भारतीय प्रौद्योगिकी संस्थान मुंबई
INDIAN INSTITUTE OF TECHNOLOGY BOMBAY

परिशिष्ट/Supplement - 4

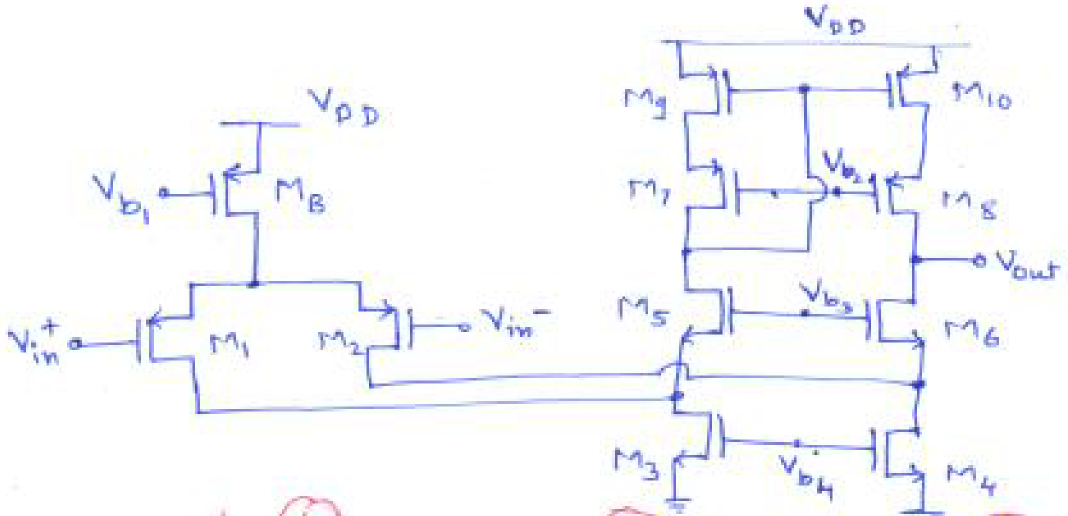
रोल नं./Roll No.	पाठ्यक्रम नाम/Course Name
शाखा/प्रभाग/Branch/Div. — शिक्षण बैच/Tutorial Batch	अनुभाग/Section
पाठ्यक्रम सं./Course No.	तिथि/Date

EE618 [ZELE]

2018 – 2019

MIDSEM SOLUTIONS

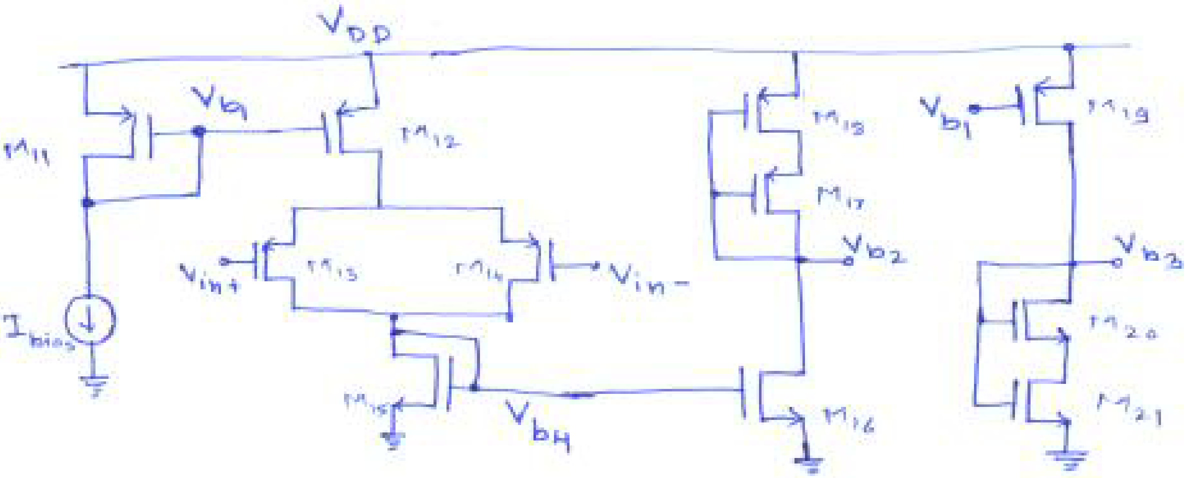
Q.1.



polarity - 1 Annotation - 0.5 Circuit 1.5

No credits if schematic is wrong.

<u>Biasing circuit</u> :→

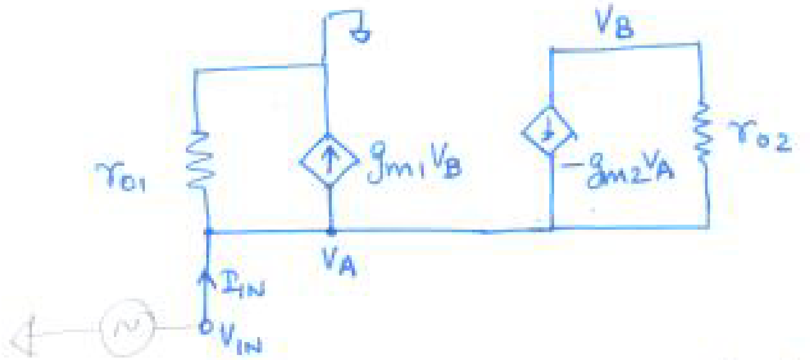


Here, M_{18} and M_{21} sizes are decided to provide a voltage drop ~~of~~ $\approx V_{dsat}$.

$V_{b1}, V_{b2}, V_{b3}, V_{b4}$ — 4×0.5

QUESTION 2

At Node A



$\underline{V_A = V_{IN}}$

1

1/2 KCL at Node B $\Rightarrow$ $(V_B - V_{IN}) = g_{m2}V_A r_{o2}$

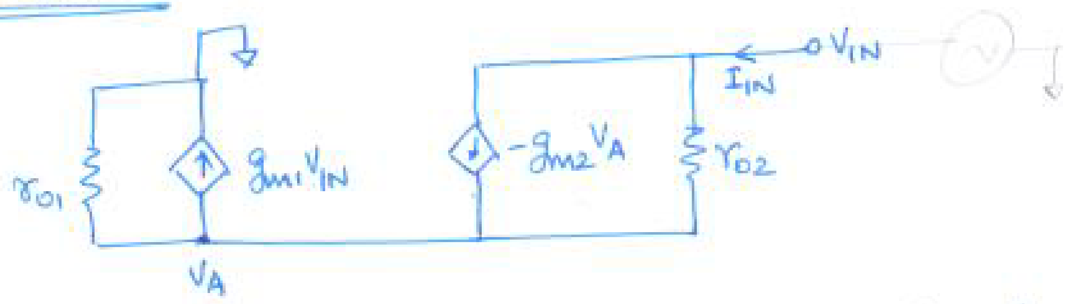
$\Rightarrow \boxed{V_B = (1 + g_{m2}r_{o2})V_{IN}}$ — 1

1/2 KCL at Node A $\Rightarrow$ $I_{IN} = g_{m1}V_B + \frac{V_{IN}}{r_{o1}}$

$= g_{m1}(1 + g_{m2}r_{o2})V_{IN} + \frac{V_{IN}}{r_{o1}}$

1 $\Rightarrow Z_{IN,A} = \frac{r_{o1}}{1 + g_{m1}r_{o1}(1 + g_{m2}r_{o2})} \approx \frac{1}{g_{m1}(g_{m2}r_{o2})}$ (Approximated $g_m r_o \gg 1$)

At Node B



KCL at input Node : $I_{IN} = -g_{m2}V_A + \frac{V_{IN} - V_A}{r_{o2}}$ — 2

KCL at Node A : $I_{IN} = g_{m1}V_{IN} + \frac{V_A}{r_{o1}}$ — 3

PROCEDURE/REASONING

0.5

Using ② and ③ to eliminate V_A,

$$\frac{V_A}{r_{01}} + \frac{V_A}{r_{02}} + g_{m2} V_A = \frac{V_{IN}}{r_{02}} - g_{m1} V_{IN}$$

$$\Rightarrow \frac{V_A}{r_{01}}\left[r_{02} + r_{01} + g_{m2} r_{01} r_{02} \right] = V_{IN}\left[1 - g_{m1} r_{02}\right]$$

$$\Rightarrow \frac{V_A}{r_{01}} = V_{IN}\left[\frac{1 - g_{m1} r_{02}}{r_{01} + r_{02} + g_{m2} r_{01} r_{02}}\right]$$

Substituting in ③

$$I_{IN} = g_{m1} V_{IN} + V_{IN}\left[\frac{1 - g_{m1} r_{02}}{r_{01} + r_{02} + g_{m2} r_{01} r_{02}}\right]$$

0.5 $$\Rightarrow \frac{V_{IN}}{I_{IN}} = \boxed{Z_{IN,B} = \frac{r_{01} + r_{02} + g_{m2} r_{01} r_{02}}{1 + g_{m1} r_{01} + g_{m1} g_{m2} r_{01} r_{02}}} \approx \frac{1}{g_{m1}}$$

(Assuming $g_m r_o >> 1$)

Rewriting $$\boxed{Z_{IN,B} = \frac{r_{01} + r_{02} + g_{m2} r_{01} r_{02}}{1 + g_{m1} r_{01} (g_{m2} r_{02} + 1)}}$$

<u>COROLLARY</u>:

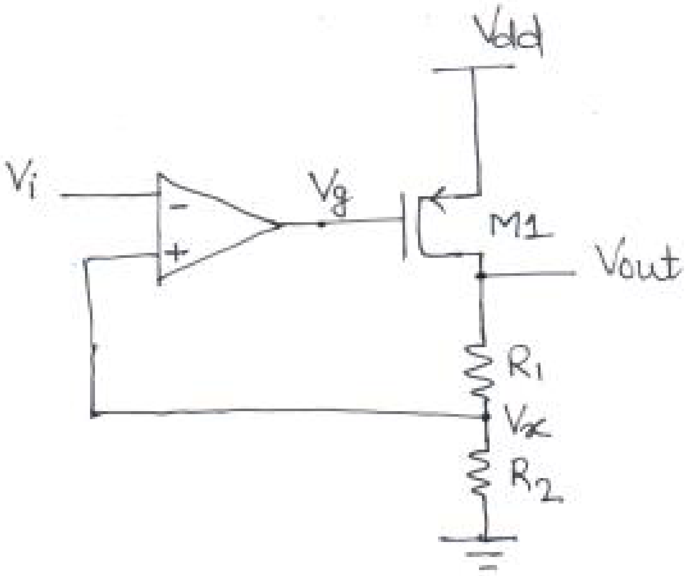
Approximately, I_{IN} from node B will flow through M2 and M1.

* V_A changes to allow I_{IN} through M2   0.5
* Hence I_{IN} is directly controlled by M1 — g_{m1}

$$\Rightarrow \boxed{Z_{IN} = 1/g_{m1}}$$ 0.5

Q.3.

a) OPAMP polarity:



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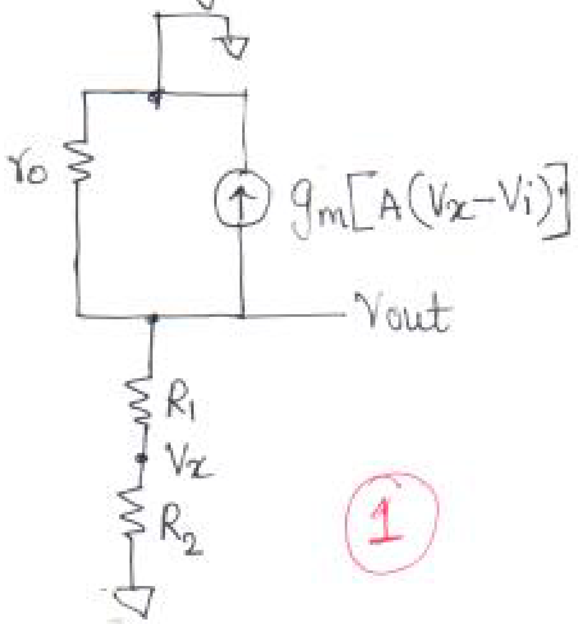
As Vi increases, Vg decreases and hence Vout and Vx increases (Due to the gate to drain inversion of M1). Hence negative feedback is established.

b) Transfer function:

Small signal model:



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0.25

$$V_x = V_{out}\left(\frac{R_2}{R_1+R_2}\right) \quad \text{— (1)}$$

Applying KCL at Vout node,

$$\frac{V_{out}}{r_o} + g_m A(V_x - V_i) + \frac{V_{out}}{R_1+R_2} = 0 \quad \text{— (2)}$$

0.25

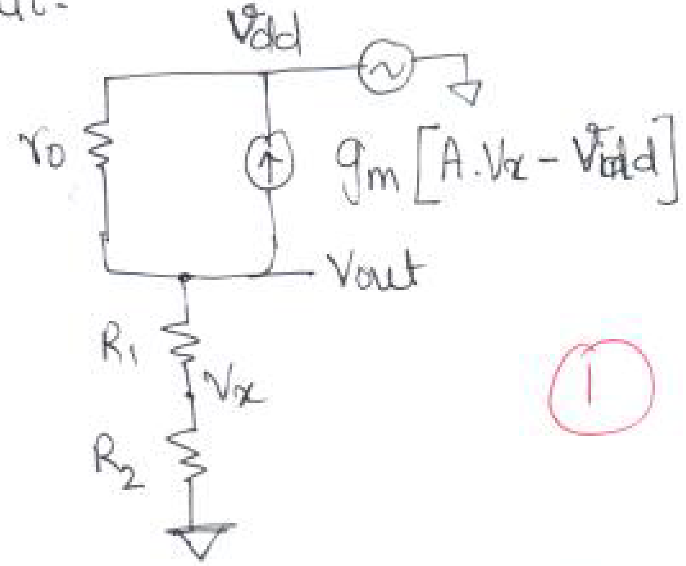
from ① & ②, we get,

$$V_{out}\left[\frac{1}{r_o} + \frac{g_m A R_2}{R_1+R_2} + \frac{1}{R_1+R_2}\right] = g_m A V_i$$

$$\Rightarrow \frac{V_{out}}{V_i} = \frac{A\cdot(g_m r_o)(R_1+R_2)}{R_1+R_2+A(g_m r_o)R_2+r_o}$$ 0.5

c) Vdd to Vout transfer function:

small signal circuit:



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$$V_x = V_{out}\cdot\frac{R_2}{R_1+R_2} \quad \text{— ①}$$ 0.25

by KCL @ Vout node gives,

$$\frac{V_{out}-v_{dd}}{r_o} + g_m\left[AV_x - v_{dd}\right] + \frac{V_{out}}{R_1+R_2} = 0 \quad \text{— ②}$$ 0.25

from ① & ②, we get

$$\frac{V_{out}}{v_{dd}} = \frac{(1+g_m r_o)(R_1+R_2)}{A(g_m r_o)R_2+R_1+R_2+r_o}$$ 0.5

d) $A \to \infty$ & $g_m r_o \to \infty$

for part b)

$$\frac{V_{out}}{V_{in}} = \frac{R_1+R_2}{\frac{R_1+R_2}{A g_m r_o} + R_2 + \frac{r_o}{A g_m r_o}} \Rightarrow \frac{V_{out}}{V_{in}} = \frac{R_1+R_2}{R_2} \text{ as } A g_m r_o \to \infty$$

1

for part c)

$$\frac{V_{out}}{V_{dd}} \approx \frac{(g_m r_o)(R_1+R_2)}{A(g_m r_o)R_2 + R_1+R_2+r_o} = \frac{R_1+R_2}{AR_2 + \frac{R_1+R_2}{g_m r_o} + \frac{1}{g_m}}$$

$= 0$ as A & $r_o \to \infty$.

1

4.1

Q.4

a) $I_5 = 10 \times I_B = 100\mu A$ 1

b) $I_7 = 50 \times I_B = 500\mu A$ 1

c) power dissipation $= (I_B + I_5 + I_7) V_{DD}$

$= 610\mu A \times 1.8V$ 1

$$\boxed{P_{diss} = 1.098\, mW}$$

d) <u>V_B</u>

$V_B = V_{DD} - V_{SG_3}$; $V_{SG_3} = |V_{Th_3}| + \sqrt{\dfrac{2 I_{D_3}}{\mu_p C_{ox} (W/L)_3}}$ 0.5

$$V_{SG_3} = 0.4 + \sqrt{\frac{2 \times 50 \times 10^{-6}}{62 \times 10^{-6} \times 32}}$$

$$V_B = 1.8V - 0.6245V$$

$$\boxed{V_B = 1.17549V}$$ 0.5

e) <u>$(W/L)_6$</u> $\Rightarrow$ for no systematic offset, $V_{dsat_3} = V_{dsat_4} = V_{dsat_6}$

$\Rightarrow V_{dsat_3} = V_{dsat_6}$ 1

$\sqrt{\dfrac{2 I_{D_3}}{\mu_p C_{ox} (W/L)_3}} = \sqrt{\dfrac{2 I_{D_6}}{\mu_p C_{ox} (W/L)_6}}$ & $I_{D_3} = \dfrac{I_5}{2}$

$I_{D_6} = I_7$

4 (2)

$$(W/L)_3 = \frac{I_5}{2 \cdot I_7}(W/L)_6 \Rightarrow \left(\frac{W}{L}\right)_6 = \frac{32 \times 500}{50}$$

$$(W/L)_6 = 320 \Rightarrow \boxed{W_6 = 320\,\mu m}$$ (1)

f.)

DC Gain:

$$A_{V_{tot}} = A_{V_1} \cdot A_{V_2} = \frac{g_{m_1}}{(g_{ds_2} + g_{ds_4})} \cdot \frac{g_{m6}}{(g_{ds_6} + g_{ds7})}$$ (1)

$\because g_{ds} = \lambda \cdot I_D$ & $\lambda_1 L_1 = \lambda_2 L_2$ for any two different lengths,

$\lambda_n = 0.48\,V^{-1}$; $\lambda_p = 0.328\,V^{-1}$ for $L = 0.18\,\mu m$

$\Rightarrow (0.48)\,0.18 = \lambda_n$ for $L = 1\,\mu m \Rightarrow \boxed{\lambda_n = 0.0864\,V^{-1}}$ (1)

$\lambda_p = (0.18)(0.328)\,V^{-1}$ for $L = 1\,\mu m \Rightarrow \boxed{\lambda_p = 0.05904\,V^{-1}}$

$$A_{V_1} = \frac{g_{m_1}}{g_{ds_4} + g_{ds_2}} = \frac{\sqrt{2 \cdot (I_5/2) \cdot K_n \cdot (W/L)_1}}{\left(\frac{I_5}{2}\right)(\lambda_n + \lambda_p)} = \sqrt{\frac{2\,\mu_n C_{ox} (W/L)_1}{(I_5/2)(\lambda_n + \lambda_p)^2}}$$

$$= \sqrt{\frac{2 \times 263 \times 16}{50}} \cdot \frac{1}{(0.0864 + 0.05904)}$$

(0.5)

$$\boxed{A_{V_1} = 89.2039}$$

$$A_{V_2} = \frac{g_{m6}}{g_{ds6} + g_{ds7}} = \sqrt{\frac{2\,\mu_p C_{ox} (W/L)_6}{I_6\,(\lambda_n + \lambda_p)^2}}$$

$$= \sqrt{\frac{2 \times 62 \times 320}{500}} \cdot \frac{1}{(0.0864 + 0.05904)} = 61.25$$

$$\boxed{A_{V_2} = 61.25}$$ (0.5)

DC Gain = $A_{V_1} \cdot A_{V_2} = 89.2039 \times 61.25$

$$\boxed{\text{DC Gain} = 5463.877 = 74.75\,dB}$$

1

g.)

V_{ICMR} (MAX)

$= V_B + V_{thn1}$

$V_{ICMR}(max) = V_{DD} - |V_{SG_3}| + V_{th_1}$ — 1

$= V_{DD} - |V_{thp}| - V_{dsat_3} + V_{thn}$

$$= V_{dd} - |V_{thp}| + V_{thn} - \sqrt{\frac{2(I_5/2)}{\mu_p C_{ox} (W/L)_3}}$$

$$= 1.8 - 0.4 + 0.48 - \sqrt{\frac{2 \times 50}{62 \times 32}}$$

$$\boxed{V_{ICMR}(max) = 1.6555\,V}$$

1

h.)

V_{ICMR} (MIN)

$V_{ICMR}(min) = V_{GS_1} + V_{dsat5}$ — 1

$= V_{thn} + V_{dsat_1} + V_{dsat5}$

$$= V_{thn} + \sqrt{\frac{2(I_5/2)}{\mu_n C_{ox} (W/L)_1}} + \sqrt{\frac{2 \cdot I_5}{\mu_n C_{ox} (W/L)_5}}$$

$$= 0.48 + \underbrace{\sqrt{\frac{2 \times 50}{263 \times 16}}}_{=0.159} + \underbrace{\sqrt{\frac{2 \times 100}{263 \times 15}}}_{=0.225}$$

$$\boxed{V_{ICMR}(min) = 0.8593\,V}$$

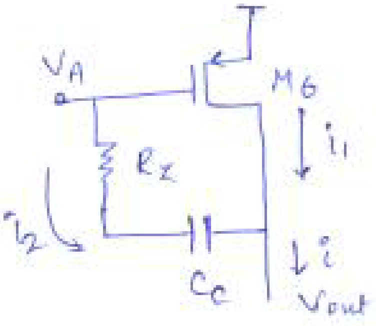
1

H 4

i) Location of 'zero'

Due to the feedforward path through R_z and C_c, between first and second stages, the OTA transfer function consists a zero

at f_z (zero frequency) $V_{out} = 0$



$$-g_{m6} V_A + \frac{V_A s C_c}{1 + s R_z C_c} = 0 \quad — 1$$

$$g_{m6} = \frac{s C_c}{1 + s R_z C_c}$$

$$s C_c (1 - g_{m6} R_z) = g_{m6}$$

$$s_z = \frac{g_{m6}}{(1 - g_{m6} R_z) C_c}$$

$$s_z = \frac{1}{\left(\frac{1}{g_{m6}} - R_z\right) C_c}$$

$$\boxed{f_{zero} = \frac{1}{2\pi C_c \left(\frac{1}{g_{m6}} - R_z\right)} \text{ Hz}} \quad 1$$

j) Rz value to achieve the best Phase margin.

to achieve the best phase margin the zero has to be placed at the non-dominant pole location.

$$\Rightarrow \quad P_2 = -\frac{g_{m6}}{C_L}$$

$$\Rightarrow \quad R_Z = \frac{1}{g_{m6}}\left(1 + \frac{C_L}{C_c}\right) \qquad 1$$

$$g_{m6} = \sqrt{2 K_p \cdot (W/L)_6 \cdot I_7} \qquad K_p = \mu_p C_{ox}$$

$$= \sqrt{2 \times 62 \times 500 \times 320}\ \mu S$$

$$g_{m6} = 4.4542 mS$$

$$R_Z = \frac{1}{4.4542 \times 10^{-3}}\left(1 + \frac{4pF}{1pF}\right)$$

$$\boxed{R_Z = 1.122536\ K\Omega} \qquad 1$$

k) Unity gain frequency:

$$f_{UGF} = \frac{g_{m1}}{2\pi C_c} \qquad 1$$

$$g_{m1} = \sqrt{2 \cdot \mu n C_{ox} \cdot (W/L)_1 \cdot \frac{I_{ss}}{2}} = 0.648 mS$$

1

$$= \frac{648.69 \times 10^{-6}}{2\pi \times 1 \times 10^{-12}} = \boxed{103.24 MHZ}$$

l) Slew rate (limited by input differential pair)

$$\frac{dV_{out}}{dt} = \boxed{\frac{I_{SS}}{C_c} = \frac{100\,\mu A}{1\,pF} = 100\,V/\mu s}$$

1 1

m) Input referred noise.

2

$$\overline{V^2_{in,eq}} = 2\left[\overline{V^2_{n_1}} + \overline{V^2_{n_3}}\left(\frac{g_{m_3}}{g_{m_1}}\right)^2\right]$$

1

$$= 2\left[4KT\gamma\frac{1}{g_{m_1}} + 4KT\gamma\frac{1}{g_{m_3}}\left(\frac{g_{m_3}}{g_{m_1}}\right)^2\right] ; \gamma = 2/3$$

Noise of the second stage can be neglected due to high gain

$$= \frac{16KT}{3}\cdot\frac{1}{g_{m_1}}\left[1 + \frac{g_{m_3}}{g_{m_1}}\right]$$

$K = 1.38 \times 10^{-23}\,J/K$

$T = 300^{\circ}K$

$$g_{m_1} = 648.69\,\mu S$$

$$g_{m_3} = \sqrt{2(\mu_p C_{ox})(W/L)_3\cdot\frac{I_{SS}}{2}}$$

$$g_{m_3} = \sqrt{62 \times 100 \times 32} = 445.42\,\mu S$$

$$\overline{V^2_{n,in}} = \frac{16 \times 1.38 \times 10^{-23} \times 300}{3 \times 0.6487 \times 10^{-3}}\left[1 + \frac{445.42}{648.69}\right]$$

$$\boxed{\overline{V^2_{n,in}} = 5.74088 \times 10^{-17}\,V^2/Hz}$$

1

$$\boxed{\overline{V^2_{n,in}} = 57.408\left(\frac{nV}{\sqrt{Hz}}\right)^2}$$

$$\boxed{\overline{V_{n,in}} = 7.57\,nV/\sqrt{Hz}}$$