

TUTORIAL 6

1. Evaluate $\int_0^{2\pi} \frac{\cos^2(3x)dx}{5-4\cos(2x)}$.
2. Evaluate $\int_{|z-2|=4} \frac{2z^3+z^2+4}{z^4+4z^2}$.
3. Show with and without using open mapping theorem that if $f(z)$ is a holomorphic function on a domain such that $|f(z)|$ is constant, then $f(z)$ is constant.
If $|f(z)|$ is constant so $f(z)$ is bounded and $f(z)$ is entire so constant
4. Show that any meromorphic function on \mathbb{C} is open. Show that if $f(z)$ is an entire function such that $f(z)^2 = \overline{f(z)}^3 z$, then $f(z)$ is identically zero.
5. How many roots does $z^9 + z^5 - 8z^3 + 2z + 1$ have in the annulus with inner and outer radii 1 and 2 resp. **6**