

Course Code : EE 614

Department: Electrical Engineering

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## Lecture 1

# Syllabus

1. Review of transmission line theory
2. Scattering parameters of two-port networks, microwave transistors
3. Matching network, Flow graph theory
4. Introduction to CAD microwave circuit simulation
5. Amplifier design techniques, unilateral and bilateral case
6. Amplifier stability
7. Low noise amplifier design
8. Broadband amplifier design
9. Power amplifier design & Linearization
10. One-port negative resistance oscillators
11. Two-port negative-resistance oscillators
12. Oscillator configurations

# Reference Books

1. Microwave Transistor Amplifier: Analysis and Design, Gonzalez Guillermo, Prentice Hall, 1984.
2. Microwave Circuit Analysis and Amplifier Design, Samuel, Y. Liao, Prentice Hall, 1987.
3. High-Frequency Amplifier, Ralph S. Carson, Wiley-Interscience, 1982.

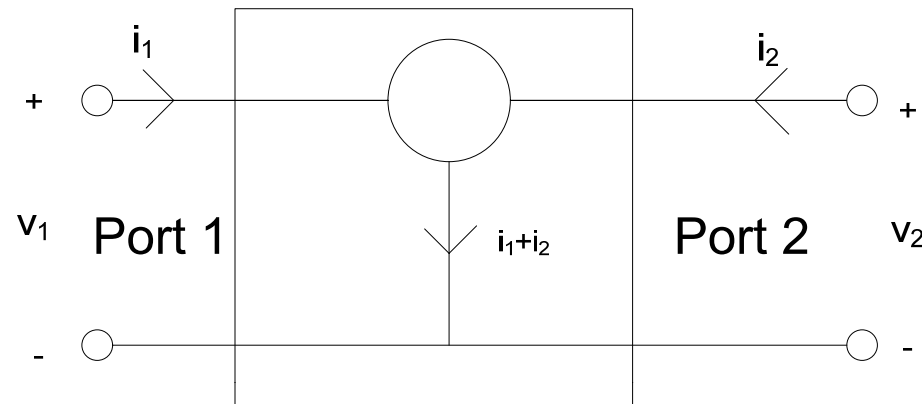
# Chapter 1

## Representation of 2 port networks

# Introduction

- The design of amplifiers and oscillators will usually be initiated using small signal parameters (linear regime: sufficiently low power applied)
- Large signal models will however be required for the design of power amplifiers and oscillators or to analyze the linearity of an amplifier
- A network analyzer is the most commonly used equipment for high frequency measurement.

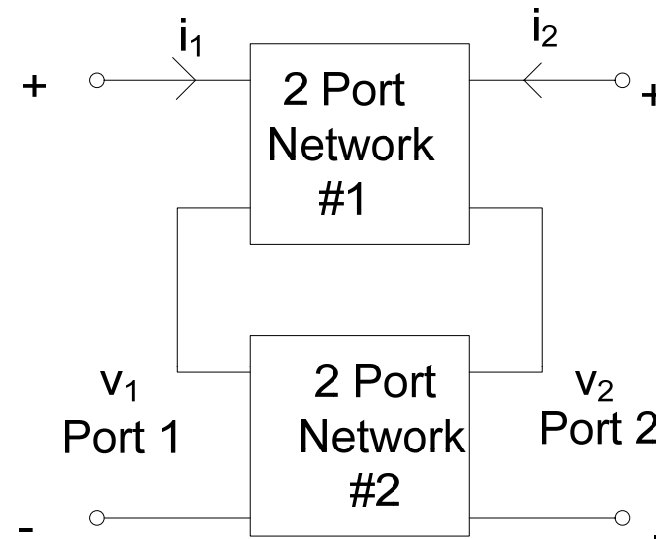
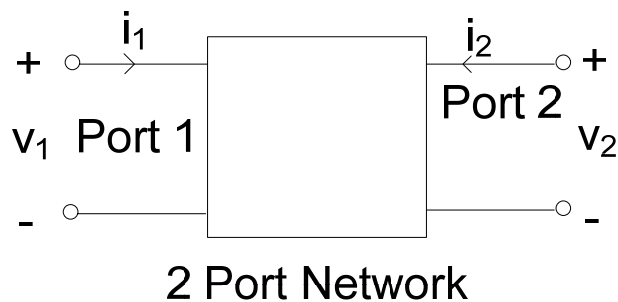
# Introduction



## 2-Port Network

- The small-signal response of a transistor (3 terminals) can be represented by a 2-port network (4 terminals). Various small-signal parameters can be used
  - z parameters
  - y parameters
  - h parameters
  - ABCD parameters

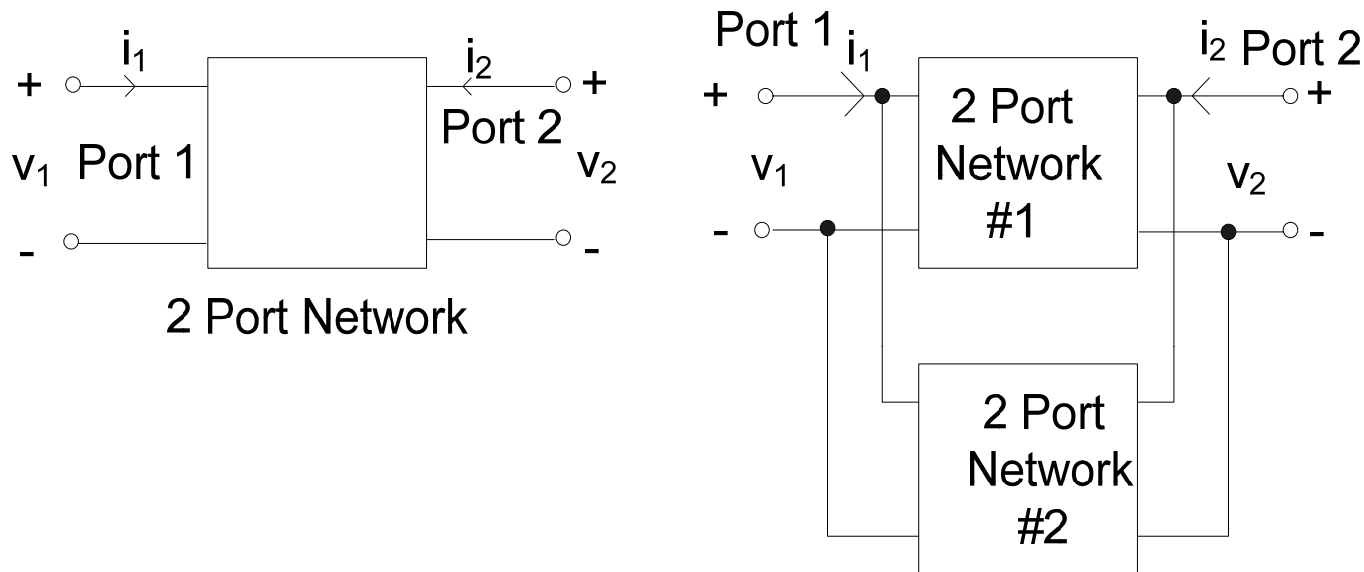
# Z Parameters



the current exiting network 1 will enter port 2 so it is a series

- $$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = Z \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$
- requires an RF open
- Useful for two 2 - port networks in series :  $Z_{1+2} = Z_1 + Z_2$

# Y parameters

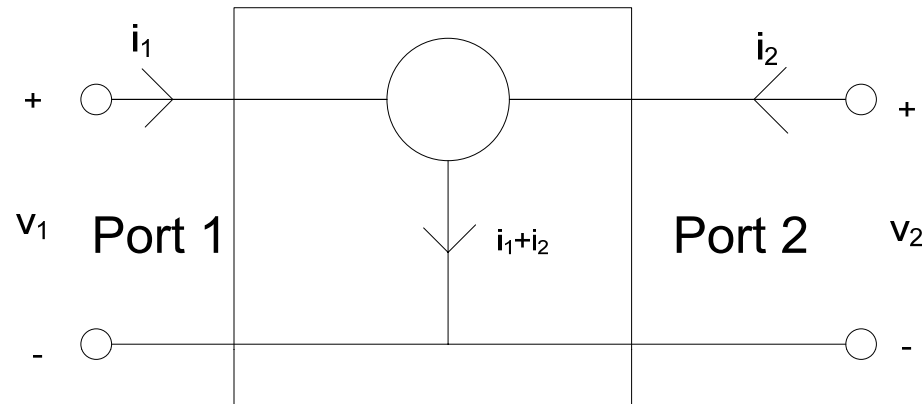


both network have same  
have same voltage  
difference so it is in  
parallel

- $$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = Y \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$
- requires an RF short
- Useful for two 2 - port networks in series :  $Y_{1+2} = Y_1 + Y_2$



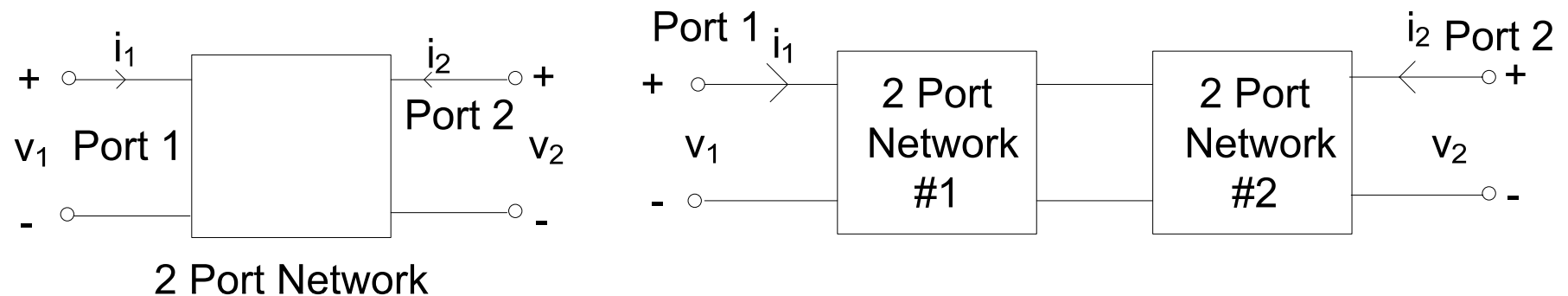
# h parameters



2-Port Network

- $$\begin{bmatrix} v_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} i_1 \\ v_2 \end{bmatrix}$$
- requires both an RF short and RF open
- traditionally used with bipolar transistors

# ABCD parameters



- $$\begin{bmatrix} v_1 \\ i_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} v_2 \\ -i_2 \end{bmatrix} = N \begin{bmatrix} v_2 \\ -i_2 \end{bmatrix}$$
- cannot be measured
- Useful for 2 - port networks in cascade :  $N_{1+2} = N_1 \times N_2$

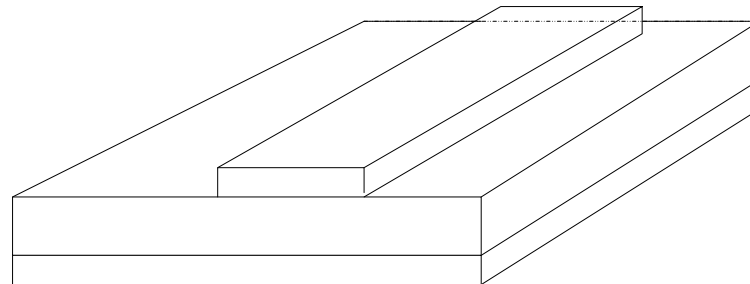
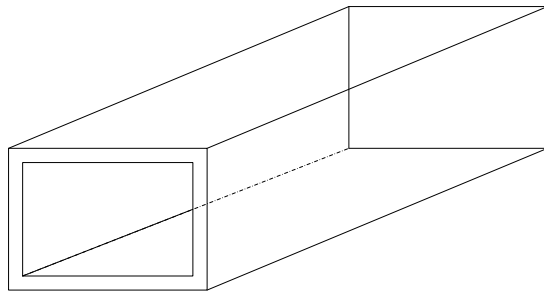
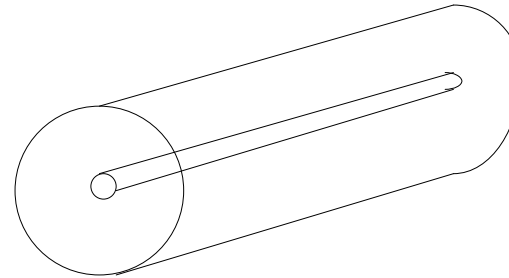
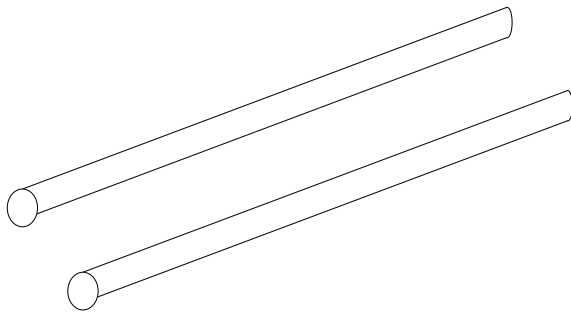
# Problems with Conventional Small-Signal Parameters

- Require an RF short or RF open which are difficult to realize at high frequency over a broad bandwidth.
- often device oscillates when not loaded with a resistive termination.

# S Parameters used since 1970

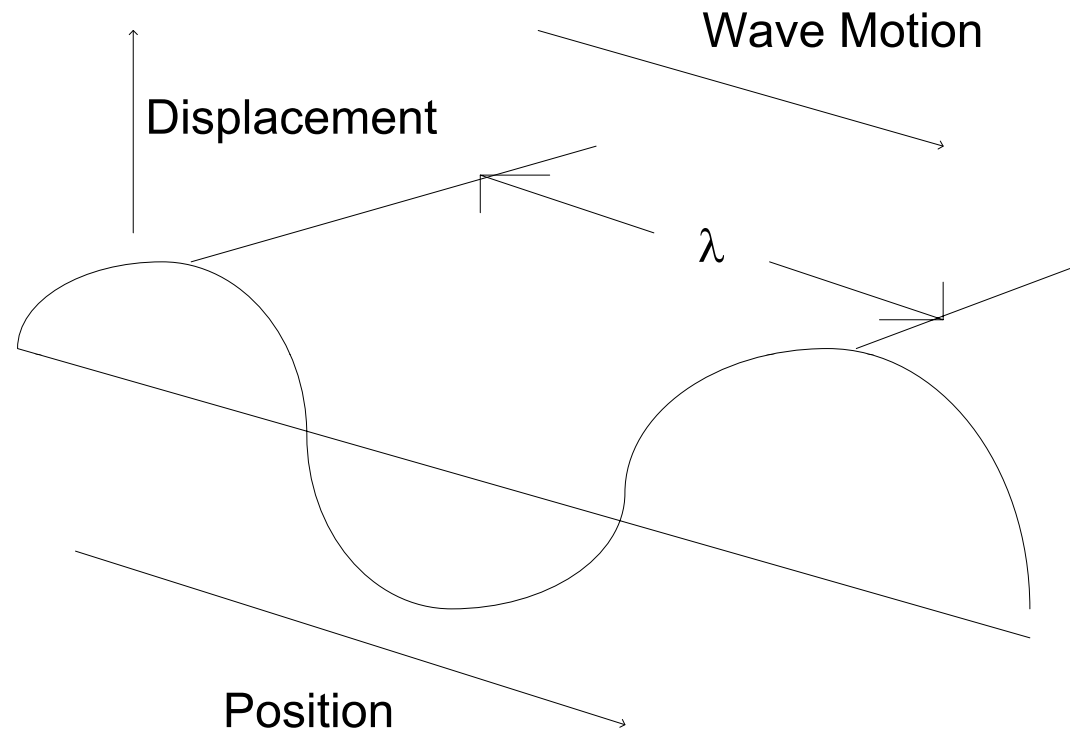
- S parameters can be measured with a network analyzer.
- They have a natural relation with the flow of power.
- S - parameters are readily represented by flow graph.
- The measurement of S - parameters relies on 50 ohm resistive terminations usually device does not oscillate for such terminations.
- Devices are measured in the medium in which they will be used e.g., microstrip lines.

# Transmission line concepts



- RF signals in large circuits need to be guided to propagate
- The return path (grounded in unbalanced case) is critical

# RF signal as waves

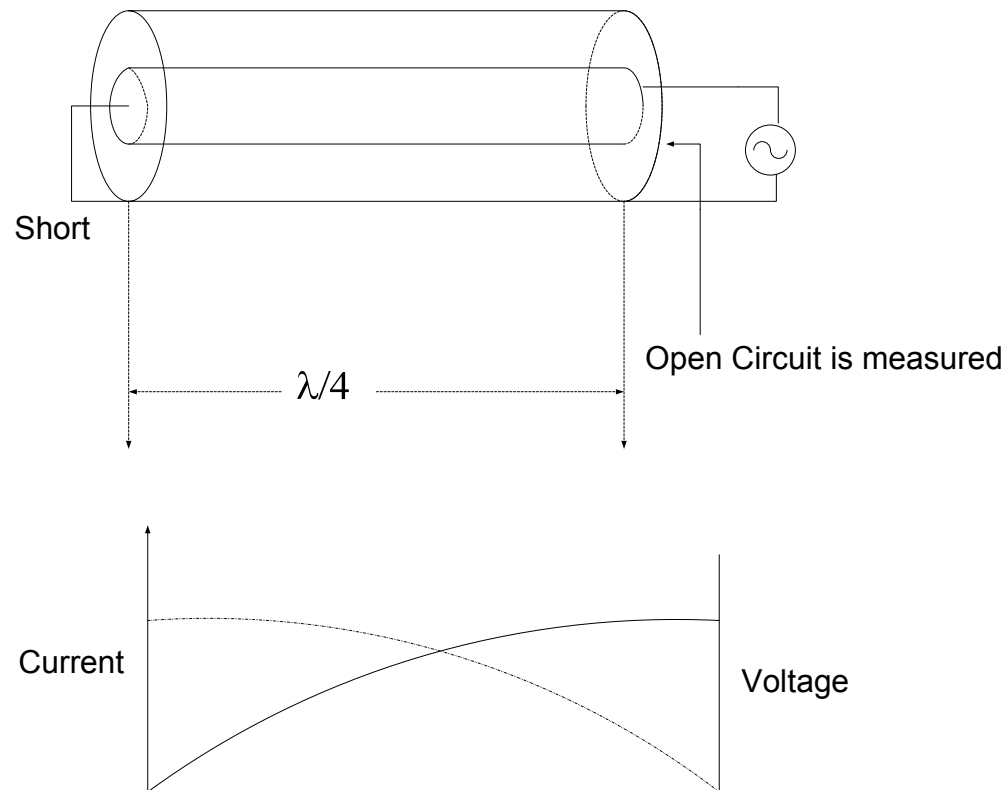


Speed of light

$$v_p = \frac{c}{\sqrt{\epsilon_r}} = \frac{\lambda}{T} = \lambda f$$

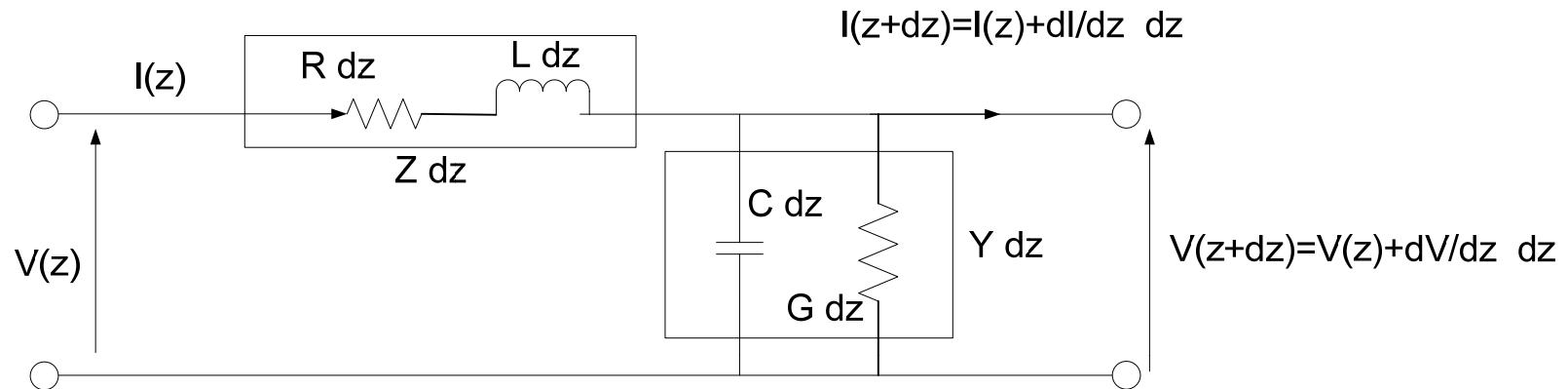
# Distributed Effects

## Coaxial line



A short can be transformed into an open

# Distributed Circuit Model



For the section  $dz$  of the above distributed circuit we can write

$$V(z + dz) = V(z) + \frac{dV}{dz} dz = V(z) - I(z) Z dz$$

$$I(z + dz) = I(z) + \frac{dI}{dz} dz = I(z) - V(z + dz) Y dz$$

Combining these two equations we obtain

$$\frac{d^2 V(z)}{dz^2} dz = ZY V(z) \text{ and } \frac{d^2 I(z)}{dz^2} = ZY I(z),$$

$$Z = R + j\omega L, Y = G + j\omega C$$



# Solution of the Telegraphist wave equation

$$V(z) = V^+ e^{-\gamma z} + V^- e^{\gamma z}$$

$$I(z) = \frac{1}{Z_0} (V^+ e^{-\gamma z} - V^- e^{\gamma z})$$

$$\text{with } \gamma = \sqrt{ZY} \text{ and with } Z_0 = \sqrt{\frac{Z}{Y}}$$

*We used the identity  $I(z) = -\frac{1}{Z} \frac{dV(z)}{dz}$  to relate  $I(z)$  to  $V(z)$*

# Low loss case

For a low loss circuit ( $R=G=0$ ) we have  $Z = j\omega L$  and  $Y = j\omega C$  so

The voltage and current waves are

$$V(x) = V^+ e^{-j\beta x} + V^- e^{j\beta x}$$

$$I(x) = \frac{1}{Z_0} (V^+ e^{-j\beta x} - V^- e^{j\beta x})$$

with

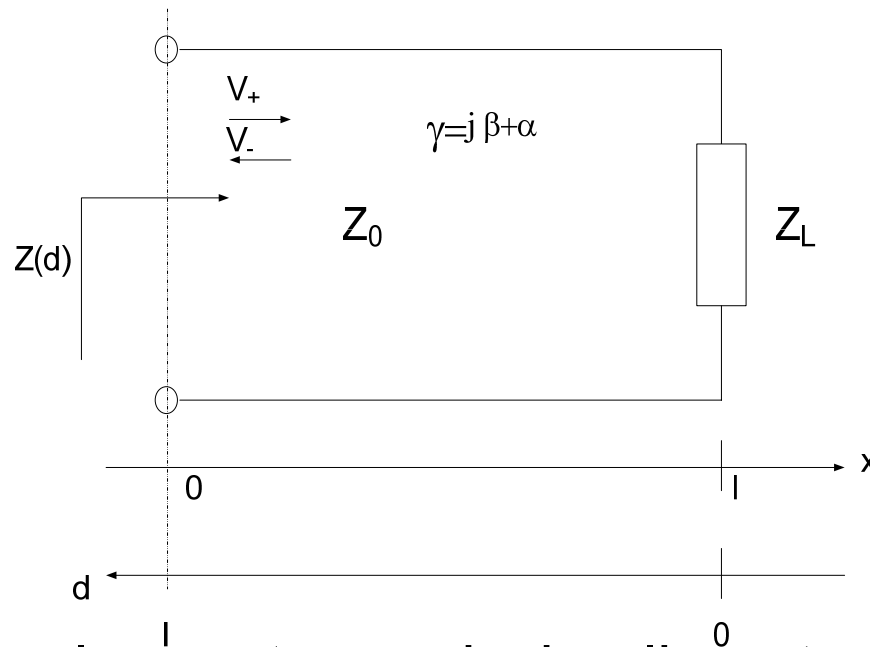
$$\gamma = \sqrt{-\omega^2 LC} = \pm j\omega\sqrt{LC} = \pm j\beta$$

and

$$Z_0 = \sqrt{\frac{L}{C}} (\Omega) \text{ (pure resistive ... for lossless case)}$$

Typical value selected for  $Z_0$  is 50 ohm

# Impedance of Loaded Transmission Lines



The impedance along a transmission line at position  $x$  is given by

$Z(x) = \frac{V(x)}{I(x)}$ , where the complex voltage  $V(x)$  and current  $I(x)$  are :

$$V(x) = V^+ e^{-\gamma x} + V^- e^{\gamma x}, \quad I(x) = \frac{V^+}{Z_0} e^{-\gamma x} - \frac{V^-}{Z_0} e^{\gamma x}$$

The reference plane for  $V^+$  and  $V^-$  is located at  $x = 0$

# Impedance Calculation

The impedance at the position  $x = l$  is the load impedance  $Z_L$

$$Z(l) = \frac{V(l)}{I(l)} = \frac{V_L}{I_L} = Z_L$$

Now from the voltage and current wave solutions we have

$$V_L = V(l) = Z_L I_L = V^+ e^{-\gamma l} + V^- e^{\gamma l} \quad (2)$$

$$I_L = I(l) = \frac{V^+}{Z_0} e^{-\gamma l} - \frac{V^-}{Z_0} e^{\gamma l} \quad (3)$$

Solving for the incident wave amplitudes  $V^+$  and  $V^-$  we obtain

$$V^+ = \frac{1}{2} (Z_L + Z_0) I_L e^{\gamma l}$$

$$V^- = \frac{1}{2} (Z_L - Z_0) I_L e^{-\gamma l}$$

Substituting the incident wave  $V^+$  and  $V^-$  amplitudes we find the input impedance ( $x = 0$ )

$$Z_{in} = \frac{V(x=0)}{I(x=0)} = Z_0 \frac{V^+ + V^-}{V^+ - V^-} = Z_0 \frac{Z_L + Z_0 \tanh(\gamma l)}{Z_0 + Z_L \tanh(\gamma l)}$$

# Lossless Case

For a loss free line we have  $\gamma=j\beta$  and the impedance reduces to:

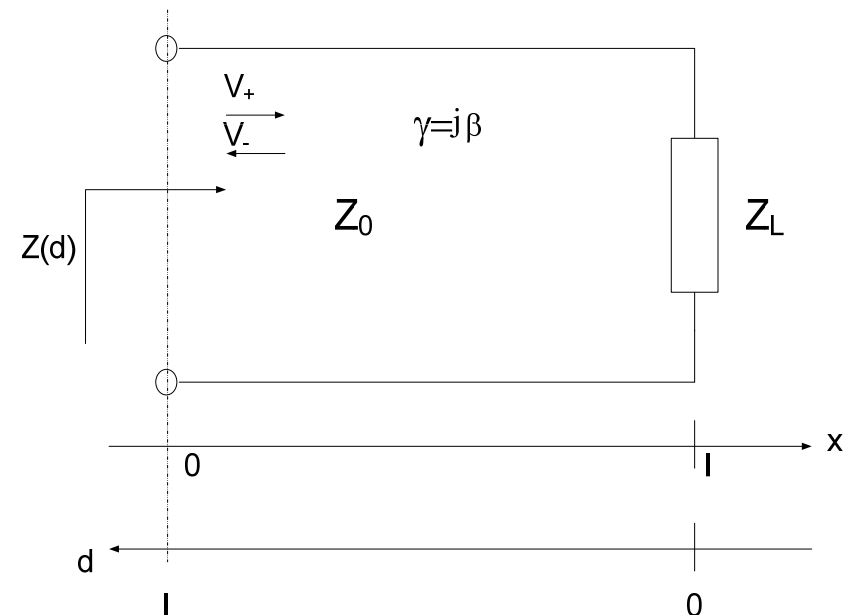
$$Z(d) = Z_0 \frac{Z_L + jZ_0 \tan(\beta d)}{Z_0 + jZ_L \tan(\beta d)}$$

The impedance  $Z$  is then periodic function of frequency and position:

- In terms of the electrical angle  $\theta=\beta d$  the impedance  $Z$  repeats every period  $\pi$
- In terms of position  $d$  it repeats every half wavelength  $\lambda/2$  since we have  $\beta d=(2\pi/\lambda)d$

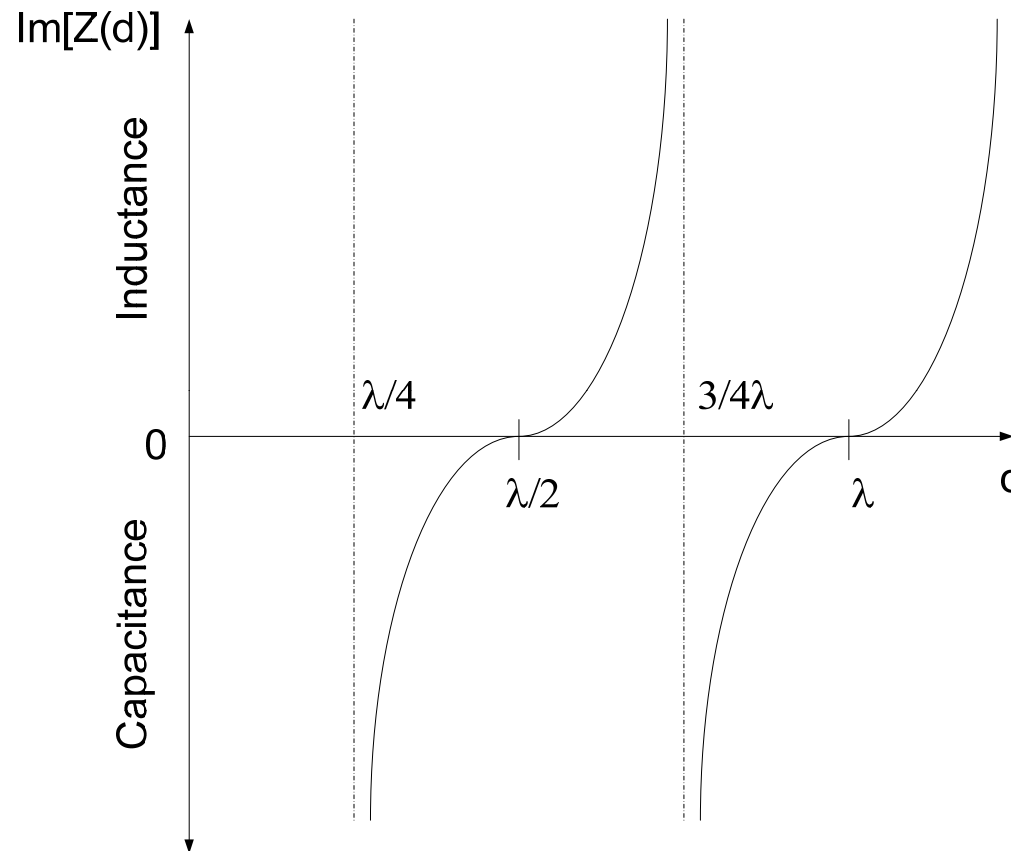
# Impedance of a Shorted Transmission Line

$$Z(d) = Z_0 \frac{Z_L + jZ_0 \tan(\beta d)}{Z_0 + jZ_L \tan(\beta d)}$$



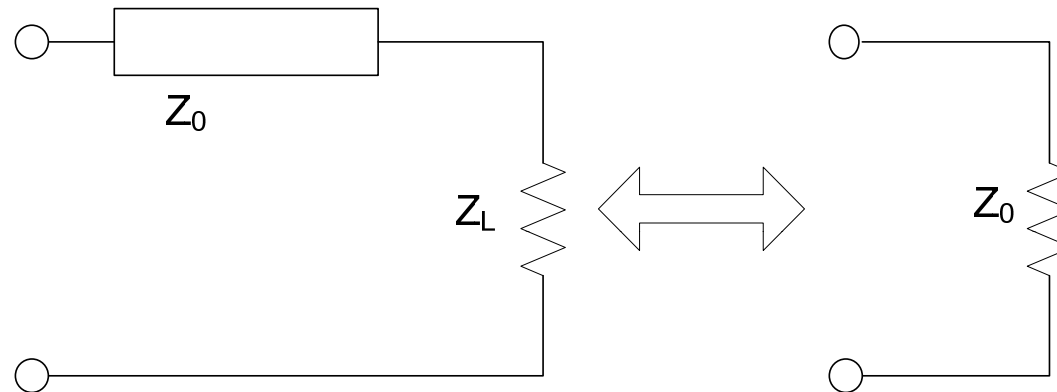
- For a short circuited line,  $Z_L=0$  and we have  $Z(d)=jZ_0\tan(\beta d)$
- For an open circuited line,  $Z_L = \infty$  and we have  $Z(d)=-jZ_0\cot(\beta d)$
- For a matched load,  $Z_L=Z_0$ , and we have  $Z(d)=Z_0$  for all values of  $d$

# Impedance of a Shorted Transmission Line



- The input impedance alternates between shorts ( $Z=0$ ) and opens ( $Z = \infty$ )
- The short is transformed into an open for  $d=\lambda/4$

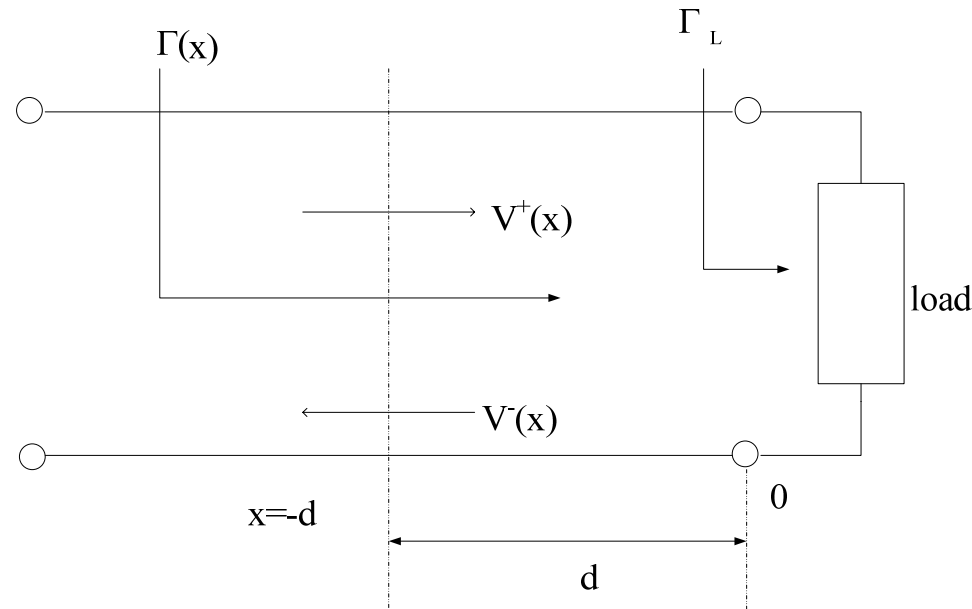
# Matched Line



- When  $Z_L = Z_0$ , the load is said to be matched



# Reflection Coefficient

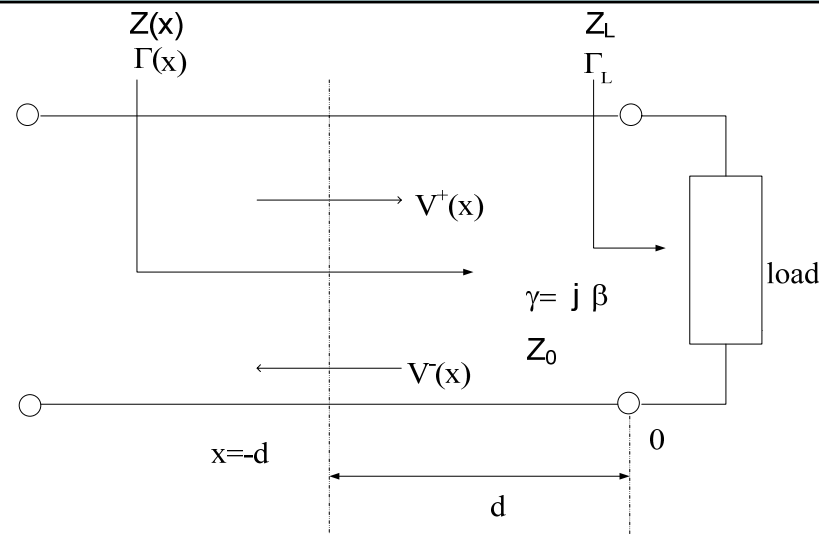


We define the reflection coefficient at a position  $x$  as the ratio of the reflected wave to the incident wave :

$$\Gamma(x) = \frac{V^- e^{\gamma x}}{V^+ e^{-\gamma x}} = \frac{V_0^-}{V_0^+} e^{2\gamma x} = \frac{V_0^-}{V_0^+} e^{2\gamma(l-d)} = \frac{V_L^-}{V_L^+} e^{-2\gamma d} = \Gamma_L e^{-2\gamma d}$$

where  $\Gamma_L = \Gamma(x = l)$

# Relation between impedance and Reflection Coefficient



$$Z(x) = \frac{V(x)}{I(x)} = \frac{V^+ e^{-\gamma x} + V^- e^{\gamma x}}{\frac{V^+}{Z_0} e^{-\gamma x} - \frac{V^-}{Z_0} e^{\gamma x}} = Z_0 \frac{1 + \Gamma(x)}{1 - \Gamma(x)}$$

Inverting :

$$\Gamma(x) = \frac{Z(x) - Z_0}{Z(x) + Z_0} \text{ and particularly at the load: } \Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$$

Note : for a matched load  $Z_L = Z_0$  and we have  $\Gamma_L = 0$  (no reflection)