Data Analysis & Interpretation

Quiz # 1.

Dt - 05/08/2017 Time - 11 am - 12.30 pm

Problem 1 [10M]: Crime data in certain locality is summarized in the following table for weekdays & weekends separately.

Time	0	3	6	9	12	15	18	21	
Frequency	80	100	20	10	0	0	0	10	week days
	20	30	5	5	2	3	4	6	week-ends

Find (separately for both data sets)

[1 each] ① mean ② median ③ mode ④ variance

⑤ Find correlation between two data sets

Problem 2 [4M]: Consider $(\Omega, \mathcal{F}, P)$ and let $A_1, A_2, \dots$ be such that $A_n \in \mathcal{F}$ and $A_n \supseteq A_{n+1}$ $\forall n$. Then show that $P(\lim_{n\to\infty} A_n) = \lim_{n\to\infty} P(A_n)$

Probem 3: Consider $(\Omega, \mathcal{F}, P)$ & let $A_1, A_2, \dots$ s.t. $A_n \in \mathcal{F}$ $\forall n$. Define $B = \bigcap_{n=1}^{\infty} \bigcup_{k=n}^{\infty} A_n$ and $C = \bigcup_{n=1}^{\infty} \bigcap_{k=n}^{\infty} A_n$.

[2M each]

① Show that B and $C \in \mathcal{F}$.

② Which of the two sets is bigger (prove your assertion).

③ Evaluate B & C. when $A_n \supseteq A_{n+1}$ $\forall n$.

EE223 : Data Analysis & Interpretation

Quiz #2

45

Date : 06/09/2017 Time : 20:30 - 22:00

———— x ———— x ———— x ———— x ————

Q.1 Let X be a discrete r.v. such that

$$P(X=k) = e^{-\lambda} \lambda^k / k!$$

for $k \in \{0, 1, 2, \ldots\}$ and $\lambda > 0$. Find $E[X^3]$.

$e^t = 1 + n + \frac{n^2}{2!} + \frac{n^3}{3!}$

———— x ———— x ————

Q.2 Let X be a Gaussian random variable with mean 5 and variance 10. Let

$$P(X > 12) = \text{Erfc}(\alpha), \text{ where}$$

$$\text{Erfc}(\alpha) = \frac{1}{\sqrt{2\pi}} \int_{\alpha}^{\infty} e^{-x^2/2}\, dx.$$

Find α.

$EY = 5$

$EX^2 - 25 = 10$

$EX^2 = 35$

———— x ———— x ————

Q.3 Let X be a continuous r.v. with strictly increasing distribution function $F_X(\cdot)$. Let $Y = F_X(X)$. Find $F_Y(\cdot)$.

———— x ———— x ————

Q.4 Let X be a r.v. with $f_X(x) = \frac{1}{2} e^{-|x|}$. Let $Y = X^2$. Find $F_Y(y \mid X \leq 5)$.

EE223: Data Analysis & Interpretation
Quiz # 3

Date: 14/09/2017 Time: 8:30-10:30

Let X be a non-negative r.v. with finite variance. Show that

(2M) ~~(a)~~ $E[\sqrt{X}] \leq \sqrt{E[X]}$ (square roots are +ve)

(2M) (b) $E\left[\frac{1}{X}\right] \geq \frac{1}{E[X]}$

Q.2 Let $X_1, \ldots, X_n$ be independent Gaussian random variables s.t. $X_k \sim G(\mu_k, \sigma_k)$.
(5M) Let $Y = \frac{1}{n}\sum_{k=1}^{n} X_k$. Find $F_Y(\cdot)$.

Hint: Moment generating fn.

Q.3 Let X_1, X_2, X_3 be i.i.d. exponential r.v.
(5M)

$$f_{X_1}(x) = \lambda e^{-\lambda x} \quad \forall x \geq 0 \text{ \& } 0 \text{ otherwise.}$$

Find $E[X_1 \mid \max\{X_1, X_2, X_3\} = X_1]$.

Q.4 Let $Y : (\Omega, \mathcal{F}, P) \to (\mathbb{R}, \mathcal{B}, P_Y)$, where
(6M) $P_Y(B) = P(\{\omega : Y(\omega) \in B\})$. Also, let $B_1 \in \mathcal{B}$ s.t. $P_Y(B_1) = 1$. Show that Y and $1_{\{Y \in B_1\}}$ are independent, where $1_{\{Y \in B_1\}}$ is