EE 210 **Spring 2017-18**

HW 4

Assigned: 13/03/18 Due: 19/03/18

You are encouraged to discuss these problems with others, but you need to write up the actual solutions alone. Start early, and come to office hours (TBA) with any doubts. Your only have to submit your solutions to **the questions marked** [†]. *Drop off your submission in the dropbox labeled EE210 in the EE office by 5.30 pm on the due date.*

1. [†] Prove the following properties of the Fourier transform. Suppose

$$x(t) \xrightarrow{\text{FT}} X(\omega), \quad y(t) \xrightarrow{\text{FT}} Y(\omega).$$

Prove the following properties.

(a) (Linearity) For complex numbers a and b, $ax(t) + by(t) \xrightarrow{\text{FT}} aX(\omega) + bY(\omega)$

(b) (Time shift) For real τ, $x(t-\tau) \xrightarrow{\text{FT}} e^{-j\omega\tau}X(\omega)$

(c) (Frequency shift) For real ω_0, $x(t)e^{j\omega_0 t} \xrightarrow{\text{FT}} X(\omega - \omega_0)$

(d) For real $x(t)$, $X(\omega) = X^*(-\omega)$. For real and even $x(t)$, $X(\omega)$ is real and even. For real and odd $x(t)$, $X(\omega)$ is purely imaginary and odd.

Note: You might want to prove Property (c) using Property (b) and the duality principle.

2. Prove that $\int_{-\infty}^{\infty} \operatorname{sinc}(t)dt = \int_{-\infty}^{\infty} \operatorname{sinc}^2(t)dt = 1$. (Recall that $\operatorname{sinc}(x) = \frac{\sin(\pi x)}{\pi x}$.)

3. Obtain the Fourier transform of the signal $x(t)$, where

$$x(t) = \begin{cases} 0 & x \in (\infty, -1.5] \\ (x+1.5) & x \in (-1.5, -0.5] \\ 1 & x \in (-0.5, 0.5] \\ 1.5 - x & x \in (0.5, 1.5] \\ 0 & x > 1.5 \end{cases}.$$

4. [†] Compute the Fourier transform corresponding to the following signals.

(a) $e^{-at}\cos(\omega_0 t)u(t) \quad (a > 0)$

(b) $e^{-3|t|}\sin(2t)$

(c) $x(t) = \begin{cases} 1 - t^2 & t \in (0,1) \\ 0 & \text{otherwise} \end{cases}$.

(d) $x(t)$ as shown in Figure 1.

5. Compute the inverse Fourier transform corresponding to the following spectra.

(a) $X(\omega) = \frac{2\sin[3(\omega - 2\pi)]}{\omega - 2\pi}$

(b) $X(\omega) = \cos(4\omega + \pi/3)$

(c) $X(\omega)$ as shown in Figure 2(a).

(d) $X(\omega)$ as shown in Figure 2(b).

6. [†] Let $p(t)$ denote the periodic triangular pulse train as shown in Fig. 6.

(a) Compute the Fourier series coefficients p_k corresponding to $p(t)$

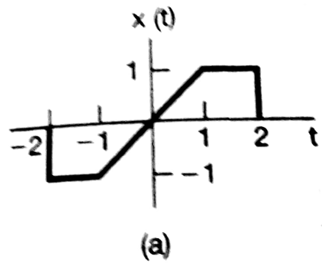

Figure 1: $x(t)$ for Problem 4(d)

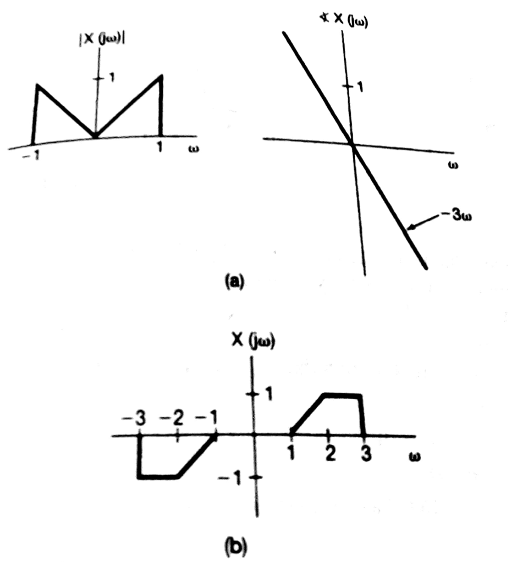

Figure 2: Spectra for Problems 5(c) and 5(d)

(b) Compute and sketch the Fourier transform $P(\omega)$ corresponding to $p(t)$

(c) Let $x(t)$ be an aperiodic signal. Define $y(t) = x(t)p(t)$. Obtain an expression for $Y(\omega)$.

(d) Sketch $Y(\omega)$ when $x(t) = \text{sinc}(t)$.

7. Let $X(\omega)$ be the Fourier transform of the signal $x(t)$ shown in Figure 4. Do the following computations without explicitly evaluating $X(\omega)$.

   (a) $X(0)$

   (b) $\int_{-\infty}^{\infty} X(\omega) d\omega$

   (c) $\int_{-\infty}^{\infty} X(\omega) e^{j\omega} d\omega$

   (d) $\int_{-\infty}^{\infty} |X(\omega)|^2 d\omega$

   (e) Sketch the inverse Fourier transform of $\text{Real}(X(\omega))$. (Here, $\text{Real}(a)$ denotes the real part of a.)

8. [†] Consider an LTI system whose response to the input $x(t) = [e^{-t} + e^{-3t}]u(t)$ is $y(t) = (2e^{-t} - 2e^{-4t})u(t)$.

   (a) Determine the frequency response of this system.

   (b) Determine the impulse response.

   (c) Find the differential equation relating the input to the output.

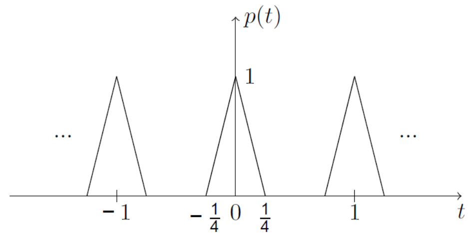

Figure 3: $p(t)$

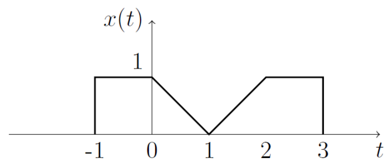

Figure 4: $x(t)$ for Problem 7

9. The goal of this problem is to understand the effect of non-linear phase response. Consider the LTI system with frequency response

$$H(\omega) = \frac{a - j\omega}{a + j\omega}.$$

Here, $a > 0$.

   (a) Sketch the magnitude and phase response of the system.

   (b) Setting $a = 1$, determine the output of the system to the input

$$\cos(t/\sqrt{3}) + \cos(t) + \cos(\sqrt{3}t).$$

10. [†] Suppose $g(t) = x(t)\cos(t)$ and the Fourier transform of $g(t)$ is

$$G(\omega) = \begin{cases} 1 & |\omega| \leq 2 \\ 0 & \text{otherwise} \end{cases}.$$

   (a) Determine $x(t)$

   (b) Specify the Fourier transform of $x_1(t)$ such that $g(t) = x_1(t)\cos(2t/3)$

11. The output $y(t)$ of a causal LTI system is related to the input $x(t)$ by the equation

$$y'(t) + 10y(t) = \int_{-\infty}^{\infty} x(s)z(t-s)ds - x(t),$$

where $z(t) = e^{-t}u(t) + 3\delta(t)$.

   (a) Find the frequency response of this system.

   (b) Find the impulse response of this system.