Problem 3.14

Validate the statement that the high-frequency components in Eq. (3.36) represent a VSB wave modulated onto a carrier of frequency $2f_c$.

Solution

The high-frequency components in Eq. (3.36) are defined by the formula

$$G(f) = \frac{1}{4}A_c A'_c[M(f-2f_c)H(f-f_c) + M(f+2f_c)H(f+f_c)] \tag{1}$$

Referring to Eq. (3.33), the spectrum of the incoming VSB modulated wave is

$$\begin{aligned} S(f) &= \frac{1}{2}A_c[M(f-f_c) + M(f+f_c)]H(f) \\ &= \frac{1}{2}A_c M(f-f_c)H(f) + \frac{1}{2}A_c M(f+f_c)H(f) \end{aligned} \tag{2}$$

Examining Eqs. (1) and (2), as labelled here, we see that (ignoring the scaling factors)

1. The first term in Eq. (1), namely, $M(f - 2f_c)H(f - f_c)$ is equal to the first term in Eq. (2), namely $M(f - f_c)H(f)$ shifted to the right by f_c.
2. By the same token, the second term in Eq. (1), namely, $M(f + 2f_c)H(f + f_c)$ is equal to the second term in Eq. (2), namely, $M(f + f_c)H(f)$ shifted to the left by f_c.

Since Eq. (2) represents a VSB wave modulated onto carrier frequency f_c, it follows that Eq. (1) represents a VSB wave modulated onto the new carrier frequency $2f_c$.