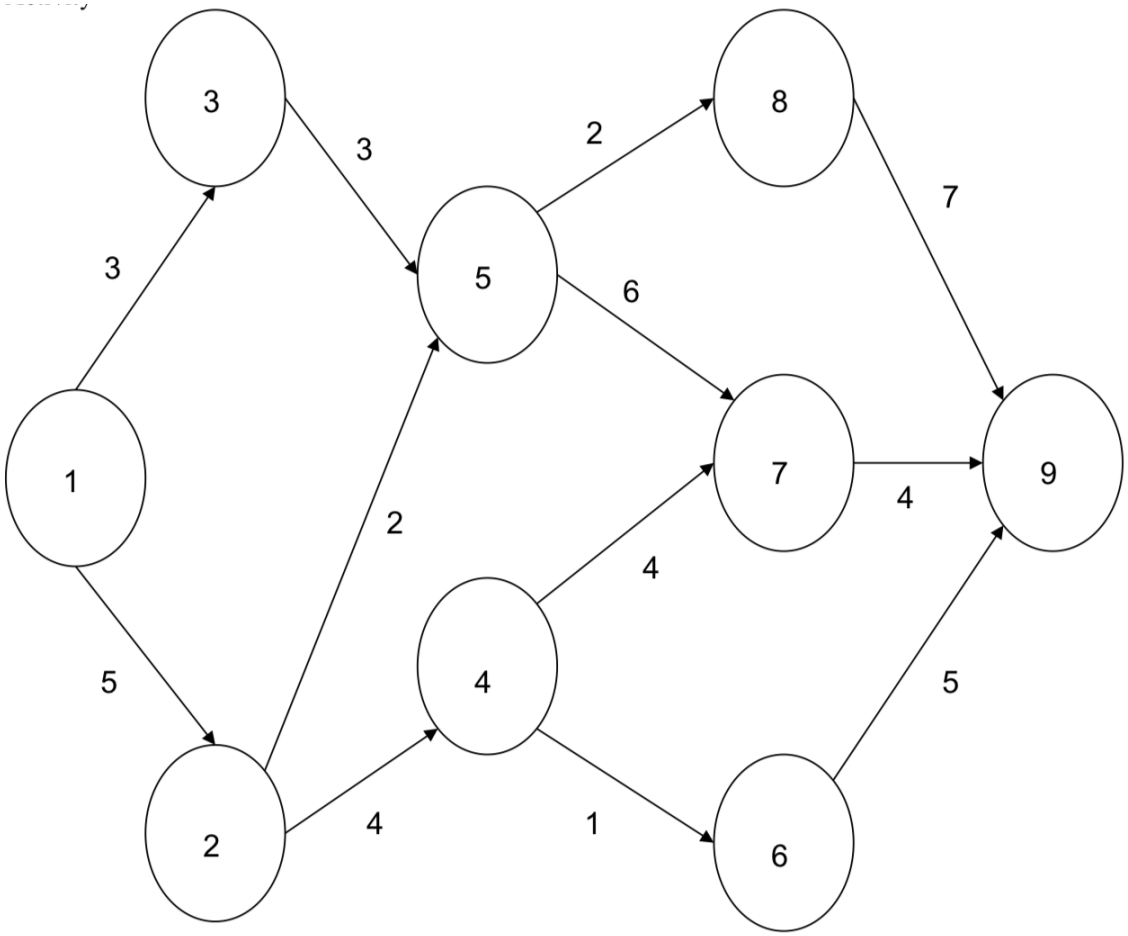
Assignment Instructions: Assignment 5

Purpose

The purpose of this assignment is to formulate and solve an Integer Programming problem.

Directions

1. Consider the following activity-on-arc project network, where the 12 arcs (arrows) represent the 12 activities (tasks) that must be performed to complete the project and the network displays the order in which the activities need to be performed. The number next to each arc (arrow) is the time required for the corresponding activity. Consider the problem of finding the longest path (the largest total time) through this network from start (node 1) to finish (node 9), since the longest path is the critical path.



Formulate and solve the binary integer programming (BIP) model for this problem using library *lpsolve* or equivalent in R.

2. **Selecting an Investment Portfolio** An investment manager wants to determine an optimal portfolio for a wealthy client. The fund has $2.5 million to invest, and its objective is to maximize total dollar return from both growth and dividends over the course of the coming year. The client has researched eight high-tech companies and wants the portfolio to consist of shares in these firms only. Three of the firms (S1 – S3) are primarily software companies, three (H1–H3) are primarily hardware companies, and two (C1–C2) are internet consulting companies. The client has stipulated that no more than 40 percent of the investment be allocated to any one of these three sectors. To assure diversification, at least $100,000 must be invested in each of the eight stocks. Moreover, the number of shares invested in any stock must be a multiple of 1000.

The table below gives estimates from the investment company's database relating to these stocks. These estimates include the price per share, the projected annual growth rate in the share price, and the anticipated annual dividend payment per share.

	Stock							
	S1	S2	S3	H1	H2	H3	C1	C2
Price per share	$40	$50	$80	$60	$45	$60	$30	$25
Growth rate	0.05	0.10	0.03	0.04	0.07	0.15	0.22	0.25
Dividend	$2.00	$1.50	$3.50	$3.00	$2.00	$1.00	$1.80	$0.00

1) Determine the maximum return on the portfolio. What is the optimal number of shares to buy for each of the stocks? What is the corresponding dollar amount invested in each stock?
2) Compare the solution in which there is no integer restriction on the number of shares invested. By how much (in percentage terms) do the integer restrictions alter the value of the optimal objective function? By how much (in percentage terms) do they alter the optimal investment quantities?

Learning Outcomes

The assignment will help you with the following course outcomes:

1. Formulate and solve Integer Programming problems.

Requirements

All assignments are due before the next class.

General Submission Instructions

All work must be your own. Copying other people's work or from the Internet is a form of plagiarism and will be prosecuted as such.

Upload an R markdown file, along with any required .lp files to your git repository. Name your file Username_#.ext, where Username is your Kent State User ID (the part before @), and # is the Assignment number.

Provide the link to your git repository in Blackboard Learn for the assignment.

Solution

Q1)

1.1 Decision Variables:

XIJ where I = activity from and J = activity to.

Therefore, we get the following variables:

X12, X13, X35, X24, X25, X46, X47, X57, X58, X69, X79, X89

1.2 Objective Function:

Maximum Z = 5X12 + 3X13 + 3X35 + 2X25 + 4X24 + 2X58 + 6X57 + 4X47 + X46 + 7X89 + 4X79 + 5X69

1.3 Constraints:

Starting Node

X13 + X12 = 1

Intermediate Nodes

X12 - X25 - X24 = 0

X13 - X35 = 0

X24 - X47 - X46 = 0

X25 + X35 - X58 - X57 = 0

X46 - X69 = 0

X47 + X57 - X79 = 0

X58 - X89 = 0

Ending Node

X89 + X79 + x69 = 1

Also, all XIJ > =0

Q2)

2.1 Decision Variables:

X1 = stocks for firm s1

X2 = stocks for firm s2

X3 = stocks for firm s3

X4 = stocks for firm h1

X5 = stocks for firm h2

X6 = stocks for firm h3

X7 = stocks for firm c1

X8 = stocks for firm c2

We get the expected rate of return using the following formula:

R = (D1/P0) + g

Return X1 = 10.25%

Return X2 = 13.3%

Return X3 = 7.51%

Return X4 = 9.2%

Return X5 = 11.7%

Return X6 = 17%

Return X7 = 29.3%

Return X8 = 25%

2.2 Objective Function:

Maximum Z = 10.25X1 + 13.3X2 + 7.51X3 + 9.2X4 + 11.7X5 + 17X6 + 29.3X7 + 25X8

2.3 Constraints:

Minimum & Total Investment:

40X1 + 50X2 + 80X3 + 60X4 + 45X5 + 60X6 + 30X7 + 25X8 <= 2500000;
40 X1 + 50 X2 + 80 X3 <= 1000000;
60 X4 + 45 X5 + 60 X6 <= 1000000;
30 X7 + 25 X8 <= 1000000;

Investment Requirements:

40X1 >= 100000;
50X2 >= 100000;
80X3 >= 100000;
60X4 >= 100000;
45X5 >= 100000;
60X6 >= 100000;
30X7 >= 100000;
25X8 >= 100000;

Stocks invested in multiples of 1000:

X1 = 1000n1;
X2 = 1000n2;
X3 = 1000n3;
X4 = 1000n4;
X5 = 1000n5;
X6 = 1000n6;
X7 = 1000n7;
X8 = 1000n8;

Note: Check R & lp files for the solution of the problem for part 1 & 2 and their respective objective functions for the answers on comparison.