→ NMOS char.

$$I_{DS} = 0 \; ; \quad V_{gs} \leq V_{Th} \; , \text{ cut-off / sub-threshold}$$

$$= \frac{\mu_n C_{ox} W}{L} \left[ (V_{gs} - V_T) V_{DS} - \frac{V_{DS}^2}{2} \right]$$

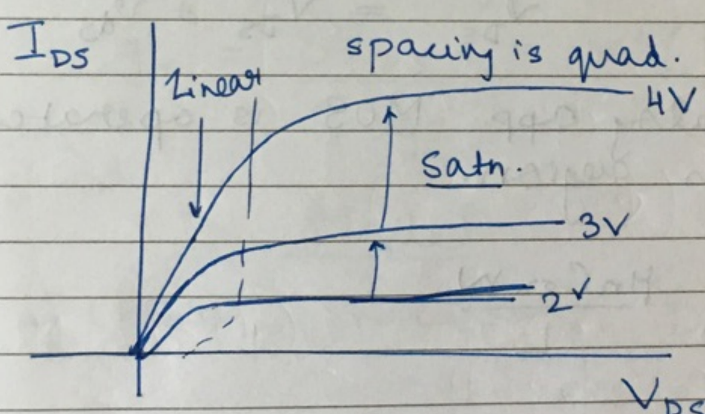
$$V_{DS} < V_{gs} - V_t$$

[Linear Reg.]

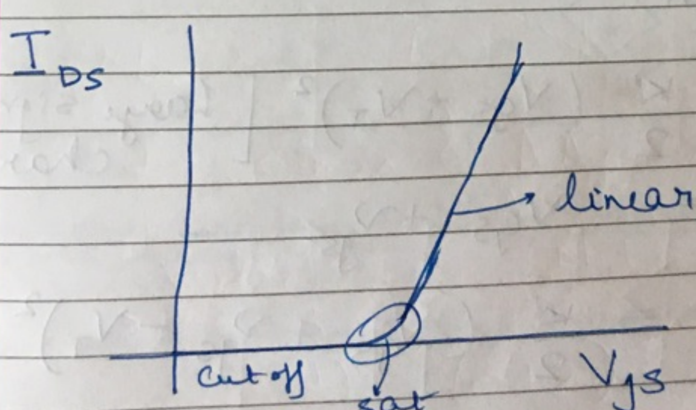
$$= \frac{\mu_n C_{ox} W}{2L} (V_{gs} - V_T)^2 \; ; \quad V_{DS} \geq V_{gs} - V_T$$

[satn]

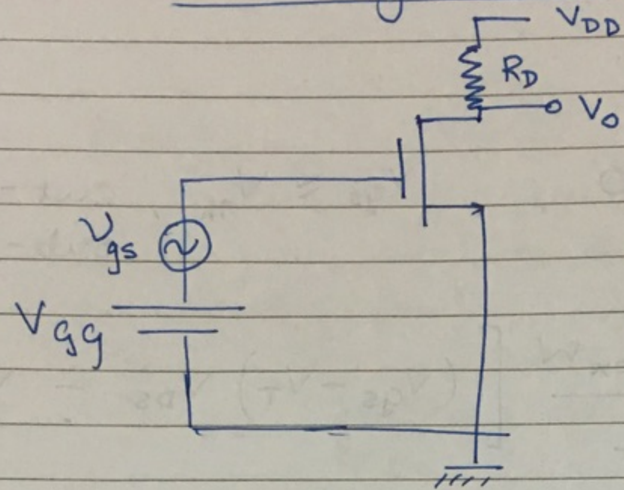
→ Char.




$V_T = 1V$ (say)



# Small Signal AC equiv. of MOSFE




$I_{DS} = f(V_{DS}, V_{gs})$

**Notation:**

DC values $\rightarrow$ $V_{gs}, V_{DS}$
AC " $\rightarrow$ $v_{gs}, v_{ds}$

DC + AC $\rightarrow$ $v_{gs} = V_{gs} + v_{gs}$
$v_{DS} = V_{DS} + v_{ds}$

- For all Analog app. MOS is operated in saturation region.

$$k' = \frac{\mu_n C_{ox} W}{L}$$

In saturation:

$$I_D = \frac{k'}{2}(V_{gs} - V_T)^2 \quad \left[\text{Large signal char.}\right]$$

$$v_{gs} = V_{gs} + v_{gs}$$

$$I_{DS} + i_{ds} = \frac{k'}{2}(V_{gs} + v_{gs} - V_T)^2$$

$$= \underbrace{\frac{K'}{2}\left[(V_{gs} - V_T)^2\right.}_{I_{DS}} \left. + 2(V_{gs} - V_T)\, v_{gs} + v_{gs}^2 \right]$$

$$i_{ds} = K'(V_{gs} - V_T)\, v_{gs} + \frac{K'}{2} v_{gs}^2$$

$$i_{ds} \approx K'(V_{gs} - V_T)\, v_{gs} \qquad (V_{gs} - V_T \gg v_{gs})$$

{Small signal Approx.}

$$i_{ds} = g_m v_{gs}$$

→ Transconductance

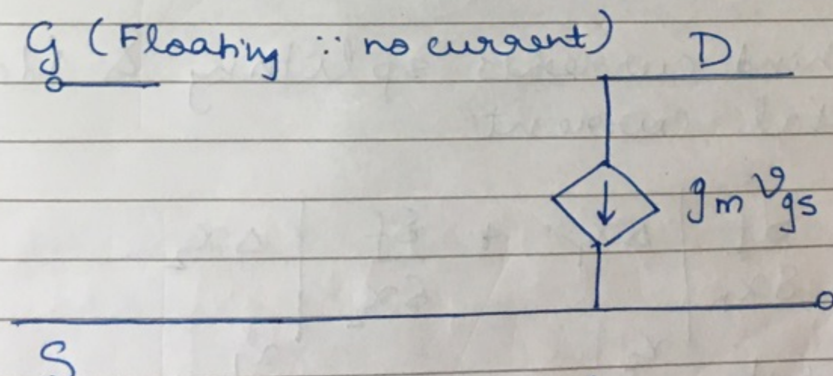
$$g_m = K'(V_{gs} - V_T)$$

→ Alternatively, using Derivative

$$I_{DS} = \frac{K'}{2}(V_{gs} - V_T)^2$$

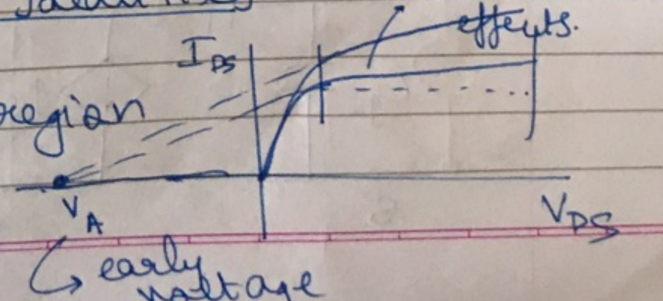
$$g_m = \frac{\delta I_{DS}}{\delta V_{gs}} = K'(V_{gs} - V_T)$$

## Model




→ Incorporating Non-Idealities

- Slope in saturation region



$$I_{DS} = \frac{k'}{2}(V_{gs} - V_T)^2\left[1 + \lambda(V_{DS} - V_{DS,sat}\right.$$

$$\rightarrow \boxed{V_{DS,sat} = V_{gs} - V_T}$$

$$\frac{\delta I_{DS}}{\delta V_{DS}} = \frac{k'}{2}(V_{gs} - V_T)^2 \lambda$$

↓ Approximation

$$\simeq \lambda I_{DS}$$

→ Ignoring $\lambda$ for DC comp.

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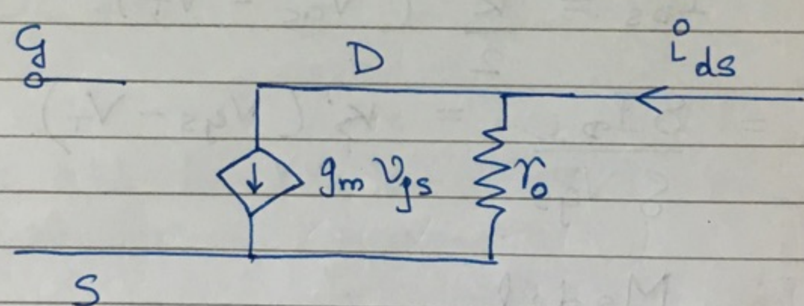
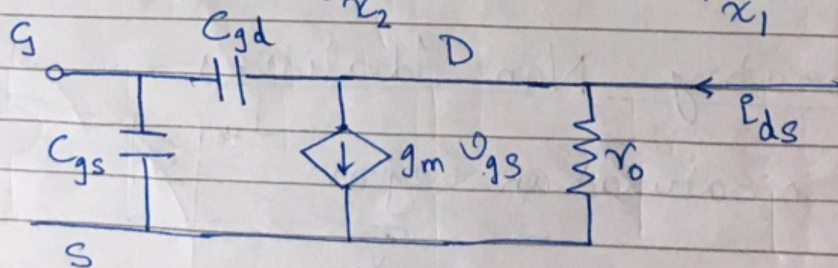
$$\frac{\delta V_{DS}}{\delta I_{DS}} = \frac{1}{\lambda I_{DS}} = r_o = \frac{V_A}{I_{DS}}$$

→ Early voltage




→ Idea behind currents splitting & $i_{ds}$ being total current

$$\Delta Y = \left.\frac{\delta f}{\delta x_1}\right|_{x_2} \Delta x_1 + \left.\frac{\delta f}{\delta x_2}\right|_{x_1} \Delta x_2$$




$I_g = 0$

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→ Results to rem

In satn.

$$C_{gs} = \frac{2}{3} C_{ox} \quad \left(\text{Due to channel bent towards Source}\right)$$

$$C_{gd} = 0$$

Linear

$$C_{gs} = C_{gd} = \frac{C_{ox}}{2} \quad \left(\text{Due to Symmetric channel}\right)$$

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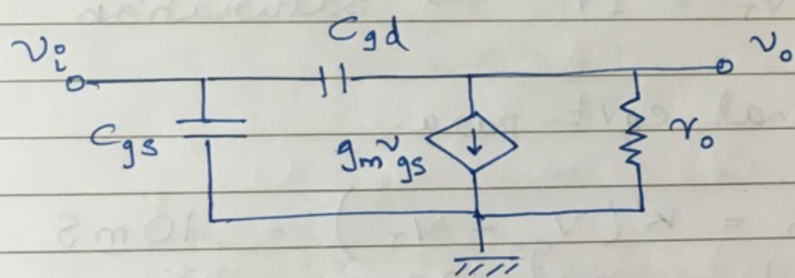
Recap

Small Signal AC eqvt ckt



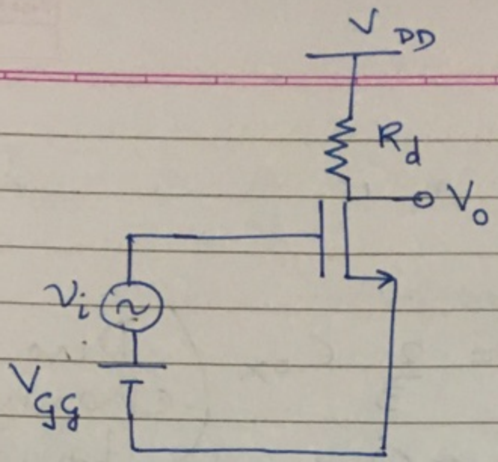

$$g_m = k'(V_{GS} - V_T) \qquad C_{gs} = \frac{2}{3} C_{ox} \times WL$$

$$r_o = \frac{1}{\lambda I_D} \qquad C_{gd} \approx 0$$

Typically $C_{gd} < 0.1\, C_{gs}$

$C_{gd}$ at high $\omega$ ⇒ Approaches short
⇒ Can't act like an amplifier

→ Low $\omega$ ⇒ $C_{gd}$ Can be assumed to be open ckt.




**Typical values**

$V_{DD} = 15V$
$V_T = 1V$
$K' = 10\,mA/V^2$
$R_d = 1K\Omega$
$V_{gg} = +2V$
$\lambda = 0.01\,V^{-1}$

$$I_D = \frac{K'}{2}(V_{gs} - V_T)^2 = 5mA$$

$$V_o = 15V - 5mA \times 1K\Omega = 10V = V_{DS}$$

→ For saturation: $V_{DS} > V_{gs} - V_T$

$V_{DS} = 10V$
$V_{gs} - V_T = 1V \Rightarrow$ Saturation

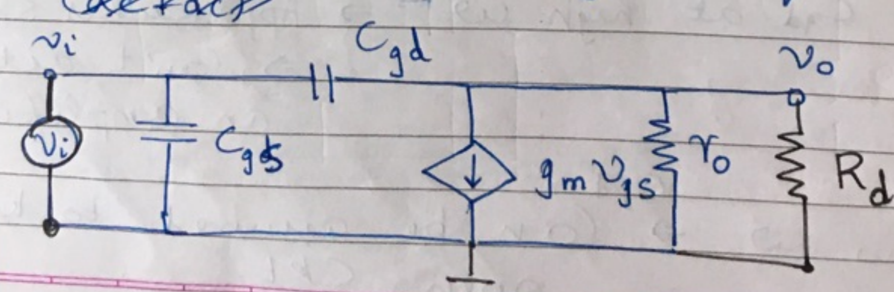
→ Small Signal eqvt para.

$$g_m = K'(V_{gs} - V_T) = 10\,mS$$

$$r_o = \frac{1}{\lambda I_D} = \frac{100}{5} = 20\,K\Omega$$

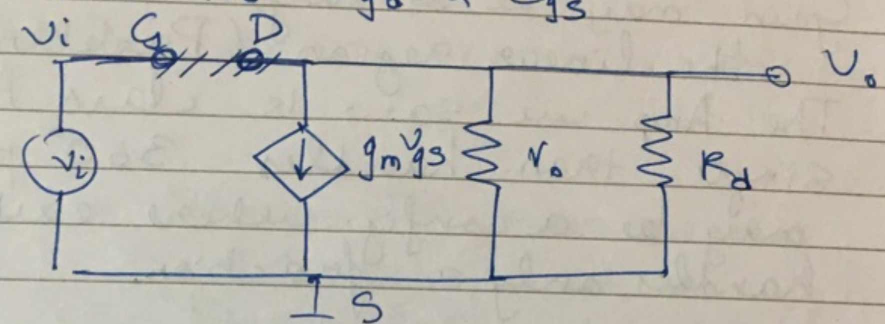
$C_{gs} \approx 0$ (for Now)
$C_{gd} \approx 0$

→ ~~Combined Equivalent Ckt.~~
→ ~~Combined~~ Small signal AC eqvt ckt.
~~(ac+dc)~~



→ $V_{DD}$ & $V_{GG}$ are shorted since DC values
→ ~~Fx~~ are set to 0 in AC equivalent ckt

- Open ckt ~~the~~ $C_{gd}$ & $C_{gs}$




$$v_o = -g_m v_{gs} (r_o // R_d)$$

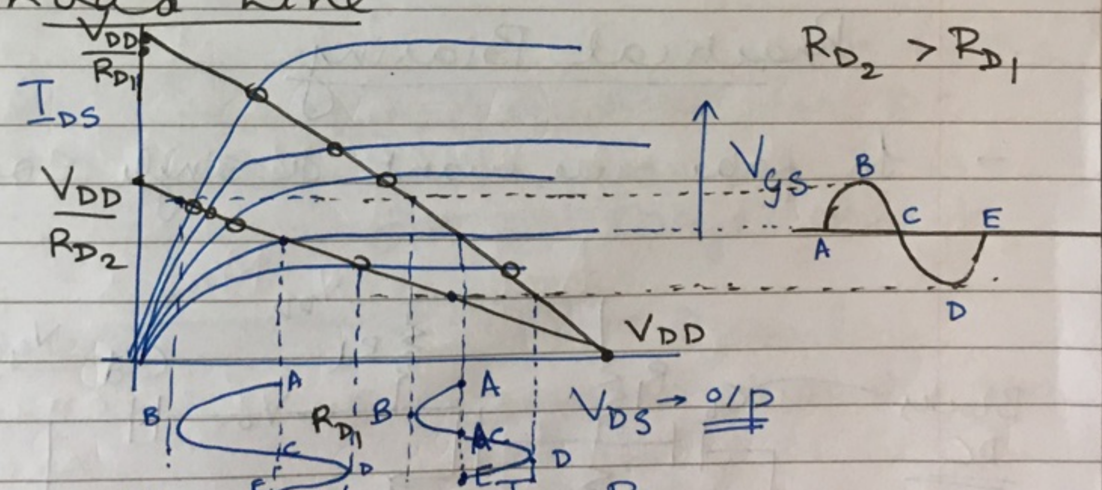
~~$= -10mS \times$~~

$$A_v = -\frac{v_o}{v_{gs}} = 10mS \times \left(\frac{r_o R_d}{r_o + R_d}\right)$$

$v_{gs} \to v_i$

$$A_v = 10 \times \frac{20 \times 1}{21} = \frac{200}{21} \approx \boxed{9.5}$$

## Load Line



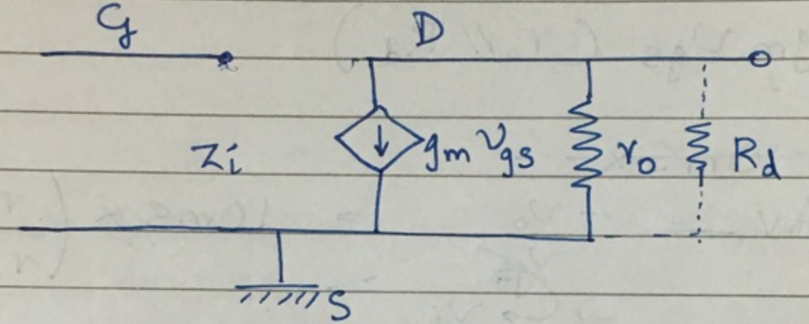

$$V_o = V_{DS} = V_{DD} - I_{DS} R_D$$

$I_{DS} = 0, \quad V_{DS} = V_{DD}$

$V_{DS} = 0, \quad I_{DS} = \frac{V_{DD}}{R_D}$

Load Line

- → With a larger value of $R_D$ we get a larger gain.
- → Gain may be so large that it hits the linear region (Problem)
- → The Amp. we saw is class A, i.e a single tran. handles $360°$ phase. There may be a config. where each tran. handles only a fraction.




Voltage Amp

$Z_i = \infty$

$Z_o = r_o$

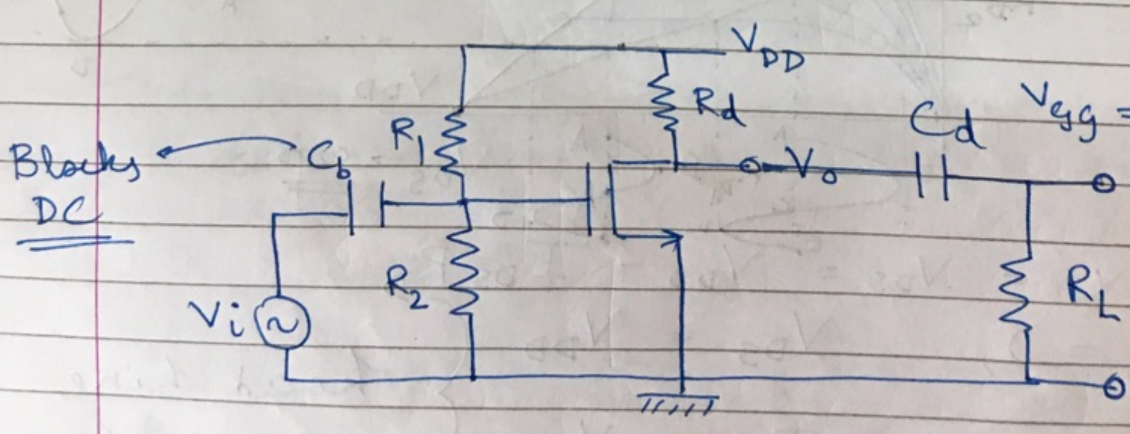
$A_v = -g_m r_o$

- → Ideal voltage Amp.

$$Z_i = \infty, \quad Z_o = 0$$

## Practical Biasing

- → In prac. we want a single source




$$V_{gg} = \frac{V_{DD}}{(R_1 + R_2)} \times R_2$$

→ As the earlier ckt stands, $R_d$ controls both Gain & Biasing

→ ⇒ Add $R_L$, and a capacitor to block DC.

→ $C_b$ ≡ Blocks DC from messing with AC or vice versa

$C_d$ ≡ "Decouples" the o/p from DC biasing

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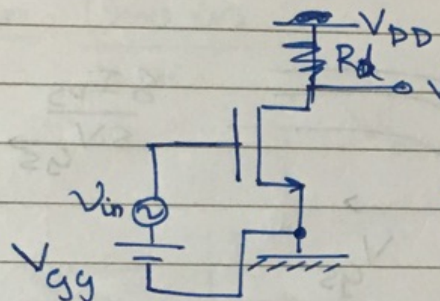
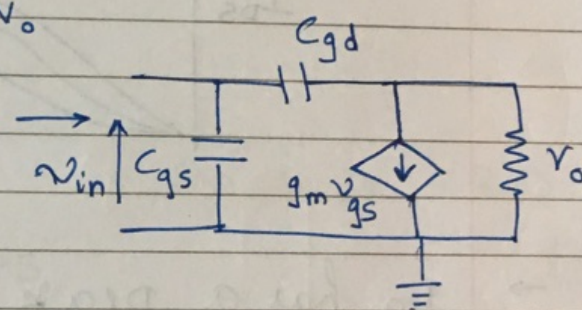
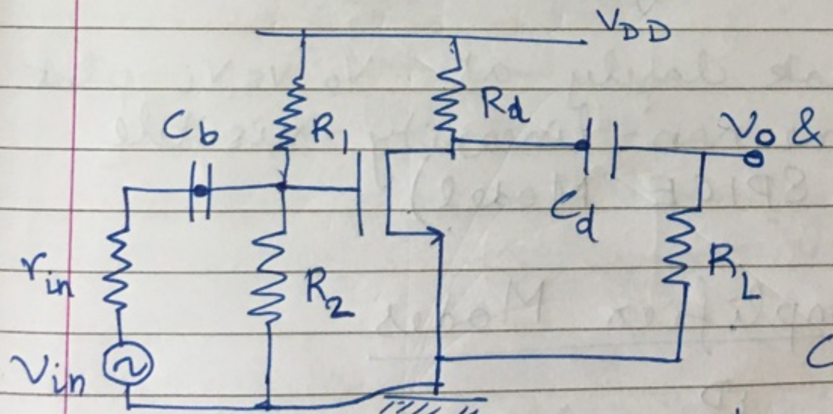


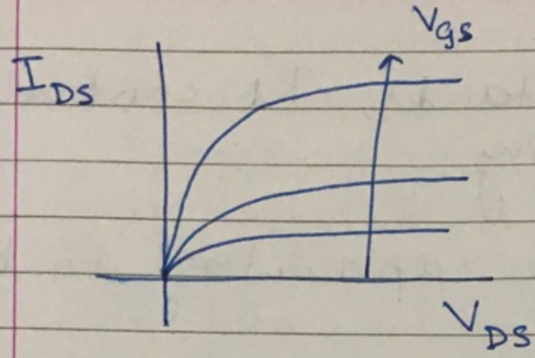



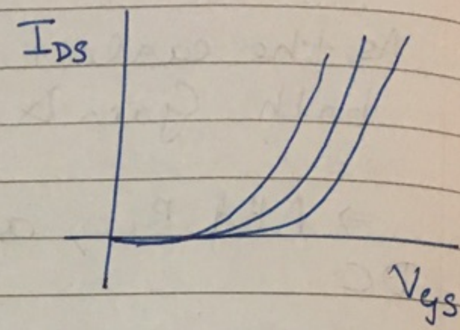

& $C_b$ is short for AC & open for DC provided it is large enough

$C_d$ → Decoupling transistor output from final o/p

$$A_V = -g_m (r_o \parallel R_d)$$

$$A_V = \frac{\delta V_o}{\delta V_{in}}$$



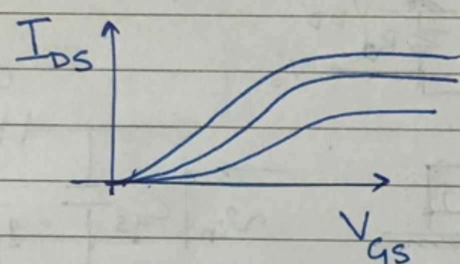




## Mobility Degradation:

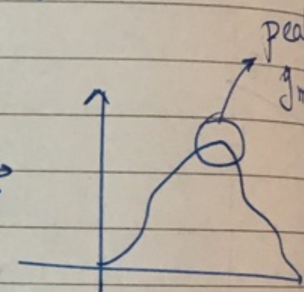
$$k' = \frac{\mu C_{ox} W}{L}$$

$\downarrow$

→ In reality in $I_D$ $V_{gs}$ plot current flattens out



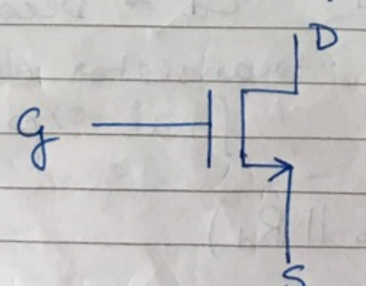

$\xrightarrow{\frac{\delta I_{DS}}{\delta V_{GS}}}$




→ $g_m$ has a peak value where we can get maximum gain

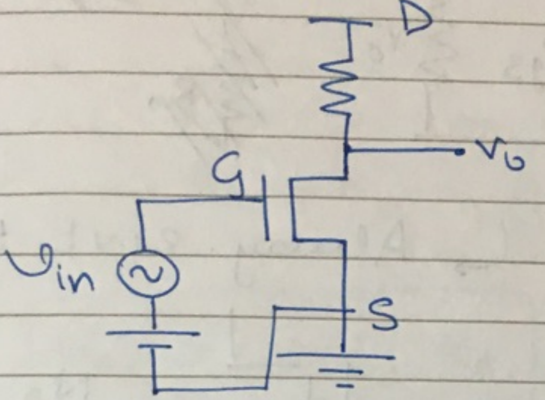
→ When we look closely at $V_o$ vs $V_i$ plot there is some non-linearity visible (Check SPICE Model)

## Amplifier Modes




- Common Source
- Common Drain
- Common Gate

→ Till Now: Common Source (CS Amp.)



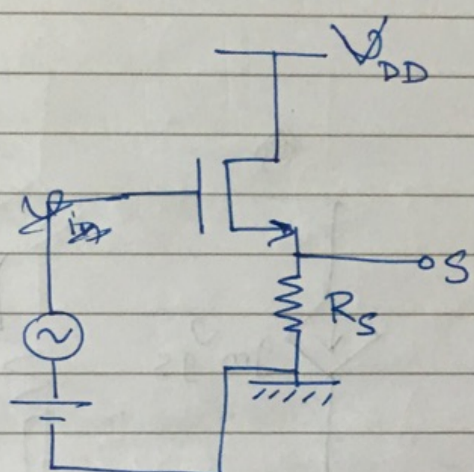

CS

$A_v = -g_m (R_d // r_o)$

★ $Z_o = r_o$ [$r_o // R_d$ if $C_d$ is conn]

$Z_i = \infty$

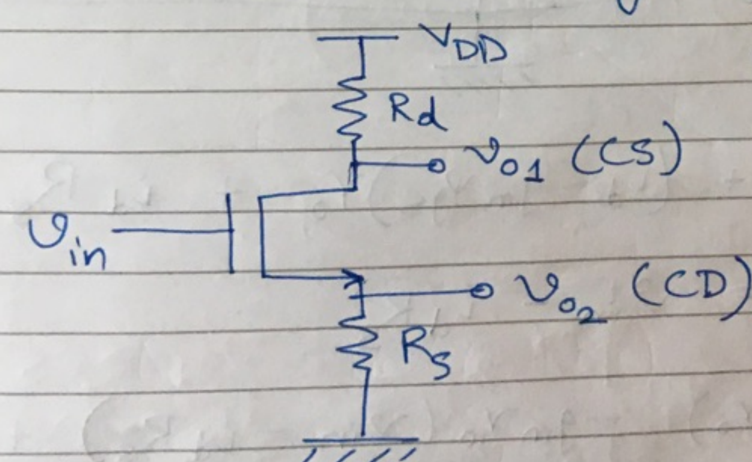
→ Common Drain



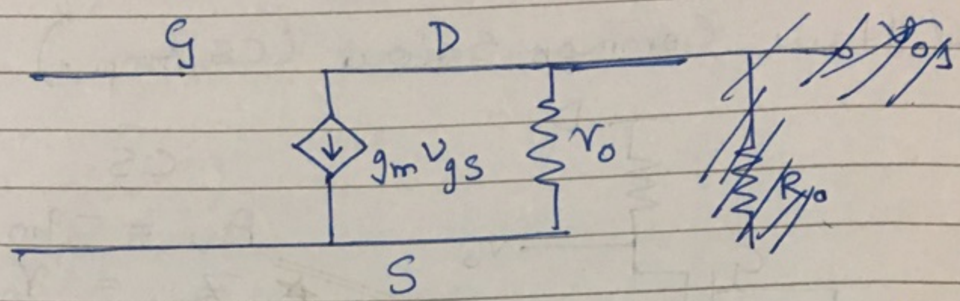

o/p b/w S & D ↳ (gnd)

i/p b/w G & D ↳ gnd

↓ Modify

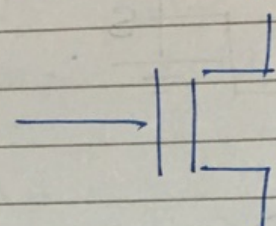



→ Now even though none of the terms CS/CD is strictly valid, we still use them due to similarity in form.



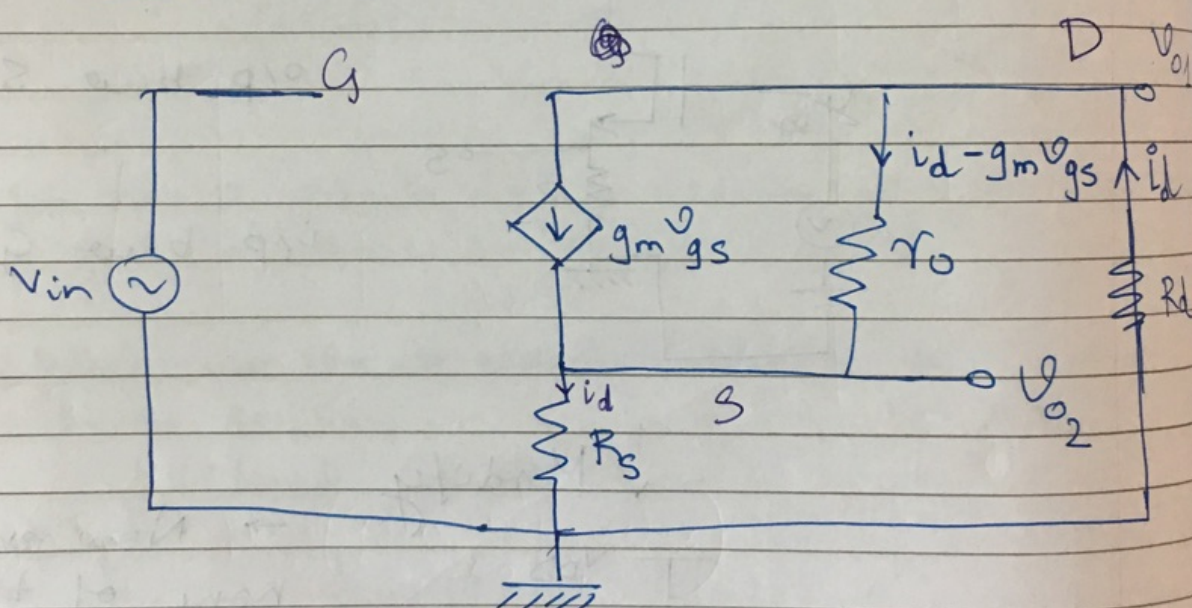

→ The above is the universally true small signal model

↳ Alway eqvt to



No matter what

→ Here




$$i_d R_d + (i_d - g_m v_{gs}) r_o + i_d R_s = 0$$

$$v_{gs} = v_{in} - i_d R_s$$

$$i_d (R_d + r_o) - g_m r_o (v_{in} - i_d R_s) + i_d R_s = 0$$

$$i_d (R_d + r_o + g_m r_o R_S + R_S) = g_m r_o v_i$$

$$i_d = \frac{g_m r_o v_i}{r_o + R_D + (g_m r_o + 1) R_S}$$

$$\boxed{v_{o1} = -i_d R_d} \qquad \boxed{v_{o2} = i_d R_S}$$

$$A_{cs} = \frac{v_{o1}}{v_i} = \frac{-R_d g_m r_o}{[r_o + R_d + (1 + g_m r_o) R_S]}$$

$$A_{CD} = \frac{v_{o2}}{v_i} = \frac{R_S g_m r_o}{[r_o + R_d + (1 + g_m r_o) R_S]}$$

Approximations;

(1) $g_m r_o >> 1$ (Gain >> 1)

$$A_{cs} \simeq \frac{-R_d g_m r_o}{r_o + R_d + \cancel{g_m R_S}\, g_m r_o R_S}$$

(2) $r_o >> R_d$ [We also said this before $A = -g_m (r_o \| R_d) \approx -g_m R_d$]

$\hookrightarrow \approx M\Omega$ ($\lambda$ is small from short channel effect)

$$A_{cs} \approx \frac{-R_d g_m r_o}{r_o + g_m r_o R_S}$$

(3) $g_m R_S >> 1$

$$\Rightarrow \approx \frac{-g_m R_D r_o}{g_m r_o R_S} \approx -\frac{R_D}{R_S}$$

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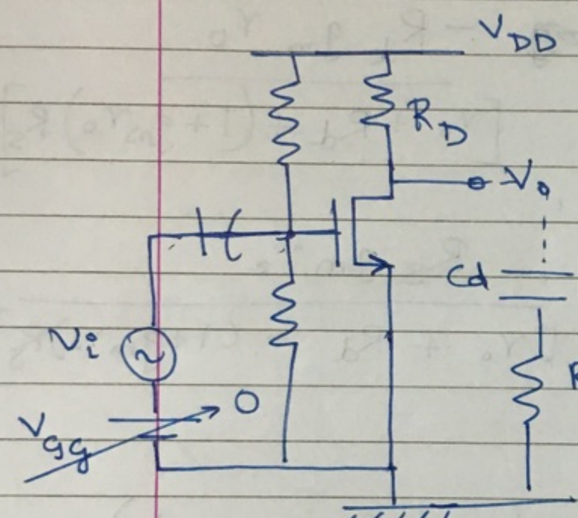
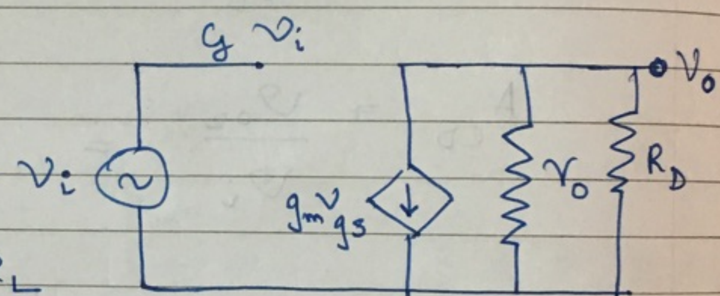
## Transistor at Low frequency



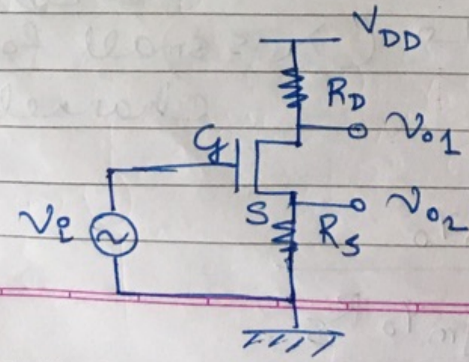




CS

$$g_m = \left.\frac{\delta I_D}{\delta V_{gs}}\right|_{BIG_{sig}} = x'(V_{gs} - V_T)$$

$g_m R_o = \mu$ (Gain Def. when no $R_L$)

→ Model for low frequency




$v_{o1} \rightarrow CS(\sim)$

$v_{o2} \rightarrow \sim CD$

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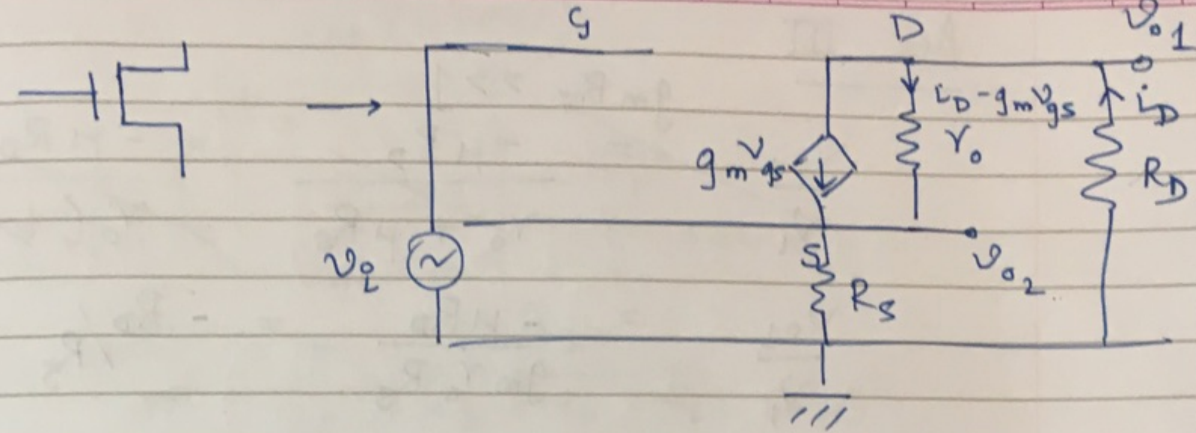



$$i_D R_D + r_o(i_D - g_m v_{gs}) + i_D R_S = 0$$

$$v_i = v_{gs} + i_D R_S$$

$$i_D R_D + r_o\left(i_D - g_m(v_i - i_D R_S)\right) + i_D R_S = 0$$

$$i_D\left(R_D + r_o + R_S \times (g_m r_o + 1)\right) = r_o g_m v_i$$

$$i_D = \frac{g_m r_o v_i}{R_D + r_o + R_S(1 + g_m r_o)} = \frac{\mu v_i}{R_D + r_o + R_S(1+\mu)}$$

$$v_{o1} = -i_d R_D \quad , \quad \frac{v_{o1}}{v_i} = \frac{-\mu R_D}{R_D + r_o + R_S(1+\mu)} = -1.587$$

Assumptions

| Typical ~~(Ignore)~~ |
|---|
| $\mu = 50$ |
| $r_o = 50 k\Omega$ |
| $g_m = 1 mS$ |
| $R_D = 10 k\Omega$ |
| $R_S = 5 k\Omega$ |

(I) Say $\mu = 50$ (typical)

$\mu >> 1$

$$\frac{v_{o1}}{v_i} = \frac{-\mu R_D}{R_D + r_o + \mu R_S} = -1.612$$

(II) ~~$\frac{v_{o1}}{v_i}$~~ $r_o >> R_D$

$$\frac{v_{o1}}{v_i} = \frac{-\mu R_D}{r_o + \mu R_S} = \frac{-\mu R_D}{r_o(1 + g_m R_S)} = -1.66$$

Ass III

$$g_m R_s >> 1$$

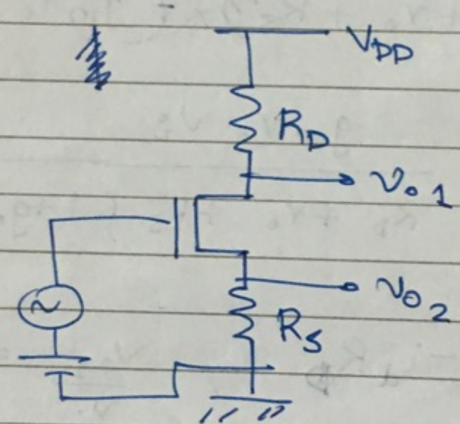
$$\frac{v_{o1}}{v_1} = \cancel{\frac{-\mu R_D}{r_o + \mu R_s}} = \cancel{\frac{-\mu R_D}{r_o(1+g_m R_s)}}$$

$$\frac{v_{o1}}{v_1} = \frac{-\mu R_D}{g_m r_o R_s} = -R_D/R_s = -2$$

→ Common Drain

$$v_{o2} = i_d R_s$$

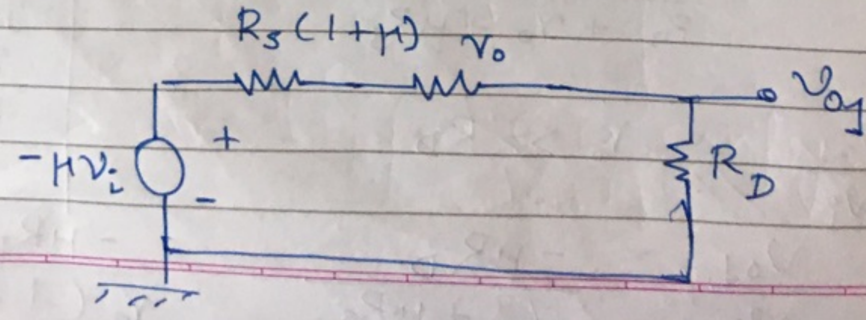
$$\frac{v_{o2}}{v_{in}} = \frac{\mu \cdot R_s}{R_D + r_o + R_s(1+\mu)}$$




→ Rewriting earlier exp.

$$v_{o1} = -\frac{\mu v_i R_D}{R_D + r_o + R_s(1+\mu)}$$

→ Eqvt CKt



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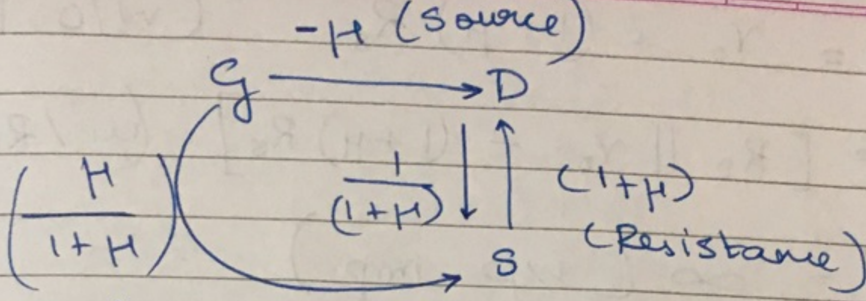
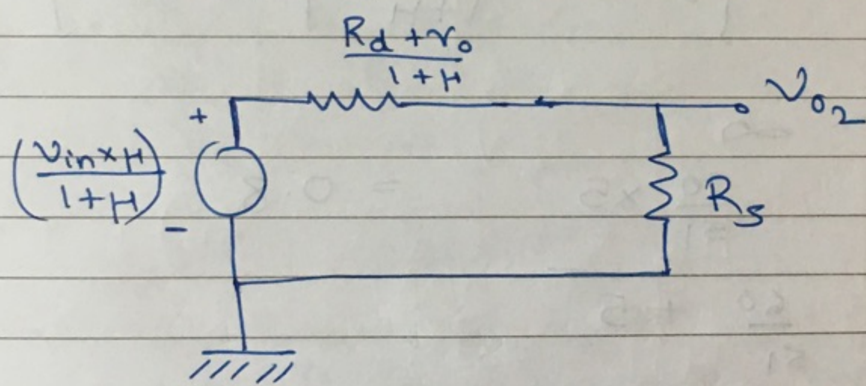


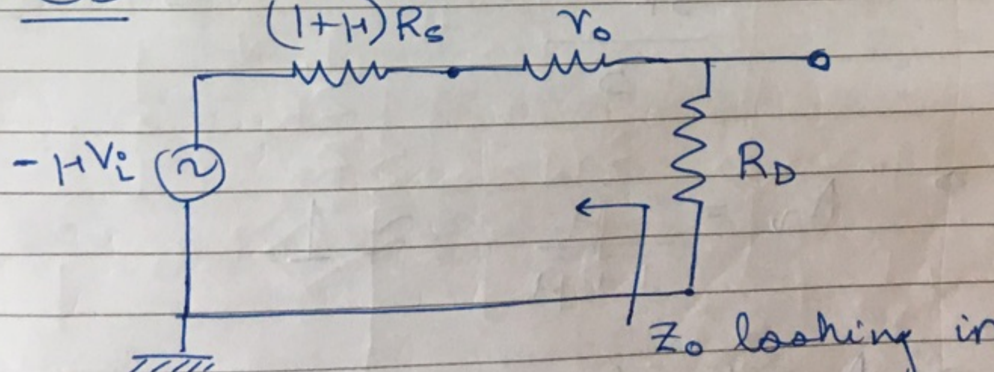

$$\frac{v_{o2}}{v_{in}} = \frac{\mu R_S}{R_D + r_o + R_S(1+\mu)}$$

$$v_{o2} = \frac{v_{in} \times \left(\frac{\mu}{1+\mu}\right) R_S}{\frac{R_D + r_o}{1+\mu} + R_S}$$

→ Eqvt ckt




CS




$$A_v = \frac{v_{o1}}{v_i} = \frac{-\mu R_D}{r_o + R_D + (1+\mu)R_S}$$

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$Z_o = r_o + (1+\mu) R_s$ (w/o $R_D$ i.e $R_D = \infty$)

$= [R_D \parallel r_o + (1+\mu) R_s]$ (w/ $R_D$)

$Z_i = \infty$ (i/p imp.)

$$A_v = \frac{v_{o2}}{v_1} = \frac{\left(\frac{\mu}{1+\mu}\right) R_s}{\left(\frac{R_D + r_o}{1+\mu}\right) + R_s}$$

$$Z_o = \frac{R_D + r_o}{1+\mu} \; [\text{w/o } R_s]$$

$$= \left[R_s \parallel \frac{R_D + r_o}{1+\mu}\right] [\text{w/ } R_s]$$

$Z_i = \infty$

$$\rightarrow \quad A_v = \frac{\frac{50}{51} \times 5}{\frac{60}{51} + 5} = 0.8$$

$$\approx \frac{\frac{50}{51} \times 5}{5} \quad \text{i.e } R_s \gg \frac{R_D + r_o}{1+\mu}$$

$$\Rightarrow \quad A_v = \frac{\mu}{1+\mu} \approx 1$$

$\Rightarrow$ <u>$A_v = 1$ for CD config. (No Gain)</u>

$v_1$

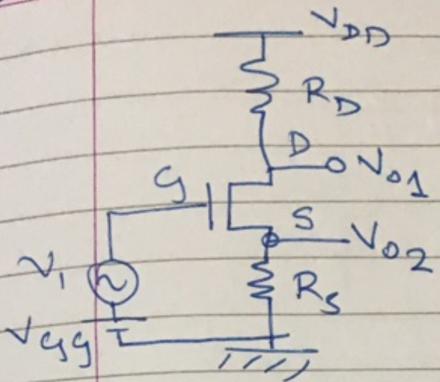
$V_{GG}$

1)

$-\mu v$

2)

16.1.18




$$\boxed{\mu = g_m r_o}$$

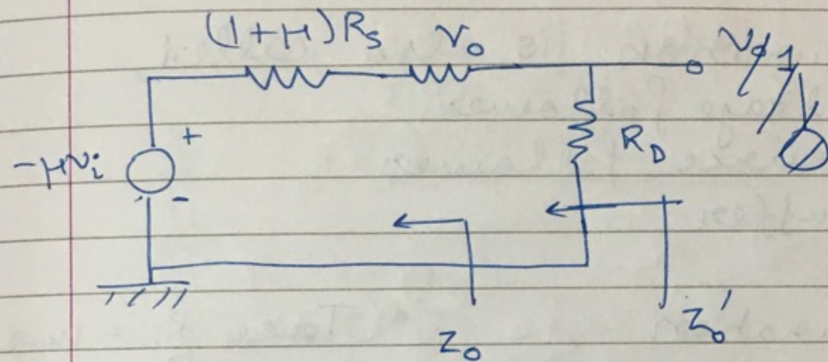
$g_m = 1mS$

$r_o = 50k\Omega$

$R_D = 10k\Omega$

CS

1)




$$A_{v_1} = \frac{v_{o1}}{v_1} = \frac{-\mu R_D}{r_o + R_D + (1+\mu)R_S}$$

$$\boxed{\simeq -R_D/R_S}$$

$Z_o = r_o + (1+\mu)R_S$

$Z_o' = R_D \parallel Z_o$

$Z_i = \infty$

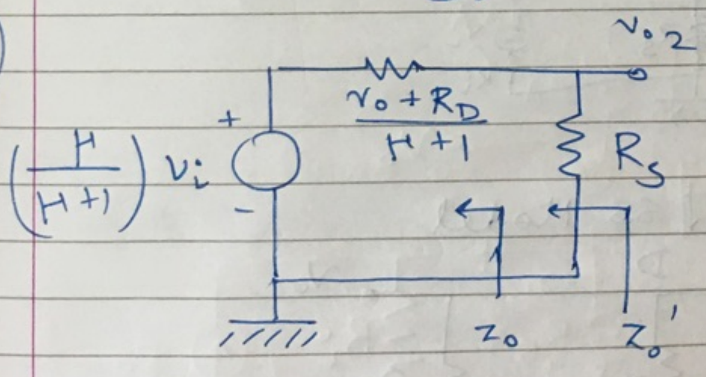
$A_{v_1} = -1.6 \cdots\to -2$

$Z_o = ? \to 305k\Omega$

$Z_o' = ? \simeq 10k\Omega$

CD

2)




We got

$A_{v_2} = 0.8 \ldots\ldots 1.0$

$Z_o = ? \approx \frac{60}{51}k\Omega = 1.2k\Omega$

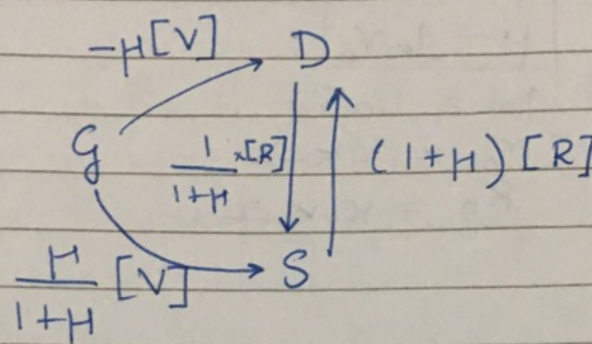
$Z_o' = ? \approx 1.2k\Omega$

$$A_{v_2} = \frac{v_{o2}}{v_i} = \frac{\left(\frac{\mu}{\mu+1}\right)R_S}{\frac{r_o + R_D}{\mu+1} + R_S} \simeq 1$$

$$Z_o = \frac{r_o + R_D}{\mu + 1}$$

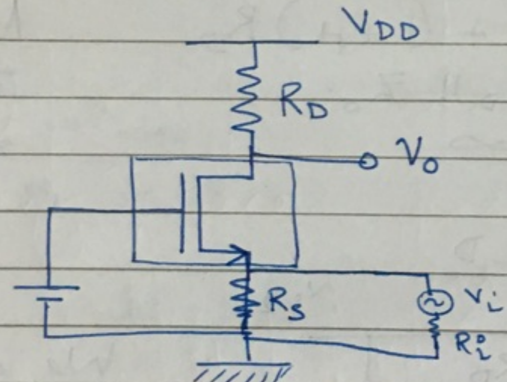
$Z_o' = R_S \parallel Z_o$

$Z_i = \infty$

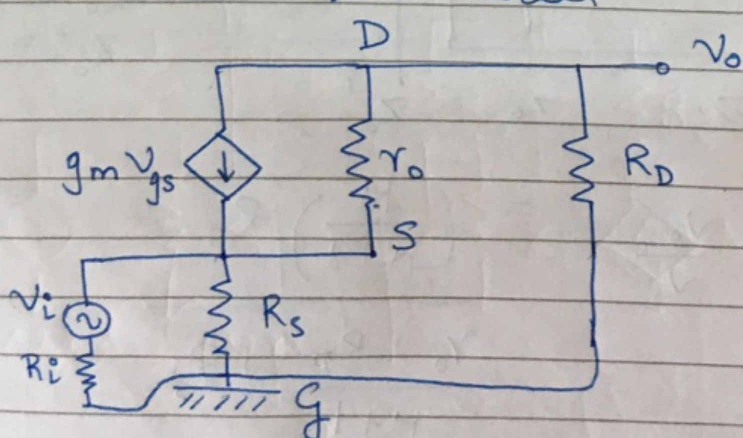


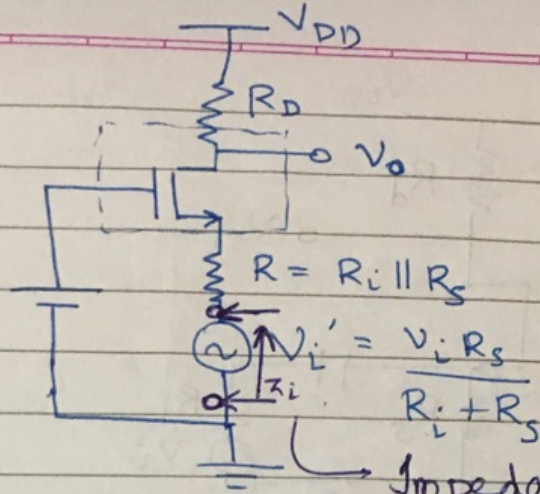

→ CD configuration is also called,
- voltage follower
- source follower
- buffer

3) CG configuration (Take $R_i = 1k\Omega$, gives us control over $Z_i$)



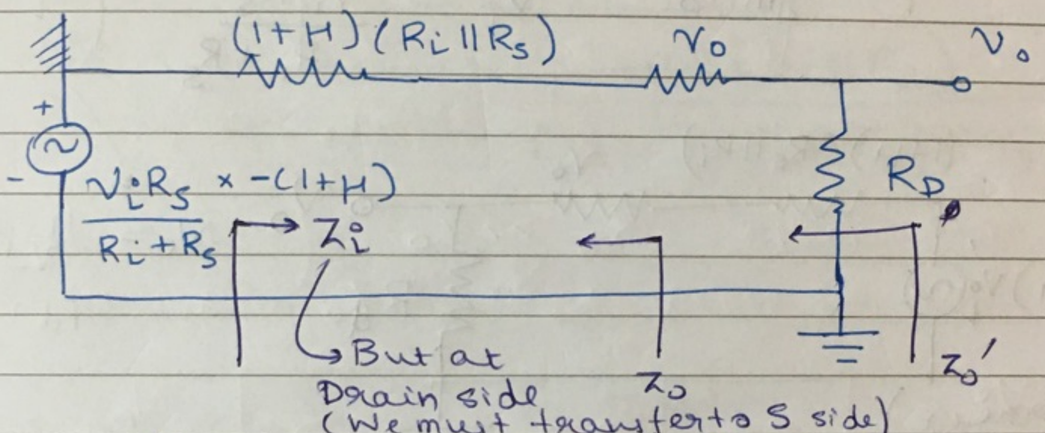

↓ SS Model






Impedance looking in from $v_i$

↓ Taking all to Drain side




$$A_v = \frac{v_o}{v_i} = \frac{-(1+\mu)\frac{R_s}{R_i + R_s} \times R_D}{(1+\mu)(R_i \| R_s) + r_o + R_D}$$

$$= -0.41$$

Approximations

$R_i \approx 0$, (Source resistance is important)

$$A_v \approx \frac{-R_D \times \cancel{\mu}^{\,g_m r_o}}{r_o} = \boxed{-g_m R_D}$$

$$Z_o = r_o + (1+\mu)(R_i \| R_s)$$

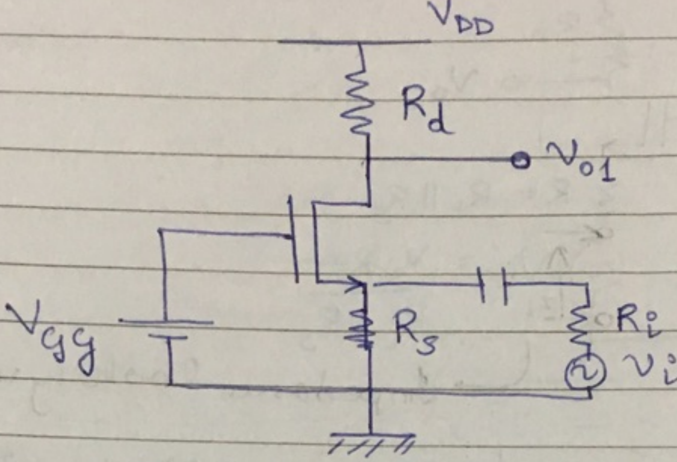
$$Z_o' = Z_o \| R_D$$

$$\boxed{A_v = \frac{-4.1}{\cancel{-0.4}}/10}$$ (We need $R_i$ to be even smaller for

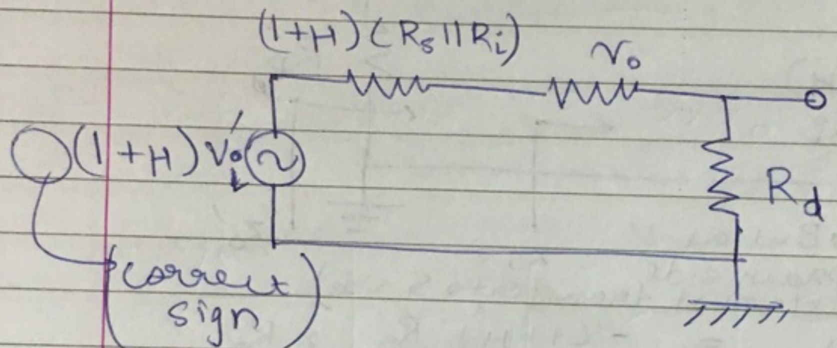
$$Z_i = \frac{R_D + r_o + (1+\mu)(R_i \| R_s)}{(1+\mu)}$$

$$= \frac{R_D + r_o}{(1+\mu)} + (R_i \| R_s)$$ approx

18.1.18



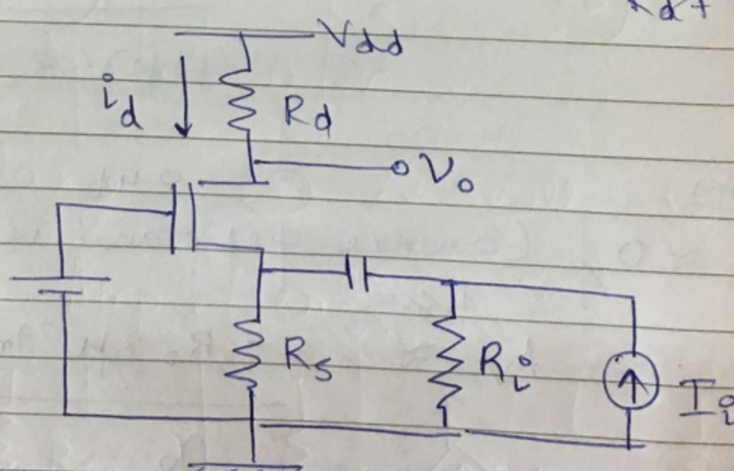

$\mu = g_m r_o$ $\qquad v_i' = \dfrac{V_i R_s}{R_i + R_s}$ $\qquad \left( i_i = \dfrac{v_i}{R_i} \right)$




$v_o = \dfrac{(1+\mu) v_i' R_d}{R_d + r_o + (1+\mu)(R_i \| R_s)}$

$v_o = \cancel{\dfrac{(1+\mu) i_i R_i R_d}{R_d + r_o + (1+\mu)(R_i \| R_s)}}$

→ Current Source




## Current Gain

$v_o = -i_d R_d$

$$-i_d = \frac{v_o}{R_d} \quad \cancel{\rightarrow (1+\mu)\, i_i}$$

$$v_o = \frac{(1+\mu)\, i_i\, (R_i \| R_s)\, R_d}{(1+\mu)(R_i \| R_s) + r_o + R_d}$$

$$-i_d = \frac{v_o}{R_d} = \frac{(1+\mu)\, i_i\, (R_i \| R_s)}{(1+\mu)(R_i \| R_s) + r_o + R_d}$$

$$A_i = \frac{i_d}{i_i} = \frac{-(1+\mu)(R_i \| R_s)}{(1+\mu)(R_i \| R_s) + r_o + R_d}$$

→ Notes: $|A_i| \leq 1$

→ $$A_i = -\frac{R_i \| R_s}{R_i \| R_s + \frac{(r_o + R_d)}{(1+\mu)}} \approx -1$$

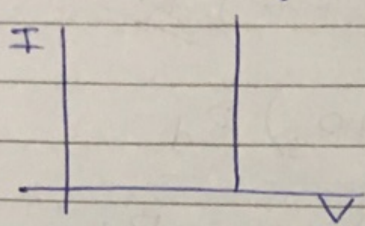
→ <u>Summary</u>

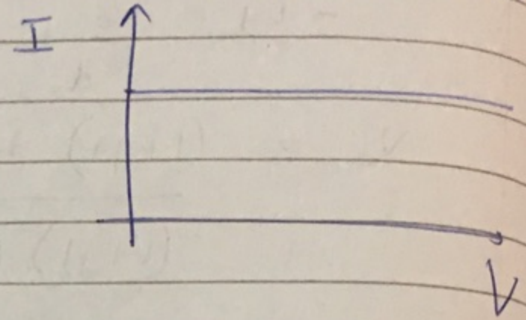
Gate current is 0 (Dielectric)

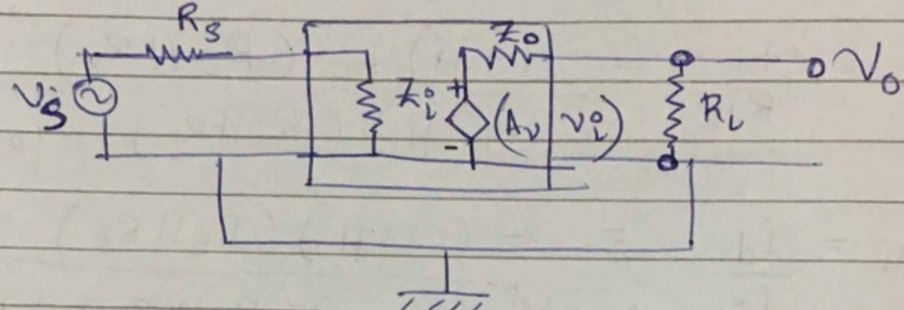
| | CS | CD | CG |
|---|---|---|---|
| $A_v$ | High | unity | High |
| $A_i$ | — | — | unity (Best Case) |
| $Z_i$ | High | High | Low |
| $Z_o$ | High | Low | High |

→ Note: High means $r_o$ or $\mu$ term.

→ Ideal Voltage source          Ideal current

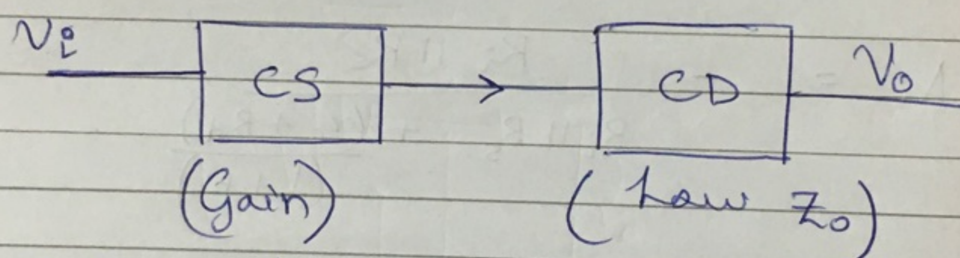



→ We want $R_L >> Z_o$ $\left( V_o = A_v v_i \times \frac{R_L}{R_L + Z_o} \right.$

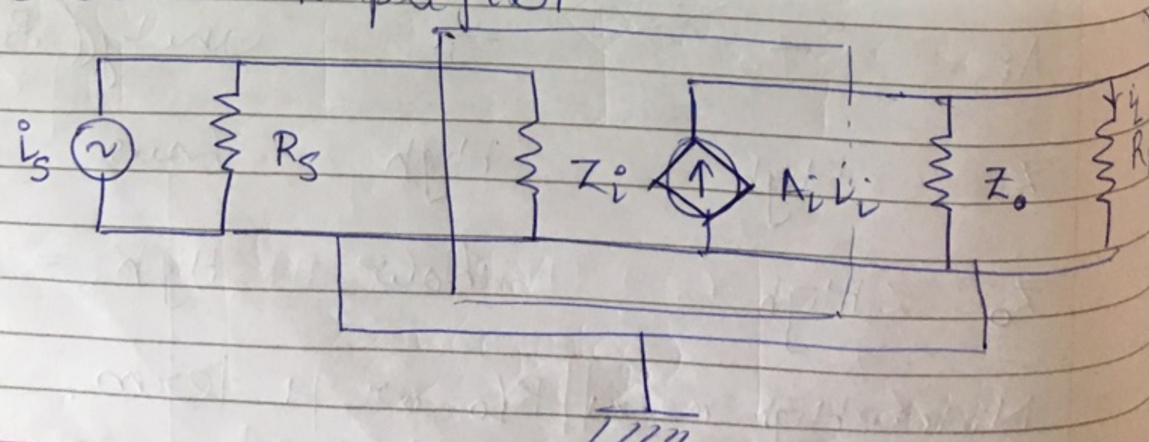
Hence we want a small $Z_o$




→ Voltage Amplification

1. $A_v$ large  2. $Z_o$ Small  3. $Z_i$ large

→ Current Amplifier

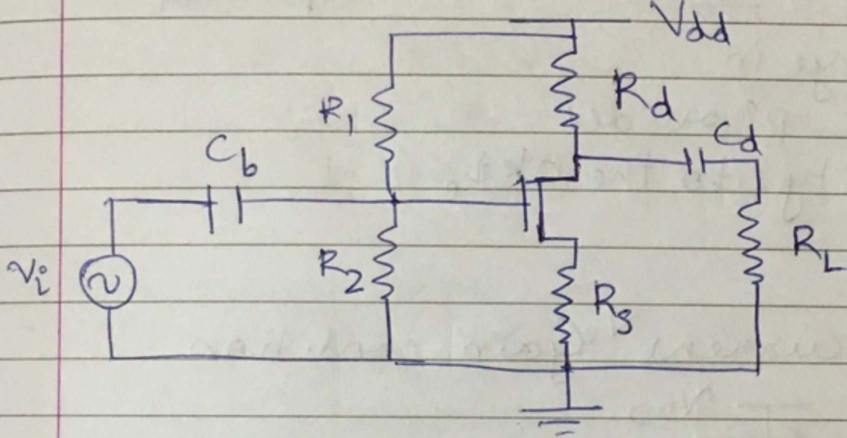


# Current Amp.

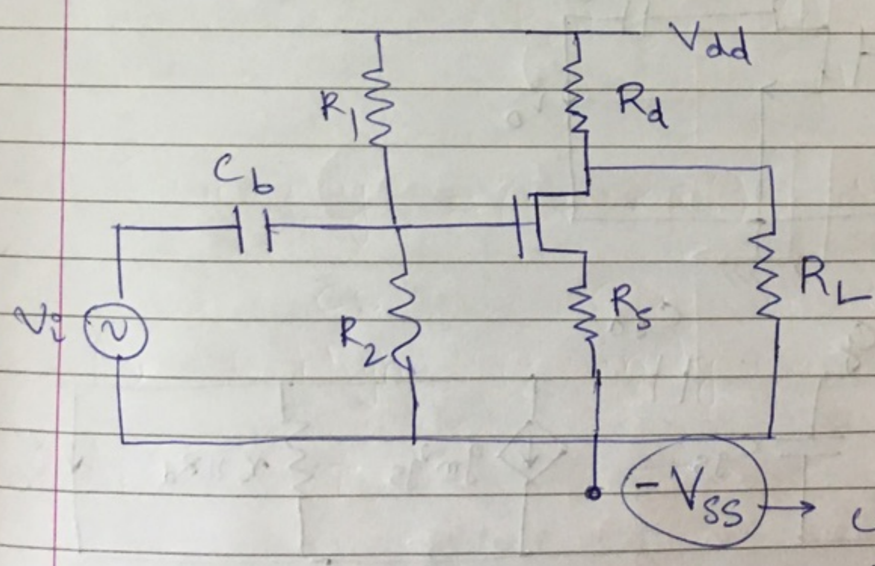
① $A_i$ Large ② $Z_o$ large ③ $Z_i$ small

→ We can't get ~~Gain~~ Current Gain with MOSFETs (We use BJTs)
Fundamentally MOSFET is $\boxed{\text{VCCS}}$

→ Details




AC coupled

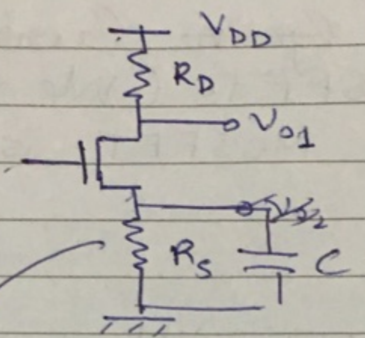



DC coupled

→ chosen such that DC bias at drain is 0

22.1.18

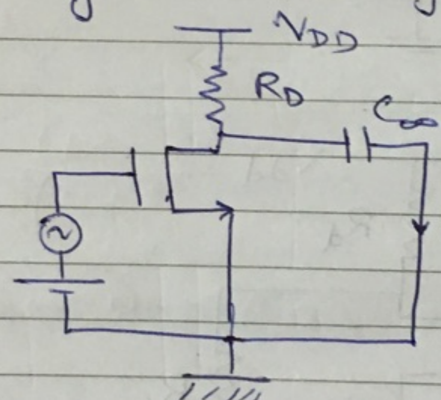
→ Source Bypass capacitor

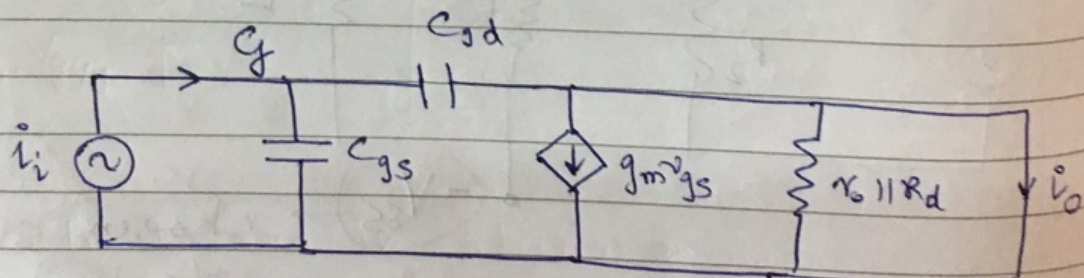



C ≡ Source By pass capacitor

No Change in Gain, but provides stability to the ckt.

→ Unity current Gain condition




$$\boxed{f_T = \frac{g_m}{2\pi (C_{gs} + C_{gd})}}$$

H.W

→ Choosing value of $R_d$

Q→ → Find value of $R_d$ that maximizes the gain.
→ Drawbacks of large $R_D$?
→ Soln. Active load, low pot. across terminals

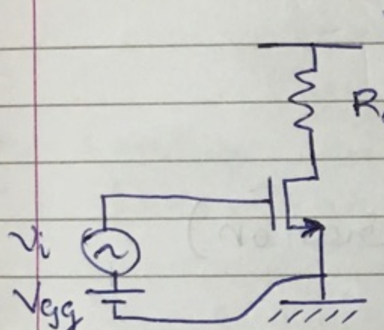
23.1.18

$K' = 1 mA/V^2$ $\qquad I_{DS} = 2mA$

$V_T = 1V$ $\qquad g_m = \sqrt{2K' I_{DS}}$

$\lambda = 0.01 V^{-1}$ $\qquad = 2m S$

$$r_o = \frac{1}{\lambda I_{DS}} = \frac{100}{2} = 50 K\Omega$$




$$\mu = g_m r_o = 100$$

$$A_v\Big|_{R_D = 10K\Omega} = \frac{-\mu R_D}{\underbrace{60}_{10K\Omega + 50K\Omega}} = \frac{-100 \times 10}{60}$$

Using $\left(A_v = \frac{-\mu R_D}{R_D + r_o}\right)$ $\qquad = -16.6$ Improvement!

$$A_v\Big|_{R_D = 50K\Omega} = \frac{-100 \times 50}{100} = -50$$

→ $V_{DD}$ calculation

$$I_{DS} = \frac{K'}{2}(V_{gs} - V_T)^2$$

$$V_{gs} = \sqrt{\frac{2 I_{DS}}{K'}} + V_T$$

$$= \sqrt{2 \times 2} + 1 = 3V$$

$V_{DS} > V_{gs} - V_T$

$> 2V$

$V_{DS} = 2 + 1 = 3V$ (Tolerance)

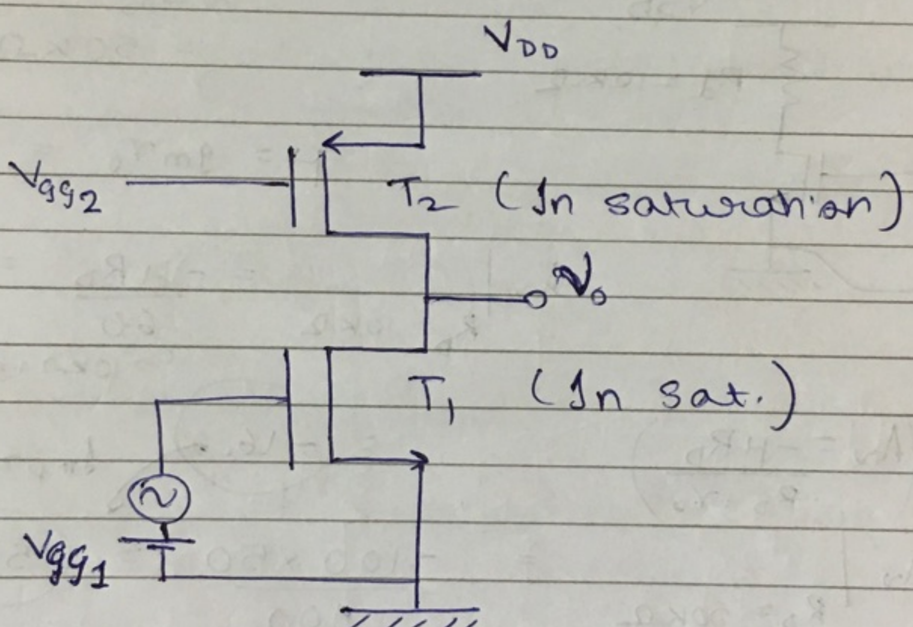
→ $R_D = 10k\Omega$, $I_D R_D + 3 = V_{DD}$

$= 2 \times 10 + 3$

$= 23V$ !

→ $R_D = 50k\Omega$, → $103V$ !

→ <u>Solution : Use a PMOS</u>




$T_1, T_2$ are in saturation

→ Parameters of $T_1$ remain the same

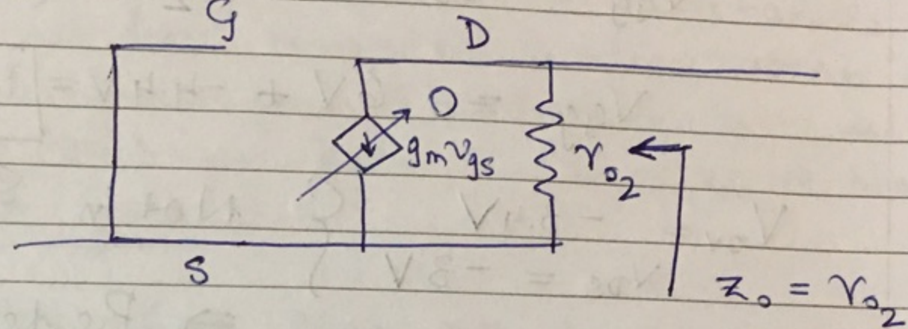
→ $T_2$

$V_{T_2} = -1V$

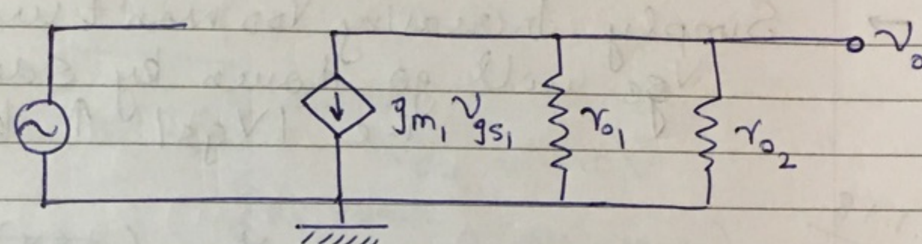
$\lambda_2 = 0.01 V^{-1}$

$K_2' = \frac{1}{3} K_1' = \frac{1}{3} mA/V^2$

→ For $T_2$




→ For $T_1$




→ Looking In from a Node ⇒ Impedance b/w that node & Ground.

→ $r_{o_2}$ is large, ⇒ Problem solved!

→ ~~Find $V_{gg_2}$~~, We know that $V_{DS} = 3V$, Pick $V_{DS}\big|_{PMOS} = -3V$ ⇒ Required $V_{DD} = 6V$

→ Find $V_{gg_2}$ if $I_{DS} = 2mA$

$$I_{DS} = \frac{K_2'}{2}(V_{gs_2} - V_{T_2})^2$$

$$|I_{DS}| = 2mA = \frac{1}{3\times 2}\cancel{(V_{gs_2} - V_{T_2})^2}\, v_{ov}^2$$

$$v_{ov} = \sqrt{12} = \pm 3.4V$$

$V_{gs} - V_T = 3.4V$ ✗ $(V_{gs} > V_T \;✗)$

$V_T - V_{gs} = 3.4V$ ✓ $(V_{gs} < V_T)$

$V_{gs} = -4.4V$

$$V_{gg} - V_{DD} = V_{gs_2} \quad (\text{Defn.}$$

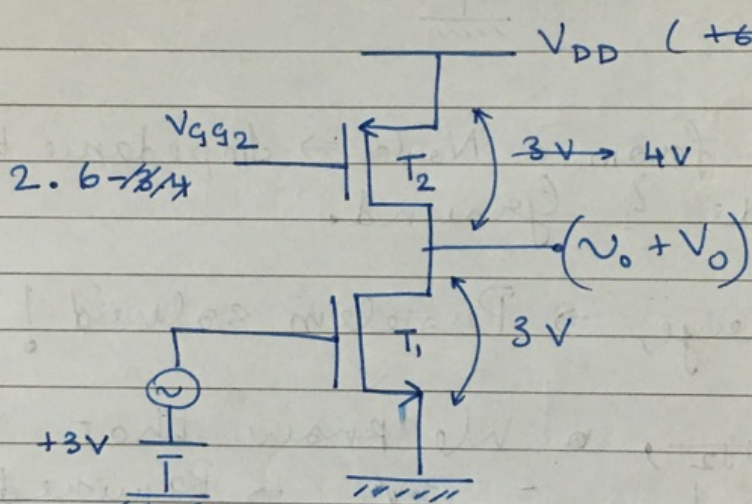
$$V_{gg} = 6V + -4.4V = \boxed{1.6V}$$

$V_{ov} = -3.4V$
$V_{DS} = -3V$ } Not in Saturation

$\Rightarrow$ Redesign

→ Simply Increasing $V_{DD}$ won't work since $V_{gs}$ will go down by same amount i.e $|V_{gs}| \uparrow$ by same.

25.1.18




$k_1' = 1\,mA/V^2$

$k_2' = \frac{1}{3}\,mA/V^2$

$r_o = 50k\Omega$

$\lambda = 0.01\,V^{-1}$

$I_{DS} = 2mA$

$$A_v = -g_{m_1}(r_{o_1} \| r_{o_2}$$

→ In simple model $V_o$ is undeterministic (In a range),

$$I_{DS} = \frac{k'}{2}(V_{gs} - V_t)^2 \rightarrow \text{Peak.}$$

$$I_{DS} = \underbrace{\frac{k'}{2}(V_{gs} - V_t)^2}(1 + \lambda V_{DS}) \rightarrow \text{deterministic}$$

over here $(V_{DS})_1 + (V_{DS})_2 = 7V$
& Since $I_{DS}$ values are the same it will be midpoint in this case.

→ Why we care about DC bias: Theoretical max headroom is $= V_{DS}$ (DC bias)



→ Advantages of PMOS
- ↓ footprint compared to resistances.
- We can have inverted configuration with NMOS load and PMOS Gain, but we loose out on both $r_o$ and $g_m$ ⇒ Not as Good.

→ HW: solve for $V_{DS}$ at junction of transistors.

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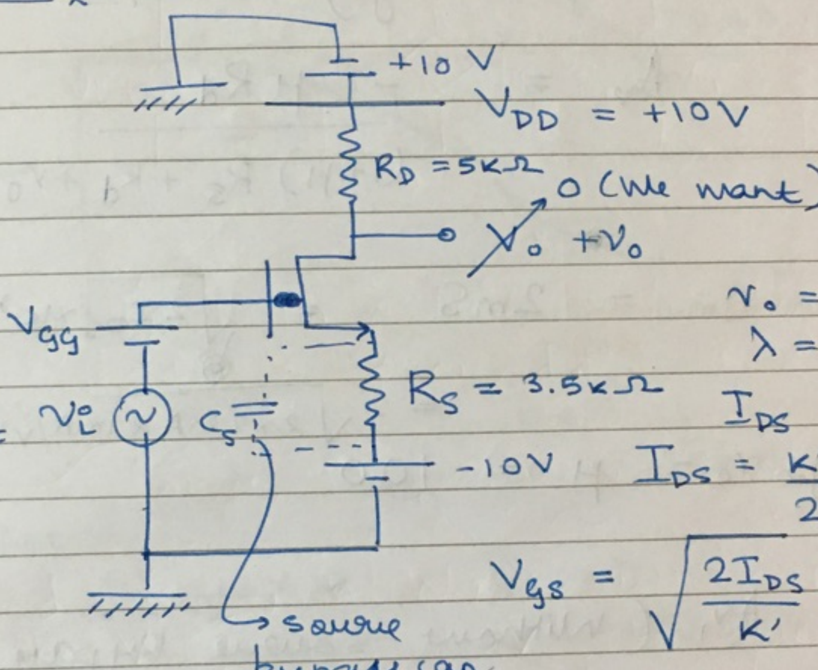



$V_{DS} > V_{gs} - V_t$
$V_{DS} > 2V$
pick, $V_{DS} = 3V$

$r_o = 50k\Omega$
$\lambda = 0.01 V^{-1}$
$I_{DS} = 2mA$

$$I_{DS} = \frac{K'}{2}(V_{gs} - V_t)^2$$

$$V_{gs} = \sqrt{\frac{2I_{DS}}{K'}} + V_t = 3V$$

→ $V_o = 0$

$$\Rightarrow V_{DS} + I_{DS} R_S = 10V$$

$$R_S = \cancel{\frac{10-3}{2}} \; \frac{10V - 3V}{2mA} = 3.5k\Omega$$

$$V_{DD} = I_D R_D$$

$$\Rightarrow R_D = 5k\Omega$$

→ Hence it is possible to get a DC coupled (No need for cap. @ o/p) Amplifier

↳ But we need a Bipolar source (Drawback)

→ Generally we use it as a middle stage amp. in op amp IC's (Avoid cap. inside package)

$$A_v = \frac{-\mu R_d}{(1+\mu) R_s + R_d + r_o}$$

$$g_m = 2mS = \sqrt{2 I_{DS} \times k'}$$

$$= \sqrt{2 \times 2mA \times 1mA/V^2} = 2mS$$

$$g_m r_o = \mu = 100$$

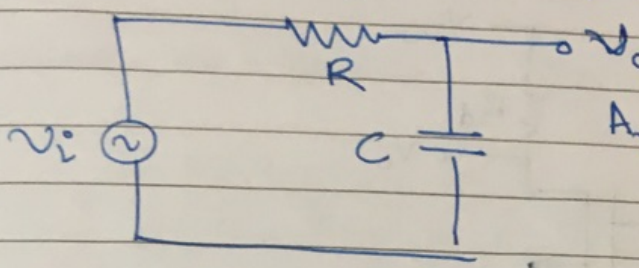
$A_{v_1}$ (without source bypass cap) $= -1.25$

$R_s$ is pulling $A_v \downarrow$

⇒ Use a source bypass cap.

$$A_{v_2} = \frac{-\mu R_d}{R_d + r_o} = \underline{\underline{-10}}$$

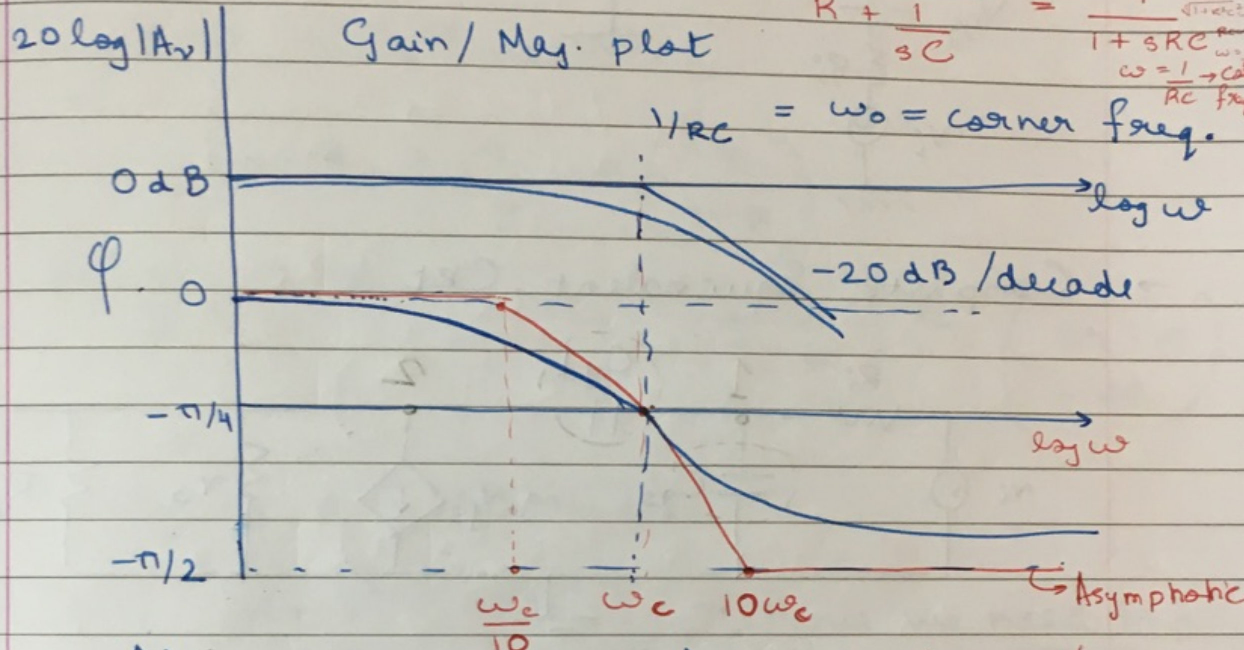
# High frequency Response




$$A_v = \frac{v_o}{v_i} = \frac{1/j\omega C}{R + 1/j\omega C}$$

$$= \frac{1/sC}{R + \frac{1}{sC}} = \frac{1}{1+sRC}$$

$\omega = \frac{1}{RC} \to$ corner freq.




At $\omega_0 = 1/RC$ $\rightarrow$ $A_v = \frac{1}{\sqrt{2}}$ (corner / 3dB freq.)

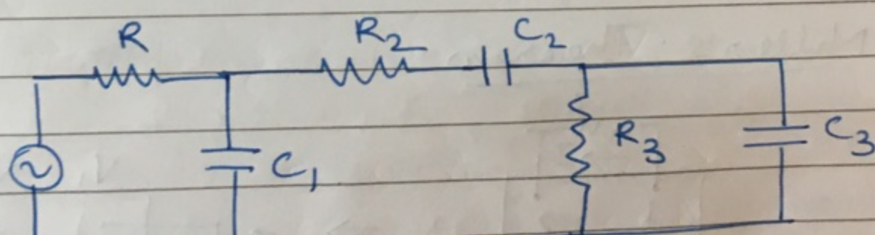
→ Phase;

$\omega = 0, \quad \varphi = 0$

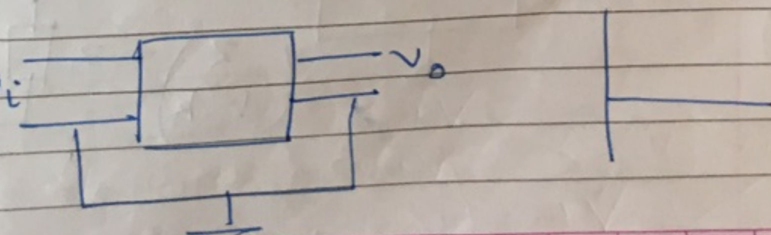
$\omega = \infty, \quad \varphi = -\pi/2$

$\omega = 1/RC, \quad \varphi = -\pi/4$

ex

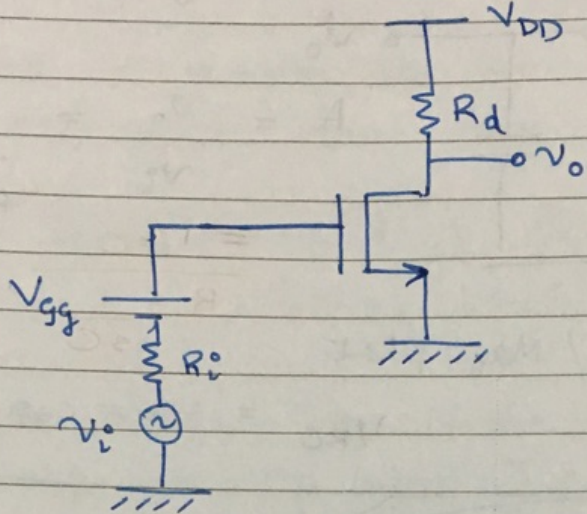



→ Consider it as a cascade of ports with transfer functions Multi.

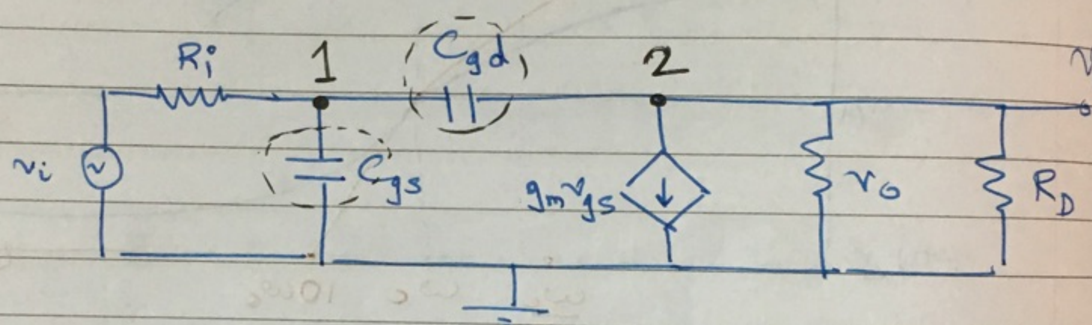


29.1.18

# Transistor at High frequency



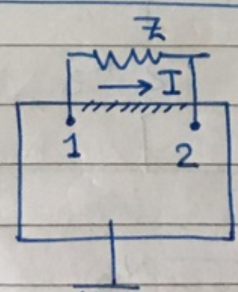

→ Complete Equivalent Ckt



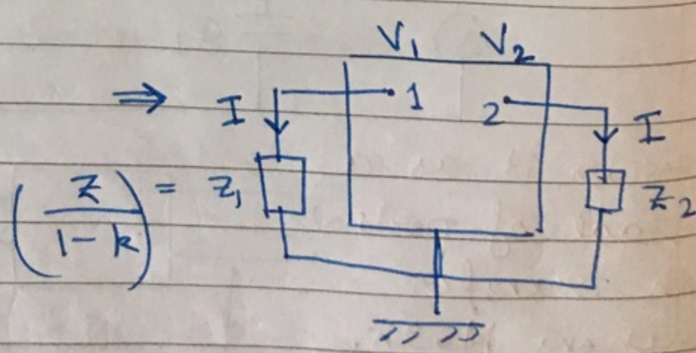

$$\frac{v_o}{v_i} = \frac{-\frac{\cancel{g_m}(R_d \parallel r_o)}{R_i}(g_m - sC_{gd})}{s^2 C_{gs} C_{gd}(r_o \parallel R_d) + s\left(C_{gs} + C_{gd} + C_{gd}(r_o \parallel R_d) \times (g_m + 1/R_i)\right) + 1/R_i}$$

$\cancel{\times (g_m + 1/R_i)}$

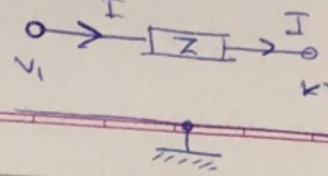
→ <u>Miller's Theorem</u>



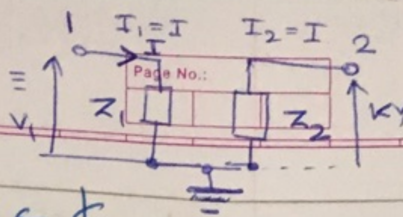

$\Rightarrow$



Better Diagram




$\equiv$




→ Suppose we know, $\frac{V_2}{V_1} = k \in \mathbb{C}$

$$I = \left(\frac{V_1 - V_2}{Z}\right) = \frac{1 - V_2/V_1}{Z/V_1} = \frac{(1-k)\,V_1}{Z}$$

~~$Z_1$~~ In 2nd model:

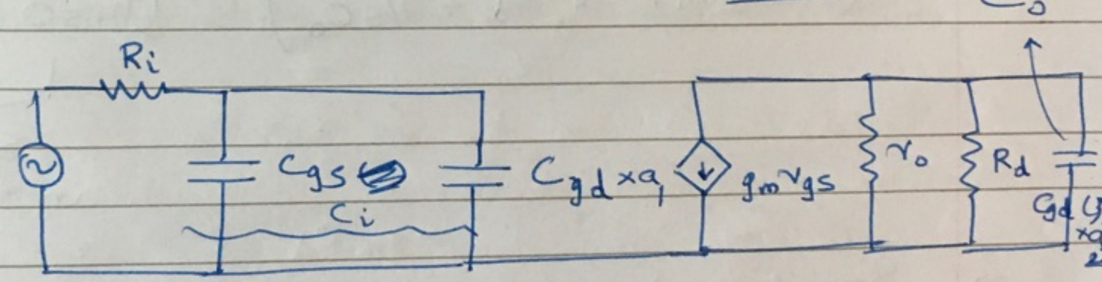
$$Z_1 = \frac{V_1}{I} = \left(\frac{Z}{1-k}\right)$$

$$\frac{kV_1}{I} = Z_2 = \left(\frac{k}{1-k}\right) Z \quad \left(\text{or its } -\text{ve}\right)$$

→ Apply to our SS Model:

$$K = \frac{V_2}{V_1} = \left(\frac{v_o}{v_{gs}}\right)$$ → Hence we need the Gain

$(r_o \parallel R_d) \times \left(g_m + \frac{1}{R_i}\right)$




$$a_1 = (1-k)$$

$$a_2 = \left(\frac{k-1}{k}\right)$$

$\left\{ Z = \frac{1}{sC} \right.$ ⇒ Inverse factor for C

Define; $C_i = C_{gs} + C_{gd}(1-k)$

$$C_o = C_{gd} \frac{(k-1)}{k}$$

$$\cancel{A_v = \frac{v_o}{v_i} = -g_m v_{gs}}$$

$$v_o = -g_m v_{gs}\left(r_o \parallel R_d \parallel \frac{1}{sC_o}\right)$$

$$v_o = \frac{-g_m\left(r_o \parallel R_d \parallel \frac{1}{sC_o}\right) \times v_i \times \frac{1}{sC_i}}{\frac{1}{sC_i} + R_i}$$

$$A_v = \frac{-g_m\left(r_o \parallel R_d \parallel \frac{1}{sC_o}\right)}{(1 + sC_iR_i)} =$$

$$\frac{-g_m}{(1+sC_iR_i)}\left(\frac{R_d' \frac{1}{sC_o}}{R_d' + 1/sC_o}\right) = \frac{-g_m \qquad R_d'}{(1+sC_iR_i)(1+sC_oR_d')}$$

Symmetric in $C_i / C_o$

→ Sample values

$g_m = 2mS$, $R_i = 1k\Omega$, $C_{gs} = 10pF$, $C_{gd} = 1pF$

$r_o = 50k\Omega$, $R_d = 10k\Omega$

Comparing Expressions

1) Original Expression (Previous page)

$$= \frac{-8.3\,(2\times10^{-3} - 5\times10^{-12})}{\cancel{s^2C_{gs}C_{gd}(r_o}\; s^2(8.3\times10^{-20}) + s(3.59\times10^{-12}) + 10^{-3}}$$

2) Our expression using ~~modified~~

$$K = \frac{V_2}{V_1} = \left(\frac{v_o}{v_{gs}}\right) \approx -g_m (r_o \parallel R_d)$$
$$= -16.6$$

value of K is used

→ Simulation Results

$$20 \log\left(\frac{V_2}{V_i}\right) = 15$$

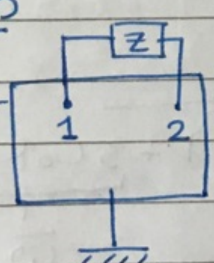
$$\Rightarrow \frac{V_2}{V_i} = 10^{15/20} = 5.6$$

$$\approx \frac{-16.6}{(1 + s \times 27.6 \times 10^{-9})(1 + s \times 1.06 \times 8.3 \times 10^{-9})}$$

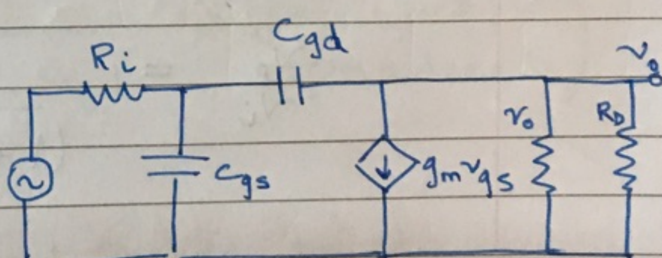
→ Our analysis shows that approximation is valid uptill 3dB Gain ⇒ It can be used.

30.1.18

Recap

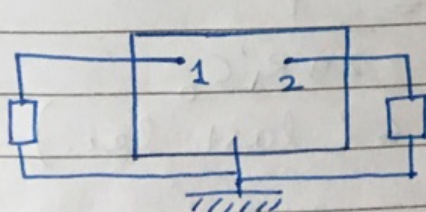



$\frac{V_2}{V_1} = K$

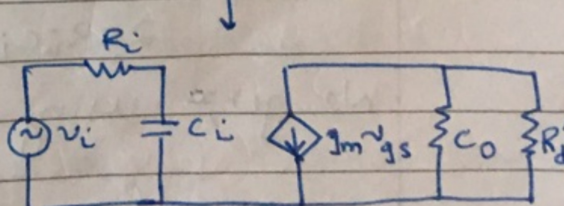



↓

$Z_1 = \frac{Z}{1-K}$




$Z_2 = \frac{KZ}{(K-1)}$




Where,

$C_i = C_{gs} + C_{gd}(1-K)$ $= 27.6\,pF$ where $R_d' = r_o \parallel R_D \approx R_D$ $= 8.33\,k\Omega$

$C_o = \left(\frac{K-1}{K}\right) C_{gd}$ $= 1.06\,pF$

$$\frac{v_o}{v_i} = \frac{-\cancel{g_m}\left(\frac{r_o \| R_D}{R_i}\right)(g_m - sC_{gd})}{s^2 C_{gs} C_{gd}(r_o \| R_D) + \frac{1}{R_i} + s\left[C_{gs} + C_{gd} + C_{gd}(r_o \| R_D)\left(g_m + \frac{1}{R_i}\right)\right.}$$

$\downarrow$ simplified version, using

$$\frac{v_o}{v_i} = \frac{-g_m R_d'}{(1 + sC_i R_i)(1 + sC_o R_d')}$$

→ <u>Miller's thm</u>

$$I = \frac{V_1 - V_2}{Z} = V_1\frac{(1-k)}{Z} = \frac{\cancel{V_1(1-k)}\, V_1}{Z/(1-k)}$$

$$\Rightarrow Z' = \left(\frac{Z}{1-k}\right)$$

$$-I = \frac{V_2 - V_1}{Z} = \frac{V_2}{Z}\left(1 - \frac{1}{k}\right) \Rightarrow Z' = \left(\frac{kZ}{k-1}\right)$$

→ <u>Bode plot</u> of <u>$A_v$ vs $f\omega$</u>

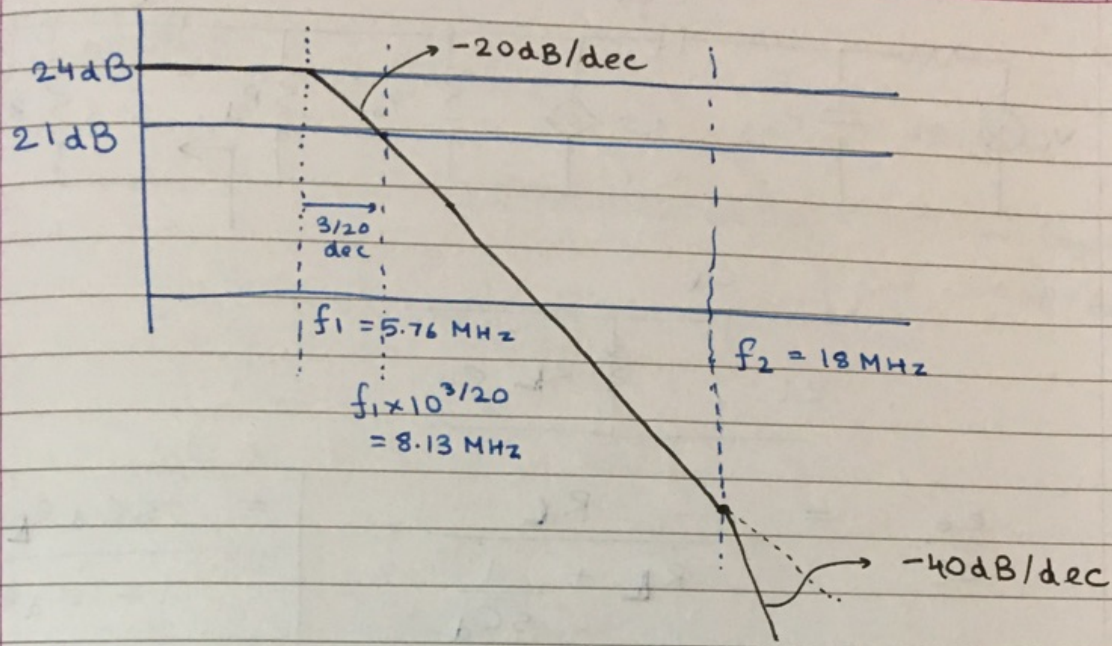
$$\frac{v_o}{v_i} = \frac{-g_m R_d'}{(1 + \underbrace{sC_i R_i}_{\frac{1}{\omega_1}})(1 + \underbrace{sC_o R_d'}_{\frac{1}{\omega_2}})}$$

$$f_1 = \frac{1}{2\pi R_i C_i}, \quad f_2 = \frac{1}{2\pi R_d' C_o}$$

We get; using $k = -16.6$ (last lec.)

$$f_1 = 5.76\,\text{MHz}, \quad f_2 = 18\,\text{MHz}$$

$$20 \log (g_m R_d') = 24\,\text{dB (peak gain)}$$




→ **Approximations**

$$A_v\Big|_{DC,\,\omega=0} = -g_m\,(r_o \parallel R_d) \approx K$$

$$f_1 = \frac{1}{2\pi R_i C_i}$$

$$f_1 \simeq \frac{1}{2\pi R_i (1-k) C_{gd}} \quad ; (1-k)C_{gd} > C_{gs}$$

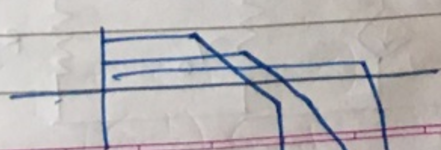
$$f_1 \approx \frac{1}{2\pi R_i |K| C_{gd}}$$

$$|A_v|\, f_1 = \frac{1}{2\pi R_i C_{gd}} \quad \text{(Gain - BW product)}$$

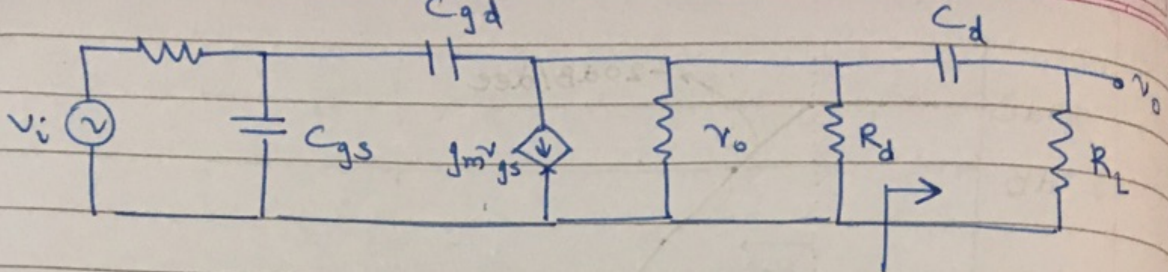
→ Qualitatively :

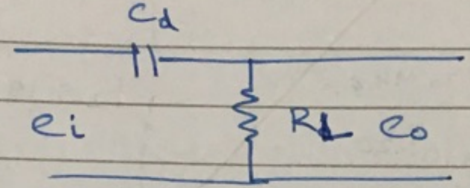
- High V across cap. $C_{gd}$ needs to change, hence can't respond at very high frequency
  $(1+A_v)\, v_i$ changes period. across cap.

  ↳ $R_i C_i \approx \tau$ ($v_o$ is being changed by this)

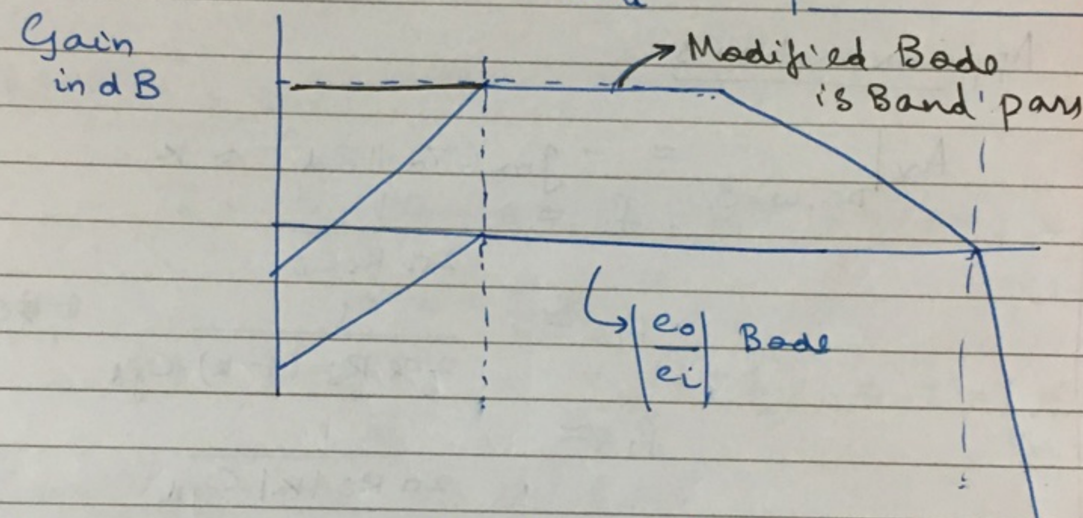


(Must sacrifice Gain for BW)



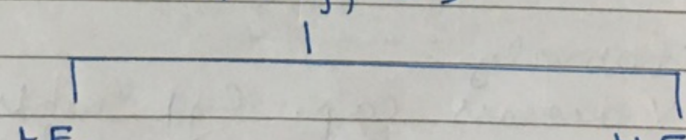

$$\frac{e_o}{e_i} = \frac{R_L}{R_L + \frac{1}{sC_d}} \quad \boxed{= \frac{sC_d R_L}{1 + sC_d R_L}}$$



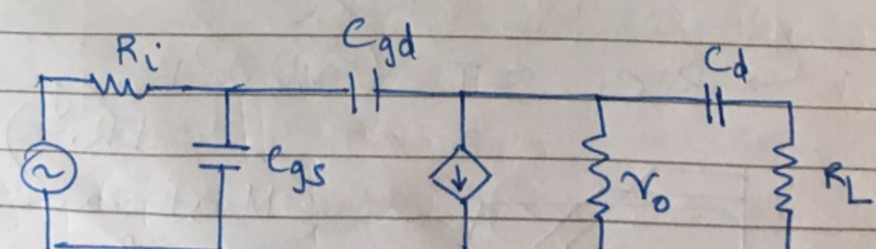

→ Hence we get a Band pass filter.

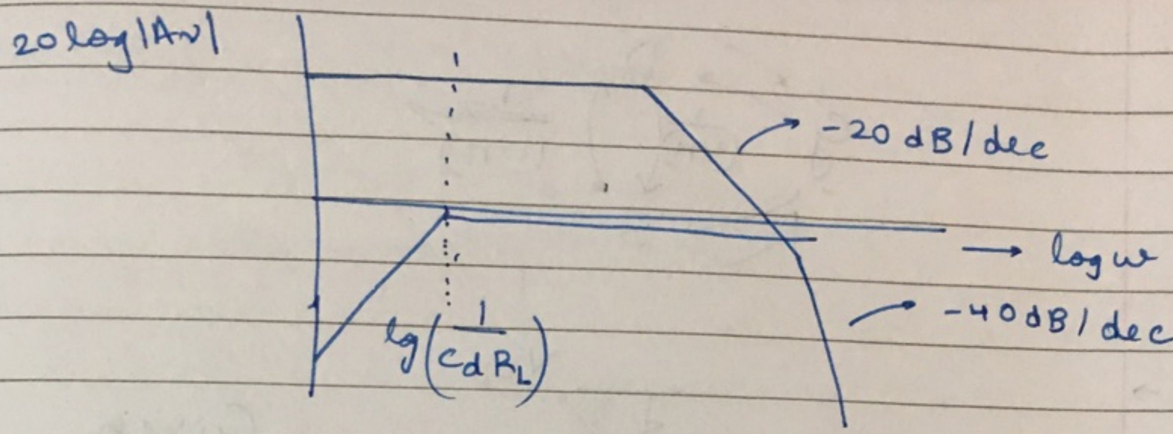
1.2.18

CS, CG, CD




→ <u>Bode plot</u> :






$$\frac{1}{1+\frac{1}{sC_dR_L}} \qquad \frac{1}{1+\frac{1}{j\omega C_dR_L}}$$

$$\omega_c = \frac{1}{C_d R_L}$$

→ No -40 dB line in simulation

$$\frac{v_o}{v_i} = \frac{-g_m(r_o \| R_d)}{(1+sR_iC_i)(1+sC_o(r_o\|R_d)r_i)}$$

$$\omega \to \infty \Rightarrow \frac{v_o}{v_i} \propto \frac{1}{\omega^2} \quad (-40\,dB)$$

$$\frac{v_o}{v_i} = \frac{-(g_m - sC_{gd})}{\alpha s^2 + \beta s + \gamma_i}$$

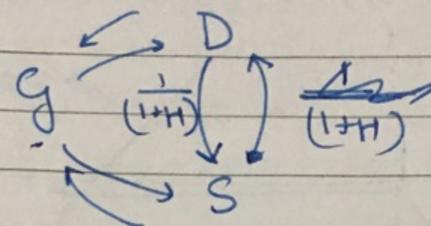
$$\omega \to \infty \Rightarrow \frac{v_o}{v_i} \propto \frac{1}{\omega} \quad (-20\,dB)$$

→ Voltage signs in referred ckt

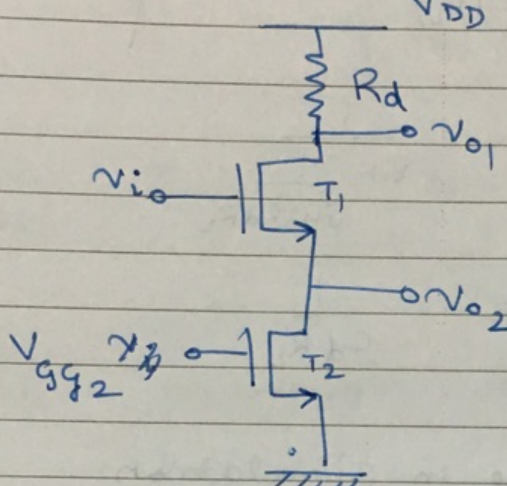
$v_i \uparrow \Rightarrow i_{ds} \uparrow \Rightarrow i_{ds}R_d \uparrow \Rightarrow v_{o1} \downarrow \rightarrow$ CS

$v_i \uparrow \Rightarrow i_{ds} \uparrow \Rightarrow i_{ds}R_s \uparrow \Rightarrow v_{o2} \uparrow \rightarrow$ CD

$v_i \uparrow \Rightarrow v_s \uparrow \Rightarrow v_{o2} \uparrow \Rightarrow v_{o1} \uparrow$



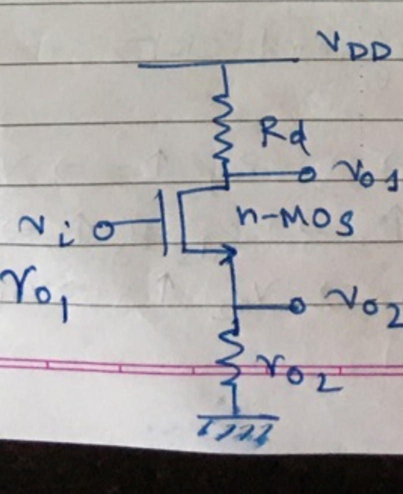

Q →




Given,
$T_1$ & $T_2$ in sat.

| CS | CD |
|---|---|
| $A_v = \dfrac{-\mu R_d}{R_d + (1+\mu) r_{o_2} + r_{o_1}}$ | $A_v = \dfrac{\left(\frac{\mu}{1+\mu}\right) R_S}{\left(\frac{R_d + r_{o_1}}{1+\mu}\right) + r_{o_2}}$ |
| $Z_i = \infty$ | $Z_i = \infty$ |
| $Z_o = R_d \parallel [r_{o_1} + (1+\mu) r_{o_2}]$ | $Z_o = r_{o_2} \parallel \left[\dfrac{r_{o_1} + R_d}{(1+\mu)}\right]$ |

↓ ac eqvt

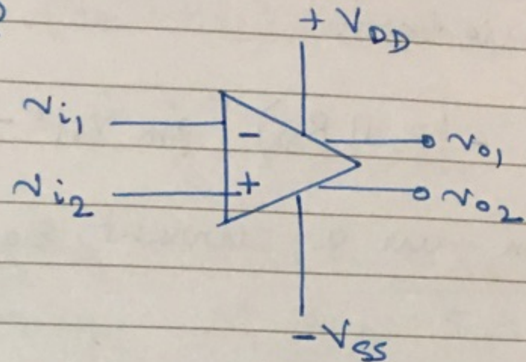



$\mu = g_{m_1} r_{o_1}$

⇒ Replace $R_s$ by $r_{o_2}$

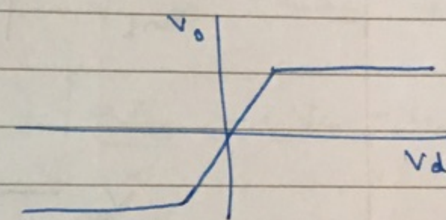
# Differential Amplifier

→ op amp

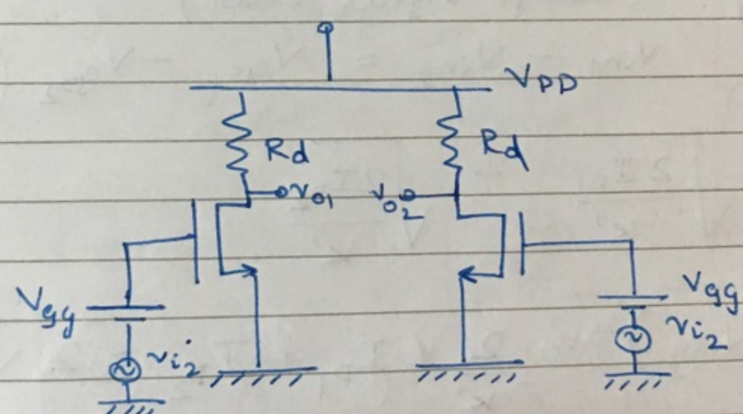



$$v_d = v_{diff,in} = (v_{i_1} - v_{i_2})$$
$$v_o = v_{diff,out} = (v_{o_1} - v_{o_2})$$

$$\left| \frac{v_o}{v_d} \right| = \text{large}$$




→ primitive difference amplifier




→ Assume completely Identical tran.

$$A_{diff} = \frac{v_{o_1} - v_{o_2}}{v_{i_1} - v_{i_2}} = \frac{-g_m(r_o \| R_d)(v_{i_1} - v_{i_2})}{(v_{i_1} - v_{i_2})}.$$

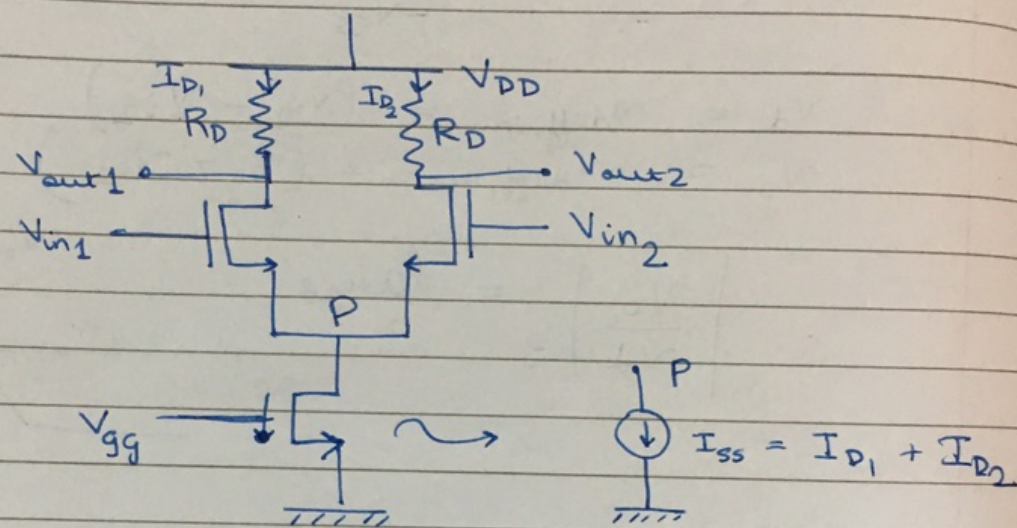
$$\boxed{A_{diff} = -g_m(r_o \| R_d)}$$

→ Major flaw: If there is slight mismatch b/w transistors,

$$A_{diff} = -(r_o \parallel R_D)\left[g_m v_{i_1} - g_m v_{i_2}\right]$$

→ solution use a current source below

6.2.18




→ Equations: (Large Signal char.)

$$V_{in_1} - V_{in_2} = V_{gs_1} - V_{gs_2} \quad \left((V_{in_1} - V_P) - (V_{in_2} - V_P)\right)$$

$$= \sqrt{\frac{2 I_{D_1}}{K'}} - \sqrt{\frac{2 I_{D_2}}{K'}}$$

$$(V_{in_1} - V_{in_2})^2 = \frac{2}{K'}\left(I_{D_1} + I_{D_2} - 2\sqrt{I_{D_1} I_{D_2}}\right)$$

$$I_D = \frac{K'}{2}(V_{gs} - V_T)^2$$

$$\Rightarrow V_{gs} = \sqrt{\frac{2 I_D}{K'}} + V_T$$

- $V_{out_1} = V_{DD} - I_{D_1} R_D$

  $V_{out_2} = V_{DD} - I_{D_2} R_D$

Page No.:

→ We want $(v_{o_1} - v_{o_2})$ in terms of $(v_{i_1} - v_{i_2})$

$$V_{o_1} - V_{o_2} = -R_D (I_{D_1} - I_{D_2})$$

$$4 I_{D_1} I_{D_2} = (I_{D_1} + I_{D_2})^2 - (I_{D_1} - I_{D_2})^2$$

$$\sqrt{I_{D_1} I_{D_2}} = \frac{1}{2} \sqrt{I_{SS}^2 - (I_{D_1} - I_{D_2})^2}$$

$\cancel{I_{D_1} - I_{D_2}} =$

$$\Rightarrow (V_{in_1} - V_{in_2}) = \frac{2}{k'} \left( I_{SS} - \sqrt{I_{SS}^2 - (I_{D_1} - I_{D_2})^2} \right)$$

$$I_{D_1} - I_{D_2} = \frac{k'}{2} (V_{in_1} - V_{in_2}) \sqrt{\frac{4 I_{SS}}{k'} - (V_{in_1} - V_{in_2})^2}$$

$$\sqrt{\frac{2}{k'}} \left( \sqrt{I_{D_1}} - \sqrt{I_{D_2}} \right)$$

→ Transconductance

$$G_m = \frac{\delta \Delta I_D}{\delta \Delta V_{in}} \quad \text{(Differential amp.)}$$

$$G_m = \frac{1}{2} k' \left( \frac{\frac{4 I_{SS}}{k'} - 2 \Delta V_{in}^2}{\sqrt{\frac{4 I_{SS}}{k'} - \Delta V_{in}^2}} \right)$$

$$\Delta V_{in} << 1, \quad G_m = \frac{k'}{2} \sqrt{\frac{4 I_{SS}}{k'}} = \sqrt{k' I_{SS}}$$

→ Regular amp. $g_m = \frac{\delta I_{DS}}{\delta V_{GS}} = k'(V_{GS} - V_T)$

$$= k' \sqrt{\frac{2 I_D}{k'}} = \sqrt{2 k' I_D}$$

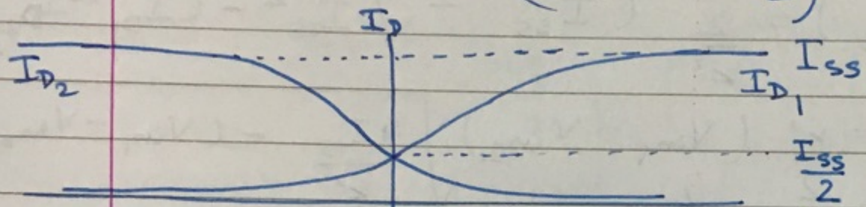
$$\rightarrow \quad \Delta V_{out} = V_{out1} - V_{out2} = (V_{DD} - I_{D_1} R_D) - (V_{DD} - I_{D_2} R_D)$$

$$\Delta V_{out} = -(I_{D_1} - I_{D_2}) R_D$$

$$A_v = \frac{\delta \Delta V_{out}}{\delta \Delta V_{in}} = \frac{\delta \Delta V_{out}}{\delta \Delta I_D} \cdot \frac{\delta \Delta I_D}{\delta \Delta V_{in}}$$

$$A_v = -G_m R_D = -\sqrt{k' I_{SS}}\, R_D$$

$$\Delta V_{in} = (V_{in_1} - V_{in_2})$$

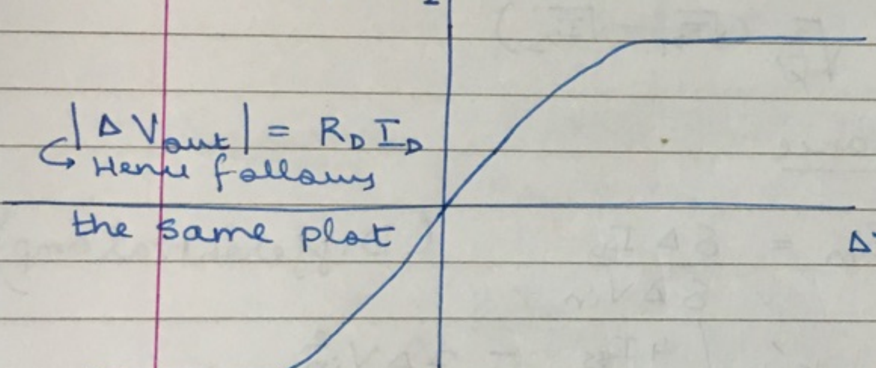



$\Delta I_D = I_{D_1} - I_{D_2}$

$\Delta V_{in} = (V_{in1} - V_{in2})$

$|\Delta V_{out}| = R_D I_D$

↳ Hence follows the same plot




→ <u>Differential Amp. key Equations</u>

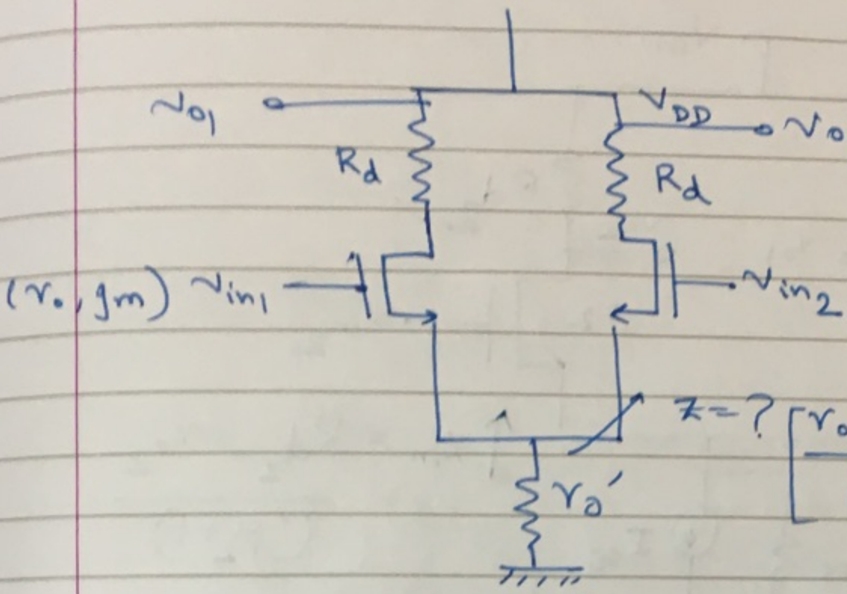
$$I_{D_1} - I_{D_2} = f(V_{in_1} - V_{in_2})$$

$$I_{D_1} + I_{D_2} = I_{SS}$$

$$G_m = \frac{\delta \Delta I_D}{\delta \Delta V_{in}}$$

$$A_v = -G_m R_D$$

→ Small Signal Char. ( Using superposition)



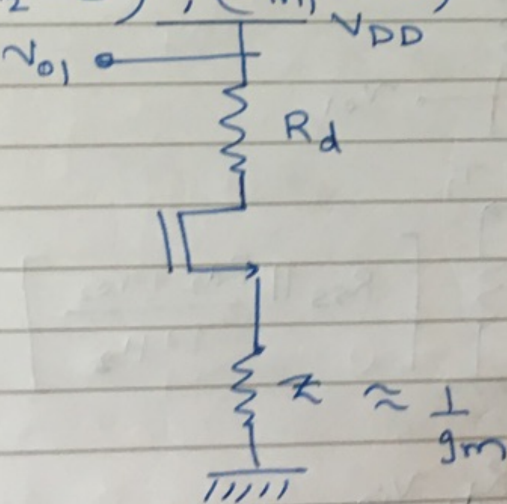

$$\left[\frac{r_o + R_d}{1+\mu}\right] \parallel r_o' = Z$$

$$\frac{r_o + R_d}{1+\mu}, \quad r_o >> R_d = \quad , \mu = g_m r_o$$

$$\Rightarrow \approx \frac{1}{g_m}$$

$$g_m \to 2mS \Rightarrow \frac{1}{g_m} \approx 500\ \Omega$$

~~$v_{in1}$~~ $(v_{in2} = 0)$ , $(v_{in1} \neq 0)$ → superposition

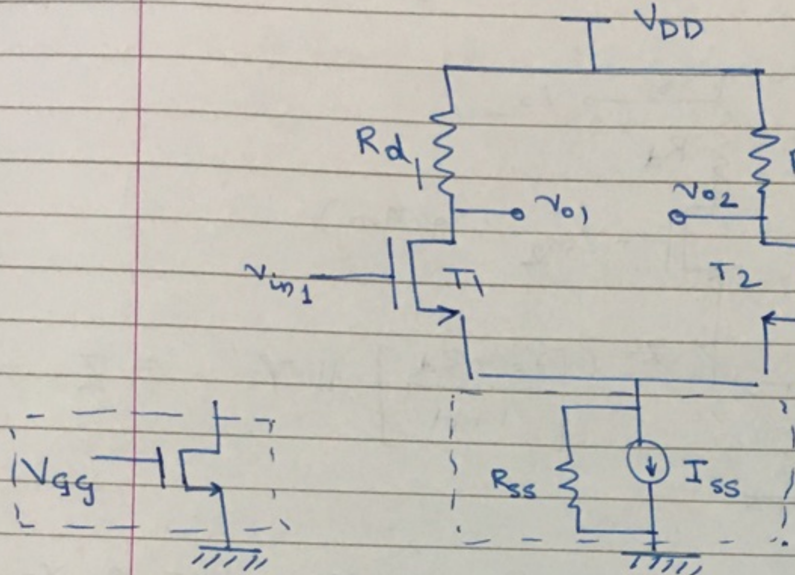



$$A_d = \frac{v_{o1} - v_{o2}}{v_{in1} - v_{in2}}$$

- <u>Bode Plots Revision</u> → Done EE225 notes.

12.2.18

→ Differential Amplifier

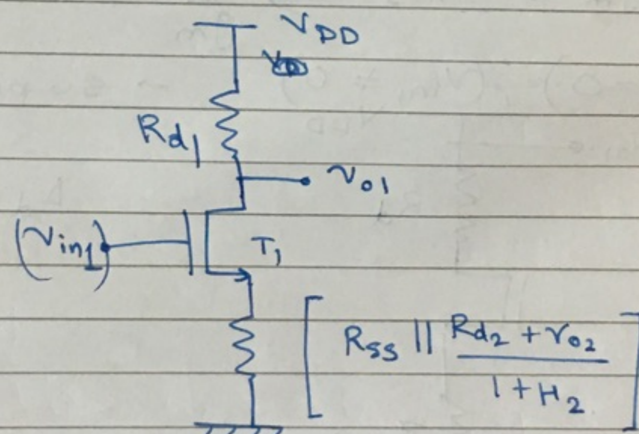



$\uparrow Z_{in} = \dfrac{R_{d_2} + r_{o_2}}{(\mu_2 + 1)}$

→ Small Signal Analysis

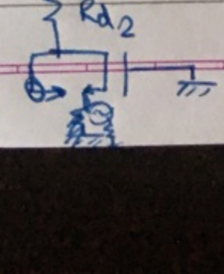
Gain, $A_{dm} = -g$

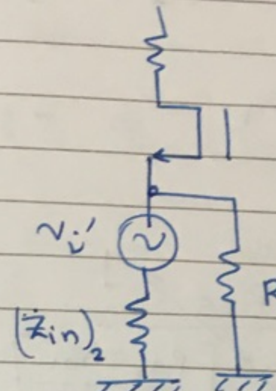
$v_{in_1} \neq 0, \; v_{in_2} = 0$




$$v_{o_1} = \frac{-\mu_1 R_{d_1} v_{in_1}}{R_{d_1} + r_{o_1} + (1 + \mu_1)\left[ R_{SS} \parallel \dfrac{R_{d_2} + r_{o_2}}{1 + \mu_2} \right]}$$

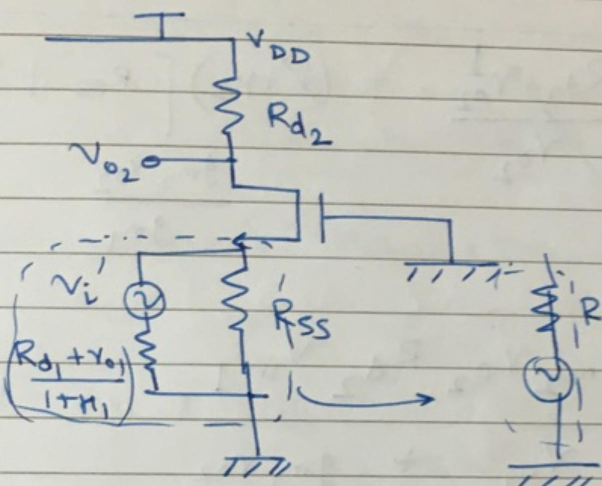
$v_{o_2} =$






$$v_i' = \left(\frac{\mu_1}{1+\mu_1}\right) v_{in_1}$$

$$R_{th} = \frac{R_{d_1} + r_{o_1}}{1+\mu_1}$$




$-g_mR_d$

$R_{th} \| R_{ss} = R_{th}'$

$$\left(\frac{R_{ss}}{R_{ss}+R_{th}}\right) v_i' = v_{th}'$$

$$v_{o_2} = \frac{(1+\mu_2)\, v_{th}'\, R_{d_2}}{(1+\mu_2) R_{th}' + r_{o_2} + R_{d_2}} = \frac{(1+\mu_2) R_{d_2} \frac{R_{ss}}{R_{ss}+R_{th}} \left(\frac{R_{d_1}+r_{o_1}}{1+\mu_1}\right) \times \left(\frac{\mu_1}{1+\mu_1}\right) v_{in_1}}{R_{d_2} + r_{o_2} + (1+\mu_2)\left[R_{ss} \| \frac{R_{d_1}+r_{o_1}}{1+\mu_1}\right]}$$

$$= \frac{(1+\mu_2) R_{d_2} \times \frac{R_{ss}}{R_{ss} + \left(\frac{R_{d_1}+r_{o_1}}{1+\mu_1}\right)} \left(\frac{\mu_1}{1+\mu_1}\right) v_{in_1}}{R_{d_2} + r_{o_2} + (1+\mu_2)\left[R_{ss} \| \frac{R_{d_1}+r_{o_1}}{1+\mu_1}\right]}$$

→ Approximations

$$\frac{r_{o2}}{g_{m2} r_{o2}} = 1/g_{m2}$$

$$v_{o_1} = \frac{-g_{m_1} r_{o_1} v_{in_1} R_{d_1}}{r_{o_1} + \frac{g_{m_1} r_{o_1}}{g_{m2}}}$$

$g_m = g_{m_1} = g_{m_2}$, $R_{d_1} = R_{d_2} = R_d$ in always

$$\Rightarrow v_{o_1} = \frac{-g_m R_d v_{in_1}}{1+1} = -\frac{g_m R_D}{2} v_{in_1}$$

$$v_{o_2} = \frac{\cancel{(1+\mu_2)} R_{d_2} \dfrac{R_{SS}}{\cancel{R_{SS} + \left[\dfrac{R_{d_1} + r_{o_1}}{1+\mu_1}\right]}}^{1} \dfrac{\mu_1}{\cancel{1+\mu_1}}^{1} v_{in_1}}{\cancel{\dfrac{R_{d_2} + r_{o_2}}{r_{o_2}}}^{1} + \underbrace{\cancel{(1+\mu_2)}\left[R_{SS} \parallel \dfrac{R_{d_1} + r_{o_1}}{1+\mu_1}\right]}_{\dfrac{g_{m_2} r_{o_2}}{g_{m_1}}}}$$

$$v_{o_2} = \frac{g_{m_2} r_{o_2} R_{d_2} v_{in_1}}{r_{o_2} + \dfrac{g_{m_2} r_{o_2}}{g_{m_1}}} = \frac{g_m r_o R_d v_{in}}{r_o + r_o}$$

$$= \frac{g_m R_d v_{in}}{2}$$

Summary

$v_{in_1} \neq 0$, $v_{in_2} = 0$, $v_{o_1} = -g_m \frac{R_D}{2} v_{in_1}$

$v_{o_2} = g_m \frac{R_d}{2} v_{in_1}$

$v_{in_1} = 0$, $v_{in_2} \neq 0$, $v_{o_1} = g_m \frac{R_D}{2} v_{in_2}$

$v_{o_2} = -g_m \frac{R_D}{2} v_{in_2}$

→ Supporpose,

$$v_{o_1} = \frac{-g_m R_D}{2}(v_{in_1} - v_{in_2})$$

$$v_{o_2} = \frac{g_m R_D}{2}(v_{in_1} - v_{in_2})$$

$$A_{dm} = \frac{v_{o_1} - v_{o_2}}{v_{in_1} - v_{in_2}} = -g_m R_D$$

$$\boxed{A_{dm} = -g_m R_D}$$

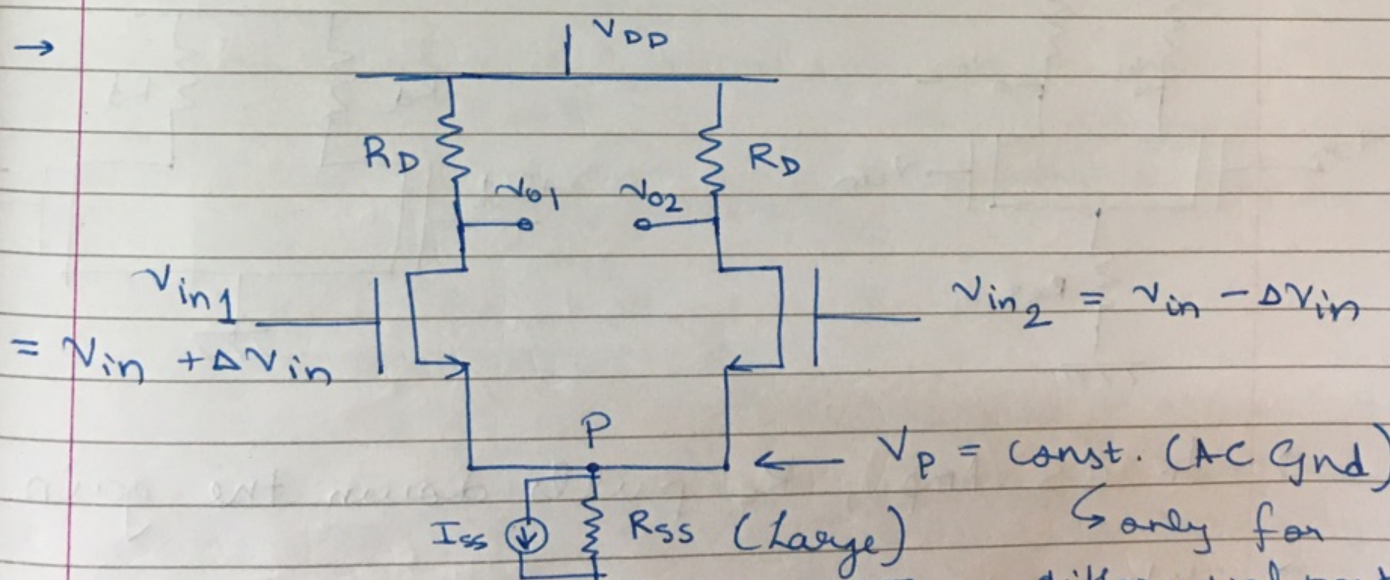
→ We converted transistor to equivalent linear model.

$$i = f(v)$$

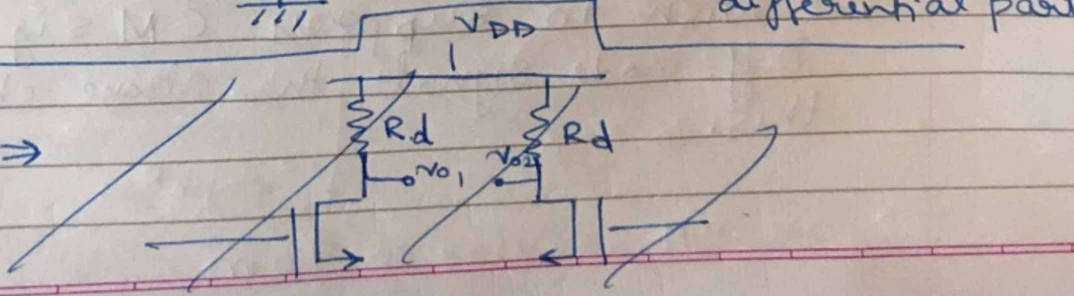
$$i_1 + i_2 = f(v_1 + v_2) = f(v_1) + f(v_2)$$

$$ai = f(av)$$

→




⇒



→ $v_{in_1} = v_{cm} + v_{dm}$

$v_{in_2} = v_{cm} - v_{dm}$

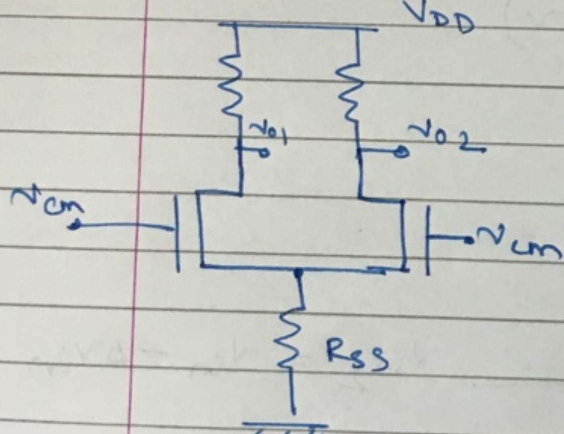
$v_{cm} = \dfrac{v_{in_1} + v_{in_2}}{2}$ , $v_{dm} = \dfrac{v_{in_1} - v_{in_2}}{2}$

hence any sig. may be written in above form.

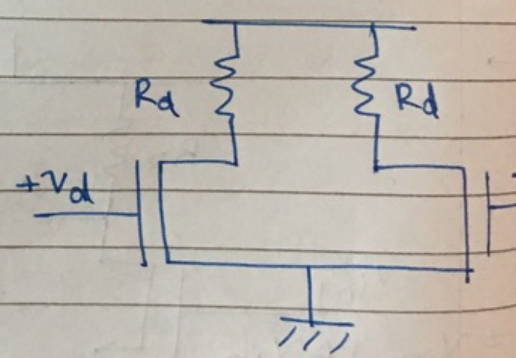
$$\left\{\begin{aligned} v_{in1} &= \left(\frac{v_{in_1} + v_{in2}}{2}\right) + \left(\frac{v_{in1} - v_{in2}}{2}\right) \\ v_{in_2} &= \left(\frac{v_{in_1} + v_{in2}}{2}\right) - \left(\frac{v_{in_1} - v_{in_2}}{2}\right) \end{aligned}\right.$$

→ P is small signal AC gnd. for differential signal/mode only.

CM Ckt




Differential Ckt




→ In CS Amp., $R_S$ pulls down the gain.

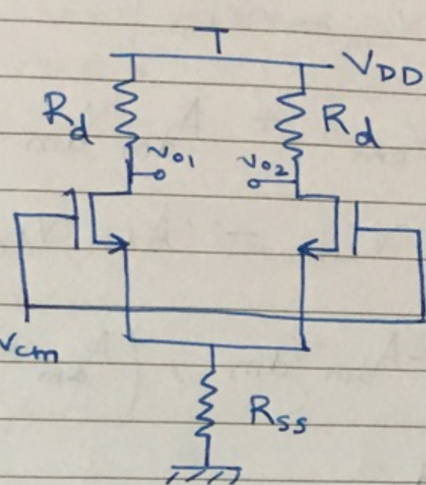
→ We wish to suppress CM sig. & Amplify Diff. mode signal, the above ckt achieves this.

13.2.18

→ CM Gain continued ....

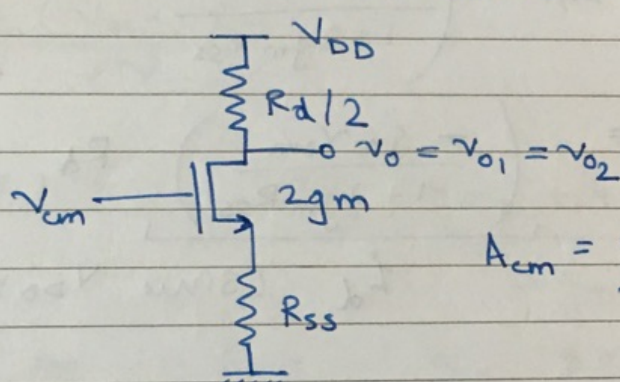
$v_{in_1} = v_{cm} + v_{dm}$

$v_{in_2} = v_{cm} - v_{dm}$




→ Small signal $v_{cm}$ applied to gates of both $T_1$ & $T_2$

→ Two Identical transistors in parallel, $v_{o_1} = v_{o_2}$




$$\mu = 2g_m \times \frac{r_o}{2}$$

$$= g_m r_o$$

$$A_{cm} = \frac{v_o}{v_{cm}} = \frac{-\mu (R_d/2)}{\frac{r_o + R_d}{2} + (1+\mu) R_{ss}}$$

$$\frac{v_o}{v_{cm}} \simeq \frac{-\mu R_d}{(r_o + R_d) + 2(1+\mu) R_{ss}} \approx \frac{-\mu R_d}{r_o + 2\mu R_{ss}}$$

$$\approx \frac{-g_m r_o R_d}{r_o + 2 g_m r_o R_{ss}}$$

$$\frac{v_o}{v_{cm}} = \boxed{\frac{-g_m R_d}{1 + 2 g_m R_{ss}} = A_{cm}}$$ → Defined this way

$$\boxed{A_{DM} = -g_m R_D}$$

$$v_{in_1} = v_{cm} + v_{dm}$$
$$v_{in_2} = v_{cm} - v_{dm}$$
$$\left.\right\} \text{Diff} = 2v_{dm}$$

$$v_{o_1} = A_{cm} v_{cm} + A_{dm} v_{dm}$$

$$v_{o_2} = A_{cm} v_{cm} - A_{dm} v_{dm}$$

$$v_{o_1} - v_{o_2} = 2A_{dm} v_{dm}, \quad \left(A_{dm} = \frac{v_{o_1} - v_{o_2}}{2v_{dm}}\right)$$

→ Mismatch in $R_d$

$$v_{o_1} \ \cancel{A_{cm}} = \left(\frac{-g_m R_d}{1+2g_m R_{ss}} \times v_{cm}\right)$$

$$v_{o_1} = \underbrace{\left(\frac{-g_m v_{cm}}{1+2g_m R_{ss}}\right)}_{i_d} R_d$$

(since $V_{DD} \equiv$ AC Gnd)

→ Let us assume mismatch in $R_D$, and ↑ in $V_{c}$ $= \Delta V_{c}$

$$\Delta V_{o_1} = \frac{-g_m \Delta V_{cm}}{1+2g_m R_{ss}} R_D$$

$$\Delta V_{o_2} = \frac{-g_m \Delta V_{cm} (R_D + \Delta R_D)}{1+2g_m R_{ss}}$$

$$\Delta(v_{o_1} - v_{o_2}) = \frac{g_m \Delta V_{cm} \Delta R_D}{1+2g_m R_{ss}}$$

$$\frac{\Delta(v_{o_1} - v_{o_2})}{\Delta V_{cm}} = \boxed{\frac{g_m \Delta R_D}{1+2g_m R_{ss}} = A_{CM-DM}}$$

→ CM-DM conversion term

→ Common Mode Rejection Ratio,

$$CMRR = \frac{A_{DM}}{A_{CM-DM}}$$

→ $$\frac{V_{o_1} - V_{o_2}}{2V_{dm}} = |A_{DM}| = \frac{R_D}{2} \frac{g_{m_1} + g_{m_2} + 4 g_{m_1} g_{m_2} R_{SS}}{1 + (g_{m_1} + g_{m_2}) R_{SS}}$$

$$A_{CM-DM} = - \frac{\Delta g_m R_D}{(g_{m_1} + g_{m_2}) R_{SS} + 1}$$

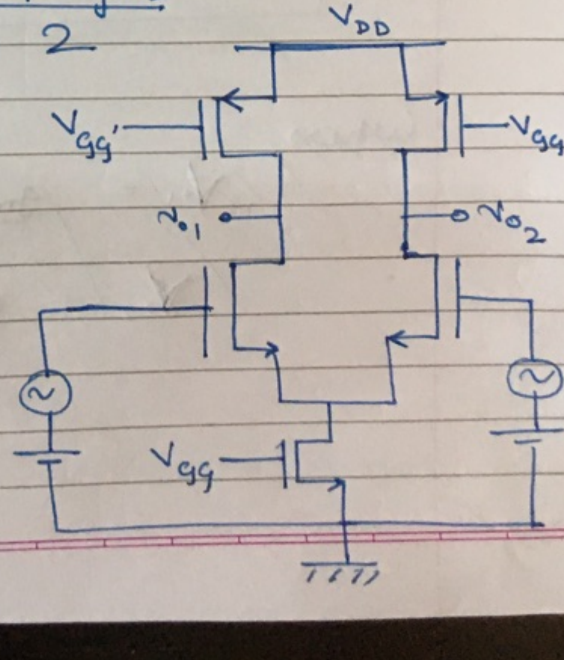
$$= \frac{\Delta (V_{o_1} - V_{o_2})}{V_{cm}}$$

$$|CMRR|\Big|_{\Delta g_m} = \frac{1}{2} \frac{g_{m_1} + g_{m_2} + 4 g_{m_1} g_{m_2} R_{SS}}{\Delta g_m}$$

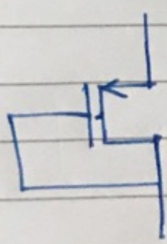
→ $$CMRR\Big|_{\Delta R_D} = \frac{A_{DM}}{A_{CM-DM}} \approx \frac{g_m R_D}{\frac{\Delta R}{2R_{SS}}} = 2 g_m R_{SS} \left( \frac{R_D}{\Delta R_D} \right)$$

→ If $\Delta g_m, \Delta R_d \neq 0$, Add the responses and take $g_m = \frac{g_{m_1} + g_{m_2}}{2}$

→ Replace $R_D$ with PMOS



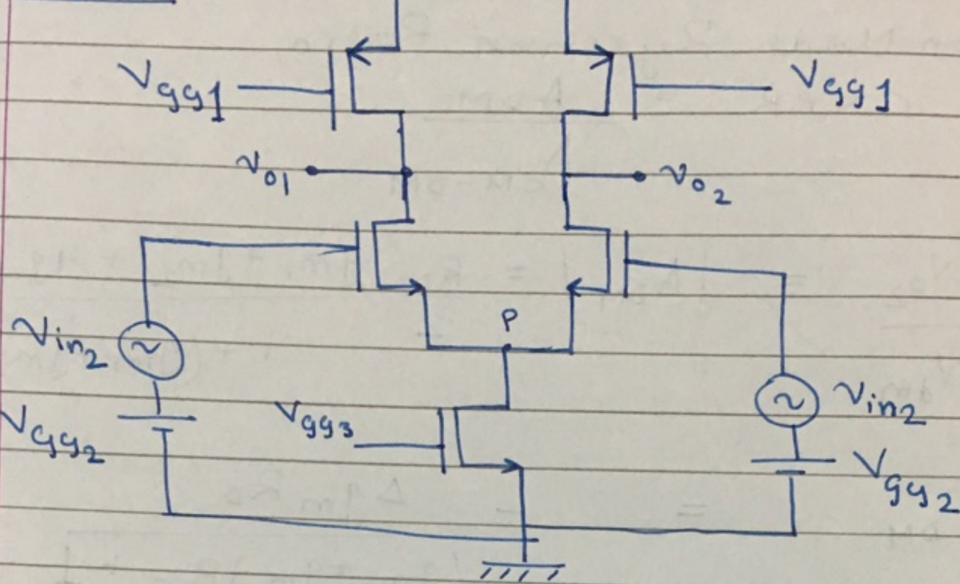

Replace PMOS by,



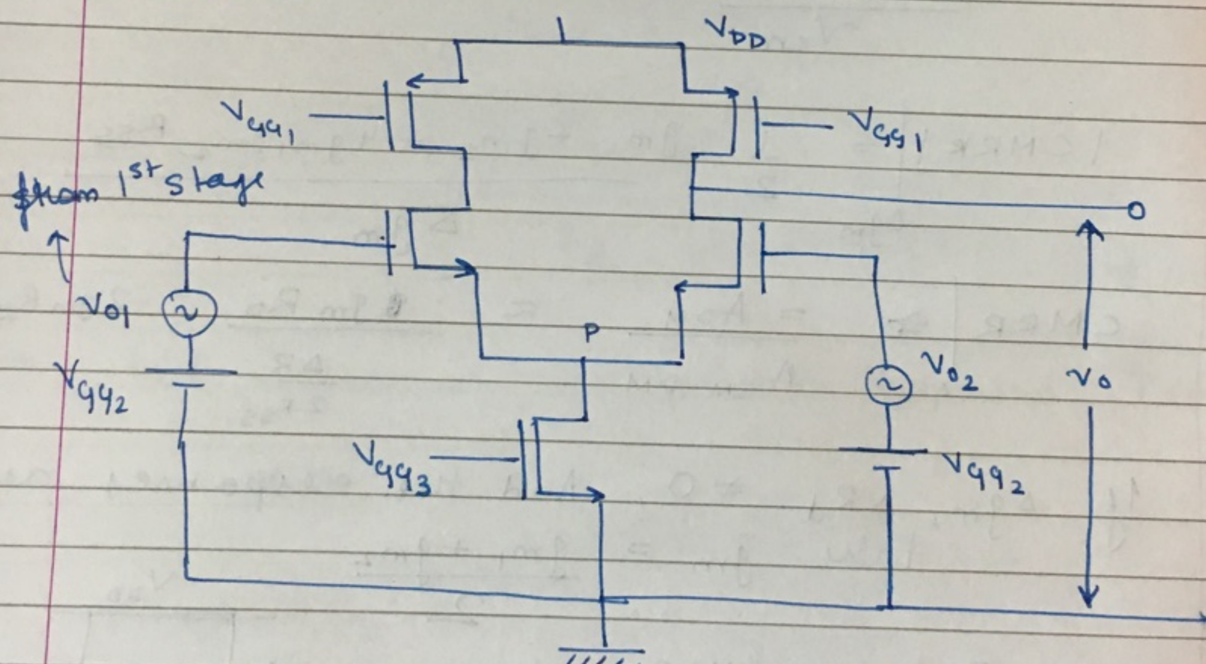
Diode connected

$V_o \to \frac{1}{g_m}$

Stage-1




Stage 2




~~where,~~

$$\cancel{v_{o1} = \frac{g_m R_D}{2}(v_{in_1} - v_{in_2})}$$

$$\cancel{v_{o2} = \frac{g_m R_D}{2}(v_{in_2} - v)}$$

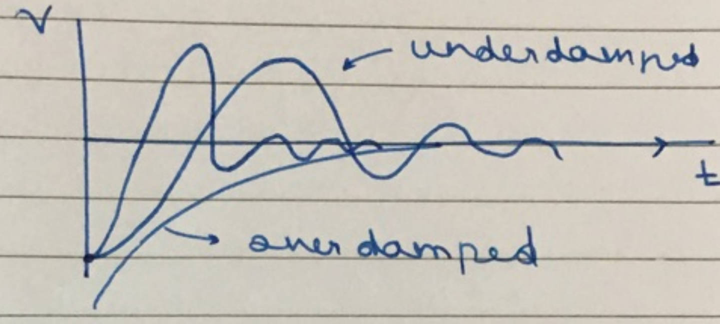
→ Symbolic python lib. (sympy)

Methods:

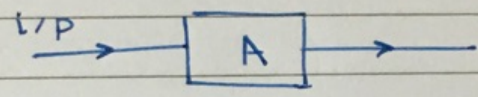
Symbol('x'), series( $\sin^2$'x' + cos 'x')

jupyter → ~~sh~~ -h ≡ shift + tab

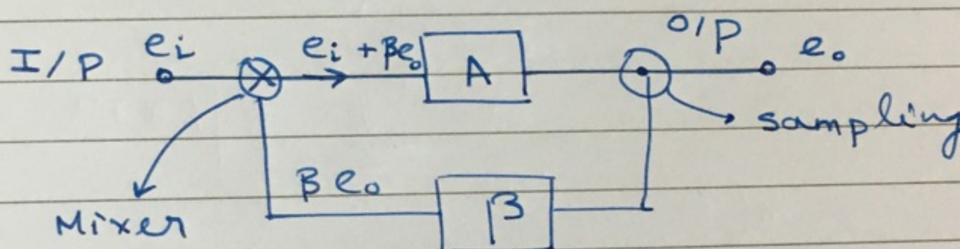
→ <u>Feedback</u>




• No feedback ≡ open loop system




→ open loop sys.




Feedback sys.

→ OL sys : → A

$$e_o = (e_i + \beta e_o) A$$

$$\Rightarrow e_o[1 - A\beta] = A e_i$$

$$\frac{e_o}{e_i} = \boxed{\frac{A}{1 - A\beta}}$$

→ closed loop sys.