Problem 6.5

Starting with the formula of Eq. (6.24) and using the definition of Eq. (6.26), demonstrate the property of Eq. (6.25).

Solution

Using $f' = f - B_0$ in Eq. (6.24), we may express the second line of Eq. (6.24) in the text for positive frequencies

$$P_v(f') = \frac{\sqrt{E}}{4B_0}\left\{1 + \cos\left[\frac{\pi(f' + B_0 - f_1)}{2(B_0 - f_1)}\right]\right\}$$

$$= \frac{\sqrt{E}}{4B_0}\left\{1 + \cos\left[\frac{\pi}{2} + \frac{\pi f'}{2(B_0 - f_1)}\right]\right\}$$

$$= \frac{\sqrt{E}}{4B_0}\left\{1 - \sin\left[\frac{\pi(f')}{2(B_0 - f_1)}\right]\right\} \qquad \text{for } f_1 - B_0 \le f' \le 0 \tag{1}$$

Similarly, we may express the third line of Eq. (6.24) as

$$P_v(f') = \frac{\sqrt{E}}{4B_0}\left\{\sin\left(\left(\left[\frac{\pi(f')}{2(B_0 - f_1)}\right]\right) - 1\right)\right\} \qquad \text{for } 0 \le f' \le B_0 - f_1 \tag{2}$$

From Eqs. (1) and (2), we readily see that

$P_v(-f') = P_v(f')$

which is the desired property.