Problem 7.1

Invoking the band-pass assumption, show that

$$\int_0^{T_b} \sin(2\pi f_c t)\cos(2\pi f_c t)dt \approx 0$$

regardless of how the bit duration T_b is exactly related to f_c so long as $f_c >> 1/T_b$.

Solution

Let

$$I(T_b) = \int_0^{T_b} \sin(2\pi f_c t)\cos(2\pi f_c t)dt$$

Using the trigonometric identity

$$\sin(A)\cos(A) = \frac{1}{2}\sin(2A)$$

we may express $I(T_b)$ as

$$\begin{aligned} I(T_b) &= \frac{1}{2}\int_0^{T_b} \sin(4\pi f_c t)dt \\ &= \frac{1}{2} \cdot \frac{1}{4\pi f_c} \cos(4\pi f_c t)\Big|_{t=0}^{T_b} \\ &= \frac{1}{8\pi f_c}[\cos(4\pi f_c T_b) - 1] \end{aligned}$$

So long as $f_c > \frac{1}{T_b}$, we may set $\cos(4\pi f_c T_b) \approx 1$, in which case, $I(T_b) \approx 0$, thereby obtaining the desired result.

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