EE308 – Communication Systems (Section 1), Autumn 2018

End-Sem Exam | November 11, 2018 (14:00 – 17:00 Hrs) | Max. Marks: 45

In this paper, you can directly use the following identities if you wish to:

$$\mathcal{F}\left(\sum_{n=-\infty}^{\infty}\delta(t-n/f_s)\right) = f_s\sum_{n=-\infty}^{\infty}\delta(f-nf_s) = \sum_{n=-\infty}^{\infty}e^{-j2\pi nf/f_s}\ ;$$

$$\mathcal{F}\left[\mathrm{rect}(t/T)\right] = T\mathrm{sinc}(fT) = \frac{T\sin(\pi fT)}{\pi fT}\ ,$$

where rect(t/T) is a rectangular pulse of unity amplitude and pulsewidth T, centered at t=0.

Please turn over for solutions.

Question 1 **[2+2+2 = 6 marks]**

1. If $X(f)$ is the Fourier transform of $x(t)$, i.e. $X(f) = \int_{-\infty}^{\infty} x(t)e^{-j2\pi ft}dt$,
   (a) Derive the Fourier transform of $x(at)$, where a is a real number.
   (b) Prove the Parseval's theorem $\int_{-\infty}^{\infty} |x(t)|^2 dt = \int_{-\infty}^{\infty} |X(f)|^2 df$.
   (c) If $X_1(\omega)$ is the Fourier transform of $x(t)$ in terms of angular frequency ω, i.e. $X_1(\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt$, write he Parseval's theorem for it in terms of $X_1(\omega)$ and ω.

<u>**Solution:**</u>

(a):

$$\begin{aligned} X(f) &= \int_{-\infty}^{\infty} x(t)e^{-j2\pi ft}dt \\ \text{Let, } \widehat{X}(f) &= \int_{-\infty}^{\infty} x(at)e^{-j2\pi ft}dt \end{aligned} \tag{1}$$

There can be two cases $a > 0$ or $a < 0$. For $a > 0$ **[1 mark]**

$$\begin{aligned} \widehat{X}(f) &= \int_{-\infty}^{\infty} x(at)e^{-j2\pi ft}dt \\ \text{Substituting } y = at, & \\ &= \frac{1}{a}\int_{-\infty}^{\infty} x(y)e^{-j2\pi f\frac{y}{a}}dy \\ &= \frac{1}{a}X(\frac{f}{a}). \end{aligned} \tag{2}$$

For $a < 0$,

$$\begin{aligned} \widehat{X}(f) &= \int_{-\infty}^{\infty} x(at)e^{-j2\pi ft}dt \\ \text{Substituting } y = at, & \\ &= \frac{1}{a}\int_{\infty}^{-\infty} x(y)e^{-j2\pi f\frac{y}{a}}dy \\ &= \frac{1}{-a}\int_{-\infty}^{\infty} x(y)e^{-j2\pi f\frac{y}{a}}dy \\ &= \frac{1}{-a}X(\frac{f}{a}). \end{aligned} \tag{3}$$

Thus fourier transform of $x(at)$ is $\frac{1}{|a|}X(\frac{f}{a})$. **[1 mark]**

(b):

$$\begin{aligned} \text{LHS } &= \int_{-\infty}^{\infty} x(t)\bar{x}(t)dt \\ &= \int_{-\infty}^{\infty} \bar{x}(t)\int_{-\infty}^{\infty} X(f)e^{j2\pi ft}df dt \\ \text{Changing order of integrals,} & \\ &= \int_{-\infty}^{\infty} X(f)\int_{-\infty}^{\infty} \bar{x}(t)e^{j2\pi ft}dt df \\ &= \int_{-\infty}^{\infty} X(f)\bar{X}(f)df \\ &= \text{ RHS} \qquad \textbf{[2 marks]} \end{aligned} \tag{4}$$

(c): We know that,

$$\begin{aligned}
X_1(\omega) &= \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt \\
X(f) &= \int_{-\infty}^{\infty} x(t)e^{-j2\pi ft}dt \\
\text{Substituting } \omega = 2\pi f, & \\
X(\frac{\omega}{2\pi}) &= \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt \\
\text{Thus, } X_1(\omega) = X(\frac{\omega}{2\pi}). \qquad & \textbf{[1 mark]}
\end{aligned} \tag{5}$$

Now looking at RHS of Parseval's relation as stated in (b)

$$\begin{aligned}
\text{RHS } &= \int_{-\infty}^{\infty} |X(f)|^2 df \\
\text{Substituting } \omega = 2\pi f & \\
&= \frac{1}{2\pi}\int_{-\infty}^{\infty} |X(\frac{\omega}{2\pi})|^2 d\omega \\
&= \frac{1}{2\pi}\int_{-\infty}^{\infty} |X_1(\omega)|^2 d\omega.
\end{aligned} \tag{6}$$

Therefore, Parseval's relation in terms of angular frequency is

$$\int_{-\infty}^{\infty} |x(t)|^2 dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} |X_1(\omega)|^2 d\omega. \qquad \textbf{[1 mark]} \tag{7}$$

Question 2 [1+1+1+2+1+2 = 8 marks]

2. Answer the following / fill in the blanks (write in your answerbook):

   (a) In analog TV signals, the video information is encoded in the form of what three independent parameters of the signal?

   (b) Why do we commonly use RRC (root raised cosine) filters at the transmitters and receivers of wireless digital communication links and not RC (raised cosine) filters?

   (c) In digital communications, the Costas loop is used for ________ synchronization. However, the Costas loop cannot remove the offset between the ________ of the transmitter and the receiver.

   (d) What does the Wiener Khintchine relationship say about the autocorrelation function of a stationary random process? How can it be useful in demodulating GPS signals?

   (e) It is always a good practice to use a/an ________ before any sort of sampling to ensure that the out-of-band ________ and ________ don't appear in-band after sampling.

   (f) Write the expansion of the FM signal $s_{FM}(t) = A_c \cos[2\pi f_c t + \beta \sin(2\pi f_m t)]$ in terms of $J_n(\beta)$'s, the n^{th} order Bessel's functions of first kind. What condition on $J_n(\beta)$'s suggests that the power in the FM modulated signal does not change with $m(t)$?

Solution:

(a) Three parameters of the signal are **carrier frequency, carrier phase, and average amplitude** (Hue, saturation, Intensity).
Marking Scheme: 1 mark if all three parametrs are given, 0.5 mark for partially correct answer.

(b) Raised cosine filter is used for **removing inter-symbol interference (ISI)** [Nyquist filter]. Using RRC at both transmitter and receiver produces matched filtering effect. Matched filtering **maxizes SNR**.
Marking Scheme: 1 mark if both the reasons are provided, 0.5 if only one reason is given.

(c) In digital communications, the Costas loop is used for **cariier phase/frequency** synchronization. However, the Costas loop cannot remove the offset between the **symbol clock** of the transmitter and the receiver.
Marking Scheme: 1 mark if both are correct, 0.5 if only one is correct.

(d) Wiener Khintchine relationship says that auto-correlation function and power spectral density forms a Fourier transform pair.
$R_X(t) \leftrightarrow$ PSD.
In GPS receivers, cross-correlation of received PRN (Gold) code and refernce code is carried out for range measurement which is computationally complex. According to Wiener Khintchine relationship, it can be computed in frequency domain by multplying FFT of received code and FFT of reference code, and then taking IFFT (computationally less complex than time domain correlation).
Marking Scheme: 2 marks if both W-K relation and reasoning are correct, 1 mark if one of them is correct.

(e) It is always a good practice to use a/an **anti-aliasing filter** before any sort of sampling to ensure that the out-of-band **noise** and **interference** don't appear in-band after sampling.
Marking Scheme: 0.5 mark for first blank + 0.5 if one or both of second and third blanks are filled correctly.

(f) Expansion:

$$s_{FM}(t) = A_c \sum_{n=0}^{\infty} J_n(\beta) \cos[2\pi(f_c + nf_m)t]$$

Relation between $J_n(\beta)$:

$$\sum_{n=0}^{\infty} J_n^2(\beta) = 1$$

Marking Scheme: 2 marks if both expansion and relation are correct, 1 mark if one of them is correct.

Question 3 [3 marks]

3. Consider the pass-band AM signal, with time varying voltage measured at the receiver being $s_{AM}(t) = A_c(1+k_a m(t))\cos(2\pi f_c t)$. Here, $m(t)$ is the message signal (with message spectrum ranging from $-W$ to $+W$) and $\mathrm{E}[m^2(t)] = P_m$. The double-sided noise PSD at the receiver is $N_0/2$, i.e. single-sided noise PSD is N_0 (use reference resistor of 1Ω for conversion to mean squared voltage). What is the SNR obtained in the demodulated signal using a coherent detector (missing factors of halves or twos will be completely unacceptable in this problem)? *[3]*

<u>Solution:</u>

A coherent detector multiplies the received signal by the carrier, and takes the Low Pass output of the resulting signal to detect the outcome. Assumption here is that the message bandwidth is much less than the carrier frequency so as not to interfere.

Signal at receiver:

$$A_c \times (1 + k_a m(t)) \times \cos 2\pi f_c t + \eta(t)$$

Noise can be written as:

$$\eta(t) = \eta_I(t) \times \cos 2\pi f_c t - \eta_Q(t) \times \sin 2\pi f_c t$$

[0.5 Mark]

Therefore, after multiplying with the cosine career wave, we get

$$A_c \times (1 + k_a m(t)) \times \cos 2\pi f_c t + \eta(t)$$

$$A_c \times (1 + k_a m(t)) \times \frac{1+\cos 4\pi f_c t}{2} + \eta_I(t) \times \frac{1+\cos 4\pi f_c t}{2} + \eta_Q(t) \times \frac{\sin 4\pi f_c t}{2}$$

Applying the LPF to this, we get

$$r(t) = \frac{A_c \times (1 + k_a m(t))}{2} + \frac{\eta_I(t)}{2}$$

[1 Mark]

Of this, $\frac{A_c \times (1+k_a m(t))}{2}$ is our relevant signal, and $\frac{\eta_I(t)}{2}$ is our noise. We do not consider the DC part of the signal as it is not relevant, and is easily removed using a capacitor.

$$P_{signal} = \frac{{A_c}^2 {k_a}^2 {P_m}^2}{4}$$

[0.5 Mark]

Noise Power: Assuming double sided Noise, we have PSD of $\eta(t) = \frac{N_0}{2}$. So, its quadrature components $\eta_I(t) and \eta_Q(t)$ will have PSD N_0. [Refer Section 8.11 Pg 352-353 from Simon Haykin for the derivation.]

Therefore,

$$P_{noise} = \frac{1}{4} \times \int_{-W}^{W} N_0 d\omega$$

$$P_{noise} = \frac{N_0 W}{2}$$

[0.5 Mark]

Hence,

$$SNR = \frac{{A_c}^2 {k_a}^2 {P_m}^2}{2N_0 W}$$

[0.5 Mark]

Question 4 **[3+3 = 6 marks]**

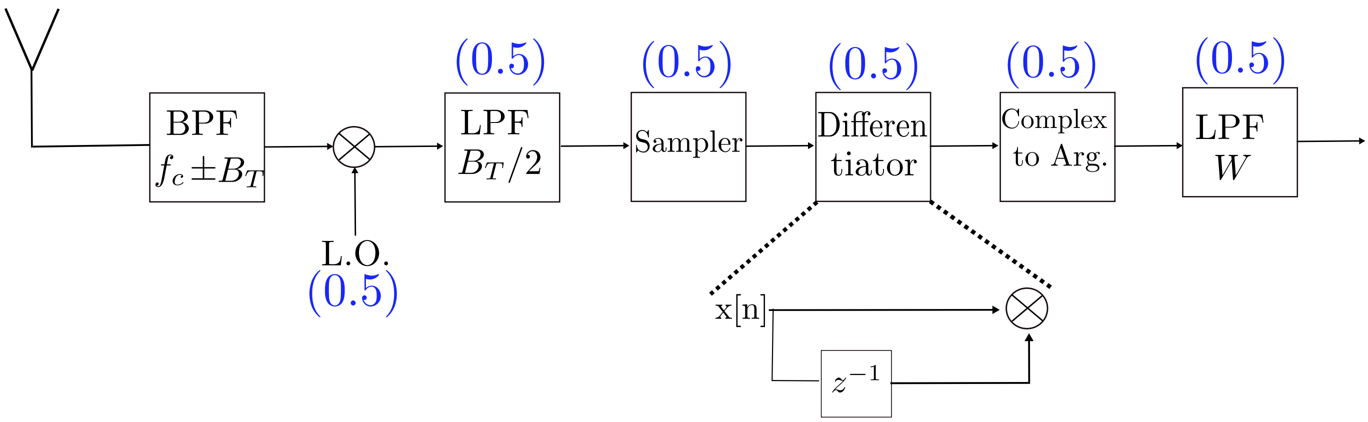
4. An FM modulated signal $s_{FM}(t) = A_c \cos[2\pi f_c t + 2\pi K_f \int m(t)dt]$ is first down converted to a complex baseband signal and then sampled at $f_s = 1\,\text{MHz}$. Adjacent samples are used to demodulate the message from the phase. Assume that the channel and the sampling process add significant amount of noise. Also, assume that the transmission bandwidth $B_T = 100\,\text{kHz}$, and the baseband signal bandwidth $W = 10\,\text{kHz}$. *[3+3]*

   (a) Neatly draw the block diagram showing how you would demodulate the message signal $m(t)$ (till the sampler and after the sampler). Draw all the basic blocks in the signal path starting from the antenna (amplifiers can be omitted). Also include filters etc. in the block diagram, which are important in ensuring good SNR, and mention parameters such as the cutoff frequencies. Ignore all non-idealities other than noise.

   (b) Now assume that the carrier frequency offset during down-conversion is 50 kHz (i.e. it is not negligible), the sampled phase has the peak noise value of ± 0.5 rad and $m(t) = \cos(10^5 \pi t)$. What is the maximum value of K_f for which the signal can be received without observing any thresholding effects.

Solution:

Part a



Marks per component is denoted by blue font (0.5)

Components like BPF after antenna, De-emphasis filters are optional and have been considered if some marked components have been left out

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Part b We have $\Delta f = 50$ kHz,

$\theta(t) = 2\pi\Delta f t + 2\pi K_f \int_0^t m(\tau)d\tau$ **(0.5 marks)**

To avoid thresholding effects, $|\theta'(t) + n_s(t)| < \pi$, $n_s(t)$ is the phase noise **(1 mark)**

If 2π is taken instead of π marks have not been deducted

Since we have a non ideal differentiator here, $\theta'(t)$ is approximated via **(1 mark)**

$\theta(nT_s) - \theta((n-1)T_s) = 2\pi\Delta f T_s + 2\pi K_f T_s \frac{(\int_0^{nT_s} m(\tau)d\tau - \int_0^{(n-1)T_s} m(\tau)d\tau)}{T_s} \approx 2\pi\Delta f T_s + 2\pi K_f m(nT_s)T_s$

Now, plugging in the values, we get

$|2\pi\Delta f T_s + 2\pi K_f m(nT_s)T_s|_{max} + |n_s(t)|_{max} < \pi$, since $m(t) = \cos(10^5\pi t)$, $|m(t)|_{max} = 1$

Hence we have $(2\pi \times 50 \times 10^3 \times 10^{-6} + 2\pi K_f \times 10^{-6} + 0.5) < \pi \implies K_f < 3.7 \times 10^5$ **(0.5 marks)**

Question 5 [2+3+1 = 6 marks]

5. Consider a random process that generates a random binary wave $X(t)$ carrying bits with bit-period T, which randomly take amplitudes $+A$ and $-A$ with equal probabilities. The first bit-period boundary after time t=0 occurs at $t = T_d$, where T_d is a random variable (for each realization of $X(t)$).

   (a) If the distribution of T_d is Unif (0,T/2), is the process stationary (in wide-sense or strict sense)? Is it ergodic? [Succinctly provide the reasons].

   (b) Now assume that the distribution of T_d is Unif (0,T). Also, we obtain $Y(t) = X(t) \times \cos(2\pi f_c t + \Theta)$, where Θ is a random variable with distribution Unif (0, 2π), f_c is a fixed carrier frequency, and Θ and T_d are independent. Find $R_Y(t_1, t_2)$.

   (c) Is $Y(t)$ stationary (in any sense)? Note that Y(t) represents an M-PSK signal. What is the value of M here?

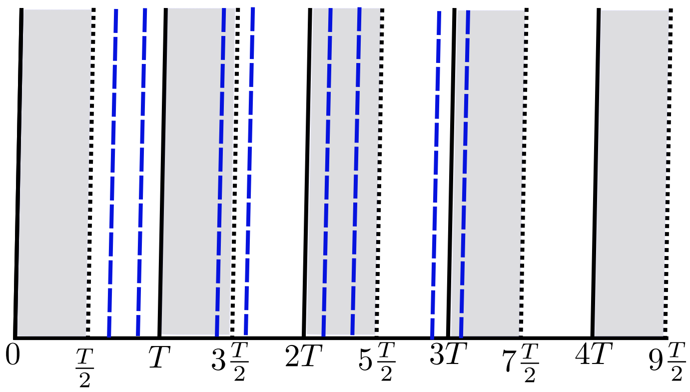
Solution:

Part a

$R_X(t_1, t_2) = \mathbb{E}[x(t_1), x(t_2)]$. Observe here that if $|t_1 - t_2| > T$, the bits corresponding to t_1, t_2 will be different and independent as per the question. Hence, $R_X(t_1, t_2) = \mathbb{E}[x(t_1)]\mathbb{E}[x(t_2)] = 0$, since $\mathbb{E}[x(t)] = 0$, for t_1, t_2 such that $|t_1 - t_2| > T$. **(0.5 marks)**

When $|t_1 - t_2| < T$, let p be the probability of getting a boundary between t_1, t_2. Hence, we have $R_X(t_1, t_2) = p \times 0 + (1 - p) \times (\frac{(A)^2}{2} + \frac{(-A)^2}{2})$, since we have $\pm A$ with equal probability. **(0.5 marks)**

Observe here that since $t_d \sim$ Unif$(0, \frac{T}{2})$, we have the boundary points (given by $t_d, t_d \pm T, t_d \pm 2T \ldots$) distributed in the following way:



The boundary points will be distributed uniformally between $(nT, nT + \frac{T}{2})$

Hence, the probability of boundary between t_1, t_2 will depend whether t_1, t_2 are as in the four possibilities as shown by blue dashed lines. Due to this, the probability depends on both the separation between t_1, t_2 and their absolute positions. Hence, $R_X(t_1, t_2)$ is niether WSS nor ergodic. **(1 mark)**

Any other appropriate explanation highlighting the above point has been awarded 1 mark

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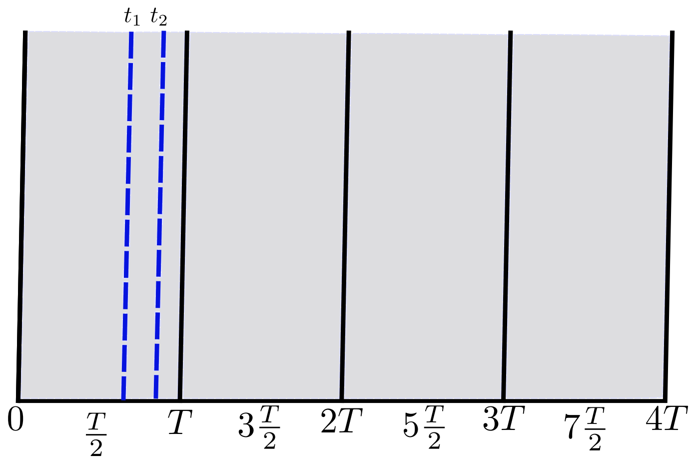
Part b

Since here $t_d \sim$ Unif$(0, T)$, we get the boundary points uniformly distributed across time, as shown below.



The boundary points are distributed across time with probability $\frac{1}{T}$

Hence, probability of boundary between t_1, t_2 is given by $p = \int_{t_1}^{t_2} \frac{1}{T} ds = \frac{t_2 - t_1}{T}$
Since, $R_X(t_1, t_2) = p \times 0 + (1-p) \times (\frac{(A)^2}{2} + \frac{(-A)^2}{2})$
the autocorrelation function is $R_X(\tau) = \begin{cases} A^2\left(1 - \frac{|\tau|}{T}\right), |\tau| < T \\ 0, |\tau| \geq T \end{cases}$ **(2 marks)**

Any other appropriate method used to obtain the above result has been awarded 2 marks

Now, $Y(t) = X(t) \times \cos(2\pi f_c t + \Theta)$, simple calculation yields $R_Y(\tau) = \frac{1}{2} R_X(\tau) cos(2\pi f_c \tau)$ **(1 mark)**

Missing constants like $A^2, \frac{1}{2}$ has been penalized with 0.5 marks

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Part c

Since Autocorrelation of Y depends only on τ the process is WSS. **(0.5 marks)**
$Y(t)$ represents a BPSK signal $(\pm A)$. **(0.5 marks)**

Question 6 **[1+4+2+2 = 9 marks]**

6. A signal is modeled as low pass stationary random process X(t) that ranges from -5 V to +5 V. Its probability density function is

$$f_X(x) = \begin{cases} Ce^{-|x|} & \text{for } |x| \leq 5; \\ 0 & \text{otherwise.} \end{cases}$$

Two 1024 level quantizers Q1 and Q2 are used independently to digitize ideal samples of X(t). Q1 is a uniform quantizer, which splits the range of -5 V to +5 V into 1024 equal quantization intervals. Q2 is a non-uniform quantizer. It splits the range -0.5 V to +0.5 V into 512 equal quantization intervals and the remaining voltage range (-5 V to -0.5 V and +0.5 V to +5 V) into another 512 quantization intervals of equal sizes. All the quantization levels are located at the centres of corresponding quantization intervals. *[1+4+2+2]*

(a) Find the value of C.

(b) What are the SQNRs (in linear scale) obtained when X(t) is quantized using Q1 and Q2?

(c) Instead, if X(t) is uniformly distributed over the entire interval [-5 V, 5 V], what will be the SQNRs obtained from Q1 and Q2?

(d) Give very short intuitive justifications for results obtained in (b) and (c). If (b) and (c) give different results, why?

Solution:

(a)

$$\int_{-5}^{5} f_X(x) = 1$$

$$\int_{-5}^{5} Ce^{-}|x| = 1$$

$$C = \frac{1}{2(1-e^{-5})}$$

[1 mark]

(b) Let S = signal power

$$\begin{aligned} S &= \int_{-\infty}^{\infty} x^2 f_X(x)dx \\ &= 2\int_{0}^{5} x^2 f_X(x)dx \\ &= 2\int_{0}^{5} C\exp(-|x|) \\ &= 2C(2-37e^{-5}) \\ &= 1.762 \end{aligned}$$

[1 mark]

$$\Delta = \frac{2Peak}{1024} = \frac{10}{1024}$$

Noise can be assumed uniform in a quantization interval. So,

$$N_{Q1} = \frac{\Delta^2}{12} = 7.94 \times 10^{-6}$$

$$SQNR_1 = \frac{S}{N_{Q1}} = 2.22{\times}10^5$$

[1 mark]

For Q2,

$$N_{Q2} = \mathbb{P}(-0.5 \leq x \leq 0.5)\ \frac{\Delta_1^2}{12} + 2\mathbb{P}(0.5 < x \leq 5)\ \frac{\Delta_2^2}{12}$$

where, **[1 mark]**

$$\begin{aligned} \mathbb{P}(-0.5 \leq x \leq 0.5)\ &= \int_{-0.5}^{0.5} f_X(x)dx \\ &= \frac{1-e^{-0.5}}{1-e^{-5}} \\ &= 0.39 \end{aligned}$$

$$\mathbb{P}(0.5 \leq x \leq 5)\ = 1 - \mathbb{P}(-0.5 \leq x \leq 0.5)\ = 0.61$$

$$\Delta_1 = \frac{2 \times 0.5}{512} = \frac{1}{512} \text{ and } \Delta_2 = \frac{2 \times 4.5}{512} = \frac{9}{512}$$

Therefore,

$$N_{Q2} = 1.56 \times 10^{-5}$$

$$SQNR_2 = \frac{S}{N_{Q2}} = 1.12 \times 10^5$$

[1 mark]

(c)

$$\begin{aligned} S &= \int_{-5}^{5} x^2 f_X(x)dx \\ &= \int_{-5}^{5} x^2 \frac{1}{10} dx \\ &= \frac{25}{3} \end{aligned}$$

[0.5 mark]

$$N_{Q1} = \frac{\Delta^2}{12} = \frac{10^2}{1024^2 \times 12}$$

$$SQNR_1 = \frac{S}{N_{Q1}} = 1024^2 = 1.04{\times}10^6$$

[0.5 mark]

For Q2,

$$N_{Q2} = \mathbb{P}(-0.5 \leq x \leq 0.5)\ \frac{\Delta_1^2}{12} + 2\mathbb{P}(0.5 < x \leq 5)\ \frac{\Delta_2^2}{12}$$

where, **[0.5 mark]**

$$\mathbb{P}(-0.5 \leq x \leq 0.5)\ = \frac{1}{10}$$

$$\mathbb{P}(0.5 \leq x \leq 5)\ = 1 - \mathbb{P}(-0.5 \leq x \leq 0.5)\ = \frac{9}{10}$$

$$\Delta_1 = \frac{2 \times 0.5}{512} = \frac{1}{512} \text{ and } \Delta_2 = \frac{2 \times 4.5}{512} = \frac{9}{512}$$

Therefore,

$$N_{Q2} = 2.32 \times 10^{-5}$$

$$SQNR_2 = \frac{S}{N_{Q2}} = 3.59 \times 10^5$$

[0.5 mark]

(d) SQNR for Q1 is more compared to Q2 for uniform distribution. this happens because Q2 represents signal range 0.5 to 5 and -0.5 to -5 with less number of bits compared to Q1 even though signal has higher probability of occurrence within that interval. **[1 mark]**

In case of (b), difference in SQNR for Q1 and Q2 is lesser compared to (c). Since signal in part (b) has higher probability of occurrence in interval -0.5 to 0.5, Q2 performs good. Increasing the quantization levels for that range will lead Q2 to outperform Q1. **[1 mark]**

Question 7 [2+3+2 = 7 marks]

7. A real signal $x(t)$ has baseband bandwidth W (i.e. its spectrum lies between $-W$ and $+W$). It is sampled using an ideal impulse train (consisting of delta-functions) of pulse repetition frequency $f_s = 10W$. The resulting samples are converted to flat-top PAM pulses of pulse-width $20/W$, and heights proportional to the samples amplitudes. *[2+3+2]*

(a) Find the magnitude spectrum of the PAM signal (consisting of the amplitude modulated flat-top pulses).

(b) Neatly draw the magnitude spectrum obtained in (a) with properly labeled frequency axis. [Consider any arbitrary shape for $X(f)$].

(c) Draw the block diagram that will be used for reconstructing the original signal $x(t)$ from the PAM signal obtained in (a). Here, amplitude scaling may be ignored. However, other parameters must be mentioned in the block diagram.

Solution:

(a) The PAM wave is

$$s(t) = \sum_{n=-\infty}^{\infty} x(nTs)p(t - nT_S)$$

where $p(t)$ is the pulse of width T with

$$P(f) = T\text{sinc}(fT)e^{-j\pi fT}$$

$s(t)$ can be written as,

$$s(t) = \left\{ x(t) \sum_{n=-\infty}^{\infty} \delta(t - nT_S) \right\} \star g(t)$$

[1 mark]

The spectrum of PAM will be

$$S(f) = X(f) \star \frac{1}{T_S} \sum_{n=-\infty}^{\infty} \delta(f - \frac{n}{T_s})P(f)$$

$$S(f) = \frac{1}{T_s} P(f) \sum_{n=-\infty}^{\infty} X(f - \frac{n}{T_s})$$

Magnitude spectrum of PAM signal is,

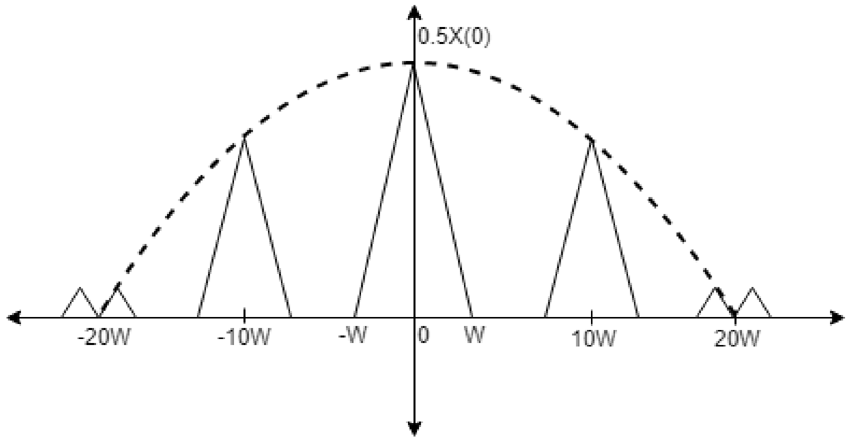
$$|S(f)| = \frac{1}{2} \sum_{n=-\infty}^{\infty} X(f - \frac{n}{T_s})\text{sinc}(fT)$$

[1 mark]

since $T = \frac{1}{20W}$ and $T_s = \frac{1}{10W}$,

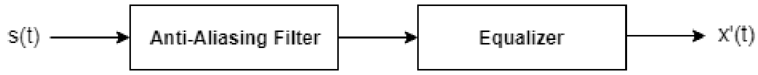
$$\frac{T}{Ts} = \frac{1}{2}$$

(b)



1 mark for showing envelope of sinc funtion, **1 mark** for correct indices, **0.5 mark** for number of copies of $X(f)$ inside envelope and **0.5 mark** for marking $S(0)$.

(c)



[0.5 mark]

Anti-aliasing filter has cutoff frequency $f_c = \frac{f_s}{2}$. **[0.5 mark]**

Equalizer has transfer function, **[0.5 mark]**

$$H(f) = \begin{cases} \frac{1}{P(f)}, & |f| \leq W \\ 0, & \text{otherwise} \end{cases}$$

0.5 mark for correct values of f_c and intervals of $H(f)$.