Problem 2.8

Considering the sinc pulse sinc(t), show that

$$\int_{-\infty}^{\infty} \mathrm{sinc}^2(t)dt = 1$$

Solution

This integral may be viewed as

$$I = \int_{-\infty}^{\infty} \mathrm{sinc}(t) \cdot \mathrm{sinc}(t)dt$$

which, in light of Rayleigh's energy theorem, may also be expressed as

$$I = \int_{-\infty}^{\infty} |\mathbf{F}[\mathrm{sinc}(t)]|^2 df$$

From Eq. (2.25) in the text, we have

$\mathbf{F}[\mathrm{sinc}t] = \mathrm{rect}(f)$

Hence,

$$\begin{aligned} I &= \int_{-\infty}^{\infty} \mathrm{rect}^2(f)df \\ &= \int_{-1/2}^{1/2} 1^2 df \\ &= 1 \end{aligned}$$