Problem 2.39

We will determine the autocorrelation function of the signal g(t) depicted in Fig. 2.45 by proceeding on a segment-by-segment basis:

- 1. The maximum value of $R_g(\tau)$ occurs at $\tau = 0$, for then g(t) and $g(t-\tau)$ overlap exactly, yielding $R_o(0) = 3(A^2)(T) = 3A^2T$
- 2. For $0 < \tau < (T/2)$, we have the picture depicted in Fig. 1. From this figure, we obtain

$$\begin{split} R_g(\tau) &= \int_{-(3T/2)+\tau}^{T/2} (+A)(+A)dt + \int_{-T/2}^{(T/2)+\tau} (+A)(-A)dt \\ &+ \int_{-(T/2)+\tau}^{T/2} (+A)(+A)dt + \int_{T/2}^{(T/2)+\tau} (-A)(+A)dt \\ &+ \int_{(T/2)+\tau}^{3T/2} (-A)(-A)dt \\ &= A^2(T-\tau) - A^2\tau + A^2(T-\tau) - A^2\tau + A^2(T-\tau) \\ &= A^2(3T-5\tau), \qquad 0 < |\tau| < (T/2) \end{split}$$

where the use of $|\tau|$ is invoked in light of the symmetric property of the autocorrelation function.

3. Next, for $(T/2) < \tau < T$, we have the picture depicted in Fig. 2, from which we obtain

$$R_g(\tau) = \int_{-(3T/2)+\tau}^{T/2} (-A)(-A)dt + \int_{-T/2}^{(T/2)+\tau} (+A)(-A)dt$$

$$+ \int_{-(T/2)+\tau}^{T/2} (+A)(+A)dt + \int_{T/2}^{(T/2)+\tau} (-A)(+A)dt$$

$$+ \int_{(T/2+\tau)}^{3T/2} (-A)(-A)dt$$

$$= A^{2}(T-\tau) - A^{2}\tau + A^{2}(T-\tau) - A^{2}\tau + A^{2}(T-\tau)$$

$$= A^{2}(3T-5\tau), \qquad \frac{T}{2} < |\tau| < T$$

4. Next, for $T < \tau < 3T/2$, we have the picture depicted in Fig. 3, from which we obtain

$$R_g(\tau) = \int_{-(3T/2)+\tau}^{T/2} (+A)(-A)dt + \int_{T/2}^{-(T/2)+\tau} (-A)(-A)dt + \int_{-(T/2)+\tau}^{3T/2} (-A)(+A)dt$$
$$= -A^2(2T - \tau) + A^2(-T + \tau) - A^2(2T - \tau)$$
$$= A^2(-5T + 3\tau) \quad \text{for } T < |\tau| < 3T/2$$

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Problem 2-39 continued

5. For $(3T/2) < \tau < 2T$ we have the picture depicted in Fig. 4, from which we obtain

$$R_g(\tau) = \int_{-(3T/2)+\tau}^{T/2} (+A)(-A)dt + \int_{T/2}^{-(T/2)+\tau} (-A)(-A)dt + \int_{-(T/2)+\tau}^{3T/2} (-A)(+A)dt$$

$$= -A^2(2T - \tau) + A^2(-T + \tau) - A^2(2T - \tau)$$

$$= A^2(-5T + 3\tau) \qquad \text{for } (3T/2) + |\tau| < 2T$$

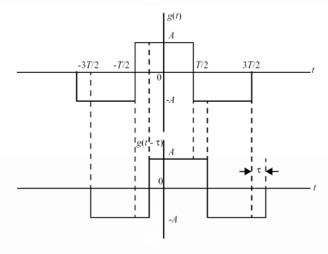


Figure 1: $-(T/2) < \tau < (T/2)$

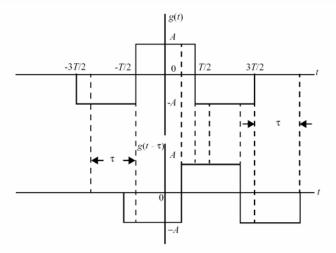


Figure 2: $(T/2) < \tau < (3T/2)$

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Problem 2-39 continued

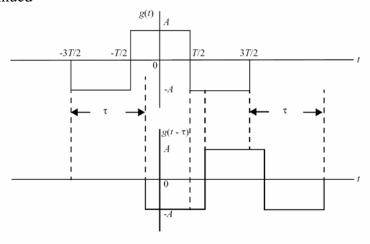


Figure 3: $T < \tau < (3T/2)$

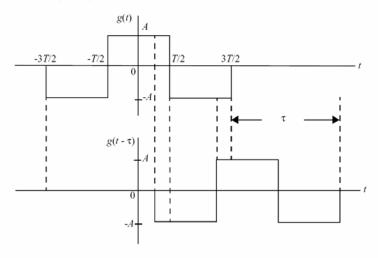


Figure 4: $(3T/2) < \tau < 2T$

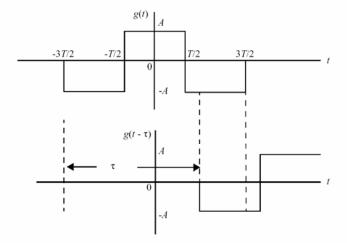


Figure 5: $2T < \tau < 3T$

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7. Finally, for $|\tau| > 3T$, we find that $R_g(\tau) = 0$. Putting all these pieces together, we get the autocorrelation function $R_g(\tau)$ plotted in Fig. 6, which is symmetric about the origin $\tau = 0$.

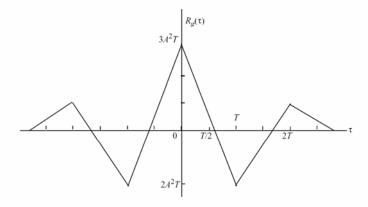


Figure 6: Plot of the autocorrelation function $R_g(t\tau)$