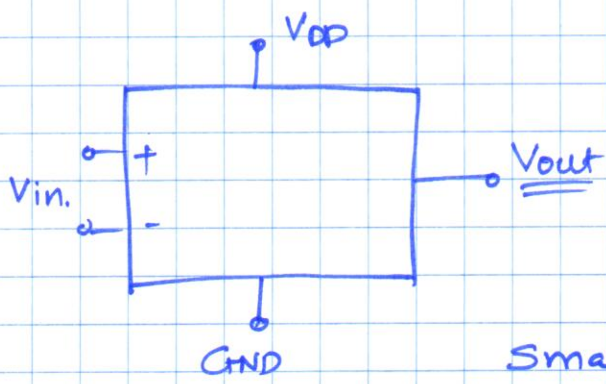
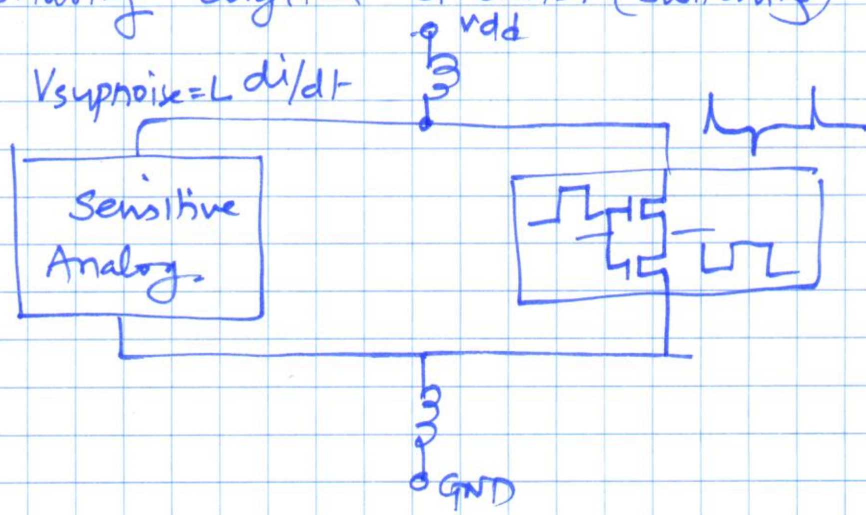
Power Supply Rejection Ratio PSRR



Supply noise either on Vdd or GND can leach into Vout - limiting smallest signal we can detect.

Source of Supply noise

* Off-chip noise electrically coupled through pins
* Noise of VDD (regulators - switching power supplies)
* Noise coupled through VDD / GND bondwires sharing digital circuits. (switching)

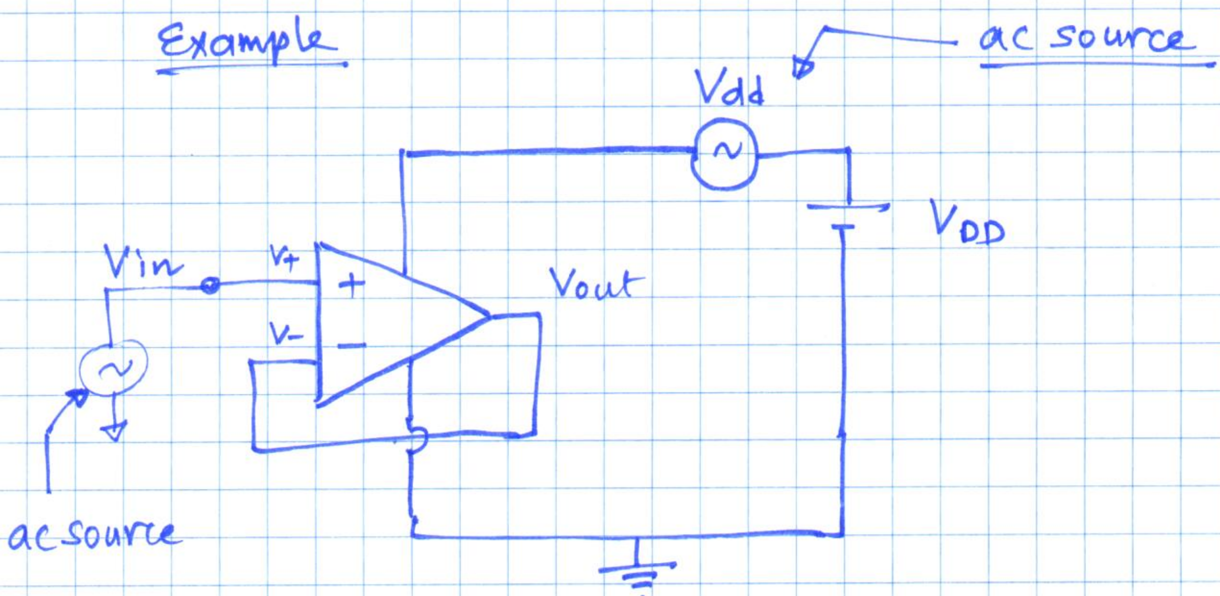


$$\text{PSRR (definition)} = \frac{A_{dm} \left(\sim V_{dd} \text{ or } \sim \text{Gnd} = 0 \right)}{A_{ps} \left(V_{in} = 0 \right)}$$

A_{ps} = AC gain from Vdd or Gnd to Vout when $V_{in} = 0$.

Measuring PSRR

Example



To calculate gain from Vdd to Vout   Vin = 0

$$V_{out} = A_{ps_{Vdd}} V_{dd} + A_{dm}(V_+ - V_-)$$

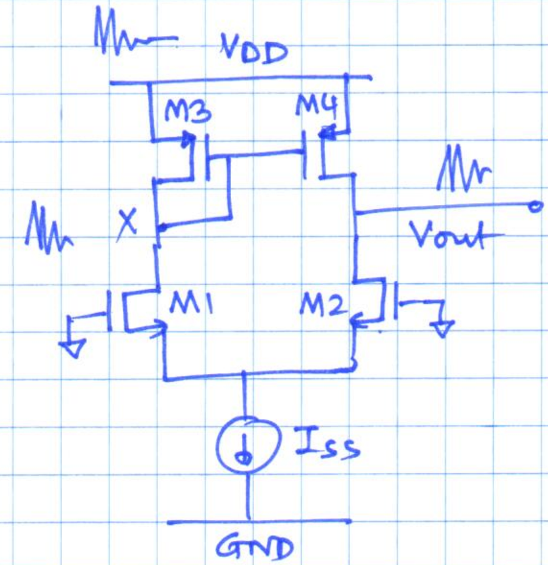
$$V_{out} = A_{ps_{Vdd}} V_{dd} + A_{dm}(0 - V_{out})$$

$$V_{out}(1 + A_{dm}) = A_{ps_{Vdd}} V_{dd}$$

$$\frac{V_{out}}{V_{dd}} = \frac{A_{ps_{Vdd}}}{(1 + A_{dm})} \cong \frac{A_{ps_{Vdd}}}{A_{dm}} = \frac{1}{PSRR}$$

Caution: During PSRR measurements, make sure all biasing circuits are included.

Insights on PSRR

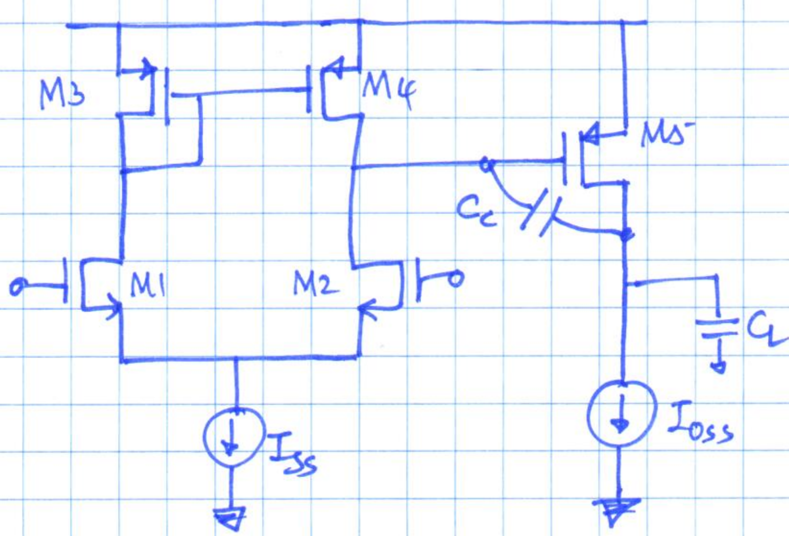


unity gain from V_{DD} to V_{out}

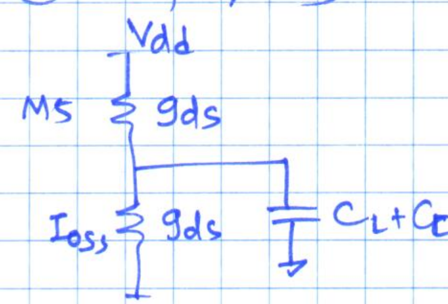
$$A_{P_{S\,VDD}} = 1$$

$$\therefore \; PSRR = \frac{g_{m_N} r_{out}}{1}$$

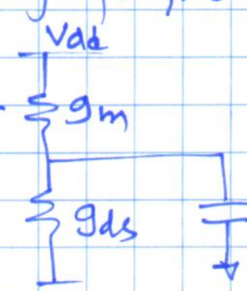
Two Stage OTA



@ Low frequency



@ High frequencies

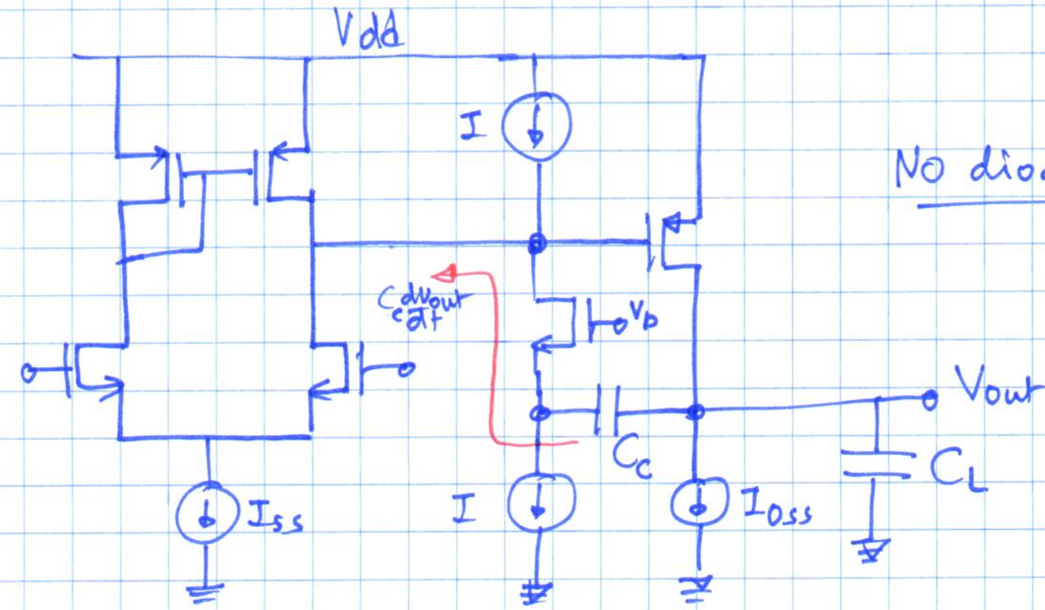


diode connected M5

Poor PSRR @ High frequencies.

To improve High freq. PSRR — C_c decoupling from V_{dd}.

<u>How?</u>



<u>No diode @ HF</u>

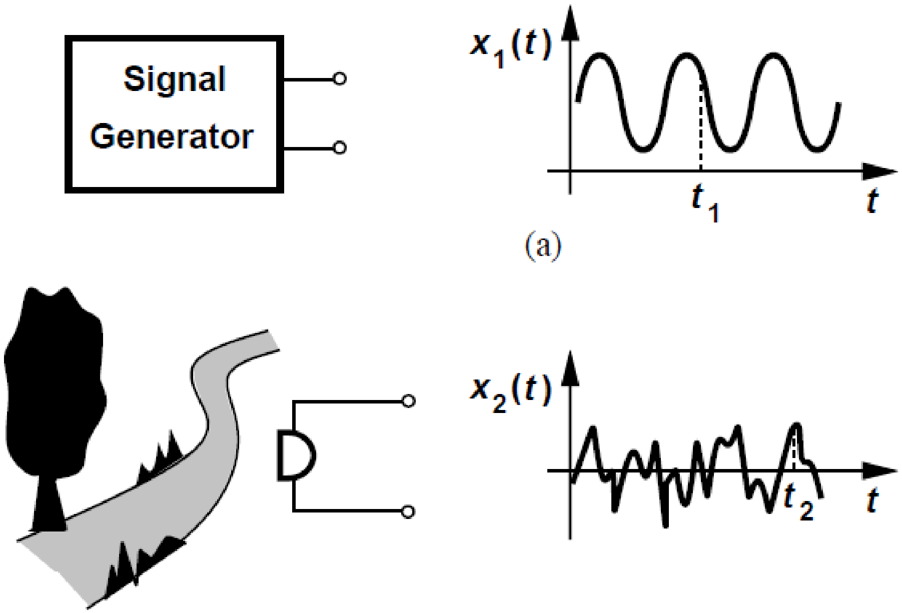
LECTURE 14

24 September 2019

NOISE

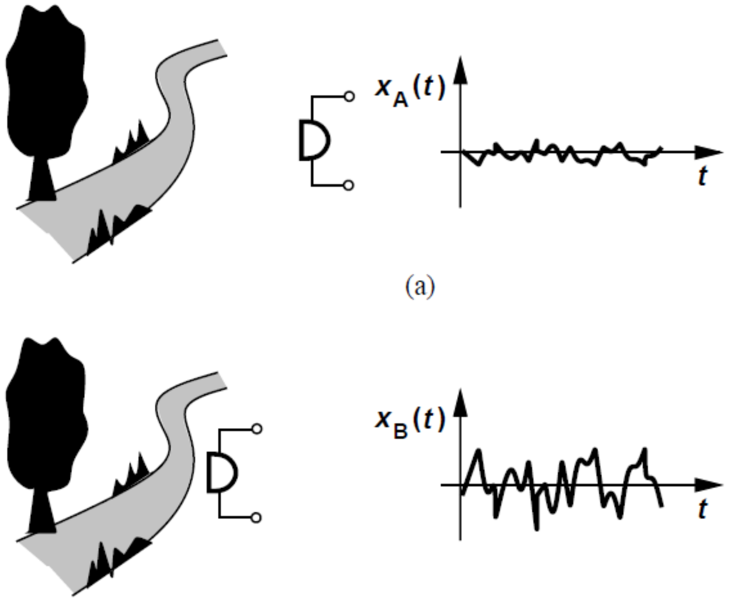
Lecture Notes adapted from Chapter 7
B. Razavi (Design of Analog CMOS ICs)

Statistical Characteristics of Noise



- **Noise - random process**
- **Noise is everywhere**
- ***$x_1(t_1)$ & $x_2(t_2)$* - Difference between deterministic and random**
- **Noise(t) unpredictable**
- **Noise can't be calibrated out**

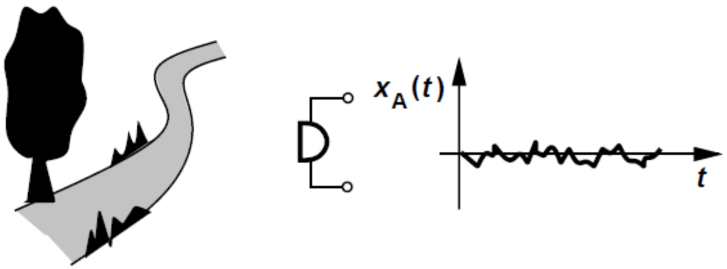
Statistical Characteristics of Noise



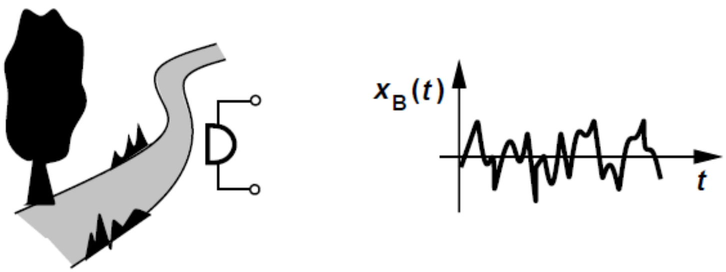
- **Average power of noise is predictable**
- **Average power delivered by a periodic voltage *v(t)* with period *T* to a load resistance R_L is defined as**

$$P_{av} = \frac{1}{T}\int_{-T/2}^{+T/2} \frac{v^2(t)}{R_L} dt$$

Statistical Characteristics of Noise



(a)



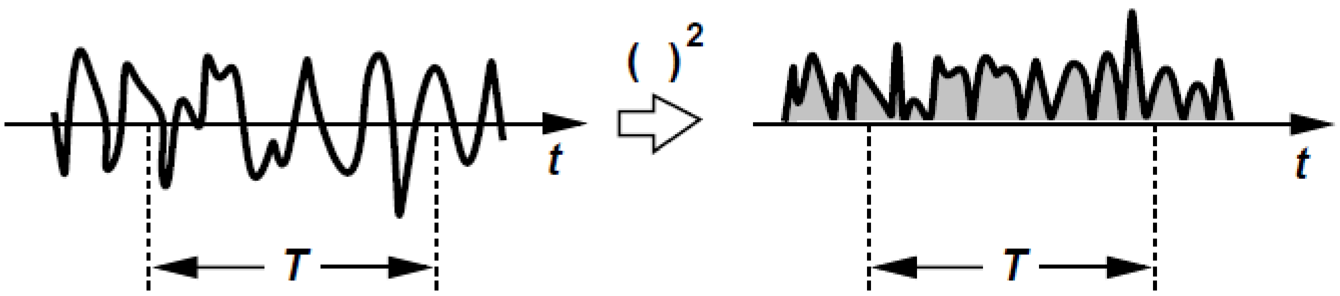
- **Random signal (aperiodic): measurement over a long time**

$$P_{av} = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{+T/2} \frac{x^2(t)}{R_L} dt$$

- ***$x_B(t)$* delivers more power than *$x_A(t)$***

Statistical Characteristics of Noise

- To calculate average power of (noise) signal *x(t)*



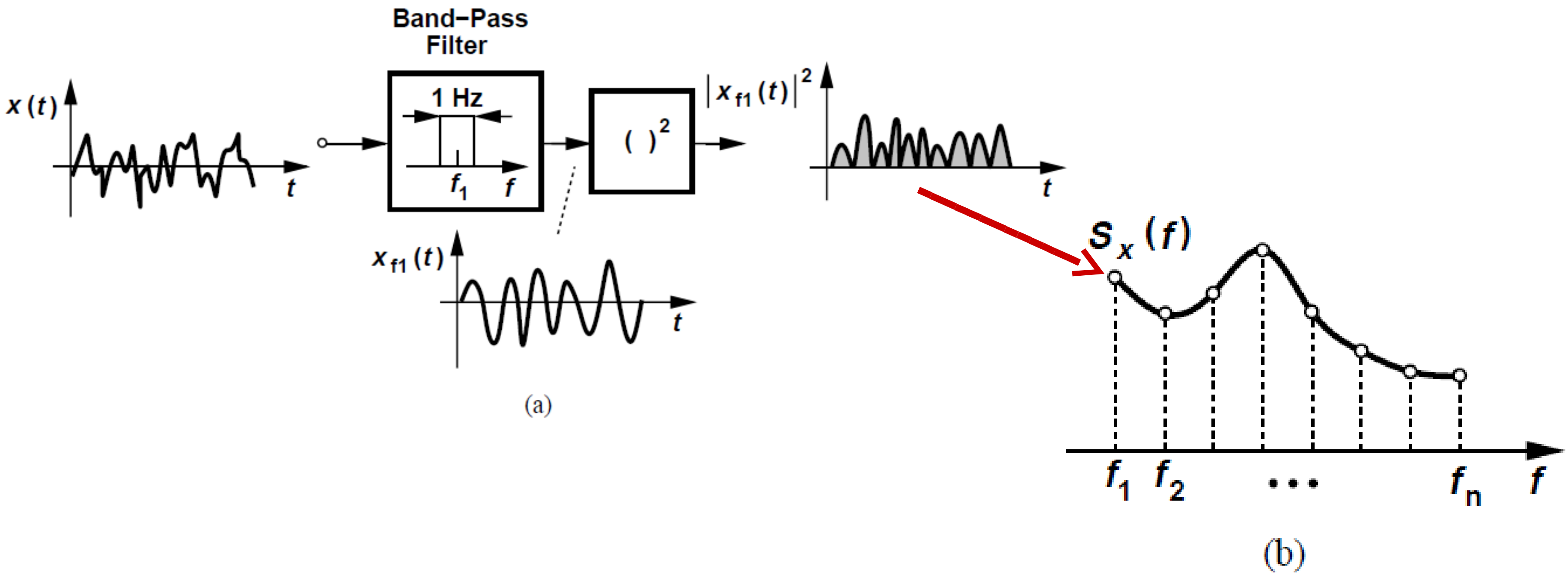
- P_{av} is defined as

$$P_{av} = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{+T/2} x^2(t)dt$$

- P_{av} is expressed in V^2 rather than *W*
- RMS voltage for noise can be defined as $\sqrt{P_{av}}$

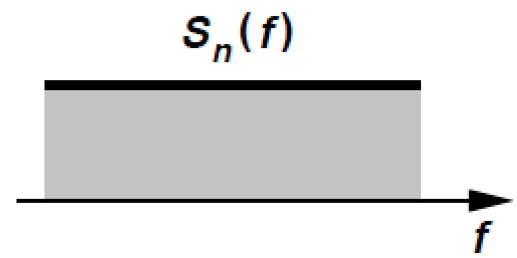
Noise Spectrum

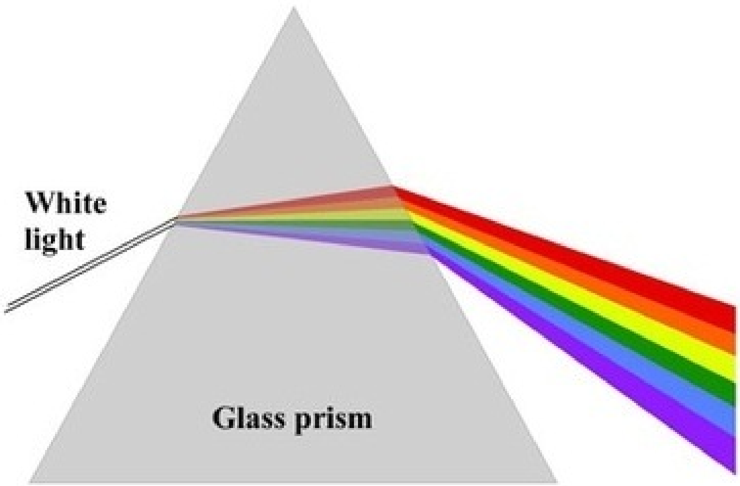
- **Power Spectral Density (PSD) $S_x(f)$**
  - **signal power at each frequency**
- **$S_x(f)$ defined as the average power carried by $x(t)$ in a 1-Hz bandwidth around f**



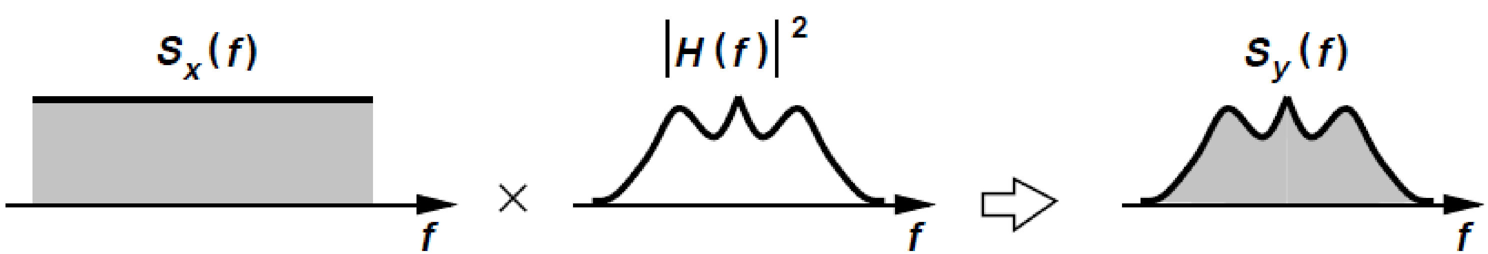
Noise Spectrum

- $S_x(f)$ -- V^2/Hz or $V/\sqrt{Hz}$
- white noise PSD -- Noise spectrum that is flat *in the band of interest* is usually called white



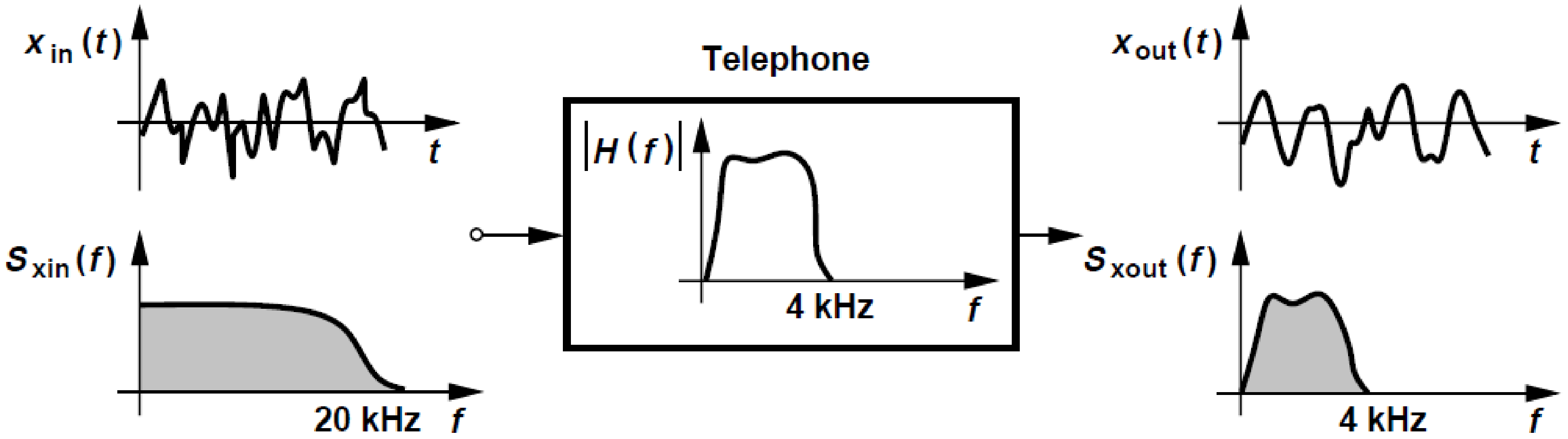


Color	λ (nm)	Freq (Hz)
Red	760-647	$4.3\text{x}10^{14}$
Orange	647-585	$4.3\text{x}10^{14}$
Yellow	585-575	$5.2\text{x}10^{14}$
Green	575-491	$5.6\text{x}10^{14}$
Blue	491-424	$6.6\text{x}10^{14}$
Violet	424-380	$7.3\text{x}10^{14}$



$$S_Y(f) = S_x(f)|H(f)|^2$$

Telephone Example



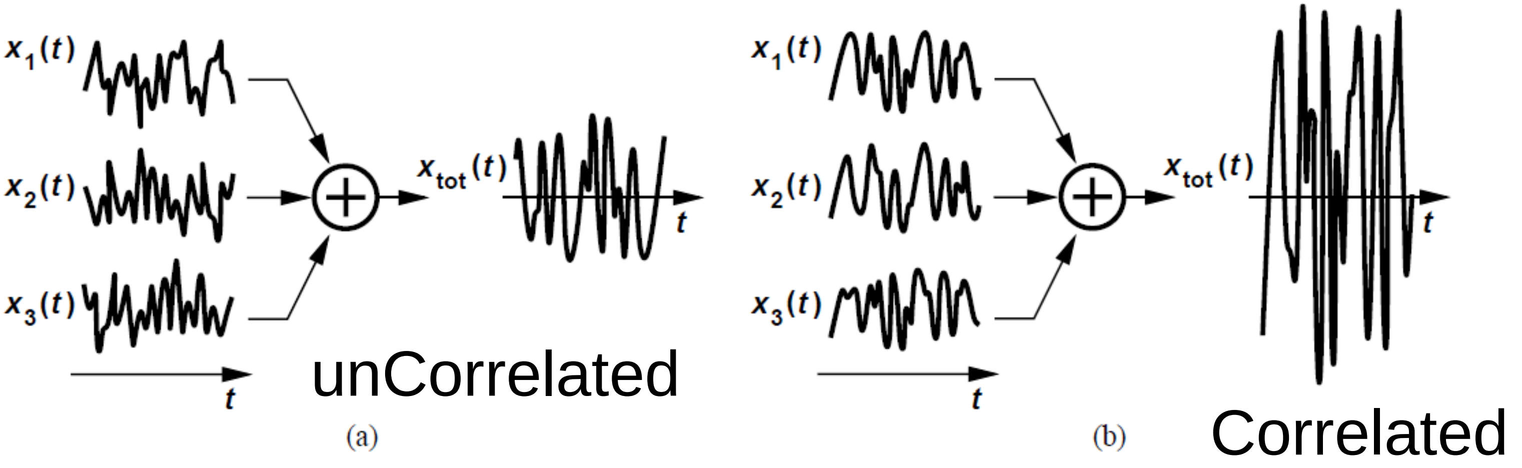
Correlated vs Uncorrelated Sources

- Superposition Principle not suitable for random noise signals
- Average noise power

$$
\begin{aligned}
P_{av} &= \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{+T/2} [x_1(t) + x_2(t)]^2 dt \\
&= \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{+T/2} x_1^2(t)dt + \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{+T/2} x_2^2(t)dt \\
&\quad + \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{+T/2} 2x_1(t)x_2(t)dt \\
&= P_{av1} + P_{av2} + \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{+T/2} 2x_1(t)x_2(t)dt,
\end{aligned}
$$

correlation

Stadium Noise



(a) (b)

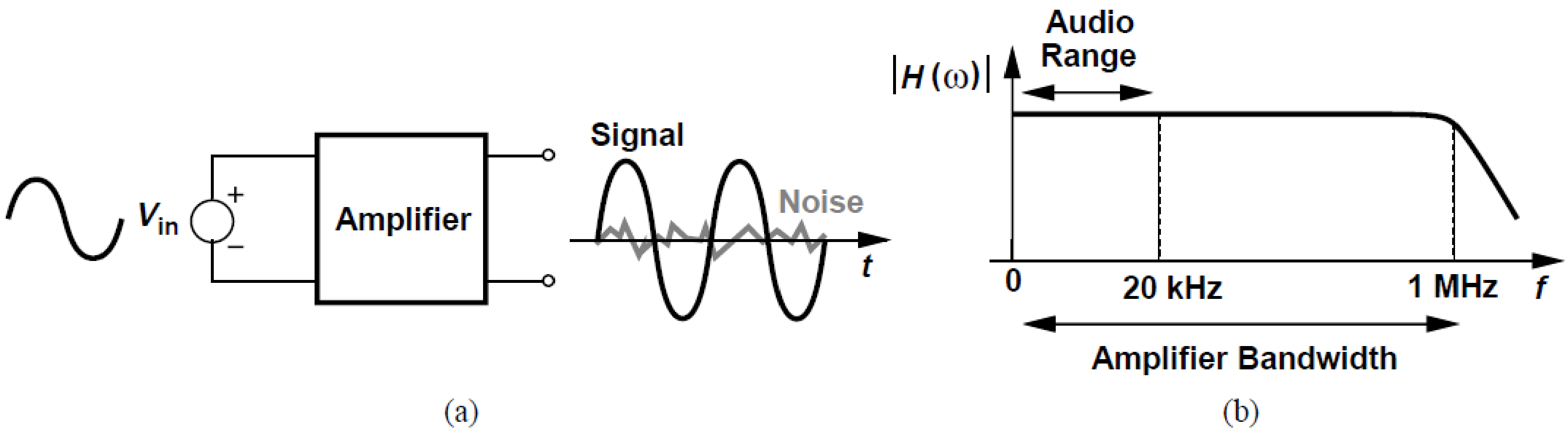
Signal-to-Noise Ratio (SNR)

- **SNR – intelligibility of the signal**

$$\mathrm{SNR} = \frac{P_{sig}}{P_{noise}}$$

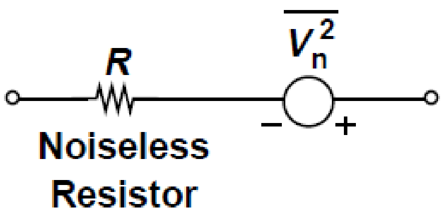
- **For a sinusoid with peak amplitude *A*, $P_{sig} = A^2/2$**
- **Noise Power**

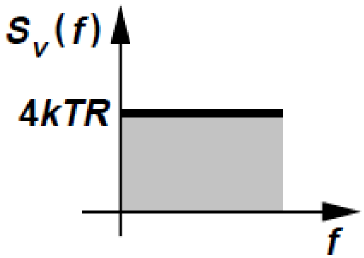
$$P_{noise} = \int_{-\infty}^{+\infty} S_{noise}(f)df.$$



Resistor Thermal Noise

- **Random motion of electrons in a conductor induces fluctuations in the voltage measured across it even though the average current is zero**



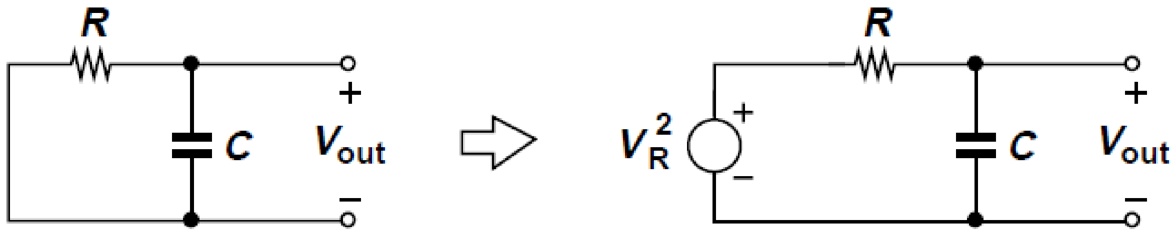


$$S_v(f) = 4kTR, \quad f \geq 0$$

$$\overline{V_n^2} = 4kTR$$

- ***k* = 1.38 X 10^{-23} J/K is the Boltzmann constant**
- **1000 Ω (300 K) → 16 X 10^{-18} *V^2/Hz*, or 4 nV/√Hz**

Resistor Thermal Noise: Example



Noise spectrum and total noise power in V_{out}

$$\frac{V_{out}}{V_R}(s) = \frac{1}{RCs+1}$$

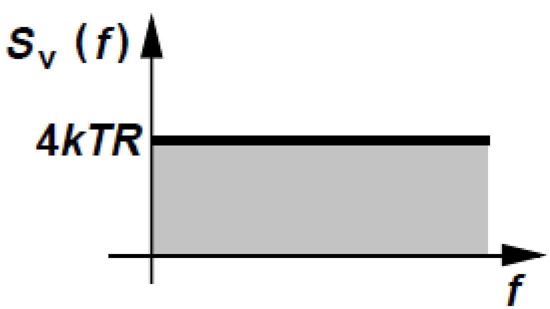
$$S_{out}(f) = S_v(f)\left|\frac{V_{out}}{V_R}(j\omega)\right|^2 = 4kTR\frac{1}{4\pi^2R^2C^2f^2+1}$$

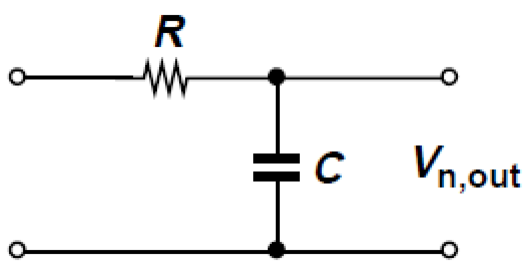
$$P_{n,out} = \int_0^\infty \frac{4kTR}{4\pi^2R^2C^2f^2+1}df$$

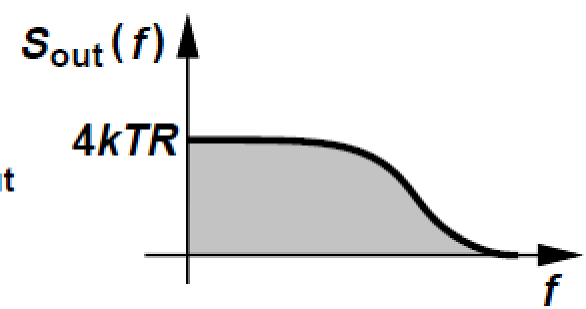
$$Put\ 2\pi RCf = u$$

$$P_{n,out} = \frac{2kT}{\pi C}\int_0^\infty \frac{du}{u^2+1}$$

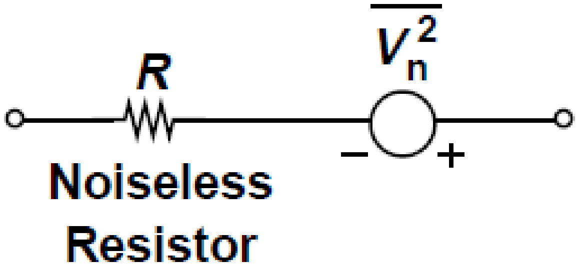
$$P_{n,out} = \frac{2kT}{\pi C}\tan^{-1}u\Big|_{u=0}^{u=\infty} = \frac{kT}{C}.$$





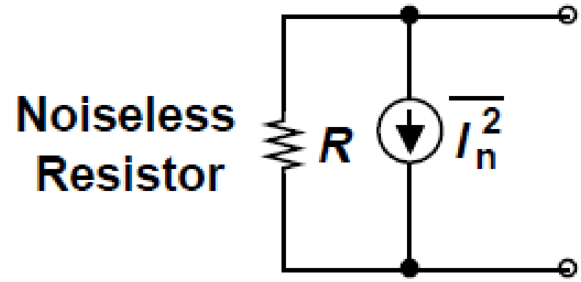


Resistor Thermal Noise



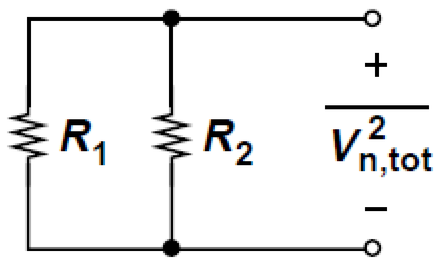
$$\overline{V_n^2} = 4kTR$$

V²/Hz

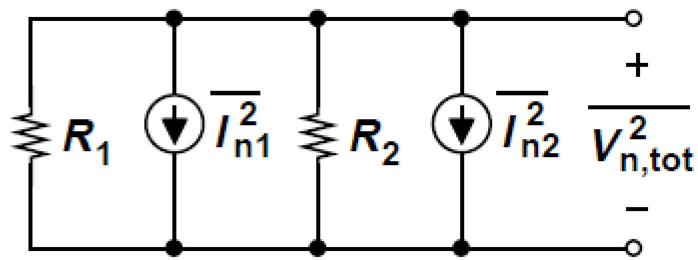


$$\overline{I_n^2} = 4kT/R$$

A²/Hz



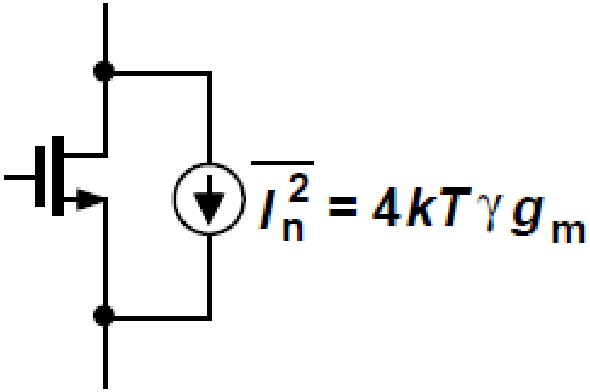
$$\begin{aligned} \overline{I_{n,tot}^2} &= \overline{I_{n1}^2} + \overline{I_{n2}^2} \\ &= 4kT\left(\frac{1}{R_1} + \frac{1}{R_2}\right) \end{aligned}$$



$$\begin{aligned} \overline{V_{n,tot}^2} &= \overline{I_{n,tot}^2}(R_1 \| R_2)^2 \\ &= 4kT(R_1 \| R_2), \end{aligned}$$

MOSFET Thermal Noise

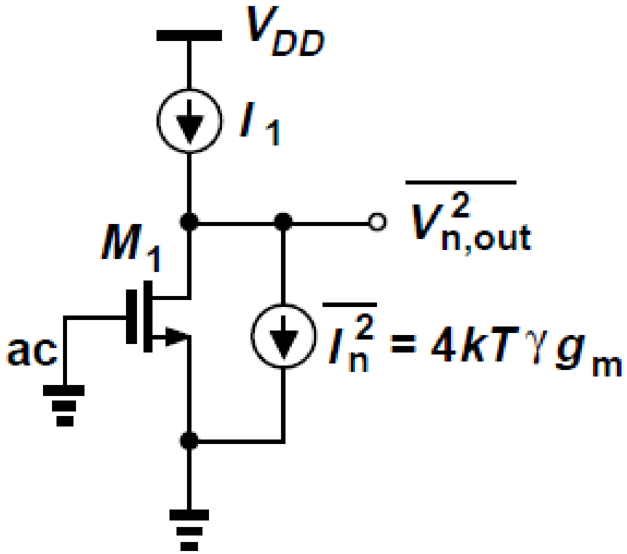
- **The Thermal noise generated in the channel**
- **For long-channel MOS devices operating in saturation,**



γ *(Excess Noise Coefficient)* = 2/3 for long-ch transistors
2-3 for short-ch transistors

Short channel device – Ecritical – Velocity saturation – extra work done by Efield – dissipation → Noise ^

MOSFET Thermal Noise: Example

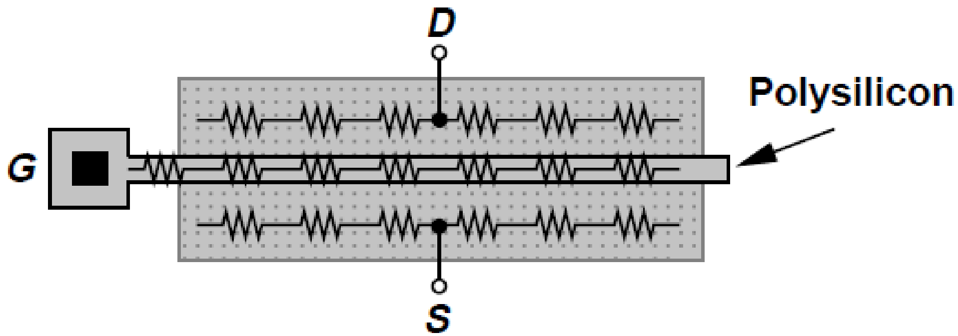


$$S_{out}(f) = S_{in}(f)|H(f)|^2$$

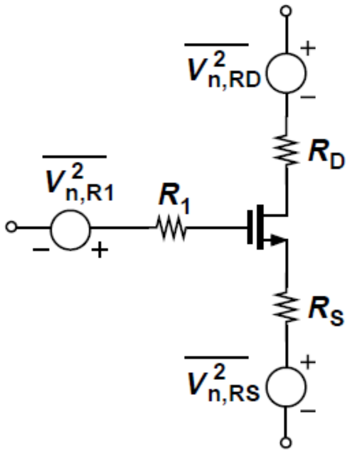
$$\begin{aligned} \overline{V_n^2} &= \overline{I_n^2} r_O^2 \\ &= (4kT\gamma g_m) r_O^2 \end{aligned}$$

- **Input is set to zero for noise calculation**
- **The output resistance r_o does not produce noise because it is not a physical resistor**

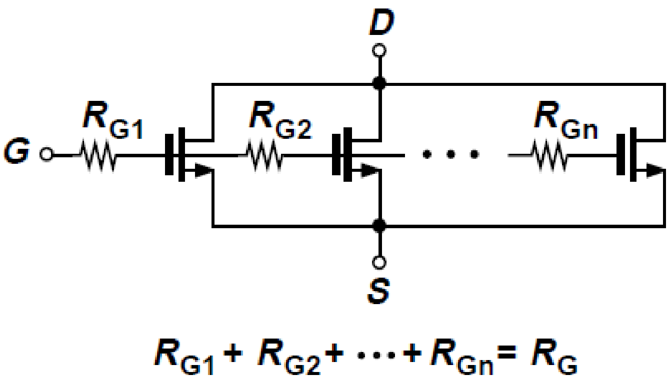
MOSFET Thermal Noise



(a)

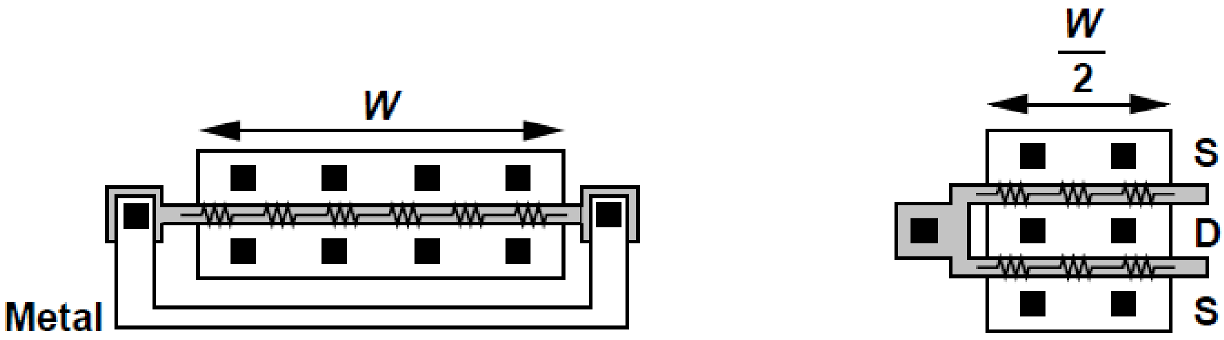


(b)



(c)

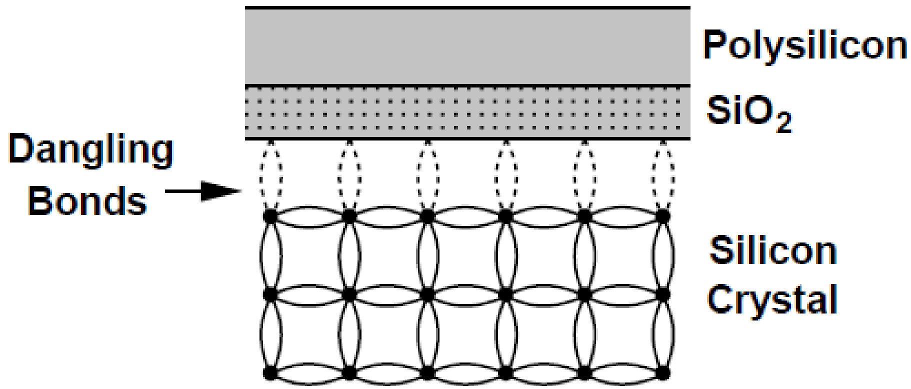
MOSFET Thermal Noise



(a) (b)

- **Effect of R_G can be reduced by proper layout**
- **For total gate resistance of R_G, equivalent noise resistance is $R_G/3$ (Distributed effects).**

Flicker Noise



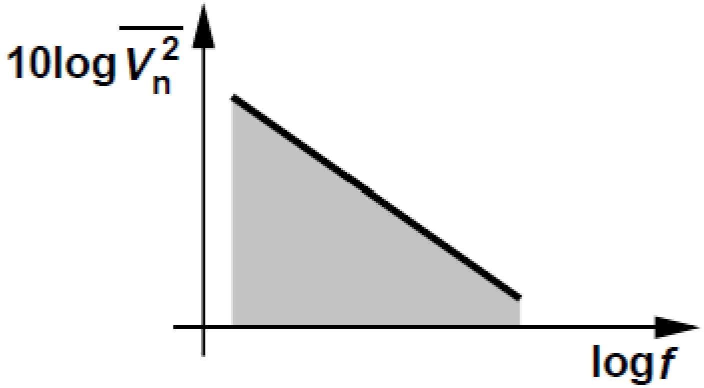
- "dangling" bonds giving rise to extra energy states
- Charge carriers moving at the interface are randomly trapped and later released by such energy states, introducing "flicker" noise in the drain current
- Depnds on cleanness of oxide-silicon interface and CMOS technology

Flicker Noise

- **Modeled as a voltage source in series with the gate and in the saturation region, is roughly given by**

$$\overline{V_n^2} = \frac{K}{C_{ox}WL} \cdot \frac{1}{f}$$

- ***K*** **is a process-dependent constant on the order of 10^{-25} V^2F**
- ***Larger Area – Averaging effect – Less Flicker***

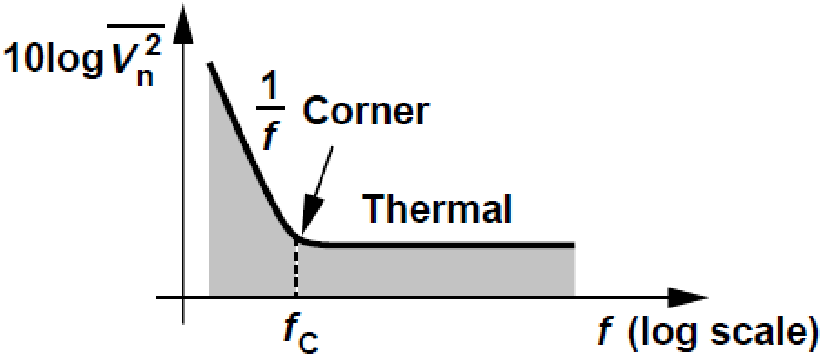


- **PMOS devices better *1/f* noise**

→ Holes are carried in a "buried" channel, below oxide-silicon interface

Flicker Noise Corner Frequency

- **At low frequencies, the flicker noise power approaches infinity**
- **Intersection point of thermal noise and flicker noise spectral densities is called "corner frequency" f_C**



At $f=f_C$,

Thermal noise = Flicker noise. i.e.,

$$4kT\gamma g_m = \frac{K}{C_{ox}WL}\cdot\frac{1}{f_c}\cdot g_m^2$$

that is,

$$f_c = \frac{K}{\gamma C_{ox}WL}\cdot g_m\frac{1}{4kT}$$

$$f_c \approx \frac{K}{\gamma}\cdot\omega_t\cdot\frac{1}{4kT}$$ and $$\omega_t \propto \frac{1}{L^2}$$

Hence, the f_C increases with technology scaling.