Problem 5.26

Let $2W$ denote the bandwidth of a narrowband signal with carrier frequency f_c. The in-phase and quadrature components of this signal are both low-pass signals with a common bandwidth of W. According to the sampling theorem, there is no information loss if the in-phase and quadrature components are sampled at a rate higher than $2W$. For the problem at hand, we have

$f_c \;\; = 100$ kHz

$2W = 10$ kHz

Hence, $W = 5$ kHz, and the minimum rate at which it is permissible to sample the in-phase and quadrature components is 10 kHz.

From the sampling theorem, we also know that a physical waveform can be represented over the interval $-\infty < t < \infty$ by

$$g(t) = \sum_{n=-\infty}^{\infty} a_n \phi_n(t) \qquad (1)$$

where $\{\phi_n(t)\}$ is a set of orthogonal functions defined as

$$\phi_n(t) = \frac{\sin\{\pi f_s(t - n/f_s)\}}{\pi f_s(t - n/f_s)}$$

where n is an integer and f_s is the sampling frequency. If $g(t)$ is a low-pass signal limited to W Hz, and $f_s \geq 2W$, then the coefficient a_n can be shown to equal $g(n/f_s)$. That is, for $f_s \geq 2W$, the orthogonal coefficients are simply the values of the waveform that are obtained when the waveform is sampled every $1/f_s$ second.

As already mentioned, the narrowband signal is two-dimensional, consisting of in-phase and quadrature components. In light of Eq. (1), we may represent them as follows, respectively:

$$g_I(t) = \sum_{n=-\infty}^{\infty} g_I(n/f_s)\phi_n(t)$$

$$g_Q(t) = \sum_{n=-\infty}^{\infty} g_Q(n/f_s)\phi_n(t)$$

Hence, given the in-phase samples $g_I\left(\frac{n}{f_s}\right)$ and quadrature samples $g_Q\left(\frac{n}{f_s}\right)$, we may reconstruct the narrowband signal $g(t)$ as follows:

$$g(t) = g_I(t)\cos(2\pi f_c t) - g_Q(t)\sin(2\pi f_c t)$$

$$= \sum_{n=-\infty}^{\infty} \left[g_I\left(\frac{n}{f_s}\right)\cos(2\pi f_c t) - g_Q\left(\frac{n}{f_s}\right)\sin(2\pi f_c t)\right]\phi_n(t)$$

where $f_c = 100$ kHz and $f_s \geq 10$ kHz, and where the same set of orthonormal basis functions is used for reconstructing both the in-phase and quadrature components.