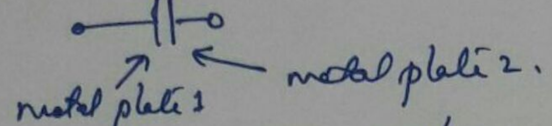
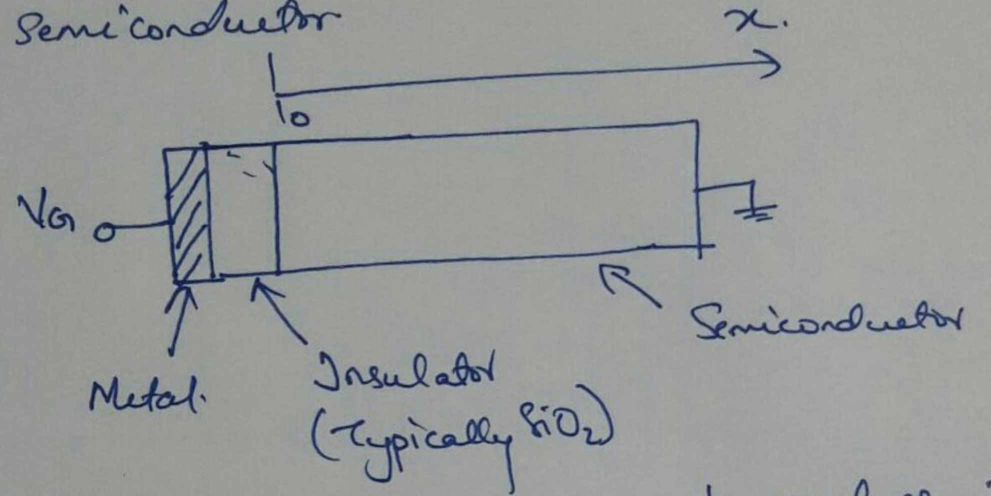
EE 207 - MOS Capacitor 23/10/2017

We know all about parallel plate capacitor.
(shown below).



Here; we replace one of the metal plates with a semiconductor



Here our challenge is to reduce the above
Such that the ~~elect~~ electrostatics could be
understood in simple terms of a capacitor
as shown below.

V_G —||— ψ_s

For the above to be successful, we need
accurate estimates for ψ_s, the potential
in semiconductor and $Q_s(\psi_s)$ the total
charge in semiconductor. This will allow
us to use the following relation.

negative charge is

$$V_g = \psi_s + V_{ox}$$
$$= \psi_s + \frac{Q_s}{\varepsilon_{ox}} \cdot t_{ox}$$

V_{ox} - potential across oxide
ε_{ox} - dielectric constant } oxide
t_{ox} - thickness

$$V_G = \Psi_s + \frac{Q_s}{C_{ox}}.$$

Now the challenge is to obtain Q_s as a function of Ψ_s.

We start with Poisson's Eqn.

$$\nabla \cdot (\varepsilon \nabla \psi) = -q(p - n - N_A).$$

The total charge in semiconductor is given by

$$Q_s = \int_{+\infty}^{0} q(p - n - N_A)\, dx.$$ (refer fig in previous page)

$$\varepsilon \frac{d^2\psi}{dx} = -q(p - n - N_A)$$

(E - Electric field)

This can be re-written as

$$\cancel{\varepsilon d}\ \varepsilon E \frac{dE}{d\psi} = -q(p - n - N_A).$$

$$\varepsilon^2 E\, dE = \frac{-q\varepsilon}{\cancel{\ }} (p - n - N_A)\, d\psi.$$

$\Rightarrow$ Integrating

$$\varepsilon^2 \int_0^{E_s} E\, dE = -q\varepsilon \int_0^{\Psi_s} (p - n - N_A)\, d\psi$$

$$\Rightarrow \quad \varepsilon^2 E_s^2 = 2q\varepsilon \int_0^{\psi_s} (p - n - N_A)\, d\psi.$$

But $(\varepsilon E_s) = Q_s$. a

$$\Rightarrow \quad Q_s^2 = 2q\varepsilon \int_0^{\psi_s} (p - n - N_A)\, d\psi.$$

The above relation allows us to express the total charge as a function of the semiconductor surface potential and the

As discussed in lectures, we have the following relation for carrier densities and

$$p = N_A e^{-q\psi/kT}$$

$$n = \frac{n_i^2}{N_A} e^{q\psi/kT}$$

Depending on the bias applied (V_G), the MOS capacitor can be in different regimes of operation, which are explained in the next section

Case 1 V_G is $-$ve. ψ_s will also be negative in this case.

The most significant charge contribution is due to the holes accumulated in the semiconductor surface. (See the Poisson eqn in previous page).

accumulation

$$\therefore \quad Q_s^2 \simeq (2\epsilon q)\int_0^{\psi_s} p \, d\psi.$$

$$= (2\epsilon q)\int_0^{\psi_s} N_A e^{-q\psi_s/kT} \, d\psi$$

$$\Rightarrow \quad Q_s \simeq \sqrt{2\epsilon q (kT/q) N_A} \; e^{-q\psi_s/2kT}.$$

Note: * In this regime, the semiconductor can support a huge charge by a small change in surface potential.

* You ~~can~~ can check this by comparing the band bending when Q_s changes ~~from~~ from $Q_s = q \times 10^{12}\,cm^{-2}$ to $Q_s = q \times 10^{13}\,cm^{-2}$.

Case 2.

V_G is +ve, ψ_s will be +ve.

This regime can have two distinct characteristics. (A) when $n \ll N_A$ &

(B) when $n \gg N_A$.

(A) when $n \ll N_A$, Poisson's eqn indicates

$$Q_s^2 = 2\varepsilon q \int_0^{\psi_s} N_A \, d\psi.$$

$$Q_s = \sqrt{2\varepsilon q N_A \psi_s} = Q_D$$

The above is the charge due to the ionized impurities (also known as depletion charge).

As we increase the V_G, the band bending increases and n increases. At a band bending of $\psi_s = 2\psi_F$, the n at the surface becomes N_A

[Substitute $\psi_s = 2\psi_F$ in the relation for n in page (3) & convince yourself].

The onset of above condition is known as INVERSION

The charge due to inversion or the minority is given ~~as~~ by, again the Poisson's eqn

$$Q_s^2 \approx 2\varepsilon q \int_0^{\psi_s} n \, d\psi$$

$$\simeq 2\varepsilon q \int_0^{\psi_s} \frac{n_i^2}{N_A} e^{+q\psi/kT} d\psi.$$

$$Q_s^2 = 2\varepsilon q \frac{kT}{q} \frac{n_i^2}{N_A} e^{q\psi_s/kT}$$

$$\Rightarrow \quad Q_s = Q_I = \left[2\varepsilon q \frac{kT}{q} \frac{n_i^2}{N_A}\right]^{1/2} e^{q\psi_s/2kT}.$$

<u>Exercise</u>: 1 Compare the inversion charge with the depletion charge at $\psi = 2\psi_F$

$$\frac{Q_I}{Q_D} \approx \left[\frac{2\varepsilon q \, kT/q \; n_i^2/N_A}{2\varepsilon q N_A \psi_s.}\right]^{1/2} e^{q\psi_s/2kT}$$

$$\approx \left[\frac{kT}{q} \frac{n_i^2}{N_A^2 \psi_s}\right]^{1/2} e^{q\psi_s/2kT.}$$

$$\approx \left[\frac{kT/q}{\psi_s}\right]^{1/2} \frac{n_i}{N_A} e^{q\psi_F/kT.}$$

$$= \left[\frac{kT/q}{2\psi_F}\right]^{1/2} \ll 1$$

& the total charge by inversion layer is still small at $\psi_s = 2\psi_F$

Exercise 2 : Compare the inversion charge & depletion charge at $\psi_s = 2\psi_F + 6kT/q$.

From the above analysis

$$\frac{Q_I}{Q_D} = \left[\frac{kT/q}{2\psi_F + 6kT/q}\right] \frac{n}{N_A} e^{q\psi_F/kT} \cdot e^{\left[\frac{6kT/q}{2kT/q}\right]}$$

$$= \left[\frac{kT/q}{2\psi_F + 6kT/q}\right] \cdot 1 \cdot e^{3}.$$

For $N_A = 10^{16}\ cm^{-3}$; $\psi_F = \frac{kT}{q} \ln\left(\frac{N_A}{n_i}\right) = \frac{kT}{q} \ln\left(\frac{10^{16}}{10^{10}}\right)$

$\psi_F \simeq 360\ mV.$

$$\therefore \frac{Q_I}{Q_D} \simeq \left[\frac{25}{720+150}\right]^{1/2} \cdot e^{3} \simeq \underline{\underline{2}}$$

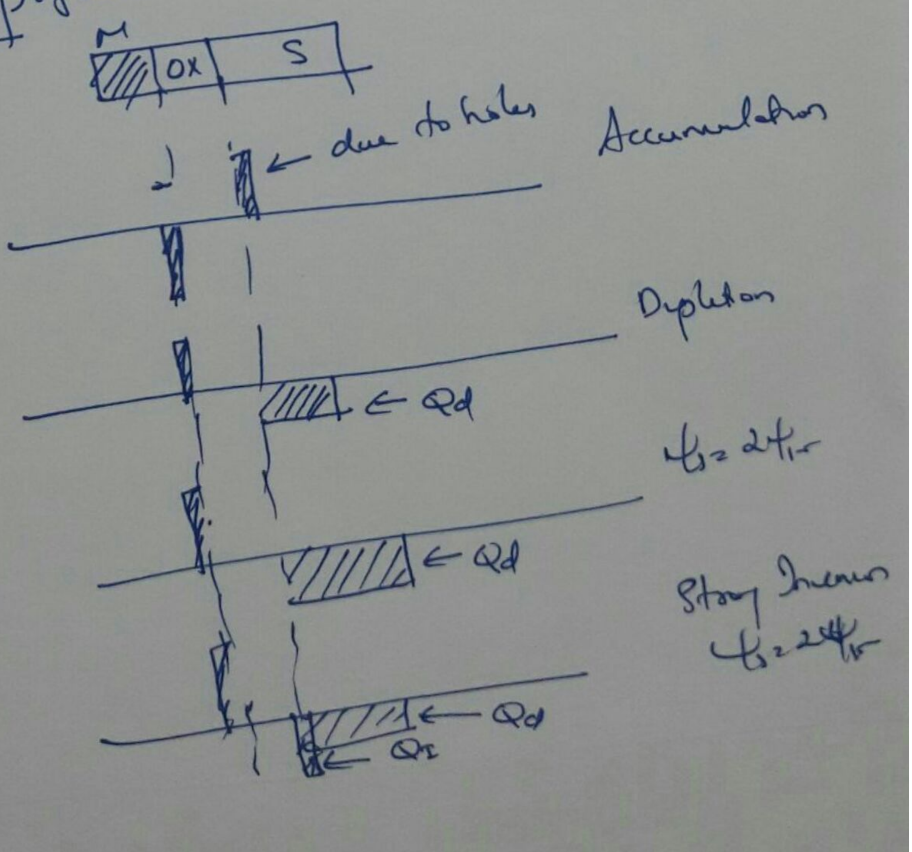
We find that the above ratio >> 1 for any further band bending.

Hence it is usually assumed that the band bending remains a constant at $\psi_s = 2\psi_F$ and any further increase in semiconductor charge is taken up by the inversion. This inversion charge is assumed to be present as a delta function at the semiconductor-oxide interface. (which is a very good assumption & we will address the validity later).

DELTA DEPLETION APPROXIMATION

* $Q_d \propto \sqrt{\psi_s}$ till inversion $\left(Q_d = [2\epsilon q N_A \psi_s]^{1/2}\right)$
* ~~$Q_d = \sqrt{2\epsilon q N_A \psi_s}$ at once~~
* $\psi_s = 2\psi_F$ ~~from~~ once inversion happens
* Q_I becomes the dominant charge once the surface inverts
* Q_d remains a constant $Q_b = (2\epsilon q N_A 2\psi_F)^{1/2}$

Charge density profile



We have

$$V_g = \psi_s + \frac{Q_s}{C_{ox}} \Rightarrow V_T = 2\psi_F + \frac{(2\epsilon q N_A 2\psi_F)^{1/2}}{C_{ox}}$$

The variation in Q_I, Q_D under various biasing regimes are as follows.

Accumulation

ψ_s -'ve.

$Q_A = C_{ox}(V_g - \psi_s)$

$Q_I = 0$

$Q_d = 0$.

Depletion

ψ_s +'ve

$Q_A = 0$

$Q_I = 0$

$Q_d = [2\epsilon q N_A \psi_s]^{1/2}$

Strong Inversion

$\psi_s = 2\psi_F$

$Q_A = 0$

$Q_I = C_{ox}(V_g - V_T)$

$Q_d = [2\epsilon q N_A 2\psi_F]^{1/2}$

Exercise

(A) A MOS capacitor with $N_A = 10^{16}\,cm^{-3}$ is under a bias $V_G = 1V$. ~~Identify the various~~ components & draw the semiconductor charge E-B diagram.

(B) Estimate the V_T of the above MOS cap.

(C) For $V_g = V_T + 2V$, estimate the various charge components.

(D) Estimate the band bending at which the $Q_I = 10\,Q_d$ [Hint: Do not use Delta depletion approx.].

(E). Estimate the approximate thickness of inversion layer. [HINT: let $Q_I = q\,n(x=0)\,W_I$, where W_I is the thickness of inversion layer].

(F). We assumed Boltzmann distribution for carrier densities. Discuss the limitations of this assumption