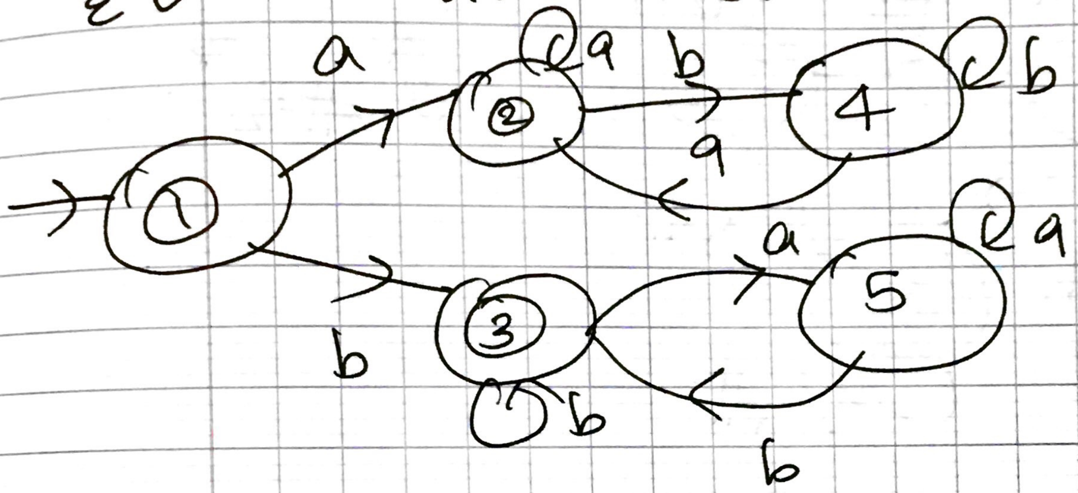
1 (a) Equal number of occurrences of ab, ba.

aaab ✗ aabaaaaa ✓

babb ✓ babaa ✗ bab ✓ baba ✗

ε ✓ a ✓ b ✓ aa ✓ bb ✓



FO definability:

$$\forall x (x \neq x) \quad \vee$$

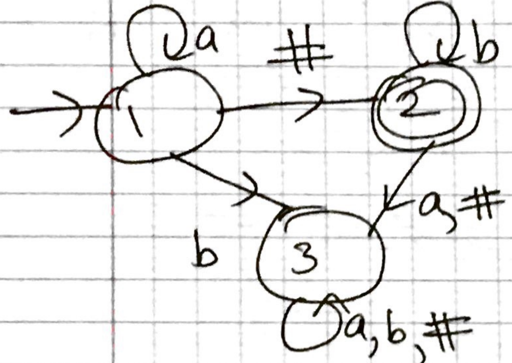
$$\left[\exists x\,(first(x) \wedge Q_a(x)) \wedge \exists y\,(last(y) \wedge Q_a(y))\right]$$

$$\vee$$

$$\left[\exists x\,(first(x) \wedge Q_b(x)) \wedge \exists y\,(last(y) \wedge Q_b(y))\right]$$

(b). $a\#b$ ✓ $bb\#a$ ✓ $\#$ ✓

$\#b$ ✓ $a\#$ ✓

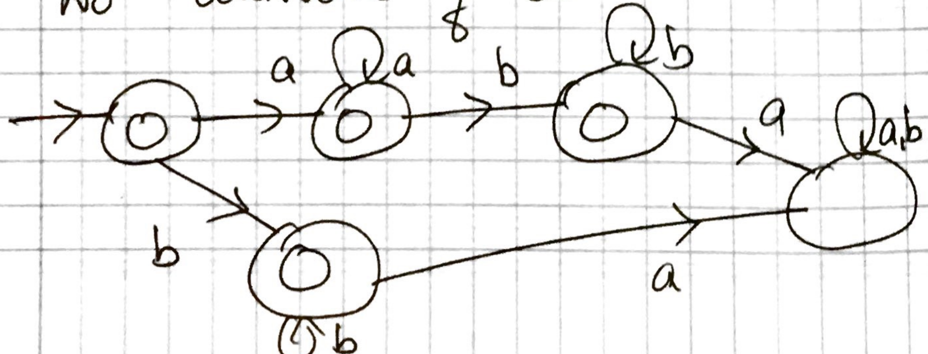


$a^* \# b^*$.

FO definability:

$$\exists x \left(Q_{\#}(x) \wedge \forall y (y < x \Rightarrow Q_a(y)) \wedge \forall z (x < z \Rightarrow Q_b(z)) \right)$$

(c) No occurrence of ba.

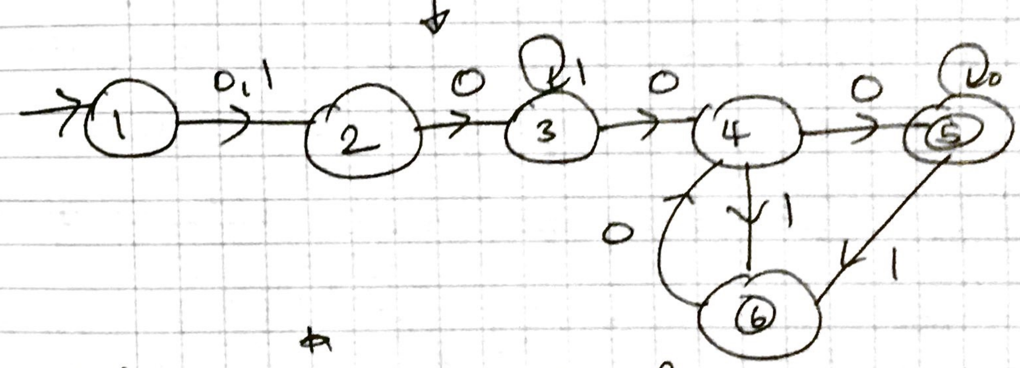


FO definability:

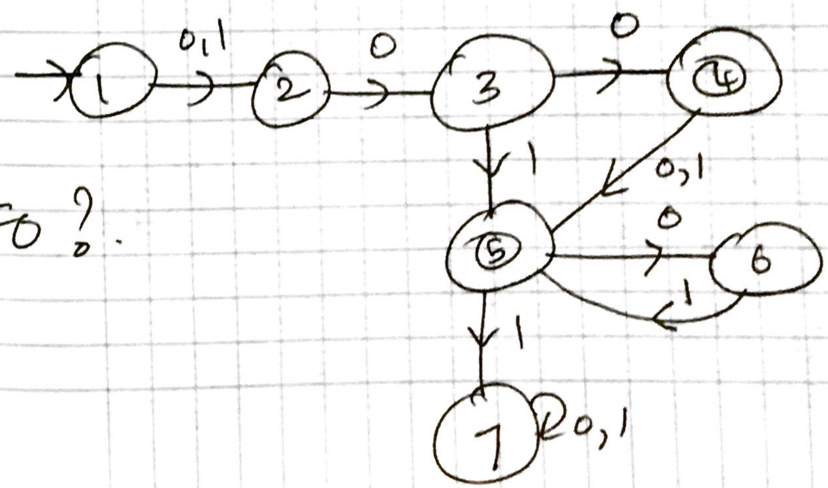
$$\neg \exists x \exists y (S(x,y) \wedge Q_b(x) \wedge Q_a(y)).$$

(d) $-0\Sigma^*0-$

ie $\Sigma 0 \Sigma^* 0 \Sigma$ or $\Sigma 0 \Sigma$



DFA for $\Sigma 0 \Sigma^* 0 \Sigma$ or $\Sigma 0 \Sigma$?



FO ?.

3.

Language accepted

$(b\,aa^*\,bb)^*\,b.$

$baa^*\underline{bb}\,baa^*\underline{bb}\,baba^*$ $\underline{bbb}$

$\exists x(\text{first}(x) \wedge \text{last}(x) \wedge Q_b(x))$ b.

$\vee$

$\exists x\big(\text{first}(x) \wedge Q_b(x) \wedge \exists y(S(x,y) \wedge Q_a(y))\big)$

$\wedge$

$\forall z(Q_a(z) \Longrightarrow \exists z_1(S(z,z_1) \wedge Q_a(z_1))$

$\vee\ \exists z_2(S(z_1,z_2) \wedge Q_b(z_2) \wedge$

$\exists z_3(S(z_2,z_3) \wedge Q_b(z_3)) \wedge$

$\exists z_4(Q_b(z_4) \wedge (\neg\text{last}(z_4) \Longrightarrow$

$\exists z_5(S(z_4,z_5) \wedge$

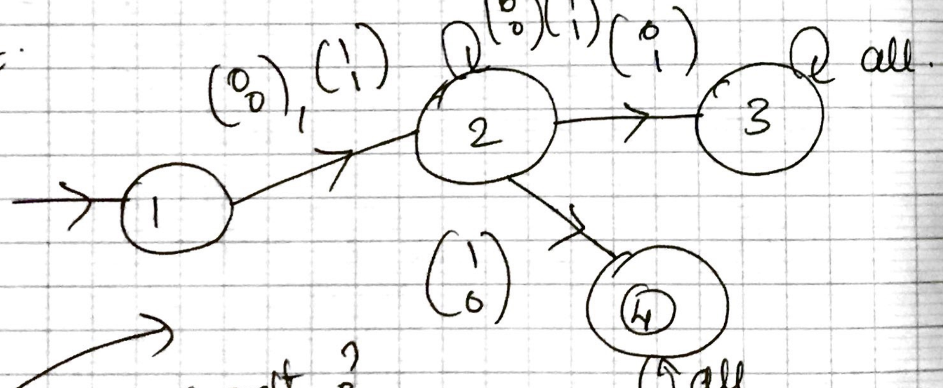
$Q_a(z_5)))))$

Idea: when you see an 'a', the next can be an 'a' (there will always be a next). If the next is not an 'a', then the next is a 'b'. (z_2) This 'b' has a 'b' successor. This b also has a (z_3) 'b' successor. Now if this is the last (z_4) position done. Else, this z_4 has an 'a' successor z_5. To z_5, your $\forall z(Q_a(z) \Longrightarrow \ldots)$ will again apply.

(e) Is $\binom{1001}{0100}$ in the language? Yes.

Is $\binom{1010}{1100}$ in the language? No.

when you read 1 in upper row, you can read 0,1 in lower row. When you read 0 in upper row, you can only read 0 in lower row, if at all positions before you have only read $\binom{0}{0}$ or $\binom{1}{1}$. if you have read $\binom{1}{0}$ somewhere, then read whatever after that.

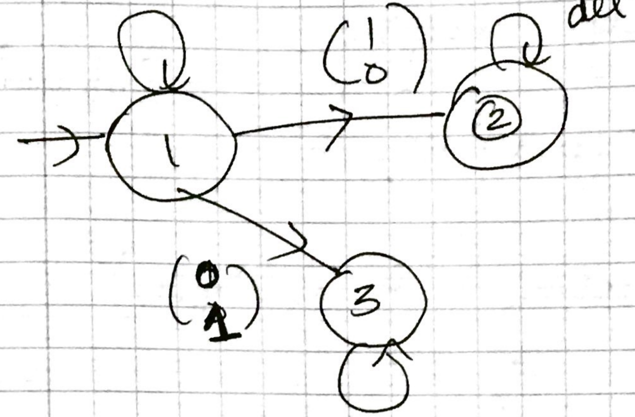


Is this correct?

Is this a DFA?

$\binom{0}{0}\binom{1}{1}$

or



$\binom{0}{0}\binom{1}{1}\binom{0}{1}\binom{1}{0}$.

Fo definability?

$\exists x \left(Q_{\binom{1}{0}}(x) \wedge \forall y \left(y < x \Rightarrow \left(Q_{\binom{0}{0}}(y) \vee Q_{\binom{1}{1}}(y) \right) \right) \right)$

2.

(1) $\forall x(x \neq x)$ $\quad \varepsilon \quad \Sigma^*$

(2) b a $\quad \Sigma^* b a^* a \Sigma^*$.

a

Typo: $\forall z(a < z < y \to Q_a(z))$

(3) a <u>last</u>.

(4) $(ab)^*$