Problem 5.30

The first-order hold corresponds to extrapolating into the future with a straight line, as shown in Fig.1.

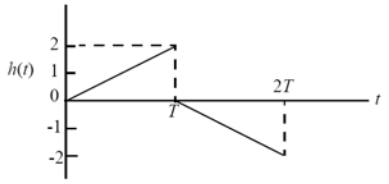

Figure 1

Specifically, the impulse response of the first-order hold may be expressed as

$$h(t) = \begin{cases} (t+T)/T & \text{for} \quad 0 \le t \le T \\ -(t-T)/T & \text{for} \quad T \le t \le 2T \\ 0 & \text{elsewhere} \end{cases} \tag{1}$$

Equivalently, we may express $h(t)$ as

$$h(t) = u(t) + \frac{t}{T}u(t) - 2u(t-T) - 2\frac{t-T}{T}u(t-T) + u(t-2T) + \frac{t-2T}{T}u(t-2T) \tag{2}$$

where $u(t)$ is the unit step function.

(a) Taking the Fourier transform of Eq. (2) and using the Fourier-transform pairs of Table A6.2, we may therefore express the frequency response of the first-order hold as

$$H(f) = \frac{1}{j2\pi f} + \frac{1}{T(j2\pi f)^2} - \frac{2}{j2\pi f}\exp(-j2\pi fT) - \frac{2}{T(j2\pi f)^2} + \frac{1}{j2\pi f}\exp(-j4\pi fT) + \frac{1}{T(j2\pi f)^2}\exp(-j4\pi fT)$$

which, after collecting and simplifying terms, yields

$$H(f) = T(1 + j2\pi fT)\left(\frac{1-\exp(-j2\pi fT)}{j2(\pi fT)}\right)^2 \tag{3}$$

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(b) Figure 2 shows the magnitude and phase responses of the first-order hold.

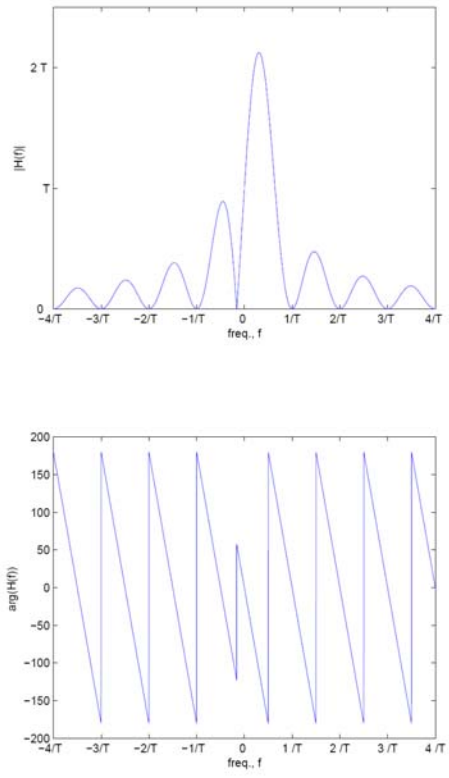

Figure 2

(c) For perfect reconstruction of the original analog signal, we need an equalizer whose transfer function is the inverse of $H(f)$ of Eq. (3), as shown by

$$H_{\text{eq}}(f) = \frac{1}{H(f)}$$

$$= \frac{1}{T(1 + j2\pi fT)}\left(\frac{j2\pi fT}{1 - \exp(-j2\pi fT)}\right)^2 \qquad (4)$$

For a duty cycle $(T/T_s) = 0.1$, the use of Eq. (4) yields

$$H_{\text{eq}}(f_s) = \frac{1}{T}(0.8732 + 0.0589)$$

(d) For the sinusoidal input

$x(t) = \cos(50t)$

and f_s = 100Hz and T - 0.01, Fig. 3(c) shows the response produced by the first-order hold. Part (b) of the figure shows the corresponding response of the sample-and-hold filter. Comparing these two parts of Fig. 3, we may make the following observations:

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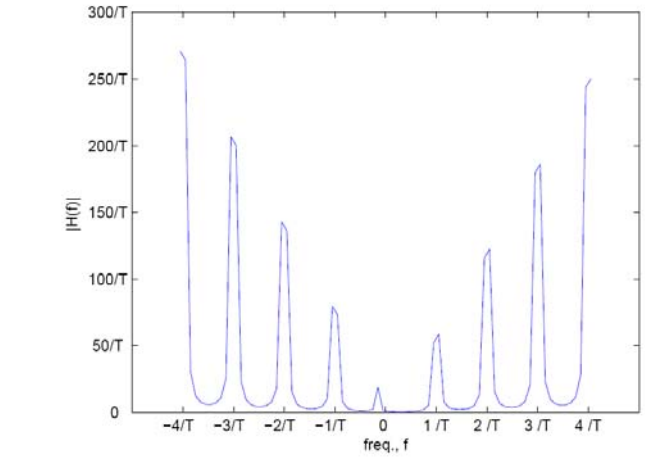
Problem 5-30 continued



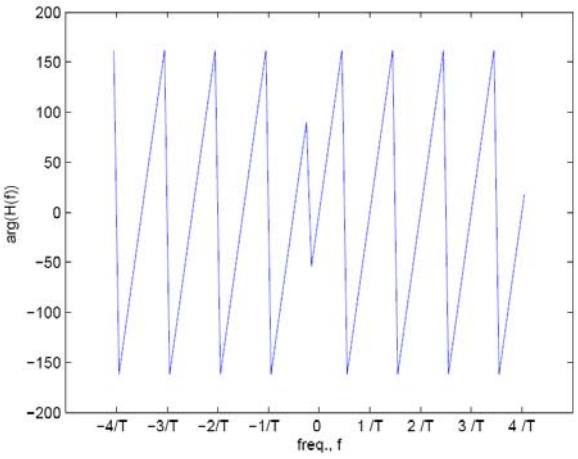


Figure 3

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