Problem 6.20

(a) When the two-level sequence embodying

$$a_k = \begin{cases} +1 \text{ if symbol } b_k \text{ is } 1 \\ -1 \text{ if symbol } b_k \text{ is } -1 \end{cases} \tag{1}$$

is applied to the duobinary conversion filter, the sequence is converted into a three-level output defined by

$$c_k = a_k + a_{k\text{-}1} \tag{2}$$

The three levels of c_k are -2, 0, and +2. One effect of transforming Eq. (1) into Eq. (2) is to produce correlated three-level sequence c_k from an uncorrelated two-level sequence a_k.

The overall transfer function of the duobinary conversion filter is therefore defined by

$$\begin{aligned} H(f) &= H_{\text{Nyquist}}(f)[1 + \exp(-j2\pi f T_b)] \\ &= H_{\text{Nyquist}}(f)[\exp(j\pi f T_b) + \exp(-j\pi f T_b)]\exp(-j\pi f T_b) \\ &= 2H_{\text{Nyquist}}(f)\cos(\pi f T_b)\exp(-j\pi f T_b) \end{aligned} \tag{3}$$

For an ideal Nyquist channel, $B_0 = 1/2T_b$. Ignoring the scaling factor $1/T_b$, we may therefore write

$$H_{\text{Nyquist}}(f) = \begin{cases} 1, & |f| \le 1/2T_b \\ 0, & \text{otherwise} \end{cases} \tag{4}$$

Substituting Eq. (4) into (3), we obtain

$$H(f) = \begin{cases} 2\cos(\pi f T_b)\exp(-j\pi f T_b), & |f| \le 1/2T_b \\ 0, & \text{otherwise} \end{cases} \tag{5}$$

(b) From the first line of Eq. (3) and the defining Eq. (4), we find that the impulse response of the duobinary conversion filter is

$$\begin{aligned} h(t) &= \frac{\sin(\pi t/T_b)}{\pi t/T_b} + \frac{\sin[\pi(t-T_b)/T_b]}{\pi(t-T_b)/T_b} \\ &= \frac{\sin(\pi t/T_b)}{\pi t/T_b} + \frac{\sin[(\pi t/T_b) - \pi]}{\pi(t-T_b)/T_b} \\ &= \frac{\sin(\pi t/T_b)}{\pi t/T_b} - \frac{\sin(\pi t/T_b)}{\pi(t-T_b)/T_b} \\ &= \frac{T_b^2 \sin(\pi t/T_b)}{\pi t(T_b - t)} \end{aligned} \tag{6}$$

(c) The original sequence may be detected from the duobinary-coded sequence using decision feedback, as shown by

$$\hat{a}_k = c_k - \hat{a}_{k\text{-}1} \tag{7}$$

A major drawback of this detection rule is that for the current detection $\hat{a}_k$ to be correct, the previous detection $\hat{a}_{k\text{-}1}$ has to be correct. If this requirement is not satisfied, we have error propagation.