EE225: Problems on Boolean Networks

November 2, 2016

1. For following Boolean networks defining the dynamical systems

$$x(k+1) = F(x(k))$$

where x is a state vector, determine, 1) all fixed points, 2) Longest orbit in state space and the period, 3) all chains of length 1 and 2 merging on to fixed points and the largest periodic orbit, 4) whether the map $F(.)$ is a permutation of the state space. In two of the cases the systems are XOR linear where $F(x) = Ax$. The matrices A are

1. $A = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}$
2. $A = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}$
3. $F(x) = (x_1x_2, x_2x_3, x_3x_4, x_4x_5, x_5x_1)^T$
4. $F(x) = (x_2, x_3, x_4, x_5, x_1 + x_3x_4)^T$
5. $F(x) = (< x_1, x_2, x_3 >, x_3, x_4, x_5, x_1)^T$
6. $F(x) = (x_2, x_3, x_4, x_5, x_1 \oplus x_5)^T$

where $< x, y, z >$ denotes the majority bit function on x, y, z and $\oplus$ is XOR.

2. Find all solutions to following Boolean system.

$$\begin{aligned} u \oplus v \oplus vw &= 0 \\ v \oplus w \oplus wx &= 1 \\ w \oplus x \oplus xy &= 0 \\ x \oplus y \oplus yz &= 1 \\ y \oplus z \oplus zw &= 0 \\ z \oplus w \oplus wx &= 1 \end{aligned}$$

3. A product of sum (POS) formula in Boolean variables u, v, w, x, y, z is given by

$$F = (u + v' + w)(v + w' + x)(w + x' + y)(x + y' + z)(y + z' + u)(z + u' + v)$$

Find all Boolean assignments to the variables such that F evaluates to True. The sum $+$ is interpreted as OR.

4. Do above problem for POS formula

$$F = (u' + v' + w)(v' + w' + x)(w' + x' + y)(x' + y' + z)(y' + z' + u)(z' + u' + v)$$

5. Find all assignments of variables such that following Boolean function evaluates to 1.

$$F = 1 \oplus w \oplus y \oplus z \oplus wx \oplus wz \oplus xy \oplus xz \oplus xyz$$