Problem 3.19

We are given

$$m(t) = \frac{t}{1+t^2} \tag{1}$$

The AM wave is therefore defined by

$$s(t) = A_c[1 + k_a m(t)] \cos(2\pi f_c t)$$

$$= A_c \left(1 + \frac{k_a t}{1 + t^2}\right) \cos\left(2\pi f_c t\right) \tag{2}$$

The message signal m(t) is plotted in Fig. 1(a) with its maximum value of 1/2 and minimum value of -1/2 at t = 1 and t = -1, respectively.

(a) Percentage modulation = 50%

$$k_a |m(t)|_{\text{max}} = 0.5$$

with $|m(t)|_{\text{max}} = \frac{1}{2}$, it follows that $k_a = 1$ for 50% modulation. For this example, Eq. (1) takes the form

$$s(t) = A_c \left(1 + \frac{t}{1 + t^2} \right) \cos(2\pi f_c t)$$
 (3)

Let t be measured in seconds. Then, for the envelope of the AM wave to be clearly visible, the period of the carrier, $1/f_c$, must be small compared to the time taken for the message signal m(t) to reach its peak value. To satisfy this requirement, we let

$$\frac{1}{f_c} = 10$$
Hz

which corresponds to

$$f_c = 10$$
Hz

Setting the carrier amplitude $A_c = 1$ volt, and $f_c = 10$ Hz, Eq. (3) is plotted in part (b) of Fig. 1.

(b) Percentage modulation = 100%

In this case, we have $k_a = 2$. Correspondingly, Eq. (1) assumes the form

$$s(t) = A_c \left(1 + \frac{2t}{1 + t^2} \right) \cos(2\pi f_c t) \tag{4}$$

Keeping $A_c = 1$ volt, and $f_c = 10$ Hz as in case (a), Eq. (4) is plotted in Fig. 1(c).

(c) Percentage modulation = 125%

In this third and final example, we have $k_a = 2.5$. Hence, Eq. (1) now assumes the form

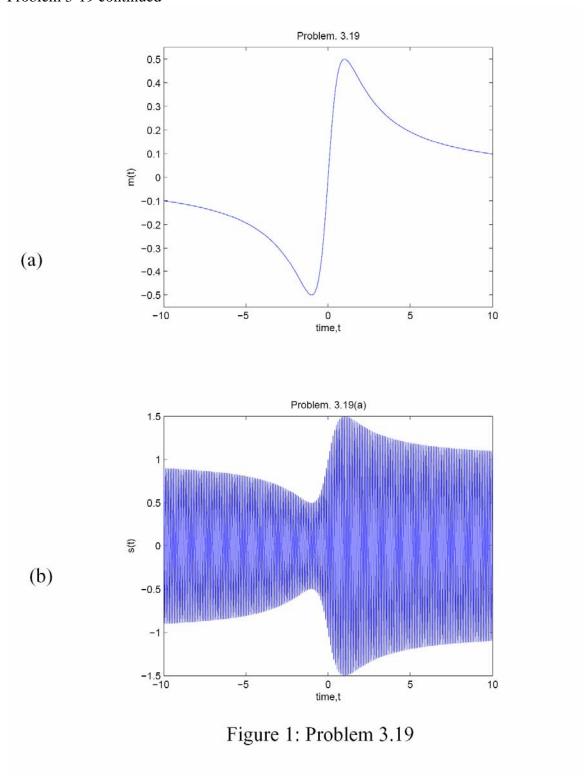
$$s(t) = A_c \left(1 + \frac{2.5t}{1 + t^2} \right) \cos(2\pi f_c t)$$
 (5)

Keeping $_c = 1$ volt, and $f_c = 10$ Hz as before, Eq. (5) is plotted in Fig. 1(d).

Comparing the AM waveforms plotted in parts (b), (c) and (d) of Fig. 1, we may make the following observations:

- The AM wave of Fig. 1(b) is undermodulated
- The AM wave of Fig. 1(c) is on the verge of overmodulation
- The AM wave of Fig. 1(d) is overmodulated

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