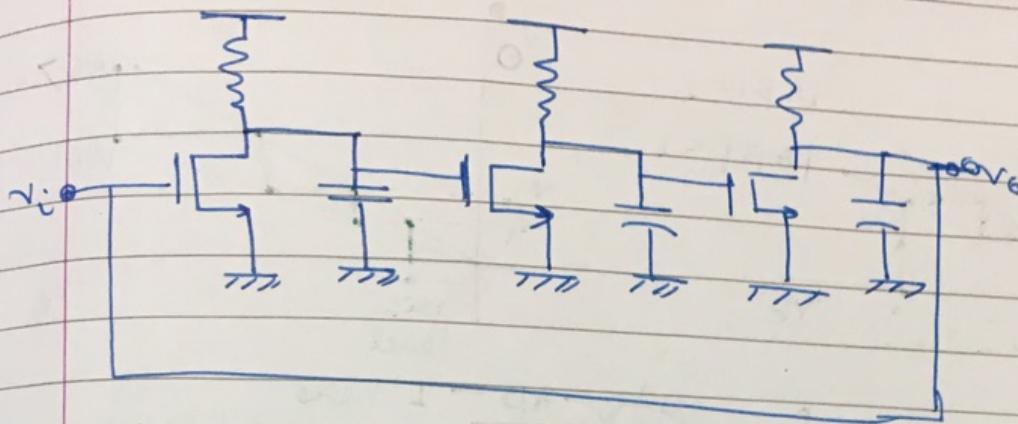


→ 3 Stage



$$A\beta = GH = \frac{v_o}{v_i} = \frac{-A^3}{(1+sRC)^3} \quad \frac{v_o}{v_i} = \pi - 3 \tan^{-1} \omega_{RC}$$

→ If we take, $\tan^{-1} \omega_{RC} = \frac{\pi}{3} \rightarrow$ Ring Oscillator

$$\tan^{-1} \omega_{RC} = \frac{\pi}{3} \Rightarrow \omega_{RC} = \sqrt{3}$$

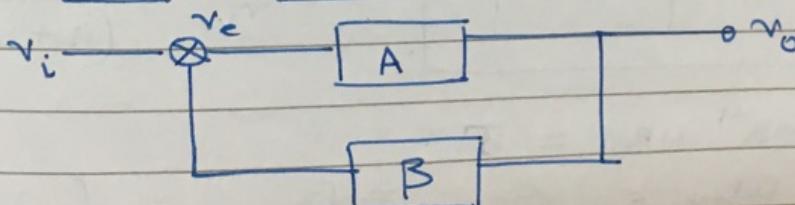
$\left. \begin{array}{l} \text{target} \\ \text{frequency} \end{array} \right\} \Rightarrow \text{pick: } RC = \frac{\sqrt{3}}{\omega_0}$

$$GH = - \frac{A^3}{\left(1 + j\frac{\omega}{\omega_0}\sqrt{3}\right)^3}$$

$$\text{At } \omega = \omega_0, \left| \frac{v_o}{v_i} \right| = \frac{A^3}{8} = 1, \Rightarrow A = 2$$

12.3.18

Positive Feedback - Oscillations



$$\frac{v_o}{v_i} = \frac{A}{1 - A\beta}$$

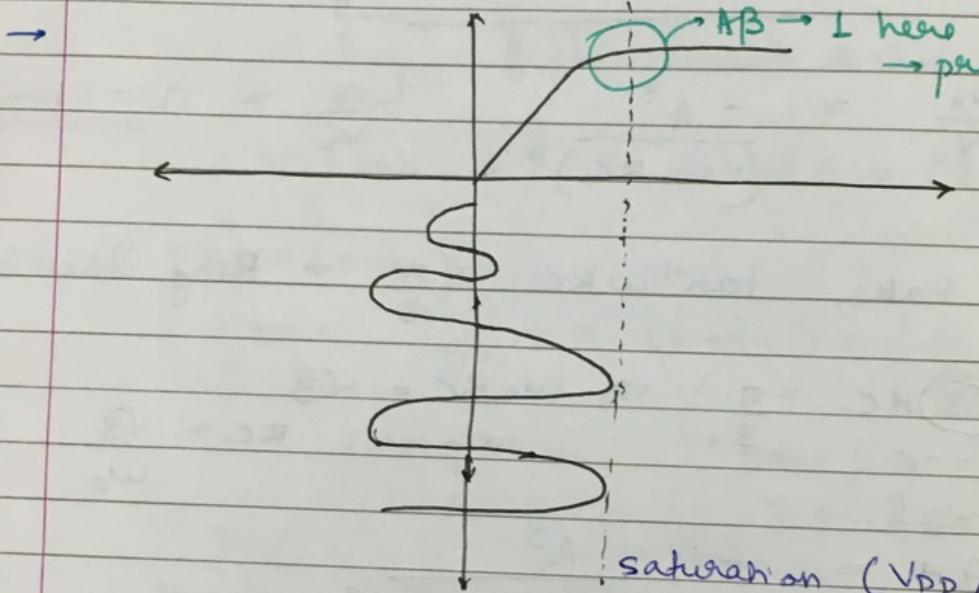
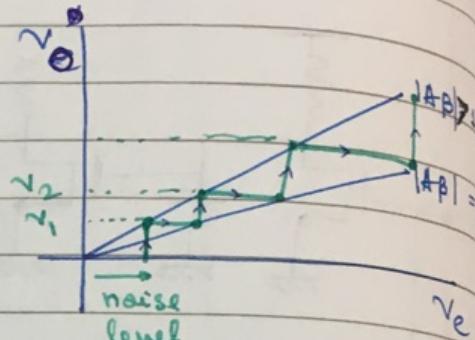
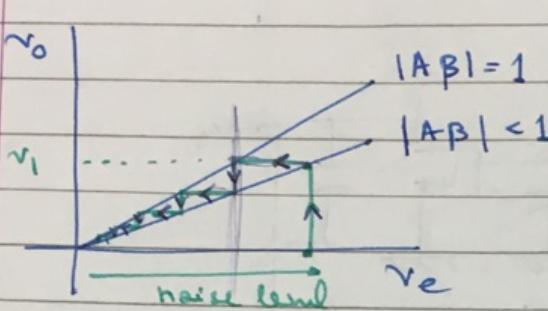
→ Criteria for oscillation,

$$|A\beta| = 1$$

$$\angle A\beta = 2n\pi$$

$v_o \neq v_e$ (Next v_e)

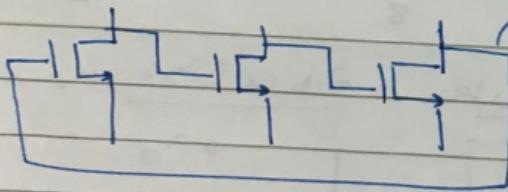
$A_B = \text{Loop Gain, } G_0 \text{ around the loop.}$



saturation (V_{DD} / V_{SS})

Example from previous lecture.

Digital
No Mixer



$$\frac{v_o}{v_i} = \frac{- (g_m R_d)^3}{(1 + sRC)^3}$$

$$3 \tan^{-1} \omega RC = \pi$$

$$\Rightarrow \omega_{osc} = \frac{\sqrt{3}}{RC} = \sqrt{3}\omega_0$$

$$\left(\frac{1}{RC} = \omega_0 \right) \hookrightarrow \text{given}$$

$$\frac{v_o}{v_i} = \frac{- (g_m R_d)^3}{(1 + j\omega/\omega_0)^3}$$

Pole II

$$\text{At } \frac{v_o}{v_i} (\omega = \omega_{osc}) = - \frac{(g_m R_D)^3}{(1 + j\sqrt{3})^3}$$

$$\left| \frac{v_o}{v_i} \right| = \frac{(g_m R_D)^3}{(\sqrt{1+3})^3} = \frac{(g_m R_D)^3}{8}$$

Hence, required cond. $\rightarrow g_m R_D = 2$

$$A_f = \frac{A}{1 - AB}$$

$$A = \frac{v_o (o_L)}{v_i} = \frac{-A_0^3}{(1 + SRC)^3}$$

$$= \frac{-A_0^3 / (1 + SRC)^3}{1 + \frac{A_0^3}{(1 + SRC)^3}}$$

this term is leading to subtraction in D↑

$$= \frac{-A_0^3}{\left(1 + \frac{s}{\omega_0}\right)^3 + A_0^3}$$

$$D_r = \left(1 + \frac{s}{\omega_0}\right)^3 + A_0^3 = \left(1 + \frac{s}{\omega_0} + A_0\right) \left(\left(1 + \frac{s}{\omega_0}\right)^2 + A_0^2 - \left(1 + \frac{s}{\omega_0}\right) A_0\right)$$

Pole I, $1 + \frac{s}{\omega_0} + A_0 = 0 \Rightarrow s = -\omega_0(A_0 + 1)$

Pole II/III $\left(1 + \frac{s}{\omega_0}\right) = \frac{A_0 \pm \sqrt{A_0^2 - 4A_0^2}}{2} = \frac{A_0}{2}(1 \pm j\sqrt{3})$

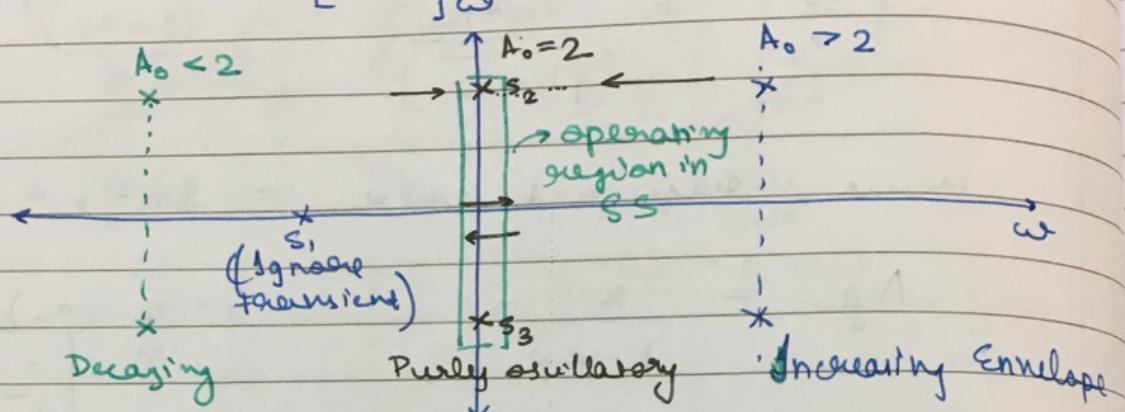
$$s_{2,3} = \left[\frac{A_0}{2} (1 \pm j\sqrt{3}) - 1 \right] \omega_0$$

$$A_f = \frac{A}{1 - AB} = \frac{k_1}{(s - s_1)} + \frac{k_2}{(s - s_2)} + \frac{k_3}{(s - s_3)}$$

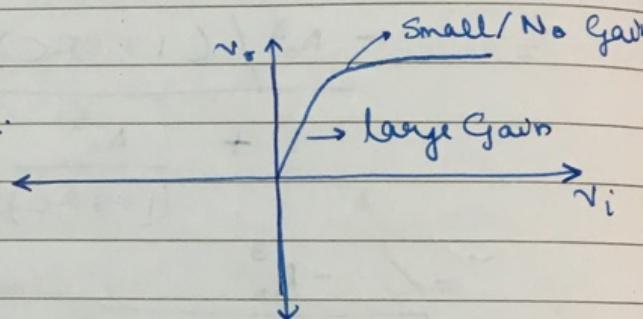
$$k_1 e^{s_1 t} + k_2 e^{s_2 t} + k_3 e^{s_3 t}$$

$$S_1 = -\omega_0 (1 + A_0) \rightarrow \text{surely Dies out}$$

$$S_{2/3} = \left[\frac{A_0}{2} (1 \pm j\sqrt{3}) - 1 \right] \omega_0$$



→ Large Signal Char.



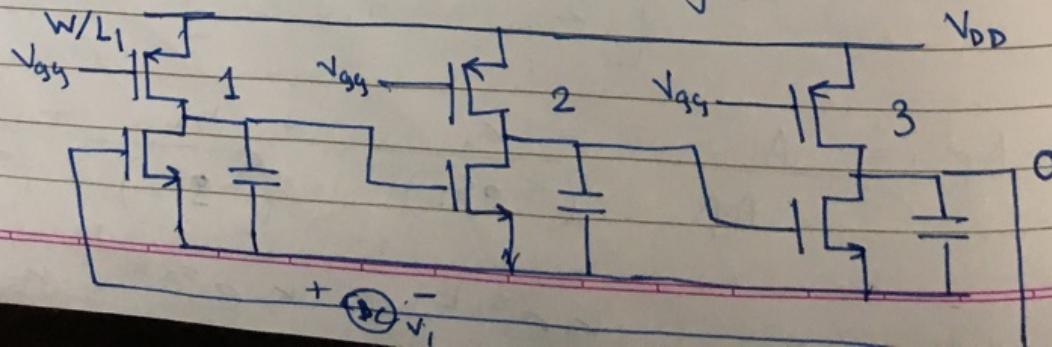
- At start if $A_0 > 2$, it grows and $A_0 \downarrow$
 - We overshoot $A_0 = 2$ & go into decaying region.
 - The decaying region will push it back since A_0 will \uparrow with decay.

→ ⚡ At start if $|AB| > 1$, then the system adjusts itself in a way that $|AB| \rightarrow 1$

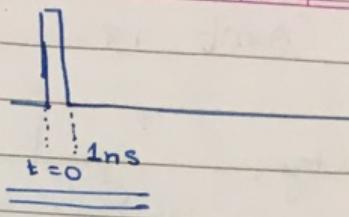
→ (Simulation Results)

Position feedback: Ring oscillator

13.3.18



Given an input:



$$\omega_{osc} = \frac{\sqrt{3}}{RC}, \quad |gmR_d| = 2$$

$$C = 20 \text{nF}$$

$$\sqrt{3} [t=0] = 5.5 \text{V}$$

Razavi on poles in Non linear EKT

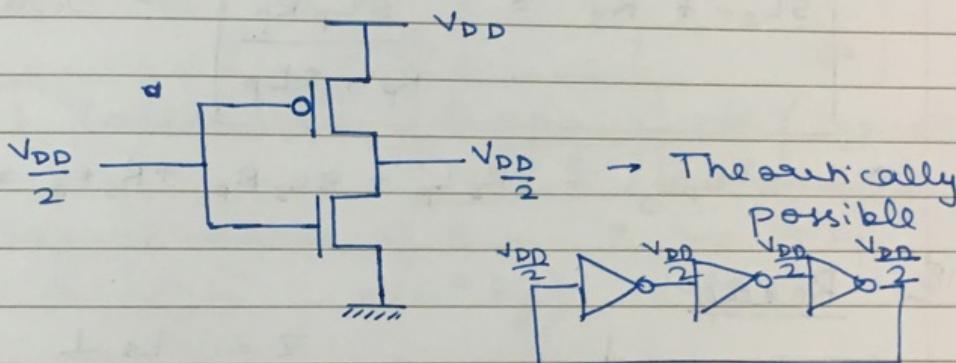
0V

→ The poles begin in the RHP & eventually move to imaginary axis to stop growth.

↓ very
& see
transient

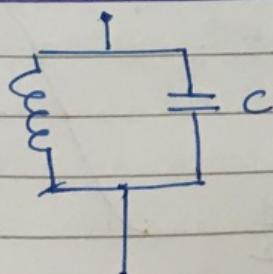
→ The Invert Loop Gain is still ≈ 1

→ Meta stable conditions



→ But practically cannot happen : like pencil on its tip.
some noise like gust of wind

→ Imp: Why doesn't Ring oscillator (Inverter based) have ~ output? (Ans: Discussion with Anandha)



→ LC oscillator / Tank oscillator

(Stores energy)

$$\frac{X_L S L}{s^2 + s^2 L C}$$

$$Z = \frac{s L S L}{s^2 L C + L^2}$$

$$\frac{S L}{s^2 + s^2 L C}$$

$$= \frac{1}{s^2 L C + \frac{1}{L^2}}$$

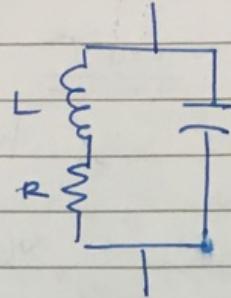
$$= \frac{S L}{1 + s^2 L C}$$

Some Approximations are valid for R_C Network

$$C \frac{1}{s} \approx \frac{1}{sR_s}$$

Conversion where Q of serving comb. is $\frac{R_p}{R_p + C_{eq}}$

→ In reality,



→ Convert:

$$\frac{1}{sL_s} + R_s$$

$$L_p \frac{1}{sC} + R_p$$

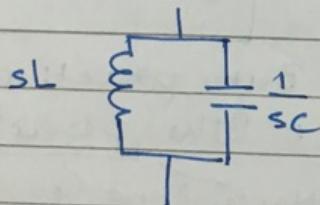
for convenience

$$SL_s + R_s = \frac{SL_p R_p}{R_p + SL_p} \rightarrow \text{Series} \leftarrow \text{Parallel conversion}$$

$$\Rightarrow S^2 L_p L_s + S R_s L_p + S L_s R_p + R_s R_p = S L_p R_p$$

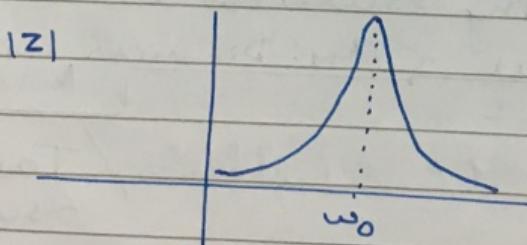
15.3.18

Recap



$$Z = \frac{L_s \frac{1}{sC}}{L_s + \frac{1}{sC}} = \frac{L_s}{s^2 L C + 1}$$

$$= \frac{j\omega L}{(1 - \frac{\omega^2}{\omega_0^2})} \quad \omega_0 = \frac{1}{\sqrt{LC}}$$

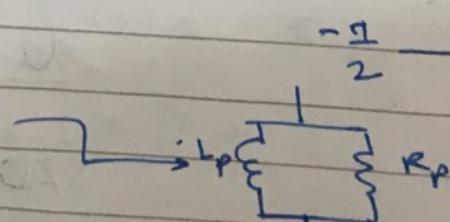


$$L_Z$$

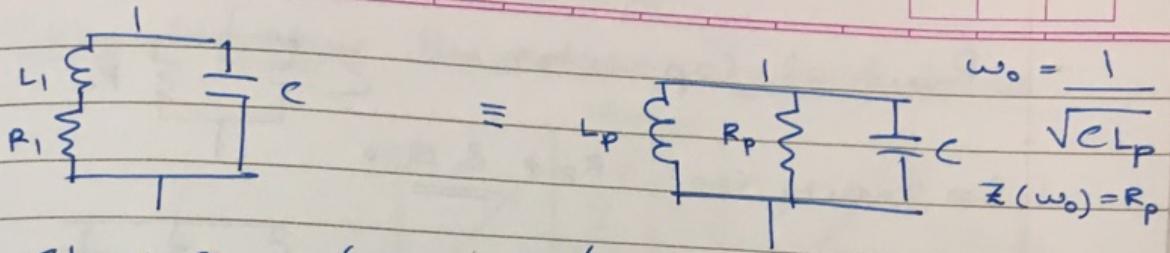
$$\omega_0 = \frac{1}{\sqrt{LC}}$$

→ Convert:

$$\frac{1}{sL_s} + R_s$$



Why?: Calc./Analysis easier.



$$sL_1 + R_1 = (sL_p R_p) / (R_p + sL_p)$$

$$\Rightarrow s^2 L_1 L_p + s R_1 L_p + s L_1 R_p + R_1 R_p = s L_p R_p$$

$$s = j\omega$$

$$\Rightarrow -\omega^2 L_1 L_p + R_1 R_p = 0 \quad \left\{ \begin{array}{l} \text{Same.} \\ R_1 L_p + L_1 R_p = L_p R_p \end{array} \right.$$

$$L_p = L_1 \left[1 + \frac{R_1^2}{\omega^2 L_1^2} \right] = L_1 \left[1 + \frac{1}{Q^2} \right]$$

→ Quality factor (Q) $Q = \frac{\omega L_1}{R_1}$ ~~approx~~ (typically $\approx \frac{3}{4}$ near resonance.)

For typical values of $R_1, L_1 \Rightarrow L_p \approx L_1$

$$R_p \approx \frac{\omega^2 L_1^2}{R_1} = (Q^2 R_1)$$

Solving,

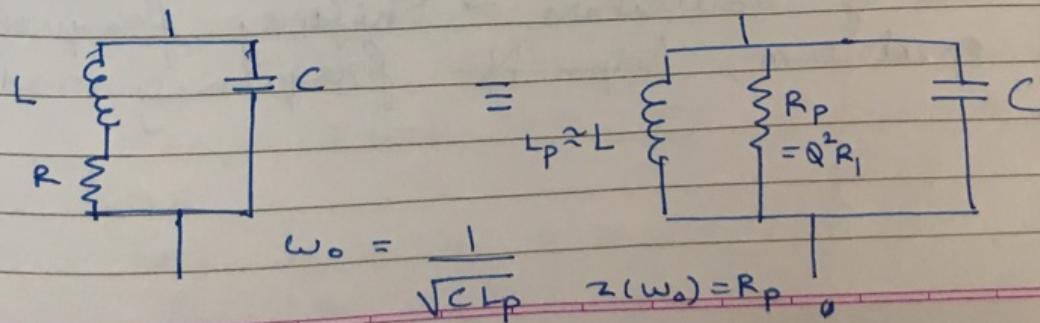
$$R_1 = \frac{(L_p R_p - L_1 R_p)}{L_p}$$

$$-\omega^2 L_1 L_p + \frac{R_p}{L_p} (L_p R_p - L_1 R_p) = 0$$

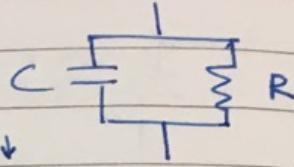
$$R_p = \frac{\omega^2 L_1 L_p^2}{R_p}$$

Quality of Inductor, Different from Quality factor

→ Approximate Qgnt Ckt.



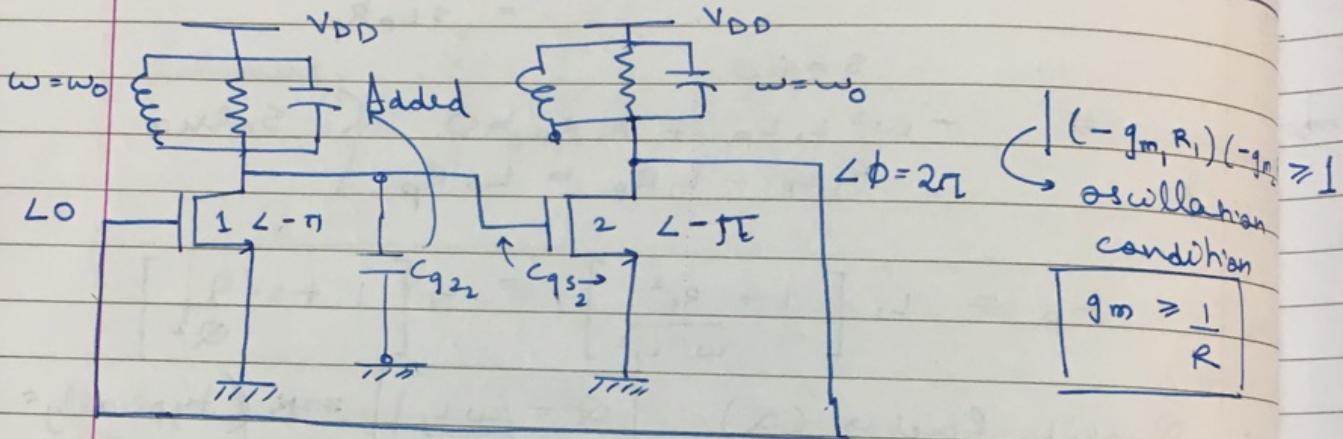
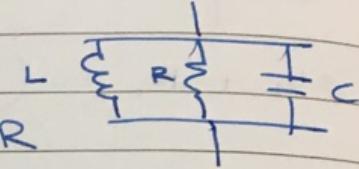
→ Practical Capacitor



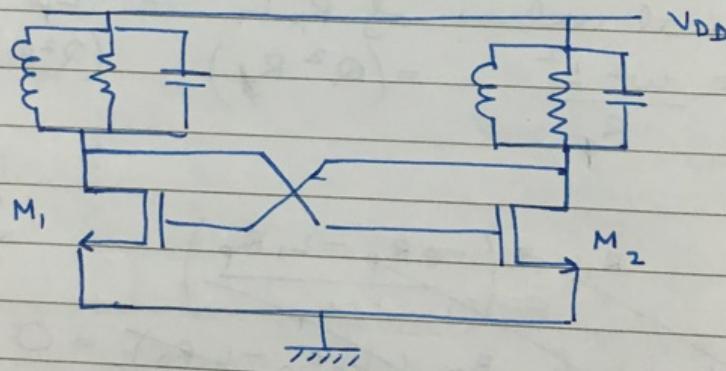
Best Quality Cap: $R_p \downarrow$ & $Q \downarrow$

→ General RCL Network:

$$\omega_0 = \frac{1}{\sqrt{LC}} \quad Z(\omega = \omega_0) = R$$



• Bistable Network



18.3.18

- Advantages
- Can Achieve oscillation in 2 stages (Just 2x)
- High o/p swing possible \Rightarrow High power applicatn
- Use of Oscillators \rightarrow Shifting frequency spectrum, Amp. & frequency modulation

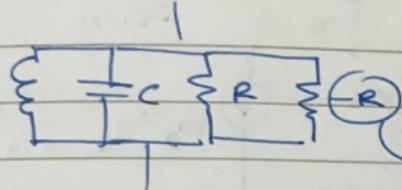
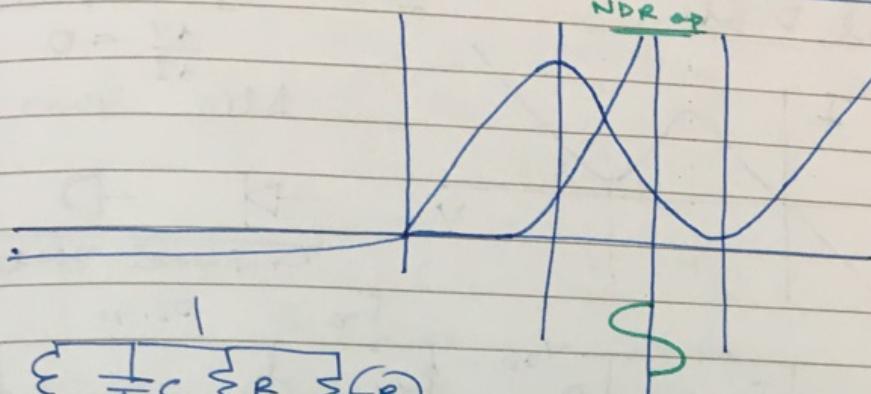
→ Using Negative Resistance - Tunnel Diode

-ve AC/Differential Resistors
(NDR)

NDR op

Resistors

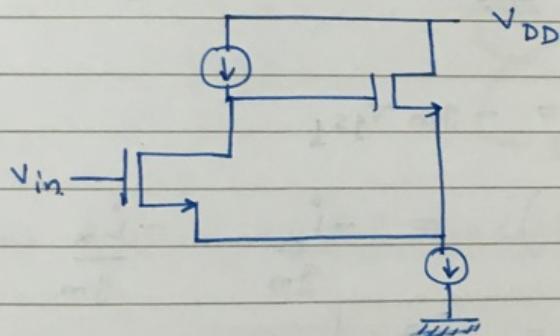
Tunnel Diode



V_o (Bias point)
from NDR

↳ Ideal Tank/
L-C

→ Another configuration



18.3.18

→ Negative F.B Examples (Self Notes)

h parameters (Series - Shunt F-B)

$$V_1 = h_{11} I_1 + h_{12} V_2$$

$$I_2 = h_{21} I_1 + h_{22} V_2$$

g para.

$$I_1 = g_{11} V_1 + g_{12} I_2$$

$$V_2 = g_{21} V_1$$

$$+ g_{22} I_2$$

z parameters.

$$V_1 = z_{11} I_1 + z_{12} I_2$$

$$V_2 = z_{21} I_1 + z_{22} I_2$$

y para.

$$I_1 = y_{11} V_1 + y_{12} V_2$$

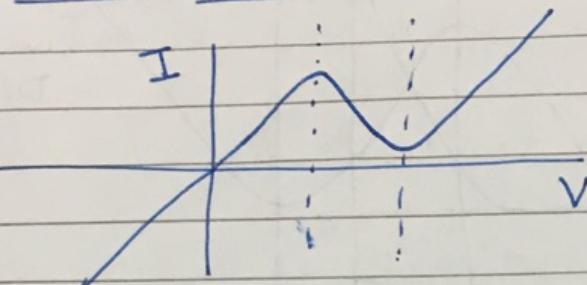
$$I_2 = y_{21} V_1 + y_{22} V_2$$

19.3.18

Negative Differential Resistors

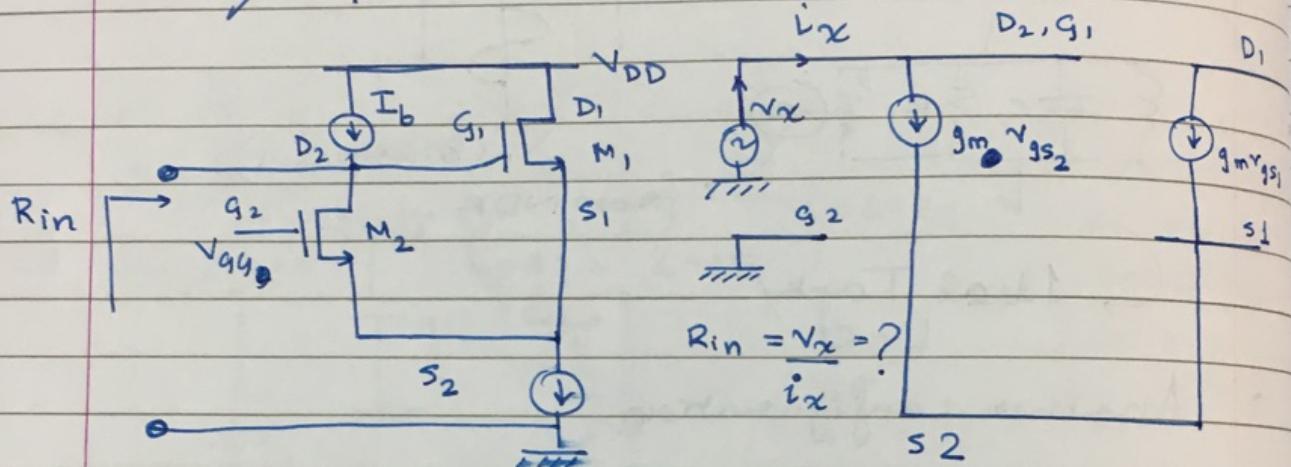
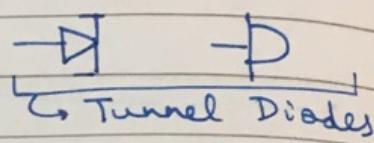
Page No.:

Tunnel Diode



$V > 0$, can't generate power.
 $\frac{dV}{dI} < 0$

Also, Gunn Diode



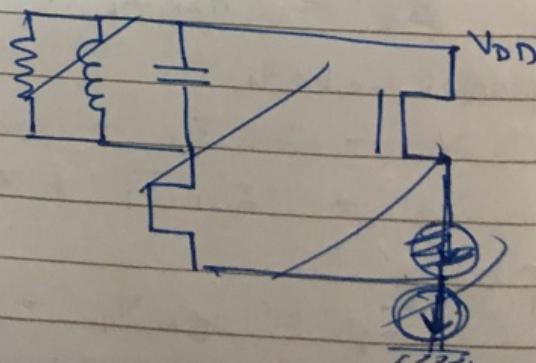
$$\dot{i}_x = g_m v_{gs_2} = -g_m v_{gs_1}$$

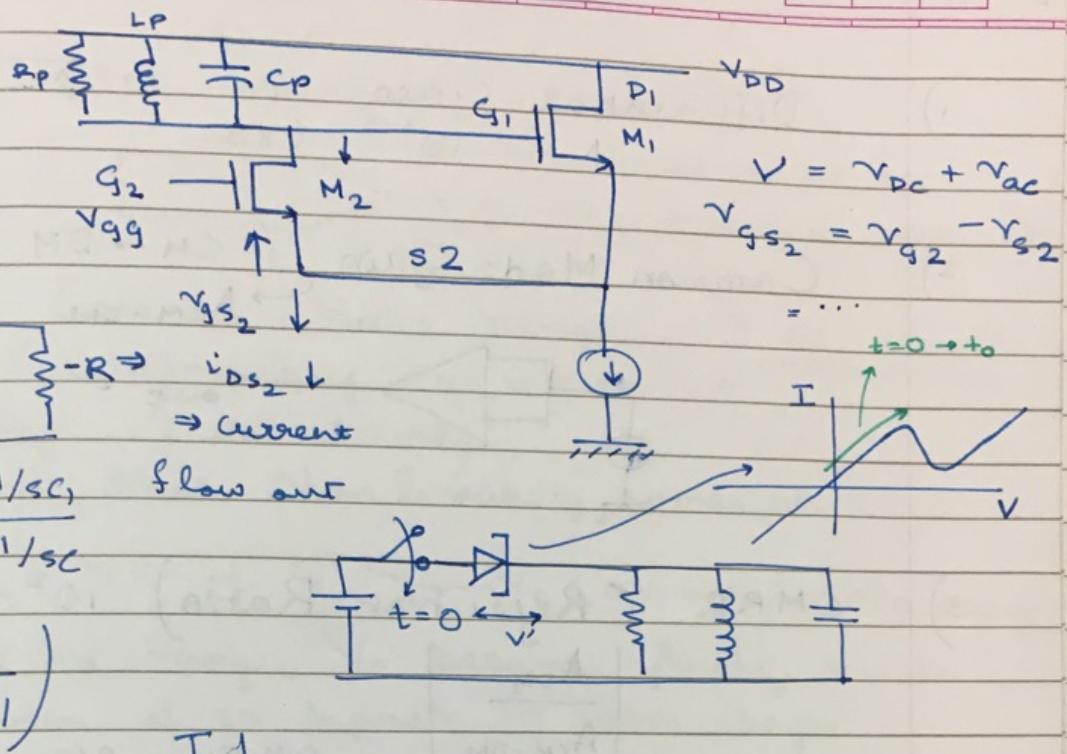
$$v_x = (v_{g_1} - v_{s_1}) = -\frac{\dot{i}_x}{g_m} - \frac{\dot{i}_x}{g_m}$$

$$\Rightarrow R_{in} = -\frac{2}{g_m}$$

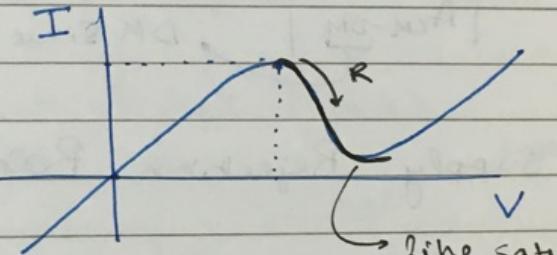
- Qualitatively: $v_x \uparrow \Rightarrow v_{g_2 s_2} \downarrow \Rightarrow I_{DS_2} \downarrow$
 Hence the excess current of I_b must go out of v_x node.

- Used in an oscillator



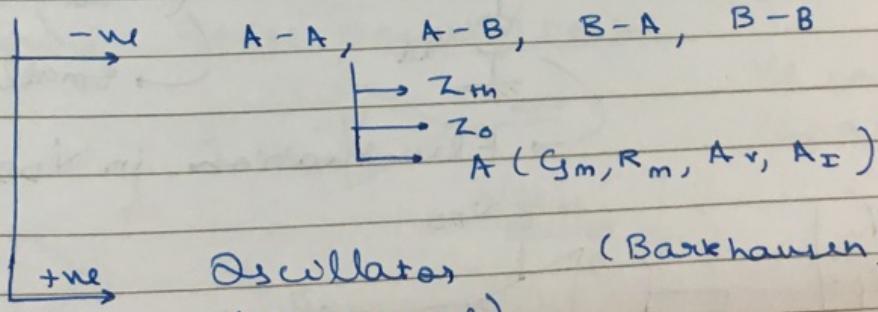


$$= \left(\frac{L_S}{S^2 L_C + 1} \right)$$

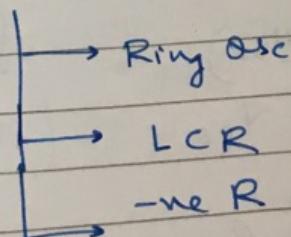


* Building Block of RF Oscillator, differential Amplifier.

Feedback



Oscillator (sinusoidal) (Barkhausen)

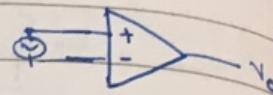


20.3.18

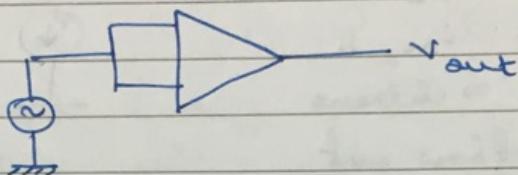
Opamp Characteristics

Page No.:

- 1) Differential Gain
 $A \approx 10^{4-5}$ (80)



- 2) Common Mode Gain ($CM \rightarrow DM$ conversion)
 $\rightarrow A_{CM-DM}$



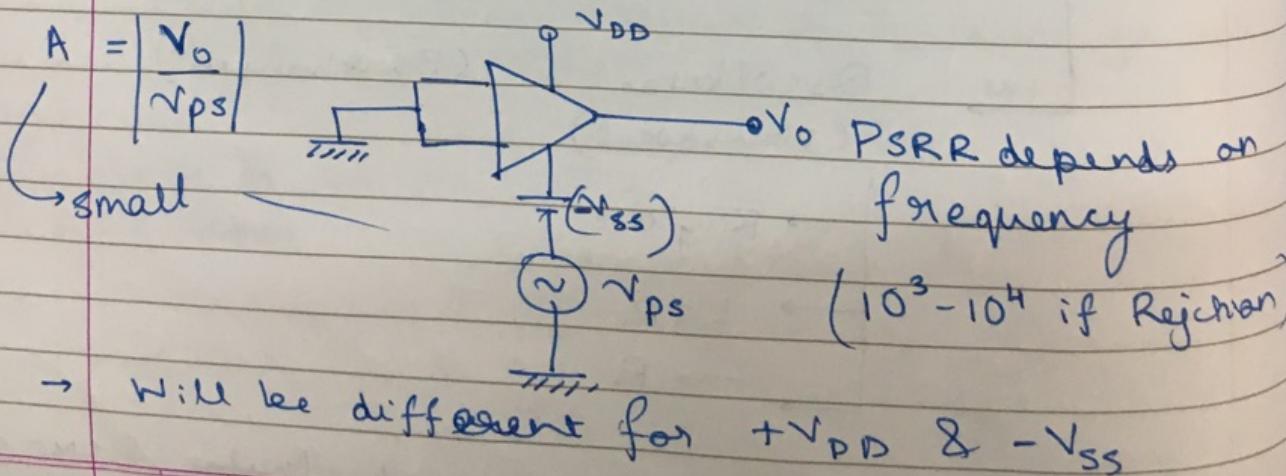
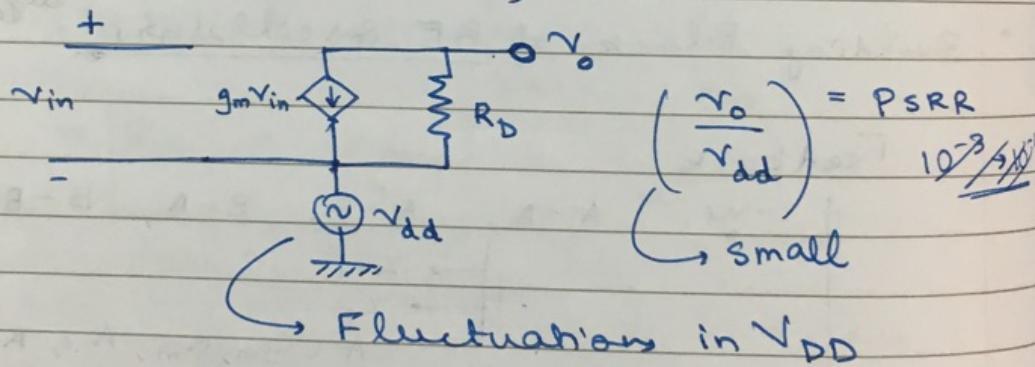
- 3) CMRR (Rejection Ratio) $10^2 - 10^4$

$$\left| \frac{A_{DM}}{A_{CM-DM}} \right|$$

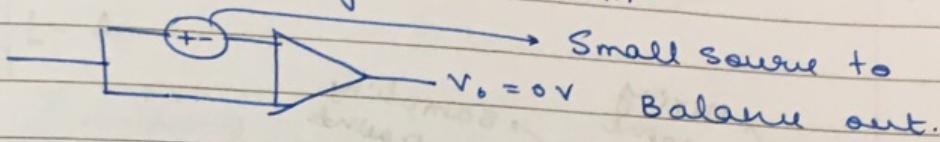
DM since o/p is b/w V_{out} & Ground

- 4) Power supply Rejection Ratio

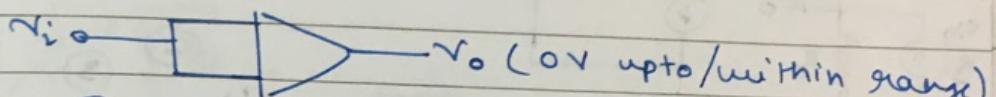
Non ideal DC sources



5) I/P offset voltage : 1-10mV

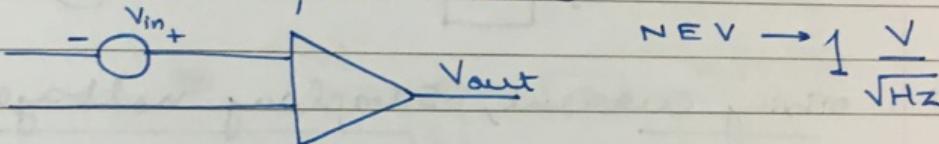


6) I/P Common-Mode range
 (Subtracting i/p offset voltage 1 at a time) $\sim 10 - 20 \mu\text{V}$
 → Beyond this Non linearity comes in.



Beyond the Range, V_o becomes finite, due to violation of ss Approx. in later stages.

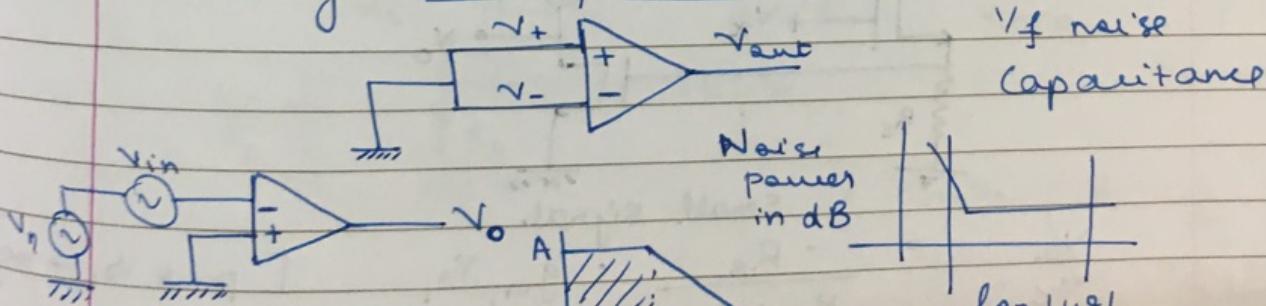
7) Equivalent Input Noise



→ If $\text{BW} = 1\text{kHz}$ 1MHz (f) \Rightarrow Noise power $= (A \times \text{NEV})^2 \times f$

typical $\text{NEV} \rightarrow 1\text{nV} / \sqrt{\text{Hz}}$ Band width

→ Measuring Noise power / noise



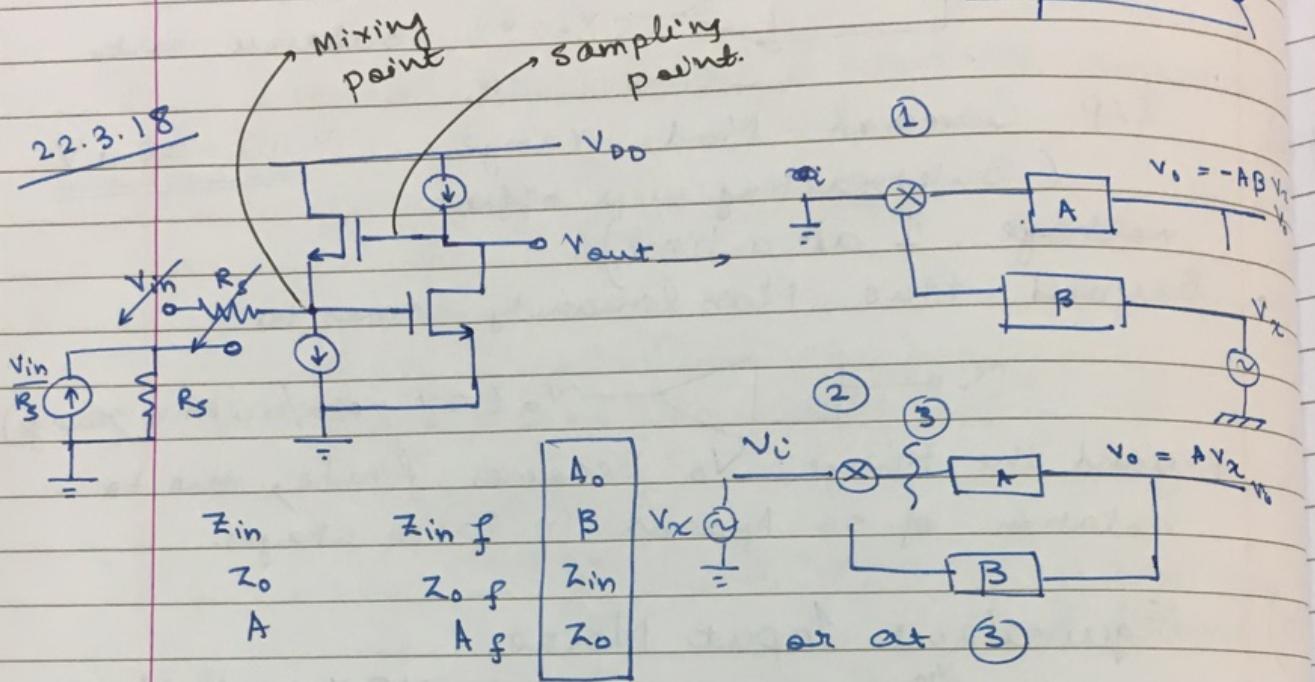
→ NEE
 NEV
 NEP

NEP is the Integral of $(A \times \text{Nev})^2$ over BW?

Typical : $\sim 100\text{nV} / \sqrt{\text{Hz}}$

$$\int_0^f \text{NEV} \times A df$$

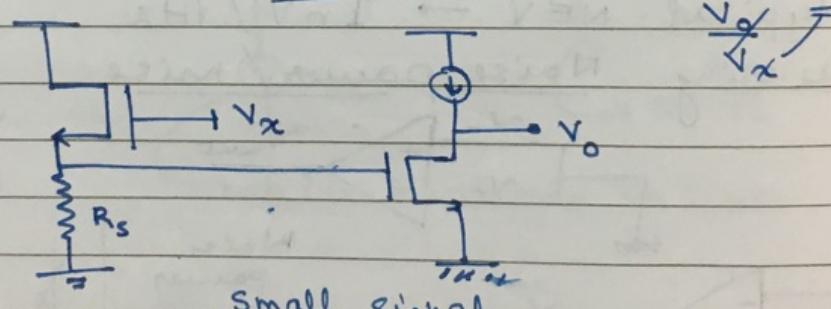
8) Unity Gain frequency



→ mixing current, sampling voltage feedback

Hence make Norton equiv.

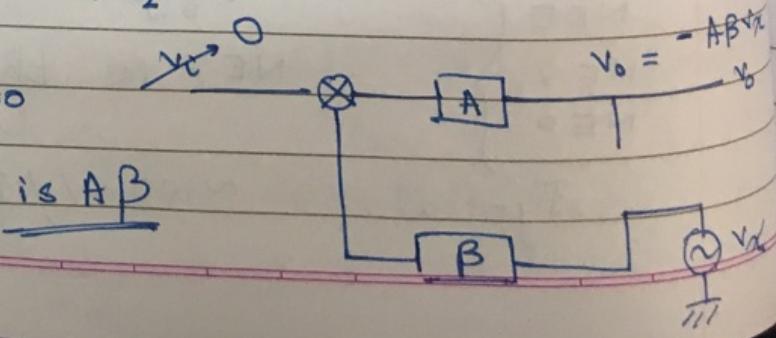
Open loop ckt Loop Gain



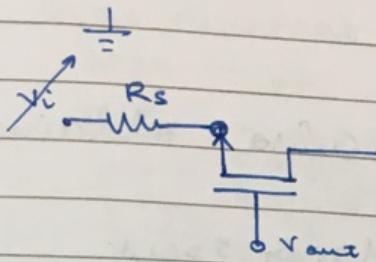
$$\frac{V_o}{V_x} = \frac{-R_f}{R_f + 1/g_m_2} \cdot g_m_1 r_o \quad [\text{put } r_o \rightarrow \infty]$$

↳ corresponds to

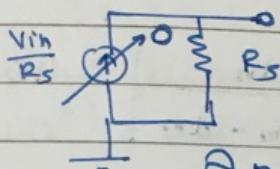
Hence A_{loop} is AB



→ Setting Input to 0

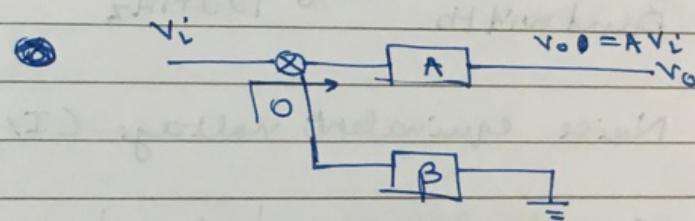


If current source, simply open

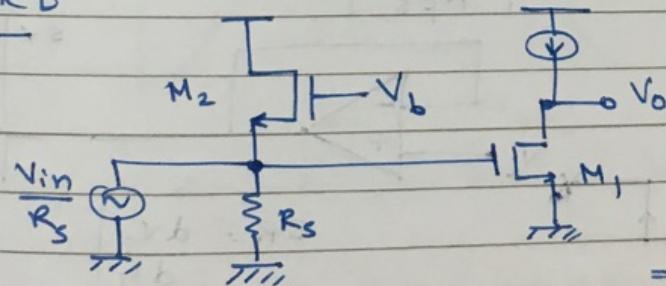


Open loop Gain

→ Get other parameters → put $\Rightarrow V_x = 0$
& put some V_2^*



→ CRB



$$\frac{V_{out}}{V_{in}/R_s} = -g_m \frac{r_o}{(R_s || \frac{1}{g_m_2})}$$

$$= -g_m r_o \left(R_s || \frac{1}{g_m_2} \right)$$

$$= G_R$$

$$A =$$

Simp. of
a MOS
when $V_o \rightarrow \infty$

$$\Rightarrow \text{Get } \beta = \frac{R_s}{\left(R_s + \frac{1}{g_m_2} \right)} \quad \left/ \frac{R_s \cdot \frac{1}{g_m_2}}{\left(R_s + \frac{1}{g_m_2} \right)} \right. = \boxed{\frac{g_m_2}{R_s}}$$

$$\text{Gain} = \frac{A}{1 - AB}$$

→ We could have also obtained B direct

26.3.18 Recap : opamp Major specs.

1) DC Gain $20 \log |A_{\text{dm}}|$ 60-80 dB (Diff. Gain at $\omega \rightarrow 0$)

2) CMRR $20 \log \left| \frac{A_{\text{dm}}}{A_{\text{dm}} - d_m} \right|$ 40-60 dB

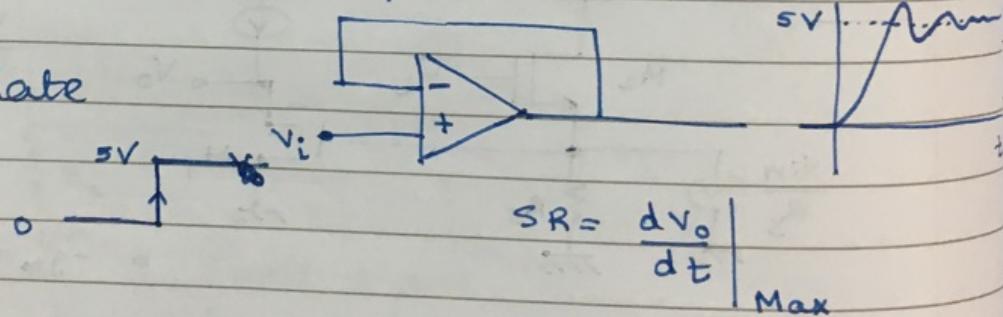
3) Offset (V_{os}) \approx 4-6 mV

4) Bandwidth $\sim 100 \text{ MHz}$

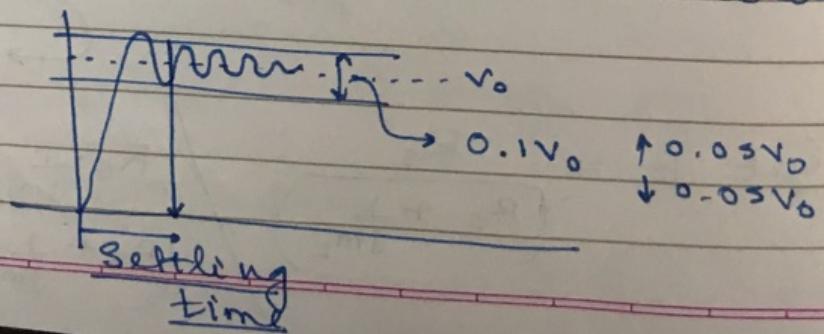
5) Noise equivalent voltage (I/P ref. Noise) 50 nV

6) PSRR $20 \log \left| \frac{A_{\text{dm}}}{A_{\text{ps}}} \right|$ 30-60 dB

7) Slew Rate



→ Settling time (5% Internal boundedness)



8)

Settling time $\approx 0.1 \text{ ms}$

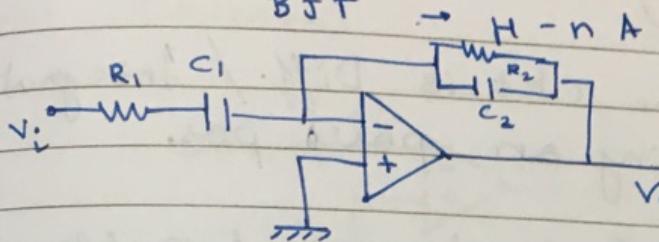
9)

Input bias current (huge difference for MOS/BJT)

MOS \rightarrow pABJT \rightarrow H-nA

- 1) Phase shifter
- 2) All pass

I



$$\begin{aligned} v_o &= A(v_+ - v_-) \\ (v_+ - v_-) &= \lim_{A \rightarrow \infty} \frac{v_o}{A} = 0 \end{aligned}$$

directly

Gains
 $\rightarrow 0$) $v_+ \approx v_-$ [Virtual Ground]

But there is a slight potential difference (causal) we can't short them



→ Transfer function:

$$\frac{v_o}{v_i} = -\frac{Z_2}{Z_1} \Rightarrow A = \frac{sC_1R_2}{(1+sC_2R_2)(1+sC_1R_1)}$$

(Ideal op-amp)

20 log |A|

20dB/dec

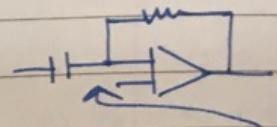
-20dB/dec

can also
be made
all-pass $y_{R_1C_1}$ $y_{R_2C_2}$

Integrator

Reap (EE112) Differentiator:

$$v_o = -RC \frac{dv_i}{dt}$$

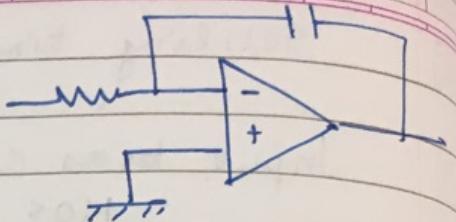


$$\Rightarrow \left| \frac{Z_2}{Z_1} \right| = sRC$$

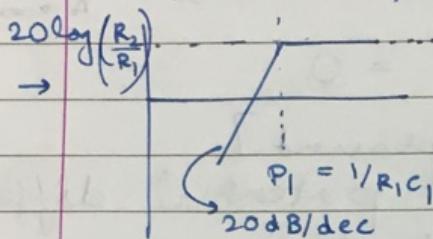
→ Push $\frac{1}{R_1C_1}$ & $\frac{1}{R_2C_2}$ to Right to get Diff.Good Diff: $\Rightarrow R_1 \downarrow$ & $C_2 \downarrow$

Good Integrator

($e_1 \uparrow$, $R_2 \uparrow$)

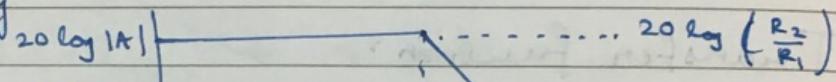


→ Hence the same ckt is Diff. / Integrator / filter depending on poles pos.

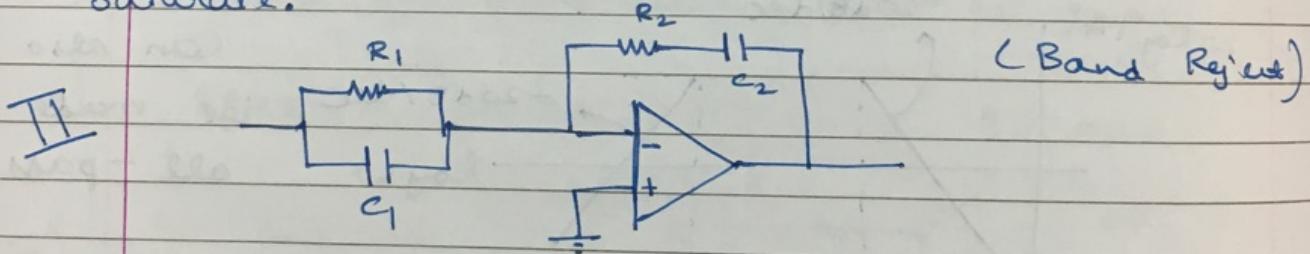


Such char. is desirable for differentiator else noise will make o/p blow up

→ Similarly

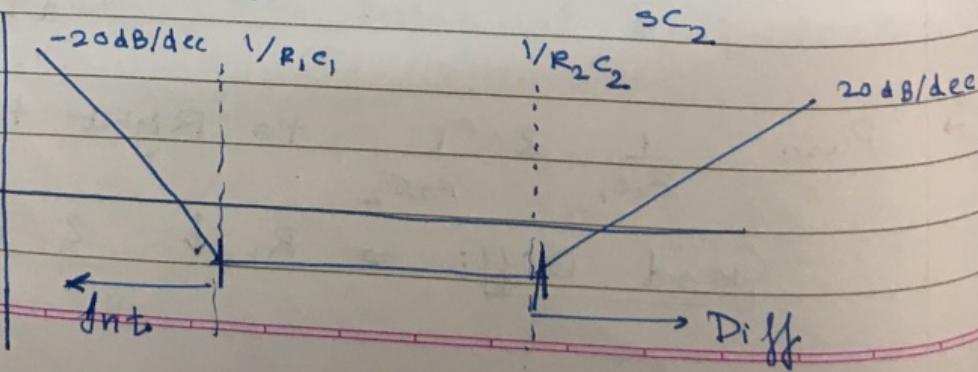


→ Since low freq. DC will cause ideal integrator to saturate.

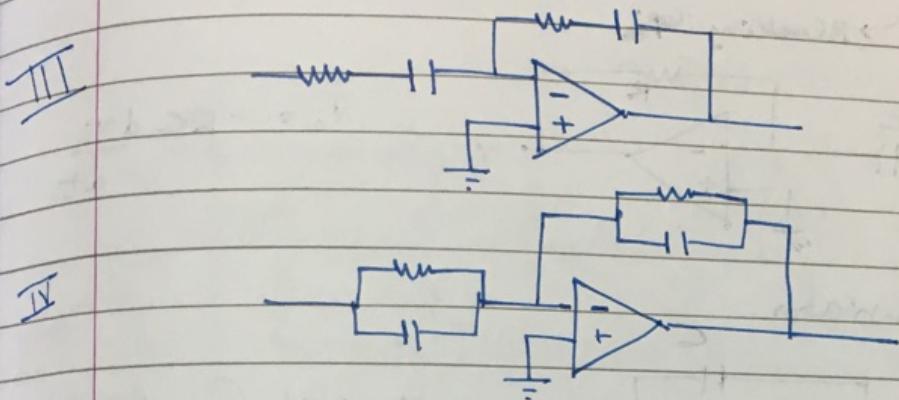


$$Z_1 = \frac{1/sC_1}{\frac{1}{sC_1} + R_1} = \frac{R_1}{1 + sC_1 R_1}$$

$$Z_2 = R_2 + \frac{1}{sC_2} = \frac{1 + sC_2 R_2}{sC_2}$$

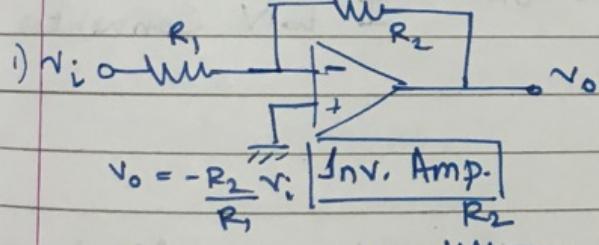


- Can be used as both Integrator & Differentiator (but will saturate because of noise)
- May saturate even in Band Reject mode. (Noise)

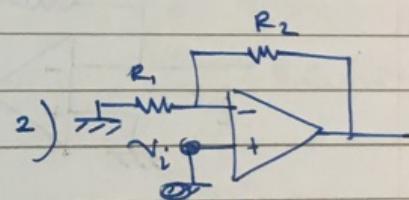


• $\frac{1}{RC}$ → Can be added to a branch to get pole/zeros

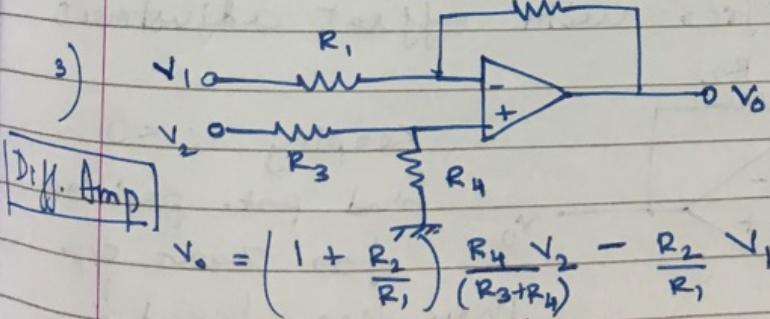
27.3.18 Op-Amp Ckts



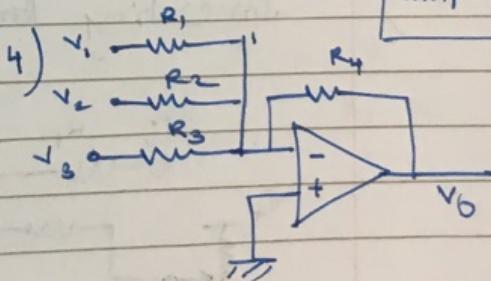
$$V_o = -\frac{R_2}{R_1} V_i \quad \boxed{\text{Inv. Amp.}}$$



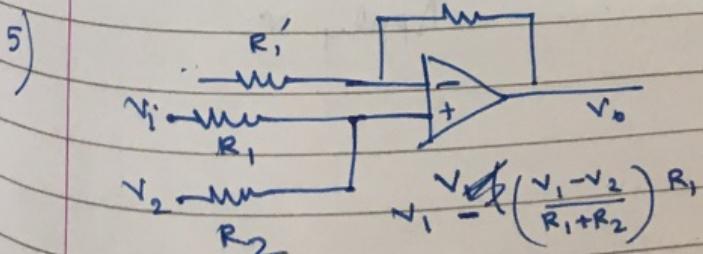
$$V_o = \left(1 + \frac{R_2}{R_1}\right) V_i \quad \boxed{\text{Non-Inv. Amp.}}$$



$$V_o = \left(1 + \frac{R_2}{R_1}\right) \frac{R_4 V_2}{(R_3+R_4)} - \frac{R_2}{R_1} V_1$$



$$V_o = -R_4 \left[\frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3} \right] \quad \boxed{\text{Inv. Summer}}$$

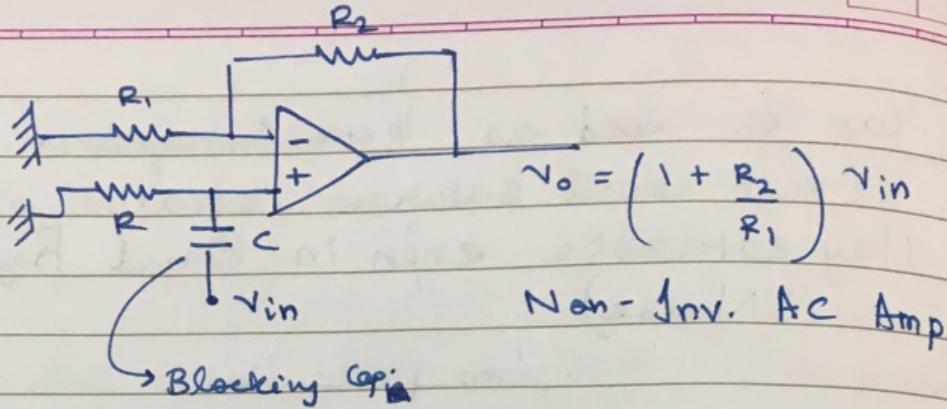


$$V_o = \left(1 + \frac{R_2}{R_1}\right) V_o' \quad V_o' = \frac{V_1 - V_2}{R_1 + R_2} R_1$$

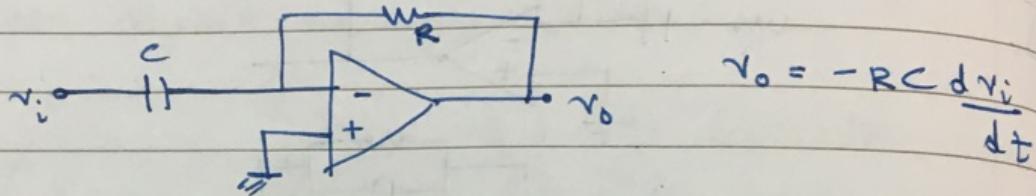
$$V_o = \left(1 + \frac{R_2'}{R_1'}\right) V_o' \quad V_o' = \frac{1}{R_1 + R_2} (V_1 R_2 + V_2 R_1)$$

Non inv. summer

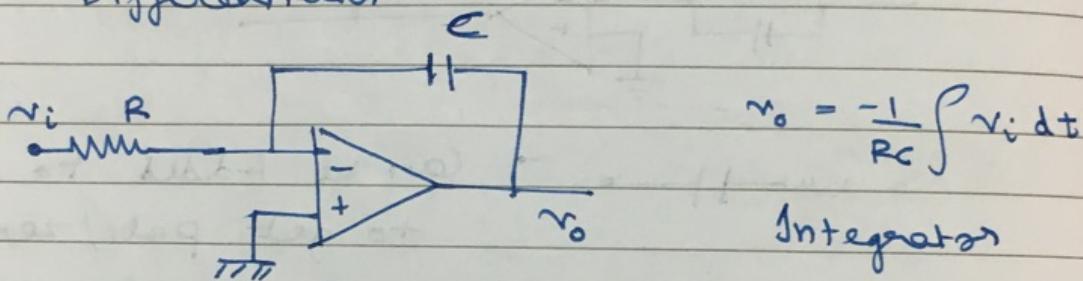
6)



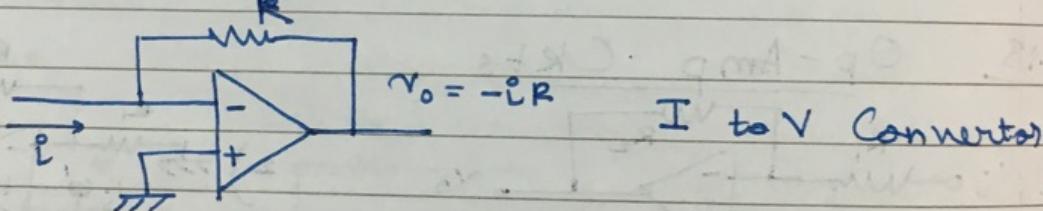
7)

**Differentiator**

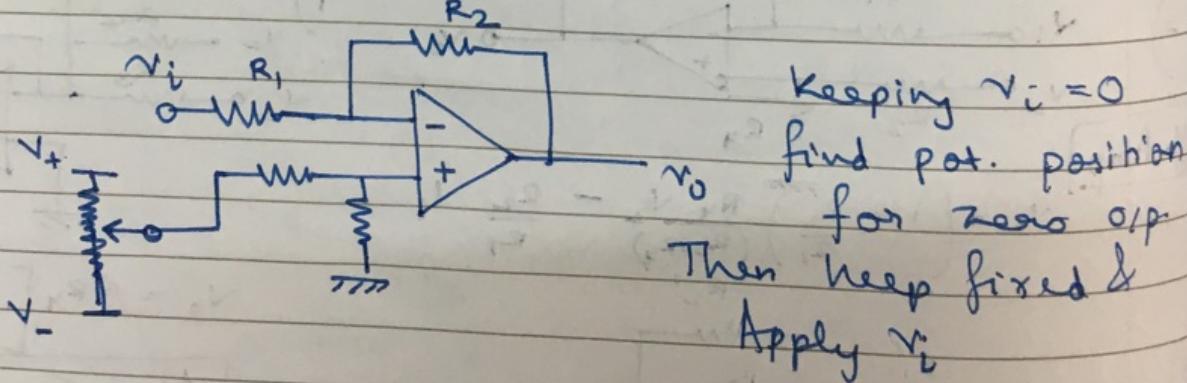
8)



9)



10)

Inverting Amplifier with offset adjustment

Multivibrators

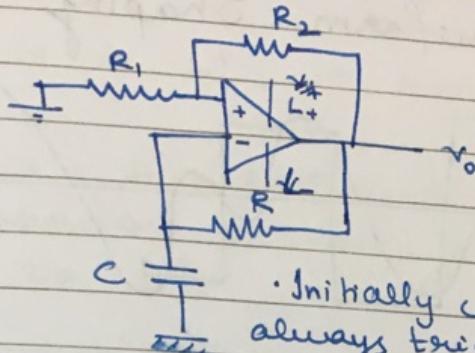
- Astable

- Bistable

- Mono-stable

→ Astable
(CLK)

$$V_- = V_{cap}$$

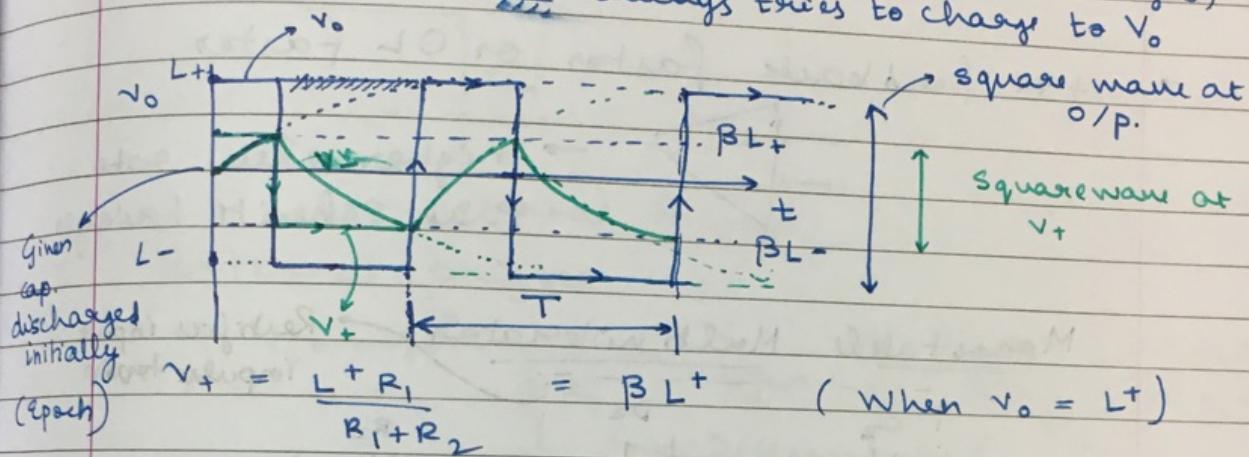


Start: Either

L^+ or L^-

∴ of +ve F.B

Initially cap. was discharged,
always tries to charge to V_0 .



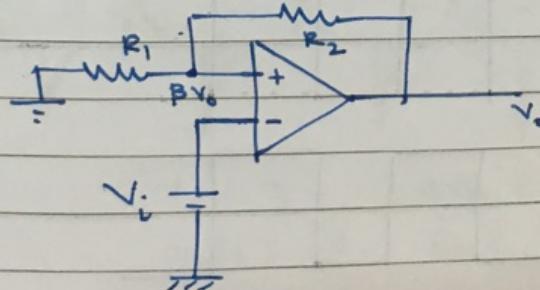
When V_0 switches to L^- (Due to $V_+ - V_-$ pol. reversal)

$$V_+ = L^- \beta \rightarrow \text{Derive}$$

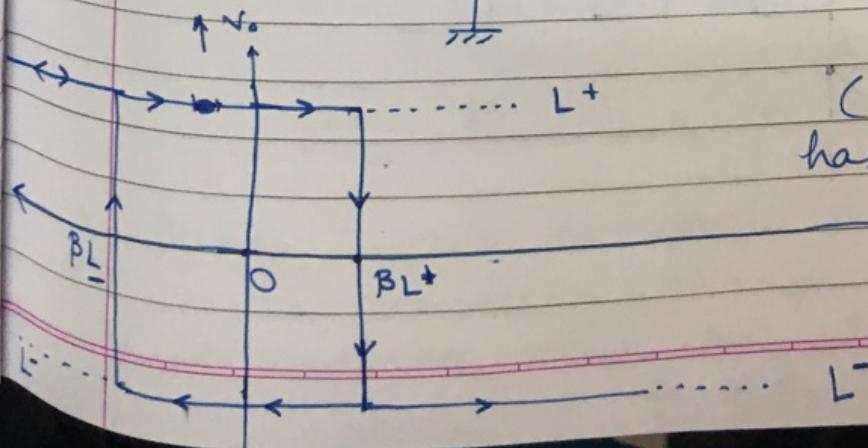
$$T = 2RC \ln\left(\frac{1+\beta}{1-\beta}\right)$$

* / What if *

→ Bistable Multivibrators

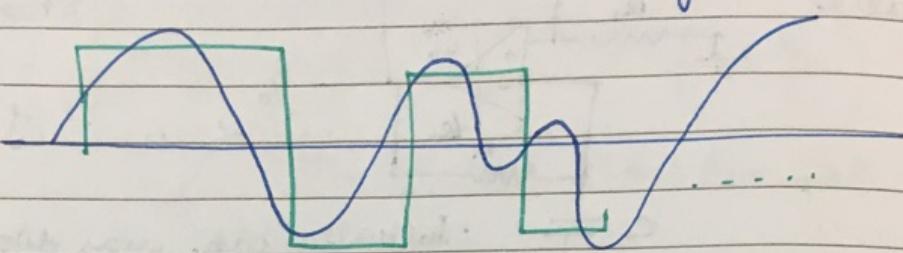


$$\beta = \frac{R_1}{R_1 + R_2}$$

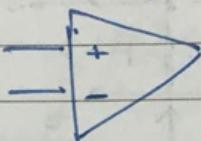


(Transfer function has hysteresis / memory)

→ Used in waveform Shaping

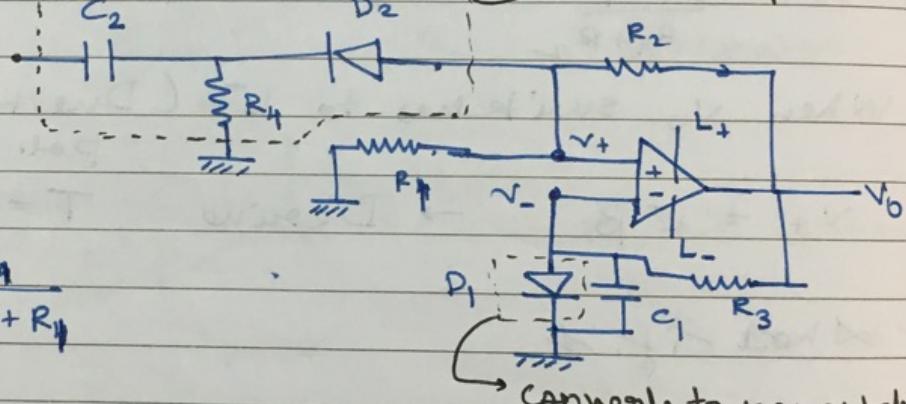


→ +ve feedback faster or OL faster



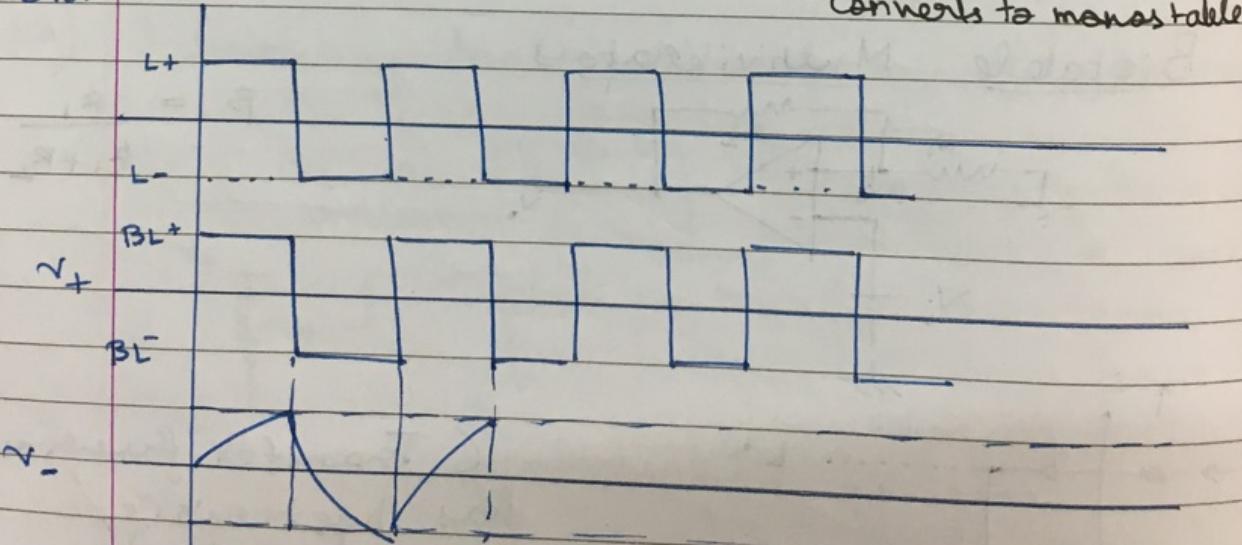
→ higher slew rate
than schmitt trigger

Monostable Multi-vibrator → Rectifies input
impulse train

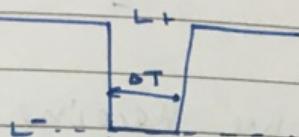


$$\beta = \frac{R_1}{R_2 + R_1}$$

Astabile
Bist



- If ext is in L^+ , it remains in L^+ till ∞
- If v_o is L^- , then it stays in L^- for a transient period and then switches back to L^+ .



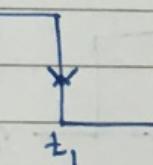
external trigger

- External trigger

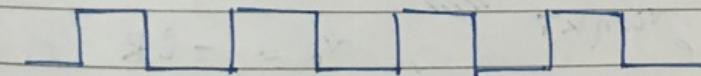
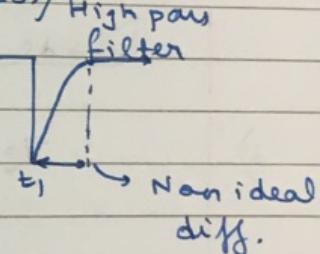
$$v_i \xrightarrow{C_2} v_o = \frac{R_4}{R_4 + \frac{1}{SC_2}} v_i \quad \left(= \frac{SC_2 R_4}{SC_2 R_4 + 1} \right)$$

Differentiator / High pass filter

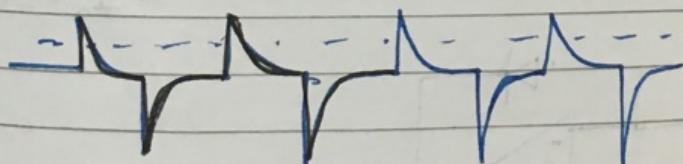
v_o



v_o



Diode Rectifies
this out



$$v_o \approx v_i SC_2 R_4 ; SC_2 R_4 \ll 1$$

$$v_o(t) \approx R_4 C_2 \left(\frac{dv_i}{dt} \right)$$

$$\text{Get } T_{\text{pulse}} = R_3 C_1 \ln \left(\frac{1}{1-\beta} \right)$$

- Diode needed to allow for (C_1, R_3) ckt to completely charge to $B L^-$ before it switches back to L^+ ensures $T_{\text{pulse}} = f(R_3, C_1)$

3.4.18

→ Closely spaced pulses

Next

Say any pulse duration is 10 s

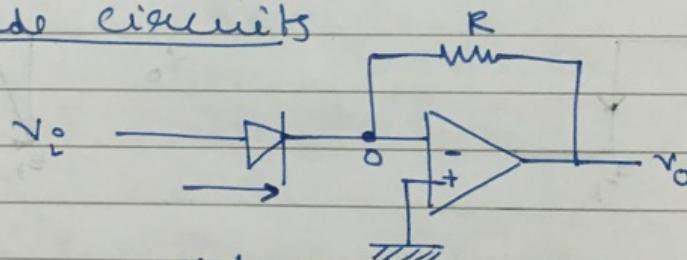
t	0 s	5 s	8 s	10 s
v_o	L^-	L^-	L^-	L^+

→ In above ex. after we receive a trigger it takes 10 s to reach L^+ again.

→ If we trigger in b/w, then we are forcing a pin to a value where it is already present, hence it has no effect on time to convergence.

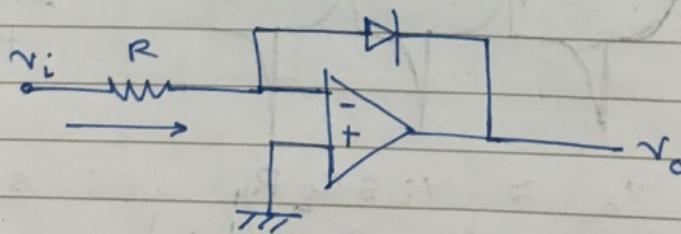
Diode circuits

1)



$$i = I_o e^{v_i / n kT}, \quad v_o = -iR = -I_o R e^{v_i / n kT}$$

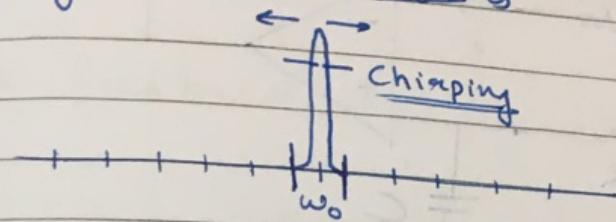
2)



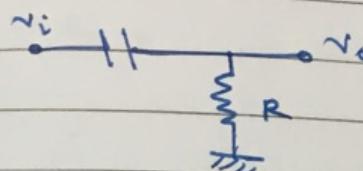
$$\frac{v_i}{R} = I_o e^{-v_o / n kT}$$

$$v_o = -n k T \ln \left(\frac{v_i}{I_o R} \right)$$

→ Used in Signal Conditioning.

Chipping in oscillators

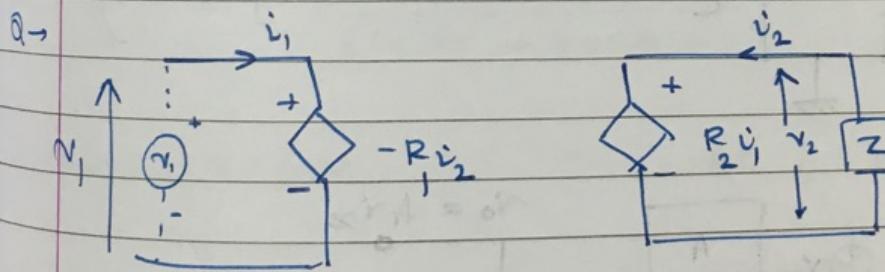
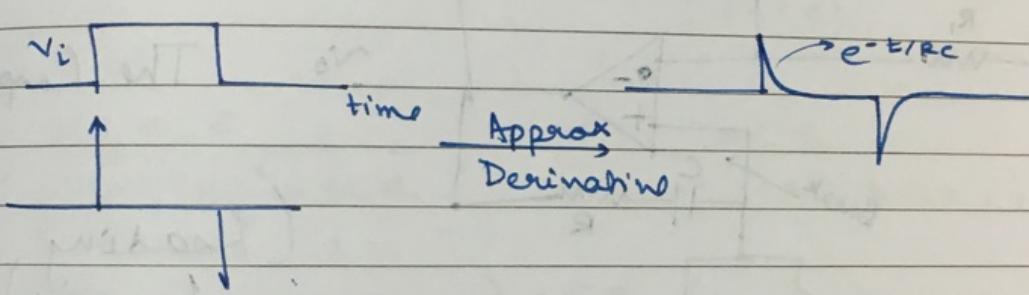
Spectrum of oscillator output



$$\frac{v_o}{v_i} = \frac{R}{R + 1/sC}$$

$$\frac{v_o}{v_i} = \frac{sCR}{1+sCR}$$

say $v_i = u(t)$, $v_o = \frac{1}{s} \cdot \frac{sCR}{1+sCR}$
 $\Rightarrow v_o(t) = e^{-t/RC}$



$$\frac{v_i}{i_1} = ?$$

$$v_1 = -R_1 i_2$$

$$i_2 = -\frac{R_2 i_1}{Z}$$

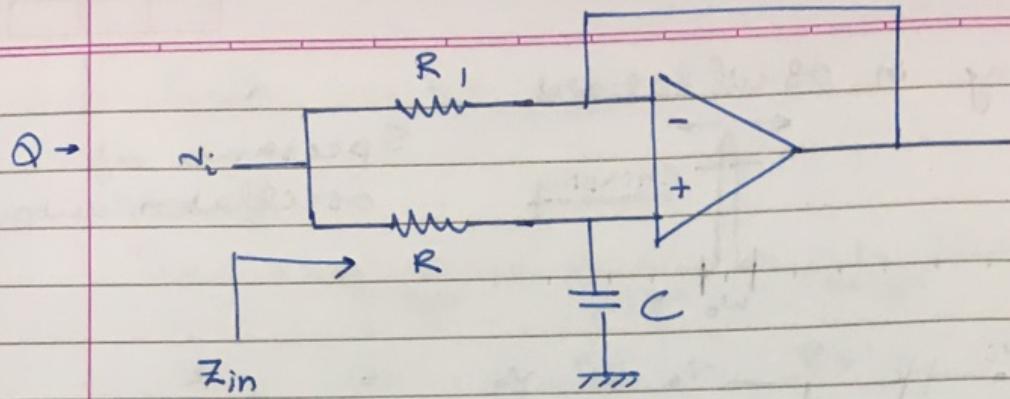
$$v_1 = -R_1 \left(-\frac{R_2 i_1}{Z} \right)$$

$$\frac{v_1}{i_1} = \frac{R_1 R_2}{Z}$$

take $Z = \frac{1}{sC}$ (Cap)

$$\Rightarrow \frac{v_1}{i_1} = SCR^2 \Rightarrow [We \ get \ L = R^2 C]$$

simulated
inductor
LS

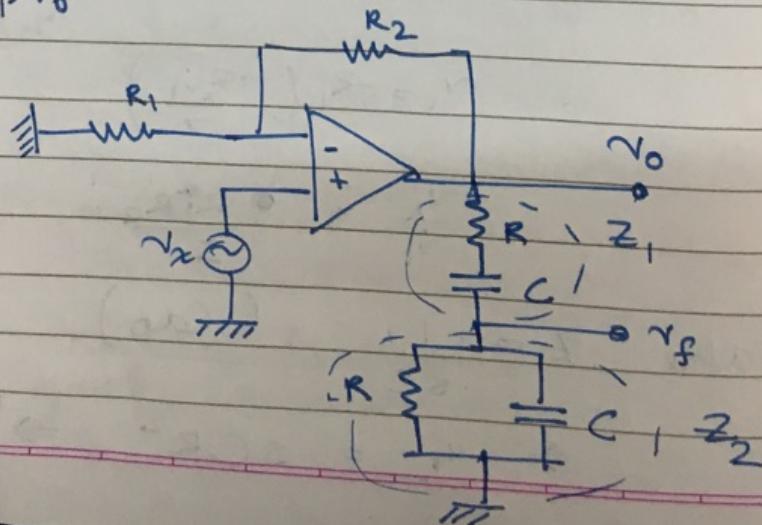
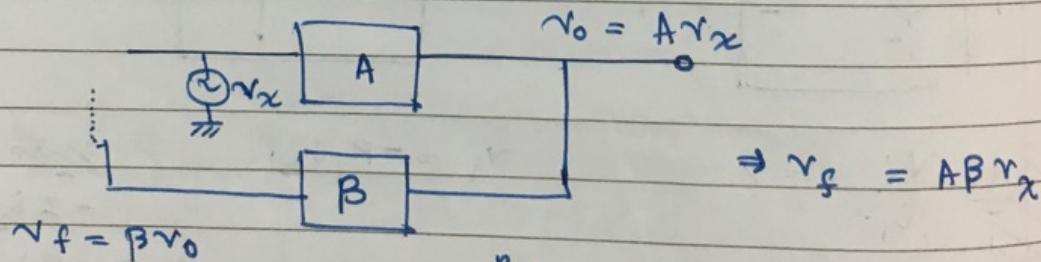
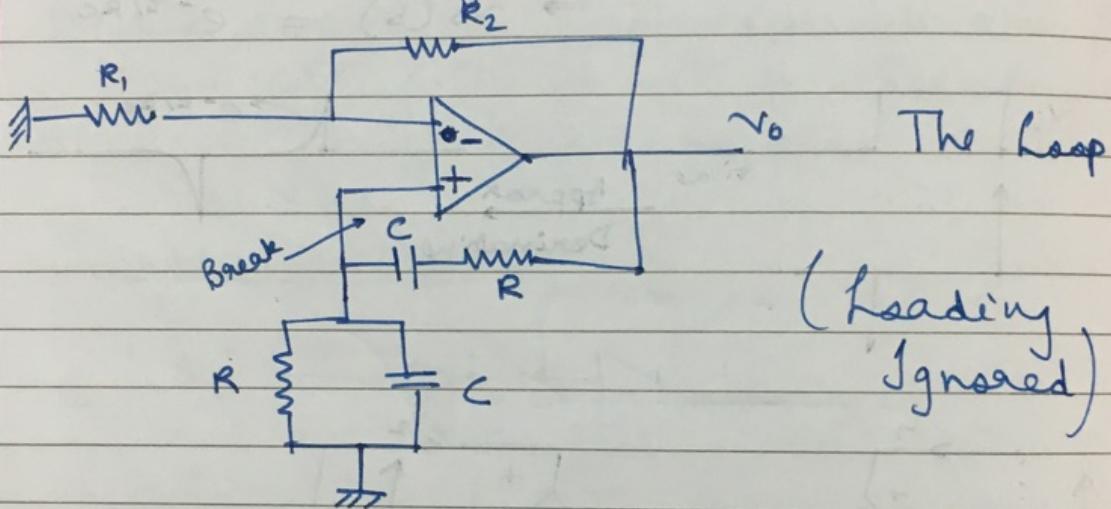


5.4.18

Op amp oscillators

15

Wien Bridge Oscillator



$$+\text{ve feedback} \rightarrow \frac{v_o}{v_i} = \frac{A}{1-AB}$$

$$-\text{ve feedback} \rightarrow \frac{v_o}{v_i} = \frac{A}{1+AB}$$

$$A = \left(1 + \frac{R_2}{R_1} \right) \quad \text{Non-Inverting Amp.}$$

$$\therefore v_o = \left(1 + \frac{R_2}{R_1} \right) v_x$$

$$\beta = \frac{z_2}{z_1 + z_2} = \frac{R / (1 + SCR)}{\frac{R}{(1 + SCR)} + \frac{SCR + 1}{SC}}$$

$$= \frac{SCR}{(1 + SCR)^2 + SCR} = \frac{SCR}{s^2 C^2 R^2 + 3SCR + 1}$$

$$AB = \frac{(1 + R_2/R_1) SCR}{s^2 C^2 R^2 + 3SCR + 1}$$

$$\text{put } s = j\omega, \quad = jWRC \left(1 + \frac{R_2}{R_1} \right) \\ \frac{3jWRC + (1 - \omega^2 R^2 C^2)}{}$$

$$\text{At } \omega = 1/RC$$

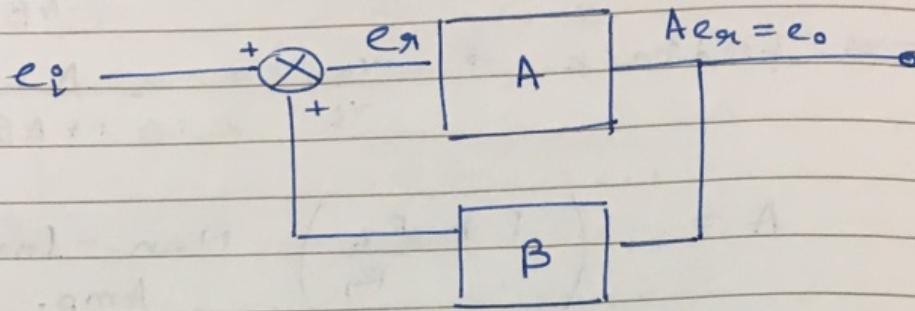
$$AB = \frac{1}{3} \left(1 + \frac{R_2}{R_1} \right)$$

$$\frac{1}{3} \left(1 + \frac{R_2}{R_1} \right) > 1$$

\Rightarrow oscillations happen at $\omega = \frac{1}{RC}$

if $R_2 > 2R_1$

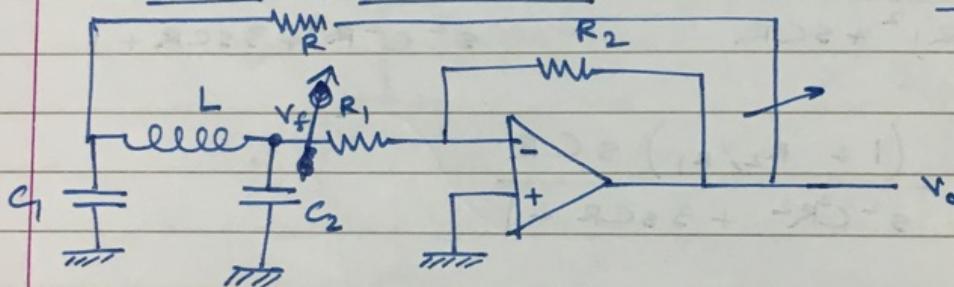
→ When is F/B +ve



$$\beta \rightarrow -\beta \quad F.B \quad +ve \rightarrow -ve$$

Note: Here, Formula used is $\frac{A}{1-AB}$
 Since we are connecting to ~~the~~ non + terminal of op amp

→ More Oscillators

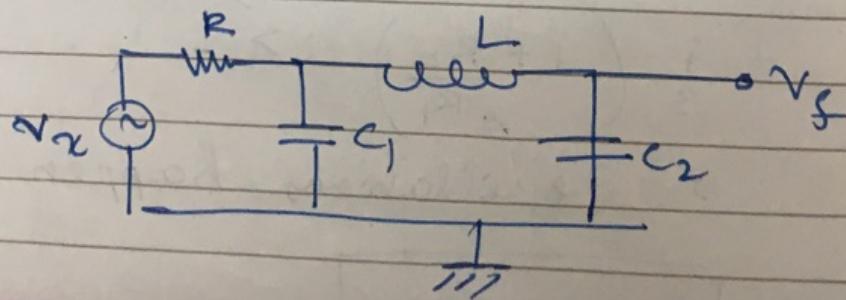


Colpitt's Oscillator

$$A = -\frac{R_2}{R_1}$$

We know $\frac{V_o}{V_f} = A = -\frac{R_2}{R_1}$ (OL gain)

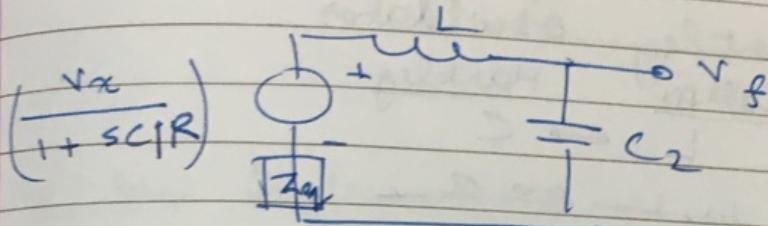
→ To get β , cut open F-B Network



$$Z_{eq} = R + \left(sL + \frac{1}{sC_2} \right) \parallel sC_1$$

\approx

\rightarrow Thevenin equivalent



$$Z_{eq} = R \parallel \frac{1}{sC_1}$$

$$\Rightarrow \beta = \frac{\left(\frac{1}{1+sC_1R} \right) \times \frac{1}{sC_2}}{\frac{R}{1+sC_1R} + LS + \frac{1}{sC_2}}$$

$$sC_1R \gg 1$$

Dx at $s = j\omega$

$$= j \left[\omega L - \frac{1}{\omega} \left(\frac{1}{C_1} + \frac{1}{C_2} \right) \right]$$

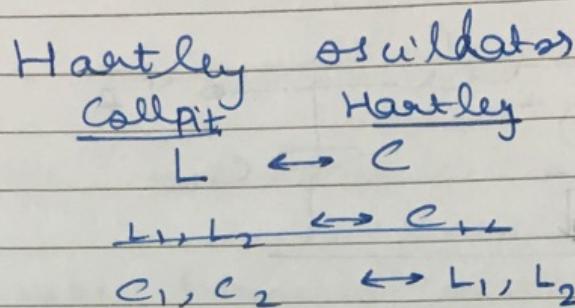
$$\omega \approx \frac{1}{\sqrt{L \left(\frac{C_1 C_2}{C_1 + C_2} \right)}} \quad \text{for } Dx = 0$$

We have $LA = \pi \Rightarrow LB = \pi$ for oscillation

\rightarrow How to get $LB = \pi$

$$\beta = \frac{\left(\frac{1}{1+sC_1R} \right) \cdot \frac{1}{sC_2}}{\frac{R}{1+sC_1R} + LS + \frac{1}{sC_2}}$$

Claim (Proof in Assignment), that
 $\angle D\alpha$ when $D\alpha \rightarrow 0 = 0$
then $\angle N\alpha \approx -\infty \Rightarrow$ we get what
we want

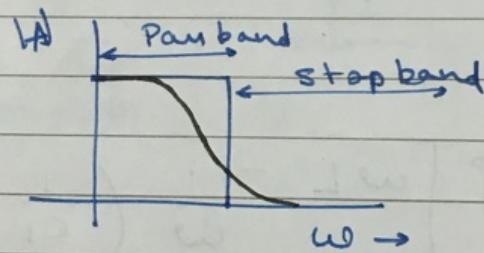


9.4.18

Filters

1) Low pass filter

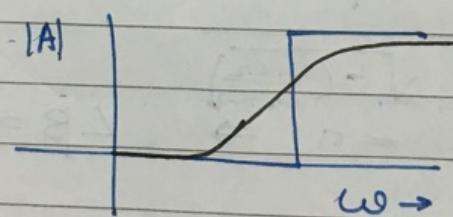
Implementation



$$\frac{V_o}{V_i} = \frac{1}{1 + SCR}$$

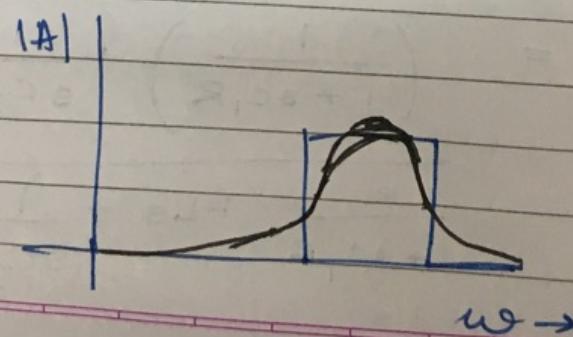
Circuit diagram for a low-pass filter: A resistor R and a capacitor C are connected in series between the input V_i and the output V_o .

2) High pass



Circuit diagram for a high-pass filter: A resistor R and a capacitor C are connected in parallel between the input V_i and ground, with the output V_o taken from the node between the resistor and the capacitor.

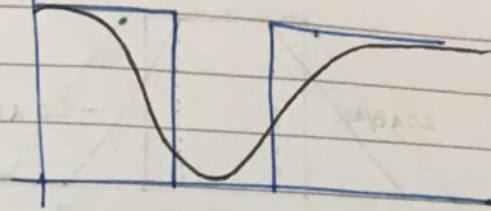
3) Band pass



$$\frac{V_o}{V_i} = \frac{RS/L}{S^2 + SR + \frac{1}{LC}}$$

4) Band Reject (Notch)

(A)



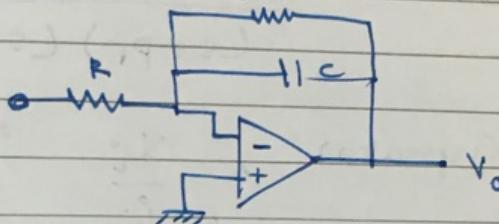
$$\frac{V_o}{V_i} = \frac{s^2 + 1/LC}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$$

$\omega \rightarrow \infty$

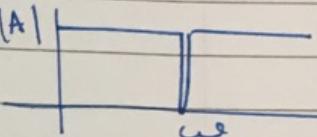
True Notch

Op-Amp Implementation R

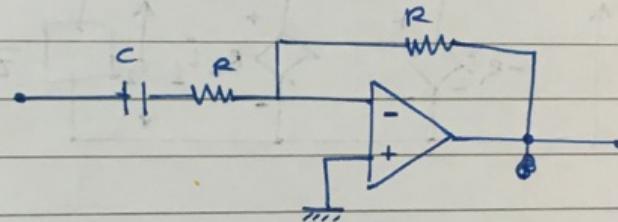
LPF :



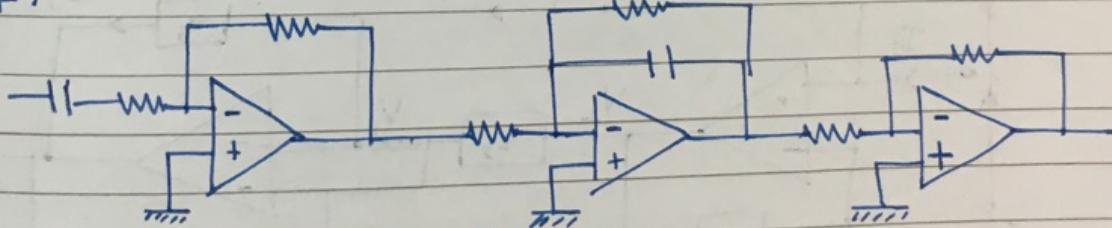
(A)



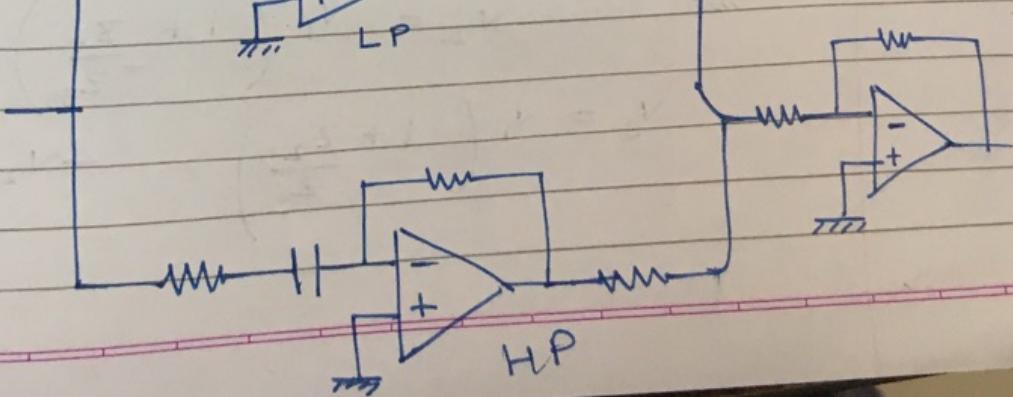
HPF :



BPF :



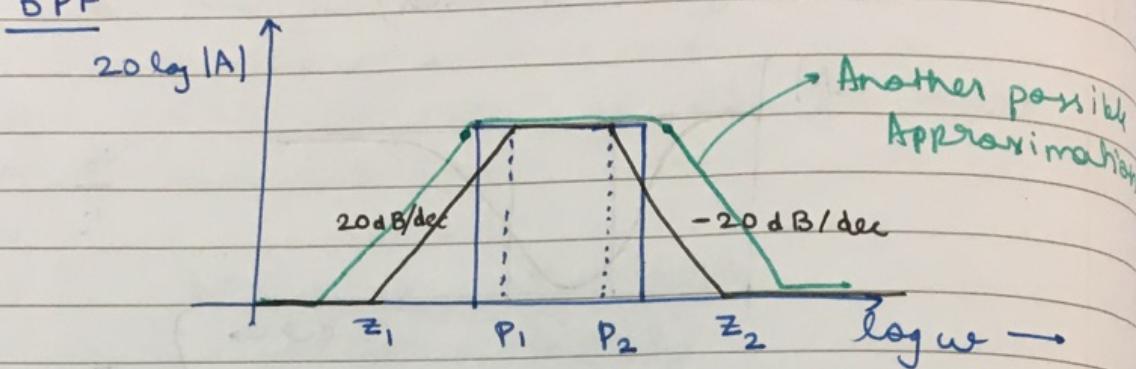
Notch :



Filter Transfer functions

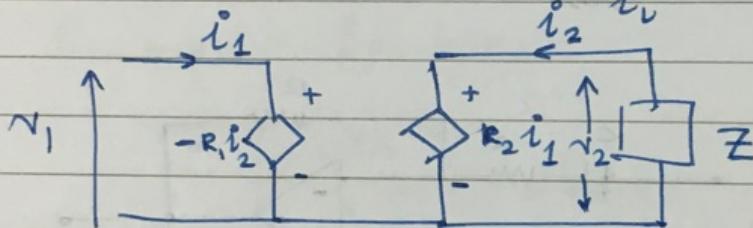
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BPF

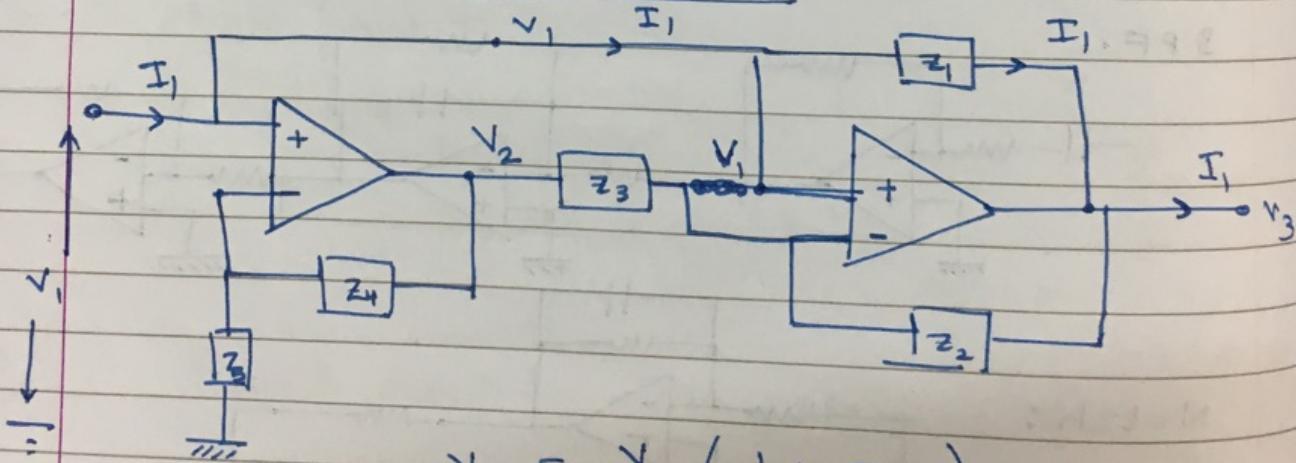


$$G(s) = \frac{k(s+z_1)(s+z_2)}{(s+p_1)(s+p_2)}$$

→ Recap: Gyrator, $\frac{v_i}{i_o} = \left(\frac{R_1 R_2}{Z} \right)$



Riordan Ckt



$$v_2 = v_1 \left(1 + \frac{z_4}{z_5} \right)$$

$$v_3 = v_1 \left(1 + \frac{z_2}{z_3} \right) - v_2 \frac{z_2}{z_3}$$

$$v_3 = v_1 \left(1 - \frac{z_2 z_4}{z_3 z_5} \right)$$

$$I_1 = \frac{v_1 - v_3}{z_1} = \left(\frac{-z_2 z_4}{z_1 z_3 z_5} \right) v_1$$

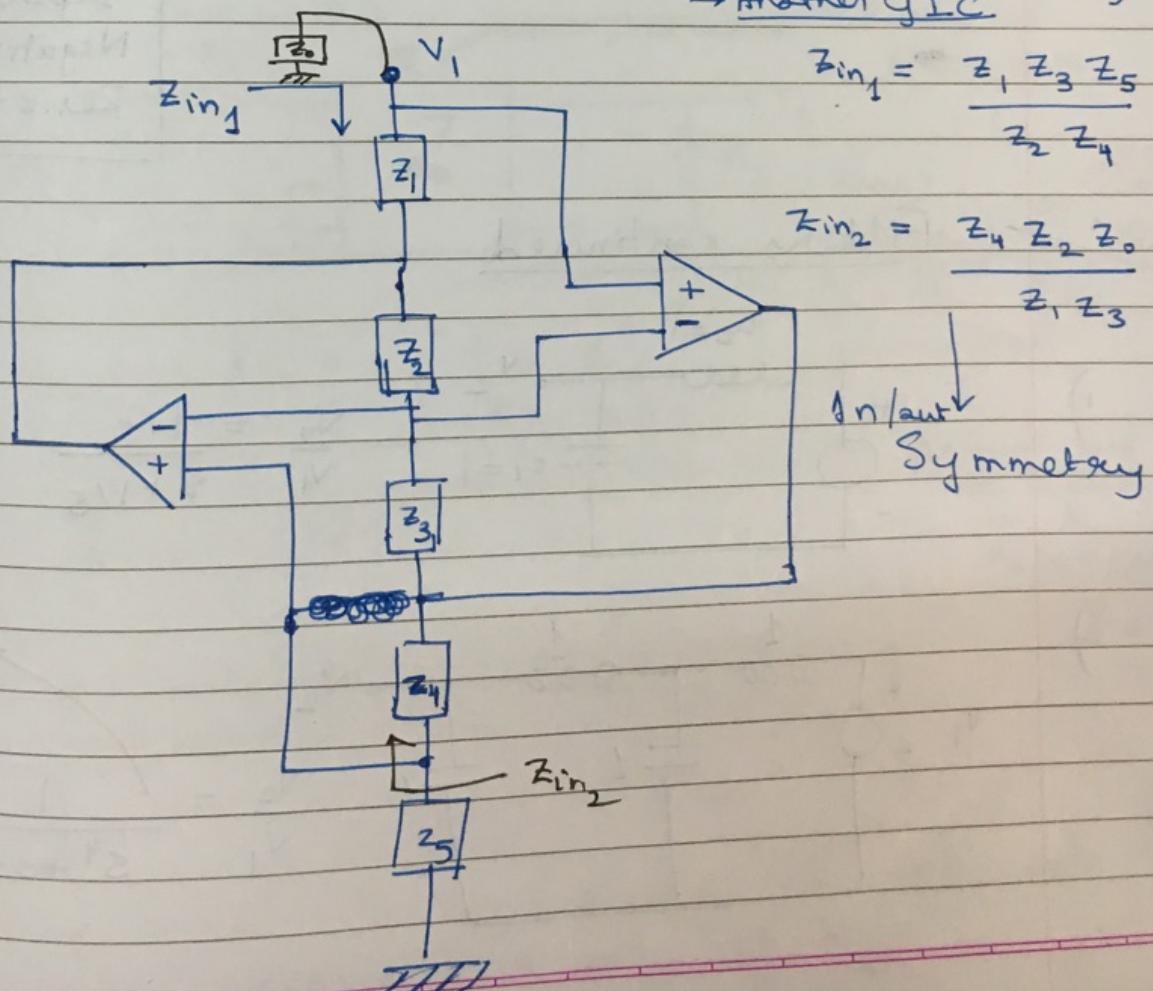
~~$\frac{I_1}{z_1} \Rightarrow$~~

$$Z_{in} = \frac{z_1 z_3 z_5}{z_2 z_4}$$

We can get any transfer function we want using this.

GIC: Generalised Impedance Converter

→ Application of GIC (Antonieu's Gyrator)
 → Another GIC



→ Pick

$$z_1 = R_p =, \quad z_3 = R_3, \quad z_5 = R_5 \\ z_2 = R_2 \quad z_4 = \frac{1}{sC_4}$$

$$Z_{in1} = \left(\frac{R_1 R_3 R_5}{R_2} \right) sC_2 = L_0 S = j\omega L_0$$

remove z_5 , take $z_0 = \frac{1}{sC_0}$, $z_4 = \frac{1}{sC_4}$
rest are resistors

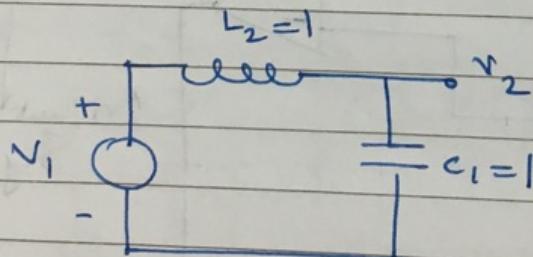
$$Z_{in2} = \frac{z_4 z_2 z_0}{z_3 z_1} = \frac{1/sC_4 R_2 \times \frac{1}{sC_0}}{R_1 R_3}$$
$$= \frac{R_2 C_4}{R_1 R_3 C_0} \cdot \frac{1}{s^2} = -\frac{\alpha}{\omega^2}$$

FDNR
Frequency
Dependant
Negative
Resistance

10.4.18

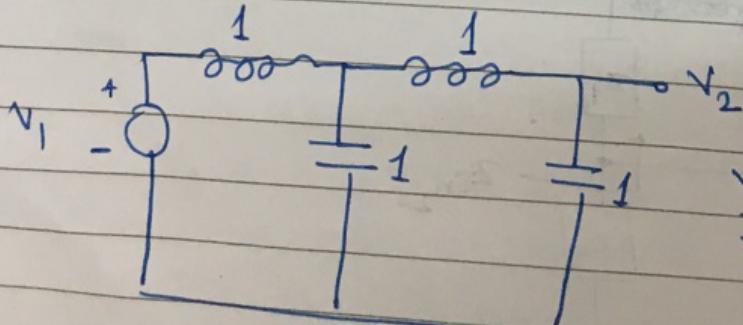
Filters continued

1)



$$\frac{v_2}{v_1} = \frac{1/s}{s + 1/s} = \frac{1}{1+s^2}$$

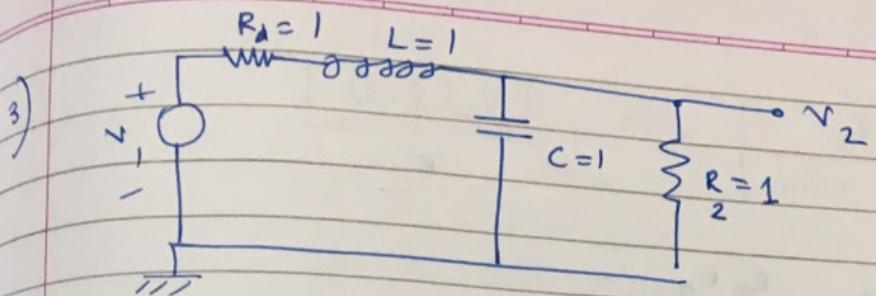
2)



$$\frac{v_2}{v_1} = \frac{1}{s^4 + 3s^2 + 1}$$

No odd power

$$V_1 = Z_{11} I_1 + Z_{12} \frac{I_2}{R_2}$$



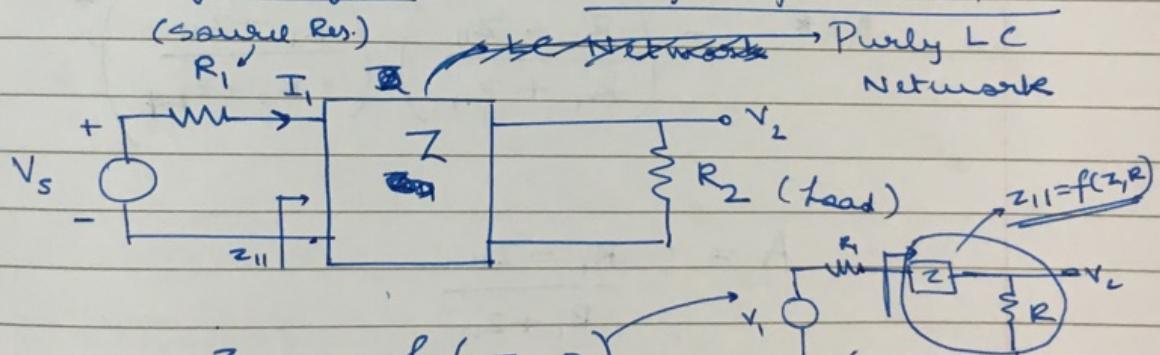
$$\frac{V_2}{V_1} = \frac{1}{s^2 + 2s + 2}$$

ζ odd powers present if R present

General, $T = \frac{k}{b_n s^n + b_{n-1} s^{n-1} + \dots + b_0}$

$$T(s=0) = \frac{k}{b_0} = \frac{R_2}{R_1 + R_2} = \frac{1}{2} \text{ for (3)}$$

→ How to get General Transfer function T



$$Z_{11} = f(z, R) = R_{11} + j X_{11}$$

only appears because we are looking in & R_2 is effectively Z_{11}

$$I_1 = \frac{V_s}{R_1 + Z_{11}} = \frac{V_s}{(R_1 + R_{11}) + j X_{11}}$$

$$|I_1(j\omega)|^2 R_{11} = \frac{|V_2(j\omega)|^2}{R_2}$$

Since Z is a purely LC Network power dissipated is only at R_2

$$R_{11} \left| \frac{V_s}{R_1 + Z_{11}} \right|^2 = \frac{|V_2(j\omega)|^2}{R_2}$$

$$|T|^2 = \frac{R_{11} R_2}{(R_1 + Z_{11})^2}$$

$$|A(j\omega)|^2 = 1 - \frac{4 R_1}{R_2} |T(j\omega)|^2$$

$$1 - \cancel{\frac{4 R_1}{R_2}} \frac{R_{11} R_2}{(R_1 + Z_{11})^2} \xrightarrow{\left(\times \frac{Z_{11}}{R_{11}} \right)}$$

$$|A(j\omega)|^2 = \left| \frac{R_1 - Z_{11}}{R_1 + Z_{11}} \right|^2$$

$$AA^* = \left(\frac{R_1 - Z_{11}}{R_1 + Z_{11}} \right) \left(\frac{R_1 - Z_{11}}{R_1 + Z_{11}} \right)^*$$

$$A = \frac{R_1 - Z_{11}}{R_1 + Z_{11}}$$

$$Z_{11} = R_1 \left(\frac{1-A}{1+A} \right)$$

→ General Butterworth filter

$$T = \frac{k}{1 + \omega^{2n}} = \frac{k}{Q(s)Q(-s)}, \text{ say}$$

take $n = 1$

$$D^n = 1 + \omega^2, \text{ then}$$

$$Q(s) = (1+s)$$

$$Q(-s) = (1-s)$$

$$Q(s)Q(-s) = 1 - s^2 : s = j\omega \Rightarrow = (1 + \omega^2)$$

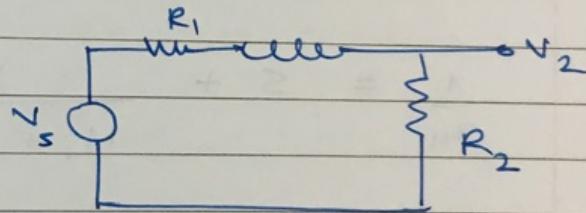
\Rightarrow pick $Q(s) = (s+1)$, take $R_1 = 1$

then pick k such that when, $R_1 = R_2 = 1$

$$A(s) = \frac{s}{s+1}$$

$$Z_{11} = \frac{1 - \frac{s}{s+1}}{\frac{1+s}{s+1}} = \frac{1}{2s+1}$$

equivalent to



$n=2$ case

$$T = \frac{k}{1 + \omega^4} = \frac{k}{Q(s)Q(-s)}$$

$$Q(s) = s^2 + \sqrt{2}s + 1$$

say

for $n=2$,

$$Z_{11} = \frac{\sqrt{2}s + 1}{s^2 + \sqrt{2}s + 1}$$

$$\frac{1}{Z_{11}} = \frac{s^2 + \sqrt{2}s + 1}{(\sqrt{2}s + 1) \cdot s\sqrt{2} + 1/2}$$

$$\frac{\sqrt{2}s + 1}{s^2 + \frac{s}{\sqrt{2}}} \int s^2 + \sqrt{2}s + 1$$

$$\frac{s\left(\frac{\sqrt{2}-1}{\sqrt{2}}\right) + 1}{\frac{\sqrt{2}s}{\sqrt{2}} + 1/2}$$

$$(s^2 + \sqrt{2}s + 1) = (\sqrt{2}s + 1) \left(\frac{s}{\sqrt{2}} + \frac{1}{2} \right) + \frac{1}{2}$$

$$\frac{1}{Z_{11}} = \left(\frac{s}{\sqrt{2}} + \frac{1}{2} \right) + \frac{1}{2(\sqrt{2}s + 1)}$$

$$\frac{1}{Z_{11}} = s + \frac{1}{M_1 + \frac{1}{s\left(\frac{\sqrt{2}-1}{\sqrt{2}}\right) + 1}}$$

