Problem 2.12

Discuss the following two issues, citing examples for your answers:

(a) Is it possible for a linear time-invariant system to be causal but unstable?

(b) Is it possible for such a system to be noncausal but stable?

Solution

(a) It is possible for a system to be causal but unstable. Causality means that the impulse response of the system $h(t)$ must be zero for negative t. Instability means that the BIBO criterion

$$\int_{-\infty}^{\infty} |h(t)|dt < \infty$$

is violated. Such a system could be represented by the impulse response

$$h(t) = \begin{cases} 0 & \text{for } t \leq 0 \\ \exp(t) & \text{for } t > 0 \end{cases}$$

(b) By the same token, it is possible for the system to be stable but noncausal. In this second case, we may cite the impulse response

$$h(t) = \begin{cases} \exp(t) & \text{for } t \leq 0 \\ 0 & \text{for } t > 0 \end{cases}$$

What does Problem 2.12 teach us?

The problem teaches us that the properties of stability and causality are independent.