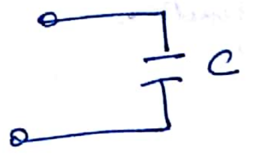
BODEE PLOTS

Transfer function: inputt → output

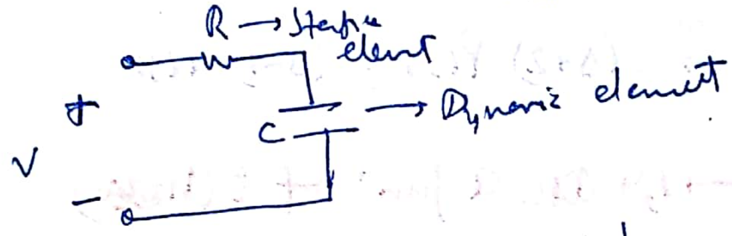


$$i(t) = C\frac{dv(t)}{dt}$$

After taking Laplace trans form

$$i(s) = CS\,V(s)$$

$$\text{impedance} = \frac{V(s)}{i(s)} = \frac{1}{sC} = Z(s)$$



$$Z(s) = R + \frac{1}{sC} = \frac{V(s)}{i(s)} = \text{Transfer function}$$

$$V_R = iR$$

$$C\frac{dV_c}{dt} = i$$

$$iR + V_c = V$$

$$RC\frac{dV_c}{dt} + V_c = V \xrightarrow[\text{L.T}]{\text{Take}} RCsV_c(s) + V_c(s) = V(s)$$

$$\Rightarrow V(s) = (RCs+1)V_c(s)$$

$$\boxed{\frac{V_c(s)}{V(s)} = \frac{1}{1+RCs}}$$

$$Y(s) = \frac{I(s)}{V(s)} = \frac{sC}{1+RCs}$$ ⇒ Admittance ~~Transfer function~~

$$\boxed{V_c(s) = \frac{I(s)}{Cs}}$$

$$u(t) \longrightarrow \boxed{G(s)} \longrightarrow y(t)$$

$$y(t) = G(s)\,u(t)$$

output's L.T

$$\frac{Y(s)}{U(s)} =: G(s) \Longrightarrow \text{Transfer function}$$

Input's L.T

If $G(s) = \frac{s-1}{s+2}$

$$\frac{Y(s)}{U(s)} = \frac{s-1}{s+2} \Rightarrow (s+2)\,Y(s) = (s-1)\,U(s)$$

$$\boxed{\dot{y} + 2y = \dot{u} - u}$$ — u, y are a fun^n of t (here)

→ For linear systems (principle of superposition)
Time invariance is involved

$$\left.\begin{array}{l} u_1 \rightarrow y_1 \\ u_2 \rightarrow y_2 \end{array}\right\} \Rightarrow \begin{array}{l} u_1 + u_2 \rightarrow y_1 + y_2 \\ k u_1 + l u_2 \rightarrow k y_1 + l y_2 \end{array}$$

u_1 (delayed) → y_1 (delayed)

→ Invariance w.r.t time

→ for linear systems ↑ time invariant systems, exponentials are "eigenfunctions".

$$[A][V] = [W]$$

$$\begin{bmatrix} 2 & 0 \\ 0 & 5 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

(2x2)x(2x1) 2x1

scaling of V by a factor of 2

$$e^{at} \longrightarrow \boxed{G(s)} \longrightarrow c\,e^{at}$$

sin & cos are also exponentials

$$\sin z = \frac{e^{iz} - e^{-iz}}{2i}, \quad \cos z = \frac{e^{iz} + e^{-iz}}{2}$$

ex: $G(s) = \dfrac{s-1}{s+3} = \dfrac{Y(s)}{U(s)}$ → Output, Input

let $u(t) = e^{-8t}$ $y(t) = ce^{-8t}$

$$G(s) = \frac{Y(s)}{U(s)} = \frac{s-1}{s+3}$$

$$\dot{y} + 3y = \dot{u} - u$$

$$-8c\cancel{e^{-8t}} + 3c\cancel{e^{-8t}} = -8\cancel{e^{-8t}} + \cancel{e^{-8t}}$$

$$-5c = 9$$

$$\therefore \boxed{c = 9/5}$$

$$\therefore y(t) = \frac{9}{5}e^{-8t}$$

ex: Given $u(t)$ and $y(t)$ ↑ Particular sol^n

$$y = \frac{9}{5}e^{-8t} + ce^{-3t} \qquad G(s) = \frac{s-1}{s+3}$$

So algo:

$u(t) = e^{at}$ output $y(t) = G(a)e^{at} + y_H$

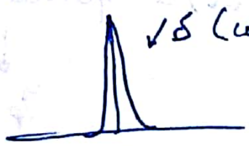
Assuming ↑ Homogeneous sol^n

$$G(s) = \frac{n(s)}{d(s)}$$

or

$u(t) \to \boxed{G(s)} \to y(t)$

$$\frac{Y(s)}{U(s)} = G(s) = \frac{1}{(s+3)(s+5)}$$



δ (continuous time)

$$\mathcal{L}(\delta) = 1 \to \mathcal{L}^{-1}(1) = \delta$$

$$Y(s) = \mathcal{L}^{-1}(G(s)\,U(s))$$

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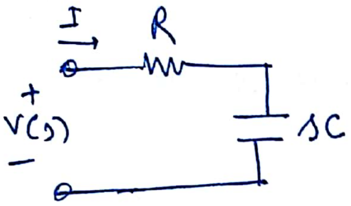
$$\frac{Y(s)}{U(s)} = G(s) = \frac{1}{(s+3)(s+5)} = \frac{a}{s+3} + \frac{b}{s+5} = \frac{a}{s-(-3)} + \frac{b}{s-(-5)}$$

$\downarrow$ take Laplace inverse $(\mathcal{L}^{-1})$

$$a e^{-3t} + b e^{-5t}$$



$$\frac{I(s)}{V(s)} = \frac{sC}{1+RCs} = G(s) =$$

$$\mathcal{L}^{-1}(G(s)) = \mathcal{L}^{-1}\left(\frac{sC}{1+RCs}\right) = \mathcal{L}^{-1}\left(\frac{1}{RC}\left(\frac{RCs+1-1}{1+RCs}\right)\right)$$

$$= \mathcal{L}^{-1}\left(\frac{1}{R} + \frac{a_2}{1+RCs}\right)$$

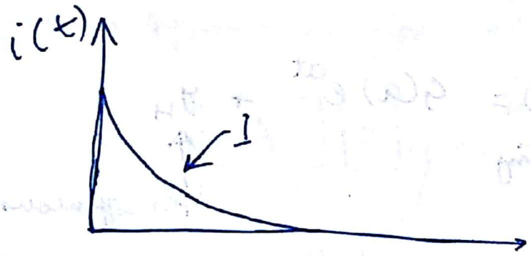
$$= \frac{\delta}{R} + a_1 e^{-t/RC}$$

$\Downarrow$

$$\mathcal{L}^{-1}\left(\frac{1}{RC}\left(\frac{1}{1+RCs}\right)\right)$$

$$\mathcal{L}^{-1}\left(\frac{1}{RC}\left(1 - \frac{1}{1+RCs}\right)\right)$$

$$\mathcal{L}^{-1}\left(\frac{1}{RC} - \frac{1}{RC+(RC)^2 s}\right)$$



$$u \longrightarrow \boxed{G(s)} \longrightarrow y$$

Output $y(t)$ = forced response + Natural response

$\downarrow$

$y_p(t)$

$$u(t) = e^{5t}$$

$$U(s) = \frac{1}{s-5}$$

$$y(s) = G(s)\,U(s) = \frac{sC}{(s-5)(1+RCs)}$$

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* If T.f $= G(s) = \dfrac{1}{s+8.0}$

$e^{-8t} \rightarrow \boxed{G(s)} \rightarrow f(t)$

output $= G(-8)\, e^{-8t}$

$\boxed{\text{huge gain}}$

Eg

* If $G(s) = \dfrac{1}{s+8}$

$e^{-8t} \rightarrow \boxed{G(s)} \rightarrow y(t)$

[output $= t\, e^{-8t}$]

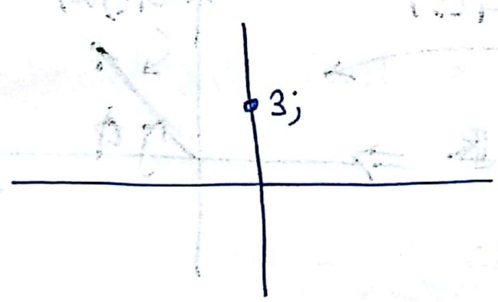
* $\sin \omega t = \dfrac{e^{j\omega t} - e^{-j\omega t}}{2j}$

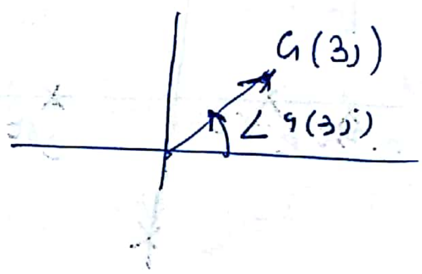
$G(s)\Big|_{s=j\omega} = G(j\omega)$

u: $\sin \omega t = \dfrac{e^{j\omega t} - e^{-j\omega t}}{2j} \longrightarrow \boxed{G(s)} \longrightarrow$ y: $|G(j\omega)| \sin(\omega t + \angle G(j\omega))$

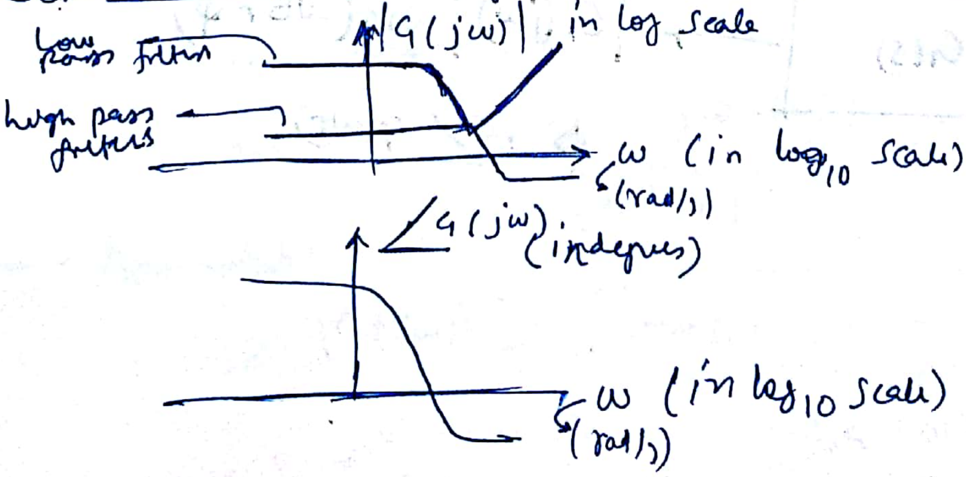
$|G(j\omega)|$: scaling factor

$\angle G(j\omega)$: shifted by this angle



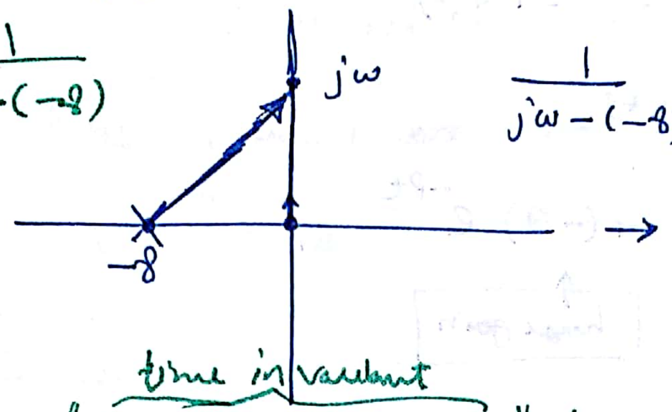


BODE PLOT — magnitude

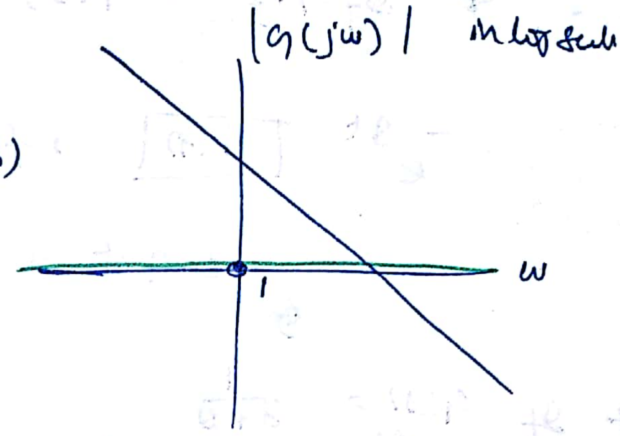


$G(s) = \frac{1}{s+8}$ roots of denominator = poles (X)

$= \frac{1}{s-(-8)}$



$\frac{1}{j\omega-(-8)}$



Linear "Constant Coefficient" (ordinary) differential eqn

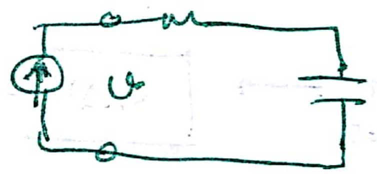
time invariant

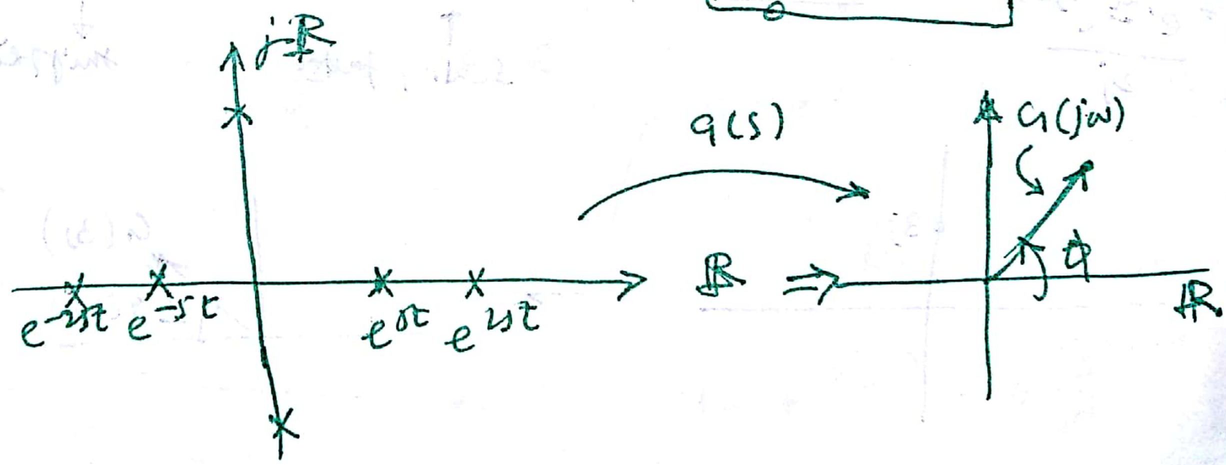
(More variables than eqn's.

we take Laplace Transform on b.s.

$$(n(s))\, V(s) = (\{d(s))\, I(s)$$

$$\frac{\text{output}}{\text{Input}} = \frac{V(s)}{I(s)} = \frac{d(s)}{n(s)}$$





$$\sin\omega t \longrightarrow \boxed{G(s)} \longrightarrow |G(j\omega)|\sin(\omega t+\phi)$$

$$\phi = \angle g(\omega t)$$

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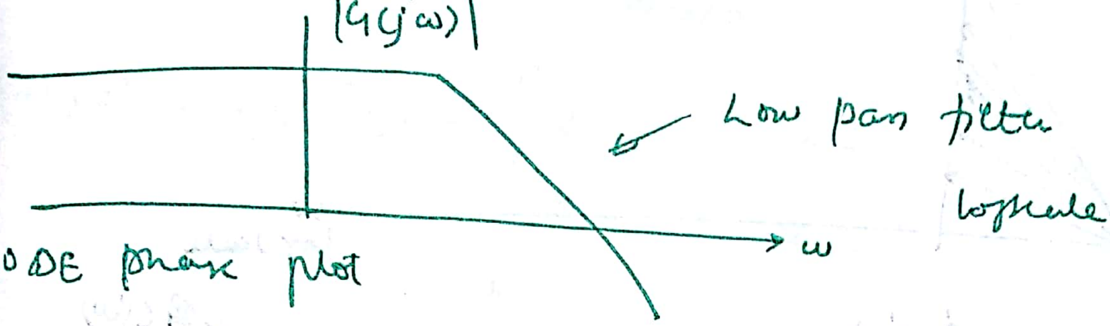
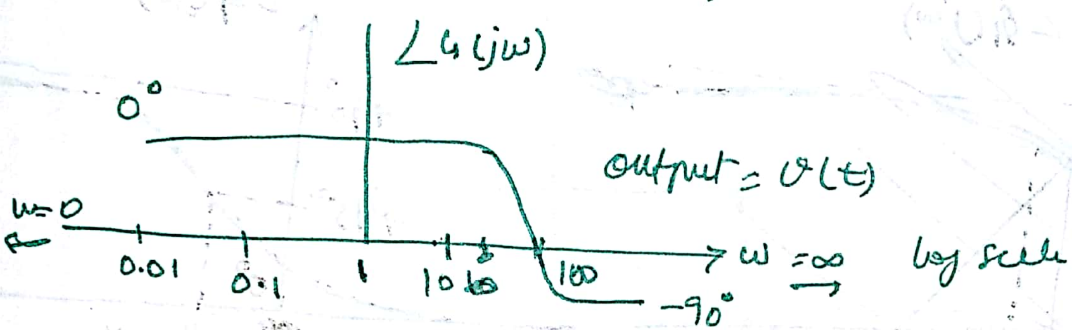
If a func^n is integrable, Fourier transform is defined

$$\int_{-\infty}^{\infty} |f(t)|\, dt < \infty$$

BODE magnitude plot



BODE phase plot



$$G(s) = \frac{(s+3)(s+8)}{(s+9)(s+6)(s+7)}$$

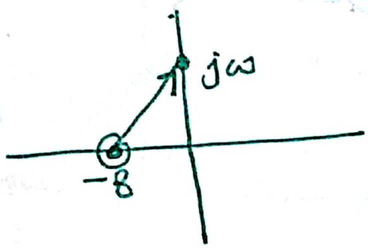
$$\log|G(j\omega)| = \log|j\omega+3| + \log|j\omega+8| - \log|j\omega+9| - \log|j\omega+6| - \log|j\omega+7|$$

$$\angle G(j\omega) = \angle(j\omega+3) + \angle(j\omega+8) - \angle(j\omega+9) - \angle(j\omega+6) - \angle(j\omega+7)$$

→ $G(s) = s+8$

$G(j\omega) = j\omega+8 = j\omega-(-8)$

$\sqrt{\omega^2+64}$

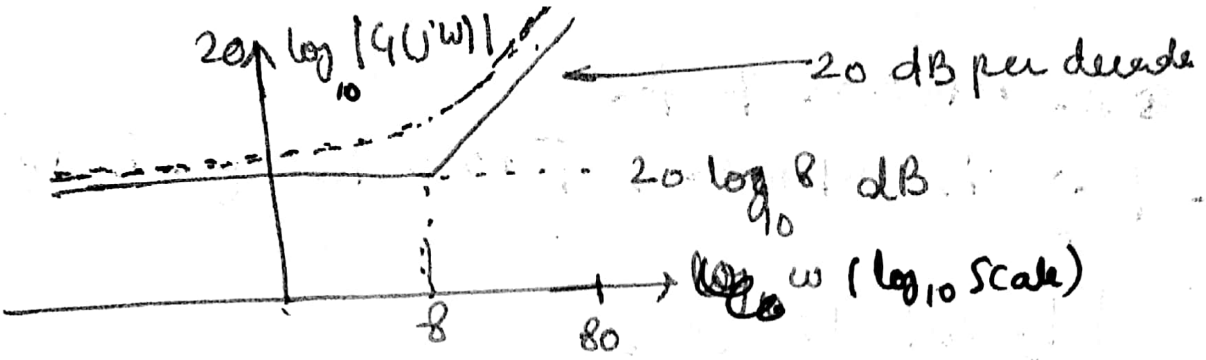


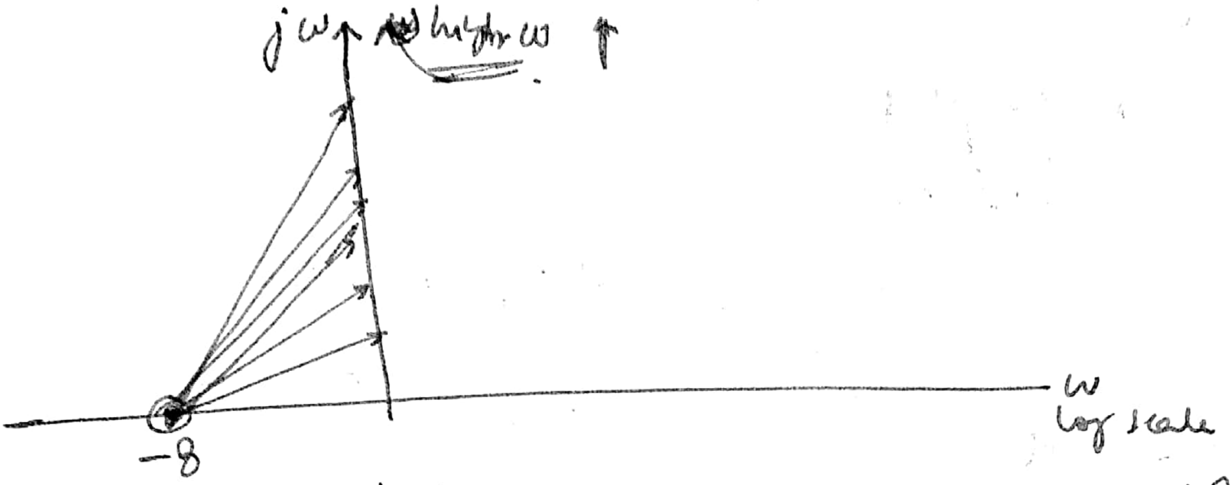
→ Asymptotic plot

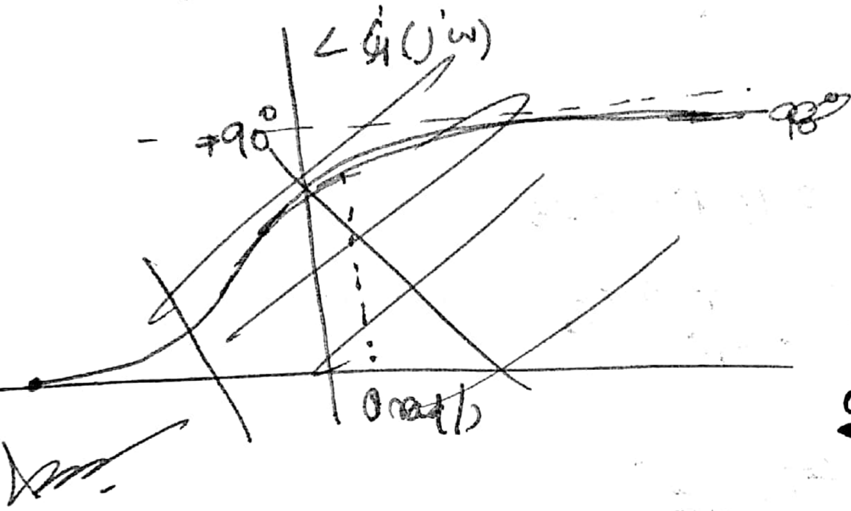
$$|G(j\omega)| = \sqrt{\omega^2+64}$$

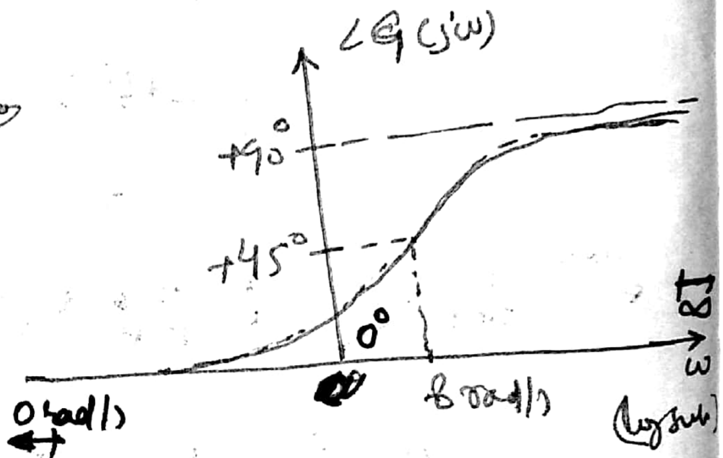
for $\omega \ll 8$ for $\omega \gg 8$

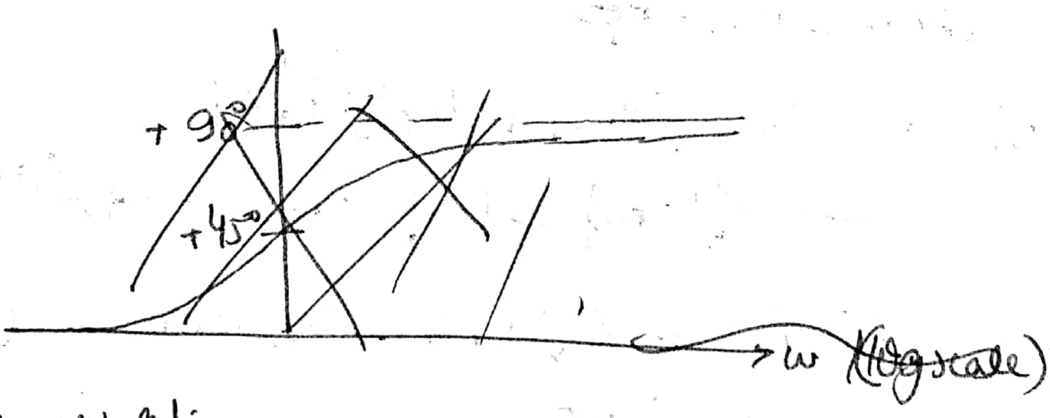
$|G(j\omega)| \approx 8$ $|G(j\omega)| \approx \omega$







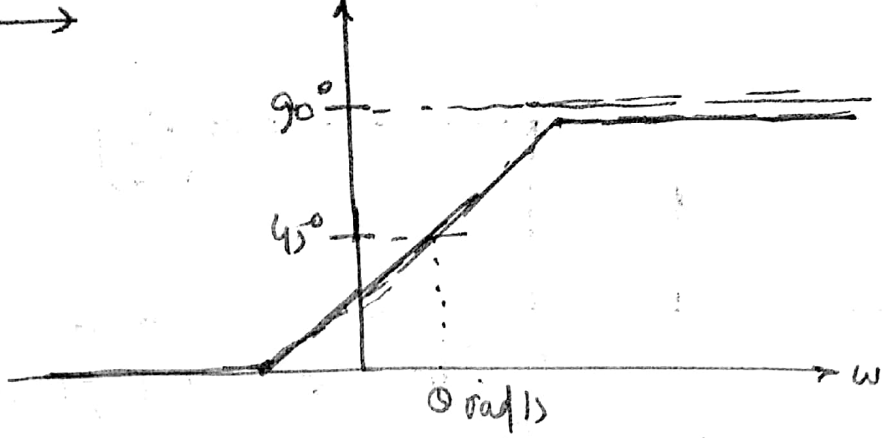




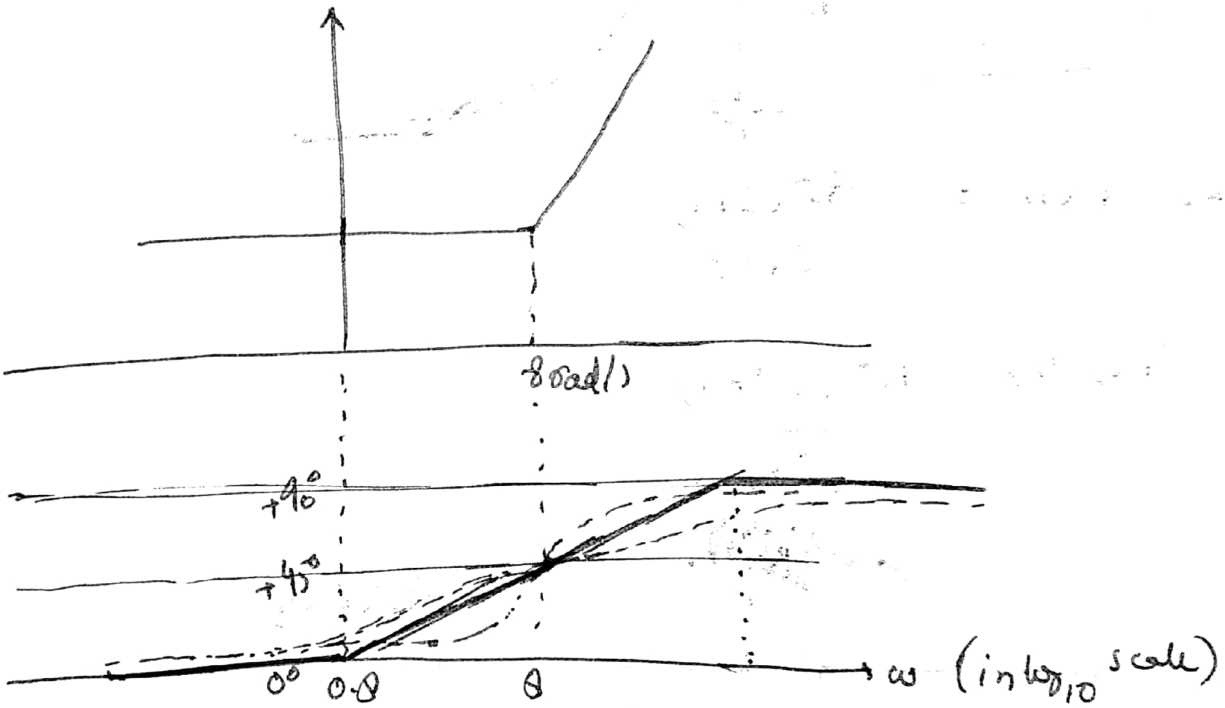
Approximation

Bode magnitude plot $\longrightarrow$ asymptotic actual

Bode phase plot $\longrightarrow$ Asymptotic actual

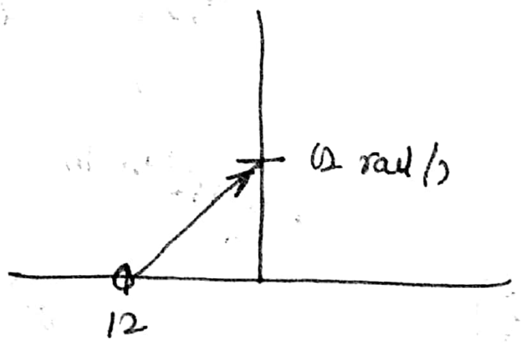


$$|G(j\omega)| = \sqrt{\omega^2+64} = \begin{cases} 8 & ; \ \omega \ll 8 \\ \omega & ; \ \omega \gg 8 \end{cases}$$

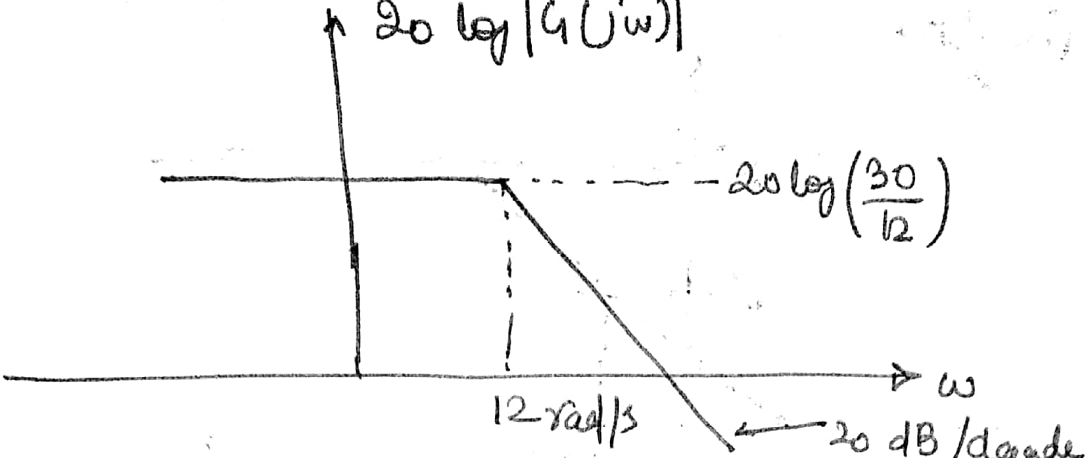


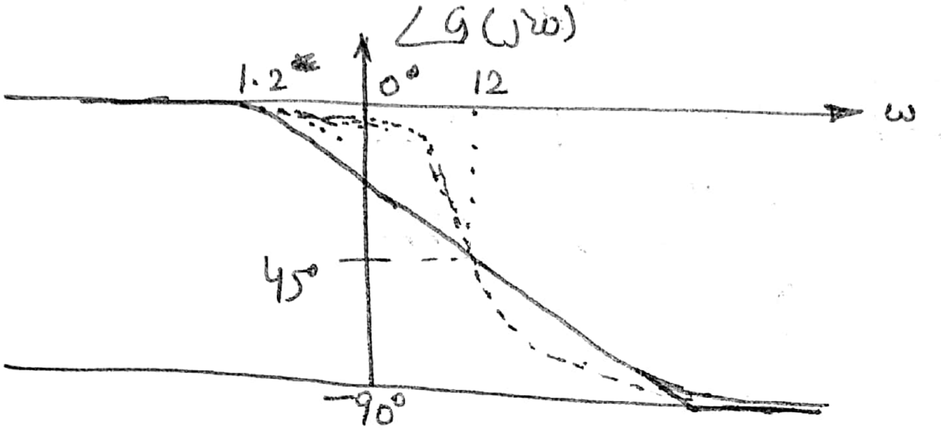
$$G(s) = \frac{30}{s+12}$$

$$G(j\omega) = \frac{30}{j\omega-(-12)}$$



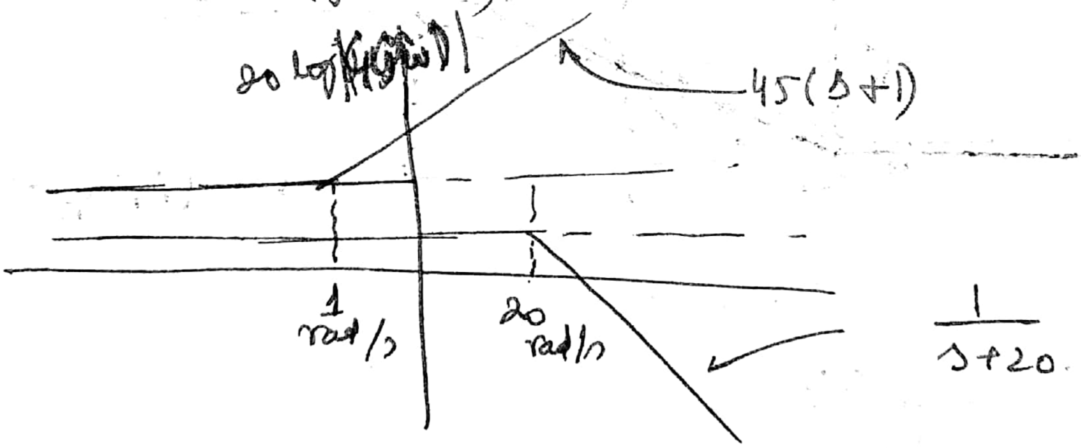
$$|G(j\omega)| = \frac{30}{\sqrt{\omega^2+(12)^2}} = \begin{cases} \frac{30}{12} & ; \ \omega \ll 12 \\ \frac{30}{\omega} & ; \ \omega \gg 12 \end{cases}$$

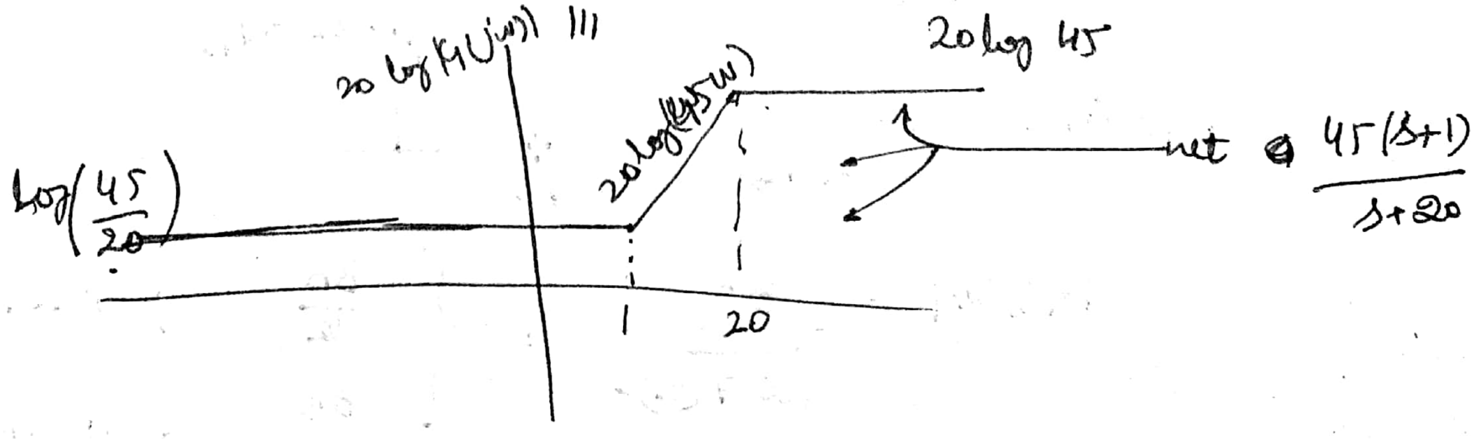


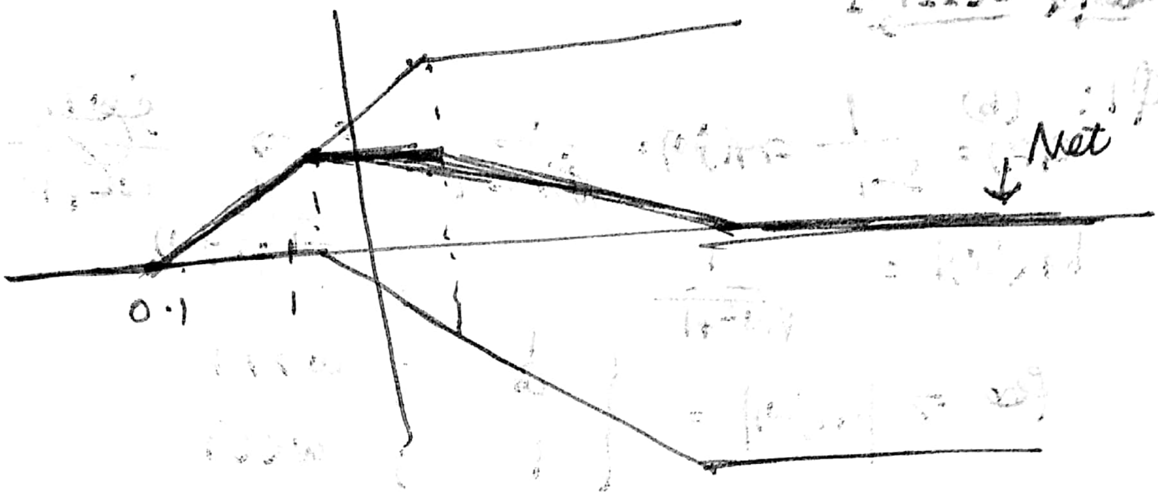


ex. $G(s) = \frac{45(s+1)}{s+20}$

$G(j\omega) = \frac{45(j\omega+1)}{(j\omega+20)}$







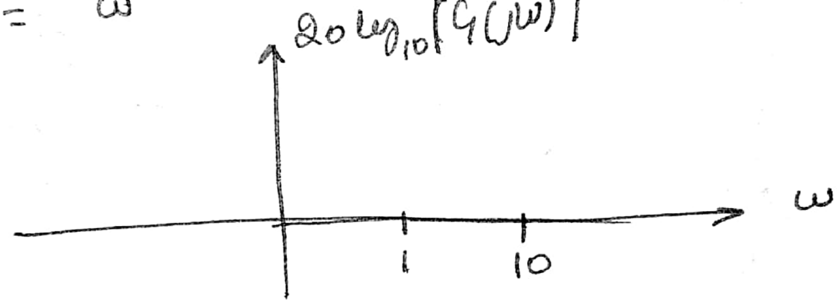
eg. $G(s) = s$

$G(j\omega) = j\omega$

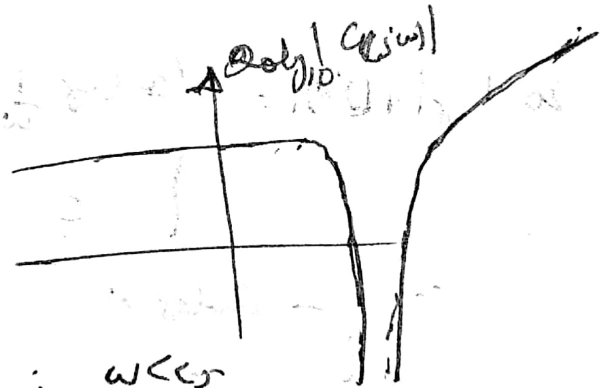
$|G(j\omega)| = \omega$

$20\log_{10}\omega$

$20\log_{10}|G(j\omega)| = 20\log_{10}\omega$



$G(s) = s^2 + 25$



$|25 - \omega^2| \approx 25$; $\omega << 5$

$|25 - \omega^2| \approx \omega^2$; $\omega >> 5$

→ Chapter 7 → V.V

Laplace Transform

$$\mathcal{L}[f(t)] = F(s) = \int_{0^-}^{\infty} f(t)\, e^{-st}\, dt$$

Ex1

$$u(t) = \begin{cases} 0 & \text{if } t<0 \\ 1 & \text{if } t \geq 0 \end{cases}$$

0

$$\mathcal{L}[u(t)] = \int_{0^-}^{\infty} 1 e^{-st}\, dt$$

$$= \int_{0^-}^{\infty} e^{-st}\, dt = \frac{1}{-s} e^{-st}\Big|_{0^-}^{\infty}$$

$$\boxed{\mathcal{L}[u(t)] = \frac{1}{s}}$$

Ex2 → $f(t) = e^{at}$

$$\mathcal{L}[f(t)] = \int_{0^-}^{\infty} e^{at} e^{-st}\, dt = \int_{0^-}^{\infty} e^{(a-s)t}\, dt$$

$$= \frac{1}{a-s} e^{(a-s)t} \Big|_{0^-}^{\infty} = \frac{1}{s-a} \qquad s > a$$

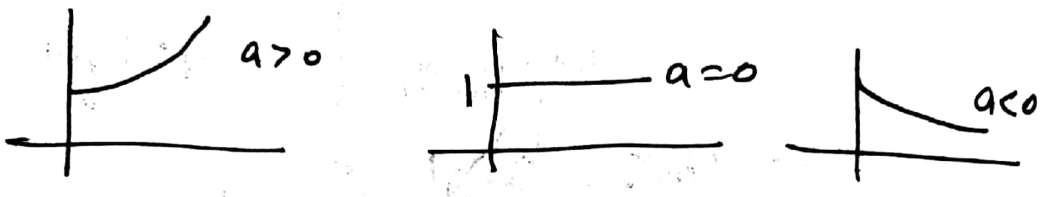
$* \; j^2 = -1 \; *$

$e^{(a+jb)t} \qquad a, b \in \mathbb{R}$

e^{jbt}

$|z| = 1$

e^{at}



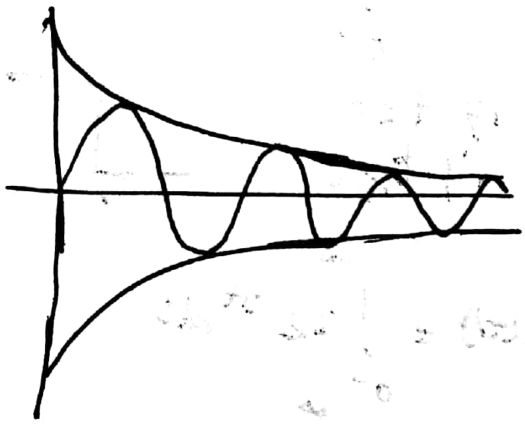
$e^{jbt} = \cos bt + j \sin bt$

$|e^{jbt}| = 1 = |\cos bt + j \sin bt| = x^2 + y^2$

$e^{\cancel{at+jbt}} \quad e^{(a+jb)t} = e^{at} e^{jbt}$

$$= e^{at}(\cos bt + j \sin bt)$$

$=$



$a<0$

Inverse Laplace transform

$$f(t) = \frac{1}{2\pi j}\int_{\sigma_1 - j\infty}^{\sigma_1 + j\infty} F(s)\, e^{st}\, ds$$

Theorems for Laplace Transform

① $\mathcal{L}[af_1 + bf_2] = aF_1(s) + bF_2(s)$

② $\mathcal{L}[\cos\omega t] = \mathcal{L}\left[\frac{e^{j\omega t} + e^{-j\omega t}}{2}\right] = \frac{s}{s^2+\omega^2}$

③ $\mathcal{L}(\sin\omega t) = \frac{\omega}{s^2+\omega^2}$

④ $\mathcal{L}\left[\frac{d}{dt} f(t)\right] = \int_{0^-}^{\infty} \frac{df(t)}{dt} e^{-st}\, dt$

$$= \int_{0^-}^{\infty} df(t)\, e^{-st} = e^{-st} f(t)\Big|_{0^-}^{\infty} + s\int_{0^-}^{\infty} f(t)\, e^{-st}\, dt$$

$$= sF(s) - f(0^-)$$

⑤ $\mathcal{L}[\delta(t)] = \mathcal{L}\left[\frac{du(t)}{dt}\right] = 1$

Laplace transform of δ function is 1

⑥ $\mathcal{L}\left[\int_0^t f(\tau)\, d\tau\right] = \frac{-e^{-st}}{s}\int_{0^-}^{t} f(\tau)\, d\tau\Big|_{0^-}^{\infty} - \frac{1}{s}\int_0^{\infty} f(t)\, e^{-st}\, dt$

$$= \frac{F(s)}{s}$$

(14) $\mathcal{L}^{-1}\left[\frac{1}{s-a}\right] = e^{at}$

$\frac{p(s)}{q(s)}$ assuming deg $(q(s)) > d(p(s))$

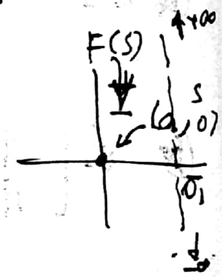
$$\frac{p(s)}{q(s)} = \sum_i \frac{c_i}{s-a_i}$$

where $q(s) = \prod_i s-a_i$

$$c_j = \frac{p(a_j)}{\prod_{i \neq j} (a_j - a_i)}$$

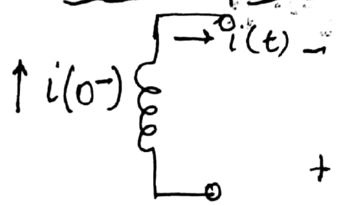
examples $\rightarrow$ chap $\rightarrow$ 7, 8

$$\mathcal{L}^{-1}(F(s)) = \frac{1}{2\pi j}\int_{\sigma_1 - j\infty}^{\sigma_1 + j\infty} F(s)e^{st}\,ds \qquad \sigma_1 > a$$



$$\mathcal{L}^{-1}(\mathcal{L}[f(t)]) = f(t)$$

Thevenin/Norton Theorems :-



$$V(t) = L\frac{d\,i(t)}{dt}$$

To solve, need $i(0^-)$

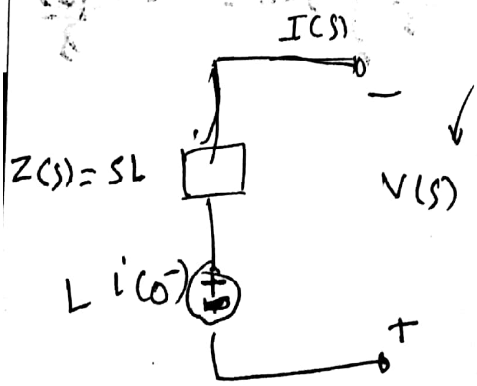
The transform circuit

$$V(s) = L\left(sI(s) - i(0^-)\right)$$

$$i(t) = \frac{1}{L}\int_{-\infty}^{t} V(\tau)\,d\tau$$

$$i(t) = \frac{1}{L}\int_{0^-}^{t} V(\tau)\,d\tau + i(0^-)$$

$$i(s) = \frac{V(s)}{sL} + \frac{i(0^-)}{s}$$



If $V(t) = V_0$

$$I(s) = \frac{V_0}{s^2 L} + \frac{i(0^-)}{s}$$

$$= \frac{V_0 + i(0^-)sL}{s^2 L}$$

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$$i(t) = i(0^-) + \frac{V_0}{L}\mathcal{L}^{-1}\frac{1}{s^2} = i(0) + \frac{V_0}{L}$$

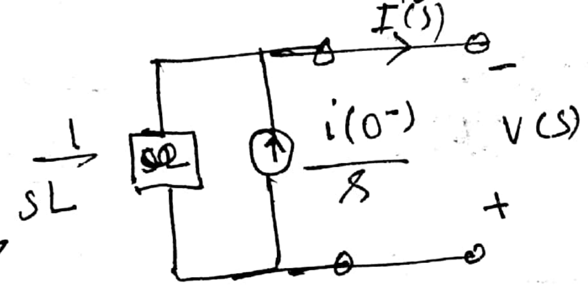
$$\int_{0^-}^{\infty} t e^{-st}\,dt \Rightarrow i(t) = i(0^-) + \frac{V_0 t}{L}$$

Algorithm

ODE $\xrightarrow{\mathcal{L}}$ Laplace eqn $\xrightarrow[\text{algebraically}]{\text{Solve}}$ get soln in s $\xrightarrow{\mathcal{L}^{-1}}$ solution in time

If I had solved an ODE

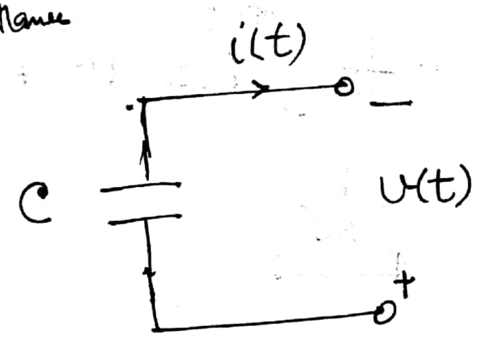
Ex.



admittance

$$I(s) = \frac{1}{sL}\left(V(s) + L\,i(0)\right)$$

Ex

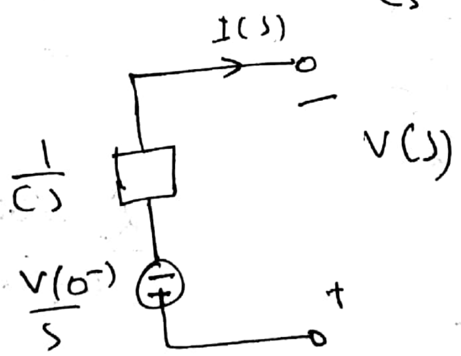


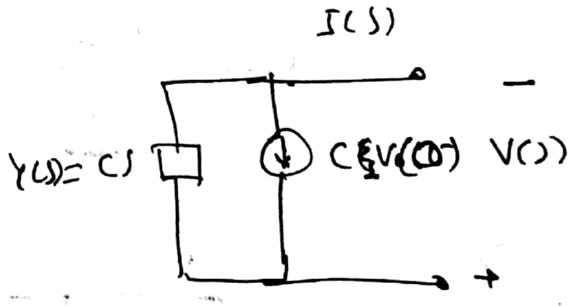
$$i(t) = C\frac{dv}{dt}$$

$$I(s) = C\left(s V(s) - V(0^-)\right)$$

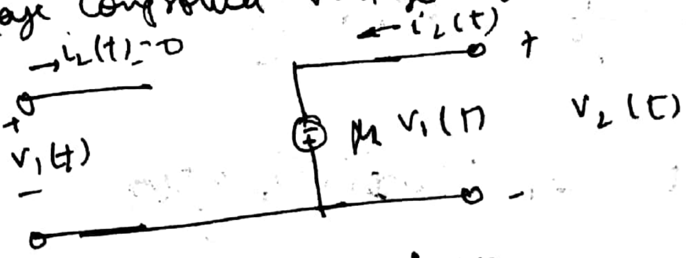
$$\frac{I(s)}{Cs} = V(s) - \frac{V(0^-)}{s}$$

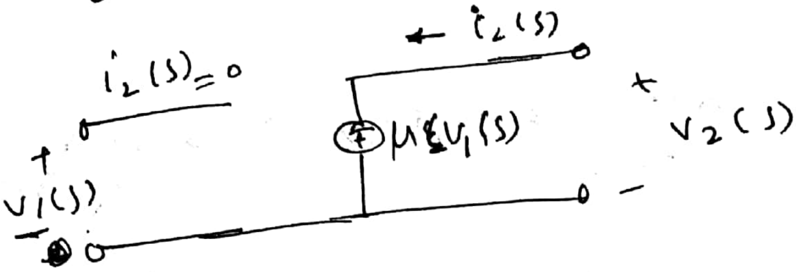
$$V(s) = \frac{I(s)}{Cs} + \frac{V(0^-)}{s}$$





Voltage Controlled Voltage Source





$V = i\left(R + \frac{1}{j\omega C}\right)$

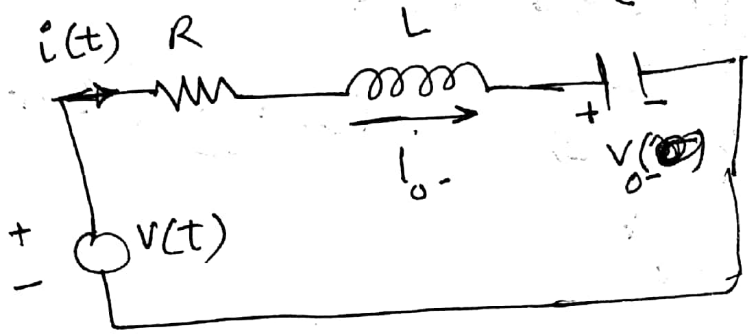
$\boxed{V = i\,(Z)}$

$i = \frac{V}{Z}$

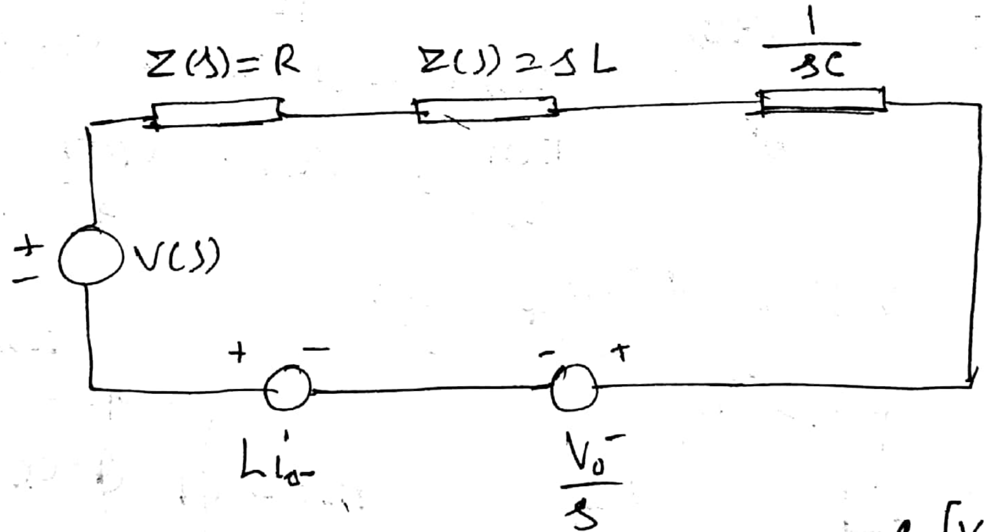
$V_C = Z_C\, i$

$V_C = \frac{V}{Z} Z_C$

Ex

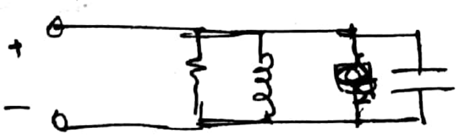


$$KVL: v(t) = R\,i(t) + L\frac{di}{dt} + \frac{1}{C}\int_0^t i(t)\,dt + V_{0^-}$$



$$\frac{\mathcal{L}[v(t)]}{\mathcal{L}[i(t)]} = Z(s)$$

Ex



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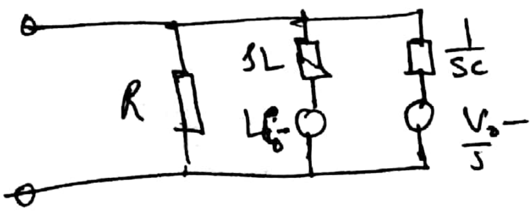
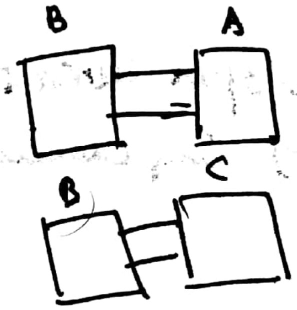
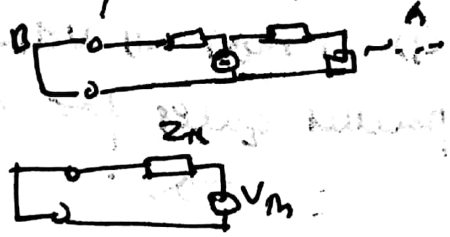
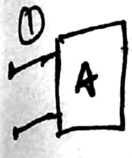


Thevenin / Norton Theorem

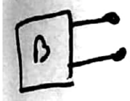




① 

Ckt A should be consisting of only linearly circuits voltage/current sources (independent). Initial condition across capacitor & inductor are possible)

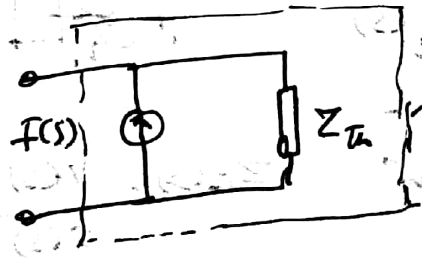
No coupling (magnetic or controlled sources).

② 

Ckt B :- NO Coupling

Thevenin's Theorem : Under conditions ① & ② there exists an impedance (Z_{Th}) ~~Circuit A is equivalent to~~ & a transform voltage (V_{Th}) s.t ckt A is ~~indistinguishable~~ equivalent to ckt C.

Norton's Theorem :-



→ Ckt D

$$I(s) = \frac{V_{Th}(s)}{Z_{Th}(s)}$$

Under cond^n ① & ② $\exists\ Z_{Th}(s),\ I_{Th}(s)$ s.t ckt D is equivalent to ckt A.

Theorem Norton $\Leftrightarrow$ Thevenin

$$y(t) = A_{L\times L}\, i(t)$$

L_2 dimension of A = n(edges) − n(nodes) + 1 = no. of window panes

R, L, C = $\mathbb{R}$, $\frac{d}{dt}$, $\int$ → A is symmetric

Controlled sources ⇒ Asymmetric 'A'.

① To find $Z_{Th}(s)$, short all voltage sources, open current sources. Use series parallel rules for combining impedances.

— or —

Reciprocity:

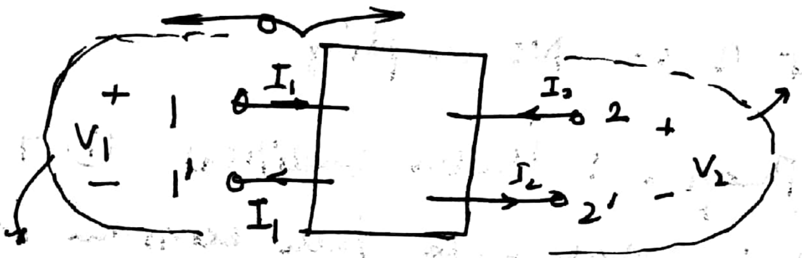
① Behaviour of i_{in} as a fun of v_{Th} is the same as behaviour of i_{Th} as a fun of v_{in}.

Shown by circuit having R, L, C, transformer, uncontrolled current & voltage sources

homework: →

Chapter 10

Port: →



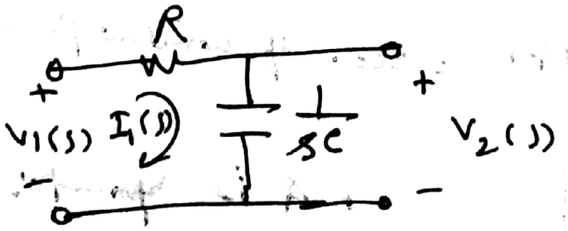
$$\underbrace{\frac{I_2(s)}{V_1(s)} = Y_{12}(s)}$$

Transfer fun $Z_{Th}(s) = \dfrac{V(s)}{I(s)}$

Numerator (output) / Denominator (Input)	$V_2(s)$	$I_2(s)$
$V_1(s)$	$g_{12}(s)$	$Y_{12}(s)$
$I_1(s)$	$Z_{12}(s)$	$\alpha_{12}(s)$

$\dfrac{\text{Output}}{\text{Input}} = \text{transfer fun}$

Ex 3



$$g_{12}(s) = \frac{V_2(s)}{V_1(s)} = \frac{I_1 1/sC}{I_1 R + I_1 1/sC} = \frac{1/sC}{R + 1/sC}$$

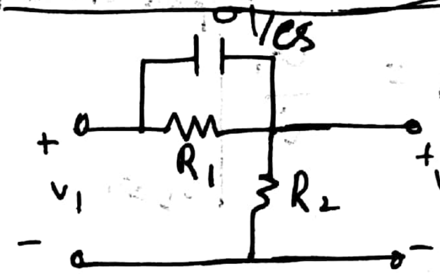
$$\boxed{g_{12}(s) = \frac{1}{(RC)s + 1}} = \frac{(1/RC)}{s + (1/RC)}$$

$$Y_{11}(s) = \frac{I_1(s)}{V_1(s)} = \frac{\cancel{I_1(s)}}{R_1 \cancel{I_1(s)} + \frac{1}{Cs}\cancel{I_1(s)}}$$

$$= \frac{Cs}{(CR)s+1}$$

$$Y_{11}(s) = \frac{s/R}{s+(1/RC)}$$

Ex 4



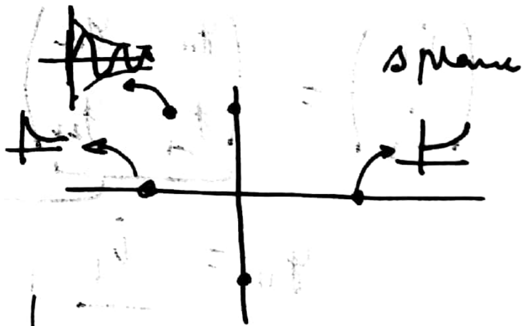
$$Z = \frac{R_1/Cs}{R_1+1/Cs} = \frac{R_1}{(R_1C)s+1}$$

$$G_{12} = \frac{V_2(s)}{V_1(s)} = \frac{i R_2}{i\frac{R_1}{R_1Cs+1} + iR_2}$$

$$G_{12} = \frac{R_2(R_1Cs+1)}{R_1 + R_2(R_1Cs+1)}$$

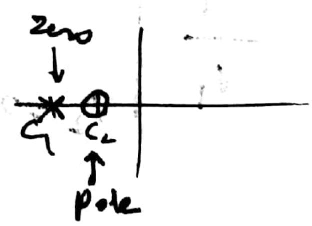
Claim → All transfer fun'n's are of the form $p(s)/q(s)$ where p & q are polynomials

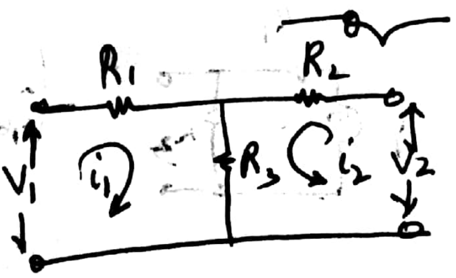
$$\cos\omega t = \frac{e^{j\omega t}+e^{-j\omega t}}{2}$$



Poles & Zeros

$$G_{12}(s) = \frac{s+c_1}{s+c_2}$$





$v_1 = i_1(R_1 + R_3) + i_2 R_2$

$v_2 = i_2(R_2 + R_3) + i_1 R_2$

$$\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \underbrace{\begin{pmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{pmatrix}}_{\text{Open Circuit impedance}} \begin{pmatrix} i_1 \\ i_2 \end{pmatrix}$$

So,

$$z_{11} = \frac{v_1}{i_1}\Big|_{i_2=0}$$

$$z_{12} = \frac{v_1}{i_2}\Big|_{i_1=0}$$

$$z_{21} = \frac{v_2}{i_2}\Big|_{i_1=0} \; ; \quad z_{22} = \frac{v_2}{i_1}\Big|_{i_2=0}$$

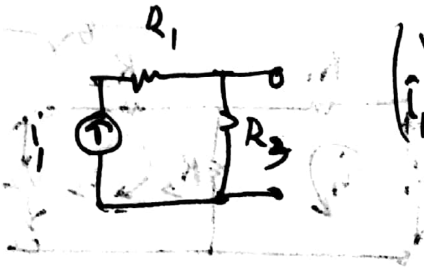
also the matrix can be written as:

$$\begin{pmatrix} i_1 \\ i_2 \end{pmatrix} = \underbrace{\begin{pmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{pmatrix}}_{\text{Short Circuit admittance}} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$$

$$y_{11} = \frac{i_1}{V_1}\Big|_{V_2=0}$$

Transmission:

$$* = \frac{i_1}{v_2}\Big|_{i_2=0} = \frac{1}{R_3}$$



$$\begin{pmatrix} v_1 \\ i_1 \end{pmatrix} = \begin{pmatrix} * & \end{pmatrix} \begin{pmatrix} v_2 \\ i_2 \end{pmatrix}$$

Inverse Transmission

$$\begin{pmatrix} v_2 \\ -i_2 \end{pmatrix} = \begin{pmatrix} & \\ & \end{pmatrix} \begin{pmatrix} v_1 \\ i_1 \end{pmatrix}$$

Some more examples

$$\begin{pmatrix} v_1 \\ i_2 \end{pmatrix} = \begin{pmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{pmatrix} \begin{pmatrix} i_p \\ v_2 \end{pmatrix}$$

↑ hybrid parameters

$$h_{11} = \frac{V_1}{i_1}\Big|_{v_2 = 0}$$

$$\begin{pmatrix} i_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{pmatrix} \begin{pmatrix} V_1 \\ i_2 \end{pmatrix}$$

$$g_{12} = \frac{i_1}{i_2}\Big|_{v_1 = 0}$$

Current Controlled Current Source : check cant

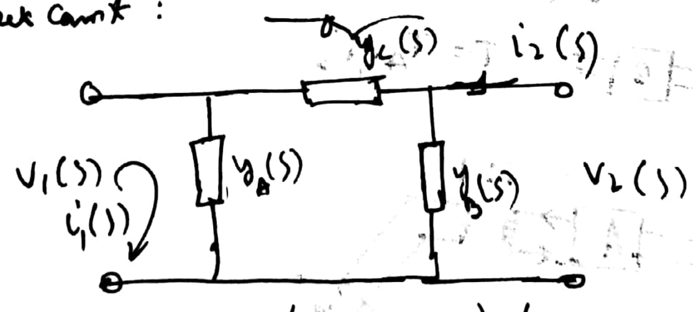
So, $v_1 = 0$

$i_2 = ki_1$

$$\begin{array}{c|cccc} & v_1 & i_1 & V_2 & i_2 \\ \hline a & 1 & 0 & 0 & 0 \\ b & 0 & -k & 0 & 1 \end{array} \begin{pmatrix} V_1 \\ i_1 \\ V_2 \\ i_2 \end{pmatrix} = 0$$

$$\begin{pmatrix} v_1 \\ i_1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -k \end{pmatrix}^{-1} \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} v_2 \\ i_2 \end{pmatrix}$$

→ Check cant :



$$\begin{pmatrix} i_1 \\ i_2 \end{pmatrix} = \begin{pmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \end{pmatrix}$$

$$y_{11} = \frac{i_1}{V_1}\Big|_{V_2 = 0} = y_A + y_C$$

$$y_{12} = \frac{i_1}{V_2}\Big|_{V_1 = 0} = -y_C$$

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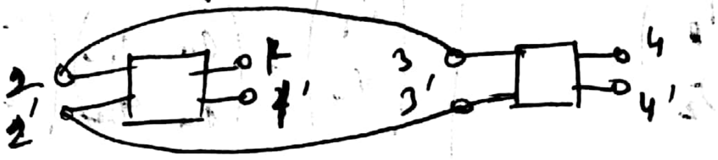
$$y_{21} = \frac{I_2}{V_1}\Bigg|_{V_2=0} = -y_c$$

$$y_{22} = y_B + y_c$$



$((2,3),(2',3'))$

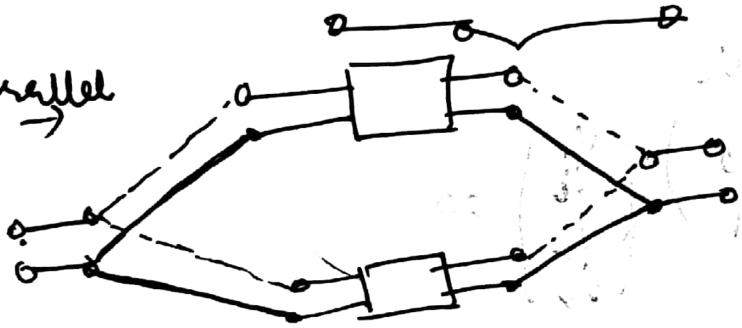
Cascade

$$\begin{pmatrix} V_1 \\ i_1 \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} V_2 \\ i_2 \end{pmatrix} \quad ; \quad \begin{pmatrix} V_2 \\ i_2 \end{pmatrix} = \begin{pmatrix} V_3 \\ -i_3 \end{pmatrix}$$

$$\begin{pmatrix} V_3 \\ i_3 \end{pmatrix} = \begin{pmatrix} A' & B' \\ C' & D' \end{pmatrix} \begin{pmatrix} V_4 \\ i_4 \end{pmatrix} \quad ; \quad \begin{pmatrix} V_1 \\ i_1 \end{pmatrix} = \begin{pmatrix} \; \end{pmatrix} \begin{pmatrix} V_4 \\ i_4 \end{pmatrix}$$

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} A' & B' \\ -C' & -D' \end{pmatrix}$$

Parallel



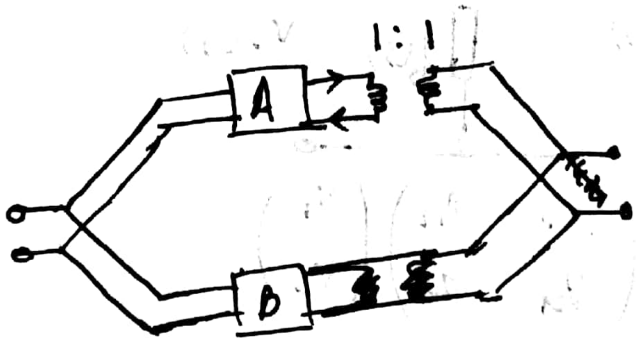
→ Anelle phone images

* How does it behave?

Cascade

$T = T_A \cdot T_B$

Parallel



$Y = Y_A + Y_B$

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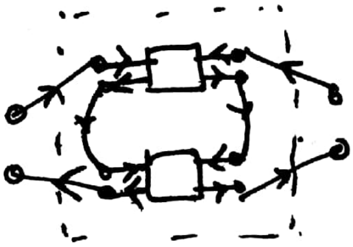
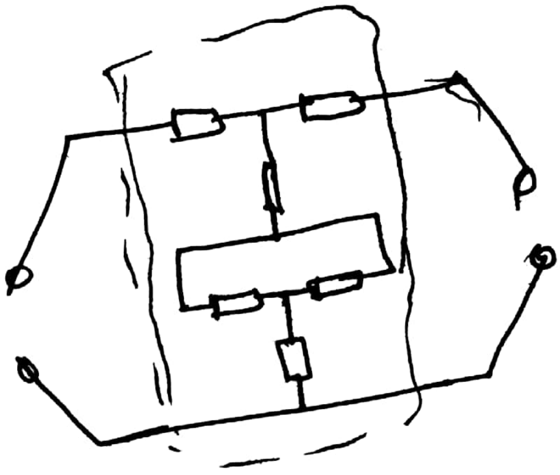
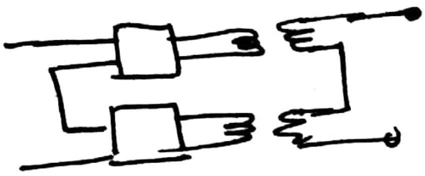
Series Chain : $Z = Z_A + Z_B$



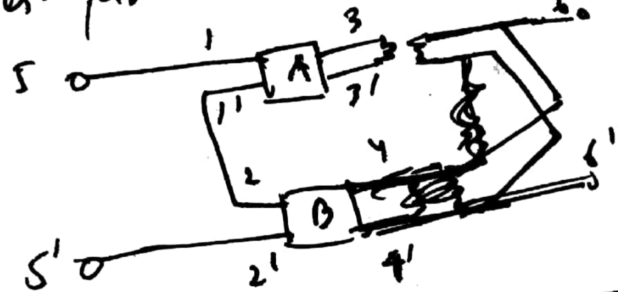
→ Not good





(Check phone)

Series - parallel



$$\begin{pmatrix} V_5 \\ I_6 \end{pmatrix} = \left(\overset{\text{Hybrid}}{H_A + H_B}\right)\begin{pmatrix} I_5 \\ V_6 \end{pmatrix}$$

Chua Chap 13.6

RECIPROCITY

$\underbrace{R, C, I}_{\text{1 port elements}}$, $\underbrace{\text{Transformer}}_{\text{2 port elmt}}$, $\underbrace{\text{Coupled inductor}}_{\text{2 port}}$ ↑ Not in syllabus

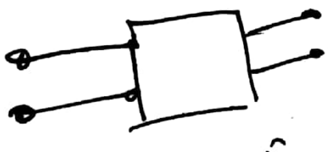
Theorem: ① If Impedance matrix exists ; it is symmetric

② " admittance " " symmetric

③ If hybrid " " anti-symmetric

④ If transmission matrix exists then its determinant is 1.

(Check phone) → To see the proof of the above 4.



V_1, I_1, V_2, I_2

$\bar{V}_1, \bar{I}_1, \bar{V}_2, \bar{I}_2$ } Claim $V_1\bar{I}_1 + V_2\bar{I}_2 = \bar{V}_1 I_1 + \bar{V}_2 I_2$

EE-225 Previously → Reciprocity Theorem.

Lemma : Claim ⇒ Reciprocity theorem proof.

$$\begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = \begin{pmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \end{pmatrix}$$

$$\begin{pmatrix} V_1 \\ I_2 \end{pmatrix} = \begin{pmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{pmatrix} \begin{pmatrix} I_1 \\ V_2 \end{pmatrix}$$

$$\begin{pmatrix} V_1 \\ I_1 \end{pmatrix} = \begin{pmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{pmatrix} \begin{pmatrix} V_2 \\ -I_2 \end{pmatrix}$$

* $V_1(\text{case 1}) I_1(\text{case 2}) + V_2(\text{case 1}) I_2(\text{case 2})$
$= V_1(\text{case 2}) I_1(\text{case 1}) + V_2(\text{case 2}) I_2(\text{case 1})$

Claim :- *

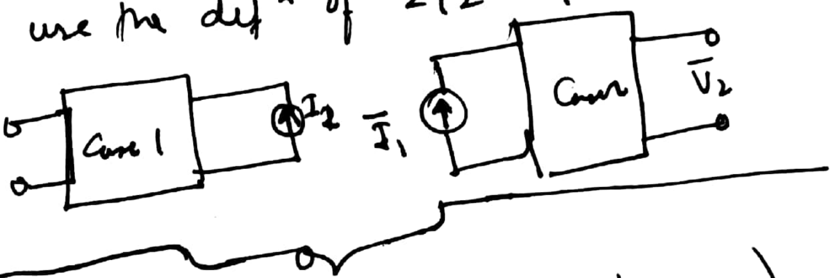
$$\left[\begin{aligned} V_1(1) I_1(2) + V_2(1) I_2(2) \\ = V_1(2) I_1(1) + V_2(2) I_2(1) \end{aligned} \right] \longrightarrow (A)$$

To show $Z_{21} = Z_{12}$

Use (A), also see photo,

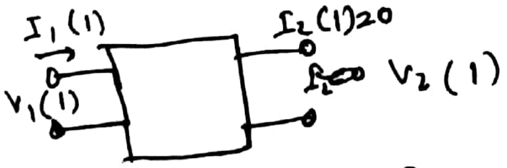
and use the def^n of Z_{12} & Z_{21} in different cases

In above case

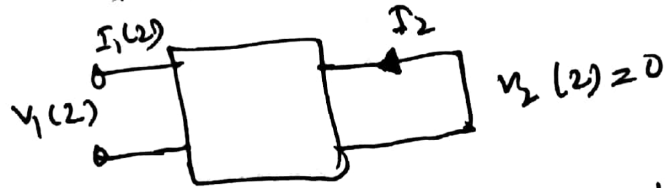


$$T = \begin{pmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{pmatrix} = \begin{pmatrix} \left.\dfrac{V_1}{V_2}\right|_{I_2=0} & \left.\dfrac{-V_1}{I_2}\right|_{V_2=0} \\ \left.\dfrac{I_1}{V_2}\right|_{I_2=0} & \left.\dfrac{-I_1}{I_2}\right|_{V_2=0} \end{pmatrix}$$

Case 1 :



Case 2 :



Now use the claim (A) *

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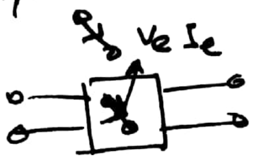
Proof Of Claim: Tellegen's Theorem

Suppose the 2 port circuit corresponds to a graph with edge set E,



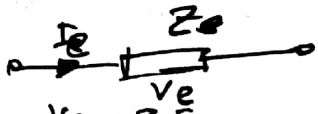
Voltages $\{V_e \mid e \in E\}$

$\{I_e \mid e \in E\}$

Tellegen's theorem:

$$V_1(1)\, I_1(2) + V_2(1)\, I_2(2) + \sum_{e \in E} V_e(1)\, I_e(2) = 0$$

$$= V_1(2)\, I_1(1) + V_2(2)\, I_2(1) + \sum_{e \in E} V_e(2)\, I_e(1)$$



$V_e = z I_e$

Claim: $\forall e \in E$

$$V_e(1)\, I_e(2) = V_e(2)\, I_e(1)$$

Proof: $I_e(1)\, z\, I_e(2) = I_e(2)\, z\, I_e(1)$

Chapter 12 Sinusoidal Steady State Analysis

$$\dot{x} = f(x)$$

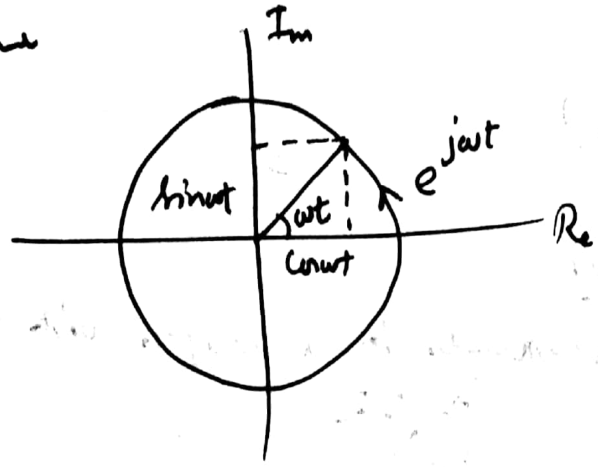
Steady State $\Rightarrow \{x \mid f(x) = 0\}$ (recurrent)

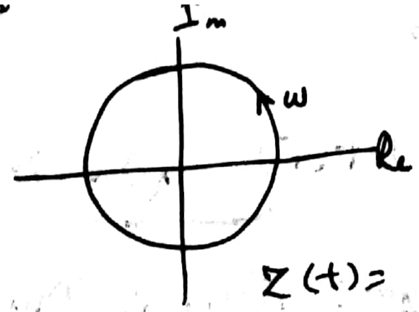
$$u(t) = A\sin(\omega t) + \overbrace{B_1 e^{-p_1 t} + B_2 e^{-p_2 t}}^{\text{transient}}$$

$$V(s) = \frac{A\omega}{s^2+\omega^2} + \frac{B_1}{s+p_1} + \frac{B_2}{s+p_2} + \text{terms}$$

$t > T$ $\quad v(t) \approx A \sin \omega t$

ℂ-plane →





$Z(t) = e^{j\omega t}$

$Z(t) = \cos\omega t + j\sin\omega t$

a general sinusoid

$A_m \sin(\omega t + \phi)$

A_m → +ve $\mathbb{R}$; ω → +ve real ; ϕ → $\in [0, 2\pi]$

If we fix ω.

$$A := A_m e^{j\phi}$$

$$A_m \cos(\omega t + \phi) = R_e\left(A\, e^{j\omega t}\right)$$

1) $R_e\left(A e^{j\omega t}\right) = R_e\left(B e^{j\omega t}\right) \iff A = B$

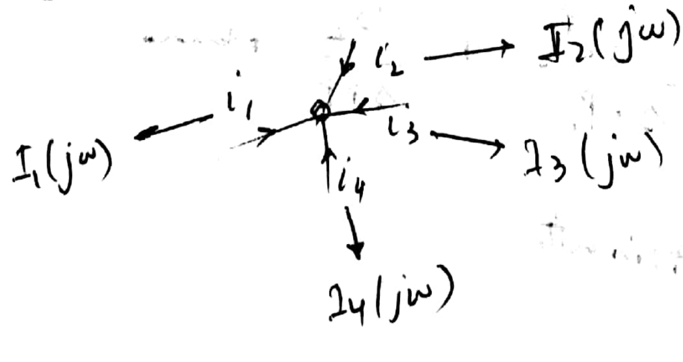
2) $a_1 R_e\left(A_1 e^{j\omega t}\right) + a_2 R_e\left(A_2 e^{j\omega t}\right)$

$$= R_e\left((a_1 A_1 + a_2 A_2)\, e^{j\omega t}\right) \quad \forall\, a_1, a_2 \in \mathbb{R}$$

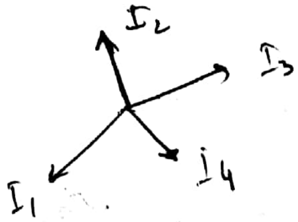
3) $\dfrac{d}{dt}\left(R_e\left(A e^{j\omega t}\right)\right) = R_e\left(j\omega A e^{j\omega t}\right)$

→ Sinusoidal Steady State at the freq. ω.

it means ⟹ $v_e(t)$, $i_e(t)$, $v_n(t)$ are sinusoidal at the same ω.



KCL: $\forall t$ $\quad i_1(t) + i_2(t) + i_3(t) + i_4(t) = 0$



$$I_1 + I_2 + I_3 + I_4 = 0$$

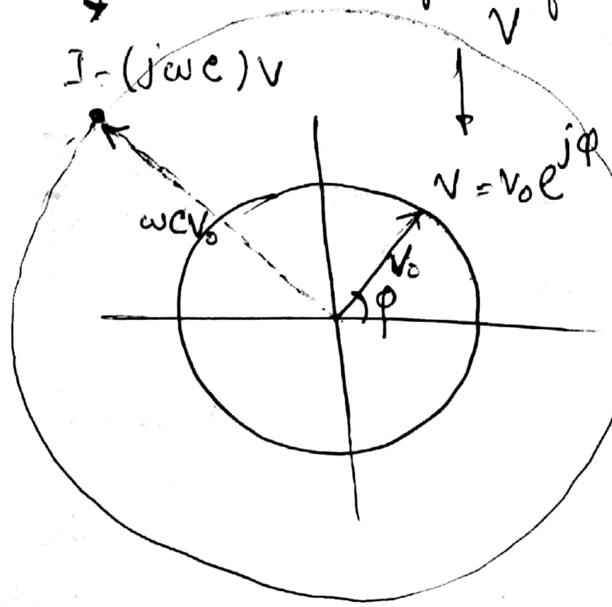
$$V = ZI$$

$$I = YV$$

Phasor of I

Phasor of V

$I = (j\omega C)V$



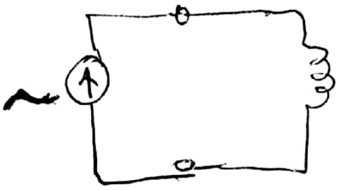
Capacitors $\rightarrow$

$$i = C\frac{dV}{dt} \Rightarrow I = Cj\omega V$$

$$\Rightarrow I = (j\omega C)V$$

$$I = j\omega C\, V_0 e^{j(\pi/2 + \varphi)}$$

Inductors $\rightarrow$

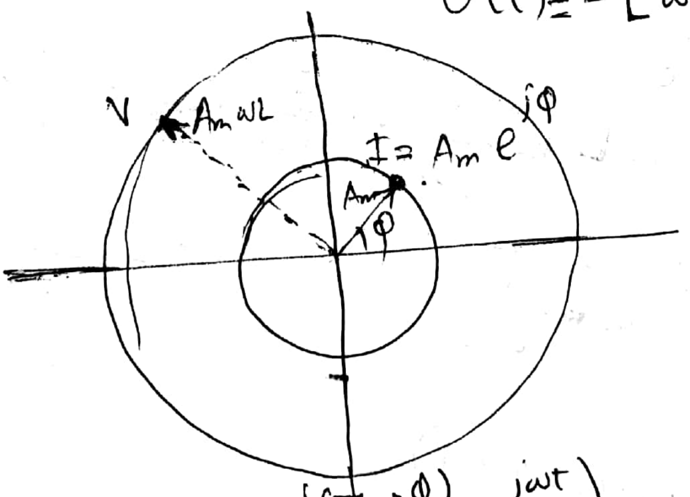


$$i(t) = A_m \cos(\omega t + \varphi)$$

$\cancel{v(t) = j\omega A_m \sin(\omega t + \varphi)}$

$$V = L\frac{di}{dt}$$

$$v(t) = -L\omega A_m \sin(\omega t + \varphi)$$



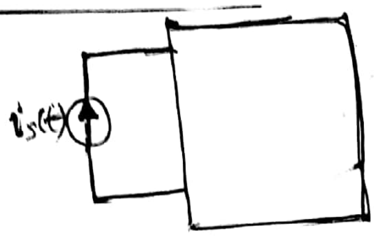
$$v(t) = R_e\left(A_m \omega L\, e^{j(\pi/2 + \varphi)} e^{j\omega t}\right)$$

$$= \omega L A_m \cos\left(\omega t + \frac{\pi}{2} + \varphi\right)$$

$$= -\omega L A_m \sin(\omega t + \varphi)$$

$=$

DRIVING POINT

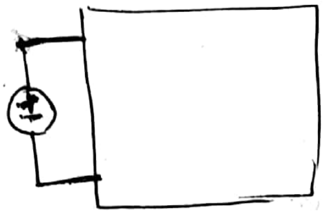


$i_s(t) = A_m \cos(\omega t + \varphi)$

when we wait for quite some time the transients die down

driving pt. impedance $= Z(j\omega) = \dfrac{V}{I_s}$

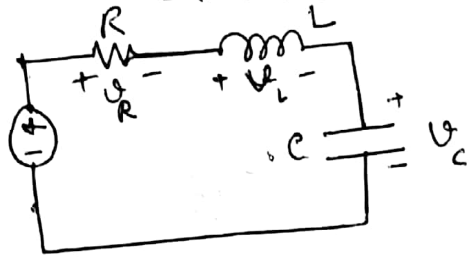
driving pt. admittance ⇒



$\boxed{Y(j\omega) = \dfrac{I}{V_s} = \dfrac{1}{Z(j\omega)}}$

driving pt. admittance

Ex RLC:

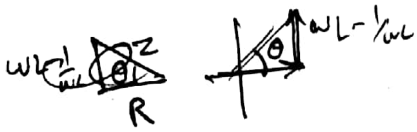


$v_1 = \cos\omega t$, $V_1 = 1$

$v_1 = v_R + v_L + v_c$

$Z(j\omega) = R + j\omega L + \dfrac{1}{j\omega C} - \dfrac{j}{\omega C} = R + j\left(\omega L - \dfrac{1}{\omega C}\right)$

$I_1 = \dfrac{V_1}{Z} = \dfrac{1}{R + j\omega L - \dfrac{j}{\omega C}}$



$\theta = \tan^{-1}\left(\dfrac{\omega L - 1/\omega C}{R}\right)$

$i_1(t) = A\cos(\omega t + \theta_1)$

$A = I_1$

$\theta = -\tan^{-1}\left(\dfrac{\omega L - 1/\omega C}{R}\right)$

$I = \dfrac{V_1\angle 0}{Z\angle\theta}$

$= \left|\dfrac{V_1}{Z}\right|\angle -\theta$

$= \dfrac{\cos\omega t}{|Z|} - \tan^{-1}\left(\dfrac{\omega L \ldots}{R}\right)$

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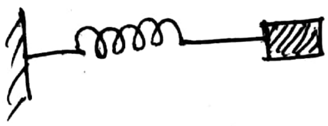
Simple Harmonic Oscillator



$$m\ddot{x} = -kx$$

$$m\ddot{\theta} = -\theta$$

LC oscillations

$$V_C + V_L = 0$$

$\Rightarrow$ ~~$cV_C = -V_L$~~

$$V_C = -V_L$$

$$C\frac{dV_C}{dt} = i \qquad \Big| \qquad V_L = L\frac{di}{dt}$$

$$\downarrow$$

$$-C\frac{dV_L}{dt} = i \qquad \qquad \frac{dV_L}{dt} = L\frac{d^2i}{dt^2}$$

$$-CL\frac{d^2i}{dt^2} = i$$