**Problem 6.22**

(a) For the modified duobinary conversion filter shown in Fig. 6.15, we have

$$c_k = a_k - a_{k\text{-}2} \qquad (1)$$

Here again, we find that a three-level sequence is generated. Specifically, for $a_k = \pm 1$, we find from Eq. (1) that $c_k$ has three possible values: 2, 0, +2.

The overall transfer function of the modified duobinary conversion filter shown i Fig. 6.15 is therefore given by

$$\begin{aligned} H(f) &= H_{\text{Nyquist}}(f)[1 - \exp(-j4\pi f T_b)] \\ &= H_{\text{Nyquist}}(f)[\exp(j2\pi f T_b) - \exp(-j2\pi f T_b)]\exp(-j2\pi f T_b) \\ &= 2jH_{\text{Nyquist}}(f)\sin(2\pi f T_b)\exp(-j\pi f T_b) \end{aligned} \qquad (2)$$

With

$$H_{\text{Nyquist}}(f) = \begin{cases} 1 & \text{for } |f| \le 1/2T_b \\ 0 & \text{otherwise} \end{cases} \qquad (3)$$

we may therefore express $H(f)$ as

$$H(f) = \begin{cases} 2j\sin(2\pi f T_b)\exp(-j2\pi f T_b) & \text{for } |f| \le 1/2T_b \\ 0 & \text{elsewhere} \end{cases} \qquad (4)$$

which is the form of a half-cycle sine function.

(b) The corresponding impulse response of the modified duobinary conversion filter follows from the first line of Eq. (2); specifically,

$$\begin{aligned} h(t) &= \frac{\sin(\pi t/T_b)}{\pi t/T_b} - \frac{\sin[\pi(t-2T_b)/T_b]}{\pi(t-2T_b)/T_b} \\ &= \frac{\sin(\pi t/T_b)}{\pi t/T_b} - \frac{\sin[\pi(t/T_b) - 2\pi]}{\pi(t-2T_b)/T_b} \\ &= \frac{\sin(\pi t/T_b)}{\pi t/T_b} - \frac{\sin[\pi t/T_b]}{\pi(t-2T_b)/T_b} \\ &= \frac{2T_b^2\sin(\pi t/T_b)}{\pi t(2T_b - t)} \end{aligned} \qquad (5)$$

(c) With the precoder in place at the front end of the modified duobinary conversion filter as shown in Fig. 6.15, we have

$$d_k = b_k \oplus d_{k\text{-}1} \qquad (6)$$

where $b_k$ is the incoming binary sequence and $d_k$ is the precoder output.

Assuming the use of a polar representation for the precoded sequence $d_k$, we find that the original data sequence $b_k$ may be detected from the encoded sequence $c_k$ by disregarding the polarity; specifically,

If $|c_k| > 1$, say symbol $b_k$ is 1
If $|c_k| < 1$, say symbol $b_k$ is 0 (7)

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Problem 6-22 continued

(d) The virtues of modified duobinary coding are two-fold:

- In the absence of channel noise, the detected binary sequence $b_k$ is exactly the same as the original data sequence $b_k$; this statement also applies to the duobinary coding with precoding.

- The use of Eq. (6) requires the addition of two extra bits to the precoded sequence $b_k$ in accordance with Eq. (6). The composition of the decoded sequence $\hat{b}_k$ using Eq. (7) is invariant to the selection made for these two additional bits.

Note: In the first printing of the book, the delay element of the precoder in Fig. 6.15 should read $2T_b$ to be consistent with Eq. 7.