Tutorial-2

1. The low-loss condition

$$\frac{\sigma}{\omega\epsilon} = \frac{10^{-6}}{2\pi \times 30\times10^{9} \times 8.854\times1.05\times10^{-12}} = 5.7\times10^{-7} << 1$$

for $\epsilon_r = 4$

$$\frac{\sigma}{\omega\epsilon} = 1.4962 \times 10^{-7} << 1.$$

(a) phase velocity in free space. $= c = 3\times10^{8}$ m/s.

phase velocity in atmosphere for $(\epsilon_r = 1.05)$ $= U_p = \frac{c}{\sqrt{1.05}}$
$= 2.928\times10^{8}$ m/s.

for $(\epsilon_r = 4) = \frac{c}{\sqrt{4}} = 1.5\times10^{8}$ m/s.

% change in phase velocity for $\epsilon_r = 1.05 \Rightarrow 2.4\%$
for $\epsilon_r = 4 \Rightarrow 50\%$

(b) $$\alpha \approx \frac{\sigma}{2}\sqrt{\frac{\mu_0}{\epsilon_0\epsilon_r}} = \frac{10^{-6}\times 377}{2\times\sqrt{1.05}} = 1.84\times10^{-4}\ \frac{NP}{m}$$

$$\alpha\Big|_{\epsilon_r=4} = 0.942\times10^{-4}\ NP/m.$$

$$\beta \approx \omega\sqrt{\mu\epsilon} = \omega\sqrt{\mu_0\epsilon_0\epsilon_r} = \omega\frac{\sqrt{\epsilon_r}}{c} = \frac{2\pi\times30\times10^{9}}{3\times10^{8}}\times\sqrt{1.05}$$
$$= 643.83 \text{ rad/m.}$$

$$\beta\Big|_{\epsilon_r=4.} = 1256.62 \text{ rad/m.}$$

for free space. $\alpha = 0.$

$$\beta_0 = \omega\sqrt{\mu_0\epsilon_0} = 628.32 \text{ rad/m.}$$

(c) $\gamma = \alpha + j\beta$.

$= 1.84 \times 10^{-4} + j\,643.83$

for free space $\gamma = j\beta_0 = j\,628.32$

(d) $\eta_0 = 377$, $\eta_n = \eta_0/\sqrt{1.05} = 367.914\ \Omega$

$\eta_n|_{\epsilon_r=4} = \eta_0/\sqrt{4} = 188.5\ \Omega$

$$\eta = \eta_n\left(1 + \frac{j\sigma}{2\omega\epsilon}\right) = 367.9 + j1.05\times10^{-4}\ \Omega.$$

$$\eta|_{\epsilon_r=4} = \eta_n\left(1 + \frac{j\sigma}{2\omega\epsilon}\right) = 188.5 + j1.41\times10^{-5}\ \Omega.$$

(e) $E = E_0 e^{-\alpha d}$

$\Rightarrow 10\times10^{-3} = E_0\, e^{-1.84\times10^{-4}\times30000} = 0.004E_0$.

$\Rightarrow E_0 = 2.5$ V/m.

2. E_y component leads the E_x component by $30° = \pi/6$ rad.

The two components of the electric field can be written as.

$$E_x = 5\cos(\omega t)$$
$$E_y = 10\cos(\omega t + \pi/6)$$

cosine funcⁿ for E_x is chosen since at $t=0$, the E_x field is maximum.

At $t=0$ $E_x = 5$

$E_y = 10\cos\pi/6 = 10\sqrt{3}/2$.

$|E| = \sqrt{E_x^2 + E_y^2} = \sqrt{25+75} = 10$ V/m.

$\theta = \tan^{-1}\left(\frac{E_y}{E_x}\right) = 60°$

at $t = 0.1$ nsec

$\omega t = 2\pi \times 2 \times 10^{9} \times 0.1 \times 10^{-9} = 0.4\pi$ rad.

$E_x = 5\cos(0.4\pi) = 1.545$ V/m.

$E_y = 10\cos(0.4\pi + \pi/6) = -2.078$ V/m.

$|E| = \sqrt{E_x^2 + E_y^2} = 2.59$ V/m.

$\theta = \tan^{-1}(E_y/E_x) = -53.37^\circ$

$E = 5\cos\omega t\ \hat{x} + 10\cos(\omega t + \pi/6)\ \hat{y}$.

$\phi = \pi/6 \Rightarrow$ linear polarization.

phase difference b/w x and y component of electric field

3. Power Transmitted $= 50$ KW.

operating freq. $= 10$ GHz.

σ = (given) $= 10^{-7}$ S/m.

attenuation in the medium $= \frac{\sigma}{2}\sqrt{\frac{\mu_0}{\epsilon_0}}$

$= \frac{10^{-7}}{2} \times 120\pi = 1.885 \times 10^{-5}$ NP/m

Distance b/w Radar antenna and airplane $= 100$ km.

The received power by the radar antenna will undergo bi-directional attenuation i.e $2\alpha = \alpha_d$.

$\Rightarrow P_r = P_t\, e^{-2(\alpha_d)d}$

$= P_t\, e^{-2 \times 2\alpha \cdot d} = 50k \times e^{-4 \times 1.885 \times 10^{-5} \times 100 \times 1000}$

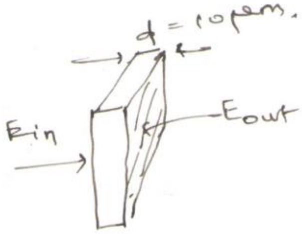
$= 26.569$ W

Actual Received power by the antenna

$= 1\% \times 26.569$

$= 0.265$ W.

4. Microwave operating freq. = 2450 MHz.



$P_{avg} = 1000\,W$.

copper foil has area = $1\,m^2$

thickness = $10\,\mu m$.

Avg. power density incident on the copper foil = $1000\ W/m^2$

⇒ incident electric field ~~1000~~ will be

$$1000 = \frac{E_{in}^2}{2\eta_0}$$

$$\Rightarrow E_{in} = \sqrt{1000 \times 2 \times 120 \times \pi} = \cancel{17.366}\ 868.32\ V/m.$$

Attenuation in the copper foil $= \alpha = \sqrt{\pi f \mu_0 \sigma}$

$$= \left(\pi \times 2450 \times 10^6 \times 4\pi \times 10^{-7} \times 5.7 \times 10^7\right)^{1/2}$$

$$= 742506.634$$

It is given that only 2% of the incident ~~electric~~ electric field enters the foil.

⇒ the amplitude of the electric field intensity just below the other surface of the foil will be

$$E_{out} = 2\%\ E_{in}\, e^{-\alpha d}$$

$$= (0.02) \times \cancel{17.3}\ 868.32 \times e^{-742506.634 \times 10 \times 10^{-6}}$$

$$= 1.03 \times 10^{-2}\ V/m.$$

5. $E = 0$ for $z < 0$ since it is the conductor region.
E for $z > 0$ lies entirely in $\hat{z}$ hence $E_{normal} = E$.
surface charge density hence is

$$\rho_s = \epsilon_0 \epsilon_r E_{normal}$$

$$\rho_s = \epsilon_0 5 \left[10 \cos(3 \times 10^8 t - 10x) \right].$$

At $x = 2m$ & $t = 0.5 \times 10^{-9} s$

$$\rho_s = 2.387 \times 10^{-10} \; c/m^2.$$

Since $\vec{J_s} = \hat{n} \times \vec{H}$, we need $\vec{H}$

so $\vec{\nabla} \times \vec{E} = \dfrac{-\partial \vec{B}}{\partial t} = -\mu_0 \mu_r \dfrac{\partial \vec{H}}{\partial t}$

$$\vec{H} = \frac{-1}{\mu_0 \mu_r} \int \vec{\nabla} \times \vec{E} \, dt$$

on solving, we get.

$$\vec{H} = \frac{-100}{\mu_0 \mu_r} \frac{\cos(3 \times 10^8 t - 10x)}{3 \times 10^8} \hat{y}.$$

$$\vec{J_s} = |\vec{H}| \hat{x}$$

And at $x = 2m$ & $t = 0.5 ns$.

$|\vec{J_s}| = 0.0072 \; A/m$. along $\hat{x}$

6. consider

$$\vec{E_1} = E_0 (\hat{x} - j\hat{y}) e^{-j\beta z}$$

$$\vec{E_2} = E_0 (\hat{x} + j\hat{y}) e^{-j\beta z} \cdot e^{j\sigma}$$

when added this gives

$$\vec{E_t} = 2E_0 \left[\cos(\sigma/2) \hat{x} + \sin(\sigma/2) \hat{y} \right] e^{-j(\beta z - \sigma/2)}.$$

linear polarized wave.