Problem 2.31

(a) The transfer function of the ith stage of the system of Fig. 2.43 is

$$H_i(f) = \frac{1}{1 + j2\pi fRC}$$

$$= \frac{1}{1 + j2\pi f\tau_0}, \qquad T_0 = RC$$

where it is assumed that the buffer amplifier has a constant gain of unity. The overall transfer function of the system is therefore

$$H(f) = \prod_{i=1}^{N} H_i(f)$$

$$= \frac{1}{(1 + j2\pi f\tau_0)^N}$$

The corresponding amplitude response is

$$|H(f)| = \frac{1}{[1 + (2\pi f\tau_0)^2]^{N/2}} \tag{1}$$

(b) Let

$$\tau_0^2 = \frac{T^2}{4\pi^2 N}$$

Then, we may rewrite Eq. (1) for the amplitude response as

$$|H(f)| = \left[1 + \frac{1}{N}(fT)^2\right]^{-N/2}$$

In the limit, as N approaches infinity we have

$$|H(f)| = \lim_{N \to \infty}\left[1 + \frac{1}{N}(fT)^2\right]^{-N/2}$$

$$= \exp\left[\frac{N}{2} \cdot \frac{1}{N}(fT)^2\right]$$

$$= \exp\left(-\frac{f^2T^2}{2}\right)$$