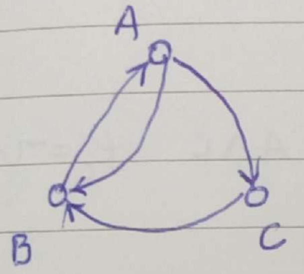
Boolean networks

Graph $G(V, E)$

node $v \in V$ has a fn. f_v of states of nodes connected to v

x_i – state of v_i



$f_A(B)$ $\quad f_B(A, C)$ $\quad f_C(A)$

$f_A(B) = B$ $\qquad$ state of $A, B, C \in \{0\ 1\}$

$f_B(A, C) = A \wedge C$

$f_C(A) = \neg A$

$\cancel{x_A(t+1) = f_A(x_A(t), x_B(t), x_C(t)) = 1}$

$A(t+1) = f_A(A(t), B(t), C(t)) = B(t)$

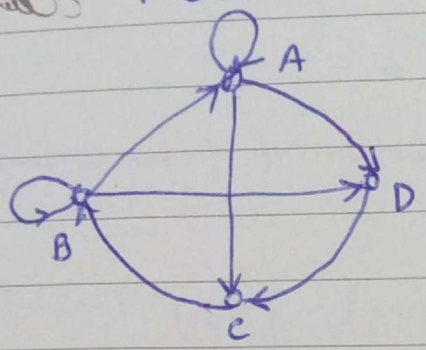
$B(t+1) = A(t) \wedge C(t)$

$C(t+1) = \neg A(t)$

~~A B C~~ A

A	B	C	A	B	C
present			next		
0	0	0	0	0	1
0	0	1	0	0	1
0	1	0	1	0	1
0	1	1	1	0	1
1	0	0	0	0	0
1	0	1	0	1	0
1	1	0	1	0	0
1	1	1	1	1	0

Monomial networks :

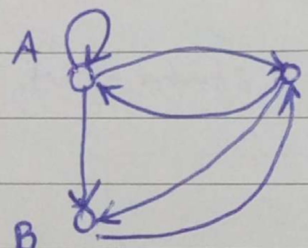


$A(k+1) = AB$

$B(k+1) = BC$

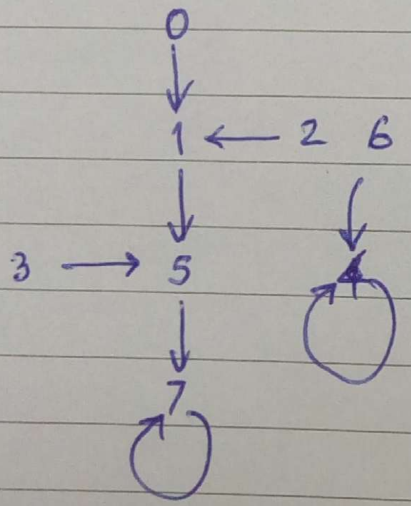
$C(k+1) = AD$

$D(k+1) = AB$

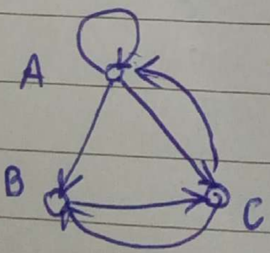


$f_A = A \vee C \quad f_B = A \wedge C \quad f_C = \neg A \vee B$

0	0	0	0	0	0	1
1	0	0	1	1	0	1
2	0	1	0	0	0	1
3	0	1	1	1	0	1
4	1	0	0	1	0	0
5	1	0	1	1	1	1
6	1	1	0	1	0	0
7	1	1	1	1	1	1



Monomial :



$f_A = AC \quad f_B = AC \quad f_C = AB$

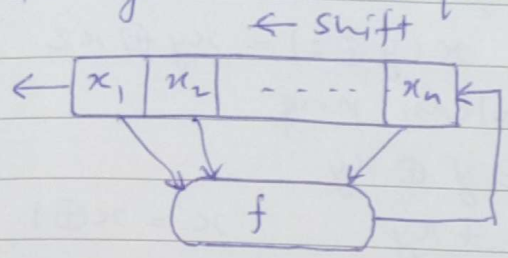
(0 0 0) ↓ (0 0 0)

(0 0 1) ↙ (0 1 0)

/ /

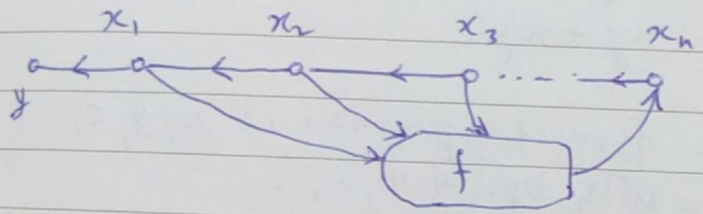
Boolean networks in digital circuits :

Shift register based sequence generators.



$$F(x_1\ x_2 \cdots x_n) = (x_2\ x_3 \cdots x_n\ f(x_1 \cdots x_n))$$

$$y = x_1$$



LFSR : ~~Fd~~

$$(x_1\ x_2 \cdots x_n)(k+1) = F(x_1\ x_2 \cdots x_n)(k).$$

$$x_2\ x_3 \cdots\cdots \oplus \sum_{i=1}^{n} a_i x_i$$

$\oplus$ is modulo 2 sum XOR.

Boolean Algebra : $B = \{B, 0, 1, +\ \cdot\ '\}$

$\vee\ \wedge\ \neg$

$$x + y = y + x \qquad xy = xy$$

$$(x+y)+z = x+(y+z) \qquad (xy)z = x(yz)$$

$$x + 0 = x \qquad x \cdot 1 = x$$

$$x + x' = 1 \qquad xx' = 0$$

Distributive Laws : $x(y+z) = xy + xz$

$\cancel{(x+y)z} = x + yz = (x+y)(x+z)$

De Morgan's Laws : $(x+y)' = x'y' \qquad (xy)' = x'+y'$

Boolean ring $B = \{B, 0, 1, \oplus, \cdot\}$

Idempotent property $x \oplus x = 0$

Distribution $x(y \oplus z) = xy \oplus xz$

Boolean Algebra $\leftrightarrow$ Boolean ring

$$x + y = x \oplus y \oplus xy$$

$$x \oplus y = x'y + xy' \qquad x' = x \oplus 1.$$

What is $x'y' + xy$? (in terms of $\oplus$?)

$$x \oplus y \oplus 1 = x \oplus y' = x'y' + xy.$$

$$(x \oplus y)' = xy + x'y'.$$

Boolean expression : formal expression in $x, y, z, \dots$ with op. of $+, \cdot, '$.

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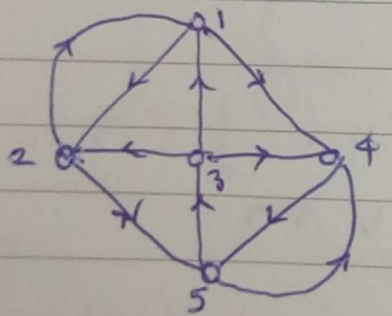
Types of Boolean systems/networks : regular functions.

Linear systems : XOR



$$F(x) = (x_2 \oplus x_3,\ x_1 \oplus x_3,\ x_5,\ x_1 \oplus x_3 \oplus x_5,\ x_2 \oplus x_4)$$

$$= \begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix}$$

$(0 \cdots 0)$ is a fixed pt.

To find a chain ending at 0 : $\quad x_5 = 0 \quad x_1 \oplus x_3 = 0 \Rightarrow x_1 = $

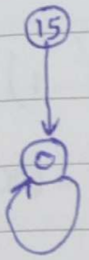
$(1 \ 1 \ 1 \ 1 \ 0)$

$x_2 \oplus x_3 = 0 \Rightarrow x_1 = x_3 = x_2$

$x_2 \oplus x_4 = 0 \Rightarrow x_2 = x_4$

$(x_1 \cdots x_5) \leftrightarrow (a_0 \cdots a_4) = (1\ 1\ 1\ 1\ 0)$

$= 1+2+2^2+2^3 = 15$



$$\begin{bmatrix}1\\1\\1\\1\\0\end{bmatrix} = \begin{bmatrix}x_2 \oplus x_3\\ x_1 \oplus x_3\\ x_5 \\ x_1 \oplus x_3 \oplus x_5 \\ x_2 \oplus x_4\end{bmatrix}$$

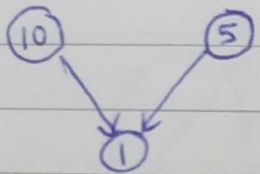
$x_5 = 1 \Rightarrow x_1 \oplus x_3 = 0$ contr. Hence only chain is length 1.

Orbit: choose a pt. $(1\ 0\ 0\ 0\ 0)$

$x_2 \oplus x_3 = 1 \qquad x_5 = 0 \qquad x_1 \oplus x_3 = 0 \qquad x_2 \oplus x_4 = 0$

$(x_2, x_3) = \{(1\ 0), (0,1)\} \quad x_1 = x_3 \qquad x_2 = x_4$

$(0\ 1\ 0\ 1\ 0) \qquad (1\ 0\ 1\ 0\ 0)$



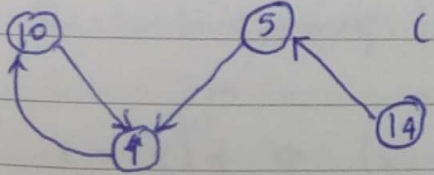
$10 = (0\ 1\ 0\ 1\ 0) = Ax$

$\Rightarrow x_5 = 0 \qquad x_1 \oplus x_3 = 1 \cancel{= x_2 \oplus x_3}$

$x_2 \oplus x_3 = 0 = x_2 \oplus x_4.$

$x_2 = x_3 = x_4 \qquad (x_1, x_3) = \{(1,0), (0,1)\}.$

$(x_1, x_3) = (1, 0) \Rightarrow x_2 = x_4 = 0 \qquad (1\ 0\ 0\ 0\ 0)$



$(x_1, x_3) = (0, 1) \qquad x_2 = x_3 = x_4 = 1$

$(0, 1, 1, 1, 0)$

$= 14$

$(0\ 1\ 1\ 1\ 0) = (x_2 \oplus x_3 \quad x_1 \oplus x_3 \quad x_5 \quad x_1 \oplus x_3 \oplus x_5, \ x_2 \oplus x_4)$

$x_5 = 1 \qquad x_1 \oplus x_3 = 0$ contr.

Consider pt. $2 = (0\ 1\ 0\ 0\ 0)$

$x_5 = 0 \qquad x_1 \oplus x_3 = 0$ contr.

Hence no predecessors of 2.

$A(2) = (1\ 0\ 0\ 0\ 1)$

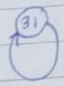
$A(17) = (0\ 1\ 0\ 0\ 0)$

$A(6) = (0\ 1\ 0\ 0\ 1)$

Monomial system:

$F = (x_2 x_3 \quad x_1 x_3 \quad x_5 \quad x_1 x_3 x_5 \quad x_2 x_4)$

Two fixed pts: $(0\ 0\ 0\ 0\ 0)$ $(1\ 1\ 1\ 1\ 1)$



$x_5 = 0$ $\quad x_1 x_3 = 0$ $\quad \{(1\ 0)\ (0\ 1)\} = (x_1\ x_3)$

$x_2 x_3 = 0$

$x_2 x_4 = 0$

$(x_1\ x_2) = (1, 0)$

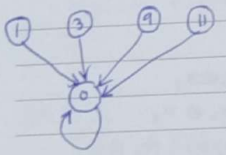
$\rightarrow x_2 = \{1, 0\}$ $\quad 1 \to x_4 = 0$, $\quad 0 \to x_4$: 0, 1

$(x_1\ x_2\ x_3\ x_4\ x_5) = (1\ 0\ 0\ 0\ 0)$

$(1\ 1\ 0\ 0\ 0)$

$(1\ 0\ 0\ 1\ 0)$

$(1\ 1\ 0\ 1\ 0)$



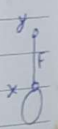
Fixed pt $\quad x = F(x)$: fixed pt = period 1 fixed pt.

Periodic orbit $\quad x = F^{(2)}(x)$: " of period 2

$x = F^{(m)}(x)$: " " " m.

Chain of length 1: $\quad x = F(y) \quad x = F(x) \Rightarrow F^{(2)}(y) = x$

at a fixed pt. $\quad y = F(z)$ has no soln.

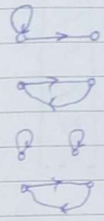


Similarly at fixed pts. of other periods.

Chain of length k: $\quad F^{(k-1)}(y) = x$. $\quad y = F(z)$ – no soln.

then the chain is $y, F(y), \dots F^{(k-1)}(y)$

Soln. of Boolean eqn:



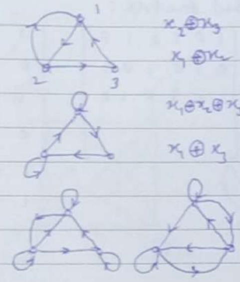
$x_1 \quad x_1$

$x_1 \quad x_1$

$x_2 \quad x_1$

$x_1 \quad x_2$

$x_2 \quad x_1$



$x_2 \oplus x_3 \quad x_1 \quad x_2$

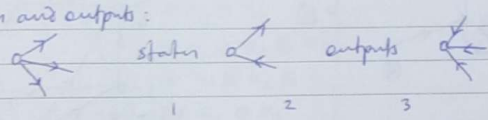
$x_1 \oplus x_2 \quad x_2 \oplus x_3 \quad x_1$

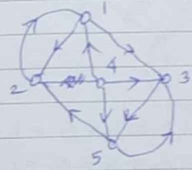
$x_1 \oplus x_2 \oplus x_3 \quad x_1 \oplus x_2 \quad x_2 \oplus x_3$

$x_1 \oplus x_3 \quad x_1 \oplus x_2 \oplus x_3 \quad x_1 \oplus x_2$

Inputs states and outputs:

Inputs $\quad$ states $\quad$ outputs





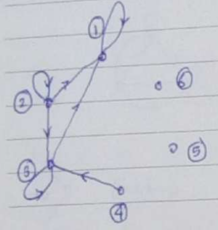
$$F = (\overset{1}{x_2 \oplus x_4},\ \overset{2}{x_1 \oplus x_4},\ \overset{3}{x_1 \oplus x_5 \oplus x_4},\ \overset{x_5}{x_3 \oplus x_4})$$

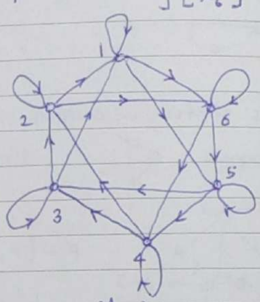
$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_5 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 1 \\ 1 \end{bmatrix} x_4$$

Reachability from zero state.

Banded matrix :

$$\begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ & & \cdot & & & \\ & & & & & \end{bmatrix} \quad \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix}$$





Eqm. for fixed pt.

$x_2 \oplus x_3 = 0$

$x_3 \oplus x_4 = 0$

$x_4 \oplus x_5 = 0$

$x_5 \oplus x_6 = 0$

$x_1 \oplus x_6 = 0$

$x_1 \oplus x_2 = 0$

$(0\ 0\ 0\ 0\ 0\ 0)$

$(1\ 1\ 1\ 1\ 1\ 1)$

chain

$x_1 \oplus x_2 \oplus x_3 = 1$

$x_2 \oplus x_3 \oplus x_4 = 1$

$x_3 \oplus x_4 \oplus x_5 = 1$

$x_4 \oplus x_6 \oplus x_6 = 1$

$x_1 \oplus x_5 \oplus x_6 = 1$

$x_1 \oplus x_2 \oplus x_6 = 1$

Gaussian elimination :

$x_1 \oplus x_2 \oplus x_3 = 1$ ~~1 1 1 0 0~~

$x_2\ 4$

$$\left[\begin{array}{cccccc|c} 1 & 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 \end{array}\right] \quad \left[\begin{array}{cccccc|c} 1 & 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{array}\right]$$

$x_4 = 1$

$x_6 = 0$

$x_5 = 0$

$x_3 = 0$

$x_2 = 0$

$x_1 = 1$

$R5 \leftarrow R5 + R1 \quad R6 \leftarrow R6 + R3$

$R5 \leftarrow R5 + R2$

$1+2 \quad 3$

$1+2+4 \quad 7$

$2^9 - 1$

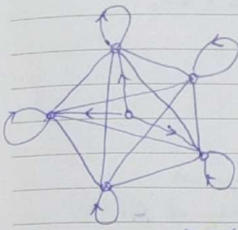
$(31) \longleftarrow (9)$

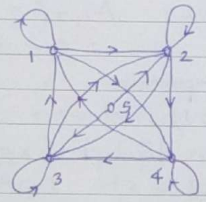
$1\ 0\ 0\ 1\ 0\ 0$

$1\ 0\ 0\ 1\ 1\ 0$

$1\ 0\ 0\ \cancel{1}\ 0\ 0$

$1 \quad 2^3$





$$\cancel{x_1(k+1)} \quad F = \begin{bmatrix} x_1 \oplus x_3 \oplus x_4 \\ x_1 \oplus x_2 \oplus x_3 \oplus u \\ x_3 \oplus x_4 \oplus x_2 \oplus u \\ x_2 \oplus x_4 \oplus x_1 \end{bmatrix}$$

Reachability : A state ~~x~~ y is reachable from x if $\exists$ an input sequence $u(0), u(1), \cdots u(k)$ s.t.

$$x(k) = y \quad \text{given} \quad x(0) = x.$$

state eqn : $x(k+1) = Ax(k) \oplus b\,u(k)$

$x(1) = Ax(0) \oplus b\,u(0)$

$x(2) = Ax(1) \oplus b\,u(1)$

$= A^2x(0) \oplus Ab\,u(0) \oplus b\,u(1)$

$\therefore$ y reachable from x iff $\exists$ k s.t.

$$y \oplus A^k x \in \text{span}\{b\ Ab\ A^2b \cdots A^{k-1}b\}.$$

Autonomous linear systems : $x(k+1) = Ax(k)$.

Fixed pts : $(A \oplus I)x = 0$ # of fixed pts : Num of solns.

Chain at a fixed pt - length 1 : $(A \oplus I)x = 0$ $x = Ay_1$ $y_1 = Ay_2$ — No soln.

Lin. tr. is a Permutation of the state space iff A is 1-1.

period k fixed pt. $(A^k \oplus I)x = 0$.

Affine map:

$$F(x) = Ax + \beta.$$

Fixed pt: $x = Ax + \beta.$

Permutation: $Ax_1 + \beta = Ax_2 + \beta. \Leftrightarrow x_1 = x_2$

$\therefore A - I - 1.$

~~Periods; powers~~

Periods: 1) $F(x) = Ax.$

k-periodic fixed pt. $x = A^k x.$

hence all k s.t. $I \oplus A^k$ is singular.

$\Leftrightarrow (\lambda^k + 1) \mid$ ch-poly. of A.

i.e. A has eigenvalue which is k^{th} root of unity.

Chains for example: $A = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$

fixed pt.

$Ax = x$

$A + I = \begin{bmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

Confluence sets:

if $x = Ay_1$ & $x = Ay_2$ then $A(y_1 - y_2) = 0$

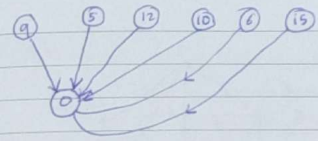
Hence to find all chains:

1. Find one soln. y s.t. $x = Ay.$
2. Find all elements of $S = \{y \mid Ay = 0\}$
3. Construct $\{y + S\}$

Chain at 0.: $Ay = 0 \Leftrightarrow y_5 = 0 \quad y_1 + y_2 + y_3 + y_4 = 0$

for $y_2\ y_3\ y_4$ ind.

$$S = \begin{bmatrix}1\\0\\0\\1\\0\end{bmatrix} \begin{bmatrix}1\\0\\1\\0\\0\end{bmatrix} \begin{bmatrix}0\\0\\1\\1\\0\end{bmatrix} \begin{bmatrix}1\\1\\0\\0\\0\end{bmatrix} \begin{bmatrix}0\\1\\0\\1\\0\end{bmatrix} \begin{bmatrix}0\\1\\1\\0\\0\end{bmatrix} \begin{bmatrix}1\\1\\1\\1\\0\end{bmatrix}$$



$A = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$ $\quad A + I = \begin{bmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

$(A+I)x = 0 \quad x_4 = 0 \quad x_3 = 0 \quad x_2 + x_5 = 0 \quad x_1 + x_5 = 0$

$x_1 = x_5 \quad x_2 = x_5$

$x_5 = 1 \quad (1\ 1\ 0\ 0\ 1) = 19$

$S = \{y \mid Ay = 0\}$

XL method: Eqn are solved for monomials of highest deg. eg in MQ: for ex.

$x_{12} = x_{13} + x_{24} + x_{26} + x_1 + x_3 + x_7$

$x_{23} = x_{13} + x_{14} + x_5 + x_6$

$x_{25} = x_{24} + x_{14} + x_1 + x_2$

The ~~no. of~~ ind. ~~highest~~ 2 deg. mon. is $x_{13}\ x_{14}\ x_{24}\ x_{26}, \underline{x_5, x_6, x_7}$ Not appearing in 2nd deg.

$\therefore$ ~~2^4 search~~. 2^7 search..

~~0 0 0 0 Undet. assignments.~~

~~for 20~~ No. of ind. substitutions: 2^4 for 2nd deg.

~~x_1 $x_{13} = 1$~~ $\Rightarrow$ 0 0 0 0

0 0 0 1 $\quad x_2 = x_6 = 1 \quad x_4 = 0$

$x_1\ x_2\ x_3\ x_4\ x_6$ — search. $\quad x_1\ x_2$

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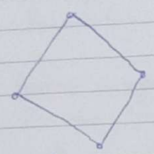
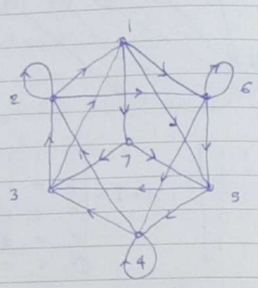
Monomial system:





Monomial map

$F = (x_2x_3,\ x_2x_3x_4,\ x_4x_5\cancel{x_7},\ x_4x_5x_6,\ x_1x_6\cancel{x_7},\ x_1x_2x_6\cancel{x_7})$ ~~x_7~~

Fixed pt. $x_1 = x_2x_3$

$x_2 = x_2x_3x_4 = x_1x_4$

$x_3 = x_4x_5\cancel{x_7} = x_2x_5$?

$x_4 = x_4x_5x_6$

$x_5 = x_1x_6\cancel{x_7}$

$x_6 = x_1x_6\cancel{x_7}$ | $x_5 = x_6 \Rightarrow x_4 = x_5$

$x_7 = x_1$ $\quad x_5 = x_6 = 1 \Rightarrow x_1 = 1 \Rightarrow x_4 = 1$

$x_5 = x_6 = 0$ $\quad \cancel{x_1 = 0}$

$x_3 = x_2x_5$

$x_4 = x_5$

$\cancel{x_5 = x_1x_5}$

Fixed pt. eqns:

$x_1 = x_2x_3 \qquad\qquad x_1 = x_2x_3$

$x_2 = x_2x_3x_4 = x_1x_4 \qquad x_2 = x_1x_4$

$x_3 = x_4x_5\cancel{x_7} \qquad x_3 = x_4x_5$

$x_4 = x_4x_5x_6 = x_3x_6 \qquad x_4 = x_3x_6$

$x_5 = x_6x_1 \qquad x_5 = x_1x_6$

$x_6 = x_1x_2x_6 = x_2x_5 \qquad x_6 = x_2x_5$

/ /

eqn. 1; $x_1 \oplus x_2x_3 = 0 \Leftrightarrow x_1'x_2x_3 + x_1(x_2' + x_3') = 0$

$\Leftrightarrow x_1'x_2x_3 + x_1x_2' + x_1x_3' = 0$

$\Leftrightarrow (x_1 + x_2' + x_3')(x_1' + x_2)(x_1' + x_3) = 1.$

~~Three possible~~ Possible assignments

$(x_1 = 0 \quad x_2 = D \quad x_3 = D)$ $\quad$ (~~$x_1 = 0$~~ x_3

eqn. 2: ~~Not SAT~~. $x_2 = 0 \quad x_4 = D.$

Rest of eqns: $x_3 = x_4x_5 \quad x_4 = x_3x_6 \quad x_5 = 0 = x_6.$

$x_3 = 0 \quad \Rightarrow x_4 = 0 \quad$ eqn. SAT.

Hence one soln: $(0, 0, 0, 0, 0, 0)$.

eqn 1: Next possible assign: $x_1 = 1 = x_2 = x_3$

2) $\Rightarrow x_4 = 1$ $\quad$ 3) $\Rightarrow x_5 = 1$ $\quad$ 4) $\Rightarrow x_6 = 1$ $\quad$ 5) sat $\quad$ 6) sat.

Hence second soln: $(1\ 1\ 1\ 1\ 1\ 1)$

No other possible soln. for eqn. 1.

Hence these are all soln.

Permutation condition: when is F 1-1.

$$F(x) = F(y) \Leftrightarrow x = y.$$

~~Hence~~ F fails to be a permutation iff there exist soln. to x, y s.t. $x \neq y$.

e.g. in the previous map: $\qquad x \oplus y = x'y + xy' = (x+y)(x'+y')$

$x_2x_3 = y_2y_3 \qquad x_2x_3x_4 = y_2y_3y_4$

$x_4x_5 = y_4y_5 \qquad x_4x_5x_6 = y_4y_5y_6$

$x_6x_1 = y_6y_1 \qquad x_1x_2x_6 = y_1y_2y_6$

$x_2x_3 \oplus y_2y_3 = 0 \qquad (x_2' + x_3')y_2y_3 + x_2x_3(y_2' + y_3') = 0$

$x_2'y_2y_3 + x_3'y_2y_3 + x_2$

$(x_2x_3 + y_2y_3)(x_2' + x_3' + y_2' + y_3') = 0$

$(x_2' + x_3')(y_2' + y_3') + x_2'x_3'y_2'y_3' = 1$

$x_2'y_2' + x_2'y_3' + x_3'y_2' + x_3'y_3' + x_2'x_3'y_2'y_3' = 1$

Implicant $x_2' y_2'$:

$x_2 x_3 x_4 = y_2 y_3 y_4$ ✓ $\quad x_4 x_5 = y_4 y_5 \quad x_4 x_5 x_6 = y_4 y_5 y_6$

$x_1 x_6 = y_1 y_6 \quad x_1 x_2 x_6 = y_1 y_2 y_6$ ✓

$x_2' y_3'$: ✓ " "

" $\quad 0 = y_1 y_2 y_6$

Fails to be permutation if there is a chain of one of the fixed pts.

$$0 = F(y)$$

$y_2 y_3 = 0 \quad y_2 y_3 y_4 = 0 \quad y_4 y_5 = 0 \quad y_4 y_5 y_6 = 0$

$y_1 y_6 = 0 \quad y_1 y_2 y_6 = 0$

$y_4 = 1 \quad y_6 = 1 \quad y_2 = 0 \quad y_3 = 1 \quad y_5 = 0 \quad y_1 = 0$

Hence not a permutation.