CS 228 : Logic in Computer Science

Krishna. S

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- What does $TS \models \varphi$ mean?

Notations for Infinite Words

- Σ is a finite alphabet
- Σ^* set of finite words over Σ
- An infinite word is written as $\alpha = \alpha(0)\alpha(1)\alpha(2)\ldots$, where $\alpha(i) \in \Sigma$
- Such words are called ω-words
- $\mathit{Inf}(\alpha) = \{a \in \Sigma \mid \alpha(i) = a \text{ for infinitely many } i\}$. $\mathit{Inf}(\alpha)$ gives the set of symbols occurring infinitely often in α.

ω-automata

An ω-automaton is a tuple $\mathcal{A} = (Q, \Sigma, \delta, q_0, Acc)$ where

- Q is a finite set of states
- Σ is a finite alphabet
- $\delta : Q \times \Sigma \to 2^Q$ is a state transition function (if non-deterministic, otherwise, $\delta : Q \times \Sigma \to Q$)
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Run

A run ρ of $\mathcal{A}$ on an ω-word $\alpha = a_1 a_2 \cdots \in \Sigma^\omega$ is an infinite state sequence $\rho(0)\rho(1)\rho(2)\ldots$ such that

- $\rho(0) = q_0$,
- $\rho(i) = \delta(\rho(i-1), a_i)$ if $\mathcal{A}$ is deterministic,
- $\rho(i) \in \delta(\rho(i-1), a_i)$ if $\mathcal{A}$ is non-deterministic,

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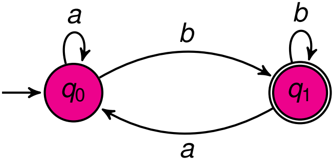
Büchi Acceptance

For Büchi Acceptance, Acc is specified as a set of states, $G \subseteq Q$. The ω-word α is accepted if there is a run ρ of α such that $Inf(\rho) \cap G \neq \emptyset$.

ω-Automata with Büchi Acceptance



$L(\mathcal{A}) = \{\alpha \in \Sigma^\omega \mid \alpha$ has a run ρ such that $Inf(\rho) \cap G \neq \emptyset$

Language accepted=Infinitely many b's.

Comparing NFA and NBA

(Non)deterministic Büchi Automata

$L(\mathcal{A}) = \{\alpha \in \Sigma^\omega \mid \alpha$ has a run ρ such that $\mathit{Inf}(\rho) \cap G \neq \emptyset\}$

(Non)deterministic Finite Automata

$L(\mathcal{A}) = \{\alpha \in \Sigma^* \mid \alpha$ has a run ρ ending in some final state $\}$

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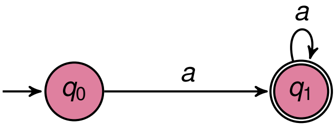

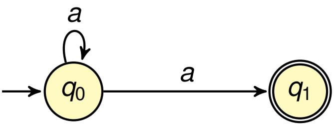

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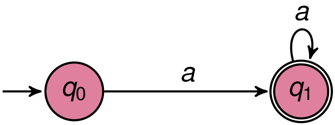

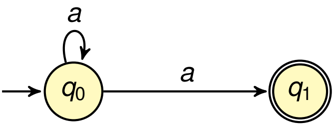

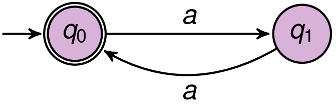

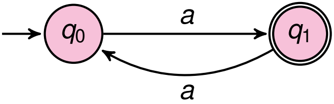

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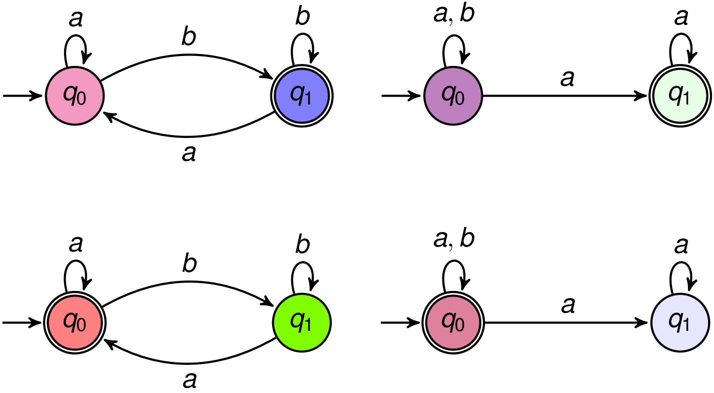


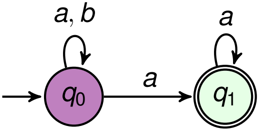
- Left (T-B): Inf many b's, Inf many a's
- Right (T-B): Finitely many b's, $(a+b)^{\omega}$

Büchi Acceptance

A language $L \subseteq \Sigma^\omega$ is called ω-regular if there exists a NBA $\mathcal{A}$ such that $L = L(\mathcal{A})$. Recall definition of regular languages and NFA/DFA acceptance.

NBA and DBA

- Is every NBA as expressible as a DBA, like in the case of DFA and NFA?
- Can we do subset construction on NBA and obtain DBA?



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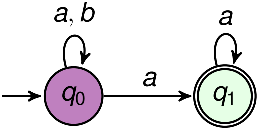

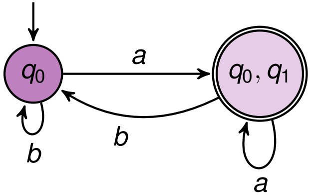

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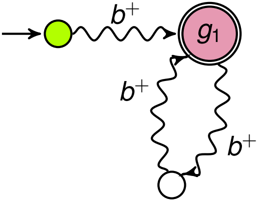

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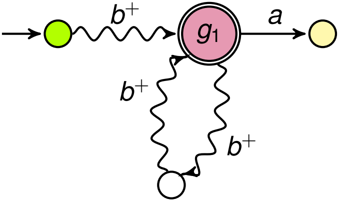

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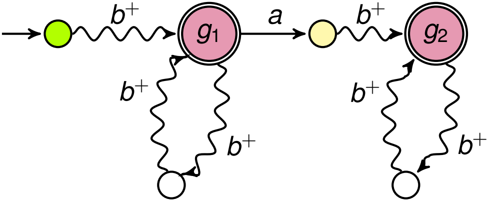

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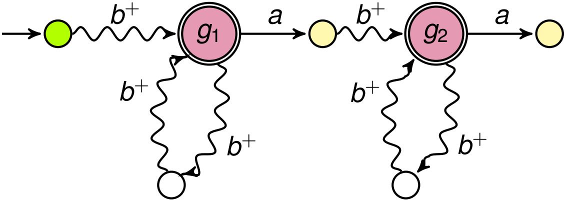

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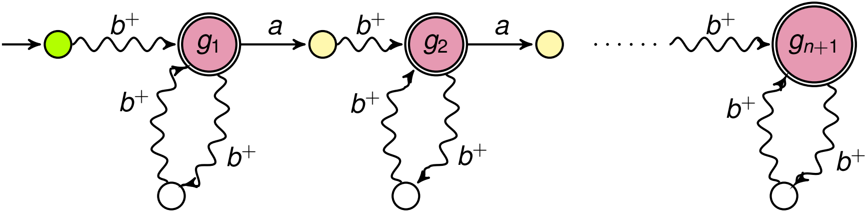
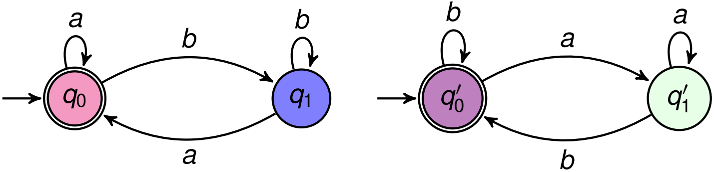


Union and Intersection of NBA



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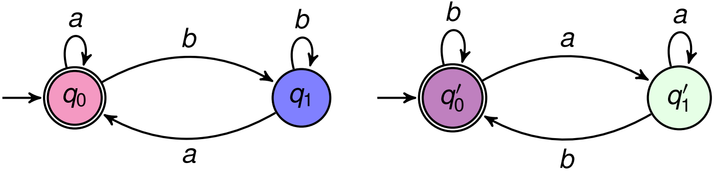

- States as $Q_1 \times Q_2 \times \{1, 2\}$, start state $(q_0, q_0', 1)$

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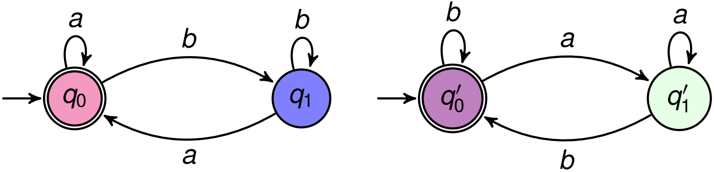

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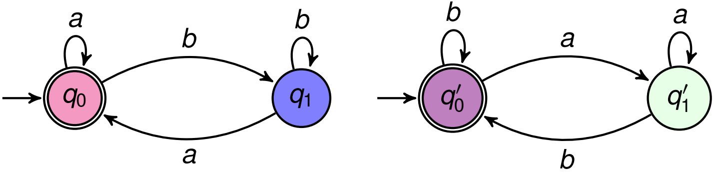

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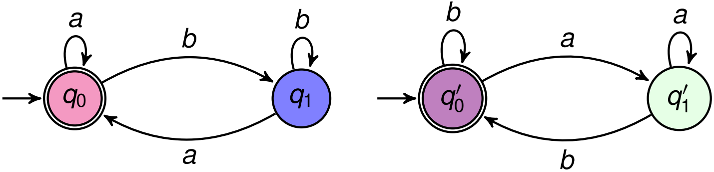

- States as $Q_1 \times Q_2 \times \{1,2\}$, start state $(q_0, q'_0, 1)$
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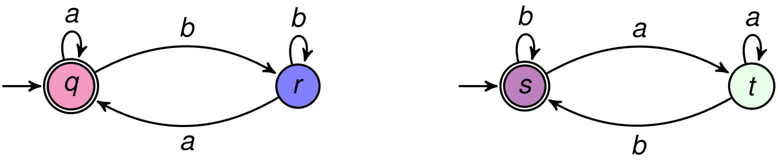
- Good states=$Q_1 \times G_2 \times \{2\}$ or $G_1 \times Q_2 \times \{1\}$

Union and Intersection of NBA



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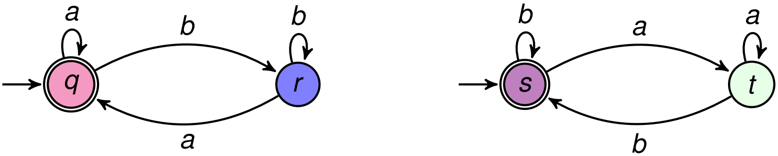

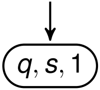

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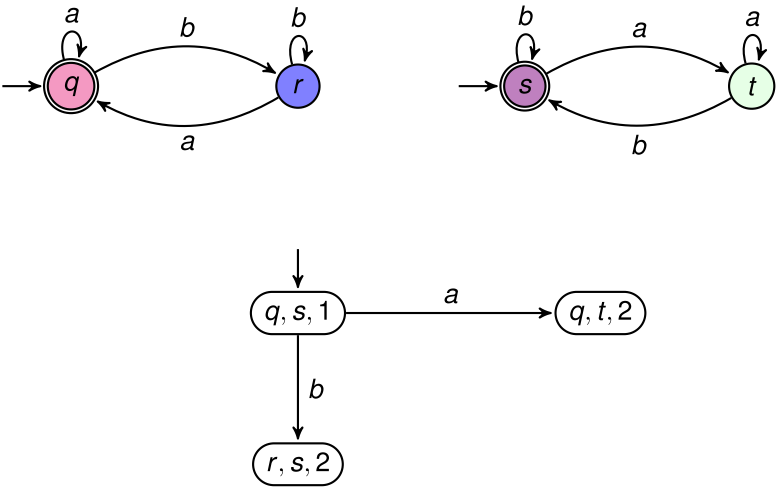

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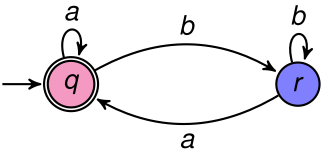

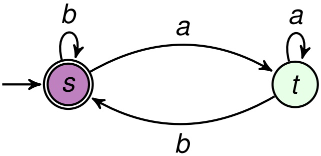

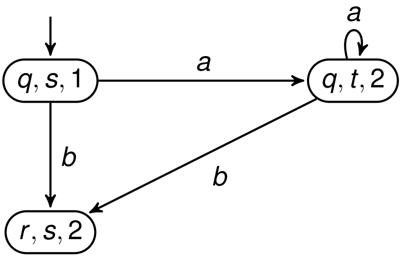

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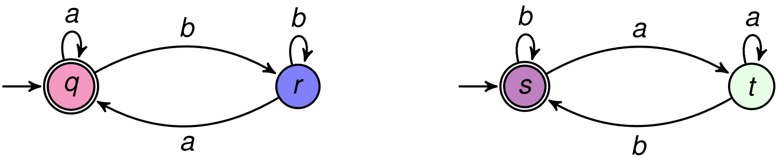

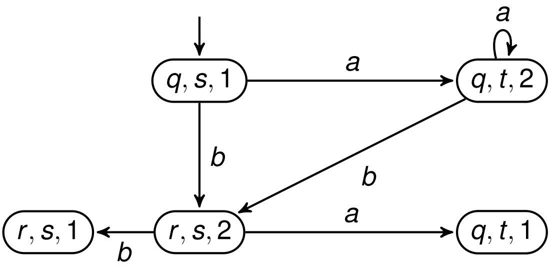

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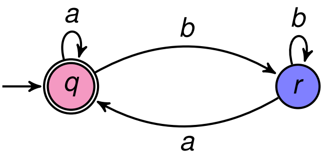

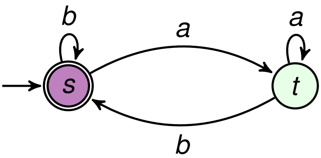

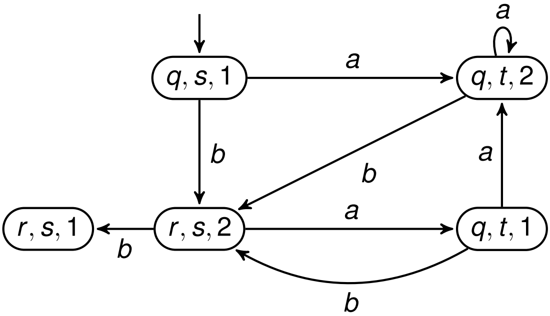

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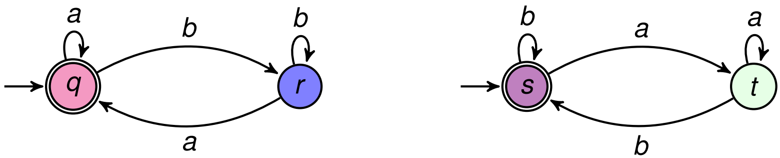

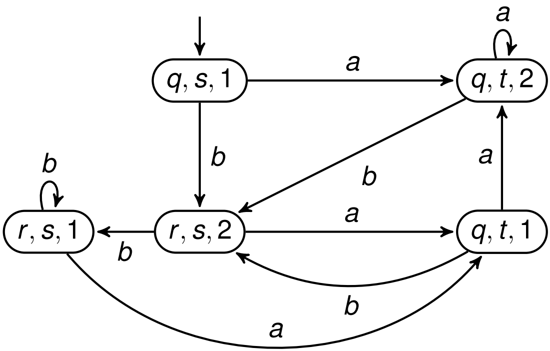

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