17.9.18

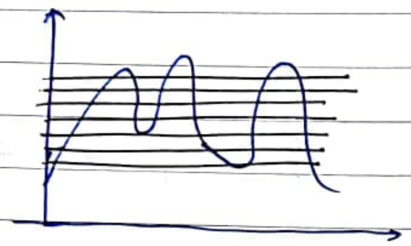
Digital Communication

- Digitized → discretized in Levels

Analog : Information is not digitized

Digital : Information is digitized

Q→ What is the point of high precision in Analog.

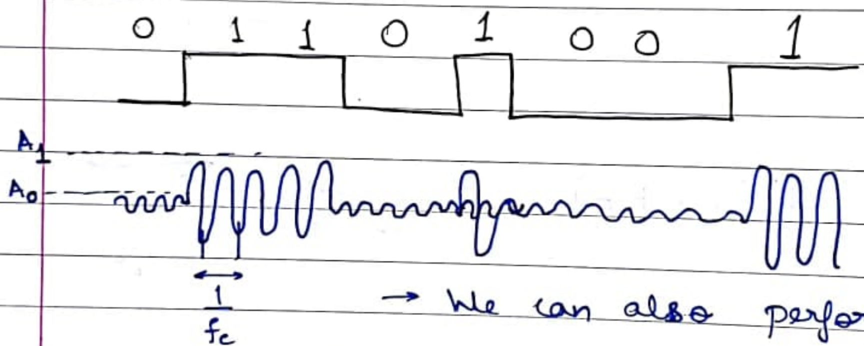


In Digital

AM → ASK (Amp. Shift keying)

FM → FSK (Freq. " ")

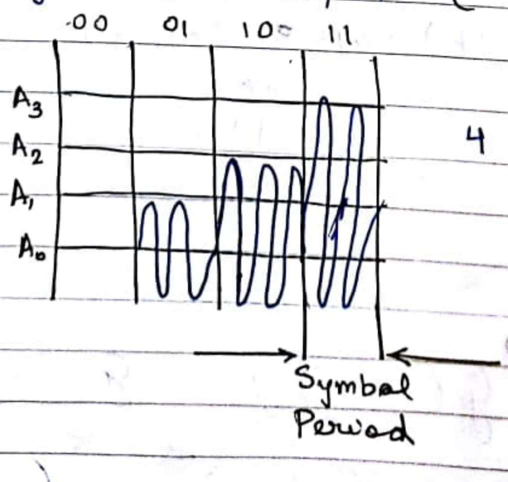
PM → PSK (Phase " ")



(ASK waveform)

→ We can also perform OOK : On-Off Keying, where Amp. = 0 for Symbol "0"

→ >2 Symbols example (4-ASK) in General M-ASK, M-PSK etc...



4 Levels or 4 possible Symbol values

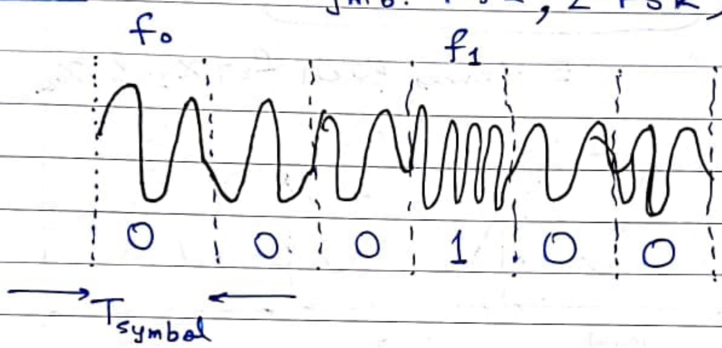
ex

Bit-rate = 1 Mbps

Symbol rate = $\dfrac{\text{Bitrate}}{\text{Bits/Symbol}}$ = 500 K sym/s for 4-ASK

Sym. rate = $\dfrac{\text{Bit-rate}}{\lceil \log_2 M \rceil}$ for M-ASK

FSK : (2 Symb. FSK, 2-FSK)



4-FSK

1-MHz	00_2
1.1-M-Hz	01_2
1.2-MHz	10_2
1.3-MHz	11_2

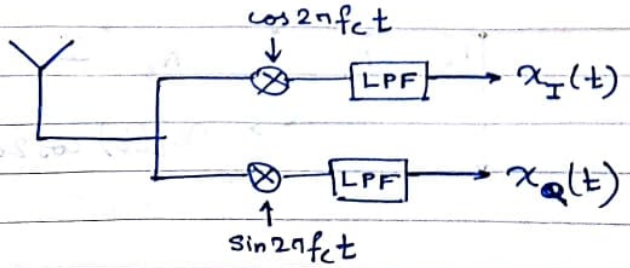
trade-off : b/w

PM : $\cos(2\pi f_c t + \phi_m(t))$

PSK $\longrightarrow$ $\in \left\{0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \pi, \frac{5\pi}{4}, \frac{3\pi}{2}, \frac{7\pi}{4}\right\}$ 8-PSK

$(000)_2$ $(001)_2$

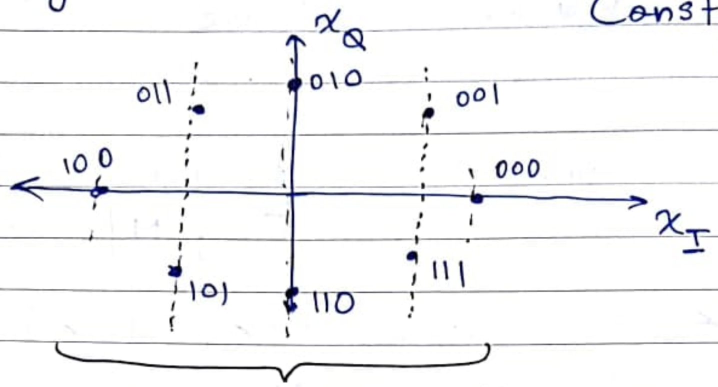
→ How to demodulate this



$$\phi_m = \tan^{-1}\left(\frac{x_Q(t)}{x_I(t)}\right)$$

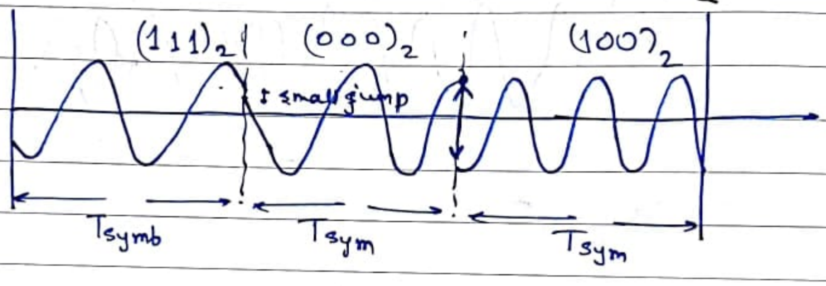
→ Encoding in x_I and x_Q

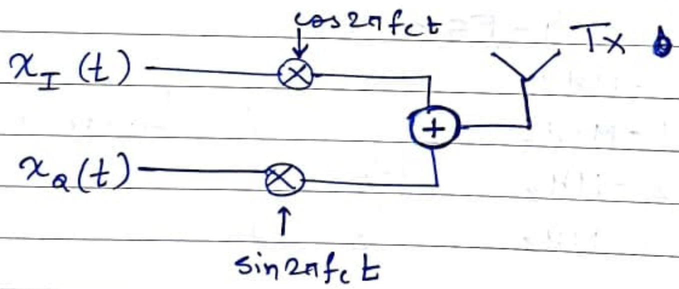
Constellation Diagram



5 - Levels each for x_I & x_Q

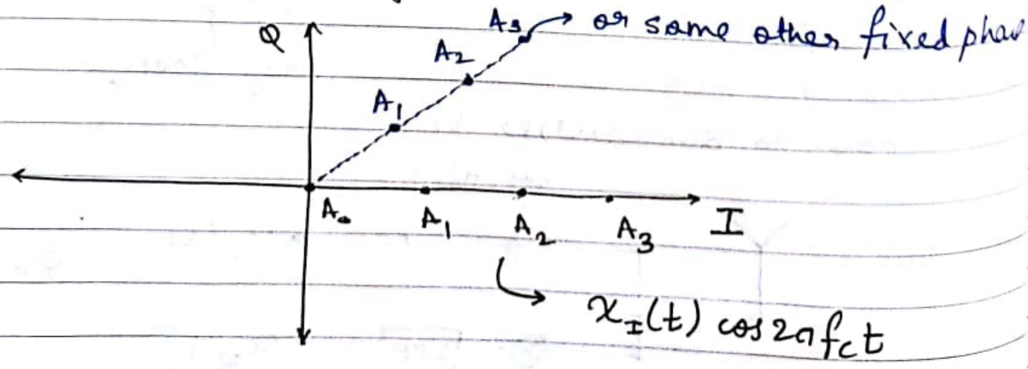
ex

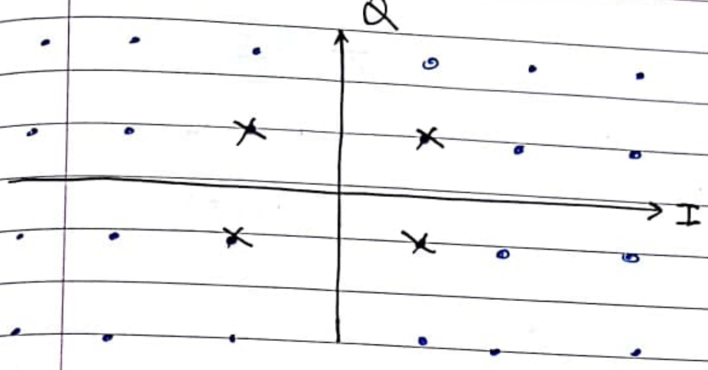




$$\text{Bandwidth efficiency} = \frac{(\text{Bits/sec})}{\text{Hz}}$$

Constellation Diagram for 4-ASK





× → 4-QPSK
• → 16-QAM

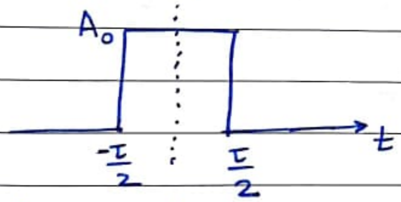
QAM : (General Case) : Quadrature Amplitude Modulation
→ General case

QPSK : Amplitude is constant
→ modified PSK

18.9.18

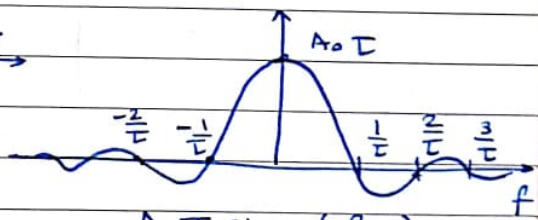
Lecture

- Recap: Digital modulation Schemes



$A\,\mathrm{rect}\left(\frac{t}{T}\right)$

$\xrightarrow{F.T}$



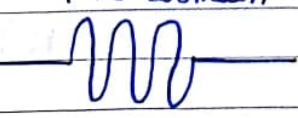
$A_0T\,\mathrm{sinc}\,(fT)$

$\mathrm{sinc}\,x$ ~~sinx~~ $\triangleq \frac{\sin x}{x}$ (w/o Normalization)

$\triangleq \frac{\sin(\pi x)}{(\pi x)}$ (Normalized)

→ ASK (Digital AM)

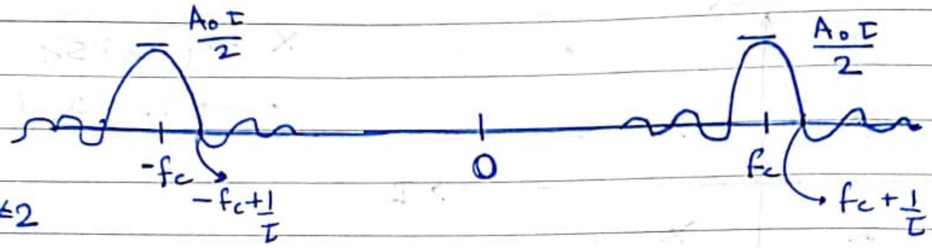
time domain

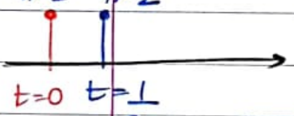


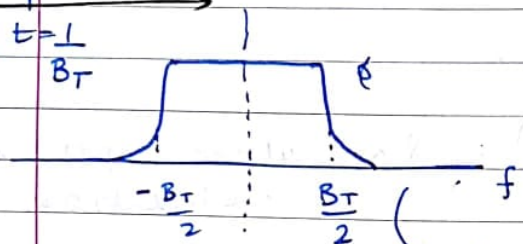
$\longrightarrow \frac{A_0T}{2}\left[\mathrm{sinc}\,(f-f_c)T + \mathrm{sinc}\,(f+f_c)T\right]$

$A_0 \cos 2\pi f_c t \times \mathrm{rect}\left(\frac{t}{T}\right)$

freq. domain

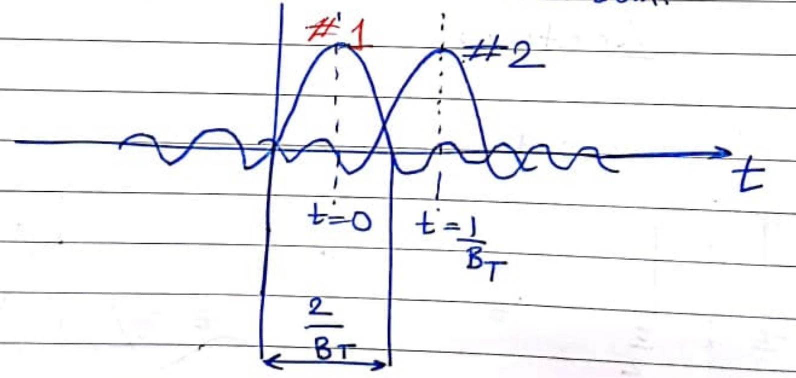




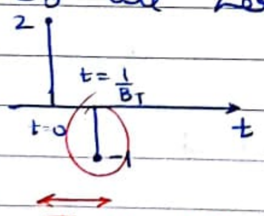


→ To make the signal transfer band limited use this "ideal brickwall filter"

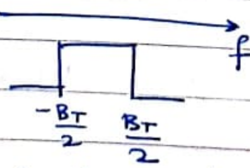
taking convolution with its time domain in time dom.



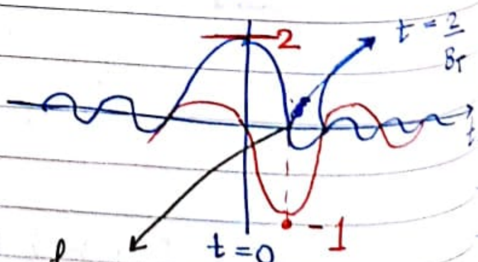
→ So use zero crossings to advantage,



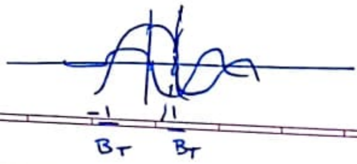
T_{symb} (Symbol period)



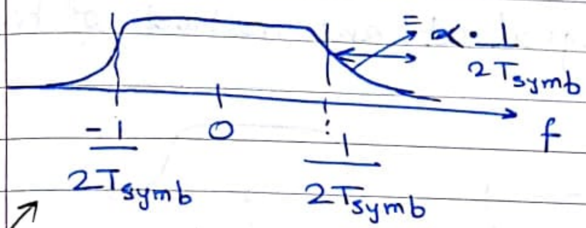
for limited/efficient spectral usage



has zero crossing at $n T_{symb}$



$$B_T = \frac{1}{T_{symb}}(1+\alpha)$$



: Pulse shaping filter (PSF)

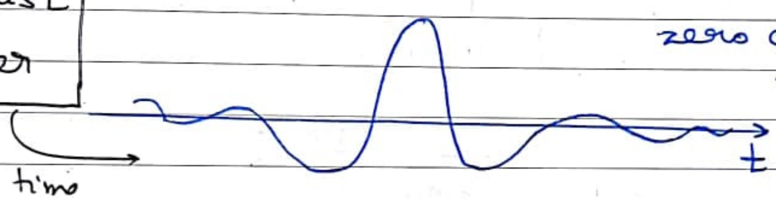
freq ex (Raised cosine filter)

Impulse response

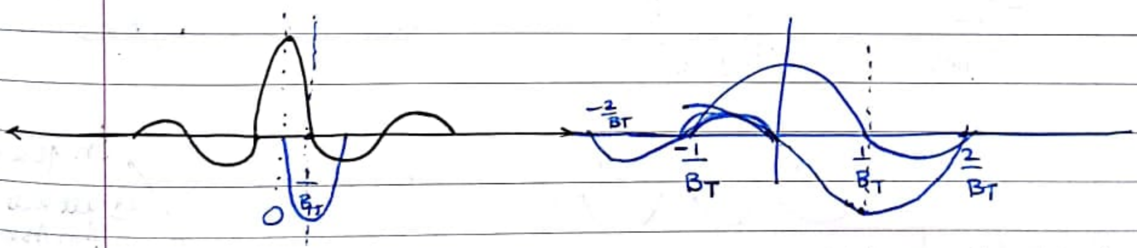
Sinc like Shape with faster Roll-offs / Decays

$\boxed{\text{Nyquist filter}}$

time



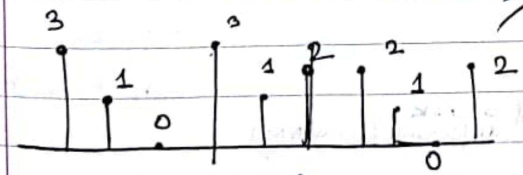
zero crossings are still at $n \cdot T_{symb}$ as before



(Discrete samples)

in AM vs Digital we simply × Impulses with carrier

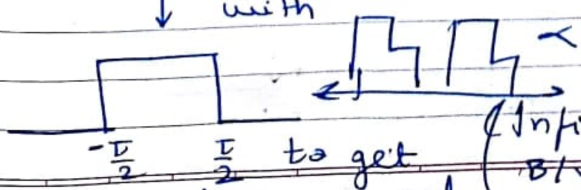
ex



PSF → × cos(2πf_ct) → ASK pass band signal

(Symbols)

can't convolve with



to get what we need (Infinite B/W)

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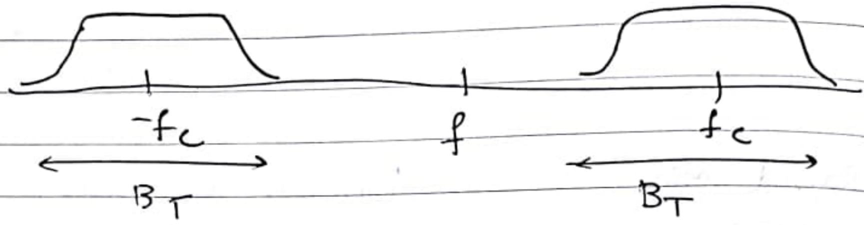
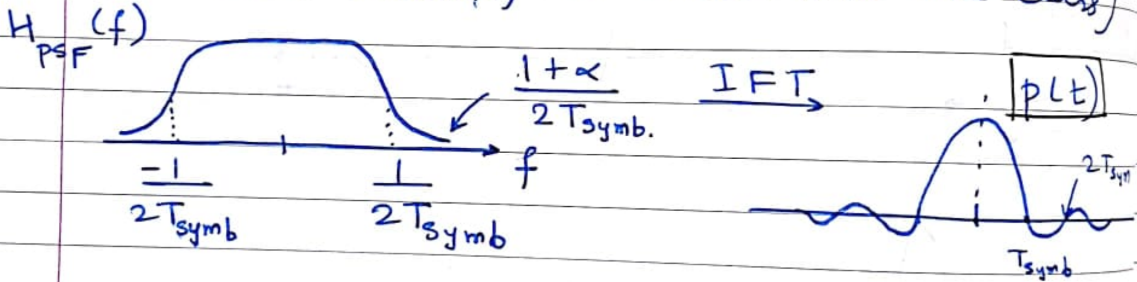
Pass - Band Signal at one instant of time



↳ Can be considered at one Instant or on an average

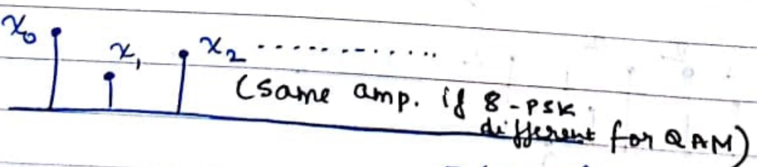
20.9.18

Crib Session: Wed 6-7pm, MS, Q1, Q2
Quiz (10 marks) on Thurs. (in the class)



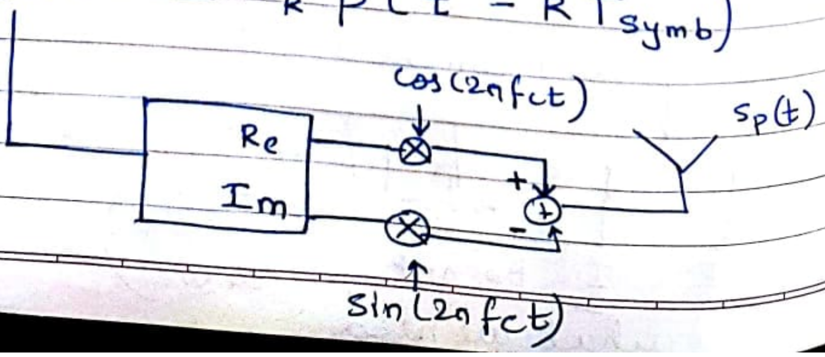
101 001 011 101 000 011

x_0 x_1 x_2 x_3 x_4 x_5 → Complex symbols (Constellation points)

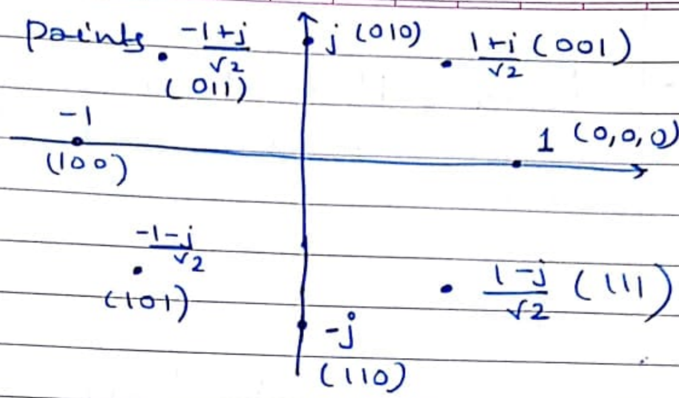


$$x_{BB}(t) = \sum x_k \, p(t - kT_{symb})$$

→ complex



ex of complex points



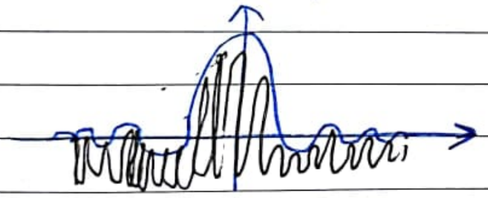
ex Transmit 101 : $\frac{-1-j}{\sqrt{2}}$

$$Re\left[\left(\frac{-1-j}{\sqrt{2}}\right) \times p(t) \times [\cos 2\pi f_c t + j \sin 2\pi f_c t]\right]$$

→ transmitted signal

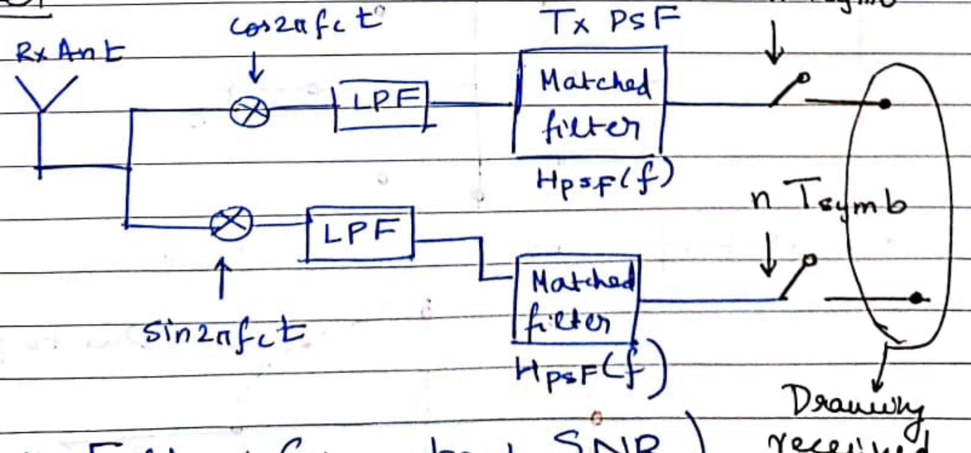
$$= \frac{p(t)}{\sqrt{2}} (-\cos 2\pi f_c t + \sin 2\pi f_c t)$$

→ envelope → determines phase



(Alignment requires pro.)

Receiver



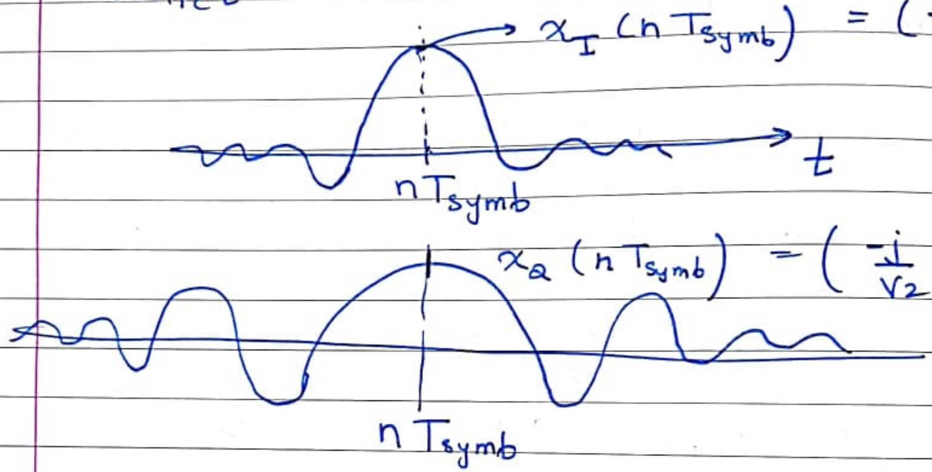
Drawing received constellation

(Matched Filter : Gives best SNR)

→ Applying Matched filter in frequency domain is convolving with same sinc like time domain waveform $(p(t))$ with itself.

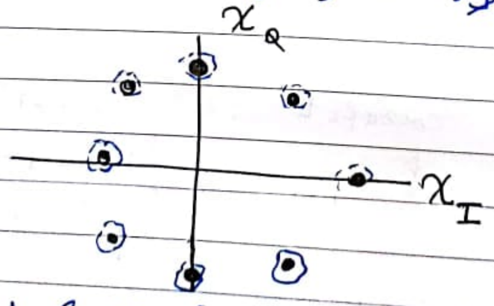
O/P

$\cos 2\pi f_c t$ Demod Channel

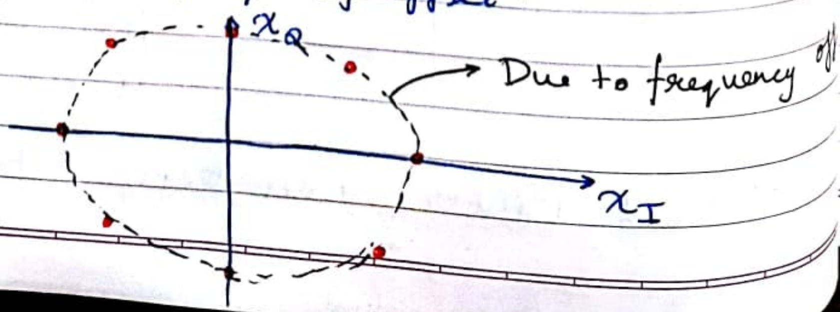


→ Received Complex Symbol

$$= x_I(nT_{symb}) + j\,x_Q(nT_{symb})$$

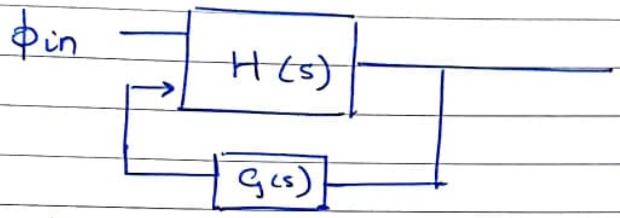


→ Effect of Small frequency offset:



- Solution: Precise Coherent detection,
  Coarse Adjustment: FLL (with Band-edge filters)
  Fine Adjustment: Costas Loop
  → Also does phase in addition to freq.

→ just like −ve feedback in op-amp

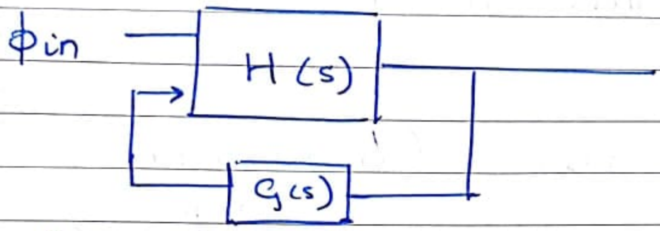


→ Sampling time issues

say $T_{sym} = 1\mu s$ & receiver → has $0.99\mu s$
even then as $n \uparrow$ nT_{sym} will have a large mismatch

→ Precise T_{symbol} matching important

- Solution : Precise Coherent detection,
  Coarse Adjustment : FLL (with Band-edge filters)
  Fine Adjustment : Costas Loop
  → Also does phase in addition to freq.
  → just like -ve feedback in op-amp



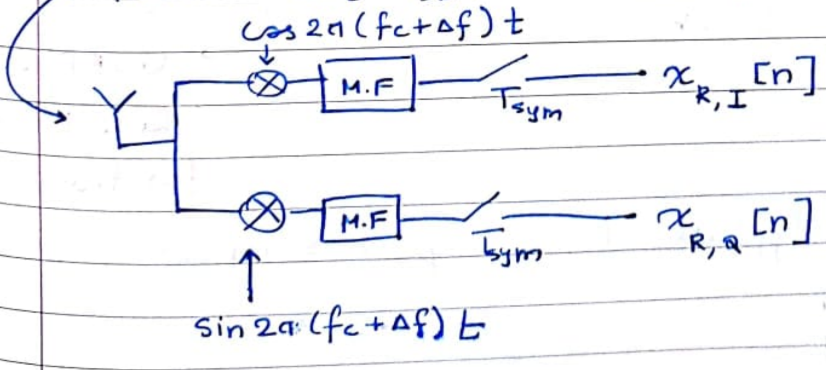
- Sampling time issues

say $T_{sym} = 1\mu s$ & receiver → has $0.99\mu s$
even then as $n \uparrow$ nT_{sym} will have a large mismatch
- Precise T_{symbol} matching important

24.9.18

Carrier Phase / Freq. Offset

$S_p(t) = Re[x_{BB}(t) \cdot e^{j2\pi f_c t}]$



At $t = nT_s$

$x_I \cos(2\pi\Delta f t) + x_Q \sin(2\pi\Delta f t)$

$x_Q \cos(2\pi\Delta f t) - x_I \sin(2\pi\Delta f t)$

$$\begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = \begin{bmatrix} \cos t & \sin t \\ -\sin t & \cos t \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \end{bmatrix}$$

- Can see it as Multiplication with Rotation Matrix

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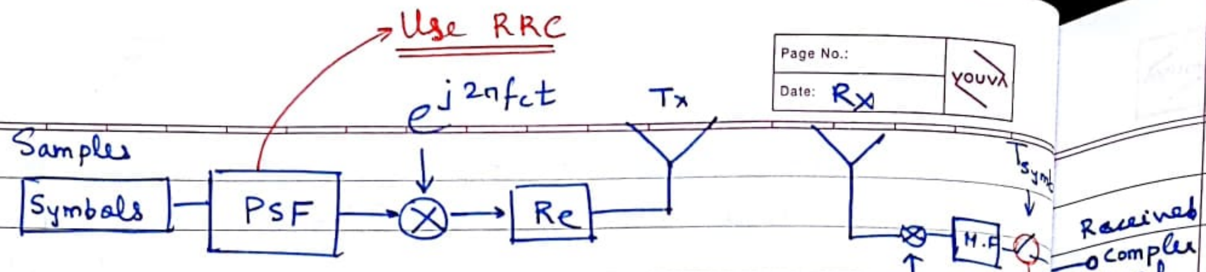
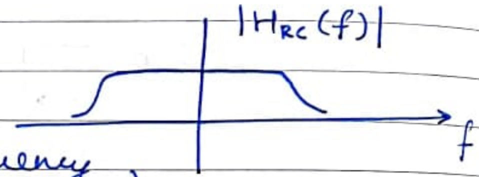


→ Issue: If we can't use a RC (raised cosine filter)

Avoid ISI: Inter symbol interference here

→ Recap: RC filter was a Nyquist filter

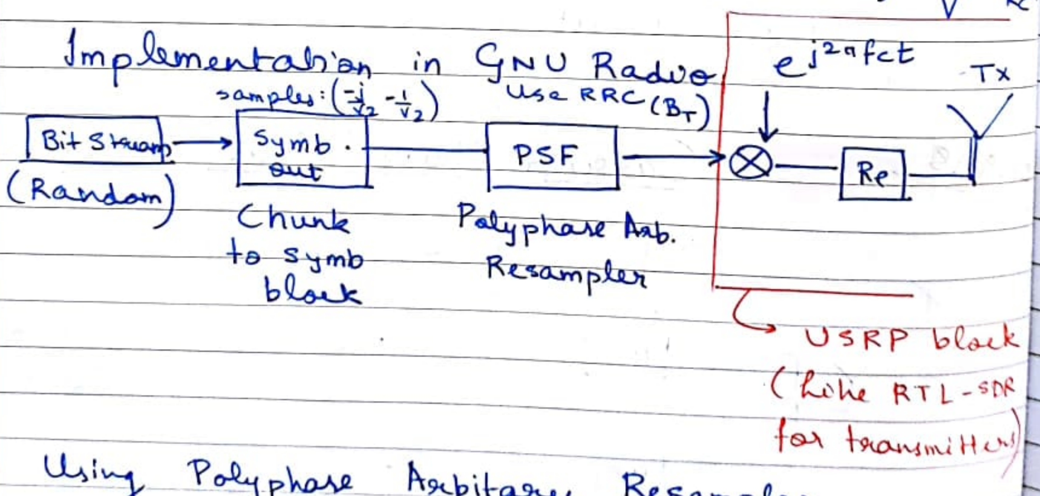


→ If we square it (In frequency domain)

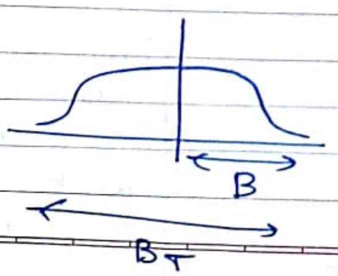
· then we loose the No Interference at sampling point property

→ Solution: Root Raised Cosine (RRC) $= \sqrt{H_{RC}(f)}$

→ Implementation in GNU Radio



→ Using Polyphase Arbitary Resampler



$f_s \geq 2B$

$f_s \geq B_T$

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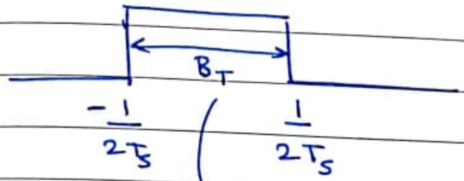
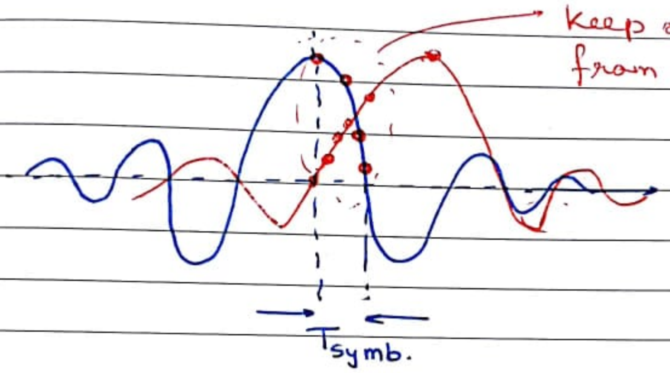


Best Case / Ideal

Keep on Adding samples from shifting ~~wave~~ sinc waveforms



→ parameter: samples/symbol (sps) = 4 here

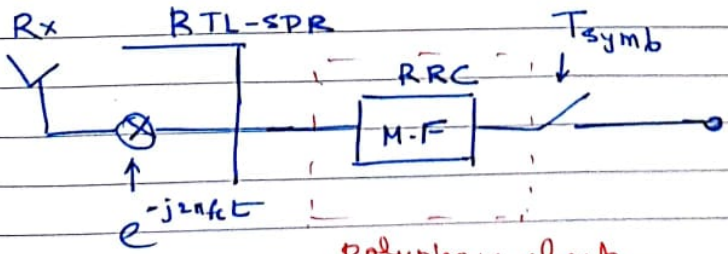
How to Choose sps = f_s

Nyquist Criterion : $f_s \geq B_T$

$$f_s \geq \frac{(1+\alpha)}{T_{symb}}$$

α : Excess Bandwidth factor

$\alpha \in (0.1, 0.4)$



polyphase clock synchronizer

25.9.18

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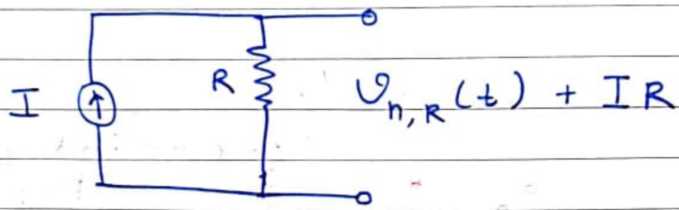
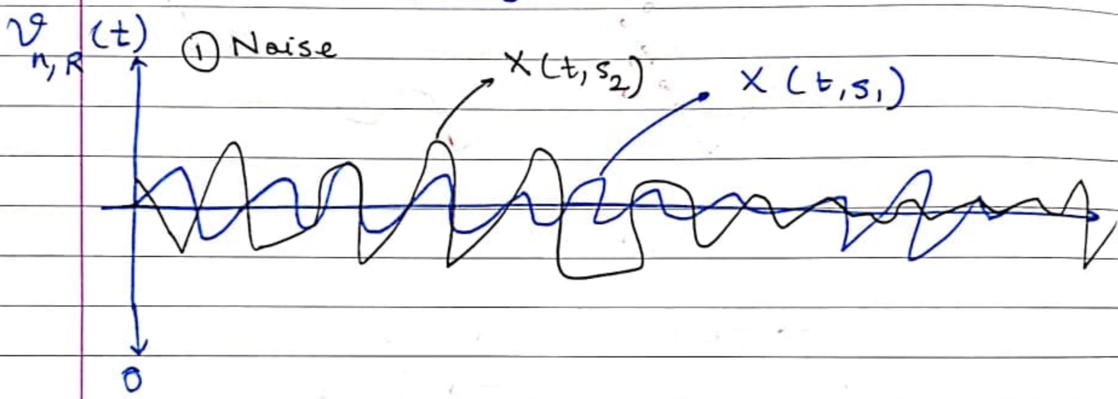
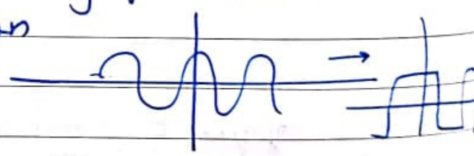
→ In Digital comm. FSK & PSK are interchangeable (Derivative is Difference reasoning)





→ Other Distorting / Degrading processes

② Non-Linear Amplification

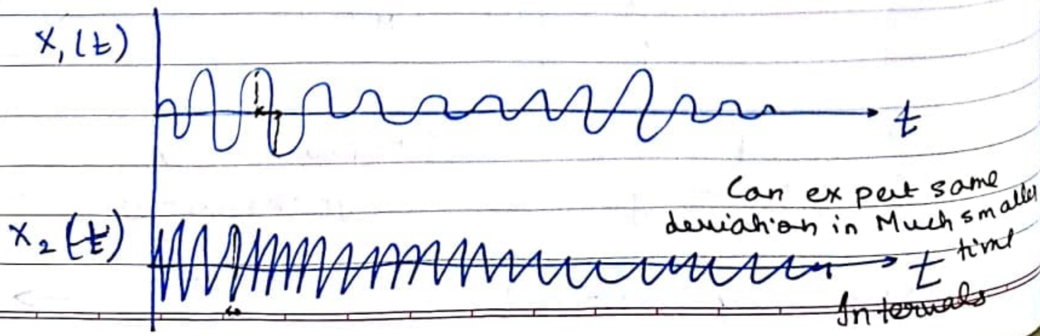


③ Interference

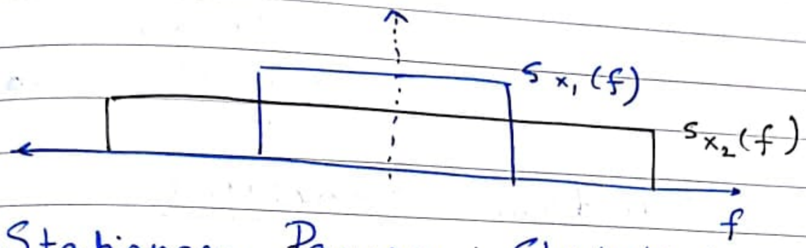
A unique RV for every time instant

• Random Variables : $\underbrace{x(t_1), x(t_2), x(t_3)}_{\text{Random var.}}$

→ Time evolution of Random variables

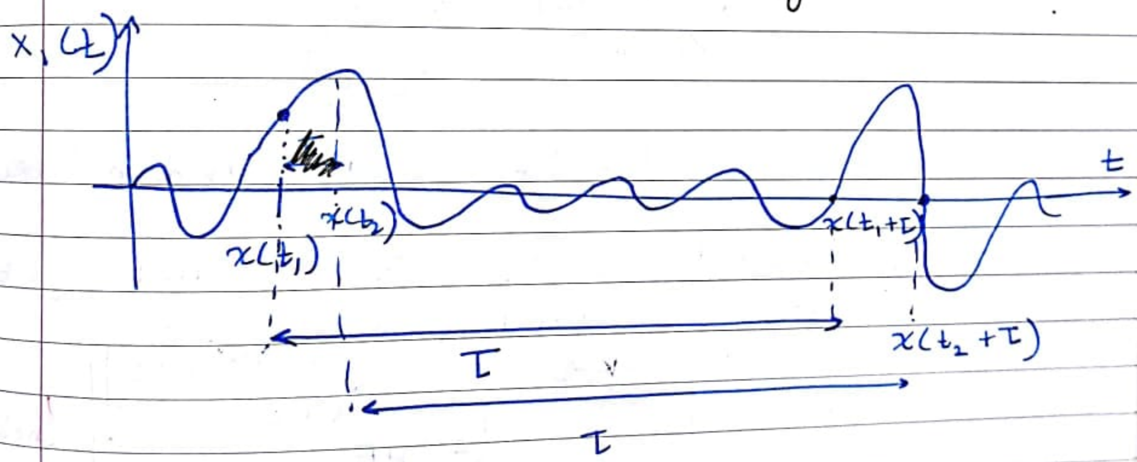


→ X_1 and X_2 may have same moments, but they are still different due to their frequency content



→ Stationary Process : Statistical properties don't change over time.

Strict Sense Stationary



$X_2(t)$: Similar:

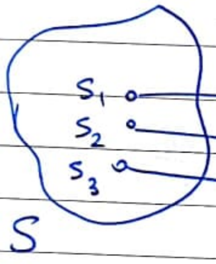
→ If the joint distribution of

$$X(t) = \sin(2\pi f_0 t + \theta)$$

If θ is a random number, then $X(t)$ is an R.P

27.9.18

Random Processes



$X(t, s) = x(t)$ — shorthand for $X(t,s)$

$X(t, s_1) = x_1(t)$

$X(t, s_2) = x_2(t)$

$X(t, s_3) = x_3(t)$

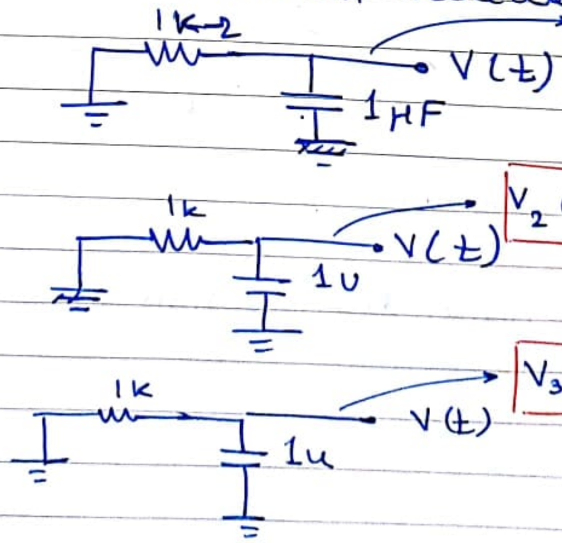
- $X(t)$ at $t = t_k$ is a Random Variable

i.e, all 3 of $x_1(t_k)$, $x_2(t_k)$ and $x_3(t_k)$ are 3 random variables

S_1 — $\boxed{V_1(t)}$

S_2 — $\boxed{V_2(t)}$

S_3 — $\boxed{V_3(t)}$



Instance of Random process $V(t)$

- $V(t_k)$: A Random variable with instances $V_i(t_k)$

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1. Each time instant is associated with a Random Variable
→ ~~$V_i(t)$~~ $X_i(t)$ is a waveform, inst. of Rand. Process.

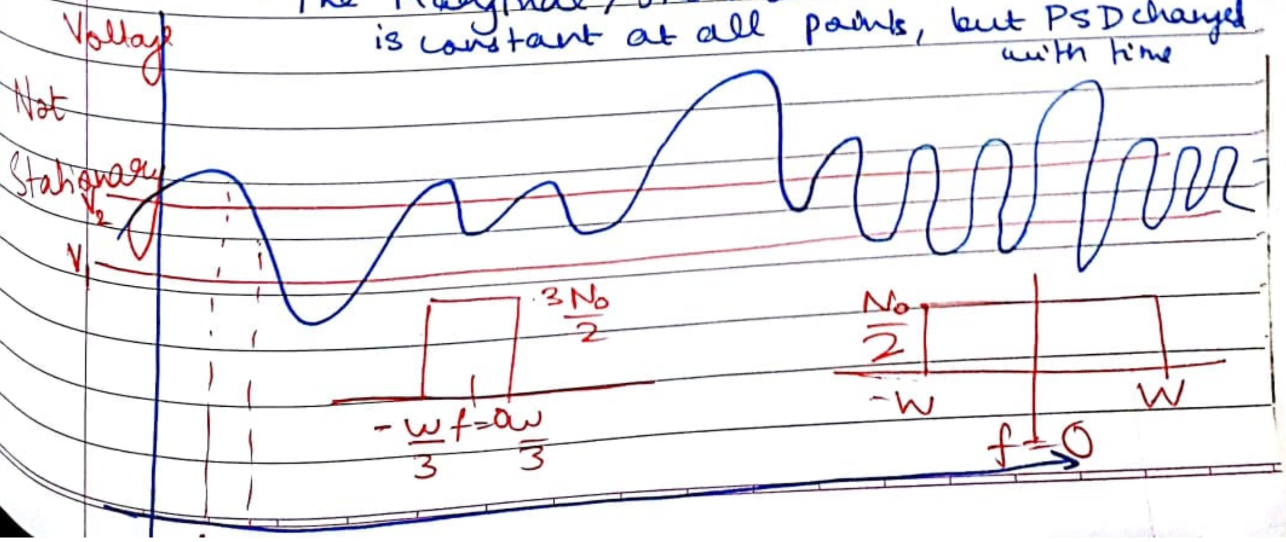
→ Density & Distribution for Random processes

→ R.V : $X(t_k)$:
$$\mu_x(t_k) = \mathbb{E}[X(t_k)]$$

· Stationary Process $\Rightarrow$ $\mu_x(t_k) = \mu_x \ \forall \ t_k$
one way implication

· ~~In General~~ : Strict Sense Stationary
$$F_{X(t_1), X(t_2), \dots, X(t_k)}(x_1, x_2, \dots, x_k) =$$
$$F_{X(t_1+\tau) X(t_2+\tau), \dots\dots X(t_k+\tau)}(x_1, x_2, \dots\dots, x_k)$$
for every τ
and $\forall k$

→ Why do we need joint Distribution?
→ The Marginal / Individual Distribution is constant at all points, but PSD changed with time



4.10.18

Random Processes

$$\mu_x(t_1) = \mathbb{E}[X(t_1)] = \mu_x$$

ind. of t_1, for a stationary process

→ R.V : $X(t_1)$

→ Autocorrelation function

$$R_x(t_1, t_2) = \mathbb{E}[X(t_1) \cdot X(t_2)]$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x_1 \cdot x_2 \, f_{X(t_1), X(t_2)}(x_1, x_2)\, dx_1 \cdot dx_2$$

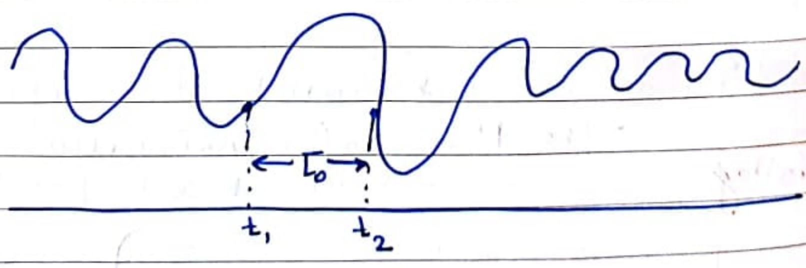
→ Stationary process ⇒ $R_x(\tau)$ is defined

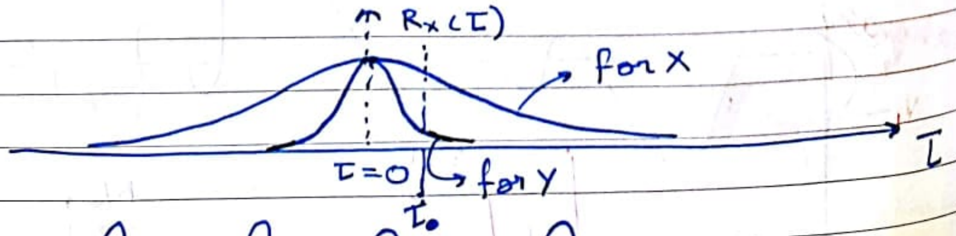
→ Hence, if,

$$\mu_x(t_1) = \mu_x$$
$$R_x(t_1, t_2) = R_x(\tau)$$
$$\tau = t_2 - t_1 \text{ or } \tau = t_1 - t_2$$

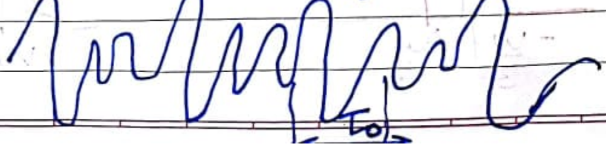
⇒ The process is Wide-sense stationary or weak-sense stationary] WSS

$X(t)$





Y



Variance : $\mathbb{E}[(X-\mu)^2]$

Covariance : $\mathbb{E}\Big[\big(X(t_1)-\mu_x(t_1)\big)\big(X(t_2)-\mu_x(t_2)\big)\Big]$

$= C_x(t_1, t_2)$

For WSS : $C_x(t_1, t_2) = C_x(\tau) = R_x(\tau) - \mu_x^2$

ex $X(t) = A\cos(2\pi f_c t + \theta)$

↳ a random process

θ is a random variable

$$f_\theta(\theta) = \begin{cases} \frac{1}{2\pi} & , -\pi \le \theta \le \pi \\ 0 & , \text{o.w} \end{cases}$$

$$R_x(\tau) = \frac{A^2}{2\pi}\int_{-\pi}^{\pi} \cos[2\pi f_c(\tau + t_1) + \theta] \cdot \cos[2\pi f_c t_1 + \theta]\, d\theta$$

$$\cos A \cos B = \frac{1}{2}\big(\cos(A-B) + \cos(A+B)\big)$$

$$= \frac{A^2}{2\pi}\int_{-\pi}^{\pi} \cdot \frac{1}{2}\left(\cos(2\pi f_c \tau) + \cos(2\pi(f_c(2t+\tau)) + 2\theta\right) d\theta$$

$$= \frac{A^2}{2\pi} \cdot \frac{2\pi}{2} \cdot \cos(2\pi f_c t)$$

$$= \frac{A^2}{2}\cos(2\pi f_c \tau)$$

8.10.18

Auto- Correlation

$$R_x(t_1, t_2) = R_x(\tau)$$

$$\tau = t_2 - t_1$$

for a WSS process

$$= E[X(t_1) \cdot X(t_2)]$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x_1 \cdot x_2 \cdot f_{X(t_1), X(t_2)} \, dx_1 \cdot dx_2$$

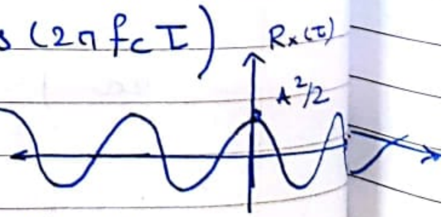
$$X(t) = \cos(2\pi f_c t + \theta)$$

$$f_\Theta(\theta) = \begin{cases} \frac{1}{2\pi} , & -\pi \le \theta \le \pi \\ 0 , & \text{o.w} \end{cases}$$

$$E[X(t_1) \cdot X(t_2)] = \int_{-\pi}^{\pi} x_1 x_2 f_\Theta(\theta) \, d\theta$$

$$= \int_{-\pi}^{\pi} A \cdot \cos(2\pi f t + \theta) \cdot A \cdot \cos(2\pi f (t+\tau) + \theta) f_\Theta(\theta) \, d\theta$$

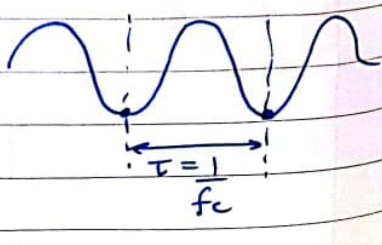
$$\Rightarrow R_x(\tau) = \frac{A^2}{2} \cos(2\pi f_c \tau)$$



→ Given: $f_\Theta(\theta)$

$$E[g(\theta)] = \int_{-\infty}^{\infty} g(\theta) \cdot f_\Theta(\theta) \, d\theta$$

→ If $\tau = \frac{n}{f_c}$

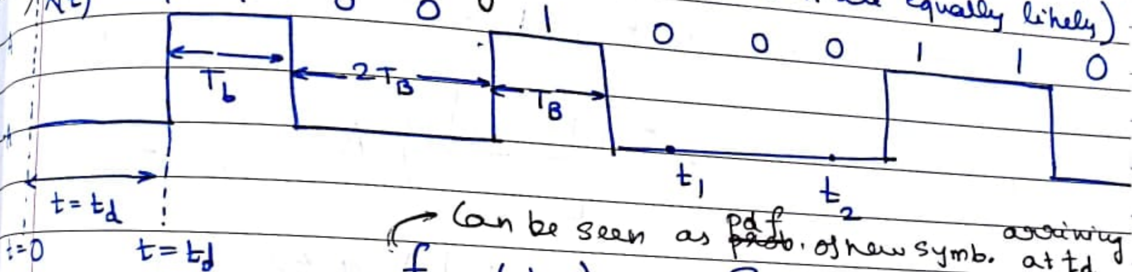


In that case, Maximum value of $R_X(\tau)$ takes its $\frac{A^2}{2}$

$$\mathbb{E}[A\cos(2\pi f_c t)\cdot A\cos(2\pi f_c t + 2n\pi)]$$

$$= \mathbb{E}[A^2\cos^2(2\pi f_c t)] = \frac{A^2}{2}$$

Ex2) Random Binary Wave (+A, −A are equally likely)



Can be seen as ~~prob.~~ pdf. of new symb. arriving at t_d

$$f_{T_d}(t_d) = \begin{cases} \frac{1}{T_B}, & 0 \le t_d \le T_b \\ 0, & \text{o.w} \end{cases}$$

Dist. for 1st pulse, for others it will be shifted version of this

$$\mathbb{E}[X(t_1)\cdot X(t_2)] = R_X(t_1, t_2)$$

① If $|t_2 - t_1| > T_b \Rightarrow \mathbb{E}[X(t_1)\cdot X(t_2)]$

$$= \frac{1}{4}[A\cdot A + A\cdot(-A) + (-A)\cdot A + (-A)\cdot(-A)]$$

$= 0$ (Since 1 & 0 are equiprobable)

② $|t_2 - t_1| \le T_b \Rightarrow \mathbb{E}[X(t_1)\cdot X(t_2)]$

No ~~transition~~ pulse bound. : → A^2

~~trans.~~ : → ~~exp~~ Same as 1st case

$\mathbb{P}($~~transition~~ pulse bound. $\in (t_1, t_2)) = p$

$\Rightarrow$ ~~exp~~ $=$ ~~$p\cdot A^2 + (1-p)A^2$~~ $=$ ~~$A^2\cdot(2$~~

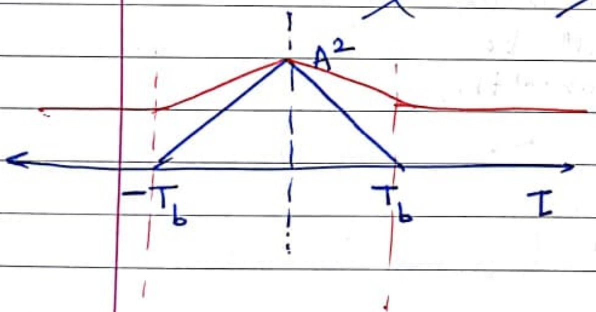
$$\mathbb{E} = \Pr(\text{~~transition~~ pulse boundary time is between } t_1 \,\&\, t_2) \cdot \mathbb{E}\left[x(t_1)\cdot x(t_2) \,\middle|\, \text{Pulse bound} \in (t_1,t_2)\right]$$

$$+ \Pr(\text{No P.B b/w } t_1 \,\&\, t_2) \cdot \mathbb{E}\left[x(t_1)\cdot x(t_2) \,\middle|\, \text{P.B} \notin (t_1,t_2)\right]$$

$$= \Pr(\text{Pulse Bound})\cdot 0 + \Pr(\text{No pulse Bound})\cdot A^2$$

$$= \left(\frac{T_b - |t_2-t_1|}{T_b}\right)\cdot A^2 = \left(1 - \frac{|t_2-t_1|}{T_b}\right)\cdot A^2$$

$$= \cancel{\left(1 - \frac{|t_2-t_1|}{T_b}\right) A^2} \quad \left(1 - \frac{|\tau|}{T_b}\right)\cdot A^2$$



$\xrightarrow{\text{if}}$ $p(1) \neq p(0)$, $\neq p$

$$\Rightarrow \mathbb{E} = A^2\cdot\left[(p^2 + p^2 - 2p + 1) - \cancel{2p + 2p^2} - 2(p)(1-p)\right]$$

$$= A^2\left[4p^2 - 4p + 1\right] \quad (= 0 \text{ for } p = \tfrac{1}{2})$$

$$\frac{|\tau|}{T_b}\cdot\left[4p^2 - 4p + 1\right] + \left(1 - \frac{|\tau|}{T_b}\right)\cdot A^2$$

$$A^2\left[(2p-1)^2 \times \frac{|\tau|}{T_b} + \left(1 - \frac{|\tau|}{T_b}\right)\right]$$

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Auto - correlation

1) $R_x(0) = \mathbb{E}[X^2(t)] \Rightarrow$ Power in the Signal

2) $R_x(\tau) = R_x(-\tau) \Rightarrow$ Symmetric (If WSS)

$\underbrace{\mathbb{E}[X(t_1)\,X(t_1+\tau)]}_{R_x(\tau)} = \mathbb{E}[X(t_2-\tau)\,X(t_2)]$

$= R_x(-\tau)$

if WSS

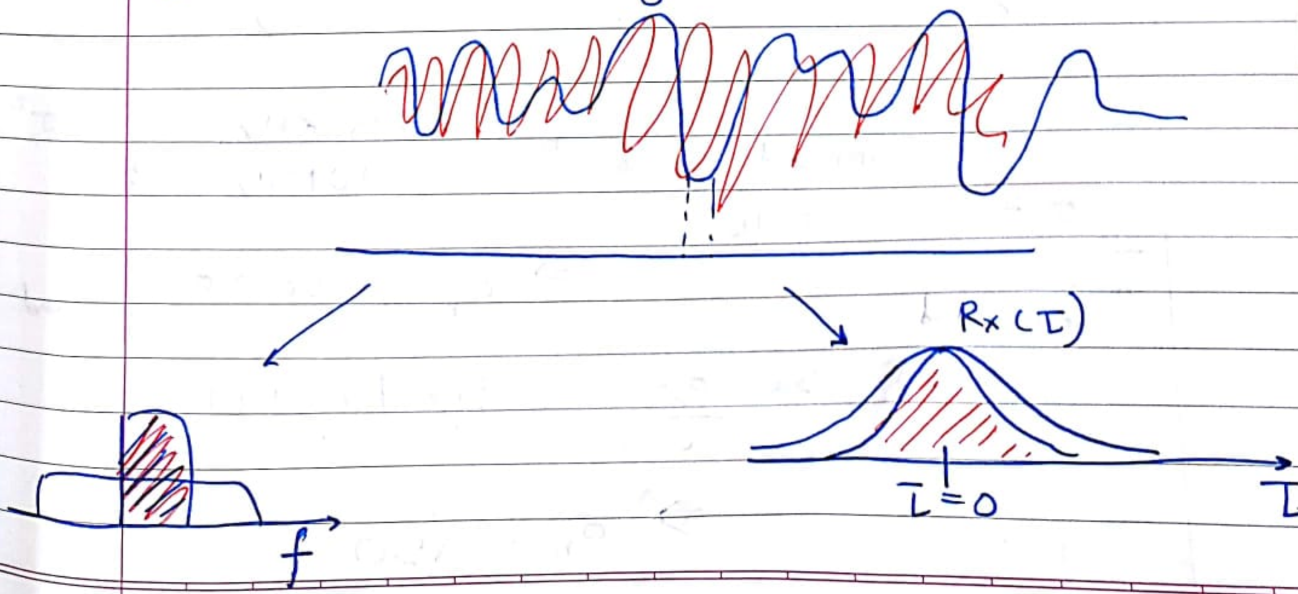
3) $|R_x(\tau)| \leq R_x(0)$

Since, $\mathbb{E}\left[[X(t_1) - X(t_1+\tau)]^2\right] \geq 0$

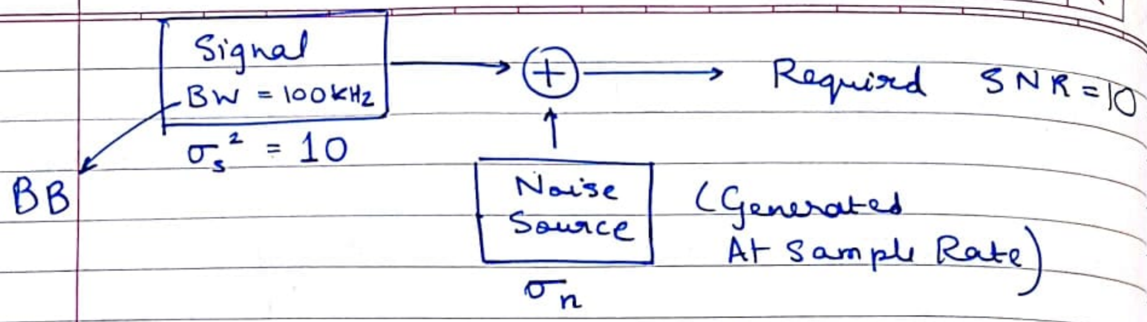
$\Rightarrow \mathbb{E}[x^2(t_1)] + \mathbb{E}[x^2(t_1+\tau)] - 2\,\mathbb{E}[X(t_1)\cdot X(t_1+\tau)] \geq 0$

$\Rightarrow 2R_x(0) - 2R_x(\tau) \geq 0$ QED

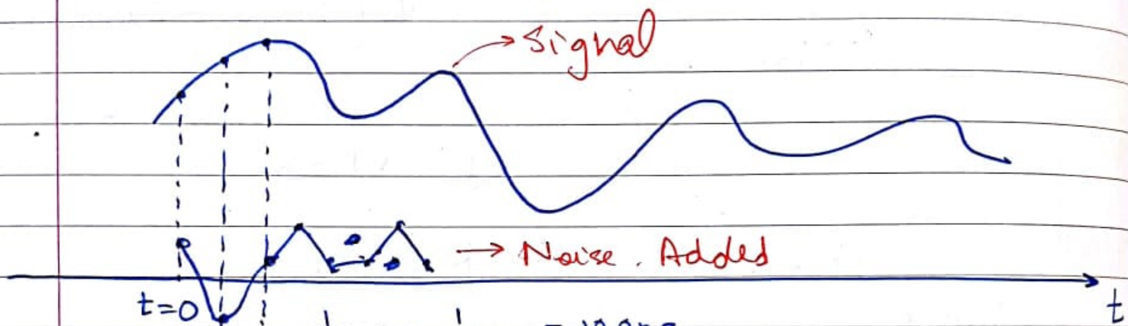
Noise & frequency



$F_s = 10\,MHz$

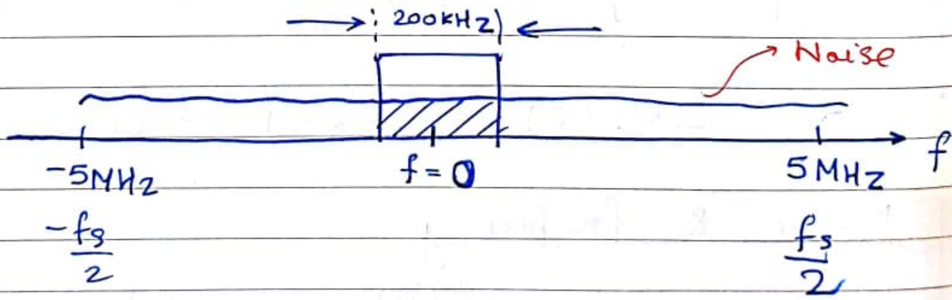


σ_s^2 = mean sq. amplitude, $\langle x^2(t) \rangle = 10$



$\frac{1}{f_s} = \frac{1}{10\,MHz} = 100\,ns$

SNR calculation,



$$\sigma_{n,\ inband}^2 = \sigma_n^2 \times \frac{200\,KHz}{10\,MHz} = \frac{1}{50} \cdot \sigma_n^2$$

$$\frac{\sigma_s^2}{\sigma_{n,\ in-band}^2} = 10 \quad \Rightarrow \sigma_n^2 = 50 \times \sigma_{n,in-band}^2$$

$$= 50 \times \frac{\sigma_s^2}{10} = 5 \times 10 = 50$$

$$\Rightarrow \sigma_n = \sqrt{50}$$

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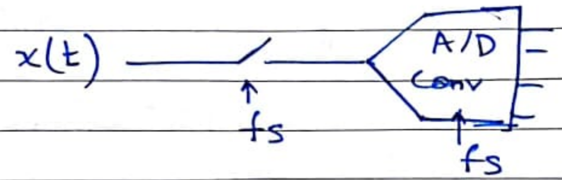
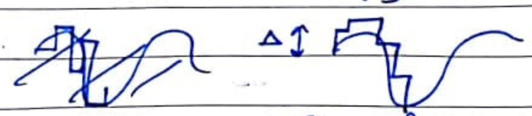


Some resolution (Least count) $= \Delta$



Noise Spectrum
Area $= \frac{\Delta^2}{12}$ (fixed)

If $f_s \uparrow$, area $\downarrow$ $\Rightarrow$ In Band Noise Reduces $\Rightarrow$ SNR $\uparrow$

15.10.18

Spectral Density

Proof H.W

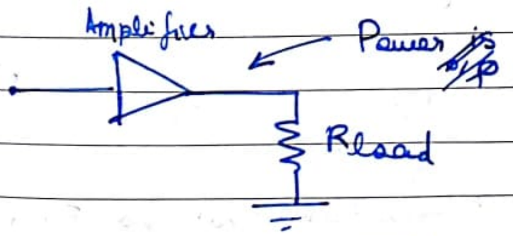
$$\int_{-\infty}^{\infty} |x^2(t)|\, dt = \int_{-\infty}^{\infty} |X^2(f)|\, df \quad : \text{Parseval Relation}$$

$$\text{Power} = \mathbb{E}\left[\,|x^2(t)|\,\right] \quad \text{units : watts}$$

$x \rightarrow$ A voltage

$$\text{Power} = \frac{\text{mean squared voltage}}{R_{load}}$$

$R_{load} \rightarrow 1\,\Omega$



$\mathbb{E}\left[\,|X^2(f)|\,\right] \triangleq f^n$ of f func. of f

Units : Energy/Hz (J-s) : Called Energy Spectral density

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• Power Spectral Density = Power / Hz

$$\text{Power} = \lim_{T\to\infty} \frac{\int_{-T}^{T} |x^2(t)|\, dt}{2T}$$

$$X_T(f) = \frac{1}{\sqrt{2T}} \int_{-T}^{T} x(t) \cdot e^{-j2\pi f t}\, dt$$

Time limited F.T

$$\langle |X_T(f)|^2 \rangle = \frac{1}{2T} E\left[\int_{-T}^{T} \int_{-T}^{T} (x^*(t_1) e^{j2\pi f_c t_1}) dt_1 \, (x(t_2) e^{-j2\pi f_c t_2}) dt_2 \right]$$

Since $\int$ is over a square region

$$\iint (\,)(\,) = \int (\,) \int (\,)$$

$$\lim_{T\to\infty} \langle |X_T(f)|^2 \rangle = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} \int_{-T}^{T} \mathbb{E}[x(t_2)\, x^*(t_1)\, e^{-j2\pi f (t_2 - t_1)}\, dt_1\, dt_2]$$

16.10.18

Let $t_2 = t_1 + \tau$ $\qquad t_1 + \tau = -T$

$\tau = -T - t_1$

$\tau = T - t_1$

Perform Change of variable

$$\lim_{T\to\infty} \int_{-T-t_1}^{T-t_1} \int_{-T}^{T} \frac{E[x^*(t_1)\, x(t_1+\tau)]\, e^{-j2\pi f_c \tau}}{2T}\, dt_1\, d\tau$$

$\hookrightarrow \tau$ $\quad \hookrightarrow t_1$

Only term that has Randomness $= E[x^*(t_1)\, x(t_1+\tau)]$

$$\lim_{T\to\infty} \int_{-T-t_1}^{T-t_1} \int_{-T}^{T} \frac{R_x(\tau)}{2T}\, e^{-j2\pi f_c \tau}\, dt_1\, d\tau$$

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$$= \lim_{T\to\infty} \int_{-T-t_1}^{T-t_1} \frac{dt_1}{2T} \int_{-T}^{T} R_x(\tau)\, e^{-j2\pi f\tau}\, d\tau$$

$$\Rightarrow \lim_{T\to\infty} \langle | X_T^2(f) | \rangle = \int_{-\infty}^{\infty} R_x(\tau)\, e^{-j2\pi f\tau}\, d\tau$$

$$= \text{F.T of } R_x(\tau)$$

$$\text{Power} = \frac{\text{Energy}}{\text{Time}} = \lim_{T\to\infty} \frac{1}{2T} \int_{-\infty}^{\infty} |x^2(t)|\, dt$$

$$= \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |x_T^2(f)|\, df \quad = \text{F.T of } R_x(\tau)$$

Units : Power / unit freq.

R.P = $X(t)$

R.V = $X(t_1)$

16.10.18

WED : 5:30 - 7pm tutorial

Recap :

PSD

$X_T(f)$ (Redefine as)

$$X_T(f) = \int_{-T}^{T} x(t)\, e^{-j2\pi ft}\, dt$$

$$\text{P.S.D} = \lim_{T\to\infty} \frac{1}{2T} \langle | X_T^2(f) | \rangle$$

$$= \int R_x(\tau)\, e^{-j2\pi f\tau}\, d\tau \triangleq S_x(f)$$

$$\mathbb{E}[X(t_1)] = \int_x x \cdot f_{X(t_1)}(x)\,dx$$

$$\mathbb{E}\left[\int_{t_1} X(t_1)\cdot g(t_1)\,dt_1\right] = \int_{t_1} E[X(t_1)]\, g(t_1)\,dt_1$$

only Random term

ex Why do we need PSD, look at just $|X(f)|^2$ instead?

if $X(t) = A\cos(2\pi f_c t + \theta)$

$$|X(f)| = \frac{A}{2}\left[\delta(f+f_c) + \delta(f-f_c)\right]$$

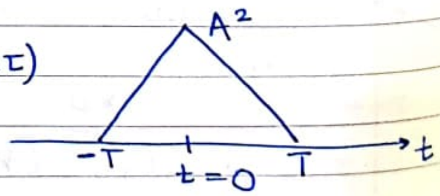
$$R_X(\tau) = \frac{A^2}{2}\cos(2\pi f\tau)$$

Einstein Wiener Klointchine Relation

$$\boxed{S_X(f) = \mathcal{F}[R_X(\tau)]} = \frac{A^2}{4}\left[\delta(f+f_c) + \delta(f-f_c)\right]$$

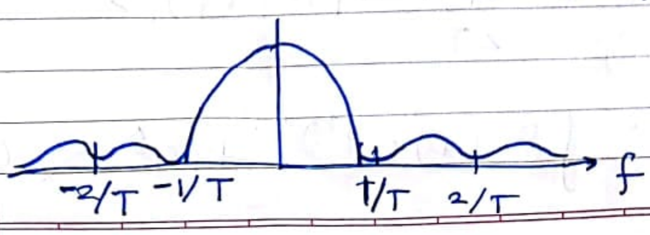
→ <u>Random Binary Wave</u>

Recap: $R_X(\tau) \xrightarrow{F.T} S_X(\tau)$



$\cancel{S\{x(\tau)\}}$ $S_X(f) = A^2 T \operatorname{sinc}^2(fT)$

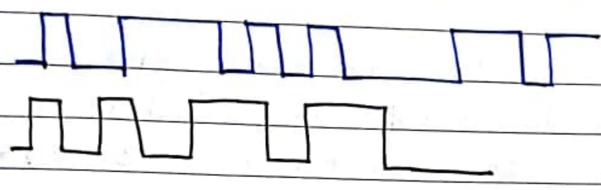
$\xrightarrow{H(f)}$ $\therefore H^2(f)\cdot S_X(f)$



Cross Spectral Density

$$S_{xy}(f) = \mathcal{F}\left[R_{xy}(\tau)\right]$$
$$= \mathcal{F}\left[E\left(x(t_1) \cdot y(t_1+\tau)\right)\right]$$

X(t)
Y(t)



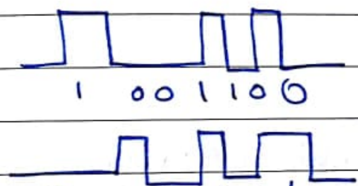
- TDMA : Same fc, share it in time
- FDMA : Different fc values.
- CDMA : Code Div. Multi. Access.

X(t)

Typ. seq. length =1023

1 0 0 1 1 0 0 × X(t) → ⊕ → Transmit Signal

0 1 0 0 1 0 1 × Y(t)



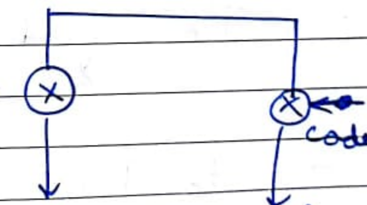
$\underbrace{\quad}_{\text{codes}}$: orthogonal (correln ~ 0)

Rx

Code1 → ⊗ ⊗ ← Code2

correln. correln.

$\underbrace{\qquad\qquad}_{\text{Compare}}$



Recap : CDMA
18.10.18

- Code Binary sequences are multiplied with the Data, sequences are orthogonal

N ∈ Z

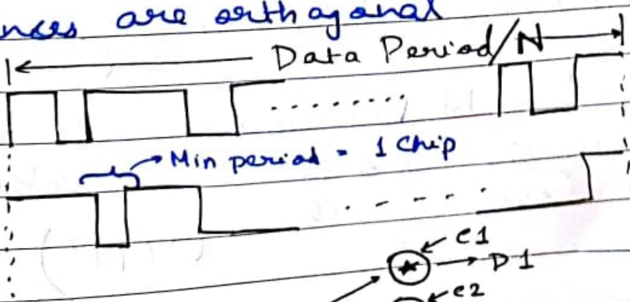
Data Period/N

c1 : × D1

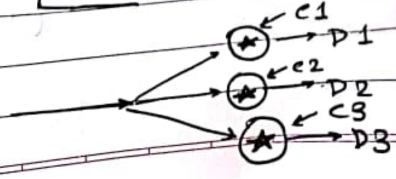
Min period = 1 Chip

c2 : × D2

→ ⊕ → Tx



Rx → ⊛ ← c1 → D1; ⊛ ← c2 → D2; ⊛ ← c3 → D3



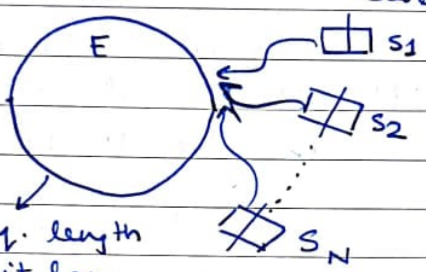
⊛ → Correlator

ex **GPS** : Code seq. length = 1023 chips (In 1 ms)

(Data) Bit period = 20 ms

→ In this case, to send one data bit we will have to send the Satellite specific code seq. 20 times $\left(\frac{20\,ms}{1\,ms} \right)$

Satellites



Code seq. length 1023 bit long for 1ms

- To send a 1, we will transmit it for 20 ms, and we will transmit code seq. × +1 20 time
- To send a 0, we will do code seq × -1 & send 20 times

→ The Receiver will have all code sequences hardcoded into it.

→ Whenever signal arrives it will Independently check for correlation with each code combination Independantly

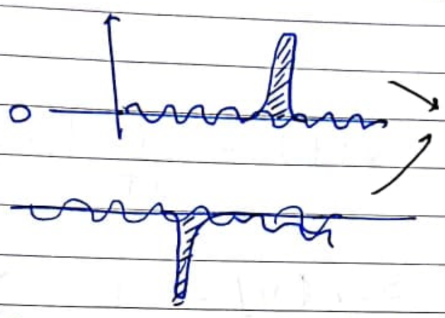
→ If N bit long code sequence, then in the worst case checking if Corr ≠ 0 is $O(N^2)$

→ Another Issue is Doppler Shift in frequencies : Ideal $f_c = 1176.45\ MHz$

$\Delta f|_{Dopp} = \pm 10\ kHz$

→ 1 transmitted ~~Bz~~ :

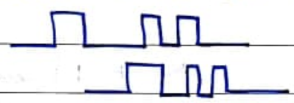
0 transmitted ~~B~~2 :



→ Ex : Check Corr. b/w C1 & C1 delayed

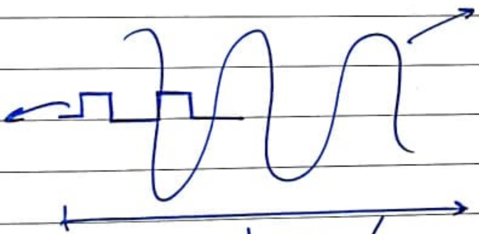
C1 :

C1 Delayed :



→ CDMA is very Robust to Noise & Interference

SNR << 1



↳ Net Signal Received Y(f)

Speed Up using FFT

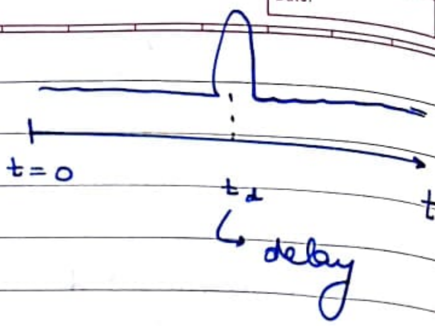
$$S_{xy}(f) = \int_{-\infty}^{\infty} E[X(t)\,Y(t-\tau)]\, e^{-j2\pi f\tau}\, df$$

$$R_{xy}(\tau) = E[X(t)\,Y(t-\tau)]$$

$$X(f) = \int c_1(t)\, e^{-j2\pi f t}\, dt$$

$$X[n] = \sum c_1[k]\, e^{-j2\pi k n/N}$$

$S_{xy}(f) \xrightarrow{IFFT}$



$$S_{xy}(f) = \frac{1}{2T} \left| X_T(f) \cdot Y_T(f) \right|$$

$$S_{xy}[n] = \frac{1}{2N} \sum_{n} X[n] \cdot Y[n]$$

$N + N \log N$

No. of Multiplications $\sim O(N \log N)$