I INDUCTANCE CALCULATION

Consider an arbitrary cable.

Total Magnetic energy $= \frac{1}{2}\int_{vol} \vec{B}\cdot H^{*}\, dv$ CS volume (length l, area S)

$$= \frac{\mu}{2}\int_{vol} \bar{H}\cdot\bar{H}^{*}\, dv$$

Time averaged Magnetic energy stored per unit length $= \frac{\mu}{4}\int_{S} \bar{H}\cdot\bar{H}^{*}\, ds$

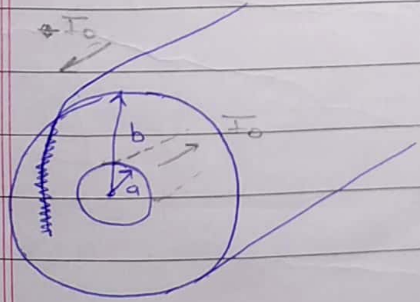
$\frac{1}{2}$ comes from $\cos^2$ from $\bar{H}\cdot\bar{H}^{*}$

This should be equal to $\frac{1}{2}\left(\frac{1}{2}L|I_0|^2\right)$

because I is sinusoidal

$$\therefore \boxed{L = \frac{\mu}{|I_0|^2}\int_{S} \bar{H}\cdot\bar{H}^{*}\, ds}$$

*



Voltage between $\rho=a$, $\rho=b$

$$\left.\boxed{\begin{aligned}\bar{E} &= \frac{V_0 e^{-\gamma z}\hat{\rho}}{\rho \ln(b/a)} \\ \bar{H} &= \frac{I_0 e^{-\gamma z}\hat{\phi}}{2\pi\rho}\end{aligned}}\right\}\text{ for } a<\rho<b$$

--- Proof later

$\rho<a$ is solid metal

$\rho=b$ is hollow metal

Outside $\rho=b$, $E = H = 0$

P.T.O

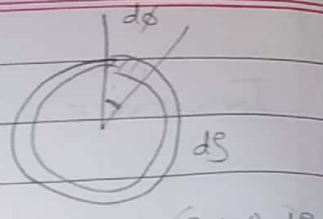
$H=0$ by Ampere's law

($I_{encl}=0$, because equal and opposite currents are flowing)

OR $\rightarrow$ Find $\phi_B = \int \vec{B} \cdot d\vec{A}$ where $dA \perp B$

Write $\phi_B = \ell L I$

Find L/ℓ



$$\text{Find } L = \frac{\mu}{|I_o|^2} \int_S \overline{H} \cdot \overrightarrow{H}^* ds$$

CS Area

$\hookrightarrow I_o = I_o e^{-\gamma z}$

$ds = (\rho d\phi) d\rho$

$$= \frac{\mu}{\cancel{I_o^2 e^{-2\alpha z}}} \int_a^b \int_0^{2\pi} \frac{\cancel{I_o^2 e^{-2\alpha z}}}{4\pi^2 \rho^2} \rho \, d\rho \, d\phi$$

$$= \frac{\mu}{2\pi} \ln(b/a)$$

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II CAPACITANCE CALCULATION

- Capacitance arises whenever there is storage of charge.

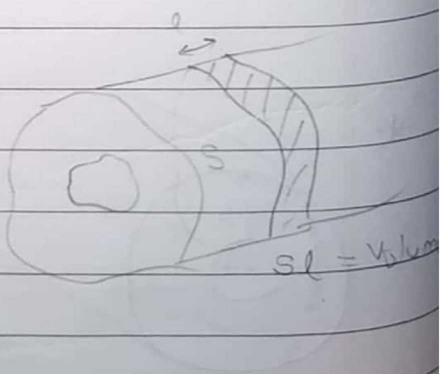
If V_o is potential difference between those charges, then

Energy stored per unit length $= W_e = \frac{1}{2} C |V_o|^2$

Multiply by ... for time avg

- We know, Energy $= \frac{1}{2} \times \frac{1}{2} \int_{vol} \vec{D} \cdot \vec{E}^* dv$

Time averaging



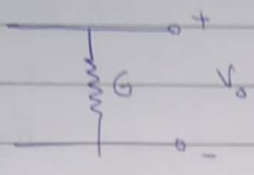
$\therefore$ Energy per unit length $= \frac{1}{2} \times \frac{1}{2} \int_S \vec{D} \cdot \vec{E}^* ds$

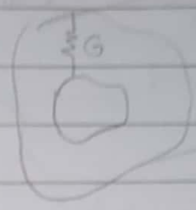
- Compare to get $C = \frac{1}{|V_o|^2} \int \vec{D} \cdot \vec{E}^* ds$

$$\boxed{C = \frac{\epsilon}{|V_o|^2} \int \vec{E} \cdot \vec{E}^* ds}$$

$$\delta \triangleq \tan^{-1} \frac{\epsilon'}{\epsilon} = \arg \epsilon''$$

III Conductance Calculation





- Power dissipated per unit length $= P_d = \frac{1}{2} |V_0|^2 G$ — time averaged

- Now, Maxwell :- $\nabla \times \vec{H} = j\omega \vec{D} + \vec{J}$

$$= j\omega \underbrace{\left(\epsilon - \frac{j\sigma}{\omega} \right)}_{\epsilon''} \vec{E}$$

$$= j\omega \, \epsilon'' \vec{E}$$

- We have defined ϵ'', which also includes the loss term.
  'Complex Permittivity'

- We can define ϵ'' only when losses are small.
  If losses are large, we can no longer say that ϵ'' is property of material

- Write $\boxed{\epsilon'' = \epsilon - j\epsilon'}$, where $\boxed{\epsilon' = \frac{\sigma}{\omega}}$ = 'Loss tangent' $\times \epsilon$

  $= \frac{\epsilon'}{\epsilon}$

- Power loss $= \frac{1}{2} \times \int_{vol} \vec{E} \cdot \vec{J}^* \, dv$ — time averaged

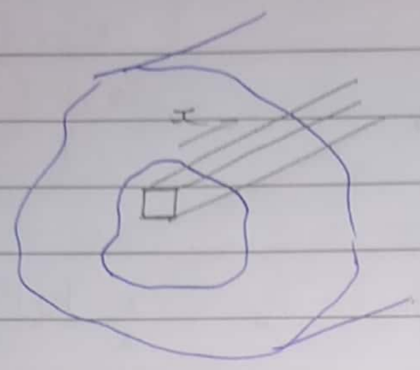
  Power loss per unit length $= \frac{\sigma}{2} \int E \cdot E^* \, ds$ — time averaged

- Compare to get $\boxed{G = \frac{\omega \epsilon'}{|V_0|^2} \int \vec{E} \cdot \vec{E}^* \, ds}$

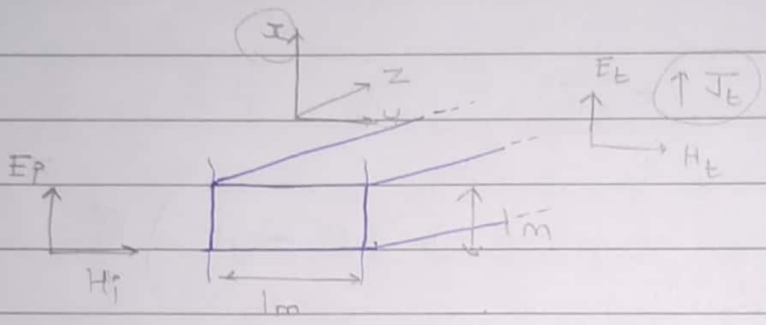
  σ

  PD b/w outer, inner

IV Resistance Calculation



Carve out a cuboidal figure in inner conductor



$$\vec{E}_p = E_0 e^{-\gamma z} \hat{x} \quad V/m$$

$$\vec{E}_t = T E_0 e^{-\gamma z} \hat{x} \quad V/m$$

$$\vec{J}_t = \sigma \vec{E}_t = \sigma E_0 T e^{-\gamma z} \hat{x} \quad \cancel{V/m} \quad A/m^2$$

$$\vec{J}_s = \int_0^{\infty} J_t \, dz$$ --- current per unit length along y

$$= \frac{\sigma E_0 T}{\gamma} \hat{x} \quad A/m$$

- Power loss in volume $1 \times 1 \times \infty$

$$= P_t = \frac{1}{2} \int \vec{E}_t \cdot \vec{J}_t \, dv$$

$$= \frac{1}{2\sigma} \int_{x=0}^{1} \int_{y=0}^{1} \int_{z=0}^{\infty} |J_t|^2 \, dz\, dy\, dx$$

$$= \frac{1}{2\sigma} \int\int\int \sigma^2 (E_0)^2 |T|^2 e^{-2\alpha z}$$

$$= \frac{\sigma \; |E_0|^2 \; |T|^2}{4\alpha}.$$

$$\cancel{\text{Now,} \quad \sigma |T|^2} \quad = |E_0|^2 \; \sigma \left| \frac{2\eta}{\eta + \eta_0} \right| \times \frac{1}{4\alpha}.$$

for metal

$$= |E_0|^2 \; \sigma \left| \frac{2\eta}{\eta_0} \right|^2 \times \frac{1}{4\alpha} \qquad \because \eta << \eta_0$$

For metal, $\eta = \frac{1+j}{\sigma \delta_s}$

$$= |E_0|^2 \; \cancel{\sigma} \times \frac{2}{\sigma^{\cancel{2}} \delta_s^{\cancel{2}} \; \eta_0^2}$$

$$= \frac{2 \; |E_0|^2 R_s}{\eta_0^2} \qquad \boxed{R_s = \frac{1}{\sigma \delta_s}} = \text{'Surface Resistance'}$$

- Consider $\frac{R_s}{2} \int |J_s|^2 \, ds$ over the same 1×1 area.

$$= \frac{R_s}{2} \int_0^1 \int_0^1 \frac{\sigma^2 \; |E_0|^2 |T|^2}{|\gamma|^2} \, dx\,dy$$

$$= \frac{\sigma^2 R_s}{2} \; \frac{|E_0|^2 |T|^2}{|\gamma|^2}$$

Consider $\frac{\sigma |T|}{|\gamma|} = \frac{\sigma \left| \frac{2\eta}{\eta + \eta_0} \right|}{\left| \frac{1+j}{\delta_s} \right|} = \frac{\sigma \left| \frac{2\eta}{\eta_0} \right|}{\sqrt{2}/\delta_s} = \sqrt{2} \, \sigma \delta_s \left| \frac{\eta}{\eta_0} \right|$

$$= \sqrt{2} \, \sigma \delta_s \left(\frac{\sqrt{2}}{\sigma \delta_s \eta_0} \right)$$

derived above

$$= 2/\eta_0$$

$$= \frac{2 \; |E_0|^2 R_s}{\eta_0^2}$$

$$= P_t \quad :)$$

$\therefore$ Power lost per unit length of the 1×1 rectangle $= P_t$

$$= \frac{R_s}{2} \int_0^1 |J_s|^2 \, dl$$

•?

$\vec{J_s} = \hat{n} \times (H_2 - \cancel{H_1})$ $\quad$ 0 because metal

$|J_s| = |H_t|$

$$= \frac{R_s}{2} \oint_0^1 |H_t|^2 \, dl$$

Total power lost per unit length = Sum over all possible rectangles

$$= \frac{R_s}{2} \oint |H_t|^2 \, dl$$

$$\frac{1}{2} |I_0|^2 R = \frac{R_s}{2} \oint \vec{H_t} \cdot \vec{H_t}^* \, dl$$

Tangential $\vec{H}$ at that point

$$\boxed{R = \frac{R_s}{|I_0|^2} \oint \vec{H} \cdot \vec{H}^* \, dl}$$

integral over both inner & outer lengths (added up)
→ Integrate wherever there is metal (C_1 & C_2)

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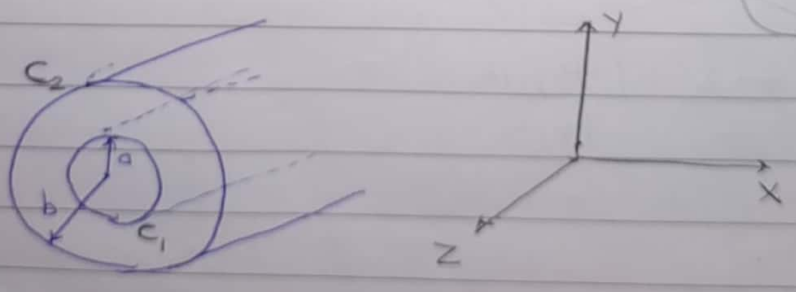
* Cylindrical Coaxial Cable (infinitely long)



1 $\quad \boxed{\vec{E} = \frac{V_0 e^{-\gamma z}}{\rho \ln(b/a)} \hat{\rho}}$ for $a < \rho < b$

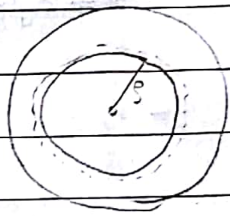
2 $\quad \boxed{\vec{H} = \frac{I_0 e^{-\gamma z}}{2\pi\rho} \hat{\phi}}$ for $a < \rho < b$

to prove (later)

3 $\boxed{L = \frac{\mu}{2\pi} \ln(b/a)}$ proven before

4 $C = \frac{\epsilon}{|V_0|^2} \int \vec{E} \cdot \vec{E}^* \, ds$ ← CS area

$= \frac{\epsilon}{V_0^2 e^{-2\alpha z}} \int_a^b \int_0^{2\pi} \frac{V_0^2 e^{-2\alpha z}}{\rho^2 \ln^2(b/a)} \rho \, d\phi \, d\rho$



$= \frac{\epsilon}{\ln^2(b/a)} \times 2\pi \times \ln(b/a)$

$= \frac{2\pi\epsilon}{\ln(b/a)}$

5 $G = \frac{\omega \epsilon''}{|V_0|^2} \int \vec{E} \cdot \vec{E}^* \, ds$ → CS Area

$= \omega\epsilon'' \times \frac{2\pi \ln(b/a)}{\ln^2(b/a)}$

$= \frac{2\pi \omega \epsilon''}{\ln(b/a)}$, $\omega\epsilon'' \rightarrow \sigma$

6 $R = \frac{R_s}{|I_0|^2} \oint_{C_1 + C_2} \vec{H} \cdot \vec{H}^* \, dl$, $C_1 + C_2 \rightarrow \phi = 0$ to 2π for both C_1 & C_2

$= \frac{R_s}{I_0^2 e^{-2\alpha z}} \int_{C_1 + C_2} \frac{I_0^2 e^{-2\alpha z}}{4\pi^2 \rho^2} \rho \, d\phi$

$= \frac{R_s}{4\pi^2} \left[\int_{C_1} \frac{d\phi}{\rho} + \int_{C_2} \frac{d\phi}{\rho} \right] = \frac{R_s}{2\pi} \left[\frac{1}{a} + \frac{1}{b} \right]$

V DERIVE TRANSMISSION LINE EQUATIONS FROM MAXWELL

$$
\boxed{\begin{aligned}
\nabla \times \vec{A} &= \hat{\rho}\left(\frac{1}{\rho}\frac{dA_z}{d\phi} - \frac{dA_\phi}{dz}\right) \\
&+ \hat{\phi}\left(\frac{dH_\rho}{dz} - \frac{dH_z}{d\rho}\right) \\
&+ \hat{z}\left(\frac{1}{\rho}\frac{d(\rho H_\phi)}{d\rho} - \frac{dH_\rho}{d\phi}\right)
\end{aligned}}
$$

* ~~sp~~ cylindrical

- Now,

$$\left.\begin{aligned} \nabla \times \vec{E} &= -j\omega\mu\vec{H} \\ \nabla \times \vec{H} &= j\omega\epsilon\vec{E} \end{aligned}\right\} \text{Lossless}$$

$$\boxed{\begin{aligned} E_z = H_z &= 0 \\ \frac{\partial E}{\partial \phi} = \frac{\partial H}{\partial \phi} &= 0 \end{aligned}}$$

(transmission line is infinite)

(symmetric in ϕ)

i $$-\hat{\rho}\frac{\partial H_\phi}{\partial z} + \hat{\phi}\frac{\partial H_\rho}{\partial z} + \hat{z}\frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho H_\phi) = j\omega\epsilon(\hat{\rho}E_\rho + \hat{\phi}E_\phi)$$

ii $$-\hat{\rho}\frac{\partial E_\phi}{\partial z} + \hat{\phi}\frac{\partial E_\rho}{\partial z} + \hat{z}\frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho E_\phi) = -j\omega\mu(\hat{\rho}H_\rho + \hat{\phi}H_\phi)$$

- Equation $\hat{z}$ components,

$$\frac{\partial}{\partial \rho}(\rho H_\phi) = 0 \qquad \text{and} \qquad \frac{\partial}{\partial \rho}(\rho E_\phi) = 0$$

$$\therefore \quad H_\phi = \frac{g(z)}{\rho} \qquad\qquad E_\phi = \frac{f(z)}{\rho}$$

g and f do not depend on ϕ, because $\frac{\partial E}{\partial \phi} = \frac{\partial H}{\partial \phi} = 0$

- Since metal cannot have tangential fields,

$$E_\phi = \cancel{H_\phi} = 0 \quad \text{at } \rho = a \text{ and } \rho = b$$

$$\therefore \quad g(z) = f(z) = 0 \qquad \forall\ z$$

$$\therefore \quad \boxed{E_\phi = \cancel{H_\phi} = 0} \quad \text{always}$$

Scanned by CamScanner

- Modifying equations ~~i~~ ~~and~~ ii with $E_\phi = \cancel{H_\phi} = 0$

$\underline{H_\rho = 0}$ $\qquad$ $\dfrac{\partial E_\rho}{\partial z} = -j\omega\mu H_\phi$ — (iii)

- From (i), $-\dfrac{\partial H_\phi}{\partial z} = j\omega\epsilon E_\rho$ — (iv)

- Since $H_\phi = \dfrac{g(z)}{\rho}$, from (iii), $E_\rho = \dfrac{h(z)}{\rho}$ — (v)

- From (iv), (v), $\dfrac{\partial g(z)}{\partial z} = -j\omega\epsilon h(z)$ — (vi)

From (iii), (v), $\dfrac{\partial h(z)}{\partial z} = -j\omega\mu g(z)$ — (vii)

- Now, $\underline{V(z) = V_a - V_b}$

$$= -\int_b^a E_\rho(\rho, z)\, dl = h(z) \ln\left(b/a\right) \quad \text{from (v)}$$

— (viii)

- By $\underline{\text{Ampere's law}}$, $\underline{I(z) = \int_{\phi=0}^{2\pi} H_\phi(\rho, z)\, \rho\, d\phi} \quad \forall \rho \in [a, b]$

$$= \int_0^{2\pi} \frac{g(z)}{\rho}\, \rho\, d\phi$$

$$= 2\pi\, g(z) \quad \text{— (ix)}$$

- From (vi), (viii), (ix), $\underline{\dfrac{\partial I(z)}{\partial z}} = -j\, \dfrac{2\pi\epsilon\, \omega\, V(z)}{\ln(b/a)}$

$$\underline{= -j\, \omega\, C\, V(z)}$$

'Transmission line equation' for lossless

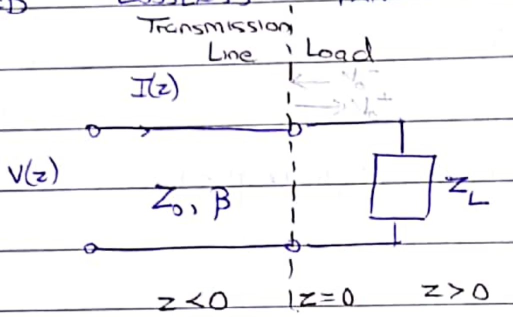
- From (vii), (viii), (ix),

$$\frac{1}{\ln(b/a)} \frac{\partial V(z)}{\partial z} = -j\omega\mu \frac{I(z)}{2\pi}$$

$$\frac{\partial V(z)}{\partial z} = -j\omega L\, I(z)$$

'Transmission Line equation' for lossless

VI TERMINATED LOSSLESS TRANSMISSION LINE



Transmission line is completely characterized by Z_0 and $\gamma = j\beta$.

$$V(z) = V_0^+ e^{-j\beta z} + V_0^- e^{j\beta z} \qquad I(z) = \frac{V_0^+ e^{-j\beta z} - V_0^- e^{j\beta z}}{Z_0}$$

$$= V^+(z) + V^-(z)$$

+z ki wave (incident) -z ki wave (reflected)

$\therefore$ $V_0^+ = V^+(0)$ and $V_0^- = V^-(0)$

- $Z_L = \frac{V(0)}{I(0)} = \left(\frac{V_0^+ + V_0^-}{V_0^+ - V_0^-}\right) Z_0$

$$\therefore \boxed{\frac{V_0^-}{V_0^+} = \frac{Z_L - Z_0}{Z_L + Z_0} = \Gamma_L} \text{ (reflection coefficient)}$$

Change of Z from Z_0 to Z_L causes reflection

→ VSWR :-

$$V(z) = V_0^+ e^{-j\beta z} + V_0^- e^{j\beta z} = V_0^+ e^{-j\beta z}\left[1 + \Gamma_L e^{j2\beta z}\right]$$

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{V_0^-}{V_0^+}$$

$$\text{Voltage SWR} = \frac{|V(z)|_{max}}{|V(z)|_{min}} = \boxed{\frac{1+|\Gamma|}{1-|\Gamma|}}$$

- $$\boxed{\Gamma(z) = \frac{V_0^-(z)}{V_0^+(z)} = \frac{V_0^-}{V_0^+} e^{j(2\beta z)}}$$

Note that (for a lossless line), $|\Gamma(z)| = \text{constant} = \Gamma_L \quad \forall z$.

- For 'well-matched load', $\Gamma_L = 0 \Rightarrow Z_L = Z_0$ (both real for lossless line)

'real' for lossless

and VSWR = 1

– In practice, VSWR < 2 is considered good enough.

→ Time averaged Power = $\boxed{P_{avg} = \frac{1}{2} \mathrm{Re}\left[V(z)\, I(z)^*\right]}$

$$= \frac{1}{2}\mathrm{Re}\left[\left(V_0^+ e^{-j\beta z} + V_0^- e^{j\beta z}\right)\left(\frac{V_0^{+*} e^{j\beta z} - V_0^{-*} e^{-j\beta z}}{Z_0}\right)\right]$$

$$= \frac{1}{2}\mathrm{Re}\left[V_0^+\left(e^{-j\beta z} + \Gamma_L e^{j\beta z}\right) \times \frac{V_0^{+*}}{Z_0}\left(e^{j\beta z} - \Gamma_L^* e^{-j\beta z}\right)\right]$$

$$= \frac{1}{2}\mathrm{Re}\left[\frac{|V_0|^2}{Z_0}\left(1 - \underbrace{\Gamma_L^* e^{-j2\beta z} + \Gamma_L e^{j2\beta z}}_{j2\,\mathrm{Im}(\Gamma_L e^{j2\beta z})} - |\Gamma_L|^2\right)\right]$$

$$= \boxed{\frac{|V_0|^2}{2Z_0}\left(1 - |\Gamma_L|^2\right)}$$

P.T.O.

- Power flowing in a transmission line depends on load Z_L
- Power flowing is same for all z (Lossless ✓)
- For well-matched circuit,

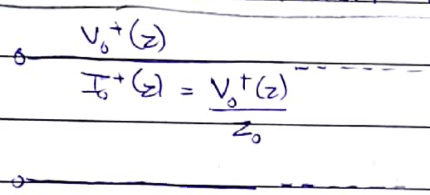
  Power entering = Power transmitted to load = $\frac{|V_0|^2}{2Z_0}$

  (no reflection of power)

- $\boxed{\text{Return Loss (RL)} \triangleq -20 \log |\Gamma|}$

  – Higher RL indicates better matching

* Infinite lossless transmission line



We assume that the transmission line never reaches load

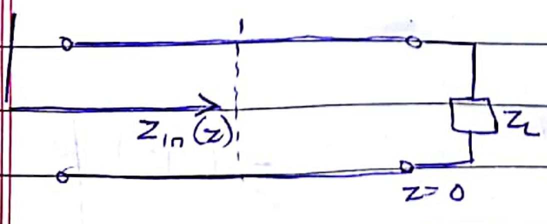
$Z_{in} = Z_0$

∴ There is no reflection and RL is very high and Γ is very small

* At any point on line, voltage is equal to sum of $V_0^+(z)$ and $V_0^-(z)$.

→ Input Impedance = $\boxed{Z_{in}(z) = \frac{V(z)}{I(z)}}$

finite line



$$= \frac{V_0^+(z) + V_0^-(z)}{\left(\frac{V_0^+(z) - V_0^-(z)}{Z_0}\right)}$$

V_0^+ & V_0^- are values of $V_0^+(z)$ & $V_0^-(z)$ at $z=0$ (near load).

$$= Z_0 \left[\frac{V_0^+ e^{-j\beta z} + V_0^- e^{j\beta z}}{V_0^+ e^{-j\beta z} - V_0^- e^{j\beta z}} \right]$$

$$= Z_0 \frac{V_0^+\left[e^{-j\beta z} + \Gamma_L e^{j\beta z}\right]}{V_0^+\left[e^{-j\beta z} - \Gamma_L e^{j\beta z}\right]}$$

$$= Z_0 \frac{\left[e^{-j\beta z} + \frac{Z_L - Z_0}{Z_L + Z_0} e^{j\beta z}\right]}{\left[e^{-j\beta z} - \frac{Z_L - Z_0}{Z_L + Z_0} e^{j\beta z}\right]}$$

$$= Z_0 \frac{\left[Z_L\left(e^{-j\beta z} + e^{j\beta z}\right) + Z_0\left(e^{-j\beta z} - e^{j\beta z}\right)\right]}{\left[Z_L\left(e^{-j\beta z} - e^{j\beta z}\right) + Z_0\left(e^{-j\beta z} + e^{j\beta z}\right)\right]}$$

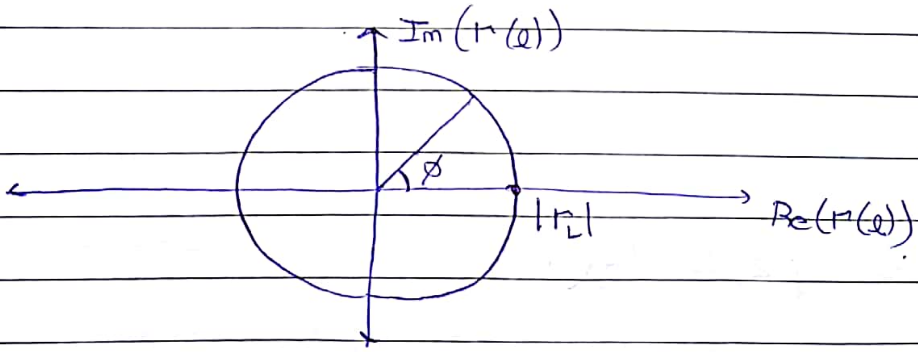
Change of co-ordinates :- $l = -z$ = distance of point from load.

$$Z_{in}(l) = Z_0 \left[\frac{Z_L\left(e^{j\beta l} + e^{-j\beta l}\right) + Z_0\left(e^{j\beta l} - e^{-j\beta l}\right)}{Z_L\left(e^{j\beta l} - e^{-j\beta l}\right) + Z_0\left(e^{j\beta l} + e^{-j\beta l}\right)}\right]$$

$$= \boxed{Z_0\left(\frac{Z_L + jZ_0\tan(\beta l)}{Z_0 + jZ_L\tan(\beta l)}\right)}$$

- Since $\Gamma(z) = \frac{V_0^-}{V_0^+} e^{j2\beta z}$, $\therefore \Gamma(l) = \frac{V_0^-}{V_0^+} e^{-j(2\beta l)}$

$\therefore |\Gamma(l)| = \text{constant} = |\Gamma_L| \quad \forall \ l$



For increasing l (moving away from load) :- ACW

decreasing towards ACW

- $V(z) = V_0^+ e^{-j\beta z}\left[1 + \Gamma e^{j2\beta z}\right]$

Substitute $\Gamma = |\Gamma| e^{j\phi}$

$\therefore V(z) = V_0^+ e^{-j\beta z}\left[1 + |\Gamma| e^{j(\phi + 2\beta z)}\right]$

For V_{max}, $\phi + 2\beta z = 2n\pi$

Distance between two consecutive maximas of $V(z)$, $I(z)$ $= \dfrac{2\pi}{2\beta} = \dfrac{\lambda}{2}$ where $\lambda = \dfrac{2\pi}{\beta}$

Distance between maxima and next minima $= \dfrac{\lambda}{4}$

A] Special Terminated Loads

1. $Z_L = Z_0$:-

$$Z_{in} = Z_0 \left(\frac{Z_{0L} + jZ_0 \tan(\beta l)}{Z_0 + jZ_L \tan(\beta l)} \right) = Z_0$$

2. $Z_L = 0$:-

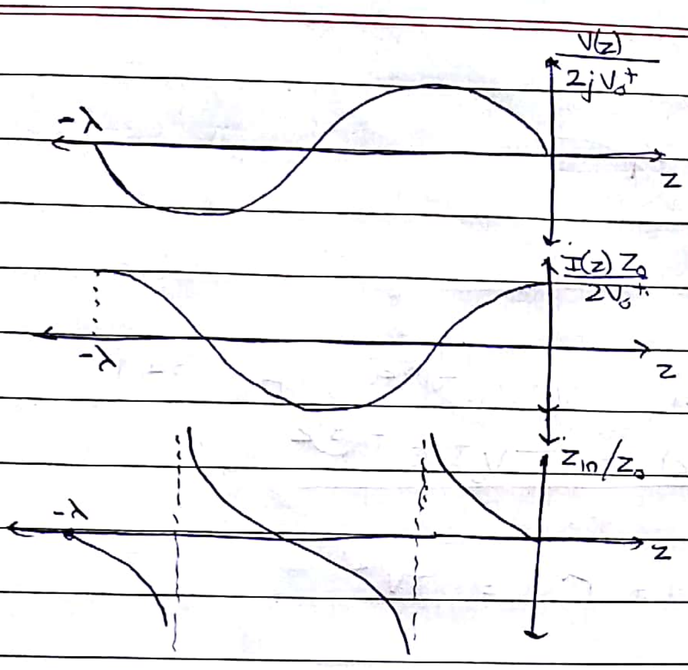
$$Z_{in}(l) = jZ_0 \tan(\beta l)$$

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = -1$$

$$V(z) = V_0^+\left[e^{-j\beta z} - e^{j\beta z}\right] = -2jV_0^+ \sin(\beta z)$$

ygb

$$I(z) = \frac{V_0^+}{Z_0}\left[e^{-j\beta z} + e^{j\beta z}\right] = \frac{2V_0^+ \cos(\beta z)}{Z_0}$$



HW 3 $Z_L = \infty$ (Open Circuit)

$$Z_{in} = -j\frac{Z_0}{\tan\beta l}$$

$$\Gamma_L = 1$$

$$V(z) = 2V_0^+ \cos(\beta z)$$

$$I(z) = -2j\frac{V_0^+}{Z}\sin(\beta z)$$

- $Z_0 = \sqrt{Z_{sc} Z_{oc}}$

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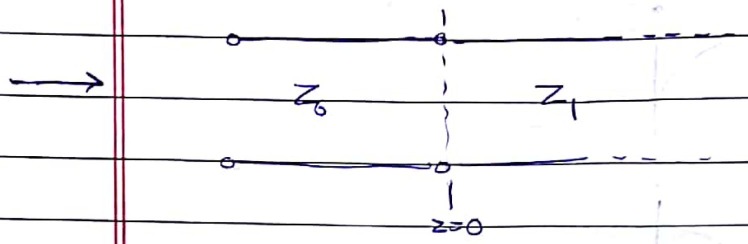

For $z<0$, $\quad V(z) = V_0^+ \left(e^{-j\beta z} + \Gamma e^{j\beta z}\right)$

$z>0$, $\quad V(z) = T V_0^+ e^{-j\beta' z}$

At $z=0$, $\quad V_0(1+\Gamma) = T V_0^+$

$\therefore \boxed{T} = 1 + \Gamma = \boxed{\dfrac{2Z_1}{Z_1+Z_0}}$ since $\Gamma = \dfrac{Z_1 - Z_0}{Z_1 + Z_0}$

- Insertion loss $= IL \triangleq \boxed{-20\log|T|}$
  - Lower is better.

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* Z_L is known as 'passive impedance' if $Re(Z_L) > 0$

- Values of $Re(Z_L)$ and $Im(Z_L)$ can vary over a large range.
  - Use Smith Chart to represent.

B] Smith Chart.

* $\Gamma_L = \dfrac{Z_L - Z_0}{Z_L + Z_0}$ is known as a 'bilinear transformation'

- Write $\boxed{\Gamma_L = \frac{z_L - 1}{z_L + 1}}$ where $\boxed{z_L = \frac{Z_L}{Z_0}}$ = '<u>Normalized impedance</u>'

$$= \frac{1+\Gamma_L}{1-\Gamma_L}$$

- $\underline{\underline{z_L = r_L + jx_L}}$

<u>If transmission line is lossless or distortionless, then Z_0 is real.</u>

$\therefore$ $r_L = \frac{R_L}{Z_0}$ $\quad$ $x_L = \frac{X_L}{Z_0}$

<u>If</u> $\Gamma_L = \Gamma_r + j\Gamma_i$, $\quad$ $r_L + jx_L = \frac{1+\Gamma_r + j\Gamma_i}{1-\Gamma_r - j\Gamma_i}$

$\therefore$ $r_L = \frac{1-\Gamma_r^2 - \Gamma_i^2}{(1-\Gamma_r)^2 + \Gamma_i^2}$ (i) $\quad$ $x_L = \frac{2\Gamma_i}{(1-\Gamma_r)^2 + \Gamma_i^2}$ (ii)

- From above two equations, eliminate x_L :-

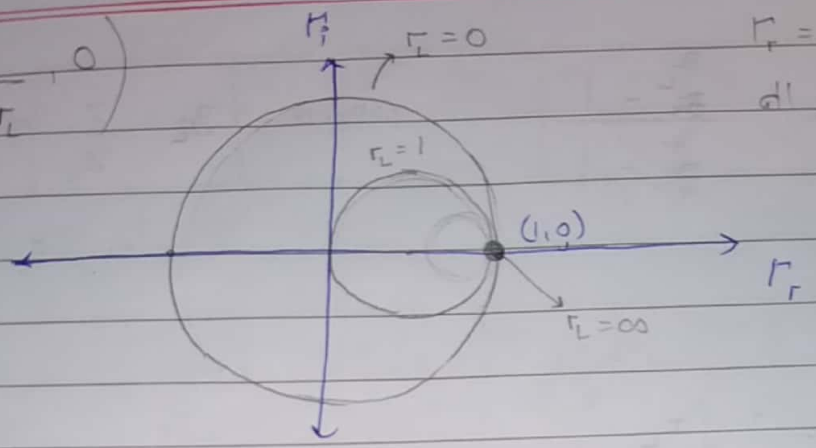
$$\boxed{\left(\Gamma_r - \frac{r_L}{1+r_L}\right)^2 + \Gamma_i^2 = \frac{1}{(1+r_L)^2}}$$ from (i)

Eliminate r_L :- $\boxed{(\Gamma_r - 1)^2 + \left(\Gamma_i - \frac{1}{x_L}\right)^2 = \left(\frac{1}{x_L}\right)^2}$ from (ii)

– Above two equations represent two circles on $\Gamma_r - \Gamma_i$ plane.

$C \equiv \left(1 - \frac{1}{1+r_L}, 0\right)$

$R \equiv \frac{1}{1+r_L}$

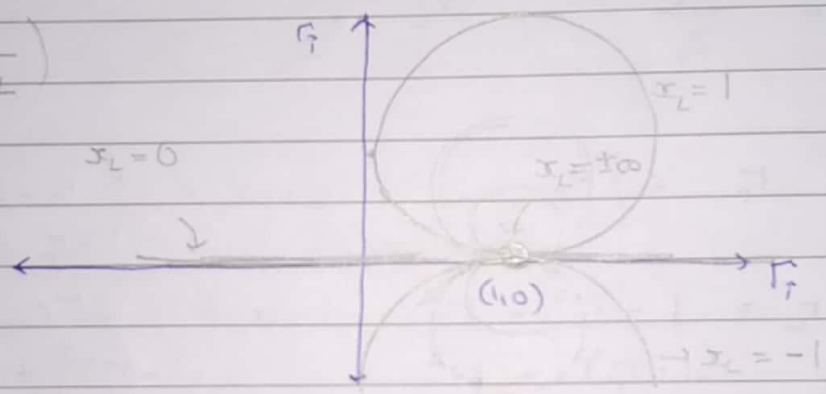


$\Gamma_r = 1$ is tangent to all circles

$$\left(\Gamma_r - \frac{r_L}{1+r_L}\right)^2 + \Gamma_i^2 = \frac{1}{(1+r_L)^2}$$

$C \equiv \left(1, \frac{1}{x_L}\right)$

$R \equiv \frac{1}{x_L}$

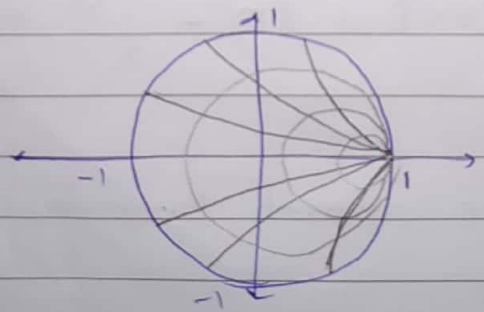


Γ_r axis is tangent to all circles

$$(\Gamma_r - 1)^2 + \left(\Gamma_i - \frac{1}{x_L}\right)^2 = \left(\frac{1}{x_L}\right)^2$$

- For both $r_L = \infty$ and $x_L = \pm\infty$, locus is the point $(1,0)$

- What a Smith chart looks like :-



Equi-r_L curves

Equi-x_L curves

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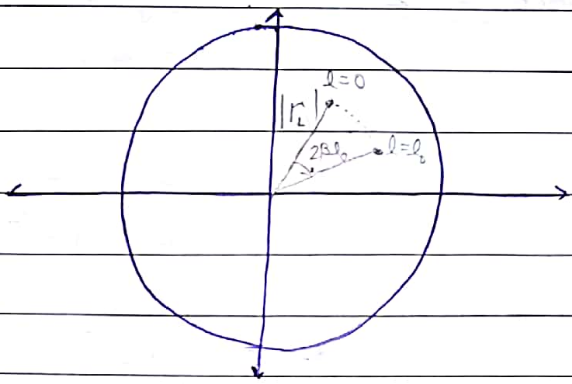
→ Application : $\Gamma(l) = \frac{V_0^-}{V_0^+} e^{-j2\beta l}$

If you move l distance away, $\Gamma(l)$ rotates clockwise by $2\beta l$



$\underline{\beta l = \text{'Electrical length'}}$

Based on Smith chart we can find out $z_L = r_L + jx_L$ at both points and hence find $Z_{IN}(l) = Z_0 z_L$

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \Gamma(0) \quad \text{where} \quad \boxed{\Gamma(l) = \frac{Z_{in}(l) - Z_0}{Z_{in}(l) + Z_0}}$$

* Any transmission line is characterized by Z_0 and β.

- If you give $\underline{DC}$ to a transmission line, all voltages and currents will be ~~DC~~ constants. Transmission line acts as shorted wire $(\beta = 0)$
  $\omega\sqrt{LC}$

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VII Lossy Transmission Line

$$R \neq 0, \quad G \neq 0.$$

- Characteristic impedance $= Z = \sqrt{\dfrac{R + j\omega L}{G + j\omega C}}$

$$= \sqrt{\frac{j\omega L}{j\omega C}\left(\frac{1 + R/j\omega L}{1 + G/j\omega C}\right)}$$

$$= Z_0 \sqrt{\frac{1 + R/j\omega L}{1 + G/j\omega C}}$$

$\sqrt{L/C}$

- Propagation constant $= \gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$

$$= j\omega\sqrt{LC}\sqrt{1 - j\left(\frac{R}{\omega L} + \frac{G}{\omega C}\right) - \frac{RG}{\omega^2 LC}}$$

$$= j\beta_0 \sqrt{1 - j\left(\frac{R}{\omega L} + \frac{G}{\omega C}\right) - \frac{RG}{\omega^2 LC}}$$

$\sqrt{LC}$

* v_p = Phase velocity $= \dfrac{\omega}{\beta}$ = Velocity at which one phase moves.

v_g = Group velocity $= \dfrac{d\omega}{d\beta}$ = Velocity at which information moves

→ Two issues ☹

1. $\alpha \neq 0$ and α varies with frequency

∴ Transmission line has non-constant frequency response

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2 For distortionless transmission, $\beta \propto \omega$
This does not happen in lossy transmission line.

→ Low loss / High frequency condition

$\frac{R}{\omega L} \ll 1$ and $\frac{G}{\omega C} \ll 1$

Ignore last term $\frac{RG}{\omega^2 LC}$ in γ.

$$\therefore \gamma = j\omega\sqrt{LC}\sqrt{1 - j\left(\frac{R}{\omega L} + \frac{G}{\omega C}\right)}$$

$$\approx j\omega\sqrt{LC}\left(1 - \frac{j}{2}\left(\frac{R}{\omega L} + \frac{G}{\omega C}\right)\right) \quad \text{.... Binomial}$$

$$\therefore \boxed{\alpha = \frac{1}{2}\left(R\sqrt{\frac{C}{L}} + G\sqrt{\frac{L}{C}}\right)} \quad \text{and} \quad \boxed{\beta = \omega\sqrt{LC}}$$

.... independent of frequency :) proportional to ω :)

→ Condition for distortionlessness :- $\boxed{\frac{R}{L} = \frac{G}{C}}$

$$\gamma = j\omega\sqrt{LC}\sqrt{1 - j\frac{2R}{\omega L} - \frac{R^2}{\omega^2 L^2}} \quad \text{.... perfect square}$$

$$= j\omega\sqrt{LC}\left(1 - j\frac{R}{\omega L}\right)$$

$$\boxed{\alpha = R\sqrt{\frac{C}{L}}} \qquad \boxed{\beta = \omega\sqrt{LC}}$$

:) :)