Problem 4.5

Strictly speaking, the FM wave of Eq. (4.15) produced by a sinusoidal modulating wave is a nonperiodic function of time t. Demonstrate this property of frequency modulation.

Solution

Starting with Eq. (4.15) we write

$$s(t) = A_c[\cos(2\pi f_c t) + \beta \sin(2\pi f_m t)] \tag{1}$$

For the FM wave $s(t)$ to be a periodic function of time, we require that the condition

$$s\left(t + \frac{1}{f_m}\right) = s(t) \tag{2}$$

be satisfied for a period equal to $1/f_m$. Replacing t with $t + (1/f_m)$ in Eq. (1), we write

$$\begin{aligned} s\left(t + \frac{1}{f_m}\right) &= A_c \cos\left[2\pi f_c\left(t + \frac{1}{f_m}\right) + \beta \sin\left(2\pi f_m t + \frac{1}{f_m}\right)\right] \\ &= A_c \cos[2\pi f_c + (2\pi f_c / f_m) + \beta \sin(2\pi f_m t + 2\pi)] \\ &= A_c \cos[2\pi f_c + (2\pi f_c / f_m) + \beta \sin(2\pi f_m t)] \end{aligned} \tag{3}$$

In general, the carrier frequency f_c is a noninteger multiple of the modulation frequency f_m. Accordingly, $s(t + (1/f_m)) \neq s(t)$ and therefore the condition of Eq. (2) for periodicity is violated.