

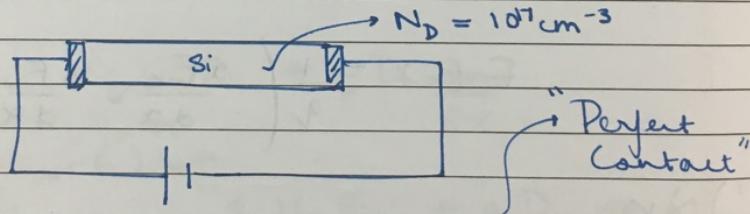
→ Before M.S.: Solid State Physics.

- Carriers, EB Diagram (Equilibrium)
- R-G (SS)  $\rightarrow$  uniform conc.
- Steady state with Spatial variation  
↓  
Diffusion                      Drift.
- Continuity Eq<sup>n</sup> + Poisson Eq<sup>n</sup>

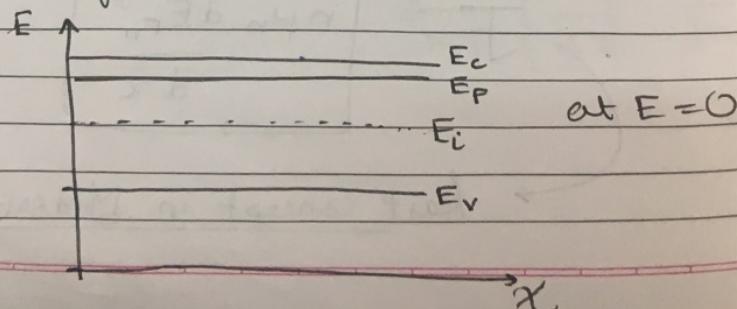
→ Note: Application to Devices.

- Resistor
  - Diodes 2 weeks
  - BJT 1 week
  - MOSFET 2 weeks.
  - New Devices 1 week
- } 7 Mini Quizzes  
More

### Resistor

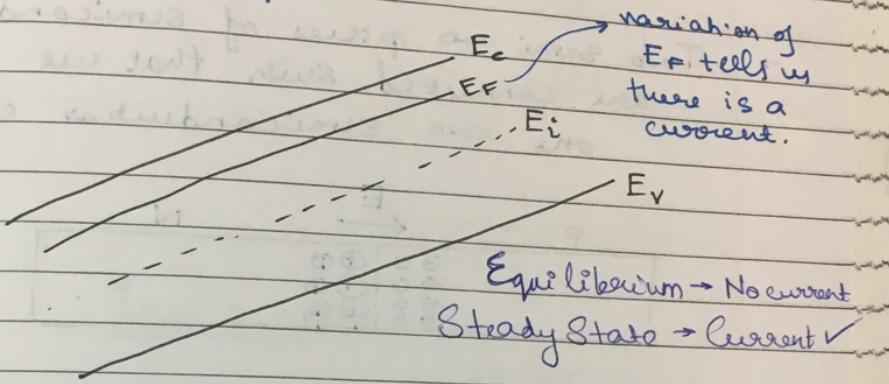


- No Diffusion.  $\rightarrow$  Metal contacts have no effect.
  - Constant electric field
- Band Diagram (At Equilibrium) ( $E = 0$ )



## Steady State, $E \neq 0$

Slope sign is imp.



### Method

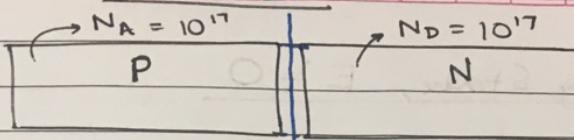
Start with  $E_c$  (or  $E_v$  just  $E$  in gen)  
 $= -q V(x)$

Get  $E$

→ Are there Quasi Fermi Levels :

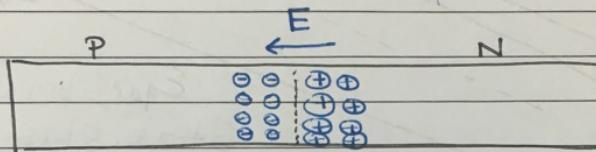
- No there is no split b/w the Quasi fermi levels.
- The n & p values are still given by the previous equations & values so we conclude no split b/w  $E_{Fn}$  &  $E_{Fp}$ .  
Despite Steady State.

# Diode



s → s As we Make crystal contact

- Two semi  $\infty$  pieces of semiconductor are connected such that we get one  $\infty$  semiconductor crystal.

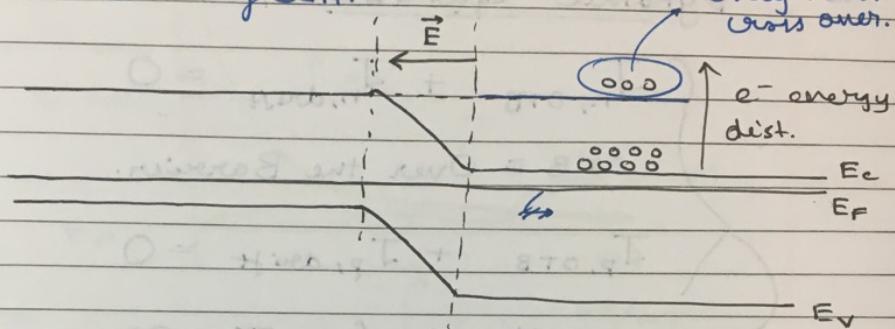


- As holes diffuse to the right and e<sup>-</sup> diffuse to the left charge build up occurs & an electric field is set up.
- Eventually the Generated Drift current offsets the existing diffusion current.

## Steps

- Diffusion
  - Generation / Uncovering of Ionic charges.
  - Electric field.
  - Equilibrium  $\Rightarrow J = 0$
- Dynamic eq v v      Static X

→ Band Diagram.



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→ Define  $I = I_0 (e^{qV/kT} - 1)$

Text → SDF Chap, 5, 6 & 7

1) Recap

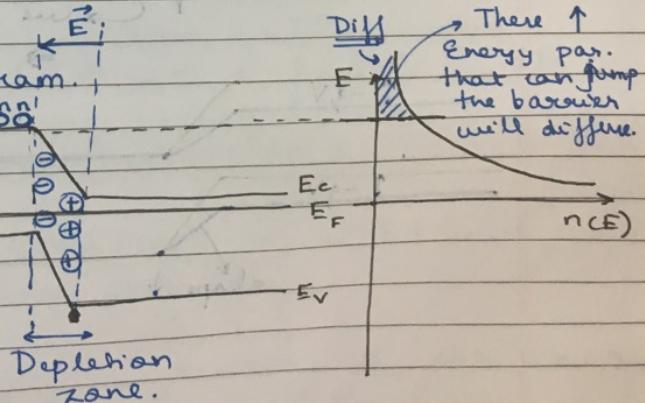
Even in Diode,  $\frac{\partial E_F}{\partial x} = 0$  since

this comes from  $J - J_N - J_P = 0$

2)

Band Diagram.

Reach the Sep. region & cross by Drift.



Where,  $n(E) = g(E) f(E)$

## → Dynamic Equilibrium

$$\left\{ \begin{array}{l} J_{n, OTB} + J_{n, drift} = 0 \\ J_{p, OTB} + J_{p, drift} = 0 \end{array} \right.$$

$OTB = \text{Over the Barrier.}$

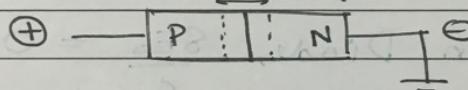
$$J_{n, OTB} + J_{n, drift} = 0$$

our Model for  $J = 0$

## → Applying Bias

In our band diagram the energy levels we plot are all  $e^-$  energies.

Dep. region.

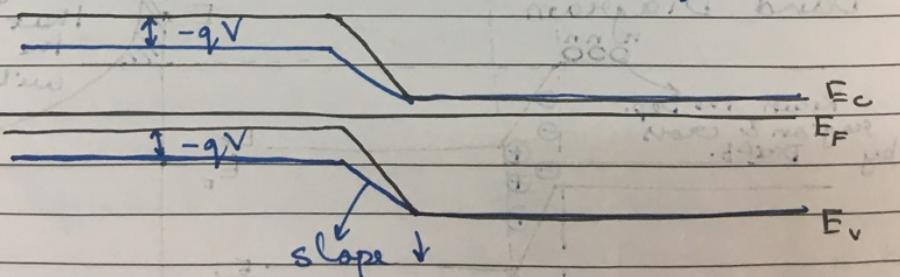


$$e^- \text{ energy}$$

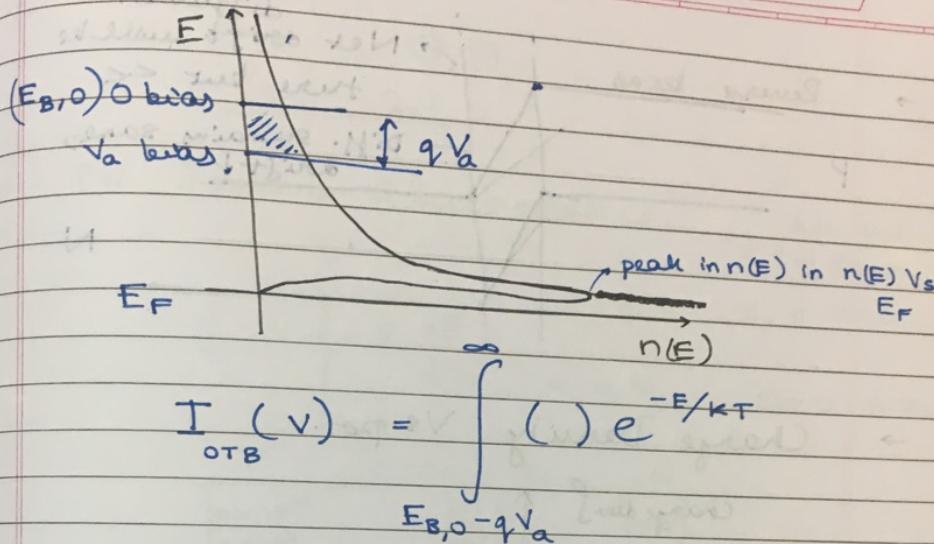
$$\epsilon = -qV$$

we apply +ve potential

$$\Rightarrow \epsilon \downarrow$$



$$(iii) EDC = (E)n_{\text{max}}$$



→ Here In drift, Minority carrier diffusion from right to left remains unchanged since it is limited by no. available.

→ Hence In drift = (In mag.) to  $J_{OTB}$  at  $E_{B,0}$

$$J(v) = \int_{E_{B,0} - qV_a}^{\infty} (\cdot) e^{-E/kT} dE - \int_{E_{B,0}}^{\infty} (\cdot) e^{-E/kT} dE$$

$OTB$  (Sted. State) -  $OTB$  (equil.)

→ Assuming constant prefactor.

$$= (\text{new prefactor}) \left( e^{E_{B,0}/kT} - e^{(E_{B,0} - qV_a)/kT} \right)$$

$$= (\cdot) \left[ e^{qV_a/kT} - 1 \right]$$

→ Reverse bias

P

Net drift will be  
there but  $\ll$   
Diff. remains same;  
drift ↓

N

→ Charge Density Vs pos.

charge den.  $\rho$

$N_A$

Ideal Depletion region,  
No mobile carriers.

→ Electric field

$\propto x^2$   $\propto x'$  (Slopes need not be same)

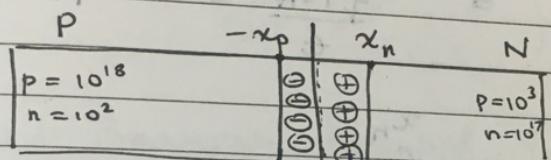
Built  
in potential

$\ominus \oplus$   
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## PN Junction Electrostatics

$$f = \begin{cases} \epsilon \frac{dE}{dx} & = -qN_A \quad -x_p < x < 0 \\ & = qN_D \quad 0 < x < x_n \end{cases}$$

left end  
of dep. region



$f = P - n + N_D - N_A$   
"Called Depletion"

Approximation of  
charge density

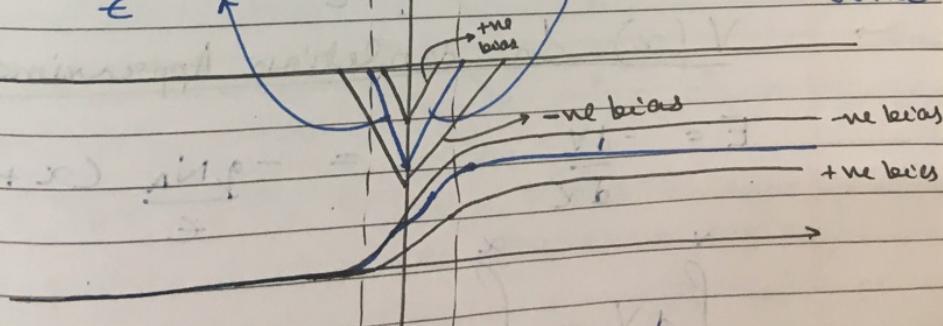
Note  
charge  
density is  
given by

Majority  
carrier  
concentration  
that 1/2

$$F = -\frac{qN_A(x+x_p)}{\epsilon}$$

$$= -\frac{qN_D(x_n-x)}{\epsilon} \quad (10^{17}/10^8)$$

$\therefore$  Ionic



→ Calculation of Internal Electric Field

$$\int dE = \int_{-x_p}^x -\frac{qN_A}{\epsilon} dx \quad \therefore f = -\frac{dE}{dx}$$

& L.H.S of  
Dep. region

$$E = -\frac{qN_A}{\epsilon} (x + x_p) \quad x < 0$$

$$\int_0^x dE = \int_x^{x_n} \frac{qN_D}{\epsilon} dx \quad E = \frac{qN_D}{\epsilon} (x_n - x)$$

$$E = \frac{qN_D}{\epsilon} (x - x_n)$$

→  $V(x)$  In depletion Approximation

$$E = -\frac{dV}{dx} \quad \therefore = -\frac{qN_A}{\epsilon} (x + x_p)$$

$$\int_0^x dV = \int_{-x_p}^x \frac{qN_A}{\epsilon} (x + x_p) dx$$

$$-x_p < x < 0$$

$$V(x) = \frac{qN_A}{2\epsilon} (x + x_p)^2 \Big|_{-x_p}$$

Other side

~~$$-\frac{qN_D}{2\epsilon} (x - x_n)^2$$~~

$$V(x) = \frac{qN_A}{2\epsilon} [(x + x_p)^2] \Big|_{-x_p}$$

Other side

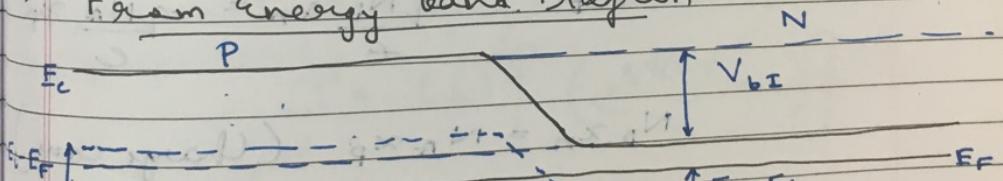
~~$$V(0) = \frac{qN_A x_p^2}{2\epsilon}$$~~

Voltages  
are relative.

→ To Get Barrier Potential  $V_{BI}$

$$\int_V^{V_{BI}} dV = \int_x^{x_n} \frac{qN_D}{\epsilon} (x_n - x) dx$$

→ Form Energy band Diagram



$$\therefore \Delta E_c = \Delta E_i \\ V_{BI} = (E_i - E_F)_{left} + (E_F - E_i)_{right}$$

(In terms of voltage)

On p side (out side depletion region)

$$p = N_A = n_i e^{(E_i - E_F)/kT}$$

$$\frac{kT}{q} \ln \left( \frac{N_A}{n_i} \right) = V_{left} \quad (\text{In terms of voltage})$$

$$\frac{kT}{q} \ln \left( \frac{N_D}{n_i} \right) = V_{right}$$

$$V_{bi} = \frac{kT}{q} \ln \left( \frac{N_A N_D}{n_i^2} \right)$$

→  $V$  inside in terms of  $x_n$  (R.H.S)

$$\int_V^{V_{bi}} dV = \int_x^{x_n} \frac{q N_D}{\epsilon} (x_n - x) dx$$

~~$V_{bi} = V$~~  & Equating E at  $x=0$

$$\frac{-q N_D}{\epsilon} (x_n + x) = -\frac{q N_A}{\epsilon} (x + x_p)$$

~~$\frac{-q N_D}{\epsilon} x_n = -\frac{q N_A}{\epsilon} x_p$~~

$$N_D x_n = N_A x_p \quad (\text{charge cons.})$$

$$V_{bi} - V = -\frac{q N_D}{2\epsilon} (x - x_n)^2$$

$$V_{bi} - V = 0 + \frac{q N_D}{2\epsilon} (x - x_n)^2$$

$$V - V$$

$$= x_n + x_p = \left[ \frac{2 \epsilon V_{bi}}{q} \left( \frac{N_A + N_D}{N_A N_D} \right) \right]^{1/2}$$

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$$V_{bi} = \frac{q N_D}{2 \epsilon} (x - x_n)^2 + \frac{q N_A}{2 \epsilon} (x + x_p)^2$$

$$V_{bi} = \frac{q}{2 \epsilon} (N_D x_n^2 + N_A x_p^2)$$

$$= \frac{q}{2 \epsilon} N_D x_n (x_n + x_p)$$

$$x_p = \frac{N_D}{N_A} x_n \quad \rightarrow \quad \frac{q N_D x_n^2}{2 \epsilon} \left( 1 + \frac{N_D}{N_A} \right) \\ = \infty V_{bi}$$

if  $\frac{N_A}{N_D} \gg 1 \Rightarrow \{ \text{Entire Depletion region lies in N side} \}$

$$\begin{cases} x_n = \sqrt{\frac{N_A}{N_A + N_D} V_{bi} \times 2 \epsilon} \\ x_p = \sqrt{\frac{N_D}{N_A + N_D} V_{bi} \times 2 \epsilon} \end{cases}$$

$$x_n + x_p = \sqrt{\frac{V_{bi} \times 2 \epsilon}{q}} \times \left( \frac{1}{N_A + N_D} \right)^{1/2} \times \left( \frac{N_A + N_D}{N_A N_D} \right)^{1/2}$$

Current Expression

From last class  $\rightarrow$  estimate  $I_0$

$$I = (I_0) \left[ (e^{qV/kT} - 1) \right]$$

$$I = I_n + I_p$$

$$I_n = \frac{q n_i^2}{T_n N_A} \sqrt{D_n T_n}$$

$$I_p = \frac{q n_i^2}{T_p N_D} \sqrt{D_p T_p}$$

$$I_0 / I = I / I_n + I_p$$

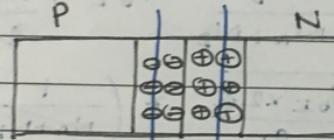
$$J = q \left( \frac{n_i^2}{N_D N_N} \sqrt{D_N T_N + h_i^2} \frac{\sqrt{P_P D_P}}{N_D T_P} \right)$$

$$(e^{qV/kT} - 1)$$

$$= J_0 ( \dots )$$

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→ P-N Junction as Capacitor.

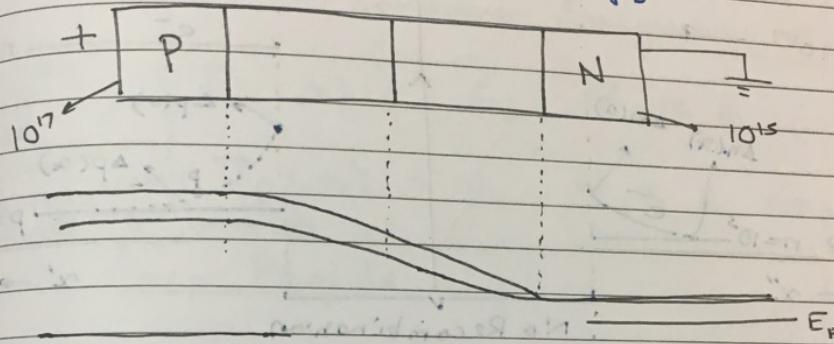


$\frac{-x_p}{2} \quad \frac{x_n}{2}$  → Assume Charge  
concentration at Mid-plane.

# Current through Diode

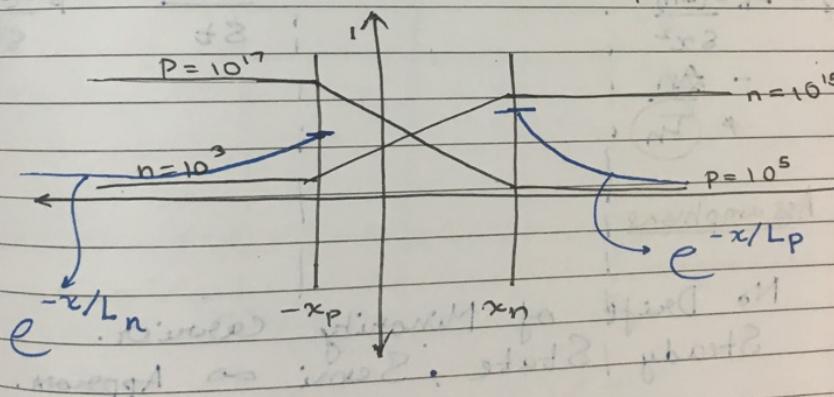
At Eq.

$$J_{n\text{diffusion}} + J_{n\text{drift}} = 0$$



$$J_p = p n \nabla E_{Fp}$$

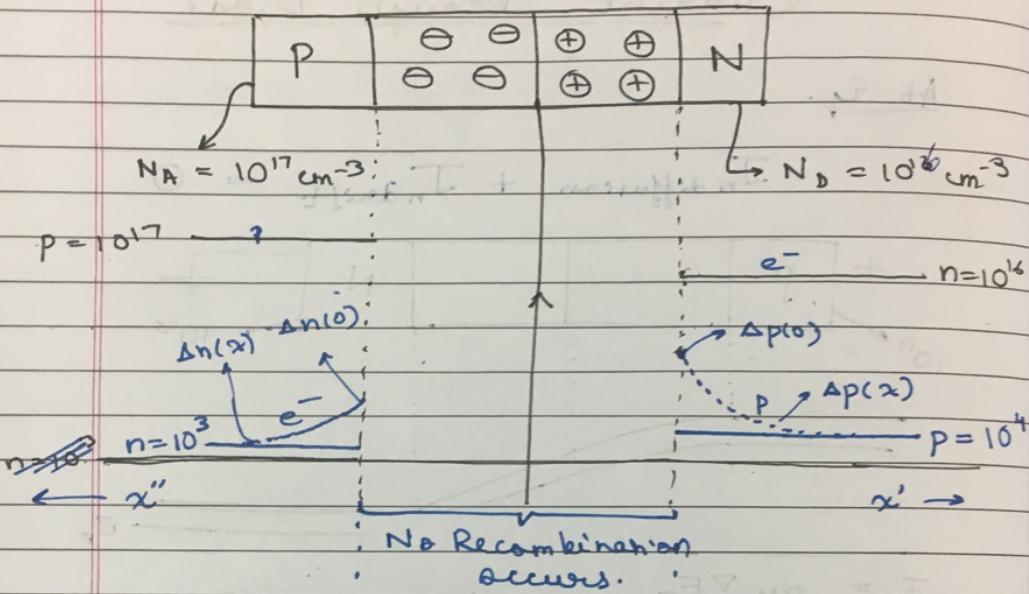
$$p \uparrow \Rightarrow \nabla E_{Fp} \downarrow$$



- e<sup>-</sup> may travel all the way to the contact or recombine in blue, in both cases it contributes to the current.

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$$\frac{\delta(\Delta n)}{st} = D_n \frac{\delta^2(\Delta n)}{\delta x^2}$$

$$-\frac{\Delta n}{L_n}$$

$$\frac{\delta(\Delta p)}{st} = D_p \frac{\delta^2(\Delta p)}{\delta x^2}$$

Assumptions

- No Drift of Minority carrier.
- Steady State; Semi  $\infty$  Approx.

$$\Delta n(x) = \Delta n(0) e^{-x''/L_n} \quad \Delta p(x) = \Delta p(0) e^{-x'/L_p}$$

Diffusion lengths.

Current Determination.

$$J = J_n(x) + J_p(x)$$

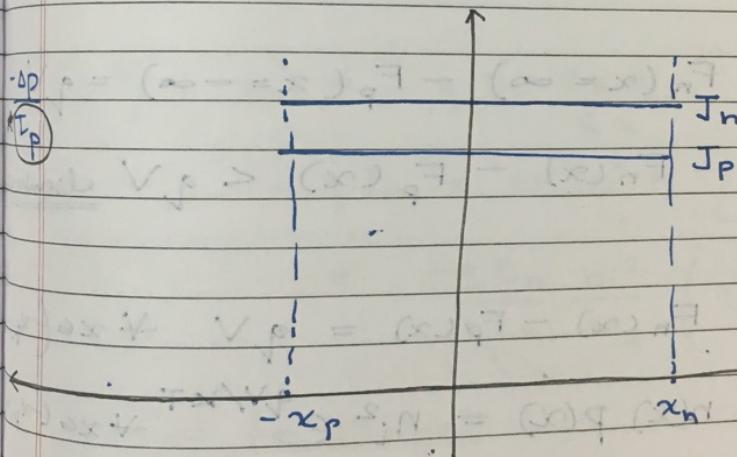
← vary ←

same / constant irrespective of  $x$

\*\*  $\begin{cases} J_n(x) = c_1 & \forall x \in (-x_p, x_n) \\ J_p(x) = c_2 & \forall x \in (x_p, x_n) \end{cases}$

No Accumulator,  
ent. resevoir  
in dep. region covered.

J<sub>n/p</sub> Profile

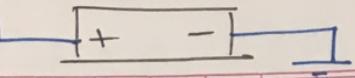


Using continuity of  $J_n / J_p$

$$J = J_p(x_n) + J_n(-x_p)$$

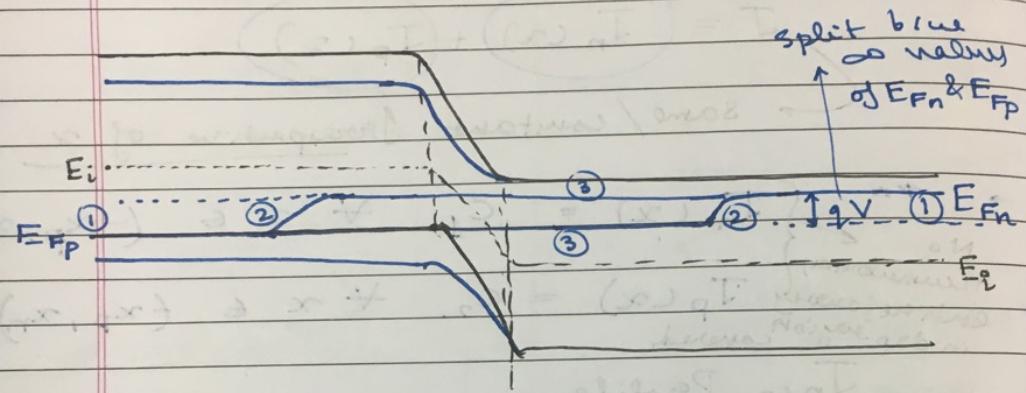
writing constant  $J$

Viewas



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## → Band Diagram in the presence of Bias



### Notes

$$F_n(x = \infty) - F_p(x = -\infty) = qV$$

$$\therefore F_n(x) - F_p(x) < qV \text{ elsewhere}$$

### Assume

$$F_n(x) - F_p(x) = qV \quad \forall x \in (x_p, x_n)$$

$$\Rightarrow n(x) p(x) = n_i^2 e^{qV/kT} \quad \forall x \in (x_p, x_n)$$

$$\text{At, } x = x_p, \quad p = N_A, \quad n = \frac{n_i^2}{N_A} e^{qV/kT}$$

$$(qV -)kT + (qV)kT = T \quad \therefore N_A$$

$$\text{At, } x = x_n, \quad n = N_D, \quad p = \frac{n_i^2}{N_D} e^{qV/kT}$$

1)

2)

3)

## Current Expression

$$J_n(-x_p) = -q D_n \frac{\delta(\Delta n)}{\delta x} \Big|_{x=-x_p}$$

$$\Delta n(-x_p) = n(x_p) - n_0(-x_p)$$

$$= \frac{n_i^2}{N_A} e^{qV/kT} - \frac{n_i^2}{N_A}$$

$$= \frac{n_i^2}{N_A} (e^{qV/kT} - 1) = \Delta n(0)$$

$$J_n(-x_p) = q D_n \frac{\delta(\Delta n)}{\delta x}$$

$$= q D_n \times \Delta n(0) \times -\frac{1}{L_n}$$

$$= -q D_n \frac{n_i^2}{L_n N_A} (e^{qV/kT} - 1)$$

Food for thought:

1) Connect Diode Across register, current?

2) Can we measure  $V_{BE}$  with a Voltmeter.

3) Reconcile the 2nd Derivation of current expression with 1st OTB derivation.

# Ideal Diode Vs. Real Diode

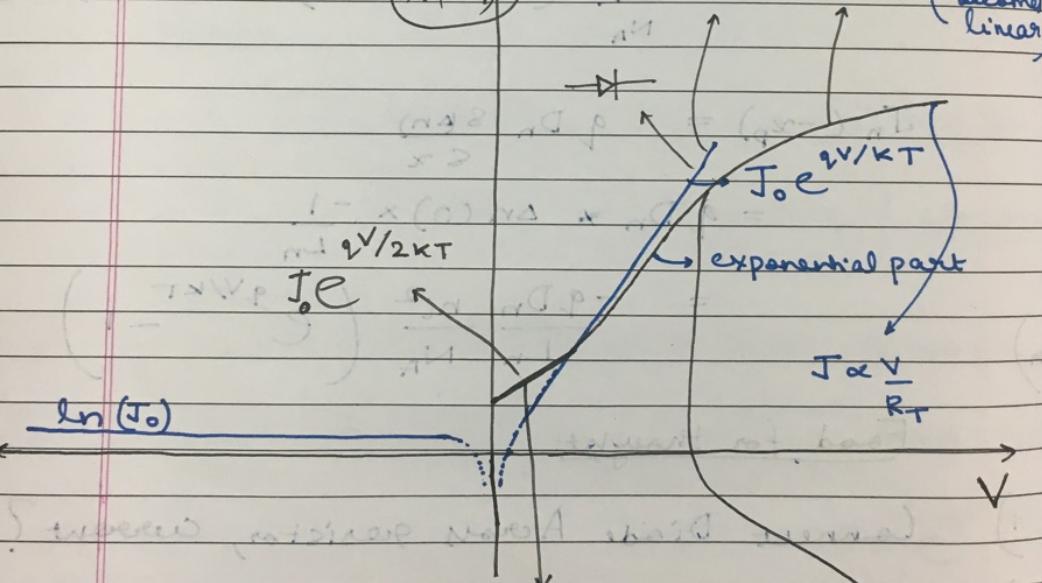
New Equations ( $W = ( )$ ,  $V_{bi} = ( )$ )

$$J = J_0 (e^{V/kT} - 1)$$

$$J_0 = q \left[ \frac{n_i^2}{N_A} \frac{D_n}{L_n} + \frac{n_i^2}{N_D} \frac{D_p}{L_p} \right]$$

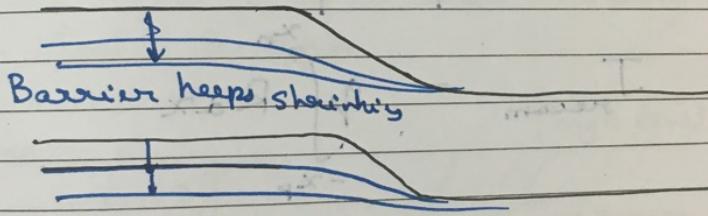
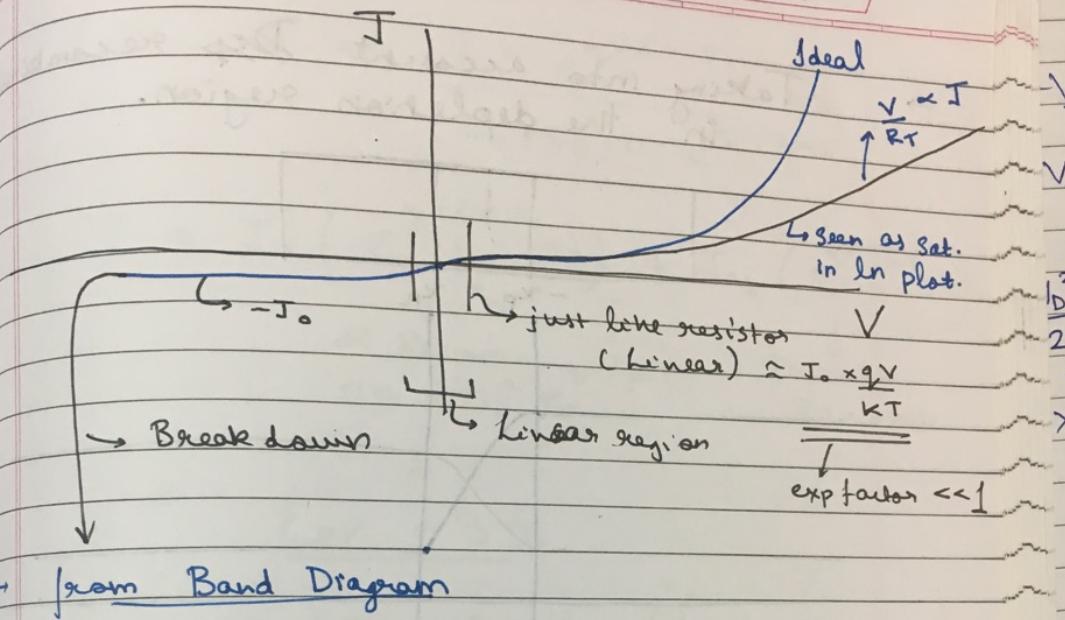
$\hookrightarrow e^- \rightarrow P_{reg.}$        $\hookrightarrow p \rightarrow N_{reg.}$

Consider  $(1 - \ln(1/J))$  for Ideal. Real (Since becomes linear)

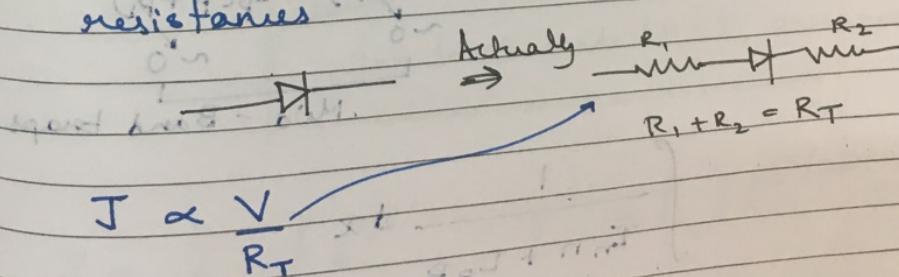


Due to the Recombination

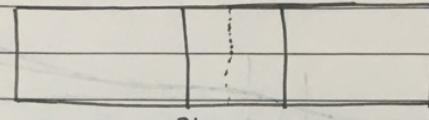
Reasons for non-ideal behavior:  
 1. If High level  
 2. If low temperature  
 3. If high voltage  
 4. If high current  
 5. If high frequency  
 6. If high speed  
 7. If high power



- As barrier keeps shrinking, all  $e^-$  can overcome the barrier and current is determined by resistances



→ Taking into account Deep recombination in the depletion region.



$-x_p \quad x_n$



negative band model.

$$J_{\text{recom}} = q \int_{-x_p}^{x_n} R dx$$

$$J_{\text{recom}} = q \int_{-\infty}^{\infty} \frac{n_i^2 e^{qV/kT} + n_i^2 e^{-qV/kT}}{E_n(n+m) + E_p(p+P)} dx$$

~0                            ~0  
Mid-Band traps.

$$\int \frac{1}{E_n(n+m) + E_p(p+P)} dx \quad V \propto T$$

$$n = p, \quad np = n_i^2 e^{qV/kT}$$

$$n = n_i e^{qV/2kT}$$

$$J = \frac{n_i^2 e^{qV/kT}}{(E_n + E_p) n_i e^{qV/2kT}}$$

$$J = J_0 (e^{qV/kT} - 1) + J_{02} (e^{qV/2kT} - 1)$$

$\downarrow$

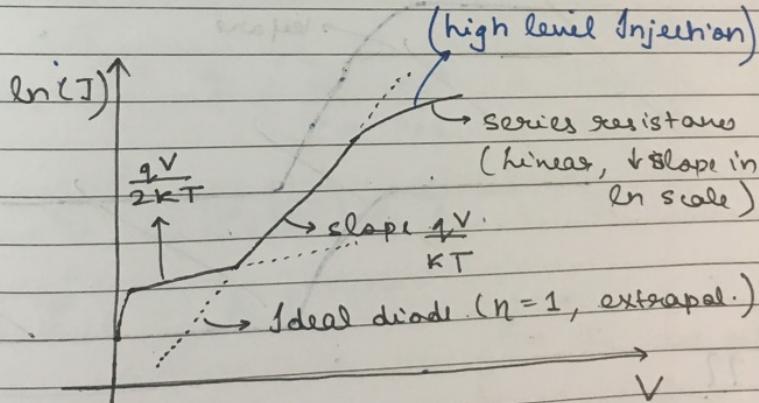
$\propto$  Recomb. Due to Recomb.

$n_p = n_i^2 e^{qV/kT}$

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Quiz: Thu 8:30 - 8:45 am (during lecture)  
 Tut: 3 - 4 pm

Recap



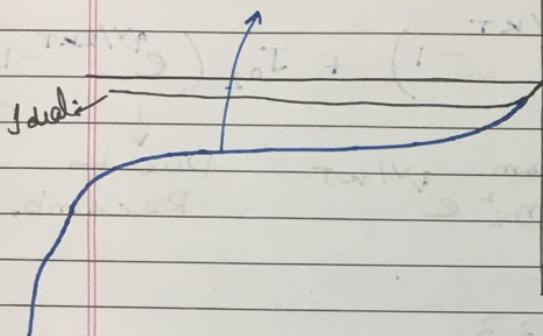
$$J = J_{01} (e^{qV/kT} - 1) + J_{02} (e^{qV/2kT} - 1)$$

$V_{on}$  / Cut-in voltage has no connection to  $V_{be}$ . Since we work in mA regime, we define  $V_{on}$  to voltage where current crosses 1mA. If we worked at μA scale value would be different.

# Revere Bias

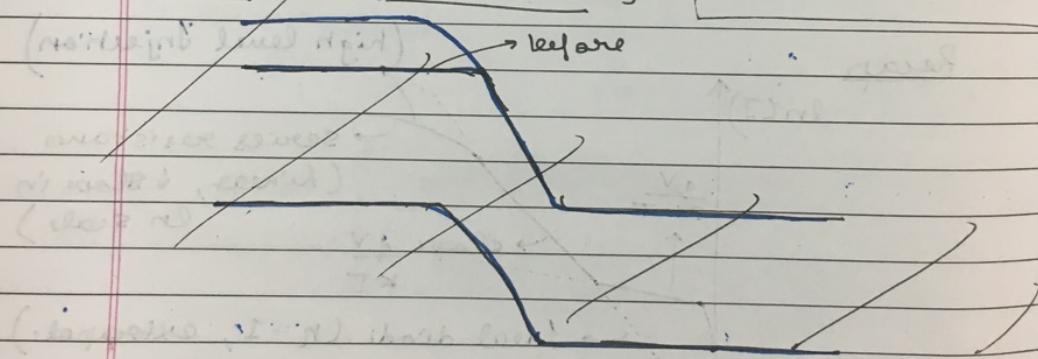
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Real, ( $J = g \times W$ )



$$W = \frac{2e}{q} \left( \frac{1}{N_A} + \frac{1}{N_D} \right) [V_{bi} - V_{applied}]^{\frac{1}{2}}$$

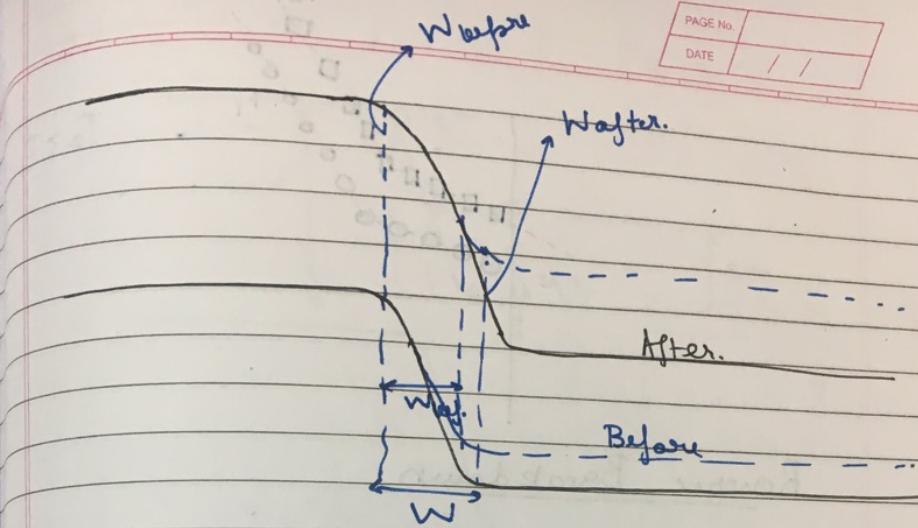
Application of Revere bias



??

$$(1 - \frac{J}{J_0}) \cdot J + (1 - \frac{J}{J_0}) \cdot J_0 = I$$

revere bias and voltage across it back  
voltage across it is same as with  $\pm 2V$  of  
reverse voltage applied at now reverse bias  
where  $A_{eff}$  is between  $10^{-12}$  to  $10^{-10} \text{ A/V}$



$$W \propto V^{1/2}_{APP}$$

Band Gap from temp. dependence

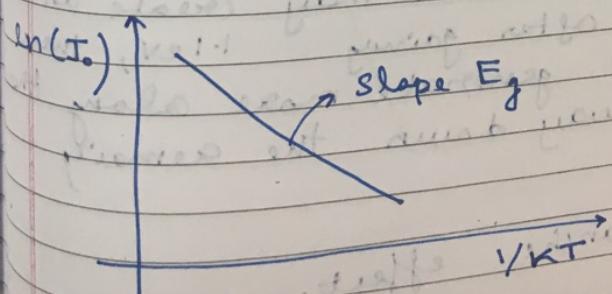
$$J_{o_1} (e^{qV/kT} - 1) + J_{o_2} (e^{qV/2kT} - 1)$$

$$2n_i^2 J_{o_1} = q \left( \frac{D_n}{L_n} \frac{n_i^2}{N_A} + \frac{D_p}{L_p} \frac{n_i^2}{N_D} \right)$$

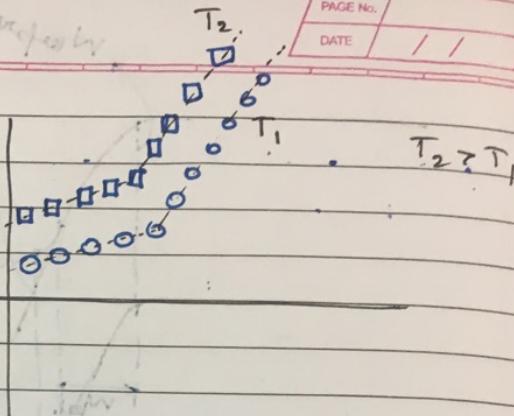
$$\text{Using } n_i^2 = (N_c N_V) e^{-E_g/kT}$$

$$J_{o_1} = A e^{-E_g/kT}$$

For  $J_{o_2}$  we get



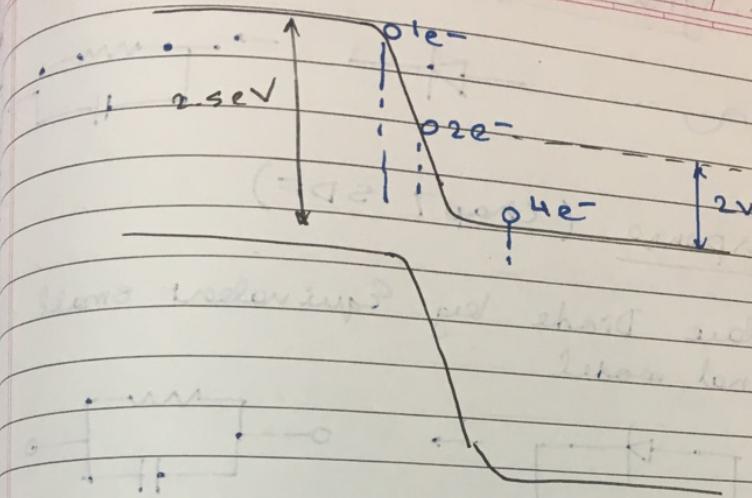
$E_g$  slope since  
it has  $n_i$  dependence  
not  $n_i^2$  depen.



## Reverse Breakdown

### 1) Avalanche Mechanism

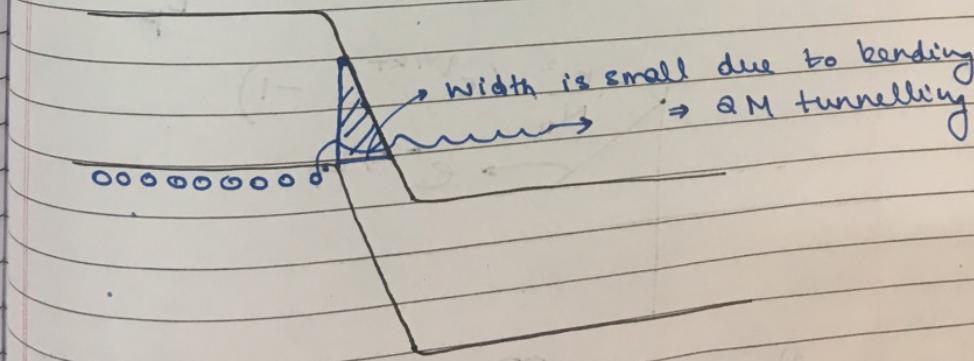
- $E_g \sim 1.1\text{ eV}$
- An  $e^-$  that goes down  $V_{bi}$  gains  $K.E. = eV_{bi}$
- This energy is less than  $F.B./E_g$  in F-B/ no way.
- But with sufficient R.B., if  $K.E. > E_g$  & it can create  $e^-$ -hole pairs
- Mid - While going down the  $V_{bi}$  say of 25V, it may create an  $e^-$  hole pair after gaining 1.1 eV, then will in turn generate more along the way down the remaining  $V_{bi}$
- ↳ Avalanche effect.



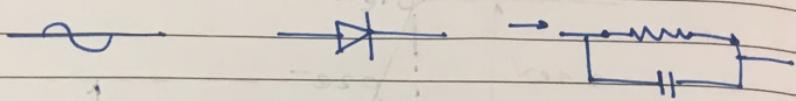
But in regular doped material this effect will be gradual & not at a fixed voltage.

### Sudden Drop (Sharp breakdown)

Heavily doped  $p^{++} \rightarrow \sim 10^{20} \text{ cm}^{-3}$   
 $n^{++} \rightarrow \sim 10^{20} \text{ cm}^{-3}$

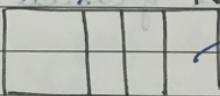
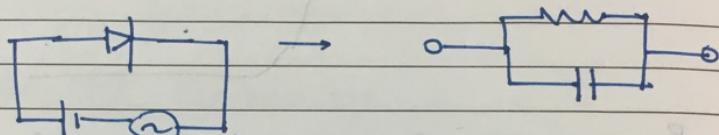


# Signal Applied to Diode



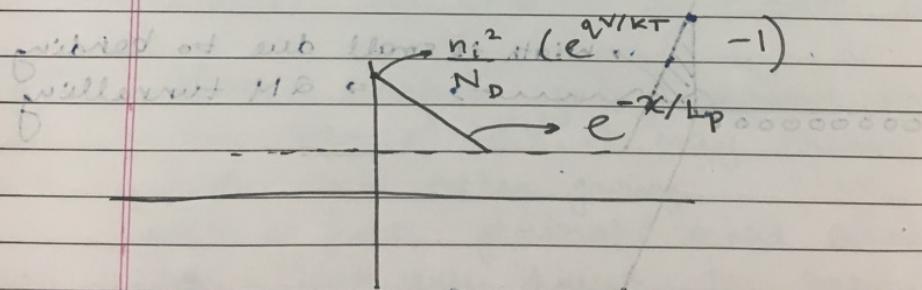
AC Response (Chap 7 SDF)

- Replace Diode by equivalent small signal model



- Continuity Eqn.

$$\frac{\delta \Delta P}{\delta t} = -\frac{1}{q} \nabla \cdot J_P - \frac{\Delta P}{I_P}$$



$$Q_P = q \int_{x_n}^{\infty} \Delta P(x) dx = ?$$

Charge due to excess Minority carriers.

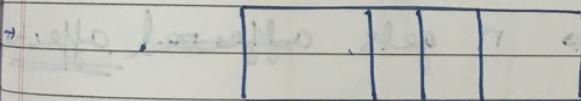
(Multiply the eqn by  $q$ )

$$q \int_{x_n}^{\infty} \frac{S(\Delta p)}{\delta t} dx = \int_{x_n}^{\infty} -\nabla \cdot J_p dx - \int_{x_n}^{\infty} \frac{q \Delta p}{\tau_p} dx$$

$$\Rightarrow \frac{Q_p}{\tau_p} = q \int_{x_n}^{\infty} \Delta p(x) dx =$$

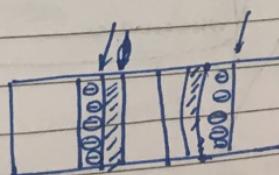
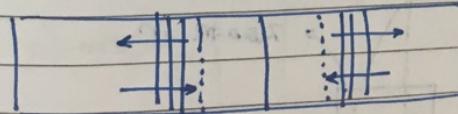
$$\frac{n_i^2}{N_D} \frac{L_D}{\tau_p} \left( e^{V/NKT} - 1 \right)$$

~~$$\frac{Q_p}{\tau_p} = \frac{\delta Q_p}{\delta t} = -\frac{Q_p}{\tau_p} = -\int_{x_n}^{\infty} (\nabla \cdot J_p) dx$$~~



Small Signal

When we apply small signal, depletion region width gets modulated.



$$C = \frac{\epsilon}{W}$$

→ We get Voltage dependent Capacitor

## → High level Injection

$$np = n_i^2 e^{qV_a/kT}$$

$$p \approx \frac{n_i^2}{N_D} e^{qV_a/kT}$$

usually valid

But at high-level injection

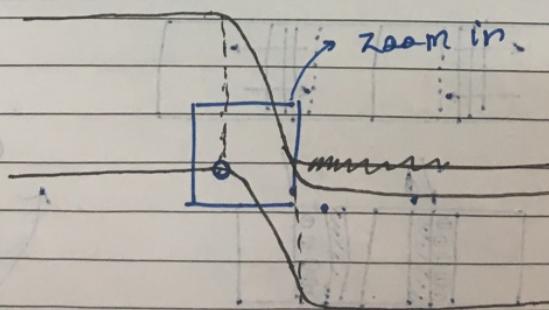
$$\bullet p - n + N_D^+ = 0$$

$$\text{if } p >> N_D^+$$

⇒ n gets affected

## → Zener Breakdown

→ Quantum tunnelling breakdown  
at elution step without resistance

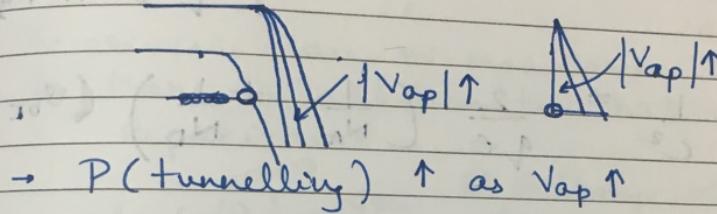


advantage: tunable without top gate

Slope  $\propto |\vec{E}|$

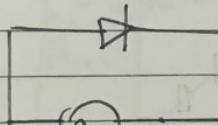
$$|\vec{E}| \propto -\alpha \frac{\sqrt{V_{ap}}}{W} \approx \sqrt{V_{ap}}^{1/2}$$

$\Rightarrow |\vec{E}| \uparrow \Rightarrow$  becomes steeper



$\rightarrow P(\text{tunnelling}) \uparrow$  as  $V_{ap} \uparrow$

### A.C Response (Rev. Bias)



$$f = p-n + N_D - N_A$$

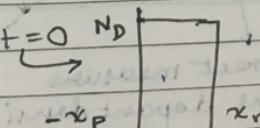
$$V_{app} = V_{DC} + v$$

$$v = 10mV \sin(\omega t)$$

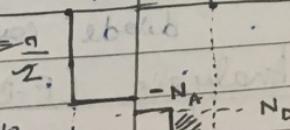
Plot  $f$

in DA

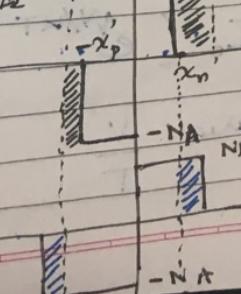
$$1) \omega t = 0$$



$$2) \omega t = \frac{\pi}{2}$$



$$3) \omega t = -\frac{\pi}{2}$$



$$\omega t = \pi/2$$

$$\omega t = -\pi/2$$

$$C = \frac{\epsilon}{W}$$

capacitance per  $\text{cm}^{-2}$

may be used as a voltage dependent capacitance

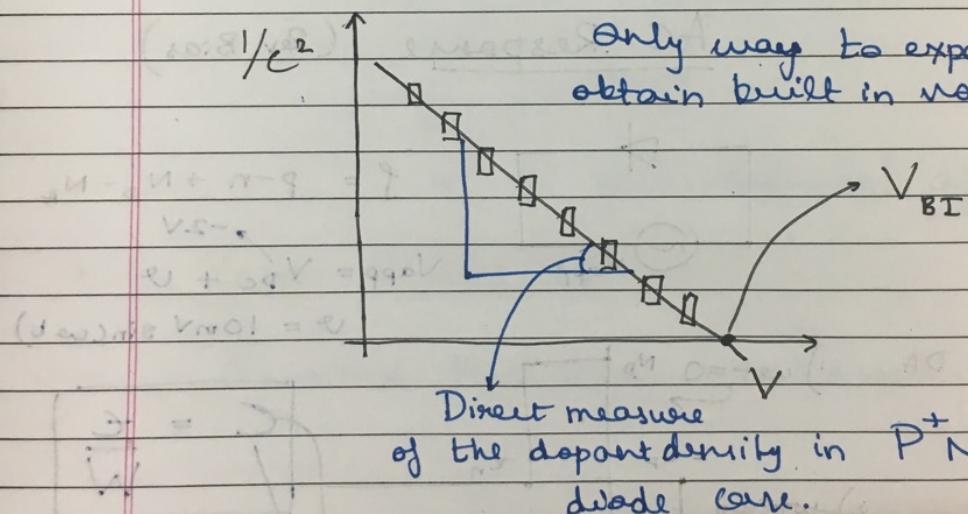
→ Built in voltage from C per cm<sup>-2</sup>

$$C = \frac{\epsilon}{W}$$

$$C = \frac{\epsilon}{W} \times \left[ \frac{2\epsilon}{q} \left( \frac{1}{N_A} + \frac{1}{N_D} \right) (V_{bi} - V_{app}) \right]$$

$$\frac{1}{C^2} = \frac{2}{q\epsilon} \left( \frac{1}{N_A} + \frac{1}{N_D} \right) (V_{bi} - V_{app})$$

$\frac{1}{C^2}$  vs  $V_{bi}$   
Only way to experimentally obtain built in voltage.



AC Analysis (F.B)

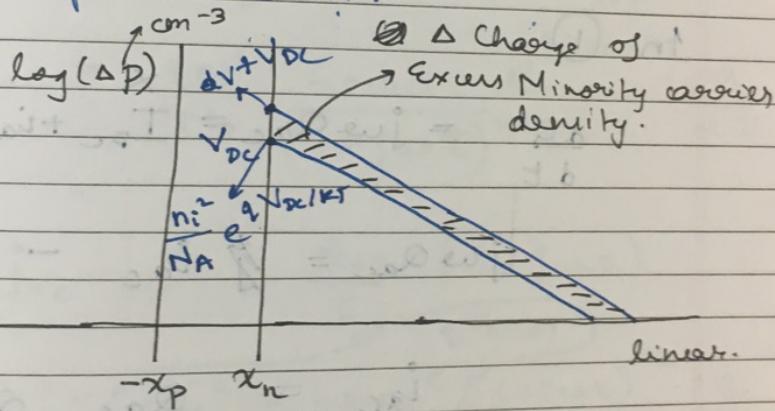
$$I = I_0 (e^{qV/kT} - 1)$$

DC resistance  $\frac{V}{I}$

$$AC \text{ Resist.} = \left( \frac{dI}{dV} \right)^{-1}$$

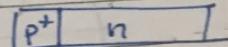
$$\frac{dI}{dV} = \frac{I_0 e^{\frac{qV}{kT}}}{\left( \frac{kT}{q} \right)} = \frac{I_{DC}}{\left( \frac{kT}{q} \right)}$$

The previous case w modulation cap. will be there, in addition to that another capacitance ....



### Derivation

Consider a  $p^+n$  junction



Charge control Eqn.

$$\frac{dQ}{dt} = i_{diff} - \frac{\alpha}{T_p} \quad (1)$$

$$Y = \frac{i_{ac}}{j\omega} \rightarrow \text{our Goal!}$$

$$j\omega Q_{ac} \rightarrow I_{dc} + i_{ac}$$

$$i = I_{dc} + i_{ac} \quad (\text{Actually } T \text{ has factor of c.s.A})$$

$$Q = Q_{dc} + Q_{ac}$$

$$\frac{dQ}{dt} = 0 + \frac{dQ_{ac}}{dt}$$

$$j\omega Q_{ac} = j\omega Q_{ac}$$

In ①

$$\frac{dQ}{dt} = j\omega Q_{ac} = I_{dc} + i_{ac} - \frac{Q_{dc} + Q_{ac}}{\tau_p}$$

$$j\omega Q_{ac} = i_{ac} - \frac{Q_{ac}}{\tau_p}$$

$$i_{ac} = \frac{Q_{ac}}{\tau_p} \quad \cancel{Q_{ac}} \left( \frac{1}{\tau_p} + j\omega \right)$$

$$i_{ac} = \frac{Q_{ac}}{\left( \frac{\tau_p}{1 + j\omega \tau_p} \right)}$$

$$\left( \frac{\tau_p}{1 + j\omega \tau_p} \right)$$

$$I_{dc} = Q_{dc} / \tau_p$$

$$i_{ac} = Q_{ac} / \left( \frac{\tau_p}{1 + j\omega \tau_p} \right)$$

$$I_{ac} = \frac{q n_i^2}{N_A} e^{qV_{DC}/kT} \left( e^{qV_{ac}/kT} - 1 \right) e^{-z/l_p}$$

$$Q_{ac} = \frac{q n_i^2}{N_A} e^{qV_{DC}/kT} \left( e^{qV_{ac}/kT} - 1 \right) L_p$$

↑ Area missing (~~Area~~ cm<sup>-2</sup> (charge density)  
like σ)

$$Q_{ac} = \frac{q n_i^2}{N_D} L_p e^{qV_{DC}/kT} \cdot \frac{qV_{ac}}{kT}$$

$$Q_{ac} = \frac{q n_i^2}{N_D} \cdot \frac{L_{p(DC)}}{(1+j\omega\tau)^{1/2}} e^{qV_{DC}/kT} \cdot \frac{qV_{ac}}{kT}$$

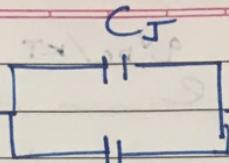
$$j_{ac} = \frac{Q_{ac}}{T} (1 + j\omega\tau)$$

$$= \frac{q n_i^2 \times \frac{L_{p(DC)}}{T} e^{qV_{DC}/kT}}{(1 + j\omega\tau)^{1/2}} \cdot \frac{qV_{ac}}{kT}$$

$$j_{ac} = I_{DC} (1 + j\omega\tau) \frac{qV_{ac}}{kT}^{1/2}$$

$$Y = \frac{I_{ac}}{V_{ac}} \Rightarrow \sim \frac{j_{ac}}{V_{ac}} = \frac{I_{DC}}{\left(\frac{kT}{q}\right)} [1 + j\omega\tau]^{1/2}$$

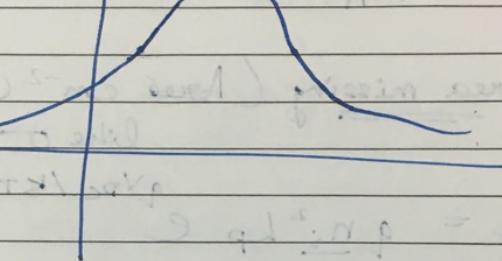
$$Y = g_0 (1 + j\omega\tau)^{1/2}$$



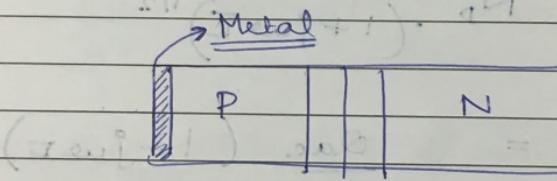
$C_J$

$C_D$

$C$



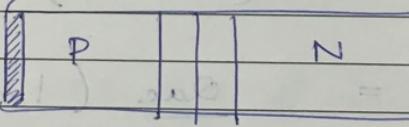
## Metal - Semiconductor Contact



Metal

P

N



Electron Affinity

$E_C$

$E_F$

Evacuum (No Inter)

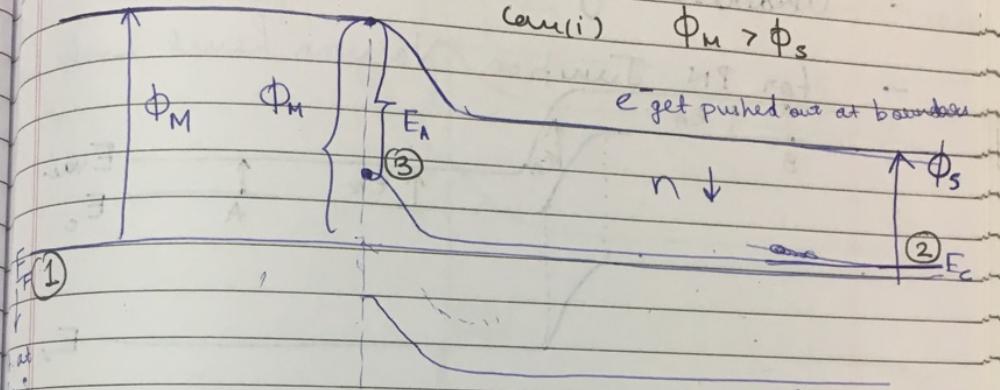
Work Function

Metal

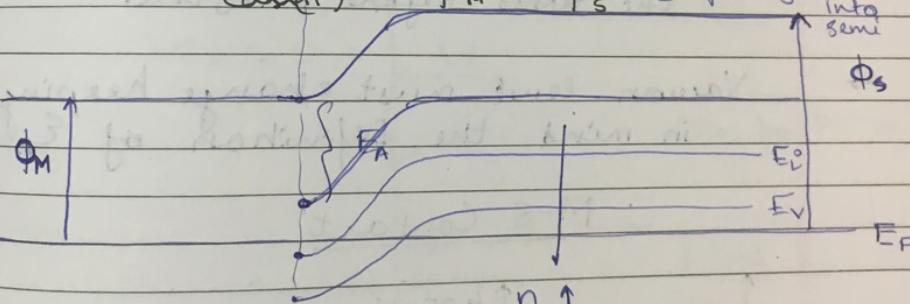
$\phi_m$

Evacuum

Combined band diagram at c.p. of metal - semi. junction.



(case ii)  $\Phi_M < \Phi_s$   $e^-$  get injected into semi



Lec - 34

Plan, 2 extra lectures

SAT - 14/10/17

SAT - 28/10/17

2 extra quizzes 8/12

Grading

Quiz - 20%

M.S - 25%

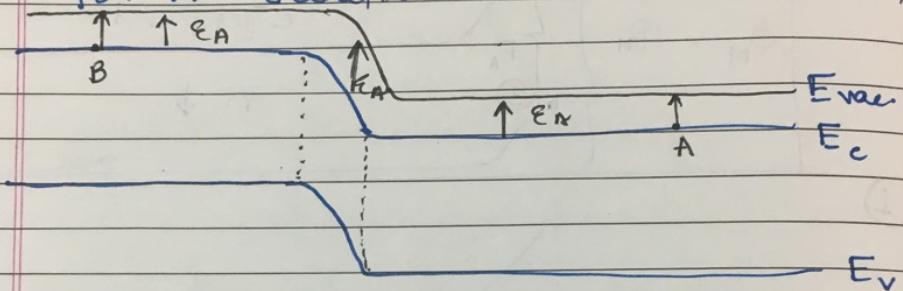
Assign. - 5%

Endsem - 50%

→ Metal - Semiconductor Contact

- Understanding Vacuum level.

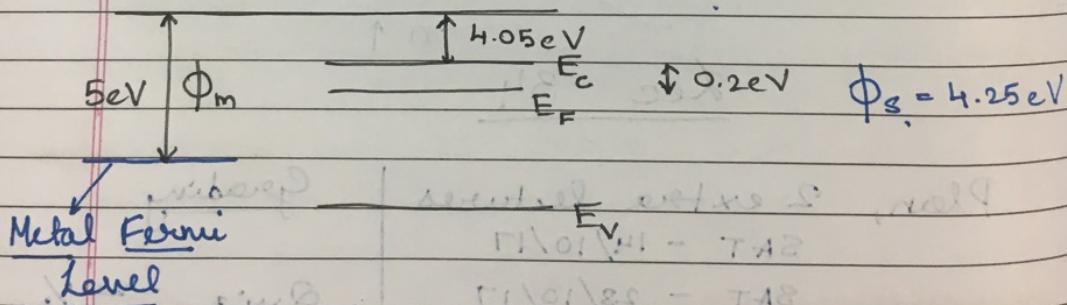
- for PN Junction Vacuum Level Looks like,



- Diff. in energy of  $e^-_A$  and  $e^-_B$  remains the same before & after.

- Vacuum level must change keeping in mind the Definition of  $E_A$ .

M-S Contact



Metal Fermi Level

$\chi_M = -2.11$

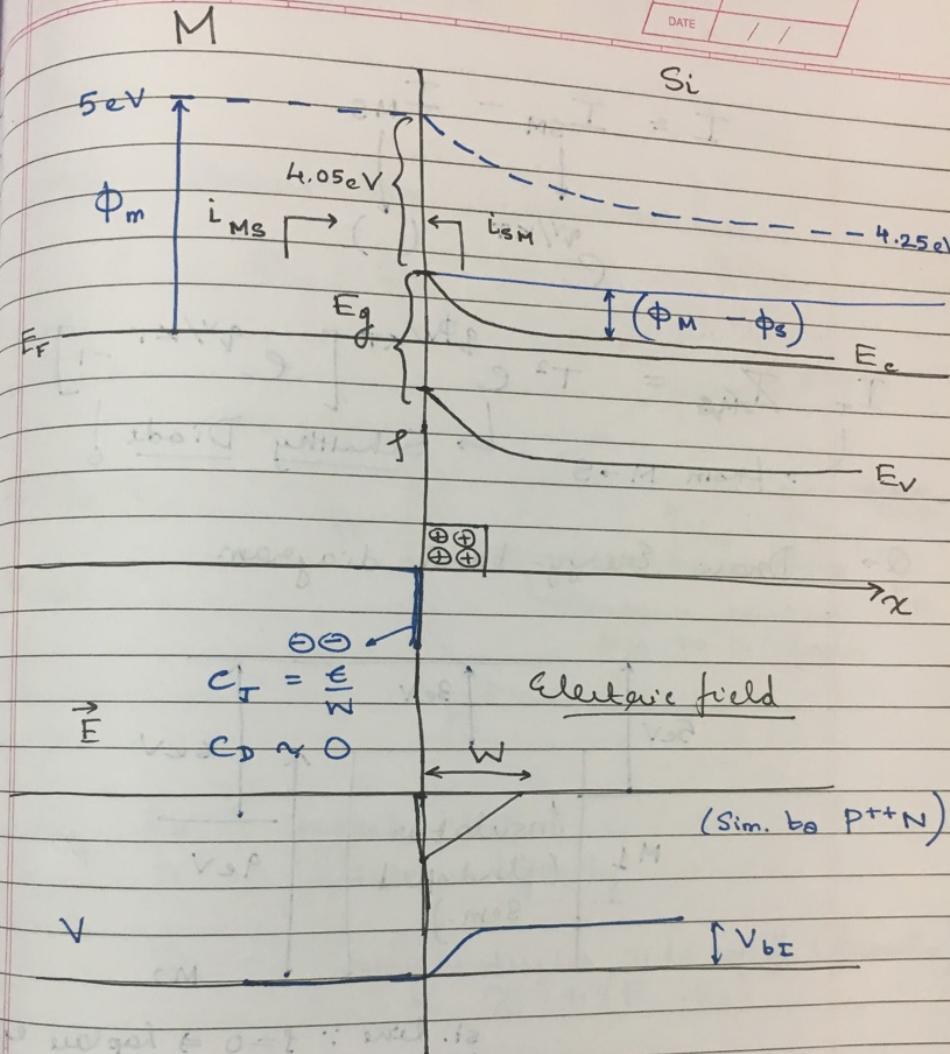
$\chi_S = \text{intrinsic}$

$\chi_{SO} = \text{metals}$

$\chi_{SO} = \text{insulating material}$

# Dep. Approx.

PAGE NO. / /  
DATE / /



→ Going from Metal to Sem. barrier is const.

→ Going from Sem. to Metal  
 -ve bias → Barrier ↑ → current ↓  
 w.r.t metal

the bias → Barrier ↑ → current ↓  
 w.r.t metal.

$$I = I_{SM} - I_{MS}$$

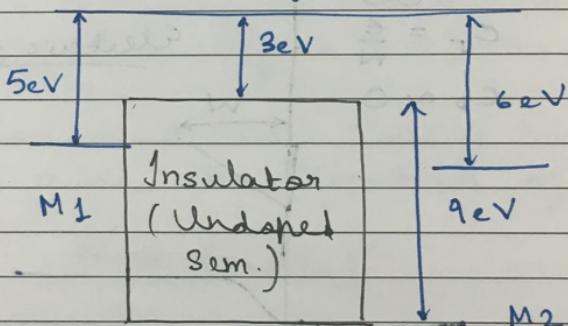
$$\downarrow \quad \downarrow$$

$$e^{\frac{qV}{kT}} \quad ( )$$

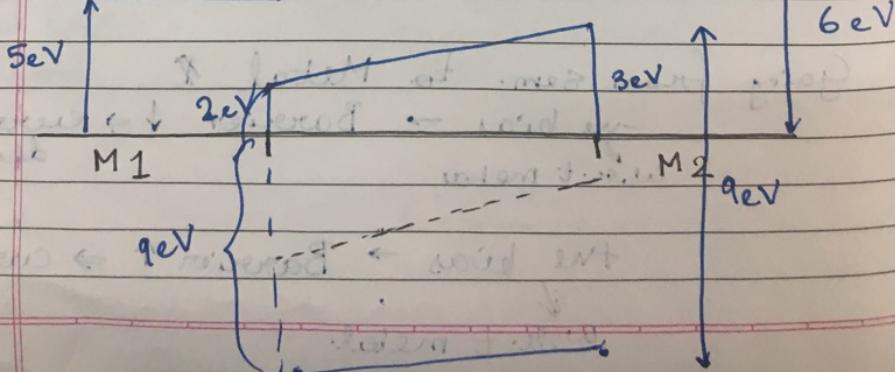
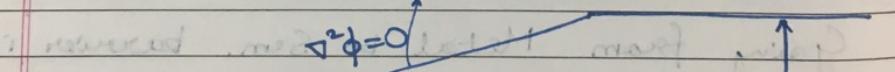
$$I_T = I_{MS} = T^2 e^{\frac{q\phi_V}{kT}} [e^{\frac{qV}{kT}} - 1]$$

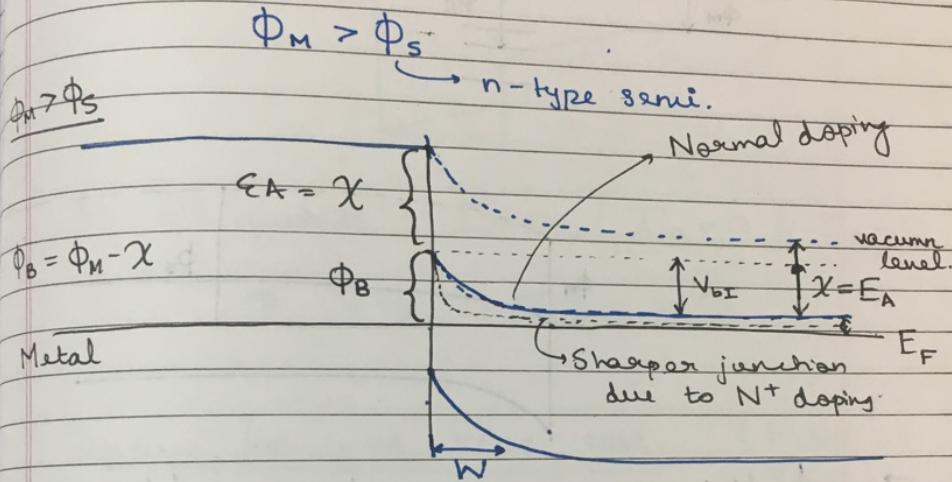
$\hookrightarrow$  from  $M \rightarrow S$        $\hookrightarrow$  Schottky Diode!

Q → Draw Energy-band diagram



st. law:  $\nabla^2\phi = 0 \Rightarrow$  Laplace eqn.



RecapMS - Metal - Semi. junction

$\Phi_M$  - metal W.F.  
 $\chi$  - Electron Affinity

$$\Phi_S = \text{Semiconductor side w.r.t (from } E_F) \\ = \chi + (E_c - E_F)$$

$$\Phi_B = \Phi_M - \chi$$

$$V_{bi} = \Phi_B - (E_c - E_F) = \Phi_M - \Phi_S$$

$$I = ( ) [e^{qV/kT} - 1]$$

Diff exp

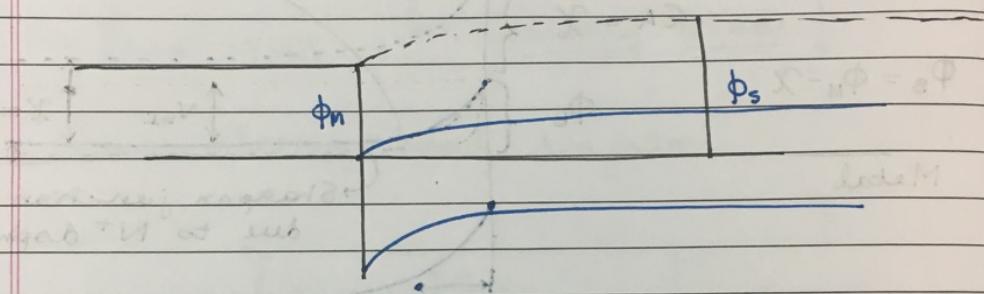
High Doping at junction, must start @ same pt. and go to a lower  $E_c$  ( $E_c - E_F \downarrow$ )

Case 2:  $\phi_m < \phi_s$

↳ N-type.

$$\phi_m$$

$$\phi_s$$



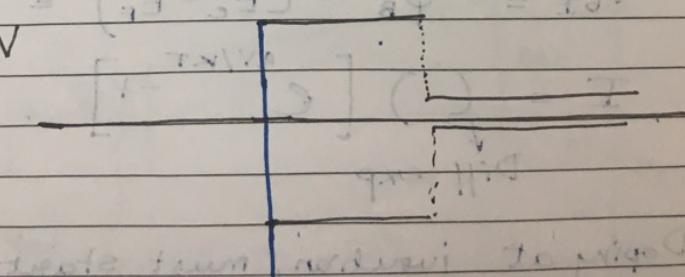
- In this case no rectification / diode action
- This is the desired behaviour but cannot be assured  $\Rightarrow$  high junction doping.

MOS Junction

$$\rightarrow \phi_m = \phi_s$$

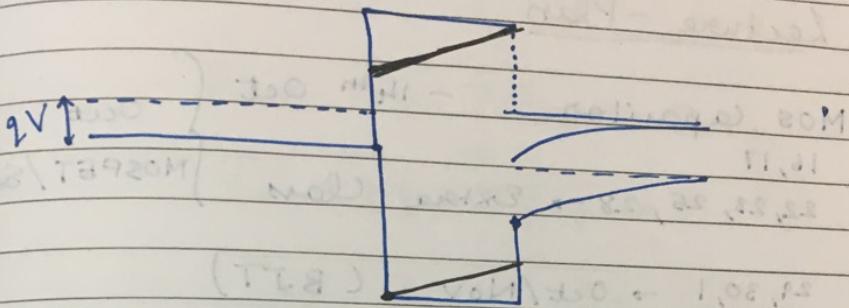
$$\phi_m - n\phi = (\beta - \beta) - n\phi = \tau V$$

$$V$$



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Part 1: +ve Voltage



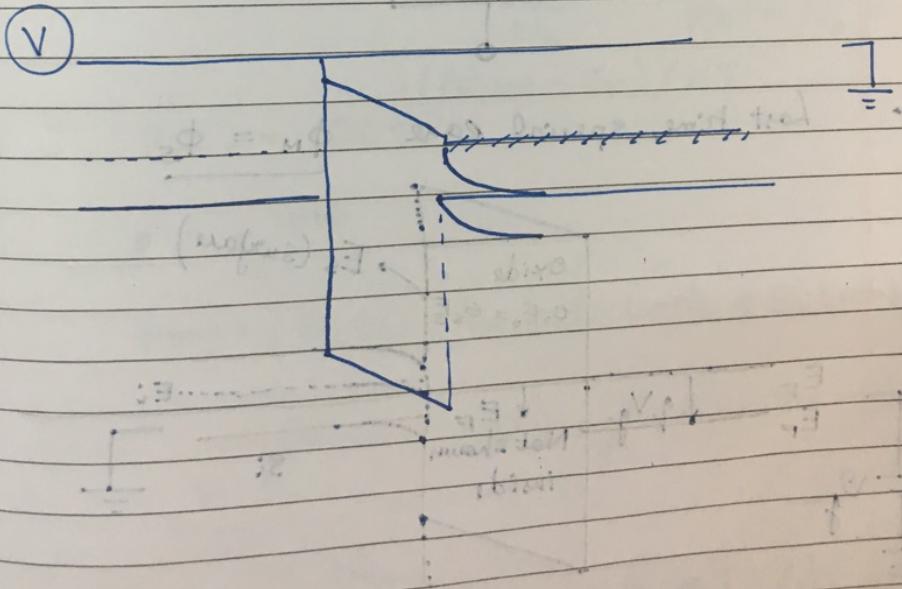
→ At all points  $I = 0$ ,  $I_n = 0$  &  $I_p = 0$

$$\Rightarrow \nabla E_{Fn} = \nabla E_{Fp} = 0$$

→ Draw for -ve bias case. (Part 2)  
( $V < 0$ )

$V$

$\frac{I}{J}$

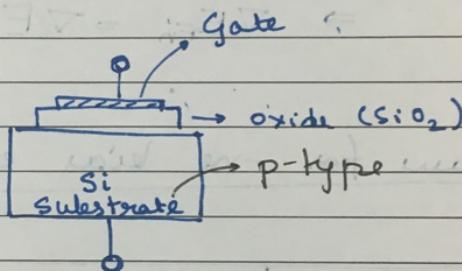


# Lec - 36

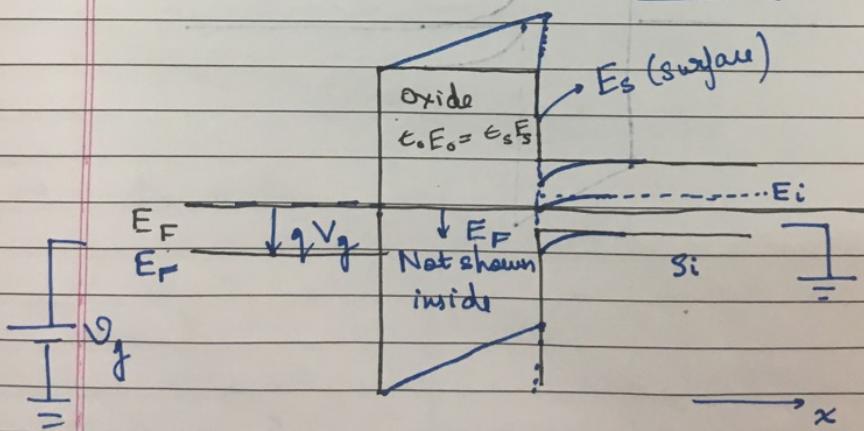
## Lecture - Plan

Mos capacitor - 14<sup>th</sup> Oct  
 16, 17  
 22, 23, 25, 28 → Extra Class      } Out  
 MOSFET / Saber / LSI  
 29, 30, 1 → Oct / Nov . (BJT)

## MOS Capacitor



→ Last time, special case  $\phi_M = \phi_S$



$$E_i(\infty) - E_i(x) = q\psi(x)$$

Current = 0

$$J_n = n \mu_n \nabla E_{Fn} = 0$$

$$J_p = p \mu_p \nabla E_{Fp} = 0$$

Analysis

$$\therefore E = -\nabla \psi$$

$$\nabla \cdot (\epsilon \nabla \psi) = -(p - n - N_A)$$

$$q\psi(x) = E_i(\infty) - E_i(x)$$

$$p(\infty) = N_A \quad n(x) = n_i^2 e^{q\psi/kT}$$

$$n(\infty) = \frac{n_i^2}{N_A} \quad p(x) = N_A e^{-q\psi/kT}$$

$$(E_i(x) - E_F)/kT$$

$$p(x) = n_i e^{(E_i(x) - E_F)/kT}$$

$$p = e^{(E_i(x) - E_F)/kT}$$

$$p_{ew} = (E_i(x) - E_F - E_i(\infty) + E_i(\infty))/kT$$

$$p(x) = N_A e^{(E_i(x) - E_i(\infty))} \quad n(x) = \frac{n_i^2}{N_A} e^{-q\psi/kT}$$

$$p(x) = N_A e^{-q\psi/kT}$$

$$n(x) = \frac{n_i^2}{N_A} e^{q\psi/kT}$$

Constant  
 $E_F$   
 $n_p = n_i^2$

$$e \frac{d^2 \psi}{dx^2} = -q \left( N_A e^{-q\psi/kT} - \frac{n_i^2}{N_A} e^{q\psi/kT} - N_A \right)$$

$$E = -\frac{d\psi}{dx}$$

$$\frac{dE}{dx} = \frac{dE}{d\psi} = \frac{dE}{dx} \times \frac{dx}{d\psi}$$

$$(11 - n - q) \Rightarrow -E \frac{dE}{d\psi} = \frac{dE}{dx}$$

$$\frac{d}{d\psi} \left( E \frac{dE}{d\psi} \right) = -q \left( N_A e^{-q\psi/kT} - \frac{n_i^2}{N_A} e^{q\psi/kT} - N_A \right)$$

$$\int_E^0 E dE = -\frac{q}{kT} \int_{-\infty}^{\psi} \left( N_A e^{-q\psi/kT} - \frac{n_i^2}{N_A} e^{q\psi/kT} - N_A \right) d\psi$$

$$\frac{E^2}{2} \neq -\frac{q}{kT} \left[ \left( \frac{N_A}{e} \right) e^{-q\psi/kT} - \frac{n_i^2}{N_A} e^{q\psi/kT} \right]$$

(cares on  $\psi$ )

1) Accumulation phase:

Apply large negative bias.  $\sim 0$

no but consider

$$\frac{E^2}{2} \neq N_A \left( e^{-q\psi/kT} - \frac{n_i^2}{N_A} e^{q\psi/kT} - N_A \right)$$

$$\frac{E_s^2}{2} = \frac{\epsilon}{\epsilon_0} \left( \left( \frac{kT}{q} \right) N_A e^{-q\psi_s/kT} \right)$$

$$E_s = \sqrt{\frac{2N_A kT}{\epsilon_0}} e^{-q\psi_s/kT}$$

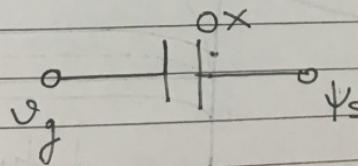
Using Gauss Law

$$E_s \cdot A = f \cdot A \times \frac{q_{\text{enclosed}}}{\epsilon_0}$$

$$E_s = \frac{Q_s}{\epsilon_0} \text{ cm}^{-2} \text{ (per unit area)}$$

$$Q_s = \sqrt{2N_A \epsilon_0 \left( \frac{kT}{q} \right) \times q \cdot e^{-q\psi_s/kT}}$$

→ Equivalent Capacitor

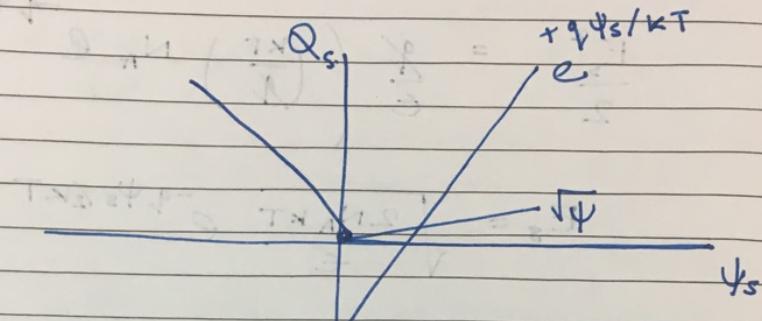


Approx: No band bending

$$q \times 10^{14} \text{ cm}^{-2}$$

$\text{SiO}_2$

If we take entire drop across  $\text{SiO}_2$ , then  $\approx 10V$  at left.

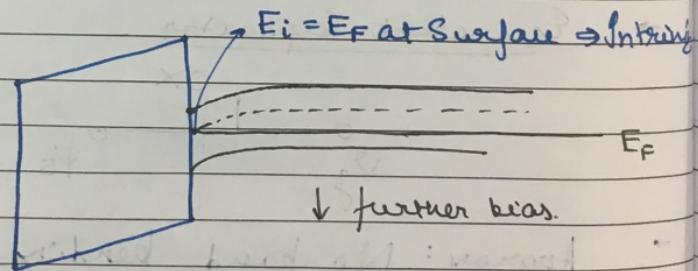


2) Depletion (Positive bias) (Low bias)

$$E^2 = \frac{-q}{\epsilon} \left( \frac{kT}{q} \frac{n_i^2}{N_A} e^{q\psi/kT} + N_A \psi \right)$$

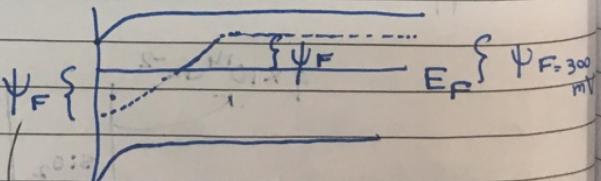
3) Inversion

Beyond a certain bias voltage, band diagram becomes



→ ~~surface Gradien~~ ~~at 0.2 mV~~  $\psi_F \{$   $300 \text{ mV}$

~~if it is V or not~~



At this minority density  
at Surface  $\rightarrow$  Majority  
density in bulk.

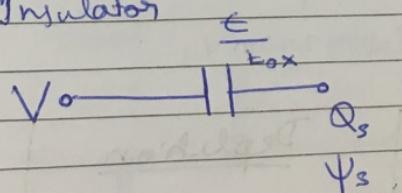
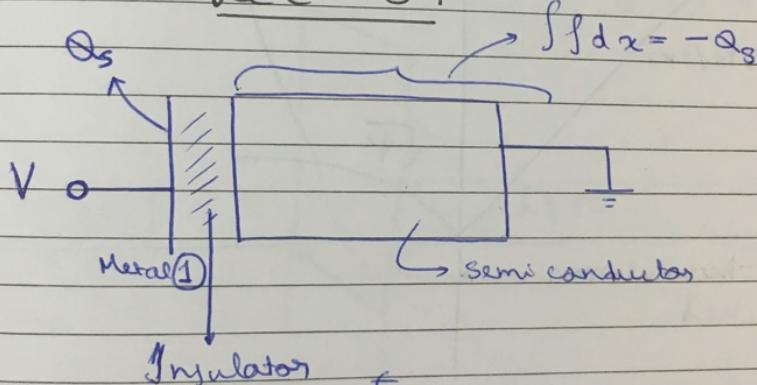
$$\frac{n_i^2}{N_A} e^{q\psi_F/kT} = N_A$$

$$\frac{n_i^2}{N_A} e^{-q(2\psi_F)/kT} = N_A$$

$$\ln\left(\frac{N_A^2}{n_i^2}\right) = \left(\frac{2q\psi_F}{kT}\right)$$

put  $\psi_s = 2\psi_F + 8$  & check variation in R.H.S

Lec - 37



$$e \frac{d^2 \psi}{dx^2} = -(p - n - N_A)$$

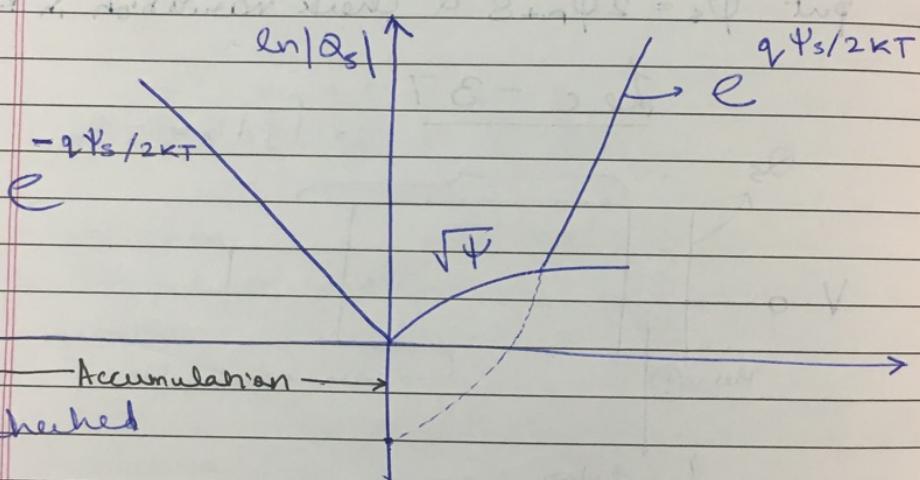
$$\frac{E^2}{2} = -\int_0^{\psi_s} \left( N_A e^{-q\psi/kT} - \frac{n_i^2}{N_A} e^{q\psi/kT} - N_A \right) d\psi$$

Majority                      Minority

$$E_s = \frac{Q_s}{\epsilon_s}$$

↓ Get the form

$$Q_s \approx \left[ \frac{N_A e^{-q\psi_s/kT}}{(kT/q)} - \frac{n_i^2}{N_A} e^{q\psi_s/kT} - N_A \psi_s \right] \frac{V_2}{V_1}$$



We checked

$$10^{14} \text{ g cm}^{-2} \Rightarrow 10 \text{ V} \quad \& \quad \psi \approx 250 \text{ mV}$$

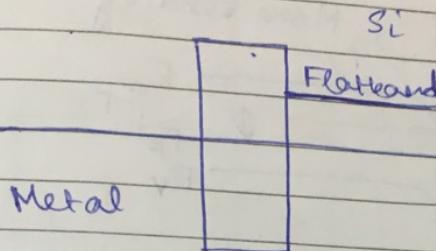
Accumulation

Depletion

Inversion

→ When  $\phi_m = \phi_s \rightarrow$  Flatband  $\rightarrow$  Equilibrium condition

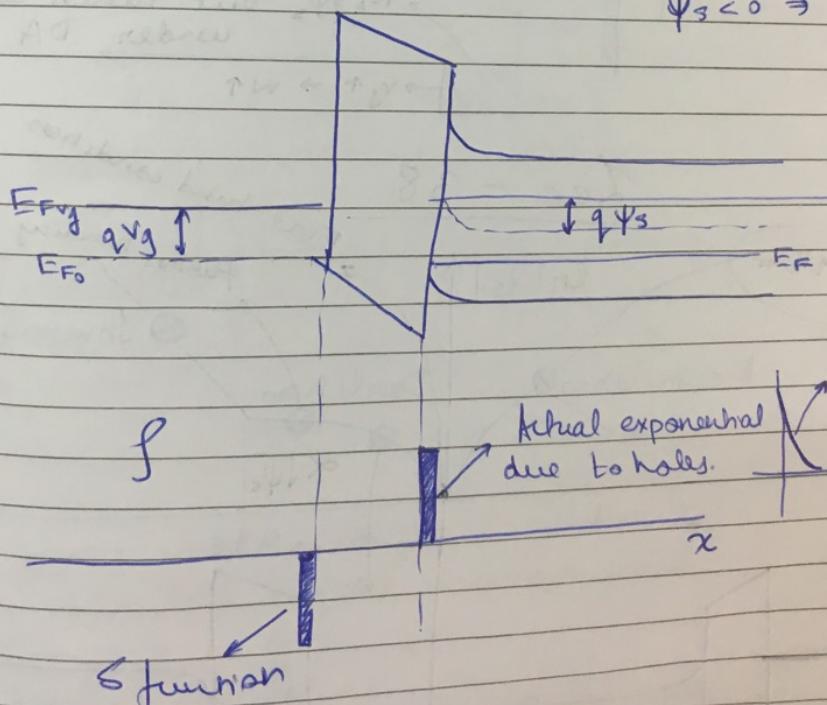
Apply  $-V_g$



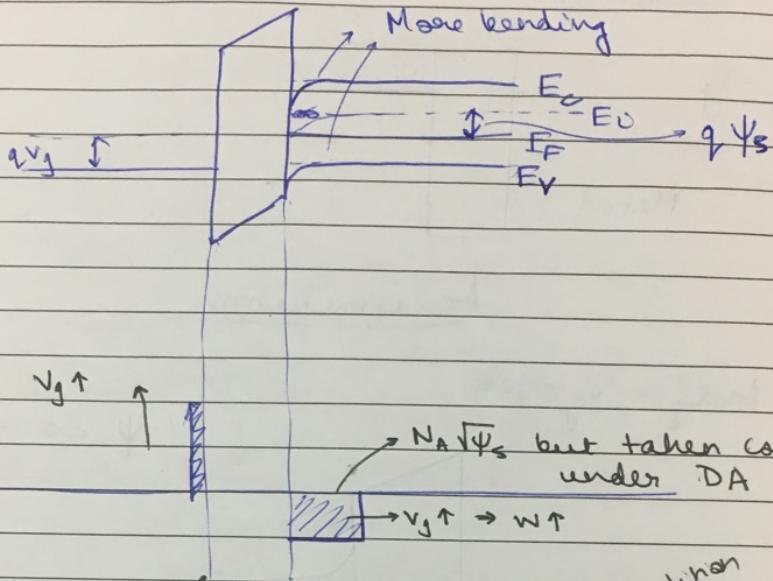
### Accumulation

→ Apply  $-V_g$  at Gate.

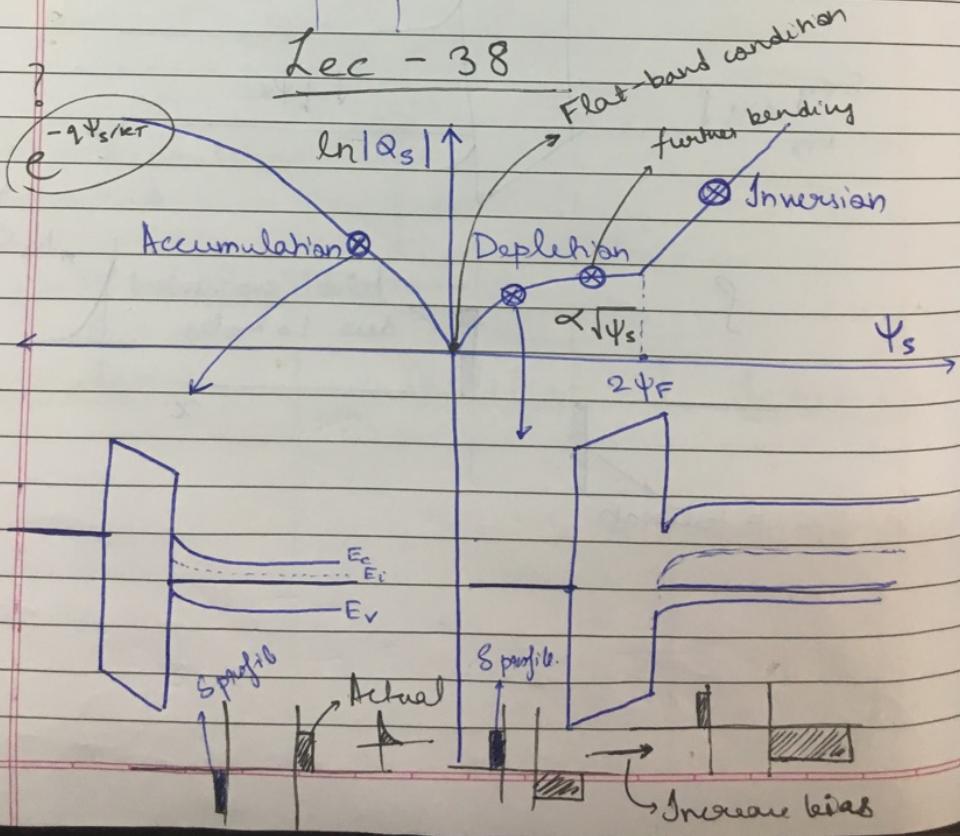
$\psi_s < 0 \Rightarrow \uparrow$  shifting.



Depletion  $V_g > 0, \psi_s > 0$  but small



Lec - 38



Inversion

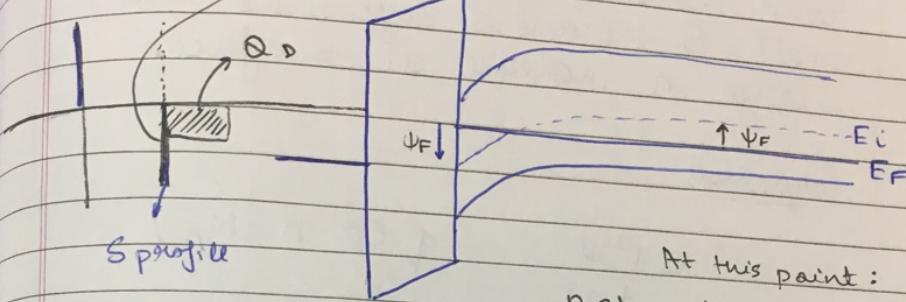
$Q_I$

of Inversion

PAGE NO.

→ Appears After the onset  
of Inversion

Small



→ Condition of Strong Inversion

$$n_{\text{surface}} = P_{\text{bulk}}, \quad n_s = P_b$$

$$\psi_F = \frac{kT}{q} \ln \left( \frac{N_A}{n_i} \right)$$

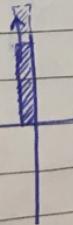
$$\Rightarrow \psi_s \text{ at Strong inversion} = \frac{2kT}{q} \ln \left( \frac{N_A}{n_i} \right)$$

Alternatively:

$$e^{q\psi_s/kT} = \frac{N_A^2}{n_i^2}$$

$$\psi_s = \frac{2kT}{q} \ln \left( \frac{N_A}{n_i} \right)$$

Further bias beyond Inversion point



→ Does not increase after onset of Inversion  
↓ S part increases.

Beyond onset  $Q_I \uparrow$   $Q_D \sim \text{constant}$   
of Inversion

→ Even though no further appreciable ↑ in  $\psi_s$  small S increase is sufficient for large n increase at surface.

### Equations

$$\epsilon \frac{d^2 \psi}{dx^2} = -q (p - n - N_A)$$

$$\epsilon E \frac{dE}{d\psi} = -q (p - n - N_A)$$

$$Q_s^2 = (\epsilon E_s)^2 = 2\epsilon q \int_0^{\psi_s} (p - n - N_A) d\psi$$

$$p = N_A e^{-q\psi/kT}$$

$$n = \frac{n_i^2}{N_A} e^{2q\psi/kT}$$

at onset of Inversion

### Gate Voltage $V_g$

$$V_{TH} (\text{MOSFET}) = V_g \text{ @ onset of Inversion}$$

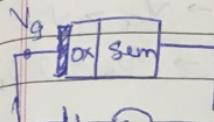
$$V_g = \psi_s + \frac{Q_s}{C_{ox}}$$

$$Q_s = (2\epsilon_q N_A \psi_s)^{1/2}$$

$$Q_s = q N_A W = (2\epsilon_q N_A \psi_s)^{1/2}$$

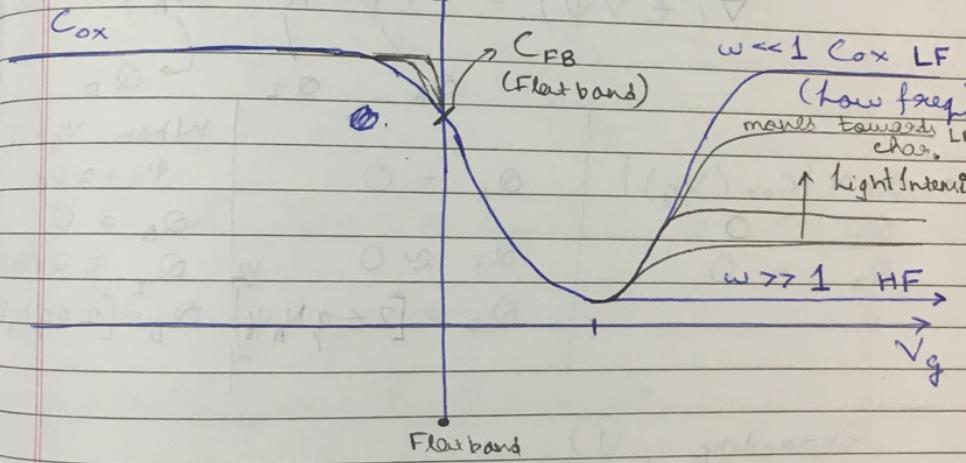
PAGE NO.	
DATE	/ /

$$V_{TH} = 2\psi_F + \frac{Q_s}{C_{ox}}$$



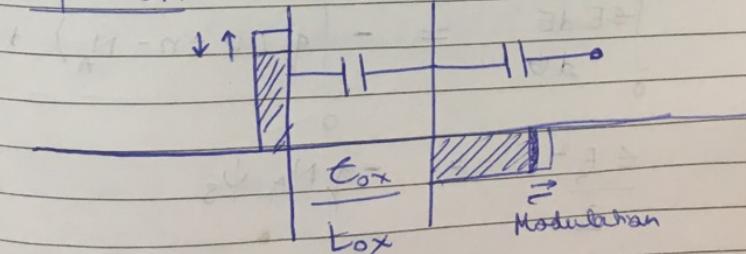
C - V<sub>DC</sub> Characteristics

V<sub>g</sub> ac ( $\omega \rightarrow 0$  &  $\omega \rightarrow \infty$  separately)



Inverter

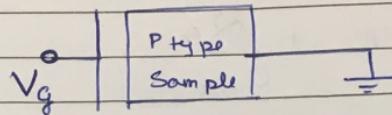
Depletion



Reference: Tariq d Ning

# MOS - CAP

PAGE No.	
DATE	/ /



(Same)

## Accumulation

- $\psi < 0, |\psi| \ll 1$
- Excess holes (Majority) carrier at surface.

## Depletion

- $\psi \ll 1$
- $\psi > 0$
- $\psi_s < 2\psi_F$
- $V_g < V_T$

## Inversion

- $V_g > V_T$
- $\psi_s = 2\psi_F$

$$\nabla(\epsilon \nabla \psi) = -(P - n - N_A) - Q_D \quad (1)$$

$$Q_A \approx |C_{ox}(V_g)|$$

$$Q_I = 0$$

$$Q_D = 0$$

$$Q_A = 0$$

$$Q_I \approx 0$$

$$Q_D = [2\epsilon q N_A \psi_s]^{1/2}$$

$$\text{When } V_g = V_T,$$

$$\psi_s = 2\psi_F$$

$$Q_A = 0$$

$$Q_I \approx 0 \text{ (small)}$$

$$Q_D = [2\epsilon q N_A^2]^{1/2}$$

Expanding (1)

$$\int_0^F \frac{\epsilon E dE}{d\psi} = - \int_0^{\psi_s} q(P - n - N_A) d\psi$$

$$\frac{\epsilon E_s^2}{2} = -q N_A \psi_s$$

$$Q_s = \epsilon |E_s| = [2\epsilon q N_A \psi_s]^{1/2}$$

$$V_T \approx V_t$$

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Date \_\_\_\_\_

Handwritten Voltage

### Strong Inversion

$$\rightarrow \psi_s \approx 2\psi_F + s \quad (s \ll 1)$$

$$V_g > V_T$$

### Formulas

$$2\psi_F = (\psi_s)_I = \frac{2kT}{q} \ln \left( \frac{N_A}{n_i} \right)$$

$$V_g = \psi_s + \frac{Q_s}{C_{ox}}$$

$$V_T (\text{At Gate}) = 2\psi_F + \frac{[4\epsilon_0 N_A \psi_F]}{C_{ox}}$$

$$V_{ox} = |\vec{E}_{ox}| \times t_{ox}$$

$$\text{and } \epsilon_s |\vec{E}_s| = \epsilon_{ox} |\vec{E}_{ox}|$$

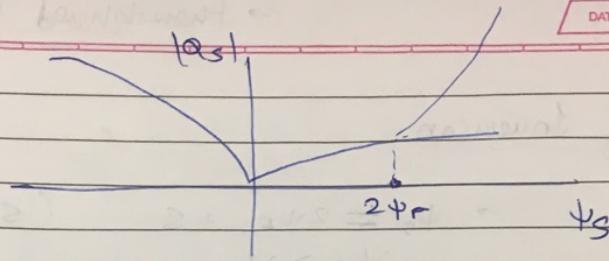
$$V_{ox} = \frac{\epsilon_s |\vec{E}_s|}{\epsilon_{ox}} t_{ox}$$

### Strong Inversion

$$Q_A = 0$$

$$* Q_I = C_{ox} (V_g - V_T) *$$

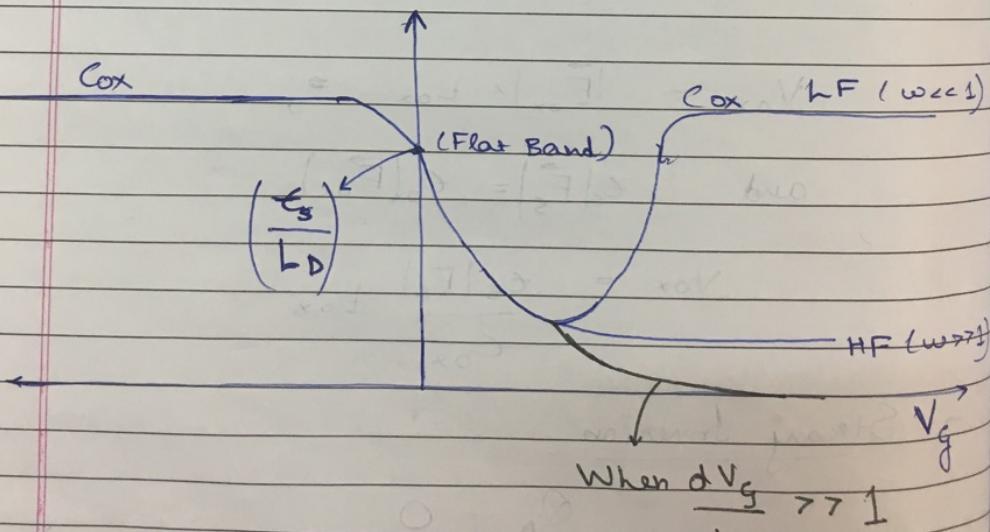
$$(Increases \ No \ further) \quad Q_D = [4\epsilon_0 N_A \psi_F]^{1/2}$$



$$\epsilon E \frac{dE}{d\psi} = \frac{n_i^2}{N_A} e^{\frac{q\psi}{kT}}$$

$$\frac{\epsilon_s^2 E_s^2}{2} =$$

### C-V Characteristics



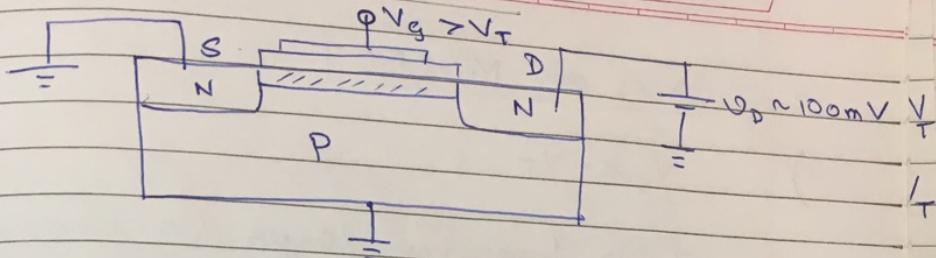
When  $\frac{dV_g}{dt} \gg 1$

- Increase in Inversion Layer can't keep up.

# MOSFET

PAGE No.  
DATE

R(n)



$$Q_{IS} = C_{ox} (V_g - V_T)$$

$$Q_{ID} = C_{ox} (V_g - V_T - V_{DS})$$

$$I = \langle Q \rangle \langle v \rangle$$

$$V = \mu \frac{V_{DS}}{L}$$

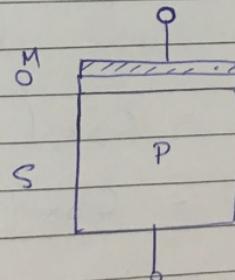
$$= \frac{(Q_{IS} + Q_{ID})}{2} \times \mu \frac{V_{DS}}{L}$$

24-10-17

Lec - 39

Office hours Wed 6-7pm  
Sat/Sun → TBD

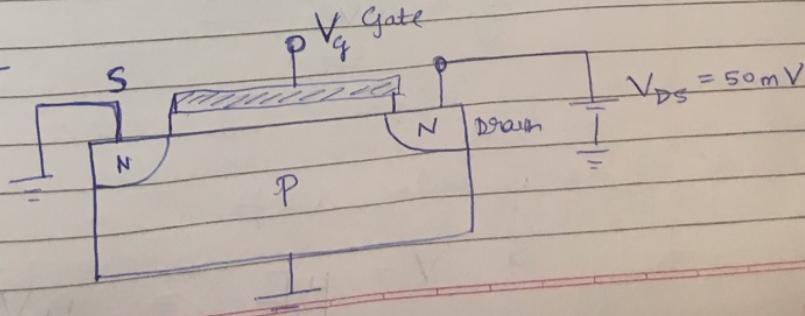
→ MOSCAP



$$V_g > V_T \text{ (Inversion)}$$

$$Q_I = C_{ox} (V_g - V_T)$$

→ MOSFET



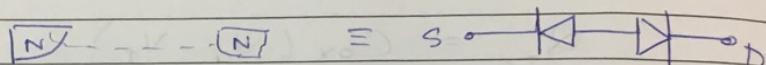
22-10-17

$V_g$   
up

→ Cases on MOSFET

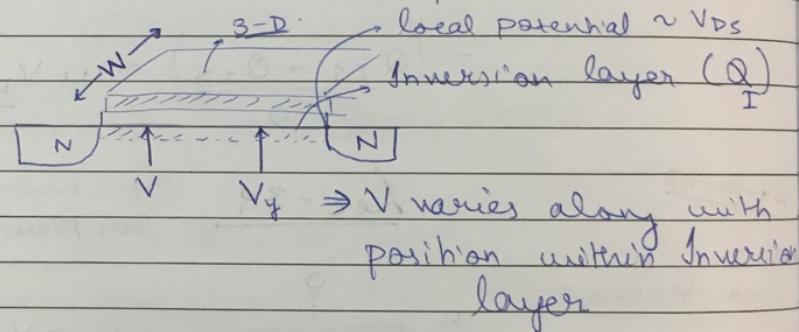
1)  $V_g \ll V_T$

→ No Inversion layer (n-channel)



2)  $V_g = V_T$  (or Greater)

→ Derive Current Expression



$$\Rightarrow Q_I(S) = C_{ox} (V_{gs} - V_T)$$

$$\Rightarrow Q_I(D) = C_{ox} (V_{gs} - V_T - V_{DS})$$

$$\frac{I}{W} = \alpha v^{\text{speed}}$$

$\curvearrowleft$  3-D

→ Effect is only drift.

$$\Rightarrow V = \mu E = \frac{\mu V_{DS}}{L}$$

$$\frac{I}{W} = \mu \times C_{ox} \left( V_{gs} - V_T - \frac{V_{ds}}{2} \right) \frac{V_{ds}}{L}$$

↓  
spatial  
average

$$I = \frac{HW}{L} C_{ox} (V_{gs} - V_T) V_{ds}$$

↓  
 $\frac{\epsilon}{t}$

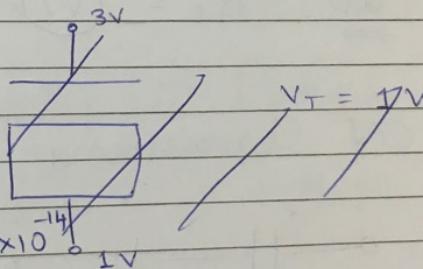
Ignoring terms  
2nd order in  $V_{ds}$

$I \uparrow$  if  $L \downarrow t \downarrow V_g \uparrow$

### Q → MOSCAP

#### Parameters

- $N_A = 10^{16} \text{ cm}^{-3}$
- $t_{ox} = 3 \text{ nm}$
- $C_{ox} = 3.9 \times 8.8 \times 10^{-14} \text{ F/V}$



#### Find

- $V_T$
- $W_{depletion}$
- $C_V$

→  ~~$V_g = \psi_s + \frac{Q_s}{C_{ox}}$~~

$$\psi_F = \frac{kT}{q} \ln \left( \frac{N_A}{n_i} \right) \approx 360 \text{ mV}$$

$$\psi_s = 2\psi_F \quad (\text{for } V_g = V_T)$$

$$Q_s \quad (\psi_s = 2\psi_F)$$

$$= [4\epsilon_0 N_A \psi_F]^{1/2}$$

$$= q$$

Get  $V_T \approx 0.744 \text{ V}$   
 $(720 + 24) \text{ mV}$

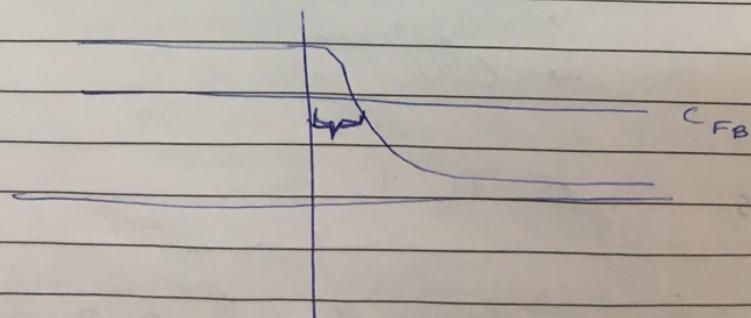
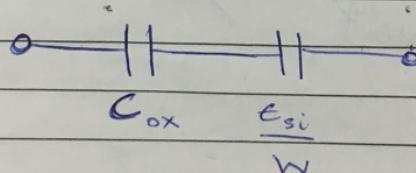
#

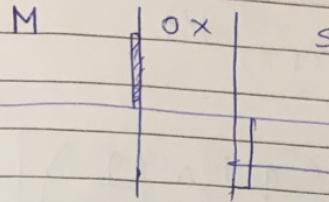
Depletion width

Here  $q_s = \frac{W}{\lambda} (qN_A)$

\*

cV Characterizing





Flat Band Condition ( $\nabla g = 0$  Ideally)

$$Q_s^2 = \epsilon_s^2 E_s^2 = -2q \epsilon \int_{-\infty}^{\psi_s} (N_A e^{-\frac{q\psi}{kT}} - N_A) d\psi$$

$$= -2q \epsilon \left[ -N_A \left( \frac{e^{-\frac{q\psi_s}{kT}} - 1}{\frac{q}{kT}} \right) - N_A \psi_s \right]$$

$$= -2q \epsilon \left[ -N_A \left[ \frac{1 - e^{-\frac{q\psi_s}{kT}} + \left(\frac{q\psi_s}{kT}\right)^2/2}{\frac{q}{kT}} \right] - N_A \psi_s \right]$$

$$\approx \frac{2q \epsilon N_A}{\chi} \left( \frac{q}{kT} \right)^2 \psi_s^2$$

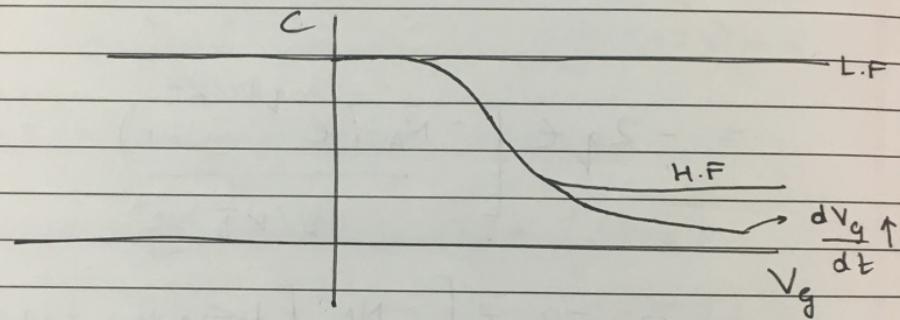
$$Q_s = \left[ \epsilon q N_A \left( \frac{q}{kT} \right) \right]^{1/2} \psi_s$$

$$\frac{dQ_s}{d\psi} = C$$

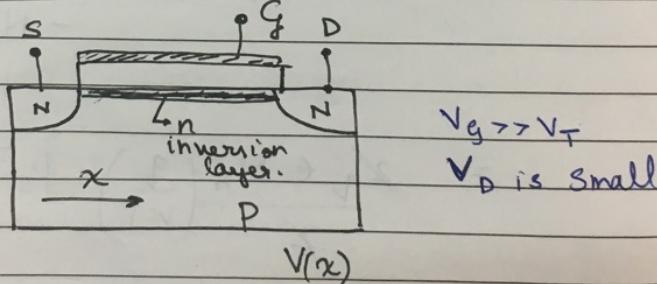
$$Q_s = C_{s_i(FB)} \psi_s$$

$$C_{FB} = \left[ \epsilon_0 N_A \left( \frac{q}{kT} \right) \right]^{1/2}$$

$$C_{FB} = \frac{1}{C_{ox}} + \frac{1}{C_{s_i(FB)}}$$



→ MOSFETs Continued



$V_g \gg V_T$ ,  $V_D$  is small

$$Q_I = C_{ox} (V_{gs} - V_T)$$

$$Q_I(x) = C_{ox} (V_{gs} - V(x) - V_T)$$

# MOSFET

Page No.

Date: / /

- Sub-threshold
- Linear

$$V_g < V_T$$

$$V_g > V_T \quad V_{DS} < V_g - V_T$$

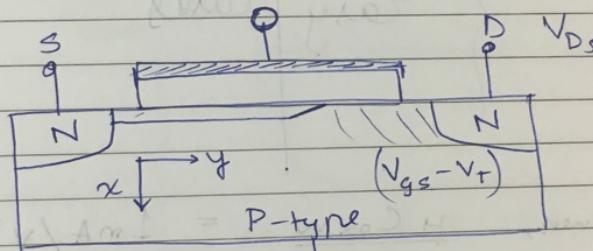
- Saturation

$$V_g > V_T, \quad V_{DS} > V_g - V_T$$

no leakage to drain at this

$$I_{(linear)} = \frac{H C_{ox} W}{L} \left( (V_g - V_T) V_{DS} - \frac{V_{DS}^2}{2} \right)$$

$$I_{saturation} = \frac{H W C_{ox}}{2} \frac{(V_{GS} - V_T)^2}{L} (1 + \lambda V_{DS})$$



$$Q_I = C_{ox} (V_{GS} - V_T - V(y))$$

Alternate derivation

$$\frac{I}{W} = q H E \int_0^{\infty} n dx \quad E \rightarrow \text{Electric field}$$

$$\frac{I}{W} = q H$$

$$I = \mu C_{ox} \left( \frac{W}{L} \right) \times \left[ (V_{GS} - V_T) V_{DS} - \frac{V_{DS}^2}{2} \right]$$

Q → Find the Regime of operation

$$V_T = 0.7V$$

$V_{DS}$	$V$	$V_{GS}$	$V$
Easy cases			

Q → Given,  $\frac{\mu C_{ox} W}{L} = 1 \text{ mA/V}^2$

Supports 5mA in linear region  
with  $\lambda = 0$

$$V_T = 0.7V$$

$$V_D = V_G \quad V_{DS} = V_{GS}$$

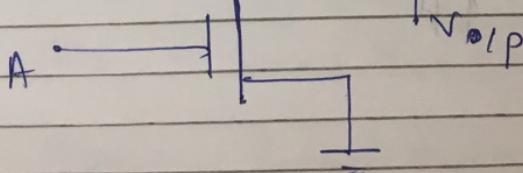
(i)

$$\Rightarrow V_{DS} > V_{GS} - V_T$$

⇒ Saturation.

$$V_{DS} = 5V$$

(ii)



$$5 - V_{DS} = 4.3V_{DS} - 0.5V_{DS}^2$$

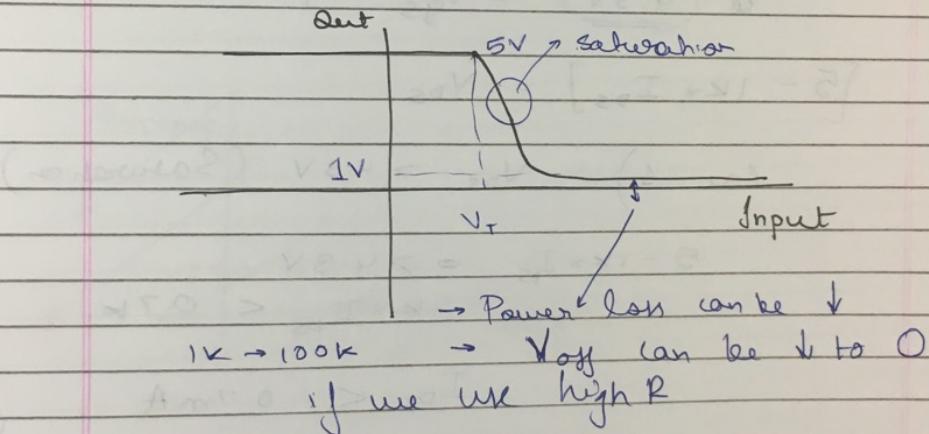
$$0.5V_{DS}^2 - 5.3V_{DS} + 5 = 0$$

$$V_{DS} = \frac{-0.5 + \sqrt{(5.3)^2 - 10}}{1}$$

$$V_{DS} = 1.04V$$

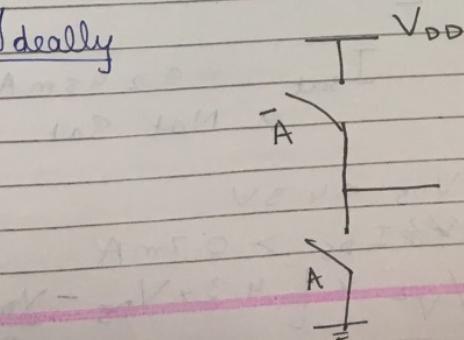
~~$V_{DS} = 3.75$~~  → check again.

→ Switching char.



→ but  $T = RC \uparrow$  and switching time ↑

→ Ideally



$$H \frac{C_0 \times W}{L} = 1 \text{ mA/V}^2$$

A	O/P	(Capacitive load)
0V	5V	
5V	$0.5V$	$\frac{1.04V}{1.04V - 0.5V}$ (Linear region)

$$V_A = 0V$$

$$V_A = 5V$$

$$V_{DS} = V_{GS} - V_T$$

$$[5 - 1k \times I_{DS}] = V_{DS}$$

$$(or -1) \quad V_{DS} > 4.3V \quad (\text{Saturation})$$

$$5 - 1k \times I_{DS} > 4.3V$$

$$1k \times I_{DS} < 0.7k$$

$$I_{DS} < 0.7mA$$

$$I_{sat} = \frac{1 \text{ mA/V}^2 \times 0.7k \times (4.3)^2}{2} (1 + \cancel{\frac{1}{2} V_{DS}})$$

$$I_{sat} = 9.245mA$$

$\Rightarrow$  Not Sat.

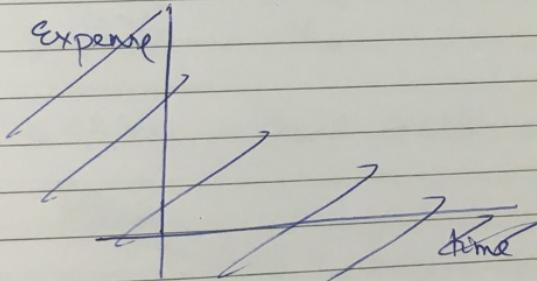
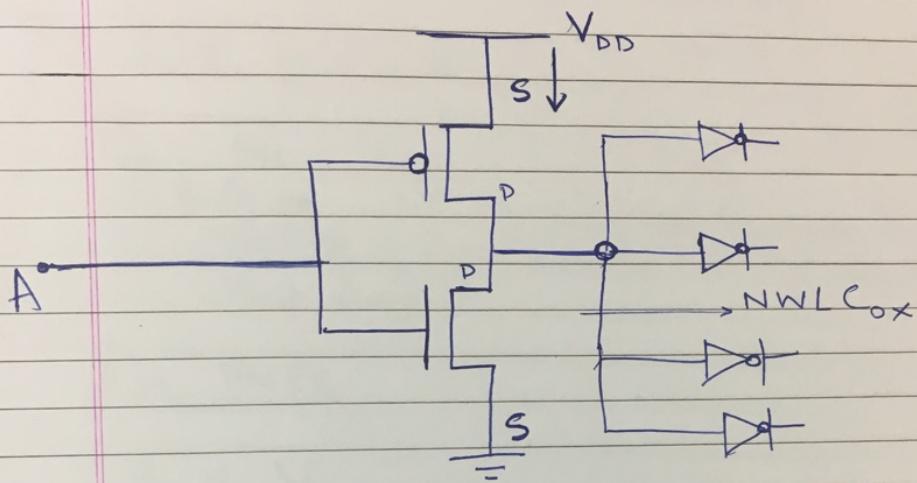
Linear

$$V_{DS} < 4.3V$$

$$I_{DS} > 0.7mA$$

$$1 \text{ mA/V}^2 \times \left[ 4.3 \times V_{DS} - \frac{V_{DS}^2}{2} \right] = 5V_{DS}$$

→ Solution  $\Rightarrow$  Use CMOS



$$Q_I = C_{ox} (V_{GS} - V_{DS} - V_T) = \frac{H V_{DS}}{2}$$

$$I = H \frac{C_{ox} W}{L} \left( (V_{GS} - V_T) V_{DS} - \frac{V_{DS}^2}{2} \right)$$

↓  
Nat

bulk  $\rightarrow$   
mobility  
(Much less)

$H \rightarrow$  Mobility near surface,  
smaller due to roughness

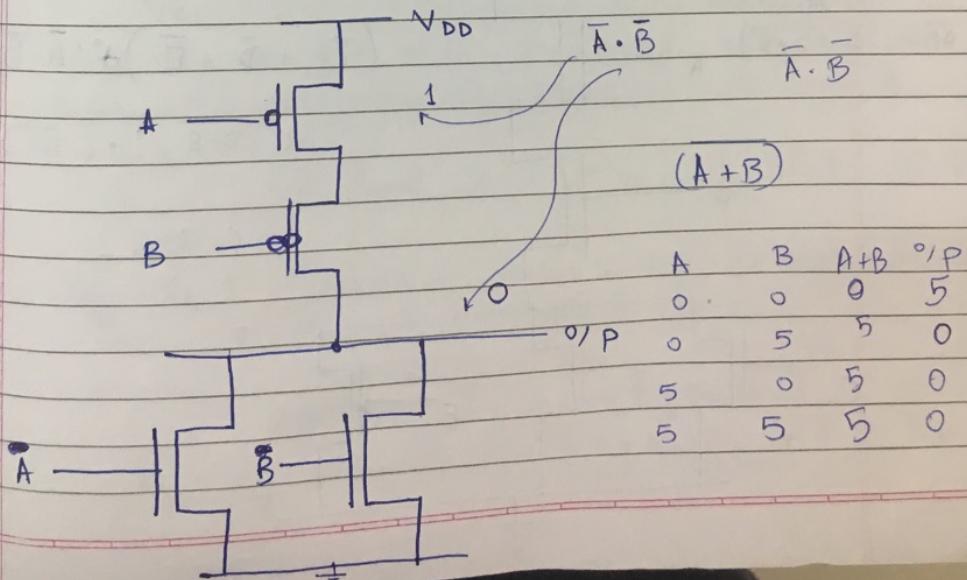
### For PMOS

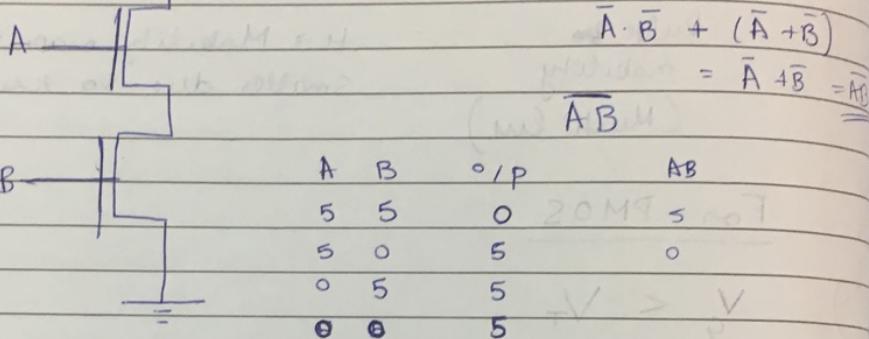
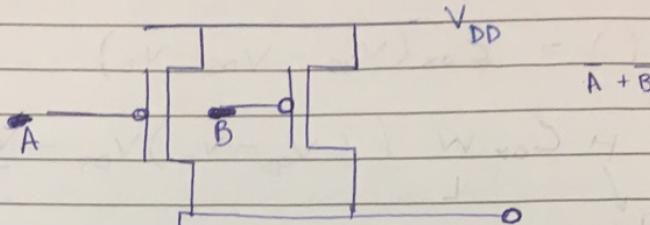
- 1)  $V_g < V_T$
- 2)  $V_g > V_T, V_{DS} \sim$
- 3)  $V_g >> V_T, V_D$  is small

30-10-12

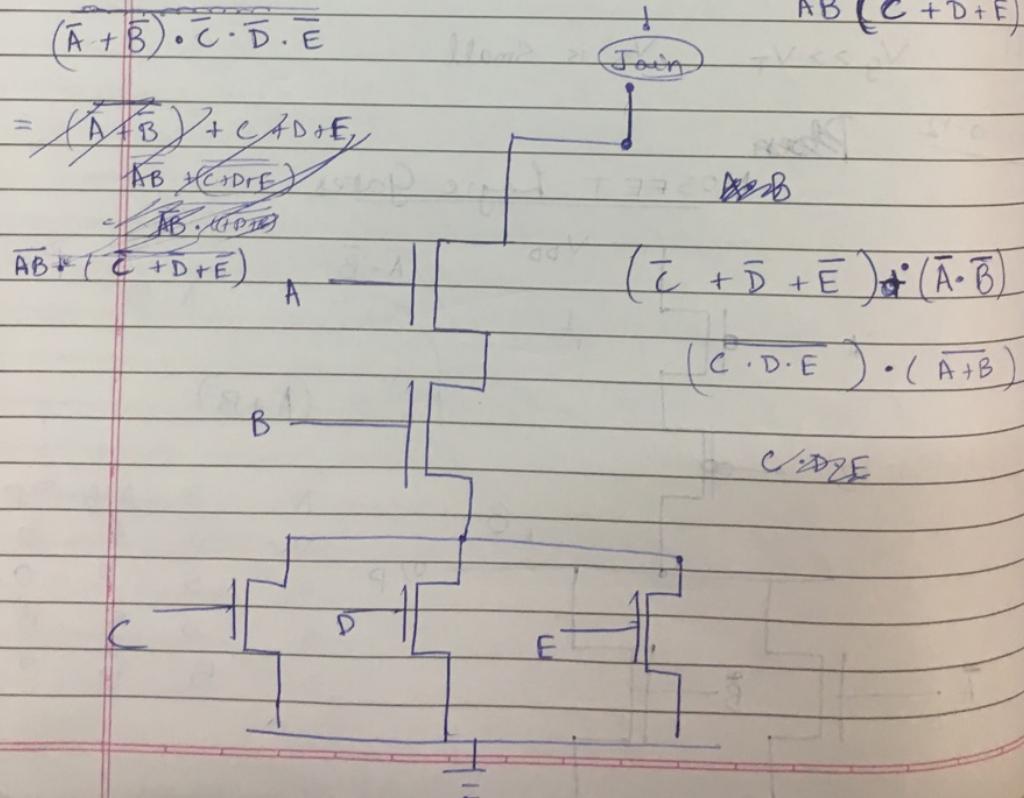
Prashan

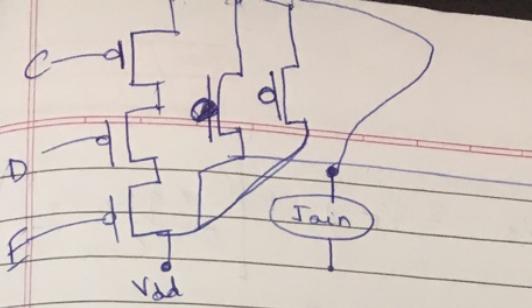
### MOSFET Logic Gates





→ We want AB(C + D + E)

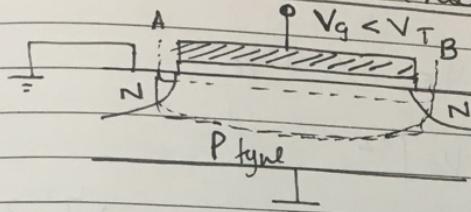




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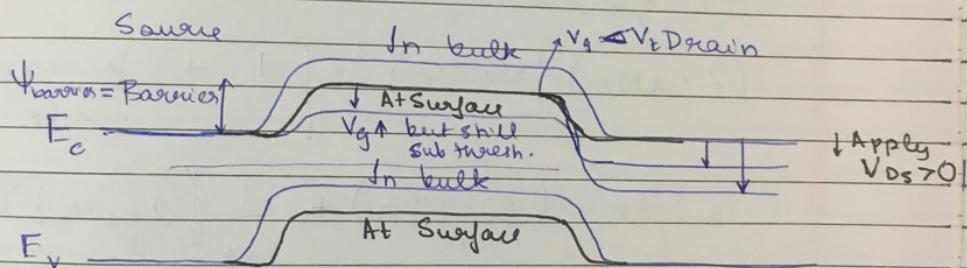
Corresponding PUN

### Sub-Threshold



(a)  $V_D = 0V$

(b)  $V_D = 0.3V$



- At  $V_{DS} = 0V \rightarrow$  Complete symmetry  $\Rightarrow I = 0$
- At  $V_{DS} > 0$  for sub-threshold, current is determined only by the barrier at left & hence, for fixed  $V_g$  ( $< V_T$ ) if  $V_D \uparrow$ ,  $I \sim k$

→ As  $V_g \uparrow$ , Barrier  $\downarrow$  and  $I \uparrow$  at sub-threshold  
&  $I \propto e^{-q\psi_{barrier}/kT}$

In this region

$$V_g - V_F = \psi_s + \frac{(2\epsilon_q N_A \psi_s)^{1/2}}{C_{ox}} - V_T$$

$$V_g - V_T = (\psi_s - 2\psi_F) + \left[ \frac{(2\epsilon_2 N_A \psi_s)^{1/2}}{C_{ox}} \right]$$

$\psi_s$  just less than  $2\psi_F$

$$\psi_s - 2\psi_F \ll 2\psi_F$$

$$[2\epsilon_2 N_A \psi_s]^{1/2}$$

$$= [2\epsilon_2 N_A (\psi_s - 2\psi_F + 2\psi_F)]^{1/2}$$

$$= (2\epsilon_2)^{1/2}$$

$$(2\epsilon_2 N_A)^{1/2} \times (2\psi_F)^{1/2} \times \left[ 1 + \frac{\psi_s - 2\psi_F}{2\psi_F} \right]^{1/2}$$

$$V_g - V_T = (\psi_s - 2\psi_F) + \frac{1}{C_{ox}} \left[ (2\epsilon_2 N_A)^{1/2} (2\psi_F)^{1/2} \right]$$

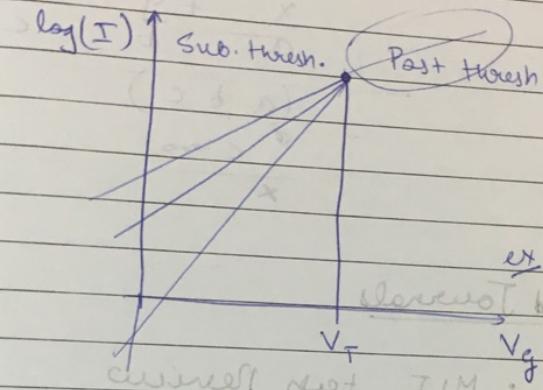
$$(\psi_s - 2\psi_F) + \frac{1}{C_{ox}}$$

$$V_g - V_T = (\psi_s - 2\psi_F) \times \left[ 1 + m \right]$$

Independent  
of  $\psi_s$

$$m = 1 + \frac{(2\epsilon_2 N_A 2\psi_F)^{1/2}}{C_{ox}}$$

$$- \left[ \frac{(2\epsilon_0 N_A 2\psi_F)^{1/2}}{C_{ox}} \right]$$



$$SS = \left( \frac{d(\log I)}{d V_g} \right)^{-1/2}$$

ex.  $60 \text{ mV/dec}$

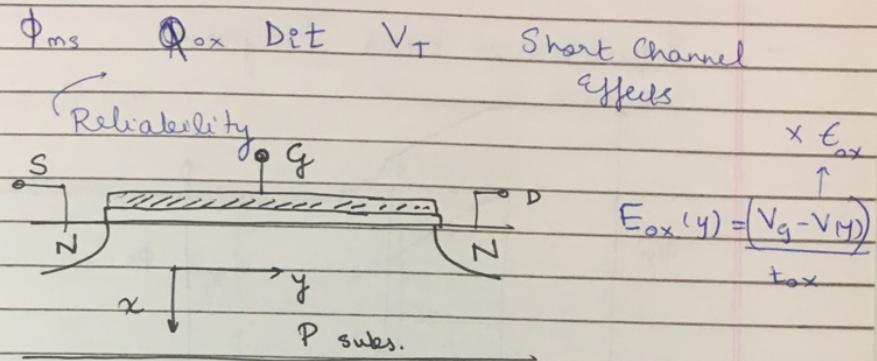
~~linear~~ ~~logarithmic~~

current not T.I.M.

$$\Delta \psi_s = q \frac{\Delta V_g}{m}$$

$$\left[ \frac{1}{4} \frac{\psi_s - 2\psi_F}{\psi_F} \right] - \left[ \frac{(2\epsilon_0 N_A 2\psi_F)^{1/2}}{C_{ox}} \right]$$

# Non-Ideal MOS

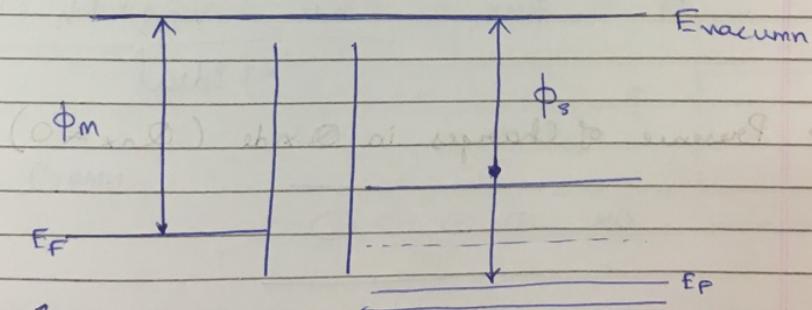


- Q → ①  $I_D$ , NMOS,  $V_T = 0.7V$ ,  $V_{DS} = 0.5V$ ,  $V_{GS} = 0.7V$   
 ②  $Q_I = C_{ox}(V_g - V_T - V(y))$

- For ① neither the Sub-threshold I exp. nor the linear region exp. are valid

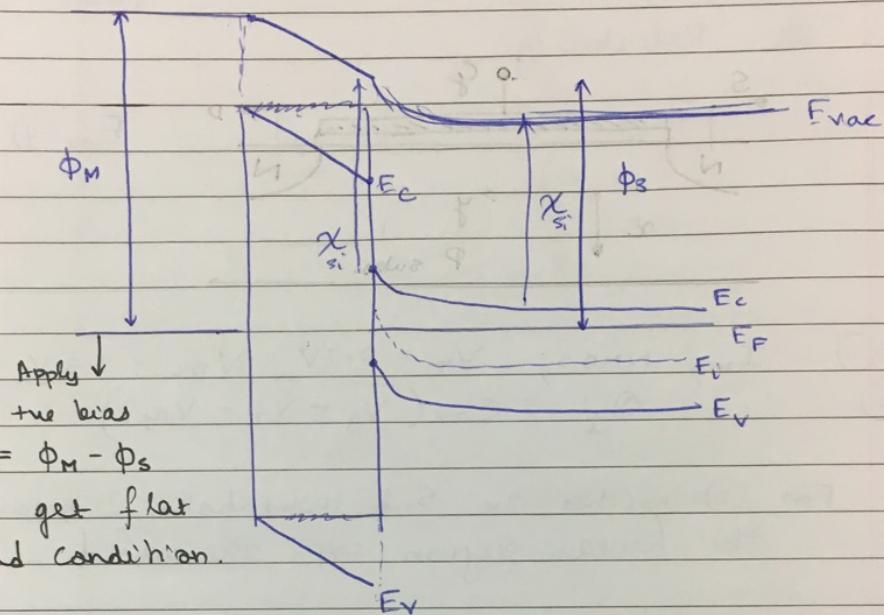
## Band Diagram

$$\phi_{ms} = \phi_m - \phi_s \neq 0$$



→ Band Diagram without 1 Metallurgical junction.

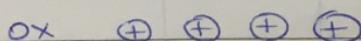
→ Equilibrium band Diagram



→ Hence all char. will shift by  $V_{fb}$   
 $\phi_{MS} = \phi_M - \phi_s \neq 0$

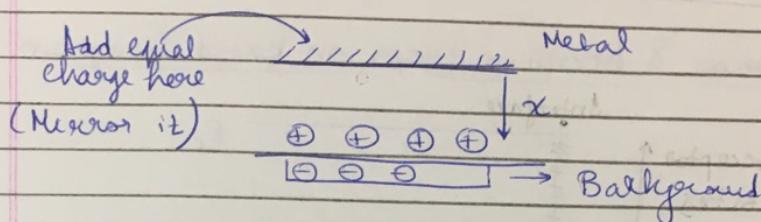
$$V_T = \phi_{MS} + \underbrace{2\psi_F + \sqrt{2q\epsilon N_A(2\psi_F)}}_{V_{T\text{ideal}}}$$

→ Presence of Charges in Oxide ( $Q_{ox} \approx 0$ )  
(NMOS)



→ Lowers  $V_T$ , since Inversion layer forms earlier due to background deposition of charge.

→ Find Shift in  $V_T$ ? What is  $V_{FB}$



$$\Delta V_g = -\frac{\sigma}{\epsilon_0} \times x$$

Metal

$\sigma \rightarrow \sigma(x)$

$V_{FB} =$

negative bias

$$\int \frac{\sigma x}{\epsilon_0} dx$$

- Immobile ions  $\rightarrow$  C-V char. shifts by  $V_{FB}$
- Mobile ions  $\rightarrow$  C-V char. further stretched out.

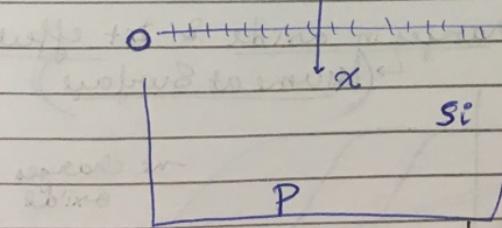
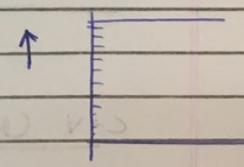
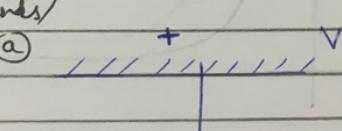
Dit : Interface state density

→ Donating bands/

traps/states @

surface

+  $V_g$

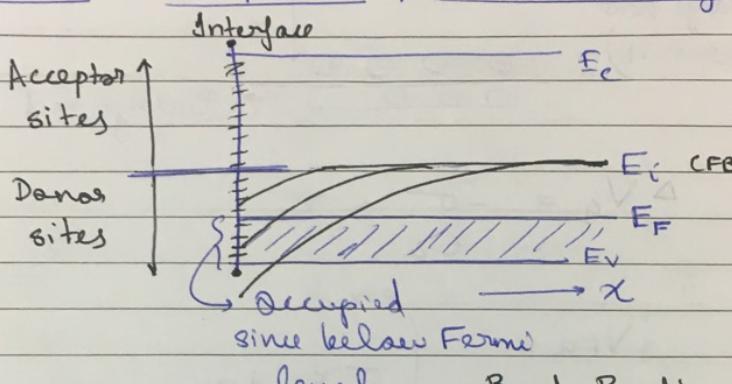


Empty states  
+ve  
0

(with  $e^-$ )  
filled states  
0  
-ve  
Acceptor  
Donor

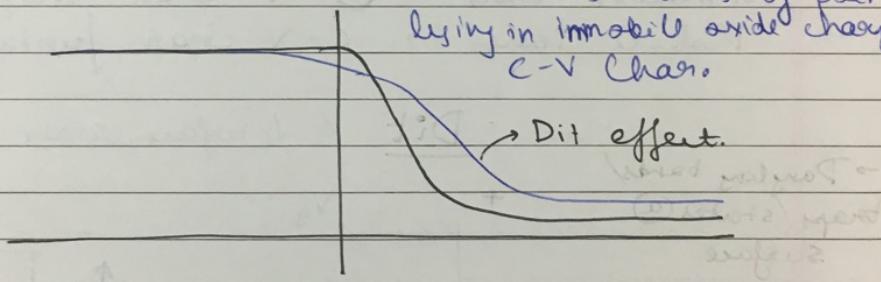
## → Effect on Band Bending

Donor & Acceptor traps on band Diagram

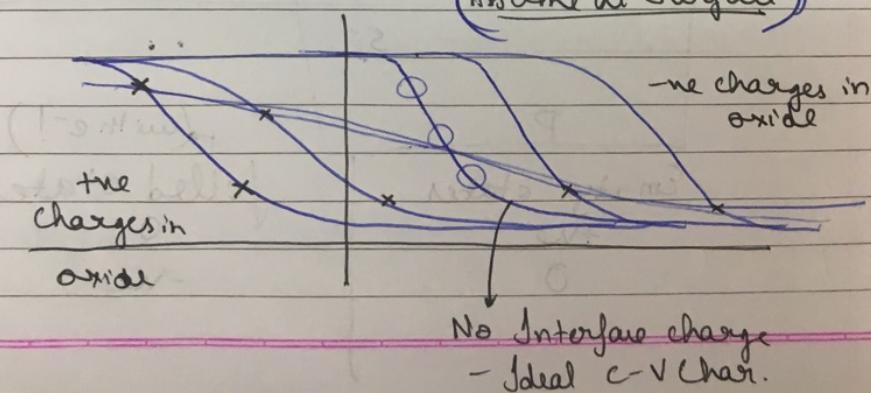


C-V Char.

As  $E_i$  tends to nature of states changes hence final C-V char is locus of points lying in immobile oxide charge C-V Char.



C-V Char. Charges in oxide & Dit effect relation  
→ Assume at Surface

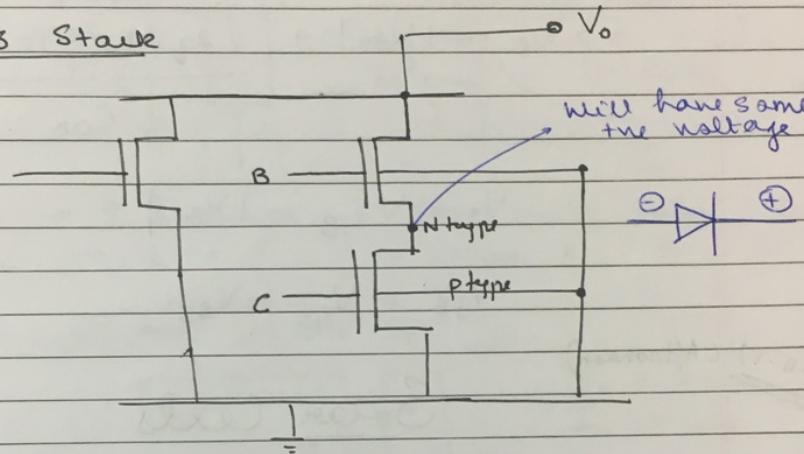


2-11-17

## → Bulk Bias

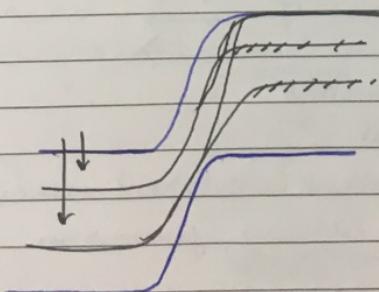
- Plays Important Role in CMOS Stacks.

NMOS Stack

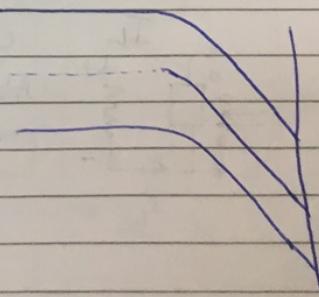


- Bodies Grounded Directly and Source/drain will have same the potential.
- This will change the threshold point (Point of Inversion)

S - B E - B Dra.



Vertical profile of Body



$$\psi_s = 2\psi_F + |V_{BS}|$$

$$V_m = 2\psi_F + |V_{BS}| + \frac{Q_0}{C_{ox}}$$

$$= 2\psi_F + |V_{BS}| + \frac{\sqrt{2q\epsilon N_A(2\psi_F + V_{BS})}}{C_{ox}}$$

$$V_T = V_{T,B} - |V_{BS}|$$

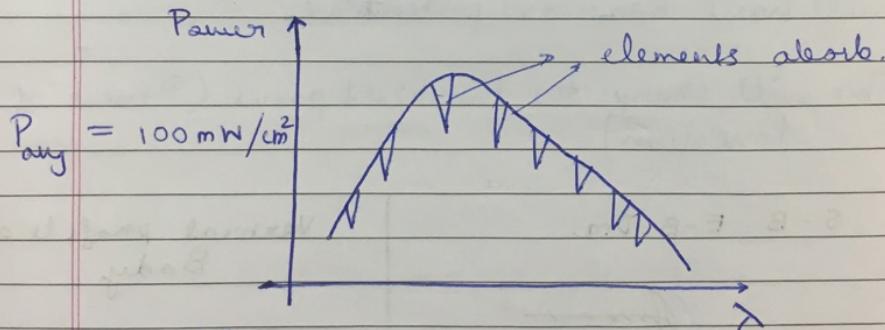
$$V_{gB} = V_{gs} - V_{BS}$$

~~2-11-17~~ (Afternoon)

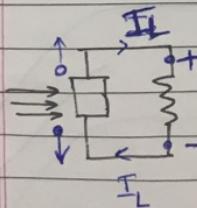
ex

## Solar Cells

### Solar Energy



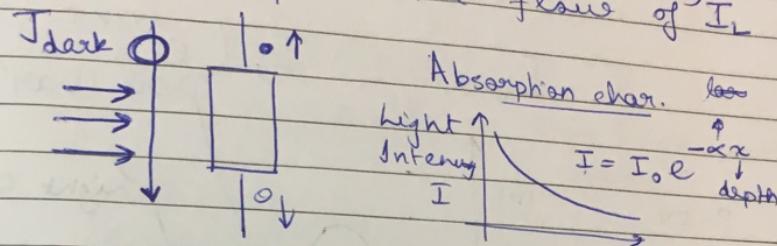
sky



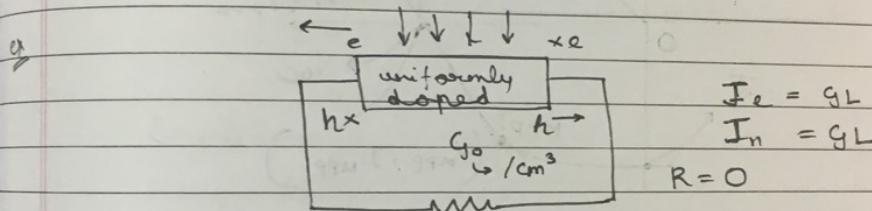
### Characteristics

- Absorbs light
- We get current in external circuit through CARRIER Generation
- free carriers not bound excitons
- Carriers should be collected at different terminals.

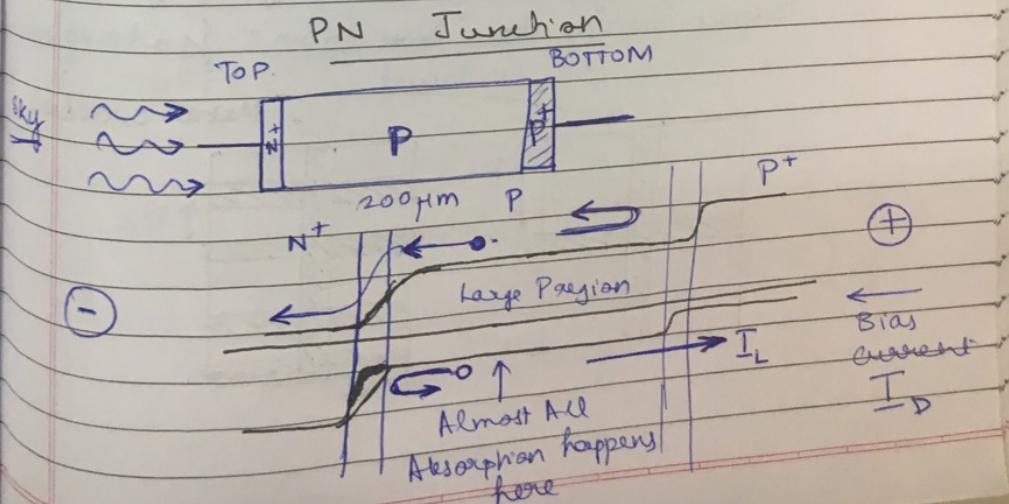
The voltage created by  $I_L$  through  $R$ , opposes the flow of  $I_L$



GaAs  $\rightarrow$  ↑ Absorption  $\sim$  1 μm thick Solar cells  
 Si  $\rightarrow$  ↓ "  $\sim$  200 μm " " "



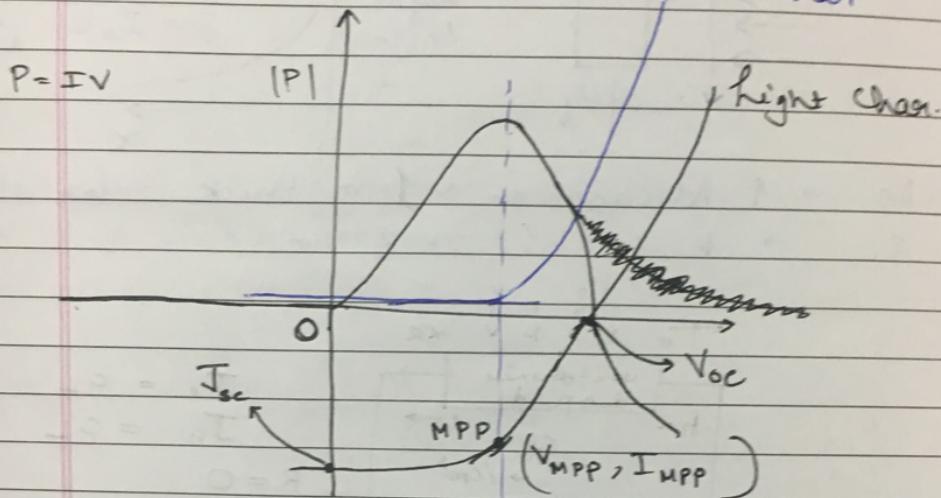
Note: If Electrodes were not special but Symmetric then No I due to  
 • Symmetry      • Recombination



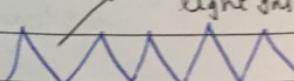
$$I_{\text{light}} = -I_L + I_o (e^{\frac{qV}{kT}} - 1)$$

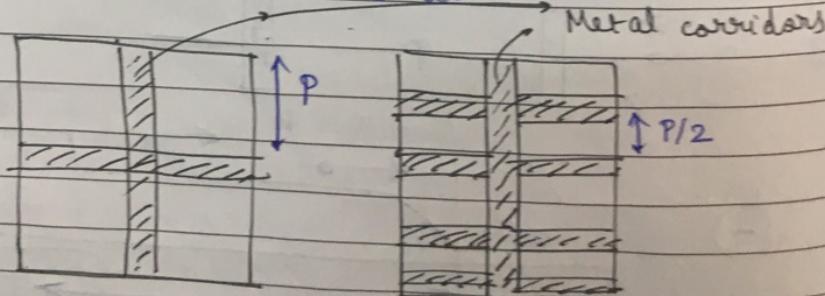
$$I_{\text{dark}} = I_o (e^{\frac{qV}{kT}} - 1)$$

Dark char



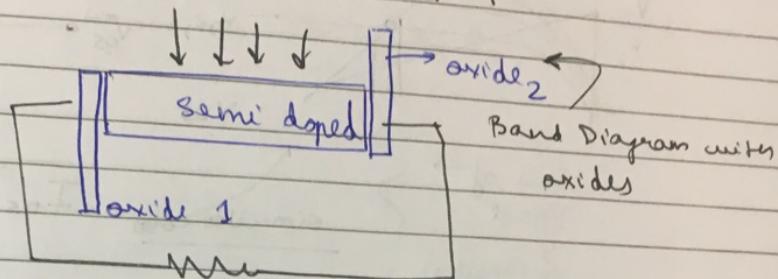
## → Solar Cell Design

- Eli Yablonovitch proposed Rough Surface →  Allows more light inside  
Expose Silicon to Non-Isothermally to etchant.



-1)

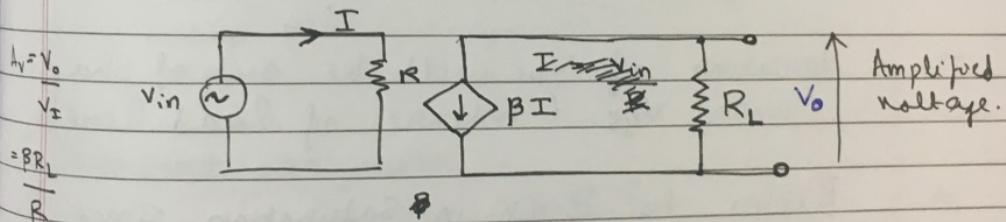
- , Oxides with easy metallization junction can be used to replace  $N^+$  &  $P^+$



→ Relation between Band Gap  $E_g$   
 $V_{oc} \downarrow E_g \downarrow \quad \left. \begin{array}{l} I \uparrow \\ E_g \downarrow \end{array} \right\}$  Hence trade-off

b11.17

→ Amplification & Ammeter.

more  
oxide

ically

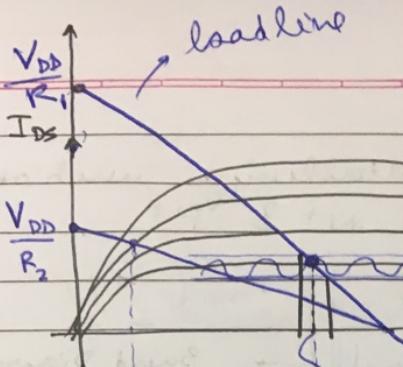
ors

MOSFET as Amplifier

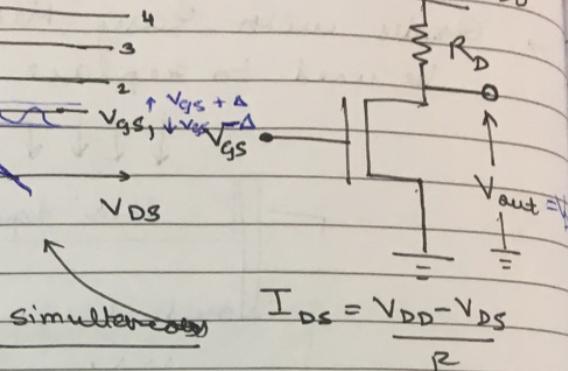
$$I_{DS} = \frac{H C_{ox} W}{L} \left[ (V_{GS} - V_T) V_{DS} - \frac{V_{DS}^2}{2} \right]$$

$$(\text{Sat.}) \quad I_{DS} = \frac{H C_{ox} W}{2L} \left[ (V_{GS} - V_T)^2 \right]$$

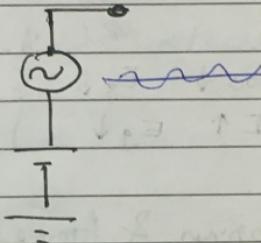
$R_2 > R_1$



Out of Phase  
(smaller)



$$V_{GS} \equiv$$



simultaneously

→ Variation of  $V_{DS}$  will be out of phase with  $V_{GS}$  (Nature of load line)

→ Better to Bias in Saturation Since we get more variation in  $V_{DS}$   
MOSFET Amps always biased in Sat.

→ Equivalent Ckt Model

$$I_{DS} + i_{DS} = I_{DS} + \frac{s}{s} \left( \right) \Big|_{V_g}$$

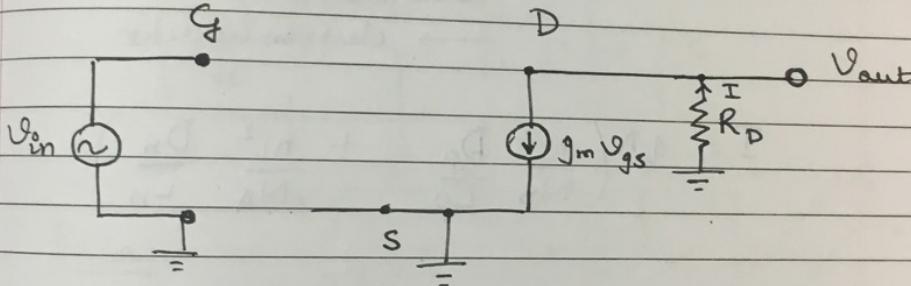
$$I_{DS} = \frac{\mu C_{ox} W}{2L} \left[ (V_{GS} - V_T)^2 \right]$$

$$i_{DS} = (?) v_{gs} \quad ??$$

$$i_{DS} = \left( \frac{S I_{DS}}{8 V_{gs}} \right) v_{ds} = g_m v_{gs}$$

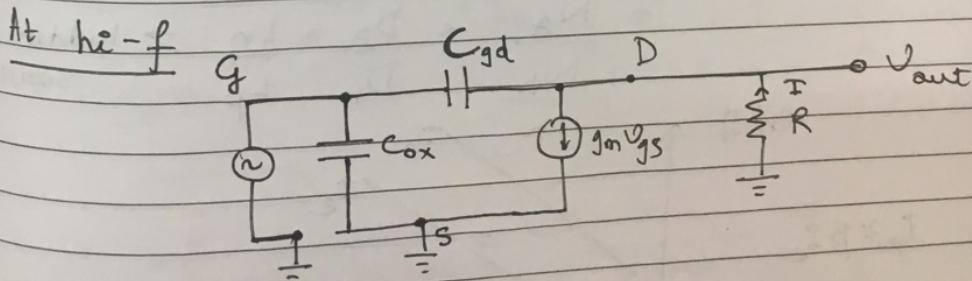
$$g_m = \frac{4 C_{ox} W}{L} (V_{gs} - V_T)$$

Model (AC. small Signal model)



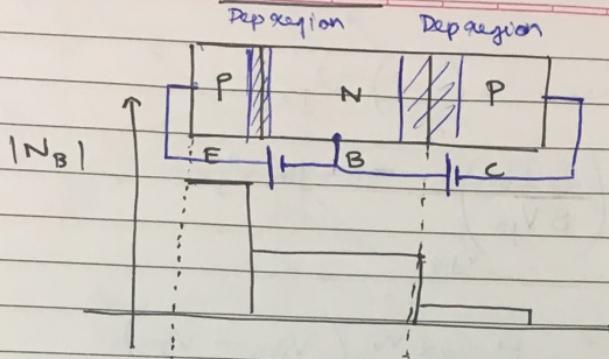
$$v_{out} = -g_m v_{gs} R_D$$

$$A_v = -g_m R_D$$



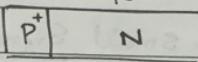
# BJT

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→ PN junction

$$10^{18} \quad 10^{16}$$



→ hole current

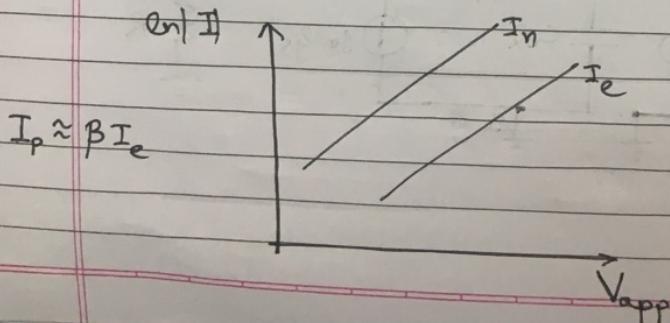
→ electron current

$$I = qA \left( \frac{n_i^2}{N_D} \frac{D_p}{L_p} + \frac{n_i^2}{N_A} \frac{D_n}{L_n} \right)$$

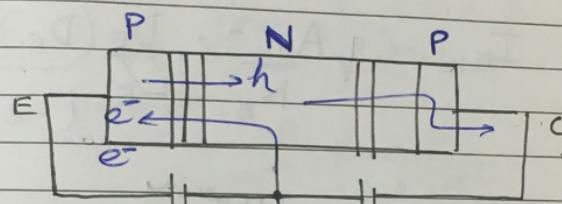
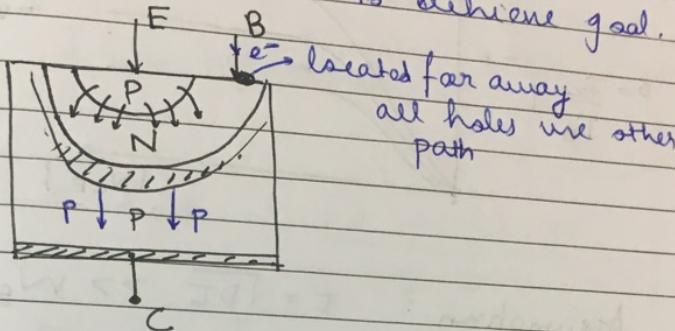
7.11.17

$$\beta = \left( \frac{I_h}{I_e} \right) = \frac{\frac{n_i^2}{N_D} D_p / L_p}{\frac{n_i^2}{N_A} D_n / L_n}$$

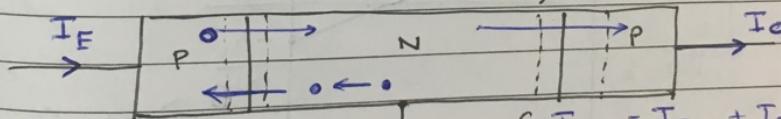
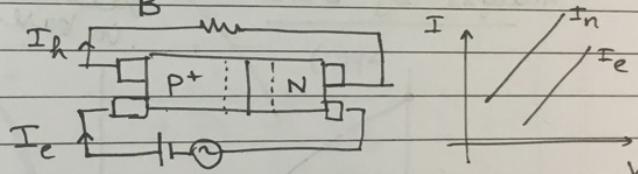
$$= \frac{N_A}{N_D} \times \frac{D_p}{D_n} \times \frac{L_n}{L_p} \rightarrow \text{hidden dependent source.}$$



, Split e & p currents to achieve goal.

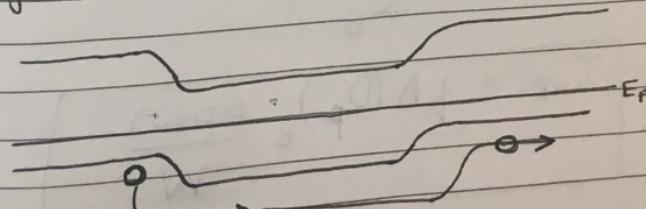


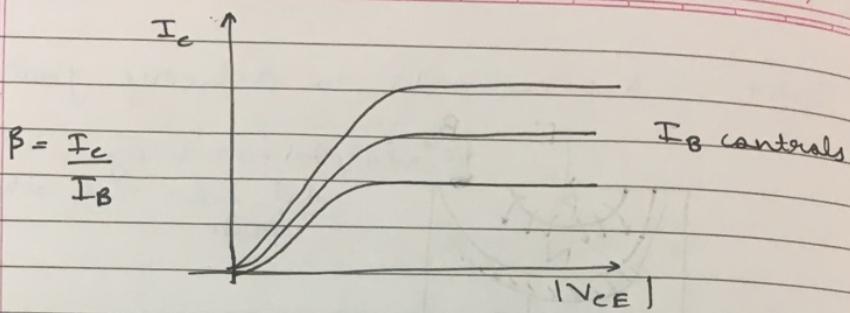
Recap



$$\left\{ \begin{array}{l} I_E = I_{EP} + I_{EN} \\ I_B = I_{en} + (\text{Recomb.}) \\ I_{CP} = I_{EP} - (\text{Recomb.}) \end{array} \right.$$

E-B Diagram

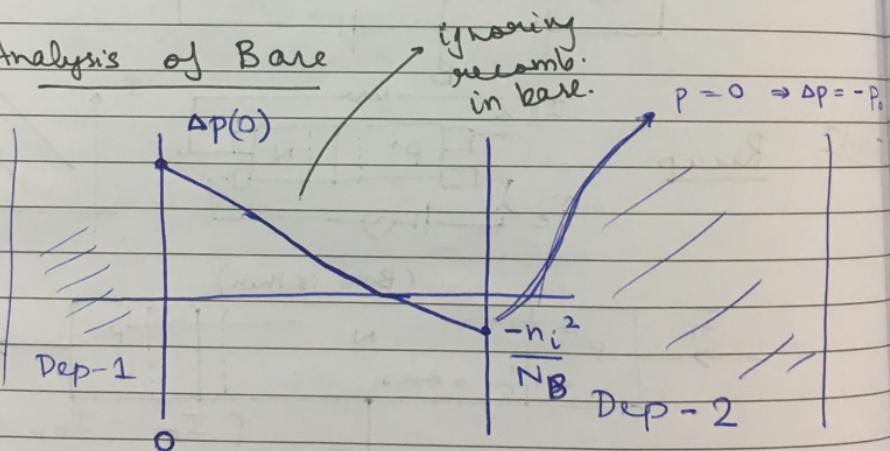




→ Assumption :  $I = \sqrt{DE} \gg W_B$

$$\Rightarrow I_{En} = q A \frac{n_i^2}{N_E} \cdot D_E \frac{(D_e)_E}{(L_e)_E} \left( e^{\frac{V_{EB}}{kT}} - 1 \right)$$

Analysis of Base



$$\Delta P(0) = \frac{n_i^2}{N_B} \left( e^{\frac{V_{EB}}{kT}} - 1 \right)$$

$$I_{cp} = q A (D_p)_B \frac{\Delta P(0)}{W}$$

Now considering recombination

$$I_{\text{recombination}} = \frac{w}{2} \int_{-W/2}^{W/2} \Delta n \frac{\Delta p(0)}{I} dx$$

Area under  
 $\Delta p/p$   
profile

$$I_{EP} = I_{CP} + \frac{qA \Delta p(0) \cdot w}{2T}$$

$$\alpha_T = \frac{I_{CP}}{I_{EP} + ( )} = \frac{D_B/w}{D_B/w + w/2T_B}$$

$$= \frac{1}{1 + \frac{w^2}{2D_B T_B}}$$

$$Y = \frac{1}{1 + \frac{D_E}{D_B} \frac{w}{L_E} \frac{N_B}{N_E}}$$

$$Y = \frac{I_{EP}}{I_E}, \quad Y = \frac{qAD_B \Delta p(0)/w}{qAD_B \Delta p(0)/w + qAn_i^2 \frac{D_E}{N_E} ( )}$$

$$I_C = \frac{I_{CP}}{I_{EP}} \cdot \frac{I_{EP}}{I_E} \cdot I_E = (Y \alpha_T) I_E$$

$$I_C = \alpha_{DC} I_E$$

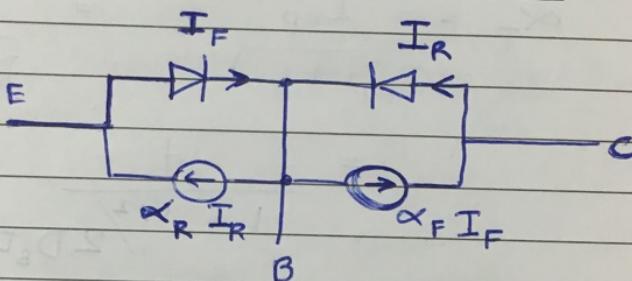
$$I_E = I_C + I_B$$

$$I_E = \alpha I_E + I_B$$

$$I_B = (1 - \alpha) I_E$$

$$\beta = \frac{I_C}{I_B} = \left( \frac{\alpha}{1 - \alpha} \right)$$

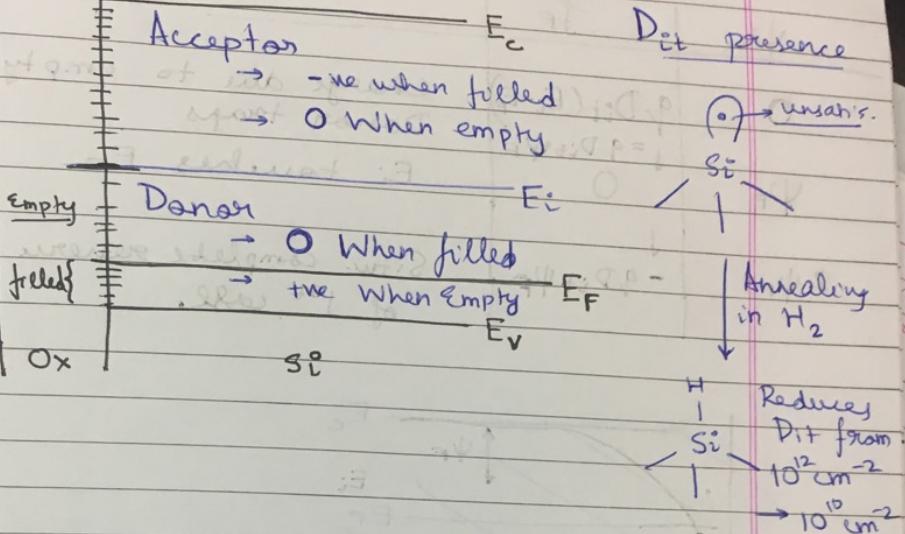
→ Ebers - Moll Model of BJT



# Interface States

$\rightarrow f_F$  (charge)

Namen.  
based on  
NA, ND  
behaviour



→ Shift in Charge.

$$V_g = \psi_s + E_{ox}x_{ox} \quad - \text{Always valid}$$

using

$$\epsilon_{ox}E_{ox} = \epsilon_{si}E_{si}, \text{ we write } \left\{ \begin{array}{l} \text{In absence} \\ \text{of Interface} \\ \text{charges.} \end{array} \right.$$

$$V_g = \psi_s + \frac{Q_{si}}{C_{ox}}$$

→ Electrostatics in presence of  $f_F$

$$\epsilon_{ox}E_{ox} = \epsilon_{si}E_{si} + f_F$$

$$= Q_{si} + f_F$$

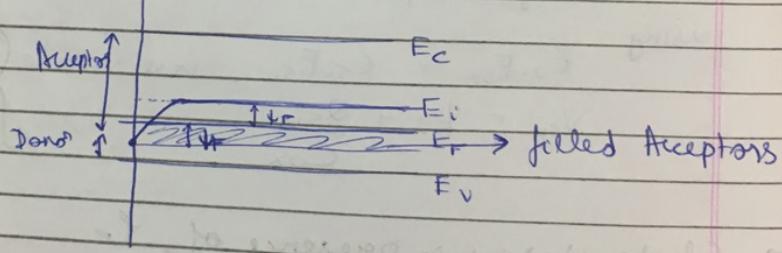
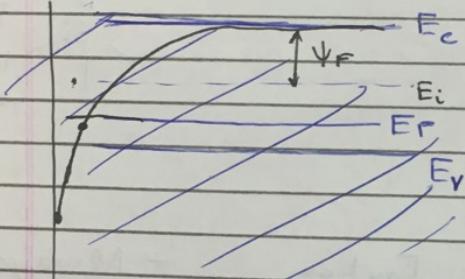
$$\Rightarrow V_g = \psi_s + \left[ \frac{Q_{si}}{C_{ox}} + \frac{f_F}{C_{ox}} \right]$$

voltage modulated

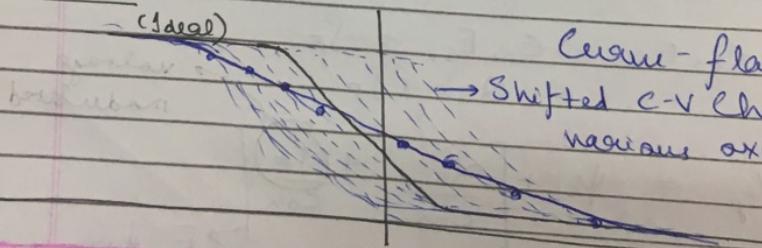
$E_{ox}x_{ox}$

$\rightarrow D_{it}$  : Taken as uniform (Approx.)  
units:  $\text{cm}^{-2} \text{eV}^{-1}$

$\psi_s$	$\psi_F$	
0	$qD_{it}(E_i - E_F)$ $\downarrow = qD_{it}\psi_F$	Charge due to empty Donor traps
$\psi_F$	0	$E_i$ touches $E_F$
$2\psi_F$	$-qD_{it}\psi_F$	Sitw. complete generation of 1 <sup>st</sup> coll.



C-V Char.



Shifted C-V Char. for  
various oxide charges

$$\Psi \rightarrow \Psi + \Delta \Psi$$

$$I \rightarrow I + \Delta I$$

$$\Delta V_g + \frac{q D_{it} \Delta \Psi}{C_{ox}}$$