Problem 3.6.

The sinusoidally modulated DSB-SC wave of Example 3.2 is applied to a product modulator using a locally generated sinusoid of unit amplitude, and which is synchronous with the carrier used in the modulation.

(a) Determine the output of the product modulator, denoted by $v(t)$.

(b) Identify the two sinusoidal terms in $v(t)$ that are produced by the upper side frequency of the DSB-SC modulated wave, and the remaining two sinusoidal terms produced by the lower side frequency.

Solution

(a) From Example 3.2, the DSB-SC modulated is defined by

$$s(t) = \frac{1}{2}A_c A_m \cos(2\pi(f_c + f_m)t) + \frac{1}{2}A_c A_m \cos(2\pi(f_c - f_m)t)$$

Applying $s(t)$ and $\cos(2\pi f_c t)$ to a product modulator yields

$$\begin{aligned} v(t) &= s(t)\cos(2\pi f_c t) \\ &= \frac{1}{2}A_c A_m \cos(2\pi(f_c + f_m)t)\cos(2\pi f_c t) + \frac{1}{2}A_c A_m \cos(2\pi(f_c - f_m)t)\cos(2\pi f_c t) \\ &= \frac{1}{4}A_c A_m[\cos(2\pi(2f_c + f_m)t) + \cos(2f_m t)] \\ &\quad + \frac{1}{4}A_c A_m[\cos(2\pi(2f_c - f_m)t) + \cos(2f_m t)] \qquad (1) \end{aligned}$$

(b) The two sinusoidal terms inside the first set of square brackets in Eq. (1) are produced by the upper side frequency at $f_c + f_m$. The other two sinusoidal terms inside the second set of brackets are produced by the lower side frequency $f_c - f_m$.

Note that with $f_c > f_m$, the first and third terms in $v(t)$, both of which relate to carrier frequency $2f_c$ are removed by a low-pass filter. This would then leave the second and fourth sinusoidal terms, both of frequency f_m, as the only output of the filter. The coherent detector thus reproduces the original modulating wave of frequency f_m, with the output consisting of two contributions, one due to the upper side frequency and the other due to the lower side frequency.