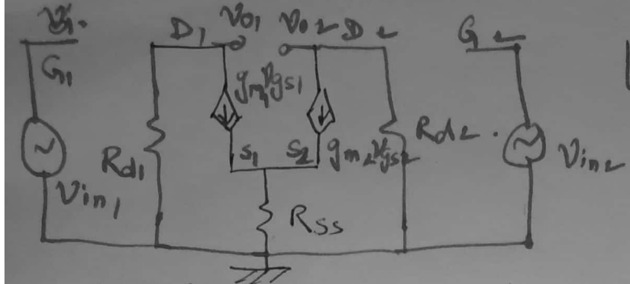

[Assuming r_{o1} & r_{o2} are very large and neglected]

$$V_S = (g_{m1}V_{gs1} + g_{m2}V_{gs2})R_{SS}$$

$$V_{gs1} = V_{in1} - V_S \qquad V_{o1} = -g_{m1}V_{gs1}R_{d1}$$

$$V_{gs2} = V_{in2} - V_S. \qquad V_{o2} = -g_{m2}V_{gs2}R_{d2}$$

$$\Rightarrow V_{gs1} = V_{in1} - g_{m1}V_{gs1}R_{SS} - g_{m2}V_{gs2}R_{SS}.$$

$$\Rightarrow V_{gs1}(1+g_{m1}R_{SS}) + g_{m2}V_{gs2}R_{SS} = V_{in1}$$

$$V_{gs2}(1+g_{m2}R_{SS}) + g_{m1}V_{gs1}R_{SS} = V_{in2}$$

$$\Rightarrow \begin{bmatrix} 1+g_{m1}R_{SS} & g_{m2}R_{SS} \\ g_{m1}R_{SS} & 1+g_{m2}R_{SS} \end{bmatrix}\begin{bmatrix} V_{gs1} \\ V_{gs2} \end{bmatrix} = \begin{bmatrix} V_{in1} \\ V_{in2} \end{bmatrix}$$

$$\Rightarrow \cancel{V_{gs1} =} \ \Delta = (1+g_{m1}R_{SS})(1+g_{m2}R_{SS}) - g_{m1}g_{m2}R_{SS}^2$$

$$= 1 + (g_{m1}+g_{m2})R_{SS}.$$

$$\Delta_1 = \begin{bmatrix} V_{in1} & g_{m2}R_{SS} \\ V_{in2} & 1+g_{m2}R_{SS} \end{bmatrix} = V_{in1}(1+g_{m2}R_{SS}) - V_{in2}g_{m2}R_{SS}$$

$$\Delta_2 = \begin{bmatrix} 1+g_{m1}R_{SS} & V_{in1} \\ g_{m1}R_{SS} & V_{in2} \end{bmatrix} = -V_{in1}g_{m1}R_{SS} + V_{in2}(1+g_{m1}R_{SS})$$

$$V_{o1} = -g_{m1}R_{d1}\frac{\Delta_1}{\Delta} = -g_{m1}R_{d1}\left[\frac{1+g_{m2}R_{SS}}{1+(g_{m1}+g_{m2})R_{SS}}\right]V_{in1} + \frac{g_{m1}g_{m2}R_{d1}R_{SS}}{1+(g_{m1}+g_{m2})R_{SS}}.$$

$$= \frac{-g_{m1}R_{d1}}{1+(g_{m1}+g_{m2})R_{SS}}\left[(1+g_{m2}R_{SS})V_{in1} - g_{m2}R_{SS}V_{in2}\right] \quad \text{—} (1)$$

$$V_{o2} = \frac{-g_{m2}R_{d2}}{1+(g_{m1}+g_{m2})R_{SS}}\left[-g_{m1}R_{SS}V_{in1} + (1+g_{m1}R_{SS})V_{in2}\right] \quad \text{—} (2)$$

Ⓐ For common mode input —

$v_{in1} = v_{in2} = v_{cm}$.

$$v_{o1} = -\frac{g_{m1} R_{d1} v_{cm}}{1+(g_{m1}+g_{m2})R_{ss}} \quad \Bigg| \quad v_{o2} = -\frac{g_{m2} R_{d2} v_{cm}}{1+(g_{m1}+g_{m2})R_{ss}}$$

(i) Assuming mismatch in R_d, $g_{m1} = g_{m2} = g_m$.

$R_{d1} = R_d$, $R_{d2} = R_d + \Delta R_D$

$$(v_{o1} - v_{o2}) = \frac{-g_m v_{cm} \Delta R_D}{1+(g_{m1}+g_{m2})R_{ss}}$$

$$\frac{\Delta(v_{o1}-v_{o2})}{\Delta v_{cm}} = \frac{-g_m \Delta R_D}{1+(g_{m1}+g_{m2})R_{ss}} = A_{CM-DM}$$ [We ~~dire~~ derived this expression based on intuition]

(ii) Assuming mismatch in R_D equally distributed in R_D as, $R_{D1} = R_D + \frac{\Delta R_D}{2}$. $R_{D2} = R_D - \Delta R_D/2$ | $R_{D1} - R_{D2} = \Delta R_D$ [As in (i)] (Ignoring sign)

$$(v_{o1} - v_{o2}) = -\frac{g_m v_{cm}}{1+(g_{m1}+g_{m2})R_{ss}} \left(\cancel{R_D} + \frac{\Delta R_D}{2} - \cancel{R_D} + \frac{\Delta R_D}{2}\right)$$

$$\frac{\Delta(v_{o1}-v_{o2})}{\Delta v_{cm}} = -\frac{g_m \Delta R_D}{1+(g_{m1}+g_{m2})R_{ss}}$$ [As in (i)]

It does not matter how the mismatch is distributed.

(iii) Assuming mismatch in g_m, $R_{D1} = R_{D2} = R_D$.
From the equations in Ⓐ g_m & R_D appear identically in the numerator for v_{o1} & v_{o2}. So, we can directly write from knowledge in (i) & (ii)

$$\frac{\Delta(v_{o1}-v_{o2})}{\Delta v_{cm}} = -\frac{R_D \Delta g_m}{1+(g_{m1}+g_{m2})R_{ss}}$$

(iv). Assuming mismatch in both g_m & R_D.

$$g_{m1} = g_m + \Delta g_m/2 \quad \Big| \quad R_{D1} = R_D + \Delta R_D/2$$
$$g_{m2} = g_m - \Delta g_m/2 \quad \Big| \quad R_{D2} = R_D - \Delta R_D/2$$

$$V_{o1} - V_{o2} = -\frac{V_{cm}}{1+2g_mR_{SS}}\left[\left(g_m + \Delta g_m/2\right)\left(R_D + \Delta R_D/2\right) - \left(g_m - \frac{\Delta g_m}{2}\right)\left(R_D - \Delta R_D/2\right)\right]$$

$$= \frac{-2V_{cm}}{1+2g_mR_{SS}}\left[\frac{\Delta g_m}{2} R_D + g_m \frac{\Delta R_D}{2}\right]$$

$$= -\frac{V_{cm}}{1+2g_mR_{SS}}\left[R_D \Delta g_m + g_m \Delta R_D\right]$$

$$\Delta A_{CM-DM} = \frac{\Delta(V_{o1}-V_{o2})}{\Delta V_{cm}} = -\left[\frac{R_D\Delta g_m + g_m \Delta R_D}{1+2g_mR_{SS}}\right]$$

(B) For differential mode input.

$$v_{in1} = -v_{in2} = v_{dm}, \quad v_{in1} - v_{in2} = 2v_{dm}.$$

From Eqn (1) & (2)

$$v_{o1} = -\frac{g_{m1}R_{d1}}{1+(g_{m1}+g_{m2})R_{ss}}(1+2g_{m2}R_{ss})v_{dm}.$$

$$v_{o2} = +\frac{g_{m2}R_{d2}}{1+(g_{m1}+g_{m2})R_{ss}}(1+2g_{m1}R_{ss})v_{dm}.$$

$$v_{o1}-v_{o2} = \frac{-v_{dm}}{1+(g_{m1}+g_{m2})R_{ss}}\left[(g_{m1}R_{d1}+g_{m2}R_{d2}) + 2R_{ss}g_{m1}g_{m2}(R_{d1}+R_{d2})\right] \quad \text{--- (3)}$$

(i) Assuming mismatch in R_D, $g_{m1}=g_{m2}=g_m$.

$$A_{dm} = \left(\frac{v_{o1}-v_{o2}}{2v_{dm}}\right) = \frac{1}{2}\cdot\frac{-g_m}{1+2g_mR_{ss}}\left[R_{d1}+R_{d2}+2g_mR_{ss}(R_{d1}+R_{d2})\right]$$

$$= -\frac{g_m}{2}(R_{d1}+R_{d2}).$$

(ii) Assuming mismatch in g_m, $R_{D1}=R_{D2}=R_D$,

$$A_{dm} = \frac{v_{o1}-v_{o2}}{2v_{dm}} = -\frac{1}{2}\,\frac{R_D.(g_{m1}+g_{m2}+4R_{ss}g_{m1}g_{m2})}{1+(g_{m1}+g_{m2})R_{ss}}.$$

(iii) When both g_m & R_D are in mismatch. Eqn. (3) can be used.

(C) Common Mode Rejection Ratio (CMRR).

$$|CMRR| = \left|\left(\frac{A_{DM}}{A_{CM-DM}}\right)\right|.$$

$$\boxed{CMRR = \frac{1}{2}\left|\frac{(g_{m1}R_{d1}+g_{m2}R_{d2})+2g_{m1}g_{m2}R_{ss}(R_{d1}+R_{d2})}{g_{m1}R_{d1}-g_{m2}R_{d2}}\right|}$$