

Assignment Instructions: Assignment 6

Purpose

The purpose of this assignment is to

- Explore the use of DEA, formulate and solve DEA problems under different assumptions.
- Compare and contrast these results
- Explore goal programming formulations and solutions

Directions

Hope Valley Health Care Association

1. The Hope Valley Health Care Association owns and operates six nursing homes in adjoining states. An evaluation of their efficiency has been undertaken using two inputs and two outputs. The inputs are staffing labor (measured in average hours per day) and the cost of supplies (in thousands of dollars per day). The outputs are the number of patient-days reimbursed by third-party sources and the number of patient-days reimbursed privately. A summary of performance data is shown in the table below.

DMU	Staff Hours per Day	Supplies per Day	Reimbursed Patient-Days	Privately Paid Patient-Days
Facility 1	150	0.2	14,000	3,500
Facility 2	400	0.7	14,000	21,000
Facility 3	320	1.2	42,000	10,500
Facility 4	520	2.0	28,000	42,000
Facility 5	350	1.2	19,000	25,000
Facility 6	320	0.7	14,000	15,000

Questions

- 1) Formulate and perform DEA analysis under all DEA assumptions of FDH, CRS, VRS, IRS, DRS, and FRH.
 - 2) Determine the Peers and Lambdas under each of the above assumptions
 - 3) Summarize your results in a tabular format
 - 4) Compare and contrast the above results
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2. The Research and Development Division of the Emax Corporation has developed three new products. A decision now needs to be made on which mix of these products should be produced. Management wants primary consideration given to three factors: total profit, stability in the workforce, and achieving an increase in the company's earnings

next year from the \$75 million achieved this year. In particular, using the units given in the following table, they want to

Maximize $Z = P - 6C - 3D$, where

P = total (discounted) profit over the life of the new products,

C = change (in either direction) in the current level of employment,

D = decrease (if any) in next year's earnings from the current year's level.

The amount of any increase in earnings does not enter into Z , because management is concerned primarily with just achieving some increase to keep the stockholders happy. (It has mixed feelings about a large increase that then would be difficult to surpass in subsequent years.)

The impact of each of the new products (per unit rate of production) on each of these factors is shown in the following table:

Factor	Unit Contribution			Goal	Units
	Product:				
	1	2	3		
Total profit	20	15	25	Maximize	Millions of dollars
Employment level	6	4	5	= 50	Hundreds of employees
Earnings next year	8	7	5	≥ 75	Millions of dollars

Questions

- 1) Define y_1^+ and y_1^- , respectively, as the amount over (if any) and the amount under (if any) the employment level goal. Define y_2^+ and y_2^- in the same way for the goal regarding earnings next year. Define x_1 , x_2 , and x_3 as the production rates of Products 1, 2, and 3, respectively. With these definitions, use the goal programming technique to express y_1^+ , y_1^- , y_2^+ and y_2^- algebraically in terms of x_1 , x_2 , and x_3 . Also express P in terms of x_1 , x_2 , and x_3 .
- 2) Express management's objective function in terms of x_1 , x_2 , x_3 , y_1^+ , y_1^- , y_2^+ and y_2^- .
- 3) Formulate and solve the linear programming model. What are your findings?

Learning Outcomes

The assignment will help you with the following course outcomes: CL2: DEA and Goal Programming

Requirements

All assignments are due before the next class.

General Submission Instructions

All work must be your own. Copying other people's work or from the Internet is a form of plagiarism and will be prosecuted as such.

Upload an R markdown file, along with any required .lp files to your git repository. Name your file Username_#.ext, where Username is your Kent State User ID (the part before @), and # is the Assignment number.

Provide the link to your git repository in Blackboard Learn for the assignment.

Solution

Q1:

Ans: Check dpetwal_6_1.rmd for code & answers to all questions

Q2:

1)

Ans:

Creating Objective Function & Constraints:

x1 = Production rates of Product 1

x2 = Production rates of Product 2

x3 = Production rates of Product 3

Objective Function:

Maximize $Z = P - 6C - 3D$

P = total (discounted) profit over the life of the new products

C = change (in either direction) in the current level of employment

D = decrease (if any) in next year's earnings from the current year's level

Definition & Constraints:

Total Profit:

Max : $20x_1 + 15x_2 + 25x_3 = P$

Employment Level:

$6x_1 + 4x_2 + 5x_3 = 50$

Earnings Next Year:

$8x_1 + 7x_2 + 5x_3 \geq 75$

Penalties:

$$C = 6(-)$$

$$D = 3(-)$$

LP Formulation:

$$y1 = 6x1 + 4x2 + 5x3 - 50$$

$$y2 = 8x1 + 7x2 + 5x3 - 75$$

We know:

$$y1 = y1+ - y1-$$

$$y2 = y2+ - y2-$$

Substituting:

$$y1+ - y1- = 6x1 + 4x2 + 5x3 - 50$$

$$y2+ - y2- = 8x1 + 7x2 + 5x3 - 75$$

Converting the constraints & P into LP:

$$6x1 + 4x2 + 5x3 - (y1+ - y1-) = 50$$

$$8x1 + 7x2 + 5x3 - (y2+ - y2-) = 75$$

$$20x1 + 15x2 + 25x3 = P$$

$$xj \geq 0, yk \geq 0, yk \geq 0$$

2)

Expressing management's objective function in terms of $x1, x2, x3, y1+, y1-, y2+, y2-$:

$$\text{Maximize } Z = P - 6C - 3D$$

$$\text{Maximize } Z = 20x1 + 15x2 + 25x3 - 6y1+ - 6y1- + 0y2+ - 3y2-$$

3)

Objective Function:

Maximize $Z = 20x_1 + 15x_2 + 25x_3 - 6y_{1+} + -6y_{1-} + -3y_{2-}$

Constraints:

$6x_1 + 4x_2 + 5x_3 - (y_{1+} - y_{1-}) = 50$

$8x_1 + 7x_2 + 5x_3 - (y_{2+} - y_{2-}) = 75$

$x_j \geq 0, y_k \geq 0, y_k \geq 0$

Decision variables:

$x_1, x_2, x_3, y_{1+}, y_{1-}, y_{2+}, y_{2-}$

Findings:

Total Profit = \$225 Million

Variable value for rate of product from 1 to 7 as follows:

0 0 15 25 0 0 0

Note: Check dpetwal_6_2.R & dpetwal_6_2.lp for solving the LP model.
(Screenshot of Execution is attached with files)