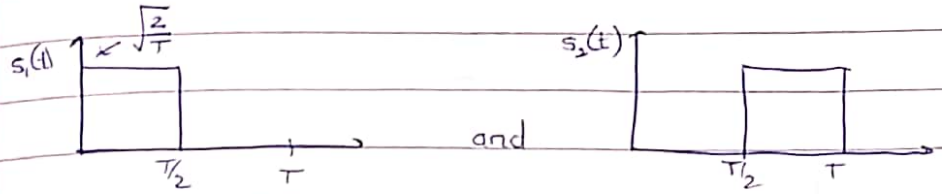
V Non-coherent Detection

eg



Coherent :- We ~~hav~~ need f_c and ϕ to do template matching

Non-coherent :- No information encoded in f_c and ϕ

In above example, find energy in $(0, T/2)$ and $(T/2, T)$, conclude symbol based on which e is higher.

- Can't use BPSK – no information in phase allowed $\because$ $-1 = 1e^{j\pi}$ (energy will not be different)
- Can't use ~~Q~~PSK at all.
- In PAM-4, can't distinguish b/w ± 3 or ± 1.

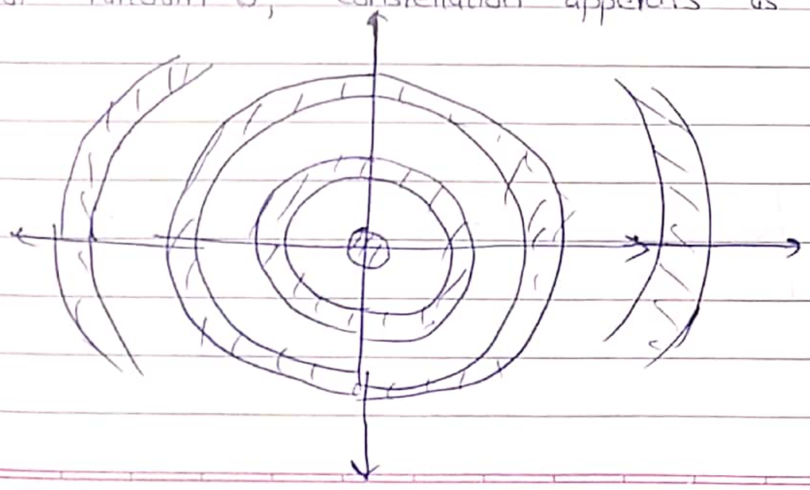
• Since we don't care about phase,

$$y = s_i e^{j\theta} + w$$

θ is random, because of f_c & ϕ mismatch.

eg $s_i(t) = i \text{ rect}\left(\frac{t - T}{T}\right)$ for $i = 0, 1, 2, 3$.

Because of random θ, constellation appears as concentric circles (bands) of radii $\approx 0, 1, 2, 3$



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:) No need to detect exact f_c and ~~θ~~ ϕ.

:(When you find energy, we ~~we~~ get ω^2 term, which is no longer Gaussian. it is chi squared distribution
- Minimum distance decoder may not be optimal.
  (still reasonable)

→ MPE

Coherent :- $\arg\max_i \langle y, s_i \rangle$

Non-coherent :- $\arg\max_i |\langle y, s_i \rangle|$

non coherent estimation is same as coherent estimation but with a phase offset. But here we know that there is no information stored in the phase so we can take mod and e^j*theta term will go away

A] Complex Gaussian.

Proper Complex Gaussian :-

U, V are complex jointly Gaussian vectors

$$E[(U - E(U))(V - E(V))] = 0$$

Xc+j*Xs

- N complex Gaussian vectors $\overset{\text{equivalent}}{\equiv}$ 2N real Gaussian vectors.
- Elements of ~~cov~~ ~~corelation~~ covariance matrix can be complex for complex Gaussian.

$$C = \begin{bmatrix} C_{cc} & C_{sc} \\ C_{cs} & C_{ss} \end{bmatrix} = E\left(\begin{bmatrix} \underline{X}_c \\ \underline{X}_s \end{bmatrix} \begin{bmatrix} \underline{X}_c & \underline{X}_s \end{bmatrix} \right).$$

Prop 4.5.4

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- If $X_{n\times 1} = \begin{bmatrix} X_{c_{n\times 1}} \\ X_{s_{n\times 1}} \end{bmatrix}$ is a $2n \times 2n$ matrix

'Covariance matrix'

$2n \times 2n$ covariance matrix $= \begin{bmatrix} C_{cc} & C_{cs} \\ C_{sc} & C_{ss} \end{bmatrix}$

- C_{cs} and C_{sc} are transposes of each other, but may not be equal.
- All four elements are real.

- $C_x \triangleq C_{cc} + C_{ss} + j(C_{sc} - C_{cs})$

Then $C_x^H = C_x$

- If $\langle s_0, s_1 \rangle = 0$, then $\langle n, s_0 \rangle$ and $\langle n, s_1 \rangle$ are independent with variance $2\sigma^2 \|s_i\|^2$

$\rightarrow$ $\text{cov}(\langle n, s_0 \rangle, \langle n, s_1 \rangle) = \langle s_0, s_1 \rangle$

- In C_x, relation between real and imaginary parts is unknown.

But if noise is 'proper', $C_{sc} = C_{cs}$ makes life chill.

$H_i : y = s_i e^{j\theta} + n$.

$Z_0 = \langle y, s_0 \rangle$

$Z_1 = \langle y, s_1 \rangle$

- $P_{e|0} = \mathbb{P}\left[|Z_1| > |Z_0| \mid H_0 \right]$

Define $\underline{Z} \triangleq \begin{bmatrix} Z_0 \\ Z_1 \end{bmatrix}$

→ Mean :-

$$E[\langle y, s_0 \rangle] = E\left[\langle s_0(l) e^{j\theta} + n(l), s_0(l) \rangle \right]$$

$$= e^{j\theta} \|s_0\|^2 + \cancel{E[\langle n, s_0 \rangle]}^{\,0}$$

* Over one symbol duration, θ is ~constant

θ is the same for both H_0 and H_1

$\therefore$ Mean of $\underline{Z} = \underline{m}_{\underline{Z}} = \begin{bmatrix} e^{j\theta} \|s_0\|^2 \\ e^{j\theta} \langle s_0, s_1 \rangle \end{bmatrix}$

→ Correlation coefficient $= \rho = \dfrac{\langle s_0, s_1 \rangle}{\|s_0\| \|s_1\|}$ ---- ρ is complex

→ $C_Z = 2\sigma^2 e^{j\theta} \begin{bmatrix} 1 & \rho^* \\ \rho & 1 \end{bmatrix}$ ☆ Verify

→ Use $\|s_0\| = \|s_1\| = \sqrt{E_s}$

$$\therefore \underline{m}_Z = e^{j\theta} \begin{bmatrix} \|s_0\|^2 \\ \langle s_0, s_1 \rangle \end{bmatrix}$$

$$= e^{j\theta} E_s \begin{bmatrix} 1 \\ \dfrac{\langle s_0, s_1 \rangle}{\|s_0\| \|s_1\|} \end{bmatrix}$$

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→ For C_z,

$$E\left[|z_0 - E_s e^{j\theta}|^2 \right]$$

$$E\left[(z_0 - E_s e^{j\theta})(z_1^* - E_s e^{-j\theta} \rho^*) \right]$$

⋮

Gives $C_z = 2\sigma^2 e^{j\theta} \begin{bmatrix} 1 & \rho^* \\ \rho & 1 \end{bmatrix}$

Z0 and Z1

* We have 2 complex RV's ⇒ 4 real RV's.
Usually we will require ${}^4C_2$ correlation values.
But because both are 'white' noises, we require only 2 correlation values ($Re(\rho)$ and $Im(\rho)$)

* $E[\langle y, s_1 \rangle \langle y, s_1 \rangle^*] = E\left[\int y(t)\, s_1^*(t)\, dt \int y^*(\tau) s_1(\tau)\, d\tau \right]$

$$= E\left[\iint s_1^*(t)\, s_1(\tau)\, y(t)\, y^*(\tau)\, dt\, d\tau \right]$$

$$= \int s_1^*(t)\, s(\tau)\, E\left[\int y(t)\, y^*(\tau) \right]$$

→ Performance depends only on $|\rho|$

$\langle s_0, s_1 \rangle = \rho E_s$ ρ = <s0 s1>/(|s0|*|s1|)

ρ has magnitude atmost 1 so can be reresented as cos~~θ~~+. sin~~θ~~

$s_1 \longrightarrow s_1 e^{j\arg(\rho)}$ ..... We will redesign s_1 as $s_1 e^{j\arg(\rho)}$

$$\therefore \langle s_0, s_1 e^{j\arg(\rho)} \rangle = \langle s_0, s_1 \rangle e^{-j\arg(\rho)}$$

$$= E_s \rho e^{-j\arg(\rho)}$$

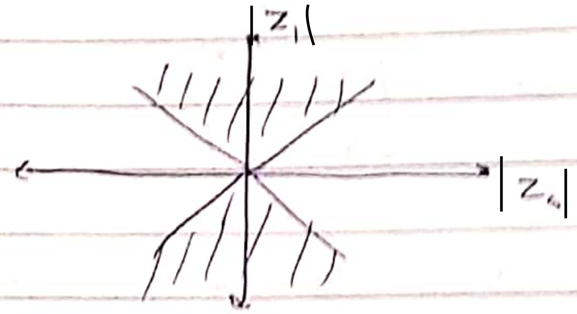
$$= E_s |\rho|$$

P.T.O.

- New $m_z = E_s e^{j\theta} \begin{bmatrix} 1 \\ |\rho| \end{bmatrix}$

$$C_z = 2\sigma^2 E_s \begin{bmatrix} 1 & |\rho| \\ |\rho| & 1 \end{bmatrix}$$

- Hypothesis : $\mathbb{P}(|z_1| > |z_0| \mid H_0)$



This BER comes out to be $e^{-\frac{E_s}{2N_0}(1-|\rho|)}$

$$\sim e^{-SNR}$$

* The Q function decays faster than e^{-SNR}.
$\therefore$ BER of coherent < BER of non-coherent

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* Mean has power terms, Covariance has power^2 terms
$\Rightarrow$ We are detecting power instead of phase, amplitude, etc.

- Transformation

$$Z \longrightarrow V$$

(Divide by $\sigma\sqrt{E_s}$)

New $m_V = \sqrt{\dfrac{E_s}{\sigma^2}} \begin{bmatrix} 1 \\ |\rho| \end{bmatrix}$

$$C_V = 2 \begin{bmatrix} 1 & |\rho| \\ |\rho| & 1 \end{bmatrix}$$

z0 = <y,s0> = |<(so*e^j +n), s0>|

- $|z_0| = |e^{j\theta} E_s + \langle n, s_0 \rangle|$

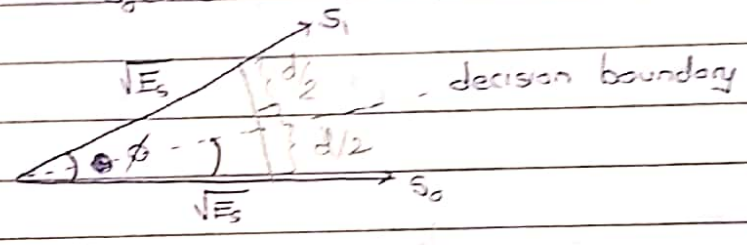
$|z_0 e^{-j\theta}| = |E_s + \langle n e^{-j\theta}, s_0 \rangle|$

$n e^{-j\theta}$ and n are identically distributed

$\therefore$ The above two expressions are equal

Put $z \rightarrow \dfrac{z}{\sigma\sqrt{E_s}}$ to obtain above V, m_v and C_v

eg $H_0 : y(t) = s_0(t) e^{j\theta} + n(t)$



* If $u(t) = \alpha s_0(t)$ and $v(t) = \beta s_1(t)$,

angle between u and v

$= \phi = \cos^{-1}\left(\dfrac{Re\langle u, v\rangle}{\|u\|\,\|v\|}\right) = \cos^{-1}(|\rho|)$

α s0*βs1

|s0|* | s1|*βα

Decision boundary is at $\phi/2$

$$\sin^2 \phi/2 = \frac{1-\cos\phi}{2} = \frac{1-|\rho|}{2}$$

$$\therefore \sin \phi/2 = \sqrt{\frac{1-|\rho|}{2}} = \frac{d}{2\sqrt{E_s}}$$

$$\text{Error rate} = Q\left(\frac{d}{2\sigma}\right) = Q\left(\sqrt{\frac{E_s(1-|\rho|)}{2N_0}}\right)$$

only dependent on $|\rho|$

→ Differential PSK

$$y = hs + \omega$$

$\hookrightarrow$ stays constant between consecutive symbols

- We can detect h from first symbol and use it for subsequent symbols
  - But h will change with time :-)

- $$y[0] = hs[0] + \omega[0] \quad \ldots \; s[0] \text{ is known}$$
  $$y[1] = hs[1] + \omega[1]$$

$$y[1]y[0]^* \propto s[1]s[0]^* \quad \ldots \text{ Phase of } h \text{ does not matter.}$$

- $$\mathbb{P}_e = \frac{1}{2} e^{-E_s/N_0 \, |h|^2}$$

- $$y[i] = hs[i] + \omega[i] \quad \ldots \; s[0] \text{ is known}$$
  $$i \in \{0, 1 \ldots N\}$$

Try all 2^N possibilities of $s[i]$'s and make an ML estimate on them.

→ Guess the $s[i]$'s which were most probably sent

- Using this, we estimate h.
  Estimation of h is better for higher N

- Performance approaches coherent detection performance.