Problem 4.19

The instantaneous frequency of the modulated wave $s(t)$ is shown in Fig. 1

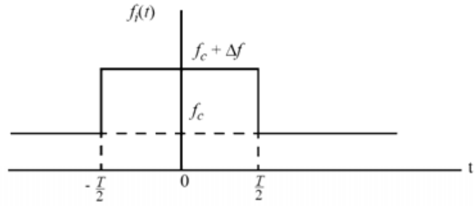

Figure 1

We may thus express $s(t)$ as follows

$$s(t) = \begin{cases} \cos(2\pi f_c t), & t < -\dfrac{T}{2} \\ \cos[2\pi(f_c + \Delta f)t], & -\dfrac{T}{2} \le t \le \dfrac{T}{2} \\ \cos(2\pi f_c t), & \dfrac{T}{2} < t \end{cases} \tag{1}$$

The Fourier transform of $s(t)$ is therefore

$$\begin{aligned} S(f) &= \int_{-\infty}^{-T/2} \cos(2\pi f_c t)\exp(-j2\pi f t)\,dt \\ &\quad + \int_{-T/2}^{T/2} \cos[2\pi(f_c + \Delta f)t]\exp(-j2\pi f t)\,dt \\ &\quad + \int_{T/2}^{\infty} \cos(2\pi f_c t)\exp(-j2\pi f t)\,dt \\ &= \int_{-\infty}^{\infty} \cos(2\pi f_c t)\exp(-j2\pi f t)\,dt \\ &\quad + \int_{-T/2}^{T/2} \{\cos[2\pi(f_c + \Delta f)t - \cos(2\pi f_c t)]\}\exp(-j2\pi f t)\,dt \end{aligned} \tag{2}$$

The second term of Eq. (2) is recognized as the difference between the Fourier transforms of two RF pulses of unit amplitude, one having a frequency equal to $f_c + \Delta f$ and the other having a frequency equal to f_c. Hence, assuming that $f_c T >> 1$, we may express the Fourier transform $S(f)$ of Eq. (2) as follows:

$$S(f) \approx \begin{cases} \dfrac{1}{2}\delta(f - f_c) + \dfrac{T}{2}\operatorname{sinc}[T(f - f_c - \Delta f)] - \dfrac{T}{2}\operatorname{sinc}[T(f - f_c)], & f > 0 \\ \dfrac{1}{2}\delta(f + f_c) + \dfrac{T}{2}\operatorname{sinc}[T(f + f_c + \Delta f)] - \dfrac{T}{2}\operatorname{sinc}[T(f + f_c)], & f < 0 \end{cases} \tag{3}$$