Problem 5.3

Show that

$$\frac{1}{2W}\int_{-W}^{W} \exp\left[j2\pi f\left(t-\frac{n}{2W}\right)\right]df = \frac{\sin(2\pi Wt-n\pi)}{(2\pi Wt-n\pi)}$$
$$= \operatorname{sinc}(2Wt-n)$$

Solution

$$\frac{1}{2W}\int_{-W}^{W} \exp\left[j2\pi f\left(t-\frac{n}{2W}\right)\right]df = \frac{1}{2W}\cdot\frac{1}{j2\pi(t-n/2W)}\cdot\exp\left[j2\pi f\left(t-\frac{n}{2W}\right)\right]_{f=-W}^{W}$$
$$= \frac{1}{j4\pi W(t-n/2W)}\cdot[\exp j\pi(2Wt-n)-\exp(-j\pi(2Wt-n))]$$
$$= \frac{\sin(\pi(2Wt-n))}{\pi(2Wt-n)}$$
$$= \operatorname{sinc}(2Wt-n)$$