...uva Harari : The Sapiens, The Homo Deus

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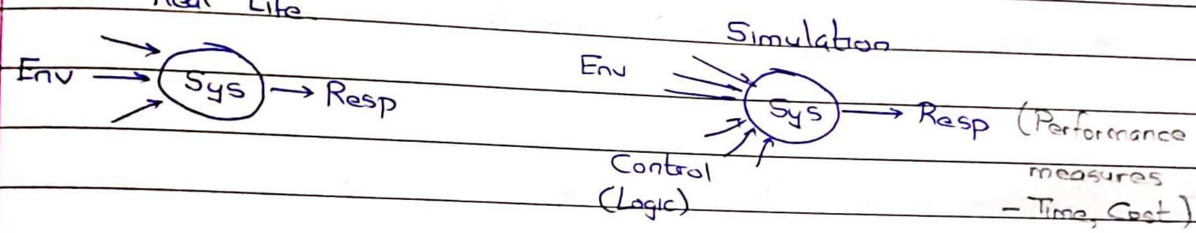
- System $\triangleq$ Combination of interdependent blocks that function differently than each independently does.
- System responds to 'environment' and 'controls' through 'responses'

'Cause'
'Effect'

- Causes < Uncontrolled / Choices

"Instruments of pleasure" xD

A] Model of a system

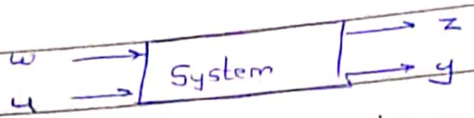


- Control logic is mostly based on feedback from performance measures
  - It may not be feedback-based, if the number of combinations of possible environmental perturbations and performance measures is finite. Control choices are directly predecided.
- System gives feedback to computer logic, and changes in control logic are physically implemented by an actuator within the system.
  - The actuator is a system by itself.

* Control by interaction
The 'compensator' works together with device to produce correct output of device.

- Applicability of simulation in real world

I CLASSIFICATION



known / unknown

w :- Exogenous inputs - reference values, approximate disturbances, etc
- cannot be controlled ★ Verify

u :- Controlled inputs -

y :- Response measurements - can be measured

z :- Regulated outputs - → determine performance

II CLASSIFICATION OF SYSTEMS

1 Memoryless - react to inputs instantaneously

2 With memory - react to present and past inputs.

8/1 3 Causal - state $x(t)$ depends only on present or past inputs

- We consider all systems to be causal.

$\dot{x} \propto \lambda_{in} - \lambda_{out}$

Take $\dot{x} = \lambda_{in} - \lambda_{out}$

$\lambda_{out} = \phi(x(t)) \approx \rho\, x(t)$ or $\lambda_{out} = 0$

$\lambda_{in} \in \{0, \lambda_{in}\}$

- State space model :- State $= x(t)$

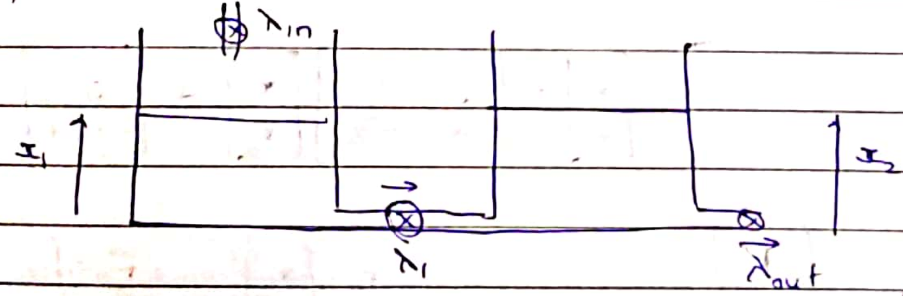
  Control input $= \lambda_{in}$

  External input $\in \{0\,(\text{OFF}), 1\,(\text{ON})\}$ for λ_{out}

  $h \le x \le H$

→ Applications :- Inventory controls, warehouse control, bank account with spending, communication networks.

B] Coupled Tank



$\dot{x}_1 = \lambda_{in} - \lambda_1$

Use $\lambda_1 = \phi(x_1 - x_2) \approx c(x_1 - x_2)$

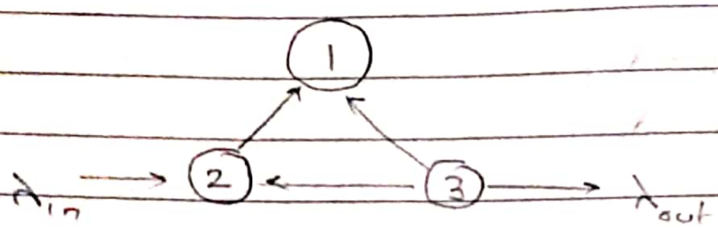
$\Rightarrow \dot{x}_1 = -c(x_1 - x_2) + \lambda_{in}$

$\dot{x}_2 = \lambda_1 - \lambda_{out}$

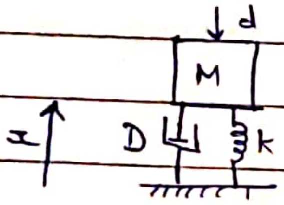
Use $\lambda_{out} \approx \rho x_2$

$$\therefore \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -c & c \\ c & -c\rho \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} \lambda_{in}$$

HW 3 tanks



C] Suspension System.

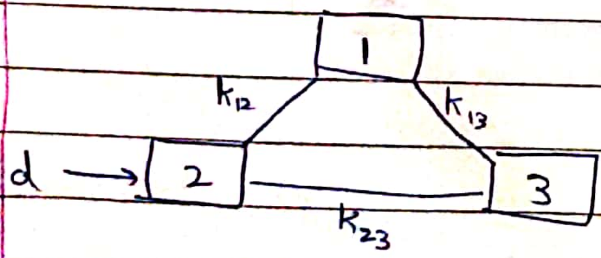


$$M\ddot{x} = -Mg - D\dot{x} - Kx + d$$

If $x_1 = x$ and $x_2 = \dot{x}$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -k/m & -D/m \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ -g+d \end{bmatrix}$$

eg



$$\vec{r}_1 = \begin{bmatrix} x_1 \\ y_1 \end{bmatrix} \quad \vec{r}_2 = \begin{bmatrix} x_2 \\ y_2 \end{bmatrix} \quad \vec{r}_3 = \begin{bmatrix} x_3 \\ y_3 \end{bmatrix}$$

$$\ddot{\vec{r}}_1 = -k_{12}\vec{r}_1$$

$$\ddot{\vec{r}}_3 = -k_{13}(\vec{r}_3 - \vec{r}_1)$$

$$\ddot{\vec{r}}_2 = d$$

- In general, state equations are of the form: (x = state vector)

$$\dot{x} = Ax + B_1 u + B_2 \omega$$

$$y = Cx + D$$

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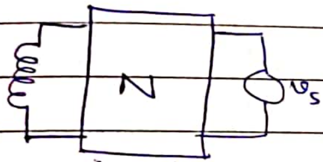
D] Circuits

- R, L, C, VCVS, VCCS, CCVS, CCCS, transformer, gyrator, independent sources
  - R, L, C: Energy storing elements
  - Resistive elements
- State circuit :- × L, C
  Dynamic ✓

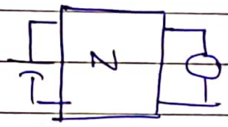
- $i_C = C\frac{dv_C}{dt}$, $v_L = L\frac{di_L}{dt}$. → State variables

→ First order circuits have first order state space equations.



'resistive'

or



'resistive'

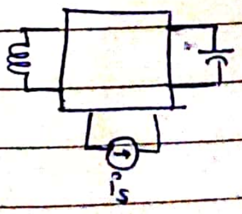
- Assume currents and directions, find voltages and polarities, or vice versa.

Write v_L = Linear function (i_L, v_{source})
i_C v_C

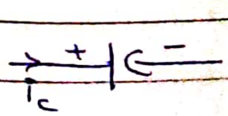
State space equations come from $C\frac{dv_C}{dt}$ and $L\frac{di_L}{dt}$

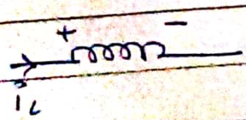
→ Second order circuits have two energy storing elements

eg -



Find $v_L = LF(i_L, v_C, i_s)$
$i_C = LF(i_L, v_C, i_s)$ } Use superposition

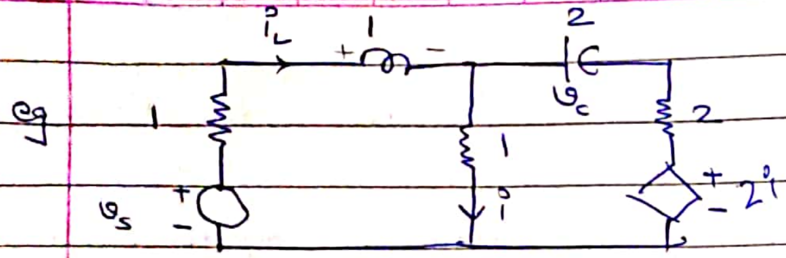
* 



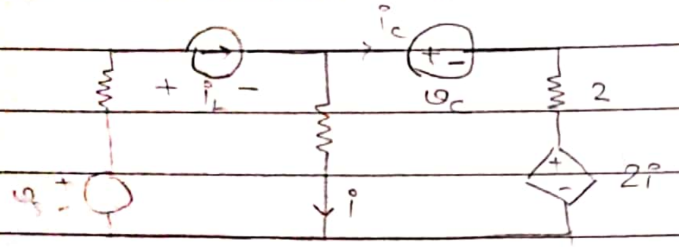
$$\frac{dv_c}{dt} = -\frac{1}{2}(v_c + i_L)$$

$$\frac{di_L}{dt} = v_s - v_c - 3i_L$$

eg



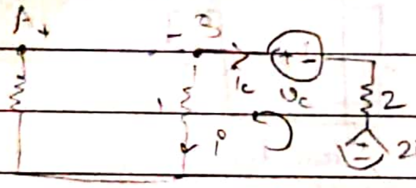
Treat i_L, v_c as so



Find v_L and i_c as functions by superposition

$\rightarrow$ v_L =

1)



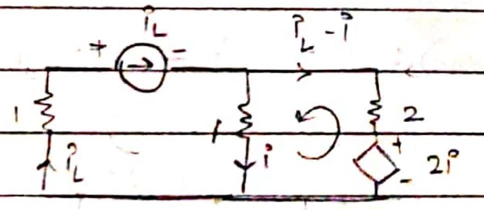
$\rightarrow$ $v_c + i_c - 2i_c + 2i_c = 0$

$\boxed{i_c = -v_c}$

$\rightarrow$ $v_A - i_c = v_B$

$\boxed{v_L = i_c = -v_c}$

2)



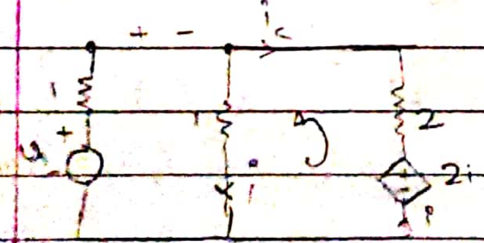
$2i + 2i_L - 2i - i = 0$

$i = 2i_L$

$\rightarrow$ $\boxed{i_c = -i_L}$

$\rightarrow$ $v_L = -2i_L - i_L = \boxed{-3i_L}$

3)

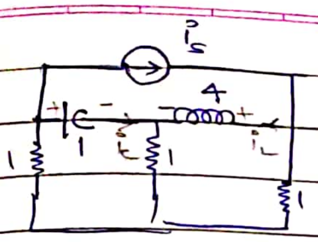


$i = 0$

$\boxed{v_L = v_s}$

$\boxed{i_c = 0}$

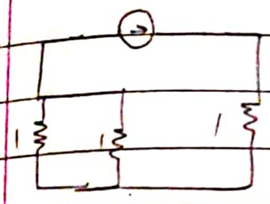
Q2



$$\frac{di_L}{dt} = \frac{1}{4}\left(\frac{3i_s}{2} - \frac{3i_L}{2} + \frac{v_c}{2}\right)$$

$$\frac{dv_c}{dt} = \frac{1}{2}\left(-i_s - v_c - i_L\right)$$

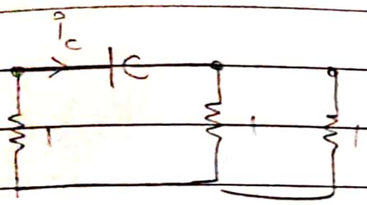
1



$$v_L = \frac{3}{2} i_s$$

$$i_c = -\frac{i_s}{2}$$

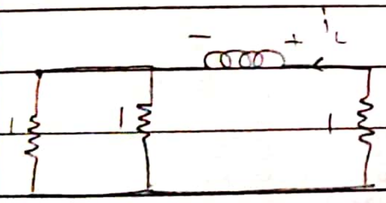
2



$$v_L = \frac{v_c}{2}$$

$$i_c = \frac{-v_c}{2}$$

3



$$v_L = -\frac{3i_L}{2}$$

$$i_c = -\frac{i_L}{2}$$

Chapter 2 examples

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15/1 MODELING OF LTI SYSTEMS

Time domain :- LCCDE

Frequency domain :- Transfer functions (Rational function of s)

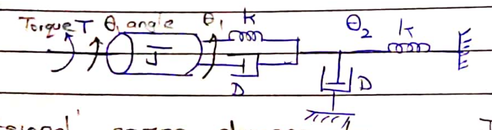
$$G(s) = \frac{q(s)}{p(s)}$$

p(s) → higher degree

- Effect of LTI system on sinusoids :- Amplitude, phase

$$A\cos\omega t \longrightarrow B\cos(\omega t + \phi)$$

- p(s) should not have roots in right half plane.

eg 

Find transfer function for T to θ_2

'Torsional' spring, damper, etc J = Moment of inertia

Time domain	Frequency domain
θ_2, T	$\theta_2(s)$, T(s)
	or $\hat{\theta}_2$, or $\hat{T}$

Torque balance 1 :- $T = J\ddot{\theta}_1 + D(\dot{\theta}_1 - \dot{\theta}_2) + k(\theta_1 - \theta_2)$

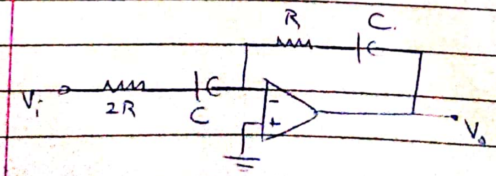
~~Torque balance 2~~ $\hat{T} = Js^2\hat{\theta}_1 + (Ds + k)(\hat{\theta}_1 - \hat{\theta}_2)$

Torque balance 2 :- $k(\theta_2 - \theta_1) + D(\dot{\theta}_2 - \dot{\theta}_1) + D\dot{\theta}_2 + k\theta_2 = 0$

$$2(Ds + k)\hat{\theta}_2 - (Ds + k)\hat{\theta}_1 = 0$$

$$\Rightarrow \hat{\theta}_1 = 2\hat{\theta}_2$$

Answer : $\dfrac{\hat{T}}{\hat{\theta}_2} = \dfrac{1}{2Js^2 + Ds + k}$

eg 

$$\frac{V_o}{V_{in}} = \frac{-\left(R + \frac{1}{sC}\right)}{2R + \frac{1}{sC}}$$

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Q

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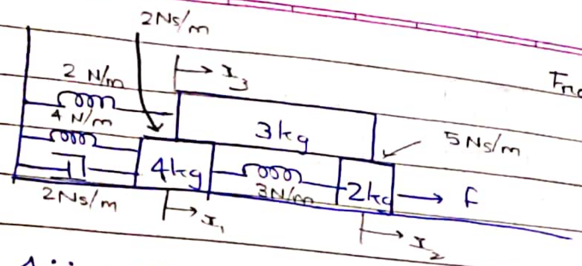
Q



Friction like damping

Find equations

Find $\dfrac{\hat{X}_2}{\hat{F}}$

For 1: $4\ddot{x}_1 + 2\dot{x}_1 + 4x_1 + 2(\dot{x}_1 - \dot{x}_3) + 3(x_1 - x_2) = 0$

2: $2\ddot{x}_2 + 5(\dot{x}_2 - \dot{x}_3) + 3(x_2 - x_1) = f$

3: $2\ddot{x}_3 + 2x_3 + 2(\dot{x}_3 - \dot{x}_1) + 5(\dot{x}_3 - \dot{x}_2) = 0$

Zero initial conditions :- $x_1(0) = x_2(0) = x_3(0) = 0$

Laplace 1 : $(4s^2 + 4s + 7)\hat{X}_1 - 3\hat{X}_2 - 2s\hat{X}_3 = 0$

$(2s^2 + 5s + 3)\hat{X}_2 - 3\hat{X}_1 - 5s\hat{X}_3 = \hat{F}$

$(3s^2 + 7s + 2)\hat{X}_3 - 5s\hat{X}_2 - 2s\hat{X}_1 = 0$

Matrix :
$$\underbrace{\begin{bmatrix} 4s^2+4s+7 & -3 & -2s \\ -3 & 2s^2+5s+3 & -5s \\ -2s & -5s & 3s^2+7s+2 \end{bmatrix}}_{\text{`P(s)'}} \begin{bmatrix} \hat{X}_1 \\ \hat{X}_2 \\ \hat{X}_3 \end{bmatrix} = \begin{bmatrix} 0 \\ \hat{F} \\ 0 \end{bmatrix}$$

Cramer's rule :- Unique solution for $\hat{X}_1, \hat{X}_2, \hat{X}_3$ can be found if $\det P(s) \neq 0$

* $P^{-1}(s) = \dfrac{\text{adj } P(s)}{\det P(s)}$

→ 2^{nd} order state space ~~eq~~ system

↳ 2 state variables.

- State space form only has first order derivative

- If a system has x_1'', then choose state variables as x_1 (for x_1, x_1') and $x_2 = x_1'$ (for x_1' and x_1'')

eg
$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -6 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u.$$

Output $y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

This is a state space system because :-

1) Derivatives are at most 1^{st} order
2) Output is a linear function of state variables.

Equations :-
$$s\hat{X}_1 = \hat{X}_2$$
$$s\hat{X}_2 = -6\hat{X}_1 - 5\hat{X}_2 + \hat{U}$$
$$\hat{Y} = \hat{X}_1$$

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Solve to get transfer function :- $\dfrac{\hat{Y}}{\hat{U}} = \dfrac{1}{s^2+5s+6}$

$$\equiv \begin{bmatrix} 0 & 1 \\ -6 & -5 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \end{bmatrix}$$

→ ABCD form :- $\dot{x} = Ax + Bu$

$y = Cx + Du$

Zero initial conditions

$X(0) = 0$

If not, then $\dot{y} \to s\hat{Y} - \ldots$

$$\left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right] = \left[\begin{array}{cc|c} 0 & 1 & 0 \\ -6 & -5 & 1 \\ \hline 1 & 0 & 0 \end{array}\right]$$

Laplace :-
$$\hat{X} = A\hat{X}_1 + B\hat{U}$$
$$\hat{Y} = C\hat{X} + D\hat{U}$$

Solution :-
$$\hat{X} = (sI - A)^{-1} B\hat{U}$$
$$\hat{Y} = (C(sI - A)^{-1} B + D)\hat{U}$$

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$$sI - A = \begin{bmatrix} s & -1 \\ 6 & s+5 \end{bmatrix}$$

$$(sI - A)^{-1} = \frac{\text{adj}(sI - A)}{|sI - A|} \qquad |sI-A| \rightarrow s^2 + 5s + 6$$

$$\text{adj}(sI - A) = \begin{bmatrix} s+5 & -6 \\ 1 & s \end{bmatrix}^T = \begin{bmatrix} s+5 & 1 \\ -6 & s \end{bmatrix}$$

$$\hat{y} = \frac{[1 \;\; 0]\begin{bmatrix} s+5 & 1 \\ -6 & s \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix}}{s^2 + 5s + 6} = \frac{1}{s^2 + 5s + 6} \checkmark$$

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eg $\dot{x} = -ax + bu \Rightarrow \frac{\hat{x}}{\hat{u}} = \frac{b}{s+a}$

- First order state space system have denominator of degree 1.

eg $\dot{y} = -ay + bu$

$$s\hat{y} - y(0) = -a\hat{y} + b\hat{u}$$

$$\hat{y} = \frac{1}{s+a} y_0 + \frac{b}{s+a} \hat{u}$$

$$\therefore y(t) = y_0 e^{-at} + L^{-1}\left(\frac{b}{s+a} \hat{u}\right)$$

- $L^{-1}\left(F(s)\, G(s)\right) = \int_0^\infty f(t-\tau)\, g(\tau)\, d\tau$

$$\mathbb{J} = \int_0^t f(t-\tau)\, g(\tau)\, d\tau \quad \text{(for causal systems to bound } g(\tau) \text{ to within } (0,t)\text{)}$$

A. We assume that $f(t) = 0 \;\; \forall \; t < 0$

$$y(t) = y_0 e^{-at} + b \int_0^t e^{-a(t-\tau)} u(\tau)\, d\tau$$

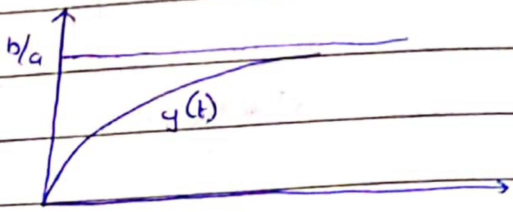
→ Response parameters

eg $\dot{y} = -ay + bu$ with $u(t) =$ ~~Step~~ Unit step

$\hat{U}(s) = \frac{1}{s}$

$$\hat{Y} = \frac{b}{s+a} \times \frac{1}{s} = \frac{b/a}{s} + \frac{b/-a}{s+a}$$

$$y(t) = \frac{b}{a}\left(1 - e^{-at}\right)$$



1 Time constant $= \frac{1}{a}$

2 Rise time $\triangleq$ Time taken to rise from 10% to 90% $= T_r$

3 Settling time $\triangleq$ Time taken to reach 2% of settling value $= T_s$

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$$\dot{x} = Ax + Bu$$
$$y = Cx + Du$$

where $x : n\times 1$, $u : m \times 1$, $y : p \times 1$

$\therefore$ $A : n \times n$, $B : n \times m$, $C : p \times n$, $D : p \times m$

- Simplified single-input single-output systems

$$\dot{x} = Ax + bu \quad \text{with } x(0) = x_0$$
$$y = cx + du$$

Represented as $\left[\begin{array}{c|c} A & b \\ \hline c & d \end{array}\right]$

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III STATE TRANSITION MATRIX $\rightarrow$ Element-wise Laplace inverse

$\rightarrow$ Define $\Phi(t) = L^{-1}\left((sI - A)^{-1}\right)$

$\therefore$ $\Phi(t)$ is a solution of $\dot{z} = Az$ with $z(0) = I$

$$s\hat{z} - I = A\hat{z}$$

$$\therefore \hat{z} = (sI - A)^{-1}$$

$$\Phi(t) \sim e^{t}$$

- Properties :-

1) $\Phi(0) = 1$
2) $\dfrac{d\Phi(t)}{dt} = A\Phi(t)$
3) $\Phi(t+s) = \Phi(t)\Phi(s)$
4) $\Phi(-t) = \Phi(t)^{-1}$
5) $\det \Phi(t) =$

eg $A = \begin{bmatrix} 0 & 1 \\ -6 & -5 \end{bmatrix}$

$$(sI - A)^{-1} = \begin{bmatrix} s & -1 \\ 6 & s+5 \end{bmatrix}^{-1}$$

$$= \frac{1}{s^2+5s+6}\begin{bmatrix} s+5 & 1 \\ -6 & s \end{bmatrix}$$

$$L^{-1}\left((sI-A)^{-1}\right) = L^{-1}\left(\frac{1}{(s+2)(s+3)}\begin{bmatrix} s+5 & 1 \\ -6 & s \end{bmatrix}\right)$$

$$= L^{-1}\left(\frac{[M]}{s+2} + \frac{[N]}{s+3}\right)$$

$$= e^{-2t}M + e^{-3t}N.$$

$$= \Phi(t)$$

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→ Matrix exponential function (e^A)

$$= I + A + \frac{A^2}{2!} \cdots$$

Define $M_n = I + A + \frac{A^2}{2!} \cdots \frac{A^n}{n!}$

Define $\|M\| = \max_n \left(\sum |m_{ij}|\right)$ = maximum absolute element.

- Satisfies triangle inequality :- $\|A+B\| \leq \|A\| + \|B\|$
- Satisfies Schwarz inequality :- $\|AB\| \leq \|A\| \, \|B\|$

Using the inequalities, $\|M_n\| \leq 1 + \|A\| + \frac{\|A\|^2}{2!} \cdots \frac{\|A\|^n}{n!}$

$$\therefore \lim_{n \to \infty} \|M_n\| \leq e^{\|A\|}$$

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Definition State Transition Matrix (STM) Φ

General solution of homogeneous equation $\dot{x} = Ax$, with $x(0) = x_0$ is:

$$x(t) = \Phi(t) x_0$$

$$\hookrightarrow \overset{\Delta}{=} \text{STM} = L^{-1}(sI - A)^{-1}$$

- $\Phi(t)$ depends only on A, not on x_0

$$s\hat{X} - X_0 = A\hat{X}$$

$$\therefore \hat{X} = (sI - A)^{-1} X_0$$

- $x(t) = \Phi(t) x_0$ is unique solution of the differential equation

• $\Phi(t)$ is the unique solution of $\dot{z} = Az$ with $z(0) = I$

This unique solution must be $M(t) = I + At + \frac{A^2 t^2}{2!} \cdots$

$$M(0) = I \qquad \dot{M}(t) = A\,M(t)$$

$$\therefore \Phi(t) = e^{At}$$

- Transition from t_0 to t_2 with $t_1 \in (t_0, t_2)$:-

$$x(t_2) = \Phi(t_2 - t_1)\,\Phi(t_1 - t_0)\,x(t_0)$$
$$= \Phi(t_2 - t_0)\,x(t_0)$$

$\therefore$ $\Phi(s+t) = \Phi(s)\,\Phi(t)$

Is $e^{A(t+s)} = e^{At} e^{As}$?

Theorem $e^{A+B} = e^{A} e^{B}$ iff $AB = BA$

→ Properties of STM

1. $\Phi(0) = I$
2. $\Phi(t+s) = \Phi(t)\,\Phi(s)$
3. $\Phi(-t) = \Phi(t)^{-1}$
4. $\det \Phi(t) \neq 0 \quad \forall t$
5. $\dot{\Phi}(t) = A\,\Phi(t)$
6. $\det \Phi(t) = e^{(\mathrm{trace}\,A)\,t}$

→ Effect of transformation (from x variable to y)

$\dot{x} = Ax$ with $x_0 = x(t_0)$

$x = Ty$ and $y = T^{-1}x$ (where T is a non-singular operation)

$$\frac{d}{dt}(Ty) = ATy$$

$$T\dot{y} = ATy$$

$$\therefore \dot{y} = (T^{-1}AT)\,y$$

~~STM~~: '$A \rightarrow T^{-1}AT$'

- Given $\Phi(t) = L^{-1}(sI - A)^{-1}$, what is $\psi(t) =$ $L^{-1}(sI - B)^{-1}$ where $B = T^{-1}AT$?

$$\psi(t) = T^{-1}\,\Phi(t)\,T$$

Write $\psi(t) = I + B + \frac{B^2}{2!} \cdots = T^{-1}\left(I + A + \frac{A^2}{2!} \cdots\right)T = T^{-1}\Phi(t)\,T$

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B is a column vector } for SISO
C row

~~IV SISO Systems~~

→ $\dot{x} = Ax + Bu$

$y = Cx + Du$

$x(t) = \Phi(t) x_0 + \int_{t_0}^{t} \Phi(t-\tau) B u(\tau) d\tau$

$y(t) = c\Phi(t) x_0 + \int_{t_0}^{t} c\,\Phi(t-\tau) B u(\tau) d\tau + D u(t)$

Transfer matrix :- $T(s) = c(sI - A)^{-1} B + D$

$L^{-1} T(s) = c\Phi(t) B$

$Y(s) = T(s) U(s)$

If $U(s)$ is scalar and not a matrix then $T(s) = \frac{\hat{Y}(s)}{\hat{U}(s)}$

→ State space from transfer function

1 $\frac{\hat{y}}{\hat{u}} = \frac{b_0}{s^2 + a_1 s + a_2}$

$\ddot{y} + a_1 \dot{y} + a_2 y = b_0 u$

State variables : $x_1 = y$, $x_2 = \dot{y}$,

$\dot{x}_2 = -a_2 x_1 - a_1 x_2 + b_0 u$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -a_2 & -a_1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ b_0 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\therefore \frac{b_0}{s^2 + a_1 s + a_2} \longleftrightarrow \left[\begin{array}{cc|c} 0 & 1 & 0 \\ -a_2 & -a_1 & b_0 \\ \hline 1 & 0 & 0 \end{array}\right]$$

2 $\frac{\hat{y}}{\hat{u}} = \frac{b_0 s + b_1}{s^2 + a_1 s + a_2}$

Consider $\hat{y} \leftarrow \boxed{b_0 s + b_1} \leftarrow \boxed{\frac{1}{s^2 + a_1 s + a_2}} \leftarrow \hat{U}$

$\hat{Y}_1 = \hat{X}$

$x_1 = y_1 \qquad x_2 = \dot{x}_1$

$\dot{x}_1 = x_2$

$\dot{x}_2 = -a_2 x_1 - a_1 x_2 + u$

$y = b_0 x_2 + b_1 x_1$

$\therefore \dfrac{\hat{y}}{\hat{u}} = \dfrac{b_0 s + b_1}{s^2 + a_1 s + a_2}$ and $\dfrac{\hat{x}}{\hat{u}} = \dfrac{1}{s^2 + a_1 s + a_2}$ and $\hat{y} = (b_0 s + b_1)\hat{x}$

$$\longleftrightarrow \left[\begin{array}{cc|c} 0 & 1 & 0 \\ -a_2 & -a_1 & 1 \\ \hline b_1 & b_0 & 0 \end{array}\right]$$

3 $\dfrac{\hat{y}}{\hat{u}} = \dfrac{b_0 s^2 + b_1 s + b_2}{s^2 + a_1 s + a_2} = b_0 + \dfrac{(b_1 - b_0 a_1)s + (b_2 - b_0 a_2)}{s^2 + a_1 s + a_2}$

$= b_0 + (2)$

$\therefore \dot{x} = Ax + bu$

$y = cx + b_0 u$

$$\left[\begin{array}{cc|c} 0 & 1 & 0 \\ -a_2 & -a_1 & 1 \\ \hline b_1 - b_0 a_1 & b_2 - b_0 a_2 & b_0 \end{array}\right]$$

4 $\dfrac{\hat{y}}{\hat{u}} = \dfrac{b_0 s^3 + b_1 s^2 + b_2 s + b_3}{s^3 + a_1 s^2 + a_2 s + a_3} \longleftrightarrow \left[\begin{array}{ccc|c} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -a_3 & -a_2 & -a_1 & 1 \\ \hline \cdot & & & b_0 \end{array}\right]$

HW

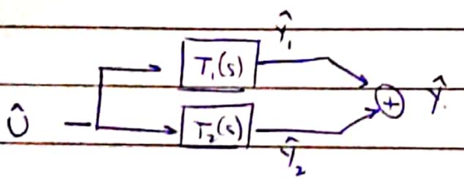
eg $\dfrac{\hat{y}}{\hat{u}} = \dfrac{s^3 + 2s + 1}{s^3 + 3s^2 + 3s + 1} = 1 + \dfrac{-3s^2 - s}{s^3 + 3s^2 + 3s + 1}$

$A + 3 = 0$

$$\hat{u} \longrightarrow \boxed{\dfrac{1}{s^3 + 3s^2 + 3s + 1}} \xrightarrow{\hat{x}} \boxed{s^3 + 2s + 1} \longrightarrow \hat{y}$$

BLOCK DIAGRAM CALCULUS IN STATE SPACE

A] Sum of two systems :-



$$T(s) = T_1(s) + T_2(s)$$

If $T_1(s) \longrightarrow \left[\begin{array}{c|c} A_1 & b_1 \\ \hline c_1 & d_1 \end{array}\right]$ with x_1 and y_1

and $T_2(s) \longrightarrow \left[\begin{array}{c|c} A_2 & b_2 \\ \hline c_2 & d_2 \end{array}\right]$ with x_2 and y_2

Then we can take $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ and $y = y_1 + y_2$

$$\dot{x} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} A_1 x_1 + b_1 u \\ A_2 x_2 + b_2 u \end{bmatrix}$$

$$y = c_1 x_1 + c_2 x_2 + (d_1 + d_2) u$$

This gives $\left[\begin{array}{cc|c} A_1 & 0 & b_1 \\ 0 & A_2 & b_2 \\ \hline c_1 & c_2 & d_1 + d_2 \end{array}\right]$

Then $T(s) = \begin{bmatrix} c_1 & c_2 \end{bmatrix} \begin{bmatrix} (sI - A_1)^{-1} & 0 \\ 0 & (sI - A_2)^{-1} \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} + d_1 + d_2$

comes out to be $T_1(s) + T_2(s)$

B] Product of two systems

$$\hat{U} \longrightarrow \boxed{T_1(s)} \longrightarrow \boxed{T_2(s)} \longrightarrow \hat{Y}$$

Take $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ $y =$