Problem 2.34

The autocorrelation function of a deterministic signal $g(t)$ is defined by

$$R_g(\tau) = \int_{-\infty}^{\infty} g(t)g(t-\tau)dt \qquad (1)$$

This formula applies to a real-valued signal, which is satisfied by all three signals specified under parts (a) through (c).

(a) $g(t) = \exp(-at)u(t)$, $\quad u(t)$: unit step function

Applying Eq. (1) yields

$$\begin{aligned} R_g(\tau) &= \int_{\tau}^{\infty} \exp(-at)\exp(-a(t-\tau))dt \\ &= \exp(a\tau)\int_{\tau}^{\infty} \exp(-2at)dt \\ &= \exp(a\tau)\left[-\frac{1}{2a}\exp(-2at)\right]_{t=\tau}^{\infty} \\ &= \frac{1}{2a}\exp(-a\tau) \end{aligned}$$

which is depicted in Fig. 1

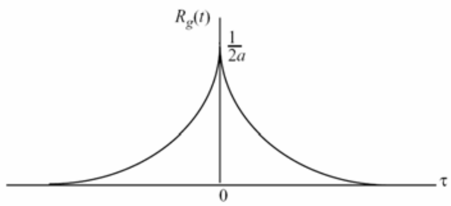

Figure 1

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Problem 2-34 continued

(b) $g(t) = \exp(-a|t|)$

which is sketched in Fig. 2(a). Part (b) of the figure sketches $g(t - \tau) = \exp(-a|t - \tau|)$

(a)

(b)

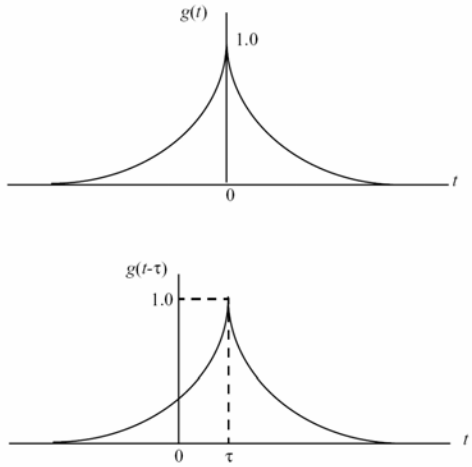

Figure 2

In light of Fig. 1, applying Eq. (1):

$$R_g(\tau) = \int_{\tau}^{\infty} \exp(-at)\exp(-a(t-\tau))dt \quad + \int_{0}^{\tau} \exp(-at)\exp(-a(t-\tau))dt$$

$$+ \int_{0}^{\tau} \exp(at)\exp(a(t-\tau))dt$$

$$= \frac{1}{2a}\exp(-at) + \tau\exp(-at) + \frac{1}{2a}\exp(-a\tau)$$

$$= \left(\frac{1}{a} + \tau\right)\exp(-a\tau)$$

which is sketched in Fig. 3.

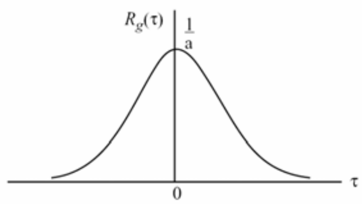

Figure 3

(c) $g(t) = \exp(-at)u(t) - \exp(at)u(-t)$

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Problem 2-34 continued

which is sketched in Fig. 4(a). Part (b) of the figure sketches $g(t - \tau)$

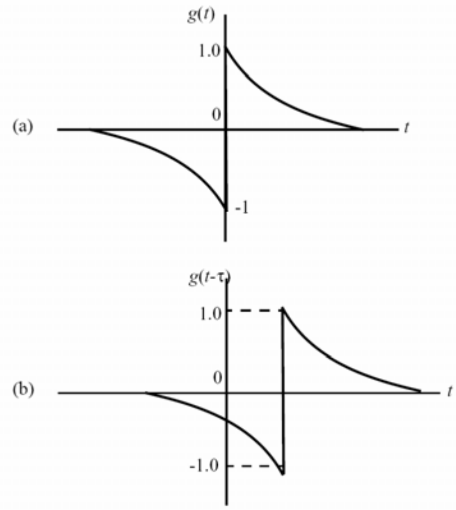

Figure 4

In light of Fig. 4, applying Eq. (1) for $\tau \geq 0$:

$$R_g(\tau) = \int_\tau^\infty \exp(-at)\exp(-a(t-\tau))dt$$

$$+ \int_0^\tau \exp(-at)[-\exp(a(t-\tau))]dt$$

$$+ \int_{-\infty}^0 [-\exp(-at)][-\exp(a(t-\tau))]dt$$

$$= \frac{1}{2a}\exp(-a\tau) - \tau\exp(-a\tau) + \frac{1}{2a}\exp(-a\tau)$$

$$= \left(\frac{1}{a} - \tau\right)\exp(-a\tau), \qquad \tau \geq 0$$

Similarly, for $\tau \leq 0$ we have

$$R_g(\tau) = \left(\frac{1}{a} + \tau\right)\exp(a\tau)$$

Summing up these two results:

$$R_g(\tau) = \begin{cases} \left(\frac{1}{a} - \tau\right)\exp(-a\tau), & \tau \geq 0 \\ \left(\frac{1}{a} + \tau\right)\exp(a\tau), & \tau \leq 0 \end{cases}$$

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Problem 2-34 continued

which is sketched in Fig. 5.

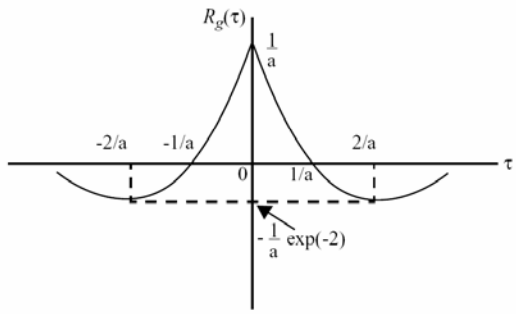

Figure 5