Partitioning and Floorplanning

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E0-285: CAD of VLSI Systems

Partitioning

- Decomposition of a complex system into smaller subsystems
  - Done hierarchically
  - Partitioning done until each subsystem has manageable size
  - Each subsystem can be designed independently
- Interconnections between partitions minimized
  - Less hassle interfacing the subsystems
  - Communication between subsystems usually costly

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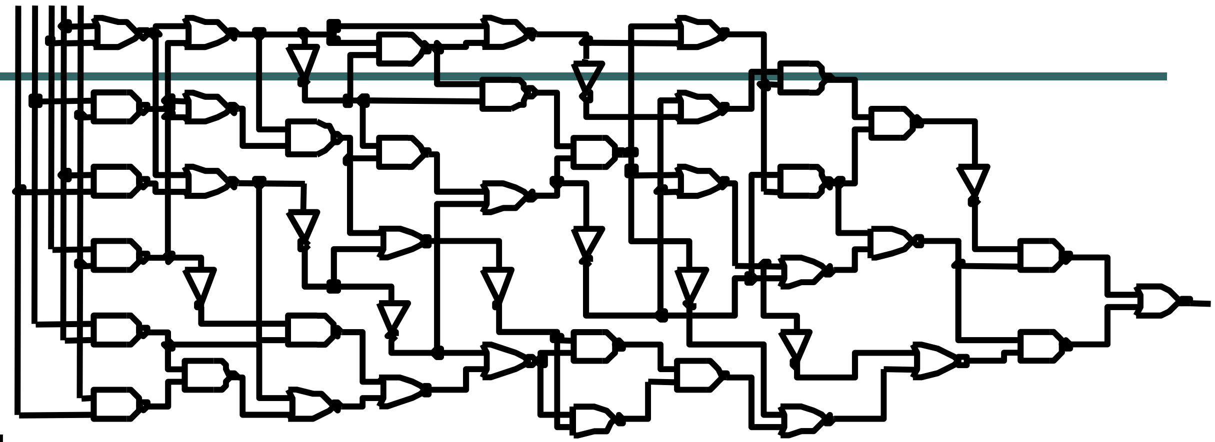
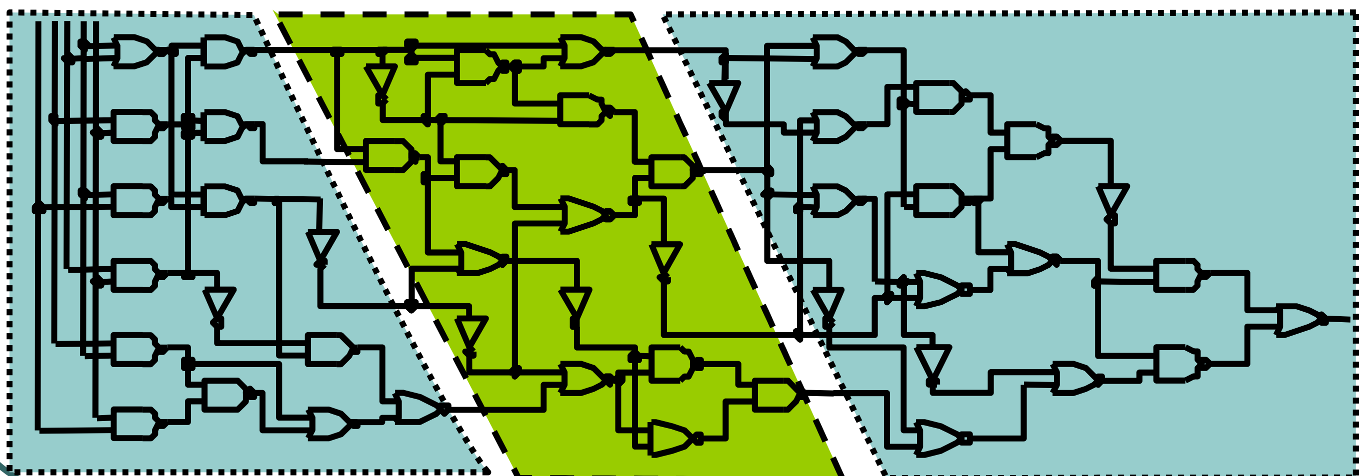
Example: Partitioning of a Circuit

Input size: 48



Cut 1=4 Cut 2=4

Size 1=15 Size 2=16 Size 3=17



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Partitioning: Formal Definition

- Input:
  - Graph or hypergraph
  - Usually with vertex weights
  - Usually weighted edges
- Constraints
  - Number of partitions (K-way partitioning)
  - Maximum capacity of each partition
    OR
    maximum allowable difference between partitions
- Objective
  - Assign nodes to partitions subject to constraints
    s.t. the cutsize is minimized
- Tractability
  - Is NP-complete ☹

Kernighan-Lin (KL) Algorithm

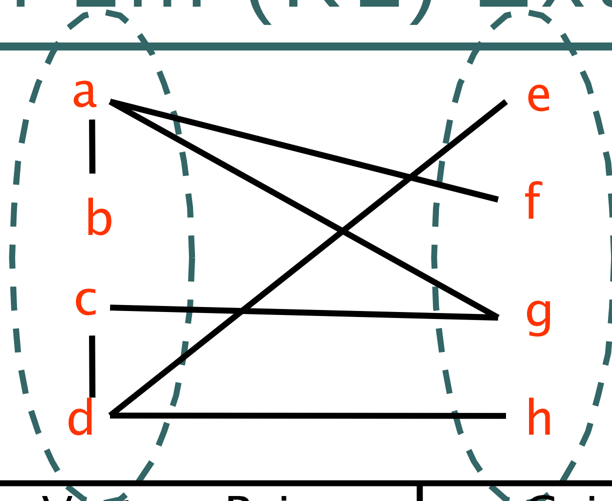
- On non-weighted graphs
- An iterative improvement technique
- A two-way (bisection) partitioning algorithm
- The partitions must be balanced (of equal size)
- Iterate as long as the cutsize improves:
  - Find a pair of vertices that result in the largest decrease in cutsize if exchanged
  - Exchange the two vertices (potential move)
  - “Lock” the vertices
  - If no improvement possible, and
    still some vertices unlocked, then
    exchange vertices that result in smallest increase in cutsize

W. Kernighan and S. Lin, Bell System Technical Journal, 1970.

Kernighan-Lin (KL) Algorithm

- Initialize
  - Bipartition G into V_1 and V_2, s.t., $|V_1| = |V_2| \pm 1$
  - $n = |V|$
- Repeat
  - for i=1 to n/2
    - Find a pair of unlocked vertices $v_{ai} \in V_1$ and $v_{bi} \in V_2$ whose exchange makes the largest decrease or smallest increase in cut-cost
    - Mark v_{ai} and v_{bi} as locked
    - Store the gain g_i.
  - Find k, s.t. $\sum_{i=1..k} g_i = Gain_k$ is maximized
  - If $Gain_k > 0$ then
    move $v_{a1},\ldots,v_{ak}$ from V_1 to V_2 and
    $v_{b1},\ldots,v_{bk}$ from V_2 to V_1.
- Until $Gain_k \leq 0$

Kernighan-Lin (KL) Example

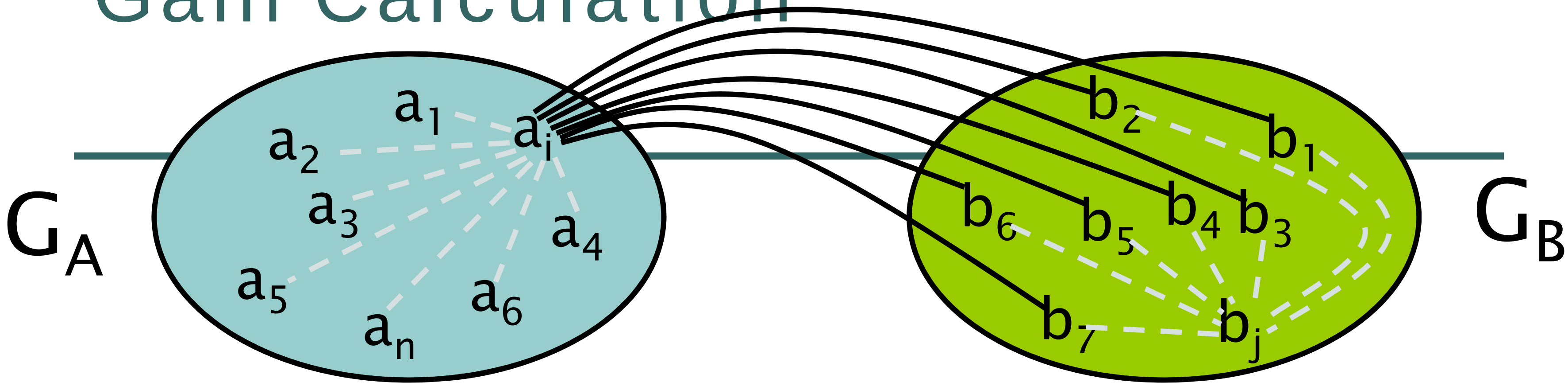


Step No.	Vertex Pair	Gain	Cut-cost
0	--	0	5
1	{ d, g }	3	2
2	{ c, f }	1	1
3	{ b, h }	-2	3
4	{ a, e }	-2	5

Kernighan-Lin (KL) : Analysis

- Time complexity?
  - Inner (for) loop
    - Iterates n/2 times
    - Iteration 1: (n/2) x (n/2)
    - Iteration i: $(n/2 - i + 1)^2$.
  - Passes? Usually independent of n
  - $O(n^3)$
- Drawbacks?
  - Local optimum
  - Balanced partitions only
  - No weight for the vertices
  - High time complexity
  - Only on edges, not hyper-edges

Gain Calculation



$$I_{a_i} = \sum_{x \in A} C_{a_i x}, \qquad E_{a_i} = \sum_{y \in B} C_{a_i y}$$

Likewise,

$$D_{a_i} = E_{a_i} - I_{a_i}$$

$$D_{b_j} = E_{b_j} - I_{b_j} = \sum_{x \in A} C_{b_j x} - \sum_{y \in B} C_{b_j y}$$

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Gain Calculation (cont.)

- Lemma: Consider any $a_i \in A$, $b_j \in B$.
  If a_i, b_j are interchanged, the gain is

$$\boxed{g = D_{a_i} + D_{b_j} - 2C_{a_i b_j}}$$

- Proof:

  Total cost before interchange (T) between A and B

$$T = E_{a_i} + E_{b_j} - C_{a_i b_j} + (\text{cost for all others})$$

  Total cost after interchange (T') between A and B

$$T' = I_{a_i} + I_{b_j} + C_{a_i b_j} + (\text{cost for all others})$$

  Therefore

$$g = T - T' = \overbrace{E_{a_i} - I_{a_i}}^{D_{a_i}} + \overbrace{E_{b_j} - I_{b_j}}^{D_{b_j}} - 2C_{a_i b_j}$$

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Gain Calculation (cont.)

- Lemma:
  - Let D_x', D_y' be the new D values for elements of $A - \{a_i\}$ and $B - \{b_j\}$. Then after interchanging a_i & b_j,

$$\boxed{\begin{aligned} D_x' &= D_x + 2C_{xa_i} - 2C_{xb_j} \ , \ x \in A - \{a_i\} \\ D_y' &= D_y + 2C_{yb_j} - 2C_{ya_i} \ , \ y \in B - \{b_j\} \end{aligned}}$$

- Proof:
  - The edge x-a_i changed from internal in D_x to external in D_x'
  - The edge y-bj changed from internal in D_x to external in D_x'
  - The x-b_j edge changed from external to internal
  - The y-a_i edge changed from external to interna

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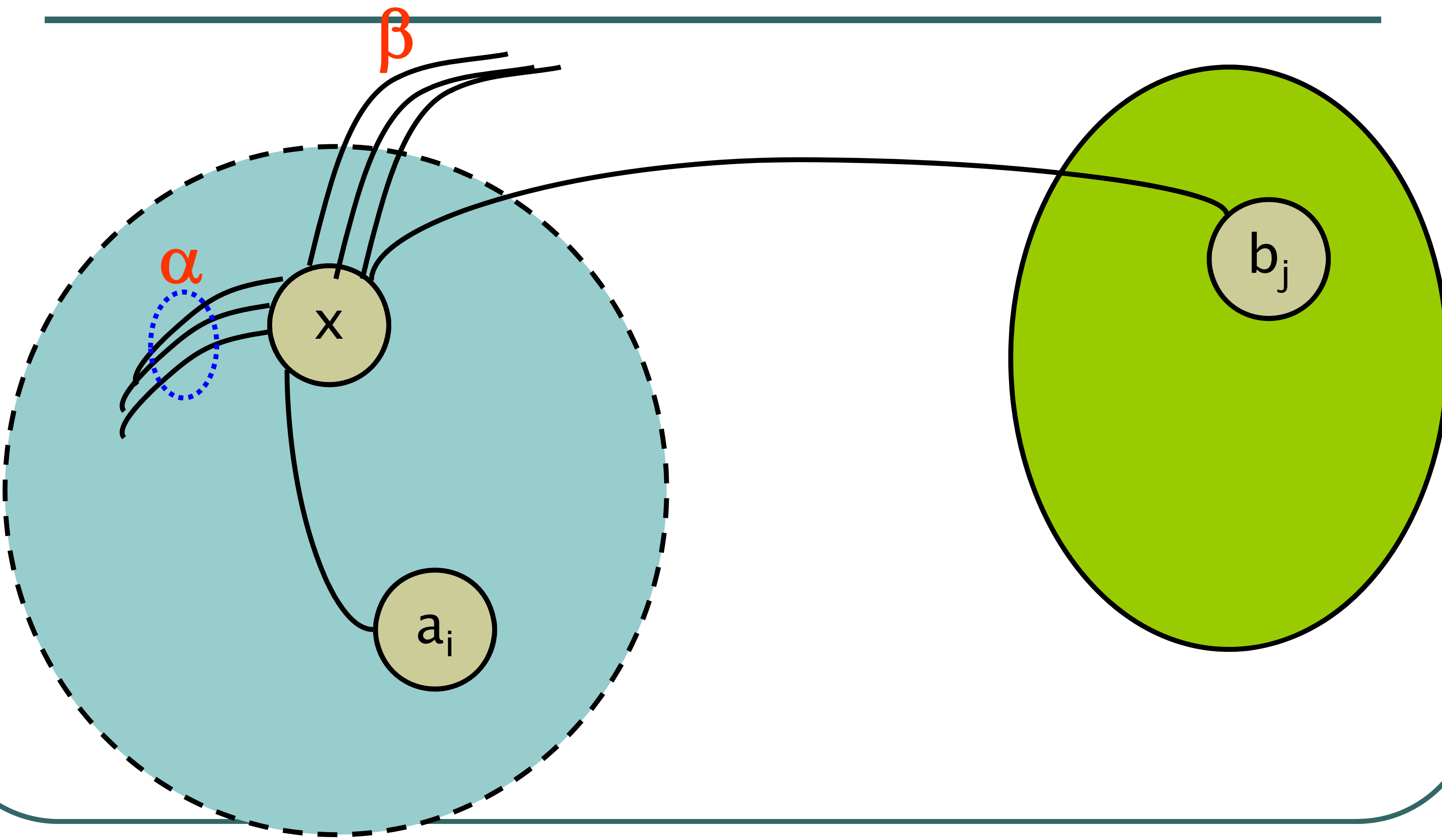
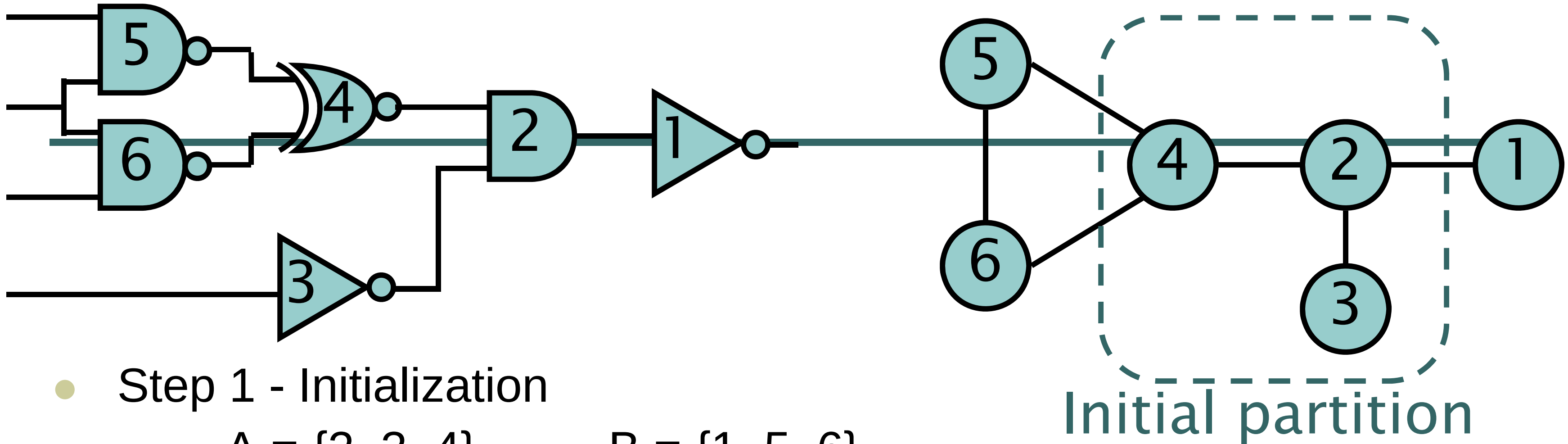
Clarification of the Lemma



Example: KL



- Step 1 - Initialization

  A = {2, 3, 4}, B = {1, 5, 6}

  A' = A = {2, 3, 4}, B' = B = {1, 5, 6}

- Step 2 - Compute D values

  $D_1 = E_1 - I_1 = 1-0 = +1$

  $D_2 = E_2 - I_2 = 1-2 = -1$

  $D_3 = E_3 - I_3 = 0-1 = -1$

  $D_4 = E_4 - I_4 = 2-1 = +1$

  $D_5 = E_5 - I_5 = 1-1 = +0$

  $D_6 = E_6 - I_6 = 1-1 = +0$

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Example: KL (cont.)

- Step 3 - compute gains

  $g_{21} = D_2 + D_1 - 2C_{21} = (-1) + (+1) - 2(1) = -2$
  $g_{25} = D_2 + D_5 - 2C_{25} = (-1) + (+0) - 2(0) = -1$
  $g_{26} = D_2 + D_6 - 2C_{26} = (-1) + (+0) - 2(0) = -1$

  $g_{31} = D_3 + D_1 - 2C_{31} = (-1) + (+1) - 2(0) = 0$
  $g_{35} = D_3 + D_5 - 2C_{35} = (-1) + (0) - 2(0) = -1$
  $g_{36} = D_3 + D_6 - 2C_{36} = (-1) + (0) - 2(0) = -1$

  $g_{41} = D_4 + D_1 - 2C_{41} = (+1) + (+1) - 2(0) = +2$
  $g_{45} = D_4 + D_5 - 2C_{45} = (+1) + (+0) - 2(+1) = -1$
  $g_{46} = D_4 + D_6 - 2C_{46} = (+1) + (+0) - 2(+1) = -1$

- The largest g value is $g_{41} = +2$

  $\Rightarrow$ interchange 4 and 1 $\boxed{(a_1, b_1) = (4, 1)}$
  A' = A' - {4} = {2, 3}
  B' = B' - {1} = {5, 6} both not empty

Example: KL (cont.)

- Step 4 - update D values of node connected to vertices (4, 1)

  $D_2' = D_2 + 2C_{24} - 2C_{21} = (-1) + 2(+1) - 2(+1) = -1$

  $D_5' = D_5 + 2C_{51} - 2C_{54} = +0 + 2(0) - 2(+1) = -2$

  $D_6' = D_6 + 2C_{61} - 2C_{64} = +0 + 2(0) - 2(+1) = -2$

- Assign $D_i = D_i'$, repeat step 3 :

  $g25 = D_2 + D_5 - 2C_{25} = -1 - 2 - 2(0) = -3$

  $g26 = D_2 + D_6 - 2C_{26} = -1 - 2 - 2(0) = -3$

  $g35 = D_3 + D_5 - 2C_{35} = -1 - 2 - 2(0) = -3$

  $g36 = D_3 + D_6 - 2C_{36} = -1 - 2 - 2(0) = -3$

- All values are equal;
  arbitrarily choose $g_{36} = -3 \Rightarrow$ $\boxed{(a2, b2) = (3, 6)}$

  $A' = A' - \{3\} = \{2\}$, $B' = B' - \{6\} = \{5\}$

  New D values are:

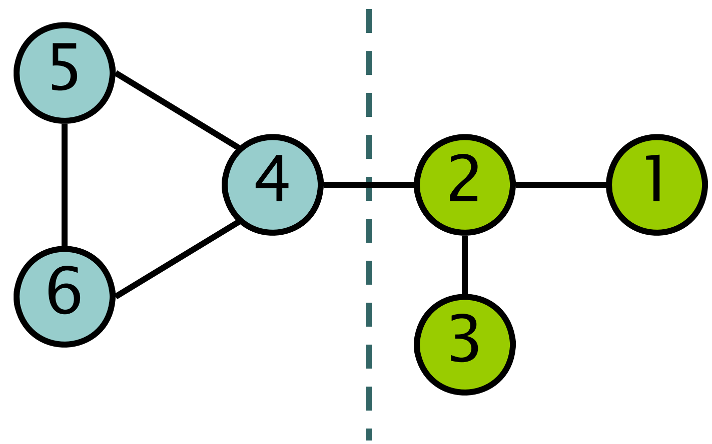
  $D_2' = D_2 + 2C_{23} - 2C_{26} = -1 + 2(1) - 2(0) = +1$

  $D_5' = D_5 + 2C_{56} - 2C_{53} = -2 + 2(1) - 2(0) = +0$

- New gain with $D_2 \leftarrow D_2'$, $D_5 \leftarrow D_5'$

  $g_{25} = D_2 + D_5 - 2C_{52} = +1 + 0 - 2(0) = +1 \Rightarrow$ $\boxed{(a3, b3) = (2, 5)}$

Example: KL (cont.)



- Step 5 - Determine the # of moves to take

  $g_1 = +2$

  $g_1 + g_2 = +2 - 3 = -1$

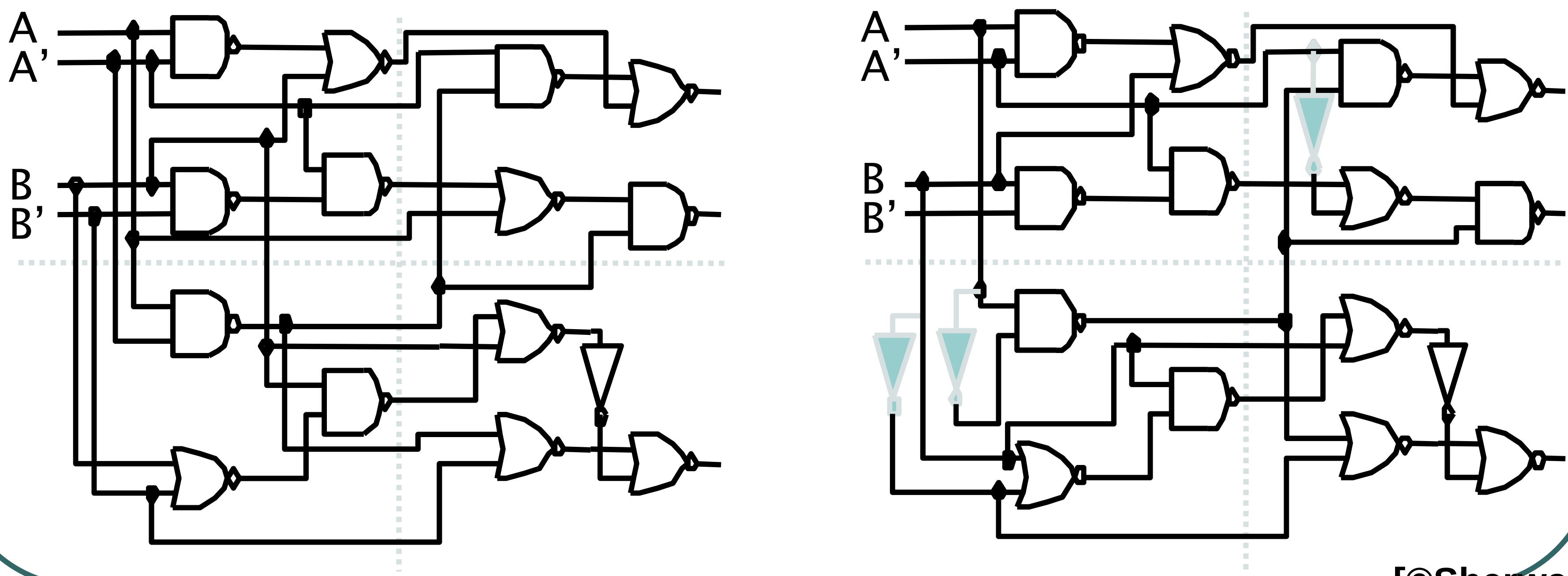
  $g_1 + g_2 + g_3 = +2 - 3 + 1 = 0$
- The value of k for max G is 1

  $X = \{a_1\} = \{4\}$, $Y = \{b_1\} = \{1\}$
- Move X to B, Y to A $\Rightarrow$ A = {1, 2, 3}, B = {4, 5, 6}
- Repeat the whole process:

  • • • • •
- The final solution is A = {1, 2, 3}, B = {4, 5, 6}

Subgraph Replication to Reduce Cutsize

- Vertices are replicated to improve cutsize
- Good results if limited number of components replicated

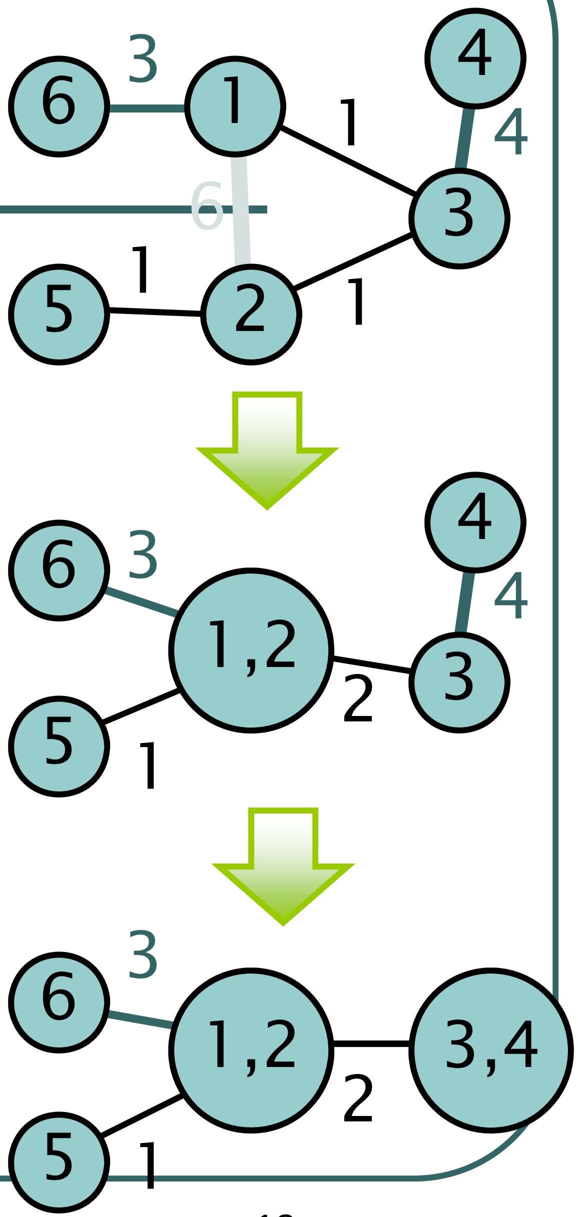


C. Kring and A. R. Newta, ICCAD, 1991.

[©Sherwani]

Clustering

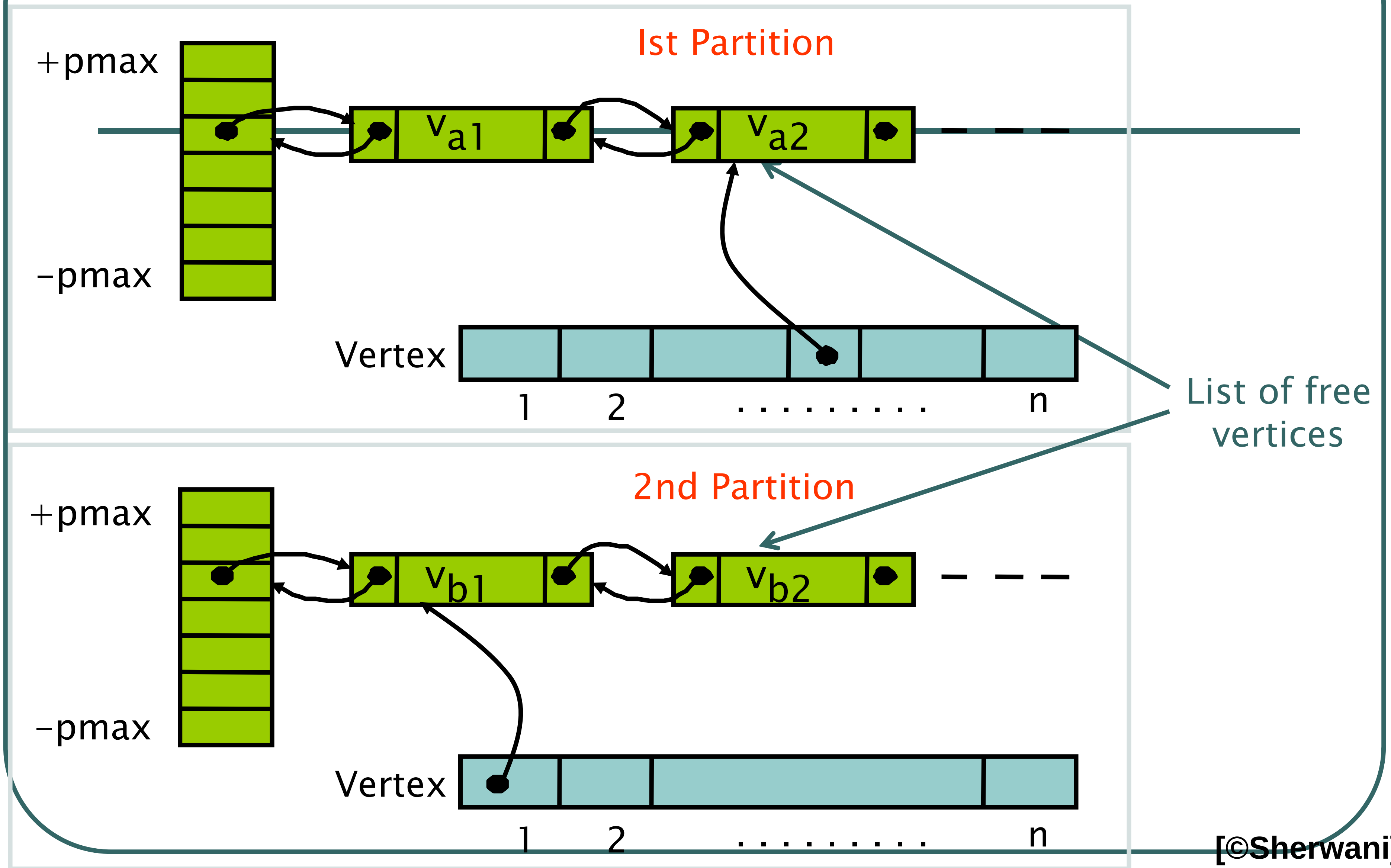
- Clustering
  - Bottom-up process
  - Merge heavily connected components into clusters
  - Each cluster will be a new "node"
  - "Hide" internal connections (i.e., connecting nodes within a cluster)
  - "Merge" two edges incident to an external vertex, connecting it to two nodes in a cluster
- Can be a preprocessing step before partitioning
  - Each cluster treated as a single node



Fiduccia-Mattheyses (FM) Algorithm

- Modified version of KL
- A single vertex is moved across the cut in a single move
  - ➔ Unbalanced partitions
- Vertices are weighted
- Concept of cutsize extended to hypergraphs
- Special data structure to improve time complexity to $O(n^2)$
  - (Main feature)
- Can be extended to multi-way partitioning

The FM Algorithm: Data Structure



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The FM Algorithm: Data Structure

- Pmax
  - Maximum gain
  - $p_{max} = d_{max} \cdot w_{max}$, where
    d_{max} = max degree of a vertex (# edges incident to it)
    w_{max} is the maximum edge weight
  - What does it mean intuitively?
- -Pmax .. Pmax array
  - Index i is a pointer to the list of unlocked vertices with gain i.
- Limit on size of partition
  - A maximum defined for the sum of vertex weights in a partition (alternatively, the maximum ratio of partition sizes might be defined)

The FM Algorithm

- Initialize
  - Start with a balance partition A, B of G
    (can be done by sorting vertex weights in decreasing order, placing them in A and B alternatively)
- Iterations
  - Similar to KL
  - A vertex cannot move if violates the balance condition
  - Choosing the node to move:
    pick the max gain in the partitions
  - Moves are tentative (similar to KL)
  - When no moves possible or no more unlocked vertices available, the pass ends
  - When no move can be made in a pass, the algorithm terminates

Other Partitioning Methods

- KL and FM have each held up very well
- Min-cut / max-flow algorithms
  - Ford-Fulkerson – for unconstrained partitions
- Ratio cut
- Genetic algorithm
- Simulated annealing

Partitioning

- **Input:**
  - A set of blocks, both fixed and flexible.
    - Area of the block $A_i = w_i \times h_i$
    - Constraint on the shape of the block (rigid/flexible)
  - Pin locations of fixed blocks.
  - A netlist.
- **Requirements:**
  - Find locations for each block so that no two blocks overlap.
  - Determine shapes of flexible blocks.
- **Objectives:**
  - Minimize area.
  - Reduce wire-length for critical nets.

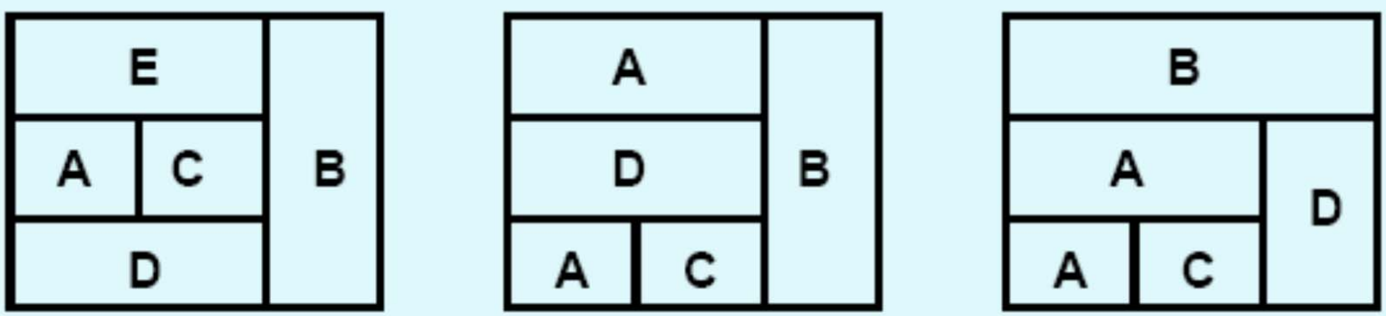
Difference between FP and Placement

- The problems are similar in nature.
- Main differences:
  - In floorplanning, some of the blocks may be flexible, and the exact locations of the pins not yet fixed.
  - In placement, all blocks are assumed to be of well-defined geometrical shapes, with defined pin locations.
- Points to note:
  - Floorplanning problem is more difficult as compared to placement.
    - Multiple choice for the shape of a block.
  - In some of the VLSI design styles, the two problems are identical.

Examples of Rigid Block

Module	Width	Height
A	1	1
B	1	3
C	1	1
D	1	2
E	2	1

Some of the Feasible Floorplans



Design Style Specific Issues

- Full Custom
  - All the steps required for general cells.
- Standard Cell
  - Dimensions of all cells are fixed.
  - Floorplanning problem is simply the placement problem.
  - For large netlists, two steps:
    - First do global partitioning.
    - Placement for individual regions next.
- Gate Array
  - Floorplanning problem same as placement problem.

Cost of Floorplanning

- The number of feasible solutions of a floorplanning problem is very large.
  - Finding the best solution is NP-hard.
- Several criteria used to measure the quality of floorplans:
  a) Minimize area
  b) Minimize total length of wire
  c) Maximize routability
  d) Minimize delays
  e) Any combination of above

Cost of FP

- **How to determine area?**
  - Not difficult.
  - Can be easily estimated because the dimensions of eac block is known.
  - Area *A* computed for each candidate floorplan.
- **How to determine wire length?**
  - A coarse measure is used.
  - Based on a model where all I/O pins of the blocks are merged and assumed to reside at its center.
  - Overall wiring length $L = \Sigma_{i,j} (c_{ij} * d_{ij})$

    where c_{ij} : connectivity between blocks i and j

    d_{ij} : Manhattan distances between the centres of rectangles of blocks i and j

Cost of FP

- **Typical cost function used:**

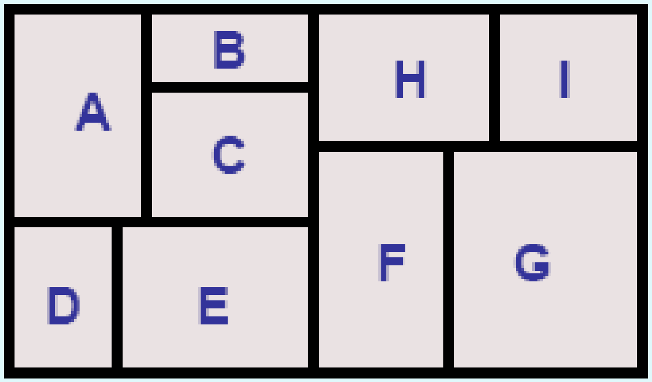
  $$Cost = w1 * A + w2 * L$$

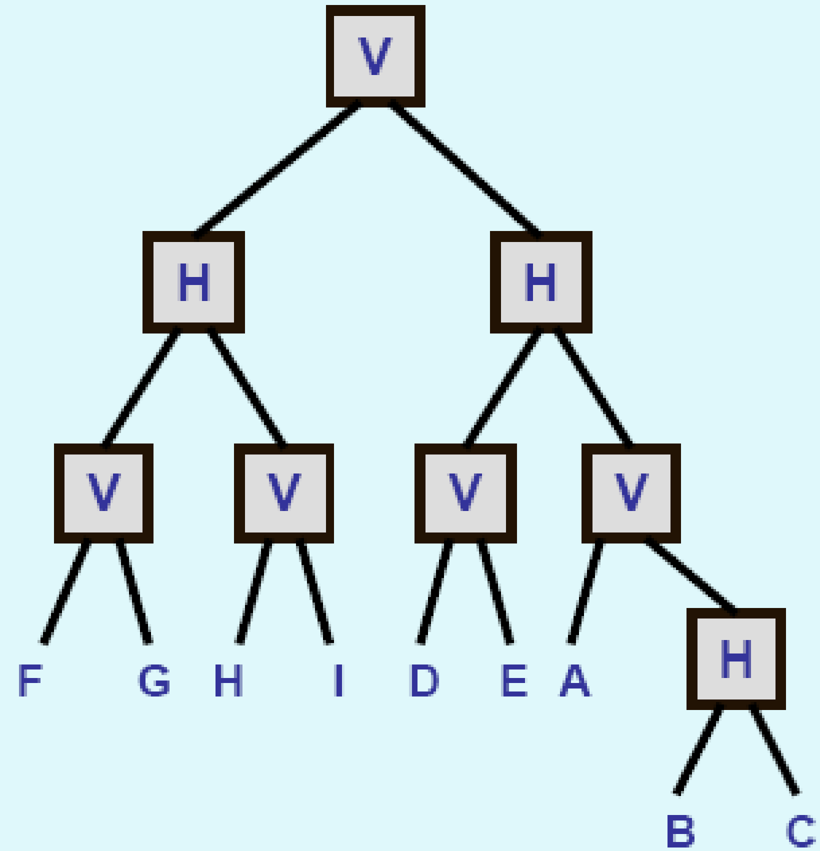
  where w1 and w2 are user-specified parameters.

Slicing Structure

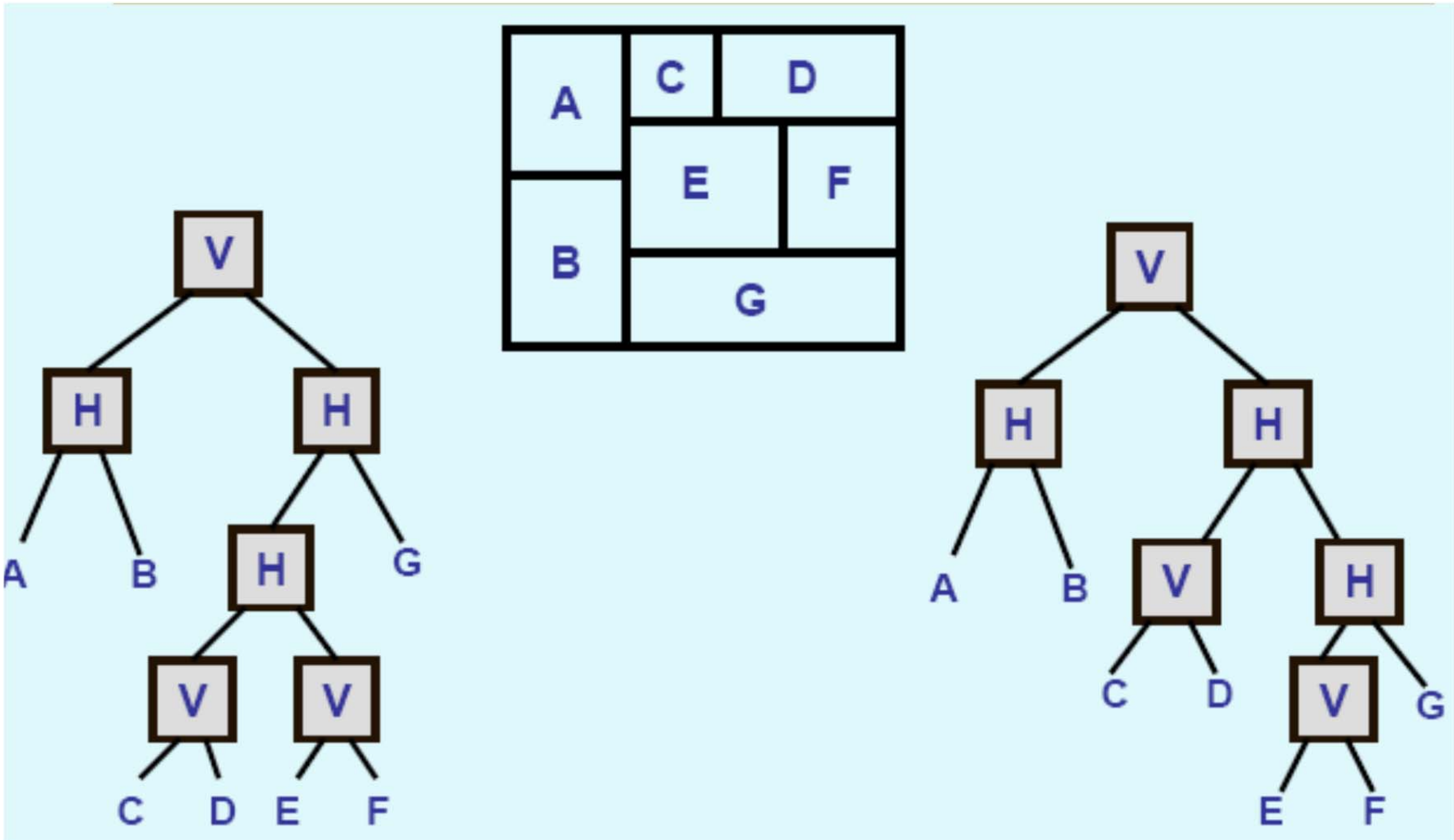
- **Definition**
  - A rectangular dissection that can be obtained by repeatedly splitting rectangles by horizontal and vertical lines into smaller rectangles.
- **Slicing Tree**
  - A binary tree that models a slicing structure.
  - Each node represents a *vertical cut line* (V), or a *horizontal cut line* (H).
    - A third kind of node called *Wheel (W)* appears for non-sliceable floorplans (discussed later).
  - Each leaf is a basic block (rectangle).

Slicing Structure





Slicing Tree is not Unique



FP Algorithms

- Several broad classes of algorithms:
  - Integer programming based
  - Rectangular dual graph based
  - Hierarchical tree based
  - Simulated annealing based
  - Other variations

ILP Formulation

- The problem is modeled as a set of linear equations using 0/1 integer variables.
- Given:
  - Set of *n* blocks $S = \{B_1, B_2, \ldots, B_n\}$ which are rigid and have fixed orientation.
  - 4-tuple associated with each block

    (x_i, y_i, w_i, h_i)