

(Q.1) Ignoring body effect and channel length modulation,

$$(A) \quad Z_{in} = \frac{1}{g_{m2} + g_{m3}} \quad (0.5)$$

$$\text{where } g_m = \sqrt{2 \mu_n C_{ox} \frac{W}{L} I_{D \times M}} ;$$

$$\text{and } I_B = I_{D2} + I_{D3} \quad \text{and } I_{D3} = 4 I_{D2} \text{ (based on multiplier ratio)}$$

$$\Rightarrow I_{D2} = \frac{I_B}{5} ; \quad I_{D3} = \frac{4 I_B}{5} \quad (0.5)$$

$$\text{So, } Z_{in} = \frac{1}{\sqrt{2 \mu_n C_{ox} \frac{W}{L}} \left[ \sqrt{\frac{I_B}{5}} + \sqrt{\frac{4 I_B}{5} \times 4} \right]}$$

$$= \frac{1}{\sqrt{2 \mu_n C_{ox} \frac{W}{L} \cdot \frac{I_B}{5}} (1+4)}$$

$$Z_{in} = \frac{1}{\sqrt{10 \cdot \mu_n C_{ox} \frac{W}{L} I_B}} \Omega \quad (0.5)$$

(B) Based on multipliers ratio,

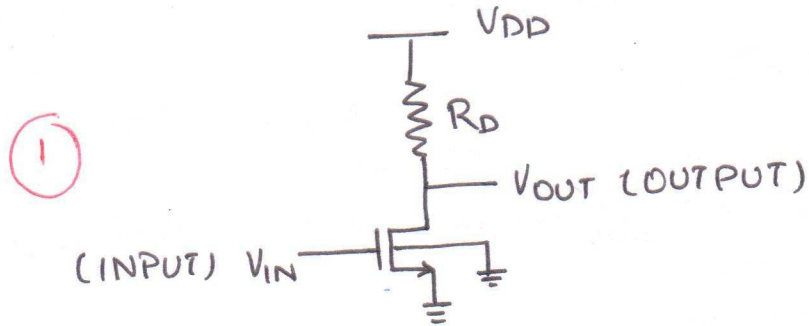
$$I_{D1} = 6 I_{D2} \quad \text{and} \quad I_{D4} = \frac{10}{4} I_{D3} - \text{Because of mirroring.} \quad (2)$$

$$V_{B1} = V_{DD} - R_D I_D = V_{DD} - \frac{6 I_B R_D}{5} \quad (1)$$

and similarly,

$$V_{B2} = V_{DD} - R_D I_D = V_{DD} - 2 R_D I_B \quad (1)$$

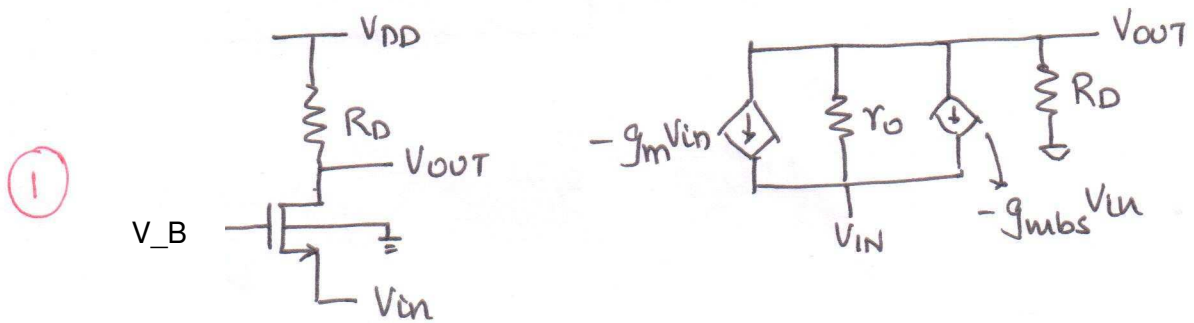
## 2 A Common-Source Amplifier



\* No Body Effect because body and source are both grounded

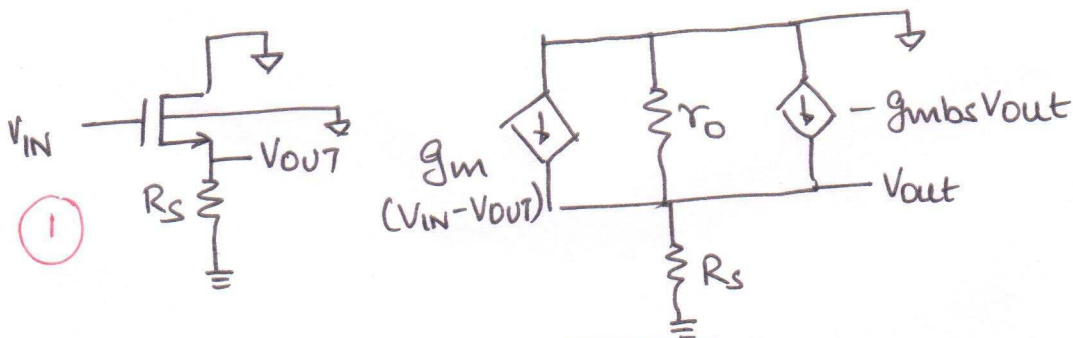
③  $A_v = -g_m (R_D \parallel r_o)$

## B Common-Gate Amplifier



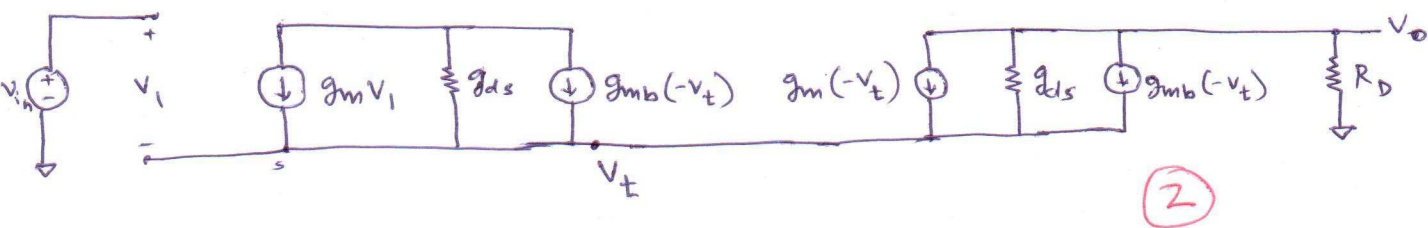
③  $A_v = \left( \frac{(g_m + g_{mbs}) r_o + 1}{r_o + R_D} \right) R_D$

## C Common-Drain Amplifier



③  $A_v = \frac{g_m R_S}{1 + R_S (g_m + g_{mbs} + g_{ds})}$

### 3. Small-Signal Model



$$V_i = V_{in} - V_t$$

Writing KCL, at node  $V_t$ :

$$g_m(V_{in} - V_t) + g_{mb}(-V_t) + g_{ds}(-V_t) = g_m V_t + g_{ds}(V_t - V_{out}) + g_{mb} V_t \quad \text{--- (1)}$$

KCL at node  $V_o$ :

$$\frac{V_o}{R_D} = g_m V_t + (V_t - V_o) \cdot g_{ds} + g_{mb} V_t \quad \text{--- (1)}$$

$$\Rightarrow V_o \cdot \left[ \frac{1}{R_D} + g_{ds} \right] = [g_m + g_{ds} + g_{mb}] \cdot V_t$$

$$\Rightarrow V_t = \frac{V_o \cdot [g_{ds} + 1/R_D]}{g_m + g_{mb} + g_{ds}} \quad \text{--- (2)}$$

Put equation (2) in (1),

$$\Rightarrow g_m V_{in} = \left[ \frac{2(g_m + g_{mb} + g_{ds}) \cdot [g_{ds} + 1/R_D]}{(g_m + g_{mb} + g_{ds})} - g_{ds} \right] V_{out}$$

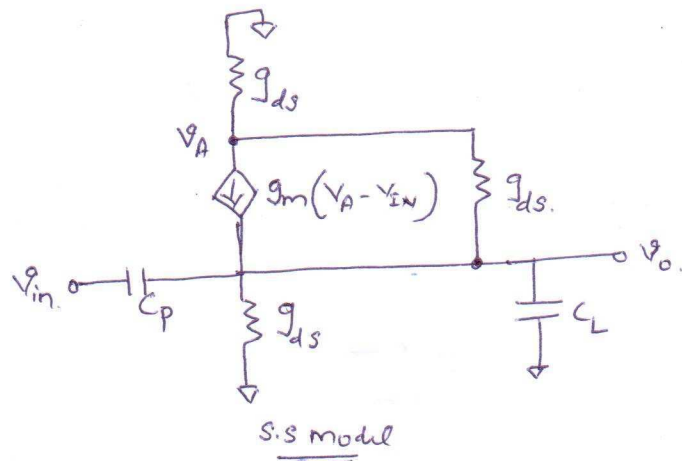
$$\Rightarrow g_m V_{in} = \left[ g_{ds} + 2/R_D \right] \cdot V_{out}$$

$$\Rightarrow \boxed{\frac{V_{out}}{V_{in}} = \frac{g_m}{g_{ds} + \frac{2}{R_D}}}$$

(1) With approximation

(2) Without approximation

4 A) Ignoring Body Effect.



(2)

B) At DC, all capacitances are open.

$$g_m(v_A - v_{in}) + (v_A - v_o)g_{ds} = v_o g_{ds} = -v_A g_{ds}$$

$$\Rightarrow v_A = -v_o$$

$$\Rightarrow g_m(-v_o - v_{in}) + (-2v_o)g_{ds} = v_o g_{ds}$$

$$\Rightarrow -g_m v_{in} = (g_m + 3g_{ds})v_o$$

$$A_v = - \left[ \frac{g_m}{g_m + 3g_{ds}} \right] \approx -1$$

(1)

$$c) g_m(-v_{in} + v_A) + (v_A - v_o)g_{ds} = -g_{ds}v_A \quad (1) \quad (1)$$

$$-g_{ds}v_A + (v_{in} - v_o)g_{ds} = v_o(g_{ds} + sC_L)$$

$$\Rightarrow -g_{ds}v_A = v_o(g_{ds} + s(C_L + C_p)) - sC_p v_{in} \quad (2) \quad (1)$$

using (1)

$$v_A = \frac{g_m v_{in} + v_o g_{ds}}{g_m + 2g_{ds}}$$

Substituting in (2)

$$-g_{ds}(g_m V_{IN} + V_o g_{ds}) = (g_m + 2g_{ds})(g_{ds} + s(C_L + C_p))V_o - sC_p V_{IN}(g_m + 2g_{ds})$$

Using approximation,

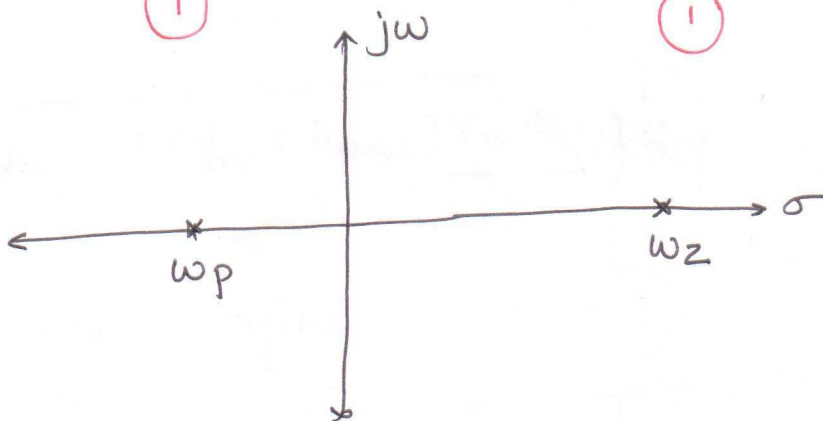
$$-g_{ds}(g_m V_{IN} + V_o g_{ds}) = (V_o(g_{ds} + s(C_L + C_p)) - sC_p V_{IN})g_m$$

$$\Rightarrow (sC_p - g_m g_{ds})V_{IN} = V_o(g_{ds}^2 + g_{ds}g_m + s g_m(C_L + C_p))$$

$$\Rightarrow (s g_m C_p - g_m g_{ds})V_{IN} \approx V_o(g_{ds}g_m + s g_m(C_L + C_p))$$

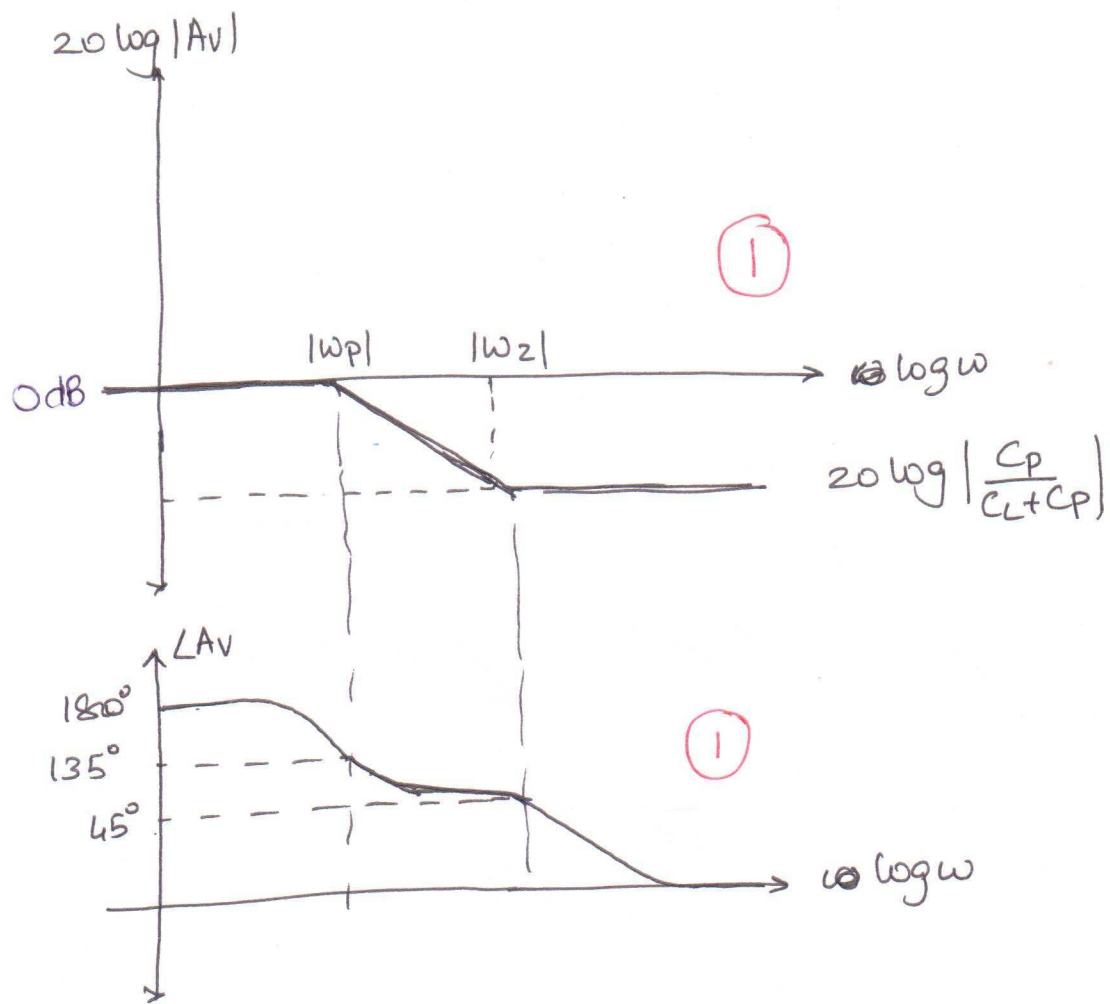
$$\Rightarrow \boxed{\frac{V_o}{V_{IN}} = \frac{-g_{ds} + sC_p}{g_{ds} + sC_L + C_p}} \quad (2)$$

$$\underline{D} \omega_z = + g_{ds}/C_p \quad ; \quad \omega_p = \frac{-g_{ds}}{C_L + C_p}$$





III



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$$A_v = \text{Intrinsic Gain} = g_m r_o$$

$$= \sqrt{2\mu_{ox}\left(\frac{W}{L}\right) I_D} \times \frac{1}{\lambda I_D}$$

$$= \underbrace{\sqrt{\frac{2\mu_{ox}(W/L)}{I_D}}}_{\text{constant}} \times \frac{1}{\lambda} \quad (1)$$

$$\therefore A_v \propto \frac{1}{\lambda} \quad \text{and} \quad \lambda \propto \frac{1}{L}$$

$$\Rightarrow A_v \propto L$$

Hence  $A_v$  scales up by the same amount as  $W$  and  $L$ . (1)