# CS 228 : Logic in Computer Science

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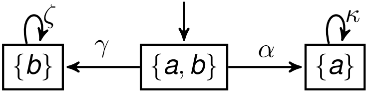
- Construct product of TS and $A_{\neg\varphi}$, obtaining a new TS, say $TS'$
- Check some very simple property on $TS'$, to check $TS \models \varphi$.

# An Example $TS \models \varphi$

- Let $\varphi = \Box(a \vee b), \neg\varphi = \Diamond(\neg a \wedge \neg b)$
- Take TS and $A_{\neg\varphi}$, and check the intersection.



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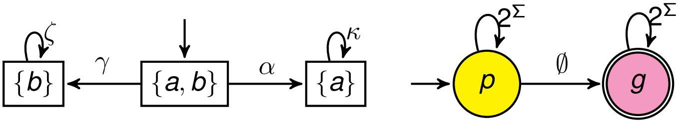

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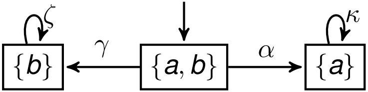


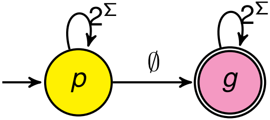


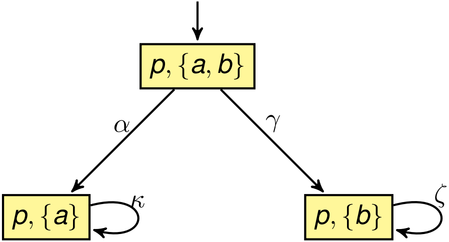

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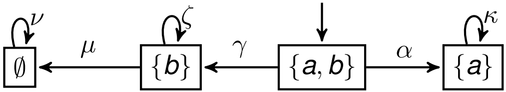

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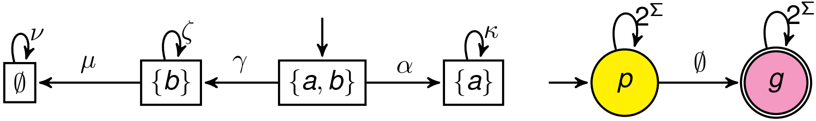

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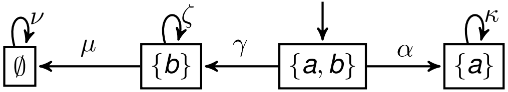
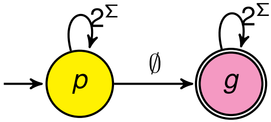
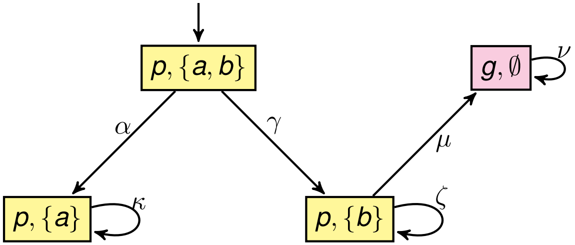


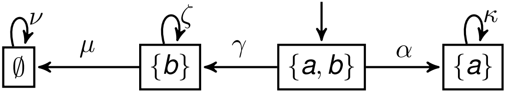
# Product of TS and NBA

Given $TS = (S, Act, I, AP, L)$ and $\mathcal{A} = (Q, 2^{AP}, \delta, Q_0, G)$ NBA.
Define $TS \otimes \mathcal{A} = (S \times Q, Act, I', AP', L')$ such that

- $I' = \{(s_0, q) \mid s_0 \in I \text{ and } \exists q_0 \in Q_0, q_0 \stackrel{L(s_0)}{\rightarrow} q\}$
- $AP' = Q$, $L' : S \times Q \rightarrow 2^Q$ such that $L'((s, q)) = \{q\}$
- If $s \stackrel{\alpha}{\rightarrow} t$ and $q \stackrel{L(t)}{\rightarrow} p$, then $(s, q) \stackrel{\alpha}{\rightarrow} (t, p)$

# Persistence Properties

Let $\eta$ be a propositional logic formula over *AP*. A persistence property $P_{pers}$ has the form $\Diamond\Box\eta$. How will you check a persistence property on a TS?

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- For example, $TS \models \Diamond\Box(a \vee (a \rightarrow b))$

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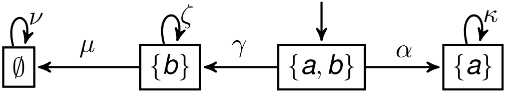


- For example, $TS \not\models \Diamond\Box(a \vee b)$
- For example, $TS \models \Diamond\Box(a \vee (a \rightarrow b))$
- $TS \not\models P_{pers}$ iff there is a reachable cycle in the TS containing a state with a label which satisfies $\neg\eta$.

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- Construct $A_{\neg\varphi}$, and let $g_1, \ldots, g_n$ be the good states in $A_{\neg\varphi}$.
- Build $TS' = TS \otimes A_{\neg\varphi}$.
- The labels of $TS'$ are the state names of $A_{\neg\varphi}$.
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**ModelChecking LTL in $TS$ = Check Persistence in $TS'$**

The following are equivalent.

- $TS \models \varphi$
- $Tr(TS) \cap L(A_{\neg\varphi}) = \emptyset$
- $TS' \models \Diamond\Box(\neg g_1 \wedge \ldots \neg g_n)$.