EE 328: Digital Comm.

Instructor: Kumar Appiah

Syllabus: Review of Random Process, Spectral Analysis, Optimum detection of Signals, Probability of Error, Baseband pulse transmission, ISI, Nyquist Criterion

Passband Digital Modulation Schemes
- Phase Shift Keying, FSK, QAM

~~Quadrature~~ → Optimum Demodulation
→ Maximum Likelihood Seq. detector (Viterbi Detector)
→ Equalization Techniques
→ Information Theory

• Pre-requisites ~~in~~ and Syllabus

→ Revision: Random Processes
Complex Baseband Equivalent
Modulation - Demod.
Synchronization, Equalization, Advanced topics

→ Book: Fundamentals of Digital Comm
→ Upamanyu Madhav
Cambridge

• Quizzes: Jan 14, Jan 28, Feb 11, Mar 11, Mar 25, Apr 8

• Evaluation:

| | | |
|---|---|---|
| Quizzes | - 20 | (Best 5 of 6) |
| Mid Sem | - 35 | |
| Endsem | - 40 | |
| Lab Exercise | - 5 | |

3.1.19

Page No.:
Date:

# Reading Assignment - 1

→ Read chapter 2 upto page 41

- Vectors : $\underline{s}$, $\underline{r}$
- Inner product : $\underbrace{\langle \underline{s}, \underline{r} \rangle}_{\text{Inner product}} = s^H r$

$$\underline{s} = \begin{bmatrix} 1 \\ j \end{bmatrix} \qquad \underline{r} = \begin{bmatrix} j \\ -1 \end{bmatrix}$$

$$\langle \underline{s}, \underline{r} \rangle = \begin{bmatrix} 1 & -j \end{bmatrix} \begin{bmatrix} j \\ -1 \end{bmatrix} = 2j$$

→ Cauchy Schwarz Inequality

$$|\langle \underline{s}, \underline{r} \rangle| \leq \|\underline{s}\| \, \|\underline{r}\|$$

where $\cancel{\|\underline{s}\|}$ $\|\underline{s}\| = \cancel{\sqrt{\langle \underline{s}, \underline{s} \rangle}}$

$\sqrt{\langle \underline{s}, \underline{s} \rangle}$

→ FT Notation

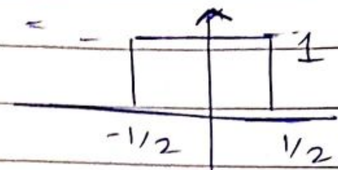
$$X(f) = \int_{-\infty}^{\infty} x(t) e^{-j2\pi f t} \, dt$$

$$x(t) = \int_{-\infty}^{\infty} X(f) e^{j2\pi f t} \, df$$

→ ~~1~~ Set Indicators

$$I_A(x) = \begin{cases} 1 & \text{if } x \in A \\ 0 & \text{else} \end{cases}$$

$$I_{[-0.5,\,0.5]}(t) = \text{rect}(t) =$$




→ Signals $s_1(t)$, $s_2(t)$ – Square Int.

i.e $\int |s_1(t)|^2 dt < \infty$

then,

Extension of Parsevals,

$$\langle s_1, s_2 \rangle = \int_{-\infty}^{\infty} s_1(t)\, s_2^*(t)\, dt$$

$$= \int_{-\infty}^{\infty} S_1(f)\, S_2^*(f)\, df$$

• Intuition: It is swap b/w bases, hence the Answer is conserved

$$s(t) = \int_{-\infty}^{\infty} s(\tau)\, \delta(t-\tau)\, d\tau$$

↕ Basis swap, Inner product doesn't change

$$= \int_{-\infty}^{\infty} S(f)\, e^{j2\pi ft}\, df$$

$$\langle v_u, u_u \rangle = \langle v_v \mid u_v \rangle$$

• More Intuition

$$\langle \underline{s_1}, \underline{s_2} \rangle = \int_{-\infty}^{\infty} s_1(t)\, s_2^*(t)\, dt = \int_{-\infty}^{\infty} S_1(f)\, S_2^*(f)\, df$$

→ Proof: $s_1(t) = \int_{-\infty}^{\infty} s_1(\tau)\, \delta(t-\tau)\, d\tau$

$$= \int_{-\infty}^{\infty} S_1(t)\, e^{j2\pi ft}\, df$$

7.1.19

Page No.:
Date:

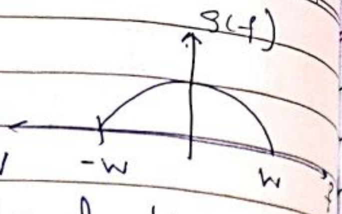
# Baseband Signal

→ $s(t)$ : Baseband

$\Downarrow$

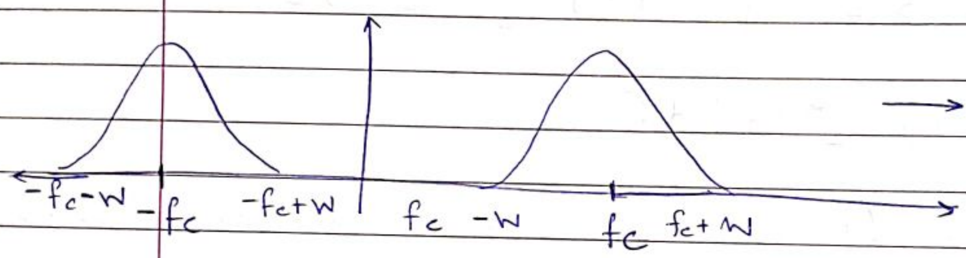
$S(f) \simeq 0$ for $|f| > W$

There is no practical signal that is both time limited and Band limited




→ Pass - Band Signal

$S(f) \simeq 0$ for $|f \pm f_c| > W$




→ Pass - Band

## Complex Base Band Representation.

- Baseband signals : $s_c(t)$ $s_s(t)$

$$s_p(t) = \sqrt{2}\, s_c(t) \cos 2\pi f_c t - \sqrt{2}\, s_s(t) \sin 2\pi f_c t$$

$$I_{[a,b]}(x) = \begin{cases} 1 & x \in [a,b] \\ 0 & \text{else} \end{cases}$$

ex $\sqrt{2}\, I_{[0,1]}(t) \cos 2\pi f_c t - \sqrt{2}\,(1-|t|)\, I_{[-1,1]}(t) \sin 2\pi f_c t$

Not Strictly BB but if $f_c$ is large enough, then fine (Approximately Baseband)

Page No.:
Date:

→ Complex Envelope (Complex BB equivalent)

$$s(t) = s_c(t) + j s_s(t)$$

$$s_p(t) = \sqrt{2}\, Re\left(s(t) e^{j 2\pi f_c t}\right)$$

→ Polar Representation

Envelope : $e(t) = \sqrt{s_s^2(t) + s_c^2(t)}$

Phase : $\theta(t) = \cancel{at}\ \tan^{-1}\left(\frac{s_s(t)}{s_c(t)}\right)$

→ $\tan^{-1}$ → atan(a) → $[0, \pi)$, can't do 1/0

→ atan2(x, y) → ~~Gives~~ Uses Ratio → $[0, 2\pi)$ → can do 1/0

$\cancel{s_p(t) = \sqrt{2} e(t)}$

$$s(t) = e(t)\, e^{j\theta(t)}$$

• Orthogonality of I and Q Components

$x_c(t) = s_c(t) \cos 2\pi f_c t$ : In phase comp.

$x_s(t) = s_s(t) \sin 2\pi f_c t$ : Quadrature comp.

$$\langle x_c, x_s \rangle = 0$$ [Proof in Book]

→ Why we need the $\sqrt{2}$ factor?

$$\|s_p\|^2 = \|s\|^2$$

$\langle s_p, s_p \rangle = \langle s, s \rangle$ : Because L.H.S has extra $\sqrt{2}$ factor

$= \langle s_c, s_c \rangle + \langle s_s, s_s \rangle$

$2 \times \cos^2()$

$\to \frac{1}{2}(1 - \cos 2())$

cancel

$S(-f_0 - f_c)$

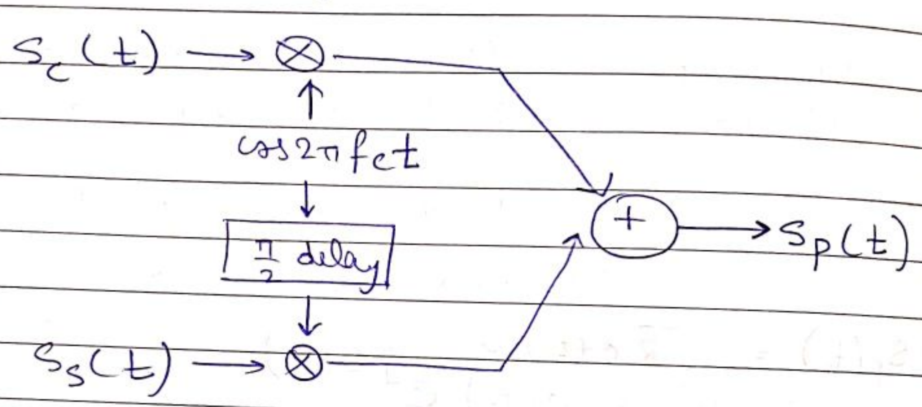
Page No.:

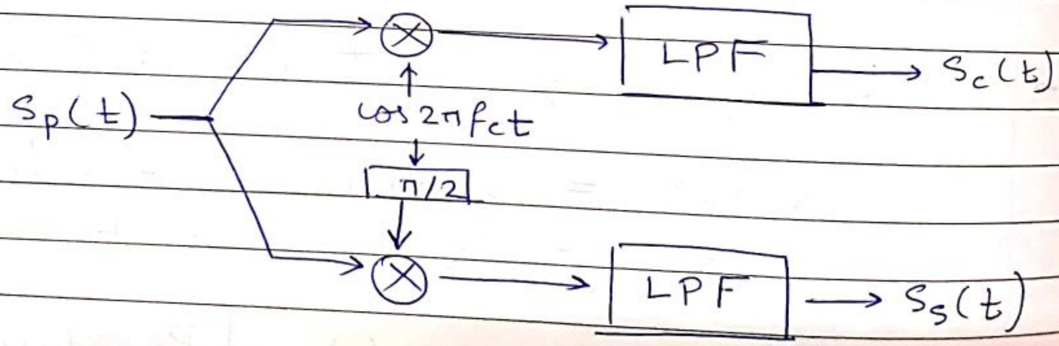
Date:

Slightly weird definition because of the way $S_p(t)$ is defined

$$\left\{ \begin{aligned} S(f) &= \sqrt{2}\, S_p^{+}(f + f_c) \\ S_p(f) &= \frac{S(f-f_c) + S^{*}(-(f+f_c))}{\sqrt{2}} \end{aligned} \right.$$

→ On the scaling factors :---



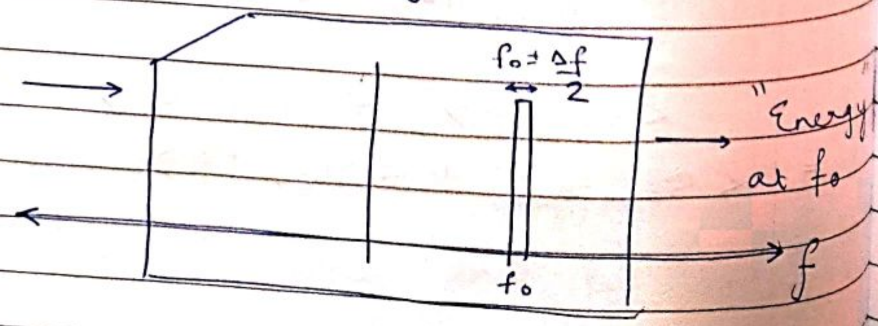




8.1.19

# Recap of Random Processes

- Power Spectral Density

Random Signal →



Page No.:
Date:

$s(t) \rightarrow$ A signal

$$s_{T_0}(t) = s(t)\, I_{\left[-\frac{T_0}{2}, \frac{T_0}{2}\right]} \rightleftharpoons S_{T_0}(f)$$

time limited version

$|S_{T_0}(f)|^2$ : Energy Spectral Density

$\lim_{T_0 \to \infty} \frac{1}{T_0} |S_{T_0}(f)|^2$ : PSD

→ Auto - correlation function

$$\hat{R}_s(\tau) = \frac{1}{T_0} \int_{-\infty}^{\infty} s_{T_0}(u)\, s_{T_0}^*(u-\tau)\, du$$

$$\bar{R}_s(\tau) = \lim_{T_0 \to \infty} \frac{1}{T_0} \int_{-\infty}^{\infty} s_{T_0}(u)\, s_{T_0}^*(u-\tau)\, d\tau$$

→ Wide Sense Stationary

$X(t)$ is WSS

$\mathbb{E}[X(t)]$ does not depend on $t$

$E[X(t)\, X^*(t+\tau)]$ ~~does~~ only depends on $\tau$

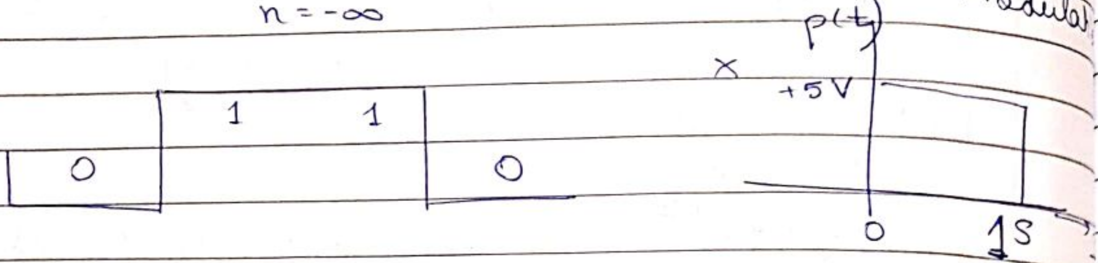
Random Binary Wave

$s\cancel{(t)} = b[n] = \begin{cases} \pm 1 & \text{w. prob } \frac{1}{2} \text{ each} \\ \text{iid} \end{cases}$

Keeping General waveform same but ~~by different numbers~~

$$s(t) = \sum_{n=-\infty}^{\infty} b[n]\, p(t - nT)$$

→ Linear Modulation




→ Sampling Theorem

- Any signal $s(t)$ band limited to $\left[-\frac{W}{2}, \frac{W}{2}\right]$ can be described completely by its samples $\left\{ s\left(\frac{n}{W}\right) \right\}_n$

$$s(t) = \sum_{n=-\infty}^{\infty} s\left(\frac{n}{W}\right) p\left(t - \frac{n}{W}\right)$$

Choosing $p(t) = \text{sinc}(Wt)$
$\Rightarrow$ No ISI

10.1.19

Inter symbol Inter.

$$s(t) = \sum_{n=-\infty}^{\infty} s\left(\frac{n}{W}\right) p\left(t - \frac{n}{W}\right)$$

→ finite set $\{0, 1\}$

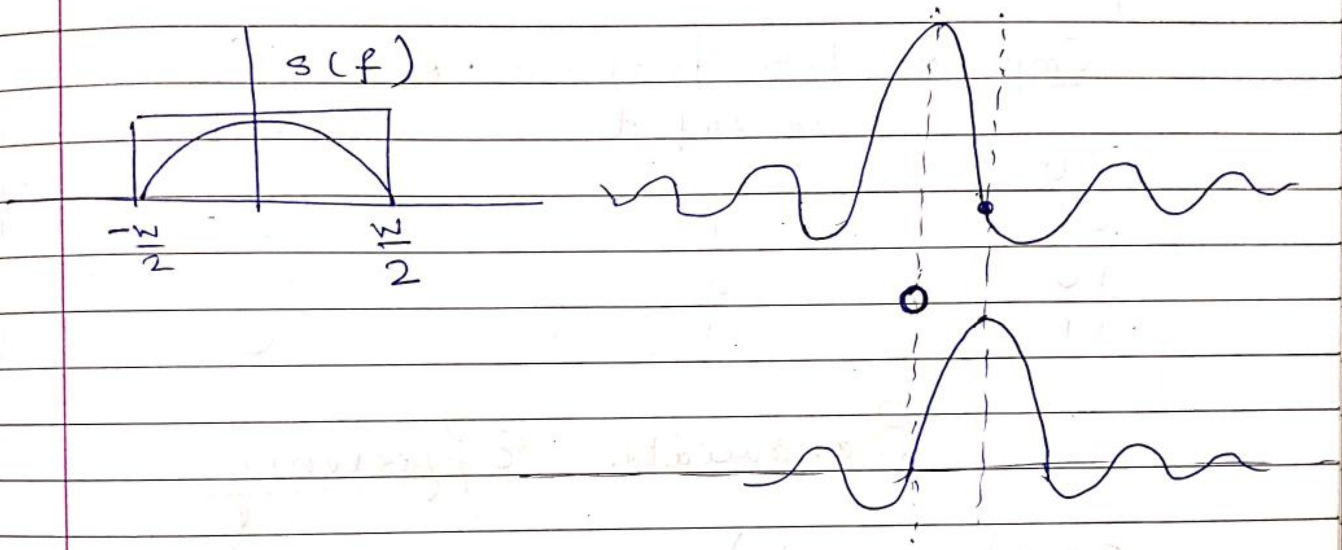
where $p(t) = \text{sinc}(Wt)$

$$\left\{ s\left(\frac{n}{W}\right) \right\}_{n \in \mathbb{N}} \rightarrow \text{finite set}$$

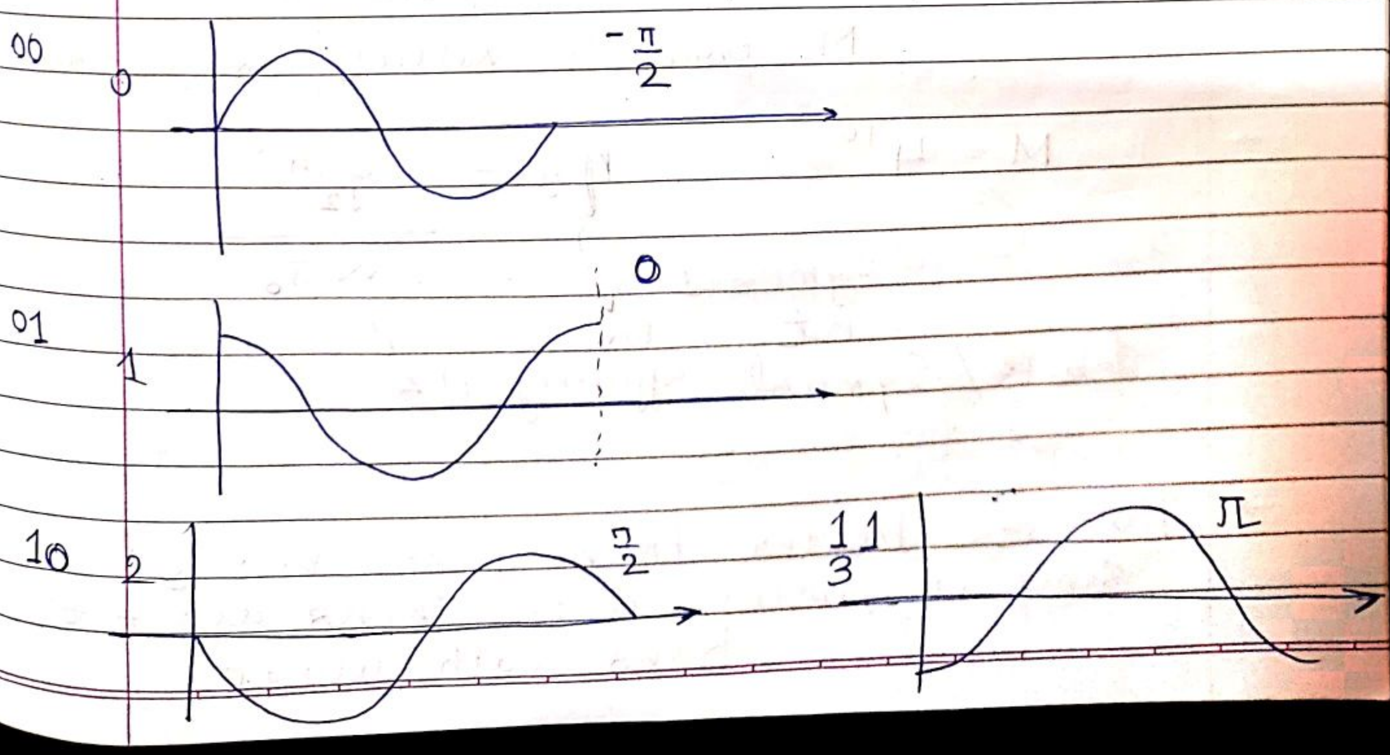
→ Say at Demod. we only sample at $t = \frac{k}{W}$ $k \in \mathbb{N}$

→ then only for ~~k≠n~~ k = k only 1 sample will be considered.

• If we restrict our consideration to a time interval "$T_0$", then our "dimension" is $W T_0$ complex values

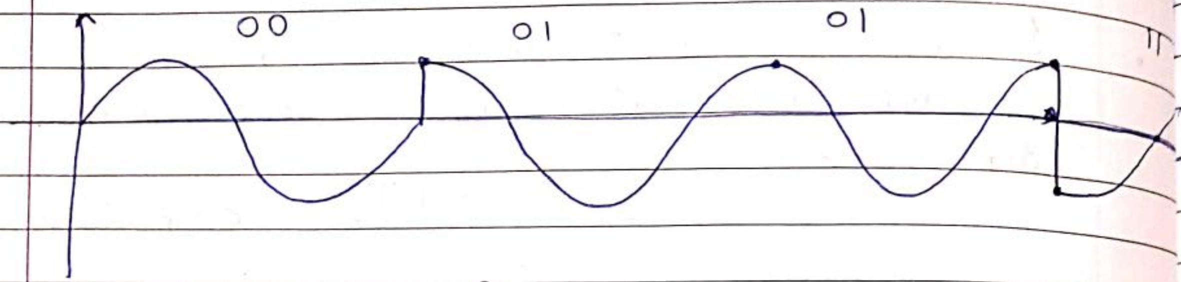



## QPSK Example



Page No.:
Date:

Q→ Find the complex BB equivalent of below signal




Cmplex BB framework

| | $\cos 2\pi f_c t$ | $\sin 2\pi f_c t$ | symbol |
|---|---|---|---|
| 00 | 0 | 1 | $-j$ |
| 01 | 1 | 0 | 1 |
| 10 | 0 | −1 | $j$ |
| 11 | −1 | 0 | −1 |

# Bandwidth Efficiency

$$\underbrace{s\left(\frac{0}{W}\right), s\left(\frac{1}{W}\right), \ldots, s\left(\frac{n}{W}\right)}_{M \text{ possible values/combinations}}$$

$M = 4^{10}$ ↳ 10 symbols sent, 2 bits/Symbol

$$\eta_B = \frac{\log_2 4^{10}}{W T_0}$$

$\eta_B$: B/W efficiency; $W$: Hz; $T_0$: sec

→ This is better than justs bits/s since it forces us to think about B/W Bandwidth usage

# Linear Modulation

$$u(t) = \sum_{n} b[n]\, g_{Tx}(t - nT)$$

$g$ → transmitter

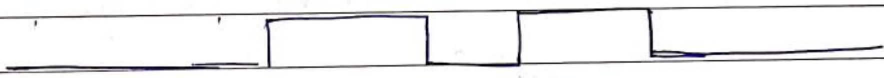
$Tx$ → Baseband waveform

$b[n]$ → Symbol from constellation

→ ~~Not~~ Example of non-linear non-Memory less system

- A 1 if there is a transition, a zero otherwise

0 0 0 1 1 0 1 1 0



13.1.19

## Summary

bits (0, 1)

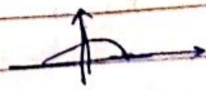
↓

Symbols (from finite set of complex numbers)

↓

Complex BB signal $p(t)$, rate, $\frac{1}{T}$, W

$$s(t) = \sum_{n=-\infty}^{\infty} b[n]\, p(t - nT)$$

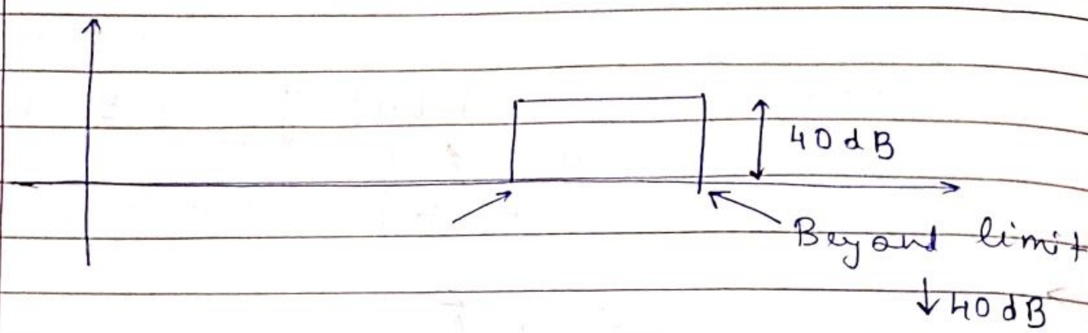


$s(nT) = b[n] \Rightarrow$ No ISI

↓ $\sqrt{2}\,\mathrm{Re}\left(s(t)\, e^{j2\pi f_c t}\right)$

Passband signal $(f_c)$ ⟶ Y

# Bandwidth Usage




$\{ b[0], b[1], \dots \}$
iid series $\Rightarrow$ uncorrelated

1. $b[n], b[m]$ if $n \neq m$ are uncorrelated
2. $\mathbb{E}[b[n]] = 0$

$$s(t) = \sum_{n=-\infty}^{\infty} b[n] \underbrace{g_{Tx}(t - nT)}_{\text{B/W Usage}}$$

→ Given above, Time averaged P.S.D is given by

$$S_u(f) = \frac{1}{T} \mathbb{E}\left[ |b[n]|^2 \right] |G_{Tx}(f)|^2$$

$G_{Tx}(f)$ is the CTFT of $g_{Tx}(t)$

→ Bandwidth $\Rightarrow$ 99% Energy containment

$$p(t) \rightleftharpoons P(f)$$

Find $W$ such that

$$\int_{-W}^{W} |P(f)|^2 \, df \Big/ \int_{-\infty}^{\infty} |P(f)|^2 \, df \geq 0.9$$

Ex 100 Mbps , QAM - 16

4 bits / symbol

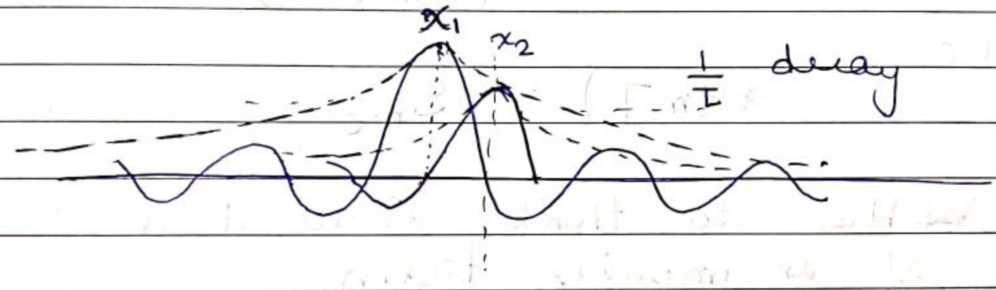
25 Msym/sec

W ?

14.1.19

Agenda

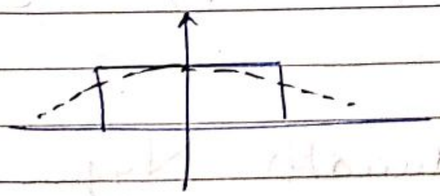
1. Limitations of sinc
2. ISI pre condition
3. Excess Bandwidth




→ If Clock synchronisation is ideal, then there is no Inter-Symbol Interference but practically not possible.

→ We need a decay faster than $\frac{1}{T}$

⇒ Non Brickwall Nyquist filter



$$\sum_{n=-\infty}^{\infty} b[n]\, g_{Tx}(t-nT)$$

$b[n] \rightarrow \boxed{g_{Tx}(t)} \rightarrow \boxed{g_c(t)} \rightarrow \boxed{g_{Rx}(t)} \rightarrow$ o/p = $x[n]$, $t = nT$

(rate $= \frac{1}{T}$)

$\sum b[n]\, \delta(t-nT)$

→ What do we get at output?

$$\sum b[n]\left(g_{Tx} * g_c * g_{Rx}\right)(t-nT)$$

→ Condition for ISI Avoidance $z[n]=$

$$x(t) = \left(g_{Tx} * g_c * g_{Rx}\right)(t)$$

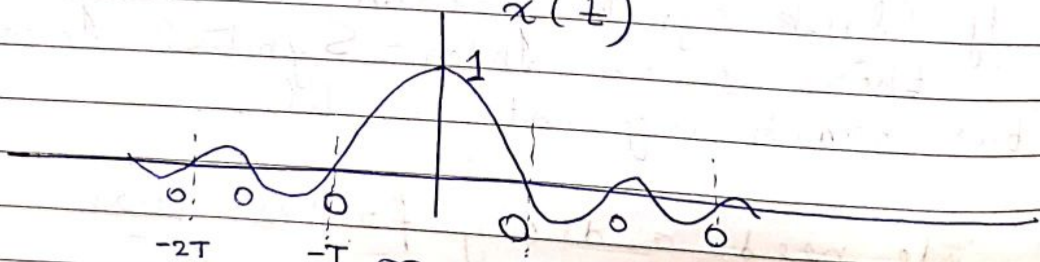
Such that,

$$x(mT) = \begin{cases} 1 & \text{if } m=0 \\ 0 & \text{else} \\ m \in \mathbb{Z} \end{cases}$$

i.e

$$x(mT) = \delta_{m0}$$

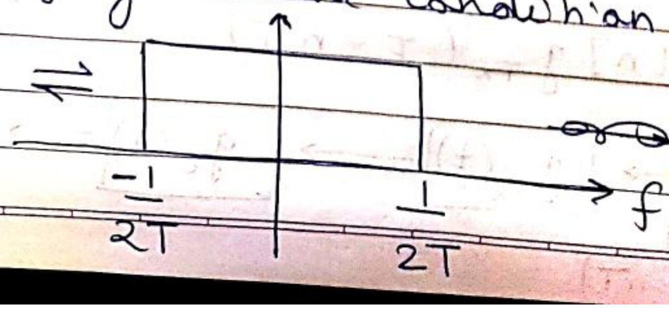
→ How to think about it in terms of an impulse train




$$x(t) \sum_{n=-\infty}^{\infty} \delta(t-nT) = \delta(t)$$

→ The minimum bandwidth Nyquist pulse satisfying above condition is,

$\text{sinc}\left(\frac{t}{T}\right) \rightleftharpoons$



Also < b.w ⇒ slower decay
⇒ ISI becomes more of a problem

→ What if we have < than this b/w

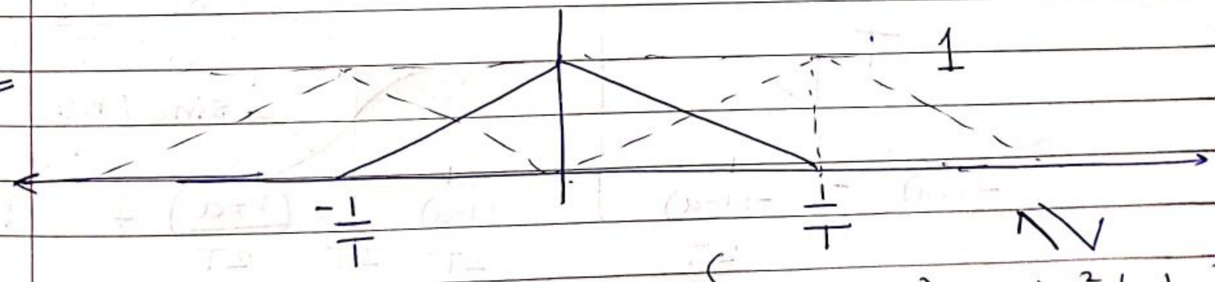
① We need impulses in time domain for perfect sampling

② That means there should be flat F.T After creating the copies as per the equation

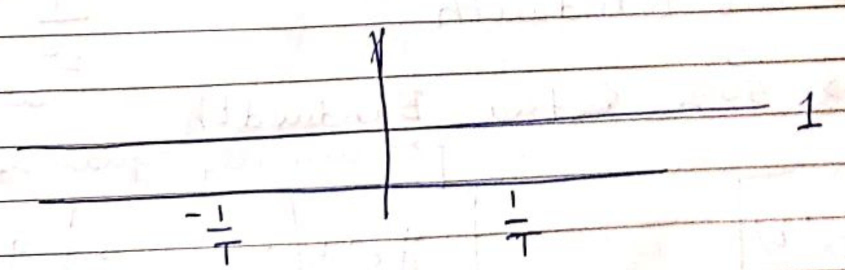
$$\frac{1}{T} \sum_{k=-\infty}^{\infty} X\left(f + \frac{k}{T}\right) = 1$$

③ Hence if we go < $\frac{1}{T}$ b/w we will have gaps ⇒ No time domain impulses ⇒ ISI ⇒ Lossy reconstruction

ex

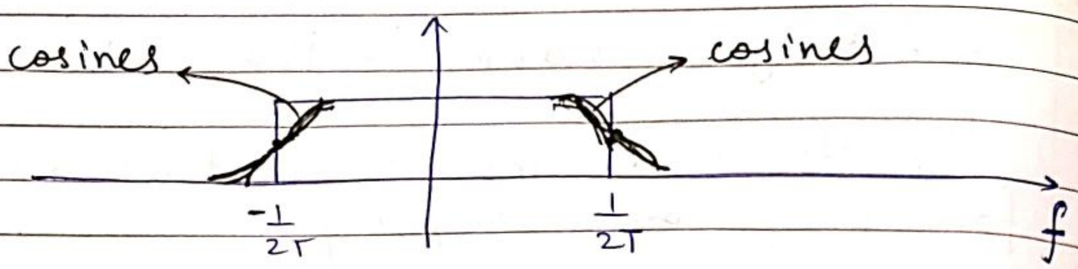


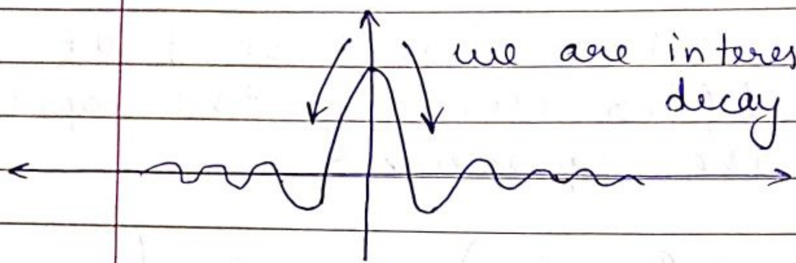

$x(t) = \sin^2\left(\frac{t}{T}\right)$




→ Here aliasing happens but its ok since we don't care about Analog waveform

# Raised Cosine Pulse



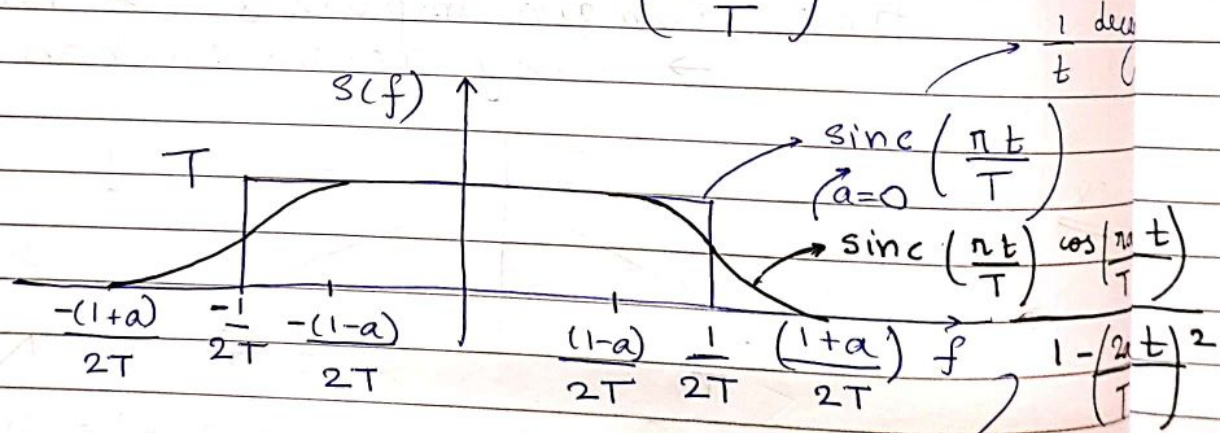



we are interested in the decay rate $\propto \frac{1}{\alpha}$

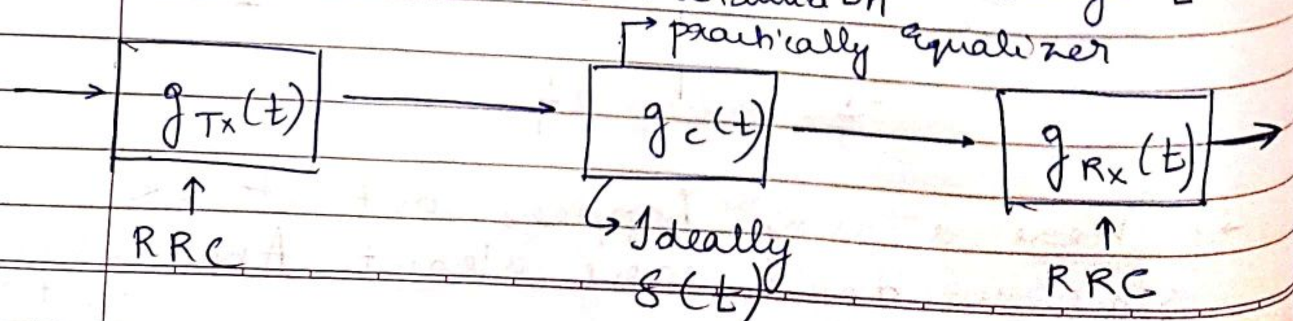
~~So~~ RC pulse in time domain,

$$s(t) = \frac{\text{sinc}\left(\frac{t}{T}\right)\cos\left(\frac{\pi a t}{T}\right)}{1-\left(\frac{2at}{T}\right)^2}$$

17.1.19

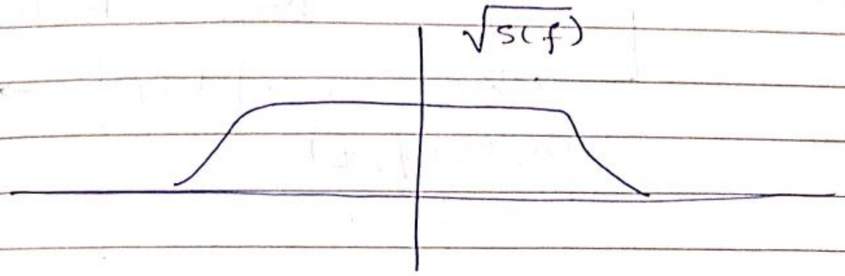



a → excess Bandwidth
a = 0.5
⇒ 50% Extra Bandwidth



→ Root raised cosine pulse : $\sqrt{S(f)}$

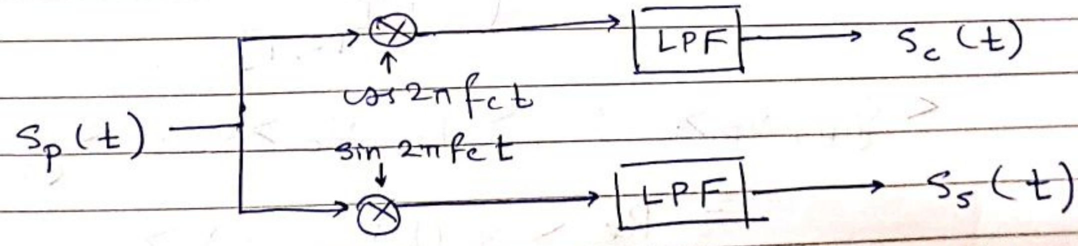
→ The Square root is in frequency domain




$$S(f) = \begin{cases} T & -\frac{(1-a)}{2T} \le f \le \frac{1-a}{2T} \\ \left[1 - \sin\left(\left(|f| - \frac{1}{2T}\right)\frac{\pi T}{a}\right)\right] \times \frac{T}{2} & \end{cases}$$

$\xrightarrow{}$ frequency & phase

**Coherent Vs Non-Coherent Demod**




→ In general there is a 0 frequency offset

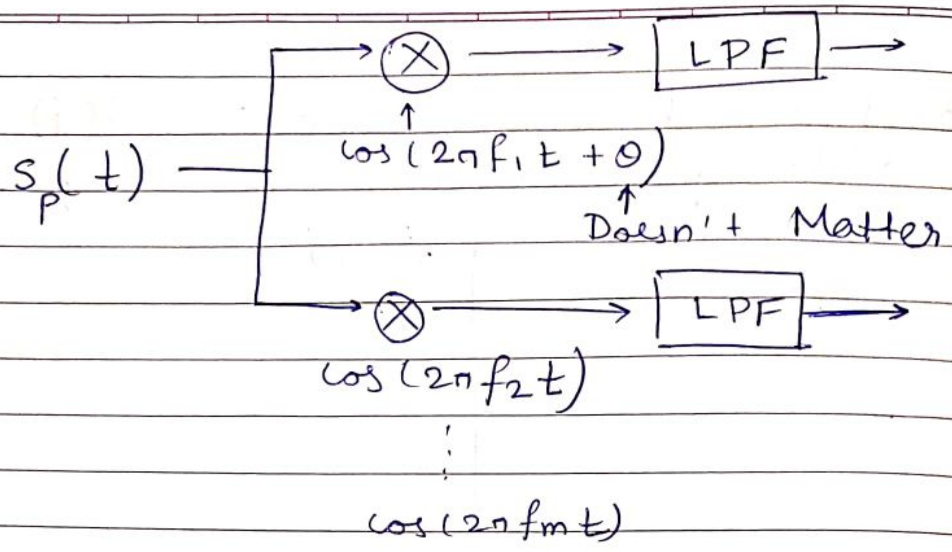
→ No frequency

**Non-Coherent Demodulation** (Phase has no Inform.)

ex M-ary FSK

$$S_i(t) = \cos(2\pi f_i t)\, I_{[0,T]}$$

$$i = 1, 2, 3, \dots, M$$




$$s_p(t) \xrightarrow{\text{Channel}} y_p(t) = s_p(t) + \text{Noise}$$

$$s_p(t) = \mathrm{Re}\{\sqrt{2}\, s(t)\, e^{j2\pi f_c t}\}$$

$$s(t) \rightarrow s_c(t) + j s_s(t)$$

→ Choices for $s(t)$

$$s(t) = \begin{cases} I_{[0,T]} & -j\, I_{[0,T]} \\ -I_{[0,T]} \\ j I_{[0,T]} \end{cases}$$

$$\langle y_p, s_p \rangle = \langle y_c, s_c \rangle + \langle y_s, s_s \rangle$$

$$= \mathrm{Re} \langle y(t), s(t) \rangle$$

⇒ Goal : Maximize

over $i$ $\quad \mathrm{Re} \langle y(t), s_i(t) \rangle$

- In non-coherent, we don't care about the phase offset,
  Even if $y(t) = s(t)e^{j\theta}$
  $\langle y, s \rangle$ will still be maximum when $y = ks$

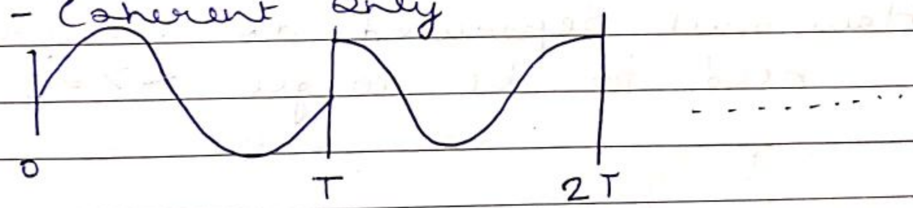
- Problem: Because $\langle y, s \rangle = \|s\|^2$ when maximum we can't get the sign

21.1.19

## Coherent Vs Non-Coherent Modulation

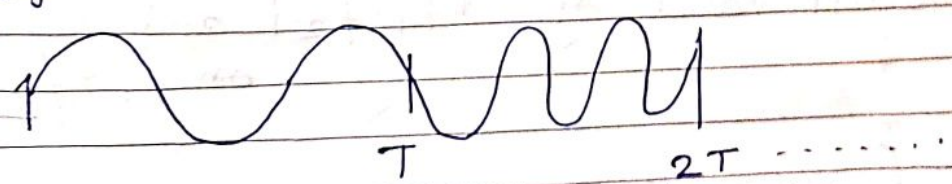
- Coherent Modulation - There can be information in the phase

PSK - Coherent only




For FSK, Detection $\Rightarrow$ find $i$ that Maximizes $\mathrm{Re}(\langle y, s_i \rangle)$

- Non-Coherent Modulation
  eg - ~~Non~~ FSK




→ FSK is good candidate for Coherent Modulation ...... because

Page No.:
Date:

→ ex Non-Coherent PAM
→ Use Envelope Detector

## Coherent FSK - 2FSK

$$0 \rightarrow s_1(t) = \cos 2\pi f_1 t \, I_{[0,T]}$$
$$1 \rightarrow s_2(t) = \cos 2\pi f_2 t \, I_{[0,T]}$$

$$f_1 \neq f_2$$

→ $y(t) = s(t) + \text{Noise}$

• To detect, ask whether $y(t)$ closer to $s_1$ or $s_2$?

$$\int_0^T y(t) s_i(t)\, dt \text{ larger for which } f?$$

→ How much seperated do the frequencies need to be to get $\langle\rangle = 0$ over T

$$\int_0^T \cos 2\pi f_1 t \cos 2\pi f_2 t \, dt$$

$$= \frac{1}{2} \int_0^T \cos(2\pi(f_1+f_2)t) + \cos(2\pi(f_1-f_2)t)\, dt$$

Smallest value of $|f_1 - f_2|$ and condition on $f_1 + f_2$

$$\int_0^T \cos(2\pi(f_1-f_2)t)\, dt = 0$$

$\Rightarrow$ ~~$f_1 \neq f_2$~~

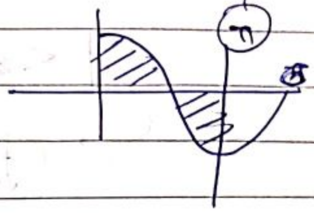
$$T_{min} = \frac{1}{2|f_1 - f_2|}$$

$$\Rightarrow |f_1 - f_2| = \frac{1}{2T}$$

$$2\pi(f_1 - f_2)T = \pi$$

Any $0 \to \pi$ Interval won't work, so assuming that start $t=0$ at zero phase

$$\int_0^{\pi} \cos x \, dx = 0$$



Also, another condition is,

$$2\pi(f_1 + f_2)T = m\pi$$

so that $\int_0^T \cos(2\pi(f_1+f_2)t)\,dt = 0$

→ In above assumption is that template to be matched is in phase with received signal

# Non-Coherent FSK

2-FSK

$0 \rightarrow s_1(t) = \cos(2\pi f_1 t + \phi_1)\, I_{[0,T]}$

$1 \rightarrow s_2(t) = \cos(2\pi f_2 t + \phi_2)\, I_{[0,T]}$

→ Templates Available,

$\left.\begin{array}{l}\cos(2\pi f_1 t\, I_{[0,T]}) \\ \cos(2\pi f_2 t\, I_{[0,T]})\end{array}\right\}$ Won't work

→ Solution, treat the incoming signal as a complex signal,

$\cos(2\pi f_1 t + \phi)\cos 2\pi f_1 t = ?$

& $\cos(2\pi f_1 t + \phi)\sin 2\pi f_1 t = ?$

$|\langle y, s_1 \rangle|$

→ effectively find this,

Now, New Requirement

$$\int_0^T \cos(2\pi f_1 t + \phi)\cos(2\pi f_2 t)\,dt = 0$$

$$|f_1 - f_2| = \frac{1}{T}$$

## Differential Modulation in PSK

- Till now, $\underbrace{\text{FSK, ASK = PAM,}}_{\text{Non-Coherent possible}}$ $\underbrace{\text{PSK, QAM}}_{\text{Coherent Detection}}$

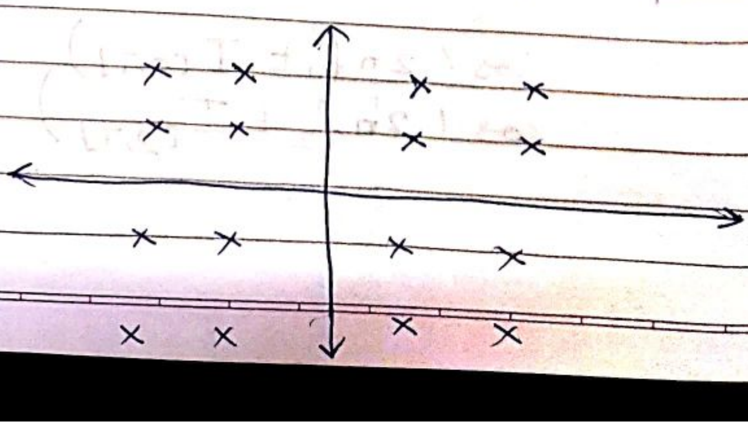
$$s_1(t) = e^{j\phi_1} I_{[0,T]}$$

$$s_2(t) = e^{j\phi_2} I_{[0,T]}$$

$$\vdots$$

$$s_M(t) = e^{j\phi_M} I_{[0,T]}$$

ex 16 - QAM



Pre Agreed about 1st Sym

Q→ For M-FSK what is the number of bits/sym.

(i) bits/sym $= \log_2 M$

(ii) What is the dimension of this space?
= M : Since there are M orthogonal vectors possible

Recap, Spectral efficiency

$$\eta_B = \frac{\log_2 M}{M} \Rightarrow \text{Don't use large } M \text{ for FSK}$$

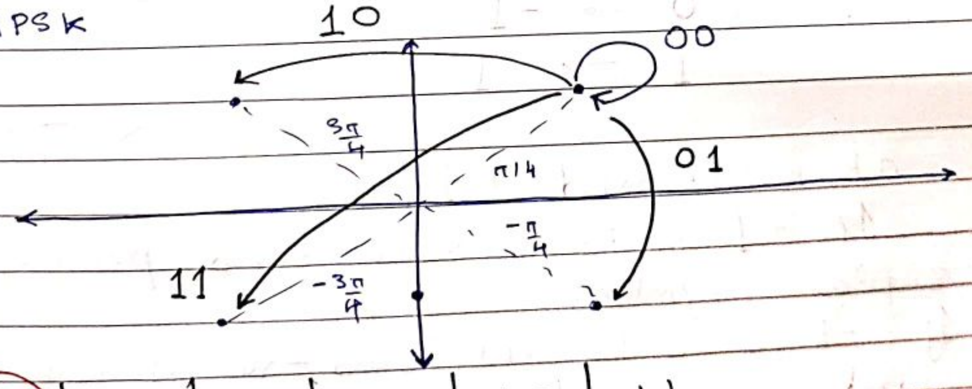
## Differential Modulation

$g_{Tx} \longrightarrow g_c \longrightarrow g_{Rx}$

$h\,\delta(t)$

↳ unknown complex number

QPSK




| 00 | 00 | 11 | 01 | 10 | 11 | |
|---|---|---|---|---|---|---|
| $e^{j\pi/4}$ | $e^{j\pi/4}$ | $e^{-j\frac{3\pi}{4}}$ | | | | ) Symbol Sent |

Pre Agreed about 1st Symbol

→ At the Receiver, perform $y^*[i-1]\, y[i]$
$= |h|^2 (\ )(\ )$

Phase of h removed

Page No.:
Date:

→ Phase of $h$ is like rotation and phase of $h$ may change over time

$y[0] = h e^{j\pi/4}$ + noise
$y[1] = h e^{j\pi/4}$ + noise
$y[2] = h e^{-j3\pi/4}$ + noise

$y^*[1]\, y[2] = |h|^2 e^{j\pi}$

22.1.19

## Differential PSK

→ BPSK, $h$ doesn't change much over 1 symbol cycle.

$b \rightarrow \boxed{g_c(t)} \rightarrow$

$y = h\, b[n]$ + noise

$0 \rightarrow -1$
$1 \rightarrow 1$

1. at $n = 0$ send $+1$
2. If $+1$ is to be sent, send previous ~~signal~~ symbol
If $-1$ is to be sent, send $-$ve of previous symbol

Added

0 →

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Bits | +1 | 0 | 1 | 1 | 0 | 1 | 0 | 1 |
| Mapping | 1 | -1 | -1 | -1 | 1 | 1 | -1 | -1 |

→ How is it equivalent to the other model?

$$y[0] = h + \text{noise}$$
$$y[1] = -h + \text{noise}$$

$$y^*[0]\, y[1] = -|h|^2$$

↪ could also have been $|h|^2$ instead

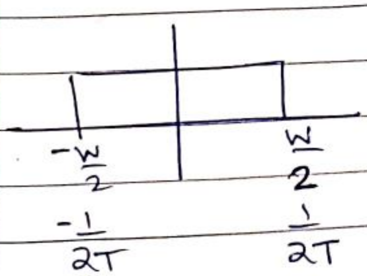
- Previous example Redone

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Bits | : | | 0 | 1 | 1 | 0 | 1 | 0 | 1 |
| Mapping Symbol | : | 1 | -1 | 1 | 1 | -1 | 1 | -1 | 1 |
| Tx symbol | : | 1 | -1 | -1 | -1 | 1 | 1 | -1 | -1 |

Mirrored

Where + means Multiplication/ Adding the phases

→ One of M symbols,
i.e send one of,
$s_1(t), s_2(t), \ldots, s_M(t)$
- power
- bandwidth




Maximum Data Rate : W
min symbol period : $\frac{1}{W}$

→ ex Consider BPSK at $\frac{1}{T}$ symbols/sec

$$s_1(t) = I_{[0,T)} \qquad s_2(t) = -I_{[0,T]}$$

Page No.:
Date:

$$\psi_1(t) = I_{[0,T]}$$

$$s_1(t) = \psi_1(t)$$
$$s_2(t) = -\psi_1(t)$$

ex Basis : $\frac{I_{[0,T]}}{\sqrt{T}}$

eg.

→ QPSK at $\frac{1}{T}$ sps

$$s_1(t) = e^{j\pi/4} I_{[0,T]} \quad (\to I)$$
$$s_2(t) = e^{j3\pi/4} I$$
$$s_3(t) = e^{-j3\pi/4} I$$
$$s_4(t) = e^{-j\pi/4} I$$

$(s_1 = -s_3)$
$(s_2 = -s_4)$
→ Hence there is some redundancy in the set

→ What can be a basis for QPSK?

$$\psi_1(t) = \frac{I_{[0,T]}}{\sqrt{T}}$$
$$\psi_2(t) = \frac{j\, I_{[0,T]}}{\sqrt{T}}$$

OR just $(s_1, s_2)$

24.1.19

$\psi_1, \psi_2, \vdots, \psi_K$ defined in $[0,T]$

Satisfy $\langle \psi_i, \psi_j \rangle = \delta_{ij}$

⇒ K dimensional vector space = 2K complex dimensions

1 waveform gives us 2 dimensions
Because of Complex Base-Band
representation

eg. $\psi_1$ "one dimensional"
$\Rightarrow$ 2 complex dimensions

$$\cancel{*} \quad \underset{\text{real}}{x_1 + j x_2} \rightarrow x_1 \psi_1(t) + j x_2 \psi_1(t)$$

$$\downarrow$$

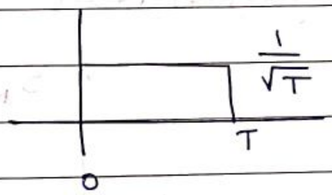
$$x_1 \psi_1(t) \cos 2\pi f_c t - x_2 \psi_2(t) \sin 2\pi f_c t$$

Representation either $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ real

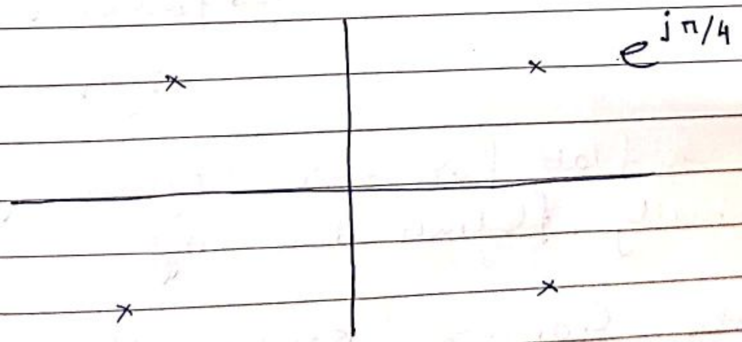
OR

$$[x_1 + j x_2]$$

Where, $\psi_1(t) =$ (say)




QPSK = QAM - 4




$$\frac{1}{\sqrt{2}} \left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \end{bmatrix} \begin{bmatrix} -1 \\ -1 \end{bmatrix} \right\}$$

$K \rightarrow 2K$ Complex

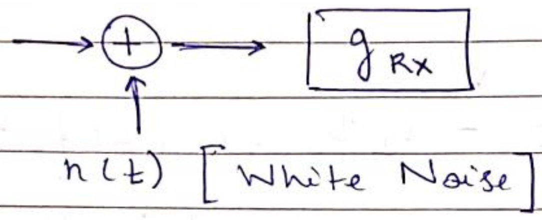
24.1.19

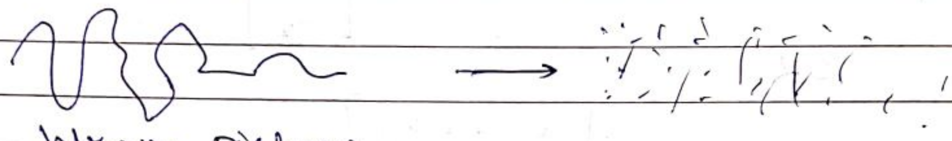
# Gaussian Random Variable / Processes

Page No.:
Date:

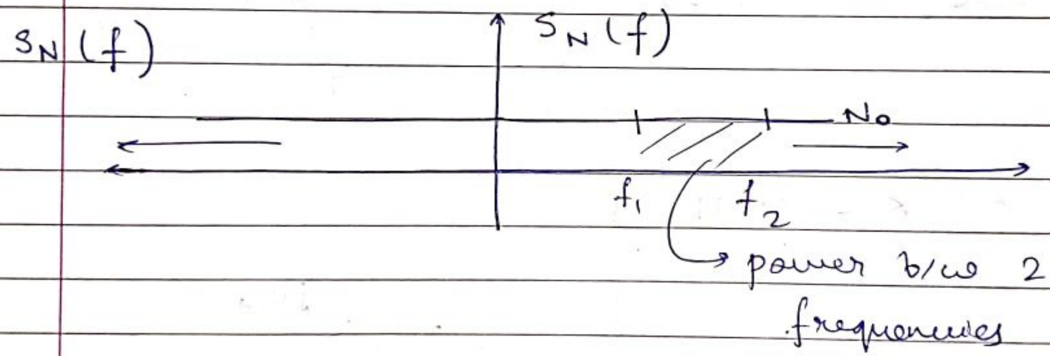
→ Gaussian Random Variable

$$f_X(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2}$$



Wrong picture of white Noise




→ PSD is flat for our region of Interest, eventually flatness dies off.

→ Jointly Gaussian Random Vector
$GR\vec{V}$

$\vec{X}$ is $GR\vec{V}$ iff

$\vec{a}^T \vec{X}$ is a GRV $\forall\ \vec{a} \neq \vec{0}$

Q → If $\left[\cancel{X}\right.$ $X = N(0,1)$
$Y = \alpha X$ $\qquad \alpha = \begin{cases} 1 & \text{w.p } 1/2 \\ -1 & \text{w.p } 1/2 \end{cases}$

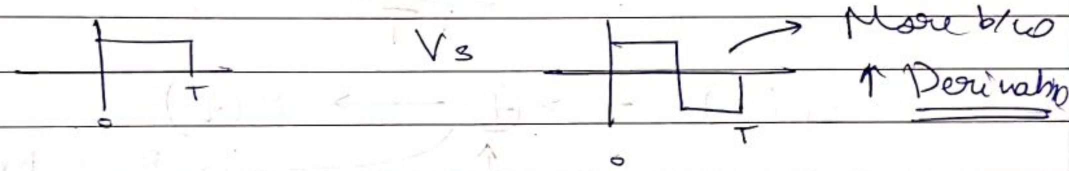
then $\begin{bmatrix} X \\ Y \end{bmatrix}$ is not $GR\vec{V}$

→ ACR function of WSS $n(t)$ : Noise

$$\cancel{\mathbb{E}} \quad \mathbb{E}[n(t)] = 0$$

$$\mathbb{E}[n(t)\, n(t-\tau)] = N_0\, \delta(\tau)$$

Side




As Carrier $\psi(t)$

BPSK

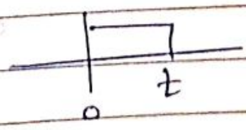
$0 \rightarrow -\sqrt{P} \xrightarrow{s_1(t)}$ $\qquad s_1(t) = \sqrt{P}\,\psi_1(t)$

$1 \rightarrow \sqrt{P} \longrightarrow$ $\qquad s_2(t) = -\sqrt{P}\,\psi_1(t)$

$$\psi_1(t) = \frac{I_{[0,T]}}{\sqrt{T}}$$

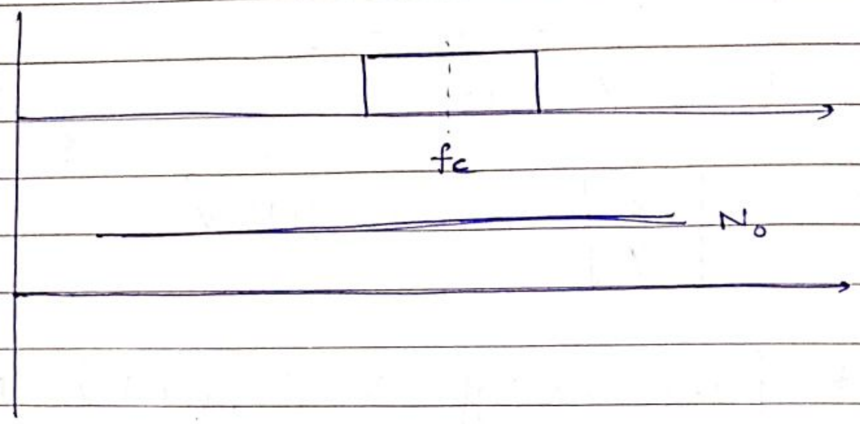
$s_1(t) \longrightarrow \oplus \longrightarrow r(t)$, with $n(t)$ into $\oplus$

→ Consider $\psi(t) =$




then Spectrum will be a sinc, but take as box

Page No.:
Date:




→ $N_0$ or $N_0/2$

$$\psi(t) = \frac{I_{[0,T]}}{\sqrt{T}}$$

$$s_1(t) \longrightarrow \oplus \longrightarrow r(t)$$

→ Real AWGN channel

$n(t)$ → real process, PSD is $N_0$

$$r(t) \longrightarrow \otimes \longrightarrow \boxed{\int_0^T} \longrightarrow \text{number}$$

$\cancel{\psi_1(T-t)}$, $\psi_1(t)$

$$\langle r, \psi_1 \rangle = \int_0^T s_i(t)\,dt + \underbrace{\int_0^T n(t)\,dt}_{\text{random variable}}$$

$$r(t) = s_i(t) + n(t)$$

Page No.:
Date:

$$\mathbb{E}\left[\left(\int_0^T n(t)\,d\tau\right)^2\right]$$

$$= \mathbb{E}\left[\int_0^T n(t_1)\,dt_1 \int_0^T n(t_2)\,dt_2\right]$$

$$= \int_0^T \int_0^T N_0\,\delta(t_1 - t_2)\,dt_1\,dt_2$$

$$= \int_0^T N_0\,dt_2 = (N_0 T)$$

## QPSK and Noise