

EE 207: Electronic Devices

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- Web resources : ECE 606 by Prof. Alam
 - ① Nanohub : "Semiconductor Devices"
- Nanohub.org
- Crystal viewer tool.
- Official text :
- * Advanced semiconductor device fundamen..
- * Semiconductor device fundamentals.

Both by Robert Pierret, Purdue univ.

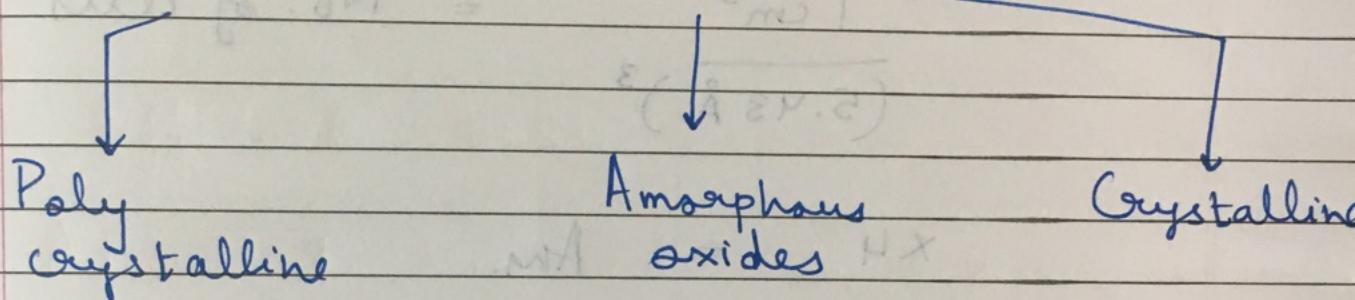
Instructor : Prof. Pradeep. R. Nair

Grading Policy

Endsem	40 - 50 %
Midsem	25 %
Quizzes	20 %

Lecture: Crystal properties and growth of semiconductors

Classification of Solids



→ MOSFET Involves all three.

Solid State

→ Unit cells

- S.C.
- B.c.c
- F.c.c

Atoms/unit cell

1

2

4

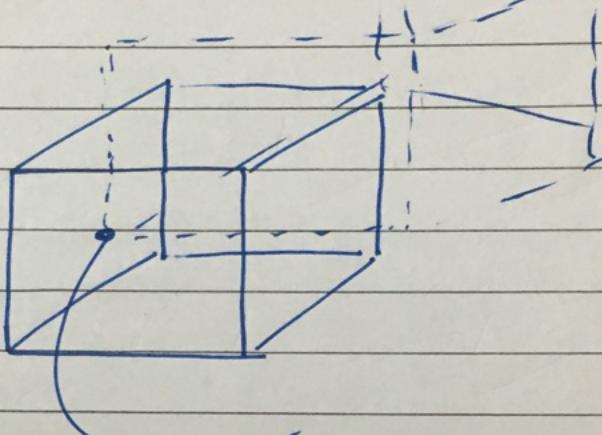
→ Tetrahedral Diamond Structure is in fact F.C.C., with some voids occupied
Visually.

Crystal Structure

→ Crystal \Rightarrow Lattice + Basis

→ Our Interest \rightarrow Si

Si Structure \rightarrow 2 Interpenetrating FCC



$$\left(\frac{a}{4}, \frac{a}{4}, \frac{a}{4} \right)$$

Corner of Shifted Cube

→ Alternatively it is like void occupying JEE cases.

→ Atoms per unit cell for Si special unit cells = 8

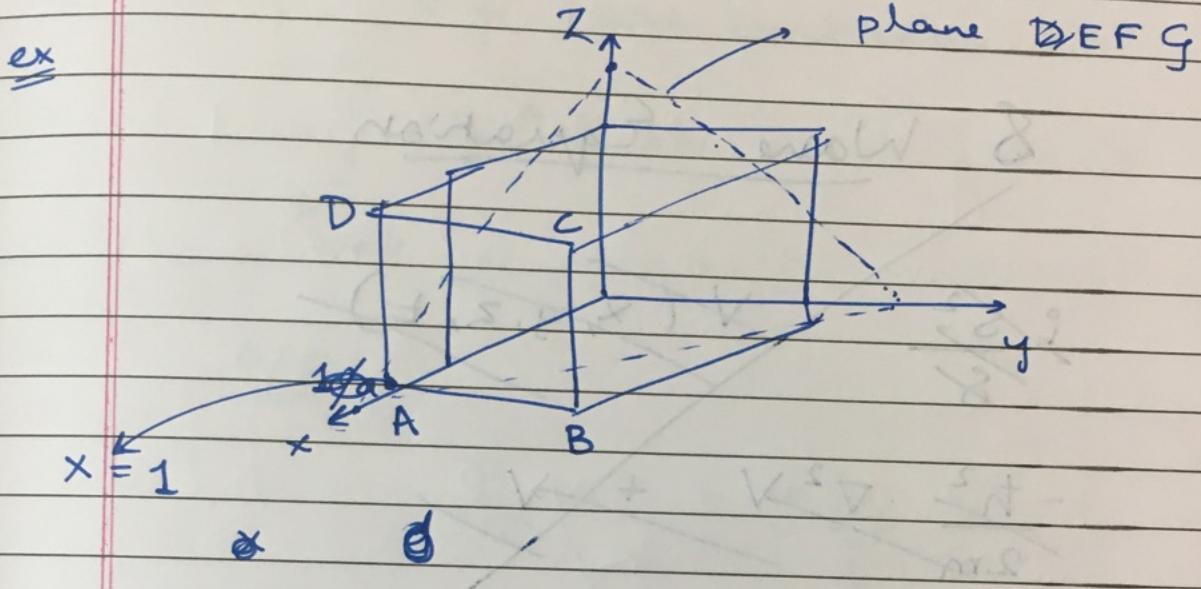
$$4 \left(\frac{1}{8} \times 8 + 6 \times \frac{1}{2} \right) + 4 \text{ atoms}$$

Completely Inside the original box
but from the shifted FCC

Miller Indices

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→ For planes



For planes ABCD

	ABCD	EF G
x	1	3
y	∞	2
z	∞	1

Inverse (Coefficients of the corresponding planes)

	ABCD	EF G
x	1	$\frac{1}{3}$ → 2
y	0	$\frac{1}{2}$ → 3
z	0	1 → 6

Family of planes

$$ax + by + cz = d$$

$(1 \ 0 \ 0) \rightarrow$ For Single plane

$\{1 \ 0 \ 0\} \rightarrow$ Family of // planes

Similarly $(2, 3, 6)$ and $\{2, 3, 6\}$

→ The atom Surface Density depends on how we cut it.

8 Wave Equation

$$\cancel{\frac{i\hbar^2}{8} \psi} + \cancel{\nabla^2 \psi} = \cancel{\nabla V(x, y, z, t)}$$

$$\Psi(x, y, z)$$

→ Wave Equation Recall

$$\Psi(x, y, z, t) \quad \nabla \Psi(x, y, z)$$

$$-\frac{\hbar^2}{2m} \nabla^2 \Psi + \nabla \Psi = \hbar i \frac{S}{St} \Psi$$

$$\left(-\frac{\hbar^2}{2m} \nabla^2 + \nabla \right) = \hat{H}, \quad \text{this is } \hat{E}$$

$$\hat{H} \Psi = \hat{E} \Psi$$

Properties

→ $\psi, \nabla \psi$ are continuous

$$\rightarrow \int_{\text{All space}} \psi^* \psi dV = 1$$

→ O/P

$$P_x = -i \frac{\hbar}{\ell} \frac{\delta}{\delta x}$$

$$H = i \hbar \frac{\delta}{\delta t}$$

$$\langle \alpha \rangle = \int_{-\infty}^{\infty} \psi^* \alpha \psi dx$$

→ Solution to ψ for Crystals.

$$i \hbar \frac{\delta}{\delta t} \psi = \hat{E} \psi \quad \psi \propto e^{-i E_b \frac{t}{\hbar}}$$

$$\text{Let, } \psi = \psi(x) \phi(t)$$

$$\langle E \rangle = E = \int \psi^* \left(-i \frac{\hbar}{\ell} \frac{\delta \psi}{\delta x} \right) dV$$

Lecture - 4

Outline

- Examples of SWE
- Kronig Penny Model (Band Structure)
- Electrons + holes, in the model.

SWE

$$-\frac{\hbar^2}{2m} \nabla^2 \psi + V \psi = \hbar i \frac{\delta \psi}{\delta t}$$

Time Independent

$$\langle E \rangle = E = \underbrace{\int \psi^* \hbar i \frac{\delta \psi}{\delta t} dV}_{\text{constant.}} =$$

$$\hbar i \frac{\delta \psi}{\delta t} = E \psi$$

$$\Rightarrow \psi = \psi(x, y, z) e^{-\frac{iEt}{\hbar}}$$

$$\phi(E)$$

~~$\hbar i \delta \psi / \delta t$~~

→ Take $V = 0$

$$\frac{\hbar^2}{2m} \nabla^2 \psi(x) + E\psi = 0$$

or

$$\frac{\hbar^2}{2m} \frac{d^2\psi}{dx^2} + E\psi = 0$$

$$\frac{d^2\psi}{dx^2} + k^2\psi = 0$$

Solution

where

$$k^2 = \frac{2mE}{\hbar^2}$$

$$\Rightarrow \psi(x) = (A e^{ikx} + B e^{-ikx})$$
$$\phi(t) = e^{-\frac{iEt}{\hbar}}$$

$$\psi(x, t) = (A e^{ikx} + B e^{-ikx}) e^{-\frac{iEt}{\hbar}}$$

→ Boundary Conditions

$\psi, \frac{d\psi}{dx}$ are continuous.

$$\int \psi^* \psi dv = 1$$

Momentum

$$\hat{p} = -\hbar i d/dx$$

$$\Rightarrow p = \hbar k$$

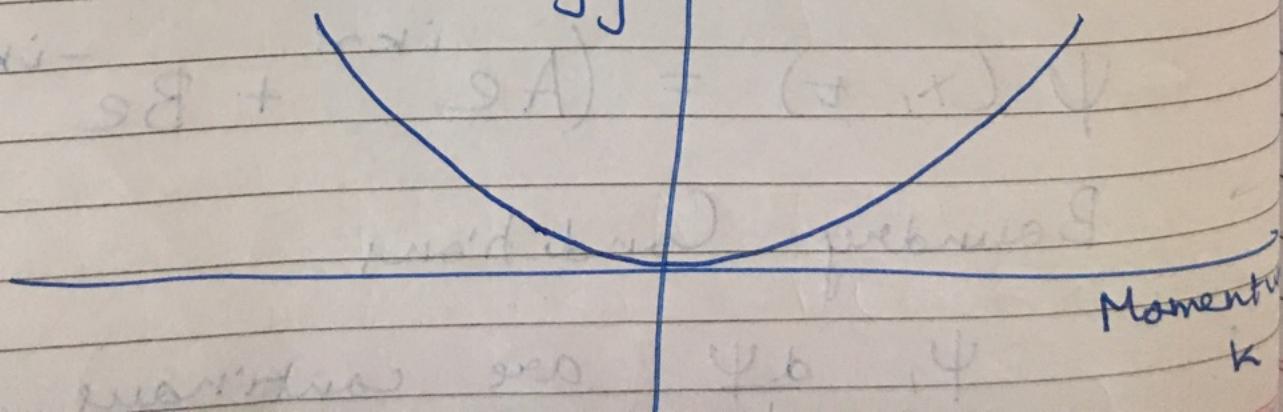
$$p_\psi = \hbar k$$

De-Broglie Idea.

$$p = \hbar k$$

$$E = \frac{\hbar^2 k^2}{2m} = \frac{p^2}{2m}$$

Energy E



→ Topics: P

→ Particle in an finite Infinite Potential well.

i.e

$$V \rightarrow \infty \quad x$$
$$V = 0$$

$$0 < x < a$$

$$V \rightarrow \infty$$

otherwise

Spatial part

$$\psi(0) = 0 \quad \psi(a) = 0$$

Boundary Condition

$$\frac{\hbar^2}{2m} \frac{d^2\psi}{dx^2} + k^2 \psi = 0 \quad 0 < x < a$$

$$k^2 = \frac{2mE}{\hbar^2}$$

$$\psi(x) = A \sin(kx) + B \cos(kx)$$

$$0 < x < a$$

$$x=0, \psi=0 \Rightarrow B=0$$

$$x=a, \psi=0 \Rightarrow A \sin(ka)=0$$

$$ka = n\pi$$

$$ka = n\pi \quad n \in \mathbb{I}/\{0\} \quad \left. \begin{array}{l} \\ \\ \text{or } n \in \mathbb{N} \end{array} \right\} \text{Dipole}$$

$$\Rightarrow k = \frac{n\pi}{a}$$

$$\psi(x) = A \sin\left(\frac{n\pi x}{a}\right)$$

Normalisation

$$A^2 \int_0^a \sin^2\left(\frac{n\pi x}{a}\right) dx$$

$$\frac{A^2}{2} \int_0^a \left(1 - \cos\left(\frac{2n\pi x}{a}\right)\right) dx = 1$$

$$\Rightarrow + \frac{A^2 a}{2} = 1 \quad \psi$$

$$\cancel{A^2} = A = \sqrt{\frac{2}{a}}$$

→ Here +n and -n are mirror images of each other.

$$\hat{P} = -i\hbar \frac{d}{dx}$$

$$\hat{P} \psi = P \psi$$

~~$$= -i\hbar \times \sqrt{\frac{2}{a}} \cos\left(\frac{n\pi x}{a}\right) \times \frac{p \rightarrow k a}{\hbar a}$$~~

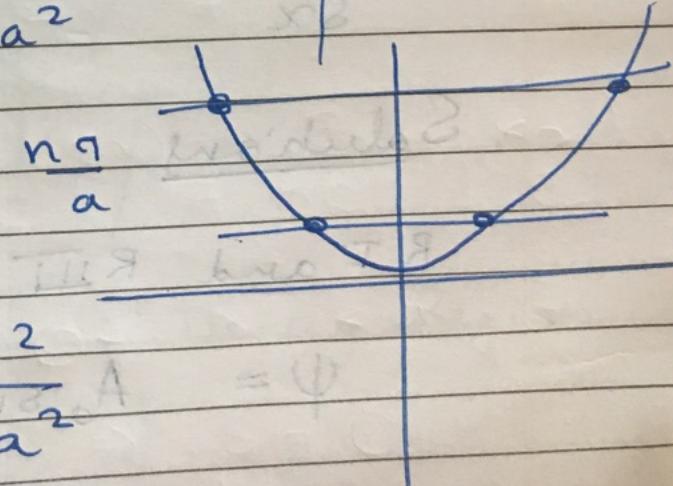
$$\langle P \rangle = \frac{2}{a} \int_0^a \sin\left(\frac{n\pi x}{a}\right) \times -i\hbar \cos\left(\frac{n\pi x}{a}\right) \times p \rightarrow K$$

$$\Rightarrow \langle P \rangle = 0$$

Quantisation
certain
values

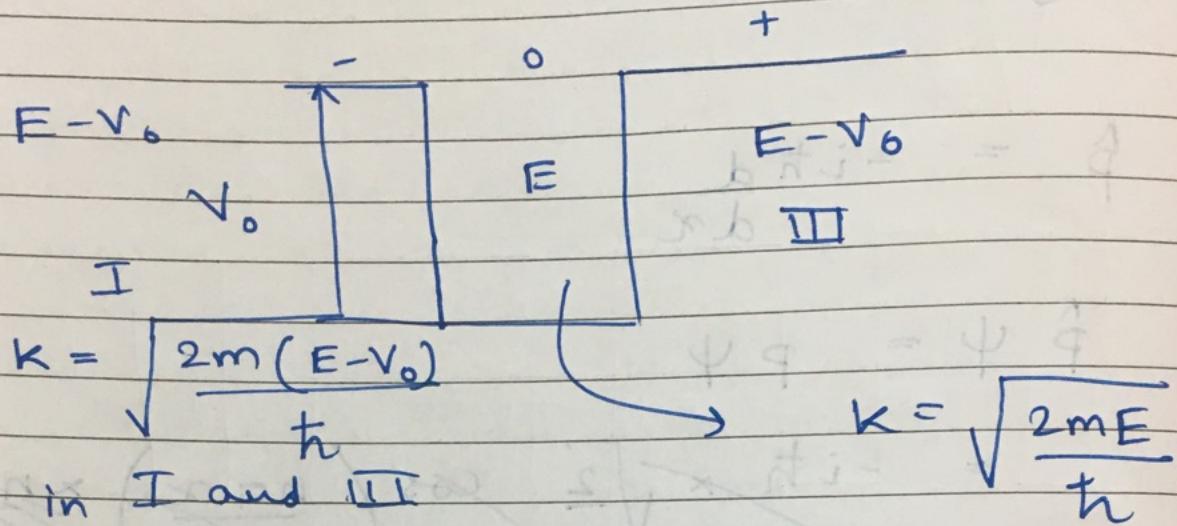
$$E = \frac{\hbar^2 k^2 a^2 n^2 \pi^2}{2ma^2}$$

$$k = \frac{n\pi}{a}$$



$$E = \frac{\hbar^2 k^2 a^2 n^2 \pi^2}{2ma^2}$$

→ Finite Potential Well.



→ Boundary conditions

$$\psi(0^-) = \psi(0^+)$$

$$\frac{s}{sx} \psi(0^-) = \frac{s}{sx} \psi(0^+)$$

$$\psi(a^-) = \psi(a^+)$$

$$\frac{s}{sx} \psi(a^-) = \frac{s}{sx} \psi(a^+)$$

Solutions

R I and R III

$$\psi = A_0 \sin kx + B_0 \cos kx$$

$$x \in (0, a)$$

$$\psi = A e^{-\alpha x} + B e^{\alpha x}$$

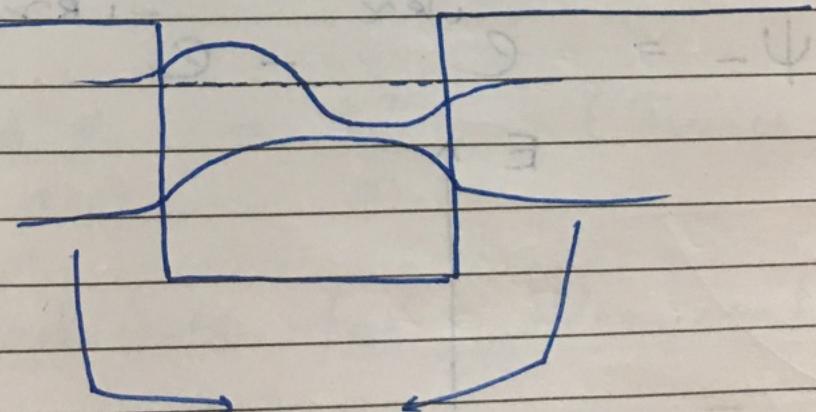
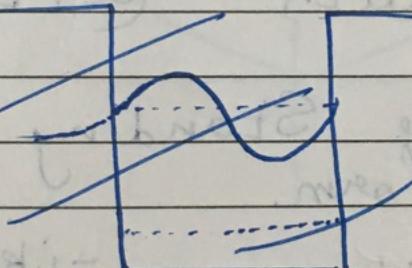
Rewriting

$$\psi_- = A_- e^{-\alpha x} + B_- e^{\alpha x}$$

$$\psi_0 = A_0 \sin kx + B_0 \cos kx$$

$$\psi_+ = A_+ e^{-\alpha x} + B_+ e^{\alpha x}$$

i.e. the wavefunction "leaky" into the classically forbidden region

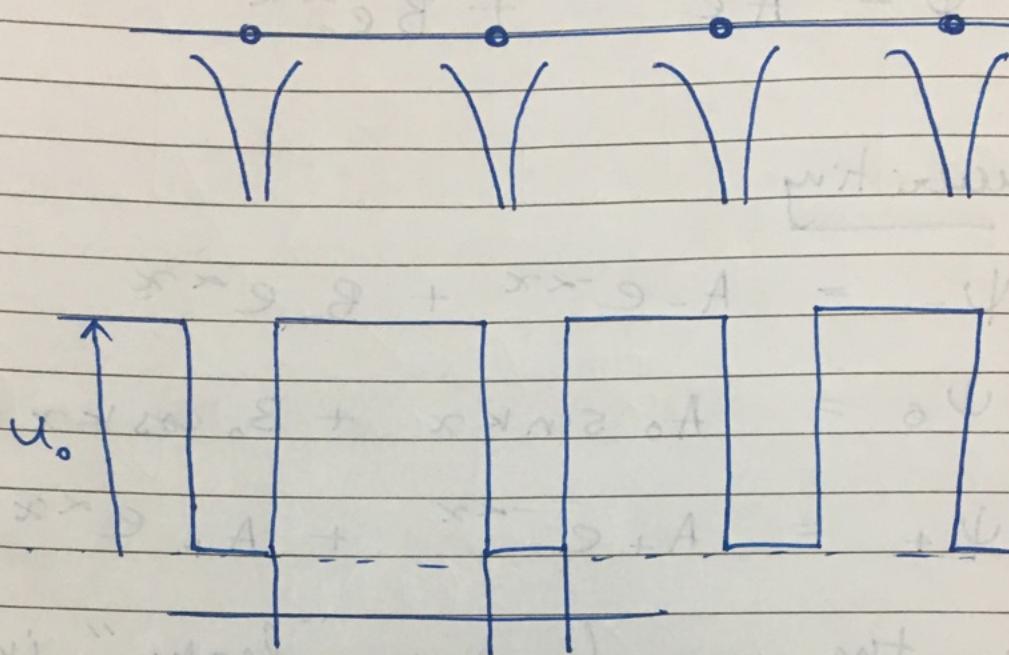


Exponential Decay Region
QM Tunneling

QM Model for Crystals

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→ Periodic with $(a+b)$

Solutions are Standing waves of the form.

$$\psi_+ = e^{ikx} + e^{-ikx}$$

$$\psi_- = e^{ikx} - e^{-ikx}$$

E ↑

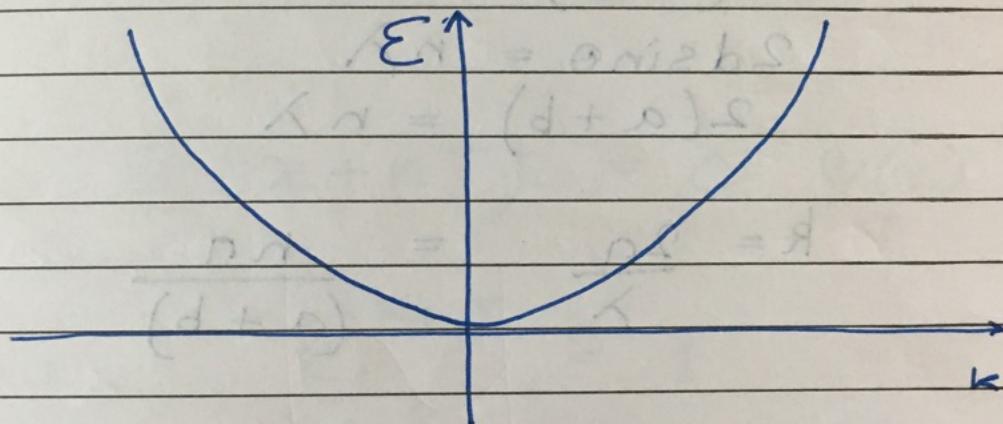
← →

→ Important points

Travelling waves reflect and superimpose to form Standing waves.

Parameters in Crystals

→ Free particle.



$$\text{--- } E = \frac{\hbar^2 k^2}{2m}$$

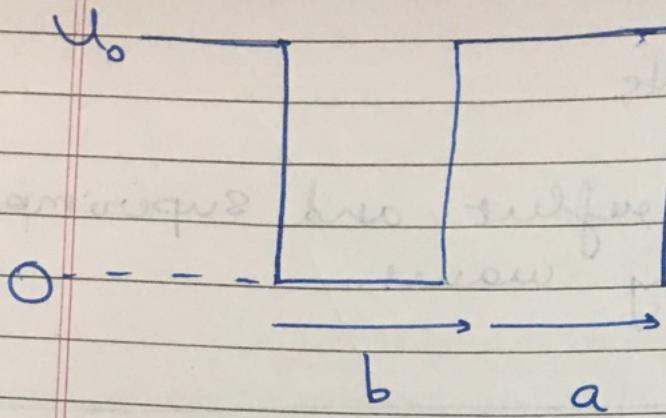
$$\frac{d^2 E}{dk^2} = \frac{\hbar^2}{m} \quad (\text{Twice Differentiate})$$

Ingredients for Crystal

1) Free particle $E - k$

2) Particle in a box

3) Bragg Reflection.



Bragg Reflection

$$2ds\sin\theta = n\lambda$$

$$2(a+b) = n\lambda$$

$$k = \frac{2n}{\lambda} = \frac{n\pi}{(a+b)}$$

→ Only Standing wave solutions are allowed.

$$v_p = \frac{\omega}{k}$$

$$v_g = \frac{d\omega}{dk} = 0 \quad (\text{for Standing waves})$$

$$E = h\nu$$

$$= \hbar\omega$$

$$\frac{dE}{dk} = \hbar \frac{d\omega}{dk}$$

- Periodic Boundary conditions

$$T = (a+b)$$

$$\Psi(x+T) = e^{ikT} \Psi(x)$$

Also

$$\Psi(x+nT)$$

Let

$$\Psi(x+nT) = c^n \Psi(x)$$

$$\Psi(x+nT) = c^n \Psi(x)$$

i.e. $e^{iknT} = 1$

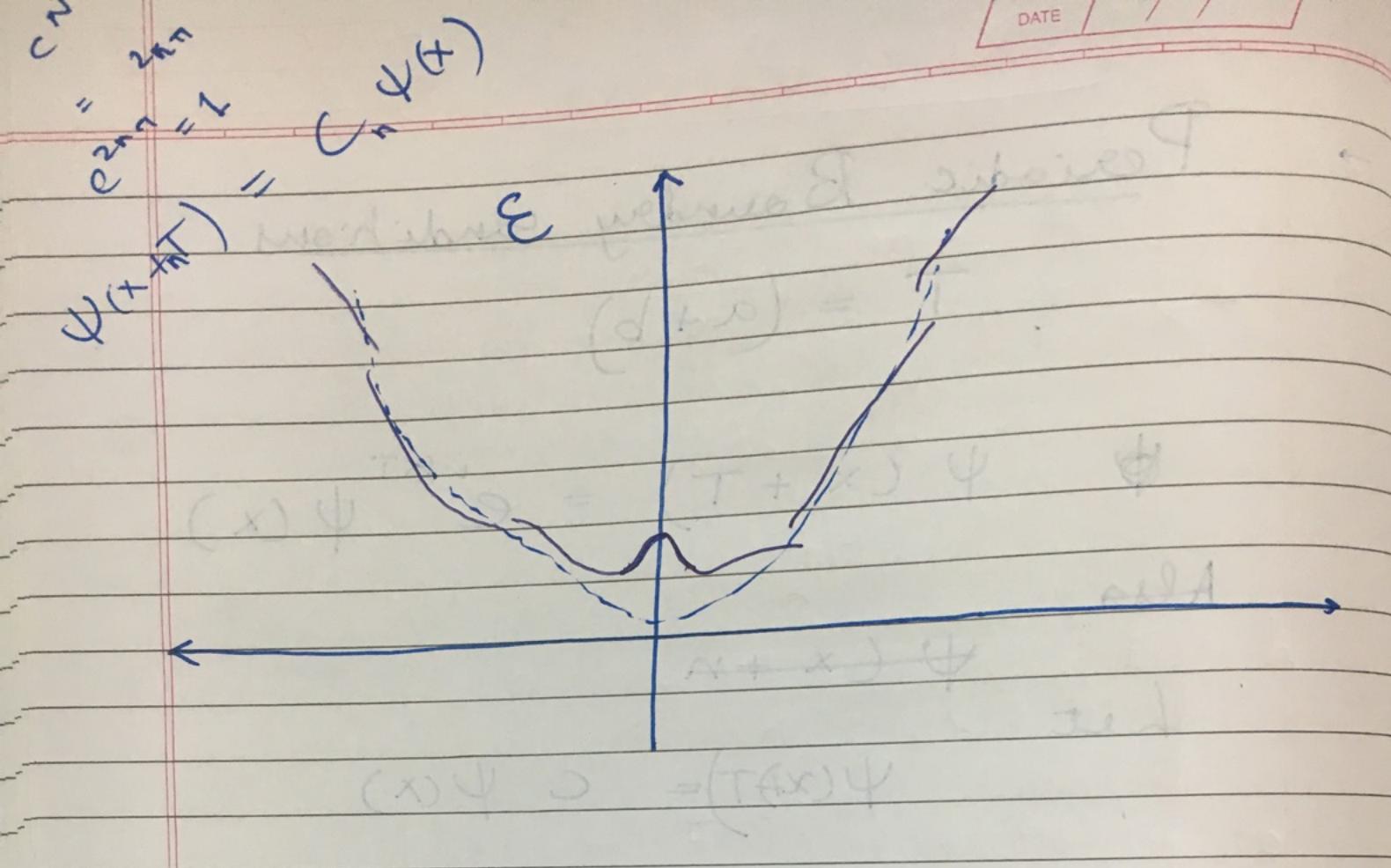
$$\Rightarrow knT = 2\pi$$

$$\frac{2\pi}{n}$$

~~$$e^{i\frac{2\pi}{n}}$$~~
$$e^{i\frac{2\pi}{kn}}$$

$$c = e^{\frac{i2\pi kn}{n}}$$

 ~~$k=0$~~



$$(d)\psi = (T+d)\psi$$

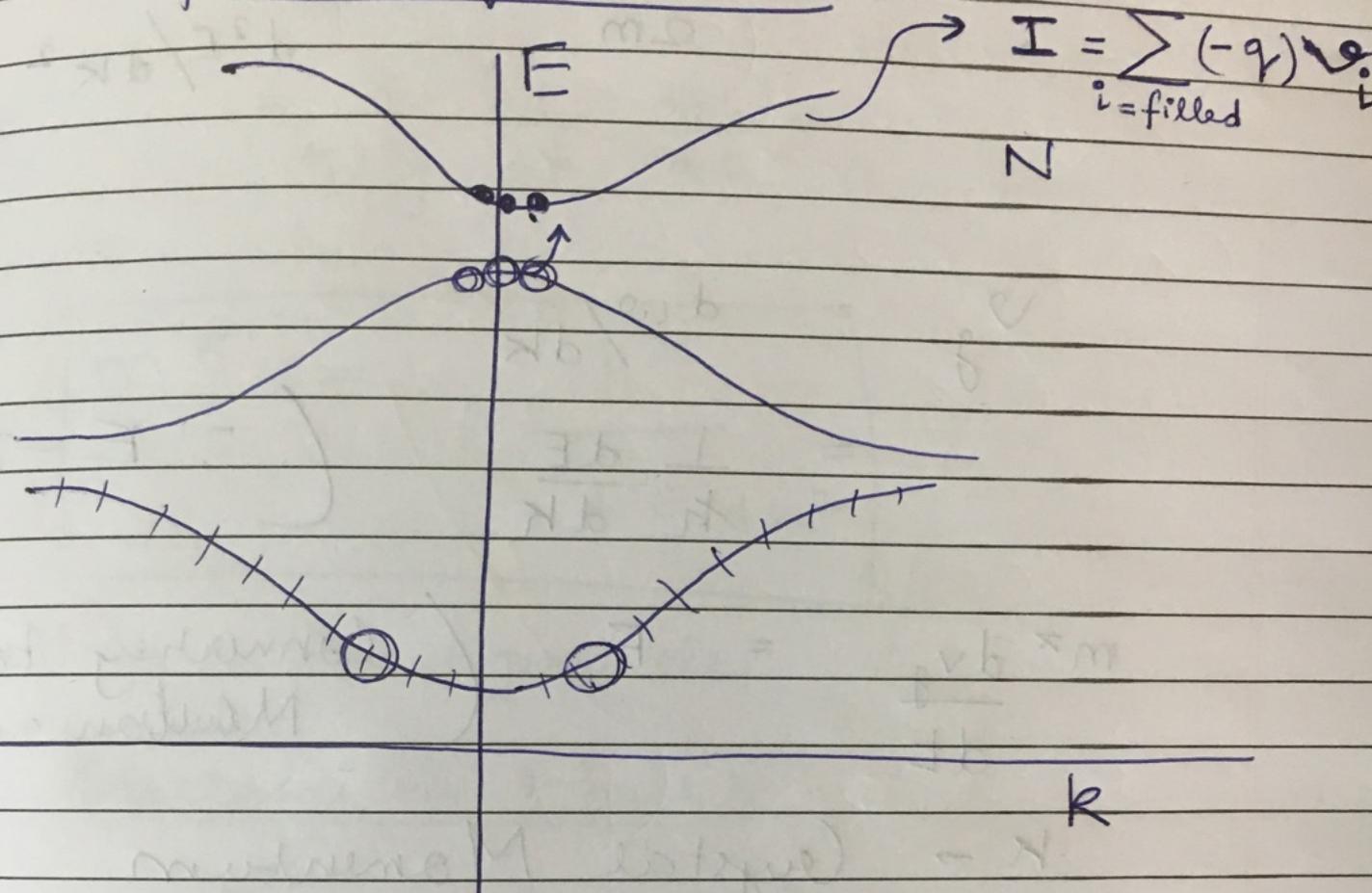
Taxi

$\Gamma = \text{Taxi}$

$\text{PS} = \text{Taxi}$

PS

Expression for Current



$$I = -q \sum_{i=1}^N v_i = 0 \rightarrow \textcircled{1}$$

from $\textcircled{1}$

$$I = q \sum_{i=1}^N v_i + (-q) \sum_{i=\text{filled}} v_i$$

$$= \boxed{q \sum_{i=\text{vacant}} v_i}$$

Effective Mass

$$E = \frac{\hbar^2 k^2}{2m} \quad m = \frac{\hbar^2}{d^2 E / dk^2}$$

$$\begin{aligned} v_g &= \frac{dk}{dt} \\ &= \frac{1}{\hbar} \frac{dE}{dk} \quad (\because E = \hbar \omega) \end{aligned}$$

$$m^* \frac{dv_g}{dt} = F \quad (\text{Converting to Newtonian})$$

$k \rightarrow$ Crystal Momentum

$$\frac{dv_g}{dt} = \frac{1}{\hbar} \frac{d^2 E}{dk^2} \frac{dk}{dE}$$

$$\frac{dv_g}{dt} = \underbrace{\frac{1}{\hbar^2} \frac{d^2 F}{dk^2}}_{\text{Dim. of force}} \times \underbrace{\frac{d(\hbar k)}{dt}}_{\text{Dim. of force}}$$

$$dE = F dx$$

$$= F \cdot v_g dt$$

$$F = \frac{1}{v_g} \frac{dE}{dL}$$

$$= \frac{1}{\hbar v_g} \frac{dE}{dk} \frac{dk}{dt}$$

$$m^* = \frac{\hbar^2}{\frac{d^2E}{dk^2}}$$

→ Holes : Quasi particles

- Electrons & Holes

$$m^* = \frac{\hbar^2}{\left(\frac{d^2E}{dk^2}\right)}$$

$$v_g = \frac{dw}{dk}$$

$$\epsilon = \frac{\hbar^2 k^2}{2m}$$

$$m^* \frac{dv_g}{dt} = F$$

here
 $k \rightarrow$ (physical)
 P

$$E = \hbar \omega$$

$$\cancel{\frac{dv_g}{dk}} = v_g = \frac{dw}{dk} \Rightarrow = \frac{1}{\hbar} \frac{dE}{dk}$$

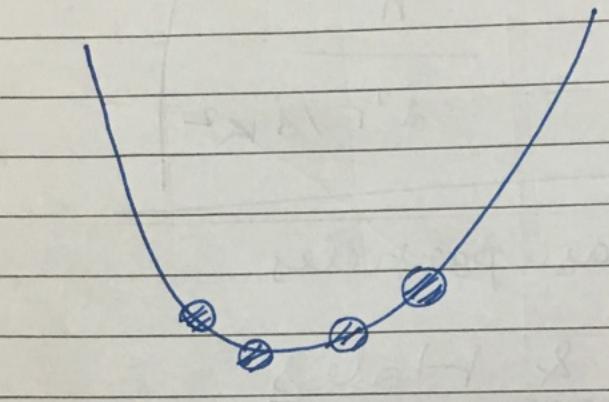
$$\frac{dv_g}{dE} = \left(\frac{1}{\hbar^2} \frac{d^2E}{dk^2} \frac{dk}{dt} \right) \rightarrow F$$

Lecture - 8

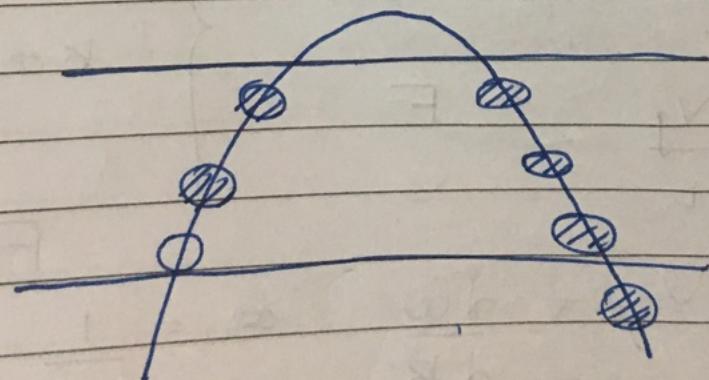
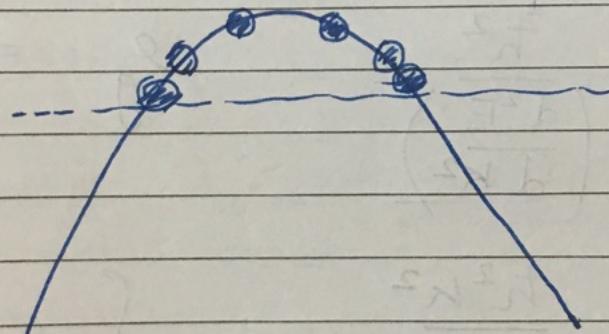
Electrons & Holes

* Effective mass

$$m^* = \frac{\hbar^2}{\frac{d^2 E}{dk^2}}$$



Absence of field



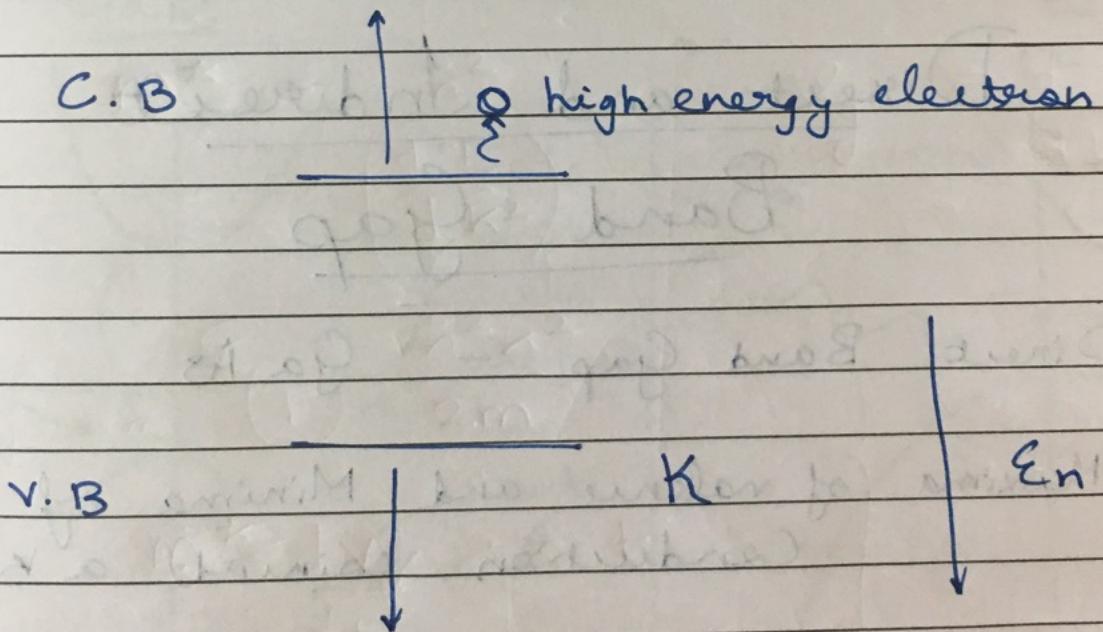
Presence of field

- * Fully filled band - zero current
- Partially filled band - non zero current

Hole - Ideas

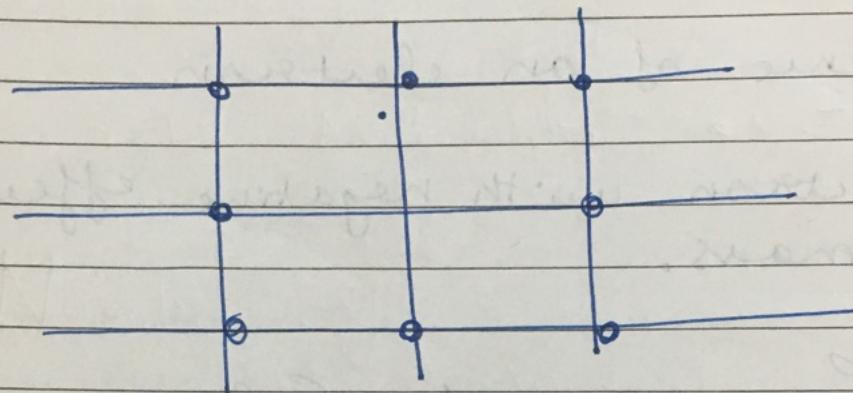
- Absence of an electron.
- Electron with negative effective mass.

Energy of Electrons



→ Brillouin zone : In blue part of crystal.

→ Wigner - Satz



Direct and Indirect Band Gap

→ Direct Band Gap - Ga As

Maxima of valence and Minima of conduction coincide at $k=0$

→ Indirect band Gap - Si, Ge

Don't coincide at all.
Valence band maxima is always at $\vec{k} = 0$

** Quiz - Aug - 3rd week **

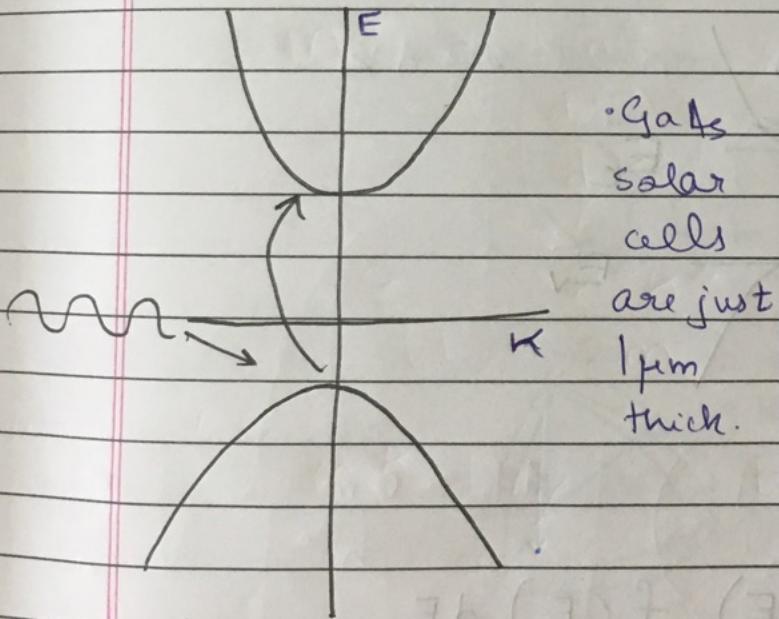
Lecture - 10 (9 actually)

- We need
 - Switch
 - and • Amplifier

- For Amplification, we need Dependant Sources.

Recap

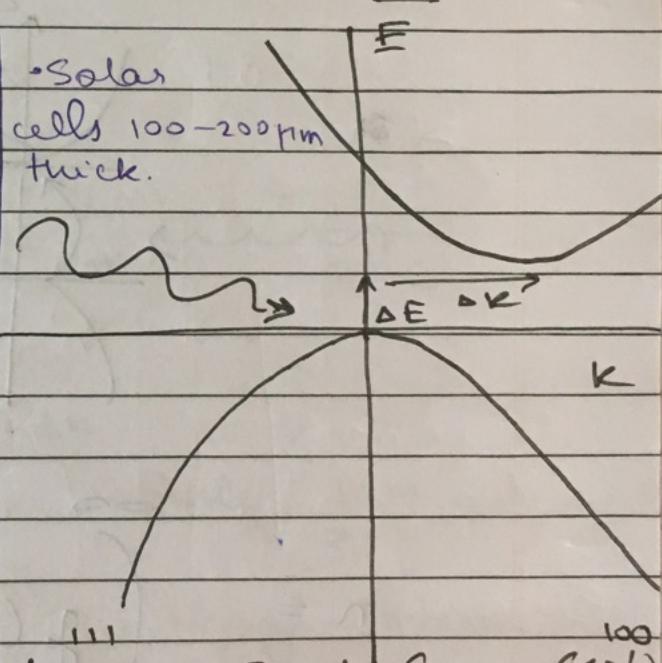
GaAs



Direct Band Gap.

- GaAs (germanium), very responsive to excitation.
- Only Energy change no K change.

Si



Indirect Band Gap. (Si/g)

- Less responsive to excitation.
- $\Delta E \rightarrow$ photon
- $\Delta k \rightarrow$ Momentum [from phonon] transfer
- Hence excitation is a 3 body (\downarrow Prob.) collision.

→ Direct band Gap, Equienergy
Surfaces are spherical

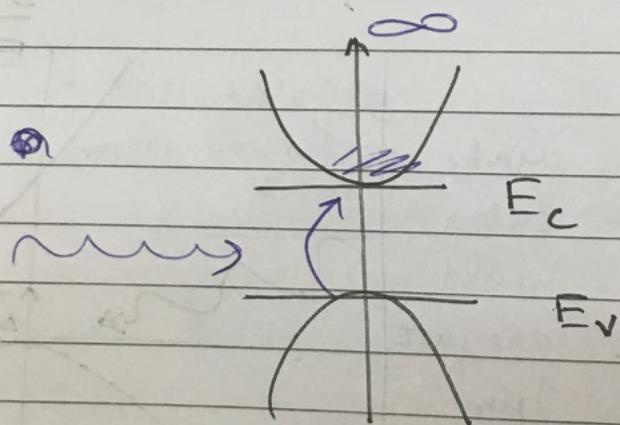
$$E = \alpha (k_1^2 + k_2^2 + k_3^2)$$

→ Indirect, same thing

$$E = \alpha k_1^2 + \beta (k_2^2 + \gamma k_3^2)$$

→ Number of Electrons / Charge carriers in Conduction band

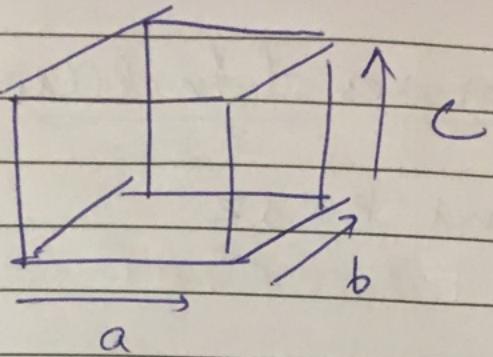
$$I \propto nV$$



$$n = \int_{E_c}^{\infty} g(E) f(E) dE$$

Available
Density of
States

Probability of
State being
occupied



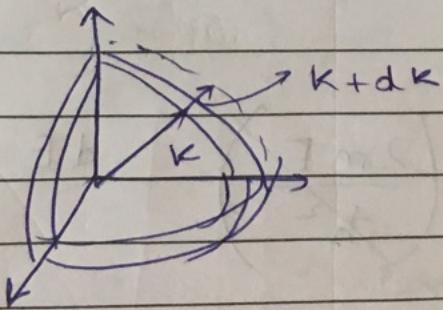
$$\psi(x) = A \sin(k_x x) \sin(k_y y) \sin(k_z z)$$

$$n \pi k_x = a$$

$$k_x = \frac{n_1 \pi}{a} \quad k_y = \frac{n_2 \pi}{b} \quad k_z = \frac{n_3 \pi}{c}$$

\Rightarrow Unit volume

$$\left(\frac{\pi}{a}\right)\left(\frac{\pi}{b}\right)\left(\frac{\pi}{c}\right) = \frac{\pi^3}{abc}$$



Only $\frac{1}{8}$ th volume
is of interest

\rightarrow Solutions per unit volume in k-space.

$$= \frac{abc}{\pi^3}$$

allowed states (Including spin)

blue

k and $k + dk$

$$= \left(\frac{4\pi k^2 dk}{8\pi} \right) \times \frac{1}{8\pi} \times \frac{abc}{\pi^2} \times \chi$$

$$= \frac{abc}{\pi^2} k^2 dk$$

→ Converting to E space.

$$E = \frac{\hbar^2 k^2}{2m}$$

$$dE = \frac{\hbar^2}{2m} (dk) dk$$

$$\sqrt{\frac{2mE}{\hbar^2}} = k$$

$$\frac{abc}{\pi^2} \times \left(\frac{2mE}{\hbar^2} \right)$$

$$dE \times \frac{2m}{\hbar^2} \times \sqrt{\frac{2mE}{\hbar^2}}$$

Dividing by volume.

of Allowed states blue E & $E + dE$
per unit volume.
(Divide by abc)

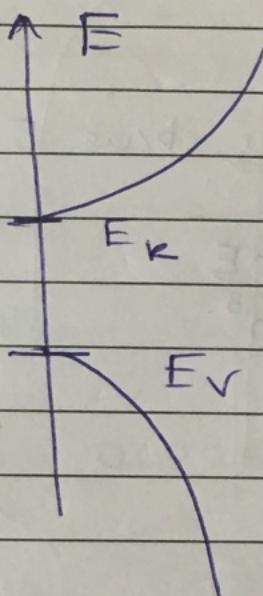
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for spherical
a diff. map for ellipsoid

$$= \frac{m}{\pi^2 h^3} \sqrt{\frac{2mE}{E}} dE$$

$$= g(E) dE$$

m is not the same as effective mass of e-/hole.



E_F (Similar analysis possible)

Assignment

$$\left. \begin{array}{l} \sum N_i = \text{const.} \\ \sum E_i N_i = \text{total energy} = \text{const.} \end{array} \right\}$$

→ find $f(E)$ for fermion

2)

Distribution of coin toss game.

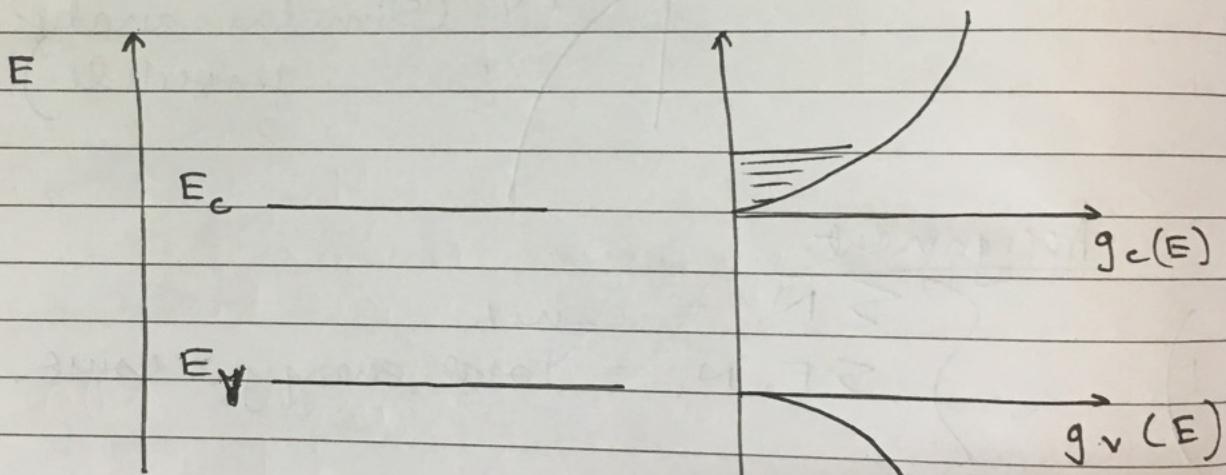
Lec. 10

Number of Charge car. in Cond. band

$$n = \int_{E_c}^{\infty} g(E) f(E) dE$$

Density of States b/w E & E+dE

$$= \frac{m \sqrt{2mE}}{\pi^2 \hbar^3}$$

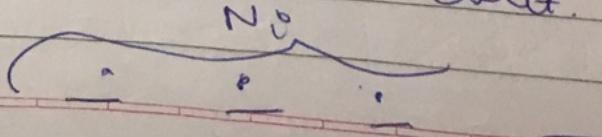


$\rightarrow \underline{f(E)}$

Constraints

$$\sum N_i = \text{constant}$$

$$\sum E_i N_i = \text{const.}$$



Si crystal

$$f(E) = \frac{N_i}{N_{\text{total}} \text{ in a state of } E}$$

↓
Derivation in
Book

f D distribution

$$f(E) = \frac{1}{1 + e^{(E - E_F)/kT}}$$

(RL)

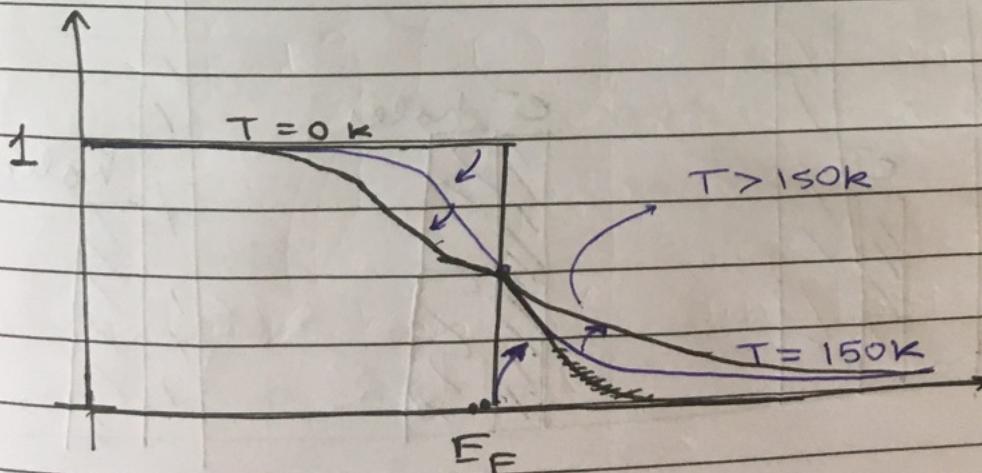
$$\frac{kT}{q} = 0.0256V = 25m\ 25.6\text{ mV}$$

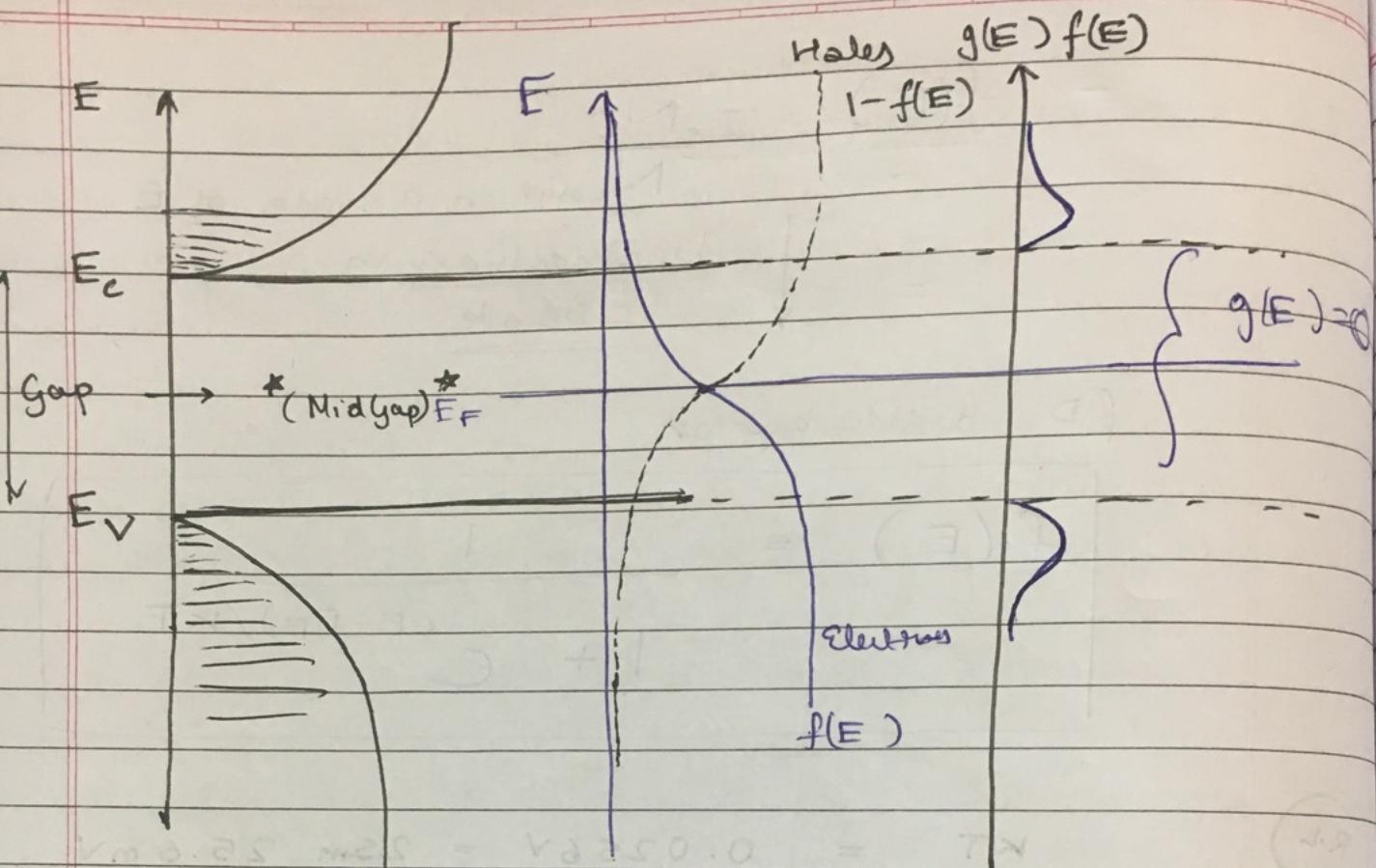
$(26\text{ mV}/25\text{ mV})$

(N.T)

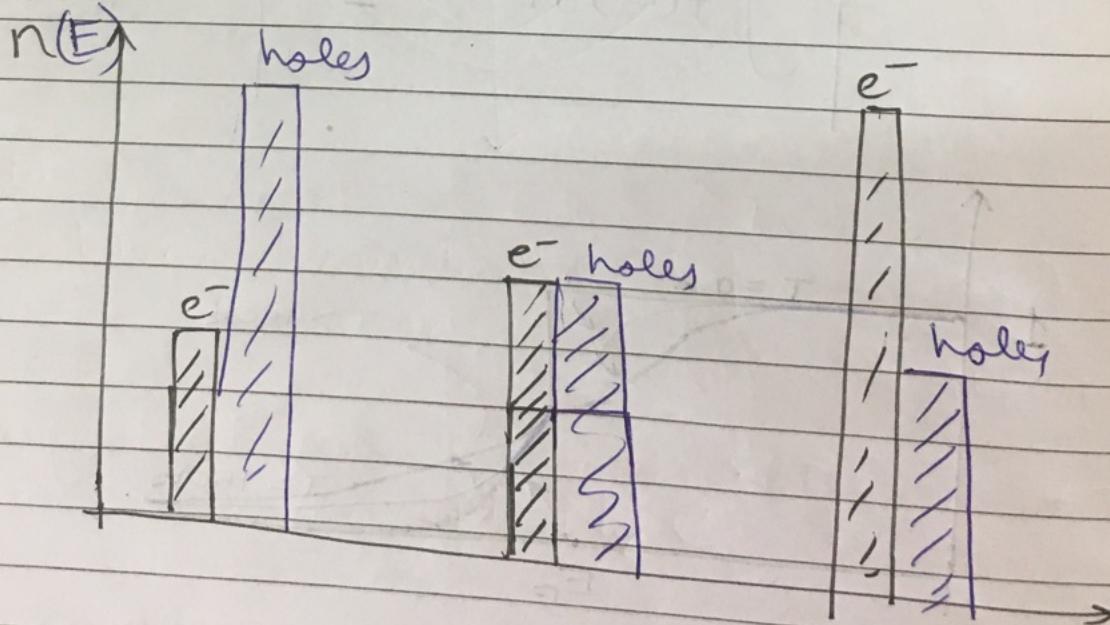
$$\int f(E)dE \neq 1$$

(Not Normalised)





- E_F is at Mid Gap for pure crystals.
- Thermal effects always keep it at mid point.



Note: Same

$$\frac{3kT}{q}$$

Approximation for E_F

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Calculation of $n(E)$

$$n = \int_{E_C}^{\infty} \frac{m \sqrt{2m(E-E_C)}}{\pi^2 \hbar^3} e^{-(E-E_F)/kT} dE$$

$$= \frac{m \sqrt{2m}}{\pi^2 \hbar^3} \int_{E_C}^{\infty} \sqrt{(E-E_C)} e^{-(E-E_F)/kT} dE$$

$$() \int_{E_C}^{\infty} \sqrt{(E-E_C)} e^{-\frac{(E-E_C)}{kT}} e^{-\frac{(E_C-E_F)}{kT}} dE$$

$$= \frac{m \sqrt{2m}}{\pi^2 \hbar^3} e^{-\frac{(E_C-E_F)}{kT}} \int_{E_C}^{\infty} \sqrt{(E-E_C)} e^{-\frac{(E-E_C)}{kT}} dE$$

↓ final

$$n = \frac{m \sqrt{2m}}{\pi^2 \hbar^3} e^{-(E_C-E_F)/kT} \cdot (kT)^{3/2} \frac{\sqrt{\pi}}{2}$$

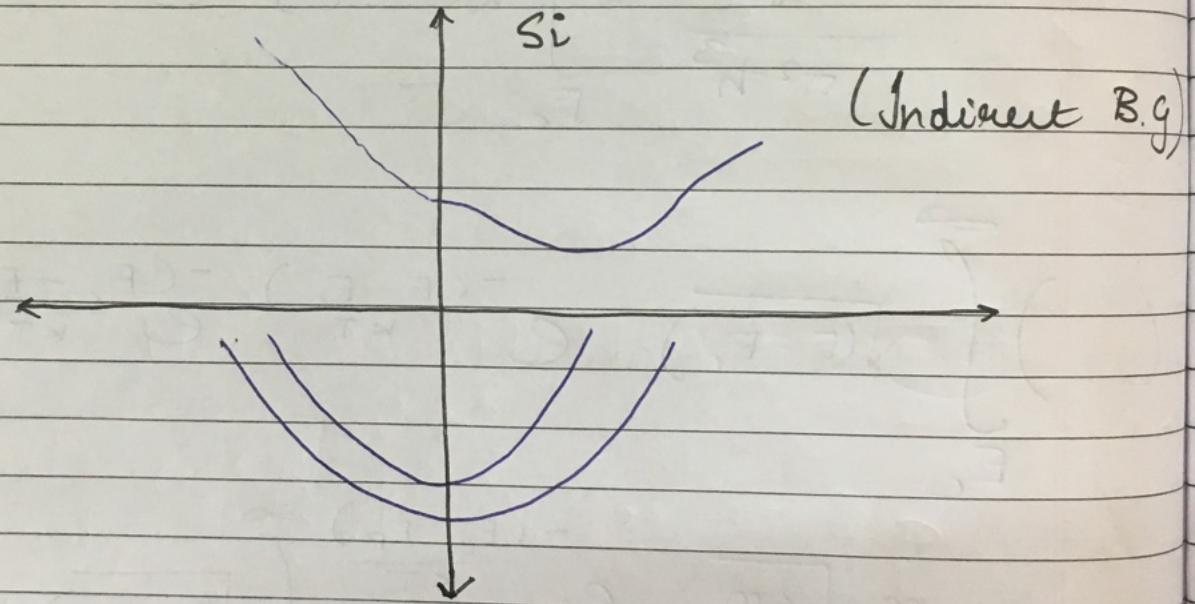
$$n = N_C e^{-(E_C-E_F)/kT}$$

lec - 11Recap

$$n = \int_{-\infty}^{\infty} g(E) f(E) dE$$

$$= \frac{m}{\pi^2 h^3} \sqrt{2m(E-E_c)} \times \frac{1}{(1 + e^{(E-E_F)/kT})}$$

*Not the simple effective mass
in General (only in GaAs)*



Non Degenerate Assumption (As long as $|E_F - E_c| > 3kT$)

$$\downarrow \frac{3kT}{q}$$

$|E_F - E_g| > 3kT$

$$n = N_c e^{-\frac{(E_c - E_F)}{kT}}$$

Here, in (207)

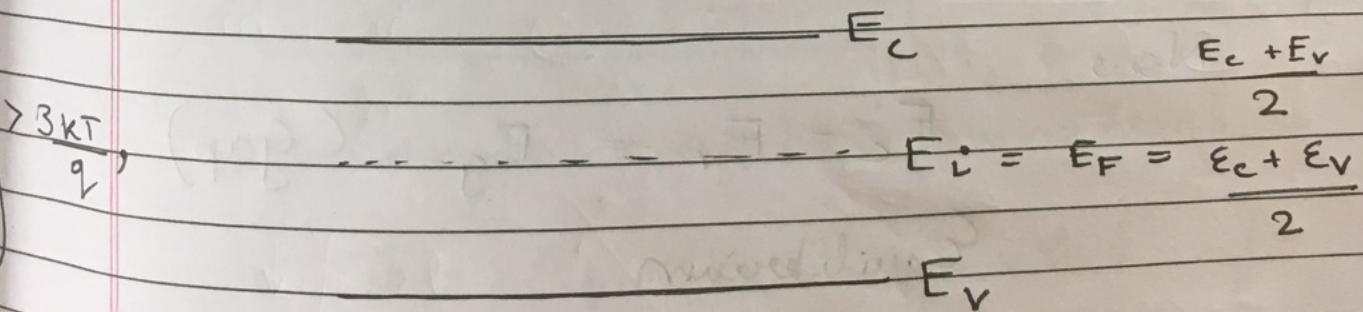
$$\boxed{n < N_c}$$

$N_c \rightarrow$ Effective no. of Available
States at E

$$e^- \quad n = N_c e^{-\frac{(E_c - E_F)}{kT}}$$

↑

$$\text{holes} \quad p = N_v e^{\frac{(E_v - E_F)}{kT}}$$



If we equate

$$n_i^o = p_i^o \Rightarrow$$

$$\ln N_c - \frac{(E_c - E_i)}{kT} = \ln N_v + \frac{(E_v - E_i)}{kT}$$

$$\frac{E_c + E_v - 2E_i^o}{kT} = \ln \frac{N_c}{N_v}$$

Say $N_c = 3.2 \times 10^{19} \text{ cm}^{-3}$ } Well known
 $N_v = 1.8 \times 10^{19} \text{ cm}^{-3}$ } for Si

$$E_i^o = \frac{E_c + E_v}{2} + \left[\frac{1}{2} kT \ln \left(\frac{N_v}{N_c} \right) \right]$$

~~correction term because~~

$$N = \frac{(m\sqrt{2m})}{\pi^2 k^3} (kT)^{3/2} \times \frac{\sqrt{\pi}}{2}$$

\hookrightarrow Different for e- & p

Now

$$E_c - E_v = E_g \quad (\text{gap})$$

Equilibrium

$$n_i p_i^o = N_c N_v e^{-(E_c - E_v)/kT}$$

$$n_i = p_i^o$$

$$n_i^2 = N_c N_v e^{-E_g / kT}$$

$$n_i = \sqrt{N_c N_v} e^{-E_g / 2kT}$$

→ Gives $n_i \approx 10^{10} \text{ cm}^{-3}$, Much lower than Atomic cm^{-3} density.

Rewriting Earlier Equations

$$n = N_c e^{-(E_c - E_F)/kT} = n_i e^{(E_F - E_i)/kT}$$

$$p = N_v e^{(E_V - E_F)/kT} = n_i e^{(E_i - E_F)/kT}$$

*
$$(np) = n_i^2$$
 * → Valid for Doped too.

Constant at a given temp.

→ This equilibrium is valid, if our Non degenerate Assumption holds.

→ This is a Dynamic Equilibrium, Analogous to Chemical Equi.

Doping

PAGE No.	
DATE	/ /

+ve $\Rightarrow P, N_D^+$

-ve $\Rightarrow n, N_A^-$

$$\nabla \cdot (\epsilon E) = f$$

$$= P - n + N_D - N_A$$

$$\boxed{P - n + N_D - N_A = 0}, np = n$$

+ve $\Rightarrow P, N_D^+$

-ve $\Rightarrow n, N_A^-$

Quiz

Aug 26 (Sat.)

Syllabus till Aug 23

lec-12

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lec-11 Formula Recap

$$n = n_i e^{(E_F - E_i)/kT}$$

$$P = n_i e^{(E_i - E_F)/kT}$$

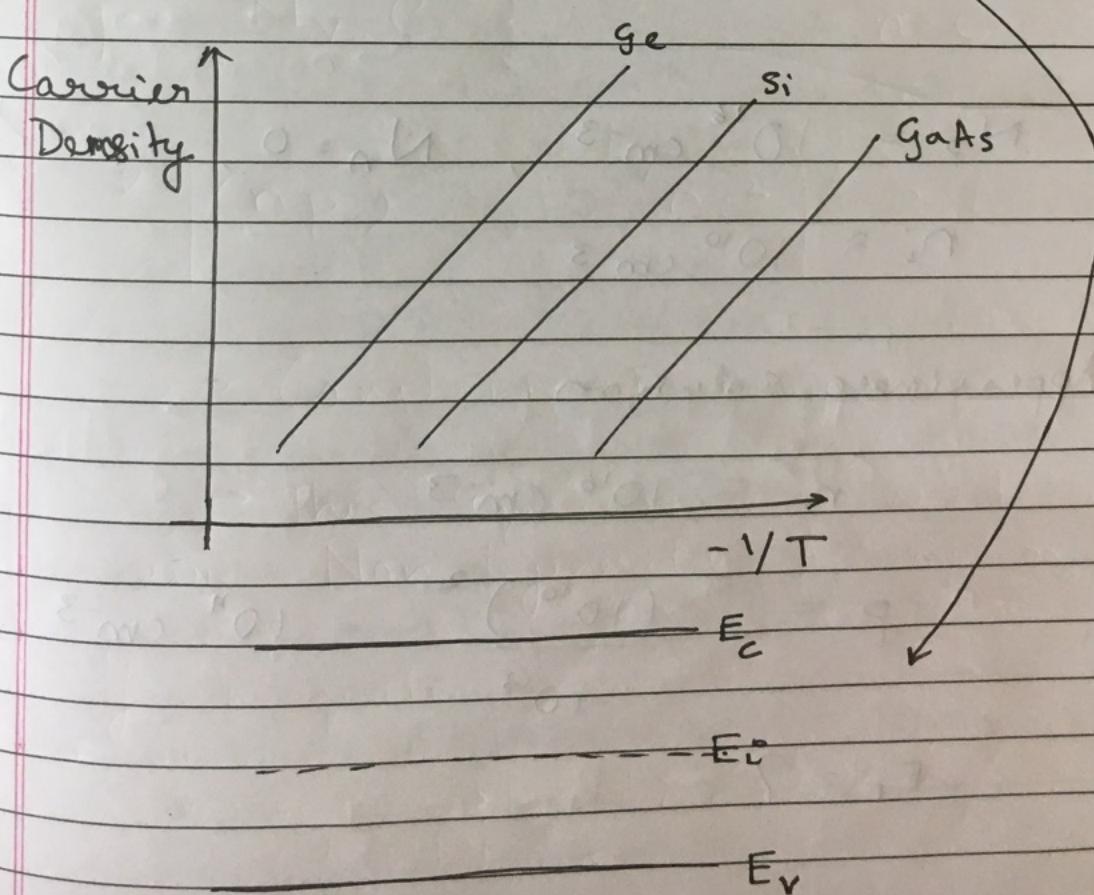
$$\boxed{np = n_i^2} \quad \text{** Equilibrium}$$

Band Gaps

$$\text{Ge} \rightarrow 0.7$$

$$\text{Si} \rightarrow 1.12$$

$$\text{GaAs} \rightarrow 1.5$$



Fermi Level (E_F) for a doped Material

$$\nabla \cdot (\epsilon E) = p - n + \underbrace{N_D^+ - N_A^-}_{\text{Intrinsic}} \quad \underbrace{\text{Dopants.}}$$

$$p - n + N_D^+ - N_A^- = 0 \quad \textcircled{2}$$

Neutrality

$$n_p = n_i^2$$

$$n_i = 10^{10} \text{ cm}^{-3}$$

Cases

① $N_D = N_A = 0 \Rightarrow E_F = E_L$

② $N_D = 10^{16} \text{ cm}^{-3}, N_A = 0, E_F = ?$

$$n_i \approx 10^{10} \text{ cm}^{-3}$$

Approximate Solution

$$n = 10^{16} \text{ cm}^{-3}$$

$$p = \frac{(10^{10})^2}{10^{12}} = 10^4 \text{ cm}^{-3}$$

$$E_F = ?$$

$$\frac{E_F - E_L}{kT} = x$$

$$10^4 = 10^{10} e^{-x}$$

- ①

$$e^{-x} = 10^{-6}$$

$$-x = 2.3 \times 6$$

$$x = 2.3 \times 6$$

$$(E_F - E_L) = 2.3 \times 6 \times 25 \times 10^{-3}$$

$$= \boxed{0.345 \text{ eV}}$$

③

$$N_D = 10^{19} \text{ cm}^{-3} \quad N_A = 0 \quad E_F = ?$$

$$n \approx 10^{19}$$

$$p \approx 10^1 / \text{cm}^3$$

$$10 = 10^{10} e^{-x}$$

$$e^{-x} = 10^{-9}$$

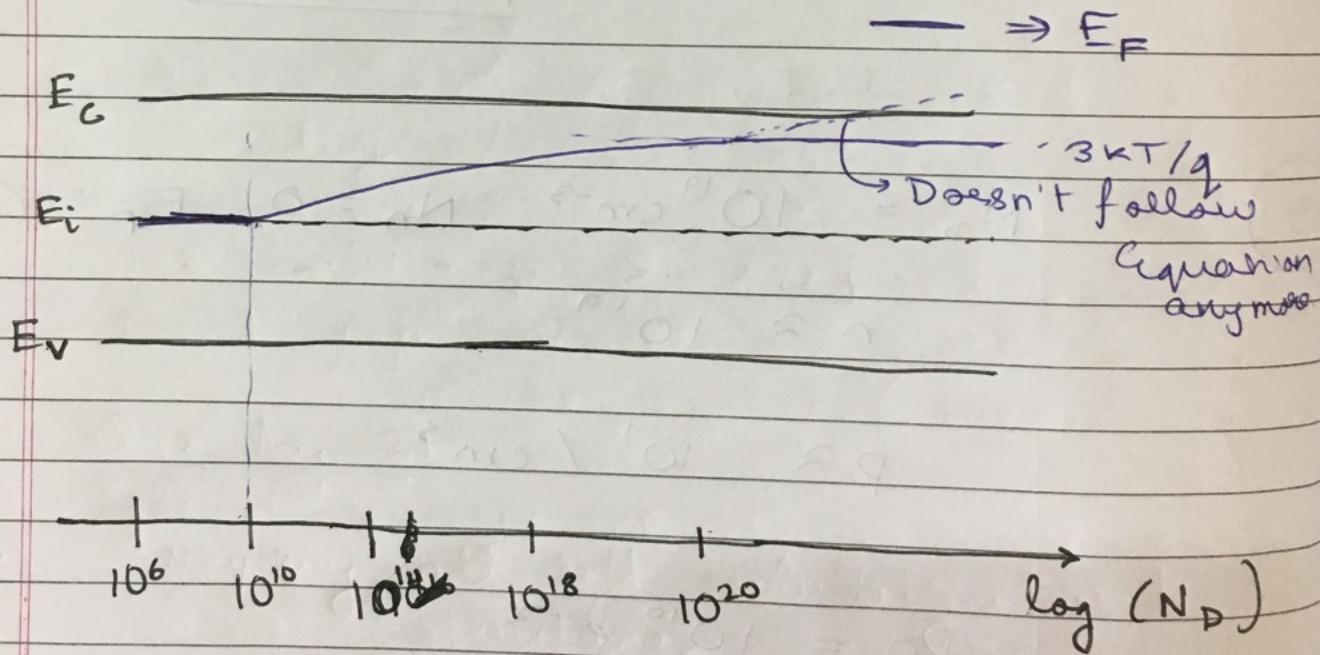
$$\frac{0.345 + 3}{2} = \boxed{0.52 \text{ eV}}$$

→ In case ③ the Ans. is not correct,
since the Doping is so high that
our approximation is not valid.

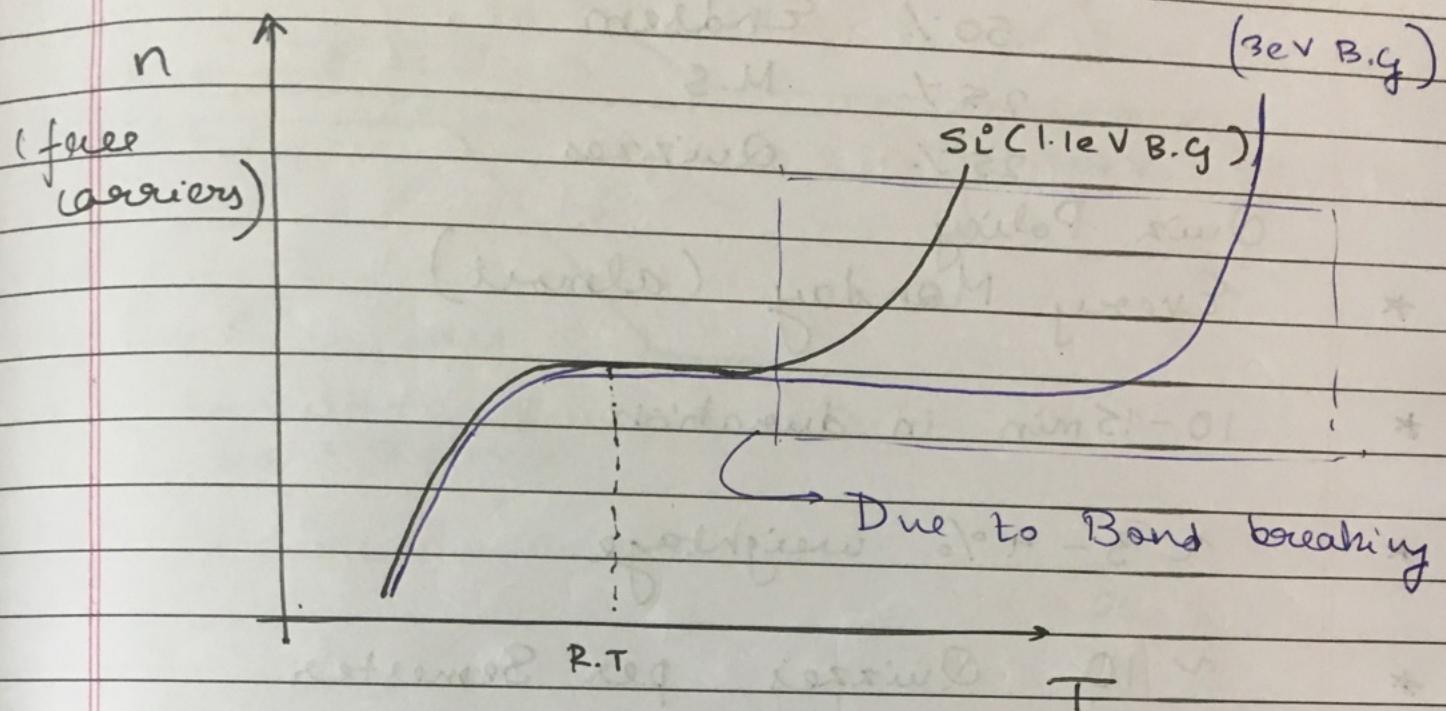
④ $N_D = 10^8/\text{cm}^3, N_A = 10^7/\text{cm}^3$

We'll have to solve Quadratic, but
here less much less than n_i , hence
 $E_i \approx E_F$

Plot



n vs T trends

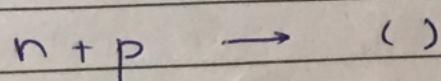
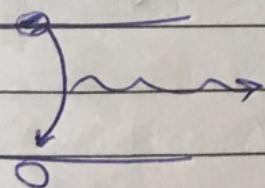


- 3eV B.G material is used for $\uparrow T$ applications

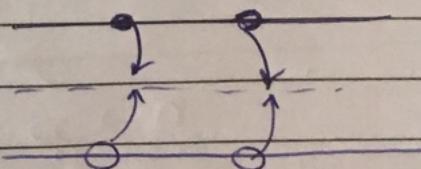
Equilibrium in Doping

→ e^- hole recombination

radiative process



non-radiative process



n, p equilibrium

$$\frac{dn}{dt} = B(n_p - n_i^2)$$

Lec - 13

New Grading Policy

50% Endsem

25% M.S

25% Quizzes

Quiz Policy

* Every Monday (almost)

* 10-15 min in duration

* ~ 3-4% weightage

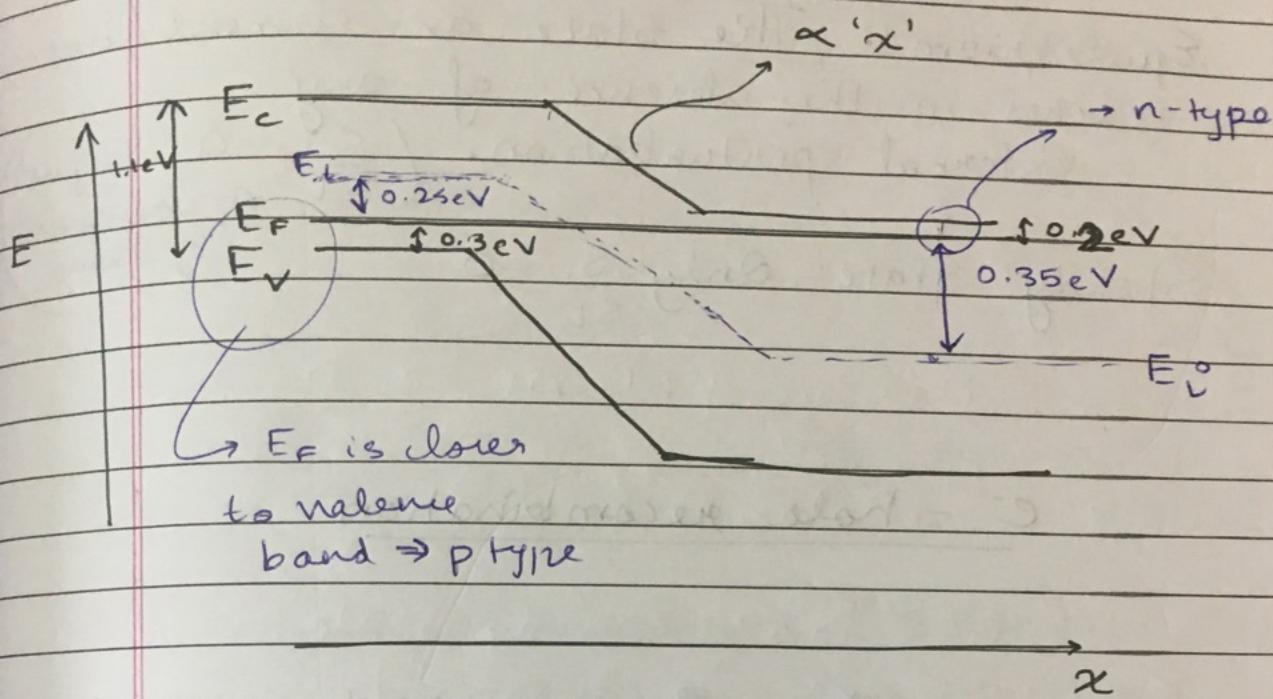
* ~ 10 Quizzes per Semester

* Best 8 or so to be counted for
Grading

* No make up quizzes

Quiz Aug - 21

→ Everything till and including band
structure.

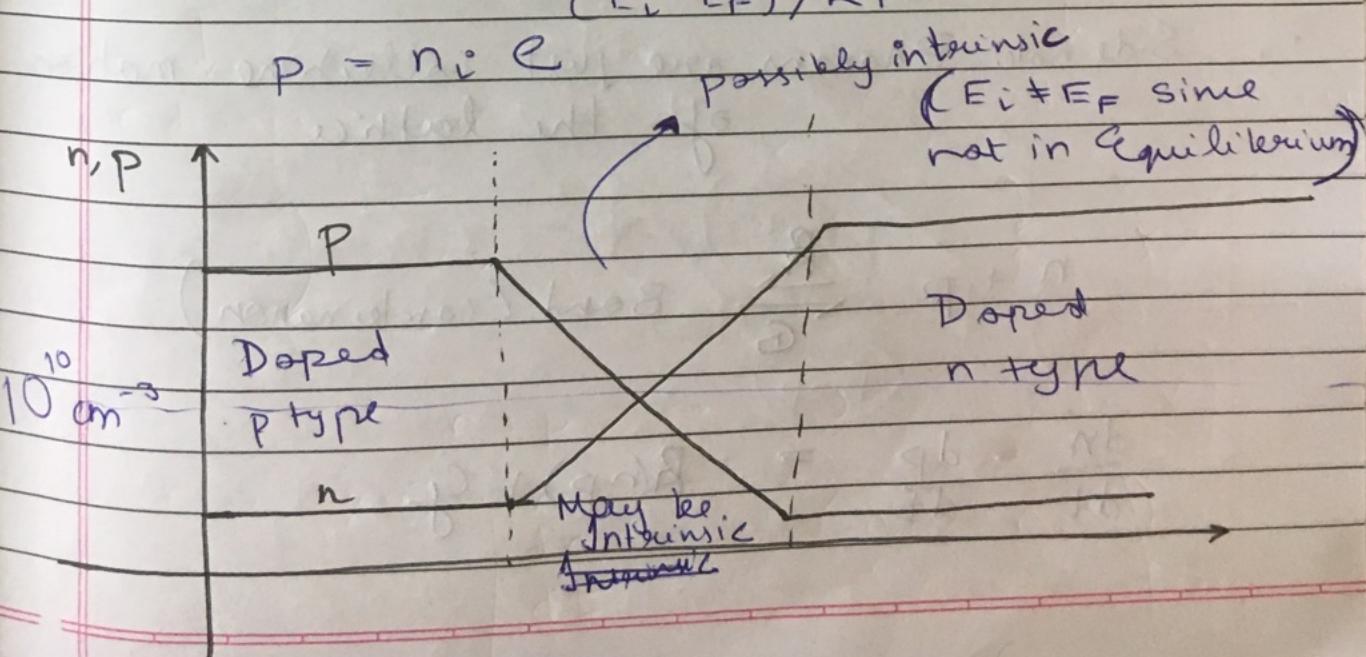


Q-1) Plot E_i Vs x (Just take M.D.)

Q-2) Plot n, p Vs x (Plotted below)

$$n = n_i e^{(E_F - E_i)/kT} \quad (n_i = 10^{10} \text{ cm}^{-3})$$

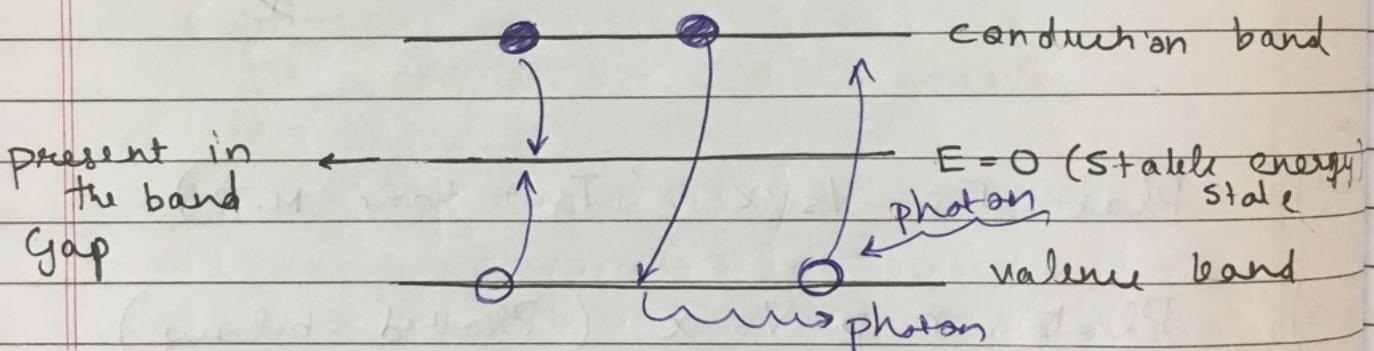
$$p = n_i e^{(E_i - E_F)/kT}$$



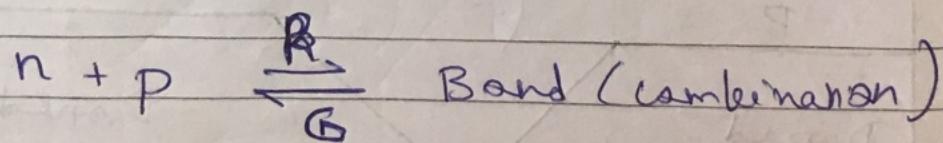
Equilibrium Vs Steady State

- Equilibrium : The state an element / dev reaches in the absence of any external perturbation. ($\frac{S}{St} = 0$ is just a consequence of absence)
- Steady State: Only $\frac{S}{St} = 0$

e^- - hole recombination



- Band Gap may have some states in an impure crystal. (Trap Levels/Defects)
- Side note: Phonons are just collective motion of the lattice



$$\frac{dn}{dt} = \frac{dp}{dt} = -R(n, p) + G$$

~~Edit: Replaced B by R
(or maybe it was B??)~~

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$$np = n_i^2 \quad - \text{equilibrium}$$

not Steady state

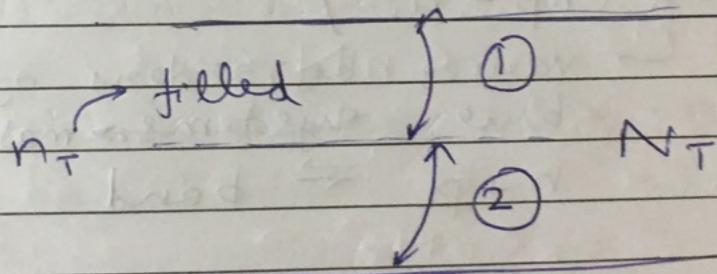
- ~~B~~

$$-Rnp + g = 0 = -Bn_i^2 + g$$

$$\frac{S_n}{S_t} = \frac{\delta p}{\delta t} = -R(np - n_i^2)$$

→ Called Shockley (SRH)

Band-Band



$$\frac{dn}{dt} = -\kappa_1 (N_T - n) n$$

$$\frac{dp}{dt} = -\kappa_2 (n_T) p$$

Qxxt 2

DATE / / /

Lec - 14

Quiz-2

Equilibrium carrier conc.

from Density of States \rightarrow Doping

Lec - 14

Equilibrium $\begin{cases} \text{Steady State} \\ \text{Transient} \end{cases}$

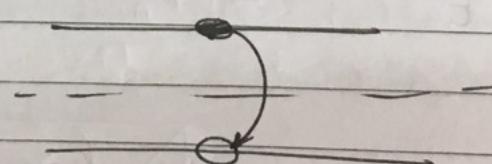
$$\frac{dn}{dt} = -k(n_p - n_i^2)$$

\hookrightarrow varies by orders of magnitude
b/w systems \rightarrow materials.

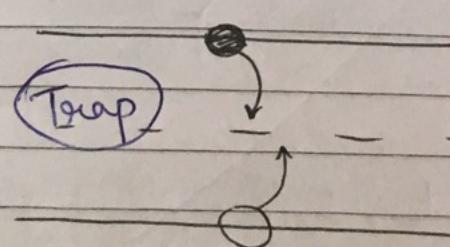
from $n + p \rightleftharpoons \text{band}$

Mechanisms

Rate eq¹)



2)



However e- should stay in the trap for long enough for a hole to reach there

→ Traps exist because crystals are not pure.

Insights

$$\rightarrow \frac{S_n}{S_t} = \frac{S_p}{S_t} = 0$$

→ Probability increases as traps are closer to mid gap.

Rate eqn for 2

$N_T \rightarrow$ Total states

$n_T \rightarrow$ total filled states

$$\frac{S_n}{S_t} \approx c_e (n_T) n$$

$$\frac{S_n}{S_t} \approx -c_e (N_T - n_T) n$$

$$\frac{S_p}{S_t} = -c_p (n_T) p$$

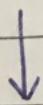
Trap located in mid-band

(Empty States)

$$c_e (N_T - n_T) n = c_p (n_T) p$$

→ In writing we neglect certain processes to simplify calc.

$$\frac{s_p}{s_t} = \frac{s_n}{s_t} = n_T p$$



Get

$$\cancel{s_p} = \frac{c_e N_T n}{c_e n + e p p}$$

Dividing N^n & D^n

$$= \frac{n p}{\frac{n}{c_p N_T} + \frac{p}{c_n N_T}}$$

$$\tau_p = \frac{1}{c_p N_T}$$

$$\tau_n = \frac{1}{c_n N_T}$$

$$R(\text{rate}) = \frac{n p}{n \tau_p + p \tau_n}$$

→ This was approx sol, correct
sol. a is

$$\left(\frac{n_p - n_i^2}{n \tau_p + p \tau_n} \right)$$

↓ Modify since it doesn't
have Near to
mid gap prob. info

$$R_{SRH} = \frac{(n_p - n_i^2)}{\tau_p(n+n_i) + \tau_n(p+p_i)}$$

SRH rate eqn

$$\left\{ \begin{array}{l} n_i = n_i e^{(\quad)} \\ p_i = p_i e^{(\quad)} \end{array} \right. \quad \left\{ \text{in book} \right.$$

→ Around Mid band, ~~n ≈ p~~,
 $n_i \approx p_i \approx n_0$

Re-Cap E_F

$$(E_F - E_i) / kT$$

equilibri. $\left\{ \begin{array}{l} n_o = n_i e^{(E_i - E_F) / kT} \\ p_o = n_i e^{(E_F - E_i) / kT} \end{array} \right.$

$$n_o p_o = n_i^2$$

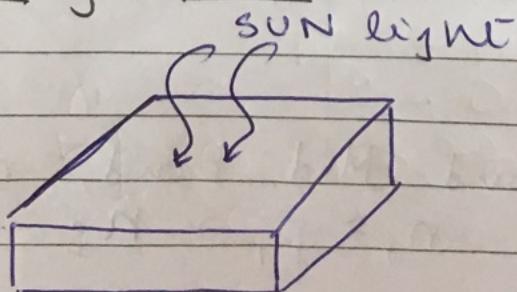
$$n_i = 10^{10} \text{ (for Si)}$$

~~Ans~~Doped Case

(Still equilibrium, but $n \neq p$)

$N_D \ll n \sim 10^{17} \rightarrow$ called Majority carriers
 $p \sim 10^3 \rightarrow$ called Minority carriers

→ Steady State (Not Equilibrium)

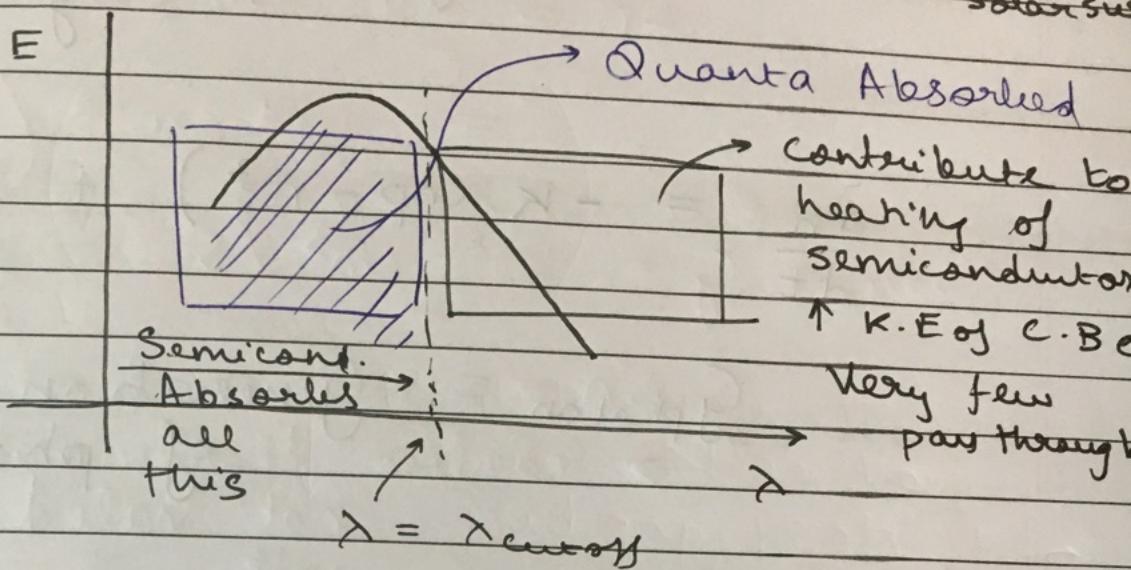


- After transition period

$$n \rightarrow n(E)$$

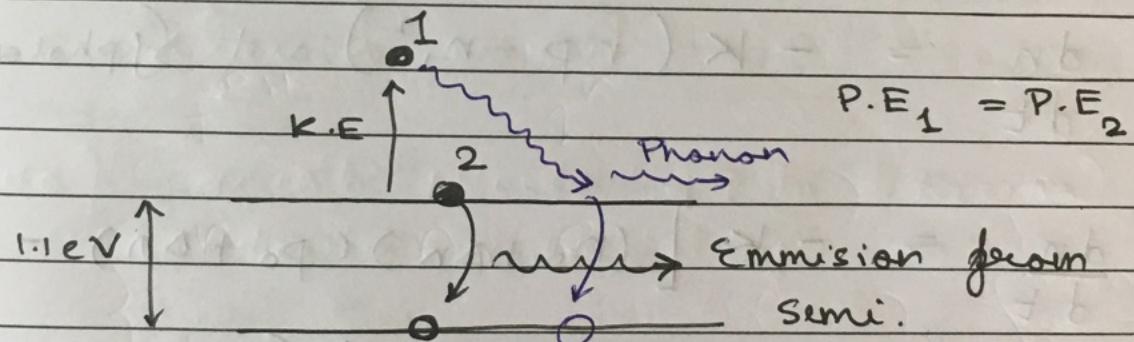
$$P \rightarrow P(E)$$

Solar Spectrum (BBR at $T = T_{\text{Solar}}$)



$$E_g(\text{eV}) = \frac{1240}{\lambda(\text{nm})}$$

Recombination & Generation



SSS Solar light

Only $\lambda \rightarrow E_g$ corres. photons
(Monochromes)

→ Number of Quanta coming out is equal (to no. of Quanta Abs.)

$\rightarrow T = T_R = G$ at Steady state
(By Def.)

$$\frac{dn}{dt} = -k(np - n_i^2) + G_{\text{photon}}$$

$G_{\text{photon}} = \text{Generation rate}$
of photons

$$n = n_0 + \Delta n$$

$$p = p_0 + \Delta p$$

$\Delta n = \Delta p$ since all processes
are pair wise

$$\frac{dn}{dt} = -k(np - n_i^2) + G_{\text{photon}}$$

$$\frac{dn}{dt} = -k [(n_0 + \Delta n)(p_0 + \Delta p) - n_i^2] + G_{\text{photon}}$$

$$= -k [(\cancel{n_0 p_0} - n_i^2) + \cancel{\Delta n(n_0 + p_0)} + (\Delta n)^2] - \cancel{\Delta n^2} + G_{\text{photon}}$$

$$\frac{Sn}{St} = -k [\Delta n(n_0 + p_0) + \Delta n \cdot \Delta n] + G_{\text{photon}}$$

Ignoring 2nd order terms...

$$\frac{S_n}{S_t} = -k(n_0 + p_0)\Delta n + G_{\text{photon}} \quad \hookrightarrow [\text{Low level irradiation}]$$

$$\bullet \frac{S(\Delta n)}{S_t} = -k(n_0 + p_0)\Delta n + G_{\text{photon}}$$

$$\frac{S(\Delta n)}{S_t} = -\frac{\Delta n}{\tau_{\text{eff}}} + G_{\text{photon}}$$

$$\tau_{\text{eff}} = \frac{1}{k(n_0 + p_0)}$$

Typical k values

$$\begin{aligned} \text{GaAs} &\rightarrow 10^{-10} \text{ (unit)} \\ \text{Si} &\rightarrow 10^{-15} \text{ (unit)} \end{aligned}$$

Say,

$$G = 10^{16} \text{ photons cm}^{-3}/\text{s}$$

$$\tau_{\text{eff}} = \frac{1}{10^{-15}(10^{17} + 10^3)} \quad \begin{array}{l} \hookrightarrow \text{for N}_D \text{ Doped case} \\ \hookrightarrow \sim 10^{17} \end{array}$$

$$\tau_{\text{eff}} = 10^{-2} \text{ s} = 10 \text{ ms}$$

Average life time post excitation
= 10 ms

$$\frac{S(\Delta n)}{S_t} = 0 \Rightarrow (\Delta n = G_{\text{photon}} \times \tau_{\text{eff}})$$

$$\boxed{\Delta n = G \tau_{\text{eff}}}$$

Here,

$$\Delta n = 10^{16} \text{ cm}^{-3}/\text{s} \times 10^{-2} \text{ s}$$

$$= 10^{14} \text{ cm}^{-3} \quad (\ll \text{ than } 10^{17} (\text{N}_A))$$

Say, $g = 10^{22} \text{ cm}^3/\text{s}$

$$\Delta n = 10^{20} \text{ cm}^{-3}$$

→ Earlier Approximation
not valid.

End Result for $g = 10^{16}$

$$\Delta n = 10^{14}$$

$$n \approx 10^{17} + 10^{14} \approx 10^{17}$$

$$p = 10^3 + 10^{14} \approx 10^{14}$$

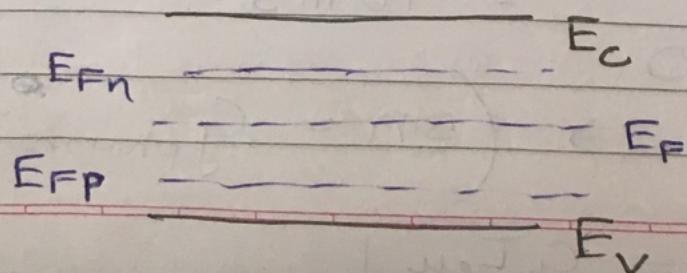
Quasi - Fermi Levels

→ Concept of E_F breaks down for non equilibrium situations

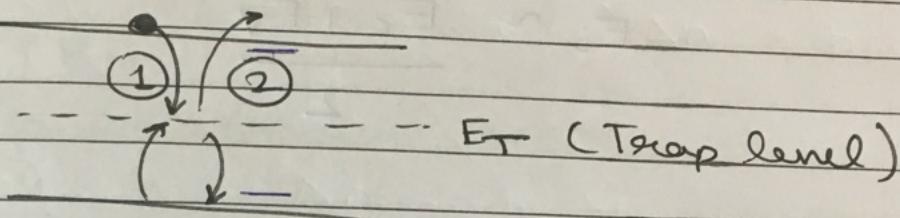
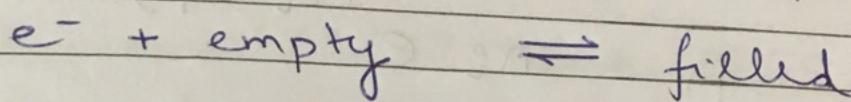
→ We use Quasi fermi levels

$$n = n_i e^{(E_{Fn} - E_i)/kT}$$

$$p = n_i e^{(E_i - E_{Fp})/kT}$$



→ The e^- & hole trap concept



$N_T \rightarrow$ Total traps (depends on impurity)

$n_T \rightarrow$ traps filled with e^-

$N_T - n_T \rightarrow$ empty traps

$$\frac{s_n}{s_t} = -\left(c_n n (N_T - n_T) \right) + e_n n_T$$

$$\frac{s_p}{s_t} = \left(-c_p p n_T \right) + e_p (N_T - n_T)$$

→ Contribution of ② is lesser (Ignore.)

Get,

$$R = \frac{-(n_p - n_i^2)}{\tau_p(n+n_1) + \tau_n(p+p_1)}$$

To account for

$R \rightarrow 0$ as Trap level is near E_C or E_V

$$(E_T - E_i) / kT$$

$$n_i = n_0 e^{(E_T - E_i) / kT}$$

$$p_i = n_0 e^{-(E_i - E_T) / kT}$$

$$E_i \sim \frac{E_c + E_v}{2}$$

$$E_T \rightarrow E_i + \frac{E_c + E_v}{2}$$

$$\rightarrow \frac{\delta(\Delta n)}{\delta t} = -\frac{\Delta n}{\tau_{\text{eff}}} + g$$

is valid for both $g \geq 0$ (Irradiation
Charging)

and $g = 0$ (Return to Equilibrium /
Discharge)

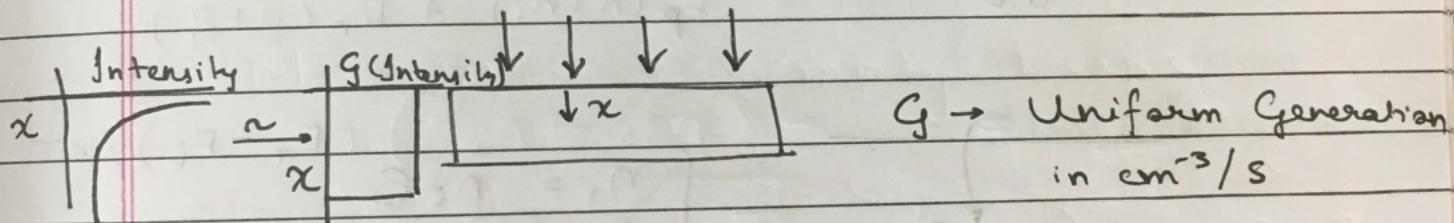
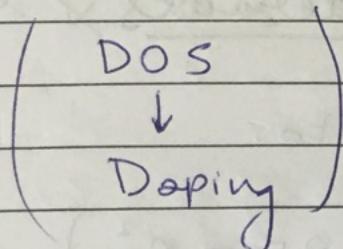
Upcoming Quizzes/tests

M.S 16/9/17, Sat, 8:30 - 10:30 am

Quiz

(Mon)

Equilibrium carrier conc.

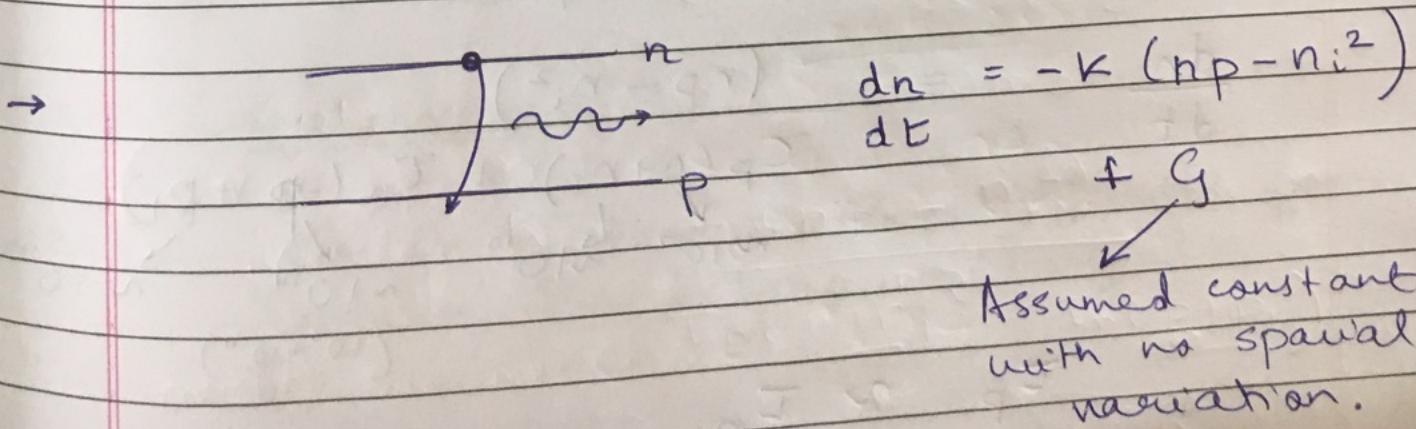


→ Equilibrium

$$\rightarrow np = n_i^2$$

$$np \neq n_i^2$$

(Non. equ.)



$$\Delta n, \Delta p \ll \frac{n_0}{p_0} (n_0, p_0)$$

Majority carrier cone.

~~Indirect Band Gap Materials~~

→ Above analysis lead to.

$$\frac{dn}{dt} = \frac{-(np - n_i^2)}{\tau_p(n+n_i) + \tau_n(p+p_i)}$$

$$E_T \left\{ \begin{array}{l} n_i = n e^{(E_T - E_i)/kT} \\ p_i = n_i e^{(E_i - E_T)/kT} \end{array} \right.$$

Analysis

$$\begin{aligned} n &\rightarrow n_0 + \Delta n \\ p &\rightarrow p_0 + \cancel{\Delta n} \end{aligned} \quad \left\{ \begin{array}{l} \therefore \Delta n = \Delta p \end{array} \right.$$

$$\frac{dn}{dt} = \frac{-(np - n_i^2)}{\tau_p(n+n_i) + \tau_n(p+p_i)}$$

$\tau_p \sim 10^{-17}$ $n_i \sim 10^{10}$ $\tau_n \sim 10^{-14}$ $(10^3 + 10^{14})$

$$\tau_p \sim \tau_n$$

Approximating

$$-\frac{\Delta n (n_0 + p_0)}{T_P n_0} + \frac{\Delta n^2}{\tau} = 0$$

\downarrow
eq. value
equilibrium value

$$\approx -\frac{\Delta n}{T_P}$$

$$R = -\frac{\Delta n}{T_P} \quad \text{for N-type semiconductors}$$

T_P

Minority carrier lifetime

$$= -\frac{\Delta n}{T_n} \quad \text{for P-type semiconductors}$$

T_n

→ Conclusion

→ B-B (Band to Band) Model for Direct Band Gap materials

$$T_{eff} = \frac{1}{k(n_0 + p_0)}$$

→ SRH → Model for Indirect B.G mat.

$$\left\{ \begin{array}{l} \tau_{eff} = \tau_p \xrightarrow{\text{N type}} \\ \tau_{eff} = \tau_n \end{array} \right\} \rightarrow P \text{ type}$$

→ Minority carrier lifetime.

→ Derivation of Intrinsic Semiconductor by SRH Model (Indirect B.G)

$$\frac{dn}{dt} = \frac{-(np - n_i^2)}{\tau_p(n+n_i) + \tau_n(p+p_i)}$$

Intrinsic Semiconductor

$$= (n_0 + \Delta n)(p_0 + \Delta p) - n_i^2$$
$$= \Delta n(n_0 + p_0) + \Delta n^2$$

This time

$\Delta n, \Delta p \gg (n_0, p_0 = n_i)$
for typical G values.

⇒ $(\Delta n)^2$ is Dominant term

$$= \frac{-(\Delta n)^2}{\tau_n \Delta n + \tau_p \Delta n}$$

$$= \frac{-(\Delta n)^2}{\tau_n \Delta n + \tau_p \Delta n}$$

$$= \frac{-(\Delta n)}{(\tau_n + \tau_p)}$$

$$\frac{d(\Delta n)}{dt} = -\frac{\Delta n}{\tau_{eff}} + g$$

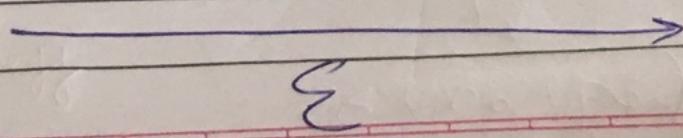
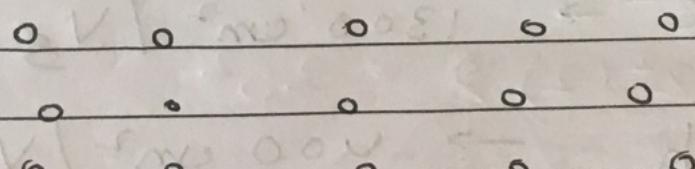
→ here $\tau_{eff} = (\tau_n + \tau_p)$

Transport

→ In presence of electric field

$$F = -qE$$

→ But in presence of regular lattice collisions.

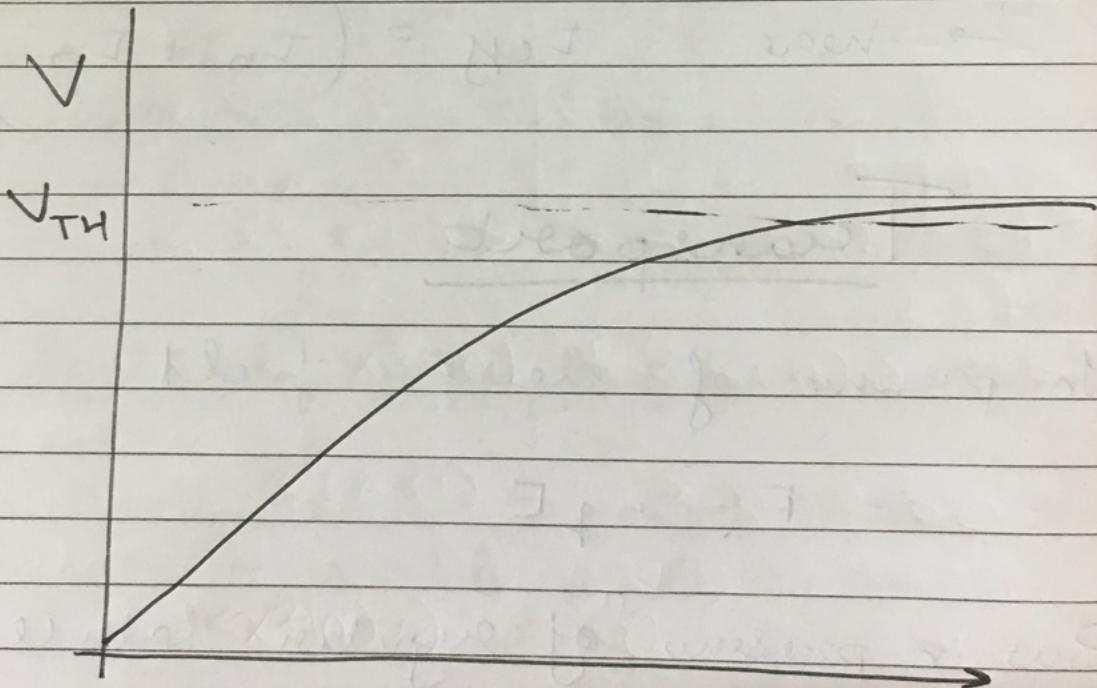


$$V = \mu \mathcal{E}$$

$$\Delta V = \frac{q E \Delta t}{m} \rightarrow \text{mobility measure.}$$

$$J_{\text{drift}} = -q n \mu_n \mathcal{E}$$

$$J_{\text{drift}} = q P \mu_p \mathcal{E}$$



$$\left. \begin{array}{l} \text{Si values.} \\ e^- \rightarrow 1300 \text{ cm}^2 / V_s \\ h \rightarrow 400 \text{ cm}^2 / V_s \end{array} \right\}$$

Next Monday
Quiz - 3

Syllabus \rightarrow Gen. - Recom.

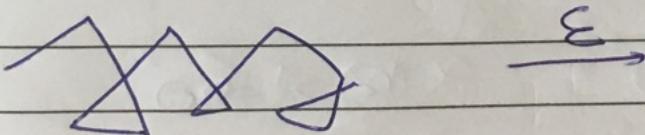
- Generation - Recombination

Email { → Radios(??)
→ SRH
→ Optical sen.
→ Quasi - Fermi

Transport

\rightarrow Phenomenon of $\vec{d}\sigma$ drift

$$\vec{E}$$



\rightarrow Get Mean collision time τ_c

$$m^* \frac{dv}{dt} = -qE$$

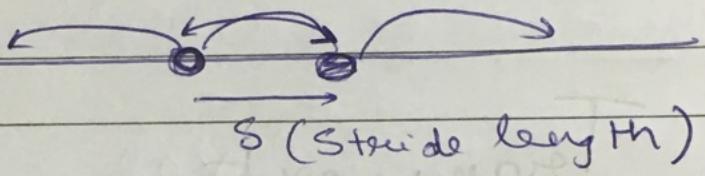
$$v = -\frac{q}{m^*} \tau_c E$$

\hookrightarrow v (Makility)

$$\mu_n = 1200 \text{ cm}^2/\text{s}$$

$$\mu_p = 400 \text{ cm}^2/\text{s}$$

Transport via Diffusion
(Random walk)



$$x(n+1) = x(n) \pm s$$

$$\langle x^2(n+1) \rangle = \langle (x(n) \pm s)^2 \rangle$$

$$= \langle x^2(n) \pm 2(x(n)s + s^2) \rangle$$

$$\langle \rangle = 0$$

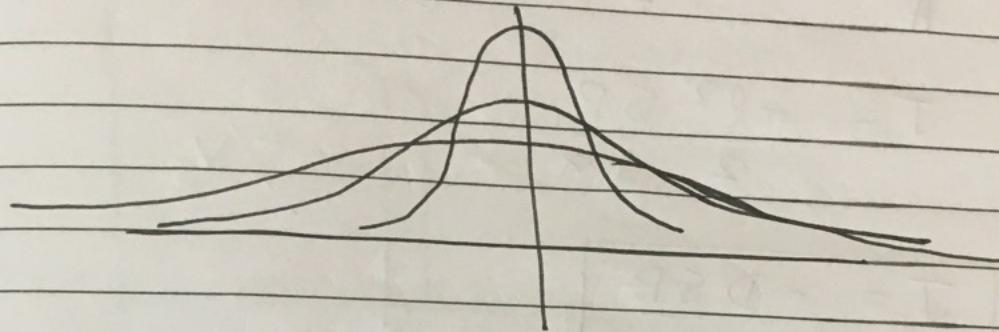
$$x^2(0) = 0$$

$$\langle x^2(1) \rangle = s^2$$

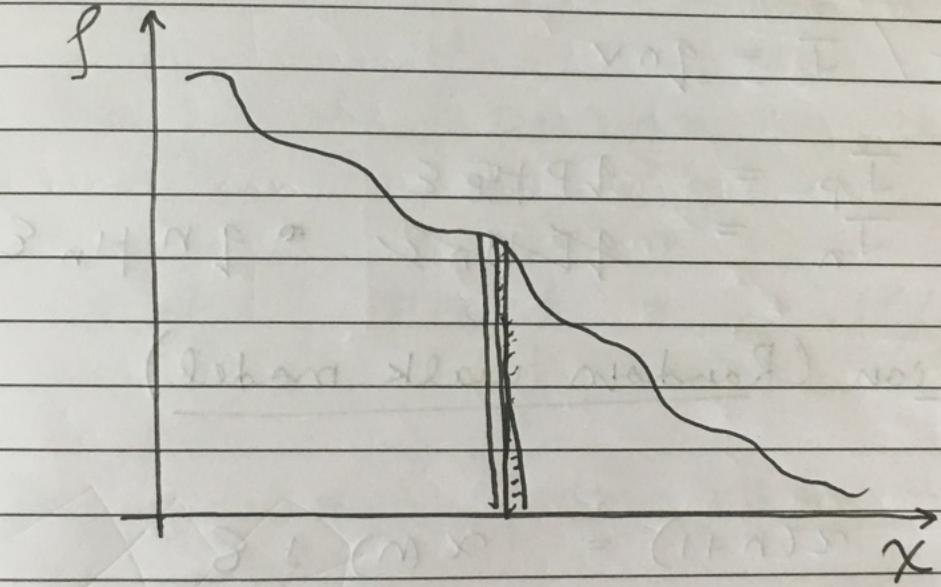
$$\langle x^2(2) \rangle = 2s^2$$

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$$\langle x^2(n) \rangle = n s^2$$



→ Diffusion can lead to net current if there is a concentration gradient



$$P(x) = P(0) + \frac{\partial P}{\partial x} \Big|_{x=0} \cdot x$$

$$I_{\text{left}} = A \times \frac{1}{2} \int_0^l P(x) dx = A \left[\frac{P(0)l}{2} + \frac{\partial P}{\partial x} \Big|_{x=0} \frac{l^2}{2} \right]$$

$$I_{\text{right}} = A \times \frac{1}{2} \int_{-l}^0 P(x) dx = A \left[\frac{P(0)(-l)}{2} - \frac{\partial P}{\partial x} \Big|_{x=0} \frac{l^2}{2} \right]$$

$$\frac{I}{A} = -\frac{l^2}{2} \frac{SP}{8x}$$

$$J = -\frac{l^2}{2} \frac{SP}{8x}$$

$$J = -D \frac{SP}{8x}$$

Carrier Transport

- Drift / $J = qnV$

$$J_p = qP \mu_p \epsilon$$

$$J_n = qE \mu_n \epsilon \cdot q n \mu_n \epsilon$$

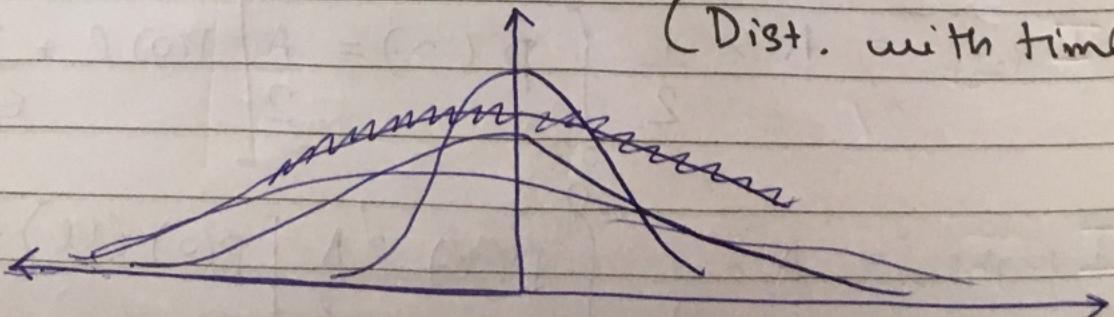
Diffusion (Random walk model)

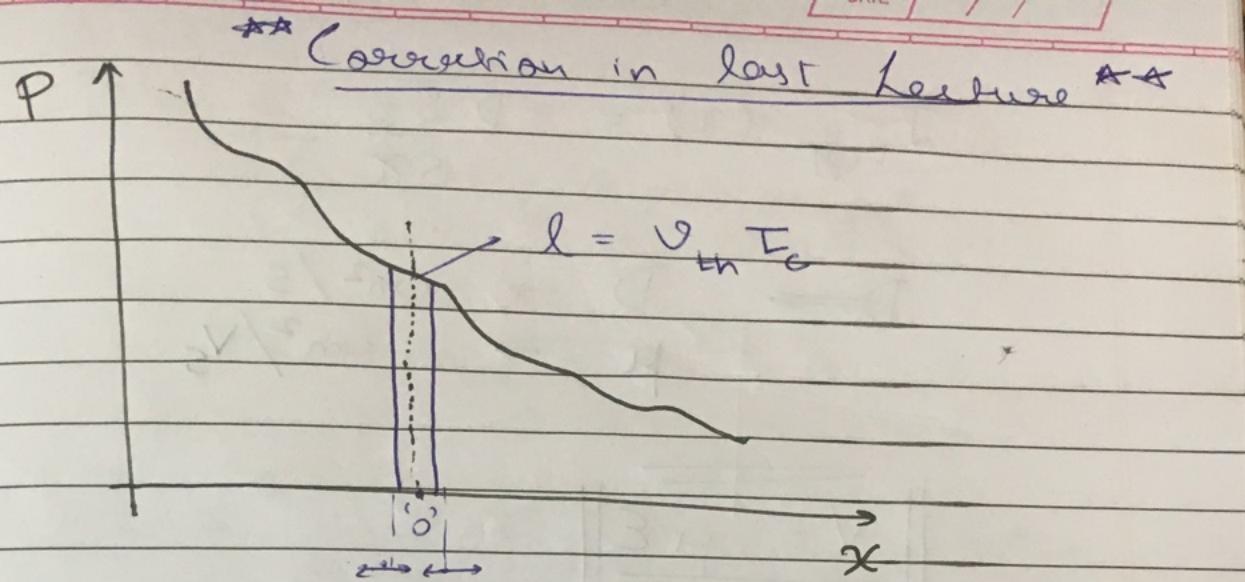
$$x(n+1) = x(n) \pm \delta$$

$$\langle x(n) \rangle = 0$$

$$\langle x^2(n) \rangle = ns^2$$

(Dist. with time)





$$P(x) = P(0) - \frac{S_p}{S_x} |_{x=0} \cdot x$$

$$\begin{aligned} N_{\text{left}} &= \frac{A}{2} \int_0^l P(x) dx \\ &= \frac{A}{2} \left[P(0)l + \frac{S_p}{S_x} \frac{l^2}{2} \right] \end{aligned}$$

$$\begin{aligned} N_{\text{Right}} &= \frac{A}{2} \int_{-l}^0 P(x) dx \\ &= \frac{A}{2} \left[P(0)l - \frac{S_p}{S_x} \frac{l^2}{2} \right] \end{aligned}$$

$$\Delta N_{\text{night}} = -\frac{A}{2} \frac{S_p}{S_x} l^2 = J \times T_c \times A$$

$$J(x) = -\frac{l^2}{2T_c} \frac{S_p}{S_x}$$

$\underbrace{\qquad}_{\qquad}$ called D

$$J_{\text{diff}}(x) = -D \frac{\delta P}{\delta x}$$

$$\begin{aligned} D &\rightarrow D \rightarrow \text{cm}^2/\text{s} \\ \mu &\rightarrow \text{cm}^2/\text{Vs} \end{aligned}$$

$$\left| \underline{\underline{V = \mu E}} \right|$$

$$F = -qE$$

$$m^* \frac{d\underline{V}}{dT} = -qE$$

$$V = \left(\frac{qE}{m} \right) E$$

$\downarrow \mu$

Diffusion

$$J_{\text{P,diffusion}} = -q D_p \frac{\delta P}{\delta x}$$

$$J_{n,\text{diffusion}} = q D_n \frac{\delta n}{\delta x}$$

Einstein Relation

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$$D = \frac{l^2}{2 \tau_c}$$

$$\mu = \frac{q \tau_c}{m^*}$$

$$l = v_{th} \tau_c$$

$$D = \frac{v_m^2 \tau_c^2}{2 \nu_c} = \frac{v_{th}^2 m^* \mu}{2q}$$

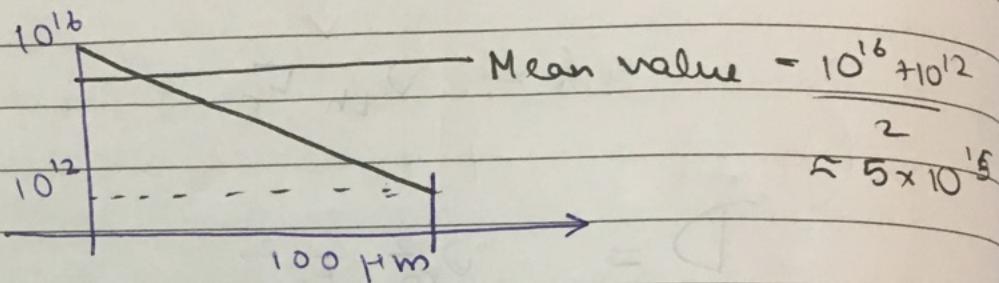
Get , $\frac{D}{\mu} = \left(\frac{1}{2} m^* v_{th}^2 \right) \times \frac{1}{q}$

$$\boxed{\frac{D}{\mu} = \frac{kT}{q}}$$

→ Relation b/w Mobility & Drift coefficient.

Non-uniformly Doped Semiconductor

Semiconductor $\rightarrow N_D(x)$



\rightarrow If diffusion distributes ~~chaos~~ dopants uniformly, there will be a charge sep. which will lead to a drift current

Equilibrium

$$J = 0$$

\rightarrow Solving for $J = 0$

Using,

$$n = n_i e^{(E_F - E_i)/kT}$$



$$\epsilon = -qV$$

$$E = -\frac{dV}{dx} = \frac{1}{q} \left[\frac{dE_x}{dx}, \frac{dE_y}{dx}, \frac{dE_z}{dx} \right]$$

Equilibrium

$$J_{\text{drift}} + J_{\text{diffusion}} = 0$$

$$\frac{s_n}{s_x} = \frac{n_i}{kT} e^{\frac{(E_F - E_i)/kT}{n}} \times \left[\frac{SE_F}{s_x} - \frac{SE_i}{s_x} \right]$$

$$q D_n \frac{s_n}{s_x} = \frac{q D_n}{kT} n_0 \left[\frac{SE_F}{s_x} - \frac{SE_i}{s_x} \right]$$

$$= \mu n_0 \left(\frac{SE_F}{s_x} - q E \right) \quad \underline{\text{from } ①}$$

$$\rightarrow J_{\text{drift}} = (q n \mu n \epsilon)$$

$$q n \mu n \epsilon + \mu n_0 \left[\frac{SE_F}{s_x} - q E \right]$$

J_{drift}

J_{diff}

$= 0$

$$\Rightarrow \boxed{\frac{SE_F}{s_x} = 0}$$

Hence

$$J = 0 + \text{Einstein Relation}$$



E_F is a constant

(Constant Fermi Level)

Re - Cap

Man Quiz \rightarrow R-G

M.S Exam \rightarrow Everything

\rightarrow Drift and Diffusion

Drift

$$J_p = (q_p \mu_p E)$$

$$J_n = (q_n \mu_n E)$$

Diffusion

$$J_p = -q_p D_p \frac{\delta p}{\delta x}$$

$$J_n = q_n D_n \frac{\delta n}{\delta x}$$

$$\frac{D}{H} = \frac{kT}{q}$$

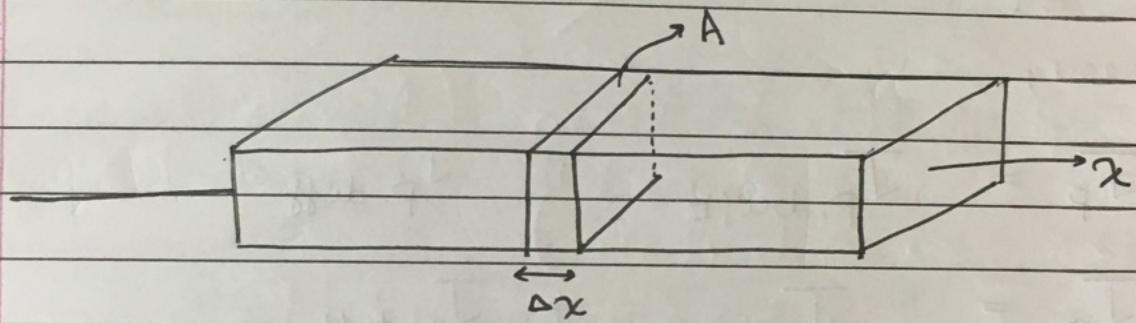
(Einstein rel.)

Equilibrium Cond.

$$\nabla E_F = 0 \quad \left(\frac{d E_F}{dx} = 0 \right)$$

$$\nabla(\varepsilon E) = (P - n + N_D^+ - N_A^-)$$

Continuity Eqn



$$N_{\text{left}} \rightarrow \rightarrow N_{\text{right}}$$

$$\Delta N = \Delta p A \Delta x = (\text{Flow}_{\text{left}} - \text{Flow}_{\text{right}}) A t$$

\downarrow
 (Δt)

Taking into account $R - g$
and writing Flow in
terms of J_p

$$\Delta N = \left\{ \begin{array}{l} \Delta p / \Delta x = 1 \left(J_{p,\text{left}} - J_{p,\text{right}} \right) \Delta t / K \\ q \\ - R (K \Delta x) \Delta t \\ + g (K \Delta x) \Delta t \end{array} \right.$$

Dividing both sides by $\Delta x \Delta t$ (In the line)

$$\frac{S_p}{S_t} = \frac{1}{q} \left(\frac{(J_p)_{\text{left}} - (J_p)_{\text{right}}}{\Delta x} \right) - R + g$$

$$\frac{S_p}{S_t} = -\frac{1}{q} \frac{\partial J_p}{\partial x} - R + g$$

Similarly

$$\frac{S_n}{S_t} = \frac{1}{q} \frac{\partial J_n}{\partial x} - R + g$$

Where,

$$J_p = J_{p,\text{drift}} + J_{p,\text{diff}}$$

$$J_n = J_{n,\text{drift}} + J_{n,\text{diff}}$$

$$n = n_i e^{(E_{F_n} - E_i)/kT}$$

$$P = P_i e^{(E_i - E_{F_p})/kT}$$

Examples

for Band Diagram & Equilibrium

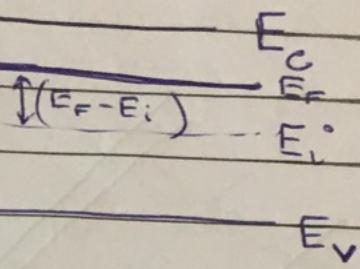
Semi. Resistor

$$N_d = 10^7 \text{ cm}^{-3}$$

a) Equilibrium

b) Steady State ($E = 10^3 \text{ V/cm}$)

Band Diagram



E_g Lec 20

Equations

$$\frac{S_n}{S_t} = \frac{1}{q} \nabla J_n - R + G$$

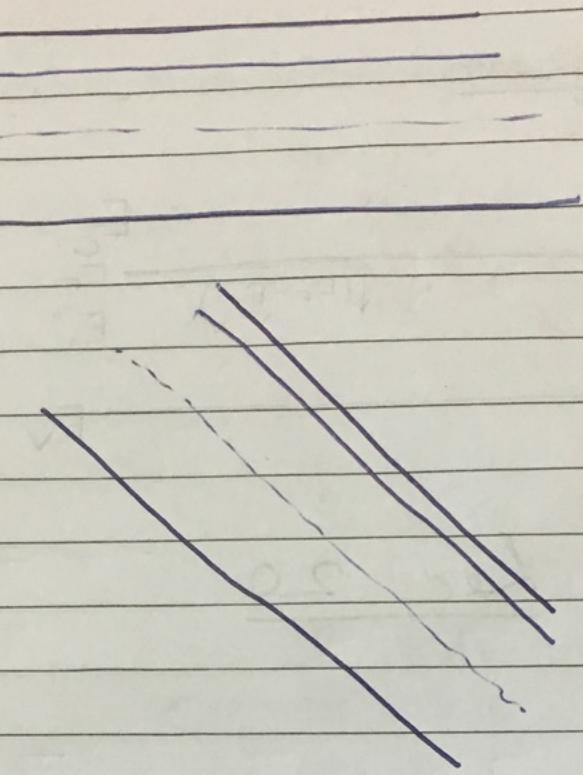
$$\frac{S_p}{S_t} = -\frac{1}{q} \nabla J_p - R + G$$

$$\nabla(\epsilon E) = \left(p' \overset{s}{\nearrow} n \overset{s}{\nearrow} + N_D^+ - N_A^- \right)$$

$$J_p = \cancel{q \mu_p E} - q \mu_p p E - q D_p \frac{S_p}{S_x}$$

$$J_n = q \mu_n n E + q D_n \frac{S_n}{S_x}$$

Constant and Uniform electric field
(E)



→ Drop Diffusion term (Assume correctly
that p & n are uniform throughout
the semiconductor)

Current

$$J = J_p + J_n$$

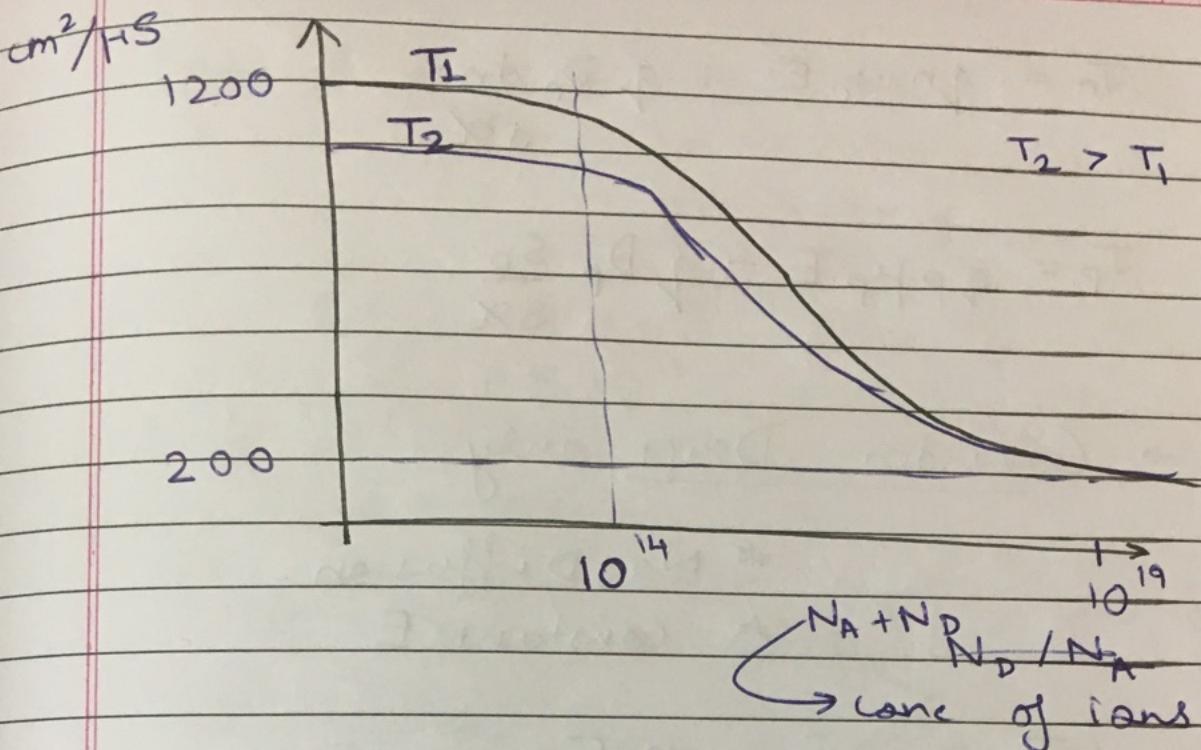
$$= q \mu_n n E + q \mu_p p E$$

$$J = q (\mu_n n + \mu_p p) E$$

↓

$$\sigma$$

(from $J = \sigma E$)



carrier
(+)

for ↑ speed

carriers spend less time near deflector.

With ~~the~~

1) Lattice atoms ($\mu \downarrow$)

2) Ionic impurities ($\mu \uparrow$)

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Recap

$$\frac{S_n}{S_F} = \frac{1}{q} \nabla \cdot J_n - R + G$$

$$\frac{S_p}{S_F} = -\frac{1}{q} \nabla \cdot J_p - R + G$$

$$\nabla \cdot (\epsilon E) = p - n + N_d^+ - N_a^-$$

$$J_n = q_n H_n E + q \frac{D_n}{\sigma} \frac{dn}{dx}$$

$$J_p = q_p H_p E - q \frac{D_p}{\sigma} \frac{dp}{dx}$$

→ Consider Drift only

- * No Diffusion
- * constant E

$$J_n = q_n H_n n E$$

$$J_p = q_p H_p p E$$

$$J = q (H_n n + H_p p) E$$

Now,

$$J = \sigma E$$

$$\sigma = q (H_n n + H_p p)$$

Take,

$$\frac{kT}{q} = 25 \text{ mV}$$

$$n_i = 10^{10} \text{ cm}^{-3}$$

$$H_n = \cancel{1000} 1200 \text{ cm}^2/\text{Vs}$$

$$H_p = 400 \text{ cm}^2/\text{Vs}$$

Band

↓ 0.2 eV

$$P \approx 10^{13} \quad n \approx 10^7$$

$$J = \sigma E$$

$$\sigma = q (H_n n + H_{PP}) \rightarrow \div 3 \text{ if } P \approx 10^{13}$$

$$\sigma \approx 1.8 \times 10^{-3} \Omega^{-1} \text{ cm}^{-1}$$

Here σ varies with Doping

→ Under Illumination

$$g = 10^{17} \text{ cm}^{-3} \text{ s}^{-1}$$

$$\tau_n = \tau_p = 1 \mu\text{s}$$

$$\Delta n = g \tau_n$$

$$\Delta n = 10^{17} \text{ cm}^{-3} \text{ s}^{-1} \times 1 \mu\text{s} \times 10^{-6}$$

$$= 10^{11} \text{ cm}^{-3} = \Delta p$$

→ Here low level Injection is valid

$$\Rightarrow \sigma = q (1200 \times 10^{11} + 400 \times 10^{13})$$

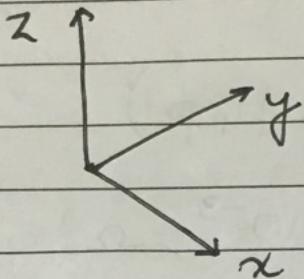
$$\sigma \approx 1.8 \times 10^{-3} \left(\begin{array}{l} \text{Still} \\ \text{unaffected} \end{array} \right)$$

$\div 3 \text{ if } P \approx 10^{13}$

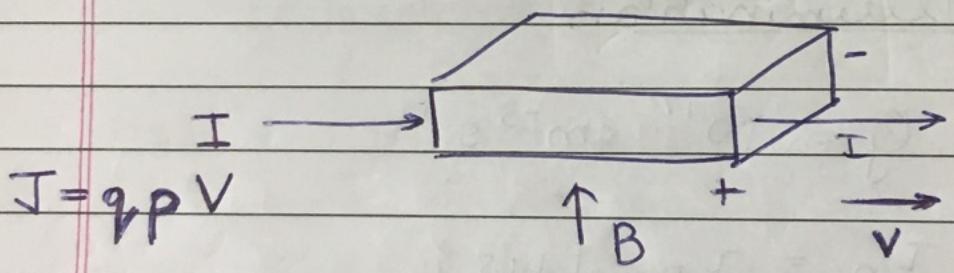
→ Say if $\Delta n \approx 10^{12} \text{ cm}^{-3}$ (Still low level injection but change in σ will be sig.)

$$\rightarrow \sigma = q (H_{nn} + H_{pp})$$

↓ ↓
splitting by hall effect



→ Use Hall effect (for Drift only \Rightarrow Majority carrier)



$$-qBv + qE_y = 0$$

$$-\frac{qBJ}{qp} + qE_y = 0$$

$$\boxed{\frac{E_y}{BJ} = \frac{1}{qp}}$$

Get p (Majority) carrier from here.

Diffusion

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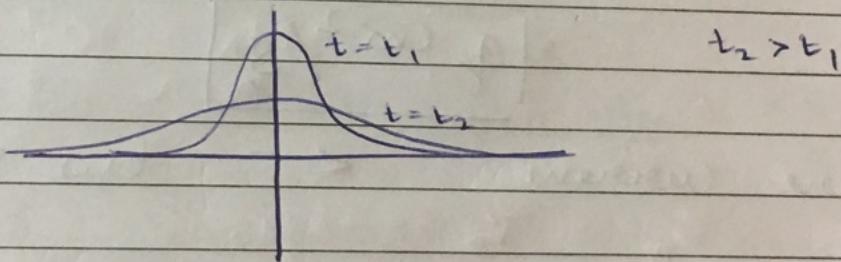
→ Take $E = 0$, No drift, only diffusion.

$$\begin{aligned} J_n &= q D_n \frac{\delta n}{\delta x} \\ J_p &= -q D_p \frac{\delta p}{\delta x} \end{aligned} \quad \left. \begin{array}{l} \text{Fick's 1st} \\ \text{Law of Diffusion} \end{array} \right.$$

$$R = G = 0 \quad (\text{Equilibrium})$$

$$\frac{\delta n}{\delta t} = \frac{1}{q} (\nabla \cdot J_n)$$

$$\frac{\delta n}{\delta t} = D_n \frac{\delta^2 n}{\delta x^2}$$

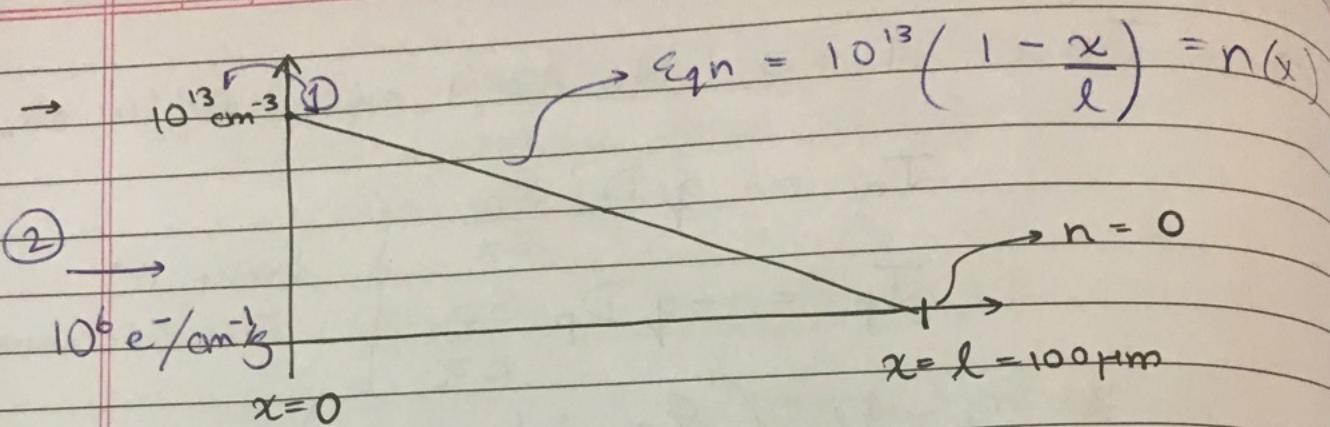


$$J_n \text{ steady state: } \frac{\delta n}{\delta t} = 0$$

$$\Rightarrow D_n \frac{\delta^2 n}{\delta x^2} = 0$$

Just like Laplace Equation $\rightarrow \nabla^2$

Here in $\rightarrow n$



$$J_n = (-q) (-D_n) \frac{S_n}{8x}$$

$$\text{Ans} \quad J = -q \times (0.025 \times 1200) \times \frac{10^{13}}{100 \mu\text{m}}$$

$$\text{Unit} = (-q) \text{ cm}^2/\text{s} \times \frac{\text{cm}^{-3}}{\text{cm}}$$

$$= 1 \text{ A cm}^{-2}/\text{s}$$

→ Here current ← but n →

Case 2

→ Steady state but constant current is being pushed through.

Now find new $n(x)$ soln.

$$n(x) = Ax + B$$

$$n(l) = 0$$

$$\left. \frac{D S_n}{8x} \right|_{x=0} = 10^6 \text{ cm}^{-2}/\text{s}$$

$$n(x) = \frac{10^6}{D} x + B = 0$$

| $n(x=l) \rightarrow$ Get B. |

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Mid-sem exam

* ASDF

* SDF - Chapter 3
Carrier Action

Diffusion continued

$$E = 0, R = G = 0 \quad \frac{s_n}{s_t} = \frac{s_p}{s_t} = 0$$

$$\ln \frac{1}{1-D}$$

$$0 = D \frac{s_n^2}{s_x^2}$$

case 1: End point conc. specified (like attaching a battery)

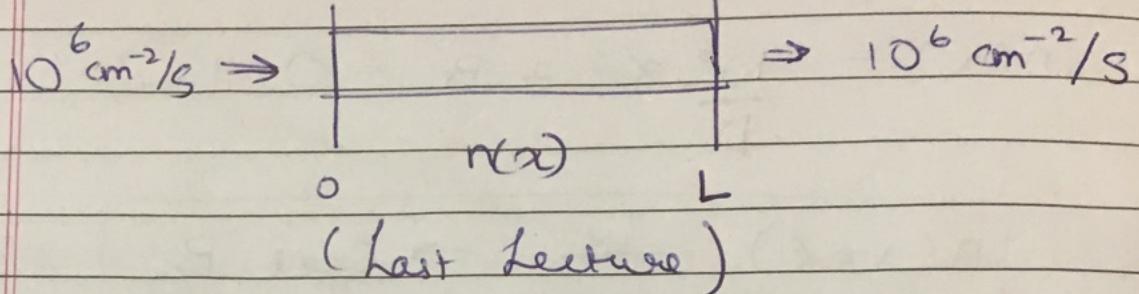
$$n(0) = n_0 \quad n(L) = 0$$

~~$n \neq 0$~~

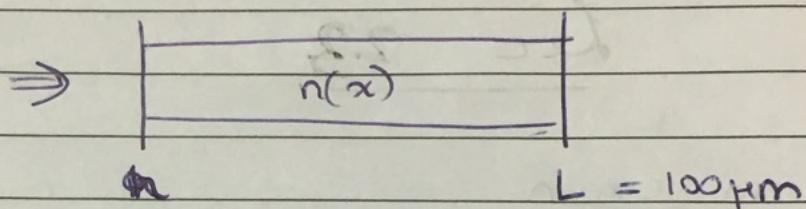
case 2: J through Semicond. (like a constant current source)

$$n(L) = 0$$

$$J_n = -D_n \frac{s_n}{s_x} \Big|_{x=0}$$



case 3



$$E = 0, \quad g = 0 \quad (R \neq 0)$$

but $\frac{S_n}{S_t} = \frac{S_p}{S_t} = 0$

P-Type semi.

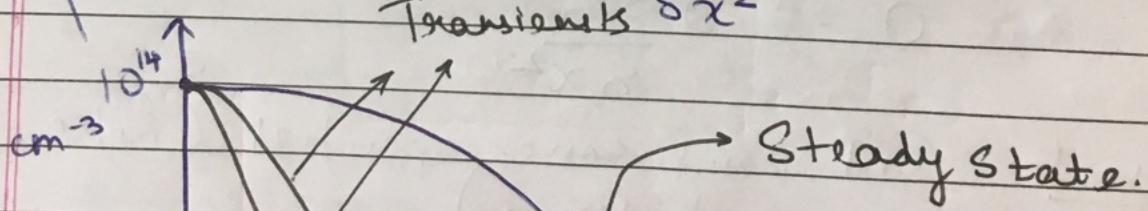
$$N_A = 10^{17} \text{ cm}^{-3}$$

$$n \approx 10^3 \text{ cm}^{-3}$$

$\text{sp.} \sim 10^3$

$I_n = I_p = 1/\mu\text{s}$

$$0 = D_n \frac{\delta^2 n}{\delta x^2} - R$$



$\approx \Delta n$

n_0

→ We have spiked the carrier density at left end, this is not illumination.

Now we shift our Analysis from $n \rightarrow \Delta n$

$$0 = D_n \frac{s^2 \Delta n}{s x^2} - \frac{\Delta n}{I_n}$$

$$\Delta n = A e^{-x/l} + B e^{x/l}$$

→ Let the semiconductor be semi ∞

$$\Delta n(0) \simeq 10^{14} \text{ cm}^{-3} = N$$

$$\Delta n(\infty) = 0$$

$$\Delta n(\infty) = 0 \Rightarrow B = 0$$

$$\Delta n(0) = N = A$$

$$\Delta n = N e^{-x/l}$$

$$l = \sqrt{D_n I_n} =$$

→ L finite can also covered

case : 4

$$E = 0, \quad g = G_0 \quad \frac{\delta n}{\delta E} = \frac{\delta p}{\delta t} = 0$$

P-type,

$$N_A = P_0 = 10^{17} \text{ cm}^{-3}$$

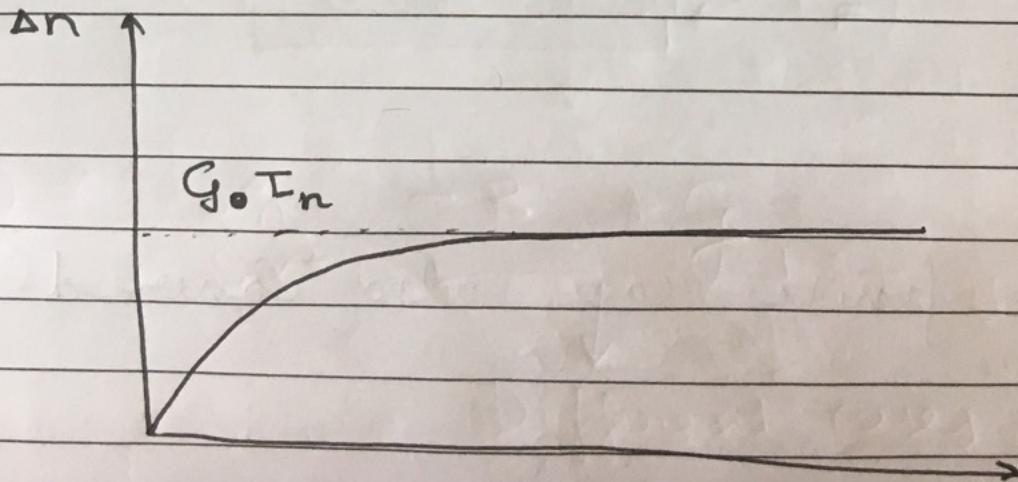
$$n_0 \sim 10^3 \text{ cm}^{-3}$$

$$\tau_n = \tau_p = 1 \mu s$$

$$n(x) = n_0 + \Delta n(x)$$

$$0 = D_n \frac{\delta^2 \Delta n}{\delta x^2} - \frac{\Delta n}{\tau_n} + G_0$$

$$\Delta n = A e^{x/\tau} + B e^{-x/\tau} + C$$



→ Friday 15/9/17

TA help session gg 001 / 102
2:30 - 4:30 pm
6 - 8:30 pm

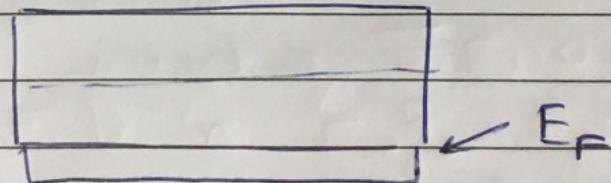
Lec 24

→ Our (Diffusion) eqⁿ

$$\frac{S_n}{S_t} = \frac{1}{q} \nabla \cdot J - R + G$$

$$J = \underbrace{q H_n n E}_{\text{Draft}} + \underbrace{q D_n \frac{dn}{dx}}_{\text{Diffusion}}$$

Steady State $\rightarrow \frac{dn}{dt} = 0$

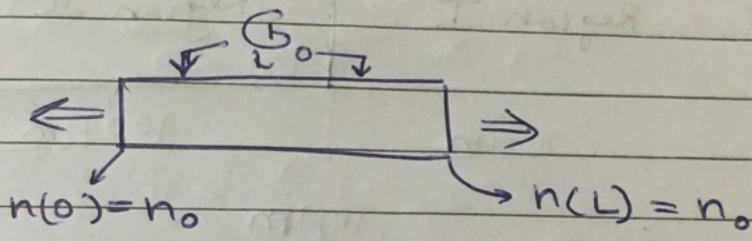


$$0 = D \frac{\delta^2 n}{\delta x^2} - R + G$$

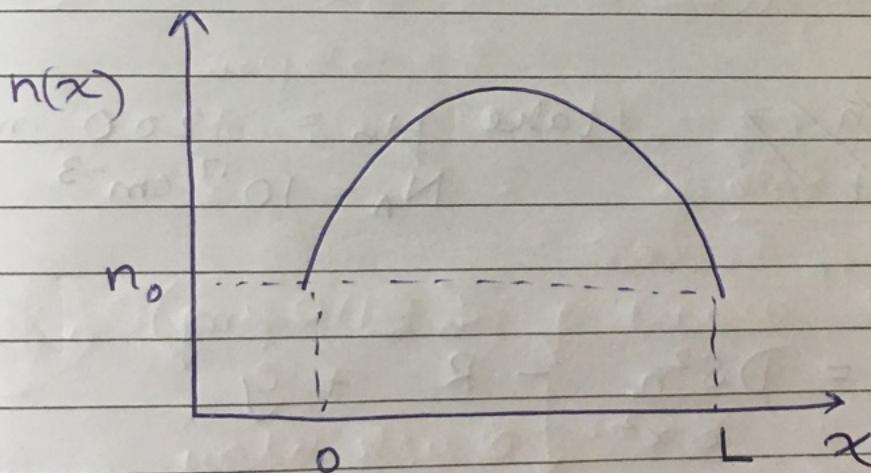
Different cases

$$n(x) = \begin{cases} Ax + B & R = g = 0 \\ \frac{G_0 x^2}{2} + Ax + B & R = 0, g \neq 0 \\ Ae^{-x/l} + Be^{x/l} & R \neq 0, g = 0 \\ & R \neq 0, g \neq 0 \end{cases}$$

→ Uniform Illumination ($G = G_0$)



n(x) profile



Note that

$$\left. -D \frac{\delta n}{\delta x} \right|_{x=0} + \left. -D \frac{\delta n}{\delta x} \right|_{x=L} = (G_0 L)$$

→ Complicated case $\left(\frac{\dot{S}_n}{\delta t} = 0 \right) \therefore$ Steady State

I

$$G_0 = 10^{17} \text{ cm}^{-3}/\text{s}$$

$$\begin{cases} \tau_n = \tau_p = \infty \\ \rightarrow \Rightarrow R \rightarrow 0 \end{cases}$$

II

$$G_0 = 0$$

$$\tau_n = \tau_p = 1 \text{ HS}$$

Non-absorb.
contact

(Blocks
flow)

(i.e $J=0$)
(or $\frac{\dot{S}_n}{\delta x} = 0$)

Region I

$$10 \mu\text{m}$$

Region II

$$100 \mu\text{m}$$

Perfect
Contact

$$n=n_0$$

as in absence
of any
contact

$$\frac{\dot{S}_n}{\delta t} \neq 0$$

take $H_n = 1200 \text{ cm}^{-2}/\text{s}$
 $N_A = 10^{17} \text{ cm}^{-3}$

$$0 = D \frac{\dot{S}_n^2}{\delta x^2} - R + G$$

Region I

$$0 = D \frac{\dot{S}_n^2}{\delta x^2} + G_0$$

$$\frac{\dot{S}_n}{\delta x^2} = -\frac{G_0}{D}$$

$$n = -\frac{g_0}{D} \frac{x^2}{2} + c_1 x + c_2$$

Region 2

$$0 = D \frac{\delta^2 n}{\delta x^2} - \frac{\Delta n}{T_n}$$

$$\Delta n = (D T_n) \frac{\delta^2 n}{\delta x^2}$$

$$\left(\frac{1}{k}\right)^2 y = \cancel{D} \frac{\delta^2 y}{\delta x^2}$$

$$y = A e^{-x/l} + B e^{x/l}$$

$$l = \sqrt{D T_n} = \sqrt{25 \times 10^{-3} \times 12 \times 10^2} \times 10^{-6}$$

$$= \sqrt{25 \times 12 \times \sqrt{10^{-7}}} \approx \sqrt{25 \times 10^{-4}} \quad \text{circled } 55 \mu\text{m}$$

$$n(x) = A e^{-x/l} + B e^{x/l} + n_0$$

n_0 $\rightarrow 0$
background
(negligible)

$$n(x) = A e^{-x/l} + B e^{x/l}$$

$$l = 55 \mu\text{m}$$

$$J(0) = 0$$

$$\rightarrow \left. \frac{\delta n}{\delta x} \right|_{x=0} = 0 \quad - \textcircled{1}$$

$$n(110 \mu m) = \cancel{0.00} \cdot 10^3 \text{ (perfect)} \quad - \textcircled{2}$$

$$n = c_1 x - \frac{c_0}{D} \frac{x^2}{2} + c_2$$

$$x \in [0, 10 \mu m]$$

$$n = A e^{-x/l} + B e^{x/l} \quad x \in [10 \mu m, 110 \mu m]$$

$l = 55 \mu m$

→ Quick soln.

We want $J(110 \mu m)$ (flux at Right side)

↓ Quick way without finding all coefficients.

$$n(x) C e^{-x/l} + D e^{x/l}$$

variable transform

$$10 \mu\text{m} \rightarrow 0 \mu\text{m}$$

$$110 \mu\text{m} \rightarrow 100 \mu\text{m}$$

$$T \mid_{x=0} = \frac{D S_n}{S_x} \mid_{x=10 \mu\text{m}} = g_0 \times 10 \mu\text{m}$$

- (1)

Uniform Generation

2nd S_n^n

$$\Delta n = n_0 \text{ at } x = 100 \mu\text{m}$$

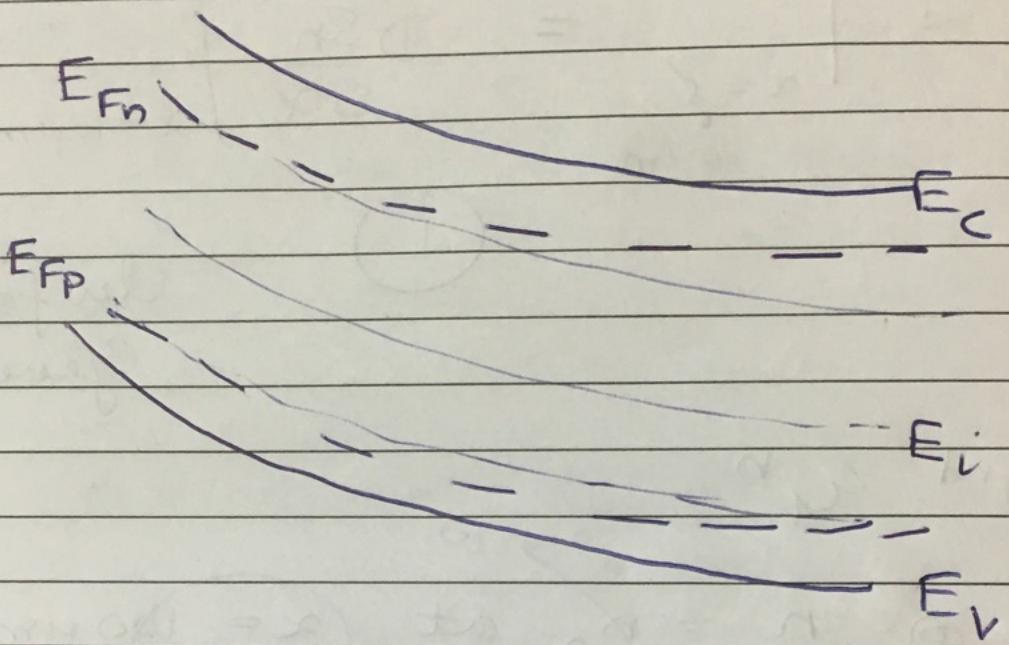
$$\Delta n = 0 \text{ at } x = 100 \mu\text{m}$$

Shifted x

Get C & D



Band Diagram Question



$$(E_{Fn} - E_i) / kT$$

$$n = n_i e^{(E_{Fn} - E_i) / kT}$$

$$(E_i - E_{Fp}) / kT$$

$$p = n_i e^{(E_i - E_{Fp}) / kT}$$

$$J_n = n q H_n E + q D_n \frac{S_n}{Sx}$$

$$n_i e^{(E_{Fn} - E_i) / kT}$$

$$q H_n E + q D_n \frac{(E_{Fn} - E_i)}{kT}$$

$$n \left(\frac{q H_n E}{kT} + \frac{q D_n}{kT} \left(\frac{\frac{E_{Fn}}{dx} - \frac{E_i}{dx}}{kT} \right) \right)$$

$$\cancel{D = \frac{kT}{q}} \\ \textcircled{D}$$

$$n \left(q \mu_n E + \mu_n \left(\frac{dE_{Fn}}{dx} - \frac{dE_i}{dx} \right) \right)$$

$$E = \frac{dV}{dx}$$

$$\cancel{\mu_e} = -q V(x)$$

$$E(x) = \frac{1}{q} \left(\frac{dE_c}{dx}, \frac{dE_v}{dx}, \frac{dE_i}{dx} \right)$$

$$-E_i') \times n$$

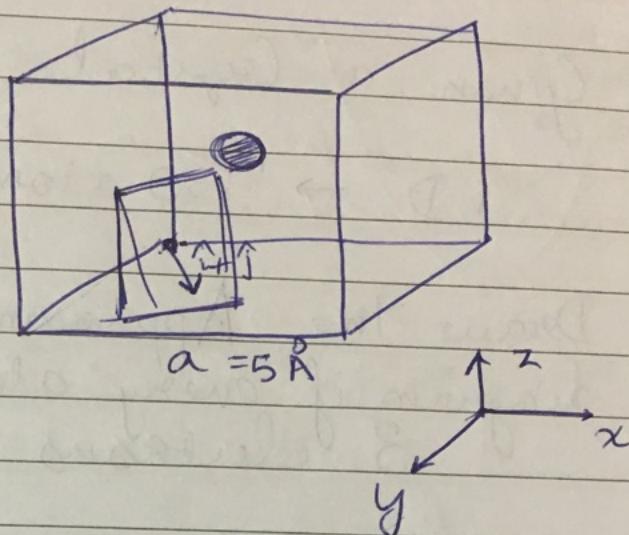
$$n \left(\mu_n \frac{dE_i}{dx} + \mu_n \left(\frac{dE_{Fn}}{dx} - \frac{dE_i}{dx} \right) \right)$$

$$J = \boxed{n \mu_n \frac{dE_{Fn}}{dx}}$$

→ Last concept in Midsem

Tut - 1

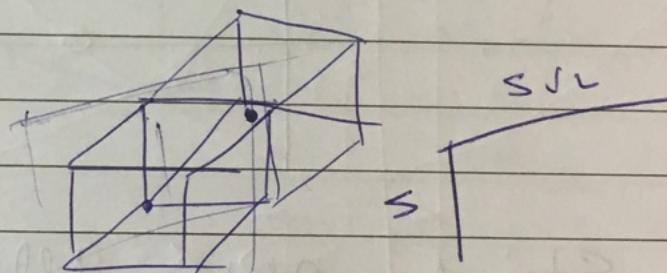
Q-1)



(110) plane density of atoms

Miller Indices, Inverse of
 $\hat{n} = \hat{i} + \hat{j}$ $\textcircled{0}$ Intercepts

1 atom per ~~$5\sqrt{2}\text{Å} \times 5\sqrt{2}\text{Å}$~~



1 atom for

$$\frac{1}{125\sqrt{2}} \times 10^{30} \text{ /m}^2$$

~~$5.66 \times 10^{27} \text{ /m}^2$~~

~~$2.82 \times 10^{28} \text{ /m}^2$~~

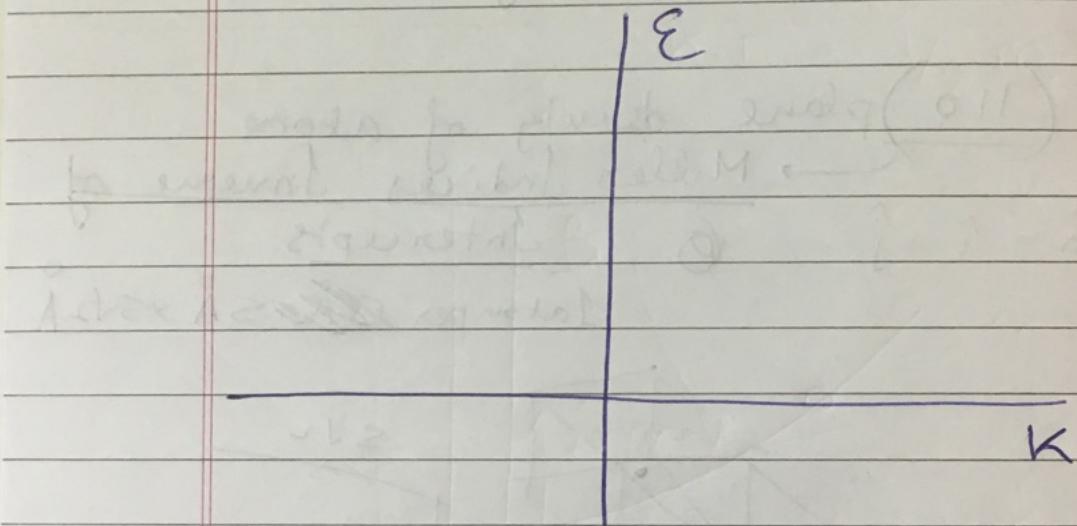
~~$5.66 \times 10^{23} \text{ /cm}^2$~~

~~$2.82 \times 10^{24} \text{ /cm}^2$~~

Q-2) Given a Crystal

1 D \rightarrow 150 atoms

Draw the Approximate E-k diagram if every atom has 3 electrons



Step 1: Obtain Allowed values of K

$$e^{ikx} = e^{ik[(Na) + x]}$$

$$e^{ikNa} = e^{i2\pi h}$$

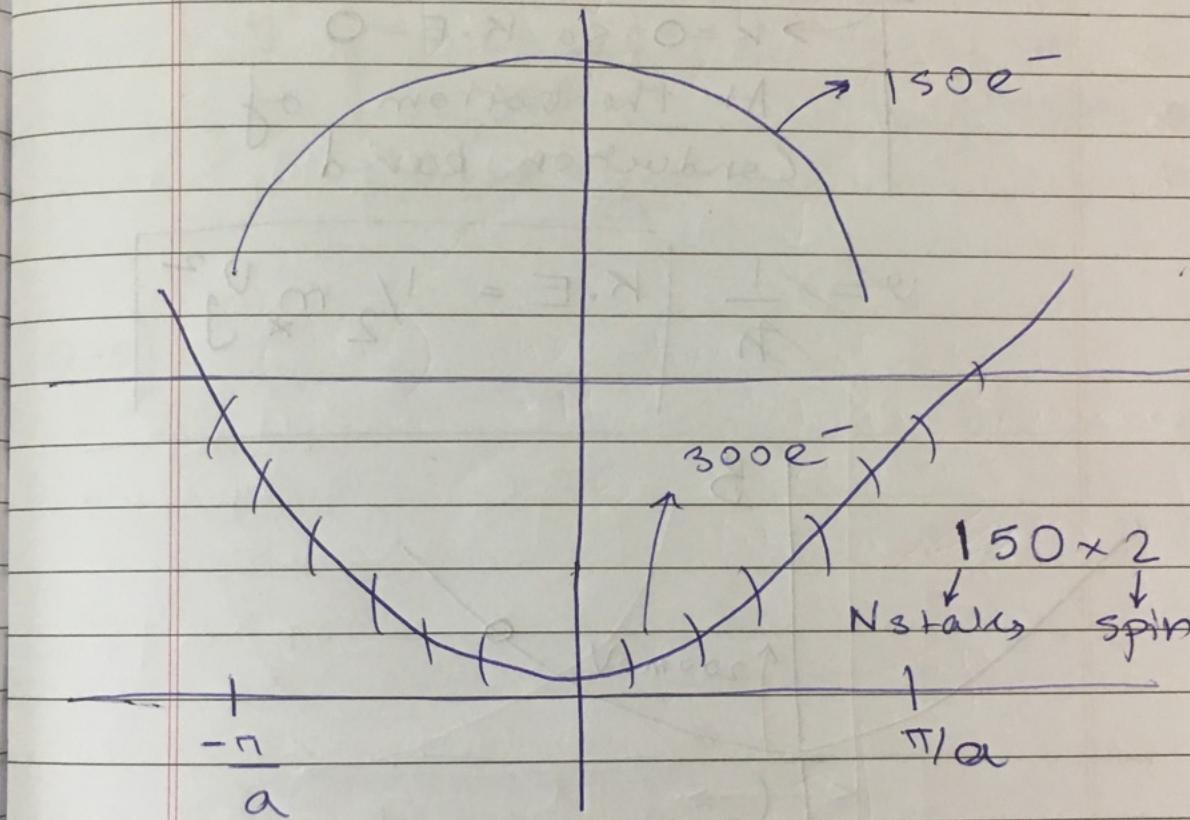
KAD

$$kNa = 2\pi h$$

$$k = \frac{p^2}{2m}$$

$$k = \frac{2\pi n}{\kappa N_a}$$

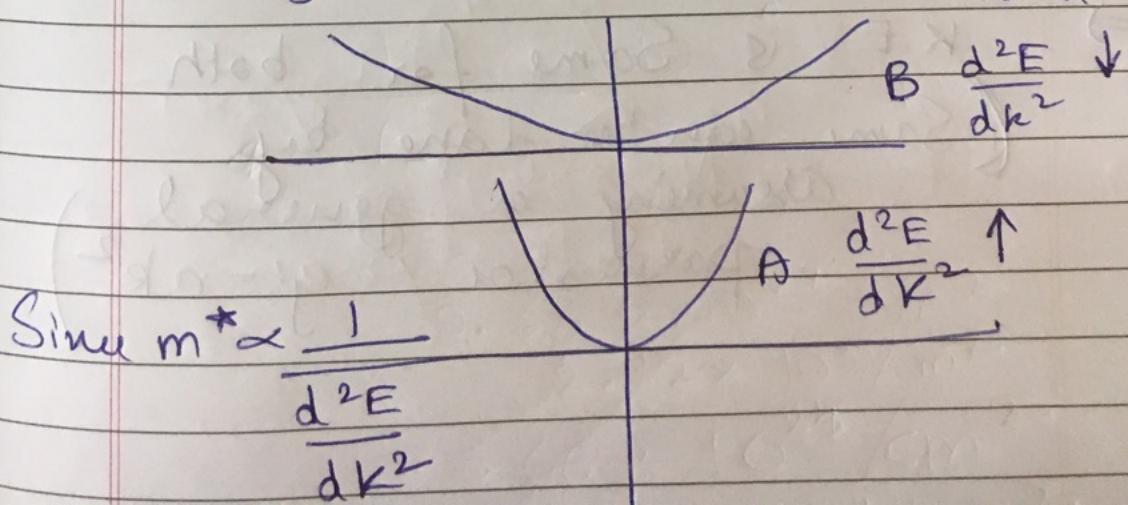
$N = 150$
total number
of atoms



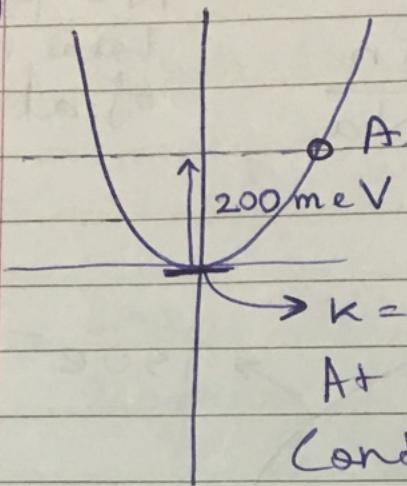
values of
Q-3)

$$\frac{m_A^*}{m_B^*} < 1 \quad \text{if}$$

Diagram
looks
like



(A)



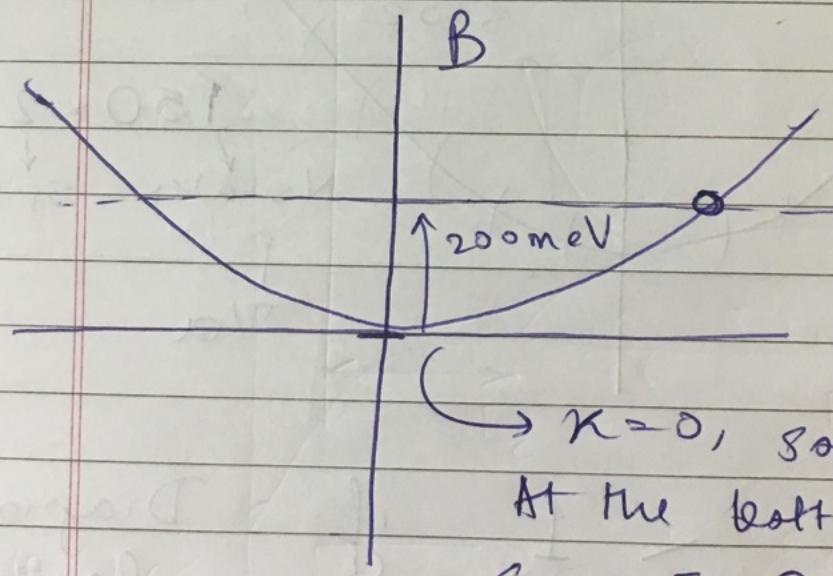
$$\vartheta = \frac{1}{\hbar} \frac{dE}{dk}$$

$\rightarrow k=0 \text{ so } K.E=0$

At the bottom of
Conduction band

$$\vartheta = \frac{1}{\hbar} \quad \boxed{K.E = \frac{1}{2} m^* v_g^2}$$

B



At the bottom,
So $K.E=0$

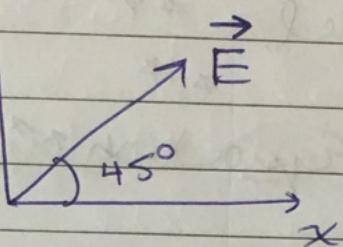
\Rightarrow K.E is Same for both

(Same can be done by
assuming a general
parabola $y = ak^2$)

Q-4)

$$\epsilon(k) = \frac{\hbar^2 k_x^2}{2m_1} + \frac{\hbar^2 k_y^2}{2m_2}$$

y



ε - k
Dispersion

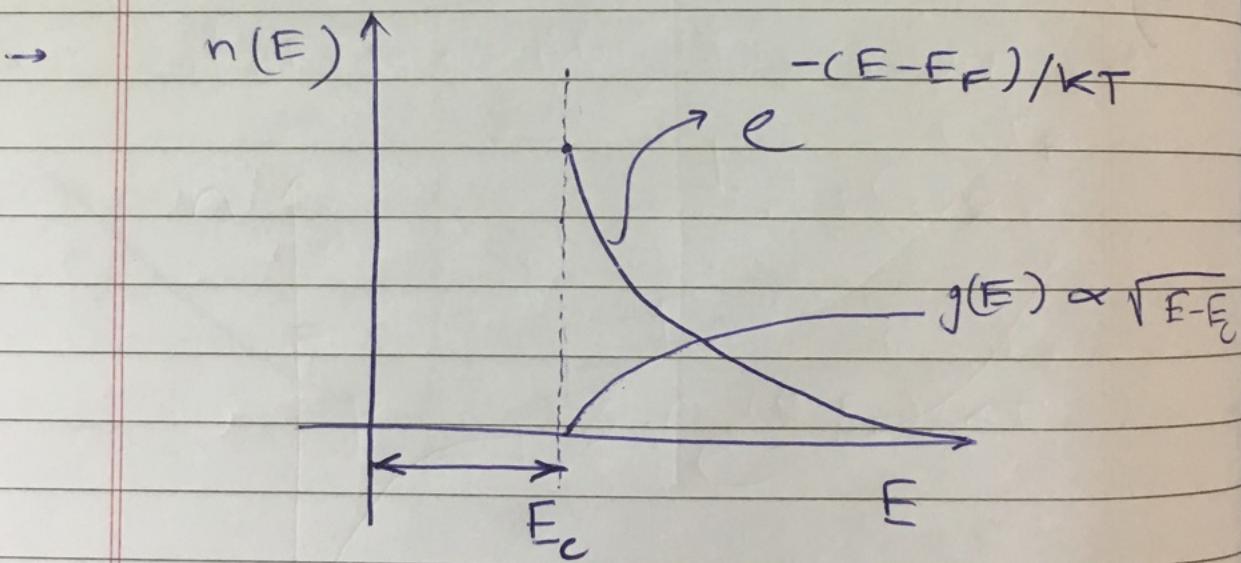
Q-5)

Equilibrium Carrier Conc.

Agenda

- * Do S ✓
- * Fermi Dirac Dist. ✓
- * Fermi Level **
- * Doping * *
- * Poisson's Equation **

Dos



$$n(E) \propto \sqrt{E - E_c} e^{-\frac{(E - E_F)}{kT}}$$

Approximating
to Boltzmann .

Q→

 E_C

Si

$$kT = 26 \text{ meV}$$

$$\frac{E_C}{E_F} \downarrow 0.2 \text{ eV}$$

 E_V 

P Doped

$$P = 10^{10} e^{-\frac{(E_F - E_V)}{26 \text{ meV}}}$$

$$P = P_i e^{\frac{(E_V - E_F)}{26 \text{ meV}}}$$

$$\frac{200}{26}$$

C

$$\Rightarrow \left[2.19 \times 10^{13} \text{ cm}^{-3} \right]$$

$$\Rightarrow N_D \approx 2.19 \times 10^{13} \text{ cm}^{-3}$$

→ But it is also possible that :

$$\left\{ \begin{array}{l} N_A = 3.19 \times 10^{13} \\ N_D = 1.00 \times 10^{13} \end{array} \right.$$

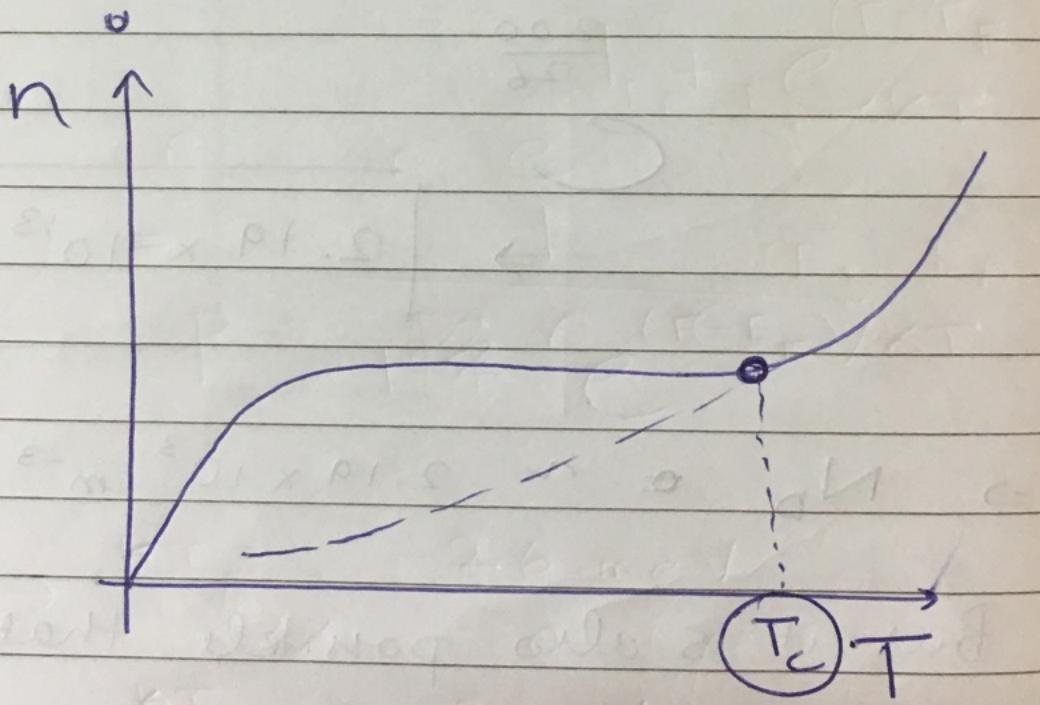
Net Result is same

→ Using Poisson's Equation

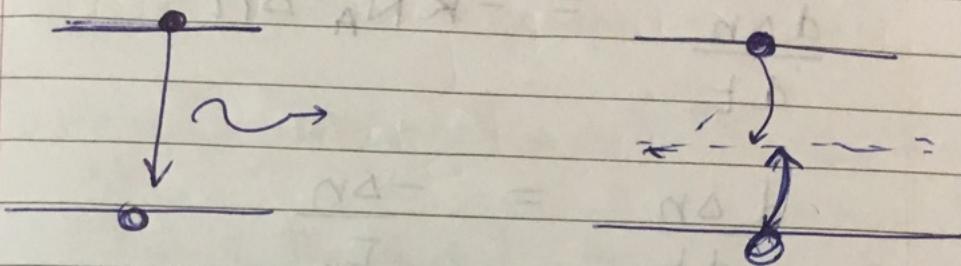
$$** (p - n) + (N_D - N_A) = 0 **$$

We can only say the total number of ~~holes~~ holes is ~~p~~ $p = N_A - N_D$ nothing else

Q → Estimate the temperature at T



$$n_{\text{total}} \propto e^{-E_g/2kT}$$

Recom - GenEquations

$$\frac{dn}{dt} = -K(n_p - n_i^2)$$

$$\frac{dn}{dt} = -\frac{(n_p - n_i^2)}{\tau_p(n + n_i) + \tau_n(p + p)}$$

Equilibrium

Ex Say,

$$\tau_n = \tau_p = 100 \text{ fs}$$

with,

$$N_d = 10^{17} \text{ cm}^{-3}$$

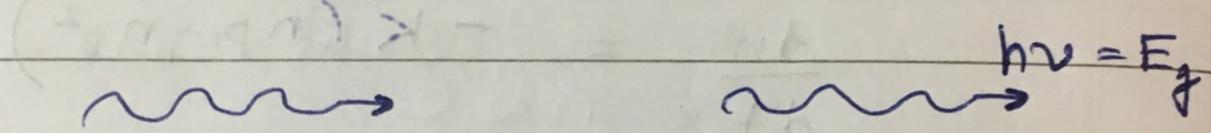
→ Rate of Recomb. = ?

→ Rate of thermal gen = ?

$$\frac{d\Delta n}{dt} = -K N_A \Delta n$$

$$\frac{d\Delta n}{dt} = -\frac{\Delta n}{\tau_n}$$

$\rightarrow Q$



$$\text{Efficiency} = \frac{N(E_g)}{N_{\text{input}}}$$

All above E_g

$$\rightarrow \frac{\text{No. of } E_g \text{ photons out}}{\text{Total input photons}}$$

→ Radiative Recom.

$$\frac{d\Delta n}{dt} = -K(n_p - n_i^2) \xrightarrow{\text{App.}} \frac{d\Delta n}{dt} = -KN_A \Delta n$$

→ SRS (Thermal) $\xrightarrow{\text{App.}} \frac{d\Delta n}{dt} = -\frac{\Delta n}{T_n}$

$$\frac{d\Delta n}{dt} = -\frac{(n_p - n_i^2)}{T_p(n+n_i) + T_n(p+p_i)}$$

↓ Combined Rate

$$\frac{d\Delta n}{dt} = -KN_A \Delta n - \frac{\Delta n}{T_n}$$

↓ Iridescence

$$= -KN_A \Delta n - \frac{\Delta n}{T_n} + G = 0$$

spills out photon

↪ No photons

In unit time

$$\text{Efficiency} = \left(\frac{KN_A}{KN_A + \frac{1}{T_n}} \right)$$

Derivation

$$n = \frac{K N_A \Delta n}{G}$$

$$= \frac{K N_A \Delta n}{K N_A \Delta n + \frac{\Delta n}{E_n}}$$

$$\boxed{\frac{K N_A}{K N_A + \frac{1}{E_n}}}$$

→ Direct B.G Mat.

$$K = 10^{-10} \text{ cm}^{-3}/\text{s}$$

Given

$$E_n = 100 \mu\text{s}$$

$$N_A = 10^{17} \text{ cm}^{-3}$$

$$= \frac{10^{-10} \times 10^{17}}{10^{-10} \times 10^{17} + 10^4} \approx 1$$

Case -II

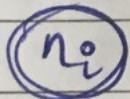
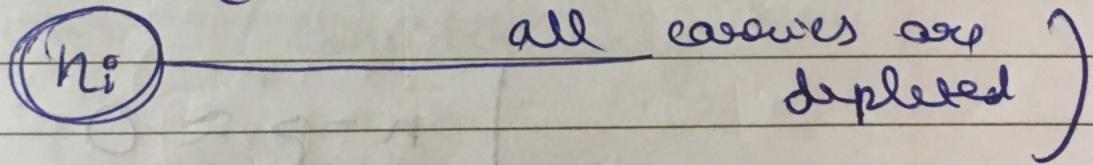
$$k = 10^{-15} \text{ cm}^{-3}/\text{s}$$

→ Indirect BG Mat.

$$\approx \frac{10^2}{10^6} \approx 0 \text{ (Law)}$$

Case III

Q → Given a semi. cont. All holes and e^- combine such that no e^- / hole left, how much time to return to equilibrium
 (Intrinsic Semicond., at $T=0$)



$$\frac{dn}{dt} = -k(n_p - n_i^2)$$

(Ans 1: Creation using only B_2B)

$$\frac{dn}{dt} = -k(n^2 - n_i^2)$$

$n \ll n_i$ for most time

$$\frac{d\Delta n}{dt} = kn_i^2$$

$\cancel{k} \approx \cancel{n_i}$

$$(kn_i^2 \Delta t) \approx n_i \quad (\rightarrow \text{Loves } \underline{\Delta t})$$

→ Using Trap Levels near mid-band

$$- (n_p - n_i^2)$$

$$T_p(n+n_i) + T_n(p+p_i)$$

$$n_i e^{-c?}$$

$$n_i e^{-c?}$$

$$\downarrow n = p \approx 0$$

$$= \cancel{n_i^2}$$

~~$T_{pn_i} + T_n p_i$~~

Around mid band

~~$n_p \approx n_i$~~

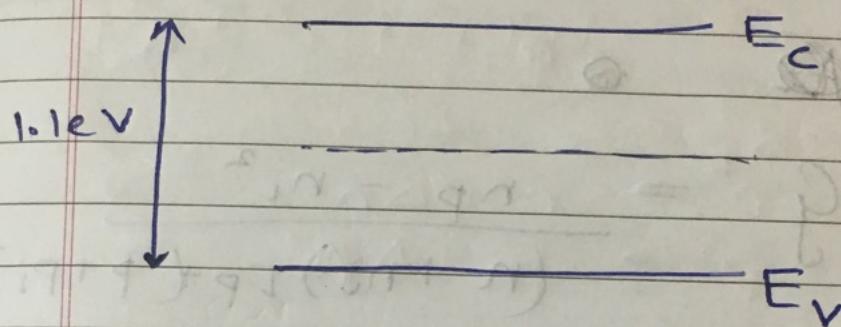
$$n_i \approx p_i \approx n_i$$

$$\frac{d(\Delta n)}{dT} = \left(\frac{n_i}{T_p + T_n} \right) \quad \checkmark$$

classmate
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Lower bound
on Δn

$\Rightarrow g = 10^{18} \text{ cm}^{-3}/\text{s}$



$$T_n = T_p = 10^{-6} \text{ s}$$

~~$E_{F_N} - E_{F_P} = ?$~~

For, (Consider only SRH)

- (a) Intrinsic
- (b) Doped with $N_d = 10^{17} \text{ cm}^{-3}/\text{s}$

(a) $\frac{d\Delta n}{dT}$

$$n = n_i e^{\frac{(E_{F_N} - E_i)}{kT}}$$

$$p = n_i e^{\frac{(E_i - E_{F_P})}{kT}}$$

Only SRH

$$\Rightarrow G = \frac{np - n_i^2}{(n + n_i) \Sigma_p + (P + P_i) \Sigma_h}$$

~~G~~ Σ

$$G = \frac{np - n_i^2}{(n + n_i) \Sigma_p + (P + P_i) \Sigma_h}$$

$$n_i = p_i \approx n_i \quad (\text{B} T_{\text{gap}} \text{ at mid gap})$$

$n = p$ (Unterseite)

$$G = \frac{n^2 - n_i^2}{(n + n_i) (\Sigma_p + \Sigma_h)}$$

$$G = \frac{n - n_i}{\Sigma_p + \Sigma_h}$$

$G \approx$

$$n = p \Rightarrow E_{Fn} - E_i^{\circ} = E_i - E_{Fp}$$

$$E_{Fn} + E_{Fp} = 2E_i^{\circ}$$

$$g(E_p + E_n) + n_i^{\circ} \xrightarrow{-10} \\ 10^{-8} \quad 10^{-6} =$$

$$\approx g(E_p + E_n)$$

$$= 10^{18} \times 2 \times 10^{-6}$$

$$= 2 \times 10^{12}$$

$$= n_i e^{(E_{Fn} - E_i^{\circ})/kT}$$

$$\ln \left(\frac{2 \times 10^{12}}{10^{-10}} \right) = \frac{E_{Fn} - E_i^{\circ}}{kT}$$

$$\boxed{\ln (2 \times 10^{12}) \times kT}$$

$$= E_{Fn} - E_i^{\circ}$$

$$2 \times \ln (2 \times 10^{12}) \times kT$$

b

Doped ($10^{17} = N_d$)

$$g = \frac{np - n_i^2}{T_p(n+n_i) + T_n(p+p_i)}$$

$$\begin{aligned} n &\rightarrow n + \Delta n \\ p &\rightarrow p + \Delta p \end{aligned}$$

$$g = \frac{(n + \Delta n)(p + \Delta p) - n_i^2}{T_p(n + \Delta n + n_i) + T_n(p + \Delta p)}$$

↓ Mid Level traps

$$\frac{\cancel{np} + \Delta n(n+p) - \cancel{n_i^2}}{T_p(n)}$$

$$\approx \frac{\Delta n}{\Delta p} = g$$

$$\Delta n = 10^{18} \times 10^{-6}$$

$$\approx 10^{12} \text{ cm}^{-3}$$

→ End Results to Remembers
with Free-dianion

$$\frac{dn}{dt} = -K(n_p - n_i^2)$$

$$\frac{dn}{dt} = \frac{-(n_p - n_i^2)}{\tau_p(n + n_i) + \tau_n(p + p_i)}$$

$$\left. \begin{array}{l} n_o = n_i e^{(E_F - E_i)/kT} \\ p_o = n_i e^{(E_i - E_F)/kT} \end{array} \right\}$$

Equilibrium Equations

→ Free-dianion

$G \rightarrow$ Rate of Absorption / Generation of photons.

$$n = n_o + \Delta n$$

$$p = p_o + \Delta p$$

$$\Delta n = \Delta p \quad (\text{Pairwise})$$

$$\frac{dn}{dt} = -\kappa(n_p - n_i^2) + G_{\text{photon}}$$

$$\frac{dn}{dt} = -\kappa[(n_0 + \Delta n)(p_0 + \Delta n) - n_i^2] + G_{\text{photon}}$$

$$= -\kappa \left[\cancel{n_0 p_0} + \Delta n(n_0 + p_0) + \cancel{\frac{(\Delta n)^2}{-h^2}} \right] + G_{\text{photon}}$$

$$\frac{\delta n}{\delta t} = -\kappa(n_0 + p_0)\Delta n$$

(2nd order term Δn

Ignored)

called low level ionization

$$\frac{\delta(\Delta n)}{\delta t} = -\kappa(n_0 + p_0)\Delta n + G_{\text{photon}}$$

\Rightarrow Rewrite

$$\frac{\delta(\Delta n)}{\delta t} = -\frac{\Delta n}{T_{\text{eff}}} + G_{\text{photon}}$$

$$\text{where } T_{\text{eff}} = \frac{1}{\kappa(n_0 + p_0)}$$

Typical κ values

$$\text{GaAs} \rightarrow 10^{-10}$$

$$\text{Si} \rightarrow 10^{-15}$$

(Indirect BG) $\frac{d\kappa}{d\lambda}$

Say,

$$G = 10^{16} \text{ photons cm}^{-3}/\text{s}$$

then,

$$\tau_{\text{eff}} = \frac{1}{10^{-15} (10^{17} + 10^3)}$$

$$\xrightarrow{\text{Doped } N_D} \sim 10^{17}$$

$$\tau_{\text{eff}} = 10 \text{ ms}$$

→ ~~Any~~ $\star\star$ Any lifetime post excitation
 $= 10 \text{ ms}$

$$\text{Steady state, } \frac{S(\Delta n)}{S_t} = 0$$

$$\Delta n = G_{\text{photon}} \times \tau_{\text{eff}}$$

$$\Delta n = G \tau_{\text{eff}}$$

$$\text{Here } = 10^{16} \times 10^{-2} = 10^4$$

$$\text{Ans} \rightarrow 10^{14} \text{ cm}^{-3}$$

High Level
Generation

Date _____
Page _____

$$G = 10^{22} \text{ cm}^{-3}/\text{s}$$

$$\hookrightarrow \text{Get } \Delta n = 10^{20} \text{ cm}^{-3}$$

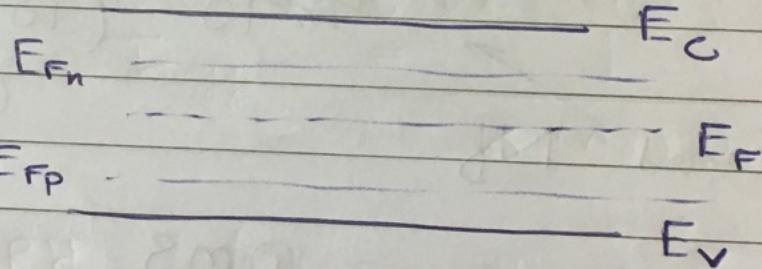
Quasi - Fermi Levels

$$(E_{F_n} - E_i) / kT$$

$$n = n_i e$$

$$(E_i - E_{F_p}) / kT$$

$$p = n_i e$$



→ SRH recombination with
Generation G

Get,

$$R = - \frac{(np - n_i^2)}{\tau_p(n+n_i) + \tau_n(p+p)}$$

$$\cancel{R \propto D \propto np}$$

$$\rightarrow \frac{S(\Delta n)}{SE} = -\frac{\Delta n}{\tau_{eff}} + g$$

$$\tau_{eff} = \frac{1}{K(n_0 + p_0)}$$

Derivation with SRH

$$n \rightarrow n_0 + \Delta n$$

$$p \rightarrow p_0 + \Delta n$$

$$\frac{dn}{dt} = -\frac{(np - n_i^2)}{\tau_p(n+n_i) + \tau_n(p+p_i)}$$

$$\tau_p \approx \tau_n \text{ (Same order)}$$

Say N_D doped

$$\frac{dn}{dt} = -\frac{(n_0^2 - n_i^2 + \Delta n(n_0 + p_0) + (\Delta n)^2)}{\tau_p(n+n_i) + \tau_n(p+p_i)}$$

$$= -\frac{\Delta n(n_0 + p_0)}{\tau_p(n+n_i) + \tau_n(p+p_i) \xrightarrow{p \approx 0}}$$

$$= -\frac{\Delta n(n_0 + p_0)}{\tau_p(n+n_i) \xrightarrow{n \approx n_0}}$$

↓ End Result

$$R = -\frac{\Delta n}{\tau_p}$$

for N-type

Minority carrier lifetime.

R.L formulas

B₂B (Direct Band Gap)

$$\tau_{eff} = \frac{1}{K(n_0 + p_0)}$$

SRH (Indirect Band Gap)

Doped { $\tau_{eff} = \tau_p \rightarrow N$ type
 $= \tau_n \rightarrow P$ type.

SRH for Intrinsic Semiconductors

$$\frac{-(\Delta n)^2}{(\tau_p + \tau_n) \Delta n}$$

$$= \frac{-(\Delta n)}{(\tau_p + \tau_n)}$$

$$\frac{s(\Delta n)}{st} = \frac{-\Delta n}{\tau_{eff}} + g$$

$$\tau_{eff} = (\tau_n + \tau_p)$$