Examples of the implicant method for solving a system of equations. Consider a system to be solved

$$\phi = 0, \psi = 0$$

Taking $f = \phi'$ and $g = \psi'$ the equivalent problem is to solve for all assignments of variables such that $F = fg = 1$.

Example 1. Let $F = fg$ where $f = wx' + w'y$, $g = w'x' + wx'y + x'yz' + y'z'$ We first compute all implicants of f which are terms in any sum of product (SOP) expression of f. In this example the set of terms $\{wx', w'y\}$ is such a set. There is another larger set called set of prime implicants which is

$$p(f) = \{t_1, t_2, t_3\} = \{wx', w'y, xy\}$$

Next we substitute the assignments of variables in the implicants which make them 1 in the next function g to compute $g/t = g(t = 1)$.

$$\begin{array}{rcll} g_1 = g/t_1 & = & y + yz' + y'z' = y + y'z' = y + z' & p(g_1) = \{y, z'\} \\ g_2 = g/t_2 & = & x' + x'z' = x' & p(g_2) = \{x'\} \\ g_3 = g/t_3 & = & 1 & p(g_3) = \{1\} \end{array}$$

when $g/t = 1$ then t is an implicant of F. If $g/t = 0$ this is a contradiction and the implicant t is discarded. Otherwise g/t is a new function in reduced variables and we continue to find its implicants and repeat the process untill all implicants of f are used or processed. We finally get all solutions in terms of a collection of implicants.

$$S(F) = \{(wx')(y), (wx')(z')\} \bigcup \{(w'y)(x')\} \bigcup \{(xy)\}$$

which are satisfying assignments of F

Yet another illustrative example is

Example 2. let $F = fg$ where $f = w' + x + z'$ and $g = wxy' + wyz' + wx'z'$. We have $p(f) = \{w', x, z'\}$.

$$\begin{array}{rcll} g/w' & = & 0 & p(g/w') = \\ g/x & = & wy' + wyz' = wy' + wyz' + wz' = wy' + wz' & p(g/x) = \{wy', wz'\} \\ g/z' & = & wxy' + wy = wxy' + wy + wx = wx + wy & p(g/z') = \{wx, wy\} \end{array}$$

hence we get

$$S(F) = \{(x)(wy'), (x)(wz')\} \bigcup \{(z')(wx), (z')(wy)\} = \{(wxy'), (wxz'), (wyz')\}$$