CS-419m: Quiz 2

Aug 17, 2018. 10:15–10:55 AM

Roll: _____

Name: _____

1. Assume that you have a sample $D = -3, -1, 4, 20$ of points generated from a Laplace distribution for which $\Pr(x|\mu, b) = \frac{1}{2b}\exp(-\frac{|x-\mu|}{b})$. Our goal is to use D to estimate the parameters μ, b of the distribution. [In answering these questions use the given sample points in writing various formulae.]

   (a) Write the maximum likelihood objective for estimating μ, b using D.

   $$\max_{\mu,b} -\frac{|-3-\mu| + |-1-\mu| + |4-\mu| + |20-\mu|}{b} - 4\log(2b) \tag{1}$$

   ..2

   (b) For the above objective, write down *all* the optimal values of

   i. μ `The objective is minimized for any value between -1 and 4.` ..2

   ii. b `Pick,` $\mu = 4$`, then the objective is` $-\frac{28}{b} - 4\log b$ `+ const. This is maximized for` $b = \frac{28}{4} = 7$`.` ..2

2. Suppose we are given this training sample with a single attribute: $(0, 1), (1, 2), (2, 1), (5, 2)$ in the form of (x_1^i, y^i) pairs. Assume the attribute x_1 follows a Gaussian distribution.

   (a) Write down all the parameters estimated when using the naive Bayes classifier on the above dataset. ..2

   $$\mu_1 = 1, \quad \sigma_1^2 = 1$$
   $$\mu_2 = 6/2 = 3, \quad \sigma_2^2 = (2^2 + 2^2)/2 = 4$$
   $$\pi_1 = 1/2 \quad \pi_2 = 1/2.$$

   (b) Write down the equation of the decision boundary for the naive Bayes classifier. That is, rewrite the criteria $\log\Pr(y = 1|x_1) > \log\Pr(y = 2|x_1)$ in terms of x_1 and the above parameters. ..3

   $$-\frac{(x-\mu_1)^2}{2\sigma_1^2} - \log\sigma_1 > -\frac{(x-\mu_2)^2}{2\sigma_2^2} - \log\sigma_2 \tag{2}$$
   $$-\frac{(x-1)^2}{2} - \log 1 > -\frac{(x-3)^2}{2*4} - \log 2 \tag{3}$$
   $$-\frac{(x-1)^2}{2} + \frac{(x-3)^2}{2*4} + \log 2 > 0 \tag{4}$$

   `Simplify this even further..`

   (c) Is the above decision boundary linear in x_1? Why? ..1 `No. Because the variance for the two classes are different.`

   (d) Write down the parameters estimated using LDA (Fisher's discriminate) on the above dataset ..2

   $$\mu_1 = 1,$$
   $$\mu_2 = 3$$
   $$\sigma^2 = (2^2 + 2^2 + 1 + 1)/4 = 2.5$$
   $$\pi_1 = 1/2 \quad \pi_2 = 1/2$$

(e) Identify a point whose predictions from the naive Bayes and LDA classifier is different. ..2 `x=-100. NB will predict this as class 1 whereas LDA will predict as class 2. Several other answers are also possible.`

(f) Suppose instead of a Gaussian distribution, we decide to use the Poisson distribution to model the distribution of x_1 in each of the classes. That is, for each class y attribute 1 follows a Poisson distribution with parameter λ_y (Recall that for a Poisson distribution $P(x = k|\lambda) = \frac{\lambda^k e^{-\lambda}}{k!}$) Write the expression for the maximum likelihood estimation of the λ_2 parameter of class 2 in terms of the above training data. ..2
$\log \Pr(x = 1|\lambda_2) + \log \Pr(x = 5|\lambda_2 = \log \lambda_2 - \lambda_2 - \log 1 + 5 \log \lambda_2 - \lambda_2 - \log 5$

(g) For what value of λ_2 is the above expression maximized? ..2 `Differentiate the above and equate to zero to get` $\lambda_2 = (5 + 1)/2$

Total: 20