Problem 2.33

The half cosine pulse in Fig. 2.33(a) is

$$g(t) = A\,\mathrm{rect}\left(\frac{t}{T}\right)\cos\left(\frac{\pi t}{T}\right)$$

Fourier transforming both sides gives

$$G(f) = AT\frac{\sin[\pi fT]}{\pi fT}\bigstar\left\{\frac{1}{2}\left[\delta\left(f-\frac{1}{2T}\right)+\delta\left(f+\frac{1}{2T}\right)\right]\right\}$$

$$= \frac{AT\sin\left[\pi fT-\frac{\pi}{2}\right]}{2\left(\pi fT-\frac{\pi}{2}\right)}+\frac{AT\sin\left[\pi fT+\frac{\pi}{2}\right]}{2\left(\pi fT+\frac{\pi}{2}\right)}$$

$$= \frac{2AT\cos(\pi fT)}{\pi(1-2fT)(1+2fT)}$$

Therefore, the energy density of $g(t)$ is

$$\Psi(f) = |G(f)|^2 = \frac{4A^2T^2\cos^2(\pi fT)}{\pi^2(1-4f^2T^2)^2} = \frac{4A^2T^2\cos^2(\pi fT)}{\pi^2(4T^2f^2-1)^2} \qquad (1)$$

Consider next the half-sine pulse in Fig. 2.33(b), which is the same as that of Fig. 2.33(b) shifted to the right by $T/2$. This time-shift corresponds to multiplication by $\exp(-j2\pi fT)$, which has unit amplitude for all f. Therefore, both pulses have exactly the same energy density defined in Eq. (1).