

7.

Region 1	Region 2
$\epsilon'_{r1}, \epsilon''_{r1}$	$\epsilon'_{r2}, \epsilon''_{r2}$
μ_{r1}	μ_{r2}

$$\epsilon = \epsilon'_r - j\epsilon'' = \epsilon'_r \left(1 - j \frac{\epsilon''}{\epsilon'_r}\right) = \epsilon'_r (1 - j \tan \delta)$$

$$\text{given. } \epsilon''_1 = \epsilon''_2 = 0. \Rightarrow \tan \delta_1 = \tan \delta_2 = 0.$$

$$\Rightarrow \sigma_1 = \sigma_2 = 0$$

$$\Rightarrow \text{lossless.}$$

$$|\Gamma|^2 = 0.2$$

$$\Rightarrow |\Gamma| = \pm 0.44$$

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{\sqrt{\frac{\mu_2}{\epsilon_2}} - \sqrt{\frac{\mu_1}{\epsilon_1}}}{\sqrt{\frac{\mu_2}{\epsilon_2}} + \sqrt{\frac{\mu_1}{\epsilon_1}}} = \frac{\frac{1}{\mu_2} - \frac{1}{\mu_1}}{\frac{1}{\mu_2} + \frac{1}{\mu_1}}$$

$$\left[\because \epsilon'_{r1} = \mu_{r1}^3 \right. \\ \left. \& \epsilon'_{r2} = \mu_{r2}^3 \right]$$

$$\left[\text{let } \mu_{r1} = \mu_1 \right. \\ \left. \& \mu_{r2} = \mu_2 \right]$$

$$= \frac{\mu_1 - \mu_2}{\mu_1 + \mu_2} = 0.44$$

$$\Rightarrow \frac{\mu_1}{\mu_2} = \frac{1.44}{0.56} = 2.5714$$

$$\Rightarrow \epsilon'_{r2}/\epsilon'_{r1} = 0.058, \text{ similarly for } \Gamma = -0.44 \Rightarrow \epsilon'_{r2}/\epsilon'_{r1} = 1.7$$

8. (a) Since $\alpha = 0$ & $\beta \neq \omega/c$, the medium is not free space but a lossless medium.

$$\beta = 0.8, \omega = 2\pi \times 10^7, \mu = \mu_0 (\text{nonmagnetic})$$

$$\epsilon = \epsilon_0 \epsilon_r.$$

$$\beta = \omega \sqrt{\mu \epsilon} = \omega \sqrt{\mu_0 \epsilon_0 \epsilon_r} = \frac{\omega}{c} \sqrt{\epsilon_r}$$

$$(\text{or}) \sqrt{\epsilon_r} = \frac{\beta c}{\omega} = \frac{0.8 (3 \times 10^8)}{2\pi \times 10^7} = 12/\pi.$$

$$\epsilon_r = 14.59$$

$$\eta = \sqrt{\frac{\mu}{\epsilon}} = \sqrt{\frac{\mu_0}{\epsilon_0 \epsilon_r}} = \frac{120\pi}{\sqrt{\epsilon_r}} = 120\pi \cdot \frac{\pi}{12} = 10\pi^2$$

$$= 98.7 \Omega.$$

$$(b) \quad P = E \times H = \frac{\epsilon_0^2}{\eta} \sin^2(\omega t - \beta z) \hat{a}_z.$$

$$P_{\text{ave}} = \frac{1}{T} \int_0^T P \cdot dt = \frac{\epsilon_0^2}{2\eta} \hat{a}_z = \frac{16}{2 \times 10\pi^2} \hat{a}_z$$

$$= 81 \text{ a.u. mW/m}^2.$$

(c) on plane $2x + y = 5$

$$\hat{a}_n = \frac{2\hat{a}_x + \hat{a}_y}{\sqrt{5}}$$

hence the total power is

$$P_{\text{total}} = \int P_{\text{ave}} \cdot d\hat{s} = P_{\text{ave}} \cdot S \hat{a}_n.$$

$$= (81 \times 10^{-3}) \hat{a}_z \cdot (100 \times 10^{-4}) \left[\frac{2\hat{a}_x + \hat{a}_y}{\sqrt{5}} \right]$$

$$= \frac{162 \times 10^{-5}}{\sqrt{5}} = 724.5 \mu\text{W}.$$