CS 228 : Logic in Computer Science

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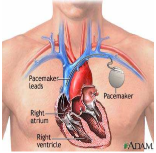
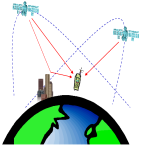
So Far

- Dwelt on classical logics : propositional logic, FO and MSO on finite words
- Words : good abstraction for capturing properties to be checked on systems built
- Moving on to Temporal logics

Safety Critical Systems



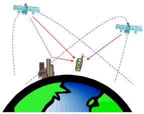



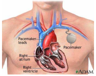


The role of Automata and Logics

- Systems modeled as certain kinds of automata
- Safety critical properties written in some logic
- Check if the property is satisfied by all runs of the system

Verification through Model Checking





System → satisfy? → specification

good/bad properties

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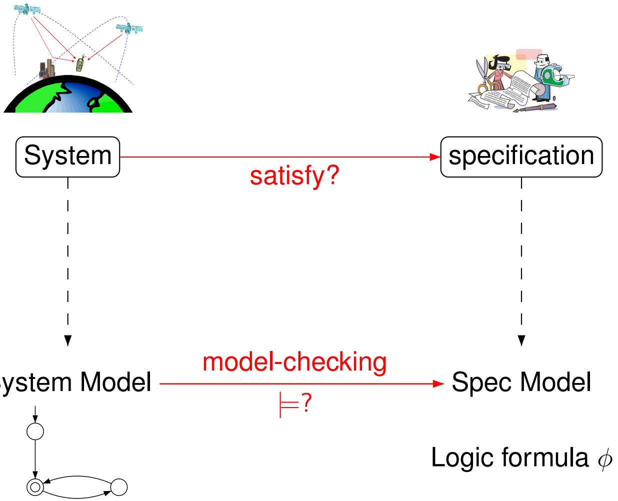
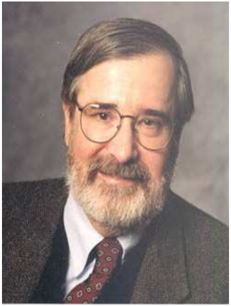
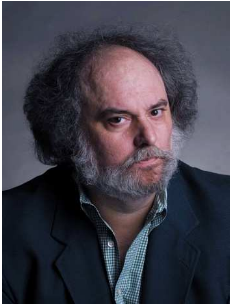


Model Checking : Pioneers


- Year 2008 : ACM confers the Turing Award to the pioneers of Model Checking: Ed Clarke, Allen Emerson, and Joseph Sifakis

Temporal Logics

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  - Turing Award 1996(Amir Pnueli)
  - For seminal work introducing temporal logic into computing science and for outstanding contributions to program and system verification.

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- Temporal Logic CTL for program correctness; introduction of model-checking (Emerson and Clarke; Sifakis, 1982)
  - Turing Award 2008 (Clarke, Emerson and Sifakis).
  - For their role in developing model-checking into a highly effective verification technology that is widely adopted in the hardware and software industries.
  - See *http://www-verimag.imag.fr/ sifakis/TuringAwardPaper-Apr14.pdf*.

Transition Systems

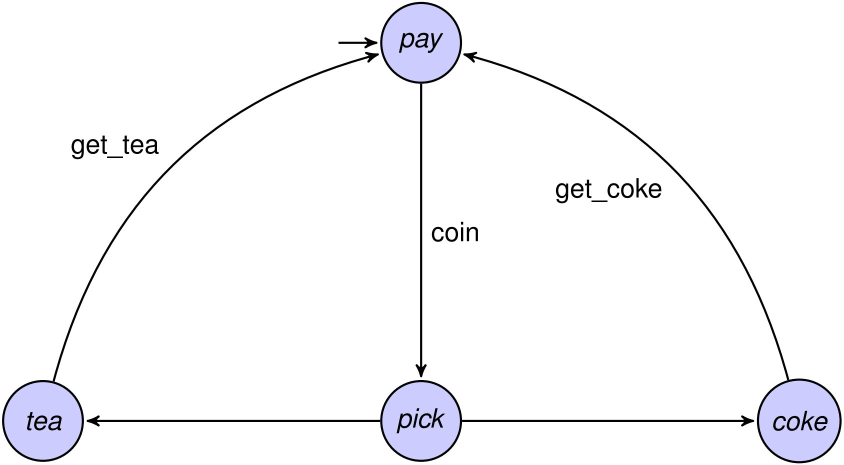
- model to capture the behaviour of systems
- Directed graph, vertices represent “states” of the system, edges represent “transitions” between states
- states? transitions? : system dependent

Transition Systems

A Transition System is a tuple $(S, Act, \rightarrow, I, AP, L)$ where

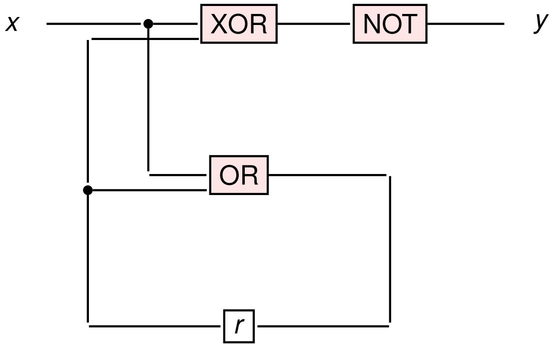
- S is a set of states
- Act is a set of actions
- $s \xrightarrow{\alpha} s'$ in $S \times Act \times S$ is the transition relation
- $I \subseteq S$ is the set of initial states
- AP is the set of atomic propositions
- $L : S \rightarrow 2^{AP}$ is the labeling function

A Model for a Vending Machine



- states, actions, transitions, initial states, atomic propositions

Sequential Circuits



- Input variable x, output variable y, register r
- Output $\neg(x \bigoplus r)$ and register evaluates to $x \vee r$

Transition System for the Circuit

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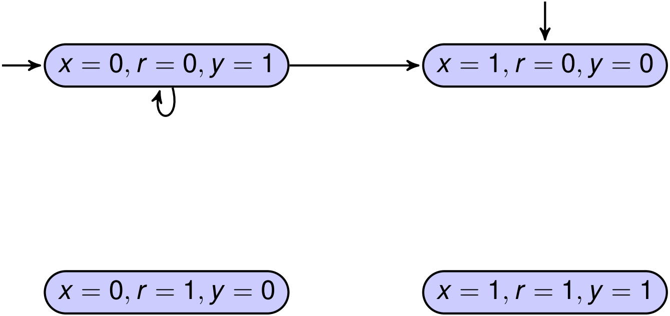

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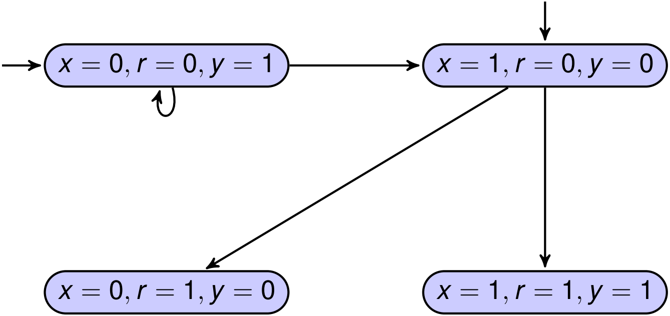

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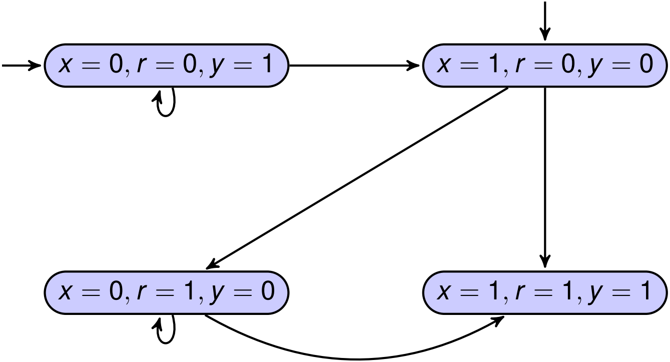

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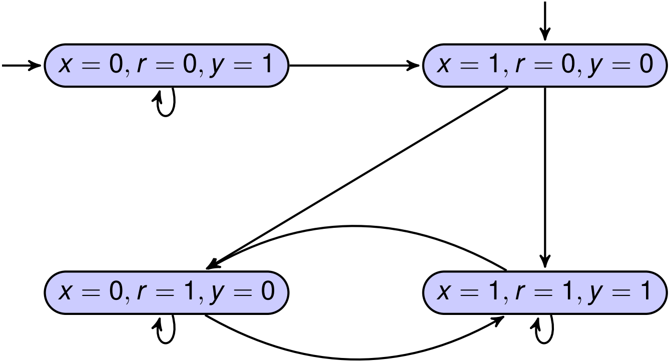

Traces of Transition Systems

- Labels of the locations represent values of all observable propositions $\in AP$
- Captures system state
- Focus on sequences $L(s_0)L(s_1)\dots$ of labels of locations
- Such sequences are called traces
- Assuming transition systems have no terminal states,
  - Traces are infinite words over 2^{AP}
  - Traces $\in (2^{AP})^\omega$

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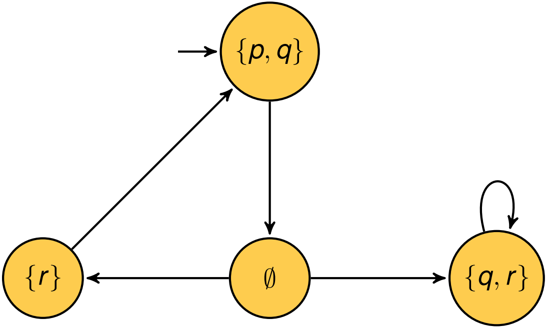
- $Traces(TS) = \bigcup_{s \in I} Traces(s)$

Example Traces



$AP = \{p, q, r, t\}$

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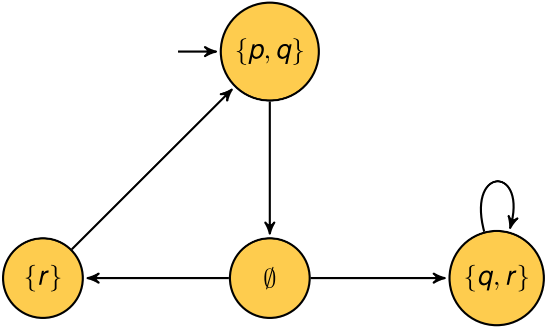

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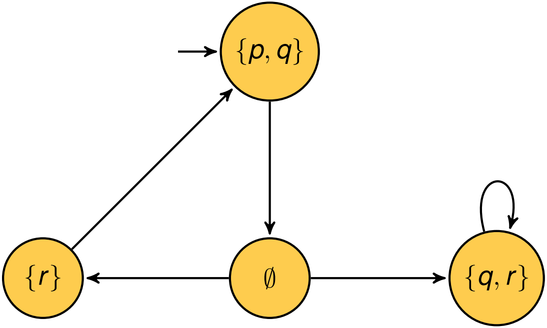

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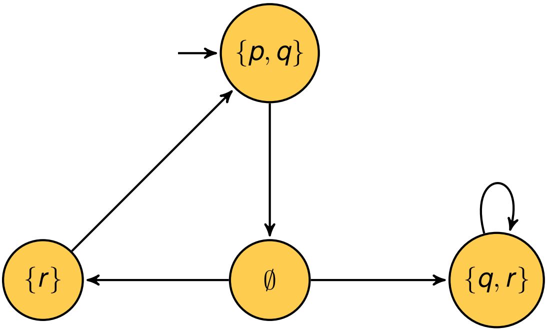

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- $(\{p, q\}\emptyset\{r\})^* \, \{p, q\}\emptyset\, \{q, r\}^\omega$

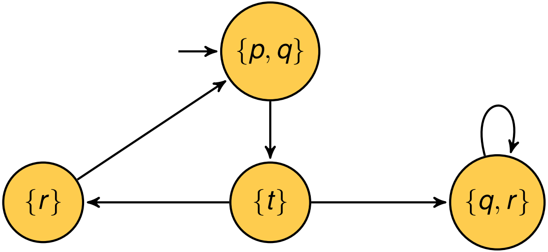
Linear Time Properties

- Linear-time properties specify traces that a *TS* must have
- A LT property P over AP is a subset of $(2^{AP})^\omega$
- *TS* over *AP* satisfies a LT property P over AP

$$TS \models P \text{ iff } Traces(TS) \subseteq P$$

- $s \in S$ satisfies LT property P (denoted $s \models P$) iff $Traces(s) \subseteq P$

Specifying Traces



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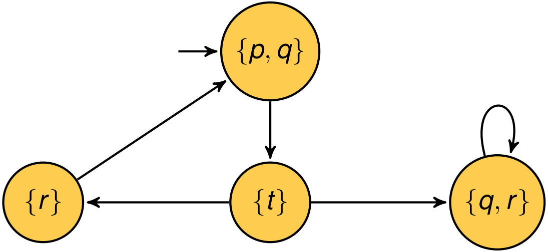

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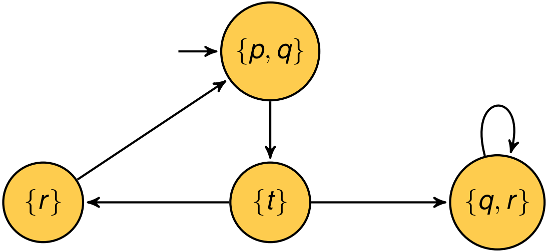

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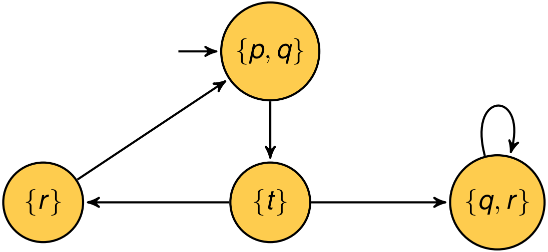

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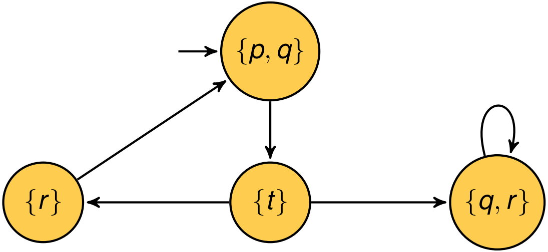

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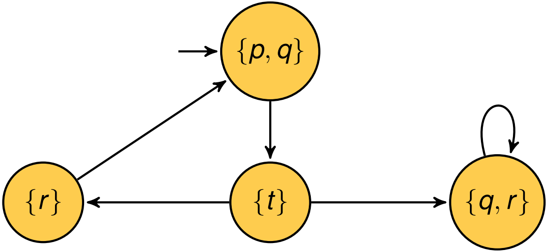

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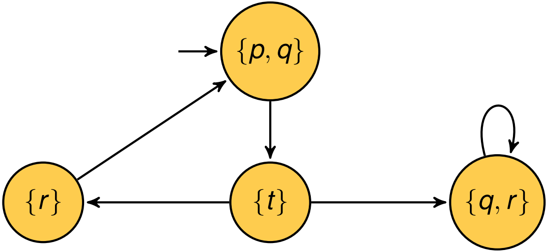

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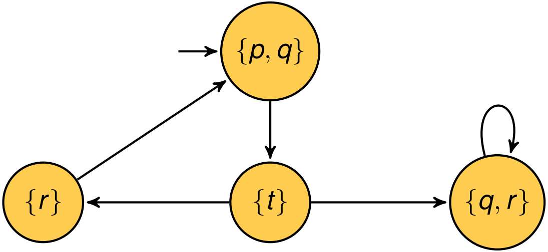

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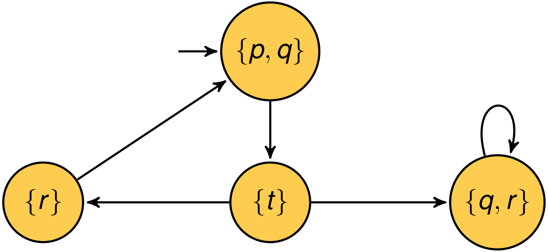

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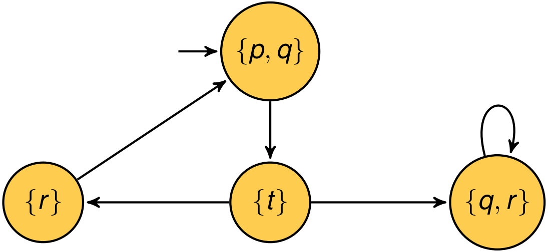

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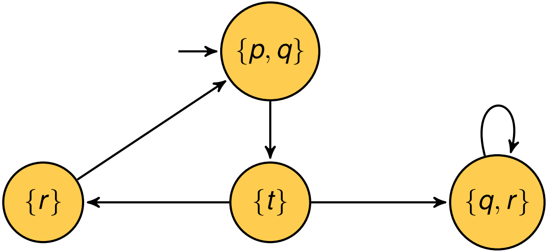

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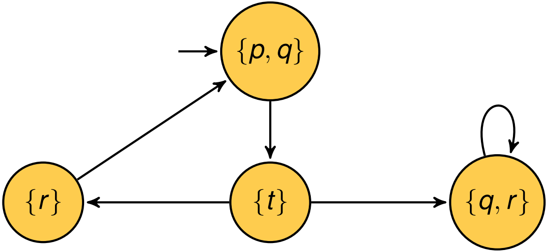

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  - $\{A_0 A_1 \cdots \mid \exists i \geqslant 0, r \in A_i, \text{ and } \forall j < i, t \notin A_j \wedge r \notin A_j\}$

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Given *AP*, a set of propositions,

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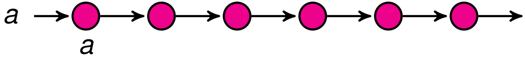
- Propositional logic formulae over *AP*
  - $a \in AP$ (atomic propositions)
  - $\neg\varphi$, $\varphi \wedge \psi$, $\varphi \vee \psi$
- Temporal Operators
  - $\bigcirc\varphi$ (Next φ)
  - $\varphi\,\mathsf{U}\psi$ (φ holds until a ψ-state is reached)
- LTL : Logic for describing LT properties

Semantics

LTL formulae over AP interpreted over words over Σ^ω, $\Sigma = 2^{AP}$



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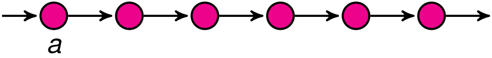
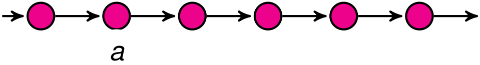


$\bigcirc a$



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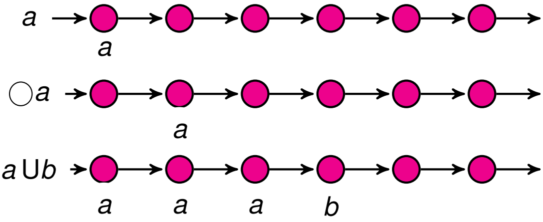

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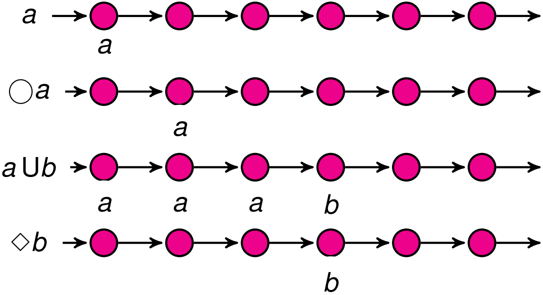

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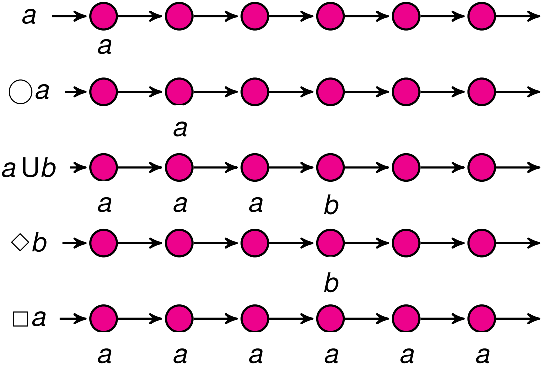

Derived Operators

- $\mathit{true} = \varphi \vee \neg\varphi$
- $\mathit{false} = \neg \mathit{true}$
- $\Diamond\varphi = \mathit{true}\, \mathsf{U} \varphi$ (Eventually φ)
- $\Box\varphi = \neg\Diamond\neg\varphi$ (Forever φ)

Precedence

- Unary Operators bind stronger than Binary
- $\bigcirc$ and $\neg$ equally strong
- U takes precedence over $\wedge, \vee, \rightarrow$
  - $a \vee b\, \mathsf{U} c \equiv a \vee (b\, \mathsf{U} c)$
  - $\bigcirc a\, \mathsf{U} \neg b \equiv (\bigcirc a)\, \mathsf{U} (\neg b)$