

BP-STDP: Approximating backpropagation using spike timing dependent plasticity

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ABSTRACT

The problem of training spiking neural networks (SNNs) is a necessary precondition to understanding computations within the brain, a field still in its infancy. Previous work has shown that supervised learning in multi-layer SNNs enables bio-inspired networks to recognize patterns of stimuli through hierarchical feature acquisition. Although gradient descent has shown impressive performance in multi-layer (and deep) SNNs, it is generally not considered biologically plausible and is also computationally expensive. This paper proposes a novel supervised learning approach based on an event-based spike-timing-dependent plasticity (STDP) rule embedded in a network of integrate-and-fire (IF) neurons. The proposed temporally local learning rule follows the backpropagation weight change updates applied at each time step. This approach enjoys benefits of both accurate gradient descent and temporally local, efficient STDP. The experimental results on the XOR problem, the Iris data, and the MNIST dataset demonstrate that the proposed SNN performs as successfully as the traditional NNs. Our approach also compares favorably with the state-of-the-art multi-layer SNNs. Thus, this method can be applied to develop deep SNNs with end-to-end STDP-based learning rules in future studies.

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1. Introduction

In the vein of neural network research, spiking neural networks (SNNs) have attracted recent and long-standing interest due to their biologically plausible realism, theoretical computation power, and power efficiency [1–5]. **Neurons in SNNs (analogous to biological neurons in the brain) communicate via discrete spike events.** An important question in implementing spiking frameworks is how these networks are trained under supervision while there is no differentiable activation function of continuous values? Multi-layer supervised learning is a crucial concept in SNNs to prove their ability to predict and classify the patterns of stimuli.

The earliest supervised learning in SNNs, SpikeProp, was proposed by Bohte et al. [6] which develops a gradient descent based learning method similar to traditional backpropagation embedded in multi layer neural networks. **SpikeProp minimizes the distance between single desired and output spikes by formulating the spike time as a function of the neuron's membrane potential.** Later, Quick-Prop and RProp [7] were introduced as faster versions of SpikeProp. Following the same approach, Booi et al. [8] and Ghosh et al. [9] proposed new SpikeProp algorithms minimizing the dis-

tance between multiple desired and output spikes to improve the model's coding ability in response to temporal patterns. Gradient descent (GD), as a popular supervised learning approach, is still being employed to develop high performance supervised learning in spiking platforms either in offline [10] or online [11,12] manner. Chronotron [13] which utilizes the Victor & Purpura (VP) metric [14] for E-learning (offline) and the synaptic current simulation for I-learning (online) is another example of GD in SNNs. As implemented in Chronotron, **the error function in GD can be defined by a difference measure between the post- and presynaptic current (or membrane potential) [15]** (online) or a spike train distance metric (offline) such as von Rossum distance [16,17] and inner product of spike trains [18].

The online gradient descent method has also attracted recent interest in multi-layer and deep SNNs [19,20]. Although the online GD methods **have been successful in developing supervised learning in multi-layer SNNs, using membrane potential and derivative based approaches are biologically implausible because SNNs only communicate via discrete spike events.** Additionally, GD is computationally expensive for multi-layer SNNs because it calculates a continuous error function and the derivative of the neuron's activation (e.g., the membrane potential formulation) at each simulation time step. Recently, Xie et al. [21] developed a normalized spiking backpropagation calculating postsynaptic spike times (which still needs expensive computations) instead of membrane potential at

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each time step to improve the algorithm's efficiency. To reduce the number of computations, recent studies have proposed efficient approaches which mostly focus on the membrane potential at the desired and undesired output times [22–24]. Although these methods improved the efficiency of gradient descent in SNNs significantly, they utilized cost functions obtained by a difference metric between the neuron's membrane potential and a firing threshold.

Another vein of research utilizes modified versions of the Widrow–Hoff learning rule for SNNs. For instance, ReSuMe [25] introduced a single-layer supervised learning approach; and later, it was extended to the multi-layer framework by Sporea et al. [26]. Recently, Zhang et. al. [27] proposed a dynamic firing threshold mechanism to significantly improve ReSuMe, especially in response to the stimuli with high level noises. SPAN [28,29] also used Widrow–Hoff by transforming the discrete spikes to corresponding analog signals. The learning methods mentioned above are more efficient but less accurate than the GD-based approaches.

Another efficient, supervised learning method belongs to the perceptron-based approaches such as Tempotron [30] where each spike event is treated as a binary tag for training the perceptron [31,32]. These models present single layer supervised learning. However, the idea of spike/no-spike classification broadens a new supervised learning category incorporating efficient spike-timing-dependent plasticity (STDP) [33–35] and anti-STDP that are triggered according to the neuron's label for single layer [36,37] and multi-layer [38] learning in SNNs. The phenomenon of STDP has been observed in many areas of the brain over a wide range of species [35]. In STDP, the presynaptic spikes occur immediately before the current postsynaptic spike strengthen the interconnecting synapses (LTP); otherwise, the synapses are weakened (LTD) [33,39,40]. Using STDP makes the SNN more biologically plausible while providing an efficient, event-based learning scheme. A question is how can the gradient descent algorithm be implemented using STDP to acquire efficient and accurate learning rules in multi-layer SNNs?

In this paper, we propose novel multi-layer, supervised learning rules to train SNNs of integrate-and-fire (IF) neurons. The proposed approach takes advantages of the both efficient, bio-inspired STDP and high performance backpropagation (gradient descent) rules. First, we show that the IF neurons approximate the rectified linear units. Then, we develop a temporally local learning approach specified by an STDP/anti-STDP rule derived from the backpropagation weight change rules that can be applied at the target/output spike times.

2. Method

Before proposing the spike-based, temporally local learning rules, we show how the biological IF neurons approximate the well-known artificial neurons equipped with the rectified linear unit (ReLU) activation function. This approximation, that is shown to be plausible in Section 2.1, takes the first step in converting rate-based learning to spatio-temporally local learning. This approximation is crucial for the learning rule derivation in Section 2.2 because the weight change rules are not derived like gradient descent; whereas they are achieved by converting the derivative results of ReLU neurons to IF neurons directly instead of taking the derivative of a loss function.

2.1. Rectified linear unit versus IF neuron

A neuron with the rectified linear unit (ReLU) activation function, $f(y)$, receiving input signals, x_h , via corresponding synaptic weights, w_h , is defined as follows:

$$f(y) = \max(0, y), \quad y = \sum_h x_h w_h \quad (1)$$

$$\frac{\partial f}{\partial y} = \begin{cases} 1, & y > 0 \\ 0, & y \leq 0 \end{cases} \quad (2)$$

Theorem 1. The IF neuron approximates the ReLU neuron. Specifically, the membrane potential of the IF neuron approximates the activation value of the ReLU neuron.

Proof. A non-leaky IF neuron integrates the temporal membrane potentials caused by its input spike trains and fires when its membrane potential, $U(t)$, reaches the neuron's threshold, θ . A simplified formulation of the IF neuron is shown in Eq. (3).

$$U(t) = U(t - \Delta t) + \sum_h w_h(t) s_h(t) \quad (3a)$$

$$\text{if } U(t) \geq \theta \text{ then } r(t) = 1, U(t) = U_{\text{rest}} \quad (3b)$$

Where $s_h(t)$ and $r(t)$ are pre- and postsynaptic spikes at time t ($s_h(t), r(t) \in \{0, 1\}$), respectively. Δt is the simulation time step (here, $\Delta t = 1$ ms). The neuron's membrane potential is reset to resting potential, U_{rest} , (assume $U_{\text{rest}} = 0$) upon firing. By formulating the presynaptic spike train, $G_h(t)$, as the sum of delta Dirac functions (Eq. (4)), the input value, $x_h \in [0, 1]$, in Eq. (1), can be determined by integrating over the spiking time interval, T , as shown in Eq. (5).

$$G_h(t) = \sum_{t_h^p \in \{t: s_h(t)=1\}} \delta(t - t_h^p) \quad (4)$$

$$x_h = \frac{1}{K} \int_0^T G_h(t') dt' \quad (5)$$

K is a constant denoting the maximum number of spikes in T ms interval.

In a short time period $(t - \alpha, t]$ between consecutive postsynaptic spikes $r(t - \alpha)$ and $r(t)$, the neuron's membrane potential, following Eq. (3), is obtained by Eq. (6). α is the time difference between two consecutive postsynaptic spikes (one spike at $t - \alpha$ and one spike at t) such that there is only one spike in $(t - \alpha, t]$. As $r(t - \alpha) = 1$, the membrane potential at $t - \alpha$ resets to zero ($U(t - \alpha) = 0$).

$$U(t) = \frac{1}{K} \sum_h w_h \left(\int_{t-\alpha}^t \sum_{t_h^p} \delta(t' - t_h^p) dt' \right) \quad (6)$$

The membrane potential calculated above ($U(t)$) is greater than the threshold, θ ($U(t) \geq \theta$). Thus, as we assumed that the IF neuron is non-leaky and its membrane potential resets to zero upon firing, R postsynaptic spikes demand an accumulated membrane potential (Eq. (7)). Since K is a constant, for the sake of simplification, we remove it from the equation and scale the threshold value in the neuron simulation accordingly. The accumulated membrane potential, U^{tot} , is obtained by linear summation over the sub-membrane potentials computed in Eq. (6). By this definition, the postsynaptic spike count, R , represents the output value, \hat{y} , that is passed through a non-linear, threshold-based activation function.

$$U^{\text{tot}} = \hat{y} = \sum_{t^f \in \{t: r(t)=1\}} U(t^f) \quad (7)$$

U^{tot} specifies the IF neuron's activity in T ms, which is proportionally related to the postsynaptic spike count, R . Therefore, the activation function of the IF neuron can be expressed as

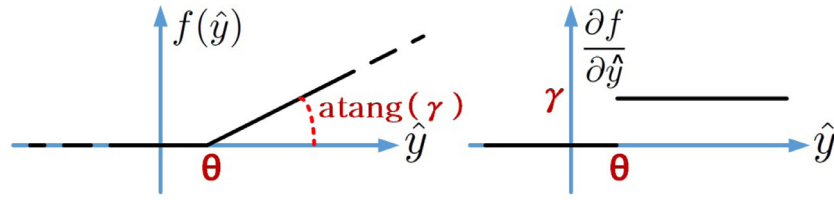


Fig. 1. IF activation function ($f(\hat{y})$) and its derivative. This function resembles a scaled version of the ReLU function that is shifted by θ .

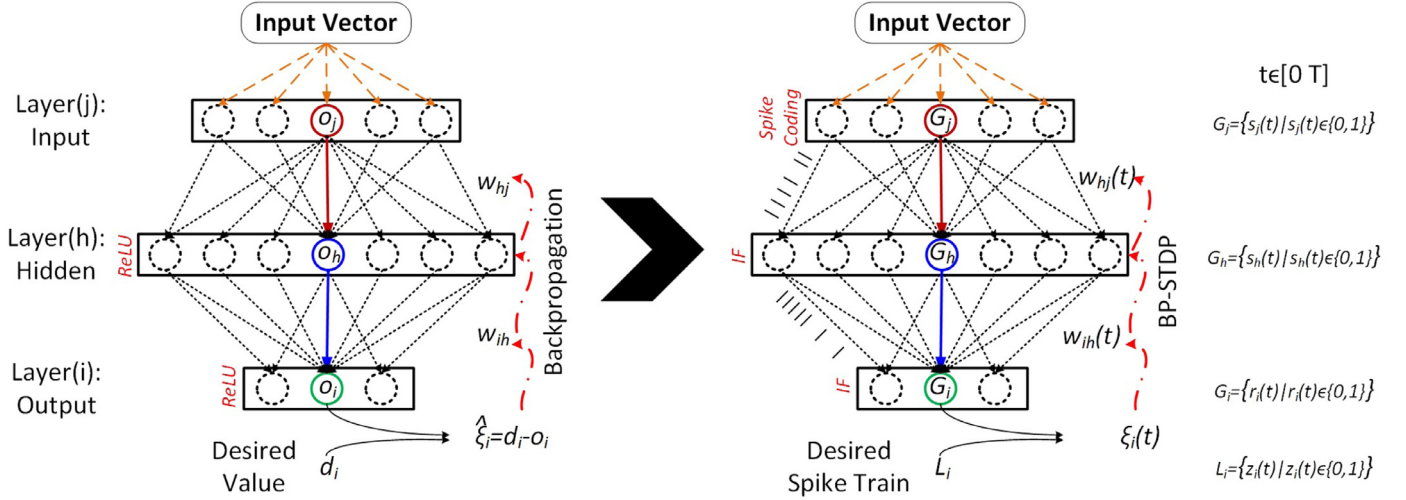


Fig. 2. The 2-Layer, conventional (left) and spiking (right) network architectures. The SNN receives spike trains representing input feature values in T ms. The learning rules and the network status in the SNN are specified by an additional term as time (t). The formulas and parameters are discussed in Eqs. (9)–(21).

$$f(\hat{y}) = \begin{cases} R = \gamma \hat{y} & \hat{y} > \theta \\ 0 & \text{otherwise} \end{cases} \quad (8)$$

where, $\gamma \propto T \cdot \theta^{-1}$ is a constant controlling the postsynaptic spike count. This activation function is similar to a linearly scaled ReLU function that is shifted to right by θ . Fig. 1 shows the activation function and its derivative.

This theorem opens a new door to develop new spike-based learning rules (applied to the spiking IF neurons) derived from the traditional learning rules (applied to the ReLU neurons). Specifically, in the next section, we use this theorem to propose an STDP-based backpropagation rule applied to the spiking IF neurons. □

2.2. Backpropagation using STDP

The proposed learning rules are inspired from the backpropagation update rules reported for neural networks that are equipped with ReLU activation function. Fig. 2 shows the network architectures and parameters used to describe the conventional and spiking neural networks in this paper. The main difference between these two networks is their data communication where the neural network (left) receives and generates real numbers while the SNN (right) receives and generates spike trains in T ms time intervals.

Non-spiking neural networks equipped with gradient descent (GD) solve an optimization problem in which the squared difference between the desired, d , and output, o , values is minimized [41,42]. A common objective function computed for M output neurons receiving N training samples is shown in Eq. (9).

$$E = \frac{1}{N} \sum_{k=1}^N \sum_{i=1}^M (d_{k,i} - o_{k,i})^2 \quad (9)$$

The weight change formula (using GD with learning rate of μ) for a linear output neuron, say i , receiving H inputs, o_h , (for a single

training sample) is achieved by

$$E = (d_i - o_i)^2 = \left(d_i - \sum_h o_h w_{ih} \right)^2 \rightarrow \frac{\partial E}{\partial w_{ih}} = -2(d_i - o_i) \cdot o_h \quad (10)$$

By reversing the sign on the derivative, we have

$$\Delta w_{ih} \propto -\frac{\partial E}{\partial w_{ih}} \rightarrow \Delta w_{ih} = \mu(d_i - o_i)o_h \quad (11)$$

By assuming d_i , o_i , and o_h as the spike counts of spike trains L_i , G_i , and G_h [43] (see Eqs. (4) and (12)), respectively, the weight change, defined above, can be re-expressed such that it computes the synaptic weight update in an SNN. This assumption is valid upon approximating the spiking IF neurons to the ReLU neurons (Theorem 1). Eq. (13) shows this update rule after $T = 50$ ms.

$$G_i(t) = \sum_{t_i^p \in \{t: r_i(t)=1\}} \delta(t - t_i^p) \quad (12a)$$

$$L_i(t) = \sum_{t_i^q \in \{t: z_i(t)=1\}} \delta(t - t_i^q) \quad (12b)$$

$$\Delta w_{ih} = \mu \int_0^T (L_i(t') - G_i(t')) dt' \cdot \int_0^T G_h(t') dt' \quad (13)$$

However, the weight change rule in Eq. (13) is not local in time. To make the learning rule local in time, we break the time interval, T , into sub-intervals such that each sub-interval contains zero or one spike. Hence, the learning rule, in a short time period of Eq. (13), is specified by Eq. (14).

$$\Delta w_{ih}(t) \propto \mu(z_i(t) - r_i(t))s_h(t) \quad (14)$$

To implement the formula above, a combination of event-based STDP and anti-STDP can be used. The proposed learning rule updates the synaptic weights using a teacher signal to switch between STDP and anti-STDP. That is, the target neuron undergoes STDP and the non-target ones undergo anti-STDP. The desired spike trains, \mathbf{z} , are defined based on the input's label. Therefore, the target neuron is represented by a spike train with maximum spike frequency (β) and the non-target neurons are silent. Additionally, the learning rule triggers at desired spike times, $z_i(t)$ (the desired spike times are the same for all the target neurons). Eq. (15) shows the weight change that is applied to the output layer of our supervised SNN.

$$\Delta w_{ih}(t) = \mu \cdot \xi_i(t) \sum_{t'=-\epsilon}^t s_h(t') \quad (15)$$

$$\xi_i(t) = \begin{cases} 1, & z_i(t) = 1, r_i \neq 1 \text{ in } [t - \epsilon, t] \\ -1, & z_i(t) = 0, r_i = 1 \text{ in } [t - \epsilon, t] \\ 0, & \text{otherwise} \end{cases} \quad (16)$$

Then, the synaptic weights of output layer are updated by

$$w_{ih}(t) = w_{ih}(t) + \Delta w_{ih}(t) \quad (17)$$

The target neuron is determined by $z_i(t)$ where $z_i(t) = 1$ denotes the target neuron and $z_i(t) = 0$ denotes the non-target neuron. The weight change scenario for the output layer starts with the desired spike times. **At desired spike time t , the target neuron should fire in a short time interval $[t - \epsilon, t]$ known as the STDP window.** Otherwise, the synaptic weights are increased proportionally by the presynaptic activity (mostly zero or one spike) in the same time interval. The presynaptic activity is denoted by $\sum_{t'=-\epsilon}^t s_h(t')$ that counts the presynaptic spikes in $[t - \epsilon, t]$ interval. On the other hand, **the non-target neurons upon firing undergo weight depression** in the same way. This scenario is inspired from the traditional GD while supporting spatio-temporal, local learning in SNNs.

The learning rule written above works for a single layer SNN trained by supervision. To train a multi-layer SNN, we use the same idea that is inspired from the traditional backpropagation rules. The backpropagation weight change rule applied to a hidden layer of ReLU neurons is shown in Eq. (18).

$$\Delta w_{hj} = \mu \cdot \left(\sum_i \hat{\xi}_i w_{ih} \right) \cdot o_j \cdot [o_h > 0] \quad (18)$$

Where $\hat{\xi}_i$ denotes the difference between the desired and output values ($d_i - o_i$). In our SNN, $\hat{\xi}_i$ is approximated by ξ_i (Eq. (16)). The value $[o_h > 0]$ specifies the derivative of ReLU neurons in the hidden layer. Using the approximation of the IF neurons to the ReLU neurons (Eq. (8)), similar to the output layer (Eq. (13)), the weight change formula can be re-expressed in terms of spike counts in a multi-layer SNN as shown in Eq. (19).

$$\Delta w_{hj} = \mu \int_0^T \left(\sum_i \xi_i(t') w_{ih}(t') \right) dt' \cdot \int_0^T \left(\sum_{t_j^p} \delta(t' - t_j^p) \right) dt' \cdot \left(\left[\int_0^T \sum_{t_h^p} \delta(t' - t_h^p) dt' \right] > 0 \right) \quad (19)$$

After dividing T into short sub-intervals $[t - \epsilon, t]$, the temporally local rule for updating the hidden synaptic weights is formulated as follows.

$$\Delta w_{hj}(t) = \begin{cases} \mu \cdot \sum_i \xi_i(t) w_{ih}(t) \cdot \sum_{t'=-\epsilon}^t s_j(t') & , s_h = 1 \text{ in } [t - \epsilon, t] \\ 0 & , \text{otherwise} \end{cases} \quad (20)$$

```

1: for  $t = \Delta t$  to  $T$  do
  - - First Layer
2:    $U_1[:, t] = s_j[:, t] * w_{jh} + U[:, t - \Delta t]$ 
3:   for  $u$  in hidden neurons do
4:     if  $U_1[u, t] > \theta_h$  then
5:        $s_h[u, t] = 1$ 
6:        $U_1[u, t] = U_{rest}$ 
  - - Second Layer
7:    $U_2[:, t] = s_h[:, t] * w_{hi} + U_2[:, t - \Delta t]$ 
8:   for  $o$  in output neurons do
9:     if  $U_2[o, t] > \theta_o$  then
10:       $r_i[o, t] = 1$ 
11:       $U_2[o, t] = U_{rest}$ 
  - - Weight Adaptation
12:  if  $t \in \mathbf{z}$  then - -  $\mathbf{z} = \{t | z(t) = 1\}$ 
13:    if  $r_i = 0$  in  $[t - \epsilon, t]$  for target neuron:
14:       $\xi_i[target] = 1$ 
15:    if  $r_i = 1$  in  $[t - \epsilon, t]$  for non-target neurons:
16:       $\xi_i[non\ targets] = [-1]$ 
17:     $derivatives_h = [s_h(t) = 1 \text{ in } [t - \epsilon, t]]$ 
18:     $\xi_h = w_{hi} * \xi_i \cdot derivatives_h$ 
19:     $w_{hi} += \text{sum}(s_h(t - \epsilon...t)) * \xi_i \cdot \mu$ 
20:     $w_{jh} += \text{sum}(s_j(t - \epsilon...t)) * \xi_h \cdot \mu$ 

```

Fig. 3. The BP-STDP algorithm applied to a multi-layer SNN consisting of input, output and one hidden layer. '*' stands for matrix multiplication. '-' starts a comment.

Table 1
Model parameters.

Parameter	Value	Parameter	Value
ϵ	4 ms	μ	0.0005
β	250 Hz	θ_o	$0.025 \times H$
θ_h	0.9	H	{10,...,1500}
Δt	1 ms	U_o	0

Finally, the synaptic weights of hidden layer are updated by

$$w_{hj}(t) = w_{hj}(t) + \Delta w_{hj}(t) \quad (21)$$

The above learning rule can be non-zero when the hidden neuron h fires (postsynaptic spike occurrence). Thus, the weights are updated according to the presynaptic ($s_j(t)$) and postsynaptic ($s_h(t)$) spike times, analogous to the standard STDP rule. Additionally, the **derivative of ReLU ($o_h > 0$) is analogous to the spike generation in the IF neurons (see the condition in Eq. (20)).** Following this scenario for the spatio-temporally synaptic weight change rule, we can build a multi-layer SNN equipped with the STDP-based Backpropagation algorithm, named BP-STDP. Fig. 3 demonstrates the BP-STDP algorithm applied to an SNN with one hidden layer.

3. Results

We ran three different experiments to evaluate the proposed model (BP-STDP) on the XOR problem, the iris dataset [44], and the MNIST dataset [45]. The parameters used in the experimental setting are shown in Table 1. The synaptic weights for all the experiments are initialized by the Gaussian random numbers with zero mean and unit standard deviation.

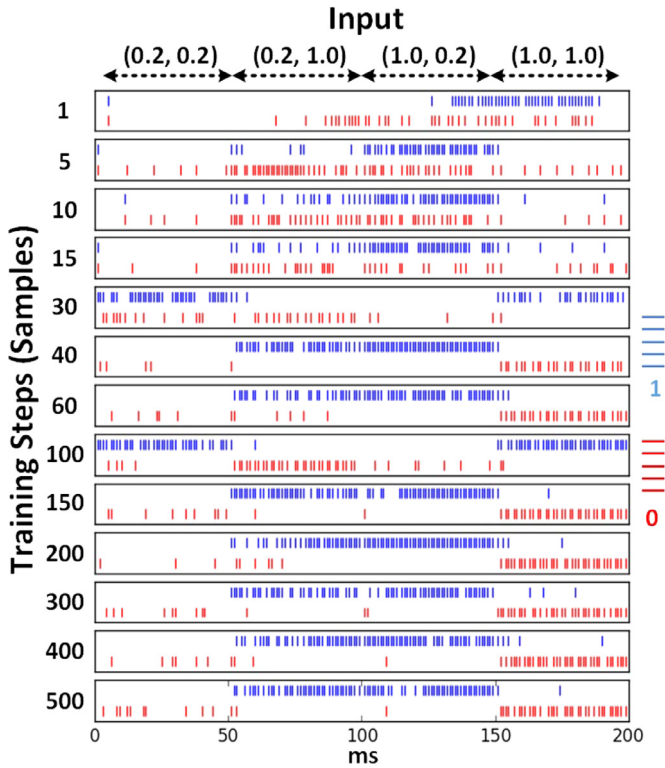


Fig. 4. Spike trains released from the output neurons in response to four pairs of spike trains representing $\{(0.2, 0.2), (0.2, 1), (1, 0.2), (1, 1)\}$ values in 1 through 500 training steps. We used high spike rates for better visualization.

3.1. XOR problem

The BP-STDP algorithm is evaluated by the XOR problem to show its ability to solve linear inseparability. The dataset contains four data points $\{(0.2, 0.2), (0.2, 1), (1, 0.2), (1, 1)\}$ and corresponding labels $\{0, 1, 1, 0\}$. We used 0.2 instead of 0 to activate the IF neurons (to release spikes). The network architecture consists of 2 input, 20 hidden, and 2 output IF neurons. The number of hidden neurons for this problem has little effect on the results. We will investigate the impact of the number of hidden neurons for the MNIST classification task. Each input neuron releases spike trains corresponding to the input values such that the value 1 is represented by a spike train with the maximum spike rate (250 Hz).

Fig. 4 shows the training process where each box represents the two output neurons' activities with respect to the four input spike

patterns determining $\{0, 1, 1, 0\}$ classes. After around 150 training steps (or 37 epochs)¹, the output neurons become selective to the input categories. The comparable studies are the works of Sporea et al. [26] and Mostafa [12] that have reported 137 and 348 average training epochs for XOR training. Fig. 5 demonstrates the learning convergence progress using the energy function defined in Eq. (22). This figure shows that the proper learning rates, μ , fall in the range $[0.01, 0.0005]$.

$$\text{MSE} = \frac{1}{N} \sum_{k=1}^N \left(\frac{1}{T} \sum_{t=1}^T \xi^k(t) \right)^2, \quad \xi^k(t) = \sum_i \xi_i^k(t) \quad (22)$$

In Eq. (22), N (here, $N = 1$) and $\xi^k(t)$ denote the training batch size and the error value of output neuron i in response to sample k .

3.2. Iris dataset

The Iris dataset consists of three different types of flowers (Setosa, Versicolour, and Virginica) represented by the length and width of both petal and sepal (4 features). After normalizing the feature values in the range $[0, 1]$, input spike trains are generated. The network architecture in this experiment consists of 4 input, 30 hidden, and 3 output IF neurons. We found, in our preliminary experiments, that using more than 10 hidden neurons does not significantly improve accuracy.

Similar to the results obtained for the XOR problem, Fig. 6 illustrates that the SNN converges to be selective to the three flower patterns through training (120 epochs). Fig. 7 shows the learning process of the SNN in terms of the MSE defined in Eq. (22). The final evaluation using 5-fold cross validation (120 training and 30 testing samples) reported 96% accuracy while the traditional neural network equipped with standard Backpropagation showed 96.7% accuracy. This result shows the success of the proposed BP-STDP algorithm to train temporal SNNs. Furthermore, Table 2 compares our results with other spike-based and traditional supervised learning methods. Table 2 also shows the learning methods and the number of training epochs. As stated earlier in Introduction, the gradient descent algorithm is computationally expensive because it calculates an error function and the derivative of the neuron's activation function at each simulation time step and does not utilize the efficient event-based scheme in SNNs. Among the STDP based methods with high accuracy rates ($> 94\%$) in Table 2,

¹ We use training step to report the number of single samples used for training. The "epoch" term shows the number of training processes done over the whole training set. For the XOR problem, the training set consists of 4 samples; so, $150/4 = 37$ epochs were counted.

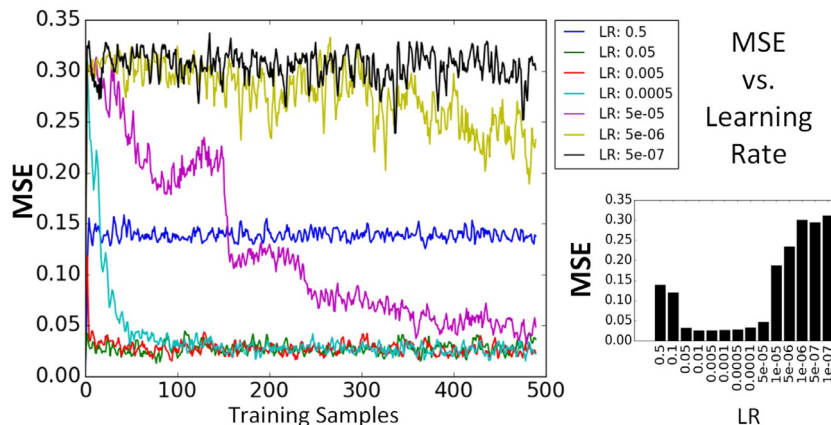
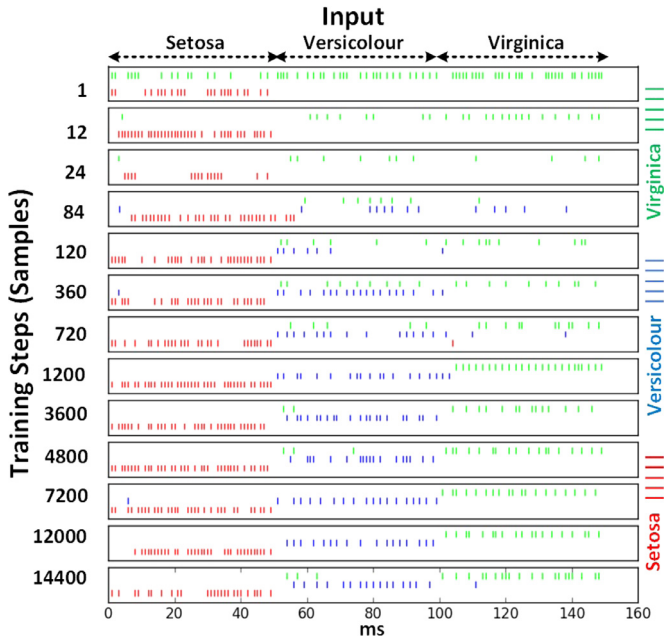


Fig. 5. MSE of the XOR learning process (left: through 500 training steps, right: after training).

Table 2

Iris classification using the BP-STDP, gradient descent (GD) based, STDP-based, and Non-spiking learning algorithms.

Model	Learning method	No. epochs	Acc(%)
SpikeProp (Bohte et al. 2002) [6]	Spiking GD	1000	96.1
MSGD; Xu et al. (2013) [10]	spiking GD	234	94.4
Xie et al. (2016) [21]	Normalized spiking GD	18	96.0
Xie et al. (2017) [23]	Spiking GD	2	95.0
Lin et al. (2017) [18]	Spiking GD	100	96.7
Wang et al. (2014) [36]	STDP	One pass	86.1
Yu et al. (2014) [31]	Tempotron	100	92.6
SWAT; Wade et al. (2010) [46]	STDP	500	95.3
multi-ReSuMe; Sporea et al. (2013) [26]	Widrow-Hoff, STDP	174	94.0
Taherkhani et al. (2018) [38]	STDP-inspired	100	95.7
BP-STDP	STDP	120	96.0
<i>Non-Spiking Examples</i>			
SVM and Bayesian [36]	Non-Spiking	–	96.0
ANN (30 hidden neurons)	Non-Spiking	–	96.7

**Fig. 6.** Spike trains released from the output neurons in response to the spike trains representing Setosa, Versicolour, and Virginica. 14,400 steps present 120 training epochs repeated over 120 training samples.

SWAT [46], multi-ReSuMe [26], and [38] have reported 95.3%, 94%, and 95.7% accuracies after 500, 174, and 100 training epochs, respectively. Although Xie et al. [21] use backpropagation, they have improved the training efficiency by changing the gradient descent method by applying the presynaptic spike jitters for adapting the corresponding synaptic weights. They achieved 96% accuracy after only 18 training epochs. They further improved the efficiency of gradient descent in SNNs by considering only the target spikes and achieved 95% accuracy after 2 training epochs [23].

3.3. MNIST dataset

To assess the proposed algorithm in solving more complex problems, we evaluate the SNN on MNIST. The MNIST dataset consists of 70,000 handwritten digits represented by 28×28 grayscale images. The SNN has 784 (28×28) input, 100 through 1500 hidden, and 10 (digits 0–9) output IF neurons equipped with BP-STDP. The SNN was trained and tested on 60k training and 10k testing samples. The input spike trains are generated by random lags with

the spike rates proportional to the normalized pixel values in the range [0, 1].

After training, the output neuron with maximum spike rate shows the target category. As the uniformly random distribution was used to generate the input spike trains, the first responding neuron shows the target category as well. Fig. 8 shows the output neurons' membrane potentials (after training) in response to the spike trains generated from three randomly selected digits. The target neuron's membrane potential grows fast and reaches the threshold while the other neurons' activities are near zero. Furthermore, this fast response (< 9 ms) reduces the network's response latency. Fig. 9 shows the learning process for 60k training samples (one pass learning). Each point in this figure shows the average MSE value after training over 50 MNIST digits. The MSE track in this plot shows the fast convergence of BP-STDP for the learning rates of 0.001 and 0.0005. Fig. 9 shows the MSE values and the accuracy rates for the SNN with 1000 hidden neurons over training. After 5000 and 45,000 training samples, the performances of 90% and 96% were achieved. To examine the impact of the number of hidden neurons on performance, we applied the BP-STDP rule to six SNNs with 100 through 1500 hidden IF neurons. Fig. 10 shows the accuracy rates over training for these SNNs. The best accuracy rates belong to the networks with more than 500 hidden neurons.

Finally, the BP-STDP algorithm was evaluated on 2-layer and 3-layer SNNs. The SNN architectures (the number of hidden neurons) are the same as the neural network architectures utilized in [47] to show a better comparison. Our model achieved $96.6 \pm 0.1\%$ and $97.2 \pm 0.07\%$ accuracy rates for the 2-layer and 3-layer SNNs, respectively. These results are comparable to the accuracy rates reported by traditional neural networks (trained by backpropagation) [47]. Table 3 compares the proposed supervised learning method (BP-STDP) with the traditional backpropagation algorithm (or GD), spiking backpropagation methods embedded in multi-layer (deep) SNNs, and recent STDP-based SNNs for MNIST classification. This comparison confirms the success of the bio-inspired BP-STDP rule applied to the temporal SNN architecture. Our model is one of the few high performance, supervised learning approaches for training the multi layer SNNs while only using STDP-based learning rules in all the layers (from the input layer to the classification layer). The SNNs introduced in [48,49] develop multi-layer STDP learning for feature extraction. However, they use a support vector machine (SVM) classifier for the final supervised layer. Although the gradient descent approaches performed successfully [12,19,50,51], they do not offer bio-inspired, power-efficient STDP learning in SNNs. Specifically, the recently developed deep SNN using backpropagation [19] showed higher accuracy than the BP-STDP's accuracy; however, it computes the derivatives of activation functions obtained by the

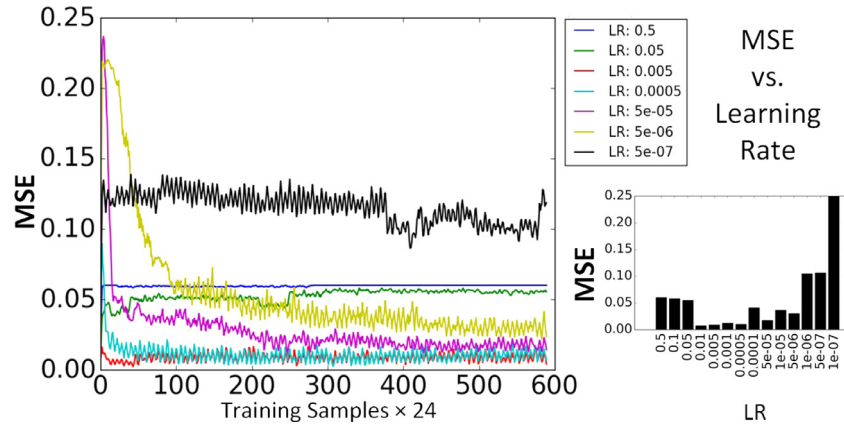


Fig. 7. MSE of the learning process applied to the Iris dataset (left: through training iterations, right: after training). To depict the training process smoother, we reported the MSE values for each 24 samples.

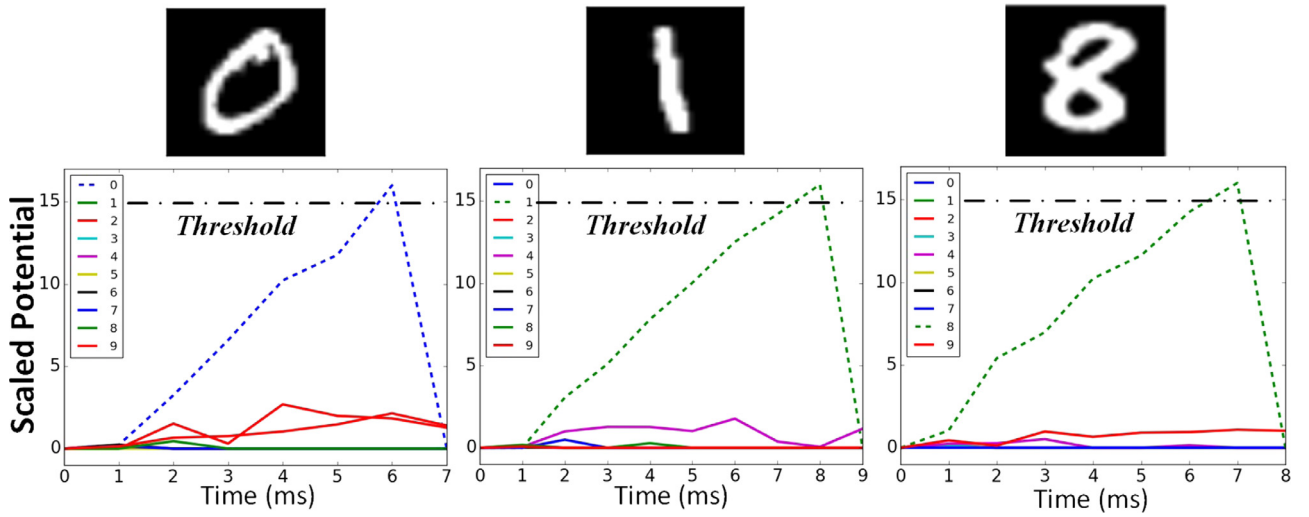


Fig. 8. Temporal membrane potentials of output neurons in response to randomly selected '0', '1', and '8' handwritten digits.

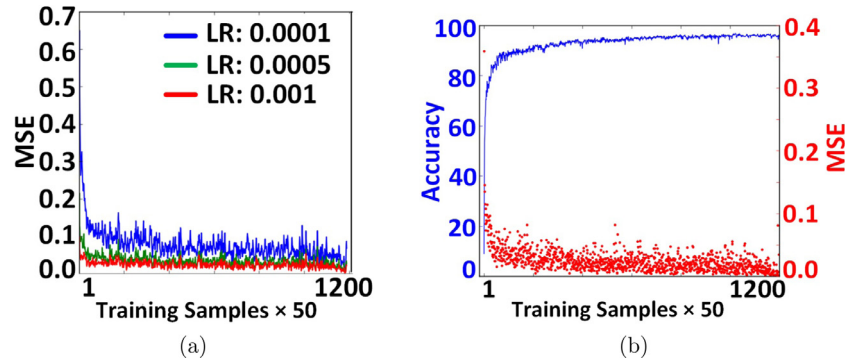


Fig. 9. (a): MSE of the SNN over training with $\mu = 0.001, 0.0005, 0.0001$. (b): MSE and accuracy of the SNN with $\mu = 0.0005$. $H = 1000$. Each point shows the average MSE/accuracy of the SNN trained by 50 images.

neurons' membrane potentials instead of using their spike events.

4. Discussion

BP-STDP introduces a novel supervised learning for SNNs. It showed promising performances comparable to the state-of-the-art conventional gradient descent approaches. BP-STDP provides bio-inspired local learning rules which take spike times into consider-

ation as well as spike rates in the subsequent layers of IF neurons. Bengio et al. [53] showed that the synaptic weight change is proportional to the presynaptic spike event and the postsynaptic temporal activity that is analogous to the STDP rule and confirms Hinton's idea that says STDP can be associated with the postsynaptic temporal rate [54]. In this paper, we showed that the backpropagation update rules can be used to develop spatio-temporally local learning rules, which implement biologically plausible STDP in SNNs.

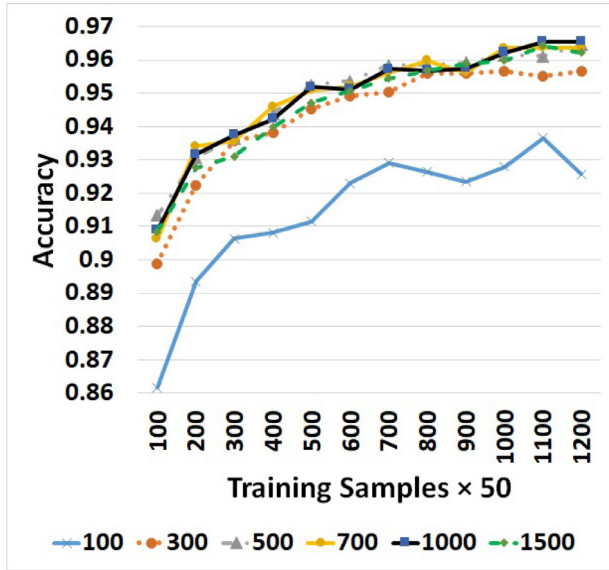


Fig. 10. Performance of BP-STDP applied to the SNNs with 100 through 1500 hidden IF neurons.

Table 3

MNIST classification performance of the proposed BP-STDP applied to SNNs in comparison with (1) traditional backpropagation applied to conventional neural networks, (2) spiking gradient descent, and (3) recent STDP-based SNNs. "*" denotes a random distortion of training set (data augmentation for improving the performance). H1 and H2 show the number of neurons in the first and second hidden layers. The STDP-based methods use one pass learning over the training set.

Model	Description	Acc. (%)
<i>Traditional Neural Networks</i>		
Lecun et al. [47]	Traditional backpropagation. 2-layer network; H1=300	95.3 96.4*
Lecun et al. [47]	Traditional backpropagation. 2-layer network; H1=1000	95.5 96.2*
Lecun et al. [47]	Traditional backpropagation. 3-layer network; H1=500; H2=150	97.1 97.6*
<i>Spiking Gradient Descent</i>		
O'Connor et al. [50]	Deep SNN. Stochastic gradient descent	96.40
O'Connor et al. [50]	Deep SNN. Fractional stochastic gradient descent	97.93
Lee et al. [19]	Deep SNN. Backpropagation	98.88
Mostafa. [12]	Membrane potential as activation value Multi-layer SNN. Gradient descent in local time intervals	97.20
<i>STDP Based Models</i>		
Diehl et al. [51]	2-Layer SNN. STDP; Example-based classifier	95.00
Tavanaei et al. [48]	Spiking CNN. Sparse coding and STDP; SVM classifier	98.36
Kheradpisheh et al. [49]	Spiking CNN. Layer-wise STDP; SVM classifier	98.40
Neftci et al. [52]	Spiking restricted Boltzmann machine. Contrastive divergence for IF neurons	91.90
BP-STDP (This paper)	STDP-based backpropagation. 2-layer SNN; H1=300	95.70
BP-STDP (This paper)	STDP-based backpropagation. 2-layer SNN; H1=1000	96.60
BP-STDP (This paper)	STDP-based backpropagation. 3-layer SNN; H1=500; H2=150	97.20

The proposed algorithm is inspired from the backpropagation update rules used for the conventional networks of ReLU neurons. However, it develops biologically plausible, temporally local learning rules in an SNN. This matter was accomplished by an initial approximation of the IF neurons to the ReLU neurons to support the spike-based communication scheme in SNNs. The spiking supervised learning rules offer a combination of STDP and anti-STDP ap-

plied to spiking neural layers corresponding to the temporal presynaptic and postsynaptic neural activities. Therefore, we take advantages of both accurate gradient descent and efficient, temporally local STDP in spiking frameworks. The main question is that how does error propagation corresponds to the spiking behavior of IF neurons in multi-layer network architectures? To answer this question, let us assume the error value as a signal that either stimulates ($\Sigma_i \xi_i > 0$) or suppresses ($\Sigma_i \xi_i < 0$) the IF neuron to fire. Stimulating (suppressing) a neuron refers to increasing (decreasing) its membrane potential that is proportionally controlled by its input weights and presynaptic spike events. Thus, as the BP-STDP update rules change the synaptic weights temporally based on the error signal and the presynaptic spike times, it manipulates the neural activities (action potentials) of hidden layers at each time step.

The experimental results showed the success of BP-STDP in implementing supervised learning embedded in multi-layer SNNs. The XOR problem proved the ability of BP-STDP to classify non-linearly separable samples represented by spike trains through T ms time intervals. The complex problems of IRIS and MNIST classification demonstrated comparable performances (96.0% and 97.2% respectively) to the conventional backpropagation algorithm and the recent SNNs while BP-STDP offers an end-to-end STDP-based supervised learning for spiking pattern classification. Using the bio-inspired and temporally local STDP rules guides us one step closer to the efficient computations that occur in the brain. To the best of our knowledge, this approach is the first high performance, STDP-based supervised learning while avoiding computationally expensive gradient descent.

5. Conclusion

This paper showed that IF neurons approximate rectified linear units, if the neurons' activities are mapped to the spike rates. Hence, a network of spiking IF neurons can undergo backpropagation learning applied to conventional NNs. We proposed a temporally local learning rule (derived from the traditional backpropagation updates) incorporating the STDP and anti-STDP rules embedded in a multi-layer SNN of IF neurons. This model (BP-STDP) takes advantages of the bio-inspired, efficient STDP rule in spiking platforms and the power of GD for training multi-layer networks. Also, converting the GD-based weight change rules to the spike-based STDP rules is much easier and computationally inexpensive than developing a spiking GD rule. The experiments on the XOR problem showed that the proposed SNN can classify non-linearly separable patterns. Furthermore, the final evaluations on the Iris and MNIST datasets demonstrated high classification accuracies comparable to the state-of-the-art, multi-layer networks of traditional and spiking neurons.

The promising results of the BP-STDP model warrants our future investigation to develop a deep SNN equipped with BP-STDP and regularization modules. The deep SNN can be utilized for larger pattern recognition tasks while preserving efficient, brain-like computations.

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