

Problem 3.11

Show that if the message signal $m(t)$ is low-pass, then the Hilbert transform $\hat{m}(t)$ is also low-pass with the same bandwidth as $m(t)$.

Solution

The Fourier transform of the Hilbert transform $\hat{m}(t)$ is defined by

$$\hat{M}(f) = -j \operatorname{sgn}(f) M(f)$$

where $M(f) = \mathbf{F}[m(t)]$. To illustrate, let the spectrum $M(f)$ be as shown in Fig. 1(a). Then, the corresponding spectrum $\hat{M}(f)$ is as shown in part (b) of the figure. The spectrum $\hat{M}(f)$ is therefore also low-pass, occupying the frequency band $-W \leq f \leq W$ just like $M(f)$.

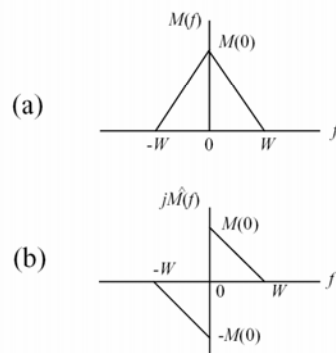


Figure 1