Problem 2.9

Determine the Fourier transform of the squared sinusoidal signals:

(i) $g(t) = \cos^2(2\pi f_c t)$

(ii) $g(t) = \sin^2(2\pi f_c t)$

Solution

(i) Using the trigonometric identity

$$\cos^2\theta = \frac{1}{2}(1 + \cos 2\theta)$$

we may express $g(t)$ as

$$g(t) = \frac{1}{2}(1 + \cos 4\pi f_c t)$$

Hence,

$$G(f) = \frac{1}{2}\delta(f) + \frac{1}{4}\delta(f - 2f_c) + \frac{1}{4}\delta(f + 2f_c)$$

(ii) Next, using the trigonometric identity

$$\sin^2\theta = \frac{1}{2}(1 - \cos 2\theta)$$

we may write

$$\sin^2(2\pi f_c t) = \frac{1}{2}(1 - \cos 4\pi f_c t)$$

Hence,

$$G(f) = \frac{1}{2}\delta(f) - \frac{1}{4}\delta(f - f_c) - \frac{1}{4}\delta(f + f_c)$$