SR. NO.	DATE	TITLE	PAGE NO.	TEACHER'S SIGN

Books For the Course

Norman Nise : Control Systems

Doyle, Francis, Tannenbaum : Feedback Control

Chua et. al. : Networks

Wolovich : Automatic Control Systems.

Endsem - 50 %
Midsem - 30 %
Quiz - 10 %
Assignment - 10 %

EE 302: Control Systems

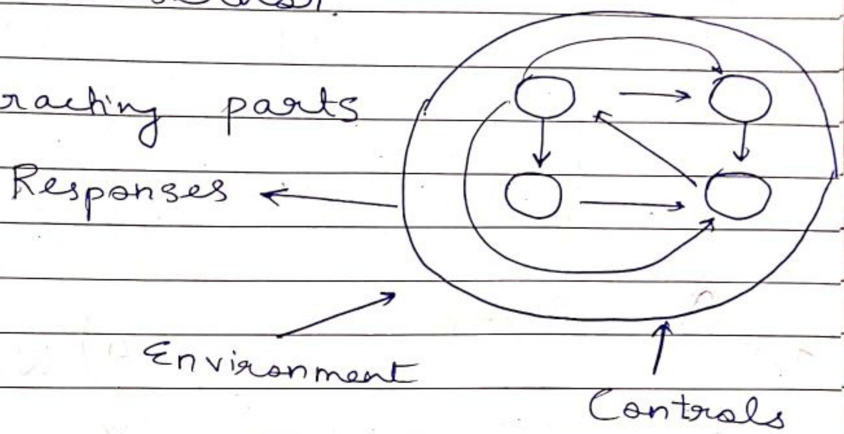
Instructor: VR Sule

Page No.:
Date:

Syllabus: OL, CL Systems, Modelling & Repre. Differential Equations, Transfer functions, Block Diagram, Flow Graph, Root locus Design, Stability Analysis, Design of Controller and Observer.

3.1.19

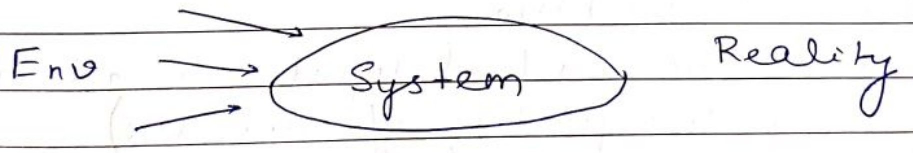
- System: Interacting parts

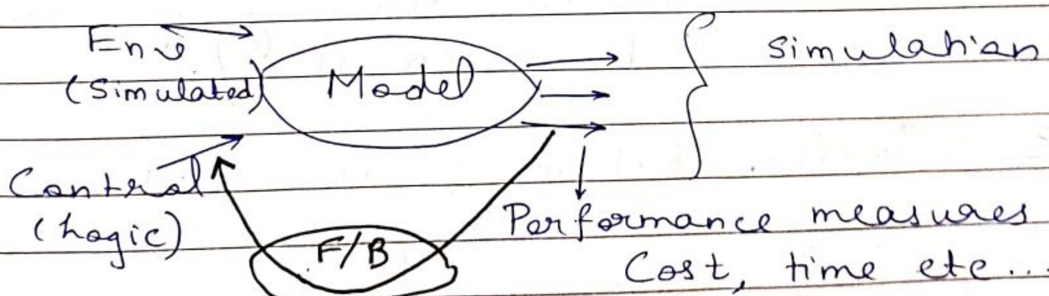


Causes → System → Effects

Uncontrolled   Choices

→ What's a Simulation?





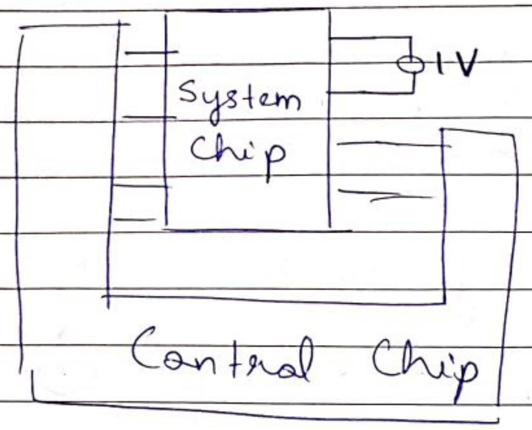
Performance measures
Cost, time etc.....

Feedback (expensive)
↳ some computer logic.

→ Feedback required when uncertainity in i/p -- o/p relations

7.1.19

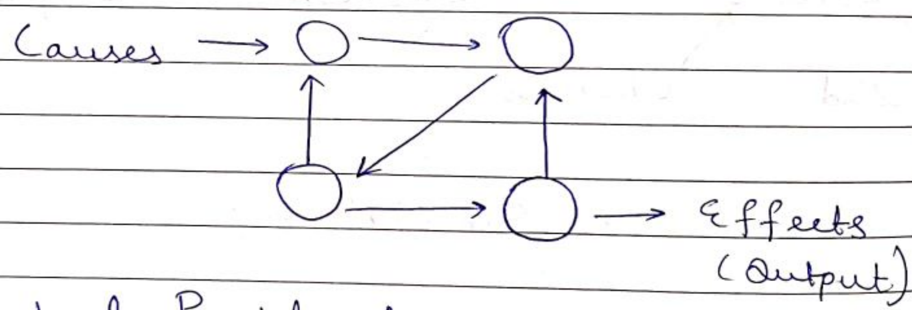
→ Recent Work: Physically Defined Control



Textbooks
1) Control Systems - Norman Nise
2) Doyle, Francis, Tannenbaum: Feedback control theory
3) Chua et al Linear & Non-Linear Networks
4) Wolovich: Automatic Control Systems

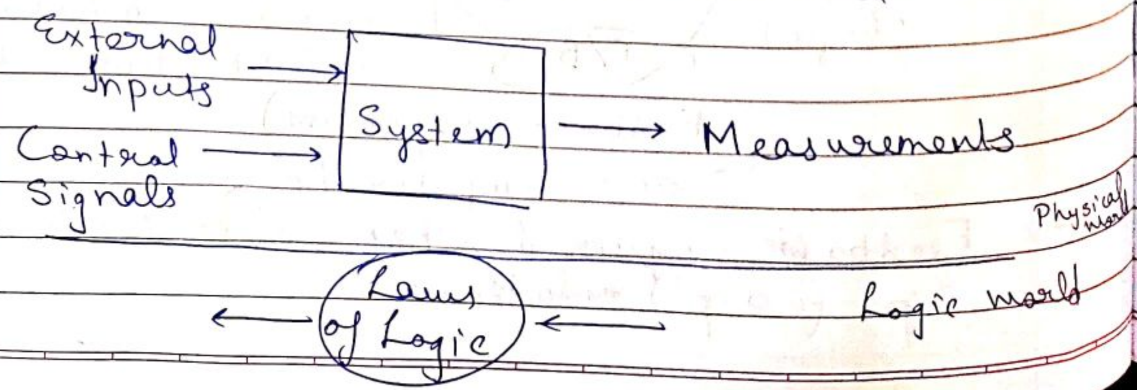
7.1.18 Recap

1. What is a System

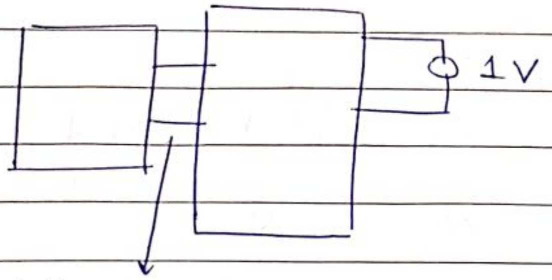


2. Control Problem:
→ Change the control inputs to get desired outputs (quality of outputs)
→ Under the uncertainity of external causes and system itself.

3. Implementation of the Control



→ Designing compensations is also a control problem



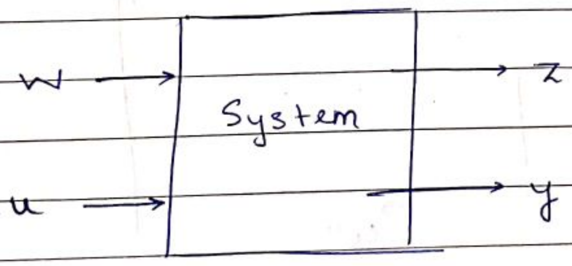
When both are connected they work well, else not

4. Model of System

→ Physical world reconstructed inside computer logic is called a Simulation

→ Model along with environmental inputs is necessary for control.

Notation



→ Labelling / Classification of IOs

overlap b/w different sets is allowed

W : exogenous inputs (ex. Ref. value)

u : Control inputs

y : Response Measurements (for Quality control)

z : Regulated Outputs. (Outputs which determine the performance)

Classes of Systems

1. Memoryless Systems :
Systems which react to inputs instantaneously

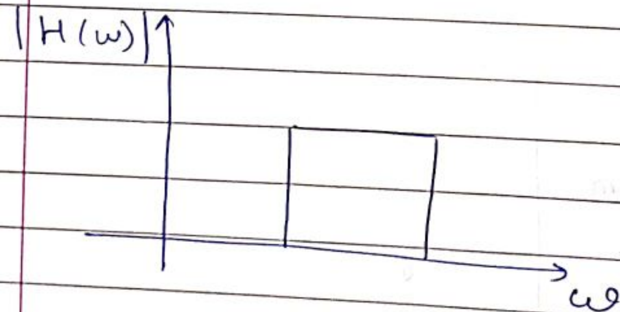
2. Systems with Memory
React to current inputs and past history of inputs
↳ Encoded as the State of the system

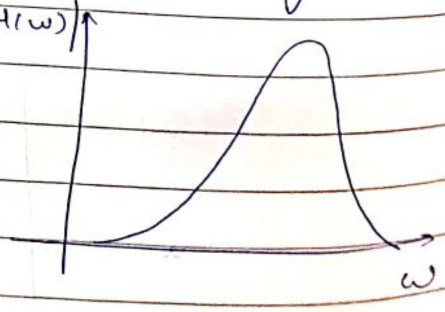
8.1.19

Causal System :

- Whose state $x(t)$ and outputs $y(t)$ are dependant only on inputs $w(\tau), u(\tau)$ for $\tau \in (-\infty, t]$

- Weiner's Conditions for causality





Transfer function : $R(s) = \dfrac{q(s)}{p(s)}$

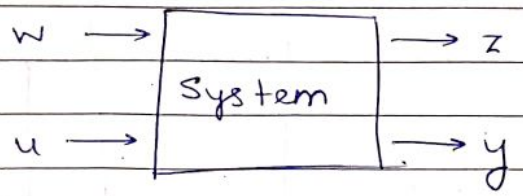
$p(s) \neq 0$ polynomial

Condition for Causality
$\deg(q(s)) \leq \deg(p(s))$

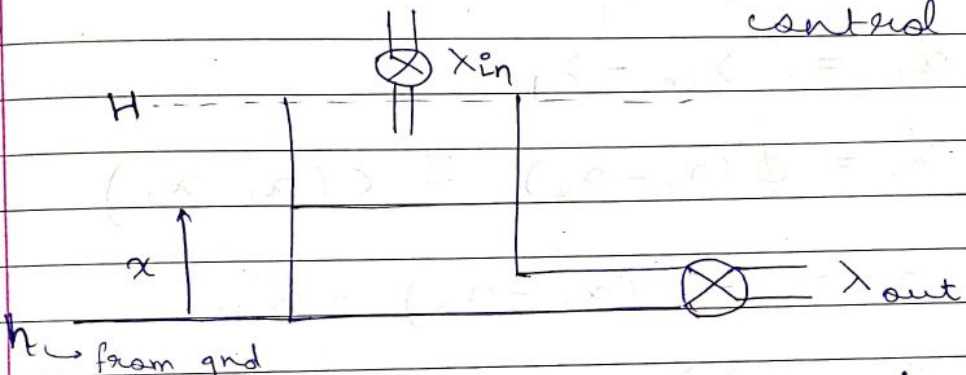
Book :

Examples of Systems

Recap : Four block model of a system



1. Dose tank / Flush tank / Water level control system

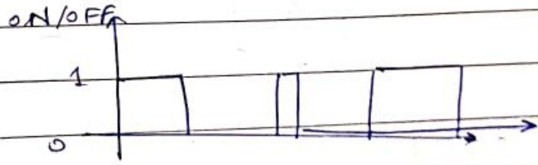


$\dot{x} \propto \lambda_{in} - \lambda_{out}$, take : $\dot{x} = \lambda_{in} - \lambda_{out}$

$\lambda_{out} = \phi(x(t)) \cong f\, x(t)$

Non linear — Apprx.

$\lambda_{in} \in \{0, \lambda_{in}\}$



State Space Model :

state $x(t)$

input λ_{in} (our control)

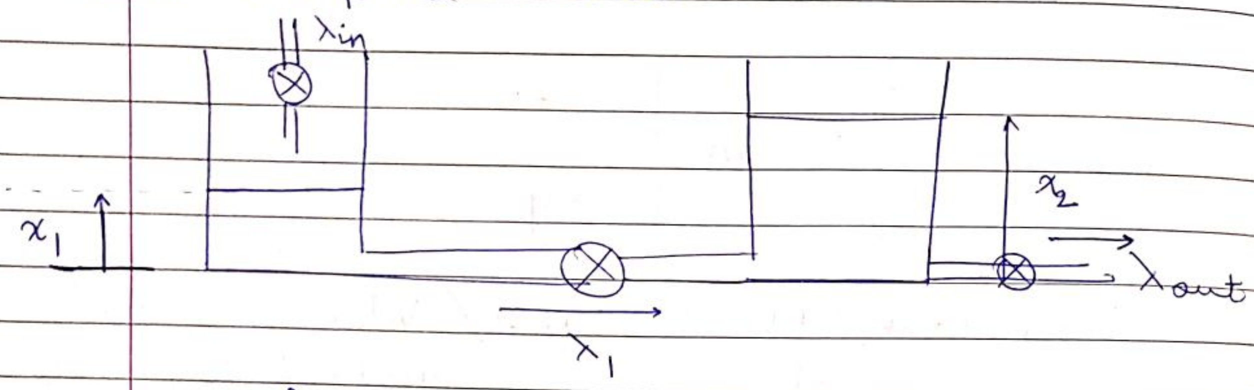
external input $\{0, 1\}$

$h \leq x(t) \leq H$

Model is applicable for :-

- Inventory Control
- Ware house Control
- Bank Account with spending
- Comm. Networks (Buffers)

2. Coupled tanks



$$\dot{x}_1 = \lambda_{in} - \lambda_1$$

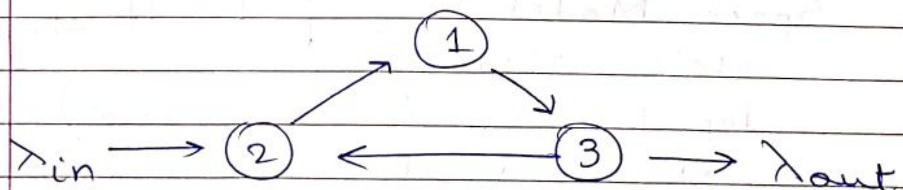
$$\lambda_1 = \phi(x_1 - x_2) \cong c(x_1 - x_2)$$

$$\dot{x}_1 = -c(x_1 - x_2) + \lambda_{in}$$

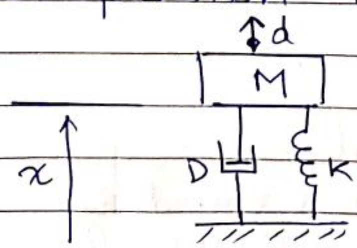
$$\dot{x}_2 = \lambda_1 - \lambda_{out}$$

$$\lambda_{out} \cong f x_2$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -c & c \\ c & -cf \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} \lambda_{in}$$



3. Suspension System

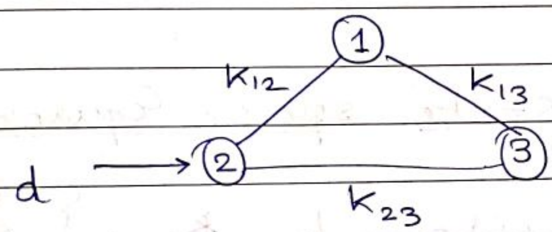


$$M\ddot{x} = -Mg - D\dot{x} - Kx + d$$

Let, $x_1 = x$ $\quad x_2 = \dot{x}_1$

$$\begin{array}{c} \dot{x} \\ \ddot{x} \end{array} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -K/M & -D/M \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ -g + d/M \end{bmatrix}$$

Coupled Mass Systems



$\vec{r}_1$ $\quad \vec{r}_3$ $\quad \vec{r}_1 = \begin{bmatrix} x_1 \\ y_1 \end{bmatrix}$ $\quad \vec{r}_3 = \begin{bmatrix} x_3 \\ y_3 \end{bmatrix}$

$$\ddot{\vec{r}}_1 = -K_{12}\vec{r}_1$$
$$\ddot{\vec{r}}_3 = -K_{13}(\vec{r}_3 - \vec{r}_1)$$
$$\ddot{\vec{r}}_2 = d$$

Four Block Model of a system

$$\dot{x} = Ax + B_1 u + B_2 w$$
$$y = C_1 x + D_1 u + D_2 w$$
$$z = C_2 x + D_3 u + D_1 w$$

The interface ... half of the band gap, with ... uniformly distributed in ... However, assume symmetric ... gap. ... /cm, kSi=12.

13.1.19

Page No.:
Date:

Background Assumed

1. Laplace transform
2. Solutions of Differential Equations
3. Impulse response, step response
4. Frequency Response plots
   → Gain
   → Phase
5. Network Theory basics

Chapter 2, Norman Nise → Also exam Numerical problems

→ **Examples of state space Equations**

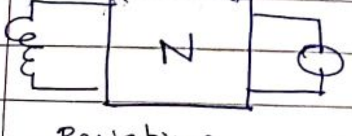
Circuits : Capacitor, L, R, $\}$ Energy storing
R, controlled sources : VCVS, VCCS, CCVS, CCCS
Transformer, Gyrator
→ "Resistive"
Independent Sources : Voltage, Current

Choose the state var. to be v_c and i_L

v_c : cap. voltage $= C\dfrac{dv_c}{dt}$

i_L : Inductor voltage $= L\dfrac{di_L}{dt}$

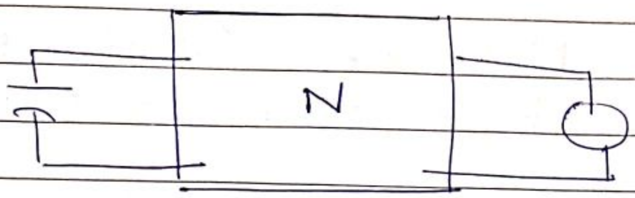
First order circuits



First order circuits

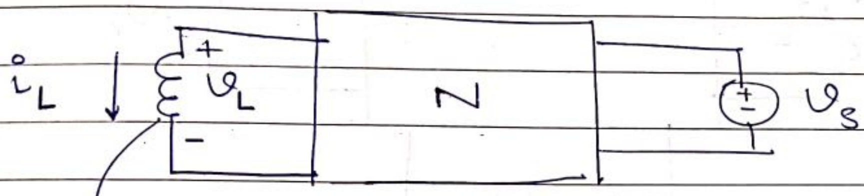
Resistive

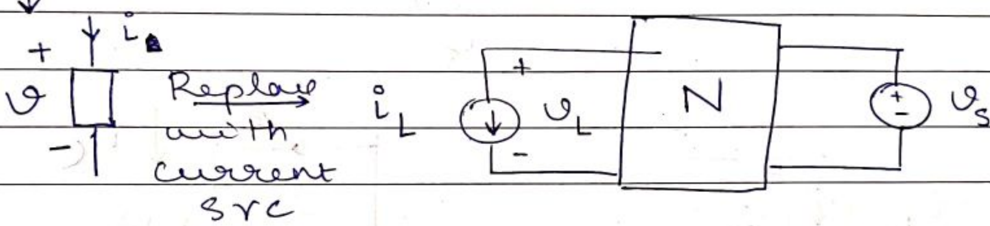
First order ⇒ Single Inductor / Capacitor



Sign Convention

1)



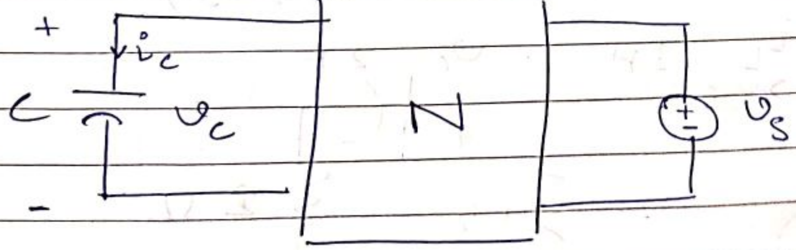


L → Linear operator

$$v_L = L(i_L, v_s)$$

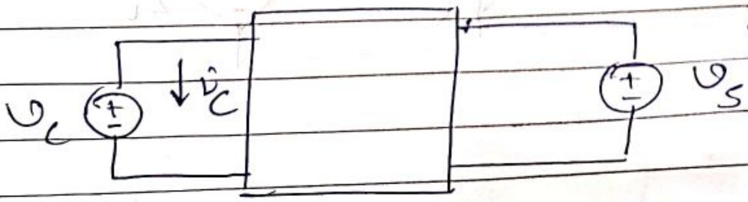
$$L\frac{di_L}{dt} = L(i_L, v_s)$$

2)



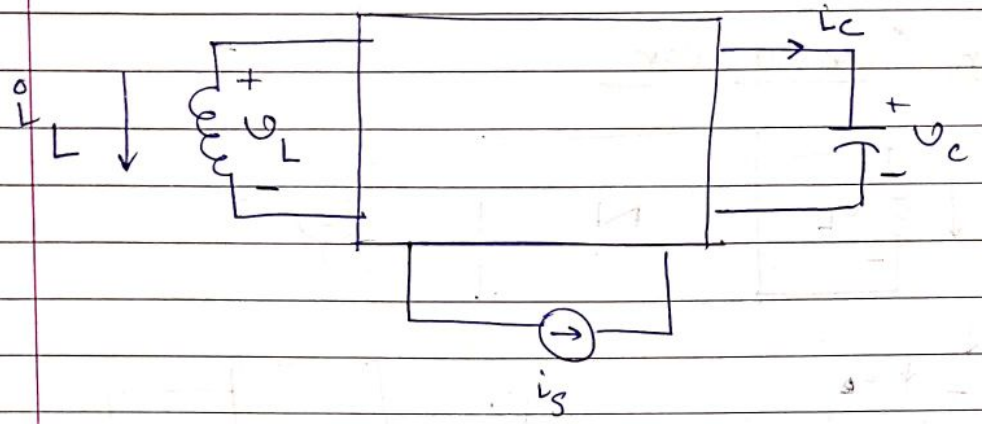
$$i_c = L(v_c, v_s)$$

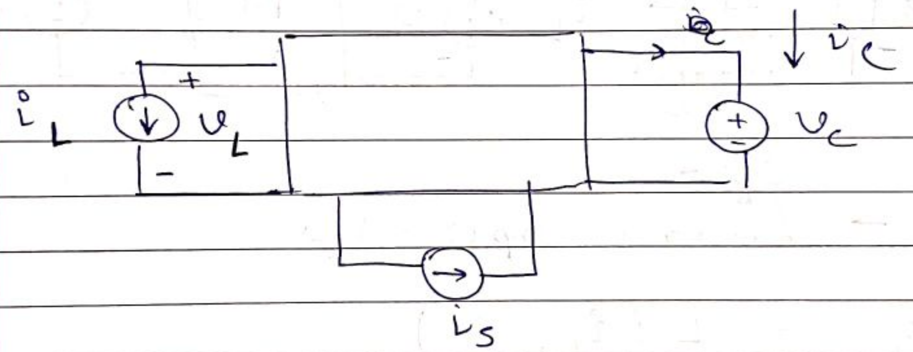
$$C\frac{dv_c}{dt} = L(v_c, v_s)$$



3) 2 port Network, Constant Source get their own port

2nd order

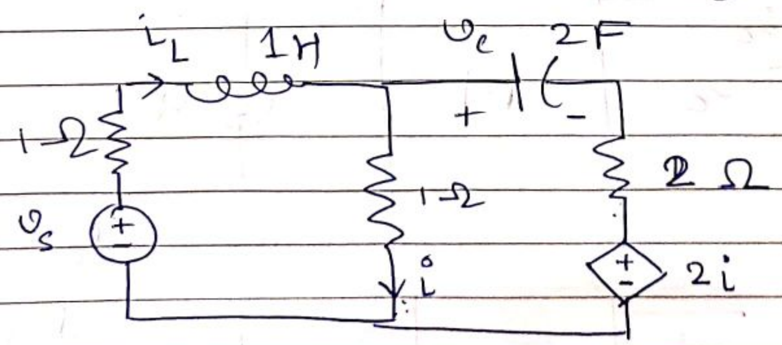




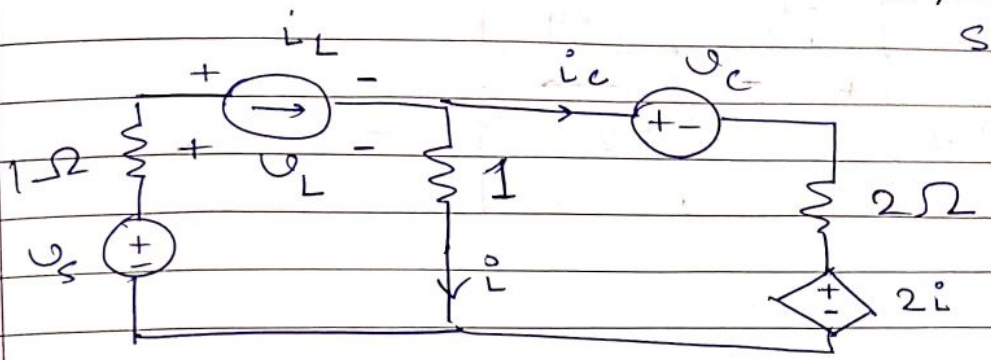
$$v_L = f_1(i_L, v_c, i_s)$$

$$i_c = f_2(i_L, v_c, i_s)$$

Q →

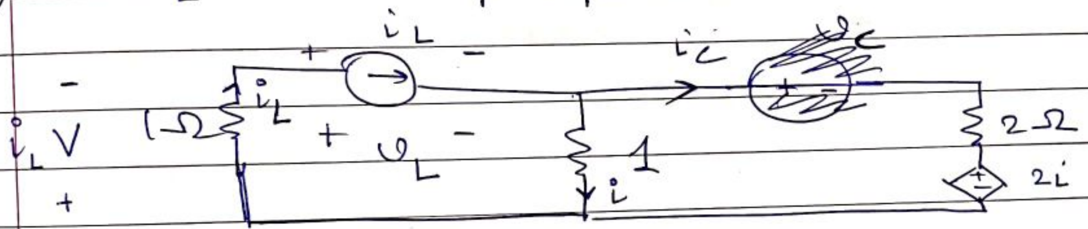


→ Convert to current src / voltage source



→ Find v_L, i_c

1) v_L : 3 Superposition K.V.L's



$$i_c + i = i_L$$

$$i = 2i_c + 2i$$

$$\cancel{-i = 2i}$$

$$\cancel{i_c + 2i_c} \qquad i = -2i_c$$

$$-i_c = i_L$$

$$i = 2i_L$$

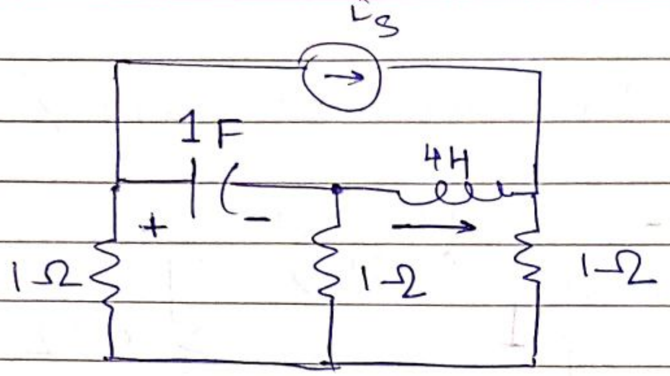
$$v_L = -i_L - i$$

Get in the form,

$$\begin{bmatrix} \frac{dv_c}{dt} \\ \frac{di_L}{dt} \end{bmatrix} = \begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \end{bmatrix} \begin{bmatrix} v_c \\ i_L \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} v_s$$

Complete this

2)



2)

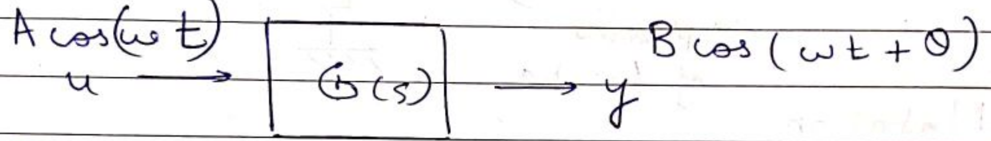
Modeling of LTI (Linear time invariant) Systems

→ Norman Nise, Chap 1, examples / classification of some common control systems

- In time domain : Linear constant coefficient differential Equations (LCCDE's)

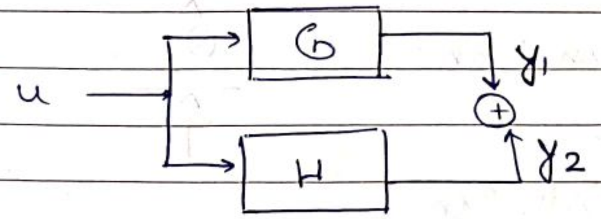
- In frequency / Laplace domain : These are transfer functions

Block Diagrams / Transfer functions

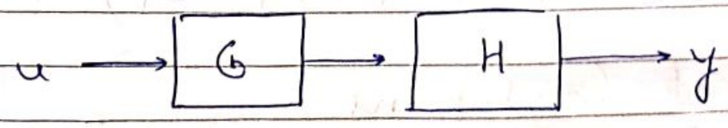


$$G(s) = \frac{q(s)}{p(s)}$$

1) Sum of Systems : ~~$\mathcal{S}$~~



2) Cascade

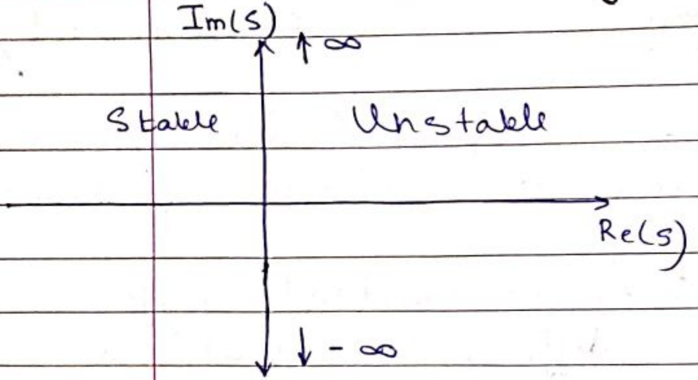


$\mathcal{S}$: Set of Stable Systems

Page No.:

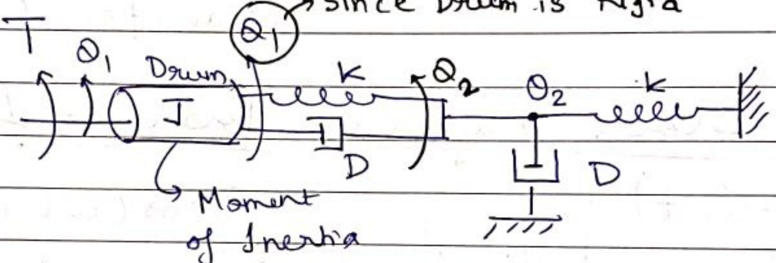
Date:

Stability & Laplace transform



Ex 1) LCCDE's → Differential systems

→ State-space equations



<u>Notation</u>

Time domain — θ_2, T

Freq domain — $\theta_2(s)$ or $\hat{\theta}_2$, $T(s)$ or $\hat{T}$

Transfer function: $\dfrac{\hat{\theta}_2}{\hat{T}}$

1. Write Diff. eqn.

$$T = J\ddot{\theta}_1 + D(\dot{\theta}_1 - \dot{\theta}_2) + K(\theta_1 - \theta_2)$$

$$K(\theta_2 - \theta_1) + D(\dot{\theta}_2 - \dot{\theta}_1) + D\dot{\theta}_2 + K\theta_2 = 0$$

Initial Conditions

$$\theta_1(0) = \theta_2(0) = 0$$

Chapter 2 Examples Norman Nise

$$\hat{T} = s^2 J \hat{\theta}_1 + sD(\hat{\theta}_1 - \hat{\theta}_2) + K(\hat{\theta}_1 - \hat{\theta}_2)$$

2 Take Laplace transforms

$$\hat{T} = s^2 J \hat{\theta}_1 + (\hat{\theta}_1 - \hat{\theta}_2)(Ds + K)$$

$$K(\hat{\theta}_2 - \hat{\theta}_1) + sD(\hat{\theta}_2 - \hat{\theta}_1) + sD\hat{\theta}_2 + K\hat{\theta}_2 = 0$$

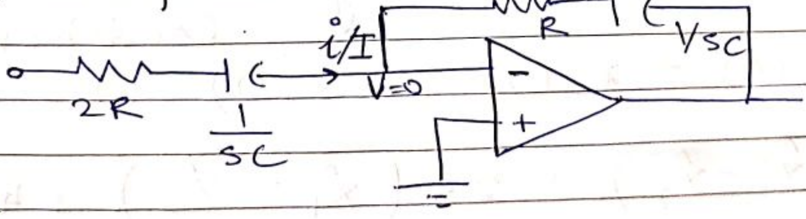
$$= 2(K + sD)(\hat{\theta}_2) - (K + sD)\hat{\theta}_1 = 0$$

$$\Rightarrow \boxed{\hat{\theta}_1 = 2\hat{\theta}_2}$$

$$\Rightarrow \hat{T} = 2s^2 J\hat{\theta}_2 + \hat{\theta}_2(Ds + K)$$

$$\Rightarrow G(s) = \frac{\hat{\theta}_2}{\hat{T}} = \left(\frac{1}{2Js^2 + Ds + K}\right)$$

2) Op-Amp Ckt Transfer function



$v_{in} \updownarrow \hat{V}_{in}$ $\quad v_o \leftrightarrow \hat{V}_o$

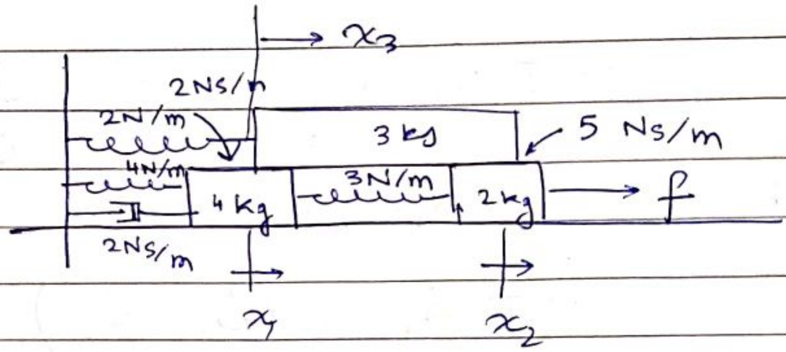
$i \longleftrightarrow I(s)$

time domain $\quad$ freq domain

find the transfer function : $\dfrac{\hat{V}_o}{\hat{V}_{in}}$

$$\frac{\hat{V}_{in}}{\left(2R + \frac{1}{sC}\right)} = \frac{-\hat{V}_o}{\left(R + \frac{1}{sC}\right)} \Rightarrow -\frac{\left(R + \frac{1}{sC}\right)}{\left(2R + \frac{1}{sC}\right)}$$

17.1.19



→ Find the transfer function $\cancel{\frac{X}{F}}$ $\frac{\hat{X}_3}{\hat{F}}$

$$\cancel{2\ddot{x}_2 = f - 3x_2 - 5(\dot{x}_2 - \dot{x}_3)}$$

Block 2 $\quad f = 2\ddot{x}_2 \cancel{+3x_2} + 5(\dot{x}_2 - \dot{x}_3) + 3(x_2 - x_1)$

Block 1 $\quad \cancel{3(x_1} \; 3(x_1 - x_2) + 2(\dot{x}_1 - \dot{x}_3) + 4x_1 + 2\dot{x}_1 + 4\ddot{x}_1 = 0$

Block 3 $\quad 2x_3 \cancel{2\dot{x}_3} + (\dot{x}_3 - \dot{x}_1) \times 2 + (\dot{x}_3 - \dot{x}_2) \times 5 + 3\ddot{x}_3$

→ Laplace transforming with zero Initial Condition (I C)

$$x_1(0) = x_2(0) = x_3(0) = 0$$

$$3(\hat{X}_1 - \hat{X}_2) + 2s(\hat{X}_1 - \hat{X}_3) + 4\hat{X}_1 + 2s\hat{X}_1 + 4s^2\hat{X}_1$$

$$= (4s^2 + 2s + 7)\hat{X}_1 - 3\hat{X}_2 - 2s\hat{X}_3 = 0$$

$$-3\hat{X}_1 + (2s^2 + 5s + 3)\hat{X}_2 - 5s\hat{X}_3 = \hat{F}$$

$$-2s\hat{X}_1 - 5s\hat{X}_2 + (3s^2 + 7s + 2) =$$

$$\begin{bmatrix} 4s^2+2s+7 & -3 & -2s \\ -3 & 2s^2+5s+3 & -5s \\ -2s & -5s & 3s^2+7s+2 \end{bmatrix} \times \begin{bmatrix} \hat{X}_1 \\ \hat{X}_2 \\ \hat{X}_3 \end{bmatrix} = \begin{bmatrix} 0 \\ \hat{F} \\ 0 \end{bmatrix}$$

→ Solution beyond this?

1) Find Determinant of $P(s)$, use Cramer's Rule
$\det(P(s)) \neq 0$

2) Invert the Matrix

$$P(s)^{-1} = \frac{\operatorname{adj} P(s)}{\det P(s)}$$

State Space Equations

- 2^{nd} order state space system

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -6 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Date:

$u \to$ input, $y \to$ output

$\dot{x}_1 = x_2$ $\qquad s\hat{x}_1 = \hat{x}_2$

$\dot{x}_2 = -6x_1 - 5x_2 + u$ $\qquad \hat{x}_2 = -6\hat{x}_1 - \hat{x}_2$

$y = x_1$ $\qquad \hat{y} = \hat{x}_1$

$$T.F = \frac{\hat{y}}{\hat{u}} \qquad X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$s\hat{x}_1 = \hat{x}_2$$
$$s\hat{x}_2 = -6\hat{x}_1 - 5\hat{x}_2 + \hat{u}$$
$$\hat{y} = \hat{x}_1$$

$$\cancel{6s\hat{x}_1 = -6\hat{x}_1 + \hat{u}}$$
$$s^2\hat{x}_1 = -6\hat{x}_1 - 5s\hat{x}_1 + \hat{u}$$
$$(s^2 + 5s + 6)\cancel{\hat{x}_1}\hat{y} = \hat{u}$$

$$\boxed{\frac{\hat{y}}{\hat{u}} = \frac{1}{s^2 + 5s + 6}} \xLeftrightarrow{\text{equivalent to}} \begin{bmatrix} 0 & 1 \\ -6 & -5 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\left[\begin{array}{c|c} A & b \\ \hline c & d \end{array}\right] = \left[\begin{array}{cc|c} 0 & 1 & 0 \\ -6 & -5 & 1 \\ \hline 1 & 0 & 0 \end{array}\right]$$

$$\left.\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx + Du \end{aligned}\right| \longrightarrow \text{After LT}$$

Page No.:
Date:

$-5\hat{x}_2 + \hat{u}$

$$\Rightarrow \dot{\hat{x}} = A\hat{x} + B\hat{u}$$
$$\hat{y} = C\hat{x} + D\hat{u}$$

$$\hat{x} = (sI - A)^{-1} B\hat{u}$$

$$\hat{y} = \left(C(sI-A)^{-1}B + D\right)\hat{u}$$

$$\updownarrow$$

$$\left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right] \longrightarrow T(s)$$

→ On why $(sI - A)^{-1}$ exists,

$$= \begin{bmatrix} s & -1 \\ 6 & s+5 \end{bmatrix}^{-1}$$

$$= \frac{adj\,(sI-A)}{det\,(sI-A)}$$

$[1 \quad 0]$

$$det\,(sI-A) = s(s+5) + 6$$
$$= s^2 + 5s + 6$$

Complete this!

$[1 \quad 0] \Big[$

Scanned by CamScanner

Page No.:
Date:

State Space Model

$$\dot{x} = Ax + bu$$
$$y = cx + du$$

zero state condition $x(0) = 0$

$\hat{X}$ - LT of $x(t)$
$\hat{Y}$ - " " $y(t)$
$\hat{U}$ - " " $u(t)$

$$\hat{X} = (sI - A)^{-1} b \hat{U}$$

$$\hat{Y} = [c(sI - A)^{-1} b + d] \hat{U}$$

$$\frac{\hat{Y}}{\hat{U}} = \left(c(sI - A)^{-1} b + d\right)$$

$$\frac{\hat{Y}}{\hat{U}} = \frac{c \operatorname{adj}(sI - A) b}{\det(sI - A)} + d$$

$$\dot{x} = -ax + bu$$

$$\hat{X} \leftarrow \boxed{\frac{b}{s+a}} \leftarrow \hat{U}$$

1^{st} order system w/o any pole-zero cancellation

$$\hat{X} \leftarrow \boxed{\frac{s+b}{s+a}} \leftarrow \hat{U}$$

→ Analysis

$$\dot{y} = -ay + bu$$
$$s\hat{Y} - y(0) = -a\hat{Y} + b\hat{U}$$

$$\hat{Y} = \frac{y(0)}{(s+a)} + \frac{b}{(s+a)}\hat{u}$$

$$y(t) = e^{-at}y(0) + \mathcal{L}^{-1}\left(\frac{b}{(s+a)}\hat{u}\right)$$

$$f(t),\ g(t) \longleftrightarrow F(s),\ G(s)$$

$$\mathcal{L}^{-1}\left(F(s)\,G(s)\right)$$

$$= \int_0^{\infty} f(t-\tau)\,g(\tau)\,d\tau$$

$$= \int_0^{t} f(t-\tau)\,g(\tau)\,d\tau$$

since $f(t) = 0$ for $t < 0$

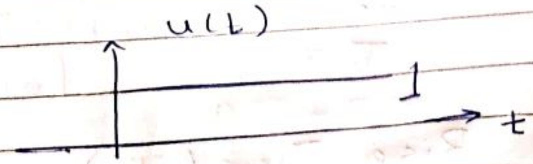
$t - \tau \geq 0 \Rightarrow \tau \leq t$ for non zero part

→ Using Inverse convolution formula

$$y(t) = \underbrace{e^{-at}y_0}_{\text{zero input response}} + \underbrace{b\int_0^t e^{-a(t-\tau)}u(\tau)\,d\tau}_{\text{zero state response}}$$

Response Parameters

→ Step response :



$$\hat{Y} = \left(\frac{b}{s+a}\right)\frac{1}{s}$$

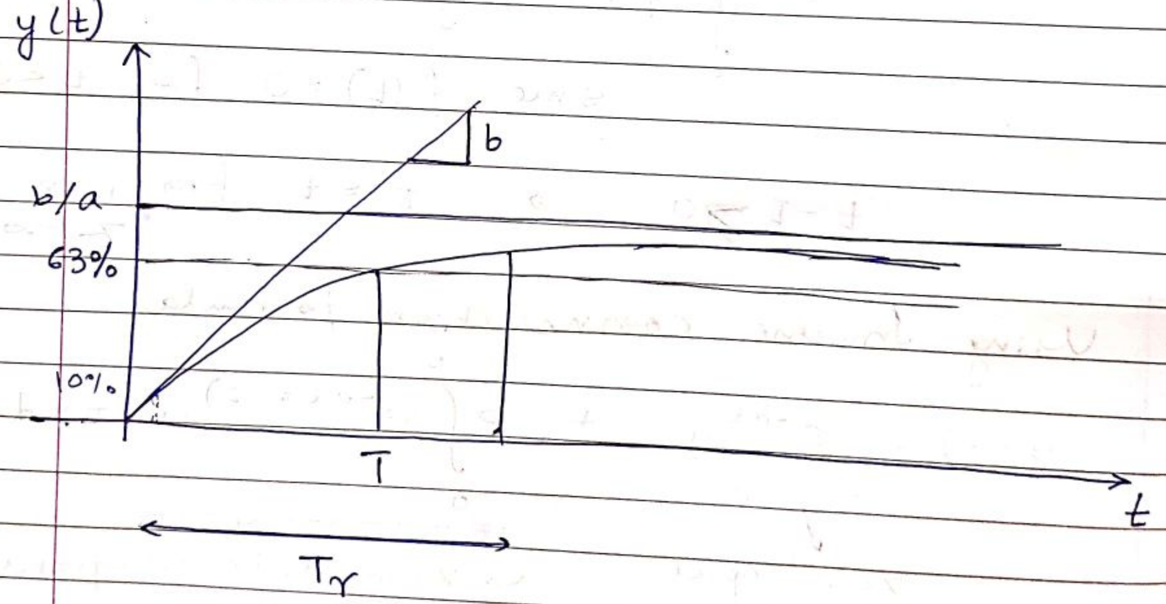
→ Using partial fraction expansion

$$\frac{b}{s(s+a)} = \frac{b/a}{s} + \frac{b/-a}{(s+a)}$$

$$\Rightarrow y(t) = \mathcal{L}^{-1}\left(\frac{b/a}{s} + \frac{b/(-a)}{s+a}\right)$$

$$= \frac{b}{a} - \frac{b}{a}e^{-at} \qquad t \geq 0$$

$$= \frac{b}{a}\left(1 - e^{-at}\right)$$



→ Parameters

① Time Constant (T)

$$T = \frac{1}{a} \quad \text{for} \quad \frac{b}{a} = 1$$

② Rise Time (T_r)

Time taken to rise from 10% to 90%

(3) Settling time (T_s)

Time taken to reach within 2% of settling value.

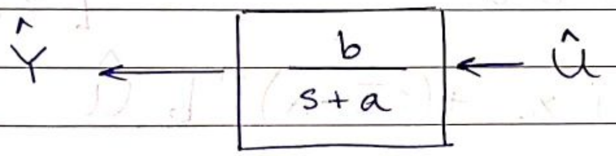
22.1.19

First order system solution:

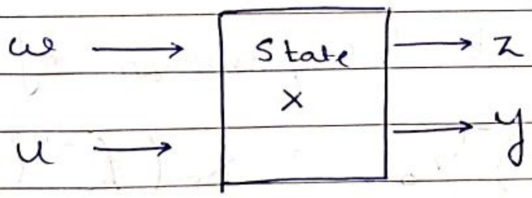
$$\dot{x} = -ax + bu$$

$$x(t) = e^{-at} y_0 + \int_0^t e^{-a(t-\tau)} u(\tau)\, d\tau$$

Step Response $y(t) = \frac{b}{a}\left(1 - e^{-at}\right)$



Notation convention



→ State-space description of above,

$$x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \qquad u = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ \vdots \\ u_m \end{bmatrix} \qquad y = \begin{bmatrix} \\ \\ \end{bmatrix}$$

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du$$

A ($n \times n$), B ($n \times m$) real Matrices

C ($p \times n$) D ($p \times m$)

→ Simplified Single Input - Single output system

$$\dot{x} = Ax + bu$$
$$y = cx + du$$

$$\left[\begin{array}{c|c} A & b \\ \hline c & d \end{array}\right]$$

$$\frac{\hat{Y}}{\hat{U}} = c(sI-A)^{-1}b + d$$

$$\dot{x} = Ax + bu \qquad \hat{x} = \begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \\ \vdots \\ \hat{x}_n \end{bmatrix}$$

$$x(0) = x_0$$

$$s\hat{X} - x_0 = A\hat{X} + b\hat{U}$$

$$\hat{X} = (sI-A)^{-1}x_0 + (sI-A)^{-1}b\hat{U}$$

$$\hat{Y} = c\hat{X} + d\hat{U}$$

→ Let $\Phi(t)$ be the function of time s.t

$$\mathcal{L}^{-1}(sI-A)^{-1}x_0 = x_0\left(\mathcal{L}^{-1}(sI-A)^{-1}\right)$$

$$\mathcal{L}(\Phi(t)) = (sI-A)^{-1}$$

$\phi(t)$ is also a solution of the Mat. eqn

$$\dot{Z} = AZ \qquad Z(0) = I$$

$$s\hat{Z} - I = A\hat{Z} \qquad \hat{Z} = (sI-A)^{-1}$$

Page No.:
Date:

→ What does the notation mean

$$Z(t) = [z_{ij}(t)]$$

$$\mathcal{L}(Z(t)) = [\mathcal{L}(z_{ij}(t))]$$

$$\dot{Z} = AZ \qquad Z(0) = I$$

↳ A homogenous differential Equation since no forcing functions

→ Properties (proved later) of $\Phi(t)$

$\Phi(t)$ satisfies

1. $\Phi(0) = I$ 2. $\dfrac{d\,\Phi(t)}{dt} = A\,\Phi(t)$

3. $\Phi(t+s) = \Phi(t)\,\Phi(s)$ 4. $\Phi(-t) = (\Phi(t))^{-1}$

5. $\det(\Phi(t)) = ?$ (Extra)

ex

$$A = \begin{bmatrix} 0 & 1 \\ -6 & -5 \end{bmatrix}$$

$$(sI - A)^{-1} = \begin{bmatrix} s & -1 \\ 6 & s+5 \end{bmatrix}^{-1}$$

$$= \frac{1}{(s^2+5s+6)} \begin{bmatrix} s+5 & 1 \\ -6 & s \end{bmatrix}$$

$$\mathcal{L}^{-1}((sI-A)^{-1}) = \mathcal{L}^{-1} \frac{1}{(s+2)(s+3)} \begin{bmatrix} s+5 & 1 \\ -6 & s \end{bmatrix}$$