Problem 4.1.
Using Eq. (4.7), show that FM waves also violate the principle of superposition.

Solution
From Eq. (4.7), the FM wave is defined by

$$s(t) = A_c \cos\left[2\pi f_c t + 2\pi k_f \int_0^t m(\tau)d\tau\right]$$

Suppose $m(t) = m_1(t) + m_2(t)$. Then,

$$s(t) = A_c \cos\left[2\pi f_c t + 2\pi k_f \int_0^t m_1(\tau)d\tau + 2\pi k_f \int_0^t m_2(\tau)d\tau\right] \quad (1)$$

Suppose next the two message signals $m_1(t)$ and $m_2(t)$ are applied individually to the frequency modulator. Then in response to $m_1(t)$, we have

$$s_1(t) = A_c \cos\left[2\pi f_c t + 2\pi k_f \int_0^t m_1(\tau)d\tau\right] \quad (2)$$

Likewise, for $m_2(t)$ we have

$$s_2(t) = A_c \cos\left[2\pi f_c t + 2\pi k_f \int_0^t m_2(\tau)d\tau\right] \quad (3)$$

From Eqs. (1) through (3), we readily see that

$$s(t) \neq s_1(t) + s_2(t)$$

In other words, the principle of superposition (basic to linear systems) is violated. Hence, frequency modulation is a nonlinear process.