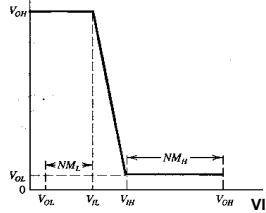
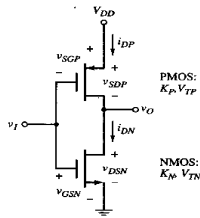
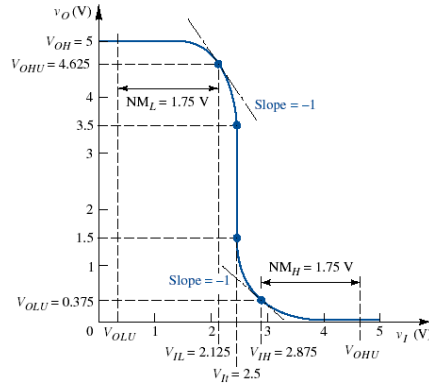


## Concept of Noise Margins



$NM_L = V_{IL} - V_{OL}$  (noise margin for low input)  
 $NM_H = V_{OH} - V_{IH}$  (noise margin for high input)



$NM_L = V_{IL} - V_{OLU}$  (noise margin for low input)  
 $NM_H = V_{OHU} - V_{IH}$  (noise margin for high input)

## Noise Margins equations

At point  $V_{IL}$  the NMOS is biased in the saturation region and PMOS is biased in the nonsaturation region

$$K_N[v_I - V_{TN}]^2 = K_P[2(V_{DD} - v_I + V_{TP})(V_{DD} - v_O) - (V_{DD} - v_O)^2] \quad (1)$$

Taking derivative with respect to  $V_I$  yields

$$2K_N[v_I - V_{TN}] = K_P \left[ -2(V_{DD} - v_O) - 2(V_{DD} - v_I + V_{TP}) \frac{dv_O}{dv_I} \right] \quad (2)$$

At  $V_{IL}$   $\frac{dv_O}{dv_I} = -1$

$$K_N[v_I - V_{TN}] = -K_P[(V_{DD} - v_O) - (V_{DD} - v_I + V_{TP}) + (V_{DD} - v_O)] \quad (3)$$

$\Rightarrow$  Solving for  $v_O$  produces

$$v_O = V_{OHU} = \frac{1}{2} \left\{ \left( 1 + \frac{K_N}{K_P} \right) v_I + V_{DD} - \left( \frac{K_N}{K_P} \right) V_{TN} - V_{TP} \right\} \quad (4)$$

Assume CMOS is symmetrical i. e.  $K_N = K_P$

$$\Rightarrow v_O = V_{OHU(K_N=K_P)} = \frac{1}{2} \{ 2v_I + V_{DD} - V_{TN} - V_{TP} \} \quad (5)$$

Substituting (5) into (1)  $\Rightarrow$

$$v_I = V_{IL(K_N=K_P)} = V_{TN} + \frac{3}{8}(V_{DD} + V_{TP} - V_{TN}) \quad (6)$$

At point  $V_{IH}$  the NMOS is biased in the nonsaturation region and PMOS is biased in the saturation region

## Noise Margins equations (cont.)

$$K_N[2(v_I - V_{TN})v_O - v_O^2] = K_P(V_{DD} - v_I + V_{TP})^2 \quad (6)$$

Taking derivative with respect to  $V_I$  yields

$$K_N \left[ 2(v_I - V_{TN}) \frac{dv_O}{dv_I} + 2v_O - 2v_O \frac{dv_O}{dv_I} \right] = 2K_P(V_{DD} - v_I + V_{TP})(-1) \quad (7)$$

Setting the derivative equal to  $-1$ , we find that

$$K_N[-(v_I - V_{TN}) + v_O + v_O] = -K_P[V_{DD} - v_I + V_{TP}] \quad (8)$$

The output voltage  $v_O$  is then

$$v_O = V_{OLU} = \frac{v_I \left( 1 + \frac{K_N}{K_P} \right) - V_{DD} - \left( \frac{K_N}{K_P} \right) V_{TN} - V_{TP}}{2 \left( \frac{K_N}{K_P} \right)} \quad (9)$$

Assume CMOS is symmetrical i. e.  $K_N = K_P$

$$v_O = V_{OLU(K_N=K_P)} = \frac{1}{2} \{ 2v_I - V_{DD} - V_{TN} - V_{TP} \} \quad (10)$$

Substituting (10) into (6)  $\Rightarrow$

$$v_I = V_{IH(K_N=K_P)} = V_{TN} + \frac{5}{8}(V_{DD} + V_{TP} - V_{TN})$$

## Summary of the noise margin of a symmetrical CMOS inverter

$$NM_L = V_{IL} - V_{OLU} \text{ (noise margin for low input)}$$

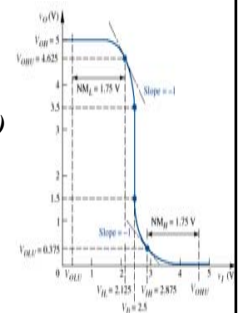
$$NM_H = V_{OHU} - V_{IH} \text{ (noise margin for high input)}$$

$$v_I = V_{IL(K_N=K_P)} = V_{TN} + \frac{3}{8}(V_{DD} + V_{TP} - V_{TN})$$

$$v_O = V_{OLU(K_N=K_P)} = \frac{1}{2} \{ 2v_I - V_{DD} - V_{TN} - V_{TP} \}$$

$$v_O = V_{OHU(K_N=K_P)} = \frac{1}{2} \{ 2v_I + V_{DD} - V_{TN} - V_{TP} \}$$

$$v_I = V_{IH(K_N=K_P)} = V_{TN} + \frac{5}{8}(V_{DD} + V_{TP} - V_{TN})$$



## Summary of the noise margin of asymmetrical CMOS inverter

$$NM_L = V_{IL} - V_{OLU} \text{ (noise margin for low input)}$$

$$NM_H = V_{OHU} - V_{IH} \text{ (noise margin for high input)}$$

$$v_I = V_{IL} = V_{TN} + \frac{(V_{DD} + V_{TP} - V_{TN})}{\left(\frac{K_N}{K_P} - 1\right)} \left[ 2 \frac{\frac{K_N}{K_P}}{\frac{K_N}{K_P} + 3} - 1 \right]$$

$$v_O = V_{OLU} = \frac{v_I \left(1 + \frac{K_N}{K_P}\right) - V_{DD} - \left(\frac{K_N}{K_P}\right) V_{TN} - V_{TP}}{2 \left(\frac{K_N}{K_P}\right)}$$

$$v_O = V_{OHU} = \frac{1}{2} \left[ \left(1 + \frac{K_N}{K_P}\right) v_I + V_{DD} - \left(\frac{K_N}{K_P}\right) V_{TN} - V_{TP} \right]$$

$$v_I = V_{IH} = V_{TN} + \frac{(V_{DD} + V_{TP} - V_{TN})}{\left(\frac{K_N}{K_P} - 1\right)} \left[ \frac{2 \frac{K_N}{K_P}}{\sqrt{3 \frac{K_N}{K_P} + 1}} - 1 \right]$$

**Example 16.11** Objective: Determine the noise margins of a CMOS inverter.

Consider a CMOS inverter biased at  $V_{DD} = 5V$ . Assume the transistors are matched with  $K_N = K_P$  and  $V_{TN} = -V_{TP} = 1V$ .

**Solution:** From Equation (16.57), the input voltage at the transition points, or the inverter switching point, is 2.5 V. Since  $K_N = K_P$ ,  $V_{IL}$  is, from Equation (16.73)

$$V_{IL} = V_{TN} + \frac{1}{2}(V_{DD} + V_{TP} - V_{TN}) = 1 + \frac{1}{2}(5 - 1 - 1) = 2.125V$$

Point  $V_{IH}$  is, from Equation (16.79)

$$V_{IH} = V_{TN} + \frac{1}{2}(V_{DD} + V_{TP} - V_{TN}) = 1 + \frac{1}{2}(5 - 1 - 1) = 2.875V$$

The output voltages at points  $V_{IL}$  and  $V_{IH}$  are determined from Equations (16.72) and (16.78), respectively. They are

$$V_{OHU} = \frac{1}{2}[2V_{IH} + V_{DD} - V_{TN} - V_{TP}]$$

$$= \frac{1}{2}[2(2.875) + 5 - 1 + 1] = 4.625V$$

and

$$V_{OLU} = \frac{1}{2}[2V_{IL} - V_{DD} - V_{TN} - V_{TP}]$$

$$= \frac{1}{2}[2(2.125) - 5 - 1 + 1] = 0.375V$$

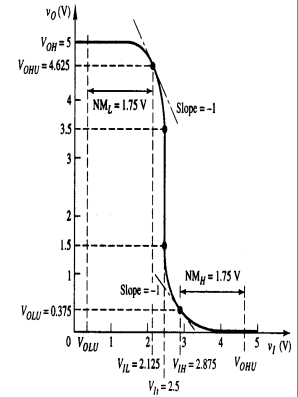
The noise margins are therefore

$$NM_L = V_{IL} - V_{OLU} = 2.125 - 0.375 = 1.75V$$

and

$$NM_H = V_{OHU} - V_{IH} = 4.625 - 2.875 = 1.75V$$

**Comment:** The results of this example are shown in Figure 16.43. Since the two transistors are electrically identical, the voltage transfer curve and the resulting critical voltages are symmetrical. Also,  $(V_{OHU} - V_{OLU}) = 0.375V$ , which is less than  $V_{TN}$ . As long as the input voltages remain within the limits of the noise margins, no logic error will be transmitted through the digital system.



## Test Your Understanding

**\*16.16** A CMOS inverter is biased at  $V_{DD} = 10V$ . The transistor parameters are:  $V_{TN} = 2V$ ,  $V_{TP} = -2V$ ,  $K_N = 200 \mu A/V^2$ , and  $K_P = 80 \mu A/V^2$ . (a) Sketch the voltage transfer curve. (b) Determine the critical voltages  $V_{IL}$  and  $V_{IH}$ , and the corresponding output voltages. (c) Calculate the noise margins  $NM_L$  and  $NM_H$ . (Ans. (b)  $V_{IL} = 3.39V$ ,  $V_{IH} = 4.86V$  (c)  $NM_L = 2.59V$ ,  $NM_H = 4.57V$ )

Review 16.16 on page 1048

Given that  $V_{DD} = 10V$ ,  $V_{TN} = 2V$ ,  $V_{TP} = -2V$ ,  $K_N = 200 \mu A/V^2$ ,  $K_P = 80 \mu A/V^2$

(a) Sketch the voltage transfer curve i.e.  $V_{IT}$ ,  $V_{OPI}$ ,  $V_{OHT}$  = ?

(b) Determine  $V_{IL}$  and  $V_{IH}$

(c) Noise margins

Sol: (a) For transition points we know eq (16.59)

$$V_{IT} = V_{DD} + V_{TP} + \sqrt{\frac{K_N}{K_P}} V_{TN}$$

$$\Rightarrow V_{IT} = 10 - 2 + \sqrt{\frac{200}{80}} \cdot 2$$

$$V_{IT} = 4.22V$$

$$V_{OPI} = V_{IT} - V_{TP} = 4.22 - (-2) = 6.22V$$

$$V_{OHT} = V_{IT} - V_{TN} = 4.22 - 2 = 2.22V$$

(b) Since  $K_P \neq K_N$  for  $V_{IL}$ , we have to use eq 16.71

$$V_{IL} = V_{TN} + \frac{(V_{DD} + V_{TP} - V_{TN})}{\left(\frac{K_N}{K_P} - 1\right)} \left[ \frac{\frac{K_N}{K_P}}{\frac{K_N}{K_P} + 3} - 1 \right]$$

$$V_{IL} = 2 + \frac{10 - 2 - 2}{\frac{200}{80} - 1} \left[ \frac{\frac{200}{80}}{\frac{200}{80} + 3} - 1 \right]$$

$$V_{IL} = 3.39V$$

For  $V_{OHU}$  use eq (16.72)

$$V_{OHU} = \frac{1}{2} \left[ (1 + \frac{K_N}{K_P}) V_{IT} + V_{DD} - \left(\frac{K_N}{K_P}\right) V_{TN} - V_{TP} \right]$$

$$V_{OHU} = \frac{1}{2} \left[ (1 + 2.5) (4.22) + 10 - 2 - 2 \right]$$

$$V_{OHU} = 9.43V$$

Similarly  $(V_{DD} + V_{TP} - V_{TN}) \left( \frac{2K_N}{K_P} - 1 \right)$

$$V_{IH} = V_{TN} + \frac{(V_{DD} + V_{TP} - V_{TN})}{\left(\frac{K_N}{K_P} - 1\right)} \left[ \frac{2 \frac{K_N}{K_P}}{\sqrt{3 \frac{K_N}{K_P} + 1}} - 1 \right]$$

$$V_{IH} = 2 + \frac{10 - 2 - 2}{\frac{200}{80} - 1} \left[ \frac{2 \left(\frac{200}{80}\right)}{\sqrt{3 \left(\frac{200}{80}\right) + 1}} - 1 \right]$$

$$V_{IH} = 4.86V$$

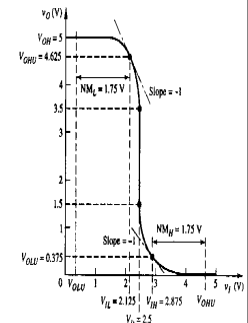
And  $V_{OLU} = V_{IH} \left(1 + \frac{K_N}{K_P}\right) - V_{DD} - \left(\frac{K_N}{K_P}\right) V_{TN} - V_{TP}$

$$V_{OLU} = 4.86 (1 + 2.5) - 10 - 2 - (-2)$$

$$V_{OLU} = 0.82V$$

(c) Noise margins

$$NM_L = V_{IL} - V_{OLU} = 3.39 - 0.82 = 2.57V$$

$$NM_H = V_{OHU} - V_{IH} = 9.43 - 4.86 = 4.57V$$


## CMOS Logic Circuits

Large scale integrated CMOS logic circuits such as watches, calculators, and microprocessors are constructed by using basic CMOS NOR and NAND gates. Therefore, understanding of these basic gates is very important for the designing of very large scale integrated (VLSI) logic circuits.

## CMOS NOR gate

CMOS NOR gate can be constructed by using two parallel NMOS devices and two series PMOS transistors as shown in the figure. In the CMOS NOR gate the output is at logic 1 when all inputs are low. For all other possible inputs, output is low or at logic 0.

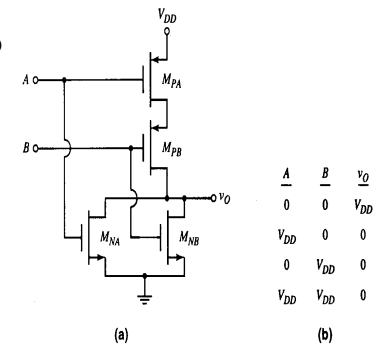


Figure 16.44 (a) Two-input CMOS NOR logic circuit and (b) truth table

## CMOS NAND gate

In CMOS NAND gate the output is at logic 0 when all inputs are high.

For all other possible inputs, output is high or at logic 1.

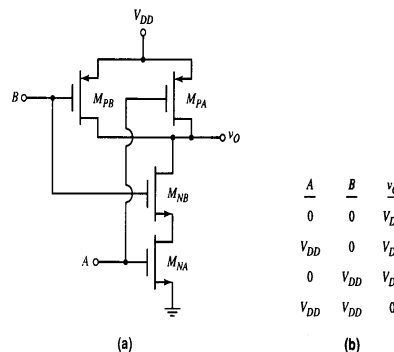


Figure 16.45 (a) Two-input CMOS NAND logic circuit and (b) truth table

## How can we design CMOS NOR symmetrical gate?

- In order to obtain symmetrical switching times for the high-to-low and low-to-high output transitions, the effective conduction (design parameters of the composite PMOS and composite NMOS device must be equal. For the CMOS NOR gate we can write as,

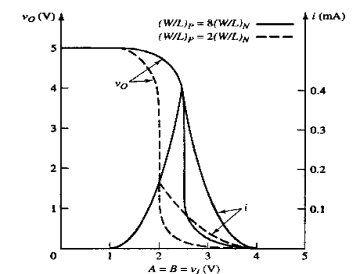
$$K_{CN} = K_{CP}$$

By recalling effective channel width and effective channel length concept, the effective conduction parameter for NMOS and PMOS for a CMOS NOR can be written as,

$$\text{Since } K'_n \sim 2K'_p \quad \frac{K'_n}{2} \left( \frac{2W}{L} \right)_N = \frac{K'_p}{2} \left( \frac{W}{2L} \right)_P$$

$$2 \left( \frac{2W}{L} \right)_N = \left( \frac{W}{2L} \right)_P \quad \text{or} \quad \left( \frac{W}{L} \right)_P = 8 \left( \frac{W}{L} \right)_N$$

For asymmetrical case switching time is longer



This implies that in order to get the symmetrical switching properties, the width to length ratio of PMOS transistor must be approximately eight times that of the NMOS device.

## Concept of effective width to length ratios

Parallel combination

Series combination

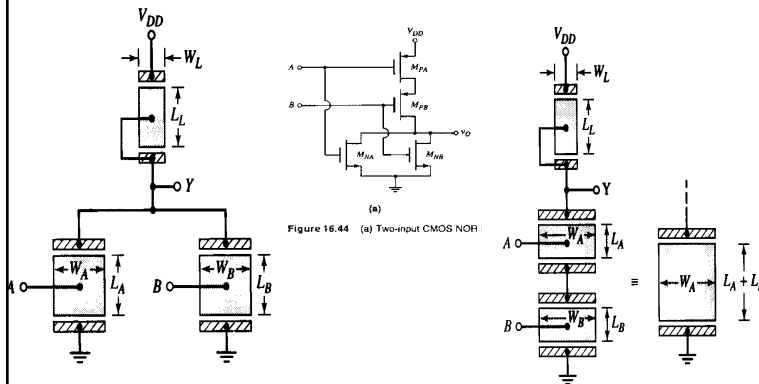


Figure 16.44 (a) Two-input CMOS NOR

**D16.19** In the two-input CMOS NAND gate in Figure 16.45(a), determine the relationship between the  $(W/L)$  ratios of the n-channel and p-channel transistors such that the composite conduction parameters of the PMOS and NMOS devices are equal. (Ans.  $(W/L)_N = 2(W/L)_P$ )

**D16.20** Design a three-input CMOS NOR logic gate such that the effective conduction parameters of the composite PMOS and NMOS transistors are equal. Determine  $(W/L)_P/(W/L)_N$ , where  $(W/L)$  is the width-to-length ratio of the individual PMOS and NMOS transistors. (Ans.  $(W/L)_P = 18(W/L)_N$ )

D16.19 on page 1051

Reference: 2-input CMOS NAND gate.

Determine relationship b/w the n-channel and p-channel transistors, such that composite conduction parameters of the PMOS and NMOS devices are equal. i.e.  $K_{N,eff} = K_{P,eff}$

Sol: Inspection of figure 16.45a

Show that the effective channel width of the parallel PMOS device is twice the individual width and effective channel length of the series NMOS transistor is twice the individual length i.e.

$$\left(\frac{K_N}{2}\right)\left(\frac{W}{2L}\right)_N = \left(\frac{K_P}{2}\right)\left(\frac{2W}{L}\right)_P$$

Since  $K_N \approx 2K_P$

$$\Rightarrow \left(\frac{2K_P}{2}\right)\left(\frac{W}{2L}\right)_N = \left(\frac{K_P}{2}\right)\left(\frac{2W}{L}\right)_P$$

$$\Rightarrow \frac{1}{2}\left(\frac{W}{L}\right)_N = K_P\left(\frac{W}{L}\right)_P$$

$$\Rightarrow \left(\frac{W}{L}\right)_N = 2\left(\frac{W}{L}\right)_P$$

16.20 on page 1051

Design a three input CMOS NOR logic gate such that

$K_{N,eff} = K_{P,eff}$

Sol: in case of the input CMOS NOR gate, we have three NMOS transistors in parallel and one PMOS transistor in series.

$$\left(\frac{K_N}{2}\right)\left(\frac{3W}{L}\right)_N = \left(\frac{K_P}{2}\right)\left(\frac{W}{L}\right)_P$$

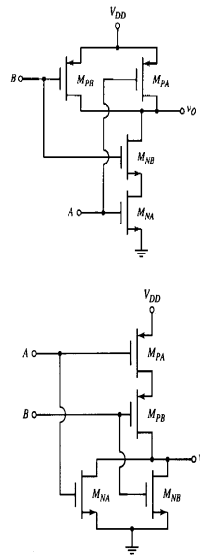
Since  $K_N \approx 2K_P$

$$\left(\frac{2K_P}{2}\right)\left(\frac{3W}{L}\right)_N = \left(\frac{K_P}{2}\right)\left(\frac{W}{L}\right)_P$$

$$3\left(\frac{W}{L}\right)_N = \frac{1}{2}\left(\frac{W}{L}\right)_P$$

$$\Rightarrow \left(\frac{W}{L}\right)_P = 18\left(\frac{W}{L}\right)_N$$

one n mos added in II and 1 pmos in series

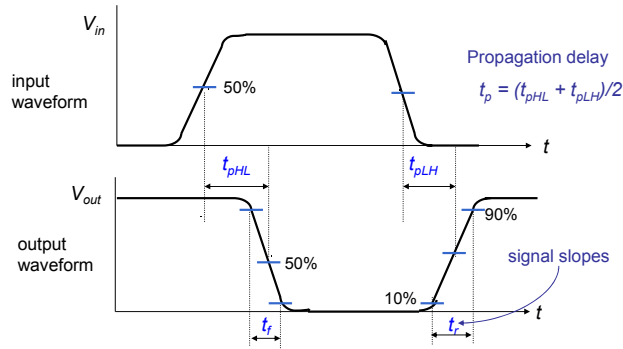
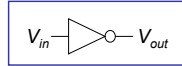


## Fan-In and Fan-Out

- The Fan-in of a gate is the number of its inputs. Thus a four input NOR gate has a fan-In of 4.
- Similarly, Fan-Out is the maximum number of similar gates that a gate can drive while remaining within guaranteed specifications.

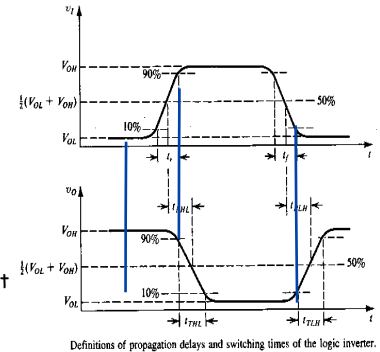
## Propagation Delay Definitions

The propagation delay  $t_p$  of a gate, defines how quickly it responds to a change at its input(s).



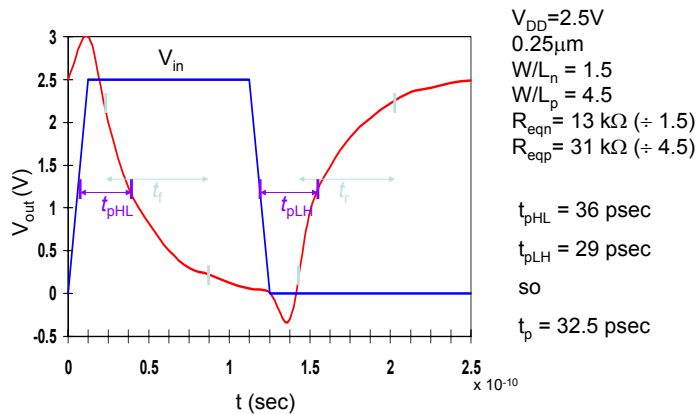
## Switching Time and Propagation Delay Time

- The dynamic performance of a logic circuit family is characterized by propagation delay of its basic inverter. The propagation delay time is defined as the *average of low-to-high propagation delay time and the high-to-low propagation delay time*.
- The propagation delay time is directly proportional to the **switching time** and increases as the Fan-out increases. Therefore, the maximum Fan-out is limited by the maximum acceptable propagation delay time.

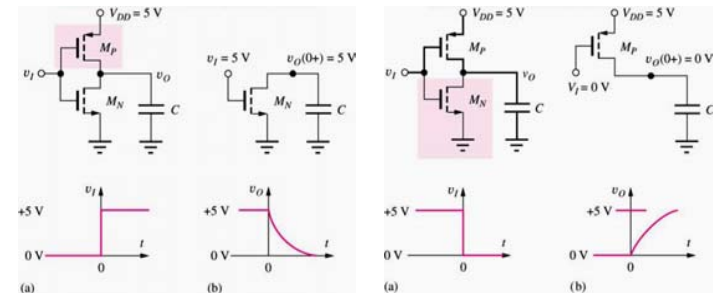


Each additional load gate increases the load capacitance they must be charge and discharge as the driver gate changes state. This places a practical limit on the maximum allowable number of load gates.

## Inverter Transient Response (input step pulse)

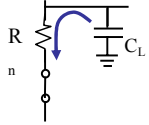


## Propagation Delay Estimate



- The two modes of capacitive charging/discharging that contribute to propagation delay

## Switch-level model



Delay estimation using switch-level model (for general RC circuit):

$$I = C \frac{dV}{dt} \rightarrow dt = \frac{C}{I} dV$$

$$I = \frac{V}{R} \rightarrow dt = \frac{RC}{V} dV$$

$$t_1 - t_0 = t_p = \int_{V_0}^{V_1} \frac{RC}{V} dV$$

$$t_p = RC [\ln(V_1) - \ln(V_0)] = RC \ln\left(\frac{V_1}{V_0}\right)$$

## Switch-level model

- For fall delay  $t_{phl}$ ,  $V_0 = V_{CC}$ ,  $V_1 = V_{CC}/2$

$$t_p = RC \ln\left(\frac{V_1}{V_0}\right) = RC \ln\left(\frac{\frac{1}{2}V_{CC}}{V_{CC}}\right)$$

$$t_p = RC \ln(0.5)$$

$$t_{phl} = 0.69 R_n C_L$$

$$t_{plh} = 0.69 R_p C_L$$

Standard RC-delay equations