TUTORIAL 3

1. Prove that:
$\sin z = \sin x \cosh y + i \cos x \sinh y$ and
$\cos z = \cos x \cosh y - i \sin x \sinh y$,

2. Letγ be the boundary of the triangle $\{0 < y < 1 - x;\ 0 \leq x \leq 1\}$ taken with the antoclockwise orientation. Evaluate:
a) $\int_\gamma Re(z)dz$
b) $\int_\gamma z^2 dz$

3. Show that $\sin, \cos : \mathbb{C} \to \mathbb{C}$ are surjective. (In particular, note the difference with real sine and cosine which were bounded by 1). How often does it attain a given value ?

4. Let γ the circle with radius R around the origin with counter-clockwise orientation. Compute the following integrals :
a) $\int_\gamma z^m dz, m \in \mathbb{Z}$ 0
b) $\int_\gamma \bar{z}^m dz, m \in \mathbb{Z}$ 2pie i m=1 rest 0
c) $\int_\gamma |z|^m dz, m \in \mathbb{Z}$ 0

5. Show that if D is a bounded domain with smooth boundary,

$$\int_{\partial D} \bar{z} dz = 2i \text{Area}(D)$$