Course on Passive Microwave ckts

EE611 - Microwave integrated circuits

Date 06/1/19

→ We can apply Maxwell's Laws but that would be too complicated.

- wave nature
- comparitive wavelength
- parasitics (L, C present)

Elements which only be realized using distributed elements

① Circulators
② Couplers
③ Tee

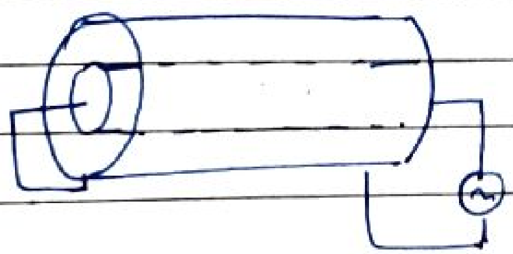
<u>Intermediate theory</u>: Somewhat like kirchoff's law with some part of Maxwell's law.

capacitance connected in shunt.

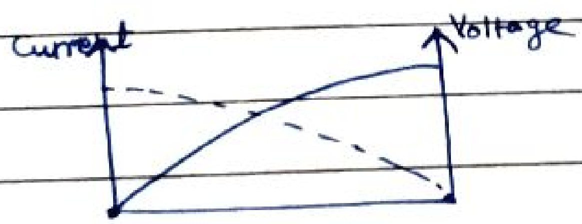
$$v_p = \frac{\lambda}{T} = \frac{\omega}{\beta}$$

$$= \underbrace{\frac{2\pi}{T}}_{\omega} \cdot \frac{\lambda}{2\pi} \frac{1}{\beta}$$

$$v_p = \frac{c}{\sqrt{\epsilon_r/\mu_r}}$$



$\longleftarrow \frac{\lambda}{4} \longrightarrow$



<u>Skin effect</u>: depth of penetration of wave in conductor decreases with increase in the frequency.

$$\delta = \sqrt{\frac{2}{\omega\mu\sigma}} \rightarrow \text{for perfect conductor } \sigma \to \infty$$

$$\Rightarrow \delta \to 0$$

$$L = \frac{\mu_r}{C\epsilon_r c^2}$$

Passive microwave devices:
devices that do not produce power of their own. They have
gain either equal to or less than 1.
eg: coupler, filters, attenuation.

EM theory with circuit theory.

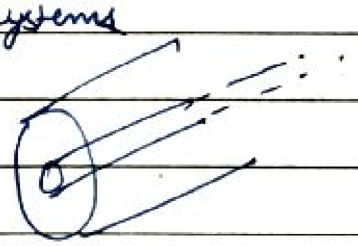
S parameters will be used

in Distributed circuit theory, we'll be dealing with.

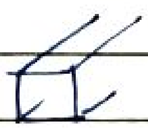
1. Transmission lines
2. open circuit stub
3. coupled lines
4. short circuit stub
5. Tapered lines

Guided wave systems: (forms the basis of any Transmission line)

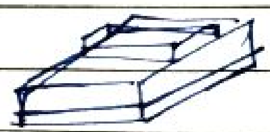
1. Two or more conductor systems
   → coaxial cable
   → Telegraph lines



2. Closed metallic wave guides consisting of hollow conductive pipes.
   → rectangular waveguides
   → cylindrical waveguides



3. Dielectric slabs enclosed by metallic sheets



→ pozar ch-2

these derivations are for cartesian system. Page

- $\boxed{\text{units important hein}}$
→ go through solution of Maxwells eqn again.

Maxwell's Laws:

$\vec{\nabla} \cdot \bar{D} = \rho$

$\bar{\nabla} \cdot \bar{B} = 0$

$\bar{\nabla} \times \vec{E} = \frac{-\partial \bar{B}}{\partial t}$

$\bar{\nabla} \times \bar{H} = \bar{J} + \frac{\partial \bar{D}}{\partial t}$

D comes from Gauss's Law

E comes from force/charge

$\bar{D} = \epsilon \vec{E}$

$\bar{\nabla} \times \vec{\nabla} \times \vec{E} = \frac{-\partial}{\partial t} (\bar{\nabla} \times \bar{B})$

$\left.\begin{array}{l} \nabla^2 \bar{E} - \mu\epsilon \frac{\partial^2 E}{\partial t^2} = 0 \\ \nabla^2 H - \mu\epsilon \frac{\partial^2 H}{\partial t^2} = 0 \end{array}\right\}$ assuming time dependence to be sinusoidal

→ $\bar{E}(x, y, z, t) = Re[\bar{E}(x, y, z)\, e^{j\omega t}]$

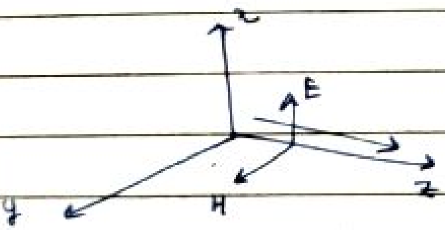
→ $\vec{H}(x, y, z, t) = Re[\bar{H}(x, y, z)\, e^{j\omega t}]$

after eliminating time dependence

$\underbrace{\nabla^2 E + \mu\epsilon\omega^2 E = 0 \quad , \quad \nabla^2 H + \mu\epsilon\omega^2 H}_{\text{Helmholtz equation}}$, $\nabla^2 = \frac{\partial}{\partial x^2} + \frac{\partial}{\partial y^2} + \frac{\partial}{\partial z^2}$

Types of Solution for Helmholtz equations:

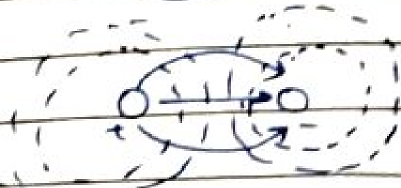
1. TEM waves, $E_z, H_z = 0$ → exists only when 2 or more conductor wave guides
eg: coaxial cable
→ micro strip line



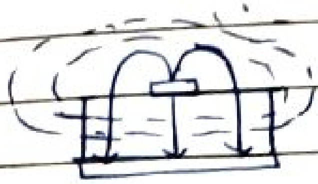
2. TE waves, $E_z = 0$, $H_z \neq 0$
3. TM waves, $H_z = 0$, $E_z \neq 0$

} → Can exist in 2 or more as well as single conductor waveguides

→ E, B in coaxial cable,

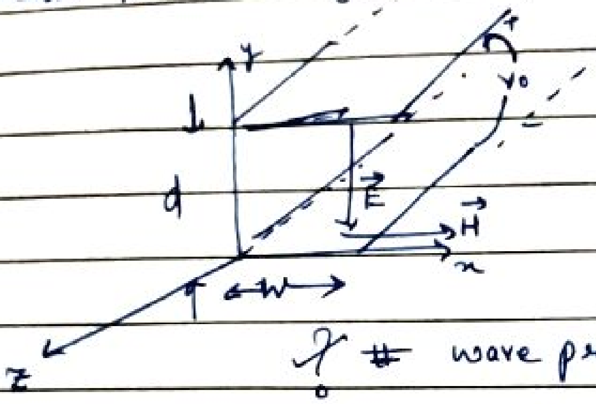


→ ——— 2 wire conductor,



→ Micro strip line,

* Parallel plate waveguide:



① TEM wave solution

$$\vec{H}(x,y,z) = \hat{x}\frac{V_0}{\eta d} e^{-jkz}$$

$$\vec{E}(x,y,z) = -\hat{y}\frac{V_0}{d} e^{-jkz}$$

if # wave propagating in + z direction

$$\eta = \sqrt{\frac{\mu_0}{\epsilon_0}} = 377\,\Omega$$ (impedance offered by atm or vacuum)

② TM wave

$$E_z(x,y,z) = A_n \sin\frac{n\pi y}{d}\, e^{-j\beta z}$$

$$H_x(x,y,z) = \frac{j\omega\epsilon}{k_c} A_n \cos\frac{n\pi y}{d}\, e^{-j\beta z}$$

, n takes
0 → solution as TM_0
1 → TM_1
2 → TM_2

$$E_y = \frac{-j\beta}{k_c} A_n \cos\left(\frac{n\pi y}{d}\right) \times e^{-j\beta z}$$

$$E_x = H_y = 0, \quad k_c = \frac{n\pi}{d}$$

$$k = \frac{2\pi}{\lambda}, \quad \beta = \sqrt{k^2 - k_c^2}$$ → +ve only for certain values of ω.

if β imaginary, sinusoidal not attained, ⇒ $\boxed{\text{evanescent modes}}$

now, $k > k_c$

$$\Rightarrow \omega\sqrt{\mu\epsilon} \geq k_c \quad \& \quad \frac{1}{\sqrt{\mu\epsilon}} = c$$

$$\frac{\omega}{c} \geq k_c$$

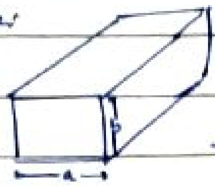
$$\frac{2\pi f}{c} \geq k_c \Rightarrow \boxed{f \geq \frac{c k_c}{2\pi}}$$

(concept of cut-off freq. ~~wav~~ exists for TM wave)

certain frequencies won't give solution
$\Rightarrow$ there exist a electro magnetic Band gap.

$\boxed{\rightarrow \text{Solve for TE modes}}$ (assignment)

→ Rectangular waveguide:



$a > b$

Solution for TE mode:

$$H_z(x,y,z) = A_{mn} \cos\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right) e^{-j\beta z}$$

$$E_x = \frac{j\omega\mu n\pi}{k_c^2 b} A_{mn} \cos\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right) e^{-j\beta z}$$

k_c^2 given by, $k_c^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$ ($\because$ m, n two dimension present

modes will be like, TE_{10}, TE_{11} etc.

mode with the lowest cut off frequency is called the dominant mode.

→ ~~which of TE_1 or TM_1 is dominant~~ (parallel)

→ in case of rect. w.g. TE_{10} is dominant mode.

$\beta = \sqrt{k^2 - k_c^2}$, $k = \omega\sqrt{\mu\epsilon}$

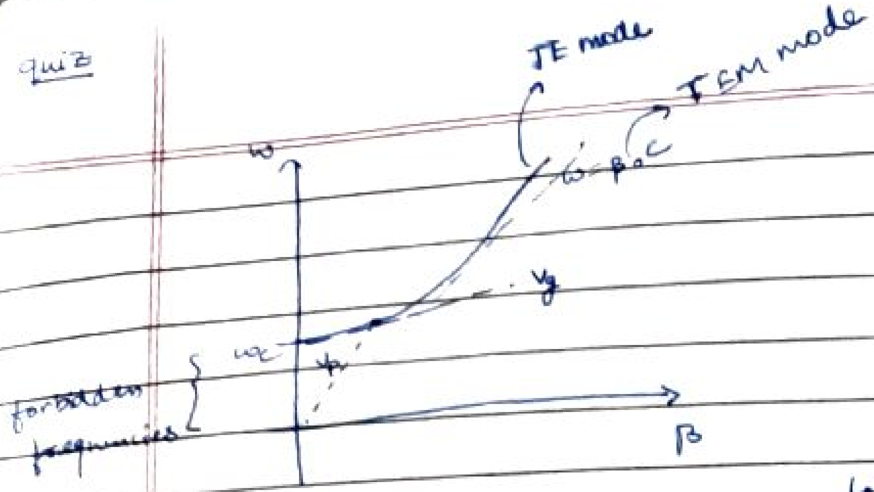
$$\beta = \sqrt{\omega^2\mu\epsilon - k_c^2} = \sqrt{\mu\epsilon}\sqrt{\omega^2 - \frac{k_c^2}{\mu\epsilon}}$$

$$= \sqrt{\mu\epsilon}\sqrt{\omega^2 - (2\pi f_c)^2} = \sqrt{\mu\epsilon}\sqrt{\omega^2 - \omega_c^2}$$

$\beta = \sqrt{\mu\epsilon}\sqrt{\omega^2 - \omega_c^2}$, β being a measure of spatial frequency

$$\beta = \frac{2\pi}{\lambda_g}$$

quiz



for TEM mode

$\omega t - kz = \text{constant}$

$\omega - k\frac{dz}{dt} = 0$

$\frac{dz}{dt}$ denotes velocity of a cons. phase point

$\Rightarrow \boxed{\frac{\partial z}{\partial t} = V_p = \frac{\omega}{k}}$

for TM or TE mode use β instead of k.

$\boxed{\frac{\partial z}{\partial t} = v_p = \frac{\omega}{\beta}}$

V_p = chordal slope

V_g = tangent $\left(\frac{\partial \omega}{\partial \beta}\right)$

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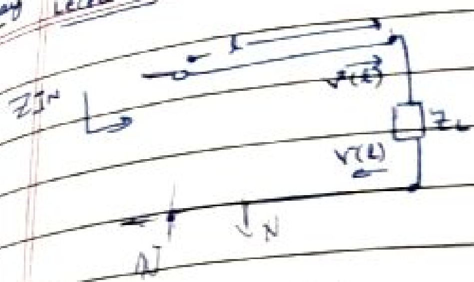
Tuesday Lecture 5



$$Z_{IN} = Z_0 \cdot \left(\frac{Z_L + Z_0 \tanh(\gamma l)}{Z_0 + Z_L \tanh(\gamma l)} \right)$$

$$\Gamma(l) = \frac{V^-(l)}{V^+(l)} = \frac{Z_L - Z_0}{Z_L + Z_0}$$

Reflection coefficient $\Gamma(x) = \frac{V^-(x)}{V^+(x)}$

for lossless case

$\gamma = j\beta, \; \alpha = 0$

$$Z_{IN} = Z_0 \cdot \left(\frac{Z_L + Z_0 \tanh(j\beta l)}{Z_0 + Z_L \tanh(j\beta l)} \right) = Z_0 \cdot \left(\frac{Z_L + jZ_0 \tan(\beta l)}{Z_0 + jZ_L \tan(\beta l)} \right)$$

$\rightarrow \beta l \rightarrow \beta l + \pi \qquad (\because \tan(\beta l))$

$\Rightarrow \frac{2\pi}{\lambda} l \rightarrow \frac{2\pi}{\lambda} l + \pi$

$\rightarrow \frac{2\pi}{\lambda}\left(l + \frac{\lambda}{2}\right) \rightarrow$ (repeats after $\frac{\lambda}{2}$) {doesn't happen with lump elements}

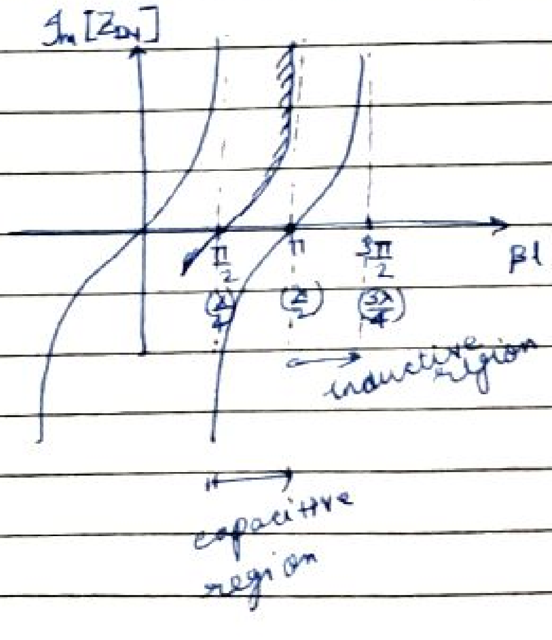
$\rightarrow \frac{2\pi f}{v_p} l \rightarrow \frac{2\pi f}{v_p} l + \pi$

$\Rightarrow \quad \rightarrow \frac{2\pi}{v_p} l \left(f + \underbrace{\frac{v_p}{2l}}_{f_0} \right)$

{lossless transmission line being periodic in frequency, electrical length as well as in length}

* Special cases of lossless transmission line:

①. Shorted line, $Z_L = 0$, $Z_{IN} = jZ_0 \tan(\beta l)$

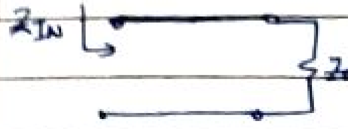


{both inductive & capacitive according to βl}

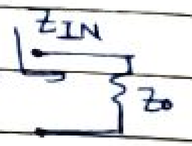
T-line

(2) open, $Z_L = \infty$, ~~$Z_{IN} = jZ_0 \tan$~~ $Z_{IN} = -jZ_0 \cot(\beta l)$

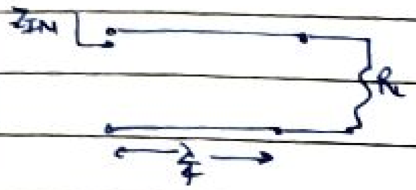
(3) Matched ~~load~~ line, $Z_L = Z_0$, $Z_{IN} = Z_0 \cdot \left(\frac{1 + j\tan\beta l}{1 + j\tan\beta l}\right) = Z_0$



$\equiv$ {transmission line doesn't exist} $\equiv$



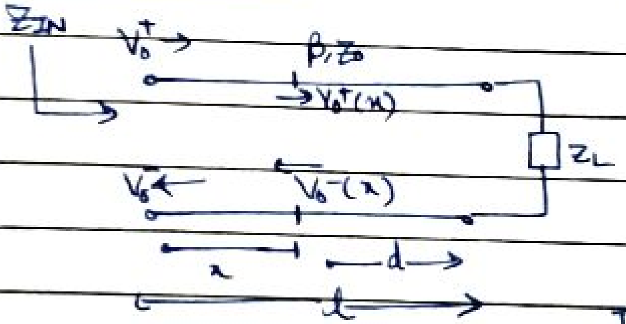
(4) Quarter wave transformer (T-line of length $\frac{\lambda}{4}$ for certain wavelength)



$\because \beta l = \frac{\lambda}{4}$, $\Rightarrow Z_{IN} = \frac{Z_0^2}{Z_L} = \frac{Z_0^2}{R_L}$

$\left\{ Z_{IN} \propto \frac{1}{R_L} \right\}$

<u>Reflection coefficients:</u>



$$\Gamma(x) = \frac{V^-(x)}{V^+(x)}$$

$$= \frac{V_0^- e^{\gamma x}}{V_0^+ e^{-\gamma x}} = \frac{V_0^-}{V_0^+} e^{+2\gamma x}$$

$$\Gamma(d) = \frac{V_0^-}{V_0^+} e^{2\gamma(l-d)}$$

$$= \frac{V_0^- e^{\gamma l}}{V_0^+ e^{-\gamma l}} e^{-2\gamma d}$$

now, $\gamma = \alpha + j\beta$

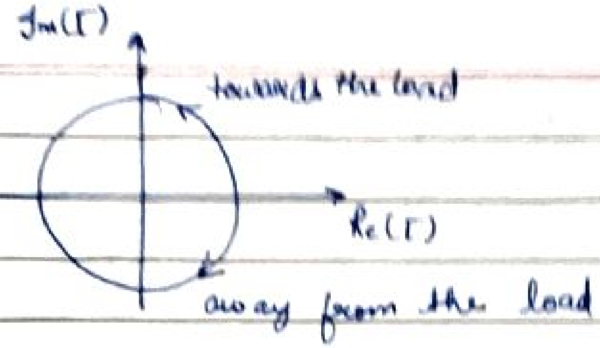
$\because$ lossless, $\therefore$ $\alpha = 0 \Rightarrow \gamma = j\beta$.

and if $\Gamma(l) = |\Gamma(l)| e^{j\alpha'}$

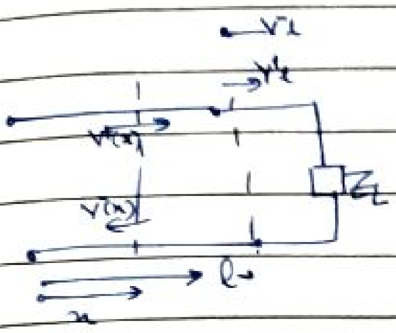
$$\boxed{\Gamma(d) = \Gamma(l) e^{-2\gamma d}}$$

Reflection coefficient at the load end

$$\boxed{\Gamma(d) = |\Gamma(l)| e^{j(\alpha' - 2\beta d)}}$$



* $V(d) = V_L^+ e^{+j\beta d} + V_L^- e^{-j\beta d}$

* 

$$Z(x) = \frac{V(x)}{I(x)} = \frac{V^+(x) + V^-(x)}{\frac{V^+(x) - V^-(x)}{Z_0}} = Z_0\left[\frac{1+\Gamma(x)}{1-\Gamma(x)}\right]$$

$$\Gamma(x) = \frac{Z(x) - Z_0}{Z(x) + Z_0}, \quad \text{at } l,\ \Gamma(l) = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$(\because Z(l) = Z_L)$$

Power delivered to load Z_L is

$$P_L = \frac{1}{2} Re\{V I^*\} = \frac{1}{2} Re\left\{(V_L^+ + V_L^-)\left(\frac{V_L^+ - V_L^-}{Z_0}\right)^*\right\}$$

$$P_L = \frac{1}{2}\left(\frac{|V_L^+|^2}{Z_0} - \frac{|V_L^-|^2}{Z_0}\right) = \underbrace{\frac{1}{2}\frac{|V_L^+|^2}{Z_0}}_{P^+}\left(1 - |\Gamma(l)|^2\right) = P^+\left(1 - |\Gamma(l)|^2\right)$$

$P^+ \to$ {power of the incident wave}

if $\Gamma(l) = 0 \Rightarrow P_L = P^+$ {Matched network}

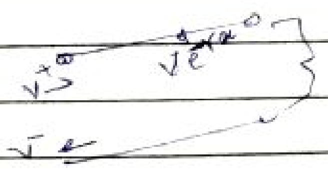
* Smith Chart: transforms Z to Γ, $\Gamma = \frac{Z - Z_0}{Z + Z_0}$ (Bilinear transformation)

$$\Gamma = \frac{\frac{Z}{Z_0} - 1}{\frac{Z}{Z_0} + 1} = \frac{z - 1}{z + 1}, \quad z = r + jx$$

$r \to$ normalized resistance

$x \to$ normalized reactance

→ by doing this all passive impedances on Right half plane of Z coordinates are mapped inside the unit circle



ratio of max to min voltage along a transmission line

(VSWR) Voltage standing wave ratio

$$VSWR = \frac{1+|\Gamma(x)|}{1-|\Gamma(x)|}$$, $VSWR < 2$ is considered as a good design

$$V(x) = V_0^+ e^{-j\beta x} + V_0^- e^{+j\beta x}$$
$$= V_0^+ e^{-j\beta x}\left(1+|\Gamma| e^{j\alpha'} e^{+2j\beta x}\right)$$

$$|V(x)|_{max} = V_0^+(1+|\Gamma|) \qquad \alpha' + 2\beta x = 2\pi$$

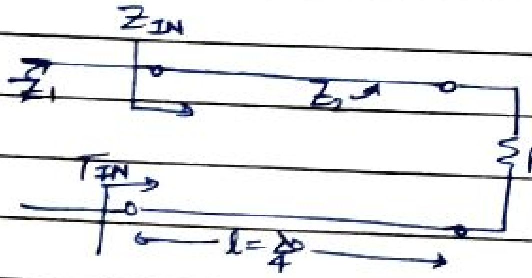
$$|V(x)|_{min} = V_0^+(1-|\Gamma|) \qquad \alpha' + 2\beta x = \pi$$

16/8/19 Lecture 6

Broadband impedance matching

narrowband matching : matching for just a single frequency

what we defined matching earlier was $\Gamma = 0$ but this is not easy to get, so we define a threshold Γ_m, such that if $\Gamma < \Gamma_m$ then we say that circuit is matched.



$l = \frac{\lambda_0}{4}$ at $f = f_0$

for matching $Z_{IN} = Z_1$

$\Rightarrow Z_2 = \sqrt{Z_1 R_L}$

ers sumption)

now above result holds only for certain frequency.

$$Z_{IN} = Z_2 \cdot \left(\frac{R_L + jZ_2 \tan\beta l}{Z_2 + jR_L \tan\beta l}\right) , \quad \beta l = \frac{2\pi}{\lambda}\cdot\left(\frac{\lambda_0}{4}\right)$$

at $\lambda = \lambda_0 \Rightarrow Z_2 = \sqrt{Z_1 R_L}$

at $\lambda \neq \lambda_0$

$$\Gamma_{IN} = \frac{Z_{IN} - Z_1}{Z_{IN} + Z_1}$$ why not Z_2? because Z_1 feed line.

$$\Gamma_{IN} = \frac{Z_2 R_L + jZ_2^2 \tan\beta l - Z_1 Z_2 - jZ_1 R_L \tan\beta l}{Z_2 R_L + jZ_2^2 \tan\beta l + Z_1 Z_2 + jZ_1 R_L \tan\beta l}$$

putting $Z_2 = \sqrt{Z_1 R_L}$

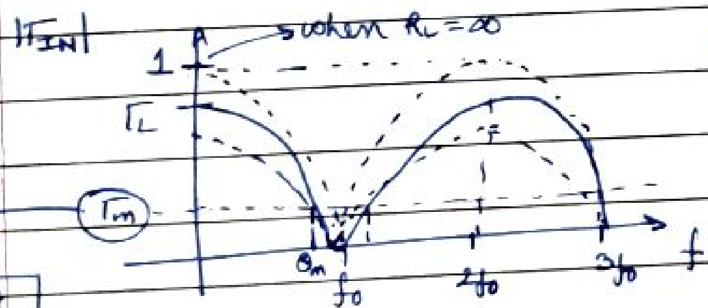
$$\Gamma_{IN} = \frac{R_L - Z_1}{R_L + Z_1 + j2\sqrt{Z_1 R_L}\,\tan(\beta l)}$$

$$|\Gamma_{IN}| = \frac{R_L - Z_1}{\sqrt{(R_L - Z_1)^2 + 4R_L Z_1 + 4R_L Z_1 \tan^2\beta l}} = \frac{R_L - Z_1}{\sqrt{(R_L - Z_1)^2 + 4R_L Z_1 \sec^2\beta l}}$$

$$|\Gamma_{IN}| = \frac{1}{\sqrt{1 + \frac{4R_L Z_1 \sec^2\beta l}{(R_L - Z_1)^2}}}, \quad \beta l = \frac{\pi}{2}\frac{f}{f_0}$$

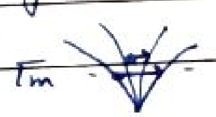
at $f = 0$, $|\Gamma_{IN}| = \frac{R_L - Z_1}{R_L + Z_1} = \Gamma_L$, assuming operation ~~near design frequency~~, we get $\beta l \approx \pi/2 \Rightarrow \sec^2\beta l >> 1$

at $f = f_0$, $|\Gamma_{IN}| = 0$. $\Rightarrow \boxed{|\Gamma_{IN}| \approx \frac{|R_L - Z_1|}{2\sqrt{Z_1 R_L}}|\cos\beta l|}$



matching threshold

* $|\Gamma_{IN}| = 1$ gives us less bandwidth



$\Delta\theta_m$ if axis on θ scale.

Bandwidth $= 2\Delta\theta$

$$\Delta\theta = \frac{\pi}{2} - \theta_m$$

$$\Gamma_{IN}(\theta = \theta_m) = \Gamma_m$$

Fractional bandwidth $= \frac{2\Delta\theta}{\theta_0 \left(= \frac{\pi}{2}\right)}$ (it is a measure of quality of matching)

$$= \frac{2\left(\frac{\pi}{2} - \theta_m\right)}{\pi/2} = \boxed{2 - \frac{4}{\pi}\theta_m}$$

$$\Gamma_{IN}(\theta) = \frac{1}{\sqrt{1 + \frac{4R_L Z_1}{(R_L - Z_1)^2}\sec^2\theta}}$$

$$\Gamma_m = \Gamma_{IN}(\theta)\Big|_{\theta=\theta_m} = \frac{1}{\sqrt{1+\frac{4R_L Z_1}{(R_L-Z_1)^2}\sec^2\theta_m}} = \frac{|R_L - Z_1|}{\sqrt{(R_L-Z_1)^2 + 4R_L Z_1 \sec^2\theta_m}}$$

$$(R_L - Z_1)^2 = \Gamma_m^2\left((R_L - Z_1)^2 + 4R_L Z_1 \sec^2\theta_m\right)$$

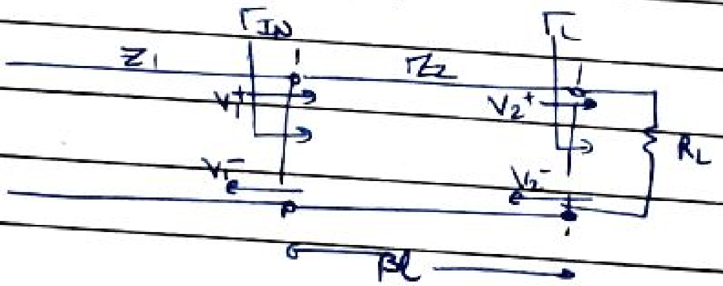
$$-4R_L Z_1 \sec^2\theta_m = (R_L - Z_1)^2\left[1 - \frac{1}{\Gamma_m^2}\right]$$

$$\sec^2\theta_m = \frac{(R_L - Z_1)^2}{4R_L Z_1}\left[\frac{1}{\Gamma_m^2} - 1\right]$$

$$\cos\theta_m = \frac{2\sqrt{Z_1 R_L}}{|R_L - Z_1|} \cdot \frac{\Gamma_m}{\sqrt{1-\Gamma_m^2}}$$

$\boxed{\theta_m \text{ can be found \& so fractional b-w can be found.}}$

Multi Section transformer:



$$V_2 = V_2^+ + V_2^-$$

$$V_1 = V_1^+ + V_1^- = V_2^+ e^{j\beta l} + V_2^- e^{-j\beta l}$$

$$\Rightarrow V_1^+\left[1 + \frac{V_1^-}{V_1^+}\right] = V_2^+ e^{j\beta l}\left(1 + \frac{V_2^-}{V_2^+}e^{-j2\beta l}\right)$$

$$\Rightarrow V_1^+\left[1 + \Gamma_{IN}\right] = V_2^+ e^{j\beta l}\left(1 + \Gamma_L e^{-j2\beta l}\right) \quad \text{—— (i)}$$

$$I_1 = \frac{V_1^+ - V_1^-}{Z_1} = \frac{V_2^+ e^{j\beta l} - V_2^- e^{-j\beta l}}{Z_2} \; ?$$

$$\frac{V_1^+}{Z_1}\left(1 - \Gamma_{IN}\right) = \frac{V_2^+ e^{j\beta l}}{Z_2}\left(1 - \Gamma_L e^{-j2\beta l}\right)$$

$$\frac{1 - \Gamma_{IN}}{1 + \Gamma_{IN}} = \frac{Z_1}{Z_2}\left(\frac{1 - \Gamma_L e^{-j2\beta l}}{1 + \Gamma_L e^{-j2\beta l}}\right) \quad \text{—— (ii)}$$

$$-\Gamma_{IN} = \frac{\frac{Z_1}{Z_2}\left(\frac{1 - \Gamma_L e^{-j2\beta l}}{1 + \Gamma_L e^{-j2\beta l}}\right) - 1}{\frac{Z_1}{Z_2}\left(\frac{1 - \Gamma_L e^{-j2\beta l}}{1 + \Gamma_L e^{-j2\beta l}}\right) + 1}$$

$1 - y = x + xy$

$y(x+1) = 1 - x$

- $$\Gamma_{IN} = \frac{Z_2 - Z_1 + \Gamma_L e^{-j2\beta l}(Z_1+Z_2)}{Z_1+Z_2 + (Z_2-Z_1)\Gamma_L e^{-j2\beta l}}$$

$$\Gamma_{IN} = \frac{\frac{Z_2-Z_1}{Z_2+Z_1} + \Gamma_L e^{-j2\beta l}}{1 + \frac{Z_2-Z_1}{Z_2+Z_1}\Gamma_L e^{-j2\beta l}} \quad , \quad \Gamma_{21} = \frac{Z_2-Z_1}{Z_2+Z_1}$$

$$\boxed{\Gamma_{IN} = \frac{\Gamma_{21} + \Gamma_L e^{-j2\beta l}}{1 + \Gamma_{21}\Gamma_L e^{-j2\beta l}}}$$

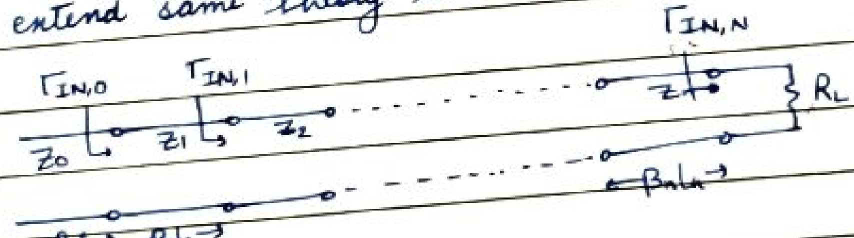
if $Z_2 = Z_1$
$\Rightarrow \Gamma_{21} = 0$
$\Rightarrow \Gamma_{IN} = \Gamma_L e^{-j2\beta l}$

now in order to ignore second order reflections, we assume $Z_2 - Z_1$ close to zero $\Rightarrow \Gamma_{21} \to 0$

$$\Rightarrow \boxed{\Gamma_{IN} \approx \Gamma_{21} + \Gamma_L e^{-j2\beta l}} \longrightarrow \text{Theory of small reflection}$$

20/8/19 **Lecture 7**

to extend same theory to n no.



let assume
$\beta_i l_i = \beta_j l_j \quad \forall i,j \in (1,$
$= \theta$

$$\Gamma_{IN,0} = \Gamma_{IN,1} e^{-j2\beta l} + \Gamma_{10}$$

$$\Gamma_{IN,0}(\theta) = \Gamma_{IN,1} e^{-j2\theta} + \Gamma_{10} \quad , \quad \Gamma_{10} = \frac{Z_1 - Z_0}{Z_1 + Z_0}$$ (measure of mismatch btw. Z_1 & Z_0

$$\Gamma_{IN,0}(\theta) = \Gamma_{1,0} + (\Gamma_{21} + \Gamma_{IN,2} e^{-j2\theta})e^{-j2\theta}$$

$$= \Gamma_{1,0} + \Gamma_{21} e^{-j2\theta} + \Gamma_{IN,2} e^{-j4\theta}$$

$$\Rightarrow \Gamma_{IN,0}(\theta) = \Gamma_{10} + \Gamma_{21} e^{-j2\theta} + \Gamma_{32} e^{-j4\theta} + \cdots + \Gamma_{N,N-1} e^{-j2(N-1)\theta} + \Gamma_L e^{-j2N\theta}$$

where, $\boxed{\Gamma_L = \frac{R_L - Z_N}{R_L + Z_N}}$

terminology changed, $\boxed{\Gamma_{k+1,k} = \Gamma_k}$

$$\Gamma_{IN,0}(\theta) = \Gamma_0 + \Gamma_1 e^{-j2\theta} + \Gamma_2 e^{-j4\theta} + \cdots + \Gamma_{N-1} e^{-j2(N-1)\theta} + \Gamma_N e^{-j2N\theta}$$

$$\Gamma_{IN,0}(\theta) = \sum_{K=0}^{N} \Gamma_K e^{-j2K\theta}$$ (self constrained solution)

$$\underset{\substack{\Downarrow \\ \theta=0}}{\Gamma_{IN,0}(f=0)} = \sum_{K=0}^{N} \Gamma_K$$ } from eqn

$$\Gamma_{IN,0}(\theta=0) = \frac{R_L - Z_0}{R_L + Z_0}$$ } from intution. ∵ zero electrical length.

$$\rightarrow \sum_{K=0}^{N} \Gamma_K = \sum_{K=0}^{N} \frac{Z_{K+1} - Z_K}{Z_{K+1} + Z_K} = \frac{Z_1 - Z_0}{Z_1 + Z_0} + \cdots - \frac{R_L - Z_N}{R_L + Z_N}$$

(✓) intution $\neq$ (x) equation (∵ we used single reflection approximation in eqn)

$$\Gamma_K = \frac{Z_{K+1} - Z_K}{Z_{K+1} + Z_K} \approx \frac{Z_{K+1} - Z_K}{2Z_K} \approx \frac{1}{2}\ln\left(1 + \frac{Z_{K+1} - Z_K}{Z_K}\right) = \frac{1}{2}\ln\left(\frac{Z_{K+1}}{Z_K}\right)$$

$$\boxed{\Gamma_K \approx \frac{1}{2}\ln\left(\frac{Z_{K+1}}{Z_K}\right)}$$ ∵ $\{ \ln(1+x) = x - \frac{x^2}{2} + \cdots$

$$\Gamma_{IN,0}(\theta) = \sum_{K=0}^{N} \Gamma_K e^{-j2K\theta}, \quad \text{at } \theta = 0 \Rightarrow \Gamma_{IN,0}(0) = \sum_{K=0}^{N} \Gamma_K$$

$$\Gamma_{IN,0}(0) = \cancel{\sum_{K=0}^{N}} = \frac{1}{2}\ln\left(\frac{\cancel{Z_1}}{Z_0} \cdot \frac{\cancel{Z_2}}{\cancel{Z_1}} \cdot \frac{\cancel{Z_3}}{\cancel{Z_2}} \cdots \frac{R_L}{Z_N}\right) = \frac{1}{2}\ln\left(\frac{R_L}{Z_0}\right)$$

$$\boxed{\Gamma_{IN,0}(0) = \frac{1}{2}\ln\left(\frac{R_L}{Z_0}\right)}$$ { multisection transform }

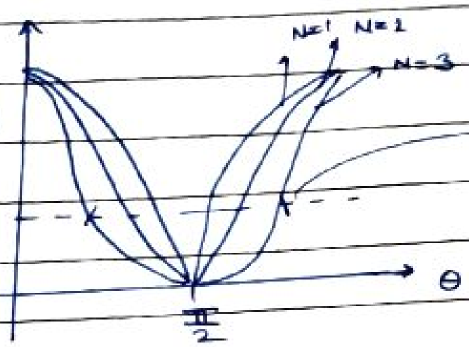
ONE PROBLEM in MIDSEM from SYNTHESIS

SYNTHESIS :

Prototype functions : functions that provide a template for how response should be

1. **Binomial function**: (Maximally flat)

$$\Gamma_{IN}(\theta) = A(1+e^{-j2\theta})^N$$



$$\frac{d(\Gamma_{IN}(\theta))}{d\theta} = A(1+e^{-j2\theta})^{N-1} \cdot e^{-j2\theta}(-j2)$$

at $\theta = \pi/2$ $\underline{\underline{= 0}}$

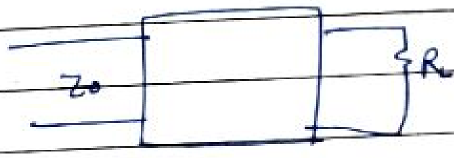
as you increase N, No. of sections, ⇒ achievable bandwidth increases.

$$\frac{d^K \Gamma_{IN}(\theta)}{d\theta^K} = 0 \quad \text{at } \theta = \pi/2$$

(Maximally flat, ∵ not only fn zero but its derivatives too zero.)

at $\theta = 0$, $\Gamma_{IN}(\theta) = \frac{R_L - Z_0}{R_L + Z_0} = \Gamma_L$

eg:



and $\Gamma_{IN}(\theta = 0) = A(2)^N$

$$\Rightarrow \boxed{A = \frac{\Gamma_L}{2^N}}$$

$$\boxed{\Gamma_{IN}(\theta) = 2^{-N}\Gamma_L(1+e^{-j2\theta})^N}$$

$$\Rightarrow |\Gamma_{IN}(\theta)| = 2^{-N}|\Gamma_L|\,|e^{-j\theta N}| \cdot |e^{j\theta} + e^{-j\theta}|^N = 2^{-N}|\Gamma_L|\,|2\cos\theta|^N$$

$$= |\Gamma_L|\,|\cos\theta|^N$$

at $\theta = \theta_m = |\Gamma_{IN}(\theta)| = \Gamma_m$, $\boxed{\Gamma_m = |\Gamma_L|(\cos\theta_m)^N}$ $\left\{ \because \theta_m < \frac{\pi}{2},\ |\cos\theta_m| = \cos\theta_m \right\}$

→ θ_m found

⇓

$$\frac{\Delta f}{f} = 2 - \frac{4\theta_m}{\pi}$$ } fractional B.W can be found.

↳ this will give us N, i.e no. of sections used from graph above, now we need to synthesize.

$$\Gamma_{IN}(\theta) = \sum_{k=0}^{N} \Gamma_k e^{-j2k\theta}$$

$$\Gamma_{IN}(\theta) = 2^{-N}\Gamma_L\left(1+e^{-j2\theta}\right)^N = 2^{-N}\Gamma_L \sum_{k=0}^{N} {}^N C_k\, e^{-j2k\theta}$$

$\Rightarrow \boxed{\Gamma_k = 2^{-N}\Gamma_L\, {}^N C_k}$ $\Rightarrow$ got this using used prototype fn.

diff. prototype fn. gives diff. values.

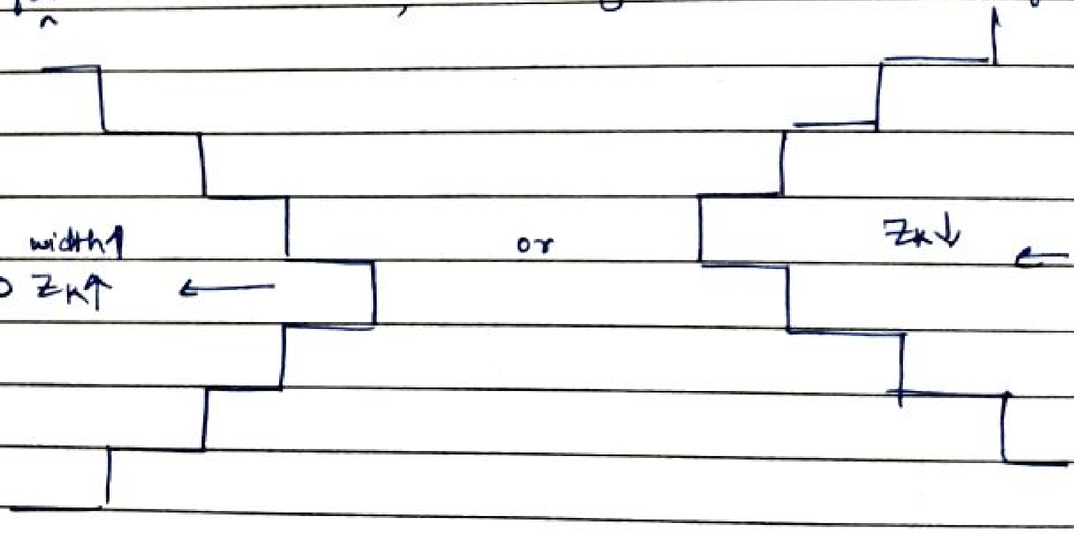
eg. say, $N=3$, $\Gamma_1 = 2^{-3}\Gamma_L\,{}^3C_1$
$\Gamma_2 = 2^{-3}\Gamma_L\,{}^3C_2$ equal.

properties:

Symmetry: $\Gamma_1 = \Gamma_{N-1}$, $\Gamma_0 = \Gamma_N$

All Γ_k's either all +ve or all -ve. , $\Gamma_k = \frac{Z_{k+1} - Z_k}{Z_{k+1} + Z_k}$

now for all +ve or -ve, matching network either of two,



* $\Gamma_k = 2^{-N}\,{}^N C_k\, \Gamma_L$, $\Gamma_k = \frac{1}{2}\ln\left(\frac{Z_{k+1}}{Z_k}\right)$ for $N=2$

$\Gamma_L = \frac{1}{2}\ln\left(\frac{R_L}{Z_0}\right)$

$$\Rightarrow \ln\left(\frac{Z_{k+1}}{Z_k}\right) = 2^{-N}\,{}^N C_k \ln\left(\frac{R_L}{Z_0}\right)$$

↓

Z_K found

$k=0, \quad \ln\left(\frac{Z_1}{Z_0}\right) = \ln\left(\frac{R_L}{Z_0}\right)^{1/4}$

$k=1, \quad \ln\left(\frac{Z_2}{Z_1}\right) = \ln\left(\frac{R_L}{Z_0}\right)^{1/2}.$

$Z_1 = Z_0\left(\frac{R_L}{Z_0}\right)^{1/4}$, $\quad Z_2 = Z_1\left(\frac{R_L}{Z_0}\right)^{1/2}$

$\boxed{Z_1 = Z_0^{3/4} R_L^{1/4}}$

$Z_2 = Z_0^{3/4} R_L^{1/4} \times R_L^{1/2} Z_0^{-1/2}$

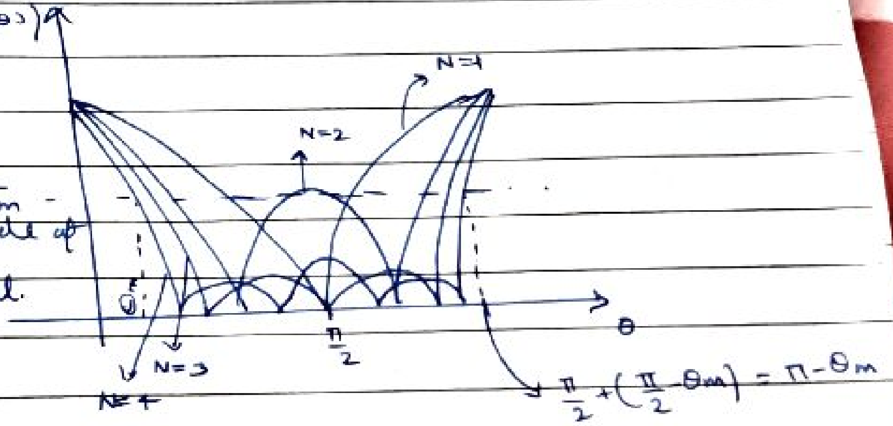
$\boxed{Z_2 = Z_0^{1/4} R_L^{3/4}}$

2. Chebyshev prototype function:

→ passband ripples allowed as long as they are less than Γ_m

→ ~~for even there is a magnitude at $\frac{\pi}{2}$, for odd there is a null.~~

→ for a given N, all ripples of the same height



B-W ⇒ $(\theta_m, \pi-\theta_m)$

Chebyshev functions:

$T_N(x) = \cosh(N\cosh^{-1}x)$

$T_1(x) = x$

$T_2(x) = 2x^2 - 1$

$T_3(x) = 4x^3 - 3x$

$T_n(x) = 2x\,T_{n-1}(x) - T_{n-2}(x)$

* $\boxed{T(\cos\theta) = \cos(n\theta)}$

for, $\theta \to (\theta_m, \pi-\theta_m)$

say $x = \frac{\cos\theta}{\cos\theta_m} = \cos\theta\sec\theta_m$ → why this divide

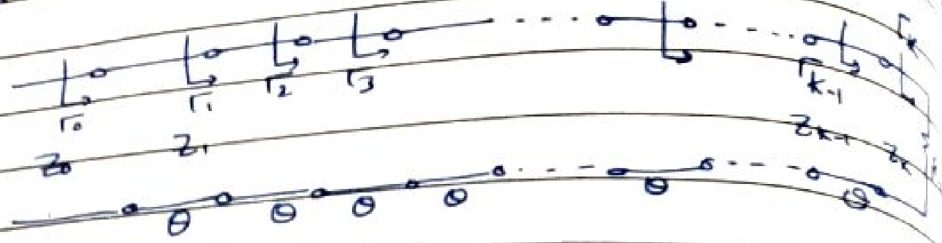
⇒ ~~Chebyshev prototype fn:~~ $\Gamma_{in}(\theta) = \underbrace{A}_{\text{magnitude flexibility}}\,\underbrace{e^{-jN\theta}}_{\text{phase flexibility}}\,T_N(\sec\theta_m\cos\theta)$

for any transformation we want to find A(mag) & N(order/no. of sections)

23/8/19

Lecture 8

Multi section X-mer



$$\Gamma_{IN}(\theta) = A e^{-jN\theta} T_N(\sec\theta_m \cos\theta)$$

when $\theta=0$, $\Gamma_{IN}(\theta=0) = A T_N(\sec\theta_m)$

$$\Gamma_{IN}(\theta=0) = \frac{R_L - Z_0}{R_L + Z_0} = \Gamma_L$$

$$\Rightarrow A = \frac{\Gamma_L}{T_N(\sec\theta_m)} \quad \text{--- (I)}$$

(poly. always real, Γ_L either +ve
So A always real)

$$\Gamma_{IN}(\theta=\theta_m) = A e^{-jN\theta}$$

$$|\Gamma_{IN}(\theta=\theta_m)| = \Gamma_m = |A| \quad \text{--- (II)}$$

(Γ_m always +ve)

$$T_N(x) = \cosh(N\cosh^{-1}(x))$$

$$\Gamma_m = |A| = \frac{|\Gamma_L|}{|T_N(\sec\theta_m)|} = \frac{|\Gamma_L|}{T_N(\sec\theta_m)}$$

$$T_N(\sec\theta_m) = \frac{|\Gamma_L|}{\Gamma_m}$$

$$\cosh(N\cosh^{-1}(\sec\theta_m)) = \frac{|\Gamma_L|}{\Gamma_m}$$

$$\Rightarrow \sec\theta_m = \cosh\left[\frac{1}{N}\cosh^{-1}\left(\frac{|\Gamma_L|}{\Gamma_m}\right)\right]$$

{ θ_m ✓ found
$\frac{\Delta f}{f} = 2 - \frac{4\theta_m}{\pi}$ can be found }

if N comes out to be 3.2 we take it as 4.

{ or you know B-W (from requirement)
you calculate θ_m
& then you calculate

Example

$N=3$

$\Gamma_{IN}(\theta) = \sum_{k=0}^{3} \Gamma_k e^{-j2k\theta}$ (from derivation) → multisection analysis

$\Gamma_{IN}(\theta) = Ae^{-jN\theta} T_3(\sec\theta_m \cos\theta)$ (from chebyshev) —— (I)

$T_3(\sec\theta_m \cos\theta) = \sec^3\theta_m (\cos 3\theta + 3\cos\theta) - 3\sec\theta_m \cos\theta$ —— (II)

put eqⁿ (II) in (I) for, $\Gamma_{IN}(\theta) = Ae^{-jN\theta}\left[\sec^3\theta_m \cos 3\theta + \cos\theta(3\sec^3\theta_m - 3\sec\theta_m)\right]$ —— (III)

also $\Gamma_{IN}(\theta) = \Gamma_0 + \Gamma_1 e^{-j2\theta} + \Gamma_2 e^{-j4\theta} + \Gamma_3 e^{-j6\theta}$

~~suppose~~ ~~symmetric~~ $\Rightarrow \Gamma_0 = \Gamma_3$, $\Gamma_1 = \Gamma_2$

$$\begin{aligned}
\Gamma_{IN}(\theta) &= \Gamma_0(1+e^{-j6\theta}) + \Gamma_1(e^{-j2\theta}+e^{-j4\theta}) \\
&= \Gamma_0 e^{-j3\theta}(e^{j3\theta}+e^{-j3\theta}) + \Gamma_1 e^{-j3\theta}(e^{j\theta}+e^{-j\theta}) \\
&= \Gamma_0 e^{-j3\theta}(2\cos 3\theta) + \Gamma_1 e^{-j3\theta}(2\cos\theta) \quad \text{—— (IV)} \\
&= 2e^{-j3\theta}\left[\Gamma_0 \cos 3\theta + \Gamma_1 \cos\theta\right]
\end{aligned}$$

from (III) & (IV)

$\Gamma_0 = \frac{A}{2}\sec^3\theta_m$,

$\Gamma_1 = \frac{3A}{2}\left[\sec^3\theta_m - \sec\theta_m\right]$, now you know $\Gamma_0, \Gamma_1, \Gamma_2, \Gamma_3$

now you calculate Z_i once Γ_i is found for $i \in (0,3)$

$\Gamma_0 = \frac{1}{2}\ln\left[\frac{Z_1}{Z_0}\right]$

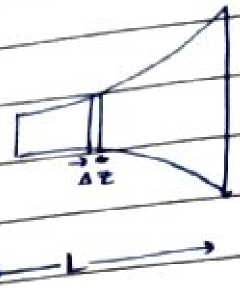
$\Gamma_1 = \frac{1}{2}\ln\left[\frac{Z_2}{Z_1}\right]$

$\Gamma_2 = \frac{1}{2}\ln\left[\frac{Z_3}{Z_2}\right]$

$\Gamma_3 = \frac{1}{2}\ln\left[\frac{\cancel{Z_4}}{Z_3}\right]$ → R_L

Taper: Structure with infine no. of multisections.

eg:



elect length $\beta l \rightarrow \beta(K\Delta z)$

instead of Γ_K we can now take $d\Gamma$

$$\Gamma_K \rightarrow \frac{Z(K+1)-Z(K)}{Z(K+1)+Z(K)} \qquad d\Gamma \rightarrow \frac{Z(z)+dZ-Z(z)}{Z(z)+dZ+Z(z)}$$

$$d\Gamma \approx \frac{dZ(z)}{2Z(z)} = \frac{1}{2}\frac{d}{dz}\left(\ln\left(\frac{Z}{Z_0}\right)\right) dz \quad \text{—— (1)}$$

$K\theta \rightarrow K(\beta\Delta z) \quad \because \left\{ \theta \rightarrow \beta l \rightarrow \beta\Delta z \right\}$

so, $$\Gamma_{IN}(\beta l) = \sum_{K=0}^{N} \Gamma_K e^{-2\beta(K\Delta z)}$$

$$\Gamma_{IN}(\beta l) = \sum_{K=0}^{N} \Delta\Gamma(K\Delta z)\, e^{-2\beta(K\Delta z)}$$

for $\Delta Z \rightarrow 0$, $$\Gamma_{IN}(L) = \int_{z=0}^{L} d\Gamma\, e^{-j2\beta z}$$

from 1

$$\boxed{\Gamma_{IN}(L) = \frac{1}{2}\int_{z=0}^{L} e^{-j2\beta z}\left[\frac{d}{dz}\ln\left(\frac{Z(z)}{Z_0}\right)\right] dz} \quad \text{—— (A)}$$

Total input reflection coeff. for Tapered

★ Prototype functions for tapered

1. **Exponential tapered:**

$$Z(z) = Z_0 e^{az}$$

$$\frac{Z(z)}{Z_0} = e^{az}$$

$$\ln\left(\frac{Z(z)}{Z_0}\right) = az$$

$$\frac{d}{dz}\left[\ln \frac{Z(z)}{Z_0}\right] = a$$

$$\Rightarrow \quad \Gamma_{IN}(L) = \frac{a}{2}\int_{z=0}^{L} e^{-2j\beta z}\, dz$$

$$= \frac{a}{2}\left[\frac{e^{-2j\beta z}}{2j\beta}\right]_{z=0}^{L}$$

$$\Gamma_{IN}(L) = \frac{a e^{-j\beta L}}{-j4\beta}\left[-2\sin(\beta L)\right] = \underline{\frac{a e^{-j\beta L}}{2j\beta}\sin(\beta L)}$$

$$Z(z=0) = Z_0$$

$$Z(z=L) = Z_0 e^{aL} = R_L$$

$$a = \frac{1}{L}\ln\left(\frac{R_L}{Z_0}\right)$$

$$\Rightarrow \quad \Gamma_{IN}(L) = \frac{1}{L}\ln\left(\frac{R_L}{Z_0}\right) e^{-j\beta L}\, \frac{\sin(\beta L)}{2j\beta}$$

$$= \ln\left(\frac{R_L}{Z_0}\right)\frac{e^{-j\beta L}}{2j}\,\mathrm{sinc}(\beta L)$$

2. **Triangular Taper:**

$$Z(z) = \begin{cases} Z_0 e^{2\left(\frac{z}{L}\right)^2 \ln\left(\frac{Z_L}{Z_0}\right)} & 0 \le z \le \frac{L}{2} \\ Z_0 e^{\left(\frac{4z}{L} - \frac{2z^2}{L^2} - 1\right)\ln\frac{Z_L}{Z_0}} & \frac{L}{2} \le z \le L \end{cases}$$

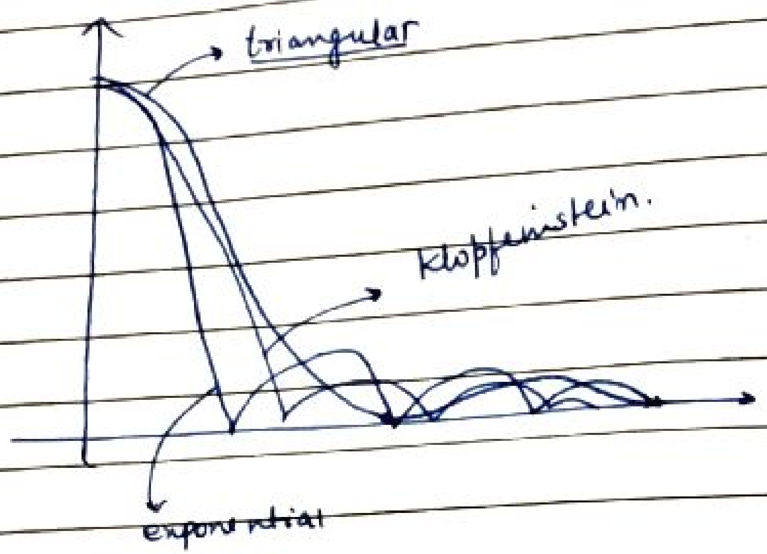
$$\ln\left(\frac{Z}{Z_0}\right) = \begin{cases} 2\left(\frac{z}{L}\right)^2 \ln\left(\frac{Z_L}{Z_0}\right) & 0 \le z \le \frac{L}{2} \\ \left(\frac{4z}{L} - \frac{2z^2}{L^2} - 1\right)\ln\frac{Z_L}{Z_0} & \frac{L}{2} \le z \le L \end{cases}$$

Putting in eq (A) we get

$$\Gamma_{IN}(L) = \frac{e^{-j\beta L}}{2}\ln\left(\frac{R_L}{Z_0}\right)\mathrm{sinc}^2\left(\frac{\beta l}{2}\right) \qquad \left\{ \text{derive it once} \right\}$$

→ instead of sinc, there is a sinc² function. Frequency of Γ to be zero is halved as compared to previous tapered considered optimum taper

* Kloffinston taper



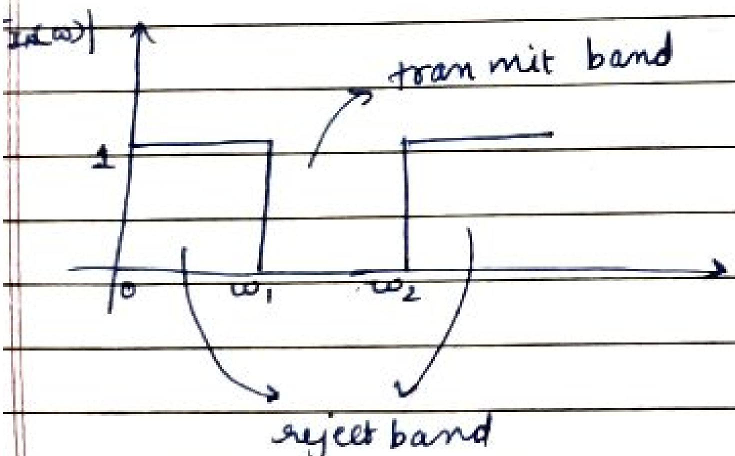
Quiz till chebyshev:

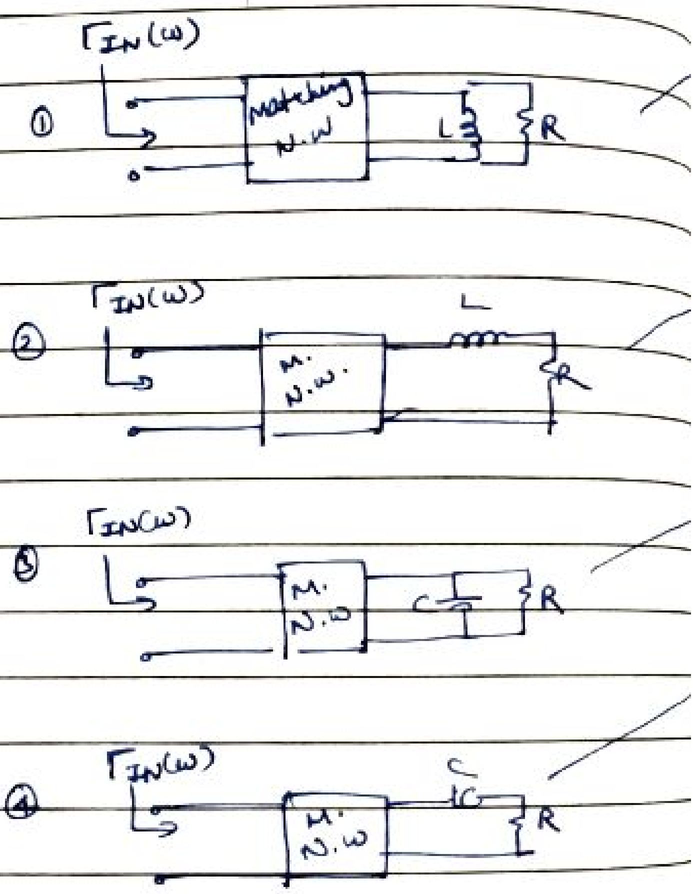
30/8/19

friday

Bode- Fanyo Critaria:

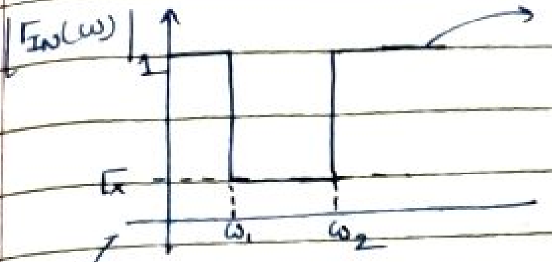
consists of 4 combinations





Bode fanno criteria says you cant achieve as last figure.

but you can achieve,



average Γ_{IN}

$|\Gamma_{IN}(\omega)|$ cannot go below a certain Γ_x

eg: ~~& say case 4 is the combination.~~

$\overset{\infty}{\underset{0}{}} (\ln 1 = 0)$

$$\int_{\omega_1}^{\omega_2} \frac{1}{\omega^2} \ln\left|\frac{1}{\Gamma_x}\right| d\omega < \pi RC \quad \Rightarrow \ln\left|\frac{1}{\Gamma_x}\right| \int_{\omega_1}^{\omega_2} \frac{1}{\omega^2} d\omega < \pi RC$$

$$\Rightarrow \ln\left|\frac{1}{\Gamma_x}\right| \cdot \left|\frac{-1}{\omega}\right|_{\omega_1}^{\omega_2} < \pi RC$$

$$\ln\left|\frac{1}{\Gamma_x}\right| \cdot \left\{\frac{1}{\omega_1} - \frac{1}{\omega_2}\right\} < \pi RC$$

$$\ln\left|\frac{1}{\Gamma_x}\right| < \frac{\pi RC}{\cancel{\omega_1 \omega_2}} \cancel{(\omega_2 - \omega_1)} \frac{(\omega_1 \omega_2)}{(\omega_2 - \omega_1)}$$

Γ_x ✓ found (minimum average value of Γ_{IN} over transmit band.

$$\to \int_0^\infty \frac{1}{\omega^2} \ln\left|\frac{1}{|\Gamma_{IN}(\omega)|}\right| d\omega < \frac{\pi L}{R}$$

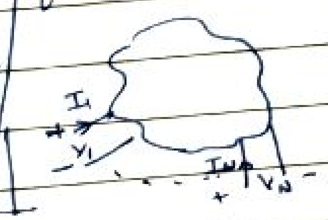
$$\to \int_0^\infty \ln \frac{1}{|\Gamma_{IN}(\omega)|} d\omega < \frac{\pi R}{L}$$

$$\to \int_0^\infty \ln\left|\frac{1}{\Gamma_{IN}(\omega)}\right| d\omega < \frac{\pi}{RC}$$

$$\to \int_0^\omega \frac{1}{\omega^2} \ln\left|\frac{1}{\Gamma_{IN}(\omega)}\right| < \pi RC$$

→ S parameter Matrix:

first about Y, Z parameters



$$\begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_N \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & \cdots & Z_{1N} \\ Z_{21} & Z_{22} & \cdots & Z_{2N} \\ \vdots & & & \\ Z_{N1} & \cdots & \cdots & Z_{NN} \end{bmatrix} \begin{bmatrix} I_1 \\ \vdots \\ I_N \end{bmatrix}$$

Zmatrix, $Z_{ij} = \frac{V_i}{I_j}\Big|_{I_k = 0 \text{ for } k \neq j}$

Similarily, $Y_{ij} = \frac{I_i}{V_j}\Big|_{V_k = 0 \text{ for } k \neq j}$

* properties:
→ ① Reciprocity: $Z_{ij} = Z_{ji}$
② Lossless N-w.

$$P_{total} = \frac{1}{2} V_1 I_1^* + \frac{1}{2} V_2 I_2^* + \dots + \frac{1}{2} V_N I_N^*$$

$$= \frac{1}{2} \sum_{n=1}^{N} V_n I_n^*$$

where $V_n = \sum_{m=1}^{N} Z_{nm} I_m$

$$P_{total} = \frac{1}{2} \sum_{n=1}^{N} \sum_{m=1}^{N} (Z_{nm} I_m) I_n^*$$

if Lossless N-W,
$\Rightarrow Real(P_{total}) = 0$

$$\Rightarrow Re\left[\frac{1}{2} \sum_{n=1}^{N} \sum_{m=1}^{N} (Z_{nm} I_m) I_n^*\right] = 0$$

Let's make an assumption, $I_i = 0 \ \forall i$, except $i = n' \Rightarrow I_{n'} \neq 0$

$$\Rightarrow Re(Z_{n'n'} I_{n'} I_{n'}^*) = 0 \Rightarrow Re(Z_{n'n'} |I_{n'}|^2) = 0$$

$$\Rightarrow |I_{n'}|^2 Re(Z_{n'n'}) = 0$$

$$\Rightarrow Re(Z_{n'n'}) = 0$$

$\Rightarrow$ ~~for a lossless only N-W all diagonal elements would be purely imaginary.~~

consider another port n''.

I have now current in n', & n''.

$$\Rightarrow Re\left\{\frac{1}{2}\left(Z_{n'n'} \overbrace{I_{n'} I_{n'}^*}^{|I_{n'}|^2} + Z_{n''n''} \overbrace{I_{n''} I_{n''}^*}^{|I_{n''}|^2} \cancel{+ Z_{n'n''} I_{n'} I_{n''}^* + Z_{n''n'} I_{n''} I_{n'}^*}\right)\right\} = 0$$

also, $Z_{n'n''} = Z_{n''n'}$ {reciprocal}

$$\Rightarrow \underbrace{\cancel{|I_{n'}|^2 Re(Z_{n'n'}) + |I_{n''}|^2 Re(Z_{n''n''})}}_{\text{already zero}}$$

$$+ Re\left[Z_{n'n''}(\underbrace{I_{n'} I_{n''}^* + I_{n''} I_{n'}^*}_{\text{purely real}})\right] = 0$$

$$\Rightarrow Re\{Z_{n'n''}\} = 0$$

$\Rightarrow$ whole matrix has purely imaginary terms.

classmate

Date
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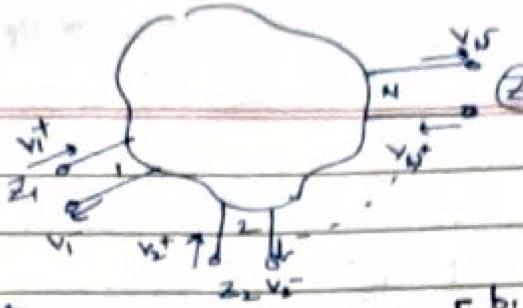
* Normalized waves:



(Z_n) → characteristic impedance at n^{th} port

$$a_n = \frac{V_n^+}{\sqrt{Z_n}}$$ → normalized incident wave

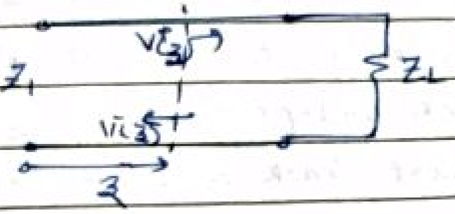
$$b = \frac{V_n^-}{\sqrt{Z_n}}$$ → normalized reflected wave

S parameter matrix

$$\begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_N \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & \cdots & S_{1N} \\ S_{21} & S_{22} & \cdots & S_{2N} \\ \vdots & & & \\ S_{N1} & \cdots & \cdots & S_{NN} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_N \end{bmatrix}$$

importance: it's like a cause & effect matrix

$$V(z) = V^+(z) + V^-(z)$$



$$\Rightarrow \frac{V(z)}{\sqrt{Z_1}} = \frac{V^+(z)}{\sqrt{Z_1}} + \frac{V^-(z)}{\sqrt{Z_1}}$$

normalized voltage, $\bar{V}(z) = a + b$

$$I(z) = \frac{V^+(z) - V^-(z)}{Z_1}$$

$$\Rightarrow \sqrt{Z_1}\, I(z) = \frac{V^+(z)}{\sqrt{Z_1}} - \frac{V^-(z)}{\sqrt{Z_1}}$$

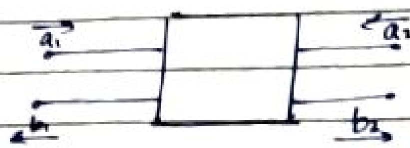
normalized current $\bar{I}(z) = a - b$

$$\begin{bmatrix} \bar{V}_1 \\ \bar{V}_2 \\ \\ \bar{V}_N \end{bmatrix} = \begin{bmatrix} \\ \bar{Z} \\ \\ \end{bmatrix} \begin{bmatrix} \bar{I}_1 \\ \bar{I}_2 \\ \vdots \\ \bar{I}_n \end{bmatrix}$$

$\bar{Z}$ → normalized impedance matrix

$$[\bar{Z}] = \begin{bmatrix} \frac{1}{\sqrt{Z_1}} & 0 & \cdots & 0 \\ 0 & \frac{1}{\sqrt{Z_2}} & & \vdots \\ \vdots & & \ddots & \\ 0 & \cdots & \cdots & \frac{1}{\sqrt{Z_n}} \end{bmatrix} [Z] \begin{bmatrix} \frac{1}{\sqrt{Z_1}} & 0 & \cdots & 0 \\ 0 & \frac{1}{\sqrt{Z_2}} & & \vdots \\ \vdots & & \ddots & \\ 0 & \cdots & \cdots & \frac{1}{\sqrt{Z_n}} \end{bmatrix}$$

2 port N-W



$S_{11} = \frac{b_1}{a_1}\Big|_{a_2=0}$ → i/p reflection coefficient

$S_{12} = \frac{b_1}{a_2}\Big|_{a_1=0}$ → reverse transmission coeff.

$S_{21} = \frac{b_2}{a_1}\Big|_{a_2=0}$ → forward transmission coeff.

$S_{22} = \frac{b_2}{a_2}\Big|_{a_1=0}$ → o/p reflection coefficient.

* S parameter matrix is impedance dependent.

if

$[S]_{Z_0=50\Omega} \rightarrow [Z/Y] \xrightarrow{Z_0 = 100\Omega}$

∵ S parameter depends on characteristic impeda[nce]
if you want to find [S] at $Z_0 = 100\Omega$
first we need to convert them into [Z]
∵ they are independent of Z_0.
then convert back to [S] at $Z_0 = 100\Omega$

* $[\bar{V}] = [a] + [b]$

$\bar{V} = \bar{Z}\,\bar{I}$

$\bar{I} = [a] - [b]$

$[a] + [b] = \bar{Z}\left([a] - [b]\right)$,

$[u]\left([a] + [b]\right) = \bar{Z}\left([a] - [b]\right)$

$\left(\bar{Z} + [u]\right)[b] = \left(\bar{Z} - [u]\right)[a]$

$$\Rightarrow \boxed{[b] = \underbrace{\left(\bar{Z} + [u]\right)^{-1}\left(\bar{Z} - [u]\right)}_{\text{S parameter matrix}}[a]}$$

S parameter matrix, ∵ $[b] = [S][a]$

$V = [a] + [b]$

$\bar{I} = [a] - [b]$

$\Rightarrow [a] = \frac{\bar{V} + \bar{I}}{2}$, $[a] = \frac{1}{2}\left\{[\bar{Z}] + [u]\right\}[\bar{I}]$

$[b] = \frac{\bar{V} - \bar{I}}{2}$, $[b] = \frac{1}{2}\left\{[\bar{Z}] - [u]\right\}[\bar{I}]$

$[\bar{I}] = 2\left\{[Z] + [u]\right\}^{-1}[a]$

$\cancel{\frac{1}{2}\{[Z] + [u]\}^{-1}}$

$[b] = \underbrace{\left\{[Z] - [u]\right\}\left\{[\bar{Z}] + [u]\right\}^{-1}}_{S}[a]$

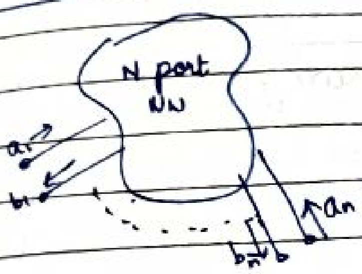
$[S] = \left\{[\bar{Z}] + [u]\right\}^{-1}\left\{[\bar{Z}] - [u]\right\}$, $S = \left\{[\bar{Z}] - [u]\right\}\left\{[\bar{Z}] + [u]\right\}^{-1}$

now, if $Z_{ij} = Z_{ji}$

$[Z] = [Z]^t$

?

*



$$P_n = |a_n|^2 - |b_n|^2$$

$$P_{i,n} = \frac{|V_n^+|^2}{Z_n} , \quad P_{r,n} = \frac{|V_n^-|^2}{Z_n}$$

P_n (total power entering n^{th} port) $= \frac{|V_n^+|^2}{Z_n} - \frac{|V_n^-|^2}{Z_n}$.

$$P_{tot} = \sum_{n=1}^{N} P_n = \sum_{n=1}^{N} \left(|a_n|^2 - |b_n|^2 \right)$$

For lossless N-W,

$P_{tot} = 0 \Rightarrow \quad \sum |a_n|^2 = \sum |b_n|^2$

$$[a]^t [a]^* = [b]^t [b]^*$$

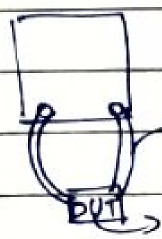
$$= [[S][a]]^t [[S][a]]^*$$

$$[a]^t [a]^* = [a]^t [S]^t [S]^* [a]^*$$

$\Rightarrow$ for a lossless N-W, $\boxed{[S]^t [S]^* = [U]}$ $\rightarrow$ unitary condition

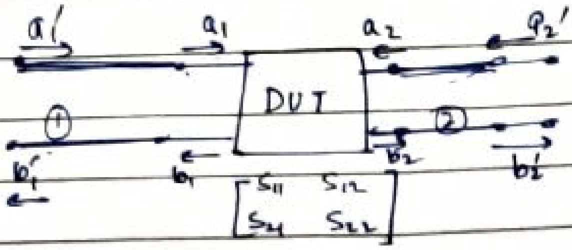
* VNA, Vector network analyzer - used to calibrate devices.

*



$\rightarrow$ Sophisticated cables (phase stabilized cable)

DUT $\rightarrow$ design/device under test



now, $a_1 = a_1' e^{-j\theta_1}$

$a_2 = a_2' e^{-j\theta_2}$

$b_1 = b_1' e^{j\theta_1}$

$b_2 = b_2' e^{j\theta_2}$

$$S_{ki}' = \frac{b_k'}{a_i'}\Bigg|_{\substack{\text{all other} \\ a_i' = 0}} = \frac{b_k e^{-j\theta_k}}{a_i e^{j\theta_i}} = \frac{b_k}{a_i} e^{-j(\theta_i + \theta_k)}$$

$$\boxed{S_{ki}' = S_{ki} e^{-j(\theta_i + \theta_k)}}$$

S_{ki} is what we want, but we're measuring shifted version of it.

$$[S'] = \begin{bmatrix} e^{-j\theta_1} & 0 & 0 & \cdots \\ 0 & e^{-j\theta_2} & & \\ 0 & & \ddots & \\ \vdots & & & e^{-j\theta_k} \end{bmatrix} [S] \begin{bmatrix} e^{-j\theta_1} & 0 & \cdots \\ 0 & e^{-j\theta_2} & \\ \vdots & & \ddots \quad e^{-j\theta_i} \end{bmatrix}$$

VNA takes inverse and all to find [S] from above eqn.

$$[S] = \begin{bmatrix} e^{j\theta_1} & 0 & \cdots & \\ 0 & \ddots & & \\ \vdots & & \ddots & \\ \vdots & & & e^{j\theta_k} \end{bmatrix} [S'] \begin{bmatrix} e^{j\theta_1} & 0 & \cdots & \\ 0 & \ddots & & \\ \vdots & & \ddots & e^{j\theta_i} \end{bmatrix}$$

$\therefore [S] = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$ if reciprocal

$\Rightarrow S_{12} = S_{21}$

if lossless

$[S]^t [S]^* = [U]$

$$\begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} S_{11}^* & S_{12}^* \\ S_{21}^* & S_{22}^* \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$|S_{11}|^2 + |S_{12}|^2 = 1$

$|S_{22}|^2 + |S_{21}|^2 = 1$ $\quad \Rightarrow |S_{12}| = \sqrt{1 - |S_{11}|^2}$

$S_{11} S_{12}^* + S_{12} S_{22}^* = 0$ $\quad |S_{22}|^2 + |S_{12}|^2 = 1 \Rightarrow |S_{11}| = |S_{22}|$

$S_{12} S_{11}^* + S_{22} S_{12}^* = 0$

now, $S_{11} = |S_{11}| e^{j\theta_1}$ 5 variables, but $|S_{12}|$ know

4 variables

$S_{22} = |S_{11}| e^{j\theta_2}$

$S_{12} = S_{21} = |S_{12}| e^{j\theta_{12}}$

$S_{11} S_{12}^* + S_{12} S_{11}^* = 0$

$|S_{11}||S_{12}| e^{j(\theta_1-\theta_{12})} + |S_{12}||S_{11}| e^{j(\theta_{12}-\theta_2)} = 0$

$\Rightarrow e^{j(\theta_1-\theta_{12})} = e^{j((\theta_{12}-\theta_2)+(2n+1)\pi)}$

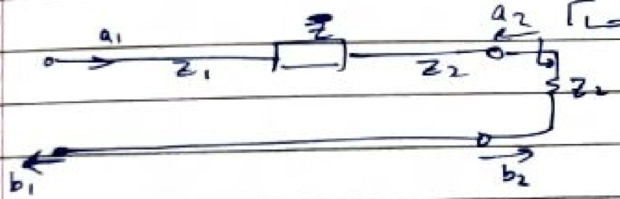
$\Rightarrow \theta_1 - \theta_{12} = \theta_{12} - \theta_2 + (2n+1)\pi$

$$\boxed{\theta_{12} = \frac{\theta_1+\theta_2}{2} - (2n+1)\frac{\pi}{2}}$$

θ_{12} ✓

3 variables $(|S_{11}|, \theta_1, \theta_2)$

$S_{11} = \frac{b_1}{a_1}\Big|_{a_2=0}$



a_2 $\Gamma_L = 0 \Rightarrow a_2 = 0$

$\bar{V}_1 = a_1 + b_1$

$\bar{I}_1 = a_1 - b_1$

$\bar{V}_2 = a_2 + b_2$

$\bar{I}_2 = a_2 - b_2$

$\bar{I}_1 = -\bar{I}_2$, *

$I_1\sqrt{Z_1} = -I_2\sqrt{Z_2}\cdot\frac{\sqrt{Z_1}}{\sqrt{Z_2}}$ $\quad a_1 - b_1 = b_2$

$a_1 - b_1 = -[a_2 - b_2]\sqrt{\frac{Z_1}{Z_2}}$ $\quad V_2 = V_1 - I_1 Z$ *

$a_1 - b_1 = b_2 \frac{\sqrt{Z_1}}{\sqrt{Z_2}}$

$a_1 - b_1 = \sqrt{\frac{Z_1}{Z_2}}\left[(a_1+b_1)\sqrt{\frac{Z_1}{Z_2}} - \frac{(a_1-b_1)Z}{\sqrt{Z_1Z_2}}\right]$ $\quad (a_2+b_2)\sqrt{Z_2} = (a_1+b_1)\sqrt{Z_1} - \frac{(a_1-b_1)}{\sqrt{Z_1}}\cdot Z$

$a_1 - b_1 = \frac{Z_1}{Z_2}(a_1+b_1) - \frac{Z}{Z_2}(a_1-b_1)$ $\quad (\overset{0}{a_2}+b_2) = (a_1+b_1)\frac{\sqrt{Z_1}}{\sqrt{Z_2}} - \frac{(a_1-b_1)}{\sqrt{Z_1Z_2}}\cdot Z$

$1 - \frac{b_1}{a_1} = \frac{Z_1}{Z_2}\left(1+\frac{b_1}{a_1}\right) - \frac{Z}{Z_2}\left(1-\frac{b_1}{a_1}\right)$ $\quad \because a_2 = 0 \Rightarrow \frac{b_1}{a_1} = S_{11}$

S_{11} ✓ $\quad S_{21} = \frac{b_2}{a_1}$ can also be found.

$\because$ reciprocal n-w, $S_{12} = S_{21}$

for S_{22} (make $a_1 = 0$)

∵ symmetric networ $S_{11} = S_{22}$ (like π, T)

only can be used for symmetric Network.