Problem 7.25

(a) The correlation coefficient of the signals $s_0(t)$ and $s_1(t)$ is

$$\rho = \frac{\int_0^{T_b} s_0(t)s_1(t)dt}{\left[\int_0^{T_b} s_0^2(t)dt\right]^{1/2}\left[\int_0^{T_b} s_1^2(t)dt\right]^{1/2}}$$

$$= \frac{A_c^2\int_0^{T_b} \cos\left[2\pi\left(f_c+\frac{1}{2}\Delta f\right)t\right]\cos\left[2\pi\left(f_c-\frac{1}{2}\Delta f\right)t\right]}{\left[\frac{1}{2}A_c^2T_b\right]^{1/2}\left[\frac{1}{2}A_c^2T_b\right]^{1/2}}$$

$$= \frac{1}{T_b}\int_0^{T_b}[\cos(2\pi\Delta ft)+\cos(4\pi f_c t)]dt$$

$$= \frac{1}{2\pi T_b}\left[\frac{\sin(2\pi\Delta f T_b)}{\Delta f}+\frac{\sin(4\pi f_c T_b)}{2f_c}\right] \qquad (1)$$

Since $f_c >> \Delta f$, then we may ignore the second term in Eq. (1), obtaining

$$\rho \approx \frac{\sin(2\pi\Delta f T_b)}{2\pi T_b\Delta f} = \mathrm{sinc}(2\Delta f T_b)$$

(b) The dependence of ρ on Δf is as shown in Fig. 1. The two signals $s_0(t)$ and $s_1(t)$ are orthogonal when $\rho = 0$. Therefore, the minimum value of Δf for which they are orthogonal is $1/2T_b$.
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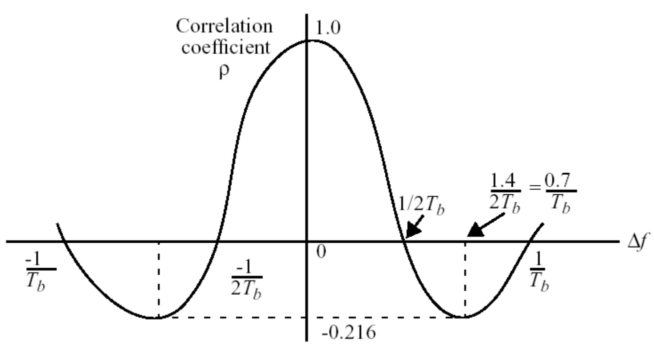

Figure 1