Tutorial-2

1. Show that the function $f(z) = z\bar{z}$ does not satisfy the Cauchy-Riemann equations (except at the origin). What about $z \cdot \mathrm{Re}(z)$ & $z \cdot \mathrm{Im}(z)$.

2. Show that if $f: \mathbb{C} \to \mathbb{C}$ takes on only real values (or only purely imaginary values) it cannot be holomorphic on $\mathbb{C}$ unless it is a constant. Can a non constant holomorphic function take on values along a given line in $\mathbb{C}$?

3. Show that if $f(z)$ is a holomorphic function on an open set Ω & $|f(z)|$ is constant then $f(z)$ is constant.

4. Determine the real & imaginary parts of the following functions:

(i) $\dfrac{1+z}{1-z}$ (ii) $z + z^{-1}$.

Verify the Cauchy Riemann equations in each case (except where the denominators vanish).

5. Show that the function $u = \dfrac{x^2+y^2-1}{x^2+y^2+1-2x}$

is harmonic and find v such that $u+iv$ is holomorphic & write the function f in terms of z.

6. Write down the Cauchy Riemann equations in polar coordinates & write the Laplace operator Δ in polar coordinates. Show that the function $\dfrac{1-r^2}{1+2r\cos\theta+r^2}$ is harmonic in the unit disc.

7. Show that if $f(z) = u(x,y) + iv(x,y)$ is holomorphic, the real & imaginary parts of $f(\bar{z})$ are harmonic. Do they

satisfy the Cauchy-Riemann equations?

8. Find the harmonic conjugate of

$e^x \cos y + e^y \cos x + xy$.