Network Theory Homeworks 6 and 7

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Homework is not to be submitted. If you prepare a solution and upload it to Moodle, you may be eligible for extra credit as per the course rules.

1. Find the inverse Laplace transform of the following functions. [Hint: Use the uniqueness prperty of the Laplace transform to find the solution, instead of performing the complex inversion integral.] The functions are: $s, s^2, 1, 1/(s-z_0)$, and $1/(s^2+as+b)$ where $z_0, a, b \in \mathbb{C}$.

2. Let $g(s)$ and $h(s)$ be polynomials with real positive coefficients. Describe an algorithm to compute the inverse Laplace transform of $g(s)/h(s)$ using the inverse Laplace transform of known functions from Q1, and the linearity of the Laplace transform. [Hint: Give a partial fraction expansion for $g(s)/h(s)$.

3. Suppose $F(s) = \frac{c(s-z_1)(s-z_2)\ldots(s-z_m)}{(s-p_1)(s-p_2)\ldots(s-p_n)}$ where $c, z_1, \ldots, z_m, p_1, \ldots, p_n \in \mathbb{C}$. Considering the three cases $m < n$, $m = n$, and $m > n$, describe the poles and zeros of F, including those at infinity. Verify that in all three cases, the number of poles equals the number of zeros. Verify that given c and the poles and zeros, $F(s)$ can be uniquely recovered.

4. We are given a one-port network with a Thevenin impedance of $Z_{th}(s)$ in the transform domain. Suppose the network is made up of resistors, capacitors and inductors, and we know its circuit diagram. Describe how to synthesize another network with Thevenin impedance of $Z_{th}(1/s)$. Show how your synthesis works on a series RLC circuit, and on a second circuit with R, L, C in parallel.

5. Compute the Laplace transform of $\cos^2 2t$.

6. Problems 7-1 to 7-20, 7-24, 7-29, 7-37, 7-49, 7-51, 7-52, 7-53 (insert an equality symbol between $\frac{f(t)}{t}$ and the integral sign), 7-54, 8-17, 8-18, 8-22, 8-25, 8-26, 8-27, 8-34, 8-36, 9-3, 9-4, 9-17, 9-18, 9-20, 9-26, 9-30, 9-33 from the textbook.