

## MA 2017, Tutorial Sheet-7

### Wave equation and Laplace equation

1. Solve the following wave equations.

- (a)  $u_{tt} = 9u_{xx}, \quad 0 < x < 1, t > 0,$   
 $u(0, t) = 0 = u(1, t), \quad t \geq 0$   
 $u(x, 0) = x(1 - x), \quad u_t(x, 0) = 0, 0 \leq x \leq 1.$
- (b)  $u_{tt} = 9u_{xx}, \quad 0 < x < 1, t > 0,$   
 $u(0, t) = 0 = u(1, t), \quad t \geq 0$   
 $u(x, 0) = 0, \quad u_t(x, 0) = x(1 - x), 0 \leq x \leq 1.$
- (c)  $u_{tt} = 4u_{xx}, \quad 0 < x < 1, t > 0,$   
 $u(0, t) = 0 = u(1, t), \quad t \geq 0$   
 $u(x, 0) = 0, \quad u_t(x, 0) = x(x^3 - 2x^2 + 1), 0 \leq x \leq 1.$
- (d)  $u_{tt} = 5u_{xx}, \quad 0 < x < \pi, t > 0,$   
 $u(0, t) = 0 = u(\pi, t), \quad t \geq 0$   
 $u(x, 0) = x \sin x, \quad u_t(x, 0) = 0, 0 \leq x \leq \pi.$
- (e)  $u_{tt} = 5u_{xx}, \quad 0 < x < 2, t > 0,$   
 $u_x(0, t) = 0 = u_x(2, t), \quad t \geq 0$   
 $u(x, 0) = 2x^2(3 - x), \quad u_t(x, 0) = 0, 0 \leq x \leq 2.$
- (f)  $u_{tt} = 5u_{xx}, \quad 0 < x < 2, t > 0,$   
 $u_x(0, t) = 0 = u_x(2, t), \quad t \geq 0$   
 $u(x, 0) = 0, \quad u_t(x, 0) = 2x^2(3 - x), 0 \leq x \leq 2.$
- (g)  $u_{tt} = 16u_{xx}, \quad 0 < x < \pi, t > 0,$   
 $u_x(0, t) = 0 = u_x(\pi, t), \quad t \geq 0$   
 $u(x, 0) = x^2(x - \pi)^2, \quad u_t(x, 0) = 0, 0 \leq x \leq \pi.$
- (h)  $u_{tt} = 16u_{xx}, \quad 0 < x < \pi, t > 0,$   
 $u_x(0, t) = 0 = u_x(\pi, t), \quad t \geq 0$   
 $u(x, 0) = 0, \quad u_t(x, 0) = x^2(x - \pi)^2, 0 \leq x \leq \pi.$

2. Solve the following Laplace equations.

- (a)  $u_{xx} + u_{yy} = 0, \quad 0 < x < 1, 0 < y < 1,$   
 $u(x, 0) = x(1 - x), \quad u(x, 1) = 0, \quad 0 \leq x \leq 1,$   
 $u(0, y) = 0, \quad u(1, y) = 0, \quad 0 \leq y \leq 1.$

- (b)  $u_{xx} + u_{yy} = 0, \quad 0 < x < 2, 0 < y < 3,$   
 $u(x, 0) = x^2(2 - x), \quad u(x, 3) = 0, \quad 0 \leq x \leq 2$   
 $u(0, y) = 0, \quad u(2, y) = 0 \quad 0 \leq y \leq 3.$
- (c)  $u_{xx} + u_{yy} = 0, \quad 0 < x < \pi, 0 < y < \pi,$   
 $u(x, 0) = x \sin x, \quad u(x, \pi) = 0, \quad 0 \leq x \leq \pi,$   
 $u(0, y) = 0, \quad u(\pi, y) = 0 \quad 0 \leq y \leq \pi.$
- (d)  $u_{xx} + u_{yy} = 0, \quad 0 < x < 2, 0 < y < 2,$   
 $u(x, 0) = 0, \quad u(x, 2) = x^2 - 4, \quad 0 \leq x \leq 2$   
 $u_x(0, y) = 0, \quad u_x(2, y) = 0, \quad 0 \leq y \leq 2$
- (e)  $u_{xx} + u_{yy} = 0, \quad 0 < x < 2, 0 < y < 1,$   
 $u_y(x, 0) = 0, \quad u_y(x, 1) = 0, \quad 0 \leq x \leq 2$   
 $u(0, y) = y^2(3 - 2y), \quad u(2, y) = 0, \quad 0 \leq y \leq 2.$
- (f)  $u_{xx} + u_{yy} = 0, \quad 0 < x < 2, 0 < y < 3,$   
 $u(x, 0) = 0, \quad u(x, 3) = 0, \quad 0 \leq x \leq 2$   
 $u_x(0, y) = 0, \quad u_x(2, y) = y(3 - y), \quad 0 \leq y \leq 3.$