Noise Analysis - AM, FM

The following assumptions are made:

- Channel model
  - distortionless
  - Additive White Gaussian Noise (AWGN)
- Receiver Model (see Figure 1)
  - ideal bandpass filter
  - ideal demodulator

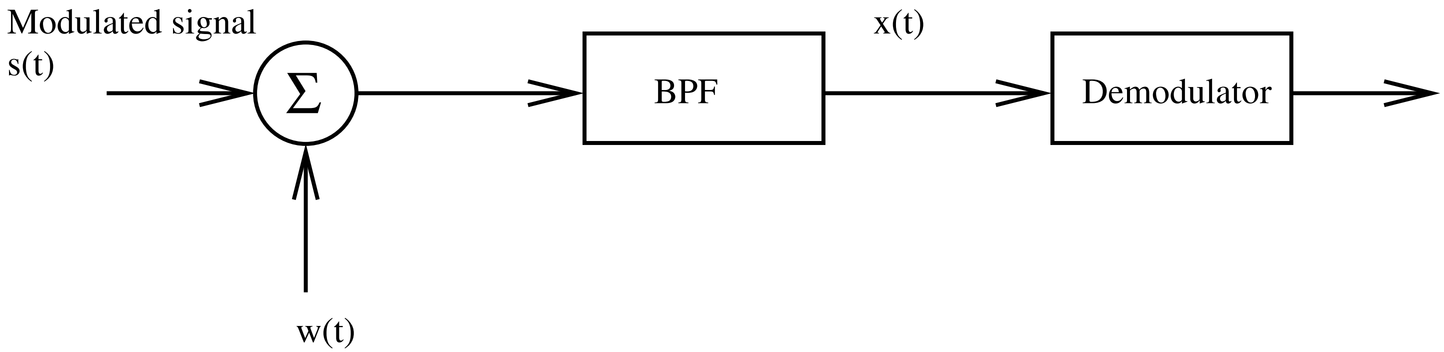

Figure 1: The Receiver Model

- BPF (Bandpass filter) - bandwidth is equal to the message bandwidth B
- midband frequency is ω_c.

Power Spectral Density of Noise

- $\frac{N_0}{2}$, and is defined for both positive and negative frequency (see Figure 2).
- N_0 is the average power/(unit BW) at the front-end of the

receiver in AM and DSB-SC.

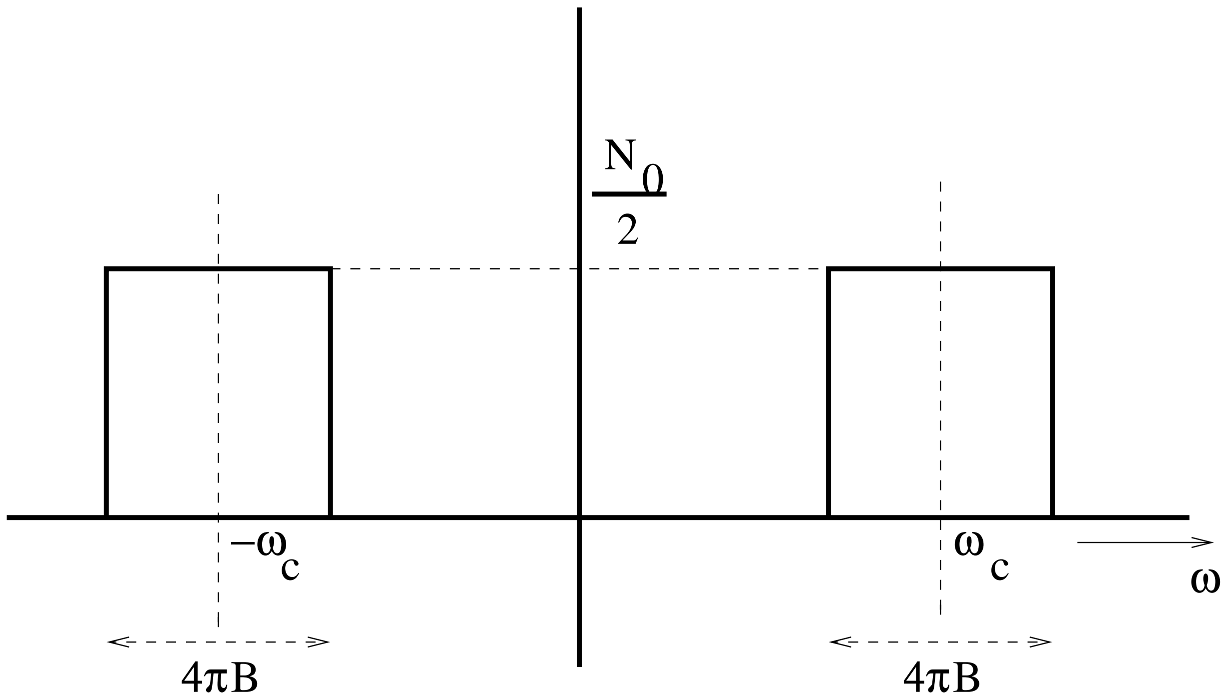

Figure 2: Bandlimited noise spectrum

The filtered signal available for demodulation is given by:

$$\begin{aligned} x(t) &= s(t) + n(t) \\ n(t) &= n_I(t)\cos\omega_c t \\ &\quad -n_Q(t)\sin\omega_c t \end{aligned}$$

$n_I(t)\cos\omega_c t$ is the in-phase component and

$n_Q(t)\sin\omega_c t$ is the quadrature component.

$n(t)$ is the representation for narrowband noise.

There are different measures that are used to define the Figure of Merit of different modulators:

- Input SNR:

$$(SNR)_I = \frac{Average\ power\ of\ modulated\ signal\ s(t)}{Average\ power\ of\ noise}$$

- Output SNR:

$$(SNR)_O = \frac{Average\ power\ of\ demodulated\ signal\ s(t)}{Average\ power\ of\ noise}$$

The Output SNR is measured at the receiver.

- Channel SNR:

$$(SNR)_C = \frac{Average\ power\ of\ modulated\ signal\ s(t)}{Average\ power\ of\ noise\ in\ message\ bandwidth}$$

- Figure of Merit (FoM) of Receiver:

$$FoM = \frac{(SNR)_O}{(SNR)_C}$$

To compare across different modulators, we assume that (see Figure 3):

- The modulated signal $s(t)$ of each system has the same average power
- Channel noise $w(t)$ has the same average power in the message bandwidth B.

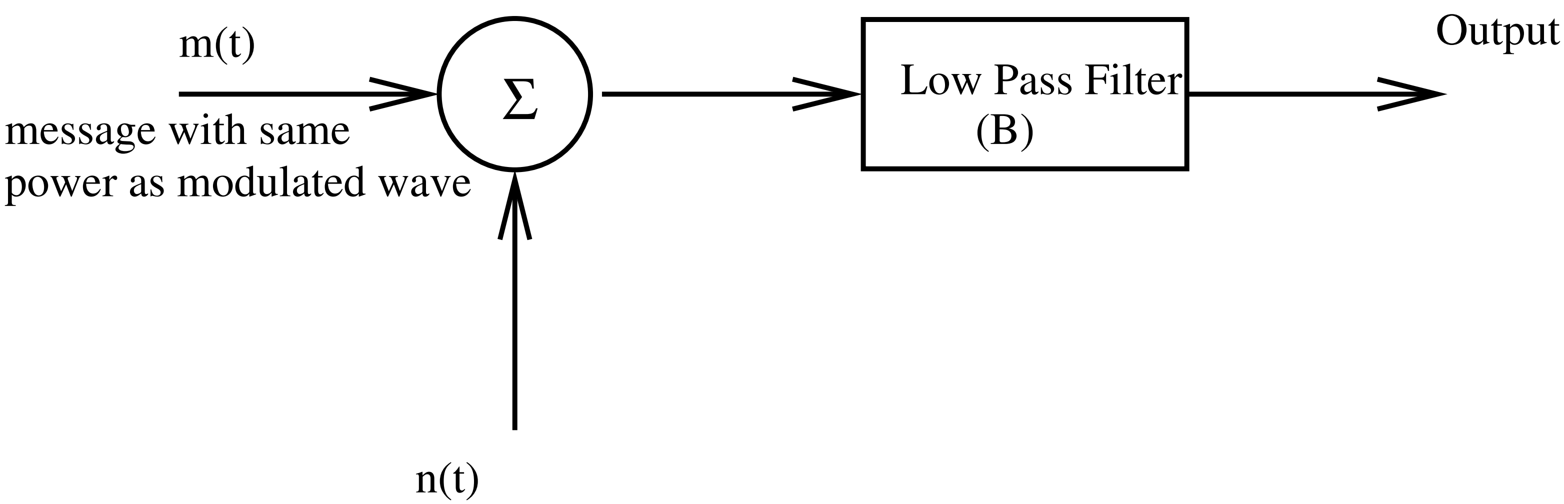

Figure 3: Basic Channel Model

Figure of Merit (FoM) Analysis

- DSB-SC (see Figure 4)

$$
\begin{aligned}
s(t) &= CA_c \cos(\omega_c t) m(t) \\
(SNR)_C &= \frac{A_c^2 C^2 P}{2BN_0} \\
P &= \int_{-2\pi B}^{+2\pi B} S_M(\omega) d\omega \\
x(t) &= s(t) + n(t) \\
& \quad CA_c \cos(\omega_c t) m(t) \\
& \quad + n_I(t) \cos \omega_c t + n_Q(t) \sin \omega_c t
\end{aligned}
$$

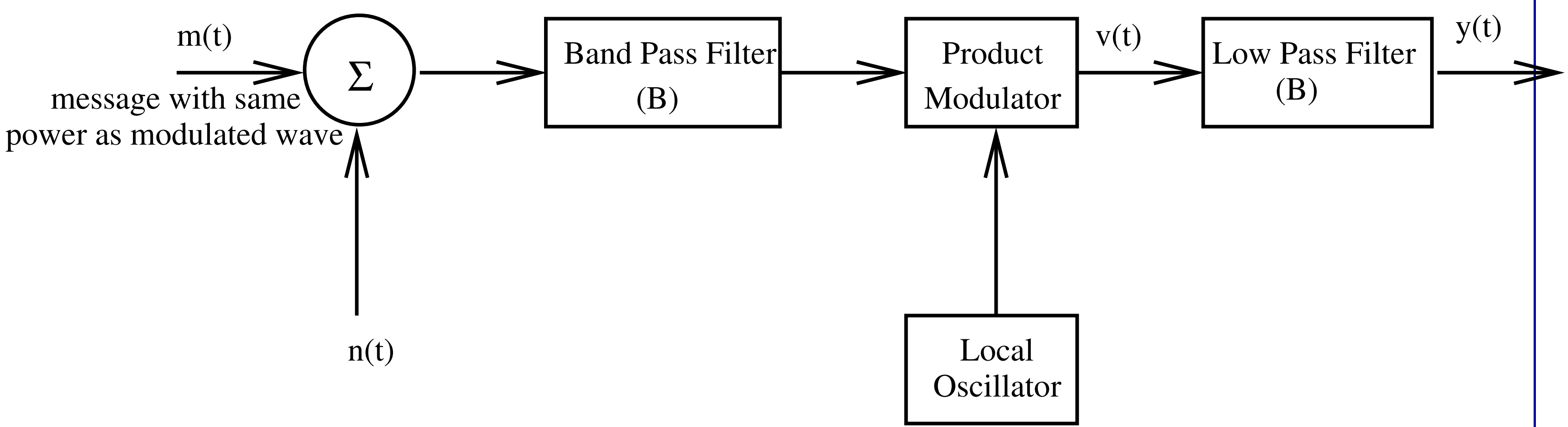

Figure 4: Analysis of DSB-SC System in Noise

The output of the product modulator is

$$
\begin{aligned}
v(t) &= x(t)\cos(\omega_c t) \\
&= \frac{1}{2}A_c m(t) + \frac{1}{2}n_I(t) \\
&\quad + \frac{1}{2}[C A_c m(t) + n_I(t)]\cos 2\omega_c t \\
&\quad - \frac{1}{2}n_Q(t)\sin 2\omega_c t
\end{aligned}
$$

The Low pass filter output is:

$$
= \frac{1}{2}A_c m(t) + \frac{1}{2}n_I(t)
$$

- $\Longrightarrow$ ONLY inphase component of noise $n_I(t)$ at the output
- $\Longrightarrow$ Quadrature component of noise $n_Q(t)$ is filtered at the output
- Band pass filter width $= 2B$

Receiver output is $\frac{n_I(t)}{2}$
Average power of $n_I(t)$ same as that $n(t)$

$$
\begin{aligned}
Average\ noise\ power &= (\frac{1}{2})^2 2BN_0 \\
&= \frac{1}{2} BN_0 \\
(SNR)_{O,DSB-SC} &= \frac{C^2 A_c^2 P/4}{BN_0/2} \\
&= \frac{C^2 A_c^2 P}{2BN_0} \\
FoM_{DSB-SC} &= \left(\frac{(SNR)_O}{(SNR)_C} \right) |_{DSB-SC} = 1
\end{aligned}
$$

- Amplitude Modulation
  - The receiver model is as shown in Figure 5

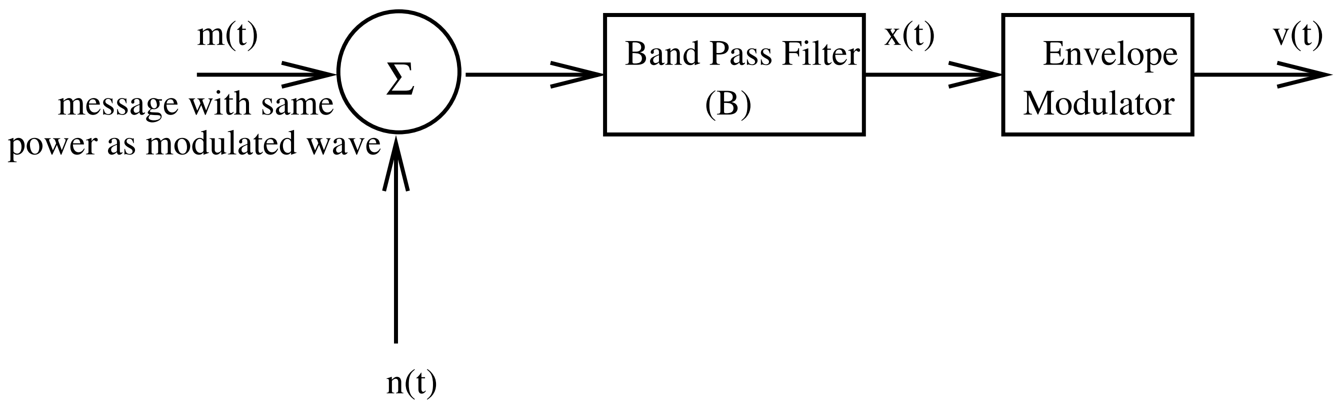

Figure 5: Analysis of AM System in Noise

$$
\begin{aligned}
s(t) &= A_c[1 + k_a m(t)] \cos \omega_c t \\
(SNR)_{C,AM} &= \frac{A_c^2(1 + k_a^2 P)}{2BN_0} \\
x(t) &= s(t) + n(t) \\
&= [A_c + A_c k_a m(t) + n_I(t)] \cos \omega_c t \\
&\quad -n_Q(t) \sin \omega_c t \\
y(t) &= \textit{envelope of } x(t) \\
&= \left[[A_c + A_c k_a m(t) + n_I(t)]^2 + n_Q^2(t) \right]^{\frac{1}{2}} \\
&\approx A_c + A_c k_a m(t) + n_I(t) \\
(SNR)_{O,AM} &\approx \frac{A_c^2 k_a^2 P}{2BN_0} \\
FoM_{AM} &= \left(\frac{(SNR)_O}{(SNR)_C} \right) |_{AM} = \frac{k_a^2 P}{1 + k_a^2 P}
\end{aligned}
$$

Thus the FoM_{AM} is always inferior to FoM_{DSB-SC}

– Frequency Modulation

∗ The analysis for FM is rather complex

∗ The receiver model is as shown in Figure 6

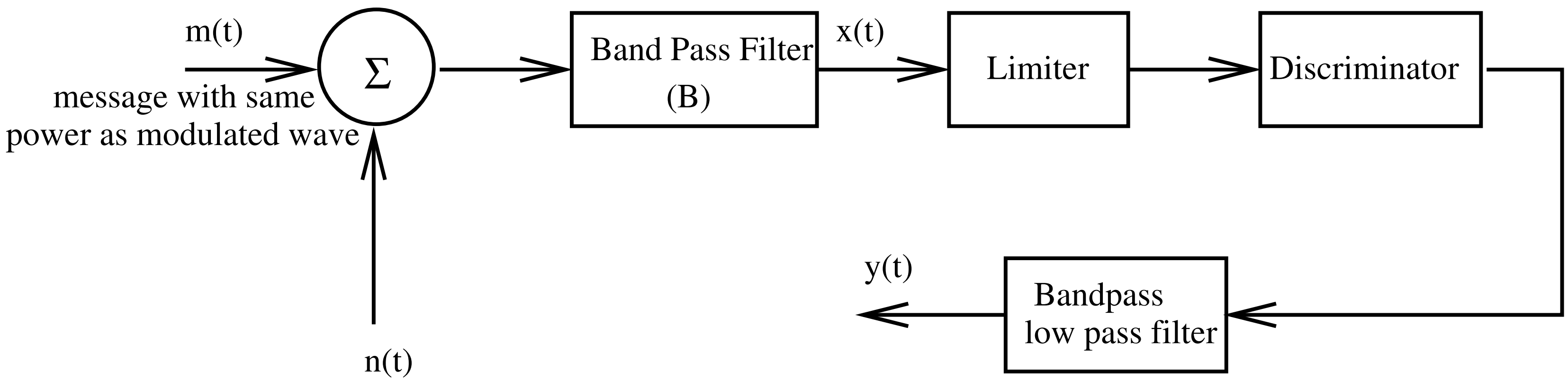

Figure 6: Analysis of FM System in Noise

$$
\begin{aligned}
(SNR)_{O,FM} &= \frac{3A_c^2k_f^2P}{2N_0B^3} \\
(SNR)_{C,FM} &= \frac{A_c^2}{2BN_0} \\
FoM_{FM} &= \left(\frac{(SNR)_O}{(SNR)_C}\right)|_{FM} = \frac{3k_f^2P}{B^2}
\end{aligned}
$$

The significance of this is that *when the carrier SNR is high, an increase in transmission bandwidth B_T provides a corresponding quadratic increase in output SNR or FoM_{FM}*