EE 338

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Group: 3

* show that polyex terms with distinct exponential parameters are linearly independent.

→ let $p(x) e^{a_1 x}$ and $q(x) e^{a_2 x}$ denote two polyex terms such that $a_1 \neq a_2$. and $p(x)$ and $q(x)$ will be polynomials in case of polyex terms.

- Now, to show that they are linearly independent, it is suffice to show that their "wronskian" is not uniformly 0.

- we'll prove this using "Proof by contradiction" method

⇒ Let's assume $W = 0$ (uniformly)

$$\Rightarrow \begin{vmatrix} p(x) e^{a_1 x} & q(x) e^{a_2 x} \\ p'(x) e^{a_1 x} + p(x) a_1 e^{a_1 x} & q'(x) e^{a_2 x} + a_2 q(x) e^{a_2 x} \end{vmatrix} = 0$$

$$\Rightarrow e^{(a_1 + a_2)x} \left[p(x) q'(x) + a_2 p(x) q(x) - p'(x) q(x) - a_1 p(x) q(x) \right] = 0$$

here $e^{(a_1+a_2)x}$ cannot be uniformly zero.

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$\Rightarrow$ $p(x)\,q'(x) - p'(x)\,q(x) = (a_1 - a_2)\,p(x)\,q(x)$

since $p(x)\cdot q(x)$ is not uniformly 0

$$\frac{q'(x)}{q(x)} = (a_1 - a_2) + \frac{p'(x)}{p(x)}$$

by integrating with respect to x both sides we get

$$\ln q(x) = (a_2 - a_1)x + \ln p(x) + c$$

$$\Rightarrow \boxed{q(x) = p(x)\,e^{(a_2-a_1)x}\,k} \quad \text{— (i)}$$

(where $k = e^c$)

- in (i) $q(x)$ and $p(x)$ has same finite polynomial terms but $e^{(a_2-a_1)x}$ has infinite polynomial expression, hense equation cannot be hold unless $a_1 = a_2$ which contradict our initial condition $(a_1 \neq a_2)$

$\Rightarrow$ two polyex terms are linealy independent.