Problem 5.9

Using Eqs. (5.23) and (5.25), respectively, derive the slope characteristics of Eqs. (5.24) and (5.26).

Solution

(a) The logarithmic law is defined by (see Eq. (5.23)

$$|v| = \frac{\log(1+\mu|m|)}{\log(1+\mu)}$$

Therefore, differentiation with respect to $|m|$ yields

$$\frac{d|v|}{d|m|} = \frac{1}{\log(1+\mu)} \cdot \frac{\mu}{1+\mu|m|}$$

Equivalently, we may write

$$\frac{dm}{d|v|} = \log(1+\mu)\frac{1+\mu|m|}{\mu}$$

(b) The A-law is defined by (see Eq. (5.25):

$$|v| = \begin{cases} \dfrac{A|m|}{1+\log A}, & 0 \le |m| \le \dfrac{1}{A} \\ \dfrac{1+\log(A|m|)}{1+\log A}, & \dfrac{1}{A} \le |m| \le 1 \end{cases}$$

Hence, differentiation of $|v|$ with respect to $|m|$ yields

$$\frac{d|v|}{d|m|} = \begin{cases} \dfrac{A}{1+\log A}, & 0 \le |m| \le \dfrac{1}{A} \\ \dfrac{A}{|m|(1+\log A)}, & \dfrac{1}{A} \le |m| \le 1 \end{cases}$$

Equivalently, we may write

$$\frac{d|m|}{d|v|} = \begin{cases} \dfrac{1+\log A}{A}\ , & 0 \le |m| \le \dfrac{1}{A} \\ \left(\dfrac{1+\log A}{A}\right)|m|, & \dfrac{1}{A} \le |m| \le 1 \end{cases}$$