Indian Institute of Technology, Bombay

EE338 – Digital Signal Processing

Filter Design Assignment

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Contents

1 Introduction

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This report documents my results and findings for the *Filter Design Assignment*, as a part of EE338 – Digital Signal Processing (Spring 2018). The report is organized as follows – section 2 describes the procedure and results in designing the a bandpass IIR filter, which is extended to an FIR design in subsection 2.10. Section 3 describes the procedure and results in designing the a bandstop IIR filter, which is extended to an FIR design in subsection 3.10. Simulations were performed on MATLAB and the frequency response and associated analysis is presented in section 4. The source code can be found attached in the supplementary material – instructions on running the code are presented in appendix A.

2 Bandpass Filter Design

2.1 Un-normalized Discrete Time Filter Specifications

Allotted filter number – 26.

f_s	300 kHz
δ	0.15
B_l	31.8 kHz
B_h	41.8 kHz
Transition width	2 kHz
Passband Nature	Monotonic
Stopband Nature	Monotonic

Table 1: Un-normalized D-t filter specifications

2.2 Normalized Digital Filter Specifications

The corresponding digital filter specification can be found by mapping the sampling frequency to 2π, i.e.

$$\omega_n = \frac{2\pi f}{f_s}$$

Thus, translating the specifications to normalized domain gives

δ	0.15
ω_l	0.212π
ω_h	0.2787π
Transition width	0.0133π

Table 2: Normalized bandpass filter specifications

Identifying the bandpass filter by its cutoff frequencies, we thus have the following specifications

ω_{s_1}	0.1987π
ω_{p_1}	0.212π
ω_{p_2}	0.2787π
ω_{s_2}	0.2920π

Table 3: Normalized edge frequencies for bandpass filter

2.3 Corresponding Analog Filter Specifications

To obtain the corresponding analog filter specifications, we use the bilinear transform, as follows

$$\Omega = \tan\frac{\omega}{2}$$

Applying the bilinear transform to the edge frequencies gives the corresponding analog filter edge frequencies as

ω	Ω
0	0
0.1987π	0.3226
0.212π	0.3459
0.2787π	0.4680
0.2920π	0.4938
π	∞

Table 4: Edge frequencies for corresponding analog filter

Since the desired discrete time filter was monotonic in passband and stopband, the corresponding analog filter must also be monotonic in passband and stopband. The tolerance of either bands remains unchanged under this transformation.

2.4 Frequency Transformation

To design the filter via its equivalent low-pass representation, we apply the following frequency transformation

$$\Omega_L = \frac{\Omega^2 - \Omega_0^2}{B\Omega}$$

where the parameters B and Ω_0 are determined as follows

$$\Omega_0 = \sqrt{\Omega_{P_1}\Omega_{P_2}} = 0.4023$$
$$B = \Omega_{P_2} - \Omega_{P_1} = 0.1221$$

2.5 Transformed Analog LPF Specifications

Applying the above transformation gives us an analogous analog lowpass filter with the following specification

Thus, the corresponding analog lowpass filter would be designed for unity cutoff frequency (Ω_p), and stopband edge (Ω_s) given by $\min(1.3591, 1.4673)$, i.e. 1.3591. Just like the BPF above, the filter would have monotonic passband and stopbands and the same tolerance (0.15).

Ω	Ω_L
0^+	$-\infty$
0.3226	−1.4673
0.3459	−1
0.4680	1
0.4938	1.3591
∞	∞

Table 5: Mapping from analog BPF to analog LPF

2.6 Analog Lowpass Transfer Function

For meet the design specifications, we need an analog filter that promises a monotonic passband as well as a monotonic stopband. This leads us to the choice of the *Butterworth filter* of appropriate parameters. To calculate the parameters, we perform the following computations

$$D_1 = \frac{1}{(1-\delta)^2} - 1 = 0.3841$$
$$D_2 = \frac{1}{\delta^2} - 1 = 43.44$$

We know that the lowest order of the Butterworth filter satisfying the D_1/D_2 requirements can be found as follows

$$N_s = \left\lceil \frac{log(D_2/D_1)}{2\,log(\Omega_s/\Omega_p)} \right\rceil$$
$$= \lceil 7.705 \rceil = 8$$

In addition, the cut-off frequency of the Butterworth design (Ω_c) must satisfy the criterion

$$\frac{\Omega_p}{D_1^{\frac{1}{2N}}} \leq \Omega_c \leq \frac{\Omega_s}{D_2^{\frac{1}{2N}}}$$
$$1.0616 \leq \Omega_c \leq 1.0737$$

We arbitrarily choose Ω_c to be 1.0677. Thus, we can see that the poles of $H_{\text{analog, LPF}}(s)^2$ lie on the circle of radius Ω_c as given by

$$s = j\Omega_c\ e^{j\frac{(2k+1)\pi}{2N}}$$

Of these, the poles of the *stable* transfer function $H_{\text{analog, LPF}}(s)$ are the poles on the open LHP, out of all the poles on the circle. Figure 1 shows the poles of the squared magnitude, and the corresponding poles of $H_{\text{analog, LPF}}(s)$. Let the 8 poles be denoted by $p_i,\ i = 1, \ldots, 8$. Then the required transfer function can be given by

$$H_{\text{analog, LPF}}(s) = \frac{\Omega_c^N}{\prod_i (s - p_i)}$$

Substituting the values of poles on the left half of the circle, we get the coefficients of the denominator of $H_{\text{analog, LPF}}(s)$ as follows

2.7 Analog Bandpass Transfer Function

To obtain the analog bandpass filter from the designed analog lowpass filter, we must apply the transformation discussed in section 2.4.

$$s_L \to \frac{s^2 + \Omega_0^2}{Bs}$$

Degree (s^k):	8	7	6	5	4	3	2	1	0
Coefficient:	0.5923	3.2414	8.8696	15.7476	19.7700	17.9506	11.5248	4.8010	1.0000

Table 6: Denominator of $H_{\text{analog, LPF}}(s)$, where numerator is normalized to 1

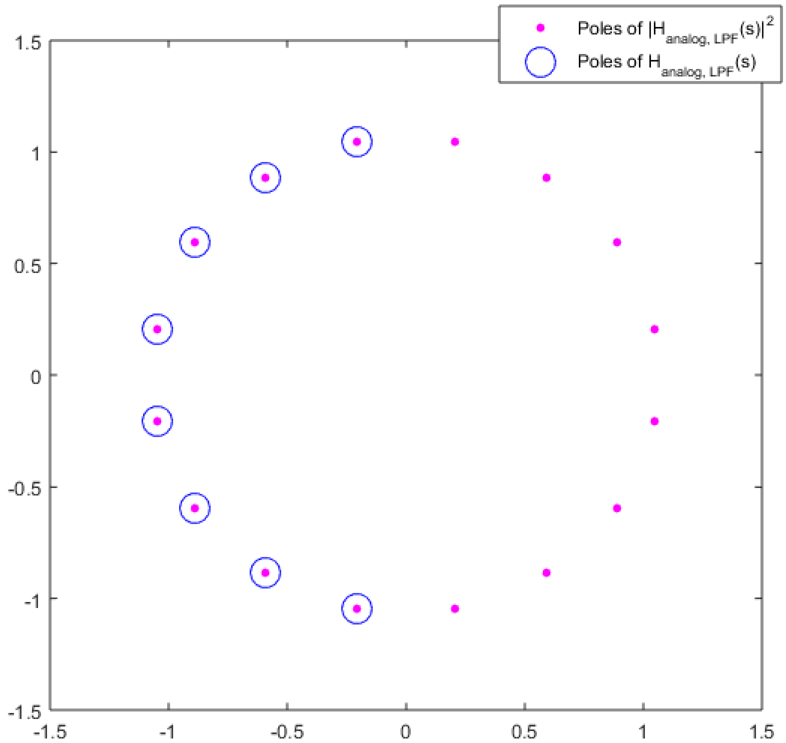

Figure 1: Visualizing poles of the analog LPF

Substituting in the transfer function obtained above, we get $H_{\text{analog, BPF}}(s) = \frac{N_{BPF}(s)}{D_{BPF}(s)}$, with the corresponding functions given below

Degree of $N_{BPF}(s)$ (s^k):	8	7	6	5	4	3	2	1	0
Coefficient:	0.832×10^{-7}	0	0	0	0	0	0	0	0

Table 7: Coefficients of $N_{BPF}(s)$

Degree of $D_{BPF}(s)$ (s^k):	16	15	14	13	12	11	10	9
Coefficient:	1	0.668	1.518	0.806	0.958	0.408	0.33	0.112

8	7	6	5	4	3	2	1	0
0.068	0.018	0.009	0.002	0.0007	0.0001	0	0	0

Table 8: Coefficients of $D_{BPF}(s)$

2.8 Discrete Time Realization

To transform the above analog BPF to its corresponding discrete time equivalent, we must use the bilinear transform (section 2.3)

$$s = \frac{1 - z^{-1}}{1 + z^{-1}}$$

Let the obtained transfer function be denoted as $H_{\text{discrete, BPF}}(z) = \frac{N_{BP}(z)}{D_{BP}(z)}$. For the sake of convenience (since the order of the system is pretty high, we represent the functions independently, as a table)

Degree of $N_{BP}(z)$ ($z^{-}k$, $\times 10^{-6}$):	0	2	4	6	8	10	12	14	16
Coefficient:	0.014	-0.113	0.396	-0.793	0.991	-0.793	0.396	-0.113	0.014

Table 9: Coefficients of $N_{BP}(z)$ (the odd coefficients are 0)

Degree of $D_{BP}(z)$ ($z^{-}k$, $\times 10^{3}$):	0	1	2	3	4	5	6	7
Coefficient:	0.001	-0.0107	0.057	-0.199	0.506	-0.99	1.538	-1.935

8	9	10	11	12	13	14	15	16
1.99	-1.676	1.154	-0.644	0.285	-0.097	0.024	-0.004	0.0003

Table 10: Coefficients of $D_{BP}(z)$

2.9 Realization using Direct Form II

The realization of the IIR filter obtained above using direct form II can be directly from the coefficients of $N_{BP}(z)$ and $D_{BP}(z)$. A graphical representation is shown in figure 2.

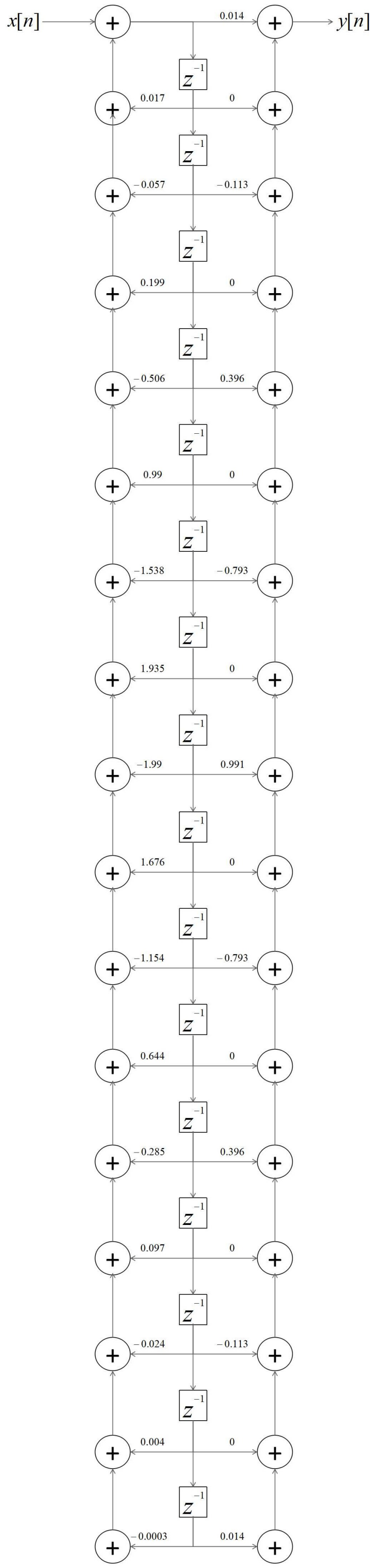

Figure 2: Direct Form - II Realization of the desired IIR bandpass filter (coefficients are appropriately scaled. Refer tables 9 & 10)

2.10 FIR Filter Using the Kaiser Window

The Kaiser window family, parametrized by length M and shape parameter β is denoted by

$$w[n] = \frac{I_0[\beta\sqrt{1-(\frac{n-\alpha}{\alpha})^2}]}{I_0(\beta)}, \qquad 0 \leq n \leq M$$

where $I_0(.)$ represents the zeroth order Bessel function of the first kind, and $\alpha = \frac{M}{2}$. The tolerance requirements of the desired FIR filter are $\delta = 0.15$, giving the stopband attenuation A as

$$A = -20 \log_{10}\delta = 16.4782$$

Using this, the value of the parameter β of the Kaiser window can be obtained as

$$\beta = \begin{cases} 0 & A < 21 \\ 0.5842(A-21)^{0.4} + 0.07886(A-21) & 21 \leq A \leq 50 \\ 0.1102(A-8.7) & A > 50 \end{cases}$$

Thus, the required value of β is 0. It is essential to note that for $\beta = 0$, the Kaiser window essentially collapses into the rectangular window of the same length. A heuristic to approximate the length M is given by

$$M > \frac{A-8}{2.285 \times \omega_T}$$

where ω_T is the minimum transition width of the desired filter (sharpest edge). Substituting $\omega_T = 0.0133\pi$ (normalized), we get

$$M > 88.6$$

Some interesting observations of the so-designed FIR filter are as follows

- The order of the corresponding FIR filter (number of delay elements) is *significantly higher* than its IIR counterpart.

- The parameters obtained using the heuristic only provide a lower bound, with no consideration of the actual values of tolerances and cut-off frequencies.

To complete this filter implementation, we increase M till the designed filter actually meets the specifications desired. Choosing $M = 121$ gives us the desired FIR filter (frequency response and plots discussed in section **??**). The Kaiser window, as mentioned above, reduces to a rectangular window, with all coefficients set to 1.

To obtain the overall FIR filter, we multiply the window, point-wise, to the ideal analog IIR filter required to satisfy the desired specifications – in our case, this could be the brick-wall bandpass filter. The coefficients of the overall FIR filter $H_{\text{FIR-BP}}(z)$ are shown in the figure below, in the form of the MATLAB output screen (figure 3), accompanied by a plot of the time domain filter (figure 4).

```
H_fir_bpf =

  Columns 1 through 16

   -0.0064    0.0008    0.0070    0.0086    0.0053   -0.0001   -0.0040   -0.0044   -0.0022   -0.0001   -0.0000   -0.0016   -0.0025   -0.0005    0.0042    0.0083

  Columns 17 through 32

    0.0079    0.0018   -0.0074   -0.0138   -0.0126   -0.0034    0.0088    0.0167    0.0150    0.0047   -0.0081   -0.0155   -0.0137   -0.0046    0.0053    0.0099

  Columns 33 through 48

    0.0077    0.0022   -0.0011    0.0000    0.0031    0.0030   -0.0034   -0.0130   -0.0179   -0.0111    0.0071    0.0272    0.0350    0.0216   -0.0091   -0.0405

  Columns 49 through 64

   -0.0521   -0.0332    0.0088    0.0508    0.0667    0.0442   -0.0064   -0.0566   -0.0765   -0.0528    0.0023    0.0572    0.0800    0.0572    0.0023   -0.0528

  Columns 65 through 80

   -0.0765   -0.0566   -0.0064    0.0442    0.0667    0.0508    0.0088   -0.0332   -0.0521   -0.0405   -0.0091    0.0216    0.0350    0.0272    0.0071   -0.0111

  Columns 81 through 96

   -0.0179   -0.0130   -0.0034    0.0030    0.0031    0.0000   -0.0011    0.0022    0.0077    0.0099    0.0053   -0.0046   -0.0137   -0.0155   -0.0081    0.0047

  Columns 97 through 112

    0.0150    0.0167    0.0088   -0.0034   -0.0126   -0.0138   -0.0074    0.0018    0.0079    0.0083    0.0042   -0.0005   -0.0025   -0.0016   -0.0000   -0.0001

  Columns 113 through 121

   -0.0022   -0.0044   -0.0040   -0.0001    0.0053    0.0086    0.0070    0.0008   -0.0064
```

Figure 3: Time domain coefficients of the FIR filter

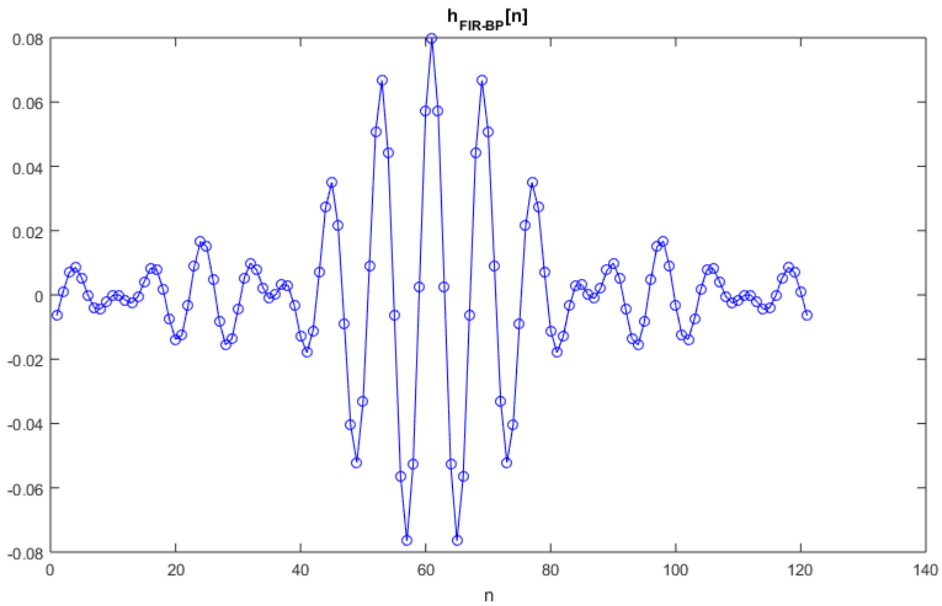

Figure 4: $h_{\text{FIR-BP}}[n]$

3 Bandstop Filter Design

3.1 Un-normalized Discrete Time Filter Specifications

Allotted filter number – 26.

f_s	200 kHz
δ	0.15
B_l	22.4 kHz
B_h	28.4 kHz
Transition width	2 kHz
Passband Nature	Equiripple
Stopband Nature	Monotonic

Table 11: Un-normalized D-t filter specifications

3.2 Normalized Digital Filter Specifications

The corresponding digital filter specification can be found by mapping the sampling frequency to 2π, i.e.

$$\omega_n = \frac{2\pi f}{f_s}$$

Thus, translating the specifications to normalized domain gives

δ	0.15
ω_l	0.224π
ω_h	0.284π
Transition width	0.02π

Table 12: Normalized bandstop filter specifications

Identifying the bandstop filter by its cutoff frequencies, we thus have the following specifications

ω_{p_1}	0.204π
ω_{s_1}	0.224π
ω_{s_2}	0.284π
ω_{p_2}	0.304π

Table 13: Normalized edge frequencies for bandstop filter

3.3 Corresponding Analog Filter Specifications

To obtain the corresponding analog filter specifications, we use the bilinear transform, as follows

$$\Omega = \tan\frac{\omega}{2}$$

Applying the bilinear transform to the edge frequencies gives the corresponding analog filter edge frequencies as

ω	Ω
0	0
0.204π	0.3319
0.224π	0.3671
0.284π	0.4783
0.304π	0.5175
π	∞

Table 14: Edge frequencies for corresponding analog filter

Since the desired discrete time filter was equiripple in passband and monotonic in stopband, the corresponding analog filter must also be equiripple in passband and monotonic in stopband. The tolerance of either bands remains unchanged under this transformation.

3.4 Frequency Transformation

To design the filter via its equivalent low-pass representation, we apply the following frequency transformation

$$\Omega_L = \frac{B\Omega}{\Omega_0^2 - \Omega^2}$$

where the parameters B and Ω_0 are determined as follows

$$\Omega_0 = \sqrt{\Omega_{P_1}\Omega_{P_2}} = 0.4144$$
$$B = \Omega_{P_2} - \Omega_{P_1} = 0.1856$$

3.5 Transformed Analog LPF Specifications

Applying the above transformation gives us an analogous analog lowpass filter with the following specification

Ω	Ω_L
0^+	$-\infty$
0.3319	1
0.3671	1.8441
0.4783	-1.557
0.5175	-1
∞	0^-

Table 15: Mapping from analog BSF to analog LPF

Thus, the corresponding analog lowpass filter would be designed for unity cutoff frequency (Ω_p), and stopband edge (Ω_s) given by $\min(1.8441, 1.557)$, i.e. 1.557. Just like the BSF above, the filter would have equiripple passband and monotonic stopband and the same tolerance (0.15).

3.6 Analog Lowpass Transfer Function

We need an analog lowpass filter with an equiripple passband and monotonic stopband – which can be easily realized using a Chebyshev filter of appropriate parameters. To choose the required parameters, we perform the following computations

$$D_1 = \frac{1}{(1-\delta)^2} - 1 = 0.3841$$
$$D_2 = \frac{1}{\delta^2} - 1 = 43.44$$

We know that the lowest order of the Chebyshev filter satisfying D_1/D_2 requirements from above can be found as follows

$$N_s = \left\lceil \frac{\cosh^{-1}(\sqrt{D_2/D_1})}{\cosh^{-1}(\sqrt{\Omega_s/\Omega_p})} \right\rceil$$
$$= \lceil 3.0191 \rceil = 4$$

For the Chebyshev design, the poles of $|H_{\text{analog, LPF}}(s)|^2$ lie on an ellipse, as can be seen from the expression below

$$s = \Omega_p \sin A_k \; \sinh B_k + j\Omega_p \cos A_k \; \cosh B_k$$
$$A_k = 2k + 1\frac{\pi}{2N}$$
$$B_k = \frac{1}{N}\sinh^{-1}(\frac{1}{\epsilon})$$

Of these, the poles of the *stable* transfer function $H_{\text{analog, LPF}}(s)$ are the poles on the open LHP, out of all the poles represented by the equation above. Figure **??** shows the poles of the squared magnitude, and the corresponding poles of $H_{\text{analog, LPF}}(s)$. Let the 4 poles be denoted by $p_i, \; i = 1, \ldots, 4$. Then the required transfer function can be given by

$$H_{\text{analog, LPF}}(s) = \frac{1}{\sqrt{1+D_1}} \prod_i \frac{-p_i}{s - p_i}$$

Note that the $\sqrt{1+D_1}$ factor comes from the fact that the desired DC gain in a Chebyshev filter is set to $\frac{1}{\sqrt{1+\epsilon^2}}$, and not 1. Substituting the values of poles on the left half of the $s-$plane, we get the coefficients of the denominator of $H_{\text{analog, LPF}}(s)$ as follows

Degree (s^k):	4	3	2	1	0
Coefficient:	1.0000	0.8342	1.3479	0.6243	0.2373

Table 16: Denominator of $H_{\text{analog, LPF}}(s)$, where numerator is 0.2017

3.7 Analog Bandstop Transfer Function

To obtain the analog bandstop filter from the designed analog lowpass filter, we must apply the transformation discussed in section 3.4.

$$s_L \rightarrow \frac{Bs}{s^2 + \Omega_0^2}$$

Substituting in the transfer function obtained above, we get $H_{\text{discrete, BSF}}(s) = \frac{N_{BSF}(s)}{D_{BSF}(s)}$, with the corresponding functions given below

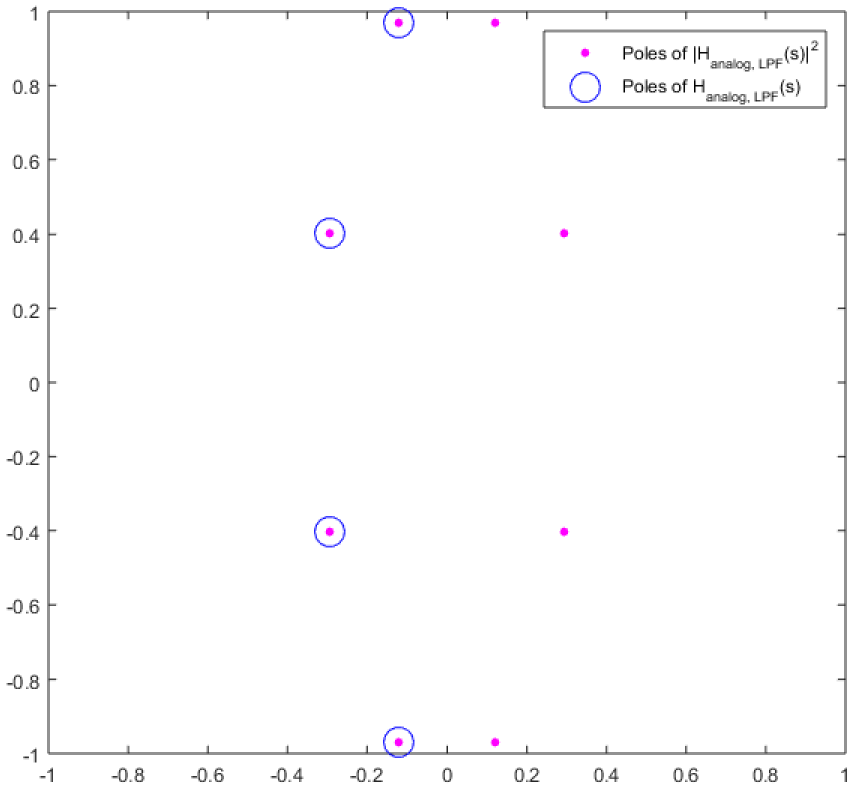

Figure 5: Visualizing poles of the analog LPF

Degree of $N_{BSF}(s)$ (s^k):	8	7	6	5	4	3	2	1	0
Coefficient:	0.85	0	0.5839	0	0.1504	0	0.0172	0	0.0007

Table 17: Coefficients of $N_{BSF}(s)$

Degree of $D_{BSF}(s)$ (s^k):	8	7	6	5	4	3	2	1	0
Coefficient:	1	0.4882	0.8826	0.274	0.2492	0.0471	0.026	0.0025	0.0009

Table 18: Coefficients of $D_{BSF}(s)$

3.8 Discrete Time Realization

To transform the above analog BPF to its corresponding discrete time equivalent, we must use the bilinear transform (section 2.3)

$$s = \frac{1 - z^{-1}}{1 + z^{-1}}$$

Let the obtained transfer function be denoted as $H_{\text{discrete, BSF}}(z) = \frac{N_{BS}(z)}{D_{BS}(z)}$. For the sake of convenience (since the order of the system is pretty high, we represent the functions independently, as tables)

Degree of $D_{BS}(z)$ (z^-k, $\times 10^3$):	0	1	2	3	4	5	6	7	8
Coefficient:	1	-4.98	12.42	-19.51	21.03	-15.85	8.23	-2.71	0.45

Table 19: Coefficients of $D_{BS}(z)$

Degree of $N_{BS}(z)$ (z^-k, $\times 10^3$):	0	1	2	3	4	5	6	7	8
Coefficient:	0.54	-3.05	8.63	-15.25	18.33	-15.25	8.63	-3.05	0.54

Table 20: Coefficients of $N_{BS}(z)$

3.9 Realization using Direct Form II

The realization of the IIR filter obtained above using direct form II can be directly from the coefficients of $N_{BS}(z)$ and $D_{BS}(z)$. A graphical representation is shown in figure 6.

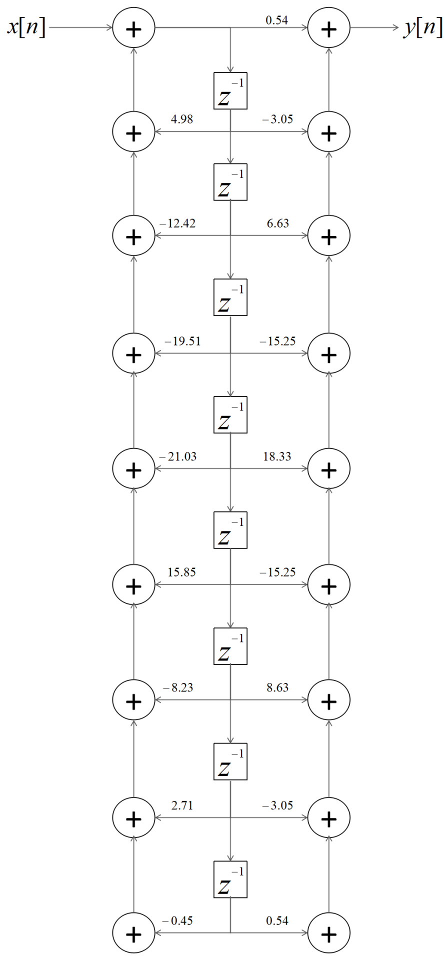

Figure 6: Direct Form - II Realization of the desired IIR bandstop filter (coefficients are appropriately scaled. Refer tables 20 & 19)

3.10 FIR Filter Using the Kaiser Window

As seen earlier for the bandpass filter, the Kaiser window family, parametrized by length M and shape parameter β is denoted by

$$w[n] = \frac{I_0[\beta\sqrt{1-(\frac{n-\alpha}{\alpha})^2}]}{I_0(\beta)}, \quad 0 \le n \le M$$

where $I_0(.)$ represents the zeroth order Bessel function of the first kind, and $\alpha = \frac{M}{2}$. The tolerance requirements of the desired FIR filter are $\delta = 0.15$, giving the stopband attenuation A as

$$A = -20\ \log_{10}\delta = 16.4782$$

Using this, the value of the parameter β of the Kaiser window can be obtained as

$$\beta = \begin{cases} 0 & A < 21 \\ 0.5842(A-21)^{0.4} + 0.07886(A-21) & 21 \le A \le 50 \\ 0.1102(A-8.7) & A > 50 \end{cases}$$

Thus, the required value of β is 0. It is essential to note that for $\beta = 0$, the Kaiser window essentially collapses into the rectangular window of the same length. A heuristic to approximate the length M is given by

$$M > \frac{A-8}{2.285 \times \omega_T}$$

where ω_T is the minimum transition width of the desired filter (sharpest edge). Substituting $\omega_T = 0.02\pi$ (normalized), we get

$$M > 59.05$$

Following the trends from the bandpass FIR, some interesting observations of the so-designed FIR filter are as follows

- The order of the corresponding FIR filter (number of delay elements) is *significantly higher* than its IIR counterpart.
- The parameters obtained using the heuristic only provide a lower bound, with no consideration of the actual values of tolerances and cut-off frequencies. To meet the specifications, some eye-balling and manual intervention is required.

To complete this filter implementation, we increase M till the designed filter actually meets the specifications desired. Choosing $M = 78$ gives us the desired FIR filter (frequency response and plots discussed in section **??**). The Kaiser window, as mentioned above, reduces to a rectangular window, with all coefficients set to 1.

To obtain the overall FIR filter, we multiply the window, point-wise, to the ideal analog IIR filter required to satisfy the desired specifications – in our case, this could be the brick-wall bandstop filter. The coefficients of the overall FIR filter $H_{\text{FIR-BP}}(z)$ are shown in the figure below, in the form of the MATLAB output screen (figure 7), accompanied by a plot of the time domain filter (figure 8).

```
H_fir_bsf =

  Columns 1 through 16

    0.0209   -0.0072   -0.0027   -0.0263   -0.0034   -0.0100    0.0208    0.0046    0.0198   -0.0110    0.0042   -0.0187    0.0090   -0.0124    0.0117   -0.0186

  Columns 17 through 32

    0.0087   -0.0133    0.0288    0.0043    0.0319   -0.0232   -0.0086   -0.0580   -0.0012   -0.0132    0.0689    0.0256    0.0597   -0.0503   -0.0202   -0.1081

  Columns 33 through 48

    0.0164   -0.0344    0.1390   -0.0182    0.1597   -0.2413    0.5630    0.5630   -0.2413    0.1597   -0.0182    0.1390   -0.0344    0.0164   -0.1081   -0.0202

  Columns 49 through 64

   -0.0503    0.0597    0.0256    0.0689   -0.0132   -0.0012   -0.0580   -0.0086   -0.0232    0.0319    0.0043    0.0288   -0.0133    0.0087   -0.0186    0.0117

  Columns 65 through 78

   -0.0124    0.0090   -0.0187    0.0042   -0.0110    0.0198    0.0046    0.0208   -0.0100   -0.0034   -0.0263   -0.0027   -0.0072    0.0209
```

Figure 7: Time domain coefficients of the FIR filter

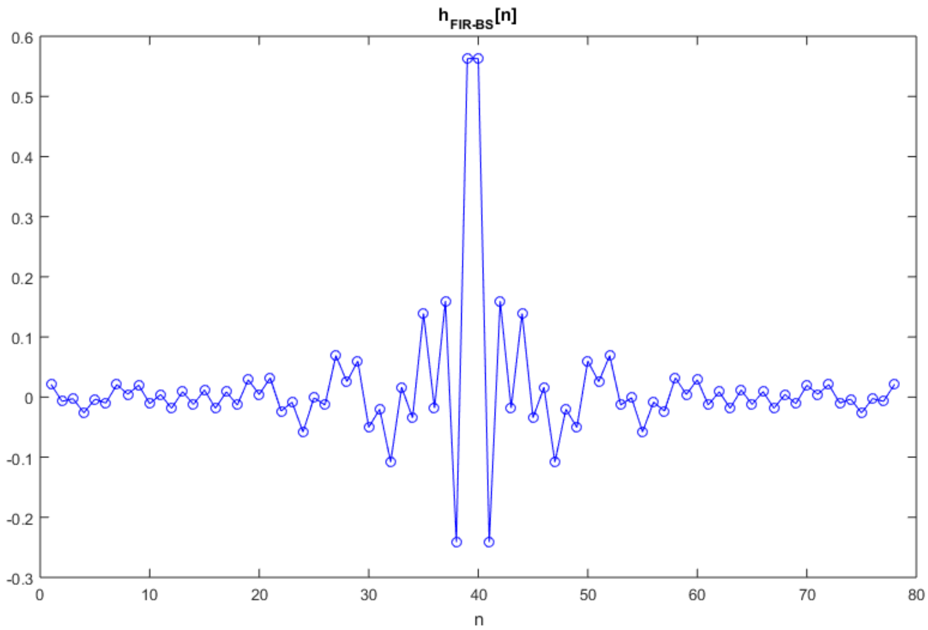

Figure 8: $h_{\text{FIR-BP}}[n]$

4 Results and Analysis

In this section, we present the results and analysis of the filters designed in the previous sections. The simulation & analysis was carried out on MATLAB R2015a.

4.1 Bandpass Filter

4.1.1 IIR Filter

The magnitude and phase response of the designed IIR filter is shown below. We notice that the passband does not feature *linear phase*, which is a critical drawback with IIR filters, introducing undesirable dispersion.

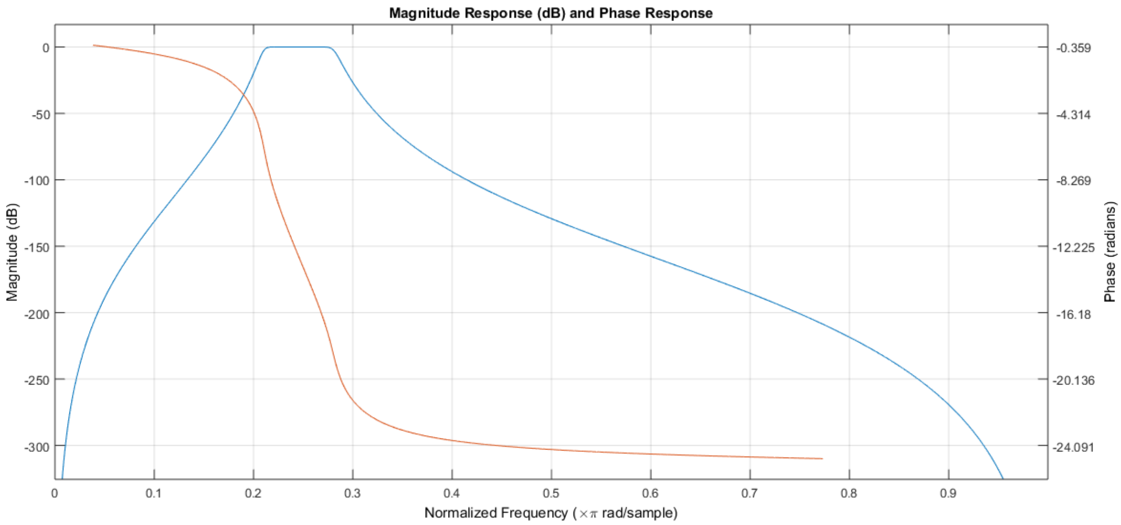

Figure 9: Magnitude and phase response of the designed IIR bandpass filter

To check the stability of the designed filter, we can look at the pole-zero positions in the $z-$plane. This plot can be found in figure 10. We see that all the poles lie inside the unit circle, and hence, the designed IIR filter is stable.

To demonstrate that the designed filter indeed meets the specifications, figure 11 identifies the magnitude response at desired edge frequencies. The IIR filter comfortably satisfies the tolerance limit and is monotonic in both passband and stopband, as desired.

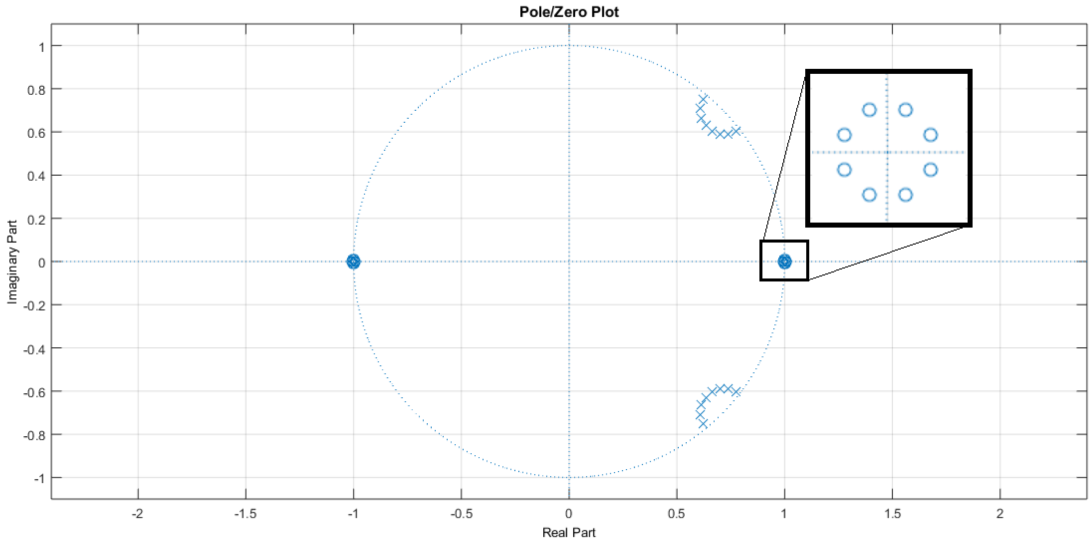

Figure 10: Pole-zero locations of the designed IIR filter

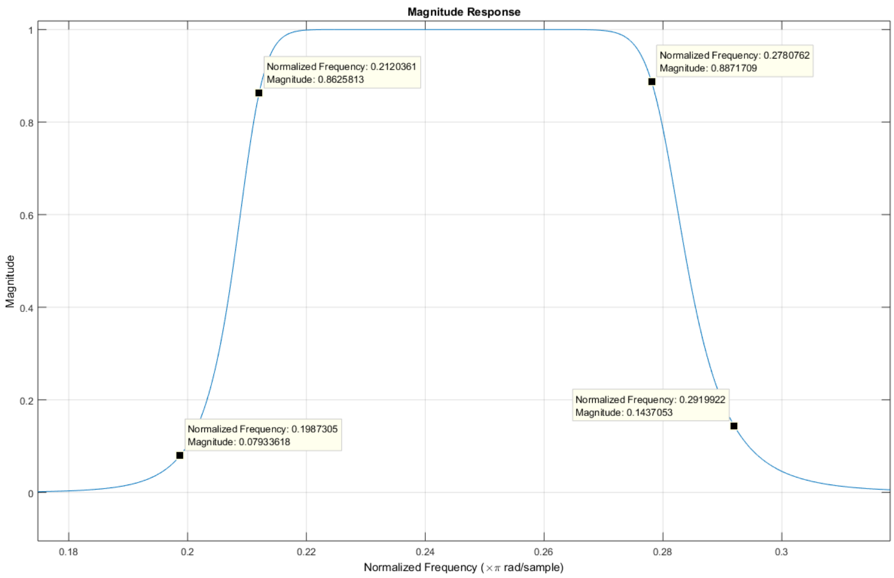

Figure 11: Magnitude response at edge frequencies of IIR bandpass filter

4.1.2 FIR Filter

The magnitude and phase response of the designed IIR filter is shown below (figure 12). We notice that the passband demonstrates *linear phase*, which is the major advantage of using an FIR filter, despite increased resource requirements.

To demonstrate that the designed filter indeed meets the specifications, figure 13 identifies the magnitude response at desired edge frequencies. We can see that the desired specifications have been met, along with providing piecewise-linear phase over the whole spectrum.

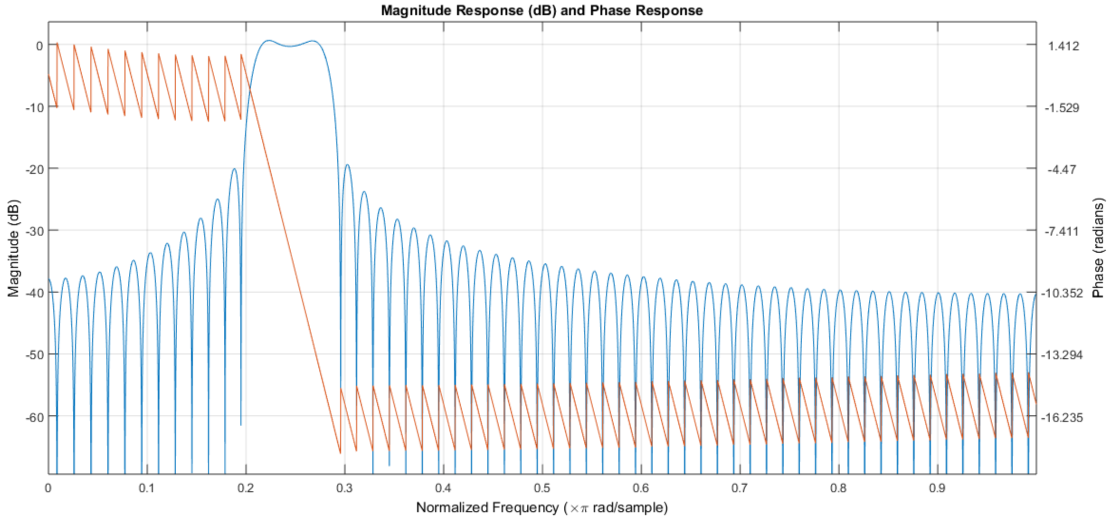

Figure 12: Magnitude and phase response of the designed FIR bandpass filter

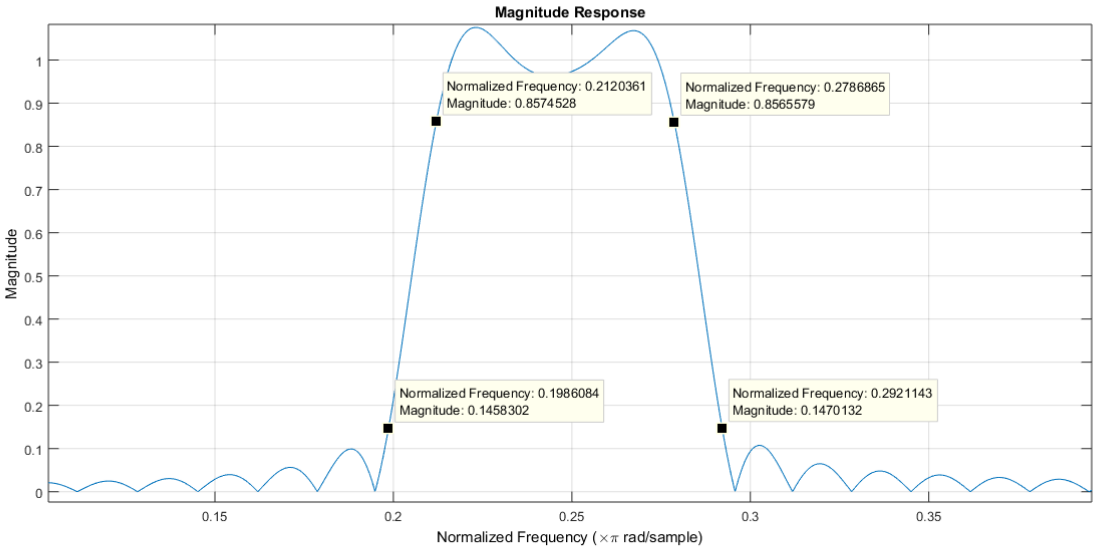

Figure 13: Magnitude response at edge frequencies of FIR bandpass filter

4.2 Bandstop Filter

4.2.1 IIR Filter

The magnitude and phase response of the designed IIR filter is shown below. We notice that the passband does not feature *linear phase*, which is a critical drawback with IIR filters, introducing undesirable dispersion.

To check the stability of the designed filter, we can look at the pole-zero positions in the $z-$plane. This plot can be found in figure 15. We see that all the poles lie inside the unit circle, and hence, the designed IIR filter is stable.

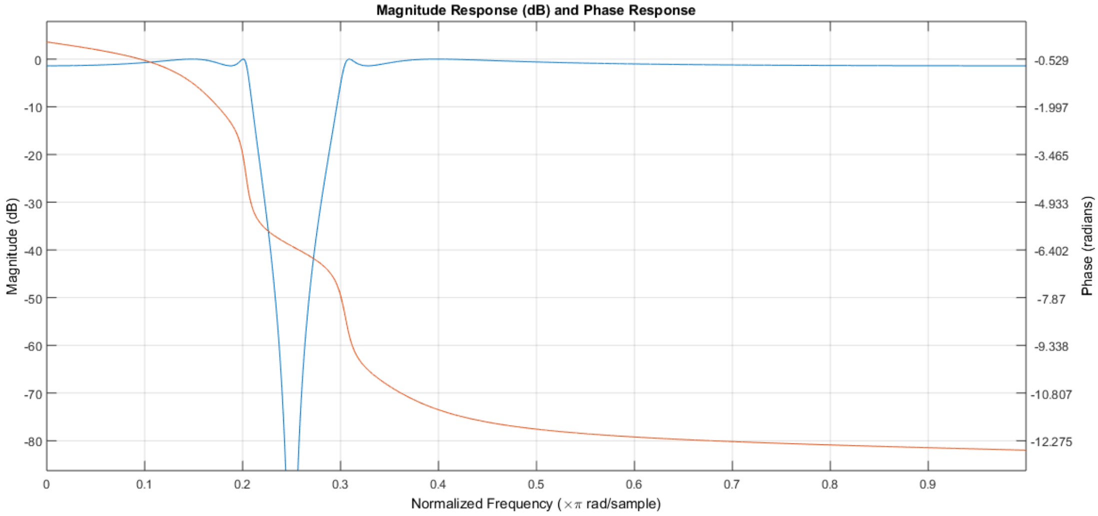

Figure 14: Magnitude and phase response of the designed IIR bandstop filter

To demonstrate that the designed filter indeed meets the specifications, figure 16 identifies the magnitude response at desired edge frequencies. The IIR filter comfortably satisfies the tolerance limit and is equiripple in passband and monotonic in stopband, as desired.

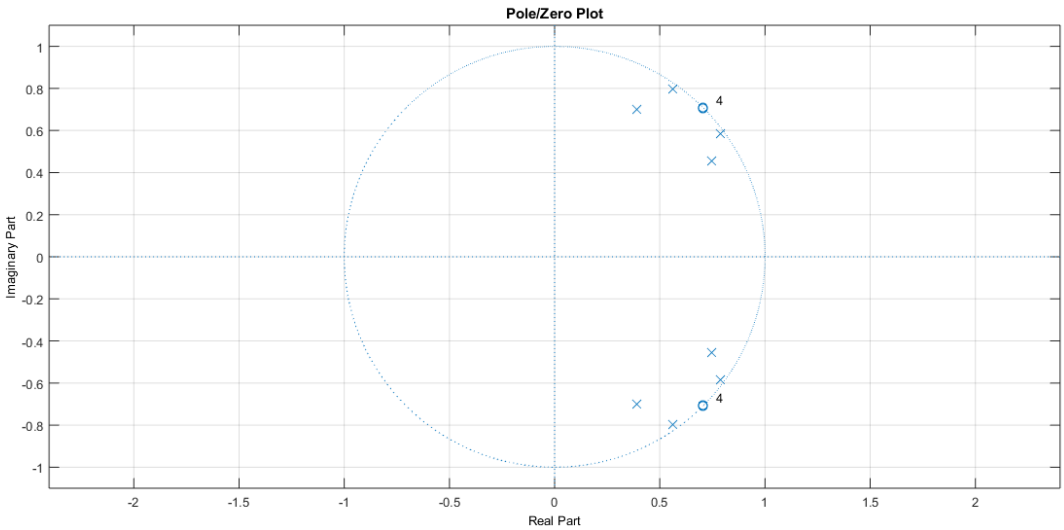

Figure 15: Pole-zero locations of the designed IIR filter

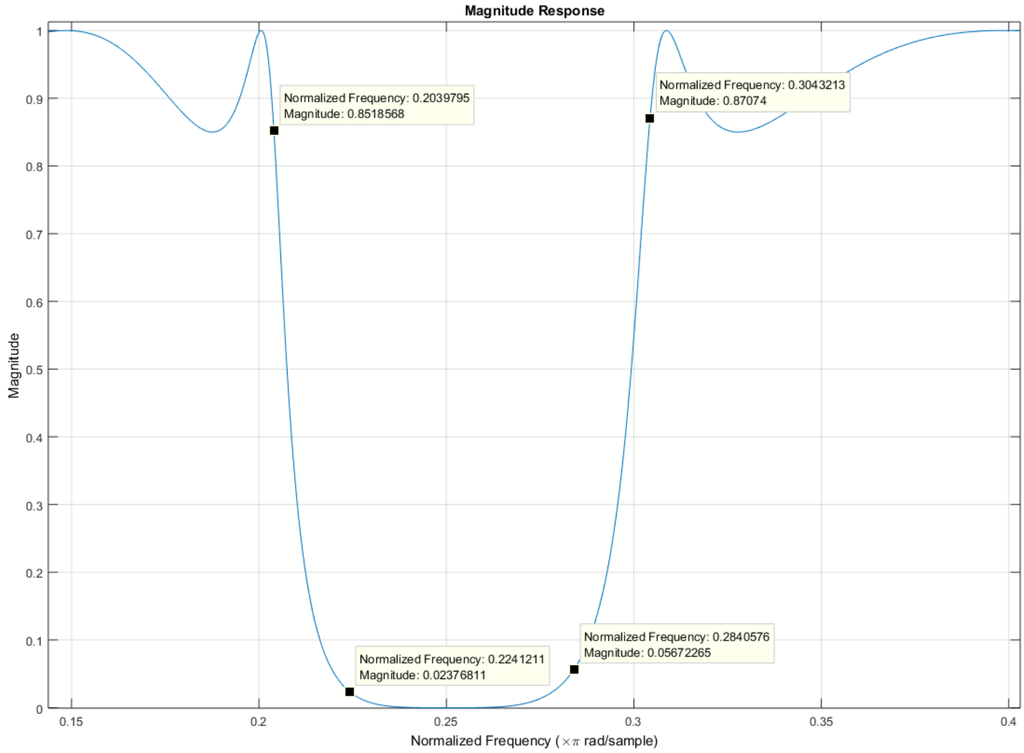

Figure 16: Magnitude response at edge frequencies of IIR bandstop filter

4.2.2 FIR Filter

The magnitude and phase response of the designed IIR filter is shown below (figure 17). We notice that the stopband demonstrates *linear phase*, which is the major advantage of using an FIR filter, despite increased resource requirements.

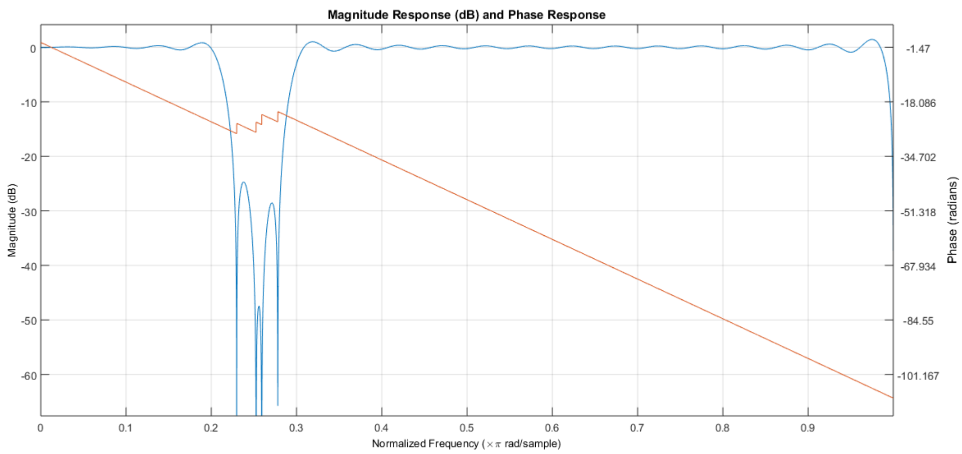

Figure 17: Magnitude and phase response of the designed FIR bandstop filter

To demonstrate that the designed filter indeed meets the specifications, figure 18 identifies the magnitude response at desired edge frequencies. We can see that the desired specifications have been met, along with providing piecewise-linear phase over the whole spectrum.

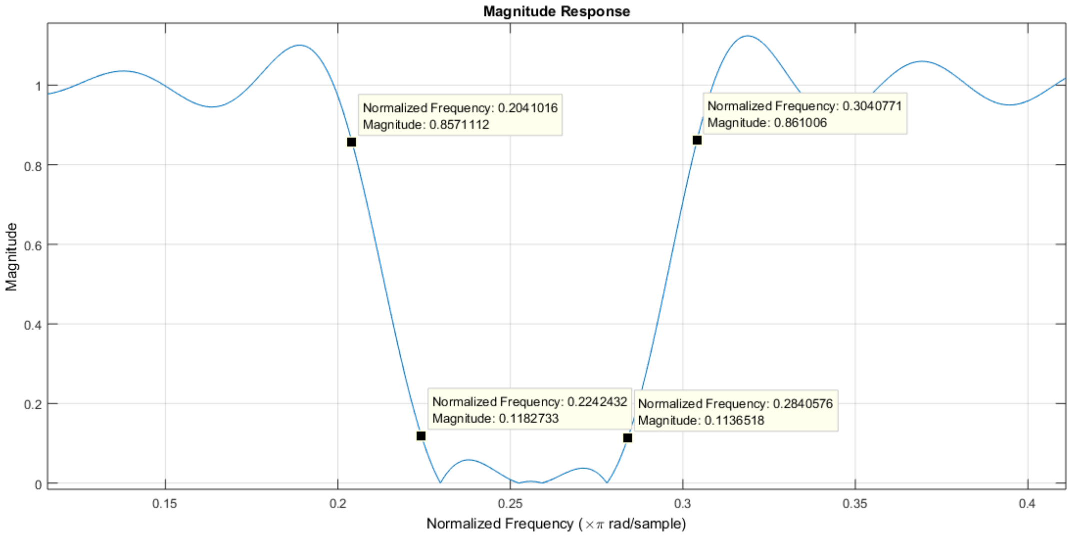

Figure 18: Magnitude response at edge frequencies of FIR bandpass filter

A Running Simulations in MATLAB R2015a

The MATLAB scripts to generate and visualize all the filters discussed above are provided in the supplementary material[1]. The files `Butterworth_BP.m`, `Chebyshev_BS.m`, `Kaiser_BP.m` and `Kaiser_BS.m` contain the code for monotonic bandpass, equiripple bandstop, FIR bandpass and FIR bandstop filters respectively. Each program generates the filter design from the specifications mentioned at the beginning – edge frequencies (or filter number, for the course EE338), tolerances, sampling frequency, transition width etc. – and is very modular, allowing tuning the filter parameters with ease.

[1]Note that the DSP Toolbox would be required to run the visualization command `fvtool` and `kaiser`, which is used to generate the Kaiser window coefficients from the parameters. The former may be circumvented by manually sweeping through frequencies with a small granularity, while the latter may be avoided by declaring the window using the definition of the Kaiser window.