Problem 3.16

Starting with the complex low-pass system depicted in Fig. 3.26(c), show that the $y(t)$ derived in Eq. (3.45) is identical to the actual output $y(t)$ in Fig. 3.26(a).

Solution

According to Fig. 3.25(a), we have

$$Y(f) = H(f)S(f) \tag{1}$$

and according to Fig. 3.25(b),

$$2\tilde{Y}(f) = \tilde{H}(f)\tilde{S}(f) \tag{2}$$

From Eq. (3.44) we note that

$$\tilde{H}(f - f_c) = 2H(f) \qquad \text{for } f > 0 \tag{3}$$

Therefore, substituting Eq. (3) into (2) and cancelling the common factor 2, we get

$$\tilde{Y}(f - f_c) = H(f)\tilde{S}(f - f_c), \qquad f > 0 \tag{4}$$

Finally, noting that for $f > 0$

$$Y(f) = \tilde{Y}(f - f_c)$$

and

$$S(f) = \tilde{S}(f - f_c),$$

we readily see that Eq. (3) is a rewrite of Eq. (1), which validates the outputs displayed in Fig. 3.26.