Homework 3

by Manoj Gopalkrishnan - Friday, 4 August 2017, 4:12 PM

1. A graph is bipartite if it has a cut consisting of all the edges. An odd cycle is a cycle with an odd number (3,5,7...) of edges.

(A) Give an example of a graph that is not bipartite.

(B) Show that a graph is bipartite if and only if it has no odd cycles.

2. Show that every tree has a node of degree less than 2. Hence show that every tree on n nodes has n-1 edges. Use this to give an inductive definition of trees

3. Fix an undirected graph G. We are going to define a graph GG whose nodes are spanning trees of G. Two spanning trees T and T' are connected if there are edges e and e' in G such that T' = T + e - e'.

a) Draw the graph GG when G is a 5-cycle, and when G is the clique on 4 nodes.

b) Prove or disprove: For all connected G, the graph GG is connected.

4. Prove/ disprove: If G is such that there is at least one path between every pair of nodes in G then G has a spanning tree.

5. Example 1 to 14 of Chapter 3 from Valkenburg.