Problem 2.29

(a) From part (b) of Problem 2.28, we have

$$G(f) = \frac{A}{\pi^2 f^2 (t_b - t_a)} \sin[\pi f(t_b - t_a)] \sin[\pi f(t_b + t_a)] \tag{1}$$

As t_b approaches t_a, we get the following result:

$$\lim_{t_b \to t_a} \frac{1}{\pi f^2 (t_b - t_a)} \sin[\pi f(t_b - t_a)] = 1$$

and

$$\lim_{t_b \to t_a} \sin[\pi f(t_b + t_a)] = \sin(\pi f t_a)$$

Accordingly, the Fourier transform of Eq. (1) approaches the limiting value

$$\begin{aligned} \lim_{t_b \to t_a} G(f) &= \frac{A}{\pi f} \sin(2\pi f t_a) \\ &= 2t_a A \frac{\sin(2\pi f t_a)}{2\pi f t_a} \\ &= 2t_a A \operatorname{sinc}(2\pi f t_a) \end{aligned} \tag{2}$$

which is the desired result.

(b) The limiting Fourier transform of Eq. (2) is recognized as the Fourier transform of a rectangular pulse of amplitude A and duration $T = 2t_a$.