

# 4530, Lecture 4

## Summary

# Body effect

- Bulk (“Body”, “Backgate”) acts as a second gate on channel

- $V_{BS}$  affects  $V_{TH}$ :  $V_{TH} = V_{TH0} + \gamma \left( \sqrt{|2\Phi_F - V_{BS}|} - \sqrt{|2\Phi_F|} \right)$

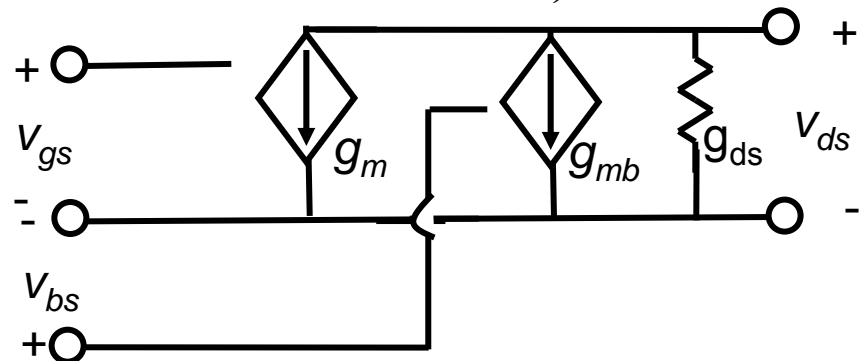
- (note book uses  $V_{SB}$ , so sign is inverted)

- For small signal model, can treat as a second transconductance,  $g_{mb}$

$$g_{mb} = \frac{\partial I_D}{\partial V_{BS}} = \frac{\partial}{\partial V_{BS}} \left( \frac{W}{L} \mu_n c_{ox} \frac{\left( V_{GS} - V_{TH0} - \gamma \left( \sqrt{|2\Phi_F - V_{BS}|} - \sqrt{|2\Phi_F|} \right) \right)^2}{2} \right)$$

$$g_{mb} = g_m \left( \frac{\gamma}{2\sqrt{|2\Phi_F - V_{BS}|}} \right)$$

- Usually  $g_{mb} \approx 0.1 g_m$



# Channel length modulation

- Under saturation
  - Higher  $V_{DS}$  increases pinch-off region
  - Effectively shorter channel:

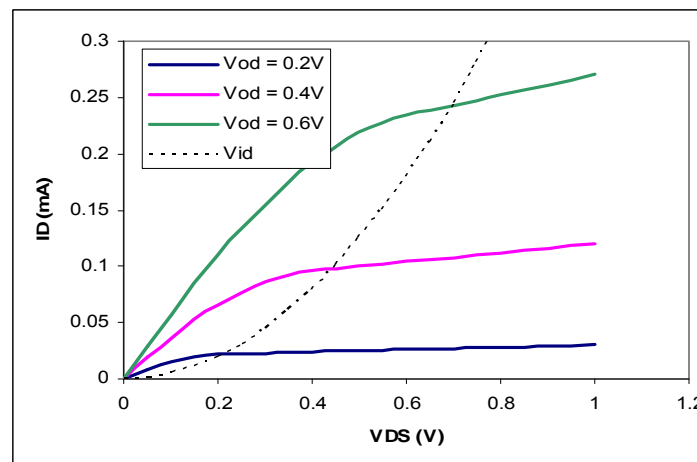
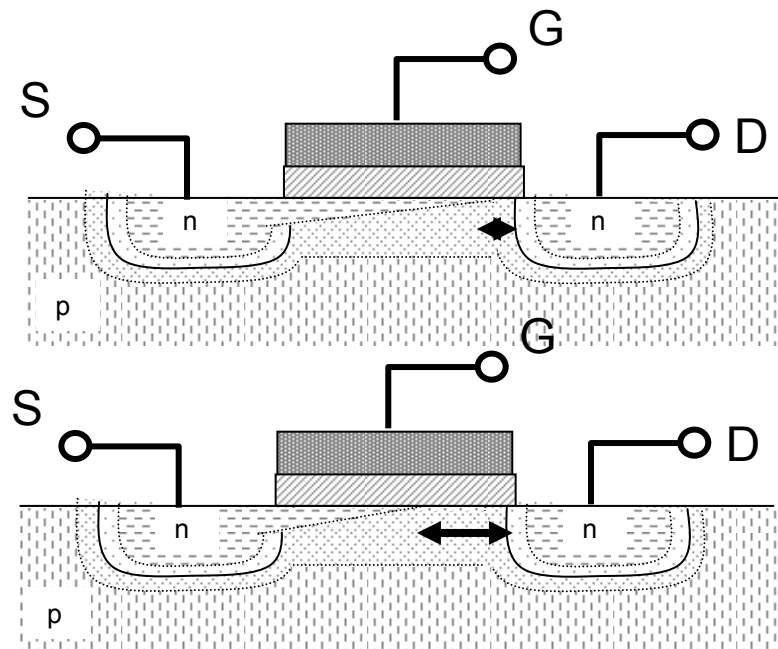
$$L_{eff} = L - L_{pinch} = L(1 - \lambda V_{DS}) \approx \frac{L}{1 + \lambda V_{DS}}$$

$$I_D = \frac{W}{L_{eff}} \mu_n c_{ox} \frac{(V_{GS} - V_{TH})^2}{2}$$

$$I_D = \frac{W}{L} \mu_n c_{ox} \frac{(V_{GS} - V_{TH})^2}{2} (1 + \lambda V_{DS})$$

$$g_{ds} = \frac{W}{L} \mu_n c_{ox} \frac{(V_{GS} - V_{TH})^2}{2} \lambda \approx \lambda I_D$$

$$r_o = \frac{1}{g_{ds}} \approx \frac{1}{\lambda I_D}$$



Note that  $\lambda$  is inversely proportional to  $L$

# Sub-threshold and velocity Saturation

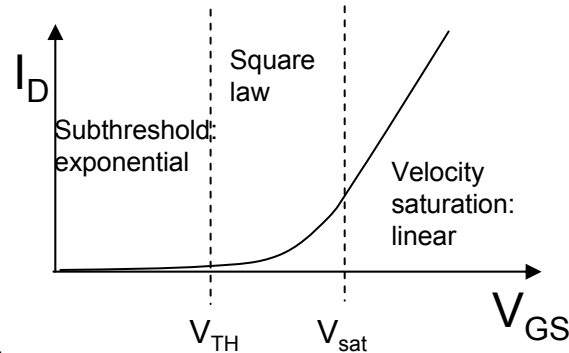
- For  $V_{GS} < V_{TH} + 100\text{mV}$ 
  - Square law breaks down
- For  $V_{GS} < V_{TH} - 100\text{mV}$ 
  - Exponential:

$$I_D = I_o e^{V_{GS} / \zeta V_T}$$

$$\zeta V_T \approx 35\text{mV} (\zeta \approx 1.25)$$

$$gm = \frac{I_o}{\zeta V_T} e^{V_{GS} / \zeta V_T} = \frac{I_D}{\zeta V_T}$$

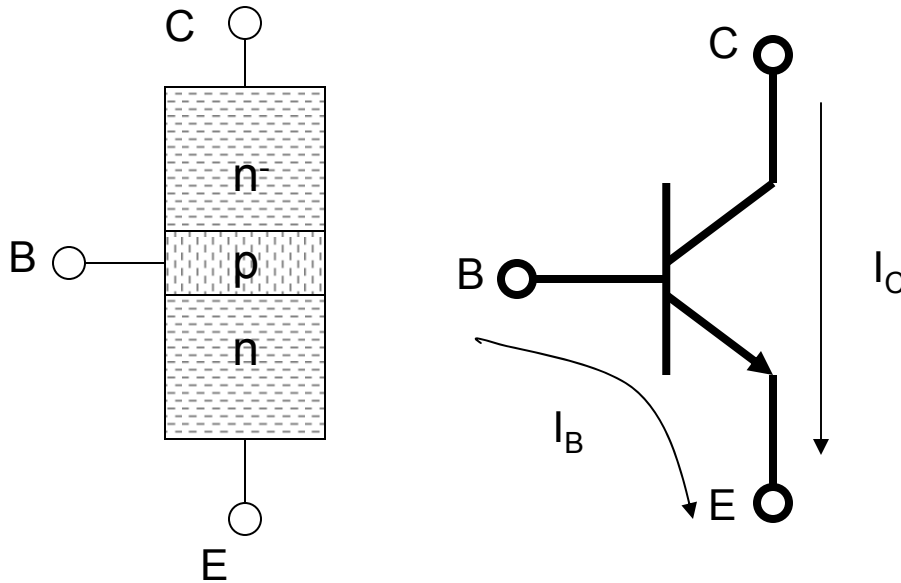
- In between smooth switch from square to exponential:
- but the math is a mess



- For short channels, large  $V_{GS}$ ,  $V_{DS}$ 
  - Electron velocity “saturates”:
  - Current not proportional to electric field
  - Response becomes linear (not square)
$$I_D = k(V_{GS} - V_{TH})$$

$$gm = k$$
  - “k” =  $W/L\mu c_{ox} V_{sat}$  where  $V_{sat}$  is the voltage where velocity saturation kicks in

# BJTs

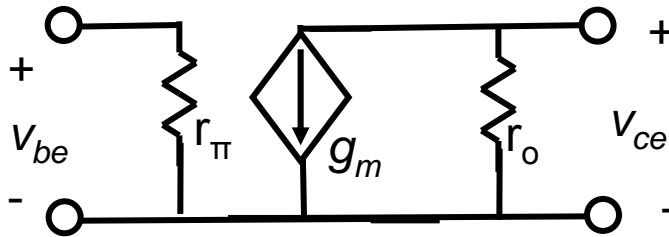


Provided in forward active ( $V_{CE} > 0.2V$ )

$$I_C = I_o e^{V_{BE}/V_T} \left( 1 + \frac{V_{CE}}{V_A} \right)$$

$$I_B = \frac{I_o}{\beta} e^{V_{BE}/V_T} \approx \frac{I_C}{\beta}$$

as  $V_{CE} \rightarrow 0$ , base-collector junction turns on, Beta decreases



$$g_m = \frac{\partial I_C}{\partial V_{BE}} = \frac{I_o e^{V_{BE}/V_T}}{V_T} \left( 1 + \frac{V_{CE}}{V_A} \right) = \frac{I_C}{V_T}$$

$$r_\pi = \left( \frac{\partial I_B}{\partial V_{BE}} \right)^{-1} = \left( \frac{I_o}{V_T \beta} e^{V_{BE}/V_T} \right)^{-1} \approx \frac{\beta}{g_m}$$

$$r_o = \left( \frac{\partial I_C}{\partial V_{CE}} \right)^{-1} = \left( \frac{I_o}{V_A \beta} e^{V_{BE}/V_T} \right)^{-1} \approx \frac{I_C}{V_A}$$

# Common Source Amplifier

- In saturation:

$$V_{out} = VDD - R_L I_D = VDD - R_L \frac{W}{L} \mu_n c_{ox} \frac{(V_{in} - V_{TH})^2}{2}$$

- Choose  $W/L$ ,  $V_{in}$  to set  $I_D$ , stay in saturation

- For saturation, choose bias:

$$I_D \times R_L < VDD - V_{OD}$$

$$\text{remember, } V_{OD} = V_{GS} - V_{TH}$$

- now can calculate gain:

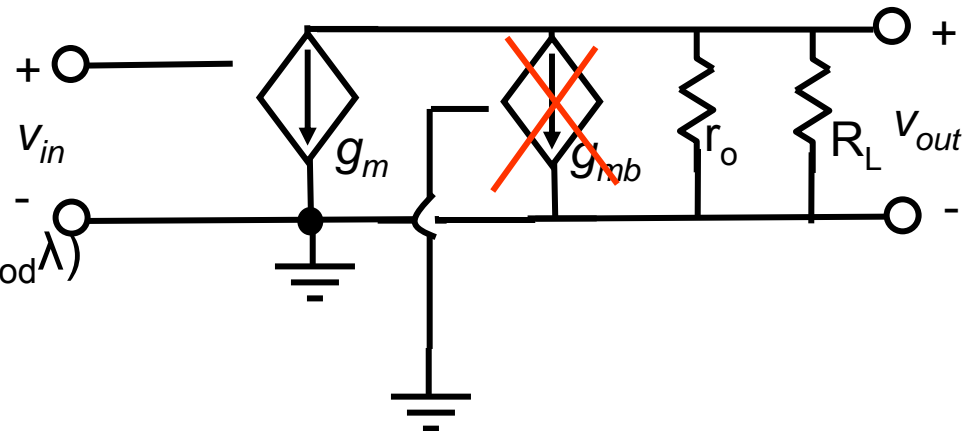
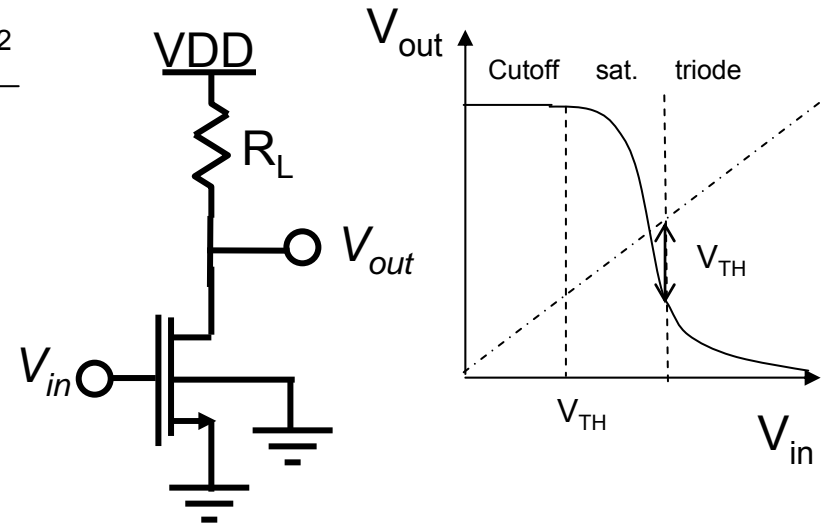
$$A = -g_m \cdot (R_L \parallel r_o)$$

- For square law:

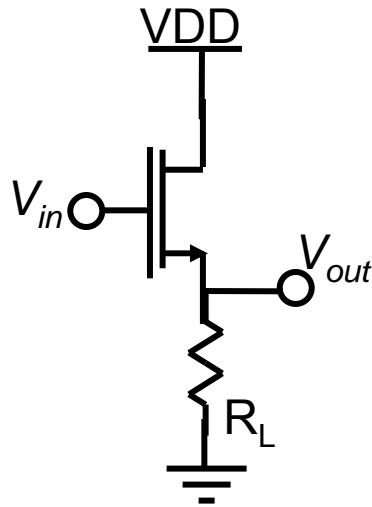
$$A = \frac{-2I_D}{V_{OD}} (R_L \parallel r_o) \approx \frac{-2I_D R_L}{V_{OD}}$$

- Max gain  $R_L \rightarrow \gg r_o$

- $A = -g_m r_o$  (square law:  $A = -2/(V_{od}\lambda)$ )



# Source Follower (common Drain)



- If bulk grounded:

$$A = \frac{g_m}{g_m + g_{mb} + \frac{1}{r_o} + \frac{1}{R_L}} \xrightarrow{R_L \rightarrow \infty} \frac{g_m}{g_m + g_{mb} + \frac{1}{r_o}} \approx 0.9$$

$$Z_{out} = \frac{1}{g_m + g_{mb} + \frac{1}{r_o} + \frac{1}{R_L}} \xrightarrow{R_L \rightarrow \infty} \frac{1}{g_m + g_{mb} + \frac{1}{r_o}} \approx \frac{1.1}{g_m}$$

- If bulk tied to source:

$$A = \frac{g_m}{g_m + \frac{1}{r_o} + \frac{1}{R_L}} \xrightarrow{R_L \rightarrow \infty} \frac{g_m}{g_m + \frac{1}{r_o}} \approx 1$$

$$Z_{out} = \frac{1}{g_m + \frac{1}{r_o} + \frac{1}{R_L}} \xrightarrow{R_L \rightarrow \infty} \frac{1}{g_m + \frac{1}{r_o}} \approx \frac{1}{g_m}$$

