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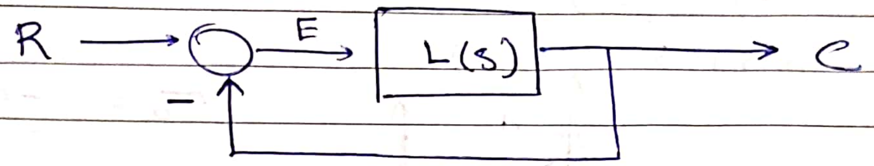
Error constants

Std. formulas for steady state error



System types

Type 0 : $L(s) = \dfrac{K \prod (s-z_i)}{\prod (s-p_i)}$

Type 1 : $L(s) = \dfrac{1}{s} L_1(s)$

→ $L_1(s)$ of type 0

Type 2 : $L(s) = \dfrac{1}{s^2} L_1(s)$

$E = \dfrac{R}{1+L(s)}$

Ref. input R **is of three type**

1. Step Input
2. Ramp Input
3. Parabolic Input

Type 0 : Step Input

R is of three type

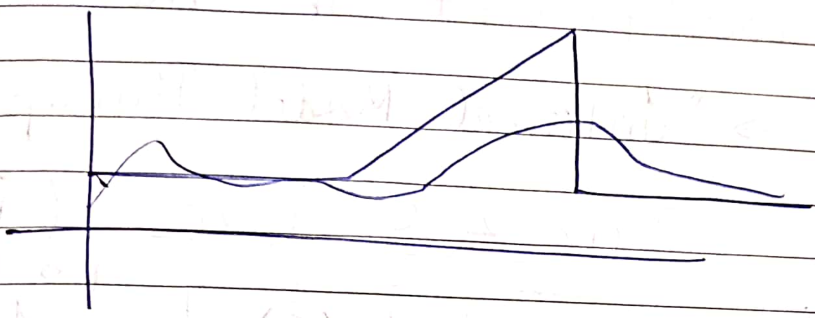
and L is of three type

nine permutation

Type 0:

Ramp input: $E = \dfrac{1/s^2}{1+L(s)}$

$e_{ss} = \infty$



Type 1 System: Step input (Always asymptotically followed)

$$E = \frac{1/s}{1+\frac{1}{s}L_1(s)}$$

$$E(s) = \frac{1}{s+L_1(s)} = \frac{s}{s+L_1(s)} = 0 \quad (s \cdot s)$$

SEs = velocity of error

~~ess~~

Type 1 system: Ramp Input

$$E = \frac{1/s^2}{1+\frac{1}{s}L_1(s)}$$

$$= \frac{1}{s^2+sL_1(s)}$$

$$sE(s) = \frac{1}{s+L_1(s)}$$

$$e_{ss} = \frac{1}{L_1(0)} \qquad k_v = L_1(0) = \lim_{s\to 0} sL(s)$$

Velocity error ← $\boxed{= \dfrac{1}{k_v}}$ → ~~Cannot~~ Cannot ~~Asy.~~ be asymptotically followed.

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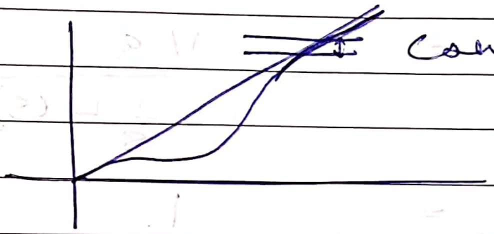
→ In general, an input can only be Asymptotically followed if the corresponding poles are present in the System.

→ "Internal Model Principle."

Type 2 System (Follows upto Ramp Input)

$$\boxed{L(s) = \frac{1}{s^2} L_1(s)}$$

→ ~~How~~

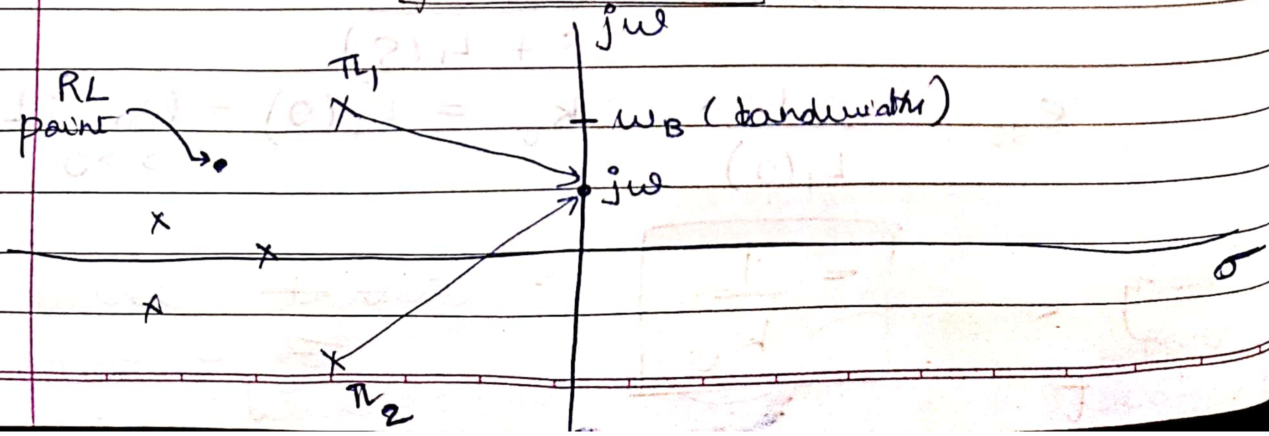


Step Input : $e_{ss} = 0$
Ramp Input : $e_{ss} = 0$

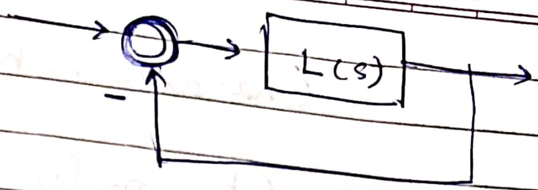
Parabolic Input : $e_{ss} = \frac{1}{k_a}$

k_a → acceleration error constant

Frequency Response Calculation from RL



$$L(s) = K L_1(s)$$



$$1 + L(s) = \frac{\prod (s - r_i)}{\prod (s - p_i)}$$

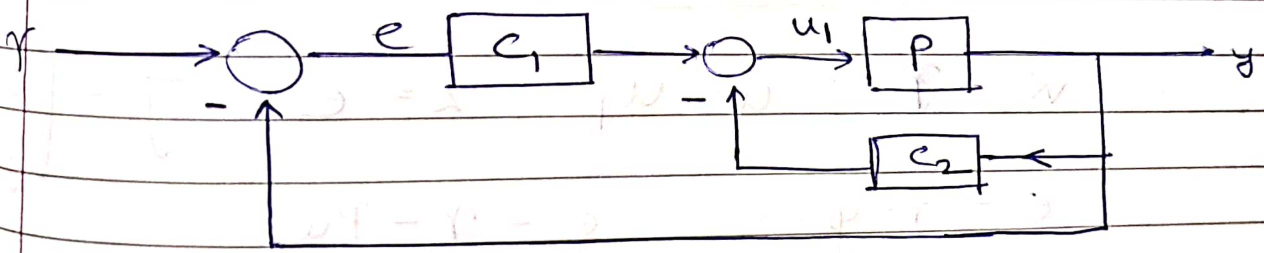
$r_i \rightarrow$ closed loop poles

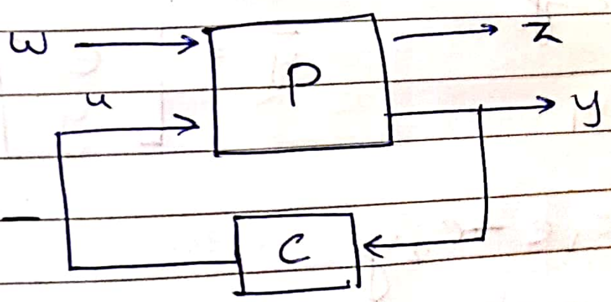
p_i poles of OL function $L(s)$

$j\omega - p_i$, $j\omega - r_i$

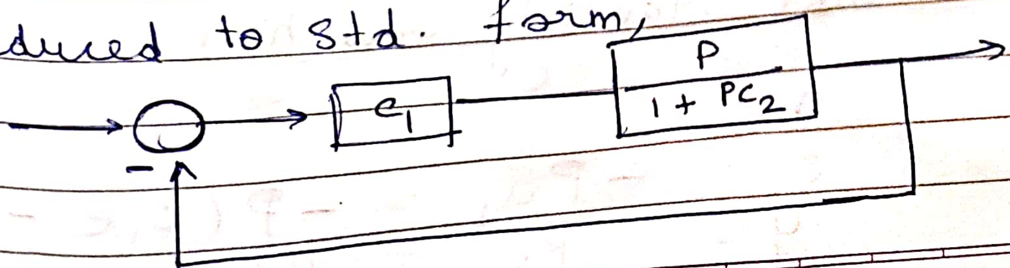
→ This is the search on $j\omega$ Axis code from Book.

Modern Control Theory Ideas





→ Reduced to std. form,



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Characteristic equation: $1 + \frac{c_1 P}{1 + c_2 P} = 0$

Ch. poly: $P = \frac{q}{p}$ $c_1 = \frac{q_1}{p_1}$ $c_2 = \frac{q_2}{p_2}$

$$\psi = 1 + \frac{(q_1 q)/p_1 p}{(p_2 p + q_2 q)/p_2 p}$$

$$= 1 + \frac{q_1 q p_2}{\cancel{p} p_1 (p_2 p + q_2 q)}$$

→ We design the whole system at once rather than 1 loop at a time.

- From 4 Block Model

$w = r$ $u = u_1$ $z = e$ $\bar{y} = \begin{bmatrix} e \\ y \end{bmatrix}$

$e = r - y$ $e = r - Pu_1$

$\bar{y} = \begin{bmatrix} e \\ y \end{bmatrix} = \begin{bmatrix} r - Pu_1 \\ Pu_1 \end{bmatrix}$

$y = Pu_1$

$u_1 = c_1 e - c_2 y$

→ Internal Stability condition

$r = 0$

$e = -Pu_1 = -P(c_1 e - c_2 y)$

$y = Pu_1 = P(c_1 e - c_2 y)$

$\bar{y} \rightarrow$ Differential Equation in elements of $\bar{y}$

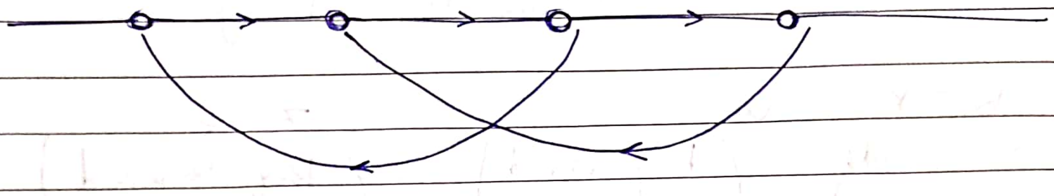
$w = r$ (Input outside our control) $u = u_1$ (Regulated i/p) $z = e$ (Reg observed) $\bar{y} = \begin{bmatrix} e \\ y \end{bmatrix}$

$$\begin{bmatrix} 1 + PC_1 & -PC_2 \\ -PC_1 & 1 + PC_2 \end{bmatrix} \begin{bmatrix} e \\ y \end{bmatrix} = 0$$

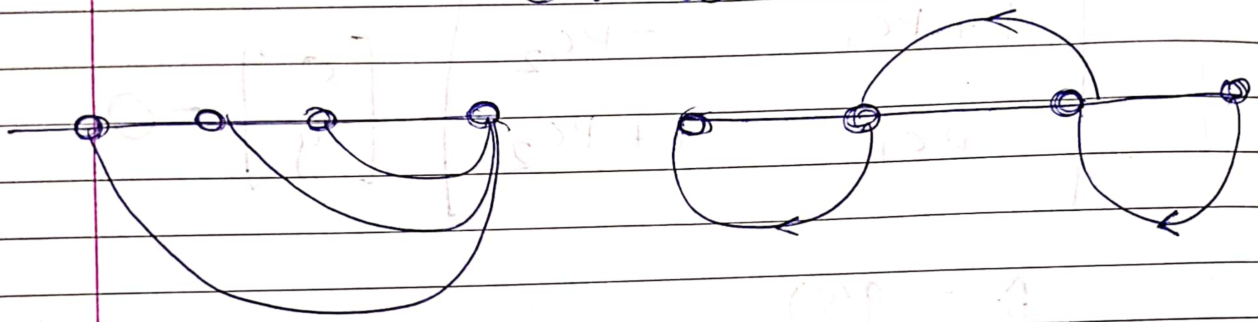
$$\hat{p} = \frac{q(s)}{p(s)}$$

$$\begin{bmatrix} \frac{pp_1 + qq_1}{pp_1} & -qq_2 \\ & \end{bmatrix}$$

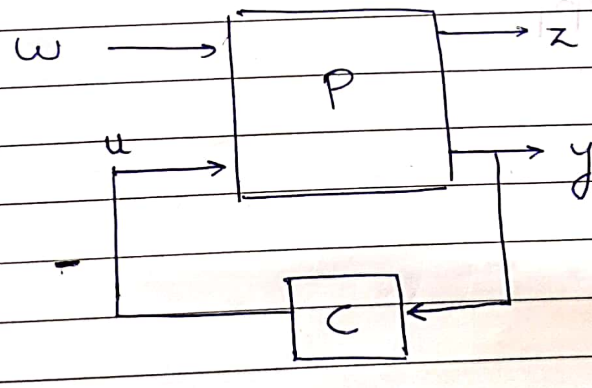
Characteristic Equation of Multiloop systems



Reduce to either hierarchy or cascade.



- Char polynomial of 4 Block Control System :



(could be vectors, each of 4 vars)

$$\begin{bmatrix} z \\ y \end{bmatrix} = \begin{bmatrix} P_1 & P_2 \\ P_3 & P_4 \end{bmatrix} \begin{bmatrix} w \\ u \end{bmatrix}$$

$$u = -Cy$$

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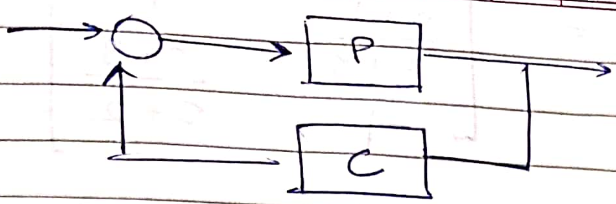

$$P = \frac{q(s)}{p(s)}$$

$$C = \frac{q_c(s)}{p_c(s)}$$

Characteristic eqn : $1 + PC = 0$

$$\Rightarrow \frac{pp_c + qq_c}{pp_c} = 0$$

Stability condition, $\psi = pp_c + qq_c$ is Hurwitz

→ Checking Internal Stability condition ($\to 0$ as $\to \infty$)

$$\Rightarrow w = 0$$

$$\begin{bmatrix} z \\ y \end{bmatrix} = \begin{bmatrix} P_2 \\ P_4 \end{bmatrix} u = -\begin{bmatrix} P_2 \\ P_4 \end{bmatrix} Cy$$

$$\Rightarrow z = -P_2 C y$$

$$y = -P_4 C y$$

$y \to 0$, $z \to 0$ as $t \to \infty$

$$M(s)\,\hat{y} = 0$$

$$(I + P_4 C)\,y = 0$$

$\cancel{\det (I + P_4 C) =}$

$$\det(M(s)) = 0$$

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$$M(s) = \begin{bmatrix} \frac{s+1}{s+3} & \frac{1}{s+3} \\ -1 & \frac{s+3}{s+2} \end{bmatrix}$$

$$\left(\frac{(s+1)}{(s+3)}\right) y_1 + y_2 = 0$$

$$-y_1 + \left(\frac{s+3}{s+2}\right) y_2 = 0$$

$$(s+1) y_1 + (s+3) y_2 = 0$$

$$-(s+2) y_1 + (s+3) y_2 = 0$$

$$\left(\frac{d}{dt} + 1\right) y_1 + \left(\frac{d}{dt} + 3\right) y_2 = 0$$

$$-\left(\frac{d}{dt} + 2\right) y_1 + \left(\frac{d}{dt} + 3\right) y_2 = 0$$

$$P\left(\frac{d}{dt}\right) \bar{y} = 0$$

$$\det\left(P\left(\frac{d}{dt}\right)\right) = 0$$

ex

→ Stability of state space system :

Homogenous system of diff. eqns in state-space form

$$\dot{x} = Ax$$

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$$x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

$A \rightarrow n \times n$ real matrix

$$\phi(t) = \exp(At)$$

$$x(t) = \phi(t)$$

$$x(t-t_0) =$$

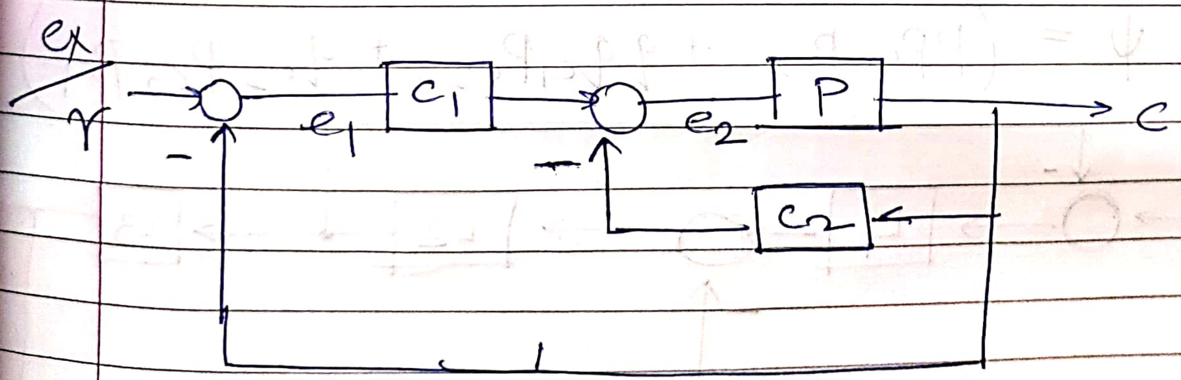
Char. polynomial $\psi = \det(sI - A)$

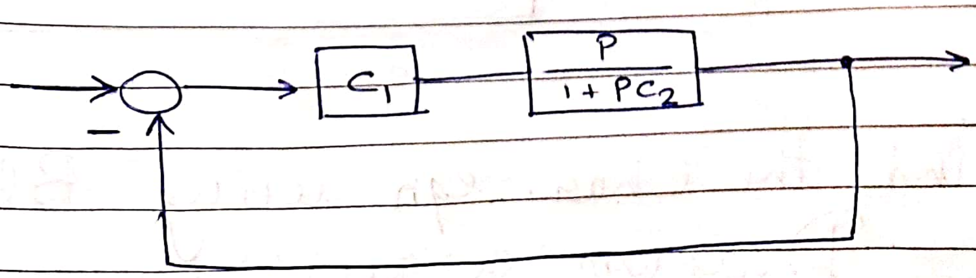
$$s\hat{X}(s) - X(0) = A\hat{x}(s)$$

$$s\hat{x}(s) - A\hat{x}(s) = X(0)$$

$$\hat{x}(s) = (sI - A)^{-1} X(0)$$

↳ Stability of State Space governed by poles of this Matrix





Char. eqn : $1 + \frac{Pc_1}{1+Pc_2} = 0$

$1 + Pc_1 + Pc_2 = 0$

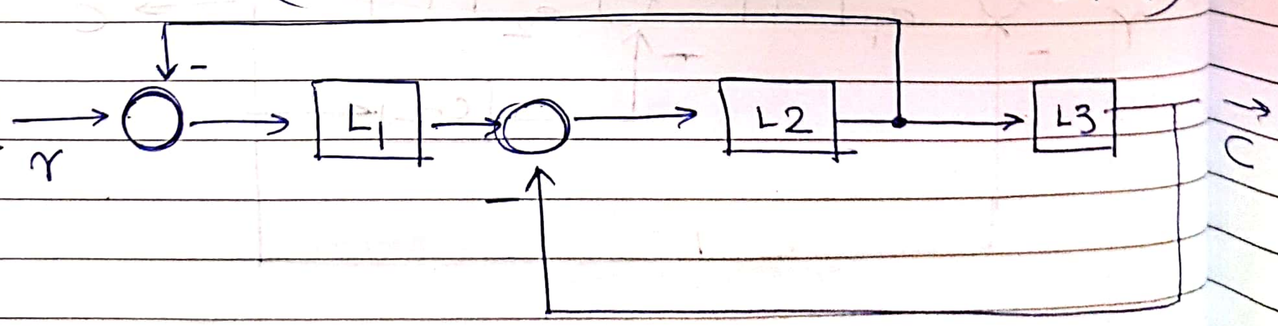
$p = \frac{q}{P}$ $c_1 = \frac{q_{c_1}}{P_{c_1}}$ $c_2 = \frac{q_{c_2}}{P_{c_2}}$

$1 + \frac{qq_{c_1}}{PP_{c_1}} + \frac{q_{c_2}P_{c_2}}{PP_{c_2}}$

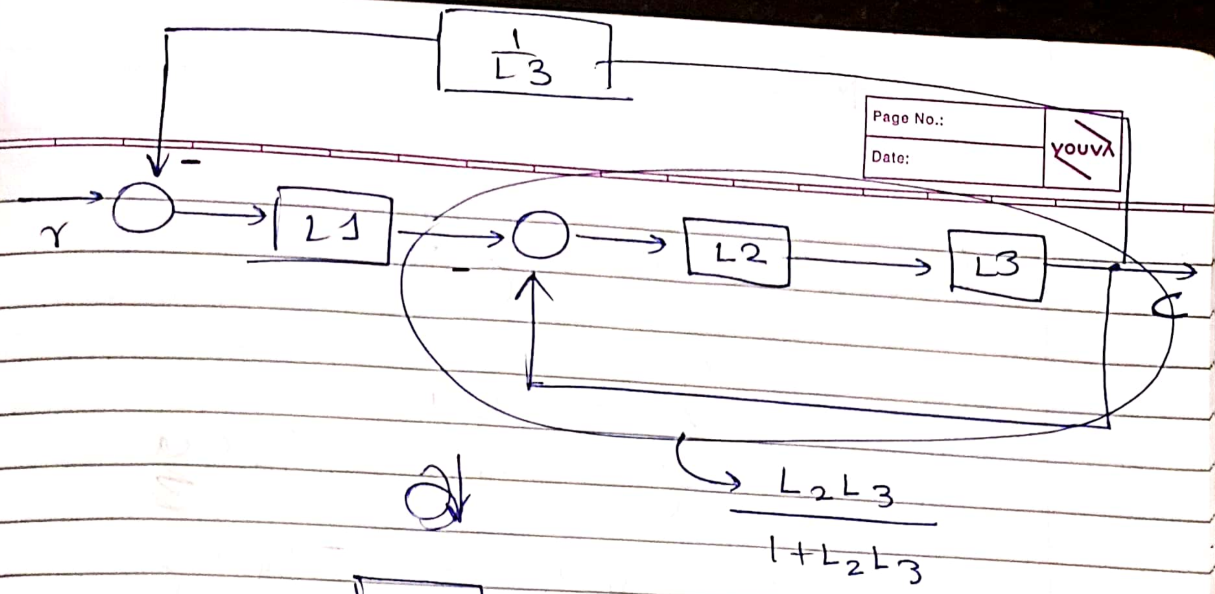
$= \frac{\cancel{PP_{c_1}PP_{c_2}}\ PP_{c_1}P_{c_2} + qq_{c_1}P_{c_2} + q_{c_2}P_{c_2}P_{c_1}}{PP_{c_1}P_{c_2}}$

$\Rightarrow \psi = (PP_{c_1}P_{c_2} + qq_{c_1}P_{c_2} + q_{c_2}P_{c_2}P_{c_1})$

ex2)



Find the char. eqn using Block Diagram reduction.



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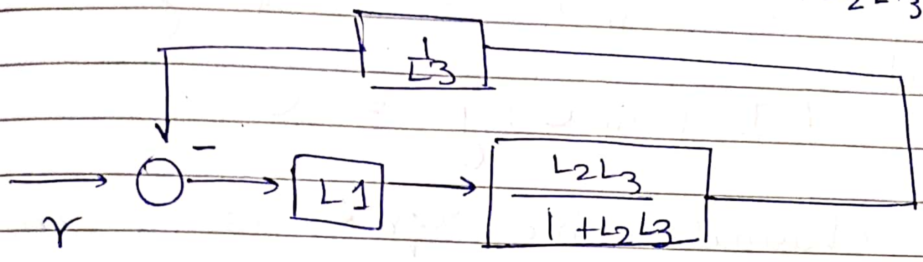

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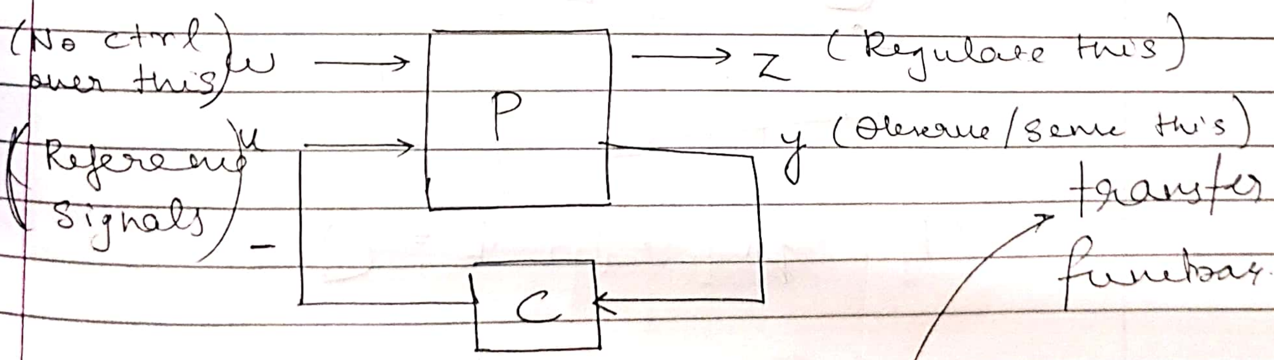
Characteristic Equations of Multiloop Control Systems

4 - Block Model:



$$\begin{bmatrix} z \\ y \end{bmatrix} = \begin{bmatrix} P_1 & P_2 \\ P_3 & P_4 \end{bmatrix} \begin{bmatrix} w \\ u \end{bmatrix}$$

$$\boxed{u = -Cy}$$

Internal Stability :

$$w = 0$$

$$z = P_2 u$$

$$\cancel{y = P_4 = -P_4 C y}$$

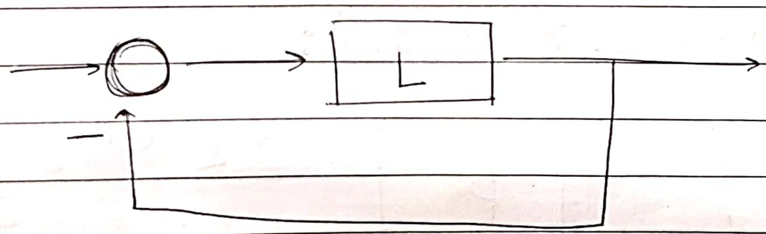
$$y = P_4 u = -P_4 C y$$

$$(I + P_4 C) y = 0$$

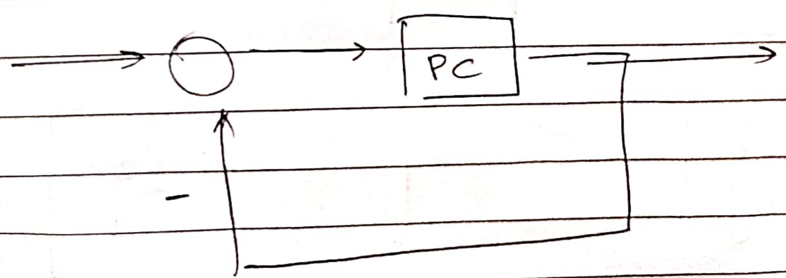
The characteristic eqn, is given by

$\det(I + P_4 C) = 0$: char. eqn

$\frac{\psi}{d}$ ψ - Hurwitz for Stability

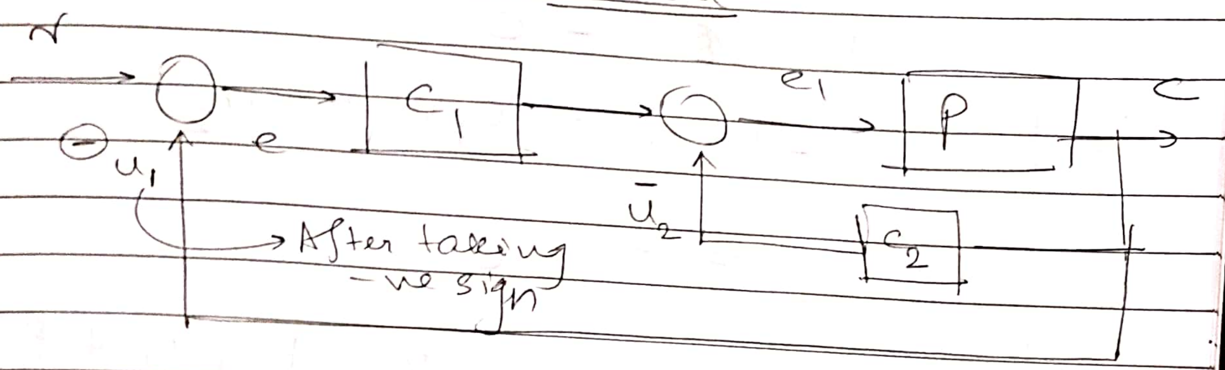


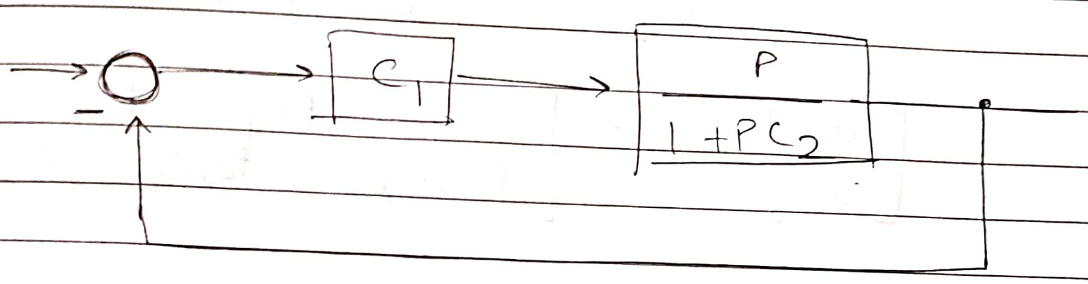
Ch. eqn $1 + L = 0$



$$1 + PC = 0$$

Examples of 4 Block Method





$$1 + \frac{PC_1}{1+PC_2} \Rightarrow \psi \text{ (Char. eqn)}$$

$$= 1 + PC_1 + PC_2$$

4 Block Method

$w : r$ $\qquad$ $u : u_1, u_2$

$y : (r, c)$ $\qquad$ $z : e$

$$e = r + u_1$$

$$r = r$$

$$c = Pe_1 = P(c_1 e + u_2)$$

$$= P(c_1 (r + u_1) + u_2)$$

$$= \left(PC_1 r + PC_1 u_1 + Pu_2\right)$$

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$$u_1 = \quad - c$$

$$= -Pc_1 r - Pc_1 u_1 - P u_2$$

$$u_2 = -c_2 c$$

$$= -c_2 (Pc_1 r + Pc_1 u_1 + P u_2)$$

~~ex 2~~

$$\cancel{\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = -\begin{bmatrix} Pc_1 & Pc_1 & P \\ Pc_1 c_2 & Pc_1 c_2 & Pc_2 \end{bmatrix} \begin{bmatrix} r \\ u_1 \\ u_2 \end{bmatrix}}$$

$$\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 0 & -c_2 \end{bmatrix} \begin{bmatrix} r \\ c \end{bmatrix}$$

$$= -\begin{bmatrix} 0 & 1 \\ 0 & c_2 \end{bmatrix} \begin{bmatrix} r \\ c \end{bmatrix}$$

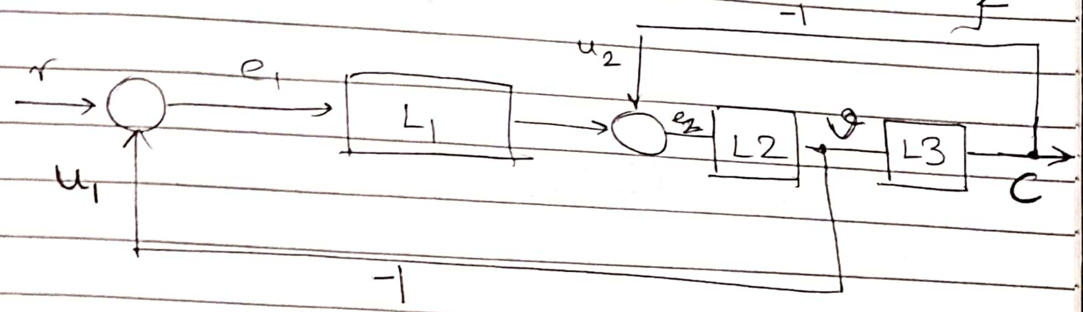
$$\begin{bmatrix} e \\ r \\ c \end{bmatrix} = \left[\begin{array}{c|cc} 1 & 1 & 0 \\ \hline 1 & 0 & 0 \\ Pc_1 & Pc_1 & P \end{array}\right] \begin{bmatrix} r \\ u_1 \\ u_2 \end{bmatrix}$$

$$= \left[\begin{array}{c|c} P_1 & P_2 \\ \hline P_3 & P_4 \end{array}\right] \begin{bmatrix} r \\ \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \end{bmatrix}$$

$$P_4 C = \begin{bmatrix} 0 & 0 \\ Pc_1 & P \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 0 & c_2 \end{bmatrix}$$

$$I + P_4 C = \begin{bmatrix} 1 & 0 \\ 0 & 1 + PC_1 + PC_2 \end{bmatrix}$$

ex2)



$$\begin{bmatrix} \frac{e_1}{r} \\ c \\ v \end{bmatrix} = \left[\begin{array}{c|cc} 1 & 1 & 0 \\ \hline 1 & 0 & 0 \\ L_1L_2L_3 & L_1L_2L_3 & L_2L_3 \\ L_1L_2 & L_1L_2 & L_2 \end{array}\right] \begin{bmatrix} \frac{r}{u_1} \\ u_2 \end{bmatrix}$$

$$P_4 = \begin{bmatrix} 0 & 0 \\ L_1L_2L_3 & L_2L_3 \\ L_1L_2 & L_2 \end{bmatrix}$$

$$\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} -v \\ -c \end{bmatrix} = -\underbrace{\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}}_{C} \begin{bmatrix} r \\ c \\ v \end{bmatrix}$$

$P_4 C$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & L_2L_3 & L_1L_2L_3 \\ 0 & L_2 & L_1L_2 \end{bmatrix}$$

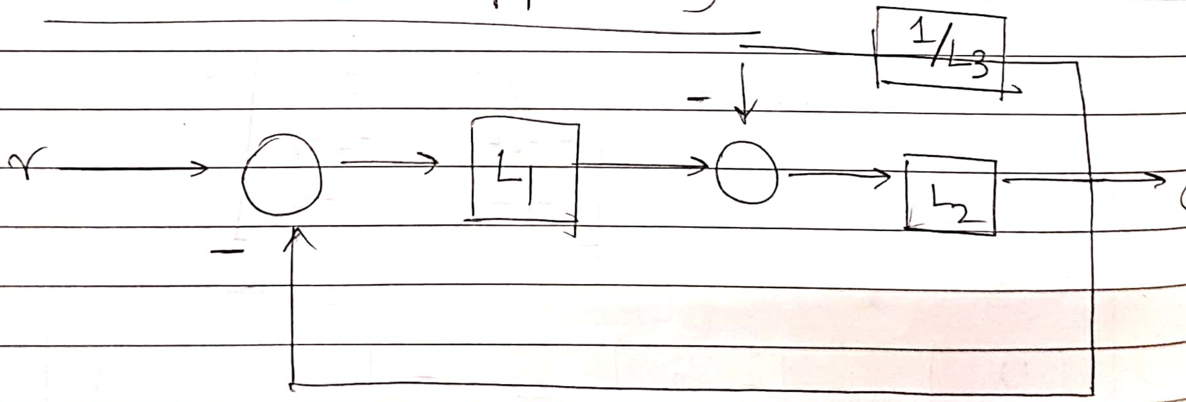
$$I + P_4 C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 + L_2 L_3 & L_1 L_2 L_3 \\ 0 & L_2 & 1 + L_1 L_2 \end{bmatrix}$$

$$\det(I + P_4 C)$$

$$= (1 + L_2 L_3)(1 + L_1 L_2) - L_1 L_2^2 L_3$$

$$= (1 + L_1 L_2 + L_2 L_3)$$

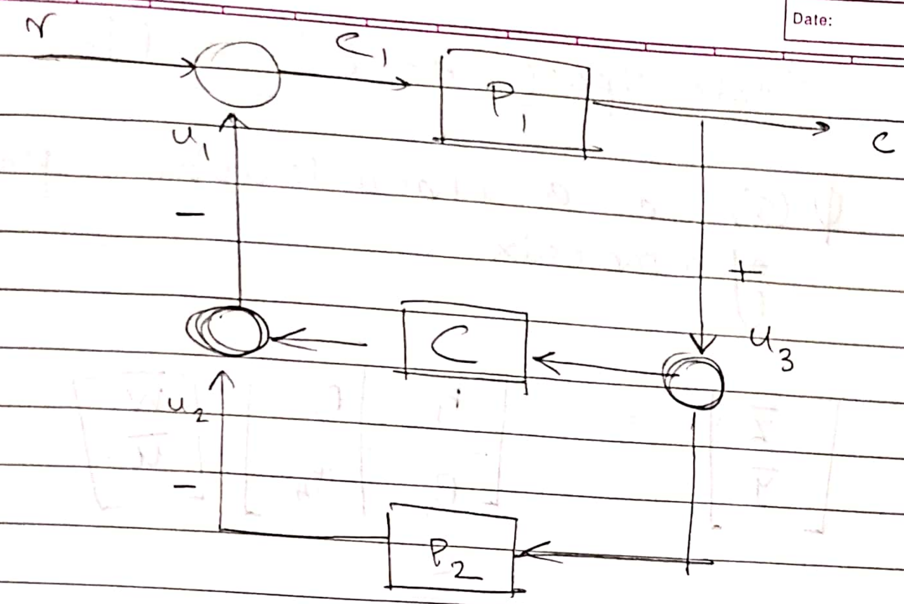
Aliter (Old Approach)



$$1 + \frac{L_2}{1 + L_2/L_3} = 1 + \frac{L_2 L_3}{1 + L_2}$$

$\Rightarrow$ Same Char. eqn.

ex 3)



$$\begin{bmatrix} e_1 \\ \hline r \\ c \end{bmatrix} = \begin{bmatrix} P_1 & P_2 \\ P_3 & P_4 \end{bmatrix} \begin{bmatrix} r \\ \hline u_1 \\ u_2 \\ u_3 \end{bmatrix}$$

$$\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = -C \begin{bmatrix} r \\ c \end{bmatrix}$$

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Computation of Characteristic polynomial

$$\det(I + P_4 C) = 0$$

$$\det M(s) = 0$$

ch. polynomial $\psi(s)$ $\qquad$ $\dfrac{\psi(s)}{p(s)}$

State space 4-Block Model

- $\psi(s)$ as a characteristic polynomial of a matrix

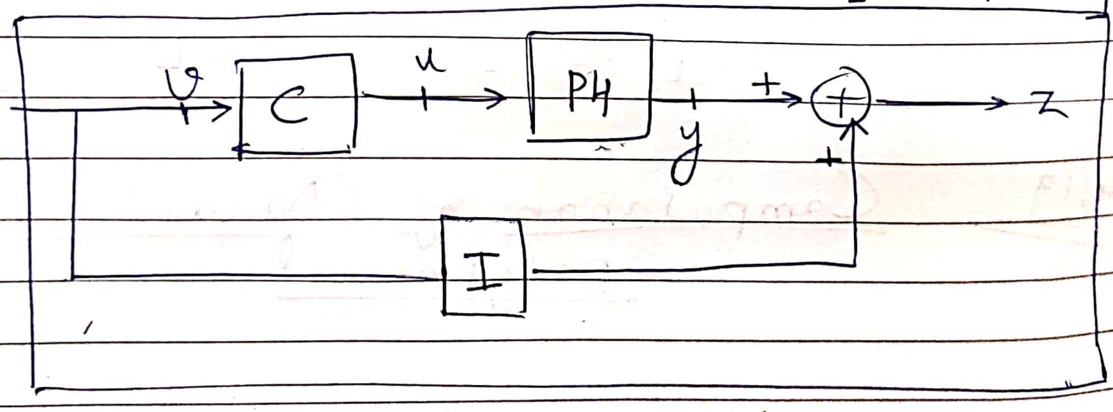
$$\begin{bmatrix} \bar{z} \\ \bar{y} \end{bmatrix} = \left[\begin{array}{c|c} P_1 & P_2 \\ \hline P_3 & P_4 \end{array}\right] \begin{bmatrix} \bar{W} \\ \bar{u} \end{bmatrix}$$

$$\bar{u} = -C\bar{y}$$

$$\begin{bmatrix} P1 & P2 \\ P3 & P4 \end{bmatrix} = \left[\begin{array}{c|cc} A & c_1 & c_2 \\ \hline B_1 & D_1 & D_2 \\ B_2 & D_3 & D_4 \end{array}\right]$$

$$P_2 = \left[\begin{array}{c|c} A & B_2 \\ \hline c_1 & D_2 \end{array}\right] \qquad P_3 = \left[\begin{array}{c|c} A & B1 \\ \hline c2 & D3 \end{array}\right]$$

$$P_4 = \left[\begin{array}{c|c} A & B2 \\ \hline c2 & D4 \end{array}\right] \qquad C = \left[\begin{array}{c|c} A_c & B_c \\ \hline c_c & D_c \end{array}\right]$$



$$\dot{x} = Ax + B_2 u \quad (1)$$

$$y = c_2 x + D_4 u \quad (2)$$

$$\dot{x}_c = A_c x_c + B_c \vartheta \qquad (3)$$

$$u = C_c x_c + B_c \vartheta \qquad (4)$$

→ Sulesihute for u in (1)

$$\dot{x} = Ax + B_2 (C_c x_c + D_c \vartheta)$$
$$= Ax + B_2 C_c x_c + B_2 D_c \vartheta$$

$$I + P_4 C = \left[\begin{array}{cc|c} A & B_2 C_c & B_2 D_c \\ 0 & A_c & B_c \\ \hline C_2 & D_4 C_c & I + D_4 D_c \end{array}\right]$$

$$\begin{bmatrix} \hat{A} & \hat{B} \\ \hat{C} & \hat{D} \end{bmatrix}^{-1} = \left[\begin{array}{c|c} \hat{A} - \hat{B}\hat{D}^{-1}\hat{C} & \hat{B}\hat{D}^{-1} \\ \hline -\hat{D}^{-1}\hat{C} & \hat{D}^{-1} \end{array}\right]$$

$$\left[\begin{array}{c|c} A - B_2 C_c & \\ \hline -(I + D_4 D_c)^{-1}(C_2 \; D_4 C_c) & (I + D_4 D_c)^{-1} \end{array}\right]$$

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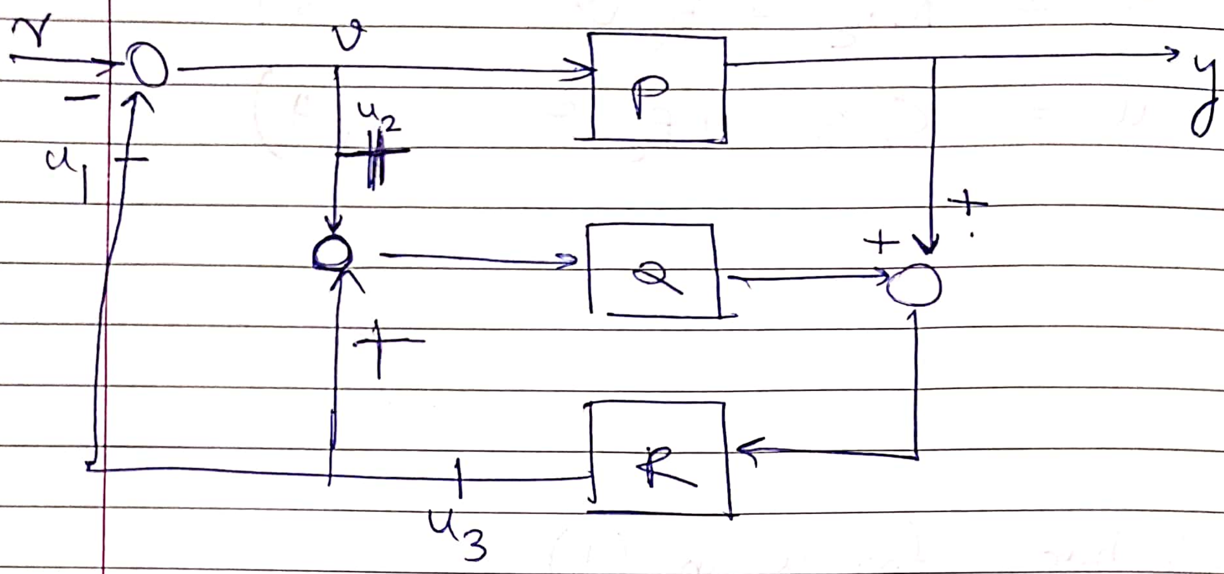
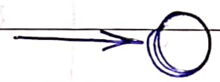
Midsem problem



- Write a 4 block closed loop system

$$w = r \qquad \bar{u} = u_1$$
$$\bar{y} = y \qquad z = v$$



$$L = \left[\begin{array}{c|c} A & b \\ \hline c & d \end{array}\right]$$

Find SS representation