$d_y = \Omega_p \cosh\left[\frac{1}{N} \sinh^{-1}\left(\frac{1}{\epsilon}\right)\right]$

$d_x = \Omega_p \sinh\left[\frac{1}{N} \sinh^{-1}\left(\frac{1}{\epsilon}\right)\right]$ $\qquad D_k = \frac{1}{N}\sinh^{-1}(1/\epsilon)$

$\therefore \quad C_k = \frac{(2k+1)\pi}{2N}$

For all poles, we take any contiguous 2N values of k

- Obtain all poles

  $H_{analog}(s)$ contains only poles in LHP.

  Find constant by $H_{analog}(0)$.

CP Sketch poles of $H_{analog}(s)$ for $N = 3, 4, 5, 6$

Theorem Poles lie on an ellipse.

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Passband	Stop-band	Filter Type
Monotonic	Monotonic	Butterworth
Equiripple	M	Chebyschev.
M	E	Inverse Chebyschev
E	E	Jacobi / Elliptic.

IV Analog Frequency Transformations

eg Transform :- $H_{analog, BPF}(s) \xrightarrow[s_L \leftarrow F(s)]{} H_{analog, LPF}(s_L)$

- We transform bandpass specifications to auxilliary specifications, design the LPF, transform back to

- The frequency transform [$s = j\Omega$ maps to $s_L = j\Omega_L$] must retain
① Stability with causality
② Rationality.

$\therefore$ $F(s)$ is an LC immitance function
$\hookrightarrow$ admittance or impedance.

$\rightarrow$ eg - LPF $\rightarrow$ HPF :- $F(s) = \boxed{s_L = \dfrac{\Omega_p}{s}}$ .... ~ C impedance or ~ L admittance.

$$s = j\Omega \Rightarrow s_L = -j\frac{\Omega_p}{\Omega} = j\Omega_L$$

$$\therefore \boxed{\Omega_L = -\frac{\Omega_p}{\Omega}}$$

Also, $F(-j\Omega) = -F(j\Omega)$

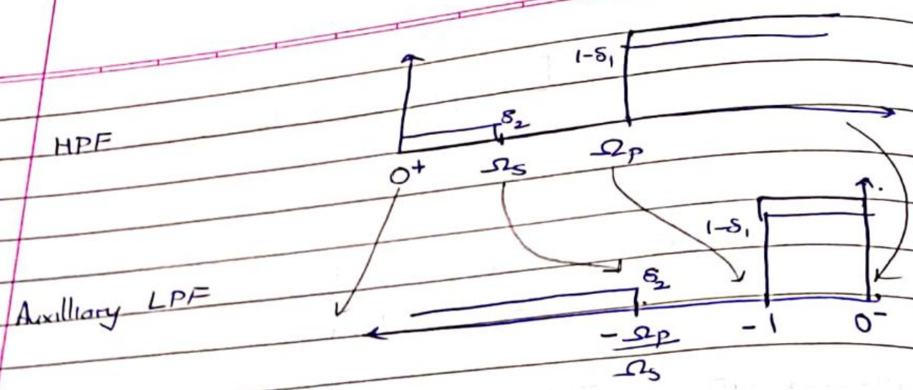
and $\dfrac{d\Omega_L}{d\Omega} = \dfrac{\Omega_p}{\Omega^2} > 0$

We observe only on positive frequencies. The result for negative images would just be a mirror image.

$\rightarrow$ Critical Points :-

Ω	Ω_L
0^+	$-\infty$
Ω_p	-1
Ω_s	$-\Omega_p/\Omega_s$
$+\infty$	0^-

P.T.O.



* We ~~could~~ could not have taken $\Omega_L = \frac{\Omega_p}{\Omega} \Rightarrow s_L = -\frac{\Omega_p}{s}$

because stability would have been compromised.

- In $F(s)$, LHP of s must be mapped to LHP of s_L
RHP RHP.

→ BPF $\xrightarrow{F(s)}$ LPF

- $F(s)$ cannot be a first order transformation.

Use —ʘʘʘ—||—

$$s_L = sL + \frac{1}{sC} = \frac{s^2LC + 1}{sC} = \frac{s^2 + \Omega_0^2}{Bs}$$

$$\Omega_0 = 1/\sqrt{LC}$$

$$B = 1/L$$

$$\boxed{\Omega_L = \frac{\Omega^2 - \Omega_0}{B\Omega}} = \frac{\Omega}{B} - \frac{\Omega_0^2}{B\Omega}$$

$$\frac{d\Omega_L}{d\Omega} = \frac{1}{B} + \frac{\Omega_0^2}{B\Omega^2}$$ ..... this mapping is also monotonic increasing.

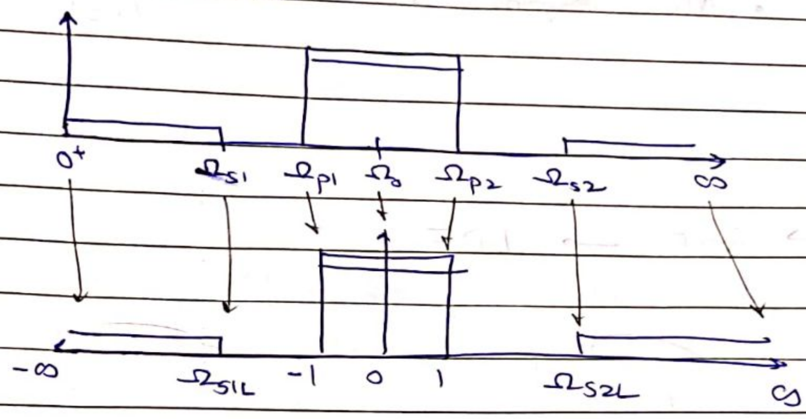
$|\Omega_L|$ is even function of Ω

• Critical Points :-

Ω	Ω_L
0^+	$-\infty$
Ω_{s1}	
Ω_{p1}	
Ω_0	0
Ω_{p2}	
Ω_{s2}	
$+\infty$	$+\infty$

BPF

LPF



• We constrain mapping of Ω_{p1} & Ω_{p2} (passband edges) to be $-1, +1$

$$\frac{\Omega_{p1}^2 - \Omega_0^2}{B\,\Omega_{p1}} = -1 \qquad\qquad \frac{\Omega_{p2}^2 - \Omega_0^2}{B\,\Omega_{p2}} = 1$$

Solve to get

$$\boxed{\begin{aligned} \Omega_0^2 &= \Omega_{p1}\Omega_{p2} \\ B &= \Omega_{p2} - \Omega_{p1} \end{aligned}}$$

* Even in RLC BPF, the resonant frequency is the GM of the two half-power frequencies.

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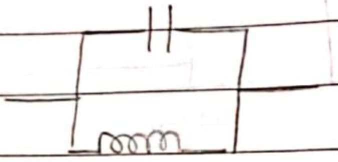
• If the two stopband tolerances are not equal, use the more stringent tolerance for the auxiliary LPF.

• If transition widths ($[\Omega_{s1L}, -1]$ and $[1, \Omega_{s2L}]$) are not equal, use the smaller of the two.

- Auxilliary LPF specifications :-
  - Passband edge = 1
  - Stopband edge = min $\{\Omega_{s2L}, -\Omega_{s1L}\}$
  - Tolerances are same as BPF *

- After designing BPF, $s \rightarrow \frac{1-z^{-1}}{1+z^{-1}}$ to get digital filter

→ BSF → LPF



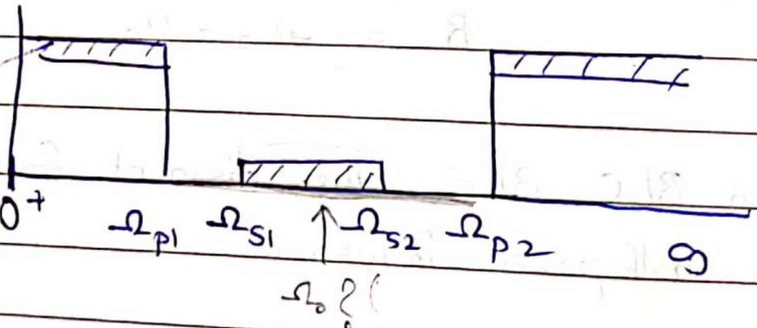
$$\frac{1}{sC + \frac{1}{sL}} = \frac{s/C}{s^2 + 1/LC}$$

- $F_{BS}(s) = \frac{Bs}{s^2 + \Omega_0^2}$ $\Rightarrow$ $\boxed{F_{BS}(j\Omega) = \frac{j\,B\Omega}{\Omega_0^2 - \Omega^2}}$

As always (since LC immitance is always an odd function),

$$\Omega_L(-\Omega) = -\Omega_L(\Omega)$$

- BSF



- Critical Points :-

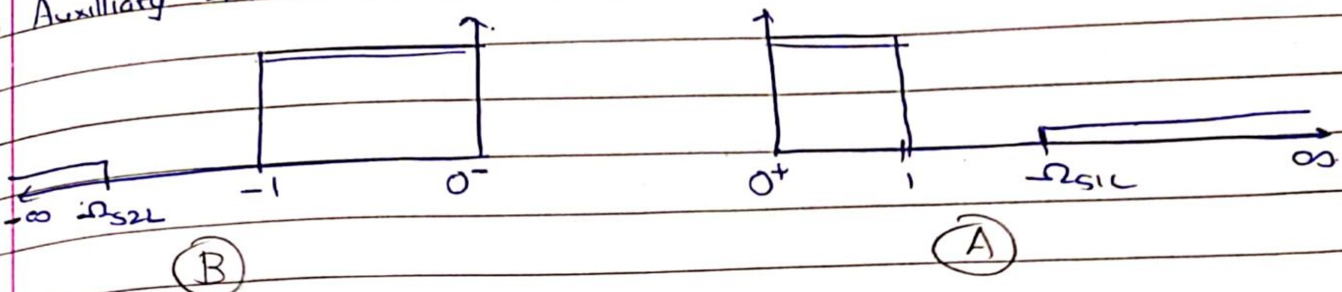
$$\boxed{\Omega_L = \frac{B\Omega}{\Omega_0^2 - \Omega^2}}$$

Ω	Ω_L	
0^+	0^+	
Ω_{p1}	$+1$	A
Ω_{s1}	Ω_{s1L}	
Ω_0^-	$+\infty$	
Ω_0^+	$-\infty$	
Ω_{s2}	Ω_{s2L}	
Ω_{p2}	-1	B
∞	0^-	

- Make Passband edge
  - Same equations as BPF.

$$\boxed{\begin{aligned}\Omega_0^2 &= \Omega_{p1}\Omega_{p2}\\ B &= \Omega_{p2}-\Omega_{p1}\end{aligned}}$$

- Auxilliary LPF :-



- Note :- $\dfrac{d\Omega_L}{d\Omega} = \dfrac{B(\Omega_0^2-\Omega^2) + 2B\Omega^2}{(\Omega_0^2-\Omega^2)^2} > 0$

$\therefore$ Ω_L is monotonically increasing
(despite it and its derivative being discontinuous at Ω_0

- Auxilliary LPF specifications.
  - Passband edge = 1
  - Stopband edge = $\min\{|\Omega_{S1L}|, |\Omega_{S2L}|\}$.
  - Tolerances are same as BPF *

- What if $\Omega_0 = \sqrt{\Omega_{p1}\Omega_{p2}}$ does not lie between $(\Omega_{S1}, \Omega$

The stopband tolerance as same?

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FINITE IMPULSE RESPONSE FILTER

- Impulse response is of finite duration $\Rightarrow$ Is absolutely summable
  $\Rightarrow$ Coefficient perturbation cannot cause instability.
- Can give linear phase response :-

$$H_{FIR}(e^{j\omega}) = |H_{FIR}(e^{j\omega})| e^{-j\omega D}$$

D is an integer or half-integers
(Any other D would make filter unrealizable)

We also generally accept

$$H_{FIR}(e^{j\omega}) = e^{-j\omega D} H_R(e^{j\omega})$$

where $H_R(e^{j\omega})$ is any real function.

- Different from above expression because H_R can be negative and hence produce a phase shift of π.

* Pseudo linear H_{FIR} can be acceptable.
  - Group delay is constant
  Phase " not.

- $H_R(e^{j\omega}) = e^{j\omega D} H_{FIR}(e^{j\omega})$ .... ~ Shifting h_{FIR} in time by D.

What if D is a half integer?

If D is an integer, we are shifting h_{FIR} by D samples.

- Since $h^r_{FIR}[n]$ is real, $H_{FIR}(e^{-j\omega}) = H^*_{FIR}(e^{j\omega})$

$$= H_R(e^{j\omega})$$

$\therefore$ H_R is even-symmetric.

* Real and even sequences have real and even DTFTs.

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- Consider $H_{FIR}(e^{j\omega}) = j e^{-j\omega D} H_R(e^{j\omega})$

$$\therefore \quad e^{j\omega D} H_{FIR}(e^{j\omega}) = j\underbrace{H_R(e^{j\omega})}_{\text{real}}$$

$\therefore \; j h_R[n]$ is real

$$\therefore \quad jH_R(e^{-j\omega}) = \left(jH_R(e^{j\omega})\right)^*$$

$$= -j\, H_R(e^{j\omega})$$

$$\therefore \quad H_R(e^{-j\omega}) = -H_R(e^{j\omega})$$

H_R is antisymmetric.

* Real and antisymmetric sequences have pure ~~and~~ imaginary and odd DTFTs

eg $\quad 1 \quad \underset{0}{\overset{0}{\uparrow}} \quad 1 \qquad$ DTFT = $2\cos\omega$.

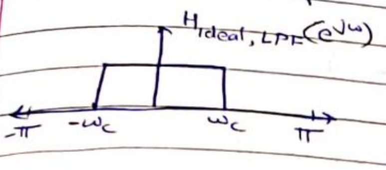
$\quad 1 \quad \underset{0}{\overset{0}{\uparrow}} \quad -1. \qquad$ DTFT = $2j\sin\omega$

Both are accepted as pseudo linear phase responses.

Theorem Pseudo linear phase requires symmetry or anti-symmetry in impulse responses

- IIR filters can never have symmetric/antisymmetric response

I Lowpass Filter



$$h_{ideal}[n] = \begin{cases} \frac{\omega_c}{\pi} & \text{for } n = 0 \\ \frac{\sin \omega_c n}{\pi n} & \text{for } n \neq 0 \end{cases}$$

$$h[-n] = h[n]$$

- Infinitely, non-causal, unstable, irrational

- Analog filters can never give us a finite length impulse response
  - Cannot have linear phase
  - No transformations possible to design FIR filters.

- We approximate ideal LPF by <u>truncating $h_{ideal}[n]$</u> to obtain finite length impulse response.
  - For linear phase response, we maintain symmetry in $h_{FIR}[n]$

eg – Choose $n \in \{-10, -9 \dots 10\}$ to make $h_{FIR}[n]$ of length 21.

13/3 – <u>Truncation</u> ~ <u>Multiplication of $h[n]$ by a window sequence</u>

$$\text{'}\underline{v[n]}\text{'} = \begin{cases} 1 & \text{for } -N \leq n \leq N \\ 0 & \text{otherwise} \end{cases}$$

'<u>Rectangular window sequence</u>'.

$$h_{FIR}[n] = h_{ideal}[n]\, v_R[n]$$

↳ R for real

$$\therefore \; H_{FIR}(e^{j\omega}) = \frac{1}{2\pi} \int_{-\pi}^{\pi} H_{ideal}(e^{j\lambda})\, V_R(e^{j(\omega-\lambda)})\, d\lambda$$

where $V_R(e^{j\omega})$ = DTFT of $v_R[n]$

$$= \sum_{n=-N}^{N} v_R[n]\, e^{-j\omega n}$$

P.T.O..

$$= \sum_{-N}^{N} e^{-j\omega n}$$

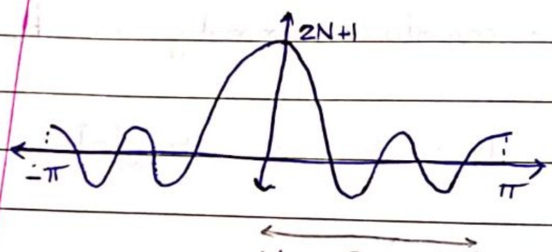
$$= \frac{e^{+j\omega N}\left((e^{-j\omega})^{2N+1} - 1\right)}{e^{-j\omega} - 1}$$

$$= \frac{e^{j\omega N}\, e^{-j\omega\left(\frac{2N+1}{2}\right)}\; 2j \sin \omega\left(\frac{2N+1}{2}\right)}{e^{-j\omega/2}\; 2j \sin\frac{\omega}{2}}$$

$$= e^{j\omega\left(N - N + \frac{1}{2} - \frac{1}{2}\right)} \frac{\sin \omega\left(\frac{2N+1}{2}\right)}{\sin\left(\frac{\omega}{2}\right)}$$

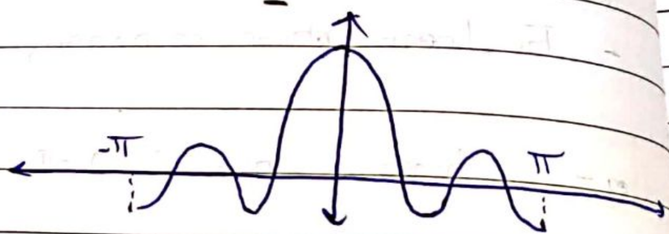
$$\boxed{V_R = \frac{\sin \omega\left(N + \frac{1}{2}\right)}{\sin\left(\frac{\omega}{2}\right)}}$$ ... even, because $v_R[n]$ was real and even

For N = even



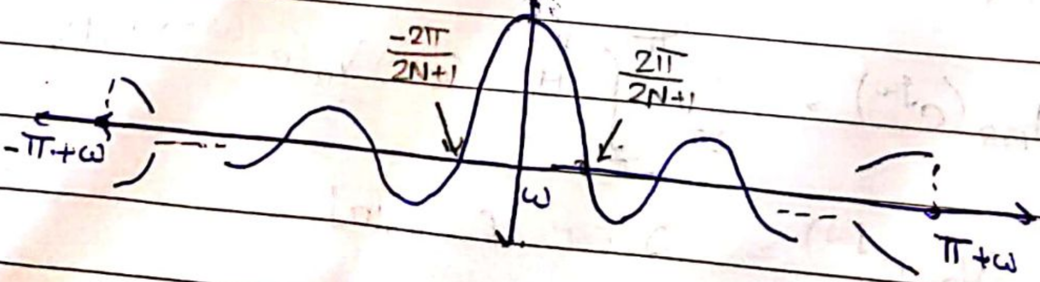
No. of oscillations = N

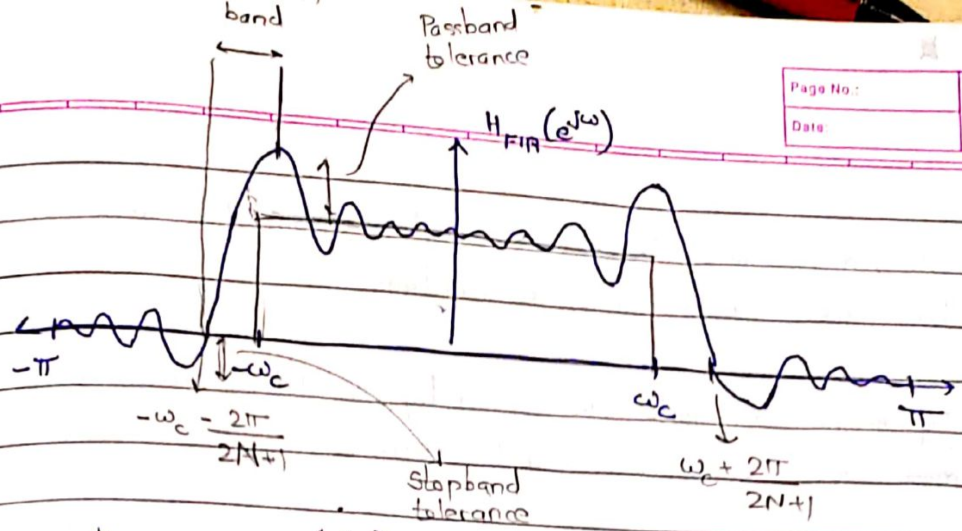
For N = odd



$$\therefore \; H_{FIR}(e^{j\omega}) = \frac{1}{2\pi} \int_{-\pi}^{\pi} H_{ideal}(e^{j\lambda})\, V_R(e^{j(\omega-\lambda)})\, d\lambda$$

$V_R(e^{j(\omega-\lambda)})$ as a function of λ





- At $\omega = \pm\omega_c$, $H_{FIR}(e^{j\omega}) \approx \frac{1}{2}$ of maximum value
- $H_{FIR}(e^{j\omega})$ is maximum at $\pm\left(\omega_c - \frac{2\pi}{2N+1}\right)$
- Passband and stopband ~~transition bands~~ tolerances are caused by side-lobes of $V_R(e^{j\omega})$
- The transition width $= \frac{4\pi}{2N+1}$

- Asymptotic $(N \to \infty)$ area under the primary side-lobe of $V_R(e^{j\omega})$

$$= \int_{\frac{2\pi}{2N+1}}^{\frac{4\pi}{2N+1}} \frac{\sin \omega\left(\frac{2N+1}{2}\right)}{\sin \frac{\omega}{2}} \, d\omega$$

$\sin \frac{\omega}{2} \to \frac{\omega}{2}$

$$\approx \frac{2}{2N+1} \int_{\pi}^{2\pi} \frac{\sin\theta}{\frac{\theta}{2N+1}} \, d\theta$$

$$\approx 2 \int_{\pi}^{2\pi} \frac{\sin\theta}{\theta} \, d\theta$$

.... independent of N

Gibb's phenomenon :- Irrespective of value of N, there is always an asymptotic deviation near the point of discon[t] even though the transition band width decrease for higher N..

- ∴ We have no control over passband and stopband tolerance.

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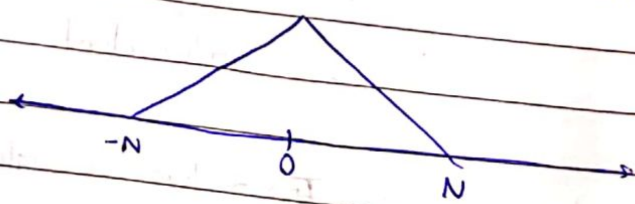
* It can similarly be shown that asymptotic area of other lobes also approaches a constant.

- The height of ripple is given by relative area between ~~main~~ side lobe and ideal 'rect' response.
  * Average height reached is caused by main lobe relative area.
- The stopband and passband tolerances are equal. (see fig) as they are both caused by ~~main~~ side lobe area.
  - They also do not depend on N for large enough N.
  - If we want to control passband and stopband tolerance, we will have to change the window sequence used.

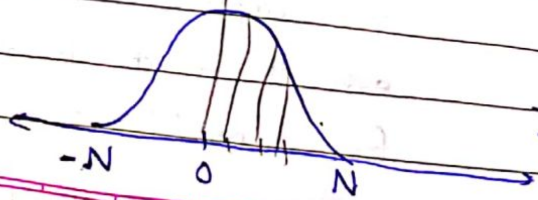
→ Other window sequences

- The passband and stopband tolerances in rectangular window case were caused because of discontinuity in the rectangular window.
  - To reduce tolerances, we try to make the window smoother.

eg Triangular

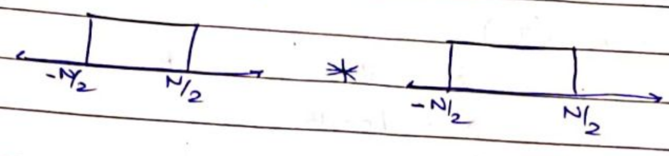


Better eg Raised Cosine (Hann window)



$$\frac{1}{2}\left(1+\cos\left(\frac{t\pi}{N}\right)\right)$$

Q Find DTFT of above window sequences

① Triangular = 

② Raised cosine :- Write cos as exponentials.

Generalized raised cosine = $\alpha + (1-\alpha)\cos\left(\frac{t}{N}\pi\right)$

for $0.5 < \alpha < 1$.

'Generalized Hamming Window'

- Smoothness of window is characterized by continuity of derivative(s). ~~Smoother is better~~

* Blackman window

$$\alpha + \beta\cos\left(\frac{t}{N}\pi\right) + \gamma\cos\left(\frac{2t}{N}\pi\right)$$

$\alpha, \beta, \gamma > 0$ and $\alpha + \beta + \gamma = 1$.

- For the same window size 'N', there is a conflict between transition width and passband/stopband tolerance.

$\underbrace{\text{transition width}}_{\text{characterized by mainlobe width}}$ and $\underbrace{\text{passband/stopband tolerance}}_{\text{characterized by relative sidelobe area.}}$

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→ Kaiser proposed a window based on modified Bes[sel] functions (I_0)

β = Shape parameter

N = Length

Window = $I_0(\beta N)$

- Kaiser window, through β and N, allows for changes in main lobe width and & relative side lobe area.

$\beta = 0$:- Rectangular Window

(High side lobe area :(

Low main lobe area :))

- Choose shape parameter based on tolerances required

Then choose length to satisfy transition width