

Problem 3.29

- (a) Using the terminated series expansion $\exp(-x) \approx 1 - x + \frac{x^2}{2} - \frac{x^3}{6}$ we may express the diode current i , normalized with respect to I_0 , as

$$\begin{aligned}\frac{i}{I_0} &= \exp\left(-\frac{v}{V_T}\right) - 1 \\ &= -\frac{v}{V_T} + \frac{1}{2}\left(\frac{v}{V_T}\right)^2 - \frac{1}{6}\left(\frac{v}{V_T}\right)^3\end{aligned}\quad (1)$$

- (b) Given

$$\begin{aligned}\frac{v}{V_T} &= \frac{0.01}{0.026}[\cos(2\pi f_m t) + \cos(2\pi f_c t)] \\ &\approx 0.385[\cos(2\pi f_m t) + \cos(2\pi f_c t)]\end{aligned}\quad (2)$$

we find that substitution of Eq. (2) into (1) yields

$$\begin{aligned}\frac{i}{I_0} &\approx -0.385[\cos(2\pi f_m t) + \cos(2\pi f_c t)] \\ &\quad + 0.074[\cos(2\pi f_m t) + \cos(2\pi f_c t)]^2 \\ &\quad - 0.0095[\cos(2\pi f_m t) + \cos(2\pi f_c t)]^3\end{aligned}\quad (3)$$

Next, using the identities

$$\begin{aligned}\cos^2\theta &= \frac{1}{2}[1 + \cos(2\theta)] \\ \cos^3\theta &= \frac{3}{4}\cos\theta + \frac{1}{4}\cos(3\theta) \\ \cos\theta\cos\phi &= \frac{1}{2}[\cos(\theta + \phi) + \cos(\theta - \phi)]\end{aligned}$$

we may rewrite Eq. (3) in the form:

$$\begin{aligned}\frac{i}{I_0} &= 0.074 - 0.406[\cos(2\pi f_m t) + \cos(2\pi f_c t)] \\ &\quad + 0.037\{\cos(4\pi f_m t) + \cos(4\pi f_c t) + \cos[2\pi(f_c + f_m)t] + \cos[2\pi(f_c - f_m)t]\} \\ &\quad - 0.0016[\cos(6\pi f_m t) + \cos(6\pi f_c t)] \\ &\quad - 0.0071\{\cos[2\pi(f_c + 2f_m)t] + \cos[2\pi(f_c - 2f_m)t] \\ &\quad + \cos[2\pi(2f_c + f_m)t] + \cos[2\pi(2f_c - f_m)t]\}\end{aligned}$$

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Problem 3-29 continued

For $f_m = 1$ kHz and $f_c = 100$ kHz, we thus find that the discrete amplitude spectrum of the diode current i (for $f \geq 0$) is as shown in Fig. 1.

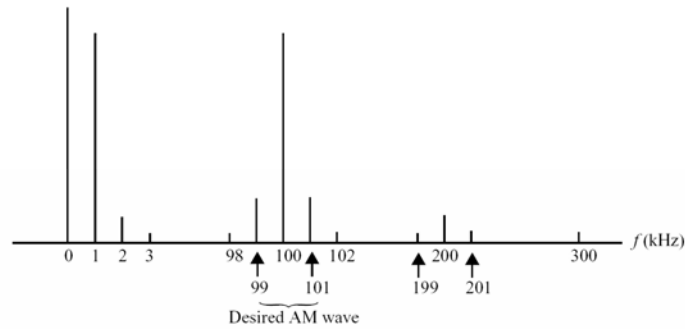


Figure 1

- (c) From the amplitude spectrum of Fig. 1 we see that in order to extract an AM wave with carrier frequency f_c from the diode current i , we need a band-pass filter that passes only the frequency components: 99, 100 and 101 kHz, corresponding to $f_c - f_m$, f_c , and $f_c + f_m$, respectively. We therefore require a band-pass filter with center frequency 100 kHz and bandwidth 2 kHz.
- (d) The resulting band-pass filter output is

$$\begin{aligned} \frac{i}{I_0} &= -0.406 \cos(2\pi f_c t) + 0.148 \cos(2\pi f_c t) \cos(2\pi f_m t) \\ &= -0.406[1 - 0.362 \cos(2\pi f_m t)] \cos(2\pi f_c t) \end{aligned}$$

The percentage modulation is therefore 36.2 percent.