

Problem 2.25

Express $y(t)$ as

$$\begin{aligned} y(t) &= x^2(t) \\ &= x(t)x(t) \end{aligned}$$

Since multiplication in the time domain corresponds to convolution in the frequency domain, we may express the Fourier transform of $y(t)$ as

$$Y(f) = \int_{-\infty}^{\infty} X(\lambda)X(f - \lambda)d\lambda$$

where $X(f)$ is the Fourier transform of $x(t)$. However, $X(f)$ is zero for $|f| > W$. Therefore,

$$Y(f) = \int_{-W}^W X(\lambda)X(f - \lambda)d\lambda$$

In this integral we note that $X(f - \lambda)$ is limited to $-W \leq f - \lambda \leq W$. When $\lambda = -W$, we find that $-2W \leq f \leq 0$. When $\lambda = W$, we find that $0 \leq f \leq 2W$. Accordingly, the Fourier transform $Y(f)$ is limited to the frequency interval $-2W \leq f \leq 2W$.