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If $\bar{E} = K\hat{x}$

$$\bar{\nabla}\times\bar{E} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ K & 0 & 0 \end{vmatrix} = 0$$

$$\Rightarrow \cancel{B_x = B_y = B_z = 0}$$

$$\Rightarrow H_x = H_y = H_z = 0$$

General solution of plane waves

$$\nabla^2\bar{E} + k^2\bar{E} = \frac{\partial}{\partial x^2}\bar{E} + \frac{\partial}{\partial y^2}E + \frac{\partial}{\partial z^2}\bar{E} + k^2\bar{E} = 0$$

$\Rightarrow$ We can obtain 3 equation

$$\frac{\partial^2 E_x}{\partial x^2} + \frac{\partial^2 E_x}{\partial y^2} + \frac{\partial^2 E_x}{\partial z^2} + k^2 E_x = 0 \quad -(1)$$

$$\frac{\partial^2 E_y}{\partial x^2} + \frac{\partial^2 E_y}{\partial y^2} + \frac{\partial^2 E_y}{\partial z^2} + k^2 E_y = 0$$

$$\frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} + \frac{\partial^2 E_z}{\partial z^2} + k^2 E_z = 0$$

Let $E_x(x,y,z) = f(x)\,g(y)\,h(z)$

Starting from eqn (1)

$$\frac{\partial^2 (fgh)}{\partial x^2} + \frac{\partial^2 (fgh)}{\partial y^2} + \frac{\partial^2 (fgh)}{\partial z^2} + k^2 E_x = 0$$

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$$\Rightarrow gh\frac{\partial^2 f}{\partial x^2} + fh\frac{\partial^2 g}{\partial y^2} + fg\frac{\partial^2 h}{\partial z^2} + k^2 \cancel{E_x = 0} fgh = 0$$

Dividing both sides by (fgh),

$$\Rightarrow f''/f + \frac{g''}{g} + \frac{h''}{h} + k^2 = 0$$

Each of f''/f, g''/g & h''/h are only fns of x, y & z respectively

Hence each should be a negative constant

Let $f''/f = -k_x^2$, $\frac{g''}{g} = -k_y^2$, $h''/h = -k_z^2$

$$\therefore k_x^2 + k_y^2 + k_z^2 = k^2.$$

Starting from,

$$\frac{f''}{f} = -k_x^2$$

We get

$$f'' + k_x^2 f = 0$$

$$\Rightarrow f(x) = \underbrace{f^+ e^{-ik_x x}}_{+\text{ve } z \text{ travelling wave}} + \underbrace{f^- e^{+ik_x x}}_{-\text{ve } z \text{ travelling wave}}$$

Similarly,

$$g(y) = g^+ e^{-ik_y y} + g^- e^{+ik_y y}$$

$$h(z) = h^+ e^{-ik_z z} + h^- e^{+ik_z z}$$

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If we only consider the +ve travelling wave

then,

$$E_x(x,y,z) = f^+(x)\, g^+(y)\, h^+(z)$$

$$= A\, e^{-i(k_x x + k_y y + k_z z)}$$

A → constant

Let us define a vector $\bar{k}$ as,

$$\bar{k} = k_x \hat{x} + k_y \hat{y} + k_z \hat{z}$$ → wavenumber vector

↓ points to the direction of propagation of wave

$\bar{k} = k\hat{n}$ → unit vector in direction of propagation

~~Let us d~~ The position vector,

$$\bar{r} = x\hat{x} + y\hat{y} + z\hat{z}$$

$$\therefore k_x x + k_y y + k_z z = \bar{k}\cdot\bar{r}$$

$$\therefore E_x(x,y,z) = A\, e^{-i(\bar{k}\cdot\bar{r})}$$

Similarly

$$E_y(x,y,z) = B\, e^{-i(\bar{k}\cdot\bar{r})}$$

$$E_z(x,y,z) = C\, e^{-i(\bar{k}\cdot\bar{r})}$$

$$\therefore \bar{E} = \hat{x}E_x + \hat{y}E_y + \hat{z}E_z$$

$$= \underbrace{(A\hat{x} + B\hat{y} + C\hat{z})}_{= \bar{E}_0}\, e^{-i(\bar{k}\cdot\bar{r})}$$

$$= \bar{E}_0\, e^{-i(\bar{k}\cdot\bar{r})}$$

$\bar{E}_0$ → constant vector

Now,

$$\bar{\nabla}\cdot\bar{E} = 0 \quad \text{(in free space)}$$

Identity $\bar{\nabla}\cdot(f\bar{A})$

$$= f(\bar{\nabla}\cdot\bar{A}) + \bar{A}\cdot(\bar{\nabla}f)$$

$$\therefore \bar{\nabla}\cdot E = e^{-i(\bar{k}\cdot\bar{r})}\underbrace{(\bar{\nabla}\cdot\bar{E}_0)}_{\text{constant hence } = 0} + \bar{E}_0\cdot\left(\bar{\nabla}e^{-i\bar{k}\cdot\bar{r}}\right)$$

$$= \bar{E}_0\cdot\left[\frac{\partial}{\partial x}e^{-i(k_x x+k_y y+k_z z)}\hat{x} + \frac{\partial}{\partial y}e^{-i(k_x x+k_y y+k_z z)}\hat{y} + \frac{\partial}{\partial z}e^{-i(k_x x+k_y y+k_z z)}\hat{z}\right]$$

$$= \bar{E}_0\cdot\left[-ik_x e^{-i\bar{k}\cdot\bar{r}}\hat{x} - ik_y e^{-i\bar{k}\cdot\bar{r}}\hat{y} - ik_z e^{-i\bar{k}\cdot\bar{r}}\hat{z}\right]$$

$$= -ie^{-i(\bar{k}\cdot\bar{r})}\bar{E}_0\cdot(\bar{k}).$$

$\because\ \bar{\nabla}\cdot\bar{E} = 0$

$$\Rightarrow -ie^{-i(\bar{k}\cdot\bar{r})}(\bar{E}_0\cdot\bar{k}) = 0$$

$$\Rightarrow \bar{E}_0\cdot\bar{k} = 0$$

$\therefore\ \bar{E}_0$ is $\perp$ to k

from which we have.

$$k_x A + k_y B + k_z C = 0$$

Hence only 2 of (A, B, C) can be chosen independently

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what about $\bar{H}$

$$(\bar{\nabla} \times \bar{E}) = -j\omega\mu\bar{H}$$

$$\Rightarrow \bar{H} = \frac{j}{\omega\mu}(\bar{\nabla} \times \bar{E})$$

$$= \frac{j}{\omega\mu}\left[\bar{\nabla} \times \bar{E}_0 e^{-j\bar{k}\cdot\bar{r}}\right]$$

Identity, $\bar{\nabla} \times (f\bar{A})$

$$= f(\bar{\nabla} \times \bar{A}) + \bar{\nabla} f \times \bar{A}$$

$$\therefore \bar{\nabla} \times \bar{E}_0 e^{-j\bar{k}\cdot\bar{r}}$$

$$= e^{-j\bar{k}\cdot\bar{r}} \underbrace{(\bar{\nabla} \times \bar{E}_0)}_{= 0 \text{ (constant)}} + \bar{\nabla}\left(e^{-j\bar{k}\cdot\bar{r}}\right) \times \bar{E}_0$$

$$\therefore \bar{H} = \frac{j}{\omega\mu}\left[\bar{\nabla}\left(e^{-j\bar{k}\cdot\bar{r}}\right) \times \bar{E}_0\right]$$

$$= \frac{j}{\omega\mu}\left[\frac{\partial}{\partial x}\left(e^{-j\bar{k}\cdot\bar{r}}\right)\hat{x} + \frac{\partial}{\partial y}\left(e^{-j\bar{k}\cdot\bar{r}}\right)\hat{y} + \frac{\partial}{\partial z}\left(e^{-j\bar{k}\cdot\bar{r}}\right)\hat{z}\right] \times \bar{E}_0$$

$$= \frac{j}{\omega\mu}\left[-jk_x e^{-j\bar{k}\cdot\bar{r}}\hat{x} - jk_y e^{-j\bar{k}\cdot\bar{r}}\hat{y} - jk_z e^{-j\bar{k}\cdot\bar{r}}\hat{z}\right] \times \bar{E}_0$$

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$$= \frac{e^{-i\bar{k}\cdot\bar{r}}}{\omega\mu}\left[\bar{k}\right] \times \bar{E_0}$$

$$= \frac{k \times \bar{E}}{\omega\mu}$$

$$= \frac{k\hat{n} \times \bar{E}}{\omega\mu}$$

$$= \frac{\cancel{\omega}\sqrt{\mu\epsilon}\,(\hat{n} \times \bar{E})}{\omega\mu}$$

$$= \frac{\hat{n} \times \bar{E}}{\eta} \qquad \eta = \sqrt{\mu/\epsilon}.$$

With time dependence

$$\bar{\mathcal{E}}(x,y,z,t) = \bar{E}(x,y,z)e^{i\omega t}$$

$$= \bar{E}_0 e^{i(\omega t - \bar{k}\cdot\bar{r})}$$

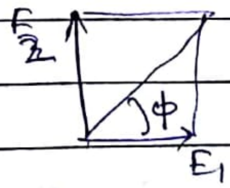
Linearly polarized because $\bar{E_0}$ direction is same

Circularly polarized

Instead of $\bar{E_0}$

Let, $\bar{E} = (E_1\hat{x} + E_2\hat{y})\,e^{-i\bar{k}\cdot\bar{r}}$

& let $\bar{k} = k_z\hat{z}$



$$\therefore \bar{E} = (E_1\hat{x} + E_2\hat{y})\,e^{-ik_z z}$$

E_1 & E_2 are non zero.

$$\phi = \tan^{-1}\left(\frac{E_2}{E_1}\right).$$

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If $E_1 = j E_2 = E_0$

then, $E_2 = -j E_0$

$\therefore \bar{E} = (E_0 \hat{x} - j E_0 \hat{y}) e^{-j k_z z}$

$= E_0 (\hat{x} - j \hat{y}) e^{-j k_z z}$

In time domain,

$$\bar{\mathcal{E}}(z,t) = E_0 (\hat{x} - j\hat{y}) e^{j(\omega t - k_z z)}$$

If $z = 0$

$$\mathcal{E}(0,t) = E_0 (\hat{x} - j\hat{y}) e^{j\omega t} = E_0 \hat{x} e^{j\omega t} - \hat{y} \cdot E_0 e^{j(\omega t + \pi/2)}$$

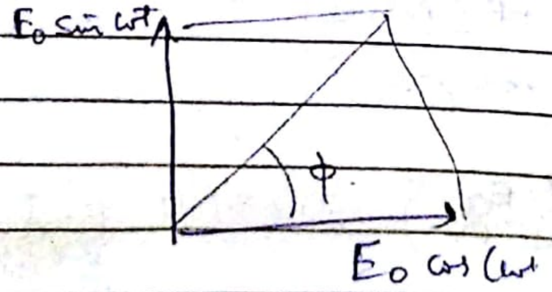
$$= E_0 \hat{x} e^{j\omega t} + \hat{y} E_0 e^{j(\omega t - \pi/2)}$$

Considering only the cosine phase. $= \cancel{E_0 \hat{x} e^{j\omega t} + \hat{y} E_0 e^{j\pi/2}}$

$\bar{\mathcal{E}}(0,t) = \cancel{E_0 (\hat{x} - j}$

$$= E_0 \hat{x} \cos(\omega t) + \hat{y} E_0 \cos(\omega t - \pi/2)$$

$$= E_0 \hat{x} \cos(\omega t) + \hat{y} E_0 \sin(\omega t)$$



$\cancel{\tan\phi = \frac{E_0 \sin\omega t}{E_0 \cos}}$

$$\tan\phi = \frac{E_0 \sin\omega t}{E_0 \cos\omega t} = \tan\omega t$$

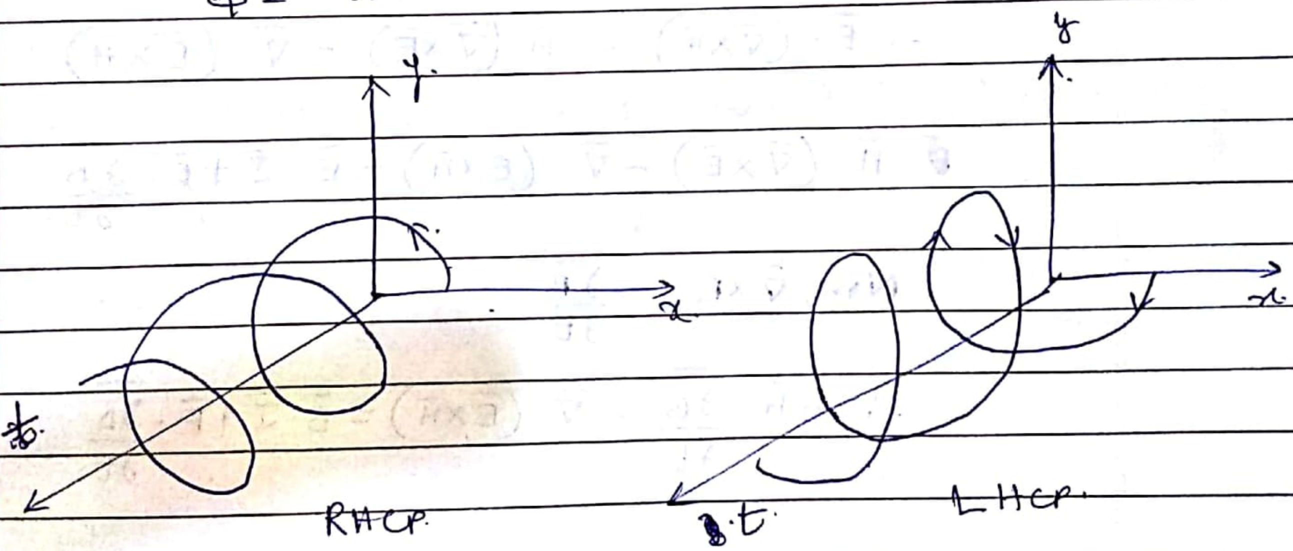
$\Rightarrow \phi = \omega t \rightarrow$ increases with time.

$\Rightarrow$ Right Hand Circularly Polarized (RHCP)

If initially, $E_0 = j E_1 = E_2$.

then, we would have got

$\phi = -\omega t$



Again, $\bar{\mathcal{E}}(0,t) = \underbrace{E_0 \cos\omega t}_{E_x}\,\hat{x} + \underbrace{E_0 \sin\omega t}_{E_y}\,\hat{y}$

Thus $E_x^2 + E_y^2 = E_0^2 \rightarrow$ locus of $\bar{\mathcal{E}}(0,t)$ vector is a circle

If $|E_1| \neq |E_2|$ but E_1 is $\pi/2$ ahead in phase.

then $E_x = E_1 \cos\omega t$ $\quad E_y = E_2 \sin\omega t$

$\Rightarrow \frac{E_x^2}{E_1^2} + \frac{E_y^2}{E_2^2} = 1 \rightarrow$ locus of $\bar{\mathcal{E}}(0,t)$ is an ellipse

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Poynting's Theorem

$$\bar{\nabla} \times \bar{H} = \bar{J} + \frac{\partial D}{\partial t} \quad \text{(Maxwell's 4th law)}$$

$$\bar{E} \cdot (\bar{\nabla} \times \bar{H}) = \bar{E} \cdot \bar{J} + \bar{E} \cdot \frac{\partial D}{\partial t}$$

Using Vector Identity

$$\bar{\nabla} \cdot (\bar{E} \times \bar{H}) = -\bar{E} \cdot \bar{\nabla} \times \bar{H} + \bar{H} \cdot \bar{\nabla} \times \bar{E}$$

$$\therefore \bar{E} \cdot (\bar{\nabla} \times \bar{H}) = \bar{H} \cdot (\bar{\nabla} \times \bar{E}) - \bar{\nabla} \cdot (\bar{E} \times \bar{H})$$

$$\therefore \bar{H} \cdot (\bar{\nabla} \times \bar{E}) - \bar{\nabla} \cdot (\bar{E} \times \bar{H}) = \bar{E} \cdot \bar{J} + \bar{E} \cdot \frac{\partial D}{\partial t}$$

Now, $\bar{\nabla} \times E = -\frac{\partial \bar{B}}{\partial t}$

$$\therefore -\bar{H} \cdot \frac{\partial \bar{B}}{\partial t} - \bar{\nabla} \cdot (\bar{E} \times \bar{H}) = \bar{E} \cdot \bar{J} + \bar{E} \cdot \frac{\partial \bar{D}}{\partial t}$$

$$\Rightarrow -\bar{\nabla} \cdot (\bar{E} \times \bar{H}) = \bar{J} \cdot \bar{E} + \epsilon \bar{E} \cdot \frac{\partial \bar{E}}{\partial t} + \mu \bar{H} \cdot \frac{\partial \bar{H}}{\partial t}$$

Now, $\epsilon \bar{E} \cdot \frac{\partial \bar{E}}{\partial t} = \frac{\partial}{\partial t}\left[\frac{1}{2} \bar{D} \cdot \bar{E}\right]$

$\mu \bar{H} \cdot \frac{\partial \bar{H}}{\partial t} = \frac{\partial}{\partial t}\left[\frac{1}{2} \bar{B} \cdot \bar{H}\right]$

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$$\therefore -\bar{\nabla}\cdot(\bar{E}\times\bar{H}) = \bar{J}\cdot E + \frac{\partial}{\partial t}\left(\frac{1}{2}\bar{D}\cdot\bar{E}\right) + \frac{\partial}{\partial t}\left(\frac{1}{2}\bar{B}\cdot\bar{H}\right)$$

Volume Integral,

$$-\int_{vol}\bar{\nabla}\cdot(\bar{E}\times\bar{H})\,dv = \int_{vol}\bar{J}\cdot\bar{E}\,dv + \int_{vol}\frac{\partial}{\partial t}\left(\frac{1}{2}\bar{D}\cdot\bar{E}\right)dv$$

$$+\int_{vol}\frac{\partial}{\partial t}\left(\frac{1}{2}\bar{B}\cdot\bar{H}\right)dv$$

Using divergence theorem

$$\Rightarrow -\oint_S(\bar{E}\times\bar{H})\cdot d\bar{s} \Big\}\ \text{Total power entering}$$

$$\text{Eqn (1)} = \underbrace{\int_{vol}(\bar{J}\cdot\bar{E})\,dv}_{\text{ohmic power loss in vol}} + \underbrace{\frac{\partial}{\partial t}\int_{vol}\frac{1}{2}(\bar{D}\cdot\bar{E})\,dv}_{\text{electric power entering}} + \underbrace{\frac{\partial}{\partial t}\int_{vol}\frac{1}{2}(\bar{B}\cdot\bar{H})\,dv}_{\text{Magnetic power entering}} \quad -(1)$$

electric energy stored in vol

Hence total power exiting,

$$P_L = \oint_S(\bar{E}\times\bar{H})\cdot d\bar{s}$$

$\bar{S} = \bar{E}\times\bar{H} \rightarrow$ Poynting vector

represent Instantaneous power density in W/m^2
shows direction of power flow.

Simplifying equation (1) assuming sinusoidal time dependence.

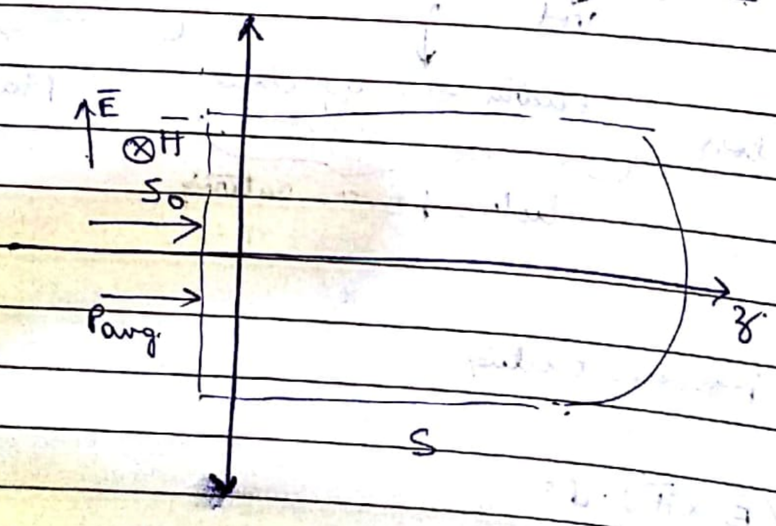
$$-\oint_S (\bar{E}\times\bar{H})\cdot d\bar{s} = \frac{\sigma}{2}\int_{vol}|E|^2 dv + \frac{j\omega\epsilon}{2}\int_{vol}|E|^2 dv + \frac{j\omega\mu}{2}\int_{vol}|H|^2 dv$$

$$\Rightarrow P_c = P_L + j\omega(W_e + W_m)$$

when, $\bar{E}$ & $\bar{H}$ are complex.

$$\bar{S} = \bar{E}\times\bar{H}^*$$

Power absorbed by good conductor



$$P_{avg} = \frac{1}{2}Re\int_{S_o+S}(\bar{E}\times\bar{H}^*)\cdot\bar{ds}$$

$$= \frac{1}{2}Re\int_{S_o}(\bar{E}\times\bar{H}^*)\cdot\hat{n}\, ds$$

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$\hat{n} = \hat{z}$

Using vector identity :. $(\bar{E} \times \bar{H}^*) \cdot \hat{z} = (\hat{z} \times \bar{E}) \cdot \bar{H}^*$

$$= \eta \bar{H} \cdot \bar{H}^*$$

$$= \eta |\bar{H}|^2.$$

$$\therefore P_{av} = \frac{1}{2} \mathrm{Re} \int_{S_0} \eta |\bar{H}|^2 ds$$

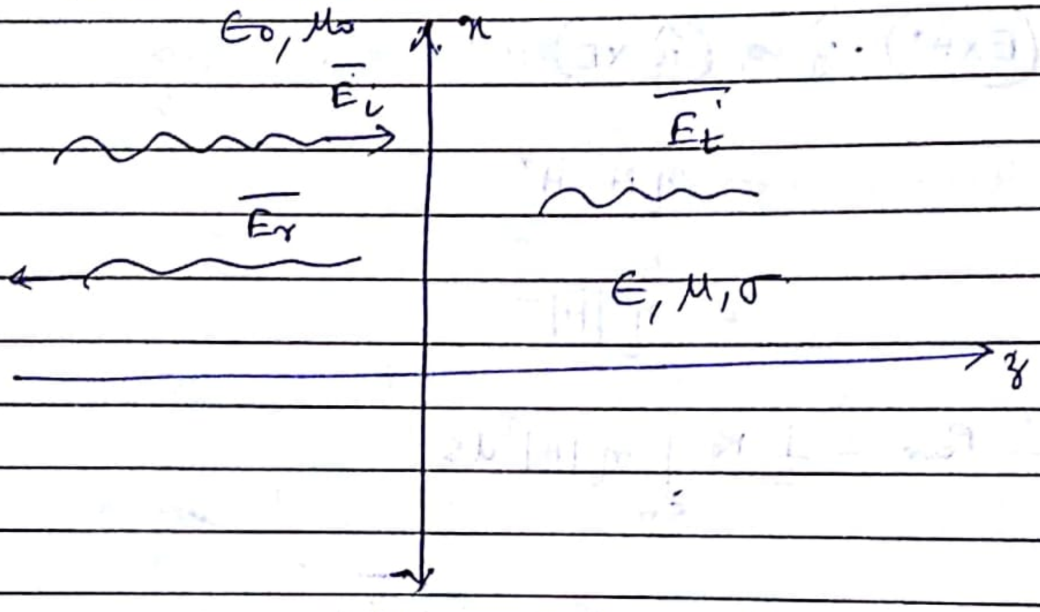
$$= \frac{R_s}{2} \int_0 |\bar{H}|^2 ds$$

$$R_s = \mathrm{Re}(\eta) = \mathrm{Re}\left[(1+j)\sqrt{\frac{\omega\mu}{2\sigma}}\right] = \sqrt{\frac{\omega\mu}{2\sigma}} = \frac{1}{\sigma\delta_s}$$

$\downarrow$

surface resistivity of the conductor

Plane wave reflection from a media interface



Let,

Incident wave for $z<0$ be

$$\bar{E_i} = \hat{x} E_0 e^{-jk_0 z}$$

$$\bar{H_i} = \hat{y} \frac{1}{\eta_0} E_0 e^{-jk_0 z} \qquad \eta_0 = \sqrt{\mu_0/\epsilon_0}$$

Reflected wave, → reflection coefficient

$$\bar{E_r} = \hat{x} \Gamma E_0 e^{jk_0 z}$$

$$\bar{H_r} = -\hat{y} \frac{\Gamma}{\eta_0} E_0 e^{jk_0 z}$$

So far we do not know whether Γ or T is complex

$$\bar{S_r} = \bar{E_r} \times \bar{H_r}^* = \cancel{-|\Gamma|^2}$$

$$= \hat{x} \Gamma E_0 e^{jk_0 z} \times \left(-\hat{y} \frac{\Gamma^*}{\eta_0} E_0^* e^{-jk_0 z}\right)$$

$$= -\hat{z} \frac{|\Gamma|^2 |E_0|^2}{\eta_0}$$ → Power flow in $-ve$ z direction

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Transmitted wave,

$$\overline{E_t} = \hat{x}\, T E_0 e^{-\gamma z}$$

$$\overline{H_t} = \hat{y}\, \frac{T E_0 e^{-\gamma z}}{\eta}$$

$\eta = \dfrac{j\omega\mu}{\gamma} \rightarrow$ complex.

$$\gamma = \alpha + j\beta = j\omega\sqrt{\mu\epsilon}\sqrt{1 - \frac{j\sigma}{\omega\epsilon}}$$

At, $z=0$, tangential components of Electric & magnetic field are continuous,

$\therefore E_i + E_r = E_t$

$\therefore E_0 e^{-jk_0 z} + \Gamma E_0 e^{jk_0 z} = T E_0 e^{-\gamma z} \Big|_{z=0}$

$\Rightarrow E_0 + \Gamma E_0 = T E_0$

$\Rightarrow 1 + \Gamma = T$ — (1)

Also,

$H_i + H_r = H_t$

$$\Rightarrow \frac{1}{\eta_0} E_0 - \frac{\Gamma}{\eta_0} E_0 = \frac{T E_0}{\eta}$$

$$\Rightarrow \frac{1-\Gamma}{\eta_0} = T/\eta \quad \text{— (2)}$$

Comparing (1) & (2)

$$\Gamma = \frac{\eta - \eta_0}{\eta + \eta_0}$$

$$\& \; T = 1 + \Gamma = \frac{2\eta}{\eta + \eta_0}$$

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Lossless medium

As $\gamma > 0$, $\sigma = 0$, ϵ & μ are real,

$$\therefore \gamma = j\omega\sqrt{\mu\epsilon} = \cancel{j k_0 \sqrt{\mu_0 \epsilon_0}}$$

$$\cancel{k_0 = \sqrt{\epsilon_r \mu_r}}$$

$$= j k_0 \sqrt{\mu_r \epsilon_r}$$

$k_0 = \omega\sqrt{\mu_0 \epsilon_0} \rightarrow$ wavenumber of plane wave in free space

Wavelength,

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{\omega\sqrt{\mu\epsilon}} = \frac{2\pi}{k_0\sqrt{\mu_r\epsilon_r}} = \frac{\lambda_0}{\sqrt{\mu_r\epsilon_r}}$$

$$\cancel{v_p = \frac{2\pi}{\beta} = \frac{2\pi}{k_0\sqrt{\mu_r\epsilon_r}} =}$$

$$v_p = \omega/\beta = \frac{\omega}{\omega\sqrt{\mu\epsilon}} = \frac{1}{\sqrt{\mu\epsilon}} = \frac{c}{\sqrt{\mu_r\epsilon_r}}$$

Wave Impedance

$$\eta = \frac{j\omega\mu}{\gamma} = \frac{j\omega\mu}{j\omega\sqrt{\mu\epsilon}} = \sqrt{\mu/\epsilon} = \sqrt{\frac{\mu_0\mu_r}{\epsilon_0\epsilon_r}} = \eta_0\sqrt{\frac{\mu_r}{\epsilon_r}}$$

$$\eta_0 = \sqrt{\mu_0/\epsilon_0}$$

$\therefore$ T & Γ are real.

$\therefore \vec{E}$ & $\vec{H}$ are in phase with each other

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For $z<0$,

$$\bar{S}^- = \bar{E}\times\bar{H}^* = (\bar{E}_i+\bar{E}_r)\times(\bar{H}_i+\bar{H}_r)^*$$

$$= \hat{z}|E_0|^2\frac{1}{\eta_0}\left(e^{-jk_0z}+\Gamma e^{+jk_0z}\right)$$

$$\times\left(e^{-jk_0z}-\Gamma^* e^{+jk_0z}\right)^*$$

$$= \hat{z}\frac{|E_0|^2}{\eta_0}\left(e^{-jk_0z}+\Gamma e^{jk_0z}\right)\times\left(e^{+jk_0z}-\Gamma^* e^{-jk_0z}\right)$$

$$= \hat{z}\frac{|E_0|^2}{\eta_0}\left[1-|\Gamma|^2+\Gamma e^{j2k_0z}-\Gamma^* e^{-j2k_0z}\right]$$

At $(z=0)$ $$= \hat{z}\frac{|E_0|^2}{\eta_0}\left[1-|\Gamma|^2+\Gamma-\Gamma^*\right]$$

$\because$ Γ is real.

$$\therefore \bar{S}^- = \hat{z}\frac{|E_0|^2}{\eta_0}\left[1-|\Gamma|^2\right] \text{ at } z=0$$

$$= \hat{z}\frac{|E_0|^2}{\eta_0}\left[1-|\Gamma|^2+2j\Gamma\sin(2k_0z)\right]$$

$$\bar{S}^+ = \bar{E}_t\times\bar{H}_t^* = \hat{x}\,T E_0 e^{-\gamma z}\times\hat{y}\,T^*\frac{E_0^* e^{\gamma^* z}}{\eta}$$

$$= \hat{z}|T|^2\frac{|E_0|^2}{\eta}$$

$$= \hat{z}|E_0|^2\frac{4\eta}{(\eta+\eta_0)^2}$$

$$= \hat{z}|E_0|^2\frac{1}{\eta_0}\left(1-|\Gamma|^2\right)$$

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Time averaged power flow through $1\,m^2$ as cross section area,

for $z<0$

$$P^- = \frac{1}{2} Re\left[\bar{S}^- \cdot \hat{z}\right]$$

$$= \frac{1}{2} \frac{|E_0|^2}{\eta_0}\left(1-|\Gamma|^2\right)$$

for $z>0$

$$P^+ = \frac{1}{2} Re\left(\bar{S}^+ \cdot \hat{z}\right)$$

$$= \frac{|E_0|^2}{2\eta_0}\left(1-|\Gamma|^2\right)$$

If we individually calculate

$$\bar{S_i} = \bar{E_i} \times \bar{H_i}^* = \hat{z} \frac{|E_0|^2}{\eta_0}$$

$$\bar{S_r} = \bar{E_r} \times \bar{H_r}^* = -\hat{z} \frac{|E_0|^2 |\Gamma|^2}{\eta_0}$$

$$\bar{S_i} + \bar{S_r} = \hat{z} \frac{|E_0|^2}{\eta_0}\left(1-|\Gamma|^2\right)$$

~~which is~~ which is not same as $\bar{S}^-$

this is due to energy stored in standing wave.

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