MOS small-signal MODEL in Saturation

water pipe analogy

☆ Transconductance $g_m = \left.\dfrac{\partial I_D}{\partial V_{GS}}\right|_{V_{DS}\text{ constant}}$

"sat".

$$I_D = \frac{\mu_n C_{ox}}{2}\frac{W}{L}(V_{GS}-V_{TH})^2(1+\lambda V_{DS})$$

Vdsat

$$g_m = \frac{\partial I_D}{\partial V_{GS}} \cong \mu_n C_{ox}\frac{W}{L}(V_{GS}-V_{TH})$$

$$g_m = \frac{2 I_D}{(V_{GS}-V_{TH})} \quad \boxed{\text{EXACT}}$$

$$g_m \cong \sqrt{2\mu_n C_{ox}\frac{W}{L}\cdot I_D}$$

IMP. to commit to memory

g_m transconductan

Insight Bipolar transistor $I_C = I_0 \cdot \exp\left(\dfrac{V_{BE}}{V_T}\right)$

Bipolar $g_m = \dfrac{\partial I_C}{\partial V_{BE}} = \dfrac{I_C}{V_T}$ ← 26mV $\quad \boxed{V_T = \dfrac{kT}{q}}$

Compare with MOS $\quad g_m = \dfrac{I_D}{(V_{GS}-V_{TH})/2} = \dfrac{I_D}{100\,mV}$

Vdsat → $\underbrace{(V_{GS}-V_{TH})}_{\sim 200mV}$

Bipolar g_m is 4x higher than MOS g_m for same current.

☆ Backgate transconductance

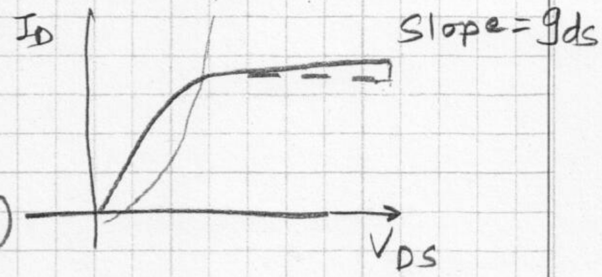
$$g_{mbs} = \frac{\partial I_D}{\partial V_{BS}} = \frac{\partial I_D}{\partial V_{TH}}\cdot\frac{\partial V_{TH}}{\partial V_{BS}} = \frac{\gamma}{2\sqrt{V_{SB}+2\phi_F}}\cdot g_m$$

$\frac{\partial I_D}{\partial V_{TH}} \downarrow -g_m$

$$= \eta\, g_m \quad (0.25)$$

$$V_{TH} = V_{TH_0} + \gamma\left(\sqrt{V_{SB}+2\phi_F} - \sqrt{2\phi_F}\right)$$

Output Conductance g_{ds}



$$I_D = \frac{\mu_n C_{ox}}{2}\frac{W}{L}(V_{GS}-V_{TH})^2(1+\lambda V_{DS})$$

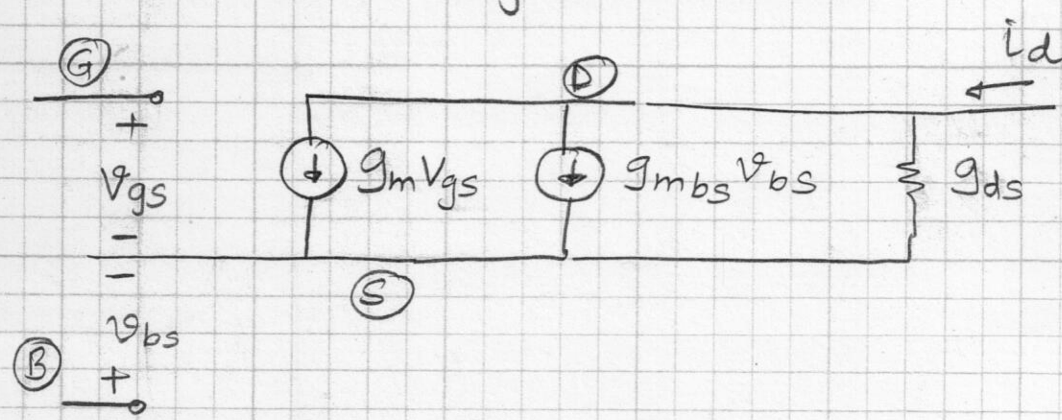
$$g_{ds} = \frac{\partial I_D}{\partial V_{DS}} = \frac{\mu_n C_{ox}}{2}\frac{W}{L}(V_{GS}-V_{TH})^2\cdot\lambda = \frac{\lambda I_D}{(1+\lambda V_{DS})}$$

$$g_{ds} \simeq \lambda I_D \quad (\text{Since } \lambda V_{DS} << 1)$$

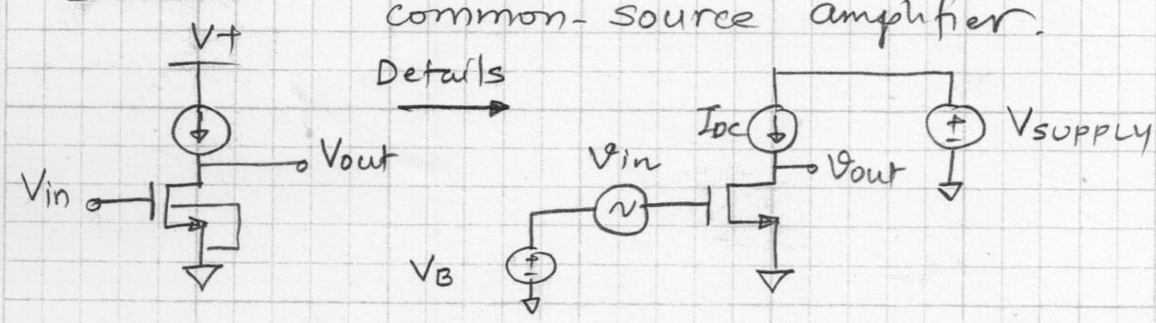
$$r_{ds} = \frac{1}{g_{ds}} = \frac{1}{\lambda I_D}$$

small signal parameter

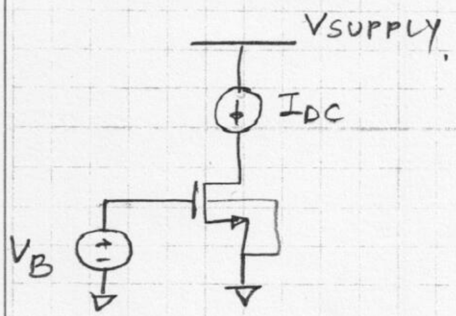
(low frequency)
MOS DC Small Signal Model.



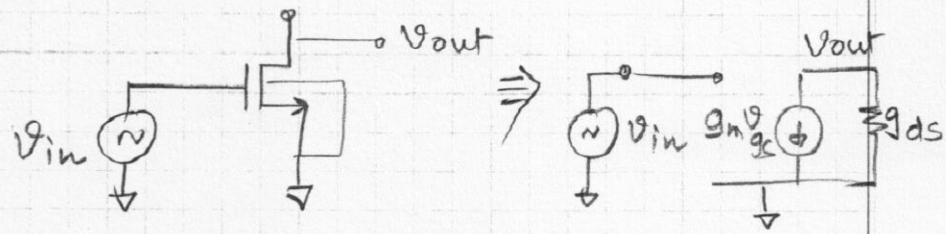
Example - small-signal L.F voltage gain for common-source amplifier.



DC circuit



AC circuit



$$v_{out} = -g_m v_{in} \cdot \frac{1}{g_{ds}}$$

open ckt gain

Self gain

$$\text{gain} \quad a_v = \frac{v_{out}}{v_{in}} = -\frac{g_m}{g_{ds}}$$

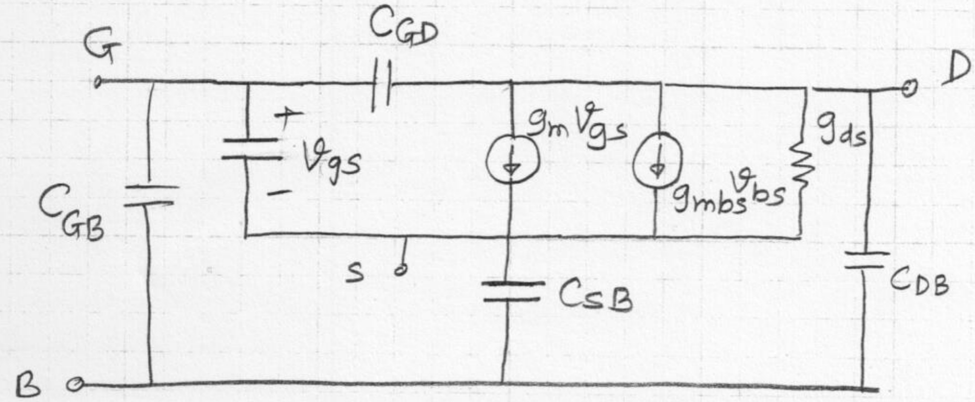
$$a_v = -\frac{g_m}{g_{ds}} = \frac{-\sqrt{2\mu_n C_{ox}(W/L) I_D}}{\lambda I_D} \propto \frac{1}{\sqrt{I_D}}$$

$$= -\frac{g_m}{g_{ds}} = \frac{2 I_D/(V_{GS}-V_{TH})}{\lambda I_D} = \frac{2}{\lambda(V_{GS}-V_T)}$$

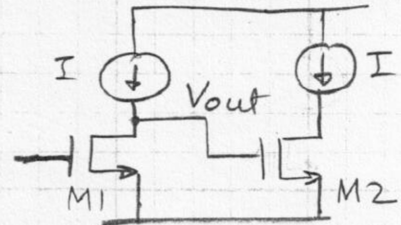
$\boxed{\text{insight}}$ Large channel length → small λ → high gain

small $(V_{GS}-V_{TH})$ → higher gain

High frequency small-signal Model



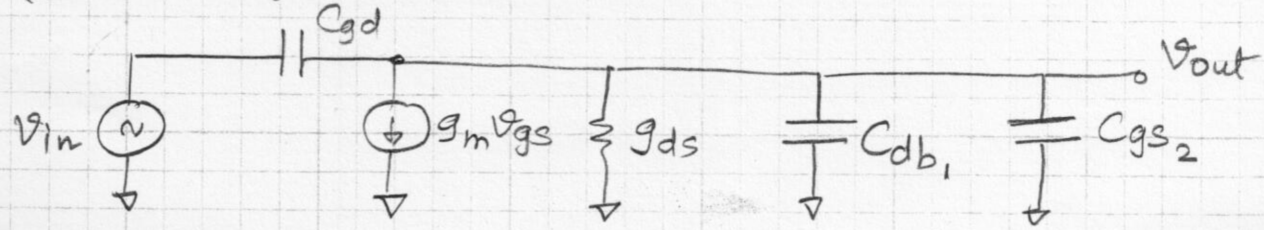
Speed of CMOS



C_{gs_1}, C_{gb_1} – ignore - source driven

g_{mbs}, C_{sb} - ignore (No backgate effect)

Small Signal model.



KCL @ Vout

$$sC_{gd}(V_{in} - v_{out}) = g_m v_{in} + g_{ds} v_{out} + s(C_{gs} + C_{db}) v_{out}$$

$$-(g_m - sC_{gd})\, v_{in} = v_{out}\left[g_{ds} + s(C_{gs} + C_{db} + C_{gd}) \right]$$

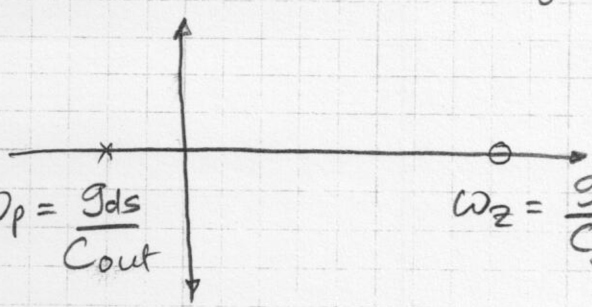
$$A_v(s) = \frac{v_{out}}{v_{in}} = -\boxed{\frac{g_m}{g_{ds}}} \frac{\left(1 - s\frac{C_{gd}}{g_m}\right)}{\left(1 + s\frac{(C_{gs} + C_{gd} + C_{db})}{g_{ds}}\right)}$$

← RHP zero

DC gain

LHP pole

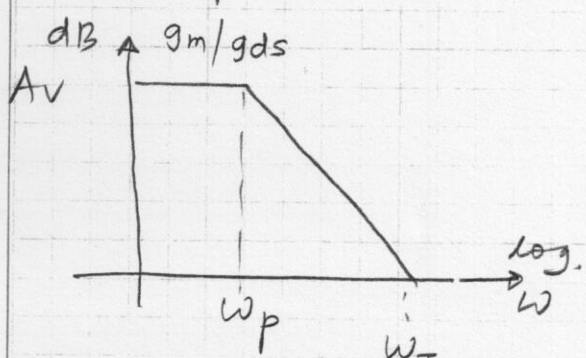
$C_{out} = C_{gs} + C_{gd} + C_{db}$



$\omega_p = \frac{g_{ds}}{C_{out}}$ $\omega_z = \frac{g_m}{C_{gd}}$

Assume $\omega_z \gg \omega_p$

& $C_{gs} \gg C_{db} + C_{gd}$.



$$A_v(s) = -\frac{g_m}{g_{ds}} \frac{1}{\left(1 + s\frac{C_{gs}}{g_{ds}}\right)}$$

Transition frequency ω_T.

ω_T = gain; BW product.

$$\omega_T = \frac{g_m}{g_{ds}} \cdot \frac{g_{ds}}{C_{gs}} = \frac{g_m}{C_{gs}} \ (\text{rad/s.})$$

$$\omega_T = \frac{g_m}{C_{gs}} = \frac{\mu_n C_{ox}(W/L)\cdot \overbrace{(V_{GS}-V_{TH})}^{V_{dsat}}}{\frac{2}{3}\, C_{ox}\cdot(W\cdot L)}$$

$$= \frac{3}{2}\cdot\frac{\mu_n (V_{GS}-V_{TH})}{L^2}$$

Example: $\mu_n = 350\ cm^2/V/s$

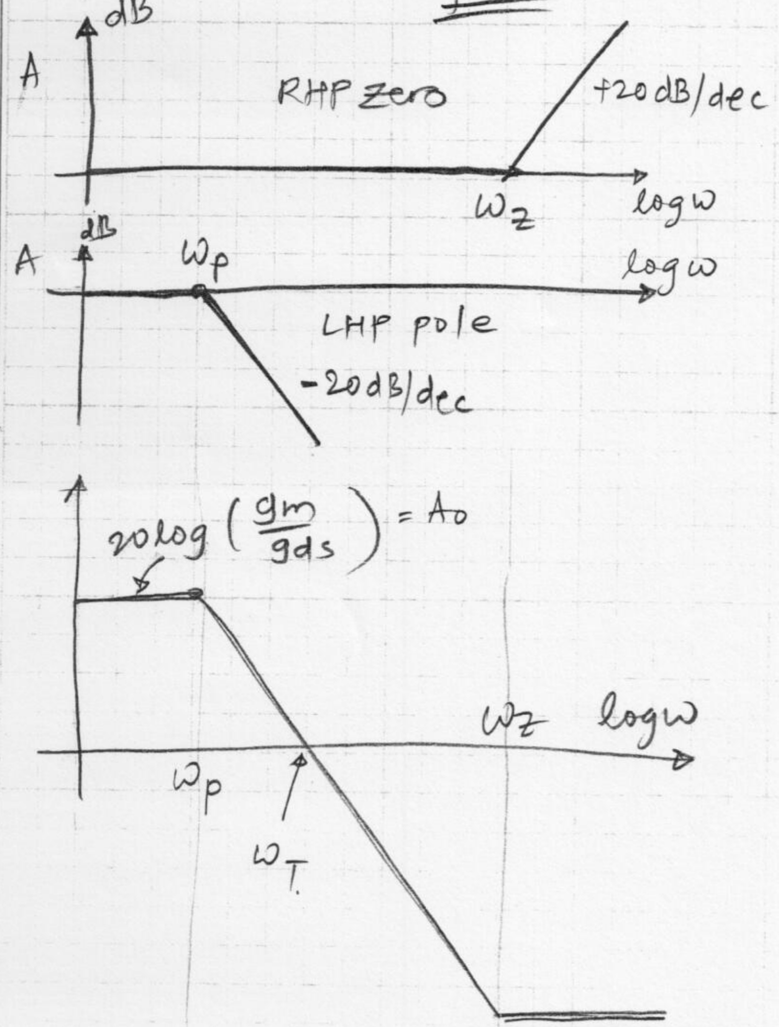
$V_{GS}-V_{TH} = 200\,mV$

$L = 100\,nm.$

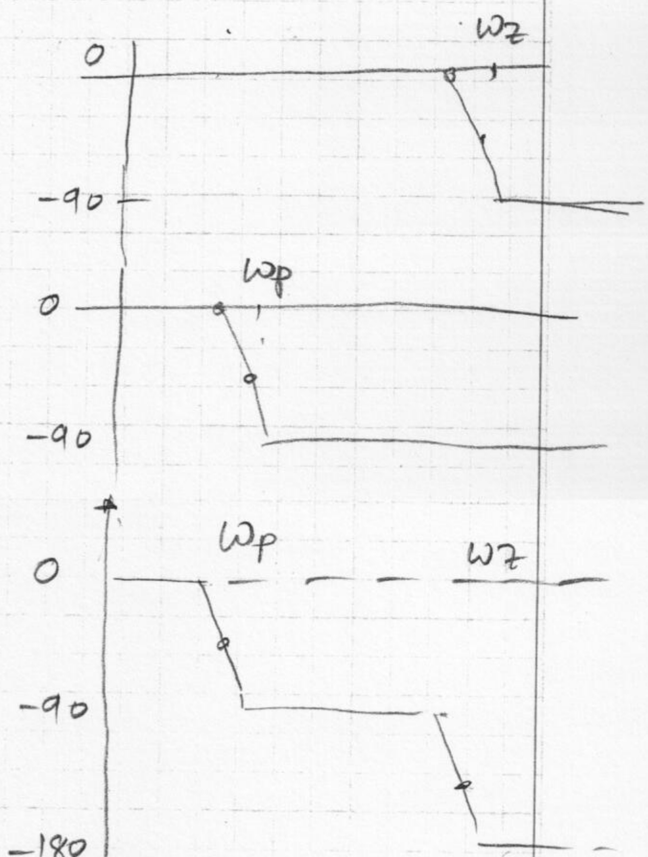
$$f_T = \frac{3}{2}\cdot\frac{350\times 0.2}{2\times\pi\times(100\times 10^{-7})^2} = \underline{167.1\ GHZ}$$

Bode Plot

Gain



Phase



Example values for all transistor parameters

Example process $L_{min} = 0.5\,\mu m$

	NMOS	PMOS
VTO (V) Threshold voltage	+0.5	−0.6
GAMMA (body Effect coef $V^{-1/2}$)	0.45	0.4
TOX (A)	90	90
UO (mobility $cm^2/V/s$)	350	100
LAMBDA (λ − ch. length modulation Coef V^{-1})	0.1	0.2
PHI ($2\phi_F$) V	0.9	0.8

$$C_{ox} = \frac{\epsilon_{ox}\,\epsilon_0}{t_{ox}} = \frac{3.97 \times 8.85 \times 10^{-12}\,(F/m)}{(90 \times 10^{-10})\,m} = 3.9 \times 10^{-3}\,F/m^2$$

$$= 3.9\,fF/\mu m^2$$

$$\lambda_n = \frac{0.1\,V^{-1} \times 0.5\,\mu m}{L_n} = \frac{0.05}{L_n}\,V^{-1}$$

L_n in μm

$$\lambda_p = \frac{0.2 \times 0.5}{L_p} = \frac{0.1}{L_p}\,V^{-1}$$

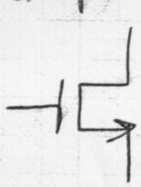
L_p in μm

$$\mu_n C_{ox} = 350\,cm^2/V/s \times 3.9\,fF/\mu m^2$$

$$= 350 \times 10^{-4} \times 3.9 \frac{\times 10^{-15}}{10^{-12}}\,A/V^2 = 136.5\,\mu A/V^2$$

$$\mu_p C_{ox} = 39\,\mu A/V^2$$

Example



$\downarrow$ 100 μA

$W = 10\,\mu m$

$L = 0.5\,\mu m$

$$g_m = \sqrt{2 \cdot \mu_n C_{ox} \frac{W}{L} \cdot I_D}$$

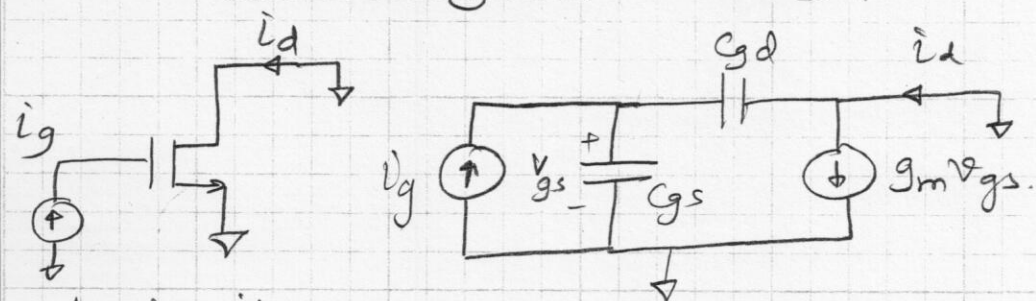
$$= \sqrt{2 \times 136.5 \underset{\times 10^{-6}}{} \times \frac{10}{0.5} \times 100 \underset{\times 10^{-6}}{}}$$

$$= 738\,\frac{\mu A}{V} = 738\,\mu S$$

$$g_{ds} = \lambda_n I_{DS} = 0.1 \times 100 \times 10^{-6} = 10\,\mu S$$

High Frequency Figure of Merit

f_T - Transition Frequency - frequency @ which current gain is unity.



AC circuit

$$v_g = s\,(C_{gs} + C_{gd})\, v_{gs} \Rightarrow v_{gs} = \frac{i_g}{s(C_{gs}+C_{gd})}$$

$$i_d = g_m v_{gs} = \frac{g_m\, i_g}{s(C_{gs}+C_{gd})}$$ { ignore C_{gd} contrib^n to o/p current

For f_T (ω_T) $$\left|\frac{i_d}{i_g}\right| = \frac{g_m}{\omega_T (C_{gd}+C_{gs})} = 1$$

$$\omega_T = \frac{g_m}{(C_{gs}+C_{gd})} \simeq \frac{g_m}{C_{gs}}$$ } $C_{gs} \gg C_{gd}$ for long-ch devices

$$f_T = \frac{\mu_n C_{ox} \frac{W}{L}(V_{GS}-V_{TH})}{2\pi \cdot \frac{2}{3} C_{ox}\, W \cdot L} = \frac{3}{4}\,\frac{\mu_n (V_{GS}-V_{TH})}{\pi L^2}$$

$f_T \propto (V_{GS}-V_{TH})$ V_{dsat}

$f_T \propto \frac{1}{L^2}$ f_T improves with process.

$L \downarrow$ $f_T \uparrow\uparrow$

$$f_T = \frac{1}{2\pi}\,\frac{g_m}{C_{gs}} = \frac{\sqrt{2\mu_n C_{ox} \frac{W}{L} I_D}}{2\pi \frac{2}{3} C_{ox} \cdot W \cdot L} = \frac{1}{2\pi}\,\frac{3}{\sqrt{2}}\sqrt{\frac{\mu_n}{C_{ox} L^2}\underbrace{\left(\frac{I_D}{W}\right)}_{\text{current density}}}$$

f_T improves with current density

EE618 (Zele)

Body Effect and Sub-threshold Conduction

Body Effect

Consider the simplified NMOS shown in Fig. 1 part (A). A voltage V_{GS} is applied between gate and source whereas a voltage V_{SB} is applied between the source and bulk of the NMOS. For this explanation, we consider that the bulk is a p-well so that we can separately bias it (NMOS in a chip may lie in substrate and substrate of the whole chip will be tied to ground). We consider the following two cases -

CASE 1: V_{GS} = V_{TH} and V_{SB} = 0 V

As shown in Fig. 1 part (B), Due to positive V_{GS} the holes in the channel region move out leaving negatively charged ions behind. The electrons that form the inversion layer comes from the n+(very high concentration) source region. The current in the device will be due to drift of these electrons when a V_{DS} is applied.

CASE 2: V_{GS} = V_{TH} and V_{SB} >0 V
In this case we are increasing V_{SB} (can be achieved by decreasing the bulk voltage keeping source voltage constant). Due to a net positive voltage on the source as compared to the bulk, some of the electrons present in the channel will be attracted back in the source as shown in of Fig. 1 part (C). Hence to achieve the same charge density as in CASE1 we need a more positive charge on the gate. Therefore we can say that increasing V_{SB} increases threshold voltage.

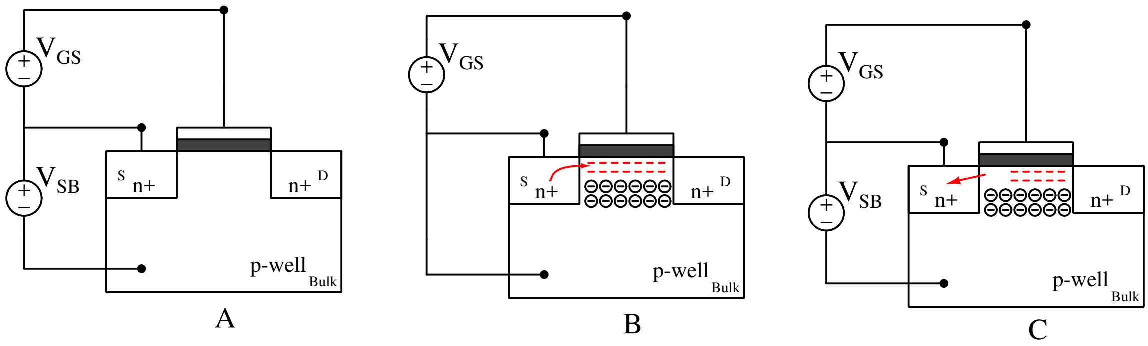


Figure 1: Body effect in MSOFETS

Effect of V_{DS} in Subthreshold Region

- The region where V_{GS} of the device is less than or close to V_{TH} is known as sub-threshold region.
- Due to diffusion of minority charge carrier electrons, some leakage current may flow in the device even when V_{GS} is less than V_{TH}.
- For a constant V_{DS}, the current equation in subthereshold region is given as

$$\mathrm{I}_{subth} = I_O \times\ exp(\frac{V_{GS}}{\zeta V_T}) \times (1 - exp(\frac{-V_{DS}}{V_T}))$$

  where ζ is a non-ideality factor greater than 1 and V_T = KT/q
- For $V_{DS} = 0$, the current $I_{subth} = 0$.
- For V_{DS} >4V_{TH} or 5V_{TH} the equation reduces to

$$\mathrm{I}_{subth} = I_O \times\ exp(\frac{V_{GS}}{\zeta V_T})$$