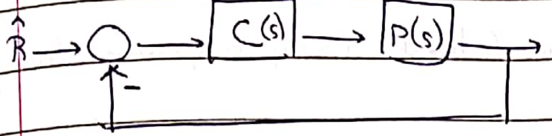
CONTROL SYSTEMS DESIGN

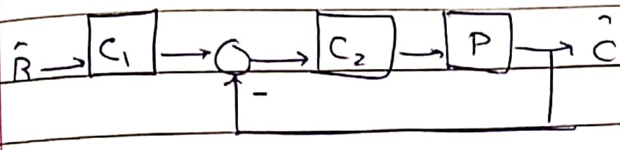
1/4

1. Mechanical / Electromechanical / Aerodynamic Systems
2. Chemical engineering systems
3. Economic systems

- Any system can be represented as standard feedback system.

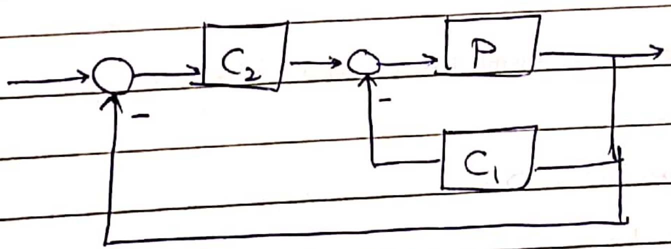
eg 

Single Degree of Freedom

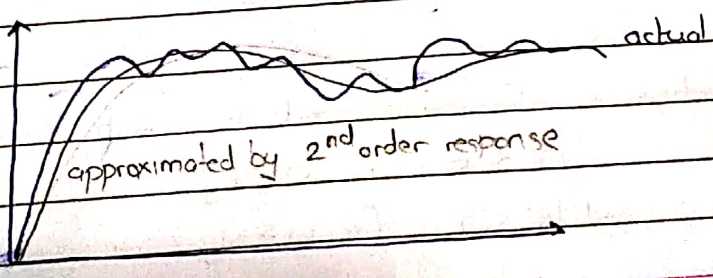


Two DoFs.

- Stability is governed by single characteristic polynomial of the transfer function of entire system.
  - Inner loops of system may be unstable. चलेगा.



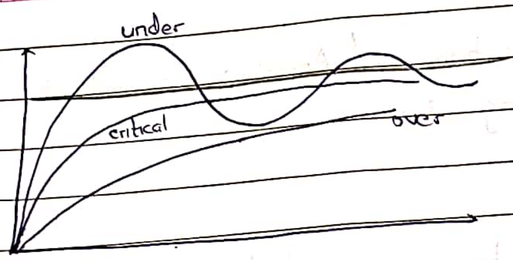
- Performance qualities
  1) Steady state error of reference signals.
  2) Model of response equivalent to 2^{nd} order response.



Rise Time

Settling Time

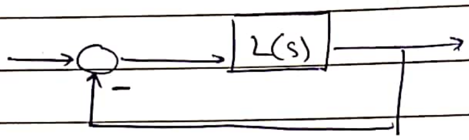
Damping factor



2nd order characteristics

*Critically damped response is the fastest rising response that has no overshoot.

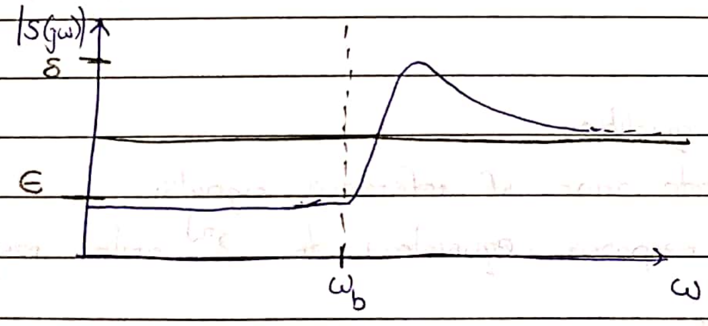
3) Sensitivity performance



$$S = \frac{1}{1 + L(s)}$$

2/4 4) ~~GM, PM~~ ~~Robustness of stability~~

Nature of $S(j\omega)$



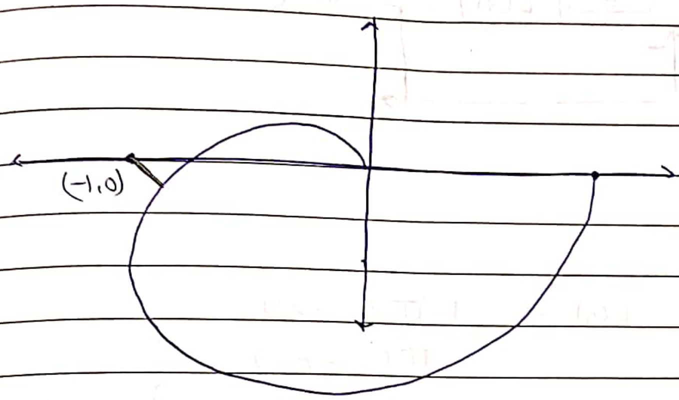
Specifications :- ϵ and δ and ω_b such that

$|S| < \epsilon$ for $\omega < \omega_b$

and $|S| < \delta$ for $\omega > \omega_b$.

* Gain-Phase Margin

= Shortest distance of Nyquist Plot from (-1, 0)



4) GM, PM .. Robustness of stability

→ Ways to design controllers

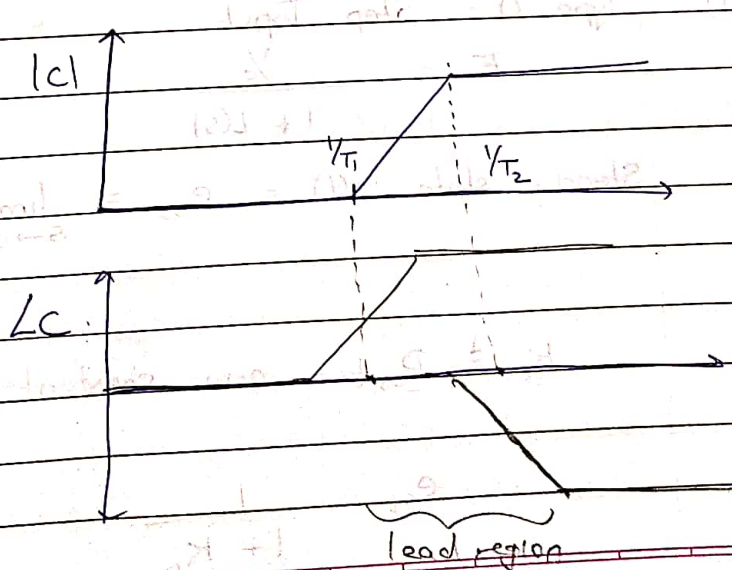
1 Classical - Controllers with fixed structure

eg - k

eg - $k_0 + \frac{k_1}{s}$

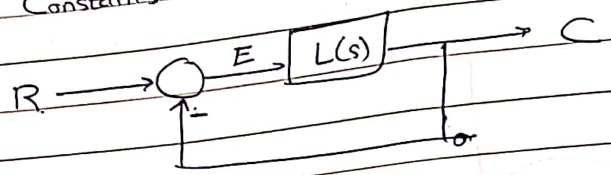
eg - $\frac{k(T_1 s + 1)}{(T_2 s + 1)}$

'Lead-Lag Compensator'



8/4

→ Error Constants



- System Types:-

Type 0 : $L(s) = \dfrac{k \prod (s - z_i)}{\prod (s - p_i)}$

Type 1 :- $L(s) = \dfrac{1}{s} L_1(s)$

Type 2 :- $L(s) = \dfrac{1}{s^2} L_1(s)$

} $L_1(s)$ is of type 0

- $E = \dfrac{R}{1 + L(s)}$

Reference input R can be step signal, ramp signal or parabolic signal.

① Type 0 : Step Input

$$E = \frac{1/s}{1 + L(s)}$$

Steady state e(t) = $e_{ss} = \lim_{s \to 0} s\,E(s) = \dfrac{1}{1 + L(0)}$

$K_p \triangleq$ Position error constant $= L(0)$

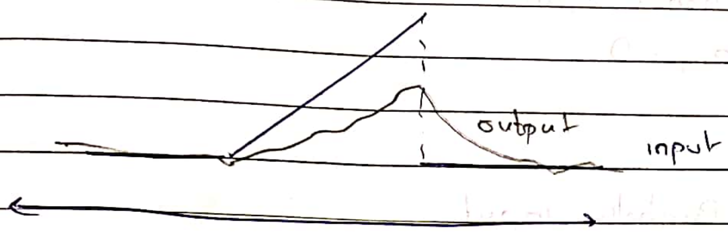
$$e_{ss} = \frac{1}{1 + K_p}$$

Page No.:
Date:

② Type 0 :- Ramp Input.

$$E = \frac{1/s^2}{1 + L(s)}$$

$$e_{ss} = \infty.$$



③ Type 1 :- Step Input.

$$E = \frac{1/s}{1 + \frac{1}{s} L_1(s)} = \frac{1}{s + L_1(s)}$$

$$e_{ss} = 0$$

④ Type 1 : Ramp Input

$$E = \frac{1/s^2}{1 + \frac{1}{s} L_1(s)} = \frac{1}{s^2 + s L_1(s)}$$

$$e_{ss} = \frac{1}{L_1(0)}$$

$$K_v \triangleq L_1(0) = \lim_{s \to 0} s L(s) = \text{Velocity error constant.}$$

$$\therefore e_{ss} = \frac{1}{K_v}$$

'Internal Model Principle'

WTH.

Page No.:
Date:

(5) Type 2 : Step input

$e_{ss} = 0$

(6) Type 2 : Ramp input

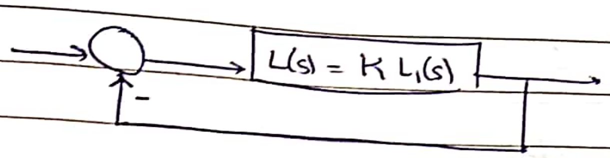
$e_{ss} = 0$

(7) Type 2 : Parabolic input $\left(R = \frac{1}{s^2}\right)$

$e_{ss} = \frac{1}{K_a}$

K_a = Acceleration error constant $\triangleq \lim_{s \to 0} s^2 L(s)$

→ Frequency Response Calculation from RL.

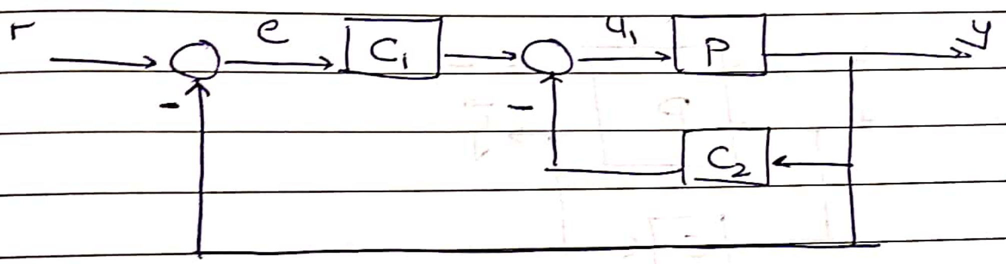


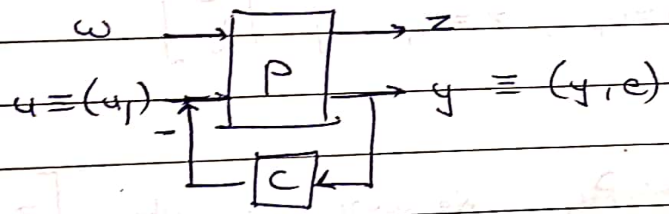
$$1 + L(s) = 1 + kL_1(s) = \frac{\prod (s - \pi_i)}{\prod (s - p_i)}$$

$p_i \equiv$ Open loop poles

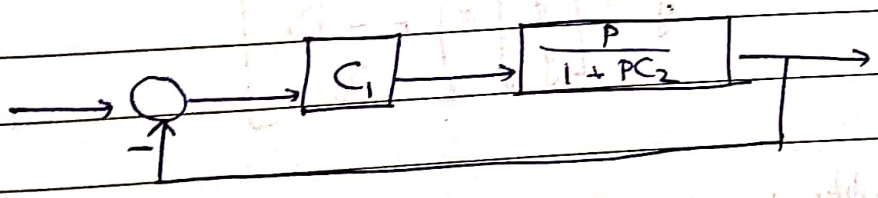
$\pi_i \equiv$ Closed loop poles (depend on K)

→ Finding characteristic equation





① Reduce to standard form.



Characteristic equation : $1 + \dfrac{C_1 P}{1 + C_2 P} = 0$

~~Ch~~

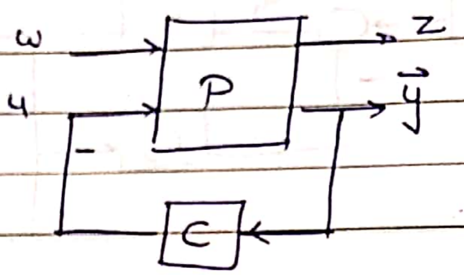
P.T.O.

Characteristic polynomial : $P = \frac{q}{p}$, $C_1 = \frac{q_1}{p_1}$, $C_2 = \frac{q_2}{p_2}$

$$\psi = \frac{\frac{qq_1}{pp_1}}{\frac{pp_2 + qq_2}{pp_2}} + 1$$

$$= 1 + \frac{q_1 q p_2}{p_1(p_2 p + q_2 q)}$$ ...... should be Hurwitz.

(2) We want to find characteristic polynomial without reducing to standard form.



$w = r$, $u = u_1$, $z = e$, $\vec{y} = \begin{bmatrix} e \\ y \end{bmatrix}$

~~$e = r - y$, $y = Pu_1$~~ We want to write: $\begin{bmatrix} z \\ \vec{y} \end{bmatrix} = \begin{bmatrix} P_1 & P_2 \\ P_3 & P_4 \end{bmatrix} \begin{bmatrix} w \\ u \end{bmatrix}$

$\therefore$ ~~$e = r - Pu_1$~~

$$\vec{y} = \begin{bmatrix} e \\ y \end{bmatrix} = \begin{bmatrix} r - Pu_1 \\ Pu_1 \end{bmatrix}$$

- Internal stability : $r = 0$

$$\begin{bmatrix} e \\ y \end{bmatrix} = \begin{bmatrix} -Pu_1 \\ Pu_1 \end{bmatrix} = \begin{bmatrix} -P(C_1 e - C_2 y) \\ P(C_1 e + C_2 y) \end{bmatrix}$$

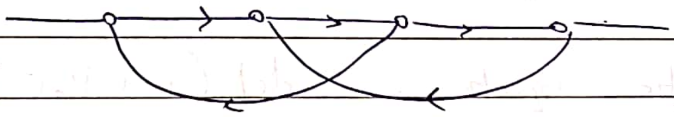
$$\begin{bmatrix} 1+PC_1 & -PC_2 \\ -PC_1 & 1+PC_2 \end{bmatrix} \begin{bmatrix} c \\ y \end{bmatrix} = 0$$

$$\begin{bmatrix} \frac{pp_1 + qq_1}{pp_1} & \frac{-qq_2}{pp_2} \\ -\frac{qq_1}{pp_1} & \frac{pp_2 + qq_2}{pp_2} \end{bmatrix} = p_1 p_2 p \begin{bmatrix} \quad \end{bmatrix}$$

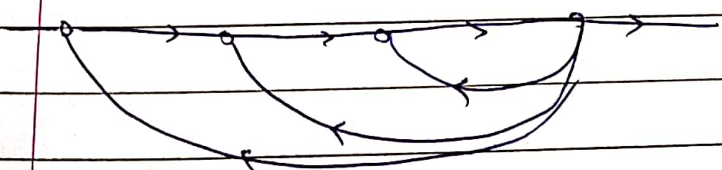
9/4

→ Computing characteristic equation of multiloop systems.

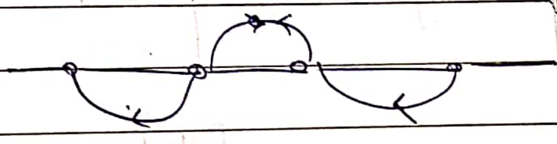
eg



– Convert to



OR



$$\begin{bmatrix} z \\ y \end{bmatrix} = \begin{bmatrix} P_1 & P_2 \\ P_3 & P_4 \end{bmatrix} \begin{bmatrix} \omega \\ u \end{bmatrix}$$

For internal stability, $\omega = 0$.

$$z = -P_2 u$$
$$y = -P_4 u$$

We want $y \to 0$ and $z \to 0$ for $t \to \infty$.

$$\begin{bmatrix} z \\ y \end{bmatrix} = \begin{bmatrix} P_2 \\ P_4 \end{bmatrix} u = -\begin{bmatrix} P_2 \\ P_4 \end{bmatrix} Cy.$$

$\therefore$ $z = -P_2 Cy$
$y = -P_4 Cy$

$\therefore$ $(I + P_4 C) y = 0$

$\therefore$ Characteristic equation: $\det(I + P_4 C) = 0$.

eg $M(s) = \begin{bmatrix} \frac{s+1}{s+3} & 1 \\ -1 & \frac{s+3}{s+2} \end{bmatrix}$

If $\begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$ are inputs,

$$\frac{s+1}{s+3} y_1 + y_2 = 0$$

$$-y_1 + \frac{s+3}{s+2} y_2 = 0$$

$$\det M(s) = \frac{(s+3)(s+1)}{(s+2)(s+3)} + 1$$

$$= \frac{(s+3)(2s+3)}{(s+3)(s+2)}$$

$\therefore$ Characteristic polynomial $= (s+3)(2s+3) = \psi(s)$

$(s+1)y_1 + (s+3)y_2 = 0 \Rightarrow \left(\frac{d}{dt}+1\right)y_1 + \left(\frac{d}{dt}+3\right)y_2 = 0$

$-(s+2)y_1 + (s+3)y_2 = 0 \Rightarrow -\left(\frac{d}{dt}+2\right)y_1 + \left(\frac{d}{dt}+3\right)y_2 = 0$

→ Stability of State Space system:-

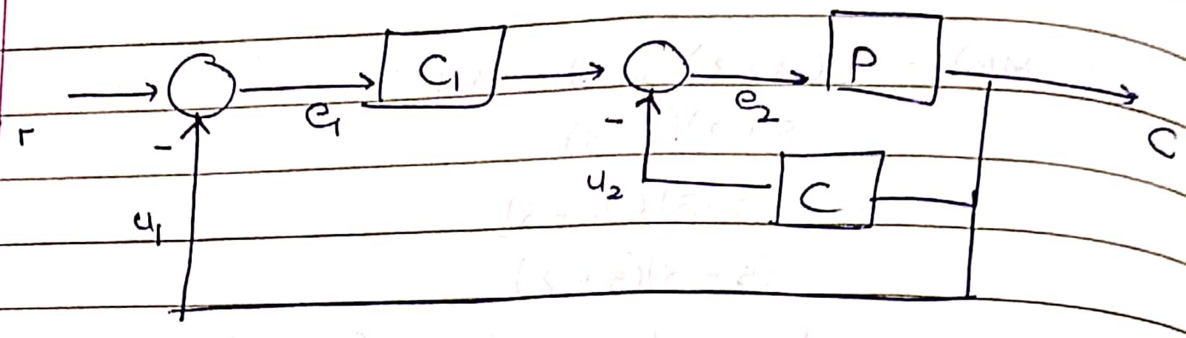
$$\dot{x} = Ax$$

$$\Phi(t) = e^{At}$$

Characteristic polynomial $= \psi = \det(sI - A)$

$$\therefore M(s) = (sI - A)^{-1}$$

eg



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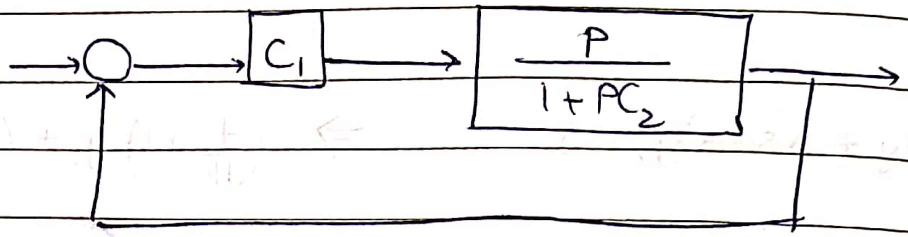

'Standard Form'

Characteristic polynomial = Numerator of $1 + \frac{PC_1}{1+PC_2}$

eg