Problem 7.9

Starting with Eq. (7.41), prove the orthogonality property of Eq. (7.42) that characterizes M-ary FSK.

Solution

From Eq. (7.41), we have

$$s_i(t) = \sqrt{\frac{2E}{T}}\cos\left[\frac{\pi}{T}(n+i)t\right] \qquad \begin{array}{l} i = 0, 1, \ldots, M\text{-}1 \\ 0 \le t \le T \end{array}$$

Applying Eq. (7.42), we therefore have

$$\int_0^T s_i(t)s_j(t)dt = \frac{2E}{T}\int_0^T \cos\left[\frac{\pi}{T}(n+i)t\right]\cos\left[\frac{\pi}{T}(n+j)t\right]dt$$

$$= \frac{E}{T}\int_0^T \left\{\cos\left[\frac{\pi}{T}(2n+i+j)t\right] + \cos\left[\frac{\pi}{T}(i-j)t\right]\right\}dt \qquad (1)$$

Let the integer $k = 2n + i + j$, and $i - j = l$ for $i \neq j$. We may then rewrite Eq. (1) as

$$\int_0^T s_i(t)s_j(t)dt = \frac{E}{T}\int_0^T \left\{\cos\left(\frac{\pi}{T}kt\right) + \cos\left(\frac{\pi}{T}lt\right)\right\}dt$$

$$= \frac{E}{T}\left[\frac{T}{k\pi}\sin\left(\frac{\pi}{T}kt\right) + \frac{T}{l\pi}\sin\left(\frac{\pi}{T}lt\right)\right]_{t=0}^{T}$$

$$= 0 \text{ for all integer } k \text{ and } l$$

which is the desired result.