Problem 2.5

Develop the detailed steps that show that the modulation and convolution theorems are indeed the dual of each other.

Solution

The modulation theorem states that

$$g_1(t)g_2(t) \rightleftharpoons \int_{-\infty}^{\infty} G_1(\lambda)G_2(f-\lambda)d\lambda \tag{1}$$

To apply the duality theorem, we say that if

$g_1(t)g_2(t) \rightleftharpoons X(f)$, then

$X(f) \rightleftharpoons g_1(-f)g_2(-f)$

For the problem at hand, we may therefore write

$$\int_{-\infty}^{\infty} G_1(\lambda)G_2(f-\lambda)d\lambda \rightleftharpoons g_1(-f)g_2(-f) \tag{2}$$

Next, we apply Eq. (2.21), which states that if $g(t) \rightleftharpoons G(f)$ then $g(-t) \rightleftharpoons G(-f)$. Hence, applying this rule to Eq. (2), we may write

$$\int_{-\infty}^{\infty} G_1(\lambda)G_2(\lambda-t)d\lambda \rightleftharpoons g_1(f)g_2(f)$$

which is a statement of the convolution theorem, with $G_1(t) \rightleftharpoons g_1(f)$ and $G_2(t) \rightleftharpoons g_2(f)$.