Problem 3.20

(a) Let the input voltage v_i consist of a sinusoidal wave of frequency $\frac{1}{2}f_c$ (i.e., half the desired carrier frequency) and the message signal $m(t)$, as shown by

$$v_i = A_c\cos(\pi f_c t) + m(t) \qquad (1)$$

Then, the output current i_o is

$$\begin{aligned} i_o &= a_1 v_i + a_3 v_i^3 \\ &= a_1[A_c\cos(\pi f_c t) + m(t)] + a_3[A_c\cos(\pi f_c t) + m(t)]^3 \\ &= a_1[A_c\cos(\pi f_c t) + m(t)] + \frac{1}{4}a_3 A_c^3[\cos 3(\pi f_c t) + 3\cos(\pi f_c t)] \\ &\quad + \frac{3}{2}a_3 A_c^2 m(t)[1 + \cos(2\pi f_c t)] + 3a_3 A_c\cos(\pi f_c t)m^2(t) + a_3 m^3(t) \end{aligned}$$

Assume that $m(t)$ occupies the frequency interval $-W \le f \le W$. Then, the amplitude spectrum of the output current i_o is as shown Fig. 1:

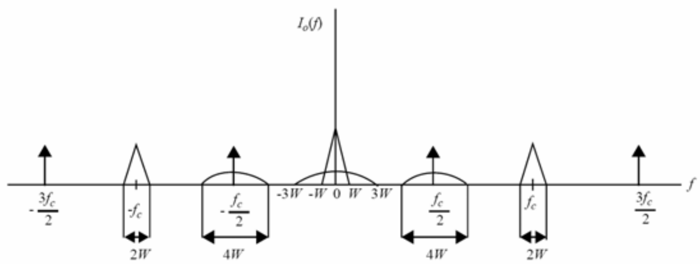

Figure 1

From this spectrum we see that in order to extract a DSB-SC wave with carrier frequency f_c from i_o, we need a bandpass filter with mid-band frequency f_c and bandwidth $2W$, the two of which satisfy the requirement:

$$f_c - W > \frac{f_c}{2} + 2W$$

that is, $f_c > 6W$

Therefore, to use the given nonlinear device as a product modulator, we may use the configuration: shown in Fig. 2.

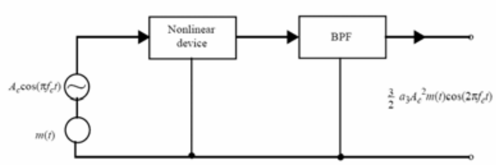

Figure 2

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Problem 3-20 continued

(b) To generate an AM wave with carrier frequency f_c, we require a sinusoidal component of frequency f_c to be added to the DSB-SC generated in the manner described under (a). To achieve this requirement, we may use a configuration involving a pair of the nonlinear devices and a pair of identical bandpass filters, as depicted in Fig. 3.

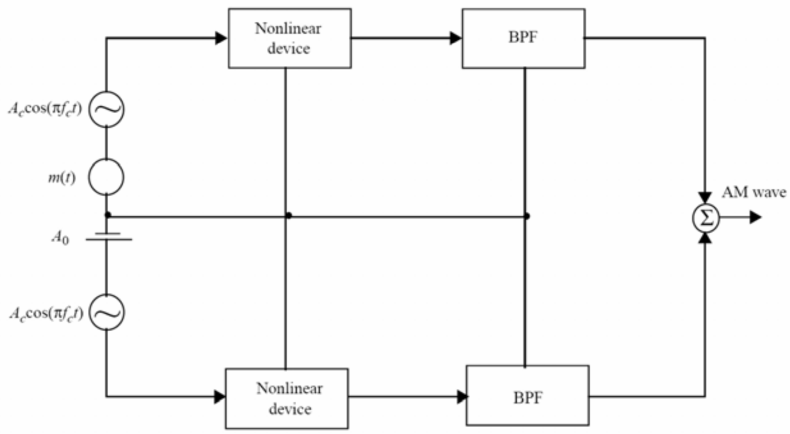

Figure 3

The resulting AM wave is therefore $\frac{3}{2}a_3 A_c^2[A_0 + m(t)]\cos(2\pi f_c t)$. Thus, the choice of the dc level A_0 at the input of the lower branch controls the percentage modulation of the AM wave.

The nonlinear device defined in Eq. (1) cannot be used for demodulation. The reason for saying so is that Eq. (1) lacks a square-law term, which is essential for demonstration (i.e., recovery of the message signal from an incoming AM wave).