1/10/19
Tue

Butler Matrix ⇒

→ $\vec{E}_{array} = \vec{E}_{\substack{single\\element}} \times AF$ → Array Factor

→ $\psi = 0$ Gives a 'θ_0' which is the direction of max...

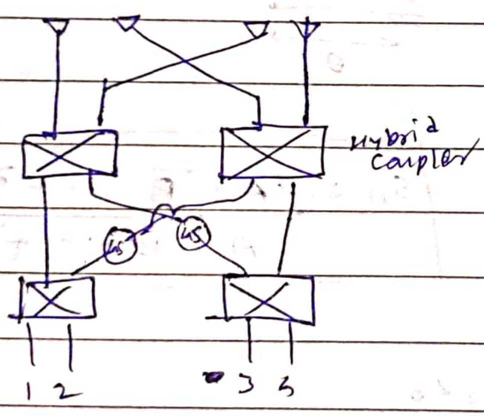
To maximize $\vec{E}_{array} \Rightarrow$ max AF $\Rightarrow \psi = 0 \Rightarrow$ some θ_0

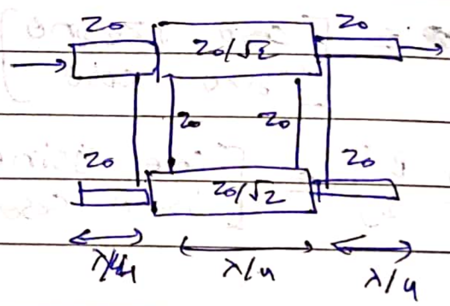
$\left(\text{as } AF = \dfrac{\sin\left(\frac{N\psi}{2}\right)}{\frac{N\psi}{2}}\right)$

& sinc( )
max at $\psi=0$

→ $\psi = kd\cos\theta + \beta = 0 \Rightarrow \beta = -kd\cos\theta$

For our choice of ~~max~~ direction of max. rad. θ we get a certain β

→ 

→ 

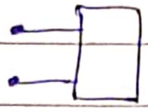
→ In ADS, for Designing Hybrid ~~Bo~~ Coupler each TLine has F which ~~must be set~~ is the frequency at which $l = \frac{\lambda}{4}$ s.t. $\theta = \beta l = \frac{2\pi}{\lambda} \cdot \frac{\lambda}{4} = \frac{\pi}{2}$ // which is required in Hybrid coupler

1/10/19 Tue

1. Ideal T-Line
2. Real T-lines MLIN, MSUBS
3. E Simulations

★ 1 port microwave circuit ⇒

$[S_{11}]$

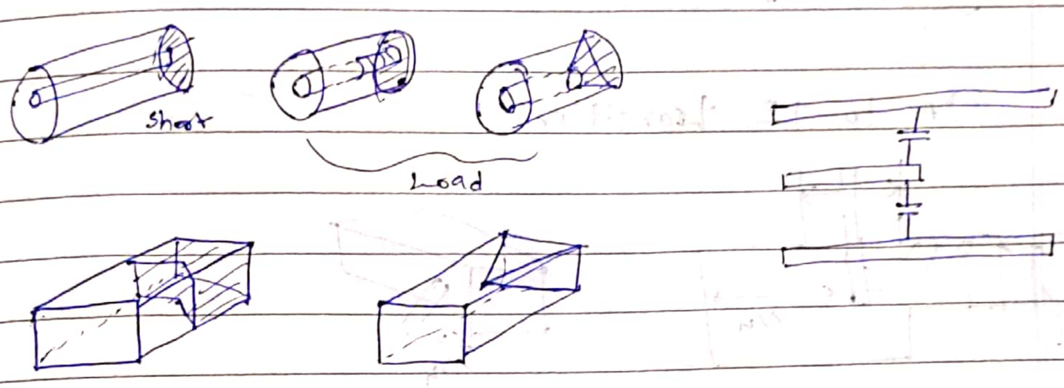


$S^t = S$

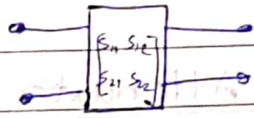
$S^* S^t = [U]$

$\Rightarrow |S_{11}| = 1$

•



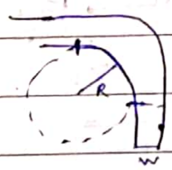
★ 2 poot Microwave Circuits ⇒



• $\begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$ $\underbrace{|S_{11}|\ \theta_1\ \theta_2}_{\text{Params.}}$

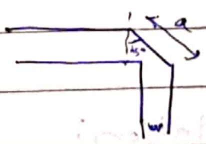
1. Transmission line
2. bends
3. discontinuities
4. Transition
5. Attenuation
6. Mode suppressor

• Radial bend:



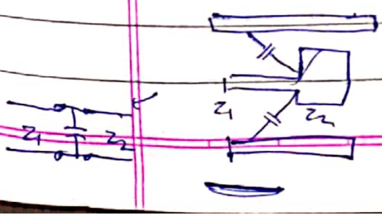
$R >> 3W$

• Miteral Bend:

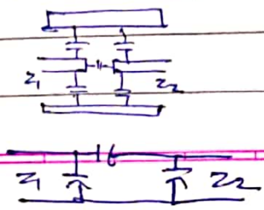


$a = 1.8W$

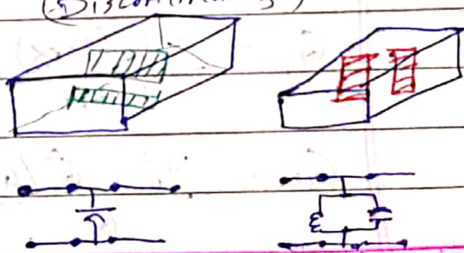
① Discontinuity



② Gap

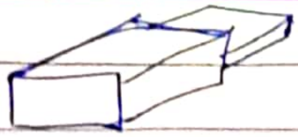


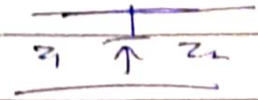
(Discontinuities)



(Strips along broader side)

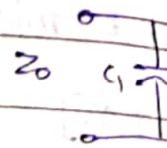
(strips along shorter side)



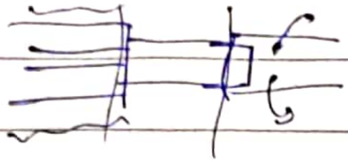


☆ SMA Launcher →

- For min. VSWR, $L = Z_0^2 (C_1 + C_2)$

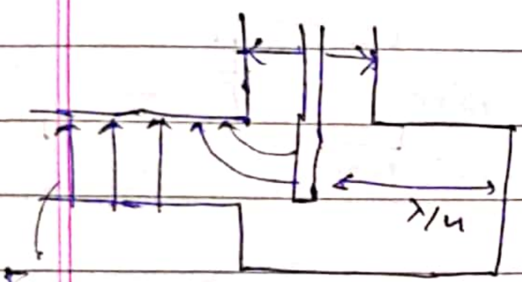


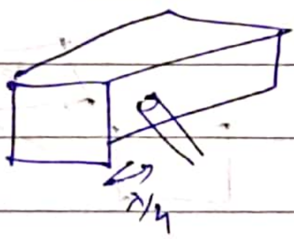
- TEM → TEM



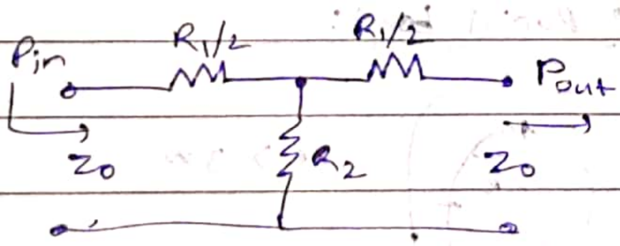
E

- TEM to TE transition





☆ Attenuator ⇒

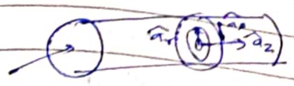


$$\alpha = \frac{P_{out}}{P_{in}} = |S_{21}|^2$$

$$R_1 = \frac{2Z_0(1-\alpha)}{1+\alpha}$$

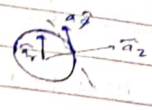
$$R_2 = Z_0 - R_1^2/4$$

☆ mode Suppressor →

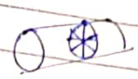


TE_{10}

$\hat{a}_\phi$ will be nullified due to metallic part



TM_{01} suppressor



☒ 3 port device →

$$\begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{21} & 0 & S_{23} \\ S_{31} & S_{32} & 0 \end{bmatrix}$$

$S_{12} = S_{21}$

$S_{13} = S_{31}$

$S_{23} = S_{32}$

$$S^* S^t = [U]$$

$$\begin{bmatrix} 0 & S_{12}^* & S_{13}^* \\ S_{12}^* & 0 & S_{23}^* \\ S_{13}^* & S_{23}^* & 0 \end{bmatrix} \begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{12} & 0 & S_{23} \\ S_{13} & S_{23} & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\Rightarrow \quad |S_{12}|^2 + |S_{13}|^2 = 1 \qquad S_{13}^* S_{23} = 0$

$|S_{12}|^2 + |S_{23}|^2 = 1 \qquad S_{12}^* S_{23} = 0$

$|S_{13}|^2 + |S_{23}|^2 = 1 \qquad S_{23}^* S_{13} = 0$

Now, it is mathematically impossible to achieve lossless, reciprocal & matched at the same time. At most we can achieve 2 out 3 properties

☆	Circulator	Tee	Power Divider
Lossless	✓	✓	
Matched at all ports	✓		✓
reciprocal		✓	✓

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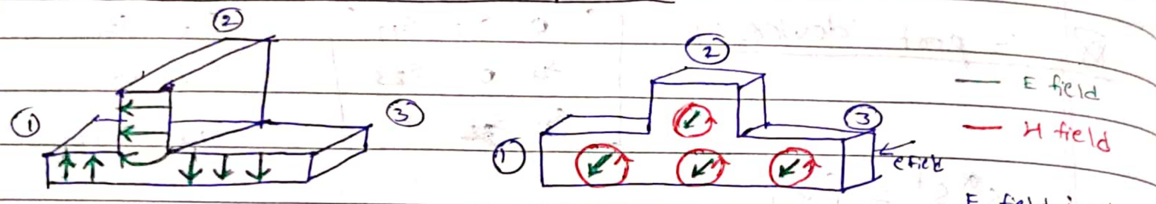
4/10/19
Fri

☆ 3 port devices →

1. Matched at all ports
2. Lossless
3. Reciprocal

1,2 → Circulant
1,3 → Power divider
2,3 → tee

☆ Tee Configuration in 3 port →



E plane Tee

H-plane Tee

$$\rightarrow \left\{ \begin{aligned} S_{12} &= -S_{32} \\ S_{11} &= S_{33} \\ S_{22} &= 0 \end{aligned} \right\} \qquad \left\{ \begin{aligned} S_{12} &= S_{32} \\ S_{11} &= S_{33} \\ S_{22} &= 0 \end{aligned} \right\}$$

for S_{ij} nature check by giving input wave at j

$$** \quad S^* S^t = [U]$$

E plane Tee

$$\Rightarrow \begin{bmatrix} S_{11}^* & S_{12}^* & S_{13}^* \\ S_{12}^* & 0 & -S_{12}^* \\ S_{13}^* & -S_{12}^* & S_{11}^* \end{bmatrix} \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{12} & 0 & -S_{12} \\ S_{13} & -S_{12} & S_{11} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow |S_{11}|^2 + |S_{12}|^2 + |S_{13}|^2 = 1$$

$$|S_{12}|^2 + |S_{12}|^2 = 1 \Rightarrow \boxed{|S_{12}| = \frac{1}{\sqrt{2}}}$$

$$S_{11}^* S_{12}^* - S_{12}^* S_{13} = 0$$

$$\Rightarrow S_{12}^* (S_{11} - S_{13}) = 0$$

$$\Rightarrow \boxed{S_{11} = S_{13}} \quad (\text{as } S_{12} \neq 0)$$

Put this in 1st eqn $\Rightarrow 2|S_{11}|^2 + \frac{1}{2} = 1 \Rightarrow \boxed{|S_{11}| = \frac{1}{2}}$

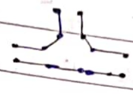
$$\therefore \; [S] = \begin{bmatrix} \frac{1}{2}e^{j\theta_{11}} & \frac{1}{\sqrt{2}}e^{j\theta_{12}} & \frac{1}{2}e^{j\theta_{11}} \\ \frac{1}{\sqrt{2}}e^{j\theta_{12}} & 0 & -\frac{1}{\sqrt{2}}e^{j\theta_{12}} \\ \frac{1}{2}e^{j\theta_{11}} & -\frac{1}{\sqrt{2}}e^{j\theta_{12}} & \frac{1}{2}e^{j\theta_{11}} \end{bmatrix}$$

→ remaining variables θ_{11}, θ_{12}

$$[S] = \begin{bmatrix} \frac{1}{2} & \frac{1}{\sqrt{2}} & \frac{1}{2} \\ \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} \\ \frac{1}{2} & -\frac{1}{\sqrt{2}} & \frac{1}{2} \end{bmatrix}$$

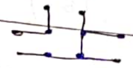
$\theta_{11} = \theta_{12} = 2n\pi$ why?

By shifting plane 1 & 2



E Plane Tee

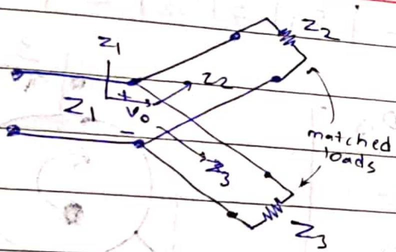
* $[S] = \begin{bmatrix} \frac{1}{2} & \frac{1}{\sqrt{2}} & \frac{1}{2} \\ \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & \frac{1}{\sqrt{2}} & \frac{1}{2} \end{bmatrix}$



→ just $S_{12} = S_{32}$ instead of $S_{12} = -S_{32}$

H Plane Tee

*



$Z_1 = Z_2 \parallel Z_3$

- For $Z_2 = Z_3$, $Z_1 = \frac{Z_2}{2}$

s.t. $Z_2 = Z_3 = 2Z_1$

- From power's perspective

$P_1 = \frac{V_0^2}{Z_1}$, $P_2 = \frac{V_0^2}{Z_2}$, $P_3 = \frac{V_0^2}{Z_3}$

(matching at 2,3) $P_1 = P_2 + P_3 = 2P_2$

$\frac{V_0^2}{Z_1} = \frac{2V_0^2}{Z_2} \Rightarrow Z_2 = 2Z_1$

- Now if Z_1 is also matched then from Z_2 we see that

$Z_2 = Z_1 \parallel Z_3$

$= Z_1 \parallel 2Z_1$ (if Z_2, Z_3 are still matched)

$= \frac{2}{3}Z_1$

Not possible as $Z_2 = 2Z_1$ for Z_2, Z_3 matching

If 3 port network is lossless all 3 matching cannot hold at same time as for one matching (2,3) we get $Z_2 = 2Z_1$ for (1,3) matching we get $Z_2 = \frac{2}{3}Z_1$ So a LOSSLESS network cannot satisfy matching at all 3 ports

* Circulators →

Satisfies
$$\begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{21} & 0 & S_{23} \\ S_{31} & S_{32} & 0 \end{bmatrix}$$

* $S^* S^t = [U]$

$$\begin{bmatrix} 0 & S_{12}^* & S_{13}^* \\ S_{21}^* & 0 & S_{23}^* \\ S_{31}^* & S_{32}^* & 0 \end{bmatrix} \begin{bmatrix} 0 & S_{21} & S_{31} \\ S_{12} & 0 & S_{32} \\ S_{13} & S_{23} & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

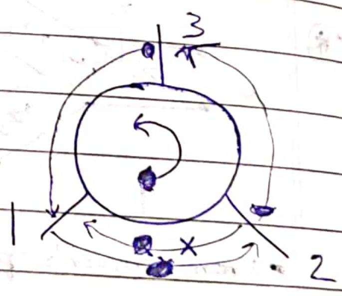
$$|S_{12}|^2 + |S_{13}|^2 = |S_{21}|^2 + |S_{23}|^2 = |S_{31}|^2 + |S_{32}|^2 = 1$$

$$S_{13}^* S_{23} = S_{21}^* S_{31} = S_{32}^* S_{12} = 0$$

Lets take, $S_{23} = S_{31} = S_{12} = 0$

This gives us, $|S_{13}| = |S_{21}| = |S_{32}| = 1$

* ⇒ $[S] = \begin{bmatrix} 0 & 0 & e^{j\theta_{13}} \\ e^{j\theta_{21}} & 0 & 0 \\ 0 & e^{j\theta_{32}} & 0 \end{bmatrix}$



For $\theta_{13} = \theta_{21} = \theta_{32} = 2\pi$

$$[S] = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

1 to 2 is possible (non zero voltage)

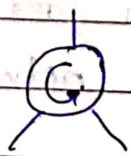
2 to 1 is not possible (zero voltage)

Showing non reciprocity

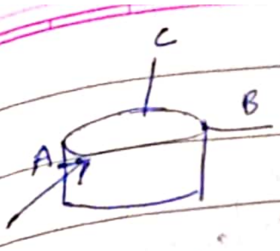
anticlockwise sol^n

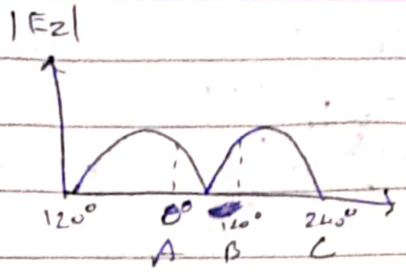
* If we took $S_{13} = S_{21} = S_{32} = 0$ we get for $\theta_{12} = \theta_{31} = \theta_{23}$

$$[S] = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$



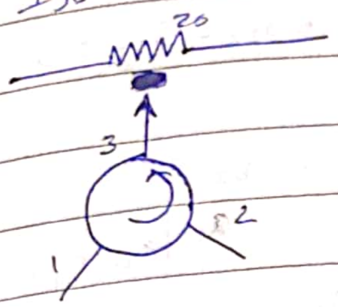
Clockwise sol^n





- No power at C (destructive interference)
- If we introduce power at port A then Port C will have no power

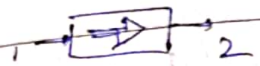
Isolation



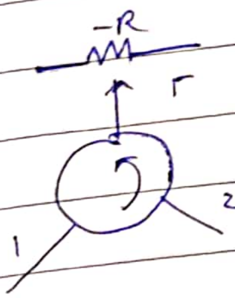
$$\begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

- If we introduce power at port 2 port 3 no longer has absence of power & port 1 will be isolated.
- We are using Circulator as a blocker to isolate.

$$\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$$



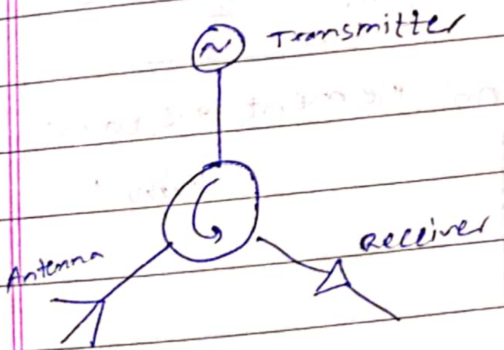
For $R<0$ we can use circulator as an amplifier



$|\Gamma| > 1$

$$\Gamma = \frac{Z-Z_0}{Z+Z_0} = \frac{R+jX-Z_0}{R+jX+Z_0} \Rightarrow |\Gamma| = \sqrt{\frac{(R-Z_0)^2+X^2}{(R+Z_0)^2+X^2}}$$

For $R<0$, $|\Gamma|>1$



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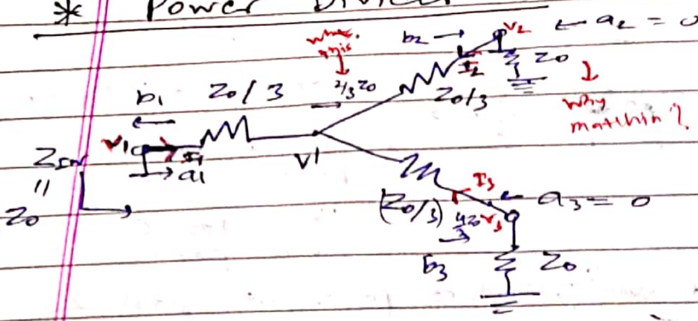
* Power Divider ⇒



- $S_{11} = \left.\frac{b_1}{a_1}\right|_{a_2 = a_3 = 0}$

$\Gamma_{IN} = \frac{Z_0 - Z_0}{Z_0 + Z_0} = 0$

- $\boxed{S_{11} = S_{22} = S_{33} = 0}$

$I_1 = \frac{V_1}{Z_0}$

$V' = V_1 - \frac{I_1 Z_0}{3}$

$V_2 = V' - \frac{I_1}{2} \cdot \frac{Z_0}{3}$

$= V_1 - \frac{I_1 Z_0}{3} - \frac{I_1 Z_0}{6}$

$V_2 = \frac{V_1}{2}$

$\Rightarrow \underset{0}{a_2} + b_2 = \frac{a_1 + b_1}{2}$

$\Rightarrow 2b_2 = a_1 + b_1$ —(2)

- For $S_{21} = \left.\frac{b_2}{a_1}\right|_{a_2 = a_3 = 0}$

$I_2 = -\frac{I_1}{2}$

$\frac{\overset{0}{a_2} - b_2}{\sqrt{Z_0}} = -\frac{1}{2}\left(\frac{a_1 - b_1}{\sqrt{Z_0}}\right)$

$\Rightarrow \frac{a_1 - b_1}{2} = b_2$ —(1)

from (1) & (2) $a_1 = 2b_2$, $\Rightarrow \boxed{\frac{b_2}{a_1} = \frac{1}{2} = S_{12}}$

from symmetry all off diagonal elements are equal

s.t. $[S] = \begin{bmatrix} 0 & 1/2 & 1/2 \\ 1/2 & 0 & 1/2 \\ 1/2 & 1/2 & 0 \end{bmatrix}$ Why?

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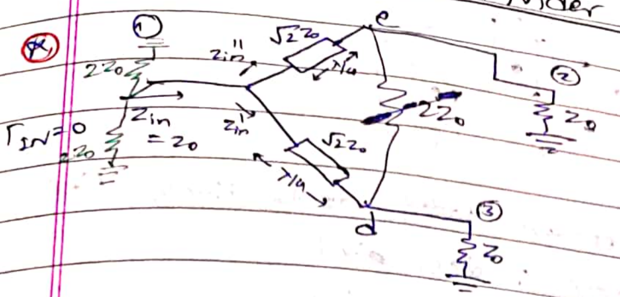
☆ Wilkinson Power Divider ⇒

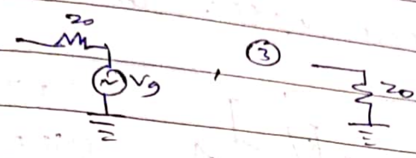


$$Z_{IN}' = \frac{(\sqrt{2}Z_0)^2}{Z_0} = 2Z_0$$

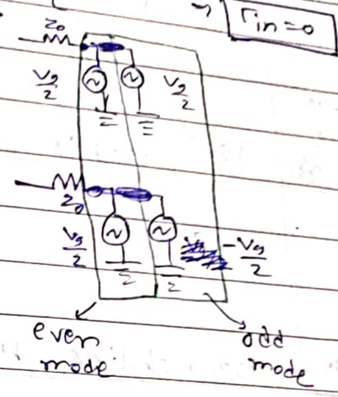
$$Z_{IN}'' = 2Z_0$$

$$\Rightarrow Z_{in} = (Z_{in}' \parallel Z_{in}'') = Z_0$$

For ② , ③



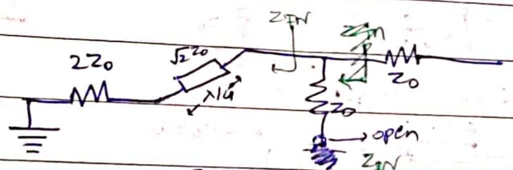
Take



$\Rightarrow \boxed{\Gamma_{in} = 0}$

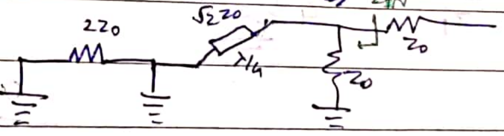
- Due to symmetry of circuit & applying same voltage ② & ③
- across $2Z_0$ we have short. i.e. $\underline{V_e = V_d}$ such that no current flows in $2Z_0$

Even Mode ckt :



$$Z_{IN} = \frac{(\sqrt{2}Z_0)^2}{2Z_0} = Z_0$$

Odd Mode ckt :



$Z_{IN} = Z_0$

$\Rightarrow \Gamma_{IN} = 0$

∴ as $\Gamma_{IN} = 0$ in ②, ③ ⇒ $S_{22} = S_{33} = 0$ also $S_{11} = 0$ from before

port ② & ③ are isolated ⇒ $S_{23} = S_{32} = 0$

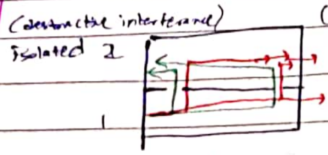
By symmetry ⇒ $\underbrace{S_{21} = S_{12}}_{\text{reciprocity}} = \underbrace{S_{13} = S_{31}}_{\text{reciprocity}}$

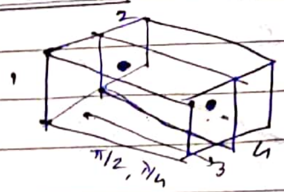
$$\therefore \boxed{[S] = \frac{1}{\sqrt{2}}\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}}$$

For equal power distribution,

$$|S_{12}| = |S_{13}| = \frac{1}{\sqrt{2}}$$

★ 4 Port Devices : Couplers ⇒





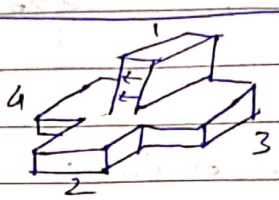
~~at ② 0° & π° phase waves are~~ added destructively Copp

at ② 0 & π phase waves : des. int

③ π & π phase waves : cons. int

16/10/19
Tue

★ 180° coupler ⇒



$S_{12} = 0 = S_{21}$

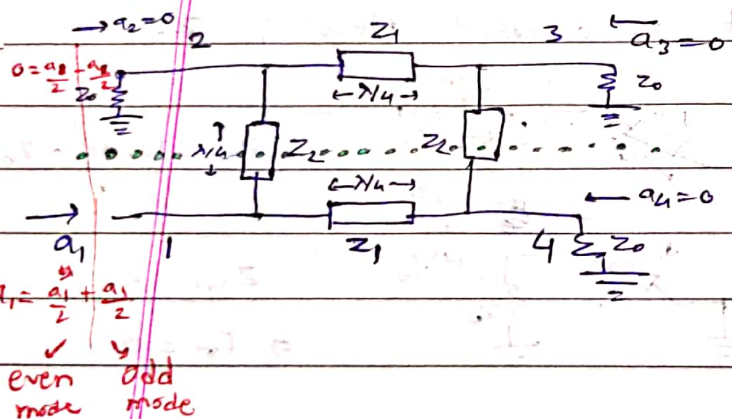
$(S_{ii} = 0)$ $i = 1, 2, 3, 4$

$S_{42} = S_{32} = \frac{1}{\sqrt{2}}$

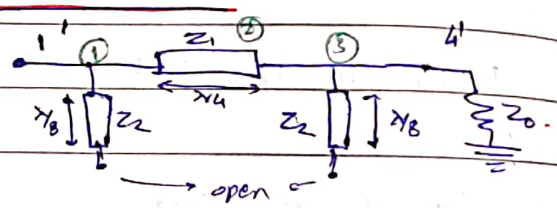
$S_{41} = -S_{31} = 1/\sqrt{2}$

$S_{34} = S_{43} = 0$

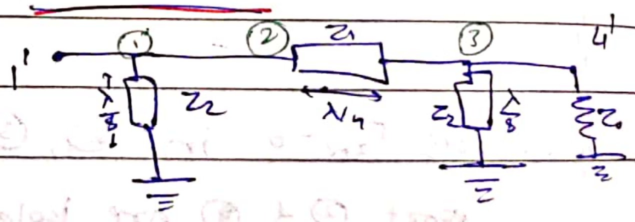
$\boxed{\text{★ Branchline Hybrid Coupler ⇒}}$



Even mode



Odd mode



(*) $b_1 = S_{11e}\frac{a_1}{2} + S_{11o}\frac{a_1}{2}$

$$\Rightarrow \boxed{S_{11} = \frac{b_1}{a_1} = \frac{S_{11e}}{2} + \frac{S_{11o}}{2}}$$

(*) $b_2 = S_{11e}\frac{a_1}{2} + S_{11o}\left(\frac{-a_1}{2}\right)$ ⟶ $S_{22} = S_{11}$ as after dividing network both sides are identical

$$\boxed{S_{21} = \frac{b_2}{a_1} = \frac{S_{11e}}{2} - \frac{S_{11o}}{2}}$$

$$b_4 = S_{4'1'e}\frac{a_1}{2} + S_{4'1'o}\left(\frac{a_1}{2}\right)$$

$$\Rightarrow \boxed{S_{41} = \frac{b_4}{a_1} = \frac{S_{4'1'e}}{2} + \frac{S_{4'1'o}}{2}}$$

$$b_3 = S_{3'1'e}\frac{a_1}{2} + S_{3'1'o}\left(\frac{-a_1}{2}\right)$$

$$\Rightarrow \boxed{S_{31} = \frac{b_3}{a_1} = \frac{S_{3'1'e}}{2} - \frac{S_{3'1'o}}{2}}$$

* $\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 0 & jZ_1 \\ jY_1 & 0 \end{bmatrix}$; for Tx line

* $\frac{1}{Y} = Z = \begin{cases} -jZ_2\cot\theta \rightarrow \text{open (even)} \\ jZ_2\tan\theta \rightarrow \text{short (odd)} \end{cases} = \begin{cases} -jZ_2 & (\text{open}) \\ +jZ_2 & (\text{short}) \end{cases}$

($\theta = \pi/4$ for $\lambda/8$ in Z_2)

* $[m]_{e/o} = \begin{bmatrix} 1 & 0 \\ \pm jY_2 & 1 \end{bmatrix}\begin{bmatrix} 0 & jZ_1 \\ jY_1 & 0 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ \pm jY_2 & 1 \end{bmatrix}$ $+ \rightarrow$ odd, $- \rightarrow$ even

$$= \begin{bmatrix} \mp Z_1/Z_2 & jZ_1 \\ jZ_1\left(\frac{1}{Z_1^2} - \frac{1}{Z_2^2}\right) & \mp\frac{Z_1}{Z_2} \end{bmatrix}$$

* $$S_{1'1'e/o} = \frac{\cancel{A_{e/o}} + \frac{B_{e/o}}{Z_0} - C_{e/o}Z_0 - \cancel{D_{e/o}}}{A_{e/o} + B_{e/o} + C_{e/o} + D_{e/o}}$$

$$= \frac{\frac{B_{e/o}}{Z_0} - C_{e/o}Z_0}{A_{e/o} + B_{e/o} + C_{e/o} + D_{e/o}}$$

$$S_{4'1'e/o} = \frac{2}{A_{e/o} + B_{e/o} + C_{e/o} + D_{e/o}}$$

for $S_{11} = S_{21}$

$\Rightarrow S_{1'1'e} = S_{1'1'o} = 0$

$\Rightarrow \boxed{\frac{B_{e/o}}{Z_0} = C_{e/o}Z_0}$

$\Rightarrow \boxed{Z_2 = \frac{Z_1}{\sqrt{1 - \frac{Z_1^2}{Z_0^2}}}}$

$$S_{31} = \frac{1}{2} S_{41}'^{e} - \frac{1}{2} S_{41}'^{o} = -\sqrt{1 - \frac{Z_1^2}{Z_0^2}}$$

$$S_{41} = \frac{1}{2} S_{41}'^{e} + \frac{1}{2} S_{41}'^{o} = -j\frac{Z_1}{Z_0}$$

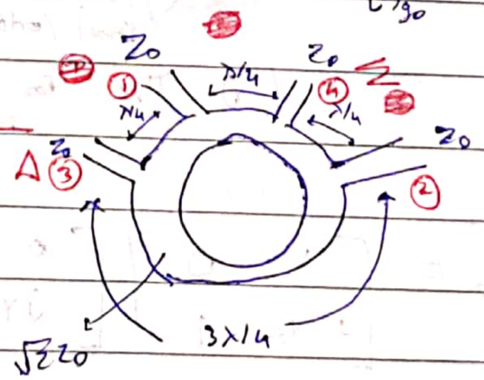
$$|S_{31}| = |S_{41}| \Rightarrow Z_1 = \frac{Z_0}{\sqrt{2}}, \quad Z_2 = Z_0.$$

$$[S] = \begin{bmatrix} 0 & 0 & -\sqrt{1-\frac{Z_1^2}{Z_0^2}} & -j\frac{Z_1}{Z_0} \\ 0 & 0 & -jZ_1/Z_0 & -\sqrt{1-\frac{Z_1^2}{Z_0^2}} \\ -\sqrt{1-\frac{Z_1^2}{Z_0^2}} & -j\frac{Z_1}{Z_0} & 0 & 0 \\ -j\frac{Z_1}{Z_0} & -\sqrt{1-\frac{Z_1^2}{Z_0^2}} & 0 & 0 \end{bmatrix}$$

After putting $Z_1 = Z_0/\sqrt{2}$, $Z_2 = Z_0$ $\Rightarrow$ $[S] = \frac{-1}{\sqrt{2}}\begin{bmatrix} 0 & 0 & 1 & j \\ 0 & 0 & j & 1 \\ 1 & j & 0 & 0 \\ j & 1 & 0 & 0 \end{bmatrix}$

" $[S]_{90}$

* RAT-RACE Coupler ⇒



3,4 → delta & Sigma po

$$[S] = \frac{-j}{\sqrt{2}}\begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & -1 & 1 \\ 1 & -1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix} = \frac{-j}{\sqrt{2}}\begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & -1 & 1 \\ 1 & -1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix}\begin{bmatrix} a_1 \\ a_2 \\ 0 \\ 0 \end{bmatrix}$$

$$b_3 = -j/\sqrt{2}\,(a_1 - a_2)$$

$$b_4 = -j/\sqrt{2}\,(a_1 + a_2)$$