'Memoryless' Linear Modulation - Waveform for a bit depends only on that bit

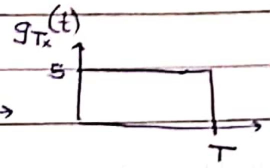
- If $b[n]$ are real → 'Pulse amplitude modulation' (PAM)
- $g_{Tx}(t) \rightarrow g_{Tx}(at) \Rightarrow G_{Tx}(f) \rightarrow G_{Tx}(f/a)$, Symbol rate : $\frac{1}{T} \rightarrow \frac{a}{T}$, BW :- $B \rightarrow aB$

  ∴ 'Normalized BW' $\triangleq BT$

## IV → Linear Modulation

$$u(t) = \sum_n b[n]\, g_{Tx}(t - nT)$$

↓ symbols from constellation ($\{1, j, -1, -j\}$ for QPSK)




eg Example of non-linear :- $u(t) = \begin{cases} 1 & \text{if bit toggles} \\ 0 & \text{if bit is same as previous} \end{cases}$

This is not linear :- Waveform is not obtained by multiplication of symbols with pulse

## * Summary

Bit-stream → Symbols → Complex baseband signal → Passband signal

- Bit-stream: $\{0, 1\}$
- Symbols: (Finite set of complex numbers)
- Complex baseband signal: ($p(t)$, rate $= 1/T$, $W$)
  - Linear Modulation
    $s(t) = \sum b[n]\, p(t - nT)$
  - $s(nT) = b[n]$
  - $W$ depends on choice of $p(t)$
- Passband signal: (at $f_c$)

$$u(t) = \sum_{n=-\infty}^{\infty} b[n]\, g_{Tx}(t - nT)$$

- TFT :- BW used to transmit $u(t)$.

- $u(t)$ is a random process because $b[n]$ are random variables

P.T.O.

...r nuclear test on 18th May, 1974.

- Because of some headers, 'Information Rate' $\lesssim$ Bit rate

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Theorem: If i) All $b[n]$ are uncorrelated
ii) $E[b[n]] = 0$

Then :- PSD of $u(t) = \frac{1}{T} E(|b[n]|^2)\,|G_{Tx}(f)|^2$

$\propto |G_{Tx}(f)|^2$

- Ideally we would use $g_{Tx}(t)$ as sinc, so that $G_{Tx}(f)$ is a perfect rect, and because sinc is zero at non-zero integers.
  - Not practical because sinc is prone to sampling errors.

- Practical BW $\triangleq$ 99% energy containment.

$$\frac{\int_{-W}^{W} |P(f)|^2 df}{\int_{-\infty}^{\infty} |P(f)|^2 df} \leq 0.99$$

Practical BW for $u(t)$ = Practical BW for $g_{Tx}(t)$

15/1

- sinc $t$ decays as $1/t$. But we want faster decay ☹
  - Slow decay increases ISI
  - If you want better decay $(1/t^2)$, use $\text{sinc}^2 t$
    - $\downarrow$ ISI ☺
    - $\uparrow$ BW ☹

A] ISI - free condition

$b[n] \rightarrow \boxed{g_{Tx}(t)} \rightarrow \boxed{g_c(t)} \rightarrow \boxed{g_{Rx}(t)} \rightarrow$ Substitute $t = nT$ to obtain $z[n]$ (Sampling)

at rate $1/T$ (Channel)

$\sum b[n]\delta(t-nT)$ $\quad \sum b[n] g_{Tx}(t-nT)$ $\quad z(t) = \sum b[n](g_{Tx} * g_c * g_{Rx})(t-nT)$

We want :- $z[n] = b[n]$ ..... 'ISI avoidance'

- All written signals are complex baseband representation

- Let $(g_{Tx} * g_c * g_{Rx})(t) = x(t)$

At $t = nT$, we want $z(nT) = z[n] = b[n]$ .....

P.T.O.

$\therefore$ Condition on $x(mT) = \begin{cases} 1 & \text{for } m = 0 \\ 0 & \text{for } m \neq 0 \end{cases}$ '$\delta_{m0}$'.

.... like sinc(t) or rect(t) or more

Then $z[n] = z(nT) = b[n]$ since $z(t) = \sum b[n]\, x(t-nT)$

- In practice, received $z[n]$ is not exactly $b[n]$. Received signal must be 'equalized'.

Equivalently, $x(t) \sum \delta(t-nT) = \delta(t)$

Fourier Transform :- $X(f) * \frac{1}{T} \sum \delta\left(f - \frac{n}{T}\right) = 1$

$$\therefore \frac{1}{T} \sum X\left(f - \frac{n}{T}\right) = 1.$$

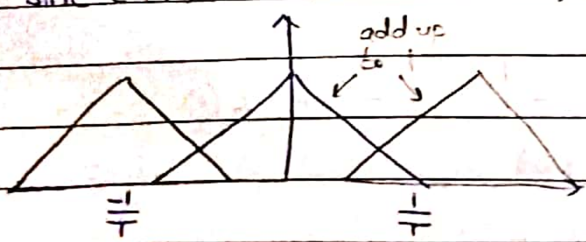
- An $x(t)$ satisfying above condition and having minimum BW is sinc(t)
- Minimum BW $= \left[\frac{-1}{2T}, \frac{1}{2T}\right]$ goes from 1 at m = 0 to 0 at m = 1 slowest

$\therefore$ If you replicate $X(f)$ at periods $1/T$, we get flat response, like $\delta(t) \rightarrow 1$

- Linear modulation using sinc takes up all degrees of freedom

## A Excess BW.

- $x(t) = \text{sinc}^2(t/T)$ ✓ Satisfies condition.

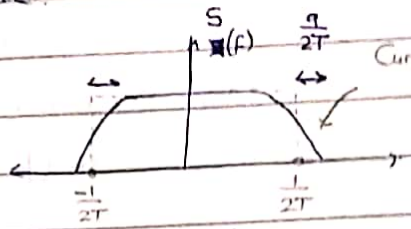


Reading – Chap 2 + Exercise Qs

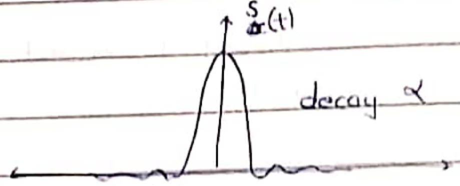
RCP $\quad s(f) = \begin{cases} T & -\frac{(1-a)}{2T} \le f \le \frac{1-a}{2T} \\ \frac{T}{2}\left(1 - \sin\left(\frac{\pi T}{a}\left(|f| - \frac{1}{2T}\right)\right)\right) & \text{for } \frac{1-a}{2T} \le |f| \le \frac{1+a}{2T} \end{cases}$

→ Raised Cosine Pulse

Family of curves :-




Time domain :-




decay $\propto 1/t^{\alpha}$ where $\alpha > 1$

Equation :- $s(t) = \dfrac{\text{sinc}\left(t/T\right)\cos\left(\frac{\pi a t}{T}\right)}{1 - \left(\frac{2at}{T}\right)^2}$ ✓ Satisfies ISI avoidance condition.

For small T, s(t) decays as $\sim 1/t^3$

17/1 $a \rightarrow$ 'Excess bandwidth' :- New BW $= \frac{1+a}{2T}$

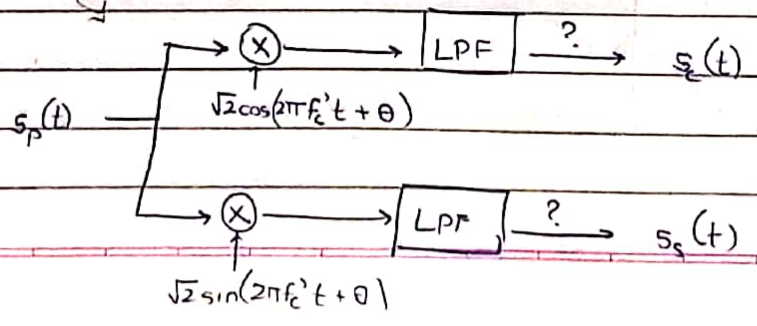
For $a = 0$, $s(t) = \text{sinc}(t/T)$

Increasing $a$ increases rate of decay of waveform, but also the BW.

→ Root Raised Cosine Pulse

- Fourier domain :- $\sqrt{S(f)}$ $\because$ S(f) is real and positive
- Choose $g_{Tx}$ and $g_{Rx}$ as RRCP

B] Coherent v/s Non-coherent Demodulation.

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Because of $\theta$ :- $s_c(t)$ is multiplied by $\cos\theta$ and also have some $s_s(t)$ component. ☹

Because of $f_c'$ :- Causes beats ☹

* M-ary FSK

$$s_i(t) = \cos(2\pi f_i t)\, I_{[0,T]} \qquad i = 1, 2 \ldots M$$

or $\log_2 M$

- Demodulation :- - Multiply received signal by all $\cos 2\pi f_i t$ and LPF.
  - Conclude the symbol that gives highest time-average after multiplication
  - $\overline{\cos 2\pi f_i t \times \cos 2\pi f_j t} = \begin{cases} 1/2 & \text{for } i = j \\ 0 & \text{otherwise} \end{cases}$

This type of averaging is unaffected by phase :)

↳ multiplication by $\cos(2\pi f_i t + \theta)$.

- Coherent :- Equal $f_c$ and phase

Non-coherent : Equal $f_c$.

~~Receive~~

→ Non-coherent demodulation :

- Received signal = $y_p(t) = s_p(t)$ + Noise

- Possible choices of $s(t)$ = (eg) $I_{[0,T]}, -I_{[0,T]}, j I_{[0,T]}, -j I_{[0,T]}$ ... 'QPSK'

- To guess which symbol was sent :-

For all choices of $s(t) = s_c(t) + j s_s(t)$,

calculate $\langle y_p, s_p \rangle = \langle y_c, s_c \rangle + \langle y_s, s_s \rangle$ ..... Real ✓

Conclude the symbol that maximizes above expression.

☆ Verify

Reason :- $\langle y_p, s_p \rangle$ is maximum when $y_p = \alpha' s_p$ (collinear)

P.T.O.



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- In non-coherent, we don't care about phase offset.
  Even if $y(t) = s(t)e^{j\theta}$,
  $\langle y, s \rangle$ will still be maximum when $y = ks$

- Problem :- Because $|\langle y, s\rangle| = \|s\|^2$ when maximum, you cannot distinguish ~~but~~ sign of symbol.

Verify from book that $|\langle y, s\rangle|$

21/1

- Coherent modulation $\xrightarrow{}$ eg - PSK – Info is stored in phase (each symbol is sent via a particular phase)

Demodulation :- Find $i$ that maximizes $\mathrm{Re}(\langle y, s_i\rangle)$

- Non-coherent modulation (eg - FSK) - Each symbol is sent via a particular frequency/amplitude.
  - We don't care about phase of modulated signal.

Demodulation :- Find $i$ that maximizes $|\langle y, s_i\rangle|$

* FSK need not be non-coherent and may embed phase information.

eg 2 - Non-coherent ASK (PAM)

~~Dem~~

Demodulation - Envelope detection (no info in phase)

* Coherent FSK (eg - 2-FSK (1 bit))

$s_1(t) = \cos 2\pi f_1 t \quad I_{[0,T]}$

$s_2(t) = \cos 2\pi f_2 t \quad I_{[0,T]}$

} Start with zero phase

- Coherent because phase difference between $s_1$ & $s_2$ is not unknown
- $y(t) = s(t) + \text{noise}$
- To find which was sent, maximise $\int_0^T y(t)\, s_i(t)\, dt$.

- We want 'orthogonal' FSK - If $s_1$ was sent, inner product with $s_2$ should give zero.

$$\int_0^T \cos 2\pi f_1 t \, \cos 2\pi f_2 t \, dt = 0 \quad \ldots \text{ Find } |f_1 - f_2|_{min} \text{ such that this holds}$$

$$\int_0^T \cos 2\pi (f_1 + f_2) t + \cos 2\pi (f_1 - f_2) t \, dt = 0$$

Condition :- $2\pi (f_1 + f_2) T = m\pi$ ✓

Condition :- $2\pi (f_1 - f_2)_{min} T = \pi$

$$\boxed{|f_1 - f_2|_{min} = \frac{1}{2T}}$$

* Non-coherent FSK :- (eg - 2-FSK)

$$s_1(t) = \cos(2\pi f_1 t + \phi_1) I_{[0,T]}$$
$$s_2(t) = \cos(2\pi f_2 t + \phi_2) I_{[0,T]}$$

- Cannot simply maximize $\int_0^T y(t) s_i(t) \, dt$.
- Find $\int_0^T \cos(2\pi f_i t + \phi) \cos 2\pi f_i t \, dt = \sim\sim \cos\phi$
  and $\int_0^T \cos(2\pi f_i t + \phi) \sin 2\pi f_i t \, dt = \sim\sim \sin\phi$ } Square and add.

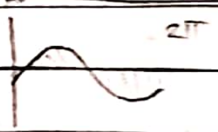
Equivalently :- Maximize $\langle y_p, s_p \rangle = \langle y_c, s_c \rangle + \langle y_s, s_s \rangle$

- We want orthogonal FSK :-
  $\int_0^T \cos(2\pi f_1 t + \phi) \cos 2\pi f_2 t \, dt$ should be zero

Condition :- $2\pi (f_1 + f_2) T = m\pi$ ✓

Condition :- $2\pi (f_1 - f_2)_{min} T = 2\pi$

$$\boxed{|f_1 - f_2|_{min} = \frac{1}{T}}$$




Tuesha Devi and Bachan ... Devi spread awareness about it among women.

* PSK :- $s_i(t) = e^{j\phi_i} I_{[0,T]}$

eg - For QPSK, $\phi_i \in \{ \pi/4, 3\pi/4, -3\pi/4, -\pi/4 \}$

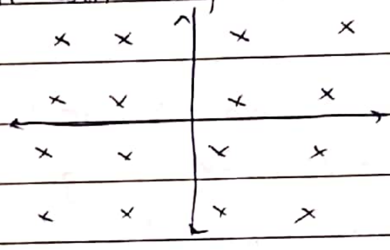
- Info is purely in phase.

* FSK

* ASK = PAM.

* QAM - Info is in both amplitude and phase.

eg - 16-QAM



* FSK, ASK could be coherent or non-coherent

PSK, QAM are coherent.

## Differential Modulation.

* Consider M-FSK.

Bandwidth efficiency = $\dfrac{\log_2 M}{M}$ (bits)

bits per symbol

As $M \to \infty$, $\eta_B \to 0$

$\therefore$ It is inefficient to pack in too many frequencies.

* Nyquist :- Minimum required BW for linear modulation = symbol rate = $1/T$

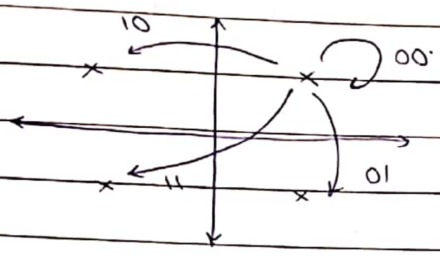
$$B_{min} = 1/T_s$$

$$\therefore \eta_B = \frac{1}{M} \times \frac{\text{Bit rate}}{B_{min}}$$

c] Differential Modulation

eg QPSK

☆ Verify




Data stream :- 00 11 01 10 11

- Start with 00. (Ask receiver to ignore first symbol)

$\underbrace{00}_{e^{j\frac{\pi}{4}}}\ \underbrace{00}_{\frac{\pi}{4}}\ \underbrace{11}_{\frac{-3\pi}{4}}\ 01\ 10\ 11$

Always start with the

- At the receiver, $y[0] = he^{j\pi/4}$. Assuming $g_c(t) = h\delta(t)$

$y[1] = he^{j\pi/4}$.

Find $y[1]y[0]^* \rightarrow = |h|^2$

$\angle y[1]y[0]^* = 0 \Rightarrow$ Move 0 phase from initial.

Now $y[2] = he^{j3\pi/4}$

Find $y[2]y[1]^* = |h|^2 e^{j\pi} \Rightarrow$ Move $\pi$ phase from where you were.

- Benefit :- Phase added by channel is immaterial.

It becomes $|h|^2$.

☆ Wrong • 'Send as is. Determine difference at receiver'.

2/1 → Differential BPSK


☆ Verify

1. Send '+1'
2. If you want to send 1 → Previous

0 → Toggle.

P.T.O.

Guessing Devi and Bachani

Devi spread awareness about it among women.

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Receiver finds phase change between consecutive symbols

$\hookrightarrow 0 : +1$

$\pi : -1$

- Benefits of DBSK :-

eg - Bitstream 0 1 1 0 1 0 1

Send +1 -1 -1 -1 +1 +1 -1 -1

1 Phase added by channel is removed, by doing $\angle(y[n]y^*[n-1])$

2 This works even if channel conditions vary with time $\equiv h(t)$

- We assume $h(t)$ does not change much over two consecutive samples.

* If you have passband BW limit of 'W', then the maximum data rate you can use is W sps. (because pulse needs to decay fast enough to be zero at next integral point — best done by sinc)

$\Rightarrow$ This minimum sampling time $= T = \frac{1}{W}$

* Basis :-

Consider QPSK at $\frac{1}{T}$ sps.

1) $s_1(t) = e^{j\pi/4} I_{[0,T]}$

2) $j3\pi/4$

3) $j - \pi/4$

4) $j - 3\pi/4$

- Basis vectors for all 4 symbols :- $\varphi_1(t) = \frac{I_{[0,T]}}{\sqrt{T}}$

'Orthonormal'

$\varphi_2(t) = j \frac{I_{[0,T]}}{\sqrt{T}}$ → to normalize

- Orthogonality :- $Re\left(\int_{-\infty}^{\infty} \varphi_1 \varphi_2 \, dt\right) = 0$

→ $s_1(t) = \cos\frac{\pi}{4} \varphi_1(t) + \sin\frac{\pi}{4} \varphi_2(t)$

- Alternate orthonormal basis :- $\psi_1(t) = s_1(t)$, $\psi_2(t) = s_2(t)$

23/1

- $k$ basis vectors $\Rightarrow$ $k$ complex dimensions $\equiv 2k$ real dimensions

This is because real part and imaginary part are orthogonal

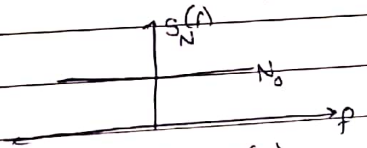
ie - In passband form, $\cos 2\pi f_c t$ and $\sin 2\pi f_c t$ are orthogonal

→ Additive White Gaussian Noise.

$$\longrightarrow \oplus \longrightarrow \boxed{g_{Rx}} \longrightarrow$$

$$\uparrow n(t)$$

$$f_X(x) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-(x-\mu)^2/2\sigma^2}$$




$R_N(\tau) = \delta(\tau)$

- Jointly Gaussian :- $\alpha_1 X_1 + \alpha_2 X_2$ is Gaussian $\forall\ \alpha_1, \alpha_2$

$X \sim N(0,1)$, $Y = \alpha X$ where $\alpha = \begin{cases} 1 & \text{wp } \frac{1}{2} \\ -1 & \text{wp } \frac{1}{2} \end{cases}$

$X, Y$ are not JG because $X+Y$ is zero wp $\frac{1}{2}$ (not zero)

$0 \rightarrow -\sqrt{P}$, $1 \rightarrow \sqrt{P}$ where $P$ = Power.

* BPSK :-

Pulse for modulation chosen as $\dfrac{I_{[0,T]}}{\sqrt{T}}$

$s_1(t) = \sqrt{P}\,\psi_1(t)$, $s_0(t) = -\sqrt{P}\,\psi_1(t)$

$$s(t) \longrightarrow \oplus \longrightarrow r(t)$$

$\uparrow n(t)$ : with PSD $N_0$

We can equivalently assume noise to be baseband, or passband.

$r(t) = s_i(t) + n(t)$

Finding $\langle r, \psi_1 \rangle$ :- $r(t) \longrightarrow \otimes \longrightarrow \boxed{\int_0^T} \longrightarrow$

$\uparrow \psi_1^*(t)$

$$= \int_0^T s_i(t)\,dt + \underbrace{\int_0^T n(t)\,dt}_{\text{'RV'}}$$

$\rightarrow E\left[\int_0^T n(t)\,dt\right] = 0$

$\rightarrow Var\left[\int_0^T n(t)\,dt\right]$

$$= E\left[\left(\int_0^T n(t_1)\,dt\right)^2\right]$$

$$= E\left[\left(\int_0^T n(t_1)\,dt_1\right)\left(\int_0^T n(t_2)\,dt_2\right)\right]$$

$$= E\left[\int_0^T\int_0^T n(t_1)\,n(t_2)\,dt_1\,dt_2\right]$$

$$= \int_0^T\int_0^T E\left[n(t_1)\,n(t_2)\right]dt_1\,dt_2$$

$$= \int_0^T\int_0^T R_n(t_1, t_2)\,dt_1\,dt_2$$

$$= \int_0^T\int_0^T N_0\,\delta(t_1 - t_2)\,dt_1\,dt_2$$

$$= N_0 T$$

We ~~&~~ don't lose information because ~~$E(\langle r, \psi_1 \rangle)$~~ $E(RV) = 0$

* QPSK :- 2 complex dimensions.

$s_i(t) \longrightarrow \oplus \longrightarrow r(t)$ Complex

Complex

$\uparrow$

$n(t)$ : PSD $N_0$

complex

$Re(n(t))$ ~~&~~ and $Im(n(t))$ are independent and each has PSD $\frac{N_0}{2}$