EE 210 **Spring 2017-18**

HW 5

Assigned: 27/3/18 Due: 5/4/18

You are encouraged to discuss these problems with others, but you need to write up the actual solutions alone. Start early, and come to office hours (TBA) with any doubts. Your only have to submit your solutions to **the questions marked** [†]. *Drop off your submission in the dropbox labeled EE210 in the EE office by 5.30 pm on the due date.*

1. Consider a signal $x(t)$ that is bandlimited in $[-\pi, \pi]$. Your pointy-haired boss samples this signal at $x(n/2)$, $n \in \mathbb{Z}$. He claims that $\phi(t) = sinc(2t)$ is the *unique interpolation* signal such that

$$x(t) = \sum_{n \in \mathcal{Z}} x(n/2) sinc(t - n/2).$$

   Is your boss correct? If not, provide another choice for $\phi(\cdot)$.

   Note: Problem designed by Prof. Animesh Kumar.

2. [†] Consider a class of signals $\mathcal{S}$ whose spectrum has finite support, such that for $x(t) \in \mathcal{S}$,

$$X(\omega) \neq 0 \text{ for } |\omega| \in (\pi, 2\pi)$$

   and $X(\omega) = 0$ otherwise.

   What is the minimal sampling rate required for perfect reconstruction of this class of signals? Come up with a corresponding interpolation formula.

   Note: Problem designed by Prof. Animesh Kumar.

3. [†] Sketch the pole-zero plot corresponding to the following causal system functions:

   (a) $\dfrac{s-2}{s^2+8s+15}$

   (b) $\dfrac{s+1}{(s+2)^2(s+3)}$

   (c) $\dfrac{2s^2+s+1}{s(s+2)}$

   (d) $\dfrac{2s+1}{(s+2)(s^2+1)^2}$

   Which of the above system functions correspond to BIBO stable systems?

4. [†] Consider LSI systems with the following transfer functions. Determine whether or not these systems are BIBO stable, causal.

   (a) $\dfrac{2s+5}{(s+2)(s+3)}; -3 < Re(s) < -2$

   (b) $\dfrac{2s-5}{(s-2)(s-3)}; 2 < Re(s) < 3$

   (c) $\dfrac{2s+3}{(s+1)(s+2)}; Re(s) > -1$

   (d) $\dfrac{2s+3}{(s+1)(s+2)}; Re(s) < -2$

5. [†] An LTI system is described by the system function $H(s) = \dfrac{s+3}{(s+2)^3}$.

(a) Find the impulse response of the system.

(b) For the input signal $x(t) = 10u(t)$, calculate the final value of the output $y(t)$ of the above system without explicitly evaluating $y(t)$.

6. [†] Consider a continuous time LTI system for which the input $x(t)$ and output $y(t)$ are related by the differential equation

$$\frac{\mathrm{d}^2 y(t)}{\mathrm{d}t^2} - \frac{\mathrm{d}y(t)}{\mathrm{d}t} - 2y(t) = x(t).$$

Let $X(s)$ and $Y(s)$ denote Laplace transforms of $x(t)$ and $y(t)$, respectively, and let $H(s)$ denote the Laplace transform of $h(t)$, the system impulse response.

(a) Determine $H(s)$ as a ratio of two polynomials in s. Sketch the pole-zero pattern of $H(s)$.

(b) Determine $h(t)$ for each of the following cases:

i. The system is stable.

ii. The system is causal.

iii. The system is neither stable nor causal.

7. [†] The system function of a causal LTI system is

$$H(s) = \frac{s+1}{s^2+2s+2}.$$

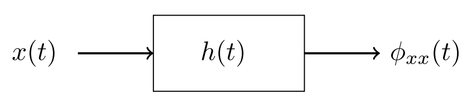
Determine and sketch the response $y(t)$ when the input is

$$x(t) = e^{-|t|}, \qquad -\infty < t < \infty.$$

8. [†] The autocorrelation function of a signal $x(t)$ is defined as

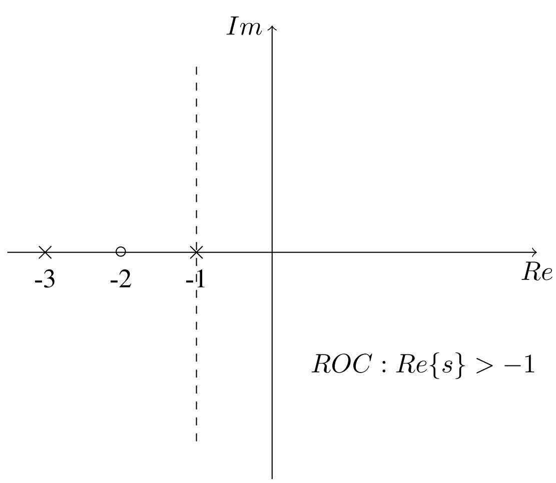
$$\phi_{xx}(\tau) = \int_{-\infty}^{\infty} x(t)x(t+\tau)dt$$

(a) Determine, in terms of $x(t)$, the impulse response $h(t)$ of an LTI system for which, when the input is $x(t)$, the output is $\phi_{xx}(t)$.



(b) From your answer in part (a), determine $\Phi_{xx}(s)$, the Laplace transform of $\phi_{xx}(\tau)$ in terms of $X(s)$.

(c) If $x(t)$ has the pole zero pattern and ROC as shown in figure, sketch the pole-zero pattern and indicate the ROC for $\phi_{xx}(\tau)$.



9. [†] Find the transfer function H(s) of the block diagrams shown in the following figures.

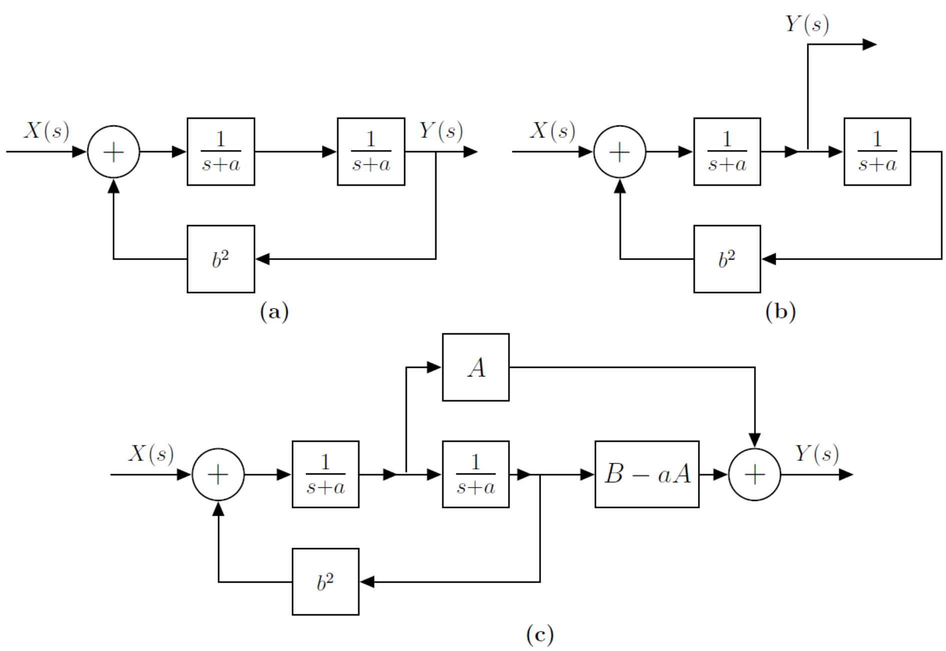


Figure 1: Problem 9