Problem 4.3

The *Cartesian baseband representation* of band-pass signals discussed in Section 3.8.1 is well-suited for linear modulation schemes exemplified by the amplitude modulation family. On the other hand, the *polar baseband representation*

$$s(t) = a(t)\cos[2\pi f_c t + \phi(t)]$$

is well-suited for nonlinear modulation schemes exemplified by the angle modulation family. The $a(t)$ in this new representation is the envelope of $s(t)$ and $\phi(t)$ is its phase.

Starting with the baseband representation [see Eq. (3.39)]

$$s(t) = s_I(t)\cos 2\pi f_c t - s_Q(t)\sin(2\pi f_c t)$$

where $s_I(t)$ is the in-phase component and $s_Q(t)$ is the quadrature component, we may write

$$a(t) = [s_I^2(t) + s_Q^2(t)]^{1/2}$$

and

$$\phi(t) = \tan^{-1}\left[\frac{s_Q(t)}{s_I(t)}\right]$$

Show that the polar representation of $s(t)$ in terms of $a(t)$ and $\phi(t)$ is exactly equivalent to its Cartesian representation in terms of $s_I(t)$ and $s_Q(t)$.

Solution

We are given

$$a(t) = [s_I^2(t) + s_Q^2(t)]^{1/2}$$

and

$$\phi(t) = \tan^{-1}\left[\frac{s_Q(t)}{s_I(t)}\right]$$

Hence, expanding the polar representation of $s(t)$, we write

$$\begin{aligned} s(t) &= a(t)\cos[\theta t] \\ &= a(t)\cos[2\pi f_c t + \phi(t)] \end{aligned}$$

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Problem 4-3 continued

$$= a(t)\cos(\phi(t))\cos(2\pi f_c t) - a(t)\sin(\phi(t))\sin(2\pi f_c t) \quad (1)$$

Since $\tan[\phi(t)] = \left[\frac{s_Q(t)}{s_I(t)}\right]$, it follows that

$$\cos\phi(t) = \frac{s_I(t)}{[s_I^2(t) + s_Q^2(t)]^{1/2}} = \frac{s_I(t)}{a(t)}$$

and

$$\sin\phi(t) = \frac{s_Q(t)}{[s_I^2(t) + s_Q^2(t)]^{1/2}} = \frac{s_Q(t)}{a(t)}$$

Hence,

$$a(t)\cos\phi(t) = s_I(t) \quad (2)$$

and

$$a(t)\sin\phi(t) = s_Q(t) \quad (3)$$

Substituting Eqs. (2) and (3) into (1), we get

$$s(t) = s_I(t)\cos(2\pi f_c t) - s_Q(t)\sin(2\pi f_c t)$$

which is the Cartesian representation of $s(t)$.