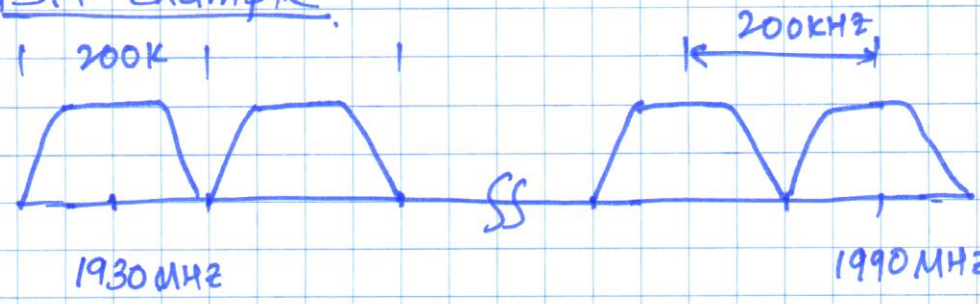
CONTINUOUS TIME FILTERS

GSM Example



AM , FM Radios – Common Medium Shared between multiple users.

Filters are used to provide frequency selectivity.

→ Keep desired spectrum (Channel)
→ Attenuate undesired channels.

→ Wireline Communication Example

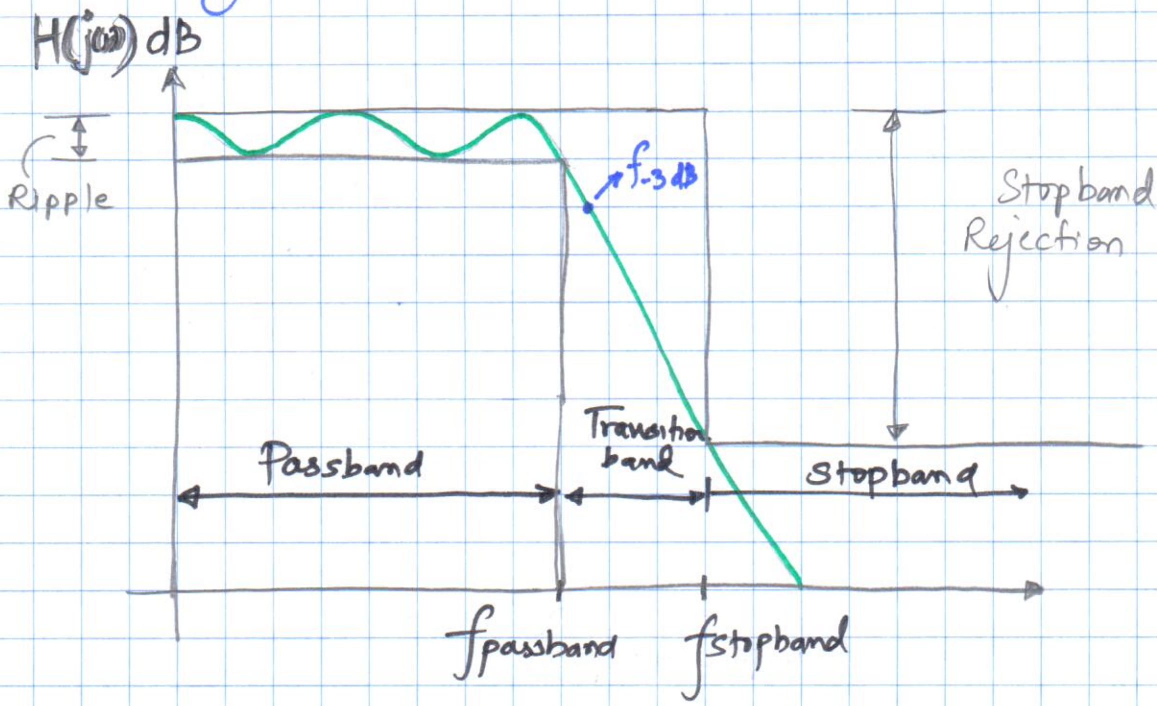
Filters used to compensate for Channel loss (frequency dependent)

→ Equalization & pulse shaping.

→ Antialiasing filters for ADC
→ Smoothing filters for DAC.

Filter Specifications

Study Low Pass filters first – Then Xform to diff types.



Filter with transfer function $H(s)$

$$H(j\omega) = |H(j\omega)| e^{j\theta(\omega)}$$

$$\text{Phase Delay} = \tau_{PD} = -\frac{\theta(\omega)}{\omega} \quad \text{(time units)}$$

$$\text{Group Delay} = \tau_{GR} = -\frac{d\theta(\omega)}{d\omega} \quad \text{(time units)}$$

$\theta(\omega) = K\omega$ No phase distortion.

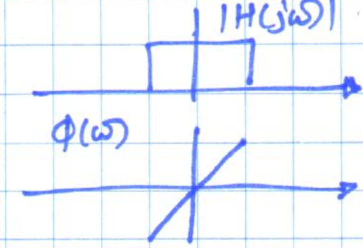
K ← Constant

$$\tau_{PD} = \tau_{GR} = -K$$

Constant group delay.

→ This property is important for pulse-shaping in Wireline applications

Filter Approximations (ideal Brickwall)



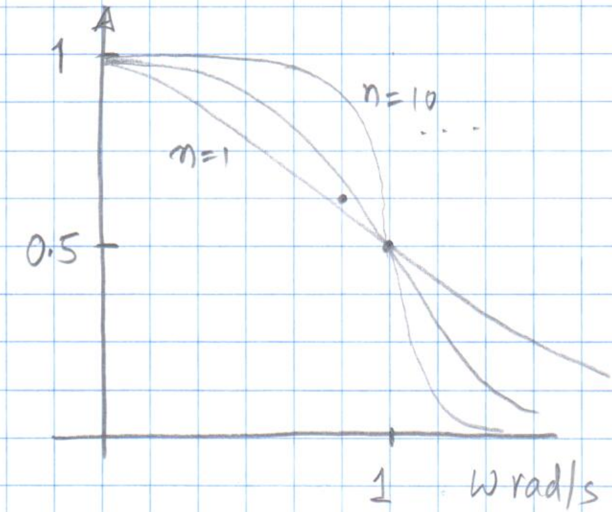
1. Butterworth Approximation

$$|H(j\omega)|^2 = \frac{1}{1+\omega^{2n}}$$

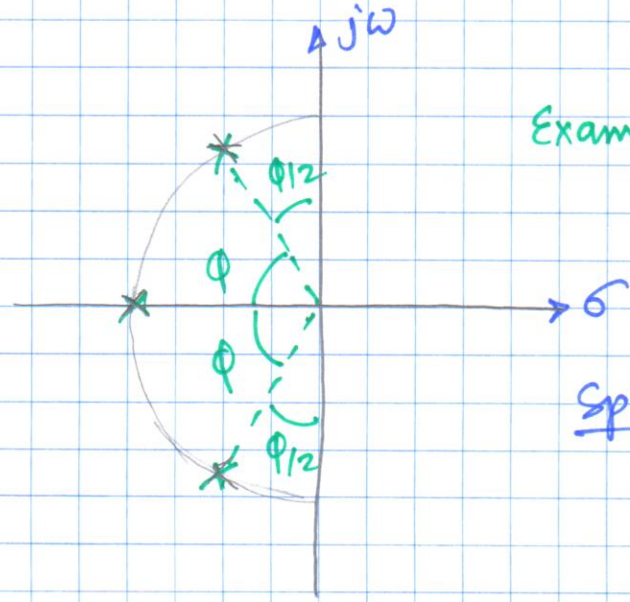
$|H(j0)|^2 = 1$; ~~$H(j0.5) =$~~

$|H(j1)|^2 = 0.5$

$|H(j\infty)|^2 = 0$



* Maximally flat passband @ $\omega = 0$.
* Moderate phase distortion. (group delay)
* Poles located on unit circle



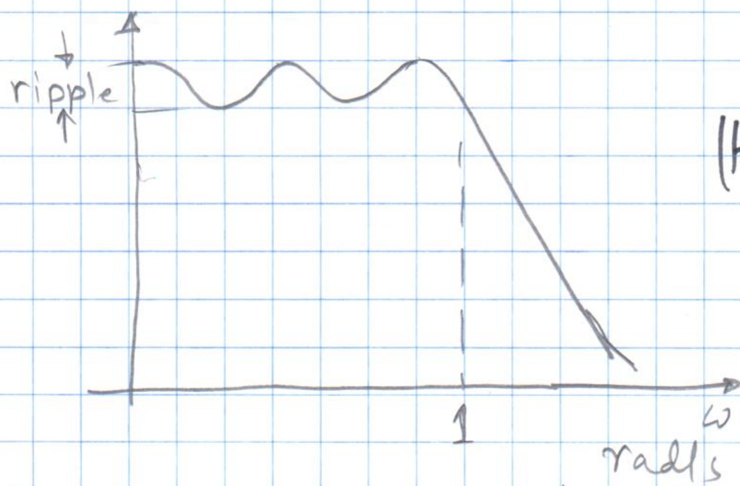
Example $n=3$

Splane

2. Chebyshev Approximation

→ keep attenuation in passband minimal
→ within Ripple spec.

→ Sharper transition band (compared to Butterw)
→ Poorer group delay distortion.
→ All poles located on Ellipse.

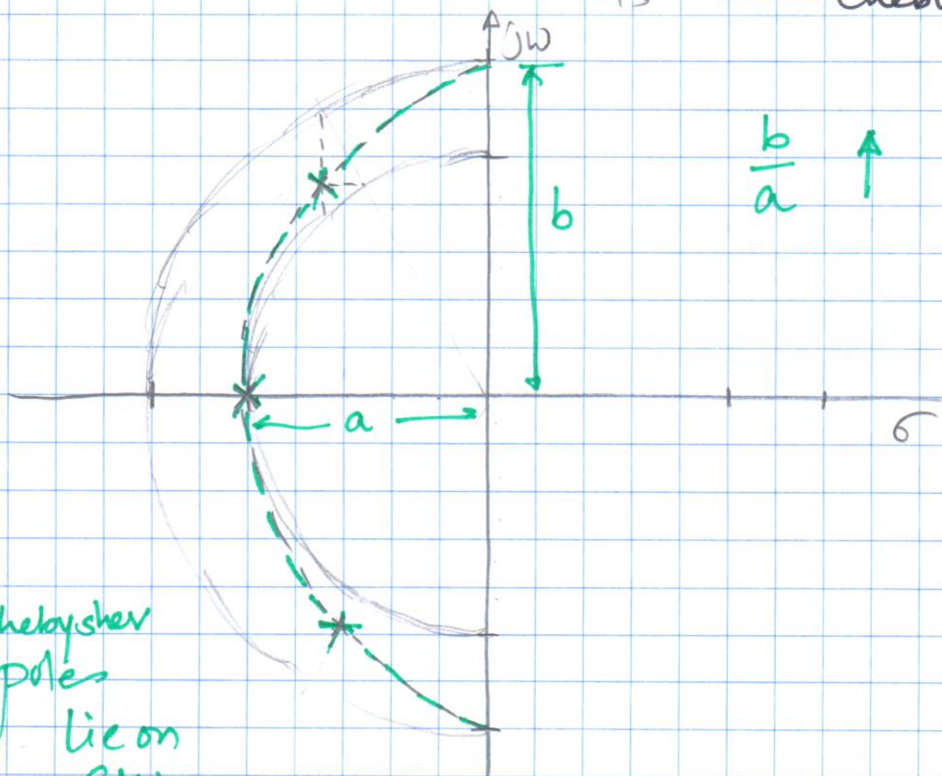


$$|H(j\omega)|^2 = \frac{1}{1+\epsilon^2 T_n^2(\omega)}$$

→ $T_n(\omega) = \cos(n \cos^{-1}\omega)$

ϵ - sets ripple amplitude

Chebyshev polynomial (nth order)

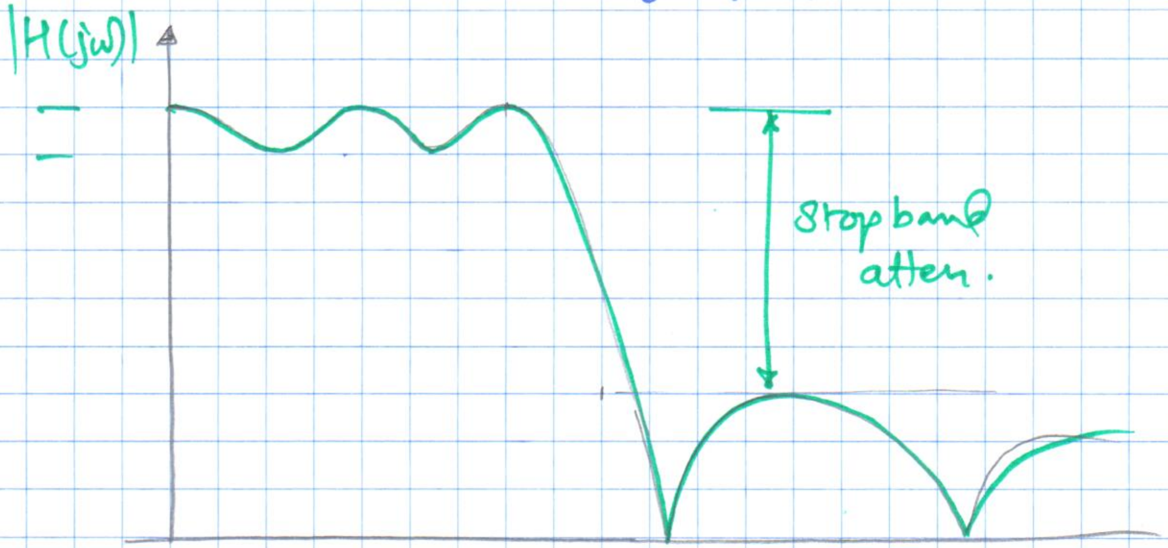


$\frac{b}{a}$ ↑ ripple ↑

Chebyshev poles lie on Ellipse

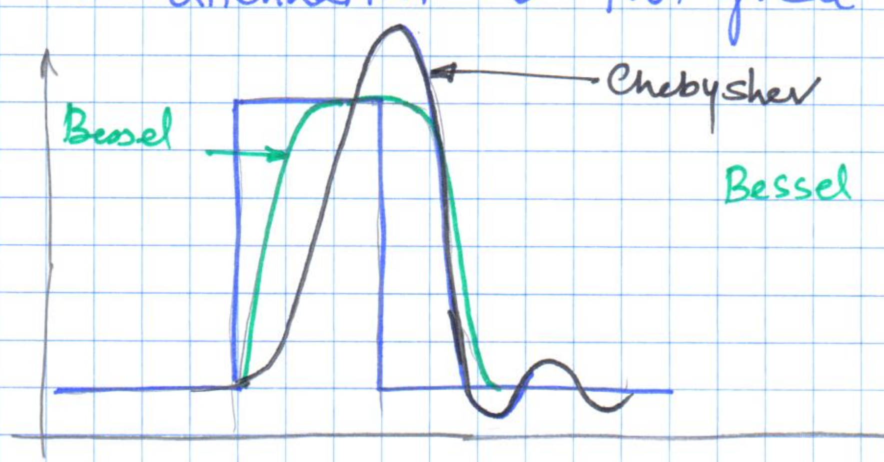
3. Elliptic filters (Cauer filters)

- → Zeros in stopband. create nulls.
- → very sharp transition band - highest atten with lowest order
- → poorest group delay performance.



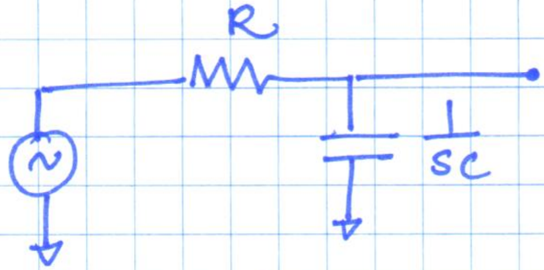
④ Bessel Approximation

- → Poles ~~out~~ outside unit circle (low Q)
- → Maximally flat group delay
- → attenuation ∼ Not great.



Bessel - minimizes [S]

RC filter - 1st order Example

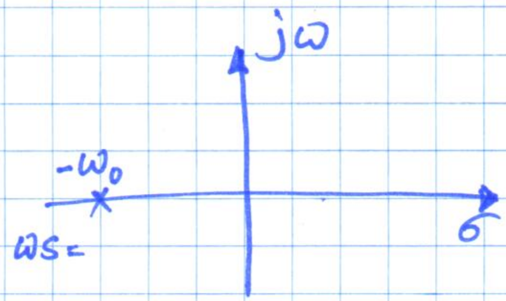


$$V_{out} = \frac{1/sC}{R + 1/sC} = \frac{1}{\left(1 + \frac{s}{1/RC}\right)}$$

pole @ $-\omega_0 = -\frac{1}{RC}$

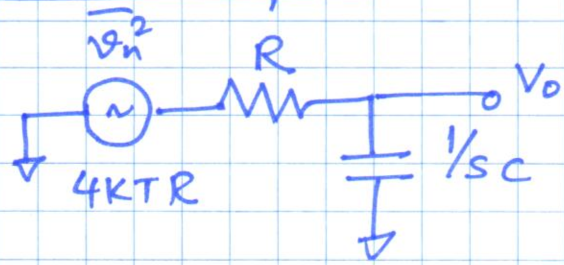
@ $s = j\omega_0$ $|H(j\omega)| = \frac{1}{\sqrt{2}}$

→ 3dB frequency



<u>Note</u>: parasitics across R will limit high freq. atten.

Noise Analysis



$$S_{out}(f) = 4KTR \cdot \frac{1}{4\pi^2 R^2 C^2 f^2 + 1}$$

Total Integrated o/p noise

$$= \int_0^{\infty} \frac{4KTR}{4\pi^2 R^2 C^2 f^2 + 1}\, df$$

$$= \frac{kT}{C}$$ ← indep of $\underline{\underline{R}}$

Example 1pF cap → 64 μVrms @ 300°K

for complex filter (higher order)

o/p noise $= x \frac{KT}{C}$

x depends on order

Also - assumption - active circuits - lower noise.

$$\text{Filter Dynamic Range} = \frac{\text{Max Voltage Swing}}{\text{Min. discernible signal}}$$

Max Voltage Swing ← VDD, Linearity

Min. discernible signal ← Noise

$$\text{Max Voltage Swing} = \frac{1}{\sqrt{2}} \frac{V_{DD}}{2} \quad \text{(Rms Voltage)}$$

$$\text{Min Voltage Swing} = \sqrt{x \frac{KT}{C}} \quad \text{(Rms Noise)}$$

$$DR = \frac{V_{DD}/2\sqrt{2}}{\sqrt{x \frac{KT}{C}}} = V_{DD} \sqrt{\frac{C}{8xKT}}$$

$$DR_{dB} = 20 \log_{10} \left(V_{DD} \sqrt{\frac{C}{8xKT}} \right)$$

$$= 20 \log_{10} \left(V_{DD} \sqrt{\frac{C}{x}} \right) + 75\,dB$$

(C in pF)

For $V_{DD} = 1V$ $C = 1pF$ $x = 1$

$$DR_{dB} = 75\,dB = (1.76 + 6.02N)$$

N ← # of bits

$N = 12$ bits Best possible

Reference - Haideh Khorramabadi Classnotes.

FILTER DESIGN EXAMPLE

Design a Chebyshev filter to meet following req.

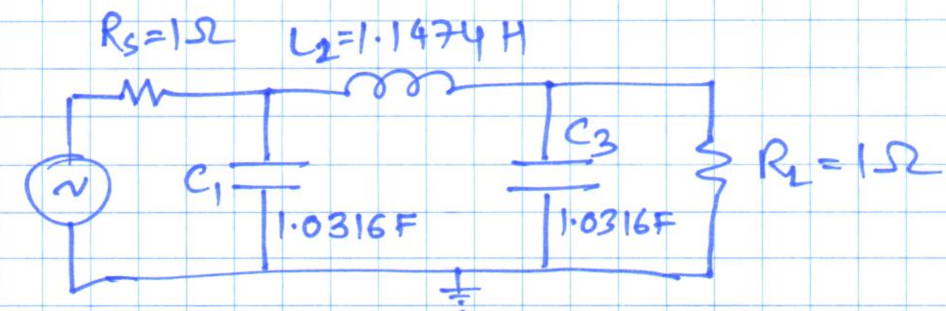
* 0.1 dB ripple
* 10 MHz ripple BW.
* Stopband atten 40 dB
* Stopband 60 MHz

→ Ripple BW / Stopband = 1/6

→ From 0.1 dB ripple Chebyshev Atten chara. (Fig 8.15) for n = 3 atten @ 6 rad/s = 42 dB.

→ 3rd order filter.

From Table 8.2

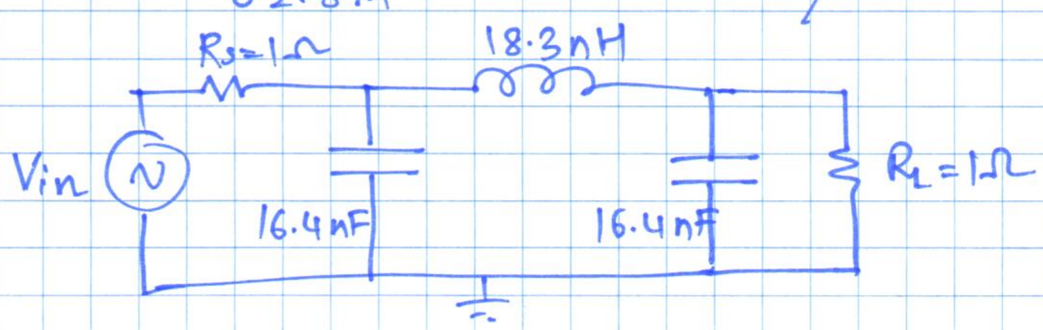


$\omega_{ripple\,BW} = 1\,rad/s$ Desired $\omega_{ripple\,B} = 2\pi \times 10\,MHz$
$= 62.8\,M\,rad/s$

$$C_1 = C_3 = \frac{1.0316\,F}{62.8\,M} = 16.4\,nF$$

$$L_2 = \frac{1.1474\,H}{62.8\,M} = 18.3\,nH$$

Impractical for IC implementation



ANALOG AND DIGITAL FILTERS: DESIGN AND REALIZATION

HARRY Y-F. LAM

Bell Telephone Laboratories, Inc.

Formerly with University of California, Berkeley

BUTTERWORTH FILTER

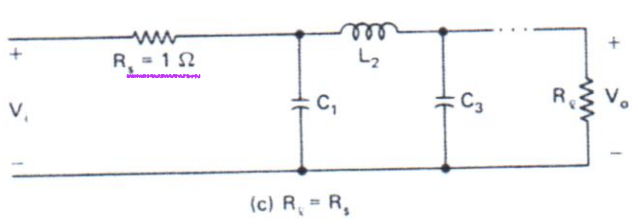


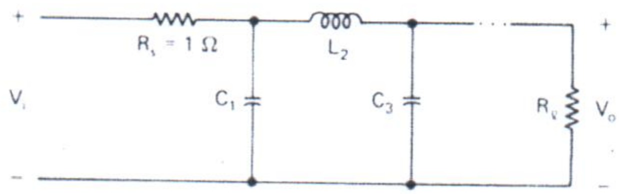
Fig. 8-9 Circuit structures of low-pass Butterworth filters.

FILTER ORDER

TABLE 8-1 Element Values for the Circuit in Fig. 8-9(c)

n	C_1	L_2	C_3	L_4	C_5	L_6	C_7	L_8	C_9
1	2.0000								
2	1.4142	1.4142							
3	1.0000	2.0000	1.0000						
4	0.7654	1.8478	1.8478	0.7654					
5	0.6180	1.6180	2.0000	1.6180	0.6180				
6	0.5176	1.4142	1.9319	1.9319	1.4142	0.5176			
7	0.4450	1.2470	1.8019	2.0000	1.8019	1.2470	0.4450		
8	0.3902	1.1111	1.6629	1.9616	1.9616	1.6629	1.1111	0.3902	
9	0.3473	1.0000	1.5321	1.8794	2.0000	1.8794	1.5321	1.0000	0.3473

Chebyshev Filter



R_L

Fig. 8-17 Circuit structure of low-pass Chebyshev filters.

R_L

TABLE 8-2 Values of Circuit Elements in Chebyshev Filters when $A_{max} = 0.1$ dB

Ripple 0.1dB

n	R_ℓ	C_1	L_2	C_3	L_4	C_5	L_6	C_7	L_8	C_9
1	1.0	0.3052								
2	0.5	1.5715	0.2880							
3	1.0	1.0316	1.1474	1.0316						
4	0.5	2.3545	0.7973	2.6600	0.3626					
5	1.0	1.1468	1.3712	1.9750	1.3712	1.1468				
6	0.5	2.5561	0.8962	3.3962	0.8761	2.8071	0.3785			
7	1.0	1.1812	1.4228	2.0967	1.5734	2.0967	1.4228	1.1812		
8	0.5	2.6324	0.9285	3.5762	0.9619	3.5095	0.8950	2.8547	0.3843	
9	1.0	1.1957	1.4426	2.1346	1.6167	2.2054	1.6167	2.1346	1.4426	1.1957

R_L

TABLE 8-3 Values of Circuit Elements in Chebyshev Filters when $A_{max} = 1$ dB

Ripple 1dB

n	R_ℓ	C_1	L_2	C_3	L_4	C_5	L_6	C_7	L_8	C_9
1	1.00	1.0177								
2	0.25	3.7779	0.3001							
3	1.00	2.0236	0.9941	2.0236						
4	0.25	4.5699	0.5428	5.3680	0.3406					
5	1.00	2.1349	1.0911	3.0009	1.0911	2.1349				
6	0.25	4.7366	0.5716	6.0240	0.5764	5.5353	0.3486			
7	1.00	2.1666	1.1115	3.0936	1.1735	3.0936	1.1115	2.1666		
8	0.25	4.7966	0.5803	6.1592	0.6005	6.1501	0.5836	5.5869	0.3515	
9	1.00	2.1797	1.1192	3.1214	1.1897	3.1746	1.1897	3.1214	1.1192	2.1797

Bessel Filter

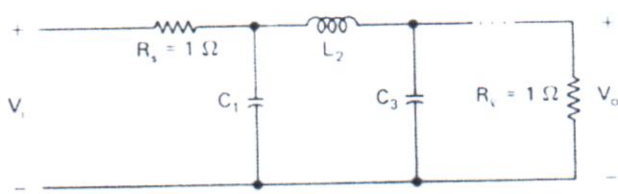


Fig. 8-23 A basic circuit structure for low-pass Bessel filters.

Filter order

TABLE 8-4 Values of Circuit Elements in Bessel Filters

n	C_1	L_2	C_3	L_4	C_5	L_6	C_7	L_8	C_9
1	2.0000								
2	1.5774	0.4226							
3	1.2550	0.5528	0.1922						
4	1.0598	0.5116	0.3181	0.1104					
5	0.9303	0.4577	0.3312	0.2090	0.0718				
6	0.8377	0.4116	0.3158	0.2364	0.1480	0.0505			
7	0.7677	0.3744	0.2944	0.2378	0.1778	0.1104	0.0375		
8	0.7125	0.3446	0.2735	0.2297	0.1867	0.1387	0.0855	0.0289	
9	0.6678	0.3203	0.2547	0.2184	0.1859	0.1506	0.1111	0.0682	0.0230

FILTER TRANSFORMATIONS

TABLE 8-5 Frequency and Element Transformations

From Normalized Low-pass Filter to:	Frequency Transformations	$R\ \Omega$	L H.	C F.
A low-pass filter with cutoff frequency ω_c	$s \longmapsto \dfrac{s}{\omega_c}$	$R\ \Omega$	$\dfrac{L}{\omega_c}$ H.	$\dfrac{C}{\omega_c}$ F.
A bandpass filter with center frequency ω_0 and bandwith B	$s \longmapsto \dfrac{s^2+\omega_0{}^2}{Bs}$	$R\ \Omega$	$\dfrac{L}{B}$ H, $\dfrac{B}{\omega_0{}^2 L}$ F.	$\dfrac{C}{B}$ F, $\dfrac{B}{\omega_0{}^2 C}$
A band-reject filter with center frequency ω_0 and rejection bandwidth B	$s \longmapsto \dfrac{Bs}{s^2+\omega_0^2}$	$R\ \Omega$	$\dfrac{1}{BL}$ F, $\dfrac{BL}{\omega_0{}^2}$ H.	$\dfrac{1}{BC}$ H, $\dfrac{BC}{\omega_0{}^2}$ F.
A high-pass filter with cutoff frequency ω_c	$s \longmapsto \dfrac{\omega_c}{s}$	$R\ \Omega$	$\dfrac{1}{L\omega_c}$ F.	$\dfrac{1}{C\omega_c}$ H.
Impedance scaling by A	No change	$AR\ \Omega$	AL H.	$\dfrac{C}{A}$ F.

Selected Plots from

ANALOG AND DIGITAL FILTERS: DESIGN AND REALIZATION -- Harry Y-F. LAM

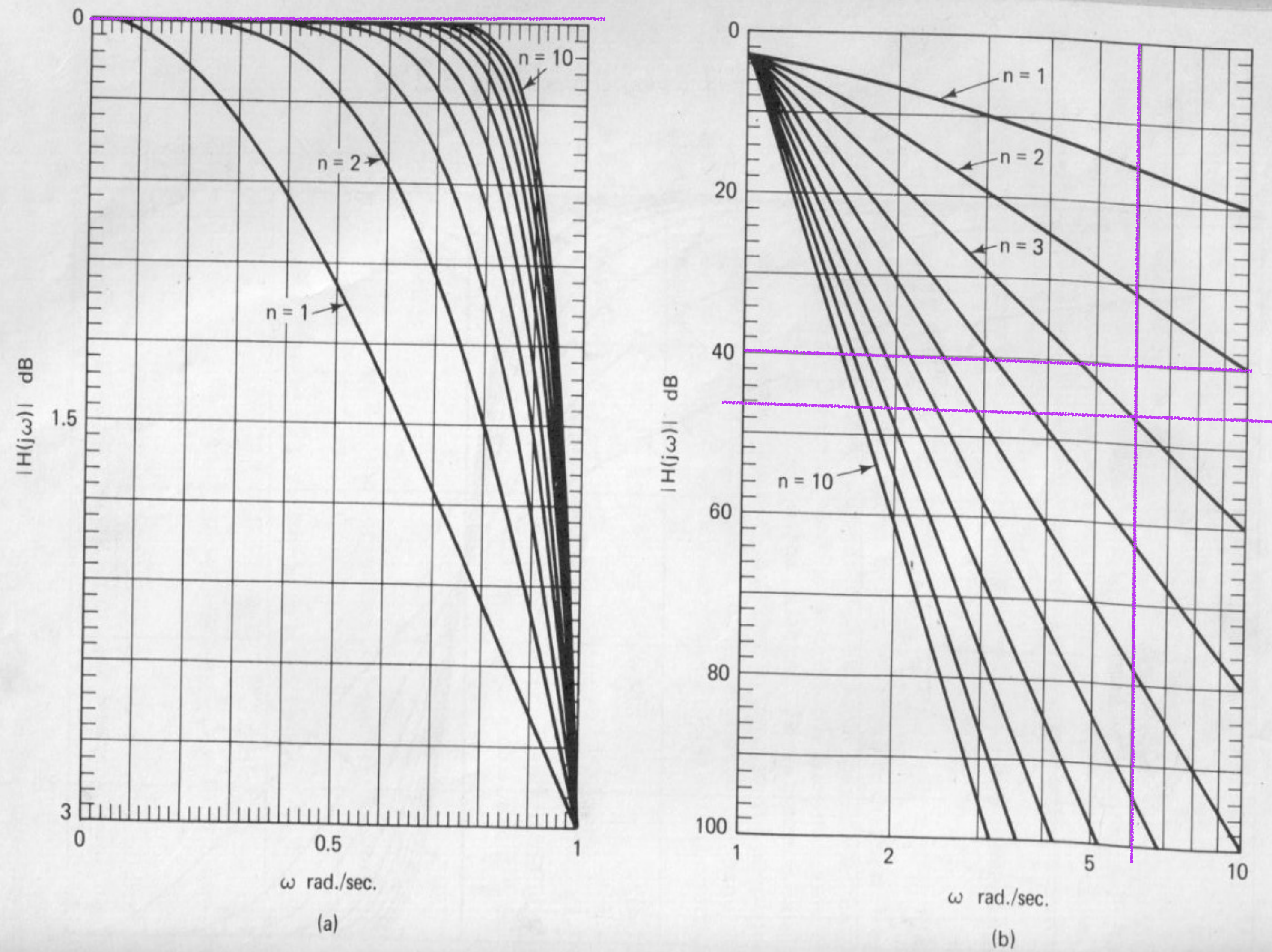


Fig. 8-4 Magnitude characteristics of Butterworth filters. (a) Passband attenuation. (b) Stopband attenuation.

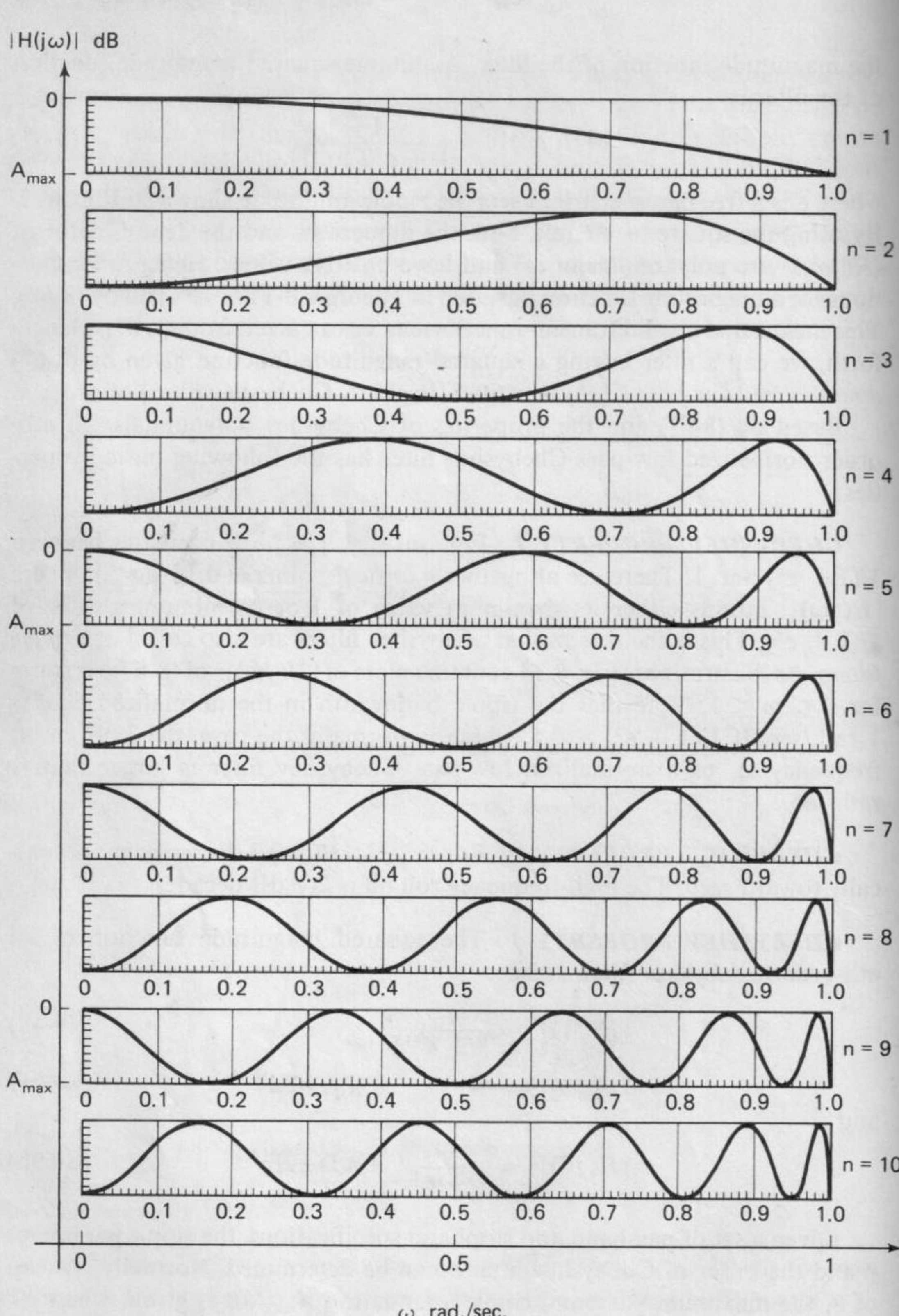
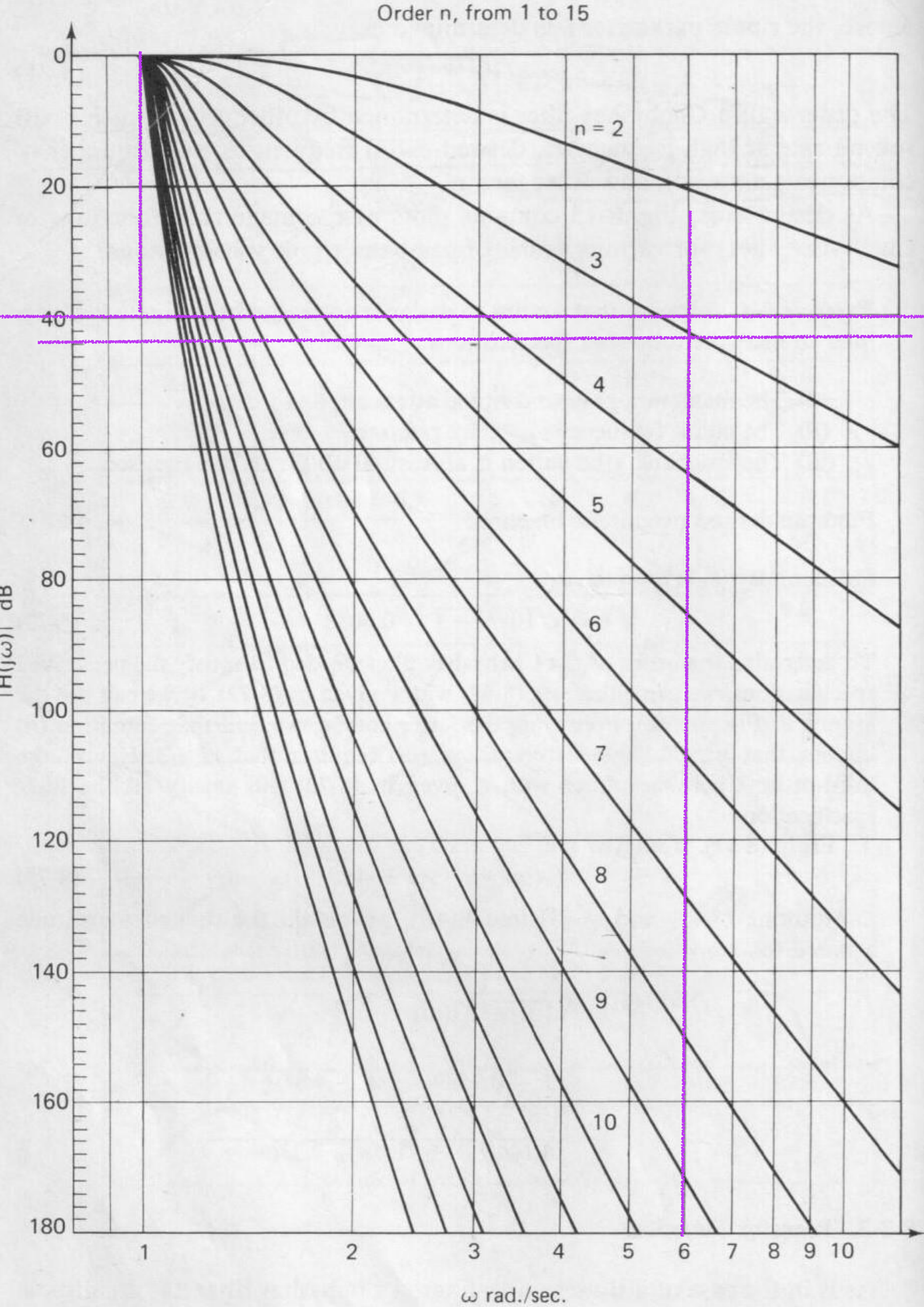
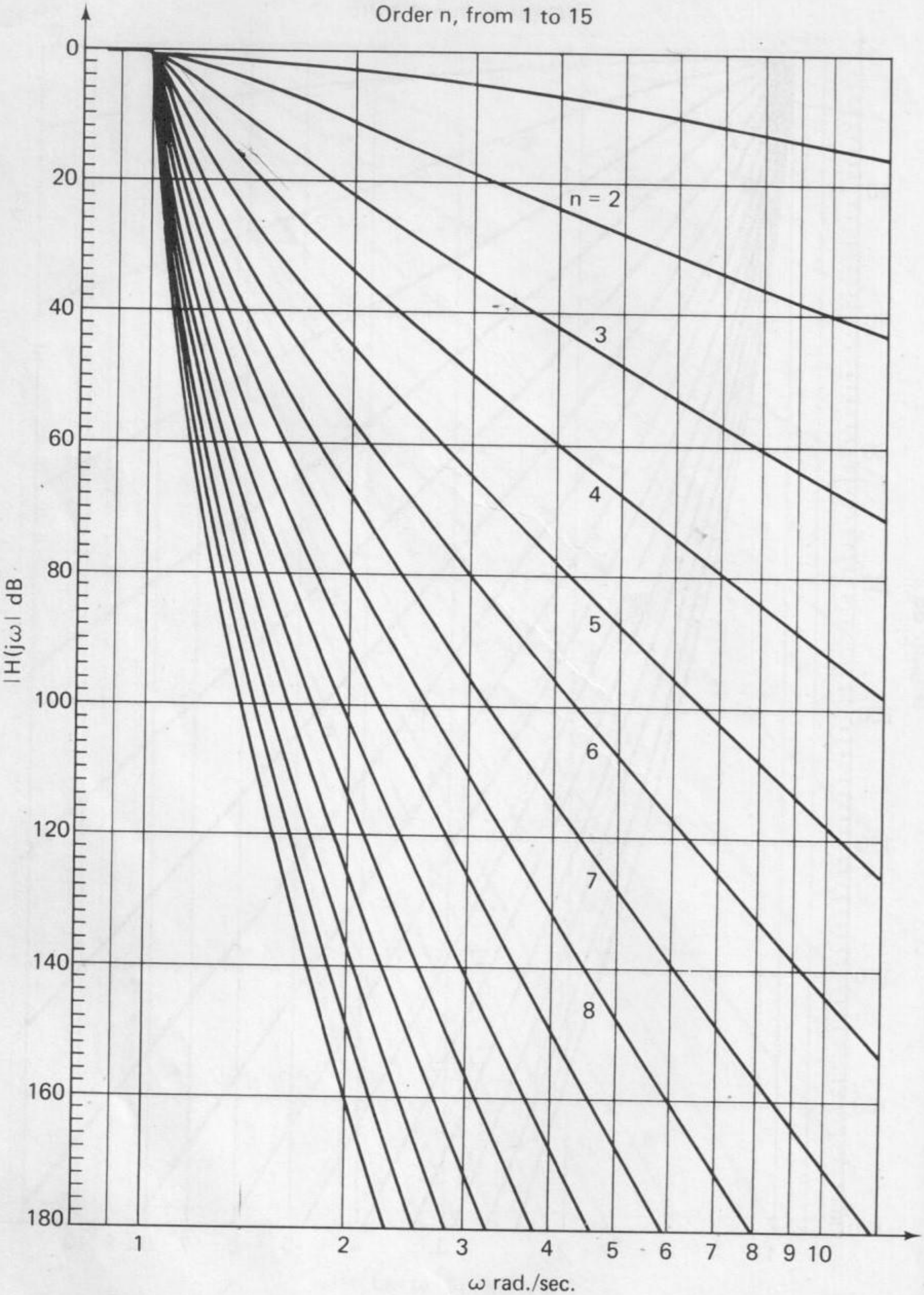


Fig. 8-14 Chebyshev passband ripples.



(a) Passband ripple = 0.1 dB

Fig. 8-15 Attenuation characteristics of Chebyshev filters.



(e) Passband ripple = 1 dB

Fig. 8-15 (continued)

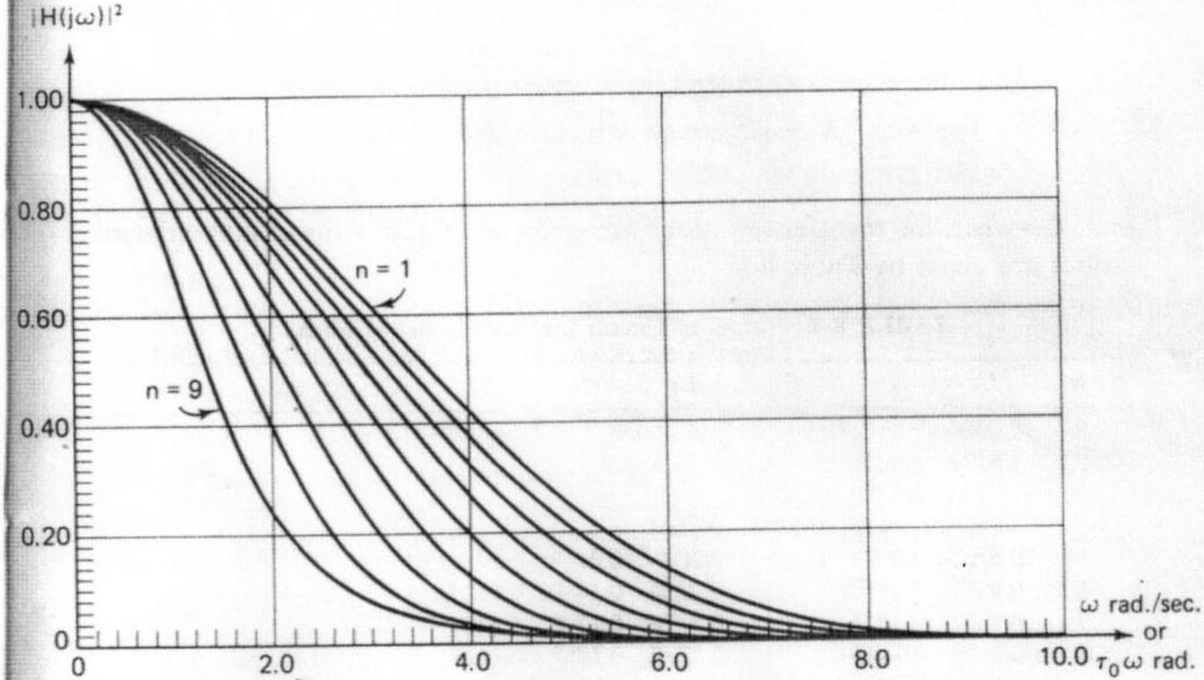


Fig. 8-22 Magnitude characteristics of Bessel filters.

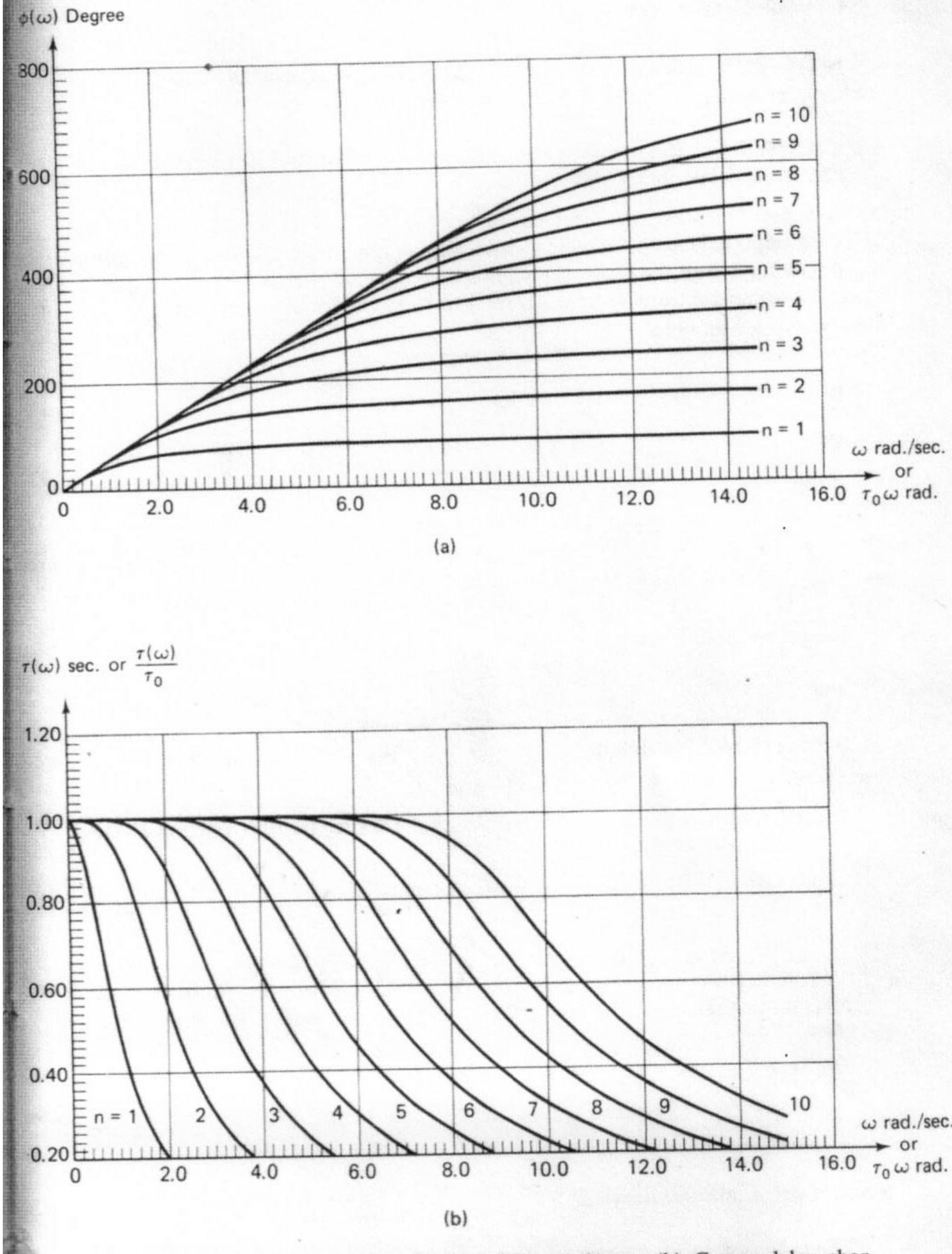


Fig. 8-21 (a) Phase characteristics of Bessel filters. (b) Group delay characteristics of Bessel filters.