CHAPTER 15

Ladder Design with Simulated Elements

In the last chapter we extolled the virtues of the doubly terminated lossless ladder: low sensitivity, the circuit for which frequency transformations are directly applicable. As design engineers we see that these advantages are obtained at a high price: we use inductors which are hard to build and heavy in weight, and which are difficult to adapt to integrated-circuit realizations. In the next three chapters we consider methods which have been discovered to get rid of inductors. We will see that it is possible, by some power akin to that of King Midas, that all inductors we touch will turn not to gold but to inductorless circuits.

15.1 THE IDEAL GYRATOR AND RIORDAN'S CIRCUIT

Some electrical devices operate by causing the roles of the electric and magnetic fields to be interchanged. Two well-known examples are the Hall-effect devices and some waveguide configurations operating at microwave frequencies. In 1948 Tellegen proposed a model for such devices which is known as the *ideal gyrator*. We define his model in terms of the currents and voltages shown in Fig. 15.1, where

$$v = Ki_2 \tag{15.1}$$

$$v_2 = -Ki_1 \tag{15.2}$$

where K is a real constant. The symbol that is used to indicate a Tellegen gyrator is shown in Fig. 15.2.

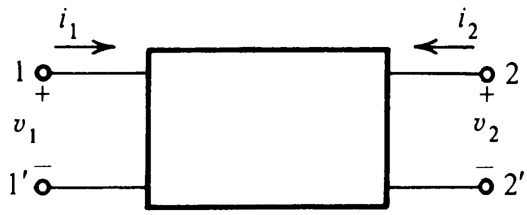


FIGURE 15.1

Now suppose that we terminate the gyrator in a capacitor, as shown in Fig. 15.3. This places a constraint between voltage v_2 and current i_2 which is that

$$i_2 = -\frac{d}{dt} C v_2 \tag{15.3}$$

If we substitute this relationship into Eq. (15.1), we have

$$v_1 = K i_2 = K\left(-\frac{d}{dt} C v_2\right) \tag{15.4}$$

We next substitute Eq. (15.2) into this equation, giving

$$v_1 = K \frac{d}{dt}(CKi_1) = \frac{d}{dt}(K^2 C i_1) = \frac{d}{dt}(L_{eq} i_1) \tag{15.5}$$

Here $L_{eq} = K^2 C$ is the value of the equivalent inductance. Thus we see that by means of a gyrator, a capacitor becomes the equivalent of an inductor. This is an important conclusion.

Unfortunately Hall-effect devices do not operate at the frequency range of our interest, and so we seek a gyrator based on an op-amp circuit. Of several that have been proposed, the most successful was given by Riordan* in 1967. This circuit is shown in Fig. 15.4 and is seen to consist of two op amps and five impedances. To analyze this circuit, we observe that

$$V_2 = V_1\left(1 + \frac{Z_4}{Z_5}\right) \tag{15.6}$$

and that

$$V_3 = V_1\left(1 + \frac{Z_2}{Z_3}\right) - V_2\left(\frac{Z_2}{Z_3}\right) \tag{15.7}$$

or

$$V_3 = V_1\left(1 - \frac{Z_2 Z_4}{Z_3 Z_5}\right) \tag{15.8}$$

* R. H. S. Riordan, "Simulated Inductors Using Differential Amplifiers," *Electron. Lett.*, vol. 3, pp. 50–51, 1967.

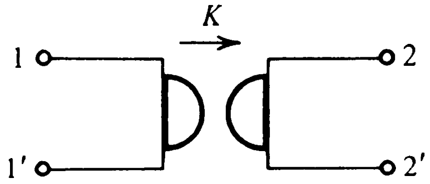


FIGURE 15.2

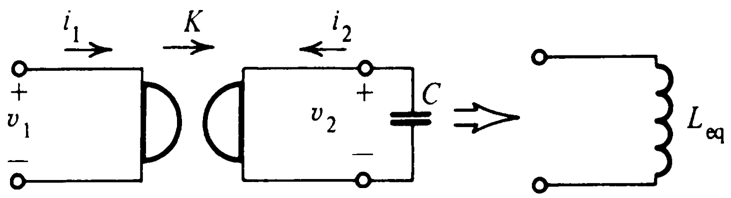


FIGURE 15.3

Now the input current is

$$I_1 = \frac{V_1 - V_3}{Z_1} = V_1 \frac{Z_2 Z_4}{Z_1 Z_3 Z_5} \tag{15.9}$$

Thus the input impedance is seen to be

$$Z_{in} = \frac{V_1}{I_1} = \frac{Z_1 Z_3 Z_5}{Z_2 Z_4} \tag{15.10}$$

From this equation we see that if either Z_2 or Z_4 are capacitors, such that $Z = 1/Cs$ and all remaining elements are resistors of value R, then the last equation becomes

$$Z_{in} = (CR^2)s = L_{eq}s \tag{15.11}$$

and the circuit behaves as if it were an inductor of value

$$L_{eq} = CR^2 \tag{15.12}$$

For example, if $C = 0.01$ μF and the four resistors have the value $R = 1$ kΩ, then at the inputs the circuit appears to be a 10-mH inductor.

The circuit of Fig. 15.5 is identical to the Riordan circuit of Fig. 15.4, as can be verified by simply tracing out the connections. Another circuit that is very similar to the Riordan circuit is given in Fig. 15.6a. The two remaining circuits of Fig. 15.6 are due to Antoniou* and these are considered extensively in the next

* A. Antoniou, "Realization of Gyrators Using Operational Amplifiers, and Their Use in RC-Active-Network Synthesis," *Proc. IEE*, vol. 116, pp. 1838–1850, 1969.

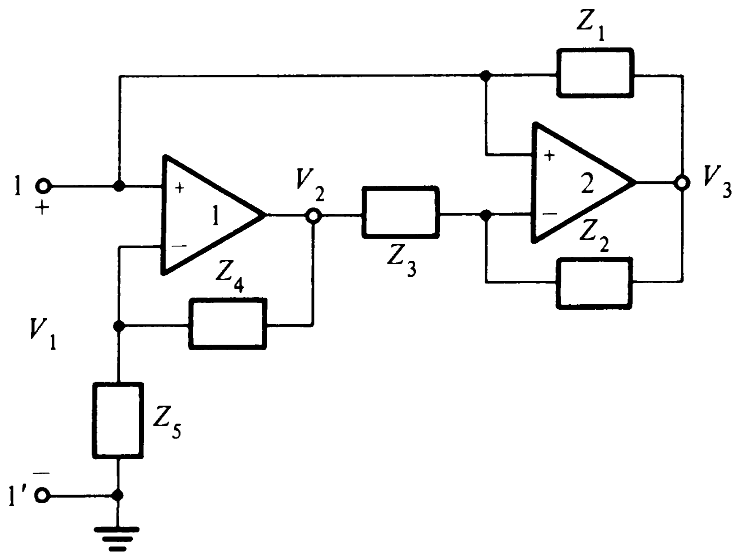


FIGURE 15.4

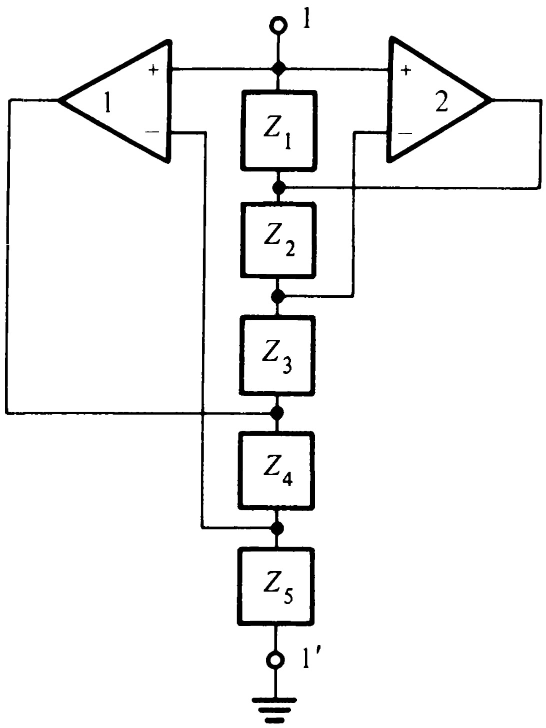


FIGURE 15.5

section. The circuits are similar in that they are all described by Eq. (15.10). Thus any of them can be used to simulate an inductor. But that is not all.

15.2 ANTONIOU'S GIC AND BRUTON'S FDNR

Of the two Antoniou circuits of Fig. 15.6 we will consider that of Fig. 15.6b, which has been shown to be the "best" circuit.† This circuit is redrawn in Fig. 15.7, with the circuit enclosed in dashed lines excluding Z_5. This part is known as a *generalized impedance converter* (GIC). We have identified part of the circuit as a GIC, but this does not change the circuit which has an input impedance as given by Eq. (15.10):

$$Z_{11'} = \frac{Z_1 Z_3}{Z_2 Z_4} Z_5 \tag{15.13}$$

However, if we remove Z_5 and identify the input terminals 2-2′ and at the same time terminate terminals 1-1′ in Z_0, then we find that

$$Z_{22'} = \frac{Z_4 Z_2}{Z_3 Z_1} Z_0 \tag{15.14}$$

these quantities being identified in Fig. 15.8.

If we now make the choice that $Z_4 = 1/C_4 s$ and that all other elements in Fig. 15.7 are resistors, then

$$Z_{11'} = \frac{R_1 R_3 R_5}{R_2} C_4 s = L_{eq} s \tag{15.15}$$

† A. S. Sedra and P. O. Brackett, *Filter Theory and Design: Active and Passive*, Matrix Publishers, Portland, Ore., 1978, sec. 8.4.

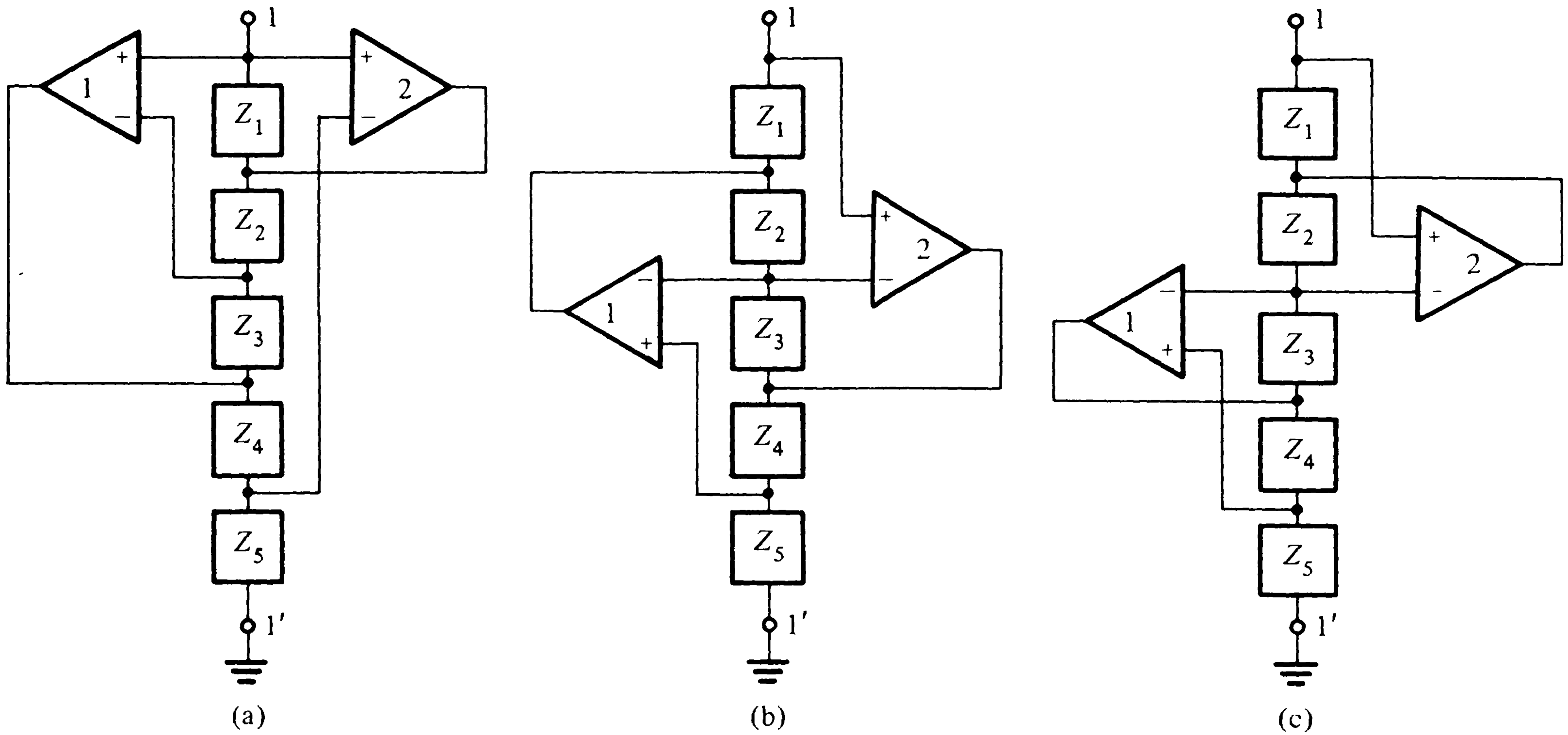


FIGURE 15.6

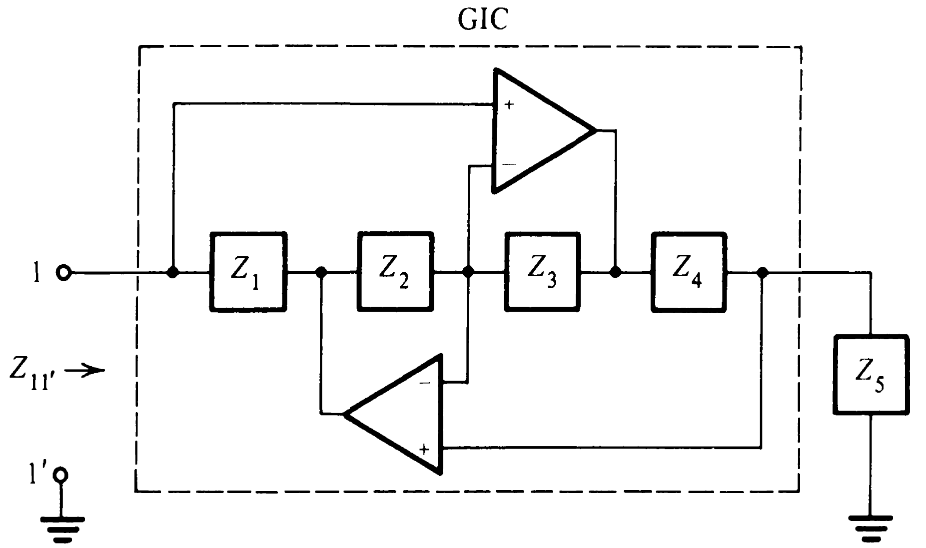


FIGURE 15.7

This is the impedance of the circuit shown in Fig. 15.9a. The result is the same as that found in the last section. Conceptually we may regard the GIC as a circuit that converts the resistor R_5 into an inductor of value $L_{eq} = R_1R_3R_5C_4/R_2$, as given by Eq. (15.15). The surprise comes when we turn the GIC end for end, remove R_5, and terminate the opposite side in a capacitor of value C_0, as shown in Fig. 15.9c. Then Eq. (15.14) gives us

$$Z_{22'} = \frac{R_2}{R_1R_3C_4C_0}\frac{1}{s^2} = \frac{1}{Ds^2} \tag{15.16}$$

This is something new. We note that when $s = j\omega$, then

$$Z_{22'}(j\omega) = \frac{-1}{D\omega^2} \tag{15.17}$$

We see that this function is negative and varies inversely with ω^2. The circuit of Fig. 15.9c is known as a *frequency-dependent negative resistor* (FDNR). The FDNR concept was introduced by Bruton* along with a method for the simula-

* L. T. Bruton, "Network Transfer Functions Using the Concept of Frequency-Dependent Negative Resistance," *IEEE Trans. Circuit Theory*, vol. CT-16, pp. 406–408, 1969.

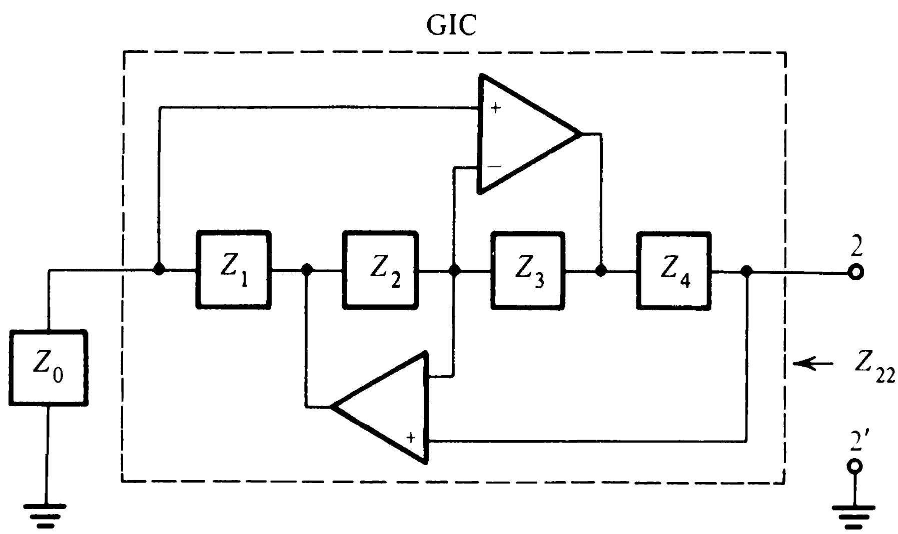


FIGURE 15.8

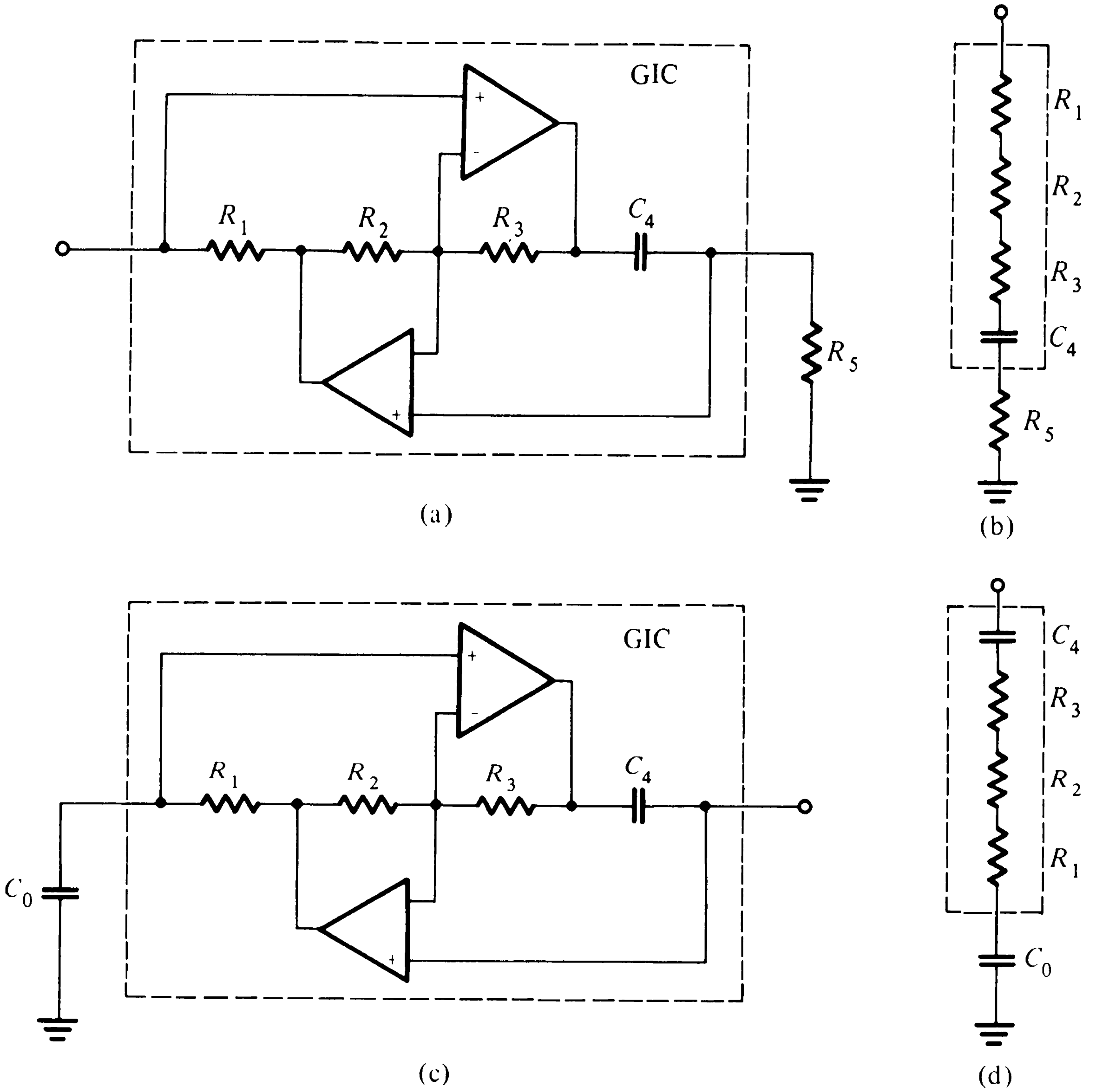


FIGURE 15.9

tion of ladder filters. Since the circuits of Fig. 15.9a and c are complicated, they may be replaced by the shorthand notation of Fig. 15.9b and d in which the op-amp connections are removed.

The other new concept introduced by Bruton is that of magnitude scaling all elements by the factor $1/s$. Past experience has shown us that circuits may be magnitude scaled without changing the transfer function $T(s)$. In the past all such scaling has been by a constant k_m. Since the elements are given by the equations

$$Z_R = R, \qquad Z_L = Ls, \qquad Z_C = \frac{1}{Cs} \tag{15.18}$$

scaling each of these by $1/s$ gives us

$$Z_R' = \frac{R}{s}, \qquad Z_L' = L, \qquad Z_C' = \frac{1}{Cs^2} \tag{15.19}$$

Thus we see that such scaling actually results in a transformation of elements: a resistor becomes a capacitor, an inductor becomes a resistor, and a capacitor becomes an FDNR. Note carefully that none of the impedances in Eq. (15.19) de-

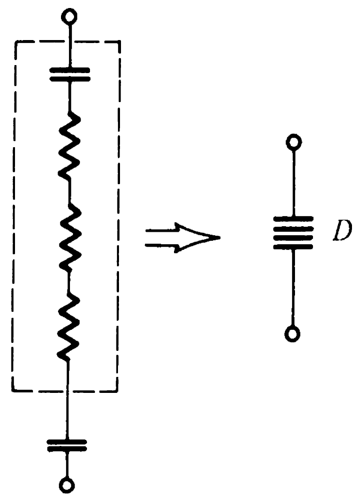


FIGURE 15.10

scribes an inductor. We can eliminate inductors, and the only price we must pay is to use FDNRs instead. These can be realized by the circuit of Fig. 15.9c. We require a new symbol for the new element just introduced. It is four parallel lines, as shown in Fig. 15.10.

In summary the elements resulting from the Bruton transformation are shown in Fig. 15.11. We now have R and L transformed into familiar elements, and a new element introduced in transforming C. The new element is, of course, the FDNR described by Eq. (15.16) which is realized by the circuit shown in Fig. 15.9c. The manner in which this element and also the simulated inductor may be realized using integrated-circuit technology is suggested by Fig. 15.12, which shows the pin connections of the AF120, a commercial implementation produced by National Semiconductor. Fig. 15.12a shows a standard gyrator module; two external capacitors makes it an FDNR, as shown in Fig. 15.12b; one external capacitor makes it into a grounded inductor, as shown in Fig. 15.12c.

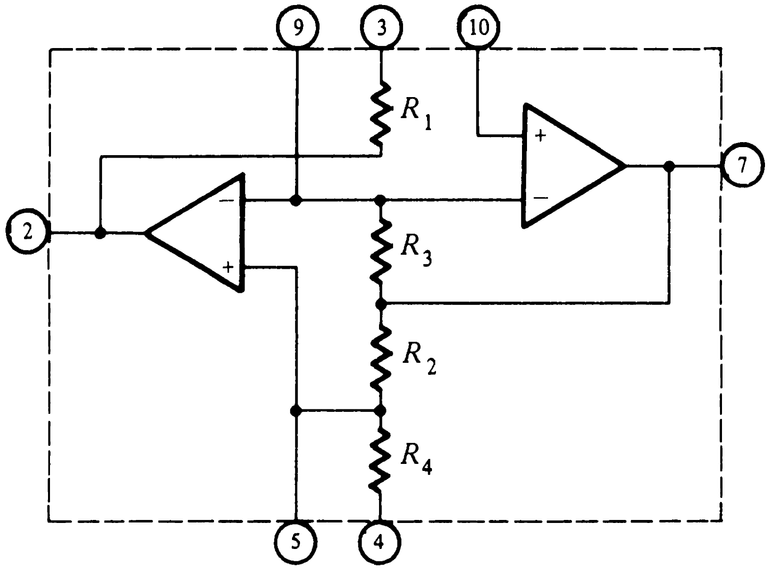
Example 15.1 Figure 15.13a shows a simple RLC series circuit arranged with the output voltage across the capacitor. This circuit has been studied extensively and is well known to be a lowpass filter. The voltage-ratio transfer function is

$$\frac{V_2}{V_1} = \frac{1/Cs}{R + Ls + 1/Cs} \tag{15.20}$$

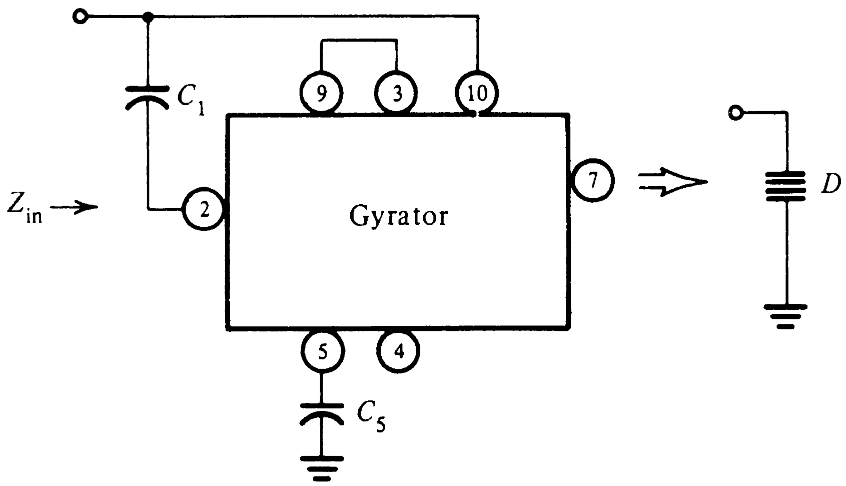
The circuit in Fig. 15.13b is obtained from that in Fig. 15.13a by applying the Bruton

Element	Bruton transformed element
R	$1/R = C$
L	$R = L$
C	$D = C$

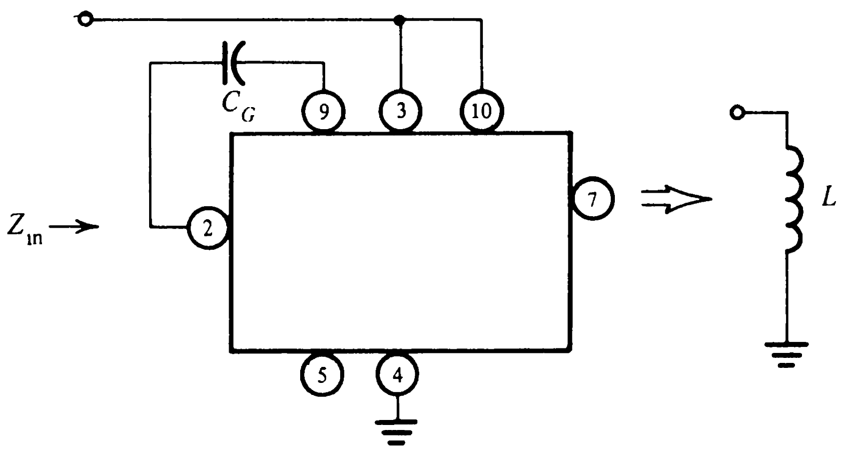
FIGURE 15.11



(a) Gyrator module



(b) FDNR connection



(c) Grounded inductor connection

FIGURE 15.12

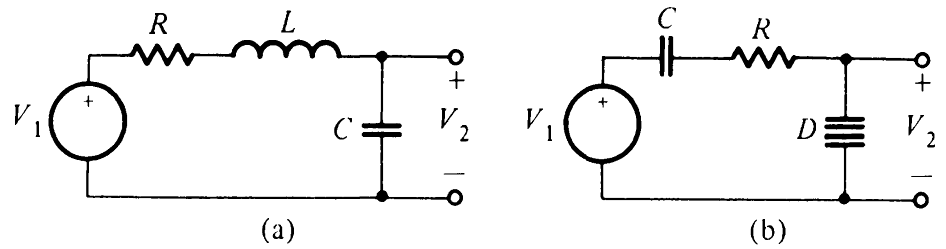


FIGURE 15.13

transformation, as shown in Fig. 15.11. The voltage-ratio transfer function for the circuit in Fig. 15.13b is

$$\frac{V_2}{V_1} = \frac{1/Cs^2}{R/s + L + 1/Cs^2} \tag{15.21}$$

which is clearly the same equation as Eq. (15.20). Thus we see that the two circuits of Fig. 15.13 have identical transfer functions. This illustrates the objective in applying the Bruton transformation.

Example 15.2 The problem to be considered is the design of a fourth-order Butterworth lowpass filter making use of FDNRs. The prototype for such a filter is found from Table 14.1, and is shown in Fig. 15.14a. Using the Bruton transformation of elements as in Fig. 15.11, we obtain the equivalent circuit of Fig. 15.14b complete with element values for R, C, and D. To realize the FDNR, we make use of the circuit of Fig. 15.9c, which is described by Eq. (15.16). We make the choices

$$C_0 = C_4 = 1\ \text{F}, \qquad R_2 = R_3 = 1\ \Omega, \qquad R_1 = D \tag{15.22}$$

and the realization of the normalized circuit of Fig. 15.14b becomes that shown in Fig. 15.15a.

Suppose that we next specify that the half-power frequency be 1000 Hz, and that the final design make use of 0.01-μF capacitors only. This means that scaling is accomplished by the choices

$$k_f = 2\pi \times 1000 \qquad \text{and} \qquad k_m = \frac{1}{(2\pi \times 1000)(0.01 \times 10^{-6})} = 1.59 \times 10^4 \tag{15.23}$$

With this scaling the circuit becomes that shown in Fig. 15.15b. It is seen that all of the

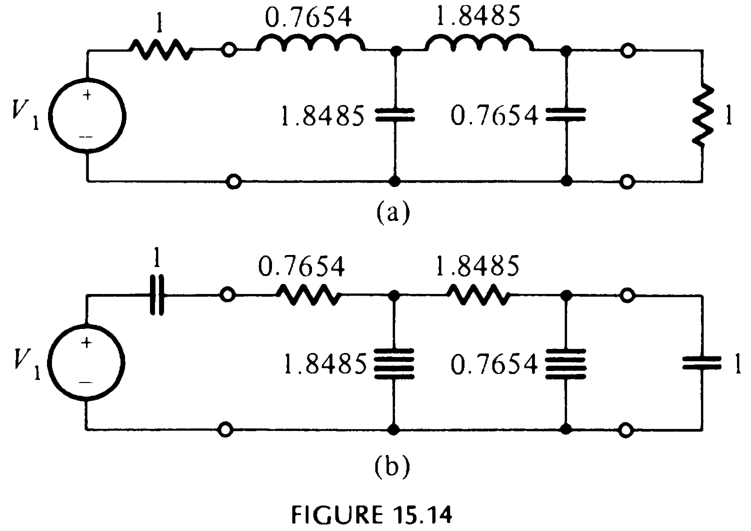
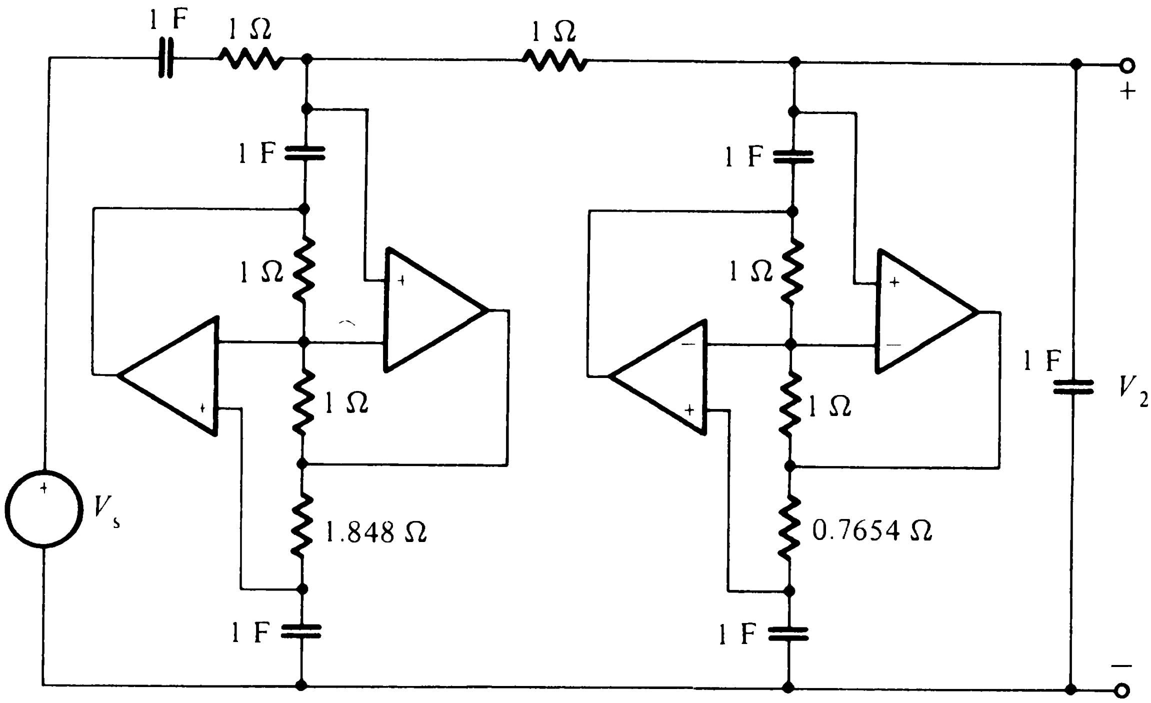
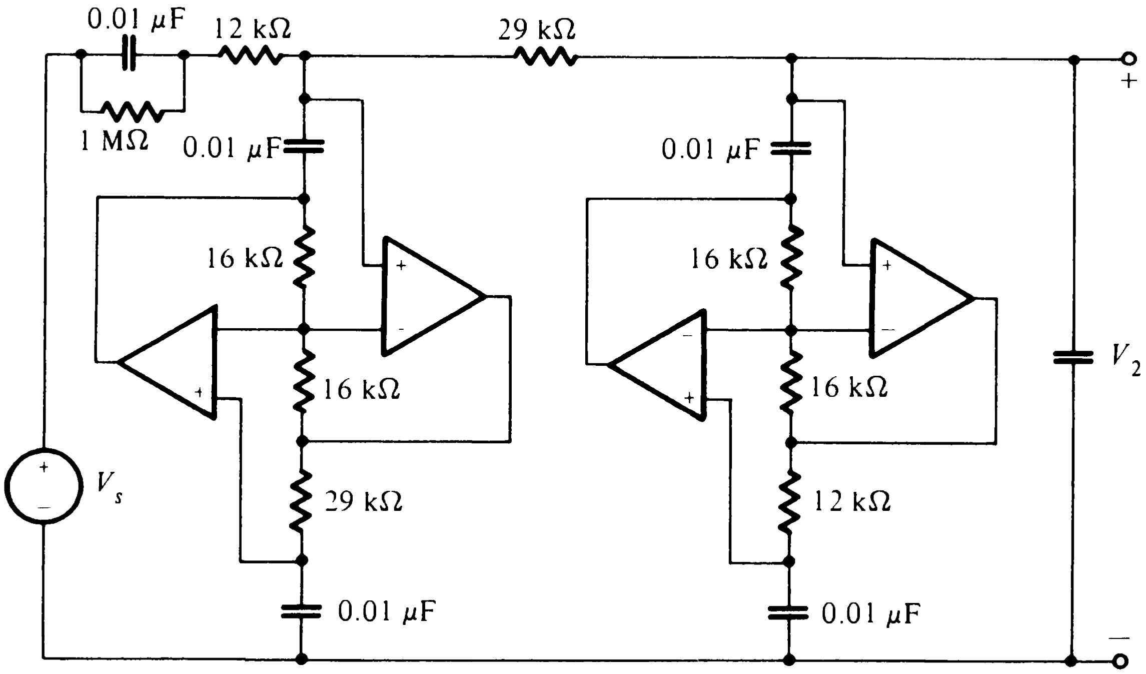


FIGURE 15.14



(a)



(b)

FIGURE 15.15

elements in the circuit are in a practical range and can be realized with ordinary op amps. Experiments on laboratory circuits such as that of Fig. 15.15b show excellent agreement between the specified Butterworth fourth-order response and that obtained. Since this circuit is completely equivalent, except for scaling, to that shown in Fig. 15.14a, this circuit will have the same low sensitivity values as those found for a passive ladder circuit. In other words, this is a very practical realization technique.

Example 15.3 We require a highpass filter with a half-power frequency of 10 Hz. If constructed using an inductor, this would be a very heavy filter, and so we wish to consider the possibility of using simulated inductors. Suppose that it is further specified that a third-order Butterworth filter be used as the prototype in this design. This prototype with element values determined from Table 14.1 is shown in Fig. 15.16a. Making use of the lowpass to highpass frequency transformation, we obtain the highpass circuit shown in Fig. 15.16b for $\omega_0 = 1$. Using the circuit of Fig. 15.9, we have the equation for impedance,

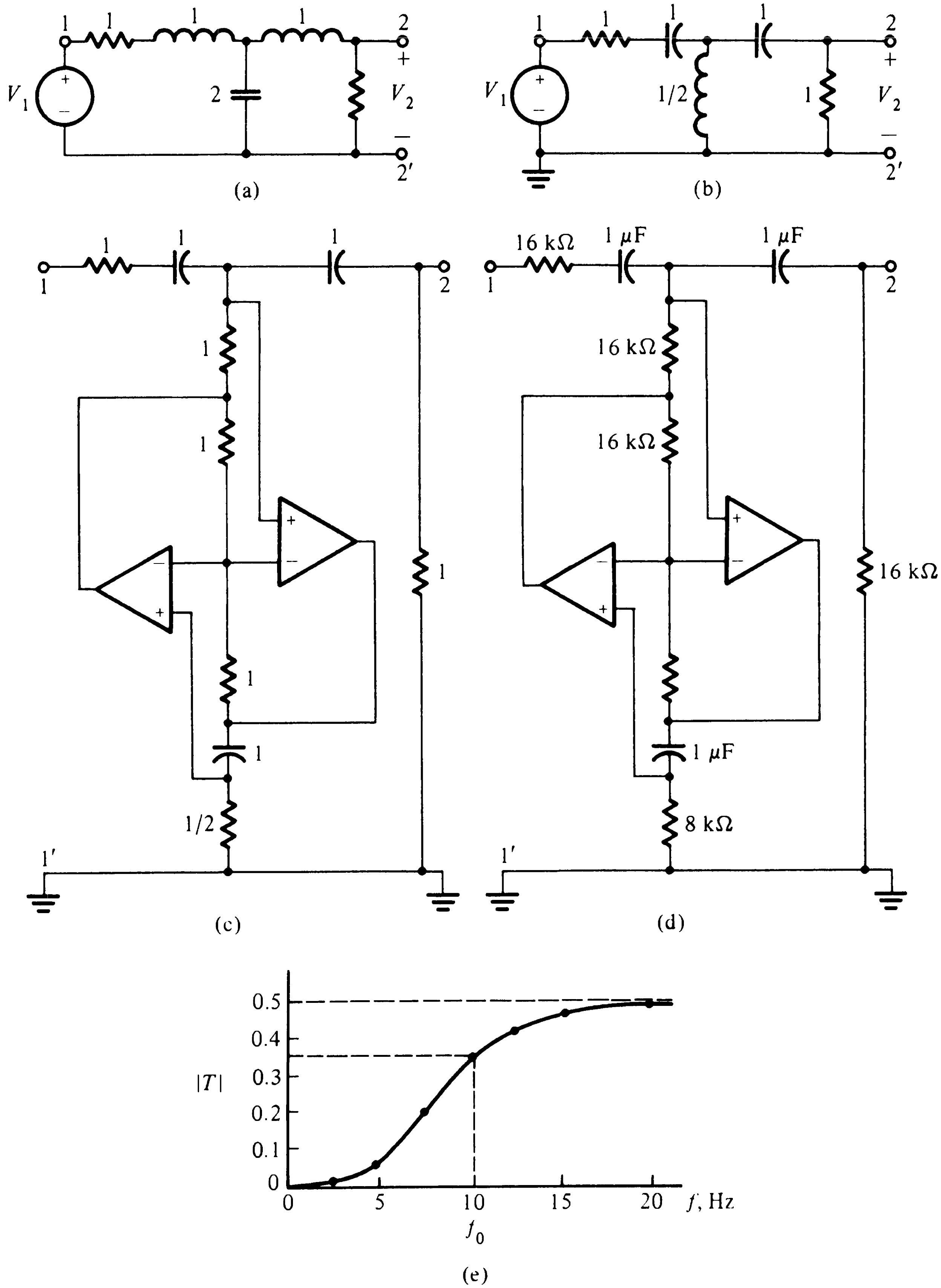


FIGURE 15.16

Eq. (15.15). In that equation let all element values be unity except for R_5, which is then equal to L_{eq}. Since we require $L_{eq} = \frac{1}{2}$ H, we obtain the circuit shown in Fig. 15.16c.

Next we scale, using the required scaling from $\omega_0 = 1$ to $f_0 = 10$, so that $k_f = 62.83$. Magnitude scaling is set by the requirement that the capacitors have the value of 1.0 μF, so that

$$k_m = \frac{1}{10^{-6} \times 20\pi} = 15{,}915 \tag{15.24}$$

The circuit resulting from this scaling is given in Fig. 15.16d, and this circuit has the response shown in Fig. 15.16e. Again, all element values are in a practical range, and this realization would be light in weight compared to one with a physical inductor.

15.3 CREATING NEGATIVE ELEMENTS

In some of our filter designs we will require negative elements. How are such elements created? To answer our question, we begin with the circuit given in Fig. 15.17, to which we will apply nodal analysis. At node 1 the Kirchhoff current law gives us

$$I_1 + \frac{V_2 - V_1}{R_1} = 0 \tag{15.25}$$

which, at node 2, is

$$\frac{V_2 - V_1}{R_2} + (0 - V_1)\frac{1}{Z_L} = 0 \tag{15.26}$$

Eliminating V_2 from these two equations, gives us

$$-I_1R_1 - V_1R_2\frac{1}{Z_L} = 0 \tag{15.27}$$

Solving for the ratio I_1/V_1, we obtain

$$\frac{V_1}{I_1} = Z_{in} = -\frac{R_1}{R_2}Z_L \tag{15.28}$$

or

$$Y_{in} = -\frac{R_2}{R_1}Y_L \tag{15.29}$$

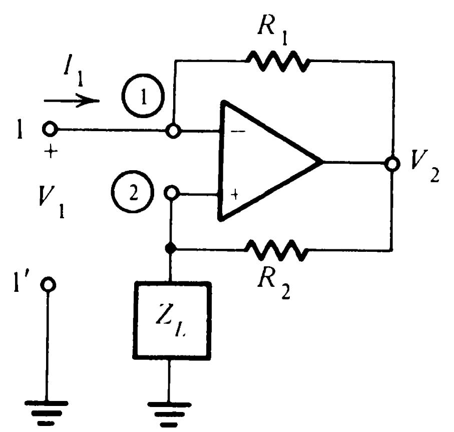


FIGURE 15.17

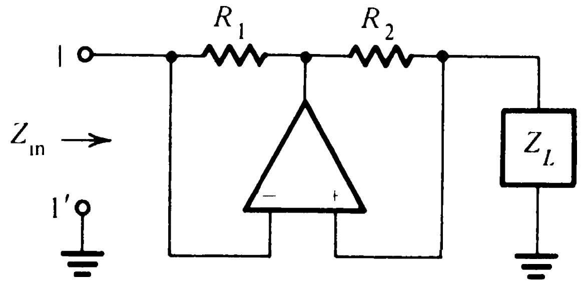


FIGURE 15.18

From these equations we see that the circuit, shown in alternative form in Fig. 15.18, creates the negative of Z_L or Y_L and also scales the value by the ratio of resistances. Thus if $Z_L = R$, then R_{in} is negative and can be thought of as a negative resistance at terminals 1-1′, as shown in Fig. 15.19. Similarly, the termination of the circuit in a capacitor makes Z_{in} appear as a negative capacitor. The circuit we have studied is one of a general class of circuits known as *negative impedance convertors* (NIC), as represented in a general form in Fig. 15.20.

To illustrate potential applications of negative elements, consider the section of a ladder circuit in Fig. 15.21a. This may be shown to be equivalent to the circuit of Fig. 15.21b.* The circuit of Fig. 15.21a may be difficult to realize because of the floating inductor L_2. The equivalent circuit of Fig. 15.21b has no floating inductor, but it does have a negative capacitor and also an ideal transformer. In general the circuit of Fig. 15.21b is easier to realize than that of Fig. 15.21a and the circuit we have studied would be used to realize the negative capacitor.

As a second example, the filter circuit shown in Fig. 15.22a, was obtained† using a sequence of circuit transformations. As given, it has problems in realization. First, it contains floating inductors, and it also contains a negative inductor. Both problems are easily solved by making use of the Bruton transformation which gives the circuit shown in Fig. 15.22b. In this circuit both FDNRs are grounded, and the negative resistor is easily realized using the circuit under study, that of Fig. 15.18.

15.4 CREATING FLOATING ELEMENTS

The floating element of our present interest is the floating inductor for which the two terminals may be at different voltages, neither equal to zero (or grounded). A representation of such an inductor is shown in Fig. 15.23. Here terminals 1 and 2 are floating, while terminals 1′ and 2′ are grounded. Any circuit we propose to replace the inductor of this figure must satisfy the following tests: (1) if terminals 2-

* H. J. Orchard and D. F. Sheahan, "Inductorless Bandpass Filters," *IEEE J. Solid-State Circuits,* vol. SC-5, pp. 108–118, 1970.
† Due to P. Geffe.

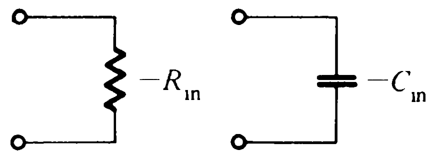


FIGURE 15.19

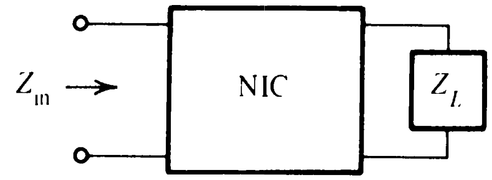


FIGURE 15.20

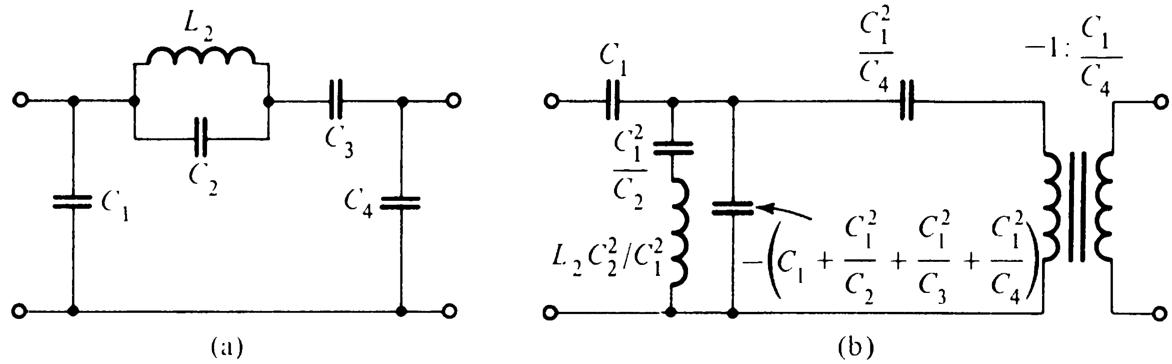


FIGURE 15.21

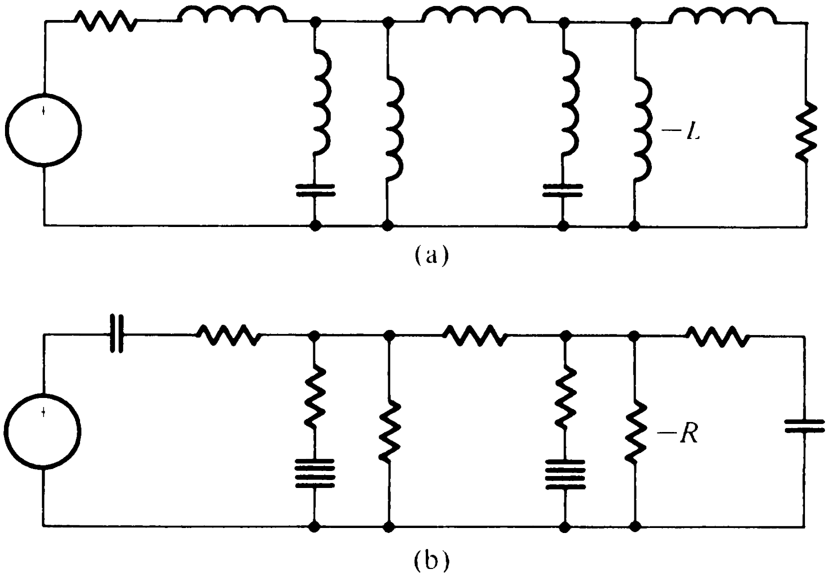


FIGURE 15.22

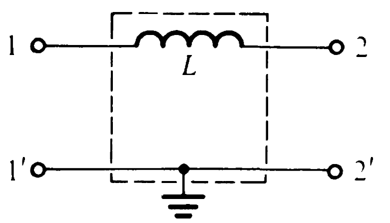


FIGURE 15.23

2′ are shorted, then $Z_{11'} = Ls$, or (2) if terminals 1-1′ are shorted, then $Z_{22'} = Ls$. These conditions are illustrated in Fig. 15.24.

One possible solution to our problem is illustrated in Fig. 15.25, which shows two gyrators, both terminated in the same capacitor C. From the basic relationship for the gyrator defined in terms of Fig. 15.26,

$$Z_{in} = K\frac{1}{Z_L} \tag{15.30}$$

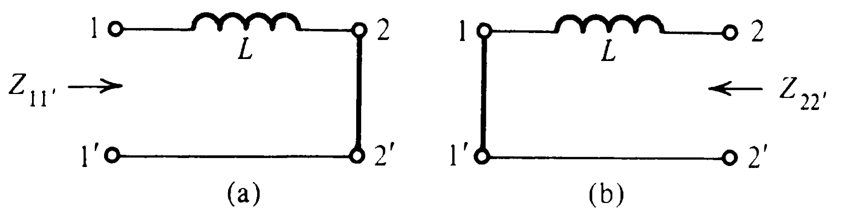


FIGURE 15.24

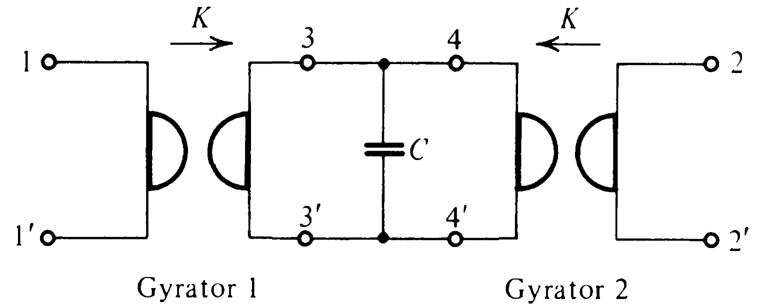


FIGURE 15.25

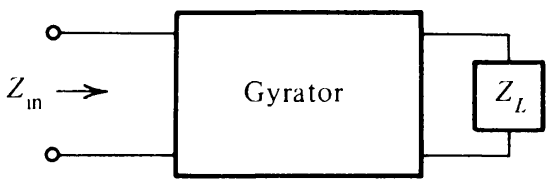


FIGURE 15.26

we see that when $Z_L = 0$, corresponding to a short circuit, then $Z_{in} = \infty$, or an open circuit. Thus if we short terminals 2-2′, then at terminal pair 4-4′ there will be an open circuit, and the input impedance at terminals 1-1′ will be as found in Eq. (15.5), that of an inductor. The same argument works in reverse, of course, so that the criterion illustrated by Fig. 15.24 is satisfied.

Even if the circuit of Fig. 15.25 does work, it turns out that there is a better way of accomplishing the same objective. The new solution was suggested by Riordan in his original paper, and can now be understood in terms of the properties of GICs as first pointed out by Gorski-Popiel.* Consider the circuit shown in Fig. 15.27, where Fig. 15.27a reminds us of the concept, and Fig. 15.27b shows us the circuit that will actually be used. First let us consider the consequences of shorting the output terminals 2-2′, as was done for the two-gyrator circuit of Fig. 15 25. We study the input impedance identified in Fig. 15.28, noting that since the voltage across the input terminals to the op amp is zero, then terminals 1 and 2 are actually connected together. Thus $Z_{in} = 0$ as shown in Fig. 15.28b. Returning to Fig. 15.27b, we see that shorting output terminals 2-2′ causes the resistor R to be shorted to ground. Then the circuit becomes the usual one for simulating an inductor, and $Z_{11'}$ is the impedance of an inductor, as required by the criterion of Fig. 15.24. Of course the same arguments work in reverse, with terminals 1-1′ shorted. Then in the circuit of Fig. 15.27 we have a floating inductor. It does take more elements than the realization of a grounded inductor, but it is no more difficult conceptually.

* J. Gorski-Popiel, "RC-active synthesis using positive-immittance converters," *Electron. Lett.*, vol. 2, pp. 381–382, Aug. 1967.

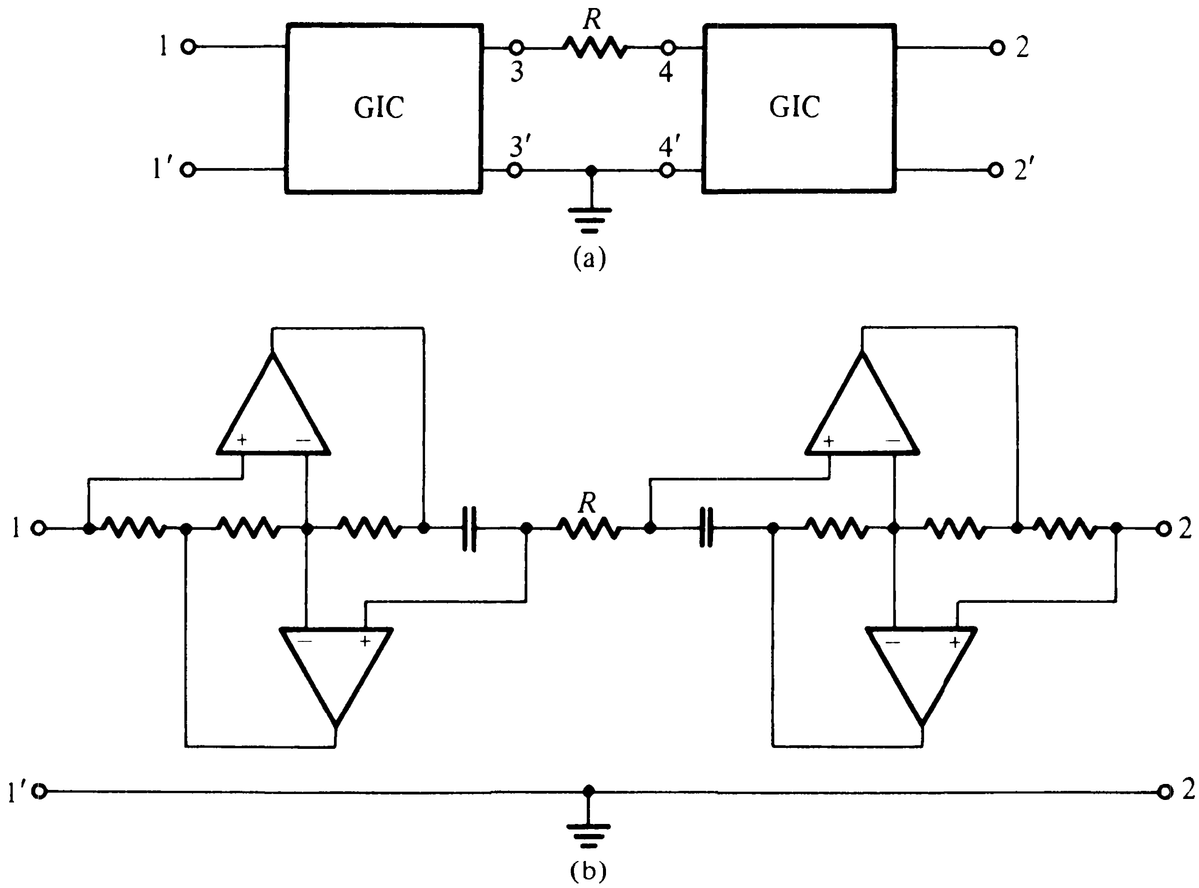


FIGURE 15.27

We can now generalize the discussion just given by noting that any circuit that is connected in place of the resistor R in Fig. 15.27b will be transformed by the NIC. The NIC we are using in the circuit for simulating floating inductors is the one that changes a resistor into an inductor. If a resistive circuit replaces R, then every element in that circuit is transformed from a resistor to an inductor. The manner in which this applies to a T-circuit and a π-circuit is illustrated by Fig. 15.29. We will soon show that the NIC can be used for other kinds of such transformations.

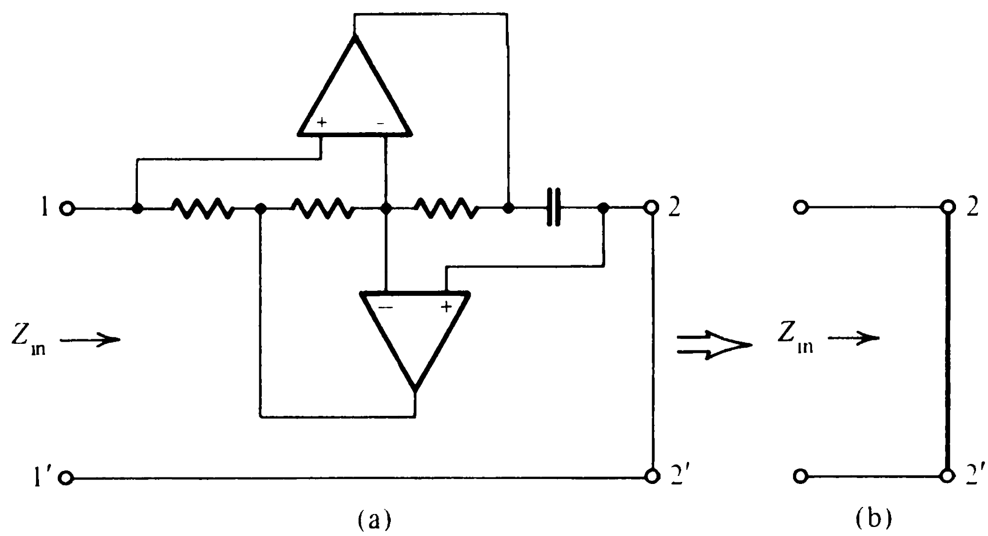


FIGURE 15.28

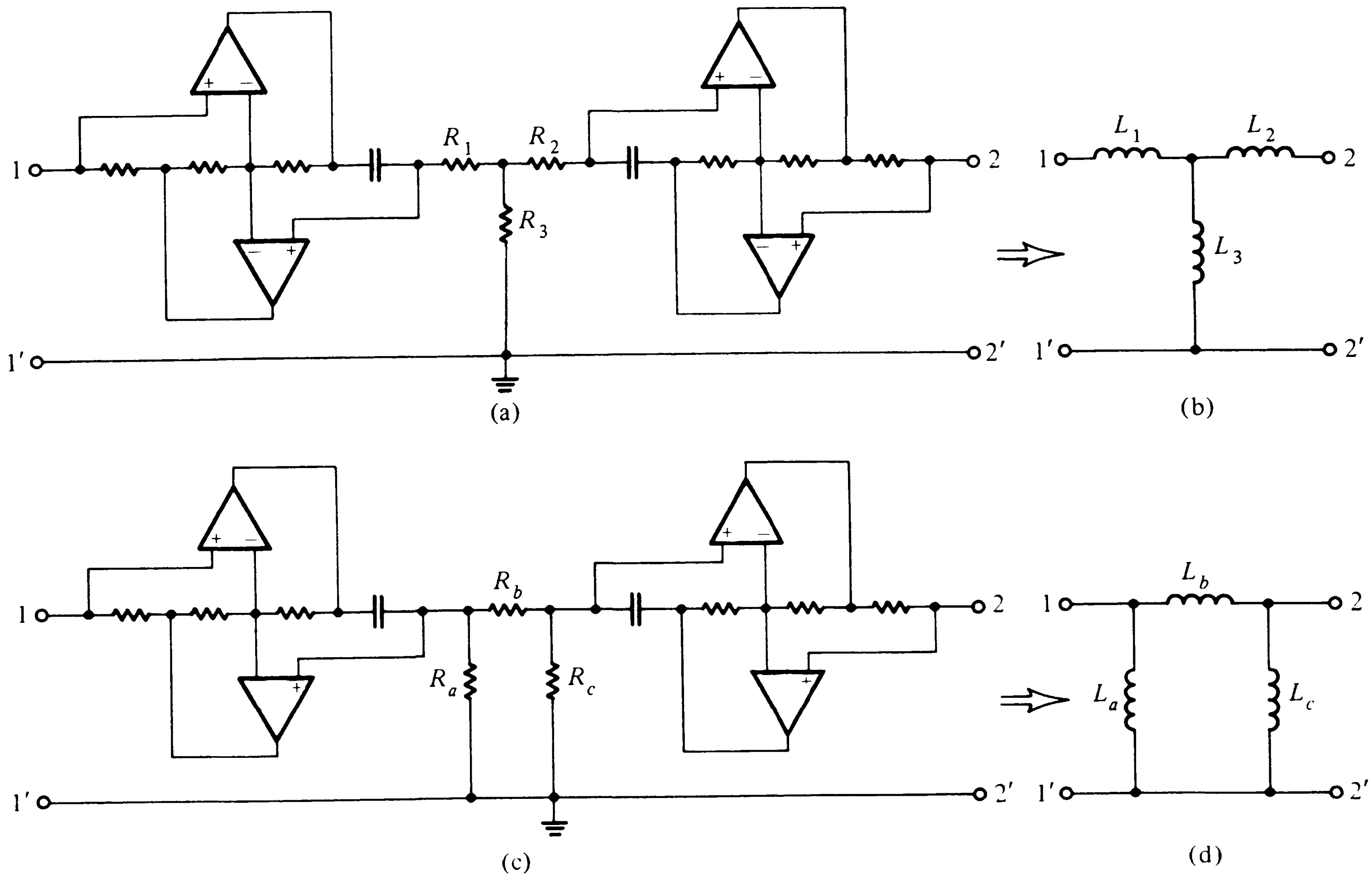
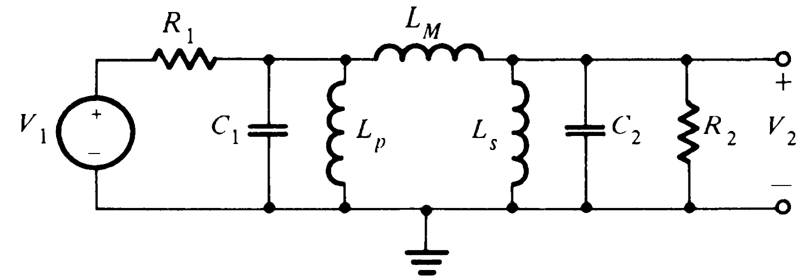


FIGURE 15.29

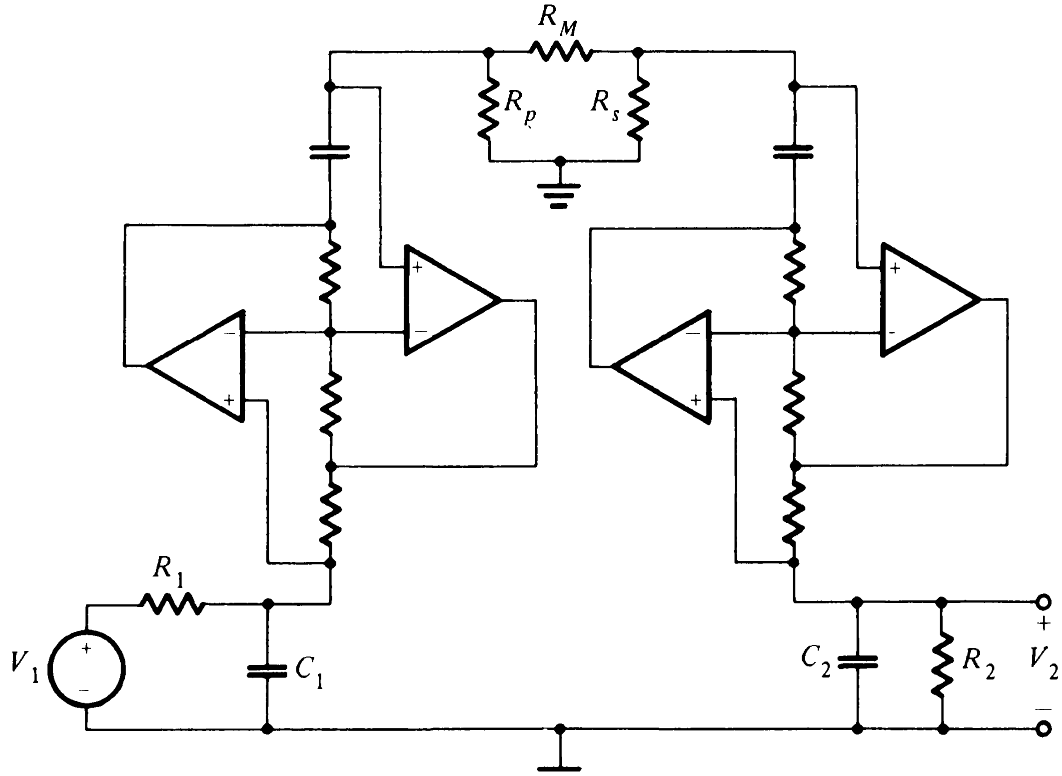
Example 15.4 The circuit shown in Fig. 15.30a is known as a double-tuned bandpass filter in which the three inductors L_p, L_s, and L_M represent the π equivalent of a transformer. We may replace the inductors by resistors, using the concepts just developed. As shown in Fig. 15.30b, the π-connection of resistors together with two GICs are equivalent to the π-connection of inductors, and so a realization may be found without inductors.

15.5 GIC BANDPASS FILTERS

The lowpass to bandpass frequency transformation studied in Chapter 14 causes shunt capacitors to become parallel LC circuits and series inductors to become series LC circuits. The bandpass filter shown in Fig. 15.31a is then the standard form for a ladder bandpass filter, and so it is the starting point in a search for a simulation of this circuit making use of GICs. We first apply the Bruton transformation to obtain the circuit shown in Fig. 15.31b. In this circuit we can identify two canonical sections, which are labeled as section A and section B. Bandpass filters of higher order will be made up of an alternation of these two kinds of sec-



(a)



(b)

FIGURE 15.30

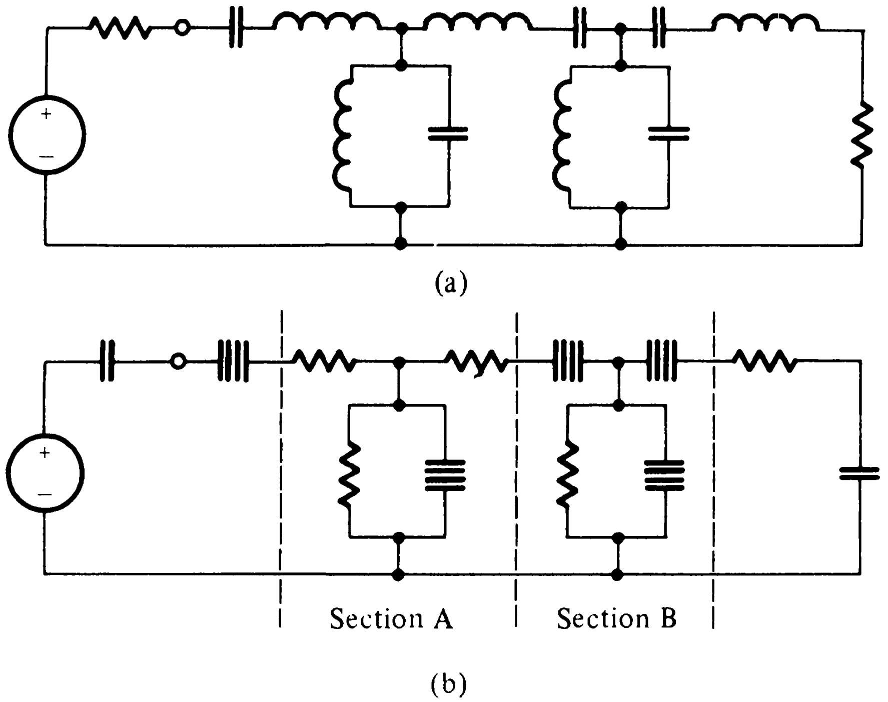


FIGURE 15.31

tions, plus the terminations shown on either end. If we understand the manner in which these two sections may be simulated, then we can simulate a bandpass filter of any order.

Figure 15.32 shows the standard section A drawn in somewhat different form than in Fig. 15.31b. Section B, which is shown in Fig. 15.33, requires further discussion. In section B we first remove the resistor from the FDNRs and then note that we have a T-section of FDNRs. We then ask the question as to what GIC would transform a T-section of resistors into a T-section of FDNRs. The answer to this question may be found by returning to Fig. 15.7, which is described by Eq. (15.13) and repeated here:

$$Z_{11'} = \frac{Z_1 Z_3}{Z_2 Z_4} Z_5 \tag{15.31}$$

and also to Fig. 15.8, which is described by Eq. (15.14):

$$Z_{22'} = \frac{Z_2 Z_4}{Z_1 Z_3} Z_0 \tag{15.32}$$

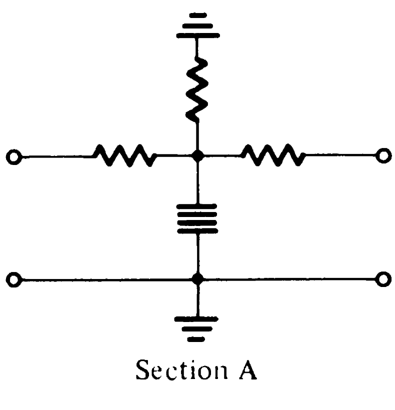


FIGURE 15.32

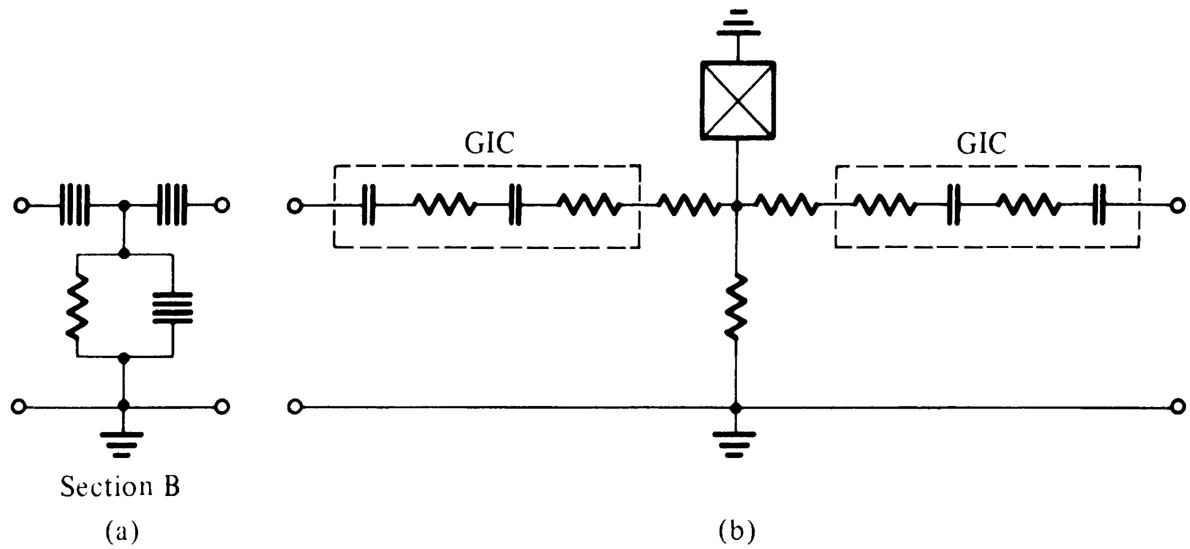


FIGURE 15.33

Thinking in terms of the resistive T-section in Fig. 15.33b, we select $Z_0 = R_0$ and also $Z_2 = Z_4 = 1/Cs$. If $Z_1 = R_1$ and $Z_3 = R_3$, then

$$Z_{22'} = \frac{R_0}{R_1 R_3 C^2} \frac{1}{s^2} \tag{15.33}$$

which is the equation for an FDNR. However, this particular FDNR converts resistors, such as R_0 in Fig. 15.34, into an FDNR, and thus it is directly applicable to the circuit of Fig. 15.33b. It does transform the T-section of resistors into the T-section of FDNRs by multiplying each element by $1/Ds^2$.

The new symbol in Fig. 15.33b represents a new element which, when multiplied by $1/Ds^2$, becomes a constant. Then it is clear that the impedance of this new element must be

$$Z_E = Es^2 \tag{15.34}$$

such that the product is

$$Es^2 \times \frac{1}{Ds^2} = \frac{E}{D} = R \tag{15.35}$$

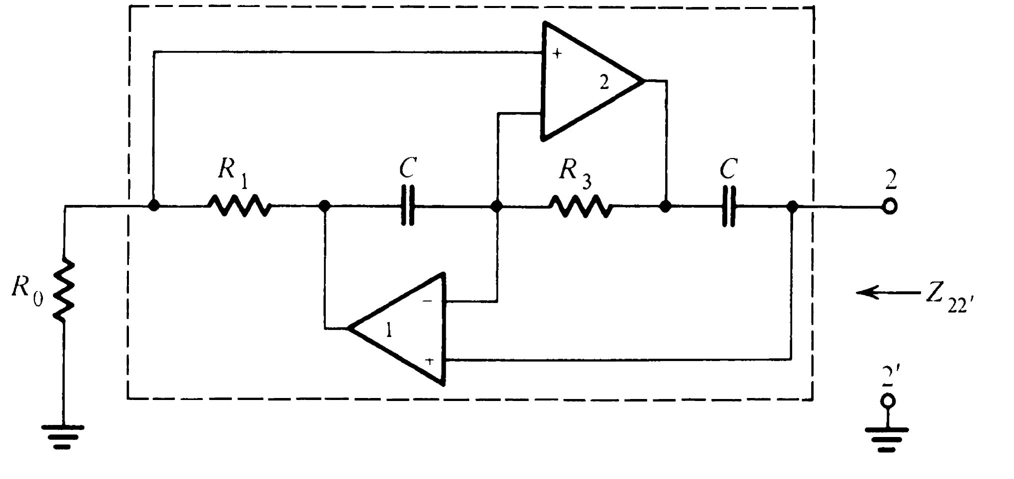


FIGURE 15.34

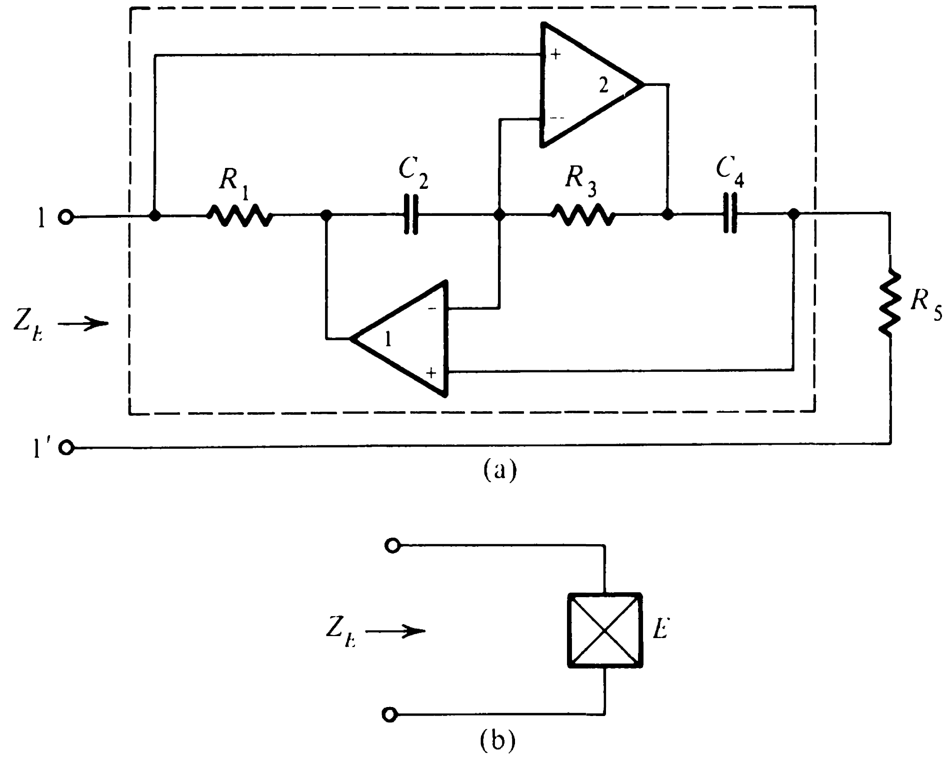


FIGURE 15.35

The circuit satisfying this specification is shown in Fig. 15.35 for which Eq. (15.31) applies. Here we let $Z_5 = R_5$, $Z_1 = R_1$, $Z_3 = R_3$, $Z_2 = 1/C_2s$, and $Z_4 = 1/C_4s$, such that Eq. (15.31) becomes

$$Z_{11'} = R_1R_3R_5C_2C_4s^2 = Es^2 \tag{15.36}$$

When $s = j\omega$, then

$$Z_{11'} = -E\omega^2 \tag{15.37}$$

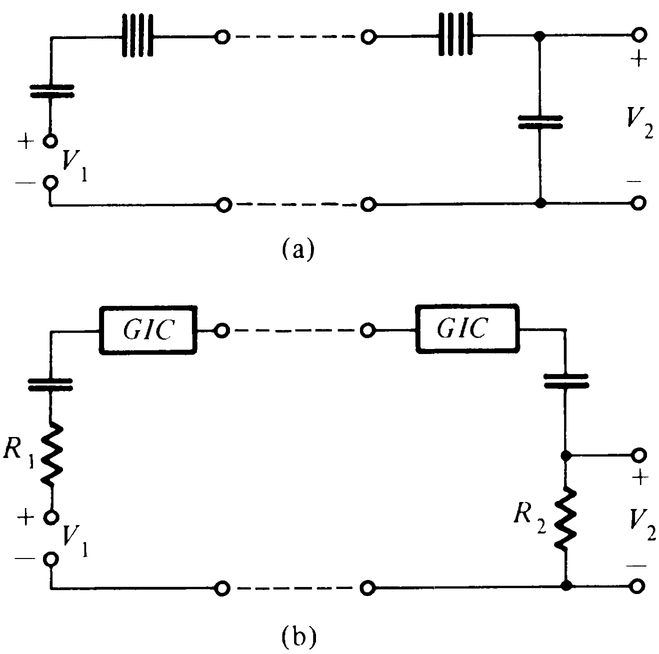
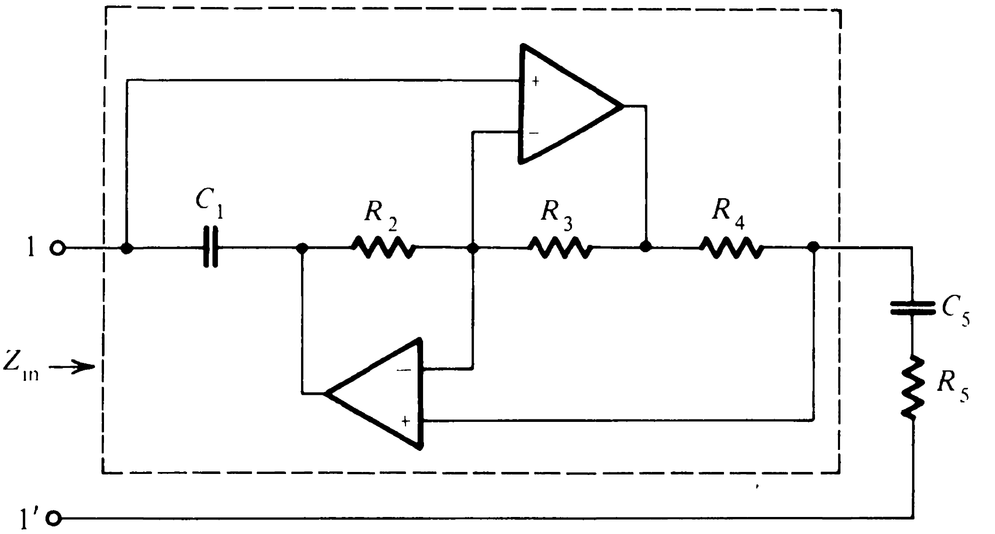


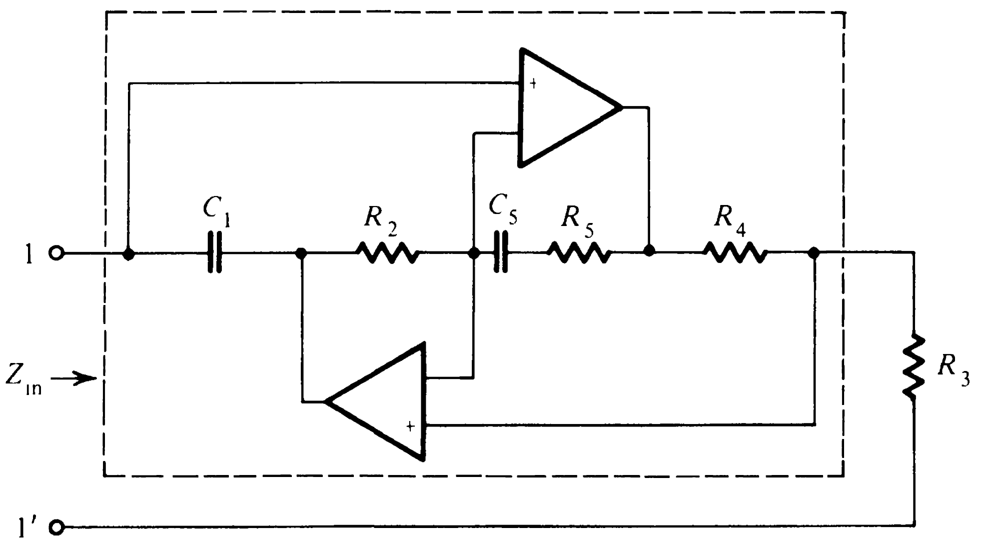
FIGURE 15.36

which shows that this too is a frequency-dependent negative resistor, but in this case the impedance varies directly with frequency squared, rather than inversely with frequency squared. Whether the × in the box element is new or not, it is routinely realized by the circuit of Fig. 15.35a and can be as easily constructed as any of the other elements in this chapter.

From the circuit of Fig. 15.31b extended to the right for higher-order cases, we see that the bandpass filter will consist of alternating A and B sections and, in addition, the termination sections marked T at both ends of the network. These termination sections are represented in Fig. 15.36a with each consisting of an FDNR and a capacitor. These elements came about through applying the Bruton transformation to R_1 and R_2 and capacitors. This is equivalent to the operation of a GIC on these circuit elements as indicated in Fig. 15.36b. One of the appropriate GIC's to perform this operation is that shown in Fig. 15.17a. The input impedance is described by Eq. (15.31). If we let the elements of the GIC have unit



(a)



(b)

FIGURE 15.37

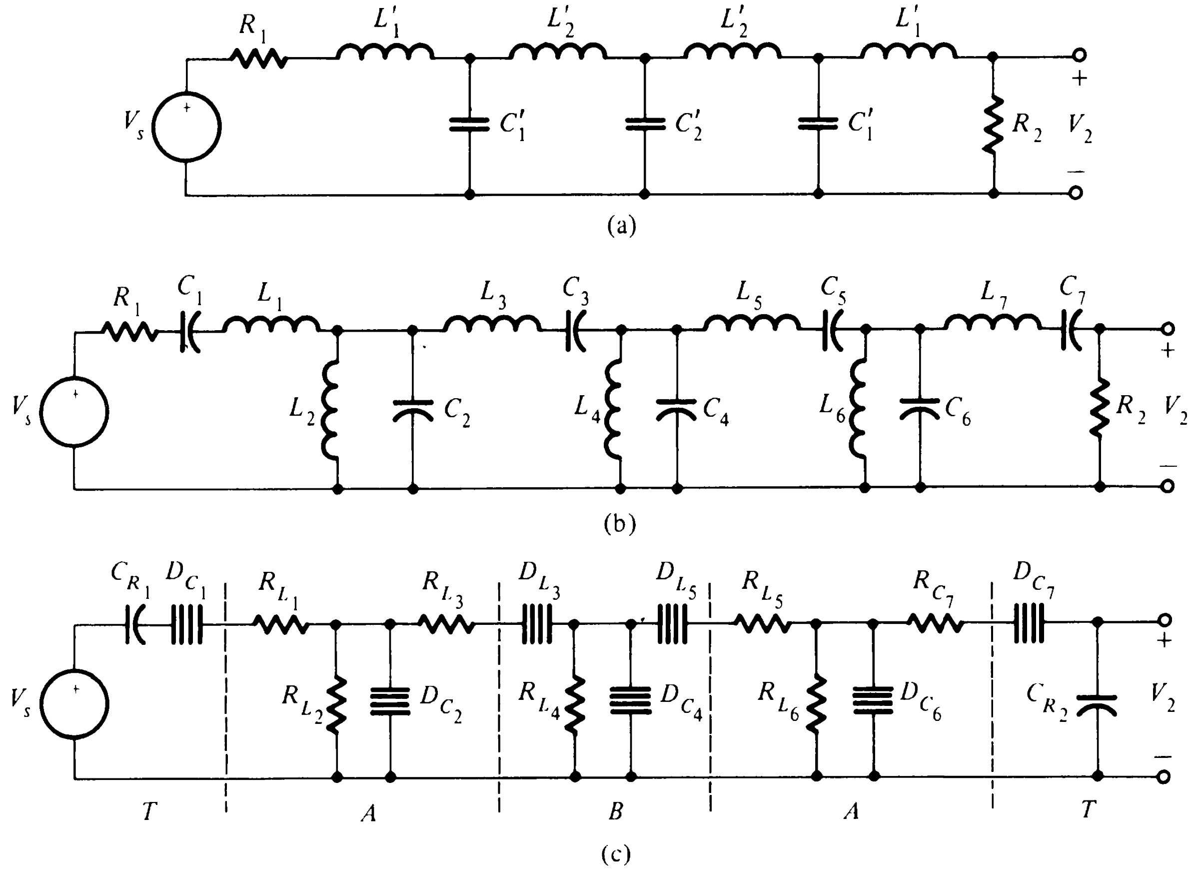


FIGURE 15.38

values, then Z_{in} represents a series FDNR and capacitor since

$$Z_{in} = \frac{Z_1Z_3Z_5}{Z_2Z_4} = \frac{1}{s}\left(\frac{1}{C_5 s} + R_5\right) = \frac{1}{Ds^2} + \frac{1}{Cs} \tag{15.38}$$

where $D = C_5$ and $C = 1/R_5$. It is interesting to note that Z_5 and Z_3 (or Z_1) can be interchanged without changing the input impedance Z_{in}. Thus the two circuits of Fig. 15.37 are equivalent. We make use of this equivalence in the next example of filter design.

Example 15.5* From the specifications given for a bandpass filter, it is found that for a Butterworth response, the prototype must be of seventh order such that the bandpass filter will be of fourteenth order. This high-order requirement suggests that there will be an advantage in the methods of this chapter when compared to using inductors. For this filter, the center frequency is to be 150 Hz, the bandwidth is 150 Hz, and the terminations should be 1000 Ω.

We will first outline the method to be used before becoming involved in actual numbers. Figure 15.38 shows (a) the seventh-order prototype, (b) the fourteenth-order bandpass filter, and (c) the Bruton-transformed filter. In terms of the parameters of the bandpass filter, the filter consisting of two A sections, one B section, and the two T sections is that shown in Fig. 15.39. The section nearest V_s has been transformed using the circuit equivalance of Fig. 15.37.

*As noted earlier, the methods of this section are due to Antoniou. This particular example follows one provided by L. T. Bruton and David Treleaven, "Active Filter Design Using Generalized Impedance Converters," *EDN*, pp. 68–75, 1973.

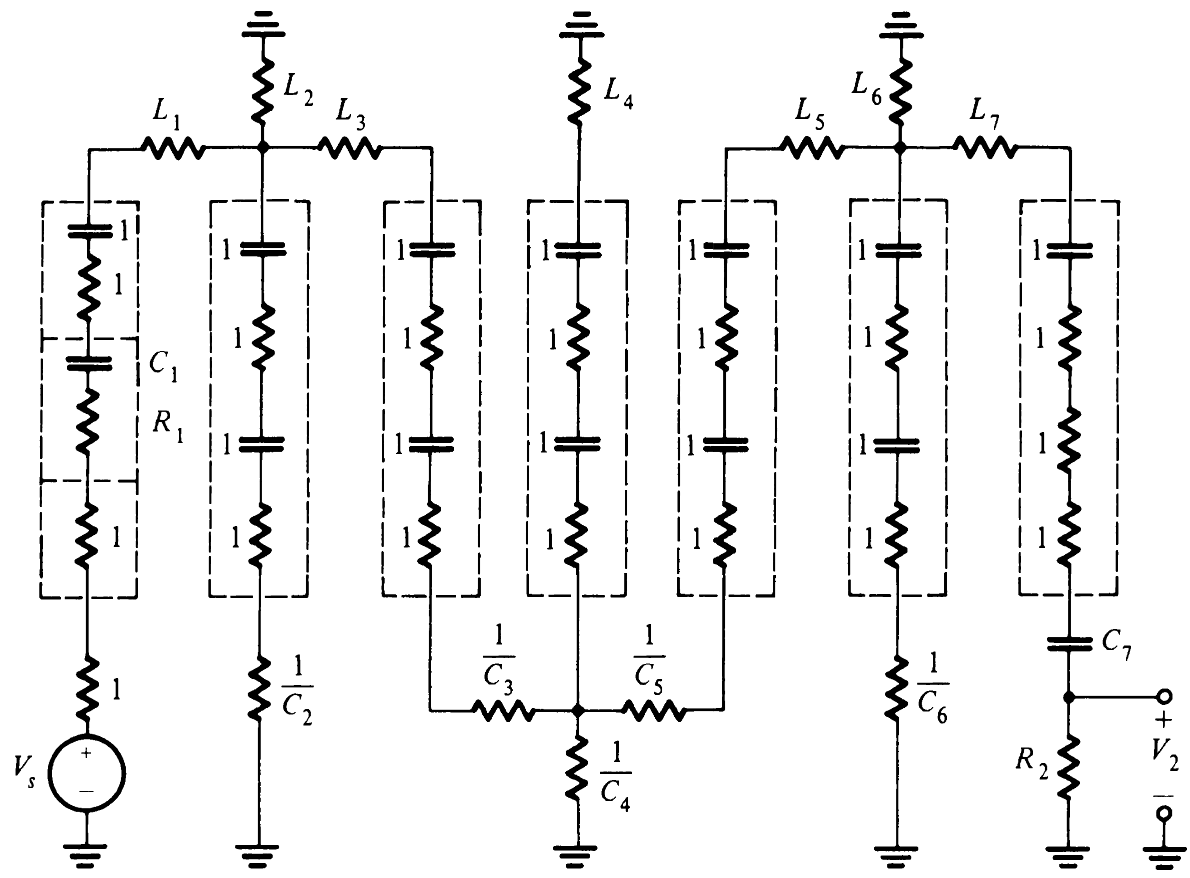


FIGURE 15.39

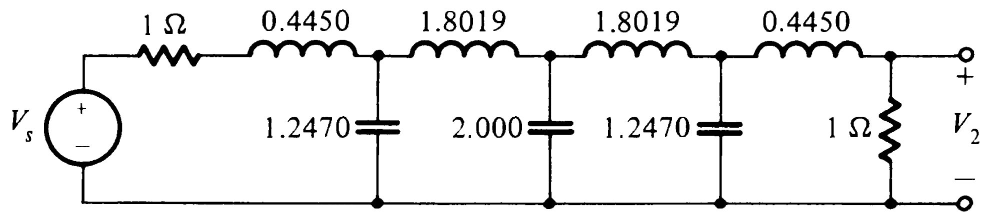
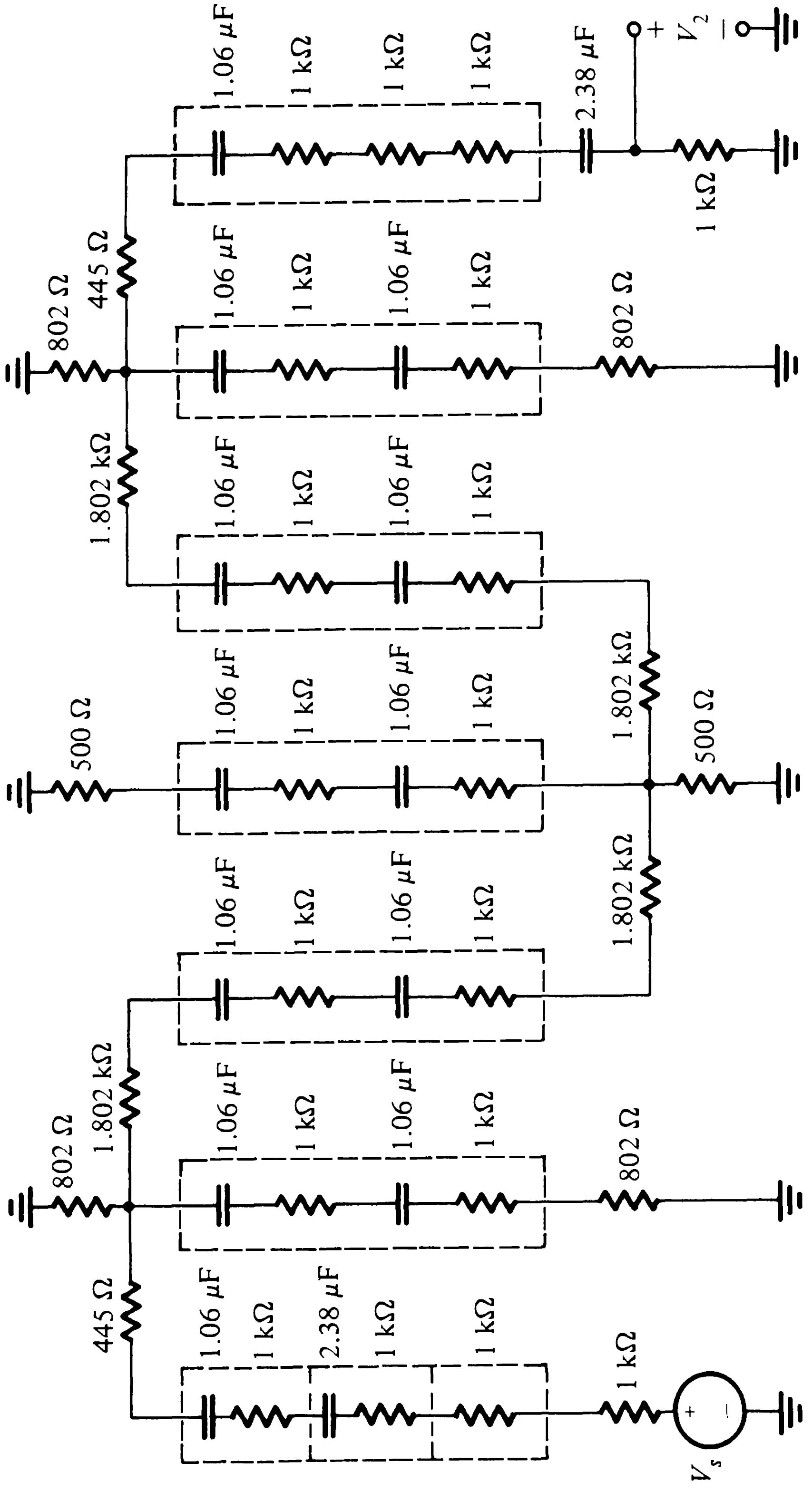


FIGURE 15.40

The prototype Butterworth filter is given in Table 14.2 and is shown in Fig. 15.40 for a bandwidth of 1 rad/s. If we frequency scale 150 Hz to 1 rad/s, then we note that $\omega_0 = 1$ and BW = 1. This particular choice greatly simplifies the frequency transformation equations, so that in transforming from Fig. 15.38a to 15.38b each inductor L_p transforms into a series-connected inductor L_p and capacitor $1/L_p$, while a capacitor transforms into a parallel-connected inductor $1/C_p$ and capacitor C_p. Thus the elements for Fig. 15.38b are routinely expressed in terms of the values from the prototype of Fig. 15.40 as follows:

$$\begin{aligned}
C_1 = C_7 = 1/L_1' &= 2.2472 \\
C_3 = C_5 = 1/L_2' &= 0.5549 \\
L_1 = L_7 = L_1' &= 0.4450 \\
C_2 = C_6 = C_1' &= 1.2470 \\
L_3 = L_5 = L_2' &= 1.8019 \\
C_4 = C_2' &= 2.0000 \\
L_2 = L_6 = 1/C_1' &= 0.8019 \\
L_4 = 1/C_2' &= 0.5000
\end{aligned} \tag{15.39}$$



1.06 μF
1 kΩ
1 kΩ
1 kΩ
2.38 μF
+
V_2
−
1 kΩ
802 Ω
445 Ω
1.06 μF
1 kΩ
1.06 μF
1 kΩ
802 Ω
1.802 kΩ
1.06 μF
1 kΩ
1.06 μF
1 kΩ
1.802 kΩ
500 Ω
1.06 μF
1 kΩ
1.06 μF
1 kΩ
500 Ω
1.802 kΩ
1.06 μF
1 kΩ
1.06 μF
1 kΩ
802 Ω
1.802 kΩ
1.06 μF
1 kΩ
1.06 μF
1 kΩ
802 Ω
445 Ω
1.06 μF
1 kΩ
2.38 μF
1 kΩ
1 kΩ
1 kΩ
V_s

FIGURE 15.41

These values along with $R_1 = R_2 = 1$ complete element values for the circuit of Fig. 15.39.

To complete this example, we scale frequency so that 1 rad/s becomes 150 Hz and 1 Ω becomes 1000 Ω. Thus

$$k_f = 2\pi \times 150 \qquad \text{and} \qquad k_m = 1000 \tag{15.40}$$

giving the final design shown in Fig. 15.41.

PROBLEMS

15.1 For a particular filtering problem, the prototype shown in Fig. P15.1 is chosen. This is a singly terminated lowpass Chebyshev filter having a ripple width of 0.9 dB and a bandwidth of 1 rad/s. For the final design, the filter is to be frequency-scaled by a factor of 1000 and magnitude-scaled to give practical element sizes. Find the filter using the methods of this chapter.

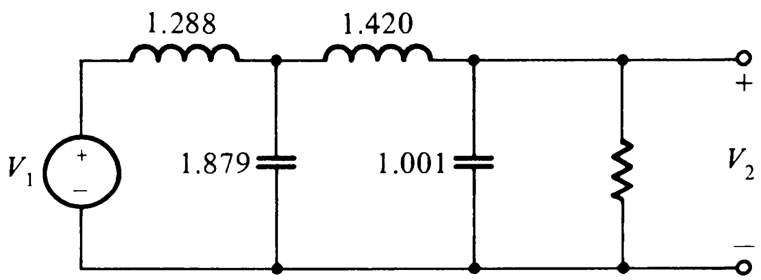


FIGURE P15.1

15.2 A filtering problem is found to require the choice of a fourth-order lowpass Butterworth filter as the prototype. The filter is to have a half-power frequency of 1000 rad/s. Magnitude scale to give practical element sizes. Design the filter making use of the methods of this chapter.

15.3 Using simulated inductors, design a doubly terminated highpass filter based on a fourth-order Butterworth prototype. The half-power frequency of the filter is to be 5 kHz and the terminating resistors are each 100 ohms.

15.4 The specifications for a highpass filter are shown in Fig. P15.4 and it is further specified that the response be Butterworth. Using the methods of this chapter, find a filter realization and magnitude scale to give element values in a practical range.

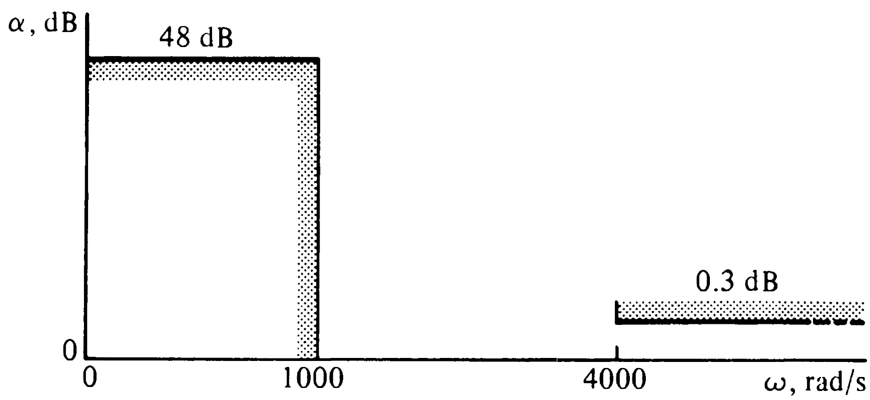


FIGURE P15.4