Problem 2.3

Suppose $g(t)$ is real valued with a complex-valued Fourier transform $G(f)$. Explain how the rule of Eq. (2.31) can be satisfied by such a signal.

Solution

With $G(f)$ being complex valued, we may express it as

$$G(f) = G_r(f) + jG_i(f)$$

where $G_r(f)$ is the real part of $G(f)$ and $G_i(f)$ is its imaginary part. Hence,

$$G(0) = G_r(0) + jG_i(0).$$

According to Eq. (2.31) in the text,

$$\int_{-\infty}^{\infty} g(t)dt = G_r(0) + jG_i(0)$$

With $g(t)$ being real valued, this condition can only be satisfied if the imaginary part $G_i(0)$ is zero.