Tutorial-4

1. Compute directly without using Cauchy's theorem the integral $\oint_\gamma f\,dz$ where $\gamma$ is the square $\pm 1 \pm i$ (traced counter clockwise) & $f$ is:

(i) $\sin z$ (ii) $1/2z+1$ (iii) $\bar{z}$ (iv) $Re(z)$

Which of these integrals can be calculated using Cauchy's theorem?

2. Show that if $\gamma$ is a simple closed curve traced counter clockwise, the integral $\oint_\gamma \bar{z}\,dz$ equals $2i$ Area($\gamma$). Evaluate $\oint_\gamma \bar{z}^m\,dz$

over a circle $\gamma$ centered at the origin.

3. Evaluate using principal values $\int_{-1}^{1} \operatorname{Log} z \, dz$

& $\int_{-1}^{1} \sqrt{z} \, dz$

4. Evaluate using Cauchy's theorem the integrals:

$\int_0^{2\pi} \exp(e^{i\theta}) \, d\theta$ & $\int_0^{2\pi} \exp(e^{i\theta} \pm i\theta) \, d\theta$

5. Given that $\int_{-\sqrt{3}}^{\sqrt{3}} \frac{dx}{x^2+1} = \frac{2\pi}{3}$, determine

the value of $\int_{\gamma} \frac{dz}{z^2+1}$, where $\gamma$ is

(i) the semi circular arc joining $-\sqrt{3}$ & $\sqrt{3}$ in the upper half plane.

(ii) the semi circular arc joining $-\sqrt{3}$ & $\sqrt{3}$ in the lower half plane.

(iii) The circle $|z| = \sqrt{3}$ traced clockwise.

6. Evaluate : (from class work)

(i) $\displaystyle\int_{|z|=6} \frac{dz}{z^3-1}$ (ii) $\displaystyle\int_{|z|=3} \frac{\cos \pi z}{z^2-1}\, dz$

(iii) Let k be a real constant. Show that

$$\int_0^{2\pi} e^{k\cos\theta} \sin(k\sin\theta)\, d\theta = 0 \quad \&$$

$$\int_0^{2\pi} e^{k\cos\theta} \cos(k\sin\theta)\, d\theta = 2\pi$$