Problem 5.29

Consider Fig. 1(a) that shows the mirror rotating counter clockwise about the horizontal axis at a rate of $2\pi f$ radians per second. At a given time t, the angular position of the position of the narrow horizontal strip on the television screen as seen in the mirror forms an angle of $2\pi ft$ with respect to the coordinate axes. The position of the narrow strip relative to the origin as seen in the mirror is described by

$$x(T_s) = \exp(j2\pi f T_s)$$

which is the sampled version of the complex exponential

$$x(t) = \exp(j2\pi ft)$$

(a) If there is exactly one revolution of the mirror between frames on the television screen, then the rotation speed of the mirror matches the sampling rate of the video signal. In this situation, the horizontal strip on the television screen does *not* appear to be rotating, as illustrated in Fig. 1(a).

(b) If however the mirror rotates at an angle less than π radians between television frames, then the rotation of the narrow strip as seen in the mirror appears like a left-to-right motion (i.e., backwards), as illustrated in Fig. 1(c). This situation implies that

$$2\pi f T_s < \pi$$

That is, with $T_s = 1/60$ seconds, the rotation rate of the mirror defined by $w = 2\pi f$ is

$$w < 60\pi \quad \text{radians/second}$$

which is one half of the television's sampling rate. If the rotation rate of the mirror satisfies this condition, then no aliasing occurs and the rotation of the mirror is visually consistent with the left-to-right motion.

On the other hand, if the mirror rotates between π and 2π radians between television frames, then the rotation of the mirror appears to be visually inconsistent with linear motion, as illustrated in Fig. 1(d). This inconsistent situation occurs when

$$\pi < 2\pi f T_s < 2\pi$$

or, with $T_s = 1/60$ seconds,

$$30 < f < 60 \quad \text{hertz}$$

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Problem 5-29 continued

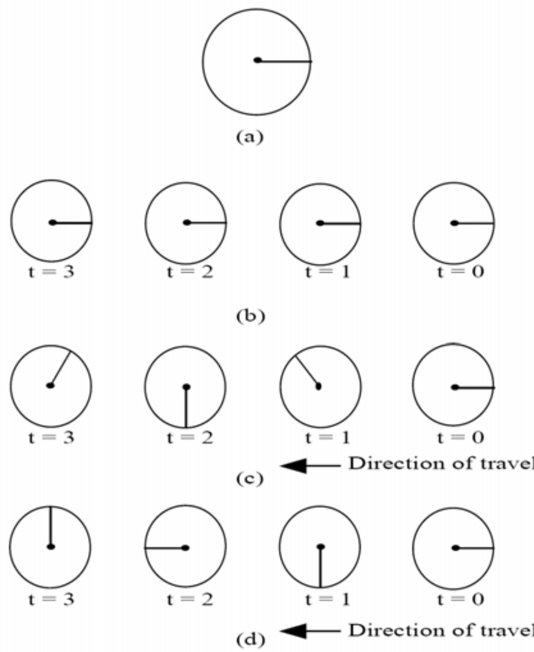

Figure 1