IIT Bombay

Course Code : EE 614

Department: Electrical Engineering

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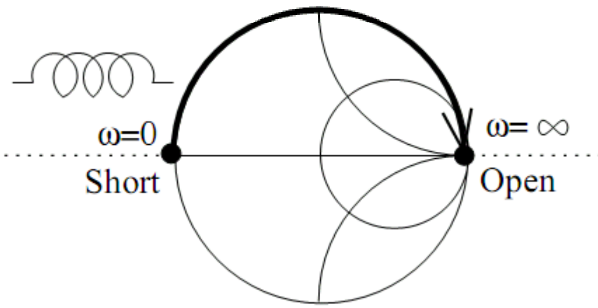
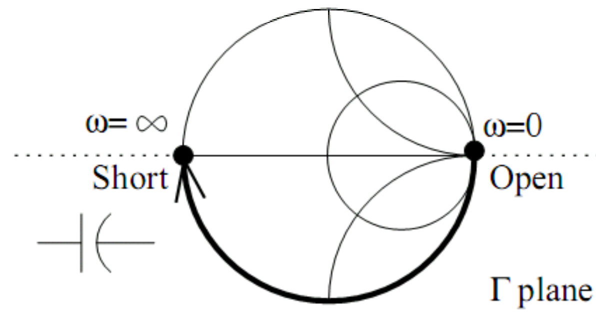
Lecture 4

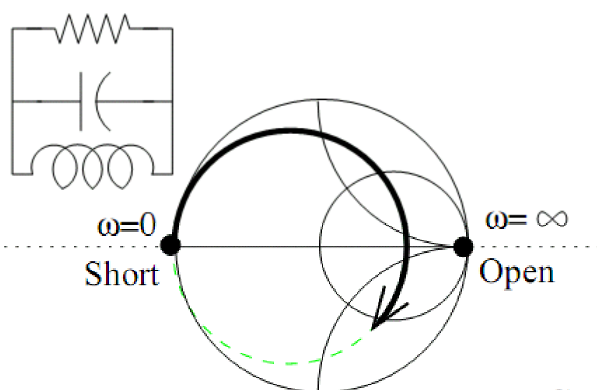
Problems Realizing LC Matching Network

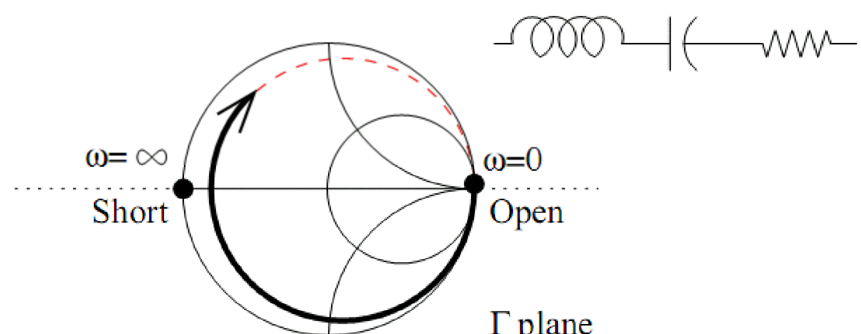


Ideal L



Ideal C





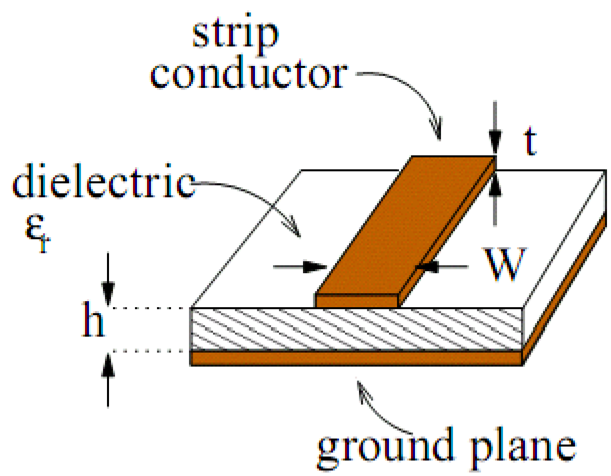
Z Smith chart

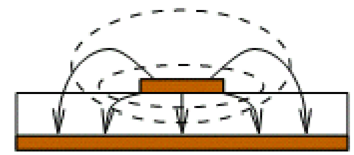
Real L

Real C

Microstrip matching network

Microstrip line:





Characteristic impedance of quasi - TEM mode :

$$Z_0 = \frac{1}{v_p C} \qquad \text{with} \qquad v_p = \frac{c}{\sqrt{\varepsilon_{r,eff}}}$$

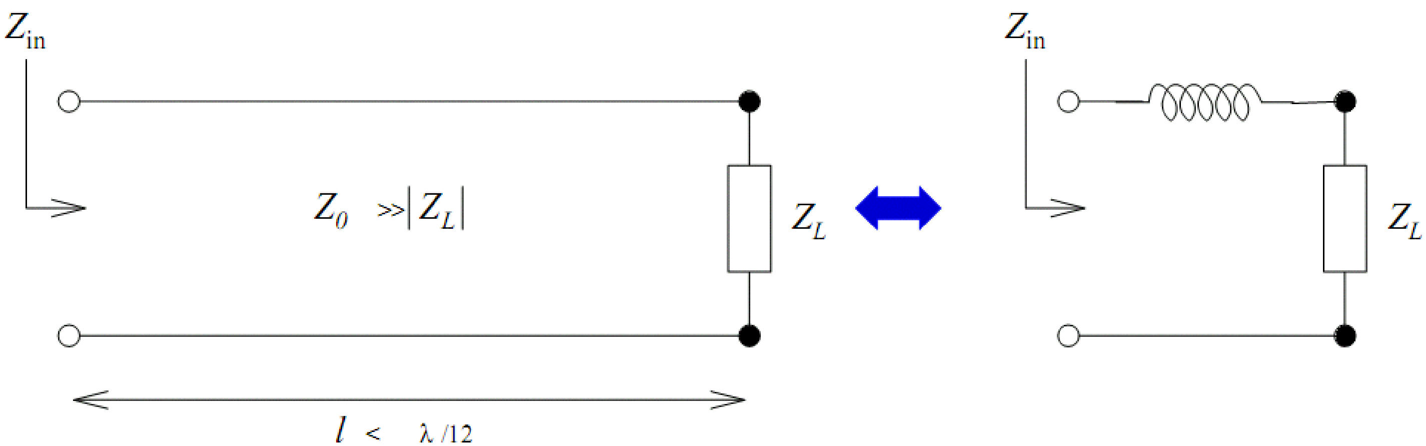
v_p is the phase velocity, ε_{eff} is the effective relative dielectric constant :

$$1 < \varepsilon_{r,eff} < \varepsilon_r$$

Various Substrates used in Microstrip lines

Material Name	Material type	Price (Rs/feet^2)	Frequency	Dielectric Constant	Dielectric Loss tangent
FR4	Epoxy	150(60mils)	0.9	4.0-4.7 (Not Stable)	0.02
ROGERS Duroid	PTFE teflor	2600	2-10	2.33	0.0012
Alumina	Al_2O_3	HIGH	10	9.5-10	0.0004
Quartz	S_iO_2	HIGH	10	3.8	0.0001
III-V RFIC	GaAs Si	HIGH	10 10	13.2 11.9	0.00056 0.004

Properties of high impedance line of Short length



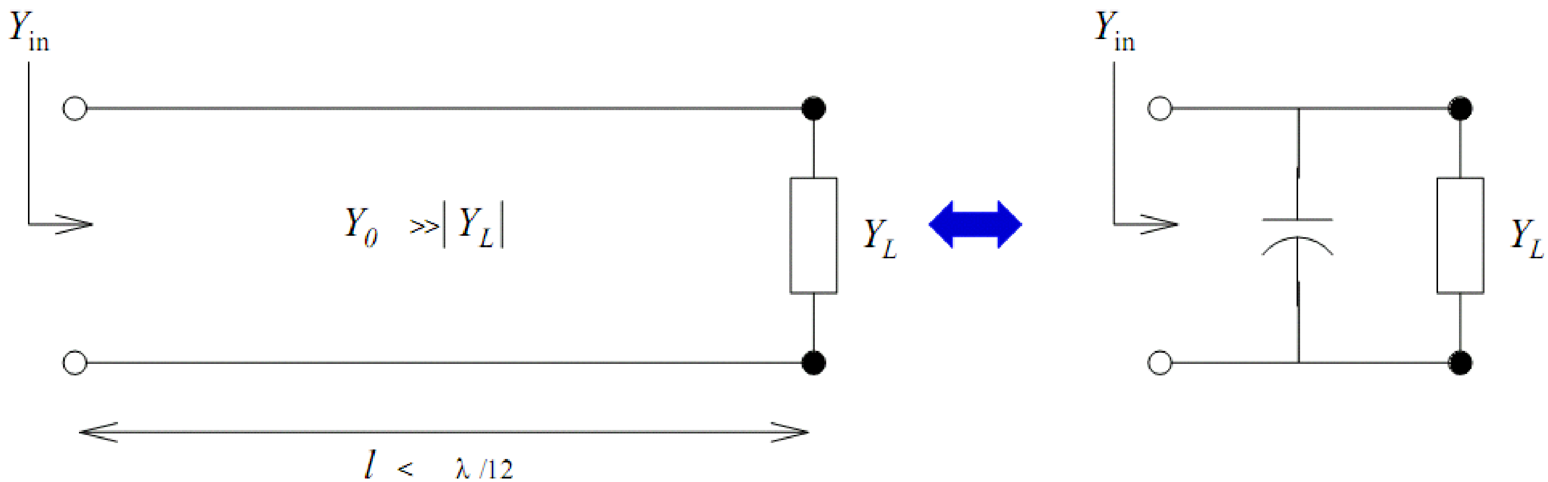
$$Z_{in} = Z_0 \frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l}$$

$$\approx Z_0 \frac{Z_L + jZ_0 \beta l}{Z_0 + jZ_L \beta l} \quad \text{for} \quad \beta l < \frac{\pi}{6} \quad \text{or} \quad l < \frac{\lambda}{12}$$

$$\approx Z_L + jZ_0 \beta l = Z_L + j\omega L \quad \text{for} \quad Z_0 > 3 \times |Z_L|$$

$$\text{where} \;\; L = \frac{Z_0 l}{v_p} \qquad \text{using} \quad \beta = \frac{\omega}{v_p}$$

Properties of Low impedance line of Short length



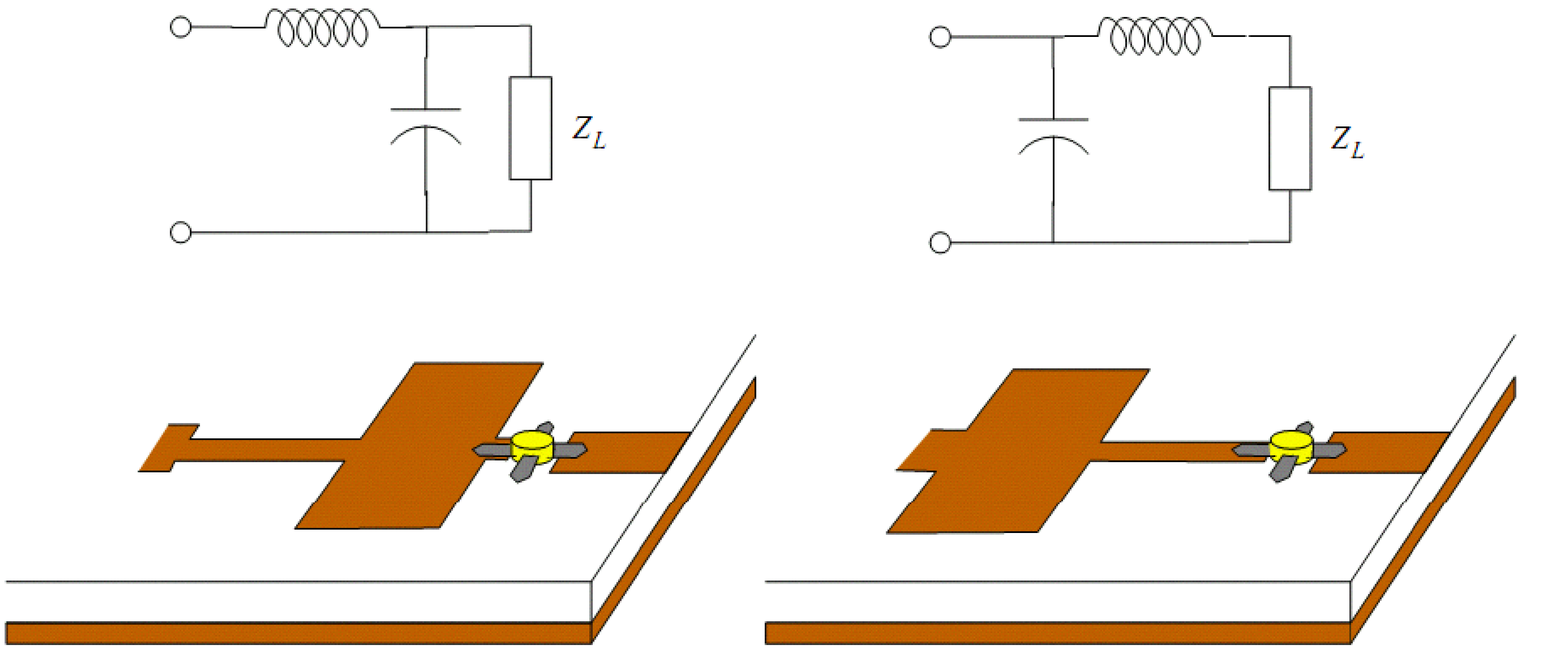
$$Y_{in} = Y_0 \frac{Y_L + jY_0 \tan \beta l}{Y_0 + jY_L \tan \beta l}$$

$$\approx Y_0 \frac{Y_L + jY_0 \beta l}{Y_0 + jY_L \beta l} \quad \text{for} \quad \beta l < \frac{\pi}{6} \quad \text{or} \quad l < \frac{\lambda}{12}$$

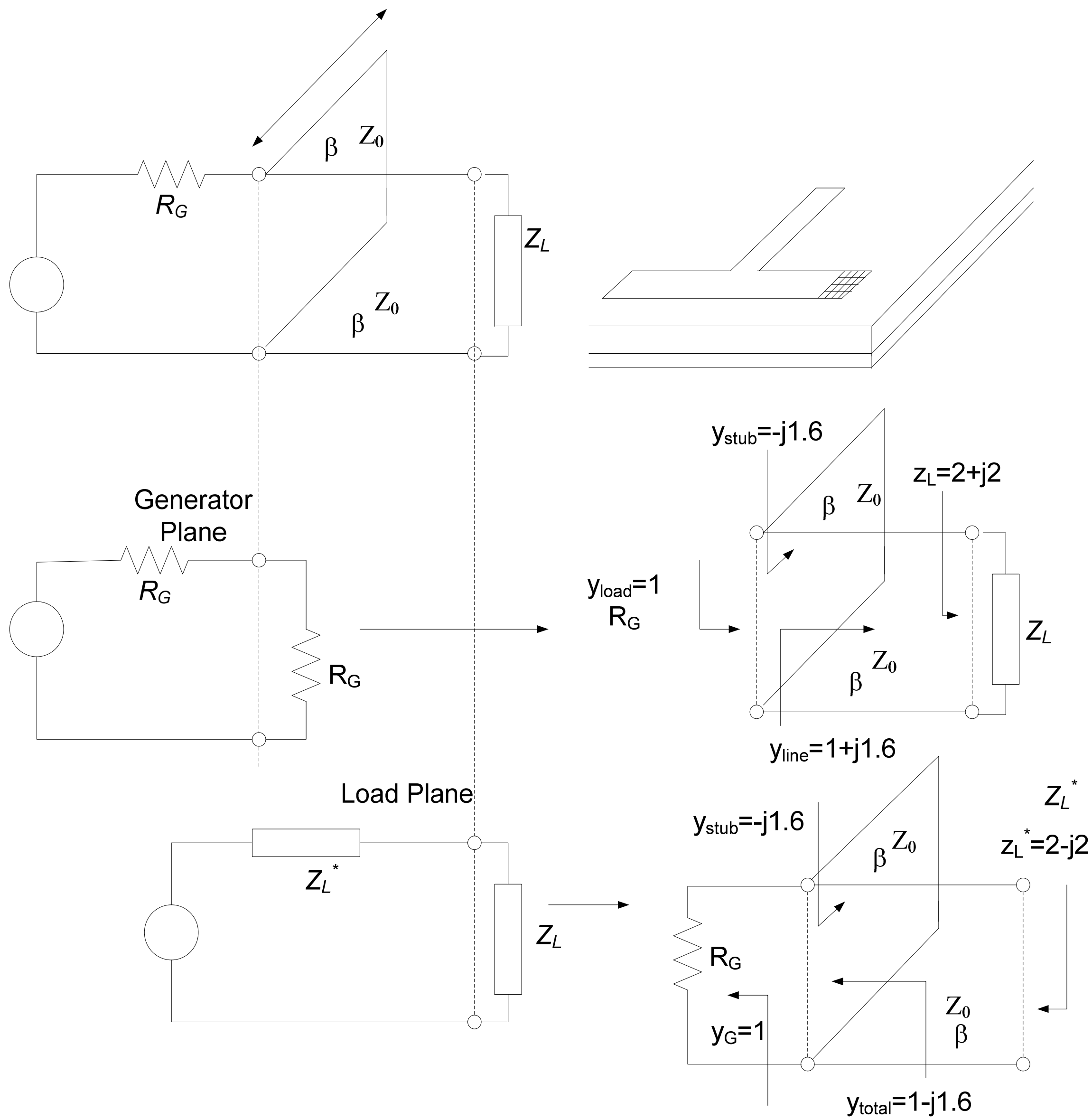
$$\approx Y_L + jY_0 \beta l = Y_L + j\omega C \quad \text{for} \quad Y_0 > 3 \times |Y_L|$$

$$\text{where } C = \frac{Y_0 l}{v_p} \quad \text{using} \quad \beta = \frac{\omega}{v_p}$$

Realization of LC Matching Network With Microstrip

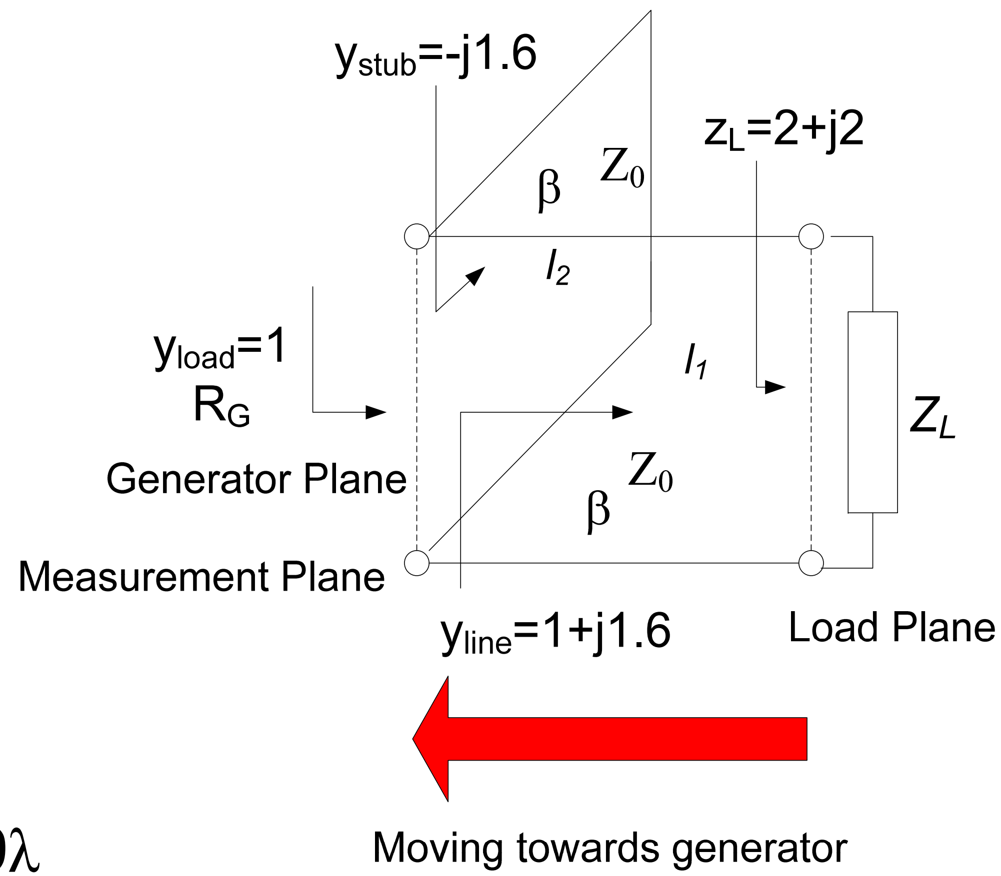


Design of a Stub Tuner



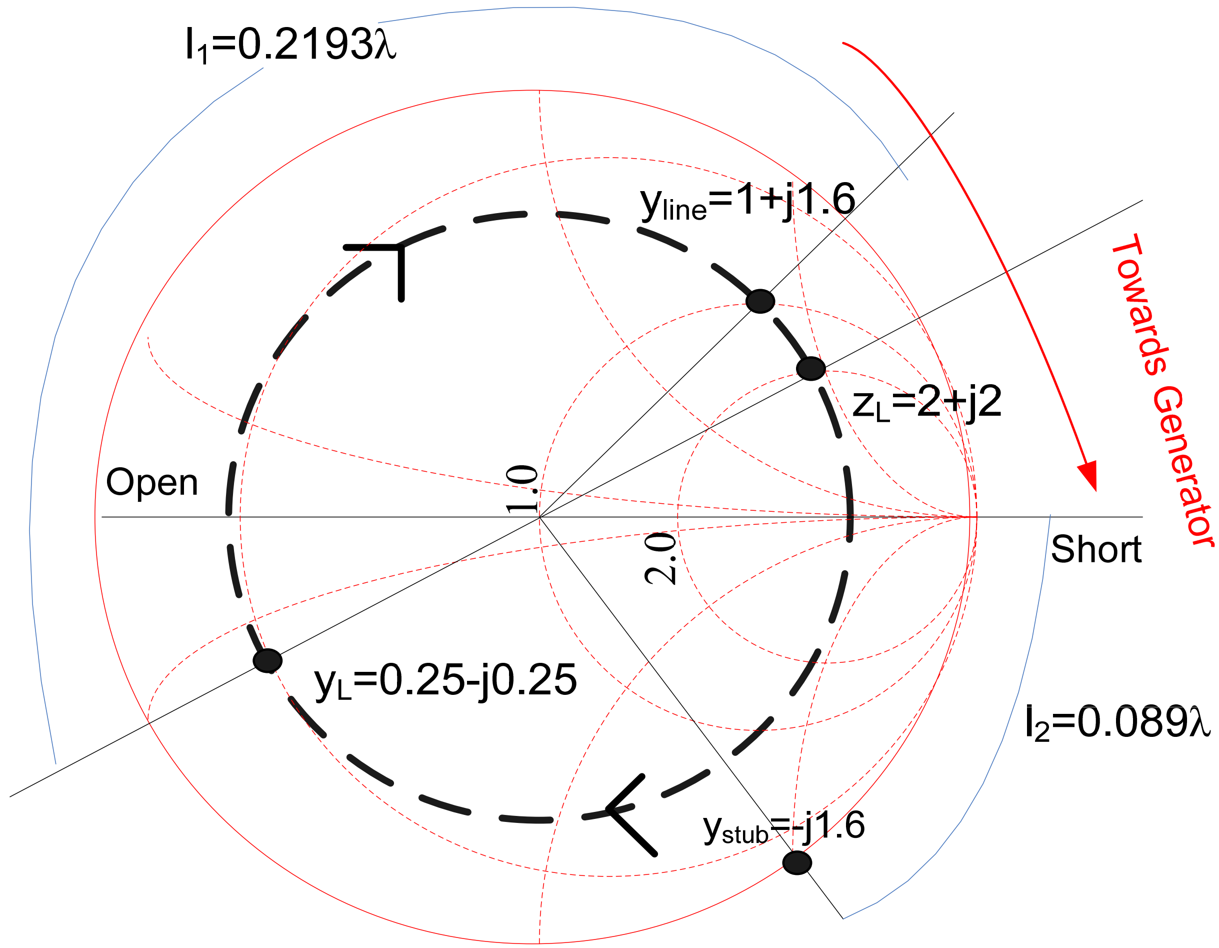
Design Using Generator Plane

Consider the load: Z_L=100+j100
Using Z_0=50 ohms we have z_L=2+j2



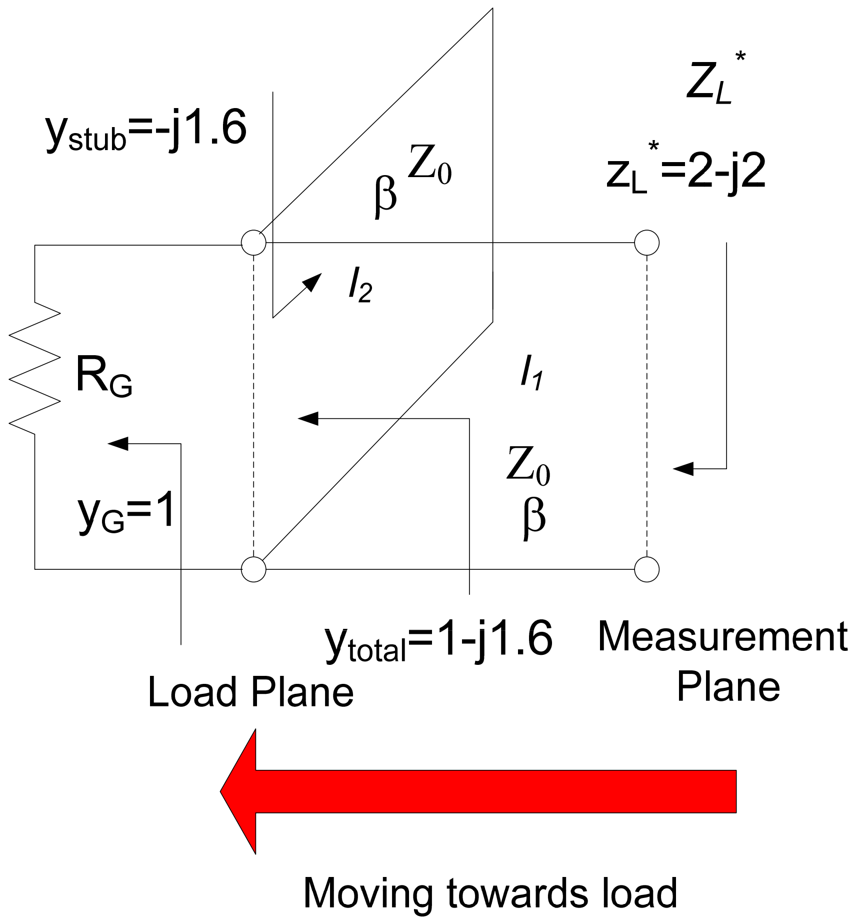
Solution
l_1=0.2193λ, l_2=0.089λ

Design Using Generator Plane



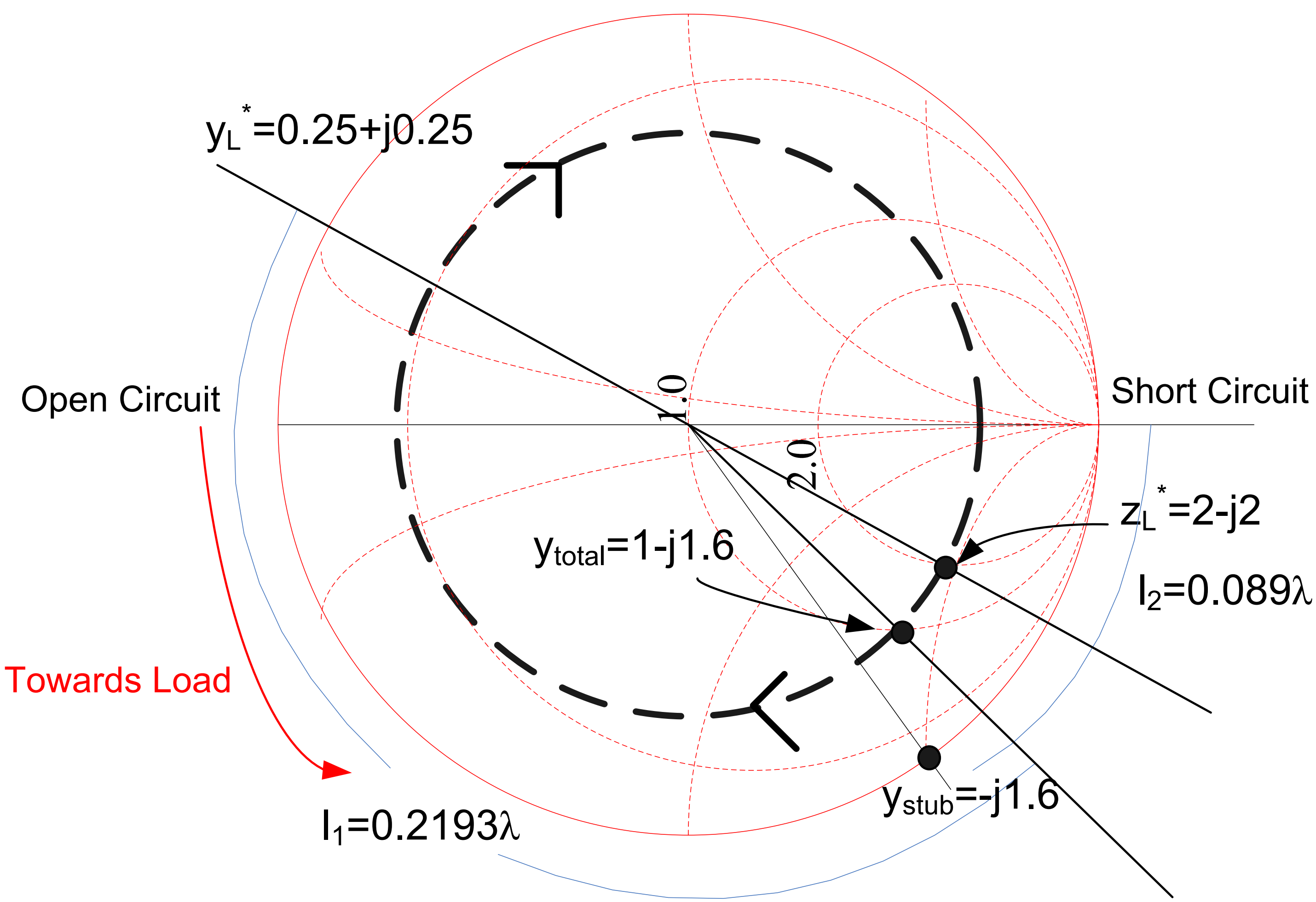
Design Using Load Plane

Consider the load: Z_L=100+j100
Using Z_0=50 ohms we have z_L^*=2-j2



Solution
l_1=0.2193λ, l_2=0.089λ

Design Using Load Plane



Signal Flow Graphs

- S-parameters were introduced in Chapter 1 to represent 2-port (can be generalized to N-port networks)

- As we design amplifiers we need to analyze bigger circuits realized with multiple building blocks

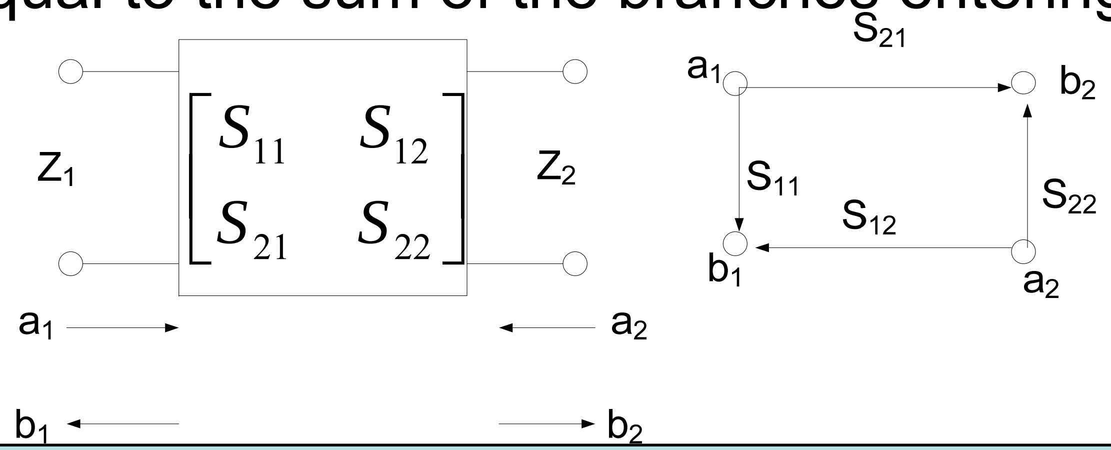
Flow Graph techniques will provide us:

- an analysis technique applicable to S parameters

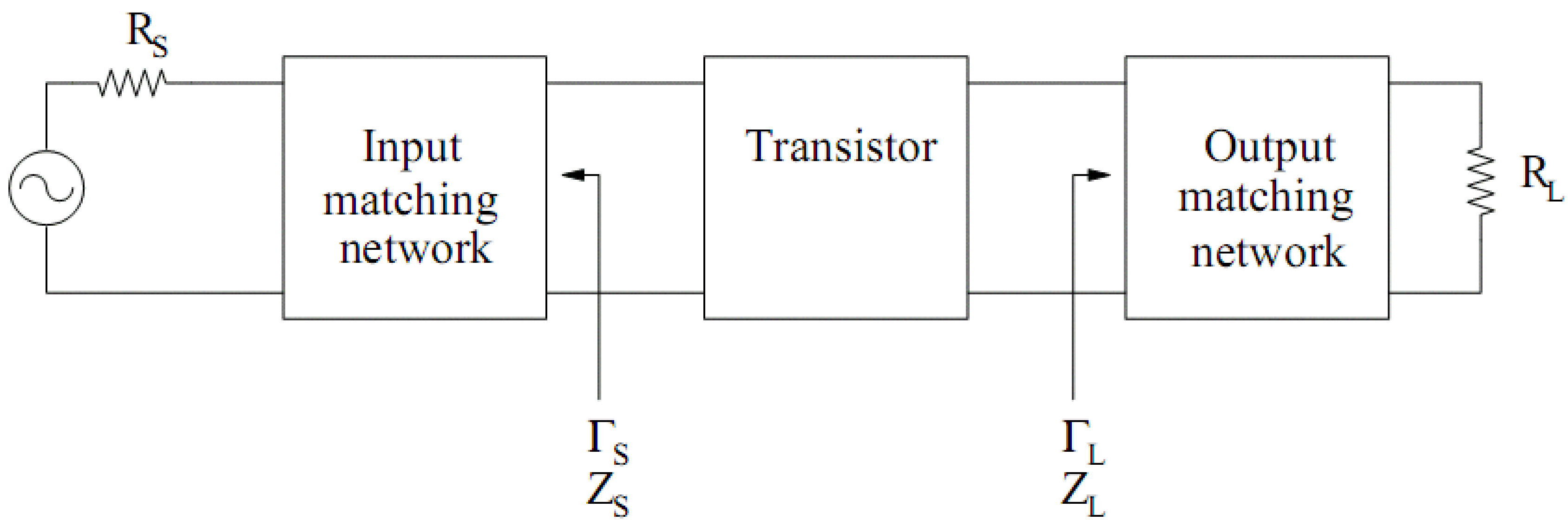
- the means to visualize the power flow in a circuit

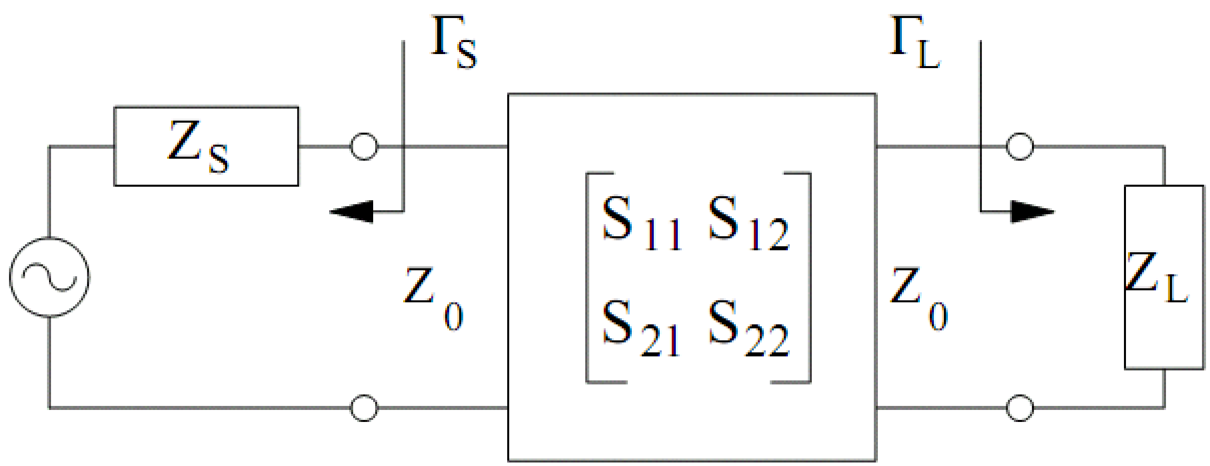
Signal Flow Graphs

- Each variable is designated as a node
- The S parameters and reflection coefficients are represented by branches
- Branches enter dependent variable nodes and emanate from independent variable nodes. The independent/dependent variable nodes are the incident/reflected waves respectively.
- A node is equal to the sum of the branches entering it



Focus on Amplifier Network

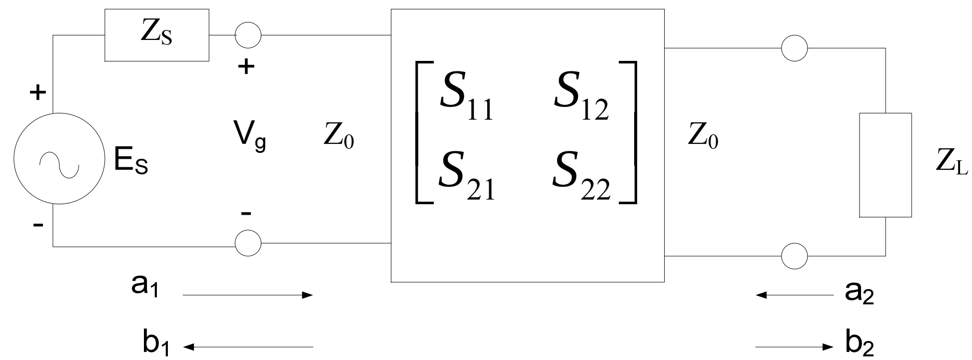




Flow Graph for a Generator

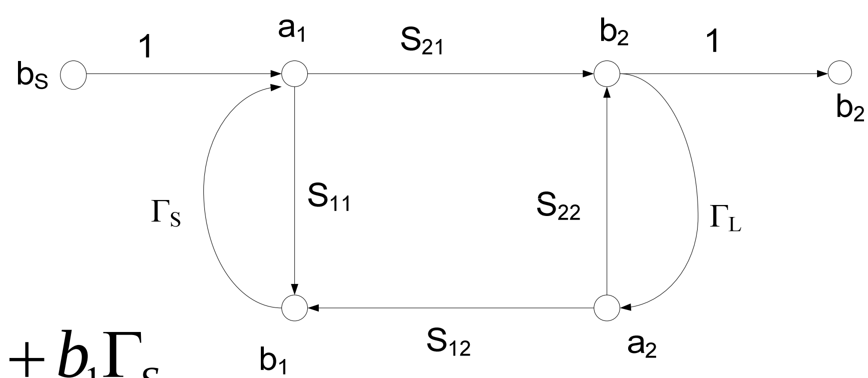
- A basic circuit we need to be analyzed is given below.

Starting from: $V_g = E_S - I_1 Z_S$ and divide by $\sqrt{Z_0}$



$$\frac{V_g}{\sqrt{Z_0}} = \frac{E_S}{\sqrt{Z_0}} - \frac{\sqrt{Z_0}\, I_1}{Z_0} Z_S$$

$$a_1 + b_1 = \frac{E_S}{\sqrt{Z_0}} - \frac{Z_S}{Z_0}\left(a_1 - b_1\right)$$



$$a_1\left[1 + \frac{Z_S}{Z_0}\right] = \frac{E_S}{\sqrt{Z_0}} + b_1\left[1 - \frac{Z_S}{Z_0}\right]$$

$$a_1 = \underbrace{\frac{\sqrt{Z_0}}{Z_0 + Z_S} E_S}_{b_S} + b_1 \underbrace{\frac{Z_S - Z_0}{Z_S + Z_0}}_{\Gamma_S} = b_S + b_1 \Gamma_S$$

Mason Semantic

Path: continuous succession of branches traversed in their indicated direction, no node being encountered more than once.

Forward Path: Path connecting input to output nodes.

Input Node: A node having only out-going branches.

Output Node: A node having only in-going branches.

Path Gain: Product of the branch multiplier.

Loop: Path which originates and terminates at the same node, no node being encountered more than once.

Loop Gain: Product of the branch multipliers around the loop.

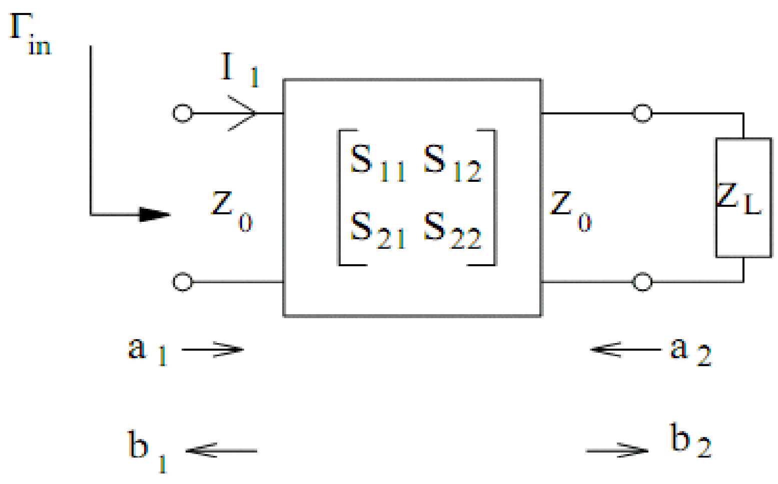
Mason's Rule

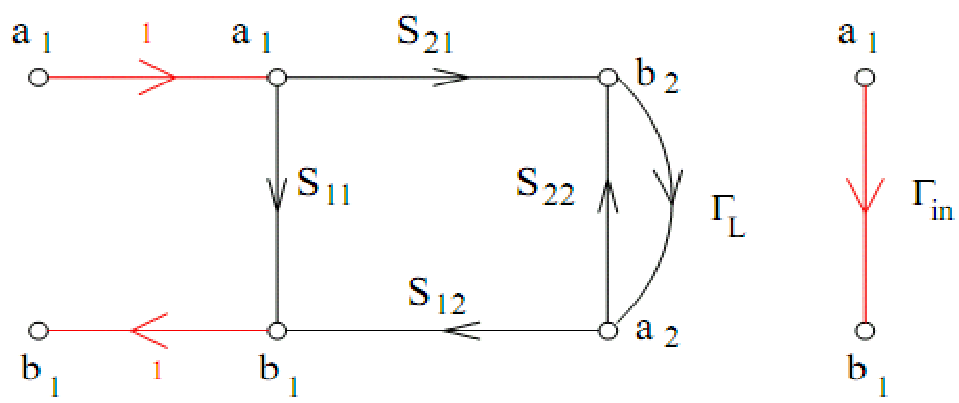
- Select input and output nodes

- The gain T from input to output is then: $T=(\Sigma_k P_k \Delta_k)/\Delta$
 with P_k the path gain of the k-th forward path and with
 Δ and Δ_k given by:
 Δ=1-(sum of all individual loop gains)
 + (sum of the loop gain products of all possible combinations of two non-touching loops)
 - (sum of the loop gain products of all possible combinations of three non-touching loops)
 + (sum of the loop gain products of all possible combinations of four non-touching loops)
 - …..

 Δ_k=value of Δ using loops not touching the k-th path

Application: Input Reflection Coefficient of a loaded 2-Port Network





- How many paths ?
- How many loops ?

Input Reflection Coefficient of a loaded 2-Port Network

$$\Gamma_{in} = \frac{b_1}{a_1} = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta}$$

$$\text{Path 1: } \mathrm{P_1 = S_{11}}$$

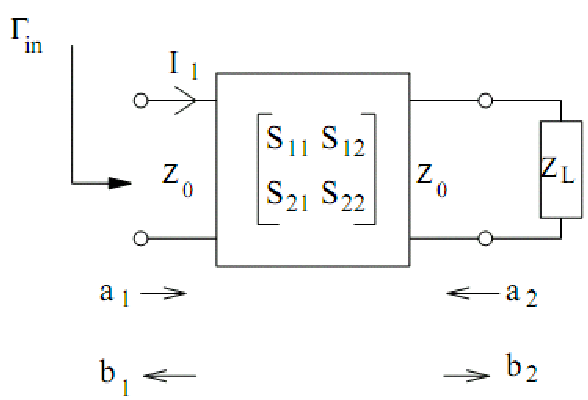
$$\text{Path 2: } \mathrm{P_2 = S_{21}\Gamma_L S_{12}}$$

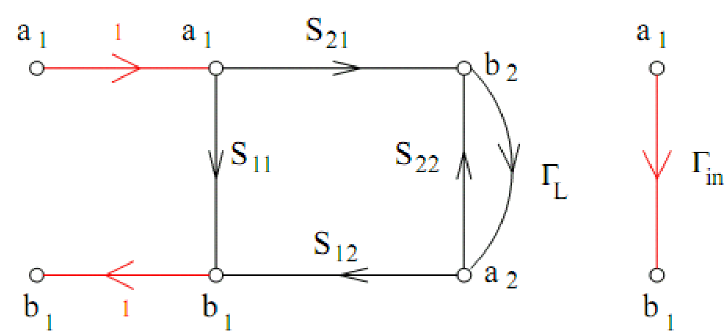
$$\text{Determinant: } \Delta = 1 - \mathrm{S_{22}\Gamma_L}$$

$$\text{Determinant 1: } \Delta_1 = 1 - \mathrm{S_{22}\Gamma_L}$$

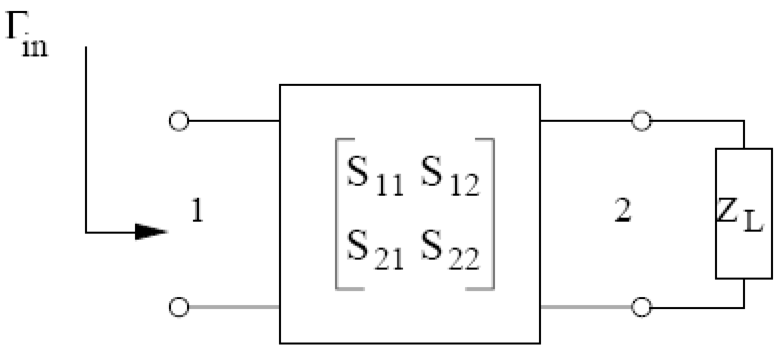
$$\text{Determinant 2: } \Delta_2 = 1$$

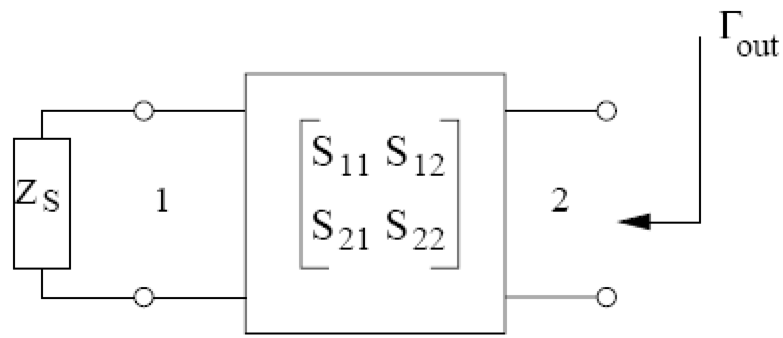
$$\Gamma_{in} = \mathrm{S_{11}} + \frac{\mathrm{S_{12}\Gamma_L S_{21}}}{1 - \mathrm{S_{22}\Gamma_L}}$$





Transmission Coefficient of a loaded 2-Port Network





$$T_{21} = \frac{b_2}{a_1} = \frac{P_1 \Delta_1}{\Delta}$$

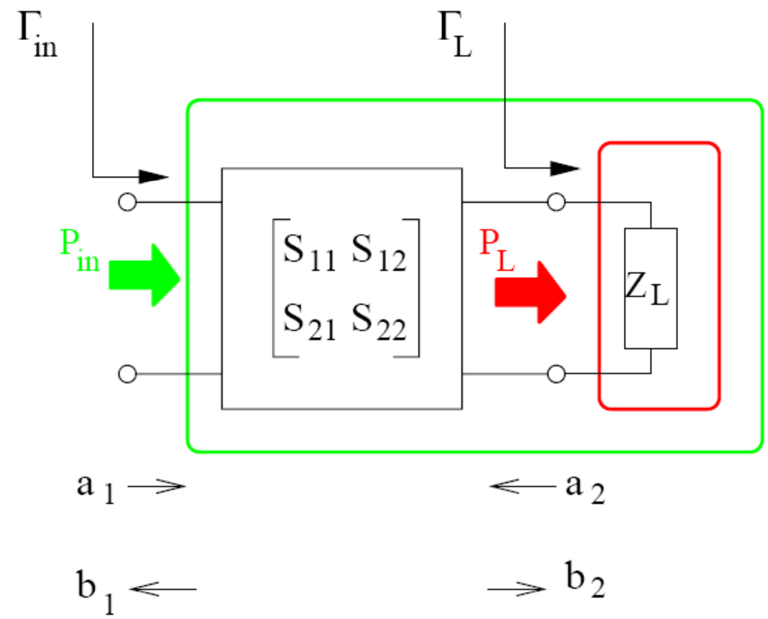
Path 1: $P_1 = S_{21}$

Determinant : $\Delta = 1 - S_{22}\Gamma_L$

Determinant 1: $\Delta_1 = 1$

$$T_{21} = \frac{b_2}{a_1} = \frac{S_{21}}{1 - S_{22}\Gamma_L}$$

Power Gain

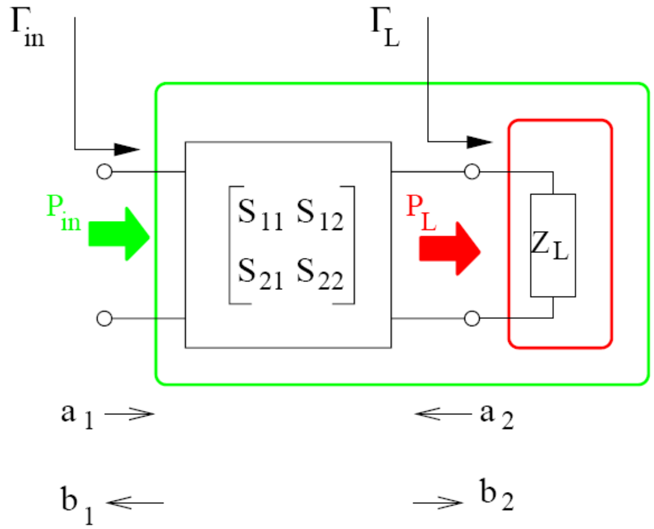


Power Gain :

$$G_P = \frac{\text{Power Delivered to the Load}}{\text{Power Delivered to the Network}} = \frac{P_L}{P_{in}} = \frac{|b_2|^2 - |a_2|^2}{|a_1|^2 - |b_1|^2}$$

$$= \left|\frac{b_2}{a_1}\right|^2 \frac{1 - \left|\frac{a_2}{b_2}\right|^2}{1 - \left|\frac{b_1}{a_1}\right|^2} = \left|\frac{b_2}{a_1}\right|^2 \frac{1 - |\Gamma_L|^2}{1 - |\Gamma_{in}|^2} = \frac{1}{1 - |\Gamma_{in}|^2} |S_{21}|^2 \frac{1 - |\Gamma_L|^2}{|1 - S_{22}\Gamma_L|^2}$$

Attributes of Power Gain



For matched loads we have $\Gamma_L = 0$ and $\Gamma_{in} = S_{11}$ and the power gain is :

$$G_P = \frac{1}{1-|\Gamma_{in}|^2}|S_{21}|^2\frac{1-|\Gamma_L|^2}{1-S_{22}\Gamma_L} = \frac{|S_{21}|^2}{1-|S_{11}|^2}$$

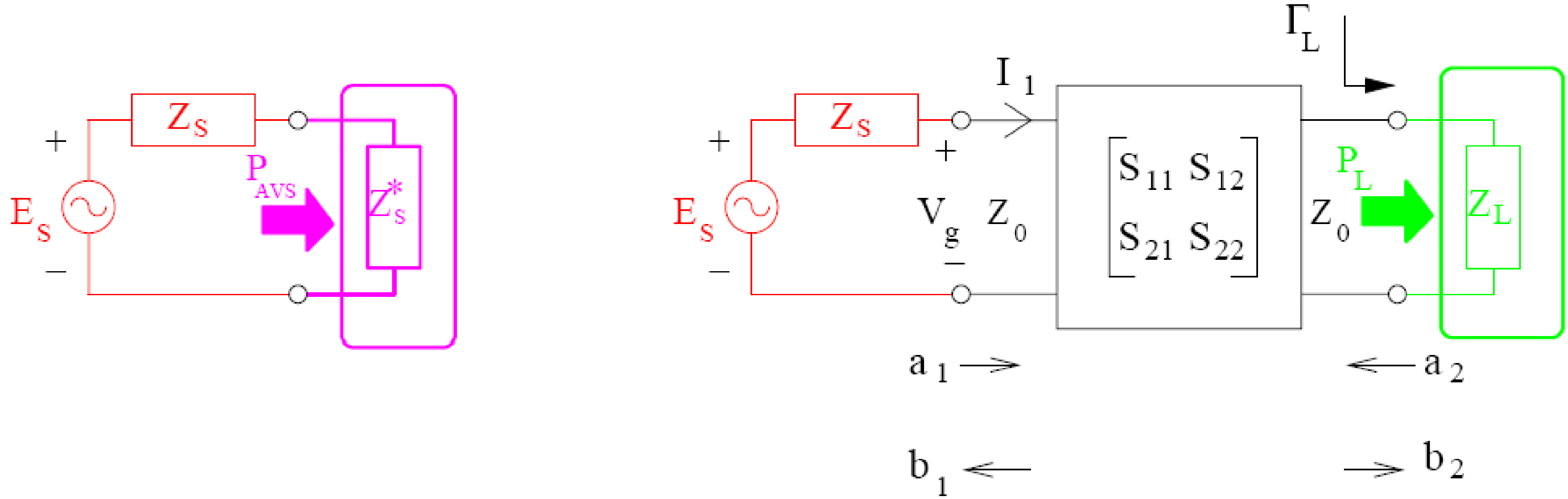
For lossless device $G_P = 1$ and we have the power conservation relation :

$$1 = |S_{21}|^2 + |S_{11}|^2$$

Features of Power Gain G_P :

- $G_P([S], \Gamma_L)$ finds applications in power amplifier design
- Problem : what if all the power is reflected at the input ?

Transducer Power Gain



Transducer Power Gain :

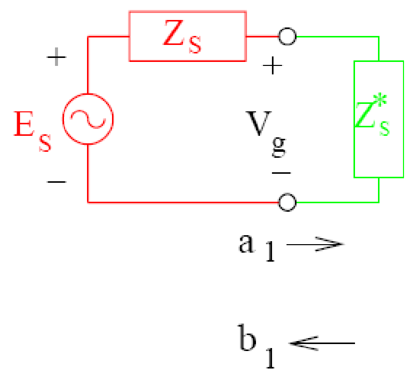
$$G_T = \frac{\text{Power delivered to the load}}{\text{Power Available from the source}} = \frac{P_L}{P_{AVS}}$$

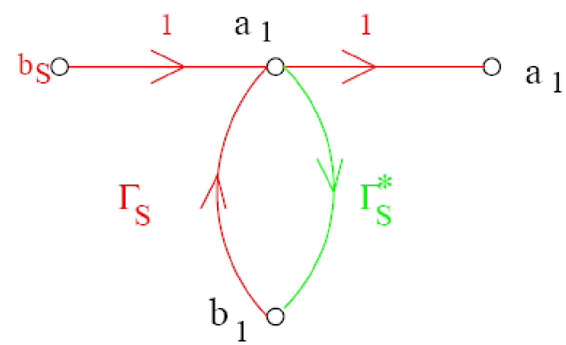
Power delivered to the load (rms)

$$P_L = |b_2|^2 - |a_2|^2 = |b_2|^2 \left(1 - |\Gamma_L|^2\right)$$

How do we get the Power Available from the source?

Available Power from the source and the resulting G_T





Flow graph solution : $\quad a_1 = \dfrac{b_s}{1 - |\Gamma_S|^2}$

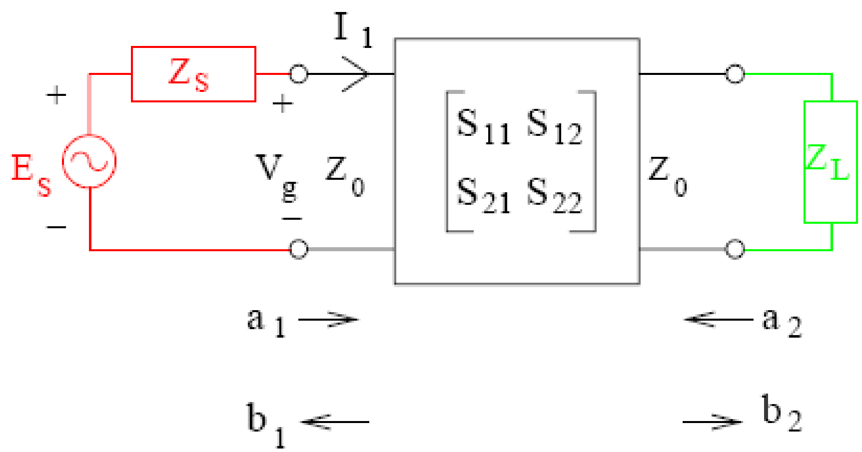
$$P_{AVS} = |a_1|^2 - |b_1|^2 = |a_1|^2\left(1 - \frac{|b_1|^2}{|a_1|^2}\right) = |a_1|^2\left(1 - |\Gamma_S|^2\right)$$

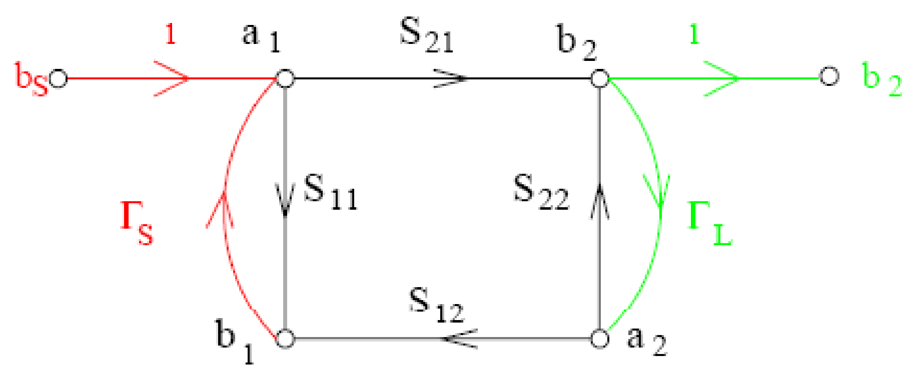
$$= \left|\frac{b_s}{1 - |\Gamma_S|^2}\right|^2\left(1 - |\Gamma_S|^2\right) = \frac{|b_s|^2}{1 - |\Gamma_S|^2} \quad \text{since } |\Gamma_S| < 1$$

Transducer Power Gain :

$$G_T = \frac{P_L}{P_{AVS}} = \left|\frac{b_2}{b_s}\right|^2\left(1 - |\Gamma_L|^2\right)\left(1 - |\Gamma_S|^2\right) \quad \text{where we need } \frac{b_2}{b_s}$$

Flow Graph for G_T calculation

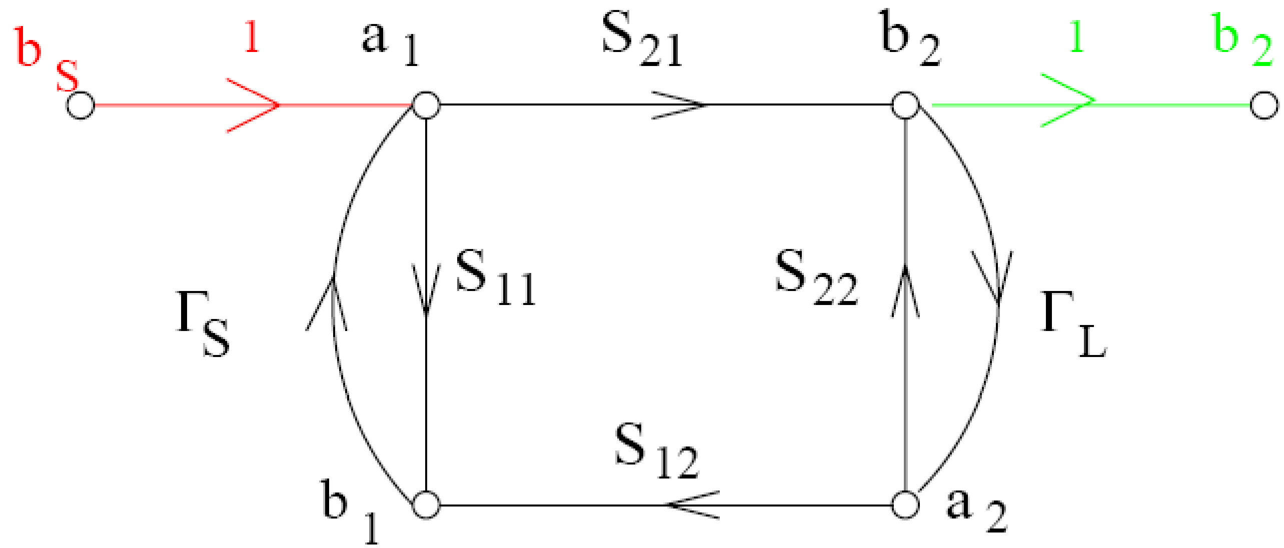




We need to calculate $\dfrac{b_2}{b_s}$

- How many paths ?
- How many loops ?

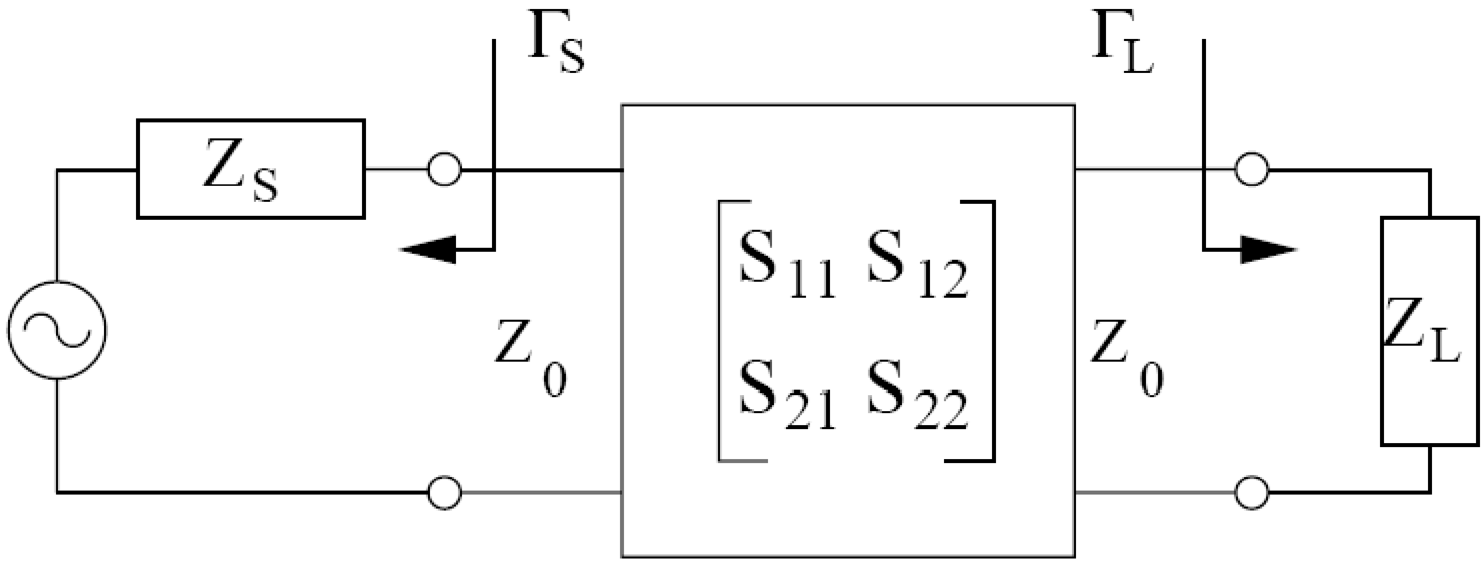
Flow Graph for G_T calculation



$$\frac{b_2}{b_s} = \frac{P_1\Delta_1}{\Delta} = \frac{S_{21}}{1-\left(\Gamma_S S_{11} + S_{22}\Gamma_L + \Gamma_S S_{21}\Gamma_L S_{12}\right) + \Gamma_S S_{11} S_{22}\Gamma_L}$$

$$G_T = \left|\frac{b_2}{b_s}\right|^2 \left(1-|\Gamma_L|^2\right)\left(1-|\Gamma_S|^2\right) = \frac{1-|\Gamma_S|^2}{|1-\Gamma_{in}\Gamma_S|^2}|S_{21}|^2\frac{1-|\Gamma_L|^2}{|1-S_{22}\Gamma_L|^2}$$

Attributes of Transducer Power Gain

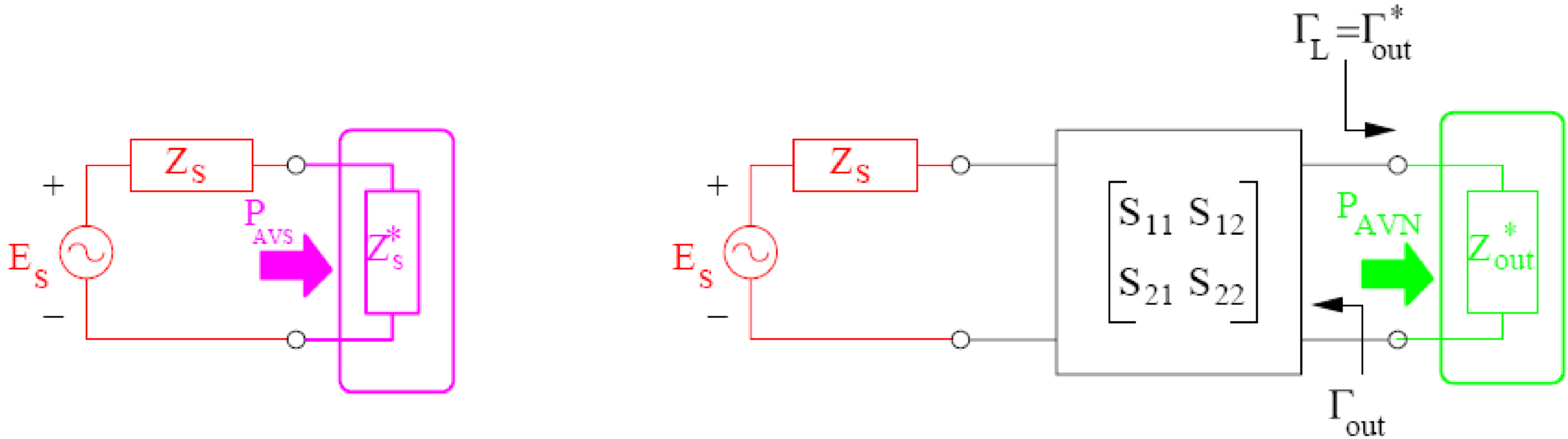


$$G_T = \left|\frac{b_2}{b_s}\right|^2 \left(1-|\Gamma_L|^2\right)\left(1-|\Gamma_S|^2\right) = \frac{1-|\Gamma_S|^2}{|1-\Gamma_{in}\Gamma_S|^2}|S_{21}|^2\frac{1-|\Gamma_L|^2}{|1-S_{22}\Gamma_L|^2} = \frac{1-|\Gamma_S|^2}{|1-S_{11}\Gamma_S|^2}|S_{21}|^2\frac{1-|\Gamma_L|^2}{|1-\Gamma_{out}\Gamma_L|^2}$$

Properties:

- $G_T([S],\Gamma_S,\Gamma_L)$ function of both source and load impedances
- $G_T = |S_{21}|^2$ for $\Gamma_S = \Gamma_L = 0$ (matched loads at input and output)

Attributes of Transducer Power Gain

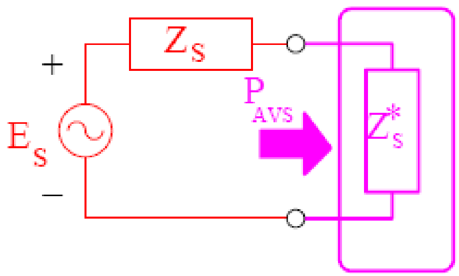


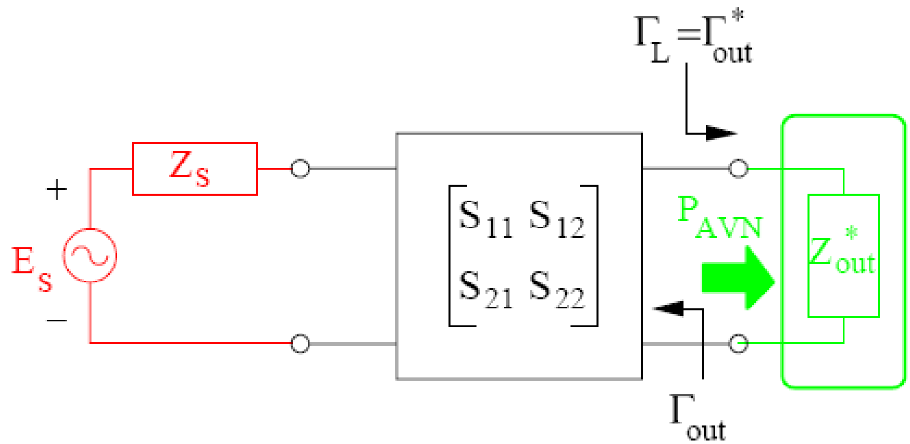
$$G_T = \left|\frac{b_2}{b_s}\right|^2 \left(1-|\Gamma_L|^2\right)\left(1-|\Gamma_S|^2\right) = \frac{1-|\Gamma_S|^2}{|1-\Gamma_{in}\Gamma_S|^2}|S_{21}|^2\frac{1-|\Gamma_L|^2}{|1-S_{22}\Gamma_L|^2} = \frac{1-|\Gamma_S|^2}{|1-S_{11}\Gamma_S|^2}|S_{21}|^2\frac{1-|\Gamma_L|^2}{|1-\Gamma_{out}\Gamma_L|^2}$$

Properties:

- $G_T([S], \Gamma_S, \Gamma_L)$ function of both source and load impedances
- $G_T = |S_{21}|^2$ for $\Gamma_S = \Gamma_L = 0$ (matched loads at input and output)

Available Power Gain





$$G_A = \frac{\text{Power Available from the Network}}{\text{Power Available from the Source}} = \frac{P_{AVN}}{P_{AVS}}$$

$$= G_T\Big|_{\Gamma_L = \Gamma_{out}^*}$$

$$= \frac{1-|\Gamma_S|^2}{|1-S_{11}\Gamma_S|^2}|S_{21}|^2\frac{1}{|1-\Gamma_{out}|^2}$$

Properties :

- $G_A([S], \Gamma_S)$ finds applications in LNA design
- *A*ssumes a conjugate match at the output