Problem 4.9

The instantaneous frequency of the mixer output is as shown in Fig. 1:
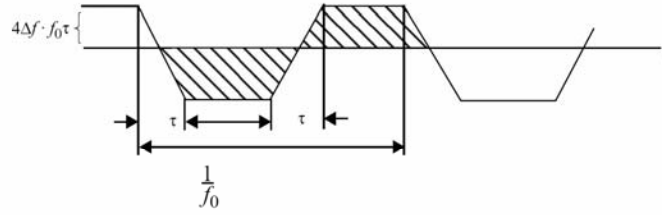

Figure 1

The presence of negative frequency merely indicates that the phasor representing the difference frequency at the mixer output has reversed its direction of rotation.

Let N denote the number of beat cycles in one period. Then, noting that N is equal to the number of shaded areas shown in Fig. 1, we deduce that

$$N = 2\left[4\Delta f \cdot f_0\tau\left(\frac{1}{2f_0} - \tau\right) + 2\Delta f \cdot f_0\tau^2\right]$$

$$= 4\Delta f \cdot \tau(1 - f_0\tau)$$

Since $f_0\tau \ll 1$, we have the approximate result

$$N \approx 4\Delta f \cdot \tau$$

Therefore, the number of beat cycles counted over one second is equal to

$$\frac{N}{1/f_0} = 4\Delta f \cdot f_0\tau.$$