Problem 3.32

We will approach the solution to this problem by showing that, as postulated in the problem, if the in-phase component $H_I(f)$ of the complex low-pass filter's transfer function and its quadrature component $H_Q(f)$ satisfy the following relations

$$H_I(f) = 1 \qquad \text{for } -W \le f \le W \tag{1}$$

and

$$H_Q(-f) = -H_Q(f) \qquad \text{for } -W \le f \le W \tag{2}$$

then, starting with the frequency-discrimination basis for generating a VSB modulated wave $s(t)$, we may express $s(t)$ containing a vestige of the lower sideband as follows:

$$s(t) = \frac{A_c}{2}m(t)\cos(2\pi f_c t) - \frac{A_c}{2}m'(t)\sin(2\pi f_c t) \tag{3}$$

where $m'(t)$ is obtained by passing the message signal $m(t)$ through the quadrature filter defined by $H_Q(f)$.

To proceed, from Eq. (3.44) in the text, recall the relation

$$\frac{1}{2}\tilde{H}(f - f_c) = H(f), \qquad f > 0 \tag{4}$$

The corresponding relation for negative frequencies is described by

$$\frac{1}{2}\tilde{H}^*(f + f_c) = H(f), \qquad f < 0 \tag{5}$$

Using frequency discrimination as the basis for generating the VSB modulated wave $s(t)$, we express the spectrum of $s(t)$ as

$$S(f) = \frac{A_c}{2}[M(f - f_c) + M(f + f_c)]H(f) \tag{6}$$

where $M(f) = \mathbf{F}[s(t)]$. Next, using Eqs., (4) and (5) in (6), we write

$$\begin{aligned} S(f) &= \frac{A_c}{4}[M(f - f_c) + M(f + f_c)][\tilde{H}(f - f_c)\tilde{H}^*(f + f_c)] \\ &= \frac{A_c}{4}M(f - f_c)\tilde{H}(f - f_c) + \frac{A_c}{4}M(f + f_c)\tilde{H}^*(f + f_c) \end{aligned} \tag{7}$$

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where it is recognized that the cross-product terms

$M(f-f_c)\tilde{H}^*(f+f_c)$ and $M(f+f_c)\tilde{H}^*(f-f_c)$ are both zero, because the individual factors in each product term occupy completely disjoint frequency bands. Setting

$$\tilde{H}(f) = H_I(f) + jH_Q(f)$$

and

$$\tilde{H}^*(f) = H_I(f) - jH_Q(f)$$

we expand Eq. (7) as

$$S(f) = \frac{A_c}{4}[M(f-f_c)H_I(f-f_c) + M(f+f_c)H_I(f+f_c)]$$
$$+ j\frac{A_c}{4}[M(f-f_c)H_Q(f-f_c) - M(f+f_c)H_Q(f+f_c)] \qquad (8)$$

Using the all-pass property of $H_I(f)$ defined in Eq. (1) and the odd-function property of $H_Q(f)$ defined in Eq. (2), we may simplify Eq. (8) as

$$S(f) = \frac{A_c}{4}[M(f-f_c) + M(f+f_c)]$$
$$+ j\frac{A_c}{4}[M(f-f_c) - M(f+f_c)]H_Q(f) \qquad (9)$$

Transforming Eq. (9) into the time domain, we obtain the formula of Eq. (3) for the VSB modulated wave $s(t)$.

As noted earlier, $m'(t)$ is obtained by passing the message signal $m(t)$ through the quadrature filter. In accordance with the description of $H_Q(f)$ depicted in the problem, we may depict the frequency response of the quadrature filter as in Fig. 1, where f_v denotes the vestigial bandwidth.

The important point to note from the solution to this problem is that Eq. (3) includes SSB modulation as a special case. Specifically, if $f_v = 0$, then the frequency response depicted in Fig. 1 simplifies to a signum function. Correspondingly, Eq. (3) reduces to a SSB modulated wave containing the upper sideband.

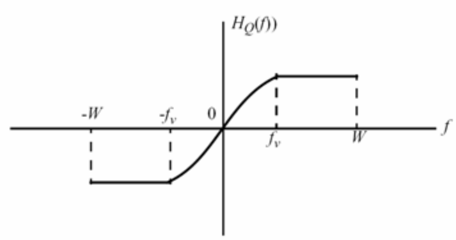

Figure 1