Problem 3.15

Derivation of the synthesizer depicted in Fig. 3.25(b) follows directly from Eq. (3.39). However, derivation of the analyzer depicted in Fig. 3.25(a) requires more detailed consideration. Given that $f_c > W$ and

$$\cos^2(2\pi f_c t) = \frac{1}{2}[1 + \cos(4\pi f_c t)]$$

and

$$\sin(2\pi f_c t)\cos(2\pi f_c t) = \frac{1}{2}\sin(4\pi f_c t),$$

show that the analyzer of Fig. 3.25(a) yields $s_I(t)$ and $s_Q(t)$ as its two outputs.

Solution

Consider first the upper channel in Fig. 3.25(a). Multiplying (see Eq. (3.39))

$$s(t) = s_I(t)\cos(2\pi f_c t) - s_Q\sin(2\pi f_c t)$$

by the carrier $2\cos(2\pi f_c t)$, we get

$$\begin{aligned} v_1(t) &= 2s(t)\cos(2\pi f_c t) \\ &= 2s_I(t)\cos^2(2\pi f_c t) - 2s_Q(t)\sin(2\pi f_c t)\cos(2\pi f_c t) \\ &= s_I(t)[1 + \cos(4\pi f_c t)] - s_Q(t)\sin(4\pi f_c t) \\ &= s_I(t) + s'(t) \end{aligned}$$

where

$$s'(t) = s_I(t)\cos(4\pi f_c t) - s_Q(t)\sin(4\pi f_c t)$$

represents a new linearly modulated signal with carrier frequency $2f_c$. Provided that both $s_I(t)$ and $s_Q(t)$ are limited to the band $-W \le f \le W$ and we pass $v_1(t)$ through a low-pass filter of cutoff frequency W as in Fig. 3.25(a), then $s'(t)$ is rejected provided that $f_c > W$.

Consider next the lower channel in Fig. 3.25(a). Multiplying $s(t)$ by $-2\sin(2\pi f_c t)$, we get

$$\begin{aligned} v_2(t) &= -2s(t)\sin(2\pi f_c t) \\ &= -2s_I(t)\sin(2\pi f_c t)\cos(2\pi f_c t) + 2s_Q(t)\sin^2(2\pi f_c t) \\ &= -s_I(t)\sin(4\pi f_c t) + [1 - \cos(4\pi f_c t)]s_Q(t) \\ &= s_Q(t) - s''(t) \end{aligned}$$

where

$$s''(t) = s_I(t)\sin(4\pi f_c t) + s_Q(t)\cos(4\pi f_c t)$$

is a new linearly modulated signal with carrier frequency $2f_c$. Hence, passing $v_2(t)$ through a low-pass filter as in Fig. 3.25(a), $s''(t)$ is rejected again provided that the cutoff frequency W of the low-pass filter satisfies the condition $f_c > W$.