Filter Design Assignment

EE-338 Digital Signal Processing

Spring 2019

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Group - 1

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1 Band-Pass Filter Design

1.1 Parameter Value Calculation

The filter assigned to me has the number **m = 109**. Using this, I have calculated the following values -

m	109
q	10
r	9
bl	55000
bh	65000
passband	equi ripple
stopband	Monotonic
transition width	2000
sampling rate	320000

1.2 Normalized Specifications

$W_{normalized} = \frac{2*\pi*W_{given}}{\omega_{sampling}}$

Pass Band	1.0799, 1.2763
Stopband	1.0407, 1.3155
Transition Width	0.0393
Tolerance	0.15

1.3 Analog Filter Specifications

In order to meet the specification I have converted the normalized parameters into analog domain.

$$\Omega_{analog} = \tan(\frac{\omega}{2})$$

Pass Band	0.5994, 0.7417
Stopband	0.7725,0.5730
Tolerance	0.15

1.4 Analog Band Pass to Analog Low Pass Filter Transformation

I have further converted the analog bandpass specification intolow pass specifications. $s = j\Omega_L$ such that,

$$\Omega_L = \frac{\Omega^2 - \Omega_0^2}{B\Omega}$$

$$\Omega_0^2 = \Omega_{p1}\Omega_{p2}$$

$$B = \Omega_{p2} - \Omega_{p1}$$

Ω_{P1}	-1
Ω_{P2}	+1
Ω_{S1}	-1.4254
Ω_{P2}	1.3856
Tolerance	0.15

1.5 Chebyshev Low-pass Specification

As the problem statement is to design a **equi-ripple** bandpass filter, therefore I have used Chebyshev design.

$$H(j\omega) * H(-j\omega) = \frac{1}{1+\epsilon^2 T_n^2(j\omega)}$$

The Chebyshev polynomial can be defined recursively. I had found the roots of the polynomial and then used it to construct H(jw). H(jw)will only have roots in Left half plane.
Poles found:

- $-0.1222 - 0.9698i$
- $-0.1222 + 0.9698i$
- $-0.2949 - 0.4017i$
- $-0.2949 + 0.4017i$

The parameters found are:

D1	0.3841
D2	43.4444
N_s	4
ϵ	0.6197
Tolerance	0.15

The corresponding analog transfer function is numerator = 0.2017

degree s^k	4	3	2	1	0
Coefficient:	1.0	0.8342	1.348	0.6243	0.2373

Table: for denominator of analog filter

1.6 Analog Bandpass Transfer Function

Formula Used to convert back to bandpass filter

$$s- > \frac{s*s+\omega_0*\omega_0}{B*s}$$

degree s^k	4	3	2	1	0
Coefficient:	$8.264\,10^{-5}$	0	0	0	0

Table: for numerator of analog band pass filter

degree s^k	8	7	6	5	4	3	2	1	0
Coefficient:	1.0	0.1187	1.805	0.1601	1.21	0.07116	0.3568	0.01043	0.03905

Table: for denominator of analog band pass filter

1.7 Discrete Time Filter Transfer Function

I have used bi linear transformation to convert back to discrete domain:

$$s- > \frac{z-1}{z+1}$$

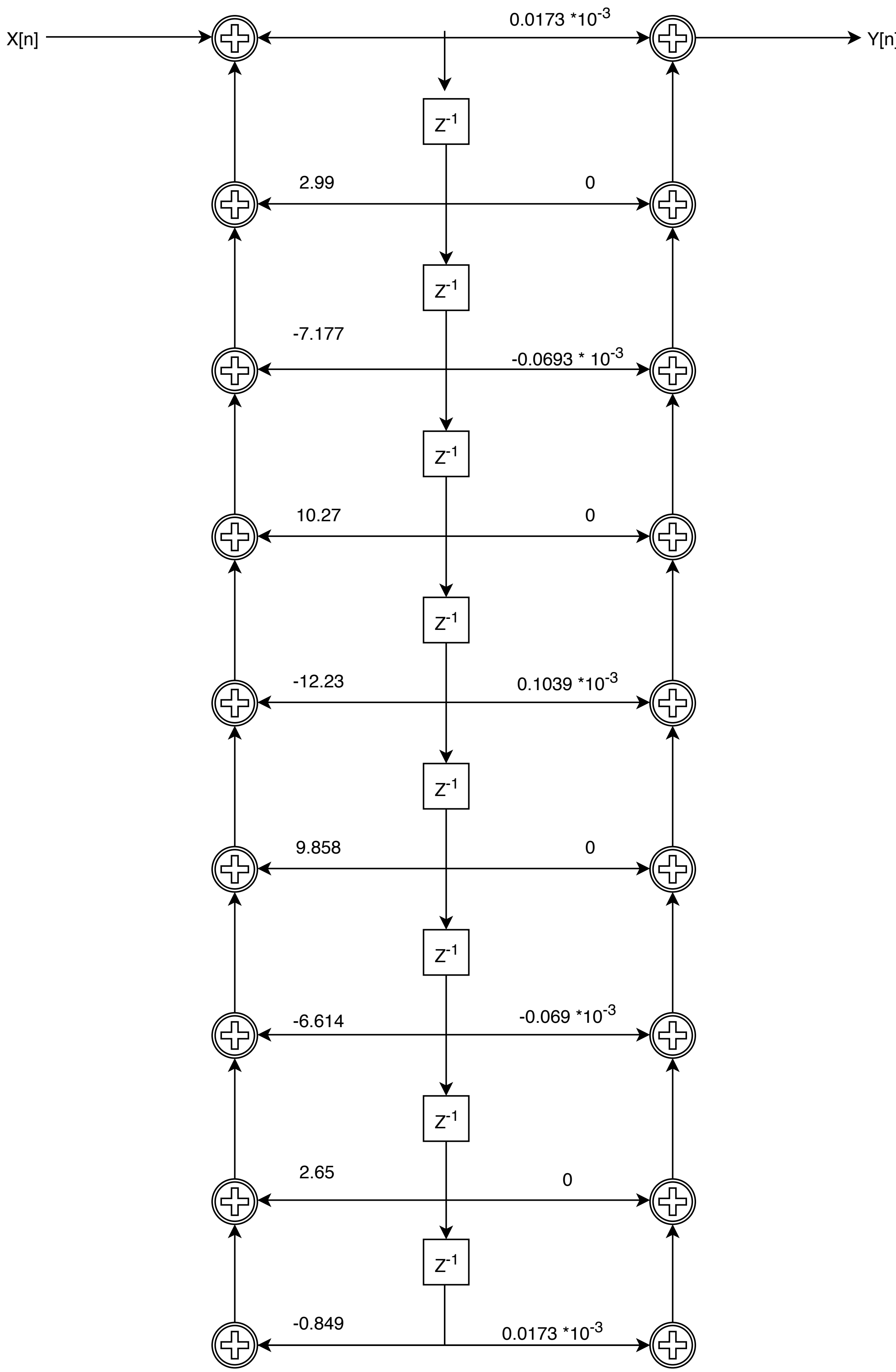
degree Z^{-k}	0	1	2	3	4	5	6	7	8
Coefficient (* 1.0e-03):	0.0173	0	-0.0693	0	0.1039	0	-0.0693	0	0.0173

Table: for numerator of discrete band pass filter

degree Z^{-k}	0	1	2	3	4	5	6	7	8
Coefficient:	1.0	-2.999	7.177	-10.27	12.23	-9.858	6.614	-2.652	0.849

Table: for denominator of discrete band pass filter

1.8 Realization using Direct Form II



X[n]
Y[n]
0.0173 *10-3
Z-1
2.99
0
-7.177
-0.0693 * 10-3
10.27
0
-12.23
0.1039 *10-3
9.858
0
-6.614
-0.069 *10-3
2.65
0
-0.849
0.0173 *10-3

1.9 FIR Filter Transfer Function using Kaiser Window

For designing the Fir band pass filter the rest of the parameters remain same as above(the filter specification).
to make a fir filter I have used kaiser window. the parameters to the kaiser window are as follows:
$A = -20 * log10(delta) = 16.4782$
And the corresponding alpha comes out to be 0.
THe N_{min} comes out to 48. which is a very loose bound.I got correct result for $N_{min} + 5$
I have used Kaiser window for design. The kaiser window basically takes the order of the fir filter and the parameter beta to construct a low pass filter of the same tolerance level. But for my filter design I needed a band pass filter. So a made band pass filter and point wise multiplied the filter with the to get the desired result. The coefficients that i got for the final fir filter is:

```
FIR_BandPass =

  Columns 1 through 17

    0.0096   -0.0062   -0.0188   -0.0082    0.0170    0.0247   -0.0000   -0.0296   -0.0245    0.0143    0.0398    0.0162   -0.0315   -0.0433    0.0000    0.0472    0.0374

  Columns 18 through 34

   -0.0209   -0.0563   -0.0221    0.0417    0.0555   -0.0000   -0.0569   -0.0439    0.0239    0.0625    0.0239   -0.0439   -0.0569   -0.0000    0.0555    0.0417   -0.0221

  Columns 35 through 51

   -0.0563   -0.0209    0.0374    0.0472    0.0000   -0.0433   -0.0315    0.0162    0.0398    0.0143   -0.0245   -0.0296   -0.0000    0.0247    0.0170   -0.0082   -0.0188

  Columns 52 through 53

   -0.0062    0.0096

>>
```

1.10 Results

1.10.1 IIR filter

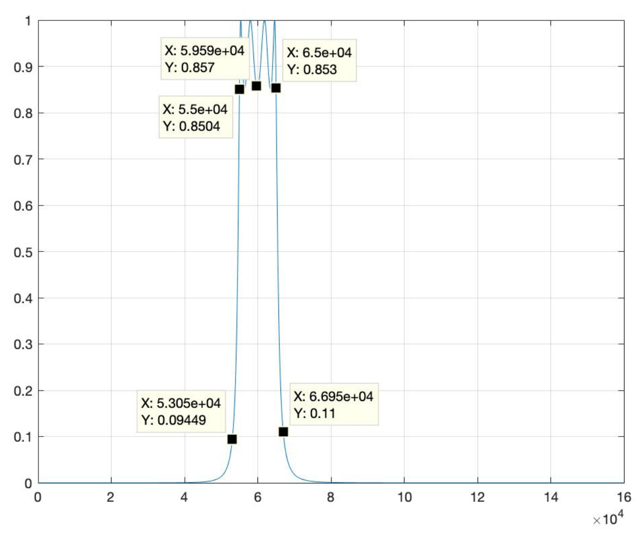


Figure 1: Magnitude plot of the Filter

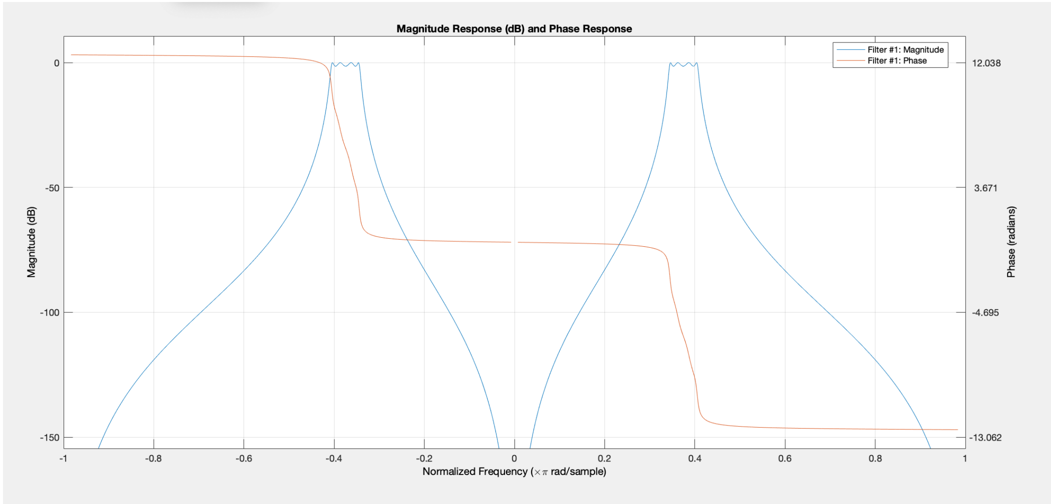


Figure 2: Normalized phase and magnitude plot in Fvtools

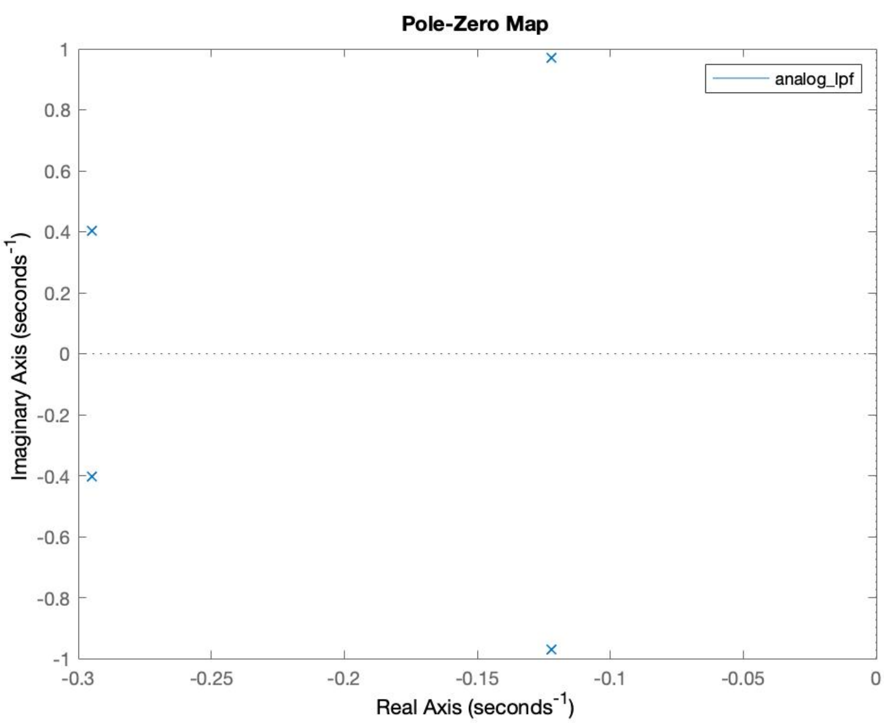


Figure 3: Pole zero plot of Analog low pass Chebyshev filter

1.10.2 FIR filter

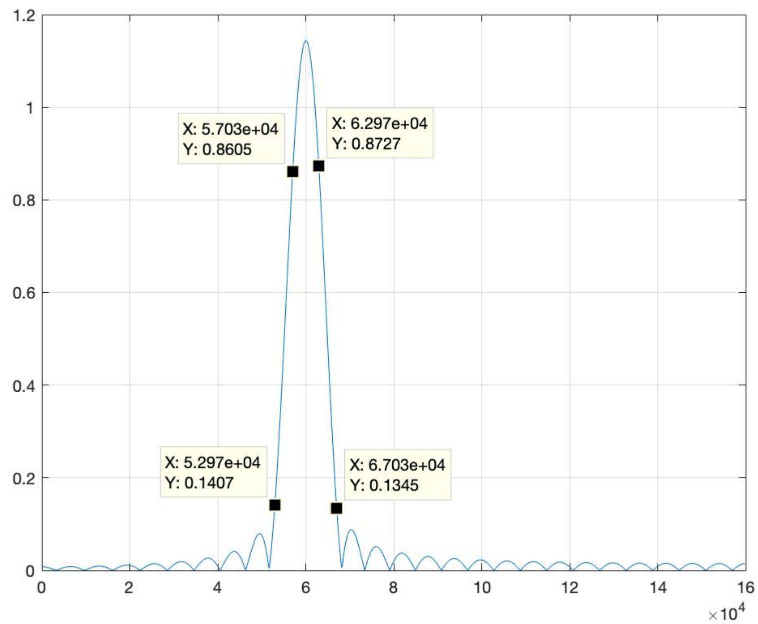


Figure 4: Magnitude plot of the Filter

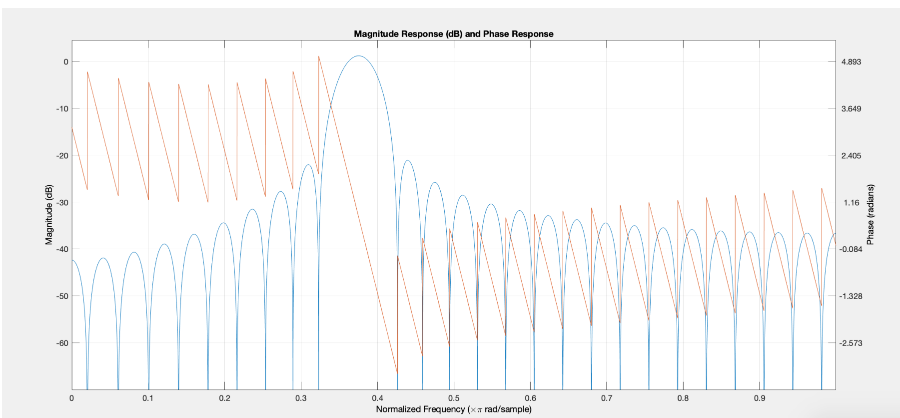


Figure 5: Normalized phase and magnitude plot in Fvtools

2 Band-Stop Filter Design

2.1 Parameter Value Calculation

The filter assigned to me has the number **m = 109**. Using this, I have calculated the following values -

m	109
q	10
r	9
bl	39500
bh	45500
passband	Monotonic
stopband	Monotonic
transition width	2000
sampling rate	250000

2.2 Normalized Specifications

$W_{normalized} = \frac{2*\pi*W_{given}}{\omega_{sampling}}$

Pass Band	1.1938, 0.9425
Stopband	1.1435, 0.9927
Transition Width	0.0503
Tolerance	0.15

2.3 Analog Filter Specifications

In order to meet the specification I have converted the normalized parameters into analog domain.

$$\Omega_{analog} = \tan(\frac{\omega}{2})$$

Pass Band	0.6796, 0.5095
Stopband	0.6435,0.5416
Tolerance	0.15

2.4 Analog Band stop to Analog Low Pass Filter Transformation

I have further converted the analog bandstop specification into low pass specifications. $s = j\Omega_L$ such that,

$$\Omega_L = \frac{B\Omega}{\Omega^2 - \Omega_0^2}$$

$$\Omega_0^2 = \Omega_{p1}\Omega_{p2}$$

$$B = \Omega_{p2} - \Omega_{p1}$$

Ω_{P1}	-1
Ω_{P2}	+1
Ω_{S1}	-1.6146
Ω_{S2}	1.7399
Tolerance	0.15

2.5 Butterworth Highpass Specification

As the problem statement is to design a **mono-ripple** bandstop filter, therefore I have used butterworth design.

$$H(j\omega) * H(-j\omega) = \frac{1}{1+\epsilon^2(\frac{\omega}{\omega_c})^{2n}}$$

D1	0.3841
D2	43.4444
N_s	5
Tolerance	0.15

$\Omega_c = [\frac{\Omega_{s2}}{d1^{\frac{1}{2n}}}, \frac{\Omega_{p1}}{d2^{\frac{1}{2n}}}]$
So I have chosen the cutoff frequency to be **1.1039**

Poles found are:

- -0.8931 + 0.6488i
- -1.1039 + 0.0000i
- -0.8931 - 0.6488i
- -0.3411 - 1.0498i
- -0.3411 + 1.0498i

The corresponding analog transfer function is numerator = **1.6391**

degree s^k	5	4	3	2	1	0
Coefficient:	1.0	3.572	6.38	7.043	4.805	1.639

Table: for denominator of analog filter

2.6 Analog Bandstop Transfer Function

Formula Used to convert back to bandpass filter

$$s -> \frac{B*s}{s*s+\omega_0*\omega_0}$$

degree s^k	10	9	8	7	6	5	4	3	2	1	0
Coefficient:	1.0	0	1.731	0	1.199	0	0.4152	0	0.07189	0	0.004978

Table: for numerator of analog band pass filter

degree s^k	10	9	8	7	6	5	4	3	2	1	0
Coefficient:	1.0	0.4986	1.856	0.7097	1.33	0.372	0.4605	0.0851	0.07705	0.007168	0.004978

Table: for denominator of analog band pass filter

2.7 Discrete Time Filter Transfer Function

I have used bi linear transformation to convert back to discrete domain: $s- > \frac{z-1}{z+1}$

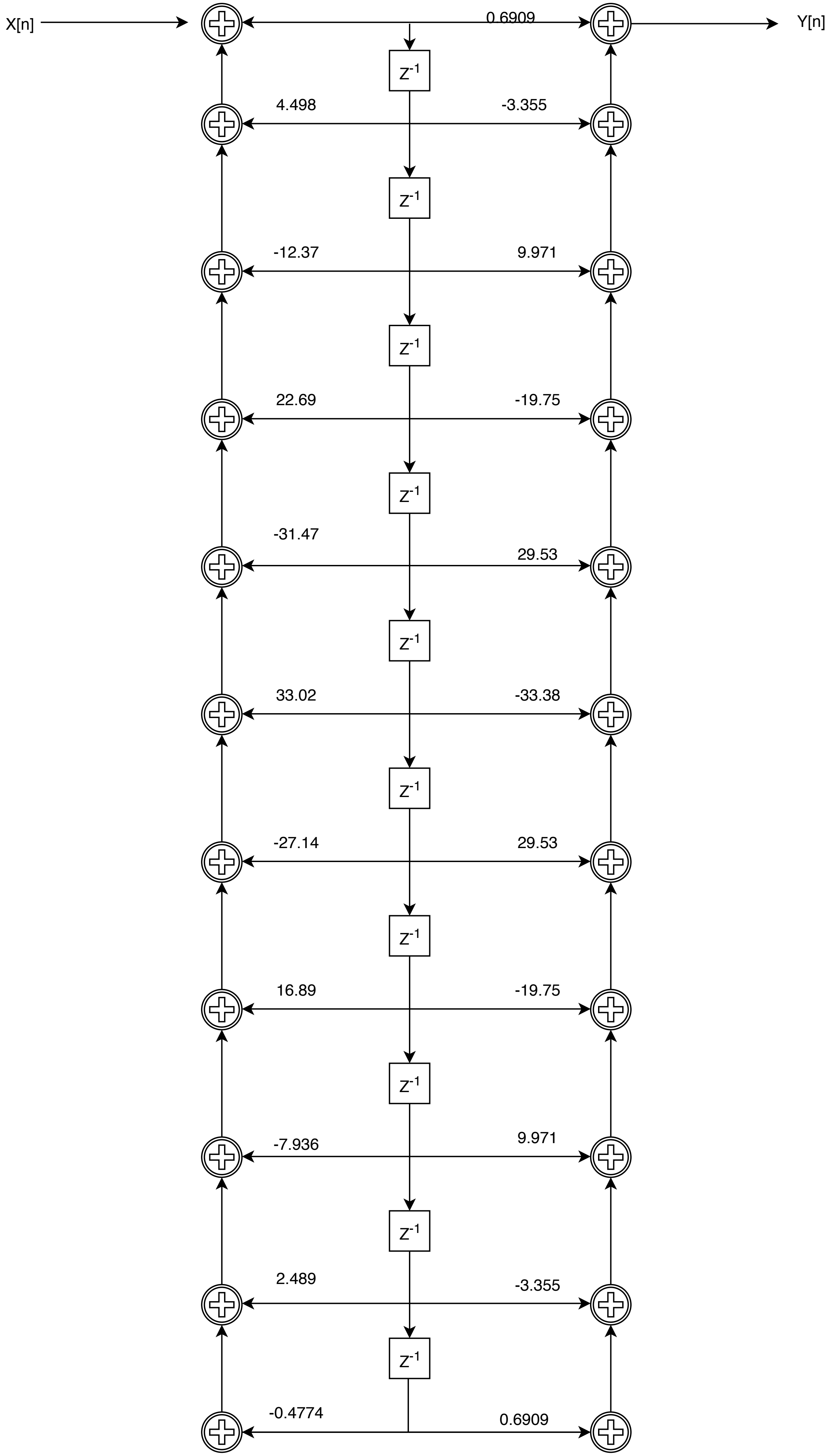
degree Z^{-k}	0	1	2	3	4	5	6	7	8	9	10
Coefficient:	0.6909	-3.355	9.971	-19.75	29.53	-33.38	29.53	-19.75	9.971	-3.355	0.6909

Table: for numerator of discrete band pass filter

degree Z^{-k}	0	1	2	3	4	5	6	7	8	9	10
Coefficient:	1.0	-4.498	12.37	-22.69	31.47	-33.02	27.14	-16.89	7.936	-2.489	0.4774

Table: for denominator of discrete band pass filter

2.8 Realization using Direct Form II



X[n]
Y[n]
0.6909
Z-1
4.498
-3.355
-12.37
9.971
22.69
-19.75
-31.47
29.53
33.02
-33.38
-27.14
29.53
16.89
-19.75
-7.936
9.971
2.489
-3.355
-0.4774
0.6909

2.9 FIR Filter Transfer Function using Kaiser Window

For designing the Fir band pass filter the rest of the parameters remain same as above(the filter specification).
to make a fir filter I have used kaiser window. the parameters to the kaiser window are as follows:
$A = -20 * log10(delta) = 16.4782$
And the corresponding alpha comes out to be 0.
THe N_{min} comes out to 38. which is a very loose bound.I got correct result for $N_{min} + 11$
I have used Kaiser window for design. The kaiser window basically takes the order of the fir filter and the parameter beta to construct a low pass filter of the same tolerance level. But for my filter design I needed a band pass filter. So a made band stop filter and point wise multiplied the filter with the to get the desired result. The coefficients that i got for the final fir filter is:

```
FIR_Bandstop =

  Columns 1 through 17

   -0.0226   -0.0231    0.0018    0.0274    0.0257   -0.0042   -0.0321   -0.0277    0.0070    0.0365    0.0288   -0.0101   -0.0404   -0.0292    0.0135    0.0436    0.0288

  Columns 18 through 34

   -0.0169   -0.0460   -0.0276    0.0201    0.0475    0.0256   -0.0231    0.9520   -0.0231    0.0256    0.0475    0.0201   -0.0276   -0.0460   -0.0169    0.0288    0.0436

  Columns 35 through 49

    0.0135   -0.0292   -0.0404   -0.0101    0.0288    0.0365    0.0070   -0.0277   -0.0321   -0.0042    0.0257    0.0274    0.0018   -0.0231   -0.0226

>> |
```

2.10 Results

2.10.1 IIR filter

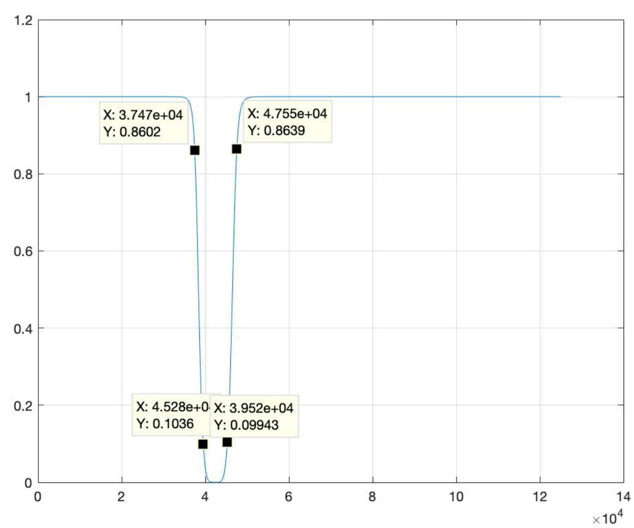


Figure 6: Magnitude response

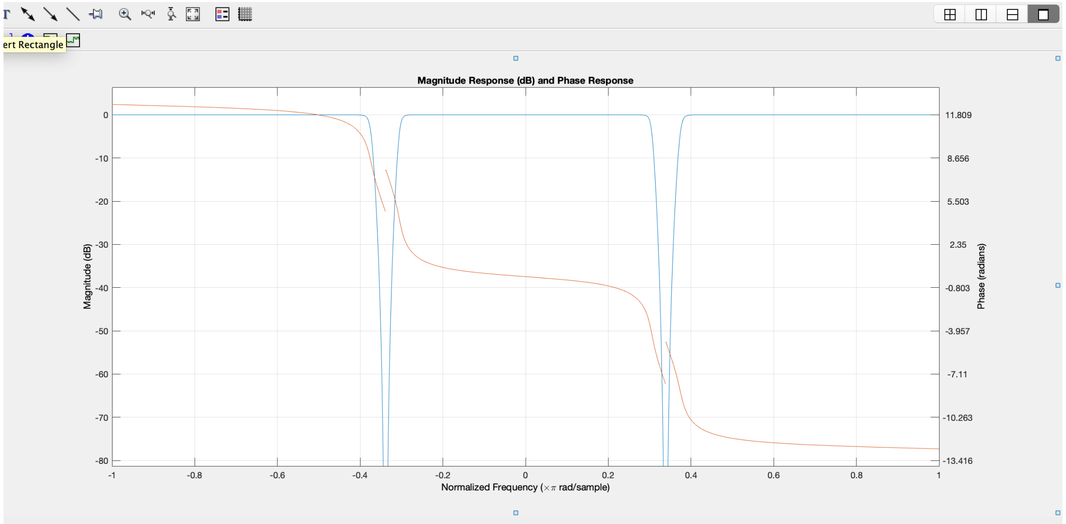


Figure 7: Normalized magnitude and phase response in Fvtool window

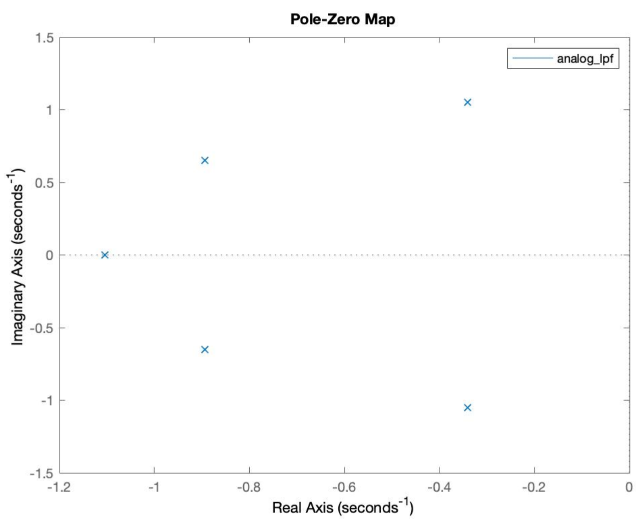


Figure 8: Pole zero plot of Analog low pass filter

2.10.2 FIR filter

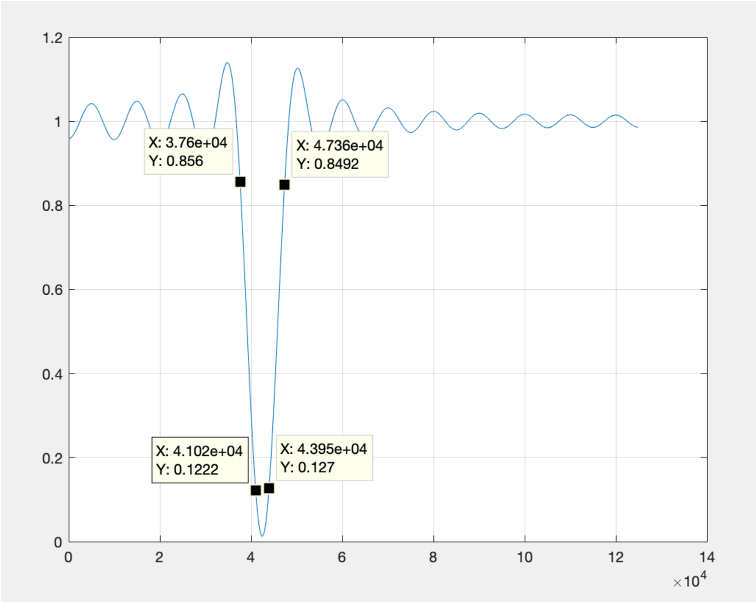


Figure 9: Magnitude response

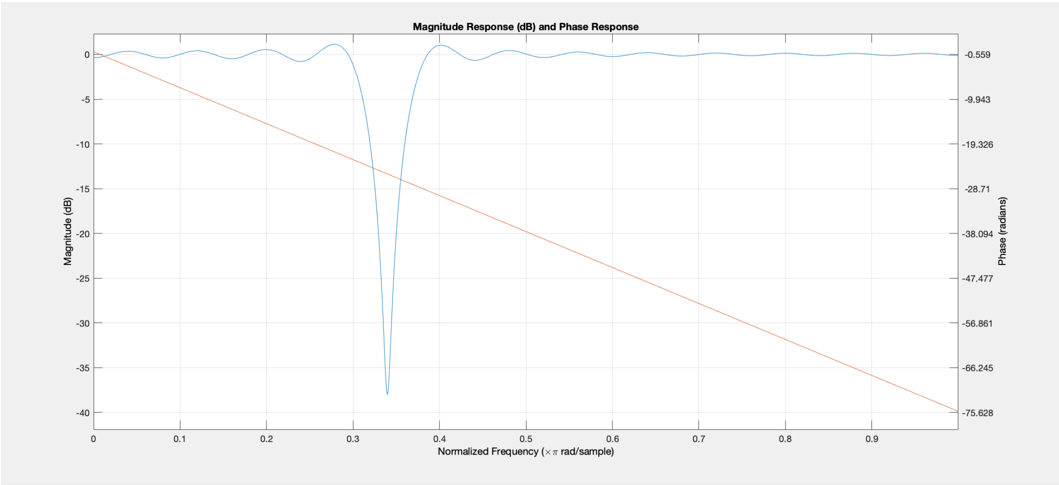


Figure 10: Normalized magnitude and phase response in Fvtool window

3 Elliptical bandpass Filter Design

3.1 Parameter Value Calculation

The filter assigned to me has the number **m = 109**. Using this, I have calculated the following values -

m	109
q	10
r	9
bl	55000
bh	65000
passband	Equiripple
stopband	Equiripple
transition width	2000
sampling rate	320000

3.2 Normalized Specifications

$W_{normalized} = \frac{2*\pi*W_{given}}{\omega_{sampling}}$

Pass Band	1.0799, 1.2763
Stopband	1.0407, 1.3155
Transition Width	0.0393
Tolerance	0.15

3.3 Analog Filter Specifications

In order to meet the specification I have converted the normalized parameters into analog domain.

$$\Omega_{analog} = \tan(\frac{\omega}{2})$$

Pass Band	0.5994, 0.7417
Stopband	0.7725,0.5730
Tolerance	0.15

3.4 Analog Band pass to Analog Low Pass Filter Transformation

I have further converted the analog bandpass specification intolow pass specifications. $s = j\Omega_L$ such that,

$$\Omega_L = \frac{\Omega^2 - \Omega_0^2}{B\Omega}$$

$$\Omega_0^2 = \Omega_{p1}\Omega_{p2}$$

$$B = \Omega_{p2} - \Omega_{p1}$$

Ω_{P1}	-1
Ω_{P2}	+1
Ω_{S1}	-1.4254
Ω_{P2}	1.3856
Tolerance	0.15

3.5 Elliptical lowpass Specification

The equiripple filter has ripples in both pass band as well as the stop band. The low pass elliptical filter has the form of:

$$H(j\omega) * H(-j\omega) = \frac{1}{1+\epsilon^2 R_n^2(\eta, j\omega)}$$

Alternatively this can also be written in the form below if poles and zeros are known:

$$H_a(s) = H_0 \left[\frac{1}{1 - s/p_{a0}} \right]^r \prod_{i=1}^{L} \left[\frac{(1 - s/z_{ai})(1 - s/z_{ai}^*)}{(1 - s/p_{ai})(1 - s/p_{ai}^*)} \right]$$

here $L = Floor(N_min/2)$
And Ho is Gp if N is even and Ho is 1 if N is odd.
Poles and zero can be found from

$$Pole(i) = \Omega_p * j * cd(ui - jv_o K, k)$$
$$zero(i) = \frac{\Omega_p j}{k * zeta_i}$$
$$V_0 = \frac{-jsn^{-1}(\frac{j}{\epsilon_p}, k_1)}{NK_1}$$
$$N = \frac{\frac{K_{1p}}{K_1}}{\frac{K_p}{K}}$$

Where K_{1p} and K_p are the complete elliptic integral of K_1 and K respectively and zeta is the value of cd elliptic function at k.

D1	0.3841
D2	43.4444
N_s	3
Tolerance	0.15

The analog transfer function that I got:

degree s^k	2	1	0
Coefficient:	0.3568	0	0.6015

Table: for numerator of analog filter

degree s^k	3	2	1	0
Coefficient:	1.0000	0.8498	1.1458	0.6015

Table: for denominator of analog filter

ref: https://www.ece.rutgers.edu/ orfanidi/ece521/notes.pdf

3.6 Analog Bandpass Transfer Function

Formula Used to convert back to bandpass filter

$$s- > \frac{s*s+\omega_0*\omega_0}{B*s}$$

degree s^k	5	4	3	2	1	0
Coefficient:	0.05077	0	0.04687	0	0.01003	0

Table: for numerator of analog band pass filter

degree s^k	6	5	4	3	2	1	0
Coefficient:	1.0	0.1209	1.357	0.1092	0.6031	0.02389	0.08784

Table: for denominator of analog band pass filter

3.7 Discrete Time Filter Transfer Function

I have used bi linear transformation to convert back to discrete domain: $s- > \frac{z-1}{z+1}$

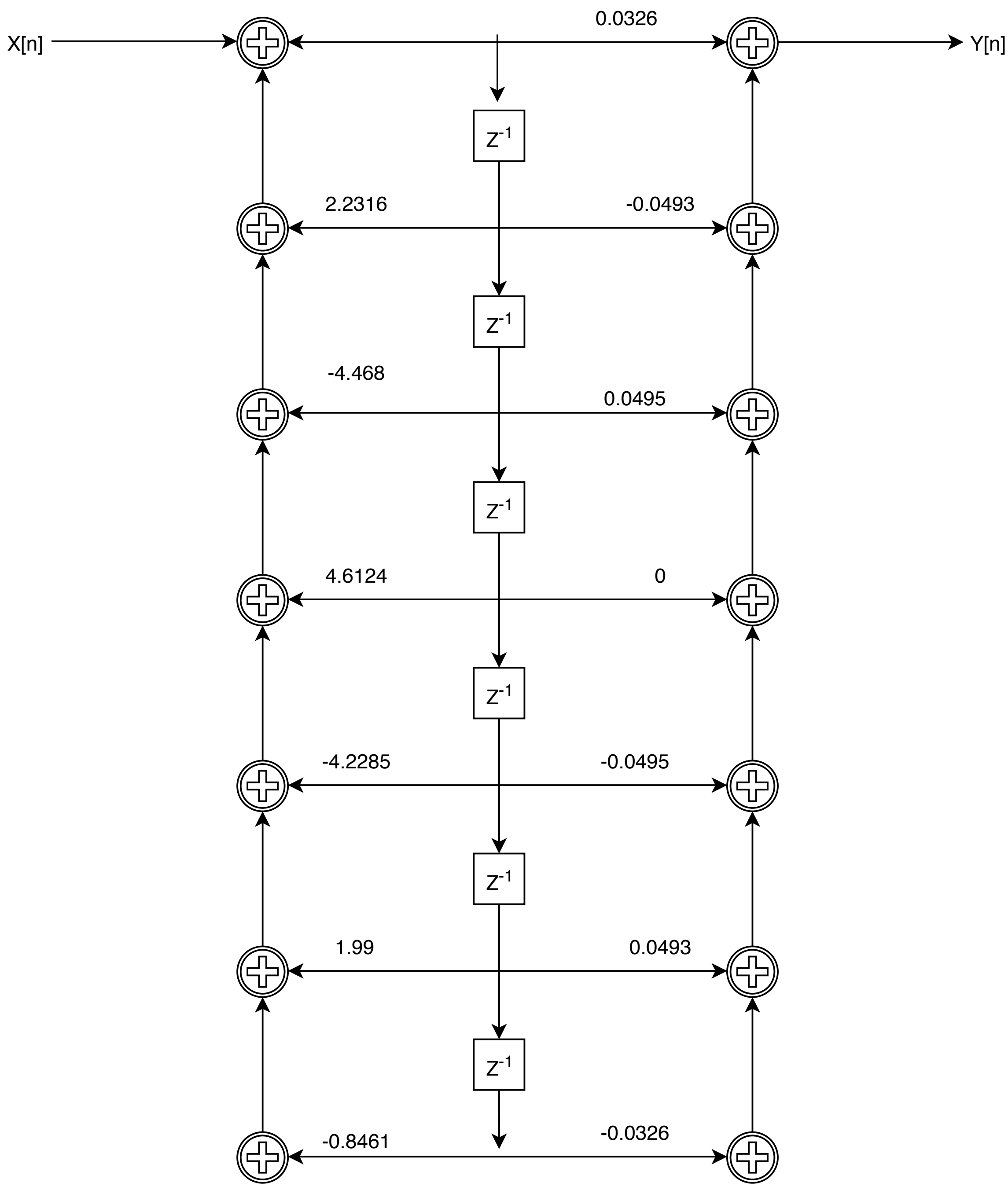
degree Z^{-k}	0	1	2	3	4	5	6
Coefficient:	0.0326	-0.0493	0.0495	0	-0.0495	0.0493	-0.0326

Table: for numerator of discrete band pass filter

degree Z^{-k}	0	1	2	3	4	5	6
Coefficient:	1.0000	-2.2316	4.4685	-4.6123	4.2285	-1.9966	0.8461

Table: for denominator of discrete band pass filter

3.8 Realization using Direct Form II


X[n]
Y[n]
0.0326
Z^-1
2.2316
-0.0493
Z^-1
-4.468
0.0495
Z^-1
4.6124
0
Z^-1
-4.2285
-0.0495
Z^-1
1.99
0.0493
Z^-1
-0.8461
-0.0326

3.9 Result

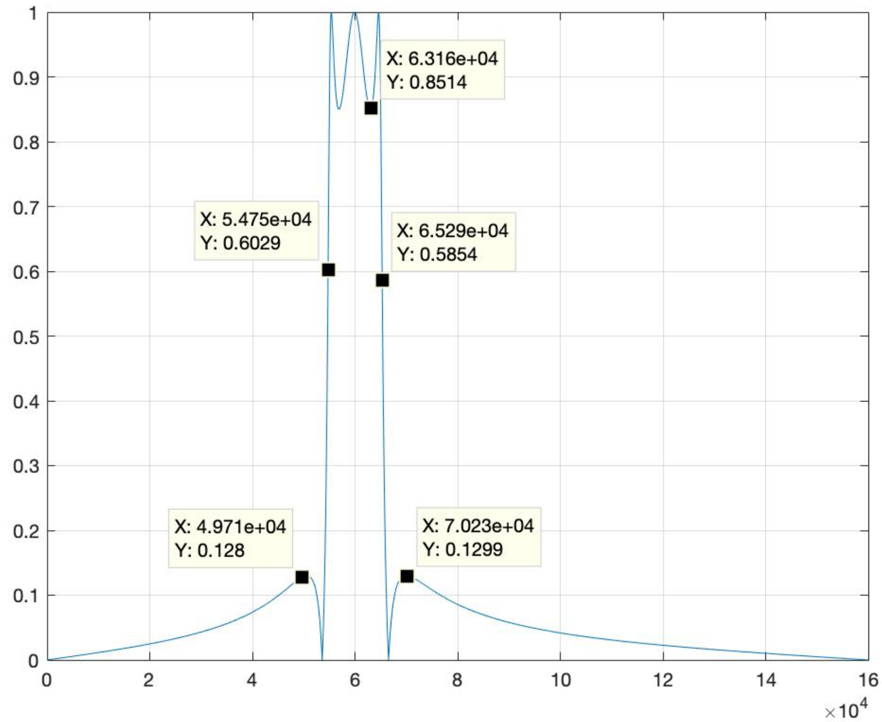


Figure 11: Magnitude Plot

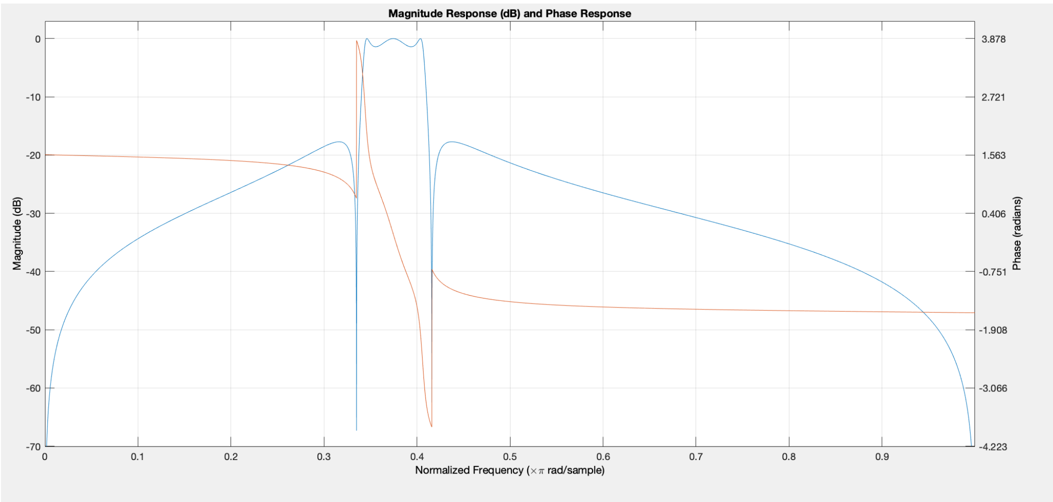


Figure 12: Normalized magnitude and phase response in Fvtool window

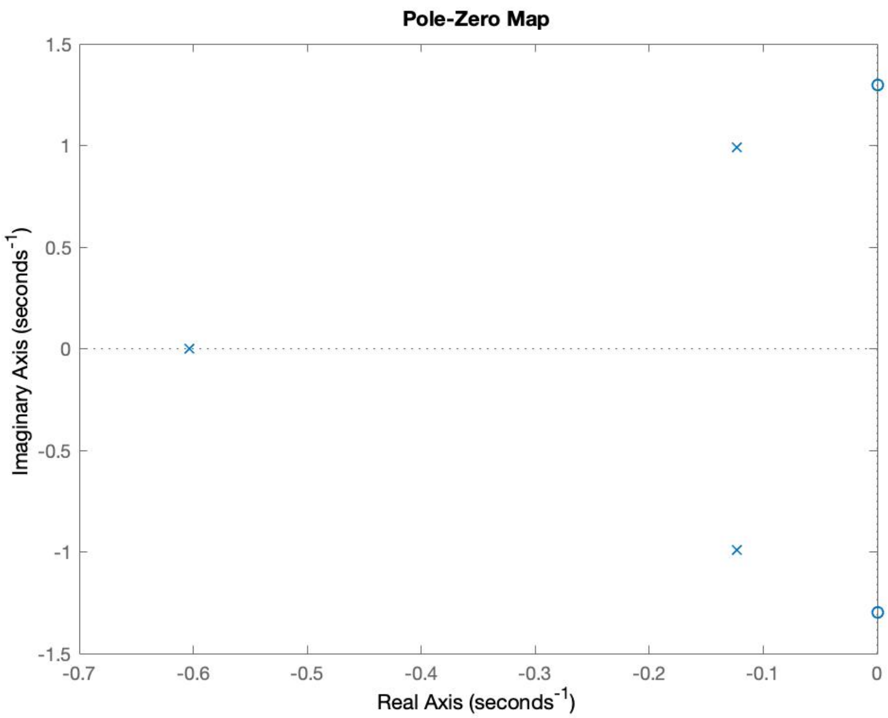


Figure 13: Pole zero plot of the analog low pass filter

4 Elliptical bandstop Filter Design

4.1 Parameter Value Calculation

The filter assigned to me has the number **m = 109**. Using this, I have calculated the following values -

m	109
q	10
r	9
bl	39500
bh	45500
passband	Equiripple
stopband	Equiripple
transition width	2000
sampling rate	250000

4.2 Normalized Specifications

$W_{normalized} = \frac{2*\pi*W_{given}}{\omega_{sampling}}$

Pass Band	1.1938, 0.9425
Stopband	1.1435, 0.9927
Transition Width	0.0503
Tolerance	0.15

4.3 Analog Filter Specifications

In order to meet the specification I have converted the normalized parameters into analog domain.

$$\Omega_{analog} = \tan(\frac{\omega}{2})$$

Pass Band	0.6796, 0.5095
Stopband	0.6435,0.5416
Tolerance	0.15

4.4 Analog Band stop to Analog Low Pass Filter Transformation

I have further converted the analog bandstop specification into low pass specifications. $s = j\Omega_L$ such that,

$$\Omega_L = \frac{B\Omega}{\Omega^2 - \Omega_0^2}$$

$$\Omega_0^2 = \Omega_{p1}\Omega_{p2}$$

$$B = \Omega_{p2} - \Omega_{p1}$$

Ω_{P1}	-1
Ω_{P2}	+1
Ω_{S1}	-1.6146
Ω_{S2}	1.7399
Tolerance	0.15

4.5 Elliptical lowpass Specification

The elliptical Low pass filter was design as before but with the updated values. the corresponding transfer function comes out to be:

degree s^k	2	1	0
Coefficient:	0.1588	0	0.5218

Table: for numerator of analog filter

degree s^k	3	2	1	0
Coefficient:	1.0000	0.8973	1.1832	0.5218

Table: for denominator of analog filter

4.6 Analog Bandstop Transfer Function

degree s^k	6	5	4	3	2	1	0
Coefficient:	1.0000	0	1.0476	0	0.3628	0	0.0415

Table: for numerator of analog band pass filter

degree s^k	6	5	4	3	2	1	0
Coefficient:	1.0000	0.3856	1.0886	0.2765	0.3769	0.0462	0.0415

Table: for denominator of analog band pass filter

4.7 Discrete Time Filter Transfer Function

I have used bi linear transformation to convert back to discrete domain: $s-> \frac{z-1}{z+1}$

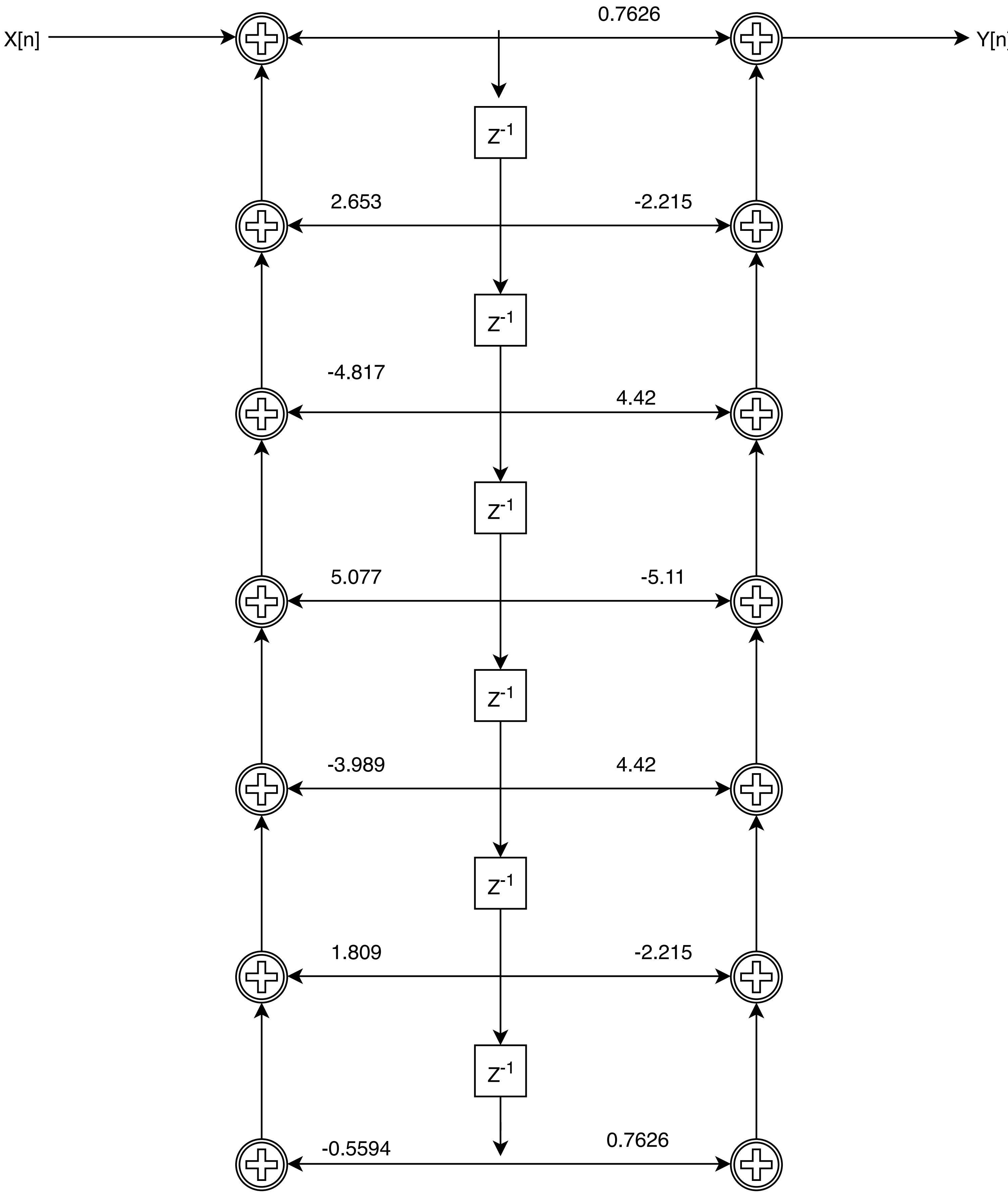
degree Z^{-k}	0	1	2	3	4	5	6
Coefficient:	0.7626	-2.215	4.42	-5.11	4.42	-2.215	0.7626

Table: for numerator of discrete band pass filter

degree Z^{-k}	0	1	2	3	4	5	6
Coefficient:	1.0	-2.653	4.817	-5.077	3.989	-1.809	0.5594

Table: for denominator of discrete band pass filter

4.8 Realization using Direct Form II


X[n]
0.7626
Y[n]
Z-1
2.653
-2.215
Z-1
-4.817
4.42
Z-1
5.077
-5.11
Z-1
-3.989
4.42
Z-1
1.809
-2.215
Z-1
-0.5594
0.7626

4.9 Result

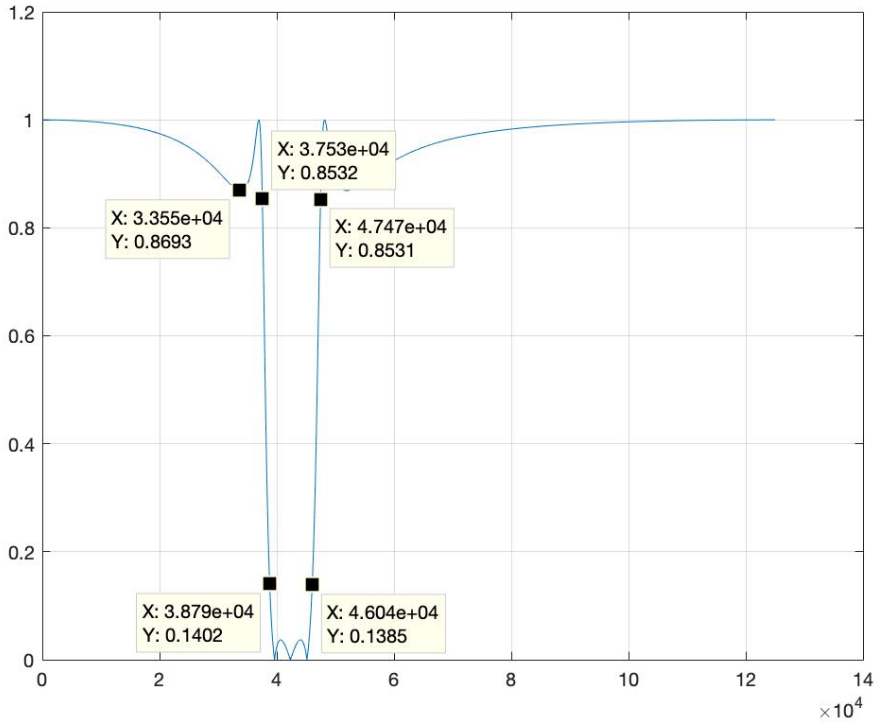


Figure 14: Magnitude plot

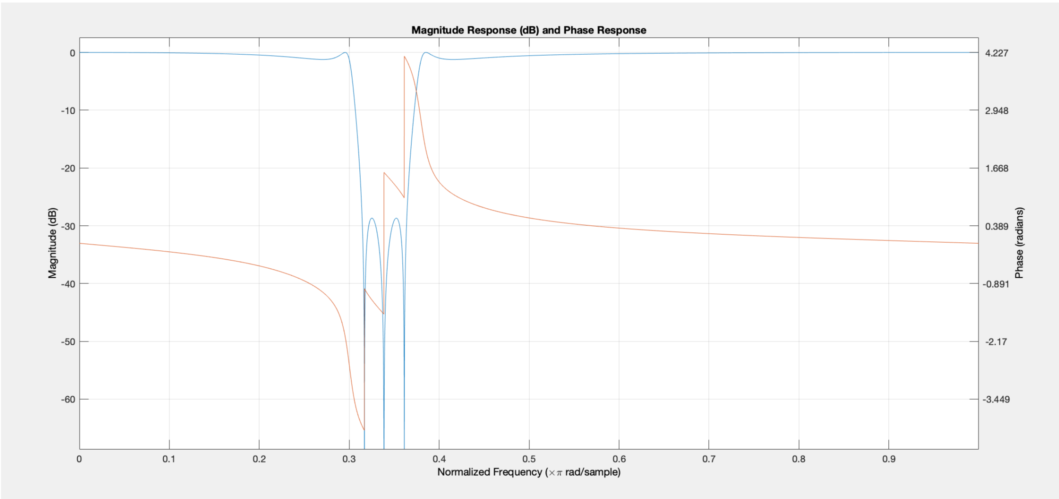


Figure 15: Normalized Magnitude and phase plot in Fvtool

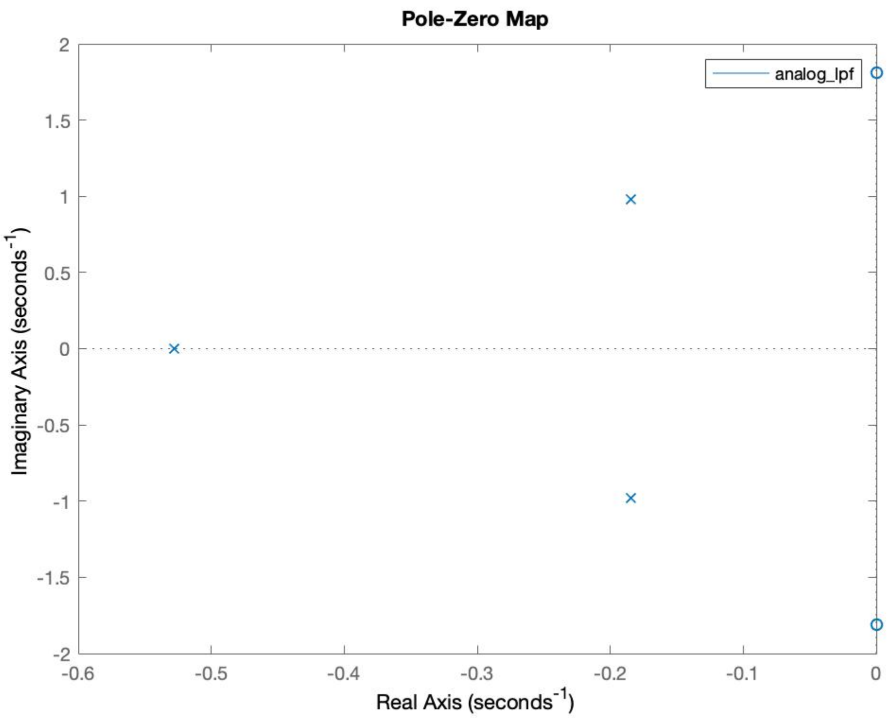


Figure 16: Pole zero plot of the analog low pass filter

5 Conclusions

5.1 Chebyshev Filter

- It requires less components(ie order of the filter) than a butterworth filter or any FIR filter.
- The phase response is not linear.
- It has lesser transition width than a butterworth filter But more than a equiripple Filter.
- Its pass band does not have a constant gain which can be a problem in certain application

5.2 Butterworth Filter

- It requires more components (ie order of the filter) than a equiripple filter
- It does have a linear phase response
- There is no ripple in ether the pass band or the stop band
- It has greater transition width than a equiripple Filter

5.3 Elliptical Filter

- It has sharpest fall from passband to stopband
- It uses least components compared to all the filter with same specification
- It does not have a linear phase response
- There are ripples both in passband and stopband.

5.4 FIR filter

- It requires more order than any IIR filter.
- To implement this We do not need ant=y buffer to store the previous value.
- It has wider transition width than IIR filter.
- Its magnitude response can overshoot.
- It requires more operations(addition and multiplication) than a corresponding IIR filter.
- It is the only filter capable of giving linear phase response