Problem 4.24

(a) We are given a nonlinear channel's input-output relation:

$$v_o(t) = a_1 v_i(t) + a_2 v_i^2(t) + a_3 v_i^3(t) \tag{1}$$

where $v_i(t)$ is the input and $v_o(t)$ is the output; a_1, a_2, and a_3 are fixed parameters. The input signal is defined by

$$v_i(t) = A_c \cos(2\pi f_c t + \phi(t)) \tag{2}$$

where

$$\phi(t) = 2\pi k_f \int_0^t m(\tau) d\tau \tag{3}$$

where $m(t)$ is the message signal and k_f is the frequency sensitivity of the frequency modulator. Substituting Eq. (2) into (1) yields

$$\begin{aligned} v_o(t) &= a_1 A_c \cos(2\pi f_c t + \phi(t)) + a_2 A_c^2 \cos^2(2\pi f_c t + \phi(t)) \\ &\quad + a_3 A_c^3 \cos^3(2\pi f_c t + \phi(t)) \end{aligned} \tag{4}$$

Using the trigonometric identities:

$$\cos^2\theta = \frac{1}{2}(1 + \cos(2\theta))$$

$$\cos^3\theta = \frac{1}{4}(1 + \cos(3\theta))$$

we may rewrite Eq. (4) as

$$\begin{aligned} v_o(t) &= \frac{1}{2} a_2 A_c^2 + \left(a_1 A_c + \frac{3}{4} a_3 A_c^3\right) \cos(2\pi f_c t + \phi(t)) \\ &\quad + \frac{1}{2} a_2 A_c^2 \cos(4\pi f_c t + 2\phi(t)) + \frac{1}{4} a_3 A_c^3 \cos(6\pi f_c t + 3\phi(t)) \end{aligned} \tag{5}$$

Equation (5) shows that the channel output consists of the following components:

- A dc component, $\frac{1}{2} a_2 A_c^2$
- Frequency modulated component of frequency f_c, phase $\phi(t)$ and amplitude $\left(a_1 A_c + \frac{3}{4} a_3 A_c^3\right)$
- Frequency modulated component of frequency $2f_c$, phase $2\phi(t)$ and amplitude $\frac{1}{2} a_2 A_c^2$
- Frequency modulated component of frequency $3f_c$, phase $3\phi(t)$ and amplitude $\frac{1}{4} a_3 A_c^3$

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Problem 4-24 continued

(b) To remove the nonlinear distortion and thereby extract a replica of the original FM signal $v_i(t)$, it is necessary to separate the FM component with carrier frequency f_c in $v_o(t)$ from the higher order FM components. Let Δf denote the frequency deviation of the original FM signal and W denote the highest frequency component of the message signal $m(t)$. Then, using Carson's rule and noting that the frequency deviation above $2f_c$ is doubled (which is the component nearest to the original FM signal), we find that the necessary condition for separating the desired FM signal with carrier frequency f_c from that with carrier frequency $2f_c$ is

$$2f_c - (2\Delta f + W) > f_c + \Delta f - W$$

or

$$f_c > 3\Delta f + 2W \tag{6}$$

(c) To extract a replica of the original FM signal $v_i(t)$, we need to pass the channel output $v_o(t)$ through a band-pass filter of midband frequency f_c and bandwidth $2(\Delta f + W)$. The resulting filter output is

$$v'_o(t) = \left(a_1 A_c + \frac{3}{4}a_3 A_c^3\right)\cos(2\pi f_c t + \phi(t)) \tag{7}$$

where $\phi(t)$ is defined by Eq. (3).