EE 308: Communication Systems (Section 1 – Autumn 2018)
Tutorial Problem Set 4

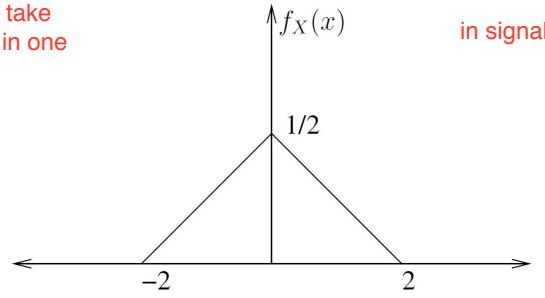
1. Specify the Nyquist rate and the Nyquist interval for each of the following signals.
   (a) $sinc(100t)$
   (b) $sinc^2(100t)$
   (c) $sinc(100t) + sinc^2(100t)$

   nyquist interval = 1/nyquist rate

2. A signal can be modeled as low pass stationary random process $X(t)$ whose probability density function is as shown inbelow figure. Let the signal be bandlimited to 5 kHz. If the signal is sampled at Nyquist rate and then quantized using 32 levels, what is the resulting SQNR?

   as there are 32 levels we take probability of x to be const in one interval

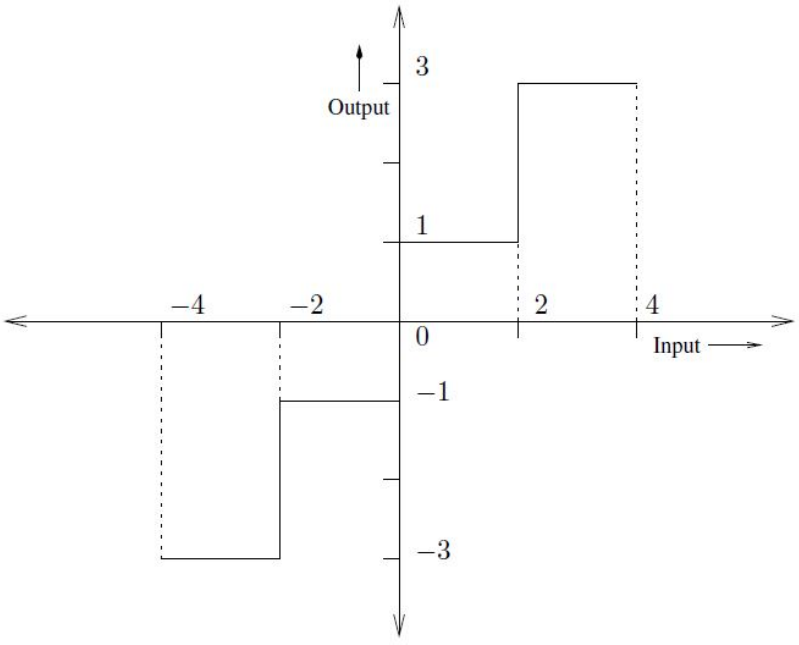
   in signal power we calculate E(x^2)



3. Consider the quantizer characteristics shown in figure below.
   Let X be a random signal input to the quantizer with probability density function, $f_X(x)$ given by:

$$f_X(x) = \begin{cases} Aexp(-|x|), & |x| \leq 4 \\ 0, & \text{otherwise} \end{cases}$$

   (a) Determine the value of A?
   (b) Determine the total quantization noise σ_Q^2
   (c) Is it same as $\frac{\Delta^2}{12}$? where Δ is the step size



4. Consider a uniform quantizer with range from -4 to +4 Volts. Assume that zero mean Gaussian distributed random variable is applied to the quantizer input.
   (a) What is the probability that the amplitude of the input lies outside the range of -4 to +4 Volts?
   (b) Calculate the SQNR.
5. Consider a system in which a signal is quantized with step sizes Δ_i where Δ_1 is the size of the first step, Δ_2 is the size of the second step and so on. p_i is the probability that the input signal amplitude lies within the i^{th} interval. Show that the mean square value of the quantization error is approximately equal to $\frac{1}{12}\sum_i {\Delta_i}^2 p_i$. Assume that the step size is small compared to the range of the input signal.
6. Derive the frequency domain expression of a PAM wave produced by the modulating signal

$$m(t) = A_m cos(2\pi f_m t)$$

   assuming a modulation frequency $f_m = 0.25Hz$, sampling period $T_s = 1$ s, and pulse duration $T = 0.45$ s.
7. In natural sampling, an analog signal $g(t)$ is multiplied by a periodic train of rectangular pulses $C(t)$. Given that the pulse repetition frequency of this periodic train is f_s and the duration of each rectangular pulse is T (with $f_s T << 1$)
   Find the spectrum of the signal $s(t)$ that results from the use of natural sampling (You may assume that time $t = 0$ corresponds to the midpoint of a rectangular pulse in $C(t)$).
8. An analog signal is sampled, quantized and transmitted using PCM. If each sample at the receiver must be known to within 0.5% of the peak-to-peak value, then how many binary digits per sample are required?
9. The SQNR of a PCM system should be minimum 22 dB. The signal amplitude varies from -10 to +10 Volts and $f_m = 2KHz$. If signal is sinusoidal and sampling rate is twice the Nyquist rate then determine:
   (a) Bits per sample required
   (b) Step size
   (c) Signal power
   (d) Noise power
   (e) Bit rate of the system
10. The pulse rate of DM system is 56000 samples/s. The input signal is $5cos(2\pi \text{x} 1000t)+2cos(2\pi \text{x} 2000t)$. Find minimum step size that will avoid slope overload distortion.