Problem 6.4

Equation (6.17) defines the raised-cosine pulse spectrum $P(f)$ as real-valued and therefore zero delay. In practice, every transmission system experiences some finite delay. To accommodate this practicality, we may associate with $P(f)$ a linear phase characteristic over the frequency band $0 \le |f| \le 2B_0 - f_1$.

(a) Show that this modification of $P(f)$ introduces a finite delay into its inverse Fourier transform, namely, the pulse shape $p(t)$.

(b) According to Eq. (6.19), $p(t)$ represents a non-causal time response. The delay introduced into $p(t)$ through the modification of $P(f)$ has also a beneficial effect, tending to make $p(t)$ essentially causal. For this to happen however, the delay must not be less than a certain value dependent on the roll-off factor α. Suggest suitable values for the delay for $\alpha = 0$, 1/2, and 1.

Solution

(a) Let the linear phase characteristic appended to $P(f)$ be

$$\theta(f) = 2\pi f\tau$$

where τ is delay to be determined. Then, the modified raised-cosine pulse spectrum is defined by

$$P_{\text{modified}}(f) = P(f)e^{-j\theta(f)}$$
$$= P(f)e^{-j2\pi f\tau}$$

Invoking the time-shifting property, we therefore have

$$p_{\text{modified}}(t) = p(t-\tau)$$

where $p(t)$ is defined by Eq. (6.19).

(b) For $p_{\text{modified}}(t)$ to be causal, it has to be zero for $t < 0$.

From Fig. 6.3(b) in the text, we deduce that we may essentially set

(i) $\tau = 5s$ for $\alpha = 0$
(ii) $\tau = 3s$ for $\alpha = 1/2$
(iii) $\tau = 2.5s$ for $\alpha = 1$

Increasing α corresponds to increasing transmission bandwidth B_T. We therefore find that as the transmission bandwidth B_T is increased, the necessary delay τ is progressively reduced, which is in accord with the inverse relationship that exists between behaviors of a function in the time- and frequency-domains.

(c) The slope of $\theta(f)$ with respect to f is

$$\frac{\partial\theta(f)}{\partial f} = 2\pi\tau$$

Hence,

(i) slope = -10π for $\alpha = 0$
(ii) slope = -6π for $\alpha = 1/2$
(iii) slope = -5π for $\alpha = 1$