$$\Rightarrow a_1 = \frac{\sqrt{Z_0}}{Z_0 + Z_s} E_s + b_1 \left(\frac{Z_s - Z_0}{Z_s + Z_0} \right)$$

$$b_s = \frac{\sqrt{Z_0}\, E_s}{Z_0 + Z_s}$$ (comes purely from generator)

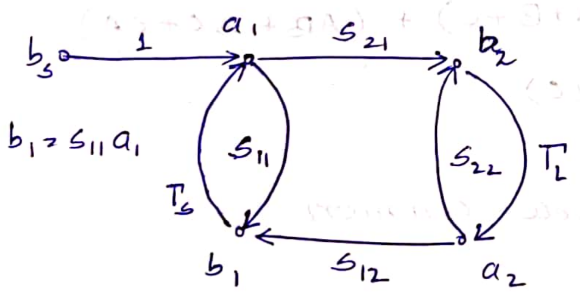
$$\Rightarrow a_1 = b_s + b_1 \Gamma_s$$

$$\Gamma_{out} = \frac{b_2}{a_2} = \frac{S_{22} + S_{12} \Gamma_s S_{21}}{1 - S_{11} \Gamma_s}$$ (find) → Γ_{out} is the dual of Γ_{IN}

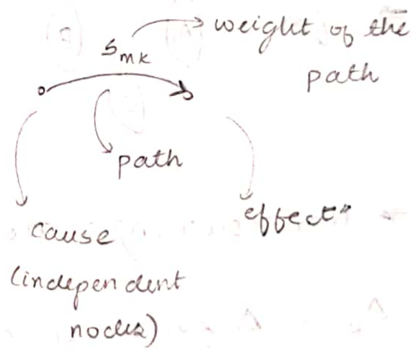
$$\left\{ \begin{array}{l} a_2 = \Gamma_L b_2 \\ b_2 = \Gamma_{out} a_2 \end{array} \right\}$$

FLOW GRAPHS

a's & b's are taken as nodes of graph.



$(a_2 = \Gamma_L b_2)$



a_1, a_2: independent variables

[Excite port 2 by a_2 we have an affect b_2]


→ loop (can't cross the same node twice)

Path should be in one direction.

Path from $a_1 \rightarrow b_1$

(i) S_{11}

(ii) $S_{21} \rightarrow \Gamma_L \rightarrow S_{12}$

Mason's rules

$$\text{Gain (ratio)} = \frac{\sum P_K \Delta K}{\Delta}$$

- where, P_K = total gain number of forward path b/w the quantities whose ratio is to be found

eg: $P_1\ (a_1 \rightarrow b_1) = S_{11}$

$P_2\ (a_1 \rightarrow b_1) = S_{21} T_L S_{12}$

- Δ = 1 − (sum of individual loop gains)
  (same for all gain ratios) + (sum of non-touching loop gain products taken 2 at a time)
  − (sum of non-touching loop gain products taken 3 at a time)

eg: $\Delta = 1 - (S_{11}T_S + S_{22}T_L + S_{21}S_{12}T_L T_S) + (S_{11}T_S S_{22}T_L)$

⇒ for A B C

$\Delta = 1 - (A+B+C) + (AB+BC+CA) - (ABC)$

→ Non-touching : no path or nodes common

- Δ_K = Δ for those loops which are not touching that forward path

→ $\Delta_1 = 1 - S_{22}T_L$

$\Delta_2 = 1$

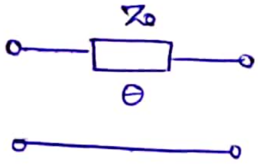
$$\Rightarrow \frac{b_1}{a_1} = \frac{S_{11}(1-S_{22}T_L) + S_{21}T_L S_{12}}{1-(S_{11}T_S + S_{22}T_L + S_{12}S_{21}T_L T_S) + (S_{11}S_{22}T_S T_L)}$$

if $T_S = 0$

$$\frac{b_1}{a_1} = \frac{S_{11}(1-S_{22}T_L) + S_{21}T_L S_{12}}{1-(S_{22}T_L)}$$

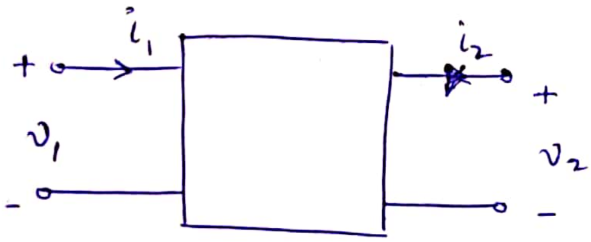
$$\frac{b_1}{a_1} = S_{11} + \frac{S_{21}\Gamma_L S_{12}}{1 - S_{22}\Gamma_L}$$ (same as before)

→ find Γ_{out}

Q. 

Find S-parameter matrix

• ABCD / TRANSMISSION PARAMETERS



(helpful in cascade network)

$$\begin{bmatrix} v_1 \\ i_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} v_2 \\ i_2 \end{bmatrix}$$