

EE 746 2018 Quiz 1: (1.5 hours); Write your answers in expressions. Then calculate values. Draw neatly diagrams (roughly to scale) to explain your ideas.

**Question 1:** The neuron is at resting potential of -60mV. (30 + 10 points)

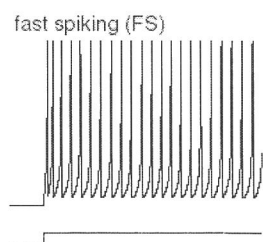
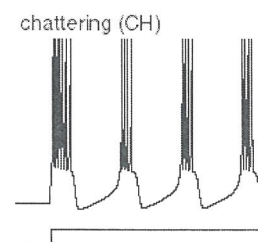
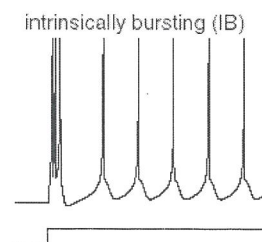
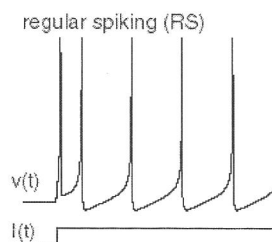
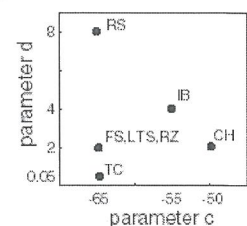
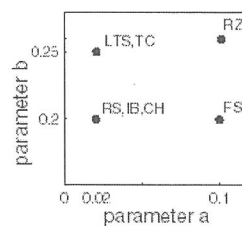
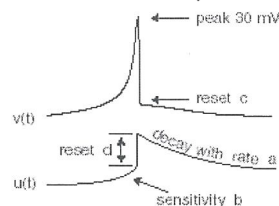
- Draw the circuit diagram for the neuron based on Hodgkin Huxley Model. Draw the time-constant  $\tau_o$  and target concentration  $X_o$  vs. membrane potential. Describe the transient m,n,h parameters that will produce a spike. Which ion channel is the cause of refractory period?
- During resting potential, which ion (K or Na) is near its equilibrium potential? Which ion is the furthest from equilibrium potential? Which total (not net) ionic current flow is dominant?

**Question 2:** For the Izhikevich Neuron use parameters for RS Neuron from the figure below.

$$v' = 0.04v^2 + 5v + 140 - u + I$$

$$u' = a(bv - u)$$

if  $v = 30$  mV,  
then  $v \leftarrow c$ ,  $u \leftarrow u + d$



Case 1: In resting state, the current is zero ( $I_{in} = 0$ );

- Analyze methodically in steps to show that there are two steady state points in the  $(u, v)$  space. Show that one is stable and the other is unstable. (20 points)
- What would be the resting values of  $u, v$  approximately? Justify your analysis. (10 points)

Case 2: Threshold current level ( $I_{in} = I_{th}$ ) for spiking (20 points)

- Methodically, show the steps to estimate the maximum constant current at which spiking should not occur. Approximately estimate the numerical value of the current (within 20% error). What is the steady state membrane potential at this current?

Case 3: Initially the current is zero from time  $t = -\infty$ ; for time  $t > 0$ , the current  $I_{in} = 2 * I_{th}$ .

- Draw the steady state for  $u' = 0$  and  $v' = 0$  separately; These two lines divide the  $(u, v)$  space into five zones. Show the direction of motion of  $(u, v)$  in these five zones approximately by arrows. (10 points)
- Given the starting point, approximately draw the trajectory for RS in  $(u, v)$  space as it hits  $v \geq 30$  mV. (10 points)
- If after 1<sup>st</sup> reset, the  $(u, v)$  falls inside the parabola of  $v'$  equation, then what will be the trajectory to the next reset i.e.  $v \geq 30$  mV. (10 points)

## Quiz 2 EE 746 2018 Instructor: Udayan Ganguly (2 hours) -

1. A signal needs to be detected by a simple sensor neuron which fires when signal  $S(t)$  exceeds the sensor threshold i.e  $V_T$ . (5+10+15 marks)

- In a system, the signal  $S(t) = \sin^2(\omega * t)$  and sensor threshold  $V_T=1.01$  and  $\omega = 2\pi f$  where  $f = 1\text{Hz}$ . Draw the signal vs time  $S(t)$  and  $V_T$ .
- A Gaussian noise is added i.e.  $p(x) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$  with zero mean ( $\mu = 0$ ) and  $\sigma$  standard deviation. For such an arbitrary  $\sigma$ , write down the method in steps to calculate the probability of spiking of the sensor in steps.
- Next, draw the approx. probability of spiking in time i.e.  $P(t)$  when (i)  $\sigma = 0$ , (ii)  $\sigma = 1$ , (iii) When  $\sigma = 100$ , Briefly compare the results about the effect of noise.

2. Assume we have a total of  $N_s = 5$  synapses driving a LIF neuron, whose connection strengths  $w_i$  (where  $i = 1 \dots N_s$ ) are Gaussian distributed, with a mean strength of  $w_o$  and standard deviation of  $\sigma_o$ . (15+5+25 marks)

(a) the  $i^{th}$  synapse produces a current due to spikes at times  $t_i^j$  where  $j = 1, 2, \dots, n$

$i(t - t_i^j) = w_i * \{ \exp(-\frac{t-t_i^j}{5\tau_o}) - \exp(-\frac{t-t_i^j}{\tau_o}) \} H(t - t_i^j)$  where  $\tau_o = 2\text{ms}$ ; Draw  $I(t)$  if  $t_i^j = 0$  with  $t$  from  $-100$  to  $200\text{ms}$ ; If  $w_i = 1$ , please estimate the total synaptic charge added to the neuron due to one spike. Is this an excitatory or inhibitory synapse?

(b) Given  $w_o = 1$  and standard deviation of  $\sigma_o = 0.01$ , a Poisson stimulus of  $\lambda = 1/\text{s}$  is supplied by each synapse for a total time of  $T = 1\text{s}$ ; What is approximately total number of spikes received by the neuron from the synapses?

(c) Assuming the total number of spikes come uniformly in time (as opposed to a Poisson distribution), an LIF neuron has a  $C=10\text{F}$ , RC time constant of  $\tau_{LIF}$  and a threshold of  $1\text{e-3 V}$ ,

(i) Write down the method in steps (1-2 sentences per step) to find the number of spikes from the neuron.

(ii) Estimate quantitatively the number of spikes if neuron has  $\tau_{LIF} = 10\text{ms}$

(iii) Estimate quantitatively the number of spikes from the neuron if neuron has  $\tau_{LIF} = 10\text{s}$

3. In biology, three synapses from pre-neurons are connected to a post-neuron's axon. Pre-neurons can be simultaneously excited to spike at a given frequency (high/low) while post-neuron can be potentiated slightly above its resting potential ( $-60\text{mV}$ ) to say  $-40\text{mV}$  without issuing a spike. Predict the results of the experiments below (1 example given). Also, write a few sentence about how biology enables this "AND" operation of pre and post neuronal activity for LTP. (16+4 marks)

Input Stimulation				Result (LTP or no change)		
Pre-neuron 1 Freq	Pre-neuron 2 Freq	Pre-neuron 3 Freq	Post-neuron Potentiation	Synapse 1	Synapse 2	Synapse 3
High				LTP	No change	No change
Low			Low			
High		Low				
Low	Low	Low				
Low		Low	Low			



1. For the C. Elegans chemotaxis to seek, find and maintain a concentration set-point of  $C_T$  (Fig. 1) the LIF neuron based incomplete neural circuit is shown,
  - a. Draw the truth table to define the input to output logic map ( 8 marks)
  - b. draw the connections (excitatory vs inhibitory) from sensors to motor neurons (8 marks)
  - c. Name the logic operations and explain briefly working principle of the neuronal circuit implementations. Are there any special requirements from the LIF neurons to enable these logic gates? (8 marks)
2. A 1000 pre-neurons are used to input a Poisson train signal (not changing in time) with a fixed 20 Hz rate into a single output IF neuron (Fig. 2); In the beginning, each synapse has same weight. A perfectly antisymmetric STDP with window  $\pm 20$ ms is applied (see fig) to evolve weights within range 0 to 1; The time evolution of latency and weights are shown. Based on the figures below, answer the following questions with brief justifications of the methods used ( include conceptual drawing as needed). State assumptions clearly.
  - a. What is the average number of spikes per pre-neuron in one epoch? (2 marks)
  - b. In the pre-neuron spike-count vs. time graph (A), what is the bin size of time used? (4 marks)
  - c. What is the initial weight per synapse – assuming uniform initial weights? (4 marks)
  - d. Initially (zero presentations/epochs), what is the total number of spikes and hence total current needed for post- neuron to fire? (6 marks)
  - e. Finally (after 2000 presentations/epochs), the latency settles to 10 ms, what is the total number of spikes needed for neuron to fire? What is the average weight of synapses for the relevant pre-neurons? How will you check for consistency based on the figures given? Alternatively, how will you estimate if these pre-neurons are providing mostly single or mostly multiple spikes to enable the post neuron to fire? (10 marks)
  - f. Suppose instead of 1000 pre-neurons firing at fixed firing rate 20 Hz, we use 10 pre-neurons firing at fixed firing rate 2000Hz, (10 marks)
    - i. What will happen to the weight evolution based on STDP for synapses for the two cases above? Compare.
    - ii. What will happen to the time evolution of the post-synaptic spike in the two cases? Compare.

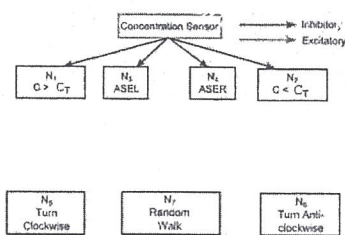


Fig. 1: An incomplete C. Elegans circuit

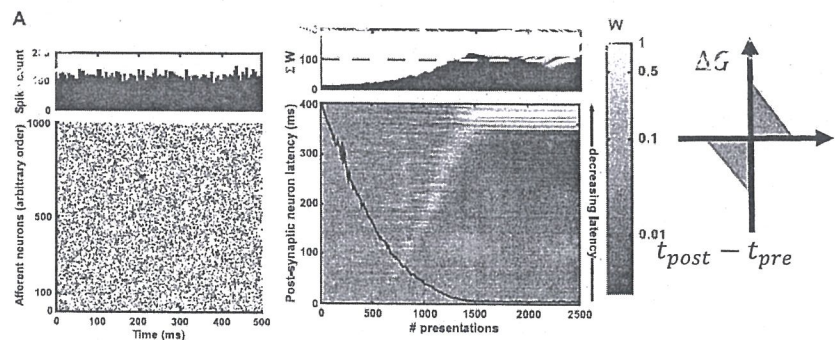
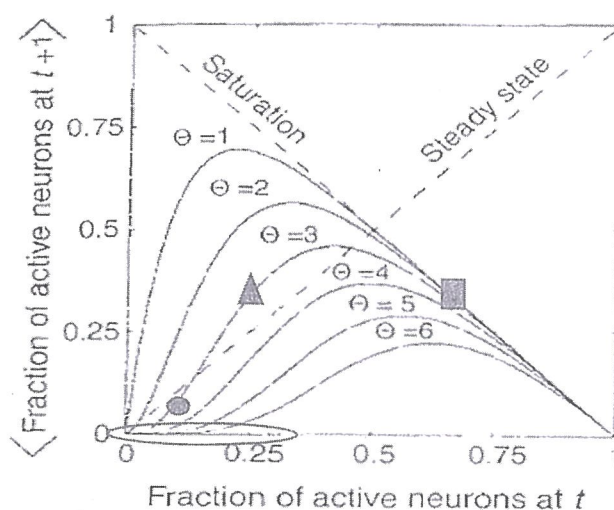


Fig. 2: (a) Input spike train (b) spike time and weight evolution (c) STDP

Quiz 4 EE 746 2018 (Total 62 marks) 1.5 hours Instructor: Udayan Ganguly

1. A  $N$  input neurons are connected to an output neuron with uniform synaptic connection strength,  $w_o$ . Each input neuron spikes with typical background cortical Activity ( $\mu_f \pm \sigma_f$ ) of  $5 \pm 3\text{Hz}$  (Gaussian Distribution) where  $p(x) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$ . (4+8 marks)
  - a. What is the spiking rate (mean and standard deviation) being received by the output neuron when (a)  $N = 100$  and (b)  $N = 10000$ ;
  - b. For  $N = 10000$ , we would like to get probability  $p$  immunity from background spikes causing the output neuron to spike by setting the weight  $w_o$  correctly – which will approximately decide an instantaneous (over discretization time of 1ms) input rate that will trigger an output firing. Write down the methodology ensure such background spiking immunity; Calculate the critical input spiking rate for (i)  $p = 0.5$  and (ii)  $p = 0.7$  (justify approximations);
2. Supposed there are total neurons ( $N = 10^{12}$ ), excitatory connections/neuron ( $C = 10^4$ ) which are random, Input ( $I = 10000$ ) neurons are made to spike; (4+5+6 marks)
  - a. What are the total neurons ( $n$ ) receiving spikes directly due to input?
  - b. Set threshold ( $Th$ ) i.e. number of pre-neuron spikes needed (at an instant) for post-neuron to fire = 3; what is the probability that a neuron receives at least 3 spike and hence fires; How many neurons will fire because of the input directly? Will this produce a sustained chain reaction?
  - c. For any  $Th$ , how to ensure sustained (but not growing) spiking by changing network size  $N$  while all other terms are constant? Derive an expression. How to ensure that runaway spiking (growing spike rate) in the network i.e. state the condition?
3. The dynamics of a randomly connected network (similar to the question 2) is shown below. Each neuron has a refractory period, i.e. if a neuron fires at time instant  $t$ , it cannot fire at  $t + 1$ ; (4+15+8+8 marks)
  - a. Explain the shapes of the following lines (i) steady state (ii) saturation
  - b. If we start from triangle, square & circle at  $\theta = 3$ , show the network dynamics to finally indicate the steady state spiking of the network. For what range of integer  $\theta$  will we never get sustained spiking;
  - c. At  $\theta = 1$  (i.e. pre-neuron spikes received to cause post neuron to fire), we see an approximately linear behavior at low  $f(t)$ ; while afterwards i.e. at  $f(t) > 0.1$ , the curve reaches a maximum and then reduces; Please explain the behavior qualitatively.
  - d. In addition to the above, as  $\theta$  increases, a threshold like behavior is observed at low  $f(t)$ ; e.g. at



$\theta = 6$ , at  $f(t) < 0.1$ ;  $f(t+1) \approx 0$  while  $f(t) > 0.1$  it increases sharply (like an exponential take-off). The threshold in  $f(t)$  increases with  $\theta$  where at  $\theta = 1$ ; threshold of  $f(t) \approx 0$  and increases with  $\theta$ . Please explain this behavior and quantitatively qualitatively; Plot  $f(t+1)$  vs.  $f(t)$  if the network did not have refractory period. Comment on it have any connection with concepts in 2(c).