Quiz 1 solutions

भारतीय प्रौद्योगिकी संस्थान मुंबई
INDIAN INSTITUTE OF TECHNOLOGY BOMBAY

परिशिष्ट/Supplement - 4

रोल नं./Roll No.

पाठ्यक्रम नाम/Course Name

शाखा/प्रभाग/Branch/Div. शिक्षण बैच/Tutorial Batch

अनुभाग/Section

पाठ्यक्रम सं./Course No.

तिथि/Date

1. State space models of P, Q:

1 P : $\left[\begin{array}{cc|c} 0 & 1 & 0 \\ -10 & -5 & 10 \\ \hline 1 & 0 & 0 \end{array}\right]$

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = -10x_1 - \cancel{10}\,5\,x_2 + 10e_1$$
$$y_1 = x_1$$

1 Q : $\left[\begin{array}{cc|c} 0 & 1 & 0 \\ -1 & 0 & 1 \\ \hline 1 & 1 & 0 \end{array}\right]$

$$\dot{\tilde{x}}_1 = \tilde{x}_2$$
$$\dot{\tilde{x}}_2 = -\tilde{x}_1 + e_2$$
$$y_2 = \tilde{x}_1 + \tilde{x}_2$$

2. Complete state space

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$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{\tilde{x}}_1 \\ \dot{\tilde{x}}_2 \end{bmatrix} = \begin{bmatrix} x_2 \\ -10x_1 - 5x_2 \\ \tilde{x}_2 \\ -\tilde{x}_1 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 10 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \end{bmatrix}$$

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -10 & -5 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \tilde{x}_1 \\ \tilde{x}_2 \end{bmatrix}$$

$e_1 = w_1 - y_2 \qquad e_2 = w_2 + y_1$

Substituting for e_1, e_2

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = -10x_1 - 5x_2 + 10(w_1 - y_2)$$

$$\dot{\tilde{x}}_1 = \tilde{x}_2$$

$$\dot{\tilde{x}}_2 = -\tilde{x}_1 + (w_2 + y_1)$$

Now substitute y_1 y_2 in term of states.

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = -10x_1 - 5x_2 + 10w_1 - 10\cancel{\tilde{x}_1}(\tilde{x}_1 + \tilde{x}_2)$$

$$\dot{\tilde{x}}_1 = \tilde{x}_2$$

$$\dot{\tilde{x}}_2 = -\tilde{x}_1 + w_2 + x_1$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{\tilde{x}}_1 \\ \dot{\tilde{x}}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -10 & -5 & -10 & -10 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \tilde{x}_1 \\ \tilde{x}_2 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 10 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \tilde{x}_1 \\ \tilde{x}_2 \end{bmatrix}$$

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3. State space model of $T_1(s)$

Since $w_2 = 0$ $\quad e_2 = y_1$ hence statespace of T_1 is

$$\left[\begin{array}{cccc|c} 0 & 1 & 0 & 0 & 0 \\ -10 & -5 & -10 & -10 & 10 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & -1 & 0 & 0 \\ \hline 1 & 0 & 0 & 0 & 0 \end{array}\right] = \left[\begin{array}{c|c} A_1 & b_1 \\ \hline c_1 & d_1 \end{array}\right]$$

Transfer function $T(s)$

sstotf$(A_1\ b_1\ c_1\ d_1)$ for $(A_1\ b_1\ c_1\ d_1)$ of T_1 above

$$= \text{---------} \quad \frac{10(s^2+1)}{20+15s+11s^2+5s^3+s^4}$$

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4. To find transfer fn. from w_2 to y_1:

State space model of $T_2(s)$ from w to y_1 is

$$\left[\begin{array}{cccc|c} 0 & 1 & 0 & 0 & 0 \\ -10 & -5 & -10 & -10 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & -1 & 0 & 1 \\ \hline 1 & 0 & 0 & 0 & 0 \end{array}\right] = \left[\begin{array}{c|c} A_2 & b_2 \\ \hline c_2 & d_2 \end{array}\right]$$

$T_2(s) = \text{sstotf}(A_2\ b_2\ c_2, d_2) =$

Step response:

inverse Laplace of $\frac{1}{s} T_2(s) = \frac{1}{s} \times \frac{-10(s+1)}{(20+15s+11s^2+5s^3+s^4)}$

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ilaplace ( )

Step response $x(t) = \left[-0.5 - e^{-2.3882t}\{0.29\cos(1.19t) + 0.609\sin(1.19t)\} + e^{-0.11178}\{0.7908\cos(1.67t) + 0.07225\sin(1.67t)\}\right]$

5. Poles of $T_1(s)$ = eigenvalues of A_1

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spec(A_1) returns eigenvalues of A_1.

The eigenvalues are $-0.11178 \pm j1.67165$

$-2.38822 \pm j1.19233$