Roll No: A

CS 228 Logic in Computer Science

Midsemester Exam: Feb 25, 2019 (11:00am - 1pm)

No books, notes, calculators, mobiles. Do rough work (using **very small font**) on back sides only. Then plan and write *concise clear* answers within the space provided. No doubts allowed.

Qn. No.	1	2	3	4	5	Total
Marks						

1. (10 marks) Uday, Varsha, Sridhar, Nutan, Bhaskar and Kavi live on an inhabited only by Devas (who always tell the truth) and Asuras (who always lie).

   Varsha claims *Kavi is an Asura and Nutan is a Deva.* Sridhar tells you *Of Bhaskar and Varsha, exactly one is a Deva.* Uday tells you *I and Bhaskar are Devas.* Nutan says *Uday would tell you that Bhaskar is an Asura.* Bhaskar says *Uday is an Asura.* Kavi tells you *Nutan is an Asura and I am a Deva.*

   Give brief reasoning above and tick the right boxes in the table below to show if each person is a Deva or an Asura.

	Uday	Varsha	Sridhar	Nutan	Bhaskar	Kavi
Deva						
Asura						
Cannot Determine						

2. *[5 marks]*
   Consider the formulae φ, ψ, η below.

$$\varphi = \forall x \forall y \forall z (P(x,y) \to (P(y,z) \to P(z,x)))$$

$$\psi = \forall x \forall y (P(x,y) \to P(y,x) \to x = y)$$

$$\eta = \forall x \exists y P(x,y) \to \exists y \forall x P(x,y)$$

   Show that there exist structures where (i) φ is false, while the other two are true, (ii) η is false, while the other two are true.

3. *[8 marks]* Say whether each of the following is true or false with a justification.

   (a) Let $\Sigma = \{0,1\}$. $L = \{xy \mid |x|_0 = |y|_1\}$. Then L is FO-definable.

   (b) For languages $L_1, L_2 \subseteq \Sigma^*$, let $L_1 - L_2$ denote the difference; that is, strings which are in L_1 but not in L_2. There exist languages L_1, L_2 such that L_1, L_2 are FO-definable, but $L_1 - L_2$ is not.

   (c) A language $L \subseteq \Sigma^*$ is co-finite if its complement is finite. If L is co-finite, then L is FO-definable.

(d) Let $L = L_1 \cup L_2$ and $L_1 \cap L_2 = \emptyset$. If L_1 is FO-definable and L_2 is not regular, then L is not FO-definable.

4. *[5+5+5=15 marks]* A language $L \subseteq \Sigma^+$ is incapable of counting iff

$$\exists n_0 \forall n \geq n_0, \forall u, v, w \in \Sigma^*, (uv^n w \in L \leftrightarrow uv^{n+1} w \in L)$$

(a) Prove or disprove : $b(a^*ba^*ba^*)^*$ is regular.

(b) Assume that a certain logician, Prof. Calculus tells you that a language is non FO-definable iff it is capable of counting (capable is the opposite of incapable). Assume Prof.Calculus is correct in his claim. Is $b(a^*ba^*ba^*)^*$ FO-definable?

(c) Prove or disprove : $(a^+ba^+b)^+$ is $FO^2[<]$ definable. $FO^2[<]$ is the 2-variable fragment of $FO[<]$. The 2-variable fragment of $FO[<]$ is one where you can use atmost 2 variables x, y. Also note that you do not have S (the successor) explicitely.