Problem 3.26

The message signal is defined by the rectangular pulse

$$m(t) = \begin{cases} A, & -T/2 \le t \le T/2 \\ 0, & \text{otherwise} \end{cases} \tag{1}$$

The SSB modulated wave is defined by

$$s(t) = \frac{A_c}{2} m(t)\cos(2\pi f_c t) \mp \frac{A_c}{2}\hat{m}(t)\sin(2\pi f_c t)$$

where $\hat{m}(t)$ is the Hilbert transform of $m(t)$. The in-phase and quadrature components of $s(t)$ are respectively defined by

$$s_I(t) = \frac{A_c}{2} m(t)$$

$$s_Q(t) = \pm\frac{A_c}{2}\hat{m}(t)$$

The envelope of $s(t)$ is therefore

$$a(t) = [s_I^2(t) + S_Q^2(t)]^{1/2}$$

$$= \frac{A_c}{2}[m^2(t) + \hat{m}^2(t)]^{1/2} \tag{2}$$

The Hilbert transform of the rectangular pulse of Eq. (1) was determined in Problem 2.52 of Chapter 2; it is reproduced here for a pulse of unit amplitude and duration T:

$$\hat{m}(t) = -\frac{1}{\pi}\ln\left|\frac{t-(T/2)}{t+(T/2)}\right| \tag{3}$$

where ln denotes the natural logarithm. From Eq. (3) we see that $\hat{m}^2(t)$ assumes an infinitely large value at $t = T/2$ and $t = -T/2$. Correspondingly, the envelope of the SSB modulator exhibits peaks at the beginning and end of the input pulse.