Problem 3.12

Starting with Eq. (3.23) for a SSB modulated wave, show that the product-modulator output in the coherent detector of Fig. 3.12 (assuming perfect synchronism with the transmitter) in response to this modulated wave contains a new SSB modulated wave with carrier frequency $2f_c$.

Solution

Suppose we focus on the upper SSB in Eq. (3.23) (i.e., use the minus sign in this formula). Then

$$s(t) = \frac{A_c}{2}m(t)\cos(2\pi f_c t) - \frac{A_c}{2}\hat{m}(t)\sin(2\pi f_c t)$$

Applying this modulated wave to the coherent detector of Fig. 3.12 with the phase error $\phi = 0$, we first get the product-modulated output

$$\begin{aligned}
v(t) &= A'_c s(t)\cos(2\pi f_c t) \\
&= \frac{A_c A'_c}{2}m(t)\cos^2(2\pi f_c t) - \frac{A_c A'_c}{2}\hat{m}(t)\sin(2\pi f_c t)\cos(2\pi f_c t) \\
&= \frac{A_c A'_c}{4}m(t)[1 + \cos(4\pi f_c t)] - \frac{A_c A'_c}{4}\hat{m}(t)\sin(4\pi f_c t) \\
&= \frac{A_c A'_c}{4}m(t) + \frac{A_c A'_c}{4}[m(t)\cos(4\pi f_c t) - \hat{m}(t)\sin(4\pi f_c t)]
\end{aligned}$$

Comparing this formula with that for $s(t)$, we see that $v(t)$ contains a new upper SSB modulated wave with carrier frequency $2f_c$. This same statement also applies to the lower SSB modulated wave $s(t)$.