

EE618: CMOS Analog IC Design

Handout: Bode Plots

Electrical Engineering, IIT Bombay

1 Making a Bode Plot: Rules

Given a transfer function in the form

$$H(s) = A \prod \frac{(1 + s/x_n)^{a_n}}{(1 + s/y_n)^{b_n}}$$

with $a_n, b_n > 0$ ¹

1.1 Amplitude Plot

- At every value of s where $\omega = |x_n|$ (a zero), be it either in RHP or LHP, increase the slope of the line by $20a_n$ dB per decade.
- At every value of s where $\omega = |y_n|$ (a pole), given y_n is positive, decrease the slope of the line by $20b_n$ dB per decade. A RHP Pole, means that the system is unstable and hence a bode plot cannot be drawn as a steady state cannot be reached
- The initial value of the graph depends on the boundaries. The initial point is found by putting the initial angular frequency ω into the function and finding $|H(j\omega)|$. Usually bode plots start from $\omega_0 = 0$, hence the magnitude begins from the DC Magnitude
- The initial slope of the function at the initial value depends on the number and order of zeros and poles that are at values below the initial value, and are found using the first two rules

1.2 Phase Plot

- If A is positive, start line (with slope 0) at 0 degrees else at 180 degrees
- At every $\omega = |x_n|$, given x_n is positive, increase the slope by $45a_n$ degrees per decade, beginning one decade before $\omega = |x_n|$ (i.e. $|x_n|/10$). For RHP zeros, at every $\omega = |x_n|$, decrease the slope by $45a_n$ degrees per decade, beginning one decade before $\omega = |x_n|$ (i.e. $|x_n|/10$).
- At every $\omega = |y_n|$, decrease the slope by $45b_n$ degrees per decade, beginning one decade before $\omega = |y_n|$ (i.e. $|y_n|/10$). Again the pole must be in the LHP for the system to be stable.
- Flatten the slope again when the phase has changed by $90a_n$ degrees (for a zero) or $90b_n$ degrees (for a pole)
- The phase angle should change by $45a_n$ degrees (for a zero) or $45b_n$ degrees (for a pole) at the exact location of the zero/pole.
- After plotting one line for each pole or zero, add the lines together to obtain the final plot

¹Replace a_n and b_n by 1 for a single zero/pole at a particular ω

2 Example ²

Consider

$$H(s) = \frac{10s}{(1 + s/10^2)(1 + s/10^5)}$$

Let $H(s) = H_1(s)H_2(s)H_3(s)H_4(s)$ where

$$H_1(s) = 10; H_2(s) = s$$

$$H_3(s) = \frac{1}{1 + s/p_1}, p = 10^2 \text{ rad/s}$$

$$H_4(s) = \frac{1}{1 + s/p_2}, p = 10^5 \text{ rad/s}$$

We can now plot the magnitude and phase of the individual components versus ω and then simply add them to obtain $|H|$ and $\angle H$.

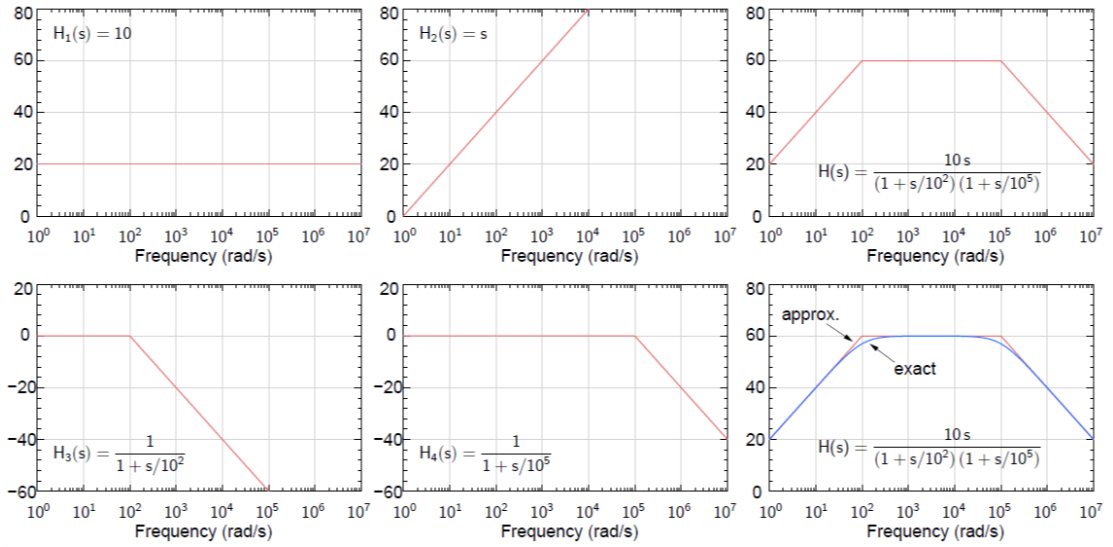


Figure 1: Bode Magnitude Plots for $H(s)$

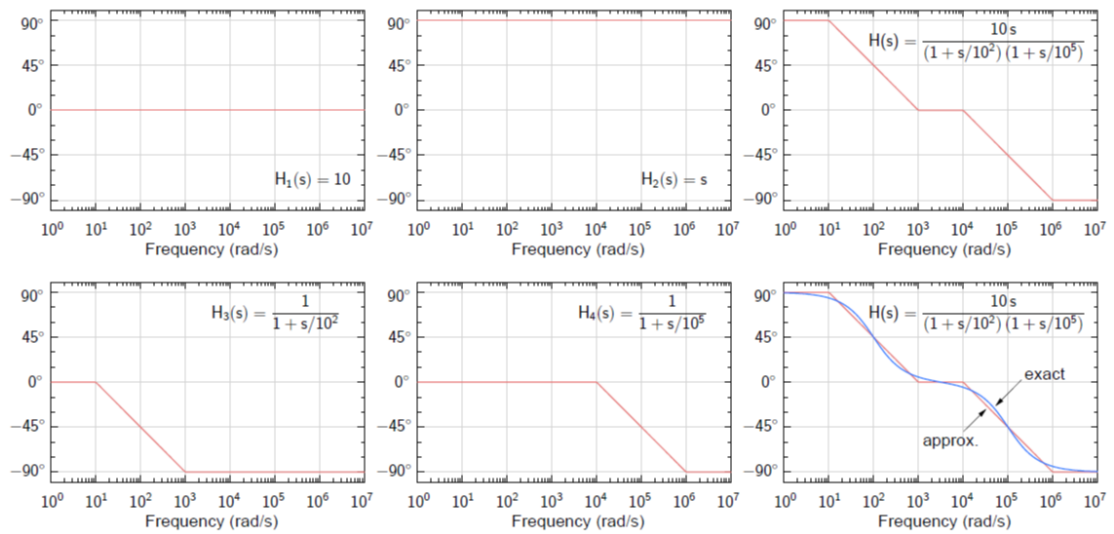


Figure 2: Bode Phase Plots for $H(s)$

²Source: EE101 Bode Plots by Prof. MB Patil, Electrical Engineering, IIT Bombay