Assignment Instructions: Assignment 3

Purpose
The purpose of this assignment is to conduct post-optimality and sensitivity analysis for an LP problem.

Directions
Consider the problem from a previous assignment.

The Weigelt Corporation has three branch plants with excess production capacity. Fortunately, the corporation has a new product ready to begin production, and all three plants have this capability, so some of the excess capacity can be used in this way. This product can be made in three sizes--large, medium, and small--that yield a net unit profit of $420, $360, and $300, respectively. Plants 1, 2, and 3 have the excess capacity to produce 750, 900, and 450 units per day of this product, respectively, regardless of the size or combination of sizes involved.

The amount of available in-process storage space also imposes a limitation on the production rates of the new product. Plants 1, 2, and 3 have 13,000, 12,000, and 5,000 square feet, respectively, of in-process storage space available for a day's production of this product. Each unit of the large, medium, and small sizes produced per day requires 20, 15, and 12 square feet, respectively.

Sales forecasts indicate that if available, 900, 1,200, and 750 units of the large, medium, and small sizes, respectively, would be sold per day.

At each plant, some employees will need to be laid off unless most of the plant's excess production capacity can be used to produce the new product. To avoid layoffs if possible, management has decided that the plants should use the same percentage of their excess capacity to produce the new product.

Management wishes to know how much of each of the sizes should be produced by each of the plants to maximize profit.

1. Solve the problem using lpsolve, or any other equivalent library in R.
2. Identify the shadow prices, dual solution, and reduced costs
3. Further, identify the sensitivity of the above prices and costs. That is, specify the range of shadow prices and reduced cost within which the optimal solution will not change.
4. Formulate the dual of the above problem and solve it. Does the solution agree with what you observed for the primal problem?

Learning Outcomes
The assignment will help you with the following course outcomes:
1. To formulate and solve an LP model, and conduct post-optimality analysis
2. To formulate and solve the dual problem

Requirements

All assignments are due before the next class.

General Submission Instructions

All work must be your own. Copying other people's work or from the Internet is a form of plagiarism and will be prosecuted as such.

- Upload an R markdown file, along with any required .lp files to your git repository. Name your file Username_#.ext, where Username is your Kent State User ID (the part before @), and # is the Assignment number.
- Note that the R markdown file allows you to add text, comments, and output as part of the file. So, all documentation should be part of the file. You can read about the R markdown file syntax here, or download the cheat sheet directly.

Provide the link to your git repository in Blackboard Learn for the assignment.

<u>Solution</u>

Q1) We have formed the linear programming model for this problem already in the previous assignment which is as follows:

1.1 Decision Variables:

L1 = number of large units produced per day at Plant 1,
M1 = number of medium units produced per day at Plant 1,
S1 = number of small units produced per day at Plant 1,
L2 = number of large units produced per day at Plant 2,
M2 = number of medium units produced per day at Plant 2,
S2 = number of small units produced per day at Plant 2,
L3 = number of large units produced per day at Plant 3,
M3 = number of medium units produced per day at Plant 3,
S3 = number of small units produced per day at Plant 3.

1.2 Objective function:

Maximize Z = 420 L1 + 360 M1 + 300 S1 + 420 L2 + 360 M2 + 300 S2 + 420 L3 + 360 M3 + 300 S3

1.3 Subject to Constraints:

1.3.1 Excess Capacity

$$L1 + M1 + S1 \leq 750$$
$$L2 + M2 + S2 \leq 900$$
$$L3 + M3 + S3 \leq 450$$

1.3.2 Square Footage

$$20\ L1 + 15\ M1 + 12\ S1 \leq 13000$$
$$20\ L2 + 15\ M2 + 12\ S2 \leq 12000$$
$$20\ L3 + 15\ M3 + 12\ S3 \leq 5000$$

1.3.3 Sales

$$L1 + L2 + L3 \leq 900$$
$$M1 + M2 + M3 \leq 1200$$
$$S1 + S2 + S3 \leq 750$$

1.3.4 Same percentage of capacity to avoid layoff

$$(1/750)\ (L1 + M1 + S1) - (1/900)\ (L2 + M2 + S2) = 0$$
$$(1/750)\ (L1 + M1 + S1) - (1/450)\ (L3 + M3 + S3) = 0$$

1.3.5 Non-Negativity

$L1 \geq 0, M1 \geq 0, S1 \geq 0, L2 \geq 0, M2 \geq 0, S2 \geq 0,$
$L3 \geq 0, M3 \geq 0, S3 \geq 0$

Therefore, we have:

Maximize $Z = 420\ L1 + 360\ M1 + 300\ S1 + 420\ L2 + 360\ M2 + 300\ S2 + 420\ L3 + 360\ M3 + 300\ S3$

Subject to:

$$
\begin{aligned}
&L1 + M1 + S1 \leq 750 \\
&L2 + M2 + S2 \leq 900 \\
&L3 + M3 + S3 \leq 450 \\
&20\ L1 + 15\ M1 + 12\ S1 \leq 13000 \\
&20\ L2 + 15\ M2 + 12\ S2 \leq 12000 \\
&20\ L3 + 15\ M3 + 12\ S3 \leq 5000 \\
&L1 + L2 + L3 \leq 900 \\
&M1 + M2 + M3 \leq 1200 \\
&S1 + S2 + S3 \leq 750 \\
&900\ L1 + 900\ M1 + 900\ S1 - 750\ L2 + 750\ M2 + 750\ S2 = 0 \\
&450\ L1 + 450\ M1 + 450\ S1 - 750\ L3 + 750\ M3 + 750\ S3 = 0
\end{aligned}
$$

Note: Please check dpetwal_3.lp & dpetwal_3.R.

Q2) Identify the shadow prices, dual solution, and reduced costs using:

Shadow prices:

0.00	0.00	0.00	12.00	20.00	60.00	0.00	0.00	0.00	-0.08	0.56

Dual solution:

0.00	0.00	0.00	12.00	20.00	60.00	0.00	0.00	0.00	-0.08	0.56

Reduced cost:

0	0	-24	-40	0	0	-360	-120	0

Note: See dpetwal_1.R to know how to get this solution.

Q3) Range of shadow prices and reduced cost within which the optimal solution will not change:

```
      price         lower         upper
 [1,]  0.00 -1.000000e+30 1.000000e+30
 [2,]  0.00 -1.000000e+30 1.000000e+30
 [3,]  0.00 -1.000000e+30 1.000000e+30
 [4,] 12.00  1.122222e+04 1.388889e+04
 [5,] 20.00  1.150000e+04 1.250000e+04
 [6,] 60.00  4.800000e+03 5.181818e+03
 [7,]  0.00 -1.000000e+30 1.000000e+30
 [8,]  0.00 -1.000000e+30 1.000000e+30
 [9,]  0.00 -1.000000e+30 1.000000e+30
[10,] -0.08 -2.500000e+04 2.500000e+04
[11,]  0.56 -1.250000e+04 1.250000e+04
```

```
     cost         lower        upper
 [1,]    0 -1.000000e+30 1.000000e+30
 [2,]    0 -1.000000e+30 1.000000e+30
 [3,]  -24 -2.222222e+02 1.111111e+02
 [4,]  -40 -1.000000e+02 1.000000e+02
 [5,]    0 -1.000000e+30 1.000000e+30
 [6,]    0 -1.000000e+30 1.000000e+30
 [7,] -360 -2.000000e+01 2.500000e+01
 [8,] -120 -4.444444e+01 6.666667e+01
 [9,]    0 -1.000000e+30 1.000000e+30
```

Note: See dpetwal_1.R to know how to get this solution.

Q4) Formulate the dual of the above problem and solve it:

4.1 Objective Function:

Minimize Z = 750 X1 + 900 X2 + 450 X3 + 13000 X4 + 12000 X5 + 5000 X6 + 900 X7 + 1200 X8 + 750 X9 + 0 X10 + 0 X11

4.2 Constraints:

X1 + 20 X4 + X7 + 900 X10 + 450 X11 ≥ 420
X1 + 15 X4 + X8 +900 X10 + 450 X11 ≥ 360
X1 + 12 X4 + X9 +900 X10 + 450 X11 ≥ 300
X2 + 20 X5 + X7 -750 X10 ≥ 420
X2 + 15 X5 + X8 -750 X10 ≥ 360
X2 + 12 X5 + X9 -750 X10 ≥ 300
X3 + 20 X6 + X7 -750 X11 ≥ 420
X3+ 15 X6 + X8 -750 X11 ≥ 360
X3 + 12 X6 + X9 -750 X11 ≥ 300

X1, X2, X3, X4, X5, X6, X7, X8, X9 ≥ 0
X10, X11 here are unrestricted.

The solution of the dual is the same as the shadow price in the primal problem. The optimal objective value is the same as that of the primal problem.

Note: See dpetwal_3_dual.lp for the dual formulation and dpetwal_3.R for solving the dual LP problem.