

### Problem 3.17

(a) We are given

$$c(t) = A_c \sin(2\pi f_c t)$$

and

$$m(t) = A_m \sin(2\pi f_m t)$$

Invoking the definition of AM wave

$$s(t) = [1 + k_a m(t)]c(t)$$

we now write

$$\begin{aligned} s(t) &= A_c [1 + k_a A_m \sin(2\pi f_m t)] \sin(2\pi f_c t) \\ &= A_c \sin(2\pi f_c t) + \mu A_c \sin(2\pi f_m t) \sin(2\pi f_c t) \end{aligned} \quad (1)$$

where

$$\mu = k_a A_m$$

is the modulation factor. Next, we use the trigonometric identity

$$\sin A \sin B = \frac{1}{2} [\cos(A - B) - \cos(A + B)]$$

Hence, we may rewrite Eq. (1) as

$$s(t) = A_c \sin(2\pi f_c t) + \frac{1}{2} \mu A_c [\cos(2\pi(f_c - f_m)t) - \cos(2\pi(f_c + f_m)t)] \quad (2)$$

The spectrum of the AM wave  $s(t)$  consists of three components:

- (i) Carrier:  $A_c \sin(2\pi f_c t)$
- (ii) Lower side-frequency:  $\frac{1}{2} \mu A_c \cos(2\pi(f_c - f_m)t)$
- (iii) Upper side-frequency:  $-\frac{1}{2} \mu A_c \cos(2\pi(f_c + f_m)t)$

This spectrum is depicted in Fig. 1.

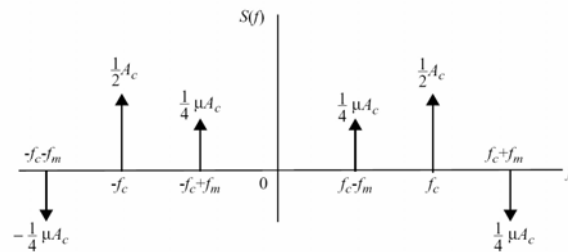


Figure 1

(b) Comparing the AM spectrum of Fig. 1 with the corresponding AM spectrum of Fig. 3.3(c) on page 105 of the text, we may make two observations:

- The frequency locations of the spectral components of these two AM waves are identical.
- The only difference between them is that the upper side-frequency  $f_c + f_m$  in Fig. 1 is the negative of the upper side-frequency  $f_c + f_m$  in Fig. 3.3(c).

**Note:** The following correction in the first printing of the book should be made. The modulating wave should read as follows:

$$m(t) = A_m \sin(2\pi f_m t)$$