Problem 7.24

(a) The noiseless PSK signal is given by

$$s(t) = A_c \cos[2\pi f_c t + k_p m(t)]$$
$$= A_c \cos(2\pi f_c t) \cos[k_p m(t)] - A_c \sin(2\pi f_c t) \sin[k_p m(t)]$$

Since $m(t) = \pm 1$, it follows that

$$\cos[k_p m(t)] = \cos(\pm k_p) = \cos(k_p)$$
$$\sin[k_p m(t)] = \sin(\pm k_p) = \pm\sin(k_p) = m(t)\sin(k_p)$$

Therefore,

$$s(t) = A_c \cos(k_p)\cos(2\pi f_c t) - A_c m(t)\sin(k_p)\sin(2\pi f_c t) \tag{1}$$

The VCO output is

$$r(t) = A_v \sin[2\pi f_c t + \theta(t)]$$

The multiplier output in the phase-locked loop is therefore

$$r(t)s(t) = \frac{1}{2}A_c A_v \cos(k_p)\{\sin[\theta(t)] + \sin[4\pi f_c t + \theta(t)]\}$$

$$- \frac{1}{2}A_c A_v m(t)\sin(k_p)\{\cos(\theta(t)) + \cos[4\pi f_c t + \theta(t)]\}$$

The loop filter removes the double-frequency components, producing the output

$$e(t) = \frac{1}{2}A_c A_v \cos(k_p)\sin[\theta(t)] - \frac{1}{2}A_c A_v m(t)\sin(k_p)\cos[\theta(t)]$$

Note that if $k_p = \pi/2$, (i.e., the carrier is fully deviated), there would be no carrier component for the PLL to track.

(b) Since the error signal tends to drive the loop into lock (i.e., $\theta(t)$ approaches zero), the loop filter output reduces to

$$e(t) = -\frac{1}{2}A_c A_v \sin(k_p)m(t)$$

which is proportional to the desired data signal $m(t)$. Hence, the phase-locked loop may be used to recover the original message $m(t)$.