Mid-Sem Examination Practice Questions

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by zero order hold or first order hold

1. (a) Based on the Shannon-Nyquist theorem, how will you reconstruct / interpolate the original continuous signal from its sampled values ? State the algorithm briefly.

   (b) Can the Fourier-transform operation, with input as an image $f(x)$ and producing output as image $Ff(w)$, be written as a convolution of the input $f(x)$ with some function $h(x)$ such that $Ff(w) = (f * h)(w)$? If so, show how. If not, prove why not ?

   (c) What is a **separable** convolution filter ? Define it.

   separable filter in image processing can be written as product of two more simple filters. Typically a 2-dimensional convolution operation is separated into two 1-dimensional filters. This reduces the cost of computing the operator.

   (d) What advantage do separable filters possess over non-separable filters ? Is seperability a necessary condition or a sufficient condition for such an advantage to hold ? Justify your answer.

   (e) Consider a transformation / operation $\mathcal{T}(\cdot)$ that operates on image $a(\cdot)$ and produces image $b := \mathcal{T}(a)$.
   (i) State the definition of the linearity of such a transform (based on what has been covered in class), in terms of superposition and scaling.
   Justify, briefly, which of the following transformations / operations are linear:
   (a) $\mathcal{T}(a) := F(a) =$ Fourier transform of a.
   (b) $\mathcal{T}(a) := a * h =$ convolution of a with a specific filter h.
   (c) $\mathcal{T}(a) :=$ histogram equalization of image a.

   (f) • For any discrete (spatially sampled and intensity quantized) grayscale image with integer-valued intensities, will histogram equalization make the intensity distribution uniform ? Prove or disprove.
   • Can such histogram equalization change the number of unique intensities in the original image ? Briefly explain why or why not.

   (g) For a denoising application, among the filters that we've covered in class, name the filter best suited for removing (i) salt-and-pepper noise and (ii) additive zero-mean Gaussian noise. Briefly justify your answer.

   (h) Suppose we have two filter masks, say, A and B, which we want to use for convolution. Suppose we have an image X. Suppose we define image $Y := (X * A) * B$, where $*$ denotes convolution. Suppose we define image $Z := (X * B) * A$. Assume full 2D convolution, where the convolved image will be of a size greater than either of the input images. Assume the intensities outside the image / mask boundaries to be zero.
   • Will image Y and image Z be the same for all masks A and B ? Why or why not ?
   • Indicate all subsets of the following set of images: $(A*X)*B$, $(A*B)*X$, $(B*X)*A$, $(B*A)*X$, $(X*A)*B$, $(X*B)*A$, where all images in that subset are identical ? Explain why.

   (i) Suppose you want to perform template matching over an image to detect a certain face in a photograph (similar to the example we covered in class, in the slides). You have access to a function, namely, conv(), for convolving an image with a filter.
   • Can you use the conv() function if you want to perform template matching by computing correlations ? Why or why not ? no as there is no normalization
   • Can you use the conv() function if you want to perform template matching by computing normalized correlations ? Why or why not ? yes cross convolution can be performed by taking conv(f(-t)*g(t)

   (j) • Describe the filtering mask that produces "bokeh" effects in photography.
   • When do we call a filter mask as "separable" ? Define this notion.
   • Is the bokeh mask separable ? If so, show how.

   (k) Suppose you need to blur an image, but aren't allowed to use any averaging directly in the spatial domain. Instead, you are restricted to the following operations (a) Fourier transform, (b) inverse Fourier transform, and (c) any operation on the Fourier transform in the frequency domain. How will you perform such a blurring using concepts of Fourier analysis ?

   take Ft of the image .multiply by ft of [1,1,1 1,1,1 1,1,1] and them inverse FT

(l) Does the Fourier transform describe a linear shift-invariant system ? Prove or disprove.
yes

(m) You want to smooth / blur an image without causing any other effects or artifacts. If you are restricted to use a filter defined in the (Fourier) frequency domain, give an example of a filter you would use ?

(n) What is the intensity transformation function underlying histogram equalization for a continuous-domain image $I(x)$ with probability density function (PDF) $P(f)$? Is this a linear function ? Why or why not ? What will the PDF of the transformed image be, if the image is: (i) continuous, (ii) discrete.

(o) What is the problem with adaptive histogram equalization (AHE), which contrast-limited AHE (CLAHE) intends to reduce ? Describe clearly and precisely (i) the motivation behind CLAHE and (ii) the CLAHE algorithm.

(p) Consider a continuous-domain bandlimited function $f(x)$ (say, in 1D), whose Fourier transform has frequency components lying within $[-L, L]$. Consider measurements of this function in the form of its discrete samples $f(x_i)$ at a rate above the Nyquist rate. Can we hope to reconstruct this continuous-domain function from the given discrete-domain data ? If not, why not ? If so, describe a way to estimate the continuous-domain function $f(x)$, given its sampled values $f(x_i)$.
convolution with sinc function

(q) What is the effect of processing a color image with the mean-shift algorithm, applied on the color and spatial feature, on the histogram of the image ? That is, describe how the image histogram changes after applying the mean-shift algorithm. Can this algorithm be applied for denoising a piecewise-smooth color image corrupted with additive independent and identically-distributed noise ? Why or why not ?

(r) An infinitely-differentiable function having finite support is bandlimited. Prove or disprove. NO

(s) Consider a 1D discrete image that is a box function. What will be the image resulting from an application of the median filter (1 pass over the image) ? Consider a 2D image that is a box function (in 2D). What will be the resulting image after application of the median filter ? Is the median filter linear ? Is the median filter edge preserving ?

2. What is the structure tensor ? Define mathematically.

   What information about the local image structure does a 2×2 structure tensor provide in its (i) two eigenvectors and (ii) two eigenvalues ? In other words, state what each eigenvector indicates and state what the pair of eigenvalues indicate, in terms of the local image structure.

3. Consider an image that is composed of two parts such that the bottom half of the image is grass and the top half is shrubs. The color histogram of the top half of the image is identical to the color histogram of the bottom half of the image. You want to design an algorithm to automatically and correctly segment the image using a call to a standard library function that implements the mean-shift algorithm. What features will you compute at each pixel for use in the mean-shift algorithm ?

4. Consider an image undergoing histogram equalization. Let the image intensities at spatial coordinates (x, y) be $f(x, y) := x$. Let the image domain be $x \in [0, 1]$ and $y \in [0, 1]$. Assume, for simplicity, that the image domain is continuous.

   (i) Derive the intensity-tansformation function for this image to achieve histogram equalization.

   (ii) What is the resulting image ?

   (iii) What is the resulting image's histogram ?

5. Consider an image undergoing histogram equalization. Let the image intensities at spatial coordinates (x, y) be $f(x, y) := x + y$. Let the image domain be $x \in [0, 1]$ and $y \in [0, 1]$. Assume, for simplicity, that the image domain is continuous.

   (i) Derive the intensity-tansformation function for this image to achieve histogram equalization.

   (ii) What the resulting image ?

   (iii) What the resulting image's histogram ?

6. (i) Is the standard 3×3 Laplacian kernel (**no** Gaussian smoothing involved here) separable ? If so, show a construction that proves seperability. If not, prove why it cannot be separable. NO

   (ii) Can you compute a Laplacian-filtered image using a sequence of two 1D convolutions on the image ? If so, show the algorithm. If not, prove why not.
   use soble to calculate dx and dy(sobel is saparable) again use sobel on the resultant to find out d/ dx2 then add

7. Consider a 2D Laplacian of Gaussian (LoG) kernel $g(\cdot)$ of size $N \times N$. Consider a 2D image $f(\cdot)$ of size $P \times Q$. Somebody asks you for a fast $O(N)$ algorithm to **exactly** compute the convolution of $f(\cdot)$ with $g(\cdot)$. Can this be done ? If so, give an algorithm. If not, prove why this isn't possible.

8. (i) For a general $N \times N$ non-separable convolution kernel, how can you find a way of performing the convolution **exactly** using only 1D convolutions with 1D kernels ?

   (ii) How can you use the answer to the previous question to attempt to design an algorithm that can speedup non-separable 2D convolutions using a sequence of 1D convolutions, such that the output of the faster algorithm is a **good approximation** to the output of the conventional slow algorithm ?

9. Consider a 1D image being denoised, where the image function is a ramp, i.e., image intensities $f(x)$, as a function of spatial coordinate $x \in (-\infty, +\infty)$, are given by $f(x) := ax + b$ for real-valued a and b.

   (i) Prove that the output of any median filter will be the same as the input.

   (ii) Prove that the output of any Gaussian filter will be the same as the input.

   (iii) Prove that the output of any bilateral filter will be the same as the input.

   Assume the image to have inifinite extent over the spatial coordinates. That is to say, ignore image boundaries.

10. Consider a sinusoidal function $f(x) := A\cos(\omega x)$. Consider a Gaussian $g(x) := \exp(-0.5x^2/\sigma^2)$. Consider a function $h(x) := f(x)g(x)$. Then, describe the Fourier transform of $h(x)$.

11. Explain the effects of the following algorithms / processes on the histogram of the image: (i) introduction of salt-and-pepper noise, (ii) introduction of zero-mean additive Gaussian noise, (iii) mean-shift filtering, (iv) global histogram equalization, and (v) filtering via Gaussian convolution. For each case, describe, for images in general, what happens to the (a) range of the intensities, (b) number of peaks, and (c) height of the peaks.

12. An amateur photographer wants to take a picture of the entrance to the KReSIT building. She fixes the camera on a tripod stand and places the stand on the footpath in front of the KReSIT building. The problem is that a few students always keep passing (walking) between the camera and the KReSIT entrance, thereby obstructing the view of the camera. After a few minutes of trying, she manages to take 20 pictures without changing the camera location, trying to time them right to avoid students coming into the photograph frame. However, none of the pictures is "perfect", i.e., there is always one or two students somewhere in the frame. Also, the locations of the students are randomly distributed over the spatial coordinates of the photograph. Given this set of "imperfect" pictures, she manages to generate a perfect picture of the entrance such that no passing student appears in the frame. Her algorithm is fully automatic (avoiding any manual intervention) and relies only on the image-processing concepts that we have discussed in the class so far.
    • What was her algorithm ? Precisely specify the assumptions on acquired images the under which such a generation of the perfect picture is possible.

13. Suppose you take a photograph of your friend in a playground such that the photograph comprises the face of your friend in the foreground and green grass and trees in the background — these are the only entities present in the scene. In the photograph, all entities, in both foreground and background, are in focus and exhibit sharp edges. Now, you decide to apply an artistic effect to this photograph such that only the background appears blurred in such a way that the amount of blur increases gradually with distance of the background pixel from the outline of the face. Inside the face, no blurring should occur.
    • Design and describe an algorithm to process the image to achieve this effect.

14. • Explain how the algorithm of mean-shift filtering can be used for denoising ? In addition, explain how to choose the kernel bandwidth parameter and how many iterations to run ?
    • Will this method work well for texture images or piecewise-constant (textureless / cartoon-like) images or both ? Explain why.

15. In geology, a ridge is a line of high ground, with the land dropping away on either side. Geometrically, this is a function that looks similar like a $\_\cap\_$ (on a 1D domain) or an inverted half-cylinder with its flat side placed on the ground (on a 2D domain). If you negate / invert this function, then we get a negative ridge that is shaped like a $^{-}\cup^{-}$.

In an image, interpreted as a function of pixel intensities over space, positive and negative ridges are important features; just like edges and corners. Consider that you want to detect ridges in an real-world image, where some amount of noise is present.

• Design and describe an algorithm for detecting ridges in an image. Specifically, the algorithm needs to find the (a) locations of the center ridgeline, i.e., the top of the $\cap$ and the bottom of the $\cup$, and (b) direction of the ridge.

16. Suppose you want to convolve a 2D image of size $N \times N$ with a large 2D mask of size $M \times M$, which isn't separable, a large number (T) of times. How can you exploit Fourier analysis to design an alternative algorithm that is more efficient (i.e., with lower algorithmic complexity) for the same task ? Derive the algorithmic complexity for each approach.

17. Prove, or disprove, that the convolution operation is: (i) symmetric, (ii) linear, (iii) associative, (iv) space invariant.

18. Prove, or disprove, that the Fourier transform is: (i) linear, (ii) space invariant.

19. Define the structure tensor, in the context of a 2D image. Is the the structure tensor symmetric ? Prove or disprove. Is the structure tensor a positive-definite matrix ? Prove or disprove. What will be the eigenvectors of the structure tensor for a pixel on an image edge ? Assume all other objects / edges lie are far from this edge.

20. Consider an image $f(x)$. Blur the image $f(x)$ by convolving it with a Gaussian $g(x; 0, a^2)$ (with mean 0 and variance a^2) to produce $f'(x)$. Blur the resulting image $f'(x)$ by convolving it with another Gaussian $g(x; 0, b^2)$ to produce $f''(x)$. Show that the final blurred image $f''(x)$ could have been obtained using a single convolution of $f(x)$ with a Gaussian $g(x; 0, c^2)$. Find the relationship between the standard deviations a, b, and c.

21. Consider two images A and B that are of the same size. Image A has intensities $x \in [0, 1]$. Assume that image B corresponds to independent and identically-distributed zero-mean noise. Let the probability mass functions (PMFs), i.e., histograms, of the intensities in A and B be $P_A(x)$ and $P_B(x)$, respectively. Construct a third image C, where the intensity at each pixel n is the sum of the intensities of the n-th pixels in A and B, i.e., $C(n) := A(n) + B(n), \forall n$. In image C, let the PMF of intensities be $P_C(x)$. Assuming that images A and B have a very large number of pixels, give an expression that well approximates the PMF $P_C(\cdot)$ in terms of the PMFs $P_A(\cdot)$ and $P_B(\cdot)$.

22. Consider a linear system whose output is defined as the discrete Fourier transform of the input, i.e., the input and output are one period of the associated periodic discrete signals. Is this system time invariant (or shift invariant) ? Provide a proof or disprove using a counter example.

23. Prove or disprove (e.g., using a counter example) the following statements:
    (a) If a function $f(x)$ is bandlimited, then $f(x)$ is infinitely differentiable (i.e., derivatives of all orders exist).
    (b) If a function $f(x)$ is infinitely differentiable, then $f(x)$ is bandlimited.

24. (a) To convolve a $2D$ image $f(x_1, x_2)$ of size $A \times A$ pixels, where $A := 2^a$, with an arbitrary $2D$ mask $g(x_1, x_2)$ of size $B \times B$ pixels, where $B < A$, what is the order of (arithmetic) operations required, in terms of the image size ?
    (b) If $g(x_1, x_2)$ is a 2D Gaussian filter mask, can the number of operations be reduced ? If so, describe the process and the reduced number of operations. If not, show why this is impossible ?
    (c) If $g(x_1, x_2)$ is a binary circular / disc filter mask (where the mask value equals 1 for all pixels within a distance of $B/2$ from the center pixel, and 0 otherwise), can the number of operations be reduced ? If so, describe the process and the reduced number of operations. If not, show why this is impossible ?