EE 210 **Spring 2017-18**

HW 2

Assigned: 30/01/18 Due: 6/02/18

You are encouraged to discuss these problems with others, but you need to write up the actual solutions alone. Start early, and come to office hours (TBA) with any doubts. Your only have to submit your solutions to **the questions marked** [†]. *Drop off your submission in the dropbox labeled EE210 in the EE office by 5.30 pm on the due date.*

1. The discrete time signals x and y have finite support. Specifically, $x[n]$ is zero outside $n_1 \leq n \leq n_2$, $y[n]$ is zero outside $n_3 \leq n \leq n_4$.

   Prove that $(x \star y)[n]$ is zero outside $n_1 + n_3 \leq n \leq n_2 + n_4$.

2. [†] Consider the continuous time system defined by the LCCDE

$$\frac{d\,y(t)}{dt} + y(t) = x(t).$$

   (a) Under the initial condition $y(0) = 1$, show that the system is neither linear nor time-invariant.
   (b) Under the initial condition $y(0) = 0$, show that the system is linear but not time-invariant.

3. Consider a continuous time system described by the LCCDE

$$\sum_{k=1}^{N} a_k \frac{d^k\,y(t)}{dt^k} = \sum_{k=1}^{M} b_k \frac{d^k\,x(t)}{dt^k}$$

   and the *initial rest* condition. The input signals x_1 and x_2 satisfy the property that $x_i(t) = 0\ \forall t < t_i$. Let y_1 and y_2 denote the corresponding output signals.

   (a) Show that the output signal corresponding to the input $\alpha x_1(t) + \beta x_2(t)$ is $\alpha y_1(t) + \beta y_2(t)$.
   (b) Show that the output signal corresponding to the input signal $x_1(t - \kappa)$ is $y_1(t - \kappa)$.

   Note: This almost verifies that the system under consideration is LTI. Why almost?

4. [†] Argue that $\delta(at) = \frac{1}{|a|}\delta(t)$.

5. [†] Perform the following convolutions where $\star$ indicates convolution.

   (a) For $u(t)$ a unit step function, find $r(t) = u(t) \star u(t)$.
   (b) Find $x(t) \star h(t)$, where $h(t) = (-e^{-t} + 2e^{-2t})u(t)$ and $x(t) = 10e^{-3t}u(t)$.
   (c) Find the output $y(t)$ of an LTI system with impulse response $h(t) = 2e^{-2t}u(t)$ when excited with an input $x(t)$ given by

$$x(t) = \begin{cases} 1, & 2 \leq t \leq 4 \\ 0, & \text{otherwise} \end{cases}.$$

6. [†] Given that $f(t) \star g(t) = y(t)$, where $\star$ denotes convolution,

   (a) Find $f(t - T_1) \star g(t - T_2)$, for some finite-valued real numbers T_1 and T_2.
   (b) Use the result of (a) and the fact that $u(t) \star u(t) = r(t)$, to find $(u(t+1) - u(t-2)) \star (u(t-3) - u(t-4))$. Verify the result graphically.

7. [†] Given $y(t) = f(t) \star g(t)$, derive a general formula to compute $f(ct) \star g(ct), c \neq 0$. Hence, if $f(t) = u(t+1) - u(t-2)$ and $g(t) = r(t)(u(t) - u(t-1))$, find $f(2t) \star g(2t)$.

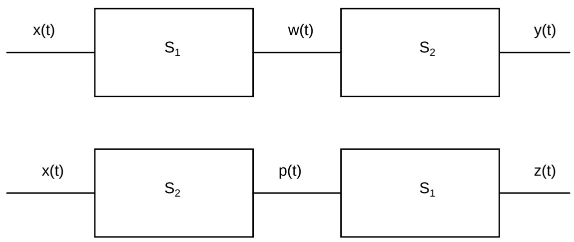
8. Given below are the impulse response of some systems. Determine whether the systems are (a) Stable (b) Causal.

(a) $h(t) = e^{-(t+2)}u(t)$.

(b) $h(t) = e^{-|t|}$.

(c) $h(t) = \delta(t) + \delta(t-3)$.

9. [†] Let $x(t) = e^{-2t}u(t)$. The system S_1 is described by $y(t) = x(2t)$ and the system S_2 has an impulse response $h(t) = e^{-t}u(t)$. Find the output for the following two cascaded connections. Are the outputs expected to be the same in both cases?



10. [†] Determine whether each of the following statements concerning LTI systems is true or false. Justify your answers.

(a) If $h(t)$ is the impulse response of an LTI system, and h(t) is periodic and nonzero, the system is unstable.

(b) The inverse of a causal LTI system is always causal.

(c) If $|h[n]| < K$ for each n, where K is a given number, then the LTI system with $h[n]$ as its impulse response is stable.

(d) If a discrete-time LTI system has a impulse response $h[n]$ of finite duration, the system is stable.

(e) If an LTI system is causal, it is stable.

(f) The cascade of a noncausal LTI system with a causal one is necessarily noncausal.

(g) A continuous-time LTI system is stable if and only if its step response $s(t)$ is absoultely integrable, that is,

$$\int_{-\infty}^{+\infty} |s(t)|dt < \infty$$

(h) A discrete-time LTI system is causal if and only if its step response $s[n]$ is zero for $n < 0$.