Course Code : EE 614

Department: Electrical Engineering

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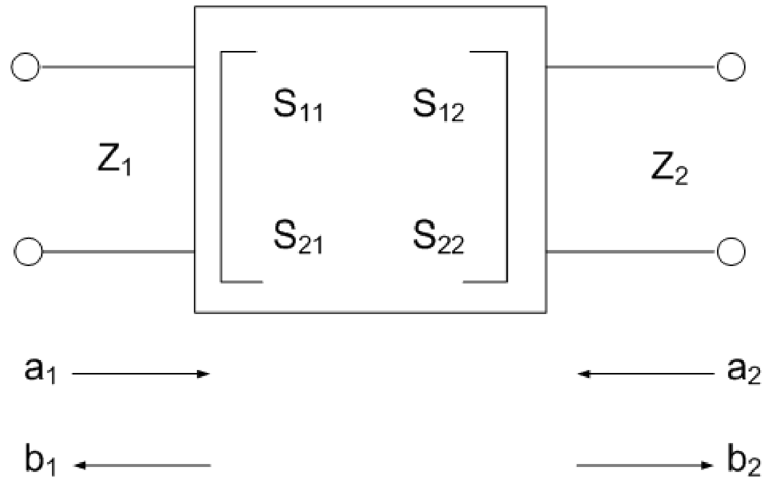
Lecture 2

The Scattering Matrix



Define normalized waves: $a_1 = \frac{V_1^+}{\sqrt{Z_1}}$ $a_2 = \frac{V_2^+}{\sqrt{Z_2}}$ with $V_j = V_j^+ + V_j^-$

$$b_1 = \frac{V_1^-}{\sqrt{Z_1}} \quad b_2 = \frac{V_2^-}{\sqrt{Z_2}} \quad \text{with } I_j = \frac{V_j^+}{Z_j} - \frac{V_j^-}{Z_j}$$

$$v_j = \frac{V_j}{\sqrt{Z_j}} = \frac{1}{\sqrt{Z_j}}\left[V_j^+ + V_j^-\right] = a_j + b_j \quad \text{defines normalized voltage at port j}$$

$$i_j = I_j\sqrt{Z_j} = \sqrt{Z_j}\left[\frac{V_j^+}{Z_j} - \frac{V_j^-}{Z_j}\right] = a_j - b_j \quad \text{defines normalized current at port j}$$

Scattering Parameters

For a linear device (small-signal) we have a linear relationship between a's and b's:

$$b_1 = S_{11}a_1 + S_{12}a_2$$

$$b_2 = S_{21}a_1 + S_{22}a_2$$

In matrix form:

$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = S \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

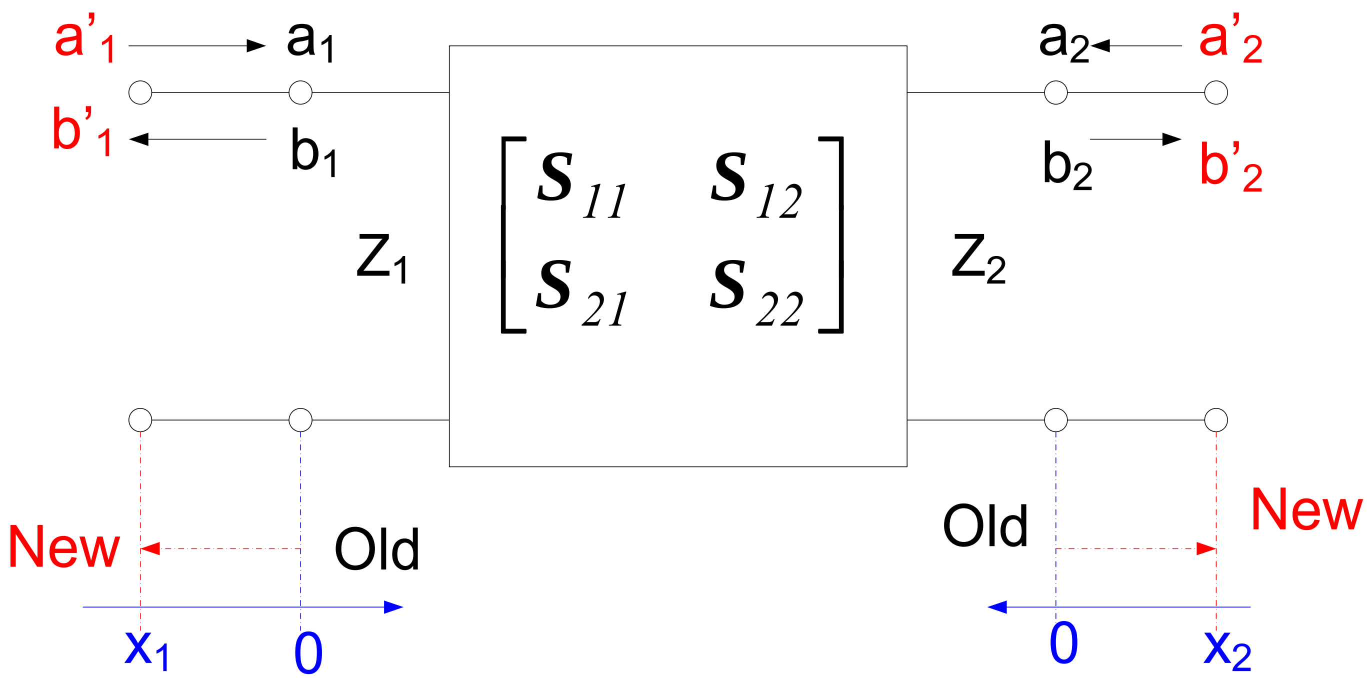
$$S_{11} = \left.\frac{b_1}{a_1}\right|_{a_2=0}$$ Input reflection coefficient with output matched

$$S_{21} = \left.\frac{b_2}{a_1}\right|_{a_2=0}$$ Forward transmission coefficient with output matched

$$S_{22} = \left.\frac{b_2}{a_2}\right|_{a_1=0}$$ Output reflection coefficient with input matched

$$S_{12} = \left.\frac{b_1}{a_2}\right|_{a_1=0}$$ Reverse transmission coefficient with input matched

Shifting reference planes for lossless lines



Shifting reference planes for lossless lines

Consider an incident wave $a_l^{'}$ at port l and reflected wave $b_k^{'}$ at port k. The new incident wave $a_l^{'}$ at port l and reflected wave $b_k^{'}$ at port k after we respectively shift the reference planes by l_l and l_k are

$$b_k^{'} = b_k e^{-j\beta_k x_k} = b_k e^{-j\theta_k}$$

with $\theta_k = \beta_k x_k$

$$a_l^{'} = a_l e^{j\beta_l x_l} = a_l e^{j\theta_l}$$

with $\theta_l = \beta_l x_l$

$$S_{kl}^{'} = \left.\frac{b_k^{'}}{a_l^{'}}\right|_{a_{i\neq l}^{'}=0} = \left.\frac{b_k e^{-j\theta_k}}{a_l e^{j\theta_l}}\right|_{a_{i\neq l}=0} = S_{kl} e^{-j(\theta_k+\theta_l)} \quad \text{(phase correction)}$$

Shifting reference planes for lossless lines

This means that our scattering parameters in terms of the primed quantities now become

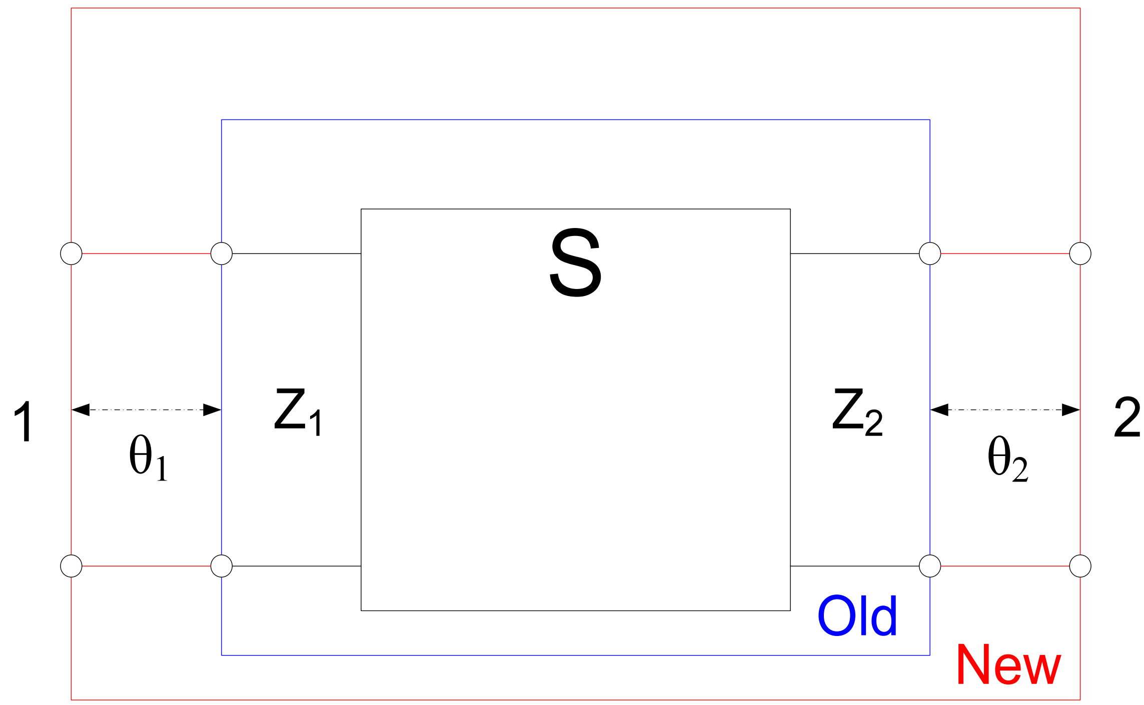
$$S'_{nm} = \left.\frac{b'_n}{a'_m}\right|_{a'_{k\neq m}=0} = \left.\frac{b_n}{a_m}\right|_{a_{k\neq m}=0} e^{-j(\beta_m x_m + \beta_n x_n)} = S_{nm} e^{-j(\theta_m + \theta_n)}$$

with $\theta_n = \beta_n d_n$

The relationship between matrices can be written more conveniently as

$$[S'] = \begin{bmatrix} e^{-j\theta_1} & 0 & \cdots & 0 \\ 0 & e^{-j\theta_2} & & 0 \\ 0 & & & 0 \\ 0 & \cdots & \cdots & e^{-j\theta_N} \end{bmatrix} [s] \begin{bmatrix} e^{-j\theta_1} & 0 & \cdots & 0 \\ 0 & e^{-j\theta_2} & & 0 \\ 0 & & & 0 \\ 0 & \cdots & \cdots & e^{-j\theta_N} \end{bmatrix}$$

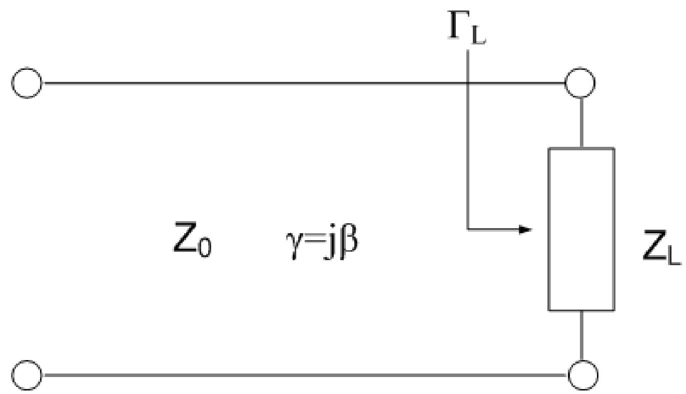
2 Port example



$$S'_{kl} = S_{kl} e^{-j(\theta_k + \theta_l)}$$

$$\begin{bmatrix} S'_{11} & S'_{12} \\ S'_{21} & S'_{22} \end{bmatrix} = \begin{bmatrix} e^{-j\theta_1} & 0 \\ 0 & e^{-j\theta_2} \end{bmatrix} \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} e^{-j\theta_1} & 0 \\ 0 & e^{-j\theta_2} \end{bmatrix}$$

Properties of Scattering Parameters



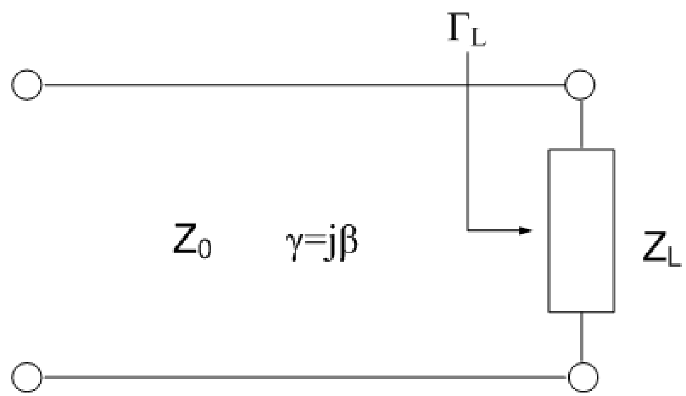
Power dissipated by the load :

$$P_L = \mathrm{Re}\{VI^*\}\Big|_{rms} = \frac{1}{2}\mathrm{Re}\{VI^*\}\Big|_{amplitude}$$

$$= \mathrm{Re}\left\{\left(V^+ + V^-\right)\left(\frac{V^+}{Z_0} - \frac{V^-}{Z_0}\right)^*\right\}$$

$$= \frac{\left|V^+\right|^2}{Z_0} - \frac{\left|V^-\right|^2}{Z_0} = P^+ - P^- \quad \text{(incident minus reflected power)}$$

Properties of Scattering Parameters



In terms of normalized waves (add 1/2 factor for amplitude)

$$P_L = \mathrm{Re}\{v\, i^*\}\big|_{rms} = |a|^2 - |b|^2 = P^+ - P^-$$

with

$$P^+ = \frac{|V^+|^2}{Z_0} = |a|^2 \quad \text{and } P^- = \frac{|V^-|^2}{Z_0} = |b|^2$$

Given $b = \Gamma_L a$ we have:

$$P_L = |a|^2\left(1 - |\Gamma_L|^2\right) = P_L = P^+\left(1 - |\Gamma_L|^2\right)$$

Two Port Network Parameter conversion

To convert Z parameters to S parameters

$$V = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = Z\,I$$

Using matrix notation ($V = V^+ + V^-$ and $I = I^+ - I^-$)

and assuming $Z_1 = Z_2 = Z_0$ *we* have, $V^+ + V^- = Z(I^+ - I^-)$

Using $I^{\pm} = \frac{1}{Z_0} V^{\pm}$ we can rearrange this equation as:

$$\left(Z + Z_0 U\right) I^- = \left(Z - Z_0 U\right) I^+$$

with U the identity matrix.

$$S = \frac{b}{a} = \frac{V-}{V+} = \frac{I-}{I+} = \left(Z + Z_0 U\right)^{-1} \left(Z - Z_0 U\right)$$

Conversely: $Z = Z_0 \left(1 + S\right)\left(1 - S\right)^{-1}$

Two Port Network Parameter conversion

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Various Types of Transistor

- Device Types: family FET's or Bipolar transistors
  - FET: C-MOSFET LDMOSFET HEMT/MODFET
  - Bipolar: BJT HBT DHBT
- Materials:
  - Si (C-MOSFET, BJT), SiGe (HBT's)
  - GaAs, AlGaAs/GaAs (1st generation of HEMTs/HBTs)
  - InP-InGaAs/AlInAs (2nd generation of HEMT's/HBT's)
  - GaN/AlGaN (NEW: high temperature, high power)
- Technology:
  - horizontal resolution (e.g. gate-length Lg) is set by lithography
  - vertical resolution is set by process or growth (MBE)

Various Types of Transistor

- Short - circuit Unity current - gain fT for BJT :

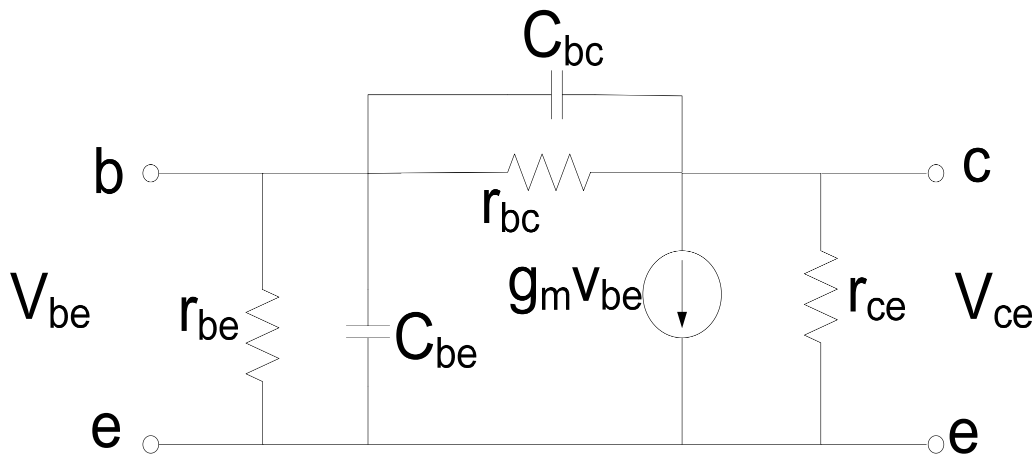
  - defined using $\left|h_{21}(\omega_T)\right| = \left|\frac{y_{21}}{y_{11}}\right|\left|\frac{z_{21}}{z_{22}}\right| = 1$

- Maximum frequency of oscillation : f_{max} :

  - defined using U(fmax) = 1 with U unilateral power gain (defined below)

- Noise Figure F (Chapter 4)
- Maximum Power Gain MAG (Chapter 3)
- Output Power P_{1dB} (Chapter 4)

BJT Model



$$h_{21}(\omega) = \frac{i_c}{i_b} = \frac{\beta}{1 + j\dfrac{\omega}{\omega_\beta}} \quad \text{with } \omega_\beta = \frac{1}{r_{be}\left(C_{be} + C_{bc}\right)}$$

From $\left| h_{21}(\omega_T) \right| = 1$ we find that the intrinsic f_T is :

$$f_T \approx \beta f_\beta \approx \frac{1}{2\pi}\frac{g_m}{C_{be}} = \frac{1}{2\pi}\frac{1}{\tau_{ec}} \quad \text{with } \tau_{ec} = \tau_b + \tau_c$$

where τ_b and τ_c are the base and collector propagation times.

Unilateral Power Gain

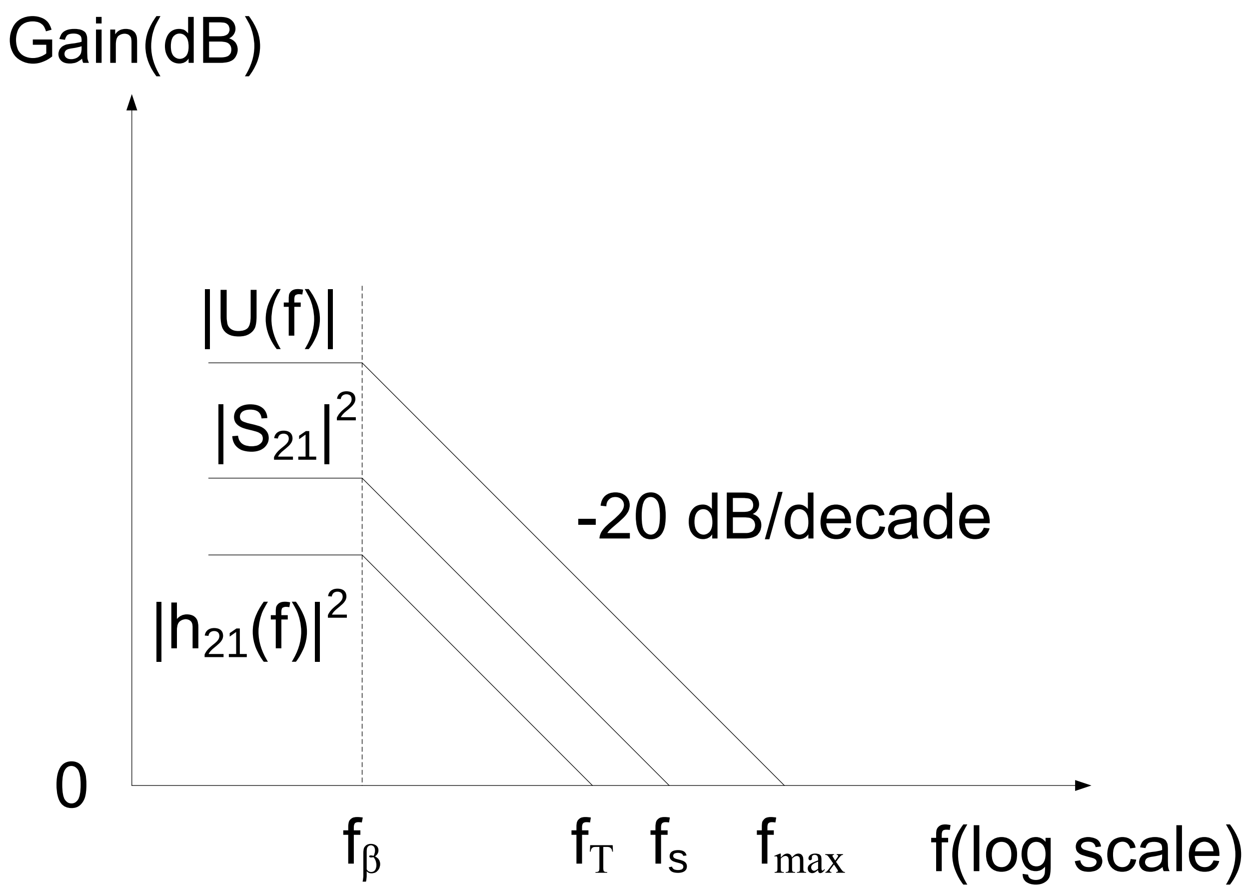
$$U = \frac{\left|y_{21} - y_{12}\right|^2}{4\left[\mathrm{Re}(y_{11})\mathrm{Re}(y_{22}) - \mathrm{Re}(y_{12})\mathrm{Re}(y_{21})\right]}$$

$$= \frac{\left|z_{21} - z_{12}\right|^2}{4\left[\mathrm{Re}(z_{11})\mathrm{Re}(z_{22}) - \mathrm{Re}(z_{12})\mathrm{Re}(z_{21})\right]}$$

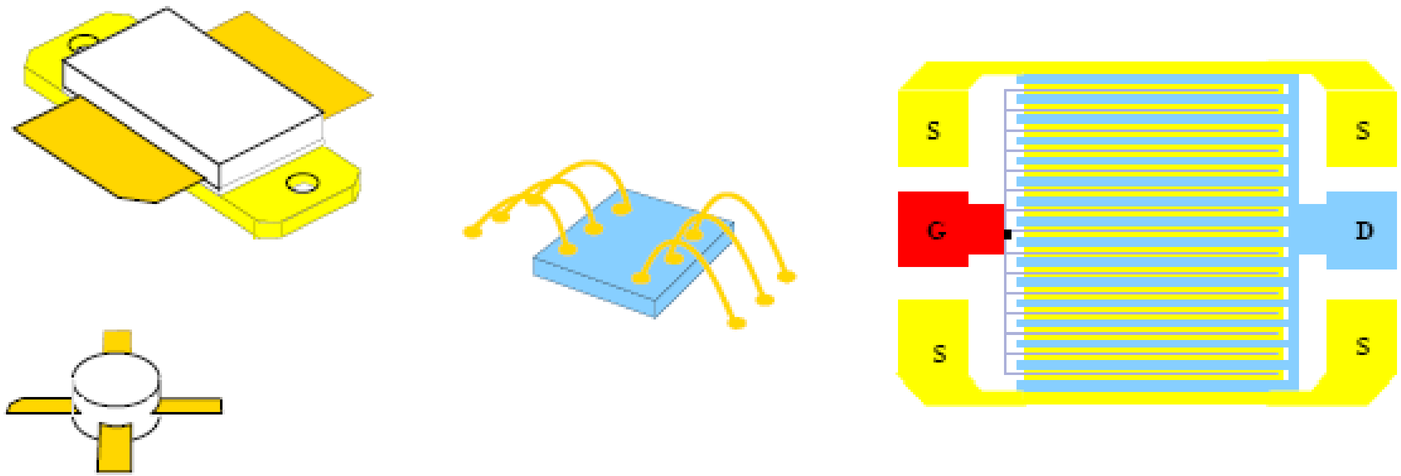
- U is the MAG (maximum available power) gain for a unilateralized device ($S_{12} = 0$)
- $U(f_{max}) = 1$
- U and therefore f_{max} are invariant under loss-less loading
- For frequencies larger than f_{max} the device becomes passive
- f_{max} is the maximum frequency of oscillation of the device
- For the intrinsic BJT, f_{max} is approximately given by:

$$f_{max} \approx \sqrt{\frac{f_T}{8\pi r_{be} C_{be}}}$$

Frequency Characteristics of U and h_{21}



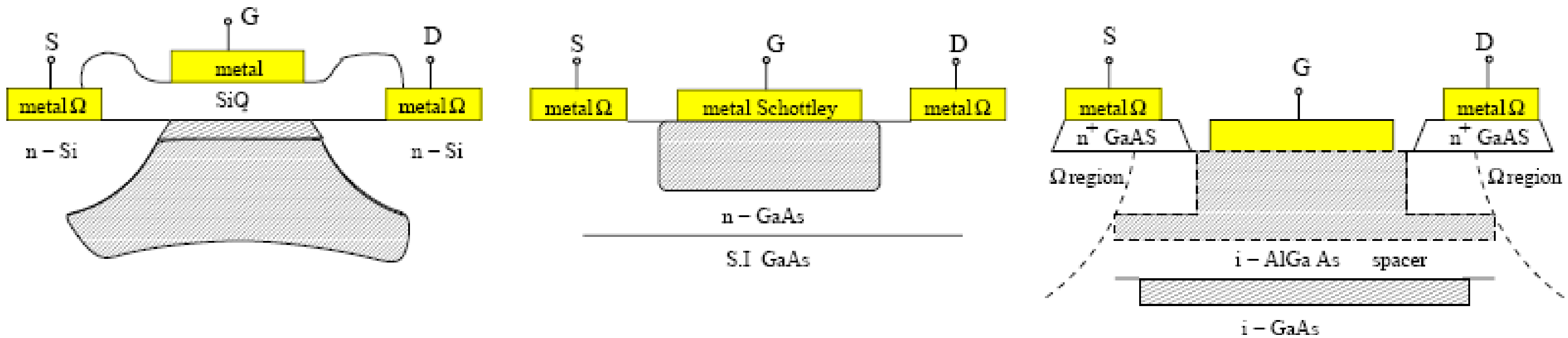
Device Packaging



Packaged Chip On–Wafer

- The device packaging impacts the device characteristics
- Direct on-wafer measurement of transistors can be used for RFIC and MMIC design
- Package or bond-wire modeling is required for packaged and chip devices respectively

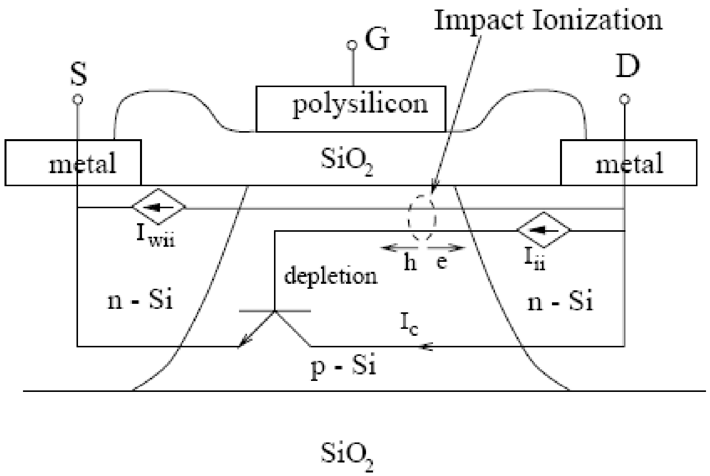
Field Effect Transistors



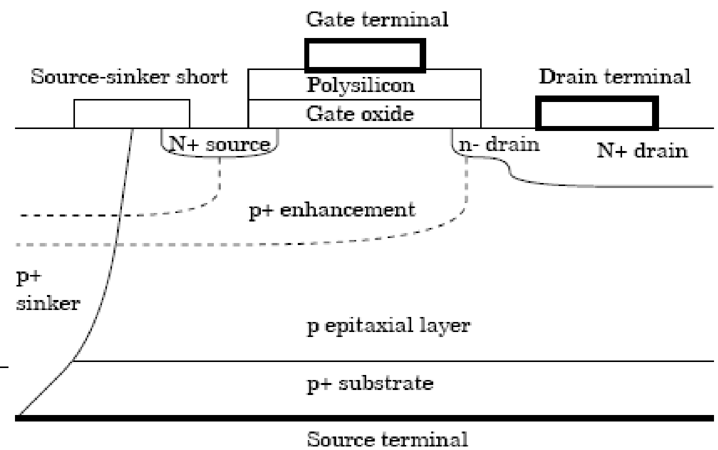
MOSFET MESFET HEMT

- C-MOSFET (Si)
- MESFET (GaAs)
- HEMT/MODFET

Field Effect Transistors



SOI-MOSFET

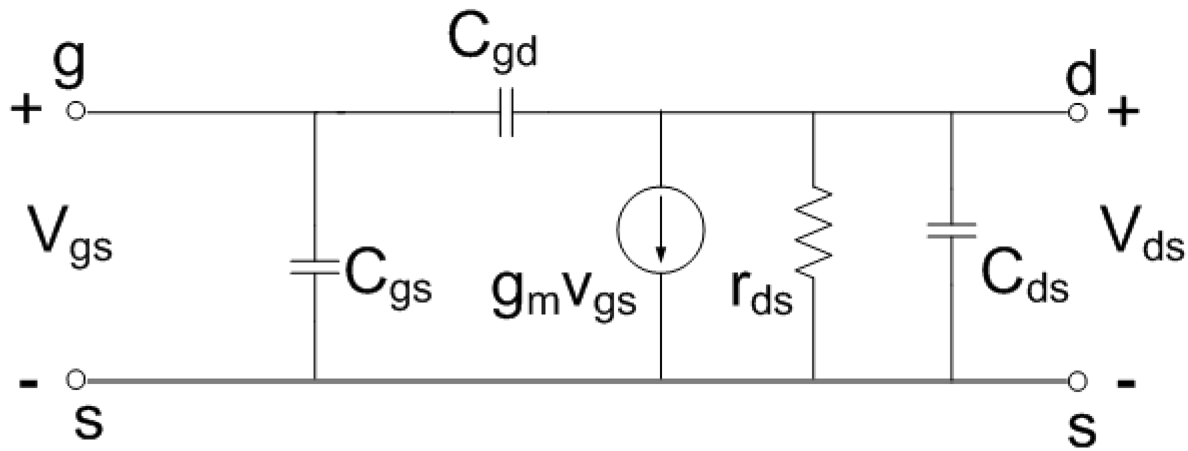


LDMOSFET

- SOI-MOSFETs provide high-frequency performance thanks to their insulated S_iO_2 substrate but are affected by a kink in their IV's due to their parasitic bipolar transistor.

- LD-MOSFETs use a laterally doped diffused drain junction to achieve high-voltage breakdown for high power applications up to 2 GHz

Field Effect Transistors



$$f_T \approx \frac{1}{2\pi}\frac{g_m}{C_{gs}} = \frac{1}{2\pi}\frac{v_s}{L_g} = \frac{1}{2\pi}\frac{1}{\tau_c}$$

where v_s is the electron saturation velocity, L_g the gate length and τ_c the channel propagation time constant.

$$f_{max} = \frac{f_T}{2}\sqrt{\frac{r_{ds}}{r_i}}$$

Comparison of Various Technologies

Technology	Circuit freq. range (GHz)	f_t (GHz)	f_{max} (GHz)	BV_{off} (V)	P_{out} (freq, bias, size)	PAE (%)	NF_{min}/G_a (dB)
Si CMOS 0.1 µm NMOS	→ 3–4	55 > 150					0.51/18.5 (2 GHz) 2.4/5.5 (10 GHz)
Si BJT		26–50 100 (R&D)	40–80	3–7			
SiGe HBT	1–30 40 GHz VCO	40–60 130 (R&D)	50–80 160? (R&D)	3–6	630 mW (2 GHz)	80	0.8/13 (2 GHz) 0.9/- (10 GHz, R&D)
GaAs HBT	0.9–64	60 170 (R&D)	100 224 (R&D)	10–30	4.33 W/mm (20 GHz,10.5 V) 0.77 W/mm (3.4 V)	66 61	0.83/16.9 (2 GHz) 1.1/11 (4 GHz) 1.7/10 (18 GHz)
InP HBT (mostly DHBT)	2–94	60–180 250 (SHBT, R&D)	90–200 800 (SHBT, R&D)	5–20	2.7 W/mm (10 GHz, SHBT) 7.5 W/mm (10 GHz) 4.9 W/mm (18 GHz) 1.9 W/mm (30 GHz)	43 54 36	0.46/- (2 GHz) 2/- (10 GHz) 3.3/- (18 GHz)

Comparison of Various Technologies

Technology	Circuit freq. range (GHz)	f_t (GHz)	f_{max} (GHz)	BV_{off} (V)	P_{out} (freq, bias, size)		PAE (%)	NF_{min}/G_a (dB)	
0.4 μm GaAs MESFET (HP)	–26	31–23	55	12	0.65 W/mm	(18 GHz,6 V)			
0.25 μm GaAs PHEMT	–50	64	> 120	10	0.65 W/mm	(40 GHz,5 V)	> 30	1.3/-	(18 GHz)
0.1–0.15 μm GaAs PHEMT	12–94	90–120	150–290		2.8 W	(77 GHz,5 V)	48	0.38/10.5	(12 GHz)
				8	0.72 W/mm	(44 GHz)	26	1.5/-	(60 GHz)
				9	0.60 W/mm	(60 GHz)	29	2.1	(94 GHz)
				5	0.40 W/mm	(94 GHz)	13		
0.2 μm InP HEMT	50–94	110	3–400?		0.45 W/mm	(60 GHz,4 V)	26		
0.1 μm InP HEMT	12–213	160–210	3–400?	5	0.40 W/mm	(40 GHz,2.5 V)		0.35/16.8	(12 GHz)
		340 (R&D)		6	0.21 W/mm	(60 GHz,2 V)		0.8/8.9	(60 GHz)
					0.34 W/mm	(60 GHz,3 V)	42	1.2/7.2	(94 GHz)
					0.53 W/mm	(60 GHz,4 V)	39		
0.15–0.25 μm (Al)GaN HEMT	4–20	73	140	→ 100	470 mW	(8 GHz,150 μm)	46	1.2/-	(10 GHz)
					3 W	(4 GHz, 2 mm, L_g = 1 μm)	30		

Non-Quasi-Static Models

- Static Model (valid at DC)

$$y_{22} = g_{22}$$

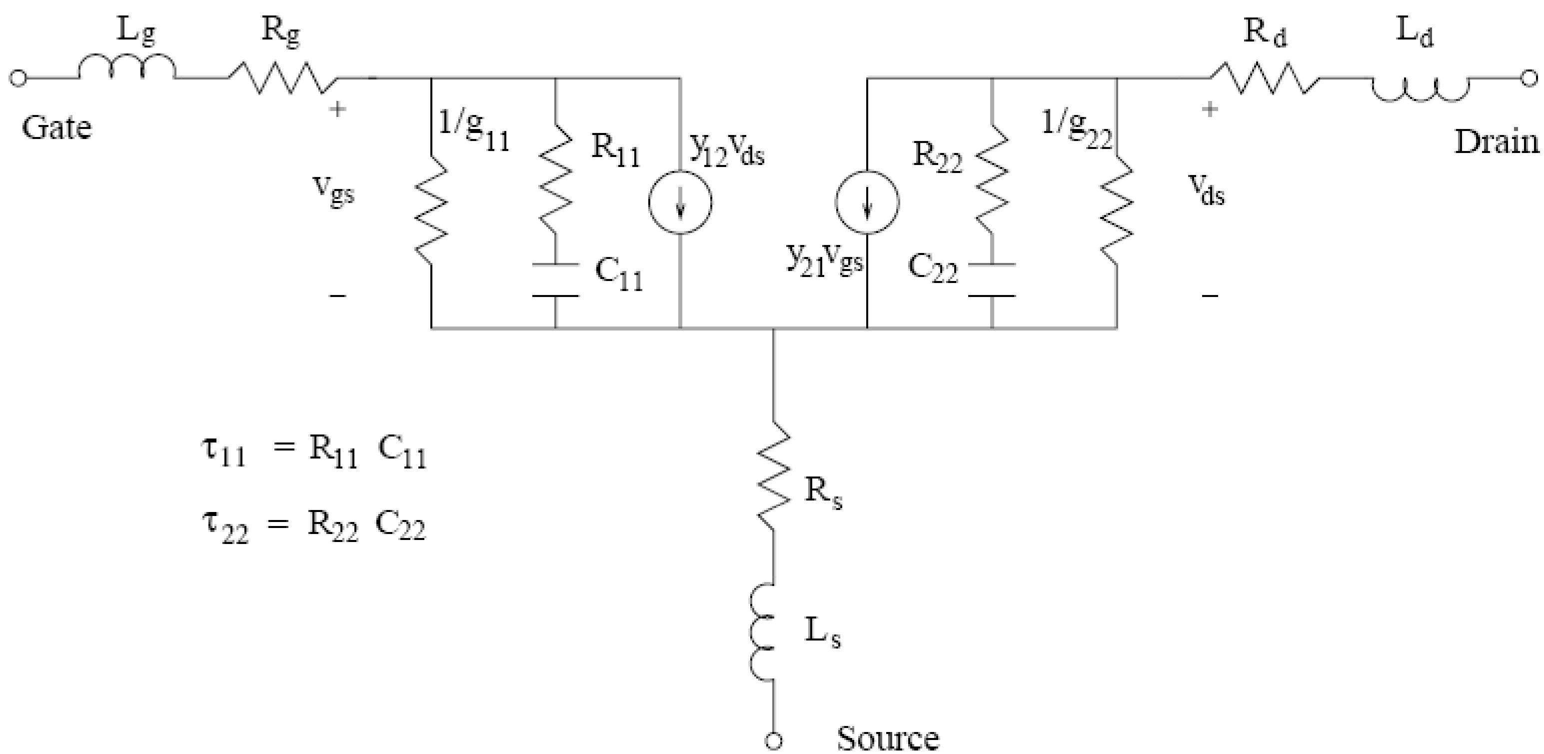
- Quasi-static model (typically valid up to f_T):

$$y_{22} = g_{22} + j\omega C_{22}$$

- Non-quasi-static model (1st order) (typically valid up to intrinsic f_{max})

$$y_{22} = g_{22} + \frac{j\omega C_{22}}{1 + j\omega\tau_{22}} \text{ with } \tau_{22} \text{ the channel charging time constant of } C_{22}$$

Non-Quasi-Static Models



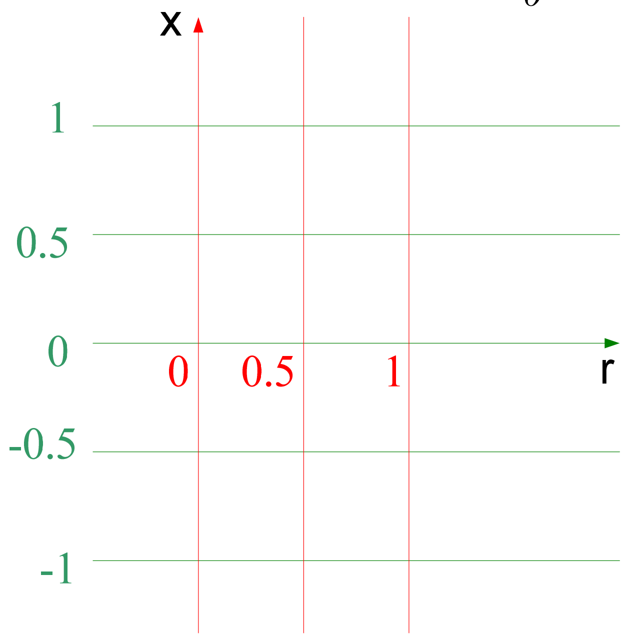
$$\tau_{11} = R_{11}\ C_{11}$$

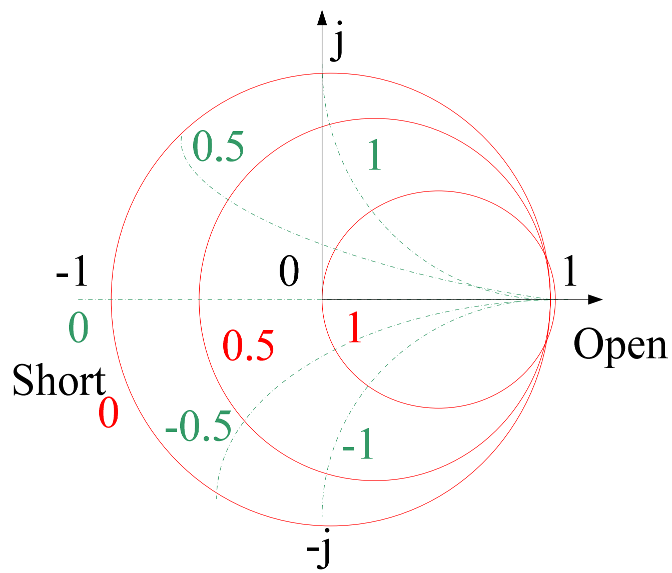
$$\tau_{22} = R_{22}\ C_{22}$$

The Smith Chart

Bilateral Transform connecting the impedance Z and the Reflection coefficient Γ . The smith chart maps the x-plane on the Γ plane

$$\Gamma = \frac{Z - Z_0}{Z + Z_0} = \frac{\frac{Z}{Z_0} - 1}{\frac{Z}{Z_0} + 1} = \frac{z - 1}{z + 1} \quad \text{with } z = \frac{Z}{Z_0} = r + jx$$





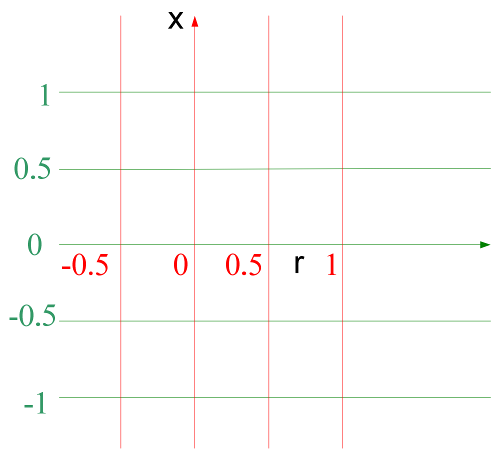
Extended Smith Chart

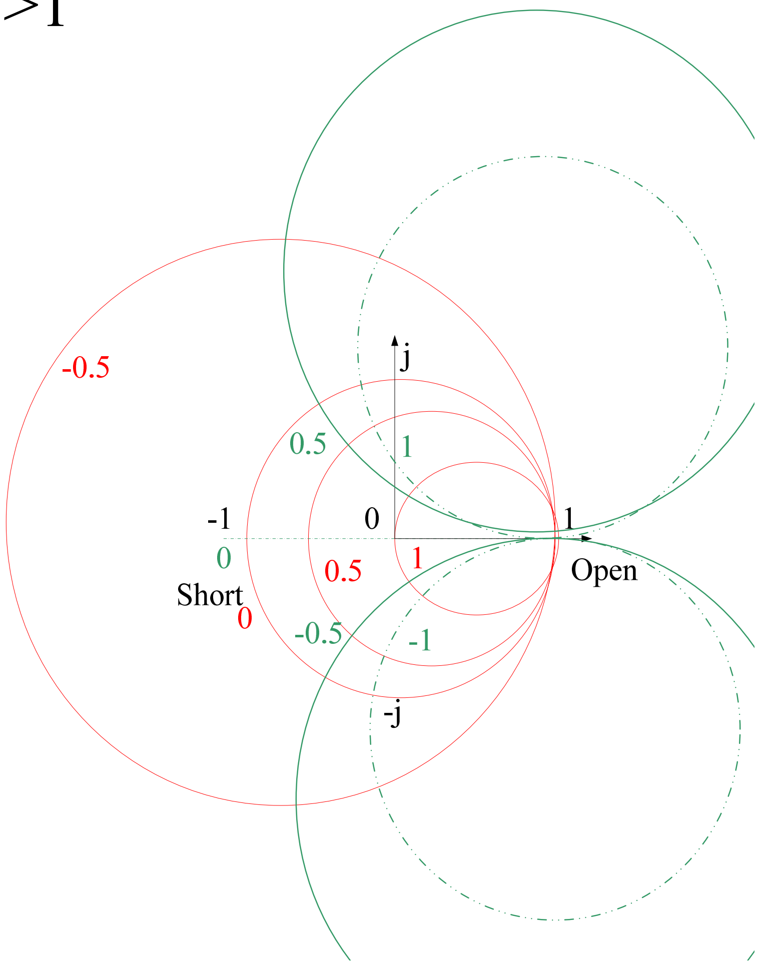
For negative resistance r<0 we have $|\Gamma|>1$

$$\Gamma = \frac{Z - Z_0}{Z + Z_0} = \frac{\frac{Z}{Z_0} - 1}{\frac{Z}{Z_0} + 1}$$

$$= \frac{z-1}{z+1}$$

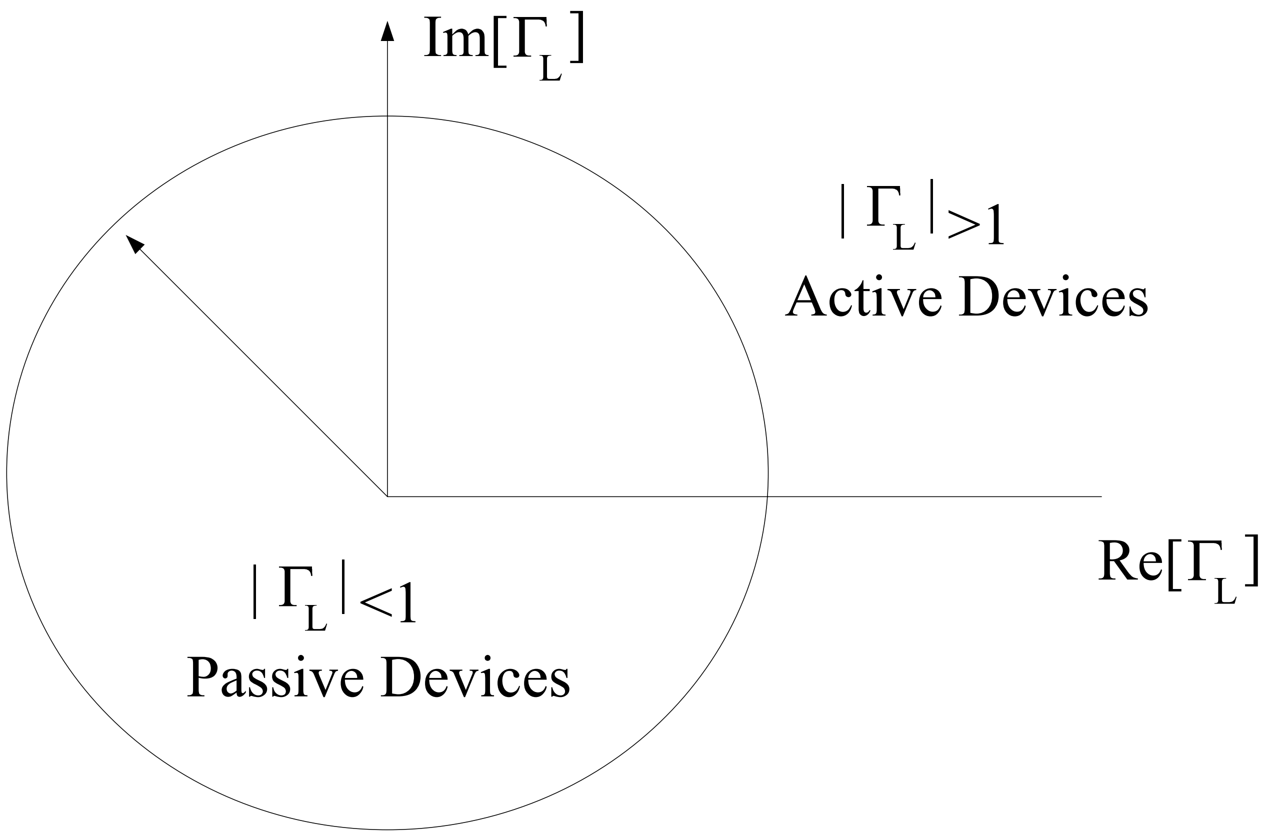
with $z = \frac{Z}{Z_0} = r + jx$





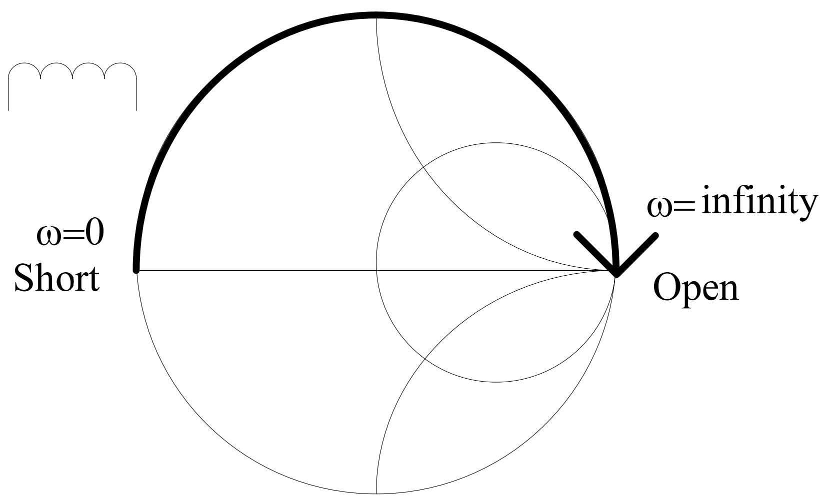
For r=-1 (Re{Z}=-50 ohms) we have Γ=infinity

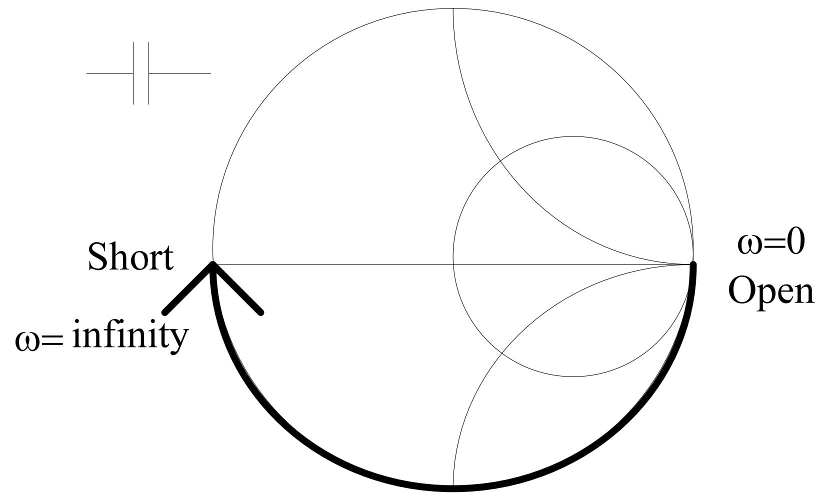
Active and Passive Load



Inductance and Capacitance on Smith Chart

Locus of the reflection coefficient for an inductor and a capacitor
In a Z Smith chart





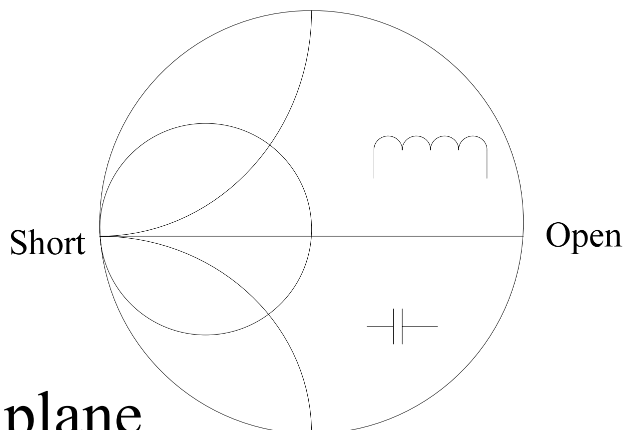
Γ plane

Z smith chart

Y Smith Chart

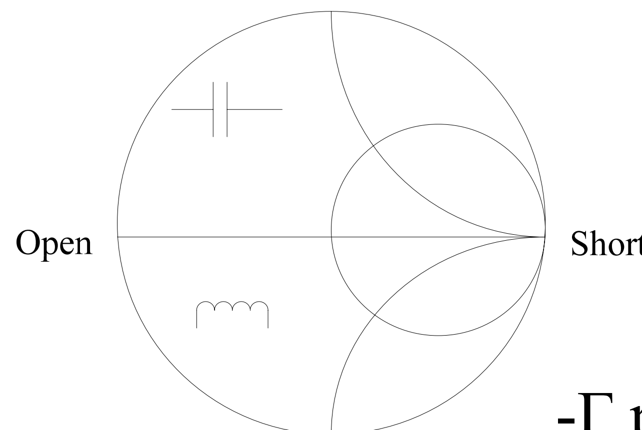
The Y - smith chart can be obtained by expressing Γ in terms of Y :

$$\Gamma_L = \frac{\frac{Z_L}{Z_0} - 1}{\frac{Z_L}{Z_0} + 1} = \frac{z-1}{z+1} = \frac{\frac{1}{y} - 1}{\frac{1}{y} + 1} = -\frac{y-1}{y+1} = -\frac{\frac{Y_L}{Y_0} - 1}{\frac{Y_L}{Y_0} + 1}$$



Γ plane

Normal



-Γ plane

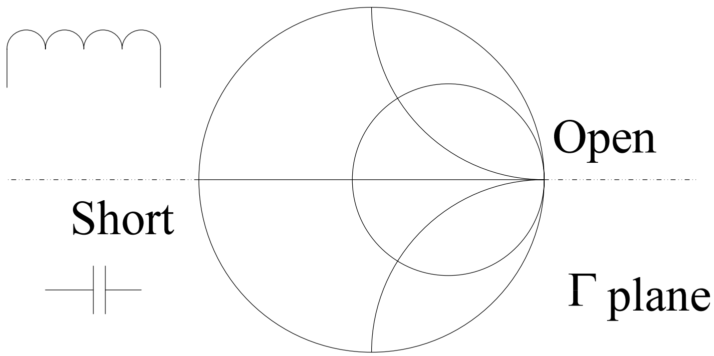
Rotated

Y Smith Chart

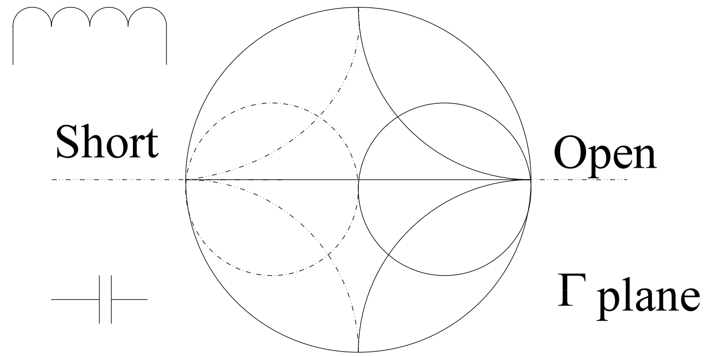
- The Y Smith Chart is obtained by inverting the Z smith chart
- In the rotated Y-Smith Chart the short and open are exchanged

Impedance and Admittance Smith Charts

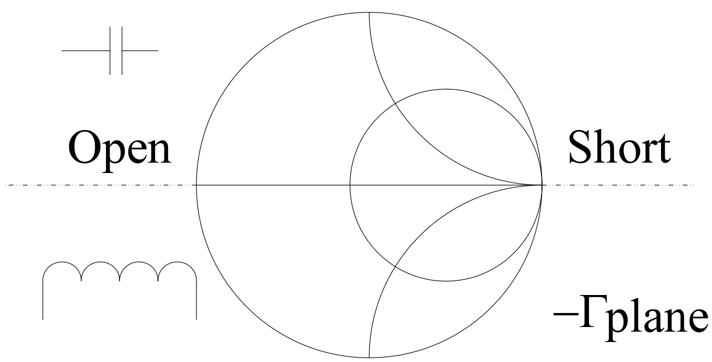
Z Smith Chart



ZY Smith Chart



Y Smith Chart



Y Smith Chart