Problem 4.14

(a) The angle of the PM wave is defined by

$$\begin{aligned}\theta_i(t) &= 2\pi f_c t + k_p m(t) \\ &= 2\pi f_c t + k_p A_m \cos(2\pi f_m t) \\ &= 2\pi f_c t + \beta_p \cos(2\pi f_m t)\end{aligned}$$

where $\beta_p = k_p A_m$. The instantaneous frequency of the PM wave is therefore

$$\begin{aligned}f_i(t) &= \frac{1}{2\pi}\frac{d\theta_i(t)}{dt} \\ &= f_c - \beta_p f_m \sin(2\pi f_m t) \qquad (1)\end{aligned}$$

Based on Eq. (1), we see that the maximum frequency deviation in a PM wave varies linearly with the modulation frequency f_m.

Using Carson's rule, we find that the transmission bandwidth of the PM wave is approximately (for the case when β_p is small compared to unity)

$$B_T \approx 2(f_m + \beta_p f_m) = 2f_m(1+\beta_p) \approx 2f_m\beta_p. \qquad (2)$$

Equation (2) shows that B_T varies linearly with the modulation frequency f_m.

(b) In an FM wave, the transmission bandwidth B_T is approximately equal to $2\Delta f$, assuming that the modulation index β is small compared to unity. Therefore, for an FM wave, B_T is effectively independent of the modulation frequency f_m.