EE_735 Assignment_3

3/09/2017

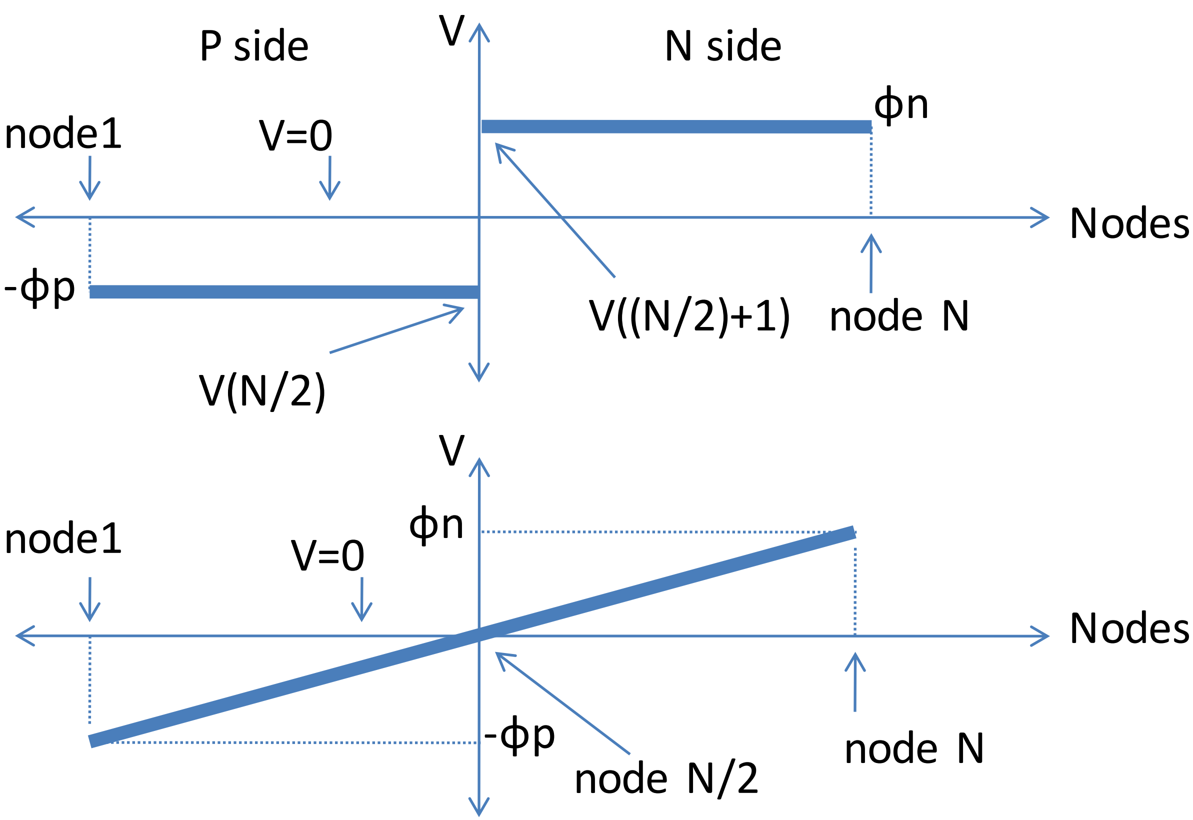
Numerical solution of PN junction diode

- Parameter values are same as given in assignment problem.
- Lets denote total device length by L cm.
- Lets divide L into N nodes. Let h be the spacing between two consecutive nodes. $h=L/(N)$. ($h=L/(N-1)$) will also do.
- Our aim is to solve : $\Delta V = - J \backslash F$. These terms have been described in the additional material.
- Lets try to derive matrix representation of J and F.
- Note: The simulation results are shown for abrupt junction case only.

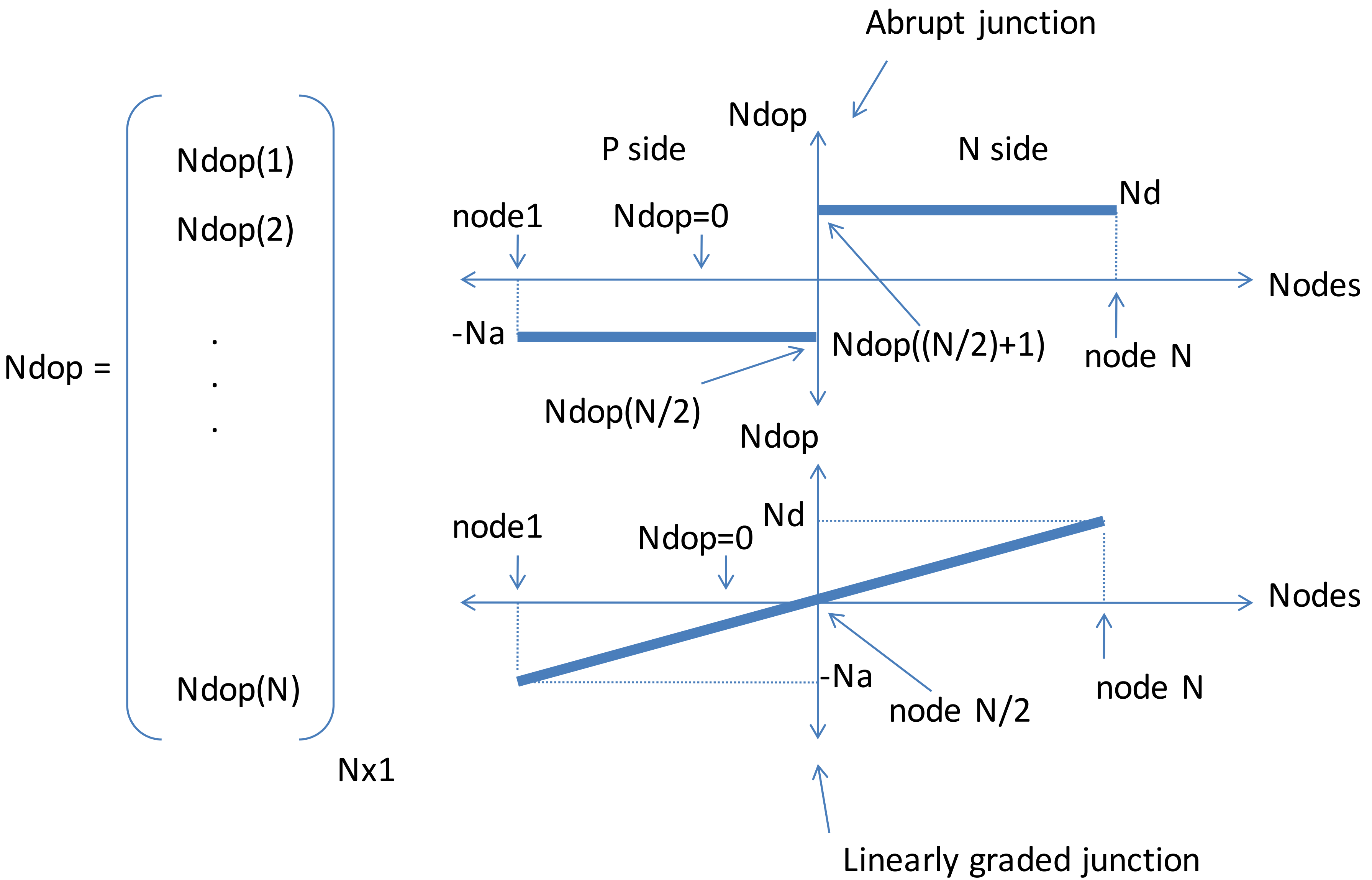
Terms and their meaning

V - potential profile; $Vt = kT/q$ is the thermal voltage; n – electron concentration; p - hole concentration; ni - intrinsic carrier concentration; Nd - donor dopant concentration; Na - acceptor dopant concentration; Ndop – net doping profile; $\phi n = Vt * \log(Nd/ni)$; $\phi p = Vt * \log(Na/ni)$; $\varepsilon = 12 * \varepsilon_0$ (where ε_0 is the permittivity of vacuum).

Some V profiles which can be used as an initial guess



Doping profile



V, n and p

$$V = \begin{pmatrix} V(1) \\ V(2) \\ \vdots \\ V(N) \end{pmatrix}_{N\times 1} \qquad n = \begin{pmatrix} ni*\exp(V(1)/Vt) \\ ni*\exp(V(2)/Vt) \\ \vdots \\ ni*\exp(V(N)/Vt) \end{pmatrix}_{N\times 1} \qquad p = \begin{pmatrix} ni*\exp(-V(1)/Vt) \\ ni*\exp(-V(2)/Vt) \\ \vdots \\ ni*\exp(-V(N)/Vt) \end{pmatrix}_{N\times 1}$$

b matrix

$$
\rho = q * \begin{pmatrix} \text{Ndop}(1)+p(1)-n(1) \\ \text{Ndop}(2)+p(2)-n(2) \\ . \\ . \\ \text{Ndop}(N/2)+p(N/2)-n(N/2) \\ \text{Ndop}((N/2)+1)+p((N/2)+1)-n((N/2)+1) \\ \\ . \\ . \\ \text{Ndop}(N-1)+p(N-1)-n(N-1) \\ \text{Ndop}(N)+p(N)-n(N) \end{pmatrix}_{Nx1} \; ; \; \Delta\rho = q * \begin{pmatrix} p(1)/(-Vt)-n(1)/Vt \\ p(2)/(-Vt)-n(2)/(Vt) \\ . \\ . \\ p(N/2)/(-Vt)-n(N/2)/Vt \\ p((N/2)+1)/(-Vt)-n((N/2)+1)/Vt \\ \\ . \\ . \\ p(N-1)/(-Vt)-n(N-1)/Vt \\ p(N)/(-Vt)-n(N)/Vt \end{pmatrix}_{Nx1}
$$

$b = -(\rho/\varepsilon)$; $\Delta b=-(\Delta\rho/\varepsilon)$

F matrix and boundary conditions

$$F = \begin{pmatrix} V(1) - b(1) \\ (V(1)-2*(V(2))+V(3))/h^2 - b(2) \\ (V(2)-2*(V(3))+V(4))/h^2 - b(3) \\ . \\ . \\ . \\ (V(N-3)-2*(V(N-2))+V(N-1))/h^2 - b(N-2) \\ (V(N-2)-2*(V(N-1))+V(N))/h^2 - b(N-1) \\ V(N) - b(N) \end{pmatrix}_{N\times 1}$$

$b(1) = V(1)$ and $b(N) = V(N)$; where $V(1)$ and $V(N)$ are fixed potentials.

$\Delta b(1) = 0$ and $\Delta b(N) = 0$;

These are the Boundary conditions and shouldn't change during subsequent iterations.

Jacobian matrix (J)

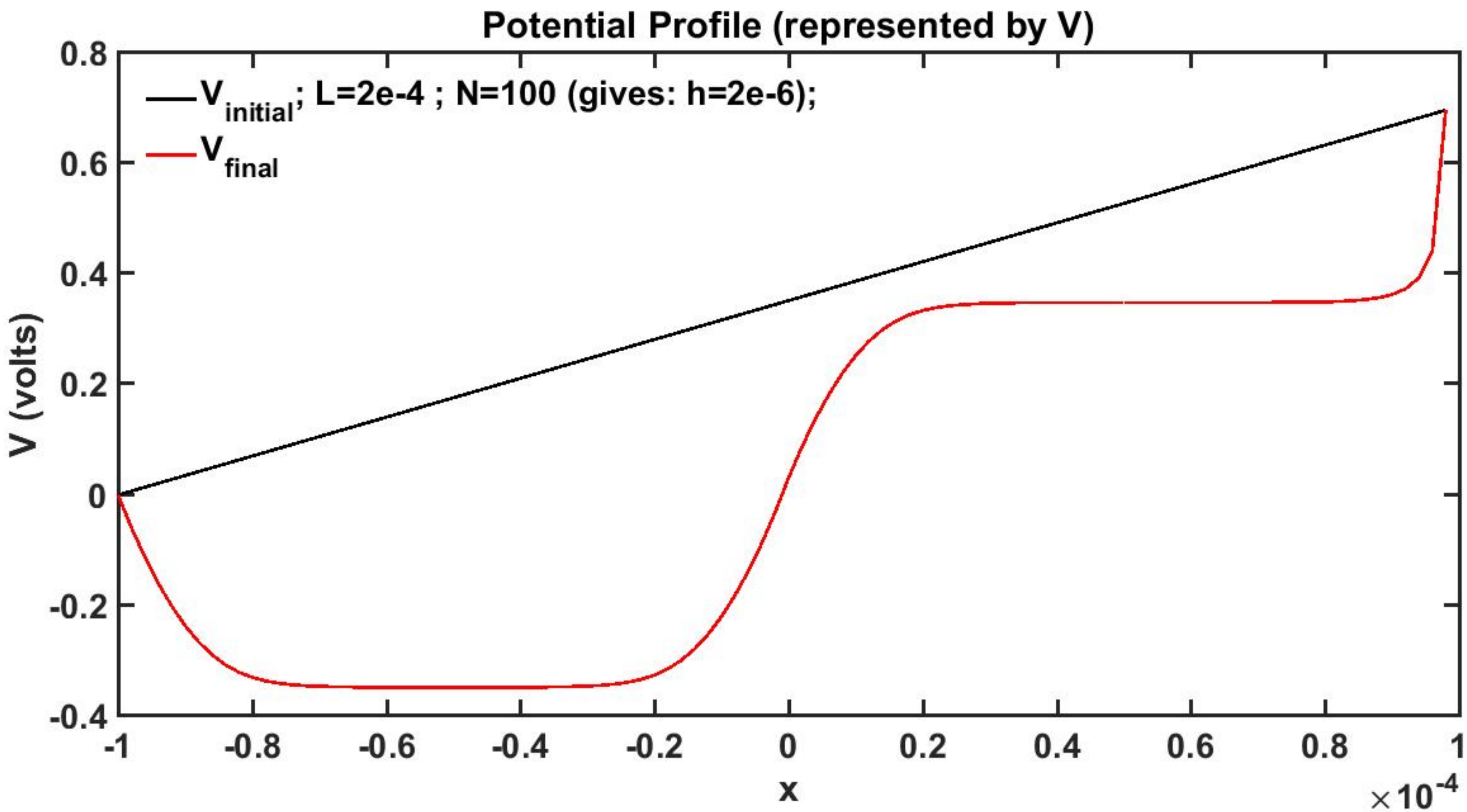
Columns

$$
\begin{array}{c|cccccccc}
 & 1 & 2 & 3 & 4 & & N-2 & N-1 & N \\
\hline
1 & 1-\Delta b(1) & 0 & 0 & 0 & & 0 & 0 & 0 \\
2 & 1/h^2 & -2/h^2-\Delta b(2) & 1/h^2 & 0 & \cdots & 0 & 0 & 0 \\
 & 0 & 1/h^2 & -2/h^2-\Delta b(3) & 1/h^2 & 0 & \cdots & 0 & 0 \\
 & & & & & \ddots & & & \\
N-2 & 0 & 0 & \cdots & 0 & 1/h^2 & -2/h^2-\Delta b(N-2) & 1/h^2 & 0 \\
N-1 & 0 & 0 & 0 & \cdots & 0 & 1/h^2 & -2/h^2-\Delta b(N-1) & 1/h^2 \\
N & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 1-\Delta b(N)
\end{array}
$$

rows

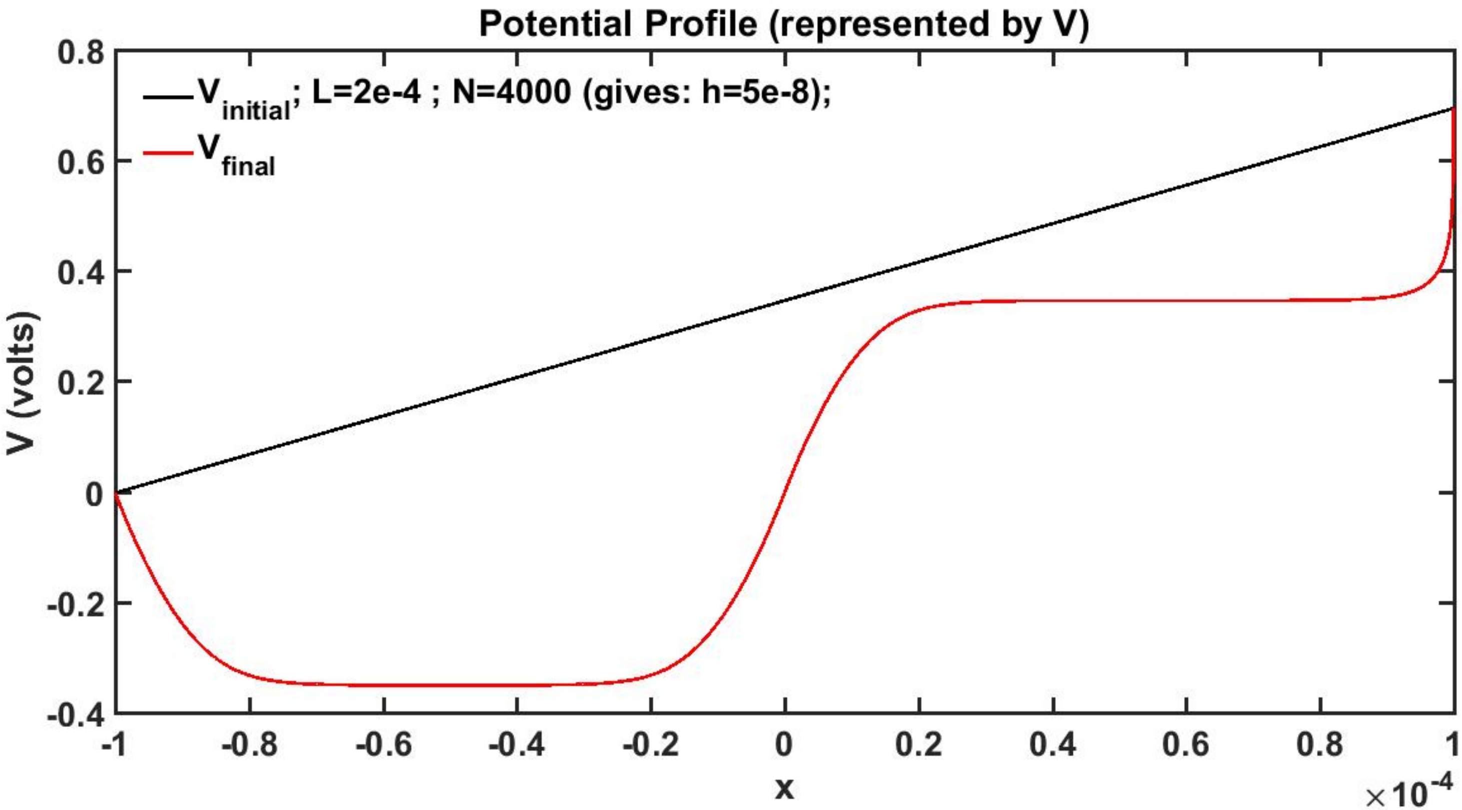
NxN

Simulation results



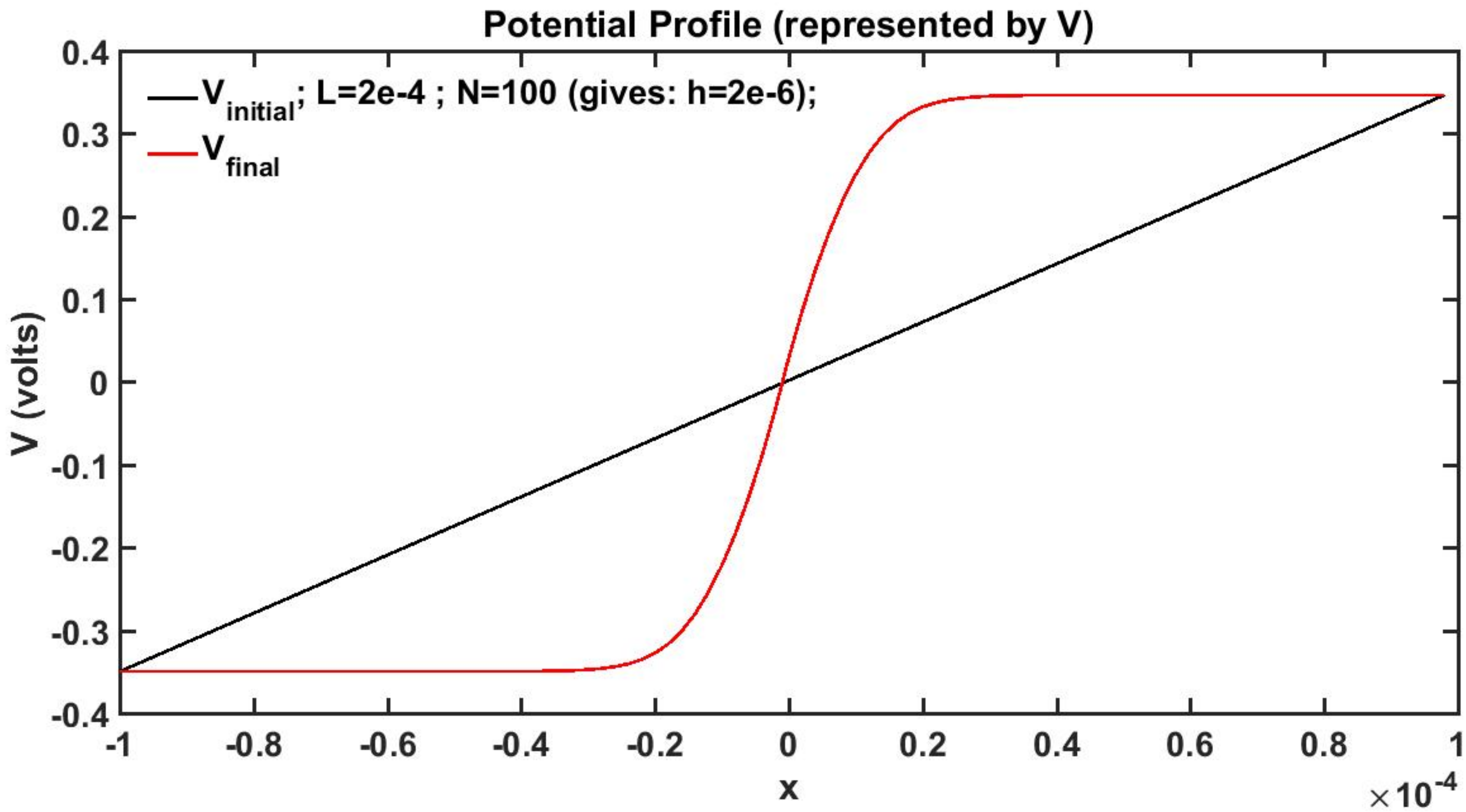
There is a problem at the boundaries. We have used p=ni * exp(-V/Vt). We have fixed V(1)=0, which yields p(1)=ni. However p(1) = Na, since it belongs to the bulk. Thus this boundary condition is not satisfying hole concentration in the bulk. Similar reasoning can be given for n. (Note : x < 0 is P type and x > 0 is N type). It took around 40 iterations to converge. h=L/N has been used.

Simulation results



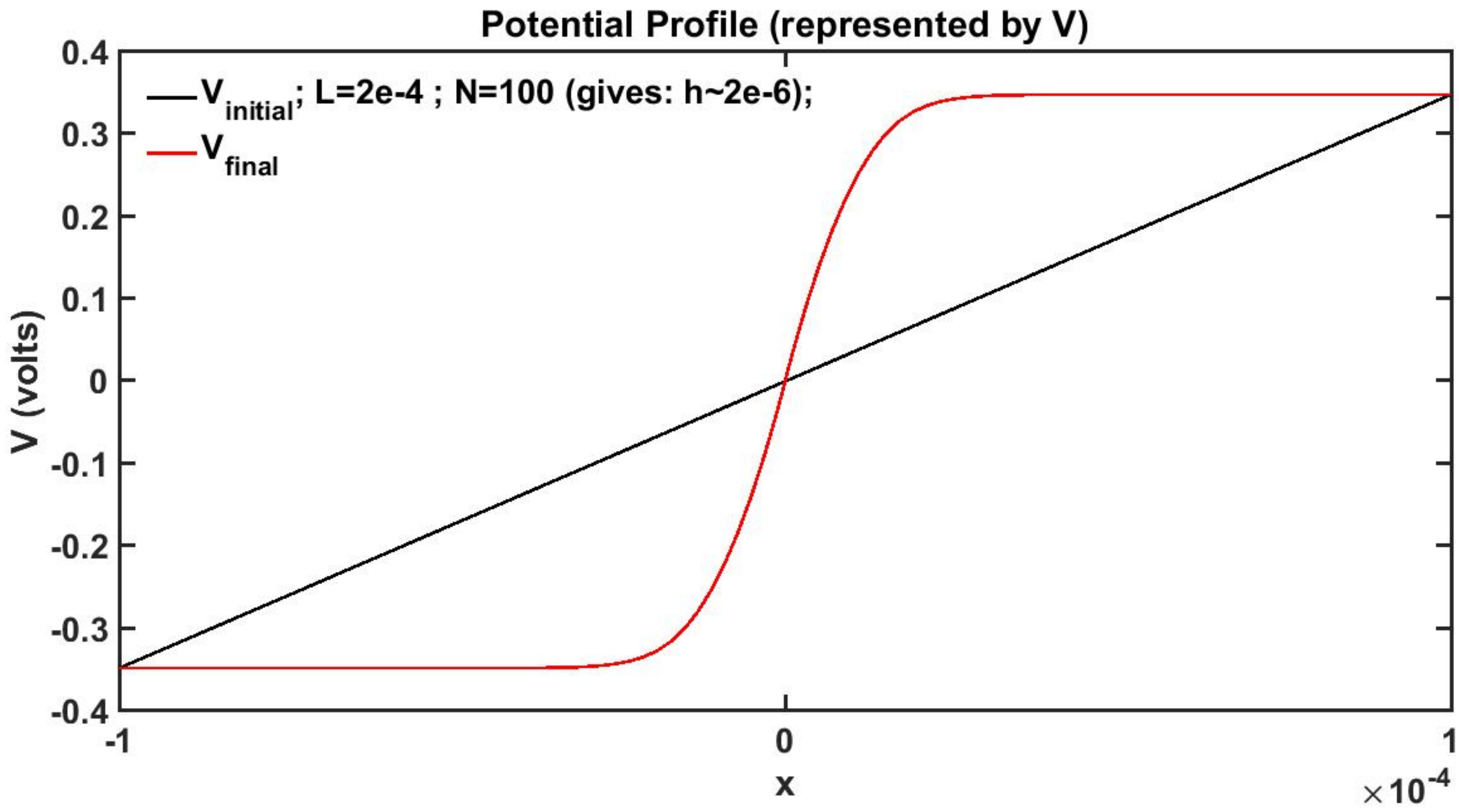
This graph is similar to the previous one. Here we have reduced the step size 'h' by increasing the number of node points 'N'. Thus varying h within the range 1e-8 cm to 1e-6 cm did not result in convergence issues. h=L/N has been used.

Simulation results



We have fixed V(1)=-ϕp and V(N)=ϕn, which yield correct values for n and p at the boundaries. It took around 50 iterations to converge. h=L/N has been used.

Simulation results



This graph is same as the previous one. h=L/(N-1) has been used.