Lecture 11
Digital Circuits (I)
THE INVERTER

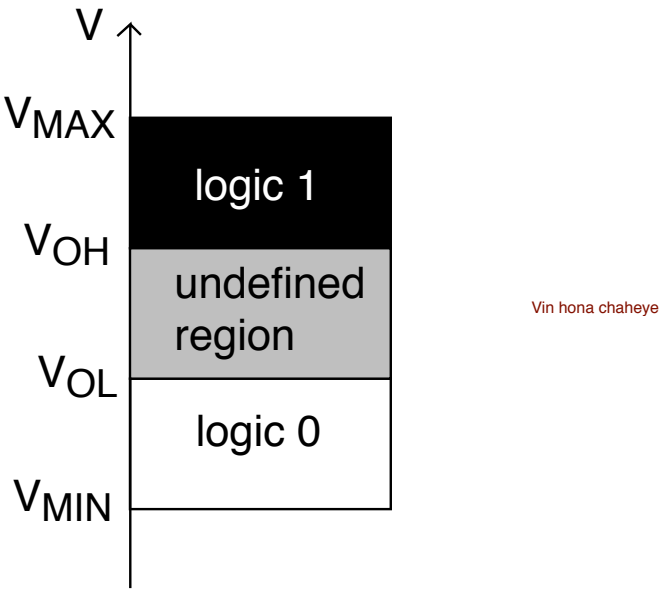
Outline

- Introduction to digital circuits
  - –The inverter
- NMOS inverter with resistor pull-up

Reading Assignment:
Howe and Sodini; Chapter 5, Sections 5.1-5.3

1. Introduction to digital circuits: the inverter

In digital circuits, digitally-encoded information is represented by means of two distinct voltage ranges:



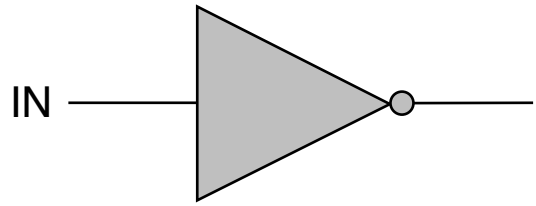
The Static Definition

- ***Logic 0:*** $V_{MIN} \le V \le V_{OL}$
- ***Logic 1:*** $V_{OH} \le V \le V_{MAX}$
- ***Undefined logic value:*** $V_{OL} \le V \le V_{OH}$

Logic operations are performed using ***logic gates***.

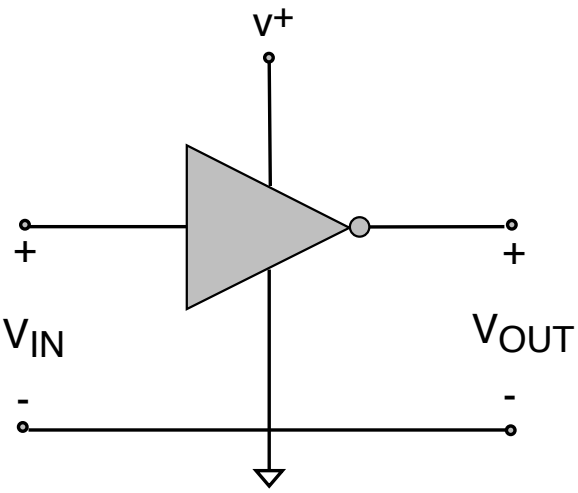
Simplest logic operation of all: ***inversion*** $\Rightarrow$ inverter

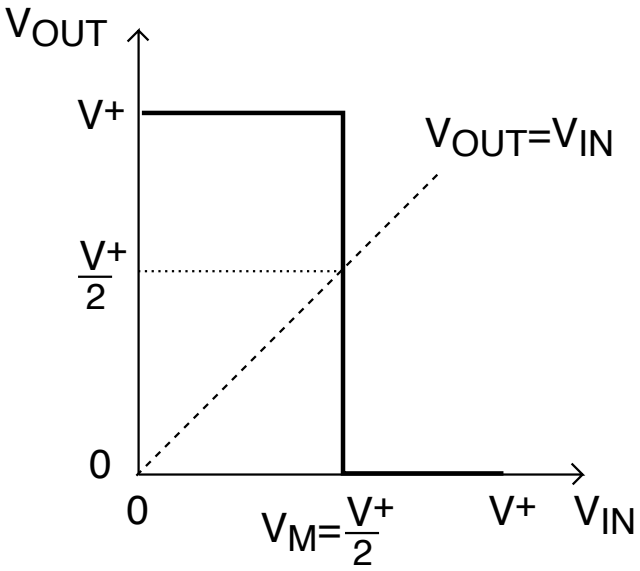
Ideal inverter



IN	OUT
0	1
1	0

Circuit representation and ideal transfer function:



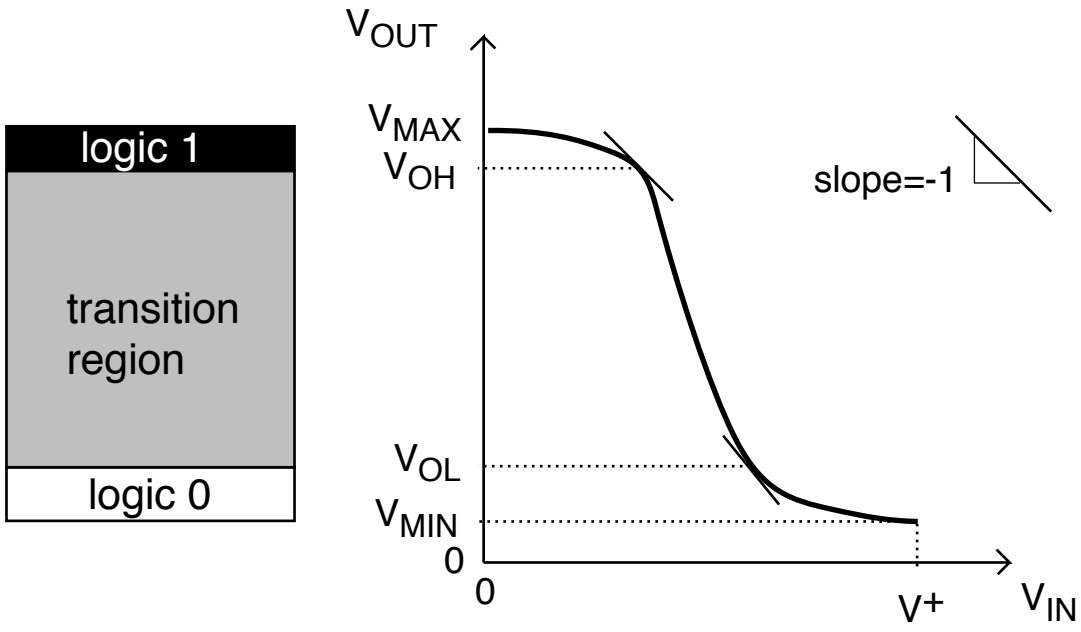


Define ***switching point*** or ***logic threshold*** :

- $V_M \equiv$ input voltage for which $V_{OUT} = V_{IN}$
  - For $0 \le V_{IN} < V_M \quad \Rightarrow V_{OUT} = V^+$
  - For $V_M < V_{IN} \le V^+ \quad \Rightarrow V_{OUT} = 0$

Ideal inverter returns well defined ~~logical outputs (0 or~~ V^+~~)~~ even in the presence of considerable noise in V_{IN} (from voltage spikes, crosstalk, etc.)
$\Rightarrow$ signal is ***regenerated***!

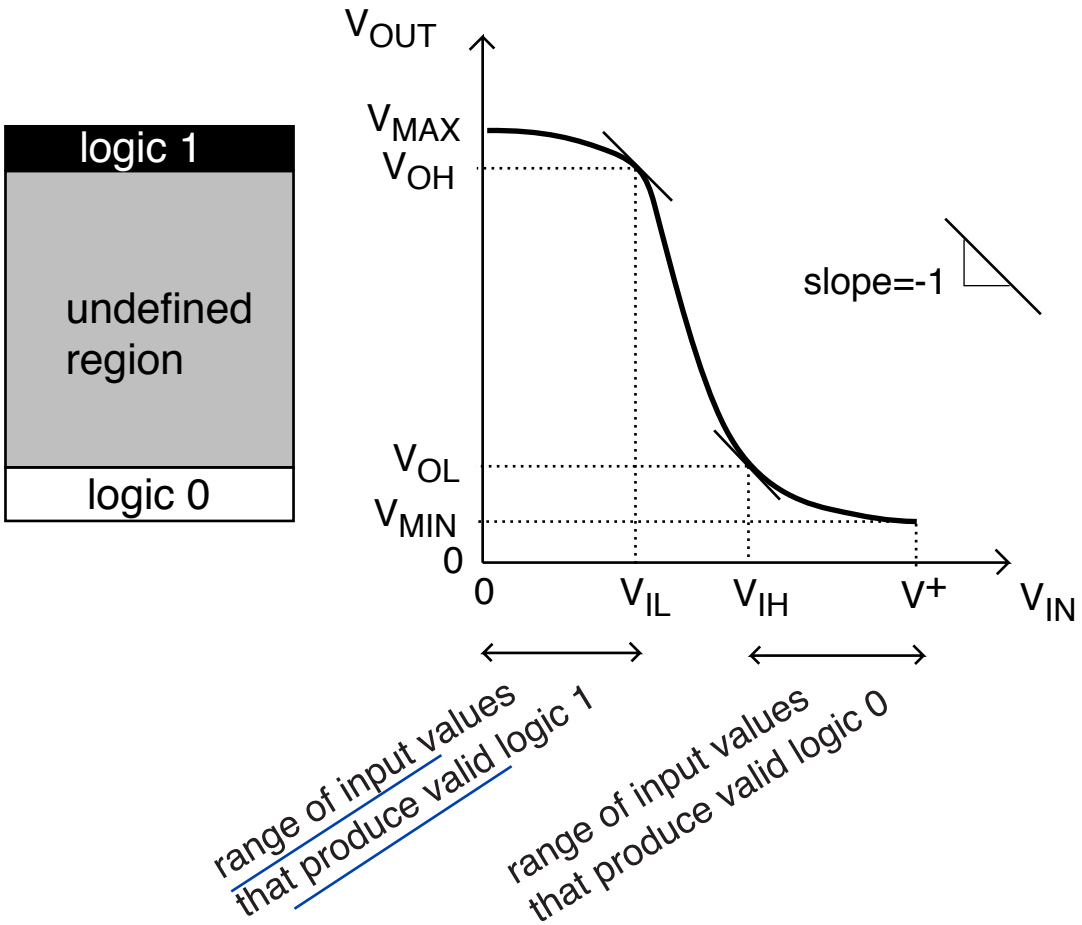
"Real" inverter



In a real inverter, valid logic levels defined as follows:

- Logic 0:
  - $V_{MIN} \equiv$ output voltage for which $V_{IN} = V^+$
  - $V_{OL} \equiv$ smallest output voltage where slope = -1
- Logic 1:
  - $V_{OH} \equiv$ largest output voltage where slope = -1
  - $V_{MAX} \equiv$ output voltage for which $V_{IN} = 0$

Two other important voltages:



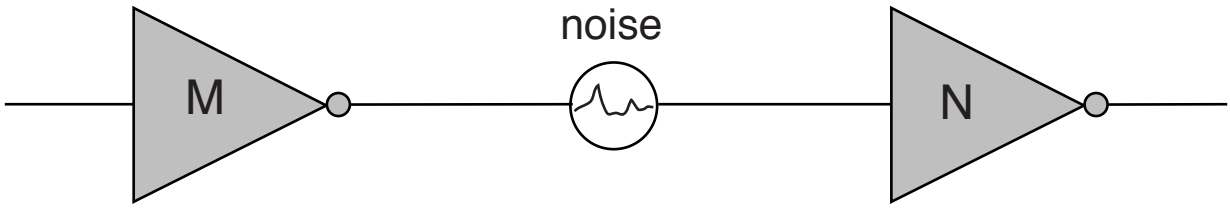
Define:

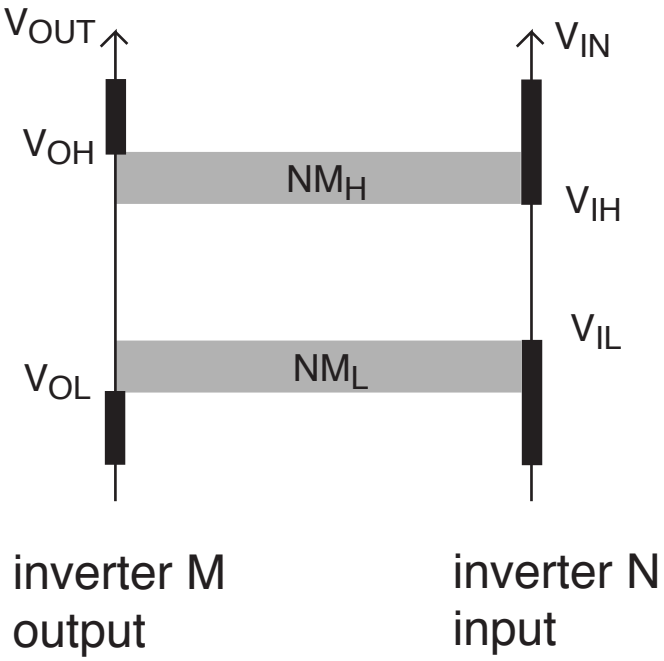
V_{IL} ≡ smallest input voltage where slope = -1
V_{IH} ≡ highest input voltage where slope = -1

If range of output values V_{OL} to V_{OH} is ***wider*** than the range of input values V_{IL} to V_{IH}, then the inverter exhibits some noise immunity. (|Voltage gain| > 1)

Quantify this through ***noise margins***.

Chain of two inverters:





Define ***noise margins***:

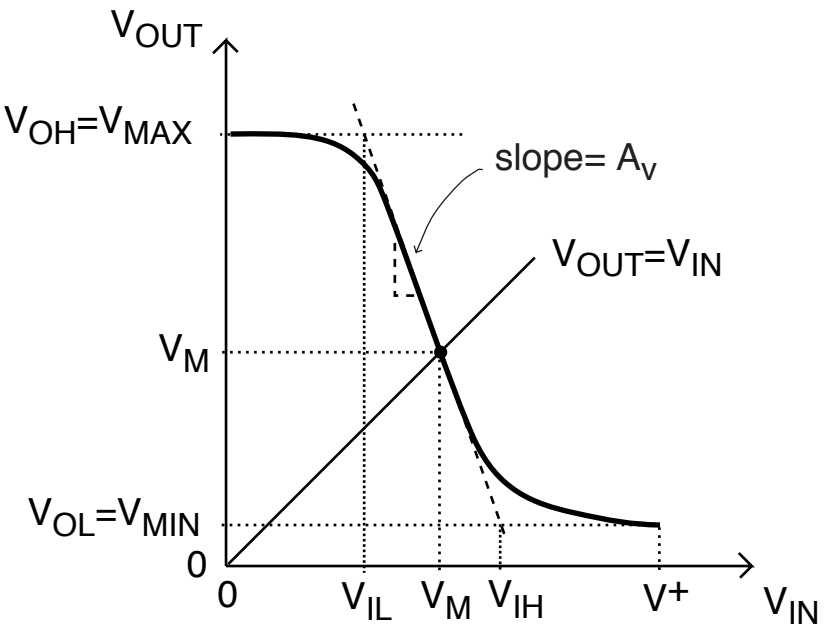
$NM_H \equiv V_{OH} - V_{IH}$ noise margin high

$NM_L \equiv V_{IL} - V_{OL}$ noise margin low

Simplifications for hand calculations:
Logic levels and noise margins

It is hard to compute points in transfer function with slope = -1.

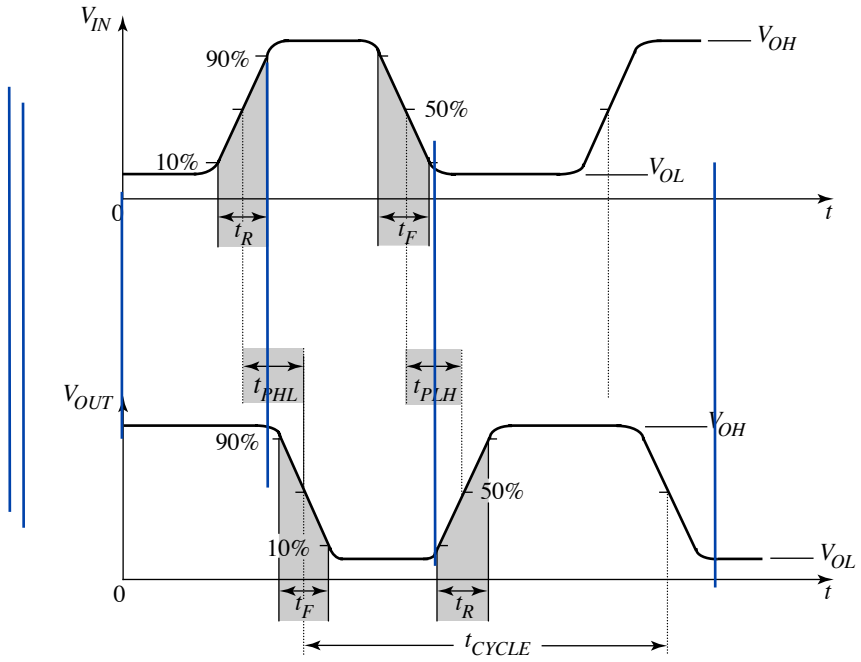
Approximate in the following way:



- Assume $V_{OL} \approx V_{MIN}$ and $V_{OH} \approx V_{MAX}$
- Trace tangent of transfer function at V_M
  - Slope = small signal voltage gain (A_v) at V_M
- $V_{IL} \approx$ intersection of tangent with $V_{OUT} = V_{MAX}$
- $V_{IH} \approx$ intersection of tangent with $V_{OUT} = V_{MIN}$

Transient Characteristics

Inverter switching in the time domain:



t_R ≡ ***rise time*** between 10% and 90% of total swing

t_F ≡ ***fall time*** between 90% and 10% of total swing

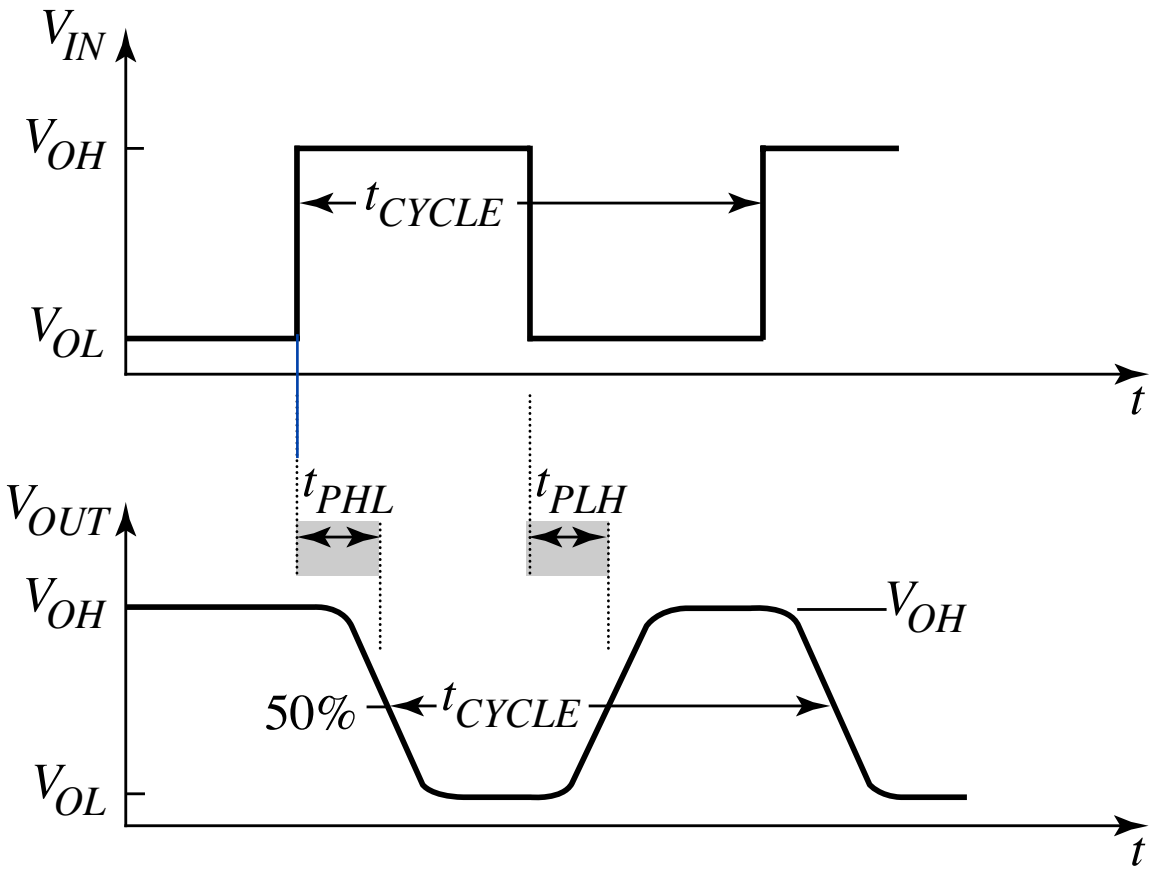
t_{PHL} ≡ ***propagation delay from high-to-low*** between 50% points

t_{PLH} ≡ ***propagation delay from low-to-high*** between 50% points

Propagation delay : $\mathbf{t_P = \frac{1}{2}(t_{PHL} + t_{PLH})}$

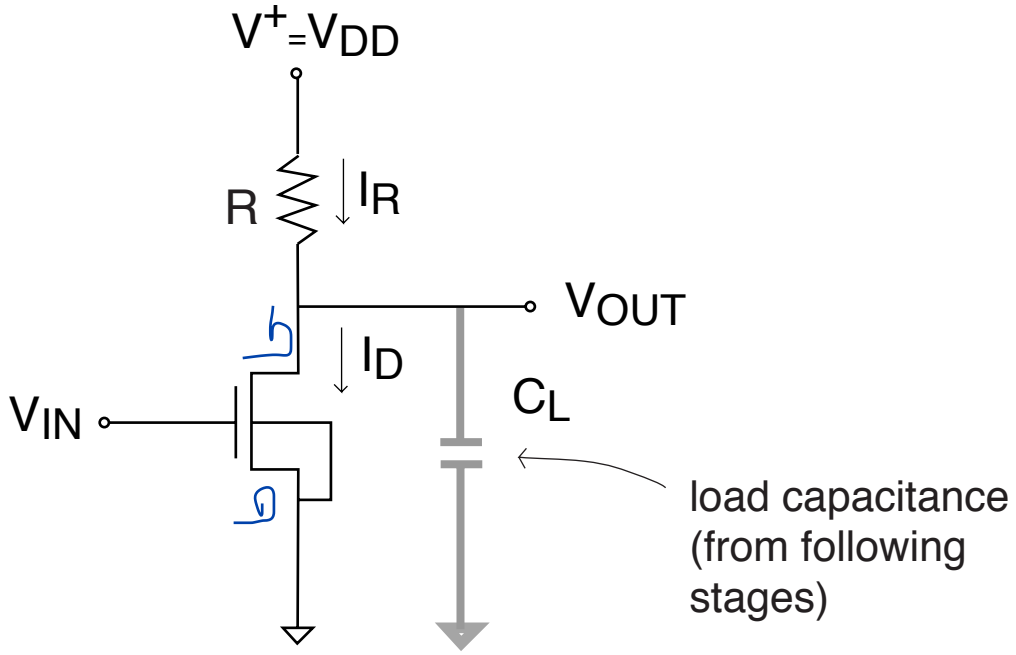
Simplifications for hand calculations:
Propagation delay

- Consider input waveform is an ideal square wave
- Propagation delay times = delay times to 50% point



- **SPICE essential for accurate delay analysis**

2. NMOS inverter with "pull-up" resistor

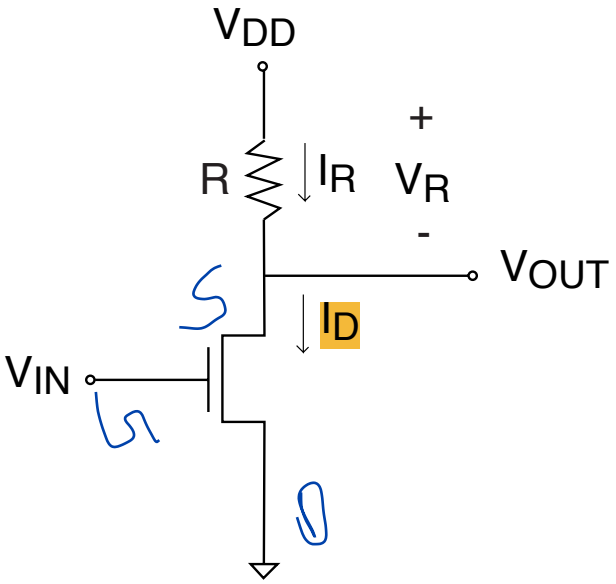


Essential features:

- $V_{BS} = 0$ (typically not shown)
- C_L summarizes capacitive loading of the following stages (other logic gates, interconnect lines, etc.)

Basic Operation:

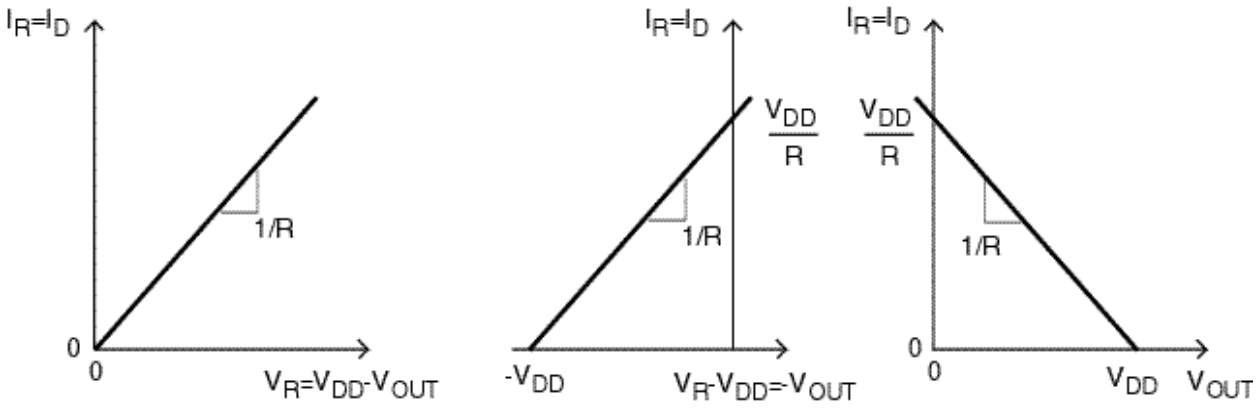
- If $V_{IN} < V_T$, MOSFET is **OFF**
  - $\Rightarrow V_{OUT} = V_{DD}$
- If $V_{IN} > V_T$, MOSFET is **ON**
  - $\Rightarrow V_{OUT}$ small
  - Value set by resistor / nMOS divider



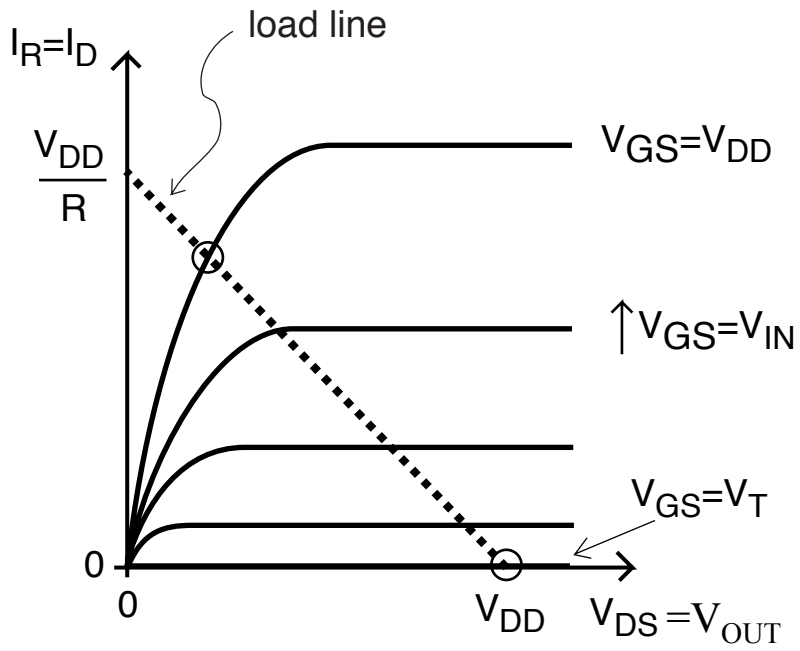
Transfer function obtained by solving:

$$I_R = I_D$$

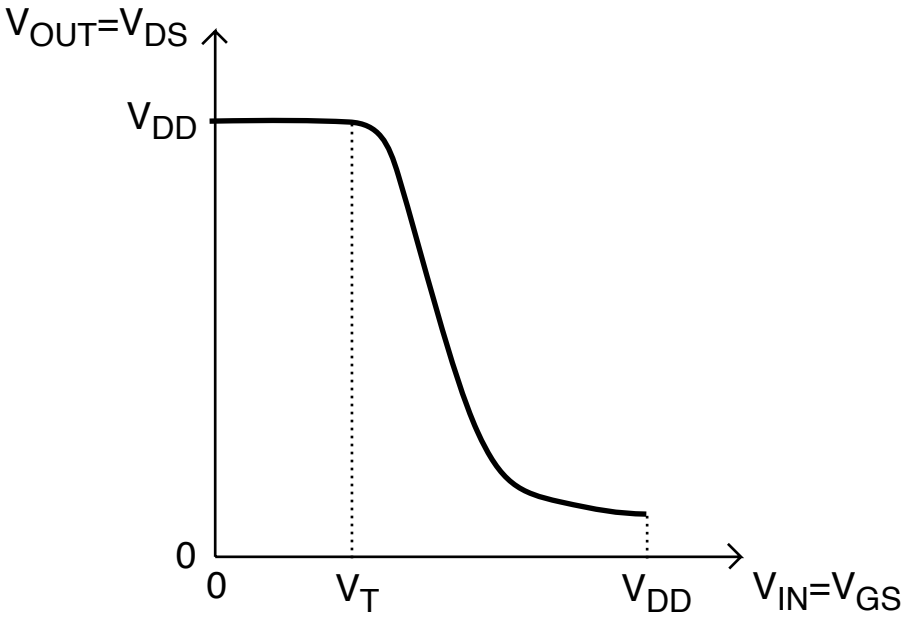
Can solve graphically: I–V characteristics of load:



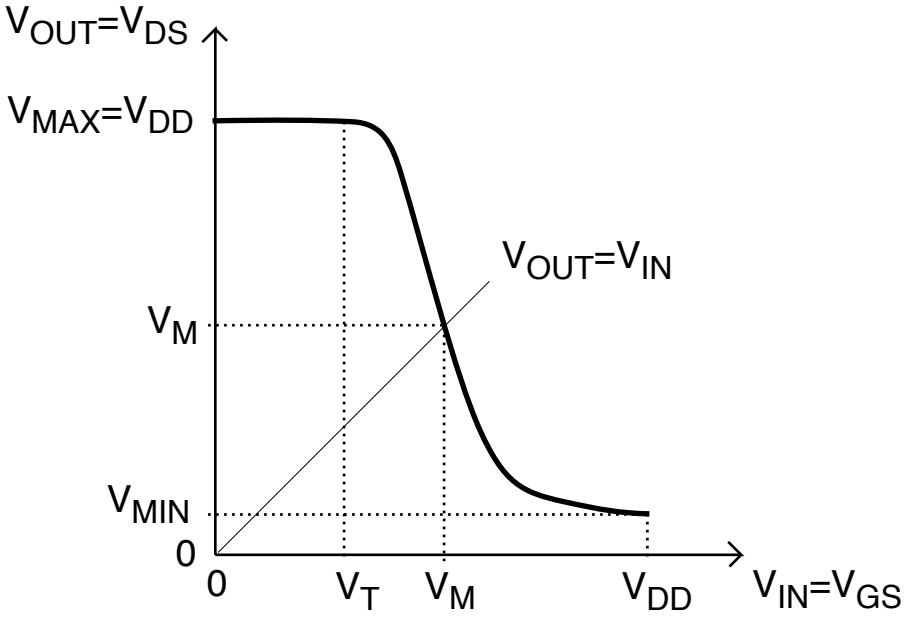
Overlap I–V characteristics of resistor pull-up on I–V characteristics of transistor:



Transfer function:



Logic levels:



For V_{MAX}, transistor is cut-off, $I_D = 0$:

$$V_{MAX} = V_{DD}$$

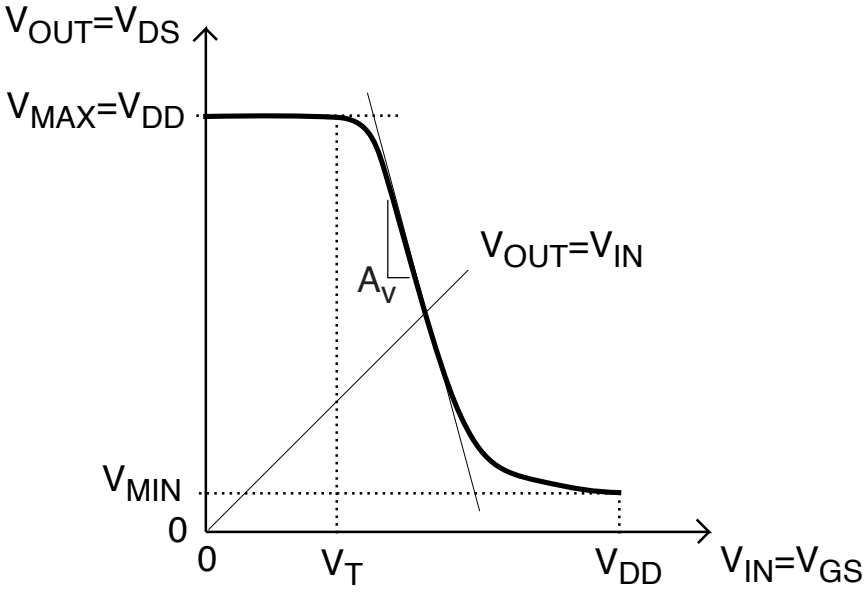
For V_{MIN}, transistor is in linear regime; solve:

$$\mathbf{I_D = \frac{W}{L}\mu_n C_{ox}\left(V_{DD} - \frac{V_{MIN}}{2} - V_T\right)V_{MIN} = I_R = \frac{V_{DD} - V_{MIN}}{R}}$$

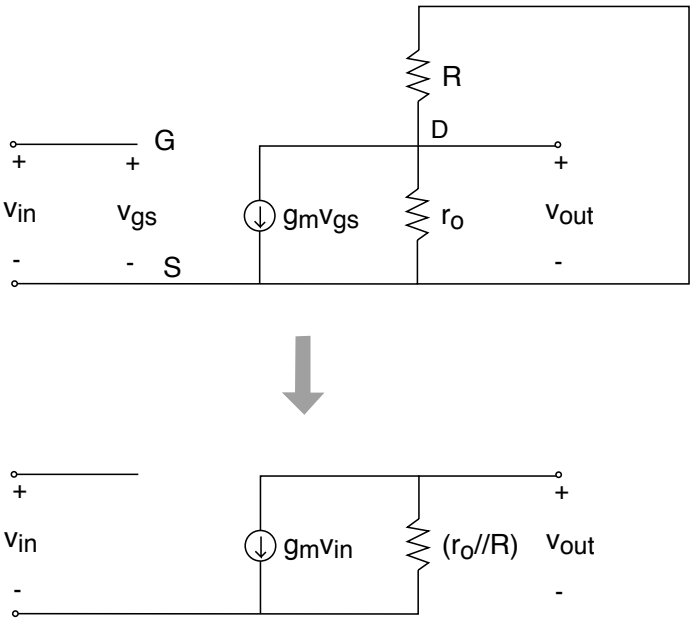
For V_M, transistor is in saturation; solve:

$$\mathbf{I_D = \frac{W}{2L}\mu_n C_{ox}\left(V_M - V_T\right)^2 = I_R = \frac{V_{DD} - V_M}{R}}$$

Noise Margins:



Small signal equivalent circuit model at V_M (transistor in saturation):



$$A_v = \frac{v_{out}}{v_{in}} = -g_m(r_o // R) \approx -g_m R$$

What did we learn today?

Summary of Key Concepts

- Logic circuits must exhibit immunity to noise in the input signal
  - ***Noise margins***
- Logic circuits must be ***regenerative***
  - Able to restore clean logic values even if input is noisy.
- ***Propagation delay***: time for logic gate to perform its function.
- Concept of ***load line***: graphical technique to visualize transfer characteristics of inverter.
- First-order solution (by hand) of inverter figures-of-merit easy if ***regions of operation*** of transistor are correctly identified.
- For more accurate solutions, use SPICE (or other CAD tool).