Problem 7.2

Show that Eq. (7.8) is *invariant* with respect to the carrier phase ϕ_c (i.e., it holds for all ϕ_c).

Solution

Assuming a carrier phase ϕ_c, the carrier is itself written as $\cos(2\pi f_c t + \phi_c)$. Then Eq. (7.7) modifies to

$$E_b = \int_0^{T_b} |s(t)|^2 dt$$

$$= \frac{1}{T_b}\int_0^{T_b} |b(t)|^2 + \frac{1}{T_b}\int_0^{T_b} |b(t)|^2 \cos(4\pi f_c t + 2\phi_c) dt$$

where we have made use of the trigonometric identity

$$\cos^2\theta = \frac{1}{2}(\cos(2\theta))$$

Hence, with $|b(t)|^2$ remaining essentially constant over one complete cycle of $\cos(4\pi f_c t + 2\phi_c)$, we have

$$\int_0^{T_b} |b(t)|^2 \cos(4\pi f_c t + \phi_c) dt \approx 0 \text{ for all } \phi_c$$

Correspondingly, we may write

$$E_b = \int_0^{T_b} |b(t)|^2 \quad \text{for all } \phi_c.$$