Problem 3.4

A *square-law modulator* for generating an AM wave relies on the use of a nonlinear device (e.g., diode); Fig. 3.8 depicts the simplest form of such a modulator. Ignoring higher order terms, the input-output characteristic of the diode-load resistor combination in this figure is represented by the *square law*:

$$v_2(t) = a_1 v_1(t) + a_2 v_1^2(t)$$

where

$$v_1(t) = A_c \cos(2\pi f_c t) + m(t)$$

is the input signal, $v_2(t)$ is the output signal developed across the load resistor, and a_1 and a_2 are constants.

(a) Determine the spectral content of the output signal $v_2(t)$.

(b) To extract the desired AM wave from $v_2(t)$, we need a band-pass filter (not shown in Fig. 3.8). Determine the cutoff frequencies of the required filter, assuming that the message signal is limited to the band-$W \le f \le W$.

(c) To avoid *spectral distortion* by the presence of undesired modulation products in $v_2(t)$, the condition $f_c > 2W$ must be satisfied; validate this condition.

Solution

The output signal is

$$\begin{aligned} v_2(t) &= a_1 v_1(t) + a_2 v_1^2(t) \\ &= a_1(A_c \cos(2\pi f_c t) + m(t)) + a_2(A_c \cos(2\pi f_c t) + m(t))^2 \\ &= [a_1 + 2a_2 m(t)] A_c \cos(2\pi f_c t) \\ &\quad + [a_1 m(t) + a_2 A_c^2 \cos^2(2\pi f_c t) + a_2 m^2(t)] \end{aligned} \tag{1}$$

(a) The expression inside the first set of square brackets defines the desired AM wave:

$$\begin{aligned} s(t) &= A_c[a_1 + 2a_2 m(t)] \cos(2\pi f_c t) \\ &= a_1 A_c\left[1 + \frac{2a_2}{a_1} m(t)\right] \cos(2\pi f_c t) \end{aligned}$$

which represents an AM wave with

$$k_a = \frac{2a_2}{a_1}$$

defining the amplitude sensitivity of the modulator.

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Problem 3.4 continued

(b) The required band-pass filter must have a passband centered on f_c and a bandwidth equal to $2W$.

(c) The expression inside the second set of square brackets of Eq. (1) defines the undesired modulated products. The terms that matter are:

- The term $a_2 m^2(t)$, whose highest frequency component is 2W.
- The term $a_2 A_c^2 \cos^2(2\pi f_c t)$, whose frequency is $2 f_c$.

To extract the desired AM wave we therefore require:
Condition 1:
$(f_c + W) < 2f_c$
or $f_c > W$
Condition 2:
$(f_c - W) > 2W$
or $f_c > 3W$
If therefore we satisfy condition 2, then condition 1 is automatically satisfied.