## Problem 4.10

By definition, the instantaneous frequency  $f_i$  is related to the phase  $\theta(t)$  as

$$f_i = \frac{1}{2\pi} \frac{d\theta}{dt}$$

which may be rewritten as

$$f_i \approx \frac{1}{2\pi} \frac{\Delta \theta}{\Delta t} \tag{1}$$

where  $\Delta\theta$  and  $\Delta t$  are small changes in the phase  $\theta(t)$  and time t. We are given

$$\theta(t + \Delta t) - \theta(t) = \pi$$

from which we infer that

$$\Delta\theta = \pi \tag{2}$$

Substituting Eq. (2) into (1) yields

$$f_i \approx \frac{1}{2\pi} \cdot \frac{\Delta \theta}{\Delta t} = \frac{1}{2\Delta t}$$

which is the desired result.