Problem 4.23

(a) Consider the FM version of angle modulation. Let the instantaneous frequency of the modulator be a linear function of the first derivative of the message signal $m(t)$, as shown by

$$f_i(t) = f_c + k_1 \frac{d}{dt} m(t)$$

Then, correspondingly, the instantaneous phase is defined by

$$\begin{aligned} \theta_i(t) &= 2\pi \int_0^t f_i(t)dt \\ &= 2\pi f_c t + 2\pi k_1 m(t) \end{aligned}$$

where it is assumed that $m(0) = 0$. In this scenario, the modulated signal is defined by

$$\begin{aligned} s(t) &= A_c \cos(\theta_i(t)) \\ &= A_c \cos[2\pi f_c t + \theta_i(t) m(t)] \end{aligned}$$

which is recognized as phase modulation.

Suppose next that $f_i(t)$ is a linear function of the second derivative of $m(t)$, as shown by

$$f_i(t) = f_c + k_2 \frac{d^2 m(t)}{dt^2}$$

Correspondingly, we have

$$\theta_i(t) = 2\pi f_c t + 2\pi k_2 \frac{dm(t)}{dt}$$

where it is assumed that $dm(t)/dt$ is zero at $t = 0$. The modulated wave assumes the new form

$$s(t) = A_c \cos\left(2\pi f_c t + 2\pi k_2 \frac{dm(t)}{dt}\right)$$

We may generalize these results by stating that if the input to a frequency modulator is the nth derivative of the message signal $m(t)$, then the corresponding modulated wave is defined by

$$s(t) = A_c \cos\left(2\pi f_c t + 2\pi k_n \frac{d^{n-1} m(t)}{dt^{n-1}}\right)$$

where it is assumed that $d^{n-1}m(t)/dt^{n-1}$ is zero at time $t = 0$.

Consider next the scenario where the input to the frequency modulator involves integrals of the message signal $m(t)$. Starting with $\int_0^t m(\tau)d\tau$ as the input to the modulator, we write

$$f_i(t) = f_c + c_1 \int_0^t m(\tau)d\tau$$

and, correspondingly,

$$\theta_i(t) = 2\pi f_c t + 2\pi c_1 \left(\int_0^\lambda m(\tau)d\tau\right) d\lambda$$

The resulting modulated signal is defined by

Continued on next slide

Problem 4-23 continued

$$s(t) = A_c \cos\left[2\pi f_c t + 2\pi c_1 \int_0^t \left(\int_0^\lambda m(\tau)d\tau\right)d\lambda\right]$$

Unlike the modulation scenario involving derivatives of $m(t)$, we can see that when considering the scenario involving integrals of $m(t)$, mathematical formulation of the modulated signal $s(t)$ becomes increasingly more complicated.

(b) There could be a practical benefit from using a frequency-modulation strategy involving integrals of the message signal $m(t)$ if $m(t)$ happens to be corrupted by additive noise. In such a scenario, the integration process tends to reduce the corruptive influence of the additive noise by smoothing it out. However, the drawback of such a modulation strategy is two-fold:

(i) Mathematical analysis of ordinary FM is complicated enough. Using integrals of the message signal as the input to the frequency modulator makes the problem even more complicated.

(ii) Likewise, designs of the transmitter and receiver become even more complicated.

Statements similar to (i) and (ii) apply to the use of second and higher derivatives of the message signal $m(t)$ as the input to the frequency modulator. The only exception here is the first derivative of $m(t)$, in which case the frequency modulator produces a phase modulated version of the signal. One other point to note is that if the message signal $m(t)$ is corrupted by additive noise, the operation of differentiation will enhance the presence of the noise component, which is undesirable.

To conclude, the "simple" forms of angle modulation exemplified by the ordinary FM and ordinary PM discussed in the chapter are good enough from a theoretical as well as practical perspective.