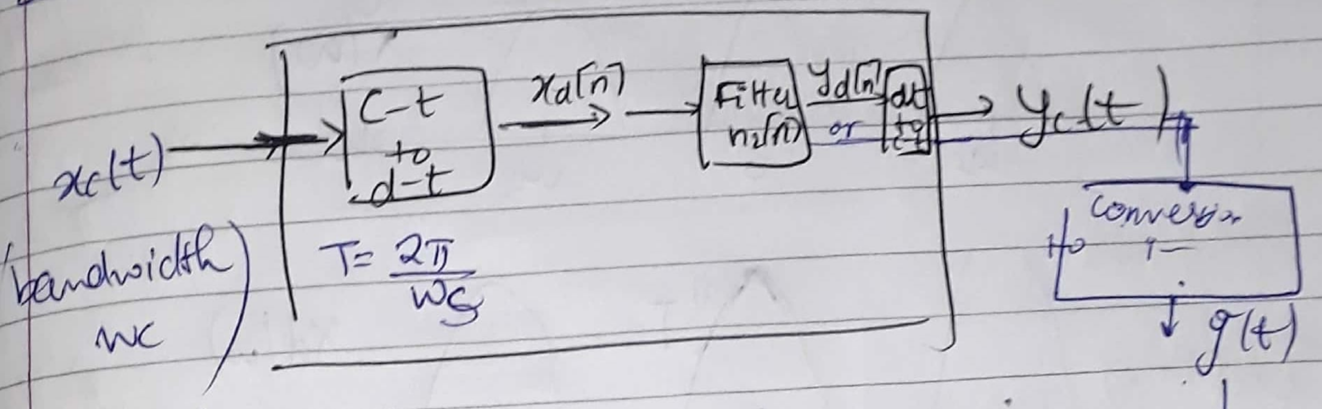
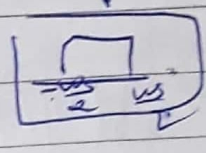
Discrete time processing of cont. time signals



$\hookrightarrow w_s > 2w_c$



$$x_d[n] = x_c[nT]$$

$$\tilde{x}(t) = x(t) \times \left(\sum_n \delta(t-nT)\right)$$

$$x_p \rightarrow \sum \underbrace{x_c(nT)}_{x_d[n]} \delta(t-nT)$$

$$= \sum x(nT)\, \delta(t-nT)$$

$$X_d = \sum^{\infty} x_d\, e^{-j\Omega n}$$

$$x_c[nT]$$

$$\tilde{X}(\omega) = \frac{1}{T} \sum_{k=-\infty}^{\infty} X(\omega - k w_s)$$

$$\tilde{X}(\omega) = \int \tilde{x}(t) e^{j\omega t} dt = \sum x_c(nT) e^{j\omega nT}$$

$\delta(t-nT) \rightarrow e^{j\omega nT}$

periodic replication in C.T.D with period w_s

$$\tilde{X}(\omega) = \sum x_c(nT) e^{j\omega nT}$$

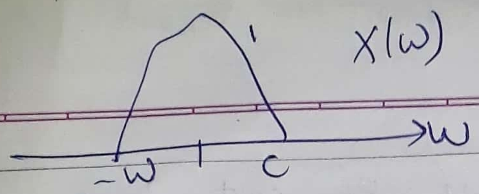
$$X_d(\Omega) = \tilde{X}\left(\frac{j\Omega}{T}\right)$$

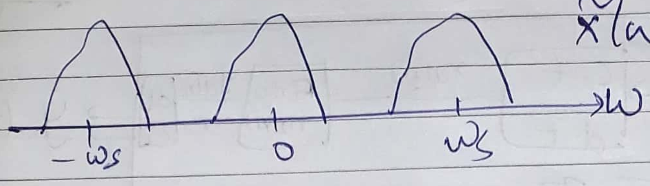
$$X_d(\omega) = \sum x_d[n] e^{-j\omega n} \quad \Big\| \quad X_d(\omega) \sum x_c(nT) e^{-j\omega n}$$

$\omega \rightarrow \omega T$

$$\boxed{X_d(\omega T) \Rightarrow \tilde{X}(\omega)}$$

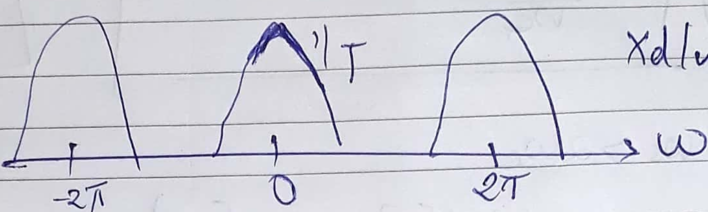
Scanned by CamScanner





period

$\omega_s = \frac{2\pi}{T}$



period

2π

$\tilde{X}(\omega) = \frac{1}{T} \sum_{-\infty}^{\infty} X_c(\omega - k\omega_s)$

$\boxed{X_d(\Omega) = X_p(\Omega/T)}$

$\boxed{X_d(\omega) = \tilde{X}(\omega/T)}$

$X_d(\omega t) = \tilde{X}(\omega)$

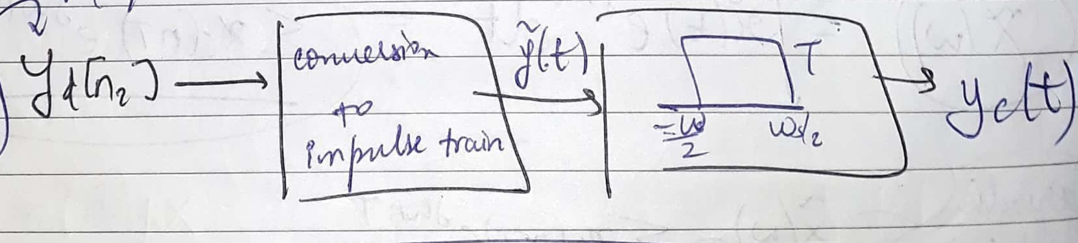
$X_d(\omega) = \tilde{X}(\omega/T)$

$$Y_d(\omega) = X_d(\omega) \cdot H_d(\omega)$$

$$Y_d(\omega) = \tilde{X}(\omega/T) \cdot H_d(\omega)$$

eq^n (equivalent)

d to ct



eg)

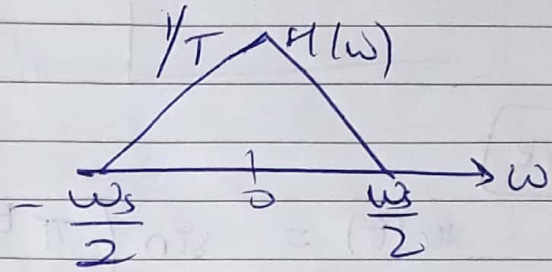
$$\boxed{\tilde{y}(t) = \sum y_d[n] \delta(t - nT)}$$

$$\tilde{Y}(\omega) = \tilde{X}(\omega) H_d(\omega T)$$

$$Y_d(\omega) = \tilde{Y}(\omega/T)$$

$$\tilde{Y}(\omega) = Y_d(\omega T)$$

$$Y_c(\omega) = \begin{cases} X(\omega) \cdot H_d(\omega T) & |\omega| \le \omega_s/2 \\ 0 & \text{else} \end{cases}$$



$$Y_c(\omega) = \begin{cases} \frac{1}{T} X(\omega) \cdot H_d(\omega T) & |\omega| < \frac{\omega_s}{2} \\ 0 & \text{else} \end{cases}$$

Discrete time processing of cont. time signal

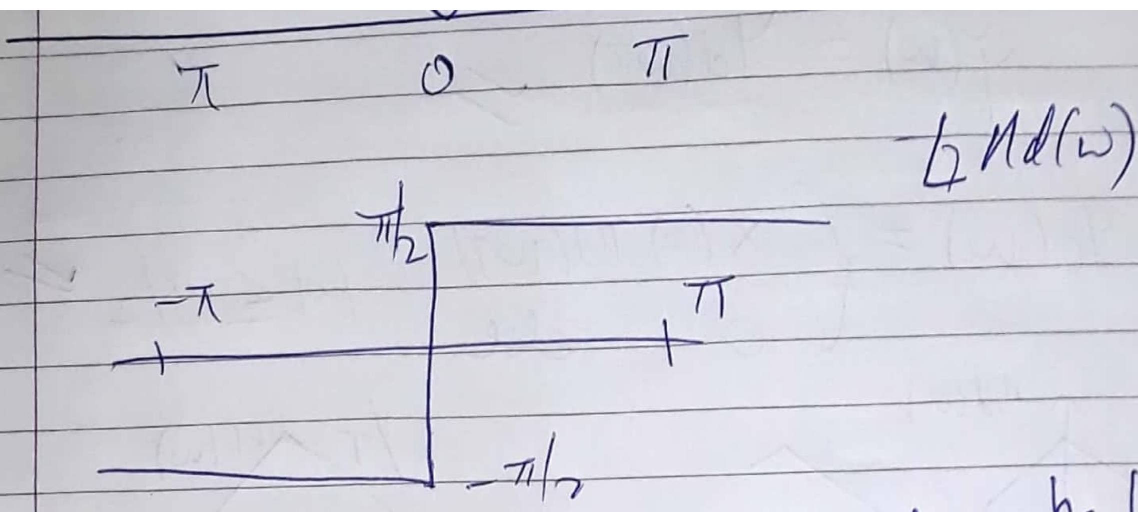
want $y_c = x_c'$

$$Y_c(\omega) = j\omega X_c(\omega)$$

$$H_c(\omega) = j\omega$$

$$H_d(\omega) = H_c\left(\frac{\omega}{T}\right) \quad |\omega| < \omega_s/2 \longrightarrow |\omega| < \pi$$

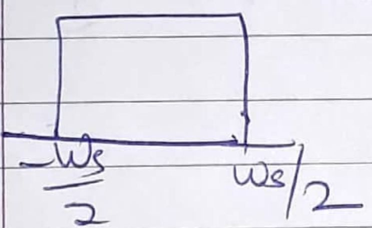
$$H_d(\pi) = H_c(\omega_s/2)$$



$\boxed{\text{Trick}}$ To get $h_2(t)$

$$x_c(t) = \frac{\sin(\pi t/T)}{\pi t} = \frac{\text{sinc}(t/T)}{T}$$

the ~~transform~~ will work on this as this input satisfies requisite condition



$$\to \frac{\cos\left(\frac{\pi t}{T}\right)}{tT} - \frac{\sin\left(\frac{\pi t}{T}\right)}{\pi t^2}$$

$$y_c(t) = x_c'(t)$$

$$x(t) \to \boxed{C\text{-}D} \to \boxed{H_d(\omega)} \to \boxed{D\text{-}C} \to x_c'(t)$$

Take $x(t)$ such that we get an impulse

$$x_d[n] = \underline{x_c[nT]} \longrightarrow \left\{\frac{1}{T}\delta[n]\right\}$$

\# $t = nT$ in $x_c(t)$ → sinc of an integer → 1 when $n = 0$
→ 0 otherwise

See book

$$\rightarrow y_d[n] = y_c(nT) = \begin{cases} 0 & n=0 \\ \dfrac{\cos(n\pi)}{nT^2} & n \neq 0 \end{cases}$$

$$h_d[n] = \begin{cases} 0 & n=0 \\ \dfrac{(-1)^n}{nT} & n \neq 0 \end{cases}$$

* $x[n] \longrightarrow X(\omega)$

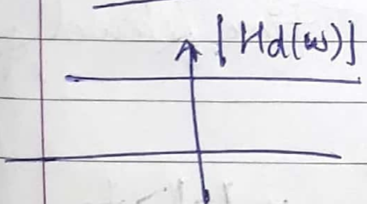
$x[n-n_0] \longrightarrow e^{-j\omega n_0} X(\omega)$

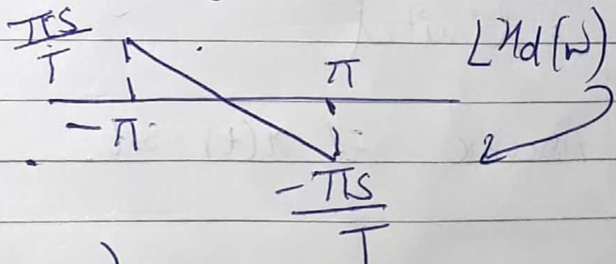
Q) what is the inverse of $e^{-j\omega/2} X(\omega)$
(half sample delay because same as $x[n-1/2]$
[delaying cont. signal with 1/2]

- Want to achieve

$\rightarrow y_c(t) = x_c(t-D)$ {delay in cont^n time is very hard to implement}

$\Rightarrow H_c(\omega) = e^{-j\omega D}$ $\Rightarrow H_1(\omega) = H_c(\omega/T)$ $(|\omega| < \pi)$





If $D = T/2$ (earlier quest^n)
same trick as before will work

$x_c(t) = \dfrac{\sin(\pi t/T)}{\pi t}$ $x_d[n] = x(nT) = \dfrac{1}{T}\delta[n]$

Gaussian Pulse !!

$$x(t) = e^{-t^2/2} \Rightarrow X(s) \overset{LT}{=} \int x(t)\, e^{-st}\, dt$$

$$\Rightarrow \int e^{-t^2/2}\, e^{-st}\, dt = e^{s^2/2} \int e^{-\frac{1}{2}(t^2 + 2st + s^2)}\, dt$$

$$= e^{s^2/2} \underbrace{\int_{-\infty}^{\infty} e^{-t^2/2}\, dt}_{C}$$

FT $\boxed{X(\omega) = C e^{-\omega^2/2}}$

To calculate c

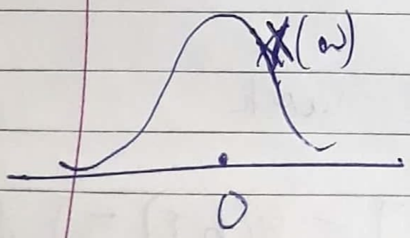
$$x(\cdot) \xrightarrow{FT} C x(\cdot) \xrightarrow{FT} C^2 x(\cdot) = 2\pi x(-\cdot)$$

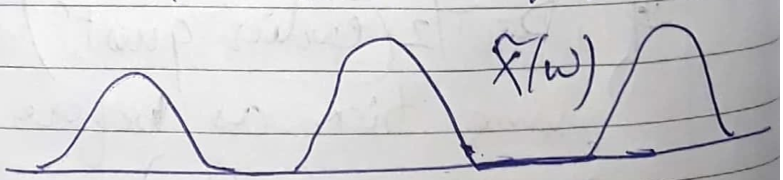
$$\boxed{C = \sqrt{2\pi}}$$

Claim: A non zero signal can't be time & band limited

Proof Assume $\exists\, x(t)$ s.t. $|x(t)| = 0 \;\; \forall\, |t| > t_0$

$|X(\omega)| = 0 \;\; \forall\, |\omega| > \omega_0$





$T = 2\pi/\omega_s$

$$\tilde{X}(\omega) = T \sum_{n=-\infty}^{\infty} x(nT)\, e^{j(2\pi/\omega_s)k\omega}$$

x time limited

$$\tilde{X}(\omega) = \underbrace{T \sum_{-n_1}^{n_1} x(nT)\, e^{jTk\omega}}_{\text{analytic}}$$

RHS can't be zero over an interval

$\Rightarrow$ contradiction

The uncertainity principle

$$\star \int_{-\infty}^{\infty} |x(t)|^2 = \frac{1}{2\pi} \int_{-\infty}^{\infty} |X(\omega)|^2 \, d\omega$$

Say $\int_{-\infty}^{\infty} |x(t)|^2 dt = 1$

can interpret

$|x(t)|^2$ or $\frac{1}{2\pi} |X(\omega)|^2$ as (extent of spread)

$\star\star$ density funct^n in time or freq^n

$$\mu_{time} = \int t \, |x(t)|^2 dt$$

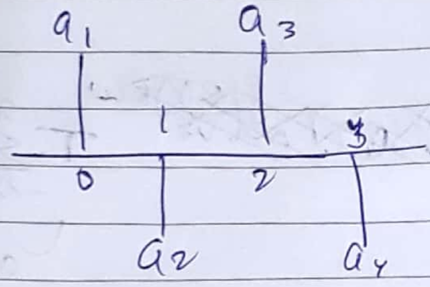
$$\sigma^2_{time} = \int (t - \mu_{time})^2 \, |x(t)|^2 dt$$

$$\mu_{freq} = \frac{1}{2\pi} \int \omega |X(\omega)|^2 d\omega \quad \Bigg|\Bigg| \quad \sigma^2_{freq} = \frac{1}{2\pi} \int (\omega - \mu_{freq})^2 \, |X(\omega)|^2 d\omega$$

Result $(\sigma^2_{time})(\sigma^2_{freq}) \geq 1/4$ (Equality for gaussian pulse)

Spread in time & freq can't simultaneously made arbitrarily small.

Q4) $x[n]$



$X(\omega) = A(\omega) + jB(\omega)$

Q) Invert $B(\omega) + A(\omega)e^{j\omega}$

$y[n]$

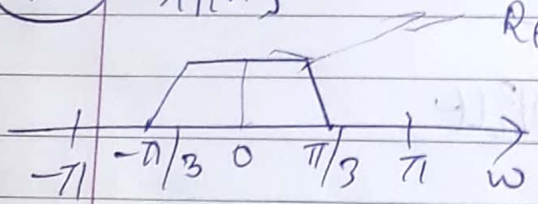
$x \rightarrow$ real

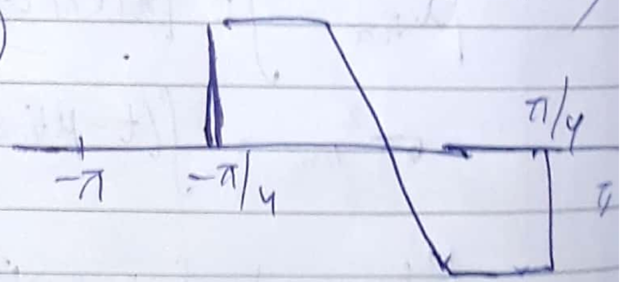
$z[n] = \dfrac{x[n] + x[-n]}{2} \rightarrow A(\omega)$ { even part of signal given real part }

$a[n] = \dfrac{x[n] - x[-n]}{2j} \rightarrow B(\omega)$

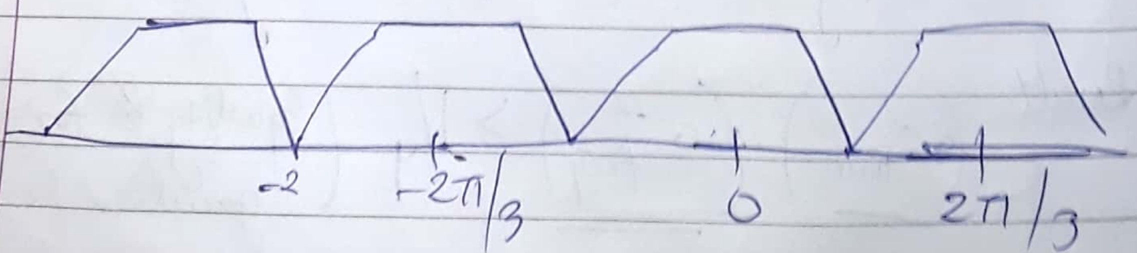
$a[n] + z[n+1] \rightarrow B(\omega) + e^{j\omega} A(\omega)$

Q6 $x_1[n]$





conjugate symmetry



ously

w) + j B(w)

part of given part

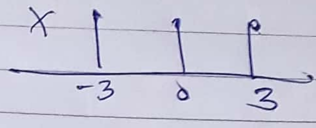
$X_1(w))$

$\pi/4$

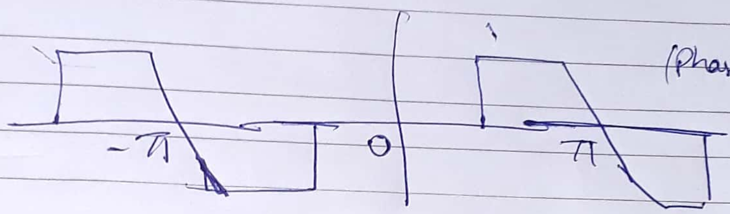
π

Q) Write $x_2[n]$ in terms of $x_1[n]$

$$x_2[n] = \left[\frac{x_1[n] + x_1[-n]}{2}\right] \times$$



$X_3(w)$



(Phase shift of π)

$$x_3[x] = (-1)^n \left(\frac{x_1[n] - x_1[-n]}{2j}\right)$$

$$(e^{j\pi n})$$