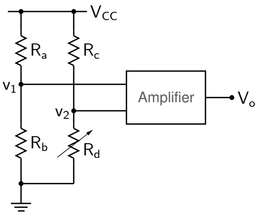
Common-mode and differential-mode voltages



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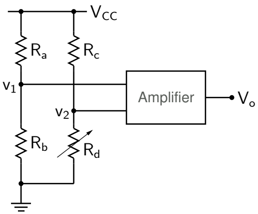

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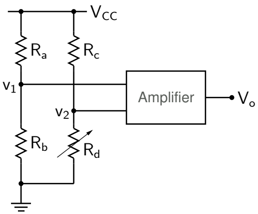
$R_d = R + \Delta R$ varies with the quantity to be measured. Typically, ΔR is a small fraction of R.



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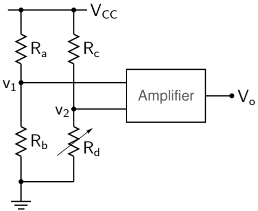

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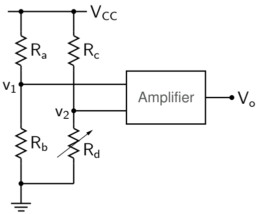

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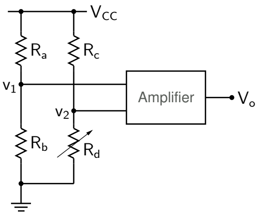

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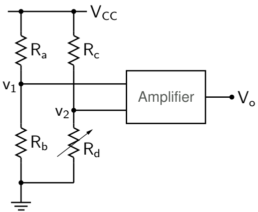
where $x = \Delta R/R$.

Common-mode and differential-mode voltages



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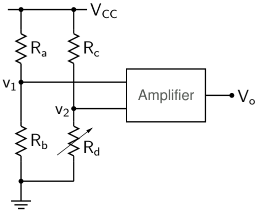
where $x = \Delta R/R$.

For example, with $V_{CC} = 15\ V$, $R = 1\,\text{k}$, $\Delta R = 0.01\,\text{k}$,

$v_1 = 7.5\ V$,

$v_2 = 7.5 + 0.0375\ V$.

Common-mode and differential-mode voltages



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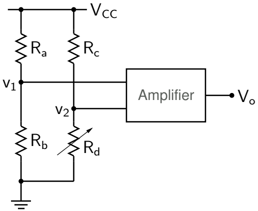
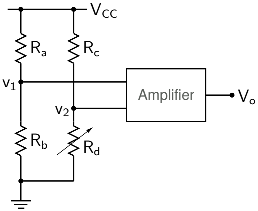


$v_1 = 7.5\ V$, $v_2 = 7.5 + 0.0375\ V$.

The amplifier should only amplify $v_2 - v_1 = 0.0375\ V$ (since that is the signal arising from ΔR).



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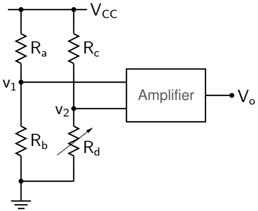

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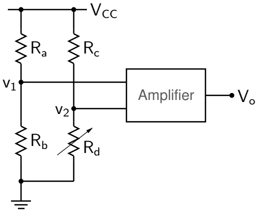

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In the above example, $v_c \approx 7.5\ V,\ \ v_d = 37.5\,\mathrm{m}V\,.$

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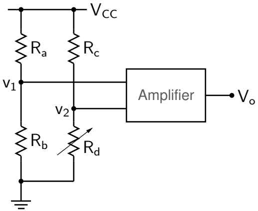

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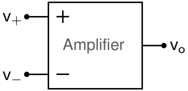
Note that the common-mode voltage is quite large compared to the differential-mode voltage.

This is a common situation in transducer circuits.



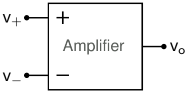
$$v_+ = v_c + v_d/2$$

$$v_- = v_c - v_d/2$$

An ideal amplifier would only amplify the difference $(v_+ - v_-)$, giving

$v_o = A_d\,(v_+ - v_-) = A_d\, v_d$,

where A_d is called the "differential gain" or simply the gain (A_V).



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In practice, the output can also have a common-mode component:

$v_o = A_d\,v_d + A_c\,v_c$,

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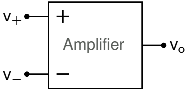

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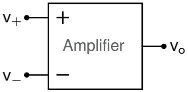
$$\boxed{\mathrm{CMRR} = \frac{A_d}{A_c}}$$



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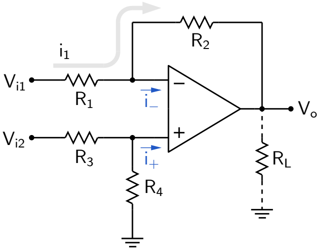
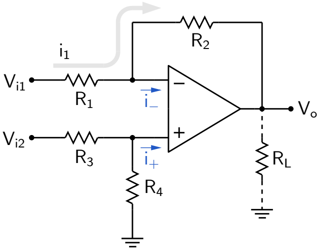
For the 741 op-amp, the CMRR is 90 dB ($\simeq$ 30,000), which may be considered to be infinite in many applications. In such cases, mismatch between circuit components will determine the overall common-mode rejection performance of the circuit.


i_1
R_2
V_{i1}
R_1
i_-
V_o
V_{i2}
R_3
i_+
R_L
R_4



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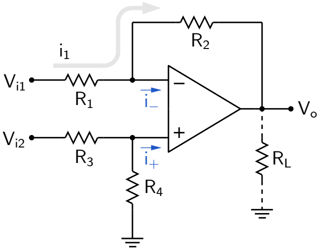
Large input resistance of op-amp $\rightarrow$ $i_+ = 0$, $V_+ = \dfrac{R_4}{R_3 + R_4}\, V_{i2}$.



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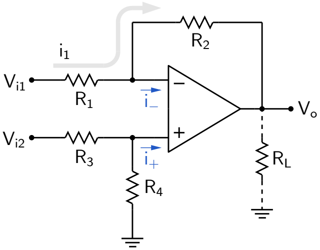

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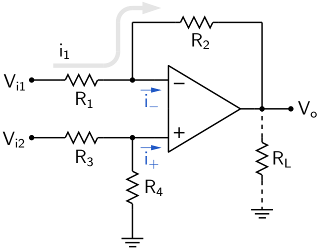
$i_- \approx 0 \rightarrow V_o = V_- - i_1\, R_2 \approx V_+ - \dfrac{R_2}{R_1}\,(V_{i1} - V_+)$.

Op-amp circuits (linear region)



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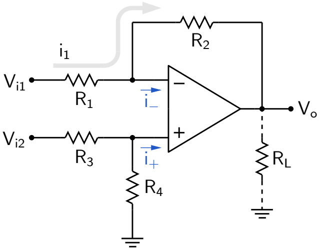
Substituting for V_+ and selecting $\dfrac{R_4}{R_3} = \dfrac{R_2}{R_1}$, we get (show this)

$V_o = \dfrac{R_2}{R_1}\,(V_{i2} - V_{i1})$.



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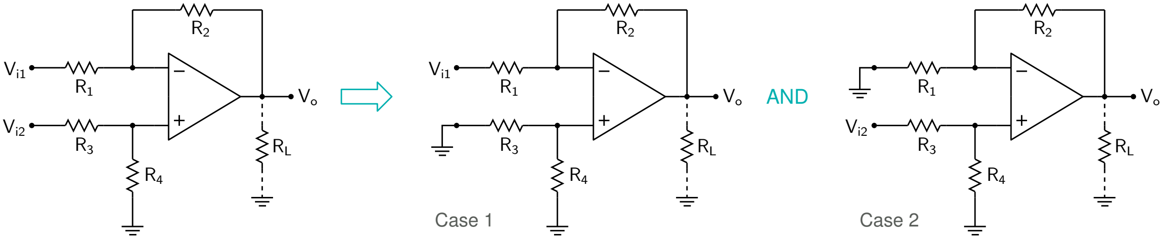
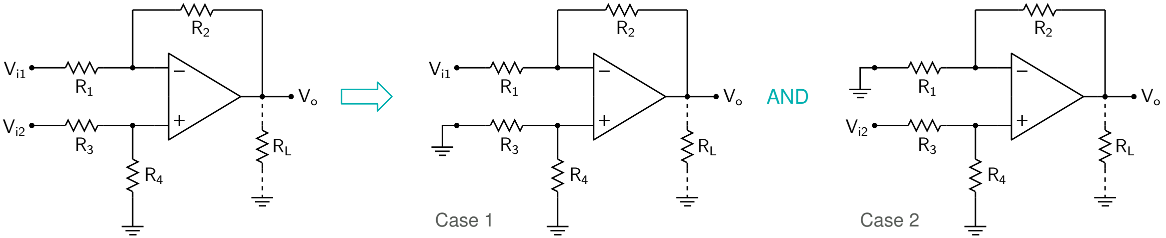
$$V_o = \frac{R_2}{R_1}\,(V_{i2} - V_{i1})\,.$$

The circuit is a "difference amplifier."



Method 2:

Since the op-amp is operating in the linear region, we can use superposition:



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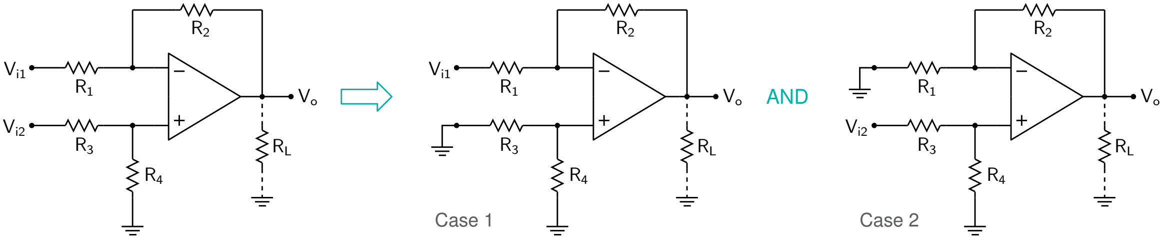
Case 1: Inverting amplifier (note that $V_+ = 0\ V$).

$$\rightarrow V_{o1} = -\frac{R_2}{R_1}\, V_{i1}\,.$$



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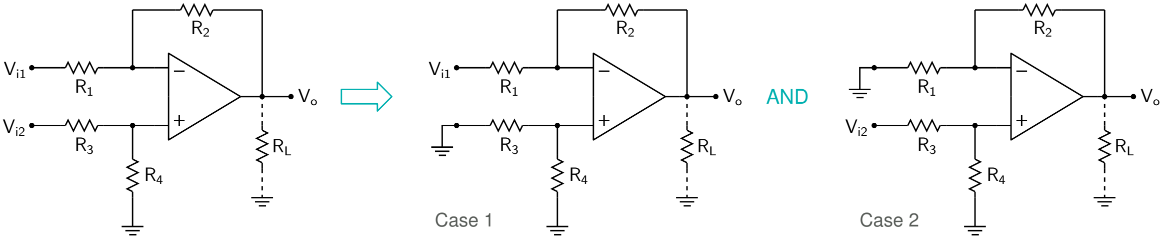
Case 2: Non-inverting amplifier, with $V_i = \dfrac{R_4}{R_3 + R_4} V_{i2}$.

$$\rightarrow V_{o2} = \left(1 + \frac{R_2}{R_1}\right)\left(\frac{R_4}{R_3 + R_4}\right) V_{i2} \,.$$



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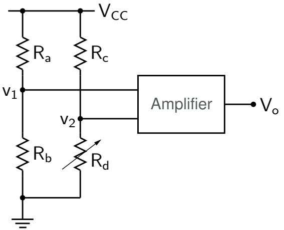
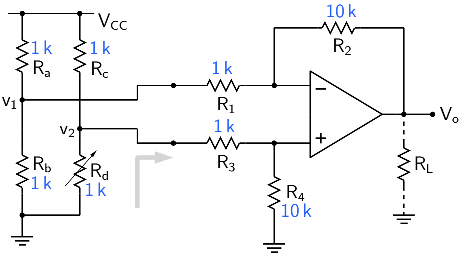
The net result is,

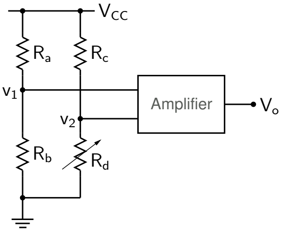
$$V_o = V_{o1} + V_{o2} = \left(1 + \frac{R_2}{R_1}\right)\left(\frac{R_4}{R_3 + R_4}\right) V_{i2} - \frac{R_2}{R_1}\, V_{i1} = \frac{R_2}{R_1}\,(V_{i2} - V_{i1})\,,\ \text{if}\ \frac{R_4}{R_3} = \frac{R_2}{R_1}.$$

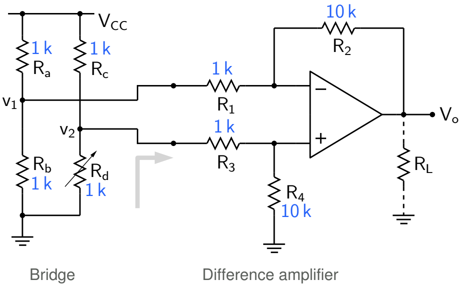


V_{CC}
R_a
R_c
v_1
v_2
Amplifier
V_o
R_b
R_d



V_{CC}
1 k
R_a
1 k
R_c
v_1
v_2
R_b
1 k
R_d
1 k
1 k
R_1
1 k
R_3
10 k
R_2
R_4
10 k
V_o
R_L
Bridge
Difference amplifier

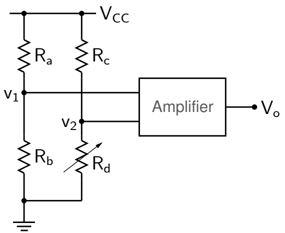


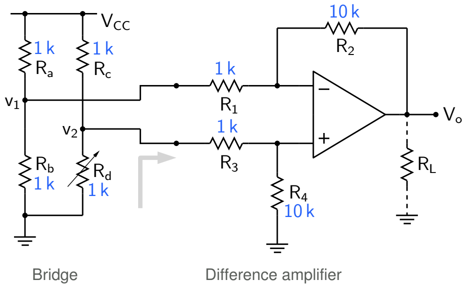


The resistance seen from v_2 is $(R_3 + R_4)$ which is small enough to cause v_2 to change.

This is not desirable.

Difference amplifier





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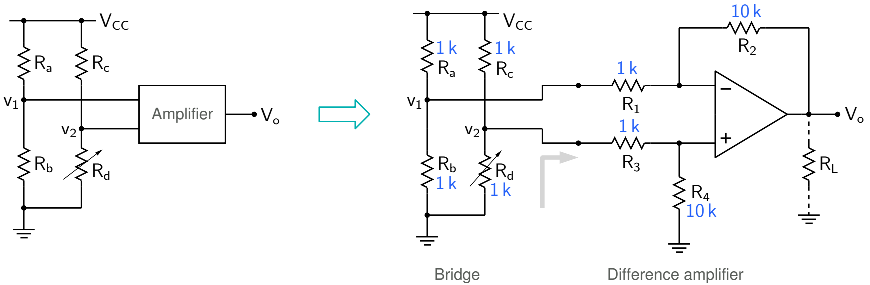
$\rightarrow$ need to improve the input resistance of the difference amplifier.

Difference amplifier



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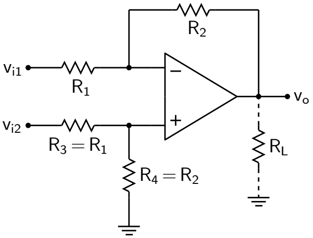
We will discuss an improved difference amplifier later. Before we do that, let us discuss another problem with the above difference amplifier which can be important for some applications (next slide).



$$v_{i1} = v_c - (v_d/2)$$

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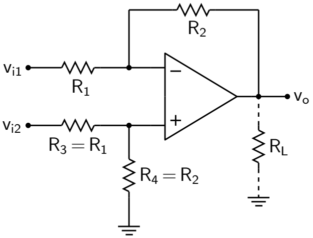

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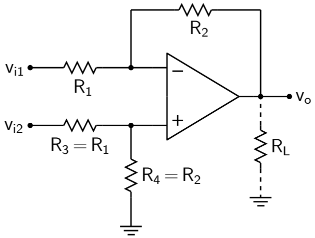
The output voltage depends only on the differential-mode signal $(v_{i2} - v_{i1})$,
i.e., A_c (common-mode gain) $= 0$.

Difference amplifier



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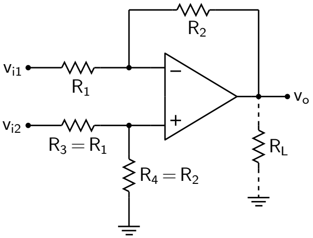

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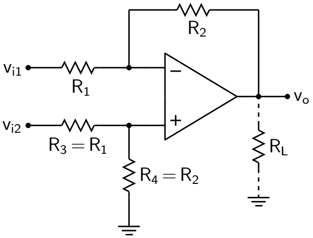

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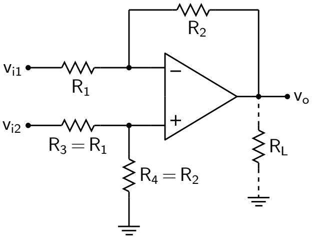
|A_c| = \frac{\Delta R}{R_1 + R_2}\,\frac{R_2}{R_1} \ll |A_d| = \frac{R_2}{R_1}.
$$



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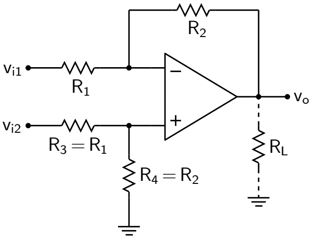
$|A_c| = \dfrac{\Delta R}{R_1 + R_2}\,\dfrac{R_2}{R_1} \ll |A_d| = \dfrac{R_2}{R_1}$. However, since v_c can be large compared to v_d, the effect of A_c cannot be ignored.

Difference amplifier



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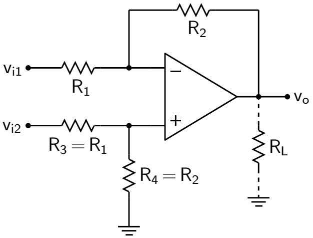

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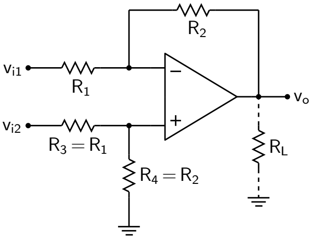

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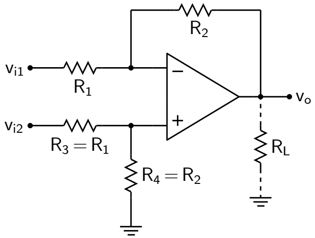
With $R_1 = 1\,\text{k}$, $R_2 = 10\,\text{k}$, $x = \dfrac{0.01\,\text{k}}{11\,\text{k}} = 0.00091 \rightarrow |A_c| = 0.00091\,\dfrac{10\,\text{k}}{1\,\text{k}} = 0.0091$, $|A_d| = \dfrac{10\,\text{k}}{1\,\text{k}} = 10$.

$|v_o^c| = |A_c\, v_c| = 0.0091 \times 7.5 = 0.068\ V$.



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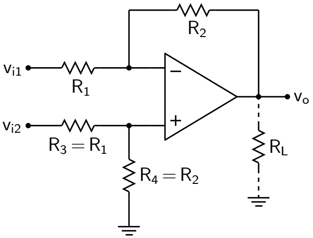

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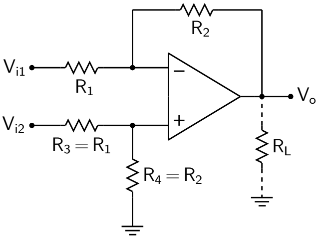
The (spurious) common-mode contribution is substantial.

If we measure v_o, we will conclude that $v_d = \dfrac{v_o}{A_d}$, but in reality, it would be different.

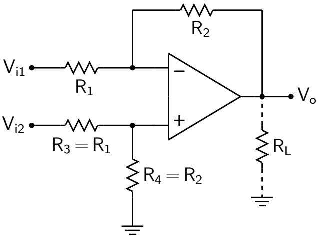
$\rightarrow$ need a circuit which will drastically reduce the common-mode component at the output.



$$A_d = \frac{R_2}{R_1}$$

$$V_o = \left(1 + \frac{R_2}{R_1}\right)\left(\frac{R_4}{R_3 + R_4}\right) V_{i2} - \frac{R_2}{R_1}\, V_{i1}$$

Difference amplifier: resistance mismatch



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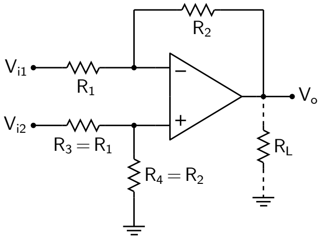

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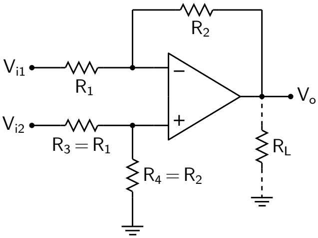

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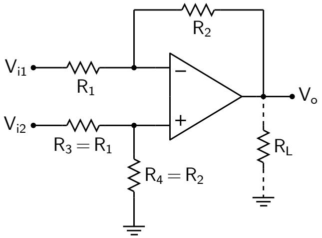

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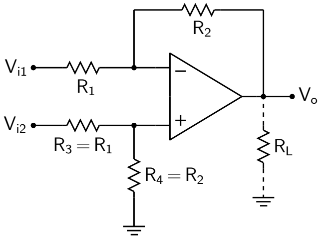

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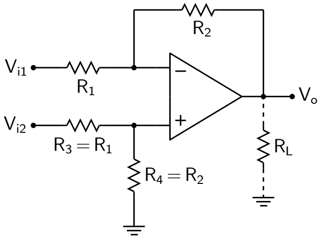

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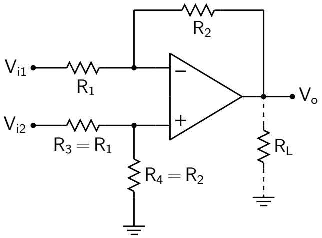
Difference amplifier: resistance mismatch



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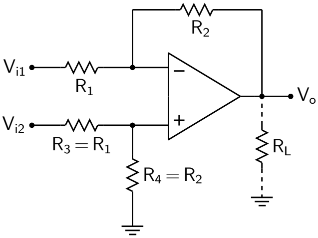

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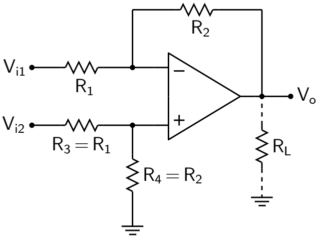

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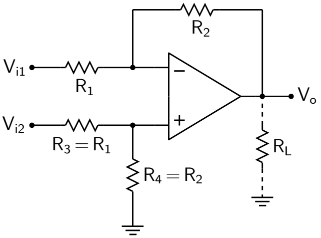

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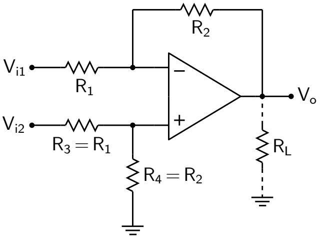

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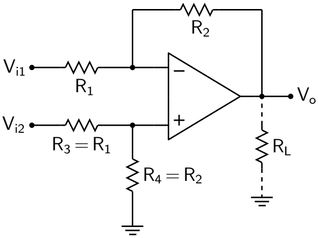
Difference amplifier: resistance mismatch



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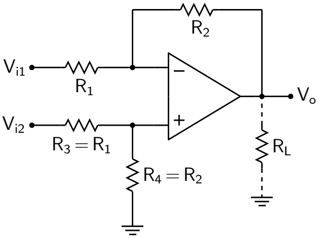
$$(1)\quad R_1^0 = R_2^0 \text{ (i.e., } R_3^0 = R_4^0\text{)}$$

$$A_c = \frac{1}{2}\,(x_1 - x_2 - x_3 + x_4)$$

$$= \frac{1}{2}\,4\,x = 2\,x \text{ (worst case)}$$



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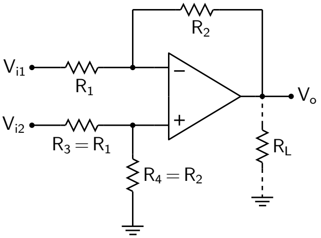

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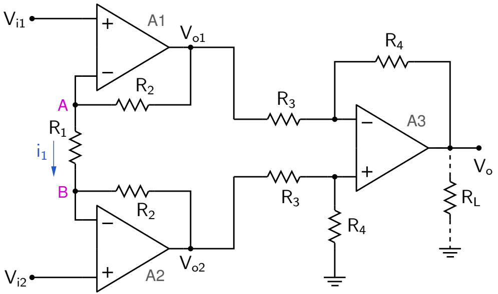
Improved difference amplifier



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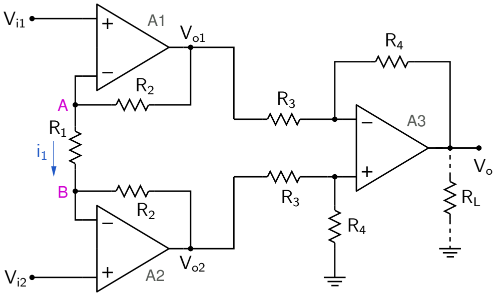

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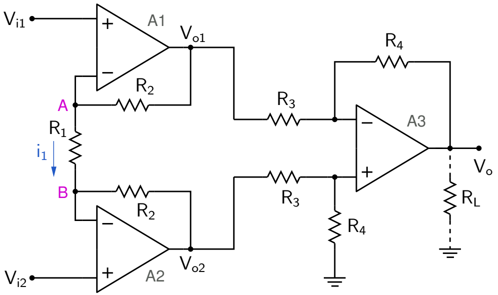

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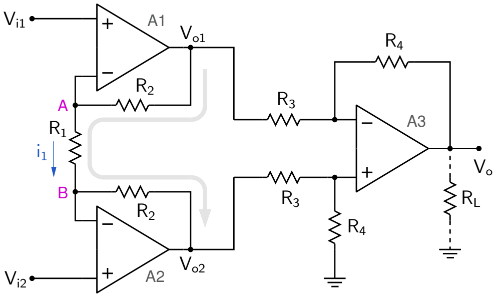

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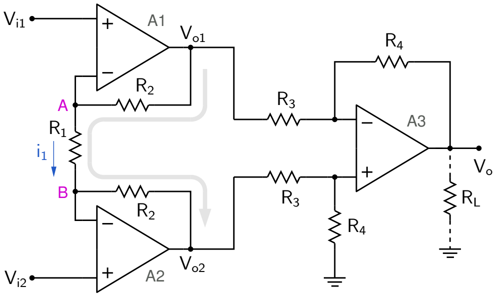

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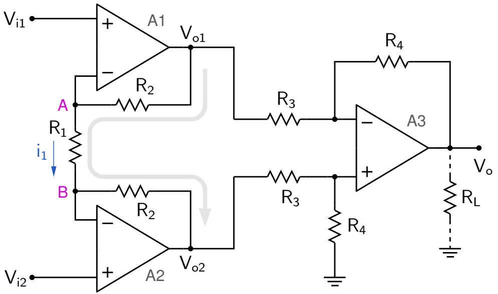

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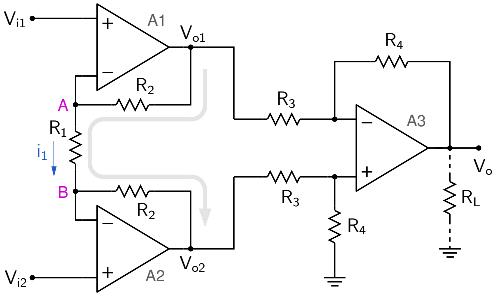

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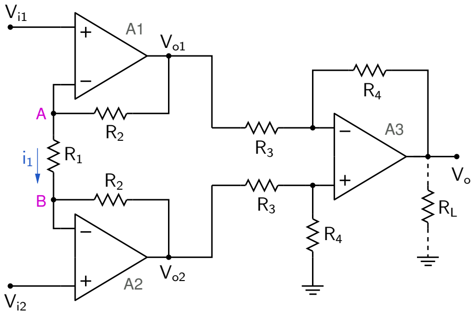
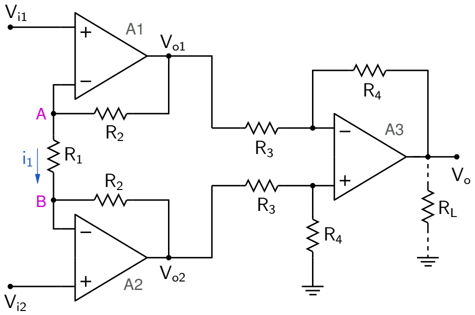
This circuit is known as the "instrumentation amplifier."

Instrumentation amplifier





The input resistance seen from V_{i1} or V_{i2} is large (since an op-amp has a large input resistance).

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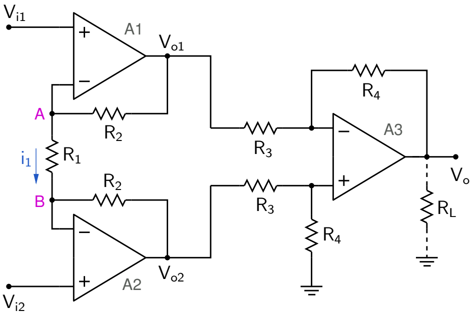

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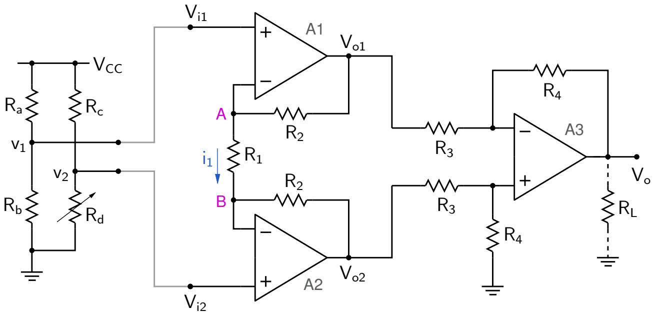

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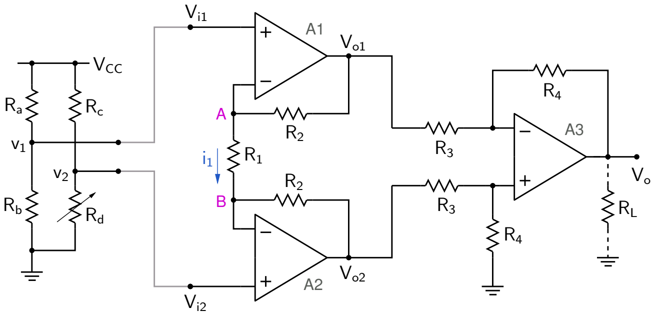

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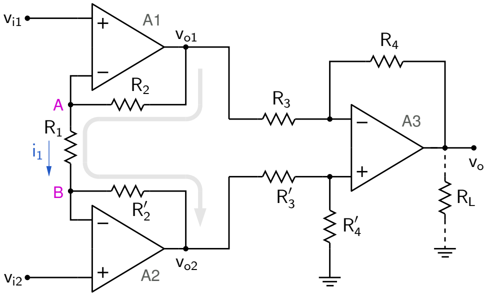
As a result, the voltages v_1 and v_2 in the bridge circuit will remain essentially the same when the bridge circuit is connected to the instrumentation amplifier.

Instrumentation amplifier: common-mode rejection



$$v_{i1} = v_c - (v_d/2)$$

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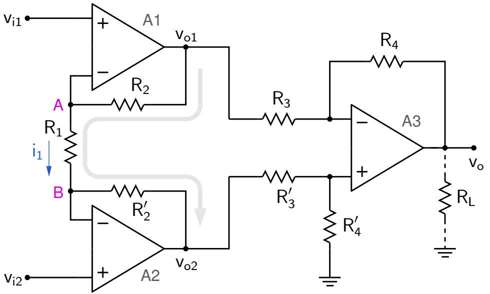

$v_{i1} = v_c - (v_d/2)$

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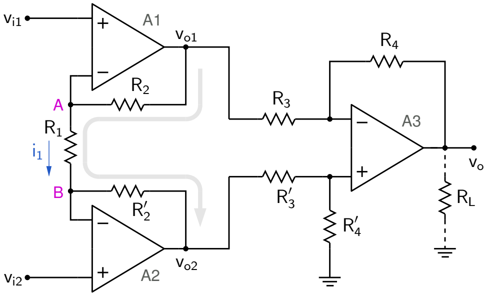
Note that v_{o1} serves as v_{i1} for the difference amplifier, and v_{o2} as v_{i2}. Let us find the differential-mode and common-mode components associated with v_{o1} and v_{o2}.

$$v'_{id} = v_{o2} - v_{o1}, \quad v'_{ic} = \frac{1}{2}\,(v_{o1} + v_{o2})$$



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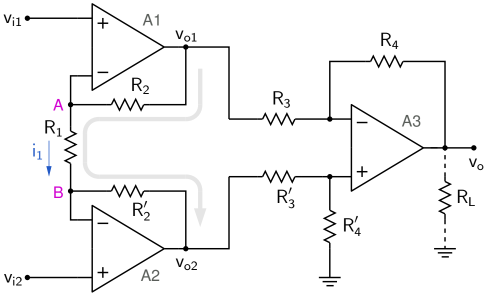

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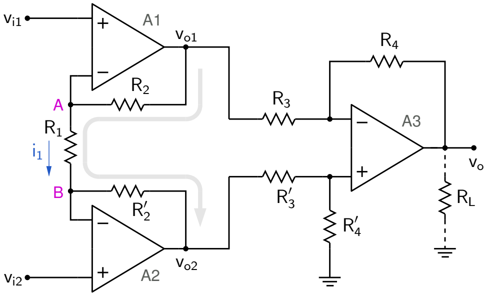

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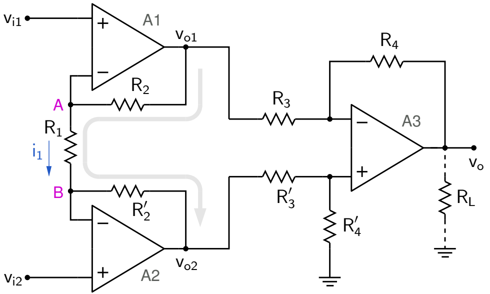

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(Note that resistor mismatch in the second stage needs to be considered, but it will have a limited effect.)

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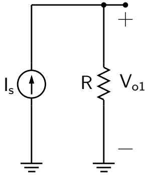

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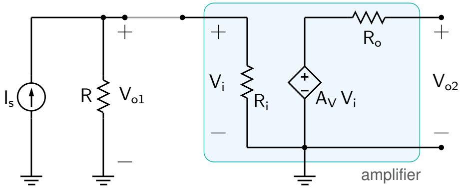
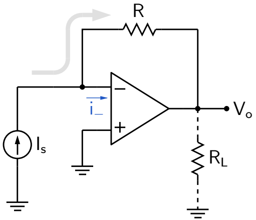


However, this simple approach will not work if the next stage in the circuit (such as an amplifier) has a finite R_i, since it will modify V_{o1} to $V_{o1} = I_s\,(R_i \parallel R)$, which is not desirable.

Current-to-voltage conversion



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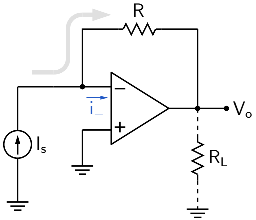

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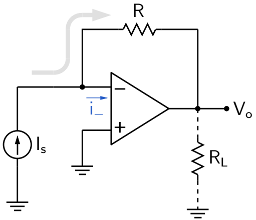

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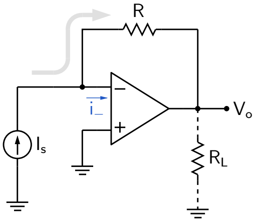

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Example: a photocurrent detector.

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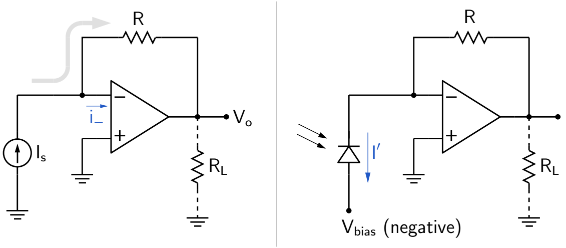

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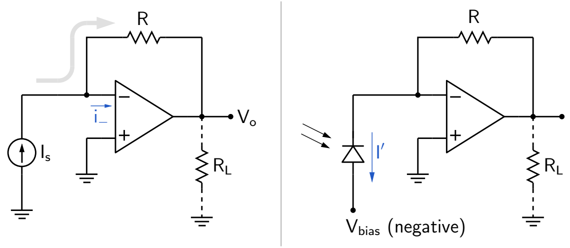

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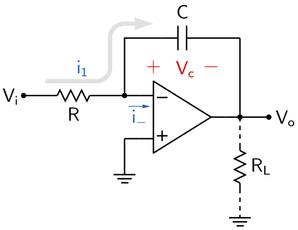
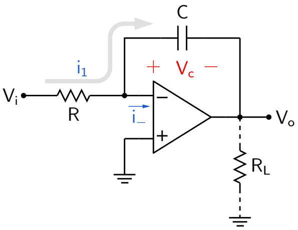
$V_o = I'\,R$. (Note: The diode is under a reverse bias, with $V_n = 0\;V$ and $V_p = V_{\text{bias}}$.)


C
i_1
+ V_c −
V_i
R
i_-
V_o
R_L



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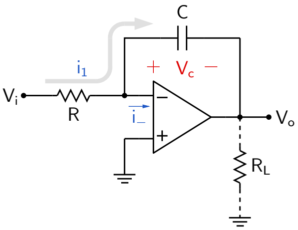

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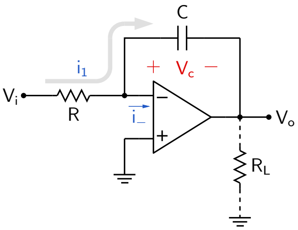
Since $i_- \approx 0$, the current through the capacitor is i_1.

$$\Rightarrow C\,\frac{dV_c}{dt} = i_1 = \frac{V_i}{R}\,.$$

Op-amp circuits (linear region)



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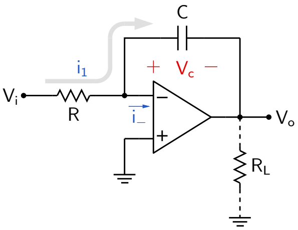

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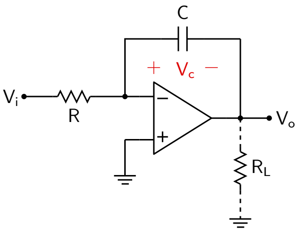
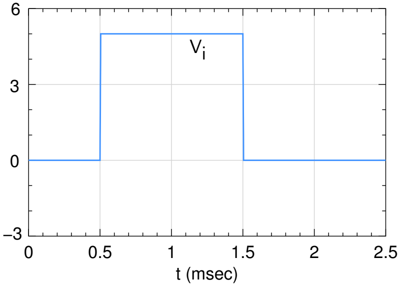
$$\boxed{V_o = -\frac{1}{RC}\int V_i\,dt}$$

The circuit works as an integrator.

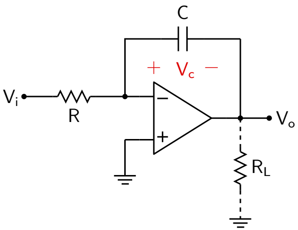
Integrator

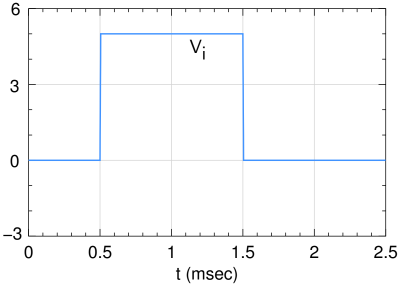




Given: $R = 10\,\text{k}$, $C = 0.2\,\mu\text{F}$.

If $V_o = 0\,\text{V}$ at $t = 0$, find $V_o(t)$ (Let $t_0 = 0.5\,\text{msec}$, $t_1 = 1.5\,\text{msec}$).





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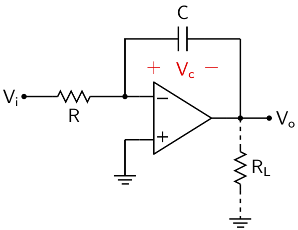
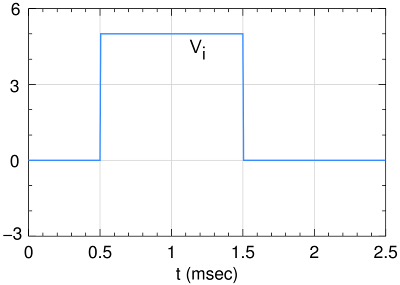
$$V_o = -\frac{1}{RC}\int V_i\,dt, \quad \tau \equiv RC = 2\,\text{msec}.$$

Integrator





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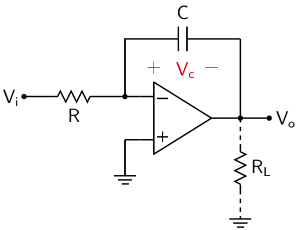

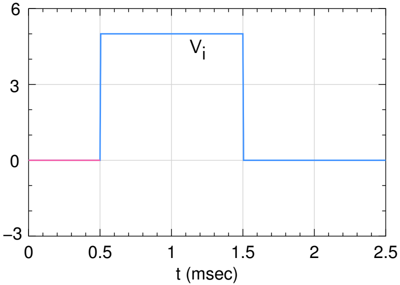

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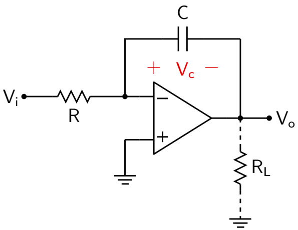
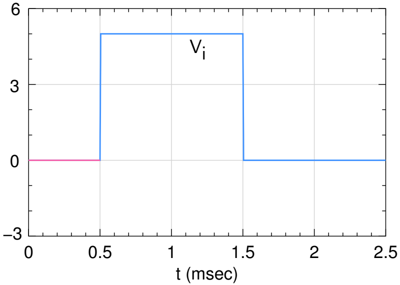
Integrator





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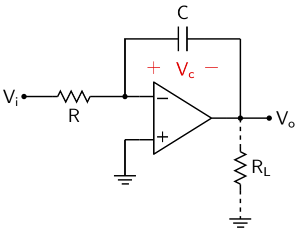
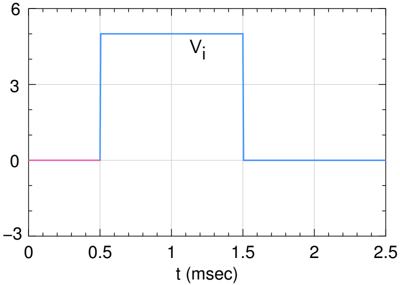
Integrator





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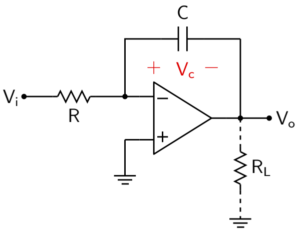

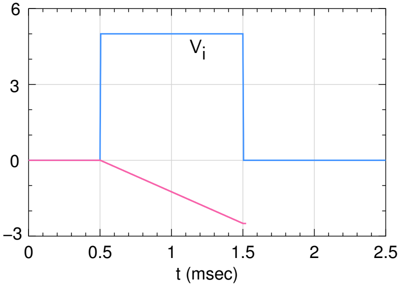

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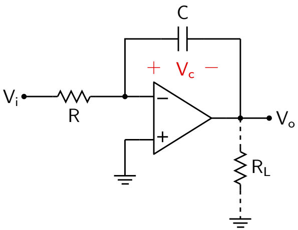
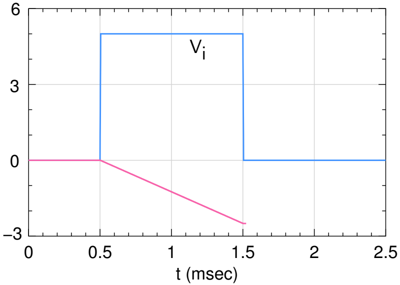
Integrator





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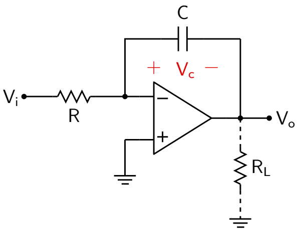

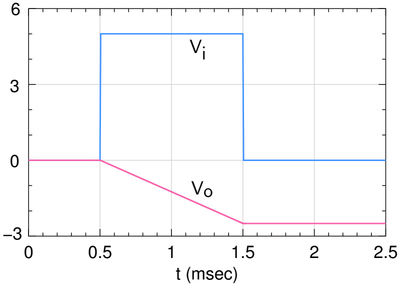

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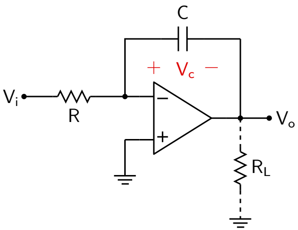

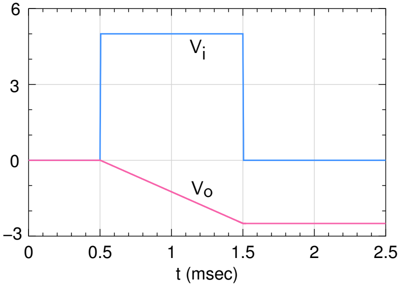

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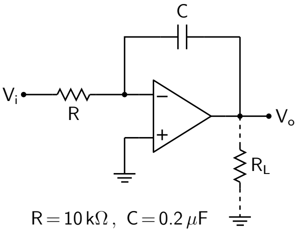
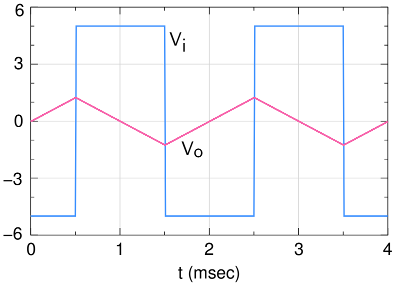
SEQUEL file: `ee101_integrator_1.sqproj`

Integrator

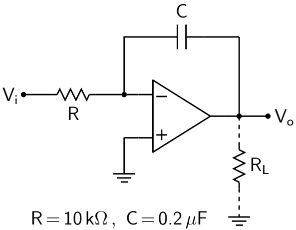


$R = 10\,k\Omega\,,\ \ C = 0.2\,\mu F$

$$V_o = -\frac{1}{RC}\int V_i\,dt$$



* An integrator can be used to convert a square wave to a triangle wave.



$R = 10\,k\Omega\,,\;\; C = 0.2\,\mu F$

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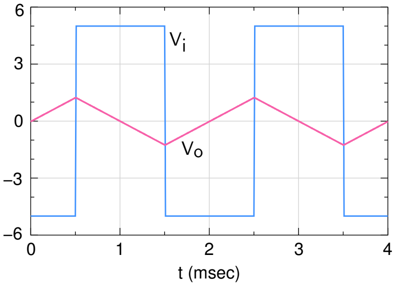
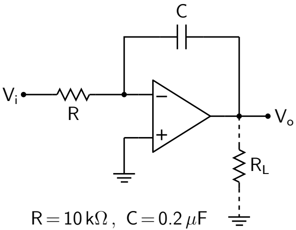


* An integrator can be used to convert a square wave to a triangle wave.
* In practice, the circuit needs a small modification, as discussed in the following.

Integrator



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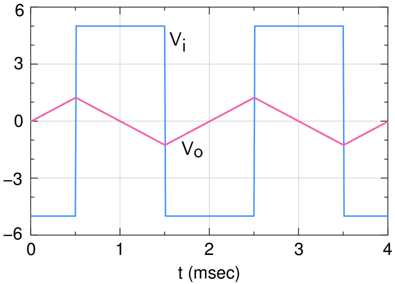
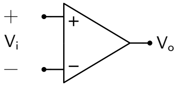
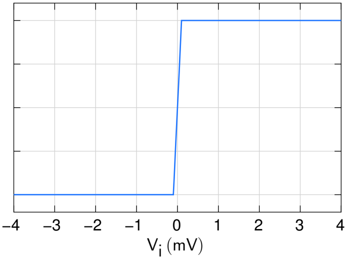


* An integrator can be used to convert a square wave to a triangle wave.
* In practice, the circuit needs a small modification, as discussed in the following.

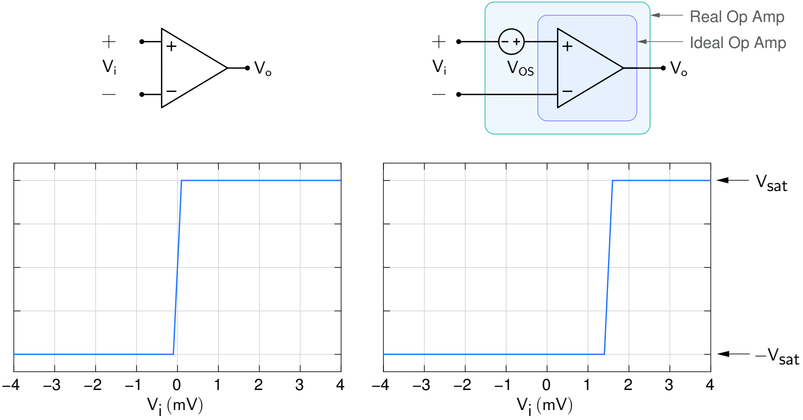
SEQUEL file: `ee101_integrator_2.sqproj`

Practical op-amps: Offset voltage

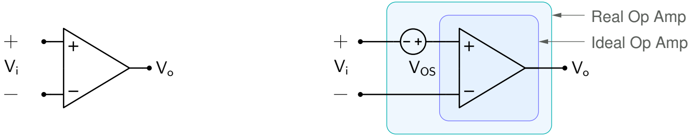


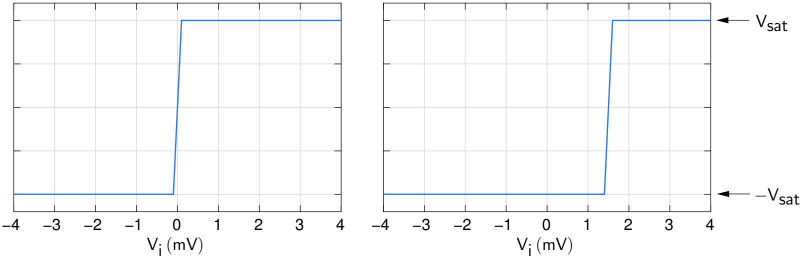


Practical op-amps: Offset voltage



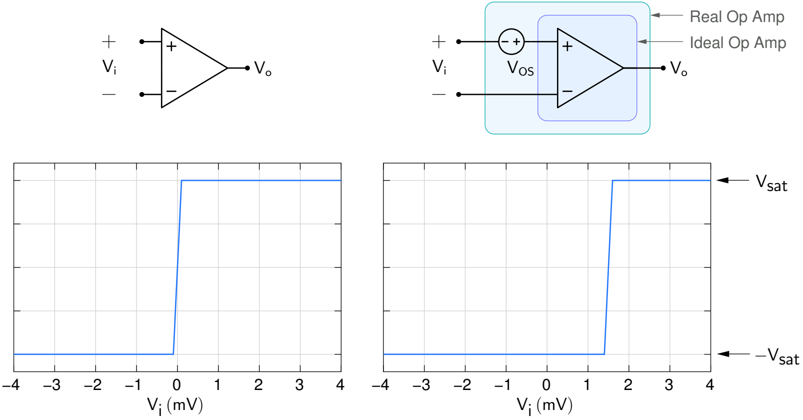
Practical op-amps: Offset voltage





For the real op-amp, $V_o = A_V((V_+ + V_{OS}) - V_-)$.

Practical op-amps: Offset voltage



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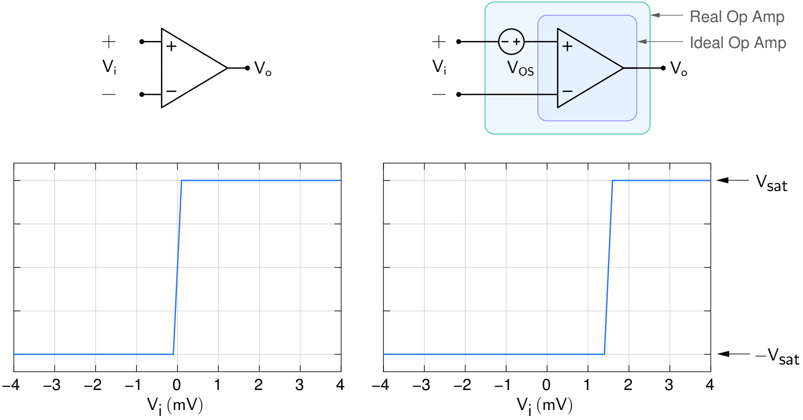

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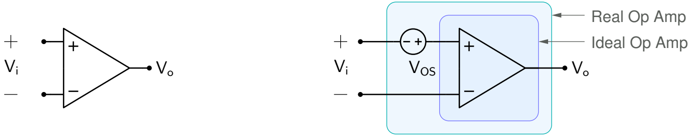
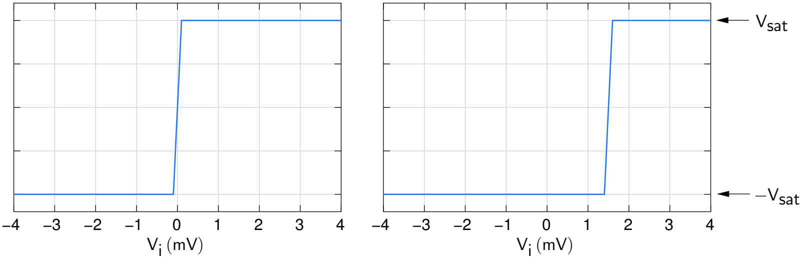
V_o versus V_i curve gets shifted (Note: V_{OS} is negative in the above example).

Practical op-amps: Offset voltage





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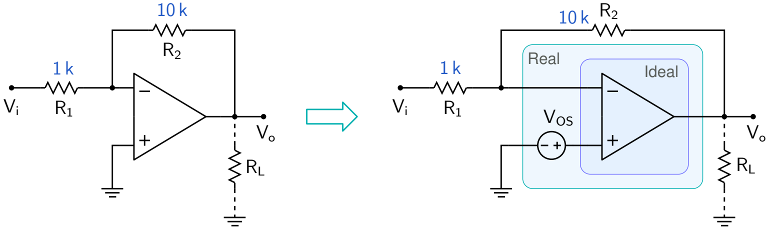
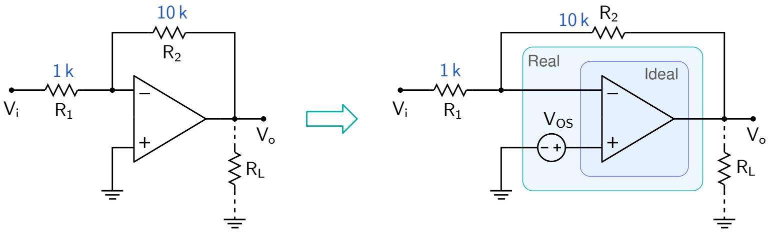
V_o versus V_i curve gets shifted (Note: V_{OS} is negative in the above example).

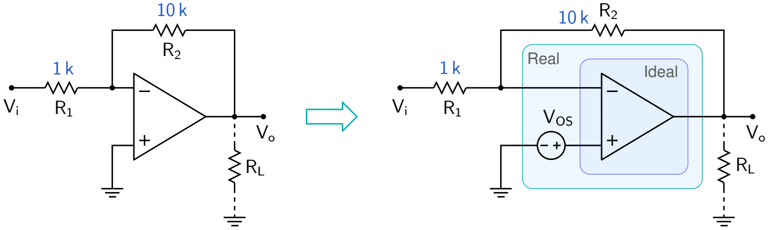
741: $-6\,\mathrm{m}V < V_{OS} < 6\,\mathrm{m}V$, OP-77: $-50\,\mu V < V_{OS} < 50\,\mu V$.


10 k
R2
1 k
Vi
R1
Vo
RL
10 k
R2
Real
Ideal
1 k
Vi
R1
VOS
Vo
RL

Effect of V_{OS}: inverting amplifier



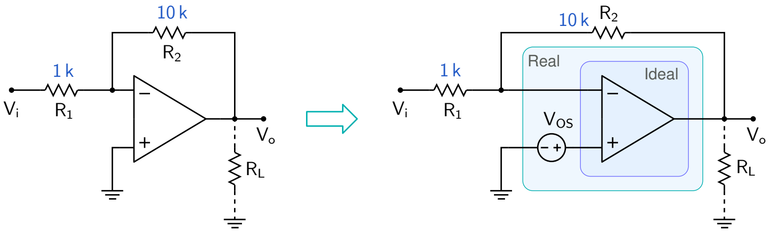
By superposition, $V_o = -\dfrac{R_2}{R_1}\, V_i + V_{OS}\left(1 + \dfrac{R_2}{R_1}\right).$



By superposition, $V_o = -\dfrac{R_2}{R_1}\, V_i + V_{OS}\left(1 + \dfrac{R_2}{R_1}\right)$.

For $V_{OS} = 2\,\mathrm{m}V$, the contribution from V_{OS} to V_o is $22\,\mathrm{m}V$,

Effect of V_{OS}: inverting amplifier

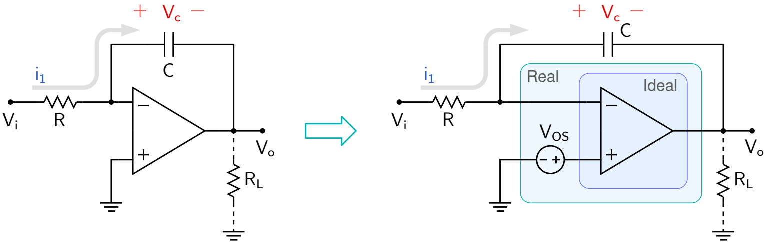


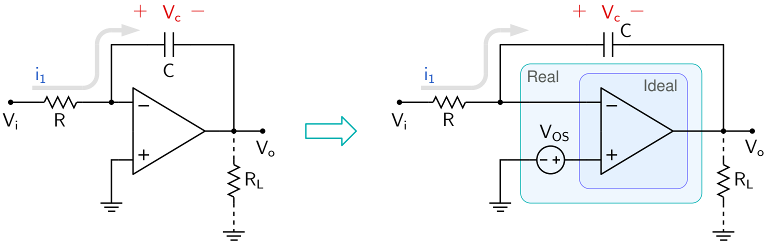
By superposition, $V_o = -\dfrac{R_2}{R_1}\,V_i + V_{OS}\left(1+\dfrac{R_2}{R_1}\right)$.

For $V_{OS} = 2\,\text{m}V$, the contribution from V_{OS} to V_o is $22\,\text{m}V$,

i.e., a DC shift of $22\,\text{m}V$.

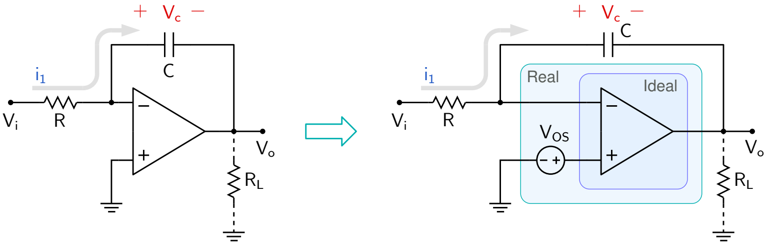
Effect of V_{OS}: integrator





$$V_- \approx V_+ = V_{OS} \rightarrow i_1 = \frac{1}{R}(V_i - V_{OS}) = C\,\frac{dV_c}{dt}\,.$$

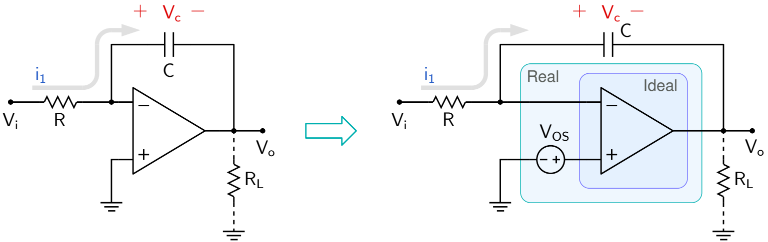
Effect of V_{OS}: integrator



$$V_- \approx V_+ = V_{OS} \rightarrow i_1 = \frac{1}{R}(V_i - V_{OS}) = C\,\frac{dV_c}{dt}\,.$$

i.e., $V_c = \dfrac{1}{RC}\displaystyle\int (V_i - V_{OS})\,dt\,.$

Effect of V_{OS}: integrator

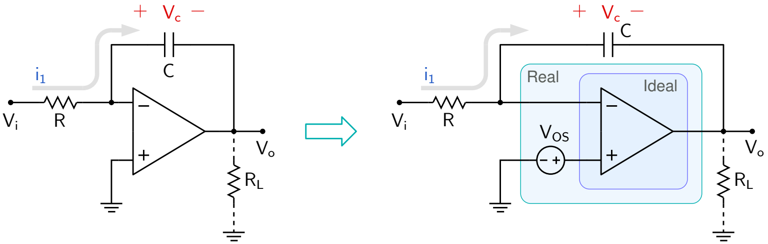


$$V_- \approx V_+ = V_{OS} \rightarrow i_1 = \frac{1}{R}(V_i - V_{OS}) = C\,\frac{dV_c}{dt}\,.$$

i.e., $V_c = \dfrac{1}{RC}\displaystyle\int (V_i - V_{OS})\,dt\,.$

Even with $V_i = 0\,V$, V_c will keep rising or falling (depending on the sign of V_{OS}).

Eventually, the Op Amp will be driven into saturation.



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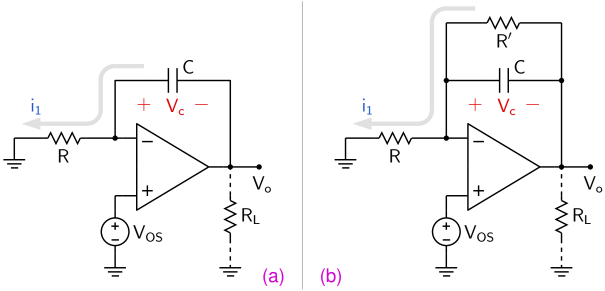
Even with $V_i = 0\,V$, V_c will keep rising or falling (depending on the sign of V_{OS}).

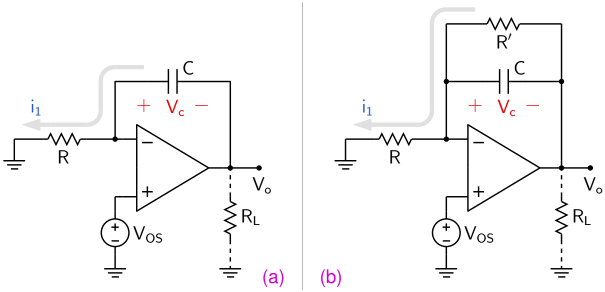
Eventually, the Op Amp will be driven into saturation.

$\rightarrow$ need to address this issue!

Effect of V_{OS}: integrator with $V_i = 0$



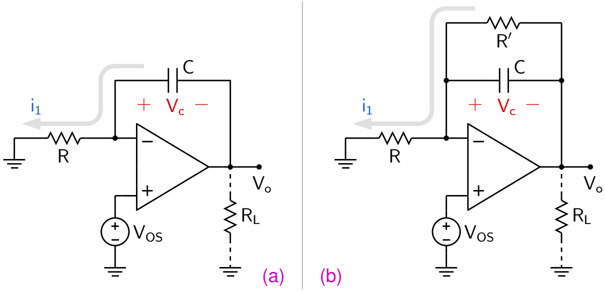
Effect of V_{OS}: integrator with $V_i = 0$



(a) $i_1 = \dfrac{V_{OS}}{R} = -C\,\dfrac{dV_c}{dt}$

$V_c = -\dfrac{1}{RC}\displaystyle\int V_{OS}\,dt \rightarrow$ op-amp saturates.

Effect of V_{OS}: integrator with $V_i = 0$



(a) $i_1 = \dfrac{V_{OS}}{R} = -C\,\dfrac{dV_c}{dt}$

$V_c = -\dfrac{1}{RC}\displaystyle\int V_{OS}\,dt \rightarrow$ op-amp saturates.

(b) There is a DC path for the current.

$\rightarrow V_o = \left(1 + \dfrac{R'}{R}\right) V_{OS}$.

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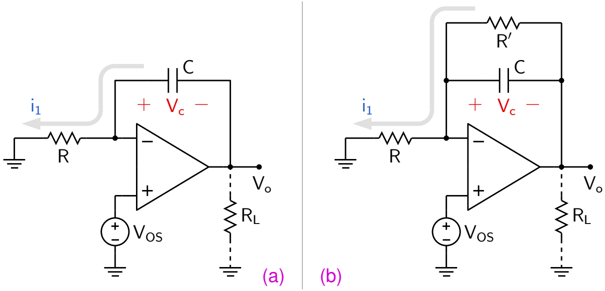

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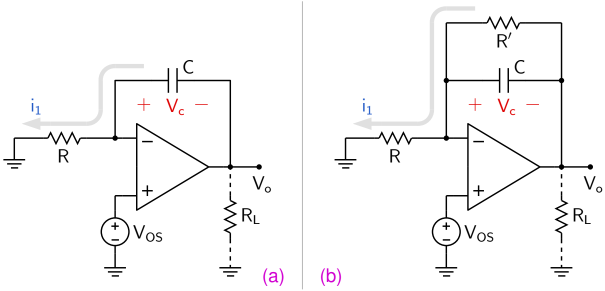
R' should be small enough to have a negligible effect on V_o.



(a) $i_1 = \dfrac{V_{OS}}{R} = -C\,\dfrac{dV_c}{dt}$

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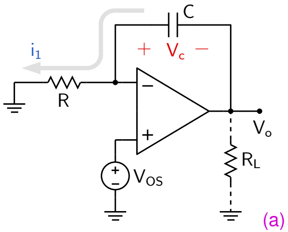
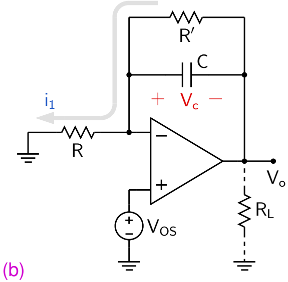
However, R' must be large enough to ensure that the circuit still functions as an integrator.

Effect of V_{OS}: integrator with $V_i = 0$





(a) $i_1 = \dfrac{V_{OS}}{R} = -C\,\dfrac{dV_c}{dt}$

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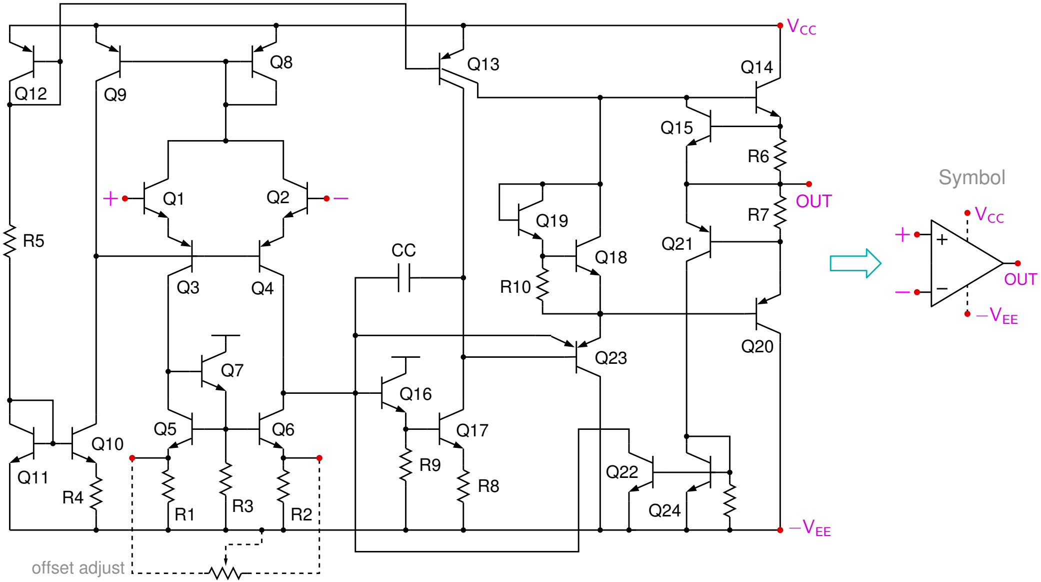
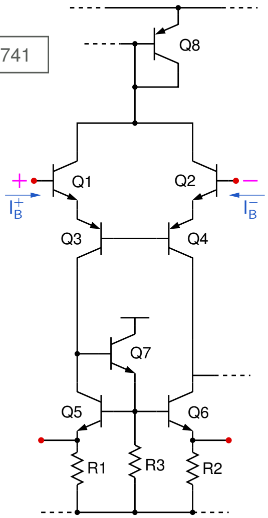
$\rightarrow R' \gg 1/\omega C$ at the frequency of interest.

Op-amp 741: offset null



Input bias currents

741



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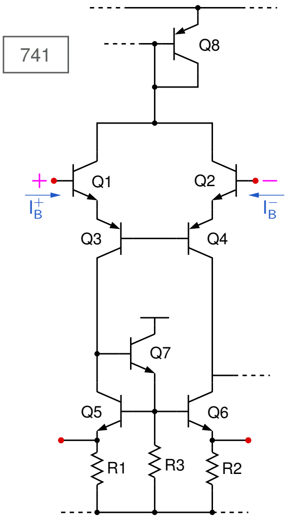

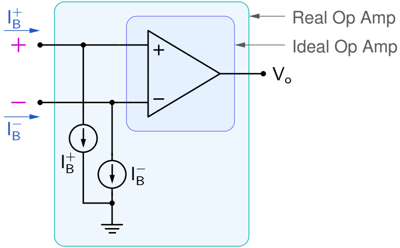

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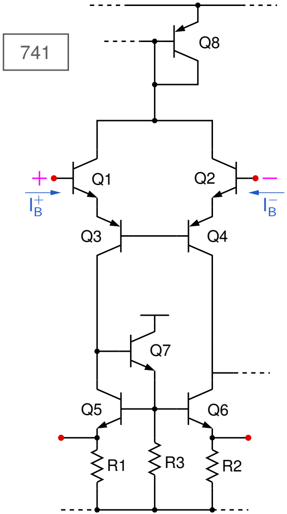
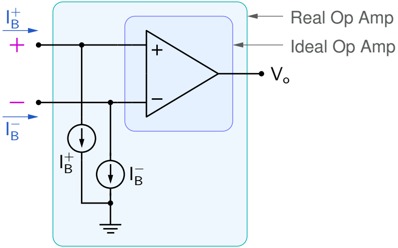
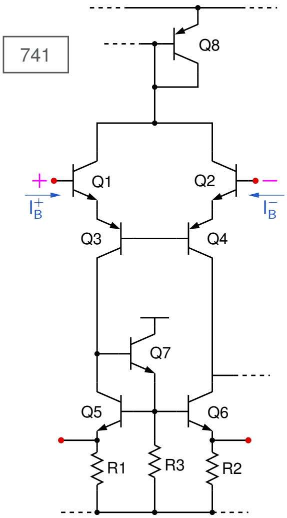


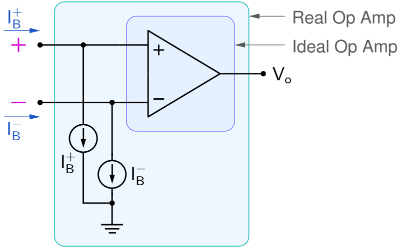


I_B^+ and I_B^- are generally not equal.

$|I_B^+ - I_B^-|$: "offset current" (I_{OS})

$(I_B^+ + I_B^-)/2$: "bias current" (I_B)





Typical values

Op–Amp	I_B	I_{OS}	V_{OS}	Type
741	80 nA	20 nA	1 mV	BJT input
OP77	1.2 nA	0.3 nA	10 μV	BJT input
411	50 pA	25 pA	0.8 mV	FET input

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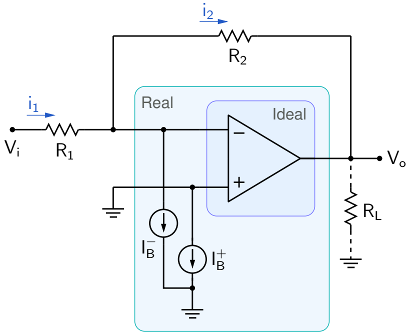
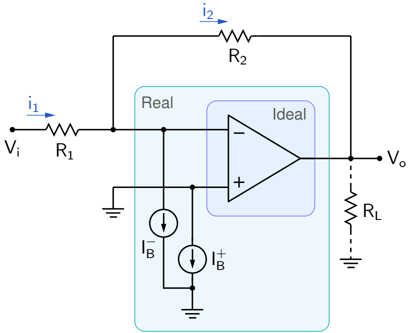
$(I_B^+ + I_B^-)/2$: "bias current" (I_B)

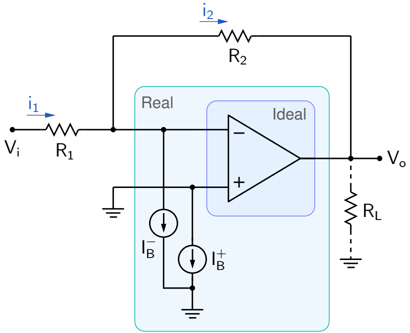


i2
R2
i1
Real
Ideal
Vi
R1
Vo
RL
IB−
IB+

Effect of bias currents: inverting amplifier



Assume that the op-amp is ideal in other respects (including $V_{OS} = 0\ V$).



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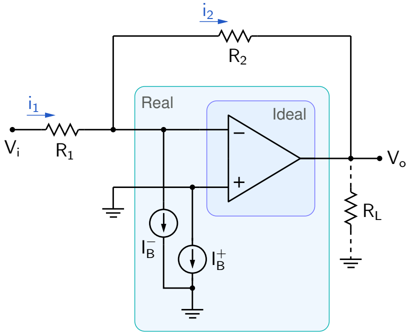
$V_- \approx V_+ = 0\ V \rightarrow i_1 = V_i / R_1$.



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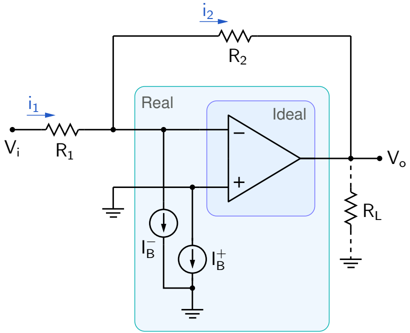

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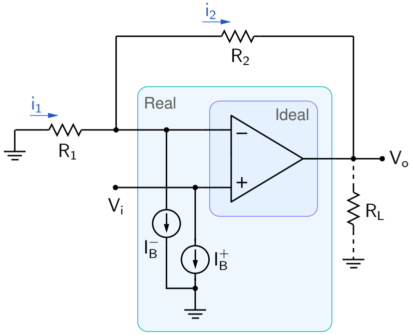
i.e., the bias current causes a DC shift in V_o.

For $I_B^- = 80\,\mathrm{n}A$, $R_2 = 10\,\mathrm{k}$, $\Delta V_o = 0.8\,\mathrm{m}V$.

Effect of bias currents: non-inverting amplifier



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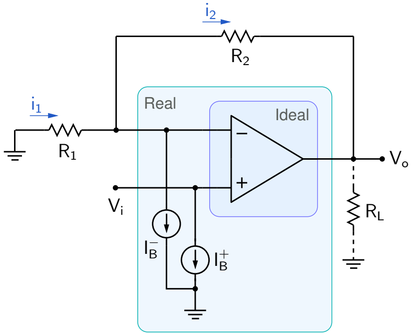

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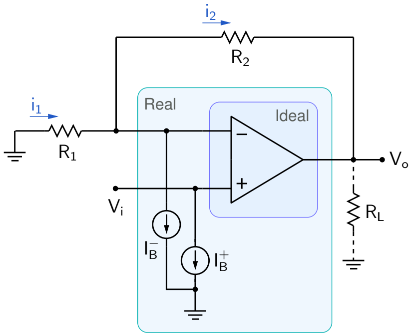

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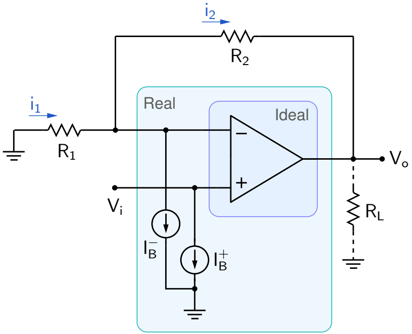

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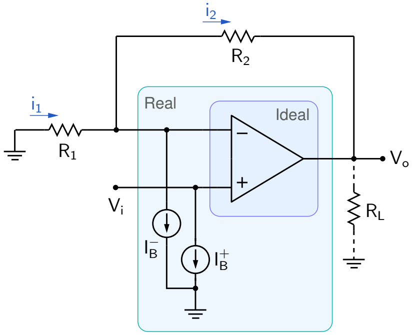

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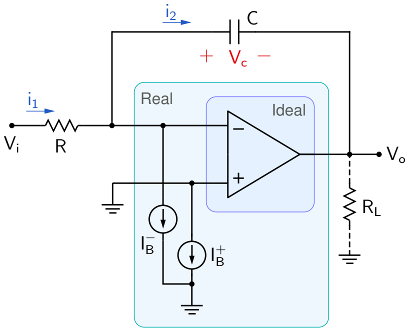
$\rightarrow$ Again, a DC shift ΔV_o.

Effect of bias currents: integrator



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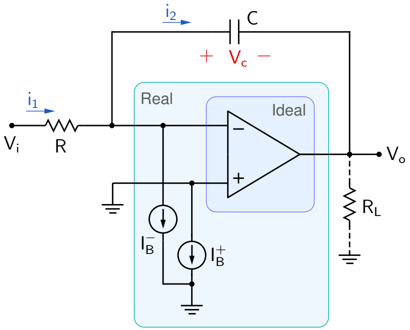
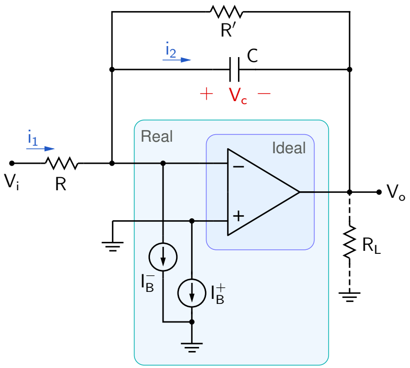


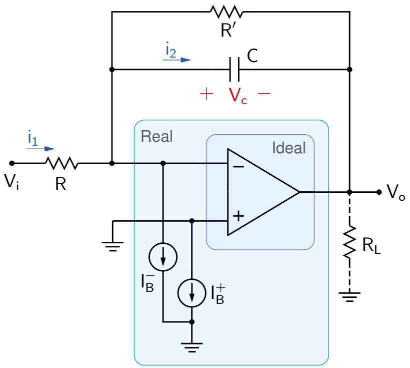
Even with $V_i = 0\,V$, $V_c = \dfrac{1}{C}\displaystyle\int -I_B^-\,dt$ will drive the op-amp into saturation.



Even with $V_i = 0\,V$, $V_c = \dfrac{1}{C}\displaystyle\int -I_B^-\,dt$ will drive the op-amp into saturation.

Connecting R' across C provides a DC path for the current, and results in a DC shift $\Delta V_o = I_B^-\,R'$ at the output.

Effect of bias currents: integrator



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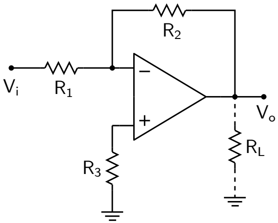
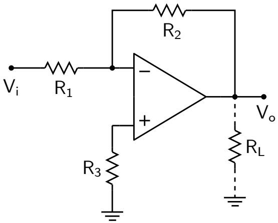
Connecting R' across C provides a DC path for the current, and results in a DC shift $\Delta V_o = I_B^- R'$ at the output.

As we have discussed earlier, R' should be small enough to have a negligible effect on V_o.

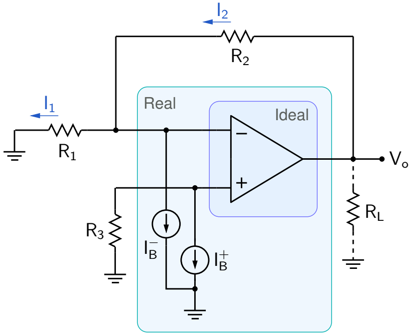
However, R' must be large enough to ensure that the circuit still functions as an integrator.

Effect of bias currents: inverting amplifier

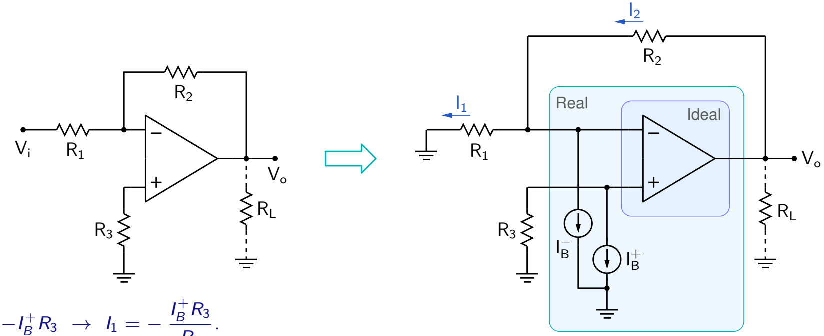




R2
Vi
R1
R3
Vo
RL



I2
R2
I1
R1
Real
Ideal
R3
Vo
RL



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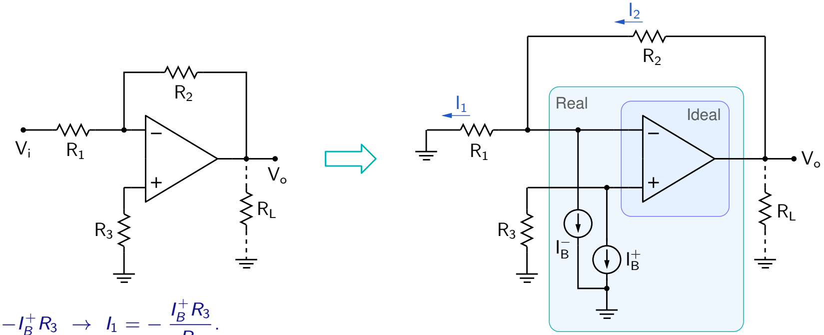
Effect of bias currents: inverting amplifier



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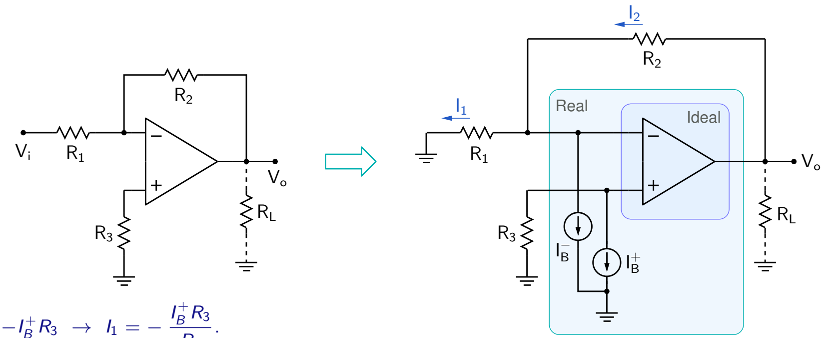

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Using $I_B = \dfrac{I_B^+ + I_B^-}{2}$, $I_{OS} = I_B^+ - I_B^-$, i.e., $I_B^+ = I_B + \dfrac{I_{OS}}{2}$, $I_B^- = I_B - \dfrac{I_{OS}}{2}$, we get

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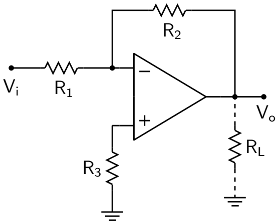

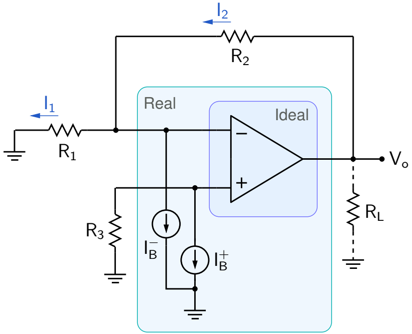

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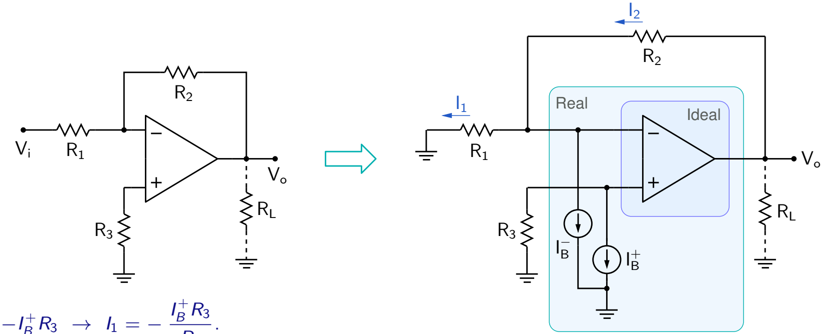
Using $I_B = \dfrac{I_B^+ + I_B^-}{2}$, $I_{OS} = I_B^+ - I_B^-$, i.e., $I_B^+ = I_B + \dfrac{I_{OS}}{2}$, $I_B^- = I_B - \dfrac{I_{OS}}{2}$, we get

$$V_o = -R_3\left(1+\frac{R_2}{R_1}\right)\left(I_B + \frac{I_{OS}}{2}\right) + R_2\left(I_B - \frac{I_{OS}}{2}\right) = \left(1+\frac{R_2}{R_1}\right)\left\{\left[\,(R_1 \,\|\, R_2) - R_3\right]\, I_B - \left[\,(R_1 \,\|\, R_2) + R_3\right]\,\frac{I_{OS}}{2}\right\}$$



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Using $I_B = \dfrac{I_B^+ + I_B^-}{2}$, $I_{OS} = I_B^+ - I_B^-$, i.e., $I_B^+ = I_B + \dfrac{I_{OS}}{2}$, $I_B^- = I_B - \dfrac{I_{OS}}{2}$, we get

$$V_o = -R_3 \left(1 + \frac{R_2}{R_1}\right)\left(I_B + \frac{I_{OS}}{2}\right) + R_2 \left(I_B - \frac{I_{OS}}{2}\right) = \left(1 + \frac{R_2}{R_1}\right) \left\{ \left[(R_1 \parallel R_2) - R_3 \right] I_B - \left[(R_1 \parallel R_2) + R_3 \right] \frac{I_{OS}}{2} \right\}$$

The first term can be made zero if we select $R_3 = R_1 \parallel R_2$.

$\rightarrow V_o = -R_2 I_{OS}$ (Compare with $V_o = R_2 I_B^-$ when R_3 is not connected.)

Should we worry about V_{OS} and I_B?

* For the integrator, V_{OS} and I_B will lead to saturation unless a DC path (a resistor) is provided.

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* In AC applications (e.g., audio), the DC shift arising due to V_{OS} or I_B is of no consequence since a coupling capacitor will block it anyway.
* A DC shift is a matter of concern when the output is expected to be a DC (or slowly varying) quantity, e.g., a temperature sensor or a strain gauge circuit.