

## TUTORIAL 4

1. Show that there is a strict inequality

$$\left| \int_{|z|=R} \frac{z^n}{z^m - 1} dz \right| < \frac{2\pi R^{n+1}}{R^m - 1}; \quad R > 1, m \geq 1, n \geq 0$$

2. A power series with center at the origin and positive radius of convergence, has a sum  $f(z)$ . If it is known that  $f(\bar{z}) = \overline{f(z)}$  for all values of  $z$  within the disc of convergence, what conclusions can you draw about the power series?

3. This is called Taylor series with remainder :

$$f(z) = f(0) + zf'(0) + \frac{z^2}{2!}f''(0) + \cdots + \frac{z^N}{N!}f^{(N)}(0) + \frac{z^{N+1}}{(N+1)!} \int_0^1 (1-t)^N f^{(N+1)}(tz) dt$$

Use this to prove the following inequalities :

a)  $\left| e^z - \sum_{n=0}^N \frac{z^n}{n!} \right| \leq \frac{|z|^{N+1}}{(N+1)!}; \quad \operatorname{Re}(z) \leq 0. \quad \text{Text}$

b)  $\left| \cos(z) - \sum_{i=0}^N \frac{(-1)^i z^{2i}}{2i!} \right| \leq \frac{|z|^{2N+2} \cosh R}{(2N+2)!}; \quad |\operatorname{Im}(z)| \leq R$

4. By computing  $\int_{|z|=1} \left(z + \frac{1}{z}\right)^{2n} \frac{dz}{z}$ , show that  $\int_0^{2\pi} \cos^{2n} \theta d\theta = \frac{2\pi}{4^n} \frac{2n!}{n!^2}$ .

5. Locate and classify the singularities of the following :

a)  $\frac{z^5 \sin(1/z)}{1+z^4}. \quad \text{4pole of order 1 and 1 essential singularity}$

b)  $\frac{z^2+z+1}{z^3-11z+13}$