Problem 2.10

Consider the function

$$g(t) = \delta\left(t+\frac{1}{2}\right)-\delta\left(t-\frac{1}{2}\right)$$

which consists of two delta functions at $t = \pm\frac{1}{2}$. The integration of $g(t)$ with respect to time t yields the unit rectangular function rect(t). Using Eq. (2.79), show that

$$\text{rect}(t) \rightleftharpoons \text{sinc}(f)$$

Solution

To begin, consider the transform pair

$$\delta(t) \rightleftharpoons 1$$

Hence, the Fourier transform of $g(t)$ is

$$G(f) = \exp(j\pi ft) - \exp(-j\pi ft)$$

from which we readily deduce that $G(0)$. Hence, applying Eq. (2.79) in the text yields

$$\begin{aligned}\mathbf{F}[\text{rect}(t)] &= \frac{1}{j2\pi f}[\exp(j\pi f) - \exp(-j\pi f)]\\ &= \frac{\sin(\pi f)}{\pi f} = \text{sinc}(f)\end{aligned}$$

where we have used the identity

$$\sin(\pi f) = \frac{1}{2j}(e^{j\pi f} - e^{-j\pi f})$$