EE 338 DIGITAL SIGNAL PROCESSING
TUTORIAL PROBLEMS - SET 4

1. If the continuous time signal x(t) is convolution of two band-limited signals $x_1(t)$ and $x_2(t)$. And x(t) is sampled to obtain s(t). where,

$$X_1(j\Omega) = 0 \qquad \text{for } |\Omega| > 2\pi \text{ x } 10000$$
$$X_2(j\Omega) = 0 \qquad \text{for } |\Omega| > 2\pi \text{ x } 20000$$

a) what is the minimum sampling frequency required to recover the x(t) from the sampled signal s(t) ?
b) Consider the squarer system $y(t) = (x_1(t))^{2.}$ If we want to implement this squarer system using a discrete time system, what is the minimum sampling frequency required to sample $x_1(t)$?

2) Consider the system for low pass filtering a continuous time signal using discrete time system as shown in Fig. 1. The sampling process can be represented as a two stage process. First stage is multiplication by an impulse train and second stage involves conversion of the impulse train to sequence. $X(j\Omega)$ is the Fourier transform of the input signal x(t). Let $H_c(j\Omega)$ is the overall frequency response of the continuous time low pass filter.

a) For what values of T the overall system with input x(t) and output y(t) behaves as a continuous time low pass filter ? What values of Ω_c should be chosen for the highest value of T ? Plot $X_s(j\Omega)$, $X(e^{jw})$ and $H_c(j\Omega)$.
b) Can I change the cutoff frequency of the overall continuous time filter without changing the discrete time filter specifications? If yes, explain the reasoning.

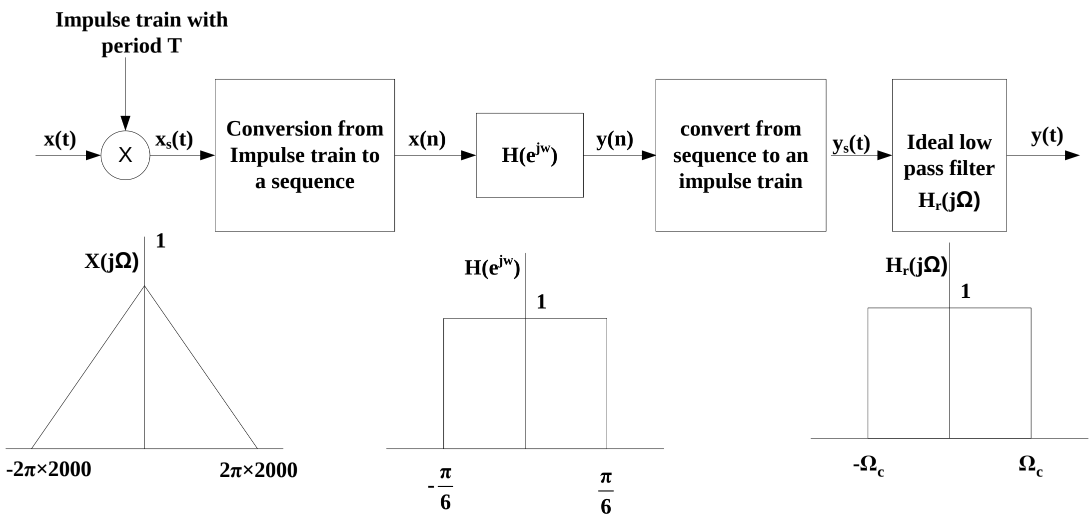


Fig. 1

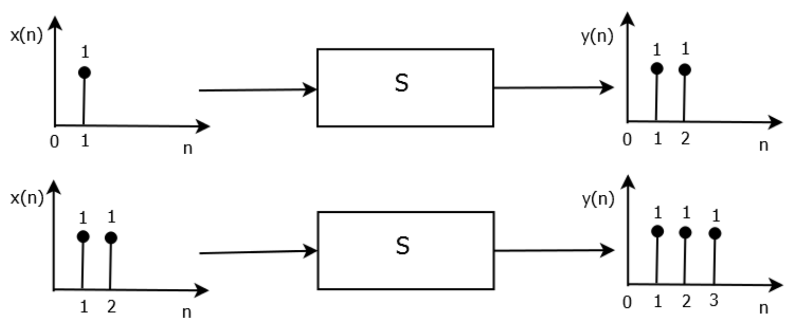
3) Consider a discrete time system represented by a linear equation $y[n] = x[n]\{g_1[n])\}+g_2[n]$. Assume we know the $g_1[n]$ and $g_2[n]$.

a) Is this system linear? What is the zero input response of this system? Can we convert the system into sum of the output of a linear system and zero input response of the system?
b) Is this system time invariant? Provide an example of $g_1[n]$ and $g_2[n]$ for which the above system is time invariant and another example where the system is time variant.

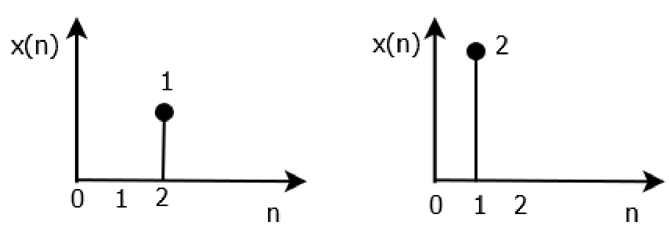
4) Consider a system $y[n] = x[n] + \beta y[n-1]$. Assume $y(-\infty) = 0$.

a) Check whether the system is (1) Linear, (2) Time invariant, (3) causal, (4) memoryless,
b) Is this system stable for $\beta = -1$? Obtain the values of β for which the system becomes stable. (Hint: Obtain the impulse response of the system)

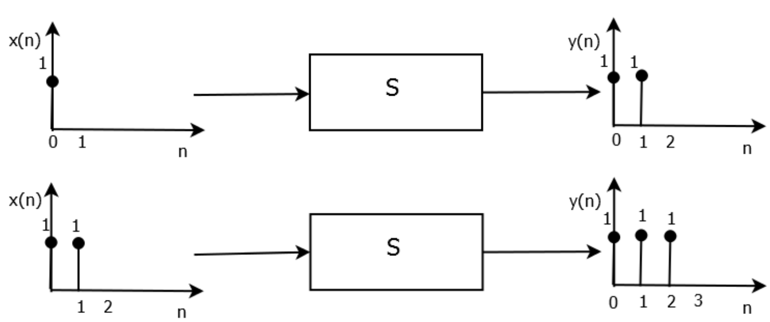
5) The given system S is Time-invariant. Two input-output pairs of the system are given



a) Is the system linear?
b) Can one find the output of the system for each of the below given inputs with the information provided. If yes, find the output. If no, state the reason.

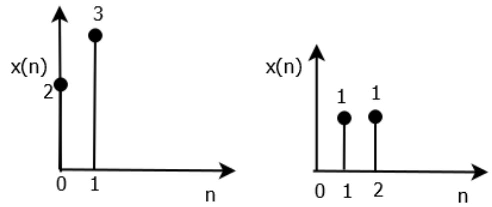


6) The given system S is Linear. Two input-output pairs of the system are given

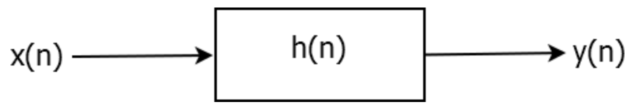


a) Is the system time-invariant?

b) Can one find the output of the system for each of the below given inputs with the information provided. If yes, find the output. If no, state the reason.



7) The system S is Linear Time-invariant with impulse response $h(n)$



$x(n) = \{\underset{\uparrow}{1},2,3,4,5,6,7,8,9\}$, $h(n) = \{\underset{\uparrow}{1},2,3\}$

a) Find $y(n) = x(n) * h(n)$

b) Let $x_1(n) = \{\underset{\uparrow}{1},2,3\}$, $x_2(n) = \{\underset{\uparrow}{4},5,6\}$ and $x_3(n) = \{\underset{\uparrow}{7},8,9\}$

Let $y_1(n) = x_1(n) * h(n)$, $y_2(n) = x_2(n) * h(n)$ and $y_3(n) = x_3(n) * h(n)$

Using the linearity and time-invariance property of the system and find out $y(n)$ using $y_1(n)$, $y_2(n)$, and $y_3(n)$

8) System identification: System S is Linear Time-invariant. Its impulse response is $h(n)$ and DTFT of $h(n)$ is $H(e^{j\omega})$

Given

i) System is causal

ii) $H\left(e^{j\omega}\right) = H^*(e^{-j\omega})$

iii) DTFT of $h(n+1)$ is real.

a) Prove that $h(n)$ has finite support and find its support

b) Given below conditions, can one find the filter $h(n)$? Is yes, find it. Else state the reason

i) $H\left(e^{j0}\right) = 2$ ii) $H\left(e^{j\pi}\right) = 0$ iii) $\frac{dH\left(e^{j\omega}\right)}{d\omega} = 0 \; at \; \omega = \pi$