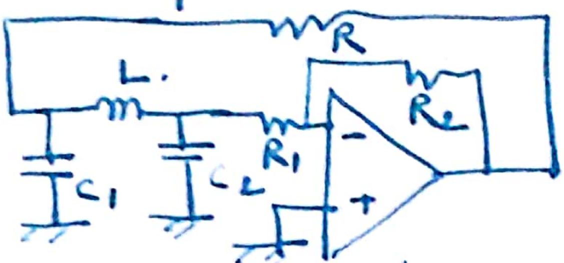
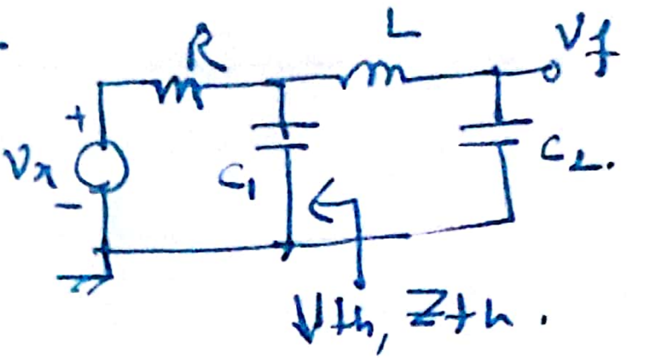
Colpitts Oscillator —





$$V_{th} = \frac{V_x \cdot 1/sC_1}{R + 1/sC_1} = \frac{V_x \cdot}{sC_1R+1}$$

$$Z_{th} = \frac{R \cdot 1/sC_1}{R + 1/sC_1} = \frac{R}{sC_1R+1}$$

$$A = -\left(R_2/R_1\right).$$

$$V_f = \frac{V_{th}\ 1/sC_2}{Z_{th} + Ls + 1/sC_2} = \frac{V_x}{(1+sC_1R)} \frac{1}{sC_2} \cdot \left[\frac{1}{\frac{R}{1+sC_1R} + Ls + 1/sC_2}\right]$$

$$\beta = \frac{V_f}{V_x}. = \frac{1}{sC_2}\left[\frac{1}{R + \left(Ls + \frac{1}{sC_2}\right)(1+sC_1R)}\right]$$

$$= \frac{1}{sC_2}\left[\frac{1}{R + Ls + 1/sC_2 + s^2C_1RL + \frac{RC_1}{C_2}}\right]$$

$$= \frac{1}{j\omega C_2}\left[\frac{1}{\left(R + R\frac{C_1}{C_2} - \omega^2 RLC_1\right) + j\left(\omega L - \frac{1}{\omega C_2}\right)}\right]$$

To satisfy the Barkhausen Criteria, β should give a ~~sto~~ phase shift of π. So we need

$$R + R\frac{C_1}{C_2} - \omega_0^2 RLC_1 = 0$$

$$\beta = -\frac{1}{\omega_0 C_2\left(\omega_0 L - 1/\omega_0 C_2\right)}$$

we ~~need~~ need to make sure/confirm $\omega_0 L > 1/\omega_0 C_2$.

$$\Rightarrow 1 + \frac{C_1}{C_2} = \omega_0^2 L C_1 \quad \left[\begin{matrix}\text{when} \\ R \neq 0\end{matrix}\right]$$

$$\Rightarrow \omega_0^2 = \frac{1}{L}\left(\frac{C_1+C_2}{C_1C_2}\right)$$

$$C_{eq} = \left(\frac{C_1 C_2}{C_1+C_2}\right).$$

$$\omega_0^2 = \frac{1}{\sqrt{LC_{eq}}}.$$

At resonance,

$$\omega_0 L = \frac{1}{\omega_0}\left(\frac{1}{C_1}+\frac{1}{C_2}\right).$$ Hence, $\omega_0 L > 1/\omega_0 C_2$.

The value of β at resonance,

$$\beta = -\frac{1}{\omega_0 C_2\left(\omega_0 L - 1/\omega C_2\right)} = -\frac{1}{\omega_0 C_2\left(\frac{1}{\omega_0 C_1}\right)}$$

$$= -\frac{C_1}{C_2}.$$

$$A\beta\,(\cancel{\text{at}}\ \omega=\omega_0) = \left(-\frac{R_2}{R_1}\right)\left(-\frac{C_1}{C_2}\right) \geq 1$$

$$\Rightarrow \quad R_2/R_1 \geq C_2/C_1$$

I have ignored the output impedance of the OPAMP.

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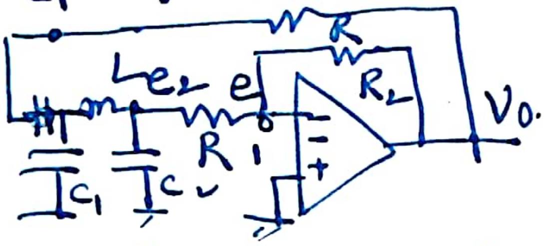
परिशिष्ट/Supplement - 4

रोल नं./Roll No. | पाठ्यक्रम नाम/Course Name
शाखा/प्रभाग/Branch/Div. | शिक्षण बैच/Tutorial Batch | अनुभाग/Section
पाठ्यक्रम सं./Course No. | तिथि/Date

Colpitts Oscillator with loading — (Brutal method).



$V_0 = -A\,e$ [A = open loop gain of the OPAMP].

$$\frac{v_0 - e_1}{R} - e_1 s c_1 + \frac{(e_2 - e_1)}{Ls} = 0 \quad --(1)$$

$$\frac{(e_1 - e_2)}{Ls} - e_2 s c_2 + \frac{e - e_2}{R_1} = 0 \quad — (2)$$

$$\frac{e_2 - e}{R_1} + \frac{v_0 - e}{R_2} = 0 \Rightarrow \quad -- (3)$$

$$e_1\left[-\frac{1}{R} - sc_1 - \frac{1}{Ls}\right] + \frac{e_2}{Ls} + \frac{v_0}{R} = 0$$

$$e_1\left[\frac{1}{Ls}\right] + e_2\left[-\frac{1}{Ls} - sc_2 - \frac{1}{R_1}\right] + v_0\left[-\frac{1}{AR_1}\right] = 0$$

$$e_1 \times 0 + e_2\left[\frac{1}{R_1}\right] + v_0\left[\frac{1}{R_2} + \left(\frac{1}{R_1} + \frac{1}{R_2}\right)\frac{1}{A}\right] = 0.$$

Further simplifying,

$$e_2 = -v_0\left[\frac{R_1}{R_2} + \frac{R_1}{A}\left(\frac{1}{R_1} + \frac{1}{R_2}\right)\right]$$

$$\frac{e_1}{Ls} = -\frac{v_0}{AR_1} + v_0\left[\frac{R_1}{R_2} + \frac{R_1}{A}\left(\frac{1}{R_1} + \frac{1}{R_2}\right)\right]\left[-\frac{1}{Ls} - sc_2 - \frac{1}{R_1}\right]$$

$$\Rightarrow e_1 = v_0\left\{\left[\frac{R_1}{R_2} + \frac{R_1}{A}\left(\frac{1}{R_1} + \frac{1}{R_2}\right)\right]\left[-1 - s^2 LC_2 - \frac{Ls}{R_1}\right] - \frac{Ls}{AR_1}\right\}$$

Use the expressions for e_1 & e_2 in (1) we get the $f \cdot r \cdot \beta$, $e_0 = A_{OL}\beta\, e_0$, Making $A_{OL}\beta = 1$ will give us the resonance condition.