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Likelihood & Log of LR
Ratio

$$\text{Sign}\left(\underbrace{\log\left(\frac{p(y|+1)}{p(y|-1)}\right)}_{\text{LLR}}\right)$$

$$y[0] = A + w[0]$$
$$\vdots$$
$$y[k-1] = A + w[k-1]$$

$$\vec{H} = a\begin{bmatrix}1\\ \vdots \\ 1\end{bmatrix} \qquad \text{Cov} = \sigma^2 I_{k\times k}$$

Lect 21.2.19

Revision Lecture

Link Budget Analysis

ex QAM - 256 (8 bits) , 1 Mb/s

⇒ Symbol Rate = $\frac{1}{8}$ Msym/s

⇒ Minimum / ideal b/w = $\frac{1}{8}$ MHz

⇒ a % excess b/w ⇒ × (1+a) factor

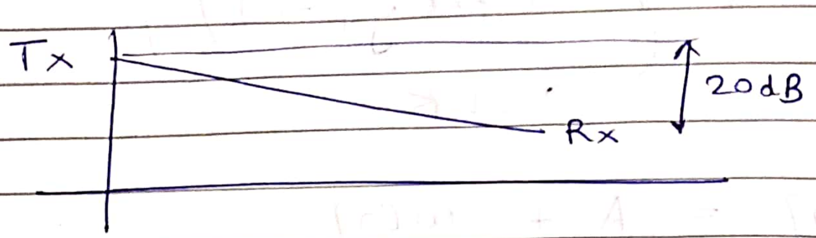
$\frac{1}{8} \times (1+a)$ → Actual

→ If we say BER < 10^{-6} = Q(....)

$\frac{E_s}{N_0}$ at receiver

→ Energy per symbol at receiver

→ Also take into account loss ....



Loss = 20dB R_x = 1mW T_x = 100mW

- BER Gives us SNR measure ......

$\left(\frac{E_s}{N_o}\right)$ At Rx is a measure of SNR

How? Since it doesn't take into account

$$\frac{E_s}{N_o} = \frac{P \cdot T}{N_o} \approx \frac{P}{W N_o} \rightarrow SNR$$

→ Find E_s, Power req. at Rx $= \frac{E_s}{T}$

⇒ $\boxed{\text{Get Tx Power}}$

~~Back to estimation Problem~~

$$y_1 = A b_1 + w_1$$

(A → unknown)

$$\vdots$$

$$y_K = A b_K + w_K$$

$$P(y \mid A = a) = f(y \mid A=a, b=+1) \times \frac{1}{2} + f(y \mid A=a, b=-1) \times \frac{1}{2}$$

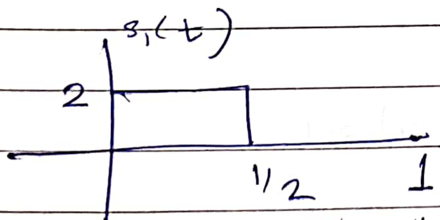
ML Gives us the formula

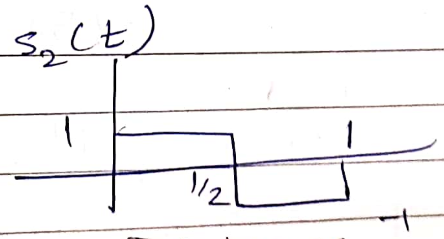
$$\boxed{a = \frac{1}{K} \sum_{k=1}^{K} y[k] \tanh\left(\frac{y_k a}{\sigma^2}\right)}$$

→ If initial guess of sign of a is right we will converge to A (true value) else we will converge to −A (Reversed)

→ Reversed is still fine, can be detected using training (Known training Sequence)

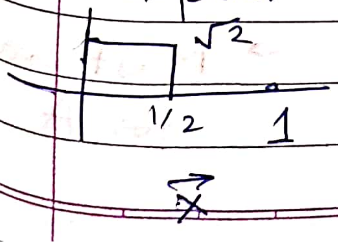
Signal Space

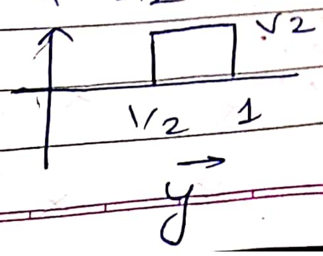




Find $\int_0^1 (s_1(t) - s_2(t))^2 dt$ ← The $\| \|^2$ of their Distance — ~~cos~~ using vectors

Express s_1 & s_2 in the basis





$$s_1 = \begin{bmatrix} \sqrt{2} \\ 0 \end{bmatrix}$$

$$s_2 = \cancel{\begin{bmatrix} \sqrt{2} \\ \sqrt{2} \end{bmatrix}} \quad \begin{bmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \end{bmatrix}$$

Norm

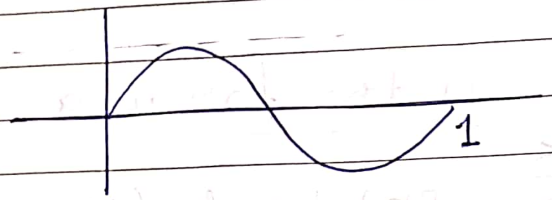
$$\left\| \begin{bmatrix} \sqrt{2} - \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix} \right\| = \cancel{2} + \frac{1}{2} - \cancel{2} + \frac{1}{2}$$

$$= 1$$

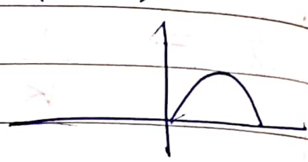
Bands

ex

$s_1(t)$



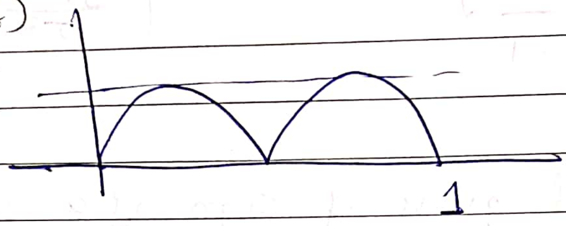
$\psi_1(t)$



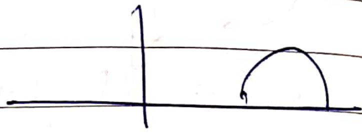
Find,

$$\int_0^1 (s_1(t) - 7 s_2(t))^2 \, dt$$

$s_2(t)$



$\psi_2(t)$



→ Whenever there are a finite number of signals we can do this.

→ Operations are Easier in Signal Space

4.3.19

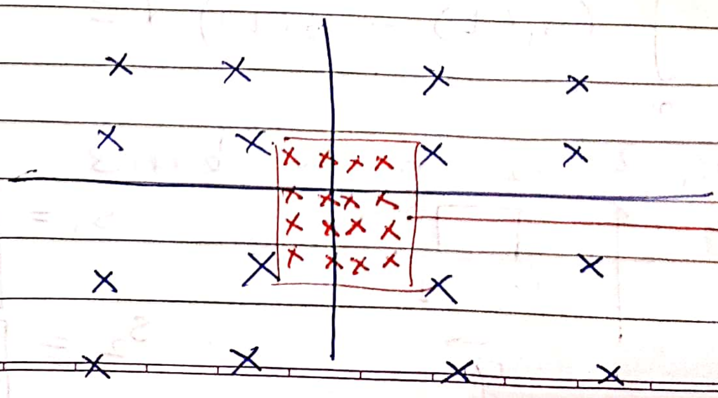
~~Find~~ $\hat{\theta}$ ~~$y(\theta)$~~

→ Recap of ML estimator

Consider QAM-16

$y = As + w$

$A = 0.1$



Must correct for A

ML estimate, $y(\theta)$, find $\hat{\theta}$ that Maximizes $\underset{\theta}{\arg\max}\ p(y(\theta)\,|\,\theta)$

Synchronization Issue

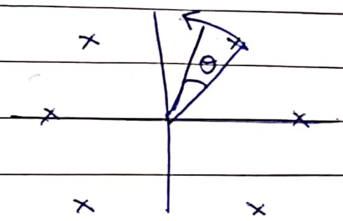
$$y = e^{j\theta} s + w$$

$$\mathrm{Re}\left(s(t)\, e^{j2\pi f_c t} + n(t) \right)$$

$$s_p(t) \longrightarrow \otimes \longrightarrow$$

$$\uparrow$$

$$e^{j(2\pi f_c t + \theta)}$$



→ How to find θ? Use ML estimator

$$\mathrm{Re}\left(\langle y, e^{j\theta} s \rangle \right) - \frac{\|s e^{j\theta}\|^2}{2}$$

$\|s e^{j\theta}\| = \|s\|$ (like a Rotation)

$$= \mathrm{Re}\left(\langle e^{j\theta} s + w, e^{j\theta} s \rangle \right) - \frac{\|S\|^2}{2}$$

Ignore

$$= \frac{\|S\|^2}{2} + \cancel{\mathrm{Re}(\langle w, s \rangle)}\ \underbrace{\mathrm{Re}\left(\langle w, e^{j\theta} s \rangle \right)}_{\theta \text{ dependent}}$$

$$\underset{\theta}{\arg\max}\ \mathrm{Re}\langle y, s e^{j\theta} \rangle = \underset{\theta}{\arg\max}\ \mathrm{Re}(\langle w, e^{j\theta} s \rangle)$$

$$\langle y, s e^{j\theta} \rangle = Z_c + j Z_s$$

$$\min \tan^{-1}\left(\frac{Z_s}{Z_c} \right) = \underset{\theta}{\arg\max}\ ?$$

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ex $\theta = \frac{\pi}{2}$, QPSK $1, -1, j, -j$

$y[0] = e^{j\theta} + w[0]$
$y[1] = e^{j\theta} + w[1]$

$Re < se^{j\theta} + w, se^{j\theta} >$

$= ||s||^2 + Re < w, se^{j\theta} >$

$\rightarrow$ Not very useful since ~~w~~ w is not known, y is

→ Sufficient Statistic

$b^*[0]\, y[0] + b^*[1]\, y[1] + \dots + b^*[N-1]\, y[N-1]$

Delay estimation

$y(t) = s(t - \tau) + w(t)$

→ Find τ that Maximizes ?

• When No delay,

$$\hat{i} = \underset{i}{\operatorname{argmax}} \int y(t)\, \phi_i(t)\, dt$$

Now take General Parameter τ

$$\underset{i,\tau}{\operatorname{argmax}} \int y(t)\, \phi_i(t-\tau)\, dt$$

5.3.

Basis functions of

$$\begin{bmatrix} \int y(t)\,\phi_1(t-\tau)\,dt \\ \vdots \\ \int y(t)\,\phi_k(t-\tau)\,dt \end{bmatrix} = \begin{bmatrix} r_1 \\ \vdots \\ r_k \end{bmatrix}$$

find τ which makes this closest to $\vec{s_i}$

5.B.1

- Assume No frequency offset → Get time offset τ

$$y(t) = A\,s(t-\tau)\,e^{j\theta} + n(t)$$

→ We can't use signal space picture here.

$$y(t) = A\,s(t-\tau)\,e^{j\theta} + n(t)$$

unknowns : $\underbrace{\tau, \theta, A}_{r}$ $\quad P(y \mid r)$

→ Assume we know which symbol $s(t)$ was sent.

Scaling + Phase Offset + Time offset

- To get time shift : Correlate with template
(OR) convolve with Matched filter

$A > 0$, Real

$$L(y|r) = \cancel{\exp\left\{\frac{1}{\sigma^2}\left(Re\langle y, s_r\rangle\right.\right.}$$

$$\exp\left(\frac{1}{\sigma^2}\left(Re\langle y, s_r\rangle - \frac{\|s_r\|^2}{2}\right)\right)$$

(OR)

Simply Maximize

$$\max\left(\underbrace{Re\langle y, s_r\rangle} - \frac{\|s_r\|^2}{2}\right)$$

s "Gamma"

$\cancel{\langle y, s_r\rangle}$,

where,

$$s_r(t) = A' s'(t-\tau) e^{j\theta'}$$

Ans $\cancel{\langle y, s\rangle}$: $((A', \theta', \tau') = (A, \theta, \tau))$

Proof

$$\langle y, s_r\rangle = \int y(t) A' e^{-j\theta'} s^*(t-\tau') \, dt$$

$$A' e^{-j\theta'} \int y(t) s^*(t-\tau') \, dt$$

$$= A' e^{-j\theta'} (y * s_{MF})(\tau')$$

Optimize,

$$\mathrm{Re}\left(A' e^{-j\theta'} \int y(t)\, s^*(t-\tau')\, dt\right) - A'^2 \frac{\|s\|^2}{2}$$

$$= J(\theta', \tau', A')$$

↳ negative cost function, Maximize this

Maximize J over θ

$$\mathrm{Re}\left(\cancel{A'} e^{-j\theta'} \int_{-\infty}^{\infty} y(t) \underbrace{s^*(t-\tau')\, dt}\right)$$

Assume some τ'

$$\Rightarrow \theta' = \arg\left\{ \int y(t)\, s^*(t-\tau')\, d\tau \right\}$$

$$J(\tau, A) = A' \left| (y * s_{MF})(\tau)' \right| - A'^2 \frac{\|s\|^2}{2}$$

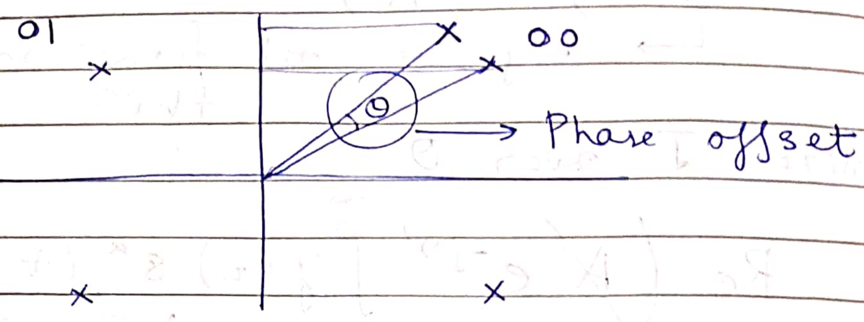
$$\frac{\delta J}{\delta A'} = 0 = \left|(y * s_{MF})(\tau')\right| - A' \|s\|^2 = 0$$

$$A' = \frac{\left|(y * s_{MF})(\tau')\right|}{\|s\|^2}$$

7.3.19

→ Why does the likelihood function take the form

$$y = e^{j\theta} + w$$



→ Effect of Clock offsets (10 - 100 ppm)

$$\frac{10}{10^6} \times 2.4 \times 10^9 \text{ Hz}$$

$$\Delta f = \boxed{24\text{ KHz}}$$

Must compensate for this amount of Δf

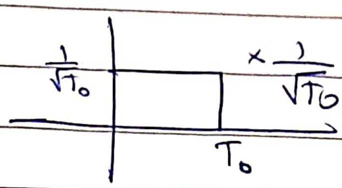
→ How to Determine frequency offset

- Unlike Amplitude/Phase 1 sample is not enough

Observe over the window $[0, T_0]$

$$s(t) = \text{rect}\left(\frac{t - T_0/2}{T_0}\right)$$

$$\|s^2\| =$$



$$\times \frac{1}{\sqrt{T_0}} = \underline{\underline{T_0}}$$

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Date: Plenty mistake.

$$L(y|\theta) = \exp\left(\frac{1}{\sigma^2}\left(\mathrm{Re}\langle y, e^{-j\theta}\rangle - \frac{T_0}{2}\right)\right)$$

$$\theta = \theta(t) : \theta(t)$$

$s(t)$ [Received waveform] =

$$\mathrm{rect}\left(\frac{t - T_0/2}{T_0}\right) e^{j\theta(t)}$$

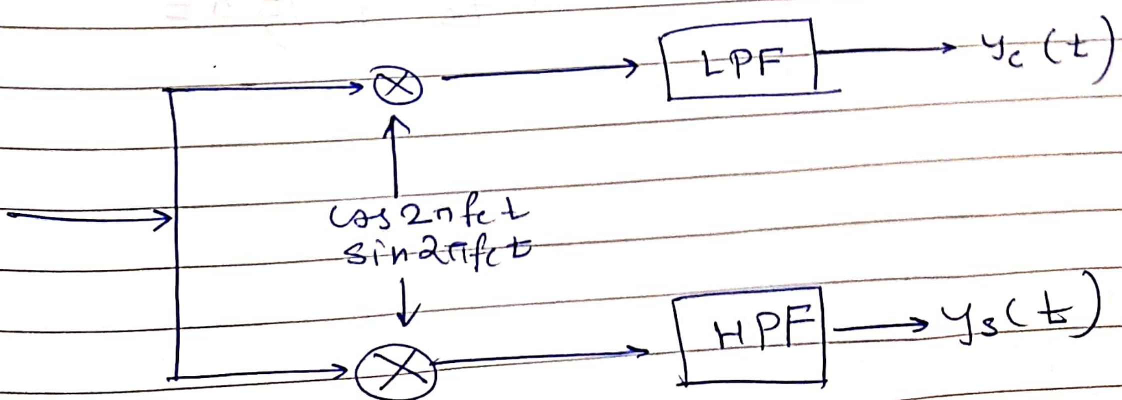
$$y(t) = s(t) + w(t) \qquad 0 \le t \le T_0$$

$$\mathrm{Re}\langle y, s\rangle - \frac{\|s\|^2}{2}$$

$$\mathrm{Re}\left\langle y, \mathrm{rect}\left(\frac{t - T_0/2}{T_0}\right) e^{j\theta(t)}\right\rangle - \frac{T_0}{2}$$

$$J(\theta) = \mathrm{Re}\langle y, e^{j\theta}\rangle = \int_0^{T_0} \left(y_c(t)\cos(\theta(t)) + y_s(t)\sin(\theta(t))\right) dt$$

$$= \int_0^{T_0} \left[y_c(t)\cos(\theta(t)) + y_s(t)\sin(\theta(t))\right] dt$$



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$$\theta(t) = 2\pi(f_c - f_c')t$$

↳ could be ~~as~~ something else (crazy variation)

$$\mathrm{Re}\langle y, e^{j\theta}\rangle = \int_0^{T_0} \left(y_c(t)\cos(\theta(t)) + y_s(t)\sin(\theta(t))\right) dt$$

$$\Big\downarrow \frac{d}{d\theta}$$

$$a\int_0^{T_0} -y_c(t)\sin(\theta(t)) + y_s(t)\cos(\theta(t))\, dt$$

$\boxed{\text{VCO}}$

↳ Use a VCO to perform updations in search for ($\theta(t)$)

ex

Say, $(f_c - f_c') = 1\,\text{Hz}$

$$\int_0^{T_0} \cos(2\pi(f_c - f_c')t)\, dt$$

$$\int_0^{T_0} -\sin(2\pi(f_c - f_c')t)\, dt$$