TUTORIAL 1

Exercises

1. A complex polynomial of degree n has exactly n roots. (Assuming fundamental theorem of algebra)

2. Show that a real polynomial that is irreducible has degree at most two. i.e., if

$$f(x) = a_0 + a_1 x + \ldots + a_n x^n, \;\; a_i \in \mathbb{R},$$

then there are non-constant real polynomials g and h such that $f(x) = g(x)h(x)$ if $n \geq 3$.

3. Show that a subset $S \subseteq \mathbb{C}$ is open if and only if $S \cap \partial(S) = \emptyset$.

4. Check for differentiability and holomorphicity:

    1. $f(z) = c$
    2. $f(z) = z$
    3. $f(z) = z^n, n \in \mathbb{Z}$
    4. $f(z) = \mathrm{Re}(z)$
    5. $f(z) = |z|$
    6. $f(z) = |z|^2$
    7. $f(z) = \bar{z}$
    8. $f(z) = \begin{cases} \frac{z}{\bar{z}} & \text{if } z \neq 0 \\ 0 & \text{if } z = 0. \end{cases}$