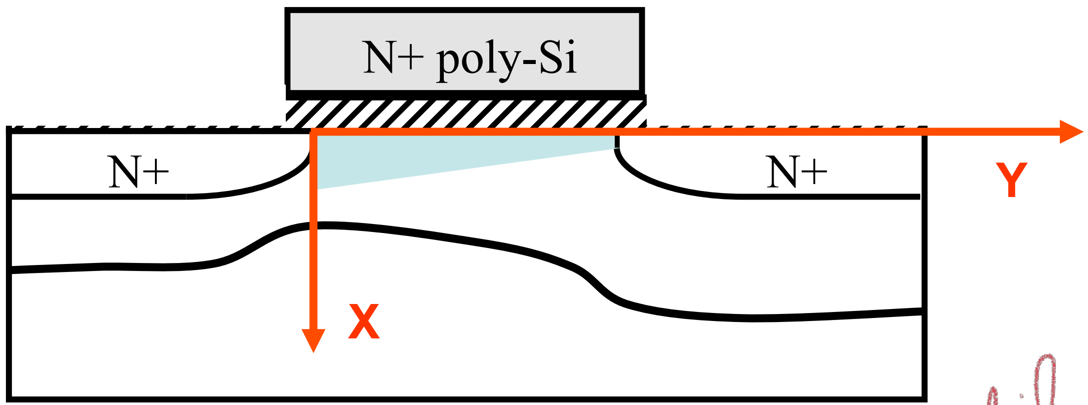
Piece-wise Model (Above Threshold)



$V_{GS}>V_T$, $V_{DS}>0$

Inversion layer exists throughout the channel

always valid

$$I_D = \mu_{eff}\frac{W}{L}\int_0^{V_{DS}}[-Q_i(V)]\,dV$$

Voltage at y

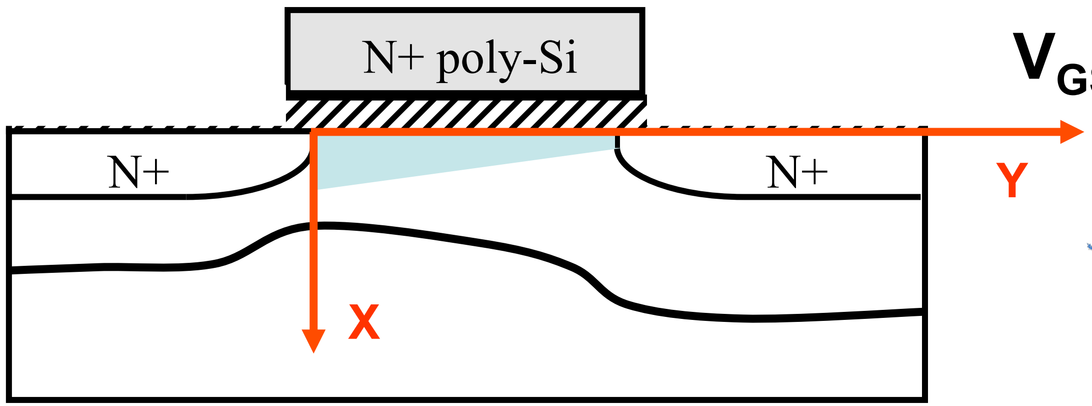
depletion approx

Valid above threshold (we have taken $\phi_s = 2\phi_F$)

$$-Q_i(V) = C_{OX}\Big[V_{GS} - V_{FB} \underbrace{-\,2\phi_F - V(y)}_{\text{Surface potential}}\Big] \underbrace{-\sqrt{2q\varepsilon_{si}N_A(2\phi_F + V(y))}}_{\text{Depletion charges}}$$

Q at y

Piece-wise Model (Above Threshold)



$V_{GS}>V_T$, $V_{DS}>0$

$I_d = \mu_{eff} \frac{W}{L} \int_0^{V_{ds}} -Q_i(V)\,dV$

$$I_D = \mu_{eff} C_{OX} \frac{W}{L}$$

$$*[(V_G - V_{FB} - 2\phi_F - \frac{V_{DS}}{2})V_{DS}$$

$$-\frac{2\sqrt{2\varepsilon_{si} q N_A}}{3C_{OX}}\{(2\phi_F + V_{DS})^{3/2} - (2\phi_F)^{3/2}\}]$$

Valid for any V_{DS} but only above threshold

Piece-wise Model (Above Threshold)

$$(2\phi_F + V_{DS})^{3/2} - (2\phi_F)^{3/2}$$

$$= (2\phi_F)^{3/2}[(1 + \frac{V_{DS}}{2\phi_F})^{3/2} - 1]$$

$$= (2\phi_F)^{3/2}[1 + \frac{3}{2}\frac{V_{DS}}{2\phi_F} + \frac{3}{8}(\frac{V_{DS}}{2\phi_F})^2 + \ldots - 1]$$

$$= \frac{3}{2}\sqrt{2\phi_F}[V_{DS} + \frac{1}{4}\frac{V_{DS}^2}{2\phi_F}]$$

Taylor expansion for small V_{DS}

Piece-wise Model (Above Threshold)

For small V_{DS}, keep 1st order term

$$I_D = \mu_{eff} C_{OX} \frac{W}{L}$$

$$*[V_G - V_{FB} - 2\phi_F - \frac{\sqrt{2\varepsilon_{si} q N_A 2\phi_F}}{C_{OX}}]V_{DS}$$

$$= \mu_{eff} C_{OX} \frac{W}{L}[V_G - V_T]V_{DS}$$

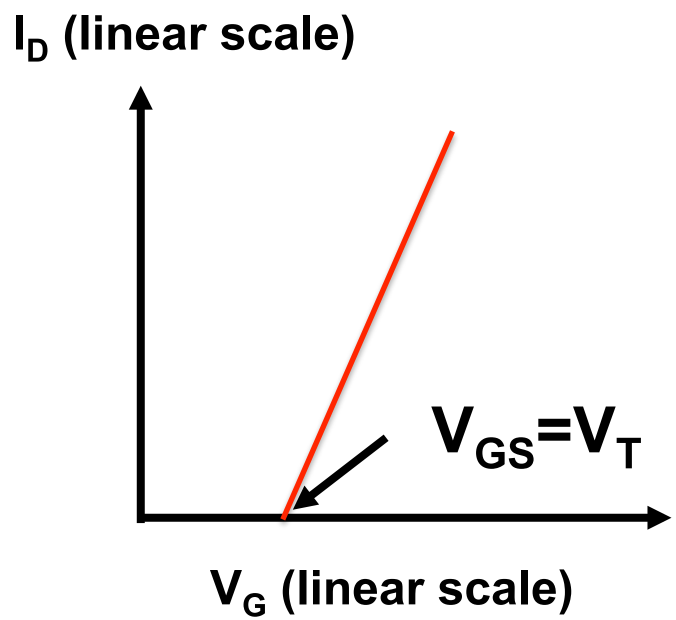
$$V_T = V_{FB} + 2\phi_F + \frac{\sqrt{2\varepsilon_{si} q N_A 2\phi_F}}{C_{OX}}$$

Linear region (V_{DS} small) – Transfer curves

Drain current linearly dependent on V_{GS} (>V_T)

$$I_D = \mu_{eff} C_{OX} \frac{W}{L_{eff}} \left[V_{GS} - V_T \right] V_{DS}$$

But the formula is only valid for Vg above V_T



Intercept with X axis gives linear $V_T \rightarrow V_{TLIN}$

Piece-wise Model (Above Threshold)

For moderate V_{DS}, keep up to 2nd order terms

$$I_D = \mu_{eff} C_{OX} \frac{W}{L}$$

$$*[(V_G - V_{FB} - 2\phi_F - \frac{V_{DS}}{2})$$

$$- \frac{\sqrt{2\varepsilon_{si} q N_A 2\phi_F}}{C_{OX}} - \frac{1}{C_{OX}}\sqrt{\frac{\varepsilon_{si} q N_A}{4\phi_F}}\frac{V_{DS}}{2}]V_{DS}$$

$$= \mu_{eff} C_{OX} \frac{W}{L}[V_G - V_T - \frac{m}{2} V_{DS}]V_{DS}$$

$$m = 1 + \frac{\sqrt{\varepsilon_{si} q N_A / 4\phi_F}}{C_{OX} = \frac{\varepsilon_{SiO2}}{T_{OX}}} = 1 + \frac{C_D}{C_{OX}} = 1 + \frac{3T_{OX}}{W_{DMAX}}$$

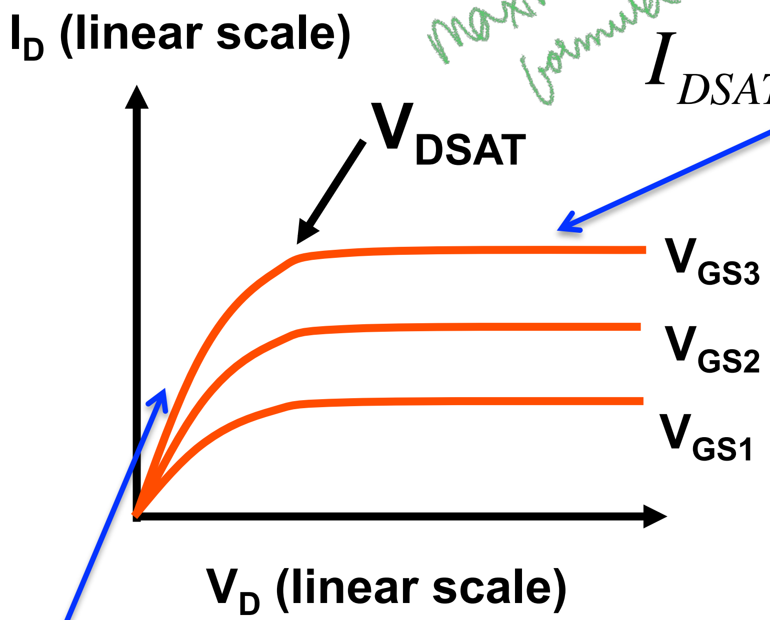
due to $\frac{\varepsilon_{SiO2}}{\varepsilon_{Si}}$

Output Characteristics

For a given V_{GS} ($>V_T$), V_{DS} is increased

maximizing Ids formula

$$I_{DSAT} = \mu_{eff} C_{OX} \frac{W}{L_{eff}} \frac{(V_{GS} - V_T)^2}{2m}$$



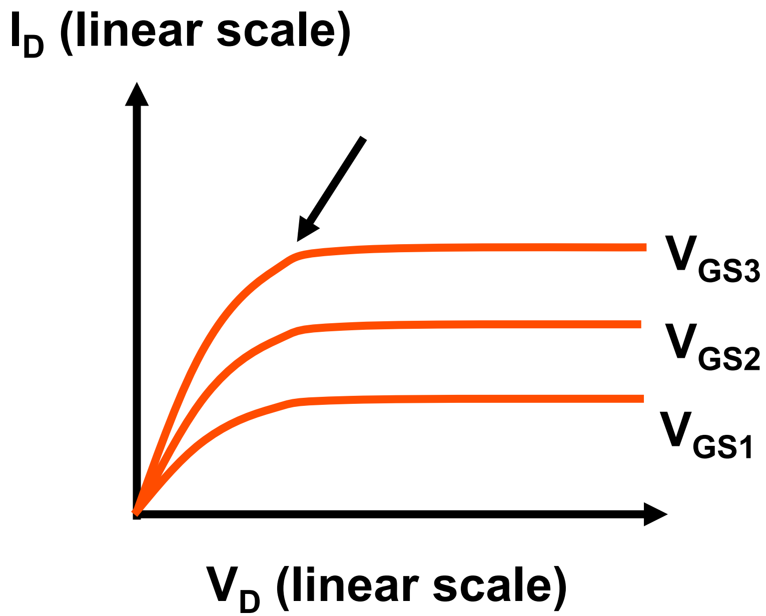
I_D increases with V_{DS} up to a value $V_{DSAT} = (V_{GS} - V_T)/m$, beyond which I_D remains constant at a value I_{DSAT}

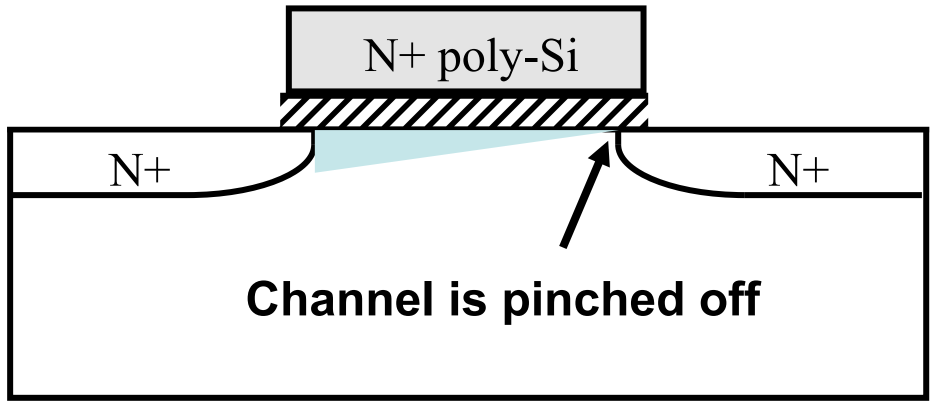
$$V_{Dsat} = \frac{V_g - V_T}{m}$$

$$I_D = \mu_{eff} C_{OX} \frac{W}{L} [V_G - V_T - \frac{m}{2} V_{DS}] V_{DS}$$

Channel Pinch Off

Channel does not exist in the drain end when $V_{DS} = V_{DSAT}$





$$Q(y) = C_{OX}\left[V_{GS} - V_T - mV(y)\right]$$

V (y=0)=0 at S end
V ($y=L_{eff}$)=V_{DS} at D end

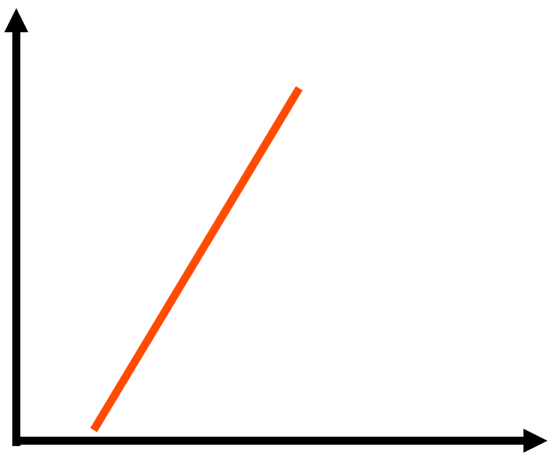
$$I_{DSAT} = \mu_{eff} C_{OX} \frac{W}{L_{eff}} \frac{(V_{GS} - V_T)^2}{2m}$$

Threshold Voltage Extraction (Saturation)

Saturation current

$$I_{DSAT} = \mu_{eff} C_{OX} \frac{W}{L_{eff}} \frac{(V_{GS} - V_T)^2}{2m}$$

I_{Dsat}^0.5 (linear scale)



V_G (linear scale)

Plot of root (I_{DSAT}) versus V_{GS} is a straight line, find V_T (saturation) from intercept

Is V_{TSAT} same as V_{TLIN}?

Better Expression for V_{DSAT}

Should not do series expansion for larger V_{DSAT}

$$I_D = \mu_{eff} C_{OX} \frac{W}{L}$$

$$*[(V_G - V_{FB} - 2\phi_F - \frac{V_{DS}}{2})V_{DS}$$

$$-\frac{2\sqrt{2\varepsilon_{si} q N_A}}{3C_{OX}}\{(2\phi_F + V_{DS})^{3/2} - (2\phi_F)^{3/2}\}]$$

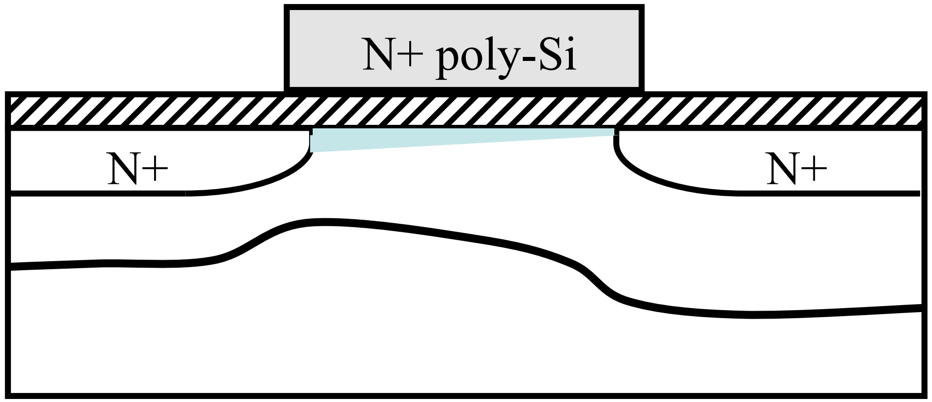
When V_D=V_{DSAT}, dI_D/dV_D=0

Better Expression for V_{DSAT}

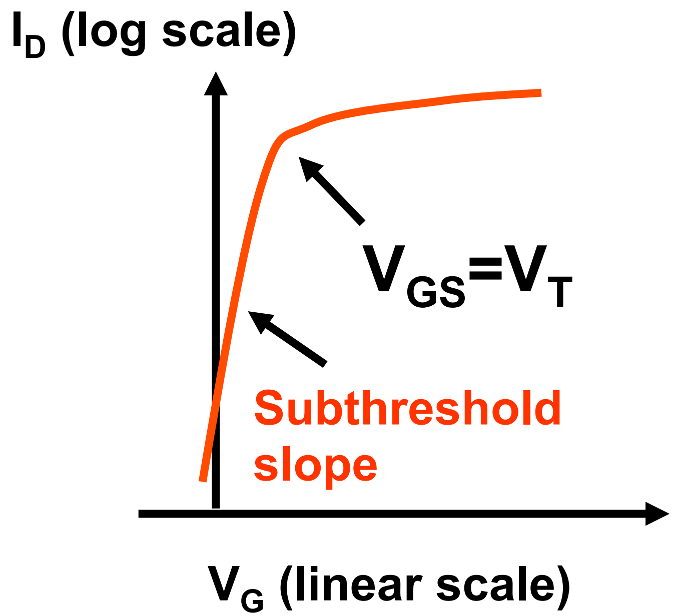
$$V_{DSAT} = V_G - V_{FB} - 2\phi_F + \frac{\varepsilon_{si} q N_A}{C_{OX}^2} - \sqrt{\frac{2\varepsilon_{si} q N_A}{C_{OX}^2}(V_G - V_{FB} + \frac{\varepsilon_{si} q N_A}{2C_{OX}^2})}$$

Compare V_{DSAT} from both methods

Current Conduction (Subthreshold)



$V_{GS}<V_T$, $V_{DS}>0$



Fewer mobile electrons (negligible drift), but strong density gradient → current dominated by diffusion of electrons form S to D

Exponential dependence on V_{GS}

Subthreshold slope → how many "V" must V_{GS} be reduced to reduce I_D by each decade below V_T

Reduction in V_T increases off current

Charges in Regime of Interest

Total charge density in depletion / inversion

from gradual channel approx

$$Q = -\sqrt{2\varepsilon_{si}kTN_A}\,[\frac{q\psi_s}{kT} + \frac{n_i^2}{N_A^2}\mathrm{e}^{q(\psi_s - V)/kT}]^{1/2}$$

$$\frac{Q_{inv}}{Q_{dep}} \ll 1$$

Expand in power series for weak inversion

$$Q = -\sqrt{2\varepsilon_{si}qN_A\psi_s}\,[1 + \frac{kT}{2q\psi_s}\frac{n_i^2}{N_A^2}\mathrm{e}^{q(\psi_s - V)/kT}]$$

Depletion charge

Inversion charge

$$Q_i = -\sqrt{\frac{\varepsilon_{si}qN_A}{2\psi_s}}\,(\frac{kT}{q})(\frac{n_i}{N_A})^2\,\mathrm{e}^{q(\psi_s - V)/kT}$$

Current Conduction (Subthreshold)

$$V_G = V_{FB} + \psi_s + \frac{\sqrt{2\varepsilon_{si} kTN_A}}{C_{OX}}[\frac{q\psi_s}{kT} + \frac{n_i^2}{N_A^2} \mathrm{e}^{q(\psi_s - V)/kT}]^{1/2}$$

For depletion region, ignore inversion term (appx.)

$$V_G = V_{FB} + \psi_s + \frac{\sqrt{2\varepsilon_{si} qN_A \psi_s}}{C_{OX}}$$

add & sub $2\phi_F$

Rewrite:

$$V_G = V_{FB} + 2\phi_F + (\psi_s - 2\phi_F) + \frac{\sqrt{2\varepsilon_{si} qN_A \{2\phi_F + (\psi_s - 2\phi_F)\}}}{C_{OX}}$$

Current Conduction (Subthreshold)

$$I_D = \mu_{eff} \frac{W}{L} \int_0^{V_{DS}} [-Q_i(V) dV$$

$$Q_i = -\sqrt{\frac{\varepsilon_{si} q N_A}{2\psi_s}} (\frac{kT}{q})(\frac{n_i}{N_A})^2 \mathrm{e}^{q(\psi_s - V)/kT}$$

$$I_D = \mu_{eff} \frac{W}{L} \sqrt{\frac{\varepsilon_{si} q N_A}{2\psi_s}} (\frac{kT}{q})^2 (\frac{n_i}{N_A})^2 \mathrm{e}^{q\psi_s/kT} [1 - e^{-qV_{DS}/kT}]$$

Current Conduction (Subthreshold)

$$V_G = V_{FB} + 2\phi_F + (\psi_s - 2\phi_F)$$
$$+ \frac{\sqrt{4\varepsilon_{si} q N_A \phi_F}}{C_{OX}} [1 + \frac{\psi_s - 2\phi_F}{2\phi_F}]^{1/2}$$
$$= V_{FB} + 2\phi_F + (\psi_s - 2\phi_F)$$
$$+ \frac{\sqrt{4\varepsilon_{si} q N_A \phi_F}}{C_{OX}} + \frac{\sqrt{\varepsilon_{si} q N_A / 4\phi_F}}{C_{OX}} (\psi_s - 2\phi_F)$$
$$= V_{FB} + 2\phi_F + \frac{\sqrt{4\varepsilon_{si} q N_A \phi_F}}{C_{OX}} + (1 + \frac{\sqrt{\varepsilon_{si} q N_A / 4\phi_F}}{C_{OX}})(\psi_s - 2\phi_F)$$
$$= V_T + m(\psi_s - 2\phi_F)$$

Assume ψ_s close to $2\Phi_F$ (appx.) contradicting assumption as if ψ_s is close to $2\phi_F$ then how $Q_{inv} << Q_{dep}$

Current Conduction (Subthreshold)

$$\psi_s = (V_G - V_T)/m + 2\phi_F$$

$$I_D = \mu_{eff}\frac{W}{L}\sqrt{\frac{\varepsilon_{si}qN_A}{2\psi_s}}\left(\frac{kT}{q}\right)^2\left(\frac{n_i}{N_A}\right)^2 e^{q\psi_s/kT}[1 - e^{-qV_{DS}/kT}]$$

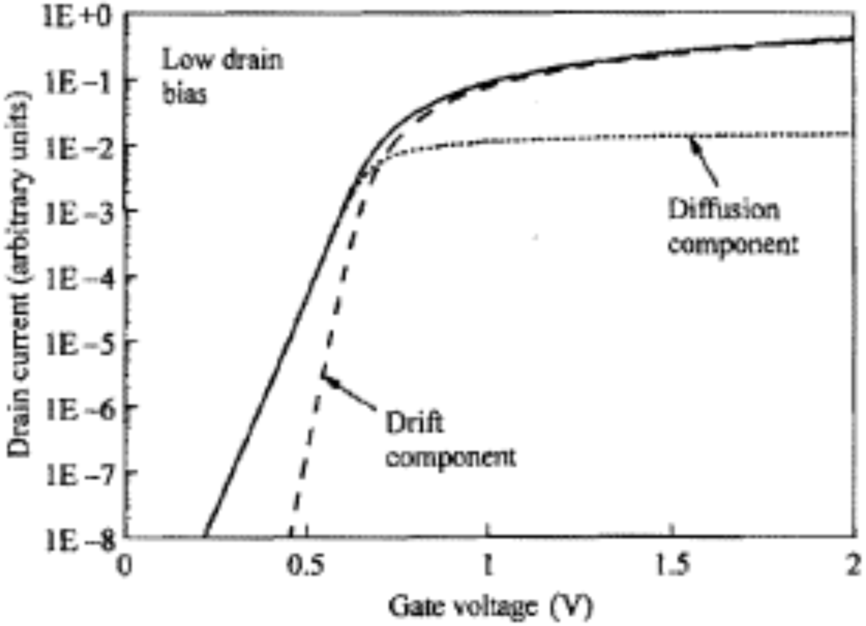
$$= \mu_{eff}\frac{W}{L}\sqrt{\frac{\varepsilon_{si}qN_A}{4\phi_B}}\left(\frac{kT}{q}\right)^2 e^{q(V_G - V_T)/mkT}[1 - e^{-qV_{DS}/kT}]$$

$m = 1 + \frac{1}{C_{ox}}\sqrt{\frac{\varepsilon_s q N_a}{4\phi_F}}$

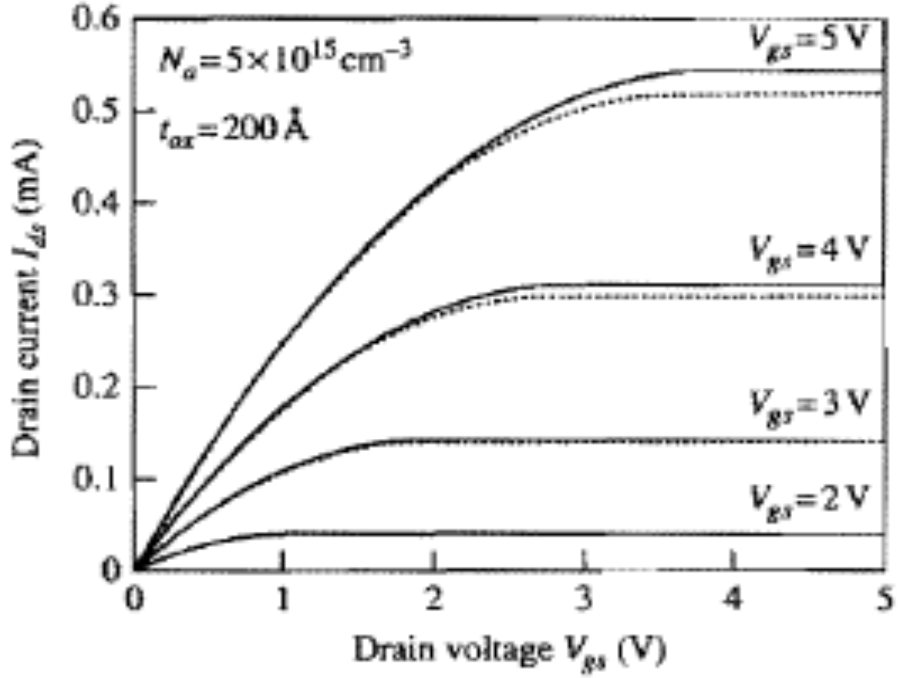
$$I_D = \mu_{eff}\frac{W}{L}C_{OX}(m-1)\left(\frac{kT}{q}\right)^2 e^{q(V_G - V_T)/mkT}[1 - e^{-qV_{DS}/kT}]$$

$$S = \left(\frac{d\log_{10} I_D}{dV_G}\right)^{-1} = 2.3m\frac{kT}{q} = 2.3\frac{kT}{q}\left(1 + \frac{C_D}{C_{OX}}\right)$$

MOSFET I-V Characteristics



Transfer curve



Output curves, calculated using full (solid, slide 30) and approximate (dashed, slide 35) equations

Ref: Taur and Ning