Problem 3.31

(a) The SSB wave $s_u(t)$ is defined by

$$s_u(t) = \frac{A_c}{2}[m(t)\cos(2\pi f_c t) - \hat{m}(t)\sin(2\pi f_c t)] \qquad (1)$$

and its Hilbert transform is defined by

$$s_u(t) = \frac{A_c}{2}[m(t)\sin(2\pi f_c t) - \hat{m}(t)\cos(2\pi f_c t)] \qquad (2)$$

In Eq. (2), we have used the following properties of the Hilbert transform:

(a) The Hilbert transform of $m(t)\cos(2\pi f_c t)$ is $m(t)\sin(2\pi f_c t)$

(b) The Hilbert transform of $\hat{m}(t)\sin(2\pi f_c t)$ is $-\hat{m}(t)\cos(2\pi f_c t)$

We may therefore use Eqs. (1) and (2) to write

$$s_u(t) = \cos(2\pi f_c t) = \frac{A_c}{2}[m(t)\cos^2(2\pi f_c t) - \hat{m}(t)\sin(2\pi f_c t)\cos(2\pi f_c t)] \qquad (3)$$

$$s_u(t)\sin(2\pi f_c t) = \frac{A_c}{2}[m(t)\sin^2(2\pi f_c t) + \hat{m}(t)\sin(2\pi f_c t)\cos(2\pi f_c t)] \qquad (4)$$

Adding Eqs. (3) and (4) and solving for $m(t)$, we get

$$m(t) = \frac{A_c}{2}[s_u(t)\cos(2\pi f_c t) + \hat{s}_u(t)\sin(2\pi f_c t)] \qquad (5)$$

Next, we use Eqs. (1) and (2) to write

$$s_u(t)\sin(2\pi f_c t) = \frac{A_c}{2}[m(t)\cos(2\pi f_c t)\sin(2\pi f_c t) - \hat{m}(t)\sin^2(2\pi f_c t)] \qquad (6)$$

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Problem 3-31 continued

$$s_u(t)\cos(2\pi f_c t) = \frac{A_c}{2}[m(t)\sin(2\pi f_c t)\cos(2\pi f_c t) + \hat{m}(t)\cos^2(2\pi f_c t)] \quad (7)$$

Subtracting Eq. (6) from Eq. (7) and then solving for $\hat{m}(t)$, we get

$$\hat{m}(t) = \frac{2}{A_c}[\hat{s}_u(t)\cos(2\pi f_c t) - s_u(t)\sin(2\pi f_c t)] \quad (8)$$

Equations (5) and (8) are the desired results for part (a) of the problem.

(b) The SSB wave $s_l(t)$ is defined by

$$s_l(t) = \frac{A_c}{2}[m(t)\cos(2\pi f_c t) + \hat{m}(t)\sin(2\pi f_c t)] \quad (9)$$

and its Hilbert transform is defined by

$$\hat{s}_l(t) = \frac{A_c}{2}[m(t)\sin(2\pi f_c t) - \hat{m}(t)\cos(2\pi f_c t)] \quad (10)$$

where again we have made use of the above-mentioned properties of the Hilbert transform. Therefore, using Eqs. (9) and (10) we write

$$s_l(t)\cos(2\pi f_c t) = \frac{A_c}{2}[m(t)\cos^2(2\pi f_c t) + \hat{m}(t)\sin(2\pi f_c t)\cos(2\pi f_c t)] \quad (11)$$

$$\hat{s}_l(t)\sin(2\pi f_c t) = \frac{A_c}{2}[m(t)\sin^2(2\pi f_c t) - \hat{m}(t)\cos(2\pi f_c t)\sin(2\pi f_c t)] \quad (12)$$

Adding Eqs. (11) and (12) and then solving for $m(t)$, we get

$$m(t) = \frac{2}{A_c}[s_l(t)\cos(2\pi f_c t) + \hat{s}_l(t)\sin(2\pi f_c t)] \quad (13)$$

Next, we use Eqs. (11) and (12) to write

$$s_l(t)\sin(2\pi f_c t) = \frac{A_c}{2}[m(t)\cos(2\pi f_c t)\sin(2\pi f_c t) + \hat{m}(t)\sin^2(2\pi f_c t)] \quad (14)$$

$$\hat{s}_l(t)\cos(2\pi f_c t) = \frac{A_c}{2}[m(t)\sin(2\pi f_c t)\cos(2\pi f_c t) - \hat{m}(t)\cos^2(2\pi f_c t)] \quad (15)$$

Subtracting Eq. (15) from Eq. (14) and then solving for $\hat{m}(t)$, we get

$$\hat{m}(t) = \frac{2}{A_c}[s_l(t)\sin(2\pi f_c t) - \hat{s}_l(t)\cos(2\pi f_c t)] \quad (16)$$

Equations (13) and (16) are the desired results for part (b) of the problem.

(c) From Eqs. (15) and (16), we see that the message signal $m(t)$ may be recovered from $s_u(t)$ or $s_l(t)$ by using the scheme shown in Fig. 1.

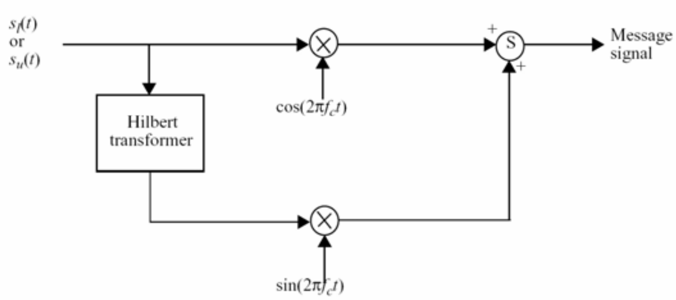

Figure 1