Problem 3.9

Verify that the outputs of the receiver in Fig. 3.17(b) are as indicated in the figure, assuming perfect synchronism between the receiver and transmitter.

Solution

The transmitted signal is

$$s(t) = A_c m_1(t)\cos(2\pi f_c t) + A_c m_2(t)\sin(2\pi f_c t)$$

Hence, the product modulator output of upper channel in Fig. 3.17(b) is

$$\begin{aligned} v_1(t) &= A'_c\cos(2\pi f_c t)s(t) \\ &= A_c A'_c m_1(t)\cos(2\pi f_c t) + A_c A'_c m_2(t)\cos(2\pi f_c t)\sin(2\pi f_c t) \\ &= \frac{1}{2}A_c A'_c m(t)[1 + \cos(4\pi f_c t)] + \frac{1}{2}A_c A'_c m(t)\sin(4\pi f_c t) \end{aligned}$$

Passing $v_1(t)$ through the low-pass filter yields $\frac{1}{2}A_c A'_c m(t)$, so long as there is no spectral overlap, that is, $f_c > W$.

Consider next the lower channel of the figure. The product-modulator output is

$$\begin{aligned} v_2(t) &= A'_c\cos(2\pi f_c t)s(t) \\ &= A_c A'_c m_1(t)\sin(2\pi f_c t)\cos(2\pi f_c t) + A_c A'_c m_2(t)\sin^2(2\pi f_c t) \\ &= \frac{1}{2}A_c A'_c m(t)\sin(4\pi f_c t) + \frac{1}{2}A_c A'_c m_2(t)[1 - \cos(4\pi f_c t)] \end{aligned}$$

Passing $v_2(t)$ through the low-pass filter yields $\frac{1}{2}A_c A'_c m(t)$, as indicated in the figure.