Problem 7.4

Show that the modulation process involved in generating Sunde's BFSK is nonlinear.

Solution

Let

$$f_1 = f_c + \frac{1}{2T_b}, \quad \text{for symbol 1}$$

and

$$f_2 = f_c + \frac{1}{2T_b}, \quad \text{for symbol 0}$$

where f_c is the unmodulated carrier frequency. We may therefore express the instantaneous frequency of Sunde's BFSK signal as

$$f_i(t) = f_c + k\frac{1}{2T_b}, \qquad 0 \le t \le T_b \tag{1}$$

where

$$k = \begin{cases} +1 \text{ for symbol } 1 \\ -1 \text{ for symbol } 0 \end{cases}$$

Correspondingly, we may define the BFSK signal itself as

$$\begin{aligned} s(t) &= \sqrt{\frac{2E_b}{T_b}}\cos[2\pi f_i t] \\ &= \sqrt{\frac{2E_b}{T_b}}\cos\left(2\pi f_c t + \frac{\pi k}{T_b}t\right) \\ &= \sqrt{\frac{2E_b}{T_b}}\cos(2\pi f_c t)\cos\left(\frac{\pi k}{T_b}t\right) - \sqrt{\frac{2E_b}{T_b}}\sin(2\pi f_c t)\sin\left(\frac{\pi k}{T_b}t\right) \\ &= \sqrt{\frac{2E_b}{T_b}}\cos(2\pi f_c t)\cos\left(\pm\frac{\pi}{T_b}t\right) - \sqrt{\frac{2E_b}{T_b}}\sin(2\pi f_c t)\sin\left(\pm\frac{\pi}{T_b}t\right) \end{aligned} \tag{2}$$

Recognizing that

$$\cos(-A) = \cos A$$

and

$$\sin(-A) = -\sin A$$

we may rewrite Eq. (2) in the new form

$$s(t) = \sqrt{\frac{2E_b}{T_b}}\cos(2\pi f_c t)\cos\left(\frac{\pi}{T_b}t\right) \mp \sqrt{\frac{2E_b}{T_b}}\sin(2\pi f_c t)\sin\left(\frac{\pi}{T_b}t\right) \tag{3}$$

where $0 \le t \le T_b$; the minus sign corresponds to symbol 0 and the plus sign corresponds to symbol 1. Equation (3) reveals the following two characteristics of Sunde's BFSK:

(i) The in-phase component of $s(t)$ is independent of the incoming binary data stream.

(ii) The incoming binary data stream only affects the quadrature component.

It is because of property (ii) that we may go on to state that Sunde's BFSK is nonlinear.