1. The number of roots of $P_{101}(x)$ lying in the open interval $(0,1)$ equals

(A) 49 (B) 50 (C) 51 (D) 52

2. For $x>0$, the equation $x^2y'' - x(1+x)y' + y = 0$ has a solution $xe^x \log x + \sum_{n=1}^{\infty} b_n H_n x^{n+1}$ with b_n equal to

(A) $\frac{-1}{(n-1)!}$ (B) $\frac{-1}{n!}$ (C) $\frac{2^n}{n!}$ (D) $\frac{1}{(n-1)!}$

3. The domain of analyticity of a real-valued function on $\mathbb{R}$ can be

(A) $\{0\}$ (B) $\bigcup_{n=1}^{\infty}\{1/n\}$ (C) $[0,1]$ (D) $(-1,1)\setminus\{0\}$

4. A pair (a,b) of real numbers is said to be good if there exists a real number p such that $aJ_p(x) + bJ_{-p}(x) = 0$ for all $x>0$. The set of all good pairs is defined by

(A) $a^2 - b^2 = 0$ (B) $a=b=0$ (C) $a-b=0$ (D) $a+b=0$

5. If $x^{50} + x^{49} = \sum_{n=0}^{50} c_n P_n(x)$, then the sum of even coefficients $c_0 + c_2 + c_4 + c_6 + \cdots + c_{50}$ equals

(A) 0 (B) 1 (C) 50/99 (D) 51/101

6. The equation $x(e^x - 1)y'' + (\sin x)y' + y = 0$ has a

(A) irregular singular point at $x=0$ (B) irregular singular point at $x=1$
(C) regular singular point at $x=0$ (D) regular singular point at $x=1$

7. In the interval $(-1,217)$, the equation $(1+x)y' = -y/2$ with $y(0)=1$ has a power series solution $\sum_{n\geq 0} a_n(x-108)^n$ with the value of $a_{207}(109)^{207}$ equal to

(A) $a_0P_{414}(0)$ (B) $a_0P_{414}(108)$ (C) $a_0P_{207}(0)$ (D) $a_0J_{207}(108)$

8. The value of ${J_0}^2(2) - {J_2}^2(2)$ equals

(A) 0 (B) $J_0(2)J_2'(2)$ (C) $J_1(2)J_1'(2)$ (D) $2J_1(2)J_1'(2)$

9. The radius of convergence of the power series $\sum_{n=0}^{\infty} \frac{(2n)!}{3^{2n}(n!)^2} x^{2n}$ equals

(A) 3 (B) 9 (C) 3/2 (D) 9/4

10. Let $g(x)$ be the quadratic polynomial with roots $\pm\sqrt{\frac{1}{3}}$ with $g(1) = 2/3$. Let $f(x)$ be the polynomial solution of the equation $((1-x^2)y')' + 6y = 0$ with $f(1) = 1$. The value of $\int_{-1}^{1} f(x)g(x)dx$ equals

(A) 0 (B) 2/3 (C) 2/5 (D) 4/15

11. The recursion obtained while solving $y'' - xy' + y = 0$ by the power series method is

(A) $(n+2)(n+1)a_{n+2} = (n-1)a_n$ (B) $(n+2)(n+1)a_{n+2} = na_n$
(C) $(n+2)(n+1)a_{n+2} = (n-1)a_{n-1}$ (D) $(n+2)(n+1)a_{n+2} = (n+1)a_{n+1} - a_n$

12. Let a and b be the number of solutions of $J_0(x) = P_0(x)$ and $J_1(x) = P_1(x)$ respectively in the interval $[0,1]$. Then (a,b) is

(A) $(0,1)$ (B) $(0,2)$ (C) $(1,1)$ (D) $(1,2)$

13. An inner product on $\mathbb{R}^2$ can be defined by setting $\langle(a_1,a_2),(b_1,b_2)\rangle$ equal to

(A) $a_1b_1 - a_2b_2$ (B) $a_1^2b_1^2 + a_2^2b_2^2$ (C) $(a_1+a_2)(b_1+b_2)$ (D) $2a_1b_1 - a_1b_2 - a_2b_1 + 5a_2b_2$

14. The set of all points where the Taylor series of the function $f(x) = \sum_{n=0}^{\infty} \frac{x^2}{(1+x^2)^n}$ around the point $x = e$ converges to $f(x)$ is

(A) $\emptyset$ (B) $(0, 2e)$ (C) $\mathbb{R} \setminus \{0\}$ (D) $\mathbb{R}$

15. The value of $\lim_{x \to 1^+} \frac{J_p(x^2 - 1)}{(x-1)^p}$ at $p = 4$ equals

(A) 0 (B) 1/24 (C) 1/120 (D) ∞

16. While solving $x^2 y'' + 2x(x-2)y' + 2(2-3x)y = 0$ by the Frobenius method around the point $x = 0$, the case encountered is that of

(A) roots not differing by an integer
(B) repeated roots
(C) roots differing by a positive integer with **no** log term
(D) roots differing by a positive integer with log term