

### Problem 4.8

For the PM case, we have by definition

$$s(t) = A_c \cos[2\pi f_c t + k_p m(t)].$$

whose angle is

$$\theta_i(t) = 2\pi f_c t + k_p m(t).$$

The instantaneous frequency is therefore

$$\begin{aligned} f_i(t) &= \frac{1}{2\pi} \frac{d\theta_i(t)}{dt}, \\ &= f_c + \frac{k_p}{2\pi} \frac{dm(t)}{dt} \\ &= f_c + \frac{Ak_p}{2\pi T_0} - \frac{Ak_p}{2\pi} \sum_n \delta(t - nT_0) \end{aligned} \quad (1)$$

which is equal to  $f_c + Ak_p/2\pi T_0$  except for the instants that the message signal has discontinuities. At these instants, the phase shifts by  $-k_p A/T_0$  radians. Accordingly, the PM wave has the waveform depicted in Fig. 1

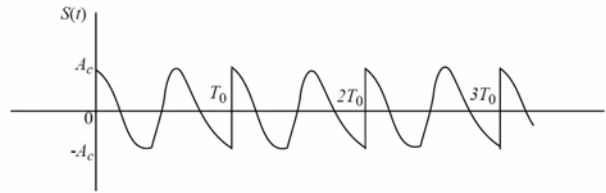


Figure 1

For the FM case, we have

$$f_i(t) = f_c + k_f m(t)$$

and the modulated wave is defined by

$$s(t) = A_c \cos\left[2\pi f_c t + 2\pi k_f \int_0^t m(\tau) d\tau\right]$$

The modulated wave is therefore depicted in Fig. 2.

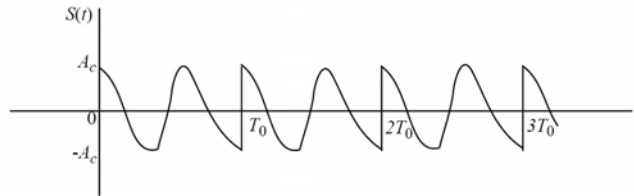


Figure 2