Problem 3.24

The noncoherent carrier is

$$c(t) = A_c \cos(2\pi f_c t + \phi)$$

and the DSB-SC modulated wave is $m(t)\cos(2\pi f_c t)$. The composite signal is therefore

$$\begin{aligned} s(t) &= A_c \cos(2\pi f_c t + \phi) + m(t)\cos(2\pi f_c t) \\ &= [A_c \cos\phi + m(t)]\cos(2\pi f_c t) - A_c \sin\phi \sin(2\pi f_c t) \\ &= s_I(t)\cos(2\pi f_c t) - s_Q(t)\sin(2\pi f_c t) \end{aligned}$$

where

$$s_I(t) = A_c \cos\phi + m(t)$$

$$s_Q(t) = A_c \sin\phi$$

Applying the composite signal $s(t)$ to an ideal envelope detector produces the output

$$\begin{aligned} a(t) &= [s_I^2(t) + s_Q^2(t)]^{1/2} \\ &= [(A_c \cos\phi + m(t))^2] + (A_c \sin\phi)^{2^{1/2}} \\ &= [A_c^2 \cos^2\phi + 2A_c \cos\phi m(t) + m^2(t) + A_c^2 \sin^2\phi]^{1/2} \end{aligned}$$

$$= [A_c^2 + 2A_c \cos\phi m(t) + m^2(t)]^{1/2} \tag{1}$$

(a) For $\phi = 0$, Eq. (1) reduces to

$$a(t) = [A_c^2 + 2A_c m(t) + m^2(t)]^{1/2}$$

$$= A_c + m(t) \tag{2}$$

which consists of the message signal $m(t)$ plus a dc bias equal to the carrier amplitude.

(b) For $\phi \neq 0$ and $|m(t)| << A_c/2$, we may approximate Eq. (1) as follows:

$$a(t) \approx [A_c^2 + 2A_c \cos\phi m(t)]^{1/2}$$

$$= A_c\left[1 + \frac{2}{A_c}\cos\phi m(t)\right]^{1/2} \tag{3}$$

With $|\cos\phi| \le 1$, and $|m(t)| << A_c/2$, we may approximate Eq. (3) further as

$$a(t) \approx A_c\left[1 + \frac{1}{A_c}\cos\phi m(t)\right]$$

$$= A_c + \cos\phi m(t) \tag{4}$$

When ϕ is close to zero, the detector output in Eq. (4) is very close to the value defined in Eq. (2). However, when ϕ approaches 90^o, $\cos\phi$ approach zero, then the envelope detector output in Eq. (4) reduces to a dc component equal to A_c with no significant trace of the message signal $m(t)$ being visible. If therefore the phase error ϕ is variable, then the envelope detector output a(t) varies in a corresponding way, which could be undesirable.