Tutorial:

Laplace transform

$F: (0, \infty) \to R$ $L(F)$ is defined as

$$L(F)(S) = \int_0^\infty e^{-st} F(t)\,dt$$

$F(S) = L(F)(S)$

integral may or may not converge

- $F(t) = 1$ $L(f(t)=1) = \int_0^\infty e^{-st} = \left[\frac{-e^{-st}}{s}\right]_0^\infty = 1/s$
- $f(t) = e^{at}$ $\int_0^\infty e^{-st} e^{at} = \int_0^\infty e^{(a-s)t}\,dt = \frac{1}{s-a}$ $s > a$
- $L(\sin at)(s) = \frac{a}{s^2+a^2}$ $s > 0$ $L(\sin hat) = \frac{a}{s^2-a^2}$ $s > a \geq 0$
- $L(\cos at)(s) = \frac{s}{s^2+a^2}$ $L(\cos hat) = \frac{s}{s^2-a^2}$ $s > a \geq 0$

- $L(af + bg) = aL(F) + bL(g)$ ↗ given $L(f), L(g)$ exists
- $L(f')(s) = sL(f) - F(0)$ f, f' piecewise cont. $[0, a]$ for $a > 0$ $|f(t)| \leq Ke^{at}$

existence of Laplace of f is possible iff f is piecewise continous and exponential order

$|F(t)| \leq Ke^{at}$ $t \geq M > 0$ $K \geq 0$ $a \in R$

and exponential order $\Rightarrow L$ exists

if $f(t)$ is piecewise continous on $[0, a]$ $\forall a>0$
and further $f(t) \leq Ke^{at}$
for $t \geq M > 0$ where $K>0$ $a, M \in R$ then $L(F)(s)$
exists in region $s > a$
These Conditions are suff. not necessary.

If $L(f) = L(g)$ then $f = g$ (if f and g are continous)

$L(f^n)$ and $L(\int_0^t f(x)dx)$

- suppose f and f' are piecewise continous on $[0,a]$ $\forall a>0$
$f(t) \leq Ke^{\alpha t}$ $t \geq M > 0$ $K>0$ $\alpha \in R$
$L(f')(s)$ exist and $= sL(f) - f(0)$ $s > \alpha$

- suppose f be piecewise wise Cont. & exponential order
$|f(t)| \leq Ke^{\alpha t}$ $\alpha \in R$ $t \geq M \geq 0$

$$L\left(\int_0^t f(x)dx\right) = \frac{1}{s} L(f) \quad s > \alpha$$

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- f is pw continous and exp order then

$$\frac{d}{ds}F(s) = -L(t\cdot f(t))$$

- f is pw Continous and of exp order. also $\lim_{t\to 0^+} f(t)/t$ exists then

$$L\left(\frac{f(t)}{t}\right) = \int_s^\infty F(x)\,dx$$

$$\int_s^\infty F(x)\,dx = \int_s^\infty \int_0^\infty e^{-xt} f(t)\,dt\,dx.$$

$$= \int_0^\infty \int_s^\infty e^{-xt} f(t)\,dx\,dt$$

$$= \int_0^\infty f(t)\left[\frac{e^{-xt}}{-t}\right]_s^\infty dt$$

$$= \int_0^\infty f(t)\frac{e^{-st}}{t}\,dt = L\left(\frac{f(t)}{t}\right)$$

- $L(f)$ $L(g)$ exist $\forall\ s>a\geq 0$
then $L(f*g) = L(f)\cdot L(g)$

$$u_c(t)\begin{cases} 0 & t<c \\ 1 & t\geq c \end{cases}$$

$$L(u_c(t))(s) = \frac{e^{-cs}}{s}$$

$$L(u_c(t)\,f(t-c)) = e^{-cs}F(s)$$

$$L(e^{ct}(f(t))) = F(s-c)$$