Problem 2.23

We are given the following inequalities:

$$|G(f)| \le \int_{-\infty}^{\infty} |g_1(t)| \; dt$$

$$|j2\pi f G(f)| \le \int_{-\infty}^{\infty} \left|\frac{dg(t)}{dt}\right| dt$$

$$\left|j2\pi f^2 G(f)\right| \le \int_{-\infty}^{\infty} \left|\frac{d^2 g(t)}{dt}\right| dt$$

Considering the triangular pulse $g(t)$ of Fig. 2.41 in the text, its first and second derivatives with respect to time t are illustrated in Fig. 1:

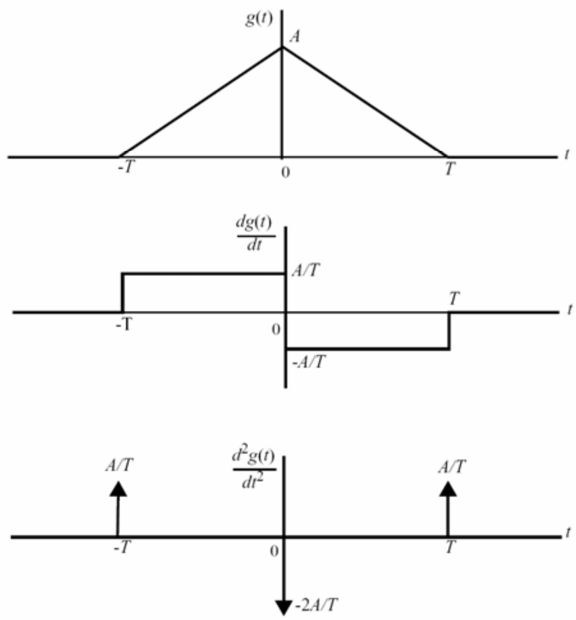

Figure 1

We thus have

$$\int_{-\infty}^{\infty} |g(t)| dt \;=\; AT$$

$$\int_{-\infty}^{\infty} \frac{dg(t)}{dt} dt \;=\; 2A$$

$$\int_{-\infty}^{\infty} \frac{d^2 g(t)}{dt} dt \;=\; \int_{-\infty}^{\infty} \frac{A}{T} |\delta(t+T) - 2\delta(t) + \delta(t-T)| dt$$

$$= \frac{4A}{T}$$

The bounds on the amplitude spectrum $|G(f)|$ are therefore as follows:

$$|G(f)| \le AT$$

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$$|G(f)| \le \frac{A}{\pi|f|}$$

$$|G(f)| \le \frac{A}{\pi^2 f^2 T}$$

which are shown plotted in Fig. 2.

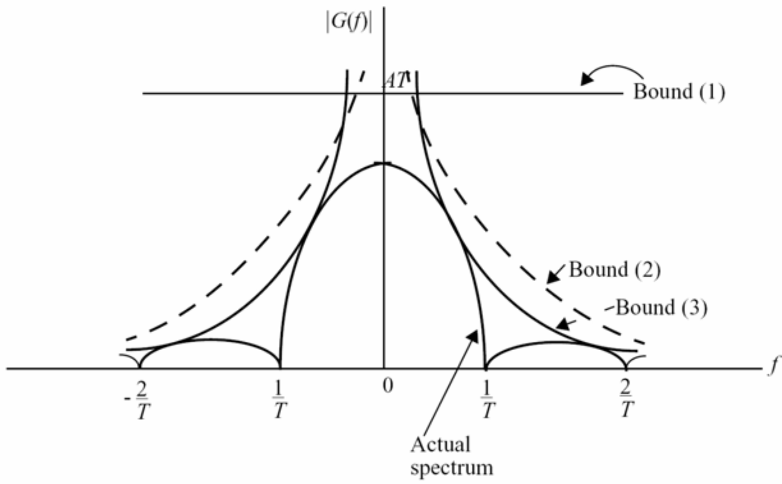

Figure 2

The actual amplitude spectrum of the triangular pulse is given by

$$|G(f)| = AT\,\mathrm{sinc}^2(fT)$$

which is also plotted in Fig. 1. From this figure we see that bounds (1) and (3) define boundaries on the actual spectrum $|G(f)|$.