1. $E_\theta = j\eta \frac{\beta I_o dl}{4\pi r} \sin\theta \left[1 - \frac{j}{\beta r} - \frac{1}{(\beta r)^2}\right] e^{-j\beta r}$

In the above expression

$E_\theta^R = j\eta \frac{\beta I_o dl}{4\pi r} \sin\theta [1] e^{-j\beta r}$ is Radiation field.

$E_\theta^i = j\eta \frac{\beta I_o dl}{4\pi r} \sin\theta \left[-\frac{j}{\beta r}\right]$ is Induction field

$E_\theta^c = j\eta \frac{\beta I_o dl}{4\pi r} \sin\theta \left[-\frac{1}{(\beta r)^2}\right]$ is ~~electrical fi~~ electrostatic field.

Magnitudes of the above three fields are equal for

$$1 = \frac{1}{\beta r} = \frac{1}{\beta r} \Rightarrow \boxed{\beta r = 1}.$$

2. Power radiated $= 40\pi^2 I_o^2 \left(\frac{dl}{\lambda}\right)^2$

$$1 \times 10^3 = 40\pi^2 I_o^2 \left(\frac{dl}{\lambda}\right)^2$$

$$\Rightarrow I_o \frac{dl}{\lambda} = \left(\frac{1\times 10^3}{40\pi^2}\right)^{1/2} = 5/\pi$$

E_θ in the farfield region is

$$E_\theta = j\eta \frac{\beta I_o dl}{4\pi r} \sin\theta\, e^{-j\beta r}$$

$$E_\theta = j\eta \frac{\cancel{2\pi}}{\lambda} (I_o dl) \frac{1}{\cancel{4\pi} r} \sin\theta\, e^{-j\beta r}$$

here $\theta = 90^\circ$ then

$$|E_\theta| = j\eta \left(\frac{I_o dl}{\lambda}\right) \frac{1}{2r} = \overset{3}{\cancel{120}}\pi \left(\frac{5}{\cancel{\pi}}\right)\left(\frac{1}{\cancel{2}\times 20\times 10^3}\right)$$

$$|E_\theta| = 15\times 10^{-3}\ V/m.$$

The Poynting vector is

$$P = \frac{|E_\theta|^2}{\eta} = \frac{\cancel{30\times}10\,(15\times10^{-3})^2}{120\pi}$$

$$= 5.96\times10^{-7}\ \mathrm{W/m^2}$$

E field is vertical since $\theta = 90^\circ$.

3. Radiation resistance (R_{rad}) $80\pi^2\left(\frac{dl}{\lambda}\right)^2$

here $dl = 0.1\lambda$

$$R_{rad} = 80\pi^2\left(\frac{0.1\lambda}{\lambda}\right)^2 = 0.8\pi^2$$

$$\text{Power radiated} = \frac{1}{2}(10)^2(0.8\pi^2)$$

$$= 394.78\ \mathrm{W}.$$

4. Directivity is $\frac{U_{max}}{U_{avg}}$

$$= \frac{4\pi E_{max}^2}{\int_{\theta=0}^{\pi/2}\int_{\phi=0}^{2\pi}|E(\theta,\phi)|^2\sin\theta\, d\theta\, d\phi}$$

$$\cancel{= 4\pi\left(\frac{1}{4}\right)\cos}$$

$$E = \frac{1}{2}\cos^4\theta$$

$$E_{max} = \frac{1}{2},\quad (E_{max})^2 = \frac{1}{4}$$

$$\int_0^{\pi}\int_0^{2\pi}\frac{1}{4}\cos^8\theta\sin\theta\, d\theta\, d\phi = \frac{1}{4}(2\pi)\left[\frac{-\cos^9\theta}{9}\right]_0^{\pi/2}$$

$$= -\frac{1}{\cancel{4}2}(\cancel{2}\pi)\left(\frac{-1}{9}\right)$$

$$U_{avg} = \frac{\pi}{2\times9} = \frac{\pi}{18}.$$

$$\therefore D = \frac{4\cancel{\pi}\left(\frac{1}{\cancel{4}}\right)}{\cancel{\pi}/18} = 18.$$

5. We know that

$$E_\theta = j\eta \frac{\beta I_o dl}{4\pi r} \sin\theta \left[1 - \frac{j}{\beta r} - \frac{1}{(\beta r)^2}\right].$$

$$H_\phi = j \frac{\beta I_o dl}{4\pi r} \sin\theta \left[1 - \frac{j}{\beta r}\right].$$

Impedance of space $(Z_s) = \frac{E_\theta}{H_\phi} = \eta \frac{1 - \frac{j}{\beta r} - \frac{1}{(\beta r)^2}}{1 - \frac{j}{\beta r}}$

if $\beta r << 1$, then

$$Z_s = \eta \left(\frac{\frac{1}{(\beta r)^2} + \frac{j}{\beta r}}{\frac{j}{\beta r}} \right)$$ high imaginary value

if $\beta r >> 1$ then

$$Z_s \to \eta.$$