Problem 6.7

Since $P(f)$ is an even real-valued function, its inverse Fourier transform may be simplified to the formula

$$p(t) = 2\int_0^{\infty} P(f)\cos(2\pi f t)\,df \qquad (1)$$

The $P(f)$ is itself defined by Eq. (6.17) which is reproduced here in the following form (ignoring the scaling factor $\sqrt{E}$ for convenience of presentation)

$$P(f) = \begin{cases} \dfrac{1}{2B_0}, & 0 < |f| f_1 \\ \dfrac{1}{4B_0}\left\{1 + \cos\left[\dfrac{\pi(|f| - f_1)}{2B_0 - 2f_1}\right]\right\}, & f_1 < f < 2B_0 - f_1 \\ 0, & |f| > 2B_0 - f_1 \end{cases} \qquad (2)$$

Hence, using Eq. (2) in (1) and recognizing that $\alpha = (B_0 - f_1)/B_0$, we may write

$$\begin{aligned}
p(t) &= \frac{1}{B_0}\int_0^{f_1}\cos(2\pi f t)\,df + \frac{1}{2B_0}\int_{f_1}^{2B_0 - f_1}\left[1 + \cos\left(\frac{\pi(f - f_1)}{2B_0\alpha}\right)\right]\cos(2\pi f t)\,df \\
&= \left[\frac{\sin(2\pi f t)}{2\pi B_0 t}\right] + \left[\frac{\sin(2\pi f t)}{4\pi B_0 t}\right]_{f_1}^{2B_0 - f_1} \\
&\quad + \frac{1}{4}B_0\left[\frac{\sin\left(2\pi f t + \dfrac{\pi(f - f_1)}{2B_0\alpha}\right)}{2\pi t + \pi/2B_0\alpha}\right]_{f_1}^{2B_0 - f_1} + \frac{1}{4B_0}\left[\frac{\sin\left(2\pi f t - \dfrac{\pi(f - f_1)}{2B_0\alpha}\right)}{2\pi t + \pi/2B_0\alpha}\right]_{f_1}^{2B_0 - f_1} \\
&= \frac{\sin(2\pi f_1 t)}{4\pi B_0 t} + \frac{\sin[2\pi t(2B_0 - f_1)]}{4\pi B_0 t} \\
&\quad - \frac{1}{4B_0}\ \frac{\sin(2\pi f_1 t) + \sin[2\pi t(2B_0 - f_1)]}{2\pi t - \pi/2B_0\alpha} + \frac{\sin(2\pi f_1 t) + \sin[2\pi t(2B_0 - f_1 t)]}{2\pi t - \pi/2B_0\alpha} \\
&= \frac{1}{B_0}[\sin(2\pi f_1 t) + \sin[2\pi t(2B_0 - f_1)]]\left[\frac{1}{4Wt} - \frac{\pi t}{(2\pi t)^2 - (\pi/2B_0\alpha)^2}\right] \\
&= \frac{1}{B_0}[\sin(2\pi B_0 t)\cos(2\pi\alpha B_0)]\left[\frac{-\pi/(2B_0\alpha)^2}{4\pi t[(2\pi t)^2 - \pi/(2B_0\alpha)^2]}\right] \\
&= \operatorname{sinc}(2B_0 t)\cos(2\pi\alpha B_0 t)\left[\frac{1}{1 - 16\alpha^2 B_0^{\,2} t^2}\right] \qquad (3)
\end{aligned}$$

Equation (3) is a reproduction of Eq. (6.19), except for the scaling factor $\sqrt{E}$ which we ignored in Eq. (2) for convenience of presentation.