## Problem 2.6

Develop the detailed steps involved in deriving Eq. (2.53), starting from Eq. (2.51).

## Solution

According to Eq. (2.51),

$$\int_{-\infty}^{\infty} g_1(\tau)g_2(t-\tau)d\tau \iff G_1(f)G_2(f)$$

According to Eq. (2.21), if  $g(t) \rightleftharpoons G(f)$ , then  $g(-t) \rightleftharpoons G(-f)$ . Hence, applying this rule to the problem at hand, we may write

$$\int_{-\infty}^{\infty} g_1(\tau)g_2(\tau - t)d\tau \rightleftharpoons G_1(f)G_2(-f)$$

Next, we note that if we complex conjugate the term  $g_2(\tau - t)$ , then the conjugation theorem of Eq. (2.22) teaches us that

$$\int_{-\infty}^{\infty} g_1(\tau) g_2^*(\tau-t) d\tau \, \rightleftharpoons \, G_1(f) G_2^*(-f)$$

which is the desired result, except for the fact that we have interchanged the roles of variables t and  $\tau$ .