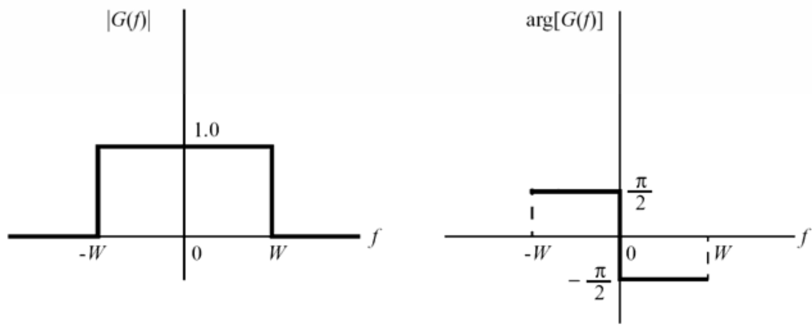
Problem 2.2

Determine the inverse Fourier transform of the frequency function $G(f)$ defined by the amplitude and phase spectra shown in Fig. 2.5.



Solution

$$g(t) = \int_{-W}^{0} e^{j\pi/2} \cdot e^{j2\pi ft} df + \int_{0}^{W} e^{-j\pi/2} e^{j2\pi ft} df$$

$$= \left[\frac{1}{j2\pi t} e^{j\left(\frac{\pi}{2} + 2\pi ft\right)}\right]_{f=-W}^{0} + \left[\frac{1}{j2\pi t} e^{j\left(-\frac{\pi}{2} + 2\pi ft\right)}\right]_{f=0}^{W}$$

$$= \frac{1}{j2\pi t}\left(e^{j\left(\frac{\pi}{2} - 2\pi Wt\right)} - e^{j\pi/2}\right) + \frac{1}{j2\pi t}\left(e^{-j\pi/2} - e^{j\left(-\frac{\pi}{2} - j2\pi Wt\right)}\right)$$

$$= \frac{1}{j2\pi t}(e^{-j\pi/2} - e^{j\pi/2}) + \frac{1}{j2\pi t} e^{-j2\pi Wt}(e^{j\pi/2} - e^{-j\pi/2})$$

$$= -\frac{1}{\pi t} + \frac{1}{\pi t} e^{-j2\pi Wt} = \frac{1}{\pi t}(e^{-j2\pi Wt} - 1)$$

Note: If we let $W \to \infty$, $G(f) \to j\,\mathrm{sgn}(t)$, the inverse of which $-\frac{1}{\pi t}$. This result agrees with the limiting value of the solution for $W = \infty$.