EE613

CMOS ANALOG IC DESIGN.

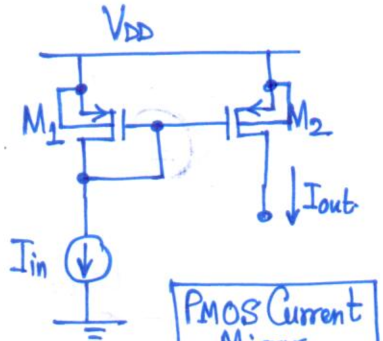
PROF. RAJESH ZELE

MIDTERM SOLUTIONS

14th SEPT 2017

---

1 (a)



PMOS Current Mirror

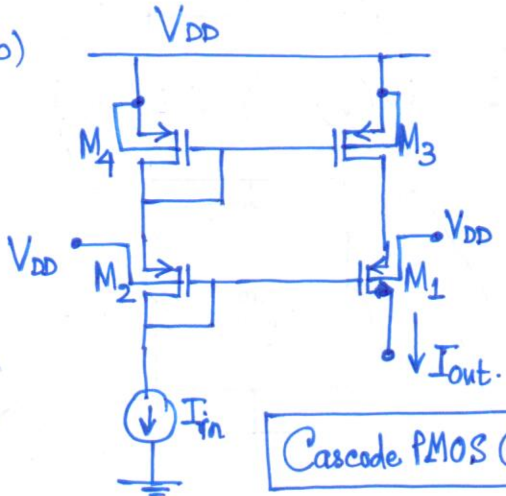
Now, $\dfrac{I_{out}}{I_{in}} = \dfrac{(W/L)_2}{(W/L)_1} = 4$

Keeping 'L's sufficiently large (so that λ becomes smaller) and for better mirroring, $L_1 = L_2$, we get,

$\boxed{W_2/W_1 = 4}$

1

(b)



Cascode PMOS Current Mirror

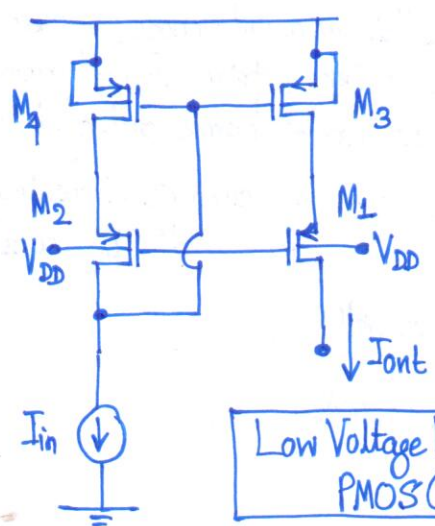
$L_3 = L_4 \implies \boxed{W_3/W_4 = 4}$

$\boxed{\dfrac{(W_1/L_1)}{(W_2/L_2)} = 4}$ Keeping, $L_1 = L_2$, $\boxed{W_1/W_2 = 4}$

(use unit cell design procedure).

1

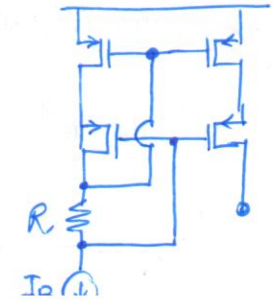
(c)



Low Voltage High Swing Cascode PMOS Current Mirror

Keeping, $L_3 = L_4 \implies \boxed{W_3/W_4 = 4}$

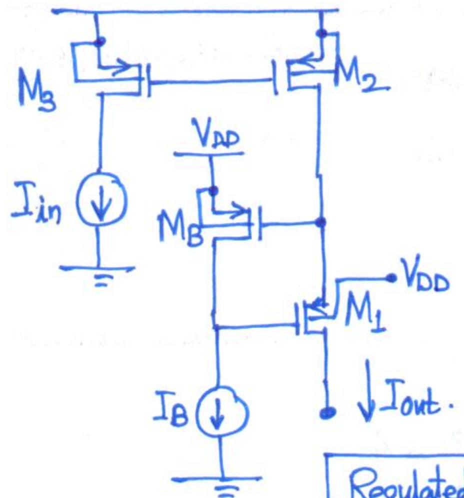
$\boxed{\dfrac{(W_1/L_1)}{(W_2/L_2)} = 4}$ Keeping, $L_1 = L_2$, $\boxed{W_1/W_2 = 4}$



Here, $I_B R > V_{DSAT_P}$

2

1 (d)

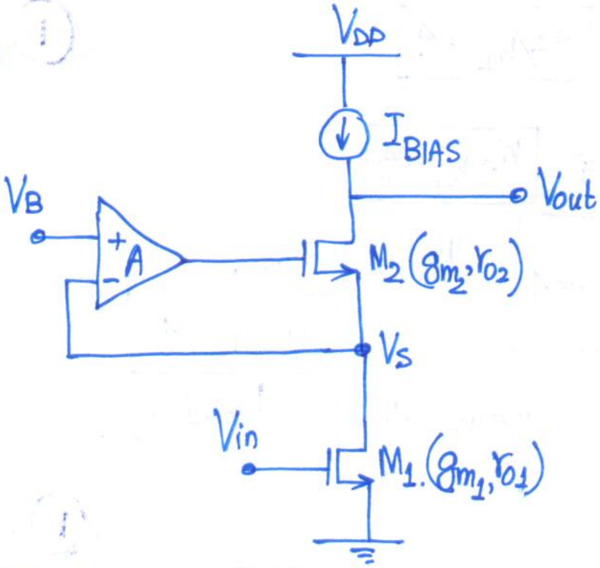


Regulated Cascode PMOS Current Mirror

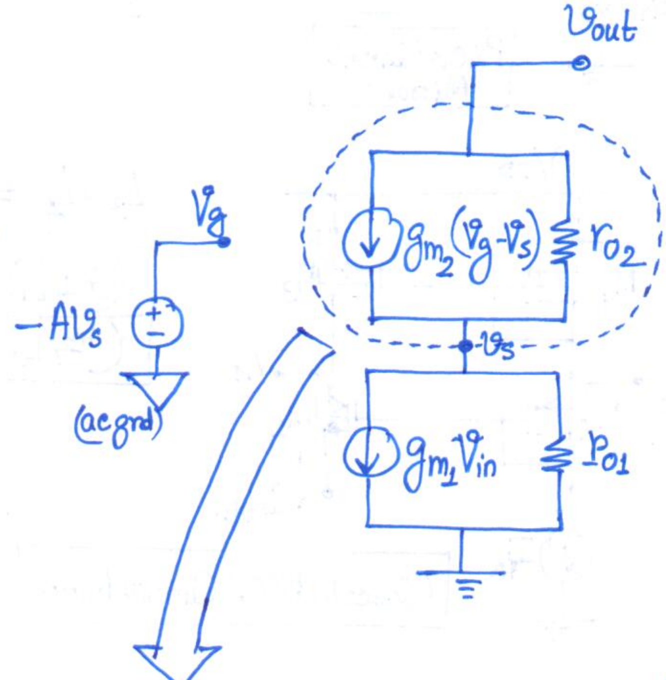
Keeping $L_3 = L_2$, we have, $W_3/W_2 = 1/4$ or $\boxed{W_2/W_3 = 4}$

2

2



(Ignoring body effect)



1

Considering this as a supernode, no current is entering from Vout-terminal. Hence, no current comes out of it as well. Thus, $g_{m_2}(v_g - v_s)$ current circulates in a loop with r_{o_2} resistance.

Therefore, same is true for $g_{m_1} v_{in}$ and r_{o_1} as well.

so, $g_{m_1} V_{in} + v_s / r_{o_1} = 0$ (KCL @ v_s node)

1

$$\boxed{v_s = -(g_{m_1} r_{o_1}) v_{in}} \quad \text{—— } 1$$

$$\frac{v_{out} - v_s}{r_{o2}} + g_{m_2}(v_g - v_s) = 0 \quad \text{(KCL @ o/p node)}$$ (1)

$$\text{so, } \frac{v_{out}}{r_{o2}} + \frac{(g_{m_1} r_{o1})}{r_{o2}} v_{in} + g_{m_2}(-A\, v_s) - g_{m_2} v_s = 0$$

$$\text{i.e. } \frac{v_{out}}{r_{o2}} + \frac{g_{m_1} r_{o1}}{r_{o2}} v_{in} - g_{m_2}(A+1) v_s = 0$$

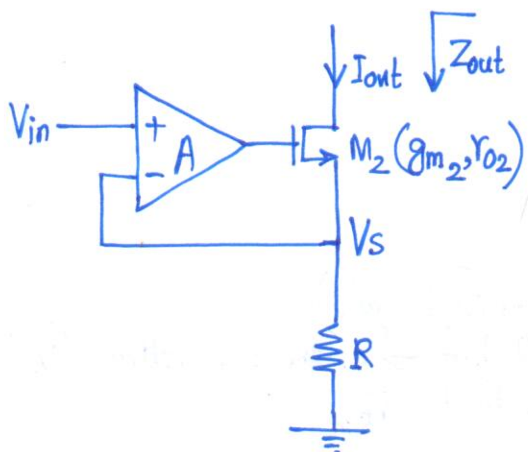
$$\text{i.e. } \frac{v_{out}}{r_{o2}} + \frac{g_{m_1} r_{o1}}{r_{o2}} v_{in} + (g_{m_1} r_{o1})\, g_{m_2}(A+1)\, v_{in} = 0$$

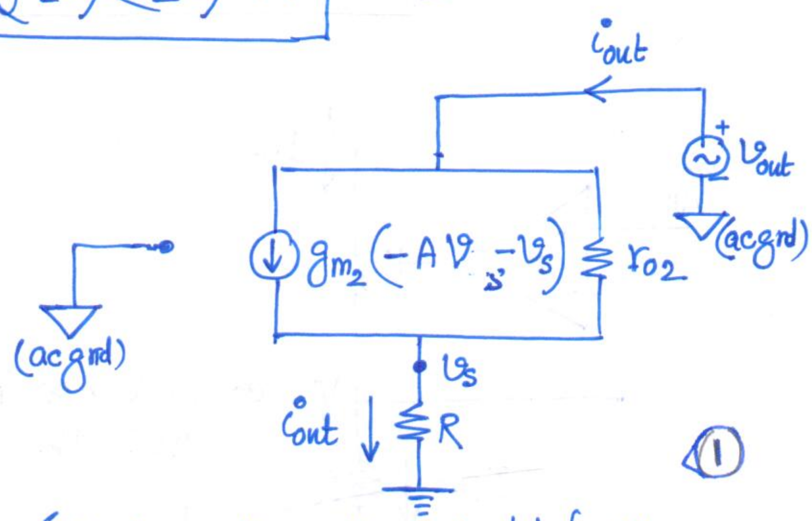
$$\text{i.e. } v_{out} = -\left[(g_{m_1} r_{o1}) + (g_{m_1} r_{o1})(g_{m_2} r_{o2})(A+1)\right] v_{in}.$$

$$\text{i.e. } \frac{v_{out}}{v_{in}} = -(g_{m_1} r_{o1})\left[1 + g_{m_2} r_{o2}(A+1)\right] \approx -(g_{m_1} r_{o1})(g_{m_2} r_{o2})(A+1)$$

$$\boxed{\frac{v_{out}}{v_{in}} \approx -(g_{m_1} r_{o1})(g_{m_2} r_{o2})(A+1)}$$ (1)

(3)





(1)

(The above is the small signal setup for Z_{out} characterisation).

Now, $v_s = i_{out} R$. (1)

$$i_{out} = -g_{m_2}(A+1) v_s + \frac{v_{out} - v_s}{r_{o2}} \quad \text{(KCL @ o/p node)}$$ (1)

$$\text{i.e. } i_{out} = -\left[g_{m_2}(A+1) + \frac{1}{r_{o2}}\right] v_s + \frac{v_{out}}{r_{o2}}$$

i.e, $$i_{out} = -\left[g_{m_2}(A+1) + \frac{1}{r_{o2}}\right] R\, i_{out} + \frac{v_{out}}{r_{o2}}$$ (replacing $v_s = i_{out} R$)

i.e, $$i_{out}\left[1 + g_{m_2}R(A+1) + \frac{R}{r_{o2}}\right] = \frac{v_{out}}{r_{o2}}$$

1

$$Z_{out} = \frac{v_{out}}{i_{out}} = r_{o2} + (g_{m_2} r_{o2})(A+1)R + R \approx (g_{m_2} r_{o2})(A+1)R$$

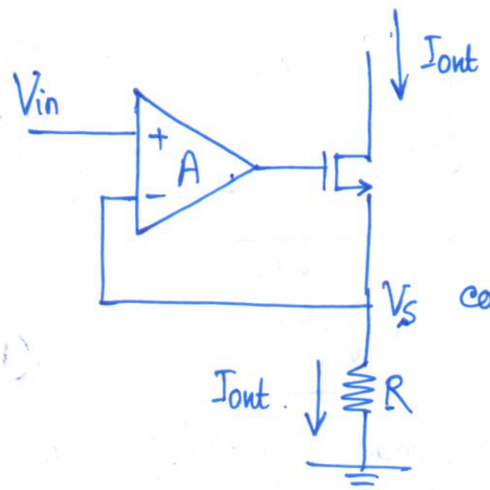
(small signal o/p impedance)

if we consider, $A \gg 1$.

$$\boxed{Z_{out} \cong (A)(g_{m_2} r_{o2})R}$$

Now, let's say, we realise 'A' using a transistor of intrinsic gain $= g_{m_1} r_{o1}$, thus,

$$\boxed{Z_{out} \approx (g_{m_1} r_{o1})(g_{m_2} r_{o2})R}$$



considering $A \gg$ (i.e, sufficiently high) the negative feedback here forces the node voltage (V_S) to track V_{in}.

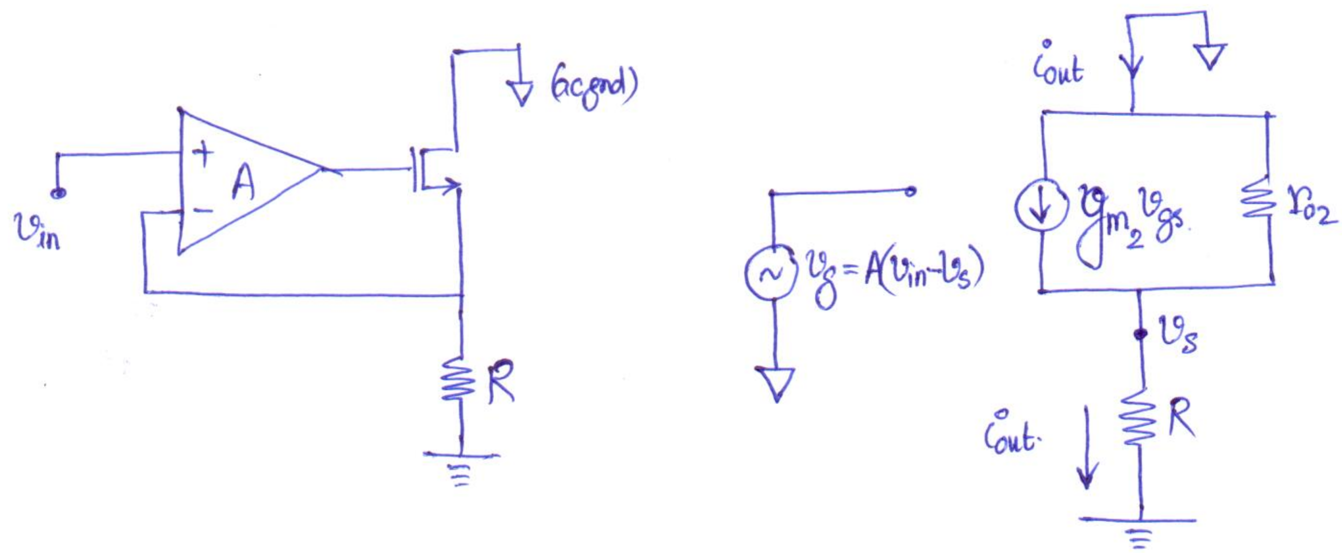
so, $V_S \approx V_{in}$.

so, clearly, $$\boxed{I_{out} = \frac{V_{in}}{R}}$$

1

Accurate analysis for your enjoyment

For finding the 'G_m' of the ckt, following small signal structure,



KCL @ o/p node,
$$i_{out} = g_{m_2}\left(A v_{in} - A v_s - v_s\right) + \frac{0 - v_s}{r_{o2}}$$
$$= g_{m_2} A v_{in} - g_{m_2}(A+1) v_s - \frac{v_s}{r_{o2}}.$$
$$= g_{m_2} A v_{in} - \left[g_{m_2}(A+1) + \frac{1}{r_{o2}}\right] v_s$$

Replacing, v_s by $i_{out} R$,
$$i_{out} = g_{m_2} A v_{in} - \left[g_{m_2}(A+1) + \frac{1}{r_{o2}}\right] R\, i_{out}$$
$$i_{out} \left[1 + g_{m_2}(A+1) R + \frac{R}{r_{o2}}\right] = g_{m_2} A v_{in}.$$
$$i_{out} = \left[\frac{(g_{m_2} r_{o2}) A}{R + r_{o2} + (g_{m_2} r_{o2})(A+1) R}\right] \cdot v_{in}. = \left\{\frac{g_{m_2} r_{o2}}{\frac{R + r_{o2}}{A} + (g_{m_2} r_{o2}) R \left(1 + \frac{1}{A}\right)}\right\} \cdot v_{in}$$

For, $A \gg$ (really high value).
$$i_{out} \approx \frac{\cancel{g_{m_2} r_{o2}}}{(\cancel{g_{m_2} r_{o2}}) R} v_{in} \implies \boxed{i_{out} \approx \frac{v_{in}}{R}}$$

EE613

CMOS ANALOG IC DESIGN.

PROF. RAJESH ZELE

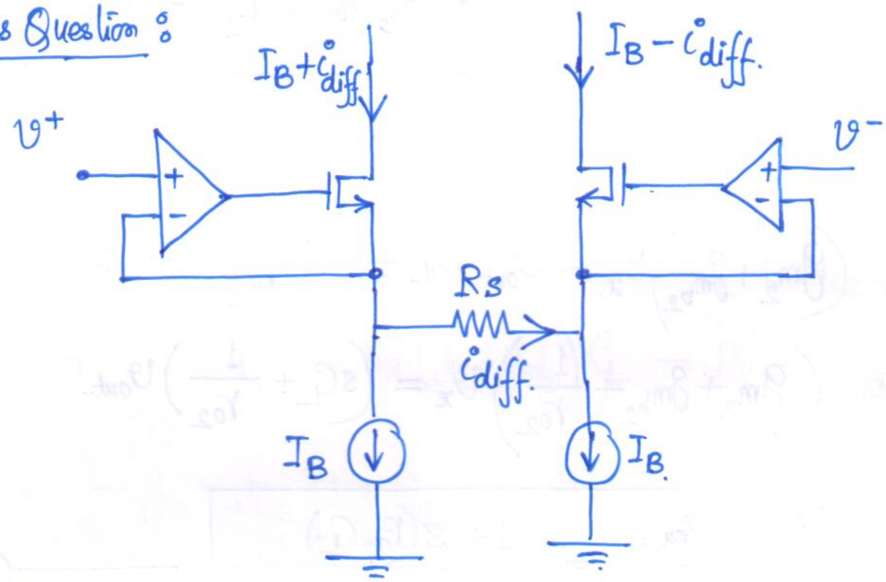
MIDTERM SOLUTIONS

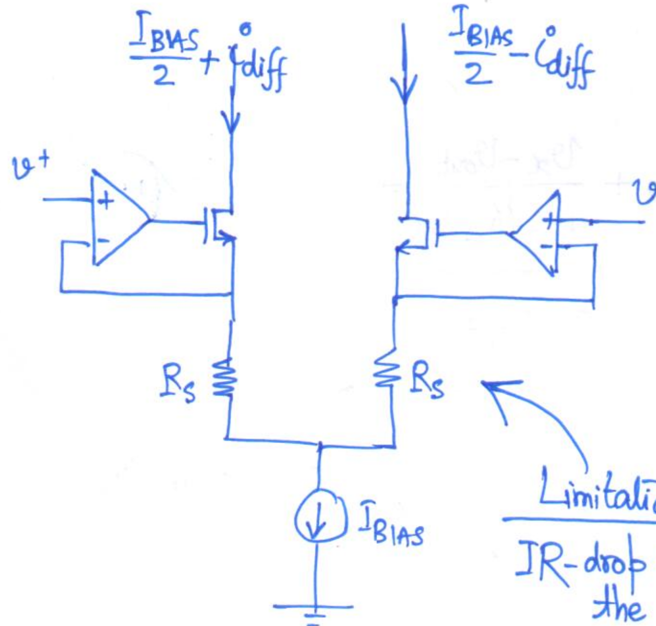
14th SEPT 2017

---

3 Bonus Question :

5



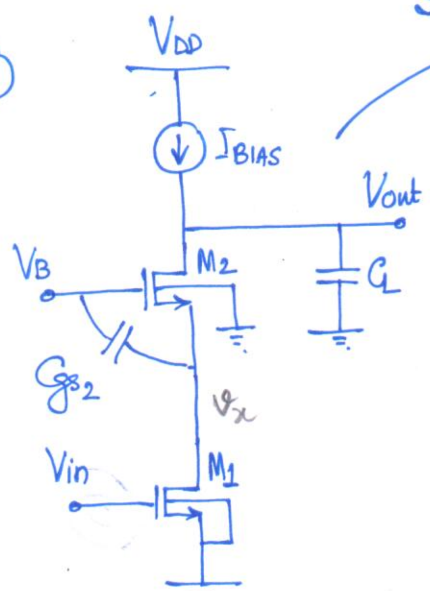


Limitation of this structure :

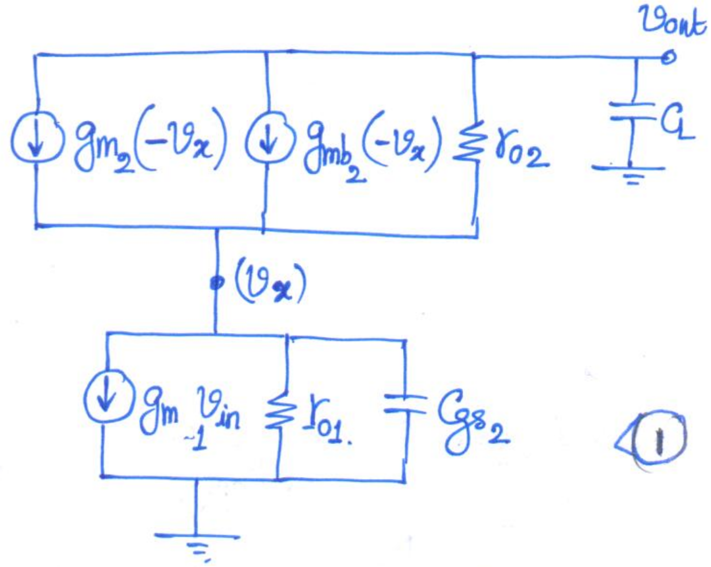
IR-drop across R_S resistor limits the o/p swing.

Note: Differential architechture means you reject any common mode signal @ the i/p.

4



Small Signal Model



1

KCL @ o/p node,

$$\left(g_{m_2}+g_{mb_2}\right)v_x - v_{out}\,sC_L + \frac{v_x - v_{out}}{r_{o2}} = 0$$ 0.5

so, $$\left(g_{m_2}+g_{mb_2}+\frac{1}{r_{o2}}\right)v_x = \left(sC_L+\frac{1}{r_{o2}}\right)v_{out}.$$

i.e, $$\boxed{\frac{v_x}{v_{out}} = \frac{1+s(r_{o2}C_L)}{1+\left(g_{m_2}+g_{mb_2}\right)r_{o2}}} \quad (1)$$

KCL @ v_x node,

$$\frac{v_x}{r_{o1}} + v_x\,sC_{gs_2} + g_{m_1}v_{in} + \left(g_{m_2}+g_{mb_2}\right)v_x + \frac{v_x - v_{out}}{r_{o2}} = 0$$ 0.5

i.e, $$v_x\left[\frac{1}{r_{o1}} + sC_{gs_2} + g_{m_2} + g_{mb_2} + \frac{1}{r_{o2}}\right] - \frac{v_{out}}{r_{o2}} + g_{m_1}v_{in} = 0$$

i.e, $$v_{out}\left\{\left[\frac{1+sr_{o2}C_L}{1+\left(g_{m_2}+g_{mb_2}\right)r_{o2}}\right]\left(\frac{1}{r_{o1}} + sC_{gs_2} + g_{m_2} + g_{mb_2} + \frac{1}{r_{o2}}\right) - \frac{1}{r_{o2}}\right\} = -g_{m_1}v_{in}$$

Multiplying both sides by $(r_{o1} r_{o2})$,

$$v_{out}\left[\left\{\frac{1+s r_{o2} C_L}{1+(g_{m2}+g_{mb2}) r_{o2}}\right\}\left(r_{o2}+s C_{gs2} r_{o1} r_{o2}+(g_{m2}+g_{mb2}) r_{o1} r_{o2}+r_{o1}\right) - r_{o1}\right] = -(g_{m1} r_{o1}) r_{o2} v_{in}$$

Multiplying both sides by $[1+(g_{m2}+g_{mb2})r_{o2}]$ term,

$$v_{out}\left[(1+s r_{o2} C_L)\left\{r_{o1}+r_{o2}+s C_{gs2} r_{o1} r_{o2}+(g_{m2}+g_{mb2}) r_{o1} r_{o2}\right\} - r_{o1}\left\{1+(g_{m2}+g_{mb2}) r_{o2}\right\}\right]$$

$$= -g_{m1} r_{o1} r_{o2}\left[1+(g_{m2}+g_{mb2}) r_{o2}\right] v_{in}$$

$$v_{out}\Big[\cancel{r_{o1}}+r_{o2}+s C_{gs2} r_{o1} r_{o2}+\cancel{(g_{m2}+g_{mb2}) r_{o1} r_{o2}}+s r_{o1} r_{o2} C_L+s r_{o2}^2 C_L+s^2 C_{gs2} r_{o1} r_{o2}^2 C_L$$
$$+ \; s r_{o1} r_{o2}^2 (g_{m2}+g_{mb2}) C_L - \cancel{r_{o1}} - \cancel{(g_{m2}+g_{mb2}) r_{o1} r_{o2}}\Big]$$

$$= -g_{m1} r_{o1} r_{o2}\left[1+(g_{m2}+g_{mb2}) r_{o2}\right] v_{in}$$

$$v_{out}\cancel{r_{o2}}\left[1+s C_{gs2} r_{o1}+s r_{o1} C_L+s r_{o2} C_L+s^2 r_{o1} r_{o2} C_{gs2} C_L+s r_{o1} r_{o2}(g_{m2}+g_{mb2}) C_L\right]$$

$$= -(g_{m1} r_{o1})\cancel{r_{o2}}\left[1+(g_{m2}+g_{mb2}) r_{o2}\right] v_{in}$$

$$\frac{v_{out}}{v_{in}} = \frac{-g_{m1} r_{o1}\left[1+(g_{m2}+g_{mb2}) r_{o2}\right]}{s^2 (r_{o1} r_{o2} C_{gs2} C_L) + s\left[r_{o1}+r_{o2}+r_{o1} r_{o2}(g_{m2}+g_{mb2}) C_L + C_{gs2} r_{o1}\right] + 1}$$

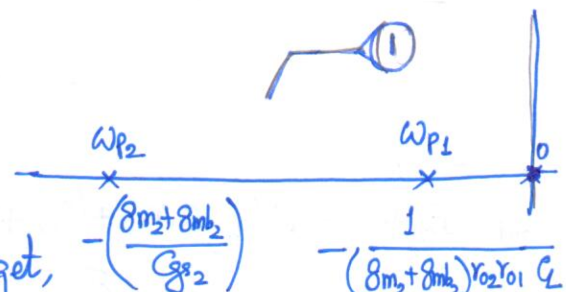
$$\frac{v_o}{v_{in}} \approx \frac{-(g_{m1} r_{o1})\left[(g_{m2}+g_{mb2}) r_{o2}\right]}{s^2 (r_{o1} r_{o2} C_{gs2} C_L) + s\left[r_{o1} r_{o2}(g_{m2}+g_{mb2}) C_L\right] + 1}$$

Now, Transfer Func^n for a 2nd order system $= \dfrac{1}{\dfrac{s^2}{\omega_{p_1}\omega_{p_2}} + \left(\dfrac{1}{\omega_{p_1}} + \dfrac{1}{\omega_{p_2}}\right)s + 1}$

Assuming, $\omega_{p_2} >> \omega_{p_1}$ (Dominant Pole approx^m)

$$T.F \approx \frac{1}{\dfrac{s^2}{\omega_{p_1}\omega_{p_2}} + \dfrac{s}{\omega_{p_1}} + 1}$$

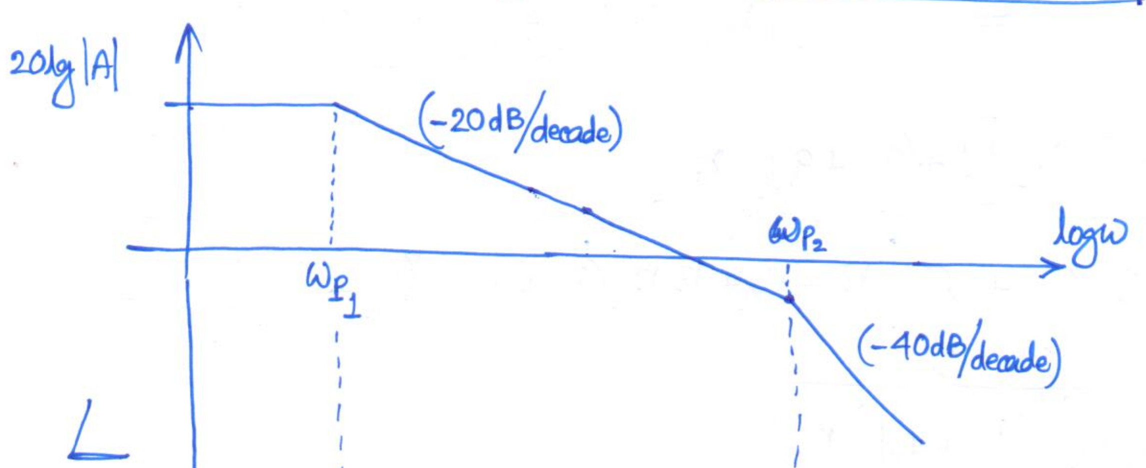
1



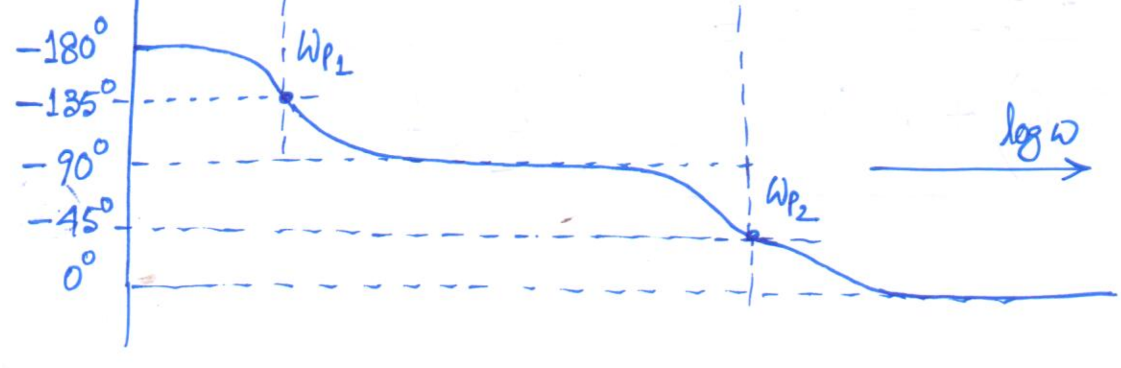
Thus, comparing above with previous expression, we get,

$$\boxed{\omega_{p_1} \approx \frac{1}{(g_{m_2}+g_{mb_2})r_{o1}r_{o2}C_L}}$$ 1

$$\omega_{p_1}\omega_{p_2} = \frac{1}{r_{o1}r_{o2}C_{gs_2}C_L} \quad \therefore \quad \boxed{\omega_{p_2} \cong \frac{(g_{m_2}+g_{mb_2})}{C_{gs_2}}}$$ 1



1.5



1.5

EE615

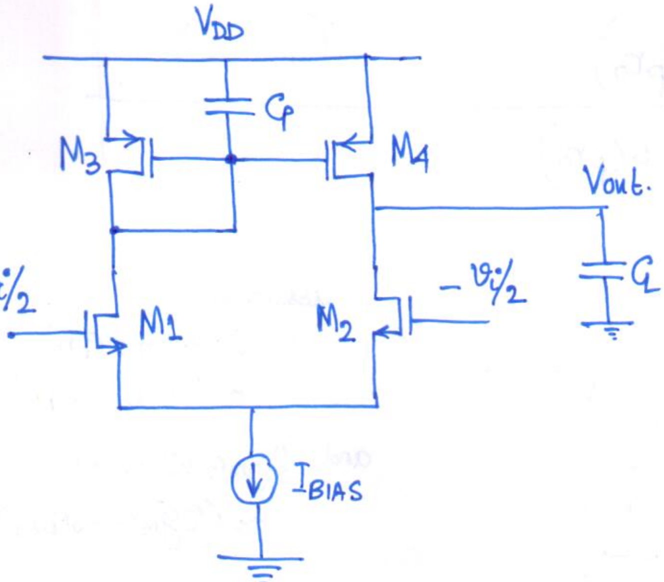
CMOS ANALOG IC DESIGN.

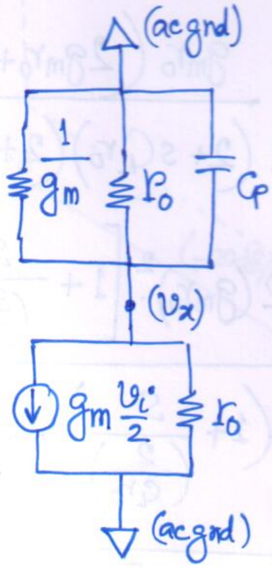
PROF. RAJESH ZELE

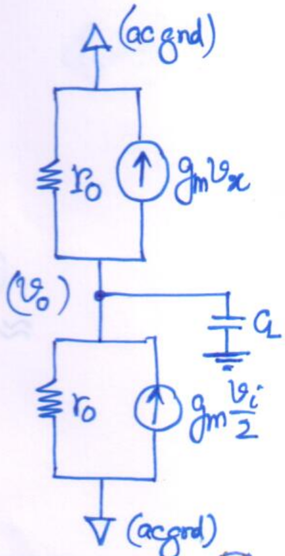
MIDTERM SOLUTIONS

14th SEPT 2017

5







1

KCL @ node v_x, $\quad v_x\left(\frac{2}{r_o}+g_m\right)+v_x sC_p+g_m\frac{v_i}{2}=0$ 0.5

$$\text{so, } v_x\left(\frac{2}{r_o}+g_m+sC_p\right)=-\frac{g_m}{2}v_i \Rightarrow v_x=-\left(\frac{\frac{g_m}{2}}{\frac{2}{r_o}+sC_p+g_m}\right)v_i$$

KCL @ node v_o, $\quad v_o\left(\frac{2}{r_o}+sC_L\right)+g_m v_x-g_m\frac{v_i}{2}=0$ 0.5

$$\text{i.e, } v_o.(2+sC_Lr_o)+(g_mr_o)v_x-(g_mr_o.)\frac{v_i}{2}=0.$$

$$\text{i.e, } v_o(2+sC_Lr_o)-\frac{\frac{g_m^2r_o}{2}}{2/r_o+sC_p+g_m}.v_i-(g_mr_o)\frac{v_i}{2}=0$$

$$\text{i.e, } v_o(2+sC_Lr_o)-\frac{(g_mr_o)^2}{2+sC_pr_o+g_mr_o}.\left(\frac{v_i}{2}\right)-g_mr_o.\left(\frac{v_i}{2}\right)=0$$

$$\text{i.e, } 2v_o(2+sC_Lr_o)=\left\{\frac{g_m^2r_o^2}{2+g_mr_o+sC_pr_o}+g_mr_o\right\}v_i$$

$$2v_o(2+sC_Lr_o) = \frac{(g_m^2r_o^2 + 2g_mr_o + g_m^2r_o^2 + sC_pg_mr_o^2)}{(2+g_mr_o+sC_pr_o)} v_i$$

$$\frac{v_o}{v_i} = \frac{2g_m^2r_o^2 + 2g_mr_o + sC_pg_mr_o^2}{2(2+sC_Lr_o)(2+g_mr_o+sC_pr_o)}.$$

$$= \frac{g_mr_o(2g_mr_o+2+sC_pr_o)}{2(2+sC_Lr_o)(2+g_mr_o+sC_pr_o)}$$

$$\approx \frac{2(g_mr_o)^2\left[1+\frac{s}{\left(\frac{2g_m}{C_p}\right)}\right]}{4\left(1+\frac{s}{\left(\frac{2}{C_Lr_o}\right)}\right)\cdot g_mr_o\cdot\left(1+\frac{s}{\left(\frac{g_m}{C_p}\right)}\right)}$$

assuming, $(g_mr_o+2+sC_pr_o) \approx (g_mr_o+sC_pr_o)$

and, $(2g_mr_o+2+sC_pr_o) \approx (2g_mr_o+sC_pr_o)$

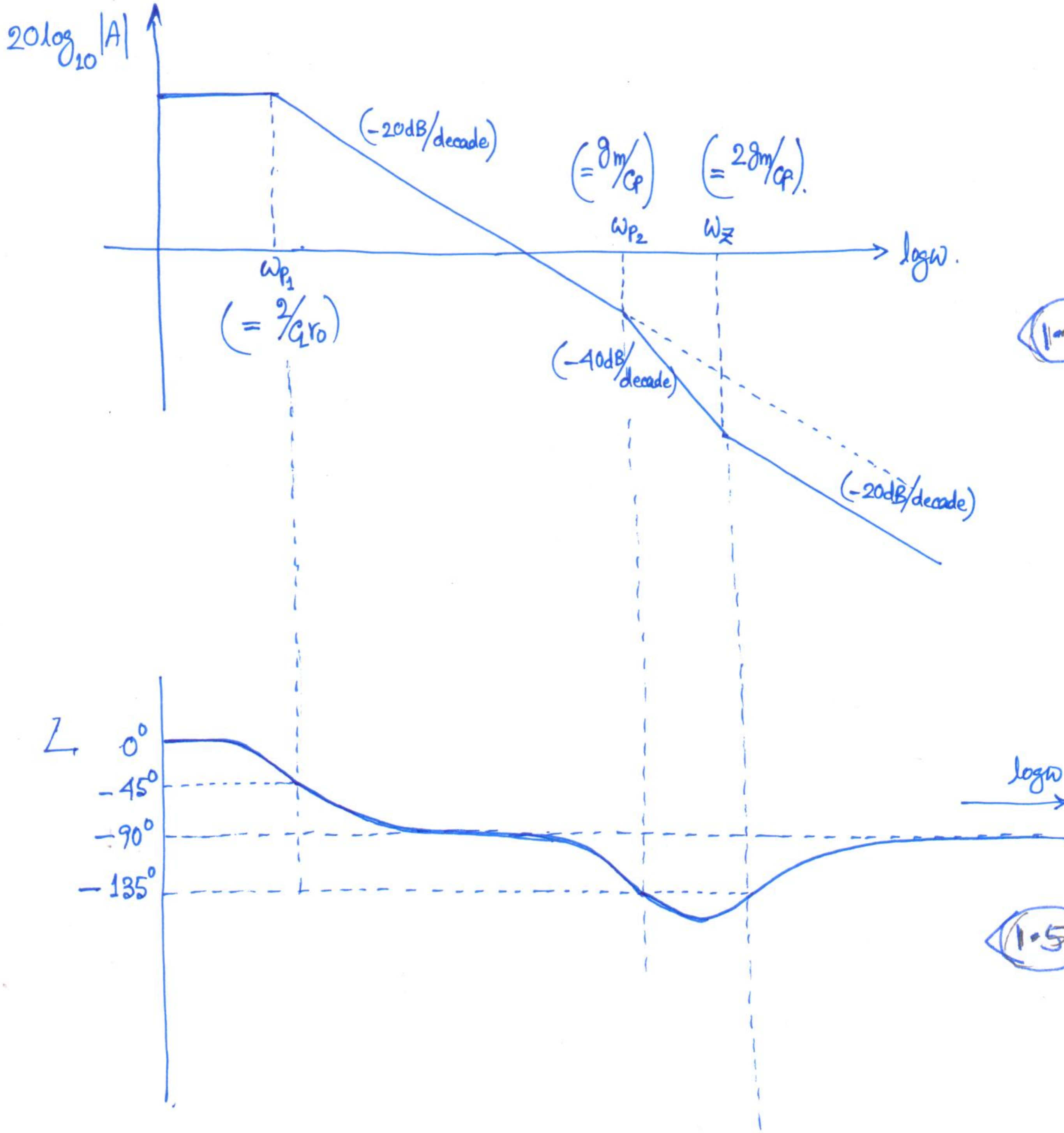
$$\boxed{\frac{v_o}{v_i}(s) \approx \frac{\frac{g_mr_o}{2}\left[1+\frac{s}{\left(\frac{2g_m}{C_p}\right)}\right]}{\left[1+\frac{s}{\left(\frac{2}{C_Lr_o}\right)}\right]\left[1+\frac{s}{\left(\frac{g_m}{C_p}\right)}\right]}}$$

← TRANSFER FUNCTION

1

so, $\boxed{\omega_{p_1} = -\frac{2}{C_Lr_o}\ ;\ \omega_{p_2} = -\frac{g_m}{C_p}.\ ;\ \omega_z = -\frac{2g_m}{C_p}}$

1

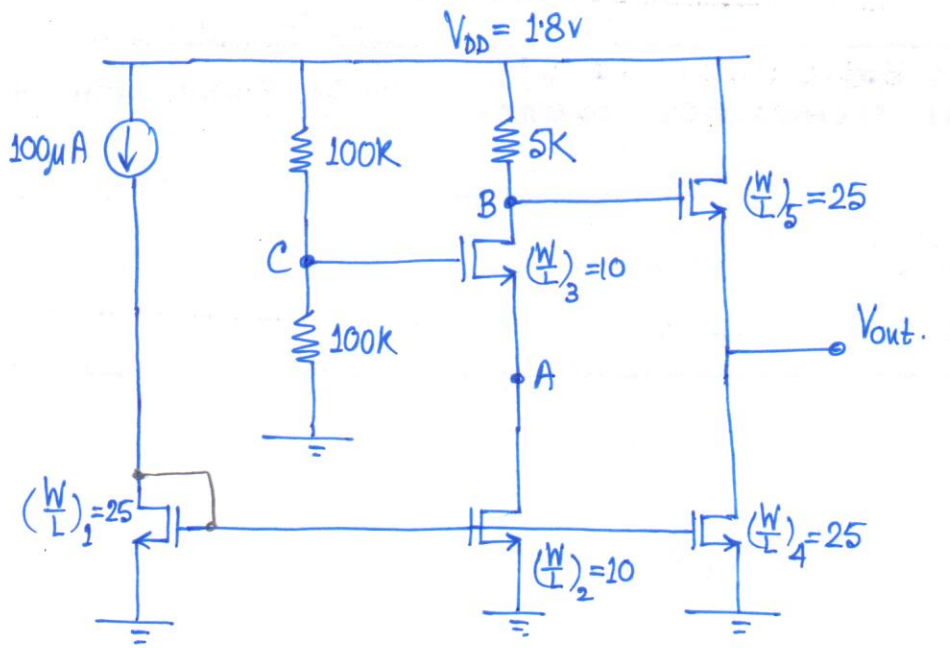


20log10|A|
(−20dB/decade)
(= gm/Cp)
(= 2gm/Cp).
ωp2
ωZ
log ω.
ωp1
(= 2/CL r0)
(−40dB/decade)
(−20dB/decade)
∠ 0°
−45°
−90°
−135°
log ω

1·5

1·5

6



DC biasing structure looks like this.

$$\left.\begin{aligned} I_{M_2} &= I_{M_3} = \left(\frac{10}{25} \times 100\right) = 40\mu A. \\ I_{M_4} &= I_{M_5} = \left(\frac{25}{25} \times 100\right) = 100\mu A \end{aligned}\right\} \text{Current mirroring.}$$

$$\boxed{V_C = \frac{1.8}{2} = 0.9\text{ v}}$$ 1

$$V_{GS_2} = V_{GS_3} = V_t + \sqrt{\frac{2 \times I_{D_{2,3}}}{\mu_n C_{ox} (W/L)_{2,3}}} = 0.4 + \sqrt{\frac{2 \times 4\cancel{0}}{200 \times 1\cancel{0}}} = 0.6\text{ v}$$

Now, $V_{GS_3} = V_C - V_A = 0.9 - V_A = 0.6$

so, $\boxed{V_A = 0.3\text{ v}}$ 1

$$\boxed{V_B = 1.8 - (5 \times 10^3)(40 \times 10^{-6}) = 1.6\text{ v}}$$ 1

$V_{GS_4} = V_{GS_2} = 0.6 = V_{GS_5}.$

so, $V_{GS_5} = V_B - V_{out} = 0.6 = 1.6 - V_{out}.$

$\boxed{V_{out} = 1\text{ v}}$ 1

Note, M_5 acts as a source follower. Therefore, $v_B = v_{out}$ (Note, here we're talking about small signal ac signal NOT the DC Bias)

M_3 acts as CS amplifier.

Node 'A' is ac gnd due to the presence of C_{dcp}.

so, $$\text{gain } A_v = -(g_{m_3}) \times (5k\Omega)$$

$$= -\sqrt{2 \mu_n C_{ox} \left(\frac{W}{L}\right)_3 I_{D3}} \times 5 \times 10^3$$

$$= -\sqrt{2 \times 200 \times 10^{-6} \times 40 \times 10^{-6} \times 10} \times 5 \times 10^3$$

$$= -2. \quad \text{———} \quad (1)$$

so, $$\boxed{\text{Inband gain } \frac{v_{out}}{v_{in}} = -2.} \quad \text{———} \quad (1)$$