Problem 2.13

The impulse response of a linear system is defined by the Gaussian function

$$h(t) = \exp\left(-\frac{t^2}{2\tau^2}\right)$$

where the parameter τ is used to adjust duration of the impulse response. Determine the frequency response of the system.

Solution

From Eq. (2.40) in the text, recall that

$$\exp(-\pi t^2) \rightleftharpoons \exp(-\pi f^2)$$

Next, from the dilation property of the Fourier transform described in Eq. (2.20), recall that if $h(t) \rightleftharpoons H(f)$, then

$$h(at) \rightleftharpoons \frac{1}{|a|} H\left(\frac{f}{a}\right)$$

where a is the dilation parameter. For the problem at hand, we have

$$a = \sqrt{\frac{1}{2\pi}} \frac{1}{\tau}$$

Accordingly, the frequency response of the system is

$$H(f) = \sqrt{2\pi}\ \tau \exp(-2\pi^2\tau^2 f^2)$$