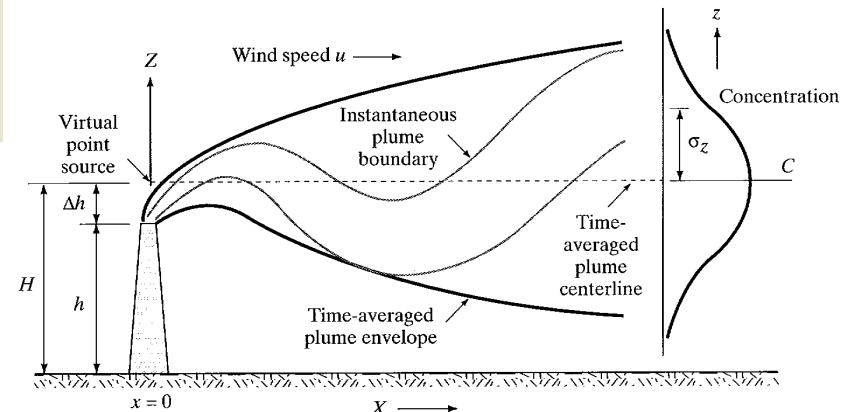


# Lecture 5

## Air Quality:

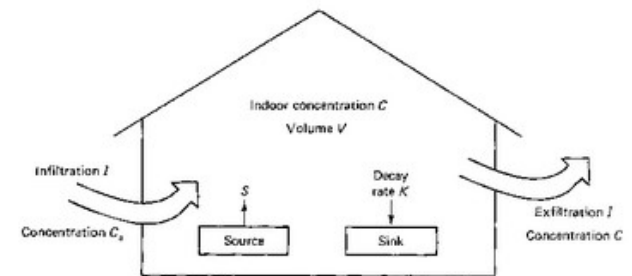
# Air Pollution Modeling



$$C(x, y, 0) = \left[ \frac{Q}{\pi \sigma_y \sigma_z u_H} \right] \left[ \exp \left[ -\frac{1}{2} \left( \frac{y}{\sigma_y} \right)^2 \right] \right] \left[ \exp \left[ -\frac{1}{2} \left( \frac{H}{\sigma_z} \right)^2 \right] \right]$$

**Harish C. Phuleria**  
CESE, IIT Bombay

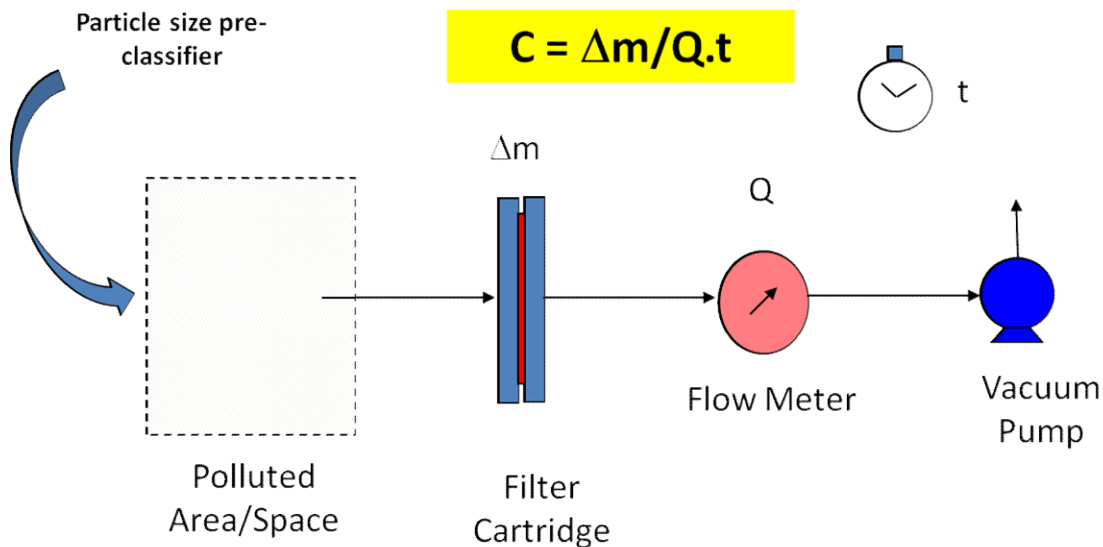
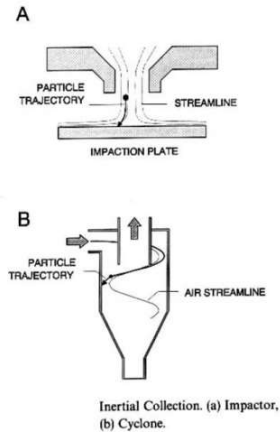
Email: [phuleria@iitb.ac.in](mailto:phuleria@iitb.ac.in)



accumulation rate = input rate + sources - output rate - decay

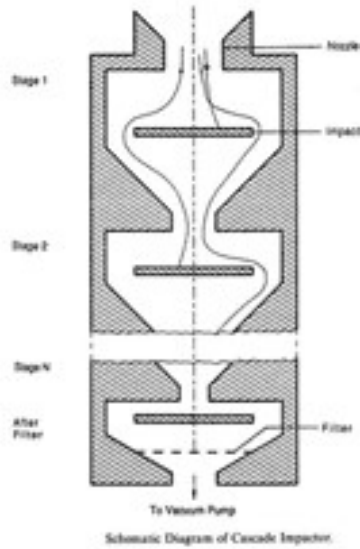
$$V \frac{dC}{dt} = S + C_a IV - CIV - KCV$$

# Recap 1

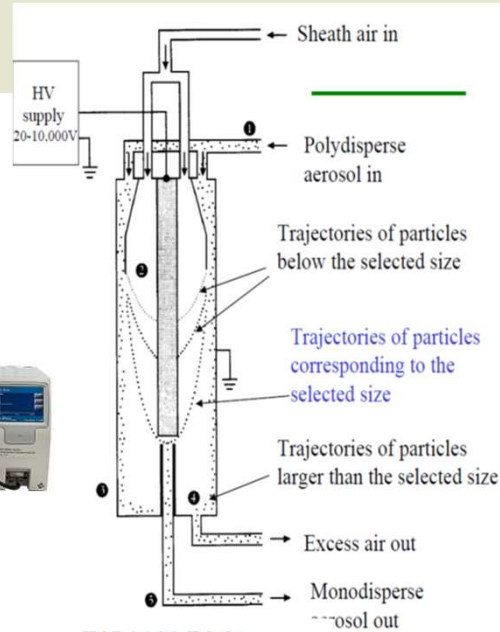


Choice of instruments is a function of its cost, intended analysis, time resolution, portability, ease of use

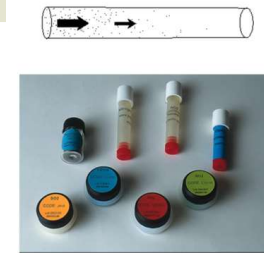
# Recap 2



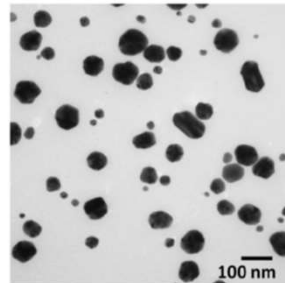
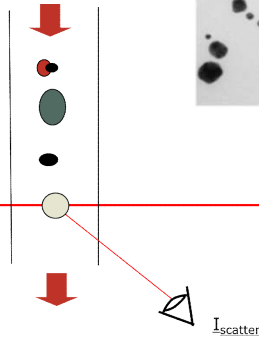
Anderson six-stage viable impactor



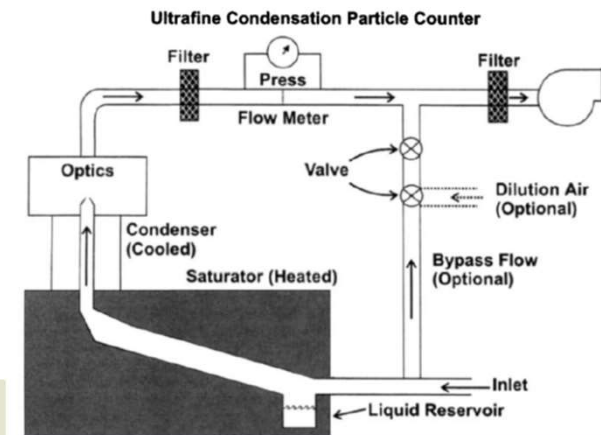
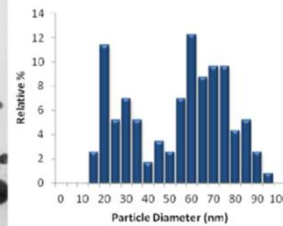
TSI 3080 DMA



Laser Source,  $\lambda$



Size Distribution

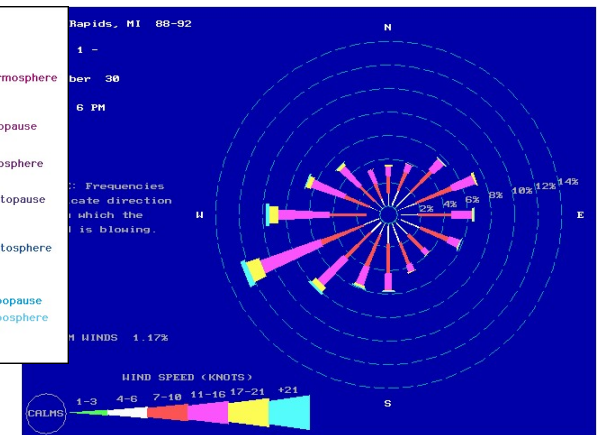
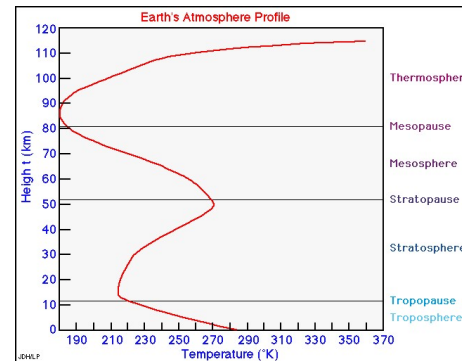


# Recap 3



- Three type of sources: point, line, & area
- Atmospheric mixing and dispersion is governed by meteorology

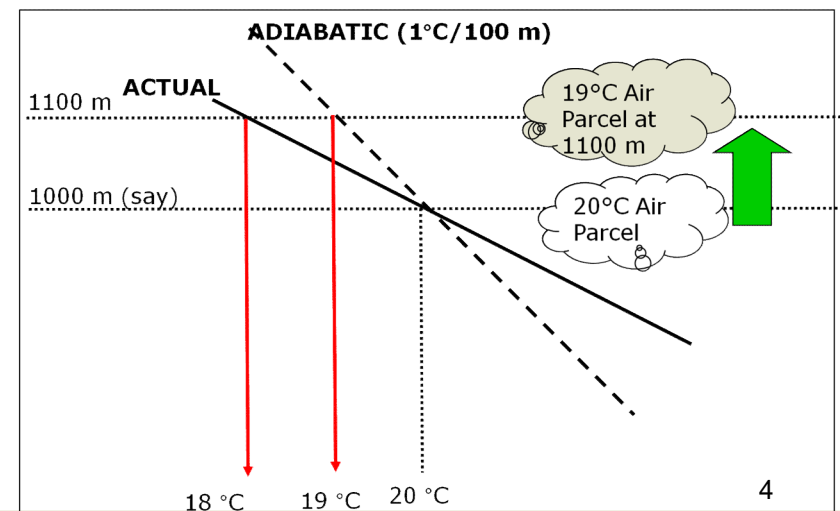
- Horizontal
  - Wind speed and direction
- Vertical
  - Temperature



- Atmosphere cools with height
  - Dry (adiabatic) lapse rate

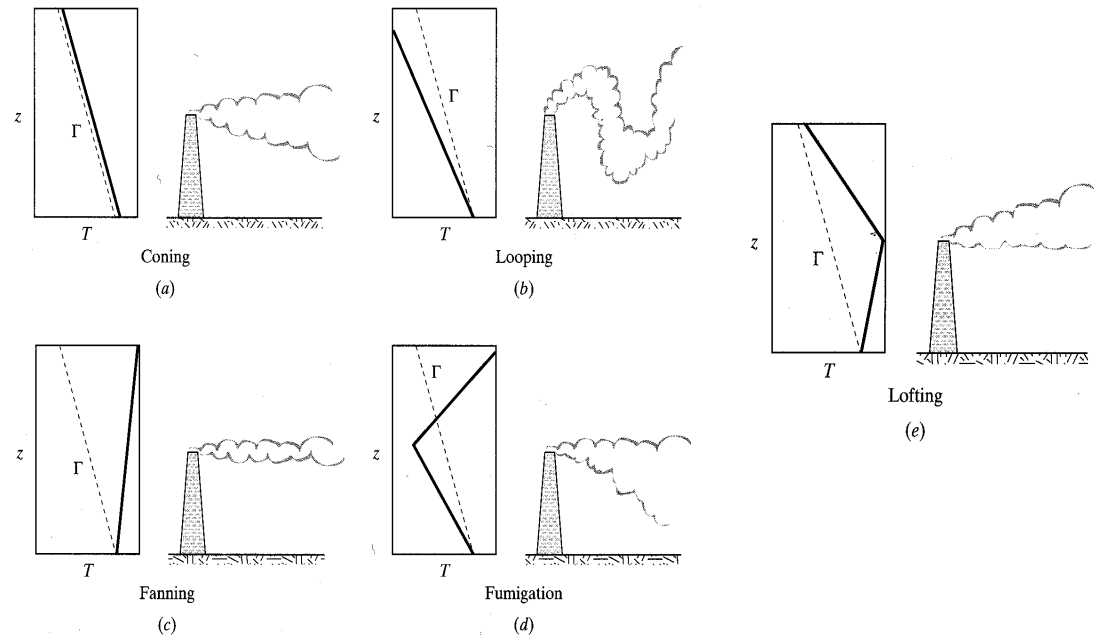
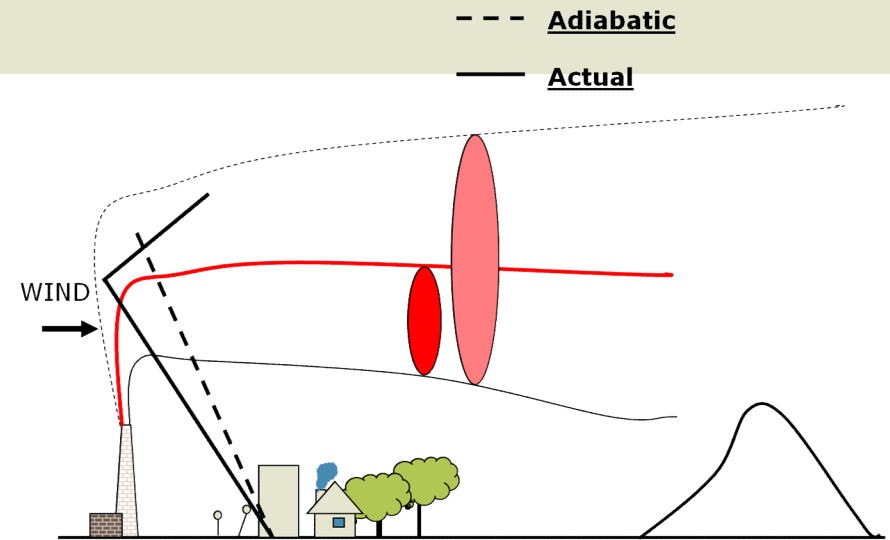
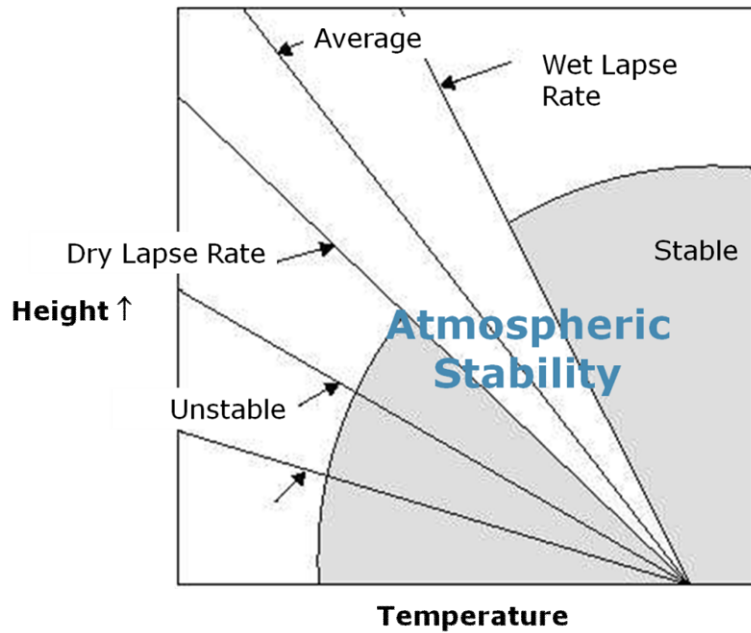
$$\Gamma = -\frac{dT}{dz} = -1.00^{\circ}\text{C}/100\text{ m}$$

- Effect of actual lapse rate on vertical mixing



# Recap 4

- Extreme case of stability when lapse rate is actually positive, leads to **temperature inversion** prevents nearly all upward mixing



# Today's Learning Objectives !

- To learn about air quality modeling methods

# Why are we doing all of this ?

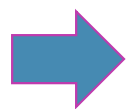
- If we want to set up a new industry, it implies adding a new source of pollutant(s)
- This source is PERMITTED to emit after it has applied the Best Available Control Technology (BACT) on their processes
- AFTER leaving the chimney, the concentrations on ground is determined by the meteorology



# Why are we doing all of this ?

Therefore,

- If we want to know WHERE to put the new industry
- If we want to know the pollution levels under the worst case scenario of STABLE conditions and low wind speeds
- If we want to know what height does the chimney need to be



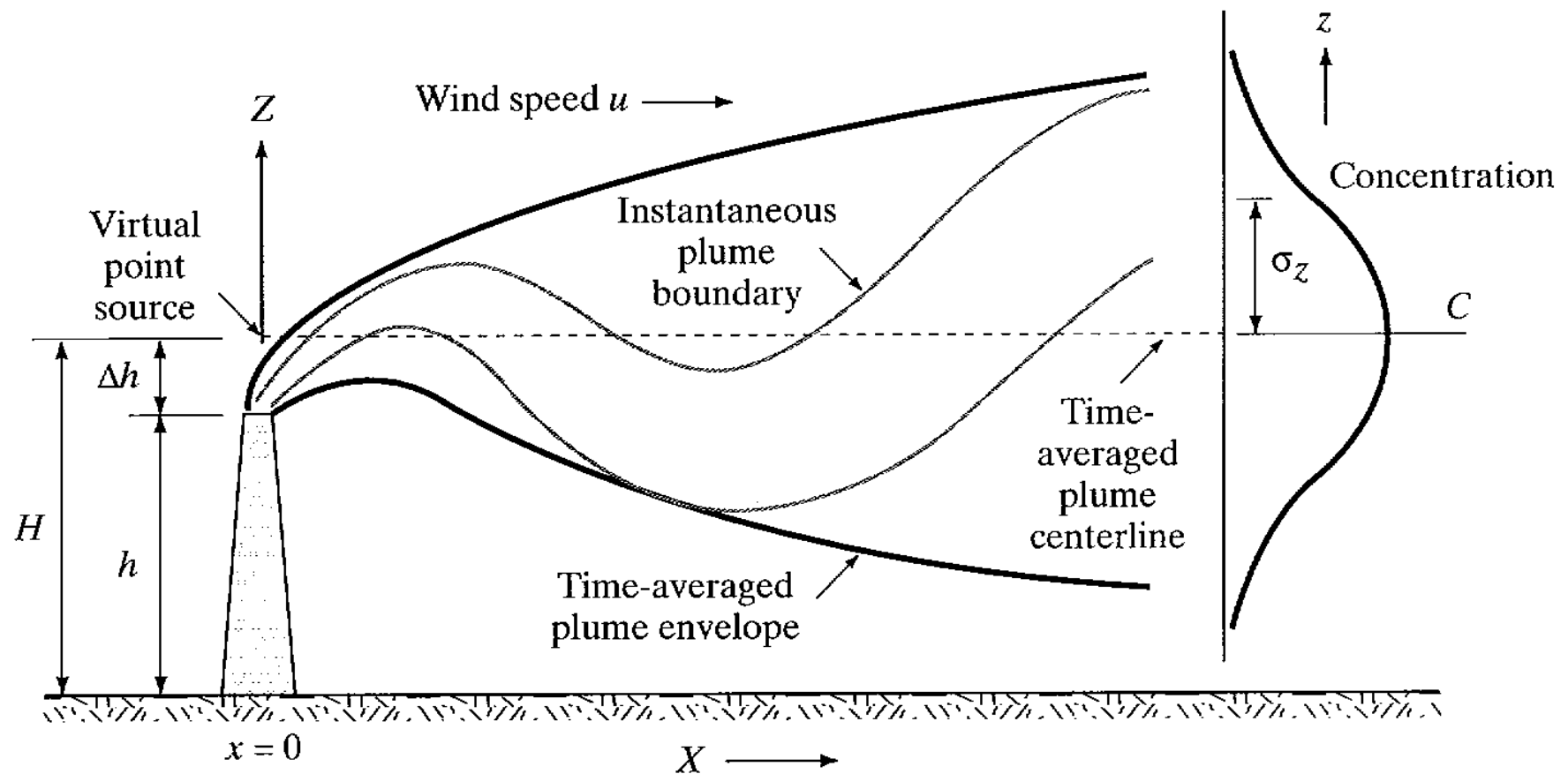
QUANTIFICATION of horizontal movement and vertical mixing becomes essential



# Modeling air quality

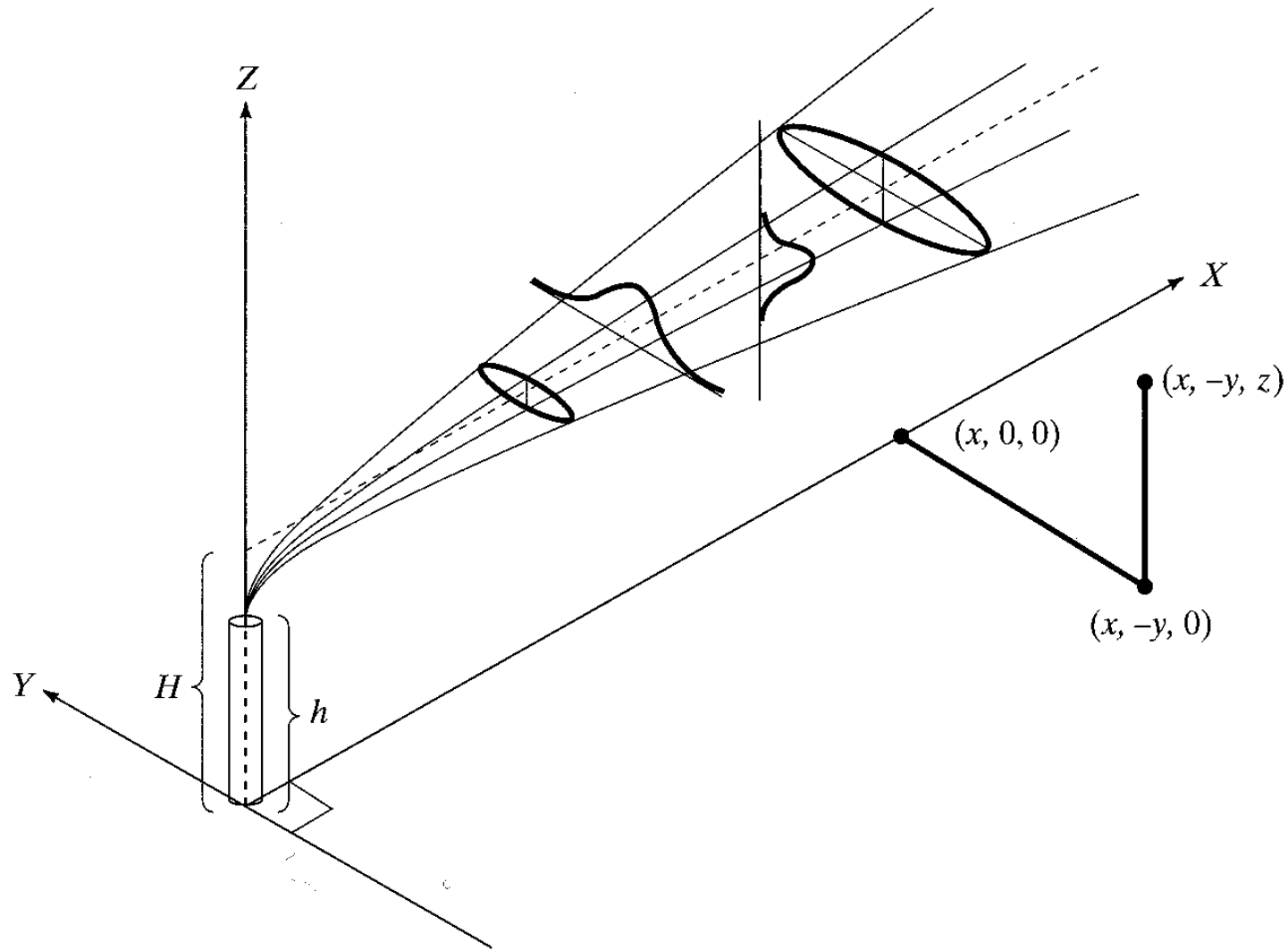
- Gaussian Plume (Dispersion) Model (GPM) is used to estimate the ground level concentrations for pollutants coming from a chimney
- Inputs to GPM
  - Height of chimney and Source strength
  - Wind rose data
  - Atmospheric stability of the region

# Point Source Gaussian Plume Model



Masters & Ela, 2008

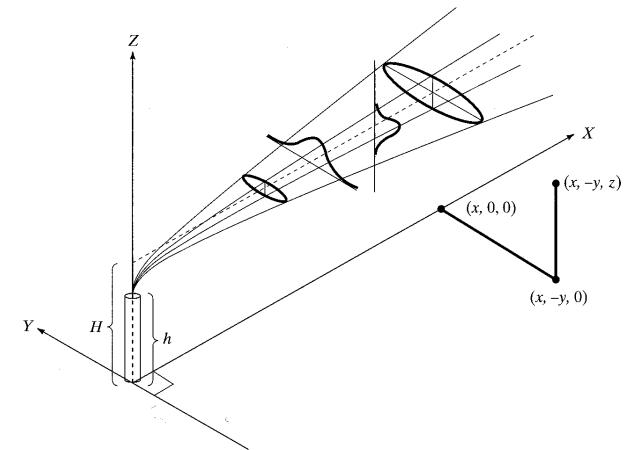
# Point Source Gaussian Plume Model



Masters & Ela, 2008

# GPM: Model Structure & Assumptions

- pollutants released from a “virtual point source”
- advective transport by wind
- dispersive transport (spreading) follows normal (Gaussian) distribution away from trajectory
- constant emission rate
- wind speed constant with time and elevation
- pollutant is conservative (no reaction)
- terrain is flat and unobstructed
- uniform atmospheric stability



# Point Source GPM: Mathematical expression

$$C(x, y, 0) = \left[ \frac{Q}{\pi s_y s_z u_H} \right] \left[ \exp \left[ -\frac{1}{2} \left( \frac{y}{s_y} \right)^2 \right] \right] \left[ \exp \left[ -\frac{1}{2} \left( \frac{H}{s_z} \right)^2 \right] \right]$$

Where,

$C$  = downwind concentration at ground level ( $\mu\text{g}/\text{m}^3$ )

$Q$  = emission rate of pollutant ( $\mu\text{g}/\text{s}$ )

$s_y, s_z$  = plume standard deviations (m)

$u$  = wind speed (m/s)

$x, y, z$  = distances (m)

$H$  = Effective Stack Height (m)

# Point Source GPM: Effective Stack Height

$$H = h + \Delta H$$

where,  $H$  = Effective stack height (m)

$h$  = height of physical stack (m)

$\Delta H$  = plume rise (m)

- Holland's formula

$$\Delta H = \frac{v_s d}{u} \left[ 1.5 + \left( 2.68 \times 10^{-3} (P) \left( \frac{T_s - T_a}{T_s} \right) d \right) \right]$$

where,  $v_s$  = stack velocity (m/s)

$d$  = stack diameter (m)

$u$  = wind speed (m)

$P$  = atmospheric Pressure, millibars

$T_s$  = stack gas temperature ( $^{\circ}\text{K}$ )

$T_a$  = air temperature ( $^{\circ}\text{K}$ )

Holland's formula, however, does not take into account the meteorology of the atmosphere, and good for certain atmospheric conditions only

# Point Source GPM: Stability Categories

**TABLE 11-6** Key to Stability Categories

Surface Wind Speed (at 10 m) (m · s <sup>-1</sup> )	Day <sup>a</sup>			Night <sup>a</sup>	
	Incoming Solar Radiation			Thinly Overcast or	
	Strong	Moderate	Slight	$\geq \frac{4}{8}$ Low Cloud	$\leq \frac{3}{8}$ Cloud
<2	A	A-B	B		
2-3	A-B	B	C	E	F
3-5	B	B-C	C	D	E
5-6	C	C-D	D	D	D
>6	C	D	D	D	D

**A** Extremely Unstable

**B** Moderately Unstable

**C** Slightly Unstable

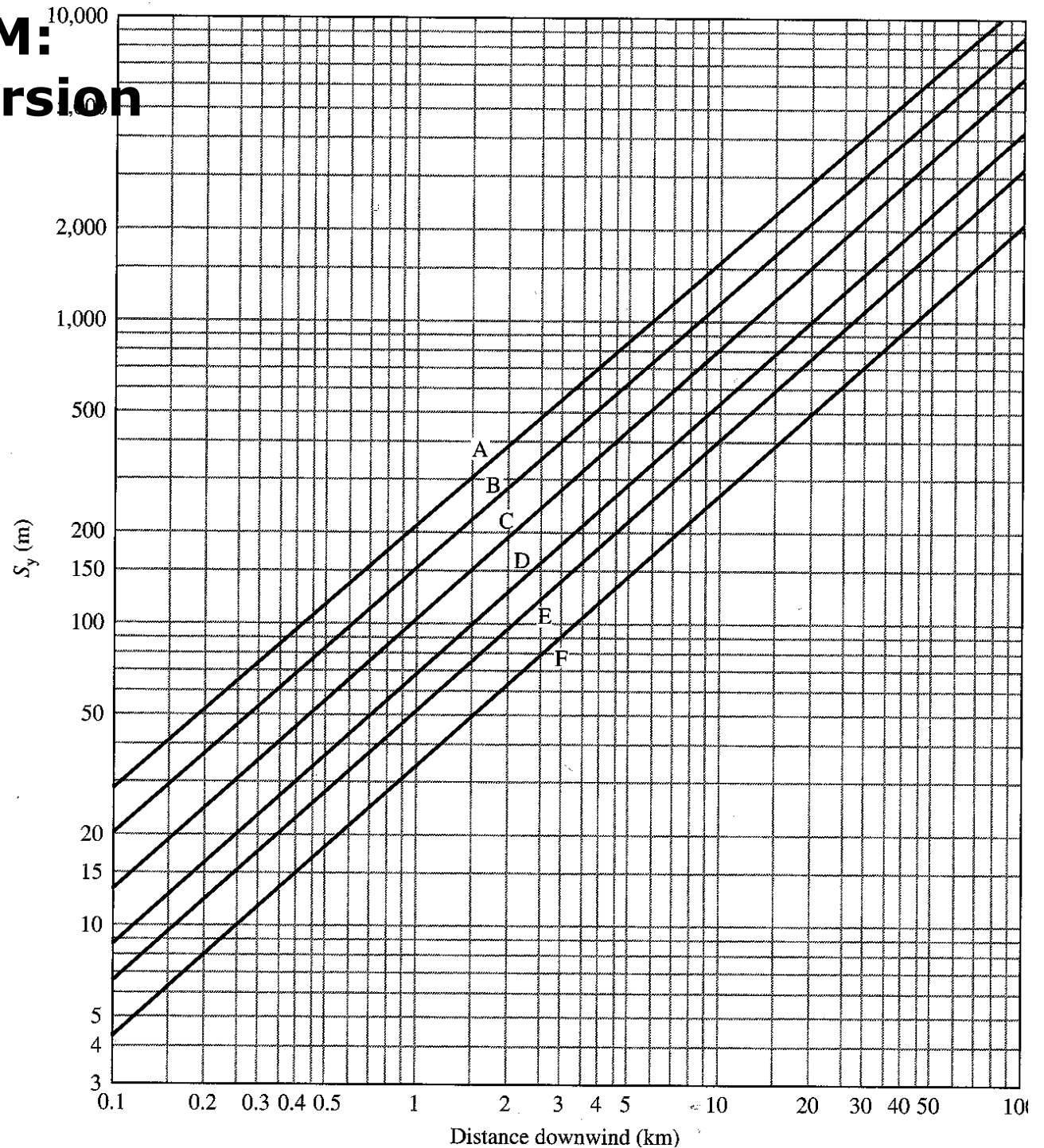
**D** Neutral

**E** Slightly Stable

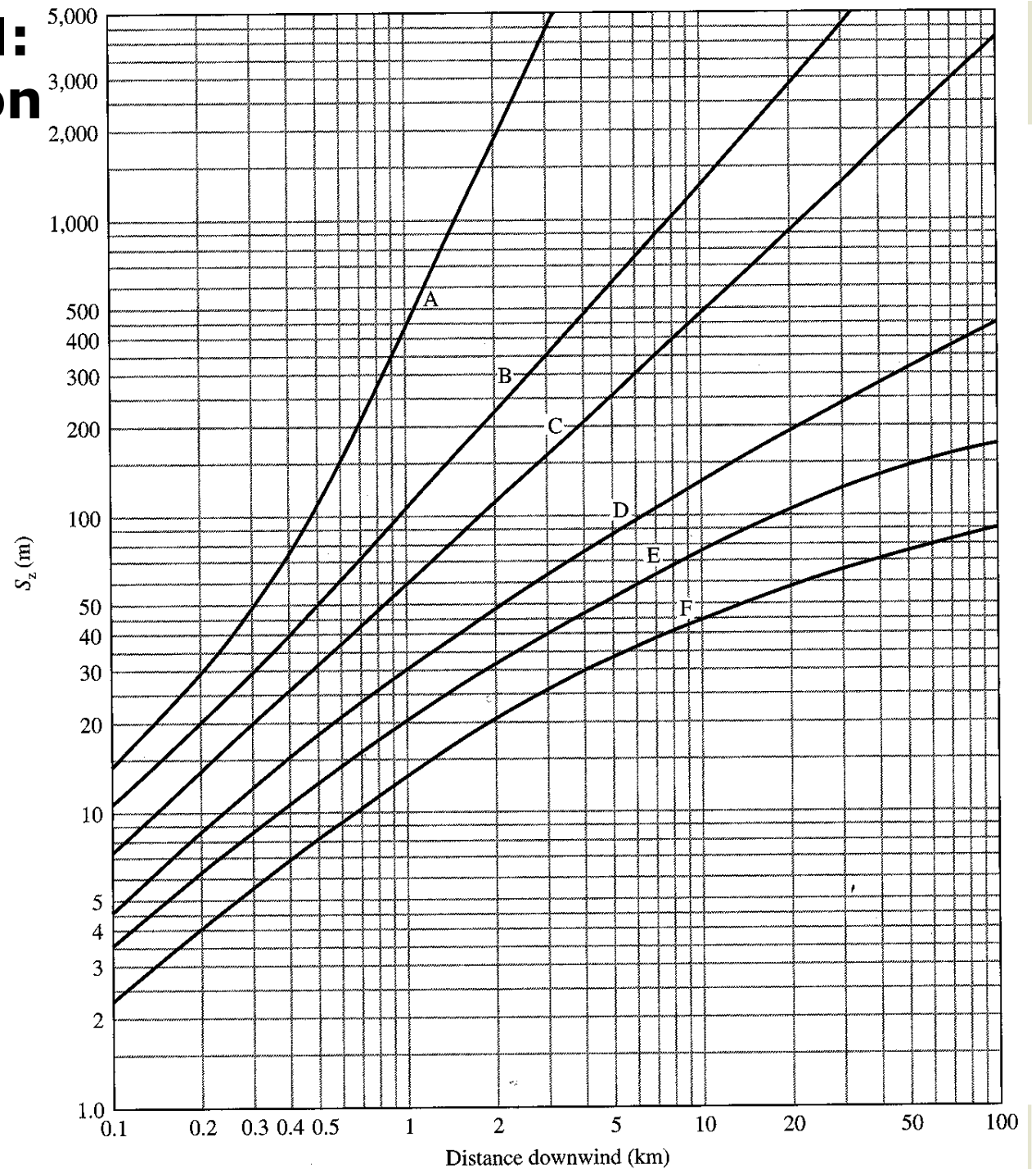
**F** Moderately Stable



# Point Source GPM: Horizontal Dispersion ( $s_y$ )



# Point Source GPM: Vertical Dispersion ( $s_z$ )



# Point Source GPM: Vertical & horizontal Dispersion ( $\sigma_y$ , $\sigma_z$ )

$$\sigma_y = a x^{0.894}$$

$$\sigma_z = cx^d + f$$

TABLE 7.8

**Values of the Constants  $a$ ,  $c$ ,  $d$ , and  $f$  for Use in (7.47) and (7.48)**

Stability	$a$	$x \leq 1 \text{ km}$			$x \geq 1 \text{ km}$		
		$c$	$d$	$f$	$c$	$d$	$f$
A	213	440.8	1.941	9.27	459.7	2.094	-9.6
B	156	106.6	1.149	3.3	108.2	1.098	2.0
C	104	61.0	0.911	0	61.0	0.911	0
D	68	33.2	0.725	-1.7	44.5	0.516	-13.0
E	50.5	22.8	0.678	-1.3	55.4	0.305	-34.0
F	34	14.35	0.740	-0.35	62.6	0.180	-48.6

*Note:* The computed values of  $\sigma$  will be in meters when  $x$  is given in kilometers.

*Source:* Martin, 1976.

# Point Source GPM: Wind Speed Correction

- Unless the wind speed at the virtual stack height is known, it must be estimated from the ground wind speed

$$u_2 = u_1 \left( \frac{z_2}{z_1} \right)^p$$

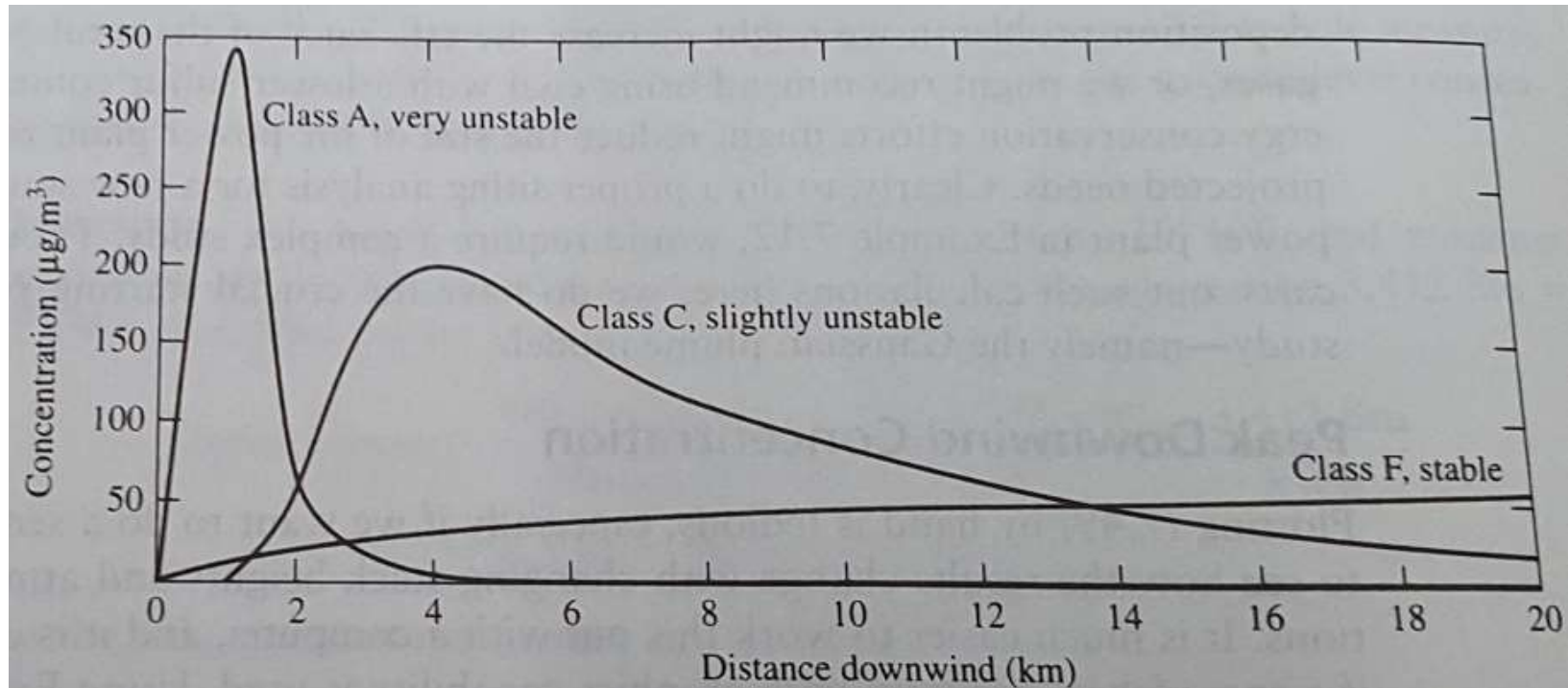
where  $u_x$  = wind speed at elevation  $z_x$   
 $p$  = empirical constant

**TABLE 11-8**

Exponent  $p$  Values for Rural and Urban Regimes

Stability Class	Rural	Urban	Stability Class	Rural	Urban
A	0.07	0.15	D	0.15	0.25
B	0.07	0.15	E	0.35	0.30
C	0.10	0.20	F	0.55	0.30

## Point Source GPM: Effect of atmospheric stability classification

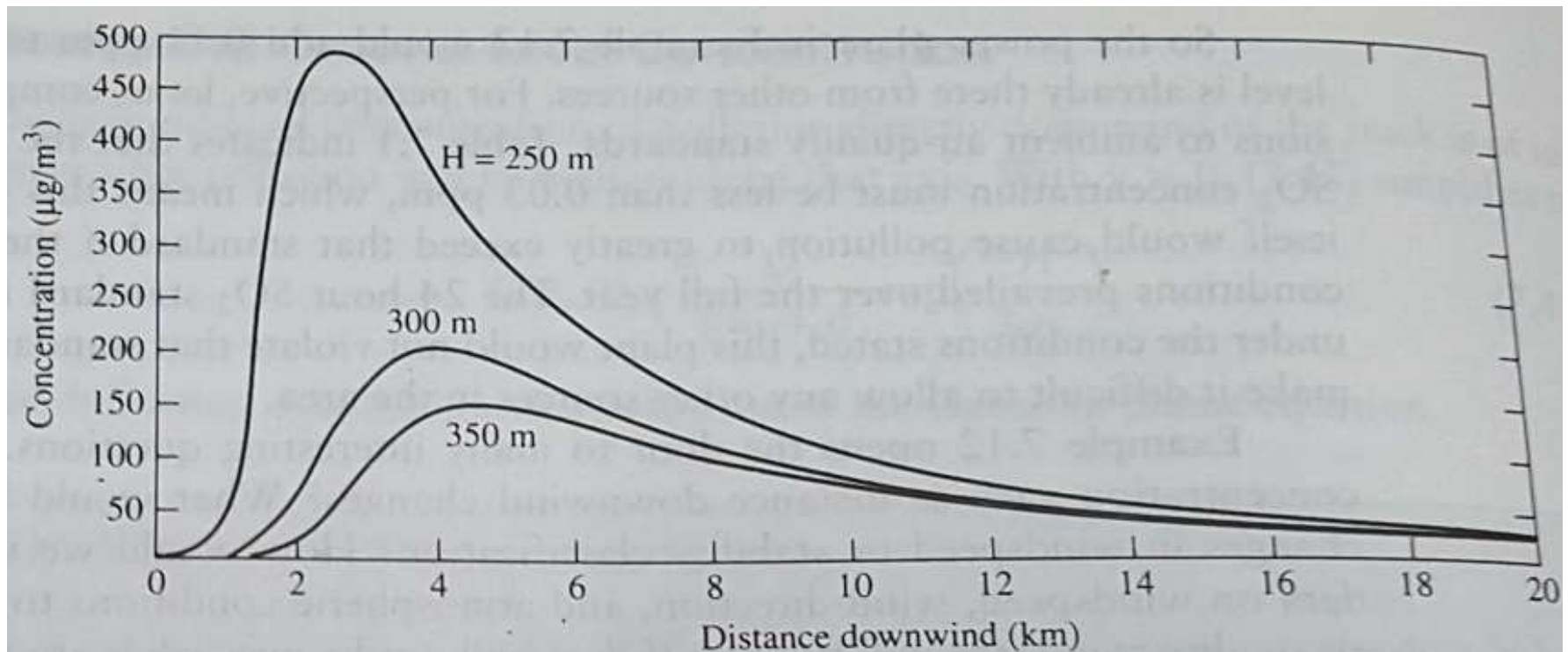


For a given stack height, maximum ground level pollutant concentration observed for very unstable atmosphere and reaches a maximum nearer to the source/earlier in time compared to other atmospheric stabilities.

**Masters & Ela, 2008**



## Point Source GPM: Effect of effective stack height



For a given atmospheric stability, maximum ground level pollutant concentration observed for lower effective stack height and reaches a maximum nearer to the source/earlier in time compared to sources with higher effective stack heights

⇒ Gaussian dispersion model estimates may have an uncertainty up to 50% due to several assumption involved

**Masters & Ela, 2008**

# Point Source GPM

## Example !

A stack in an urban area is emitting 80 g/s of NO. It has an effective stack height of 100m. The wind speed is 4 m/s at 10 m. It is a clear summer day with the sun nearly overhead. Estimate the ground level concentration at:

- a. 2 km downwind on the centerline, and
- b. 2 km downwind, 0.1 km off the centerline.



# Point Source GPM: Example !

## 1. Determine stability class

Assume wind speed is 4 m/s at ground surface.  
Description suggests strong solar radiation.

hence *stability class* is **B**

clear summer day with the sun nearly overhead; wind speed 4 m/s at 10 m.

**TABLE 11-6** Key to Stability Categories

Surface Wind Speed (at 10 m) (m · s <sup>-1</sup> )	Day <sup>a</sup>			Night <sup>a</sup>	
	Incoming Solar Radiation			Thinly Overcast or	
	Strong	Moderate	Slight	$\geq \frac{4}{8}$ Low Cloud	$\leq \frac{3}{8}$ Cloud
<2	A	A-B	B		
2-3	A-B	B	C	E	F
3-5	<b>B</b>	B-C	C	D	E
5-6	C	C-D	D	D	D
>6	C	D	D	D	D

# Point Source GPM: Example !

2. Estimate the wind speed at the effective stack height

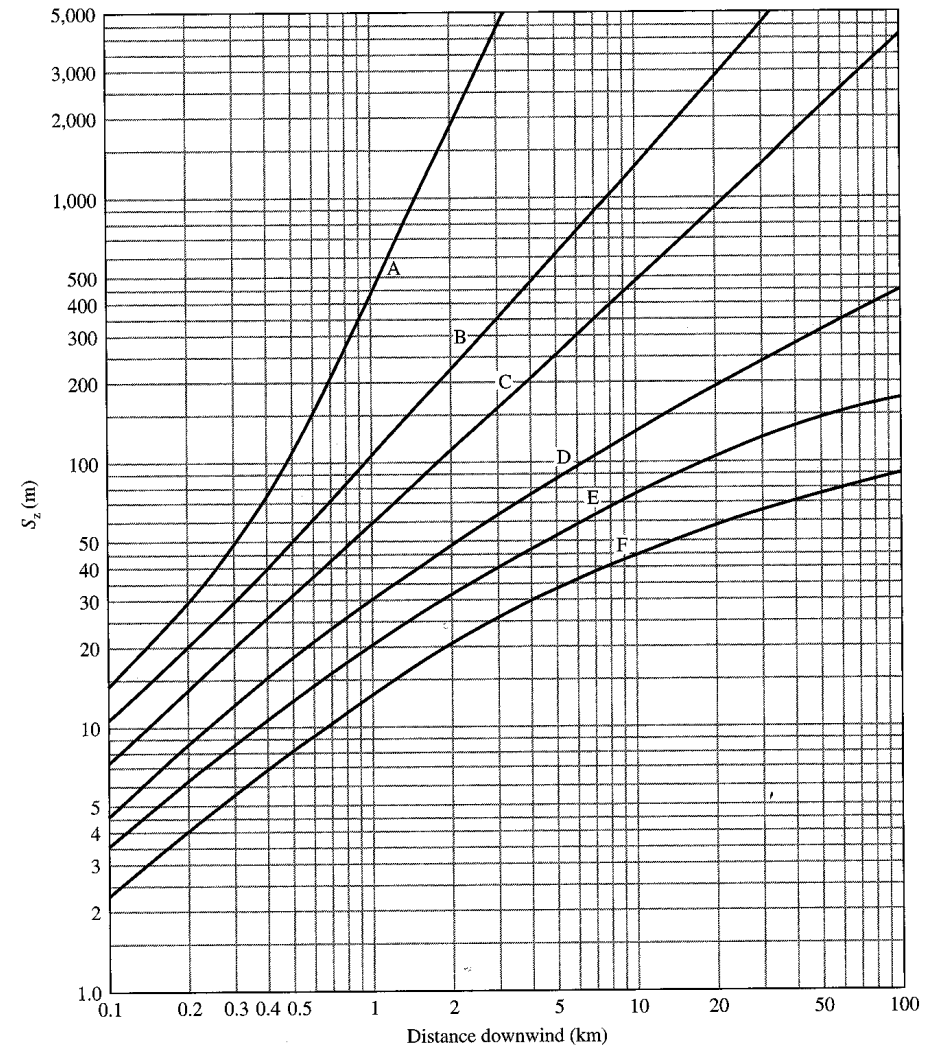
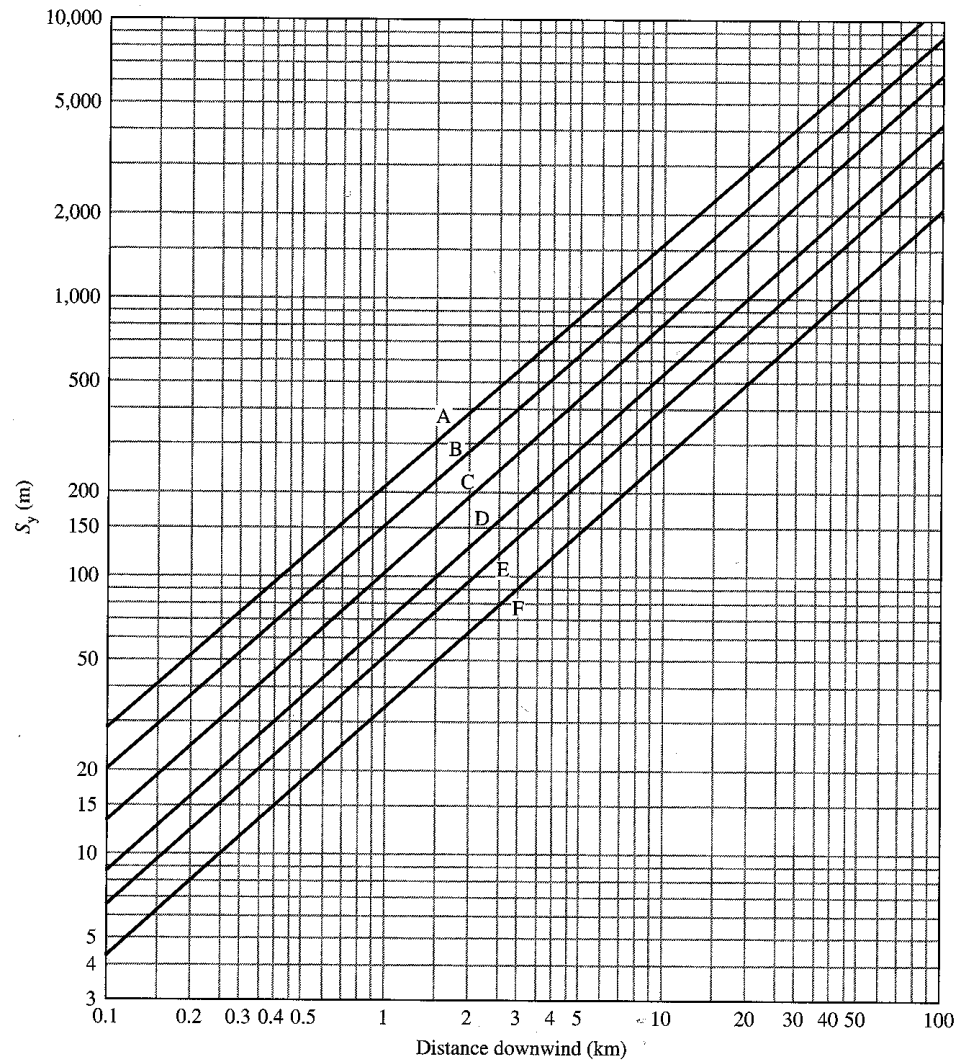
Note: effective stack height given – no need to calculate using Holland's formula

<b>TABLE 11-8</b> Exponent $p$ Values for Rural and Urban Regimes					
<b>Stability Class</b>	<b>Rural</b>	<b>Urban</b>	<b>Stability Class</b>	<b>Rural</b>	<b>Urban</b>
A	0.07	0.15	D	0.15	0.25
B	0.07	0.15	E	0.35	0.30
C	0.10	0.20	F	0.55	0.30

$$u_2 = u_1 \left( \frac{z_2}{z_1} \right)^p = 4 \left( \frac{100}{10} \right)^{0.15} = 5.65 \text{ m/s}$$

# Point Source GPM: Example !

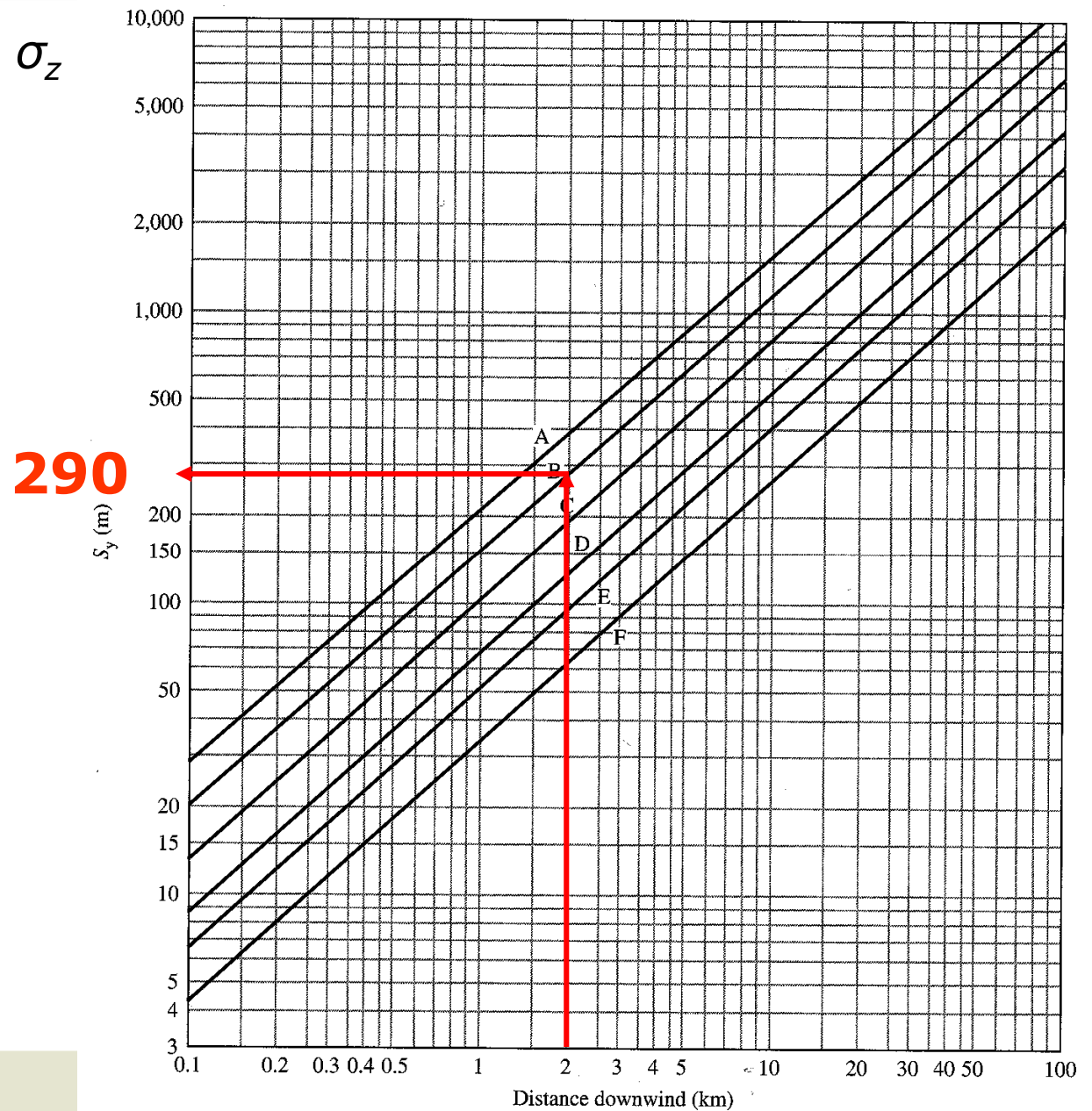
## 3. Determine $\sigma_y$ and $\sigma_z$



# Point Source GPM: Example !

3. Determine  $\sigma_y$  and  $\sigma_z$

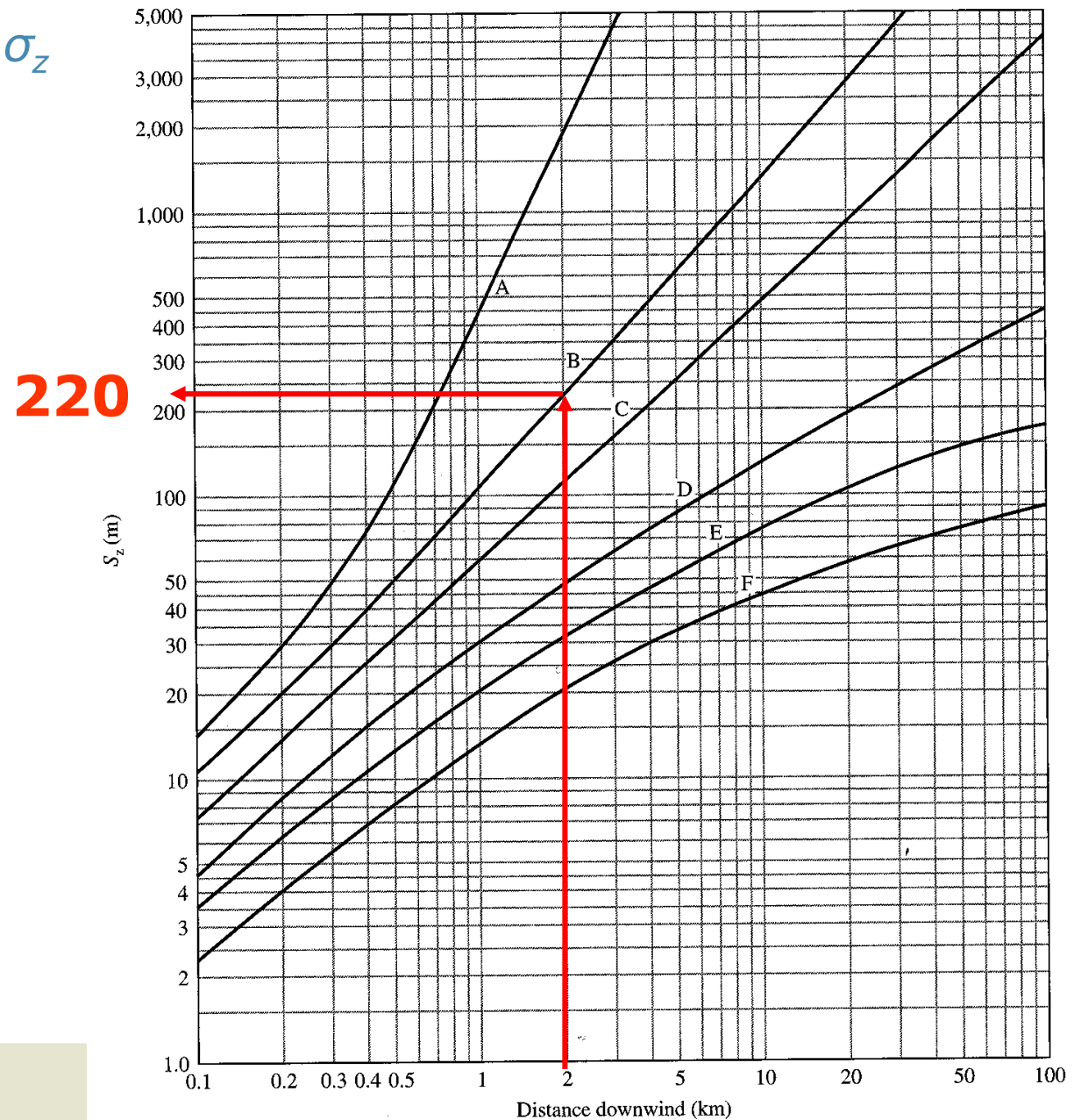
$$\sigma_y = 290$$



# Point Source GPM: Example !

3. Determine  $\sigma_y$  and  $\sigma_z$

$$\sigma_z = 220$$



## Point Source GPM: Example !

4. Determine concentration using the Eqn.

a.  $x = 2000, y = 0$

$$C(2000,0) = \frac{80}{\pi(290)(220)(5.6)} \exp\left[-\frac{1}{2}\left(\frac{0}{290}\right)^2\right] \exp\left[-\frac{1}{2}\left(\frac{100}{220}\right)^2\right]$$

$$C(2000,0) = 6.37 \times 10^{-5} \text{ g/m}^3 = 63.7 \text{ } \mu\text{g/m}^3$$

## Point Source GPM: Example !

4. Determine concentration using the Eqn.

b.  $x = 2000$ ,  $y = 0.1$  km (= 100m)

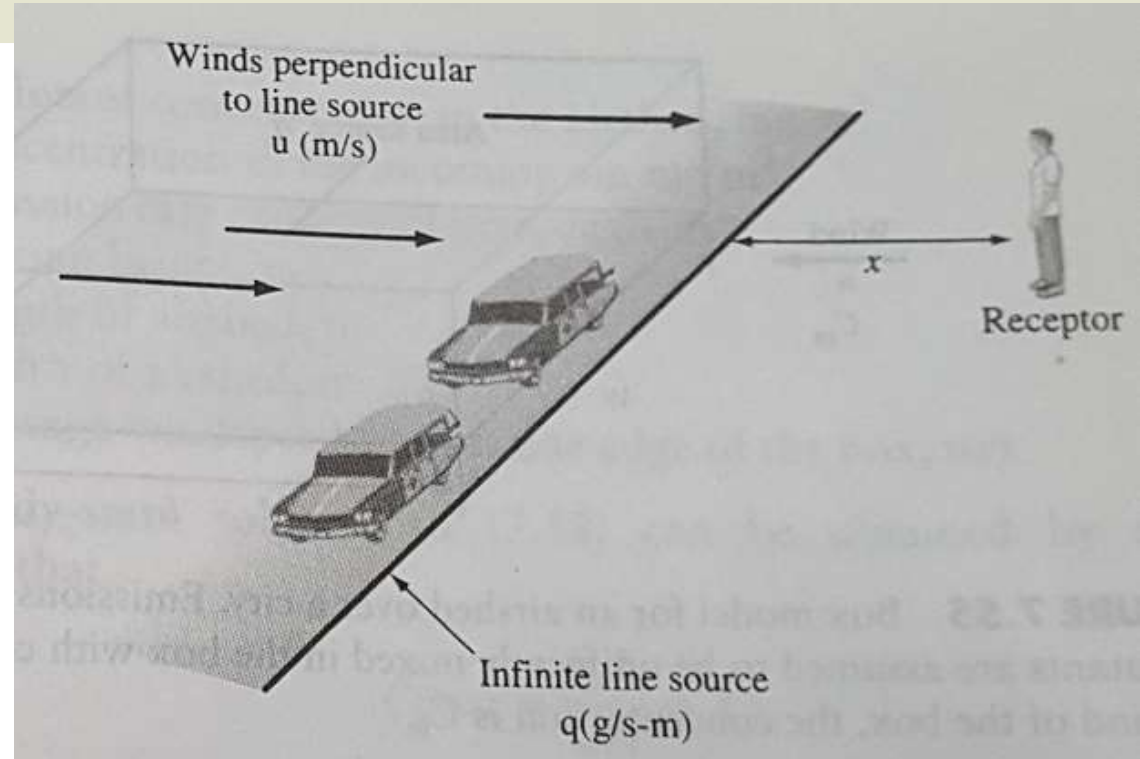
$$C(2000,100) = \frac{80}{\pi(290)(220)(5.6)} \exp\left[-\frac{1}{2}\left(\frac{100}{290}\right)^2\right] \exp\left[-\frac{1}{2}\left(\frac{100}{220}\right)^2\right]$$

$$C(2000,0) = 6.00 \times 10^{-5} \text{ g/m}^3 = 60 \text{ } \mu\text{g/m}^3$$



# Line Source Dispersion Model

- Useful in certain conditions where source is distributed along a line with continuous emissions
- Hence, ground level conc. of pollutant at a distance perpendicular distance  $x$  can be obtained by:



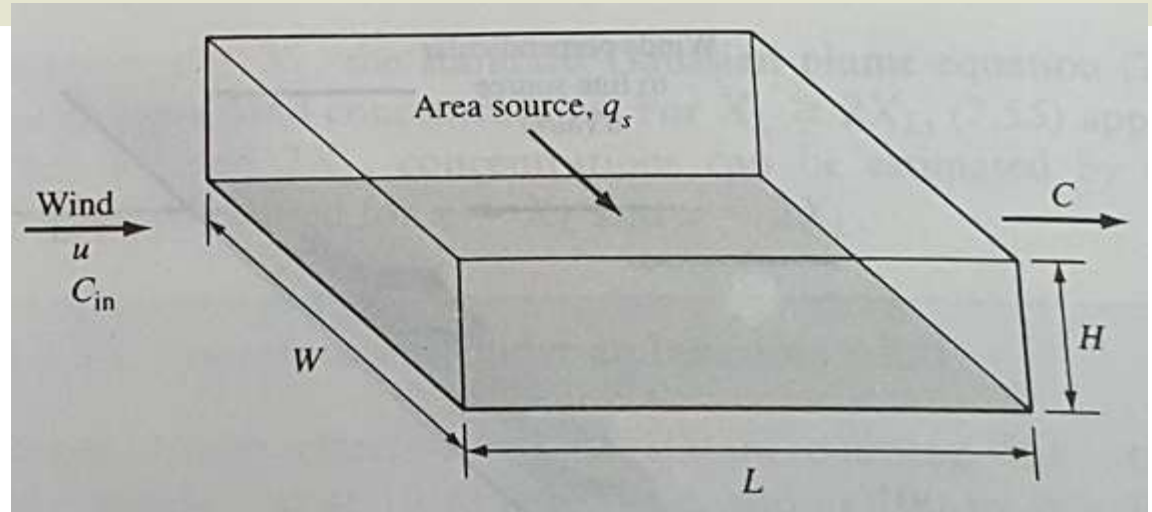
$$C(x) = \frac{2q}{\sqrt{2\pi} \sigma_z u}$$

Where

$q$  = emission rate per unit distance along the line (g/m-s)

# Area Source Model

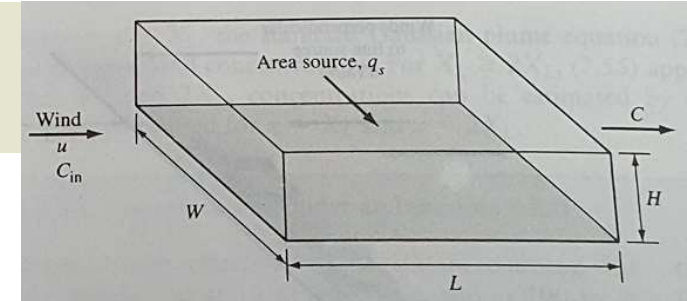
- Useful for distributed sources spread over an area
- e.g. Box model for an airshed over a city
- Pollutants are assumed to be uniformly mixed in the box
- If we assume the pollutant is conservative, then



$$\left( \begin{array}{c} \text{Rate of change of} \\ \text{pollution in the box} \end{array} \right) = \left( \begin{array}{c} \text{Rate of pollution} \\ \text{entering the box} \end{array} \right) - \left( \begin{array}{c} \text{Rate of pollution} \\ \text{leaving the box} \end{array} \right)$$

# Area Source Model

- If we assume the pollutant is conservative, then



$$LWH \frac{dC}{dt} = q_s LW + WHu C_{in} - WHu C$$

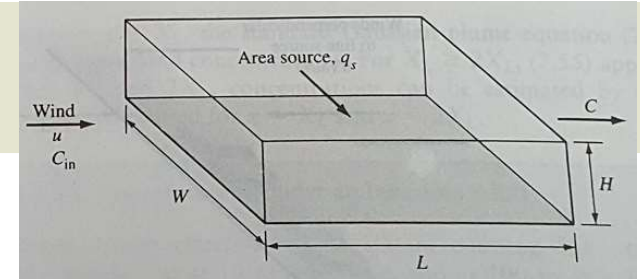
where

$C$  = pollutant concentration in the airshed,  $\text{mg/m}^3$   
 $C_{in}$  = concentration in the incoming air,  $\text{mg/m}^3$   
 $q_s$  = emission rate per unit of area,  $\text{mg/s-m}^2$   
 $H$  = mixing height,  $\text{m}$   
 $L$  = length of airshed,  $\text{m}$   
 $W$  = width of airshed,  $\text{m}$   
 $u$  = average windspeed against one edge of the box,  $\text{m/s}$

- Hence the steady state solution of the above equation, by simply setting  $dC/dt=0$ , is:

$$C(\infty) = \frac{q_s L}{uH} + C_{in}$$

# Area Source Model



$$LWH \frac{dC}{dt} = q_s LW + WHu C_{in} - WHu C$$

- The time-dependent change in the pollutant concentration in the city can be obtained by following:

$$C(t) = \left( \frac{q_s L}{uH} + C_{in} \right) (1 - e^{-ut/L}) + C(0) e^{-ut/L}$$

where,  $C(0)$  is the concentration in the airshed at  $t=0$

- If we assume incoming wind blow relatively no pollution in the box,  $C(0)=0$  then:

$$C(t) = \frac{q_s L}{uH} (1 - e^{-ut/L})$$

# Reminder !!!

**Quiz next week**

**Syllabus: All covered until Mon, 14.08**

# Home work !!!

- Do you have an air quality monitoring station in your city? If not, in the nearest city? What is the current status of air quality there?
- In the last 5 years or decade has the air quality improved or worsened? Why?
- Please do this **by Thu, 17.08**; we will discuss in the class!

**Please spend 3 mins. and fill the google form that was shared over moodle; discussion will be held on Mon, 21.08!**