Problem 5.3

Show that

$$\frac{1}{2W} \int_{-W}^{W} \exp\left[j2\pi f \left(t - \frac{n}{2W}\right)\right] df = \frac{\sin(2\pi Wt - n\pi)}{(2\pi Wt - n\pi)}$$
$$= \sin(2Wt - n)$$

Solution

$$\frac{1}{2W} \int_{-W}^{W} \exp\left[j2\pi f\left(t - \frac{n}{2W}\right)\right] df = \frac{1}{2W} \cdot \frac{1}{j2\pi(t - n/2W)} \cdot \exp\left[j2\pi f\left(t - \frac{n}{2W}\right)\right]_{f = -W}^{W}$$

$$= \frac{1}{j4\pi W(t - n/2W)} \cdot \left[\exp j\pi(2Wt - n) - \exp(-j\pi(2Wt - n))\right]$$

$$= \frac{\sin(\pi(2Wt - n))}{\pi(2Wt - n)}$$

$$= \sin c(2Wt - n)$$

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