Problem 4.6

Using a well-known trigonometric identity involving the product of the sine of an angle and the cosine of another angle, demonstrate the two results just described under points 1 and 2.

Solution

The incoming FM wave is defined by (see Eq. (4.57))

$$s(t) = A_v \cos[2\pi f_c t + \phi_1(t)] \tag{1}$$

The internally generated output of the VCO is defined by (see Eq. (4.59))

$$r(t) = A_v \cos[2\pi f_c t + \phi_2(t)] \tag{2}$$

Multiplying $s(t)$ by $r(t)$ yields

$$s(t)r(t) = A_c A_v \sin[2\pi f_c t + \phi_1(t)] \cos[2\pi f_c t + \phi_2(t)] \tag{3}$$

Using the trigonometric identity

$$\sin A \cos B = \frac{1}{2}[\sin(A+B) + \sin(A-B)]$$

we may rewrite Eq. (3) as

$$\begin{aligned} s(t)r(t) &= \frac{1}{2} A_c A_v \sin[4\pi f_c t + \phi_1(t) + \phi_2(t)] \\ &\quad + \frac{1}{2} A_c A_v \sin[\phi_1(t) - \phi_2(t)] \end{aligned} \tag{4}$$

Except for a scaling factor, the first term of Eq. (4) defines the double-frequency term (identified under point 1 on page 179) and the second term of the equation defines the difference-frequency term (identified under point 2 of the same page).