Problem Set 2

Data Analysis and Interpretation (EE 223)

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1. $\mathcal{B}$ is the smallest σ-field containing collection $\mathcal{A} = \{(-\infty, x] : x \in \mathcal{R}\}$ show that:
   (a) $(x, \infty) \in \beta \; \forall \; x$ A belong then a compliment gs to
   (b) $(-\infty, x) \in \beta \; \forall \; x$
   (c) $\{x\} \in \beta \; \forall \; x$
   (d) $(x_1, x_2] \in \mathcal{B} \; \forall \; x_1 < x_2$
   (e) $[x_1, x_2] \in \mathcal{B} \; \forall \; x_1 < x_2$
   (f) $(x_1, x_2) \in \mathcal{B} \; \forall \; x_1 < x_2$

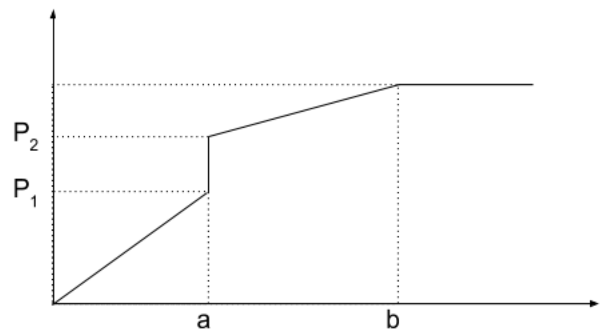
2. Consider $\Omega = \{\omega_1, \omega_2, ..., \omega_n\}$, $n < \infty$ and $\mathcal{F}$ is the power set of Ω. Argue that assigning values to singletons $\{\omega_k\}$ is enough to describe a probability measure $\mathcal{P}$ completely.
   Text
   (a) Let X be any random variable, then show that $\{\mathcal{A} : \exists \; B \in \mathcal{B} \; s.t. \; X^{-1}(B) = A\} = X^{-1}(B)$ is a σ-field on Ω.
   (b) Find $X^{-1}(B)$ when
      i. $X = \mathbb{1}_A$ - indicator random variable
      ii. $X = \sum_{k=1}^{\infty} \alpha_k \mathbb{1}_{A_k}$ where $\alpha_k \in \mathcal{R}$ & $\{A_1, ..., A_n\}$ partition Ω.

3. Find the mean and variance of
   (a) A Gamma RV
   (b) A Geometric RV

4. Find the pdf of
   (a) $Y = e^X$, where X is a normal RV given by,
   $$f_X(x) = \frac{1}{\sqrt{2\sigma^2}} e^{-\frac{x^2}{2}}$$
   (b) $Y = aX^2$, where X is a uniformly distributed RV between [1 4].

5. Consider X to be a Poisson RV given by,
   Text
   $$f_X(x) = \frac{e^{-\lambda}\lambda^k}{k!}$$
   and $Y = ba^X$, where a & b are positive constants. Find $E[Y]$.

6. Let $F_X(.)$ is as shown below. Write X as a linear combination of the continuous and discrete random variables.



7. Show that if $X > 0$, then $E(X) = \int_{-\infty}^{\infty} (1 - F_X(x)) dx$.