Problem Set 5

Data Analysis and Interpretation (EE 223)

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1. Let $X_1, \ldots, X_n$ be i.i.d. Poisson(λ). Find UMVUE for λ. £x/n

2. Let $X_1, \ldots, X_n$ be i.i.d. Poisson(λ).

   (a) Find $E_\lambda[X_1^2]$. lam+lam ^2

   (b) Find $E_\lambda[X_1^2 / \sum_{i=1}^n X_i = y]$. y/n)^2-y/n +y/n^2

   (c) Find $\psi(\lambda)$ s.t. $E_\lambda[X_1^2 / \sum_{i=1}^n X_i]$ is UMVUE for $\psi(\lambda)$. lam+lam ^2

3. Let $X_1, \ldots, X_n$ be i.i.d. Gaussian(μ, σ^2), where μ is known. Consider the following family of estimators for σ^2,

$$\delta_K(X_1, \ldots, X_n) = \frac{1}{K} \sum_{i=1}^n (X_i - \bar{X})^2, \tag{1}$$

   where $\bar{X}$ is the sample mean.

   (a) Find MSE for $\delta_K(\cdot)$.

   (b) Find the optimal value of K for which MSE is the minimum.

4. Let $X_1, \ldots, X_n$ be i.i.d. RVs with $f_\lambda(\cdot)$ s.t.

$$f_\lambda(x) = \frac{1}{\lambda} e^{-\frac{x}{\lambda}}, \quad \forall\, x \geq 0\,,\ \lambda \in (0, \infty) \tag{2}$$

   unbiased: g(sumx)=(sum x)^2/ n(n+1).
   Let $\psi(\lambda) = \lambda^2$. Find UMVUE for $\psi(\cdot)$. sufficient: sum(x)

5. Let $X_1, \ldots, X_n$ be i.i.d. RVs with $f_\theta(\cdot)$ where

$$\begin{aligned} f_\theta(x) &= 2x/\theta^2, \quad 0 < x < 0, \\ &= 0, \qquad\quad\ \ otherwise. \end{aligned}$$

   Let $\delta_c(\bar{x}) = c\, max\{x_1, \ldots, x_n\}$.

   (a) Find MSE for $\delta_c(\bar{x})$.

   (b) Find c that minimizes MSE. (2n+2)/(2n+1)

6. Let $X \sim N(\theta, \theta^2)$ and $\theta \in [0, \infty)$. Find MLE for θ^2.see the range of theta

7. Let X_1, ...,X_n be i.i.d. RVs. Find MLE for θ.

   (a) Bernoulli distribution with parameter θ

   (b) Geometric distribution

$$f_\theta\,(x) = (1-\theta)^{x-1}\theta$$

   (c) Poisson distribution

$$f_\theta(x) = \frac{\theta^x e^{-\theta}}{x!}$$

   sum X/n

   (d) Binomial distribution

$$f_\theta(x) = \frac{n!}{x!(n-x)!}\theta^x (1-\theta)^{n-x}$$

   sum X/n2

(e) Negative Binomial distribution

$$f_\theta(x) = \binom{x-1}{r-1} \cdot \theta^r \cdot (1-\theta)^{x-r}$$

(f) Exponential distribution

$$f_\theta(x) = \frac{1}{\theta} e^{-\frac{x}{\theta}}, \qquad x \geq 0$$

(g) Gaussian distribution

$$f_\theta(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(x-\theta)^2/2\sigma^2}$$ £x/n

(h) Rayleigh distribution

$$f_\theta(x) = \frac{r}{\theta^2} exp\left(\frac{-r^2}{2\theta^2}\right), \qquad x > 0$$

(i) Gamma distribution

$$f_\theta(x) = \frac{\theta^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\theta x}, \qquad x > 0, \quad \theta, \alpha > 0.$$ alpha n /£X

(j) Pareto distribution

$$f_\theta(x) = \theta \frac{\beta^\theta}{x^{\theta+1}}, \qquad x \geq \beta, \quad \theta, \beta > 0.$$

8. Let $X_1, \ldots, X_n$ be i.i.d. Poisson(λ). Find MLE for $(1-\lambda)^2$.