

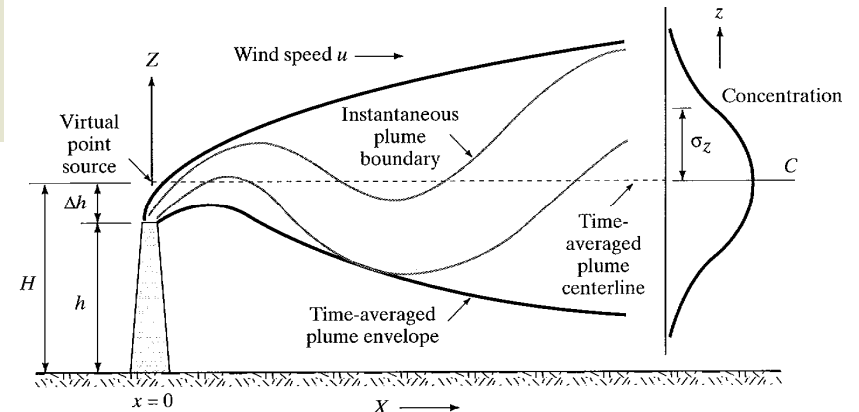
# Lecture 6

## Air Quality:

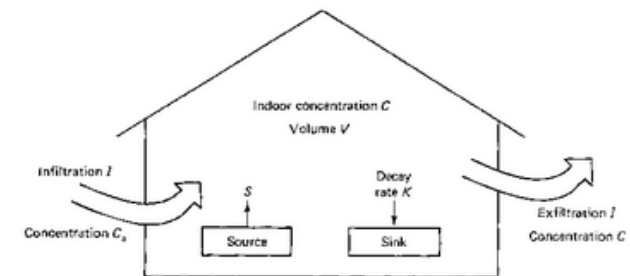
# Air Pollution Modeling

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$$C(x, y, 0) = \left[ \frac{Q}{\pi \sigma_y \sigma_z u_H} \right] \left[ \exp \left[ -\frac{1}{2} \left( \frac{y}{s_y} \right)^2 \right] \right] \left[ \exp \left[ -\frac{1}{2} \left( \frac{H}{s_z} \right)^2 \right] \right]$$

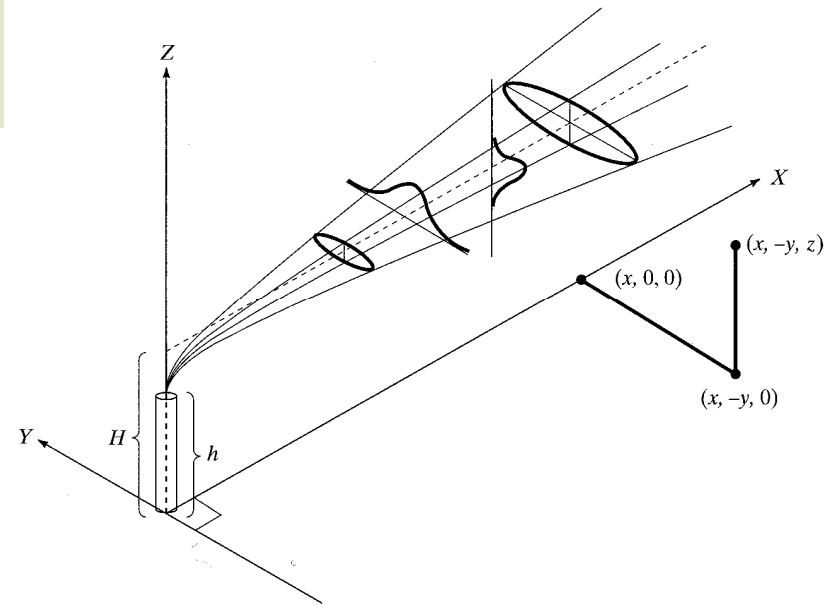


accumulation rate = input rate + sources - output rate - decay

$$V \frac{dC}{dt} = S + C_a IV - CIV - KCV$$

# Recap 1

- Air pollution monitoring is crucial for true understanding of the state of the environment but impossible to have wide spatial coverage, hence air quality modeling is needed
- **Gaussian Plume (Dispersion) Model (GPM)** is used to estimate the ground level concentrations for pollutants coming from a chimney
- Inputs to GPM are: height of chimney; source emission strength; wind rose data; atmospheric stability of the region



$$\chi(x, y, 0, H) = \left[ \frac{E}{\pi s_y s_z u} \right] \left[ \exp \left[ -\frac{1}{2} \left( \frac{y}{s_y} \right)^2 \right] \right] \left[ \exp \left[ -\frac{1}{2} \left( \frac{H}{s_z} \right)^2 \right] \right]$$

# Recap 2

- Assumption
  - pollutants released from a “virtual point source”
  - advective transport by wind
  - dispersive transport (spreading) follows normal (Gaussian) distribution away from trajectory
  - constant emission rate
  - wind speed constant with time and elevation
  - pollutant is conservative (no reaction)
  - terrain is flat and unobstructed
  - uniform atmospheric stability
- Effective stack height

$$H = h + \Delta H$$

$$\Delta H = \frac{v_s d}{u} \left[ 1.5 + \left( 2.68 \times 10^{-3} (P) \left( \frac{T_s - T_a}{T_s} \right) d \right) \right]$$

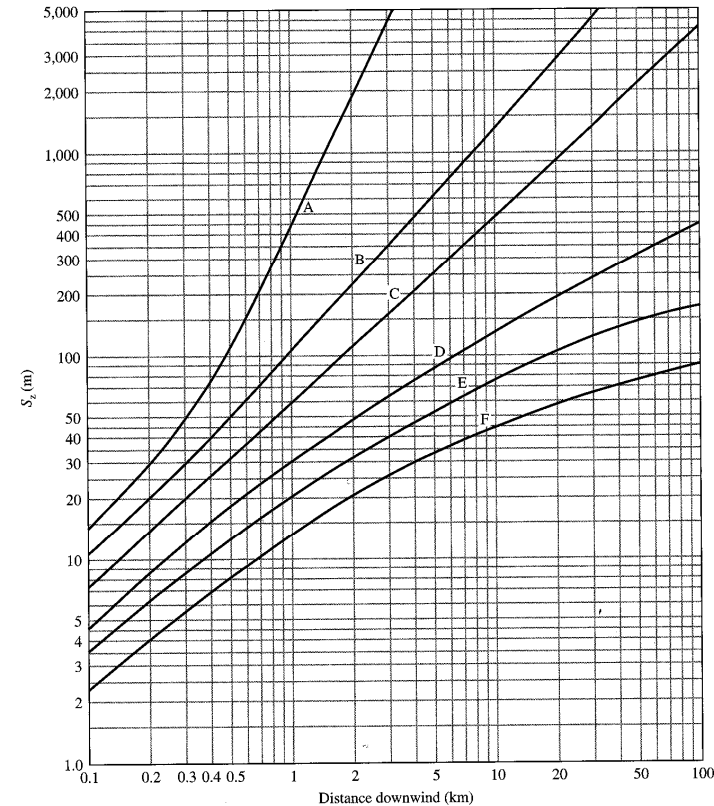
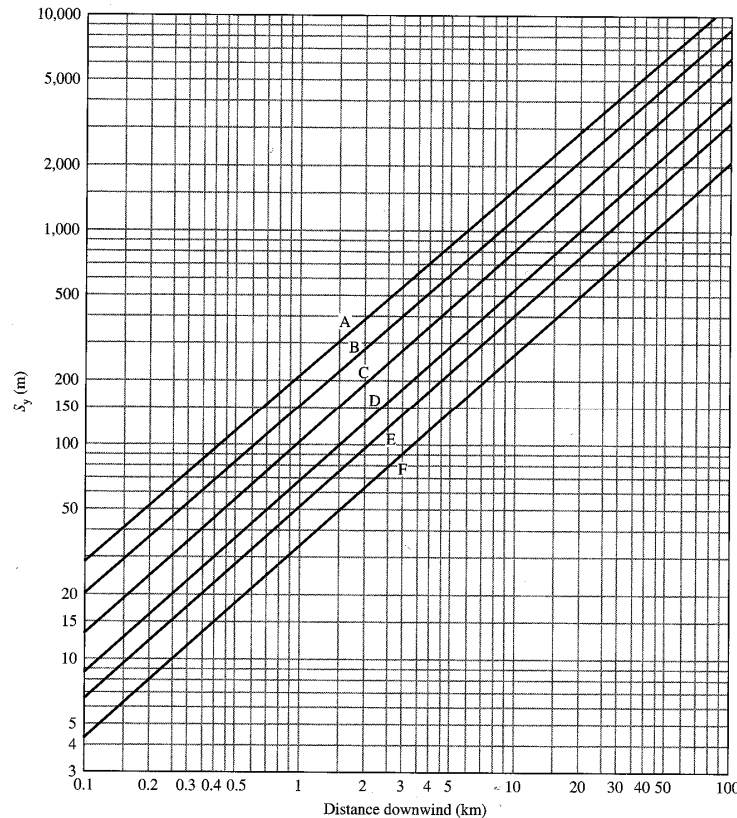
Holland's formula

# Recap 3

$$\sigma_y = a x^{0.894}$$

$$\sigma_z = cx^d + f$$

- Estimation of horizontal dispersion ( $s_y$  and  $s_z$ )



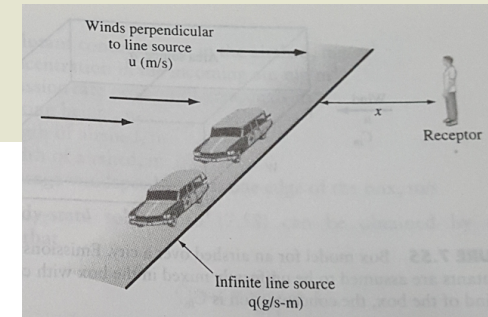
- Estimation of wind speed at stack height
- Model uncertainty could be up to 50%

$$u_2 = u_1 \left( \frac{z_2}{z_1} \right)^p$$

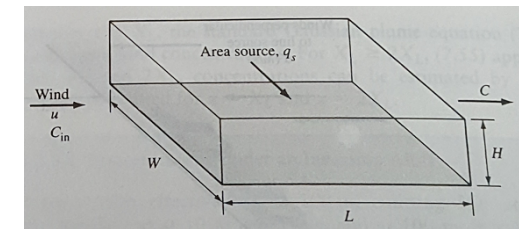
# Recap 4

- When source is distributed along a line with continuous emissions, ground level conc. of pollutant at a perpendicular distance  $x$  can be obtained by:

$$C(x) = \frac{2q}{\sqrt{2\pi} \sigma_z u}$$



- For distributed sources spread over an area, **box model** can be used, assuming uniform mixing in the box, rate of change of pollutants in the box:



$$LWH \frac{dC}{dt} = q_s LW + WHu C_{in} - WHu C$$

$$C(\infty) = \frac{q_s L}{uH} + C_{in}$$

$$C(t) = \left( \frac{q_s L}{uH} + C_{in} \right) (1 - e^{-ut/L}) + C(0)e^{-ut/L}$$

# Today's Learning Objective !

- To learn about air quality modeling methods

## Box Model: Example !

In a square city of 15 km a side, there are 20,000 cars on the road, each driven 30 km between 4-6pm, and each emitting 3 g/km of CO. It's a clear winter evening with radiation inversion restricting mixing height to 20 m. The wind is bringing clean air at a steady rate of 1.0 m/s, along an edge of the city. Using box model estimate CO at 6 pm if there was no CO in air at 4 pm, and only source of CO is cars. Assume CO is conservative and uniform mixing in the box.

First, calculate the CO emissions,  $q_s$  (in units of mass/area-time):

$$q_s = \frac{2000 \text{ cars} \times 30 \text{ km} / \text{car} \times 3 \text{ g} / \text{km}}{(15 \times 10^3 \text{ m})^2 \times 3600 \text{ s} / \text{hr} \times 2 \text{ hr}}$$
$$= 1.1 \times 10^{-5} \text{ g} / \text{s} - \text{m}^2$$

## Box Model: Example !

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$$C(t) = \left( \frac{q_s L}{uH} + C_{in} \right) (1 - e^{-ut/L}) + C(0)e^{-ut/L}$$

Hence,

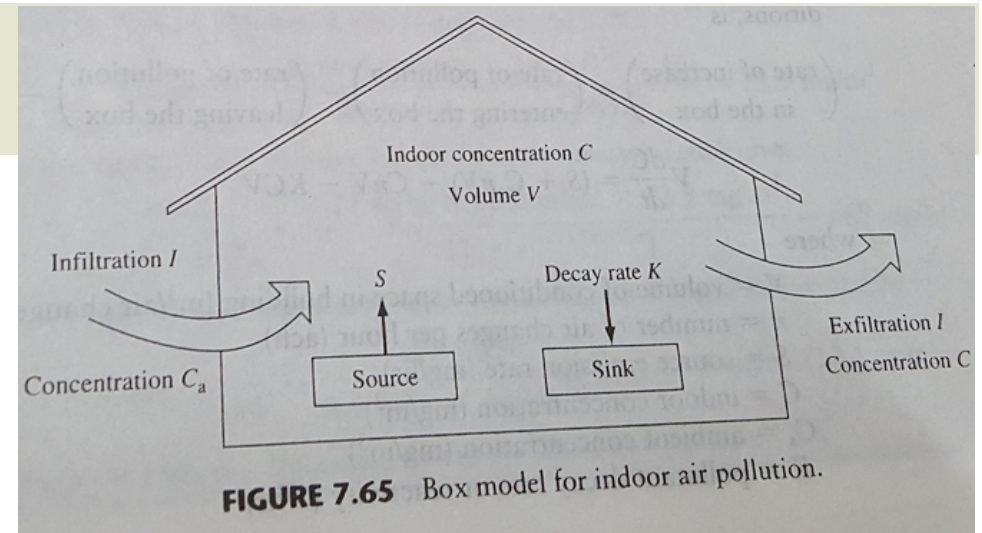
$$C(t) = \frac{q_s L}{uH} (1 - e^{-ut/L})$$

$$C(2hr) = \frac{1.1 \times 10^{-5} \text{ g / s - m}^2 \times 15 \times 10^3 \text{ m}}{(15 \times 10^3 \text{ m})^2 \times 3600 \text{ s / hr} \times 2 \text{ hr}} \left[ 1 - \exp \left( \frac{-1.0 \text{ m / s} \times 7200 \text{ s}}{15 \times 10^3 \text{ m}} \right) \right]$$
$$= 3.2 \times 10^{-3} \text{ g / m}^3$$



# Indoor Box Model

- Similar to urban airshed, a box model can be used for indoor environments
- e.g. a basic mass balance for pollution in the building/room, assuming well-mixed conditions:



$$\left( \begin{array}{c} \text{rate of increase} \\ \text{in the box} \end{array} \right) = \left( \begin{array}{c} \text{rate of pollution} \\ \text{entering the box} \end{array} \right) - \left( \begin{array}{c} \text{rate of pollution} \\ \text{leaving the box} \end{array} \right) - \left( \begin{array}{c} \text{rate of decay} \\ \text{in the box} \end{array} \right)$$

$$V \frac{dC}{dt} = (S + C_a n V) - C n V - K C V$$

where

- $V$  = volume of conditioned space in building ( $\text{m}^3/\text{air change}$ )
- $n$  = number of air changes per hour (ach)
- $S$  = source emission rate ( $\text{mg/hr}$ )
- $C$  = indoor concentration ( $\text{mg/m}^3$ )
- $C_a$  = ambient concentration ( $\text{mg/m}^3$ )
- $K$  = pollutant decay rate or reactivity ( $1/\text{hr}$ )

# Indoor Box Model

- Steady-state indoor pollutant conc., by setting  $dC/dt = 0$ , will be:

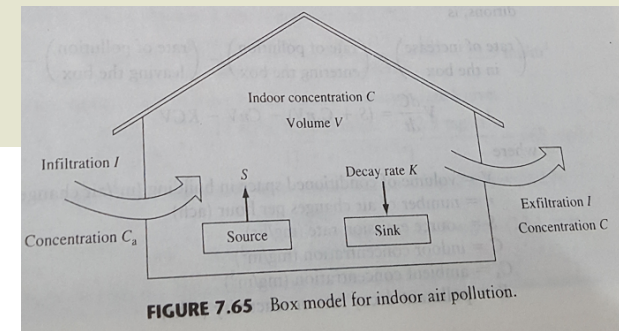
$$C(\infty) = \frac{(S/V) + C_a n}{n + K}$$

- Time-dependant variation of indoor pollutant conc. can be obtained by:

$$C(t) = \left[ \frac{(S/V) + C_a n}{n + K} \right] \left[ 1 - e^{-(n+K)t} \right] + C(0)e^{-(n+K)t}$$

- For pollutants such as CO and NO<sub>2</sub>, which can be treated as conservative (K=0), also, if ambient conc. is negligible (C<sub>a</sub>=0), and before a source is operated indoors (C(0)=0), then:

$$C(t) = \left( \frac{S}{nV} \right) (1 - e^{-nt})$$



**Quiz time !!!**