Problem 3.19

We are given

$$m(t) = \frac{t}{1+t^2} \tag{1}$$

The AM wave is therefore defined by

$$\begin{aligned} s(t) &= A_c[1 + k_a m(t)]\cos(2\pi f_c t) \\ &= A_c\left(1 + \frac{k_a t}{1+t^2}\right)\cos(2\pi f_c t) \end{aligned} \tag{2}$$

The message signal $m(t)$ is plotted in Fig. 1(a) with its maximum value of 1/2 and minimum value of -1/2 at $t = 1$ and $t = -1$, respectively.

(a) Percentage modulation = 50%

$$k_a|m(t)|_{\max} = 0.5$$

with $|m(t)|_{\max} = \frac{1}{2}$, it follows that $k_a = 1$ for 50% modulation. For this example, Eq. (1) takes the form

$$s(t) = A_c\left(1 + \frac{t}{1+t^2}\right)\cos(2\pi f_c t) \tag{3}$$

Let t be measured in seconds. Then, for the envelope of the AM wave to be clearly visible, the period of the carrier, $1/f_c$, must be small compared to the time taken for the message signal $m(t)$ to reach its peak value. To satisfy this requirement, we let

$$\frac{1}{f_c} = 10\text{Hz}$$

which corresponds to

$$f_c = 10\text{Hz}$$

Setting the carrier amplitude $A_c = 1$ volt, and $f_c = 10$Hz, Eq. (3) is plotted in part (b) of Fig. 1.

(b) Percentage modulation = 100%

In this case, we have $k_a = 2$. Correspondingly, Eq. (1) assumes the form

$$s(t) = A_c\left(1 + \frac{2t}{1+t^2}\right)\cos(2\pi f_c t) \tag{4}$$

Keeping $A_c = 1$ volt, and $f_c = 10$Hz as in case (a), Eq. (4) is plotted in Fig. 1(c).

(c) Percentage modulation = 125%

In this third and final example, we have $k_a = 2.5$. Hence, Eq. (1) now assumes the form

$$s(t) = A_c\left(1 + \frac{2.5t}{1+t^2}\right)\cos(2\pi f_c t) \tag{5}$$

Keeping $_c = 1$ volt, and $f_c = 10$Hz as before, Eq. (5) is plotted in Fig. 1(d).

Comparing the AM waveforms plotted in parts (b), (c) and (d) of Fig. 1, we may make the following observations:

- The AM wave of Fig. 1(b) is undermodulated
- The AM wave of Fig. 1(c) is on the verge of overmodulation
- The AM wave of Fig. 1(d) is overmodulated

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Problem 3-19 continued

(a)

(b)

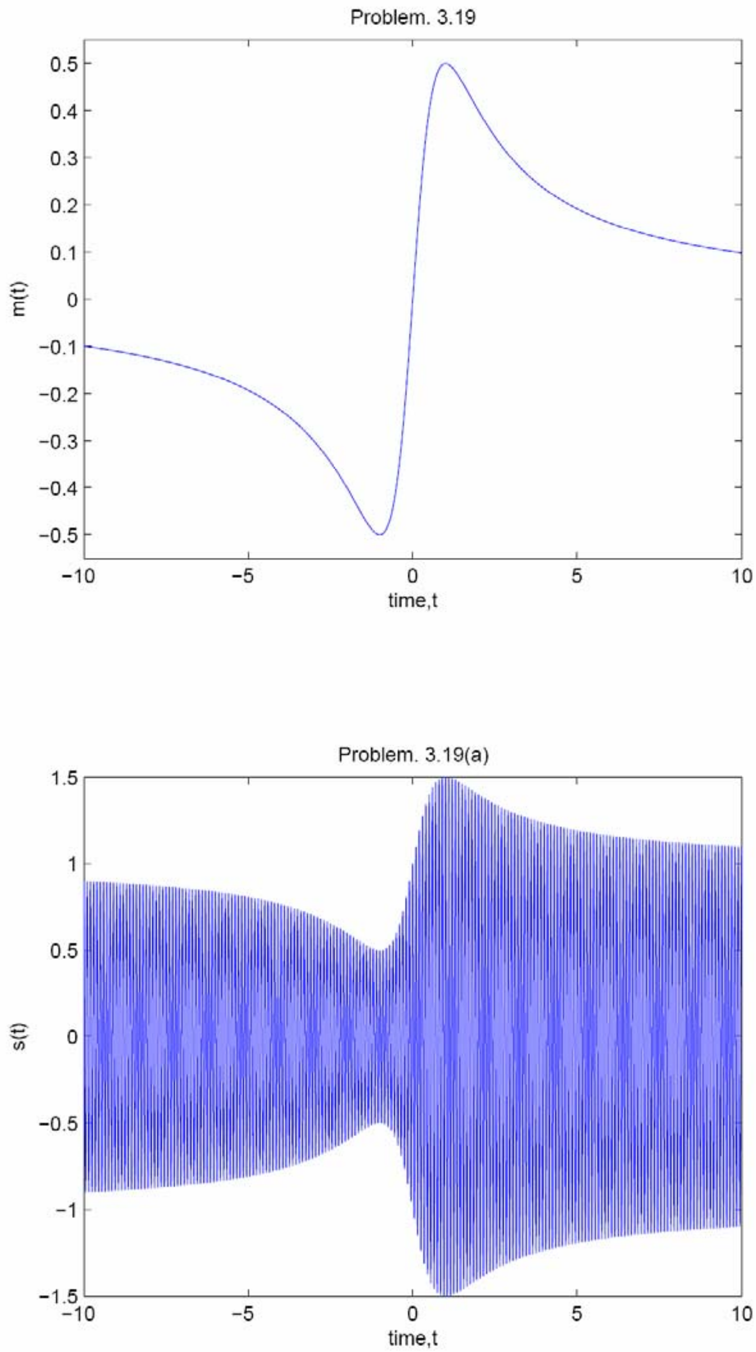

Figure 1: Problem 3.19

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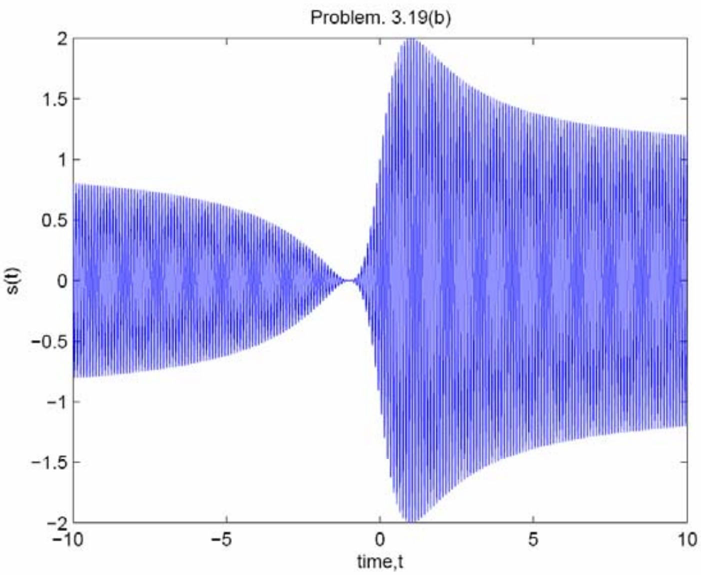
Problem 3-19 continued

(c)



(d)

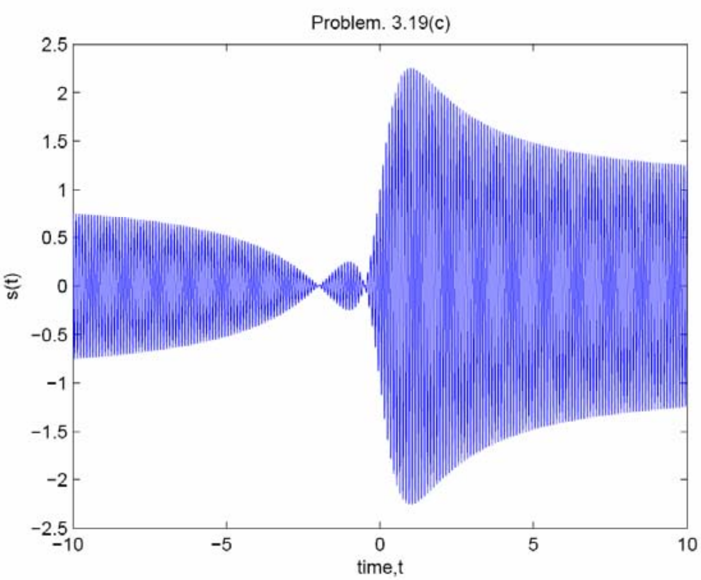


Figure 1 (continued): Problem 3.19