Problem 5.12

(a) The PAM wave is defined by

$$s(t) = \sum_{n=-\infty}^{\infty} [1 + \mu m'(nT_s)] g(t - nT_s), \tag{1}$$

where $g(t)$ is the pulse shape, $m'(t) = m(t)/A_m = \cos(2\pi f_m t)$ and μ is the modulation factor. The PAM wave is equivalent to the convolution of the instantaneously sampled signal $[1 + \mu m'(t)]$ and the pulse shape $g(t)$, as shown by

$$s(t) = \left\{ \sum_{n-\infty}^{\infty} [1 + \mu m'(nT_s)] \delta(t - nT_s) \right\} \star g(t)$$

$$= \left\{ 1 + \mu m'(t) \sum_{n=-\infty}^{\infty} \delta(t - nT_s) \right\} \star g(t) \tag{2}$$

Let $m'(t) \rightleftharpoons M'(f)$, $g(t) \rightleftharpoons G(f)$, and $s(t) \rightleftharpoons S(f)$.

The spectrum of the PAM wave is therefore,

$$S(f) = \left\{ [\delta(f) + \mu M'(f)] \star \frac{1}{T_s} \sum_{m=-\infty}^{\infty} \delta\left(f - \frac{m}{T_s} \right) \right\} G(f)$$

$$= \frac{1}{T_s} G(f) \sum_{m=-\infty}^{\infty} \left[\delta\left(f - \frac{m}{T_s} \right) + \mu M'\left(f - \frac{m}{T_s} \right) \right] \tag{3}$$

For a rectangular pulse $g(t)$ of duration $T = 0.45$s and with $AT = 1$, we have

$$G(f) = AT \operatorname{sinc}(fT)$$
$$= \operatorname{sinc}(0.45 f)$$

For $m'(t) = \cos(2\pi fmt)$ and $f_m = 0.25$ Hz, we have

$$M'(f) = \frac{1}{2}[\delta(f - 0.25) + \delta(f + 0.25)]$$

For $T_s = 1s$, the ideally sampled spectrum is

$$S_\delta(f) = \sum_{m=-\infty}^{\infty} [\delta(f - m) + \mu M'(f - m)] \tag{4}$$

which is plotted in Fig. 2(c).

The actual sampled spectrum is defined by

$$S(f) = \sum_{m=-\infty}^{\infty} \operatorname{sinc}(0.45 f)[\delta(f - m) + \mu M'(f - m)] \tag{5}$$

which is plotted in Fig. 1(b).

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Problem 5-12 continued

(b) The ideal reconstruction filter would retain the centre 3 delta functions of $S(f)$. With no aperture effect, the two outer delta functions would have amplitude $\mu/2$. The aperture effect distorts the reconstructed signal by attenuating the high-frequency portion of the message signal.

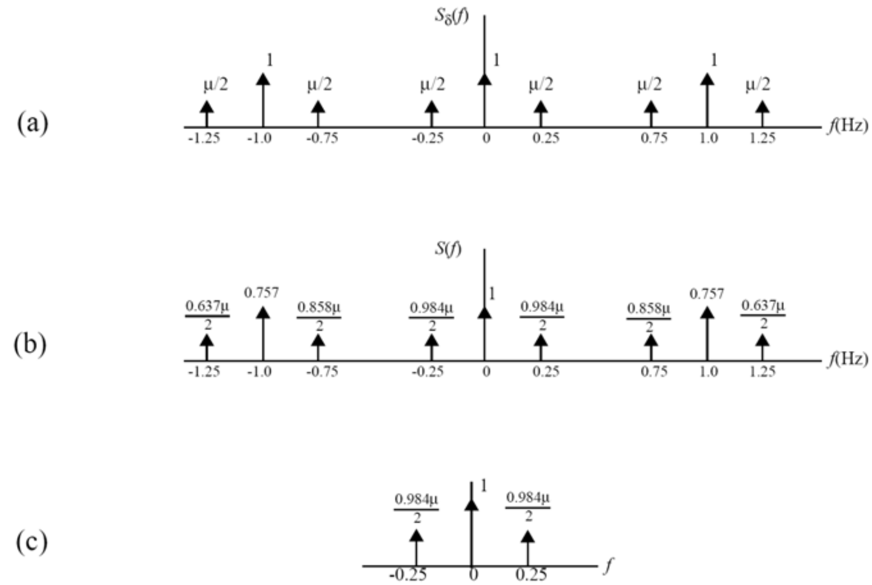

Figure 1