Problem 5.1

(a) Using the material presented in Section 2.5, justify the mathematical relationships listed at the bottom of the left-hand side of Table 5.1, which pertain to ideal sampling in the frequency domain.

(b) Applying the duality property of the Fourier transform to part (a), justify the mathematical relationships listed at the bottom of the right-hand side of this table, which pertain to ideal sampling in the time-domain.

Solution

1. Entry 1 on the left-hand side of Table 5.1:
   - The relationship

     $$\sum_{m=\infty}^{\infty} g(t-mT_s) = f_s \sum_{n=-\infty}^{\infty} G(nf_s)e^{j2\pi nf_st}$$

     where $g(t) \rightleftharpoons G(f)$ and $f_s = 1/T_s$, is a rewrite of Eq. (2.87) with one trivial change, namely, the replacements of T_o and f_o by T_s and f_s, respectively.
   - The Fourier transform pair

     $$\sum_{m=\infty}^{\infty} g(t) - m(T_s) \rightleftharpoons f_s \sum_{n=-\infty}^{\infty} G(nf_s)\delta(f-f_s)$$

     is also a rewrite of Eq. (2.88) except for the replacement of T_o and f_o with T_s and f_s, respectively.
2. Entry 2 on the right-hand side of Table 5.2:
   - The relationship

     $$\sum_{n=\infty}^{\infty} g(nT_s)e^{j2\pi nf_st} = f_s \sum_{m=-\infty}^{\infty} G(nf_sf-mf_s)$$

     is an exact reproduction of the equality in Eq. (5.2).
   - The Fourier-transform pair

     $$\sum_{n=\infty}^{\infty} g(nT_s)\delta(t-nT_s) \rightleftharpoons f_s \sum_{m=-\infty}^{\infty} G(f-mf_s)$$

     is an exact reproduction of the Fourier-transform pair listed in Eq. (5.2).