EE 207. LECTURE NOTES - PROF. P.R. NAIR

EE, IITB.

TOPIC: CARRIER TRANSPORT.

OVERVIEW: We first define drift & diffusion based transport. Then the continuity equations are derived. Further analysis is done based on minority carrier diffusion equation.

A * DRIFT based carrier transport: ~~Here~~ The free carriers in a semiconductor sample are under random thermal motion. As such this do not lead to a directed flow in any direction. Once an external electric field is applied, the carriers will acquire a velocity accordingly. However, they still undergo random collision which prevent them from accelerating too much.

If τ_c is the mean time between two consecutive collisions, then the velocity (mean) acquired during this period is

$$\langle v \rangle = \frac{q \tau_c E}{2m^*}$$

m^* - effective mass.

μ - mobility

But $\langle v \rangle = \mu E$

$$\Rightarrow \mu = \frac{q \tau_c}{2m^*}$$

With the above definition of velocity, the current is given as

$$J_{p,drift} = q\mu_p \cdot p \cdot E$$

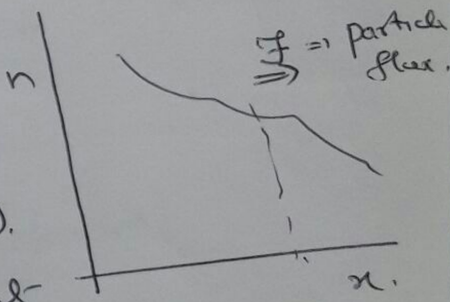
$$J_{n,drift} = q\mu_n \cdot n \cdot E$$

μ_p - hole mobility

μ_n - electron mobility

B DIFFUSION BASED TRANSPORT

Diffusion, by definition, denotes random walk of particles. However, if a concentration gradient exists, there could be a net flow. Following the derivation in ADSF, we find,



$$\mathcal{F} = -D\frac{\partial n}{\partial x}$$

where $D = \frac{v_{th}^2 \tau_c}{2}$ (see ADSF).

is the diffusion coefficient

v_{th}, the thermal velocity

& τ_c the mean time between collision

Accordingly, the diffusion current is given as

$$J_{p,diffusion} = -q D_p \partial p/\partial x$$

$$J_{n,diffusion} = q D_n \partial n/\partial x$$

The total current is then given as

$$J = J_{drift} + J_{diffusion}$$

$$J_n = q n \mu E + q D_n \frac{\partial n}{\partial x}$$

$$J_p = q p \mu_p E - q D_p \partial p/\partial x$$

$$J = J_n + J_p.$$

C EINSTEIN RELATION

We have $\mu = \frac{q\tau_c}{2m^*}$ and $D = \frac{v_{th}^2 \tau_c}{2}$.

Eliminating τ_c between the two relations, we get

$$\frac{D}{\mu} = \left(\frac{v_{th}^2}{m}\right) \times \frac{1}{q}.$$

However, equipartition theorem indicates that $\frac{1}{2}m^* v_{th}^2 = \frac{kT}{2}$, this allows us to re-write the above relation as:

$$\frac{D}{\mu} = \frac{kT}{q}$$ which is known as Einstein's relation.

D FERMI LEVEL AND EQUILIBRIUM

The Einstein relation allows us to obtain crucial insights on the spatial variation of E_F at Equilibrium. Specifically, we have

$$n = n_i e^{(E_F - E_i)/kT}$$

$$p = n_i e^{(E_i - E_F)/kT}$$

With $J_n = q n \mu_n E + q D_n \partial n/\partial x$.

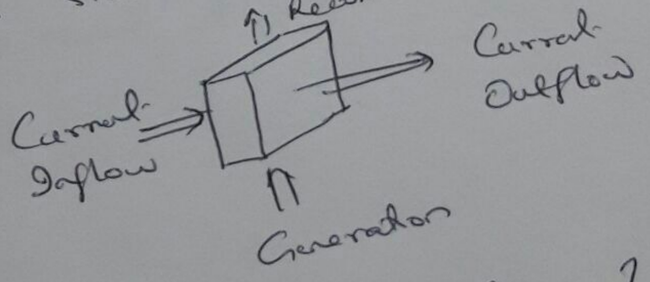
and $E = \frac{1}{q}\frac{dE_i}{dx}$, we obtain

$$\frac{dE_F}{dx} = 0$$ at Equilibrium Conditions

E. CONTINUITY EQUATIONS

The drift-diffusion equations for electron and hole are coupled to each other through the electric field and ~~the~~ explicitly through the Poisson equation. In addition, the generation recombination processes also influence the carrier density and a formalism is needed to account for all the processes.

Specifically, if we consider an incremental volume of semiconductor, then the conservation of particles should be valid. This leads to



$$\left.\begin{aligned}\frac{\partial n}{\partial t} &= \frac{1}{q}\nabla\cdot J_n - R + G\\ \frac{\partial p}{\partial t} &= -\frac{1}{q}\nabla\cdot J_p - R + G\end{aligned}\right\}$$ Continuity equations.

Also, the Poisson's equation is equally important

$$\nabla\cdot(\varepsilon E) = p - n + N_d^+ - N_a^-$$

F. QUASI FERMI LEVELS AND CURRENT

We have $n = n_i e^{(E_{Fn}-E_i)/kT}$

and $p = n_i e^{(E_i-E_{Fp})/kT}$.

With $J_n = qn\mu E + q \cdot D_n \frac{\partial n}{\partial x}$ and

D/μ the Einstein relation and $E = \frac{1}{q}\frac{\partial E_i}{dx}$

We obtain

$$J_n = qn\mu_n \frac{\partial E_{Fn}}{\partial x} \quad \text{and}$$

$$J_p = qp\mu_p \frac{\partial E_{Fn}}{\partial x}$$

i.e., the slope of Quasi-Fermi levels act as the driving force for carrier transport in steady state condition (accounts for both drift + diffusion)

G.

SPECIFIC EXAMPLES

A ⊛ RESISTOR. – Drift based transport.

We have $J_{drift} = qn\mu_n E + q\mu_p p E$.

In the absence of diffusion, the above is the only current component. Hence we obtain

$$J = \sigma E$$

where $\sigma = q(\mu_n n + \mu_p p)$.

The above equation indicates that conductivity scales with carrier density and hence the importance of doping. However, mobility is a function of both doping and ~~ee~~ Temperature. The details ~~of the~~ on the above dependence of mobility on carrier density & Temperature is provided in ADSF.

The combination $\mu_n n$ & $\mu_p p$ always appear together in any current calculation and hence ~~ee~~ both mobility & carrier density cannot be uniquely obtained through current - voltage measurements. In this regard Hall measurements are immensely useful. (details provided in ADSF).

* The photoconductivity is an associated topic for discussion

B. DIFFUSION BASED Transport

In the absence of any electric field, and with $R = G = 0$, Under steady state condition, the ~~indicate that~~ continuity equations indicate that

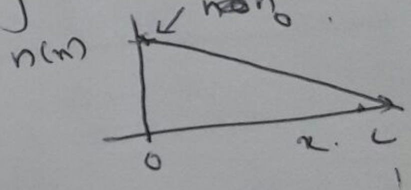
$$0 = D_n \frac{\partial^2 n}{\partial x^2}.$$

(Similarly $0 = D_n \partial^2 p/\partial x^2$)

We now list a few examples with the above system

B1. Sample with constant source diffusion

Let 'L' be the length of a sample with "Dirichlet" boundary condition



$n(0) = n_0$

$n(L) = 0$

the solution of steady state equation is linearly varying with 'x'. (find the coefficients).

* Find out the diffusion current
* Is the current the same irrespective of "x"? What does this indicate.

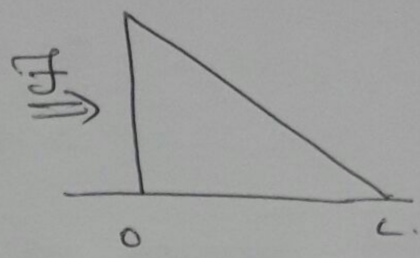
yes this indicate that is constant

* ~~Bo~~

B2. The above system with a constant current (or particle flux)

Boundary condition

$$-D_n \frac{\partial n}{\partial x}\Big|_{x=0} = J$$



$$n(L) = 0.$$

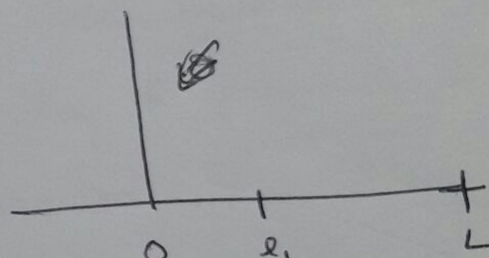
* Find the particle profile.
* Evaluate the current

B3. The above sample with constant generation over a certain region

$R = 0 \quad 0 < x < L.$

$G = G_0 \quad 0 < x < l_1$

$G = 0 \quad l_1 < x < L$



* Write the steady state eqn. & B. Conditions
* Find the particle profile
* Find the particle current
* Can the above be solved without ?

B4. Same sample with recombination and no generation

$R = \frac{n}{\tau} \quad 0 < x < L.$

$G = 0 \quad 0 < x < L$

~~Repeat~~ * Attempt the sub question of B3.

* Is there any characteristic length?

B5. The same sample with both R & G.

$R = n/2 \quad 0 < x < l$

$G = G_0 \quad 0 < x < L.$

~~B6~~ ~~Solve~~ (*) Attempt the previous sub question

B6. Analyze the above problems for

(*) Semi-infinite samples

(*) Infinite samples.

B7. Extend the above to minority carrier diffusion equation with R given by SRH recombination mechanism.