Problem 7.3

Although QPSK and OQPSK signals have different waveforms, their magnitude spectra are identical; but their phase spectra differ by a nonlinear phase component. Justify the validity of this two-fold statement.

Solution

In QPSK, the modulated signal is defined by (see Eq. (7115))

$$s_{\text{QPSK}}(t) = \sqrt{\frac{2E}{T}}\cos\left[(2i-1)\frac{\pi}{4}\right]\cos(2\pi f_c t) - \sqrt{\frac{2E}{T}}\sin\left[(2i-1)\frac{\pi}{4}\right]\sin(2\pi f_c t) \qquad (1)$$

where $0 \le t \le T$ the index $i = 1,2,3,4$, depending on which particular dibit is sent. For a specific index i, the in-phase component of $S_{\text{QPSK}}(t)$ is therefore

$$s_{I,\,\text{QPSK}}(t) = \sqrt{\frac{2E}{T}}\cos\left[(2i-1)\frac{\pi}{4}\right], \qquad 0 \le t \le T \qquad (2a)$$

and its quadrature component is

$$s_{Q,\,\text{QPSK}}(t) = \sqrt{\frac{2E}{T}}\sin\left[(2i-1)\frac{\pi}{4}\right], \qquad 0 \le t \le T \qquad (2b)$$

In OQPSK, the in-phase component is left intact but the quadrature component is delayed by $T/2$ (half symbol period). Accordingly, for the same index i in QPSK, we may express the in-phase component of OQPSK as

$$s_{I,\,\text{OQPSK}}(t) = \sqrt{\frac{2E}{T}}\cos\left[(2i-1)\frac{\pi}{4}\right], \qquad 0 \le t \le T \qquad (3a)$$

and its quadrature component as

$$s_{Q,\,\text{OQPSK}}(t) = \sqrt{\frac{2E}{T}}\sin\left[(2i-1)\frac{\pi}{4}\right], \qquad \frac{T}{2} \le t \le \frac{3}{2}T \qquad (3b)$$

Let $b_I(t)$ denote a rectangular pulse of duration T, representing the in-phase component of the QPSK signal and $b_Q(t)$ denote the corresponding quadrature component. Then, in light of Eqs. (2) and (3), we may express the complex envelope of QPSK as

$$\tilde{s}_{\text{QPSK}}(t) = b_I(t) + jb_Q(t), \qquad 0 \le t \le T \qquad (4)$$

and

$$\tilde{s}_{\text{OQPSK}}(t) = b_I(t) + jb_Q\left(t - \frac{T}{2}\right), \qquad 0 \le t \le T \qquad (5)$$

Applying the Fourier transform to Eqs. (4) and (5), we correspondingly have

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Problem 7-3 continued

$$\tilde{s}_{\text{QPSK}}(f) = B_I(f) + jB_Q(f) \tag{6}$$

and

$$\begin{aligned}\tilde{s}_{\text{OQPSK}}(f) &= B_I(f) + jB_Q(f)\exp(-j\pi f\tau)\\ &= B_I(f) + jB_Q(f)[\cos(\pi f\tau) j\sin(\pi f\tau)]\\ &-[B_I(f)B_Q(f)\sin(\pi f\tau)] + jB_Q(f)\cos(\pi f\tau)\end{aligned} \tag{7}$$

From Eqs. (6) and (7), it therefore follows that for the QPSK

$$|\tilde{s}_{\text{QPSK}}(f)|^2 = B_I^2(f) + B_Q^2(f) \tag{8a}$$

and

$$\arg[\tilde{s}_{\text{QPSK}}(f)] = \tan^{-1}\left(\frac{B_Q(f)}{B_I(f)}\right) \tag{8b}$$

Similarly, for the OQPSK

$$\begin{aligned}\tilde{s}_{\text{OQPSK}}(f)|^2 &= [B_I(f) \ - \ B_Q\sin(\pi f\tau)]^2 + [B_Q(f)\cos(\pi fT)]^2\\ &= B_I^2(f) + B_Q^2(f) - 2B_I(f)B_Q(f)\sin(\pi fT)\end{aligned} \tag{9a}$$

and

$$\arg[\tilde{s}_{\text{OQPSK}}(f)] = \tan^{-1}\left[\frac{B_Q(f)\cos(\pi fT)}{B_I(f) - B_Q\sin(\pi fT)}\right] \tag{9b}$$

For a square wave input, we typically find that the cross-product term $2B_I(f)B_Q(f)\sin(\pi fT)$ is small compared to the composite term $B_I^2(f) + B_Q^2(f)$. Accordingly, from Eqs. (8a) and (9a), it follows that for all practical purposes, the magnitude spectra $|S_{\text{QPSK}}(f)|$ and $|S_{\text{OQPSK}}(f)|$ are identical. In direct contrast, however, from Eqs. (8b) and (9b), we find that the corresponding phase spectra are not only different but the difference between them is a nonlinear function of frequency f.

Note: In the problem statement, the following correction should be made:
The term "linear phase component" is replaced by "nonlinear phase component".