Digital Image Processing

Mean-Shift Segmentation

Suyash P. Awate

Segmentation

- Partitioning an image into regions
  - Labeling
- Criteria
  - **Small variation** of intensities / patterns **within** segment
  - **Large variation** of intensities / patterns **between** segments

Mean-Shift Segmentation

- References
  - https://en.wikipedia.org/wiki/Mean-shift
  - Mean Shift: A Robust Approach Toward Feature Space Analysis.
    D Comaniciu and P Meer.
    IEEE Transactions on Pattern Analysis and Machine Intelligence 2002. 24(2):603-619

Mean-Shift Segmentation

- **Goal**: Estimate probability density function (PDF) f(x) given observations x_i (d-dimensional) drawn from f(x)
- **Nonparametric / kernel density estimation**
- **Strategy**: Superpose kernel functions placed at x_i

$$f(\mathbf{x}) = \frac{1}{nh^d}\sum_{i=1}^{n} K\left(\frac{\mathbf{x}-\mathbf{x}_i}{h}\right)$$

$$\int_{R^d} K(\mathbf{x})\mathrm{d}\mathbf{x} = 1$$

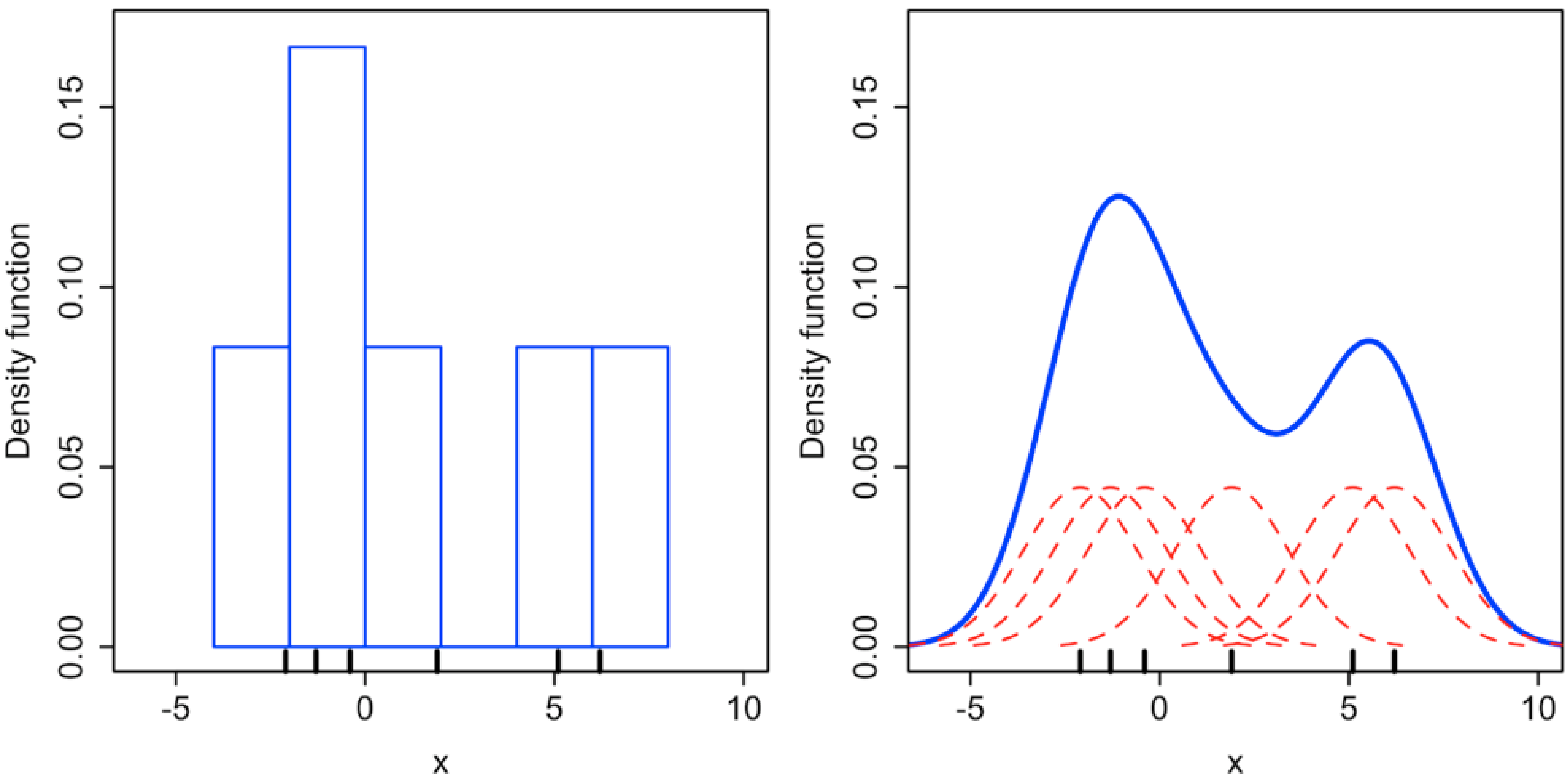
$$\int_{R^d} \mathbf{x}K(\mathbf{x})\mathrm{d}\mathbf{x} = 0$$

$$\lim_{\|\mathbf{x}\|\to\infty} \|\mathbf{x}\|^d K(\mathbf{x}) = 0$$

  - K(.) = **kernel**
    - Non-negative, integrates to 1, mean 0, finite valued, decays to 0 sufficiently fast
    - Typically, radially symmetric $K(\mathbf{x}) = c_{k,d}k(\|\mathbf{x}\|^2)$
      - k(.) = non-increasing, c = normalization constant
  - h = **bandwidth** parameter

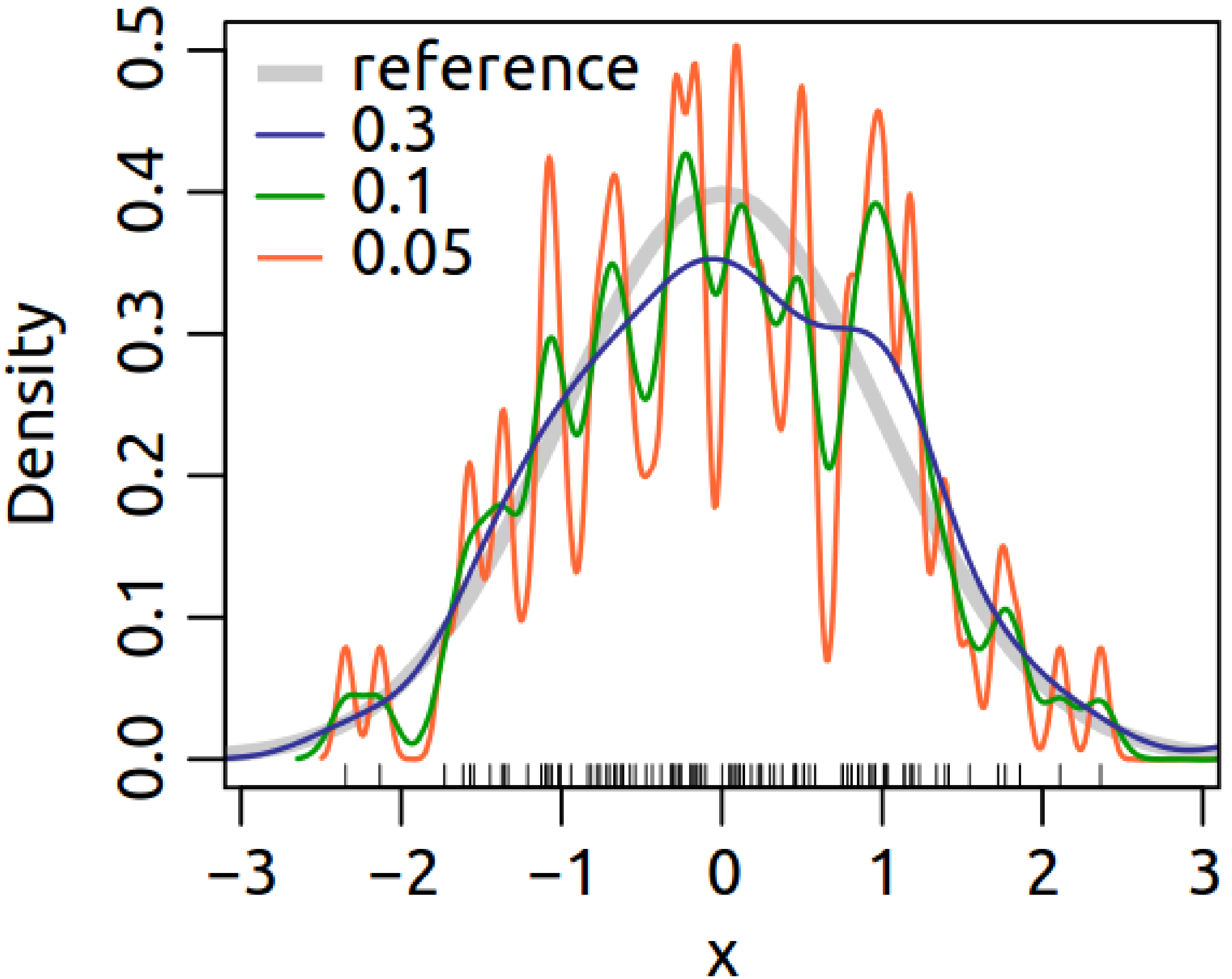
Mean-Shift Segmentation

- Nonparametric / kernel density estimation
  - Histogram (left) versus Kernel density estimate (right)



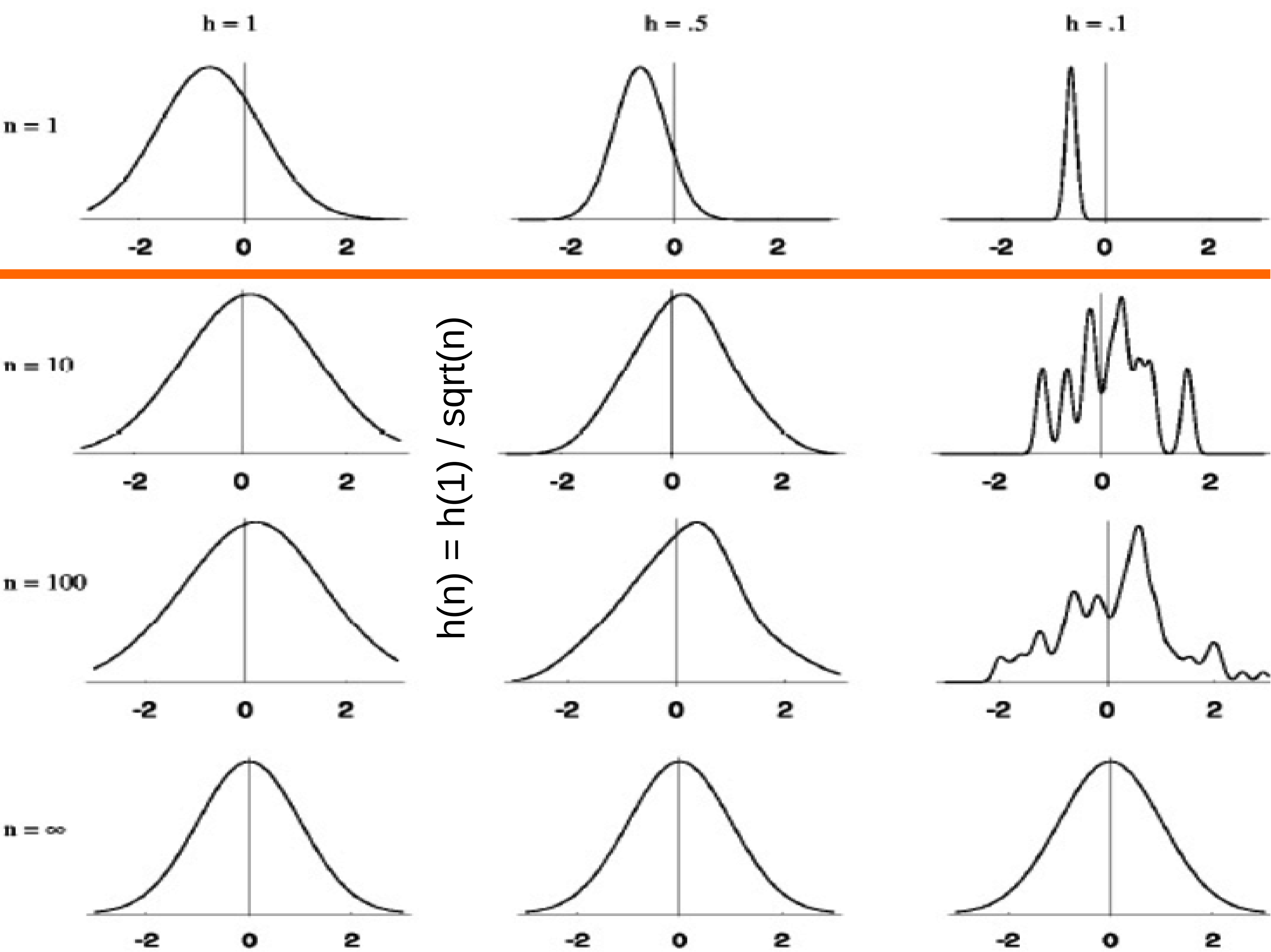
Mean-Shift Segmentation

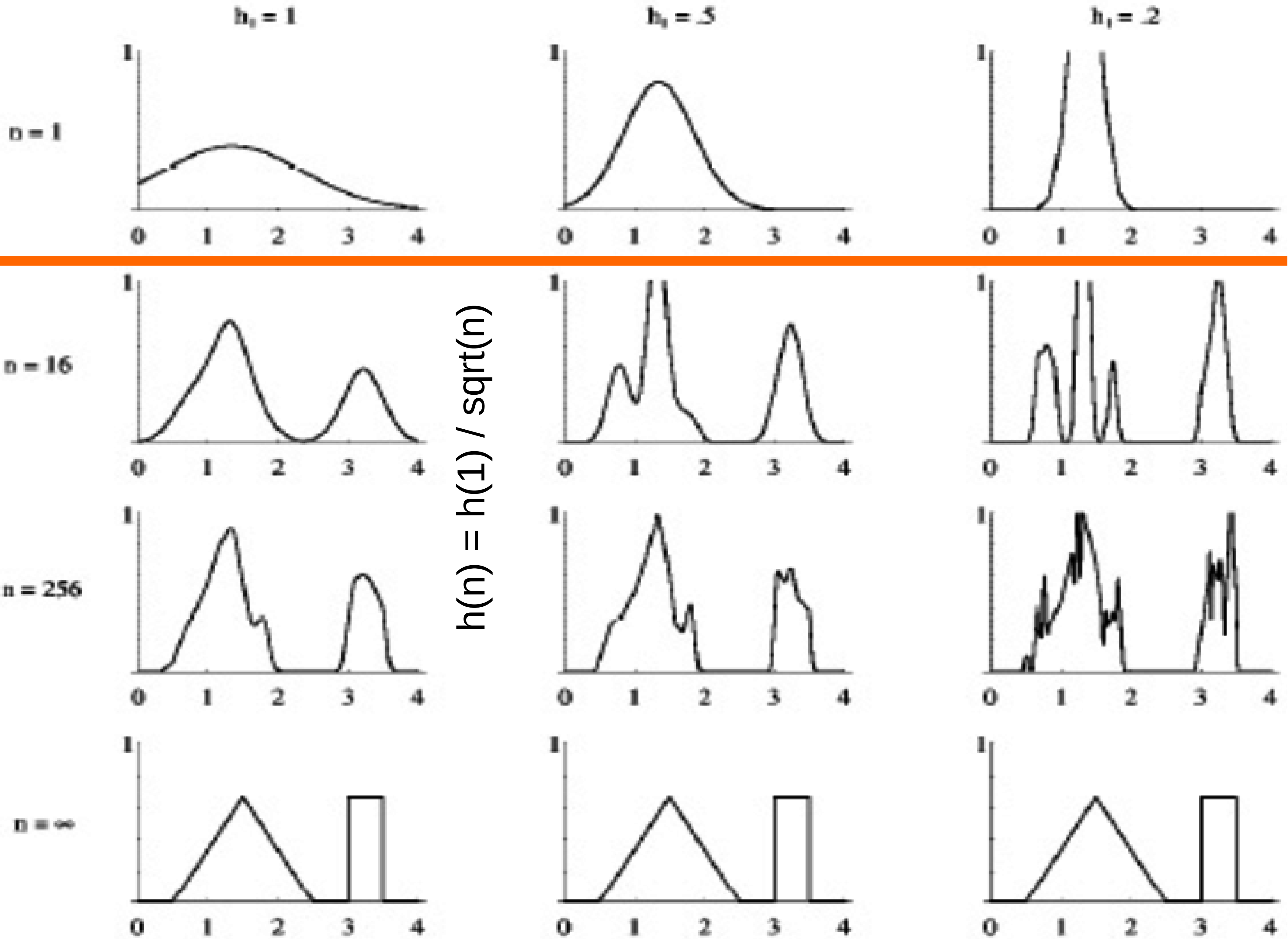
- Nonparametric / kernel density estimation
  - **Bandwidth** (h) **selection**
    - Too small: orange
    - Small: green
    - Desirable: blue
    - True: **gray**
    - What if h is too large ?
    - What if $h \rightarrow 0$?



Mean-Shift Segmentation

- Nonparametric / kernel density estimation
- Density estimate convergence as sample size $n \to \infty$
  - Bandwidth h must reduce to 0 with increasing n
    - Sufficiently fast
    - Sufficiently slow
    - $\text{Lim}_{n \to \infty} \; [\, h(n)\,]^d = 0$
      $\text{Lim}_{n \to \infty} \; n\, [\, h(n)\,]^d = \infty$
    - Example for 1D case (d=1),
      h(n) = h(1) / sqrt(n)
  - Guaranteed convergence
    - $P_n(x)$ is estimate of density p(x) at 'x', using sample size 'n'
    - $\lim_{n \to \infty} E \; [\, P_n(x)\,] = p(x)$
    - $\lim_{n \to \infty} \text{Var}\, [\, P_n(x)\,] = 0$ (convergence in mean square)


h = 1
h = .5
h = .1
n = 1
n = 10
n = 100
n = ∞
h(n) = h(1) / sqrt(n)
-2
0
2



h1 = 1
h1 = .5
h1 = .2
n = 1
n = 16
n = 256
n = ∞
h(n) = h(1) / sqrt(n)

Mean-Shift Segmentation

- Application to images
  - Left: color image, 3 color components
  - Right: Scatter plot of color 3-tuples
  - Assumption: Each object → cluster of color values
  - *Color can be replaced with any other feature*