Problem 2.11

Using the Euler formula

$$\cos x = \frac{1}{2}\exp[(jx) + \exp(-jx)]$$

reformulate Eqs. (2.91) and (2.92) in terms of cosinusoidal functions.

Solution

$$\sum_{n=-\infty}^{\infty} \exp(j2\pi n f_0 t) = \sum_{n=1}^{\infty} \exp(j2\pi n f_0 t) + 1 + \sum_{n=-\infty}^{-1} \exp(j2\pi n f_0 t)$$

$$= 1 + \sum_{n=1}^{\infty} [\exp(j2\pi n f_0 t) + \exp(-j2\pi n f_0 t)]$$

$$= 1 + 2\sum_{n=1}^{\infty} \cos(2\pi n f_0 t)$$

We may therefore reformulate Eq. (2.91) as

$$\sum_{m=-\infty}^{\infty} \delta(t - mT_0) = f_0 + 2f_0 \sum_{n=1}^{\infty} \cos(2\pi m f_0 t)$$

where $f_0 = 1/T$.

Similarly, we may write

$$\sum_{n=-\infty}^{\infty} \cos(j2\pi m f_0 t) = 1 + 2\sum_{m=1}^{\infty} \cos(2\pi m f_0 t)$$

Hence, we may reformulate Eq. (2.92) as

$$1 + 2\sum_{m=1}^{\infty} \cos(2\pi m f_0 t) = T_0 \sum_{n=-\infty}^{\infty} \delta(f - n f_0)$$