EE 735

Assignment-4

Diffusion

Due on 19 September, 2019

1 Numerical solution of steady state continuity equation: For each of the cases listed below, provide analytical solutions and compare with numerical results, if possible. Assume D=30cm2/s, unless otherwise stated. Use

$$D\frac{d^2n}{dx^2} = \frac{n}{\tau}$$

- a. Consider diffusive transport of particles from point A to point B and the separation between these points being 100µm. The concentration of particles at A is n=10¹² cm⁻³, and at B is n=0cm⁻³. Assume τ =10⁻⁷ s. Find the particle profile from A to B. What is the particle flux from A to B?
- b. For the configuration in part (a), assume that the boundary condition at B is such that J = kn, where J is the particle flux (outgoing), $k = 10^3 \ cm/s$, and n is the particle density. Assume $\tau = 10^{-7}$ s Find the particle profile from A to B and the particle flux at B. Explore the implications of this change in boundary conditions at B.
- c. For the configuration in part (a), assume that a particle flux is introduced at x=30um at the rate of 10^{12} cm⁻² /s. Assume that the particle density at A and B are held constant at n=0 and $\tau = 10^{-7}$ s. Find the particle profile from A to B, and the flux at A and B.
- 2 Consider a region of length 10 μ m. Assume perfectly absorbing boundary conditions at x=0 and x=10, at time t = 0, assume that particles are injected at x=5 μ m such that the density is 10⁶ cm⁻³ (i.e., the injection is a delta function in both space and time). Using the formalism described, plot the evolution of particle density over the specified domain (use D = 10⁻⁴ cm² /s). Compare with analytical results. Explore the significance of the parameter \sqrt{Dt} .
- 3 Consider a region of length 100um. Assume that the region is devoid of any particles at time t=0. Also assume perfectly absorbing boundary condition at x=100um. Solve for the diffusion of particles from the side x=0 as a function of time under the assumption that n(x=0,t)=1000. Plot the space and temporal evolution of the particle density profile. (Note that this scenario is very similar to doping of a semiconductor to form a PN junction diode). Compare the numerical solution with the analytical solution.

4 Ion—implantation is a standard technique for doping in semiconductor materials, and accurate estimation of ion-implanted profile is necessary for process development. The main parameters associated with the profile are depth, straggle, and dose and peak concentration. The simplest function used to defined ion-implantation profile is a Gaussian distribution expressed in terms of depth and straggle.

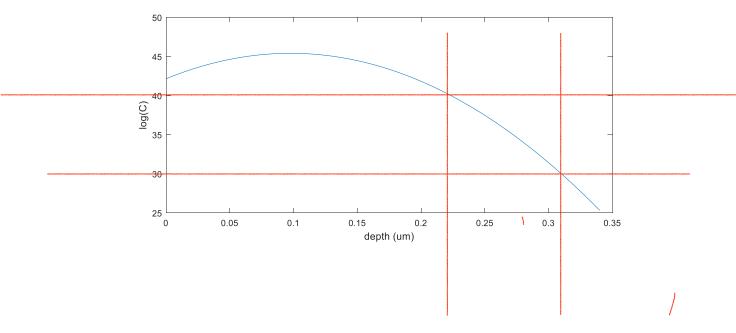
For formation of shallow P-N junctions we implant the substrate with less energy ions of some particular dose. Dose is responsible for the peak concentration and the ion energy is responsible for depth of penetration. Thermal budget is defined as the product of Diffusion constant (a function of temperature) and time to reach a particular junction depth and abruptness. The slope roll-off of concentration is defined as the length it takes for the concentration to fall of one decade (junction abruptness).

$$N(x) = N_m exp(-\frac{(x - R_p)^2}{4(\frac{(\Delta R_p)^2}{2})})$$

Where Nm is peak concentration determined by implantation dose.

Consider the profile given below of boron implant in silicon with energy of 30keV and dose of 10^{15} cm⁻². The depth of penetration is 0.1um and straggle is 0.0374um. Design the thermal budget for a slope roll-off of 40nm/decade at the junction depth and plot the spatial and temporal evolution of the profile. Take standard parameters for silicon (parameters taken should be realistic). Substrate doping = 10^{17} cm⁻³.

The junction formed should be shallow, steep and with highest peak.



Data of the figure

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0.1768	6.12501E+18
0.1802	5.07468E+18
0.1836	4.17137E+18
0.187	3.40186E+18
0.1904	2.75247E+18
0.1938	2.20952E+18
0.1972	1.7597E+18
0.2006	1.39043E+18
0.204	1.09E+18
0.2074	8.47761E+17
0.2108	6.54165E+17
0.2142	5.00806E+17
0.2176	3.80381E+17
0.221	2.8664E+17
0.2244	2.143E+17
0.2278	1.58956E+17
0.2312	1.16977E+17
0.2346	8.5406E+16
0.238	6.18652E+16
0.2414	4.44603E+16
0.2448	3.17005E+16
0.2482	2.24248E+16
0.2516	1.57383E+16
0.255	1.09586E+16
0.2584	7.57043E+15
0.2618	5.18865E+15
0.2652	3.52822E+15
0.2686	2.38027E+15
0.272	1.59317E+15
0.2754	1.05796E+15
0.2788	6.97015E+14

0.2822	4.556E+14
0.2856	2.95456E+14
0.289	1.90095E+14
0.2924	1.21343E+14
0.2958	7.68474E+13
0.2992	4.82848E+13
0.3026	3.00995E+13
0.306	1.86156E+13
0.3094	1.14225E+13
0.3128	6.95369E+12
0.3162	4.19988E+12
0.3196	2.51667E+12
0.323	1.49618E+12
0.3264	8.82487E+11
0.3298	5.16418E+11
0.3332	2.99821E+11
0.3366	1.727E+11
0.34	1E+11