Assignment Instructions: Assignment 2

Purpose
The purpose of this assignment is to
- continue deepening your ability of modeling a problem with linear programming;
- solve a linear programming problem graphically.

Directions

1. **(Computer Center Staffing)** You are the Director of the Computer Center for Gaillard College and responsible for scheduling the staffing of the center. It is open from 8 am until midnight. You have monitored the usage of the center at various times of the day and determined that the following numbers of computer consultants are required.

Time of day	Minimum number of consultants required to be on duty
8 am–noon	4
Noon–4 pm	8
4 am–8 pm	10
8 am–midnight	6

Two types of computer consultants can be hired: full-time and part-time. The full-time consultants work for eight consecutive hours in any of the following shifts: morning (8 am – 4 pm), afternoon (noon – 8 pm), and evening (4 pm – midnight). Full-time consult- ants are paid $14 per hour.

Part-time consultants can be hired to work any of the four shifts listed in the table. Part-time consultants are paid $12 per hour. An additional requirement is that during every time period, at least one full-time consultant must be on duty for every part-time consultant on duty.

a) Determine a minimum-cost staffing plan for the center. In your solution, how many consultants will be paid to work full time and how many will be paid to work part time? What is the minimum cost?
b) After thinking about this problem for a while, you have decided to recognize meal breaks explicitly in the scheduling of full-time consultants. In particular, full-time consultants are entitled to a one-hour lunch break during their eight-hour shift. In addition, employment rules specify that the lunch break can start after three hours of work or after four hours of work, but those are the only alternatives. Part-time consultants do not receive a meal break. Under these conditions, find a minimum-cost staffing plan. What is the minimum cost?

Hint: for this problem, you only need to formulate the LP problem without solving it.

2. Consider the problem from the previous assignment.

Back Savers is a company that produces backpacks primarily for students. They are considering offering some combination of two different models—the Collegiate and the Mini. Both are made out of the same rip-resistant nylon fabric. Back Savers has a long-term contract with a supplier of the nylon and receives a 5000 square-foot shipment of the material each week. Each Collegiate requires 3 square feet while each Mini requires 2 square feet. The sales forecasts indicate that at most 1000 Collegiates and 1200 Minis can be sold per week. Each Collegiate requires 45 minutes of labor to produce and generates a unit profit of $32. Each Mini requires 40 minutes of labor and generates a unit profit of $24. Back Savers has 35 laborers that each provides 40 hours of labor per week. Management wishes to know what quantity of each type of backpack to produce per week. Solve this problem graphically.

3. **(Weigelt Production)** The Weigelt Corporation has three branch plants with excess production capacity. Fortunately, the corporation has a new product ready to begin production, and all three plants have this capability, so some of the excess capacity can be used in this way. This product can be made in three sizes--large, medium, and small-- that yield a net unit profit of $420, $360, and $300, respectively. Plants 1, 2, and 3 have the excess capacity to produce 750, 900, and 450 units per day of this product, respectively, regardless of the size or combination of sizes involved.

   The amount of available in-process storage space also imposes a limitation on the production rates of the new product. Plants 1, 2, and 3 have 13,000, 12,000, and 5,000 square feet, respectively, of in-process storage space available for a day's production of this product. Each unit of the large, medium, and small sizes produced per day requires 20, 15, and 12 square feet, respectively.

   Sales forecasts indicate that if available, 900, 1,200, and 750 units of the large, medium, and small sizes, respectively, would be sold per day.

   At each plant, some employees will need to be laid off unless most of the plant's excess production capacity can be used to produce the new product. To avoid layoffs if possible, management has decided that the plants should use the same percentage of their excess capacity to produce the new product.

   Management wishes to know how much of each of the sizes should be produced by each of the plants to maximize profit.
   a. Define the decision variables
   b. Formulate a linear programming model for this problem.
   c. Solve the problem using *lpsolve*, or any other equivalent library in R.

Learning Outcomes
The assignment will help you with the following course outcomes:
   1. To formulate and solve an LP model

Requirements
All assignments are due before the next class.

General Submission Instructions

All work must be your own. Copying other people's work or from the Internet is a form of plagiarism and will be prosecuted as such.

- Upload a pdf file to your git repository. Name your file Username_#.ext, where Username is your Kent State User ID (the part before @), and # is the Assignment number. In this case, 2.

Provide the link to your git repository in Blackboard Learn for the assignment.

Solution

Q1)

   a) Let's have the no of fulltime workers in the three shifts be Xi where i= 1,2, 3.
Therefore:

Number of fulltime workers from:
8 am – 4 pm = X1
12pm – 8 pm = X2
4pm – 12 am = X3

For each 4 hr shift the part time workers be Yi, where j= 1, 2 ,3, 4.
Therefore:

Number of part time workers from:
8am – 12pm = Y1
12pm – 4pm = Y2
4pm – 8pm = Y3
8pm – 12am = Y4

We must minimize it:

Min Z = ($14 / hr) (8hr) [X1+X2+X3] + ($12 / hr) (4 hr) [Y1+Y2+Y3+Y4]

S.T.

X1+Y1 >= 4
X1+X2+Y2 >= 8
X2+X3+Y3 >= 10
X3+Y4 >= 6

Now, for 1 part-time at least 1 full-time must be on duty:

X1 >= Y1
X1+X2 >= Y2
X2+X3 >= Y3
X3 >= Y4

For all Xi >= 0 & Yj >= 0

Now we can answer the following questions by plugging values in solver:

Minimum daily cost: $ 1456
Total full-time staff: 7
Total part-time staff: 14

b)

In this we have following types of workers:

Full time workers with a break after 3-hour work
Full time with a break after 4-hour work
Part-time

Let's have it denoted as X_{il} where i= 1,2,3 & l = 3, 4 (no of hours before break)

Timings:
8am – Noon
Noon – 4pm
4am – 8 pm
8am – Midnight

We must minimize it:

Min Z = ($14 / hr) (8hr) $[X_{13}+ X_{14}+ X_{23}+ X_{24}+ X_{33}+ X_{34}]$ + ($12 / hr) (4 hr) $[Y1+Y2+Y3+Y4]$

S.T.

X1+Y1 >= 4
X1+X2+Y2 >= 8
X2+X3+Y3 >= 10
X3+Y4 >= 6

Now, for 1 part-time at least 1 full-time must be on duty:

X1 >= Y1
X1+X2 >= Y2
X2+X3 >= Y3
X3 >= Y4

For all Xi >= 0 & Yj >= 0

Q2) The following are the equations & constraints involved as we have solved this problem in the previous assignment:

$45x+40y <= 84000$
$3x+2y <= 5000$

$x <= 1000$
$y <= 1200$

$x > 0$
$y > 0$

Lets plot the following equations on the graph using the online tool:
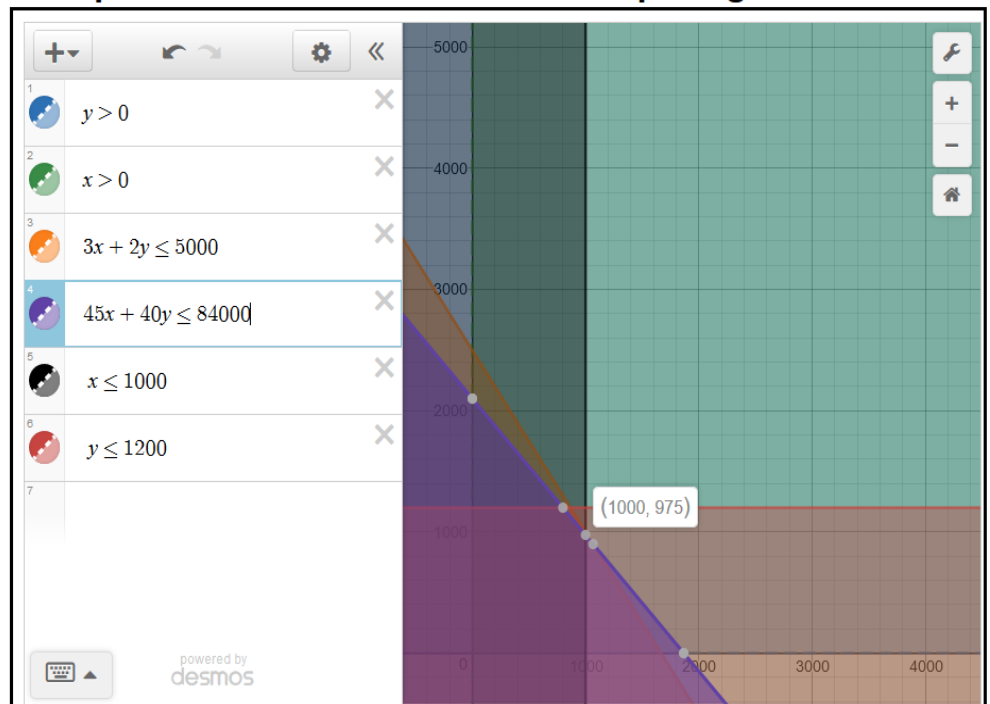https://www.transum.org/Maths/Activity/Graph/Desmos.asp

We find out that every point on the line from (800, 1200) to (1000, 975) is optimal as indicated in the images below.

Graph Plotter :: An Online Graphing Calculator

1 $y > 0$

2 $x > 0$

3 $3x + 2y \leq 5000$

4 $45x + 40y \leq 84000$

5 $x \leq 1000$

6 $y \leq 1200$

(800, 1200)

powered by
desmos

Graph Plotter :: An Online Graphing Calculator

1 $y > 0$

2 $x > 0$

3 $3x + 2y \leq 5000$

4 $45x + 40y \leq 84000$

5 $x \leq 1000$

6 $y \leq 1200$

(1000, 975)

powered by
desmos
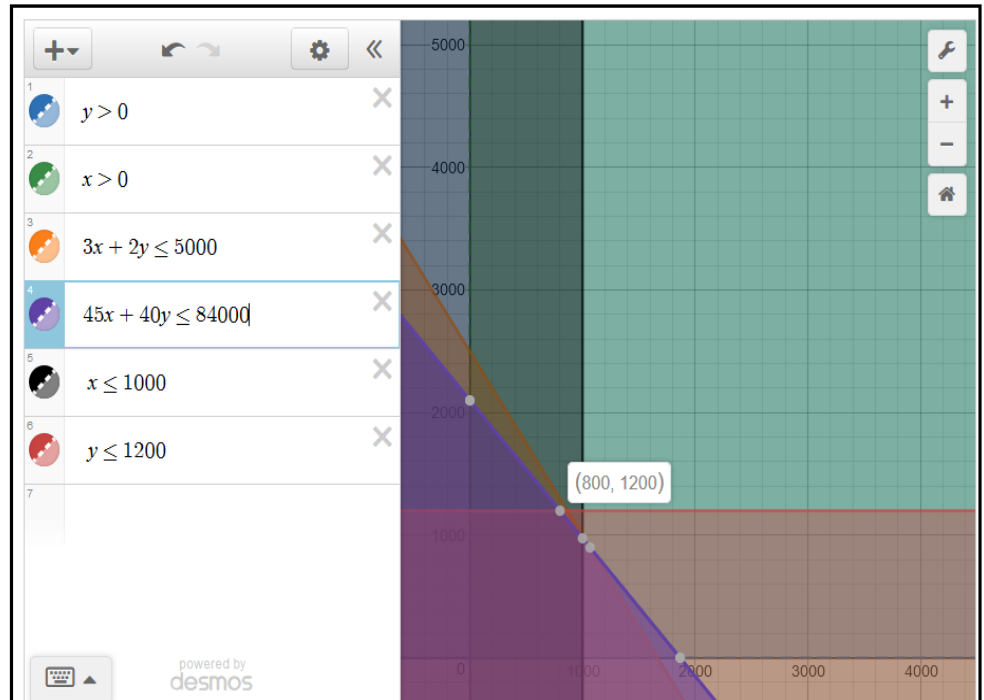
Q3)

a) Define decision variables:

The decision variables can be denoted and defined as follows:

Lets have X_{ij} where i= Plant and j=Size

X_{1L} = number of large units produced per day at Plant 1
X_{1M} = number of medium units produced per day at Plant 1
X_{1S} = number of small units produced per day at Plant 1
X_{2L} = number of large units produced per day at Plant 2
X_{2M} = number of medium units produced per day at Plant 2
X_{2S} = number of small units produced per day at Plant 2
X_{3L} = number of large units produced per day at Plant 3
X_{3M} = number of medium units produced per day at Plant 3
X_{3S} = number of small units produced per day at Plant 3

b) Formulate a linear programming model for this problem:

In this problem we have to maximize the total net profit per day, thus the linear programming model for this problem is:

Maximize $Z = 420\ X_{1L} + 360\ X_{1M} + 300\ X_{1S} + 420\ X_{2L} + 360\ X_{2M} + 300\ X_{2S} + 420\ X_{3L} + 360\ X_{3M} + 300\ X_{3S}$

Constraints:

$X_{1L} + X_{1M} + X_{1S} <= 750$
$X_{2L} + X_{2M} + X_{2S} <= 900$
$X_{3L} + X_{3M} + X_{3S} <= 450$

$20\ X_{1L} + 15\ X_{1M} + 12\ X_{1S} <= 13000$
$20\ X_{2L} + 15\ X_{2M} + 12\ X_{2S} <= 12000$
$20\ X_{3L} + 15\ X_{3M} + 12\ X_{3S} <= 5000$

$X_{1L} + X_{2L} + X_{3L} <= 900$
$X_{1M} + X_{2M} + X_{3M} <= 1200$
$X_{1S} + X_{2S} + X_{3S} <= 750$

$1/750\ (X_{1L} + X_{1M} + X_{1S}) - 1/900\ (X_{2L} + X_{2M} + X_{2S}) = 0$
$1/750\ (X_{1L} + X_{1M} + X_{1S}) - 1/450\ (X_{3L} + X_{3M} + X_{3S}) = 0$
For all $X_{ij} >= 0$

c) For lpsolve check the R attachment on github