SUMMARY OF WINDOW FUNCTIONS IN FIR FILTER DESIGN.

Reference: "Digital Filters: Theory and Applications", N. K. Bose, Elsevier Science Publishing Co., Inc., New York, 1985.

WINDOW TYPE	WINDOW FUNCTION $W(K)$ $-N \leq K \leq N$ $W(K) = 0 \ \forall \ \lvert k \rvert > N$.
RECTANGULAR	1
HANN	$\frac{1}{2}[1 + \cos((2\pi k)/(2N))]$
HAMMING	$0.54 + 0.46\cos((2\pi k)/(2N))$
GENERALIZED HAMMING	$\alpha + (1-\alpha)\cos((2\pi k)/(2N)) \quad 0<\alpha<1$
FEJER - CESARO / BARTLETT / TRIANGULAR	$1 - ((2\lvert k \rvert)/(2N))$
LANCZOS	$\left\{\sin[(2k\pi)/(2N)] / [(2k\pi)/(2N)]\right\}^{L}_{L>0}$
DOLPH - CHEBYSHEV (Fourier Transform)	$W(e^{j\omega}) = \dfrac{\cos[(2N)\cos^{-1}(\alpha\cos(\omega/2))]}{\cosh(2N\cosh^{-1}\alpha)}$ $\alpha > 0$
PAPOULIS	$\frac{1}{\pi}[\lvert\sin[(2\pi k)/(2N)]\rvert] + \frac{2\lvert k \rvert}{2N}\cos\frac{2\pi k}{2N}$
KAISER	$\dfrac{I_0[\beta N\sqrt{1-(k/N)^2}]}{I_0(\beta N)} \quad \beta > 0.$ $I_0(x)$ = modified Bessel function of first kind and order 0 in x. $I_0(x) \triangleq 1 + \sum_{l=1}^{\infty}\left[\frac{(x/2)^{l}}{l!}\right]^{2}.$
TUKEY	$1 \quad \forall \ \lvert k \rvert \leq \alpha N \quad 0<\alpha<1$ $\frac{1}{2}[1 + \cos\{[(k-\alpha N)\pi]/[(1-\alpha)N]\}]$ $\alpha N \leq \lvert k \rvert \leq N.$
PARZEN	$m \triangleq \frac{1}{2} k/N.$ $1 - 24m^2 + 48\lvert m \rvert \quad \lvert m \rvert \leq 1/4$ $\frac{1}{2}[1 - 2\lvert m \rvert] \quad 1/4 \leq \lvert m \rvert \leq 1/2$
BLACKMAN	$0.42 + 0.5\cos\left(\frac{2\pi k}{2N}\right) + 0.08\cos\left(\frac{4\pi k}{2N}\right)$

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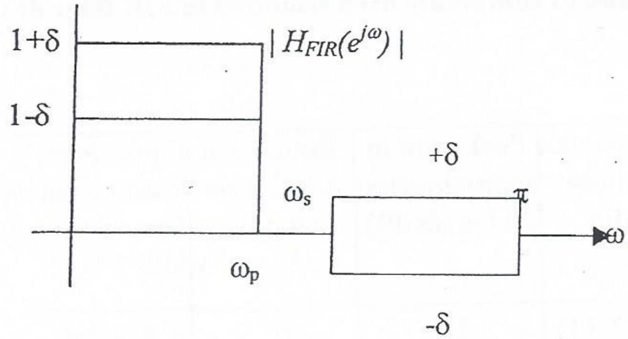
- **Window functions - Comparison of commonly used windows in FIR filter design**

Window Type (Name)	Peak relative sidelobe amplitude (dB)	Approximate mainlobe width	Peak error in approximation $20.\log_{10}\delta$(dB)	Equivalent α in Kaiser window *	Equivalent Kaiser window Transition width **
Rectangular	- 13	$4\pi/(2N+1)$	- 21	0	$1.81\pi/2N$
Bartlett	- 25	$4\pi/N$	- 25	1.33	$2.37\pi/2N$
Hann	- 31	$4\pi/N$	- 44	3.86	$5.01\pi/2N$
Hamming	- 41	$4\pi/N$	- 53	4.86	$6.27\pi/2N$
Blackman	- 57	$6\pi/N$	-74	7.04	$9.19\pi/2N$

δ = peak ripple in passband and stopband
$\alpha = \beta.N$ in Kaiser window

* : to get the same δ as the corresponding window
** : accordingly $\Delta\omega_T$ from empirical equations below.

- **Empirical design equations for Kaiser window**



$\Delta\omega_T = \omega_s - \omega_p$

$A = -20.\log_{10}\delta$

$\alpha = \beta.N$

Fig. : Prototype Specifications for Low Pass FIR filter

Reference : "Discrete – Time Signal Processing", Oppenheim and Schafer, pp.450-454

Design steps :

1. Choose N according to $\boxed{(2N+1) \geq 1 + \frac{A-8}{2.285\Delta\omega_T}}$

2. Now choose α and hence β according to

$$\alpha = \begin{cases} 0.1102\,(A - 8.7) & \text{for all } A > 50 \\ 0.5842\,(A - 21)^{0.4} + 0.07886\,(A - 21) & \text{for } 21 \leq A \leq 50 \\ 0 & \text{for } A < 21 \end{cases}$$

Remember $\beta = \alpha/N$

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- **Park Mc Clellan Algorithm for FIR filters (odd length, symmetric)**

To determine optimum filter of the form, $H_{FIR}(e^{j\omega})$ as below,
Optimum filter is bound to satisfy - Eqn 1 below

$$H_{FIR}(e^{j\omega}) = \sum_{n=-N}^{N} h_{FIR}[n].e^{-j\omega n}$$

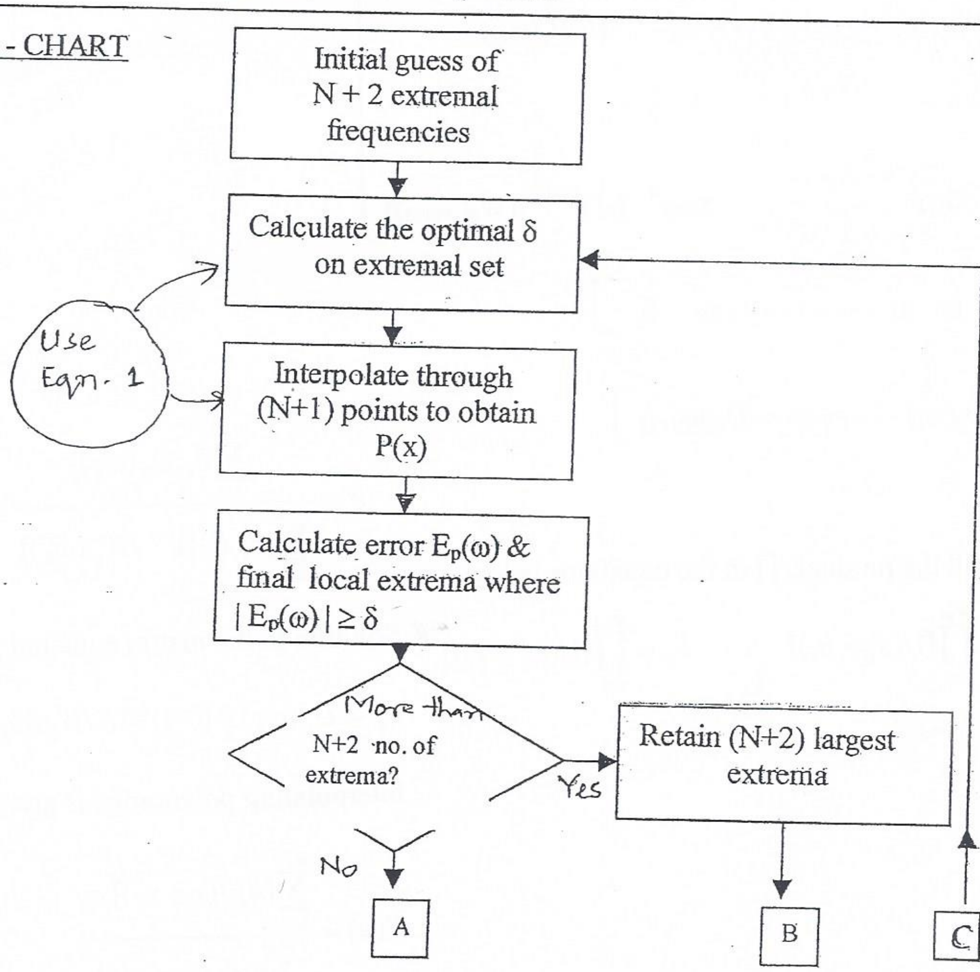
A

$$Eqn.1: E_p(\omega_i) = W_p(\omega_i).[D_p(\omega_i) - P(\cos\omega_i)] = (-1)^{i+1}\delta \qquad i = 1, -----, N+2.$$

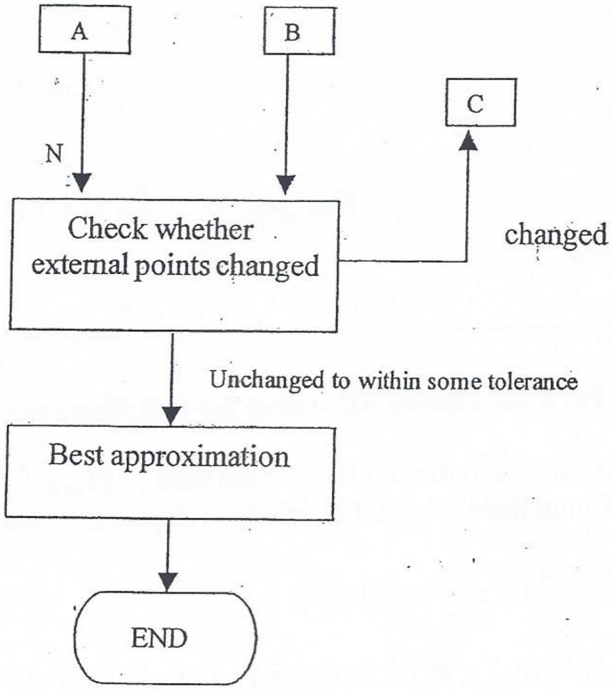
$$H_{FIR}(e^{j\omega}) = P.(\cos\omega) = P(x) = \sum_{k=0}^{N} a_k.x^k \qquad x = \cos\omega$$

where ω_i, $i = 1, \ldots, N+2$ are the external frequencies

FLOW - CHART



Flow - chart (contd...)



References:
Discrete Time Signal Processing
"Oppenheim and Schafer" pp. 468 - 480

Eqn 1) gives the system

$$x.A = H$$

$$x = \begin{pmatrix} 1 & x_1 & x_1^2 & \cdots & x_1^N & 1/(W_p(\omega_1)) \\ 1 & x_2 & x_2^2 & \cdots & x_2^N & -1/(W_p(\omega_2)) \\ \cdot & & & & & \\ \cdot & & & & & \\ \cdot & & & & & \\ \cdot & & & & & \\ 1 & x_{N+2} & & & x_{N+2}^N & (-1)^{1+N+2}/(W_p(\omega_{N+2})) \end{pmatrix}$$

$$A = \begin{bmatrix} a_0 & a_1 & \cdots & a_N & \delta \end{bmatrix}^T$$

$$H = \begin{bmatrix} D_p(\omega_1) & \cdots & D_p(\omega_{N+2}) \end{bmatrix}$$

$$\delta = \frac{\sum_{k=1}^{N+2} b_k . D_p(\omega_k)}{\sum_{k=1}^{N+2} [b_k . (-1)^{k+1} / W_p(\omega_k)]}$$

$i \neq k$ for all the products $\prod$ in the equations below

$$d_k = \prod_{i=1}^{N+1} [1/(x_k - x_i)] \qquad b_k = \prod_{i=1}^{N+2} [1/(x_k - x_i)]$$

Solution to the matrix equation given by $x.A = H$ is :

$$c_k = D_p(\omega_k) - [(-1)^{k+1} \delta / W_p(\omega_k)]$$

Interpolating polynomial is given by

$$P(x) = \frac{\sum_{k=1}^{N+1} [d_k / (x - x_k)] . c_k}{\sum_{k=1}^{N+1} [d_k / (x - x_k)]}$$