \sin 2\pi f_0 \right.$$

If we want to ~~transmit~~ transmit $m_1(t)$ & $m_2(t)$ simultaneously

$$m_c(t) = m_1(t) + j\, m_2(t)$$

$$s_p(t) = \mathrm{Re}\left[m_c(t) \cdot e^{j2\pi f_0 t} \right]$$

→ Send this to the Antenna

→ For real $x(t)$, $X(f)$ is necessarily Hermitian

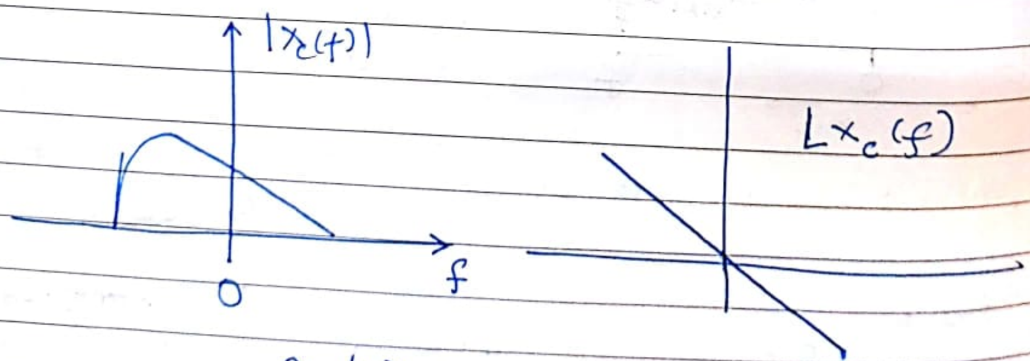
$$\overline{X(f)} = X(-f) \rightarrow |X(f)| = |X(-f)|$$

ex

cos → purely Real
sin → purely im
} Hermitian

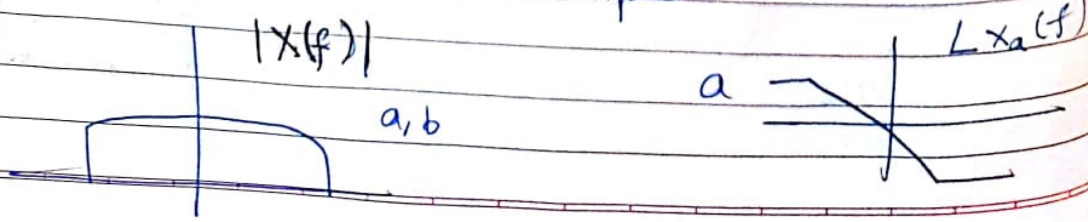
ex 1) $x_c(t)$

1)



$x_c(t)$ is complex

2)



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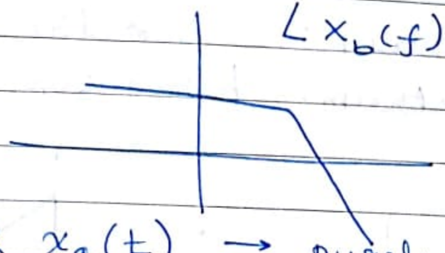
b

$\angle x_b(f)$



$x_a(t) \rightarrow$ purely real
$x_b(t) \rightarrow$ complex.

→ Condition can be called
$|X|$ is even
$\angle X$ is odd

5) Derivative property :

$$\frac{d^n x(t)}{dt^n} \xrightarrow{F.T} (j2\pi f)^n \cdot X(f)$$

$$t^n x(t) \xrightarrow{F.T} \left(\frac{j}{2\pi}\right)^n \frac{d^n X(f)}{df^n}$$

6) Multiplication / Convolution

$$x(t) \times y(t) \xrightarrow{FT} X(f) * Y(f)$$
$$X(f) \cdot Y(f) \xrightarrow{IFT} x(t) * y(t)$$

$* \Rightarrow \int_{-\infty}^{\infty} x(\tau)\, y(t-\tau)\, d\tau = x * y$

Lec - 4

23.7.18

Signals : Complex Vs Real

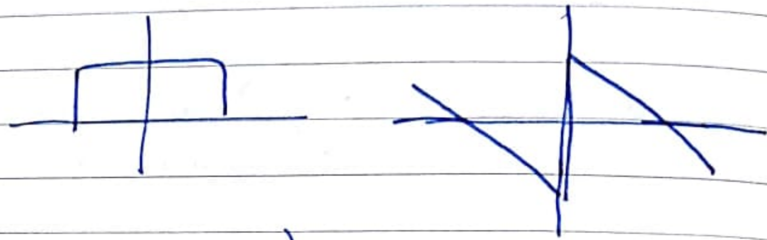
Real Signal : $m(t)$

→ properties of transform

$M(f) = M^*(-f) = |M(-f)| e^{-\angle M(-f)}$

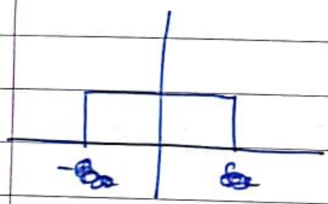
for a Real sig. { Magnitude Spectrum : Symmetric
Phase spectrum : Anti - Symmetric

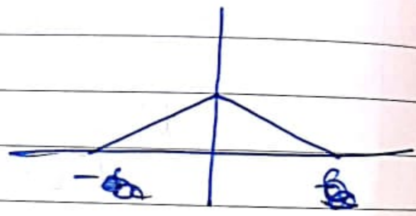
ex

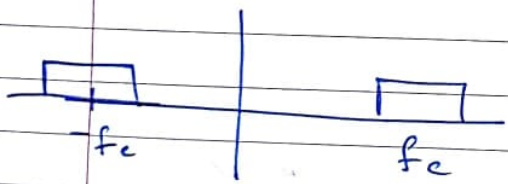


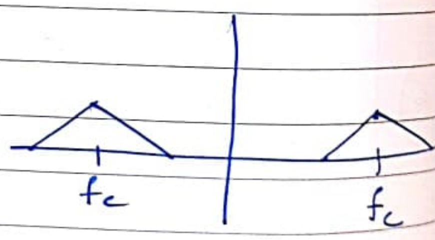
$s_p(t) = \{ m_1(t) \times \cos(2\pi f_c t) - m_2(t) \sin(2\pi f_c t) \}$

$S_p(f)$ is asym. $M_1(f)$ (symm.) $M_2(f)$ (symm.)

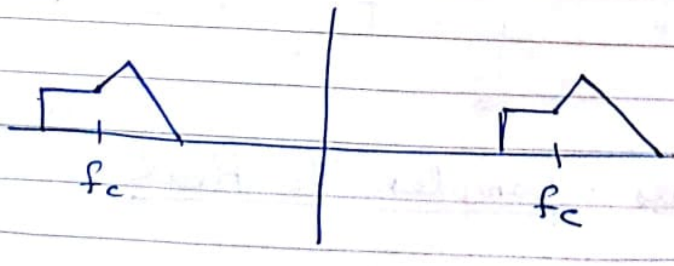








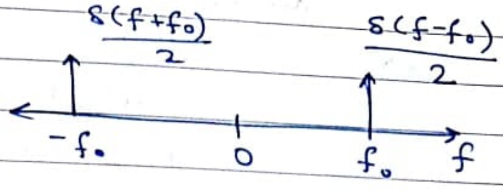
$S_p(f)$



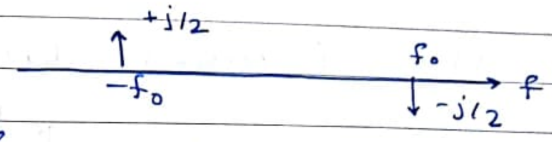
$m_1(t) + j m_2(t) \longrightarrow$

→ **Positive & Negative frequencies**

$\cos(2\pi f_0 t)$

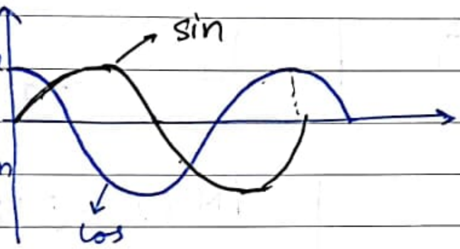


$\sin(2\pi f_0 t)$



→ **To identify +ve/−ve frequencies**

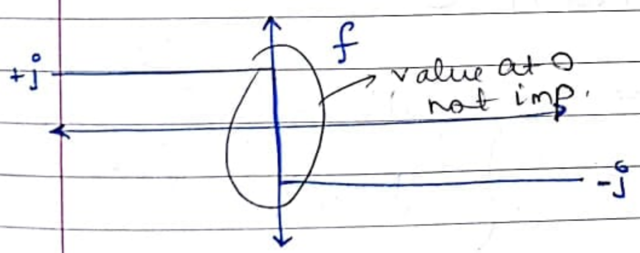
→ We will have to split ~~ø~~ signal into cos/sin comp.



if cos leads sin ⇒ +ve freq.
& if sin leads cos ⇒ −ve freq.

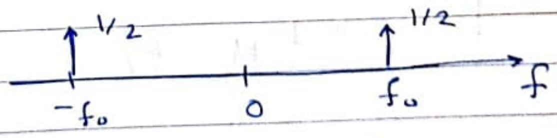
→ **Hilbert transform**

$$H_h(\underset{\text{freq}}{f}) = -j\,\text{sign}(\underset{\text{freq.}}{f})$$



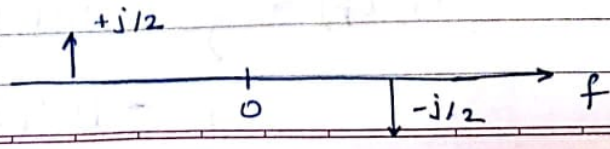
In frequency domain it is simply Multiplication with $H_h(f) : -j\,\text{sign}(f)$

ex $\cos(2\pi f_0 t)$



$\downarrow H_h(f)$

$\sin(2\pi f_0 t) \longleftarrow$



Hilbert Transform

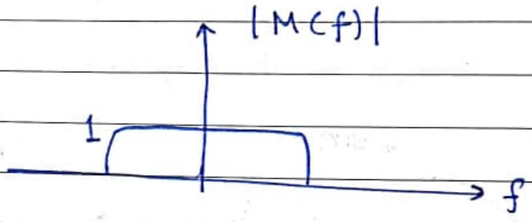
→ $\boxed{H_h(f)}$ introduces a phase delay of 90° in f.
domain

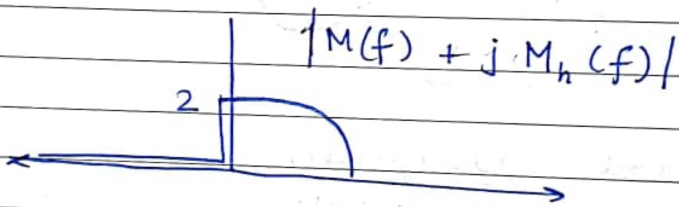
ex $m(t) \xrightarrow{H.T} m_h(t)$

$$m(t) = \cos 2\pi f_0 t$$

$$m(t) + j m_h(t) = e^{+j2\pi f_0 t}$$

ex (General Case) : Real $m(t)$



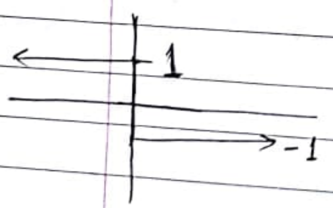


proof

$$M(f) = |M(f)| \cdot e^{j\phi(f)}$$

$\cancel{M_h(f)} \; M(f) =$ $\cancel{\begin{cases} |M(-f)| \cdot e^{j\phi(f)} & f<0 \\ |M(f)| \cdot e^{j\phi f} & f>0 \end{cases}}$

$\cancel{M(f) =}$



$$M_h(f) = \begin{cases} |M(f)| \cdot \left[e^{j\phi(f) - \frac{j\pi}{2}}\right] & f>0 \\ |M(f)| \cdot \left[e^{j\phi(f) + j\pi/2}\right] & f<0 \end{cases}$$

$$j M_h(f) = \begin{cases} |M(f)| \cdot \left[e^{j\phi(f)}\right] & f>0 \\ -|M(f)| \cdot \left[e^{j\phi(f) \cancel{+ j\pi}}\right] & f<0 \end{cases}$$

24.7

Add & get

$$M(f) + jM_h(f) = \begin{cases} 2|M(f)|\cdot e^{j\phi(f)} & f>0 \\ 0 & f<0 \end{cases}$$

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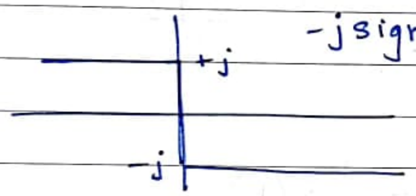
Lecture 5

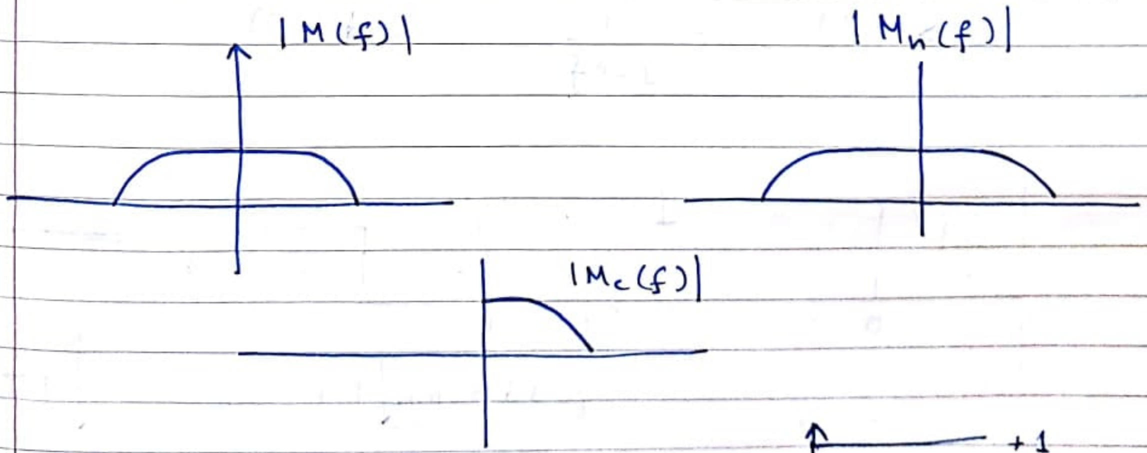
Recap : Hilbert Transform



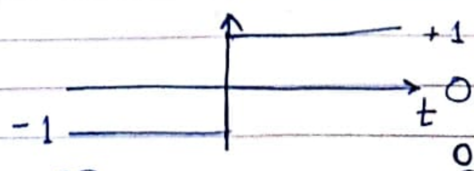
At $f=0$: can take $0, +j$ or $-j$ doesn't matter

$m(t)$: Real
$m_h(t)$: Real

$$\begin{cases} m_c(t) = m(t) + j\, m_h(t) \\ M_c(f) = M(f) + j\, M_h(f) \end{cases}$$



→ FT of sgn(t)



$$\mathcal{F}[\mathrm{sgn}(t)] = \lim_{a\to 0^+} \int_0^{\infty} e^{-at} e^{-j2\pi f t}\, dt - \int_{-\infty}^{0} e^{+at} e^{-j2\pi f t}\, dt$$

$$= \lim_{a \to 0^+} \frac{1}{a + j2\pi f} - \frac{1}{a - j2\pi f}$$

$$= \lim_{a \to 0^+} \frac{-j4\pi f}{a^2 + (2\pi f)^2} = \boxed{\frac{1}{j\pi f}}$$

Now, $u(t) = \frac{1}{2}\,\text{sgn}(t) + \frac{1}{2}$

$$\mathcal{F}[u(t)] = \frac{1}{j2\pi f} + \frac{\delta(f)}{2}$$

$\parallel$

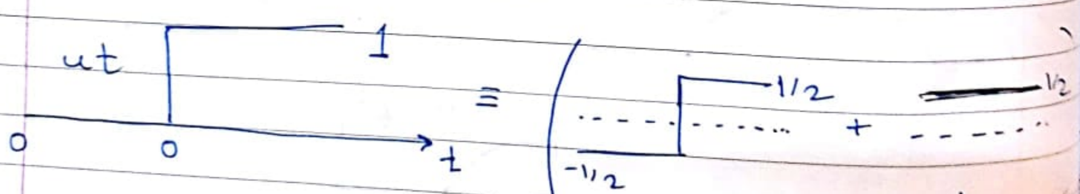
$U(f)$

same issue refer EE210

$$\frac{d}{dt}[u(t) + c] = \delta(t)$$

$$j2\pi f\,[U(f) + c\cdot\delta(f)] = 1$$

$$\Rightarrow U(f) = \frac{1}{j2\pi f} - c\delta(f)$$



(odd symm) ↓ FT

Purely imaginary comp.

↓ FT (even symm)

Purely real

↓

$\frac{\delta(t)}{2}$

$\Rightarrow c = \frac{1}{2}$

$$\text{sgn}(t) \rightarrow \frac{1}{j\pi f}$$

$$h_h(t) : \mathcal{F}^{-1}[-j\,\text{sign}(f)]$$

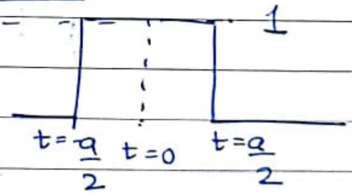
Impulse Response of Hilbert transform

using Duality,

$$\left.\begin{array}{l} x(t) \xrightarrow{F.T} X(f) \\ X(t) \xrightarrow{F.T} x(-f) \end{array}\right] \text{Duality property}$$

$$= -j \cdot \frac{1}{j\pi(-t)} = \boxed{\frac{1}{\pi t}} = h_h(t)$$

→ FT of $\text{rect}\left(\frac{t}{a}\right)$



$$= u\left(t + \frac{a}{2}\right) - u\left(t - \frac{a}{2}\right)$$

$$\downarrow F.T$$

$$\left[\frac{1}{j2\pi f} + \frac{\delta(f)}{2}\right]\left[e^{+j2\pi f \cdot \frac{a}{2}} - e^{-j2\pi f \cdot \frac{a}{2}}\right]$$

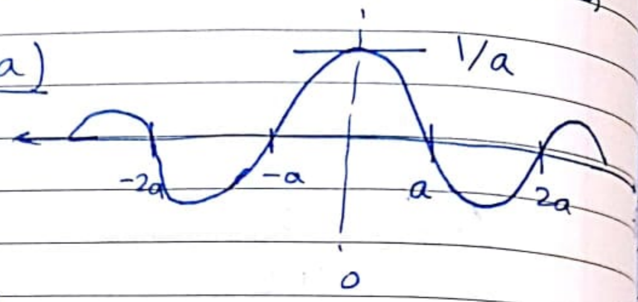
$$= \frac{\delta(f)}{2}\cancel{[1-1]}^{0} + \frac{1}{j2\pi f}\left[\sin(\pi f a)\right]$$

$$\boxed{= a\,\text{sinc}(af)}$$

Scanned by CamScanner

or, if we started with rect(at), then

sinc (f/a)

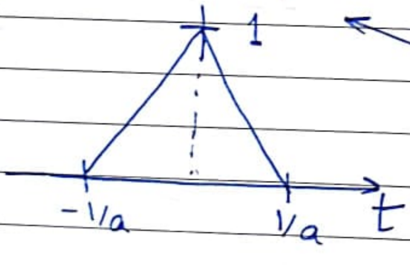


Amplitude / DC component can also be obtained by,

$$X(0) = \int_{-\infty}^{\infty} x(t)\, dt$$

~~Tri~~ tri(t/a)

tri(at) = a rect$(at) * $ rect(at) $\longleftrightarrow$ $\frac{1}{a}\,\text{sinc}^2\left(\frac{f}{a}\right)$

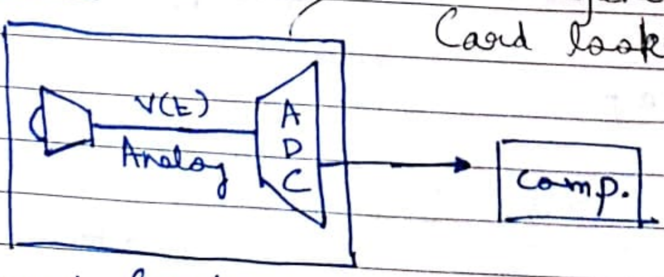


26.7.18

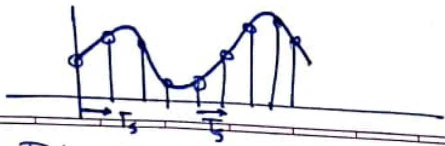
Lecture 6

GNU radio

What a Generic Sound Card looks like



Sound Card



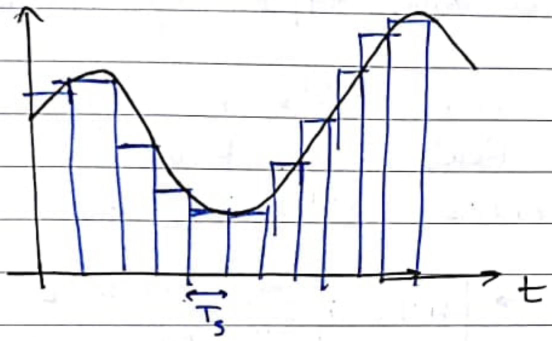
→ Before Digitizing a signal we ~~discretise~~ it.
discretize

$$x[n] = x(nT_s)$$

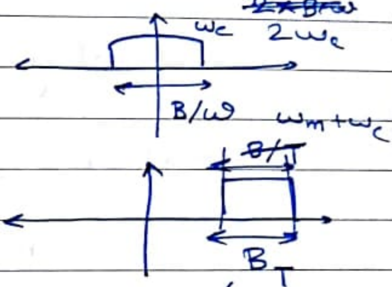
discretisation

Quantize

Whole Number (bits) + {Quantization Noise of ZOH}



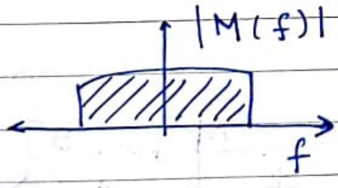
Sampling Rates

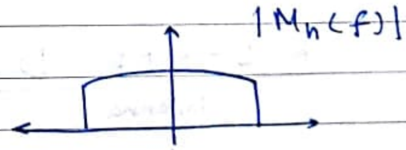


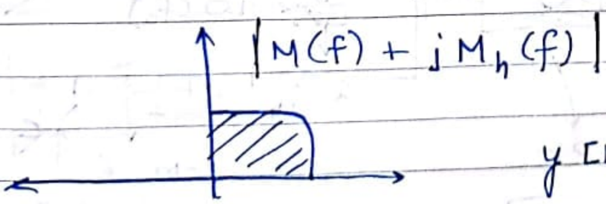
we can do it with $2(\omega_m+\omega_c)$
ΣB.W

Recap: Hilbert

(Real) $m(t) \xrightarrow{H.T} m_h(t)$







What does a digital filter look like,

$$y[n] = a_0 x[n] + a_1 x[n-1] + a_2 x[n-2]$$

$$h_h(t) = \frac{1}{\pi t} \rightarrow h_h[n] = \{a_0, \ldots, a_n\}$$

$$a_n = nT_s$$

→ Why Noise doesn't peak at 1

→ Power is spread over entire spectrum

→ no peaks as tall as a pure sinusoid in FT

$$\int_{-\infty}^{\infty} |x(t)|^2 dt = \int_{-\infty}^{\infty} |X(f)|^2 df$$

→ In and out ~~to~~ var. types of Hilbert Trans.

Ideally : $m(t) \xrightarrow{HT} m_h(t)$

Real → Real

But in GNU radio Hilbert : Real → Comp.

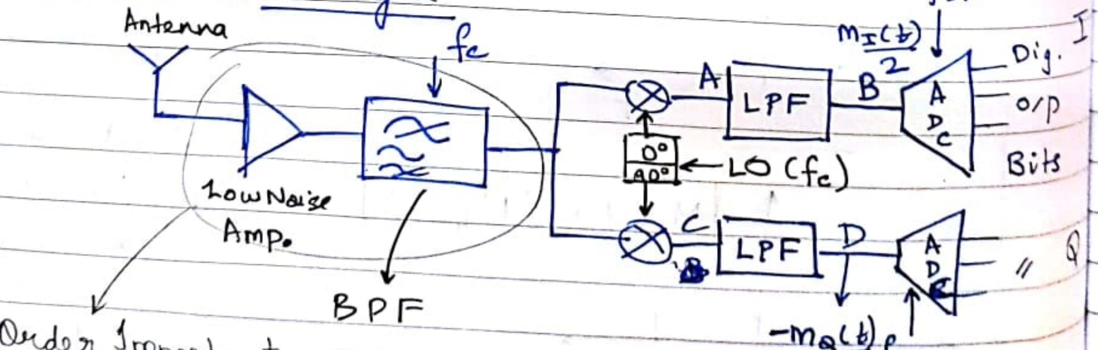
$m(t) \rightarrow \boxed{\text{Hilbert}}$ $\xrightarrow{Re}$ $m(t-\tau)$, $\xrightarrow{Im}$ $m_h(t-\tau)$

↳ o/p is $m(t) + j\,m_h(t)$

28.7.18

Lec. 7

RTL-SDR Dongle



Order Important, Reason: RF Systems course

LO ⇒ Local oscillator

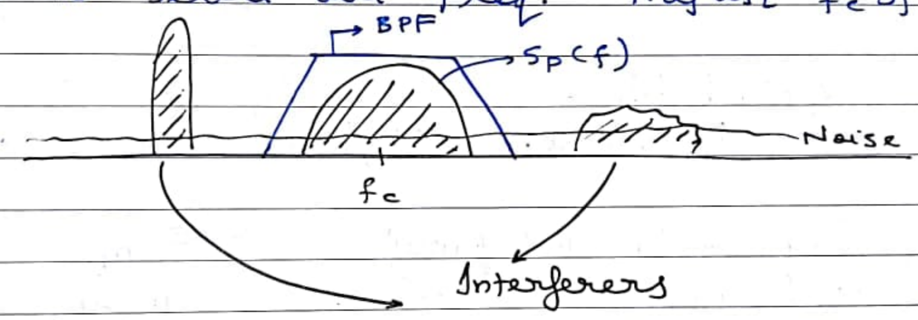
$$\text{Noise} + \text{Inter} + \mathrm{Re}\left[m_c(t)\, e^{j2\pi f_c t}\right] = S_p(t)$$: Captured by Antenna

$$m_c(t) = \underset{\text{In-phase}}{m_I(t)} + j\, \underset{\text{Quadrature}}{m_Q(t)}$$

$$S_p(t) = m_I(t)\cos(2\pi f_c t) - m_Q(t)\sin(2\pi f_c t)$$

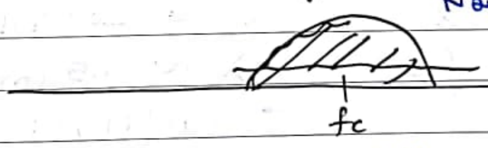
→ RTL-SDR Block parameters:
Sample Rate : f_{SR}

→ How to select out freq. : Adjust f_c of BPF



↓

Now $= m_I(t)\cos(2\pi f_c t) - m_Q(t)\sin(2\pi f_c t)$



→ Multiplication with LO

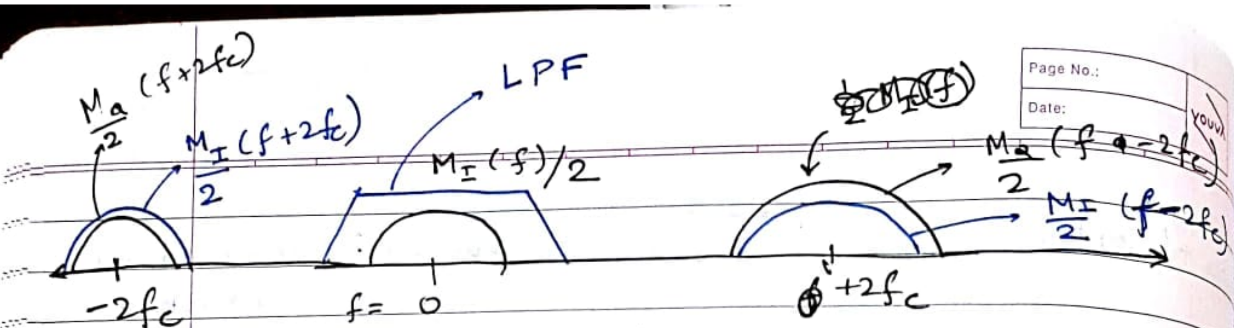
$\times \cos 2\pi f_c t$
$\boxed{\begin{matrix} 0^\circ \\ 90^\circ \end{matrix}}$
$\times \sin 2\pi f_c t$

A: $S_p(t) \times \cos(2\pi f_c t) \rightarrow \dfrac{m_I(t)}{2} + \dfrac{m_I(t)}{2}\cos(4\pi f_c t) - \dfrac{m_Q(t)}{2}\sin(4\pi f_c t)$

↓ LPF

$= \dfrac{m_I(t)}{2}$

B: $S_p(t) \times \sin(2\pi f_c t) \rightarrow \dfrac{m_I(t)}{2}\sin(4\pi f_c t) - \dfrac{m_Q(t)}{2}(1 - \cos 4\pi f_c t)$

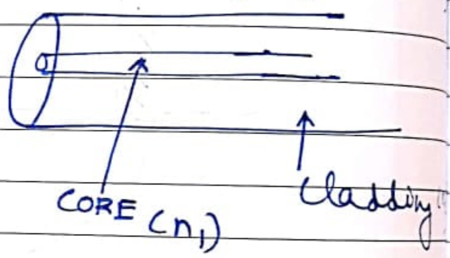


→ Practical Issue: The LO oscillates not exactly at f_c but some $f_c + \Delta f$: Must be solved using signal processing.

Communication Modes

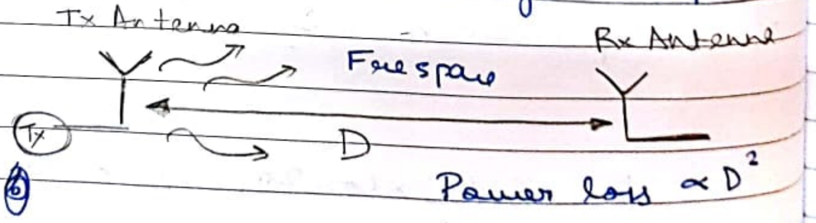
1. Free space
   → Radio Waves (Range of EM wave)

2. Optical Fiber
   → ↑ efficient: ~0.18 dB/km



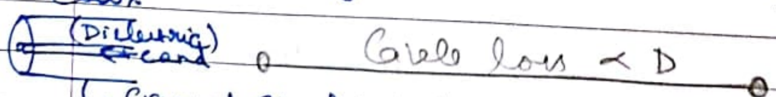
$n_2 > n_1$
→ Uses TIR
$f_c \sim 200$ THz
$\lambda \sim 1.5\,\mu m$
→ Challenge: Dispersion

3. Copper (metal wires)
   → Lossy, but loss is less than free space for short dis. & > free space for long distance
   → Reason:



Power loss $\propto D^2$

- Types
  → twisted pair
  → Coax



Cable loss $\propto D$

4. Wave Guides: Idea similar to optical fiber
   1 GHz – 100 GHz
   No glass, lower frequency, (GHz ...)

30.7.18

5. Solids / Liquids : Sound waves & SONAR

Formulas till now

$$\text{rect}\left(\frac{at}{\cancel{a}}\right) \xrightarrow{FT} \frac{\text{Sinc}(f/a)}{a}$$

$$\text{tri}(at) \xrightarrow{FT} \frac{1}{a}\,\text{sinc}^2\left(\frac{f}{a}\right)$$

$$H_h(f) = -j\,\text{sig}(f)$$

$$h_h(t) = \frac{1}{\pi t}$$

$$M(f) + j M_h(f) = \begin{cases} 2|M(f)|\, e^{j\phi(f)} & f>0 \\ 0 & f<0 \end{cases}$$

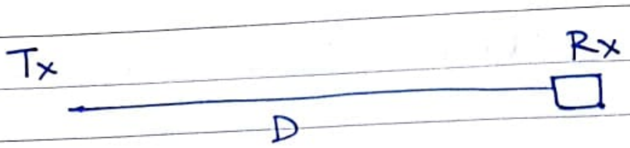
Conditions Apply

$$\text{Hermitian} \Rightarrow \overline{X(f)} = X(-f)$$

30.7.18

Lecture 8

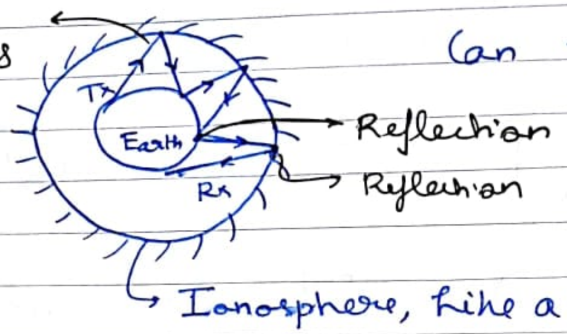
- Slides
  1) Effect of Scattering & Absorption on light in Optical fibers
     - Raleigh Scattering
     - OH^- Res. absorption
  2) Atmospheric Opacity
     - Ionosphere reflects EM rad.



Atten $\propto D^2$

Where?

Called Sky-Waves



Can transmit the freq.

Range:

3 MHz – 30 MHz

$$\underset{(\text{bits/s})}{\text{Capacity}} = \overset{\text{Bandwidth}}{B} \cdot \log_2 \left(1 + \text{SNR}\right)$$

$$\text{SNR} = \left(\frac{\text{Signal Power}}{\text{Noise Power}}\right)$$

→ <u>Problems with Sky Waves</u>

1) Limited B/W
2) Large Antenna Size

WiFi
- → 802.11 b/g (2.4 GHz)
- → 802.11 a (5 GHz)

→ Newer comm. technology ⇒ Move to high center frequency for ↑ Bandwidth

5G : ↑ GHz Range

→ Extremely high frequency : (millimetre wave) (MM)

30 GHz – 300 GHz

λ = 10 mm – 1 mm

Modulation

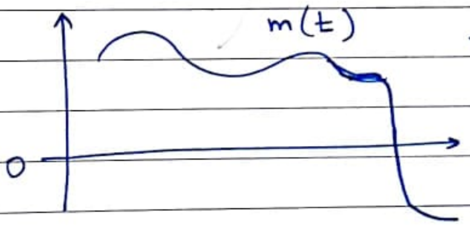
→ If only $f > 0$ spectrum is shown, it means spectrum is Symmetric

→ Message to be trans. by modulation $= m(t)$

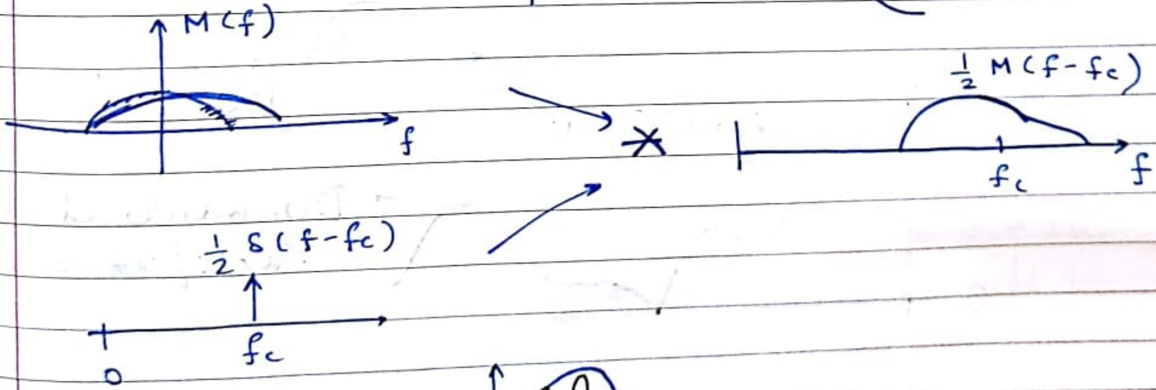
$$\cos 2\pi f_c t \begin{cases} \xrightarrow{AM} A_m \cdot m(t) \cdot \cos(2\pi f_c t) \\ \xrightarrow{PM} A_c \cos(2\pi f_c t + k_\phi m(t)) \\ \longrightarrow A_c \cos\left(2\pi f_c t + K_f \int^{t} m(t)\,dt\right) \end{cases}$$

AM: types

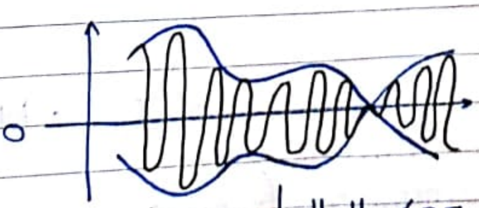
$m(t)$: real



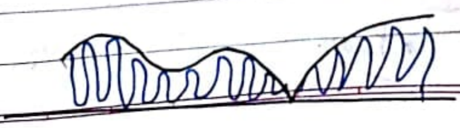
$\times \cos 2\pi f_c t$



Result :

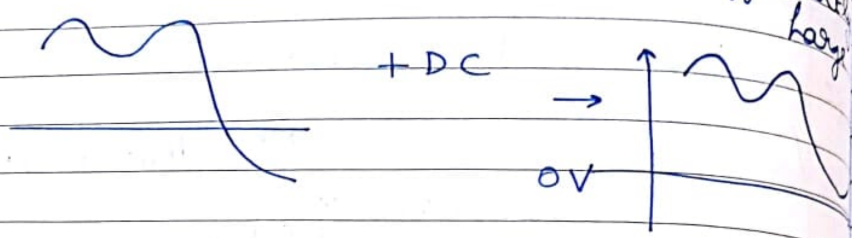


$\downarrow$ $\|\cdot\|$ LPF (Take abs. & then LPF)



→ Information lost

→ Hence choose : $m(t) + DC$
↳ sufficiently large

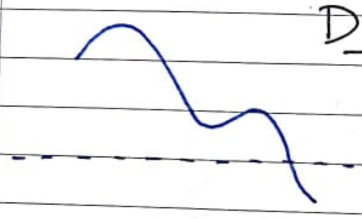


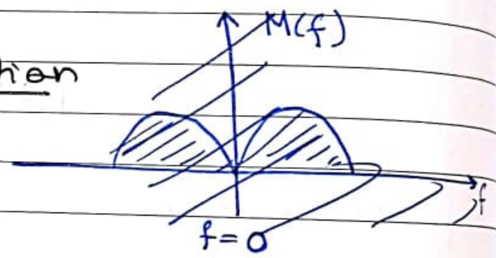
31.7.18

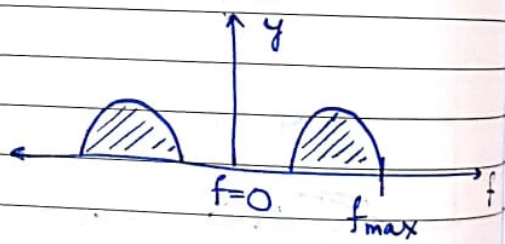
Lec 9

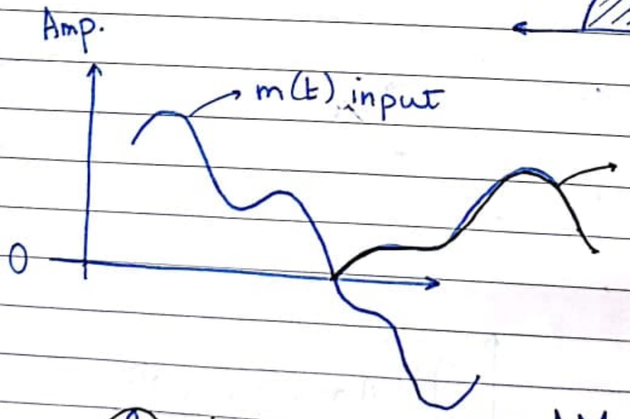
Amplitude Modulation

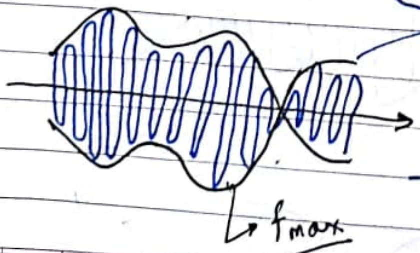
Direct Detection





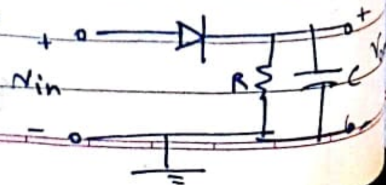


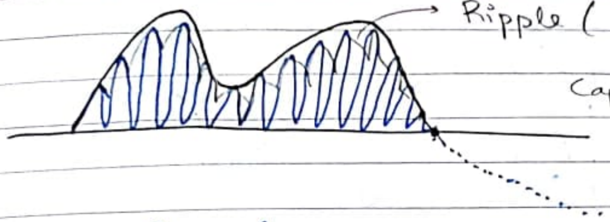




→ AM signal $= A_c \cdot k_a\, m(t) \cos(2\pi f_c t)$

Rectify →





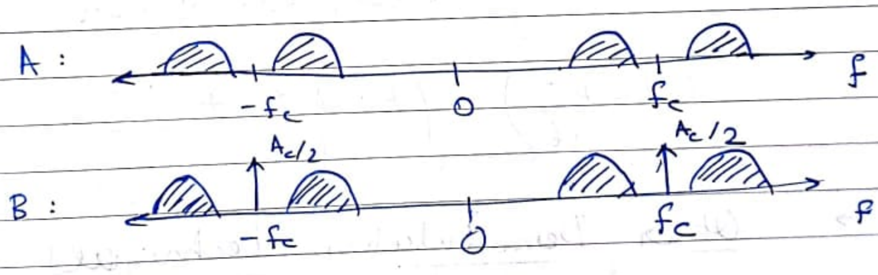
(i) $f_c >> f_{max}$

(ii) $f_{max} << \frac{1}{2\pi RC} << f_c$

→ Keyword : $\boxed{\text{Envelope Detector}}$: Rectifier based
↳ m(t)

AM signal : $= A_c\, k_a\, m(t) \cdot \cos(2\pi f_c t)$ — (A)
↳ Double side band with suppressed carrier (DSB-SC)

$A_c\,(1 + k_a\, m(t)) \cdot \cos(2\pi f_c t)$ — (B)
↳ " " " " Full carrier (DSB-FC)



→ m(t) : Normalized s.t peak amplitude = 1 unit
→ Hence in B, choice of k_a (M.I) should be
↳ (DSB-FC)

~~$k_a > 1$:~~
$k_a \leq 1$: (Envelope m(t) has no zero crossing)

→ ~~We must do this since we have no control over~~

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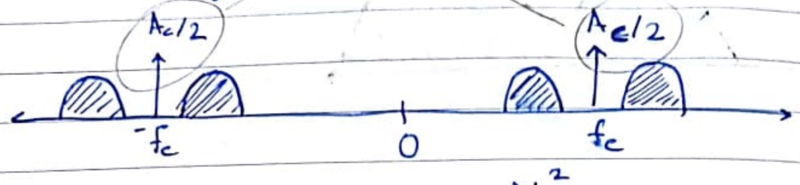
Disadvantage → Transmission requires ↑ Power



Power comparison : $\frac{V_{RMS}^2}{R}$

↳ $Z \approx 50\,\Omega$ for Antenna's

ex $k_a = 0.5$

Tx power in A : (RMS Amplitude^2)

$\Rightarrow$ for $m(t)$: $A_c^2 \times (0.5)^2 \times \frac{1}{2} \times \frac{1}{2} = \frac{A_c^2}{16}$

Proof : $\frac{1}{T}\int_0^T \cos^2(2\pi f_1 t) \cdot \cos^2(2\pi f_2 t)\, dt = \frac{1}{4}$

$\left(A : \frac{A_c^2}{16}\right) \quad \left(B : \frac{A_c^2}{16} + \frac{A_c^2}{2}\right)$

→ **Other Demodulation techniques**

Coherent Detection (Done before, Multiplier based)

Rx: $A_c k_a \cos 2\pi f_c t \cdot m(t) \times \underbrace{\cos(2\pi f_c t)}_{@\ Rx}$

$\downarrow$

$\frac{A_c k_a}{2}\left[1 + \cos(2\pi \cdot 2 f_c t)\right] m(t)$

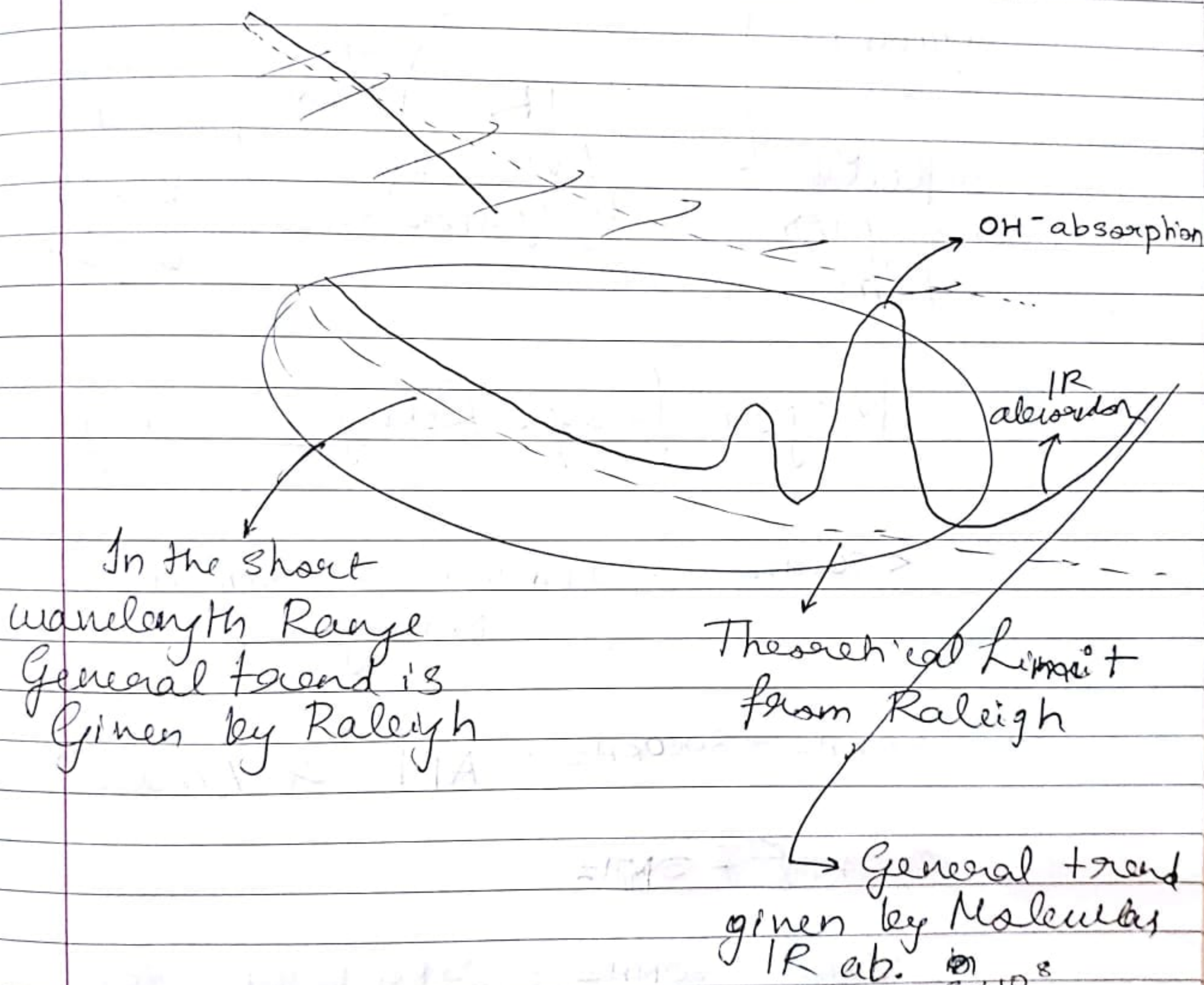
$\downarrow$ LPF

$\frac{A_c k_a}{2} m(t)$

$\frac{3\times10^{8}}{10^{-2}}$ 3×10^{10} 30 GHz

Quiz - 1 preparation

- Attenuation in Optical fibers

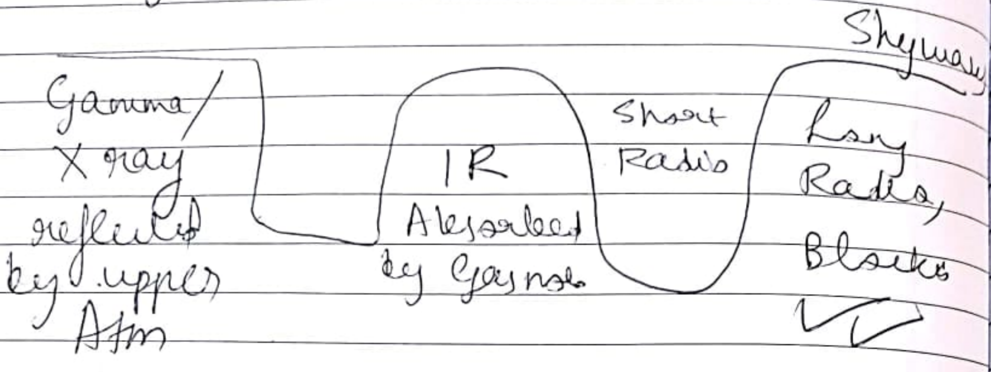


$\frac{3\times10^{8}}{3\times10^{6}}$ 100 m

→ Sky waves

- Propagation of Radio waves Ref/Refracted back to earth.
- From ionosphere
- Used for Short wave high freq. transmission in the range 3 MHz - 30 MHz

Opacity of Atmosphere towards diff. Ranges



Major freq. Ranges (EM)

2.8.18

< 30KHz : Underwater Communication, Navigation

30KHz – 300KHz : AM & Amateur radio

~~300KHz – 3MHz~~

3MHz – 30MHz : Sky waves for Aviation, Radio etc

30 – 300MHz : FM

300MHz – 3GHz : Television, oven, Mobile phones, Wifi, Bluetooth

3 - 30 GHz : More Wifi

30 GHz - 300 GHz : ~~As~~ Radio Astronomy

> 300 GHz : Medical Imaging

Formulas: $\text{rect}(at/a) \xrightarrow{FT} \text{sinc}(f/a)$

$\text{tri}(at) \xrightarrow{FT} 1/a\ \text{sinc}^2(f/a)$

$H_h(f) = -j\,\text{sign}(f)$

$h_h(t) = \frac{1}{\pi t}$

$$M(f) + jM_h(f) = \begin{cases} 2|M(f)|e^{j\phi(f)} & f>0 \\ 0 & f<0 \end{cases}$$

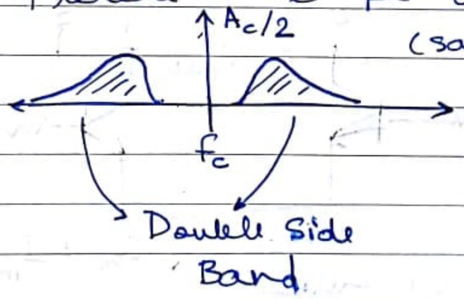
2.8.18

Lec 10

Recap: AM (Amplitude Modulation)

Method 1) Present envelope 0 crossing

DSB - FC



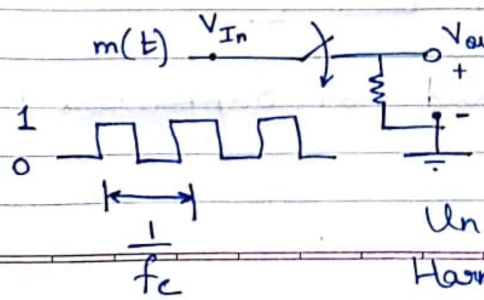
$1 + K_a m(t) \longrightarrow \otimes \longrightarrow$ AM

$\cos(2\pi f_c t)$

How can we Multiply?

↓

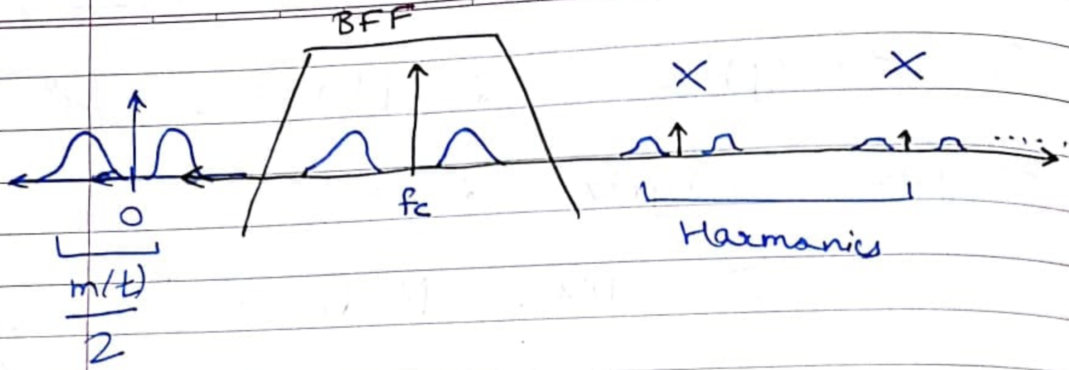
Explained



$V_{out} = m(t) \times$ square wave

$= m(t)\left[\frac{1}{2} + \underbrace{K\cos(2\pi f_c t)}_{\text{Desired carrier}}\right.$

Undesired Harmonic carriers $\left\{ + \frac{K}{5}\cos(2\pi \cdot 5 f_c t) + \ldots \right.$



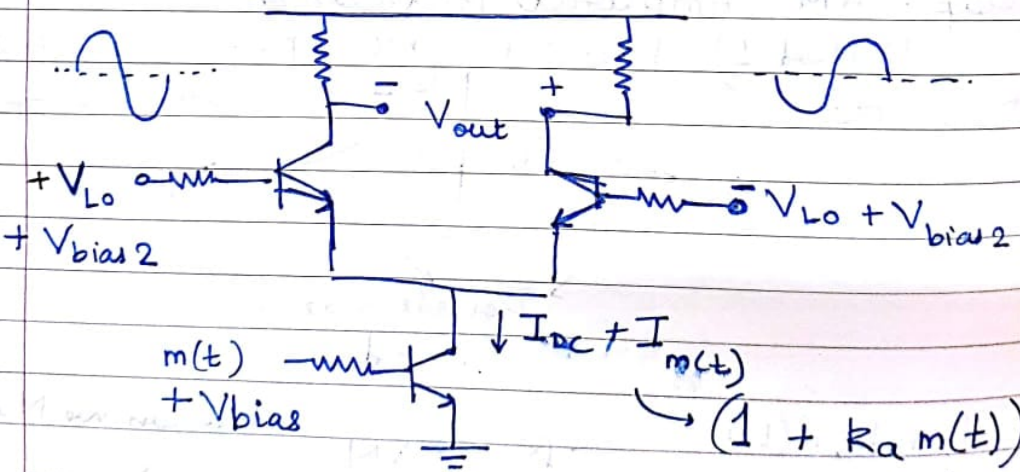
→ Implementation 1

- A simple diode implementation

$m(t) + \cos(2\pi f_c t)$

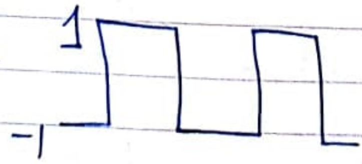
V_{out} → Has DC offset & other new har[monics]

→ 2) ~~Gilbert Cell~~ & Diff. Amp Based →



$(1 + k_a m(t))$ ⇒ DSB - FC

$m(t) \times$ Gain $\times$



→ This was the method for ~~Suppressed~~ FC Modulation

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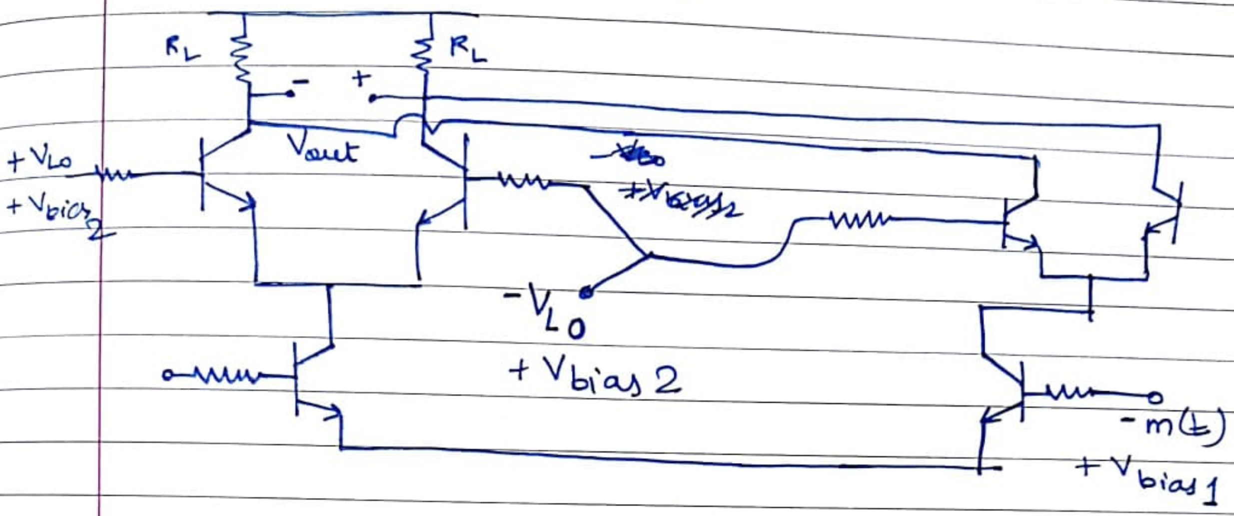
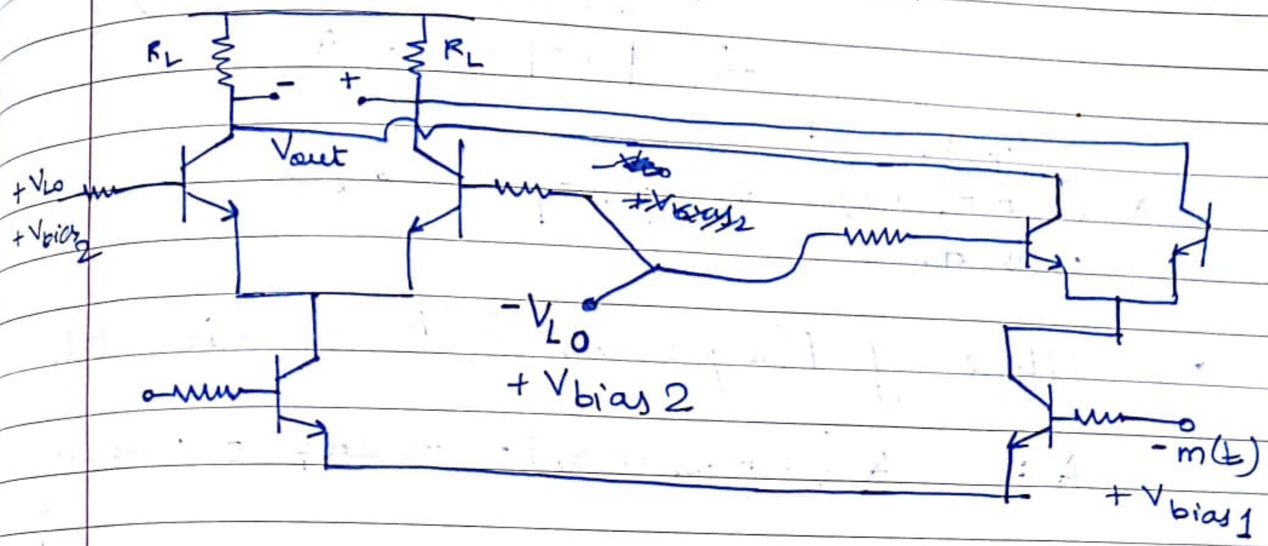
→ For SC modulation, we must use Gilbert cell based Multiplication



→ For SC modulation, we must use Gilbert cell based Multiplication



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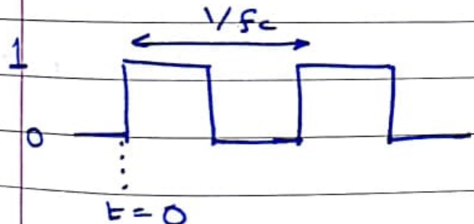
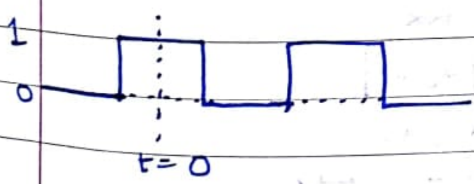
Lecture 11



$$= \frac{1}{2} + \frac{2}{\pi}\sum_{k=1}^{\infty} \frac{\sin\left(2\pi f_c t\cdot(2k-1)\right)}{(2k-1)}$$

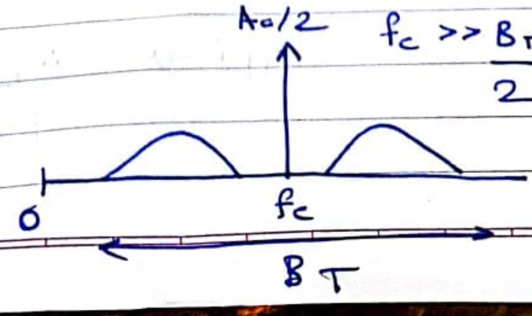


$$= \frac{1}{2} + \frac{2}{\pi}\sum_{k=1}^{\infty} \frac{(-1)^{k+1}\cos\left(2\pi f_c t\cdot(2k-1)\right)}{(2k-1)}$$

AM Receivers

1) Envelope Detector

Recap: Condition on f_c, B_T, R & C of Rectifier

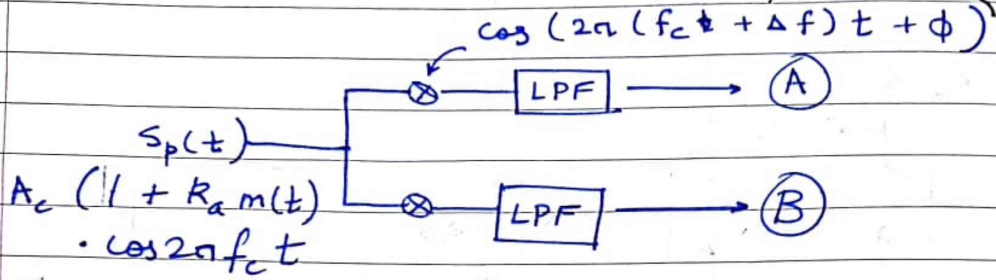


$f_c >> \frac{B_T}{2}$

2. Coherent Detection

- Use the transmit. freq. f_c + phase

$$\cos(2\pi(f_c + \underbrace{\Delta f) t + \phi}_{\text{undesired}})$$



→ Effect of Frequency & Phase mismatch

A : $A_c(1 + k_a m(t)) \cdot \cos 2\pi f_c t \cdot \cos(2\pi f_c t + \Delta f t + \phi)$

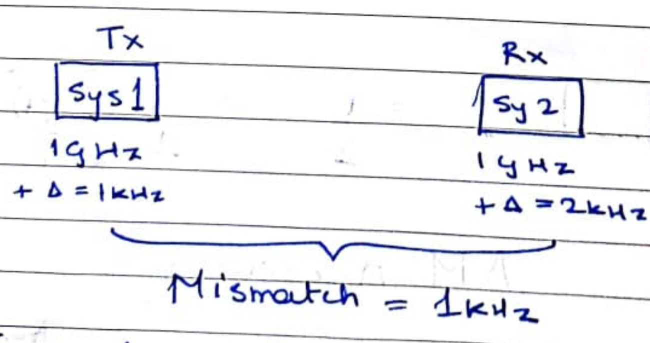
$= \cancel{A_c(1 + k_a}$

Verify :

$$\text{(A)} : \frac{A_c}{2}(1 + k_a m(t)) \cdot \underbrace{\cos(2\pi \Delta f t + \phi)}_{\text{Ideally } = 1}$$

$$\text{(B)} : \frac{A_c}{2}(1 + k_a(m(t))) \cdot \underbrace{\sin(2\pi \Delta f t + \phi)}_{\text{Ideally } = 0}$$

→ Typical Errors



Typical $\frac{\Delta f}{f} = 10\,ppm$

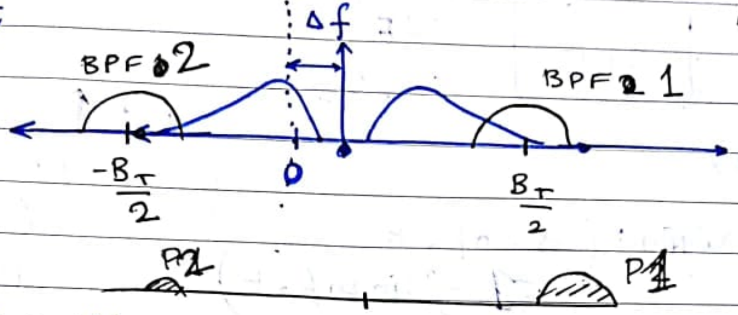
i.e 10 kHz at 1 GHz trans.

→ We Assume signal to be $A + jB$

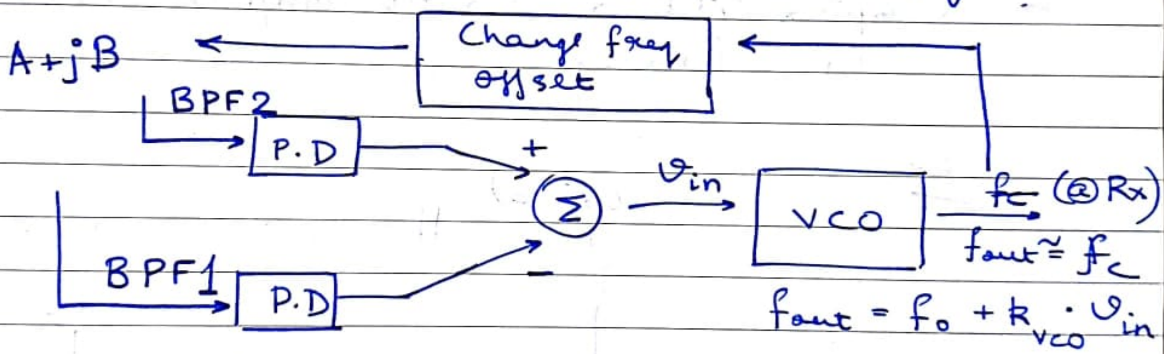
$$A + jB = \frac{A_c}{2}\left(1 + k_a m(t)\right) \cdot e^{j(2\pi \Delta f t + \phi)}$$

<u>Result</u>

$\Delta f > 0$ here.



→ We will compare power output at Rx and Change $(f_c)_{Rx}$ accordingly.



- P.D ≡ Power detector. Converts Input power to a voltage
- BP1 1 and 2 are Band edge loops
- Process is frequency locked loop (FLL)
- Issues
  1) Phase offset
  2) Mismatch in BPFs
  3) Signal may not be real ⇒ Not Centered at 0

ex $m_1(t)\cos(2\pi f_c t) + m_2(t)\sin(2\pi f_c t)$

→ Meaning of -ve frequency BPF

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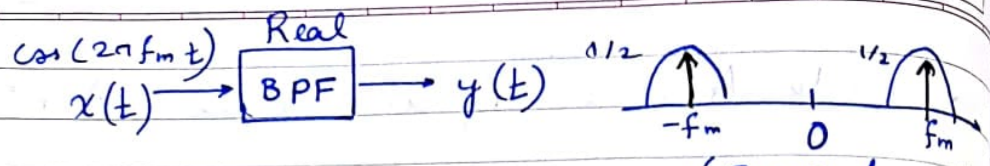
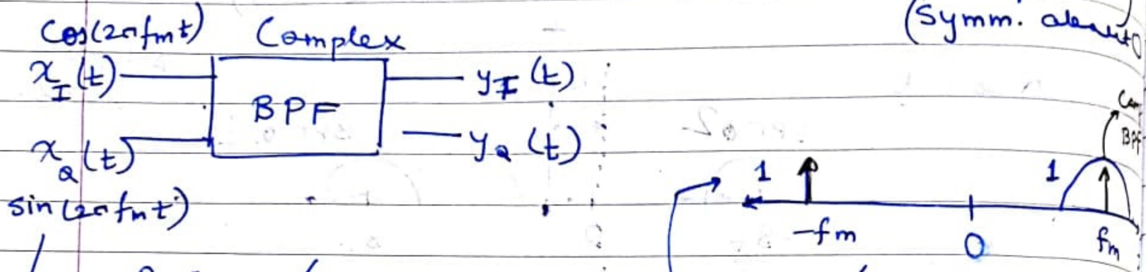


(Real filter)
(Symm. about 0)

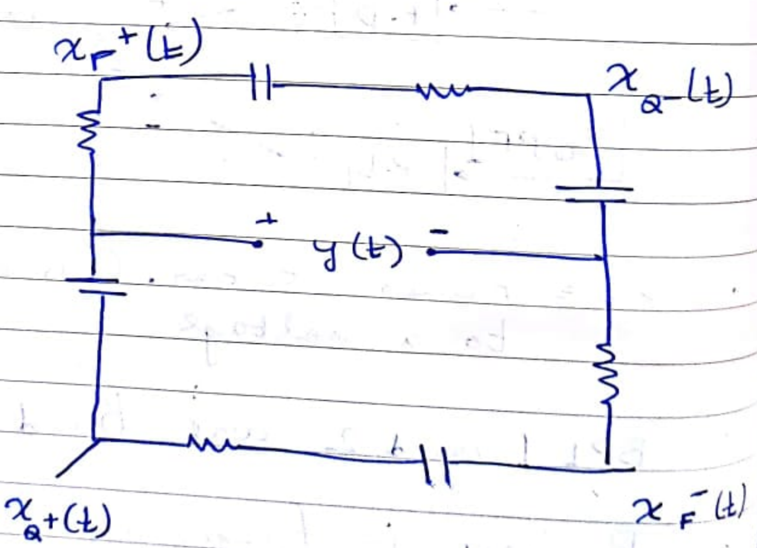


→ Origin: $(\cos 2\pi f_m t - j\sin 2\pi f_m t)$

(Complex filter)
(Not Symm. about 0)

→ Complex filters can be Implemented in Analog

ex

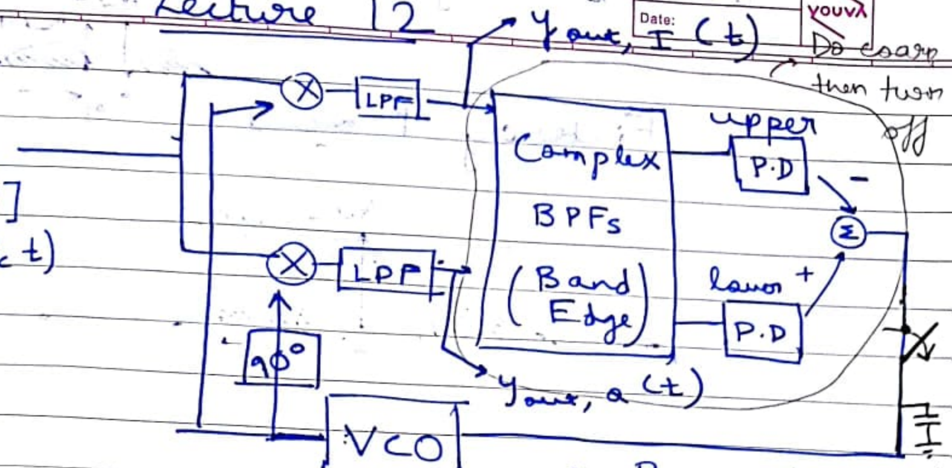


7.8.18

Lecture 12

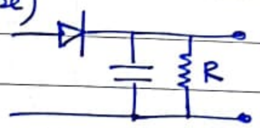
FLL

$S_p(t) = [1 + k_a m(t)] \cdot \cos(2\pi f_c t)$



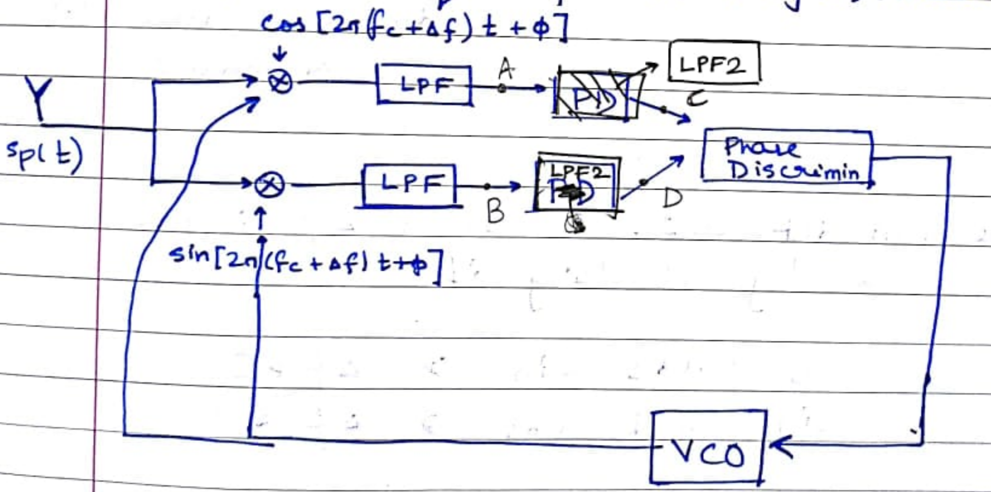
→ Coarse freq. Adjustment

PD: Power detector / pk. detector (1 type)



Costas Loop

→ (fine freq. + phase Adjustment)



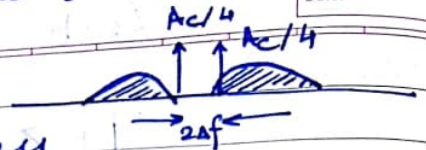
$A = \frac{A_c}{2}[1 + k_a m(t)] \cos(2\pi \Delta f_c t + \phi)$

$B =$ ~~Ac~~ Result after sin ⊗ and taking LPF

$C = \frac{A_c}{2} \cos(2\pi \Delta f_c t + \phi)$ + Residual $m(t)$

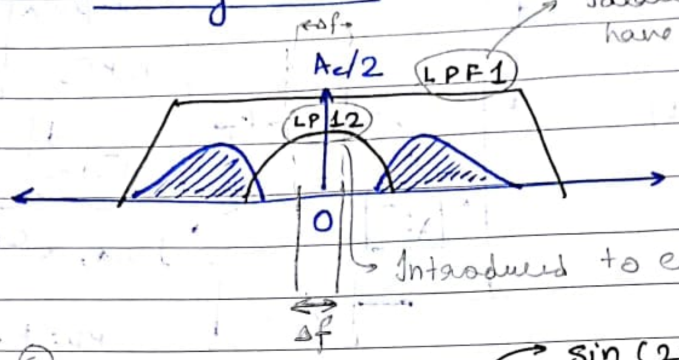
$D = \frac{A_c}{2} \sin(2\pi \Delta f t + \phi)$

At A/B: Signal looks like

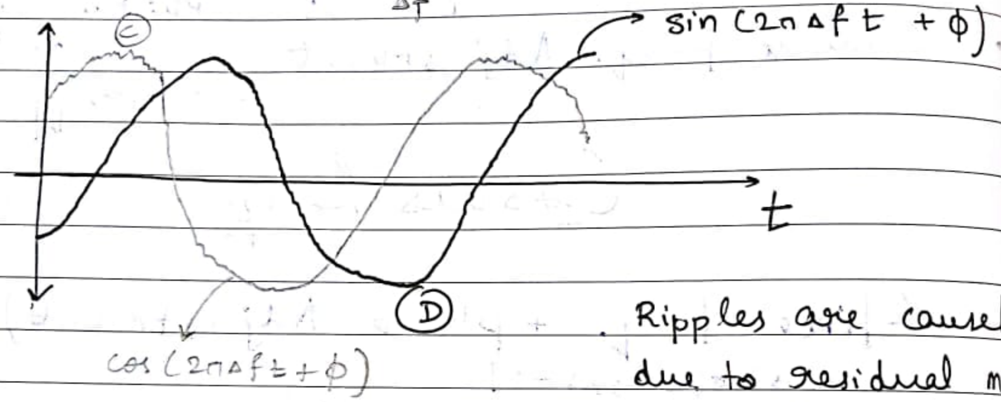


Filtering Process

Ideally this should have been enough



Introduced to extract Δf comp.



Ripples are caused due to residual m(t) component

→ We can determine whether Δf is +ve or -ve by checking if

$$\cos \text{ leads } \sin \Rightarrow \Delta f > 0$$
$$\sin \text{ leads } \cos \Rightarrow \Delta f < 0$$

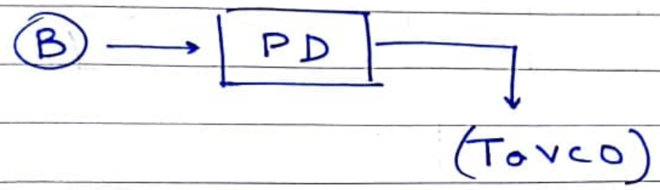
- After ~~Costa~~ This loop converges A and B will stabilise to (loop removes Δf)

$$A = \frac{A_c}{2}\left[1 + k_a m(t)\right]\cos\phi$$

$$B = \frac{A_c}{2}\left[1 + k_a m(t)\right]\sin\phi$$

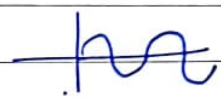
Phase (ϕ) correction

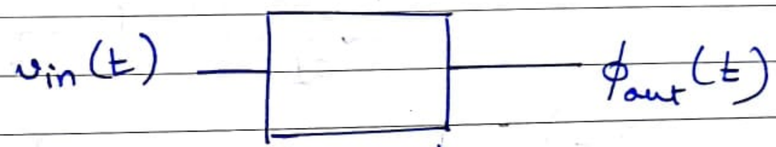
- The VCO also permits a control on the phase shift



As long as $P_B > 0$, correct for ϕ

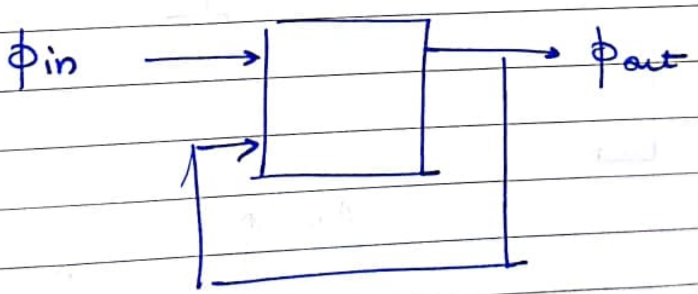
→ More on VCO :





$$\phi_{out} = 2\pi f_0 t + k_{vco} \int_0^t v_{in}(t)\, dt$$

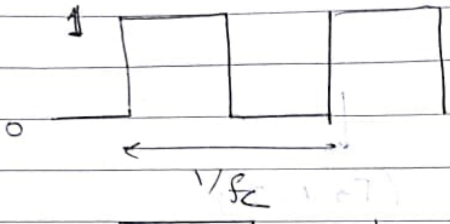
PLL



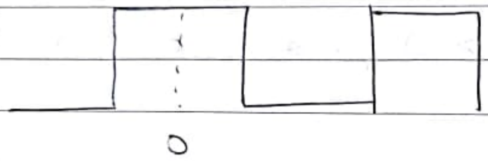
Quiz 2 Preparation

- **Mixers for AM**

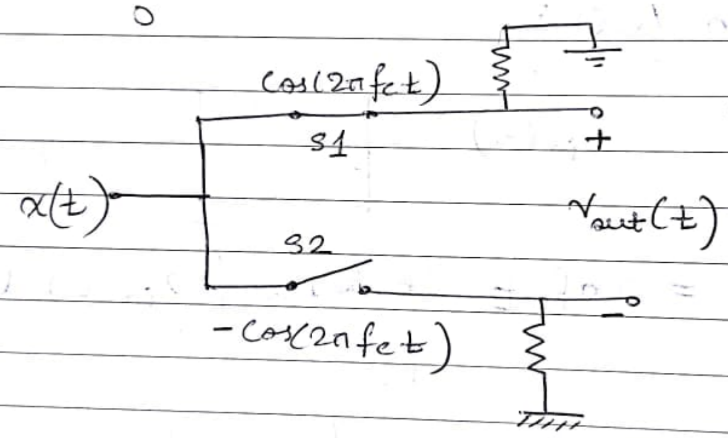
General idea : Use Square wave.

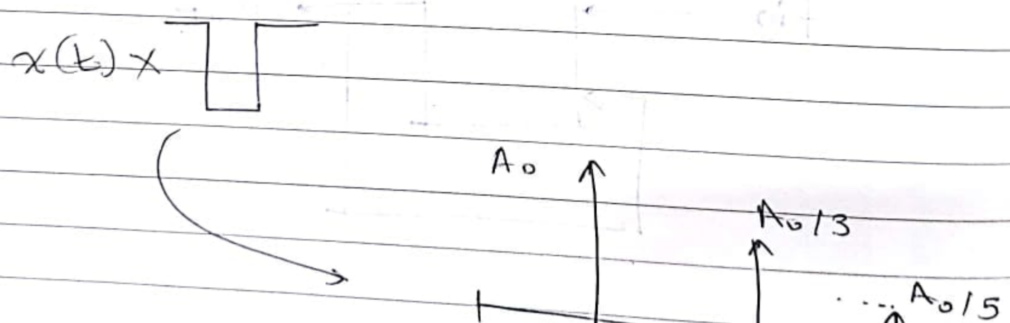


$$= \frac{1}{2} + \sum_{k=1}^{\infty} \frac{\sin(2\pi f_c t \cdot (2k-1))}{(2k-1)}$$



$$= \frac{1}{2} + \sum_{k=1}^{\infty} \frac{(-1)^{k+1} \cos(2\pi f_c t\,(2k-1))}{(2k-1)}$$

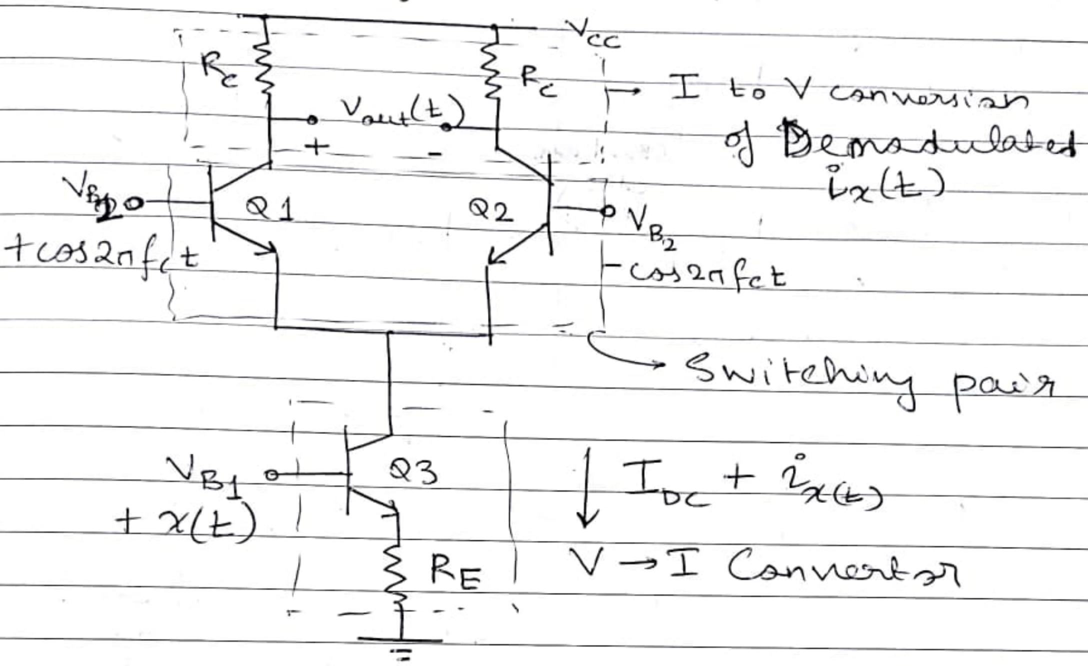




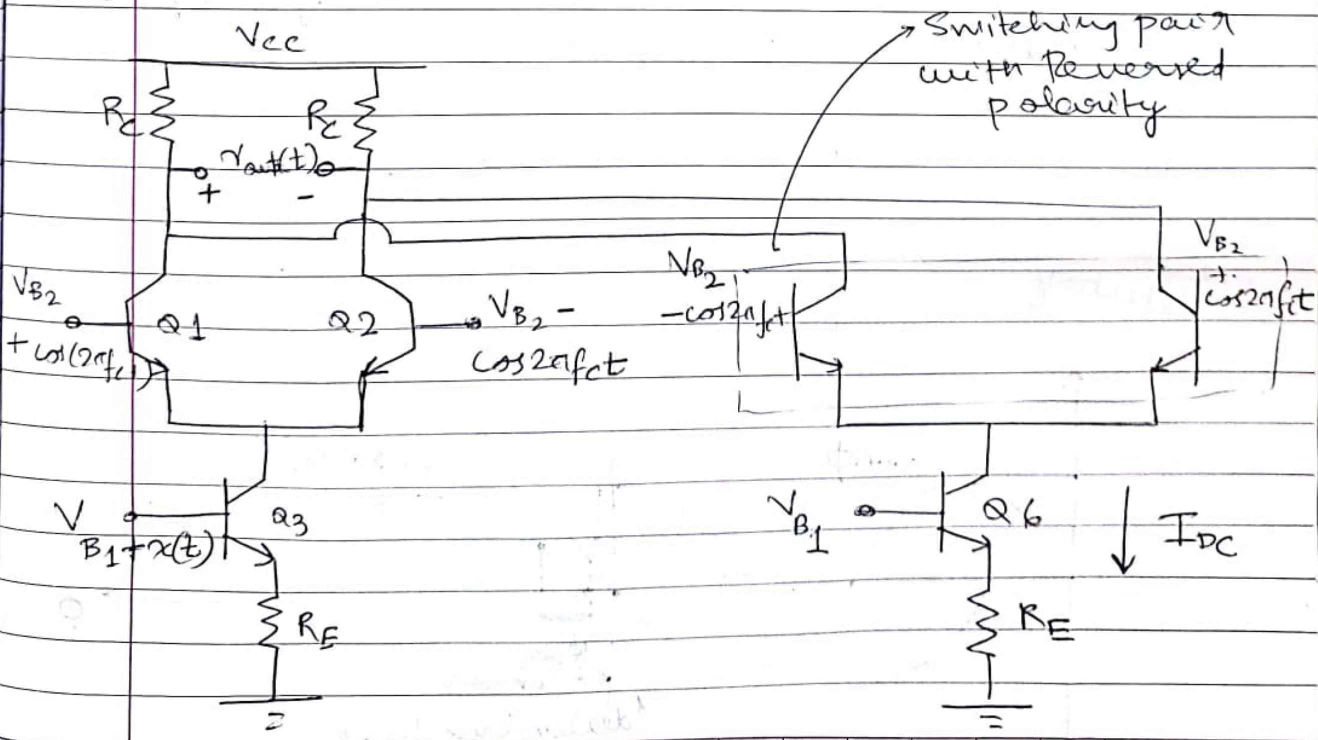
→ Harmonics cause noise and force ineffective utilisation of Spectrum

Implementation for DSB-FC

(a) Single Balanced Mixer DSB-FC



(b) Double Balanced Mixer DSB-SC



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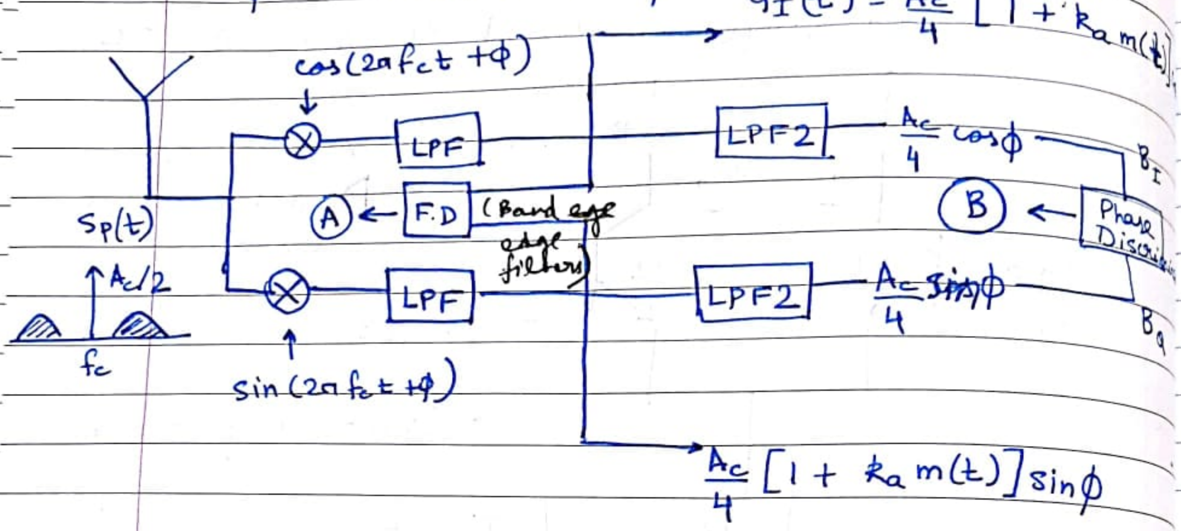
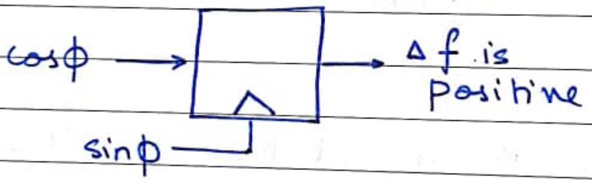
9.8.18 Thu

Lecture 13

$\phi := 2\pi \Delta f t + \Delta\phi$



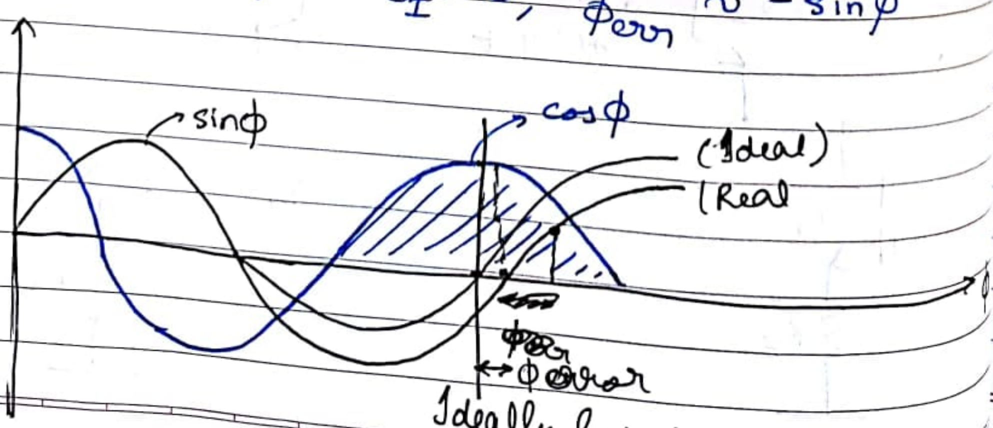
Implementation of Phase discriminator



if on the rising edge of $\sin\phi$
if $\cos\phi = 1$
then Δf is +ve
if 0 Δf is -ve

Goal: $\cos\phi = 1, \ \sin\phi = 0$

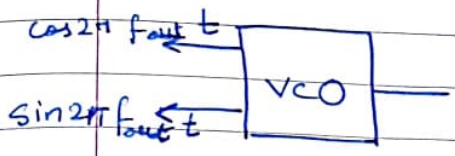
Better than power detector:
1. Sample B_I, B_Q
2. If $B_I > 0$, $\phi_{err} \sim \sin\phi$
   If $B_I < 0$, $\phi_{err} \sim -\sin\phi$

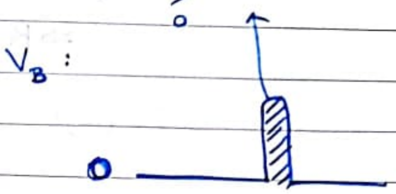


Ideally lock here

VCO output phase

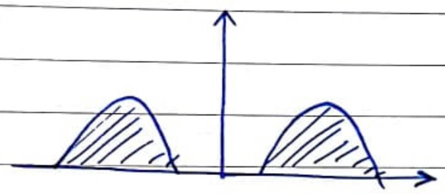
$$(f_0 + k_{vco} \cdot V_A)\,t \quad + k_\phi \int_0^t V_B \cdot dt$$





Impulse adds finite Integral for correction

DSB – RC



$A_c [1 + k_a m(t)] \cdot \cos 2\pi f_c t$

$k_a < 1 \Rightarrow$ DSB – FC

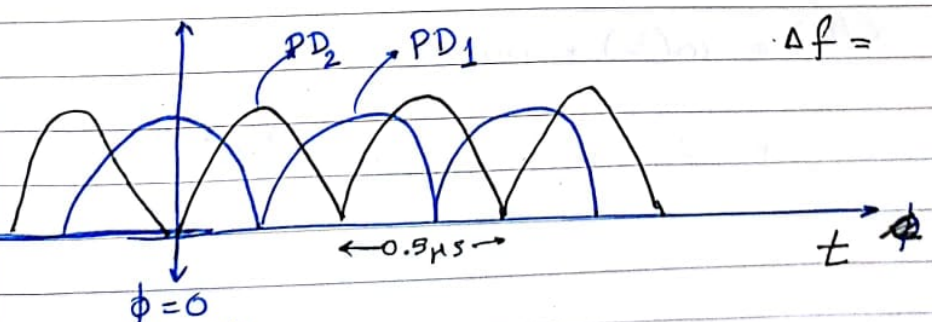
$A_c [0.1 + k_a m(t)] \cdot \cos 2\pi f_c t$: DSB – RC

→ We can still do coherent detection

→ Coherent Detection cannot be done

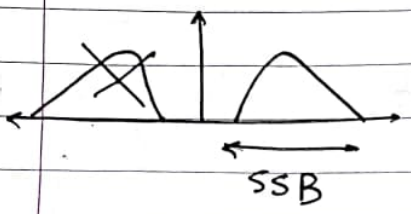
$A_c k_a m(t) \cdot \cos 2\pi f_c t$: DSB – SC

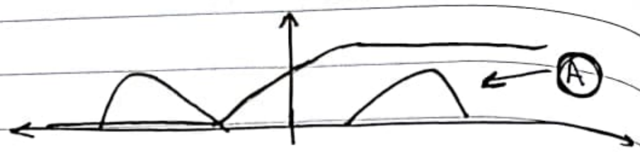
$\Delta f = 1\,MHz$



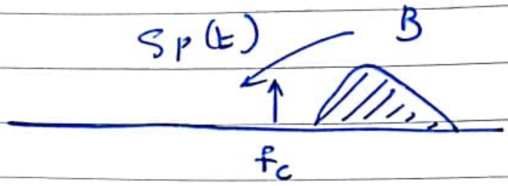
Use Above technique in place of coherent

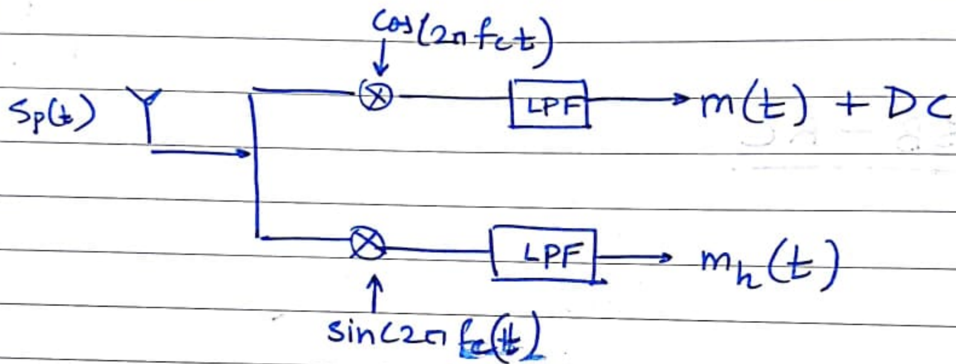
SSB





$s_p(t)$





$$s_p(t) = m(t) \cdot \cos 2\pi f_c t - m_h(t) \cdot \sin 2\pi f_c t$$
$$+ A_c \cos 2\pi f_c t$$

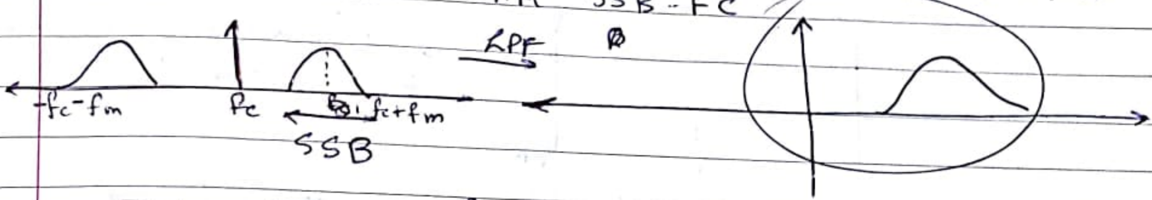
$s_p(t) =$

$$x(t) = m(t) + j m_h(t)$$

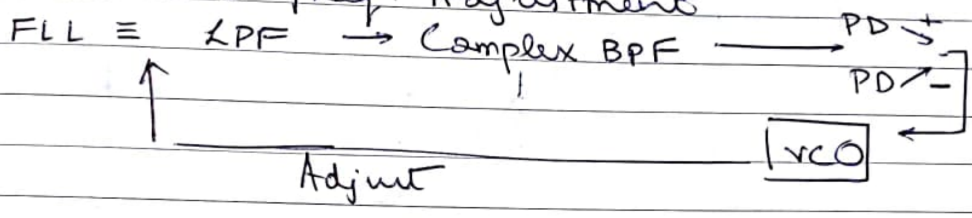
$$s_p(t) = Re\left(x(t)\, e^{j2\pi f_c t} \right)$$

Quiz 2 Prep.

- How to transmit AM SSB-FC



- FLL: Coarse freq. Adjustment



- Costas Loop: fine freq. / Phase Adjustment

13.8.18

Lecture - 14

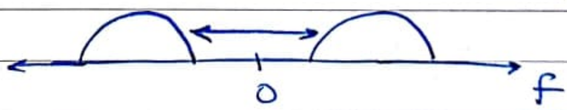
Quiz 2 on Mon
F.T, H.T, AM

- Vestigial - Sideband Modulation (VSB)

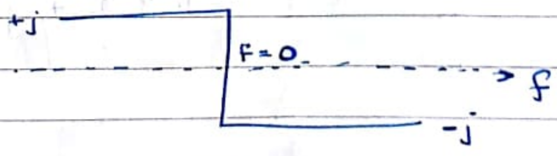
→ Combines low power (Reduced / Sup. carrier) and ↓ B/w hoggy : SSB type char.

SSB :

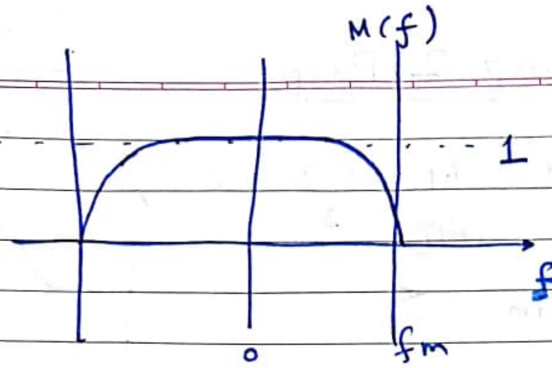
M(f) (Assumption : No DC)



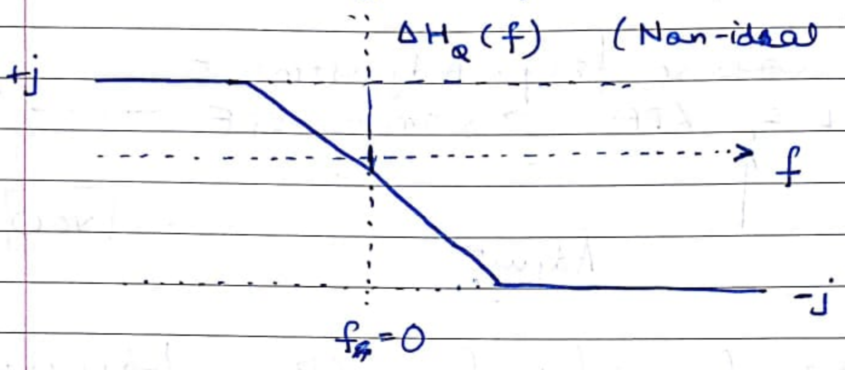
Hilbert Tf :



OPC, Ideal filter.

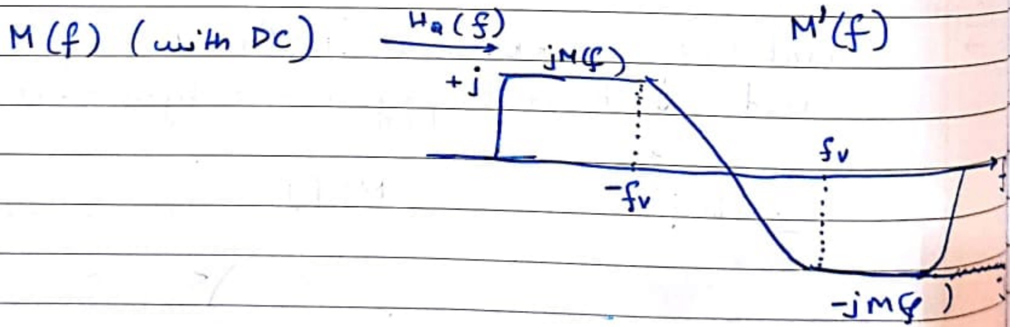


· Finite DC with non ideal filter will have same non Idealities

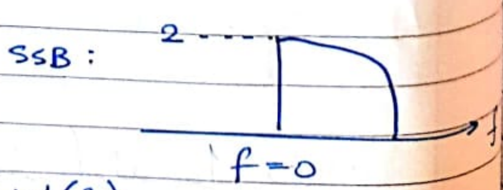


SSB: $M(f) + j M_h(f)$

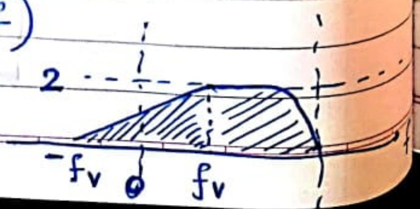
VSB: $M(f) + j M'(f)$



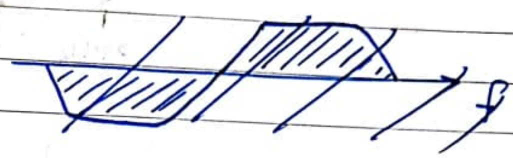
SSB: $M(f) + j M_h(f)$ (Ideal)

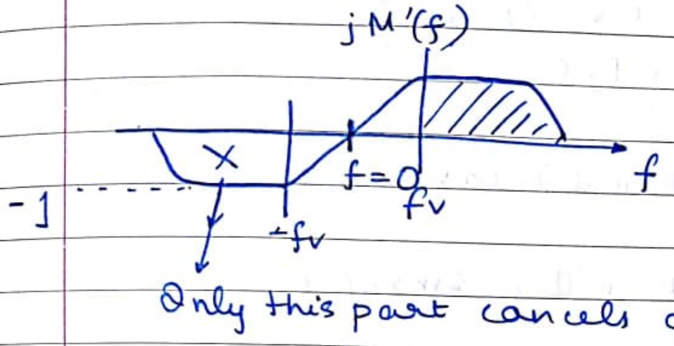


VSB: $M(f) + j M'(f)$



$$M'(f) \xrightarrow{\times j} jM'(f)$$



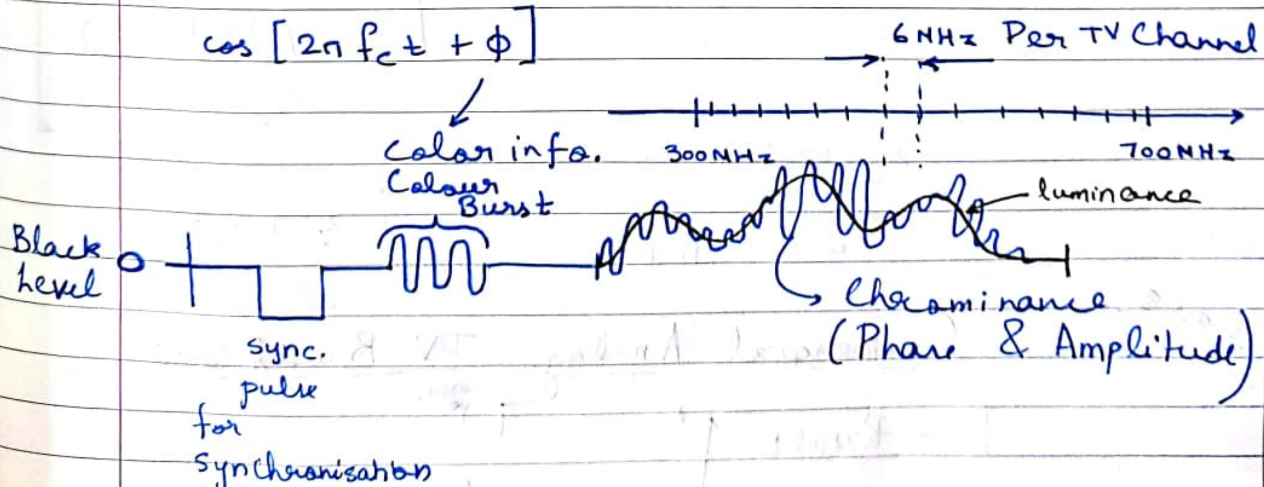


→ Result if W = Baseband B/W (fm-0)
then,

$$B_T = W + f_v \quad \left(W \triangleq \text{Baseband } b/w \right)$$

B_T → Transmission B/W

TV Signals : 6 MHz Bands



1-D TV signal / One line on screen

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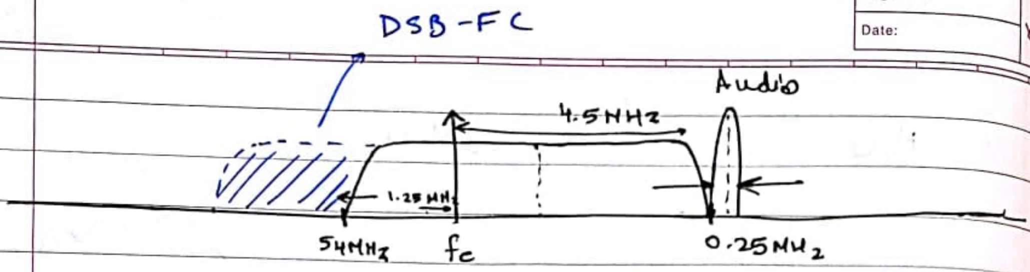


T.V. Channel

N.S.B + F.C

VSB + FC: $\frac{A_c}{2} k_a m(t) \cdot \cos 2\pi f_c t$

$\frac{A_c}{2} k_a m'(t) \cdot \sin 2\pi f_c t$

$+ A_c \cos 2\pi f_c t$

Envelope Detection:

$$A_c^2 \left[\left(1 + \frac{k_a}{2} m(t)\right)^2 + \left(\frac{k_a}{2} m'(t)\right)^2 \right]^{1/2}$$

$$= A_c^2 \left(1 + k_a m(t)\right) \left[1 + \frac{\left(k_a m'(t)\right)^2}{\left(1 + \frac{k_a m(t)}{2}\right)^2} \right]$$

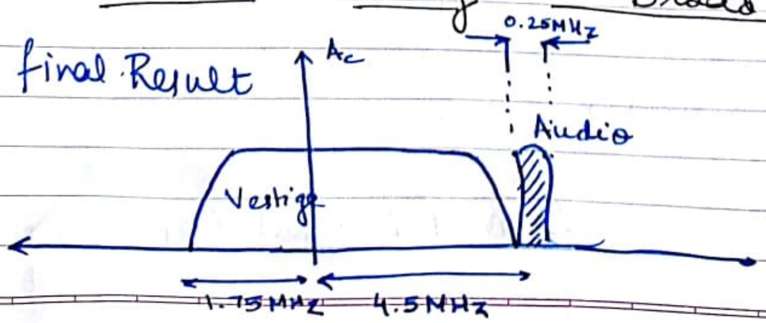
$x_I \cos 2\pi f_c t$

$x_Q \sin 2\pi f_c t \rightarrow \sqrt{x_I^2 + x_Q^2}$

14.8.18

Commercial Analog TV Broadcast

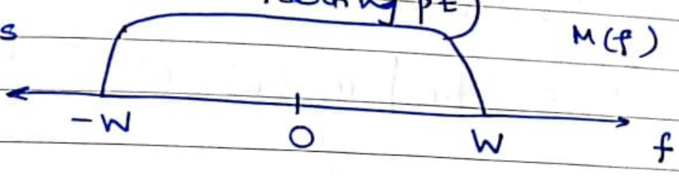
final Result



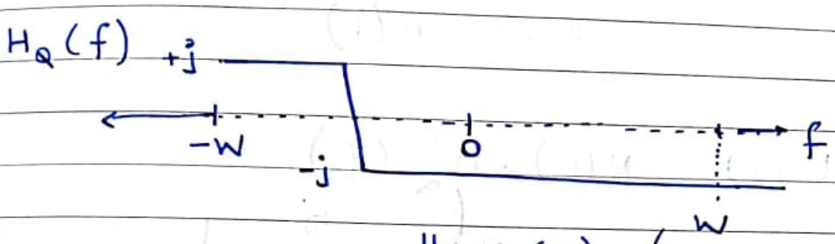
→ Audio is treated seperately (Ignore for now)

(Starting pt)

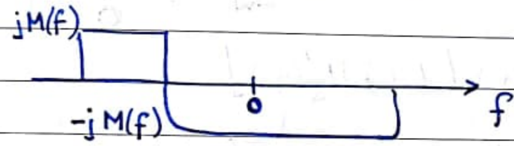
Information is not lost



W = 4.5 MHz

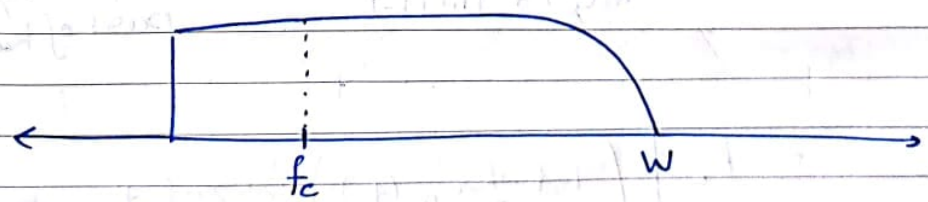


$\Downarrow$ $M_1(f)$ (Not purely real in time domain)



→ $H_Q(f)$ Applied to $M(f)$ ~~we get~~ and Multiplied by carrier in time domain

$$\{M(f) + \underbrace{jH_Q(f)\,M(f)}_{jM_1(f)}\} * e^{j2\pi f_c t}$$



Can we write,

$$s_p(t) = \cos(2\pi f_c t)\cdot m(t) - m_1(t)\sin 2\pi f_c t$$

→ No Since the Hilbert transform used was not symmetric about 0

$$s_p(t) = \cos(2\pi f_c t)\cdot m(t) - m'(t)\sin 2\pi f_c t$$

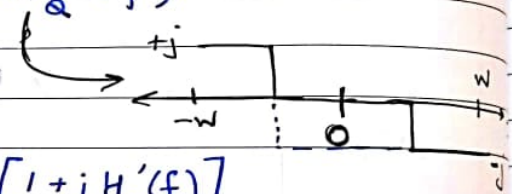
Where,

We have Redefined (or rather recalled the def. of $s_p(t)$ as)

$$s_p(t) = Re\left[m_c(t) \cdot e^{j2\pi f_c t} \right]$$

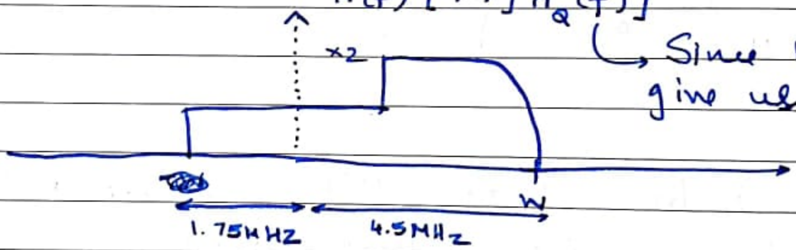
$$m_c(t) \to m(t) + jm'(t)$$

$$M'(f) = M(f) \cdot H_Q'(f)$$



$M(f)\,[1 + j H_Q'(f)]$

↳ Since $H_Q(f)$ didn't give us what we need



$$s_p(t) = Re\left[m_c(t) \cdot e^{j2\pi f_c t} \right] \cdot \frac{A_c k_a}{2} + A_c \cos 2\pi$$

$m_c(t) \to m(t) + jm'(t)$

Acc. for x2 / part of k_a

$$= A_c \left[\left(1 + \frac{k_a}{2} m(t) \right) \cos 2\pi f_c t - \frac{k_a}{2} m'(t) \sin 2\pi \right] \quad -(1)$$

$\frac{1}{2}$ → Replace by $\frac{1}{\sqrt{\alpha^2 + \beta^2}}$

$$A_c\left(1+\frac{k_a}{2}m(t)\right)\left[1+\left(\frac{m'(t)\cdot k_a/2}{1+\frac{k_a m(t)}{2}}\right)^2\right]^{1/2}$$

Since Magnitude of ~~$\alpha \sin$~~ $\alpha \cos + \beta \sin = \sqrt{\alpha^2+\beta^2}$

$$A_c\left[\left[\left(1+\frac{k_a}{2}m(t)\right)^2+\left(\frac{k_a}{2}m'(t)\right)^2\right]\right]^{1/2}$$

$$= A_c\left(1+\frac{k_a}{2}m(t)\right)\left[1+\underbrace{\left(\frac{m'(t)\,k_a/2}{1+\frac{k_a}{2}m(t)}\right)^2}_{\text{Distortion}}\right]^{1/2}$$

To Reduce Distortion:

(1) $\downarrow k_a$

(2) $\downarrow m'(t)$

{ from (1) or phasors

→ Sat. 2:30pm Tutorial

Angle Modulation

Angle Modulation → FM, PM

$A_c \cos(2\pi f_c t + \phi)$

↳ message info

$\phi = k_\phi\, m(t)$: phase mod.

$\Delta \text{freq} = \frac{d\phi}{2\pi dt}$: $k_f\, m(t)$: freq. mod.

w

→ f

−j

didn't

true

need

$\cos 2\pi f_c t$

$\sin 2\pi f_c t$

−(1)

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Angle Modulation

$$s(t) = A_c \cos[2\pi f_c t + \phi_m(t)]$$

$$\phi_m(t) = k_\phi \cdot m(t) \quad : \quad PM$$

$$f_i(t) = \frac{1}{2\pi}\frac{\delta \phi_m}{\delta t} = k_f\, m(t) \Rightarrow FM$$ (Ignoring the carrier freq)

$$\Rightarrow \phi_m(t) = 2\pi k_f \int_0^t m(\tau)\, d\tau$$

→ If amp. of $m(t) = 1$
$\Rightarrow k_\phi$: Modulation Index

AM : $A_c[1 + k_a m(t)] \cos(2\pi f_c t)$
↳ MI of AM

Say : $\boxed{m(t) = A_m \cos 2\pi f_m t}$

$$\phi_m(t) = \frac{2\pi k_f \cdot A_m}{2\pi f_m} \cdot \sin(2\pi f_m t) = \frac{k_f A_m}{f_m} \sin(2\pi f_m t)$$

Added to phase

→ Max freq. deviation, from $f_i(t) = k_f m(t)$

$$\Delta f_{max} = \boxed{|k_f \cdot A_m|}$$

Mod. index for FM $\triangleq \frac{\Delta f_{max}}{f_m}$ (rad.)

$$\boxed{\beta = \frac{\Delta f_{max}}{f_m} = \frac{k_f A_m}{f_m}}$$

Scanned by CamScanner

→ These Quantities are not defined for Multiple tones

$$s_{p,FM}(t) = A_c \cos[2\pi f_c t + \underbrace{\beta}_{\left|\frac{k_f A_m}{f_m}\right|} \sin(2\pi f_m t)]$$

Assuming $\beta << 1$ ∴ $\cos(A+\delta) \approx \cos A - \sin A \cdot \delta$

$$s_{p,FM}(t) \cong A_c \cos(2\pi f_c t) - \beta A_c \sin(2\pi f_c t)\sin(2\pi f_m t)$$

$$= A_c \cos(2\pi f_c t) + \frac{\beta A_c}{2}\left[\cos 2\pi(f_m+f_c)t - \cos 2\pi(f_c - f_m)t\right]$$

↳ Approximate.

Comparison with AM

AM : $A_c(1 + k_a m(t)) \cdot \cos(2\pi f_c t)$

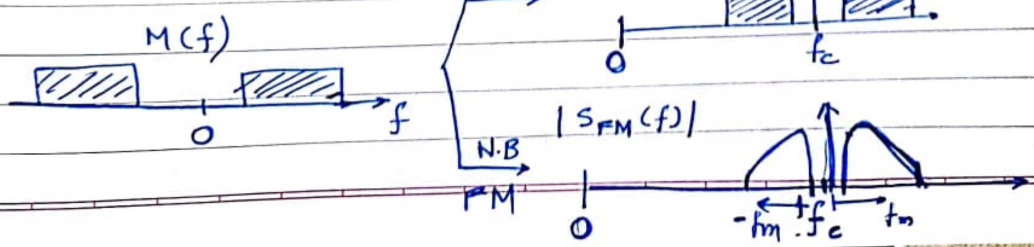
$$= A_c[1 + \underset{\text{mod. ind}}{\beta_{AM}} \underbrace{\cos 2\pi f_m t}_{m(t)}] \cos 2\pi f_c t$$

$m(t) = A_m \cos 2\pi f_m t$ $\quad \beta_{AM} = k_a \cdot A_m$

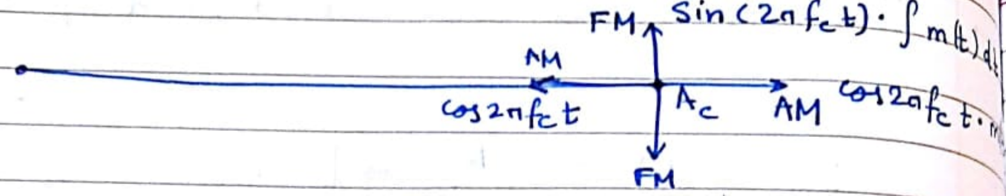
$$= A_c \cos 2\pi f_c t + \frac{\beta_{AM} A_c}{2}\left[\cos 2\pi(f_c+f_m)t + \cos 2\pi(f_c - f_m)t\right]$$

↑ exact

→ The expression for FM is approximate
$\beta << 1$: Condition for Narrow Band FM



→ Understand Amplitudes with Phasors



→ Comparison: Amplitude Mod. & Angle Mod.

Adv of ϕM

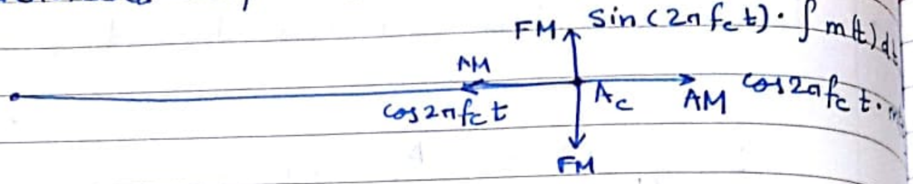
→ More Immune to additive noise
→ Can use Non-linear Power Amp(PA) in Tx which is more Power efficient.

→ PA η : 15% - ~~60%~~ 70%
Non linear → Better

Disad.

→ Spectrally inefficient

→ Understand Amplitudes with Phasors



→ Comparison: Amplitude Mod. & Angle Mod.

Adv of ϕM

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→ PA η : 15% - ~~60%~~ 70%

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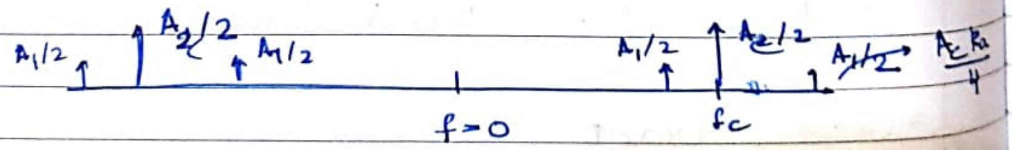
Disad.

→ Spectrally ineffuceint

20.8.18

→ Power in Sidebands $A_1\cos_1 + A_2\cos_2 + A_3\cos$



$$C.P = \frac{A_c^2}{2} \times 2$$

$$S.P = 4 \cdot \frac{A_c^2 k_a^2}{16}$$

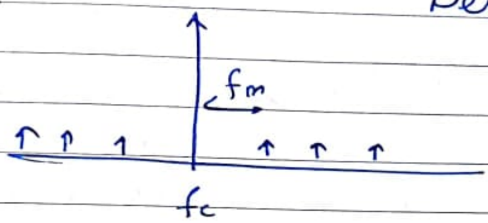
Angle Modulation

$$S_{P,FM}(t) = A_c \cos(2\pi f_c t + \beta \cdot \cos 2\pi f_m t)$$

$$= Re\left[A_c e^{-j2\pi f_c t} \cdot s(t)\right] e^{-j2\pi f_m t \cdot \beta}$$

$$\tilde{S}(t) = \sum c_n \cdot e^{j\beta 2\pi f_m t \cdot n}$$

↳ Bessel function

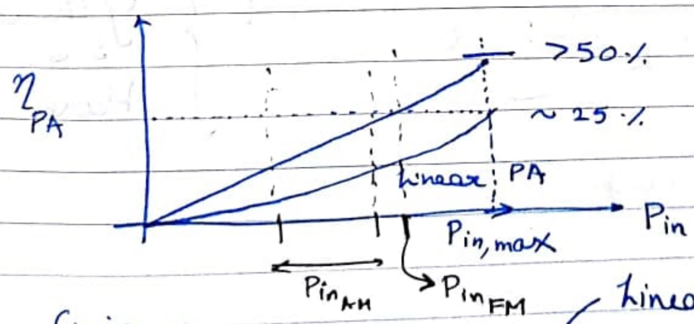


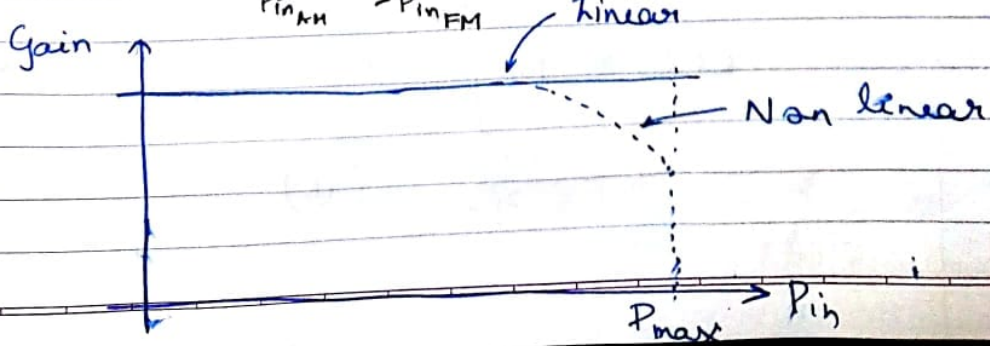
21.8.18

Lecture

Frequency Modulation

- → Advantages: Power efficient Tx (Power Amp.)
- → Can use a Nonlinear Power Amplifier
- → Immune to additive noise
- → Disadvantage: More B/W Required





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→ In case of Angle modulation, there is a single Input power level ⇒ No Distortion

$$s_p(t) = \mathrm{Re}\left[\tilde{s}(t)\cdot e^{j2\pi f_c t}\right]$$

$\tilde{s}(t) = m(t)$: Real ⇒ AM (Angle $= 0°$)

$\;\;\;\;\;\; = e^{j\phi_m(t)}$: const. amplitude complex] Angle mod.

⇒ $\tilde{s}(t)$ is called complex envelope

$$m(t) = A_m \cos 2\pi f_m t$$

$$\phi_{FM}(t) = \beta \sin 2\pi f_m t \, ; \quad \beta = \frac{\boxed{k_{FM}} \cdot A_m}{f_m}$$

$$e^{j\beta\sin(2\pi f_m t)} = \sum_{n=0}^{\infty} \frac{(j\beta \sin 2\pi f_m t)^n}{n!} = \cdots$$

$$\cdots \sum_{k=-\infty}^{\infty} \underbrace{C_k(\beta)}_{\text{complex coefficients}} \cdot e^{jk2\pi f_m t}$$

{ Generalised Sum of all Harmonics

where,

$$C_k(\beta) = f_m \int_{-\frac{1}{2f_m}}^{\frac{1}{2f_m}} \underbrace{e^{j\beta \sin 2\pi f_m t}}_{\tilde{s}(t)} \cdot e^{-jk2\pi f_m t}\, dt$$

$$x = 2\pi f_m t$$

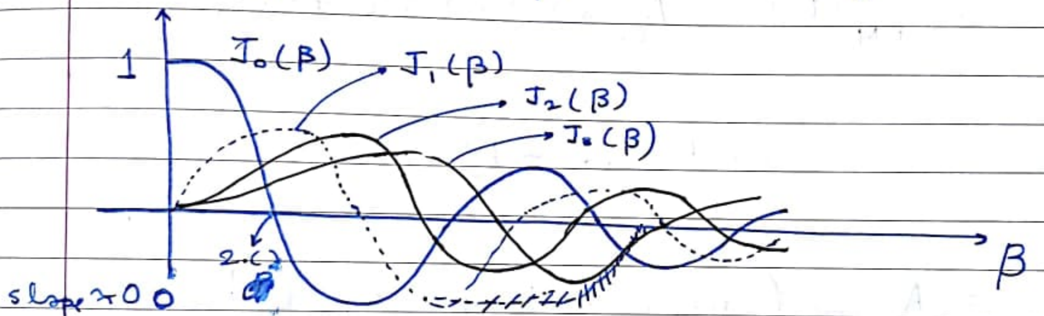
$$C_k(\beta) = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{j\beta \sin x} e^{-jkx}\,dx$$

$$\Vert \Delta$$

$J_n(\beta)$: Bessel's function of first kind of n^{th} order

$\Uparrow$

Orthogonal for different values of n



1) for $\beta << 1$, $J_0(\beta) \simeq 1$
$J_1(\beta) \simeq \beta/2$
$J_k(\beta) \approx 0 \quad k \geq 2$

2) $$J_n(\beta) = (-1)^n J_{-n}(\beta) \quad \forall \beta$$

3) $$\sum_{n=-\infty}^{\infty} J_n^2(\beta) = 1$$

$$S_{P,FM}(t) = A_c \cdot Re\left[e^{j\beta \sin(2\pi f_m t)} \cdot e^{j2\pi f_c t}\right]$$

$$= A_c \sum_{n=-\infty}^{\infty} J_n(\beta) \cos[2\pi(f_c + f_m)t]$$

NB FM $(\beta < 0.3)$: $A_c \cos(2\pi f_c t) + \frac{A_c \beta}{2} \cos 2\pi (f_m + f_c) t$

$$- \frac{\beta A_c}{2} \cos 2\pi (f_c - f_m) t$$

WBFM : $\beta > 0.3$

→ ~~W get other harmonics~~

→ We get other harmonics

23.8.18

Lecture

$$S_{P,FM}(t) = A_c \cos\left(2\pi f_c t + 2\pi k_f \int_0^t \underbrace{A_m \cos 2\pi f_m \tau}_{m(\tau)}\right)$$

$$\rightarrow \text{Power} = \frac{A_c^2}{2}$$ (1)

$$= A_c \cos\left(2\pi f_c t + \beta \sin(2\pi f_m t)\right)$$

$$\beta = \left(\frac{k_f A_m}{f_m}\right)$$

$$= A_c \cdot \sum_{n=-\infty}^{\infty} J_n(\beta) \cdot \cos\left(2\pi(f_c + n f_m)t\right)$$

Where, $\sum_n J_n^2(\beta) = 1$

$$\text{Power} = \frac{A_c^2}{2} \sum_n J_n^2(\beta) = \frac{A_c^2}{2}$$ (2)

$$f_i = \frac{1}{2\pi} \cdot \frac{d\phi}{dt} = f_c + \underbrace{k_f\, m(t)}_{max = k_f \cdot A_m}$$

→ Carson's Rule :

$$B_T = 2(\Delta f + f_{max}) \quad : 98\% \text{ power}$$

f_{max} ↓ Max. freq. contained in BB signal

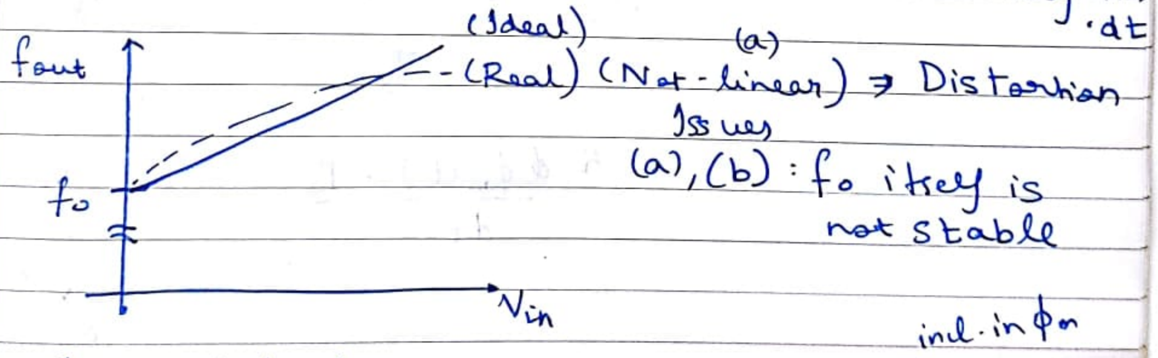
Δf: Max. freq. deviation
$= A_m \cdot k_f$
$= \beta \cdot f_m$

FM generation

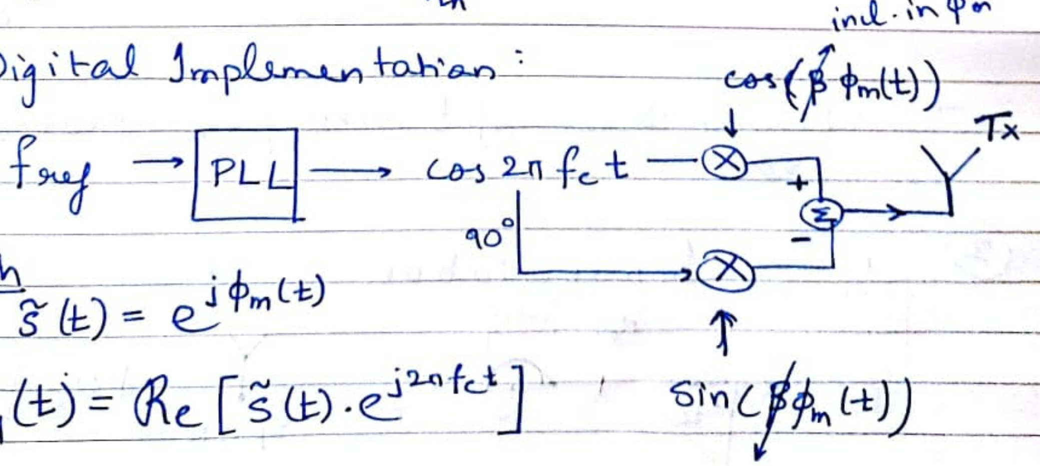
(1) Vin to a VCO (Analog Implementation)

$V_{in} \rightarrow \boxed{VCO} \rightarrow f_{out}$

$\cos\left[2\pi f_0 t + 2\pi \cdot k_{VCO} \int u_{in}(t) \cdot dt\right]$



(2) Digital Implementation :



<u>Math</u>

$$\tilde{s}(t) = e^{j\phi_m(t)}$$

$$S_{P,FM}(t) = Re\left[\tilde{s}(t) \cdot e^{j2\pi f_c t}\right]$$

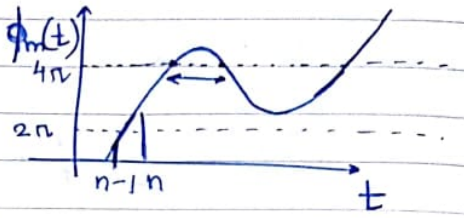
All addresses ... contain!

$$Arg\left[e^{j\phi[n]} - e^{j\phi[n-1]}\right] \approx 0.2 \checkmark$$

$$Arg\left[e^{j\phi[n]}\right] - Arg\left[e^{j\phi[n-1]}\right] = 2\pi - 0.2 \checkmark$$

$0.2 - 2\pi$

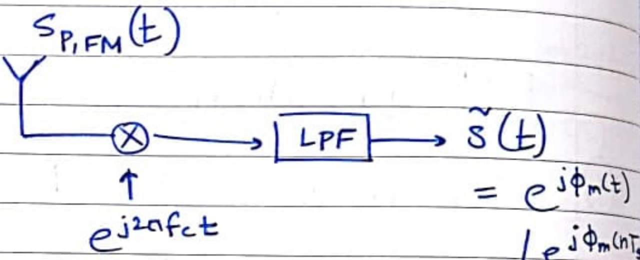
$$\phi_m(t) =$$

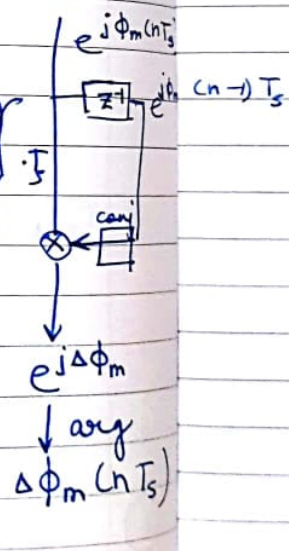


$$\phi_m = 2\pi k_f \int_0^t m(\tau)\, d\tau$$

→ FM Receiver

① Digital



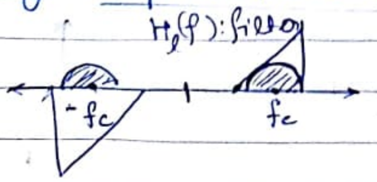


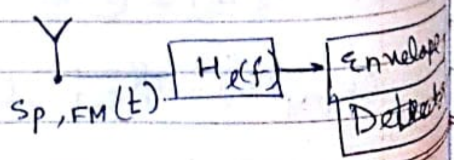
$$\Delta\phi_m(nT_s) = \left\{\frac{\phi_m(nT_s) - \phi_m((n-1)T_s)}{T_s}\right\} \cdot T_s$$

$$\approx \frac{d\phi_m(t)}{dt} \cdot T_s$$

→ Changing Order of Blocks: fixes issue creates above.

② Analog Implementation





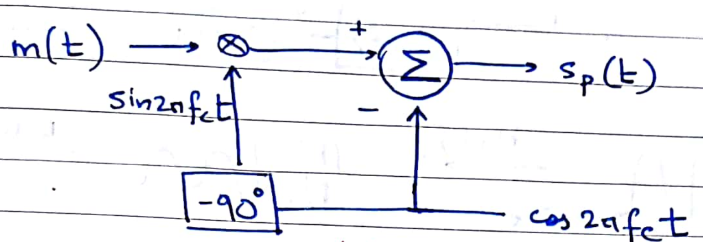
f (2π) · (2π − 0.1) ← 2π + 0.1

27.8.18

Lecture

sin(2πf$_c$t

Q →

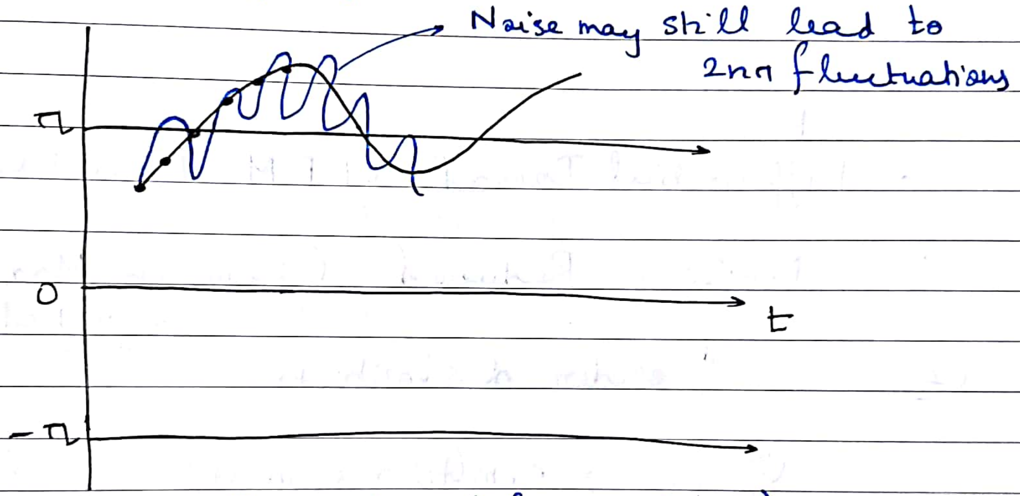


AM/FM/PM ?

$m(t)\sin 2\pi f_c t - 1\cdot\cos 2\pi f_c t = s_p(t)$

cosB sinA − cosAsinB sin(A−B)

→ Recap: 2nπ jumps

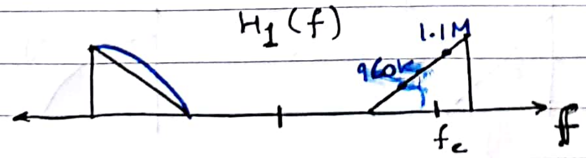


(n−1) T$_s$

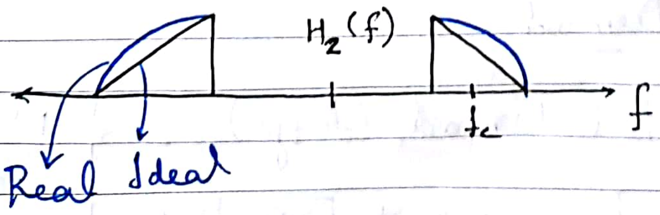
$Arg(\phi_m) \in [-\pi, \pi)$ (from Block)

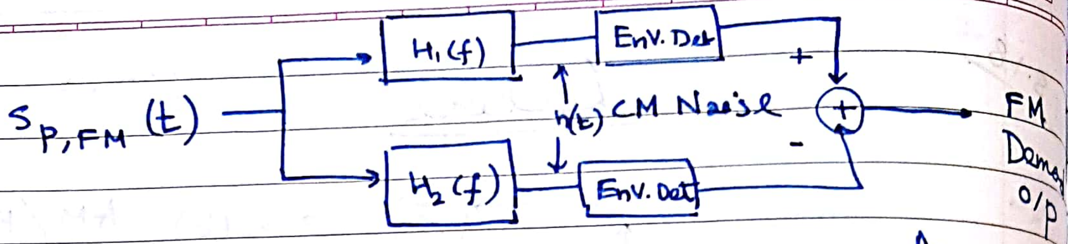
Do : $e^{j\phi_m[n]} e^{-j\phi_m[n-1]}$ → (Correct way)

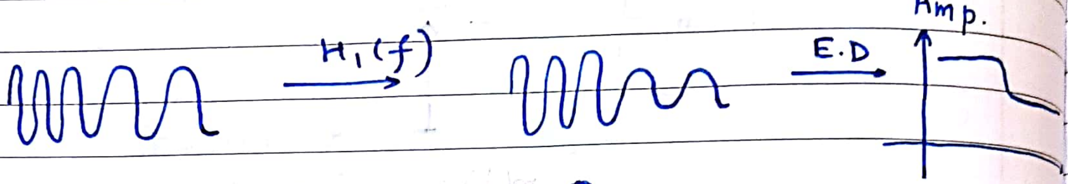
FM Demod



Amplitude proportional to freq.







Output Amplitude $\propto f_i(t) - \cancel{f_0}\, f_c \underbrace{\propto m(t)}$

$\Rightarrow$ Get back $m(t)$

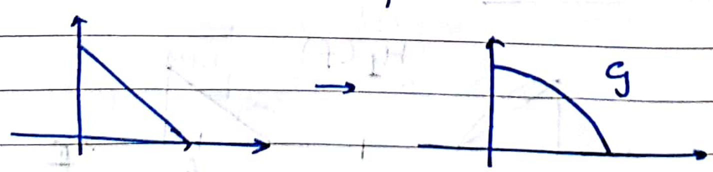
→ Differential Demod of FM Advantages

1. Noise is Reduced (Common Mode Noise Rejected)
2. 2^{nd} order distortion

$$v_{out1} = c_1 m(t) + c_2 m^2(t) + c_3 m^3(t) + \ldots$$
$$v_{out2} = c_1(-m(t)) + c_2(-m(t))^2 + c_3(-m(t))^3$$

$$v_{out} = v_{out1} - v_{out2} = c_1 \cdot m(t) + c_3 m^3(t) + \ldots$$

Why: Since we can't implement an Ideal filter



$G(\Delta f) = c_1 \Delta f + c_2 \Delta f^2 + c_3 \Delta f^3$

→ FM Demod

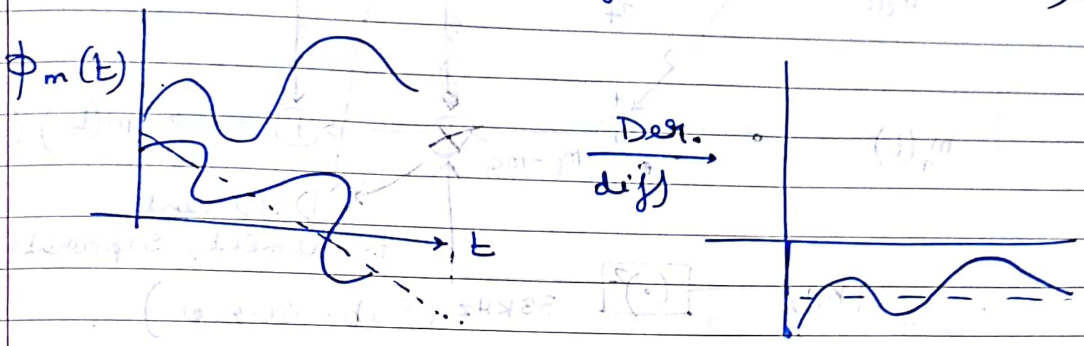
1. Digital Demod (Arg)
2. Freq. Dep. Gain + Env. Detc

Non-Coherent

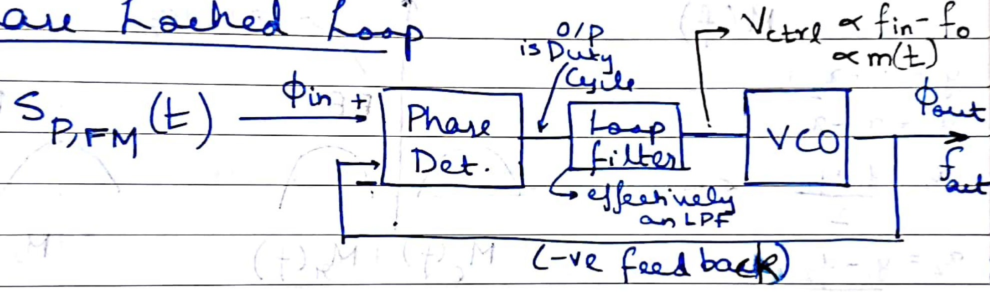
The previous 2 Approaches were Non-Coherent i.e We did not try to get a precisely Matching freq. comp.

(3) Coherent Approach : Use a PLL

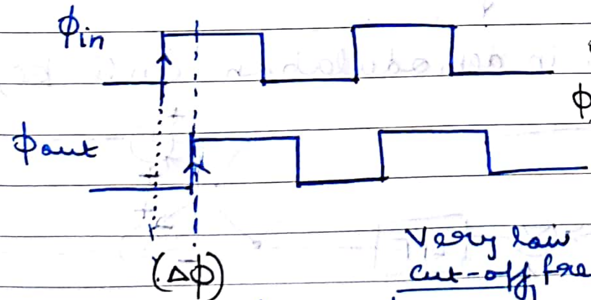
→ Received with Δf offset (Non Coherent)



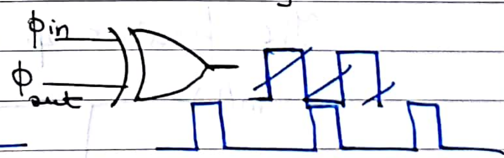
Phase Locked Loop



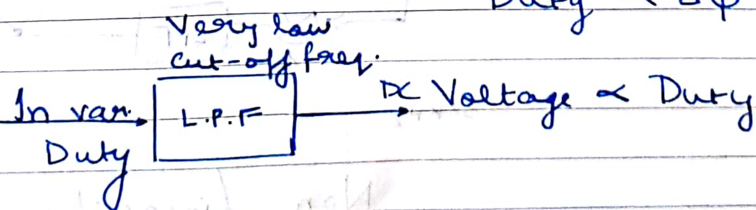
→ Phase Detector

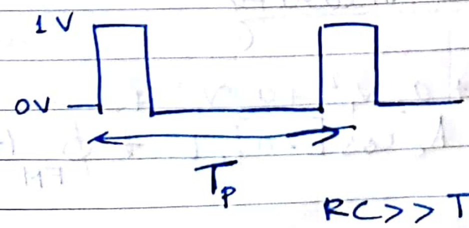


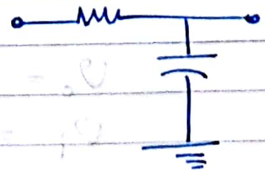
ex XOR Gate



Duty $\propto \Delta\phi$





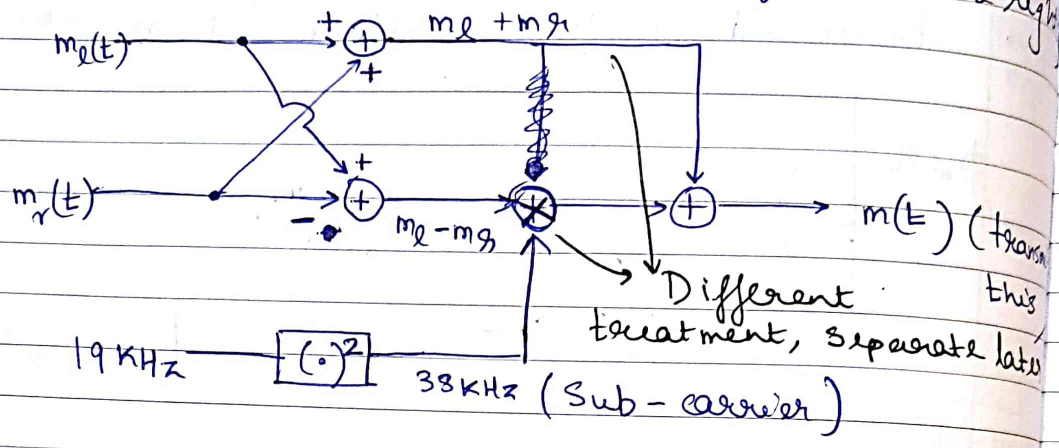


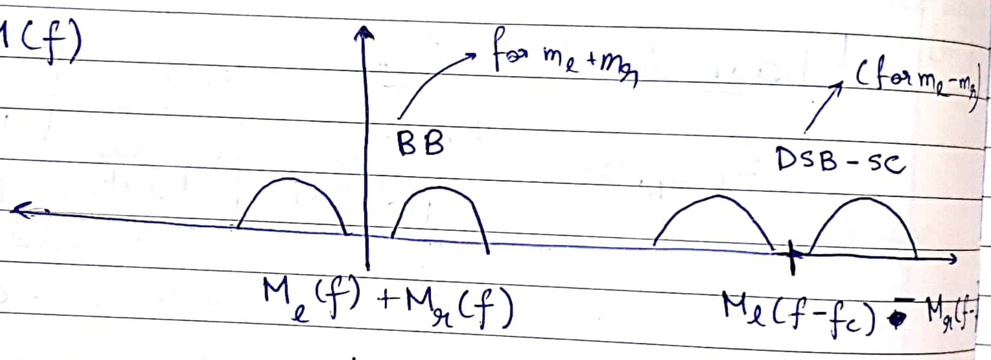
$RC >> T_p$ (All harmonics filtered out, DC Remains)

28.8.18

Lecture

→ Stereo (2 Channel) FM   left (l) and right





$y_1 = x_1 + x_2$

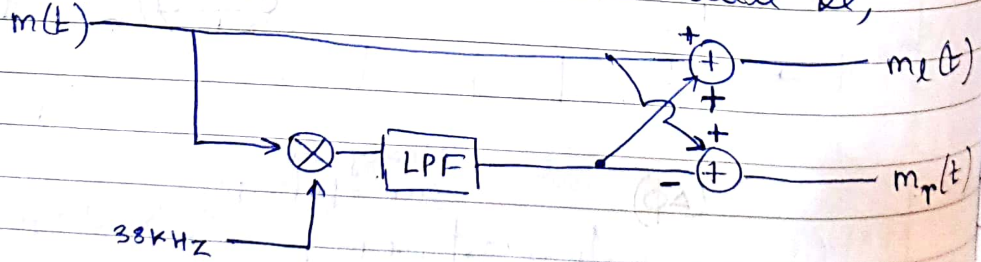
$y_2 = x_1 - x_2$

$x_1 = \frac{y_1 + y_2}{2}$

$x_2 = \frac{y_1 - y_2}{2}$

↓ Demod.

The first step in demodulation will be,



Non-Linearity

$$v_o = a_1 v_i + a_2 v_i^2 + a_3 v_i^3 + \underbrace{\ldots}_{\text{weaker}}$$

$$v_i = A_c \cos[2\pi f_c t + \phi_{FM}(t)]$$

$\sin 3x = 3\sin x - 4\sin^3 x$

$\cos 3x = 4\cos^3 x - 3\sin x$

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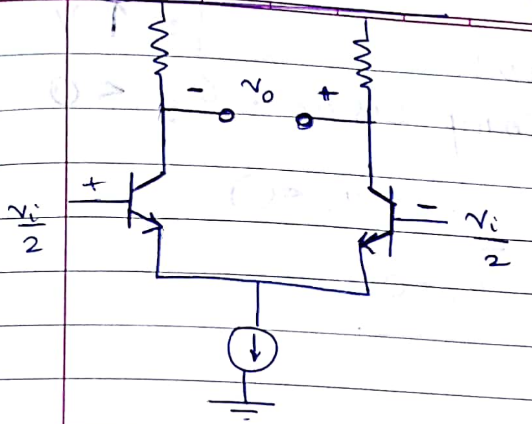
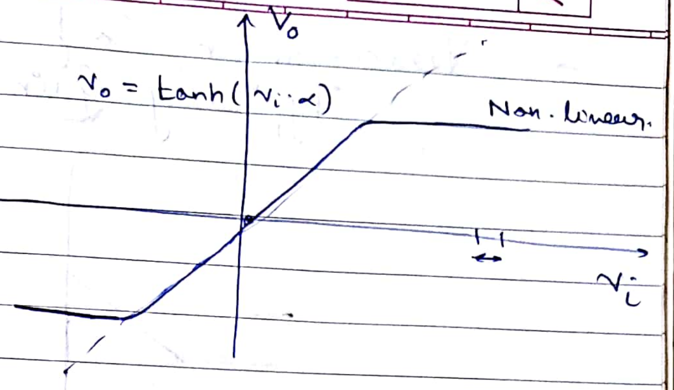




original signal with slight change in Amp.

Fundamental

$$v_o = A_c\left[a_1 + \frac{3}{4}a_3 A_c^2\right]\cos(2\pi f_c t + \phi_{FM}(t))$$

Linear gain $= a_1$

$$+ \frac{a_2 A_c^2}{2} + \frac{a_2 A_c^2}{2}\cos(4\pi f_c t + 2\phi_{FM}(t))$$

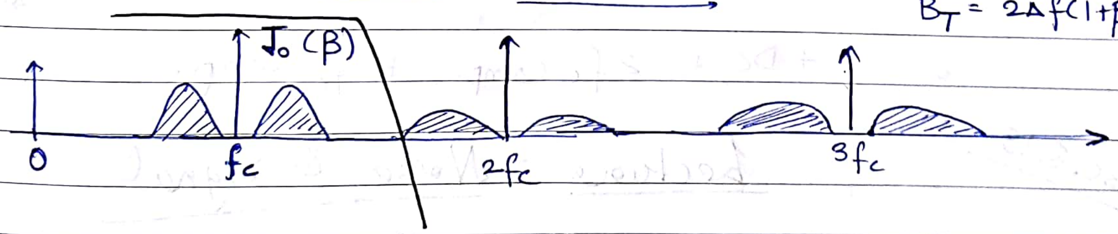
DC

2^{nd} harmonic

$$+ \frac{a_3 A_c^3}{4}\cdot\cos(6\pi f_c t + 3\phi_{FM}(t)) :\rightarrow 3^{rd}\ \text{harmonic}$$

Spectrum of v_o

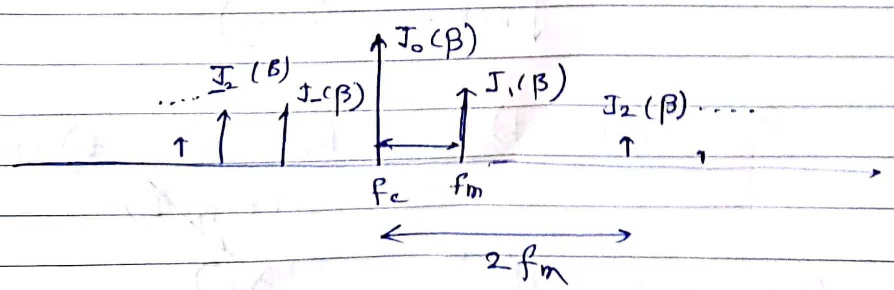
B/w Increases since $B_T = 2\Delta f(1+\beta)$



LPF to recover

$m(t) = \cos(2\pi f_m t)$

$$\cos(2\pi f_c t + \beta\sin 2\pi f_m t) = \sum_{n=-\infty}^{\infty} J_n(\beta)\cos(2\pi(f_c + n f_m)t)$$



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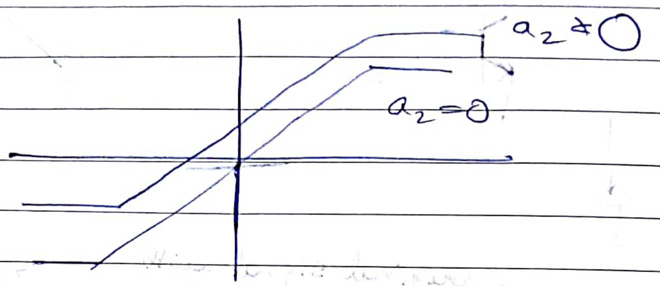
→ Variation in Coefficients: Saturation and for ↑ V

Typically Gain compression : $a_3 < 0$



→ FM is not distorted only Amplitude of fund. change

$\phi m(t)$ remains intact.

Non-linearity in AM

$$v_i = A_c(1 + k_a m(t)) \cdot \cos 2\pi f_c t$$

$$V_o = A_c(1 + k_a m(t)) \cdot \left[a_1 + \frac{3a_3 A_c^2 (1 + k_a m(t))^2}{4} \right] \cos 2\pi f_c t$$

$+ DC + 2f_c$ comp. $+ 3f_c$ comp.

30.8.18

Lecture - Noise & Signal

Noise :

$$s_{P,Tx}(t) = A_c \cos(2\pi f_c t) \cdot m(t) + \cancel{n_{Tx}(t)}$$ ≈ 0

↓ DSB-SC, (Say)

$$s_{P,Rx}(t) = C \cdot A_c \cos(2\pi f_c t) \cdot m(t) + n_p(t) + \cancel{n_{Tx}(t)}$$

≈ 0

Added by Channel + Rx H/w

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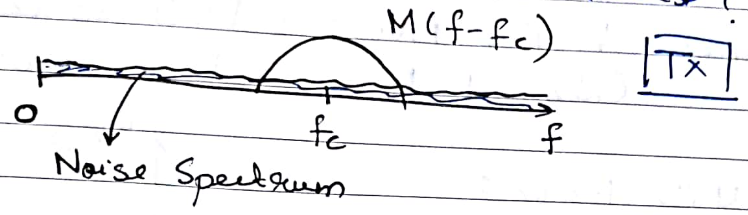
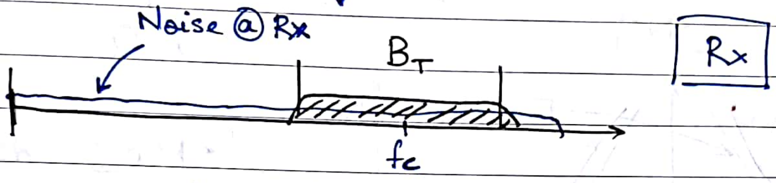
→ Transmission does not add Noise?



Transmitted to Mars

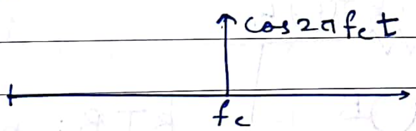


→ The noise within the Signal B/w is the problem

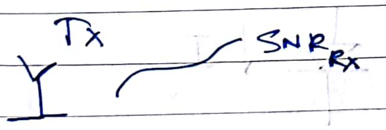
In-band Noise Power @ Receiver $= \frac{N_0}{2} \times B_T \times 2$ (2 bands)
(Only this affects SNR)

$$\boxed{= B_T N_0}$$

→ Solution: Decrease B/W : We can't send one harmonic at a time since Mod. intro. harm.



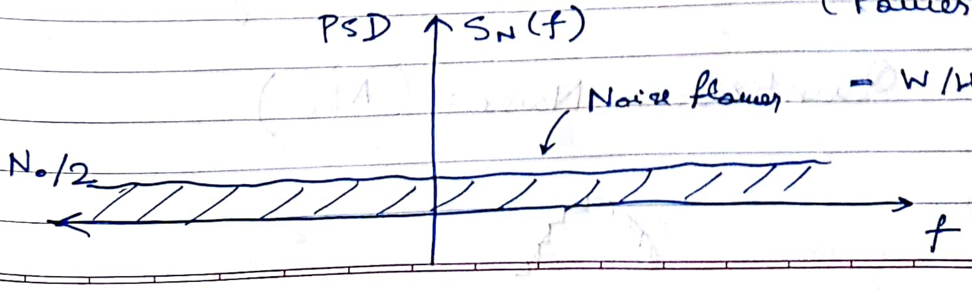
Trade off → Less Info: $C = B \log_2 (1 + SNR)$
-Shanon



$$SNR_{Rx} = \frac{P_{Tx} \cdot C^2}{B_T \cdot N_0} = \frac{\text{Sig. Power}}{\text{Noise Power}}$$

Where N_0 is from power spectral density,

(Power / unit BW) - W/Hz (J)

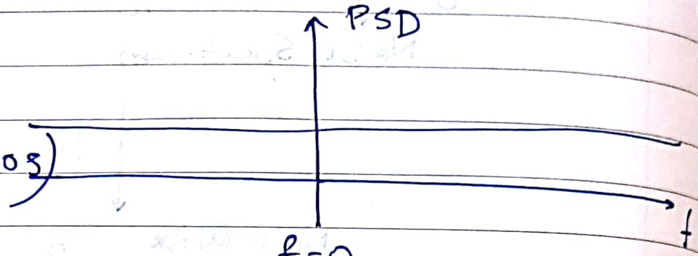


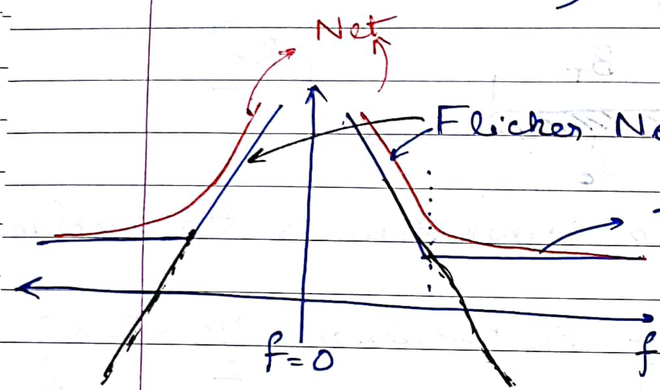
Noise

3.9.18

→ Types : White Noise : PSD is const. with freq.
Coloured Noise : PSD varies with freq.

White Noise :
- Channel
- Electronics (non MOS)





- MOS devices

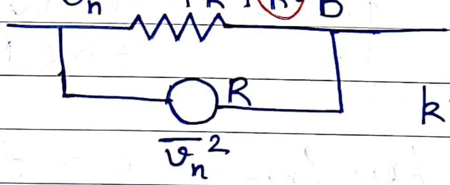
Sources of Noise (White Noise)

① Thermal Noise

Added by a Resistor

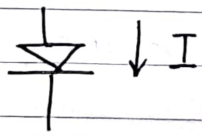
$\overline{v_n^2} = 4kTRB$ — Resistance

$k \rightarrow$ Boltzman Const.
$1.38 \times 10^{-23}\ J/K$



kTB : min. amount of Noise power

② Shot Noise :



$\overline{i_n^2} = 2qIB$

q → elect. charge

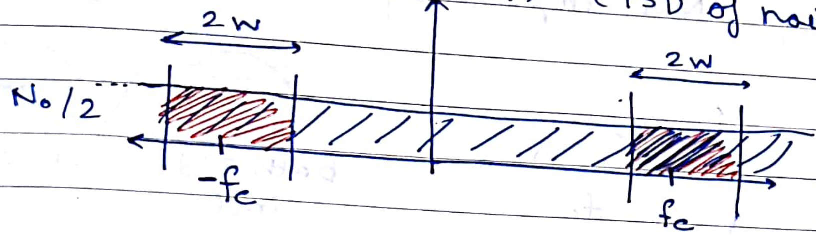
③ Quantization Noise : (ADC)

3.9.18

Noise in AM

$$x_{P,DSB-SC}(t) = \underbrace{C}_{\text{Amp. Attenuation}} A_c\, m(t) \cdot \cos 2\pi f_c t + n(t)$$

$S_N(f)$ (PSD of noise)



$N_0 = kT$ for thermal Noise

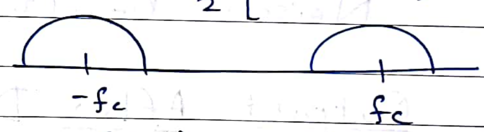
PSD: units same as energy (J)

pass-band noise

$$\boxed{n(t) = \underbrace{n_I(t)}_{\text{Baseband noise levels}} \cos 2\pi f_c t - \underbrace{n_Q(t)}_{} \sin 2\pi f_c t} \quad -(1)$$

Baseband noise levels

→ For DSB-SC $\quad \frac{A_c}{2}\left[M(f-f_c) + M(f+f_c)\right]$



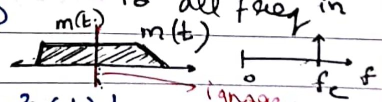
→ Signal to Noise Ratio (Rx)

1) Signal Power at Rx $= \langle |C A_c m(t) \cdot \cos(2\pi f_c t)|^2 \rangle$

$$= \boxed{C^2 A_c^2 \cdot P \cdot \frac{1}{2}}$$

$\because \cos 2\pi f_c t$ is orthogonal to all freq in $m(t)$

Where $\langle |m^2(t)| \rangle = P$



→ Noise power at receiver : $\langle |n^2(t)| \rangle$

from (1) $\quad \dfrac{\langle n_I^2(t) \rangle}{2} + \dfrac{\langle n_Q^2(t) \rangle}{2} \quad = \langle n_I^2(t) \rangle = \langle n_Q^2(t) \rangle$ (AWGN)

$$= (4W) \times \frac{N_0}{2}$$

$$= 2WN_0$$

$$\boxed{SNR_{Rx} = \frac{C^2 A_c^2 P}{4WN_0}}$$

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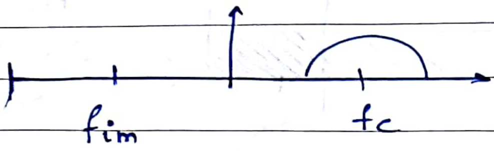
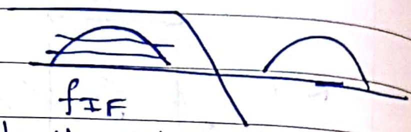
→ $\langle |\,|^2 |\,|^2 \rangle = \langle |\,|^2 \rangle \langle |\,|^2 \rangle$ When the frequency comp. are orthogonal except finitely many (Non impulse) points ($f=0$)





both sides of noise incl.

Even here both sides of noise come into picture

$$s_p(t) \rightarrow \otimes \rightarrow \boxed{LPF} \rightarrow \frac{1}{2} m(t) \cos(2\pi f_{IF} t)$$
$\cos 2\pi f_{LO} t$

Noise : $n_{IM}(t) \cos(2\pi f_m t) \cos(2\pi f_{LO} t)$

→ Output After Demodulation

Demod o/p : $x_{p, DSB-FC}(t) \times \cos 2\pi f_c t$

$\downarrow$ LPF

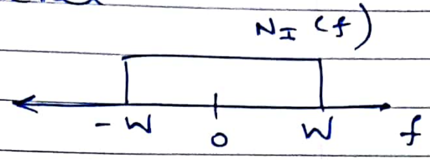
$$m(t) \cdot c A_c \cdot \left(1 + \cancel{\cos 4\pi f_c t}\right) \times \frac{1}{2}$$
$$+\ n_I(t) \left(1 + \cancel{\cos 4\pi f_c t}\right) \times \frac{1}{2}$$
$$+\ n_Q(t) \cdot \sin \frac{4\pi f_c t}{2}$$ ~~zero~~ LPF cancels

$$= \frac{1}{2}\left(m(t) \cdot A_c \cdot c + n_I(t)\right)$$

$$SNR_{out} = \frac{A_c^2 c^2 \langle m^2(t) \rangle / 4}{W N_0 / 2} = \frac{A_c^2 c^2 P}{2 W N_0}$$

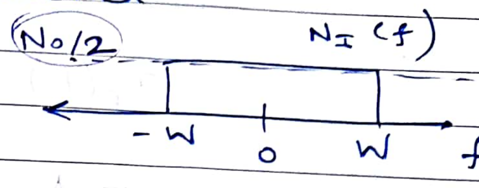
→ SNR × 2 since out of phase noise does not contribute to Noise Level

effective noise PSD



→ SNR x 2 since out of phase noise does not contribute to Noise Level

effective noise PSD

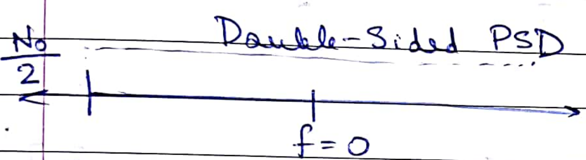


17.9.18

Lecture

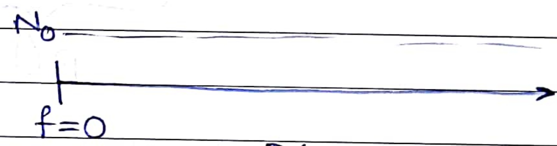
Noise PSD

Double sided PSD



BW 2W

Single sided PSD



B/W W

→ $SNR_c = \dfrac{\text{Modulated Signal Power}}{\text{Base-band signal Noise Power}}$

$$\underbrace{SNR_{c,\,DSB\text{-}SC}}_{\text{Notional}} = \frac{c^2 A_c^2 \cdot P/2}{WN_0} = \frac{c^2 A_c^2 P}{2WN_0}$$

$WN_0 \to 2W \times \frac{N_0}{2}$

$$SNR_{o,\,DSB\text{-}SC} = \frac{c^2 A_c^2\, P/4}{WN_0/2} = \frac{c^2 A_c^2 P}{2WN_0}$$

o → Rx output (After demod)

→ Define figures of Merit

$$\text{Figure of merit} = \frac{SNR_o}{SNR_c} = 1 \quad (DSB\text{-}SC)$$

DSB-FC:

$$x_p(t) = s_p(t) + n(t)$$

$$= A_c(1 + k_a m(t)) \cdot \cos 2\pi f_c t + n_I(t)\cos(2\pi f_c t) - n_Q(t)\sin 2\pi f_c t$$

→ Env. Det. o/p

$$= \left\{\Big[\underbrace{A_c[1 + k_a m(t)]}_{p(t)} + n_I(t)\Big]^2 + n_Q^2(t)\right\}^{1/2}$$

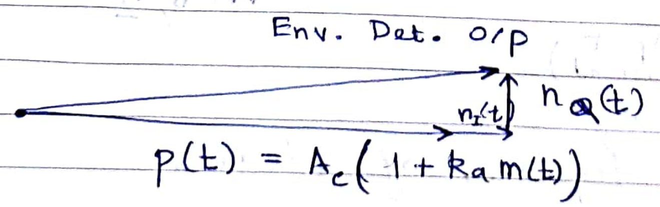
$$= p(t)\left[\left(1 + \frac{n_I(t)}{p(t)}\right)^2 + \left(\frac{n_Q(t)}{p(t)}\right)^2\right]^{1/2}.$$

~~p(t) using Bin~~

$$= p(t)\left[1 + \frac{2n_I(t)}{p(t)} + \frac{n_I^2(t) + n_Q^2(t)}{p(t)}\right]^{1/2}$$

$\cong$ (Binomial) $\cong$ $p(t) + n_I(t)$

$$\approx cA_c(1 + k_a m(t)) + n_I(t)$$



$$SNR_{O,\,Env.\,Det} = \frac{c^2 A_c^2 k_a^2 P}{2WN_0}$$

$\langle n_I^2(t)\rangle = 2WN_0$

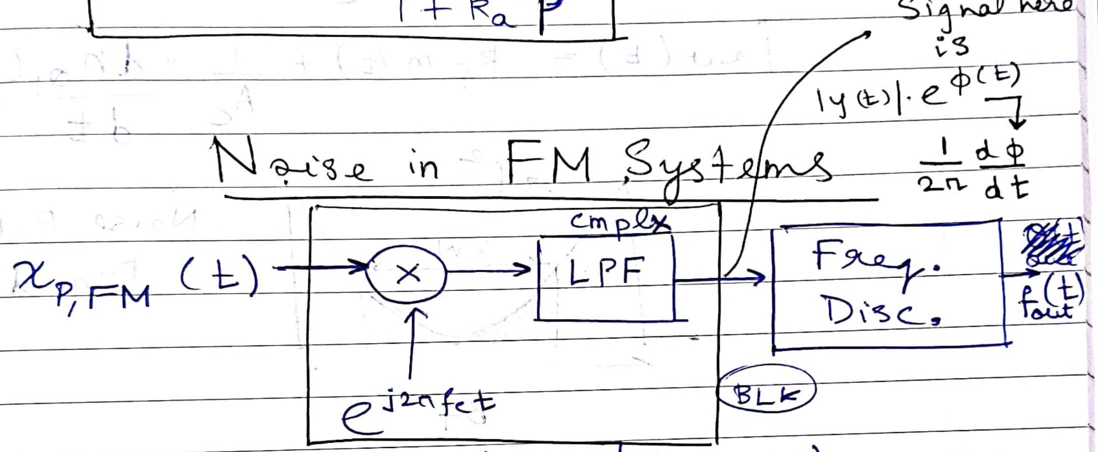
$$= \frac{\langle n_I^2\rangle}{2} + \frac{\langle n_Q^2\rangle}{2} = \langle n_I^2(t)\rangle$$

$$SNR_{C, DSB-FC} = \frac{\overbrace{c^2 \frac{A_c^2}{2} \cdot (1 + k_a^2 P)}^{\text{Mod. Signal power}}}{W N_0}$$

(Notional)

$$\boxed{FOM = \frac{k_a^2 P}{1 + k_a^2 P}}$$

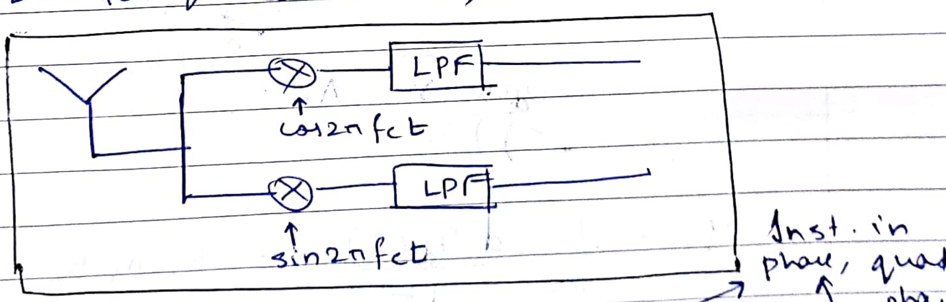
Noise in FM Systems



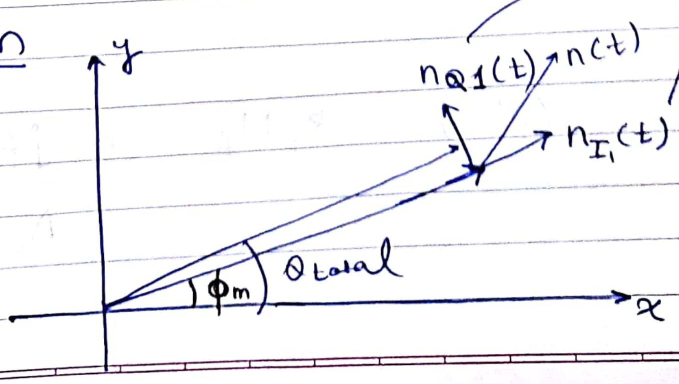
$$A_c \cos[2\pi f_c t + \phi_m(t)] + n_I(t) \cos 2\pi f_c t - n_Q(t) \sin 2\pi f_c t$$

$$\phi_m = 2\pi k_f \int_{-\infty}^{t} m(\tau)\, d\tau$$

The BLK is equivalent to,



Phasor Diagram



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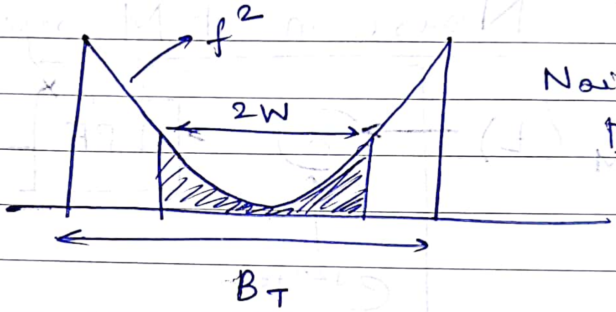
$$\theta_{tot}(t) = \phi_m(t) + \tan^{-1}\left(\frac{n_{Q_1}(t)}{A_c + n_I(t)}\right)$$

$$\simeq \phi_m(t) + \frac{n_{Q_1}(t)}{A_c} \qquad |n_I(t)| << A_c$$

$$f_{out}(t) = k_f\, m(t) + \frac{1}{A_c} \cdot \frac{d n_{Q_1}(t)}{dt}$$

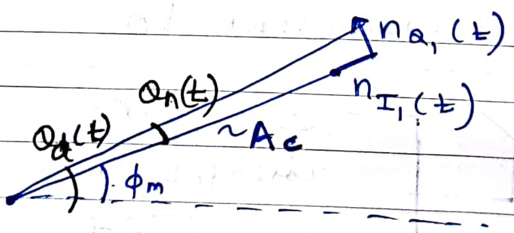


Noise PSD at Demod o/p

$|2\pi i f|^2$

6.9.18

Noise in FM



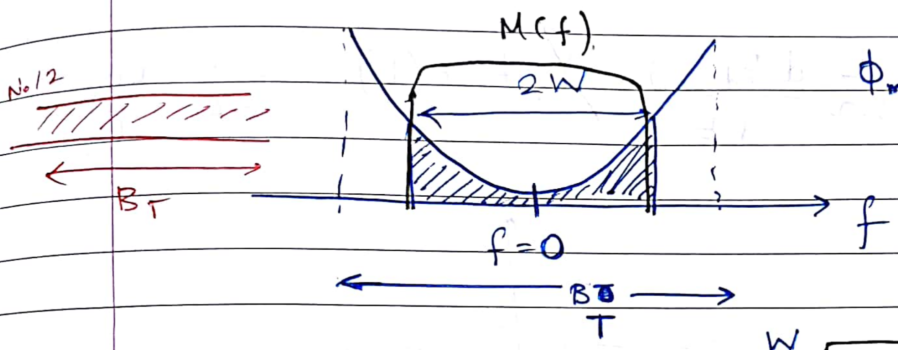
$$y_o = \frac{1}{2\pi}\frac{d\theta_d(t)}{dt} \simeq k_f\, m(t) + \frac{1}{2\pi}\frac{d n_{Q_1}(t)}{A_c\, dt}$$

$$Y_o(f) = k_f M_f + \frac{if}{A_c} N_{Q_1}(f)$$

$$S_{N_Q}(f) \xrightarrow{H(f)} |H(f)|^2 \, S_{N_Q}(f)$$

Here a diff.

So, here $H(f) = \dfrac{2\pi j f}{2\pi A_c}$ $\quad \boxed{(f\,t) = \dfrac{1}{2\pi A_c}\dfrac{d}{dt}}$



$$\phi_m = 2\pi k_f \int^{t} m(\tau)\, d\tau$$

$$\frac{1}{2\pi}\frac{d\phi_m}{dt} = k_f m(t)$$

$$\text{Noise power in } Y_0 = \int_{-W}^{W} \frac{\overbrace{S_{NQ_1}(f)}^{N_0\,(B.B)} \cdot f^2 \, df}{A_c^2} = \frac{2W^3 N_0}{3A_c^2}$$

FoM Calculation

$$SNR_0 = \frac{k_f^2 P \cdot 3A_c^2}{2W^3 N_0}$$

$$FoM = \frac{SNR_0}{SNR_c} = \frac{SNR_0}{\frac{A_c^2}{2} \times \frac{1}{WN_0}} = \frac{3k_f^2 P}{W^2}$$

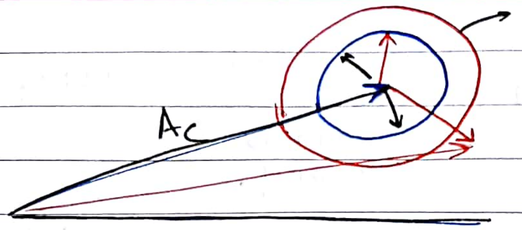
$$m(t) = A_m \cos 2\pi f_m t \; ; \quad \beta = \frac{k_f A_m}{f_m}$$

$$\boxed{FoM = \frac{3}{2}\beta^2}$$ → Seems that FoM improves with β (MI)

- If no restriction on Bandwidth utilisation, then Increase β (and hence B_T) ~~and~~ to Improve? NO!

Scanned by CamScanner

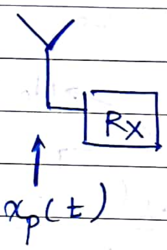
→ Since,

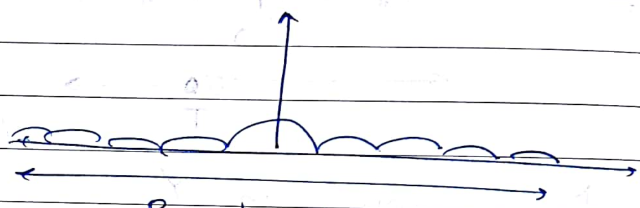


$\theta_n \simeq \tan^{-1}\left(\frac{n_{Q}}{A_c}\right)$

the Approx,

$$y_0 = \frac{1}{2\pi}\frac{d\theta_d(t)}{dt} \cancel{\simeq} k_f\, m(t) + \frac{1}{2\pi A_c}\cdot\frac{dn_{Q}(t)}{dt}$$



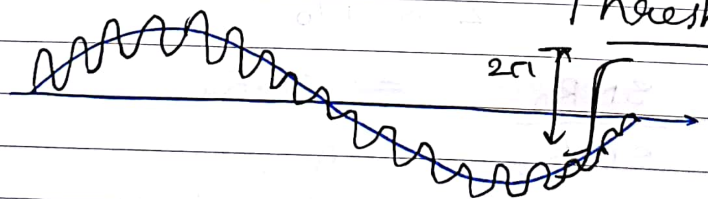


B_T by Carson's Rule

→ Issue is, $\quad CNR = \dfrac{A_c^2/2}{N_0 \cdot B_T}$

Assump: CNR is high $> 13\,dB$

Thresholding effect



2π jumps

→ FoM Comparison

For, DSB-FC AM : if $k_a = 1$

$\Rightarrow FOM = \dfrac{1}{3}$

FM : $\quad FoM = \dfrac{3}{2}\beta^2$ (today)

$(FoM)_{AM,\,DSB\text{-}FC} = (FoM)_{FM,\,DSB\text{-}FC} \Rightarrow \boxed{\beta = \dfrac{\sqrt{2}}{3} \simeq 0.471}$

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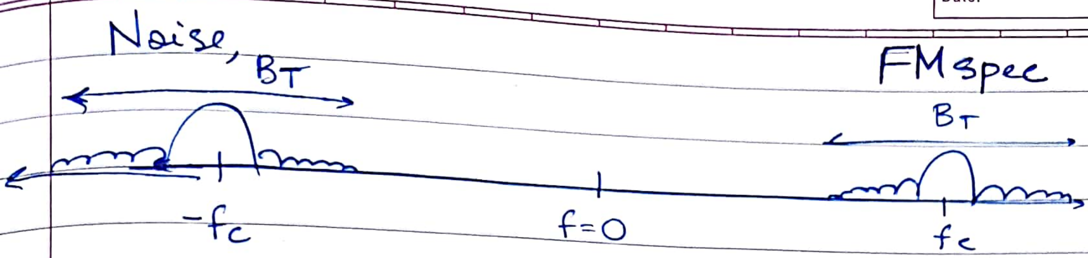
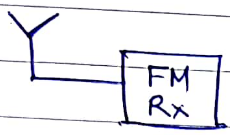
Date:





Noise power in FM $= 2 \cdot \frac{N_0}{2} \times B_T$