Problem 2.14

A tapped-delay-line filter consists of N weights, where N is odd. It is symmetric with respect to the center tap, that is, the weights satisfy the condition

$$w_n = w_{N-1-n}, \qquad 0 \le n \le N-1$$

(a) Find the amplitude response of the filter.

(b) Show that this filter has a linear phase response. What is the implication of this property?

Solution

The impulse response of the filter is

$$h(t) = \sum_{n=0}^{N-1} w_n \delta(t - n\Delta\tau)$$

Hence, the frequency response of the filter is

$$H(f) = \sum_{n=0}^{N-1} w_n \exp(-j2\pi nf\Delta\tau)$$

To illustrate, consider the example of $N = 5$. Then

$$\begin{aligned} H(f) &= w_0 + w_1 \exp(-j2\pi f\Delta\tau) + w_2 \exp(-j4\pi f\Delta\tau) + w_3 \exp(-j6\pi f\Delta\tau) + w_4 \exp(-j8\pi f\Delta\tau) \\ &= \exp(-j4\pi f\Delta\tau)[w_0 \exp(j4\pi f\Delta\tau) + w_1 \exp(j2\pi f\Delta\tau) + w_2 + w_3 \exp(-j2\pi f\Delta\tau) \\ &\qquad + w_4 \exp(-j4\pi f\Delta\tau)] \end{aligned} \tag{1}$$

For this example, the symmetry condition

$$w_n = w_{N-1-n} \quad \text{for } 0 \le n \le N\text{-}1$$

reads as

$$w_n = w_{4-n} \quad \text{for } 0 \le n \le 4$$

Hence, $w_0 = w_4$ and $w_1 = w_3$. Accordingly, we may rewrite Eq. (1) as

$$\begin{aligned} H(f) &= \exp(-j4\pi f\Delta\tau)[w_0 \exp(j4\pi f\Delta\tau) + \exp(-j4\pi f\Delta\tau) \\ &\qquad + w_1(\exp(j2\pi f\Delta\tau)) + \exp(-j2\pi f\Delta\tau) \\ &\qquad + w_2] \\ &= \exp(-j4\pi f\Delta\tau)[2w_0 \cos(4\pi f\Delta\tau) + 2w_1(2\pi f\Delta\tau) + w_2] \end{aligned}$$

We may therefore generalize this result as

$$H(f) = \exp\left(-j2\pi\left(\frac{N-1}{2}\right)f\Delta\tau\right)\left[w_{\frac{N-1}{2}} + 2\sum_{n=0}^{\frac{N-1}{2}-1} w_n \cos(2\pi nf\Delta\tau)\right]$$

(a) The amplitude response of the filter is therefore

$$|H(f)| = w_{\frac{N-1}{2}} + 2\sum_{n=0}^{\frac{N-1}{2}-1} w_n \cos(2\pi nf\Delta\tau)$$

(b) The phase response of the filter is therefore

$$\arg(H(f)) = \exp\left(-j2\pi\left(\frac{N-1}{2}\right)f\Delta\tau\right)$$

which is linear with respect to the frequency f. The implication of this condition is that except for a delay, there is no phase distortion produced by the filter.

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