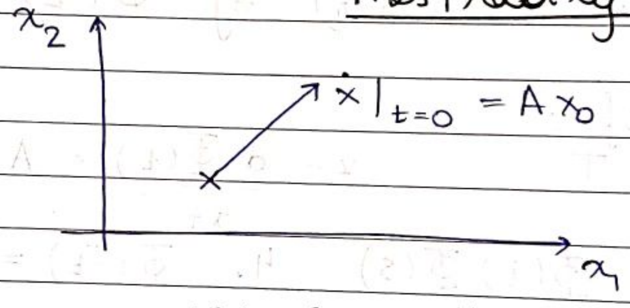
$$\mathcal{L}^{-1}\left(\frac{\begin{bmatrix}2 & 1\\ -6 & -3\end{bmatrix}}{(s+2)} + \frac{\begin{bmatrix}3 & 1\\ -6 & -2\end{bmatrix}}{(s+3)}\right)$$

$$= e^{-2t}\begin{bmatrix}2 & 1\\ -6 & 3\end{bmatrix} + e^{-3t}\begin{bmatrix}3 & 1\\ -6 & -2\end{bmatrix}$$

$$\underline{\Phi(t)} = \begin{bmatrix}2e^{-2t} + 3e^{-3t} & e^{-2t} + e^{-3t}\\ -6(e^{-2t} + e^{-3t}) & 3e^{-2t} - 2e^{-3t}\end{bmatrix}$$

<u>Abstractly</u>

$x^{\circ} = Ax$



→ e^{A} — Matrix exponential form (A: n×n square Matrix)

$$e^{A} = I + A + \frac{1}{2!}A^2 + \frac{1}{3!}A^3 + \dots\dots$$

→ M (Should converge)

M_n = partial sum $= I + A + \frac{1}{2!}A^2 + \dots + \frac{1}{n!}A$

$\|M\|$ → ~~mod of~~ $\max\limits_{i,j} |m_{ij}|$

where $M = [m_{ij}]$

<u>Correction</u>

$\|M\|_{n\times n} = \max\limits_{1\le i\le n} \sum\limits_{1\le j\le n} |m_{ij}|$ The Maximum sum of mod of elements in a row

Using this Norm e^{At} exists for any t

→ Triangle Inequality holds for this notion of Modulus, hence $||M||$ can be used as a distance from the $[0]$ mat

$$||A+B|| \leq ||A|| + ||B||$$

$$||M_n|| \leq 1 + ||A|| + \frac{1}{2!}||A||^2 + \frac{1}{3!}||A||^3 + \cdots + \frac{1}{n!}||A||^n$$

Since, $||AB|| \leq ||A|| \cdot ||B||$

$$\Rightarrow ||A^2|| \leq ||A||^2$$

$$\lim_{n\to\infty} ||M_n|| \leq 1 + ||A|| + \frac{1}{2}||A||^2 + \cdots$$

$$= e^{||A||}$$

$$\Phi(t) = e^{At} : \text{~~Hence proved~~}$$

~~Correction in these~~

24.1.19

State Transition Matrix (STM)

→ General solution of the homogenous eqn.

$$\dot{x} = Ax \qquad x(0) = x_0$$

in terms of operators on x_0

STM is a matrix $\Phi(t)$: $\underbrace{x(t) = \Phi(t)x_0}$

Once initial conditions are given, $\Phi(t)$ is unique: Existence - Uniqueness theorem

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$$x_0 \xrightarrow{\Phi(t)} X(t)$$

$$s\hat{X} = x_0 + A\hat{X}$$

$$X(t) = \Phi(t)\, x_0 = \left(\mathcal{L}^{-1}(sI-A)^{-1}\right) x_0$$

— valid for any x_0 hence,

$$\Phi(t) = \mathcal{L}^{-1}(sI-A)^{-1}$$

→ How to find this solution?

$$\Phi(t) = \mathcal{L}^{-1}(sI-A)^{-1}$$

$\Phi(t)$ is the unique solution $Z(t)$ of the matrix equation.

$$\dot{Z} = AZ \qquad Z(0) = I$$

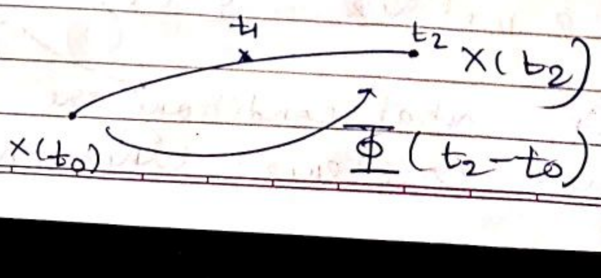
$$Z(t) = I + At + \frac{1}{2}A^2t^2 + \frac{1}{3!}A^3t^3 + \dots = M$$

$$\frac{d}{dt}M(t) = A(\dots\dots) = AM(t)$$

$$\dot{x} = Ax \qquad x_0 = X(0) = x(t_0) \quad \text{Initial time}$$

$$X(t) = \Phi(t)\, x_0$$

$$X(t) = \Phi(t-t_0)\, x_0$$



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$$X(t_2) = \Phi\big((t_2 - t_1) + (t_1 - t_0)\big)\, X(t_0)$$
$$= \Phi(t_2 - t_1)\, X(t_1)$$
$$= \Phi(t_2 - t_1)\, \Phi(t_1 - t_0)\, X(t_0)$$

→ Since this state transition relation holds for any x_0

$$\Phi(t_2 - t_0)\, x_0 = \Phi(t_2 - t_1)\, \Phi(t_1 - t_0)\, x_0$$

$$\Leftrightarrow \quad \Phi(t_2 - t_0) = \Phi(t_2 - t_1)\, \Phi(t_1 - t_0)$$

In General,

$$\Phi(t+s) = \Phi(t) \times \Phi(s)$$

So,

$$\Phi(t) = e^{At}$$

$$e^{A(t+s)} = e^{At} \cdot e^{As}$$

↳ But how is this correct from power series convergence point of view

Theorem : $e^{A+B} = e^A \cdot e^B$
iff $AB = BA$

ex $e^{A(t+s)} = e^{At+As} = e^{At} \cdot e^{As}$

Properties of STM $\Phi(t)$

1. $\Phi(0) = I$
2. $\Phi(t+s) = \Phi(t) \cdot \Phi(s)$
3. $\Phi(-t) = [\Phi(t)]^{-1}$

...$=M(t)$

time

4. $\det \Phi(t) \neq 0 \quad \forall t$

5. $\frac{d}{dt} \Phi(t) = A \Phi(t)$

6. $\det \Phi(t) = e^{(\mathrm{tr} A) t}$

Effect of Transformations

$\dot{x} = Ax \qquad x_0 = x(t_0)$

$x = Ty \Rightarrow y = T^{-1} x$

T is a non- ~~sig~~ singular transformatn.

$\frac{d}{dt}(Ty) = A(Ty)$ Since $x = Ty$

$T\dot{y} = (AT) y$

$\dot{y} = (T^{-1}AT) y$

If, ~~B~~

$B = (T^{-1}AT)$

What is $\psi(t) = \mathcal{L}^{-1}(sI - B)^{-1}$

given $\Phi(t) = \mathcal{L}^{-1}(sI - A)^{-1}$

~~$\psi(t) = T^{-1}\Phi(t)\,T$~~ ??

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Method 1)

$$\mathcal{L}^{-1}(sI - B)^{-1}$$

$$B = \overset{\phi}{(T^{-1}AT)}$$

$$s(T^{-1}T) - T^{-1}AT$$

$$\mathcal{L}^{-1}\left(T^{-1}(sT - AT)^{-1}\right)$$

$$= \mathcal{L}^{-1}\left(T^{-1}(sI-A)\,T\right)$$

$$\psi(t) = T^{-1}\left[\mathcal{L}^{-1}(sI-A)\right]T$$

$$T\,\psi(t)\,T^{-1} = \Phi(t)$$

mation.

$$\boxed{\psi(t) = T^{-1}\,\phi(t)\,T}$$

→ More Math,

$$\dot{x} = Ax + Bu$$

$$\dot{y} = Cx + Du$$

$$x(t) = \Phi(t)\,x_0 + \int_{t_0}^{t} \Phi(t-\tau)\,B\,u(\tau)\,d\tau$$

$$y(t) = c\,\Phi(t)\,x_0 + \int_{t_0}^{t} C\,\Phi(t-\tau)\,B\,u(\tau)\,d\tau + D\,u(t)$$

→ Transfer function Matrix

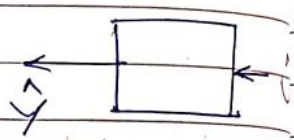
$$T(s) = C(sI-A)^{-1}B + D$$

$$Y(s) = T(s)\,U(s)$$

$$\mathcal{L}^{-1}(T(s)) = C\phi(t)B$$

→ State space from transfer function

1. $$\frac{\hat{Y}}{\hat{U}} = \frac{b_0}{s^2 + a_1 s + a_2}$$



$$\ddot{y} + a_1\dot{y} + a_2 y = b_0 u$$

state variables,

$$x_1 = y \qquad x_2 = \dot{y} \qquad y = x_1$$

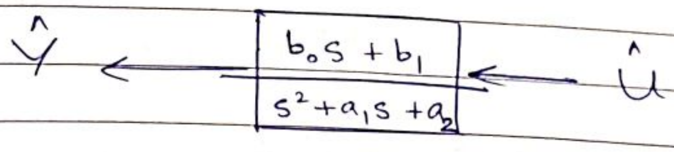
$$\dot{x}_2 = -a_2 x_1 - a_1 x_2 + b_0 u$$

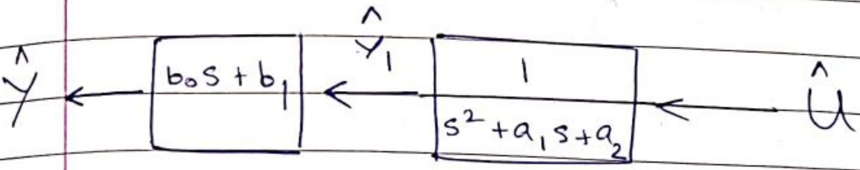
$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -a_2 & -a_1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ b_0 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\frac{b_0}{s^2 + a_1 s + a_2} \longleftrightarrow \left[\begin{array}{cc|c} 0 & 1 & 0 \\ -a_2 & -a_1 & b_0 \\ \hline 1 & 0 & 0 \end{array}\right]$$

2. $$\frac{\hat{Y}}{\hat{U}} = \frac{b_0 s + b_1}{s^2 + a_1 s + a_2}$$





State Space Representation

$x_1 = y_1 \qquad x_2 = \dot{x}_1$

$\dot{x}_1 = x_2 \qquad \dot{x}_2 = \cancel{a x_2} - a_2 x_1 - a_1 x_2 + u$

$y_1 = x_1$

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Recap: State Space Model

MIMO (Multi-Input, Multi-output)

$\dot{x} = Ax + b_1 u_1 + b_2 u_2$ — ~~single input u~~

$y_1 = c_1 x + d_1 u_1 + d_2 u_1$ — ~~single output y~~

$y_2 = c_2 x + d_3 u_1 + d_4 u_2$ — ~~?~~

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du \iff \left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right]$$

For single Input - Single output (SISO)
Multiple Multiple (MIMO)

→ $B, C \rightarrow$ vector, $D \rightarrow$ scalar.

SISO State space Model

$$\left[\begin{array}{c|c} A & \vec{b} \\ \hline \vec{c} & d \end{array}\right]$$

$$\frac{\hat{Y}}{\hat{U}} = c(sI - A)^{-1} b + d = T(s)$$

Trip to GMRT Narangaon

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→ MIMO transfer Matrix

$$T(s) = C(sI - A)^{-1}B + D$$

$$\hat{Y} = T(s)\,\hat{U}$$

Since, $\frac{\hat{Y}}{\hat{U}}$ does not make sense

→ Home-paper Assignment

• Write-up on a control System

ex Line of Sight Clarity Control System

ex $\frac{\hat{Y}}{\hat{U}} = \frac{b_0}{s^2 + a_1 s + a_2}$ $\ddot{y} + a_1\dot{y} + a_2 y = b_0 u$

$x_1 = y_1$ $x_2 = \dot{x}_1$

$\dot{x}_2 = -a_2 x_1 - a_1 x_2 + b_0 u$

$y = x_1$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -a_2 & -a_1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ b_0 \end{bmatrix} u$$

$$y = [1 \quad 0]\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + (0)u$$

$$\left[\begin{array}{cc|c} \begin{bmatrix} 0 & 1 \\ -a_2 & -a_1 \end{bmatrix} & & \begin{matrix} 0 \\ b_0 \end{matrix} \\ \hline 1 & 0 & 0 \end{array}\right]$$

1)

2)

$$\frac{\hat{Y}}{\hat{U}} = \frac{b_0 s + b_1}{s^2 + a_1 s + a_2}$$

$$\frac{\hat{X}}{\hat{U}} = \frac{1}{s^2 + a_1 s + a_2}$$

$$\hat{Y} = (b_0 s + b_1)\hat{X}$$

$$\hat{X} \longleftarrow \boxed{\frac{1}{s^2 + a_1 s + a_2}} \longleftarrow \hat{u}$$

$$\hat{Y} \longleftarrow \boxed{b_0 s + b_1} \longleftarrow \hat{X}$$

$$x_1 = x \qquad x_2 = \dot{x}_1$$

$$y_1 = x$$

$$y = b_0 \dot{x} + b_1 x$$
$$= b_0 x_2 + b_1 x_1$$

$$\left[\begin{array}{cc|c} 0 & 1 & 0 \\ -a_2 & -a_1 & 1 \\ \hline b_1 & b_0 & 0 \end{array}\right]$$

→ <u>Increase Complication</u>

1) $$\frac{\hat{Y}}{\hat{u}} = \frac{b_0}{s^2 + a_1 s + a_2}$$ Diff State Space

2) $$\frac{\hat{Y}}{\hat{u}} = \frac{b_0 s^2 + b_1 s + b_2}{s^2 + a_1 s + a_2}$$
$$= b_0 + \frac{(b_1 - b_0 a_1)s + (b_2 - b_0 a_2)}{s^2 + a_1 s + a_2}$$

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→ Putting it into State Space form

$$\dot{x} = Ax + bu$$
$$y = Cx + b_0 u$$

$$\left[\begin{array}{cc|c} 0 & 1 & 0 \\ -a_2 & -a_1 & 1 \\ \hline b_2 - b_0 a_2 & b_1 - b_0 a_1 & b_0 \end{array}\right]$$

H.W 1) $$\frac{\hat{Y}}{\hat{U}} = \frac{b_0 s^3 + b_1 s^2 + b_2 s + b_3}{a_0 s^3 + a_1 s^2 + a_2 s + a_3}$$

$$\left[\begin{array}{ccc|c} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -a_3 & -a_2 & -a_1 & 1 \\ \hline ? & ? & ? & b_0 \end{array}\right] \rightarrow \cancel{b}$$

2) $$\frac{\hat{Y}}{\hat{U}} = \left(\frac{s^3 + 2s + 1}{s^3 + 3s^2 + 3s + 1}\right)$$ Convert to State sp...

$$c(sI - A)^{-1} b + d$$

$$\hat{Y} = \left(\frac{s^3 + 2s + 1}{s^3 + 3s^2 + 3s + 1}\right) \hat{U}$$

$$\left(s^3+3s^2+3s+1\right)\hat{Y} = \left(s^3+2s+1\right)\hat{U}$$

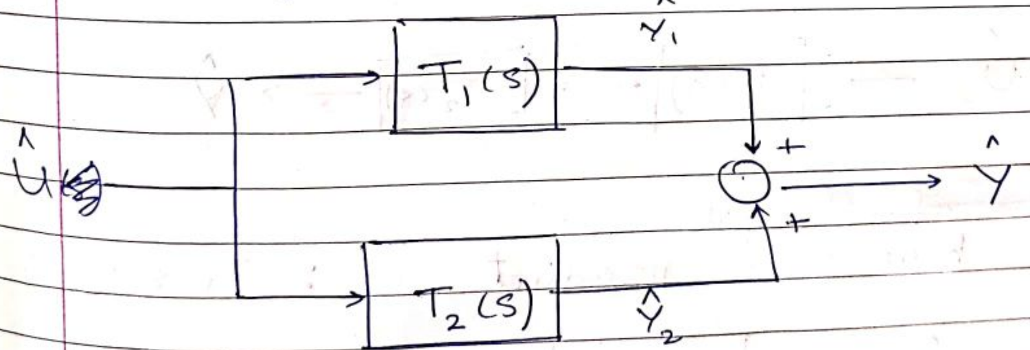
$$1 + \frac{\cancel{s^3}+2s+1-\cancel{s^3}-3s^2-3s-1}{(\qquad)}$$

$$1 + \frac{-3s^2-s}{\left(s^3+3s^2+3s+1\right)}$$

$$1 - \frac{s(3s-1)}{\left(s^3+3s^2+3s+1\right)}$$

Block Diagram Calculus in State Space

→ Sum of two systems



$$T(s) = T_1(s) + T_2(s)$$

T.F State-Space

$$T_1(s) \longleftrightarrow \left[\begin{array}{c|c} A_1 & b_1 \\ \hline c_1 & d_1 \end{array}\right] \quad - x_1, y_1$$

$$T_2(s) \longleftrightarrow \left[\begin{array}{c|c} A_2 & \\ \hline & \end{array}\right] \quad - x_2, y_2$$

→ We cannot directly add 2 ABCD Matrices

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$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \qquad y = y_1 + y_2$$

$$\dot{x} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} A_1 x_1 + b_1 u \\ A_2 x_2 + b_2 u \end{bmatrix}$$

$$y = c_1 x_1 + c_2 x_2 + (d_1 + d_2) u$$

$$\left[\begin{array}{cc|c} A_1 & & b_1 \\ & A_2 & b_2 \\ \hline c_1 & c_2 & d_1 + d_2 \end{array}\right] \qquad T(s) = [c_1 \ \ c_2] \begin{bmatrix} (sI - A_1)^{-1} & \\ & (sI - A_2)^{-1} \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} + ($$

Product of two Systems

$$\hat{u} \longrightarrow \boxed{T_1(s)} \longrightarrow \boxed{T_2(s)} \longrightarrow \hat{Y}$$

~~Sum of 2 syst~~

- Show that product is : $T_1(s) T_2(s)$

$$\hat{u}_1 \longrightarrow \boxed{T_1(s)} \longrightarrow \hat{Y}_1 \qquad \text{Ex 1)}$$

$$\hat{u}_2 \longrightarrow \boxed{T_2(s)} \longrightarrow \hat{Y}_2 \qquad \text{Ex 2)}$$

$$\dot{x} = Ax + bu$$
$$y = Cx + du$$

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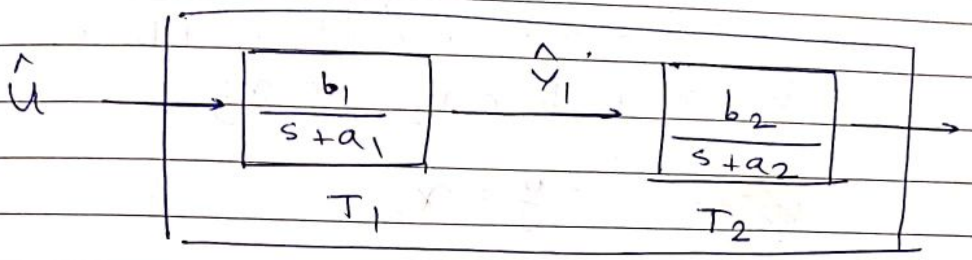
State Space Model of Product



$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} + (d_1 + d_2)$

S.S

$T_1 \longleftrightarrow \dot{x}_1 = -a_1 x_1 + b_1 u$

$y_1 = x_1$

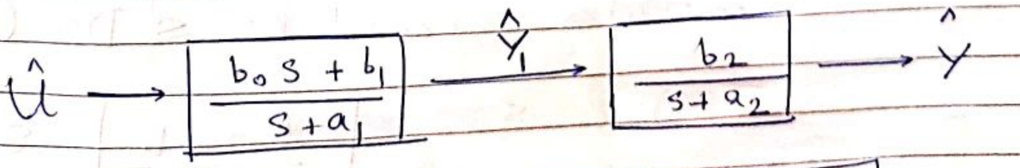
$T_2 \longleftrightarrow \dot{x}_2 = -a_2 x_2 + b_2 y_1$

$y_2 = x_2$ → final y

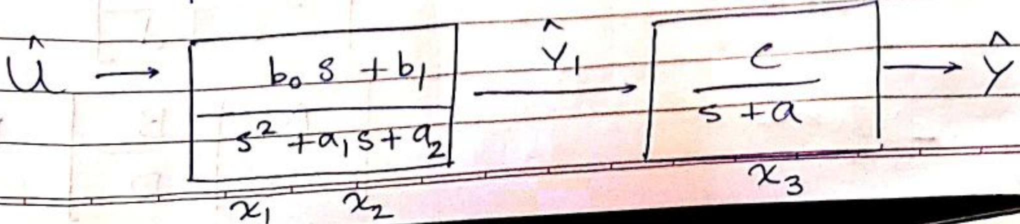
$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -a_1 & 0 \\ b_2 & -a_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} b_1 \\ 0 \end{bmatrix} u$$

$$y = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Ex 1)



Ex 2)



$s^2 x_1 = -a_2 x_1 - a_1 s x_1 + u$

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Definition

$y_1 = b_1 X_1 + s b_0 X_1$

$\hat{x}_1 (s^2 + a_1 s + a_2) = u$

$\frac{b_1 + s b_0}{s^2 + a_1 s \dots}$

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = -a_2 x_1 - a_1 x_2 + u$$

$$y_1 = b_1 x_1 + b_0 x_2 \quad , \quad y = x_3$$

$$\dot{x}_3 = -a x_3 + c y_1$$

$$= -a x_3 + c(b_1 x_1 + b_0 x_2)$$

$$= b_1 c x_1 + b_0 c x_2 - a x_3$$

$$y = x_3$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ -a_2 & -a_1 & 0 \\ b_1 c & b_0 c & -a \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} u$$

$$y = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

→ $\left[\begin{array}{c|c} A & b \\ \hline c & d \end{array}\right]$ = State Space Representation

→ ~~Pro~~ Completed

(i) Sum formula (ii) Product formula

→ State Space limitation: Degree of Numerator should be $\leq$ Dr degree

$T(s) \longleftrightarrow \left[\begin{array}{c|c} A & b \\ \hline c & d \end{array}\right]$ SISO

$\left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right]$ MIMO

$$T(s) = C(sI - A)^{-1}B + D$$

$$T(s) = \frac{b_0 s^2 + b_1 s + b_2}{s^2 + a_1 s + a_2}$$

(-1)

$$\left[\begin{array}{c|c} A & b \\ \hline c & d \end{array}\right] \longleftrightarrow$$ Some argument about Not possible .....

$$c(sI - A)^{-1}b + d$$

→ State Space (A, B, C, D) of Inverse

$$T(s)^{-1} = \frac{s^2 + a_1 s + a_2}{b_0 s^2 + b_1 s + b_2}$$

What is the formula for $T(s)^{-1}$ when it exists?

In : $T(s) = \underbrace{c(sI - A)^{-1}b}_{\text{Represent a proper, } N_r < D_r \text{ (deg) type transfer function.}} + d$

$d = b_0$ In Original $T(s)$

So, In General, when is a system Invertible

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du$$

and if D^{-1} exists

$$D^{-1}y = D^{-1}Cx + u \Rightarrow u = D^{-1}y - D^{-1}Cx$$

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$$\dot{x} = Ax + Bu$$
$$= Ax + B\left(D^{-1}y - D^{-1}Cx\right)$$

$$u = \cancel{D^{-1}Cx}\ D^{-1}y - D^{-1}Cx$$

$\hookrightarrow$ ~~Input from o/p~~

$$\left[\begin{array}{c|c} A - BD^{-1}C & BD^{-1} \\ \hline -D^{-1}C & D^{-1} \end{array}\right]$$

Ex

$$T(s) = \frac{2s^2+3s+1}{s^2+2s+1} = \frac{\hat{y}}{\hat{u}}$$

$d = 2$, $\cancel{2 + \left(\frac{-s}{\ }\right)}$

$$\frac{2s^2+4s+2-s-1}{s^2+2s+1}$$

$$= 2 - \frac{(s+1)}{(s^2+2s+1)}$$

$$2 + \frac{(-s-1) - \cancel{(s+1)}}{(s^2+2s+1)} \qquad s \neq -1$$

$\hookrightarrow (s+1)^2$

$$2 - \frac{1}{(s+1)}$$

$$\hat{u} \longrightarrow \boxed{\frac{-1}{(s+1)}} \longrightarrow \hat{y}$$

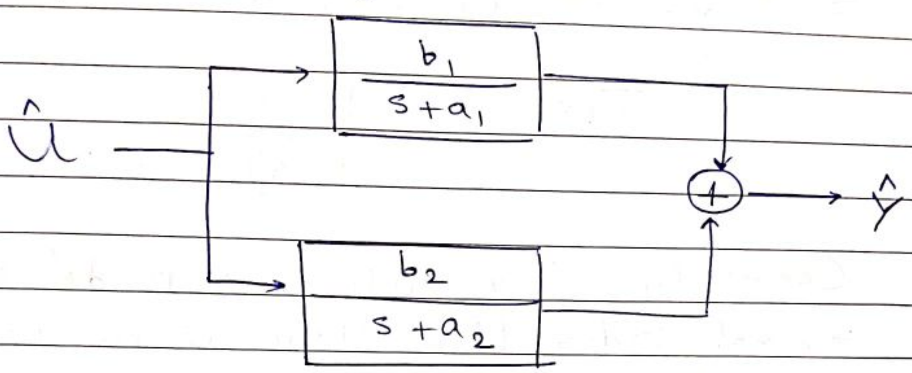
Ex

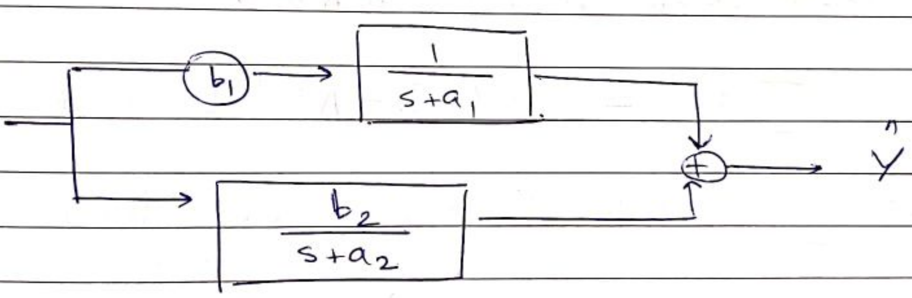
Minimal State Space ⇒ Use least No. of States

$$\frac{\hat{Y}}{\hat{U}} = \frac{-1}{(s+1)}$$

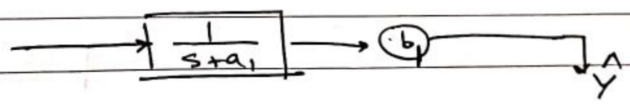
$$\dot{y} + y = -u$$

Ex





OR



Consider,

$$\frac{b_1}{s+a_1} + \frac{b_2}{s+a_2} = \frac{b_1 s + b_1 a_2 + b_2 s + b_2 a_1}{(s+a_1)(s+a_2)}$$

$$= \frac{b_1(s+a_2) + b_2(s+a_1)}{(s+a_1)(s+a_2)}$$

if $b_1 \to 0$ there is a pole - zero cancellation

→ When $T(s)$ has NO pole - zero cancellation then the numbers state variables (order of the State Space system) = deg of denominator of $T(s)$

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Pole - Zero Cancellation

- If a tr. fn $T(s)$ has no pole zero cancella[tion] then the order of state space representation of $T(s)$

= Degree of denominator polynomial of $T(s)$

$$T(s) = \frac{q(s)}{p(s)}$$

Conversely if a state space model of $T(s)$ has minimal order then there is no pole-zero cancellation in the formula

$$T(s) = c(sI - A)^{-1} b + d$$

Say we modify the way we have written $T(s)$ as,

$$T(s) = \frac{q(s)}{p(s)} \frac{(s+1)}{(s+1)}$$

$$T(s) \longleftrightarrow \left[\begin{array}{c|c} A & b \\ \hline c & d \end{array}\right]$$

How can we check if A is of minimal order

→ Test for minimality of State Space (A, b, c, d) representing a transfer function

- from the system of vectors :

$$b,\ Ab,\ A^2 b,\ A^3 b, \dots,\ A^{n-1} b$$

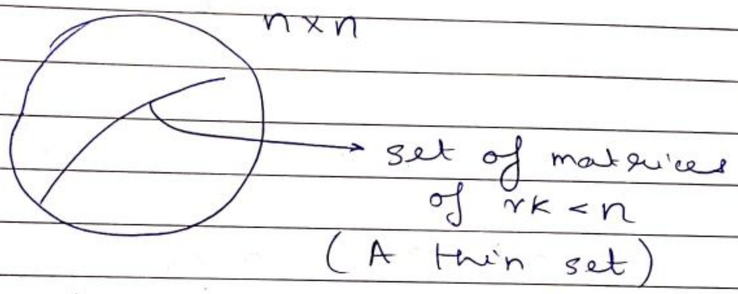
and $C^T,\ A^T C^T,\ (A^T)^2 C^T, \dots, (A^T)^{n-1} C^T$

then for no pole-zero cancellation / minimality (iff)

$$rk\,[b,\ Ab,\ A^2b,\ \dots,\ A^{n-1}b]_{n\times n} = n$$

$$rk\,[c^T,\ A^Tc^T,\ A^{T2}c^T,\ \dots,\ A^{T\,n-1}c^T]_{n\times n} = n$$

→ Consider the set of all $n\times n$ Matrices



- It is a thin set since the Matrices lie in a Subspace which is ~~a d~~ of a lower dimension.

$$\dot{x} = Ax + bu$$
$$y = cx + du$$

$$x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \sum x_i e_i$$

$$T(s) \leftrightarrow \left[\begin{array}{c|c} A & b \\ \hline c & d \end{array}\right]$$

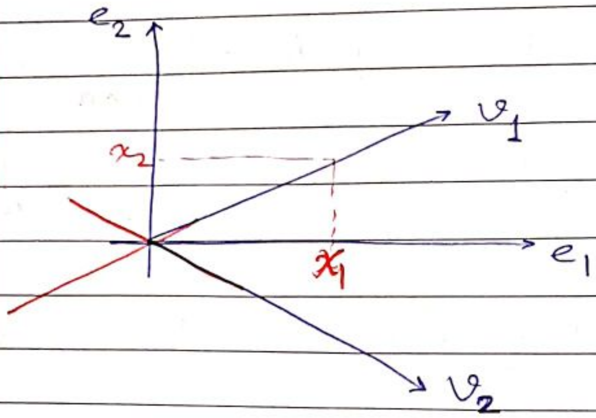
where

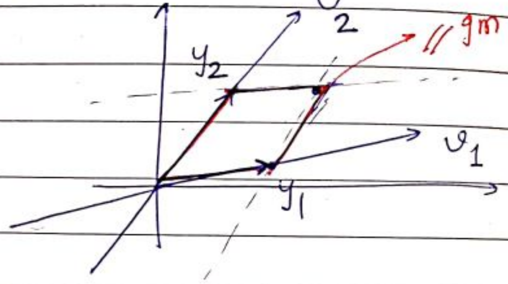
$$e_i = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \\ 0 \\ \vdots \end{bmatrix} \to i^{th}\text{ place}$$

$$T(s) = c(sI - A)^{-1}b + d$$

→ Project into a different Basis

$x = [x_1 \quad x_2]$ in $[e_1 \quad e_2]$





$$x = x_1 e_1 + x_2 e_2$$

$$x = y_1 v_1 + y_2 v_2 \quad , \; v_1, v_2 \text{ are LI}$$

$\cancel{\dot{x} = V\dot{y}}$ $\qquad x = Vy$ where $V = [v_1 \quad v_2]$

$$y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$$

$$\dot{x} = V\dot{y}$$

$$x = Ax + bu$$
$$= AVy + bu$$

$$\begin{cases} \dot{y} = (V^{-1}AV)y + (V^{-1}b)u \\ w = (CV)y + du \end{cases}$$

Here y is Actually "transformed input"

$$\dot{x} = Ax + bu$$
$$y = cx + du$$

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$$\left[\begin{array}{c|c} V^{-1}AV & V^{-1}b \\ \hline cV & d \end{array}\right]$$

Can do nice things like Diagonalizing

Same ~~when~~ T(S) when we Multiply out

$$G(s) = cV\,(sI - V^{-1}AV)^{-1}\,V^{-1}b + d$$

$$= c\,(sI - A)^{-1}\,b + d$$

$$= T(s)$$

Solution by Transformation

$\dot{x} = Ax$, using the transform $x = Vy$

v_1, v_2 — eigen vectors

$$Av_1 = \lambda_1 v_1, \quad Av_2 = \lambda_2 v_2$$

$$x = y_1 v_1 + y_2 v_2$$

$$\dot{x} = \dot{y}_1 v_1 + \dot{y}_2 v_2$$

$$\dot{x} = Ax + bu$$
$$= A(y_1 v_1 + y_2 v_2) + bu$$

$$= y_1 \lambda_1 v_1 + y_2 \lambda_2 v_2 + bu$$