Economies of Scope from Shared Inputs*

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June 12, 2025

Abstract

The prevailing explanation for large, multi-product firms is economies of scope driven by shared inputs across production lines. Using data from the Federal Trade Commission's Line of Business Surveys, which detail both line-specific and shared inputs, we show that US manufacturing firms report substantial shared inputs for both capital and management/marketing expenses. The use of shared inputs is positively correlated with firm size and scope. We estimate a nested CES production function between private and shared inputs and find that they are substitutes with an elasticity of substitution of 2.6. Shared inputs provide significant economies of scope: reducing shared inputs by 50% would decrease output by 3.4% for the average multi-product firm. Finally, we find small average merger synergies from greater economies of scope generated by pooling shared inputs in merger simulations.

Keywords: Economies of scope, shared inputs, multiproduct firms, production function, productivity

JEL Codes: D24, L23, L40, L60

*We dedicate this article to the memory of Mike Scherer, chief economist at the FTC from 1974 to 1976, who led efforts to implement the Line of Business survey program. The views expressed in this article are those of the authors. They do not necessarily represent those of the Federal Trade Commission or any of its Commissioners. We thank Dan Ackerberg, Bram De Rock, Mert Demirer, Paul Grieco, Dan Henderson, Subal Kumbhakar, Miguel Leon-Ledesma, Glenn Magerman, Emir Malikov, Ezra Oberfield, Ariel Pakes, Ted Rosenbaum, Dave Schmidt, David Slichter, Chad Syverson, Philip Ushchev, and various seminar participants for their comments as well as Scott Orr, Steve Puller, Eddie Watkins, and Yingyan Zhao for discussing this paper. This paper was originally circulated as "Economies of Scope and Common Inputs in Multi-Output Production."

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1 Introduction

Large, multi-product firms account for a significant fraction of U.S. output (Gabaix, 2011). On average, they are more productive than smaller firms, with shifts in their product mix contributing to aggregate productivity growth (Bernard et al., 2010; Goldberg et al., 2010). They conduct a disproportionate share of R&D, driving innovation and productivity growth (Anderson, 2024). Idiosyncratic shocks to these firms explain a sizable portion of business cycle fluctuations (Carvalho and Grassi, 2019; Di Giovanni et al., 2014). The rise of "superstar" firms may also help explain rising market concentration and markups as well as declining labor shares (Autor et al., 2020).

The leading explanation for the presence of such firms is *economies of scope*, defined as cost savings that arise when a firm produces multiple products (Panzar and Willig, 1981; Baumol et al., 1982). Panzar and Willig (1981) show that economies of scope imply the use of shared inputs (also referred to as "common," "public," or "joint" inputs) across different production lines. For example, the same manufacturing equipment can produce multiple products; knowledge of one product may lower the cost of producing related ones; a single management team can oversee multiple production lines; and advertising a common brand can boost demand across the product range.

Despite a rich theoretical literature on shared inputs, empirical studies of multi-output production have largely overlooked them due to data limitations. While researchers sometimes have access to product-level output data, they rarely have information on how inputs are allocated to production lines, or whether and how inputs are shared across such lines (De Loecker and Syverson, 2021).

This article examines how inputs shared across product lines contribute to economies of scope, using microdata on large U.S. manufacturing firms from the 1970s collected through the Federal Trade Commission's Line of Business Surveys. As detailed in Section 2, these surveys report revenue and input data at the line-of-business level and include questions on whether certain inputs were specific to a line of business or general to the firm. Because the data distinguish between shared and line-specific inputs within firms, we can estimate production functions at the firm-line-of-business level while accounting for common inputs. We then evaluate the contribution of these shared inputs to economies of scope.

In Section 3, we present stylized facts on the two shared inputs observed in the data—capital and management/marketing expenses. The use of shared inputs is widespread: two-thirds of firms and three-quarters of firm-line of business pairs report positive amounts of shared capital and management inputs. Second, firms spend substantial resources on shared inputs: for the average line of business with shared inputs, shared capital exceeds private capital by 20%, and shared management input exceeds its private counterpart by 270%. Finally, we find that the scalability ratio—the ratio of shared to private input—is positively correlated with firm size and scope, consistent with the theoretical predictions of Argente et al. (2024).

These empirical patterns motivate a production function that explicitly incorporates both private and shared inputs, which we develop in Section 4. We model output using a nested CES production function, where private and shared inputs are each represented by Cobb-Douglas aggregates of sub-inputs. The key parameters capturing economies of scope are the elasticity of substitution between private and shared inputs and the distribution weights assigned to each. Since we observe revenue rather than physical output, we estimate a revenue production function derived from a CES demand framework.

Our product-level input and output data allow us to adapt the single-output production function estimation approach of Gandhi et al. (2020) to the multi-output setting. Section 5 outlines our identification and estimation strategy. We focus on firms that report positive shared inputs and identify the production function using two sets of moments. First, we derive moments from input share equations based on first-order conditions with respect to flexible inputs. Second, we also incorporate dynamic panel moments after assuming Markov processes for unobserved productivity and demand shocks.

In Section 6, we present our production function estimates and analyze the resulting revenue elasticities with respect to inputs. We find that private and shared inputs are substitutes, with an estimated elasticity of substitution of 2.6 and a distribution weight of 4.2% on shared inputs. Shared inputs matter for revenue: the median revenue elasticities at the line-of-business level are 0.01 for shared capital and 0.03 for shared management.

Since increases in shared inputs raise revenue across all of a firm's lines of business, we also compute firm-level elasticities. We find substantially higher aggregate revenue elasticities from common inputs at the firm level: for the median firm, the elasticities are 0.08 for shared capital and 0.11 for shared management. However, these are smaller than the elasticities from increasing only private counterparts—0.12 for private capital and 0.21 for private management. These firm-level private input elasticities are similar to estimates obtained using two common approaches when line-of-business input data are unavailable: aggregating inputs to the firm level to estimate a firm-wide production function or allocating inputs to business lines based on revenue shares.

In Section 7, we analyze the distribution of revenue productivities based on our production function estimates. We find substantial variation in productivity across business lines within firms; a variance decomposition reveals that 76% of productivity differences occur within firms rather than between them. This pattern aligns with core competence theory (Eckel and Neary, 2010; Mayer et al., 2014), which posits that firms are most productive in their core products. We estimate the product-level efficiency ladder; larger firms tend to exhibit higher productivity in their core products and lower productivity in peripheral ones. Moreover, the gap between the most and least productive products widens as firms expand into more non-core varieties.

To measure the significance of economies of scope, we conduct two counterfactual analyses in Section 8. First, we simulate the effect of reducing shared inputs on revenue. On average, a 50% reduction in shared inputs lowers firm revenue by 3.4%. The impact is larger for firms with more lines of business: a 50% cut in shared inputs reduces revenue by 2.3% for firms with 2 to 3 lines, compared to 4.0% for those with 10 or more.

Finally, we explore potential merger synergies arising from economies of scope with a merger simulation. We simulate all possible mergers of firms with no overlapping production lines, assuming that merged firms can pool shared inputs either by taking the maximum or the sum of their existing common inputs. The results suggest modest synergies: on average, expanded access to shared inputs raises total revenue by 1.5% to 2.4%.

Related Literature Our work contributes to the burgeoning literature on multi-output production functions. Although shared inputs play a central role in multi-output production, data limitations have often led researchers to abstract away from them. Many studies assume that firm inputs are fully allocated to individual products, effectively ruling out economies of scope from shared inputs (Foster et al., 2008; Collard-Wexler and De Loecker, 2015; De Loecker et al., 2016; Gong and Sickles, 2021; Itoga, 2019; Orr, 2022; Valmari, 2023). Others estimate a transformation function that maps firm-level inputs into multiple outputs (Diewert, 1973; Lau, 1976; Grieco and

McDevitt, 2017; Maican and Orth, 2021; Malikov and Lien, 2021; Dhyne et al., 2022, 2023). In contrast, our data provide direct evidence on how large multi-product firms employ shared inputs across production lines.[1]

Our findings complement the growing literature that studies the importance of shared inputs. Khmelnitskaya, Marshall and Orr (2024) find evidence of substantial marginal cost and price savings due to economies of scope from shared inputs in the US beer industry. They develop a novel demand-based approach that relies on standard product market demand data rather than production data. Cairncross, Morrow, Orr and Rachapalli (2024) show that product-level markups are not identifiable when there are shared inputs across production lines and within-firm productivity differences across products; we document the presence and significance of both.

A recent literature in macroeconomics also examines shared inputs and economies of scope. Ding (2023) builds a model where shared inputs allow the firm to develop knowledge, which can then be allocated across industries, and uses the model to quantify aggregate economies of scope from knowledge inputs in US manufacturing. Boehm, Dhingra and Morrow (2022) find that economies of scope arising from complementarities for inputs jointly used across products are important determinants of product market entry. Kleinman (2024) studies spatial scope arising from the non-rival output of the labor hired at firm headquarters.

Our model is most closely related to the framework in Argente et al. (2024) in which firm productivity depends on a CES function of shareable and private inputs. The key parameter in their model is the elasticity between the shareable and private input. The authors argue that the empirically relevant case is when the shareable input and private input are substitutes, as we find. Their model predicts that the shareable input to private input ratio will positively correlate with size and scope and that firms with larger size or scope will be more sensitive to demand shocks. Finally, they show how economies of scope from shared inputs can amplify the effects of greater productivity on firm revenue.

To our knowledge, we are the first to estimate firms' elasticity of substitution between private and public inputs. The empirical literature has focused on the micro and macro elasticity between capital and labor, with most estimates of these elasticities below one (see, e.g., Doraszelski and

[1] Nichols (1989) provides an early attempt to study economies of scope with the FTC Line of Business Survey data using a reduced-form correlation analysis. We take a structural approach.

Jaumandreu (2018), Raval (2019), Zhang (2019), and Oberfield and Raval (2021)).[2] An, Kangur and Papageorgiou (2019) finds that private and public capital are complements, but they focus on government-owned capital for their measure of public capital.

2 Data

2.1 Background to the Line of Business Survey Program

The Federal Trade Commission has the authority under Section 6(b) of the FTC Act to require firms to provide "annual or special ... reports or answers in writing to specific questions" providing information about their "organization, business, conduct, practices, management, and relation to other corporations, partnerships, and individuals," and can make such information public if it deems doing so to be in the public interest.[3] Over the years, the FTC has released dozens of industry studies using its 6(b) authority, with ongoing inquiries on physician group and healthcare facility mergers as well as Pharmacy Benefit Managers (PBMs).[4]

The largest data collection effort by the FTC in the immediate post-war period was to provide information on corporate profitability through the Quarterly Financial Report (QFR).[5] The FTC viewed data on the profitability of industries as crucial to inform potential entrants of which industries to enter and thus encourage more efficient allocation of resources across sectors (Scherer, 1990). However, the conglomerate wave of the 1960s and early 1970s made it difficult to learn about the profitability of individual industries from firm-level data on increasingly complex firms.

The FTC's response to this challenge was to develop the Annual Line of Business Survey program. Beyond providing "sunlight" on industry profitability to firms, the survey program would inform antitrust enforcers of industries with potential competition problems, as seen through the lens of the then dominant Structure-Conduct-Performance (SCP) paradigm (Panhans, 2024). In addition, data from the Line of Business surveys would allow researchers to assess emerging Chicago

[2] Gechert et al. (2022), Knoblach et al. (2020), and Raval (2017) provide meta-analyses of this elasticity.

[3] See `https://www.ftc.gov/about-ftc/mission/enforcement-authority`.

[4] See `https://www.ftc.gov/news-events/news/press-releases/2022/06/ftc-launches-inquiry-prescription-drug-middlemen-industry` and `https://www.ftc.gov/enforcement/competition-matters/2021/04/physician-group-healthcare-facility-merger-study`.

[5] The QFR was originally a collaboration with the SEC and only included (public and non-public) manufacturing corporations. In 1971, the FTC was given full responsibility over the QFR, which was extended to some non-manufacturing sectors; it was given to the Census in 1983.

School critiques of the SCP approach (e.g., Demsetz (1973)).

The FTC designed the Line of Business Survey program to measure the concentration and profitability of manufacturing industries by collecting disaggregated data on revenue and costs from the largest US manufacturing firms at the "line-of-business level" (Ravenscraft and Wagner, 1991).[6] To measure profitability at the line-of-business level, the FTC asked firms about several types of firm costs at the line-of-business level, including payroll, materials costs, and R&D expenditures, as well as the degree to which certain assets and expenses were traceable to a given line of business. The FTC piloted the survey in 1973 and ran four annual waves from 1974 to 1977.

Firms were less enthusiastic about this new data collection effort; 180 corporations sued to stop it on the grounds that the FTC lacked statutory authority or a recognizable need for the program, that proper procedures were not followed, and that compliance would be too costly, among other complaints (Whipple, 1979). The FTC eventually prevailed in litigation in the DC District Court in 1977 as well as on appeal at the DC Circuit Court of Appeals in 1978, although continuing litigation delayed the release of the reports.

However, the then General Accounting Office (GAO) recommended a cost-benefit analysis of the program, and the Reagan Transition Team went further by recommending that the FTC end the program if a cost-benefit analysis did not find substantial benefits relative to costs. The FTC paused data collection pending the outcome of this cost-benefit analysis; the FTC finally discontinued any future data collection in 1984 on a 4-1 vote after concluding that the costs of the Line of Business surveys outweighed their benefits.

The recent debate over antitrust policy has led to renewed interest by policymakers in the Line of Business survey program. Senator Amy Klobuchar argued in favor of restarting this data collection program in her recent book *Antitrust* (Klobuchar, 2021), writing that:

> The FTC used to collect industry data on lines of business in an effort to make sure particular sectors did not become too concentrated, and antitrust officials today also have a need to get accurate information so that they can closely monitor industries for monopoly power and consolidation. Although the data collection program was stopped

[6] We refer the reader to U.S. Federal Trade Commission (1985) for the details of the program, including survey forms, instructions, glossary of terms, and the list of lines of business and participating companies. Ravenscraft and Wagner (1991) provides a comprehensive overview of the data.

in the mid-1980s, if antitrust agencies are adequately funded, they will be better able to use modern-day technology to effectively track anticompetitive or exclusionary conduct.

2.2 Lines of Business

The FTC designed 259 lines of business in manufacturing to reflect the economic realities and operations of diversified firms and their competition.[7] For example, in the glass industry, flat glass (SIC 321), glass containers (SIC 3221), pressed and blown glass not classified elsewhere (SIC 3229), and products made from purchased glass (SIC 323) are each distinct lines of business. The data also include information on 14 non-manufacturing lines of business, defined at a roughly one- or two-digit SIC level of aggregation (e.g., "construction," "retail trade," or "services").

Table 1 reports the number of firms, total lines of business, and manufacturing lines of business in the data. Each annual wave in the sample has between 436 and 469 firms. There are between 4,291 and 4,650 lines of business in total. Most business lines are in the manufacturing category. On average, firms operate in about 10 lines of business, of which 7 to 8 are in manufacturing.

Table 1: Number of Firms and Lines of Business Per Year

Year	Firms	Firm-LB	Firm-LB in Manufacturing
1974	436	4,291	3,383
1975	469	4,507	3,536
1976	466	4,572	3,598
1977	456	4,650	3,693

We observe large differences in the number of lines of business that firms operate. Figure 1 depicts the distribution of manufacturing lines of business across all firms and years. This distribution is quite skewed. The modal firm has four lines of business, and the median firm has five lines of business. Only 8% of firms operate a single line of business; 25% of firms have 10 lines of business or more, and 5% have over 20 lines of business.

2.3 Survey Variables

The data includes information on revenues, distinguishing between sales to outside parties and transfers across a firm's lines of business. The survey also collects line-of-business-level data on

[7] See Appendix E in U.S. Federal Trade Commission (1985) for the complete list of FTC lines of business and corresponding SIC codes.

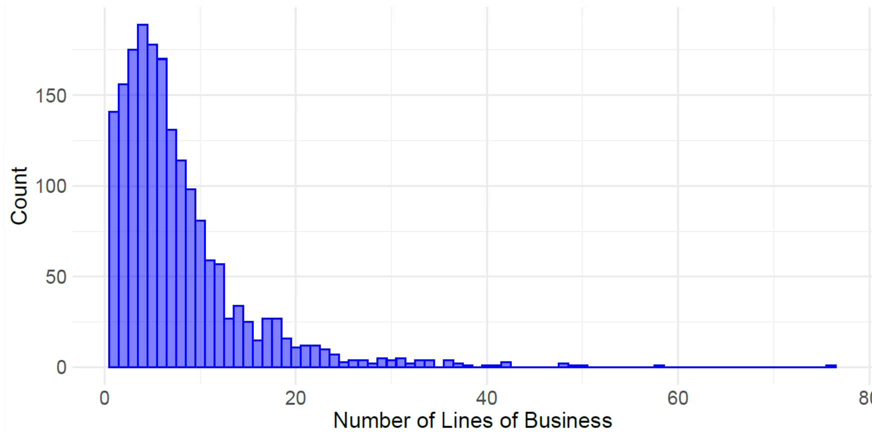

Figure 1: Distribution of Manufacturing Lines of Business

materials, payroll, and capital (net plant, property, and equipment). However, we do not observe physical quantities of inputs and outputs or their prices. In addition, the survey reports three categories of additional expenses at the line-of-business level—advertising, other selling, and general/administrative expenses—which we group as management and marketing expenses, or simply "management" for short. We defer discussion of additional information on firm-level inputs—capital and management expenses not "traceable" to a specific line of business—to the next section.

While our sample has less than 500 firms per year, these firms collectively represent a significant share of the manufacturing industry. When compared to the 1977 NBER Productivity Database (Bartelsman and Gray, 1996; Becker et al., 2021), the firms in our data account for 47–53% of the manufacturing revenue, 49% of materials, and 53% of payroll. The firms also account for 73–84% of the manufacturing gross capital in the 1977 Census of Manufactures (U.S. Department of Commerce, 1981).[8]

We clean the data by dropping all non-manufacturing lines of business and all observations with zero or negative records for sales, payroll, materials, traceable capital, and traceable management/marketing expenses. The latter restriction removes about 6% of all observations in the data.

[8]For revenue calculations, the lower bound is based on revenue from external transactions, while the upper bound includes within-firm transfers. For capital calculations, the lower bound is based only on capital traceable to a given line of business, while the upper bound also includes non-traceable capital. We base the capital comparison on gross capital stocks across databases to ensure the best "apples-to-apples" comparison, although we use the net capital stock to construct capital input measures.

2.4 Summary Statistics

We present summary statistics that illustrate the heterogeneity in the line of business data along several dimensions in Table 2; we report the mean and standard deviation as well as the 10th, 25th, 50th, 75th, and 90th percentiles of several variables at the firm-business line level.

Table 2: Summary Statistics for Line of Business Data

				Percentiles				
	Count	Mean	SD	10th	25th	50th	75th	90th
Size and Growth								
Share of Firm Revenue	13,360	11.0%	18.0%	0.6%	1.5%	3.9%	11.6%	29.3%
Share of Overall LB Sales	13,360	7.8%	11.4%	0.4%	1.2%	3.5%	9.3%	20.4%
Real Yearly Sales Growth	9,073	5.0%	58.7%	-26.0%	-11.2%	1.5%	13.7%	30.1%
Inputs and Profits								
Labor Share of Flexible Input Cost	13,360	31.7%	16.2%	12.0%	19.8%	29.9%	41.9%	53.6%
Flexible Input Share of Sales	13,360	68.9%	21.9%	46.7%	59.2%	69.6%	78.9%	88.0%
Operating Income Share of Sales	13,360	6.4%	26.4%	-3.9%	2.1%	7.3%	12.6%	18.6%
Technological Sophistication								
Value Added Per Worker Dollar	13,360	4.04	14.84	1.50	1.88	2.46	3.63	6.28
Share of Sales Internal	13,360	8.1%	16.3%	0%	0%	1.4%	8.0%	23.1%
Capital to Labor Cost Ratio	13,360	1.93	14.88	0.29	0.51	0.90	1.71	3.51
R&D to Sales Ratio	13,360	1.9%	5.6%	0%	0%	0.5%	1.8%	4.2%

Notes: Observations are at the firm-line-of-business-year level.

Size and Growth We first examine three measures of the size and growth of a given firm's line of business. The average line of business is a small share of firm revenue, with a 11% share for the mean business line and a 3.9% share for the median business line. We next calculate the firm-line of business's share of the overall line of business revenue reported in the data in "Share of LB Revenue."[9] The average firm has a small share of the overall line of business revenue among all firms in our data, at 7.8% for the average firm and 3.5% for the median firm. Finally, we measure real yearly sales growth for lines of business after the first year. The average firm-line of business sees little real sales growth with an average of 5.0% growth and a median of 1.5%. However, there are large differences across firm-line pairs in sales growth, with a standard deviation of 58.7%, a 10th percentile of -26.0%, and a 90th percentile of 20.4%.

[9] We caution that these estimates are not market shares, as they exclude smaller firms not in the line of business surveys. With further assumptions, however, one can construct market shares by merging line of business data with aggregated Census data on shipments. See Ravenscraft (1983) for more details.

Inputs and Profits Next, we look at three variables related to firm inputs and profits. In our baseline production function estimates, we assume that both labor and materials are flexible inputs. Firm-line pairs vary substantially in the labor share of flexible input cost. The average firm-line has a 31.7% share and the median firm-line has a 29.9% share, with the 10th percentile at a 12.0% share and the 90th percentile at a 53.6% share. We next examine the flexible input cost (payroll and materials) as a share of sales; variable costs account for most of the firm sales. The average firm-line pair has 68.9%, and the median firm-line pair has 69.6%, with the 10th percentile at 46.7% and the 90th percentile at 88.0%.

Finally, we examine profitability, the major output for the original line of business program, through the ratio of operating income to sales.[10] Operating income margins also vary considerably across firm-lines, with a a standard deviation of 26.4% compared to a mean of 6.4% and a median of 7.3%. The 10th percentile is below zero, indicating that many business lines operate at a loss.

Technological Sophistication We next examine four measures of the technological sophistication of the firm. We first examine value added per worker dollar, a simple way to assess revenue productivity. The average line of business produces $4.04 in value added per dollar spent on payroll, with the median line of business at $2.46 of value added per worker dollar. We find substantial differences in value added per worker dollar across firm-lines, with a 90-10 ratio of 4.2 and 75-25 ratio of 1.5. Our next variable measures the degree of vertical integration through the share of revenue that is not sold to outside parties ("Share of Sales Internal"); that is, the share of revenue attributable to internal transfers.[11] The share of internal sales is low for most firm-business lines; the average is 8.1%, with the median firm-line of business reporting only 1.4% of revenue as transfers and the 25th percentile having no revenue from internal transfers.

Our final two measures examine capital intensity and R&D. We examine capital intensity using the capital-to-labor cost ratio. The average line of business has about $1.9 in capital per worker

[10] For an early study examining profitability through this measure, see Ravenscraft (1983). Operating income was defined as sales minus materials, payroll, advertising, other selling expenses, general and administrative expenses, and depreciation.

[11] The data report three sources of transfers: to other lines of business in the dataset, to foreign parts of the firm, and to domestic parts of the firm regulated by other federal regulators (These are domestic corporations included in the Total Reporting Company, but not in the LB Reporting Section because either: (1) a corporation was primarily engaged in banking, finance, or insurance; or (2) a corporation was required to file annual financial statements with the Interstate Commerce Commission, Civil Aeronautics Board, Federal Communications Commission, or Federal Power Commission.). We combine all three sources for these estimates.

dollar and the median \$0.90. However, capital-labor cost ratios vary substantially across firm-line pairs, with the 10th percentile having a capital-labor cost ratio of \$0.29 and the 90th percentile a ratio of \$3.51. We measure R&D intensity through the R&D to sales ratio. The average firm-business line spends about 1.9% of sales on R&D, while the median line spends 0.5% of sales on R&D. Here, too, we find large differences across business lines; the 25th percentile spends nothing on R&D, whereas the 90th percentile spends over 4% of sales on R&D.

3 Stylized Facts on Shared Inputs

The most unique feature of the FTC's Line of Business Surveys is that firms report a "traceable" portion of inputs for each line of business they operate. In this section, we describe how the survey asked firms about shared inputs and then depict the distribution of shared inputs for two such inputs—capital (net plant, property, and equipment) and management (advertising, general/administrative, and other selling expenses)—through the statistic of the scalability ratio. We also examine how shared inputs vary by firm size and scope.

3.1 Survey Questions

For both capital and management, the Line of Business surveys distinguished between assets or expenditures "traceable" to the line of business and those that are not specific to a line of business. The FTC defined "traceable" as "Those costs and assets which a company can directly attribute to a line of business or which can be assigned to a line of business by use of a reasonable allocation method developed on the basis of operating level realities." Specifically, the FTC referred firms to the Financial Accounting Standards Board's Standard No. 14 (FAS 14): For expenses, traceable could be compared to the terms "directly traceable" and "allocated on a reasonable basis" in FAS 14, and non-traceable expenses to "general corporate expenses" in FAS 14; For assets, traceable could be compared to the term "identifiable" in FAS 14 and non-traceable assets to the term "assets maintained for general corporate purposes."

We use this distinction of "traceable" or "non-traceable" to separate common inputs across the firm's lines of business from inputs specific to a given line of business. A major limitation of our analysis is that the survey only distinguishes between product-specific and general-purpose inputs;

thus, we cannot identify inputs shared by some, but not all, production lines.

Most firms report using positive amounts of shared inputs for both capital and management. Across firms, 75% report positive shared management expenses, 70% positive shared capital, and 66% positive amounts of both shared inputs. Firms with more lines of business are more likely to report positive common inputs. Examining data at the firm-line of business level, we find 83% of lines of business have positive shared management, 78% positive shared capital, and 74% positive values of both inputs.

3.2 Scalability Ratio

To understand how firms' usage of shared inputs vary with their size and scope, we next examine the *scalability ratio* (Argente et al., 2024), defined as the ratio of the shareable input to the private input for a given firm and product line.[12] Argente et al. (2024) identifies the scalability ratio as a sufficient statistic characterizing the degree to which a firm's input (expertise in their case) can be leveraged across its products.

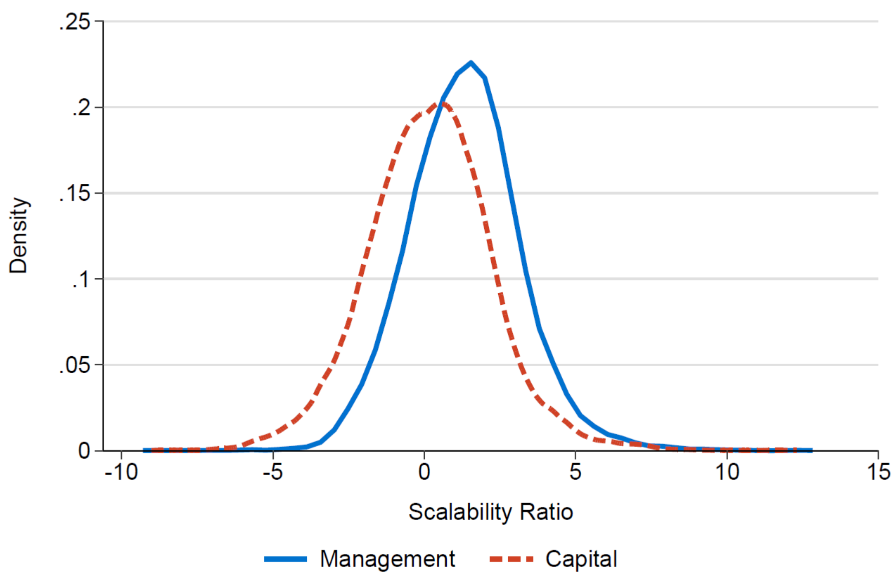

Figure 2: Density of Scalability Ratio

In Figure 2, we depict the density of the log scalability ratio for each input across firms and business lines with positive amounts of the given shareable input. Both densities are approximately

[12]Formally, the scalability ratio of input X for firm i and line of business j is calculated as $SR_{ij}^{X} \equiv \frac{X_i^C}{X_{ij}^P}$, where X_i^C is the shareable input for firm i and X_{ij}^P is the private input for firm i and line of business j.

symmetric, with little difference between the median and mean. For capital, the median firm-line of business has 20% (log ratio of 0.18) more shareable capital than private capital; for management expenses, the median firm-line of business has 270% (log ratio of 1.31) higher shareable management than private management expenses. Thus, management inputs appear more scalable than capital.

Argente et al. (2024) predict that firms with greater size and scope should have a higher scalability ratio. We test this prediction with our data. We measure firm size and scope as the firm's revenue and number of lines of business, respectively. We then estimate the following regression equation:

$$\log(SR^X_{ijt}) = \beta_0 + \beta_1 \log(Size_{it}) + \beta_2 \log(Scope_{it}) + Controls_{it} + \epsilon_{ijt}, \tag{1}$$

where the additional controls include the technological sophistication of the firm-line of business through the log capital-to-labor cost ratio and log R&D-to-sales ratio.

Table 3 displays the estimation results of (1). The first four columns examine management expenses, and the last four columns examine capital; within each set of columns, the first column examines size alone, the second column examines scope alone, the third column both size and scope, and the fourth column includes the additional technological sophistication controls.

Table 3: Relationship between the Scalability Ratio and Firm Size and Scope

	Management				Capital			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Log Size	0.44		0.30	0.29	0.48		0.25	0.42
	(0.06)		(0.07)	(0.08)	(0.08)		(0.09)	(0.11)
Log Scope		0.60	0.39	0.37		0.79	0.63	0.42
		(0.11)	(0.12)	(0.14)		(0.12)	(0.14)	(0.14)
Obs.	11,085	11,085	11,085	10,696	10,367	10,367	10,367	9,987
Controls	No	No	No	Yes	No	No	No	Yes
R^2	0.06	0.05	0.07	0.10	0.05	0.08	0.09	0.09

Notes: Table reports the estimates of regression equation (1). Standard errors are clustered at the firm level. Additional controls include the log capital-to-labor cost ratio and the log R&D-to-sales ratio at the firm-line of business level.

We find robust evidence that the scalability ratio is positively associated with both size and scope, with stronger estimates for scope compared to size. These patterns hold for both capital and management.[13] For example, specification (3) includes both size and scope together, and finds

[13]We also find that scope, but not size, is a strong predictor of the presence of shared inputs. When we regress

that a 10% increase in size is associated with an increase in the scalability ratio for management by 3.0%, and a 10% increase in scope with a 3.9% increase in the ratio for management. Specification (7) examines capital; a 10% increase in size is associated with an increase in the scalability ratio for capital by 2.5%, and a 10% increase in scope with a 6.3% increase in the ratio. The positive associations between the scalability ratio and firms' size and scope remain even after controlling for the technological sophistication of the firm-business line, as shown in columns (4) and (8). Thus, our estimation results are consistent with Argente et al. (2024)'s predictions that size and scope positively correlate with the scalability ratio, and imply that shared inputs are particularly important for the production structure of larger multi-product firms.

Finally, we find that the scalability ratios for management and capital are positively correlated. An increase in the scalability ratio for capital by 10% is associated with a 5.2% (standard error of 0.24%) increase in the scalability ratio for management. Similarly, firms with positive shareable input for capital are also more likely to have positive shareable input for management. Having a positive shareable input for capital is associated with a 55% higher probability (standard error of 5.5%) of having positive shareable input for management.

4 Model

Are private and shared inputs substitutes or complements? How important are the shared inputs for multi-product firm production? To answer these questions, we build a revenue production function that we can estimate with the Line of Business survey data by assuming a nested CES production function between private and shared inputs and a CES demand function.

4.1 Quantity Production Function

Firms produce via the following production function:

$$Y_{jt} = A_{jt} F(H_{jt}, C_{jt}; \theta), \tag{2}$$

an indicator for positive shared input on size and scope by replacing the outcome variable in (1) with the indicator variable, we find that the coefficients on scope are statistically positive for both capital and management, but the coefficients on size are statistically insignificant.

where j indexes the line of business and t the year. For simplicity, we use "product" and "line of business" interchangeably. Y_{jt} is physical output, A_{jt} Hicks-neutral productivity, H_{jt} an index of private inputs, and C_{jt} an index of shareable inputs such that $C_{jt} = C_{j't}$ for all products j and j' being produced by the same firm. Finally, $F_{jt} \equiv F(H_{jt}, C_{jt}; \theta)$ is a function that relates inputs to physical output. Production function (2) exhibits economies of scope if $\partial F_{jt}/\partial C_{jt} > 0$ and $C_{jt} > 0$ (Panzar and Willig, 1981).

We further assume that F_{jt} is a CES function

$$F(H_{jt}, C_{jt}; \theta) = (\alpha \tilde{H}_{jt}^{\rho} + (1-\alpha)\tilde{C}_{jt}^{\rho})^{\frac{\gamma}{\rho}}, \tag{3}$$

where $\tilde{X}_{jt} \equiv X_{jt}/\overline{X}$ is the input variable normalized by its geometric mean $\overline{X}$.[14] Parameter α is the distribution parameter that represents the importance of $\tilde{H}_{jt}$ relative to $\tilde{C}_{jt}$, and $\rho \equiv \frac{\sigma-1}{\sigma}$ with σ representing the elasticity of substitution.[15] The production function exhibits economies of scope when $\alpha > 0$.

We construct the $\tilde{H}_{jt}$ and $\tilde{C}_{jt}$ as Cobb-Douglas indices so that the overall production function is a nested CES:

$$\begin{aligned} \tilde{H}_{jt} &= \tilde{M}_{jt}^{\beta_m} \tilde{L}_{jt}^{\beta_l} \tilde{K}_{jt}^{\beta_k} \tilde{E}_{jt}^{\beta_e}, \\ \tilde{C}_{jt} &= \tilde{\mathcal{K}}_{jt}^{\delta} \tilde{\mathcal{E}}_{jt}^{1-\delta}, \end{aligned} \tag{4}$$

where $\tilde{M}_{jt}$, $\tilde{L}_{jt}$, $\tilde{K}_{jt}$, and $\tilde{E}_{jt}$ represent (normalized) private materials, labor, capital, and management expenses, and $\tilde{\mathcal{K}}_{jt}$ and $\tilde{\mathcal{E}}_{jt}$ represent (normalized) public capital and management expenses. The Cobb-Douglas parameters β_m, β_l, β_k, and β_e are non-negative and sum to one.

This nested CES production function between private and public inputs is similar to the production function in Argente et al. (2024), except that private and public inputs are indices of multiple sub-inputs in our model.[16] Our model generalizes the Cobb-Douglas production function

[14] Variable normalization is standard for CES production function estimation (Klump et al., 2012; Grieco et al., 2016). First, it removes the effect of units on the parameters. Second, as we show in Appendix A.1, it allows us to interpret the distribution parameters α and $(1-\alpha)$ as capturing the marginal returns to inputs (in normalized units) for a firm with the geometric mean level of inputs and productivity.

[15] Having $\rho > 0$ (resp. $\rho < 0$) means the inputs are gross substitutes (resp. complements). The CES function includes three special cases: (i) if $\rho \to 0$ ($\sigma \to 1$), then the elasticity of substitution is fixed at unity, and $Y_{jt} = A_{jt}(\tilde{H}_{jt}^{\alpha}\tilde{C}_{jt}^{1-\alpha})^{\gamma}$; (ii) if $\rho \to -\infty$ ($\sigma \to 0$), then the inputs are perfect complements and $Y_{jt} = A_{jt} \min\{\tilde{H}_{jt}, \tilde{C}_{jt}\}^{\gamma}$; (iii) if $\rho \to 1$ ($\sigma \to \infty$), then the inputs are perfect substitutes and $Y_{jt} = A_{jt}[\alpha\tilde{H}_{jt} + (1-\alpha)\tilde{C}_{jt}]^{\gamma}$.

[16] In Argente et al. (2024), one component of productivity (expertise) is modeled as a CES function of private and

specifications in Cairncross et al. (2024) and Khmelnitskaya et al. (2024), who build on Baumol et al. (1982), by allowing for a more flexible relationship between private and public inputs.

4.2 Productivity

We specify the Hicks-neutral productivity shock as $A_{jt} \equiv \exp(\omega_{jt} + \varepsilon_{jt})$, where ω_{jt} is the persistent productivity shock, known to the firm before making its period t decision, and ε_{jt} is the independently and identically distributed ex-post productivity shock realized only after period t decisions. Let $\mathcal{I}_{jt}$ be the information set of product j's producer when making period t decisions on inputs. Then, by definition, $\omega_{jt} \in \mathcal{I}_{jt}$ whereas $\varepsilon_{jt} \notin \mathcal{I}_{jt}$. The shock ε_{jt} is assumed to be independent of the within period variation in the information set $\mathbb{P}(\varepsilon_{jt}|\mathcal{I}_{jt}) = \mathbb{P}(\varepsilon_{jt})$. Without loss of generality, we normalize the mean of ε_{jt} to be zero.

4.3 Revenue Production Function

We only have data on revenue and not the quantity of output produced. Thus, we build a revenue production function by assuming a CES demand function of the form

$$\frac{P_{jt}}{P_t} = \left(\frac{Y_{jt}}{Y_t}\right)^{\frac{1}{\eta}} e^{\chi_{jt}}, \tag{5}$$

where P_{jt} is the output price of product j, P_t is the industry price index, Y_t is the quantity index that plays the role of an aggregate demand shifter, $\eta < 0$ represents the elasticity of demand, and χ_{jt} is a demand shock observed by the producer (Klette and Griliches, 1996; Grieco et al., 2016; Gandhi et al., 2020).

The revenue production function is then

$$R_{jt} = A_{jt}^{\zeta} F_{jt}^{\zeta} \Lambda_t e^{\chi_{jt}}, \tag{6}$$

where $R_{jt} \equiv P_{jt} Y_{jt}$ is the annual revenue from product j in year t, $\zeta \equiv \frac{\eta+1}{\eta}$, and $\Lambda_t \equiv P_t Y_t^{1-\zeta}$. Taking the log of (6) gives

$$r_{jt} = \zeta f_{jt} + \lambda_t + \nu_{jt} + \zeta \varepsilon_{jt}, \tag{7}$$

public inputs, and production is linear in labor input.

where $\lambda_t \equiv \log \Lambda_t$, and $\nu_{jt} \equiv \zeta\omega_{jt} + \chi_{jt}$.

5 Identification and Estimation

Although our firms produce across many different lines of business, we can apply econometric techniques designed for single-output production functions since we observe input allocations at the product level. We cannot identify all production function parameters without output price data (Klette and Griliches, 1996; De Loecker, 2011; Kasahara and Sugita, 2020; Kirov et al., 2023). However, we can identify a subset of parameters required to study economies of scope, including the elasticity between private and public inputs and the returns to scale to the revenue production function.

We follow Gandhi et al. (2020) and identify the production function parameters based on the moment conditions derived from two sets of assumptions. First, we use the firms' static profit maximization condition with respect to flexible inputs to derive moments from input share equations.[17] Second, we impose a Markov assumption on the stochastic process of the unobserved productivity shock to add dynamic panel moments. Our econometric approach allows us to identify revenue production function parameters while abstracting from the complex dynamic optimization problem of choosing shared inputs.

5.1 Input Share Moments

We assume materials and labor are flexible inputs. To choose the optimal level of flexible inputs given its information set, the producer of product j solves a static profit maximization problem

$$\max_{X_{jt}} \mathbb{E}[P_{jt}Y_{jt}|\mathcal{I}_{jt}] - X_{jt}, \tag{8}$$

where X_{jt} is the expenditure on the flexible input.[18] As we show in Appendix A.2, rearranging the first-order conditions with respect to X_{jt} gives the input share equations

$$s^X_{jt} = \log\zeta + \log\xi^X_{jt} + \log\mathbb{E}[e^{\zeta\varepsilon_{jt}}] - \zeta\varepsilon_{jt},$$

[17] Grieco et al. (2016) develop a similar procedure for estimating parametric production functions.

[18] We express the profit maximization problem in terms of expenditure to avoid introducing extra notation for input prices since we observe inputs in dollar units.

where $s_{jt}^X \equiv \log \frac{X_{jt}}{R_{jt}}$ is the log of expenditure on input X relative to revenue, and $\xi_{jt}^X \equiv \frac{\partial Y_{jt}}{\partial X_{jt}} \frac{X_{jt}}{Y_{jt}}$ is the output elasticity with respect to input X. Our nested CES production function yields the output elasticity with respect to flexible private input $X_{jt} \in \{M_{jt}, L_{jt}\}$ as $\xi_{jt}^X = \gamma \beta_X \frac{\alpha \tilde{H}_{jt}^\rho}{\alpha \tilde{H}_{jt}^\rho + (1-\alpha)\tilde{C}_{jt}^\rho}$.

Our assumption on the unexpected productivity shock ε_{jt} ensures

$$\mathbb{E}[\varepsilon_{jt} | \mathcal{I}_{jt}] = 0, \tag{9}$$

so variables that are functions of period t information set $\mathcal{I}_{jt}$ serve as valid instruments. An estimate of $\mathbb{E}[e^{\zeta \varepsilon_{jt}}]$ can be obtained using residuals from a preliminary estimation.[19]

5.2 Dynamic Panel Moments

We construct a second set of moments by assuming that ν_{jt} follows a linear first-order Markov stochastic process

$$\nu_{jt} = \mu_0 + \mu_1 \nu_{jt-1} + \eta_{jt}, \tag{10}$$

where the error term η_{jt} satisfies $\mathbb{E}[\eta_{jt} | \mathcal{I}_{jt-1}] = 0$.[20] Plugging in $\nu_{jt} = r_{jt} - \zeta f_{jt} - \lambda_t - \zeta \varepsilon_{jt}$ from the log revenue production function (7) and rearranging gives

$$r_{jt} = \zeta f_{jt} + \mu_1 (r_{jt-1} - \zeta f_{jt-1}) + \phi_t + \eta_{jt}^*, \tag{11}$$

where $\phi_t = \mu_0 + \lambda_t - \mu_1 \lambda_{t-1}$, and $\eta_{jt}^* = \eta_{jt} + \zeta \varepsilon_{jt} - \mu_1 \zeta \varepsilon_{jt-1}$. To estimate the production function parameters via the dynamic panel equation (11), we assume

$$\mathbb{E}[\eta_{jt}^* | \mathcal{I}_{jt-1}, \phi_t] = 0, \tag{12}$$

[19] For example, one may obtain preliminary estimates of production function parameters after assuming $\varepsilon_{jt} = 0$ almost surely, so that $\log \mathbb{E}[e^{\zeta \varepsilon_{jt}}] = 0$, and use the corresponding residuals to calculate $\mathbb{E}[\widehat{e^{\zeta \varepsilon_{jt}}}]$. We obtain an estimate of $\mathbb{E}[e^{\zeta \varepsilon_{jt}}]$ by running a flexible second-order polynomial regression of input shares on private and common inputs.

[20] More specifically, we assume that $\mathbb{P}(\nu_{jt} | \mathcal{I}_{jt-1}) = \mathbb{P}(\nu_{jt} | \nu_{jt-1})$ so that $\nu_{jt} = \mathbb{E}[\nu_{jt} | \nu_{jt-1}] + \eta_{jt}$, where η_{jt} is the innovation term that satisfies $\mathbb{E}[\eta_{jt} | \mathcal{I}_{jt-1}] = 0$. We then impose $\mathbb{E}[\nu_{jt} | \nu_{jt-1}] = \mu_0 + \mu_1 \nu_{jt-1}$ to obtain (10). When price data are absent, researchers have typically assumed that the weighted sum of productivity and demand shock is Markov (see, e.g., De Loecker (2011) and Gandhi et al. (2020)).

which is satisfied if the aggregate prices and outputs are realized independently from product j-specific variables.[21] Then, together with time-fixed effects, any variables that are functions of $\mathcal{I}_{jt-1}$ serve as valid instruments.

5.3 Combined Moments

Imposing the nested CES production function assumption yields the input share and dynamic panel equations as

$$s_{jt}^X = \log\psi + \log\beta_X + \log\left(\frac{\alpha\tilde{H}_{jt}^\rho}{\alpha\tilde{H}_{jt}^\rho + (1-\alpha)\tilde{C}_{jt}^\rho}\right) + \log\mathbb{E}[e^{\zeta\varepsilon_{jt}^X}] - \zeta\varepsilon_{jt}^X, \tag{13}$$

$$r_{jt} = \frac{\psi}{\rho}\log\left(\alpha\tilde{H}_{jt}^\rho + (1-\alpha)\tilde{C}_{jt}^\rho\right) + \mu_1\left(r_{jt-1} - \frac{\psi}{\rho}\log\left(\alpha\tilde{H}_{jt-1}^\rho + (1-\alpha)\tilde{C}_{jt-1}^\rho\right)\right) + \phi_t + \eta_{jt}^*, \tag{14}$$

where $\psi \equiv \zeta\gamma$ is the returns-to-scale parameter on the revenue production function.[22] The identifiable parameters are $(\beta_m, \beta_l, \beta_k, \beta_e, \psi, \mu_1, \alpha, \delta, \rho)$ and time fixed effects.

To identify the production function parameters using GMM, we form the unconditional moment equations as

$$\mathbb{E}[\zeta\varepsilon_{jt}^X\tilde{Z}_{jt}^1] = 0,$$
$$\mathbb{E}[\eta_{jt}^*\tilde{Z}_{jt}^2] = 0.$$

Instruments $\tilde{Z}_{jt}^1$ include variables that are functions of $\mathcal{I}_{jt}$; we include the log of (normalized) private and common inputs and their quadratic terms. Instruments $\tilde{Z}_{jt}^2$ includes time-fixed effects and variables that are functions of $\mathcal{I}_{jt-1}$; we include the log of contemporaneous non-flexible private inputs and common inputs, lagged revenue (net of the unexpected productivity shock), lagged private and common inputs, and their quadratic terms.

[21] If $\phi_t \perp \eta_{jt}^*|\mathcal{I}_{jt-1}$, $\mathbb{E}[\eta_{jt}^*|\mathcal{I}_{jt-1}, \phi_t] = \mathbb{E}[\eta_{jt}^*|\mathcal{I}_{jt-1}] = 0$, where the last equality follows from our assumptions that ensure $\mathbb{E}[(\eta_{jt}, \varepsilon_{jt}, \varepsilon_{jt-1})|\mathcal{I}_{jt-1}] = 0$.

[22] Without output quantity data, we cannot separately identify the returns to scale parameter on output γ and ζ $(= \frac{1}{\eta} + 1)$, which depends on demand elasticity η.

5.4 Measurement of Variables

Revenue is the total sales and transfers at the line-of-business level. Materials is the total cost of materials. Labor is the total payroll. Capital is the net plant, property, and equipment. Management expenses are the sum of general, administrative, media advertising, and other selling expenses. For capital and management expenses, we measure the private and public inputs as the traceable and non-traceable parts of the inputs, respectively.

We deflate the values of all variables to 1977 dollars. For output and materials, we match shipment and materials deflators from the NBER Productivity Database (Bartelsman and Gray, 1996; Becker et al., 2021) using line of business to SIC 1977 and SIC 1977 to SIC 1987 concordances. For capital, we use a combined deflator of the investment deflator from the NBER Productivity Database with the ratio of the current cost to the historical cost of fixed assets, available from the Bureau of Economic Analysis (BEA) at the 2-digit SIC level (U.S. Bureau of Economic Analysis, 2025a,b). We deflate labor and management expenses using the CPI. Finally, we drop observations that have zero common inputs. Table 4 provides the summary statistics for each variable entering the production function in the log of real 1977 dollars.

Table 4: Summary Statistics for Production Variables

	Line-of-Business Level					Firm Level				
	Count	Mean	SD	Min	Max	Count	Mean	SD	Min	Max
Revenue	9,856	10.95	1.32	3.49	17.16	1,193	13.46	1.11	8.34	17.44
Private Inputs										
Materials	9,856	10.13	1.43	1.18	16.99	1,193	12.80	1.20	7.83	17.18
Labor	9,856	9.20	1.34	0.06	15.60	1,193	12.48	1.35	6.77	16.07
Capital	9,856	9.66	1.68	0.44	15.90	1,193	12.48	1.35	6.77	16.07
Management	9,856	8.61	1.51	0.21	14.60	1,193	11.61	1.60	2.08	16.23
Common Inputs										
Capital	9,856	9.83	1.57	3.43	14.53	1,193	11.49	1.91	3.56	17.38
Management	9,856	9.94	1.22	1.39	14.03	1,193	11.61	1.60	2.08	16.23

Notes: The values represent the log of 1977 dollars at the firm-line of business level.

6 Estimation Results

Given our relatively small sample size, we first estimate the production function, assuming the same production parameters across all lines of business. We then examine heterogeneity in these estimates across a variety of dimensions. Finally, we measure revenue elasticities at the line-of-business and firm levels using the production function estimates.

6.1 Production Function Estimates

Table 5 reports the production function parameters estimates based on the approach detailed in Section 5. The first column reports the estimates from our baseline specification that assumes material and labor are flexible inputs, consistent with previous work in which researchers have assumed both inputs are flexible (Raval, 2023).

Table 5: Production Function Estimates

	(1)	(2)	(3)
β_m	0.478	0.463	0.592
	(0.052)	(0.107)	(0.034)
β_l	0.210	0.255	0.258
	(0.021)	(0.061)	(0.015)
β_k	0.116	0.111	0.051
	(0.035)	(0.031)	(0.026)
β_e	0.196	0.171	0.099
	(0.054)	(0.046)	(0.040)
δ	0.207	0.028	0.319
	(0.184)	(0.190)	(0.234)
ψ	1.001	1.016	0.760
	(0.087)	(0.144)	(0.046)
μ_1	0.790	0.735	0.975
	(0.069)	(0.080)	(0.024)
α	0.958	0.964	0.974
	(0.012)	(0.015)	(0.013)
ρ	0.613	0.644	0.000
	(0.229)	(0.181)	-
σ	2.584	2.809	1.000
Obs.	6,524	6,524	6,524
Flexible	$\{M, L\}$	$\{M\}$	$\{M, L\}$
Function	CES	CES	CD

Notes: Nonparametric bootstrap standard errors ($B = 999$, resampled at the firm-line-of-business level) are shown in parentheses. Row σ reports the elasticity of substitution implied by the estimated ρ.

Materials have the highest weight amongst the Cobb-Douglas parameters for the line-of-business-specific inputs ($\beta_m = 0.478$), followed by labor, management, and capital. For public inputs,

$\delta = 20.7\%$ of the weight is on capital and $1 - \delta = 79.3\%$ on management. Thus, our estimates suggest that management plays a significant role in firm production, which is consistent with the literature on the role of management in production (see, e.g., Mefford (1986); Ichniowski et al. (1997); Ichniowski and Shaw (1999); Bloom and Van Reenen (2007); Bloom et al. (2013)).

The returns to scale parameter on the revenue production function ψ is estimated to be unity, indicating the revenue production function exhibits constant returns to scale. Given that $\psi = \zeta\gamma = \frac{\eta+1}{\eta}\gamma$, so long as demand is elastic ($\eta < -1$), the output production function would have increasing returns to scale ($\gamma > 1$). We estimate that 4.2% of the weight in the CES function is on the common input ($\alpha = 0.958$), so the production function exhibits economies of scope from shared inputs. Finally, we estimate $\rho > 0$, indicating that private and public inputs are substitutes; the implied elasticity of substitution is $\sigma = \frac{1}{1-\rho} = 2.58$.

We also estimate the production function under the alternative assumption that only materials are flexible in the second column of Table 5. We find broadly consistent estimates with this specification, with an elasticity of substitution of 2.81 and 3.6% of the weight on the public input.

Finally, the third column of Table 5 reports an estimate from a Cobb-Douglas specification that imposes $\rho = 0$ (i.e., $\sigma = 1$) a priori, as Cairncross et al. (2024) and Khmelnitskaya et al. (2024) assume. While we continue to estimate α as less than 1 at 0.974, we cannot reject the null hypothesis that $\alpha = 1$. In addition, we find lower estimates of returns to scale ($\psi = 0.760$) and a higher weight on capital for the public input ($\delta = 0.319$). Thus, imposing a Cobb-Douglas functional form may lead to substantial bias in the estimates of production function parameters and economies of scope.

Heterogeneity

Production lines may vary in factor intensities depending on the types of products or the size and scope of the firm. To account for potential differences in production function parameters across the firms and lines of business, we consider three cuts of the data and examine how our estimates vary. First, we consider durable and non-durable products.[23] Second, we compare firms with high and low scope, defining high-scope firms as those having ten or more lines of business. Finally, we compare

[23] We define non-durables as lines of business with two-digit SIC codes 20, 21, 22, 23, 26, 27, 28, 29, 30, or 31 and durables as those with two-digit SIC codes 24, 25, 32, 33, 34, 35, 36, 37, 38, or 39.

firms with high and low sizes, defining high-size firms as those with higher than median firm-level revenue. We assume that materials and labor are flexible inputs for all of these specifications.

Table 6 reports the production function estimates across these specifications. Overall, we find limited heterogeneity in the production function parameters, as the estimates across these sub-samples are quantitatively and qualitatively similar to our baseline estimates with the full sample. For example, we find α ranges around 0.93–0.98, which indicates that common inputs have small but positive distribution weights in the production function, consistent with economies of scope from shared inputs. Moreover, common management expenses have higher weights than common capital, as δ consistently remains below one-half. We also find that private and shared inputs are substitutes (i.e., $\sigma > 1$) for all specifications except for the non-durables sample, although this last estimate is quite imprecise.

Table 6: Heterogeneity in Production Function Estimates

	(1) Durables	(2) Non-durables	(3) High Scope	(4) Low Scope	(5) High Size	(6) Low Size
β_m	0.531	0.600	0.520	0.477	0.608	0.497
	(0.031)	(0.061)	(0.034)	(0.069)	(0.044)	(0.032)
β_l	0.262	0.185	0.214	0.220	0.222	0.244
	(0.016)	(0.020)	(0.015)	(0.029)	(0.015)	(0.015)
β_k	0.062	0.107	0.080	0.130	0.047	0.099
	(0.036)	(0.049)	(0.031)	(0.060)	(0.029)	(0.027)
β_e	0.144	0.108	0.186	0.174	0.123	0.160
	(0.033)	(0.056)	(0.028)	(0.083)	(0.052)	(0.040)
δ	0.249	0.335	0.338	0.000	0.377	0.003
	(0.166)	(0.272)	(0.214)	(0.245)	(0.309)	(0.158)
ψ	0.891	0.817	0.977	1.005	0.784	0.987
	(0.055)	(0.086)	(0.057)	(0.105)	(0.070)	(0.054)
μ_1	0.950	0.956	0.851	0.822	0.958	0.786
	(0.065)	(0.052)	(0.066)	(0.067)	(0.039)	(0.063)
α	0.952	0.985	0.959	0.936	0.977	0.929
	(0.013)	(0.026)	(0.013)	(0.029)	(0.018)	(0.017)
ρ	0.466	-0.993	0.576	0.604	0.219	0.476
	(0.133)	(1.074)	(0.167)	(0.353)	(0.635)	(0.131)
σ	1.873	0.502	2.358	2.525	1.280	1.908
Obs.	3,589	2,935	3,302	2,906	3,197	3,131

Notes: Nonparametric bootstrap standard errors ($B = 999$, resampled at the firm-line-of-business level) are shown in parentheses. Row σ reports the elasticity of substitution implied by the estimated ρ.

6.2 Revenue Elasticities

We now examine how revenues respond to marginal changes in public inputs by estimating revenue elasticities. We begin by estimating revenue elasticities at the line-of-business level. However, firms deciding how much of a given shared input to employ will care about its effects on firm revenue i.e. revenue aggregated across all of their production lines. Thus, we also derive aggregate elasticities at the firm level.

Line-of-Business-Level Revenue Elasticities

Given revenue production function (6), the line-of-business-level revenue elasticities with respect to inputs are

$$\frac{\partial \log R_{jt}}{\partial \log X_{jt}} = \begin{cases} \psi \beta_X \frac{\alpha \tilde{H}_{jt}^{\rho}}{\alpha \tilde{H}_{jt}^{\rho} + (1-\alpha)\tilde{C}_{jt}^{\rho}} & \text{if } X_{jt} \in \{M_{jt}, L_{jt}, K_{jt}, E_{jt}\}, \\ \psi \delta \frac{(1-\alpha)\tilde{C}_{jt}^{\rho}}{\alpha \tilde{H}_{jt} + (1-\alpha)\tilde{C}_{jt}^{\rho}} & \text{if } X_{jt} = \mathcal{K}_{jt}, \\ \psi (1-\delta) \frac{(1-\alpha)\tilde{C}_{jt}^{\rho}}{\alpha \tilde{H}_{jt} + (1-\alpha)\tilde{C}_{jt}^{\rho}} & \text{if } X_{jt} = \mathcal{E}_{jt}, \end{cases} \tag{15}$$

all of which are identified.[24]

Table 7 reports the distribution of line-of-business-level revenue elasticities across inputs. Materials exhibit the highest revenue elasticities with a mean of 0.45. Revenue elasticities with respect to common inputs appear smaller than those for private inputs, with a median elasticity of 0.01 for shareable capital and 0.03 for shareable management. However, changes in common inputs affect the revenue of all lines of business within a firm, so the aggregate effect on total firm revenue will generally be larger. We thus examine aggregate revenue elasticities below.

Aggregate Revenue Elasticities

The aggregate effect of an increase in an input on firm revenue will depend on whether the change stems from a private or shared portion of the input. For a given firm and year, let $R = \sum_j R_j$ be the aggregate revenue summed across the firm's lines of business indexed by j. Let $X = \sum_j X_j^P + X^C$,

[24] From (6), we have $\frac{\partial \log R_{jt}}{\partial \log X_{jt}} = \zeta \frac{\partial \log Y_{jt}}{\partial \log X_{jt}}$. Since $\zeta > 1$, the revenue elasticities are the upper bounds of output elasticities.

Table 7: Distribution of Revenue Elasticities at the Line-of-Business Level

				Percentiles				
Input	Count	Mean	SD	10th	25th	50th	75th	90th
Private Inputs								
Materials	9,856	0.45	0.03	0.42	0.44	0.46	0.47	0.47
Labor	9,856	0.20	0.01	0.18	0.19	0.20	0.21	0.21
Capital	9,856	0.11	0.01	0.10	0.11	0.11	0.11	0.11
Management	9,856	0.18	0.01	0.17	0.18	0.19	0.19	0.19
Common Inputs								
Capital	9,856	0.01	0.01	0.00	0.00	0.01	0.02	0.03
Management	9,856	0.05	0.05	0.01	0.02	0.03	0.06	0.10

where X_j^P is the product-specific input and X^C is the common input. Finally, assume that

$$\begin{aligned} dX_j^P &= \pi_j^P dX, \\ dX^C &= \pi^C dX, \\ \sum_j \pi_j^P + \pi^C &= 1, \end{aligned} \tag{16}$$

where the proportionality coefficients π_j^P and π^C capture how much X_j^P and X^C increase to create an increase of one unit of aggregate input X. In Appendix A.3, we derive the aggregate elasticity as

$$\frac{\partial \log R}{\partial \log X} = \sum_j s_j^R \left(\left(\frac{\pi_j^P}{s_j^{X,P}} \right) \frac{\partial \log R_j}{\partial \log X_j^P} + \left(\frac{\pi^C}{s^{X,C}} \right) \frac{\partial \log R_j}{\partial \log X^C} \right), \tag{17}$$

where $s_j^R \equiv R_j/R$, $s_j^{X,P} = X_j^P/X$, and $s^{X,C} = X^C/X$. The aggregate elasticity is the weighted sum of the product-level elasticity with respect to private and common inputs, but the weights depend on the configuration of the proportionality coefficients π_j^P's and π^C.

We consider three assumptions on the proportionality coefficients π_j^P's and π^C. In the first case, we assume that the private inputs and common input increase in proportion to their share (i.e., $\pi_j^P = s_j^{X,P}$ and $\pi^C = s^{X,C}$), so that the percentage change for each input is the same across private and public inputs. In the second case, we assume that only the public input increases but private inputs do not (i.e., $\pi_j^P = 0$ and $\pi^C = 1$). In the last case, we assume that all private inputs increase in proportion to their share, so they all have the same percentage change in the input, but the common input does not increase (i.e., $\pi_j^P = s_j^{X,P}/(1 - s^{X,C})$ and $\pi^C = 0$).

Table 8 reports the distribution of aggregate elasticities under each assumption. We highlight two findings. First, we find substantially higher elasticities for inputs at the aggregate level relative to the line-of-business level. For example, in the first case of proportional changes in private and public inputs, the median aggregate revenue elasticities for capital and management from increasing the common input are 0.12 and 0.21, compared to 0.01 and 0.03 at the line-of-business level.

Table 8: Distribution of Aggregate Revenue Elasticities at the Firm Level

				Percentiles				
Input	Count	Mean	SD	10th	25th	50th	75th	90th
Case 1: $\pi_j^P = s_j^{X,P}$, $\pi^C = s^{X,C}$								
Capital	1,193	0.12	0.00	0.12	0.12	0.12	0.12	0.12
Management	1,193	0.21	0.01	0.20	0.20	0.21	0.22	0.23
Case 2: $\pi_j^P = 0$, $\pi^C = 1$								
Capital	1,193	0.20	0.58	0.02	0.04	0.08	0.17	0.35
Management	1,193	0.14	0.18	0.04	0.06	0.11	0.17	0.26
Case 3: $\pi_j^P = s_j^{X,P}/(1 - s^{X,C})$, $\pi^C = 0$								
Capital	1,193	0.15	0.18	0.12	0.12	0.12	0.13	0.15
Management	1,193	0.38	0.94	0.20	0.21	0.23	0.28	0.40

Notes: Firm-level aggregate elasticities are obtained using equation (17) for the given assumption on the proportionality coefficients.

Second, aggregate elasticities are sensitive to whether private or public inputs drive the change. The median aggregate elasticities are 0.12 and 0.23 for capital and management when the private input for each line of business increases in proportion to their shares, compared to 0.08 and 0.11 when only the public input increases and 0.12 and 0.21 when all inputs, public and private, increase in proportion to their shares.

6.3 Production Function Estimation without Product-Level Input Data

Since researchers typically lack access to product-level input data, we assess the potential bias associated with missing disaggregated inputs from two approaches that one can implement in standard datasets. First, we estimate product-level production functions using an input allocation rule that distributes firm-level inputs across products in proportion to their revenue shares. Such rules are common in the literature; for example, Foster et al. (2008) allocate inputs to products using revenue shares, and Orr (2022) shows the set of assumptions under which inputs can be allocated to business lines based upon cost weights, which correspond to revenue weights with the same markups across products. Second, we aggregate revenues and inputs to the firm level and estimate firm-level production functions, as in De Loecker et al. (2020). In both cases, we estimate a Cobb-Douglas production function.

Table 9: Production Function Estimates Without Product-Level Input Data

	Input Allocation		Firm Level	
	(1)	(2)	(3)	(4)
β_m	0.489	0.487	0.438	0.457
	(0.005)	(0.006)	(0.022)	(0.029)
β_l	0.212	0.133	0.195	0.189
	(0.003)	(0.035)	(0.011)	(0.067)
β_k	0.101	0.157	0.130	0.111
	(0.011)	(0.031)	(0.032)	(0.042)
β_e	0.198	0.223	0.236	0.243
	(0.011)	(0.023)	(0.038)	(0.041)
ψ	1.001	0.976	1.068	1.005
	(0.005)	(0.013)	(0.033)	(0.047)
μ_1	0.719	0.888	0.850	0.786
	(0.043)	(0.072)	(0.058)	(0.070)
Obs.	6,524	6,524	852	852
Flexible	$\{M, L\}$	$\{M\}$	$\{M, L\}$	$\{M\}$

Notes: Nonparametric bootstrap standard errors (B = 999, resampled at the firm-line-of-business level) for columns (1) and (2) and firm level for columns (3) and (4)) are shown in parentheses.

Table 9 reports production function estimates from the alternative approaches, allowing labor and materials or just materials to be flexible inputs. The production function parameter estimates reported in all specifications are quite similar to our baseline estimates in Table 5. For example, the coefficient on materials ranges from 0.44 to 0.49, compared to 0.48 in our baseline specification.

Next, we compare the implied revenue elasticities to our baseline estimates. Under the Cobb-Douglas specification, these elasticities correspond to the production function parameters. Estimates from the alternative approaches closely align with those from our baseline, which are based on proportional increases in both line-of-business-specific and common inputs (see Table 8). For capital, the alternative elasticities range from 0.10 to 0.16, compared to 0.12 in the baseline. For management, they range from 0.20 to 0.24, versus 0.21 in the baseline. These findings suggest that researchers without product-specific input data can still recover meaningful aggregate elasticities.

However, these estimates are much lower than the aggregate revenue elasticities from increasing only the common input, which are 0.08 for capital and 0.11 for management at the median firms. Thus, while the revenue elasticities in Table 9 closely match the median revenue elasticities with respect to private inputs, they do not reflect the elasticities associated with changes in the common input alone.

7 Productivity Differences

Economists have long documented large and persistent productivity differences across firms (Bartelsman and Doms, 2000; Syverson, 2011). In this section, following Orr (2022), we show that similarly large productivity differences exist *within* firms across their lines of business—patterns consistent with a core competency model of multiproduct firms.

7.1 Within-Firm Heterogeneity in Revenue Productivity

We first document large differences in revenue productivity within firms. Recall from (6) that the revenue production function is

$$R_{ijt} = \tilde{A}_{ijt}\left(\alpha \tilde{H}_{ijt}^{\rho} + (1-\alpha)\tilde{C}_{ijt}^{\rho}\right)^{\frac{\psi}{\rho}} \tag{18}$$

for firm i and line of business j, where $\tilde{A}_{ijt} \equiv A_{ijt}^{\zeta}\Lambda_t e^{\chi_{ijt}}$. In this section, we make the firm subscript i explicit. We can recover the revenue productivity term (or "revenue-based TFP") $\tilde{A}_{ijt}$ using our production function estimates.[25]

For each firm i with multiple lines of business, we compute $\log(\tilde{A}_{it}^{max}/\tilde{A}_{it}^{min})$, where $\tilde{A}_{it}^{max}$ and $\tilde{A}_{it}^{min}$ represent the maximum and minimum of $\tilde{A}_{ijt}$ across all products j produced by firm i in year t. Figure 3 shows the distribution of this within-productivity gap $\log(\tilde{A}_{it}^{max}/\tilde{A}_{it}^{min})$ for firms in 1977. We observe large within-firm differences in revenue productivity. The median value is 0.58, indicating that the median firm's most productive line of business has approximately $\exp(0.58) \approx 1.79$ times the productivity of its least productive line. The distribution is right-skewed, with a mean of 0.69, implying that, on average, the most productive line is nearly twice as

[25] Our measure of revenue productivity is the residual from a revenue production function and differs from "TFPR," which Foster et al. (2008) defined as the product of TFPQ and price. See also Foster et al. (2016) and Haltiwanger (2016).

productive (1.99x) as the least productive one.[26]

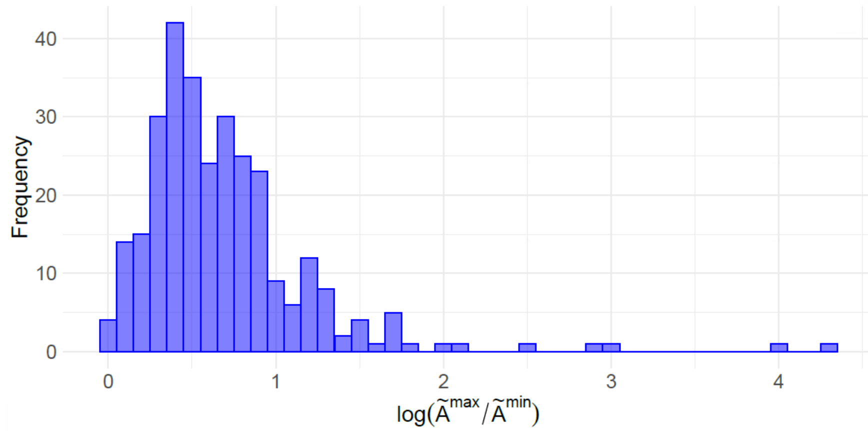

Figure 3: Within-Firm Heterogeneity in Revenue Productivity

Next, we follow Orr (2022) and decompose the productivity variance into within- and between-firm productivity differences. Letting $\tilde{a}_{ij} = \log \tilde{A}_{ij}$, we have $\text{Var}(\tilde{a}_{ij}) = \text{Var}_{\text{within}}(\tilde{a}_{ij}) + \text{Var}_{\text{across}}(\tilde{a}_{ij})$, where $\text{Var}(\tilde{a}_{ij})$ is the total variance of $\tilde{a}_{ij}$, $\text{Var}_{\text{within}}(\tilde{a}_{ij})$ is the within-firm variance, and $\text{Var}_{\text{across}}(\tilde{a}_{ij})$ is the cross-firm variance.[27] Table 10 reports the result of the variance decomposition: 75.8% of the variance is due to within-firm heterogeneity in productivity.

Table 10: Variance Decomposition of Revenue Productivity

	Across	Within	Total
Variance	0.024	0.074	0.098
Percentage	0.242	0.758	1.000

Notes: Table reports the result of cross- vs. within-firm variance decomposition using the 1977 sample.

The within-firm share of productivity variation in our data is substantially larger than that reported in Orr (2022), who finds that within-firm differences account for 36% of total productivity variance across products in Indian manufacturing plants. However, our estimates are not directly comparable for several reasons. First, our sample includes only large manufacturing firms, whereas Orr's data covers both small and large plants—potentially increasing cross-firm variation in his

[26] To compare results across producers, Syverson (2004) estimate a 90-10 ratio for revenue productivity of 1.92 for manufacturing plants in the average US 4 digit US industry. Hsieh and Klenow (2009) find larger differences in the 90-10 ratio of revenue productivity for manufacturing at 5.0 for India and 4.9 for China, compared to 3.3 for the US.

[27] Letting $\tilde{a}_{ij} = (\frac{1}{|J_i|}\sum_{j\in J_i} \tilde{a}_{ij}) + (\tilde{a}_{ij} - \frac{1}{|J_i|}\sum_{j\in J_i} \tilde{a}_{ij})$, the variance of the first term is the cross-firm variance, and the variance of the second term is the within-firm variance. The two terms are uncorrelated by construction, so the total variance is the sum of the variances of the two terms.

setting. Second, we analyze productivity differences across lines of business, while Orr focuses on differences across products within a business line. Finally, Orr measures physical productivity, while we examine revenue productivity.

7.2 Core Competence Models

Core competence models (Eckel and Neary, 2010; Mayer et al., 2014; Arkolakis et al., 2021) posit that firms have their highest productivity (alternatively, lowest marginal cost) for their "core" product, and that their product level productivity declines as they expand into alternative products. In this section, we first estimate an efficiency ladder for firm productivities across lines of business following Orr (2022) to examine how productivity declines, on average, when moving from core to peripheral products. We then examine predictions from core competency models.

Let $\tilde{A}_{it}^{r}$ be the rth highest revenue productivity after ordering all lines of business of firm i from highest to lowest productivity. We consider an efficiency ladder of the form

$$\log \tilde{A}_{it}^{r} - \log \tilde{A}_{it}^{1} = -H(r). \tag{19}$$

We estimate (19) with three specifications. The first two make parametric assumptions on $H(r)$: a linear form as in Eckel and Neary (2010), where $H(r) = \beta(r-1)$, and a log-linear form as in Mayer et al. (2014) and Arkolakis et al. (2021), where $H(r) = \beta \log(r)$. The third is a nonparametric specification that regresses $\log \tilde{A}_{it}^{r} - \log \tilde{A}_{it}^{1}$ on rank indicators.

Table 11 presents the estimated coefficient β under each parametric specification. The linear model (column (1)) implies that each line of business is approximately 4.8% worse than the one ranked just above it.[28] The log-linear model (column (2)) implies that the second-best line of business is about 18.9% less productive than the core (best) line, while the tenth-best line is about 3.1% worse than the ninth-best. A firm's tenth-best line of business is 35.7% worse than the best business line under the linear specification and 48.5% less productive under the log-linear model. These results point to substantial productivity gaps between a firm's core and non-core business lines.[29]

[28] We can translate these coefficients to statements on relative efficiency as follows. For $r \geq k$, we have $\log \tilde{A}_{it}^{r} - \log \tilde{A}_{it}^{k} = -(H(r) - H(k))$, so the relative size of $\tilde{A}_{it}^{r}$ compared to $\tilde{A}_{it}^{k}$ is $\frac{\tilde{A}_{it}^{r} - \tilde{A}_{it}^{k}}{\tilde{A}_{it}^{k}} = \exp\left(-(H(r) - H(k))\right) - 1$.

[29] We do find smaller differences than those in Orr (2022) for products within Indian manufacturing plants, as Orr

Table 11: Efficiency Ladder for Revenue Productivity

	(1) Linear	(2) Log-Linear
$r-1$	-0.049	
	(0.001)	
$\log(r)$		-0.302
		(0.002)
Obs.	8,663	8,663

Notes: Lines of business with rank $r = 1$ excluded. Standard errors are clustered at the firm level.

Figure 4 compares the predicted values of $\log \tilde{A}_{it}^{r} - \log \tilde{A}_{it}^{1}$ based on the coefficient estimates for the three specifications. The predicted values of the log-linear model align closely to those of the nonparametric specification, especially until rank $r = 20$, although very few firms operate more than 20 lines of business (see Figure 1). Thus, we find that the log-linear specification provides a good approximation to the nonparametric model.

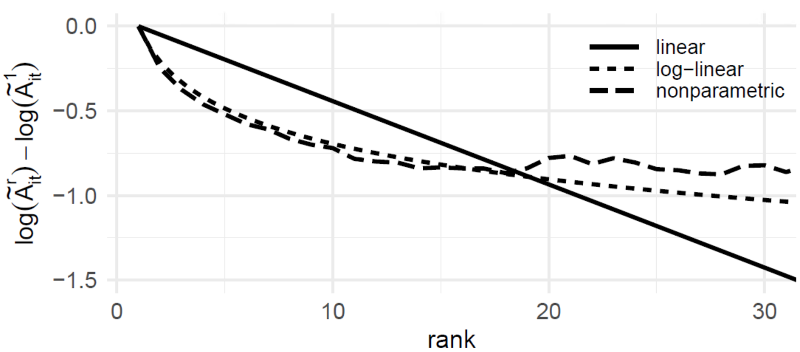

Figure 4: Comparison of Core Competence Models

Core competence models predict that, as firms increase product scope, the average firm productivity should fall, and the productivity gap between the best and worst products should rise. In addition, conditional on scope, larger firms should have higher revenue productivity. We test these predictions in our data by examining how revenue productivity relates to a firm's size and scope. Let $\tilde{a}_{ijt} \equiv \log \tilde{A}_{ijt}$ be the log revenue productivity for firm i and line of business j in year t. Let $\tilde{a}_{it}^{\max} \equiv \max_{j \in J_{i(j)t}} \tilde{a}_{ijt}$ be the log revenue productivity of firm i's best product at t; $\tilde{a}_{it}^{\min}$ is defined similarly. Let $\tilde{a}_{it}^{\text{mean}}$ be the mean of log revenue productivity weighted by flexible private input

(2022) finds $\beta = -0.4773$ for the linear specification and $\beta = -1.2378$ for the log-linear specification.

costs (sum of material and labor expenditures). We then regress different revenue productivity measures on (the log of) firm size and scope as measured by total revenue and the number of lines of business.

Table 12 reports the coefficients from these regressions. First, columns (1) and (2) show that product-level productivities and their within-firm means are positively correlated with size but negatively and more strongly correlated with scope.[30] Second, column (3) shows that the productivity of firms' core product is positively correlated with firm size and scope, although the coefficient on log scope is statistically insignificant. Third, column (4) shows that the productivity of firms' worst products are negatively correlated with scope but uncorrelated with size. Finally, column (5) shows that the within-firm spread of productivity, measured as the difference between the log productivity of the best and worse products, is positively correlated with size and scope.

Table 12: Revenue Productivity Regression

	(1) $\tilde{a}_{ijt}$	(2) $\tilde{a}_{it}^{\text{mean}}$	(3) $\tilde{a}_{it}^{\text{max}}$	(4) $\tilde{a}_{it}^{\text{min}}$	(5) $\tilde{a}_{it}^{\text{max}} - \tilde{a}_{it}^{\text{min}}$
Log Size	0.028	0.079	0.112	0.003	0.109
	(0.010)	(0.011)	(0.029)	(0.014)	(0.031)
Log Scope	-0.086	-0.099	0.063	-0.217	0.280
	(0.014)	(0.015)	(0.034)	(0.024)	(0.040)
Obs.	9,856	1,193	1,193	1,193	1,193
Adj. R^2	0.026	0.168	0.121	0.235	0.265

Notes: Standard errors are clustered at the firm level.

The results in Table 12 are largely consistent with the model of core competence: Larger firms are those with stronger competitiveness in their core products, but have lower productivity for varieties outside their core products. The gap between the productivity of the best and worse product increases as the number of non-core varieties increases.

8 Counterfactuals

We quantify the importance of economies of scope from common inputs through two counterfactual exercises. First, we simulate the revenue loss from a reduction in common input. Second, we

[30] The coefficients on log size and log scope are 0.041 and -0.091 when we use the unweighted mean of productivity, with an adjusted R^2 of 0.098.

simulate the gains from mergers where firms with non-overlapping production lines pool their common inputs.

8.1 Revenue Loss from Reduction in Common Input

We assess the degree of economies of scope by calculating the counterfactual loss in revenue following a reduction in common input. Let R^*_{jt} be the counterfactual revenue associated with a ceteris paribus change in common input to $C^*_{jt} = (1-\phi)C_{jt}$, where $\phi \in [0,1]$ denotes the percentage reduction in common input. Given the revenue function (6), the change in revenue when reducing C_{jt} by a fraction ϕ is $\%\Delta R_{jt} = R^*_{jt}/R_{jt} - 1$, or

$$\%\Delta R_{jt} = \left(\frac{\alpha \tilde{H}^{\rho}_{jt} + (1-\alpha)\tilde{C}^{*\rho}_{jt}}{\alpha \tilde{H}^{\rho}_{jt} + (1-\alpha)\tilde{C}^{\rho}_{jt}}\right)^{\psi/\rho} - 1. \tag{20}$$

We can calculate (20) using our estimates of the production function together with data on firm-line-of-business inputs.

Reductions in the common input can cause firms to lose substantial revenue. Table 13 reports the average revenue loss for different percentage reductions in the common input for firms in our data, both for all firms and broken out by categories of the number of lines of business of the firm. For the average firm, the expected loss in revenue is 0.6% for a 10% reduction in the public input, 3.4% for a 50% reduction, and 7.4% for a 90% reduction.

Table 13: Expected Revenue Loss by the Number of Lines of Business

	Reduction in Common Input				
LOB	10%	25%	50%	75%	90%
1	0.3%	0.9%	1.8%	3.0%	3.9%
2 – 3	0.4%	1.1%	2.3%	3.8%	5.0%
4 – 6	0.5%	1.3%	2.7%	4.4%	5.8%
7 – 9	0.5%	1.4%	3.0%	4.9%	6.4%
10+	0.7%	1.9%	4.0%	6.5%	8.6%
All	0.6%	1.6%	3.4%	5.6%	7.4%

Notes: Table reports the expected revenue loss following a reduction in common inputs. Row "All" reports the unconditional average across all firms and lines of business.

As one would expect from economies of scope, the decline in revenue is larger for firms with greater scope as measured by more lines of business. For example, after reducing the common input by 50%, we estimate an average decline of 1.8% for firms with one line of business, 2.3% for firms with two to three lines of business, 2.7% for firms with four to six lines of business, 3.0% for firms with seven to nine lines of business, and 4.0% for firms with ten or more lines of business. The positive correlation between the number of lines of business and the expected revenue loss is because firms with greater scope have more common inputs.

8.2 Merger Efficiencies

A central concern for merger analysis is whether a given merger would generate efficiencies that enhance welfare and how such efficiencies should be weighed against potential harm from reduced competition (Williamson, 1968). One potential source of efficiency – of particular interest for conglomerate mergers – arises from the consolidation of shared inputs. Williamson (1969) emphasizes organizational structure as a source of such efficiency, arguing that "the conglomerate organization-or, more generally, that the multidivision form-often permits the realization of substantial managerial economies," while he acknowledges that such managerial economies are often "dismissed as *de minimis*." Similarly, Bork (1978) notes that conglomerate mergers may generate efficiencies through "improvement of managerial efficiency," including by "superior data retrieval or financial control systems," and "transfer of technical and marketing know-how across traditional industry lines," which could stem from shared inputs of management, marketing, or physical capital.

We quantify the potential merger synergies from economies of scope by simulating mergers between firms in our data. Merger efficiencies arise from an increase in the stock of shared inputs, which enhances scope economies. To isolate the role of shared inputs, we focus on mergers between firms with no overlap in their lines of business.

Suppose that firms A and B with no overlap in production lines merge. We assume the post-merger revenue from pooling common inputs is

$$R_{jt}^{\text{post}} = \tilde{A}_{jt}^{\text{pre}} \left(\alpha (\tilde{H}_{jt}^{\text{pre}})^{\rho} + (1-\alpha)(\tilde{C}_{jt}^{\text{post}})^{\rho} \right)^{\psi/\rho}. \tag{21}$$

Thus, our counterfactual post-merger revenue in (21) assumes that the revenue productivity and product-specific private inputs are fixed, but that common inputs are aggregated to $\tilde{C}_{jt}^{\text{post}} =$

$(\tilde{\mathcal{K}}_{jt}^{\text{post}})^{\delta}(\tilde{\mathcal{E}}_{jt}^{\text{post}})^{1-\delta}$. We consider two models of common input aggregation. First, we consider $\tilde{\mathcal{X}}_{jt}^{\text{post}} = \max\{\tilde{\mathcal{X}}_{jt}^{A}, \tilde{\mathcal{X}}_{jt}^{B}\}$ for $\tilde{\mathcal{X}}_{jt} \in \{\tilde{\mathcal{K}}_{jt}, \tilde{\mathcal{E}}_{jt}\}$; such an assumption would be valid if the firm adopts the best of two alternatives and pooling of common resources is impossible for technological reasons.[31] Second, we consider $\tilde{\mathcal{X}}_{jt}^{\text{post}} = \tilde{\mathcal{X}}_{jt}^{A} + \tilde{\mathcal{X}}_{jt}^{B}$, which would be valid if the firms' common inputs are perfect substitutes to each other.

We use the 1977 cross-section to simulate two-firm mergers. We consider all possible pairwise mergers of firms with positive shared inputs but no overlap in production lines.[32] Our simulation exercise asks how the merging firms' total revenues would change if they could pool their resources. However, it is not an equilibrium exercise as we do not allow firms to re-optimize their level of shared input.

Table 14 reports the distribution of predicted percentage changes in total revenue under both assumptions on the aggregation of common inputs. In our simulations, a merger of firms with no overlap in production lines increases total revenue by 1.5–2.4% for the average merger and 0.9–1.8% for the median merger. Thus, while merging firms may boost their revenues by pooling shared inputs due to economies of scope, the overall effect on revenue is modest.

Table 14: Predicted Percentage Change in Total Revenue from Mergers

				Percentiles				
Common Input Aggregation	Count	Mean	SD	10th	25th	50th	75th	90th
$\mathcal{X}^{\text{post}} = \max\{\mathcal{X}^A, \mathcal{X}^B\}$	33,018	1.5%	2.1%	0.3%	0.5%	0.9%	1.7%	3.1%
$\mathcal{X}^{\text{post}} = \mathcal{X}^A + \mathcal{X}^B$	33,018	2.4%	2.2%	0.8%	1.2%	1.8%	2.8%	4.3%

Notes: The table reports the distribution of percentage change in total revenue of the merging firms for 33,018 mergers of firms with no horizontal overlap in lines of business.

The above counterfactual measured the additional revenue from pooling the common inputs of the merging firms. Another potential efficiency would be that the merged firm reduces its spending on the common input. For example, in the specification where the merged firm used the maximum of the common inputs from each of the merging firms, it would reduce its spending by the minimum of the common inputs. We formalize these cost savings by calculating reduced common input expenditure as $\min\{\mathcal{X}^A, \mathcal{X}^B\}$. Such savings are small at 0.6% of pre-merger total

[31] For example, the merging firms are likely to adopt the best of two alternative employee training programs without needing to maintain both.

[32] Given 305 firms with positive common inputs in 1977, considering all pairs of firms gives $\binom{305}{2} = 46{,}360$ merger simulations. Limiting the scope to only those with no horizontal overlap leaves us with 33,018 pairwise mergers.

revenue for the average firm and 0.4% for the median firm for both capital and management.[33]

9 Conclusion

In this article, we have examined the degree of economies of scope using data on large manufacturing firms from the FTC's Line of Business survey data. With this data, we could examine inputs at the line-of-business level as well as shared or common inputs to the firm as a whole. We found that firms report substantial amounts of shared inputs, and that the ratio of the shared input to private input was positively associated with firm size and scope.

These facts motivated a nested CES model of production that included common inputs. We estimated this model using the line-of-business data and found that the common inputs have positive output elasticities and are substitutable with line-of-business-specific inputs. After estimating revenue productivity, we found that three-quarters of productivity differences are within firms rather than across firms, and support for models of core competency. Finally, we found considerable revenue declines following a reduction in common inputs and modest merger synergies from increased economies of scope from pooling shared inputs.

We see several avenues for future research. First, our econometric strategy was agnostic about how firms choose how much shared input to employ. Future research could examine what triggers firms to incorporate common inputs in their production lines as well as how firms adjust common inputs in response to demand and supply shocks. Second, researchers could examine how mergers and other changes in the structure of the firm affect shared inputs and economies of scope.

More broadly, we examined economies of scope from shared inputs in manufacturing in the 1970s. Manufacturing itself has changed considerably in the intervening decades with the rise of global supply chains (Antràs, 2015). Outside manufacturing, retail trade has seen a massive increase in the importance of national firms (Hsieh and Rossi-Hansberg, 2023), which may indicate large economies of scope from operating in different geographic and product markets. Despite immense policy interest in large digital platforms, we know little about the degree of economies of scope across their business lines, or what factors generate scope economies in such firms.

[33] The capital effects would be even smaller if the sale price for capital is significantly less than the purchase price, as the literature has found (Abel and Eberly, 1994; Dixit and Pindyck, 1994). As with the post-merger revenue calculation above, we do not consider post-merger reallocation of resources from re-optimization.

A Technical Appendix

A.1 Interpretation of CES Distributional Parameters

Output Production Function

We show normalizing variables allows the researchers to interpret the CES distributional parameters as marginal returns to inputs (up to a return to scale parameter), as Grieco et al. (2016) point out. Let

$$Y_{jt} = A^{*}_{jt}\overline{Y}\left(\alpha\left(\frac{H_{jt}}{\overline{H}}\right)^{\rho} + (1-\alpha)\left(\frac{C_{jt}}{\overline{C}}\right)^{\rho}\right)^{\frac{\gamma}{\rho}},$$

where $\overline{X}$ denotes the geometric mean of the variable. The marginal products of Y_{jt} with respect to H_{jt} and C_{jt} are

$$\frac{\partial Y_{jt}}{\partial H_{jt}} = A^{*}_{jt}\overline{Y}\frac{\gamma}{\rho}(\cdot)^{\frac{\gamma}{\rho}-1}\alpha\rho\left(\frac{H_{jt}}{\overline{H}}\right)^{\rho-1}\frac{1}{\overline{H}},$$
$$\frac{\partial Y_{jt}}{\partial C_{jt}} = A^{*}_{jt}\overline{Y}\frac{\gamma}{\rho}(\cdot)^{\frac{\gamma}{\rho}-1}(1-\alpha)\rho\left(\frac{C_{jt}}{\overline{C}}\right)^{\rho-1}\frac{1}{\overline{C}},$$

Evaluating the expressions at $A^{*}_{jt} = 1$, $H_{jt} = \overline{H}$, and $C_{jt} = \overline{C}$ gives

$$\frac{\partial Y_{jt}}{\partial H_{jt}} = \frac{\overline{Y}\gamma\alpha}{\overline{H}},$$
$$\frac{\partial Y_{jt}}{\partial C_{jt}} = \frac{\overline{Y}\gamma(1-\alpha)}{\overline{C}}.$$

Thus, the marginal products are expressed in normalized units, and they are proportional to the distributional parameters.

Revenue Production Function

A similar derivation applies to the revenue production function. Under the CES demand assumption, we can express the revenue production function as

$$R_{jt} = A^{**}_{jt}\overline{R}\left(\alpha\left(\frac{H_{jt}}{\overline{H}}\right)^{\rho} + (1-\alpha)\left(\frac{C_{jt}}{\overline{C}}\right)^{\rho}\right)^{\frac{\psi}{\rho}},$$

where $\psi = \zeta\gamma$. Repeating the logic above shows that the marginal revenues with respect to inputs for a firm with $A_{jt}^{**} = 1$, $H_{jt} = \overline{H}$, and $C_{jt} = \overline{C}$ are

$$\begin{aligned}\frac{\partial R_{jt}}{\partial H_{jt}} &= \frac{\overline{R}\psi\alpha}{\overline{H}},\\ \frac{\partial R_{jt}}{\partial C_{jt}} &= \frac{\overline{R}\psi(1-\alpha)}{\overline{C}}.\end{aligned}$$

A.2 Derivation of Input Share Equations

Consider the static profit maximization problem (8). Given the i.i.d. asssumption on ε_{jt}, we can express the expected revenue given information set $\mathcal{I}_{jt}$ as

$$\begin{aligned}\mathbb{E}[P_{jt}Y_{jt}|\mathcal{I}_{jt}] &= \mathbb{E}[A_{jt}^{\zeta}F_{jt}^{\zeta}\Lambda_t e^{\chi_{jt}}|\mathcal{I}_{jt}]\\ &= A_{jt}^{\zeta}F_{jt}^{\zeta}\Lambda_t e^{\chi_{jt}}\frac{\mathbb{E}[A_{jt}^{\zeta}|\mathcal{I}_{jt}]}{A_{jt}^{\zeta}}\\ &= A_{jt}^{\zeta}F_{jt}^{\zeta}\Lambda_t e^{\chi_{jt}}\frac{\mathbb{E}[e^{\varepsilon_{jt}}]}{e^{\varepsilon_{jt}}}.\end{aligned}$$

Thus, the static profit maximization problem with respect to input X_{jt} can be rewritten

$$\max_{X_{jt}} Y_{jt}^{\zeta}\Lambda_t e^{\chi_{jt}}\frac{\mathbb{E}[e^{\zeta\varepsilon_{jt}}]}{e^{\zeta\varepsilon_{jt}}} - X_{jt}.$$

The first-order condition with respect to X_{jt} gives

$$\zeta Y_{jt}^{\zeta-1}A_{jt}\frac{\partial F_{jt}}{\partial X_{jt}}\Lambda_t e^{\chi_{jt}}\frac{\mathbb{E}[e^{\zeta\varepsilon_{jt}}]}{e^{\zeta\varepsilon_{jt}}} = 1.$$

Rearranging the above using (6) gives

$$\zeta\underbrace{\Lambda_t Y_{jt}^{\zeta}e^{\chi_{jt}}}_{=R_{jt}}\underbrace{\left(\frac{\partial F_{jt}}{\partial X_{jt}}\frac{X_{jt}}{F_{jt}}\right)}_{=\xi_{jt}^{X}}\underbrace{\frac{A_{jt}F_{jt}}{Y_{jt}}}_{=1}\frac{1}{X_{jt}}\frac{\mathbb{E}[e^{\zeta\varepsilon_{jt}}]}{e^{\zeta\varepsilon_{jt}}} = 1.$$

Rearranging produces the input share equation

$$S_{jt}^{X} = \zeta\xi_{jt}^{X}\frac{\mathbb{E}[e^{\zeta\varepsilon_{jt}}]}{e^{\zeta\varepsilon_{jt}}},$$

where $S^X_{jt} = \frac{X_{jt}}{R_{jt}}$ is the input expenditure share relative to revenue, and $\xi^X_{jt} \equiv \frac{\partial F_{jt}}{\partial X_{jt}} \frac{X_{jt}}{F_{jt}}$ is the output elasticity with respect to input X. By taking the log, we obtain

$$s^X_{jt} = \log \zeta + \log \xi^X_{jt} + \log \mathbb{E}[e^{\zeta \varepsilon_{jt}}] - \zeta \varepsilon_{jt}, \quad X \in \{M, L\},$$

where $s^X_{jt} \equiv \log S^X_{jt}$. The above equations represent the share equations from firms' static profit maximization conditions.

A.3 Derivation of Aggregate Revenue Elasticity

We derive expressions for aggregate revenue elasticities. Let $R = \sum_j R_j$ be the aggregate revenue of a given firm, summed across products indexed by j. Let $X = \sum_j X^P_j + X^C$ be the aggregate input, where X^P_j is product-specific input and X^C is common input. We want to characterize the elasticity of R with respect to X (i.e., $\partial \log R / \partial \log X$), recognizing that the aggregate revenue elasticity with respect to aggregate input may be different depending on which component of X drives the change.

Let

$$dX^P_j = \pi^P_j dX,$$
$$dX^C = \pi^C dX,$$
$$\sum_j \pi^P_j + \pi^C = 1,$$

where the proportionality coefficients π^P_j and π^C characterize how much each of X^P_j and X^C

increase to create an increase of X by one unit. We have

$$\begin{aligned}
\frac{\partial R}{\partial X} &= \sum_j \frac{\partial R_j}{\partial X} \\
&= \sum_j \left(\sum_l \frac{\partial R_j}{\partial X_l^P} \frac{dX_l^P}{dX} + \frac{\partial R_j}{\partial X^C} \frac{dX^C}{dX} \right) \\
&= \sum_j \left(\frac{\partial R_j}{\partial X_j^P} \frac{dX_j^P}{dX} + \frac{\partial R_j}{\partial X^C} \frac{dX^C}{dX} \right) \\
&= \sum_j \left(\frac{\partial R_j}{\partial X_j^P} \pi_j^P + \frac{\partial R_j}{\partial X^C} \pi^C \right).
\end{aligned}$$

Then

$$\begin{aligned}
\frac{\partial \log R}{\partial \log X} &= \frac{\partial R}{\partial X} \frac{X}{R} \\
&= \frac{X}{R} \sum_j \left(\frac{\partial R_j}{\partial X_j^P} \pi_j^P + \frac{\partial R_j}{\partial X^C} \pi^C \right) \\
&= \frac{X}{R} \sum_j \left(\frac{\partial R_j}{\partial X_j^P} \frac{X_j^P}{R_j} \frac{R_j}{X_j} \pi_j^P + \frac{\partial R_j}{\partial X^C} \frac{X^C}{R_j} \frac{R_j}{X^C} \pi^C \right) \\
&= \sum_j \left(\frac{R_j}{R} \right) \left(\frac{\partial \log R_j}{\partial \log X_j^P} \left(\frac{X}{X_j^P} \right) \pi_j^P + \frac{\partial \log R_j}{\partial \log X^C} \left(\frac{X}{X^C} \right) \pi^C \right).
\end{aligned}$$

Thus, the aggregate revenue elasticity with respect to the aggregate input is

$$\frac{\partial \log R}{\partial \log X} = \sum_j s_j^R \left(\left(\frac{\pi_j}{s_j^{X,P}} \right) \frac{\partial \log R_j}{\partial \log X_j^P} + \left(\frac{\pi^C}{s^{X,C}} \right) \frac{\partial \log R_j}{\partial \log X^C} \right),$$

where $s_j^R \equiv R_j/R$, $s_j^{X,P} \equiv X_j^P/X$, and $s^{X,C} = X^C/X$.

- Case 1: If $\pi_j^P = s_j^{X,P}$ and $\pi^C = s^{X,C}$, then

$$\frac{\partial \log R}{\partial \log X} = \sum_j s_j^R \left(\frac{\partial \log R_j}{\partial \log X_j^P} + \frac{\partial \log R_j}{\partial \log X^C} \right).$$

- Case 2: If $\pi_j^P = 0$ and $\pi^C = 1$, then

$$\frac{\partial \log R}{\partial \log X} = \sum_j \frac{s_j^R}{s^{X,C}} \frac{\partial \log R_j}{\partial \log X^C}.$$

- Case 3: If $\pi_j^P = \frac{s_j^{X,P}}{1-s^{X,C}}$ and $\pi^C = 0$, then

$$\frac{\partial \log R}{\partial \log X} = \sum_j \frac{s_j^R}{1 - s^{X,C}} \frac{\partial \log R_j}{\partial \log X_j^P}.$$

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