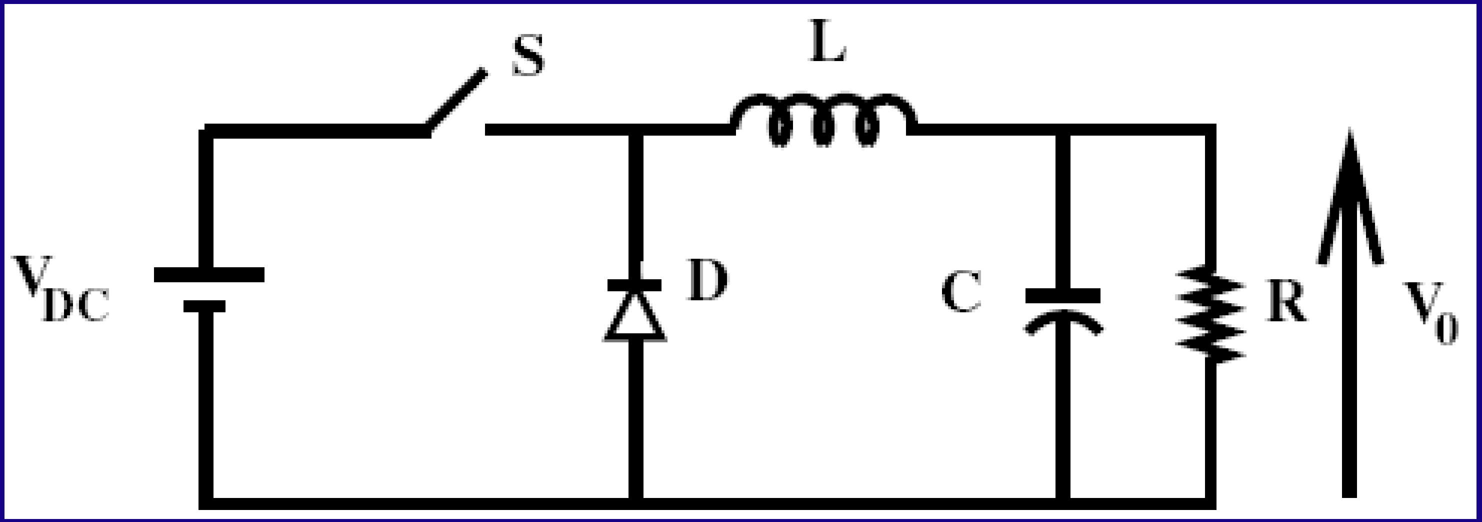
Buck converter :

$L \rightarrow$ Filter inductor.

$C \rightarrow$ Filter capacitor.

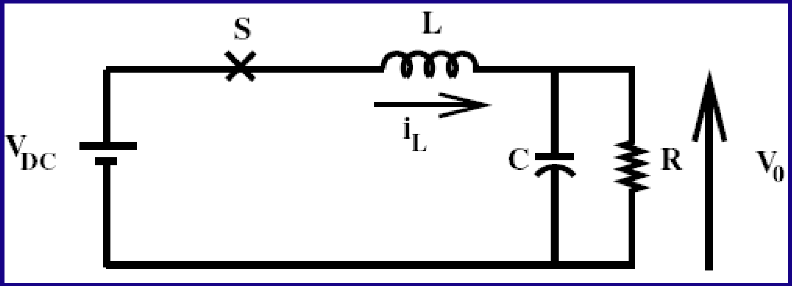
$'V_0'$ is assumed to remain constant.

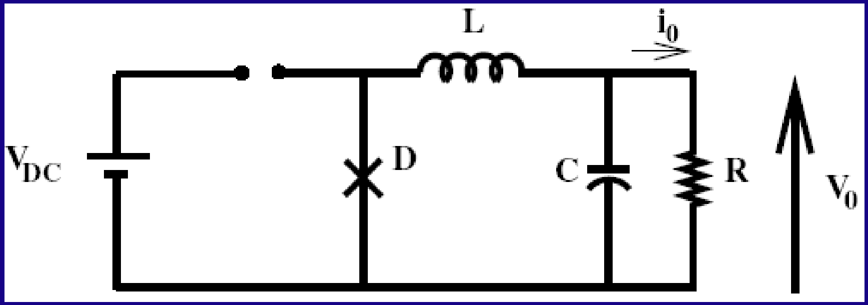


'S' is switched at a very high frequency.

S – ON for DT

– OFF for $(1-D)T$





$$V_L = V_{DC} - V_0 \quad 0 < t < DT$$

$$= \text{Constant}$$

$i_L \uparrow$ Linearly

$$i_L = C\frac{dV_0}{dt} + \frac{V_0}{R}$$

$$V_D = -V_{DC}$$

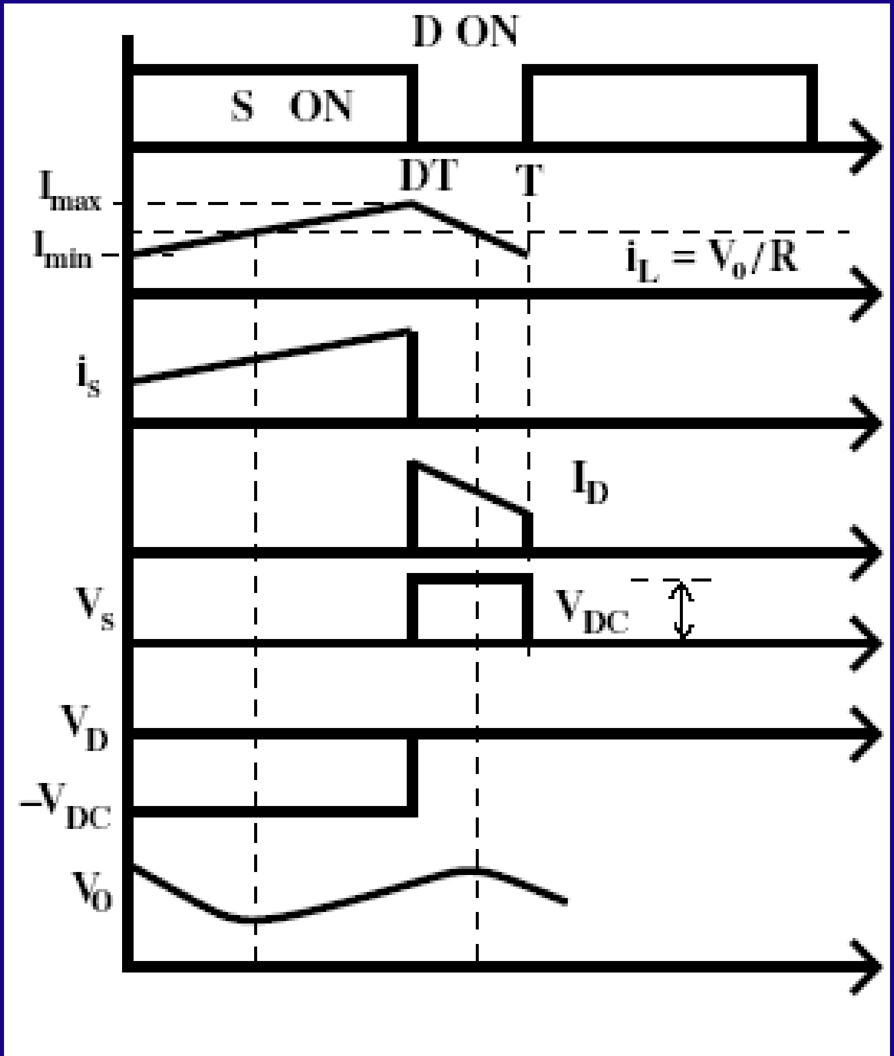
$$V_L = -V_0 \qquad (1-D)T < t < T$$

$i_L \downarrow$ Linearly

$$i_L = C\frac{dV_0}{dt} + \frac{V_0}{R}$$

$$V_S = V_{DC}$$

Assume i_L is continuous.



Neglect losses

Input power = Output power.

$V_{DC} \, I_S = V_0 \, I_0$

$= D \, V_{DC} I_0$

$\therefore \; I_S = D \, I_0$

Avg. source current < Avg. load current.

$\Rightarrow$ Similar to step-down transformer.

Source current waveform jumps from peak to zero.

$\Rightarrow$ peak value of $i_S > I_S$

$\Rightarrow$ L-C filter at the input side.

Expression for current ripple in i_L :

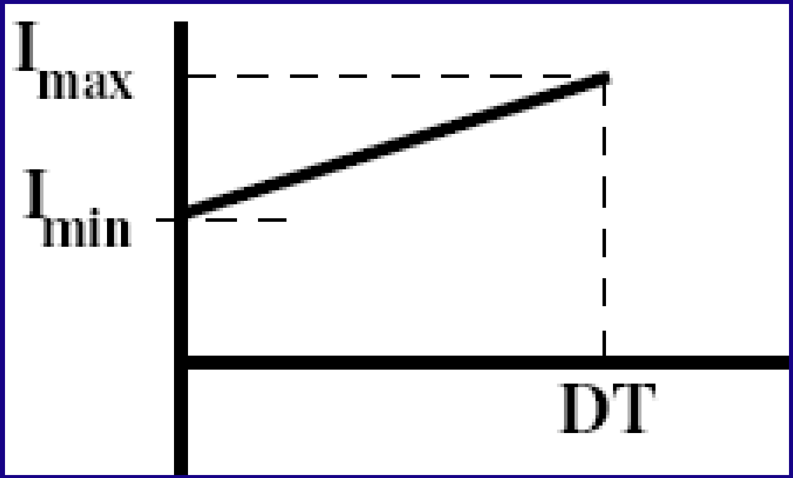
Assume that V_0 is constant (neglect the voltage ripple)

$$\frac{di_L}{dt} = \frac{V_{DC} - V_0}{L}$$

$$= \frac{V_{DC} - DV_{DC}}{L}$$

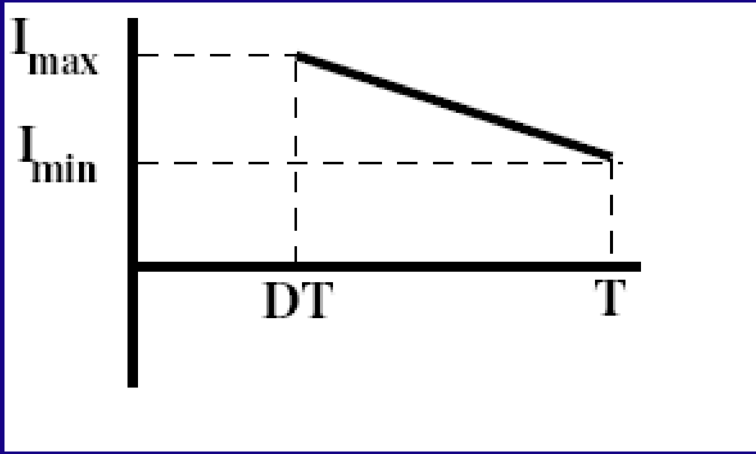
$$= I_{min} + \frac{V_{DC}}{L}(1-D)t$$

$$I_{max} = I_{min} + \frac{V_{DC}}{L}(1-D)DT$$



For $DT < t < T$

$$\frac{di_L}{dt} = \frac{-V_0}{L} = \frac{-DV_{DC}}{L}$$



$$\therefore i_L = I_{max} - \frac{DV_{DC}}{L}\left(t - DT\right)$$

$$i_L = I_{min} \text{ at } t = T$$

$$\therefore \Delta i_L = I_{max} - I_{min} = \frac{V_{DC}}{L}\left(1 - D\right)DT$$

$$\Delta i_L\Big|_{max} \text{ when } D = 0.5 = \underline{\underline{\frac{V_{DC}T}{4L}}}$$

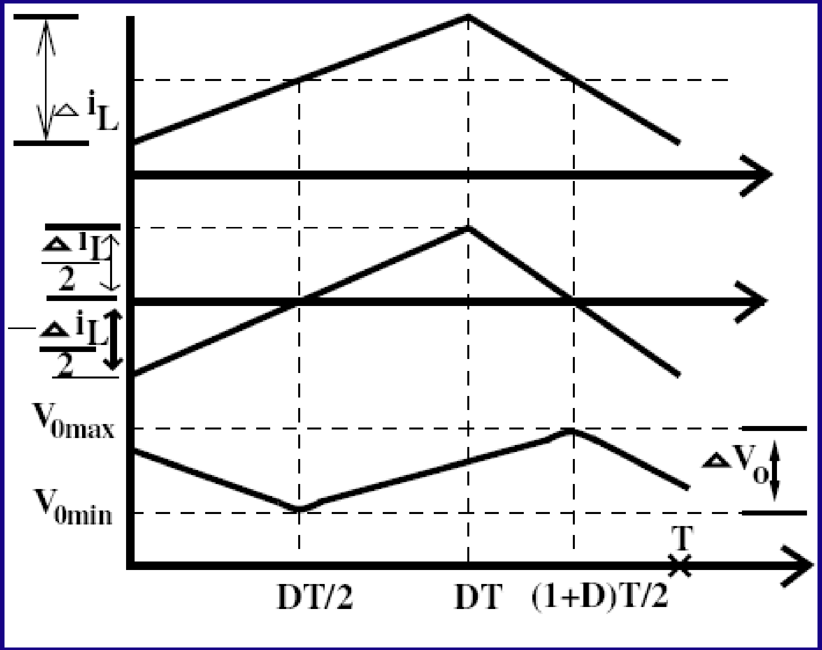
Capacitor voltage ripple :

for $0 < t < DT$

$$i_c = \frac{-\Delta i_L}{2} + \Delta i_L \frac{t}{DT}$$

for $DT < t < T$

$$i_c = \frac{-\Delta i_L}{2} - \Delta i_L \frac{t - DT}{T - DT}$$



$$\Delta V_0 = V_{0\max} - V_{0\min} = \frac{1}{C} \int_{DT/2}^{(1+D)T/2} i_c dt$$

$$= \frac{1}{C} \Delta i_L \frac{T}{8}$$

$$\therefore \Delta V_0 = \frac{V_{DC}}{8LC}(1-D)DT^2$$

for constant L, C & T, $\frac{\Delta V_0}{V_{DC}}$ is maximum

for $D = \underline{\underline{0.5}}$

Discontinuous Conduction:

Inductor current 'i_L' and NOT I_0.

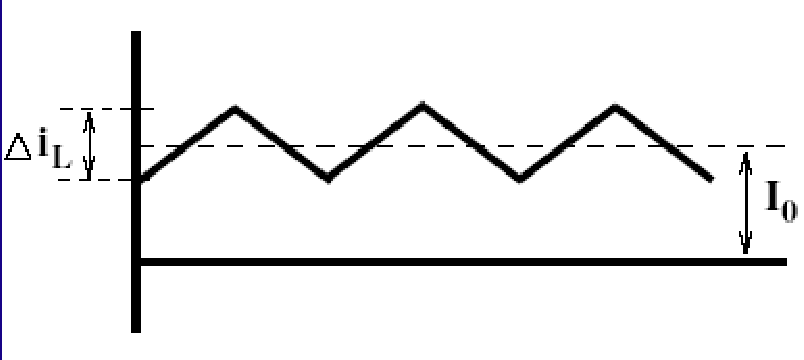
$$I_0 = \frac{V_0}{R}.$$

Now i_L is continuous if

$$\frac{V_0}{R} \geq \frac{\Delta i_L}{2}$$

$$\frac{D\,V_{DC}}{R} \geq \frac{V_{DC}}{2L}(1-D)\,DT$$

$$\therefore\ R \leq \frac{2L}{(1-D)T} = R_{CR}$$



If load $R > R_{CR}$,

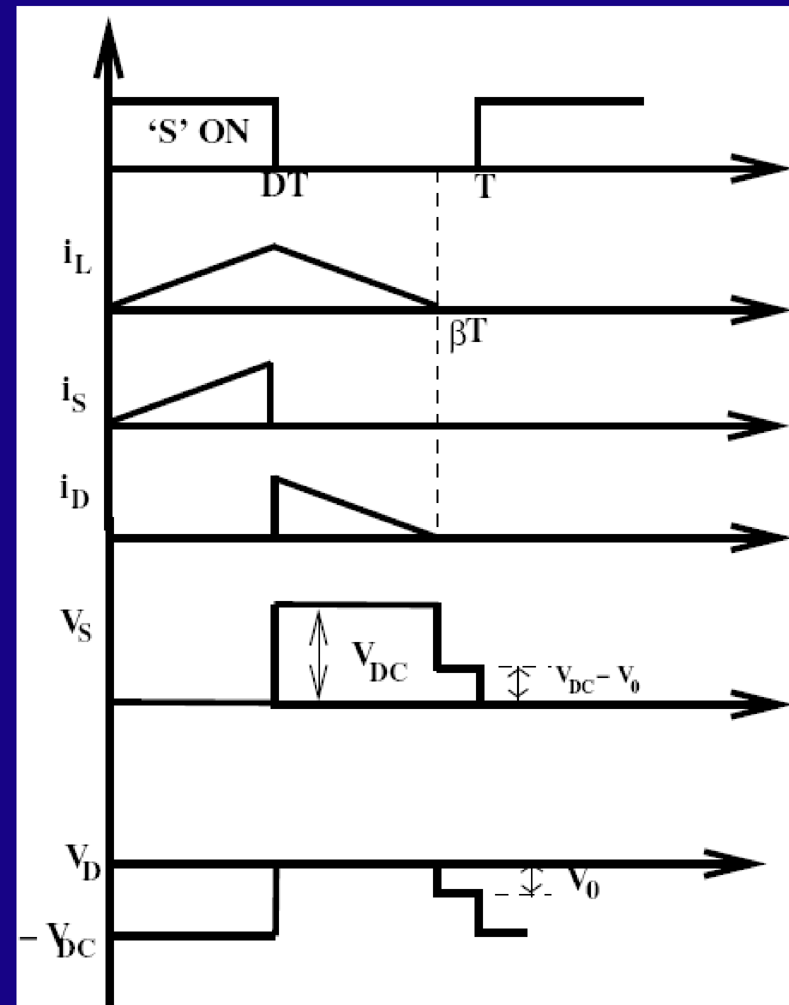
i_L is DISCONTINUOUS.

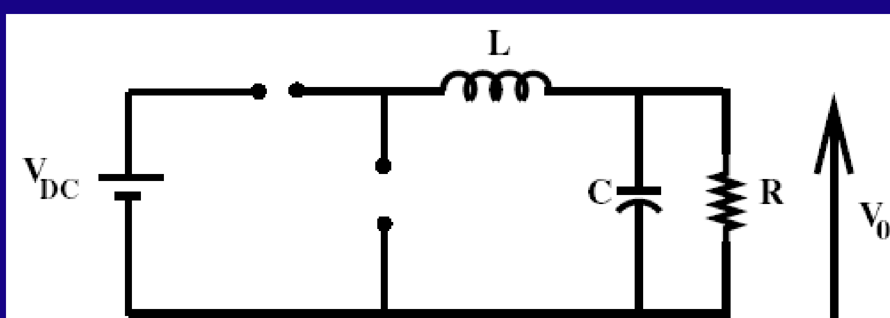
$\Rightarrow$ $i_L = 0$ for finite time.

i_S starts from zero.

If i_L is continuous, $V_0 = D\,V_{DC}$

$\Rightarrow$ Independent of I_0





Circuit Equation:

For $0 < t < DT$

$$\frac{di_L}{dt} = \frac{V_{DC} - V_0}{L} \qquad \therefore\ i_L = \frac{V_{DC} - V_0}{L}\, t.$$

For $DT < t < \beta T$

$$\frac{di_L}{dt} = \frac{-V_0}{L} \qquad \therefore\ i_L = \frac{V_{DC} - V_0}{L} DT - \frac{V_0}{L}(t - DT).$$

initially at t=Dt il =

For $\beta T < t < T$ $\qquad i_L = 0.$

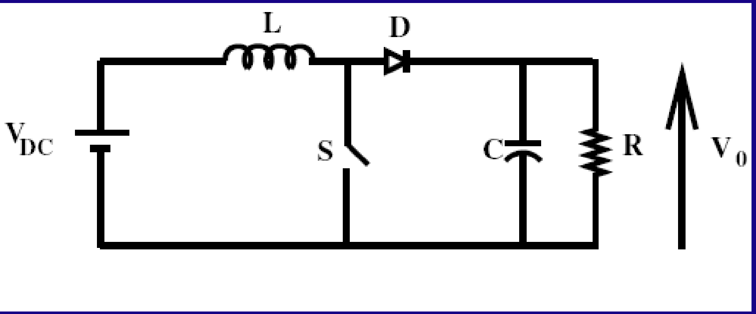
at $t = \beta T$, $\qquad i_L = 0.$

$$\frac{V_{DC} - V_0}{L} DT = \frac{V_0}{L}(\beta T - DT)$$

$$\therefore\ V_0 = \frac{D\, V_{DC}}{\beta}, \qquad \beta < 1$$

$\Rightarrow$ $\boxed{V_0 \text{ is higher than } D\, V_{DC} \text{ IF } i_L \text{ is discontinuous.}}$

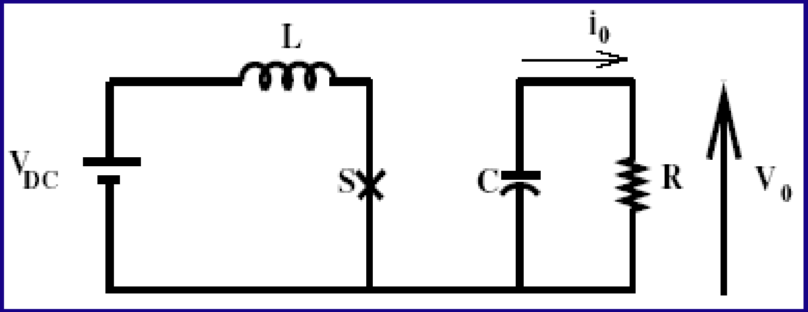
Boost Converter



All components are ideal.

V_0 & V_{DC} are constant and ripple free.

Close S : for DT

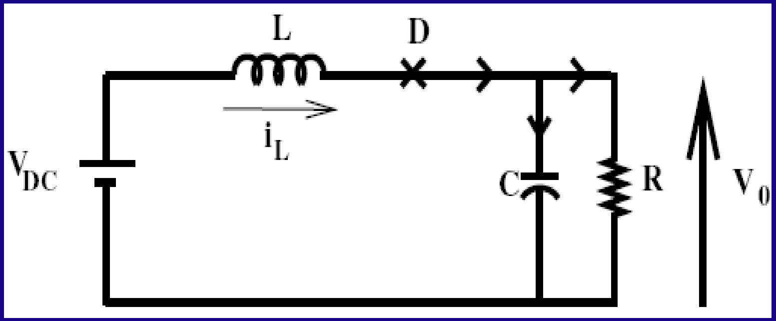


$V_L = V_{DC}$

$\therefore$ $i_L \uparrow$ linearly.

$$i_0 = -C\frac{dV_0}{dt} = \frac{v_0}{R}$$

Open S



$v_L = V_{DC} - V_0$

$$C\frac{dV_C}{dt} + \frac{V_0}{R} = i_L$$

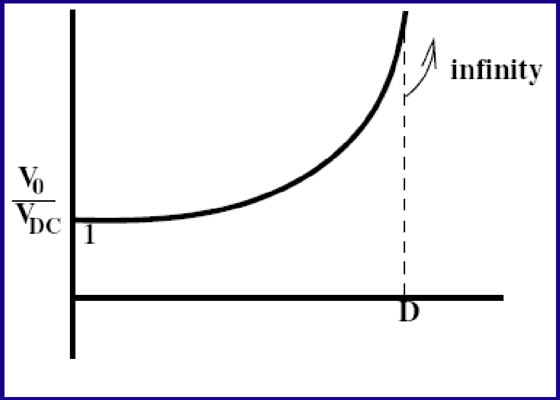
Capacitor supplies power to the load.

$V_S = 0$

$V_D = -V_0$

avg. voltage across 'L' = 0.

$$V_0 = \frac{V_{DC}}{(1-D)}$$



System is loss-less.

$$V_{DC}\ I_S = V_0\ I_0$$

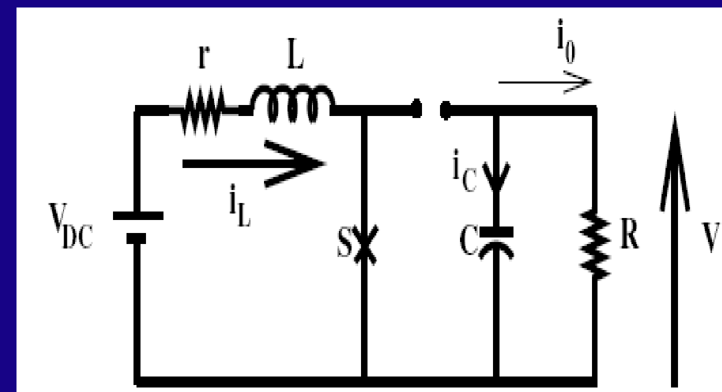
$$\therefore\ I_S = \frac{V_0}{V_{DC}} * I_0 = \frac{I_0}{(1-D)}$$

Effect of 'r':

$$V_{DC} = r\,i_L + L\frac{di_L}{dt}$$

$$C\frac{dv_0}{dt} + \frac{v_0}{R} = 0$$

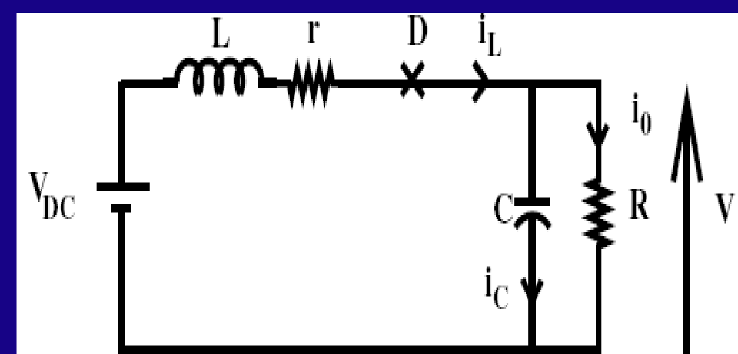
$$0 < t < DT$$



$$V_{DC} = r\,i_L + L\frac{di_L}{dt} + v_0$$

$$C\frac{dv_0}{dt} + \frac{v_0}{R} = i_L$$

$$DT < t < T$$



Let the avg. values of v_0 & i_L be V_0 & I_L respectively.

$$V_{DC} = rI_L + L\left(\frac{di_L}{dt}\right)_{av} + \frac{1}{T}\int_{DT}^{T} v_0 \, dt.$$

$$C\left(\frac{dv_0}{dt}\right)_{av} + \frac{V_0}{R} = \frac{1}{T}\int_{\alpha T}^{T} i_L \, dt$$

Average values of $\frac{dv_0}{dt}$ & $\frac{di_L}{dt}$ are zero at steady state.

Also, variation of v_0 and i_L is assumed to be linear.

$\Rightarrow$ avg. values of v_0 & i_L during (DT, T) are equal to avg. values of them varying on the whole cycle.

$$V_{DC} = rI_L + (1-D)V_0 \rightarrow (1)$$

$$\frac{V_0}{R} = (1-D)\,I_L \rightarrow (2)$$

multiply (1) by $(1-D)$

$$V_{DC}\,(1-D) = rI_L\,(1-D) + (1-D)^2\,V_0$$

using (2)

$$V_{DC}\,(1-D) = r\,\frac{V_0}{R} + (1-D)^2\,V_0$$

$$\therefore\ V_0 = \frac{V_{DC}\,(1-D)}{\frac{r}{R} + (1-D)^2} \rightarrow (3)$$

$$I_L = \frac{V_0}{R\,(1-D)} = \frac{I_0}{(1-D)} = \frac{V_{DC}}{r + (1-D)^2 R}$$

$$\text{If } r \to 0,\ V_0 = \frac{V_{DC}}{(1-D)}$$

$$\therefore \quad I_L = \frac{V_0}{R\,(1-D)}$$

$$= \frac{I_0}{(1-D)}$$

$$= \frac{V_{DC}}{(1-D)^2 R}$$

Power Electronics

$$D = 0,\ V_0 = \frac{V_{DC}}{r + R} * R$$

$\Rightarrow$ As $D \uparrow$, $V_0 \uparrow$.

From (3), $V_0 = 0$! when $D = 1$

($V_0 \to \infty$ at $D = 1$ when $r = 0$)

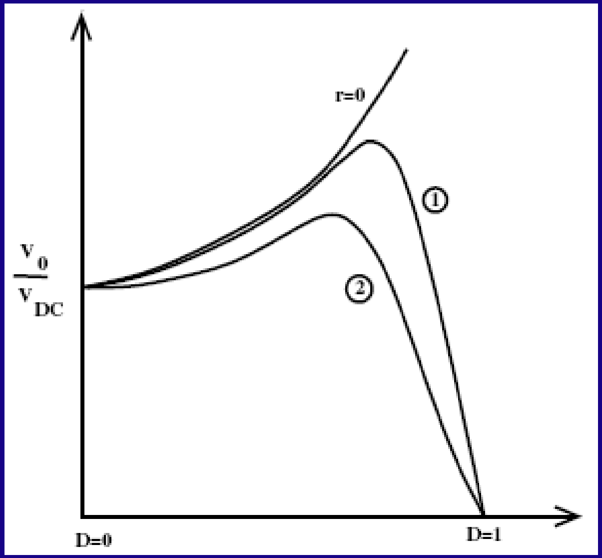
Let $D = D_{max}$ at $V_0 = V_{max}$

$$\frac{dV_0}{dD} = 0. \quad D = 1 - \sqrt{\frac{r}{R}}$$

$$\therefore\ V_{0\,(max)} = \frac{V_{DC}}{2}\sqrt{\frac{R}{r}}$$

$\Rightarrow$ Depends on $\frac{R}{r}$ ratio.

$\frac{R}{r}$ for curve 1 > $\frac{R}{r}$ for curve 2



avg. value of $I_L = \frac{V_{DC}}{R + r}$ for D = 0

avg. value of I_L $\uparrow$ with D

$= \frac{V_{DC}}{r}$ when D = 1