Problem 5.2

Show that as the sampling period T_s approaches zero, the formula for the discrete-time Fourier transform $G_\delta(f)$ approaches the Fourier transform $G(f)$.

Solution

From Eq. (5.3), we have

$$G_\delta(f) = \sum_{n=-\infty}^{\infty} g\left(\frac{n}{2W}\right)\exp\left(-\frac{j\pi nf}{W}\right)$$

The sampling period $T_s = -1/(2W)$. We may therefore rewrite this equation as

$$G_\delta(f) = \sum_{n=\infty}^{\infty} g(nT_s)\exp(-j2\pi nT_s f)$$

In the limit, as T_s approaches zero, the discrete time nT_s, approaches continuous time t. Moreover, the summation over n approaches the integral

$$\int_{-\infty}^{\infty} g(t)\exp(-j2\pi tf)dt$$

Correspondingly, $G_\delta(f)$ approaches the continuous Fourier transform $G(f)$. We may therefore state that the formula for the discrete Fourier transform $G_\delta(f)$ given in Eq. (5.3) approaches the formula for the Fourier transform:

$$G(f) = \int_{-\infty}^{\infty} g(t)\exp(-j2\pi tf)dt$$

as the sampling period T_s approaches zero.