Problem 5.27

(a) The commutator at the output of the bipolar chopper switches between the direct path and inverted path at the frequency f_s. In effect, every $1/f_s$ seconds, the output of the chopper consists of the input $x(t)$ -- via the direct path -- for $1/2f_s$ seconds followed by the inverted version of $x(t)$ -- via the inverted path -- for the remaining $1/2f_s$ seconds of the commutation period. For one period of the commutation process, we may thus write

$$y(t) = \begin{cases} x(t) & \text{for} \quad 0 \le t \le 1/(2f_s) \\ -x(t) & \text{for } 1/(2f_s) \le t \le 1/f_s \end{cases} \tag{1}$$

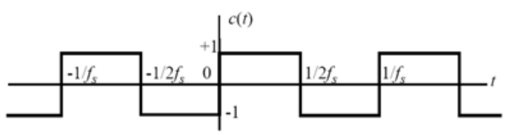
Equation (1) repeats itself every $1/f_s$ seconds.

(b) Equation (1) may be equivalently expressed as follows:

$$y(t) = c(t)x(t) \tag{2}$$

where $c(t)$ consists of the square wave (see Fig. 1)

$$c(t) = \begin{cases} 1+ & \text{for} \quad 0 \le t \le 1/(2f_s) \\ -1 & \text{for } 1/(2f_s) \le t \le 1/f_s \end{cases} \tag{3}$$



By inspection, we may make three observations from Fig. 1:

(i) The dc component of $c(t)$ is zero.

(ii) The Fourier series representation of $c(t)$ consists of sine components with a fundamental frequency f_s.

(iii) The even harmonic components of $c(t)$ are all zero.

Accordingly, we may represent $c(t)$ by the Fourier series:

$$c(t) = b_1 \sin(2\pi f_s t) + b_3 \sin(6\pi f_s t) + b_5 \sin(10\pi f_s t) + \dots \tag{4}$$

where b_n is defined by

$$\begin{aligned} b_n &= f_s \int_0^{1/f_s} c(t)\sin(2\pi n f_s t)dt \\ &= f_s \int_0^{1/2f_s} \sin(2\pi n f_s t)dt - f_s \int_{-1/2f_s}^{1/f_s} \sin(2\pi n f_s t)dt \\ &= \frac{-1}{2\pi n}[\cos(2\pi n f_s t)]_{t=0}^{1/2f_s} + \frac{1}{2\pi n}[\cos(2\pi n f_s t)]_{t=(1/2f_s)}^{1/f_s} \\ &= -\frac{1}{2\pi n}(\cos(n\pi) - 1) + \frac{1}{2\pi n}(\cos(n\pi) - \cos(n\pi)) \\ &= \begin{cases} \dfrac{2}{\pi n} & \text{for } n = 1, 3, 5, \dots \\ 0 & \text{for } n = 0, 2, 4, \dots \end{cases} \end{aligned} \tag{5}$$

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Problem 5-27 continued

We may thus express the Fourier series of the commutation function $c(t)$ as

$$c(t) = \frac{2}{\pi}\sin(2\pi f_s t) + \frac{2}{3\pi}\sin(6\pi f_s t) + \frac{2}{5\pi}\sin(10\pi f_s t) + \dots \qquad (6)$$

Using Eq. (6) in (2) yields

$$y(t) = \frac{2}{\pi}\sin(2\pi f_s t)x(t) + \frac{2}{3\pi}\sin(6\pi f_s t)x(t) + \frac{2}{5\pi}\sin(10\pi f_s t) + \dots \qquad (7)$$

The Fourier transform of $y(t)$ is therefore defined by

$$Y(f) = \frac{1}{j\pi}[X(f-f_s) - X(f+f_s)]$$

$$+ \frac{1}{j3\pi}[X(f-3f_s) - X(f+3f_s)]$$
$$+ \frac{1}{j5\pi}[X(f-5f_s) - X(f+5f_s)] + \dots \qquad (8)$$

where $X(f)$ is the Fourier transform of the input $x(t)$.

Figure 2 displays the relationship between the two Fourier transforms: $X(f)$ and $Y(f)$. Note that $X(f)$ can only be recovered from $Y(f)$ only through a band-pass filter with bandwidth $2W$ centered on f_s.

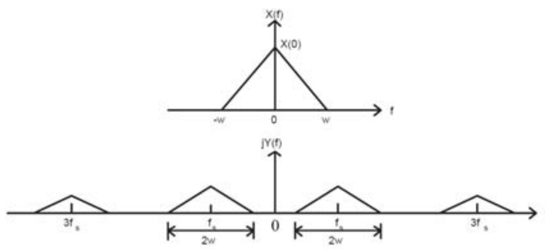

Figure 2