TUTORIAL 2

1. Show that $f(z) = e^x(\cos y + \imath \sin y)$ is holomorphic throughout $\mathbb{C}$.

2. Show that the CR equations take the form

$$u_r = \frac{1}{r} v_\theta \ \& \ v_r = -\frac{1}{r} u_\theta$$

in polar coordinates.

3. If u and v are harmonic conjugates of each other, show that they are constant functions.

4. Show that $u = XY + 3X^2Y - Y^3$ is harmonic and find its harmonic conjugate.

5. Find the radius of convergence of the following power series :

a) $\sum_{k=1}^{\infty} kz^k$

b) $\sum_{p \ prime} z^p$

c) $\sum_{k=1}^{\infty} k!z^k/k^k$

6. Give an example of a series which can be shown to be convergent by root test but not by ratio test.