Problem 4.2

Suppose that the linear modulating wave

$$m(t) = \begin{cases} at & \text{for } t \geq 0 \\ 0 & \text{for } t < 0 \end{cases}$$

is applied to the scheme shown in Fig. 4.3(a). The phase modulator is defined by Eq. (4.4). Show that if the resulting FM wave is to have exactly the form as that defined in Eq. (4.7), then the phase-sensitivity factor k_p of the phase modulator is related to the frequency sensitivity factor k_f in Eq. (4.7) by the formula

$$k_p = 2\pi k_f T$$

where T is the interval over which the integration in Fig. 4.3(a) is performed. Justify the dimensionality of this expression.

Solution

According to Fig. 4.3(a), the FM wave is defined by

$$s(t) = A_c \cos\left[2\pi f_c t + \frac{1}{T} k_p \int_0^t m(\tau)d\tau\right] \tag{1}$$

where T is an integration constant.

According to Eq. (4.7), the FM wave is defined by

$$s(t) = A_c \cos\left[2\pi f_c t + 2\pi k_f \int_0^t m(\tau)d\tau\right] \tag{2}$$

If Eqs. (1) and (2) are to be identical, then we require that

$$k_p = 2\pi k_f T$$

Dimensionality of this expression is justified as follows:

1. k_f is measured in hertz per volt. Therefore, $2\pi k_f T$ has the dimensions of cycles, per volt and therefore radians per volt.
2. k_p is itself measured in radians per volt.