Problem 3.25

(a) The effect of a frequency error Δf in the local oscillator used in the coherent detector shows itself as follows:

$$c'(t) = \cos(2\pi(f_c + \Delta f)t)$$

Applying the DSB-SC modulated wave $s(t)$

$$s(t) = A_c \cos(2\pi f_c t) m(t)$$

to a coherent detector employing $c'(t)$ yields the product modulator output (see Fig. 1)

$$\begin{aligned} v(t) &= s(t)c'(t) \\ &= A_c \cos(2\pi f_c t)\cos(2\pi f_c t + 2\pi\Delta f t) m(t) \end{aligned} \tag{1}$$

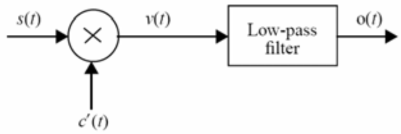

Figure 1

Using the trigonometric identity

$$\cos A \cos B = \frac{1}{2}[\cos(A+B) + \cos(A-B)]$$

we may rewrite Eq. (1) as

$$v(t) = \frac{1}{2}A_c[\cos(4\pi f_c t + 2\pi\Delta f t) + \cos(2\pi\Delta f t)]m(t) \tag{2}$$

Next, passing $v(t)$ through the low-pass filter in Fig. 1 removes the high-frequency component, producing the output

$$o(t) = \frac{1}{2}A_c \cos(2\pi\Delta f t) m(t) \tag{3}$$

which exhibits beats at the error frequency Δf.