MOS Transistor (Basics)

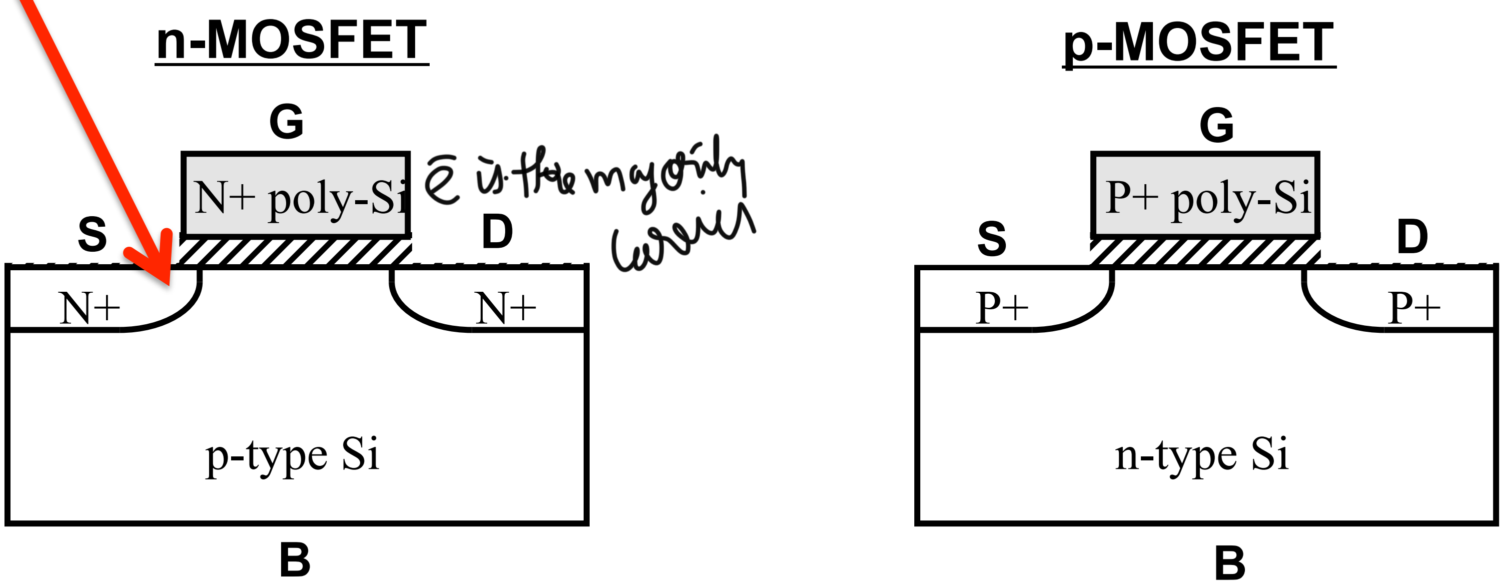
A) Overview
B) Drift + Diffusion model for I-V (Pao-Sah and Brews)
C) Piecewise I-V model − linear and saturation
D) Piecewise I-V model − subthreshold
E) MOSFET parameter extraction
F) MOSFET mobility, split CV method
G) Determination of channel length, series resistance
H) Gate Induced Drain Leakage

Refs: Books by Sze, Taur and Ning, Pierret

MOSFET Schematic

S and D diffusion (minority carrier source / sink) added to MOS capacitor

Note the overlap between gate and S / D diffusion

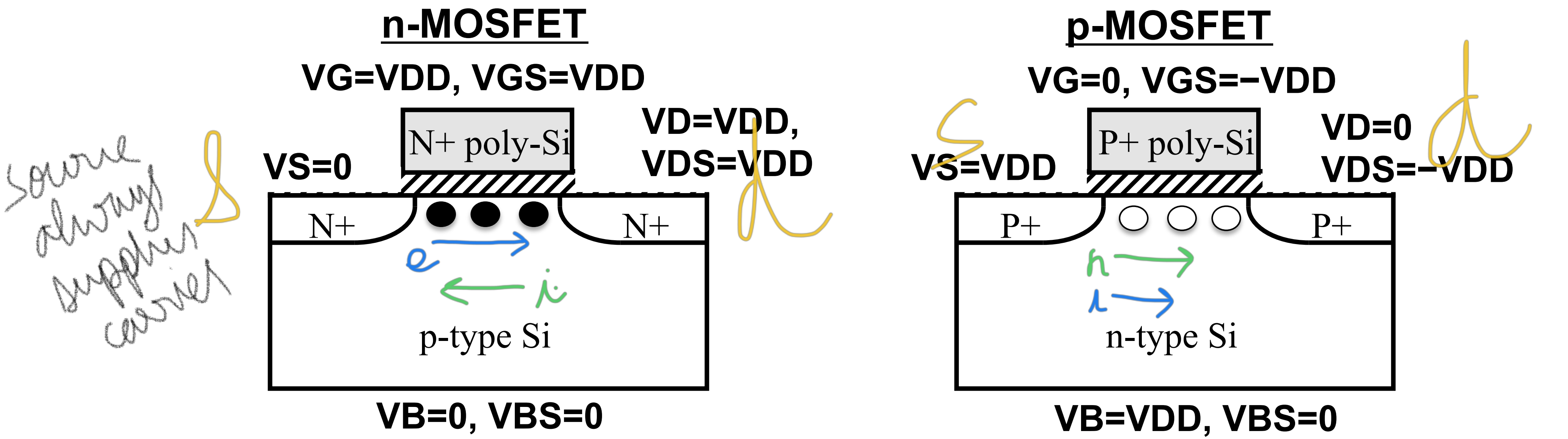


Applied voltages: V_{GS}, V_{DS}, and V_{BS}

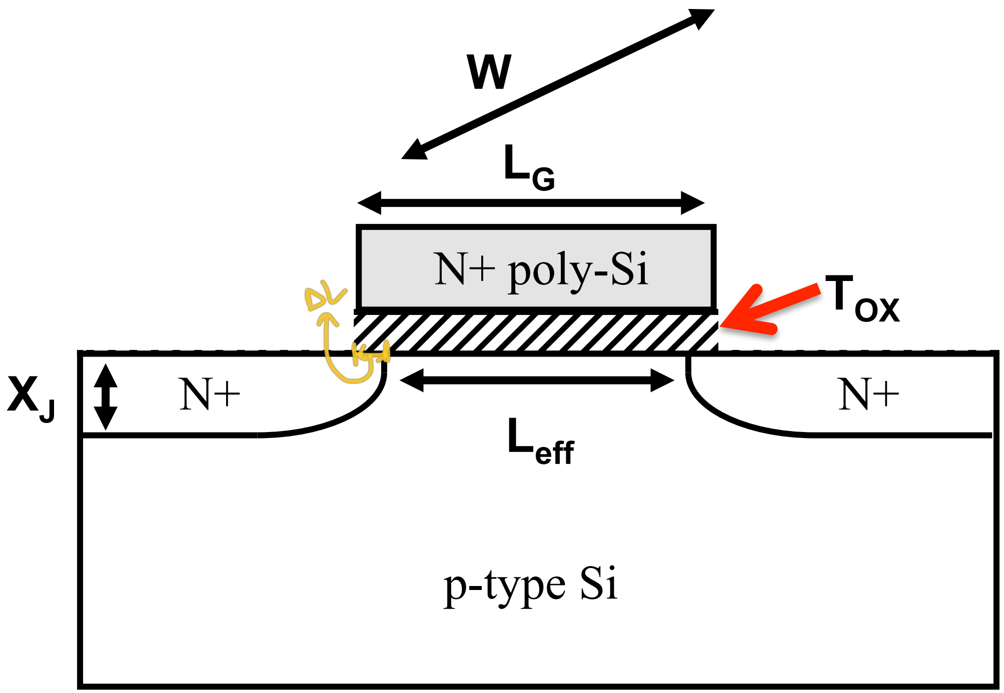
MOSFET Biasing (ON State)

NMOSFET: Electrons flow from S to D, current flows D to S

PMOSFET: Holes from from S to D, same as current flow



Device Structural Parameters



L_G: Gate length

X_J: Junction depth ~ 1/3 L_G

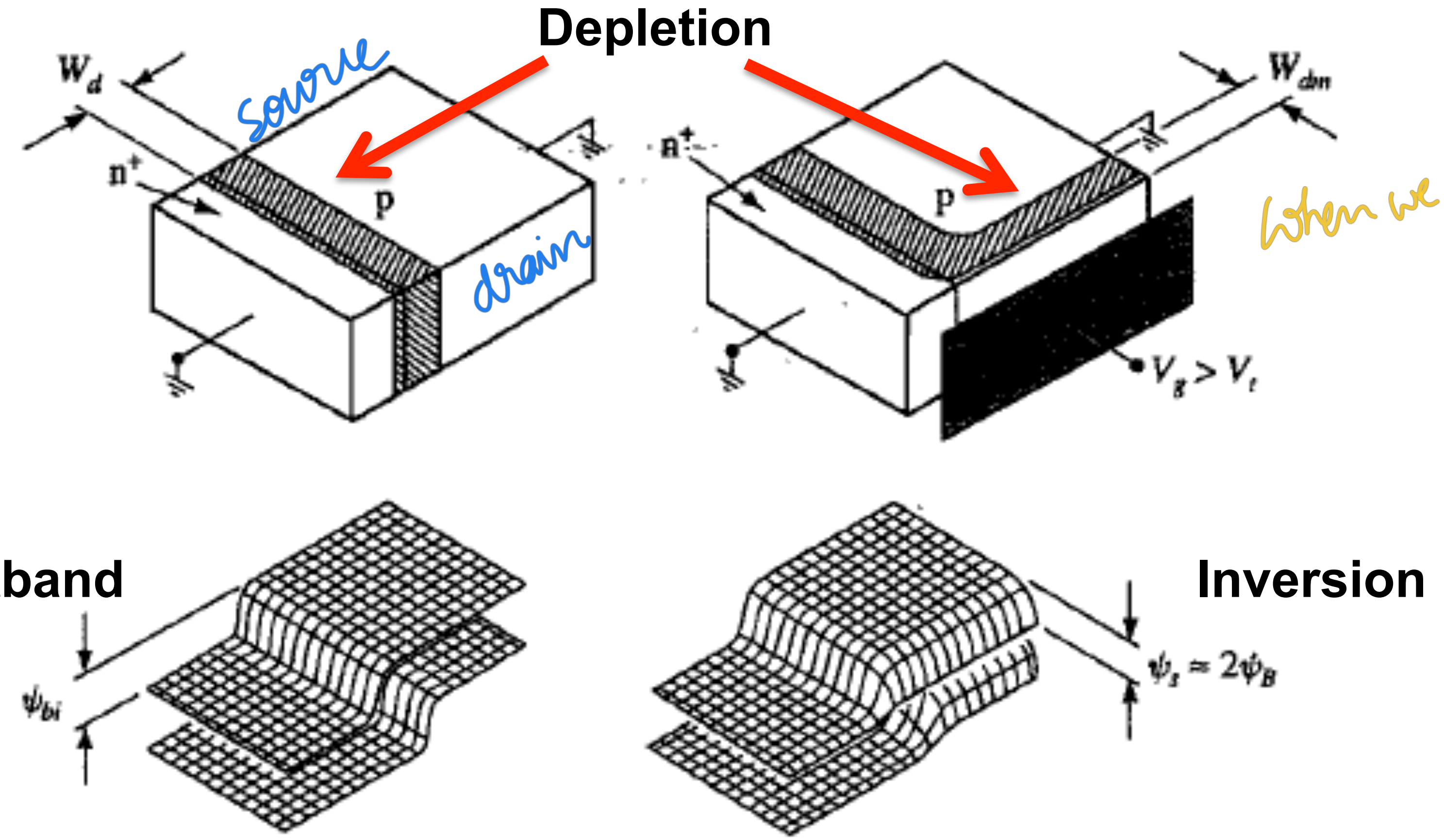
$L_{eff} = L_G - 2\Delta L$, Effective channel length (L_G – lateral diffusion of S/D), ΔL ~1/5 X_J

W: Width

T_{OX}: Oxide thickness

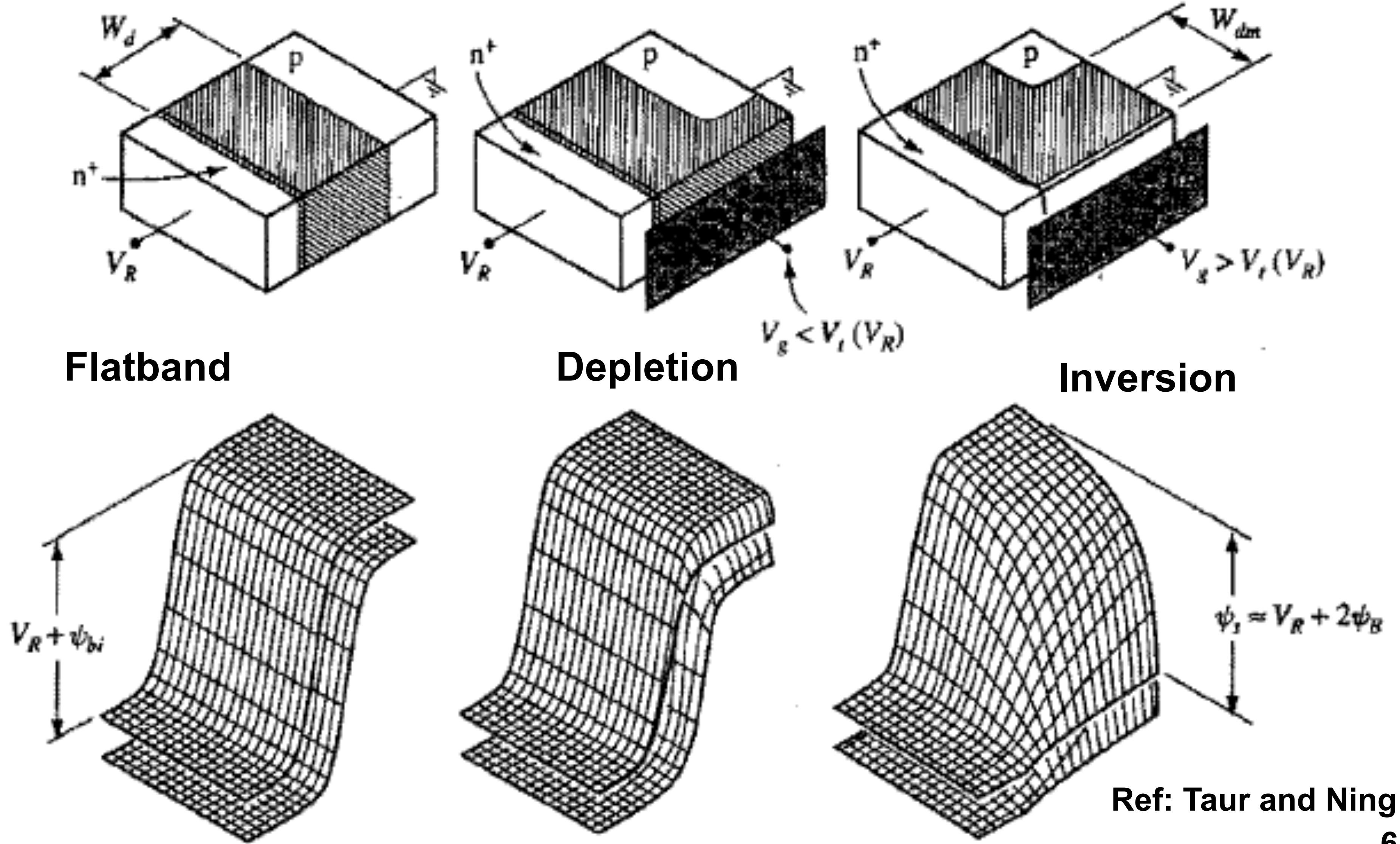
Energy Bands (Gated Diode, V_D=0)

Gated diode: Half of MOSFET (PN+ junction, plus Gate)



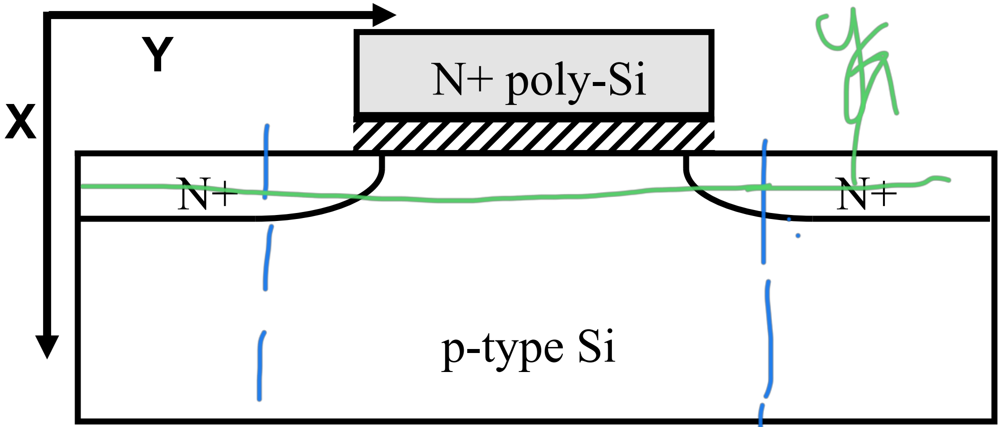
Ref: Taur and Ning

Energy Bands (Gated Diode, V_D Applied)

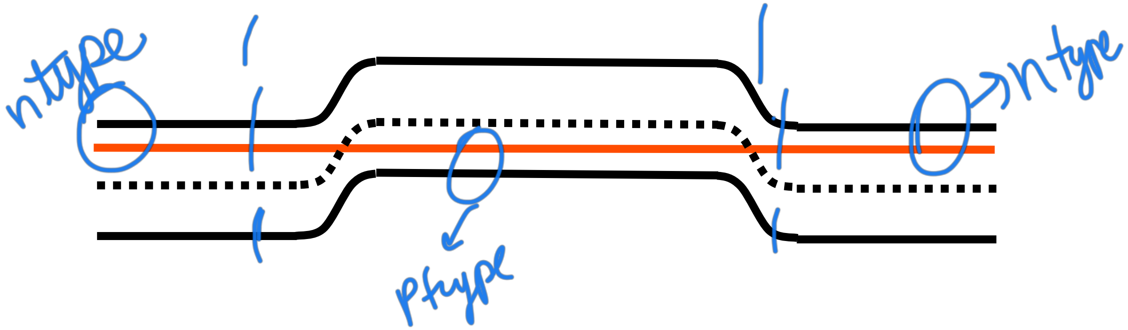


Ref: Taur and Ning

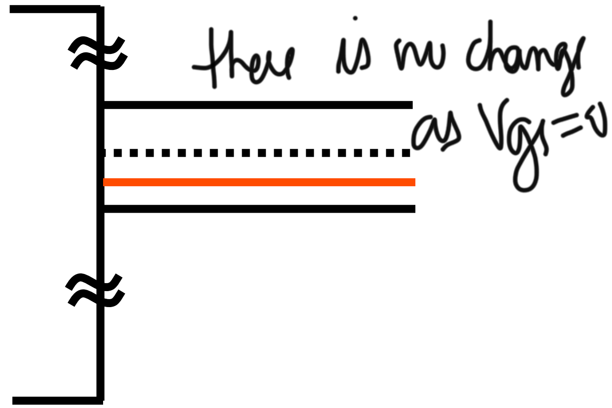
1D Energy Band Diagram ($V_{GS}=V_{FB}$, $V_{DS}=0$)



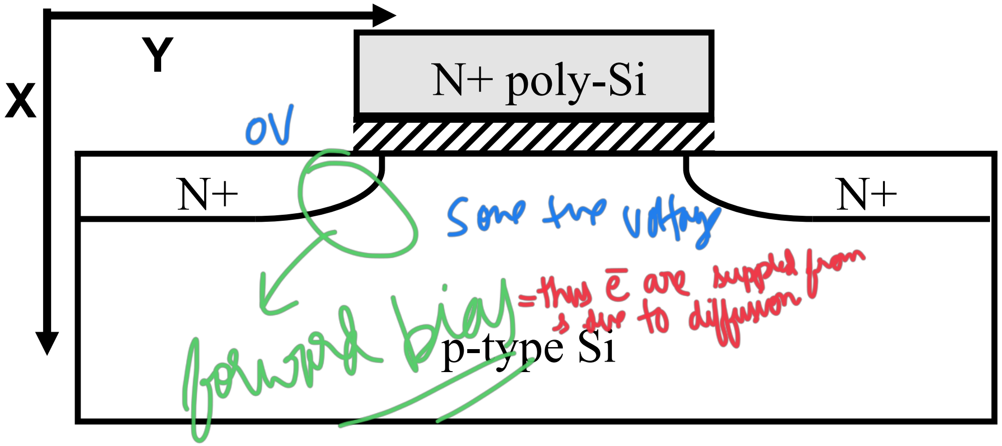
Along Y (S – channel – D)



Along X (channel, interface to bulk)

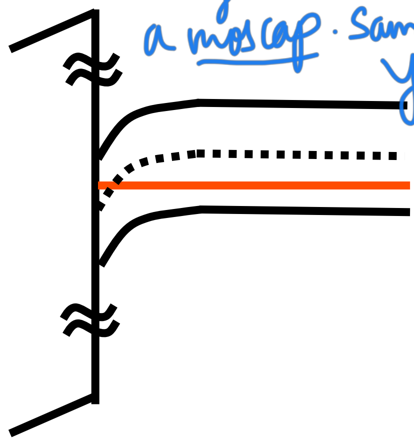


1D Energy Band Diagram (V_{GS}-V_{FB}>0, V_{DS}=0)



Along X (channel, interface to bulk)

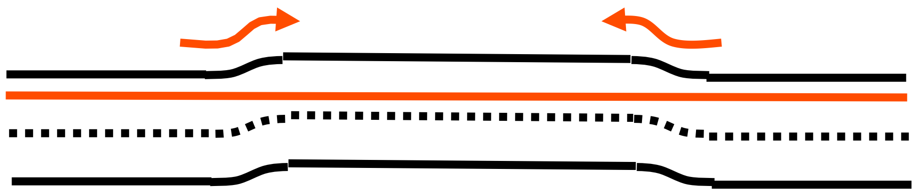
Along x it is just a moscap. Same throughout y direction



Depletion and inversion

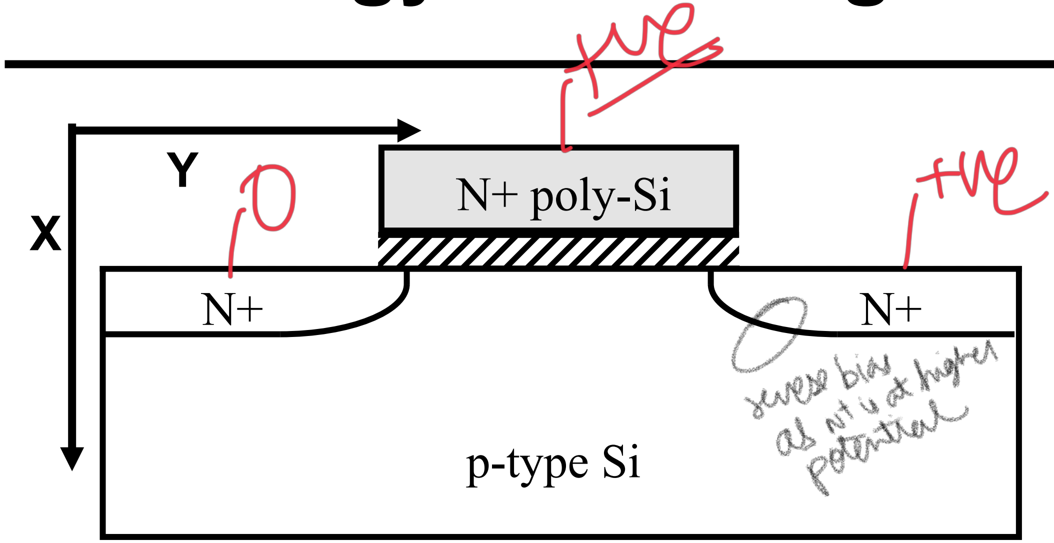
Along Y (S – channel – D)

When we apply Vg then band corresponding to Vg lowers

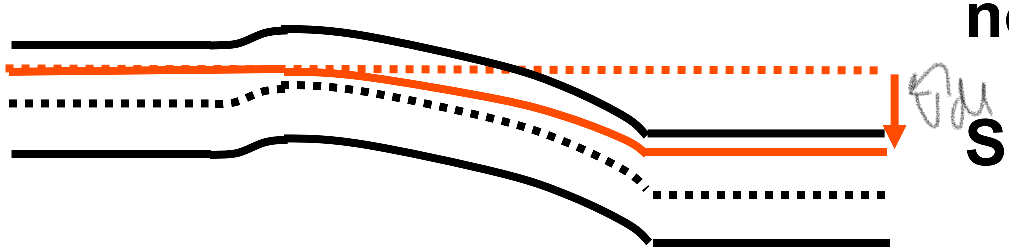


Gate induced lowering of barrier at S / D → Effective forward biasing of S / D junction → Supply of electrons from S / D to channel

1D Energy Band Diagram (V_{GS}-V_{FB}>0, V_{DS}>0)



Along Y (S – channel – D)



Drain voltage drops from D (=V_{DS}) to S (=0)

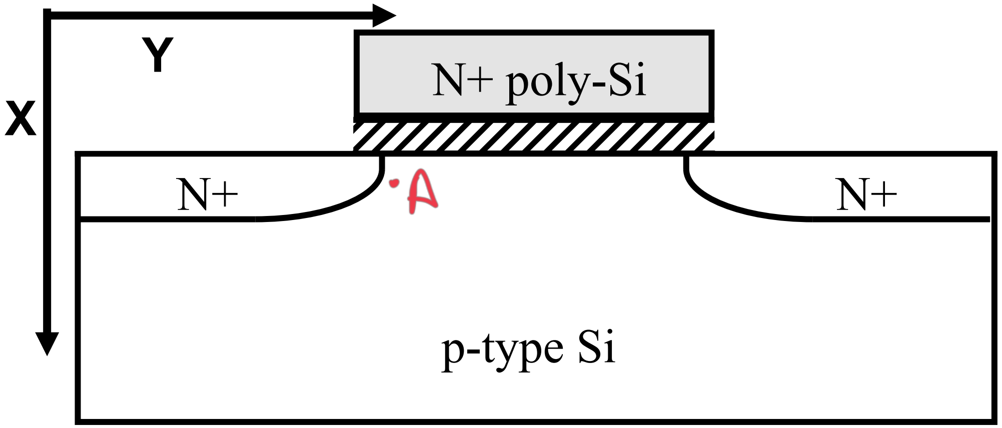
D – channel junction in reverse bias

S – channel junction not affected by V_{DS}

Splitting of Fermi level

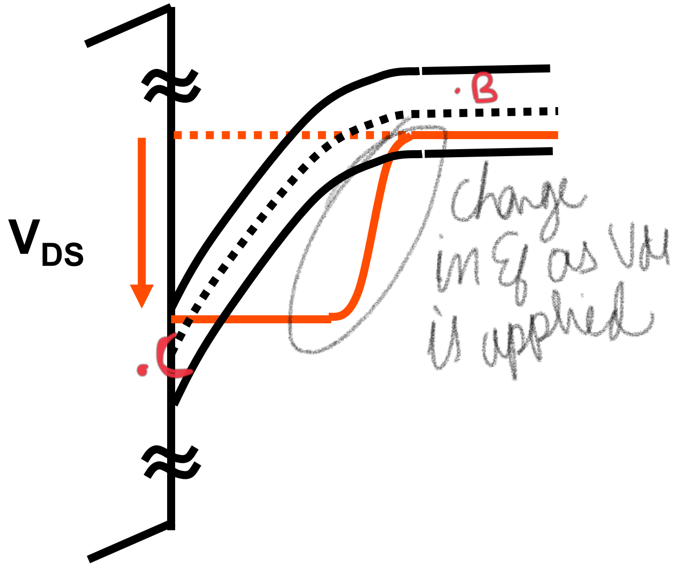
Tilt in band due to lateral (D to S) field → flow of electrons from S to D

1D Energy Band Diagram (V_{GS}-V_{FB}>0, V_{DS}>0)



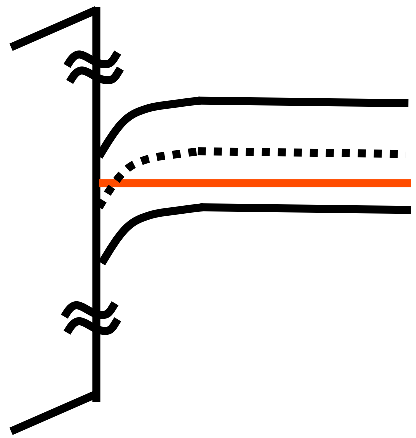
Along X (channel, interface to bulk)

Near Drain



Near Source

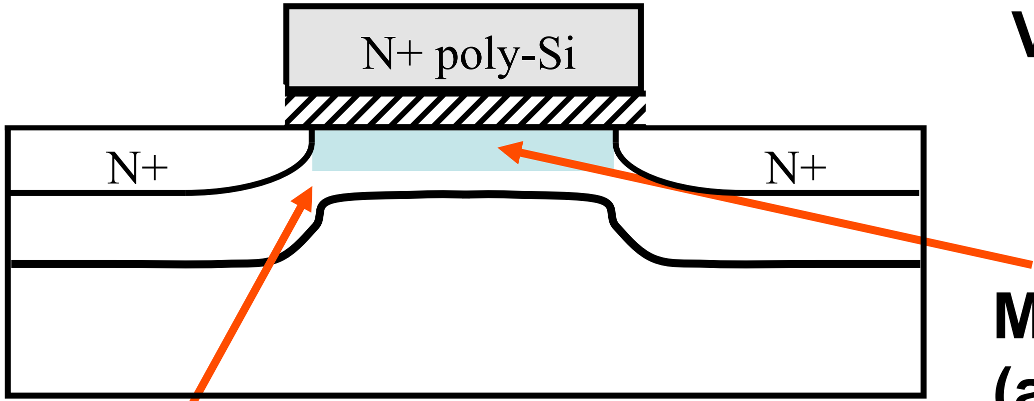
there is not much change near source as v at A same as before



Band bending must be 2ϕ_B over V_{DS} to have inversion at the D side of the channel E_f is lowered by V_{DS} & ψ_s lowered by $V_{DS}+2\phi_F$

At B E_i is ϕ_F above E_f and at C it is ϕ_F below E_f and E_f dropped by V_{ds} so

Charges in MOSFET channel



V_{GS}-V_{FB}>0, V_{DS}=0

Depletion layer (ionized acceptors)

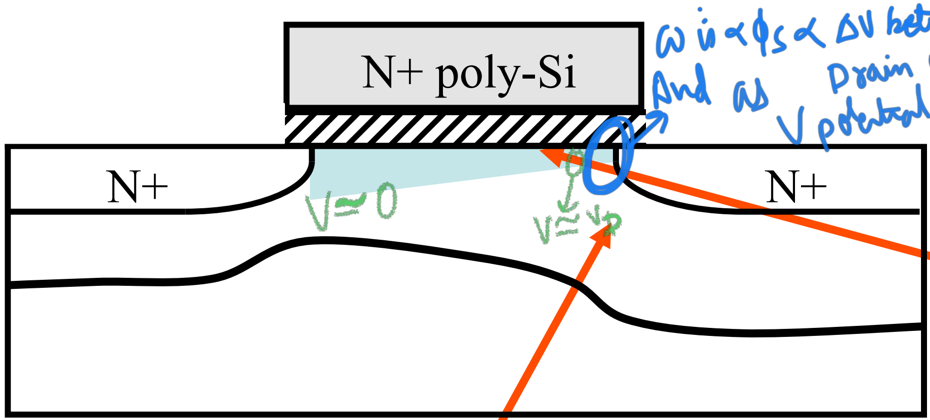
Mobile electrons (always present, number increases exponentially as V_{GS} goes to V_T and linearly for V_{GS}>V_T)

Depletion approximation: Ignores mobile electrons before threshold

Charges in MOSFET channel

$V_{GS}-V_{FB}>0,\ V_{DS}>0$

ω is ∝ ϕ_s ∝ ΔV between metal and semiconductor
And as Drain is at V potential point just below it cannot be of 0 voltage

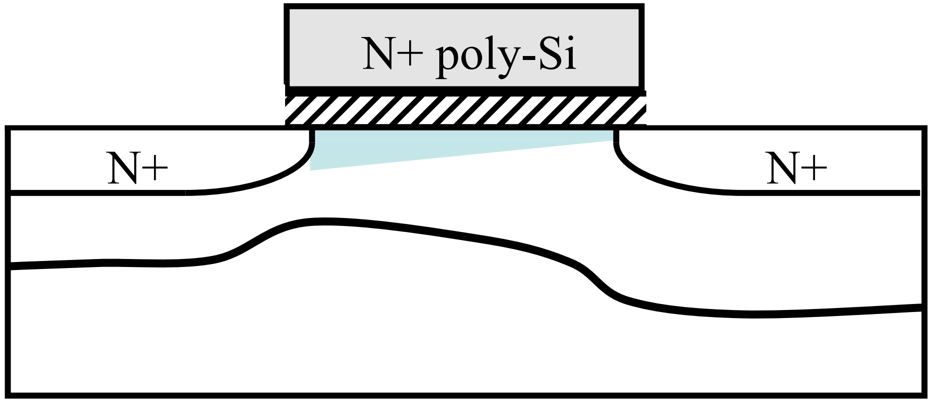


Mobile electrons reduce near the D side as vertical field drops due to increase in surface potential

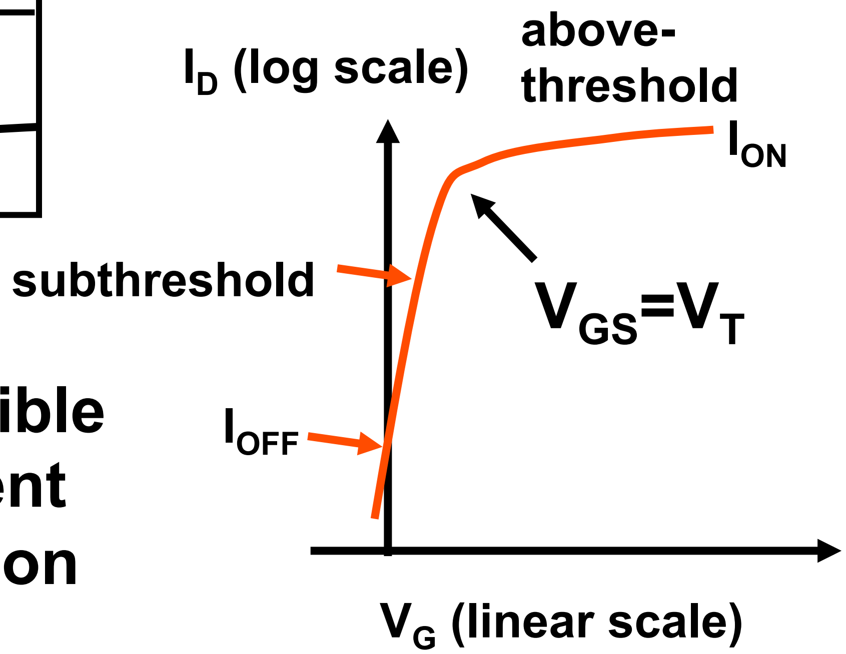
Depletion layer increases near the D side as D junction is reverse biased

Drain current → In general due to drift and diffusion of electrons (from S to D)

Current Conduction (Subthreshold)



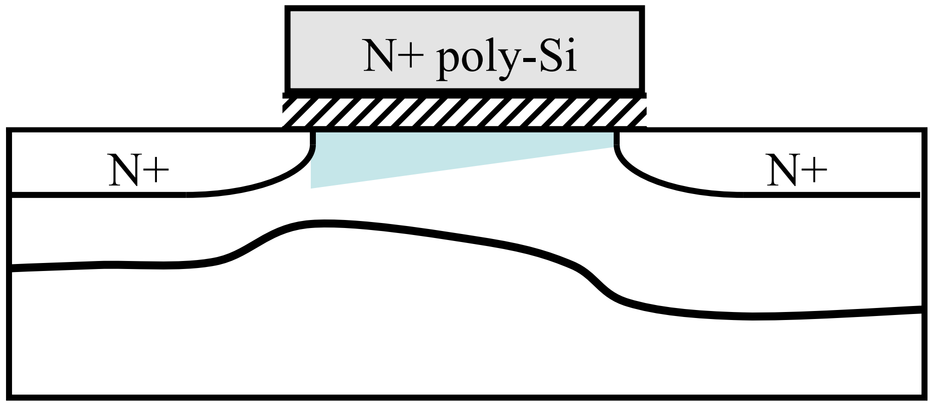
$V_{GS}<V_T$, $V_{DS}>0$



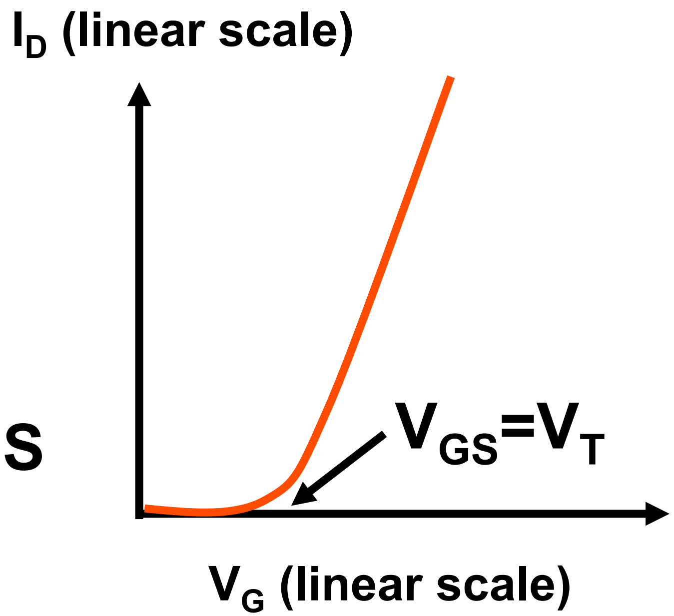
Fewer mobile electrons (negligible drift), but strong density gradient → current dominated by diffusion of electrons form S to D

Exponential dependence on V_{GS} (up to V_T)

Current Conduction (Above-Threshold)



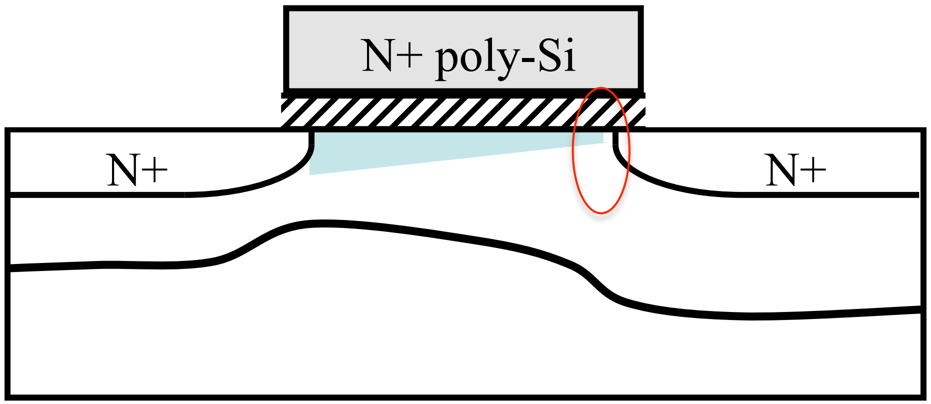
$V_{GS}>V_T$, $V_{DS}>0$



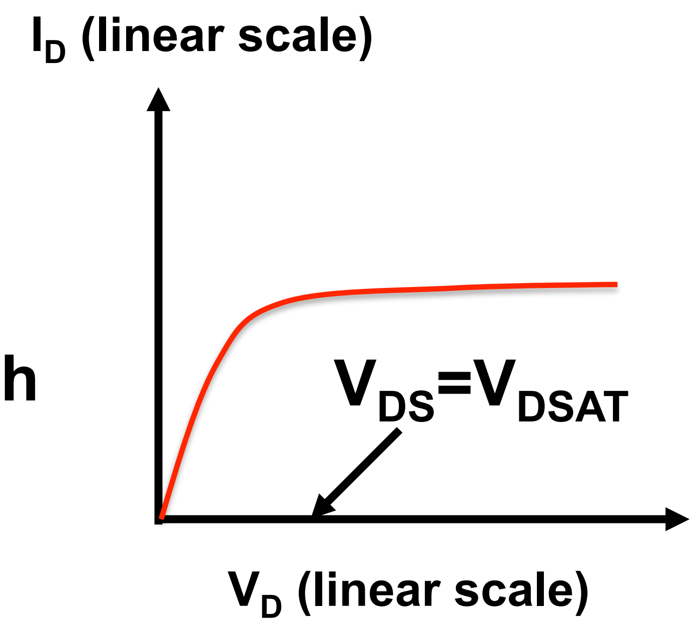
Lots of mobile electrons → current dominated by drift of electrons form S to D (diffusion is present, but negligible)

Linear dependence on V_{GS}

Current Conduction (Above-Threshold)



$V_{GS}>V_T$, $V_{DS}>0$



Linear region: Current increases with VDS

Saturation region: Inversion layer cease to exist near D end, current saturation

Recap: MOS charge density (V_D=0)

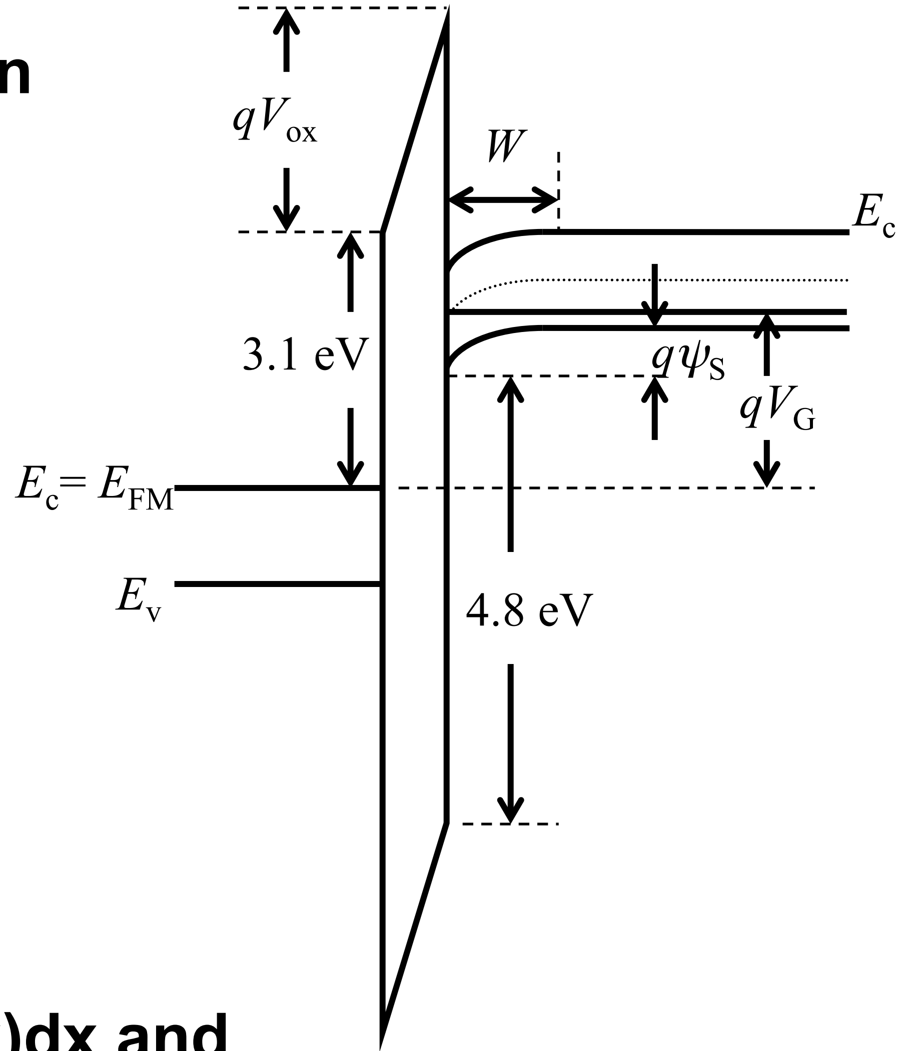
Solution of Poisson equation

$$\frac{d^2\psi}{dx^2} = -\frac{q}{\varepsilon_{si}}[p - n + N_D - N_A]$$

$$p = N_A\,\mathrm{e}^{-q\psi/kT}, \quad n = \frac{n_i^2}{N_A}\mathrm{e}^{q\psi/kT}$$

$$\frac{d^2\psi}{dx^2} = -\frac{q}{\varepsilon_{si}}[N_A(\mathrm{e}^{-q\psi/kT} - 1)$$

$$-\frac{n_i^2}{N_A}(\mathrm{e}^{q\psi/kT} - 1)]$$



Multiply both sides by (dψ/dx)dx and integrate from bulk towards surface

Recap: MOS charge density (V_D=0)

$$\int_0^{d\psi/dx} \frac{d\psi}{dx}\, d\left(\frac{d\psi}{dx}\right) = -\frac{q}{\varepsilon_{si}} \int_0^{\psi} [N_A(\mathrm{e}^{-q\psi/kT} - 1) - \frac{n_i^2}{N_A}(\mathrm{e}^{q\psi/kT} - 1)]\, d\psi$$

$$\left(\frac{d\psi}{dx}\right)^2 = \frac{2kTN_A}{\varepsilon_{si}}[(\mathrm{e}^{-q\psi/kT} + \frac{q\psi}{kT} - 1) + \frac{n_i^2}{N_A^2}(\mathrm{e}^{q\psi/kT} - \frac{q\psi}{kT} - 1)]$$

Applying Gauss law for total substrate charge (use surface potential and field values)

$$Q_S = -\varepsilon_{si}\left(-\frac{d\psi_s}{dx}\right) = \pm\sqrt{2\varepsilon_{si}kTN_A}[(\mathrm{e}^{-q\psi_s/kT} + \frac{q\psi_s}{kT} - 1)$$

$$+\frac{n_i^2}{N_A^2}(\mathrm{e}^{q\psi_s/kT} - \frac{q\psi_s}{kT} - 1)]^{1/2}$$

Ψ_S and $d\psi_S/dx$ are band bending and electric field at oxide/semiconductor interface

MOSFET Charge Density (V_D Applied)

Under V_D, the minority and majority quasi Fermi levels split up, Poisson equation is 2D

ψ_s is the total band bending but in n formula diff between E_f & E_i is taken

$$\frac{d^2\psi(x,y)}{dx^2} + \frac{d^2\psi(x,y)}{dy^2} = -\frac{q}{\varepsilon_{si}}[p - n + N_D - N_A]$$

$$\Rightarrow \frac{d^2\psi(x,y)}{dx^2} = -\frac{q}{\varepsilon_{si}}[p - n + N_D - N_A]$$

Gradual channel approximation, y derivative is small

As both fermilevel and Ei have came down

$$p = N_A \mathrm{e}^{-q\psi/kT}, \quad n = \frac{n_i^2}{N_A}\mathrm{e}^{q(\psi - V)/kT}$$

MOSFET Charge Density (V_D Applied)

Follow same procedure as before

$$\frac{d^2\psi}{dx^2} = -\frac{q}{\varepsilon_{si}}[N_A(\mathrm{e}^{-q\psi/kT} - 1) - \frac{n_i^2}{N_A}(\mathrm{e}^{q(\psi - V)/kT} - 1)]$$

Add & Sub $\frac{q\psi}{kT}$

$$(\frac{d\psi}{dx})^2 = \frac{2kTN_A}{\varepsilon_{si}}[(\mathrm{e}^{-q\psi/kT} + \frac{q\psi}{kT} - 1) + \frac{n_i^2}{N_A^2}\{e^{-qV/kT}(\mathrm{e}^{q\psi/kT} - 1)\} - \frac{q\psi}{kT}]$$

$Q = \varepsilon_0 E$

$= \varepsilon \varepsilon_0 \frac{d\psi'}{dx}$

$$Q_S = \pm\sqrt{2\varepsilon_{si}kTN_A}[(\mathrm{e}^{-q\psi_s/kT} + \frac{q\psi_s}{kT} - 1)$$
$$+ \frac{n_i^2}{N_A^2}\{e^{-qV/kT}(\mathrm{e}^{q\psi_s/kT} - 1)\} - \frac{q\psi_s}{kT}]^{1/2}$$

Field and Charge in Regime of Interest

$$(-\frac{d\psi}{dx}) = \sqrt{\frac{2kTN_A}{\varepsilon_{si}}}[\frac{q\psi}{kT} + \frac{n_i^2}{N_A^2}\mathrm{e}^{q(\psi-V)/kT}]^{1/2}$$

We are only interested in depletion or inversion region

Total charge density in depletion / inversion, includes ionized acceptors + mobile electrons

$$Q_S = -\sqrt{2\varepsilon_{si}kTN_A}[\frac{q\psi_s}{kT} + \frac{n_i^2}{N_A^2}\mathrm{e}^{q(\psi_s-V)/kT}]^{1/2}$$

during inversion second term dominates this q total ……..different from q(y)

Recap – Drift and Diffusion (1D)

$$J_n = qn\mu_n \mathcal{E} + qD_n \frac{dn}{dx}$$ ⇒ drift + diffusion

electric field

$$n = n_i e^{(F_N - E_i)/kT};\ \mathcal{E} = \frac{1}{q}\frac{dE_i}{dx};\ \boxed{\frac{D}{\mu} = \frac{kT}{q}}$$ Einstein eqn

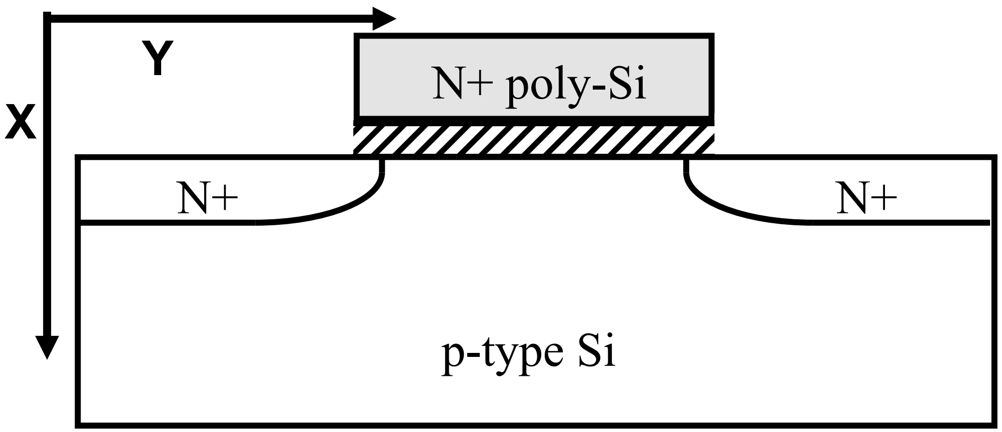
$$\frac{dn}{dx} = \underbrace{n_i e^{[\ ]}}_{n} * [F' - E']$$

$$J_n = n\mu_n \frac{dE_i}{dx} + \cancel{kT}\mu_n \frac{n}{\cancel{kT}}\left(\frac{dF_N}{dx} - \frac{dE_i}{dx}\right)$$

$$\Rightarrow J_n = n\mu_n \frac{dF_N}{dx}$$

Non-zero gradient of Quasi-Fermi level indicates total current transport (drift + diffusion)

MOSFET Current Calculation



W is channel width

Current direction

Split in Fermi level: qV(y)

$$J(x,y) = -q\mu_n n(x,y)\frac{dV(y)}{dy}$$

Integrate over inversion layer thickness

$$I_D(y) = qW\int_o^{x_I} \mu_n n(x,y)\frac{dV(y)}{dy}\,dx$$

MOSFET Current Calculation

Inversion charge density at a given y

$$Q_i(y) = -q\int_o^{x_I} n(x,y)\,dx; \quad \mu_{eff} = \frac{\int_0^{X_I} \mu_n n(x)\,dx}{\int_0^{X_I} n(x)\,dx}$$

Effective mobility, integrate over inversion layer thickness

$$I_D(y) = -\mu_{eff} W Q_i(y)\frac{dV}{dy} = -\mu_{eff} W Q_i(V)\frac{dV}{dy}$$

length

Change of variable

$$\int_0^L I_D(y)\,dy = \mu_{eff} W \int_0^{V_{DS}} [-Q_i(V)\,dV$$

q at a particular y integrated throughout Y

$$\boxed{I_D = \mu_{eff}\frac{W}{L}\int_0^{V_{DS}} [-Q_i(V)\,dV}$$

in amp.

Generic expression, drift and diffusion, need to find Q

MOSFET Current Calculation (Pao-Sah)

$$Q_i(y) = -q\int_o^{x_I} n(x,y)\ dx \Rightarrow Q_i(V) = -q\int_{\psi_S}^{\delta} n(\psi,V)\ \frac{dx}{d\psi} d\psi$$

$$n(x,y) = n(\psi,V) = \frac{n_i^2}{N_A} e^{q(\psi-V)/kT}$$

Finite, small but non-zero potential δ at edge of inversion layer

$$-Q_i(V) = q\int_{\delta}^{\psi_S} \frac{n_i^2 / N_A e^{q(\psi-V)/kT}}{(-d\psi / dx)} d\psi$$

$$(-\frac{d\psi}{dx}) = \sqrt{\frac{2kTN_A}{\varepsilon_{si}}}[\frac{q\psi}{kT} + \frac{n_i^2}{N_A^2} \mathrm{e}^{q(\psi-V)/kT}]^{1/2}$$

Keeping only relevant terms

gradual channel approximation

MOSFET Current Calculation (Pao-Sah)

$$I_D = q\mu_{eff}\frac{W}{L}\int_0^{V_{DS}}\left(\int_\delta^{\psi_S}\frac{n_i^2/N_A e^{q(\psi-V)/kT}}{(-d\psi/dx)}d\psi\right)dV$$

$$(-\frac{d\psi}{dx}) = \sqrt{\frac{2kTN_A}{\varepsilon_{si}}}\left[\frac{q\psi}{kT}+\frac{n_i^2}{N_A^2}\mathrm{e}^{q(\psi-V)/kT}\right]^{1/2}$$

$$V_G = V_{FB} + \psi_s + \frac{\sqrt{2\varepsilon_{si}kTN_A}}{C_{OX}}\left[\frac{q\psi_s}{kT}+\frac{n_i^2}{N_A^2}\mathrm{e}^{q(\psi_s-V)/kT}\right]^{1/2}$$

Numerical solution, valid for below and above threshold

Find Ψ_S at S and D ends for different V_G, V_D using this expression

MOSFET Current Calculation (Brews)

thin invertion layer approximation

$$I_D = \mu_{eff} \frac{W}{L} \int_0^{V_{DS}} [-Q_i(V) dV = \mu_{eff} \frac{W}{L} \int_{\psi_{SS}}^{\psi_{SD}} [-Q_i(\psi_s)] \frac{dV}{d\psi_s} d\psi_s$$

$$V_G - V_{FB} - \psi_s = \frac{-Q_s}{C_{OX}} = \frac{-(Q_i + Q_d)}{C_{OX}}$$

$$-Q_i = C_{OX}(V_G - V_{FB} - \psi_s) - \sqrt{2\varepsilon_{Si} q N_A \psi_s}$$

Surface potential at Source and Drain end of channel, obtain for a given V_G, V_D

Inversion layer has zero thickness

$$V_G = V_{FB} + \psi_s + \frac{\sqrt{2\varepsilon_{si} kTN_A}}{C_{OX}} [\frac{q\psi_s}{kT} + \frac{n_i^2}{N_A^2} e^{q(\psi_s - V)/kT}]^{1/2}$$

V=0 at Source and V=V_D at Drain

gradual channel approximation to find q_total

MOSFET Current Calculation (Brews)

$$V_G = V_{FB} + \psi_s + \frac{\sqrt{2\varepsilon_{si} kTN_A}}{C_{OX}} [\frac{q\psi_s}{kT} + \frac{n_i^2}{N_A^2} \mathrm{e}^{q(\psi_s - V)/kT}]^{1/2}$$

$$V = \psi_s - \frac{kT}{q} \ln\left[\frac{N_A^2}{n_i^2}\left\{\frac{C_{\mathrm{OX}}^2 \left(V_G - V_{FB} - \psi_s\right)^2}{2\varepsilon_{si} kTN_A} - \frac{q\psi_s}{kT}\right\}\right]$$

$$\frac{dV}{d\psi_s} = 1 + \frac{2kT}{q} \frac{C_{\mathrm{OX}}^2 \left(V_G - V_{FB} - \psi_s\right) + \varepsilon_{si} qN_A}{C_{\mathrm{OX}}^2 \left(V_G - V_{FB} - \psi_s\right)^2 - 2\varepsilon_{si} qN_A \psi_s}$$

Calculation of derivative, hint: d(lnf(x))/dx=1/f(x).df(x)/dx

MOSFET Current Calculation (Brews)

$$I_D = \mu_{eff} \frac{W}{L} \int_{\psi_{SS}}^{\psi_{SD}} [-Q_i(\psi_s)] \frac{dV}{d\psi_s} d\psi_s$$

$$= \mu_{eff} \frac{W}{L} \int_{\psi_{SS}}^{\psi_{SD}} \left[\begin{array}{l} C_{OX}(V_G - V_{FB} - \psi_s) - \sqrt{2\varepsilon_{Si} q N_A \psi_s} \\ + \dfrac{2kT}{q} \dfrac{C_{\mathrm{OX}}^2 \left(V_G - V_{FB} - \psi_s\right) + \varepsilon_{si} q N_A}{C_{\mathrm{OX}} \left(V_G - V_{FB} - \psi_s\right) + \sqrt{2\varepsilon_{si} q N_A \psi_s}} \end{array} \right] d\psi_s$$

Unlike Pao-Sah model, here only a single integral needs to be evaluated (numerically), but valid for below and above threshold