Problem 4.11

The phase-modulated wave is defined by

$$
\begin{aligned}
s(t) &= A_c \cos[2\pi f_c t + k_p A_m \cos(2\pi f_m t)] \\
&= A_c \cos[2\pi f_c t + \beta_p \cos(2\pi f_m t)], \qquad \beta_p = k_p A_m \\
&= A_c \cos(2\pi f_c t)\cos[\beta_p \cos(2\pi f_m t)] - A_c \sin(2\pi f_c t)\sin[\beta_p \cos(2\pi f_m t)] \qquad (1)
\end{aligned}
$$

If $\beta_p \le 0.3$, then for all time t we approximately have

$$\cos[\beta_p \cos(2\pi f_m t)] \approx 1$$

$$\sin[\beta_p \cos(2\pi f_m t)] \approx \beta_p \cos(2\pi f_m t)$$

Correspondingly, we may approximate Eq. (1) as follows:

$$
\begin{aligned}
s(t) &\approx A_c \cos(2\pi f_c t) - \beta_p A_c \sin(2\pi f_c t)\cos(2\pi f_m t) \\
&= A_c \cos(2\pi f_c t) - \frac{1}{2}\beta_p A_c \sin[2\pi(f_c + f_m)t] - \frac{1}{2}\beta_p A_c \sin[2\pi(f_c - f_m)t] \qquad (2)
\end{aligned}
$$

The spectrum of $s(t)$ is therefore

$$
\begin{aligned}
S(f) \approx {} & \frac{1}{2}A_c[\delta(f - f_c) + \delta(f + f_c)] \\
& -\frac{1}{4j}\beta_p A_c[\delta(f - f_c - f_m) - \delta(f + f_c + f_m)] \\
& -\frac{1}{4j}\beta_p A_c[\delta(f - f_c + f_m) - \delta(f + f_c - f_m)]
\end{aligned}
$$